

THREE ESSAYS ON EQUALIZATION TRANSFERS IN A FISCAL FEDERALISM

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사랑하는 아버지 어머니 그리고 진경이에게 이 논문을 드립니다

To my dad, Byuk Woon Kim, mom, Hyun Sook Cho, and sister, Jin Kyung Kim.

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ABSTRACT

This doctoral thesis contains three essays on equalization transfers in a fiscal federalism.

In Chapter One, we study the impact of equalization transfers in a fiscal federalism on the policies of the regional governments. This chapter presents a dynamic general equilibrium model of a fiscal federalism in which two asymmetric regions provide their residents with non-productive public expenditures (a flow) and public capital (a stock). In our model, each regional government behaves strategically in choosing its policies to maximize the discounted welfare of its own residents, under the equalization transfer scheme. Our analysis indicates that the tax on the use of the private capital input is equal to zero in the steady state. In addition, we observe that the only change induced by the equalization transfer scheme is an increase in the non-productive public expenditures in less-endowed region (Quebec) with an offsetting fall in the non-productive public expenditures in more-endowed region (Ontario). The results of the numerical exercise we carry out also suggest that an equalization scheme in a federal state lowers the welfare gap between a rich and a poor region.

In Chapter Two, we investigate how the equalization transfer formula is determined and how the equalization transfer program affects a region's policies. This chapter presents a political economy model of equalization payments in a fiscal federalism in which asymmetric regional governments, who care about the welfare of its own residents, lobby the (incumbent) federal government, who takes into consideration both the welfare of the federation and the political support it receives from the states when allocating equalization transfers. It is shown that if the federal government allows politics to distort its economic policy it actively implements an equalization transfer program that is different from the one it would implement if it behaved like a benevolent dictator. The equalization transfer scheme implemented by the federal government induces a fall in the investment of public capital in both regions, and if the political power of the poor region is sufficiently higher than that of the rich province, then the equalization transfer scheme induces a higher level of the non-productive public good in the poor region than in the rich region. A numerical example is provided to illustrate this result.

Chapter Three presents a model of equalization transfers in a federation in which each regional government has private information on its own technology for public service delivery. The aim of the federal government is to design an equalization transfer scheme that is Bayesian incentive compatible and satisfies the interim participation constraint in order to achieve the goal of providing residents of a poor region with at least a certain level of utility without imposing an excessive burden on the giving region. We show that the equalization transfers allow the recipient region to raise its private consumption above the level it would have attained in the absence of equalization transfers because some of the transfer is allocated to raise private consumption. Furthermore, it is shown that the equalization transfers are also lower if the federal government can observe the type of the poor region.

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INTRODUCTION TO THE THESIS

There are in general three types of governmental forms around the world, depending on the assignments of the powers on different levels of government by constitution. The first type is a unitary state in which the central government has direct control of all levels of government mainly by delegating authority to the sub-national governments. This unitary form of government emphasizes on a uniform level of public service, rather than on its diversity, across sub-national governments. The second type of governmental form is a federal state in which the lower levels of government has a constitutional assurance of sovereignty over some policies. This federal form of government promotes decentralized decision-making in order for the sub-national governments to be able to provide its own residents with public services that well match their tastes or preferences. Lastly, a confederation is a union of a number of countries – usually by a treaty – where the central government is formed by the member states.

Over the last thirty years, more and more nations have decentralized fiscal responsibilities to the lower levels of government (or have turned into federal states). In the United States, significant level of federal dominion has been moved to the states for some major programs, such as welfare, Medicaid, legal service etc. In developed countries, we have also observed a shift in political structure in favour of a federal state. In the developing countries, such fiscal decentralization has been partly due to the failure of centralized planning to bring sustained growth. The essential argument on behalf of fiscal decentralization is that regional governments are in a better position than the central government to accommodate the preferences of their own citizens for local public goods. In addition, there are other claims in support of decentralization: (i) the fact that mobile households are able to migrate to jurisdictions that provide public services better fitted to their preferences raises the potential benefits from the decentralized provision of public services, and (ii) (fierce) competition among decentralized levels of government provides an environment for the efficient provision of local public goods.

In a federal state, it is prevalent to decentralize the provision of public services throughout health care, education and welfare while the federal government is usually responsible to deliver national public good such as the national defense, immigration, foreign affairs, foreign trade, and the legal system. Although it is different from country to country, it is generally observed that federal governments collect more revenues than they need while the sub-national governments are *not* financially self-sufficient to pay for the public services that they are supposed to deliver to their own constituencies. This disproportionate assignment of revenue raising capacities and expenditure responsibilities between two levels of government is known as vertical fiscal imbalance.

The sub-national governments, which are allowed to raise their own revenues to some extent, are *not*, however, equally endowed with natural resources and populations or have same industrial foundation etc. Hence, sub-national governments have different capabilities to raise revenues from their own tax bases to finance the provision of public services. This disparity in the revenue raising ability of regional governments is known as horizontal fiscal imbalance in a literature of fiscal federalism.

Thus, fiscal decentralization necessarily and inevitably leaves states or provinces with not only insufficient revenues but also different financial capacities to provide *assigned* public services to their own residents. Equity consideration requires that a citizen, regardless of where she resides in a federation, should have access to comparable levels of public services. The federal governments in federal states, such as Canada, Australia, Belgium, Germany, Switzerland, and the United States, employ an equalization transfer scheme as well as conditional inter-governmental grants to minimize or rectify the vertical and horizontal imbalances. Although the design and extent of equalization transfers is not the same across countries, the main goal of equalization transfers is to allow the per capita revenues of poorer jurisdictions to approach to that of richer jurisdictions so that the citizens in the federation have access to same standard of public services and living conditions, regardless of where they reside.

In the present era of highly mobile capital and labor, sub-national governments in a federal country are assumed to compete for private capital by offering favorable tax rates on corporate income or by making investments in public capital. However, the fiscal competition for private

capital is known to generate distortions and externalities, and the role of equalization transfers is to limit the harmful effects of fiscal competition as well as to promote the efficient tax and public good policies for the federation, *besides* correcting the fiscal imbalances across jurisdictions. In most of the literature on fiscal federalism, the analysis of equalization transfers is conducted under a *static* framework. Furthermore, a considerable number of studies are devoted to the problem of allocating a *fixed* stock of private or public capital across regions. The first essay of this dissertation tackles this issue in a dynamic general equilibrium model of the fiscal competition for private capital in a federation that is composed of two asymmetric regions and that implements an equalization transfer, with the evolution of private and public capital accumulation being explicitly modeled. In our model, the residents of a region derive utility from the consumption of a private good, the stock of public capital, and a non-productive public good within their own jurisdiction at every instant. Each regional government provides its residents with non-productive public expenditures and the stock of public capital. In each region competitive firms produce a private good using labor, private capital, and public capital, where public capital is complementary to private capital. A regional government taxes the use of the private capital input within its own jurisdiction, and uses the tax revenues to finance the non-productive public expenditures and the investments in public capital. Given the revenue-sharing scheme, each regional government behaves strategically in choosing its policies to maximize the discounted welfare of its own residents. We calibrate the model by specifying the values of parameters obtained from the data for Ontario and Quebec. Our analysis indicates that the tax on the use of the private capital input is equal to zero in the steady state without equalization transfers, which supports the theoretical result established by Gross (August 2013) that the optimal capital income tax is zero in a world made up of two large economies. In addition, the zero tax rate of the private capital input still holds in the long-run equilibrium under the revenue-sharing scheme. Our analysis also indicates that the only change induced by the equalization transfer scheme is an increase in the non-productive public expenditures in less-endowed region (Quebec) with an offsetting fall in the non-productive public expenditures in more-endowed region (Ontario); the stocks of public capital and the wages in both regions remain the same as well as the capital asset held by a resident, the labor supply by a resident and her private consumption in each region remains the same. The results of the numerical exercise we carry out

also suggest that an equalization scheme in a federal state lowers the welfare gap between a rich and a poor region. An equalization payments scheme hence might be justified on equity grounds.

The rationale for equalization transfers in the traditional approach on fiscal federalism is to raise both efficiency and equity. They are implemented to correct distortions generated by harmful fiscal competitions as well as to provide needed help to poorer regions. The presumption in these traditional studies is that each level of government aims to maximize the social welfare of its residents within its own jurisdiction, i.e., a government – federal or regional – is assumed to be a benevolent social planner. However, if one subscribes to the idea, borrowed from the field of public choice, that public-decision makers have their own objective function that they seek to maximize, then it is worthwhile to inquire whether equalization transfers are distributed or used for political purposes. Indeed, there are numerous empirical analyses that address this issue, and their main findings confirm that interregional transfers, including unconditional transfers, are distributed to buy votes from states or for other political reasons. In Chapter Two, we present a political economy model of equalization transfers in a fiscal federalism. We adopt the special-interest group approach propounded by Dixit, Grossman, and Helpman (1997),¹ first employed in the field of international trade. In our model, there are two asymmetric regional governments in the federation: each sub-national government, who cares about the utilities of its own residents, lobbies the (incumbent) federal government by promising political support for a favorable equalization transfer formula. Our main finding is that if the federal government behaves like a benevolent dictator and adopts the utilitarian philosophy of maximizing the sum of utilities of all the citizens in the federation it does not implement any equalization payment scheme. However, if the federal government allows politics to distort its economic policy, it actively implements an equalization payment program that transfers a positive fraction of the tax revenues collected by one regional government to the other regional government, and vice versa. Therefore, we assert that politics matters for the distribution of equalization transfers. In addition, under our political economy framework, we show that it is possible for the poorer region to provide its residents with a higher level of public service than the richer region if the

¹ Avinash Dixit, Gene M. Grossman, and Elhanan Helpman (1997): “Common Agency and Coordination: General Theory and Application to Government Policy Making,” *Journal of Political Economy*, vol. 105, no.41, 752-769.

political power of the poor region is sufficiently higher than that of rich province and if the total factor productivity of the poor region is much lower than that of the rich region. This finding confirms the evidences found in Canada and Germany that the equalization transfer program might make it possible for the have-not provinces to provide a level of non-productive public good higher than that of the have provinces. Furthermore, our model demonstrates that the have-not provinces might be inclined to invest less in infrastructure.

Recently, one strand of the literature in fiscal federalism attempts to consider the problems of asymmetric information. The essential presumption is that it is hard for the federal government to observe some of the exogenous regional circumstances, which are private information for the regional government itself and which change over time. Furthermore, it is difficult to design an equalization transfer scheme that incorporates all the possible contingencies. In this setting of asymmetric information, the optimal procedures implemented by the federal government are likely to be quite different from those in the settings of perfect information. Thus, to deal with these information issues, researchers in the field of fiscal federalism have adapted the works from industrial organization and microeconomic theory that have investigated these information problems. In Chapter Three, we consider asymmetric information on the productivity of the technology for public service provision in each region. In each region, a consumption good is produced using labor, private capital, and public capital. The cost-efficiency parameter that characterizes the delivery of public services in each region is private information. A region invests in public capital, provides public services, and imposes a tax on the use of private capital to finance two types of public goods. The federal government (or mechanism designer) determines the equalization transfer scheme that is Bayesian incentive compatible and satisfies the interim participation constraint so as to attain the goal of providing residents of a poor region with at least a certain level of utility at a minimum expected value of equalization transfer. The important feature that differentiates our model from other theoretical studies on equalization transfers is that the regions in our model are asymmetric in terms of endowments and population sizes in contrast while other theoretical studies which often assume that the regions are symmetric. We show the equalization transfers and the tax rate on private capital are lower, but the public capital investment is higher under incomplete information than under complete

information. Hence, asymmetric information has an influence on the qualitative properties of regional fiscal policies. One of the central results of this paper is that, under the asymmetric information, the equalization transfers enable the have-not province to increase its private consumption above the level it would have achieved in the absence of equalization transfers since part of the equalization transfers is used to help increase private consumption by allowing the region to impose a lower tax rate on private capital and making more public capital investment while the entire equalization transfer received by recipient government is spent on public service provision under perfect information. The only difference for the rich region is that the resources assigned to public service delivery now decreases by the amount it contributes to the equalization transfer scheme.

Chapter 1: EQUALIZATION TRANSFERS AND COMPETITION FOR CAPITAL FLOWS IN A FISCAL FEDERALISM

1. INTRODUCTION

In a federal system of government, there are several levels of government, each with its own powers to raise revenues and provide public goods and services. The levels of government in a federation can be divided into three broad categories: central government, regional government, and local government. However, discussions on the problems of fiscal federalism concern mainly the central government and regional governments. In a fiscal federalism, it is expected that the central government and the regional governments should have access to sufficient revenues to carry out their responsibilities effectively without financial strain. Such a happy situation, when it is achieved, is referred to as vertical balance. It is also expected that each region should be able to provide within its own borders a level of public goods and services that is comparable to that available in other regions. This happy situation, when it is achieved, is referred to as horizontal balance. Because regions are not equally endowed with natural resources or do not have the same industrial base, a well-endowed regions will have a higher GDP per capita than a less-endowed region, and thus can provide a higher level of public goods and services to its own residents than the latter region. In federal states, such as Canada, Australia, Belgium, Germany, Switzerland, and the United States, the central governments make cash payments – known as equalization payments – to provincial or state governments to bring the per capita revenues of poorer regions closer to that of richer regions. The purpose of an equalization transfer scheme is to rectify the fiscal imbalances among richer and poorer regions.

In the present age of highly mobile capital, a country attracts foreign investments by offering a favorable tax rate on corporate income. Every other thing equal, a country with a lower corporate income tax rate attracts more capital inflows than one with a higher corporate income tax rate. In the public debate on tax competition in the EU, it is often asserted that the consequence of the tax competition among the member states is a “race to the bottom,” with each member state

cutting its corporate income tax rate either to induce capital inflows or to prevent capital outflows.

There are two different views on the theories of tax competition. The traditional view is that tax competition is harmful: if independent jurisdictions behave non-cooperatively in setting the tax rates on the use of the private capital within their own borders, then the tax rate they set are inefficiently low, with the ensuing consequence of a low level of public good that they can provide to their own citizens. The main source of the inefficiency comes from the fact that a rise in one region's tax rate causes perfectly mobile capital to relocate to other regions, raising the tax revenues of those regions because capital constitutes part of their tax bases.² Michael Keen and Maurice Marchand (1996) build on this idea, and show that the competition for private capital might distort governments' spending choices between public input and non-productive public good, leading them to invest too much in public input and infrastructure, but too little in non-productive public good.

The other view maintains that tax competition is beneficial. Tiebout (1956) argues that tax competition is beneficial because it induces governments to impose efficient tax rates on their own residents for the provision of public services. Some economists claim that (international) tax competition is beneficial because it tends to limit the power of governments to levy taxes³ and punishes wasteful or corrupted governments.⁴

Another dimension in the competition for capital involves the provision of public capital – infrastructures, enforcement of property rights, labor training, public R&D – which has a positive impact on the productivity of private capital. Because public capital raises the productivity of private capital (which increases the inflow of the private capital), corporate income taxation can be seen as a return on public investments. There is thus an intrinsic relationship between the corporate tax and the stock of public capital. In an empirical analysis on the competition for capital by the member states of the EU, Gomes and Pouget (2008) found a linkage between these two variables, and that the linkage is positive: the corporate tax and the stock of public capital respond positively to each other. Thus, firms might be willing to pay a higher corporate tax in a

²Zodrow and Mieszkowski (1986), Mintz and Tulkens (1986), and Wilson (1986).

³ Edwards and M. Keen (1996).

⁴ Qian and Roland (1998).

country that provides good infrastructures. Bénassy-Quéré et al. (2007) quantified the trade-off between a “high-tax, high public capital” strategy and a “low-tax, low public capital strategy,” and concluded that raising the stock of public capital through an increase in the corporate tax rate reduces inward FDI and that tax competition might lead to an under-provision of public capital. On the whole, these researchers found that tax competition might be more complex than a simple race to the bottom.

The process of competition for foreign investments observed at the level of nation states is also observed at the regional level in a federal state. Because a large number of federal states adopt various equalization transfer schemes that allow the central governments to address the fiscal imbalances across jurisdictions, the revenues received by a region in a federal state also depend on the tax sharing scheme, and this dimension in the competition for capital – in addition to corporate taxation and public investments – must also be considered. Another purpose of equalization payments, according to Boadway and Flatters (1982), is that interregional equalizing transfers lessen the inefficiency for labor allocation by internalizing the fiscal externalities. The theoretical literature on the competition for capital in a federal state concentrates mostly on the efficiency aspect of a tax revenues sharing scheme. Köthenbürger (2002) showed that transfer mechanisms with an explicit redistributive character do not always impair efficiency. Hindriks et al. (2008) demonstrated that tax competition lowers public investments and that equalization transfers discourage public investments, with a little effect on taxes. These researchers also showed that tax-sharing schemes are beneficial for the federation, and when the degree of regional asymmetry is low, are also beneficial for each region. Bucovetsky and Smart (2006) showed how an equalization transfer scheme in a federal state can limit tax competition among various regions, correct fiscal externalities, and increase the expenditures on public goods.

All the models just mentioned share a common feature: the analysis is carried out under a static framework. Under a static framework, the time horizon is reduced to a single period, which means that there is no scope for inter-temporal substitution. Chamley (1986) demonstrated – for a closed economy – that when individuals have infinite lives and a utility function of a fairly general form, the optimal tax rate on capital income is equal to zero in the steady state. Gross (August 2013, October 2013) opened the economy of Chamley and showed that for a world

made up of two large economies, the optimal tax rate on capital income is still equal to zero in long-run equilibrium. If one is willing to accept the result proved by Gross, then the debate on “the race to the bottom” becomes moot: the optimal tax rate on capital income is zero, not positive, and the problem of setting the optimal income tax rate ceases to exist.

In this paper, we formalize a dynamic model of the competition for capital in a federal state that is made up of two asymmetric regions and that adopts an equalization transfer scheme. There are two reasons why we choose to formalize our ideas under the framework of a dynamic general equilibrium model.

One reason is that in the traditional theories of tax competition, there is no capital accumulation – public or private, and a major part of the analysis is concentrated on how a fixed stock of private capital is allocated among the various regions. Furthermore, in order to make the models tractable, these authors have employed highly simplifying assumptions, such as not allowing the residents of the regions being analyzed to even own capital. Also, these static models mostly ignore individual’s optimal sequence of consumption and investment choice, which also affect the government’s optimal policy, and vice versa. According to Arrow and Kurz (1970, p. 10), an investment policy – of an individual or a government – is a simultaneous choice of present and future investments, and these optimal choices at different times are interrelated. Furthermore, the returns to investments are available for future consumption, individual saving (i.e., private investment), or government investment. All these issues can only be properly dealt with in a dynamic model. The equilibrium of a static model, such as that of Keen and Marchand (1997), which includes both public capital and public goods can only be coherently interpreted as are the stationary equilibrium of a differential game played by a central governments and various regional governments. This interpretation can be explained as follows. First, because public capital helps raise the productivity of private capital, it must be available before production begins. Second, according to the central government’s budget constraint – total revenues equal expenditures on public goods plus public capital investment – the funds needed for public capital investment are only available to the central government after production. There is thus some circularity in the central government’s sequence of actions if one incorporates public capital in a static model of a closed economy in which a given stock of capital is allocated among various regions. The problem of circularity can be avoided in a static model if one allows the economy to

be open so that capital from the rest of the world can flow in.

The other reason is to follow Chamley (1986) in allowing the consumers to have a broader scope for inter-temporal substitution. Chamley (1986) demonstrated that under some reasonable assumptions on preferences, the economy converges to the long-run equilibrium if it begins not too far from the long-run equilibrium. We expect that the convergence to the long-run equilibrium for a closed economy also holds for model of two large economies, and will simply assume the convergence in the computations of the long-run equilibrium.

In our model, the two regions differ in their production technologies as well as in their populations.⁵ The process of capital accumulation – both public and private – is explicitly modeled. The government of each region makes investments in infrastructure, such as roads, bridges, sewers..., as well as non-productive public expenditures, such as recreational activities, social services... At each instant, the residents of a region derive utility from the consumption of a private good and a public good. The objective of each regional government is to maximize the present value of the stream of welfare enjoyed by its own residents. In pursuing this objective, a regional government behaves strategically in setting the values of four policy instruments at each instant: the tax on the private capital input used within its own borders, the tax rate on labor income, the investments in public capital, and the non-productive public expenditures. A strategy for a regional government is thus a time path for these four policy instruments. A pair of strategies – one for each regional government – induces a competitive equilibrium on the economy made up of the two regions and a fortiori a level of discounted welfare for each region. Our model thus has the structure of a differential game.

The paper is organized as follows. In Section 2, the model is presented. The best response of each regional government to the strategy chosen by the strategy of the other regional government

⁵ There are several previous papers that analyze the asymmetric tax competition in a static setting. Bucovetsky (1991) presented a model in which smaller populated country levies a lower equilibrium tax rates and its residents are better off than those in a large populated jurisdiction. Wilson (1991) examined that smaller populated region is better off than residents in a large populated region in a quite general model. Kanbur and Keen (1993) use two country-model with commodity taxes and transportation costs and show that the geographically small country sets the lower tax rate than the large country.

is analyzed in Section 3. The model is calibrated in Section 4. Section 5 contains the results on the long-run equilibrium of the numerical exercise and a discussion of these results. Some concluding remarks are given in Section 6.

2. THE MODEL

Time is continuous and denoted by $t, t \geq 0$. A federation is constituted by two regions, called region 1 and region 2. There are two levels of government in the federation: a central government and two regional governments. The two regions participate in an equalization transfer scheme administered by the central government. Each regional government is responsible for the welfare of the citizens who live within its own borders.

In the model, one consumption good is produced by competitive firms in each region, using three inputs: labor, private capital, and public capital. The consumption good can also be used as investment goods to augment the stock of private capital and the stock of public capital. Public capital has a positive impact on the productivity of private capital. Following Gomes and Pouget, *op. cit.*, and Bénassy-Quéré et al., *op. cit.*, we also include public capital as an argument in the utility function of the citizens of the federation because most stocks of public capital used by firms, such as highways, bridges, water supply systems, public education, police and justice, are also likely to raise households' utility. Based on our judgment that consumers also value the stock of public capital, the exclusion of the stock of public capital from the utility function might undermine the incentive of the regional governments' to invest in public capital. Within the boundaries of each region, there is also a local public good produced by the regional government. The local public good corresponds to items, such as recreational facilities, social services, or transfers to the poor.

The revenues of a regional government come from taxing the use of the private capital input within its own borders and taxing the labor income of its own residents. A regional government justifies the taxing of the private capital input by appealing to the infrastructure that it provides within its own borders. We assume that the central government administers a simple equalization transfer scheme under which a fraction, say $\tau, 0 \leq \tau \leq \frac{1}{2}$, of the tax revenues collected by one

regional government is transferred to the other regional government. The revenues that are available to a regional government thus consist of the remaining fraction plus the transfer it receives from the other region under the equalization transfer scheme. These revenues are used by a regional government to finance the production of the local public good and the accumulation of public capital within its own borders.

There is no population growth in the model, and all the individuals in a region are assumed to be identical and live forever. In what follows, the number of individuals in region i is denoted by $N_i, i = 1, 2$. We assume that private capital is free to cross regional frontiers, but labor is immobile.

2.1. Preferences and Technology

An individual in region i owns an amount of private capital asset $k_i[t]$ at time t , and has a time endowment at each instant that is normalized to be 1. The utility obtained by an individual in region $i, i = 1, 2$, at an instant t is assumed to be given by

$$(1) \quad \text{Log} c_i[t] + \sigma \text{Log}[1 - \ell_i[t]] + \beta \text{Log} G_i[t] + \delta \text{Log} H_i[t],$$

where $c_i[t]$, $\ell_i[t]$, $G_i[t]$, and $H_i[t]$ denote, respectively, her consumption of the private good, her labor supply, the non-productive public good, and the stock of public capital – all at time t . Also, $\sigma > 0$, $\beta > 0$, and $\delta > 0$ are parameters that characterize, respectively, the intensities of her preferences for leisure, the non-productive public good, and public capital relative to private consumption. The preferences specified by (1) thus provide a clear-cut distinction between productive and non-productive public expenditures.

For each $i = 1, 2$, the consumption good in region i is produced according to the following Cobb-Douglas technology:

$$(2) \quad Y_i = A_i H_i^\gamma K^\alpha L^{1-\alpha},$$

where A_i is a parameter that characterizes the endowment of region i , and H_i represents the stock of public capital in this region. As for K and L , they represent, respectively, the private capital

input and the labor input used by the competitive firms that produce the consumption good in this region. Also, γ and α are parameters that satisfy the following conditions: $\gamma > 0, 0 < \alpha < 1, \gamma + \alpha < 1$. Note that the augmented Cobb-Douglas production function represented by (2) displays constant returns to scale with respect to the two private inputs. Also, note that the assumption $\gamma + \alpha < 1$ implies that there are diminishing returns when inputs of public capital and private capital both rise.⁶

2.2. The Competitive Equilibrium Induced by a Strategy Profile

A strategy for regional government i is a map: $S_i: t \rightarrow (q_i[t], \bar{w}_i[t], h_i[t], G_i[t]), t \geq 0$, where $q_i[t]$, $\bar{w}_i[t]$, $h_i[t]$, and $G_i[t]$ represent, respectively, the tax on the use of the private capital input, the net wage rate (the wage rate after tax), the investment in infrastructure, and the non-productive public expenditures – all in region i at time t .

At each instant, the state of the system is represented by the list $(H_i[t], k_i[t])_{i=1,2}$, with $H_i[t]$ being the stock of public capital in region i at time t , and $k_i[t]$ being the asset owned by a resident in region i at time t . For a given strategy S_i , the motion the stock of public capital $H_i[t]$ is governed by the following differential equation:

$$(3) \quad \frac{dH_i}{dt} = h_i[t], \quad H_i[0] \text{ is given.} \quad (i = 1, 2).$$

To find the competitive equilibrium induced by a strategy profile (S_1, S_2) , we need to find the time paths of (i) the rental rate of capital – the rate that the competitive firms pay the owners of private capital for the use of this asset – that clears the global market for capital at each instant, (ii) the gross wage rate (the wage rate before tax) that clears the labor market in each region at each instant, (iii) the demand for labor and capital of the representative firm in each region, and (iv) the decisions on consumption, labor supply, and asset accumulation of a resident in each region.

⁶ This is a variant of the production function used by Barro and Sala-i Martin (1995).

2.2.1. Utility Maximization

Let (S_1, S_2) be a strategy profile. In the model, the consumption good at each instant is taken as the numéraire. Let $r[t]$ denote the rental rate of private capital at time t received by an individual in the federation.

At each instant, an individual rents her capital assets and sells part of her time endowment – as labor supply – to the representative firm in her own region. More specifically, an individual in region i solves the following utility maximization problem: Find a time path of private consumption cum labor supply $(c_i[t], \ell_i[t]), t \geq 0$, to

$$(4) \quad \max_{(c_i[t], \ell_i[t])_{t \geq 0}} \int_0^\infty e^{-\rho t} (\text{Log} c_i[t] + \sigma \text{Log}[1 - \ell_i[t]]) dt$$

subject to

$$(5) \quad \frac{dk_i}{dt} = r[t]k_i[t] + \bar{\omega}_i[t] \ell_i[t] - c_i[t], \quad k_i[0] \text{ is given,}$$

where ρ is the discount rate she uses to discount future utilities.

The solution of the maximization problem constituted by (4) and (5) is straightforward. First, write the current Hamiltonian as

$$\text{Log} c_i[t] + \sigma \text{Log}[1 - \ell_i[t]] + \lambda_i[t](r[t]k_i[t] + \bar{\omega}_i[t] \ell_i[t] - c_i[t]),$$

with $\lambda_i[t]$ being the current shadow price of asset at time t .

The optimal consumption at time t is given by

$$(6) \quad c_i[t] = \frac{1}{\lambda_i[t]}.$$

The optimal labor supply at each instant is given by

$$(7) \quad \ell_i[t] = 1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}.$$

The differential equation that governs the motion of the shadow price of asset is

$$(8) \quad \frac{d\lambda_i}{dt} = (\rho - r[t])\lambda_i[t].$$

Together, (5)-(8) constitute the first-order conditions that characterize the optimal consumption cum labor supply at each instant of an individual in region i .

For our own use later, let

$$(9) \quad v_i[\bar{\omega}_i[t], \lambda_i[t]] = \text{Log} c_i[t] + \sigma \text{Log}[1 - \ell_i[t]] \\ = -(1 + \sigma) \text{Log}[\lambda_i[t]] - \sigma \text{Log}[\bar{\omega}_i[t]] + \sigma \text{Log}[\sigma]$$

denote the optimal utility derived from private consumption and leisure at each instant – as a function of the net wage rate and the current shadow price of the private capital asset.

2.2.2. Profit Maximization

At each instant t , the representative firm in region i solves the following profit maximization problem:

$$(10) \quad \max_{(K,L)} A_i(H_i[t])^\gamma K^\alpha L^{1-\alpha} - (r[t] + q_i[t])K - \omega_i[t]L,$$

where $\omega_i[t]$ represents the gross wage rate (wage rate before tax) at time t in region i .

The following first-order conditions characterize the solution of (10):

$$(11) \quad \alpha A_i(H_i[t])^\gamma K^{\alpha-1} L^{1-\alpha} - (r[t] + q_i[t]) = 0,$$

$$(12) \quad (1 - \alpha) A_i(H_i[t])^\gamma K^\alpha L^{-\alpha} - \omega_i[t] = 0.$$

2.2.3. The Equilibrium Rental Rate of Capital and the Equilibrium Gross Wage Rate in Each Region

Using (7), we obtain the following expression for the market supply of labor in region i at time t :

$$(13) \quad N_i \ell_i[t] = N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]} \right).$$

Substituting the expression of $N_i \ell_i[t]$ for L in (11), and then solve this first-order condition for K , we obtain the following expression for the demand for private capital by the representative firm in region i at time t :

$$(14) \quad K_i^+[t] = N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}\right) \left(\frac{\alpha A_i(H_i[t])^\gamma}{r[t] + q_i[t]}\right)^{\frac{1}{1-\alpha}}.$$

The equilibrium condition in the global market for capital is then given by

$$(15) \quad \sum_{i=1}^2 K_i^+[t] - \sum_{i=1}^2 N_i k_i[t] = \sum_{i=1}^2 N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}\right) \left(\frac{\alpha A_i(H_i[t])^\gamma}{r[t] + q_i[t]}\right)^{\frac{1}{1-\alpha}} - \sum_{i=1}^2 N_i k_i[t] = 0.$$

Note that the first expression on the left-hand side of the second equality in (15) is strictly decreasing in $r[t]$. Furthermore, it tends to infinity when $r[t] \downarrow -q_i[t]$ and to 0 when $r[t] \rightarrow \infty$. Hence there exists a unique value of $r[t]$, say

$$(16) \quad r[t] = r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t])_{i=1,2}],$$

which might be negative, such that the market-clearing condition (15) holds.

