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Mathematical Model of the Sprint Relay Race

by

Stephen D. Murphy

Thesis submitted to the
School of Graduate Studies and Research
in partial fulfilment of the requirements for the
Master of Science degree in Kinanthropology

University of Ottawa



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to Pauline and Dunc
a.k.a. Mom and Dad

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Abstract

The purpose of this investigation was to develop and validate mathematical models of the sprint relay race. Two approaches, a classical exponential approach and a “new” polynomial approach, were chosen for implementation.

Archival film of a 100 m sprint was used to gather displacement data for the first 60 m of the race. Filming had been performed with a single highspeed 16 mm cine camera (LOCAM) at 50 fps.

The coordinates were digitized, scaled and filtered using a low pass, critically damped, 4th order, zero-lag Butterworth digital filter with a 1 Hz cutoff frequency. Linear velocities were calculated using finite differences.

A sprinter was modelled in two ways. The first was an Exponential Model which required as input a personal best time for the 100 m race and the sprinter’s maximum constant velocity. The second was a Polynomial Model which required as input the parameters mentioned in the Exponential Model and, additionally, the displacement coefficients for the first 60 m of the 100 m sprint. Relay software was developed to piece the sprint relay together using the corresponding exponential or polynomial approach.

The results indicated that the relay software reasonably simulated the kinematic and temporal quantities of a 4x100 m relay and can be used by coaches to gain insight into the sprint relay without risking injury to their athletes. Furthermore, the Exponential Model, using less information, described the sprinter’s kinematics better than the Polynomial Model.

Introduction

The relays are “...the climax of most athletic matches,...” and “...generally prove to be the most exciting event...” (Marlow, 1972). The first record of a relay was at the University of Pennsylvania in 1893. In the London Olympics of 1908 the medley relay of 1600 metres (200 x 200 x 400 x 800 m) was included but was mainly dependent upon the quality of the 800 m runner. This motivated the exclusion of the medley relay in the Stockholm Olympics of 1912 in favour of the 4x100 m and 4x400 m relays which are still present in the Olympic program today (Marlow, 1972).

One of the difficult aspects of the 4x100 m relay is how the coach decides which leg of the race each member of the team will run. Once the assignments have been made the next problem concerns the exchanges. When should the departing sprinters leave so that the exchanges are performed near the ends of the exchange zones? And if so, what will be the effect on the final time? Since the elite sprinter cannot have peak performances in practice what adjustments must be made so that the team performs well in competition? All of these questions are difficult to answer and do not have immediately obvious answers. A solution is to analyze the sprint relay through computer simulation.

Computer simulation can provide insight into the nature of various problems. It is especially useful for human movement studies because it does not require the subjects to perform potentially injurious activities. For example, Wood *et al.* (1987) developed a computer simulation of the leg recovery during a sprint using an optimization approach. Wood *et al.* found that a “likely performance criterion” (objective function) was based on using the minimization of the average muscle power generated or absorbed at the hip and

knee joints. This study would have been difficult to attempt with sprinters since they rarely perform maximally except in competition where data collection may be prohibited.

The current study used a computer model to simulate the 4x100 m relay race. The software provides two different approaches to simulate the 4x100 m race. The first approach, which gives an optimal solution, accepts as input sprinter specific information (for the construction of the sprinter model) and certain race parameters. It then predicts the final time for the race as well as the distance at which the receiving sprinter should depart to achieve an exchange at a preselected exchange position. The second approach accepts similar inputs but predicts the exchange position and the final time.

The Sprint

The straight 100 m sprint is an “all-out” sprinting effort from a motionless “set” position to the 100 m finish line. Four phases have been identified for most sprinters: reaction time, acceleration, maximum velocity and deceleration (Mero *et al.*, 1992). The sprint run on the bend is also composed of the same four phases mentioned above but takes longer to run than the straight 100 m sprint. A description of each of the above phases follows.

Reaction Time. Reaction time is the time that elapses between the sound of the starter’s gun and the moment the athlete is able to exert a certain amount of horizontal pressure on the starting blocks (Moravec *et al.*, 1988; Mero, 1988). It includes the duration for sound to travel from the starter’s gun to the sprinter, the reaction time of the sprinter and the mechanical delay inherent in the starting blocks. Mero and Komi (1990) attempted to isolate two aspects of reaction time defined by the time between the gun signal and EMG activity, and concluded the “reaction time” was approximately 120 ms. Reaction time has been studied extensively but is not a key portion of this research since it can be included as zero displacement in the acceleration phase.

Acceleration. Once the athlete begins to move from the blocks, at the end of the reaction phase, a top sprinter accelerates for approximately 30-60 m until maximum velocity is reached, though in some instances the maximum velocity is reached as late as 80-100 m (Moravec *et al.*, 1988). Based on the results from the World Track Championships in Rome, the finishing positions for the 100 m sprint were the same as those recorded at the 50 m point (Moravec *et al.*, 1988).

Maximum Velocity. Once the acceleration phase is complete the athlete faces the task of maintaining maximum velocity for as long as possible or until the end of the 100 m race. A high maximum velocity is also very important in the 100 m sprint since half of the race may be run at maximum velocity.

Deceleration. The existence of the deceleration phase in the 100 m sprint exists for non-elite sprinters but not for elite sprinters. Older studies by Henry (1954b), Ikai (1968) and Volkov and Lapin (1979) agreed that sprinters had a decline in velocity before crossing the 100 m finish line. Recent studies by Moravec *et al.* (1988) and Lemaire and Robertson (1989) concluded that the elite sprinter maintains maximum velocity until the end of the 100 m sprint.

Based on the above description of the sprint phases the requirements for an elite sprinter model can be outlined. The model should consist of an acceleration phase and a constant maximal velocity phase. The acceleration phase includes the reaction time as zero displacement (zero acceleration). Since elite sprinters do not decelerate before the end of the 100 m sprint the acceleration phase should be followed by a constant maximal velocity phase.

In the 4x100 m relay, sprinters 1, 2, 3 and 4 can potentially sprint all-out for 110, 130, 130 and 120 m, respectively, assuming the exchanges occur at the extreme end of the exchange zones. Since the receiving sprinter has a maximum of 30 m to accelerate

and perform the exchange, s/he must receive the baton while accelerating. At the same time, the approaching sprinter will be at or near maximum velocity.

The approaching sprinter may therefore be required to decelerate to perform a successful exchange and not overtake the receiving sprinter. No data have been published to confirm this possibility. For modelling purposes it will be assumed that the incoming sprinters do not decelerate during the exchange. This may be a source of error since the sprinters are required to sprint longer than 100 m. The sprinter model is also limited in that it would not be appropriate for non-elite sprinters since they may decelerate before reaching the 100 m distance.

Sprinter Models

To simulate the sprint a mathematical model of the sprinter's displacement is required. Hill (1927) was the first researcher to model the sprint mathematically. Hill's model included two horizontal forces with one being a constant propulsive force (F_p) and the other a resistive force (F_r) caused by muscular viscosity. Assuming that the positive horizontal direction was in the same direction the sprinter was running, the application of Newton's second law yields

$$F_p - F_r = ma \quad (1)$$

where m is the mass of the sprinter and a is the sprinter's acceleration in the positive horizontal direction. The propulsive force can be written as

$$F_p = f m \quad (2)$$

where f is the force per unit mass of the athlete and m is the sprinter's mass. Furthermore the model assumes that the total sum of resistive forces acting on the sprinter can be represented by

$$F_r = k m v \quad (3)$$

where k is a constant of proportionality, m is the mass of the sprinter and v is the sprinter's velocity. Substituting (2) and (3) into (1), dividing out the sprinter's mass (m) and rearranging terms yields a first-order linear differential equation in standard form

$$\frac{dv(t)}{dt} + kv(t) = f \quad (4)$$

The solution to (4) may be obtained analytically (see Spiegel, 1981, for additional information on first-order linear differential equations) and by remembering that $v(t) = ds/dt$, solutions for $v(t)$ and $s(t)$ may be obtained subject to the initial conditions $v(0) = 0, s(0) = 0$.

$$v(t) = \frac{f}{k} (1 - e^{-kt}) \quad (5)$$

$$s(t) = \frac{f}{k^2} (kt + e^{-kt} - 1) \quad (6)$$

From (5) the limiting velocity, v_{max} , will be equal to f/k . Due to the assumptions made in the construction of Hill's model there was no deceleration phase so the sprinter approached maximum velocity for the duration of the sprint.

Hill never determined whether the model truly described a real sprinter. Henry and Trafton (1951) used the model proposed by Hill and established that the model reasonably represented the velocity of the sprinter at any time t , prior to the attainment of maximum velocity during a 50 yard sprint.

In an effort to model the fatigue factor Henry (1954b) described the propulsive force (2) as being proportional to an exponential term and consequently extended the model in a convenient manner to accommodate distances up to 400 m provided the sprinter makes an “all out” effort. The first order linear differential equation of (4) becomes another first order linear differential equation of the form

$$\frac{dv(t)}{dt} + k_1 v(t) = f e^{-k_2 t} \quad (7)$$

$v(t)$ can be found using the same techniques as above but this time yields a sum of exponentials.

$$v(t) = v_o(e^{-k_2 t} - e^{-k_1 t}) \quad (8)$$

It should be noted that if k_2 , the rate constant determining the reduction of running efficiency due to fatigue, equals zero then (7) becomes the same equation as (4). The solution for the velocity, $v(t)$, increases sharply for about the first 6 s to approximately the maximum velocity and almost immediately begins to decay at a much slower rate as t increases. This new model was validated using unconditioned students and a varsity “track squad” that was not in peak condition.

Volkov and Lapin (1979) expanded upon the research of Henry (1954b) by developing methods of determining the parameters of the exponential velocity equation developed from (7). The subjects used to provide the required data were not elite athletes since their times for the 100 m sprint varied from 11.661 to 15.448 s. There was no constant maximum velocity phase in Volkov and Lapin’s model which does not correspond to the reality of the performance of the present day sprinter.

Ikai (1968) determined that the velocity curve in sprint running was related to age and sex and that the velocity was significantly higher in trained versus untrained

sprinters. The existence of a constant maximum velocity phase for elite sprinters was further supported by the single elite athlete studied which revealed "...an uneven plateau of speed after 70 meters to 100."

Henry (1954a, 1955) further extended the model for distances from sprints to marathons by supposing that the propulsive force (2) declined in a more detailed manner according to a sum of exponentials. This extension of the sprinter model did not appear to be based on any theoretical platform. Morton (1985), furthermore, suggested "...there (was) no indication that it (the velocity equation) was derived any other way than by expediently extending equations..." (5) or (8).

Lloyd (1966) developed a possible competitor to the exponential equations of (5) and (8) by developing a non-linear differential equation that included oxygen debt which decreased exponentially, and a constant representing the blood-borne oxygen power supply. Lloyd found this equation "...difficult to integrate..." but Morton (1985) was able to obtain a solution using simplifying assumptions regarding the oxygen uptake and numerical methods. Morton concluded that even though initially (for 200 m) the solution appeared reasonable, the solution was mathematically "inappropriate" since the solution became irrational and "...furthermore (was) not based on any well known properties of the muscular system." This formulation required knowledge of the sprinter's caloric cost and a method for numerical integration since the solution could not be obtained analytically.

A second competitor to the exponential equations (5 and 8) was developed by Alexandrov and Lucht (1981) who predicted a runner's performance for a 100 m sprint run on the curve for different lanes. Once again the model was based upon the resistive force (3) and the propulsive force (2) but the propulsive force "...must supply a component that gives rise to the centripetal acceleration of magnitude v^2 / R ." The resulting non-linear differential equation was of the form

$$\frac{dv(t)}{dt} + kv(t) = \sqrt{f^2 - \frac{v(t)^4}{R^2}} \quad (9)$$

where R was the constant radius (m) for lane n given by

$$R(n) = \frac{100}{\pi} + (n-1)1.22 \quad (10)$$

when the lanes were 1.22 m wide. Using approximations to simplify (9) the displacement on the curve was given by

$$s_c(t) = v_{max}(1 - \beta)t - \frac{v_{max}}{k}(1 - \frac{37}{12}\beta) \quad (11)$$

where

$$\beta = \frac{1}{2} \left[\frac{f}{Rk^2} \right]^2$$

Using (11) the new perturbed personal best time (P.B.) for the 100 m sprint on the curve can be determined. Additionally, they found that the sprinter in lane 8 was at an advantage of 0.1 s over the sprinter in lane 1 for a 100 m race with a staggered start on the curve.

