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## Model Uncertainty and the Direction of Fit of the Postwar U.S. Phillips Curve(s)\*

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## ***Abstract***

*This paper proposes a model uncertainty framework that accounts for the uncertainty about both the specification of the Phillips curve and the identification assumption to be used for parameter estimation. More specifically, the paper extends the framework employed by Cogley and Sargent (2005) to incorporate uncertainty over the direction of fit of the Phillips curve. I first study the evolution of the model posterior probabilities, which can be interpreted as a measure of the econometrician's real-time beliefs over the prevailing model of the Phillips curve. I then characterize the optimal policy rule within each model, and I analyze alternative policy recommendations that incorporate model uncertainty. As expected, different directions of fit of the same model of the Phillips curve imply very different optimal policy choices, with the "Classical" specifications typically suggesting low and stable optimal inflation rates. I also find that allowing rational agents to incorporate model uncertainty in their expectations does not change the optimal or robust policies. On the other hand, I show that the models' fit to the data and the robust policy recommendations are affected by the specific price index that is used to measure inflation.*

**Key words:** *Phillips curve, Model Uncertainty, Robust Policy, Bayesian Model Averaging, Expectations.*

**JEL Classification:** C52, E37, E52, E58.

# 1 Introduction

This paper is concerned with the issue of model uncertainty in a Phillips curve representation of the economy, and in particular with the uncertainty about the direction of fit of the empirical model to be estimated. More specifically, I propose a framework that incorporates uncertainty about both the specification of the Phillips curve and the identification assumption to be employed for parameter estimation. I use this framework to address three questions. First, I assess the extent to which the different models of the Phillips curve considered in the analysis fit the data over time. Second, I discuss the role that different forms of model uncertainty might play in policy decisions. Third, I study the impact of private agents' perception of model uncertainty on the models' fit to the data and on the robust policy recommendations.

The negative correlation between inflation and unemployment was first noted by Phillips (1958) in the U.K. data, and has subsequently been studied for many different countries and different time periods. For the U.S., Samuelson and Solow (1960) showed the existence of a negative trade-off similar to the one studied by Phillips (1958), and suggested an empirical model that was able to reproduce this relationship. However, a few years later, the Phillips curve model advanced by Samuelson and Solow (1960) seemed to break down when the rise in inflation that emerged in the late 1960s was not followed by a decrease in unemployment. The failure of the Samuelson-Solow model to rationalize the behavior of the data in these years encouraged other researches to develop alternative representations of the Phillips curve (in particular, this paper will consider the models introduced by Solow, 1968; Tobin, 1968; Lucas, 1972; Sargent, 1973). According to Cogley and Sargent (2001) and Sims (2001), it should have become clear shortly thereafter that the model of the Phillips curve offering the best fit to the data was one incorporating the natural rate hypothesis.<sup>1</sup>

One of the fundamental characteristics of the Phillips curve is that it was discovered as a purely empirical relationship. As such, the original model does not imply any assumptions about the direction of causality between inflation and unemployment. However, in order to be able to estimate the parameters of the Phillips curve equation, the econometrician will need to make an assumption about the direction of fit of the empirical model. In other words, the econometrician will need to decide whether to use inflation or unemployment as the left-hand side variable in the model to be

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<sup>1</sup>Sims (2008) offers an interesting and thought provoking discussion of the Phillips curve and its evolutions over time.

estimated. In the absence of an underlying structural framework, this choice is, to a large extent, arbitrary. This issue is well known in the economic literature, and several works have discussed the impact that the direction of fit of the Phillips curve has on the estimation and interpretation of the trade-off between inflation and unemployment (see, for instance, King and Watson, 1994; Sargent, 2001; Cogley and Sargent, 2005).

This paper provides a contribution to this literature in a new direction. More specifically, this work proposes a framework in which the uncertainty over the direction of fit of the Phillips curve is included in the analysis in the same way as the uncertainty about the version of the model that best approximates the true data generating process. The models of Phillips curve that I consider are the same as in Cogley and Sargent (2005), but two of the models, namely the Samuelson-Solow and the Solow-Tobin, will be estimated using different directions of fit, each of which will be treated as a separate model. To my knowledge, this is the first paper that considers the direction of fit of the Phillips curve as a form of model uncertainty, and that incorporates it in a model uncertainty type of environment. In this sense, this work also adds to the literature on model uncertainty in economics, and more specifically to the branch following the Bayesian Model Averaging approach introduced by Brock, Durlauf and West (2003, 2007).<sup>2</sup>

The model uncertainty framework that I define in the first part of the paper is then be used for three purposes. First, I study the patterns of the posterior probabilities of each specification of the Phillips curve using U.S. data for the postwar period. Following the standard approach of the literature on uncertainty and learning in economic policy<sup>3</sup>, these probabilities are computed in each period based on the data available up to that point, and they are updated over time as additional observations become available. This implies that the models' posteriors can be interpreted as real-time estimates of the extent to which each model approximates the true data generating process. Second, I add a monetary policy equation to the baseline framework, and I use it to study the optimal policy rule within each model and alternative policy choices under model uncertainty. Third, I examine the role of private agents' expectations, and in particular I investigate whether allowing rational agents to account for model uncertainty affects the models' posteriors and the robust policy recommendations.

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<sup>2</sup>An alternative approach to model uncertainty is the Minimax Robust Control method developed by Hansen and Sargent (2008).

<sup>3</sup>See, among the others, Cogley and Sargent (2005), Primiceri (2006), Sargent, Williams and Zha (2006).

The results of the paper are as follows. In terms of model posterior probabilities, I find that after the mid 1960s, the data quickly favors models of the Phillips curve that imply a very costly trade-off between inflation and unemployment (i.e. a very steep or vertical Phillips curve), and that recommend low and stable inflation rates. However, I show that the exact specification that becomes predominant over time depends on whether the uncertainty over the direction of fit of the Phillips curve is incorporated into the analysis. In the policy exercise, I verify the claim of Cogley and Sargent (2005) that adding uncertainty over the direction of fit does not alter the robust policy if the more unstable models of the economy (the "worst-case" scenarios) were already included in the original model space. I also confirm their conclusion that in a model uncertainty environment, a robust policy type of argument can rationalize the pattern of inflation in the postwar U.S. data, while an "average" policy approach cannot do so. I show that this result extends to the period 2002 – 2016, which was not covered in their original study. However, I find that this result is not robust to the measure of inflation employed in the analysis, as it holds for PCE and CPI inflation, but not for inflation computed from the GDP deflator. For this last measure, starting from the late 1980s the robust policy becomes very similar to the optimal policy in the model with the highest posterior, and it recommends a stable inflation rate set at the value of the target rate. Finally, I show that allowing rational agents to incorporate model uncertainty in their expectations does change the patterns of the models' posteriors, but it does not change the robust policy rules.

Overall, the main take-away of this paper is that accounting for the uncertainty over the direction of fit of the Phillips curve is important for our understanding of the data in real-time and, as a consequence, for our ability to interpret the current behavior of the variables and to predict their future patterns. However, if our main interest is the analysis of robust policy-making under model uncertainty, then the larger model space might be unnecessary, as the recommended policy choices will be affected almost exclusively by the specifications of the Phillips curve that are more difficult to stabilize.

Before moving on with the discussion, I would like to remark that this paper is in the spirit of the literature on economic decisions in real-time, and the analysis is performed from the perspective of an econometrician (or policymaker) who has to form his opinions based on the currently available information. The main purpose of this work is to discuss some possible approaches to decision-making in a model uncertainty environment that econometricians can employ using the data that

they observe, and without having to make any assumptions about the true model of the economy. This implies, among other things, that the patterns of the models' posteriors that I report in the paper are the econometrician's estimates of the model that best approximates the data over time, and they do not (directly) measure changes in the underlying true model of the Phillips curve. Thus, even if these estimates can change as new data becomes available, no assumption is made on whether the underlying true model of the Phillips curve is actually time-varying.<sup>4</sup>

The remainder of the paper is organized as follows. Section 2 describes the models of the Phillips curve considered in the analysis and characterizes the model space. Section 3 studies the patterns of the models' posterior probabilities in the postwar U.S. data. Section 4 discusses the impact of the uncertainty over the direction of fit of the Phillips curve on policy decisions, and analyses the role of private agents' expectations. Section 5 concludes.

## 2 Models of the Phillips Curve

I assume that the true model of the economy is unknown, and I consider a small number of alternative empirical models, which represent possible approximations to the true data generating process. More specifically, the model space that I consider in this paper is composed of five alternative empirical models, which originate from three different specifications of the Phillips curve. The true data generating process does not need to be among the specifications included in the model space and, as mentioned above, the different models considered in the analysis will all be treated as just approximations to the true underlying representations of the world.

The three approximating models of the Phillips curve that I employ in this paper are the same as in Cogley and Sargent (2005). These models are in the spirit of the empirical frameworks previously adopted in the macroeconomic literature addressing similar questions (King, Stock and Watson, 1995; Rudebusch and Svensson, 1999; Primiceri, 2006; Brock, Durlauf and West, 2007). Each model postulates its own version of the relationship between the inflation rate  $\pi_t$ , and the unemployment rate  $u_t$ . The first model is a version of the Phillips curve proposed by Samuelson and Solow (1960),

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<sup>4</sup>Some works assuming a time-varying model of the Phillips curve are Conway and Gill (1991), Stock and Watson (2010), Matheson and Stavrev (2013), Blanchard, Cerutti and Summers (2015), and Blanchard (2016).

and allows for a long-run trade-off between unemployment and inflation:

$$\pi_t = \gamma_0^{SS} + \gamma_\pi^{SS}(L) \pi_{t-1} + \gamma_u^{SS}(L) u_t + \eta_t^{SS} \quad (1)$$

The shock  $\eta_t^{SS}$  is assumed to be i.i.d.  $N(0, \sigma_{SS}^2)$ .

The second model is inspired by Solow (1968) and Tobin (1968):

$$\Delta\pi_t = \gamma_\pi^{ST}(L) \Delta\pi_{t-1} + \gamma_u^{ST}(L) (u_t - u_t^*) + \eta_t^{ST} \quad (2)$$

Here,  $u_t^*$  is the natural rate of unemployment and  $\eta_t^{ST}$  is i.i.d.  $N(0, \sigma_{ST}^2)$ . This model is a special case of (1), and is characterized by a short-run trade-off between inflation and unemployment, and long-run neutrality.

