

Different Approaches to Model Cover-Cracking of RC Structures due to Corrosion

by

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To those who fight for education

Abstract

This thesis presents three different approaches to model corrosion-induced crack propagation in reinforced concrete structures. The first approach is solved numerically using finite differences to model the softening behaviour of concrete in tension. The second approach idealizes the concrete cover as either a brittle elastic or an elastoplastic material so that it may be solved using a closed-form solution. Both approaches are based on a thick-walled cylinder (TWC) analogy and consider rust compressibility and rust diffusion into cracks. The third approach uses finite element modelling to validate the application of the TWC and perform a parametric study. The results obtained using each approach are compared against each other as well as against experimental results. The TWC was found to be an appropriate analogy for the geometries and reinforcement configurations considered. Analytical models were found to provide upper and lower limits to the results based on the numerical model. The experimental data found in the literature showed reasonable agreement with predictions from the numerical and elastoplastic models.

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Nomenclature

α	Angle between principle tensile strain and the element's side (if none of the sides is perpendicular to the crack)
α_E	Aggregate type factor
Δr_b	Uniform radial deformation at the reinforcement and concrete interface (inner boundary of the thick-walled cylinder) imposed by the corrosion expansion
Δr_{bp}	Change in the rebar's radius due to an applied pressure
ΔV_p	Change in volume of the corrosion products due to an applied pressure
ΔV_s	Volume of consumed steel due to corrosion
ε_θ	Hoop strain
ε_{cr}	Crack strain (tensile strain corresponding to the tensile strength of concrete)
ε_r	Radial strain
ε_t	Tensile strain
ε_u	Ultimate strain of concrete in tension
ν_c	Poisson's ratio of concrete
ν_s	Poisson's ratio of steel
σ_θ	Hoop stress
σ_b	Radial compressive stress at the interface of the two cylinders
σ_c	Compressive stress
σ_r	Radial stress
B_r	Bulk modulus of the corrosion products

C	Cover depth
C_v	Crack volume coefficient
d_a	Maximum aggregate size
d_b	Rebar's diameter
E_θ	Secant moduli of concrete along the hoop direction
E_c	Young's modulus of concrete
E_r	Secant moduli of concrete along the radial direction
E_s	Young's modulus of steel
E_0	Initial Young's modulus of concrete
f'_c	Axial compressive strength of concrete
f'_t	Tensile strength of concrete
G_f	Fracture energy of concrete
h	Element's width perpendicular to the crack
i_{cor}	Current density
P	Pressure at the reinforcement/concrete interface due to expansion of corrosion products.
P_c	Inner pressures applied to the un-cracked cylinder
P_e	Outer pressures applied to the un-cracked cylinder
R_v	Rust expansion coefficient
r	Radial coordinate
r_b	Rebar's radius

r_c	Crack front
r_i	Inner radius of the cracked cylinder
r_r	Rust front, the radial coordinate of the cracked cylinder's inner boundary after deformation has taken place.
r_{rg}	Radial coordinate of the inner boundary of the cracked cylinder if the corrosion products were rigid.
r_{rb}	Radius of the corroded rebar
S	Centre to centre space between the steel reinforcements
S'	Clear space between the steel reinforcements
t	Time
t_p	Thickness of porous zone
u	Radial displacement at r
u_b	Radial deformation at the crack front
u_c	Total radial compressive deformation of the cracked cylinder
V_{cr}	volume of corrosion products within the cracks
V_r	Available space around the corroding rebar per unit length of rebar
V_{rr}	Expected volume of corrosion products if they could expand freely
V_t	The total available volume per length of rebar
V_{uct}	Uncompressed total volume of the corroded rebar and the rust around it (it does not consider the compressibility and does not include the volume of the corrosion products within the cracks).
w_c	Width of the crack band front (characteristic width of the advancing micro-crack zone)

w_{cr}	Total crack width at the reinforcement/concrete interface
x	Attack penetration
x_{cr}	Critical attack penetration
x_{vcr}	Visual critical attack penetration

Chapter 1

Introduction

1.1 Motivation

Reinforced concrete (RC) structures are typically an economical and practical building solution, which has resulted in concrete becoming the most used man-made material in the world. However, reinforcement corrosion in RC structures is a major and common problem nowadays especially since a lot of structures are aging (Broomfield 2007). According to Tilly (2007), reinforcing steel corrosion is the most common mode of degradation of RC structures, comprising 55% of deterioration cases in Europe. Since the presence of chloride ions boosts the corrosion rate (Glass et al. 1991), due to the excessive use of deicing salts, reinforcement corrosion is an even more important phenomenon in Canada. According to Environment Canada, approximately 6-9 million tons of de-icing salts were sold in Canada annually from 2005 to 2009 (Environment Canada 2012).

Corrosion can affect serviceability as well as the ultimate strength of a structure dramatically by reducing the (i) cross section and/or (ii) bond strength (by the cracking or spalling of the concrete cover) of the steel reinforcement. It can even lead to a catastrophic failure. As an example, the southern roof of the Berlin Congress Hall collapsed in May 1980 due to corrosion of the post-tensioned tendons. One person was killed and another severely injured (Figure 1.1) (Crane et al. 1983).

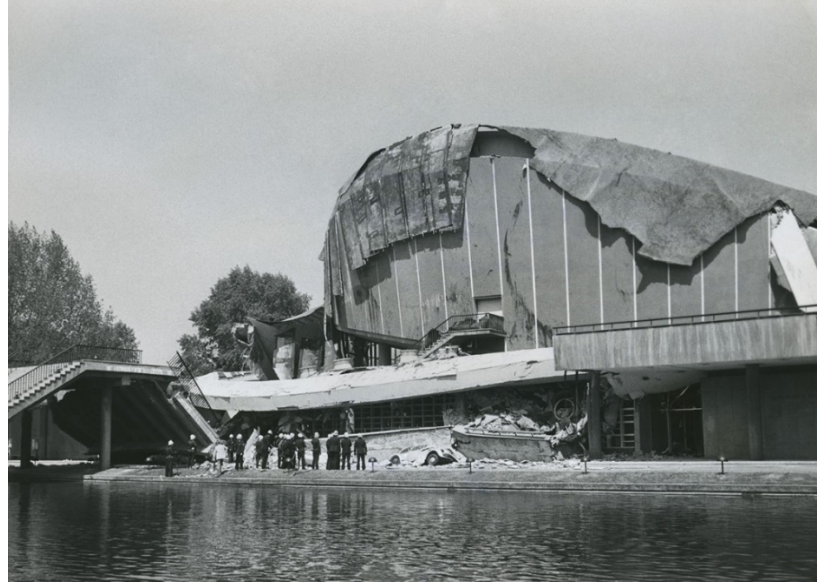


Figure 1.1: Collapsed Berlin Congress Hall (reproduced from Scherer, 2016)

Unfortunately, not only can structures deteriorate and eventually fail before reaching their intended service life in aggressive environments, but the rehabilitation of corroded structures is also not always effective. About 20% of rehabilitations in Europe failed in only 5 years, 55% failed in 10 years, and only 10% survived more than 25 years. The longest repair life of a rehabilitated structure was reported as 52 years (Tilly 2007).

Fortunately, the corrosion process in RC is generally slow and noticeable enough that fatal failures are preventable in most cases. Nonetheless, the cost of rebuilding or rehabilitating corroded structures is still a major concern (Broomfield 2007). The annual direct cost of corrosion-induced damage for highway bridges in the United States has been estimated around \$8.3 billion, while the indirect cost has been estimated to be around 10 times the direct cost (Koch et al. 2005).

One of the questions that an engineer needs to answer is whether or not the designed structure needs to be rehabilitated during its service life. If the answer to the question is positive, then the second question would be the cost of the rehabilitations during the service life.

Engineers can possibly eliminate or delay future rehabilitations due to reinforcement corrosion through design, such as increasing cover depth (Alonso et al. 1998; Liu and Weyers 1998; Vu et al. 2005), or using FRP or epoxy-coated reinforcement instead of

conventional steel reinforcement. While the long-term costs may be reduced, the initial cost of these solutions is generally higher. In order to select an optimal design, an engineer needs to estimate total rehabilitation costs by estimating the rehabilitation intervals which are a function of the rate of deterioration of the RC structure or element.