A third competitor was the revised velocity equation derived by Morton (1985). The resistive force (3) was separated into two components—a muscle viscosity component and an aerodynamic drag component. The muscle viscosity component was term (3) developed by Hill (1927) and the drag component was proportional to the square of the running velocity which yielded the following non-linear differential equation

$$\frac{dv(t)}{dt} + k_1v(t) + k_2 v(t)^2 = f \quad (12)$$

Morton obtained a “Riccati type” solution and used data from Hill’s (1927) book to determine that k_1 was “...not significantly different from zero...” and can therefore be set to zero. This simplification yielded the following solutions

$$v(t) = \sqrt{\frac{f}{k_2}} \tanh(\sqrt{f k_2} t) \quad (13)$$

$$s(t) = \frac{\ln(\cosh(\sqrt{f k_2} t))}{k_2} \quad (14)$$

Hill’s exponential displacement equation (6) fit very well ($p < 0.001$) but the lack of fit component was also significant ($p < 0.001$). Equation (14) fit Hill’s data better than the simple exponential equation (6) and “...there (was) no significant lack of fit...”. Both (6) and (14) were then fitted to Henry’s and Trafton’s (1951) data which contained duplicated times for 25 inexperienced sprinters from a university physical education program timed at 5 yard intervals over a 50 yard sprint. Morton found that equation (14) fit the data better (higher R^2) for 17 of the 25 subjects when compared to the fit of equation (6). Morton concluded that even though equation (6) was still “...a good fit *per se*, one should not be reticent to try new approaches as more information becomes available.”

In summary, several authors have presented models of the sprint velocity curve. Choosing an appropriate model must include an acceleration phase and a prolonged constant maximal velocity phase. The model should also have parameters that can be easily obtained to facilitate its use in the field. For these reasons Hill’s equations (5) and (6) were chosen. From equation (5) the limiting (maximal) velocity was $v_{max} = f / k$. Assuming that the sprinters know their maximal velocity, k can be determined from (6) by using the sprinter’s best time for the 100 m sprint. Since sprinters 1 and 3 run the

bend, equation (11), developed by Alexandrov and Lucht (1981), was used to determine the increase in time for the sprinter's best time for the 100 m sprint on the curve.

The purpose of the article by Morton was to renew interest in the mathematical analysis of velocity equations for sprint running. With this goal in mind, a new approach was taken which utilized a "best fit" polynomial to the raw displacement data of the acceleration phase. The motivation behind using this approach was due to the varied acceleration patterns found in the literature (Moravec *et al.*, 1988) and the belief that the polynomial approach would directly reflect a particular sprinter's displacement characteristics. This polynomial was pieced together with a linear displacement portion having a slope corresponding to the sprinter's known maximal velocity.

Validation

For the biomechanical analysis of human motor behaviour validation proves to be a difficult task since it is often difficult to get the required data. Often the researcher is left with indirect methods of validating a particular method or model. The researcher must make use of the available data to validate his/her model.

While the sprint relay is regularly performed, complete quantitative descriptions of the race do not appear in the literature. A partial validation can be achieved, however, by using the data from individual performances in the 100 m sprint race. For example, each sprinter knows his/her personal best time for the 100 m sprint, this can be used in the model to assure that the displacement curve is "forced" to fit this personal best time. This will also achieve a level of customization for each sprinter.

Secondly, a comparison of the models' results to those obtained by the world record holders of the sprint relay can be made. Data from Moravec *et al.* (1988) combined with the known personal best times for the 100 m sprint can be used to closely approximate the individual sprinter models that can be pieced together to form the relay.

Purpose

The main purpose of this investigation was to develop and validate mathematical models of the sprint relay race. Two approaches, a classical exponential approach and a “new” polynomial approach, were chosen for implementation.

Methodology

Filming Procedures

Archival film of a 100 m sprint with 6 participating male athletes was used to gather the displacement data. Filming had been performed with a single highspeed 16 mm cine camera (LOCAM) at 50 frames per second (fps) for the first 60 m (approximately) of the 100 m sprint. The cine camera was positioned perpendicularly, at approximately the midpoint of the 60 m.

Data Reduction

The cinefilm was projected frame-by-frame onto a digitizer (Hewlett-Packard 9874A). The sprinter closest to the cine camera was digitized every 5th frame from the 0 to 60 m portion of the race. The head was used as the body marker since the waist (approximate centre of mass) was often blocked by hurdles positioned at the side of the track, between the camera and the sprinter. Markers were identified, in the sprinter's lane, to recognize the zero metre and the 53 m point which was used as the linear scale factor during digitizing. The start of the race was identified by the flame from the starter's gun. The digitization process was repeated a second time immediately following the first digitizing session to check reliability.

Kinematics

The digitizer coordinates were scaled to provide the displacement coordinates in metres. It was found by means of trial and error that at least 3 decimal places were necessary for the precision required. Linear extrapolation based on the slope of the last two points was used to pad the end of the data set with 7 artificial points. The beginning of the data set was not padded since the values were very close to zero. The raw

displacement coordinates in the horizontal (x) direction were then smoothed using a low pass, critically damped, 4th order, zero-lag Butterworth digital filter with a cutoff frequency of 1 Hz (Winter, 1990). From the filtered displacement data the linear velocities were calculated using finite differences (Miller and Nelson, 1973). The process of digitally filtering cinefilm coordinate data and then applying finite differences was shown to be an appropriate solution for obtaining kinematics by Pezzack *et al.* (1977). Both sets of raw displacement data were compared to determine which would best represent the sprinter's displacement.

Sprinter Models

Two approaches to sprinter modelling have been implemented. The first was the polynomial approach and the second was the classical exponential approach. An outline of each model follows.

Polynomial Model. The displacement of a typical elite sprinter was characterized as having an acceleration phase and a constant maximal velocity phase. The displacement in the acceleration phase was modelled using a best fit polynomial of the form

$$p(t) = c_0 + \sum_{i=1}^n c_i t^i$$

The constant maximal velocity phase was modelled as linear displacement with the slope given by the athlete's known maximal velocity (v_{max}) and the height (b) given by the position at which maximum velocity was attained (determined suitably by the model)

$$l(t) = v_{max}t + b$$

Both the acceleration and linear displacement portions were pieced together such that a P.B. for the 100 m sprint would be achieved (see Figure 9). The necessary sprinter information entered into the system were the sprinter's name, P.B. time for a 100 m sprint, maximum velocity and the displacement coefficients. The acceleration portion of the model assumed that the displacement data had been obtained for the first 60 m of the sprint. The sprinter may use his/her personal displacement coefficients, if they have been previously calculated, otherwise the displacement coefficients obtained in this study would be used as the default values.

Exponential Model. The displacement of a typical elite sprinter was modelled as having an acceleration phase and a constant maximal velocity phase. Equations (5) and (6) were used to determine the sprinter's velocity and displacement, respectively. Since equation (5) implies that $v_{max} = f / k$, equation (6) can be expressed as

$$s(t) = \frac{v_{max}}{k} (kt + e^{-kt} - 1) \quad (16)$$

The exponential model, as does the polynomial model, assumes that v_{max} is known for each sprinter. The k parameter, which describes the sprinter's acceleration, can be determined numerically by using the sprinter's P.B. for the 100 m sprint.

Both of the above models should make adjustments for sprinters 1 and 3 since they run the bends. Alexandrov's and Lutch's (1981) equation (11) will be used to increase the sprinter's time for a 100 m sprint on the bend.

Optimal Model

The Optimal Model calculates the distances from the approaching sprinters at which the receiving sprinters should depart, called the "Depart Distance", to produce the optimal race. The model, by default, assumes that the optimal exchange occurs at 5 m before the end of the exchange zone which means that the optimal exchanges occur at

105, 205 and 305 m from the starting line (see Figure 1). Vilkov and Turin (1990) suggested the above Exchange Positions as the furthest safe distance to accomplish a successful exchange. The Exchange Positions can be modified to see the effect on the final time of the 4x100 m relay. For example, if the exchanges are required to occur at 5 m before the 100 m point then the coach enters -5 m.

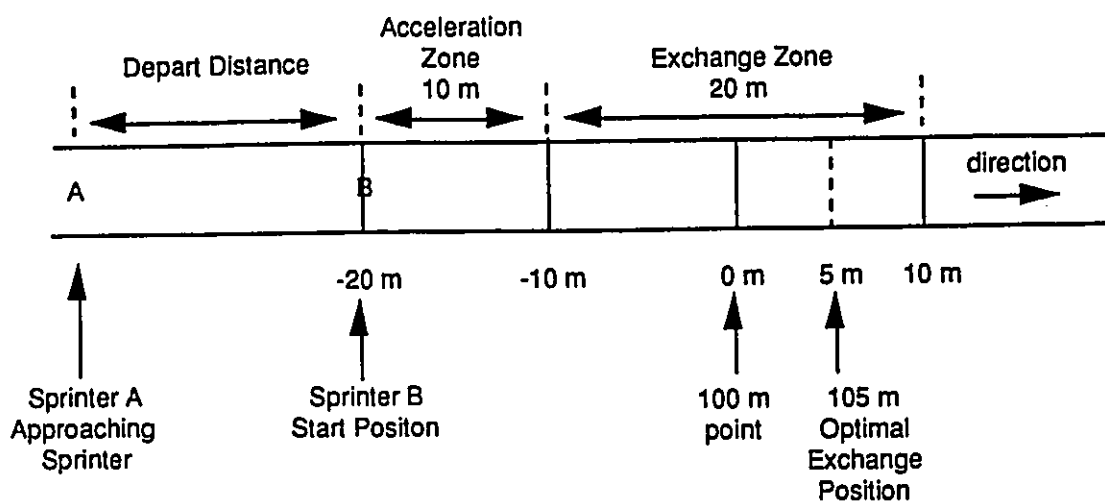


Figure 1: Defines the parameters utilized by the Optimal Model. The Depart Distance is the distance between the approaching sprinter A and the receiving sprinter B that triggers B to start sprinting such that the exchange occurs at the Optimal Exchange Position (105 m).

The model accepts two sets of inputs: sprinter specific information that permits construction of the Sprinter Models (Polynomial and Exponential) described above and optimal parameters that include the Start Position, the Exchange Distance and the Exchange Position (see Figures 1 and 2). The Start Position identifies the position from which the receiving sprinter will start relative to the 100 m point of the exchange zone. For example if the sprinter started 20 m before the 100 m point then this position was identified as -20 m. The Start Position will typically be -20 m for the Optimal Model but

was made flexible to permit injured sprinters to run shorter distances. The Exchange Distance, typically 1 m, is the distance between the approaching and receiving sprinter that constitutes a successful exchange. This allows for the fact that in a well executed exchange the baton is ideally passed ahead of the incoming sprinter and received behind the outgoing sprinter.

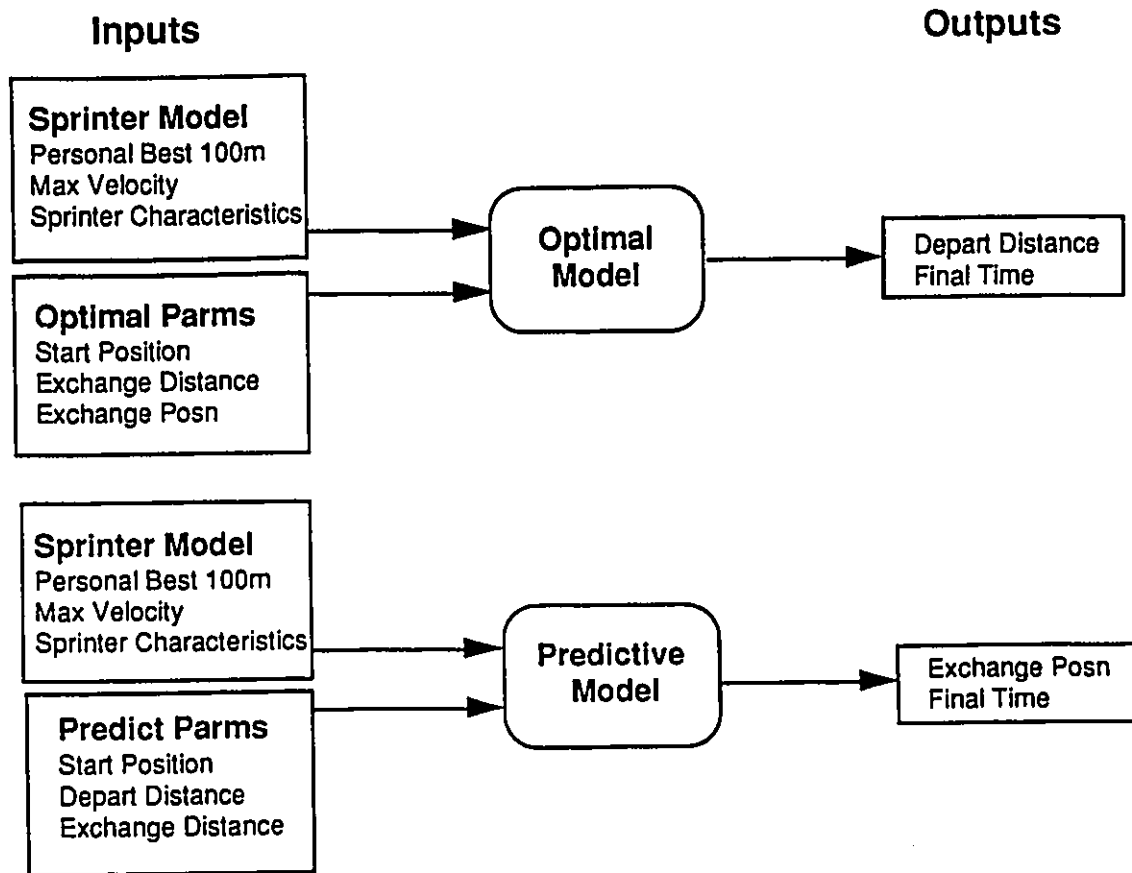


Figure 2: Outline of Inputs and Outputs for both the Optimal and Predictive Models. The Sprinter Characteristics listed in the Sprinter Model will reflect whether the Polynomial or Exponential model was being used.