When

$$\sum_{i=1}^2 N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}\right) \left(\frac{\alpha A_i(H_i[t])^\gamma}{q_i[t]}\right)^{\frac{1}{1-\alpha}} - \sum_{i=1}^2 N_i k_i[t] < 0,$$

we have $r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t])_{i=1,2}] < 0$, i.e., there is no non-negative value of $r[t]$ that clears the global capital market, i.e., On the other hand, if

$$\sum_{i=1}^2 N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}\right) \left(\frac{\alpha A_i(H_i[t])^\gamma}{q_i[t]}\right)^{\frac{1}{1-\alpha}} - \sum_{i=1}^2 N_i k_i[t] \geq 0,$$

then $r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t])_{i=1,2}] \geq 0$, and a non-negative rental rate of private capital exists that clears the global capital market.

Using the definition of $r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t])_{i=1,2}]$ in (14), we obtain the following expression for the demand for private capital by the representative firm in region i at time t – as a function of the tax policies of the two regional governments and the state of the system at that instant:

$$(17) \quad K_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] \\ = N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}\right) \left(\frac{\alpha A_i (H_i[t])^\gamma}{r^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] + q_i[t]} \right)^{\frac{1}{1-\alpha}}, \quad (i = 1, 2).$$

Using (13) and (17) in (12), we obtain the following expression for the equilibrium gross wage rate in region i at time t :

$$(18) \quad \omega_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] \\ = (1 - \alpha) A_i (H_i[t])^\gamma \left(\frac{\alpha A_i (H_i[t])^\gamma}{r^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] + q_i[t]} \right)^{\frac{\alpha}{1-\alpha}}, \quad (i = 1, 2).$$

2.3. Equalization Transfers and the Strategies Chosen by a Regional Government

Assuming that each regional government adopts a balanced budget policy, we obtain the following expression for the budget constraint of a regional government at each instant:

$$(19) \quad h_i[t] + G_i = \Re_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}], \quad (i = 1, 2),$$

where we have let

$$(20) \quad \Re_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] = \\ (1 - \tau) \left(+ N_i \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]}\right) (\omega_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] - \bar{\omega}_i[t]) \right)$$

$$+ \tau \left(+N_j \left(1 - \frac{\sigma}{\lambda_j[t] \bar{\omega}_j[t]} \right) (\omega_j^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] - \bar{\omega}_j[t]) \right),$$

($i = 1, 2$),

denote the total revenues at the disposal of regional government i at time t for financing the provision of the non-productive public good and the accumulation of public capital.

Note that the first expression on the second line of the equality in (20) represents the part of the tax revenues it collects on its own and is allowed to keep, while the expression on the third line represents the equalization transfer it receives under the tax-sharing scheme.

It follows from (19) that the expenditure on the non-productive good in region i at each instant is given by

$$(21) \quad G_i[t] = \Re_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] - h_i[t], \quad (i = 1, 2).$$

With the expenditure on the non-productive public good in a region given by (21), a strategy for a regional government is reduced to a map: $S_i: t \rightarrow (q_i[t], \bar{\omega}_i[t], h_i[t]), t \geq 0$, which specifies its tax policy and its investment in public capital.

2.4. Definition of the Nash Equilibrium

For each $i = 1, 2$, let $S_i: t \rightarrow (q_i[t], \bar{\omega}_i[t], h_i[t]), t \geq 0$, be a strategy for regional government i . A strategy profile (S_1, S_2) is said to be admissible if it induces a competitive equilibrium under which the equilibrium rental rate of capital is non-negative. In what follows, we only consider admissible strategy profiles.

Consider an admissible strategy profile (S_1, S_2) . Let $(c_i[t], \ell_i[t], k_i[t]), t \geq 0$ and $\lambda_i[t], t \geq 0$, denote, respectively, the time path of the lifetime plan and the time path of the current shadow price of asset of an individual in region i . Also, let

$$r[t] = r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t])_{i=1,2}]$$

be the rental rate of capital under the competitive equilibrium induced by (S_1, S_2) .

Suppose that the payoff for the government of a region is the discounted utilities of all the residents in its own region. Because the number of residents in a region is fixed, the payoff for the government of region i under the strategy profile (S_1, S_2) can be taken as the lifetime utility of a resident of that region, namely

$$(22) \quad \begin{aligned} \Gamma_i[S_1, S_2] &= \int_0^\infty e^{-\rho t} (v_i[\bar{\omega}_i[t], \lambda_i[t]] + \beta \text{Log} G_i[t] + \delta \text{Log} H_i[t]) dt \\ &= \int_0^\infty e^{-\rho t} \left(\begin{aligned} &-(1 + \sigma) \text{Log}[\lambda_i[t]] - \sigma \text{Log}[\bar{\omega}_i[t]] + \sigma \text{Log}[\sigma] \\ &+ \beta \text{Log} \left[\mathfrak{R}_i^+[(q_{i'}[t], \bar{\omega}_{i'}[t], H_{i'}[t], k_{i'}[t], \lambda_{i'}[t])_{i'=1,2}] - h_i[t] \right] + \delta \text{Log} H_i[t] \end{aligned} \right) dt \end{aligned}$$

DEFINITION: An admissible strategy profile (S_1, S_2) is said to constitute a Nash equilibrium if

$$\Gamma_i[S_i, S_j] \geq \Gamma_i[\tilde{S}_i, S_j], \quad (i, j = 1, 2, i \neq j),$$

for all strategies $\tilde{S}_i$, such that $(\tilde{S}_i, S_j)$ is admissible.

3. THE BEST RESPONSE OF A REGIONAL GOVERNMENT

Consider an admissible strategy profile (S_1, S_2) . To find the best response of regional government i to $S_j, i \neq j, i, j = 1, 2$, we use the first-order approach.

3.1. The Best Response of Regional Government i to S_j

The best response of regional government i is found in two stages. In the first stage, regional government i chooses $(\lambda_i^i[0], \lambda_j^i[0])$, the initial shadow price of assets for a resident of region i and the initial shadow price of assets for a resident of region j . In the second stage, taking $(\lambda_i^i[0], \lambda_j^i[0])$ as given, the government of region i solves the second-best problem constituted by (23)-(30), the optimal payoff of which is denoted by $\psi^i[\lambda_i^i[0], \lambda_j^i[0]]$. Finally, the best response of regional government i to the strategy chosen by regional government j is then found by

choosing $(\lambda_i^i[0], \lambda_j^i[0])$ to maximize $\psi^i[\lambda_i^i[0], \lambda_j^i[0]]$. Note the superscript, which is i in the present second-best problem, refers to the regional government that owns and tries to solve this problem.

The second-best problem in the second stage can be stated formally as follows:

$$(23) \quad \psi^i[\lambda_i^i[0], \lambda_j^i[0]] = \max_{(q_i[t], \bar{\omega}_i[t], h_i[t])_{t \geq 0}} \int_0^\infty e^{-\rho t} \left(\begin{aligned} & -(1 + \sigma) \text{Log}[\lambda_i[t]] - \sigma \text{Log}[\bar{\omega}_i[t]] + \sigma \text{Log}[\sigma] \\ & + \beta \text{Log} \left[\mathfrak{R}_i^+ \left[\begin{aligned} & (q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), \\ & (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t]) \end{aligned} \right] - h_i[t] \right] \\ & + \delta \text{Log} H_i[t] \end{aligned} \right) dt$$

subject to

$$(24) \quad r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] \geq 0,$$

$$(25) \quad \begin{aligned} \frac{dk_i}{dt} = & r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] k_i[t] \\ & + \bar{\omega}_i[t] \ell_i[t] - \frac{1}{\lambda_i[t]}, \end{aligned}$$

$k_i[0]$ is given,

$$(26) \quad \begin{aligned} \frac{dk_j}{dt} = & r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] k_j[t] \\ & + \bar{\omega}_j[t] \ell_j[t] - \frac{1}{\lambda_j[t]}, \end{aligned}$$

$k_j[0]$ is given,

$$(27) \quad \frac{d\lambda_i}{dt} = (\rho - r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])]) \lambda_i[t],$$

$\lambda_i^i[0]$ is given,

$$(28) \quad \frac{d\lambda_j}{dt} = (\rho - r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])]) \lambda_j[t],$$

$\lambda_j^i[0]$ is given,

$$(29) \quad \frac{dH_i}{dt} = h_i[t],$$

$H_i[0]$ is given,

and

$$(30) \quad \frac{dH_j}{dt} = h_j[t],$$

$H_j[0]$ is given.

Observe that (24) represents the constraint that the rental rate of capital received at each instant by an individual in the federation is non-negative. As for (25)-(28), they represent the first-order conditions of the lifetime utility maximization problems for two individuals – one in region i and one from region j . Also, equations (29) and (30) represent, respectively, the motion of the stocks of public capital in the two regions.

To solve the second-best problem constituted by (23)-(30), write the current-value Hamiltonian⁷

$$(31) \quad \mathcal{H}^i = \left(\begin{aligned} & -(1 + \sigma) \text{Log}[\lambda_i[t]] - \sigma \text{Log}[\bar{\omega}_i[t]] + \sigma \text{Log}[\sigma] \\ & + \beta \text{Log} \left[\mathfrak{R}_i^+ \left[\begin{aligned} & (q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), \\ & (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t]) \end{aligned} \right] - h_i[t] \right] \\ & + \delta \text{Log} H_i[t] \end{aligned} \right) \\ & + v^i[t] r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] \\ & + \kappa_i^i[t] \left(\begin{aligned} & r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] k_i[t] \\ & + \left(1 - \frac{\sigma}{\lambda_i[t] \bar{\omega}_i[t]} \right) \bar{\omega}_i[t] - \frac{1}{\lambda_i[t]} \end{aligned} \right) \\ & + \kappa_j^i[t] \left(\begin{aligned} & r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] k_j[t] \\ & + \left(1 - \frac{\sigma}{\lambda_j[t] \bar{\omega}_j[t]} \right) \bar{\omega}_j[t] - \frac{1}{\lambda_j[t]} \end{aligned} \right) \end{aligned}$$

⁷ Again, the superscript, which is i in the present case, refers to the regional government that owns and tries to solve the second-best problem.

$$\begin{aligned}
& +\mu_i^i[t](\rho - r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])])\lambda_i[t] \\
& +\mu_j^i[t](\rho - r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])])\lambda_j[t] \\
& +\eta_i^i[t]h_i[t] \\
& +\eta_j^i[t]h_j[t].
\end{aligned}$$

In (31), $\nu^i[t][t]$, $\kappa_i^i[t]$, $\kappa_j^i[t]$, $\mu_i^i[t]$, $\mu_j^i[t]$, $\eta_i^i[t]$, and $\eta_j^i[t]$ are the multipliers associated, respectively, with the constraints (24)-(30) of the second-best problem.

The differential equations that govern the motion of the co-states are given by

$$(32) \quad \frac{d\kappa_i^i}{dt} = \rho\kappa_i^i[t] - \frac{\partial \mathcal{H}^i}{\partial \kappa_i[t]},$$

$$(33) \quad \frac{d\kappa_j^i}{dt} = \rho\kappa_j^i[t] - \frac{\partial \mathcal{H}^i}{\partial \kappa_j[t]},$$

$$(34) \quad \frac{d\mu_i^i}{dt} = \rho\mu_i^i[t] - \frac{\partial \mathcal{H}^i}{\partial \lambda_i[t]},$$

$$(35) \quad \frac{d\mu_j^i}{dt} = \rho\mu_j^i[t] - \frac{\partial \mathcal{H}^i}{\partial \lambda_j[t]},$$

$$(36) \quad \frac{d\eta_i^i}{dt} = \rho\eta_i^i[t] - \frac{\partial \mathcal{H}^i}{\partial H_i[t]},$$

$$(37) \quad \frac{d\eta_j^i}{dt} = \rho\eta_j^i[t] - \frac{\partial \mathcal{H}^i}{\partial H_j[t]},$$

The value of $q_i[t]$, $\bar{\omega}_i[t]$, and $h_i[t]$ that maximize the Hamiltonian satisfy, respectively, the following first-order condition:

$$(38) \quad \frac{\partial \mathcal{H}^i}{\partial q_i[t]} = 0,$$

$$(39) \quad \frac{\partial \mathcal{H}^i}{\partial \bar{\omega}_i[t]} = 0,$$

$$(40) \quad \frac{\partial \mathcal{H}^i}{\partial h_i[t]} = 0.$$

3.2. The Best Response of Regional Government j to S_i

The manner used to find the best response of regional government i can be repeated to find the best response of regional government j . In the first stage, regional government j chooses $(\lambda_i^j[0], \lambda_j^j[0])$, the initial shadow price of assets for a resident of region i and the initial shadow price of assets for a resident of region j . In the second stage, taking $(\lambda_i^j[0], \lambda_j^j[0])$ as given, the government of region i solves the second-best problem constituted by (41)-(48), the optimal payoff of which is denoted by $\psi^j[\lambda_i^j[0], \lambda_j^j[0]]$. Finally, the best response of regional government j to the strategy chosen by regional government i is then found by choosing $(\lambda_i^j[0], \lambda_j^j[0])$ to maximize $\psi^j[\lambda_i^j[0], \lambda_j^j[0]]$.

The second-best problem in the second stage can be stated formally as follows:

$$(41) \quad \psi^j[\lambda_i^i[0], \lambda_j^i[0]] = \max_{(q_j[t], \bar{\omega}_j[t], h_j[t])_{t \geq 0}} \int_0^\infty e^{-\rho t} \left(\begin{aligned} & -(1 + \sigma) \text{Log}[\lambda_j[t]] - \sigma \text{Log}[\bar{\omega}_j[t]] + \sigma \text{Log}[\sigma] \\ & + \beta \text{Log} \left[\mathfrak{R}_j^+ \left[\begin{aligned} & (q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), \\ & (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t]) \end{aligned} \right] - h_j[t] \right] \\ & + \delta \text{Log} H_j[t] \end{aligned} \right) dt$$

subject to

$$(42) \quad r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] \geq 0,$$

$$(43) \quad \frac{dk_i}{dt} = r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] k_i[t] \\ + \bar{\omega}_i[t] \ell_i[t] - \frac{1}{\lambda_i[t]},$$

$k_i[0]$ is given,

$$(44) \quad \frac{dk_j}{dt} = r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])]k_j[t] \\ + \bar{\omega}_j[t]\ell_j[t] - \frac{1}{\lambda_j[t]},$$

$k_j[0]$ is given,

$$(45) \quad \frac{d\lambda_i}{dt} = (\rho - r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])])\lambda_i[t],$$

$\lambda_i^j[0]$ is given,

$$(46) \quad \frac{d\lambda_j}{dt} = (\rho - r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])])\lambda_j[t],$$

$\lambda_j^j[0]$ is given,

$$(47) \quad \frac{dH_i}{dt} = h_i[t],$$

$H_i[0]$ is given,

and

$$(48) \quad \frac{dH_j}{dt} = h_j[t],$$

$H_j[0]$ is given.

To solve the second-best problem constituted by (41)-(48), write the current-value Hamiltonian⁸

$$(49) \quad \mathcal{H}^j = \left(\begin{array}{c} -(1 + \sigma)\text{Log}[\lambda_j[t]] - \sigma\text{Log}[\bar{\omega}_j[t]] + \sigma\text{Log}[\sigma] \\ + \beta\text{Log} \left[\mathfrak{R}_j^+ \left[\begin{array}{c} (q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), \\ (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t]) \end{array} \right] - h_j[t] \right] \\ + \delta\text{Log}H_j[t] \end{array} \right) \\ + \nu^j[t]r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])]$$

⁸ Note that the superscript j refers to the regional government that owns and tries to solve the second-best problem. The superscript is needed because the multipliers associated with the constraints in the second-best problems of the two regional governments are not identical.

$$\begin{aligned}
& +\kappa_i^j[t] \left(r^+[(q_i[t], \bar{w}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{w}_j[t], H_j[t], k_j[t], \lambda_j[t])]k_i[t] \right. \\
& \quad \left. + \left(1 - \frac{\sigma}{\lambda_i[t]\bar{w}_i[t]} \right) \bar{w}_i[t] - \frac{1}{\lambda_i[t]} \right) \\
& +\kappa_j^j[t] \left(r^+[(q_i[t], \bar{w}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{w}_j[t], H_j[t], k_j[t], \lambda_j[t])]k_j[t] \right. \\
& \quad \left. + \left(1 - \frac{\sigma}{\lambda_j[t]\bar{w}_j[t]} \right) \bar{w}_j[t] - \frac{1}{\lambda_j[t]} \right) \\
& +\mu_i^j[t](\rho - r^+[(q_i[t], \bar{w}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{w}_j[t], H_j[t], k_j[t], \lambda_j[t])])\lambda_i[t] \\
& +\mu_j^j[t](\rho - r^+[(q_i[t], \bar{w}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{w}_j[t], H_j[t], k_j[t], \lambda_j[t])])\lambda_j[t] \\
& +\eta_i^j[t]h_i[t] \\
& +\eta_j^j[t]h_j[t].
\end{aligned}$$

In (49), $v^j[t]$, $\kappa_i^j[t]$, $\kappa_j^j[t]$, $\mu_i^j[t]$, $\mu_j^j[t]$, $\eta_i^j[t]$, and $\eta_j^j[t]$ are the multipliers associated, respectively, with the constraints (41)-(48) of the second-best problem.

The differential equations that govern the motion of the co-state variables are given by

$$(50) \quad \frac{d\kappa_i^j}{dt} = \rho\kappa_i^j[t] - \frac{\partial \mathcal{H}^j}{\partial k_i[t]},$$

$$(51) \quad \frac{d\kappa_j^j}{dt} = \rho\kappa_j^j[t] - \frac{\partial \mathcal{H}^j}{\partial k_j[t]},$$

$$(52) \quad \frac{d\mu_i^j}{dt} = \rho\mu_i^j[t] - \frac{\partial \mathcal{H}^j}{\partial \lambda_i[t]},$$

$$(53) \quad \frac{d\mu_j^j}{dt} = \rho\mu_j^j[t] - \frac{\partial \mathcal{H}_i}{\partial \lambda_j[t]},$$

$$(54) \quad \frac{d\eta_i^j}{dt} = \rho\eta_i^j[t] - \frac{\partial \mathcal{H}^j}{\partial H_i[t]},$$

$$(55) \quad \frac{d\eta_j^j}{dt} = \rho\eta_j^j[t] - \frac{\partial \mathcal{H}^j}{\partial H_j[t]},$$

The value of $q_j[t]$, $\bar{\omega}_j[t]$, and $h_j[t]$ that maximize the Hamiltonian satisfy, respectively, the following first-order condition:

$$(56) \quad \frac{\partial \mathcal{H}^j}{\partial q_j[t]} = 0,$$

$$(57) \quad \frac{\partial \mathcal{H}^j}{\partial \bar{\omega}_j[t]} = 0,$$

$$(58) \quad \frac{\partial \mathcal{H}^j}{\partial h_j[t]} = 0.$$

4. CALIBRATION

In this section, we calibrate the model, with Ontario as region 1 and Québec as region 2. The first step in the numerical exercise is to calibrate the model by specifying the values of its parameters. Recall that the utility obtained at each instant t by a resident of region i is given by

$$(1) \quad \text{Log} c_i[t] + \sigma \text{Log}[1 - \ell_i[t]] + \beta \text{Log} G_i[t] + \delta \text{Log} H_i[t].$$

We set $\sigma = 1.3$ so that in the steady state the labor supply of an individual is around 25% of her time endowment, the number suggested by Prescott (2006).⁹ For public goods, we set $\beta = 0.2$, $\delta = 0.05$. These intensities for the preferences of the non-productive public goods and public capital – although they lead to a rather high ratio of public expenditures over GDP – make the computations of the steady state more bearable. As for the rate of time preferences, we set $\rho = 0.05$, which is close to the rate often found in the economic literature.

As for the production functions, recall that the production function of region i is

$$(2) \quad Y_i = A_i H_i^\gamma K^\alpha L^{1-\alpha}.$$

We set $\alpha = 0.31$, the usual estimate of the elasticity of output with respect to private capital. The estimate of the elasticity of public capital with respect to output is controversial. Following Ryan

⁹ E.C. Prescott (2006): Nobel Lecture: The Transformation of Macroeconomic Policy and Research, Journal of Political Economy 114, 203-235.

MacDonald (2008),¹⁰ who carried out an empirical study on the rate of return on public capital for Canada, we set $\gamma = 0.06$.

As for the total factor productivity A_i , its rate of growth, which represents the rate of technical progress, has been extensively estimated for Canada and its ten provinces.¹¹ Because it is the impact of the equalization transfer scheme – not technological progress – on the strategic behavior of the two regional governments, we shall assume that A_i remains the same over time, and thus need its value for a particular year to be able to carry out the numerical simulation. To this end, we need the value of Y_i, H_i, K_i, L_i for a particular year. We choose 2007 as the year to calibrate A_i .

In 2007, the stocks of public capital in Ontario and Québec were 93,267 *million dollars* and 68,103 *million dollars*, respectively.¹² Here public capital covers five public assets: highways and roads, bridges and overpasses, water supply systems, wastewater treatment facilities and sanitary and storm sewers.

In 2007, the capital services¹³ – that we assume for simplicity to be the same as the capital stocks – in Ontario and Québec were 160,829 and 82,435 million of 2002 dollars, respectively.¹⁴ In terms of 2007 dollars,¹⁵ these capital stocks amount to $1.104 \times 160,829 = 177,555$ million and $82,435 \times 1.129 = 93,069$ million.

¹⁰ Ryan MacDonald (2008): An Examination of Public Capital's Role in Production, Micro-economic Analysis Division Statistics Canada, 11F0027MIE No. 050.

¹¹ See the study entitle "Estimates of Labour, Capital, and Multifactor Productivity by Provinces and Industry, 1997-2010, The Centre for the Study of Living Standards, March 7, 2012.

¹² See Mychèle Gagnon, Valérie Gaudreault, and Donald Overton (2008): "Age of Public Infrastructure: A Provincial Perspective," Table 2, Statistics Canada, Catalogue No: 11-621-MIE2008067.

¹³ For a distinction between the capital stock and capital service, see Sharpe et al. (2009): "New Estimates of Multifactor Productivity Growth for the Canadian Provinces," Centre for the Study of Living Standards.

¹⁴ These numbers are from Table 8 Capital Services (\$2002), by Two-Digit NAICS Industry, 1997-2010, in "Estimates of Labour, Capital and Multifactor Productivity by Province and Industry, 1997-2010," The Centre for the Study of Living Standards.

¹⁵ The GDP deflator for Québec and Ontario are obtained from Table 7 GDP Deflator (Index, 2002=100), by Two-Digit NAICS Industry, 1997-2008, in "Estimates of Labour, Capital and Multifactor Productivity by Province and Industry, 1997-2010," The Centre for the Study of Living Standards.

According to the 2006 Census of Statistics Canada,¹⁶ the employed labor forces in Québec and Ontario were, respectively, 3,375,505 and 6,164,245. We assume that the labor forces did not change much in 2007, and accept these numbers as the labor forces in 2007 for the two provinces. Assuming the usual participation rate of about 65%, the numbers of individuals looking for work in Québec and in Ontario in 2007 were thus 2194078 and 4006759, respectively. Also, assuming an unemployment rate of 7%, we obtain 3726286 and 2040492 as the numbers of employed workers in Québec and Ontario, respectively.

In our theoretical model, we assume that N_i , the population of region i , $i = 1, 2$, is also its labor force and that every individual in the labor force works. For our numerical example, we take the number of employed workers to represent the value of N_i , i.e., $N_i = 3726286$ for Ontario and $N_i = 2040492$ for Québec.

In 2007, the numbers of hours worked (in millions) in Ontario and Québec were 9721 and 5110, respectively.¹⁷ The number of hours worked by a worker in Ontario was then $\frac{9721 \times 10^6}{3726286} = 2609$ hours. Because the total yearly time endowment of a worker is $24 \times 365 = 8760$ hours, the labor supply of an employed worker in Ontario – as a fraction of her yearly time endowment – is $\ell_1 = \frac{2609}{8760} = 0.30$. The number of hours worked by a worker in Québec was $\frac{5110 \times 10^6}{2040492} = 2504$ hours. The labor supply of an employed worker in Québec – as a fraction of her yearly time endowment – is $\ell_2 = \frac{2504.3}{8760} = 0.28$.

With the yearly time endowment of an employed worker normalized at 1, the total labor supply in Ontario in 2007 was $L_1 = 0.30 \times N_1 = 0.30 \times 3726286 = 1117886$; the corresponding number for Québec was $L_2 = 0.280 \times N_2 = 0.28 \times 2040492 = 571338$.

In 2007, nominal output for Ontario was 415,631 million dollars. The corresponding number for Quebec was 208,217 million dollars.¹⁸

¹⁶ Statistics Canada, CANSIM, table 282-0087 and Catalogue no. 71-001-XIE.

¹⁷ Table 1 Hours Worked (millions) by Two-Digit NAICS Industry, 1997-2010, in “Estimates of Labour, Capital and Multifactor Productivity by Province and Industry, 1997-2010,” The Centre for the Study of Living Standards.

¹⁸ Table 9 Nominal Output, by Two-Digit NAICS Industry, 1997-2008, in “Estimates of Labour, Capital and Multifactor Productivity by Province and Industry, 1997-2010,” The Centre for the Study of Living Standards.

The following table summarizes the results just computed:

Table 1

The values of the multifactor productivity of Ontario and Québec for 2007

Region	Y_i (in millions of \$)	H_i (in millions of \$)	K_i (in millions of \$)	L_i (in units of time endowment)	$A_i = \frac{Y_i}{H_i^\gamma K^\alpha L^{1-\alpha}}$ (calibrated)
Ontario	415,631	93,267	177555	1117886	0.33
Québec	208,217	68,103	93069	571338	0.32

Observe from Table 1 that in 2007 the total factor productivity of Québec was lower than that of Ontario by about 3%. Also, observe that we have chosen to model the production heterogeneity of the two regions as the difference in their total factor productivity. Another way of modeling regional heterogeneity is to allow for α and γ to vary across regions.

5. THE STEADY STATE

The Nash equilibrium is characterized by two sets of equations, with each set of equations characterizing the best response of regional government.

The best response of regional government i is characterized by the following set of equations:

- Equations (25), (26), (27), (28), (29), (30), which govern the motion, respectively, of the state variables $k_i[t]$, $k_j[t]$, $\lambda_i[t]$, $\lambda_j[t]$, $H_i[t]$, $H_j[t]$.
- Equations (32), (33), (34), (35), (36), (37), which govern the motion, respectively, of the co-state variables associated with the state variables $k_i[t]$, $k_j[t]$, $\lambda_i[t]$, $\lambda_j[t]$, $H_i[t]$, $H_j[t]$ in the second-best problem faced by regional government i .

- Equations (38), (39), (40), which characterize, respectively, the optimal values of $q_i[t], \bar{\omega}_i[t], h_i[t]$.

In total, the best response of regional government i is characterized by 15 equations.

Similarly, the best response of regional government j is characterized by the following set of equations:

- Equations (43), (44), (45), (46), (47), (48), which govern the motion, respectively, of the state variables $k_i[t], k_j[t], \lambda_i[t], \lambda_j[t], H_i[t], H_j[t]$.
- Equations (50), (51), (52), (53), (54), (55), which govern the motion, respectively, of the co-state variables associated with the state variables $k_i[t], k_j[t], \lambda_i[t], \lambda_j[t], H_i[t], H_j[t]$ in the second-best problem faced by regional government j .
- Equations (56), (57), (58), which characterize, respectively, the optimal values of $q_j[t], \bar{\omega}_j[t], h_j[t]$.

In total, the best response of regional government j is characterized by 15 equations.

Observe that the equations that govern the motion of the state variables $k_i[t], k_j[t], \lambda_i[t], \lambda_j[t], H_i[t], H_j[t]$ appear in both the best response of regional government i and in the best response of regional government j , the actual number of equations that characterize the Nash equilibrium is only 24 after the double counting is eliminated. These 24 equations govern the motion of 24 variables:

- Six state variables $k_i[t], k_j[t], \lambda_i[t], \lambda_j[t], H_i[t], H_j[t]$.
- Six co-state variables for regional government i : $\kappa_i^i[t], \kappa_j^i[t], \mu_i^i[t], \mu_j^i[t], \eta_i^i[t], \eta_j^i[t]$.
- Six co-state variables for regional government j : $\kappa_i^j[t], \kappa_j^j[t], \mu_i^j[t], \mu_j^j[t], \eta_i^j[t], \eta_j^j[t]$.
- Three policy choices for regional government i : $q_i[t], \bar{\omega}_i[t], h_i[t]$.
- Three policy choices for regional government j : $q_j[t], \bar{\omega}_j[t], h_j[t]$.

Also, under a Nash equilibrium the initial shadow prices of assets chosen by the two regional governments must be the same, i.e., we must have $(\lambda_i^i[0], \lambda_j^i[0]) = (\lambda_i^j[0], \lambda_j^j[0])$.

In the long-run equilibrium, the state and co-state variables are constant, and their time derivatives all vanish, which yields a nonlinear system of 24 equations in 24 unknowns. Note that because $\lambda_i[t], \lambda_j[t]$ are constant in the long run, their time derivatives must vanish when $t \rightarrow \infty$, which in turn imply that the rental rate of capital must be equal to the rate of time preferences:

$$(59) \quad \lim_{t \rightarrow \infty} r^+[(q_i[t], \bar{\omega}_i[t], H_i[t], k_i[t], \lambda_i[t]), (q_j[t], \bar{\omega}_j[t], H_j[t], k_j[t], \lambda_j[t])] = \rho.$$

Now recall that the equilibrium rental rate of capital at each instant can only be defined implicitly by (15), the market-clearing condition for the global capital market. Using (59), we can assert that in the long-run equilibrium, (15) is reduced to

$$(60) \quad \sum_{i=1}^2 N_i \left(1 - \frac{\sigma}{\lambda_i \bar{\omega}_i}\right) \left(\frac{\alpha A_i (H_i)^\gamma}{\rho + q_i}\right)^{\frac{1}{1-\alpha}} - \sum_{i=1}^2 N_i k_i = 0.$$

In (60), a variable without the time variable t indicates its long-run equilibrium value.