Corrosion is a very slow phenomenon which takes many years to develop in the field. Even artificially accelerated experimental tests often take months to be completed (Alonso et al. 1998; Andrade et al. 1993; Liu and Weyers 1998). Therefore, any model which is able to provide accurate predictions of time to the onset of corrosion-induced damage can introduce significant cost and time savings. Jamali et al. (2013) investigated some of the previously developed models to predict corrosion-induced cracking and found that they were only applicable for the experimental tests from which they were calibrated. Obviously, more efforts are needed to understand and model this phenomenon properly.

The main goal of this study is to introduce one or multiple practical models which can model the corrosion-induced cracking in RC members. The models must be able to provide results which are sufficiently close to experimental findings reported by different researchers.

1.2 Objective and Scope of the Study

Despite ongoing efforts to model the crack propagation in RC structures caused by corrosion of the embedded rebar, many of the modelling approaches used have not been quite successful in simulating the problem and providing the same results as the experimental data. There is a need for a comprehensive comparison of different modelling approaches and a detailed investigation of the effect and relative importance of different parameters on corrosion-induced cover cracking (particularly those which are typically ignored). The validity of commonly accepted simplifying assumptions, such as the use of the thick-walled cylinder analogy, also requires critical examination. This thesis aims to address these issues through a thorough and multi-faceted study of the cover cracking problem in RC structures subjected to corrosion-induced deterioration. The main objectives of this thesis are summarized as follows:

1. Evaluating the state-of-the-art in the literature and identifying current gaps with respect to corrosion-induced cover cracking in RC structures.
2. Identifying the influential parameters and investigating their effects. In particular, this study investigates the importance of several parameters which are usually neglected by other researchers (e.g., compressibility of corrosion products and diffusion of corrosion products within the cracks).
3. Investigating different modelling approaches (i.e., analytical, numerical, FEM) to model corrosion-induced cracking in RC structures, and comparing their results against each other as well as experimental data provided by a broad range of researchers.
4. Validating the use of the thick-walled cylinder analogy for modelling this problem. Although it has been used by several researchers, to the author's knowledge no comprehensive studies are available to demonstrate its validity.

1.3 Organization of the Thesis

This study is divided into 7 chapters, including this one:

- Chapter 2 discusses the state-of-the-art in the literature. The first part of the chapter provides some background to the corrosion phenomenon and process in RC structures. The second part investigates the effect of influential parameters on reported experimental tests as well as input parameters on previous models. If applicable, it explores the reasonable range of values for those parameters. The third part of the chapter provides a summary of the modeling approaches found in the literature.
- In Chapter 3, a program was developed using MATLAB to solve corrosion-induced crack propagation numerically using finite differences.
- Chapter 4 discusses two analytical approaches which provide an upper and a lower limit to the numerical approach presented in Chapter 3. Both analytical models can be solved using a closed-form solution.

- In Chapter 5, The results obtained from the numerical and analytical models discussed in Chapters 3 and 4, respectively, are compared to experimentally measured values found in the literature.
- In Chapter 6, finite element analysis is used to simulate the problem. The results have been compared against the analytical and numerical results presented in Chapter 5. This chapter also investigates the influence of some other parameters which were not discussed in the previous chapters. The aim is to verify some of the simplifying assumptions used in the previous chapters.
- Finally, Chapter 7 summarizes the main results and findings of the study. It also provides some suggestions for future work.

Chapter 2

Literature Review

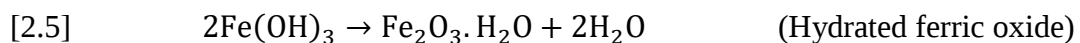
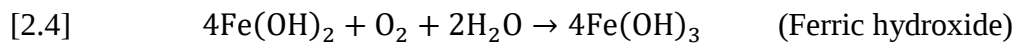
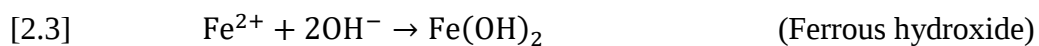
2.1 Background

2.1.1 Corrosion Phenomenon

It is a well-known fact that corrosion of steel reinforcement in concrete is an electrochemical process. It consists of an anodic reaction (Equation 2.1) and a cathodic reaction (Equation 2.2) (Broomfield 2007):



The two electrons released through the anodic reaction are consumed in the cathodic reaction in the presence of water and oxygen. However, Fe^{2+} is soluble and further oxidation is required to produce stable oxide products, such as ferrous hydroxide, ferric hydroxide, hydrated ferric oxide, etc. (Broomfield 2007) (Equations 2.3-2.5). Each of these products has a different relative volume to consumed iron (Caré et al. 2008) (Figure 2.1).



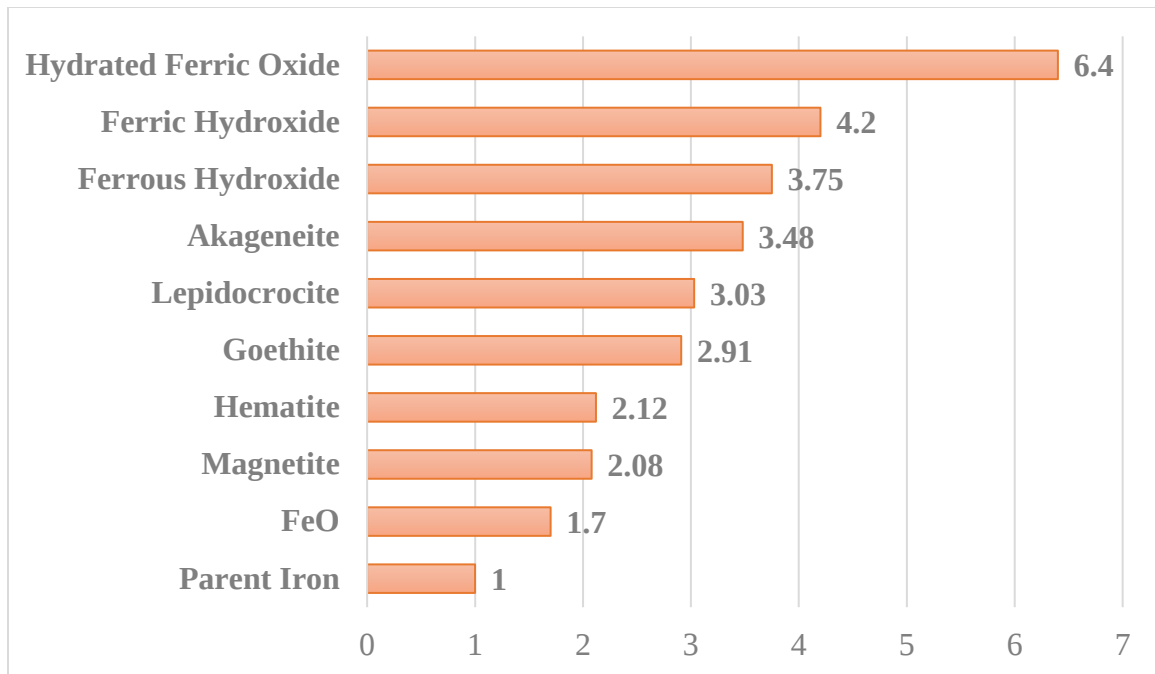


Figure 2.1: Relative volume of corrosion products to parent iron

Different steel bars, or different regions of a single rebar, may act as anodes, cathodes and electrical conductors, and the concrete pore solution, which contains dissolved salts, acts as an electrolyte (Figure 2.2).

