VIRTUAL PATENT EXTENSION, CANNIBALIZATION, AND COMPULSORY LICENSING

by

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Major paper presented to the Departement of Economics

In partial fulfillment of the requirement of the M.A. Degree

University of Ottawa

Supersivor : Professor Vicky Barham

November 12, 2001

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Nov. 12, 2001

1. Introduction

The problem of optimal patent regulation is one that has been struggled with for many years. As summarized by Mazzoleni and Nelson (1998), the most well known argument in favour of the granting of patents is that a patent allows the inventing firm to reap the rewards of their invention. The anticipation of a patent therefore provides motivation for invention, and society is made better off by a corresponding increase in useful inventions. The cost of granting patents however, is that dead-weight losses are incurred, since the firm which is granted a patent is given monopoly power in the sale of the new invention. Output of the new invention is lower and prices are higher than if competition were allowed to take place.

These costs of patents are generally considered to be outweighed by the benefits of increased inventiveness, and most developed countries therefore have strong intellectual property rights laws. Certainly, there is widespread agreement that patents are warranted for such inventions as new televisions, computer technology, movies and music. However, in the case of pharmaceuticals, which have the potential to save lives, the costs of restricted output and increased prices can be too high. Recently for example, the Brazilian government, faced with an AIDS crisis, decided that the costs of abiding by patents on AIDS drugs were too high. The decision was made to abandon patent protection and allow the manufacturing of generic AIDS drugs. These generics were to be supplied to infected citizens at prices much lower than for what the patented drugs could be supplied (see Rich and Peterson (2001)). The pharmaceutical companies involved have fought back determinedly, arguing that without them the firms had no incentive to invent new drugs.

Developed countries have also had to make similar decisions about pharmaceutical patents. In Canada for example, full patent protection was not provided to pharmaceutical companies until 1987. Rather than increasing the amount of drugs invented world-wide however, the reason for granting patents was to provide incentives for pharmaceutical companies to conduct research in Canada. The goal of the Canadian government was to "stimulate growth in the Canadian pharmaceutical industry while maintaining its policy objective of moderating drug costs." (Consumer and Corporate Affairs Canada, 1983) Prior to 1987, Canada had a policy of compulsory licensing for imported pharmaceuticals. Recently, Pazderka (1999) showed that a change in the trend in Canadian pharmaceutical R&D expenditures has indeed resulted since the 1987 legislation granting full patent protection. However, few, if any studies have been done to examine the extent to which drug prices have increased during this period.

The goal of this paper is to examine the theoretical role of compulsory licensing as a policy instrument. Results show that by allowing competing firms to produce generic drugs while paying a pre-set royalty rate to the patentee can increase competitiveness, thus decreasing drug prices, while at the same time providing some reward, and thus incentive, to the inventing firm. By increasing the royalty rate, a policy maker increases the rewards to the inventing firm but increases drug prices. Conversely, by decreasing the royalty rate, generic drug prices are decreased but the reward to the inventing firm is lessened. As such, a regime of compulsory licensing allows, in theory, the policy maker to trade off the benefits of increased competition with the benefits of rewarding inventing firms. Compulsory licensing is therefore found to be a more flexible policy tool than the choice between full patent protection and full competition. This policy instrument may be of particular interest to developing countries who wish to respect western property rights but also want to provide cheap generic drugs to their poor citizens.

An example is also given which demonstrates how a developed country might use a longer period of compulsory licensing in lieu of patent protection. It should be possible to extend the period of compulsory licensing long enough to make the inventing firm indifferent between compulsory licensing and patent protection. However, consumers would be made better off, assuming that they had a higher discount rate than the firm. This is because the price of pharmaceuticals is decreased immediately whereas the effects of the longer duration of compulsory licensing are not felt until later.

This paper extends a theoretical model of the behaviour of pharmaceutical firms developed by Kamien and Zang (1999) (henceforth K&Z). They showed that a pharmaceutical firm, when faced with the expiration of its patent on a drug, finds it optimal to cannibalize the sale of their own patented drug by introducing a generic product of its own. By doing so, the firm becomes a Stackelberg leader in subsequent periods, when it faces competition from entrants producing competing generic drugs. K&Z find that consumers are no worse off due to this strategy of the firm.

This paper adds a period of compulsory licensing to the K&Z model. The analysis is therefore conducted over three periods, as compared to two for K&Z, and in the second period the patent is still in force but the generic-producing entrants must pay a fixed royalty rate to the patentee for each unit of product sold. I am therefore able to compare three scenarios in period 2 of each model: patent expired (competition); patent in force (monopoly for inventing firm); and compulsory licensing (competition with a royalty paid to the inventing firm.) Under the first three models, these three scenarios are compared assuming that the inventing firm produces only the brand-name product. Under the last three models, the inventing firm produces both the brand-name drug as well as their own generic version, in order to obtain a Stackelberg leadership position in the sale of generic drugs in subsequent periods.

Section 2 of this paper presents a description of the models and the notation to be used. Section 3 presents the first three of six models (CC, CP, CCL) where the inventing firm produces only the brand-name drug. Section 4 presents the last three models (SC, SP, SCL) where the inventing firm produces its own generic version in order to obtain a Stackelberg leadership position in subsequent periods of the models. A summary of the results and discussion are presented in Section 5. Section 6 gives conclusions and presents areas for future research. Attached as an Appendix is Section 7 which contains the comparisons of the models.

2. Description of the Models

2.1. Periods 1, 2, and 3

In the third period, the patent has expired in all three scenarios. In models CC.3, CP.3, and CCL.3, the ex-patentee continues to produce only the brand-name product and competes with the entrants under Cournot competition. This models therefore follows K&Z's Model C2. In models SC.3, SP.3, and SCL.3 the ex-patentee continues to produce both the brand-name and the generic drugs. Therefore, the ex-patentee continues to be (or becomes) the Stackelberg leader to the n entrant firms due to having started producing generics prior to them. This model follows K&Z's model S2.

In the second period, three alternative scenarios are considered. In the first (CC.2 and SC.2), the patent has expired. Competition occurs between the ex-patentee and n identical competitors who produce generic drugs. In model CC, the ex-patentee continues to produce only the brand-name drug, and engages in Cournot competition with the entrants. This corresponds to K&Z's model Cournot, period 2 (C2). In model SC, the ex-patentee produces both the brand-name drug and continues to produce a generic drug. The ex-patentee is the Stackelberg leader due to having produced generics in period one. This corresponds to K&Z's model Stackelberg, period 2 (S2).

In the second scenario (CP.2 and SP.2), the patent continues to be in force as in the case of period 1. In Model CP.2, the patentee continues to produce only the brand-name drug. This period also corresponds to K&Z's model C1. In Model SP.2, the patentee produces both the brand-name and the generic drugs, which corresponds to K&Z's S2 .

The third scenario (CCL.2 and SCL.2), is the one which is new to this paper. In this scenario (compulsory licensing), the patentee is forced to licence the production of the drug to n entrants who pay a predetermined royalty rate, σ, to the patentee for each unit of generic drug they produce. In Model CCL.2, the patentee continues to produce only the brand-name drug. The model is designed like K&Z's Model Cournot period 2, with the exception of the royalty rate being paid by the entrants to the patentee. In Model SCL.2 the patentee continues to produce both the brand-name and the generic drug. Therefore, the patentee has a Stackelberg leadership position in the sale of the generic drug by virtue of having produced the generic drug in period 1. The model therefore is designed like K&Z's Model Stackelberg period 2, with the exception of the royalty rate being paid by the entrants to the patentee.

Tables 1 and 2 describe the models. Each model involves three periods, such that model CC, period 2 will be denoted model CC.2. In the first period of all models, the patent is in force for the brand-name producer and so only this firm may produce pharmaceuticals. In models CC.1, CP.1, and CCL.1, the patentee produces only the brand-name product, corresponding with K&Z's Cournot Model, Period 1 (C1).

In models SC.1, SP.1, and SCL.1, the patentee produces both the brand-name product, as well as introducing its own generic product. The monopolist does this in order to obtain a Stackelberg leadership position in the sale of generic drugs in subsequent periods. These models, in period 1, correspond to K&Z's model Stackelberg, period 1 (S1).

	Model CC - Cournot Competition	Model CP - Cournot Patent	Model CCL - Cournot Compulsory Licensing
Period 1 - Patent in force	Patentee produces the brand-name product as a monopolist. (C.1)	Patentee produces the brand-name product as a monopolist. (C.1)	Patentee produces the brand-name product as a monopolist. (C.1)
Period 2	Patent expired. Ex-patentee produces the brand-name product. There are n Cournot competitors producing the generic product. (C.2)	Patentee produces the brand-name product as a monopolist. (C.1)	Compulsory Licensing. Ex-patentee produces the brand-name product. There are n Cournot competitors producing the generic product. Followers pay royalty rate, s, to leader for each unit of generic product produced.
Period 3 - Patent expired	Patent expired. Ex-patentee produces the brand-name product. There are n Cournot competitors producing the generic product. (C.2)	Patent expired. Ex-patentee produces the brand-name product. There are n Cournot competitors producing the generic product. (C.2)	Patent expired. Ex-patentee produces the brand-name product. There are n Cournot competitors producing the generic product. (C.2)

(Table 1)

	Model SC - Stackelberg Competition	Model SP - Stackelberg Patent	Model SCL - Stackelberg Compulsory Licensing
Period 1 - Patent in force	4.1 Patentee produces both the brand-name and the generic products as a monopolist. (S.1)	5.1 Patentee produces both the brand-name and the generic products as a monopolist. (S.1)	6.1 Patentee produces both the brand-name and the generic products as a monopolist. (S.1)
Period 2	4.2 Ex-patentee produces the brand-name product and is a Stackelberg leader to n followers who produce only the generic product. (S.2)	5.2 Patent in force. Patentee produces both the brand-name and the generic products as a monopolist. (S.1)	6.2 Patent in Force. Compulsory Licensing. Ex-patentee produces the brand-name product and is a Stackelberg leader to n followers who produce only the generic product. Followers pay royalty rate, σ, to leader for each unit of generic product produced.
Period 3 - Patent Expired	4.3 Ex-patentee produces the brand-name product and is a Stackelberg leader to n followers who produce only the generic product. (S.2)	5.3 Ex-patentee produces the brand-name product and is a Stackelberg leader to n followers who produce only the generic product. (S.2)	6.3 Ex-patentee produces the brand-name product and is a Stackelberg leader to n followers who produce only the generic product. (S.2)

(Table 2)

2.2. Notation

Following the notation of K&Z, I denote by B and G the overall quantities produced of the branded and generic products, respectively. Also, g_L and $g(g_L, B)$ represent the quantity of generic product produced by the ex-patentee (the Stackelberg leader) and each of n entrants respectively. The marginal cost of production is assumed to be the same for the branded and generic drugs and is assumed to be a constant marginal cost of c. In the cases of compulsory licensing, the royalty rate is denoted by σ, which is a fixed dollar amount for each unit of generic product produced by the entrants. Producer profits are denoted by π. The inverse demand functions are given by:

$$P_B = a - B - \gamma G \tag{2.1}$$

and

$$P_G = a - G - B \tag{2.2}$$

where P_B and P_G are the prices of the brand-name and generic prices, and the parameter γ, satisfying $0 \leq \gamma < 1$, is a measure of the perceived substitutability of the two products. It is assumed that $a > c$ holds. The demand functions can be derived from equations 2.1 and 2.2, as follows:

$$B = \begin{cases} \frac{a(1-\gamma)+\gamma P_G - P_B}{1-\gamma}, \text{ if } P_B > P_G, \\ a - P_B, \text{ if } P_B < P_G \end{cases}$$

$$G = \begin{cases} \frac{P_B - P_G}{1-\gamma}, \text{ if } P_B > P_G, \\ 0, \text{ if } P_B < P_G \end{cases}.$$

As seen in equation 2.1, the loyal buyers of the brand-name product believe the generic product to be an inferior substitute, as measured by γ. The buyers of the generic drug, the price-sensitive segment view both products to be perfect substitutes, as is seen in equation 2.2. See K&Z for a description of the underlying utility functions which are represented by the demand functions.

3. Models CC, CP, and CCL

3.1. Model CC

3.1.1. Model CC.3 - K&Z C.2

In this period, firm behaviour is identical to that of period CC.2 (K&Z's model C.2) as described below.

3.1.2. Model CC.2 - KZ C.2

In this period, also derived by K&Z (called C.2), the patent has expired. The ex-patentee continues to produce the brand-name drug, but now faces Cournot competition from n identical entrants, who produce a generic alternative product. The quantities are provided here for comparative purposes. For a complete derivation of these quantities see Kamien and Zang (1999).

The output of the brand-name drug, by the ex-patentee, is

$$B_{(CC.2)} = \frac{(a-c)(1+n-n\gamma)}{2+2n-n\gamma}. \tag{3.1}$$

The n entrants collectively produce a total of generic product of

$$G_{(CC.2)} = \frac{n(a-c)}{2+2n-n\gamma}. \tag{3.2}$$

These outputs result in prices of brand-name and generics, respectively, of

$$P_{B(CC.2)} = \frac{a(1+n-n\gamma)+c(1+n)}{2+2n-n\gamma} \tag{3.3}$$

and

$$P_{G(CC.2)} = \frac{a+c(1+2n-n\gamma)}{2+2n-n\gamma}. \tag{3.4}$$

The profit of the ex-patentee is

$$\pi_{L(CC.2)} = \frac{(a-c)^2(1+n-n\gamma)^2}{(2+2n-n\gamma)^2}. \tag{3.5}$$

An individual entrant has profits of

$$\pi_{E(CC.2)} = \frac{(a-c)^2}{(2+2n-n\gamma)^2} \tag{3.6}$$

Summing equations 3.5 and n*3.6 yields total producer profits of

$$\pi_{T(CC.2)} = \frac{(a-c)^2\left[n^2(\gamma-1)^2+n(3-2\gamma)+1\right]}{(2+2n-n\gamma)^2}. \tag{3.7}$$

From the output of brand-name and generic drugs given in equations 3.1 and 3.2, and using the same method as in period CC.1, we obtain that consumer surplus is equal to

$$CS_{(CC.2)} = \frac{(a-c)^2\left[n^2(3-2\gamma)+n(3-\gamma)+1\right]}{2(2+2n-n\gamma)^2}. \tag{3.8}$$

Adding consumer surplus (equation 3.8) and producer profits (equation 3.7) yields total surplus

$$TS_{(CC.2)} = \frac{(a-c)^2\left[n^2(2\gamma^2-6\gamma+5)+n(3-\gamma)+1\right]}{2(2+2n-n\gamma)^2}. \tag{3.9}$$

3.1.3. Model CC.1 - KZ C.1

For completeness, the results derived by K&Z are reported below. In this period, the patent is in force. The patentee produces only the brand-name product as a monopolist. The quantities are provided here for comparative purposes. For a complete derivation of these quantities see Kamien and Zang (1999).