The third model is in the spirit of Lucas (1972) and Sargent (1973):

$$u_t - u_t^* = \gamma_\pi^{LS} (\pi_t - E_{t-1}(\pi_t)) + \gamma_u^{LS}(L) (u_{t-1} - u_{t-1}^*) + \eta_t^{LS} \quad (3)$$

The variable  $E_{t-1}(\pi_t)$  represents private agents' rational expectations of time  $t$  inflation, given the information available at time  $t - 1$ , and  $\eta_t^{LS}$  is i.i.d.  $N(0, \sigma_{LS}^2)$ . This model is characterized by long-run neutrality and by the assumption that only unexpected inflation is able to affect the unemployment rate. Notice that in order to be able to estimate the fit of the Lucas-Sargent model to the data, I will need to compute the value of  $E_{t-1}(\pi_t)$ , which in turns means that I will need to make further assumptions about the way in which  $\pi_t$  is determined in this model. Following the previous literature using similar models (Sargent, 2001; Cogley and Sargent, 2005; Sargent, Williams and Zha, 2006), I will assume that the monetary authority is able to control the inflation rate to some extent, and that private agents will set their expectations based on this information. Further details will be provided in section 3.

## 2.1 The Direction of Fit of the Phillips Curve

As previously mentioned, the Phillips curve was first discovered as an empirical relationship between inflation and unemployment. While several models of the Phillips curve have been proposed over time, most of them remain empirical relationships, and do not aim at making explicit statements

about the direction of causality between these two variables. This implies that the econometrician will need to decide which orthogonality assumption to impose in order to be able to estimate the parameters of the model. Using the notation introduced above, the Phillips curve could be estimated using  $\pi_t$  as the dependent variable and assuming that the shock  $\eta_t^i$  is orthogonal to  $u_t$ , or vice-versa using  $u_t$  as the dependent variable and assuming that the shock  $\eta_t^i$  is orthogonal to  $\pi_t$ . Sargent (2001) called these two alternative identification assumptions "Keynesian" and "Classical". In the Keynesian identification assumption, the shock  $\eta_t^i$  is assumed to be orthogonal to current unemployment (in addition to all the other right hand side variables, such as past inflation and unemployment). On the contrary, in the Classical identification assumption the shock  $\eta_t^i$  is assumed to be orthogonal to current inflation (again, in addition to all the other right hand side variables).

The choice of the Keynesian or Classical identification assumption in a Phillips curve framework is not without consequences. As previously mentioned, King and Watson (1994) and Sargent (2001) show that the identification assumption used to estimate the parameters of the Phillips curve affects the nature of the trade-off between inflation and unemployment, and implies a different optimal policy response to changes in the variables of interest. In addition, Cogley and Sargent (2005) show that the assumption used to identify the coefficients of the Phillips curve affects the estimated sacrifice ratios, which measure the cost of reducing inflation in terms of unemployment. In more detail, they show that the Samuelson-Solow and Solow-Tobin models described by (1) and (2) produce high sacrifice ratios if estimated using the Keynesian assumption, while they suggest a much lower cost of disinflation if estimated using the Classical assumption. This in turn implies that the optimal inflation rate recommended by these two models will typically be lower and more stable when they are estimated using the Classical direction of fit instead of the Keynesian.

In the analysis of the robust policy choice in the face of model uncertainty, Cogley and Sargent (2005) argue that it is unnecessary to account for the uncertainty over the direction of fit of the Phillips curve if the specifications that represent the worst-case scenario of the economy are already included in the model space. Section 4 of this paper does indeed confirm this argument. However, researchers might be interested in adopting a model uncertainty approach to address other type of questions. For instance, they might want to investigate which specification of the Phillips curve provides the best fit to the data given the information that they have available. As it will be discussed below, this exercise might offer interesting insights about the true underlying model of the economy.

In addition, the knowledge of the model that best describes the data at each point in time might be useful to interpret the behavior of inflation and unemployment and to predict their future patterns. Finally, the monetary authorities might be interested in analyzing the optimal policies recommended by alternative specifications of the Phillips curve, and they might want to study how far apart these policies are.<sup>5</sup>

This paper suggests that, for the reasons just mentioned, it might be of interest to researchers and policymakers to consider a model space that incorporates uncertainty over the direction of fit of the Phillips curve in addition to specification uncertainty. Thus, the space of models of the Phillips curve under study in this paper will be composed of 5 model specifications: the Samuelson-Solow model, estimated using the Keynesian and the Classical identification assumptions (from now on denoted as  $SS - K$  and  $SS - C$ ), the Solow-Tobin model estimated using the Keynesian and the Classical identification assumptions (denoted as  $ST - K$  and  $ST - C$ ), and finally the Lucas-Sargent model (denoted as  $LS$ ).<sup>6</sup> In the  $SS - K$  model, equation (1) will be estimated using inflation as the dependent variable, with the identification assumption  $E(u_t \eta_t^{SS-K}) = 0$ . On the other hand, in the  $SS - C$  model the same equation will be estimated using unemployment as the dependent variable, with the identification assumption  $E(\pi_t \eta_t^{SS-C}) = 0$ . Similarly, in the  $ST - K$  model equation (2) will be estimated using the identification assumption  $E(u_t \eta_t^{ST-K}) = 0$ , while in the  $ST - C$  model the same equation will be estimated under the assumption  $E(\pi_t \eta_t^{ST-C}) = 0$ .

To provide some further intuition about the relationship between the Keynesian and Classical versions of the same model of the Phillips curve, I conclude this section with an example which uses the Samuelson-Solow model described by (1).<sup>7</sup> Let assume that inflation admits the following Moving Average (MA) representation:  $\pi_t = f(L) v_t$ , where  $v_t$  could be a combination of structural shocks of the economy, with  $E(v_t) = 0$ .<sup>8</sup> Under the Classical identification assumption,  $E(\pi_t \eta_t^{SS-C}) = 0$  implies that  $E(v_t \eta_t^{SS-C}) = 0$ . On the other hand, under the Keynesian identification assumption  $E(\pi_t \eta_t^{SS-K}) \neq 0$ , which implies that  $E(v_t \eta_t^{SS-K}) \neq 0$ . Let

$$\eta_t^{SS-K} = \rho v_t + \tilde{\eta}_t^{SS-C}$$

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<sup>5</sup>Brock, Durlauf and West (2007) call this type of analysis "action dispersion".

<sup>6</sup>In the  $LS$  model, only the unexpected part of inflation is able to affect the unemployment rate, so this model can only be estimated using the Classical assumption.

<sup>7</sup>This example is very much in the spirit of the discussion presented in King and Watson (1994) and Sargent (1976).

<sup>8</sup>This MA representation for  $\pi_t$  could be interpreted, for instance, as originating from the policy rule that the monetary authority uses to control the inflation rate.

where  $\rho = cov(v_t, \eta_t^{SS-K})/var(v_t)$ . Notice that, by construction,  $E(v_t \tilde{\eta}_t^{SS-C}) = 0$ , which also implies that  $E(\pi_t \tilde{\eta}_t^{SS-C}) = 0$ . Thus, we can rewrite:

$$\pi_t = \gamma_0^{SS-K} + \gamma_\pi^{SS-K}(L) \pi_{t-1} + \gamma_u^{SS-K}(L) u_t + \rho v_t + \tilde{\eta}_t^{SS-C}$$

and we can further substitute  $v_t = \tilde{f}(L) \pi_t$ , where  $\tilde{f}(L) = f^{-1}(L)$ , to obtain:

$$\pi_t = \gamma_0^{SS-K} + \gamma_\pi^{SS-K}(L) \pi_{t-1} + \gamma_u^{SS-K}(L) u_t + \rho \tilde{f}(L) \pi_t + \tilde{\eta}_t^{SS-C} \quad (4)$$

Finally, we can rewrite (4) in a Classical form as:

$$u_t = \tilde{\gamma}_0^{SS-C} + \tilde{\gamma}_\pi^{SS-C}(L) \pi_t + \tilde{\gamma}_u^{SS-C}(L) u_{t-1} + \gamma_\eta \tilde{\eta}_t^{SS-C} \quad (5)$$

where  $E(\pi_t \tilde{\eta}_t^{SS-C}) = 0$ , and all the parameters can be obtained as a function of  $\rho$  and the parameters in (4).

This example shows that if inflation has the MA representation assumed above, then from a model of the Phillips curve estimated using the Keynesian identification assumption, we can obtain an equivalent model that has the same characteristics of the Classical version of the same Phillips curve. This conclusion can be extended to more general representations for the processes of inflation and unemployment (for more details, see King and Watson, 1994). Yet, as I will show later on, the Classical and Keynesian versions of the same model are very different in terms of fit to the data and policy implications.

## 2.2 Empirical Approach

The empirical approach that I use to estimate the parameters of each model and to compute their posterior probabilities is the same as in Cogley and Sargent (2005). For each Phillips curve specification, I estimate the model parameters using Bayesian methods. Given the selected identification assumption, (1), (2) and (3) are simple regression models. For each model, the prior distribution of the parameters is assumed to be of the Normal-Inverse Gamma family. The Phillips curve residuals  $\eta_t^i$  in (1), (2) and (3) are assumed to be i.i.d. and conditionally normal given the regressors for each of the models. These assumptions ensure that the conditional likelihood function is Gaussian. Given a

Normal-Inverse Gamma prior and a Gaussian conditional likelihood, the posterior joint distribution will also be of the Normal-Inverse Gamma family, with parameters that can be updated recursively. More details, together with the updating formulas for the parameters, are given in Appendix 2.

The fit of each specification of the Phillips curve to the data is evaluated based on the model's posterior probability. Let  $i = \{SS - K, SS - C, ST - K, ST - C, LS\}$ . The posterior probability of model  $i$  given data up to time  $t$  can be defined according to Bayes's theorem as:

$$p(M_i | Y^t, X^t) \propto m_{it} \cdot p(M_i) \quad (6)$$

where  $p(M_i)$  and  $p(M_i | Y^t, X^t)$  are the prior and posterior probabilities of model  $i$ , while  $m_{it}$  is the marginalized likelihood function for model  $i$  at time  $t$ . Here,  $X^t$  represents the history up to time  $t$  of the right-hand variables of the model, and  $Y^t$  the history up to time  $t$  of the left-hand variable. Notice that the set of right-hand variables and left-hand variables will be different for each of the specifications of the Phillips curve under analysis.

The expression for the models' posteriors can be used to compute the normalized posterior probabilities, defined as:

$$\alpha_{it} = \frac{w_{it}}{\sum_i w_{it}} \quad (7)$$

where  $w_{it} \equiv m_{it} \cdot p(M_i)$ . Given the assumptions on the posterior distribution of the parameters of each model, the marginalized likelihood function  $m_{it}$  can be computed analytically, and the values of  $w_{it}$  can be updated recursively.<sup>9</sup> Again, Appendix 2 provides more details and reports the updating formulas.

