Short-term Forecasting by Univariate Time Series Models with Application

by

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0. INTRODUCTION

In all situations of life, every choice or decision has future implications. Good decision-making can become better or worse, depending on the quality of the forecasts which underline the decision-making process. At present two types of forecasting techniques are widely used: Econometric Modelling and Time Series Analysis. Econometric models are systems of statistical equations which specify relationships among economic variables based on economic theories. The coefficients of the equations are estimated from past data. The equations are used to quantify assumed structural relationships. They form the basis of forecasts of what will happen to the economy if certain conditions are fulfilled.

The time series analysis approach basically consists of describing a variable as a function of its own past behaviour. One particular form of time series modelling, the Auto-Regressive Integrated Moving Average (ARIMA) model, developed by Box and Jenkins (1970) is widely applied. Although time series models have been studied for many years, Box and Jenkins popularized their use, demonstrated how to extend their application to seasonal data, and made the methodology operational. Consequently, ARIMA models are often referred to as Box-Jenkins models. These models are univariate time series models and do not use explanatory variables or causal variables that are constrained by economic theory (Vandaele, 1983).

This paper will introduce the technique of ARIMA modelling as outlined by Box and Jenkins, and forecast GDP values for 1989. Comparisons will also be made regarding forecast accuracy between ARIMA models and other popular forecasting methods. The paper is divided into three sections. The first section deals with the theoretical aspects of time series modelling, the second section applies this methodology to forecast gross domestic product, and the third section presents other forecasting models and compares their forecasts with those obtained from time series models. While the theoretical section is generally easy to follow, some highly technical aspects of the discussion are given in the Appendix.

1. MODELLING ARIMA PROCESSES.

Univariate Box-Jenkins (B-J from here on) is a time series modelling process which describes a single series as a function of its own past values. The purpose of the B-J process is to find the equation that reduces a time series with underlying structure to white noise (Box and Jenkins, 1970). Since the equation accounts for the predictable portion of the time series, it can be used to forecast future values of the series.

The modelling procedure itself is a three stage iterative process :

a. Identification:

Choosing a tentative model form by examining a plot of the series and several key statistics.

b. Estimation:

Fitting an appropriate model through some non-linear estimation procedure to the time series under study.

c. Forecasting:

Using the fitted model to predict future values of the time series.

The ARIMA models used for forecasting the time series are of the general multiplicative type (Box,G.E.P. and Jenkins,G.M.,1970), that is:

(1.0) $$\phi_p(B)\Phi_P(B^s)\ \nabla^d\nabla^D_s Z_t = \theta_q(B)\Theta_Q(B^s)a_t$$

Where s denotes periodicity of the seasonal component (equal to 4 for our series); B denotes the backward operator, i.e:

$$BZ_t=Z_{t-1};\ B^sZ_t=Z_{t-s};$$

$\nabla^d=(1-B)^d$ is the ordinary difference operator of order d; $\nabla^D_s=(1-B^s)^D$ is the seasonal difference operator of order D; $\phi_p(B)$ and $\Phi_P(B^s)$ are stationary autoregressive operators (they are polynomials in B of degree p and in B^s of degree P, respectively); $\theta_q(B)$ and $\Theta_Q(B^s)$ are invertible moving average operators (they are polynomials in B of degree q and in B^s of degree Q, respectively); a_t is a purely random process.

The general multiplicative model (1.0) is said to be of order $(p,d,q)\ (P,D,Q)_s$.

1.1 Model Identification

The identification phase entails examining the time series in order to choose a tentative model form. There are several key statistical tools used during this phase. The two most important tools are the autocorrelation function and the partial autocorrelation function of a time series (formal definition of these functions are provided in the appendix). The first step in identification is to make the series stationary (i.e. the mean of the time series, its variance, and its covariance with other values of the series do not depend on time). This implies two types of methods for inducing stationarity. Applying the appropriate differencing factor to a series may create a mean stationary series. By applying the correct power transformation, a variance stationary series may be obtained.

The autocorrelations of a time series provides an indication as to the appropriate level of differencing that is required since the autocorrelations provide us with a means of how much correlation or interdependency there is between neighbouring data points in the series. The need for a power transformation can be ascertained by examining plots of both the original series and the transformed series (Cryer, 1986). If the autocorrelation function starts out high and decays slowly, it usually implies the need for differencing. To determine the order of the differencing, the number of time periods between the relatively high autocorrelation is usually a good indicator (see figure 1a for example). On the other hand if the series shows a variance that changes over time, transforming the original series may provide a stationary variance series.

The transformation can be obtained from a flexible family of transformations introduced by Box and Cox (1964,Vandaele,1983). For a given value of λ, the transformation:

$$\begin{aligned} g(x) &= x^{\lambda}-1 / \lambda \quad \lambda \neq 0 \\ &= \log(x) \quad \lambda = 0 \end{aligned}$$

note if λ equals 1, it implies the original series, a value of -1 implies the inverse (of the original series) and so on (see appendix also).

The Box-Cox method is more accurate than just observing the plot of a series. If the type of software allows for Box-Cox test one can estimate a simple mean model with lambda determination option used (AFS, 1986).

Once stationarity is obtained, the autocorrelation and partial autocorrelation functions of the transformed and stationary series (ARMA) will be studied so as to determine the order of the autoregressive and/or moving average process.

1.2 ESTIMATION

From the identification stage a tentative ARIMA model is specified for the data generating process on the basis of the estimated autocorrelation and partial autocorrelation (Box and Jenkins, 1970). The following are some possible results:

a. For an MA(q) process the autocorrelation $Q_k = 0$ for $k>q$ and the partial autocorrelations taper off. To determine a cut off point to the autocorrelation function the sample autocorrelations are used.

b. For an AR(p) the partial autocorrelations $\phi_{kk} = 0$ for $k>p$ and the autocorrelations taper off. A cutoff point of the partial autocorrelation function may be determined by comparing the estimates with T, since $(1/ T)^{.5}$ is the approximate standard deviation of the estimators ϕ_{kk} for $k>p$.

c. If neither the autocorrelation nor the partial autocorrelations have a cutoff point, an ARIMA model may be adequate. The AR and the MA degree have to be inferred from the particular pattern of the autocorrelation and partial autocorrelation.

Once the identification is completed, a non-linear least squares or a maximum likelihood estimation based on the Marquardt Algorithm, is employed (Box-Jenkins,1970,pp 504 - 505). A pure AR(1) process sometimes known as a "random walk" model can be estimated by linear methods. The non-linear estimation, whether by minimizing least squares or maximizing a likelihood function, makes an appropriate computer software necessary to lessen the labour input and computational time.

The next step after estimation is diagnostic checking. These tests are for necessity, invertibility and sufficiency. Each parameter included in the model should be statistically significant (necessity) and each factor must be invertible (see appendix for detail). In addition, the residuals from the estimated models should be white noise (model sufficiency).

The test for necessity is performed by examining the T-ratios for the individual parameter estimates. Parameters with non-significant coefficients may be deleted from the model in order to have a parsimonious model.

Invertibility is determined by extracting the roots from each factor in the model. All the roots must lie outside of the unit circle. If one of the factors is non-invertible, then the model must be adjusted. The appropriate adjustment is dictated by the type of the factor that is non-invertible. For example, a non-invertible autoregressive factor usually indicates under-differencing, while a non-invertible moving average factor may indicate over differencing. A non-invertible moving average factor could also represent the presence of a deterministic factor. Since the model fixup is not really clear-cut, the overall model must be considered when adjusting for non-invertibility.

The residuals are tested for white noise by studying the autocorrelation and partial autocorrelation of the residuals. Furthermore, a test statistics Q or "portmanteau test" is performed on the residuals autocorrelations of all lags (the test is formally introduced in the appendix). If the model is misspecified or inappropriately fitted, the Q test tends to be inflated (Box-Jenkins, 1970; Vandaele, 1983).

1.3 FORECASTING

The forecasting function of the general multiplicative model (1.0) can be expressed in different forms. For computational purpose, the difference equation form is the most useful. Thus, at time t+r the ARIMA model (1.0) may be written:

$$(1.1)\quad Z_{t+r} = \psi_1 Z_{t+r-1} + \ldots + \psi_m Z_{t+r-m} - a_{t+r} - \pi_r a_{t+r-1} - \ldots - \pi_n a_{t+r-n}$$

where $m=p+(s.P)+d+(s.D)$ and $n=q+(s.Q)$; $\psi(B)=\phi_p(B^s)\nabla^d\nabla^D_s$ is the general autoregressive operator; $\pi(B)=\theta_q(B)\Theta_Q(B)$ is the general moving average operator. For example, if the ARIMA model is of order $(0,1,1)(0,1,1)_{12}$ the difference equation that generates the observations Z_{t+r}, is:

$$(1.2)\quad Z_{t+r} = Z_{t+r-1} + Z_{t+1-12} - Z_{t+r-13} + a_{t-r} - \theta a_{t+r-1}$$
$$-\Theta a_{t+r-12} - \theta\Theta a_{t+r-13}$$

Standing at origin t, to make a forecast $Z^f_t(r)$ of Z_{t+r}, the conditional expectation of (1.1) is taken at time t with the following assumptions:

(1.3) $E_t(Z_{t+j}) = Z_{t+j},\ j=0$; $E_t(Z_{t+j}) = Z^f_t(j),\ j>0$

(1.4) $E_t(a_{t+j}) = a_{t+j},\ j\leq 0$; $E_t(a_{t+j}) = 0,\ j>0$

where $E_t(Z_{t+j})$ is the conditional expectation of Z_{t+j} taken at origin t. Thus, the forecasts $Z^f_t(r)$ for each lead time are computed from previous observed Z's, previous forecasts of Z's and current and previous random shocks a's. The unknown a's are replaced by zeroes.

In general, if the moving average operator $\pi(B)=\theta(B)\Theta(B^s)$ is of degree q+(s.Q), the forecast equations for $Z^f_t(1)$, $Z^f_t(2)$, ..., $Z^f_t(q+(s.Q))$ will depend directly on the a's but forecasts at longer lead times will not. The latter will receive indirectly, the impact of the a's by means of the previous forecasts. In effect, $Z_t(q+(s.Q)+1)$ will depend on the q+(s.Q) previous Z_t, which in turn will depend on the a's.

From the view point of studying the nature of the forecasts, it is important to consider the explicit form of the forecasting function. For r>n=q+(s.Q), the conditional expectation of (1.1) at time t is:

(1.5) $Z^f_t(r)-\psi_1 Z^f_t(r-1)-\ldots-\psi_m Z^f_t(r-m)=0 \quad r>m$

and the solution of this difference equation is:

(1.6) $Z^f_t(r)= b_0(t) f_0(r)+b_1^{(t)} f_1(r)+\ldots+b_{m-1}^{(t)} f_{m-1}^{(r)} \quad r>n-m$

This function is called the "eventual forecast function", eventual because when n>m, it supplies the forecasts only for lead times r>n-m.

In (1.6), $f_0(R)$, $f_1(R)$, ...$f_{m-1}(R)$ are functions of the lead time R and in general they include polynomials, exponentials, sines and cosines, and products of these functions. For a given origin t, the coefficients $b_j^{(t)}$ are constants applying for all lead time R but they change from one origin to the next, adapting themselves to the particular part of the series being considered. It is important to point out that it is the general autoregressive operator $\psi(B)$ defined above , which determines the mathematical form of the forecasts function, i.e., the nature of the f's. In other words, it determines whether the forecasting function is to be a polynomial, a mixture of sines and cosines, a mixture of exponentials or some combinations of these functions.

2. EMPIRICAL INVESTIGATION

To begin the empirical investigation of the ARIMA modelling and forecasting, we choose the Gross Domestic Product (GDP) in seasonally adjusted form. The choice of GDP is based on its importance for micro-economic decision makers such as firms and macro-economic agents like governments and labour unions.

For governments a higher level of growth in GDP implies a strong economy, and hence tighter fiscal or monetary policy may be advisable in order to halt the economy from "over-heating". Firms, on the other hand, will be concerned about higher inflation rates if the GDP growth is more than anticipated. Labour unions also have keen interest about the level of GDP, especially if they are going to negotiate wage contracts in the near future.

The gross domestic product is published every quarter by Statistics Canada both in real (constant 1981 prices) or nominal (market prices) terms. For the type of policy evaluations mentioned above real GDP is often consulted and it is a much preferred indicator of economic growth than the nominal GDP (obviously, nominal GDP movements are "contaminated" by price level developments). However, for illustrative purposes both the nominal and real GDP are discussed here.

2.2 THE MODELLING PROCEDURE

2.2.A. NOMINAL GDP

As mentioned earlier the time series under consideration are the GDP at market and constant prices for the period 1976 first quarter to 1988 fourth quarter, a total of 52 observations. From the plot of the data (see figure 1) the variance of this series (GDP at market price) appears to be roughly constant through time while the mean seems to increase. This may imply an estimated autocorrelation function (acf) that decays slowly to zero indicating a non-stationary mean and the need for differencing.