The optimal output of the brand-name product is given by

$$B_{(CC.1)} = \frac{a-c}{2}. \tag{3.10}$$

The optimal quantities increase as the choke price, a, increases, and decrease as the marginal cost of production, c, increases.

There is no output of generic product in this period. The case where the patentee produces a generic product prior to the expiration of the patent is examined under model SC.1. Therefore

$$G_{(CC.1)} = 0. \tag{3.11}$$

The price of both generic and brand-name product are

$$P_{B(CC.1)} = P_{G(CC.1)} = \frac{a+c}{2}. \tag{3.12}$$

We can see that the price increases as either the choke price or the marginal cost of production increases.

This results in profit to the patentee of

$$\pi_{L(CC.1)} = \frac{(a-c)^2}{4}. \tag{3.13}$$

Consumer surplus can be calculated by the quantities and prices of the two products. The area of the consumer surplus triangle is given by $1/2B(B+\gamma G)$

for the brand-name drug and $1/2G(B+G)$ for the generic drug. By using the quantities in equations 3.10 and 3.11, and summing the two expressions, we obtain that consumer surplus equals

$$CS_{(CC.1)} = \frac{(a-c)^2}{4}. \tag{3.14}$$

Finally, by summing consumer surplus (equation 3.14) and producer profits (equation 3.13) we obtain total surplus of

$$TS_{(CC.1)} = \frac{(a-c)^2}{2}. \tag{3.15}$$

3.2. Model CP

3.2.1. Model CP.3 - K&Z C.2

In this period, firm behaviour is identical to that of period CC.2 (K&Z's model C.2) as described above.

3.2.2. Model CP.2 - K&Z C.1

In this period, firm behaviour is identical to that of period CC.1 (K&Z's model C.1) as described above.

3.2.3. Model CP.1 - K&Z C.1

In this period, firm behaviour is identical to that of period CC.1 (K&Z's model C.1) as described above.

3.3. Model CCL - Monopolist Produces only Branded Product, Compulsory Licensing in Second period

3.3.1. Period CCL.3 - K&Z (C.2)

In this period, firm behaviour is identical to that of period CC.2 (K&Z's model C.2) as described above.

3.3.2. Period CCL.2 - Compulsory Licensing

In this period the patent continues to be in force. The patentee continues to produce only the brand-name product. However, entrants are allowed to compete with the patentee provided they pay a royalty rate, σ, to the patentee for each unit of generic product they produce. The n identical entrants engage in Cournot competition with the patentee and produce generic substitutes to the brand-name drug.

Each firm is assumed to act as a Nash-Cournot profit maximizer. Consequently, the patentee solves

$$max_B\,(a - B - \gamma G - c)\,B + \sigma G. \tag{3.16}$$

which yields a first order (necessary) condition for profit maximization:

$$a - 2B - \gamma G - c = 0. \tag{3.17}$$

Solving for G, we obtain

$$G^* = \frac{(a - 2B - c)}{\gamma}. \tag{3.18}$$

The generic producer solves

$$max_{g_j}\left(a - \sum G_i - g_j - B - c - \sigma\right) g_j. \tag{3.19}$$

but $G_i = g_j = G/n$, yielding a first order condition of

$$a - G - G/n - B - c - \sigma = 0. \tag{3.20}$$

Solving for G yields

$$G^* = \frac{(a - c - B - \sigma)\,n}{n + 1}. \tag{3.21}$$

Using equations 3.18 and 3.21, and solving for the output of brand-name product, we obtain

$$B_{(CCL.2)} = \frac{(a - c)\,(n - n\gamma + 1) + n\gamma\sigma}{-n\,(\gamma - 2) + 2}. \tag{3.22}$$

From this we see that, ceteris paribus, as the royalty rate rises, the patentee increases its production of the brand-name product.

Now by substituting equation 3.22 back into 3.18 we obtain the quantity of generic drug

$$G_{(CCL.2)} = \frac{n\,(a - c - 2\sigma)}{-n\,(\gamma - 2) + 2}. \tag{3.23}$$

Observe that $dG/d\sigma < 0$. Therefore, we see that as the royalty rate increases, the output of generics fall. Indeed, as the royalty rate rises, quantities of generics fall faster than the quantity of brand-name rises (since $2n\sigma > n\gamma\sigma$), which accounts for the falling consumer surplus as we shall see shortly. From this point on, we shall assume that $(a - c - 2\sigma) > 0$, so that entrants can only produce a positive output of generics.

Now by substituting the quantities of brand-name and generic found in equations 3.22 and 3.23 into the inverse demand functions (equations 2.1 and 2.2), we obtain the prices of brand-name and generic drugs, respectively of

$$P_{B(CCL.2)} = \frac{a\,(n - n\gamma + 1) + c\,(n + 1) + n\gamma\sigma}{-n\,(\gamma - 2) + 2} \tag{3.24}$$

and

$$P_{G(CCL.2)} = \frac{a + c - n\,(c + \sigma)\,(\gamma - 2)}{-n\,(\gamma - 2) + 2}. \tag{3.25}$$

From these two expression, it is straightforward to calculate that $dP_B/da > 0$, $dP_B/dc > 0$, $dP_B/d\sigma > 0$, $dP_G/da > 0$, $dP_G/dc > 0$, and $dP_G/d\sigma > 0$, so that increases in the choke price, the marginal cost of production, and the royalty rate all increase the price of both the brand-name and generic drugs.

By substituting the quantities of brand-name and generic drugs found in equations 3.22 and 3.23 into 3.16, we obtain the profits of the patentee

$$\pi_{P(CCL.2)} = \frac{(a - c)^2\,[eq.1] + n\sigma\,(a - c)\,(\gamma + 1)\,[eq.2] + n\sigma^2\,(eq.3)}{[n\,(\gamma - 2) - 2]^2} \tag{3.26}$$

where eq.1 (positive)
$n^2\,(\gamma - 1)^2 - 2n\,(\gamma - 1) + 1$
eq.2 (positive)
$-n\,(\gamma - 2) + 2$
eq.3 (negative)
$n\,(\gamma^2 + 2\gamma - 4) - 4.$

The patentee's profits increase with increases in $(a - c)$. Profits continue to increase with the royalty rate, as seen in the positive expression beside $n\sigma\,(a - c)$, which outweighs the negative expression beside $n\sigma^2$, since we assumed that $(a-c) > 2\sigma$. However, as $2\sigma \rightarrow (a-c)$, the negative on $n\sigma^2$, the output of generics approaches zero, so no profits accrue to the patentee from the royalty rate.

By substituting the quantities of brand-name and generic drugs found in equations 3.22 and 3.23 into 3.19, we obtain the profit of an individual entrant of

$$\pi_{E(CCL.2)} = \frac{(a-c-2\sigma)^2}{[n(\gamma-2)-2]^2}. \tag{3.27}$$

As can be seen, as the royalty rate increases, the profits of the entrants fall.

Summing the profits of the patentee and the n entrants (equation 3.26 and n*3.27) we obtain total producer profits of

$$\pi_{T(CCL.2)} = \frac{(a-c)^2[eq.1] + \sigma(a-c)(eq.2) + n^2\sigma^2(eq.3)}{[n(\gamma-2)-2]^2} \tag{3.28}$$

where eq.1 (positive)
$n^2(\gamma-1)^2 - n(2\gamma-3)+1$
eq.2 (positive)
$-n^2(2\gamma^2-\gamma-2) - 2n(\gamma-1)$
eq.3 (negative)
$n^2(\gamma^2+2\gamma-4)$.

The expression beside $\sigma(a-c)$ is positive and outweighs the negative on σ^2 since $(a-c) > 2\sigma$. Therefore, ceteris paribus, increasing the royalty rate increases total producer profits.

Using the quantities of brand-name and generic drugs found in equations 3.22 and 3.23, we can obtain consumer surplus from the same method as that in model CC.1:

$$CS_{(CCL.2)} = \frac{(a-c)^2[eq.1] - \sigma(a-c)[eq.2] + n^2\sigma^2(-\gamma^2-2\gamma+4)}{2[n(\gamma-2)-2]^2} \tag{3.29}$$

where eq.1 (positive)
$n^2(3-2\gamma) + n(3-\gamma)+1$
eq.2 (positive)
$n^2(-\gamma^2-3\gamma+6)+2n$.

The expression beside $-\sigma(a-c)$ is positive and outweighs the negative on σ^2 since $(a-c) > 2\sigma$. Therefore, as the royalty rate is increased, ceteris paribus, the consumer surplus falls. This is as a result of the fact that, as we saw earlier, as the royalty rate increases, output of generics falls faster than the output of brand-name product increases.

By adding the consumer surplus (equation 3.29) and the total producer profits (equation 3.28) we obtain total surplus:

$$TS_{(CCL.2)} = \frac{(a-c)^2 (eq.1) + n\sigma (a-c) (eq.2) + n^2\sigma^2 (eq.3)}{2\left[n(\gamma-2)-2\right]^2} \tag{3.30}$$

where eq.1 (positive)
$n^2 (2\gamma^2 - 6\gamma + 5) - n(5\gamma - 9) + 3$
eq.2 (negative if $\gamma < 2/3$, can be positive if $\gamma > 2/3$ and n high)
$-n(3\gamma-2)(\gamma-1) + 2(2\gamma-3)$
eq.3 (negative)
$\gamma^2 + 2\gamma - 4$.

Like consumer surplus, total surplus falls as the royalty rate is increased, as can be seen by the (usually) negative sign on the expression beside $\sigma(a-c)$ and the negative sign beside $n^2\sigma^2$. Therefore, the loss to consumers by increasing the royalty rate is greater than the gain to total producer profits. In some circumstances however, $\gamma > 2/3$, n high, and σ low, profit increases to producers may outweigh decreases to consumer surplus, so an increase to a very small royalty rate may increase total surplus.

3.3.3. Period CCL.1 - K&Z (C.1)

In this period, firm behaviour is identical to that of period CC.1 (K&Z's model C.1) as described above.

4. Models SC, SP, and SCL

4.1. Model SC

4.1.1. Period SC.3 - K&Z S2

In this period, firm behaviour is identical to that of period SC.2 (K&Z's model S.2) as described below.

4.1.2. Period SC.2 - K&Z S.2

In this period, derived by K&Z, the patent is no longer in force. However, the ex-patentee has assumed a Stackelberg leadership position by virtue of having produced the generic drug in the previous period. The quantities are provided here for comparative purposes. For a derivation of these quantities, see Kamien and Zang (1999).

The optimal quantity of the brand-name drug is given by

$$B_{(SC.2)} = \frac{(a-c)(2n+1)}{3+4n+\gamma}. \tag{4.1}$$

The optimal total quantity of generic drugs produced by both the ex-patentee and the n entrants ($G = g_l + ng_i$) is also

$$G_{(SC.2)} = \frac{(a-c)(2n+1)}{3+4n+\gamma}. \tag{4.2}$$

The price of the brand-name and generic drugs are, respectively,

$$P_{B(SC.2)} = \frac{2a(1+n-n\gamma)+c(2n+1)(1+\gamma)}{3+4n+\gamma} \tag{4.3}$$

and

$$P_{G(SC.2)} = \frac{a(1+\gamma)+2c(2n+1)}{3+4n+\gamma}. \tag{4.4}$$

Again we see that increases in the choke price, a, or the marginal cost of production, c, increase both prices.

The generic output produced by the ex-patentee, who is the Stackelberg leader is

$$g_{L(SC.2)} = \frac{(a-c)(1+n-n\gamma)}{3+4n+\gamma}. \tag{4.5}$$

The entrants, having observed the quantities of generic and branded product made by the leader, produce the following quantity of generic drug:

$$g(g_L, B)_{(SC.2)} = \frac{(a-c)(1+\gamma)}{3+4n+\gamma}. \tag{4.6}$$

Given the quantities produced and their respective prices, the profits of the ex-patentee (the leader) are given by

$$\pi_{L(SC.2)} = \frac{(a-c)^2(1+n-n\gamma)}{3+4n+\gamma}. \tag{4.7}$$

The profits of an individual entrant are given by

$$\pi_{E(SC.2)} = \frac{[(a-c)(1+\gamma)]^2}{(3+4n+\gamma)^2}. \tag{4.8}$$

It is clear that the individual entrant firm's profits decrease as the number of entrants, n, increases.

By using the quantities of generic and brand-name drugs (equations 4.1, and 4.2), and the inverse demand functions (equations 2.1, and 2.2), we obtain consumer surplus by the same process as in model CC.1

$$CS_{(SC.2)} = \frac{(2n+1)^2(a-c)^2(3+\gamma)}{2(3+4n+\gamma)^2}. \tag{4.9}$$

Again, consumer surplus increases with the choke price and decreases with marginal costs.

4.1.3. Period SC.1 - K&Z S.1

In this period, derived by K&Z, the patent is in force. The patentee produces both the brand-name and the generic drugs in order to achieve a Stackelberg leadership position in the sale of generic drugs in subsequent periods. The quantities are provided here for comparative purposes. For a derivation of these quantities, see Kamien and Zang (1999).

The optimal output of both the brand-name and generic product is given by

$$B_{(SC.1)} = G_{(SC.1)} = \frac{a-c}{3+\gamma}. \tag{4.10}$$

The optimal quantities increase with a, the choke price, and decrease with c, the marginal cost. Quantities also decrease as the substitutability of the two products, measured by γ, approaches one.

The price of the brand-name and generic drugs respectively, are

$$P_{B(SC.1)} = \frac{2a+c(1+\gamma)}{3+\gamma} \tag{4.11}$$

and

$$P_{G(SC.1)} = \frac{a(1+\gamma)+2c}{3+\gamma}. \tag{4.12}$$

Both prices, ceteris paribus, increase with both the choke price and the marginal cost of production. It is clear that if $\gamma = 1$, the price of both generic and brand-name drugs are equal.