Predictive Model

The Predictive Model calculates the positions at which the exchanges would occur (Exchange Positions) for a particular set of conditions. As is the case with the Optimal Model, the Predictive Model accepts two sets of inputs: sprinter specific information that permits construction of the Sprinter Models described above (Polynomial and Exponential) and predictive parameters that include the Start Position, the Depart Distance and the Exchange Distance (see Figure 2). Once the Optimal Model has been executed the Predictive Model can be used to compare the effects of using non-optimal values for the input parameters.

Relay Software

Software was developed on a 386, 33 MHz PC in the Turbo "C" programming environment and with the CWORDY library. The software permits the user to create a sprinter database with the appropriate sprinter related information. The coach can then choose the sprinters that will run each leg of the race. Once the team has been chosen either the Optimal or the Predictive Model can be executed. Each model has an input screen to permit the coach to enter the relevant parameters. Once either model has been successfully executed a report can be printed to summarize the results of the chosen sprint relay team and their final times.

Testing

Testing the Relay Software was performed both functionally and interactively. Functional testing was performed to insure that the system operated as the user would expect. Functional testing assured that the flow of menus was correct, that the sprinter database could be updated, modified and deleted, and that all fields, of all input forms, could accept the correct input. Functional testing also verified that the reports were printed successfully and in the proper format on the printer.

The Turbo "C" programming environment has an interactive debugger (Turbo Debugger) that affords the programmer the luxury of viewing all variables and structures that are being updated at a particular instance in time while the system is in normal operation. A walkthrough of the code for both the Optimal and Predictive Models using the interactive debugger ensured that all calculations were performed correctly.

Validation

Direct validation could not be performed since the data required from a complete sprint relay to test both the Optimal and Predictive Models (see Figure 2) was not available. Consequently an indirect approach was utilized and focussed on the construction of the Polynomial and Exponential Sprinter Models described above.

A key element of the Polynomial Model was the construction of the piecewise sprinter displacement curve. Since most sprinters do not have knowledge of their displacement coefficients, default coefficients must be used in conjunction with their known constant maximal velocity and P.B. for a 100 m sprint to approximate a particular sprinter's displacement curve. The Exponential Model can be customized simply by knowing an athlete's maximum velocity and P.B. time for a 100 m sprint. The final time for a 4x100 m relay, with data obtained from Moravec *et al.* (1988), will be used to compare the results obtained from the models.

Results and Discussion

The scaled raw displacement data along with the filtered displacement and velocity data (padded and unpadded) can be found in the Appendix. All Figures can be found at the end of this section.

Figure 3 displays how well both digitizing sessions corresponded to each other. Since there was not enough data to permit any form of averaging, the displacement data from the first digitizing session (trial 1) was chosen to represent a typical displacement history. This choice was based on the fact that the film was slightly damaged by the projector after the first digitizing session. Though the film was repaired the potential for imprecisions was higher in the second digitizing session.

The data from trial 1 were filtered with and without padding at the end of the data set (Figure 4). Though the differences between the padded and unpadded data seemed fairly small these differences were magnified, for the last 5 points, once the data were differentiated once to calculate the velocity (Figure 5). The data padded using linear extrapolation were a smoother representation of the displacement. Smith (1989) suggested using a reflection approach rather than a linear extrapolation approach to pad the data artificially but his study assumed that accelerations were required. The Sprinter, Optimal and Predictive Models of this study used the displacement curve only. Furthermore the velocities and accelerations were never calculated by either model and consequently linear extrapolation sufficed as the approach to padding data.

The correspondence between the raw displacement data and the filtered (padded) displacement data was very close (Figure 6). Since the numerical processing steps must be kept to a minimum, without sacrificing accuracy, the raw data can be used for the polynomial displacement curve fitting. See Table 1 for the results of the "best fit" for

different orders (1 to 10) of the approximating polynomial. As the order of the approximating polynomial increased the root mean square (RMS) error decreased. Note that for the 6th to 10th order approximating polynomials the RMS error remained constant at about 0.0500. The 5th order polynomial of best fit was chosen to minimize the numerical data required by the user because it corresponded very well with the raw displacement data (see Figure 7). The Exponential Model also fit to the raw data very well using $v_{max} = 11.87$ m/s and $s(t = 7.0 \text{ s}) = 58.803$ m to calculate $k = 0.47061$ using equation (16).

Table 1 RMS Error for Best Fit Polynomial: Contains the RMS error for each order (1 to 10) of the best fit polynomial for trial 1. Note that for orders 6 through 10 the RMS error remains near 0.0500.

Order	RMS Error
1	2.4239
2	0.6589
3	0.1961
4	0.0648
5	0.0609
6	0.0503
7	0.0502
8	0.0502
9	0.0500
10	0.0495

Best fit polynomials can only be utilized over the range for which they were originally constructed since there is no guarantee that the interpolating polynomial will behave in the same reasonable manner outside of the range. The polynomial of best fit in this case has a displacement range of 0 to 60 m and a time domain of 0 to 7.0 s. See Figure 8 for a plot outside of these limits. Notice that this particular polynomial and exponential increased rapidly for $-6 \leq t < 0$ which was not of serious consequence since the domain for time (t) must include only positive values ($t \geq 0$ s). Furthermore, the raw displacement data were gathered for only the first 7.0 s; consequently, there was no

guarantee that the behaviour displayed by the polynomial after 7.0 s matched that of the sprinter. The Exponential Model on the other hand is much more consistent for $t > 7.0$ s.

The acceleration phase of the Polynomial Model was modelled as a 5th order polynomial. This portion of the model must be combined with the linear (constant velocity) portion of the model to attain the sprinter's P.B. for the 100 m sprint. Figure 9 displays the intersection of the linear and polynomial portions of the model for a male sprinter who had a maximum velocity of 11 m/s and a personal best of 10.53 s (Lemaire and Robertson, 1989). Since the intersection did not occur between 30 and 60 m the model forced a "jump" to the distance along the line at the time the sprinter reached 30 m (see Figure 10). This approach was justified since the displacement coefficients utilized were the default values obtained from this study and not the sprinter's personal displacement coefficients. Though the displacement was no longer continuous the prescribed time was achieved. The Exponential Model achieved the same result but adjusted the acceleration phase of the displacement to reflect the sprinter's potential capabilities. The Exponential Model consistently modified the acceleration phase of the displacement curve while the Polynomial Model makes the corresponding adjustments only when the user has entered new displacement coefficients (see Figure 11). This renders the Polynomial Model less flexible when compared to the adaptive capabilities of the Exponential Model.

The approach outlined above permitted any sprinter to simulate their acceleration phase using the Exponential Model. The model reflected the natural displacement pattern of sprinters. They accelerate slowly when receiving the baton and approach maximum velocity before handing off the baton. See Figure 12 for the displacement curves of each sprinter for the 4x100 relay as predicted by the Optimal Model (generated from the results from Figure 17).

The Optimal and Predictive Models produced very similar results when the “same” inputs were utilized. The Exponential Optimal Model was executed first and the Depart Distances were produced (see Figure 13). The Exponential Predictive Model was in turn executed using the Depart Distances produced by the Optimal Model (see Figure 14). When the Predictive Model was executed the Turbo Debugger was utilized to view the contents of the Exchange Position variables. The Depart Distances were modified until the Exchange Position values were as close as possible to the required 105, 205 and 305 m Exchange Positions. The results produced by the Optimal and Predictive Models corresponded extremely well (< 0.1% error).

Typical applications of the Relay Software involved using the Optimal and Predictive Models. The first application required the execution of the Optimal Model using the same sprinters as in Figure 13 with positions 2 and 4 swapped (see Figure 15). Note the decrease in the overall time of 0.02 s. This was due to the simple fact that the faster sprinter (F) ran a longer distance (125 m instead of 120 m) and the slower sprinter ran a shorter distance (120 m instead of 125 m). This could lead the coach to conclude that the faster sprinters should run the longer legs of the race (legs 2 and 3). However, this was only true if the faster sprinters were efficient at receiving and handing off the baton otherwise the gains made due to their speed would be lost in the exchange zone. A second factor was the sprinter’s ability to run the curves well (leg 3). If the faster sprinter cannot run the curves well then the gains made due to his/her speed would be lost in the curve.

A second application involved the coach using non-optimal Depart Distances (15.0, 17.0 and 15.0 m, see Figure 16) which were less than those predicted by the Optimal Model (15.8, 17.7 and 15.8 m, see Figure 13). This approach increased the overall time of the race by 0.19 s since the sprinters received the baton earlier in the exchange zones.

A third application used a sprint relay team which was created with 3 sprinters possessing identical maximum velocities and best times and a fourth sprinter who was faster. The Optimal Model was executed four separate times with the fastest sprinter running a different leg each time (see Table 2). For this particular team the fastest sprinter should run the first leg.

Table 2 Position of Fastest Sprinter and Final Times: A sprint relay team was created that had 3 sprinters of identical calibre and a fourth sprinter that was faster. The Optimal Model predicted the following final times when the fastest sprinter ran different legs of the race.

Leg	Final Time (s)
1	37.10
2	37.13
3	37.14
4	37.15

A fourth application uses the team from Figure 17 and required the execution of the Optimal Model four separate times with 5, 0, -5 and -10 m as the Exchange Positions (see Table 3). It was interesting to note that the difference in the final times between making the exchange at the 5 and -10 m Exchange Positions was 0.77 s. The final times displayed in Table 3 provide a quantitative measure of the consequences of not performing the exchanges at the optimal Exchange Position (5 m).

Table 3 Various Exchange Positions: The Optimal Model was executed four separate times using the Exchange Positions given below and the sprint relay team from Figure 17. When 5 m and -10 m were used as the optimal Exchange Positions the differences between the final times was 0.77 s.

Exchange Position (m)	Final Time (s)
5	35.99
0	36.15
-5	36.39
-10	36.76

Finally, a comparison of the results produced by the Optimal Model was made to the team that held the world record in the sprint relay of 37.50 s, set in Tokyo, September, 1991 (Figure 17). Assumptions were made about the maximum velocity achieved by each sprinter (Moravec *et al.*, 1988) since these data were not available. The Optimal Model produced a 35.99 s final time compared to the 37.50 s actually achieved. The Optimal Model was necessarily faster by nature of its mathematical design.

A factor that accounts for part of the 1.51 s difference is the simplistic manner in which the model deals with the actual handoff of the baton. The model assumed that when the sprinters were a certain distance apart (Exchange Distance), which can be modified by the user, a successful exchange occurred. The model also assumes that the approaching sprinter does not decelerate due to fatigue or to facilitate the handoff.

A second factor that affects the results was the method used to obtain the P.B.'s for the 100 m on the curve outlined in Alexandrov and Lucht (1980). Their method increased the time to run the 100 m curve from 10.1 s for the straight 100 m to 10.35 s. The implementation of the same formula provided an increase of over 1.5 s for the models developed as part of this study. Alexandrov and Lucht assumed that the values for f , k and v_{max} were satisfactory but were unable to truly validate their results. Consequently a choice was made, based on the data of Alexandrov and Lucht, to increase the time by 0.3 s for the sprinters in legs 1 and 3.

It is also important to mention that the models used in this study assume that the sprinters running legs 2 and 4 were sprinting a straight 125 m race but this is not the case since part of their sprint is on the curve. Since the sprinters are slower when running the curve a portion of the 1.51 s difference can be attributed to the modelling of the sprinters in legs 2 and 4.

A third factor that accounts for part of the 1.51 s difference is that the sprint relay team most likely did not perform optimal exchanges at the 5 m point as assumed by the

Optimal Model. The actual Exchange Positions are required to improve the accuracy of the results.

Though more data are required to improve the model it nonetheless provided an effective tool for understanding the intricacies of the sprint relay. The coach can modify the parameters in both the Optimal and Predictive Models and study the effects of these modifications consequently gaining insight into the effects of the different parameters. A major advantage of the modelling approach was that the coach does not require the sprinters to actually run the relay and possibly incur or aggravate an injury. Another advantage of using the modelling approach was that regardless of the weather or the condition of the athletes the computer model always performed consistently. Finally, all possible combinations of ordering the sprinters can be simulated to determine which produces the best final time.