Identification I: To start the identification procedure the acf and partial autocorrelation function (pacf) of the undifferenced data (see figure 1a, 1b) are analyzed. The estimated

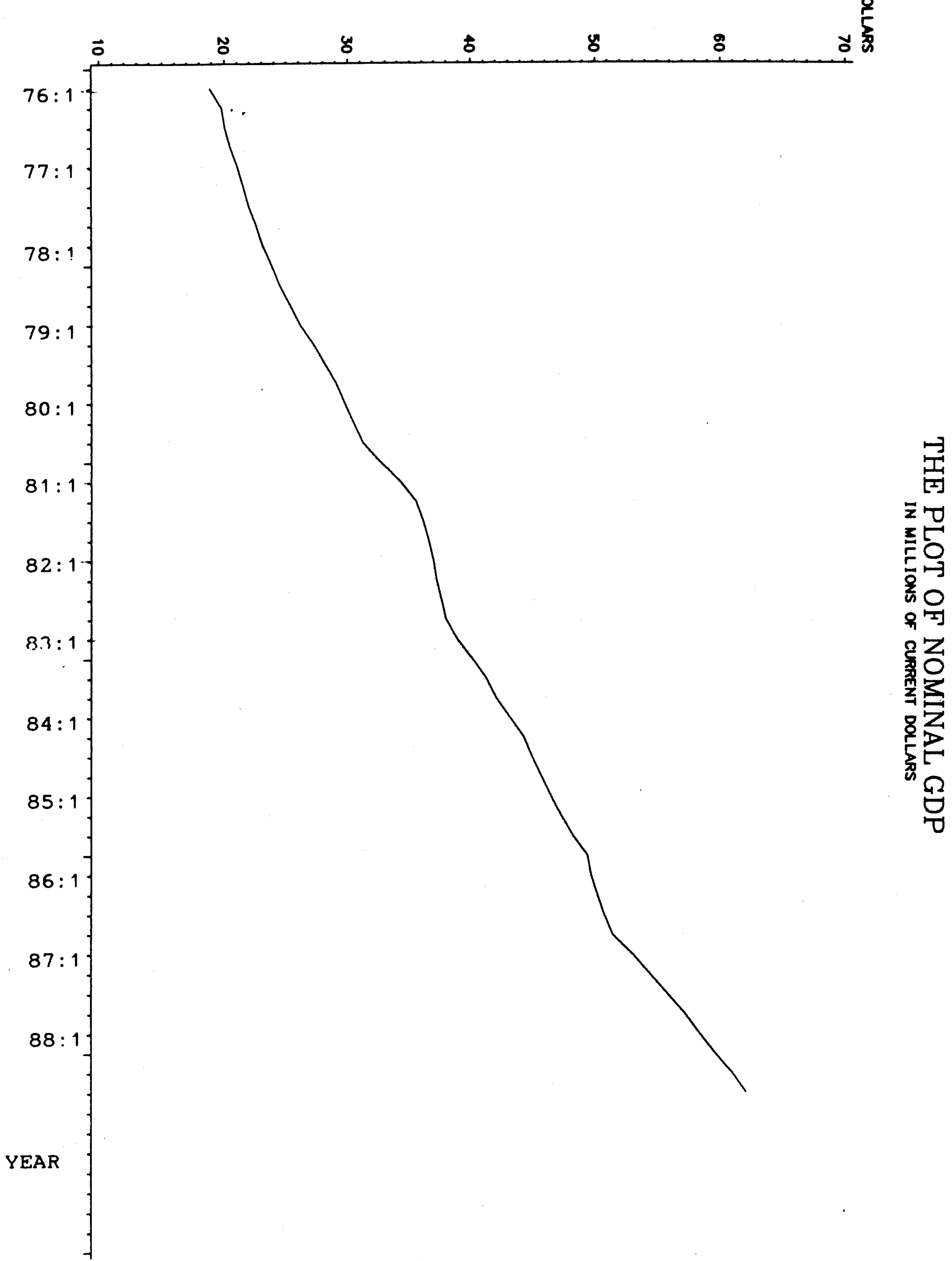
THE PLOT OF NOMINAL GDP
IN MILLIONS OF CURRENT DOLLARS
DOLLARS
10
20
30
40
50
60
70
76:1
77:1
78:1
79:1
80:1
81:1
82:1
83:1
84:1
85:1
86:1
87:1
88:1
YEAR

Figure 1

```
                         ARIMA IDENTIFICATION PROCEDURE                 9:58 THURSDAY,
                                                *
                                ARIMA PROCEDURE

                       NAME OF VARIABLE      =       GDPS
                       MEAN OF WORKING SERIES=     381442
                       STANDARD DEVIATION    =     125338
                       NUMBER OF OBSERVATIONS=         52
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1         STD
  0  1.571E+10     1.00000 |                    |********************|          0
  1  1.476E+10     0.93982 |              .     |******************* |   0.138675
  2  1.383E+10     0.88039 |           .        |******************  |   0.230657
  3  1.290E+10     0.82085 |        .           |****************    |   0.288121
  4  1.196E+10     0.76159 |       .            |***************     |   0.330044
  5  1.104E+10     0.70306 |      .             |**************      |   0.362267
  6  1.015E+10     0.64592 |    .               |*************  .    |    0.38762
  7 9270559144     0.59012 |    .               |************   .    |   0.407794
  8 8421900880     0.53610 |   .                |***********     .   |   0.423898
  9 7608813681     0.48434 |   .                |**********      .   |   0.436742
 10 6795257881     0.43255 |  .                 |*********        .  |   0.446952
 11 5977807049     0.38052 |  .                 |********         .  |   0.454931
 12 5160075673     0.32847 |  .                 |*******          .  |   0.461011
 13 4331418447     0.27572 | .                  |******            . |    0.46549
 14 3548770772     0.22590 | .                  |*****             . |    0.46862
 15 2790693435     0.17764 | .                  |****              . |    0.47071
 16 2052686375     0.13066 | .                  |***               . |   0.471997
 17 1327454245     0.08450 | .                  |**                . |   0.472692
 18  616127903     0.03922 | .                  |*                 . |   0.472983
 19  -85317522    -0.00543 | .                  |                  . |   0.473045
 20 -728138303    -0.04635 | .                 *|                  . |   0.473046
 21 -1.300E+09    -0.08275 | .                **|                  . |   0.473134
 22 -1.825E+09    -0.11617 | .                **|                  . |   0.473412
 23 -2.312E+09    -0.14714 | .               ***|                  . |    0.47396
 24 -2.756E+09    -0.17543 | .              ****|                  . |   0.474838
                           '.' MARKS TWO STANDARD ERRORS
```

FIGURE 1a

* Nominal GDP identification

```
                    ARIMA IDENTIFICATION PROCEDURE                9:58 THURSDAY,

                                          *
                       PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.93982  |             .     |*******************|
  2    -0.02467  |             .     |     .             |
  3    -0.03243  |             .    *|     .             |
  4    -0.03033  |             .    *|     .             |
  5    -0.02734  |             .    *|     .             |
  6    -0.02236  |             .     |     .             |
  7    -0.02313  |             .     |     .             |
  8    -0.01968  |             .     |     .             |
  9    -0.01558  |             .     |     .             |
 10    -0.03503  |             .    *|     .             |
 11    -0.03829  |             .    *|     .             |
 12    -0.03763  |             .    *|     .             |
 13    -0.04476  |             .    *|     .             |
 14    -0.01545  |             .     |     .             |
 15    -0.02658  |             .    *|     .             |
 16    -0.02990  |             .    *|     .             |
 17    -0.03486  |             .    *|     .             |
 18    -0.03569  |             .    *|     .             |
 19    -0.03872  |             .    *|     .             |
 20    -0.01294  |             .     |     .             |
 21    -0.00437  |             .     |     .             |
 22    -0.01541  |             .     |     .             |
 23    -0.01992  |             .     |     .             |
 24    -0.01784  |             .     |     .             |
                    ARIMA IDENTIFICATION PROCEDURE                9:58 THURSDAY,

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6  219.70  6  0.000  0.940  0.880  0.821  0.762  0.703  0.646
 12  305.10 12  0.000  0.590  0.536  0.484  0.433  0.381  0.328
 18  318.77 18  0.000  0.276  0.226  0.178  0.131  0.084  0.039
 24  326.02 24  0.000 -0.005 -0.046 -0.083 -0.116 -0.147 -0.175
```

FIGURE 1b

* pacf of Nominal GDP

acf falls to zero slowly, indicating that the mean of the data is non-stationary and that non-seasonal differencing is required. (It is important to mention, at this point, that a decaying acf implies that the estimated autocorrelations at longer lags are still statistically significant, based on a t-test. According to figure 1a the autocorrelations at lag 1 through 5 are significantly different from zero at the 5% significance level, this level is indicated by the dot ".", the remaining lags are also significant although at a higher significance level).

At this point one can proceed with the differencing of the series to attain stationarity. For more illustrative purposes, however, one can also confirm this finding by going through the estimation stage. Suppose differencing is not required, then according to the pacf (figure 1b) the model for estimation is an AR(1) since this is consistent with the combination of a decaying pattern in the estimated acf and the cutoff to zero after lag 1 in the estimated pacf. For a pure MA model the process would be just the opposite: spikes followed by a cutoff in the acf, and a tailing off of the pacf. An AR(1) is called for rather then a higher order AR model because the pacf has a spike at lag 1 only. Therefore the model is

$(1-\phi B)Q_t= a_t$ where Q_t denotes GDP at time t and a_t denotes a purely random (white noise) process.

<u>Estimation and Diagnostic Checks:</u> Output 1 and 1a show the estimated results and the corresponding residual acf and pacf. According to the estimation result ϕ is equal to 0.998925 with a

```
          THE UNDIFFERNCED NOMINAL GDP
          ESTIMATION RESULT


               Maximum Likelihood Estimation

                                     Approx.
     Parameter     Estimate      Std Error     T Ratio  Lag
     MU            35.60961      115.16660        0.31    0
     AR1,1          0.99292      0.0091773      108.63    1

     Constant Estimate   = 0.03828467

     Variance  Estimate = 0.88362877
     Std Error Estimate =  0.9400153
     AIC                = 149.239493
     SBC                =  153.14198
     Number of Residuals=         52


                Correlations of the Estimates


            Parameter              MU         AR1,1

            MU                  1.000         0.987
            AR1,1               0.987         1.000


            Autocorrelation Check of Residuals

 To    Chi                                 Autocorrelations
Lag   Square DF   Prob
  6   211.51  5   0.000   0.889   0.840   0.789   0.739   0.716   0.704
 12   375.56 11   0.000   0.672   0.655   0.633   0.638   0.619   0.622
 18   526.46 17   0.000   0.589   0.581   0.578   0.575   0.557   0.550
 24   625.51 23   0.000   0.498   0.442   0.408   0.400   0.391   0.399
                              OUTPUT 1
```

```
                    THE UNDIFFERENCED NOMINAL GDP


                      Autocorrelation Plot of Residuals

 ag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.883629    1.00000  |                    |********************|
  1   0.785430    0.88887  |              .     |******************  |
  2   0.742117    0.83985  |           .        |*****************   |
  3   0.696787    0.78855  |         .          |****************    |
  4   0.652853    0.73883  |       .            |***************     |
  5   0.632257    0.71552  |      .             |**************      |
  6   0.621900    0.70380  |     .              |**************.     |
  7   0.594030    0.67226  |    .               |*************  .    |
  8   0.578934    0.65518  |   .                |*************   .   |
  9   0.577068    0.65307  |  .                 |*************    .  |
 10   0.563595    0.63782  |  .                 |*************    .  |
 11   0.547072    0.61912  | .                  |************      . |
 12   0.549578    0.62196  |.                   |************       .|
 13   0.520565    0.58912  |.                   |************       .|
 14   0.513374    0.58098  |                    |************        |
 15   0.510753    0.57802  |                    |************        |
 16   0.507984    0.57488  |                    |***********         |
 17   0.492039    0.55684  |                    |***********         |
 18   0.485785    0.54976  |                    |***********         |
 19   0.440181    0.49815  |                    |**********          |
 20   0.390402    0.44162  |                    |*********           |
 21   0.360341    0.40780  |                    |********            |
 22   0.353289    0.39982  |                    |********            |
 23   0.345480    0.39098  |                    |********            |
 24   0.352660    0.39910  |                    |********            |
                           Partial Autocorrelations

           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.88887  |              .     |******************  |
             2    0.23706  |              .     |*****.              |
             3    0.04183  |              .     |*    .              |
             4   -0.00284  |              .     |     .              |
             5    0.11370  |              .     |**   .              |
             6    0.12201  |              .     |**   .              |
             7   -0.04263  |              .    *|     .              |
             8    0.03213  |              .     |*    .              |
             9    0.12227  |              .     |**   .              |
            10    0.01769  |              .     |     .              |
            11   -0.03973  |              .    *|     .              |
            12    0.10171  |              .     |**   .              |
            13   -0.07957  |              .   **|     .              |
            14    0.04393  |              .     |*    .              |
            15    0.05532  |              .     |*    .              |
            16    0.06123  |              .     |*    .              |
            17   -0.06603  |              .    *|     .              |
            18    0.00232  |              .     |     .              |
            19   -0.17606  |              . ****|     .              |
            20   -0.17009  |              .  ***|     .              |
            21   -0.02305  |              .     |     .              |
            22    0.15235  |              .     |***  .              |
            23    0.04574  |              .     |*    .              |
            24    0.02664  |              .     |*    .              |
```

OUTPUT 1A

high t-statistic. The estimate is almost 1.0, which contradicts the stationarity condition for an AR(1) model. Furthermore, the estimated mean is highly correlated with the parameter (r = 0.987), which indicates inappropriate differencing. This conclusion is reinforced by estimates that are unstable as indicated in output 1. A practical rule is to difference when we have serious doubts about whether the mean of a series is stationary which is the case under question. Now we proceed with the modelling of the differenced series.

Identification II: Figure 2 shows the plot of the first differences of the original data. This series seems to fluctuate around a constant mean and does not show upward movement as the original series in figure 1. The estimated acf for the first differences in figure 2a indicates no further differencing is required since after two lags all the autocorrelations are not significant at the 5% level. The pacf (see figure 2b) has one spike at lag one and the rest are insignificant. The autocorrelation check for white noise shows a chi- square value 41.94 indicating the series is not white noise. (See figure 2a, 2b). Based on this information an AR(1) model seems to be an adequate model to estimate.