The patentee's profits are given by

$$\pi_{L(SC.1)} = \frac{(a-c)^2}{3+\gamma}. \tag{4.13}$$

Profits increase, ceteris paribus, as $a-c$ increases, and decrease as γ increases.

Though not derived by K&Z, by using the quantities of generic and branded drugs produced (equation 4.10) and the inverse demand functions (equations 2.1, and 2.2), we can obtain consumer surplus using the same process as in period CC.1:

$$CS_{(SC.1)} = \frac{(a-c)^2}{2(3+\gamma)}. \tag{4.14}$$

Consumer surplus also increases with the choke price, a, and decreases with marginal costs, c, and the perceived substitutability between brand-name and generics, γ.

4.2. Model SP

4.2.1. Period SP.3 - K&Z S2

In this period, firm behaviour is identical to that of period SC.2 (K&Z's model S.2) as described above.

4.2.2. Period SP.2

In this period, firm behaviour is identical to that of period SC.1 (K&Z's model S.1) as described above.

4.2.3. Period SP.1 - K&Z S1

In this period, firm behaviour is identical to that of period SC.1 (K&Z's model S.1) as described above.

4.3. Model SCL

4.3.1. Period SCL.3 - K&Z S2

In this period, firm behaviour is identical to that of period SC.2 (K&Z's model S.2) as described above.

4.3.2. Period SCL.2 - Compulsory Licensing with Patentee as Stackelberg Leader and Cannibalization

In this period, the patent continues to be in force. However, entrants are allowed to compete with the leader provided they pay a royalty rate, σ, for each unit of generic product they produce. The patentee is a Stackelberg leader by virtue of having produced the generic product in the last period. Given the quantities of B and g_L produced by the leader the jth entrant solves

$$max_{g_j}\left(a-\sum g_i - g_j - g_L - B - c - \sigma\right) g_j \tag{4.15}$$

where g_i is the amount of generic drug produced by the ith entrant. The first order (necessary) condition yields a symmetric solution where $g_j = g\left(g_L, B\right)$ for all j which is

$$g(g_L, B) = \frac{a-c-g_L-B-\sigma}{n+1}. \tag{4.16}$$

The leader solves

$$\begin{aligned} &\max_{g_L, B}\left[a-B-\gamma n g\left(g_L, B\right)-\gamma g_L-c\right] B \\ &+\left[a-n g\left(g_L, B\right)-g_L-B-c\right] g_L+\sigma n g\left(g_L, B\right). \end{aligned} \tag{4.17}$$

By substituting 4.16 into 4.17 gives first order (necessary) conditions

$$\frac{\delta \pi}{\delta B}=\frac{(a-c-2B)(1+n+n\gamma)+g_L(2n+1+\gamma)+n\sigma(1-\gamma)}{n+1}=0 \tag{4.18}$$

and

$$\frac{\delta \pi}{\delta g_L}=\frac{(a-c-2g_L)(2n+1)-B(2n+1+\gamma)-2n\sigma}{n+1}=0. \tag{4.19}$$

From 4.18 and 4.19, we obtain

$$B_{(SCL.2)} = \frac{(a-c)\left[-4n^2\gamma - 2n\left(2\gamma-1\right) - \gamma + 1\right] + \sigma\left[-4n^2\left(\gamma-2\right) + 4n\right]}{4n^2\left(2\gamma+1\right) + 8n - \left(\gamma+3\right)\left(\gamma-1\right)} \quad (4.20)$$

from which we can see that as the royalty rate increases, so does production of brand-name drugs. Furthermore, if $\gamma \geq 0.5$, then the higher is $a-c$, the less branded product is produced (ie. production is focussed on the generic). However, if $\gamma \leq 0.5$, the effect of increasing $a-c$ becomes uncertain; if γ and n are sufficiently low, production of the branded product may increase.

Also from 4.18 and 4.19, we obtain:

$$g_{L(SCL.2)} = \frac{(a-c)\left[eq.1\right] - \sigma\left[2n^2\left(\gamma+3\right) - n\left(\gamma^2-5\right)\right]}{4n^2\left(2\gamma+1\right) + 8n - \left(\gamma+3\right)\left(\gamma-1\right)} \quad (4.21)$$

where eq.1 - (positive)
$2n^2\left(3\gamma+1\right) + n\left(\gamma^2+2\gamma+3\right) + 1 - \gamma$.

From this we see that the higher the royalty rate, the less the leader produces generics (production is concentrated on the branded drug). However, the higher is $a-c$, the more the leader produces generics.

Finally, by substituting 4.20 and 4.21 into 4.16, we obtain:

$$g(g_L, B)_{(SCL.2)} = \frac{(a-c)\left[eq.1\right] - \sigma\left[eq.2\right]}{(n+1)\left[4n^2\left(2\gamma+1\right) + 8n - \left(\gamma+3\right)\left(\gamma-1\right)\right]} \quad (4.22)$$

where eq.1 (positive)
$2n^2\left(3\gamma+1\right) - n\left(\gamma+1\right)\left(\gamma-3\right) - \left(\gamma+1\right)\left(\gamma-1\right)$
eq.2 (positive)
$2n^2\left(\gamma+3\right) + n\left(\gamma^2+7\right) - \left(\gamma+3\right)\left(\gamma-1\right)$.

Here we see that the higher the royalty rate, the less generics are produced by the entrants. However, the higher is $(a-c)$, the more generics they produce.

Using the three quantities given in equations 4.20,4.21, and 4.22, and substituting into equation 2.1, we obtain the price of the brand-name drug:

$$P_{B(SCL.2)} = \frac{2a\left[eq.1\right] + c\left[eq.2\right] + \sigma\left[eq.3\right]}{(n+1)\left(4n^2 + 8n - 2\gamma - \gamma^2 + 3 + 8n^2\gamma\right)} \quad (4.23)$$

where eq.1 (positive)
$-2n^3\left(3\gamma+1\right)\left(\gamma-1\right) - n^2\left(5\gamma^2-4\gamma-5\right) - n\left(\gamma^2+\gamma-4\right) - \left(\gamma-1\right)$
eq.2 (positive)

$12n^3\gamma^2 + 2n^2(5\gamma^2+1) + n(\gamma^2+3) - (\gamma+1)(\gamma-1)$
eq.3 (uncertain; positive with high γ)
$4n^3(\gamma^2+4\gamma-2) + 2n^2(\gamma^2+11\gamma-6) - 2n(\gamma^3+\gamma^2-4\gamma+2)$.

From this we see that the price of the brand-name drug increases with a and c. With high levels of substitutability, γ, between the generic and brand-name products, increasing the royalty rate also increases the price of the branded product. However, with a sufficiently low level of substitutability between products, an increased royalty rate can actually decrease the price of the brand-name product.

By substituting the three quantities given in equations 4.20,4.21, and 4.22, into 2.2, we obtain:

$$P_{G(SCL.2)} = \frac{a\,[eq.1] + 2c\,[eq.2] + 2n\sigma\,[eq.3]}{(n+1)(4n^2+8n-2\gamma-\gamma^2+3+8n^2\gamma)} \tag{4.24}$$

where eq.1 (positive)
$2n^2(3\gamma+1) - n(\gamma+1)(\gamma-3) - (\gamma+1)(\gamma-1)$
eq.2 (positive)
$2n^3(2\gamma+1) + 2n(\gamma+2) - (\gamma+1)(\gamma-1)$
eq.3 (positive)
$2n^2(2\gamma+1) + 3n(\gamma+1) - (\gamma+2)(\gamma-1)$.

From this we see that increases in a, c, or the royalty rate, σ, all increase the price of the generic drug.

Substituting equations 4.20,4.21, and 4.22 into 4.15, we obtain the profits for an individual entrant:

$$\pi_{E(SCL.2)} = \left[\frac{(a-c)\,[eq.1] + \sigma\,[eq.2]}{(n+1)(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma)}\right]^2 \tag{4.25}$$

where eq.1 (negative)
$-2n^2(3\gamma+1) + n(\gamma-3)(\gamma+1) + (\gamma+1)(\gamma-1)$
eq.2 (positive)
$2n^2(\gamma+3) + n(\gamma^2+7) - (\gamma-1)(\gamma+3)$.

We can see that the royalty rate and $a-c$ work in opposite directions. Assuming that $(a-c)\,[eq.1] > \sigma\,[eq.2]$entrant profits are positive by virtue of the entire equation being squared, but decreasing as the royalty rate increases.

Now substituting equations 4.20,4.21, and 4.22 into 4.17, we obtain the profits of the patentee:

$$\pi_{L(SCL.2)} = \frac{(a-c)^2[eq.1] - n\sigma^2[eq.2] + 2n\sigma(a-c)[eq.3]}{(n+1)(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma)^2} \tag{4.26}$$

where eq.1 (positive but first term, profits increase on $a-c$)
$16n^5\gamma(3\gamma+1)(\gamma-1)+4n^4(22\gamma^3-13\gamma^2-4\gamma+3)$
$+4n^3(15\gamma^3-10\gamma^2-2\gamma+9)+n^2(-\gamma^4+12\gamma^3-10\gamma^2-24\gamma+39)$
$-n(\gamma-1)(\gamma+3)(\gamma^2-3\gamma+6)+(\gamma+3)(\gamma-1)^2$
eq.2 (positive, profits decrease on squared royalty rate)
$16n^4(\gamma^3+4\gamma^2-3\gamma+7)+4n^3(2\gamma^3+17\gamma^2-32\gamma+65)$
$+4n^2(-2\gamma^4-3\gamma^3-\gamma^2-33\gamma+55)+n(\gamma-1)(\gamma+9)(\gamma-3)(\gamma+3)$
$+(\gamma-1)^2(\gamma+3)^2$
eq.3 (positive, profits increase with royalty rate)
$8n^4(2\gamma^3-12\gamma^2+10\gamma+3)+4n^3(2\gamma^3-31\gamma^2+36\gamma+15)$
$+2n^2(2\gamma^4-2\gamma^3-51\gamma^2+40\gamma+27)+3n(\gamma-1)(\gamma^3+\gamma^2-11\gamma-7)$
$+(\gamma-1)^2(\gamma+3)(\gamma+1)$.

We can see from equation 4.26 that the patentee earns a greater profit as $a-c$ increases. Profits also increase with the royalty rate since $-n\sigma^2[eq.2] < 2n\sigma(a-c)[eq.3]$ by virtue of the fact that $a-c>\sigma$.

By adding the patentee's profits (equation 4.26) to the entrants profits multiplied by n (equation 4.25*n), we obtain total producer profits of:

$$\frac{(a-c)^2(eq.1)+4n^2\sigma(a-c)(eq.2)+4n^2\sigma^2(eq.3)}{(n+1)^2(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma)} \tag{4.27}$$

where eq.1 (positive, but first term)
$16n^6\gamma(3\gamma+1)(\gamma-1)+8n^5(17\gamma^3-6\gamma^2-\gamma+2)$
$+4n^4(34\gamma^3-18\gamma^2+5\gamma+15)+8n^3(7\gamma^3-7\gamma^2-\gamma+11)$
$+n^2(3\gamma-7)(3\gamma^2+2\gamma-9)+2n(\gamma-1)(\gamma^2+2\gamma-11)$
$+(\gamma+3)(\gamma-1)^2$
eq.2 (positive)
$4n^4(2\gamma^3-6\gamma^2+10\gamma+3)+4n^3(3\gamma^3-23\gamma^2+23\gamma+9)$
$+n^2(2\gamma^4-113\gamma^2+82\gamma+41)+n(4\gamma^4+\gamma^3+57\gamma^2+31\gamma+21)$
$+(\gamma-1)(2\gamma^3+3\gamma^2-9\gamma-4)$
eq.3 (negative)
$-4n^4(\gamma^3+4\gamma^2-3\gamma+7)-2n^3(3\gamma^3+16\gamma^2-25\gamma+42)$
$+n^2(2\gamma^4+2\gamma^3-13\gamma^2+72\gamma-99)+2n(\gamma-1)(\gamma+3)(\gamma^2-2\gamma+9)$
$-(\gamma-1)(\gamma+3)(\gamma^2+2\gamma-4)$.

From this we can see that total profits increase as $(a-c)$ increases. Also, from the positive sign on $4n^2\sigma(a-c)$, total profits increase as the royalty rate increases. This increases outweighs the decrease on $4n^2\sigma^2$ since $(a-c)>\sigma$.

By using the quantities of the brand-name and generic drugs (equations 4.20,4.21, and 4.22) and the inverse demand functions (equations 2.1, and 2.2), we obtain consumer surplus in the same manner as in the previous models:

$$CS_{(SCL.2)} = \frac{(a-c)^2(eq.1) - 2n\sigma(a-c)(eq.2) + 4n^2\sigma^2(eq.3)}{2(n+1)^2(4n^2+8n-2\gamma-\gamma^2+3+8n^2\gamma)^2} \tag{4.28}$$

where eq.1 (positive)
$16n^6(-3\gamma^3+6\gamma^2+5\gamma+1)+8n^5(-17\gamma^3+16\gamma^2+28\gamma+9)$
$+4n^4(-36\gamma^3+9\gamma^2+53\gamma+34)+2n^3(-33\gamma^3-13\gamma^2+25\gamma+69)$
$-n^2(7\gamma^3+15\gamma^2+41\gamma-79)+4n(\gamma-1)(\gamma-2)(\gamma+3)$
$+(\gamma+3)(\gamma-1)^2$
eq.2 (positive)
$16n^5(\gamma^3-\gamma^2+7\gamma+2)+4n^4(6\gamma^3-32\gamma^2+71\gamma+27)$
$+4n^3(\gamma^4-3\gamma^3-56\gamma^2+69\gamma+37)+n^2(8\gamma^4-13\gamma^3-153\gamma^2+101\gamma+148)$
$+n(\gamma-1)(\gamma+3)(\gamma+1)(5\gamma-13)+(\gamma+3)(\gamma+2)(\gamma-1)^2$
eq.3 (positive)
$4n^4(\gamma^3+4\gamma^2-3\gamma+7)+2n^3(3\gamma^3+16\gamma^2-25\gamma+42)$
$+n^2(-2\gamma^4-2\gamma^3+13\gamma^2-72\gamma+99)-2n(\gamma-1)(\gamma+3)(\gamma^2-2\gamma+9)$
$+(\gamma-1)(\gamma+3)(\gamma^2+2\gamma-4)$.