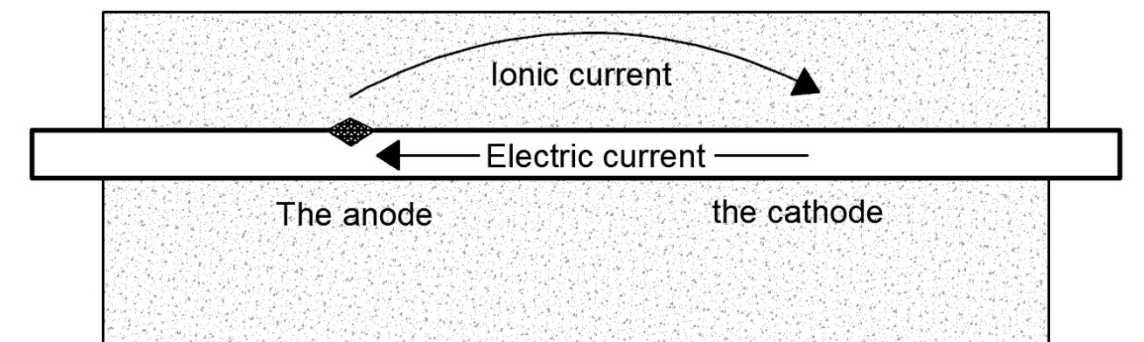


Figure 2.2: Corrosion diagram (adapted from Broomfield, 2007)

2.1.2 Corrosion Process and Service Life in RC Structures

Reinforcement corrosion can affect the serviceability as well as the ultimate strength of an RC structure, and it can eventually lead to failure in a structure with respect to one or more limit states. Since structural failure can cause both bodily injury and property damage, engineers need to define an end time for a structure's life.

The service life span of a corroded RC structure is typically divided into two stages (phases) using the conceptual model presented by Tuutti (1981), as illustrated in Figure 2.3. The two stages are an initial stage and a propagation stage. The length of the first stage is defined as the time required for the corrosion process to initiate. The second stage is the period after the initiation in which corrosion-induced damage accumulates up to a predefined end of service life.

In general, the whole life span of a structure, from the time it is constructed to the time it fails, and affected by reinforcement corrosion, is divided into several phases (assuming there is no rehabilitation): (i) start of service life, (ii) initiation of corrosion, (iii) creation of the first crack at the reinforcement and concrete cover interface, (iv) propagation of the first crack to either the concrete surface or adjacent rebar, (v) crack widening to a predefined width, (vi) spalling or delamination of the concrete cover, and (vii) significant loss of the reinforcement cross section, which usually corresponds to the ultimate limit state of the structure (Figure 2.3). However, significant loss of the reinforcement cross section can happen at different times depending on the corrosion type and the definition of significant loss (Angst et al. 2012) (Section 2.1.2.7). With the exception of the first milestone, which is the start point of the lifespan, each of the other milestones can theoretically be defined as the end of service life; however, considering each of them as the end of service life has some benefits and drawbacks. The potential milestones are further explained in the next subsections.

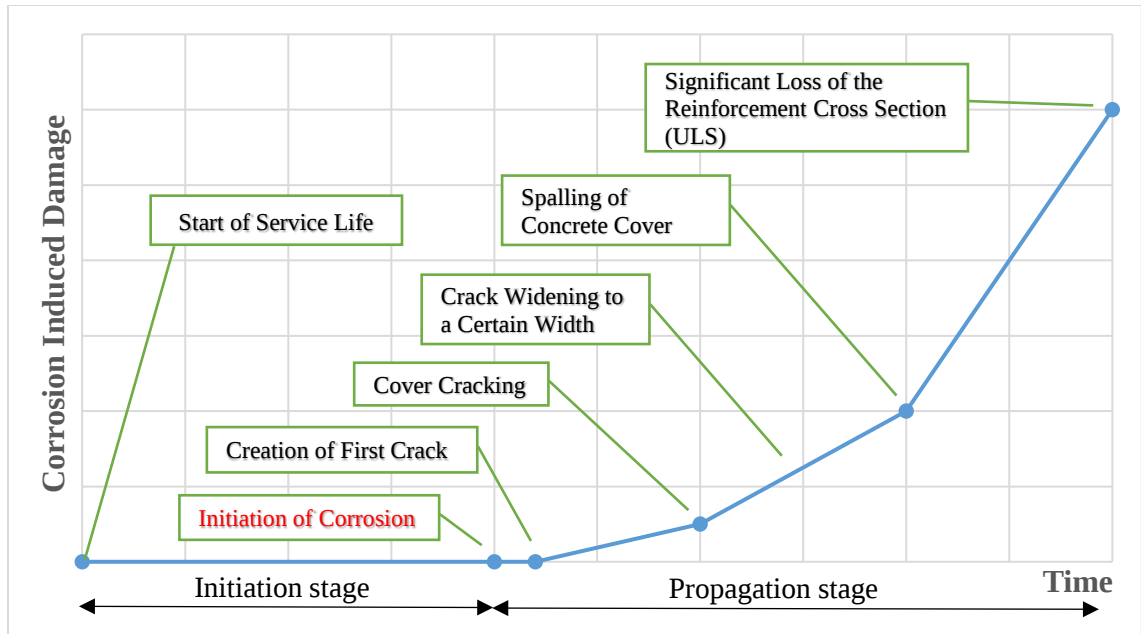


Figure 2.3: Corrosion process and service life

2.1.2.1 Start of Service Life

Although steel reinforcement can be exposed to a corrosive environment even before it is embedded in concrete, the effects are negligible as the rust products formed do not exert any pressure on the concrete, which could lead to cover cracking in the structure. Therefore, it is legitimate to define the start of service life as the moment the steel is embedded in the concrete.

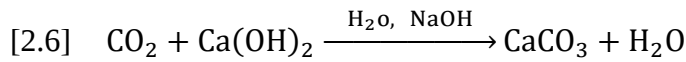
2.1.2.2 Initiation of Corrosion

The hydration of Portland cement increases the pH level of concrete to around 13, mostly by producing $\text{Ca}(\text{OH})_2$. Consequently, a very thin (few nanometers thick) oxide film (passive film) is formed on the steel reinforcement, which protects it from further corrosion. This process is known as passivation. The protective passive film can be destroyed mainly by one of two ways, namely carbonation of the concrete cover or the presence of chloride ions in sufficient quantities (Bertolini et al. 2013). However, it is not

limited to those. As an example, contact with acids may also damage passivation and cause corrosion initiation.

Carbonation

The reaction of carbon dioxide present in the atmosphere with the alkaline hydrated cement paste is called carbonation. It causes the pH of the concrete pore solution to drop from its initial value of around 13 to approximately 9 (Neville 2011). The most dominant carbonation reaction is between CO_2 and Ca(OH)_2 as given in Equation 2.6 (Bertolini et al. 2013):



Carbonation begins at the exterior surface of the concrete and propagates inward over time, known as a “carbonation front.” The depth of carbonation can be estimated conservatively by Equation 2.7 (Bertolini et al. 2013):

$$[2.7] \quad d = K\sqrt{t}$$

where “d” is the depth of the carbonation front (mm), “t” is time (years), and “K” depends on the environment to which the concrete is exposed (i.e., humidity, temperature, CO_2 concentration, etc.) as well as concrete quality (i.e., permeability). The value of “K” ranges from 2 to 15 for real structures exposed to the atmosphere but protected from rain, $2 < K < 6$ for low porosity concrete, $6 < K < 9$ for medium porosity concrete, and $K > 9$ for highly porous concrete.

The carbonation rate (Equation 2.8) becomes negligible as “t” increases, making carbonation the non-dominant type of corrosion-inducing mechanisms when the cover depth is large enough.

$$[2.8] \quad \dot{d} = \frac{d}{dt} [d] = \frac{K}{2\sqrt{t}}$$

Carbonation is a very slow process in high-quality un-cracked concrete. For instance, the depth of carbonation would be about 10 mm after 20 years for a 35 MPa concrete (Collins and Mitchell 1997).

When the carbonation front reaches the steel rebar, it depassivates the oxide film allowing the corrosion process to initiate in the presence of water and oxygen. In this case, both the anodic and the cathodic reactions will occur homogeneously over the steel surface. Carbonation usually leads to corrosion being uniformly distributed around the perimeter and along the length of reinforcing bars (Figure 2.4 and Figure 2.5).