Estimation and Diagnostic Checks: The estimation results and the residuals appear in output 2 and 2a. All the pertinent statistics indicate that the model is adequate. The estimated AR coefficient is significant according to the value of the t-

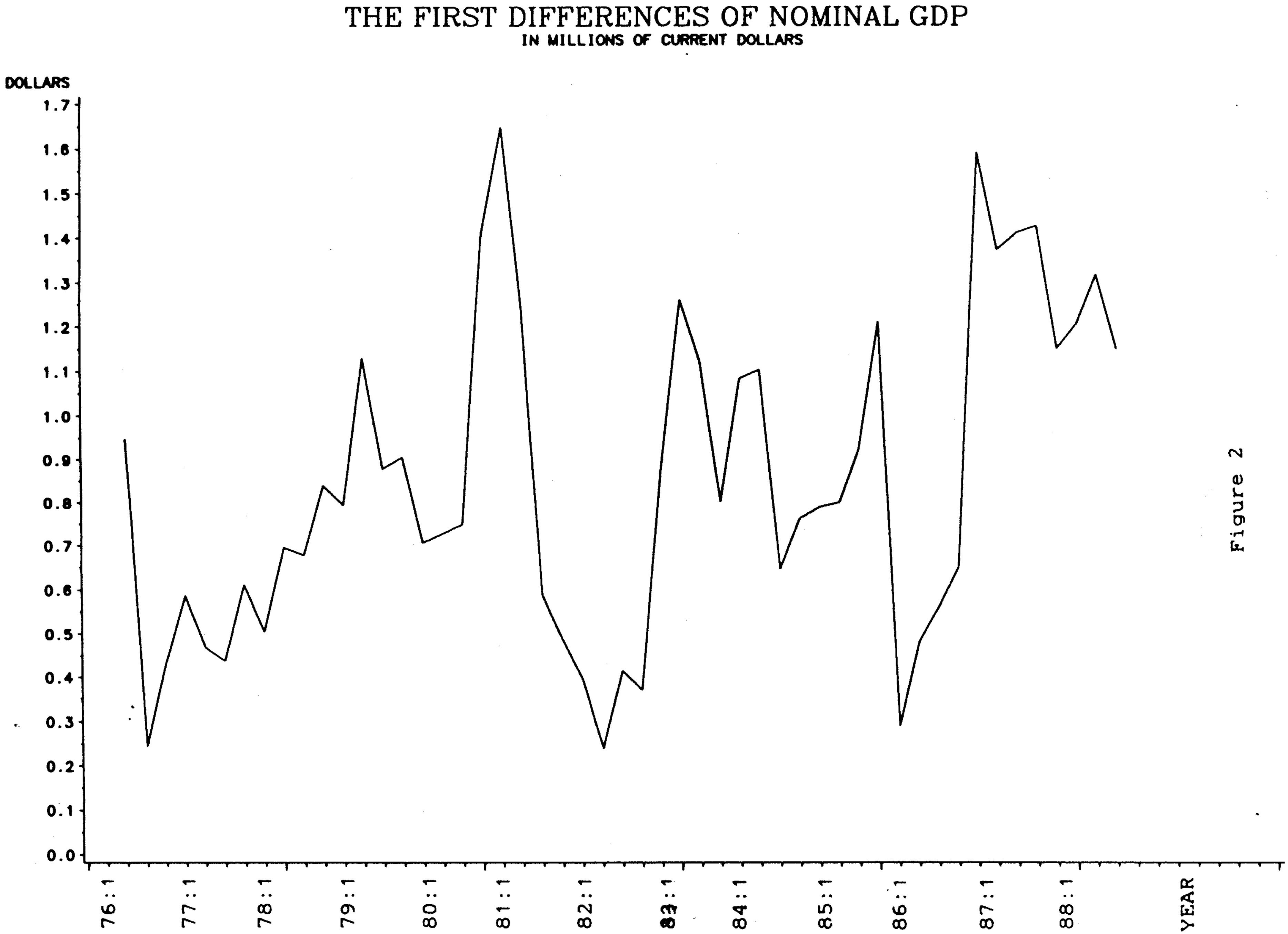
THE FIRST DIFFERENCES OF NOMINAL GDP
IN MILLIONS OF CURRENT DOLLARS
DOLLARS
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
76:1
77:1
78:1
79:1
80:1
81:1
82:1
83:1
84:1
85:1
86:1
87:1
88:1
YEAR

Figure 2

```
                                 ARIMA IDENTIFICATION PROCEDURE                 9:58 THURSDAY

                                        ARIMA PROCEDURE  *

                             NAME OF VARIABLE        =      GDPS
                             PERIOD(S) OF DIFFERENCING=1.
                             MEAN OF WORKING SERIES=   8455.14
                             STANDARD DEVIATION     =   3627.36
                             NUMBER OF OBSERVATIONS=         51
                                      AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          STD
  0   13157734     1.00000 |                    |********************|           0
  1    8228334     0.62536 |              .     |*************       |    0.140028
  2    4158759     0.31607 |             .      |******.             |    0.186934
  3    1366791     0.10388 |            .       |**     .            |    0.197134
  4   -1187570    -0.09026 |            .     **|       .            |    0.198204
  5   -1585225    -0.12048 |            .     **|       .            |    0.199009
  6    -918784    -0.06983 |            .      *|       .            |    0.200434
  7   -1446720    -0.10995 |            .     **|       .            |     0.20091
  8    -945612    -0.07187 |            .      *|       .            |    0.202087
  9    7904.51     0.00060 |            .               .            |    0.202587
 10    -676488    -0.05141 |            .      *|       .            |    0.202587
 11   -1265874    -0.09621 |            .     **|       .            |    0.202843
 12    -706199    -0.05367 |            .      *|       .            |    0.203736
 13    -532688    -0.04048 |            .      *|       .            |    0.204013
 14   -12196.4    -0.00093 |            .               .            |     0.20417
 15    1208866     0.09187 |            .       |**     .            |     0.20417
 16    1733614     0.13176 |            .       |***    .            |    0.204979
 17    1292129     0.09820 |            .       |**     .            |    0.206633
 18    1634827     0.12425 |            .       |**     .            |    0.207546
 19     939343     0.07139 |            .       |*      .            |       0.209
 20    -449599    -0.03417 |            .      *|       .            |    0.209477
 21   -2020028    -0.15352 |            .    ***|       .            |    0.209586
 22   -2236380    -0.16997 |            .    ***|       .            |     0.21178
 23   -1608396    -0.12224 |           .      **|        .           |    0.214438
 24    -305058    -0.02318 |           .        |        .           |      0.2158
                           '.' MARKS TWO STANDARD ERRORS
```

FIGURE 2a

* First differences of
Nominal GDP

```
                    ARIMA IDENTIFICATION PROCEDURE                    9:58 THURSDAY

                       PARTIAL AUTOCORRELATIONS *

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.62536  |              .     |*************       |
  2   -0.12318  |              .   **|     .              |
  3   -0.06853  |              .    *|     .              |
  4   -0.16045  |              .  ***|     .              |
  5    0.06365  |              .     |*    .              |
  6    0.04351  |              .     |*    .              |
  7   -0.14590  |              .  ***|     .              |
  8    0.04851  |              .     |*    .              |
  9    0.06023  |              .     |*    .              |
 10   -0.13072  |              .  ***|     .              |
 11   -0.07847  |              .   **|     .              |
 12    0.08387  |              .     |**   .              |
 13    0.01354  |              .     |     .              |
 14   -0.00950  |              .     |     .              |
 15    0.07940  |              .     |**   .              |
 16    0.07610  |              .     |**   .              |
 17   -0.06599  |              .    *|     .              |
 18    0.07021  |              .     |*    .              |
 19   -0.01339  |              .     |     .              |
 20   -0.08051  |              .   **|     .              |
 21   -0.19075  |              . ****|     .              |
 22    0.08878  |              .     |**   .              |
 23    0.08343  |              .     |**   .              |
 24   -0.00790  |              .     |     .              |
                    ARIMA IDENTIFICATION PROCEDURE                    9:58 THURSDAY

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                                AUTOCORRELATIONS
LAG  SQUARE  DF   PROB
  6   28.87   6  0.000  0.625  0.316  0.104 -0.090 -0.120 -0.070
 12   30.94  12  0.002 -0.110 -0.072  0.001 -0.051 -0.096 -0.054
 18   35.06  18  0.009 -0.040 -0.001  0.092  0.132  0.098  0.124
 24   41.91  24  0.013  0.071 -0.034 -0.154 -0.170 -0.122 -0.023
```

FIGURE 2b

* PACF OF THE FIRST DIFFERENCES
OF NOMINAL GDP

```
                                ARIMA:PRELIMINARY ESTIMATION  *


                                       INITIAL AUTOREGRESSIVE ESTIMATES
                                         1         0.62536

                            CONSTANT TERM ESTIMATE=   3167.62
                            WHITE NOISE VARIANCE EST=   8012055
                                ARIMA IDENTIFICATION PROCEDURE                      12:45 FRID

                            ARIMA: CONDITIONAL LEAST SQUARES ESTIMATION

                        ITERATION    SSE         MU     AR1,1     CONSTANT    LAMBDA
                          0     405027051  8455.14 0.625361      3167.62  1.0E-05
                          1     404739630  8624.85  0.63432      3153.93  1.0E-06
                          2     404739071  8631.45 0.634919      3151.18  1.0E-07

                                                     APPROX.
                          PARAMETER     ESTIMATE    STD ERROR    T RATIO  LAG
                          MU              8631.45      1038.46       8.31    0
                          AR1,1          0.634919     0.111699       5.68    1

                          CONSTANT ESTIMATE  =   3151.18

                          VARIANCE  ESTIMATE =   8259981
                          STD ERROR ESTIMATE =   2874.02
                          AIC                =   958.965*
                          SBC                =   962.829*
                          NUMBER OF RESIDUALS=        51
                          * DOES NOT INCLUDE LOG DETERMINANT

                                  CORRELATIONS OF THE ESTIMATES

                                                 MU     AR1,1
                                  MU          1.000     0.059
                                  AR1,1       0.059     1.000


                               AUTOCORRELATION CHECK OF RESIDUALS

                    TO    CHI                               AUTOCORRELATIONS
                   LAG   SQUARE  DF     PROB
                     6     3.60   4   0.463   0.071 -0.021   0.011 -0.195 -0.101   0.091
                    12     6.30  10   0.789  -0.104 -0.010   0.132 -0.021 -0.113  -0.020
                    18     8.63  16   0.928  -0.021 -0.051   0.076  0.106 -0.039   0.091
                    24    12.79  22   0.939   0.076  0.022  -0.150 -0.104 -0.061   0.049
                                  OUTPUT 2
```

OUTPUT 2

* Estimation result of the
  first differences of Nominal GDP

```
                           AUTOCORRELATION PLOT OF RESIDUALS *

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1            STD
  0    8259981    1.00000  |                    |********************|               0
  1     590541    0.07149  |              .     |*    .              |        0.140028
  2    -176105   -0.02132  |              .     |     .              |        0.140742
  3    94583.6    0.01145  |              .     |     .              |        0.140805
  4   -1614364   -0.19544  |              . ****|     .              |        0.140824
  5    -836283   -0.10125  |              .   **|     .              |        0.146045
  6     748347    0.09060  |              .     |**   .              |        0.147415
  7    -857359   -0.10380  |              .   **|     .              |        0.148503
  8   -84806.6   -0.01027  |              .     |     .              |        0.149919
  9    1086533    0.13154  |              .     |***  .              |        0.149932
 10    -174098   -0.02108  |              .     |     .              |        0.152179
 11    -932012   -0.11283  |              .   **|     .              |        0.152236
 12    -163758   -0.01983  |              .     |     .              |        0.153867
 13    -172283   -0.02086  |              .     |     .              |        0.153917
 14    -420661   -0.05093  |              .    *|     .              |        0.153972
 15     631115    0.07641  |              .     |**   .              |        0.154302
 16     878642    0.10637  |              .     |**   .              |        0.155042
 17    -320075   -0.03875  |              .    *|     .              |        0.156467
 18     752242    0.09107  |              .     |**   .              |        0.156655
 19     625403    0.07571  |              .     |**   .              |         0.15769
 20     178812    0.02165  |              .     |     .              |        0.158401
 21   -1239218   -0.15003  |              .  ***|     .              |        0.158459
 22    -855658   -0.10359  |              .   **|     .              |         0.16122
 23    -502369   -0.06082  |             .     *|      .             |         0.16252
 24     408682    0.04948  |             .      |*     .             |        0.162965
                           '.' MARKS TWO STANDARD ERRORS

                              PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.07149  |              .     |*    .              |
            2   -0.02657  |              .    *|     .              |
            3    0.01500  |              .     |     .              |
            4   -0.19915  |              . ****|     .              |
            5   -0.07437  |              .    *|     .              |
            6    0.09660  |              .     |**   .              |
            7   -0.12009  |              .   **|     .              |
            8   -0.02655  |              .    *|     .              |
            9    0.10053  |              .     |**   .              |
           10   -0.00945  |              .     |     .              |
           11   -0.14468  |              .  ***|     .              |
           12   -0.04036  |              .    *|     .              |
           13    0.05033  |              .     |*    .              |
           14   -0.05588  |              .    *|     .              |
           15    0.00507  |              .     |     .              |
           16    0.11290  |              .     |**   .              |
           17   -0.03336  |              .    *|     .              |
           18    0.04118  |              .     |*    .              |
           19    0.07693  |              .     |**   .              |
           20    0.10844  |              .     |**   .              |
           21   -0.20794  |              . ****|     .              |
           22   -0.10028  |              .   **|     .              |
           23    0.06010  |              .     |*    .              |
           24    0.06406  |              .     |*    .              |
                              OUTPUT 2A
```

* ACF AND PACF OF THE RESIDUALS
OF THE FIRST DIFFERENCES OF
NOMINAL GDP

statistic. The invertibility (stationary) condition is also satisfied since ϕ is less than 1.

According to the residual act and pacf in output 2a, we accept the null hypothesis that the innovations of the model are not correlated. At 5% level of significance the p-value for the first 24 lags is 0.939 (the p-value is the probability of obtaining a sample chi-square value as large as the calculated chi-square).

<u>2.2.A.1 Alternative Models:</u> From the acf and pacf of the first differenced series we may assume an MA (2) or an ARMA (1,1) model as possible candidates. However, earlier estimates rendered the models inadequate (either due to insignificant parameters or high chi- square statistics for the residuals).

The plot of the first difference suggests, as pointed out earlier, that the mean of the series is stationary. However, there seems to be some instability of the variance given the rather high oscillation in the late periods compared to the earlier quarters. To verify this observation, we used the Box-Cox sum of squares test to determine the optimal lambda (see appendix) value for the identified model.

The appropriate transformation according to the Box-Cox test is the square root of the original series. As can be seen from the plot of the transformed series (figure 4), the variance has relatively stabilized. The identification and estimation of the transformed model gave us an autoregressive process of order one.