Adding (60) to the set of equations that characterize the Nash equilibrium in the long run, we obtain (25) equations in (24) unknowns. However, when (60) holds, the steady state versions of the two differential equations (27) and (28) automatically hold. Thus, we now have 23 independent equations in (24) unknowns. There is thus one unknown more than the number of independent equations. The system of equations that characterize the long-run equilibrium seems to be under-identified, and thus there is the open possibility that the system has multiple solutions. This problem does not arise when the two regions are symmetric. When the two regions are identical, the symmetric equilibrium is characterized by the best response of a single region. Furthermore, when the symmetric version of the market-clearing condition (60) is added to the system of equations that characterize the best response of a region, say region i , we are able to remove the steady state versions two differential equations – one for λ_i and one for λ_j – are removed. We then have 14 equations in 14 unknowns, i.e., the number of equations and the number of variables are equal.

In our attempt to find a solution to the system of equations that characterize the long-run equilibrium, we set $q_j = 0$,¹⁹ and in this manner, we manage to reduce the number of variables by one. We now have a nonlinear system of 23 equations in 23 unknowns. A *Mathematica* program is written to solve this non-linear system of 23 equations in 23 unknowns. In this manner, we obtain a solution to the system of equations that characterize the long-run equilibrium. The solution involves $q_i = q_j = 0$.

We have also attempted to assign several non-zero values for q_j . However, the command *FindRoot* of *Mathematica* failed to converge each time, and we are led to believe that the equations that characterize the long-run equilibrium cannot be satisfied if either regional government taxes the use of the private capital input within their own borders. Therefore, we accept the solution obtained by setting $q_j = 0$ to reduce the number of unknowns by one as the long-run equilibrium of the system.

Exploiting the brute-force symbolic as well as numerical computational capacity of *Mathematica*, we obtain the long-run equilibrium of the system first for the case of no equalization transfers, and then for the case the central government administers a tax-revenue sharing scheme under which a fraction $\tau = 0.10$ of the tax revenues collected by one region is transferred to the other region, and vice versa. The long-run equilibrium without equalization transfers is presented in Table 2. The equilibrium with equalization transfers is represented in Table 3. Table 4, which is computed from Tables 2 and 3, summarizes the impact of the equalization transfers on resource allocation and welfare.

Table 2

¹⁹ This action is not needed under a symmetric equilibrium. The *Mathematica* program written to compute the symmetric equilibrium for two regions identical to Ontario finds that the use of the private capital input is not taxed in the long run, i.e., $q_i = q_j = 0$ under the symmetric equilibrium in the long run.

The steady state: no equalization transfers

Region	Ontario	Québec
H_i (public capital, in millions of dollars)	2,306,652	1,106,804
k_i (asset held per resident, in millions of dollars)	1.140	1.337
q_i (tax on private capital input)	0	0
ω_i^+ (gross wage rate, in millions of dollars)	0.594	0.550
$\bar{\omega}_i$ (net wage rate, in millions of dollars)	0.157	0.194
$\frac{\omega_i^+ - \bar{\omega}_i}{\omega_i^+}$ (tax rate on labor income)	74%	60%
$N_i k_i$ (private capital assets held by all the residents in their own region, in millions of dollars)	4,249,707	2,727,905
K_i^+ (demand for private capital input by the representative firm, in millions of dollars)	4,554,736	2,422,875
$K_i^+ - N_i k_i$ (capital import, in millions of dollars)	305,029	-305,029
G_i (non-productive public expenditures, in millions of dollars)	373,174	174,466
Y_i (GDP, in millions of dollars)	734,635	390,786
ℓ_i (labor supply per individual)	0.229	0.240
c_i (private consumption per individual, in millions of dollars)	0.093	0.113
u_i (welfare per resident)	0.584	0.576

It can be seen from Table 2 that the tax rate on capital income is equal to zero in the long run. Our numerical solution thus supports the theoretical result established by Gross (August 2013,

October 2013) that in a world made up of two large economies the optimal capital income tax is zero.

The stock of public capital and the non-productive expenditures are both higher in Ontario than in Québec. The reasons behind these results are not hard to see. First, because Ontario has a higher total factor productivity and a much higher population, its tax base is larger than that of Québec, and thus the tax revenues collected by the government of this region are higher than that of Québec, which makes it feasible for Ontario to have a higher level of public capital and a higher level of non-productive public expenditures. Second, both public capital and non-productive public expenditures enter the utility function of each individual in the federation, and the preferences for these two types of public goods reinforce the technological advantage and the large population of Ontario. As can be seen from Table 2, the stock of public capital in Ontario is about twice that in Québec, while the non-productive public expenditures are more than double that of Québec.

The gross wage rate is higher in Ontario than in Québec as expected. However, the net wage rate is lower in Ontario than in Québec. This result is a little surprising. However, when one takes into consideration that in the long run capital is not taxed, the non-productive expenditures must be financed entirely from the tax on labor income, and this explains the extremely high tax rate on wages. As can be seen from Table 2, wages are taxed at 74% in Ontario and 60% in Québec. Because Ontario has a higher level of the non-productive public expenditures, it needs more tax revenues to finance this type of expenditures, and this explains why the tax rate on wages is higher than that in Québec.

Another result that seems surprising at first is that the capital asset owned by a resident is lower in Ontario than in Québec. However, in the literature on social security it is well known that the pay as you go system, which taxes the earnings of young workers to finance the retirement of older individuals, has a depressing effect on the capital labor ratio in the long run because it leaves a young individual with less income to save.

The lower capital asset owned by a resident of Ontario coupled with a higher technological level, a larger population, and a larger stock of public capital raises the productivity of private capital in this region, inducing an inflow of private capital. As can be seen from line 10 of Table 2,

capital flows from Québec in Ontario, and this flows amounts to 305,629 million dollars in each period.

An individual in Ontario uses only .22.9% of her time endowment for working. The corresponding number for Québec is slightly higher at 24%.

The lower net wage rate and the lower labor supply of a resident in Ontario leave her with fewer resources to spend on private consumption, and this can be seen from the second line from the bottom of Table 2. Indeed, most of the utilities enjoyed by a resident of Ontario come from the consumption of the public goods.

When the central government administers an equalization transfer scheme under which a region transfers a proportion of $\tau = 10\%$ of the tax revenues it collects to the other region, the results – as shown in Table 3 – are quite surprising. Under this equalization transfer scheme, Ontario sends to Québec 19,871 million dollars as equalization transfers in each period, as can be seen from Table 4.

Table 3

Steady state with equalization transfers: $\tau = 0.10$

Region	Ontario	Québec
H_i (public capital, in millions of dollars)	2,306,652	1,106,804
k_i (asset held per resident, in millions of dollars)	1.140	1.337
q_i (tax on private capital input)	0	0
ω_i^+ (gross wage rate, in millions of dollars)	0.594	0.550
$\bar{\omega}_i$ (net wage rate, in millions of dollars)	0.157	0.194

dollars)		
$\frac{\omega_i^+ - \bar{\omega}_i}{\omega_i^+}$ (tax rate on labor income)	74%	60%
$N_i k_l$ (private capital assets held by all the residents in their own region, in millions of dollars)	4249707.	2,727,905
K_i^+ (demand for private capital input by the representative firm, in millions of dollars)	4,554,736	2,422,875
$K_i^+ - N_i k_l$ (capital import, in millions of dollars)	305,029	-305029
G_i (non-productive public expenditures, in millions of dollars)	353,303	194,337
Y_i (GDP, in millions of dollars)	734,635	390,786
ℓ_i (labor supply per individual)	0.229	0.240
c_i (private consumption per individual, in millions of dollars)	0.093	0.113
u_i (welfare per resident)	0.573	0.598

When one compares Tables 2 and 3, one discovers that the implementation of the equalization induces no changes other than the levels of non-productive public expenditures in the two regions: the stocks of public capital in both regions remain the same; the gross and net wages in both regions are the same; the capital asset held by a resident in each region remains the same; the labor supply by a resident in each region as well as her private consumption remain the same. The only change induced by the equalization transfer scheme is a rise in the non-productive public expenditures in Québec with an offsetting fall in the non-productive public expenditures in Ontario. The end result is a rise in utilities of about 3.7% for a resident of Québec and a fall in utilities of 0.2% for a resident of Ontario, as can be seen from the last line of Table 4.

Table 4

The impact of an equalization transfer scheme

Region	Ontario	Québec
Equalization transfers (in millions of dollars)	−19,871	19,871
ΔG_i (variation in non-productive good expenditures, in millions of dollars)	−19,871	19,871
$\frac{\Delta G_i}{G_i}$ (variation in non-productive good expenditures, in per cent)	−5%	11%
$\frac{\Delta u_i}{u_i}$ (variation in individual utility)	−0.2%	3.7%

We have varied the value of τ , and have found that each time the change in the long-run equilibrium involves only the change in the levels of non-productive public expenditures, and this leads us to believe that the equalization transfers are spent entirely on the non-productive public good, not on public capital accumulation. In a related paper,²⁰ we have formalized a game-theoretic model along the special-interest group approach propounded by Dixit, Grossman, and Helpman (1997) to explain the observation made by David MacKinnon (2011)²¹ that through the equalization transfer program, the have-not provinces of Canada have managed to provide a higher level of public service than the economically more efficient provinces.

6. CONCLUDING REMARKS

In this paper, we formalize a dynamic model of the competition for capital in a federal state that is made up of two asymmetric regions and that adopts an equalization transfer scheme, with the

²⁰ Jin-Woong Kim (2013): “The Political Economy of Equalization Transfers in a Fiscal Federalism,” manuscript.

²¹ See the report entitled “Dollars and Sense: A Case for Modernizing Canada’s Transfer Agreements,” released by the Ontario Chamber of Commerce in February 2011.

evolution of private and public capital accumulation being explicitly modeled. In our model, the residents of a region derive utility from the consumption of a private good, her own leisure, the stock of public capital, and a non-productive public good within their own jurisdiction at every instant. Each regional government provides its residents with non-productive public expenditures and the stock of public capital. In each region competitive firms produce a private good using labor, private capital, and public capital, where public capital is complementary to private capital. A regional government taxes the use of the private capital input and the wage rate within its own jurisdiction, and uses these two tax revenues to finance the non-productive public expenditures and the investments in public capital. Given the revenue-sharing scheme, each regional government behaves strategically in choosing its policies to maximize the discounted welfare of its own residents. We calibrate the model by specifying the values of parameters obtained from the data for Ontario and Quebec.

Our analysis indicates that the the tax on the use of the private capital input is equal to zero in the steady state without equalization transfers, which supports the theoretical result established by Gross (August 2013) that the optimal capital income tax is zero in a world made up of two large economies. In addition, the zero tax rate of the private capital input still holds in the long-run equilibrium under the revenue-sharing scheme.

We observe from our analysis that the only change induced by the equalization transfer scheme is an increase in the non-productive public expenditures in less-endowed region (Quebec) with an offsetting fall in the non-productive public expenditures in more-endowed region (Ontario); the stocks of public capital and the wages in both regions remain the same as well as the capital asset held by a resident, the labor supply by a resident and her private consumption in each region remains the same. The results of the numerical exercise we carry out also suggest that an equalization scheme in a federal state lowers the welfare gap between a rich and a poor region. An equalization payments scheme hence might be justified on equity grounds.

Finally, in this paper our analysis concentrates mainly on the impact of a tax revenues sharing scheme on the steady state. The next step is to carry out an analysis of the convergence to the steady state and the transition from one steady state to another steady state.

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Chapter 2: The Political Economy of Equalization Transfers in a Fiscal Federalism

1. INTRODUCTION

When asked why the have-not provinces of Canada, in general, have higher government public services than the economically more efficient provinces, one of the answers lays the blame at the door of the equalization payments program. In the report entitled “Dollars and Sense: A Case for Modernizing Canada’s Transfer Agreements,” released by the Ontario Chamber of Commerce in February 2011, David MacKinnon, the author of the study, laid out the astonishing evidence for how far the equalization program has gone astray from the path it was initially designed for more than fifty years ago. On average, Alberta, Ontario, and British Columbia – the economically and fiscally competitive provinces – have 24% fewer registered nurses for every 100,000 people than the equalization receiving provinces of New Brunswick, P.E.I., Nova Scotia, and Manitoba. The three economic engines that drive the Canadian economy – Alberta, Ontario, and British Columbia – have the lowest number of nursing home beds per capita. Québec, which is the largest recipient of equalization payments – \$8.5 billion out of \$14.3 billion in total 2010-11 equalization transfers – subsidizes daycare at the rate of \$7 per day, and has the lowest university tuition fees among the provinces. Strikingly, according to the author of the study, Ontario, which is the province that until recently was the engine of growth of the Canadian economy, has probably the least accessible public sector of all Canadian provinces. The Canadian experience is not an isolated incident. According to Horst Seehofer, premier of Bavaria, Germany’s richest state, 10 percent of the state’s budget flows out to other states as equalization transfers to pay for things that Bavaria cannot afford.²² For example, those who go to university in Bavaria have to pay tuition fees while those in the recipient states do not. The recipient states are also more generous than Bavaria in paying for childcare.²³

The equalization payments program has also created a culture of dependency among the have-not provinces. After more than fifty years of massive transfers, the performance of the have-not

²² “German states oppose stupid wealth transfers,” by Wolfgang Dick/cd, Deutsche Welle, 04.03.2013.

²³ “The World from Berlin: Bavaria Mulls an End to Solidarity,” Spiegel OnLine International, July 18, 2012 – 02:35 PM.

provinces in terms of income per capita in the Canadian rankings has changed little. In “Manitoba: The Suppliant Society,” the first of a series of articles written by Bryan Schwartz, Professor of International Trade and Business Law at the University of Manitoba, for the Winnipeg Free Press²⁴ and the Frontier Centre for Public Policy, asserted that despite its strengths, Manitoba lags the other provinces in lower income, higher taxes, and lower productivity. The massive equalization payments the province receives from the federal government encourage waste and mismanagement on the part of the provincial government. The province uses its \$1.2 billion equalization transfer to subsidize electric power consumption and to inflate the public sector. Under the current equalization payments program, the transfer that Manitoba receives from the federal government will be reduced if the province becomes more efficient. That the province is stagnant is the unintended consequence of its rational response to the incentive structure of the equalization payments program. In Germany, Article 72 of the constitution mandates “equal living conditions” to the citizens of the federal state, and Germany’s complicated system of equalization transfers has been designed to fulfill this mandate. Despite the 1.4 trillion Deutsche marks of equalization transfers to the Eastern states,²⁵ the economies of this region still suffer from serious unemployment and unsustainable budget deficits. The governments of the *Länder* of Saarland and Bremen have consistently run large deficits in recent decades, sued the federal government for bailouts at the Federal Constitution Court, and won.²⁶

Traditional theories of fiscal federalism often presume that intergovernment grants should be made to achieve efficiency and equity goals. However, recent empirical studies from the perspective of public choice suggest that the distribution of these transfers is heavily influenced by political calculations. The methodology used in these empirical studies is to estimate an econometric equation with intergovernmental grants as the dependent variables and a set of efficiency/equity and political variables as independent variables. A significant coefficient for a political variable in the estimated equation is then taken as *prima facie* evidence that intergovernmental grants are influenced by politics. Grossman (1994) formulated a model in which federal grants are used to buy political capital from state politicians and special-interest groups,

²⁴ The Winnipeg Free Press, January 22, 2012.

²⁵ Spahn, P. B. and Franz, O. (2002).

²⁶ Rodden, J. A. (2003).

which can be used to increase the political support of state voters for the federal politicians. In an econometric study, Wright (1974) found evidence that the distribution of federal welfare spending during the New Deal is strongly correlated with electoral votes per capita across states. According to Wallis (1996), federal grants to the states are influenced by both economic and political factors. This researcher suggested that the enormous growth in federal grants might not be the result of an effort to increase economic efficiency or equity, but huge “pork-barrel boondoggles” carried out for the political benefits of incumbent Presidents and Congressmen. Porto and Sanguinetti (2001) – in their empirical analysis of intergovernmental grants in Argentina – found evidence that more intergovernmental grants flow to provinces that are better represented both at the senate and at the lower chamber than provinces which are more populous and less represented. In Australia, equalization transfers are supposed to be determined by the Commonwealth Grant Commission (CGC), a neutral body of technocrats, and it would seem that there is no room for politics to be involved in the allocation of transfers. However, Worthington and Dollery (1998) found evidence that specific purpose payments, which are made outside the scope of equalization transfers and thus not under the control of CGC, have been manipulated by the federal government to purchase political capital that can be used to enhance its electoral success.

The public choice literature on intergovernmental grants – while considering both economic and political variables – concentrates only on the demand side of the political market. The supply side, which offers political capital for sale, and the strategic interactions between the central government and the states are not considered. In the conclusion of his study, Wallis, *op cit.* suggested that an endogenous model of grant allocations is called for because economic and political variables may influence both sides of the political markets.

In theoretical studies on fiscal federalism, a government – federal or regional – is often assumed to be a benevolent social planner whose objective is to maximize the welfare of the residents within its own jurisdiction.²⁷ In these studies, regional governments – in their efforts to improve the welfare of their own residents – engage in fiscal competition to attract private capital. The fiscal competition generates distortions, and the federal government – as a benevolent dictator –

²⁷ There are some theoretical studies on fiscal competition in which governments are assumed to be partially self-interest. See, for example, Jeremy Edwards and Keen (1996) and Cai and Treisman (2005).

employs equalization transfers as a tool to mitigate the harmful effects of these distortions. The analysis of equalization grants in this strand of research is conducted under this standard fiscal competition framework, and seeks to answer the following questions:

- To what extent can equalization schemes mitigate fiscal externalities caused by the fiscal competition and, thus, promote both equity and efficiency?²⁸
- Do tax-base equalization grants give an incentive for regional governments to enhance tax effort and implement efficient policy choices?²⁹
- What impact does the equalization scheme create on public investments and tax rate on private capital, and whether the equalization scheme is beneficial for the federation and regional governments?³⁰

In these theoretical studies, the role played by the equalization scheme is mainly to correct or limit the harmful externalities generated by the fiscal competition and to promote efficient tax and public-good policies for regional governments and achieve a certain level of equity for the federation.³¹ However, these studies all assume that intergovernmental transfers are distributed according to a certain formula (rule), or are assigned to achieve a certain level of social welfare for the federation. Little attention is paid to the possibility of whether the intergovernmental transfers are used and distributed for political reasons. In this paper, we attempt to fill part of this lacuna in the theoretical literature, using the theoretical machinery of the first-price menu auction developed by Bernheim and Whinston (1986) and the common agency theory developed by Dixit, Grossman, and Helpman (1997).³²

²⁸ See Köthenbürger (2002).

²⁹ Bucovetsky and Smart (2006).

³⁰ Hindriks et al. (2008).

³¹ There are several papers that address the issues related to equalization transfer *not* under the standard fiscal competition framework. Boadway and Flatters (1982) showed, under the assumption of labor mobility across jurisdictions, that interregional equalizing transfers lessen the inefficiency for labor allocation by internalizing the fiscal externality. Dahlby and Wilson (1994), under the assumption that there are neither interregional spillovers of benefits from public services nor fiscally induced flows or migrations, showed that a federal government that maximizes the sum of utilities of the representative citizen of each region should design the intergovernmental transfer in such a way that marginal cost of raising revenues across regions is equalized. Smart (1998), under the framework of single local jurisdiction and a federal agent, investigated the effect of the equalization scheme on the tax effort of the regional governments and the implication for the optimal design of transfer program.

³² The common agency theory of Dixit Grossman, and Helpman is built upon the theoretical machinery of a first price menu auction provided by Bernheim and Whinston, op. cit.

In the political economy model of equalization transfers we formalize, a federation is made up of two regions, and has two orders of government – a central government, also called the federal government, and two regional governments. In the model, an incumbent federal government makes equalization transfers to the two regions and the two regional governments try to sway the incumbent federal government into making favorable transfers to their own regions by offering political support – the political capital in the terminology of the public choice approach to inter-governmental grants. The political support offered by a regional government to the federal government is valued by the latter because it enhances the latter's chance of electoral success. The political support that a regional government offers to the central government might be useful to the latter in achieving objectives other than electoral success. In making the equalization transfers, the central government is not devoid of ethical considerations; social welfare also enters its utility function. As for the regional governments, they behave like benevolent dictators and maximize the welfare of the residents within their own borders.

That the regional governments try to influence the central governments in the process of equalization transfers is not surprising. State politicians are motivated to secure a “fair share of federal grants” for their constituents because this demonstrates their work for the states they represent. According to the Canadian constitution of 1982, the federal government is mandated to make equalization payments to the poorer provinces so that each province has sufficient revenues to ensure that a Canadian citizen, regardless of the province of residence, has access to the same level of public service. Basically, the federal government collects taxes from each of the provinces, and then redistributes the pool of money in unconditional block grants to the poorer provinces according to a formula set out in federal legislation. This legislation is renewed every five years, at which time the federal government and the provinces work together to negotiate any changes to the program. In its lifetime, the equalization payments program has redistributed \$267 billion, mostly from Canadians in Ontario and Alberta, to the governments of other provinces. Many provincial governments see equalization payments as entitlements, and never cease to make increasing demand for more. These provinces back up their demand for more equalization payments by threatening to withdraw political support for the federal government. Their threats are credible because of the peculiarity of the Canadian electoral map, which gives the have-not provinces a disproportionate allocation of seats in Parliament. An incumbent federal government, in its effort to buy votes for the coming federal election, often

finds that it is in its own interest to accommodate the demands of the equalization receiving provinces. Because the funds to satisfy these demands come from the economically more efficient provinces, this creates resentment in these provinces. In Germany, with the equalization transfer scheme solidly anchored in the constitution, one might think that there is no room for strategic behaviour by the states. This presumption is far from being borne out in reality. Germany's poor and small states exploit the uniform living condition enshrined in the constitution to run enormous budget deficits, and it is the citizens of these states who lobby the federal government to bail them out.³³ In its effort to win the support of Bremen for a reform tax package, the federal government has promised to retain the weights used in the "weighing" of population figures to compute specific burdens for the city states.³⁴ Pitlik, Schmid and Strotmann (2000) investigated whether political factors have an impact on the distribution of the intergovernmental transfer across 10 German Länder (states) in the German Länderfinanzausgleich using data from 1979–90. Applying the idea that the state's bargaining power comes from the (population) size of a state and that the vote of the smaller size state can be bought more inexpensively by the federal government, they confirmed that small states with a low number of residents receive more (favorable) intergovernmental transfers. In the US, state and local officials lobby the federal government and Congress for inter-government grants. Wallis, *op cit*, found that in presidential elections incumbents delivered more federal grants to swing states presumably because the political support thus bought has a high marginal productivity in electoral votes.

Our model also goes further than the traditional theoretical studies in two aspects. The first aspect involves the influence of equalization transfers on the provision of local public goods while the second aspect concerns the unintended consequence of equalization transfers on a region's investment on infrastructure. In a fiscal federalism, states cater to the preferences of their constituents and differentiate themselves on the bases of taxes, regulations, and the local public goods they provide. Those who prefer more local public goods should be willing to pay more taxes and move to the regions that accommodate their preferences. This is the traditional explanation for the different levels of public goods being provided in different states. There is no resentment by an individual who resides in state that provides a lower level of public services as

³³ Rodden, J. *op. cit.*

³⁴ Paul Bernard Spahn and Oliver Franz, *op. cit.*

long as she pays lower taxes. Our political economy model demonstrates that *ceteris paribus* a state receiving equalization transfers might provide a higher level of public services than a donor state, and this creates resentment among the states whose resources are used to finance the equalization transfers. Our model thus provides another plausible explanation for the observation that a poor state in a federation might provide a higher level of public service than a rich state. As for the second aspect, our model demonstrates that the recipient of equalization transfers (as well as the have-provinces) might be inclined to invest less in infrastructure.

To focus on the strategic behavior of the central and regional governments in the determination of equalization transfers, we have made some simplifying assumptions. First and foremost, we assume that the regions are small open economies and that there is free flow of capital from the rest of the world across regional frontiers. This assumption removes the strategic behavior of the two regions for private capital because the world rate of interest is given. The assumption that a region cannot influence the rental rate of capital is made in the theoretical literature by some authors by assuming that there is a large number of regions in the federation; see, for example, Bucovetsky et al. (1998). Allowing the rental rate of capital to be influenced by the tax rate that a regional government imposes on this factor of production complicates the analysis, but – we believe – does not change the main message of the paper. Second, we follow Hindriks et al. (2008) in modeling equalization transfers by assuming that the two regions participate in a revenue-sharing scheme according to which a fraction of the tax revenues collected in one region is transferred to the other region, and vice versa. In reality, the equalization transfers in federations, such as Canada, Australia, and Germany are often calculated according to complicated preset formulae. The formula we use for calculating equalization transfers, although simplistic, does capture the essence of an equalization transfer scheme.

The paper is organized as follows. In Section 2, the political economy model of equalization payments is presented.³⁵ The properties of the equilibrium of the game are presented in Section 3. To illustrate the main results of the paper, a numerical example is presented in Section 4. Some concluding remarks are given in Section 5.

³⁵ All the symbolic computations as well as all the numerical computations in this paper have been carried out by *Mathematica*.

2. THE MODEL

Consider a federation which is made up of two regions - called region 1 and region 2. The federation is assumed to be a small open economy. Let N_i denote the population of region $i, i = 1, 2$. There are two levels of government in the federation: a federal government and two regional governments. The two regions participate in a tax-revenues sharing scheme under which a proportion of the tax revenues collected by one region is transferred to the other region, and vice versa. In its effort to obtain an equalization payments formula that is favorable to its own region, a regional government lobbies the (incumbent) federal government by promising political support for the latter government in the coming election.

2.1. Technologies and Preferences

While private capital is allowed to cross regional frontiers, labor is completely immobile. There is a consumption good, which is produced in each region with the help of private capital, labor, and public capital. In what follows, the consumption good is taken to be the numéraire. The technology for producing the consumption good in region i is represented by the production function

$$(1) \quad Y_i = A_i H_i^\gamma K_i^\alpha L_i^{1-\alpha}, \quad (i = 1, 2),$$

where Y_i is the output; H_i is the stock of public capital.; K_i is the input of private capital; and L_i is the labor input. Also, A_i is a parameter which characterizes the endowments of the region, such as climate, soil quality, availability of natural resources... In the conventional literature, A_i is called the total factor productivity of the production technology in region i . Following Cai and Treisman (2005), we shall interpret A_i as the effect of the region's endowments on output. As for α and γ , these are positive parameters that satisfy $\alpha + \gamma < 1$. The stock of public capital H_i represents public infrastructure – roads, sewer systems, public R&D, and so on. The public investment is assumed to raise the productivity of the firms engaged in the production of the consumption good. The condition $\alpha + \gamma < 1$ captures the idea that for a given level of labor input, the technology exhibits decreasing returns to scale jointly with respect to public and private capital. On the other hand, the production technology exhibits constant returns to scale with respect to private capital and labor inputs.

For each $i = 1, 2$, let $\bar{k}_i$ denote the private capital endowment of an individual in region i . All individuals in the federation, regardless of the region of residence, have the same preferences, which depend on private consumption and the consumption of the non-productive public good in their region of origin. More specifically, the preferences of an individual in region i are assumed to be represented by the following linear utility function

$$(2) \quad u_i = \beta_i G_i + c_i, \quad (i = 1, 2).$$

where c_i is her private consumption; G_i the non-productive public good produced in region i ; β a parameter that characterizes her preferences for the non-productive public good.

The linear structure of preferences together with the assumption that the federation is a small open economy make the choice of the tax rate on the use of private capital and the choice of public investments by a region independent of those made by the other region. These assumptions greatly simplify the computations, and allow us to obtain a closed-form solution. Preferences that are represented by a strictly concave utility function are more realistic, but, we believe, will not change the main message of the paper: a region with more political power will be able to lobby the federal government to implement an equalization transfer scheme more favorable to itself.

The amount of the private good consumed by each resident in the region i is determined from the private budget constraint

$$(3) \quad c_i = \omega_i + r\bar{k}_i,$$

where ω_i is the wage rate prevailing in region i , and r is the world rate of interest.

2.2. The Extensive Form of the Game

The incumbent federal government has one policy instrument: $\tau, 0 \leq \tau \leq 1$, the fraction of revenues it collects from a region, and then transfers to the other region.³⁶ The value of τ represents the formula for equalization payments that it chooses to implement.

A regional government, say regional government i , has three policy instruments: a tax rate q_i on the use of private capital within its own borders, a level of public investment H_i , and a level of non-productive public expenditures G_i . A strategy for regional government i is a list (q_i, H_i, G_i) . In choosing its strategy, a regional government takes τ as given, and lobbies the federal government to induce the latter government into choosing an equalization payments formula that is favorable to its own region. In lobbying the federal government, a regional government promises to exert effort in furthering the re-election of the federal government. The link between effort and political support of a region, say i , is assumed to be represented by the following production function:

$$(4) \quad e_i \rightarrow \mu_i + \lambda_i e_i, \quad 0 \leq e_i \leq e_i^{max},$$

where e_i is the effort spent by this regional government in generating the political support for the federal government. In the political support production function, $e_i^{max} > 0$ represents the maximum amount of effort that the government of region i is able to exert, while $\lambda_i > 0$ and $\mu_i > 0$ are two parameters that characterize their political support production function.

The parameter λ_i represents the marginal product of effort. As for μ_i , this parameter can be interpreted as the political support from the base of the federal party that is currently in power. Because of their alignment with the political philosophy of the federal party currently in power, the voters in the base always vote for the federal government. Hence the federal government will obtain at least the amount of political support μ_i from region i even if the government of this

³⁶ Hindriks et al. assume that $0 \leq \tau \leq \frac{1}{2}$. In practice, it might not be politically feasible for the federal government to choose a value of τ that is less than $\frac{1}{2}$, but substantially greater than 0. In our model, we allow τ to take on any value in the interval $[0,1]$, which makes the analysis less cumbersome. We expect that the solution of the model dictates a value of τ much lower than $\frac{1}{2}$.

region exerts no effort in furthering the re-election of the federal government. The term $\lambda_i e_i$ represents the political support from the voters that need some persuasion on the part of the regional government. Thus, λ_i captures the political power of region i . The political support that the incumbent federal government receives from a regional government has an important impact on the number of votes it receives in the coming election. Furthermore, because of the special nature of the electoral map, a small region might have a disproportionate influence on the electoral success of a federal political party. Thus, in its choice of an equalization transfer formula, the incumbent federal government – with an eye on the coming election – values the political support from different regions in different ways. The value of λ_i , assumed to be exogenously given, reflects the political power of region i within the federation, and it plays a prominent role in the political calculus of the federal government. The political support might serve the central government in ways other than the desire for electoral success, as we have already mentioned in the introduction the support given by Bremen to Germany's central government in its negotiations with other states for new equalization transfers arrangements.