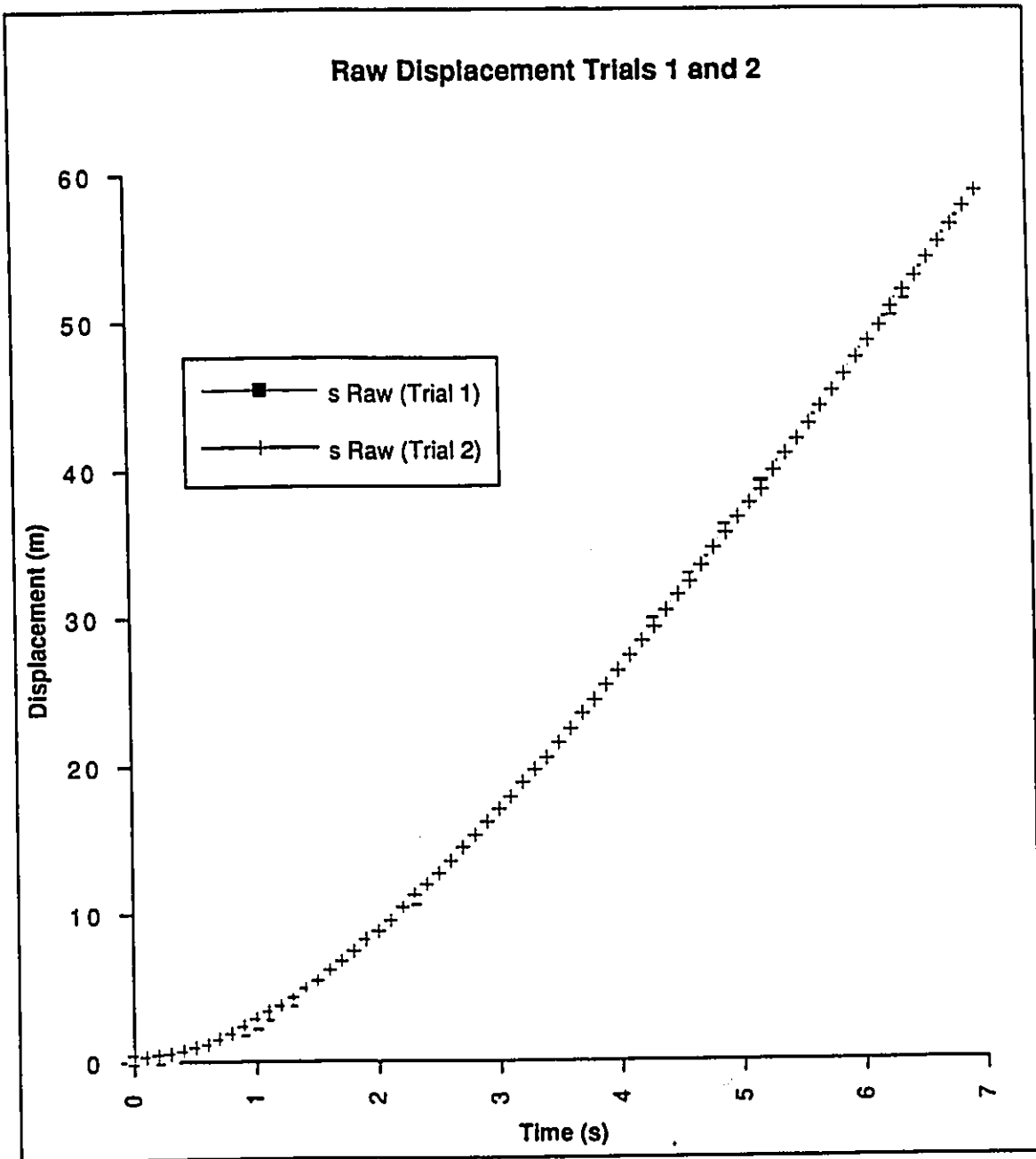


Figure 3: Raw displacement data from digitizing sessions 1 and 2.

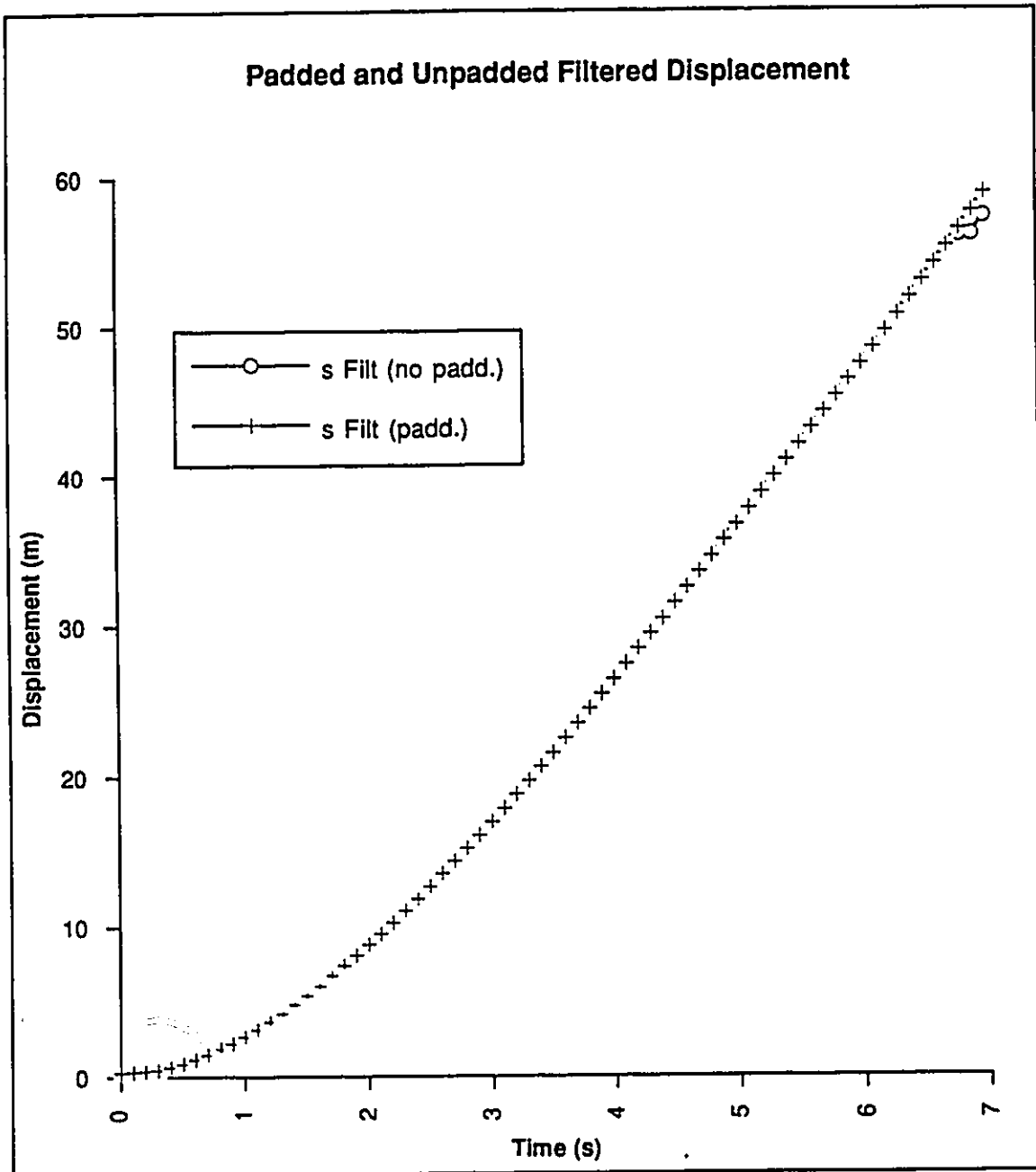


Figure 4: Padded and unpadded filtered displacement data from trial 1.

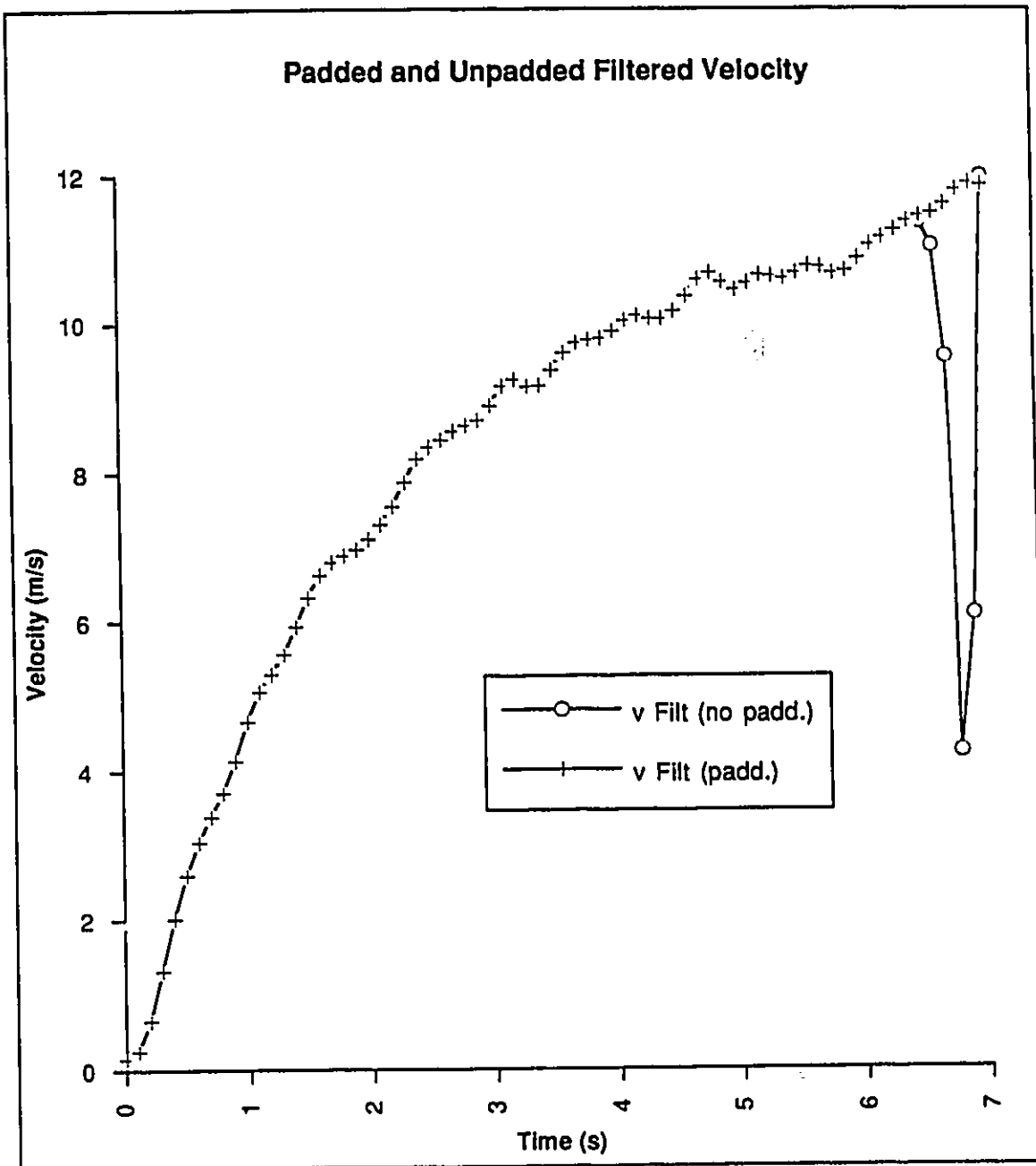


Figure 5: Padded and unpadded filtered velocity data from trial 1.