From the expression beside $(a-c)^2$, we see that consumer surplus increases as the choke price increases or marginal production costs decrease. From the expression beside $-2n\sigma(a-c)$ we see, ceteris paribus, that consumer surplus decreases as the royalty rate increases. This decrease on $-2n\sigma(a-c)$ outweighs the increase to consumer surplus on $4n^2\sigma^2$, since $(a-c)>\sigma$.

Finally, by adding the total producer profits (equation 4.27) to the consumer surplus (equation 4.28) we obtain total surplus:

$$\frac{(a-c)^2(eq.1) + 2n\sigma(a-c)(eq.2) + 4n^2\sigma^2(eq.3)}{2(n+1)^2(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma)} \tag{4.29}$$

where eq.1 (positive)
$16n^6(3\gamma^3+2\gamma^2+3\gamma+1)+8n^5(17\gamma^3+4\gamma^2+26\gamma+13)$
$+4n^4(32\gamma^3-27\gamma^2+63\gamma+64)+2n^3(23\gamma^3-69\gamma^2+17\gamma+157)$

$+n^2 (11\gamma^3 - 45\gamma^2 - 123\gamma + 205) + 4n (\gamma - 1) (2\gamma^2 + 3\gamma - 17)$
$+3 (\gamma + 3) (\gamma - 1)^2$
eq.2 (low γ, (+,+,+,-,-,-), $\gamma \to 1$, (0,-,-,-,0,0)
$16n^5 (\gamma - 1) (\gamma^2 - 4\gamma - 1) + 12n^4 (2\gamma^3 - 20\gamma^2 + 7\gamma + 3)$
$+4n^3 (\gamma^4 + 3\gamma^3 - 57\gamma^2 + 13\gamma + 4) + n^2 (8\gamma^4 + 17\gamma^3 - 75\gamma^2 + 23\gamma - 21)$
$+n (\gamma - 1) (3\gamma^3 + 5\gamma^2 + \gamma + 23) - (\gamma + 3) (\gamma + 2) (\gamma - 1)^2$
eq.3 (negative)
$-4n^4 (\gamma^3 + 4\gamma^2 - 3\gamma + 7) - 2n^3 (3\gamma^3 + 16\gamma^2 - 25\gamma + 42)$
$+n^2 (-2\gamma^4 - 2\gamma^3 + 13\gamma^2 - 72\gamma + 99) + 2n (\gamma - 1) (\gamma + 3) (\gamma^2 - 2\gamma + 9)$
$- (\gamma - 1) (\gamma + 3) (\gamma^2 + 2\gamma - 4)$.

From this expression we see that increases to $(a - c)$ increase total surplus. If γ is sufficiently high, then increases to the royalty rate decrease total surplus. Therefore losses to consumer surplus outweigh gains to producer profits. Under some circumstances, with a high n, low γ, and low σ, increases to the royalty rate may increase total surplus, meaning that increases to a low royalty rate may increase producers' profits more than the loss to consumer surplus.

4.3.3. Period SCL.1 - K&Z S.1

In this period, firm behaviour is identical to that of period SC.1 (K&Z's model S.1) as described above.

5. Summary and Discussion

5.1. Summary of Models CCL.2 and SCL.2

Effect of Increasing Royalty Rate in Model CCL.2

	On B	On G	On Price B	On Price G	On π_P	On π_E	On π_T	On CS	On TS
Increasing σ	↑	↓	↑	↑	↑	↓	↑	↓	↓

(Table 3)

Effect of Increasing Royalty Rate in Model SCL.2

	On B	On g_L	On g(g_L,B)	On Price B	On Price G	On π_P	On π_E	On π_T	On CS	On TS
Increasing σ	↑	↓	↓	↑ if γ high, ↓ if γ low	↑	↑	↓	↑	↓	↓

(Table 4)

Tables 3 and 4 summarize the effect of increasing the royalty rate on the prices and output of brand-name and generic drugs as well as on the profits of patentee and entrants. Decreasing the royalty rate would have the opposite effect. In model CCL.2, where the patentee produces only the brand-name drug, increasing the royalty rate which the entrants must pay to the patentee results in an increase in the prices of both brand-name and generic drugs. As a result, the patentee increases production of the brand-name drug. However, the entrants reduce their production of generic drugs since their marginal production costs have effectively increased. Not surprisingly, the patentee makes a greater profit as the royalty rate increases, while the entrants' profits decrease. The patentee gains more than the entrants lose however, so total producer profits increase with the royalty rate. Consumers are worse off due to the increasing prices, so consumer surplus falls. Total surplus also falls, as consumer surplus falls further than the gain to producer profits.

Results are similar when the patentee produces both the brand-name and generic products in model SCL.2. As the royalty rate increases, the patentee increases its production of the brand-name drug, and decreases its production of generics. The entrants, as before, decrease their production of generics due to the increase in the cost of production. The price of generics increases with the royalty rate, but the effect on the price of brand-name drugs depends on the perceived substitutability of brand-name and generic drugs. Again, patentee and total profits increase, while entrant profits decrease. Consumer and total surplus again decrease as the royalty rate increases.

5.2. Comparison of Models

Rankings of Cournot Models (assuming $0 < \sigma < (a - c)/2$)

	B	G	B+G	Price B	Price G	π_P	π_E	π_T	CS	TS
CC.2	3	1	1	3	2	3	1	3	1	1
CP.2	1	-	3	1	-	1	-	1	3	3
CCL.2	2	2	2	2	1	2	2	2	2	2

(Table 5)

Rankings of Stackelberg Models (assuming $0 < \sigma < (a - c)/2$)

	B	G	$g(g_L, B)$	Price B	Price G	π_P	π_E	π_T	CS	TS
SC.2	2	1	1	3	3	1	1	3	1	1
SP.2	3	3	-	1	1	3	-	1	3	3
SCL.2	1	2	2	2	2	2	2	2	2	2

(Table 6)

Tables 5 and 6 rank the models in terms of which is greatest for each quantity or price. A rank of 1 indicates that the model has the highest output or price. For example, from table 6, we see that the price of generics is greatest under patent protection (SP.2) and least under competition (SC.2). These rankings were derived from the comparison of the models located in the Appendix of this paper. In order to make comparisons of compulsory licensing and competition, it was assumed that the number of entrants, n, was the same under both scenarios. This, admittedly, is a rather dubious assumption but was necessary in order to make comparisons between the two scenarios. Since there are no entrants under patent protection however, this problem was not relevant when comparing compulsory licensing or competition to patent protection.

Tables 5 and 6 indicate that total output is greatest under competition (CC.2 and SC.2), and least under Patent Protection (CP.2 and SP.2). Compulsory Licensing (CCL.2 and SCL.2) fits in the middle of the two; at low royalty rates is very close to competition , while at high royalty rates is very close to Patent Protection. Conversely, prices are greatest under Patent Protection, and least under Competition. Patent Protection provides the greatest profit to the patentee, while Competition provides the least. Consumer and Total surplus are greatest under Competition. Compulsory Licensing again ranks in the middle, close to competition at low royalty rates and close to Patent Protection at high royalty rates.

When comparing competition and patent protection (Models CC.2 and CP.2 or SC.2 and SP.2) , it is clear that competition is far superior to patent protection in terms of consumer and total surplus. This is primarily because competition forces drug prices down from the monopoly level and drug quantities thereby increase. However, before we conclude that patent protection is a poor policy tool, we must remember its original purpose: to reward and therefore provide incentive to firms to invent new products. Under competition, the inventing firm makes far less profits than under patent protection and may not be able to recoup expensive R&D costs.

Models SCL.2 and SCL.2, compulsory licensing, introduce an alternative way to allow the inventing firm to get some reward for their innovation, while at the same time allowing competition to drive down drug prices. At low royalty rates, compulsory licensing is essentially similar to competition. However, as the royalty rate is increased, the model becomes more similar to patent protection. Increases in the royalty rate cause increases in drug prices, and decreases in output, consumer surplus and total surplus. On the positive side, however, the higher the royalty rate, the higher the profits to the inventing firm, and therefore, presumably, a higher incentive to innovate in the first place.

Policy makers then, after choosing a regime of compulsory licensing could, in theory, adjust the royalty rate according to need. If there were a pressing need for more research and development in the country, increasing the royalty rate would make sense. If however, the primary concern were to decrease the cost of pharmaceuticals, lowering the royalty rate would be the solution.

This policy tool may be attractive to developing countries who wish to pacify western pharmaceutical companies concerns about patent rights but at the same time keep drug prices low through competition. The royalty rate provides some reward to the inventing firm, albeit less than patent protection.

Alternatively, developed countries may view this policy tool as another option in their patent laws. In theory, the period of compulsory licensing could last longer than the alternative period of patent protection at a royalty rate which, over the duration of the compulsory licensing period, would allow the firm to make the same profit (at present discounted value) as if it were given patent protection. The firm would therefore be indifferent between the longer period of compulsory licensing and the shorter period of patent protection. If it were assumed that consumers had a higher discount rate than firms (due to their need to access the drug immediately), then it may be efficient to allow compulsory licensing now to lower drug prices and increase consumer surplus. The consumers will face a period in the future where the patent protection would have expired but the compulsory licensing is still in effect, but due to their high discount rate are made better off.

The limitation to this argument, however, is that in comparing consumer surplus now and in the future, the consumers of the pharmaceuticals are not likely to be the same (depending on the length of the patent and length of use of the drug.) Therefore, consumers who purchase the drugs after the time when a patent would have expired, but prior to the expiration of compulsory licensing, are made worse off.

An Example

To illustrate the above argument, consider the following example. Suppose under models CC.2, CP.2, and CCL.2 the following parameter values existed: $n = 3$, $\gamma = 0.75$, $a = 10$, $c = 2$, $\sigma = 1$.

Under patent protection (from equations 3.13, and 3.14), the patentee's profits would be 16. Consumer surplus would also equal 16.

Under competition (from equations 3.5 and 3.8), the ex-patentee's profits would be only 5.93, while consumer surplus would increase to 20.56.

Finally, under compulsory licensing (from equations 3.26 and 3.29), patentee profits would be 12.21, while consumer surplus would equal 16.63.

Next, for simplicity, let us assume that the patentee has a discount rate of 0 percent. Consumers, who place a higher value on the present, discount the future at an annual rate of 10 percent. (For the example to work, it is not important that the patentee has a discount rate of 0; only that it be lower than the discount rate for consumers.) Finally, let us assume that the duration of patent protection is for twenty five years. During this time, the patentee has an income stream of 16 each year for twenty-five years followed by an income stream of 5.93. Our first task then is to determine the additional number of years, over 25, that compulsory licensing would have to be in effect in order for the patentee to be indifferent between it and patent protection. Since there is a 0 discount rate for the patentee, this problem can be solved by:

$$16 * 25 + 5.93x = 12.21\,(25 + x) \tag{5.1}$$

where x is the addition number of years beyond 25. The solution is for x is approximately equal to 15 years. This means that, under these conditions, the patentee is be indifferent between patent protection for 25 years and compulsory licensing for 40 years.

Consumers, on the other hand, would be faced with a choice between two consumer surplus streams. The first, under patent protection, would consist of 25 years of consumer surplus of 16, followed by 15 years of consumer surplus of 20.56. The second, under compulsory licensing would be 40 years of consumer surplus of 16.63. With a discount rate of 10 percent, the stream under patent protection has a present value of 158.35, while the stream under compulsory licensing has a present value of 162.63. Consumers are therefore made better off under compulsory licensing, while the patentee is indifferent.

5.3. Discussion of Limitations of Model

One major limitation of the compulsory licensing model, as it is currently developed, is that there is no attempt to identify the optimal choice of σ, the royalty rate. In order to do so, one would have to consider the effects of the royalty rate on the likelihood of future useful inventions by firms. Presumably, as the royalty rate increases, the increased potential profits to the patentee act as incentive to spend money on research and development, much as the promise of patent protection is thought to. As the royalty rate increases, however, consumer surplus falls. Finding the optimal level, therefore requires balancing these two effects.

Likewise, no attempt has been made to determine the optimal number of entrants under compulsory licensing. If the potential patentee knew that the government would restrict the number of compulsory licensees, this might also provide additional profit to the patentee and thus an incentive to invent. Again, however, restricting the amount of competition would likely lower consumer surplus.

Finally, as mentioned above, when comparing compulsory licensing to competition, the assumption that the number of entrants is the same under each scenario is unlikely to be realistic. However, this assumption does not affect the results in terms of the effects of increasing or decreasing the royalty rate, nor does it effect the comparisons with patent protection.

6. Conclusions

Extending a model derived by Kamien and Zang (1999), I have shown how compulsory licensing can be used as an alternative and flexible policy tool as compared with patent protection or the lack thereof (competition). The effect of increasing the royalty rate under compulsory licensing is to provide the inventing firm with greater profits and thus greater incentive for further inventiveness. Meanwhile however, the increase in royalty rate decreases competitiveness and lowers consumer surplus. Directions for future research are clear: find ways to optimally determine the royalty rate and the number of entrants under compulsory licensing.

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7. Appendix

7.1. Comparisons of Models CC.2, CP.2, and CCL.2

7.1.1. Quantities of Brand-name Drugs

Comparing Competition and Patent Protection (CC.2 and CP.2)

By subtracting the output of brand-name drugs under competition (equation 3.1) from the output under patent protection (equation 3.10) we obtain (CP.2 - CC.2)

$$\frac{n\gamma\,(a-c)}{2\,(2+2n-n\gamma)}. \tag{7.1}$$

This expression is positive, so the output of brand-name drugs is greater under monopoly than under competition.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

By subtracting the output of brand-name drugs under competition (equation 3.1) from the output under compulsory licensing (equation 3.22), we obtain (CCL.2 - CC.2)

$$\frac{n\gamma\sigma}{2+2n-n\gamma}. \tag{7.2}$$

This expression is also positive, so the output of brand-name drugs is greater under compulsory licensing than competition. As the royalty rate increases under compulsory licensing, the output of brand-name drugs increases, and therefore so does the difference from the competitive level.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

We now compare the difference in output of brand-name drugs between patent protection and compulsory licensing by subtracting the output under compulsory licensing (equation 3.22) from the output under patent protection (equation 3.10) (CP.2 - CCL.2):

$$\frac{n\gamma(a-c-2\sigma)}{2(2+2n-n\gamma)}. \tag{7.3}$$

This expression is positive, so less brand-name product is produced under compulsory licensing than under patent protection. However, as the royalty rate rises under compulsory licensing, the output of brand-name product increases, and so the difference from patent protection disappears when $2\sigma = (a-c)$.