By a contingent effort function for regional government i we mean a continuous map

$$(5) \quad e_i: [0,1] \rightarrow [0, e_i^{max}],$$

where $e_i[\tau]$, $0 \leq \tau \leq 1$, is the effort that it will exert in furthering the re-election of the federal government, contingent upon the implementation of τ as the equalization payments formula.

Given a contingent effort function e_i , the amount of political support that the federal government receives from regional government i , as a function of the equalization payments formula that it implements, is given by

$$(6) \quad \tau \rightarrow \mu_i + \lambda_i e_i[\tau], 0 \leq \tau \leq 1.$$

The extensive form of the game is as follows. In the first stage, each regional government announces a contingent effort function to the federal government. The announcements by the two regional governments are simultaneous. In the second stage, the federal government chooses the equalization payments formula, taking into account the contingent effort functions announced by

the two regional governments. In the third stage, the two regional governments simultaneously set the tax rates they impose on the use of private capital within their own borders, their public investments, and their expenditures on the non-productive public good. In the fourth stage, private agents carry out their own activities – producers maximize profits and consumers maximize utilities – taking the policies set by the regional governments as given. The competitive equilibrium that is induced by the strategies chosen by the two levels of governments determines the wage rates, which in turn determine the profits of producers and the incomes of consumers. In the fifth stage, consumers in each region consume the private good and the non-productive public good, while each regional government generates the political support it promises the federal government.

To solve the game, we use backward induction.

2.3. Profit Maximization

The representative firm in region i solves the following profit maximization problem:

$$(7) \quad \max_{(K_i, L_i)} A_i H_i^\gamma K_i^\alpha L_i^{1-\alpha} - (r + q_i)K_i - \omega_i L_i.$$

The solution of this problem, using the market-clearing condition for labor $L_i = N_i, i = 1, 2$, yields the following demand for private capital by the representative firm and the equilibrium wage rate in region i :

$$(8) \quad K_i[q_i, H_i] = N_i H_i^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_i}{r+q_i} \right)^{\frac{1}{1-\alpha}},$$

and

$$(9) \quad \omega_i[q_i, H_i] = (1 - \alpha) A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha}{r+q_i} \right)^{\frac{\alpha}{1-\alpha}}.$$

Note that the demand for private capital in a region declines when the government of this region raises the tax rate that it imposes on the use of private capital within its own borders. On the other hand, the demand for private capital in a region rises with its population (N_i), its

endowment (A_i), and its stock of public capital (H_i). Observe also that the wage rate in a region rises with its endowment and its stock of public capital. However, it falls as the tax rate on the use of private capital rises. Because $\alpha + \gamma < 1$, we have $0 < \frac{\gamma}{1-\alpha} < 1$, which in turn implies that the private capital input and the wage rate in a region rises with the stock of public capital, but at a decreasing rate.

2.4. The Best Responses of the Regional Governments to the Equalization Formula

The payoff for a regional government is the utilities of all the residents in its own region minus the disutility of the effort devoted to generating political support for the incumbent federal government. More precisely, for each $i = 1, 2$, the payoff for the government of region i is given by

$$(10) \quad N_i(\beta G_i + c_i) - e_i[\tau] = N_i(\beta G_i + \omega_i[q_i, H_i] + r\bar{k}_i) - e_i[\tau].$$

Suppose that the regional governments have announced their contingent political effort functions to the federal government and that the latter government has chosen to implement the equalization transfer formula τ . At this stage of the game, the government of region i solves the following welfare maximization problem:

$$(11) \quad \max_{(q_i, H_i, G_i)} N_i(\beta G_i + \omega_i[q_i, H_i] + r\bar{k}_i)$$

subject to the non-negative constraints $q_i \geq 0, H_i \geq 0, G_i \geq 0$, and the public budget constraint:

$$(12) \quad G_i + H_i = (1 - \tau)q_i K_i[q_i, H_i] + \tau q_j K_j[q_j, H_j] \\ = (1 - \tau)q_i N_i H_i^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_i}{r + q_i} \right)^{\frac{1}{1-\alpha}} + \tau q_j N_j H_j^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_j}{r + q_j} \right)^{\frac{1}{1-\alpha}}, \quad (i, j = 1, 2, i \neq j).$$

Note that under the equalization transfer scheme, each region obtains a proportion τ of the revenues collected by the other region from its tax on the use of private capital within its own borders, and keeps a fraction, namely $1 - \tau$, of its own tax revenues. The right-hand side of (12)

represents the total revenues available to the government of region i to be spent on the provision of the public good (G_i) and the financing of infrastructure. Note also that each regional government is assumed to adopt a balanced-budget policy.

Using the budget constraint (12), we can reduce the preceding maximization problem to

$$(13) \quad \max_{(q_i, H_i)} N_i \left(\beta \left((1 - \tau) q_i N_i H_i^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_i}{r+q_i} \right)^{\frac{1}{1-\alpha}} + \tau q_j N_j H_j^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_j}{r+q_j} \right)^{\frac{1}{1-\alpha}} - H_i \right) \right. \\ \left. + (1 - \alpha) A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha}{r+q_i} \right)^{\frac{\alpha}{1-\alpha}} + r \bar{k}_i \right)$$

subject to the conditions $q_i \geq 0$, $H_i \geq 0$, and the condition

$$(14) \quad (1 - \tau) q_i N_i H_i^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_i}{r+q_i} \right)^{\frac{1}{1-\alpha}} + \tau q_j N_j H_j^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_j}{r+q_j} \right)^{\frac{1}{1-\alpha}} - H_i \geq 0.$$

Note that (14) represents the constraint that the expenditure on infrastructure investment cannot exceed the total revenues at the disposal of regional government i so that the expenditure on public good provision in this region is non-negative. Because $0 < \frac{\gamma}{1-\alpha} < 1$, there is a unique value of H_i , say $H_i^+[\tau] > 0$, such that

$$(15) \quad (1 - \tau) q_i N_i (H_i^+[\tau])^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_i}{r+q_i} \right)^{\frac{1}{1-\alpha}} + \tau q_j N_j H_j^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_j}{r+q_j} \right)^{\frac{1}{1-\alpha}} - H_i^+[\tau] = 0.$$

As defined, $H_i^+[\tau]$ represents the upper bound on the feasible public capital investment in region i so that the non-negative constraint $G_i \geq 0$ is satisfied.

The following first-order condition characterizes the optimal tax rate that the government of region i imposes on the use of private capital within its own borders:

$$(16) \quad \frac{1}{1-\alpha} \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} (r + q_i)^{-\frac{2}{1+\alpha}} \left(\begin{array}{c} (-1 + \alpha)(r + q_i)^{\frac{1}{1-\alpha}} \\ + \beta(-1 + \tau) N_i (r + q_i)^{\frac{\alpha}{1-\alpha}} (r(-1 + \alpha) + \alpha q_i) \end{array} \right) \leq 0,$$

with equality holding if $q_i > 0$.

As can be seen from (16), the tax rate on the use of private capital is independent of its own endowment due to linear preferences. For an interior solution, (16) becomes an equality and can be solved for q_i , and the tax rate on the use of private capital in region i is then given by

$$(17) \quad q_i[\tau] = \frac{r(1-\alpha)(\beta(1-\tau)N_i-1)}{1-\alpha+\alpha\beta(1-\tau)N_i}.$$

The denominator of the expression on the right-hand side of (17) is positive. In order for $q_i[\tau]$ to be non-negative, it is necessary that the numerator of this expression is positive, i.e., $-1 + \beta(1 - \tau)N_i \geq 0$, or

$$(18) \quad \tau \leq 1 - \frac{1}{\beta N_i}.$$

Because the marginal product of public capital is infinite when H_i tends to 0, the optimal investment in public capital is always positive. However, it is possible – due to linear preferences – that under the equalization scheme implemented by the federal government the optimal investment in public capital will exhaust all the revenues available to the government of a region so that no more resources are left for public good provision. If the government of region i chooses the level of public capital investment without taking into consideration the constraint (14), then the following first-order condition characterizes the optimal level of public capital investment:

$$(19) \quad \beta \left(-1 - r\alpha^{\frac{\alpha}{1-\alpha}}\gamma(-1 + \tau)A_i^{\frac{1}{1-\alpha}}H_i^{-1+\frac{\gamma}{1-\alpha}}N_i \left(\frac{r\beta(-1+\tau)N_i}{-1+\alpha+\alpha\beta(-1+\tau)N_i} \right)^{\frac{1}{-1+\alpha}} \right) = 0.$$

Solving (19) for H_i , we obtain

$$(20) \quad \hat{H}_i[\tau] = \left(\frac{\alpha^{\alpha}\gamma^{1-\alpha}A_i(1-\alpha+\alpha\beta(1-\tau)N_i)}{\beta((1-\tau)rN_i)^{\alpha}} \right)^{\frac{1}{1-\alpha-\gamma}}.$$

The optimal level of infrastructure investment in region i is then given by

$$(21) \quad H_i[\tau] = \begin{cases} \left(\frac{\alpha^{\alpha}\gamma^{1-\alpha}A_i(1-\alpha+\alpha\beta(1-\tau)N_i)}{\beta((1-\tau)rN_i)^{\alpha}} \right)^{\frac{1}{1-\alpha-\gamma}} & \text{if } \hat{H}_i[\tau] \leq H_i^+[\tau], \\ H_i^+[\tau], & \text{otherwise,} \end{cases}$$

and the non-productive public good consumption by

$$(22) \quad G_i[\tau] = (1 - \tau)q_i N_i (H_i[\tau])^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_i}{r+q_i} \right)^{\frac{1}{1-\alpha}} + \tau q_j N_j H_j^{\frac{\gamma}{1-\alpha}} \left(\frac{\alpha A_j}{r+q_j} \right)^{\frac{1}{1-\alpha}} - H_i[\tau],$$

($i = 1, 2$).

The following figure depicts the levels of the non-productive public good provided in the two regions, as a function of the equalization formula implemented.

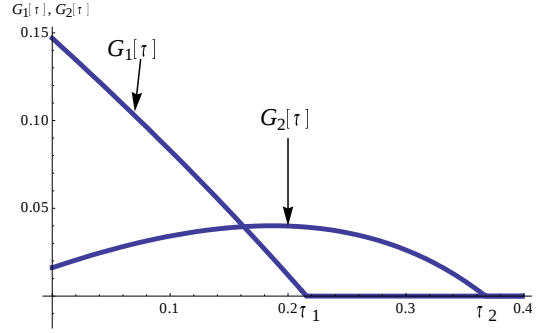


Figure 1. The consumption of the non-productive public good in each region, as a function of the equalization transfer formula

As depicted in Figure 1, the levels of the non-productive public good provided in both regions are positive at $\tau = 0$, with that of the have-region being higher than that of the have-not region (Lemma 3). As τ rises above 0, the consumption of the non-productive public good in the have-region falls, reaches 0 at τ_1 , and then remains at 0 after that. As for the consumption of the non-productive public good in the have-not region, it begins at $\tau = 0$ below that of the have region; rises as τ rises; reaches a maximum; descends to 0 at τ_2 ; and remains at 0 after that. As depicted in Figure 1, the curve $\tau \rightarrow G_2[\tau]$ crosses the curve $\tau \rightarrow G_1[\tau]$ at a point where the consumption of the non-productive public good is positive for both regions. Figure 1 also shows that when the consumption of the non-productive public good in the have region descends to 0, that of the have-not region is positive, capturing the economic contents of Proposition 7.

In what follows, we assume that $G_i[0] > 0, i = 1, 2$; that is, in the absence of equalization transfers, each region provide a positive level of the non-productive public good to its own residents. Let $\tau_i = \inf\{\tau | G_i[\tau] = 0\}, i = 1, 2$, be the least upper bound of the equalization

transfer formulas for which the level of the non-productive public good provided in region i is equal to 0. We shall assume that τ_i satisfies the condition represented by (18), and let

$$(23) \quad \bar{\tau} = \min\{\tau_1, \tau_2\}.$$

To keep the problem more realistic, we shall only consider the equalization transfer formulas that satisfy the condition $0 \leq \tau \leq \bar{\tau}$, so that the tax rate on the private capital input and the level of the non-productive public good provided are positive in each region for $0 \leq \tau < \bar{\tau}$.

Using (17) and (21) in (8) and (9), we obtain, respectively, the following expression for the input of private capital and that for the prevailing wage rate in a region under the competitive equilibrium induced by the best response to of the government of this region to the equalization transfer formula τ :

$$(24) \quad K_i[q_i[\tau], H_i[\tau]] = \left(\frac{\alpha^{1-\gamma} \gamma^\gamma A_i (1-\alpha + \alpha\beta(1-\tau)N_i)}{\beta N_i^\alpha ((1-\tau)r)^{1-\gamma}} \right)^{\frac{1}{1-\alpha-\gamma}},$$

and

$$(25) \quad \omega_i[q_i[\tau], H_i[\tau]] = (1-\alpha) \left(\frac{\alpha^\alpha \gamma^\gamma A_i (1-\alpha + \alpha\beta(1-\tau)N_i)^{\alpha+\gamma}}{\beta^{\alpha+\gamma} ((1-\tau)r N_i)^\alpha} \right)^{\frac{1}{1-\alpha-\gamma}}.$$

To keep the notations from becoming too burdensome, we shall from now on simply write $K_i[\tau]$ in place of $K_i[q_i[\tau], H_i[\tau]]$ and $\omega_i[\tau]$ in place of $\omega_i[q_i[\tau], H_i[\tau]]$.

The following lemma gives some results concerning the influence of a region's endowment and population on the tax rate it imposes on the use of private capital within its own borders, its public capital investment, and the private capital input used within its own borders in the sub-game after the equalization transfer formula τ has been implemented.

LEMMA 1: *For any given equalization payments formula τ , the following results hold:*

(i) *The tax rate imposed by a regional government on the use of private capital within its borders does not depend on its endowment. However, it rises with its own population.*

(ii) *The public investment made by a regional government rises with its own population and the endowment of the region.*

(iii) *The private capital input in a region under the competitive equilibrium induced by τ and the best response to τ by the government of the region rises with its own endowment and its own population.*

PROOF: To prove (i) of Lemma 1, note that $\frac{\partial q_i[\tau]}{\partial A_i} = 0$ and $\frac{\partial q_i[\tau]}{\partial N_i} > 0$. To establish (ii) of Lemma 1, note that $\frac{\partial \text{Log}[H_i[\tau]]}{\partial A_i} > 0$, $\frac{\partial \text{Log}[H_i[\tau]]}{\partial N_i} > 0$. As for (iii) of Lemma 1, it follows from the facts that that $\frac{\partial \text{Log}[K_i[\tau]]}{\partial A_i} > 0$, $\frac{\partial \text{Log}[K_i[\tau]]}{\partial N_i} > 0$.

■

The following lemma presents some comparative static results on the tax rate on private capital and the wage rate in a region as well as the tax revenues collected by the government of a region as the equalization formula τ varies.

LEMMA 2: *Under the competitive equilibrium induced by the implementation of an equalization transfer formula τ and the best response to τ by the regional governments, the following results hold:*

(i) *The tax rate on the use of private capital, $q_i[\tau]$, and the public investment, $H_i[\tau]$, fall as τ rises.*

(ii) *The wage rate, namely $\omega_i[\tau]$, falls as τ rises within the interval $0 \leq \tau < 1 - \frac{1-\alpha}{\gamma} \frac{1}{\beta N_i}$, and rises as τ rises within the interval $1 - \frac{1-\alpha}{\gamma} \frac{1}{\beta N_i} \leq \tau \leq \bar{\tau}$.*

(iii) *The tax revenues collected by each regional government, namely $q_i[\tau]K_i[\tau]$, falls as τ rises.*

PROOF: It is trivial to show (i) and (ii) of Lemma 2 by differentiating $q_i[\tau]$, $H_i[\tau]$, and $\omega_i[\tau]$ logarithmically with respect to τ . For (iii) of Lemma 2, note that direct computations yield

$$\frac{\partial}{\partial \tau} \text{Log}[q_i[\tau]K_i[\tau]] = \frac{-1+\alpha+\gamma-\alpha\gamma-\alpha\gamma\beta(1-\tau)N_i(-2+\beta(1-\tau)N_i)}{(1-\alpha-\gamma)(1-\tau)(-1+\beta(1-\tau)N_i)(1-\alpha+\alpha\beta(1-\tau)N_i)}.$$

The denominator of the preceding derivative is positive. We claim that its numerator is negative. To see why, note that the derivative of the numerator with respect to τ is given by $2\alpha\gamma\beta N_i(-1 + \beta(1 - \tau)N_i)$, which is positive due to the condition (18). Hence the numerator is an increasing function of τ . Furthermore, when τ assumes the value of $\tau = 1 - \frac{1}{\beta N_i}$, the numerator is reduced to $-1 + \alpha + \gamma < 0$. Hence the derivative is negative for all the allowed values of τ .

■

As (i) of Lemma 2 shows, when τ increases both the tax rate on the use of private capital and the public capital investment decrease in each region. A decrease in the tax rate on the use of private capital input induces a rise in the demand for private capital. Given the assumption of a small open economy, the increase in the demand for private capital induces an inflow of foreign private capital into both regions. In addition, there is a flow of private capital from one region to another region, depending on the differences on the final tax rates on private capital between two regions. Also, the increase in the demand for private capital induces a rise in the wage rate in both regions. Private capital continues to flow into both regions until the representative firm in each region achieves the profit-maximization point, given the fixed labor force in each region. On the other hand, a decrease in the public capital investment shifts the demand for private capital downward. This downward shift induces an outflow of private capital from each region. Given a fixed labor force in each region, (with the assumption of full employment), the wage rate in both regions falls as a result of the decrease in the demand for private capital. Hence, as τ increases these two opposite effects interact to determine whether the wage increases or decreases in both regions. In our model, as τ increases, the wage rates in both regions initially decrease because the impact of the decrease in public investment dominates the impact of the decrease in tax rate on private capital; however, when τ becomes high enough, the wage rate starts to increase because the impact of the latter starts to dominate that of the former.

LEMMA 3: *If $\tau = 0$, then (i) $\frac{\partial G_i[0]}{\partial A_i} > 0$ and (ii) $\frac{\partial G_i[0]}{\partial N_i} > 0, i = 1, 2$. That is, if the federal government does not carry out any equalization transfers, then the consumption of the non-productive public good in a region rises with its own endowment and its own population.*

PROOF: The proof of Lemma 3 is a little technical, and is thus relegated to the Appendix.

The economic contents of Lemma 3 are quite intuitive. When there are no equalization transfers, a region that is well endowed (high A_i) has a high output level, and thus can afford to provide more of the non-productive public good to its own residents. Also, by its own nature, a public good can be enjoyed by any number of individuals, and thus, a region with a high population can obtain a high level of regional social welfare for the same level of public good provided. Thus, ceteris paribus, a region with a high population will have a high level of public good provision. According to Lemma 3, a region that is poorly endowed, and has a low population will provide a lower level of public good than a better-endowed and more populous region, and this result constitutes the basis for equalization transfers.

Now for $\tau \in (0, \bar{\tau})$, let

$$(26) \quad U_i[\tau] = N_i \left(\beta \left((1 - \tau)q_i[\tau]K_i[\tau] + \tau q_j[\tau]K_j[\tau] - H_i[\tau] \right) + \omega_i[\tau] + r\bar{k}_i \right),$$

($i = 1, 2$),

denote the total utilities – obtained from the consumption of the private good and the non-productive public good – of all the residents in region i , given the equalization transfer formula τ and the best responses to τ of the two regional governments. Applying the envelope theorem to the regional welfare maximization problem constituted by (13) and (14), we obtain

$$(27) \quad \frac{\partial}{\partial \tau} U_i[\tau] = N_i \beta \left(-q_i[\tau]K_i[\tau] + q_j[\tau]K_j[\tau] + \tau \frac{\partial}{\partial \tau} (q_j[\tau]K_j[\tau]) \right).$$

2.5. The Best Response of the Federal Government

Given the contingent effort function that it announces to the federal government, and the equalization transfer formula chosen by the latter government, the net payoff – net of the disutility of effort – for regional government i is $U_i[\tau] - e_i[\tau]$, $i = 1, 2$. The payoff for the federal government is assumed to be a weighted sum of national social welfare and the political support offered by each of the regional government, say

$$(28) \quad \epsilon \sum_{i=1}^2 (U_i[\tau] - e_i[\tau]) + \sum_{i=1}^2 (\mu_i + \lambda_i e_i[\tau]),$$

where $\epsilon, 0 < \epsilon < \lambda_i, i = 1, 2$, is the weight assigned to national social welfare. Carrying a positive transformation of (28), we can also represent the preferences of the federal government by the following utility function:

$$(29) \quad \sum_{i=1}^2 (U_i[\tau] - e_i[\tau]) + \sum_{i=1}^2 \frac{\lambda_i}{\epsilon} e_i[\tau] = \sum_{i=1}^2 U_i[\tau] + \sum_{i=1}^2 \left(\frac{\lambda_i}{\epsilon} - 1 \right) e_i[\tau].$$

Note that the assumption $0 < \epsilon < \lambda_i$ implies that $\left(\frac{\lambda_i}{\epsilon} - 1 \right) > 0$, and this means that the effort expended by a regional government to generate political support for the federal government is valued by the latter. If we consider the change of variable $\phi_i[\tau] = \left(\frac{\lambda_i}{\epsilon} - 1 \right) e_i[\tau]$, then $\phi_i[\tau]$ can be interpreted as the *effective effort* offered by regional government i , contingent upon the equalization transfer formula implemented by the federal government. An alternative way for representing a contingent effort function is to specify a non-negative continuous map $\phi_i: \tau \rightarrow \phi_i[\tau] \leq \left(\frac{\lambda_i}{\epsilon} - 1 \right) e_i^{max}, 0 \leq \tau \leq 1$. With this change of variable, the payoff for the federal government becomes

$$(30) \quad \sum_{i=1}^2 U_i[\tau] + \sum_{i=1}^2 \phi_i[\tau].$$

Using the definition of $\phi_i[\tau]$, we can express the net payoff for a regional government, after a positive transformation, by

$$(31) \quad \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau] - \phi_i[\tau].$$

To find the equilibrium of the political economy model of equalization transfers in a fiscal federalism, it is more convenient to use (30) as an alternative representation of the preferences of the federal government and (31) as an alternative representation of the preferences of a regional government.

For any list of contingent effective effort functions $(\phi_i)_{i=1}^2$, let

$$\mathcal{R}[\phi_1, \phi_2] = \operatorname{argmax}_{0 \leq \tau \leq \bar{\tau}} \sum_{i=1}^2 U_i[\tau] + \sum_{i=1}^2 \phi_i[\tau].$$

As defined, $\mathcal{R}[\phi_1, \phi_2]$ represents the set of best responses of the federal government to $(\phi_i)_{i=1}^2$.

2.6. Definition of the Nash Equilibrium

DEFINITION: A list $(\phi_1^*, \phi_2^*, \tau^*)$, where ϕ_i^* is the contingent effective effort function of region $i, i = 1, 2$, and τ^* is the equalization transfer formula implemented by the federal government, is said to constitute a Nash equilibrium for the political economy model of equalization transfers in a fiscal federalism if the following conditions are satisfied:

- (a) $\tau^* \in \mathcal{R}[\phi_1^*, \phi_2^*]$;
- (b) For a regional government, say i , and any contingent effective effort function ϕ_i that it might announce to the federal government, we have

$$\sup_{\tau \in \mathcal{R}[\phi_i, \phi_j^*]} \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau] - \phi_i[\tau] \leq \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau^*] - \phi_i^*[\tau^*].$$

Condition (a) asserts that τ^* is the best response for the federal government to (ϕ_1^*, ϕ_2^*) , while condition (b) asserts that if regional government j chooses the contingent effective effort function ϕ_j^* , then regional government i cannot improve its net payoff by deviating from ϕ_i^* .

3. THE EQUILIBRIUM

In finding an equilibrium for the first-price menu auction problem that they formulated, Bernheim and Whinston (1986), restricted their search to a class of strategies that they labeled “truthful strategies,” and called the Nash equilibrium under which the lobbies use truthful strategies a *truthful Nash equilibrium*. A *truthful strategy*, for a regional government, say regional government i , is a contingent effective effort function of the following form:

$$(32) \quad \phi_i[\tau] = \max \left\{ \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau] - \Gamma_i, 0 \right\},$$

where Γ_i is a nonnegative number. In announcing such a contingent effective effort function, regional government i informs the federal government that it only aims for a payoff, net of the disutility of effort spent in generating the political support it promises, equal to Γ_i . If the gross payoff it obtains under the equalization payments formula implemented is less than or equal to Γ_i , it will offer no political support. On the other hand, if the gross payoff is higher than Γ_i , then it will generate all the political support up to the level where the payoff net of the disutility of effort is reduced to Γ_i . Truthful strategies and truthful Nash equilibria have a particularly simple structure. Sometimes, they might even be focal. Berheim and Whinston (1986) offered the following reasons for accepting a truthful equilibrium as the solution of the game. First, for any given feasible strategy adopted by region j , the set of best response for region i contains a truthful strategy. Second, there exists a truthful equilibrium that is Pareto efficient. Third, a truthful equilibrium is coalition-proof.

In what follows, we refer to the federal government as player 0, and regional government $i, i = 1, 2$, as player i . Now let

$$(33) \quad \Gamma_0^{max} = \max_{0 \leq \tau \leq \bar{\tau}} (U_1[\tau] + U_2[\tau]).$$

As defined, Γ_0^{max} represents the maximum payoff for the federal government when social welfare is the only concern in its preferences.

Next, for each $j = 1, 2$, let

$$(34) \quad \begin{aligned} \Gamma_{\{0,j\}}^{max} &= \max_{0 \leq \tau \leq \bar{\tau}} \left(\sum_{i=1}^2 U_i[\tau] + \left(\frac{\lambda_j}{\epsilon} - 1 \right) U_j[\tau] \right) \\ &= \max_{0 \leq \tau \leq \bar{\tau}} \left(U_i[\tau] + \frac{\lambda_j}{\epsilon} U_j[\tau] \right), \end{aligned} \quad (i, j = 1, 2, i \neq j).$$

As defined, $\Gamma_{\{0,j\}}^{max}$ represents the maximum joint payoff for the federal government and regional government j when regional government i does not make any political contribution.

Also, let

$$\begin{aligned}
(35) \quad \Gamma_{\{0,1,2\}}^{max} &= \max_{0 \leq \tau \leq \bar{\tau}} \left(\sum_{i=1}^2 U_i[\tau] + \sum_{i=1}^2 \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau] \right) \\
&= \max_{0 \leq \tau \leq \bar{\tau}} \sum_{i=1}^2 \frac{\lambda_i}{\epsilon} U_i[\tau].
\end{aligned}$$

As defined, $\Gamma_{\{0,1,2\}}^{max}$ represents the maximum joint payoff for the federal government and the two regional governments.

Let $\mathcal{E}$ denote the set of ordered pairs (Γ_1, Γ_2) that satisfies the following conditions:

- (i) $\Gamma_1 \geq 0, \Gamma_2 \geq 0$,
- (ii) $\Gamma_1 + \Gamma_2 \leq \Gamma_{\{0,1,2\}}^{max} - \Gamma_0^{max}$,
- (iii) $\Gamma_1 \leq \Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,2\}}^{max}, \Gamma_2 \leq \Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,1\}}^{max}$.

We can interpret (Γ_1, Γ_2) as the vector of possible net payoffs – net of the disutility of effort – for the two regional governments in the political economy model of equalization payments in a fiscal federalism. Under this interpretation, (i) asserts that net payoffs of the two regional governments are non-negative. Condition (ii) asserts that the sum of the net payoffs of the two regional governments cannot exceed $\Gamma_{\{0,1,2\}}^{max} - \Gamma_0^{max}$. Indeed, if (ii) is not true, then the payoff for the federal government will be strictly less than Γ_0^{max} , the payoff it obtains by simply maximizing national social welfare. As for condition (iii), it asserts that the net payoff for a regional government cannot exceed the difference between the maximum joint payoff for the three players and the maximum joint payoff for the federal government and the other regional government if they form a coalition.

An element $(\Gamma_1, \Gamma_2) \in \mathcal{E}$ is said to be efficient if there is no element $(\Gamma_1', \Gamma_2') \in \mathcal{E}$ such that $\Gamma_1' \geq \Gamma_1, \Gamma_2' \geq \Gamma_2$, with at least one strict inequality holding. The set of efficient vectors in $\mathcal{E}$ is called the Pareto frontier of $\mathcal{E}$. Because $\mathcal{E}$ is a closed and bounded set in $\mathbb{R}^2$, it contains its Pareto frontier.

The following proposition is Theorem 2 of Bernheim and Whinston (1986) restated in the context of the political economy model of equalization transfers in a fiscal federalism.

PROPOSITION 1: Let (Γ_1, Γ_2) be a point on the Pareto frontier of $\mathcal{E}$, and

$$(36) \quad \phi_i^*[\tau] = \max \left\{ \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau] - \Gamma_i, 0 \right\}, \quad (i = 1, 2).$$

Also, let

$$(37) \quad \begin{aligned} \tau^* &= \operatorname{argmax}_{0 \leq \tau \leq \bar{\tau}} \left(\sum_{i=1}^2 U_i[\tau] + \sum_{i=1}^2 \left(\frac{\lambda_i}{\epsilon} - 1 \right) U_i[\tau] \right) \\ &= \operatorname{argmax}_{0 \leq \tau \leq \bar{\tau}} \left(\sum_{i=1}^2 \frac{\lambda_i}{\epsilon} U_i[\tau] \right). \end{aligned}$$

The combination of strategies $(\phi_1^*, \phi_2^*, \tau^*)$ constitutes a truthful Nash equilibrium for the political economy model of equalization transfers in a fiscal federalism.

As indicated by (37), the equalization transfer scheme implemented by the federal government – when it allows politics to distort its economic policy – is the value of τ that maximizes the weighted average of regional welfare, with the weight assigned to a region being proportional to its political power.

In a fiscal federalism, it is often the case that some regions are poorer than other regions. It is this asymmetry that is invoked to justify the implementation of an equalization transfer scheme. We introduce this asymmetry into the political economy model of equalization transfers by assuming that $A_1 > A_2$ and $N_1 > N_2$, i.e., region 1 is better endowed and has a larger population than region 2.

PROPOSITION 2: If the federal government behaves like a benevolent dictator and adopts the utilitarian philosophy of maximizing the sum of utilities of all the individuals in the federation, then it will not carry out any equalization transfers; that is, the optimal value for τ is $\tau = 0$.