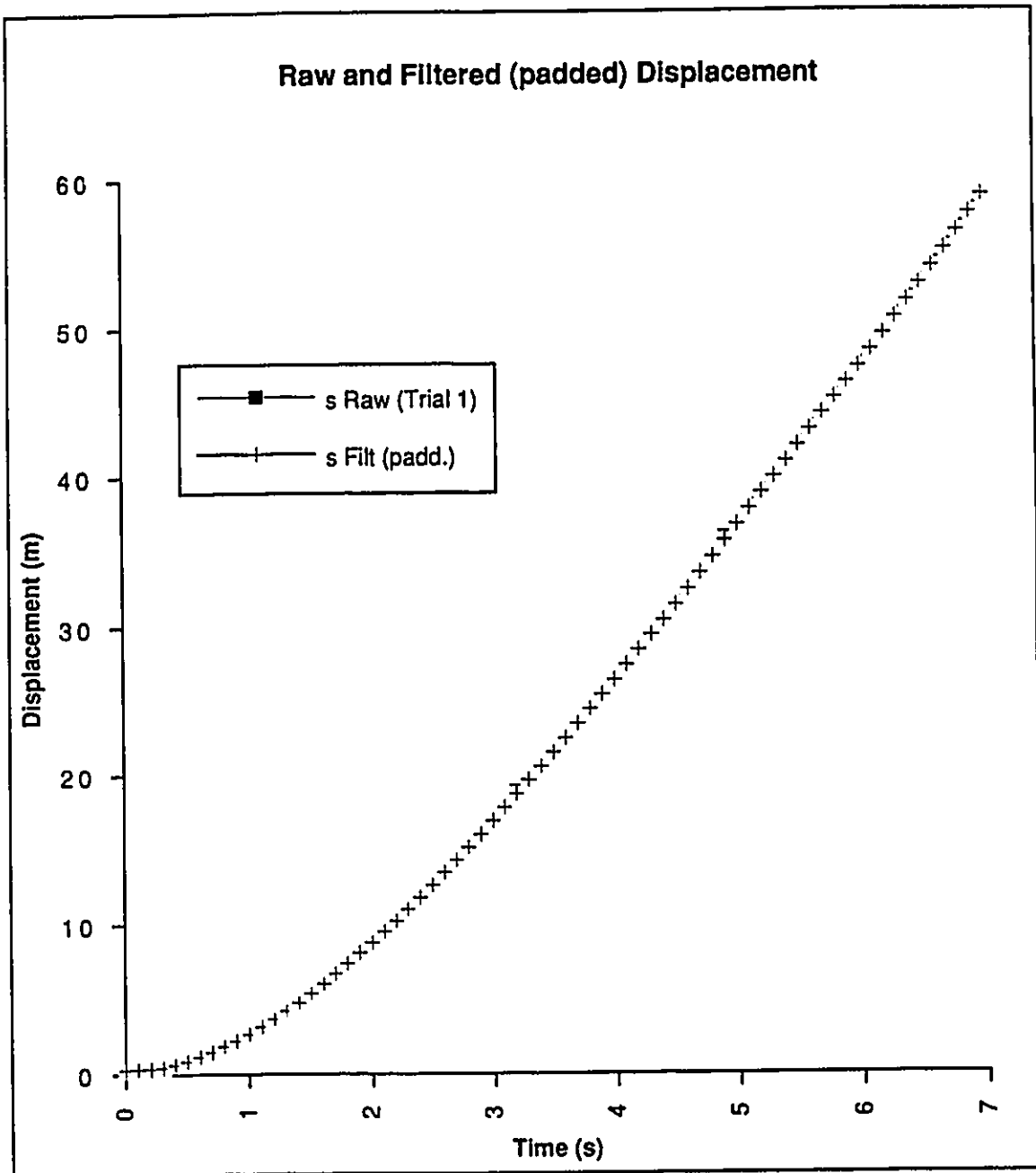


Figure 6: The raw and filtered (padded) displacement curves from trial 1.

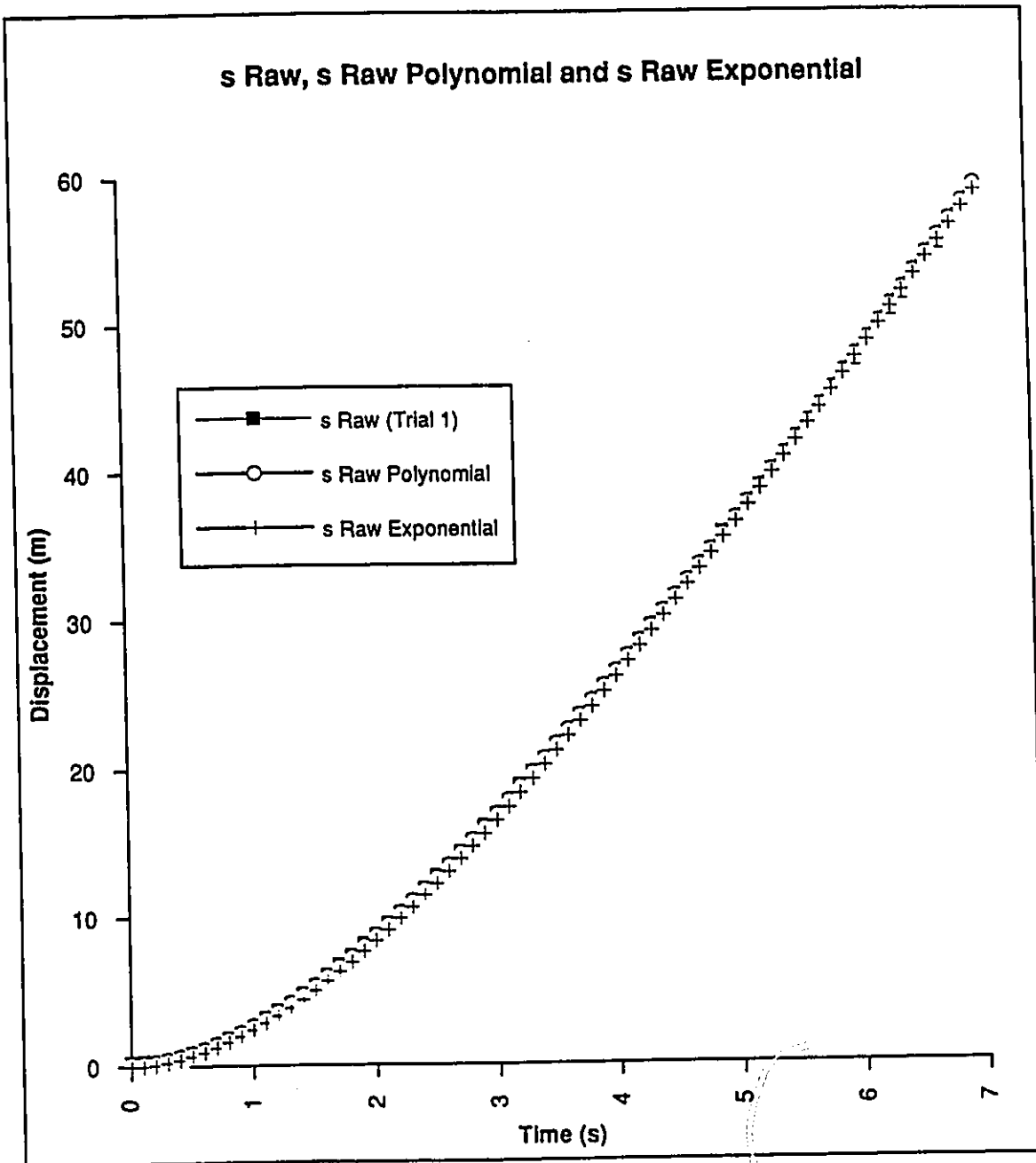


Figure 7: The raw data, raw polynomial (best fit) and raw exponential ($v_{max} = 11.87$, $k = 0.47061$) displacement curves from trial 1.

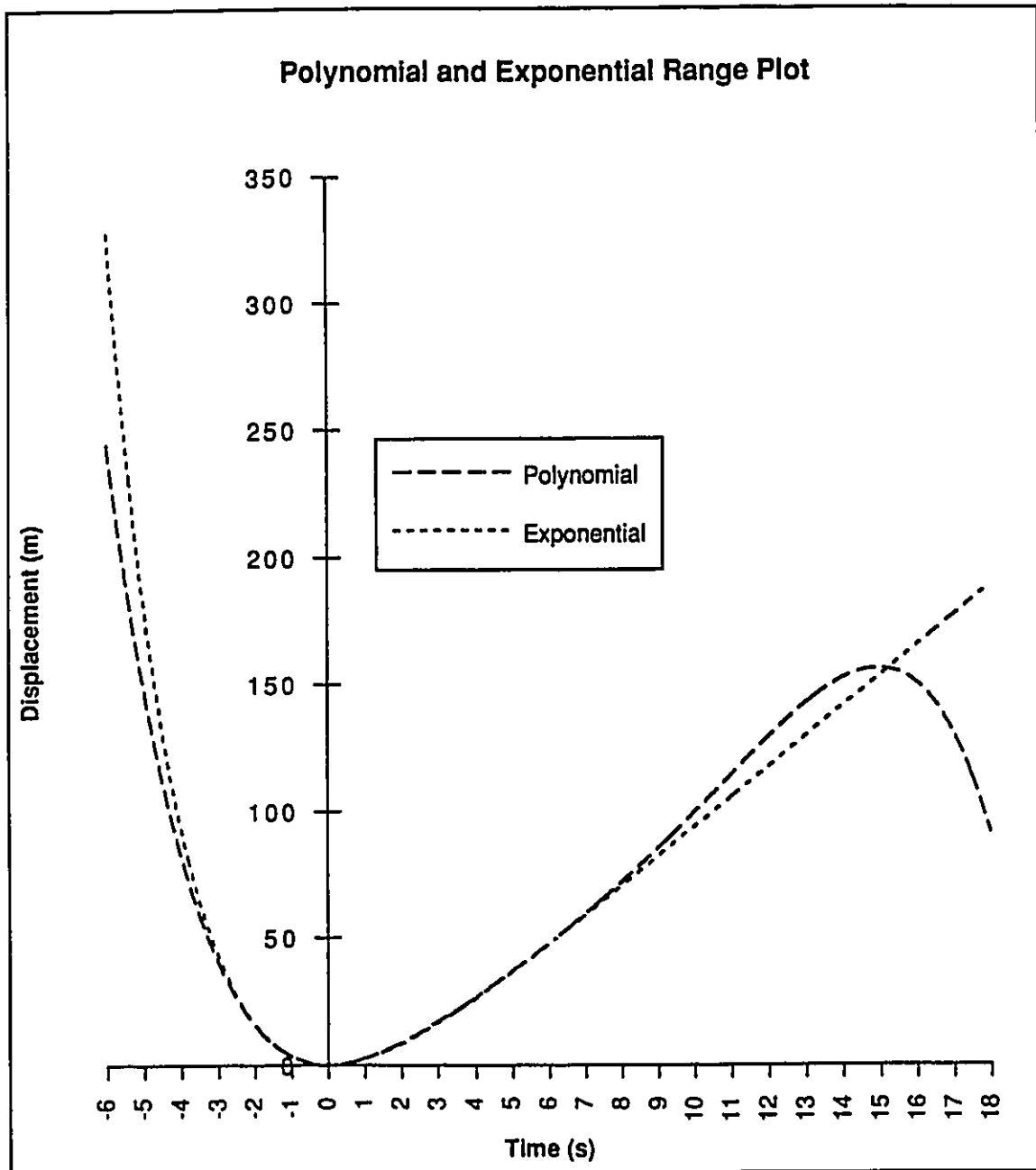


Figure 8: The raw polynomial and exponential displacement curves (for trial 1) plotted for values well beyond $0 \leq t \leq 7.0$ s. Note that the exponential displacement is more consistent for $t > 7.0$ s.

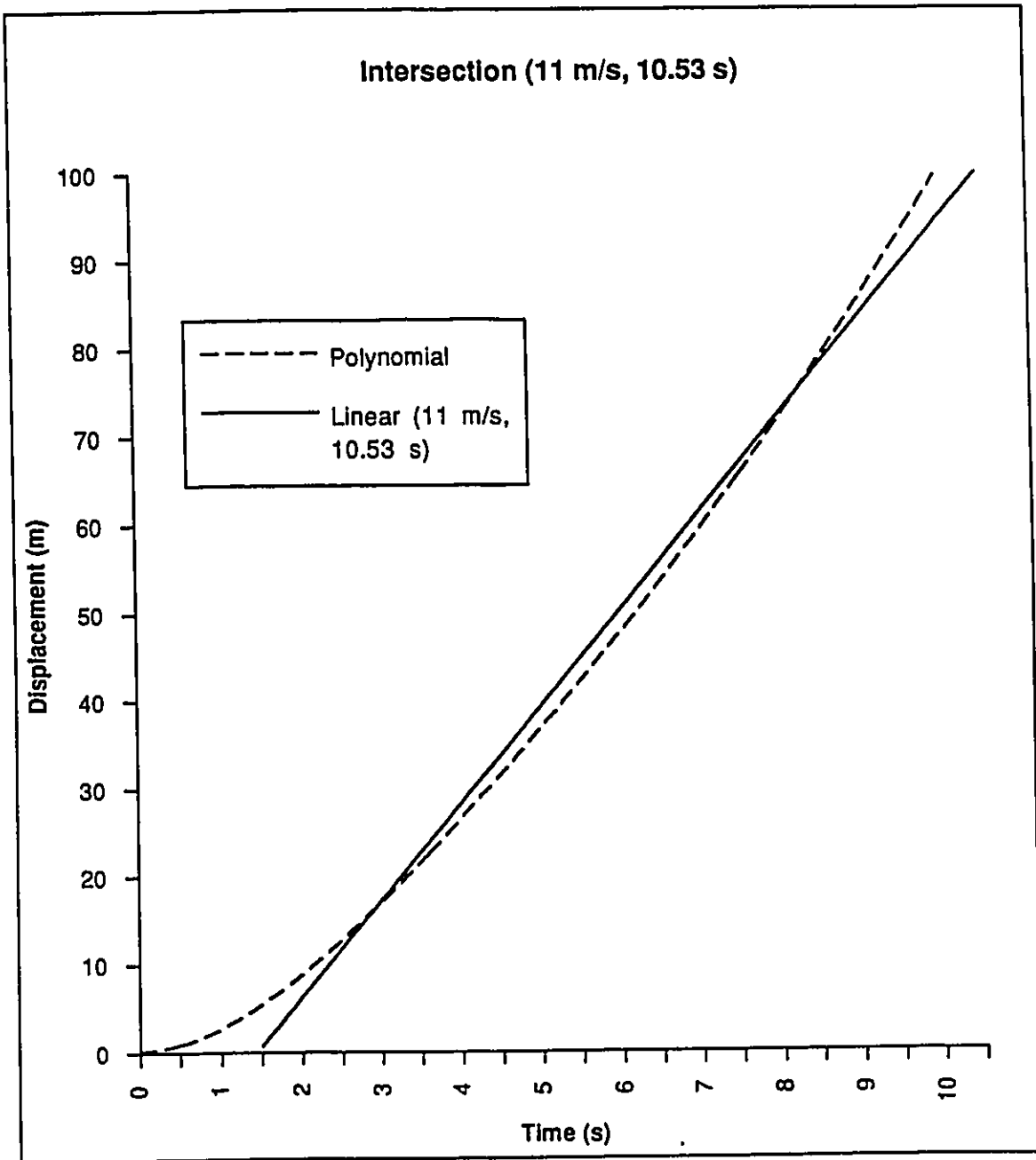


Figure 9: Displays the intersection of the acceleration (displacement polynomial) and maximum velocity (linear displacement) phases of the sprinter model. The sprinter information provided (Lemaire and Robertson, 1989) required a P.B. of 10.53 s with a maximum velocity of 11 m/s.