Figure 2.4: Uniform corrosion (reproduced from Angst et al., 2012), (ruler's unit: cm)

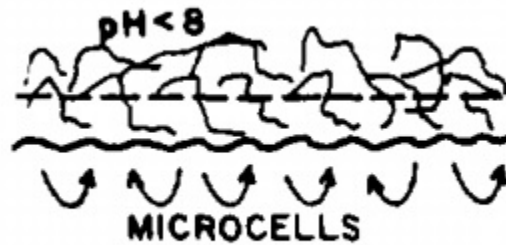


Figure 2.5: Uniform corrosion (reproduced from Andrade and Alonso, 1996)

Presence of Chloride Ions

When a sufficient quantity of chloride ions reach the surface of the steel reinforcement, the passive film protecting the rebar is destroyed and corrosion is initiated (Jamali et al. 2013).

Furthermore, the risk of chloride-induced corrosion is higher where there are pre-existing cracks (e.g., due to loading, shrinkage).

Chloride-induced corrosion breaks the oxide film locally. The unprotected areas act as anodes, and the rest of the rebar acts as a cathode. Since the anodic and the cathodic reactions are separated in this type of corrosion, the products of the corrosion process are usually formed locally. Consequently, chloride-induced corrosion mostly develops as a localized corrosion (Bertolini et al. 2013; Jamali et al. 2013) (Figure 2.6 and Figure 2.7).

Chlorides can be introduced from different sources. Some chlorides exist in the concrete from the beginning, which can be as high as 200,000 ppm where calcium chloride accelerating admixtures have been used. All concrete mix ingredients contain a certain amount of chlorides: cement 50 to 100 ppm, potable water less than 250 ppm, aggregates 10 to 400 ppm, non-chloride water-reducing admixtures 100 to 800 ppm (Gaynor 1985). However, only the soluble portion of chlorides (1/2 to 3/4) contributes to corrosion. The other portion is chemically bound to the hydrating cement (Gaynor 1985).

CSA 23.1/CSA 23.2 (2014) limits the amount of water-soluble chloride ions by mass of the cementing material in the concrete before exposure to 0.06%, 0.15%, and 1.0% for pre-stressed concrete, RC exposed to a moist environment and/or chlorides, and RC exposed to neither moist environment nor chlorides, respectively.

External chlorides (e.g., salt water, deicing salts) may also penetrate the concrete substrate. These chlorides are generally more critical to inducing corrosion. Therefore, the quality and water/cement ratio of the mixture are important as they relate to the porosity and permeability of concrete. The diffusion of external chlorides can be explained by Fick's second law of diffusion (Equation 2.9) (Crank 1975).

$$[2.9] \quad \frac{\partial \phi}{\partial t} = D \frac{\partial^2 \phi}{\partial x^2}$$

where ϕ is the concentration (mol/m³), t is time (s), D is the diffusion coefficient (m²/s), and x is the distance to the concrete surface (m).



Figure 2.6: Localized corrosion (reproduced from Angst et al., 2012), (ruler unit: cm)

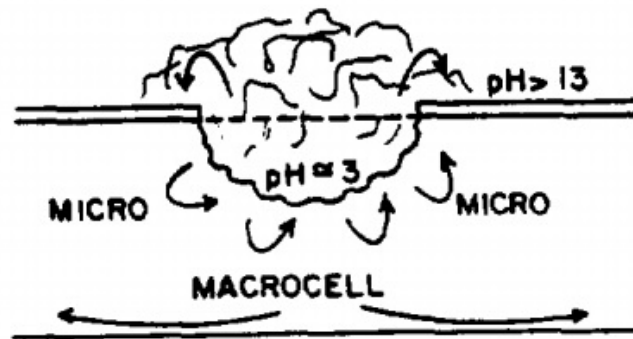


Figure 2.7: Localized corrosion (reproduced from Andrade and Alonso, 1996)

Most of the models reported in the literature present a uniform corrosion assumption. This is due to the fact that uniform corrosion is easier to model (Balafas and Burgoyne 2011; Bazant 1979; Bhargava et al. 2006; Li et al. 2006; Molina et al. 1993; Pantazopoulou and Papoulia 2001; Val and Chernin 2012; Wang and Liu 2004; Y. Liu and R. E. Weyers 1998).

2.1.2.3 Creation of First Crack

When corrosion starts, the formation of corrosion products around the corroded reinforcement generates an internal pressure. This is because corrosion products occupy more volume than the original iron (see Figure 2.1). The internal expansion causes tensile hoop strains in the concrete cover, which are maximum at the concrete-reinforcement interface (Bazant 1979). When the maximum tensile stress reaches the tensile strength of concrete, the concrete cover cracks (Figure 2.8).

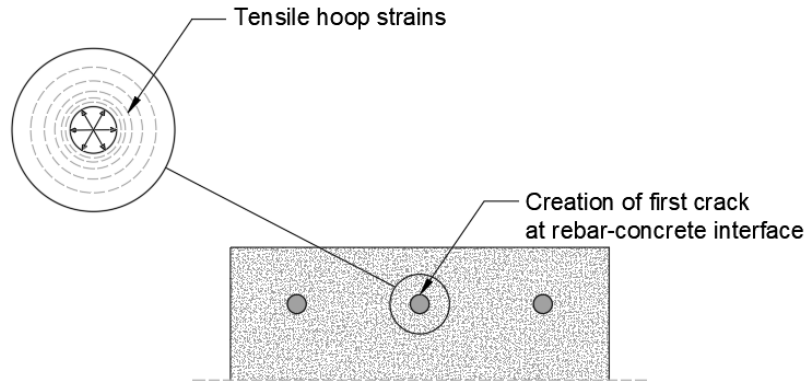


Figure 2.8: Creation of first crack

2.1.2.4 Appearance of First Crack

As corrosion continues, more cracks will form, and they will propagate either through the concrete cover or to an adjacent reinforcing bar, depending on the bar spacing (S) and cover-to-rebar diameter (C/d_b) ratio. Although there is usually more than one crack, one of the cracks generally propagates as the main crack (Andrade et al. 1993; Merino 2014) (Figure 2.9, Figure 2.10).

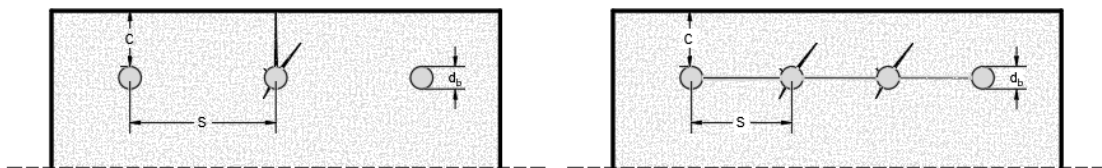


Figure 2.9: Crack propagation

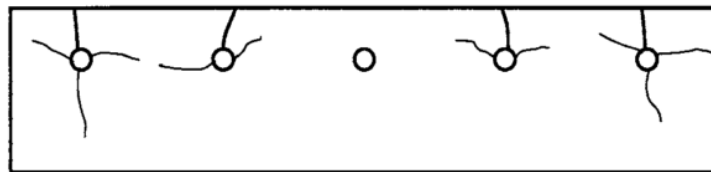


Figure 2.10: Typical cracking pattern caused by reinforcement corrosion (reproduced from Vu et al., 2005)

The attack penetration (x) is defined as the radius of the original rebar minus the radius of the corroded rebar (Figure 2.11), which is used as an indication of the steel mass loss and the extent of the corrosion. The critical attack penetration (x_{cr}) is defined as the attack penetration corresponding to the first crack appearance.

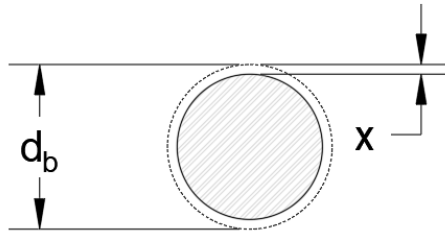


Figure 2.11: Attack penetration

2.1.2.5 Crack Widening to a Certain Width

If a steel rebar embedded in concrete continues to be corroded after the first crack has reached the surface, the crack width will increase over time (Figure 2.12) (Alonso et al. 1998; Andrade et al. 1993; Vu et al. 2005).