PROOF: Under the hypothesis of Proposition 2, the federal government solves the following welfare maximization problem:

$$(38) \quad \max_{0 \leq \tau \leq \bar{\tau}} U_1[\tau] + U_2[\tau].$$

Differentiating the objective function of the preceding national social welfare maximization problem with respect to τ , and using (27), we obtain

$$(39) \quad \frac{\partial U_1[\tau]}{\partial \tau} + \frac{\partial U_2[\tau]}{\partial \tau} = -(N_1 - N_2)\beta(q_1[\tau]K_1[\tau] - q_2[\tau]K_2[\tau]) \\ + \tau\beta \left(N_1 \frac{\partial}{\partial \tau}(q_1[\tau]K_1[\tau]) + N_2 \frac{\partial}{\partial \tau}(q_2[\tau]K_2[\tau]) \right) < 0.$$

The strict inequality in (39) can be established as follows. Invoking Lemma 1.(i) and using the assumption $N_1 > N_2$, we have $q_1[\tau] > q_2[\tau]$. Invoking Lemma 1.(iii) and using the assumptions $A_1 > A_2$ and $N_1 > N_2$, we obtain $K_1[\tau] > K_2[\tau]$. Thus, using the assumed asymmetry that region 1 is better endowed and more populous than region 2, we can assert that the first expression on the right-hand side of the equality in (39) is negative. According to Lemma 2.(iii), we have $\frac{\partial}{\partial \tau}(q_i[\tau]K_i[\tau]) < 0, i = 1, 2$. Hence the second expression on the right-hand side of the equality in (39) is negative, Hence $\tau = 0$ is the solution of (38).

■

The economic contents of Proposition 2 are rather subtle. Because preferences are linear and a fortiori quasi-linear, a transfer of the numéraire from one region to another – when there are no public capital – will make the giving region worse-off and the receiving region better-off, but leaving the sum of the utilities of all the citizens in the federation unchanged. However, in our model, a transfer of the resources from the richer and more populous region to the less endowed and less populous region leaves fewer resources for the former region to spend on public capital and public good provision. A lower level of public capital investment in region 1, the economically more efficient region, in turn lowers the productivity of private capital in this region, and leads to an overall production inefficiency for the federation as a whole, with the ensuing loss of national social welfare.

Proposition 2 will cease to hold if preferences are strictly convex (the utility function of a citizen of the federation is strictly concave) even if all individuals in the federation are given the same weight in the national Benthamite social welfare function. The concavity of the utility function builds in a bias for equalization transfers because at the social optimum, marginal utilities will be

equalized across individuals in the federation, and this calls for transferring resources from poor to richer regions. Proposition 2 will also cease to hold – and this is the message of Proposition 3 – if preferences are linear, but are assigned different weights (due to the lobbying efforts of regional governments) in the objective function of the federal government.

It is well known that the first-price menu auction model of Bernheim and Whinston (1986) might possess multiple truthful Nash equilibria. Such an unpleasant possibility does not arise in the political economy model of equalization transfers in a fiscal federalism, as asserted by the following proposition:

PROPOSITION 3: *The Pareto frontier of $\mathcal{E}$ consists exactly of one point, namely*

$$(40) \quad E = (\Gamma_{\{0,1,2\}}^{\max} - \Gamma_{\{0,2\}}^{\max}, \Gamma_{\{0,1,2\}}^{\max} - \Gamma_{\{0,1\}}^{\max}),$$

which represent the unique equilibrium net payoffs for the two regional governments in the political economy model of equalization payments in a fiscal federalism.

PROOF: The contents of Proposition 3 are depicted in Figure 2, with Γ_1 on the horizontal axis and Γ_2 on the vertical axis.

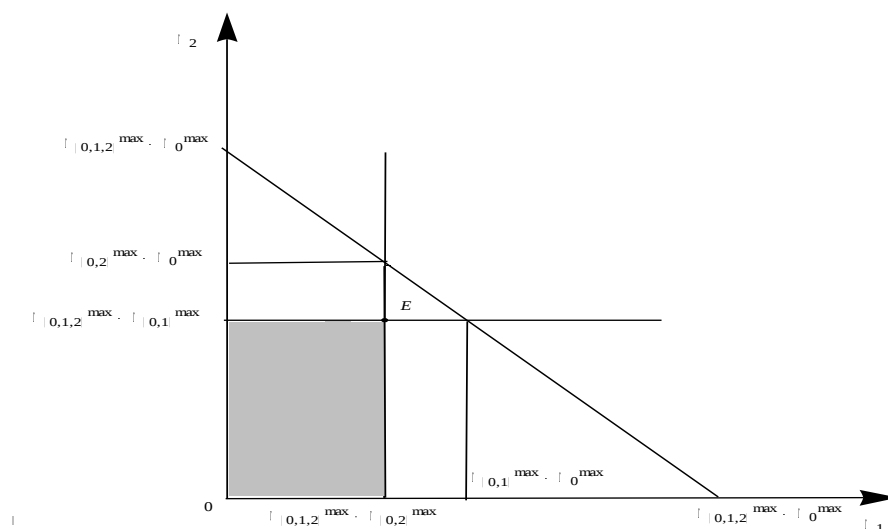


Figure 2. The Pareto frontier of $\mathcal{E}$

The shaded area represents $\mathcal{E}$, which is the set of net payoffs for the two regional governments that satisfy the following three conditions:

- (i) $\Gamma_1 \geq 0, \Gamma_2 \geq 0$,
- (ii) $\Gamma_1 + \Gamma_2 \leq \Gamma_{\{0,1,2\}}^{max} - \Gamma_0^{max}$,
- (iii) $\Gamma_1 \leq \Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,2\}}^{max}, \Gamma_2 \leq \Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,1\}}^{max}$.

In Figure 1, and this is what we shall prove, the point represented by (40) is below the line $\Gamma_2 = (\Gamma_{\{0,1,2\}}^{max} - \Gamma_0^{max}) - \Gamma_1$. That is, the point E is the Pareto frontier of $\mathcal{E}$.

According to (34), we have

$$(41) \quad \Gamma_{\{0,1\}}^{max} = \max_{\tau} \left(\left(\frac{\lambda_1}{\epsilon} - 1 \right) U_1[\tau] + U_1[\tau] + U_2[\tau] \right).$$

According to (27), we have

$$\frac{\partial}{\partial \tau} U_1[\tau] = N_1 \beta \left(-q_1[\tau] K_1[\tau] + q_2[\tau] K_2[\tau] + \tau \frac{\partial}{\partial \tau} (q_2[\tau] K_2[\tau]) \right) < 0,$$

where the strict inequality is due to the fact that $q_1[\tau] > q_2[\tau], K_1[\tau] > K_2[\tau]$, and $\frac{\partial}{\partial \tau} (q_i[\tau] K_i[\tau]) < 0$, already explained in the proof of the strict inequality in (39). Furthermore, according to (39), $(U_1[\tau] + U_2[\tau])$ falls as τ rises. Thus, the value of τ that solves the maximization problem in (41) is $\tau = 0$, and

$$(42) \quad \Gamma_{\{0,1\}}^{max} = \frac{\lambda_1}{\epsilon} U_1[0] + U_2[0].$$

Again, according to (34), we have

$$(43) \quad \Gamma_{\{0,2\}}^{max} = \max_{0 \leq \tau \leq \bar{\tau}} \left(U_1[\tau] + \frac{\lambda_2}{\epsilon} U_2[\tau] \right).$$

Because τ^* is a possible value for τ that can be considered in solving the maximization problem in (43), we obviously have

$$(44) \quad \Gamma_{\{0,2\}}^{max} \geq \left(U_1[\tau^*] + \frac{\lambda_2}{\epsilon} U_2[\tau^*] \right).$$

To show that the Pareto frontier of $\mathcal{E}$ consists of the single point represented by point E in the figure, we now show

$$(45) \quad (\Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,2\}}^{max}) + (\Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,1\}}^{max}) - (\Gamma_{\{0,1,2\}}^{max} - \Gamma_0^{max}) \\ = \Gamma_{\{0,1,2\}}^{max} + \Gamma_0^{max} - \Gamma_{\{0,1\}}^{max} - \Gamma_{\{0,2\}}^{max} < 0.$$

To this end, note that

$$(46) \quad \Gamma_{\{0,1,2\}}^{max} + \Gamma_0^{max} - \Gamma_{\{0,1\}}^{max} - \Gamma_{\{0,2\}}^{max} \\ = \sum_{i=1}^2 \frac{\lambda_i}{\epsilon} U_i[\tau^*] + \sum_{i=1}^2 U_i[0] - \left(\frac{\lambda_1}{\epsilon} U_1[0] + U_2[0] \right) - \Gamma_{\{0,2\}}^{max} \\ \leq \sum_{i=1}^2 \frac{\lambda_i}{\epsilon} U_i[\tau^*] + \sum_{i=1}^2 U_i[0] - \left(\frac{\lambda_1}{\epsilon} U_1[0] + U_2[0] \right) - \left(U_1[\tau^*] + \frac{\lambda_2}{\epsilon} U_2[\tau^*] \right) \\ = \left(\frac{\lambda_1}{\epsilon} - 1 \right) U_1[\tau^*] - \left(\frac{\lambda_1}{\epsilon} - 1 \right) U_1[0] < 0.$$

In (46), the third line follows from (44); the second equality is obtained by simplifying the expression on the third line of (46); and the second inequality follows from the fact that $U_1[0] > U_1[\tau^*]$. ■

The following proposition is the main result of the paper. It asserts that if the relative political power of the have-not region (λ_2/λ_1) is greater than the relative population of the have region (N_1/N_2), then the have-not region will manage to induce the federal government into implementing an equalization transfer program to transfer resources from the have region to itself.

PROPOSITION 4: *If $\frac{\lambda_2}{\lambda_1} > \frac{N_1}{N_2}$, then $\tau^* > 0$, i.e., the federal government will choose to implement an equalization payments program that transfers a positive fraction of the tax revenues collected by a region to the other region, and vice versa.*

PROOF: If the federal government allows politics distort its economic policy, then according to Proposition 1, it will implement the equalization payments formula that solves the following maximization problem:

$$(47) \quad \max_{0 \leq \tau \leq \bar{\tau}} \frac{\lambda_1}{\epsilon} U_1[\tau] + \frac{\lambda_2}{\epsilon} U_2[\tau].$$

Differentiating the objective function in (47) with respect to τ , and using (27), we obtain

$$(48) \quad \frac{\lambda_1}{\epsilon} \frac{\partial U_1[\tau]}{\partial \tau} + \frac{\lambda_2}{\epsilon} \frac{\partial U_2[\tau]}{\partial \tau} = \frac{\lambda_1}{\epsilon} N_1 \beta \left(-q_1[\tau] K_1[\tau] + q_2[\tau] K_2[\tau] + \tau \frac{\partial}{\partial \tau} (q_2[\tau] K_2[\tau]) \right) \\ + \frac{\lambda_2}{\epsilon} N_2 \beta \left(-q_2[\tau] K_2[\tau] + q_1[\tau] K_1[\tau] + \tau \frac{\partial}{\partial \tau} (q_1[\tau] K_1[\tau]) \right).$$

When $\tau = 0$, (48) is reduced to

$$(49) \quad \frac{1}{\epsilon} (\lambda_2 N_2 - \lambda_1 N_1) \beta (q_1[0] K_1[0] - q_2[0] K_2[0]),$$

which is positive if $\frac{\lambda_2}{\lambda_1} > \frac{N_1}{N_2}$. Thus, the value of the equalization transfer formula that solves the maximization in (42) is $\tau^* > 0$.

■

We have already suggested that if preferences are strictly convex, instead of being linear, then national social welfare maximization will dictate a transfer of resources from the have region to the have-not region. In this case, with much more efforts, we believe, Proposition 4 can be generalized to assert that the lobbying efforts of the have-not region will induce the federal government to implement an equalization transfer formula higher than that which maximizes national social welfare.

Critics of the equalization payments program in Canada often claim that the equalization payments program is at the heart of the dependency and stagnation of the poorer provinces. This claim is confirmed in the following proposition, which asserts that the equalization payments program induces the poorer region to reduce its investment in public capital and lower the tax rate it imposes on the use of private capital within its own borders. Furthermore, the tax revenues

collected by a regional government from the use of private capital within its own borders also falls as a result of the equalization payments program. What is even more striking is that the equalization payments program also induces a fall in the investment of public capital, the tax rate imposed on the use of private capital in the richer region, and a fall in the tax revenues collected by this region on the use of private capital within its own borders.

PROPOSITION 5: If the federal government allows politics to distort its economic policy, then the equalization transfer program that it chooses to implement will induce a fall in the tax rate on the use of private capital, a fall in the public investments in each region, and a fall in the tax revenues collected from the use of private capital within its own borders by each region – the have-region as well as the have-not region.

PROOF: If the federal government only cares about national welfare, then it sets $\tau = 0$. On the other hand, if it allows politics to distort its economic policy, the equalization payments formula rises from 0 to $\tau^* > 0$, and this induces a fall in both the tax rate on private capital and the public investment in each region, as asserted by Lemma 2.(i). As for the fall in tax revenues collected by a region from the use of private capital within its own borders, it follows from Lemma 2.(iii).

■

The following proposition confirms the intuition that the greater the political power of the have-not region, the higher will be the equalization transfer formula implemented by the federal government.

PROPOSITION 6: If the federal government allows politics to distort its economic policy, then the equalization transfer formula it implements is an increasing function of the political power of the have-not region. That is, a higher value of λ_2 induces a higher value of τ^ .*

PROOF: The following first-order condition characterizes the equilibrium equalization payments formula:

$$(50) \quad \frac{\lambda_1}{\epsilon} \frac{\partial}{\partial \tau} U_1[\tau^*] + \frac{\lambda_2}{\epsilon} \frac{\partial}{\partial \tau} U_2[\tau^*] \geq 0,$$

with equality holding if $\tau^* < \bar{\tau}$. Furthermore, the following second-order condition must also be satisfied:

$$(51) \quad \frac{\lambda_1}{\epsilon} \frac{\partial^2}{\partial \tau^2} U_1[\tau^*] + \frac{\lambda_2}{\epsilon} \frac{\partial^2}{\partial \tau^2} U_2[\tau^*] < 0.$$

Because $\frac{\partial}{\partial \tau} U_1[\tau^*] < 0$, it follows from (50) that $\frac{\partial}{\partial \tau} U_2[\tau^*] > 0$. To study the comparative statics of a rise in λ_2 , there are two possibilities to consider: (i) $\tau^* = \bar{\tau}$ and (ii) $\tau^* < \bar{\tau}$. Under possibility (i), we have a corner solution, with τ^* taking on the maximum value $\bar{\tau}$ allowed, and this corner solution still prevails after the rise in λ_2 . Under the second possibility, we have an interior solution, and (50) holds with equality. In this case, totally differentiating (50), we obtain

$$(52) \quad \frac{\lambda_1}{\epsilon} \frac{\partial^2}{\partial \tau^2} U_1[\tau^*] d\tau^* + \frac{\lambda_2}{\epsilon} \frac{\partial^2}{\partial \tau^2} U_2[\tau^*] d\tau^* + \frac{1}{\epsilon} \frac{\partial}{\partial \tau} U_2[\tau^*] d\lambda_2 = 0,$$

from which follows

$$(53) \quad \frac{\partial \tau^*}{\partial \lambda_2} = - \frac{\frac{1}{\epsilon} \frac{\partial}{\partial \tau} U_2[\tau^*]}{\frac{\lambda_1}{\epsilon} \frac{\partial^2}{\partial \tau^2} U_1[\tau^*] + \frac{\lambda_2}{\epsilon} \frac{\partial^2}{\partial \tau^2} U_2[\tau^*]} > 0.$$

■

In the introduction of this paper, we have cited the evidence that the equalization transfers program makes it possible for the have-not provinces of Canada as well as the poorer states of Germany to provide a level of non-productive public good higher than that of the have provinces. The following proposition provides theoretical support for this finding. The economic logic of this proposition is quite intuitive, and can be explained as follows. If the political power of the have-not region is sufficiently higher than that of the have province, then it will induce a high equalization payments formula, which transfers a high proportion of the tax revenues collected by the have-region to itself. Furthermore, because the have-not region is less endowed than the have-region, it will invest less in public capital and leaves more resource for the provision of the non-productive public good.

PROPOSITION 7: *Suppose that the federal government allows politics to distort its economic policy. Also, suppose that A_2 is much smaller than A_1 , but λ_2 is significantly higher than λ_1 . Then the equalization transfer formula implemented by the federal government induces a higher level of the non-productive public good in the have-not region than in the have-region.*

PROOF: For each $i = 1, 2$, it follows from the definition of τ_i that $G_i[\tau_i] = 0$ and $G_i[\tau] > 0$ if $\tau \in [0, \tau_i)$. There are two possibilities to consider: (i) $\tau_1 < \tau_2$ and (ii) $\tau_2 \leq \tau_1$.

Under possibility (i), we have $\bar{\tau} = \tau_1$. Furthermore, because $\tau_1 < \tau_2$, we have $G_2[\tau_1] > 0$. Thus, under possibility (i), we have $G_2[\tau_1] = G_2[\bar{\tau}] > 0$, which in turn implies $G_2[\bar{\tau}] > G_1[\bar{\tau}] = 0$. Next, invoking Proposition (6), we can assert that τ^* , the equilibrium equalization transfer formula is strictly increasing in λ_2 , and this means that when λ_2 is sufficiently large, τ^* will be in a left neighborhood of $\bar{\tau}$. Using $G_2[\bar{\tau}] > G_1[\bar{\tau}] = 0$ we can then assert – by continuity – that $G_2[\tau^*] > G_1[\tau^*] \geq 0$, and Proposition 7 is proved.

Under possibility (ii), we have $\bar{\tau} = \tau_2$. According to (24), we have

$$(54) \quad H_i[\tau_2] = \left(\frac{\alpha^\alpha \gamma^{1-\alpha} A_i (1-\alpha + \alpha\beta(1-\tau_2)N_i)}{\beta((1-\tau_2)rN_i)^\alpha} \right)^{\frac{1}{1-\alpha-\gamma}}, \quad (i = 1, 2),$$

and

$$(55) \quad (1 - \tau_2)q_2[\tau_2]K_2[\tau_2] + \tau_2 q_1[\tau_2]K_1[\tau_2] = H_2[\tau_2].$$

It follows from (54) that

$$(56) \quad \frac{H_2[\tau_2]}{H_1[\tau_2]} = \left(\frac{A_2}{A_1} \right)^{\frac{1}{1-\alpha-\gamma}} \left(\frac{N_1}{N_2} \right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{1-\alpha + \alpha\beta(1-\tau_2)N_2}{1-\alpha + \alpha\beta(1-\tau_2)N_1} \right)^{\frac{1}{1-\alpha-\gamma}}.$$

Observe that the expression on the right-hand side of (56) will be small if A_1 is much larger than A_2 . Hence the ratio $\frac{H_2[\tau_2]}{H_1[\tau_2]}$ will be small if A_1 is much larger than A_2 .

Next, note that

$$(57) \quad \frac{(1-\tau_2)q_2[\tau_2]K_2[\tau_2] + \tau_2 q_1[\tau_2]K_1[\tau_2]}{(1-\tau_2)q_1[\tau_2]K_1[\tau_2] + \tau_2 q_2[\tau_2]K_2[\tau_2]} = \frac{H_2[\tau_2]}{G_1[\tau_2] + H_1[\tau_2]} < \frac{H_2[\tau_2]}{H_1[\tau_2]}.$$

Hence the expression on the left-hand side of (57) will be small if A_1 is much larger than A_2 .

On the other hand,

$$(58) \quad \frac{(1-\tau_2)q_2[\tau_2]K_2[\tau_2]+\tau_2q_1[\tau_2]K_1[\tau_2]}{(1-\tau_2)q_1[\tau_2]K_1[\tau_2]+\tau_2q_2[\tau_2]K_2[\tau_2]} > \frac{\tau_2q_1[\tau_2]K_1[\tau_2]}{(1-\tau_2)q_1[\tau_2]K_1[\tau_2]+\tau_2q_2[\tau_2]K_2[\tau_2]} = \frac{\tau_2}{(1-\tau_2)+\tau_2\frac{q_2[\tau_2]K_2[\tau_2]}{q_1[\tau_2]K_1[\tau_2]}} > \tau_2,$$

with the strict inequality follows from the fact that $q_2[\tau_2]K_2[\tau_2] < q_1[\tau_2]K_1[\tau_2]$ according to Lemma 1.

Together, (57) and (58) allow us to write

$$(59) \quad \tau_2 < \frac{(1-\tau_2)q_2[\tau_2]K_2[\tau_2]+\tau_2q_1[\tau_2]K_1[\tau_2]}{(1-\tau_2)q_1[\tau_2]K_1[\tau_2]+\tau_2q_2[\tau_2]K_2[\tau_2]} < \frac{H_2[\tau_2]}{H_1[\tau_2]}.$$

Now if we let A_1 grow indefinitely large and at the same time also let λ_2 grow to keep up with A_1 so that $\tau_2 = \tau^*$ and that τ^* remains outside a right neighborhood of 0, then the two strict inequalities in (59) imply that

$$\frac{(1-\tau_2)q_2[\tau_2]K_2[\tau_2]+\tau_2q_1[\tau_2]K_1[\tau_2]}{(1-\tau_2)q_1[\tau_2]K_1[\tau_2]+\tau_2q_2[\tau_2]K_2[\tau_2]}$$

is bounded below and away from 0 and tends to 0 at the same time, and this is not possible. Thus, Proposition 7 is also proved under possibility (ii).

■

Intuitively, we expect that the net payoff of a region rises with its own political power, but falls as the political power of the other region rises. The following proposition confirms this intuition.

PROPOSITION 8: *Suppose that the federal government allows politics to distort its economic policy. If $\frac{\lambda_2}{\lambda_1} > \frac{N_1}{N_2}$, then, for $i, j = 1, 2$, $i \neq j$, Γ_i is an increasing function of λ_i and a decreasing function of λ_j .*

PROOF: According to Proposition 3, the equilibrium net payoff for region 2 is

$$(60) \quad \Gamma_2 = \Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,1\}}^{max}.$$

When λ_2 rises, $\Gamma_{\{0,1,2\}}^{max}$ rises with λ_2 , but $\Gamma_{\{0,1\}}^{max}$ remains unchanged. Thus, the higher is the political power of the have-not region, the higher will be its net payoff.

As for the have region, its net payoff is

$$(61) \quad \Gamma_1 = \Gamma_{\{0,1,2\}}^{max} - \Gamma_{\{0,2\}}^{max}.$$

Differentiating (61) with respect to λ_2 , and using the envelope theorem, we obtain

$$(62) \quad \frac{\partial \Gamma_1}{\partial \lambda_2} = \frac{\partial \Gamma_{\{0,1,2\}}^{max}}{\partial \lambda_2} - \frac{\partial \Gamma_{\{0,2\}}^{max}}{\partial \lambda_2} = \frac{1}{\epsilon} U_2[\tau^*] - \frac{1}{\epsilon} U_2[\tau_{\{0,2\}}],$$

where we have let

$$(63) \quad \tau_{\{0,2\}} = \operatorname{argmax}_{0 \leq \tau \leq \bar{\tau}} \left(U_1[\tau] + \frac{\lambda_2}{\epsilon} U_2[\tau] \right).$$

The following first-order condition characterizes the solution of the maximization problem in (63):

$$(64) \quad \frac{\partial}{\partial \tau} U_1[\tau_{\{0,2\}}] + \frac{\lambda_2}{\epsilon} \frac{\partial}{\partial \tau} U_2[\tau_{\{0,2\}}] \geq 0,$$

with equality holding if $\tau_{\{0,2\}} < \bar{\tau}$. Also, recall the first-order condition (50), which characterizes the equilibrium equalization transfer formula, and use the fact $\frac{\partial}{\partial \tau} U_1[\tau] < 0$ to obtain

$$(65) \quad \frac{\partial}{\partial \tau} U_1[\tau^*] + \frac{\lambda_2}{\epsilon} \frac{\partial}{\partial \tau} U_2[\tau^*] > 0,$$

i.e., $\left(U_1[\tau] + \frac{\lambda_2}{\epsilon} U_2[\tau] \right)$ is increasing in a right neighborhood of τ^* . Thus, if we presume that the values of τ that satisfy the first-order conditions (50) and (64) are unique, then we must have $\tau_{\{0,2\}} \geq \tau^*$ and $U_2[\tau_{\{0,2\}}] > U_2[\tau^*]$. It follows from the preceding inequality that the derivative in (62) is negative, i.e., the net payoff for the have region falls as the political power of the have-not region rises.

In the same manner, we can demonstrate that a rise in the political power of region 1 induces a rise in its own net payoff, but a fall in the net payoff of region 2.

■

It is instructive to see the impact of the increase in the political power of the region 2 on the equilibrium level of the effort expended by each region to generate political support for the federal government.

PROPOSITION 9: *The higher the productivity of the political support function of the less endowed and less populous region,*

(i) the higher will be the effective level of effort it devotes to generating political support for the federal government. However, it is not clear whether the real effort level that yields the effective effort is higher or lower;

(ii) the higher will be the real effort level that the better endowed and more populous region devotes to generating political support for the federal government if the political support technology of this region is not very productive.

PROOF: (i) According to (36) and (40), the equilibrium level of effective effort chosen by regional government 2 is given by

$$(66) \quad \phi_2^*[\tau^*] = \left(\frac{\lambda_2}{\epsilon} - 1\right) U_2[\tau^*] - \Gamma_2.$$

Differentiating (66) with respect to λ_2 , we obtain

$$(67) \quad \begin{aligned} \frac{\partial \phi_2^*[\tau^*]}{\partial \lambda_2} &= \left(\frac{\lambda_2}{\epsilon} - 1\right) \frac{\partial U_2[\tau^*]}{\partial \lambda_2} + \frac{1}{\epsilon} U_2[\tau^*] - \frac{\partial \Gamma_2}{\partial \lambda_2} \\ &= \left(\frac{\lambda_2}{\epsilon} - 1\right) \frac{\partial U_2[\tau^*]}{\partial \lambda_2} + \frac{1}{\epsilon} U_2[\tau^*] - \frac{1}{\epsilon} U_2[\tau^*] = \left(\frac{\lambda_2}{\epsilon} - 1\right) \frac{\partial U_2[\tau^*]}{\partial \lambda_2} > 0. \end{aligned}$$

Thus, the equilibrium effective effort level of region 2 rises with λ_2 . However, it is not clear whether $e_2[\tau^*] = \frac{\phi_2^*[\tau^*]}{\frac{\lambda_2}{\epsilon} - 1}$ rises or falls when λ_2 rises. Part (i) of Proposition 9 is proved.

The equilibrium level of effective effort for region 1 is given by

$$(68) \quad \phi_1^*[\tau^*] = \left(\frac{\lambda_1}{\epsilon} - 1\right) U_1[\tau^*] - \Gamma_1.$$

Differentiating (68) with respect to λ_2 , we obtain

$$(69) \quad \begin{aligned} \frac{\partial \phi_1^*[\tau^*]}{\partial \lambda_2} &= \left(\frac{\lambda_1}{\epsilon} - 1\right) \frac{\partial U_1[\tau^*]}{\partial \lambda_2} - \frac{\partial \Gamma_1}{\partial \lambda_2} \\ &= \left(\frac{\lambda_1}{\epsilon} - 1\right) \frac{\partial U_1[\tau^*]}{\partial \lambda_2} + \frac{1}{\epsilon} (U_2[\tau_{\{0,2\}}] - U_2[\tau^*]), \end{aligned}$$

where the second line in (69) has been obtained with the help of (64). The sign of (69) is ambiguous because $\frac{\partial U_1[\tau^*]}{\partial \lambda_2} < 0$, and $(U_2[\tau_{\{0,2\}}] - U_2[\tau^*]) > 0$. However, if λ_1 is slightly greater than ϵ , the first expression on the right-hand side of (69) is negligible, and we have $\frac{\partial \phi_1^*[\tau^*]}{\partial \lambda_2} > 0$. That is, $\phi_1^*[\tau^*]$ rises with λ_2 and a fortiori $e_1[\tau^*] = \frac{\phi_1^*[\tau^*]}{\frac{\lambda_1}{\epsilon} - 1}$ also rises with λ_2 .

■

4. A NUMERICAL EXAMPLE

In the numerical example, we set the weight assigned to social welfare to $\epsilon = 1$. Also, we maintain the political power of region 1 at $\lambda_1 = 1.1$, while we allow the political power of region 2 to take on successively the following values: $\lambda_2 = 1.5, 2.5, 3.0, 3.5$. The values for the remaining parameters are as follows:

$$\alpha = 0.35, \gamma = 0.1, A_1 = 1, A_2 = 0.5, N_1 = 2, N_2 = 1.75, \beta = 0.95, r = 0.05, \bar{k}_1 = 0.5, \bar{k}_2 = 0.4.$$

The base case – the equilibrium without equalization payments – is presented in Table I:

TABLE I–THE EQUILIBRIUM WITHOUT EQUALIZATION TRANSFERS ($\tau = 0$)

q_1	0.022
q_2	0.017
H_1	0.264
H_2	0.072

K_1	18.460
K_2	5.061
G_1	0.147
G_2	0.016
c_1	1.263
c_2	0.382
U_1	2.806
U_2	0.696

When there are no equalization transfers, the tax rate imposed by the regional government 1 on the private capital input is 0.022, and the corresponding value for region 2 is 0.017. The stock of public capital in region 1 is 0.264, which is larger than 0.072, the stock of public capital that in region 2. The non-productive public expenditure in region 1 is 0.147, which is much larger than that in region 2 (0.016). The demand for private capital in region 1 and in region 2 are 18.460 and 5.061, respectively. Note that the demand for private capital in region 1 is much larger than that in region 2, and this result comes from the fact that region 1 has a higher total factor productivity ($A_1 = 1, A_2 = 0.5$), a larger population ($N_1 = 2, N_2 = 1.75$), and a higher stock of public capital than region 2.

Given that the supply of private capital in region 1 and region 2 are $N_1 \bar{k}_1 = 1$ and $N_2 \bar{k}_2 = 0.7$ and that the demand for private capital in region 1 is much higher than that in region 2, some of the stock of private capital in region 2 moves into region 1, and, in addition, there is a large volume of foreign capital entering both regions. A resident in region 1 enjoys 1.263 units of consumption good, which is much higher than 0.383 units of the consumption good enjoyed by a resident in region 2. The total utility enjoyed by residents in region 1 is 2.806, which is much higher than 0.696, the corresponding number for region 2.

Table II gives the equilibrium with equalization payments.