Adding this extra step in the identification and estimation stage, as illustrated in table 1, gives better forecast values and

THE SQUARE ROOT OF NOMINAL GDP
IN MILLIONS OF CURRENT DOLLARS

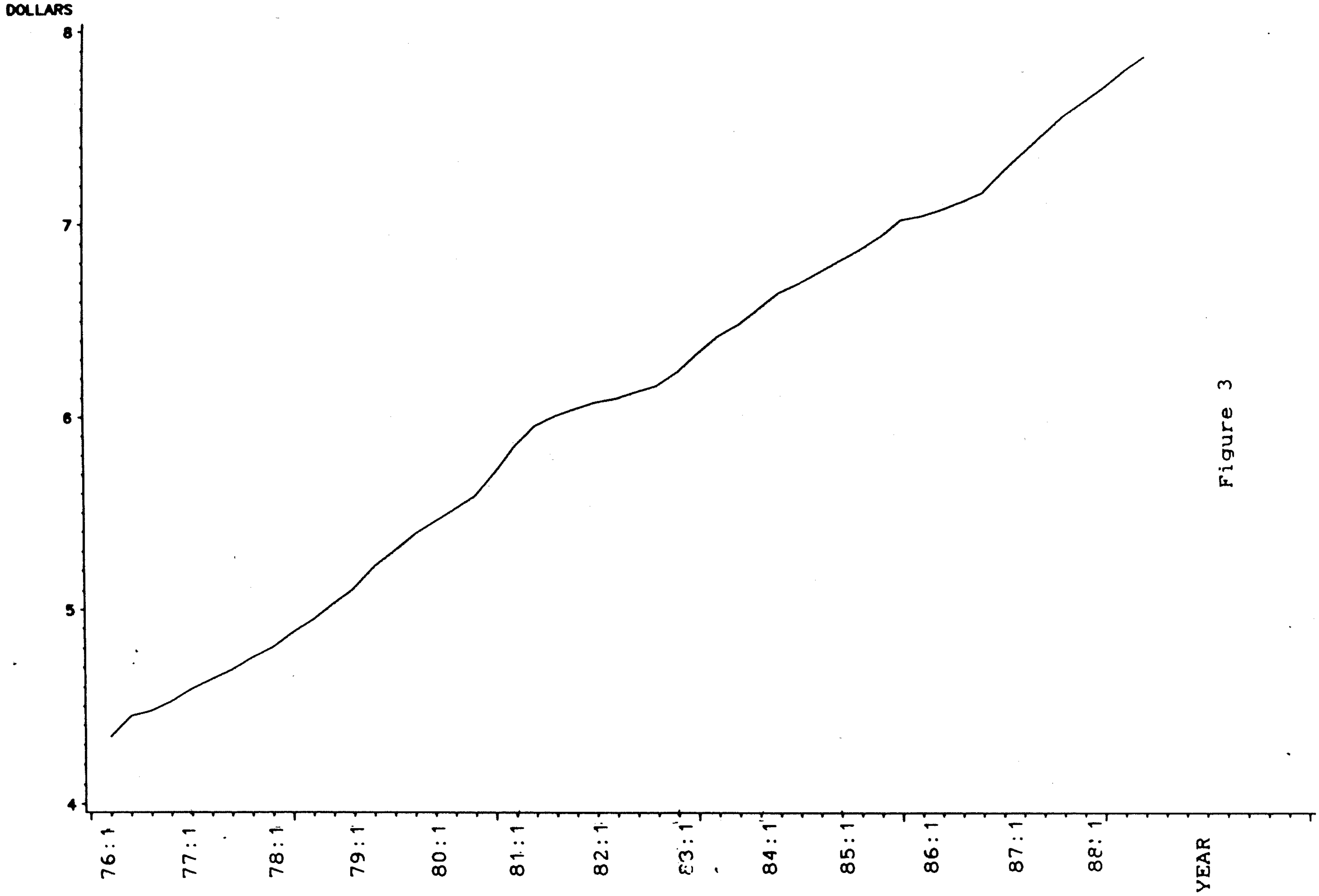

Figure 3

THE FIRST DIFFERENCES OF NOMINAL GDP (SQRT)*

IN MILLIONS OF CURRENT DOLLARS

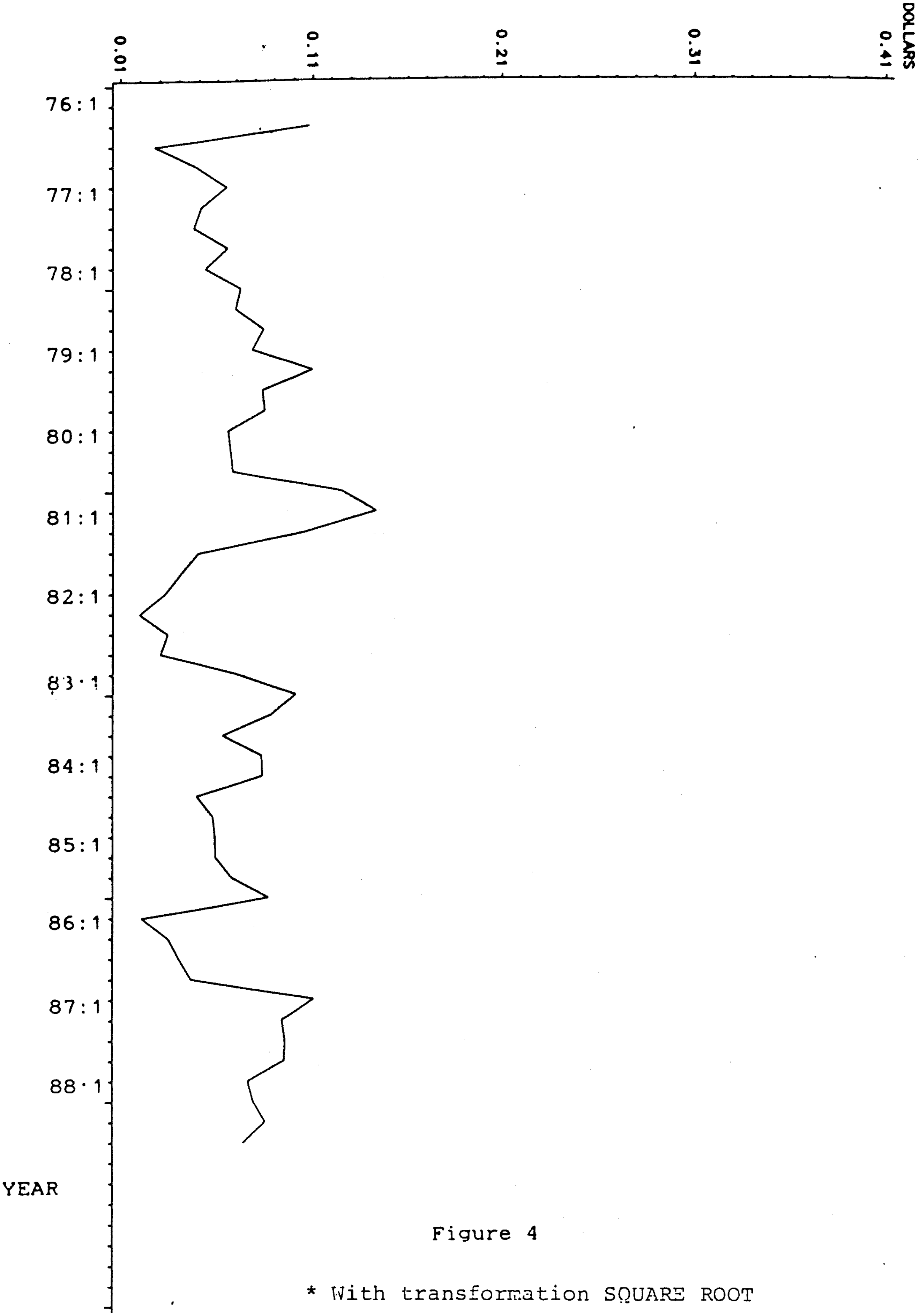

Figure 4

* With transformation SQUARE ROOT

TABLE 1

FORECAST COMPARISONS BETWEEN TENTATIVELY IDENTIFIED AND ESTIMATED ARIMA MODELS FOR NOMINAL GDP LEVELS

ERROR MEASURES	FORECAST MODEL	
	ARIMA(TRAN)	ARIMA
% ERROR		
1989:1	-0.55	-0.67
1989:2	-0.27	-0.51
1989:3	0.38	-0.18
1989:4	0.19	-0.66
MSPE*	0.14	0.29
RMSPE**	0.37	0.54

NOTE:

ARIMA(TRAN) => (1,1,0) TRANSFORMATION(SQUARE ROOT)

ARIMA => (1,1,0) NO TRANSFORMATION

* Mean Square Percentage Error (see page 48)

** Root Mean Square Percentage Error (see page 48) emphasises the point that a simple eye-balling of a plot of a series is not sufficient.

Seasonality:

According to the residual acf and pacf (output 2a or 4a,b), there is no significant autocorrelation for all lags. However, the plot shows some "relatively" high spikes at the seasonal lags (lags 4,8,12...) as compared to the other lags. This seems to indicate that there still may be some residual seasonality left in the already seasonally adjusted GDP.

One possible explanation for the presence of residual seasonality is that GDP is an aggregate economic variable, the seasonal adjustment of which is made through the seasonal adjustment of several components. Such aggregation by itself could create autocorrelation at the seasonal and other lags. Furthermore, some of the variables that make up the GDP, for one reason or another, may enter into the aggregate, as not seasonally adjusted. Finally, another possible explanation is that the seasonal adjustment method may have inadequately adjusted the series. However, the results from the X11ARIMA (Dagum,1988) (see appendix) indicate that there is no significant stable or moving seasonality in the seasonally adjusted series.

We have also fitted a seasonal autoregressive and moving average components to our tentatively identified model and the parameter were statistically insignificant. The final estimated model based on our modelling process is the first differences of the square root of GDP with an autoregressive component at lag 1

```
                                        OUTPUT FROM THE IDENTIFICATION
                                        & ESTIMATION OF THE SQUARE ROOT
                                                  OF GDP *

                           NAME OF VARIABLE       =     SQGDP
                           PERIOD(S) OF DIFFERENCING=1.
                           MEAN OF WORKING SERIES= 0.0692251
                           STANDARD DEVIATION    = 0.0268283
                           NUMBER OF OBSERVATIONS=        51
                                     AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0 .000719759     1.00000 |                    |********************|         0
  1 .000369117     0.51283 |              .     |**********          |  0.140028
  2 .000134563     0.18696 |             .      |****  .             |  0.172978
  3 -2.702E-05    -0.03754 |             .     *|      .             |  0.176896
  4 -.00016469    -0.22881 |             . *****|      .             |  0.177052
  5 -.00018705    -0.25988 |             . *****|      .             |  0.182758
  6 -0.0001025    -0.14241 |            .    ***|       .            |  0.189866
  7 -.00011656    -0.16194 |            .    ***|       .            |  0.191949
  8 -4.243E-05    -0.05895 |            .      *|       .            |  0.194609
  9 .000012426     0.01726 |            .       |       .            |  0.194959
 10 -3.059E-05    -0.04250 |            .      *|       .            |  0.194989
 11 -.00010719    -0.14892 |            .    ***|       .            |  0.195171
 12 -7.786E-05    -0.10818 |            .     **|       .            |  0.197386
 13 -7.360E-05    -0.10226 |            .     **|       .            |  0.198545
 14 -4.317E-05    -0.05998 |            .      *|       .            |  0.199575
 15 .000028084     0.03902 |            .       |*      .            |  0.199929
 16 0.00007254     0.10078 |            .       |**     .            |  0.200078
 17 .000040577     0.05638 |            .       |*      .            |  0.201071
 18 .000082725     0.11493 |            .       |**     .            |   0.20138
 19 .000068027     0.09451 |            .       |**     .            |  0.202663
 20 -2.990E-05    -0.04155 |            .      *|       .            |  0.203525
 21 -.00015975    -0.22194 |            .   ****|       .            |  0.203691
 22 -.00015816    -0.21974 |            .   ****|       .            |  0.208379
 23 -.00010428    -0.14488 |           .     ***|        .           |  0.212874
 24 -1.958E-05    -0.02720 |           .       *|        .           |  0.214799
                           '.' MARKS TWO STANDARD ERRORS
```

OUTPUT 4a

* (NOMINAL GDP)

OUTPUT FROM THE IDENTIFICATION
& ESTIMATION OF THE SQUARE ROOT
OF GDP *

```
                                                              12:47 SUND

                  PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.51283  |             .     |**********         |
  2    -0.10318  |             .   **|     .             |
  3    -0.12397  |             .   **|     .             |
  4    -0.19263  |             . ****|     .             |
  5    -0.05533  |             .    *|     .             |
  6     0.05985  |             .     |*    .             |
  7    -0.17533  |             . ****|     .             |
  8     0.04388  |             .     |*    .             |
  9    -0.00960  |             .     |     .             |
 10    -0.12737  |             .  ***|     .             |
 11    -0.18757  |             . ****|     .             |
 12     0.01750  |             .     |     .             |
 13    -0.03637  |             .    *|     .             |
 14    -0.06858  |             .    *|     .             |
 15     0.00659  |             .     |     .             |
 16     0.02941  |             .     |*    .             |
 17    -0.09071  |             .   **|     .             |
 18     0.04927  |             .     |*    .             |
 19     0.02167  |             .     |     .             |
 20    -0.13478  |             .  ***|     .             |
 21    -0.28573  |             ******|     .             |
 22    -0.01229  |             .     |     .             |
 23     0.06453  |             .     |*    .             |
 24    -0.07349  |             .    *|     .             |
                           SAS                                12:47 SUND

             AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                      AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   24.42  6  0.000  0.513  0.187 -0.038 -0.229 -0.260 -0.142
 12   28.70 12  0.004 -0.162 -0.059  0.017 -0.043 -0.149 -0.108
 18   31.94 18  0.022 -0.102 -0.060  0.039  0.101  0.056  0.115
 24   43.88 24  0.008  0.095 -0.042 -0.222 -0.220 -0.145 -0.027
```

OUTPUT 4b

* (NOMINAL GDP)

OUTPUT FROM THE IDENTIFICATION & ESTIMATION OF THE SQUARE ROOT OF GDP*

```
                                                          12:47 SUND

              ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
           PARAMETER     ESTIMATE    STD ERROR    T RATIO  LAG
           MU           0.0701098   0.00670152      10.46    0
           AR1,1         0.524553     0.120379       4.36    1

           CONSTANT ESTIMATE  = 0.0333335

           VARIANCE  ESTIMATE =.000543563
           STD ERROR ESTIMATE = 0.0233144
           AIC                =  -236.372
           SBC                =  -232.509
           NUMBER OF RESIDUALS=        51

                  CORRELATIONS OF THE ESTIMATES

                                 MU     AR1,1
                  MU          1.000     0.005
                  AR1,1       0.005     1.000


               AUTOCORRELATION CHECK OF RESIDUALS

   TO    CHI                          AUTOCORRELATIONS
  LAG  SQUARE DF   PROB
    6    4.39  4  0.356  0.058 -0.014 -0.036 -0.184 -0.184  0.054
   12    7.85 10  0.644 -0.135  0.004  0.104  0.017 -0.153 -0.021
   18   10.11 16  0.861 -0.054 -0.061  0.042  0.108 -0.057  0.078
   24   16.16 22  0.808  0.104  0.023 -0.192 -0.116 -0.068  0.027
```