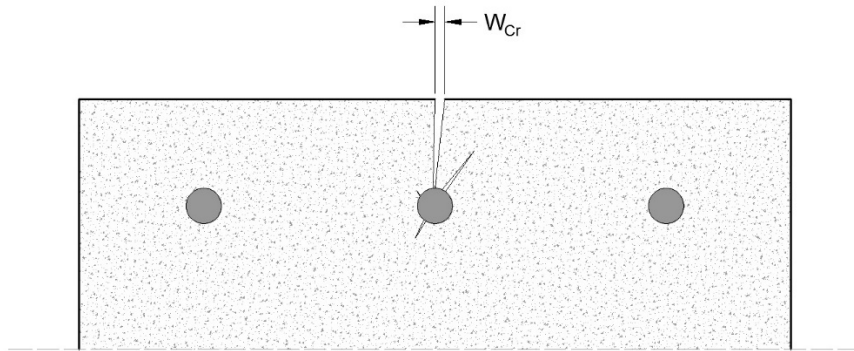


Figure 2.12: Crack widening

There are several reasons that make the accurate prediction of the effects of the corrosion process more difficult once the crack has reached the concrete surface. Some of the reasons are as follows:

- (i) The presence of a crack or cracks usually increases the corrosion rate, since it increases the permeability of the concrete (discussed in detail in Section 2.2.1.1).
- (ii) Although the influence of the corrosion rate is insignificant with respect to the attack penetration corresponding to cover cracking or first visual crack, this is no longer the case after the appearance of the first crack. Alonso et al. (1998) found that after cracking, the relationship between crack width and attack penetration is dependent on the corrosion rate. This means that only a model which considers this effect is able to properly and accurately predict the corrosion process after the first crack reaches the surface.
- (iii) More corrosion products will migrate to the outside of the concrete cover and its quantity is very unpredictable.

The end of the service life can be defined when a certain value of crack width has been reached.

2.1.2.6 Spalling/Delamination of Concrete Cover

When multiple cracks have formed and are connected to each other, a portion of the cover will be disconnected from the rest of the RC member (Figure 2.13-2.15). Consequently, the disconnected concrete will spall off and directly expose the reinforcement to the corrosive environment. Since there is no more cover to protect the reinforcement, the corrosion rate increases. It also means that there is less concrete around the reinforcement, which decreases the bond between the reinforcement and the concrete cover dramatically.

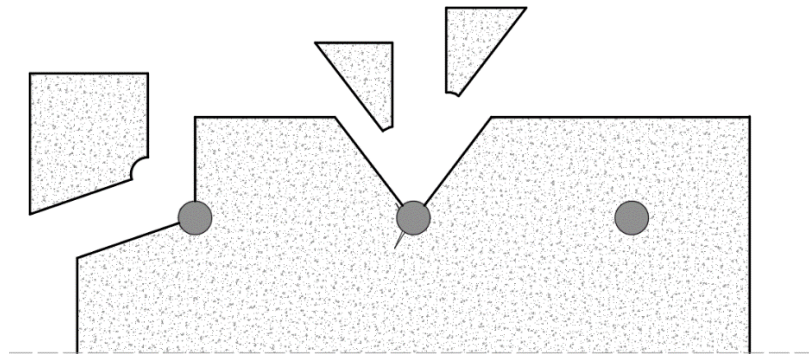


Figure 2.13: Cover spalling



Figure 2.14: Cover delamination on cap beam of Gardiner Expressway - Toronto, ON



Figure 2.15: Cover delamination on pier of Gardiner Expressway - Toronto, ON

2.1.2.7 Significant Loss of the Reinforcement Cross Section

Significant loss of the reinforcement cross section means that the reinforcement has lost a great portion of its strength. In most cases, it happens after spalling has occurred, so the

reinforcement has lost its bond to concrete as well. Therefore, the structure has lost a large portion of its capacity by this time and may be unsafe.

Although the concrete cover usually becomes cracked before the reinforcement loses a significant portion of its cross-section in the case of uniform corrosion, it might not be the case for localized corrosion. A reinforcing bar could lose a considerable amount of its cross section before the corrosion symptoms (e.g., cracks, rust stains) become visible from the outside (Angst et al. 2012). Experimental studies have shown that the maximum penetration depth in the case of localized corrosion is about 4 to 8 times that of an equivalent uniform corrosion in terms of total mass lost (González et al. 1995).

In general, later milestones are harder to predict due to (i) cumulative errors and (ii) increasing number of influential parameters. In addition, later milestones mean losing more of the structure's capacity due to (i) loss of cross section and (ii) loss of bond. On the other hand, earlier milestones may occur in a relatively short period of time (Figure 2.3). Consequently, it may be very expensive to define the end of the service life at the early stages.

2.2 Parameters Influencing Reinforcement Corrosion

There are several parameters which may influence the rate of the corrosion process as well as its consequences. These parameters can be divided into four general categories as described in the following subsections.

2.2.1 Corrosion Rate and Localization

2.2.1.1 Corrosion Rate (i_{cor})

The electrical resistivity of concrete and the availability of oxygen usually control the corrosion rate. Consequently, the corrosion rate is related to environmental conditions (e.g., oxygen availability, moisture, temperature, alkalinity and amount of chlorides in the

environment), concrete quality (e.g., porosity and permeability, cracks, chlorides in concrete components), and dimensions such as cover depth. Therefore, the corrosion rate is not the same in different structures exposed to different environments, nor is it constant throughout the service life or at different regions of the same structure (Jamali et al. 2013). The appearance of a crack or cracks makes the concrete more permeable to chlorides. Consequently, the corrosion rate for both chloride and carbonation-induced corrosion may increase in cracked regions where chloride ions penetrate from external sources. Glass et al. (1991) found that 1% chloride contamination increased the corrosion rate due to carbonation by approximately 10 times in a 40% relative humidity environment (Figure 2.16). This phenomenon can be explained by the hygroscopic nature and the ionic conductivity of the chlorides.

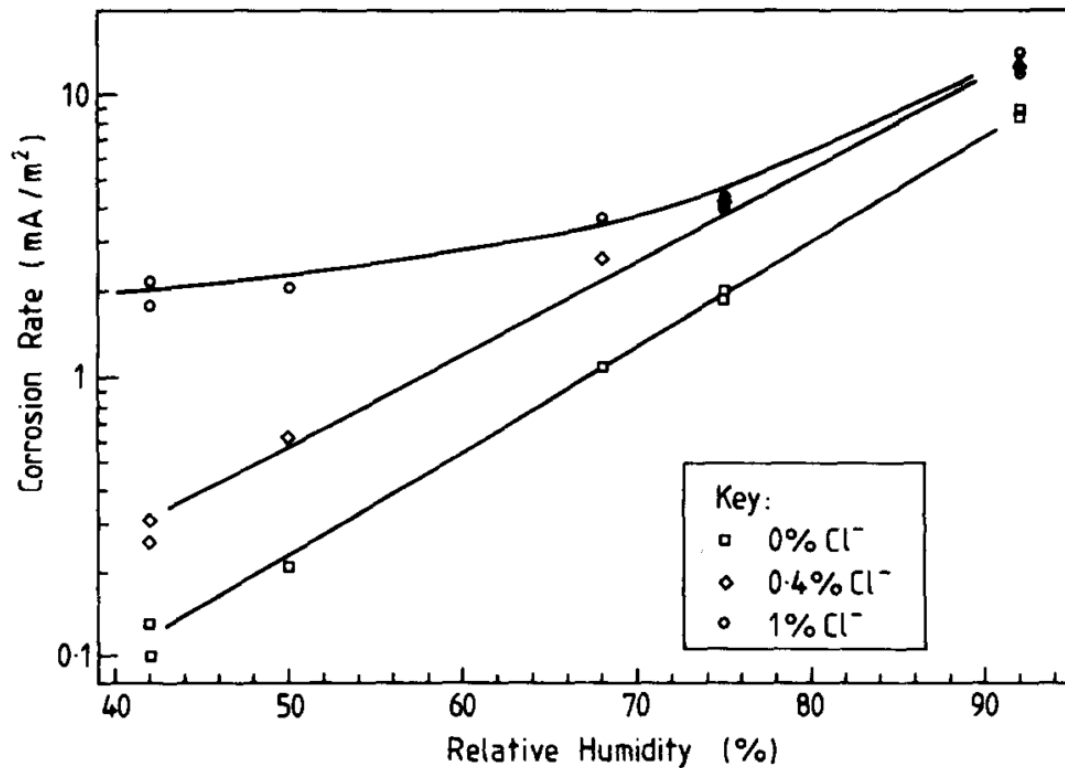


Figure 2.16: Effect of chloride ions on corrosion rate due to carbonation (reproduced from Glass et al., 1991)

The appearance of cracks has a negligible effect on corrosion rate in cases where the chloride ion penetration from external sources does not occur, regardless of whether the corrosion process is dominated by carbonation or the presence of chlorides. In both cases, the crack increases oxygen availability and decreases the wetness period (if rain or other periodic external water sources are responsible for the wetness), which each offset the effect of the other (Andrade et al. 1993).