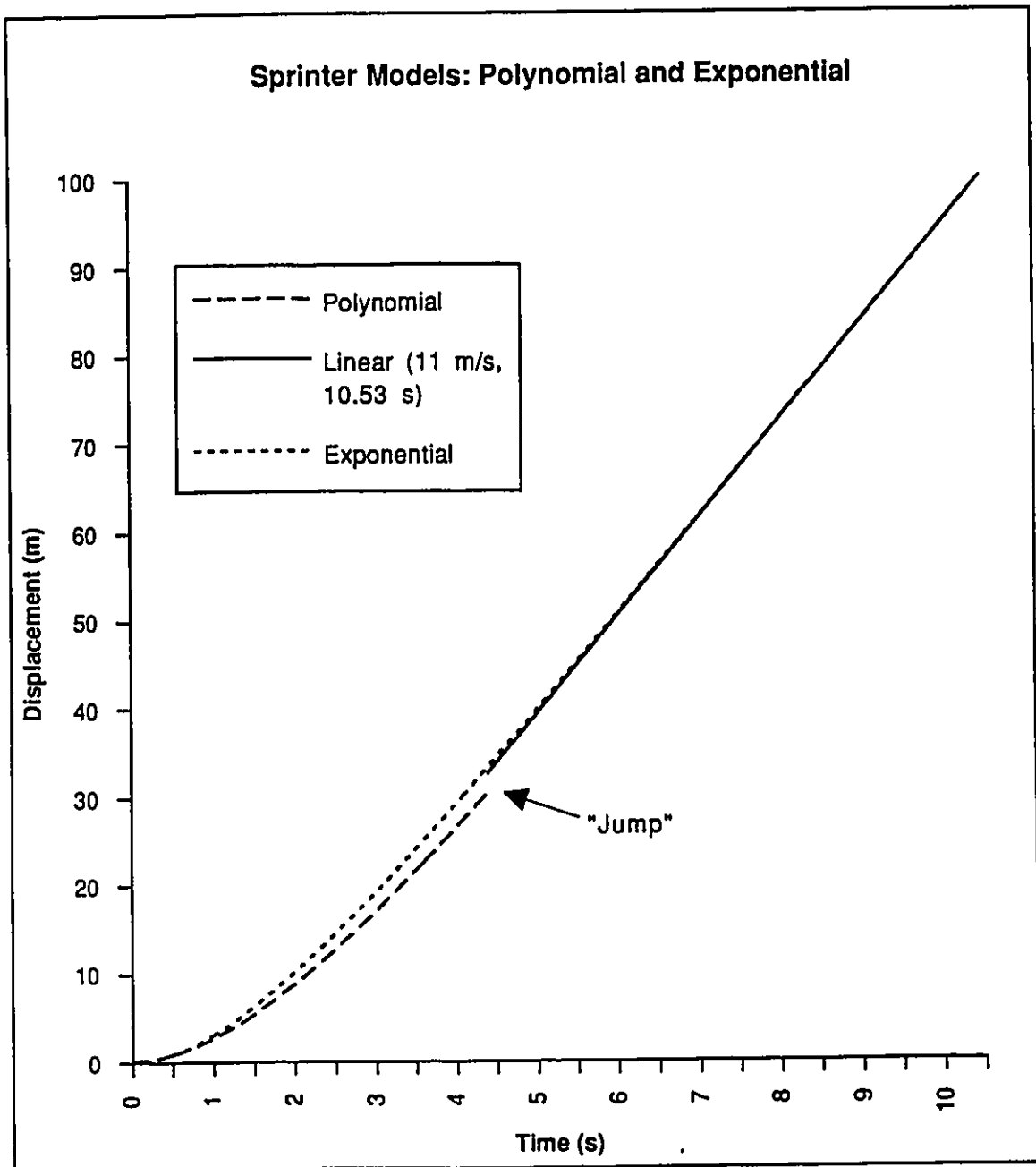


Figure 10: Note that for the Polynomial Model the displacement “jumped” to the linear portion of the model at the 30 m point. The model achieved a P.B. of 10.53 s with a maximum velocity of 11 m/s. The Exponential Model adjusts the acceleration portion of the displacement to reflect the sprinter’s potential capabilities.

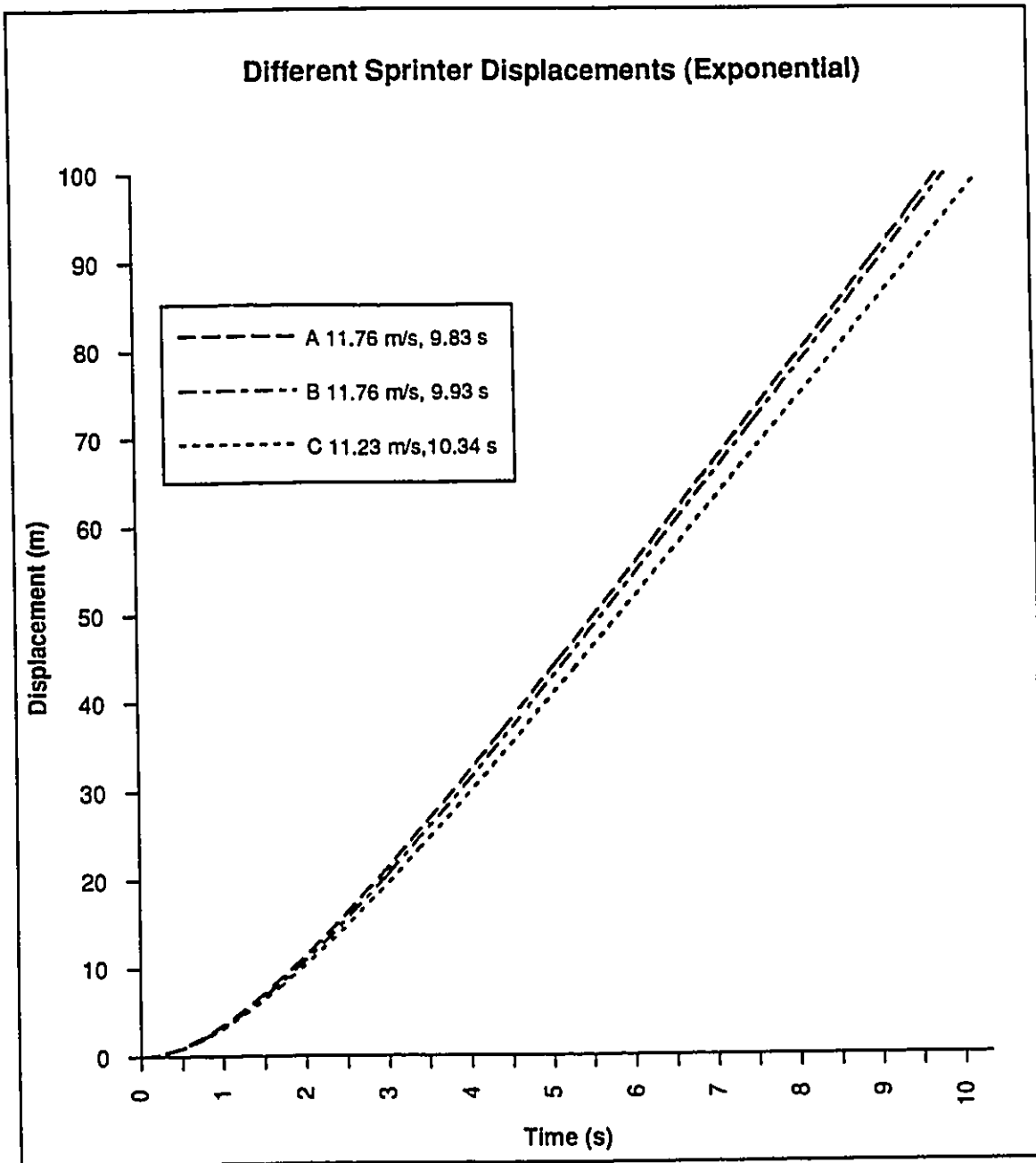


Figure 11: The displacement curves based on the exponential model for three different sprinters at the 1987 World Track Championships. Note how the acceleration phase adjusts for each sprinter.

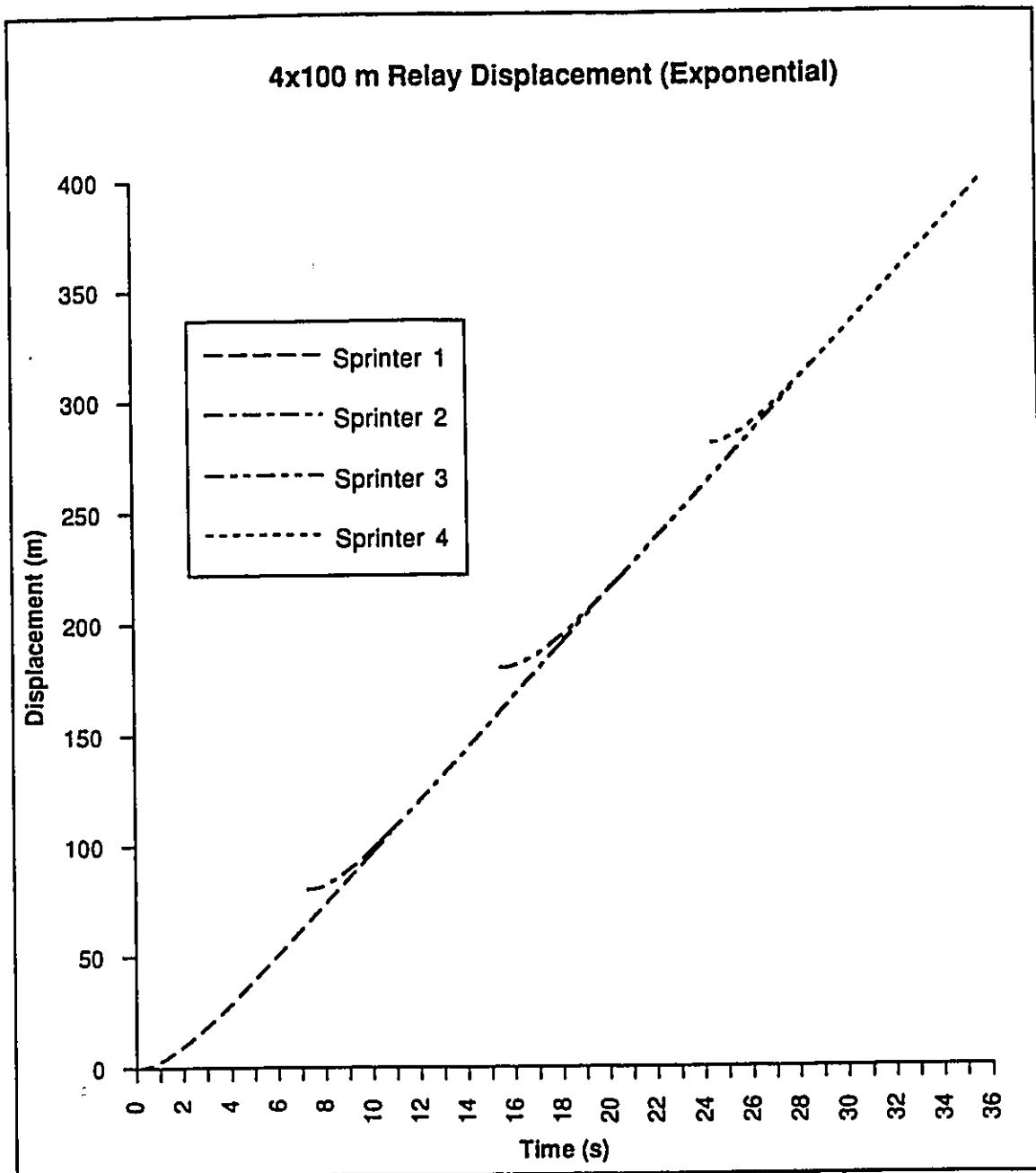


Figure 12: The results from the sprint relay team from Figure 17 were plotted (35.99 s). The information was obtained by executing the Exponential Optimal Model .