### 3 The Postwar U.S. Phillips Curve(s)

I start by employing the model uncertainty framework presented in the previous section to study the patterns of the model posterior probabilities in the U.S. data from 1960 to 2016. The purposes of this exercise are two. First, I want to examine whether accounting for the uncertainty over the direction of fit of the Phillips curve affects the pattern of the model posteriors. Cogley and Sargent (2005) studies a model space that only includes the  $SS - K$ ,  $ST - K$  and  $LS$  Phillips curves, and report

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<sup>9</sup>See Cogley and Sargent (2005) for a more extensive explanation of this result.

the normalized posterior probabilities for these models in the period 1960 – 2002. By comparing the results obtained in this paper with those arising from the model space of Cogley and Sargent (2005), we can verify whether the model of the Phillips curve that best fits the data over time changes when the larger model space is used for the analysis. Second, I am interested in investigating the patterns of the posterior probabilities during the last few years of the sample, and in particular during the 2008 – 2009 recession. A recent literature in economics suggests that there might have been a change in the true underlying relationship between inflation and unemployment during the global financial crisis (see, for instance, Coibon and Gorodnichenko, 2015; Friedrich, 2014). If this is the case, then it is possible that the patterns of the models’ posteriors will show larger adjustments around 2008.

For the interpretation of the results, it is worth remarking that the normalized posterior probabilities reported in this paper are a measure of the models’ fit to the data up to the point in time at which they are computed. In other words, for each time  $t$ , they measure the models’ relative ability to describe the data in the sample period going from time 1 to time  $t$ . This implies that these posteriors will change over time because the additional data that becomes available will improve the estimates of each model. However, the posteriors could also change because of changes in the underlying true model of the economy. As mentioned above, one of the central features of the model uncertainty approach adopted in this paper is that no assumptions is made on the actual data generating process, and the specifications of the Phillips curve considered in the analysis could all be just approximations of the true model of the economy. As a consequence, different specifications could fit the data better in some periods rather than others, especially if the underlying data generating process is not stable over time. If this is the case, then the relative posterior probabilities will keep adjusting. Notice that, for this reason, it is not necessarily the case that there will be one dominant model for which the posterior probability will converge to one in the long run.

### **3.1 The Prevailing Model of the Phillips Curve**

I estimated the model parameters and posterior probabilities using U.S. quarterly data for inflation and unemployment. Unemployment is the quarterly average of the monthly Civilian Unemployment rate. In the baseline exercise, inflation is computed from the Personal Consumption Expenditure (PCE) chain-type price index, which is the primary price measure used by the Federal Reserve

for policy decisions.<sup>10</sup> However, I also report the results obtained using the GDP chain-type price index, the GDP implicit price deflator, and the CPI. The data employed in the estimation goes from 1960 : *I* to 2016 : *IV*, while observations from 1948 : *I* to 1959 : *IV* are used to set the parameters in the prior distributions and the initial values in the updating formulas. Appendix 1 gives further details about the assumptions and the initial settings used in the estimation procedure.

The Keynesian and Classical versions of the *SS* and *ST* models are the same in everything, except for the direction of fit of the equation that is estimated. The *ST* and *LS* models include the variable  $u_t^*$ , which denotes the natural rate of unemployment. As known, this variable is not observed by the econometrician, and needs to be either estimated or approximated. While I believe that it would be very interesting to treat  $u_t^*$  as unobservable, and estimate its history jointly with all the other parameters of the models, this approach would significantly complicate the computations, particularly for the model posteriors.<sup>11</sup> For this reason, in this paper I decided to approximate  $u_t^*$  using the same formula as in Cogley and Sargent (2005)

$$u_t^* = u_{t-1}^* + g(u_t - u_{t-1}^*) \tag{8}$$

with gain parameter set as  $g = 0.075$ . The last part of this section will provide a discussion of the sensitivity of the results to alternative approximating formulas for  $u_t^*$ .

As discussed above, in order to estimate equation (3) I will need to make some additional assumptions about the variable  $E_{t-1}(\pi_t)$ . Following the previous literature using similar models of the Phillips curve, I assume that the monetary authority is able to control the inflation rate to some extent, and that private agents will set their expectations based on this information. Given this assumption, two possible approaches could be followed in a model uncertainty environment. The approach that I adopt in the baseline analysis is the same as in Cogley et al. (2011), and it is based on the idea that expectations are formed "internally" within the model. In other words, private agents in the *LS* model are assumed to believe that the true model of the economy is the *LS* model and that the value of inflation will be equal to the optimal rate based on this model. Thus, I will set

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<sup>10</sup>In the February 2000 "Monetary Policy Report to the Congress", the Federal Open Market Committee (FOMC) stated that its main measure of inflation was changing from CPI inflation to inflation computed using the PCE chain-type price index.

<sup>11</sup>In this case, we would not be able to use the analytical formulas reported in Appendix 2 to compute the model weights.

$E_{t-1}(\pi_t) = x_{t|t-1}^{LS}$ , where  $x_{t|t-1}^{LS}$  is the optimal policy choice for inflation in the *LS* model. In a model uncertainty environment, an alternative to this assumption is the approach employed by Cogley and Sargent (2005), which set  $E_{t-1}(\pi_t) = x_{t|t-1}^E$ . Here,  $x_{t|t-1}^E$  is the "Encompassing" policy choice, which takes into account policymakers' uncertainty over the true model of the economy. Under this second approach, private agents' expectations will incorporate model uncertainty through its impact on the decisions of policymakers. My choice of setting  $E_{t-1}(\pi_t) = x_{t|t-1}^{LS}$  in the *LS* model was motivated by a preference for leaving all the models self-contained in their implications and predictions, in order to let the data discriminate more clearly between them. However, in order to have a better understanding of the impact of private agents' expectations on the conclusions of this paper, I will later also discuss the results and the policy implications under the alternative approach  $E_{t-1}(\pi_t) = x_{t|t-1}^E$ .

For the choice of the models' priors, I followed the same reasoning as in Cogley and Sargent (2005). Since the *ST* and *LS* models were developed after 1960 : *I*, I set the prior to 0.98 for the *SS* model, and to 0.01 each for the *ST* and *LS* models. For the *SS* and *ST* models, I split the prior equally between the Classical and the Keynesian specifications. Thus, the *SS* – *K* and *SS* – *C* models received a prior of 0.49 each, the *ST* – *K* and *ST* – *C* models a prior of 0.005 each, and the *LS* model a prior of 0.01. Again, I will discuss the sensitivity of the results to this choice.

Figure 1 reports the patterns of the model posteriors in the 1960 – 2016 period for the full model space considered in this paper. Figure 2 shows the same patterns for the model space of Cogley and Sargent (2005), which does not include the uncertainty over the direction of fit of the Phillips curve.<sup>12</sup> The story that Figure 1 and Figure 2 tell is to some extent consistent. In both figures, the *SS* model estimated under the Keynesian assumption remains the predominant model in the first few years of the sample, while the *LS* Phillips curve becomes the model with the highest posterior starting from around 1970. However, there are also some other interesting insights that we can observe from comparing these two figures. The first one is the importance of the Classical versions of the *SS* and *ST* models. The posterior attached to the *SS* – *C* model is substantial in the first part of the sample, while the *ST* – *C* model exhibits a relatively high posterior from the mid 1970s until today. Overall, the Classical models of the Phillips curve, i.e. those estimated with unemployment as the

<sup>12</sup>The posteriors shown in Figure 2 are slightly different from those reported by Cogley and Sargent (2005). The differences are due to the different price index used to measure inflation in the two papers (PCE versus GDP chain-type price deflator), and to the different way in which unemployment is computed (quarterly average of the monthly rates versus rate in the middle month of the quarter).

left-hand side variable, seem to fit the data better than their Keynesian counterparts in almost the entire sample period under analysis.

One second observation is that both figures confirm the conclusion of Sims (2001) and Cogley and Sargent (2001, 2005) that it should have become clear, already in the early 1970s, that the model of Phillips curve providing the best fit to the data was one incorporating the assumption of long-run neutrality. However, Figure 1 provides some additional details in this respect. Indeed, this figure shows that if we take into account different directions of fit of the Phillips curve, then the data will support, at least to some extent, the Solow-Tobin version of the long-run neutrality in addition to the version implied by the Lucas-Sargent model. From a policy perspective, it might not be important to know which Classical model ( $SS - C$ ,  $ST - C$ , or  $LS$ ) is prevailing over time, as all of them will deliver very similar policy recommendations (this statement will be explained and discussed in more detail in section 4). But for the interpretation of the past data, or to predict the way in which inflation and unemployment will affect each other in future periods, it seems to be important to consider a model space that also includes the Classical direction of fit of the alternative models of the Phillips curve.

A third interesting result that we can observe from Figure 1 is that the patterns of the model posteriors do change considerably in the years of the 2008-2009 crisis. More specifically, right after the beginning of the crisis the posterior attached to the  $ST - C$  model increased rapidly at the expenses of the  $LS$  model. After this sudden change, the posteriors of the two models have been slowly moving back towards their previous values.

One possible interpretation for the behavior of the posterior probabilities for the  $ST - C$  and  $LS$  Phillips curves is in terms of private agents' expectations of inflation. Let consider the  $LS$  Phillips curve reported in (3). We can relax the assumption of rational expectations and assume instead that expectations are formed using some general rule in the form:

$$E_{t-1}(\pi_t) = \gamma\pi_t + (1 - \gamma) \gamma^e(L) \pi_{t-1} \quad (9)$$

with  $\left[ \gamma + (1 - \gamma) \sum_{j=0}^J \gamma_j^e \right] = 1$ .<sup>13</sup> Under this alternative assumption, it is straightforward to see that

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<sup>13</sup>This condition enforces expectations to be correct in the long-run.

the *LS* model in (3) can be rewritten as:

$$u_t - u_t^* = \tilde{\gamma}_\pi^{LS}(\pi_t - \pi_{t-1}) + \tilde{\gamma}^e(L)(\pi_{t-1} - \pi_{t-2}) + \gamma_u^{LS}(L)(u_{t-1} - u_{t-1}^*) + \eta_t^{LS} \quad (10)$$

where  $\tilde{\gamma}_\pi^{LS}$  and  $\tilde{\gamma}^e(L)$  can be obtained from  $\gamma_\pi^{LS}$ ,  $\gamma$  and  $\gamma^e(L)$ . Notice that the assumption on the parameters  $\gamma$  and  $\gamma^e(L)$  in the expression for private agents' expectations implies that  $\left[ \tilde{\gamma}_\pi^{LS} + \sum_{j=0}^J \tilde{\gamma}_j^e \right] = 0$ . Thus, if expectations are formed according to (9), equation (3) becomes exactly the same as the equation for the *ST* model estimated under the Classical identification assumption.

The purpose of this discussion is to point out that, in fact, the *ST* – *C* and the *LS* specifications can be interpreted as two different versions of the same model. In one of these versions (*LS*) rational expectations are imposed, while in the other (*ST* – *C*) a more general expectation formation process is assumed instead.<sup>14</sup>

In the light of this argument, we can attempt some conjectures about the changes in the true underlying data generating process based on the patterns of the model posteriors reported in Figure 1. The posteriors for the *ST* – *C* and *LS* models suggest that, starting from the mid 1970s, the data has been favoring a model of the economy that resembles (3), but in which expectations were not always rationally formed. The assumption of rational expectations seems to fit the data slightly better for a long period of time, specifically from the mid 1970s until around 2007. However, during the recent financial crisis, a version of (3) in which private agents were allowed to form expectations using a more general rule was able to describe the data to a much better extent. The study the true model of the Phillips curve and its possible changes over time is not among the main purposes of this paper; nonetheless, the patterns of the model posteriors reported in Figure 1 provides some interesting insights that researchers could use to investigate this issue further.