Although researchers generally agree that the electrical current inside the steel rebar is representative of the corrosion rate, there is no consensus on the precise relationship between the current and the rate of the corroding rebar mass loss. The most common way to relate electric current to attack penetration is through Faraday's law, which is a linear relation (Equation 2.10):

$$[2.10] \quad \frac{dm_s}{dt} = \frac{M}{zF} \times I_{cor}$$

where m_s is the mass loss (g), M is the metal molar mass of iron (55.85 g/mol), F is Faraday's constant ($F=96,485$ C/mol), z is the valence of the ion formed as a result of ion oxidation, which is taken equal to 2 (assuming the corrosion product is $Fe(OH)_2$, $Fe \rightarrow Fe^{2+} + 2e^-$), and I_{cor} is the corrosion current (A).

Substituting these values in Equation 2.10 gives us Equation 2.11:

$$[2.11] \quad \frac{dm_s}{dt} = \frac{55.85 \frac{g}{mol}}{2 \times 96485 \frac{C}{mol}} I_{cor} = 2.894 \times 10^{-4} I_{cor}$$

The corrosion current density is defined as Equation 2.12:

$$[2.12] \quad i_{cor} = I_{cor} / (\pi d_b L_b)$$

where d_b and L_b are respectively the diameter and length of the steel reinforcement in meters.

Substituting Equation 2.11 into Equation 2.12 gives the steel mass consumption rate for a unit length (Equation 2.13):

$$[2.13] \quad \frac{dm_s}{dt} = 2.894 \times 10^{-7} \pi d_b i_{cor}$$

Consequently, the consumed mass of steel rebar per unit length of rebar would be:

$$[2.14] \quad m_s = 2.894 \times 10^{-7} \pi d_b i_{cor} t$$

where d_b is in meters, i_{cor} is in A/m^2 , m_s is in kg/m , and t is in seconds (since the onset of corrosion).

The attack penetration ($x(t)$) of the steel rebar per unit length can be calculated as Equation 2.15:

$$[2.15] \quad x(t) = \frac{2.894 \times 10^{-7} i_{cor} t}{\rho_s} = \frac{2.894 \times 10^{-7}}{7.85 \times 10^3} i_{cor} t = 3.687 \times 10^{-11} i_{cor} t$$

where ρ_s is the density of iron, which is equal to 7.85 g/cm^3 .

Converting the unit of time (t) to years, the corrosion current density (i_{cor}) to $\mu A/cm^2$, and the attack penetration $x(t)$ to mm gives Equation 2.16:

$$[2.16] \quad x(t) = 0.0116 i_{cor} t$$

Unlike Faraday's law, which assumes a constant rate of attack penetration over time for a constant corrosion current density, Liu and Weyers (1998) introduced another model which defines a non-linear relationship between corrosion current density and attack penetration. The authors state that the rate of corrosion decreases as corrosion products accumulate around the rebar, due to the fact that the iron ions have to diffuse through the layer of corrosion products to further oxidize. Later, Balafas and Burgoyne (2010) modified the

Liu and Weyers model by combining it with Faraday's law, since they believed Weyers' model gives unrealistic high corrosion rates at the beginning of the corrosion process.

Alonso et al. (1998) measured the actual gravimetric loss at the end of experimental tests and compared them to the nominal values calculated by Faraday's law. The authors found that the experimental values were higher than the predicted values in 18 out of 21 cases, and explained the discrepancy of the results with two reasons: (i) additional corrosion due to acidification developed as a result of the corrosion itself, and (ii) some of the un-corroded iron is spalled out due to the oxidization of the surrounding iron. Merino (2014) also found the experimental gravimetric loss to be greater than Faraday's law estimations. The models presented by Balafas and Weyers would estimate an even lower nominal iron loss than Faraday's law; since their models tended to underestimate the cracking time, they tried to increase the predicted time by decreasing the corrosion rate. Hence, Faraday's law probably provides the most realistic results out of the three approaches when it comes to converting time to mass loss (or attack penetration).

Although tests by Melchers and Jeffrey (2005) have shown a non-linear relation between time and attack penetration, it does not necessarily justify a non-linear relation between corrosion current density (i_{cor}) and attack penetration ($x(t)$). The non-linearity could be also justified by a non-linear corrosion current density (i_{cor}) itself. Even the reason Liu and Weyers (1998) use to justify a non-linear relation between corrosion current density and corrosion attack penetration (that the iron ions have a longer distance to diffuse through) justifies a decrease in corrosion current density instead of the non-linear relation between corrosion current density and attack penetration. The current density is usually constant in the experimental sets since it is applied artificially.

Figure 2.17 shows the typical ranges of corrosion rate in different environmental situations (Bertolini et al. 2013). The corrosion rate in a real situation might range from 0.01 to $100 \frac{\mu A}{cm^2}$, but it is usually less than $10 \frac{\mu A}{cm^2}$. Andrade and Alonso (1996) measured on-site corrosion rates between 0.1 to $10 \frac{\mu A}{cm^2}$; however, the corrosion rate measured in the field

mostly fell below $1 \frac{\mu A}{cm^2}$, and the environments causing corrosion rates higher than $1 \frac{\mu A}{cm^2}$ are considered highly corrosive.

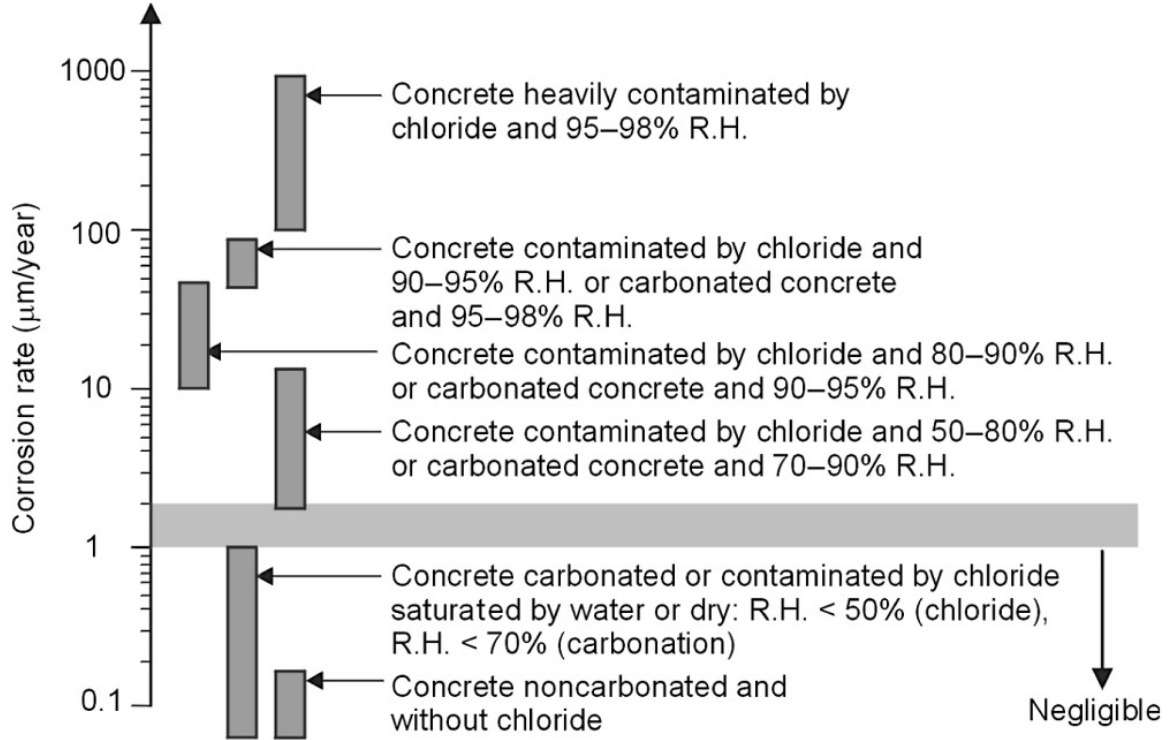


Figure 2.17: Typical corrosion rate in different environments (reproduced from Bertolini et al., 2013)