TABLE II– THE EQUILIBRIUM WITH EQUALIZATION TRANSFERS

	$\lambda_2 = 1.5$	$\lambda_2 = 2.5$	$\lambda_2 = 3.0$	$\lambda_2 = 3.5$
τ^*	0.084	0.183	0.201	0.212
q_1	0.019	0.015	0.014	0.0137
q_2	0.014	0.01	0.0095	0.009
H_1	0.258	0.251	0.250	0.2496

H_2	0.071	0.070	0.0695	0.0694
K_1	19.696	21.548	21.933	22.196
K_2	5.429	5.98	6.095	6.174
G_1	0.093	0.024	0.011	0.002
G_2	0.032	0.040	0.0398	0.0394
c_1	1.289	1.326	1.333	1.339
c_2	0.391	0.403	0.405	0.407
U_1	2.755	2.700	2.688	2.682
U_2	0.737	0.772	0.776	0.778
e_1	0.009	0.003	0.002	0.002
e_2	0.029	0.029	0.025	0.021

When the two regions are allowed to lobby the federal government and $\frac{\lambda_2}{\lambda_1} > \frac{N_1}{N_2}$ the latter government is induced to implement a positive equalization transfer formula according to Proposition 4. As λ_2 rises, the equalization transfer formula τ^* implemented by the federal government rises with λ_2 , which can be seen from the second row of Table II. As λ_2 increases (therefore, τ^* rises), the tax rates imposed by the regional government 1 and regional government 2 on the private capital input declines, which can be seen from the third and fourth rows of Table II, respectively. The stock of public capital in each region, regardless of whether the region is rich or not, declines because it has less incentive to attract private capital: as τ^* rises, the region is only allowed to keep a smaller fraction of the revenues it obtains by taxing private capital. Note that the stock of public capital in region 1 declines more rapidly than that in region 2 as τ^* increases. This result might reflect the idea that region 1 has higher level of tax revenues, and thus has less incentive to raise its stock of public capital because much of the fruit of the investment will flow to the other region under the equalization transfer program. The seventh and eighth rows of Table II indicates that when τ^* rises the demand for private capital in both regions increases, but that the demand for private capital for region 1 rises more rapidly than that for region 2. This result might imply that some stock of private capital in region 2 moves into region 1 and that foreign capital will enter both regions. The ninth and tenth rows of Table II indicate that as τ^* rises, the non-productive public good in region 1 declines and that in region 2 initially increases, reaches the maximum, and then declines after that. Observe that when $\tau^* = 0.183$ region 2 provides a higher level of non-productive public good than region 1. This result has been established in Proposition 7. The eleventh and twelfth rows of Table II

indicate that as τ^* rises, each resident of the both regions consume more and more of the private good. As shown by the thirteenth and fourteenth rows of Table II, the total utility of the residents in region 1 (resp. in region 2) declines (resp. increases) as τ^* rises. Thus, residents in the richer (resp. poorer) region are worse-off (resp. better-off) when τ^* rises. For residents of region 2, total welfare improves as τ^* rises due to the increase in the consumption of the private good and the rise in the non-productive public good provision. As for residents of region 1, total welfare deteriorates as τ^* increases. This is because the rise in the consumption of the private good is not sufficient enough to offset the decline in the non-productive public good.

5. CONCLUDING REMARKS

Traditional theories of fiscal federalism often presume that public decision makers are benevolent social planners whose objective is social welfare maximization and that inter-government grants should be made to achieve efficiency and equity goals. However, casual empiricism reveals that economic policy is not decided by benevolent social planners, but by politicians who have their own interests, usually with an eye on their re-election prospects. In this paper, we have formulated a political economy model of equalization transfers in a fiscal federalism in which the state politicians lobby the central government – by promising political support to the latter government – in order to induce the central government into implementing an equalization transfer scheme that is favorable to their own states. The force that drives the lobbying activities of state officials is the political map that gives different states different levels of political power. When the central government allows politics to distort its economic policy, it will implement an equalization transfer scheme that is more favorable to the state with more political power. The equalization transfer scheme thus implemented has unintended consequences. The rational behavior of the states lead them to investing less in infrastructure and taxing capital at a lower rate, with the ensuing consequence of lower tax receipts in all the states. Another unintended consequence is that a receiving state might provide a higher level of public services to its own residents than a giving state, creating resentment in the latter state. In contrast with the traditional theories of equalization transfers, our model is positive, not normative, and it is meant to explain how politics interacts with economics in determining inter-governmental grants.

APPENDIX

The Proof of Lemma 3

If the federal government does not carry out any equalization payments, then the consumption of the non-productive public good in region i is given by

$$(A.1) \quad G_i[0] = q_i[0]K_i[0] - H_i[0]$$

$$= ((1 - \alpha)\alpha(\beta N_i - 1) - \gamma(1 - \alpha + \alpha\beta N_i)) \left(\frac{\alpha^\alpha \gamma^\gamma A_i (1 - \alpha + \alpha\beta N_i)^{\alpha + \gamma}}{\beta (r N_i)^\alpha} \right)^{\frac{1}{1 - \alpha - \gamma}}.$$

We presume that $G_i[0] > 0$, and this requires the expression on the second line of (A.1) to be positive, i.e.,

$$((1 - \alpha)\alpha(\beta N_i - 1) - \gamma(1 - \alpha + \alpha\beta N_i)) > 0,$$

or

$$(A.2) \quad N_i > \frac{(1 - \alpha)(\alpha + \gamma)}{\alpha(1 - \alpha - \gamma)\beta}.$$

Because $1 - \alpha - \gamma > 0$, it is clear that $G_i[0]$ is strictly increasing in A_i , and (i) of Lemma 2 is proved.

To prove (ii) of Lemma 3, differentiate $G_i[0]$ logarithmically with respect to N_i to obtain

$$(A.3) \quad \frac{\partial \text{Log}[G_i[0]]}{\partial N_i} = \left(\frac{\alpha}{1 - \alpha - \gamma} \right) \left(\frac{(1 - \alpha)(-(1 - \alpha)(\alpha + \gamma) + \beta N_i((1 - \alpha)(-1 + 2\alpha + 2\gamma) - \alpha(1 - \alpha - \gamma)\beta N_i))}{N_i(1 - \alpha + \alpha\beta N_i)((1 - \alpha)(\alpha + \gamma) - \alpha\beta N_i(1 - \alpha - \gamma))} \right).$$

On the right-hand side of (A.3), the denominator of the expression inside the second grand parentheses is negative due to the condition (A.2). As for the numerator of the same expression, its derivative with respect to N_i is

$$(1 - \alpha)\beta((1 - \alpha)(-1 + 2\alpha + 2\gamma) - 2\alpha(1 - \alpha - \gamma)\beta N_i)$$

which negative if

$$(A.4) \quad N_i > \frac{(1 - \alpha)(-\frac{1}{2} + \alpha + \gamma)}{\alpha(1 - \alpha - \gamma)\beta}.$$

Note that when (A.2) is satisfied, (A.4) is also satisfied. Hence the numerator of the expression on the right-hand side of (A.3) is decreasing in N_i when (A.2) is satisfied. Finally, when the numerator of this same expression, when evaluated at $N_i = \frac{(1-\alpha)(\alpha+\gamma)}{\alpha(1-\alpha-\gamma)\beta}$, is equal to $-\frac{(1-\alpha)^2(\alpha+\gamma)}{\alpha} < 0$. Therefore, $\frac{\partial G_i[0]}{\partial N_i} > 0$, as desired.

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Chapter 3: Equalization Transfers with Asymmetric Information on the Technology of Public Services Provision

1. INTRODUCTION

At the present time, there are 196 countries in the world. In terms of the forms of government that they assume, these countries can be classified as unitary states, federal states, or confederations.

A unitary state is governed as a single political unit in which the central government is supreme. Any administrative division in a unitary state exercises only the powers that are delegated by the central government. The majority of the countries in the world are unitary states. The city-states of Singapore and Monaco are unitary states. China, Japan, Indonesia, France, Egypt... are unitary states.

A federal state is a political entity constituted by the union of several partially self-governing regions. In a federal state, there are two orders of government: a federal government and constituent-unit governments, each of which has some autonomy from the other. The governments at each level are primarily accountable to their own electorates. The number of people who live in federal states accounts for about 40% of the world's population. Some small countries, such as Switzerland, Belgium, Bosnia and Herzegovina, are federal states. Some federal states, such as Canada, USA, Australia, and Brazil, have vast territories. India, Pakistan, and Nigeria are federal states with large populations. Germany, with its sixteen Länder, is a federal state.

A confederation is a union of a number of countries – usually by a treaty – whose main purpose is to deal with critical issues, such as defence, foreign affairs, or a common currency. A confederation has a central government formed by the member states, but not elected directly. The European Union and the Commonwealth of Independent States are confederations. In a confederation, the central government is required to provide support to all the member states.

Together with globalization and the progress made in information technology, a gradual shift in governance structure from unitary state to federal state and confederacy has been taking place. Even in unitary states, more and more economic issues are now handled by lower levels of governments. The main argument in support of decentralization centers on the provision of local public goods. It is argued that local public goods, such as parks, roads, sewage treatment, which provide collective benefits to a localized population, are better provided by lower levels of governments. First and foremost, local governments, being close to the people they serve, are likely to be in a better position than the central government in matching the provision of these public goods with the needs and preferences of local residents. Public services – health care, education, and welfare – must be administered by agencies close to those they serve. Second, local governments are also in a better position to provide local public goods and services at a lower cost because there are fewer levels of bureaucracy, and this reduces agency cost. In addition, local governments are in a better position to identify and contract suppliers who provide the inputs used in the production of local public goods.

In a federation, the federal government is responsible for national defence, foreign affairs, foreign trade, immigration, and the legal system. For stabilization purposes, the federal government is in control of the central bank and the currency. The states, which constitute the federation, are responsible for the delivery of public services, such as health care, education (at all levels, including post-secondary and manpower training), and welfare. As for revenues-raising capacities,³⁷ federal governments collect the lion's share of taxes in their own federations. In Mexico, Russia, and Malaysia, federal revenues stand at 90% of the total. The number is between 70% and 85% in Argentina, Australia, Belgium, and Brazil. In Germany, Austria, Spain, and India, the federal governments' share is between 60% and 65%. In the US and Canada, the numbers are 55% and 47%, respectively. In light of these statistics, one should not be surprised to learn that federal governments raise more revenues than they need. The states that constitute a federation, on the other hand, do not generate enough revenues on their own to pay for the public services that they are supposed to deliver to their own constituencies. There is thus a gap in the

³⁷ Anderson, G. (2010): *Fiscal Federalism: A Comparative Introduction*, p.21. Printed and bound in Canada.

revenues-raising capacities of the two orders of government known as *vertical fiscal imbalance* in the literature of fiscal federalism.

Another kind of fiscal imbalance – known as *horizontal fiscal imbalance* – involves the disparities in the revenues raising capacity of constituent-unit governments. Regions in a federal state differ in terms of their climate, their endowments, and their populations. Regions with a good endowment, such as oil, have a high capacity for raising revenues from royalties. Another source of disparities among regions in a federation involves the cost of providing public services. Regions with a higher proportion of their population in remote areas have a higher cost of providing public services, and regions with a more aged population require more health care. All these factors lead to horizontal fiscal imbalance, which makes it hard pressed for the poor states in the federation to find enough revenues to finance the public services they are required to provide.

Equity consideration dictates that all the residents of a federal state, should be treated the same. In particular, a resident, regardless of where she resides, should have access to the same level of public services. Fundamentally, an equalization transfer scheme is a zero-sum game of redistribution, with resources being transferred from richer to poorer provinces. An equalization transfer scheme is intended to rectify the vertical and horizontal fiscal imbalances. Section 36(2) of the Canadian constitution requires that equalization payments be made to the poorer provinces so that they have sufficient revenues to provide “reasonably comparable levels of public services at reasonably comparable levels of taxation.” In Germany, Article 72 of the constitution mandates “equal living conditions” to the citizens of the federal state. In the Commonwealth of Australia, the states should receive grants from the Commonwealth so that if they make the same effort to raise revenues from their own tax bases and operate at the same level of efficiency, then they would have the capacity to provide public services to their own residents at the same standard. In the European Union, a small part of its budget is set aside for transfer to less prosperous member states, such as Greece, Portugal, and Ireland.

In Canada,³⁸ the federal government collects taxes from each of the provinces, and then redistributes the pool of money in unconditional payments – according to a formula set out in federal legislation – to the provinces with revenue-generating capacity below the national average. The legislation is renewed every five years, and represents a funding mechanism, which is the outcome of negotiations between the federal government and the provinces. The equalization formula involves two key calculations. First, the fiscal capacity of each province is determined, with fiscal capacity being measured using five tax bases: personal income tax, business income tax, consumption tax, property tax and natural resources.³⁹ The fiscal capacity of each province thus calculated helps determine the revenue per capita that each province is able to raise. The revenues per capita will not be uniform across provinces. Second, these per capita revenue figures are compared with a national standard based on the average incomes of all 10 provinces. The provinces with per capita revenues below the national standard will receive equalization payments to reach the national standard. Those provinces with per capita revenues above the national standard are not eligible for equalization payments.

In the Commonwealth of Australia,⁴⁰ the main tax bases – customs and excise, income taxes, and the GST – are all under the control of the central government, while the states have control over domestic policy. There is thus a substantial vertical fiscal imbalance, and the Commonwealth Grants Commission (CGC), a non-partisan neutral body of technocrats, is in charge of the transfer of the revenues raised by the Commonwealth to the states. In calculating the equalization transfers to a state, the CGC follows the principle of horizontal equity. First, the CGC tries to assess the expenditure needs of each state, taking into consideration all the factors outside the state's control. A state's relative expenditure requirements are based on demand for and cost of services, which in turn depend on factors, such as population size and age structure, income, size of the indigenous population, education level, community size and remoteness... After the CGC has determined the states' expenditure requirements and fiscal capacities, it calculates grant entitlements using the following formula: a per capita share of the total pool of funds, plus expenditure needs, plus revenue needs, plus needs for Specific Purpose Payments. Because the

³⁸ Boadway, R and Watts, R (2000)

³⁹ In a concession to the provinces endowed with rich natural resources, 50% of natural resource revenues are excluded from the calculation of a province's fiscal capacity as well as in the calculation of the national standard.

⁴⁰ McLean, L (2002): Fiscal Federalism in Australia.

data needed to implement the formula are not available in the year the grants are allocated, the calculations for a state are based on a rolling five-year average of its needs and fiscal capacity. The CGC updates its rolling five-year data estimates every year and carries out a comprehensive review of its methods every five years.

In Germany, intergovernmental transfers⁴¹ fall into two broad categories: specific grants, and equalization transfers. Specific grants are made by the federal government to the Länder to pay for joint projects, federally mandated expenditures, and specific projects aimed at creating uniform living conditions in the federation. Under the equalization transfers scheme, there is a revenue pool into which richer Länder pay and from which poorer Länder draw according to a preset formula. The equalization entitlements received by a state are calculated in terms of the difference between its adjusted fiscal capacity and its individual equalization standard. A poor state also receives supplementary payments from the federal government.

There is a large literature on equalization transfers. Although equalization transfer schemes are largely designed to serve equity goals, not to improve efficiency, most theoretical studies on the subject try to explain how inter-governmental grants can help internalize the inter-regional spillovers from public goods⁴² or to eliminate inefficient tax competition aimed at attracting mobile capital.⁴³ One researcher⁴⁴ explains how inter-governmental grants can be used for risk sharing among the regions of a federation which might experience idiosyncratic shocks in regional incomes.

A reasonable equalization transfer scheme – for redistributive or insurance purposes – should incorporate the characteristics of all the regions in the federation. Furthermore, it is generally recognized that local governments know more about those they serve and about the cost of local public goods than the central government, and this presents a serious problem for designing an equalization transfer scheme. In Canada, property tax enters the calculations of equalization transfers. Although statutory tax rates on real estate are readily observable, local governments

⁴¹ Watts, R. and Hobson, P (2000).

⁴² Oats, W. E. (1972).

⁴³ Wildasin, D. (1989).

⁴⁴ Lockwood, B (1999).

usually have better information about the true value of the homes in their regions than the central government. We have also mentioned that in the Commonwealth of Australia, the CGC has to estimate the various parameters of each state on a rolling five-year basis. These parameters change over time, and it might be difficult to observe or incorporate them as verifiable contingencies into the formula used to calculate the equalization transfers. In these matters, regional governments definitely know more about the parameters that characterize their own regions. Last but not least, and as we have already mentioned on our discussion in support of decentralization, a local government knows more than the central government about the suppliers of inputs used in producing local public goods. This is the source of the information asymmetry on the provision of public goods.

Until recently, most theoretical studies on equalization transfers took for granted that the designer of an equalization transfer scheme has complete information on these characteristics. Lately, some researchers have begun incorporating asymmetric information into theoretical models of equalization transfers. The approach used by these researchers is that of the two-type principal agent model with hidden information. In a model with two regions in which each region might have a low or a high taste for public goods, Bucovetsky et al. (1998) argued that a regional government knows more about the preferences for public goods of the residents in its own region than the central government because presumably that is why it is elected. These researchers explained how this asymmetry of information has implication for the design of an inter-governmental grant that induces a regional government to choose an efficient mix of public and private consumption. Bordignon et al. (2001) formulated a two-type model of equalization transfers with asymmetry of information over the size of regional tax bases. Cornes et al. (2002) formalized a model of equalization transfers with two regions in which the cost of producing the public goods for one region is common knowledge, while that of the other region is private information. Each region can lower the marginal cost of the public good by devoting some costly effort to improve its technology. The information asymmetry resides in the fact that the effort of one region is not observable. Their model is thus that of a principal agent model in which the action of one agent is hidden. In their model, in addition to the incentive compatibility constraint, the participation constraint that requires at least a reservation utility for each region is also incorporated. These researchers also analyzed the case of a continuum of cost types.

In this paper, we present a model of equalization transfers for a federal state, which is made up of two regions and which has two orders of government – a federal government and two regional governments. In each region, a consumption good is produced using labor, private capital, and public capital. The government of each region invests in public capital and provides public services – that we also call the non-productive public good. To finance the public capital investment and the provision of public services, a regional government imposes a tax on the use of private capital within its own borders. The technology used by a regional government to produce the non-productive public good is linear and uses the consumption good as the only input. The information asymmetry in the model resides in the fact that the productivity of the technology for public service provision in a region is the private information of the regional government.

To capture the equity dimension of an equalization transfer scheme, we insist that such a scheme must induce at least a certain expected level of utility for a resident of the federation that is guaranteed by the constitution regardless of where she resides. Our model thus stands in sharp contrast with the theoretical studies, which are mainly preoccupied with efficiency, in that it encompasses both the equity and the efficiency dimensions. Another important feature that differentiates our model from other theoretical studies on equalization transfers is that those theoretical studies often assume that the regions are symmetric,⁴⁵ while our model begins with an asymmetry among the two regions.⁴⁶ The asymmetry between the two regions reflects their different endowments and different population sizes: some regions are rich and some are poor; some regions have a high density population, while other regions are sparsely populated. The two regions also differ in how efficient they are in delivering public services to their own residents. And this is why equalization transfers are made for equity reasons.

In the model we formalize, even before the game begins, the central government has some idea about which region is rich and which region is poor. The asymmetry of information resides in the

⁴⁵ In the model of Bordinon et al., the two regions are symmetric ex ante. Only after Nature has made a random draw for their types do they become asymmetric.

⁴⁶ If the two regions are symmetric, the only justification for an equalization transfer scheme is insurance against idiosyncratic shocks to regional incomes, not for equity reason.

fact that although the federal government knows the range over which the parameter that characterizes the productivity of the technology for public service provision in a region, it does not know exactly the location of this parameter within its range. Only the regional government knows the exact position of this parameter. One might imagine that the range over which the productivity parameter of the poor region is located to the left of that of the rich region on the real line, and that the two ranges are overlapping or non-overlapping. With asymmetric regions – a poor and a rich region – we have eschewed a two-productivity-type model because in such a model, and knowing from the start which region is poor, the federal government knows which one has the high productivity type and which one has the low productivity type. Thus, we allow the productivity of the technology for public service provision in each region to be a continuous variable. Finally, except for Bucovetsky et al., who incorporate tax competition for mobile capital in their model, researchers on the subject of equalization transfers do not consider this policy choice as a means to raise revenues to finance the provision of public goods. Our model goes farther than that of Bucovetsky et al. in considering also the role played by public capital investment to attract mobile capital.

Because an equalization transfer scheme is a zero-sum game, it is a transfer of resources from richer to poorer regions, and this creates resentment among the regions that are the sources of the inter-governmental grant flows.⁴⁷ Thus, instead of maximizing a social welfare function of the entire federation, we shall assume that the objective of the designer of the equalization scheme is to provide the residents of a less fortunate region with at least a certain level of utility judged equitable without imposing an excessive burden on the richer regions.

The paper is organized as follows. In Section 2, the model is presented. Section 3 provides an analysis of a benchmark: equalization transfers with complete information. The analysis of equalization transfers with asymmetric information is presented in Section 4. The impact of an

⁴⁷ “Bavaria, Germany’s richest state, is tired of paying billions each year to the country’s poorer states. On Tuesday, the state’s governor announced it planned to challenge Germany’s complicated transfer system in court.” The World from Berlin: Bavaria Mulls an End to Solidarity, Spiegel Online International, July 18, 2012 – 02:35 PM.

equalization scheme on the welfare of the richer region is analyzed in Section 5. Some concluding remarks are given in Section 6.

2. THE MODEL

2.1. *Preferences and Technologies*

Suppose that a federation consists of two regions – region 1 and region 2. Let N_i denote the population of region i , $i = 1, 2$. The federation is assumed to be a small open economy, and there is free flow of capital across the borders of each region from the rest of the world. There is a single consumption good, which is produced in each region using inputs of labor, private capital, and public capital. The consumption good can also be used as investment goods to accumulate capital – public as well as private. Besides the consumption good, there is another good – public services – such as health, education, and welfare – that we also call non-productive public good. In what follows, the consumption good is taken as the numéraire. The federal government implements an equalization transfer program to ensure that each citizen, regardless of where she resides in the federation, has access to at least a certain level of public services.

For simplicity, we assume that the preferences of all residents within a region, say region i , are identical and represented by the following linear utility function:

$$(1) \quad u_i = \beta_i G_i + c_i, \quad (i = 1, 2),$$

where G_i and c_i represent, respectively, the consumption of public services and the consumption of the private good. Also, β_i is a positive parameter, which characterizes her preferences for public services relative to the consumption good. The budget constraint for a resident of region i is given by

$$(2) \quad c_i = \omega_i + r\bar{k}_i,$$

where ω_i is her labor income (the wage rate prevailing in region i); r is the world rate of interest; and $\bar{k}_i$ is the private capital endowment of an individual in region i .

For each $i = 1, 2$, the consumption good is produced in region i according to the following Cobb-Douglas technology:

$$(3) \quad Y_i = A_i K_i^\alpha L_i^{1-\alpha} H_i^\gamma,$$

where Y_i is the output; K_i is the input of private capital; L_i is the labor input; and H_i is the stock of public capital. Also, A_i is a parameter which characterizes the endowments of the region i . As for α and γ , they are positive parameters, which satisfy the condition $\alpha + \gamma < 1$.

We assume that private capital is perfectly mobile across the two regions, but labor is completely immobile. The stock of public capital represents public infrastructure – roads, sewer systems, public R&D, and so on. The condition $\alpha + \gamma < 1$ captures the idea that the production technology exhibits decreasing return to scale jointly with respect to private and public capital for a given level of labor input. On the other hand, the production technology exhibits constant returns to scale with respect to the labor input and private capital.

The technology for producing public services in a region, say region i , is linear, which produces θ_i units of public services from each unit of the consumption good. The value of θ_i represents the productivity of region i 's technology for producing public services, and is the private information of regional government i . It is assumed that θ_i lies in an interval $[\underline{\theta}_i, \bar{\theta}_i]$ of the real line, with $0 < \underline{\theta}_i < \bar{\theta}_i, i = 1, 2$.

To model the asymmetric information on the productivity of the technology for producing public services, we suppose that the type profile (θ_1, θ_2) is the outcome of a random draw by Nature from the rectangle $[\underline{\theta}_1, \bar{\theta}_1] \times [\underline{\theta}_2, \bar{\theta}_2]$, with $\Phi[\theta_1, \theta_2]$ as its distribution. After Nature has made the random draw, she informs the government of region i the value of θ_i obtained from the draw. In this manner, θ_i becomes the private information of the government of region $i, i = 1, 2$. The distribution function $\Phi[\theta_1, \theta_2]$ is assumed to be common knowledge. We shall let $\Phi_i[\theta_i]$ denote the marginal distribution of θ_i and assume that its density $\phi_i[\theta_i]$ is continuous and positive for all θ_i in $[\underline{\theta}_i, \bar{\theta}_i]$. We also assume that θ_1 and θ_2 are independent.

2.2. An Equalization Transfer Scheme

An equalization transfer scheme – or a social choice function – is a map

$$(4) \quad \mathfrak{F}: (\theta_1, \theta_2) \rightarrow \mathfrak{F}[\theta_1, \theta_2] = (q_i[\theta_1, \theta_2], H_i[\theta_1, \theta_2], G_i[\theta_1, \theta_2], T_i[\theta_1, \theta_2])_{i=1,2},$$

where $q_i[\theta_1, \theta_2]$, $H_i[\theta_1, \theta_2]$, $G_i[\theta_1, \theta_2]$, and $T_i[\theta_1, \theta_2]$ represent, respectively the tax rate on the use of private capital, the public capital investment, the level of public services provided – all in region i – and the equalization transfer to region i , $i = 1, 2$. Here we follow the convention that $T_i[\theta_1, \theta_2] \geq 0$ indicates that region i is a recipient region and $T_i[\theta_1, \theta_2] < 0$ a contributing region in the equalization transfer scheme. To capture the idea that the purpose of an equalization transfer scheme is redistributive, it is required to satisfy the following zero-sum condition:

$$(5) \quad T_1[\theta_1, \theta_2] + T_2[\theta_1, \theta_2] = 0.$$

2.3. The Case for Equalization Transfers

In the absence of equalization transfers, each regional government must finance its own expenditures from the tax revenues it raises within its own borders. A regional government, say regional government i , has three policy choices: (i) q_i , the tax rate it imposes on the use of private capital within its own borders, (ii) H_i , the public capital investment, and (iii) G_i , the provision of public services. However, using the budget constraint for a regional government, one can find G_i once q_i and H_i have been set; that is, a regional government has actually only two policy choices, namely q_i and H_i .

Let (q_i, H_i) be the policy choice of regional government i . Taking (q_i, H_i) as given, the representative firm in region i solves the following profit maximization problem:

$$(6) \quad \max_{(K_i, L_i)} A_i K_i^\alpha L_i^{1-\alpha} H_i^\gamma - (r + q_i)K_i - \omega_i L_i.$$

The following first-order conditions characterize the solution of the preceding profit maximization problem:

$$(7) \quad \alpha A_i K_i^{\alpha-1} L_i^{1-\alpha} H_i^\gamma - (r + q_i) = 0,$$

$$(8) \quad (1 - \alpha)A_i K_i^\alpha L_i^{-\alpha} H_i^\gamma - \omega_i = 0.$$

The labor market clearing condition in region i is

$$(9) \quad L_i = N_i.$$

Solving the system constituted by (7), (8), and (9), we obtain, respectively, the following expressions for the demand for private capital by the representative firm and the equilibrium wage rate in region i .

$$(10) \quad K_i = \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} N_i (r + q_i)^{-\frac{1}{1-\alpha}},$$

and

$$(11) \quad \omega_i = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} (r + q_i)^{-\frac{\alpha}{1-\alpha}}.$$

Note that the demand for private capital in a region declines when the government of this region raises the tax rate that it imposes on the use of private capital within its own borders. On the other hand, the demand for private capital in a region increases with its stock of public capital, H_i , its population, N_i , and its endowment, A_i . As for the equilibrium wage rate in a region, it falls as the tax rate on the use of private capital increases, but rises with the endowment and the stock of public capital in the region.

The private consumption of a resident in region i is equal to her income, which is the sum of her labor income and capital income:

$$(12) \quad c_i = \omega_i + r \bar{k}_i = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} (r + q_i)^{-\frac{\alpha}{1-\alpha}} + r \bar{k}_i.$$

The tax revenues obtained by regional government i , as a function of (q_i, H_i) , is then given by

$$(13) \quad q_i K_i = \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} N_i q_i (r + q_i)^{-\frac{1}{1-\alpha}}.$$

Using its budget constraint, we obtain the following level of public services provided by the government of region i , once it has chosen (q_i, H_i) :

$$(14) \quad G_i = \theta_i(q_i K_i - H_i) = \theta_i \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} N_i q_i (r + q_i)^{-\frac{1}{1-\alpha}} - H_i \right).$$

The utility of a resident in region i – as a function of the chosen policy (q_i, H_i) and the type θ_i of the region – is then given by

$$(15) \quad u_i[q_i, H_i, \theta_i] = \beta_i \theta_i \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} N_i q_i (r + q_i)^{-\frac{1}{1-\alpha}} - H_i \right) \\ + (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} (r + q_i)^{-\frac{\alpha}{1-\alpha}} + r \bar{k}_i.$$

The problem of the government in region i – in the absence of equalization transfers – is to find a policy (q_i, H_i) to maximize u_i , the utility of a resident in this region. That is, the government of region i solves the following maximization problem:

$$(16) \quad \max_{(q_i, H_i)} u_i[q_i, H_i, \theta_i].$$

The solution of (16) is given by

$$(17) \quad q_i^0[\theta_i] = \frac{r(1-\alpha)(-1+N_i\beta_i\theta_i)}{1-\alpha+\alpha N_i\beta_i\theta_i},$$

$$(18) \quad H_i^0[\theta_i] = r^{-\frac{\alpha}{1-\alpha-\gamma}} \alpha^{\frac{\alpha}{1-\alpha-\gamma}} \gamma^{\frac{1-\alpha}{1-\alpha-\gamma}} A_i^{\frac{1}{1-\alpha-\gamma}} N_i^{-\frac{\alpha}{1-\alpha-\gamma}} \beta_i^{-\frac{1}{1-\alpha-\gamma}} \left(\frac{1-\alpha}{\theta_i} + \alpha N_i \beta_i \right)^{\frac{1}{1-\alpha-\gamma}}.$$

The level of public services that emerges from the solution of (16) is given by

$$(19) \quad G_i^0[\theta_i] = r^{-\frac{\alpha}{1-\alpha-\gamma}} \alpha^{\frac{\alpha}{1-\alpha-\gamma}} \gamma^{\frac{\gamma}{1-\alpha-\gamma}} A_i^{\frac{1}{1-\alpha-\gamma}} N_i^{-\frac{\alpha}{1-\alpha-\gamma}} \beta_i^{-\frac{1}{1-\alpha-\gamma}} \left(\alpha N_i \beta_i + \frac{1-\alpha}{\theta_i} \right)^{\frac{\alpha+\gamma}{1-\alpha-\gamma}} \times \\ ((-1 + \alpha)(\alpha + \gamma) - \alpha(-1 + \alpha + \gamma) N_i \beta_i \theta_i).$$

In order for $q_i^0[\theta_i]$ to be positive, it is necessary to assume that

$$(20) \quad N_i \beta_i \theta_i > 1.$$

In order for $G_i^0[\theta_i]$ to be positive, it is necessary to assume that

$$(21) \quad ((-1 + \alpha)(\alpha + \gamma) - \alpha(-1 + \alpha + \gamma) N_i \beta_i \theta_i) > 0, \quad \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i,$$

or equivalently that

$$(22) \quad N_i > \frac{(1-\alpha)(\alpha+\gamma)}{\alpha(1-\alpha-\gamma)\beta_i\theta_i}, \quad \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i.$$

Note that when (22) is satisfied, (20) is automatically satisfied.