Exponential
Optimal Solution Report

Leg Sprinter Names	INPUTS					OUTPUTS					
	P.B. 100m (s)	Max Vel (m/s)	Start Posn (m)	Exchng Dist (m)	Exchng Posn (m)	Depart Dist (m)	Exchng Time (s)	Max Vel (m)	Distance Run (m)	Time (s)	100m Splits (s)
1 E Elastname	10.25	11.23	0.0	1.0	105.0	0.0	10.91	N/A	105.0	10.91	10.55
2 F Flastname	10.34	11.23	-20.0	1.0	205.0	15.8	19.84	N/A	125.0	12.48	9.08
3 G Glastname	10.08	11.62	-20.0	1.0	305.0	17.7	28.58	N/A	125.0	12.45	8.89
4 H Hlastname	9.93	11.76	-20.0	0.0	400.0	15.8	36.79	N/A	120.0	11.63	8.68

Final Time: 36.79 s

Figure 13: This report was generated by the Optimal Exponential Model. Note the good correspondence with the Predictive Model results generated in Figure 14.

Exponential
Predict Solution Report

Leg Sprinter Names	INPUTS					OUTPUTS					
	P.B. 100m (s)	Max Vel (m/s)	Start Posn (m)	Depart Dist (m)	Exchng Dist (m)	Exchng Posn (m)	Exchng Time (s)	Max Vel (m)	Distance Run (m)	Time (s)	100m Splits (s)
1 E Elastname	10.25	11.23	0.0	0.0	1.0	105.0	10.90	39.8	105.0	10.90	10.55
2 F Flastname	10.34	11.23	-20.0	15.8	1.0	205.0	19.84	29.0	125.0	12.48	9.08
3 G Glastname	10.08	11.62	-20.0	17.7	1.0	305.0	28.58	30.0	125.0	12.44	8.89
4 H Hlastname	9.93	11.76	-20.0	15.8	0.0	400.0	36.79	40.7	120.0	11.63	8.68

Final Time: 36.79 s

Figure 14: This report was generated by the Exponential Predictive Model. Note the good correspondence with the Optimal Model results generated in Figure 13.

Exponential
Optimal Solution Report

Leg Sprinter Names	INPUTS					OUTPUTS					
	P.B. 100m (s)	Max Vel (m/s)	Start Posn (m)	Exchng Dist (m)	Exchng Posn (m)	Depart Dist (m)	Exchng Time (s)	Max Vel Posn (m)	Distance Run (m)	Time (s)	100m Splits (s)
1 E Elastname	10.25	11.23	0.0	1.0	105.0	0.0	10.91	N/A	105.0	10.91	10.55
2 H Hlastname	9.93	11.76	-20.0	1.0	205.0	14.5	19.45	N/A	125.0	11.97	8.68
3 G Glastname	10.08	11.62	-20.0	1.0	305.0	19.6	28.19	N/A	125.0	12.45	8.89
4 F Flastname	10.34	11.23	-20.0	0.0	400.0	17.1	36.77	N/A	120.0	12.12	9.08

Final Time: 36.77 s

Figure 15: The overall time for this race was less than in Figure 13 since sprinter H was running a longer distance (125 m instead of 120 m) and the slower sprinter F was running a shorter distance (120 m instead of 125 m).



Exponential
Predict Solution Report

Leg Sprinter Names	INPUTS					OUTPUTS					
	P.B. 100m (s)	Max Vel (m/s)	Start Posn (m)	Depart Dist (m)	Exchng Dist (m)	Exchng Posn (m)	Exchng Time (s)	Max Vel (m)	Distance Run (m)	Time (s)	100m Splits (s)
1 E Elastname	10.25	11.23	0.0	0.0	1.0	98.6	10.34	39.8	105.0	10.34	10.55
2 F Flastname	10.34	11.23	-20.0	15.0	1.0	199.9	19.46	29.0	125.0	12.02	9.08
3 G Glastname	10.08	11.62	-20.0	17.0	1.0	298.2	28.12	30.0	125.0	11.86	8.89
4 H Hlastname	9.93	11.76	-20.0	15.0	0.0	400.0	36.98	40.7	120.0	11.63	8.68

Final Time: 36.98 s

Figure 16: The use of non-optimal Depart Distances increased the overall time by 0.19 s when compared to the Optimal results from Figure 13 because the receiving sprinter received the baton earlier in the exchange zone than in the Optimal case.

Exponential
Optimal Solution Report

Leg Sprinter Names	INPUTS					OUTPUTS					
	P.B. 100m (s)	Max Vel (m/s)	Start Posn (m)	Exchng Dist (m)	Exchng Posn (m)	Depart Dist (m)	Exchng Time (s)	Max Vel Posn (m)	Distance Run (m)	Time (s)	100m Splits (s)
1 A Alastname	10.00	11.68	0.0	1.0	105.0	0.0	10.64	N/A	105.0	10.64	10.30
2 B Blastname	9.88	11.76	-20.0	1.0	205.0	15.5	19.18	N/A	125.0	11.92	8.67
3 C Clastname	9.91	11.76	-20.0	1.0	305.0	18.7	27.80	N/A	125.0	12.25	8.77
4 D Dlastname	9.86	11.76	-20.0	0.0	400.0	15.6	35.99	N/A	120.0	11.56	8.66

Final Time: 35.99 s

Figure 17: The world record for the sprint relay was 37.50 s. The Optimal Model predicted a time of 35.99 s (4.03 % difference).

Conclusion

Based on the results obtained the following conclusions are warranted:

- 1) The Relay Software reasonably simulates the kinematic and temporal quantities of a 4x100 m relay and can be used by coaches to gain insights into the sprint relay without risking injury to the athletes.
- 2) The Exponential Model, using less information, describes the sprinter's kinematics better than the Polynomial Model.

Recommendations

Based on the understanding gained from this study the following recommendations are suggested:

- 1) More data from different elite sprinters are required to get a proper representation of the elite sprinting population.
- 2) More research is required to obtain data on sprinting the 100 m curve as compared to a standard 100 m sprint.
- 3) The manner in which exchanges are handled in the Optimal and Predictive Models should possibly be modified to include deceleration by the approaching sprinter.

- 4) If possible a complete validation of the 4x100 m relay including filming and the recording of all the parameters required for validating the Optimal and Predictive Models should be performed.

- 5) The Relay Software should be utilized by a committed group of elite sprinters as an aid to analyzing the relay, improving the team's understanding of the relay and finally reducing the time required to run the 4x100 m relay.

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Appendix: Data

Filename: ST100T08.CN1

English Filename: Start 100 m Trial 1, unpadding

Description: File contains x (horizontal) and y (vertical) displacement coordinates in metres (m) and also contains the frame count for a total of 71 frames.

x	y	Frame
0.278	0.673	ST101 -3
0.34	0.645	ST101 -2
0.291	0.656	ST101 -1
0.427	0.888	ST101 0
0.534	1.044	ST101 1
0.878	1.11	ST101 2
1.1	1.218	ST101 3
1.457	1.252	ST101 4
1.839	1.395	ST101 5
2.165	1.461	ST101 6
2.567	1.544	ST101 7
3.185	1.641	ST101 8
3.678	1.589	ST101 9
4.142	1.572	ST101 10
4.802	1.582	ST101 11
5.329	1.586	ST101 12
6.051	1.634	ST101 13
6.751	1.606	ST101 14
7.355	1.558	ST101 15
8.181	1.624	ST101 16
8.722	1.679	ST101 17
9.537	1.669	ST101 18
10.283	1.735	ST101 19
10.943	1.586	ST101 20
11.834	1.683	ST101 21
12.722	1.728	ST101 22
13.441	1.773	ST101 23
14.395	1.724	ST101 24
15.179	1.773	ST101 25
16.133	1.707	ST101 26
16.92	1.697	ST101 27
17.767	1.78	ST101 28
18.877	1.759	ST101 29
19.738	1.717	ST101 30
20.567	1.717	ST101 31
21.549	1.756	ST101 32
22.451	1.766	ST101 33
23.488	1.783	ST101 34
24.435	1.821	ST101 35
25.445	1.801	ST101 36
26.381	1.735	ST101 37

27.377	1.808	ST101 38
28.425	1.839	ST101 39
29.466	1.825	ST101 40
30.403	1.846	ST101 41
31.426	1.783	ST101 42
32.45	1.821	ST101 43
33.483	1.856	ST101 44
34.538	1.759	ST101 45
35.777	1.905	ST101 46
36.637	1.88	ST101 47
37.702	1.839	ST101 48
38.816	1.894	ST101 49
39.874	1.894	ST101 50
40.95	1.901	ST101 51
41.945	1.825	ST101 52
43.052	1.79	ST101 53
44.18	1.964	ST101 54
45.214	1.929	ST101 55
46.286	1.936	ST101 56
47.299	1.832	ST101 57
48.461	1.901	ST101 58
49.575	1.905	ST101 59
50.667	1.922	ST101 60
51.792	1.825	ST101 61
52.968	1.762	ST101 62
54.126	1.901	ST101 63
55.205	1.825	ST101 64
56.399	1.828	ST101 65
57.627	1.846	ST101 66
58.803	1.835	ST101 67

Filename: ST100T08.KN1

English Filename: Start 100 m Trial 1, unpadding, kinematics

Description: File contains raw (horizontal) displacement coordinates in metres.
Also contains the filtered (unpadding) displacement, velocity and acceleration as well as the 1st and 2nd integrals.

Input filename was:ST100T08.CN1
Array processed: 1 of 71 elements.
Sampling rate was: 10.00 Hz
Sampling time was: 7.00 s
Filter cutoff frequency was: 1.00 Hz
Number of filter passes was: 1
Filter was critically damped and zero-lag.

Time	Raw	Filtered	1st deriv.	2nd deriv.	1st integ.	2nd integ.
0.000	0.278	0.324	0.151	1.100	0.000	0.000
0.100	0.340	0.339	0.259	2.200	0.033	0.002
0.200	0.291	0.376	0.663	5.900	0.069	0.007
0.300	0.427	0.472	1.321	7.200	0.111	0.016
0.400	0.534	0.640	2.016	6.700	0.167	0.030
0.500	0.878	0.875	2.598	5.000	0.243	0.050
0.600	1.100	1.160	3.040	3.900	0.344	0.080
0.700	1.457	1.483	3.387	3.100	0.477	0.121
0.800	1.839	1.837	3.711	3.400	0.643	0.177
0.900	2.165	2.225	4.146	5.300	0.846	0.251
1.000	2.567	2.666	4.671	5.200	1.090	0.348
1.100	3.185	3.160	5.063	2.600	1.382	0.471
1.200	3.678	3.679	5.298	2.100	1.724	0.627
1.300	4.142	4.219	5.570	3.400	2.118	0.819
1.400	4.802	4.793	5.941	4.000	2.569	1.053
1.500	5.329	5.407	6.325	3.600	3.079	1.336
1.600	6.051	6.058	6.622	2.300	3.652	1.672
1.700	6.751	6.732	6.794	1.100	4.292	2.069
1.800	7.355	7.417	6.885	0.700	4.999	2.534
1.900	8.181	8.109	6.968	1.000	5.776	3.073
2.000	8.722	8.810	7.110	1.900	6.621	3.692
2.100	9.537	9.531	7.306	2.000	7.539	4.400
2.200	10.283	10.272	7.546	2.700	8.529	5.204
2.300	10.943	11.040	7.872	3.800	9.594	6.110
2.400	11.834	11.846	8.177	2.300	10.739	7.127
2.500	12.722	12.675	8.333	0.800	11.965	8.262
2.600	13.441	13.513	8.431	1.200	13.274	9.524
2.700	14.395	14.362	8.550	1.200	14.668	10.921
2.800	15.179	15.223	8.630	0.400	16.147	12.461
2.900	16.133	16.088	8.693	0.900	17.712	14.154
3.000	16.920	16.961	8.887	3.000	19.365	16.008
3.100	17.767	17.865	9.162	2.500	21.106	18.032
3.200	18.877	18.794	9.247	-0.800	22.939	20.234
3.300	19.738	19.714	9.151	-1.100	24.865	22.624