7.1.2. Compare Total Output

Comparing Competition and Patent Protection (CC.2 and CP.2)

By subtracting the total output under competition (equations 3.1 and 3.2) from the output under patent protection (equation 3.10), we obtain the difference in total output between competition and patent protection (CP.2 - CC.2):

$$\frac{n(a-c)(\gamma-2)}{2(2+2n-n\gamma)}. \tag{7.4}$$

This expression is negative. Therefore, total output is greater under competition than under patent protection. The introduction of generic products by the entrants has more than offset the decline in production of the brand-name product by the patentee.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

When comparing compulsory licensing and competition, we can compare both the output of generic drugs as well as the total output.

By subtracting the output of generics under competition (equation 3.2) from the output under compulsory licensing (equation 3.23) we obtain (CCL.2 - CC.2)

$$\frac{-2n\sigma}{2+2n-n\gamma}. \tag{7.5}$$

This expression is negative, so compulsory licensing lowers the output of generic drugs from the competitive level. The output of generics continues to shrink as the royalty rate grows under compulsory licensing, so the difference from the competitive level increases.

By subtracting the total output under competition (equations 3.1 and 3.2) from the output under compulsory licensing (equations 3.22 and 3.23) we obtain (CCL.2 - CC.2)

$$\frac{n\sigma\left(\gamma-2\right)}{2+2n-n\gamma}. \tag{7.6}$$

This expression is also negative, so compulsory licensing lowers total output from competitive level. Total output continues to shrink as the royalty rate grows under compulsory licensing so the difference from the competitive level grows.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

By subtracting the total output under compulsory licensing (equations 3.22 and 3.23) from the total output under patent protection (equation 3.10), we obtain (CP.2 - CCL.2)

$$\frac{n\left(a-c-2\sigma\right)\left(\gamma-2\right)}{2\left(2+2n-n\gamma\right)}. \tag{7.7}$$

With a small royalty rate, this expression is negative, so output is greater under compulsory licensing than patent protection. However, as the royalty rate increases under compulsory licensing, total output shrinks, and the difference from patent protection decreases. Once $2\sigma = (a-c)$, total output is the same under compulsory licensing and patent protection since generic output hits zero and the patentee maximizes as it would have under patent protection.

7.1.3. Compare Prices

Comparing Competition and Patent Protection (CC.2 and CP.2)

By subtracting the price of brand-name drugs under competition (equation 3.3) from the price under patent protection (equation 3.12) we obtain (CP.2 - CC.2)

$$\frac{n\gamma\,(a-c)}{2\,(2+2n-n\gamma)}. \tag{7.8}$$

This expression is positive, so the price of brand-name products is lower under competition than under patent protection. This is in spite of the fact that the output of brand-name products is greater under patent protection. The output of generic drugs under competition forces down the price of the brand-name drug from the level under patent protection.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

When comparing competition and compulsory licensing, we can compare both the price of generics and the price of brand-name drugs.

By subtracting the price of generics under competition (equation 3.4) from the price under compulsory licensing (equation 3.25), we obtain (CCL.2 - CC.2)

$$\frac{n\sigma\,(2-\gamma)}{2+2n-n\gamma}. \tag{7.9}$$

This expression is positive, so the price of generics is higher under compulsory licensing than competition. As the royalty rate increases under compulsory licensing, the price of generics continue to rise, so the difference from the competitive price increases.

By subtracting the price of brand-name drugs under competition (equation 3.3) from the price under compulsory licensing (equation 3.24) we obtain (CCL.2 - CC.2)

$$\frac{n\gamma\sigma}{2+2n-n\gamma}. \tag{7.10}$$

This expression is positive. Therefore, the price is higher under compulsory licensing than competition. As the royalty rate increases under compulsory licensing, the price of brand-name drugs increases, so the difference from the competitive level grows.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

By subtracting the price of brand-name drugs under compulsory licensing (equation 3.24) from the price under patent protection (equation **??**), we obtain (CP.2 - CCL.2)

$$\frac{n\gamma\,(a-c-2\sigma)}{2\,(2+2n-n\gamma)}. \tag{7.11}$$

This expression is positive. This means that compulsory licensing lowers the price of the brand-name drug from patent protection. However, as the royalty rate rises under compulsory licensing, so does the price of the brand-name drug, so the difference from the price under patent protection shrinks. Once $2\sigma = (a-c)$, the price of brand-name drugs under compulsory licensing is the same as under patent protection.

7.1.4. Compare Patentee Profits

Comparing Competition and Patent Protection (CC.2 and CP.2)

We compare patentee profits under patent protection and competition by subtracting the patentee's profits under competition (equation 3.5) from the profits under patent protection (equation 3.13) (CP.2 - CC.2)

$$\frac{n\gamma\,(a-c)^2\,(4+4n-3n\gamma)}{4\,(-2-2n+n\gamma)^2}. \tag{7.12}$$

This expression is positive, so the (ex)-patentee profits are lower under competition than under patent protection.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

By subtracting the patentee's profits under competition (equation 3.5) from the profits under compulsory licensing (equation 3.26) we obtain (CCL.2 - CC.2)

$$\frac{n\sigma\,[(a-c)\,(\gamma+1)\,(2n-n\gamma+2)-\sigma\,[-n\,(\gamma^2+2\gamma-4)+4]]}{(-2-2n+n\gamma)^2}. \tag{7.13}$$

This expression is positive since $(a-c-2\sigma)>0$, so the patentee's profits are greater under compulsory licensing than under competition. As the royalty rate grows under compulsory licensing, so do the patentee's profits, and the difference from the competitive level increases.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

By subtracting the profits of the patentee under compulsory licensing (equation 3.26) from their profits under patent protection (equation 3.13), we obtain (CP.2 - CCL.2)

$$\frac{n\left[(a-c)^2(eq.1)-4\sigma(a-c)(eq.2)+4\sigma^2(eq.3)\right]}{4(-2-2n+n\gamma)^2} \tag{7.14}$$

where eq.1 (positive)
$4+4n-3n\gamma$
eq.2 (positive)
$n(-2\gamma^2+\gamma+2)+2(\gamma+1)$
eq.3 (positive)
$-n(\gamma^2+2\gamma-4)+4.$

This expression is positive on $(a-c)^2$, so at low royalty rates, the patentee's profits are lower under compulsory licensing than patent protection. However, from the expression beside $-4\sigma(a-c)$, as the royalty rate increases under compulsory licensing, so do profits, and the difference from the level under patent protection shrinks. This increase on $-4\sigma(a-c)$ outweighs the decrease in profits on $4\sigma^2$ since $(a-c-2\sigma)>0$.

7.1.5. Compare Total Producer Profits

Comparing Competition and Patent Protection (CC.2 and CP.2)

To compare total profits under patent protection and competition, we subtract the total profits under competition (equation 3.7) from the total profits under patent protection (equation 3.13) (CP.2 - CC.2):

$$\frac{n(a-c)^2[n\gamma(-3\gamma+4)+4(\gamma-1)]}{4(-2-2n+n\gamma)^2}. \tag{7.15}$$

This expression is most likely positive (if $n\gamma>1$), so total profits are greater under patent protection than under competition. However, with low n and γ, the expression can be negative, so higher total profits can be achieved under competition than patent protection. This is due to the fact that the competing firms can now produce both generics and brand-name products whereas under patent protection only brand-name products are produced. If n is small, profits are not driven down very far, and the firms reap the benefits of producing both generics and brand-name products.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

When comparing competition and compulsory licensing, both entrants' profits and total profits can be compared.

By subtracting the total entrants' profits under competition (equation 3.6) from the profits under compulsory licensing (equation 3.27) we obtain (CCL.2 - CC.2)

$$\frac{-4n\sigma\left(a-c-\sigma\right)}{\left(-2-2n+n\gamma\right)^{2}}. \tag{7.16}$$

This expression is negative, so entrants' profits decrease under compulsory licensing from competition. This loss in profits becomes greater as the royalty rate increases under compulsory licensing, so the difference from the competitive level increases.

In order to compare total profits under compulsory licensing and competition, we subtract the total profits under competition (equation 3.7) from the profits under compulsory licensing (equation 3.28) (CCL.2 - CC.2):

$$\frac{n\sigma\left[\left(a-c\right)\left[-n\left(\gamma+1\right)\left(\gamma-2\right)+2\left(\gamma-1\right)\right]+n\sigma\left(\gamma+2\right)\left(\gamma-2\right)\right]}{\left(-2-2n+n\gamma\right)^{2}}. \tag{7.17}$$

This expression is positive, from the positive expression on $(a-c)$, which outweighs the negative expression on $n\sigma$, since $(a-c-2\sigma)>0$. Therefore, total profits are higher under compulsory licensing than competition. Total profits continue to increase as the royalty rate increases so the difference from the competitive level increases. If the royalty rate is zero, there is no difference from the competitive level.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

By subtracting total profits under compulsory licensing (equation 3.28) from the total profits under patent protection (equation 3.13) we obtain the difference in total profits (CP.2 - CCL.2)

$$\frac{n\left[\left(a-c\right)^{2}(eq.1)-4\sigma\left(a-c\right)(eq.2)+4n\sigma^{2}(eq.3)\right]}{4\left(-2-2n+n\gamma\right)^{2}} \tag{7.18}$$

where eq.1 (positive if $n\gamma>1$)
$n\gamma\left(-3\gamma+4\right)+4\left(\gamma-1\right)$
eq.2 (positive)
$n\left(-2\gamma^{2}+\gamma+2\right)+2\left(\gamma-1\right)$
eq.3 (positive)

$-\gamma^2 - 2\gamma + 4$.

This expression is most likely positive on $(a-c)^2$, so total producer profits are greater under monopoly than under compulsory licensing at low royalty rates. However, from the expression beside $-4\sigma(a-c)$, as royalty rate increases under compulsory licensing so do total profits. this increase outweighs the decrease on $4n\sigma^2$ since $(a-c-2\sigma) > 0$. Therefore as the royalty rate increases under compulsory licensing, the difference from the patent protection level of total profits decreases.

7.1.6. Comparing Consumer Surplus

Comparing Competition and Patent Protection (CC.2 and CP.2)

By subtracting the consumer surplus under competition (equation 3.8) from the consumer surplus under patent protection (equation 3.14), we obtain (CP.2 - CC.2)

$$\frac{(a-c)^2\left[n^2\left(\gamma^2-2\right)+2n\left(1-\gamma\right)+2\right]}{4\left(-2-2n+n\gamma\right)^2}. \tag{7.19}$$

This expression is negative if $n > 2$, so consumer surplus is higher under competition than under patent protection patent protection. However, if $n = 1$, under competition, the expression becomes positive and consumer surplus is higher under patent protection than competition. This is due to the fact that with only one entrant, prices are not driven down very far, but consumer surplus is increased since generics are now available while they weren't under patent protection.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

By subtracting the consumer surplus under competition (equation 3.8) from the consumer surplus under compulsory licensing (equation 3.29) we obtain (CCL.2 - CC.2)

$$\frac{n\sigma\left[(a-c)\left[n\left(\gamma^2+3\gamma-6\right)-2\right]+n\sigma\left(-\gamma^2-2\gamma+4\right)\right]}{2\left(-2-2n+n\gamma\right)^2}. \tag{7.20}$$

This expression is negative, since the expression beside $(a-c)^2$ greater than that beside σ and $(a-c-2\sigma) > 0$. Therefore consumer surplus is greater under competition than compulsory licensing, and the difference grows larger as the royalty rate increases. If the royalty rate is zero, consumer surplus is exactly the same as under competition.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

To compare the consumer surplus under patent protection and compulsory licensing, we subtract the consumer surplus under compulsory licensing (equation 3.29) from the consumer surplus under patent protection (equation 3.14) (CP.2 - CCL.2) to obtain:

$$\frac{(a-c)^2\,(eq.1) + 2n\sigma\,(a-c)\,(eq.2) + 2n^2\sigma^2\,(eq.3)}{4\,(-2-2n+n\gamma)^2} \tag{7.21}$$

where eq.1 (negative if $n > 1$)
$n^2(\gamma^2-2) + 2n(1-\gamma) + 2$
eq.2 (positive)
$n(-\gamma^2-3\gamma+6) + 2$
eq.3 (negative)
$\gamma^2 + 2\gamma - 4.$

This expression is negative on $(a-c)^2$, so at low royalty rates consumer surplus is higher under compulsory licensing than patent protection. But, from $2n\sigma(a-c)$ consumer surplus decreases as the royalty rate under compulsory licensing increases and the difference from patent protection shrinks. This decrease in consumer surplus on $2n\sigma(a-c)$ outweighs the increase on $2n^2\sigma^2$ since $(a-c-2\sigma) > 0$.

7.1.7. Compare Total Surplus

Comparing Competition and Patent Protection (CC.2 and CP.2)

By subtracting the total surplus under competition (equation 3.9) from the total surplus under patent protection (equation 3.15) we obtain (CP.2 - CC.2)

$$\frac{(a-c)^2\left[-n^2(\gamma-1)^2 + n(\gamma-1) + 1\right]}{2\,(-2-2n+n\gamma)^2}. \tag{7.22}$$

This expression is negative unless n is very low and γ very high. Therefore, under most circumstances, total surplus is increased under competition from patent protection. However, if n is very low, prices don't get driven down very far. Also if γ is very high, consumers see no difference between generics and brand-name products. Therefore, the introduction of competition does not increase total surplus under these circumstances.

Comparing Competition and Compulsory Licensing (CC.2 and CCL.2)

By subtracting the total surplus under competition (equation 3.9) from the total surplus under compulsory licensing (equation 3.30) we obtain (CCL.2 - CC.2)

$$\frac{n\sigma\left[(a-c)\left[eq.1\right]+n\sigma\left(eq.2\right)\right]}{2\left(-2-2n+n\gamma\right)^{2}} \tag{7.23}$$

where eq.1 (negative if $\gamma < 2/3$, can be positive if $\gamma > 2/3$ and n high)
$-n\left(3\gamma-2\right)\left(\gamma-1\right)+2\left(2\gamma-3\right)$
eq.2 (negative)
$\gamma^{2}+2\gamma-4.$

From this expression, we see that total surplus falls as the royalty rate is increased under compulsory licensing, as can be seen by the (usually) negative sign on the expression beside $(a-c)$ and the negative sign beside $n\sigma^{2}$. Therefore, total surplus is less under compulsory licensing than competition. Under some circumstances however, increasing the royalty rate under compulsory licensing may increase total surplus. See the previous description of equation 3.30 for details.