OUTPUT 4c

* (NOMINAL GDP)

```
                                OUTPUT FROM THE IDENTIFICATION
                                & ESTIMATION OF THE SQUARE ROOT
                                           OF GDP*
                                                                          12:47 SUND

                       AUTOCORRELATION PLOT OF RESIDUALS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0 .000543563     1.00000 |                    |********************|         0
  1 .000031468     0.05789 |              .     |*    .              |  0.140028
  2 -7.774E-06    -0.01430 |              .     |     .              |  0.140497
  3 -1.983E-05    -0.03648 |              .    *|     .              |  0.140525
  4 -9.994E-05    -0.18386 |              . ****|     .              |  0.140711
  5 -.00010024    -0.18442 |              . ****|     .              |  0.145345
  6 .000029445     0.05417 |              .     |*    .              |  0.149863
  7 -.00007356    -0.13533 |              .  ***|     .              |  0.150246
  8 .000001929     0.00355 |              .     |     .              |  0.152618
  9 .000056697     0.10431 |              .     |**   .              |  0.152619
 10  9.177E-06     0.01688 |              .     |     .              |  0.154011
 11 -8.319E-05    -0.15304 |              .  ***|     .              |  0.154047
 12 -1.160E-05    -0.02133 |              .     |     .              |     0.157
 13 -2.956E-05    -0.05438 |              .    *|     .              |  0.157057
 14 -3.292E-05    -0.06057 |              .    *|     .              |  0.157425
 15 .000022915     0.04216 |              .     |*    .              |  0.157882
 16 .000058923     0.10840 |              .     |**   .              |  0.158102
 17 -3.073E-05    -0.05654 |              .    *|     .              |  0.159553
 18 .000042258     0.07774 |              .     |**   .              |  0.159945
 19 .000056781     0.10446 |              .     |**   .              |  0.160685
 20 .000012466     0.02293 |              .     |     .              |  0.162011
 21 -.00010437    -0.19201 |              . ****|     .              |  0.162074
 22 -6.284E-05    -0.11561 |             .    **|      .             |  0.166475
 23 -3.672E-05    -0.06756 |             .     *|      .             |  0.168042
 24 .000014908     0.02743 |             .      |*     .             |  0.168574
                           '.' MARKS TWO STANDARD ERRORS
                            PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.05789 |              .     |*    .              |
             2    -0.01771 |              .     |     .              |
             3    -0.03474 |              .    *|     .              |
             4    -0.18083 |              . ****|     .              |
             5    -0.17183 |              .  ***|     .              |
             6     0.06461 |              .     |*    .              |
             7    -0.16806 |              .  ***|     .              |
             8    -0.02901 |              .    *|     .              |
             9     0.04242 |              .     |*    .              |
            10    -0.01461 |              .     |     .              |
            11    -0.20140 |              . ****|     .              |
            12    -0.07233 |              .    *|     .              |
            13    -0.02520 |              .    *|     .              |
            14    -0.09075 |              .   **|     .              |
            15    -0.04801 |              .    *|     .              |
            16     0.05140 |              .     |*    .              |
            17    -0.09341 |              .   **|     .              |
            18    -0.01885 |              .     |     .              |
            19     0.07965 |              .     |**   .              |
            20     0.06547 |              .     |*    .              |
            21    -0.25051 |              .*****|     .              |
            22    -0.16788 |              .  ***|     .              |
            23     0.02244 |              .     |     .              |
            24    -0.00014 |              .     |     .              |
```

OUTPUT 4d

* (NOMINAL GDP)

and a deterministic trend.

2.2.B. REAL GDP

We begin our identification process of seasonally adjusted real GDP by examining the plot (figure 5). The mean of the series is not stationary since there is a persistent upward trend. However, the trend is not as dominant as the one from the nominal GDP we saw earlier. Recall also from figure 1 that the nominal GDP does not show the recession of 1981-82 at all. In fact due to the high inflation and interest rate of the period the national output increased slightly!

<u>Identification I</u> According to the estimated acf (output 5a,5b) the autocorrelations drop to zero after four lags (at 5% significance level). While this may imply a stationarity mean, fitting an AR(1) model (based on a decaying acf a spike at lag one in the pacf) gave us a non-invertible parameter which is an indication of under-differencing. Therefore, we difference the series once to attain stationarity.

Figure 6 shows the plot of the first differences of real GDP (1981 dollars) and output 6a and 6b show the plot of the estimated acf and pacf of the first differences. After lag one all the autocorrelations and partial autocorrelations are insignificant; this indicates the mean to be stationarity and also an AR(1) or MA(1) to be possible models to estimate. Since the pattern in the pacf seem to resemble a damped sine wave pattern we choose an MA(1)

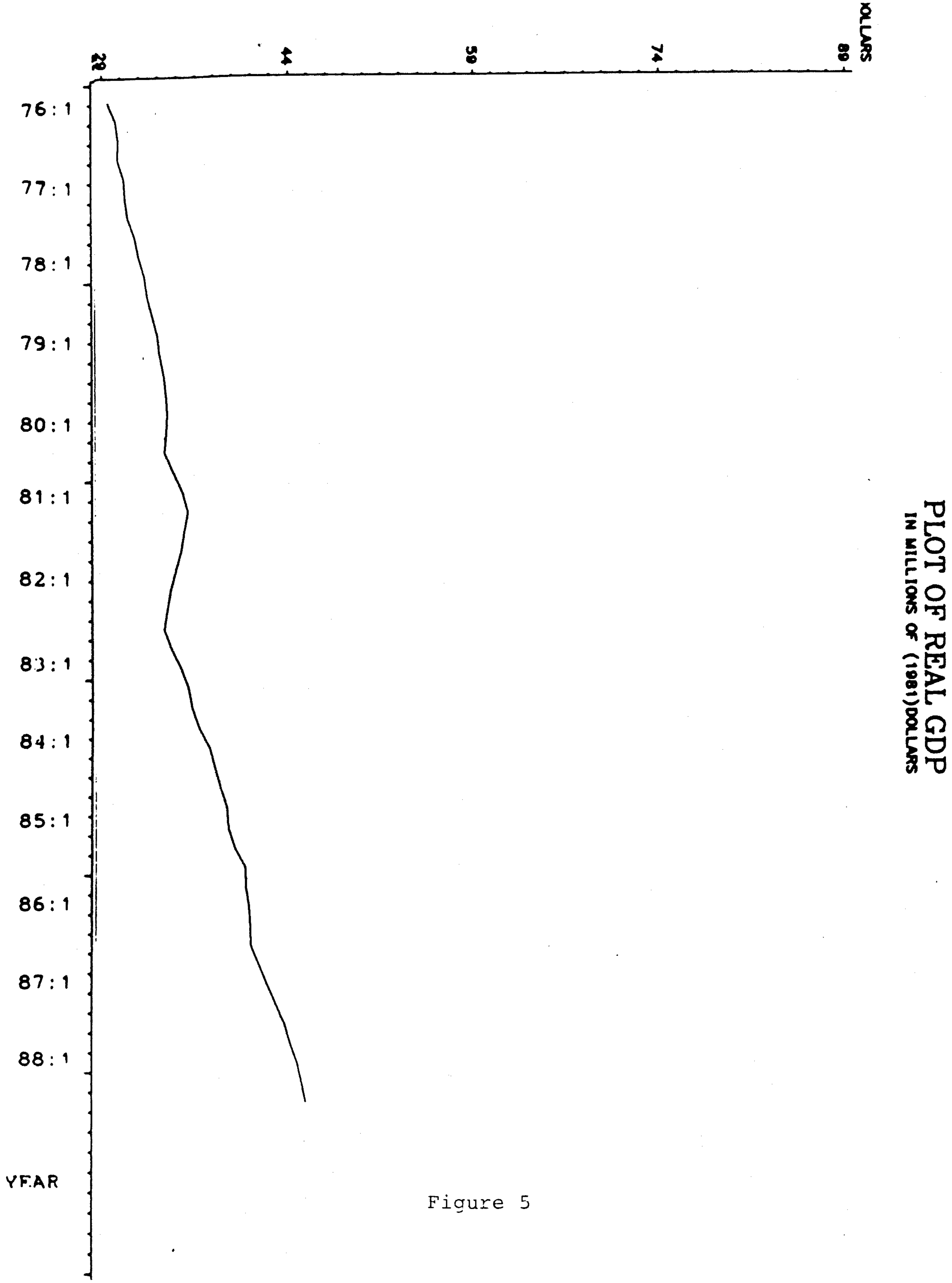
PLOT OF REAL GDP
IN MILLIONS OF (1981)DOLLARS
DOLLARS
29
44
59
74
89
76:1
77:1
78:1
79:1
80:1
81:1
82:1
83:1
84:1
85:1
86:1
87:1
88:1
YEAR

Figure 5

```
                                OUTPUT FROM THE IDENTIFICATION          14:45 SATURD
                                & ESTIMATION OF GDP(1981)

                           NAME OF VARIABLE       =     GDP81
                           MEAN OF WORKING SERIES=    36.3977
                           STANDARD DEVIATION     =   4.31162
                           NUMBER OF OBSERVATIONS=         52
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          STD
  0    18.5901      1.00000 |                    |********************|         0
  1    17.2551      0.92819 |              .     |******************* |  0.138675
  2    15.9209      0.85642 |           .        |*****************   |  0.228838
  3    14.5669      0.78358 |         .          |****************    |   0.28386
  4    13.2162      0.71093 |       .            |**************      |  0.322788
  5    11.9346      0.64199 |      .             |*************.      |  0.351612
  6    10.7001      0.57558 |     .              |************  .     |  0.373474
  7    9.54708      0.51356 |    .               |**********     .    |   0.39016
  8    8.51945      0.45828 |    .               |*********      .    |  0.402951
  9    7.58498      0.40801 |   .                |********        .   |  0.412852
 10     6.6487      0.35765 |   .                |*******         .   |  0.420535
 11     5.6863      0.30588 |   .                |******          .   |  0.426344
 12    4.75824      0.25596 |   .                |*****           .   |  0.430544
 13    3.82736      0.20588 |   .                |****            .   |   0.43346
 14    3.01416      0.16214 |   .                |***             .   |  0.435337
 15    2.27591      0.12243 |   .                |**              .   |  0.436496
 16    1.52982      0.08229 |   .                |**              .   |  0.437156
 17   0.823561      0.04430 |   .                |*               .   |  0.437454
 18   0.134353      0.00723 |  .                 |                 .  |   0.43754
 19  -0.561249     -0.03019 |  .                *|                 .  |  0.437543
 20   -1.07248     -0.05769 |  .                *|                 .  |  0.437583
 21   -1.42501     -0.07665 |  .               **|                 .  |  0.437729
 22    -1.6913     -0.09098 |  .               **|                 .  |  0.437987
 23   -1.91959     -0.10326 |  .               **|                 .  |   0.43835
 24   -2.06162     -0.11090 |  .               **|                 .  |  0.438818
                            '.' MARKS TWO STANDARD ERRORS
```

OUTPUT 5a

```
                                                                             14:45
                              OUTPUT FROM THE IDENTIFICATION
                              & ESTIMATION OF GDP(1981)

                          PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.92819  |              .     |******************* |
           2   -0.03691  |              .    *|     .              |
           3   -0.04642  |              .    *|     .              |
           4   -0.03953  |              .    *|     .              |
           5   -0.01541  |              .     |     .              |
           6   -0.02367  |              .     |     .              |
           7   -0.01078  |              .     |     .              |
           8    0.00739  |              .     |     .              |
           9   -0.00231  |              .     |     .              |
          10   -0.03796  |              .    *|     .              |
          11   -0.04789  |              .    *|     .              |
          12   -0.02448  |              .     |     .              |
          13   -0.03832  |              .    *|     .              |
          14    0.00673  |              .     |     .              |
          15   -0.00719  |              .     |     .              |
          16   -0.03940  |              .    *|     .              |
          17   -0.02476  |              .     |     .              |
          18   -0.03228  |              .    *|     .              |
          19   -0.04125  |              .    *|     .              |
          20    0.03349  |              .     |*    .              |
          21    0.03147  |              .     |*    .              |
          22    0.00601  |              .     |     .              |
          23   -0.01424  |              .     |     .              |
          24    0.00661  |              .     |     .              |
                                    SAS                                      14:45 SATURD

                    AUTOCORRELATION CHECK FOR WHITE NOISE

       TO   CHI                          AUTOCORRELATIONS
      LAG  SQUARE DF   PROB
        6  198.23  6  0.000  0.928  0.856  0.784  0.711  0.642  0.576
       12  258.52 12  0.000  0.514  0.458  0.408  0.358  0.306  0.256
       18  265.34 18  0.000  0.206  0.162  0.122  0.082  0.044  0.007
       24  269.28 24  0.000 -0.030 -0.058 -0.077 -0.091 -0.103 -0.111
```