3.400	20.567	20.624	9.169	1.500	26.881	25.212
3.500	21.549	21.548	9.376	2.600	28.990	28.005
3.600	22.451	22.499	9.607	2.000	31.192	31.014
3.700	23.488	23.470	9.744	0.800	33.491	34.248
3.800	24.435	24.448	9.783	0.000	35.887	37.717
3.900	25.445	25.426	9.803	0.400	38.380	41.431
4.000	26.381	26.409	9.899	1.500	40.972	45.398
4.100	27.377	27.406	10.047	1.400	43.663	49.630
4.200	28.425	28.418	10.111	-0.200	46.454	54.136
4.300	29.466	29.428	10.065	-0.700	49.346	58.926
4.400	30.403	30.431	10.055	0.500	52.339	64.010
4.500	31.426	31.439	10.167	1.700	55.433	69.399
4.600	32.450	32.464	10.368	2.300	58.628	75.102
4.700	33.483	33.513	10.592	2.200	61.927	81.130
4.800	34.538	34.583	10.681	-0.400	65.332	87.493
4.900	35.777	35.649	10.553	-2.200	68.843	94.201
5.000	36.637	36.693	10.445	0.000	72.460	101.267
5.100	37.702	37.738	10.537	1.800	76.182	108.699
5.200	38.816	38.801	10.651	0.500	80.009	116.508
5.300	39.874	39.868	10.636	-0.800	83.942	124.706
5.400	40.950	40.928	10.605	0.100	87.982	133.302
5.500	41.945	41.989	10.678	1.300	92.128	142.308
5.600	43.052	43.064	10.774	0.600	96.381	151.733
5.700	44.180	44.144	10.759	-0.900	100.741	161.589
5.800	45.214	45.216	10.682	-0.600	105.209	171.887
5.900	46.286	46.281	10.710	1.200	109.784	182.636
6.000	47.299	47.358	10.878	2.200	114.466	193.849
6.100	48.461	48.456	11.052	1.300	119.257	205.535
6.200	49.575	49.568	11.151	0.700	124.158	217.706
6.300	50.667	50.686	11.243	1.200	129.171	230.372
6.400	51.792	51.817	11.352	1.000	134.296	243.545
6.500	52.968	52.957	11.351	-1.000	139.534	257.237
6.600	54.126	54.087	11.032	-5.400	144.886	271.458
6.700	55.205	55.163	9.535	-24.500	150.349	286.220
6.800	56.399	55.994	4.250	-81.200	155.907	301.533
6.900	57.627	56.013	6.071	117.600	161.507	317.403
7.000	58.803	57.208	11.953	58.800	167.168	333.837

Filename: ST100T08.CN0

English Filename: Start 100 m Trial 1, 7 points padded at end using linear extrapolation

Description: File contains x (horizontal) and y (vertical) displacement coordinates in metres (m) and also contains the frame count for a total of 71 frames.

x	y	Frame
0.278	0.673	ST101 -3
0.34	0.645	ST101 -2
0.291	0.656	ST101 -1
0.427	0.888	ST101 0
0.534	1.044	ST101 1
0.878	1.11	ST101 2
1.1	1.218	ST101 3
1.457	1.252	ST101 4
1.839	1.395	ST101 5
2.165	1.461	ST101 6
2.567	1.544	ST101 7
3.185	1.641	ST101 8
3.678	1.589	ST101 9
4.142	1.572	ST101 10
4.802	1.582	ST101 11
5.329	1.586	ST101 12
6.051	1.634	ST101 13
6.751	1.606	ST101 14
7.355	1.558	ST101 15
8.181	1.624	ST101 16
8.722	1.679	ST101 17
9.537	1.669	ST101 18
10.283	1.735	ST101 19
10.943	1.586	ST101 20
11.834	1.683	ST101 21
12.722	1.728	ST101 22
13.441	1.773	ST101 23
14.395	1.724	ST101 24
15.179	1.773	ST101 25
16.133	1.707	ST101 26
16.92	1.697	ST101 27
17.767	1.78	ST101 28
18.877	1.759	ST101 29
19.738	1.717	ST101 30
20.567	1.717	ST101 31
21.549	1.756	ST101 32
22.451	1.766	ST101 33
23.488	1.783	ST101 34
24.435	1.821	ST101 35

25.445	1.801	ST101 36
26.381	1.735	ST101 37
27.377	1.808	ST101 38
28.425	1.839	ST101 39
29.466	1.825	ST101 40
30.403	1.846	ST101 41
31.426	1.783	ST101 42
32.45	1.821	ST101 43
33.483	1.856	ST101 44
34.538	1.759	ST101 45
35.777	1.905	ST101 46
36.637	1.88	ST101 47
37.702	1.839	ST101 48
38.816	1.894	ST101 49
39.874	1.894	ST101 50
40.95	1.901	ST101 51
41.945	1.825	ST101 52
43.052	1.79	ST101 53
44.18	1.964	ST101 54
45.214	1.929	ST101 55
46.286	1.936	ST101 56
47.299	1.832	ST101 57
48.461	1.901	ST101 58
49.575	1.905	ST101 59
50.667	1.922	ST101 60
51.792	1.825	ST101 61
52.968	1.762	ST101 62
54.126	1.901	ST101 63
55.205	1.825	ST101 64
56.399	1.828	ST101 65
57.627	1.846	ST101 66
58.803	1.835	ST101 67
59.979	1.835	ST101 67
61.155	1.835	ST101 67
62.331	1.835	ST101 67
63.507	1.835	ST101 67
64.683	1.835	ST101 67
65.859	1.835	ST101 67
67.035	1.835	ST101 67

Filename: ST100T08.KN0

English Filename: Start 100 m Trial 1, padded, kinematics

Description: File contains raw (horizontal) displacement coordinates in metres.
Also contains the filtered (padded) displacement, velocity and acceleration as well as the 1st and 2nd integrals.

Input filename was:ST100T08.CN0
Array processed: 1 of 78 elements.
Sampling rate was: 10.00 Hz
Sampling time was: 7.70 s
Filter cutoff frequency was: 1.00 Hz
Number of filter passes was: 1

Filter was critically damped and zero-lag.

Time	Raw	Filtered	1st deriv.	2nd deriv.	1st integ.	2nd integ.
0.000	0.278	0.324	0.151	1.100	0.000	0.000
0.100	0.340	0.339	0.259	2.200	0.033	0.002
0.200	0.291	0.376	0.663	5.900	0.069	0.007
0.300	0.427	0.472	1.321	7.200	0.111	0.016
0.400	0.534	0.640	2.016	6.700	0.167	0.030
0.500	0.878	0.875	2.598	5.000	0.243	0.050
0.600	1.100	1.160	3.040	3.900	0.344	0.080
0.700	1.457	1.483	3.387	3.100	0.477	0.121
0.800	1.839	1.837	3.711	3.400	0.643	0.177
0.900	2.165	2.225	4.146	5.300	0.846	0.251
1.000	2.567	2.666	4.671	5.200	1.090	0.348
1.100	3.185	3.160	5.063	2.600	1.382	0.471
1.200	3.678	3.679	5.298	2.100	1.724	0.627
1.300	4.142	4.219	5.570	3.400	2.118	0.819
1.400	4.802	4.793	5.941	4.000	2.569	1.053
1.500	5.329	5.407	6.325	3.600	3.079	1.336
1.600	6.051	6.058	6.622	2.300	3.652	1.672
1.700	6.751	6.732	6.794	1.100	4.292	2.069
1.800	7.355	7.417	6.885	0.700	4.999	2.534
1.900	8.181	8.109	6.968	1.000	5.776	3.073
2.000	8.722	8.810	7.110	1.900	6.621	3.692
2.100	9.537	9.531	7.306	2.000	7.539	4.400
2.200	10.283	10.272	7.546	2.700	8.529	5.204
2.300	10.943	11.040	7.872	3.800	9.594	6.110
2.400	11.834	11.846	8.177	2.300	10.739	7.127
2.500	12.722	12.675	8.333	0.800	11.965	8.262
2.600	13.441	13.513	8.431	1.200	13.274	9.524
2.700	14.395	14.362	8.550	1.200	14.668	10.921
2.800	15.179	15.223	8.630	0.400	16.147	12.461
2.900	16.133	16.088	8.693	0.900	17.712	14.154
3.000	16.920	16.961	8.887	3.000	19.365	16.008
3.100	17.767	17.865	9.162	2.500	21.106	18.032
3.200	18.877	18.794	9.247	-0.800	22.939	20.234
3.300	19.738	19.714	9.151	-1.100	24.865	22.624
3.400	20.567	20.624	9.169	1.500	26.881	25.212

3.500	21.549	21.548	9.376	2.600	28.990	28.005
3.600	22.451	22.499	9.607	2.000	31.192	31.014
3.700	23.488	23.470	9.744	0.800	33.491	34.248
3.800	24.435	24.448	9.783	0.000	35.887	37.717
3.900	25.445	25.426	9.803	0.400	38.380	41.431
4.000	26.381	26.409	9.899	1.500	40.972	45.398
4.100	27.377	27.406	10.047	1.400	43.663	49.630
4.200	28.425	28.418	10.111	-0.200	46.454	54.136
4.300	29.466	29.428	10.065	-0.700	49.346	58.926
4.400	30.403	30.431	10.055	0.500	52.339	64.010
4.500	31.426	31.439	10.167	1.700	55.433	69.399
4.600	32.450	32.464	10.368	2.300	58.628	75.102
4.700	33.483	33.513	10.592	2.200	61.927	81.130
4.800	34.538	34.583	10.681	-0.400	65.332	87.493
4.900	35.777	35.649	10.553	-2.200	68.843	94.201
5.000	36.637	36.693	10.445	0.000	72.460	101.267
5.100	37.702	37.738	10.537	1.800	76.182	108.699
5.200	38.816	38.801	10.651	0.500	80.009	116.508
5.300	39.874	39.868	10.636	-0.800	83.942	124.706
5.400	40.950	40.928	10.605	0.100	87.982	133.302
5.500	41.945	41.989	10.678	1.300	92.128	142.308
5.600	43.052	43.064	10.774	0.600	96.381	151.733
5.700	44.180	44.144	10.759	-0.900	100.741	161.589
5.800	45.214	45.216	10.682	-0.600	105.209	171.887
5.900	46.286	46.281	10.710	1.200	109.784	182.636
6.000	47.299	47.358	10.878	2.200	114.466	193.849
6.100	48.461	48.456	11.052	1.300	119.257	205.535
6.200	49.575	49.568	11.151	0.700	124.158	217.706
6.300	50.667	50.686	11.246	1.200	129.171	230.372
6.400	51.792	51.817	11.367	1.200	134.296	243.545
6.500	52.968	52.960	11.435	0.200	139.535	257.237
6.600	54.126	54.104	11.470	0.500	144.888	271.458
6.700	55.205	55.254	11.595	2.000	150.356	286.220
6.800	56.399	56.423	11.779	1.700	155.939	301.535
6.900	57.627	57.610	11.868	0.100	161.641	317.414
7.000	58.803	58.797	11.838	-0.700	167.461	333.869

Filename: ST100T09.CN1

English Filename: Start 100 m Trial 2, unpadding

Description: File contains x (horizontal) and y (vertical) displacement coordinates in metres (m) and also contains the frame count for a total of 71 frames.

x	y	Frame
0.487	0.682	ST101 -3
0.397	0.653	ST101 -2
0.48	0.66	ST101 -1
0.573	0.757	ST101 0
0.721	1.046	ST101 1
0.985	1.078	ST101 2
1.183	1.313	ST101 3
1.526	1.324	ST101 4
1.897	1.417	ST101 5
2.366	1.547	ST101 6
2.867	1.67	ST101 7
3.383	1.692	ST101 8
3.755	1.684	ST101 9
4.339	1.692	ST101 10
4.934	1.72	ST101 11
5.468	1.728	ST101 12
6.16	1.778	ST101 13
6.762	1.609	ST101 14
7.44	1.616	ST101 15
8.23	1.645	ST101 16
8.775	1.616	ST101 17
9.511	1.753	ST101 18
10.409	1.807	ST101 19
11.249	1.706	ST101 20
11.938	1.648	ST101 21
12.71	1.652	ST101 22
13.507	1.681	ST101 23
14.448	1.634	ST101 24
15.296	1.684	ST101 25
16.172	1.702	ST101 26
17.056	1.742	ST101 27
17.874	1.699	ST101 28
18.855	1.796	ST101 29
19.732	1.623	ST101 30
20.493	1.655	ST101 31
21.524	1.713	ST101 32
22.448	1.655	ST101 33
23.526	1.767	ST101 34
24.421	1.774	ST101 35
25.416	1.699	ST101 36
26.354	1.663	ST101 37

27.385	1.666	ST101 38
28.359	1.771	ST101 39
29.289	1.655	ST101 40
30.408	1.742	ST101 41
31.45	1.771	ST101 42
32.341	1.782	ST101 43
33.426	1.771	ST101 44
34.631	1.774	ST101 45
35.623	1.742	ST101 46
36.69	1.832	ST101 47
37.653	1.987	ST101 48
38.519	1.912	ST101 49
39.85	1.814	ST101 50
40.953	1.98	ST101 51
41.927	1.93	ST101 52
43.002	1.875	ST101 53
44.192	1.926	ST101 54
45.227	1.865	ST101 55
46.331	1.893	ST101 56
47.434	1.901	ST101 57
48.563	1.908	ST101 58
49.584	1.832	ST101 59
50.857	2.027	ST101 60
51.99	1.893	ST101 61
52.92	1.868	ST101 62
54.186	1.829	ST101 63
55.225	1.764	ST101 64
56.401	1.771	ST101 65
57.638	1.767	ST101 66
58.712	1.836	ST101 67