### 3.2 The Slope of the Phillips Curve

Figure 3 illustrates the evolution of the Phillips curve slope over the sample period under analysis for each of the empirical models included in the model space. As in Primiceri (2006), the slope is computed as the sum of the coefficients on unemployment in the estimated model re-written so

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<sup>14</sup>We could even extend this interpretation to the *SS* – *C* model, which can be viewed as a version of (3) where expectations are formed based on (9) but the condition  $\left[ \gamma + (1 - \gamma) \sum_{j=0}^J \gamma_j^e \right] = 1$  does not hold.

that inflation is expressed on the left-hand side. For each model, Figure 3 reports the median, and the 25th and 75th percentiles, of the values of the slope computed using 10,000 draws from the distributions of the model's parameters.

Figure 3 confirms that the slope is very different depending on the specification of the Phillips curve under consideration, and in particular on the direction of fit used to estimate the model. By construction, the slope of the *LS* model is infinite, as this model implies no trade-off between inflation and unemployment (i.e. the Phillips curve is vertical). For this reason, this model is not reported in Figure 3. The slopes of the Classical specifications at the beginning of the sample are very large and very volatile, while the slopes of the Keynesian models are more precisely estimated and much smaller in magnitude (notice that scale of the vertical axis in the two panels of Figure 3 is different). The patterns reported in Figure 3 change over time because of the updating of the estimated model parameters.<sup>15</sup>

The slope of the Phillips curve is a measure of the trade-off between inflation and unemployment, and provides information about the possibility to "exploit" the Phillips curve for policy purposes. The remarkable difference in the slopes of the Classical and Keynesian models is due to the different way in which the contemporaneous correlation between inflation and unemployment affects the estimated parameters in the two models. A low correlation, as the one observed in the data starting from the early 1970s, implies a flat trade-off between inflation and unemployment in a Keynesian Phillips curve, but a steep one in a Classical Phillips curve. In this context, a Keynesian Phillips curve would suggest that inflation can only be reduced at the expenses of very high unemployment, while a Classical Phillips curve would imply that the cost of reducing inflation in terms of unemployment is significantly lower. Clearly, this difference will have consequences on the optimal policy choice in each of these models.

Cogley and Sargent (2005) report sacrifice ratios instead of the slope of the Phillips curve as a measure of the trade-off between inflation and unemployment. These sacrifice ratios are computed as the cumulative output loss associated with a reduction in inflation by 1% sustained for 8 quarters. The *SS* and *ST* models estimated using the Keynesian assumption are shown to generate very large sacrifice ratios, while these ratios are close to zero for the Classical versions of the same models. A

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<sup>15</sup>For the Classical models, computing the slope involves taking a ratio of the estimated parameters. This is the reason why the error bands are much larger for these specifications, especially at the beginning of the sample when the estimates of the parameters are less precise.

flat Phillips curve implies high sacrifice ratios, as a decrease in inflation will be associated with a large increase in unemployment, while the opposite reasoning stands for a flat Phillips curve. As a consequence, the slopes reported in Figures 3 are consistent with the sacrifice ratios computed by Cogley and Sargent (2005). The patterns reported in Figure 3 are also consistent with the results of Blanchard (2016), who shows that the slope of the true underlying model of the Phillips curve has remained almost unchanged since the 1980s. In fact, the slopes estimated in this paper are stable after the mid 1980s for all the specifications of the Phillips curve under analysis.

One last remark can be made by observing the patterns reported in Figure 3 together with the model posteriors of Figure 1. Figure 3 implies that the estimated parameters of each model, and their distributions, have remained relatively stable in the last 30 years, and in particular they seem to exhibit only minor changes during the 2008 – 2009 economic crisis. This result seems to suggest that the sharp shift in the relative model posteriors at the end of 2007 and beginning of 2008 was mostly due to a change in the models' conditional likelihoods rather than to a change in the distributions of the models' parameters.<sup>16</sup> This conclusion seems to support the explanation proposed above that the shift in the models' fit to the data that happened around 2007 might have been caused by a change in some of the characteristics of the underlying data generating process.

### 3.3 Sensitivity Analysis

The patterns reported in Figures 1 to 3 are robust to changes in several of the assumptions used in the empirical implementation of the model uncertainty framework. Different values of the parameter  $g$  in (8) did not change the pattern of the posterior probabilities reported in Figures 1 and 2. Replacing (8) with the updating formulas employed by Primiceri (2006) did not substantially alter the results either.<sup>17</sup> In general, alternative assumptions generating a smoother series for  $u_t^*$  resulted in the  $SS - C$  model maintaining a high posterior for a longer period at the beginning of the sample, but the patterns of the posterior probabilities in the later portion of the sample were never affected in a significant way. Similarly, changes in the models' priors resulted in some changes in the first part of the sample (where the priors have a relatively larger impact, since the number of observations is still small), but the normalized posteriors were again very similar to those reported in Figure 1 in the

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<sup>16</sup>See Appendix 2 for the specific formulas used to compute the model posteriors.

<sup>17</sup>Primiceri (2006) estimates the current value of the natural rate of unemployment using a constant gain learning approach. For the specific formulas, see Primiceri (2006).

later part of the sample. The results were also unaffected by changes in the length of the training sample, and in the approach used to approximate  $E_{t-1}(\pi_t)$  in the *LS* model within the training sample (see Appendix 1 for more details).

### 3.3.1 Measuring Inflation

There is no widespread consensus about which measure of inflation should be used to estimate the Phillips curve. The main measure used in this paper is the PCE chain-type index which, as mentioned, is the primary indicator used by the Federal Reserve for policy purposes. However, in order to assess the robustness of the results and to compare them with those of previous contributions in this area, I repeated the analysis using a few alternative measures of inflation. These measures are the GDP deflator chain-type index (as in Cogley and Sargent, 2005), the GDP implicit price deflator (as in Primiceri, 2006), and the CPI for all items (as in Blanchard, 2016). Figure 4 compares the model posteriors computed using these alternative measures with the baseline values obtained using the PCE index and reported in Figure 1.

The patterns of the models' posteriors are similar for all measures except for the GDP implicit price deflator, for which the specification of the Phillips curve that fits the data best starting from the mid 1970s is the *ST - C* rather than the *LS* model. These two specifications of the Phillips curve are very similar in terms of policy implications but, as I will discuss in the next section, some of the differences in the patterns of the models' posteriors might actually originate different robust policy recommendations in a model uncertainty framework. A second consideration that we can make from Figure 4 is that the shift in posteriors that happened around 2008 is the most evident when the PCE index is used to compute inflation. The shift is still present, but not as pronounced, when the CPI or the GDP deflator chain-type index are used, and it is very small when the GDP implicit price deflator is employed.

Overall, even if there are some differences in the four panels shown in Figure 4, the main message that they all convey is still the same: starting from the mid 1970s, the model of the Phillips curve that best fits the data is one that implies long-run neutrality and a very steep (or vertical) short-run slope. In this model the incentives to exploit the unemployment-inflation trade-off are low, because a reduction in unemployment would require a very large cost in terms of inflation.

## 4 Policy Implications

In a model uncertainty environment, there are several directions in which policy analysis can be discussed. In this section, I will first focus on the optimal policy choices within each of the models of the Phillips curve described in the first part of the paper, and then I will move into the analysis of policies that account for model uncertainty. The main approach that I will use in this second group of policies is the robust regulator problem proposed by Cogley and Sargent (2005).

One thing that is worth remarking is that, in a model uncertainty environment, the monetary authorities will need to consider, at least to some extent, the effects of policy changes in all the models that they believe to be possible approximations of the underlying data generating process. In this sense, a model uncertainty framework will urge policymakers to address the issue of Lucas' critique. A model of the Phillips curve with time-varying parameters will perhaps be more flexible and it might be able to capture changes in the data to a better extent, but it will probably be unable to incorporate the impact of policy changes as precisely. In a model uncertainty framework, on the other hand, policymakers will be able to explicitly take into account that alternative model specifications imply different assumptions on the channels through which policy affects the economy and on the set of parameters that remains unaffected by policy changes.

In order to be able to discuss policy implications, the models of the Phillips curve described in section 2 need to be augmented of a monetary policy equation. As mentioned before, the previous literature using similar models of the Phillips curve assumes that the central bank can control the inflation rate to some extent. I will follow this literature, and in particular Cogley and Sargent (2005) and Sargent, Williams and Zha (2006), and I will assume that the policymaker can set the value of the policy instrument  $x_{t|t-1}$ , which is related to inflation according to the expression:

$$\pi_t = x_{t|t-1} + \xi_t \tag{11}$$

where  $\xi_t$  is an i.i.d. normal shock with mean zero and variance  $\sigma_\xi^2$ . The subscript on the policy variable  $x$  implies that the policymaker will choose the value of the policy instrument for time  $t$  using the information available at time  $t - 1$ . Thus, the value of  $x_{t|t-1}$  is assumed to be decided in real-time, based on the observable data and on the value of the parameters estimated using this data.

I assume that the policymaker chooses the value of the policy instrument optimally by solving a linear quadratic dynamic programming problem. More specifically, the policymaker aims at minimizing the loss function:

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} \beta^j \left[ (u_{t+j} - u^*) + \lambda (\pi_{t+j} - \pi^*)^2 \right] \quad (12)$$

where  $\beta$  is the discount factor,  $u^*$  and  $\pi^*$  are the target values of the variables, and  $\lambda$  represent the weight attached to inflation relative to unemployment. As common in the literature on learning in macroeconomics (see, for instance, Evans and Honkapohja, 2001), the policymaker is assumed to consider the estimated parameters of each model as if they were true values, which will not change over time. This assumption allows to disregard parameter uncertainty, and makes the decision rules depend only on point estimates rather than on the whole posterior distributions of the coefficients. In addition, it allows the policymaker to engage in a process of learning that is passive, in the sense that it takes into account the new information that becomes available but ignores the effects of current decisions on the future values of the variables of interest.<sup>18</sup> This approach has been called "anticipated utility" decision-making by Kreps (1998).

The method used to compute the optimal and robust policy rules is described in detail in Appendix 3. All the exercises performed in this section use a target inflation rate  $\pi^* = 2\%$ , and a target unemployment rate equal to the value of  $u_t^*$  in the period in which the optimal policy is computed. However, none of these assumptions matters for the results, as the main conclusions of the policy analysis remain unaltered if the values of the targets are changed in a reasonable manner, or if the targets are eliminated altogether.