OUTPUT 5b

PLOT OF THE FIRST DIFFERENCES OF REAL GDP
IN MILLIONS OF (1981)DOLLARS

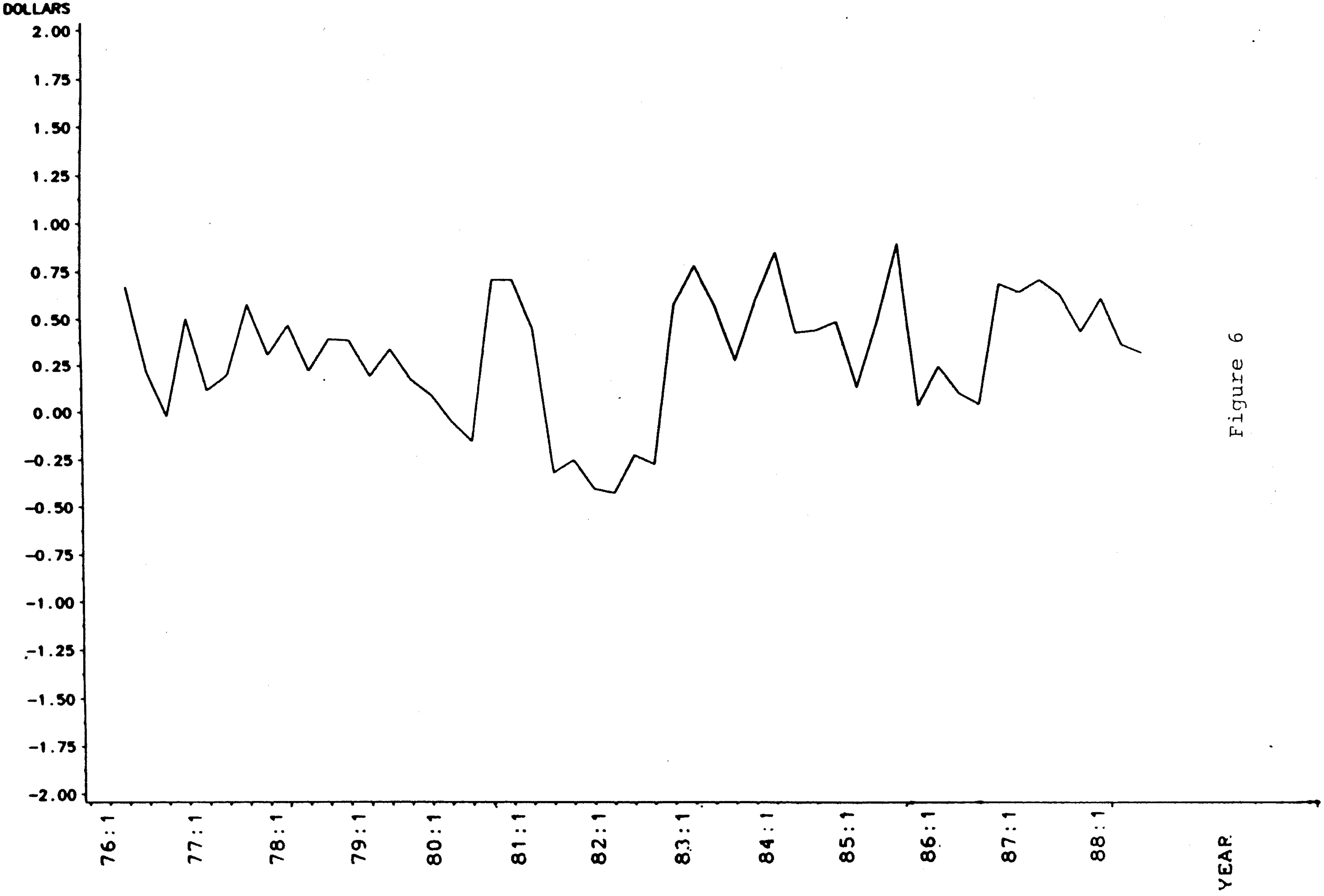

Figure 6

```
                                            OUTPUT FROM THE IDENTIFICATION
                                            & ESTIMATION OF FIRST DIFFERENCE OF
                                                   GDP(1981)

                              NAME OF VARIABLE      =     GDP81
                              PERIOD(S) OF DIFFERENCING=1.
                              MEAN OF WORKING SERIES=  0.311553
                              STANDARD DEVIATION    =  0.331897
                              NUMBER OF OBSERVATIONS=        51
                                    AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1           STD
  0   0.110156     1.00000 |                   |********************|             0
  1  0.0535509     0.48614 |             .     |**********          |      0.140028
  2  0.0212392     0.19281 |            .      |****  .             |      0.169928
  3  0.0104193     0.09459 |            .      |**    .             |      0.174165
  4  -0.011891    -0.10795 |            .    **|      .             |       0.17517
  5 -0.0050625    -0.04596 |            .     *|      .             |      0.176469
  6 -0.0117609    -0.10677 |            .    **|      .             |      0.176704
  7 -0.0105205    -0.09551 |            .    **|      .             |      0.177964
  8 -.00261669    -0.02375 |            .      |      .             |      0.178966
  9 0.00464206     0.04214 |            .      |*     .             |      0.179028
 10 0.00636418     0.05777 |            .      |*     .             |      0.179222
 11 -.00557378    -0.05060 |            .     *|      .             |      0.179587
 12 -.00423314    -0.03843 |            .     *|      .             |      0.179867
 13 0.00144066     0.01308 |            .      |      .             |      0.180027
 14 -.00410689    -0.03728 |            .     *|      .             |      0.180046
 15 -.00281978    -0.02560 |            .     *|      .             |      0.180197
 16 .000621421     0.00564 |            .      |      .             |      0.180269
 17 -.00348002    -0.03159 |            .     *|      .             |      0.180272
 18 0.00376661     0.03419 |            .      |*     .             |      0.180381
 19 0.00841511     0.07639 |            .      |**    .             |      0.180508
 20 -.00357804    -0.03248 |            .     *|      .             |      0.181141
 21 -0.0181372    -0.16465 |            .   ***|      .             |      0.181255
 22 -0.0229536    -0.20837 |            .  ****|      .             |      0.184164
 23 -0.0245721    -0.22307 |           .   ****|       .            |       0.18873
 24  -0.010769    -0.09776 |           .     **|       .            |      0.193831
                           '.' MARKS TWO STANDARD ERRORS
```

OUTPUT 6a

```
                                      OUTPUT FROM THE IDENTIFICATION
                                      & ESTIMATION OF FIRST DIFFERENCE OF
                                             GDP(1981)

                                 PARTIAL AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1     0.48614  |             .     |**********         |
       2    -0.05699  |             .    *|     .             |
       3     0.03050  |             .     |*    .             |
       4    -0.20715  |             . ****|     .             |
       5     0.12764  |             .     |***  .             |
       6    -0.16235  |             .  ***|     .             |
       7     0.05976  |             .     |*    .             |
       8    -0.03189  |             .    *|     .             |
       9     0.13124  |             .     |***  .             |
      10    -0.07431  |             .    *|     .             |
      11    -0.08786  |             .   **|     .             |
      12     0.01532  |             .     |     .             |
      13     0.07158  |             .     |*    .             |
      14    -0.08480  |             .   **|     .             |
      15     0.00723  |             .     |     .             |
      16     0.03898  |             .     |*    .             |
      17    -0.05042  |             .    *|     .             |
      18     0.05392  |             .     |*    .             |
      19     0.03960  |             .     |*    .             |
      20    -0.09870  |             .   **|     .             |
      21    -0.19341  |             . ****|     .             |
      22    -0.05694  |             .    *|     .             |
      23    -0.07838  |             .   **|     .             |
      24     0.12684  |             .     |***  .             |
                                     SAS                                  14:45 SATURD

                        AUTOCORRELATION CHECK FOR WHITE NOISE

      TO    CHI                         AUTOCORRELATIONS
     LAG  SQUARE DF   PROB
       6   16.81  6  0.010  0.486  0.193  0.095 -0.108 -0.046 -0.107
      12   18.02 12  0.115 -0.096 -0.024  0.042  0.058 -0.051 -0.038
      18   18.36 18  0.432  0.013 -0.037 -0.026  0.006 -0.032  0.034
      24   31.19 24  0.148  0.076 -0.032 -0.165 -0.208 -0.223 -0.098
```

OUTPUT 6b

model for estimation.

Estimation and Diagnostic Checks: The estimation results indicate (output 6c,6d) that an MA(1) with a deterministic constant is an adequate model. The estimated MA coefficient is invertible and statistically significant, the residuals are non-correlated (the p-value for the chi-square statistics is .937) and the constant term is also statistically significant. Preliminary Box-Cox test on the original series did not show a need for transforming and the first differences on the original series is still the appropriate model.

However, the pacf of the residuals (output 6d) now show a high autocorrelation at lag 4 (seasonal lag). Thus, we include a seasonal moving average parameter to the estimated model.

Identification II: The inclusion of the seasonal moving average (output 6e,6f) parameter improves the residual autocorrelations and, though not shown here, gives better forecast values than the MA(1) model. One reason why this residual seasonality was not observed for the nominal series may be the fact that the trend is very dominant in the latter and consequently obscures the identification of the small amount of seasonality left in. The final estimated model, therefore, is the first differences of real GDP with a moving average components at lag 1 and 4 plus a deterministic constant. Using the Box-Jenkins notation, it results in an

OUTPUT FROM THE IDENTIFICATION
& ESTIMATION OF FIRST DIFFERENCE OF
GDP(1981)

```
                ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                    APPROX.
          PARAMETER     ESTIMATE    STD ERROR    T RATIO  LAG
          MU            0.315229    0.0637052       4.95    0
          MA1,1        -0.562176     0.120244      -4.68    1

          CONSTANT ESTIMATE  =  0.315229

          VARIANCE  ESTIMATE = 0.0866444
          STD ERROR ESTIMATE =  0.294354
          AIC                =   22.3282
          SBC                =   26.1918
          NUMBER OF RESIDUALS=        51

                    CORRELATIONS OF THE ESTIMATES

                                 MU    MA1,1
                    MU        1.000   -0.008
                    MA1,1    -0.008    1.000


                 AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF    PROB
  6    6.15  4   0.189 -0.001  0.149  0.117 -0.201  0.105 -0.141
 12    7.12 10   0.714 -0.024 -0.019  0.015  0.084 -0.076 -0.026
 18    7.87 16   0.953  0.047 -0.045 -0.024  0.041 -0.053  0.026
 24   13.13 22   0.929  0.074 -0.014 -0.121 -0.082 -0.169 -0.033
```

OUTPUT 6c

```
                                                                    14:45 SATURD
                            OUTPUT FROM THE IDENTIFICATION
                            & ESTIMATION OF FIRST DIFFERENCE OF
                                       GDP(1981)

                         AUTOCORRELATION PLOT OF RESIDUALS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          STD
  0  0.0866444     1.00000 |                    |********************|          0
  1 -5.914E-05    -0.00068 |              .     |     .              |   0.140028
  2  0.0128979     0.14886 |              .     |***  .              |   0.140028
  3  0.0100969     0.11653 |              .     |**   .              |   0.143097
  4  -0.017405    -0.20088 |              . ****|     .              |   0.144946
  5 0.00906722     0.10465 |              .     |**   .              |   0.150306
  6 -0.0122172    -0.14100 |              .  ***|     .              |   0.151728
  7 -.00205885    -0.02376 |              .     |     .              |   0.154276
  8 -.00161026    -0.01858 |              .     |     .              |   0.154347
  9 0.00134136     0.01548 |              .     |     .              |   0.154391
 10 0.00725332     0.08371 |              .     |**   .              |   0.154422
 11 -0.0066084    -0.07627 |              .   **|     .              |   0.155309
 12 -.00221539    -0.02557 |              .    *|     .              |   0.156042
 13 0.00407679     0.04705 |              .     |*    .              |   0.156124
 14 -.00386284    -0.04458 |              .    *|     .              |   0.156402
 15 -.00208498    -0.02406 |              .     |     .              |   0.156651
 16 0.00352127     0.04064 |              .     |*    .              |   0.156723
 17 -.00455838    -0.05261 |              .    *|     .              |    0.15693
 18 0.00222656     0.02570 |              .     |*    .              |   0.157275
 19 0.00643733     0.07430 |              .     |*    .              |   0.157357
 20 -.00118037    -0.01362 |              .     |     .              |   0.158044
 21 -0.0104435    -0.12053 |              .   **|     .              |   0.158067
 22 -0.0070887    -0.08181 |              .   **|     .              |   0.159859
 23 -0.0146066    -0.16858 |              .  ***|     .              |   0.160678
 24 -.00287097    -0.03314 |             .     *|      .             |   0.164109
                           '.' MARKS TWO STANDARD ERRORS
                                       SAS                              14:45 SATURD.

                               PARTIAL AUTOCORRELATIONS

                LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                  1    -0.00068 |              .     |     .              |
                  2     0.14886 |              .     |***  .              |
                  3     0.11937 |              .     |**   .              |
                  4    -0.22910 |              .*****|     .              |
                  5     0.07426 |              .     |*    .              |
                  6    -0.09497 |              .   **|     .              |
                  7     0.00069 |              .     |     .              |
                  8    -0.05062 |              .    *|     .              |
                  9     0.09536 |              .     |**   .              |
                 10     0.03930 |              .     |*    .              |
                 11    -0.08054 |              .   **|     .              |
                 12    -0.08634 |              .   **|     .              |
                 13     0.09626 |              .     |**   .              |
                 14    -0.00967 |              .     |     .              |
                 15    -0.07134 |              .    *|     .              |
                 16     0.05120 |              .     |*    .              |
                 17    -0.00600 |              .     |     .              |
                 18    -0.01909 |              .     |     .              |
                 19     0.06398 |              .     |*    .              |
                 20     0.02203 |              .     |     .              |
                 21    -0.18515 |              . ****|     .              |
                 22    -0.09834 |              .   **|     .              |
                 23    -0.12356 |              .   **|     .              |
                 24     0.05376 |              .     |*    .              |
```

OUTPUT 6d

```
                                   ESTIMATION RESULT OF GDP(1981)       15:04 SATURD
                                   WITH A SEASONAL PARAMETER INCLUDED

                    ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
              PARAMETER    ESTIMATE   STD ERROR   T RATIO  LAG
              MU           0.311267   0.0509748     6.11    0
              MA1,1       -0.565995    0.122303    -4.63    1
              MA2,1        0.208595    0.144525     1.44    4

              CONSTANT ESTIMATE  =  0.311267

              VARIANCE  ESTIMATE = 0.0846775
              STD ERROR ESTIMATE =  0.290994
              AIC                =   22.3331
              SBC                =   28.1286
              NUMBER OF RESIDUALS=        51

                      CORRELATIONS OF THE ESTIMATES

                                  MU     MA1,1    MA2,1
                    MU         1.000    -0.013   -0.055
                    MA1,1     -0.013     1.000    0.103
                    MA2,1     -0.055     0.103    1.000

                   AUTOCORRELATION CHECK OF RESIDUALS

  TO    CHI                              AUTOCORRELATIONS
 LAG  SQUARE DF    PROB
   6    3.90  3   0.273  0.041  0.160  0.123 -0.012  0.122 -0.105
  12    4.88  9   0.845 -0.020 -0.032  0.041  0.057 -0.088 -0.026
  18    5.81 15   0.983  0.043 -0.047 -0.035  0.034 -0.076 -0.010
  24   12.09 21   0.937  0.028 -0.029 -0.143 -0.102 -0.181 -0.038
```

OUTPUT 6e

15:04 SATUR

```
                                    ESTIMATION RESULT OF GDP(1981)
                                    WITH A SEASONAL PARAMETER INCLUDED

                       AUTOCORRELATION PLOT OF RESIDUALS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1           STD
  0  0.0846775     1.00000 |                    |********************|            0
  1 0.00344283     0.04066 |              .     |*    .              |     0.140028
  2  0.0135681     0.16023 |              .     |***  .              |     0.140259
  3  0.0104106     0.12294 |              .     |**   .              |     0.143804
  4 -.00103003    -0.01216 |              .     |     .              |      0.14585
  5  0.0103187     0.12186 |              .     |**   .              |      0.14587
  6 -.00887718    -0.10484 |              .   **|     .              |     0.147853
  7 -.00172494    -0.02037 |              .     |     .              |     0.149303
  8 -.00269637    -0.03184 |              .    *|     .              |     0.149358
  9 0.00345512     0.04080 |              .     |*    .              |     0.149491
 10 0.00478557     0.05652 |              .     |*    .              |     0.149709
 11 -.00746705    -0.08818 |              .   **|     .              |     0.150127
 12 -.00216795    -0.02560 |              .    *|     .              |     0.151139
 13 0.00362928     0.04286 |              .     |*    .              |     0.151224
 14 -.00398596    -0.04707 |              .    *|     .              |     0.151462
 15 -.00293244    -0.03463 |              .    *|     .              |     0.151748
 16 0.00283684     0.03350 |              .     |*    .              |     0.151903
 17 -.00639669    -0.07554 |              .   **|     .              |     0.152048
 18 -.00080929    -0.00956 |              .     |     .              |     0.152782
 19 0.00238477     0.02816 |              .     |*    .              |     0.152794
 20 -.00245984    -0.02905 |              .    *|     .              |     0.152896
 21 -0.0121305    -0.14326 |              .  ***|     .              |     0.153004
 22 -.00866093    -0.10228 |              .   **|     .              |     0.155612
 23 -0.0153422    -0.18118 |              . ****|     .              |     0.156924
 24 -.00323411    -0.03819 |              .    *|     .              |     0.160974
                           '.' MARKS TWO STANDARD ERRORS
                            PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.04066 |              .     |*    .              |
             2     0.15884 |              .     |***  .              |
             3     0.11407 |              .     |**   .              |
             4    -0.04540 |              .    *|     .              |
             5     0.08936 |              .     |**   .              |
             6    -0.12038 |              .   **|     .              |
             7    -0.04200 |              .    *|     .              |
             8    -0.02213 |              .     |     .              |
             9     0.08838 |              .     |**   .              |
            10     0.05600 |              .     |*    .              |
            11    -0.08647 |              .   **|     .              |
            12    -0.06409 |              .    *|     .              |
            13     0.06345 |              .     |*    .              |
            14    -0.03852 |              .    *|     .              |
            15    -0.04488 |              .    *|     .              |
            16     0.07726 |              .     |**   .              |
            17    -0.06070 |              .    *|     .              |
            18    -0.04859 |              .    *|     .              |
            19     0.04677 |              .     |*    .              |
            20     0.00516 |              .     |     .              |
            21    -0.17506 |              . ****|     .              |
            22    -0.09840 |              .   **|     .              |
            23    -0.15253 |              .  ***|     .              |
            24     0.04325 |              .     |*    .              |
```

OUTPUT 6f

$(0,1,1)\ (0,0,1)_4$ model.