Comparing Patent Protection and Compulsory Licensing (CP.2 and CCL.2)

In order to compare the total surplus under patent protection and compulsory licensing, we subtract the total surplus under compulsory licensing (equation 3.30) from the total surplus under patent protection (equation 3.15) (CP.2 - CCL.2) to obtain:

$$\frac{\left(a-c\right)^{2}\left(eq.1\right)-n\sigma\left(a-c\right)\left(eq.2\right)+n^{2}\sigma^{2}\left(-\gamma^{2}-2\gamma+4\right)}{2\left(-2-2n+n\gamma\right)^{2}} \tag{7.24}$$

where eq.1 (negative unless n really low or γ really high)
$-n^{2}\left(\gamma-1\right)^{2}+n\left(\gamma-1\right)+1$
eq.2 (likely negative)
$n\left(\gamma-1\right)\left(3\gamma-2\right)+2\left(2\gamma-3\right).$

This expression is most likely negative on $\left(a-c\right)^{2}$, so at low royalty rates, total surplus is greater under compulsory licensing than patent protection. However, as in the comparison of consumer surplus under patent protection and competition (equation 7.22) if n is very low and γ very high, total surplus may be greater under patent protection.

The expression generally is positive on $n\sigma\left(a-c\right)$ and $n^{2}\sigma^{2}$, so total surplus decreases as the royalty rate increases under compulsory licensing. Therefore, total surplus becomes less under compulsory licensing, and the difference from the level under patent protection decreases. Under some circumstances however, increasing the royalty rate under compulsory licensing may increase total surplus. See the previous description of equation 3.30 for details.

7.2. Comparisons of Models SC.2, SP.2, and SCL.2

7.2.1. Quantities of Brand-name drug

Comparing Competition and Patent Protection (SC.2 and SP.2)

First, we will compare the output of the drugs in the three scenarios. By subtracting the output of brand-name drug under competition (equation 4.1) from the output of brand-name drug under patent protection (equation 4.10), we obtain (SP.2 - SC.2)

$$\frac{-2n\,(a-c)\,(1+\gamma)}{(3+\gamma)\,(3+4n+\gamma)}. \tag{7.25}$$

By virtue of the fact that this value is negative, we know that the competition quantity of brand-name product is greater than the quantity under patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

Similarly, by subtracting the output of brand-name drug under competition (equation 4.1) from the output of brand-name drug under compulsory licensing (equation 4.20) we obtain (SCL.2 - SC.2):

$$\frac{2n\,[(a-c)\,(eq.1)+\sigma\,(eq.2)]}{-\left[-4n^2\,(2\gamma+1)-8n+(\gamma-1)\,(\gamma+3)\right][3+4n+\gamma]} \tag{7.26}$$

where eq.1(negative)

$-4n^2\,(4\gamma+1)-2n\,(\gamma^2+9\gamma+3)-(\gamma^2+5\gamma+2)$

eq.2 (positive)

$-8n^2\,(\gamma-2)-2n\,(\gamma^2+\gamma-10)+2\,(\gamma+3)$.

The denominator of this expression is positive. Therefore the fact that the equation next to $a-c$ is negative means that ceteris paribus, at low levels of royalty rates, compulsory licensing causes a decrease in output of the branded product compared to competition. However, as the royalty rate grows, the compulsory licensing quantity of brand-name product grows and the difference between the competitive output decreases.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

Finally, by subtracting the output of brand-name drug under compulsory licensing (equation 4.20) from the output of brand-name drug under patent protection (equation 4.10), we obtain (SP.2 - SCL.2):

$$\frac{2n\left[(a-c)\left[eq.1\right]+2\sigma\left[eq.2\right]\right]}{-\left(3+\gamma\right)\left[-4n^2\left(2\gamma+1\right)-8n+\left(\gamma-1\right)\left(\gamma+3\right)\right]} \tag{7.27}$$

where eq.1

$2n\left(\gamma^2+5\gamma+1\right)+2\gamma^2+5\gamma+1$

eq.2

$n\left(\gamma+3\right)\left(\gamma-2\right)-\left(\gamma+3\right)$.

Similarly to the preceding comparison, when the royalty rate is low,this expression is positive, so less brand-name product is produced under compulsory licensing as compared to patent protection. However, as the royalty rate grows, the compulsory licensing quantity of brand-name product grows and the difference between the patent protection output decreases.

7.2.2. Quantities of Generic Drug

Comparing Competition and Patent Protection (SC.2 and SP.2)

We now compare quantities of the generic drugs. By subtracting the quantity of generic drug produced under competition (equation 4.2) from the quantity produced under patent protection (equation 4.10), we obtain (SP.2 - SC.2)

$$\frac{-2n\left(a-c\right)\left(1+\gamma\right)}{\left(3+\gamma\right)\left(3+4n+\gamma\right)}. \tag{7.28}$$

This expression is negative, so the competition quantity of generic product is greater than the output under patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

In comparing compulsory licensing and competition, we are able to compare both the generic output of the leader and of the entrants. By subtracting the generic output of the leader under competition (equation 4.5) from its output under compulsory licensing (equation 4.21, we obtain (SCL.2 - SC.2):

$$\frac{n\left[2\left(a-c\right)\left(eq.1\right)+\sigma\left(eq.2\right)\right]}{-\left[-4n^2\left(2\gamma+1\right)-8n+\left(\gamma-1\right)\left(\gamma+3\right)\right]\left[3+4n+\gamma\right]} \tag{7.29}$$

where eq.1 (positive)

$\left(2n^2+1\right)\left(2\gamma^2+5\gamma+1\right)+n\left(5\gamma^2+14\gamma+3\right)$

eq.2 (negative)
$-8n^2(\gamma+3)+2n(\gamma^2-6\gamma-19)+(\gamma+3)(\gamma^2-5)$.

From this we see that, ceteris paribus, at low royalty rates, the positive equation beside $a-c$ indicates that the leader produces more generic drugs under compulsory licensing than competition. However, as the royalty rate rises, the leader decreases its production of generics under compulsory licensing and the difference from the competitive level falls.

The entrants production of generics was compared by subtracting the level of output under competition (equation 4.6) from the level of output under compulsory licensing (equation 4.22) which gives us (SCL.2 - SC.2):

$$\frac{(a-c)[eq.1]-\sigma[eq.2]}{-[n+1][-4n^2(2\gamma+1)-8n+(\gamma-1)(\gamma+3)][3+4n+\gamma]} \tag{7.30}$$

where eq.1 (positive)
$-4n^3(2\gamma^2-3\gamma-1)-2n^2(3\gamma^2-4\gamma-3)-2n(\gamma+1)(\gamma-1)$
eq.2 (positive) increase in royalty rate causes drop in output of generics
$8n^3(\gamma+3)+2n^2(3\gamma^2+6\gamma+23)$
$+n(\gamma+3)(\gamma^2-4\gamma+11)-(\gamma-1)(\gamma+3)^2$.

From this we see that with a low royalty rate, compulsory licensing increases the generic output of the entrants. However, as the royalty rate rises, the entrants decrease their production under compulsory licensing, and the difference between the competitive output decreases.

Somewhat surprisingly then, both the leader and the entrants behave the same way, so that the introduction of compulsory licensing increases generic output from the competitive level, but the greater the royalty rate, the lower the output of generic drugs.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

Finally, we compare the output of generic drugs under patent protection versus compulsory licensing. By subtracting the generic produced under compulsory licensing (equations 4.21 and 4.22) from the generic output under patent protection (equation 4.10) we obtain (SP.2 - SCL.2):

$$\frac{(a-c)[eq.1]+\sigma[eq.2]}{-(3+\gamma)(n+1)[-4n^2(2\gamma+1)-8n+(\gamma-1)(\gamma+3)]} \tag{7.31}$$

where eq.1 (negative)

$-2n^3(3\gamma^2+6\gamma+1)-n^2(\gamma^3+17\gamma^2+41\gamma+9)$
$-2n(2\gamma^2+9\gamma+5)+(\gamma^2-1)(\gamma+3)$
eq.2 (positive)
$(\gamma+3)[2n^3(\gamma+3)-n^2(\gamma^2-4\gamma-17)+12n-(\gamma-1)(\gamma+3)]$.

From this we see that at a low royalty rate compulsory licensing causes the output of generics to increase. However, this difference gets smaller as the royalty rate increases.

7.2.3. Prices of Brand-name Drugs

Comparing Competition and Patent Protection (SC.2 and SP.2)

We now compare the price of the brand-name drug under the three models. First, by subtracting the brand-name price under competition (equation 4.3) from the price under patent protection (equation 4.11) we obtain (SP.2 - SC.2)

$$\frac{2n(a-c)(1+2\gamma+\gamma^2)}{(3+\gamma)(3+4n+\gamma)}. \tag{7.32}$$

This expression is positive, so we see that competition lowers the brand-name price as compared to patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

By subtracting the brand-name price under competition (equation 4.3) from the price under compulsory licensing (equation 4.23), we obtain (SCL.2 - SC.2):

$$\frac{2n[(a-c)(eq.1)+\sigma(eq.2)]}{(n+1)(4n^2+8n-2\gamma-\gamma^2+3+8n^2\gamma)(3+4n+\gamma)} \tag{7.33}$$

where eq.1 (positive)
$-4n^3(4\gamma+1)(\gamma-1)-2n^2(3\gamma^3+13\gamma^2-13\gamma-5)$
$-n(\gamma^3+16\gamma^2-18\gamma-8)-2(\gamma-1)(\gamma^2+3\gamma+1)$
eq.2 (positive with high γ, negative with low γ)
$-8n^3(\gamma^2-4\gamma+2)-2n^2(-\gamma^3-9\gamma^2-32\gamma+18)$
$-n(3\gamma^3-10\gamma^2-43\gamma+26)-(\gamma-1)(\gamma+3)(\gamma^2+2\gamma-2)$.

From the equation beside $a-c$, we see that, if the royalty rate is low, compulsory licensing increases the brand-name price compared to the competitive price. As the royalty rate increases, the price of the branded product continues to rise if the brand-name and generic products are seen as close substitutes. However, if the generic drug is seen as far inferior to the brand-name, the price of the brand-name drug may decline with increasing royalty rates, so that the difference from the competitive price declines.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

By subtracting the price of the brand-name product under compulsory licensing (equation 4.23) from the price under patent protection (equation 4.11), we obtain (SP.2 - SCL.2):

$$\frac{2n\left[(a-c)\left[eq.1\right]+\sigma\left[eq.2\right]\right]}{-(3+\gamma)(n+1)\left[-4n^2(2\gamma+1)-8n+(\gamma-1)(\gamma+3)\right]} \tag{7.34}$$

where eq. 1 (negative if low γ; positive if high γ)

$2n^2\left(3\gamma^3+7\gamma^2-3\gamma-1\right)+n\left(5\gamma^3+11\gamma^2-9\gamma-3\right)+(\gamma-1)\left(\gamma^2+4\gamma+1\right)$

eq. 2 (positive if low γ; negative with high γ and sufficiently high n)

$(\gamma+3)\left[-2n^2\left(\gamma^2+4\gamma-2\right)-n\left(\gamma^2+11\gamma-6\right)+(\gamma-1)\left(\gamma^2+2\gamma-2\right)\right]$.

From this we see that if the products are seen as close substitutes, and the royalty rate is low, compulsory licensing lowers the price of the brand-name drug. As the royalty rate increases, this price difference decreases. If however, the generic drug is seen as far inferior to the brand-name, and the royalty rate is low, compulsory licensing increases the price of the brand-name drug. In this case, as the royalty rate increases, the price under compulsory licensing decreases, lowering the difference from the price under patent protection.

7.2.4. Prices of Generic Drugs

Comparing Competition and Patent Protection (SC.2 and SP.2)

By subtracting the price of generic drugs under competition (equation 4.4) from the price under patent protection (equation 4.12), we obtain (SP.2 - SC.2):

$$\frac{4n(a-c)(1+\gamma)}{(3+\gamma)(3+4n+\gamma)}. \tag{7.35}$$

This expression is positive, so it is clear that competition lowers the generic price as compared to patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

Comparing generic prices under competition and compulsory licensing by subtracting equation 4.4 from equation 4.24, yields (SCL.2 - SC.2)

$$\frac{2n\left[(a-c)(eq.1)+\sigma(eq.2)\right]}{(n+1)\left(4n^2+8n-2\gamma-\gamma^2+3+8n^2\gamma\right)(3+4n+\gamma)} \tag{7.36}$$

where eq.1 (positive)

$2n^2(-2\gamma^2+3\gamma+1)-n(3\gamma^2-4\gamma-3)-(\gamma-1)(\gamma+1)$
eq.2 (positive)
$8n^3(2\gamma+1)+2n^2(2\gamma^2+13\gamma+9)-n(\gamma^2-8\gamma-17)$
$-(\gamma-1)(\gamma+3)(\gamma+2)$.

This expression is positive, meaning that compulsory licensing increases the generic price compared to the competitive price. This price difference increases as the royalty rate increases.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

By subtracting the price of generics under compulsory licensing (equation 4.24) from the price under patent protection (equation 4.12) we obtain (SP.2 - SCL.2):

$$\frac{2n\left[(a-c)(eq.1)+\sigma(eq.2)\right]}{-(3+\gamma)(n+1)\left[-4n^2(2\gamma+1)-8n+(\gamma-1)(\gamma+3)\right]} \tag{7.37}$$

where eq.1 (positive)
$2n^2(2\gamma+1)(\gamma+1)+n(\gamma^2+3)-(\gamma+1)(\gamma-1)$
eq.1 (negative)
$(\gamma+3)\left[-2n^2(2\gamma+1)-3n(\gamma+1)+(\gamma+2)(\gamma-1)\right]$.

From this expression we see that at low royalty rates, compulsory licensing lowers the generic price as compared to the price under patent protection. However, as the royalty rate rises, so does the price under compulsory licensing, so the difference from the price under patent protection falls.