Figure 5 reports the optimal policy recommendations for each specifications of the Phillips curve included in the model space. The middle panel focuses on the Keynesian specifications, while the bottom panel shows the optimal policies for the Classical specifications (including the *LS* model). The patterns of the posterior probabilities already presented in Figure 1 are also reported in the top panel to facilitate the interpretation of the results. As already discussed before, in the *LS* model the optimal value of  $x_{t|t-1}^{LS}$  is the target inflation rate. As expected, the other Classical models

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<sup>18</sup>In a learning environment, the connection between today's decisions and tomorrow's information might create the incentive for experimentation, which is not allowed in this framework. Cogley, Colacito and Sargent (2007) and Svensson and Williams (2007) discuss the benefits that policymakers could obtain from experimentation by exploiting this connection.

also recommend low inflation rates, which are very close to the 2% target. On the other hand, the optimal policy rates for the Keynesian models are much higher. As discussed extensively in Cogley and Sargent (2005), the  $SS - K$  and  $ST - K$  specifications are unstable in some parts of the sample if low inflation policies are implemented (this is true, in particular, in the first part of the sample until the mid 1980s). The patterns of  $x_{t|t-1}^{SS-K}$  and  $x_{t|t-1}^{ST-K}$  reflect the fact that policymakers will need to accept higher inflation rates in order to stabilize these models.

As shown in Figure 5, the empirical models of the Phillips curve included in the model space imply very different optimal policy recommendations. So, what specific policy should be selected in a model uncertainty environment? Figure 6 provides three possible answers to this question. This figure reports three different policy measures. The top panel shows the weighted average optimal policy, computed as the sum of the optimal policies weighted by the models' posteriors. The middle panel reports the optimal policy for the model with the highest posterior in each quarter. Finally, the bottom panel reports the robust policy recommendation obtained using the approach proposed by Cogley and Sargent (2005).<sup>19</sup> In all panels, the actual inflation rate is also depicted for comparison.

A large literature in economics has focused on policymaker's changing beliefs over the true model of economy as the main explanation for the rise and fall of the U.S. inflation in the postwar period (see, for instance, DeLong, 1997; Sargent, 2001; Cogley and Sargent, 2005; Sargent and Williams, 2005; Primiceri, 2006). The results reported in Figures 5 and 6 are related to this literature in four important directions.

First, several papers in this area have pointed out a puzzle in the timing of the decrease in inflation in the mid 1980s. As discussed above, the literature agrees that, by looking at the data, policymakers should have realized already in the early/mid 1970s that models of the Phillips curve recommending a low inflation rate were a better representation of the economy. Figures 1, 2, and 4, do confirm this conjecture, as they all show that the posteriors attached to the  $ST - C$  model and/or the  $LS$  model become predominant quite early in the sample. These figures also show that this conclusion is very robust, as it holds under alternative model spaces, and for different measures of inflation. The problem is that if policymakers were indeed aware of these posterior probabilities, or more generally of the models' fit to the data, then the fact that inflation was decreased only several years later is difficult to rationalize. This puzzle is clearly shown in Figure 5: if we consider

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<sup>19</sup>Again, see Appendix 3 for more details about this approach.

the patterns of the models' posteriors reported in the top panel together with the optimal policies shown in the middle and bottom panels, then we are left to wonder why the inflation rate was not reduced already in the early/mid 1970s.

A second direction in which this work is related to the previous literature in this area is in the analysis of the possible explanations for this puzzle. This paper confirms that the puzzle might be difficult to reconcile. Some contributions have proposed an argument based on the monetary authority's incorrect beliefs about the natural rate of unemployment, while Cogley and Sargent (2005) provide an interpretation that is based on policymakers' quest for a robust policy in the face of model uncertainty. The bottom panel of Figure 6 confirms that in a model uncertainty environment, a robust optimal control type of approach does provide a possible explanation for the puzzle. However, this figure also shows that some other possible ways of accounting for model uncertainty (for instance considering an average policy, or the policy of the model with the highest posterior), are not able to rationalize the pattern of inflation in the U.S. during the postwar period.

Third, this work offers a clear picture of the extent to which the direction of fit used to estimate the *SS* and *ST* models matters for monetary policy. As previously discussed, the identification assumption used to compute the parameters of the Phillips curve has a large impact on the perceived cost of disinflation and, consequently, on the monetary authority's optimal policy decisions. This argument is evident in the middle and bottom panels of Figure 5, which show that for the same specification of the Phillips curve, a different direction of fit of the estimated empirical model implies completely different policy recommendations.<sup>20</sup>

Last, this paper confirms that, in a model uncertainty environment, the model (or models) that represents the worst-case scenario of the economy is going to affect the robust policy choice to a very large extent. As the Keynesian versions of the Samuelson-Solow and Solow-Tobin models are less stable and imply significantly higher sacrifice ratios<sup>21</sup> than the corresponding Classical versions, Cogley and Sargent (2005) suggest that these specifications are the models against which the policymaker will want to protect. For this reason, they argue that including the Classical versions of the same models in the optimal control problem would not alter the robust policy recommendations. The results reported here do indeed support this argument, as the policies depicted in the bottom

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<sup>20</sup>Figure 5 is consistent with the statement of Cogley and Sargent (2005) that for the results of their paper, "the direction of fit matters more than the qualitative nature of the tradeoff" (Cogley and Sargent, 2005, p. 547).

<sup>21</sup>In terms of the increase in unemployment required to reduce inflation.

panel of Figure 6 are very similar to the robust policies obtained by Cogley and Sargent (2005) for a reduced model space that consists of the  $SS - K$ ,  $ST - K$ , and  $LS$  models only. In this sense, we can conclude that what matters the most to the robust policymaker is the magnitude of the trade-off between inflation and unemployment in the model(s) in which the cost to reduce inflation in terms of unemployment is the highest.

#### 4.1 Private Agents' Expectations

The policies shown in Figure 5 and Figure 6 were computed under the assumption that in the  $LS$  model private agents set  $E_{t-1}(\pi_t) = x_{t|t-1}^{LS}$ , where  $x_{t|t-1}^{LS}$  is the optimal policy within the  $LS$  model. As I explained above, this assumption implies that private agents form expectations internally within the model, without taking model uncertainty into account. Cogley and Sargent (2005), on the other hand, assume that private agents in the  $LS$  model form expectations by setting  $E_{t-1}(\pi_t) = x_{t|t-1}^E$ , where  $x_{t|t-1}^E$  is the robust policy under model uncertainty, computed using the approach described in Appendix 3. This assumption implies that private agents account for model uncertainty in their expectations through its impact on policymakers' decisions.

In order to investigate the impact of private agents' expectations on the conclusions of the paper, I computed the patterns of the posterior probabilities again using  $E_{t-1}(\pi_t) = x_{t|t-1}^E$  instead of  $E_{t-1}(\pi_t) = x_{t|t-1}^{LS}$ . I repeated this exercise for all the measures of inflation that were employed to produce Figure 4. The results are reported in Figure 7.

The different way in which agents are assumed to form expectations can potentially change the fit to the data of the  $LS$  model and, as a consequence, the normalized posterior probabilities. In fact, for some of the measures of inflation employed in the analysis, the normalized posteriors reported in Figure 7 are different from those shown in Figure 4. In particular, assuming that private agents account for model uncertainty in their expectations improves the fit of the  $LS$  model when inflation is measured using the GDP implicit price deflator or the CPI, while it makes it worse when inflation is measured from the GDP chain-type index. The posteriors are almost unchanged in the case of PCE inflation. Despite the different patterns of the posterior probabilities reported in Figure 7, the story that all these figures suggests is again the same: starting from the early 1970s, the data clearly favors models of the Phillips curve which imply a very steep trade-off between inflation and unemployment, and that recommend low inflation rates. In terms of robust policy, it is possible to show that for

all the measures of inflation considered in Figures 4 and 7, the robust policy recommendations are almost unaffected if private agents' expectations are assumed to be set as  $E_{t-1}(\pi_t) = x_{t|t-1}^E$  instead of  $E_{t-1}(\pi_t) = x_{t|t-1}^{LS}$ .<sup>22</sup>

## 4.2 Measuring Inflation and Robust Policymaking

One last question that we might be interested in addressing is whether all the measures of inflation considered in the analysis generate the same robust policy recommendations. I will answer this question using the posterior probabilities reported in Figure 7 but, as just discussed, the results would be the same if the posteriors shown in Figure 4 were employed instead. I am also going to focus on the sub-period 1985 – 2016, for two reasons. First, the period 1960 – 1985 has already been extensively studied in the literature, and a robust policy analysis of the inflation rate in these years has already been developed by Cogley and Sargent (2005). On the other hand, the sample used in Cogley and Sargent (2005) ends in 2002, so it is actually interesting to examine the robust policy recommendation in the more recent years. Second, during the 1985 – 2016 period the data very strongly favors two models, the *ST* – *C* and the *LS* Phillips curves, while all the other models have posterior probabilities that are basically zero.<sup>23</sup> The *ST* – *C* and *LS* models recommend a low and stable inflation rate, but the robust policy rule could still be affected by the models that imply large losses under low inflation policies, even if their posterior probability is very close to zero. Thus, it is interesting to investigate whether this is indeed the case under all the measures of inflation employed in Figure 7.

Figure 8 reports the robust policy under model uncertainty for inflation measured using the PCE index, the GDP deflator chain-type index, the GDP implicit price deflator, and the CPI, for the period 1985 – 2016. It is clear from Figure 8 that the robust policy recommendation under model uncertainty is quite different depending on the price index used to compute the inflation rate. In particular, the GDP measures of inflation seem to suggest a robust inflation rate that is equal to the

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<sup>22</sup>In computing the robust policy rule, I always assumed that policymakers acted as if their policies were ineffective in the *LS* model, regardless of the way in which private agents formed expectations. This assumption was motivated by the fact that policy ineffectiveness is one of the main characteristics of the *LS* model, which I wanted to maintain. However, this assumption is likely the reason why the robust policy is almost unaffected by the way in which private agents' expectations are defined.

<sup>23</sup>As an example, the normalized posteriors for the last quarter of 2016 in the models estimated using PCE inflation were: 0.4305 for the *ST* – *C* model, 0.5695 for the *LS* model,  $3.15 \times 10^{-36}$  for the *SS* – *K* model,  $2.13 \times 10^{-5}$  for the *SS* – *C* model, and  $5.92 \times 10^{-37}$  for the *ST* – *K* model.

target inflation rate of 2% in most periods, while the PCE and the CPI imply a robust inflation rate that is a lot more volatile (and that seems to track more closely the pattern of the actual inflation rate during the same period). These results are interesting, and they show that the robust policy choice will depend on the models of the Phillips curve included in the analysis, but also on the specific measure of inflation that the monetary authority employs for policy decisions.<sup>24</sup>

## 5 Concluding Remarks

This paper studied a model uncertainty framework that explicitly accounts for the uncertainty about the specification of the Phillips curve and the identification assumption to be used for the estimation of its parameters. This framework was employed to analyze the changes in the fit of the different models over time, and to understand the implications of these changes for monetary policy.