3. FORECAST COMPARISONS

This section compares the forecast performance of ARIMA models for real and nominal GDP with two other forecasting methods, namely, the naive forecast model and a macro-econometric model from the Conference Board of Canada.

3.1 Naive Forecast Model:

The "naive" forecast model implies that the forecasts for time t+1 is equal to the value observed at time t. The naive forecast is often used as a point of reference in the sense that, given its simplicity, any other forecasting method has to produce superior results. The naive forecasting method is then defined by:

(3.1.1) $Z_t = Z_{t-1}$

$\Rightarrow Z_{t+1} = Z_t$

(3.1.2) $Z_{t+1} = Z_{t-1}$

3.2 Econometric Models

The Econometric model chosen for this study is the quarterly macro-econometric model of the Conference Board of Canada (CBC). This model consists of 849 equations including disaggregated sectors of consumption, investment, exports, imports, etc. The method of estimation is ordinary least squares (OLS) on each equation. The exogenous variables are generated from the U.S. Warton Econometric model, population and demographics, energy investment, oil prices, tax rates, and government spending. The GDP is estimated from the aggregates of the following sectors of the Keynesian identity:

$$Q = C + I + G + X - M \qquad (3.2.1)$$

Further details about the model are not provided for the public.

Even though the information on the Econometric model is limited the following observations can be made:

a) The choice of OLS estimation versus 2SLS (two stage least squares) may be a source of serious concern from a theoretical point of view since 2SLS is unbiased asymptotically (for large sample size). From the practical point of view, however, it is quite impossible to have a large sample size to begin with and thus for the small sample size the practitioner is justified in using OLS. It is appropriate to mention at this stage that preliminary studies done on other quarterly macro-econometric models seem to suggest 2SLS to be a better estimator (Bodkin, 1989).

b) At the forecasting stage of an econometric model, forecasts of the exogenous variables are required. These forecasts are usually obtained, as mentioned above, from other econometric models (Warton) or equations estimated outside of the systems of equations under study. In cases like this, the large econometric model may introduce errors from other models or equations that can seriously distort the forecasts. At the same time, there is clearly a delay in generating forecasts if the future values of the exogenous variables.are unavailable.

Forecasts from a simultaneous equation model are obtained from the following structural form equations:

$$(3.2.2) \quad BY_i + \Gamma X_i = e_i$$

where Y_i denotes the vector of G endogenous variable at the ith observation, B denotes the parameters of endogenous variables. (a non-singular matrix), X_i denotes the exogenous or lagged endogenous variables at the ith observation, Γ denotes the parameters of predetermined variable and e_i denotes a vector of G stochastic disturbance term at the ith observation. Thus the estimates will have the following form:

$$(3.2.3) \quad Y_i = -B^{-1}\Gamma x_i + B^{-1}e_i$$

$$Y_i = \pi x_i + U_i$$

$(3.2.4) \quad Y^e{}_{i+h} = \pi^e x^e{}_{i+h} + U^e{}_{i+h}$ where Y^e, π^e, X^e and U^e denote estimated values.

Equation (3.2.3) is the reduced form equation where π denotes a matrix of reduced form coefficients and U_i denotes the reduced form stochastic disturbance term.

3.3 Forecast Error Measures

Four types of error measures are used to indicate the forecasting performance of the three models used to extrapolate the two series analyzed here.

percentage (standardized) difference between the actual and the forecasted value for each quarter, and is defined by:

$$(3.3.1) \qquad \frac{(X_t^f - X_t)}{(X_t)} * 100$$

Where X_t denotes actual and X_t^f denotes forecast value.

ii) Mean Square Percentage Error (MSPE): The mean square percentage error is the sum of the square of the percentage error (3.3.1) divided by the number of forecasts calculated, symbolically:

$$(3.3.2) \qquad MSPE = 1/n \sum_{t=1}^{n} (((X_t^f - X_t)/(X_t))*100)^2$$

iii) Root Mean Square Percentage Error (RMSPE): Same as (ii) but the square root of equation 3.3.2 is used.

iv) Theil's Inequality Coefficient: If $X_t\%$ denotes the percentage growth and $X^f{}_t\%$ denote the predicted percentage growth, then

Theil's inequality coefficient (Theil,1966) U_t is defined as:

$$(3.3.3) \qquad U_T{}^2 = \frac{1/n \sum_{t=1}^{n} (X_t\% - X^f{}_t\%)^2}{1/n \sum_{t=1}^{n} (x_t\%)^2}$$

$$U_t = [\,U_t{}^2]^{1/2}$$

3.4 Analysis of the Results

Table 2a and table 2b show the results of the three measures for the level forecasts of the nominal and real GDP respectively, while table 3a and 3b show the results of the four measures for the growth rates of the nominal and real GDP. Both the ARIMA and the econometric models use information up to and including the fourth quarter of 1988 to forecast one year ahead (the forecasts are not reinitialized).

In forecasting the level of nominal GDP (see table 2a) the ARIMA model performed better than the econometric model. This is also true for Real GDP (table 2b). In tracking the growth rate of nominal GDP, the CBC econometric model does a better job (table 3a) but for Real GDP (table 3b) the forecast from the ARIMA model are closer.

Since GDP is a slow changing variable, a fairly large simultaneous equation model like the CBC model (which is built with many purposes in mind, including the studies of the many possible economic effects of changes in government policies and exogenous variables), the short term forecasting may be affected.

Summarizing the information provided in the tables above, we can conclude that both the econometric and the ARIMA models provide superior results than does the naive forecast model as one would expect. Furthermore, in forecasting the levels of nominal and real GDP, the ARIMA model is superior to the CBC econometric model, while in forecasting the growth rate of nominal GDP, the econometric model does a better job.

TABLE 2a

FORECAST ERRORS OF NOMINAL GDP LEVELS

ERROR MEASURE	FORECAST MODEL		
	CBC	ARIMA	NAIVE
% ERROR			
1989:1	-0.91	-0.55	2.3
1989:2	-0.86	-0.27	3.7
1989:3	-0.81	0.38	4.7
1989:4	-0.75	0.19	6.5
MSPE	0.7	0.14	20.8
RMSPE	0.84	0.37	4.56

NOTE:

CBC => CONFERENCE BOARD OF CANADA

ARIMA => (1,1,0)TRAN(SQUARE ROOT)

TABLE 2b

FORECAST ERRORS OF REAL GDP LEVELS

ERROR MEASURE	FORECAST MODEL		
	CBC	ARIMA	NAIVE
% ERROR			
1989:1	-0.85	0.11	-0.6
1989:2	-0.77	0.2	-1
1989:3	-1.27	0.26	-1.55
1989:4	-1.2	0.13	-2.31
MSPE	1.09	0.03	5.1
RMSPE	1.04	0.18	2.26

NOTE:

CBC => CONFERENCE BOARD OF CANADA

ARIMA => $(0,1,1)(0,0,1)_4$

TABLE 3a

FORECAST ERRORS OF NOMINAL GDP GROWTH RATES

ERROR MEASURE	FORECAST MODEL		
	CBC	ARIMA	NAIVE
% ERROR			
1989:1	0.12	-0.57	-2.37
1989:2	0.04	0.29	-1.46
1989:3	0.05	0.65	-1.06
1989:4	0.07	-0.19	-1.89
MSPE	0.01	0.22	3.1
RMSPE	0.08	0.47	1.76
Ut	0.04	0.26	1

NOTE:

CBC => CONFERENCE BOARD OF CANADA

ARIMA => (1,1,0)TRAN(SQUARE ROOT)

TABLE 3b

FORECAST ERRORS OF REAL GDP GROWTH RATES

ERROR MEASURE	FORECAST MODEL		
	CBC	ARIMA	NAIVE
% ERROR			
1989:1	0.7	0.11	-0.6
1989:2	0.09	0.09	-0.41
1989:3	-0.51	0.06	-0.56
1989:4	0.07	-0.13	-0.77
MSPE	0.19	0.01	0.36
RMSPE	0.43	0.1	0.6
Ut	0.37	0.09	1

NOTE:

CBC => CONFERENCE BOARD OF CANADA

ARIMA => $(0,1,1)(0,0,1)_4$

3.5 Limitations of ARIMA Forecasting Models.

i) The most identifiable shortcoming of univariate time series models is their limitation in providing information beyond a forecast value. These models, for example, cannot provide us with information regarding the effect of government expenditure or the impact of free trade on the forecasts provided for 1989.

ii) Given the type of identification, estimation and forecasting technique used in this paper, the time series models cannot indicate or predict sudden movements away from the historical path (e.g. turning points) of a series under study. This is a serious limitation if a recession of some depth is expected to hit the economy, and some information regarding the timing and/or severity is required to foretell.

iii) Although the ARIMA class of models are quite broad, they are restricted to stationary time series or those series that exhibit stationarity after differencing. Since the prediction of future values are constrained to be linear functions of the observations, we also have to assume linearity of the models. Such restriction, often sufficient approximation of reality, does not cover every aspect of real life situation and thus a broader class of models should be entertained.

It should also be pointed out here that the assumption of linearity was somewhat relaxed in our case study earlier by applying the Box-Cox transformation to our model. According to table 1, using the square root of the original series has provided us with a better forecast.

iv) Since stationarity is an important factor in ARIMA modelling, increasing trends are usually differenced. Such trends may be caused by systematic components that consequently should be explained appropriately through some explicit economic theory.

v) Whereas, the statistical tools such as the acf and pacf help the researcher in choosing a tentative model for estimation and forecasting, most of the time he or she has to rely on personal (subjective) judgements. Unless the researcher has extensive knowledge and experience in time series modelling, such judgements can be inaccurate and may lead to inferior forecast values.

It is desirable, therefore, for the researcher to seek expert opinion from other researchers who have done more work on that particular time series or related data. Furthermore, fitting a series of simple ARIMA and seasonal ARIMA models and selecting the one with desirable statistics is a good starting point if the identification stage gives inadequate information.

Fortunately, as indicated by authors such as Wecker (1981), Newbold (1983), and Priestley (1988), current developments in time series analysis show that some of the limitations may be overcome by new sets of time series models. One interesting contribution is that of Harvey and Todd (1983) which introduces "structural time series models" as alternatives to ARIMA models.

In their article, the above authors point out that structural time series models are formulated in terms of the usual time series decomposition of trend, seasonal, irregular and other underlying components and thus facilitate alternative model selection strategy. Furthermore, according to their empirical findings, the structural time series models perform as well as, and in some cases (especially in cases were sudden changes occur in the data), better than univariate ARIMA models.

As verified by this study, time series models produce relatively superior forecast values than competing models. Subsequently applications of time series models have extended into econometric modelling realms as substitutes for large econometric models (such as Vector Autoregressive models) or as a complement to regression models through the modelling of the stochastic term (Pindyck and Rubenfeld,1981). Both of these applications require extensive theoretical exposition and empirical verification to include in this paper, and thus, interested readers are referred to Pindyck and Rubenfeld(1981) and Litterman(1986) for a detailed discussion.

4. CONCLUSIONS:

This investigation has shown that the univariate ARIMA model forecast of nominal and Real GDP for the period first quarter 1989 to the fourth quarter 1989 has performed better than the macro-econometric model of the Conference Board of Canada (CBC) for three of the four cases discussed. The process leading to an ARIMA forecasting stage, however, was shown to be rather complex and at times subjective. It was suggested, therefore, to exercise caution and on occasion to re-estimate the models and compare the forecasting performance of each model to acquire an adequate model.

While current research in time series is expected to introduce better models that can improve some of the drawbacks of ARIMA modelling, existing models were found to be limited in providing prediction of turning points and in decomposing components of time series adequately or efficiently. Furthermore, the restriction of stationarity and linearity are an inadequate approximation of reality and a broader class of models (including non-linear models as well) should be considered.

Since in most business firms and government agencies quick and reliable information regarding macro-economics time series is needed, time series models are justifiably useful.

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APPENDIX

Invertibility:

As mentioned earlier the invertibility condition is satisfied if the roots of the characteristic equation are outside the unit circle (Proof in B-J 1970). To illustrate consider the following examples:

i) $\phi_1 = 0.8 \quad \phi_2 = -0.15$

$$(1 - 0.8B + 0.15B^2) = (1 - .5B)(1 - .3B) = 0$$

The characteristic roots in this case are all greater than one (i.e, they are outside the unit circle) and thus the stationarity condition is satisfied.

ii) $\phi_1 = 1.5 \quad \phi_2 = -.5$

$$(1 - 1.5B + 0.5B^2) = (1 - B)(1 - 0.5B)$$

This example has one root at 1 and consequently the stationarity condition is not satisfied.