7.2.5. Profits of Leader

Comparing Competition and Patent Protection (SC.2 and SP.2)

We now turn to comparing the profits of the leader under the different models. By subtracting the leader's profit under competition (equation 4.7) from its profit under patent protection (equation 4.13), we obtain (SP.2 - SC.2):

$$\frac{n(a-c)^2(1+2\gamma+\gamma^2)}{(3+\gamma)(3+4n+\gamma)}. \tag{7.38}$$

This expression is positive, so we know that competition lowers the leader's profits as compared to patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

By subtracting the leader's profits under competition (equation 4.7) from its profits under compulsory licensing (equation 4.26), we obtain (SCL.2 - SC.2):

$$\frac{n\left[4n(a-c)^2[eq.1]+2\sigma(a-c)[eq.2]-\sigma^2[eq.3]\right]}{(n+1)\left(4n^2+8n-2\gamma-\gamma^2+3+8n^2\gamma\right)^2(3+4n+\gamma)^2} \tag{7.39}$$

where eq. 1 (negative)
$4n^4(\gamma-1)(4\gamma+1)^2+4n^3(3\gamma^4+33\gamma^3-16\gamma^2-18\gamma-3)$
$+n^2(18\gamma^4+107\gamma^3-49\gamma^2-67\gamma-13)+n(\gamma-1)(10\gamma^3+51\gamma^2+33\gamma+6)$
$+(\gamma-1)(2\gamma^3+8\gamma^2+5\gamma+1)$
eq.2 (positive)
$32n^5(2\gamma^3-6\gamma^2+10\gamma+3)+8n^4(2\gamma^4+4\gamma^3-70\gamma^2+105\gamma+39)$
$+4n^3(6\gamma^4-29\gamma^3-159\gamma^2+203\gamma+99)$
$+2n^2(2\gamma^5+10\gamma^4-57\gamma^3-185\gamma^2+171\gamma+123)$
$+n(\gamma-1)(\gamma+3)(3\gamma^3-7\gamma^2-33\gamma-25)+(\gamma-1)^2(\gamma+1)(\gamma+3)^2$
eq. 3 (positive)
$64n^5(\gamma^3+4\gamma^2-3\gamma+7)+16n^4(\gamma^4+9\gamma^3+26\gamma^2-34\gamma+86)$
$+4n^3(-6\gamma^4+11\gamma^3+15\gamma^2-163\gamma+415)$
$-8n^2(\gamma+3)(\gamma^4+\gamma^3-2\gamma^2+33\gamma-41)$
$+n(\gamma-1)(\gamma+3)^2(\gamma^2+10\gamma-31)+(\gamma-1)^2(\gamma+3)^2.$

From the expression beside $(a-c)^2$ we see the structure of compulsory licensing actually decreases the leader's profits from the competitive level. However, profits increase with increases to the royalty rate as seen in the expression beside $2\sigma(a-c)$. This increase in profits due to an increased royalty rate outweighs the decrease from the equation beside σ^2, since we assume that $a-c>\sigma$ holds.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

We now compare leader profits under compulsory licensing and patent protection by subtracting equation 4.26 from equation 4.13 to obtain (SP.2 - SCL.2):

$$\frac{(a-c)^2[eq.1]-2n\sigma(\gamma+3)(a-c)[eq.2]+n\sigma^2(\gamma+3)[eq.3]}{(3+\gamma)(n+1)\left[-4n^2(2\gamma+1)-8n+(\gamma-1)(\gamma+3)\right]^2} \tag{7.40}$$

where eq.1 (positive)
$16n^5(-3\gamma^4-7\gamma^3+11\gamma^2+7\gamma+1)+4n^5(-22\gamma^4-53\gamma^3+59\gamma^2+57\gamma+11)$
$+4n^3(-15\gamma^4-39\gamma^3+22\gamma^2+37\gamma+11)$
$+n^2(\gamma-1)(\gamma^4-8\gamma^3-50\gamma^2-52\gamma-19)$
$+n(\gamma+3)(\gamma-1)^2(\gamma+1)^2$
eq. 2 (positive)

$8n^4(2\gamma^3-6\gamma^2+10\gamma+3)+4n^3(2\gamma^3-31\gamma^2+36\gamma+15)$
$+2n^2(2\gamma^4-2\gamma^3-51\gamma^2+40\gamma+27)+3n(\gamma-1)(\gamma^3+\gamma^2-11\gamma-7)$
$+(\gamma+1)(\gamma+3)(\gamma-1)^2$
eq. 3 (positive)
$16n^4(\gamma^3+4\gamma^2-3\gamma+7)+4n^3(2\gamma^3+17\gamma^2-32\gamma+65)$
$-4n^2(2\gamma^4-3\gamma^3+\gamma^2+33\gamma-55)+n(\gamma-1)(\gamma+3)(\gamma+9)(\gamma-3)$
$+(\gamma+3)(\gamma-1)^2.$

From the equation beside $(a-c)^2$ we see that, at low royalty rates, the leader makes less profit under compulsory licensing than under patent protection. However, from the equation beside $-2n\sigma(\gamma+3)(a-c)$, we see that as the royalty rate rises under compulsory licensing, so do the leader's profits and the difference from its profits under patent protection fall. This increase on $-2n\sigma(\gamma+3)(a-c)$ outweighs the decrease on $n\sigma^2(\gamma+3)$ since $(a-c)>\sigma$.

7.2.6. Profits of Entrants

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

When comparing compulsory licensing and competition, we are also able to compare the profits of the entrants (there are no entrants under patent protection, model SP.2) By subtracting the profits under competition (equation 4.8) from the profits under compulsory licensing (equation 4.25), we obtain (SCL.2 - SC.2):

$$\frac{4n(a-c)^2[eq.1]+2\sigma(a-c)[eq.2]+\sigma^2[eq.3]}{(n+1)^2(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma)^2(3+4n+\gamma)^2} \tag{7.41}$$

where eq.1 (positive)
$-4n^5(2\gamma^2-3\gamma-1)(2\gamma^2+9\gamma+3)+8n^4(-4\gamma^4-13\gamma^3+20\gamma^2+27\gamma+6)$
$+n^3(4\gamma^5-9\gamma^4-132\gamma^3+62\gamma^2+240\gamma+75)$
$+n^2(\gamma-3)(\gamma+1)(7\gamma^3+23\gamma^2-27\gamma-19)$
$+n(\gamma-1)(\gamma+1)(4\gamma^3+9\gamma^2-24\gamma-21)+(\gamma+3)(\gamma-1)^2(\gamma+1)^2$
eq.2 (negative)
$-64n^6(\gamma+3)(3\gamma+1)-32\gamma^5(5\gamma^3+19\gamma^2+63\gamma+25)$
$-4n^4(7\gamma^4+28\gamma^3+114\gamma^2+652\gamma+351)$
$+4n^3(\gamma^5+12\gamma^4+60\gamma^3+50\gamma^2-429\gamma-334)$
$+n^2(\gamma+3)(\gamma^5+9\gamma^4+26\gamma^3+186\gamma^2-107\gamma-243)$
$-8n(\gamma+1)(\gamma-1)(\gamma-3)(\gamma+3)^2$
$-(\gamma+1)(\gamma-1)^2(\gamma+3)^3$
eq.3 (positive)

$$64n^6(\gamma+3)^2 + 32n^5(3\gamma^3+15\gamma^2-41\gamma+69)$$
$$+4n^4(13\gamma^4+44\gamma^3+158\gamma^2+396\gamma+925)$$
$$+4n^3(\gamma+3)(3\gamma^4-10\gamma^3+12\gamma^2-38\gamma+289)$$
$$+n^2(\gamma+3)^2(\gamma^4-20\gamma^3+26\gamma^2-156\gamma+213)$$
$$-2n(\gamma-1)(\gamma+3)^3(\gamma^2-4\gamma+11)+(\gamma-1)^2(\gamma+3)^4.$$

From the equation beside $(a-c)^2$, we see that the structure of compulsory licensing actually makes the entrants better off than competition. However, from the expression beside $2\sigma(a-c)$, increasing the royalty rate under compulsory licensing causes the entrants profits to fall and they are made worse off as compared competition. This decrease on profits on $2\sigma(a-c)$ outweighs the increase on σ^2 since $(a-c)>\sigma$.

7.2.7. Total Producer Profits

Comparing Competition and Patent Protection (SC.2 and SP.2)

By subtracting the profits of the n entrants (equation 4.8*n) and leader (equation 4.7) under competition from the leader's profits under patent protection (equation 4.13), we can compare the total producer profits under competition and patent protection (SP.2 - SC.2):

$$\frac{4n^2(a-c)^2(1+2\gamma+\gamma^2)}{(3+\gamma)(3+4n+\gamma)^2}. \tag{7.42}$$

This expression is positive, so we can see that competition lowers the total producer profits as compared to patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

Now, by subtracting the profits of the n entrants (equation 4.8*n) and leader (equation 4.7) under competition from the leader's and entrant profits under compulsory licensing (equations 4.26 and 4.25) , we can compare the total producer profits under compulsory licensing and patent protection (SCL.2 - SC.2):

$$\frac{4n^2\left[(a-c)^2(eq.1)+\sigma(a-c)(eq.2)+\sigma^2(eq.3)\right]}{(n+1)^2(3+4n+\gamma)^2(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma)^2} \tag{7.43}$$

where eq.1 (negative with low γ, positive as $\gamma\longrightarrow 1$)

$$16n^6(\gamma-1)(4\gamma+1)^2+32n^5(3\gamma^4+28\gamma^3-13\gamma^2-13\gamma-2)$$
$$+4n^4(3\gamma^5+80\gamma^4+336\gamma^3-170\gamma^2-164\gamma-25)$$
$$+2n^3(17\gamma^5+216\gamma^4+532\gamma^3-342\gamma^2-289\gamma-38)$$
$$+n^2(35\gamma^5+289\gamma^4+471\gamma^3-413\gamma^2-306\gamma-28)$$

$+4n(\gamma-1)(4\gamma^4+29\gamma^3+58\gamma^2+24\gamma+1)$
$+\gamma(\gamma-1)(\gamma+3)(3\gamma^2+9\gamma+4)$
eq.2 (positive)
$64n^6(2\gamma^3-6\gamma^2+10\gamma+3)+32n^5(2\gamma^4+6\gamma^3-54\gamma^2+79\gamma+27)$
$+4n^4(2\gamma^5+38\gamma^4-120\gamma^3-811\gamma^2+1060\gamma+407)$
$+4n^3(7\gamma^5+23\gamma^4-310\gamma^3-802\gamma^2+959\gamma+411)$
$+n^2(2\gamma^6+44\gamma^5+41\gamma^4-1012\gamma^3-1796\gamma^2+1976\gamma+937)$
$+n(\gamma-1)(\gamma+3)(4\gamma^4+33\gamma^3-13\gamma^2-249\gamma-95)$
$+(\gamma-1)(\gamma+3)^2(2\gamma^3+3\gamma^2-9\gamma-4)$
eq.3 (negative, but fifth term which is mostly negative)
$-64n^6(\gamma^3+4\gamma^2-3\gamma+7)-32n^5(\gamma^4+10\gamma^3+25\gamma^2-27\gamma+63)$
$-4n^4(\gamma^5+14\gamma^4+122\gamma^3+169\gamma^2-405\gamma+963)$
$+2n^3(\gamma+3)(5\gamma^4-\gamma^3-123\gamma^2+497\gamma-666)$
$+n^2(\gamma+3)^2(2\gamma^5+24\gamma^4-23\gamma^3+49\gamma^2+597\gamma-793)$
$+2n(\gamma^6+2\gamma^5-17\gamma^4+227\gamma^2+174\gamma-372)$
$-(\gamma-1)(\gamma+3)^3(\gamma^2+2\gamma-4)$.

From the expression beside $(a-c)^2$, we see that with a low substitutability between the brand-name and generic products, γ, and a low royalty rate, compulsory licensing lowers total producer profits from the competitive level. However, as $\gamma \rightarrow 1$, at a low royalty rate, compulsory licensing increases total producer profits from the competitive level.

As the royalty rate increases however, compulsory licensing increases total producer profits from the competitive level. This increase in total profits on $\sigma(a-c)$ outweighs the decrease on σ^2, since $(a-c)>\sigma$.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

By subtracting the profits of the n entrants (equation 4.25 *n) and leader (equation 4.26) under compulsory licensing from the leader's profits under patent protection (equation 4.13), we can compare the total producer profits under compulsory licensing and patent protection (SP.2 - SCL.2):

$$\frac{4n^2\left[(a-c)^2(eq.1)-\sigma(a-c)(\gamma+3)(eq.2)+\sigma^2(eq.3)\right]}{(\gamma+3)(n+1)^2\left[-4n^2(2\gamma+1)-8n+(\gamma-1)(\gamma+3)\right]^2} \tag{7.44}$$

where eq.1 (positive)
$-4n^4(3\gamma^4+7\gamma^3-11\gamma^2-7\gamma-1)-2n^3(17\gamma^4+45\gamma^3-70\gamma^2-66\gamma-12)$
$-n^2(34\gamma^4+88\gamma^3-55\gamma^2-58\gamma-13)-2n(\gamma-1)(7\gamma^3+25\gamma^2+15\gamma+3)$
$-(\gamma-1)(2\gamma^3+8\gamma^2+5\gamma+1)$

eq.2 (positive)
$4n^4(2\gamma^3 - 6\gamma^2 + 10\gamma + 3) + 4n^3(3\gamma^3 - 23\gamma^2 + 23\gamma + 9)$
$+n^2(2\gamma^4 - 113\gamma^2 + 82\gamma + 41) + n(\gamma - 1)(4\gamma^3 + 5\gamma^2 - 52\gamma - 21)$
$+(\gamma - 1)(2\gamma^3 + 3\gamma^2 - 9\gamma - 4)$
eq.3 (positive)

$$(\gamma + 3)\left[\begin{array}{c} 4n^4(\gamma^3 + 4\gamma^2 - 3\gamma + 7) \\ +2n^3(3\gamma^3 + 16\gamma^2 - 25\gamma + 42) \\ -n^2(2\gamma^4 + 2\gamma^3 - 13\gamma^2 + 72\gamma - 99) \\ -2n(\gamma + 3)(\gamma - 1)(\gamma^2 - 2\gamma - 9) \end{array}\right]$$

$+(\gamma - 1)(\gamma^4 + 8\gamma^3 - 6\gamma^2 - 6\gamma - 36).$

From the expression beside $(a-c)^2$ we can see that at low royalty rates, total profits decrease from patent protection levels under compulsory licensing. However, from the expression beside $\sigma(a-c)$, as the royalty rate increases, so too do total producer profits and the difference from the patent protection level decreases. This increase in total profits due to the increase in royalty rate on $\sigma(a-c)$ outweighs the decrease on σ^2 since $(a-c) > \sigma$.