The main messages of the paper are three. First, different directions of fit of the same specification of the Phillips curve fit the postwar U.S. data in a very different way, imply very different trade-offs between inflation and unemployment, and suggest very different optimal policies. In more detail, the "Classical" versions of the models fit the data significantly better than their Keynesian counterparts, imply a very steep (or vertical) slope of the Phillips curve, and recommend optimal policies in which the inflation rate is low and stable. Second, in the rational expectations model of the Phillips curve (the *LS* model), assuming that private agents incorporate policymakers' model uncertainty in their expectations can change the fit of the model to the data, but in a model uncertainty framework, it does not alter the robust policy recommendation. Third, the price index used to compute the inflation rate matters for the fit of the models to the data, and for the robust policy choices.

The analysis developed in this paper presents several suggestions for future research. I am particularly interested in extending the discussion in two directions. First, I believe that the issue of private agents' expectations in a model uncertainty environment should be investigated further. The literature in this area has focused on policymakers' uncertainty about the true model of the economy, while private agents are typically assumed to disregard model uncertainty (as in Cogley et al., 2011)

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<sup>24</sup>The dominant eigenvalue under no policy response to inflation or unemployment (i.e. for  $x_{t|t-1}$  equal to a constant value, for instance the target rate  $\pi^*$ ) is typically higher than the stability threshold of  $\beta^{-1/2}$  for the Keynesian models, and lower than this threshold for the Classical models. However, it is possible to show that, for the Keynesian specifications, the dominant eigenvalue is much higher when the PCE or CPI are used, relative to the case in which one of the measures of the GDP deflator is employed to compute inflation. This might be a possible explanation for the differences in the robust policies reported in Figure 8.

or to incorporate it through their observation of policy decisions (as in Cogley and Sargent, 2005). A different approach is the one of Brock, Durlauf and West (2007), in which the form of model uncertainty that the monetary authorities face is precisely the uncertainty about the way in which private agents form expectations. I think that it would be very interesting to extend this analysis towards the study of environments in which private agents are also assumed to be uncertain about the true model of the economy. In this context, I believe that it would be of great relevance to assess the impact of monetary policy in different scenarios in which private agents might or might not share the same form of model uncertainty as policymakers.

The second direction in which I would like to extend the framework presented in this paper is in the analysis of the role of the natural rate of unemployment. As mentioned, the literature has suggested that the monetary authority's incorrect beliefs about the natural rate of unemployment is a possible explanation of the high inflation rates in the U.S. in the late 1970s and early 1980s. For this reason, I believe that it would be interesting to develop a framework in which the natural rate of unemployment is treated as an unobservable variable that needs to be estimated together with the parameters of the model. However, in a model uncertainty environment this assumption requires some additional considerations, as it implies that the estimated natural rate of unemployment could be different in each of the alternative models of the Phillips curve included in the model space.

## Appendix 1

### *Data description and initial settings*

Unemployment  $u_t$  is measured from the monthly Civilian Unemployment rate, averaged to obtain quarterly data. The baseline measure used to compute the inflation rate  $\pi_t$  is the quarterly Personal Consumption Expenditure (PCE) chain-type price index. The alternative measures employed in the paper are the GDP chain-type price index, the GDP implicit price deflator, and the quarterly CPI. The sample goes from 1948 : *I* to 2016 : *IV*; the data from 1960 : *I* to 2016 : *IV* is used in the empirical exercises, while the observations from 1948 : *I* to 1959 : *IV* are used as training sample to set the initial values in the updating recursions and the parameters in the prior distributions. All the data was obtained from the Federal Reserve Bank of St. Louis website (FRED).

The natural rate of unemployment is approximated as:

$$u_t^* = u_{t-1}^* + g(u_t - u_{t-1}^*)$$

where the gain parameter is set as:  $g = 0.075$ . The value of  $u_0^*$  was set as:  $u_0^* = u_0$ , where  $t = 0$  is 1948 : *I*.

The number of lags used in the estimation of the models is as in Cogley and Sargent (2005). The *SS* model included 4 lags of inflation and 2 lags of unemployment in addition to the contemporaneous values of the variables. The *ST* model included 3 lags of inflation and 2 lags of the unemployment gap in addition to the contemporaneous values of the variables. The *LS* model included 2 lags of the unemployment gap in addition to the difference between inflation and its expected value.

The parameters for (3), and (1) and (2) for both the Keynesian and the Classical identification assumptions, were estimated using Bayesian methods. For each model  $i$ , let  $\sigma_i^2$  denote the variance of the Phillips curve residuals and  $\theta_i$  the vector of coefficients of the model. The prior distribution of the parameters is assumed to be a Normal-Inverse Gamma distribution:

$$\begin{aligned} p(\theta_i, \sigma_i^2) &= p(\theta_i | \sigma_i^2) p(\sigma_i^2) \\ &= N\left(\theta_{i,0}, \sigma_i^2 P_{i,0}^{-1}\right) IG(s_{i,0}, v_{i,0}) \end{aligned} \quad (13)$$

The parameters of this distribution were computed using data from the training sample. More

specifically, for each model  $i$ , the vector  $\theta_{i,0}$  was set as the point estimate of the coefficients obtained from an OLS regression. The initial value of the other variables in the prior distributions of the parameters was set as follows:  $P_{i,0} = X'_{i,T_0} X_{i,T_0}$ , where  $X_{i,T_0}$  is a matrix including the training sample observations of the right-hand variables for model  $i$ ;  $s_{i,0}$  is the sum of squared residuals from the initial regression;  $v_{i,0}$  is the difference between the number of observations and the number of estimated coefficients in the initial regression. After the initial beliefs on  $\theta_{i,0}$ ,  $P_{i,0}$ ,  $s_{i,0}$  and  $v_{i,0}$  are set, 1959 – IV becomes  $t = 0$ , and the model parameters and weights are updated starting from 1960 – I using the approach described below in Appendix 2.

For the *LS* model, the initial settings require to approximate the evolution of  $E_{t-1}(\pi_t)$  in the training sample. In the exercises in which I assumed  $E_{t-1}(\pi_t) = x_{t|t-1}^{LS}$  (i.e. the optimal policy in the *LS* model), I set  $E_{t-1}(\pi_t) = \pi^* = 2\%$ , as this is the optimal policy choice within this model. In the exercises in which I assumed  $E_{t-1}(\pi_t) = x_{t|t-1}^E$  (i.e. the optimal policy in the model uncertainty environment), I followed Cogley and Sargent (2005) and I approximated  $E_{t-1}(\pi_t)$  by the value  $x_t$ , which was obtained by exponentially smoothing the current inflation rate according to the formula:

$$x_t = x_{t-1} + 0.075(\pi_t - x_{t-1})$$

with  $x_0 = \pi_0$ .

In the policy exercises performed in section 4, the discount factor  $\beta$  was set so that the annual discount rate is 4%, and the weight on inflation was set to  $\lambda = 16$ , reflecting an equal weight with unemployment.

## Appendix 2

### *Parameters updating*

Let  $\sigma_i^2$  denote the variance of the residuals and  $\theta_i$  the vector of coefficients for each specification of the Phillips curve included in the model space,  $i = \{SS - K, SS - C, ST - K, ST - C, LS\}$ . Let  $Z^t$  summarize the joint history of both right-hand and left-hand variables in each of the models up to time  $t$ . At time  $t$ , the econometrician is assumed to have information on the variables up to time  $t - 1$ , while the data for time  $t$  is still unknown. The prior on the model's parameters at time  $t$

is  $p(\theta_i, \sigma_i^2 | Z_i^{t-1})$ , while the posterior after the data for time  $t$  is observed is  $p(\theta_i, \sigma_i^2 | Z_i^t)$ . As in (13), we have:

$$p(\theta_i, \sigma_i^2 | Z_i^{t-1}) = p(\theta_i | \sigma_i^2, Z_i^{t-1}) p(\sigma_i^2 | Z_i^{t-1})$$

$$p(\theta_i | \sigma_i^2, Z_i^{t-1}) = N(\theta_{i,t-1}, \sigma_i^2 P_{i,t-1}^{-1}) \quad (14)$$

$$p(\sigma_i^2 | Z_i^{t-1}) = IG(s_{i,t-1}, v_{i,t-1}) \quad (15)$$

The values  $\theta_{i,t-1}$ ,  $P_{i,t-1}$ ,  $s_{i,t-1}$  and  $v_{i,t-1}$  are estimated based on data through period  $t-1$ . In each period, before the data is observed, the value of  $\theta_i$  can be estimated by  $\theta_{i,t-1}$ , while the value of  $\sigma_i^2$  can be estimated by  $s_{i,t-1}/v_{i,t-1}$ . When information about data at time  $t$  becomes available, the econometrician will update the parameters in (14) and (15) to obtain the posterior distribution  $p(\theta_i, \sigma_i^2 | Z_i^t)$ :

$$P_{i,t} = P_{i,t-1} + X_{i,t} X_{i,t}'$$

$$\theta_{i,t} = P_{i,t}^{-1} (P_{i,t-1} \theta_{i,t-1} + X_{i,t} Y_{i,t})$$

$$s_{i,t} = s_{i,t-1} + Y_{i,t}' Y_{i,t} + \theta_{i,t-1}' P_{i,t-1} \theta_{i,t-1} - \theta_{i,t}' P_{i,t} \theta_{i,t}$$

$$v_{i,t} = v_{i,t-1} + 1$$

Here  $X_{i,t}$  is the vector of right-hand variables and  $Y_{i,t}$  is the left-hand variable for model  $i$ . The posterior for date  $t$  becomes the prior for date  $t+1$ , and given the new data that becomes available at  $t+1$ , the same procedure can be used to compute  $P_{i,t+1}$ ,  $\theta_{i,t+1}$ ,  $s_{i,t+1}$  and  $v_{i,t+1}$ .

### *Model posteriors*

The approach used to compute the normalized model posteriors is the same as in Cogley and Sargent (2005). Given the assumptions on the distribution of the parameters, the posterior (6) can be obtained analytically, and  $w_{it}$  can be written recursively as:

$$\log w_{i,t+1} = \log w_{i,t} + \log p(Y_{i,t+1} | X_{i,t+1}, \theta_i, \sigma_i^2) - \log \frac{p(\theta_i, \sigma_i^2 | Z_i^{t+1})}{p(\theta_i, \sigma_i^2 | Z_i^t)}$$

where  $p(Y_{i,t+1} | X_{i,t+1}, \theta_i, \sigma_i^2)$  is the conditional likelihood for observation  $t+1$  and  $p(\theta_i, \sigma_i^2 | Z_i^t)$  is the posterior density computed using observations up to time  $t$ ,  $Z_i^t = (Y_i^t, X_i^t)$ . Analytical expressions for all these terms are available from Cogley and Sargent (2005). The recursion for  $w_{it}$  was started by setting  $w_{i,0} = \alpha_{i,0}$ , with the values of  $\alpha_{i,0}$  reported in the main text:  $\alpha_{SS-K,0} =$

$\alpha_{SS-C,0} = 0.98/2$ ;  $\alpha_{ST-K,0} = \alpha_{ST-C,0} = 0.01/2$ , and  $\alpha_{LS,0} = 0.01$ .