Box-Cox Transformation:

Box-Cox transformations for forecasting purposes have two uses (Box-Cox,1964). The first is to make a series variance stationary while the other is to assist in determining the relationship between two or more variables (for example, between dependent and explanatory variables).

$$Z_{t\,(\lambda,m)} = \frac{(z_t+m)^{\lambda} - 1}{\lambda} \qquad \text{suppose } m=0$$

ii

case a: $\lambda=0$ => $$\frac{(Z_t)^0 - 1}{0} \quad \text{indeterminate}$$

by L'ho^pital's rule:

$$\lim_{\lambda \to 0} \frac{\partial[(Z_t)^\lambda - 1]}{\partial\lambda}$$

$$\lim_{\lambda \to 0} \frac{Z_t^\lambda \operatorname{Ln} Z_t}{1} = \operatorname{Ln} Z_t$$

case b: $\lambda = 1$ $$\frac{(Z_t) - 1}{1} = Z_t - 1$$

case c: $\lambda = -1$ $$\frac{(Z_t)^{-1} - 1}{-1} = -(Z_t)^{-1} + 1$$

Portmanteau (Q) Statistics:

Recall that $\rho_k = \gamma_k / \gamma_0 \quad \forall k\ 0,1,2,...$

For an ARMA (p,q) process:

$$(\phi)Z_t = (\theta)n_t \text{ thus,}$$

$$[1+ \alpha_1\phi + \alpha_2\phi^2 + ... + \alpha_p\phi^p]Z_t = [1+ \beta_1\theta + \beta_2\theta^2 + ... + \beta_q\theta^q)n_t$$

If this process is invertible and the estimates are significant, then the residuals are estimated for white noise property:

$\hat{n}_t$ from $[\hat{(\phi)}/\hat{(\theta)}]$

$$r_k = \frac{(\Sigma^T_{t=k+1} \hat{n}_t \hat{n}_{t-k})}{(\Sigma^T_{t=1} \hat{n}_t)^2} \quad \forall\ k\ 1,2,...$$

According to Box and Pierce (1970):

$$T\, \Sigma^n_{k=1} r_k^2 \sim X^2_{m-p-q}$$

Ho: r_k is white noise when n_t is $n \sim (0,\sigma^2)$

Autocorrelation Function (ACF):

An estimate of the theoretical autocorrelation function is given by:

$$r_k = \frac{\Sigma^n{}_{t=k+1} (x_t - \tilde{x})\ (x_{t-k} - \tilde{x})}{\Sigma^n{}_{t=1} (x_t - \tilde{x})^2} \quad \text{where } \tilde{x} \text{ is the sample mean}$$

For uncorrelated observations, the variance of r_k is approximately:

$$V(r_k) \sim= 1/n \quad \text{where n is number of observations}$$

For the general case, however, the Bartlett (Box-jenkins,1970; 34-35) approximation is used to calculate the standard deviation:

$$s(r_k) = (1 + 2\ \Sigma_{j=1}{}^{k-1} r_j)^{½}\ n^{½} \quad \forall\ r_j > k-1$$

The theoretical autocorrelation is also calculated as the ratio of the covariance and variance of the time series at specific lags. The following are three simple ARMA models and their theoretical acf.

CASE 1 AR(1)

$e_t = \phi e_{t-1} + n_t$ where n_t is white noise

$E(n_t) = 0$
$E(n_t)^2 = \sigma^2$
$E(n_{t-i} n_{t-j}) = 0 \ \forall\ i \neq j$

$$\begin{aligned} e_t &= \phi(\phi e_{t-2} + n_{t-1}) + n_t \\ &= \phi^2(\phi e_{t-3} + n_{t-2}) + \phi n_{t-1} + n_t \\ &= \phi^3(\phi e_{t-4} + n_{t-3}) + \phi^2 n_{t-2} + \phi n_{t-1} + n_t \\ &= \ldots \\ &= \ldots. \\ &= \ldots. \\ &=.\ \phi^j e_{t-j} + n_t + \phi n_{t-1} + \phi^2 n_{t-2} + \ldots. + \phi^{j-1} n_{t-j+1}, \end{aligned}$$

and for $j \to \infty$,

$$e_t = n_t + \phi n_{t-1} + \phi^2 n_{t-2} + \ldots$$

$$E(e_t) = E(n_t + \phi n_{t-1} + \phi^2 n_{t-2} + \ldots) = 0$$

$$E(e_t)^2 = E(n_t + \phi n_{t-1} + \phi^2 n_{t-2} + \ldots)^2$$

$$= E(n_t)^2 + \phi^2 E(n_{t-1})^2 + \phi^4 E(n_{t-2})^2 \ldots$$

$$= \sigma^2 + \phi^2\sigma^2 + \ldots.$$

since $E(n_{t-i}n_{t-j})=0$ all cross terms are ignored.

$$= \sigma^2(1 + \phi^2 + \phi^4 + \ldots)$$

$$E(e_t)^2 = \frac{\sigma^2}{1-\phi^2} \qquad \text{(A1.1)}$$

$$E(e_t e_{t-1}) = E(n_t + \phi n_{t-1} + \phi^2 n_{t-2} + \ldots)(n_{t-1} + \phi n_{t-2} + \phi^2 n_{t-3} \ldots)$$

$$= \phi\sigma^2 + \phi^3\sigma^2 + \phi^5\sigma^2 + \ldots$$

$$= \phi\sigma^2 (1 + \phi^2 + \phi^4 + \ldots)$$

$$= \frac{\phi\sigma^2}{1-\phi^2} \qquad \text{(A1.2)}$$

$$\rho_1 = \frac{(e_t e_{t-1})}{(e_t)^2} = \frac{\phi\sigma^2/1-\phi^2}{\sigma^2/1-\phi^2} = \phi \qquad \text{(A1.3)}$$

$$E(e_t e_{t-k}) = E(n_t + \phi n_{t-1} + \ldots + \phi^k n_{t-k} + \phi^{k+1} n_{t-k+1} \ldots)(n_{t-k} + \phi n_{t-k+1} + \ldots)$$

$$= (\phi^k\sigma^2 + \phi^{k+1}\phi\sigma^2 + \phi^{k+2}\phi^2\sigma^2 + \ldots)$$

$$= \phi^k\sigma^2(1 + \phi^2 + \phi^4 + \ldots)$$

$$= \frac{\phi^k\sigma^2}{1-\phi^2} \qquad \text{(A1.4)}$$

$$\rho_k = \frac{E(e_t e_{t-k})}{E(e_t)^2} = \frac{\phi^k\sigma^2/1-\phi^2}{\sigma^2/1-\phi^2} \quad \phi^k \qquad \text{(A1.5)}$$

CASE 2 MA(1)

$e_t = n_t + \theta n_{t-1}$ (again n_t is white noise)

$$E(e_t) = 0$$

$$E(e_t)^2 = E(n_t + \theta n_{t-1})^2$$

$$= \sigma^2 + \theta^2 \sigma^2$$ (all cross terms are zero)

$$= \sigma^2 (1 + \theta^2) \qquad (A2.1)$$

$$E(e_t e_{t-1}) = E(n_t + \theta n_{t-1})(n_{t-1} + \theta n_{t-2})$$

$$= \theta \sigma^2 \qquad (A2.2)$$

$$\rho_1 = \frac{(e_t e_{t-1})}{(e_t)^2} = \frac{\theta \sigma^2}{\sigma^2 (1+\theta^2)}$$

$$\rho_1 = \frac{\theta}{(1+\theta^2)} \qquad (A2.3)$$

$$E(e_t e_{t-k}) = E(n_t + \theta n_{t-1})(n_{t-k} + \theta n_{t-k+1})$$

$$= 0$$

$$\rho_k = 0$$

CASE 3 ARMA(1,1)

$$e_t = \phi e_{t-1} + n_t + \theta n_{t-1}$$

$$= \phi(e_{t-2} + n_{t-1}) + n_t + \theta n_{t-1}$$

$$\phi^2 (e_{t-3} + n_{t-2}) + n_t + (\phi + \theta) n_{t-1}$$

....

....

$$= \phi^j e_{t-j} + n_t + (\theta + \phi) n_{t-1} + \phi(\theta + \phi) n_{t-2} + \phi^2 (\theta + \phi) n_{t-3} + .. + \phi^{j-2} n_{t-j+1},$$

again for $j \to \infty$,

$$e_t = n_t + (\theta+\phi)n_{t-1} + \phi(\theta+\phi)n_{t-2} + \phi^2(\theta+\phi)n_{t-3} + \ldots$$

$$E(e_t) = 0$$

$$E(e_t)^2 = E(n_t + (\theta+\phi)n_{t-1} + \phi(\theta+\phi)n_{t-2} + \ldots)^2$$

$$= \sigma^2 [\, 1 + (\theta+\phi)^2 + \phi^2(\theta+\phi)^2 + \phi^4(\theta+\phi)^2 + \ldots]$$

$$= \sigma^2 \{\, 1 + (\theta+\phi)^2 [\, 1 + \phi^2 + \phi^4 + \phi^6 + \ldots]\}$$

$$= \sigma^2 \{\, 1 + [(\theta+\phi)^2/(1-\phi^2)]\}$$

$$E(e_t)^2 = \frac{(1 + \theta^2 + 2\phi\theta)\sigma^2}{1-\phi^2} \qquad \text{(A3.1)}$$

$$E(e_t e_{t-1}) = E(n_t + (\theta+\phi)n_{t-1} + \phi(\theta+\phi)n_{t-2} + \ldots)(n_{t-1} + (\theta+\phi)n_{t-2} + \ldots)$$

$$= [(\theta+\phi)\sigma^2 + \phi(\theta+\phi)^2\sigma^2 + \phi^3(\theta+\phi)^2\sigma^2 + \ldots]$$

$$= \sigma^2(\theta+\phi)\ [1 + \phi(\theta+\phi) + \phi^3(\theta+\phi) + \ldots]$$

$$= \sigma^2(\theta+\phi)\{\, 1 + \phi(\theta+\phi)[\, 1 + \phi^2 + \phi^4 + \ldots]\}$$

$$= \sigma^2(\theta+\phi)\{\, 1 + [\phi(\theta+\phi)/(1-\phi^2)]\}$$

$$= \frac{(1+\phi\theta)(\phi+\theta)\sigma^2}{1-\phi^2} \qquad \text{(A3.2)}$$

$$\rho_1 = \frac{(1+\phi\theta)(\phi+\theta)\sigma^2}{1-\phi^2} \cdot \frac{(1-\phi^2)}{(1 + \theta^2 + 2\phi\theta)\sigma^2}$$

$$= \frac{(1+\phi\theta)(\phi+\theta)}{(1 + \theta^2 + 2\phi\theta)} \qquad \text{(A3.3)}$$

$$E(e_t e_{t-k}) = (\ldots + \phi^{k-1}(\theta+\phi)n_{t-k} + \phi^k(\theta+\phi)n_{t-k+1}\ldots)(\ldots + n_{t-k} + (\phi+\theta)n_{t-k+1} + \phi(\phi+\theta)n_{t-k+2}\ldots)$$

$$= \phi^{k-1}(\theta+\phi)\sigma^2 + \phi^k(\theta+\phi)^2\ \sigma^2 + \phi^{k+2}(\theta+\phi)^2 + \ldots$$

$$= \phi^{k-1}(\theta+\phi)\sigma^2[\, 1 + \phi(\phi+\theta) + \phi^3(\theta+\phi)^2 + \ldots]$$

$$= \phi^{k-1}(\theta+\phi)\sigma^2 \{ 1 + \phi(\phi+\theta) [1+ \phi^2+ \phi^4+...]\}$$

$$= \phi^{k-1}(\theta+\phi)\sigma^2 \{ 1 +[\phi(\phi+\theta) /(1-\phi^2)]\}$$

$$= \frac{\sigma^2 (\phi^{k-1})(1+\theta\phi)(\theta+\phi)}{1-\phi^2} \qquad \text{(A3.4)}$$

$$\rho_k = \frac{\sigma^2 (\phi^{k-1})(1+\theta\phi)(\theta+\phi)}{1-\phi^2} \cdot \frac{1-\phi^2}{(1+ \theta^2 +2\phi\theta)\sigma^2}$$

$$= \frac{(\phi^{k-1})(1+\theta\phi)(\theta+\phi)}{(1+ \theta^2 +2\phi\theta)}$$

Partial Autocorrelation Function:

The correlation between two random variables, in some cases, is due to the correlation of these two variables to the same third variable. To adjust for this correlation the partial autocorrelation function is used. The pacf essentially measures the additional correlations between two lags after adjusting for the intermediate lags.

To calculate the sample pacf one can fit autoregressive models of increasing order; the estimate of the last coefficient of each model is the sample pacf.

$$\phi_{kk} = \frac{r_k - \Sigma^{k-1}{}_{j=1}\, \hat{\phi}_{k-1,j} r_{k-j}}{1- \Sigma^{k-1}{}_{j=1}\, \hat{\phi}_{k-1,j} r_j}$$

INTEGRATION

The original variable Z_t and a differenced variable W_t are linked deterministically by the differencing operator $(1-B)^d$.

$$W_t = (1-B)^d Z_t$$

This relationship between Z_t and W_t is very important because, after building an ARIMA model for the stationary series W_t, we often want to forecast the original nonstationary series Z_t. Suppose d=1 then,

$$Z_t = (1-B)^{-1} W_t$$

$(1-B)^{-1}$ can be written as infinite series:

$$(1+B+B^2+B^3+\ldots)$$

$$\begin{aligned} Z_t &= (1+B+B^2+B^3+\ldots) W_t \\ &= W_t + W_{t-1} + W_{t-2} + \ldots \\ &= {}^{t}\Sigma_{i=\infty} W_i \end{aligned}$$

Since the Z's are sums of the W's, we can get to Z by integrating.

REFERENCES (For Appendix)

Box, G.E.P. and D.R. Cox (1964), "An Analysis of Transformations", **J. Roy.Statist.Soc.**,Ser B,26,211-243.

Box, G.E.P. and G.Jenkins (1970), **Time Series Analysis: Forecasting and Control**. Holdenday, San Fransisco.

Box, G.E.P. and D.A.Pierce (1970), "Distribution of Residual Autocorrelations in Autoregressive moving average Time Series Models", **J.Amr Statist.Assoc.**,65,1509-1526.

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  TEST FOR THE PRESENCE OF SEASONALITY ASSUMING STABILITY

                          SUM OF      DGRS.OF        MEAN
                          SQUARES     FREEDOM       SQUARE          F-VALUE
BETWEEN QUARTERS      6202894.0444       3     2067631.34814         1.792
        RESIDUAL     55393658.8271      48     1154034.55890
           TOTAL     61596552.8715      51

          NO EVIDENCE OF STABLE SEASONALITY AT THE 0.1 PER CENT LEVEL

  NONPARAMETRIC TEST FOR THE PRESENCE OF SEASONALITY ASSUMING STABILITY

                   KRUSKAL-WALLIS    DEGREES OF    PROBABILITY
                     STATISTIC        FREEDOM         LEVEL

                       7.8055            3            5.021%

          NO EVIDENCE OF SEASONALITY AT THE ONE PERCENT LEVEL

  MOVING SEASONALITY TEST
                         SUM OF      DGRS.OF         MEAN
                         SQUARES     FREEDOM        SQUARE           F-VALUE
  BETWEEN YEARS      8919456.1667       12     743288.013888          1.012
          ERROR     26432910.7019       36     734247.519498

          NO EVIDENCE OF MOVING SEASONALITY AT THE FIVE PERCENT LEVEL

  COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY

          IDENTIFIABLE SEASONALITY NOT PRESENT
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X

Test for the Presence of
stable and moving seasonality
of the GDP series