Corrosion rate in $\left(\frac{\mu m}{year}\right)$ can be converted to $\left(\frac{\mu A}{cm^2}\right)$ by Faraday's law as follows:

$$\begin{aligned} \text{Corrosion rate} \left(\frac{mm}{year}\right) &= 0.0116 \times i_{cor} \left(\frac{\mu A}{cm^2}\right) \times 1(year) \\ \rightarrow i_{cor} \left(\frac{\mu A}{cm^2}\right) &= \frac{\text{Corrosion rate} \left(\frac{mm}{year}\right) \times \frac{1}{1000} \left(\frac{\mu m}{mm}\right)}{0.0116} \\ [2.17] \quad \rightarrow i_{cor} \left(\frac{\mu A}{cm^2}\right) &\cong \frac{\text{Corrosion rate} \left(\frac{\mu m}{year}\right)}{10} \end{aligned}$$

Researchers generally apply higher current rates in lab experiments than those found in the field to decrease the duration of experimental tests. However, since the current rate on-site is much lower than what researchers apply in experimental conditions, there is a concern

that tests might not represent the real conditions properly. Table 2.1 shows the artificial electrical currents that have been used in some of the experimental tests reported in the literature:

Table 2.1: Applied electrical rates in the experimental tests

No.	Experimental Test	Current Rate ($\mu\text{A}/\text{cm}^2$)
1	Andrade et al. (1993)	10, 100
2	Al-Saadoun and Al-Gahtani (1992)	3000, 10000, 15000, 20000
3	Alonso et al. (1998)	3, 10, 100
4	Vu et al. (2005)	100
5	Torres-Acosta and Sagues (2004)	100
6	Al-Sulaimani et al. (1990)	2000

A higher corrosion rate means that less time is needed for a certain amount of iron to be corroded. Moreover, Alonso et al. (1998) found that the corrosion rate also has a significant influence on the amount of corrosion products (or attack penetration) which is needed to cause a certain crack width. Their tests showed that if a higher current is applied to steel reinforcement, a higher attack penetration is also needed to get the same crack width. For example, it was found for a concrete of 20-mm cover that about three times greater attack penetration is needed when $100 \mu\text{A}/\text{cm}^2$ is applied instead of $10 \mu\text{A}/\text{cm}^2$ to cause the same crack width of 0.5 mm. This effect has not been considered in any of the investigated models. Fortunately, it has also been found that this effect is negligible up to the appearance of the first crack (0.05 mm).

2.2.1.2 Corroded Length

In the case of localized corrosion, concentrated corrosion occurs at certain points instead of throughout the length or the circumference of the rebar. Torres-Acosta and Sagues (2004) conducted a series of experimental tests by limiting the corrosion length (L_{cor}) in an effort to simulate localized corrosion. They found that as the ratio of the corrosion length

(L_{cor}) to the total length of the rebar (L_{tot}) decreases (more localized corrosion), a higher attack penetration (x) is needed to crack the concrete cover.

2.2.2 Concrete Quality and its Mechanical Properties

2.2.2.1 Water to Cement Ratio (w/c)

Increasing the w/c ratio of the concrete increases its porosity and permeability (Petre-Lazar 2000). Likewise, it is expected that the corrosion initiation time is decreased for concretes with higher w/c ratios due to higher permeability. Al-Saadoun and Al-Gahtani (1992) found that w/c ratios of 0.40 and 0.45 increase the corrosion initiation time 2.35 and 1.78 times, respectively, compared to a w/c ratio of 0.65.

On the other hand, increasing porosity provides more space to accommodate the rust products as they are formed, which increases the time between corrosion initiation and cracking of the concrete cover.

2.2.2.2 Tensile Strength of Concrete (f'_t)

According to Jamali et al. (2013), there is a contradiction between different experimental results reported in the literature pertaining to the effect of concrete tensile strength on corrosion-induced cracking. While some of the experimental results have shown that the critical attack penetration increases with increasing tensile strength of the concrete, other experimental results have shown the opposite. There are also some experimental results which have not shown any significant influence of f'_t on the critical attack penetration.

This is probably due to the fact that a lower tensile strength of concrete usually associates with a higher water to cement ratio.

2.2.2.3 Porous Zone (t_p)

Not all of the corrosion products cause expansive pressure. Some of the corrosion products fill the pores available in the nearby concrete. Liu and Weyers (1998) were the first

researchers to account for the higher concrete porosity present around the rebar (Figure 2.18). They assumed a completely void and uniform region around the perimeter of the bar with a thickness t_p . The void area has been referred to as “diffusion zone”, “porous zone”, or “corrosion accommodating region” (Jamali et al. 2013).

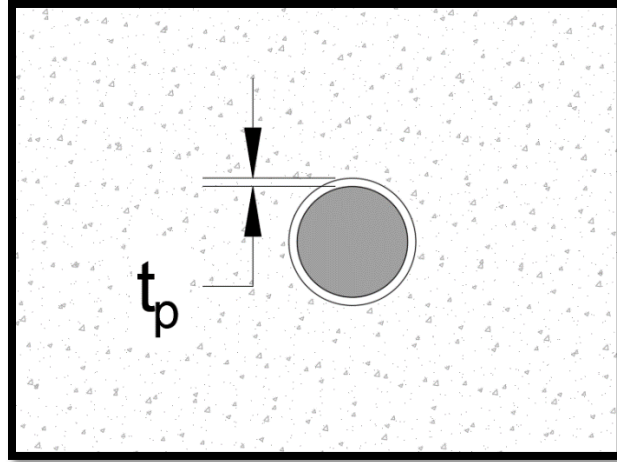


Figure 2.18: Porous zone around a rebar

As reported by Val and Chernin (2012), a porous area around the steel reinforcement was observed by Petre-Lazar (2000) using a scanning electron microscope. The authors also proposed a formula to estimate t_p , which gives values between 2 to 8 micrometers. However, researchers have usually used higher values of t_p in their analyses (10-20 μm or even higher in some cases) (Jamali et al. 2013). This is due to the fact that t_p is usually used as an unknown parameter to fit the results to the experimental observations, and it may also account for the other porosities within the concrete cover (i.e., shrinkage cracks, radial cracks, etc.)

2.2.2.4 Poisson's Ratio of the Concrete (ν_c)

Poisson's ratio of concrete is usually between 0.10 and 0.25, and the effect of temperature and moisture on Poisson's ratio is negligible (Shoukry et al. 2011). The examination of various theoretical models by Jamali et al. (2013) shows a negligible effect of Poisson's ratio on the appearance of the first crack in different models.

2.2.3 Geometric Dimensions

2.2.3.1 Cover Depth (C), Rebar Diameter (d_b), (C/d_b) Ratio

The effect of these parameters will be discussed in three time phases:

Start of Service Life to Initiation of Corrosion

Increasing the cover depth (C) delays the corrosion initiation for both carbonation and chloride-induced corrosion. For example, for a concrete with a w/c ratio of 0.45, a cover of 50.8 mm delayed the corrosion initiation time 12 and 6 times compared to 12.7 mm and 19 mm covers, respectively. The same cover depth delayed the initiation time 6 and 3 times in a concrete with w/c ratio of 0.65 and 12.7 mm and 19 mm covers, respectively (Al-Saadoun and Al-Gahtani 1992).