### Appendix 3

Given the Phillips curve equations (1) – (3) and the policy equation (11), each of the models included in the model space can be written in state-space representation as:

$$S_{i,t} = A_i S_{i,t-1} + B_i x_{t|t-1} + C_i \eta_t^i \quad (16)$$

For the state vectors,  $S_{SS-K,t} = S_{SS-C,t} = S_{SS,t}$  and  $S_{ST-K,t} = S_{ST-C,t} = S_{ST,t}$ . The state vectors are defined as:

$$\begin{aligned} S_{SS,t} &= [u_t \quad u_{t-1} \quad \pi_t \quad \pi_{t-1} \quad \pi_{t-2} \quad \pi_{t-3} \quad 1]' \\ S_{ST,t} &= [(u_t - u_t^*) \quad (u_{t-1} - u_{t-1}^*) \quad \pi_t \quad \pi_{t-1} \quad \pi_{t-2} \quad \pi_{t-3} \quad 1]' \\ S_{LS,t} &= [(u_t - u_t^*) \quad (u_{t-1} - u_{t-1}^*) \quad \pi_t \quad 1]' \end{aligned}$$

For the  $SS - K$ ,  $ST - K$  and  $LS$  models, the vectors and matrices in (16) are defined exactly as in Cogley and Sargent (2005). For the  $SS - C$  Phillips curve, the model is estimated as:

$$\begin{aligned} u_t &= \gamma_0^{SS-C} + \gamma_1^{SS-C} \pi_t + \gamma_2^{SS-C} u_{t-1} + \gamma_3^{SS-C} u_{t-2} + \\ &+ \gamma_4^{SS-C} \pi_{t-1} + \gamma_5^{SS-C} \pi_{t-2} + \gamma_6^{SS-C} \pi_{t-3} + \gamma_7^{SS-C} \pi_{t-4} + \eta_t^{SS-C} \end{aligned}$$

which has the state-space representation:

$$A_{SS-C}^0 S_{SS,t} = A_{SS-C}^1 S_{SS,t-1} + A_{SS-C}^2 x_{t|t-1} + \eta_{SS-C,t}$$

The vector of parameters  $A_{SS-C}^2$  and the vector of innovations  $\eta_{SS-C,t}$  are the same as in the  $SS - K$

model. On the other hand, the matrices  $A_{SS-C}^0$  and  $A_{SS-C}^1$  are defined as:

$$A_{SS-C}^0 = \begin{bmatrix} 1 & 0 & -\gamma_1^{SS-C} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{SS-C}^1 = \begin{bmatrix} \gamma_2^{SS-C} & \gamma_3^{SS-C} & \gamma_4^{SS-C} & \gamma_5^{SS-C} & \gamma_6^{SS-C} & \gamma_7^{SS-C} & \gamma_0^{SS-C} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The state space representation can then be rewritten in the form (16), in which:

$$A_{SS-C} = (A_{SS-C}^0)^{-1} A_{SS-C}^1$$

$$B_{SS-C} = (A_{SS-C}^0)^{-1} A_{SS-C}^2$$

$$C_{SS-C} = (A_{SS-C}^0)^{-1}$$

The  $ST - C$  Phillips curve is estimated as:

$$(u_t - u_t^*) = \gamma_0^{ST-C} + \gamma_1^{ST-C} \Delta\pi_t + \gamma_2^{ST-C} (u_{t-1} - u_{t-1}^*) + \gamma_3^{ST-C} (u_{t-2} - u_{t-2}^*) + \gamma_4^{ST-C} \Delta\pi_{t-1} + \gamma_5^{ST-C} \Delta\pi_{t-2} + \gamma_6^{ST-C} \Delta\pi_{t-3} + \eta_t^{ST-C} \quad (17)$$

The state-space representation is:

$$A_{ST-C}^0 S_{ST,t} = A_{ST-C}^1 S_{ST,t-1} + A_{ST-C}^2 x_{t|t-1} + \eta_{ST-C,t} \quad (18)$$

The vector of parameters  $A_{ST-C}^2$  and the vector of innovations  $\eta_{ST-C,t}$  are the same as in the  $ST-K$  model. On the other hand, the matrices  $A_{ST-C}^0$  and  $A_{ST-C}^1$  are defined as:

$$A_{ST-C}^0 = \begin{bmatrix} 1 & 0 & -\gamma_1^{ST-C} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{ST-C}^1 = \begin{bmatrix} \gamma_2^{ST-C} & \gamma_3^{ST-C} & (\gamma_4^{ST-C} - \gamma_1^{ST-C}) & (\gamma_5^{ST-C} - \gamma_4^{ST-C}) & (\gamma_6^{ST-C} - \gamma_5^{ST-C}) & -\gamma_6^{ST-C} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus:

$$A_{ST-C} = (A_{ST-C}^0)^{-1} A_{ST-C}^1$$

$$B_{ST-C} = (A_{ST-C}^0)^{-1} A_{ST-C}^2$$

$$C_{ST-C} = (A_{ST-C}^0)^{-1}$$

The loss function (12) can be written using (16) as:

$$\mathcal{L}_i = E_t \sum_{j=0}^{\infty} \beta^j \left( S'_{i,t+j} M'_i Q M_i S_{i,t+j} + x'_{t+j|t-1} R x_{t+j|t-1} \right) \quad (19)$$

where  $R$  is the cost of using the policy instrument, which is set equal to 0.001.<sup>25</sup> The matrix  $M_i$  selects  $u_t$  and  $\pi_t$  from the state vector and introduces the target values of the variables, while the matrix  $Q$  attaches the weight  $\lambda$  to inflation. More specifically:

$$M_{SS-K} = M_{SS-C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -u^* \\ 0 & 0 & 1 & 0 & 0 & 0 & -\pi^* \end{bmatrix}$$

$$M_{ST-K} = M_{ST-C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\pi^* \end{bmatrix}$$

$$M_{LS} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\pi^* \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}$$

The optimal policy rule for each specification of the Phillips curve is computed by minimizing (19) with respect to  $\{x_{t+j|t-1}\}_{j=0}^{\infty}$ . This is a standard optimal control problem, and the solution to this problem can be obtained using well known techniques. Under standard conditions, the solution will take the form:  $x_{t+j|t-1} = -F^i S_{i,t+j-1}$ , where  $F^i$  is a vector of constant policy parameters. As the estimated values of the parameters of each model will change over time based on the updating procedure described in Appendix 2, the vector  $F^i$  will be computed again in each period using the new set of parameters for each model.

The robust policy under model uncertainty is obtained by implementing the same approach as in Cogley and Sargent (2005). The policymaker is assumed to consider a composite ("encompassing") problem constructed from the models of Phillips curve included in the model space, each of them

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<sup>25</sup>It is standard to include this variable, mostly for computational reasons. Its inclusion does not affect the results of the optimization problem.

weighted based on its normalized posterior probability. The loss function for the composite problem is:

$$\mathcal{L}_E = E_t \sum_{j=0}^{\infty} \beta^j \left( S'_{E,t+j} Q_{E,t} S_{E,t+j} + x'_{t+j|t-1} R x_{t+j|t-1} \right) \quad (20)$$

The composite state space representation is

$$S_{E,t} = A_E S_{E,t-1} + B_E x_{t|t-1} + C_E \eta_{E,t} \quad (21)$$

with:

$$S_{E,t} = [S'_{SS-K,t} \ S'_{ST-K,t} \ S'_{SS-C,t} \ S'_{ST-C,t} \ S'_{LS,t}]'$$

and

$$\eta_{E,t} = [\eta_t^{SS-K} \ \eta_t^{ST-K} \ \eta_t^{SS-C} \ \eta_t^{ST-C} \ \eta_t^{LS}]'$$

The matrix  $A_E$  is a block diagonal matrix with blocks composed by the individual matrices  $A_i$ , and similarly  $C_E$  is a block diagonal matrix composed of the individual matrices  $C_i$ . The vector  $B_E$  is defined as:

$$B_E = \begin{bmatrix} B_{SS-K} \\ B_{ST-K} \\ B_{SS-C} \\ B_{ST-C} \\ B_{LS} \end{bmatrix} \quad (22)$$

Finally, the matrix  $Q_{E,t}$  is a block diagonal matrix composed of the elements  $\alpha_{i,t} M'_i Q M_i$ , where  $M_i$  and  $Q$  are as previously defined, and  $\alpha_{i,t}$  are the models' normalized posteriors computed from (7) and updated as described in Appendix 2. The posteriors  $\alpha_{i,t}$  and the matrices  $A_E$ ,  $B_E$  and  $C_E$  will change from period to period as the policymaker updates his estimates using the new data that becomes available.

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FIGURES

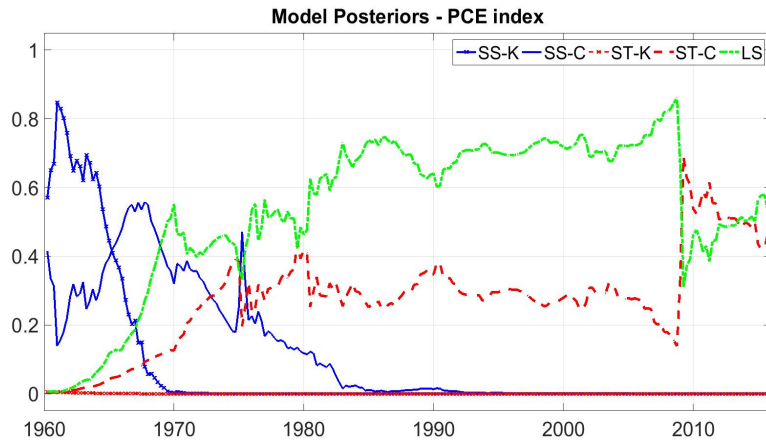


Figure 1 - Posterior probabilities over the period 1960 : *I* – 2016 : *IV* for the 5 model specifications of the Phillips curve included in the model space. The posterior probabilities are normalized so that they add up to one in each quarter.