LEMMA 1: *In the absence of equalization transfers, the tax rate imposed by the government of a region on the use of private capital within its own borders, $q_i^0[\theta_i]$, does not depend on A_i . However, it is increasing in N_i and θ_i .*

PROOF: We have

$$\frac{\partial q_i^0}{\partial A_i} = 0,$$

$$\frac{\partial q_i^0}{\partial N_i} = \frac{r(1-\alpha)\beta_i\theta_i}{(1-\alpha+\alpha N_i\beta_i\theta_i)^2} > 0,$$

and

$$\frac{\partial q_i^0}{\partial \theta_i} = \frac{r(1-\alpha)N_i\beta_i}{(1-\alpha+\alpha N_i\beta_i\theta_i)^2} > 0.$$

■

LEMMA 2: *In the absence of equalization transfers, the public capital investment made by the government of a region, $H_i^0[\theta_i]$, is increasing in A_i and N_i , but decreasing in θ_i .*

PROOF: We have

$$\frac{\partial \text{Log}[H_i^0]}{\partial A_i} = \frac{1}{(1-\alpha-\gamma)A_i} > 0,$$

$$\frac{\partial \text{Log}[H_i^0]}{\partial N_i} = \frac{(1-\alpha)\alpha(-1+N_i\beta_i\theta_i)}{(1-\alpha-\gamma)N_i(1-\alpha+\alpha N_i\beta_i\theta_i)} > 0,$$

and

$$\frac{\partial \text{Log}[H_i^0]}{\partial \theta_i} = -\frac{1-\alpha}{(1-\alpha-\gamma)\theta_i(1-\alpha+\alpha N_i\beta_i\theta_i)} < 0.$$

■

LEMMA 3: *In the absence of equalization transfers, the private capital input in a region falls as the technology it uses for public service provision become more productive.*

PROOF: As θ_i rises, the public capital investment in a region falls according to Lemma 2 while the tax rate on the use of private capital in the region rises according to Lemma 1, and these two results together imply that less private capital will flow into the region.

■

Using (17) and (18) in (16), we obtain the following expression for the utility of a resident of region i in the absence of equalization transfers:

$$(23) \quad u_i^0[\theta_i] = r\bar{k}_i + r^{-\frac{\alpha}{1-\alpha-\gamma}} \alpha^{\frac{\alpha}{1-\alpha-\gamma}} (1-\alpha-\gamma) \gamma^{\frac{\gamma}{1-\alpha-\gamma}} A_i^{\frac{1}{1-\alpha-\gamma}} \beta_i^{1-\frac{1}{1-\alpha-\gamma}} \kappa_i[\theta_i],$$

where we have let

$$(24) \quad \kappa_i[\theta_i] = N_i^{-\frac{\alpha}{1-\alpha-\gamma}} \theta_i^{1-\frac{1}{1-\alpha-\gamma}} (1-\alpha+\alpha N_i \beta_i \theta_i)^{\frac{1}{1-\alpha-\gamma}}.$$

Obviously, $u_i^0[\theta_i]$ rises with θ_i , as can be seen from the objective function in (16). It is also clear from (23) that $u_i^0[\theta_i]$ rises with A_i . As for the impact of N_i on $u_i^0[\theta_i]$, we have

$$(25) \quad \frac{\partial \text{Log}[\kappa_i[\theta_i]]}{\partial N_i} = \frac{(1-\alpha)\alpha(-1+N_i \beta_i \theta_i)}{(1-\alpha-\gamma)N_i(1-\alpha+\alpha N_i \beta_i \theta_i)} > 0,$$

due to (20); that is, $u_i^0[\theta_i]$ also rises with N_i .

LEMMA 4: *In the absence of equalization transfers, the level of public services provided in a region, say i , rises with (i) its endowment and (ii) its population. Furthermore, when the technology for public service provision in the region becomes more efficient, the input used in providing public services and a fortiori the level of public services provided will rise with θ_i .*

PROOF: To establish (i), note that

$$\frac{\partial \text{Log}[G_i^0]}{\partial A_i} = \frac{1}{(1-\alpha-\gamma)A_i} > 0.$$

To establish (ii), we claim

$$\frac{\partial \text{Log}[G_i^0]}{\partial N_i} = \frac{(-1+\alpha)\alpha((-1+\alpha)(\alpha+\gamma)+N_i\beta_i\theta_i(-(-1+\alpha)(-1+2\alpha+2\gamma)+\alpha(-1+\alpha+\gamma)N_i\beta_i\theta_i))}{(-1+\alpha+\gamma)N_i(1-\alpha+\alpha N_i\beta_i\theta_i)(-(-1+\alpha)(\alpha+\gamma)+\alpha(-1+\alpha+\gamma)N_i\beta_i\theta_i)} > 0.$$

The claim is supported by exploiting the symbolic computational capability of *Mathematica* to carry out the required brute-force computations. The results are presented in Appendix A.

To prove that the input used in the provision of public services in a region rises with the productivity of the public service production function and a fortiori the level of public services provided in region i rises with θ_i . Indeed, the public capital investment in this region falls – according to Lemma 2 – and the private capital flowing into this region declines – according to Lemma 3 – as θ_i rises, and this means that at the optimum the equilibrium wage rate in region i decreases when its technology for public services becomes more productive. The fall in wages implies a fall in private consumption. If the input used in the provision of public services declines, then repeating at θ_i the policy carried out in a left neighborhood of θ_i , say at $\theta_i - \epsilon$, will yield a higher private consumption and a higher input for public service provision. Together, these two results imply a level of utility higher than that at the optimum with θ_i as the type of region i , a contradiction. ■

We note in passing that as θ_i rises, the equilibrium wage rate in region i declines because the public and private capital inputs both decline according to Lemmas 2 and 3, and this means that private consumption is displaced by the consumption of public services. This result follows from the hypothesis that preferences are linear in the consumption of the private and public goods, i.e., the two goods are perfect substitutes.

The comparative static analysis of u_i^0 and G_i^0 suggest that the disparities in welfare and in the levels of public services provided in the two regions lie in the disparities in their endowments, population sizes, and their technologies for producing public services. Thus, resources must be transferred from the richer to the poorer region to close the gap in welfare and public service provisions between the two regions, and this is the case for equalization transfers.

2.4. Equalization Transfers under Asymmetric Information

To introduce the asymmetry that lies at the heart of the argument for equalization transfers, we shall from now on assume that one region, say region i , has a lower endowment, a lower population, and a less productive technology for public service provision than the other region, say region j , and these disparities suggest that resources from region j flow to region i as equalization transfers. An equalization transfer scheme is now simply a map

$$\mathfrak{F}: \theta_i \rightarrow \mathfrak{F}[\theta_i] = (q_i[\theta_i], H_i[\theta_i], G_i[\theta_i], T_i[\theta_i]),$$

where $q_i[\theta_i]$, $H_i[\theta_i]$, $G_i[\theta_i]$, and $T_i[\theta_i]$ represent, respectively, the tax rate on the use of private capital, the public capital investment, the level of public services – all in region i – and the equalization transfer received by region i . The equalization transfer $T_i[\theta_i]$ also represents the resource flow from region j to region i that is dictated by $\mathfrak{F}$.

The mechanism for equalization transfers varies from one federation to another. In Canada – as already mentioned in the introduction – equalizations transfers are computed using a preset formula that uses five tax bases as inputs. Whenever the formula is revised, and this occurs every five years, the provinces present their own data and argue strenuously for favorable equalization transfers. In the Commonwealth of Australia, the states put forth data on their own needs and disabilities before the CGC. In Germany, interstate equalization transfers are calculated according to a preset formula using as inputs Land taxes and the Land share of revenues from income and corporation taxes collected by the revenues authorities within each Länder. Given the asymmetric information, each mechanism induces a game with incomplete information, and a Bayesian Nash equilibrium for such a game that implements a social choice function yields an equalization transfer formula.

Because there is a multitude of mechanisms that one can adopt, there is no limit to our imagination in designing an equalization transfer scheme. Fortunately, there is an important result known as the *revelation principle*,⁴⁸ which asserts that if a mechanism implements a social choice function $\mathfrak{F}$ in Bayesian Nash equilibrium, then it is also truthfully implemented in

⁴⁸ See Chapter 23 in MasColell et al. (1995).

Bayesian Nash equilibrium. The revelation principle thus allows us to restrict our search for an equalization transfer scheme to the direct revelation mechanism. Under the direct revelation mechanism, the federal government asks each regional government to reveal its type, namely the cost of producing the non-productive public good in its region, and then uses the information thus obtained to implement the social choice dictated by $\mathfrak{F}$.

2.4.1. The Incentive Compatible Condition

Under the direct revelation mechanism, a strategy for regional government i is a map $s_i: \theta_i \rightarrow s_i[\theta_i]$, where $s_i[\theta_i]$ the type that the government of region i reveals to the federal government when its type is θ_i . If $s_i[\theta_i] = \theta_i$ for all θ_i in $[\underline{\theta}_i, \bar{\theta}_i]$, then the government of region i tells the truth. Otherwise, the response of the government of region i is not truthful. If the social choice function $\mathfrak{F}$ is not well structured, then it might not be in the interest of a region to reveal its type truthfully. The social choice function $\mathfrak{F}$ is said to be *Bayesian incentive compatible* if the strategy profile under which regional government i reveals its type truthfully constitutes a Bayesian Nash equilibrium for the game with incomplete information induced by the direct revelation mechanism.

Let s_i be a strategy for the government of region i . Let θ_i be the type of region i , and $\hat{\theta}_i = s_i[\theta_i]$ be the type declared by the government of region i under s_i . The equalization transfer scheme $\mathfrak{F}$ will then implement the following social choice:

$$(26) \quad \mathfrak{F}[\hat{\theta}_i] = (q_i[\hat{\theta}_i], H_i[\hat{\theta}_i], G_i[\hat{\theta}_i], T_i[\hat{\theta}_i]).$$

Given $(\hat{q}_i, \hat{H}_i) = (q_i[\hat{\theta}_i], H_i[\hat{\theta}_i])$, the demand for private capital and the equilibrium wage rate in region i are given, respectively, by

$$(27) \quad \hat{K}_i = \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \hat{H}_i^{\frac{\gamma}{1-\alpha}} N_i (r + \hat{q}_i)^{-\frac{1}{1-\alpha}},$$

and

$$(28) \quad \hat{\omega}_i = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \hat{H}_i^{\frac{\gamma}{1-\alpha}} (r + \hat{q}_i)^{-\frac{\alpha}{1-\alpha}}.$$

The total revenues – tax revenues from the use of private capital plus the equalization transfer $\hat{T}_i = T_i[\hat{\theta}_i]$ – for regional government i are then given by

$$(29) \quad \hat{q}_i \hat{K}_i + \hat{T}_i = \hat{q}_i \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \hat{H}_i^{\frac{\gamma}{1-\alpha}} N_i (r + \hat{q}_i)^{-\frac{1}{1-\alpha}} + \hat{T}_i.$$

The level of public services provided in region i is then given by

$$(30) \quad \hat{G}_i = \theta_i (q_i \hat{K}_i + \hat{T}_i - \hat{H}_i) = \theta_i \left(\hat{q}_i \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \hat{H}_i^{\frac{\gamma}{1-\alpha}} N_i (r + \hat{q}_i)^{-\frac{1}{1-\alpha}} + \hat{T}_i - \hat{H}_i \right).$$

The private consumption for a resident of region i is

$$(31) \quad \hat{c}_i = r \bar{k}_i + \hat{\omega}_i = r \bar{k}_i + (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \hat{H}_i^{\frac{\gamma}{1-\alpha}} (r + \hat{q}_i)^{-\frac{\alpha}{1-\alpha}}.$$

Thus, under the equalization transfer scheme $\mathfrak{F}$, the utility enjoyed by a resident of region i , as a function of $\hat{\theta}_i$, the type announced by region i , and θ_i , the true type of this region, is given by

$$(32) \quad \begin{aligned} v_i[\hat{\theta}_i|\theta_i] &= \beta_i \hat{G}_i + \hat{c}_i \\ &= \beta_i \theta_i \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} N_i q_i[\hat{\theta}_i] (r + q_i[\hat{\theta}_i])^{-\frac{1}{1-\alpha}} + T_i[\hat{\theta}_i] - H_i[\hat{\theta}_i] \right) \\ &\quad + (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} + r \bar{k}_i. \end{aligned}$$

The utility for a resident of region i , when the type of this region is θ_i , and when the governments of both regions tell the truth is then given by

$$(33) \quad \begin{aligned} v_i[\theta_i|\theta_i] &= r \bar{k}_i + (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}} \\ &\quad + \beta_i \theta_i \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i[\theta_i] (r + q_i[\theta_i])^{-\frac{1}{1-\alpha}} + T_i[\theta_i] - H_i[\theta_i] \right). \end{aligned}$$

In light of (30), which allows us to compute the level of public services, from $q_i[\theta_i]$, $H_i[\theta_i]$, and $T_i[\theta_i]$, it is redundant to include $G_i[\theta_i]$ in the specification of an equalization transfer scheme. Thus, from now on we shall define an equalization transfer scheme simply as a map

$$\mathfrak{F}: \theta_i \rightarrow \mathfrak{F}[\theta_i] = (q_i[\theta_i], H_i[\theta_i], T_i[\theta_i]).$$

PROPOSITION 1: *Let*

$$\mathfrak{F}: \theta_i \rightarrow \mathfrak{F}[\theta_i] = (q_i[\theta_i], H_i[\theta_i], T_i[\theta_i])$$

be an equalization transfer scheme. Next, let $v_i[\hat{\theta}_i|\theta_i]$ be the utility of a resident of region i – as defined by (32) – when $\mathfrak{F}$ is implemented, given that (i) θ_i is the type of this region, (ii) the government of this region declares its type to be $\hat{\theta}_i$. The following conditions are necessary and sufficient for $\mathfrak{F}$ to be Bayesian incentive compatible:

(a) *The curve*

$$\theta_i \rightarrow (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}(r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}, \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i,$$

is decreasing.

(b) *For each $\theta_i, \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i$, we have*

$$\frac{v_i[\theta_i|\theta_i] - r\bar{k}_i}{\theta_i} = \frac{v_i[\underline{\theta}_i|\underline{\theta}_i] - r\bar{k}_i}{\underline{\theta}_i} - \int_{\underline{\theta}_i}^{\theta_i} \frac{1}{s^2} (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[s])^{\frac{\gamma}{1-\alpha}}(r + q_i[s])^{-\frac{\alpha}{1-\alpha}} ds.$$

PROOF: See Appendix B.

Observe that the image of the map in (a) of Proposition 1 is nothing other than the wage rate induced by the equalization transfer scheme $\mathfrak{F}$. Condition (a) asserts that as θ_i rises, the wage rate declines.

2.4.2. The Formal Statement of the Equalization Transfer Problem.

Let

$$\mathfrak{F}: \theta_i \rightarrow \mathfrak{F}[\theta_i] = (q_i[\theta_i], H_i[\theta_i], T_i[\theta_i])$$

be an equalization transfer scheme that is Bayesian incentive compatible. If θ_i is the type of region i , then the utility obtained by a resident of region i under the equalization transfer scheme $\mathfrak{F}$ is given by (33), and solving (33) for $T_i[\theta_i]$, we obtain

$$(34) \quad T_i[\theta_i] = \frac{v_i[\theta_i|\theta_i] - r\bar{k}_i}{\beta_i\theta_i} - \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}(r+q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i\theta_i} \\ - \left(\alpha^{\frac{1}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}N_iq_i[\theta_i](r+q_i[\theta_i])^{-\frac{1}{1-\alpha}} - H_i[\theta_i] \right).$$

Using (b) of Proposition 1 in (34), we obtain

$$(35) \quad T_i[\theta_i] = \frac{v_i[\theta_i|\theta_i] - r\bar{k}_i}{\beta_i\theta_i} - \int_{\underline{\theta}_i}^{\theta_i} \frac{1}{\beta_i s^2} (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[s])^{\frac{\gamma}{1-\alpha}}(r+q_i[s])^{-\frac{\alpha}{1-\alpha}} ds \\ - \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}(r+q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i\theta_i} \\ - \left(\alpha^{\frac{1}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}N_iq_i[\theta_i](r+q_i[\theta_i])^{-\frac{1}{1-\alpha}} - H_i[\theta_i] \right).$$

The expected value of the equalization transfer received by region i under $\mathfrak{F}$ is then given by

$$(36) \quad \int_{\underline{\theta}_i}^{\bar{\theta}_i} T_i[\theta_i] \phi_i[\theta_i] d\theta_i = \frac{v_i[\theta_i|\theta_i] - r\bar{k}_i}{\beta_i\theta_i} \\ - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \left(\int_{\underline{\theta}_i}^{\theta_i} \frac{1}{\beta_i s^2} (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[s])^{\frac{\gamma}{1-\alpha}}(r+q_i[s])^{-\frac{\alpha}{1-\alpha}} ds \right) \phi_i[\theta_i] d\theta_i \\ - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}(r+q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i\theta_i} \phi_i[\theta_i] d\theta_i \\ - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \left(\alpha^{\frac{1}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}N_iq_i[\theta_i](r+q_i[\theta_i])^{-\frac{1}{1-\alpha}} - H_i[\theta_i] \right) \phi_i[\theta_i] d\theta_i.$$

Using integration by parts to evaluate the double integral in (36), and then using the result thus obtained, we can rewrite (36) as

$$\begin{aligned}
(37) \quad & \int_{\underline{\theta}_i}^{\bar{\theta}_i} T_i[\theta_i] \phi_i[\theta_i] d\theta_i = \frac{v_i[\underline{\theta}_i|\underline{\theta}_i] - r\bar{k}_i}{\beta_i \underline{\theta}_i} \\
& - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \frac{1}{\beta_i \theta_i^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}} (1 - \Phi_i[\theta_i]) d\theta_i \\
& - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \frac{(1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i \theta_i} \phi_i[\theta_i] d\theta_i \\
& - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i[\theta_i] (r + q_i[\theta_i])^{-\frac{1}{1-\alpha}} - H_i[\theta_i] \right) \phi_i[\theta_i] d\theta_i \\
& = \frac{v_i[\underline{\theta}_i|\underline{\theta}_i] - r\bar{k}_i}{\beta_i \underline{\theta}_i} \\
& - \int_{\underline{\theta}_i}^{\bar{\theta}_i} \left((1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}} \left(\frac{1 - \Phi_i[\theta_i]}{\theta_i \phi_i[\theta_i]} + 1 \right) \frac{1}{\beta_i \theta_i} \right. \\
& \quad \left. + \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i[\theta_i] (r + q_i[\theta_i])^{-\frac{1}{1-\alpha}} - H_i[\theta_i] \right) \right) \phi_i[\theta_i] d\theta_i.
\end{aligned}$$

We assume that the designer of the equalization transfer scheme wishes to give a resident of region i at least a level of utility $\bar{u}$ judged to be equitable without imposing an excessive burden on region j whose resources are used to finance the equalization scheme. Thus, the problem of the designer of an equalization transfer program is to find a Bayesian incentive compatible equalization transfer scheme $\mathfrak{F}$ to minimize (37) subject to the following equity constraint:

$$(38) \quad v_i[\theta_i|\theta_i] \geq \bar{u}, \forall \theta_i \in [\underline{\theta}_i, \bar{\theta}_i].$$

To obtain sharper results, we shall assume that $u_i^0[\bar{\theta}_i] < \bar{u}$. That is, in the absence of equalization transfers, the utility of a resident of region i – even when its technology for public service provision is at its most productive level – falls short of the level judged equitable by the designer of the equalization transfer scheme. In solving the cost minimization problem constituted by (37) and (38), we search for an equalization transfer scheme that is continuous.

3. THE BENCHMARK EQUILIBRIUM: EQUALIZATION TRANSFERS WITH COMPLETE INFORMATION

Suppose that information is complete; that is, the federal government can observe θ_i , the productivity of region i 's technology for public service provision. If (q_i, H_i, T_i) is the social choice that the federal government chooses to implement, then the utility obtained by a resident of region i – according to (33) – is given by

$$\begin{aligned}
 (39) \quad & r\bar{k}_i + (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}H_i^{\frac{\gamma}{1-\alpha}}(r + q_i)^{-\frac{\alpha}{1-\alpha}} \\
 & + \beta_i\theta_i \left(\alpha^{\frac{1}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}H_i^{\frac{\gamma}{1-\alpha}}N_iq_i(r + q_i)^{-\frac{1}{1-\alpha}} + T_i - H_i \right) \\
 & = u_i[q_i, H_i, \theta_i] + \beta_i\theta_iT_i.
 \end{aligned}$$

The problem faced by the designer of the equalization transfer scheme under complete information is to

$$(40) \quad \min_{(q_i, H_i, T_i)} T_i$$

subject to the equity constraint that (39) is greater than or equal to $\bar{u}$, i.e.,

$$(41) \quad u_i[q_i, H_i, \theta_i] + \beta_i\theta_iT_i \geq \bar{u}.$$

At the optimum, the equity constraint (41) must be binding, and we have

$$(42) \quad T_i = \frac{\bar{u} - u_i[q_i, H_i, \theta_i]}{\beta_i\theta_i}.$$

The minimum equalization transfer that satisfies the equity constraint is then obtained by choosing (q_i, H_i) to maximize $u_i[q_i, H_i, \theta_i]$, and this welfare maximization – stated as (16) – is the welfare maximization solved by regional government i in the absence of equalization transfers. Thus, if we let $(q_i^\#[\theta_i], H_i^\#[\theta_i], T_i^\#[\theta_i])$ be solution of the constrained minimization problem constituted by (40) and (41), then

$$(43) \quad q_i^\#[\theta_i] = q_i^0[\theta_i],$$

$$(44) \quad H_i^\#[\theta_i] = H_i^0[\theta_i],$$

and

$$(45) \quad T_i^\#[\theta_i] = \frac{\bar{u} - u_i^0[\theta_i]}{\beta_i \theta_i}.$$

The following proposition is immediate.

PROPOSITION 2: *If the productivity of region i 's technology for public service provision is observable, then the equalization transfer is given by (45), which is just enough to raise the utility of an individual in this region to $\bar{u}$, the minimum level judged equitable by the designer of the equalization scheme. The entire equalization transfer is added on top of the input that the government of this region allocates to the provision of public service to produce this non-productive public good. Furthermore, the equalization transfer falls as the technology for public service provision becomes more efficient.*

4. EQUALIZATION TRANSFERS UNDER ASYMMETRIC INFORMATION

To solve the problem for the designer of an equalization transfer scheme with asymmetric information, note that the equity constraint (38) at $\theta_i = \underline{\theta}_i$ is

$$(46) \quad v_i[\underline{\theta}_i | \underline{\theta}_i] \geq \bar{u}.$$

We claim that if (46) holds, then (38) must hold. Indeed, according to (b) of Proposition 1, we have

$$(47) \quad \frac{d}{d\theta_i} \left[\frac{v_i[\theta_i | \theta_i] - r\bar{k}_i}{\theta_i} \right] = -\frac{1}{\theta_i^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}},$$

i.e.,

$$(48) \quad \frac{\theta_i \frac{d}{d\theta_i} [v_i[\theta_i | \theta_i] - r\bar{k}_i] - (v_i[\theta_i | \theta_i] - r\bar{k}_i)}{\theta_i^2} = -\frac{1}{\theta_i^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}},$$

from which we obtain

$$(49) \quad \theta_i \frac{d}{d\theta_i} [v_i[\theta_i|\theta_i] - r\bar{k}_i] = v_i[\theta_i|\theta_i] - r\bar{k}_i - (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}(r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}} \\ = \beta_i\theta_i \left(\alpha^{\frac{1}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i[\theta_i])^{\frac{\gamma}{1-\alpha}}N_iq_i[\theta_i](r + q_i[\theta_i])^{-\frac{1}{1-\alpha}} + T_i[\theta_i] - H_i[\theta_i] \right) > 0.$$

Note that the second equality in (49) has been obtained by using (33). It follows from (49) that $v_i[\theta_i|\theta_i]$ rises with θ_i , and, therefore, the equity constraint (38) will be satisfied if (46) is satisfied. Thus, in solving the equalization transfer designer's problem we can clearly set

$$(50) \quad v_i[\underline{\theta}_i|\underline{\theta}_i] = \bar{u}$$

Once we have chosen (50), the constrained minimization problem constituted by (37) and (38) becomes that of maximizing the integral on the last line of (37), which in turn is reduced to the following maximization problem:

$$(51) \quad \max_{(q_i, H_i)} \left(\left(\frac{(1 - \Phi_i[\theta_i])}{\theta_i \phi_i[\theta_i]} + 1 \right) \frac{1}{\beta_i \theta_i} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i)^{\frac{\gamma}{1-\alpha}} (r + q_i)^{-\frac{\alpha}{1-\alpha}} \right. \\ \left. + \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i)^{\frac{\gamma}{1-\alpha}} N_i q_i (r + q_i)^{-\frac{1}{1-\alpha}} - H_i \right) \right).$$

In what follows, we shall let

$$(52) \quad \mu_i[\theta_i] = \left(\frac{(1 - \Phi_i[\theta_i])}{\theta_i \phi_i[\theta_i]} + 1 \right) \frac{1}{\beta_i \theta_i},$$

and assume that $\mu_i[\theta_i], \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i$, is a decreasing function of θ_i .

If we imagine – from the perspective of reliability theory – that $\Phi_i[\theta_i]$ represents the probability a piece of machinery fails within the time interval $[\underline{\theta}_i, \theta_i]$, then $\frac{\phi_i[\theta_i]}{1 - \Phi_i[\theta_i]}$, known as the failure rate, is the rate the piece of machinery will fail at time θ_i , given that it has not failed before. A common presumption is that the failure rate increases with time; that is, if the piece of machinery has not failed, then it is more likely to fail as time goes on. An increasing failure rate implies that $\frac{1 - \Phi_i[\theta_i]}{\phi_i[\theta_i]}$ is a decreasing function of θ_i , and (52) will be satisfied. If the distribution function

$\Phi_i[\theta_i]$ is log-concave, i.e., if $\theta_i \rightarrow \text{Log}[\Phi_i[\theta_i]]$ is concave, then $\frac{\phi_i[\theta_i]}{1-\Phi_i[\theta_i]}$ will be increasing in θ_i .⁴⁹ In auction theory, it is often assumed that $\theta_i - \frac{1-\Phi_i[\theta_i]}{\phi_i[\theta_i]}$ is an increasing function of θ_i , and this assumption is certainly satisfied if the failure rate is increasing with θ_i .⁵⁰

The solution of (51) is given by

$$(53) \quad q_i^*[\theta_i] = r \left(\frac{N_i}{\alpha N_i + (1-\alpha)\mu_i[\theta_i]} - 1 \right),$$

$$(54) \quad H_i^*[\theta_i] = r^{\frac{\alpha}{-1+\alpha+\gamma}} \alpha^{\frac{\alpha}{-1+\alpha+\gamma}} \gamma^{\frac{-1+\alpha}{-1+\alpha+\gamma}} A_i^{\frac{1}{-1+\alpha+\gamma}} N_i^{\frac{\alpha}{-1+\alpha+\gamma}} (\alpha N_i + (1-\alpha)\mu_i[\theta_i])^{\frac{1}{1-\alpha-\gamma}}.$$

The tax rate on the use of private capital $q_i^*[\theta_i]$ and the public capital investment $H_i^*[\theta_i]$ – as defined by (53) and (54) – induce the following equilibrium wage rate in region i :

$$(55) \quad \omega_i^*[\theta_i] = (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^*[\theta_i]^{\frac{\gamma}{1-\alpha}} (r + q_i^*[\theta_i])^{-\frac{\alpha}{1-\alpha}}.$$

LEMMA 5: *The wage rate $\omega_i^*[\theta_i]$, as defined by (55), is a decreasing function of θ_i .*

PROOF: We have

$$\frac{d\text{Log}[\omega_i^*[\theta_i]]}{d\theta_i} = \frac{(1-\alpha)(\alpha+\gamma)\mu_i'[\theta_i]}{(1-\alpha-\gamma)(\alpha N_i + (1-\alpha)\mu_i[\theta_i])} < 0.$$

■

Now let $v_i^*[\theta_i|\theta_i]$ be defined by

$$(56) \quad \frac{v_i^*[\theta_i|\theta_i] - r\bar{k}_i}{\theta_i} = \frac{\bar{u} - r\bar{k}_i}{\theta_i} - \int_{\theta_i}^{\theta_i} \frac{1}{s^2} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[s])^{\frac{\gamma}{1-\alpha}} (r + q_i^*[s])^{-\frac{\alpha}{1-\alpha}} ds.$$

Then $v_i^*[\theta_i|\theta_i]$ satisfies (b) of Proposition 1.

⁴⁹ See Bergstrom, T and Bagnoli, M (2005).

⁵⁰ See MasColell et al. (1995), p.903.

Next, let

$$\begin{aligned}
(57) \quad T_i^*[\theta_i] = & \frac{v_i^*[\underline{\theta}_i|\underline{\theta}_i] - r\bar{k}_i}{\beta_i \underline{\theta}_i} - \int_{\underline{\theta}_i}^{\theta_i} \frac{1}{\beta_i s^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[s])^{\frac{\gamma}{1-\alpha}} (r + q_i^*[s])^{-\frac{\alpha}{1-\alpha}} ds \\
& - \frac{(1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i^*[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i \theta_i} \\
& - \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i^*[\theta_i] (r + q_i^*[\theta_i])^{-\frac{1}{1-\alpha}} - H_i^*[\theta_i] \right).
\end{aligned}$$

PROPOSITION 3: *The equalization transfer scheme*

$$\mathfrak{F}^*: \theta_i \rightarrow \mathfrak{F}^*[\theta_i] = (q_i^*[\theta_i], H_i^*[\theta_i], T_i^*[\theta_i]), \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i,$$

where $q_i^*[\theta_i]$, $H_i^*[\theta_i]$, and $T_i^*[\theta_i]$ are defined, respectively, by (53), (54), and (57), is the solution to the problem faced by the designer of the equalization transfer scheme.