7.2.8. Consumer Surplus

Comparing Competition and Patent Protection (SC.2 and SP.2)

By subtracting the consumer surplus under competition (equation 4.9) from the consumer surplus under patent protection (equation 4.14), we obtain (SP.2 - SC.2):

$$\frac{-2n(a-c)^2[n(\gamma+5)(\gamma+1) + (\gamma+3)(\gamma+1)]}{(3+\gamma)(3+4n+\gamma)^2}. \tag{7.45}$$

This expression is negative. Therefore, we know that consumer surplus is greater under competition than under patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

By subtracting the consumer surplus under competition (equation 4.9) from the consumer surplus compulsory licensing (equation 4.28), we obtain (SCL.2 - SC.2):

$$\frac{n\left[(a-c)^2(eq.1) - \sigma(a-c)(eq.2) + 2n\sigma^2(eq.3)\right]}{(n+1)^2(-4n^2 - 8n + 2\gamma + \gamma^2 - 3 - 8n^2\gamma)^2(3+4n+\gamma)^2} \tag{7.46}$$

where eq.1 low γ, high n, can be positive, negative as $\gamma \longrightarrow 1$,
$-32n^7(\gamma - 1)(4\gamma + 1)^2 - 32n^6(6\gamma^4 + 52\gamma^3 - 22\gamma^2 - 21\gamma - 3)$

$-8n^5 (3\gamma^5 + 76\gamma^4 + 300\gamma^3 - 143\gamma^2 - 84\gamma - 4)$
$-4n^4 (17\gamma^5 + 206\gamma^4 + 465\gamma^3 - 361\gamma^2 - 42\gamma + 51)$
$-2n^3 (37\gamma^5 + 294\gamma^4 + 385\gamma^3 - 605\gamma^2 + 58\gamma + 167)$
$-n^2 (\gamma + 3) (39\gamma^4 + 116\gamma^3 - 194\gamma^2 - 4\gamma + 75)$
$-2n (\gamma - 1)^2 (\gamma + 3) (5\gamma^2 + 20\gamma + 12) - (\gamma - 1)^2 (\gamma + 3)^2 (\gamma + 1)$
eq.2 (positive)
$256n^7 (\gamma^3 - \gamma^2 + 7\gamma + 2) + 64n^6 (2\gamma^4 + 10\gamma^3 - 24\gamma^2 + 117\gamma + 39)$
$+16n^5 (\gamma^5 + 21\gamma^4 - 30\gamma^3 - 239\gamma^2 + 831\gamma + 328)$
$+4n^4 (14\gamma^5 + 36\gamma^4 - 639\gamma^3 - 1239\gamma^2 + 3157\gamma + 1551)$
$+4n^3 (\gamma + 3) (\gamma^5 + 16\gamma^4 - 71\gamma^3 - 457\gamma^2 + 426\gamma + 373)$
$+n^2 (\gamma + 3) (8\gamma^5 + 51\gamma^4 - 160\gamma^3 - 710\gamma^2 + 344\gamma + 659)$
$+n (\gamma - 1) (\gamma + 3)^2 (5\gamma^3 + 15\gamma^2 - 29\gamma - 55)$
$+ (\gamma - 1)^2 (\gamma + 3)^3 (\gamma + 2)$
eq.3 (positive, but fifth term which is mostly positive)
$64n^6 (\gamma^3 + 4\gamma^2 - 3\gamma + 7) + 32n^5 (\gamma^4 + 10\gamma^3 + 25\gamma^2 - 27\gamma + 63)$
$+4n^4 (\gamma^5 + 14\gamma^4 + 122\gamma^3 + 169\gamma^2 - 405\gamma + 963)$
$-2n^3 (\gamma + 3) (5\gamma^4 - \gamma^3 - 123\gamma^2 + 497\gamma - 666)$
$-n^2 (\gamma + 3)^2 (2\gamma^5 + 24\gamma^4 - 23\gamma^3 + 49\gamma^2 + 597\gamma - 793)$
$-2n (\gamma^6 + 2\gamma^5 - 17\gamma^4 + 227\gamma^2 + 174\gamma - 372)$
$+ (\gamma - 1) (\gamma + 3)^3 (\gamma^2 + 2\gamma - 4)$.

From the equation beside $(a - c)^2$, we see that if the royalty rate is low, and the substitutability between brand-name and generics is high, then compulsory licensing lowers consumer surplus from the competitive level. This continues to be the case unless γ is very low and n is high, in which case compulsory licensing can increase consumer surplus from the competitive level.

Increasing the royalty rate however, from the expression beside $-\sigma (a - c)$, lowers the consumer surplus. This decrease in total surplus on $-\sigma (a - c)$ outweighs the increase on σ^2 since $(a - c) > \sigma$.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

By subtracting the consumer surplus under compulsory licensing (equation 4.28) from the consumer surplus under patent protection (equation 4.14) we obtain (SP.2 - SCL.2):

$$\frac{n \left[(a - c)^2 (eq.1) + \sigma (a - c) (\gamma + 3) (eq.2) - 2n\sigma^2 (\gamma + 3) (eq.3) \right]}{(3 + \gamma) (n + 1)^2 (-4n^2 - 8n + 2\gamma + \gamma^2 - 3 - 8n^2\gamma)^2} \tag{7.47}$$

where eq.1 (negative)

$8n^5(3\gamma^4+3\gamma^3-19\gamma^2-12\gamma-2)+4n^4(17\gamma^4+35\gamma^3-60\gamma^2-61\gamma-15)$
$+2n^3(36\gamma^4+95\gamma^3-74\gamma^2-105\gamma-44)$
$+n^2(33\gamma^4+96\gamma^3-34\gamma^2-64\gamma-63)+2n(\gamma-1)(2\gamma^3+8\gamma^2+11\gamma+11)$
$-(\gamma+3)(\gamma+1)(\gamma-1)^2$

eq.2 (positive)

$16n^5(\gamma^3-\gamma^2+7\gamma+2)+4n^4(6\gamma^3-32\gamma^2+71\gamma+27)$
$+4n^3(\gamma^4-3\gamma^3-56\gamma^2+69\gamma+37)$
$+n^2(8\gamma^4-13\gamma^3-153\gamma^2+101\gamma+105)$
$+n(\gamma+3)(\gamma-1)(\gamma+1)(5\gamma+13)+(\gamma+3)^2(\gamma+2)(\gamma-1)^2$

eq.3 (positive)

$4n^4(\gamma^3+4\gamma^2-3\gamma+7)$
$+2n^3(3\gamma^3+16\gamma^2-25\gamma+42)$
$-n^2(2\gamma^4+2\gamma^3-13\gamma^2+72\gamma-99)$
$-2n(\gamma+3)(\gamma-1)(\gamma^2-2\gamma+9)+(\gamma+3)(\gamma-1)(\gamma^2+2\gamma-4).$

From the expression beside $(a-c)^2$, we see that at low royalty rates, compulsory licensing increases consumer surplus as compared to under patent protection. However, from the expression beside $\sigma(a-c)(\gamma+3)$, as the royalty rate increases under compulsory licensing, consumer surplus decreases and the difference from the patent protection level decreases. This decrease'on $\sigma(a-c)(\gamma+3)$ outweighs the increase on $-2n\sigma^2(\gamma+3)$ since $(a-c)>\sigma$ holds.

7.2.9. Total Surplus

Comparing Competition and Patent Protection (SC.2 and SP.2)

By subtracting the total profits and consumer surplus under competition (equations 4.8, 4.7 and4.9) from the total profits and consumer surplus under patent protection (equations 4.13 and 4.14), we obtain the difference in total surplus between patent protection and competition (SP.2 - SC.2):

$$\frac{2n(a-c)^2[n(\gamma-3)(\gamma+1)-(\gamma+3)(\gamma+1)]}{(3+\gamma)(3+4n+\gamma)^2}. \tag{7.48}$$

This expression is negative. This means that total surplus is higher under competition than it is under patent protection.

Comparing Competition and Compulsory Licensing (SC.2 and SCL.2)

By subtracting the total profits and consumer surplus under competition (equations 4.8, 4.7 and4.9) from the total profits and consumer surplus under compulsory licensing (equations 4.25*n, 4.26 and 4.28), we obtain the difference in total surplus between patent protection and compulsory licensing (SCL.2 - SC.2):

$$\frac{n\left[(a-c)^2(eq.1)+\sigma(a-c)(eq.2)-2n\sigma^2(eq.3)\right]}{(n+1)^2\left(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma\right)^2(3+4n+\gamma)^2} \tag{7.49}$$

where eq.1 (negative)

$32n^7(\gamma-1)(4\gamma+1)^2+32n^6(\gamma-1)(6\gamma^3+66\gamma^2+36\gamma+5)$
$+8n^5(3\gamma^5+84\gamma^4+372\gamma^3-197\gamma^2-244\gamma-46)$
$+4n^4(17\gamma^5+226\gamma^4+599\gamma^3-323\gamma^2-536\gamma-127)$
$+2n^3(33\gamma^5+284\gamma^4+557\gamma^3-221\gamma^2-670\gamma-223)$
$+n^2(25\gamma^5+167\gamma^4+310\gamma^3+42\gamma^2-431\gamma-241)$
$+2n(\gamma-1)(\gamma+3)(\gamma^3+3\gamma^2+16\gamma+12)-(\gamma-1)^2(\gamma+3)^2(\gamma+1)$

eq.2 negative as $\gamma\to 1$ positive if very low γ, and high n

$256n^7(\gamma-1)(\gamma^2-4\gamma-1)+64n^6(2\gamma^4+2\gamma^3-84\gamma^2+41\gamma+15)$
$+16n^5(\gamma^5-17\gamma^4-90\gamma^3-572\gamma^2+229\gamma+79)$
$+4n^4(14\gamma^5+56\gamma^4-601\gamma^3-1969\gamma^2+679\gamma+93)$
$+4n^3(\gamma^6+25\gamma^5+64\gamma^4-342\gamma^3-851\gamma^2+325\gamma-182)$
$+n^2(\gamma+3)(8\gamma^5+65\gamma^4-24\gamma^3-234\gamma^2+272\gamma-279)$
$+n(\gamma-1)(\gamma+3)^2(3\gamma^3-3\gamma^2-7\gamma+39)$
$-(\gamma-1)^2(\gamma+3)^3(\gamma+2)$

eq.3 (positive, but fifth term which is mostly positive)

$64n^6(\gamma^3+4\gamma^2-3\gamma+7)+32n^5(\gamma^4+10\gamma^3+25\gamma^2-27\gamma+63)$
$+4n^4(\gamma^5+14\gamma^4+122\gamma^3+169\gamma^2-405\gamma+963)$
$-2n^3(\gamma+3)(5\gamma^4-\gamma^3-123\gamma^2+497\gamma-666)$
$-n^2(\gamma+3)^2(2\gamma^5+24\gamma^4-23\gamma^3+49\gamma^2+597\gamma-793)$
$-2n(\gamma^6+2\gamma^5-17\gamma^4+227\gamma^2+174\gamma-372)$
$+(\gamma-1)(\gamma+3)^3(\gamma^2+2\gamma-4)$.

From the expression beside $(a-c)^2$, we see that at low royalty rates, compulsory licensing lowers total surplus from the competitive level. From the expressions beside $\sigma(a-c)$ and $-2n\sigma^2$, we see that increasing the royalty rate further decreases total surplus from the competitive level.

Comparing Patent Protection and Compulsory Licensing (SP.2 and SCL.2)

By subtracting the total profits and consumer surplus under compulsory licensing (equations 4.25*n, 4.26, and 4.28) from the total profits and consumer surplus under patent protection (equations 4.13 and 4.14), we obtain the difference in total surplus between patent protection and competition (SP.2 - SCL.2):

$$\frac{n\left[(a-c)^2(eq.1)-\sigma(a-c)(\gamma+3)(eq.2)+2n\sigma^2(\gamma+3)(eq.3)\right]}{(3+\gamma)(n+1)^2\left(-4n^2-8n+2\gamma+\gamma^2-3-8n^2\gamma\right)^2} \tag{7.50}$$

where eq.1 (negative as $\gamma \to 1$ (-,-,-,+,0,-); with low γ and high n could be positive)

$-8n^5\gamma\left(3\gamma^3+11\gamma^2-3\gamma-2\right)-4n^4\left(17\gamma^4+55\gamma^3-10\gamma^2-5\gamma+3\right)$
$-2n^3\left(32\gamma^4+81\gamma^3-36\gamma^2-11\gamma+18\right)$
$-n^2(\gamma+1)\left(23\gamma^3+25\gamma^2-71\gamma-39\right)-2n(\gamma-1)^2\left(2\gamma^2+10\gamma+9\right)$
$-(\gamma+3)(\gamma+1)(\gamma-1)^2$

eq.2 low γ (+,+,+,-,-,-) $\gamma \to 1$ (0,-,-,-,0,0)

$16n^5(\gamma-1)\left(\gamma^2-4\gamma-1\right)+12n^4\left(2\gamma^3-20\gamma^2+7\gamma+3\right)$
$+4n^3\left(\gamma^4+3\gamma^3-57\gamma^2+13\gamma+4\right)$
$+n^2\left(8\gamma^4+17\gamma^3-75\gamma^2+23\gamma-21\right)$
$+n(\gamma-1)\left(3\gamma^3+5\gamma^2+\gamma+23\right)-(\gamma+2)(\gamma-1)^2(\gamma+3)^2$

eq.3 (positive)

$4n^4\left(\gamma^3+4\gamma^2-3\gamma+7\right)$
$+2n^3\left(3\gamma^3+16\gamma^2-25\gamma+42\right)$
$-n^2\left(2\gamma^4+2\gamma^3-13\gamma^2+72\gamma-99\right)$
$-2n(\gamma+3)(\gamma-1)\left(\gamma^2-2\gamma+9\right)+(\gamma+3)(\gamma-1)\left(\gamma^2+2\gamma-4\right).$

From the expression beside $(a-c)^2$ we see that, at low royalty rates, compulsory licensing increases total surplus from the level under patent protection.

From the equations on $-\sigma(a-c)(\gamma+3)$ and $+2n\sigma^2(\gamma+3)$ we see that as the royalty rate increases under compulsory licensing, the total surplus decreases. Therefore, the difference from total surplus under patent protection decreases.