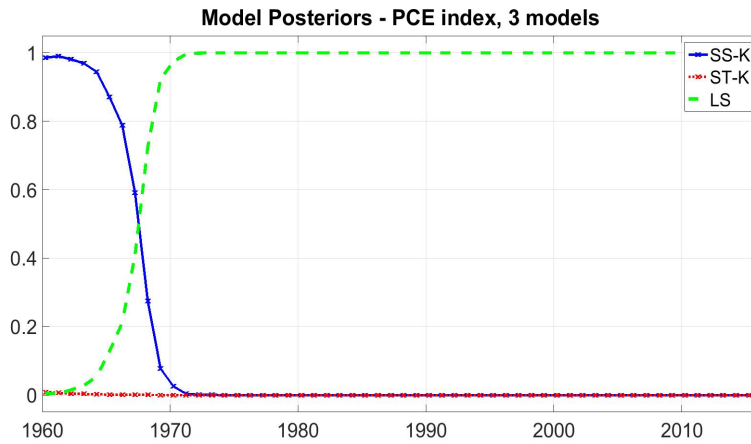


Figure 2 - Posterior probabilities over the period 1960 : *I* – 2016 : *IV* for the reduced model space ( $SS - K$ ,  $ST - K$ ,  $LS$ ). The posterior probabilities are normalized so that they add up to one in each quarter.

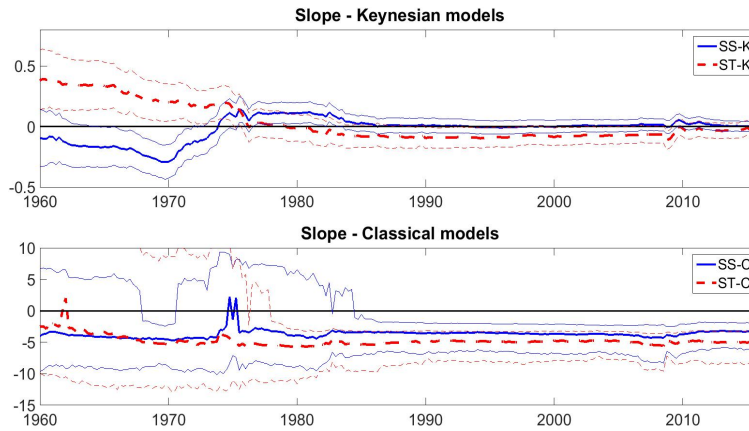


Figure 3 - Slope of the Phillips curve over the period 1960 : *I* – 2016 : *IV* for the 5 model specifications of the Phillips curve included in the model space. The slope is computed as the sum of the coefficients on unemployment in the Phillips curve equation.

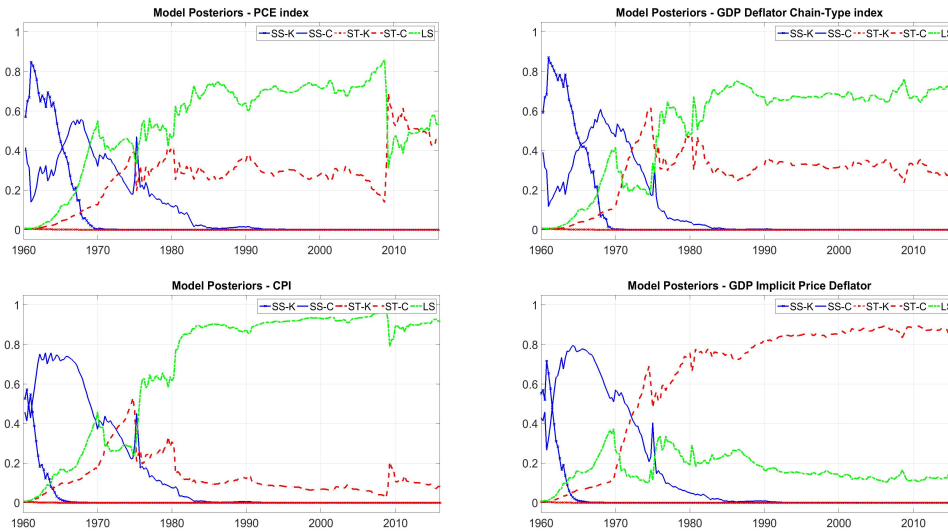


Figure 4 - Posterior probabilities over the period 1960 : *I* – 2016 : *IV* for the 5 specifications of the Phillips curve included in the model space. The posterior probabilities are normalized so that they add up to one in each quarter. The top-right panel is obtained using the GDP deflator chain-type index. The bottom-left panel is obtained using the CPI for all items. Finally, the bottom-right panel is obtained using the GDP implicit price deflator. For comparison, the top-left panel reports the posteriors using the PCE chain-type index, as in Figure 1.

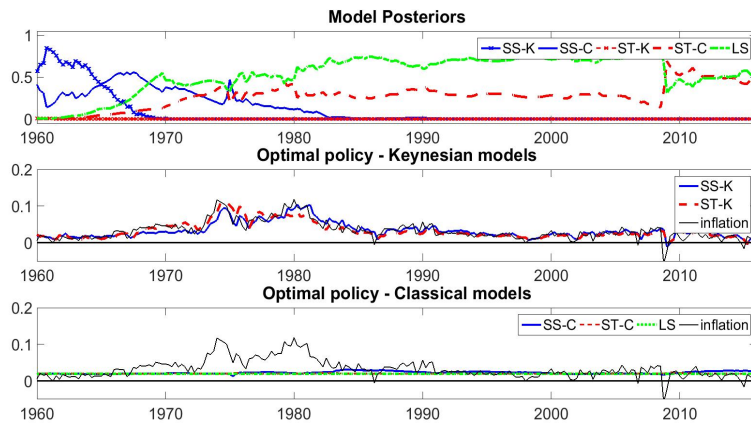


Figure 5 - The top panel reports the posterior probabilities already shown in Figure 1. The middle and bottom panels report the optimal policy recommendations for the Keynesian and Classical models respectively, obtained using the approach described in Appendix 3. The actual inflation rate during the same period is also shown for comparison.

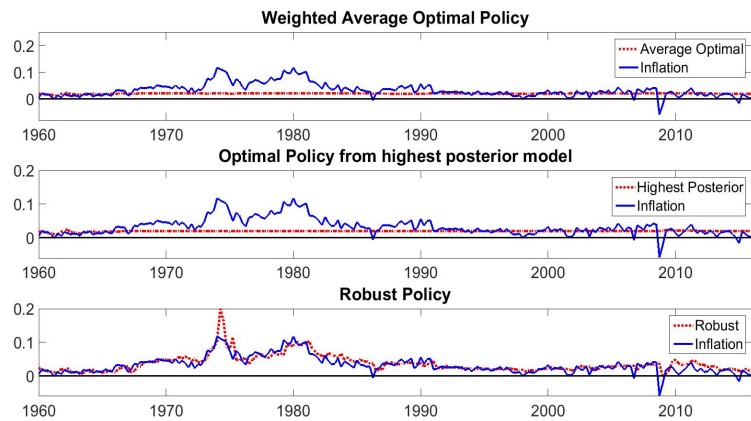


Figure 6 - The top panel shows the weighted average optimal policy computed as the sum of the optimal policies weighted by the models' posteriors. The middle panel reports the optimal policy for the model with the highest posterior in each quarter. The bottom panel reports the robust policy under model uncertainty obtained using the approach proposed by Cogley and Sargent (2005).

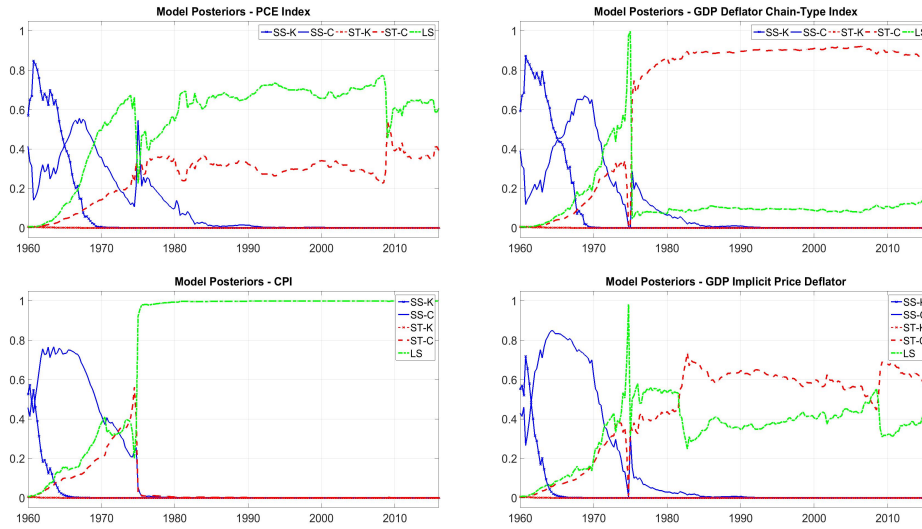


Figure 7 - Posterior probabilities over the period 1960 :  $I$  – 2016 :  $IV$  for the 5 model specifications of the Phillips curve included in the model space, under the alternative assumption  $E_{t-1}(\pi_t) = x_{t|t-1}^E$ . The posterior probabilities are normalized so that they add up to one in each quarter. The top-left panel is obtained using the PCE chain-type index. The top-right panel is obtained from the GDP deflator chain-type index. The bottom-left panel is obtained using the CPI. Finally, the bottom-right panel is obtained using the GDP implicit price deflator.

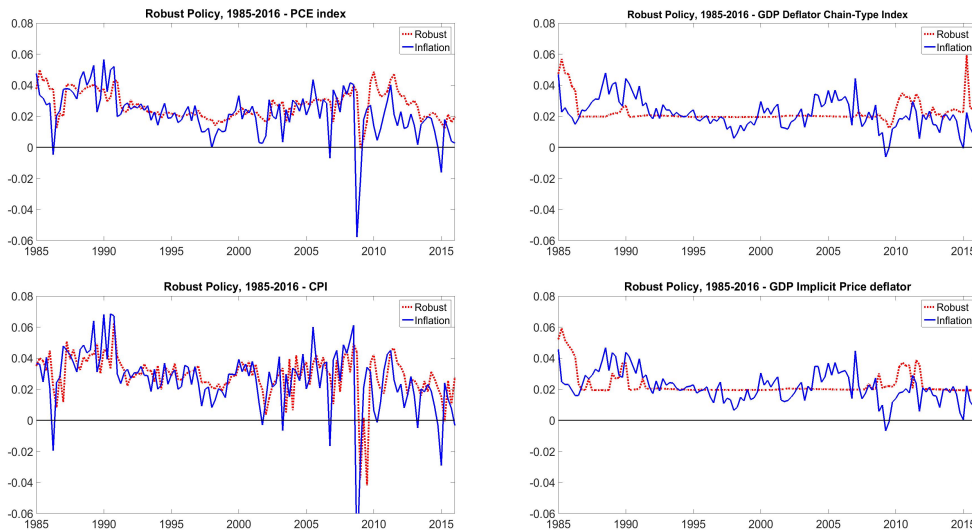


Figure 8 - Robust policy under model uncertainty during the period 1985 :  $I$  – 2016 :  $IV$ . The top-left panel is obtained using the PCE chain-type index. The top-right panel is obtained from the GDP deflator chain-type index. The bottom-left panel is obtained using the CPI. Finally, the bottom-right panel is obtained using the GDP implicit price deflator.