PROOF: The equalization transfer scheme satisfies both (a) and (b) of Proposition 1, and is, therefore, Bayesian incentive compatible. By its construction, it minimizes the expected equalization transfer received by region i under the equity constraint (38). ■

Having computed the optimal equalization transfer scheme under asymmetric information, we now present some of its properties.

PROPOSITION 4: *As θ_i rises from $\underline{\theta}_i$ to $\bar{\theta}_i$, the utility of an individual in region i rises with θ_i , and the ascent begins at $\bar{u}$, the minimum level of utility judged to be equitable.*

PROOF: According to (56), we have

$$v_i[\underline{\theta}_i|\underline{\theta}_i] = \bar{u}.$$

According to the definition of $v_i^*[\theta_i|\theta_i]$ and (49), $v_i^*[\theta_i|\theta_i]$ rises with θ_i . ■

In contrast with the case of complete information in which the inter-government grant is just sufficient to maintain the utility of an individual in region i at the equitable level $\bar{u}$, regardless of

the type of this region, the utility of an individual in this region in the case of asymmetric information exceeds the equitable level and rises as the technology for delivering public services in this region is more productive. The gap $(v_i^*[\theta_i|\theta_i] - \bar{u}) > 0$ represents the information rent that the region must obtain in order for its government to reveal its type truthfully.

PROPOSITION 5: As θ_i rises from $\underline{\theta}_i$ to $\bar{\theta}_i$, the tax rate on private capital, $q_i^*[\theta_i]$, rises with θ_i , but the public capital investment, $H_i^*[\theta_i]$, and the equalization transfer, $T_i^*[\theta_i]$, both decline. Furthermore, as θ_i rises, the public services displace private consumption and rise with θ_i .

PROOF: As θ_i rises, $\mu_i[\theta_i]$ declines, and it is clear from (53) and (54) that $q_i^*[\theta_i]$ rises and $H_i^*[\theta_i]$ fall. As for $T_i^*[\theta_i]$, its derivative with respect to θ_i is

$$\begin{aligned} \frac{d}{d\theta_i} T_i^*[\theta_i] &= -\frac{1}{\beta_i \theta_i} \frac{d}{d\theta_i} \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i^*[\theta_i])^{-\frac{\alpha}{1-\alpha}} \right] \\ &\quad - \frac{d}{d\theta_i} \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i^*[\theta_i] (r + q_i^*[\theta_i])^{-\frac{1}{1-\alpha}} - H_i^*[\theta_i] \right). \\ &= \frac{1}{(1-\alpha-\gamma)\beta_i \theta_i^2 \phi_i[\theta_i]} r^{\frac{\alpha}{-1+\alpha+\gamma}} (1-\alpha)^2 \alpha^{\frac{\alpha}{-1+\alpha+\gamma}} \gamma^{\frac{\gamma}{-1+\alpha+\gamma}} (\alpha + \gamma) \times \\ &\quad A_i^{\frac{1}{-1+\alpha+\gamma}} N_i^{\frac{\alpha}{-1+\alpha+\gamma}} (\alpha N_i + (1-\alpha)\mu_i[\theta_i])^{-2-\frac{1}{-1+\alpha+\gamma}} (1 - \phi_i[\theta_i]) \mu_i'[\theta_i] < 0. \end{aligned}$$

To prove the last statement of Proposition 4, recall that the equilibrium wage rate and a fortiori private consumption falls as θ_i rises. The decline in private consumption and the rise in $v_i^*[\theta_i|\theta_i]$ imply a rise in the consumption of public services. ■

According to Proposition 5, the behavior of the equalization transfer with asymmetric information is qualitatively similar to that with complete information.

LEMMA 6: We have

$$(58) \quad \frac{(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i^*[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i \theta_i}$$

$$\begin{aligned}
& + \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i^*[\theta_i] (r + q_i^*[\theta_i])^{-\frac{1}{1-\alpha}} - H_i^*[\theta_i] \right) \\
& \leq \max_{(q_i, H_i)} \left(\left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} N_i q_i (r + q_i)^{-\frac{1}{1-\alpha}} - H_i \right) \right. \\
& \quad \left. + \frac{(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} H_i^{\frac{\gamma}{1-\alpha}} (r + q_i)^{-\frac{\alpha}{1-\alpha}}}{\beta_i \theta_i} \right) = \frac{u_i^0[\theta_i] - r \bar{k}_i}{\beta_i \theta_i},
\end{aligned}$$

with equality holding when $\theta_i = \bar{\theta}_i$, and strict inequality holding otherwise.

PROOF: First, note that the inequality in (58) is obvious. Second, observe that the maximization problem in (58) is nothing other than the welfare maximization problem (16) expressed in an alternative form, and the equality on the last line of (58) follows from the solution of (16). Third, note that when $\theta_i = \bar{\theta}_i$, the maximization problem (51) is reduced to the maximization problem in (58), and the inequality in (58) becomes an equality. Otherwise, for $\theta_i \neq \bar{\theta}_i$, (51) is distinct from the maximization problem in (58), and the inequality in (58) is strict. ■

PROPOSITION 6: For all $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$, we have $T_i^*[\theta_i] > T_i^\#[\theta_i]$. That is, the equalization transfer received by region i is higher under asymmetric information than under complete information.

PROOF: According to (56) and (57), we have

$$(59) \quad T_i^*[\theta_i] = \frac{v_i^*[\theta_i | \theta_i] - r \bar{k}_i}{\beta_i \theta_i} - \left(\frac{(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i^*[\theta_i])^{-\frac{\alpha}{1-\alpha}}}{\beta_i \theta_i} + \left(\alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i^*[\theta_i])^{\frac{\gamma}{1-\alpha}} N_i q_i^*[\theta_i] (r + q_i^*[\theta_i])^{-\frac{1}{1-\alpha}} - H_i^*[\theta_i] \right) \right).$$

Because $v_i^*[\theta_i | \theta_i] \geq \bar{u}$, with strict inequality holding if $\theta_i > \underline{\theta}_i$, and because according to Lemma 6 the expression between the pair of grand parentheses in (59) is strictly less than $\frac{u_i^0[\theta_i] - r \bar{k}_i}{\beta_i \theta_i}$, the right-hand side of (59) must be greater than

$$(60) \quad \frac{\bar{u} - r \bar{k}_i}{\beta_i \theta_i} - \frac{u_i^0[\theta_i] - r \bar{k}_i}{\beta_i \theta_i} = \frac{\bar{u} - u_i^0[\theta_i]}{\beta_i \theta_i} = T_i^\#[\theta_i],$$

for all $\theta_i \in [\underline{\theta}_j, \bar{\theta}_i]$. ■

According to Proposition 6, it is the equalization transfer that provides region i with the needed information rent to induce thei economic agent into revealing its type truthfully.

PROPOSITION 7: *For all $\theta_i \in [\underline{\theta}_j, \bar{\theta}_i]$, we have $q_i^*[\theta_i] < q_i^\#[\theta_i]$, but $H_i^*[\theta_i] > H_i^\#[\theta_i]$. That is, in region i the tax rate on private capital is lower, but the public capital investment is higher under incomplete information than under complete information.*

PROOF: Let η be a positive parameter, and consider the following variant of (51):

$$(61) \quad \max_{(q_i, H_i)} \left(\eta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i)^{\frac{\gamma}{1-\alpha}}(r+q_i)^{-\frac{\alpha}{1-\alpha}} + \left(\alpha^{\frac{1}{1-\alpha}}A_i^{\frac{1}{1-\alpha}}(H_i)^{\frac{\gamma}{1-\alpha}}N_iq_i(r+q_i)^{-\frac{1}{1-\alpha}} - H_i \right) \right).$$

The solution of (61) is given by

$$(62) \quad q_i^+[\eta] = r \left(-1 + \frac{N_i}{(1-\alpha)\eta + \alpha N_i} \right),$$

and

$$(63) \quad H_i^+[\eta] = r^{\frac{\alpha}{-1+\alpha+\gamma}} \alpha^{\frac{\alpha}{1-\alpha-\gamma}} \gamma^{\frac{-1+\alpha}{-1+\alpha+\gamma}} A_i^{\frac{1}{1-\alpha-\gamma}} N_i^{\frac{\alpha}{-1+\alpha+\gamma}} ((1-\alpha)\eta + \alpha N_i)^{\frac{1}{1-\alpha-\gamma}}.$$

Clearly, $q_i^+[\eta]$ is decreasing, but $H_i^+[\eta]$ is increasing in η . Furthermore, note that

$$q_i^+ \left[\frac{1}{\beta_i \theta_i} \right] = q_i^0[\theta_i], H_i^+ \left[\frac{1}{\beta_i \theta_i} \right] = H_i^0[\theta_i]$$

and

$$q_i^+ \left[\left(\frac{1-\Phi_i[\theta_i]}{\theta_i \phi_i[\theta_i]} + 1 \right) \frac{1}{\beta_i \theta_i} \right] = q_i^*[\theta_i], H_i^+ \left[\left(\frac{1-\Phi_i[\theta_i]}{\theta_i \phi_i[\theta_i]} + 1 \right) \frac{1}{\beta_i \theta_i} \right] = H_i^*[\theta_i].$$

Hence

$$q_i^*[\theta_i] < q_i^0[\theta_i] = q_i^\#[\theta_i], H_i^*[\theta_i] > H_i^0[\theta_i] = H_i^\#[\theta_i].$$

■

PROPOSITION 8: For all $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$, the consumption of the private good in region i is higher under incomplete information than under complete information. As for the consumption of public services, it is lower when the technology for public service delivery is least productive and higher when the technology for public service delivery is most productive under incomplete information than under complete information.

PROOF: Let η be a positive parameter, and suppose that $q_i^+[\eta]$ is the tax rate on private capital and $H_i^+[\eta]$ is the public capital investment – both in region i . This policy choice induces the following equilibrium wage rate in region i :

$$(64) \quad \omega_i^+[\eta] = r^{-\frac{\alpha\gamma}{(-1+\alpha)(-1+\alpha+\gamma)}} (1-\alpha)\alpha^{-\frac{\alpha}{-1+\alpha+\gamma}} \gamma^{-\frac{\gamma}{-1+\alpha+\gamma}} A_i^{-\frac{1}{-1+\alpha+\gamma}} N_i^{-\frac{\alpha\gamma}{(-1+\alpha)(-1+\alpha+\gamma)}} \times$$

$$\left(\frac{rN_i}{\eta(1-\alpha) + \alpha N_i} \right)^{-\frac{\alpha}{1-\alpha}} (\eta(1-\alpha) + \alpha N_i)^{\frac{\gamma}{(1-\alpha)(1-\alpha-\gamma)}},$$

which is clearly increasing in η . Thus,

$$(65) \quad \omega_i^+ \left[\left(\frac{1-\Phi_i[\theta_i]}{\theta_i \phi_i[\theta_i]} + 1 \right) \frac{1}{\beta_i \theta_i} \right] = \omega_i^*[\theta_i] > \omega_i^+ \left[\frac{1}{\beta_i \theta_i} \right] = \omega_i^0[\theta_i],$$

where $\omega_i^0[\theta_i]$ is the wage rate that prevails in region i in the absence of equalization transfers, and this wage rate is also the wage rate that prevails under complete information. We have just proved the first statement of Proposition 8.

To prove the second statement of Proposition 8, note that at $\theta_i = \underline{\theta}_i$ the utility of an individual in region i is equal to $\bar{u}$ under complete as well as under incomplete information. Also, we have just shown that private consumption is higher under incomplete information. Thus, when the technology for public service delivery is least productive, the consumption of public services is lower under incomplete information than under complete information. By continuity, this result still holds in a right neighborhood of $\underline{\theta}_i$. Next, note that at $\theta_i = \bar{\theta}_i$, the tax rate on private capital and the public capital investment are the same under complete information as well as under incomplete information, and this means that the wage rate and a fortiori the private consumption

is the same under both complete and incomplete information. Furthermore, the utility of an individual in region i is higher under incomplete information than under complete information. Thus, the consumption of public services is higher under incomplete than under complete information when the technology for public service delivery is in its most productive range.

■

According to Proposition 2, when the federal government can observe the type of region i , the entire equalization transfer received by the government of this region is spent on public service provision. What Proposition 8 asserts is that under asymmetric information, part of the equalization transfer is used to raise private consumption above the level under complete information by allowing the region to impose a lower tax rate on private capital and making more public capital investment (Proposition 7). The effort to raise private consumption by the equalization transfer scheme under asymmetric information is more pronounced when the region's technology for public service delivery is least productive: it dictates a lower level of public services than when information is complete. Only, when the region's technology for public service is more productive will the level of public services delivered be higher under incomplete than under complete information.

5. THE IMPACT OF EQUALIZATION TRANSFERS ON THE RICHER REGION

Until now, we have focused exclusively on the behavior of the poor region and the behavior of the federal government. The richer region, region j , is coerced by the federal government into contributing to the funds to finance the equalization transfers. The welfare of the residents of region j obviously depends on the productivity of the technology it uses for public service delivery as well as that of region i , the recipient of equalization transfers. Let $T_i[\theta_i]$ denote the resource flow from region j to region i as equalization transfer when θ_i is the type of region i . If θ_i is the type of region i and θ_j is the type of region j , and if the government of the latter region chooses q_j as the tax rate on private capital and H_j as its public capital investment, then it induces a competitive equilibrium in its own region under which the wage rate is

$$(66) \quad \omega_j = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} H_j^{\frac{\gamma}{1-\alpha}} (r + q_j)^{-\frac{\alpha}{1-\alpha}},$$

The private consumption given by

$$(67) \quad c_j = \omega_j + r\bar{k}_j = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} H_j^{\frac{\gamma}{1-\alpha}} (r + q_j)^{-\frac{\alpha}{1-\alpha}} + r\bar{k}_j.$$

The tax revenues obtained by regional government j , as a function of (q_j, H_j) , is given by

$$(68) \quad q_j K_j = \alpha^{\frac{1}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} H_j^{\frac{\gamma}{1-\alpha}} N_j q_j (r + q_j)^{-\frac{1}{1-\alpha}}.$$

Using its budget constraint, we obtain the following level of public services provided by the government of region j , given the equalization transfer $T_i[\theta_i]$ and once it has chosen (q_j, H_j) :

$$(69) \quad G_j = \theta_j (q_j K_j - H_j - T_i[\theta_i]) = \theta_j \left(\alpha^{\frac{1}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} H_j^{\frac{\gamma}{1-\alpha}} N_j q_j (r + q_j)^{-\frac{1}{1-\alpha}} - H_j - T_i[\theta_i] \right).$$

The utility of a resident in region j – as a function of the chosen policy (q_j, H_j) and the type profile (θ_i, θ_j) is then given by

$$(70) \quad u_j[q_j, H_j, \theta_i, \theta_j] = \beta_j \theta_j \left(\alpha^{\frac{1}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} H_j^{\frac{\gamma}{1-\alpha}} N_j q_j (r + q_j)^{-\frac{1}{1-\alpha}} - H_j - T_i[\theta_i] \right) \\ + (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} H_j^{\frac{\gamma}{1-\alpha}} (r + q_j)^{-\frac{\alpha}{1-\alpha}} + r\bar{k}_j.$$

The problem of the government in region j is to find a policy (q_j, H_j) to maximize u_j , the utility of a resident in this region. That is, the government of region j solves the following maximization problem:

$$(71) \quad \max_{(q_j, H_j)} u_j[q_j, H_j, \theta_i, \theta_j].$$

The solution of (71) is given by

$$(72) \quad q_j^0[\theta_i, \theta_j] = \frac{r(1-\alpha)(-1+N_j\beta_j\theta_j)}{1-\alpha+\alpha N_j\beta_j\theta_j},$$

$$(73) \quad H_j^0[\theta_i, \theta_j] = r^{-\frac{\alpha}{1-\alpha-\gamma}} \alpha^{\frac{\alpha}{1-\alpha-\gamma}} \gamma^{\frac{1-\alpha}{1-\alpha-\gamma}} A_j^{\frac{1}{1-\alpha-\gamma}} N_j^{-\frac{\alpha}{1-\alpha-\gamma}} \beta_j^{-\frac{1}{1-\alpha-\gamma}} \left(\frac{1-\alpha}{\theta_j} + \alpha N_j \beta_j \right)^{\frac{1}{1-\alpha-\gamma}}.$$

The level of public services that emerges from the solution of (71) is given by

$$(74) \quad G_j^0[\theta_i, \theta_j] = r^{-\frac{\alpha}{1-\alpha-\gamma}} \alpha^{\frac{\alpha}{1-\alpha-\gamma}} \gamma^{\frac{\gamma}{1-\alpha-\gamma}} A_j^{\frac{1}{1-\alpha-\gamma}} N_j^{-\frac{\alpha}{1-\alpha-\gamma}} \beta_j^{-\frac{1}{1-\alpha-\gamma}} \left(\alpha N_j \beta_j + \frac{1-\alpha}{\theta_j} \right)^{\frac{\alpha+\gamma}{1-\alpha-\gamma}} \times \\ \left((-1 + \alpha)(\alpha + \gamma) - \alpha(-1 + \alpha + \gamma) N_j \beta_j \theta_j \right) - \theta_j T_i[\theta_i].$$

Note that the policy carried out by the government of region j when it is coerced into contributing to the equalization transfer scheme is the same as the policy it would carry out in the absence of equalization transfers. The only difference is that the resources allocated to public service delivery now falls by the amount it contributes to the equalization transfer scheme. Because the equalization transfers it sends to region i falls as the technology for public service delivery in region i is more productive, the welfare of the contributing region rises when θ_i rises. Region j is also better-off under complete than under incomplete information.

6. CONCLUDING REMARKS

In this paper, we have formalized a model of equalization transfers in a federation. The model explains how disparities in endowments, populations, and public service delivery technologies among the various regions of a federation give rise to disparities in welfare and levels of public services provided in different regions of the federation. Because the constitution of a federation often guarantees its citizens a more or less uniform standard of living, regardless of where they reside in the federation, the disparities among the various regions of the federation have been addressed by means of an equalization transfer scheme through which resources flow from richer to poorer regions. The model we formulate is a model with asymmetric information in which a regional government knows more about its own technology for public service delivery than the central government. In our model, the designer of the equalization transfer scheme wishes to achieve the goal of providing residents of a poor region with at least a certain level of utility judged to be equitable without imposing an excessive burden on the giving region. In order to

induce the recipient region into revealing its type truthfully, the equalization scheme offers the residents of this region a level of utility above the level judged equitable. The equalization transfers allow the recipient region to raise its private consumption above the level it would have attained in the absence of equalization transfers. Although the purported objective of an equalization transfer scheme is to ensure that a state is able to provide a comparable level of public services comparable with its capacity to raise revenues, not all of the transfer is allocated to achieve this objective; part of the equalization transfer is used to help raise private consumption.

Our analysis suggests that the equalization transfers received by a poor region decline when this region's technology for public service delivery is more productive. The equalization transfers are also lower if the federal government can observe the type of the poor region. Because rich regions in a federation often resent sending their own resources to poor regions, and because equalization transfer is often alleged to be the cause of dependency and stagnation in poor regions, the central government should obtain more information about the technology for public service delivery in poor regions and to provide incentives for these regions to innovate.

APPENDIX A

The Proof of Lemma 3

We have

$$(A.1) \quad \frac{\partial \text{Log}[G_i^0[\theta_i]]}{\partial N_i} = \frac{(\alpha-1)\alpha((-1+\alpha)(\alpha+\gamma)+N_i\beta_i\theta_i(-(-1+\alpha)(-1+2\alpha+2\gamma)+\alpha(-1+\alpha+\gamma)N_i\beta_i\theta_i))}{-(1-\alpha-\gamma)N_i(1-\alpha+\alpha N_i\beta_i\theta_i)(-(-1+\alpha)(\alpha+\gamma)+\alpha(-1+\alpha+\gamma)N_i\beta_i\theta_i)} = \frac{m_1}{m_0}.$$

Using condition (21), we can assert that $m_0 > 0$. Hence to show $\frac{\partial \text{Log}[G_i^0]}{\partial N_i} > 0$ we need to show that $m_1 > 0$. To this end, first note that when it is evaluated at the value of N_i given by the right-hand side of (22), m_1 assumes the following form:

$$(A.2) \quad m_1|_{N_i = \frac{(1-\alpha)(\alpha+\gamma)}{\alpha(1-\alpha-\gamma)\beta_i\theta_i}} = (-1+\alpha)^2(\alpha+\gamma) > 0.$$

Furthermore,

$$(A.3) \quad \frac{\partial m_1}{\partial N_i} = -(-1+\alpha)^2\alpha(-1+2\alpha+2\gamma)\beta_i\theta_i + 2(-1+\alpha)\alpha^2(-1+\alpha+\gamma)N_i\beta_i^2\theta_i^2,$$

i.e., m_1 is increasing in N_i . It follows from (A.2) and (A.3) that $m_1 > 0$ for all $N_i > \frac{(1-\alpha)(\alpha+\gamma)}{\alpha(1-\alpha-\gamma)\beta_i\theta_i}$,

and we have just shown that $\frac{\partial \text{Log}[G_i^0[\theta_i]]}{\partial N_i} > 0$. ■

APPENDIX B

The Proof of Proposition 1

Necessity: Suppose that $\mathfrak{F}$ is Bayesian incentive compatible. Pick two values, say θ_i and θ_i' , for the possible productivities of the technology for public service provision in region i . If θ_i is the type of region i and its government declare θ_i' as its type, then we must have $v_i[\theta_i|\theta_i] \geq$

$v_i[\theta_i'|\theta_i]$ because that $\mathfrak{F}$ is Bayesian incentive compatible. In this case the following inequality must hold:

$$(B.1) \quad \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} \geq \frac{v_i[\theta_i'|\theta_i]-r\bar{k}_i}{\theta_i} = \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'} + \frac{v_i[\theta_i'|\theta_i]-r\bar{k}_i}{\theta_i} - \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'}.$$

Next, using (32), we can write

$$(B.2) \quad \begin{aligned} \frac{v_i[\theta_i'|\theta_i]-r\bar{k}_i}{\theta_i} - \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'} &= \left(\frac{1}{\theta_i} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}} \right. \\ &\quad \left. + \beta_i \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} N_i q_i[\theta_i'] (r + q_i[\theta_i'])^{-\frac{1}{1-\alpha}} \right) \\ &\quad - \left(\frac{1}{\theta_i'} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}} \right. \\ &\quad \left. + \beta_i \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} N_i q_i[\theta_i'] (r + q_i[\theta_i'])^{-\frac{1}{1-\alpha}} \right) \\ &= \left(\frac{1}{\theta_i} - \frac{1}{\theta_i'} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Using (B.2), we can rewrite (B.1) as

$$(B.3) \quad \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} \geq \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'} + \left(\frac{1}{\theta_i} - \frac{1}{\theta_i'} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}},$$

or

$$(B.4) \quad \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} - \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'} \geq \left(\frac{1}{\theta_i} - \frac{1}{\theta_i'} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}}.$$

Interchanging θ_i and θ_i' in (B.4), we obtain

$$(B.5) \quad \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'} - \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} \geq \left(\frac{1}{\theta_i'} - \frac{1}{\theta_i} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}.$$

Together, (B.5) and (B.4) imply

$$(B.6) \quad \left(\frac{1}{\theta_i'} - \frac{1}{\theta_i} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}} \leq \frac{v_i[\theta_i'|\theta_i']-r\bar{k}_i}{\theta_i'} - \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i}$$

$$\leq \left(\frac{1}{\theta_i'} - \frac{1}{\theta_i}\right) (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}}.$$

If we choose $\theta_i < \theta_i'$, then it follows from (B.6) that

$$(B.7) \quad (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i'])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i'])^{-\frac{\alpha}{1-\alpha}} \\ \leq (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}},$$

and this means the curve

$$(B.8) \quad \theta_i \rightarrow (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}$$

is decreasing. We have just establish (a) of Proposition 1.

Dividing (B.6) by $(\theta_i' - \theta_i)$, and then letting $\theta_i' \downarrow \theta_i$, we obtain

$$(B.9) \quad \frac{d}{d\theta_i} \left[\frac{v_i[\theta_i|\theta_i] - rk_i}{\theta_i} \right] = -\frac{1}{\theta_i^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\theta_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\theta_i])^{-\frac{\alpha}{1-\alpha}}.$$

Integrating (B.9), we obtain

$$(B.10) \quad \frac{v_i[\theta_i|\theta_i] - rk_i}{\theta_i} = \frac{v_i[\underline{\theta}_i|\underline{\theta}_i] - rk_i}{\underline{\theta}_i} - \int_{\underline{\theta}_i}^{\theta_i} \frac{1}{s^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[s])^{\frac{\gamma}{1-\alpha}} (r + q_i[s])^{-\frac{\alpha}{1-\alpha}} ds,$$

which is (b) of Proposition 1.

Sufficiency: To prove that if (a) and (b) of Proposition 1 are satisfied, then $\mathfrak{F}$ is Bayesian incentive compatible, suppose that θ_i is the type of region i , but the government of this region declares $\hat{\theta}_i \neq \theta_i$ as its type. The gain obtained by doing so is $v_i[\hat{\theta}_i|\theta_i] - v_i[\theta_i|\theta_i]$.

$$(B.11) \quad \frac{v_i[\hat{\theta}_i|\theta_i] - rk_i}{\theta_i} - \frac{v_i[\theta_i|\theta_i] - rk_i}{\theta_i} = \frac{v_i[\hat{\theta}_i|\theta_i] - rk_i}{\theta_i} - \frac{v_i[\hat{\theta}_i|\hat{\theta}_i] - rk_i}{\hat{\theta}_i} - \frac{v_i[\theta_i|\theta_i] - rk_i}{\theta_i} + \frac{v_i[\hat{\theta}_i|\hat{\theta}_i] - rk_i}{\hat{\theta}_i}.$$

Using (32), we can write

$$\begin{aligned}
\text{(B.12)} \quad \frac{v_i[\hat{\theta}_i|\theta_i]-r\bar{k}_i}{\theta_i} - \frac{v_i[\hat{\theta}_i|\hat{\theta}_i]-r\bar{k}_i}{\hat{\theta}_i} &= \left(\frac{1}{\theta_i} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \right. \\
&\quad \left. + \beta_i \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} N_i q_i[\hat{\theta}_i] (r + q_i[\hat{\theta}_i])^{-\frac{1}{1-\alpha}} \right) \\
&\quad - \left(\frac{1}{\hat{\theta}_i} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \right) \\
&\quad \left. + \beta_i \alpha^{\frac{1}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} N_i q_i[\hat{\theta}_i] (r + q_i[\hat{\theta}_i])^{-\frac{1}{1-\alpha}} \right) \\
&= \left(\frac{1}{\theta_i} - \frac{1}{\hat{\theta}_i} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}}.
\end{aligned}$$

Using (b) of Proposition 1, we can write

$$\text{(B.13)} \quad \frac{v_i[\hat{\theta}_i|\hat{\theta}_i]-r\bar{k}_i}{\hat{\theta}_i} - \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} = - \int_{\theta_i}^{\hat{\theta}_i} \frac{1}{s^2} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[s])^{\frac{\gamma}{1-\alpha}} (r + q_i[s])^{-\frac{\alpha}{1-\alpha}} ds.$$

Using (B.13) and (B.12) in (B.11), we obtain

$$\begin{aligned}
\text{(B.14)} \quad \frac{v_i[\hat{\theta}_i|\theta_i]-r\bar{k}_i}{\theta_i} - \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} &= \left(\frac{1}{\theta_i} - \frac{1}{\hat{\theta}_i} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \\
&\quad - \int_{\theta_i}^{\hat{\theta}_i} \frac{1}{s^2} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[s])^{\frac{\gamma}{1-\alpha}} (r + q_i[s])^{-\frac{\alpha}{1-\alpha}} ds.
\end{aligned}$$

Now suppose that regional government i overstates its type: $\hat{\theta}_i > \theta_i$. Using (a) of Proposition 1, we can assert that

$$\begin{aligned}
\text{(B.15)} \quad (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[s])^{\frac{\gamma}{1-\alpha}} (r + q_i[s])^{-\frac{\alpha}{1-\alpha}} \\
\geq (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}}, \quad \theta_i \leq s \leq \hat{\theta}_i.
\end{aligned}$$

Using (B.15) in (B.14), we can then write

$$\begin{aligned}
\text{(B.16)} \quad \frac{v_i[\hat{\theta}_i|\theta_i]-r\bar{k}_i}{\theta_i} - \frac{v_i[\theta_i|\theta_i]-r\bar{k}_i}{\theta_i} &\leq \left(\frac{1}{\theta_i} - \frac{1}{\hat{\theta}_i} \right) (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \\
&\quad - \int_{\theta_i}^{\hat{\theta}_i} \frac{1}{s^2} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} ds
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{1}{\theta_i} - \frac{1}{\hat{\theta}_i}\right) (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \\
&\quad - \left(\frac{1}{\hat{\theta}_i} - \frac{1}{\theta_i}\right) (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} = 0.
\end{aligned}$$

That is, overstating its type does not pay for region i .

Now suppose that regional government i understates its type: $\hat{\theta}_i < \theta_i$. Using (a) of Proposition 1, we can assert that

$$\begin{aligned}
\text{(B.17)} \quad &(1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[s])^{\frac{\gamma}{1-\alpha}} (r + q_i[s])^{-\frac{\alpha}{1-\alpha}} \\
&\leq (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}}, \hat{\theta}_i \leq s \leq \theta_i.
\end{aligned}$$

Using (B.17) in (B.14), we can then write

$$\begin{aligned}
\text{(B.18)} \quad &\frac{v_i[\hat{\theta}_i|\theta_i] - r\bar{k}_i}{\theta_i} - \frac{v_i[\theta_i|\theta_i] - r\bar{k}_i}{\theta_i} \leq \left(\frac{1}{\theta_i} - \frac{1}{\hat{\theta}_i}\right) (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \\
&\quad + \int_{\hat{\theta}_i}^{\theta_i} \frac{1}{s^2} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} ds \\
&= \left(\frac{1}{\theta_i} - \frac{1}{\hat{\theta}_i}\right) (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} \\
&\quad - \left(\frac{1}{\hat{\theta}_i} - \frac{1}{\theta_i}\right) (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} (H_i[\hat{\theta}_i])^{\frac{\gamma}{1-\alpha}} (r + q_i[\hat{\theta}_i])^{-\frac{\alpha}{1-\alpha}} = 0.
\end{aligned}$$

That is, understating its type does not pay for region i . We have just demonstrated that truth telling is best for region i , i.e., $\mathfrak{F}$ is Bayesian incentive compatible.

■

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