Initiation of Corrosion to the Appearance of the First Crack

Using a larger rebar diameter (d_b) for the same cover depth (C) decreases the time to the first crack appearance. Al-Saadoun and Al-Gahtani (1992) found that for a constant cover of 31.8 mm (1.25 in), the time to the first crack appearance at the concrete surface for No. 4 (12.7 mm) and No. 3 (9.5 mm) steel bars were 3 and 4.5 times, respectively, that of a No. 8 (25.4 mm) rebar.

However, not all the theoretical models capture this effect (Al-Harthi et al. 2011). This is mostly due to the fact that the volume of void areas within the cover is calculated as the multiplication of t_p by the bar surface area ($\pi \times d_b$). Consequently, a larger diameter means a larger porous zone around the rebar to be filled before the internal pressure is generated, although this porous zone is more related to the w/c ratio of the concrete than the rebar diameter (Jamali et al. 2013).

Increasing the cover depth also delays the time period of this phase. Researchers tend to combine the effect of d_b and C in a new parameter, C/d_b ratio. Al-Saadoun and Al-Gahtani (1992) found that 15 times more corrosion products are needed to induce a cover crack to the surface for a C/d_b ratio of 4.0 compared to a C/d_b ratio of 1.0.

Alonso et al. (1998) also found a linear relation between the C/d_b ratio and the attack penetration needed to cause the first visual crack.

Crack Widening

The C/d_b ratio is not highly influential after the appearance of the first crack (Alonso et al. 1998). This is due to the fact that the concrete cover cannot sustain any tensile stresses over wide cracks.

2.2.3.2 Space between the Reinforcing Bars (S)

The spacing between the steel reinforcing bars (S) can define the mode of damage mechanism. Corrosion causes horizontal delamination when the spacing (S) is small. If the spacing (S) is large enough, spalling of the concrete cover governs (Figure 2.19) (Dagher and Kulendran 1992; Zhou et al. 2005).

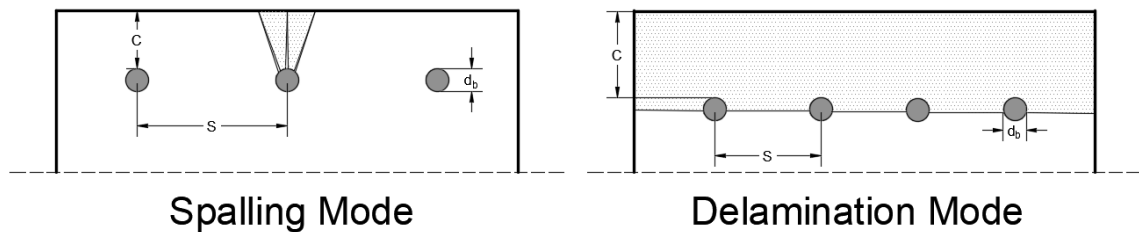


Figure 2.19: Spalling and delamination modes

Dagher and Kulendran (1992) analyzed FEM models with a cover of 50.8 mm, reinforcement diameter of 19.05 mm, and bar spacings of 152, 203 and 254 mm. Delamination was the damage mode for the first two cases ($S = 152$ and 203 mm) and spalling governed the last one ($S = 254$ mm).

Zhou et al. (2005) later conducted further parametric finite element analysis of slabs with different S values and suggested the following equations to predict the damage mode:

$$[2.18] \quad \text{Delamination: } S < 3.828C + 2.414d_b$$

$$[2.19] \quad \text{Spalling: } S \geq 3.828C + 2.414d_b$$

The Zhou et al. model confirms the experimental findings of Dagher and Kulendran (1992):

$$[2.20] \quad S_{cr} = 3.828(50.8) + 2.414(19.05) = 240.45 \text{ mm}$$

However, the effect of S is usually neglected in most theoretical and empirical works.

2.2.4 Rust Properties

2.2.4.1 Bulk Modulus (Compressibility of Rust Products)

It has been observed in experimental tests that the inner layers of the rust products are denser than the outer ones (Melchers 2003). This may be a result of the compaction of the rust products under pressure. Some researchers have tried to model the compressibility of the rust products by incorporating its deformation capacity in analyses through its bulk modulus.

Molina et al. (1993) used the bulk modulus of water (2.0 GPa) in a numerical model in the absence of more accurate data. Balafas and Burgoyne (2011) used a bulk modulus of 0.5 GPa in their model with reference to Konopka (2005). According to Balafas and Burgoyne (2011), Konopka (2005) also showed that rust products do not expand greatly after decompression.

2.2.4.2 Rust Expansion Coefficient (R_v)

The volume of corrosion products is always higher than the consumed iron. The volumetric ratio of rust products to parent iron is called the rust expansion coefficient (R_v). The rust expansion coefficient is related to the type of rust products formed, which is a function of environmental factors such as humidity and oxygen supply (Zhao et al. 2011). As humidity and oxygen supply increase, " R_v " also increases. Table 2.2 lists the different rust products and the corresponding " R_v ".

Table 2.2: Rust expansion coefficients

Name	Chemical Formula	Rust Expansion Coefficient (R_v)
	FeO	1.7
Hematite	$\frac{1}{2} \text{Fe}_2\text{O}_3$	2.12 (Caré et al. 2008)
Magnetite	$\frac{1}{3} \text{Fe}_3\text{O}_4$	2.08 (Caré et al. 2008)
Goethite	$\alpha - \text{FeOOH}$	2.91 (Caré et al. 2008)
Akageneite	$\beta - \text{FeOOH}$	3.48 (Caré et al. 2008)
Lepidocrocite	$\gamma - \text{FeOOH}$	3.03 (Caré et al. 2008)
Ferrous Hydroxide	$\text{Fe}(\text{OH})_2$	3.75
Ferric Hydroxide	$\text{Fe}(\text{OH})_3$	4.2
Hydrated Ferric Oxide	$\text{Fe}(\text{OH})_3, 3\text{H}_2\text{O}$	6.4

However, most of the studies that analyze corrosion-induced cracking of the concrete cover use an average rust expansion coefficient between 2.0 and 3.0. Zhao et al. (2011) found “ R_v ” to lie between 2.64 and 3.24 in experimental tests. Marcotte and Hansson (2007) found that “ R_v ” mostly falls between 2 and 3. Suda et al. (1993) recommended values from 2.9 to 3.2.

2.2.4.3 Rust Diffusion into Cracks

Some researchers accounted for the diffusion of corrosion products into the cracks in their models. For example, Pantazopoulou and Papoulia (2001) assumed triangular cracks, whereas Berra et al. (2003) and Coronelli (2002) assumed rectangular cracks. They all assumed that all of the available space within the cracks is filled by corrosion products.

2.3 Modelling Approaches

Previous research conducted to model corrosion-induced cracking of the concrete cover can be divided into three types of solutions based on their approach: (i) empirical, (ii) analytical, and (iii) numerical models.

2.3.1 Empirical Models

This approach is basically a fitted formula or diagram based on experimental results. Therefore, these models do not explain the cracking phenomenon and apply only to the experimental sets from which they were derived. Table 2.3 provides some of the empirical equations proposed by various researchers. These models try to capture the influence of geometric parameter C/d_b .

Table 2.3: Empirical equations

Author	Equation
Torres-Acosta and Sagues (2004)	$x_{cr}^{(1)} = 0.011 \left(\frac{C}{d_b} \right) \left(\frac{C}{L_{cor}^{(2)}} + 1 \right)^{2.0} \times 10^3$
Alonso et al. (1998)	$x_{cr}^{(1)} = 7.53 + 9.32 \left(\frac{C}{d_b} \right)$
Rodriguez et al. (1996) reported by Jamali et al. (2013)	$x_{cr}^{(1)} = (83.8 + 7.4 \left(\frac{C}{d_b} \right) - 22.6 \times f_t'^{(3)})$

(1) x_{cr} : Critical attack penetration in μm

(2) L_{cor} : Length of the corroding zone

(3) f_t' : Tensile strength of concrete in MPa.

2.3.2 Analytical Models

Analytical models can be used to predict concrete cover cracking due to corrosion-induced expansion. However, they generally require simplifying assumptions including the

assumptions to simplify the material's behaviour. Consequently, they can be solved using a closed-form solution. All of the reviewed models are similar in some aspects: they all considered corrosion-induced concrete cracking as a two dimensional problem based on the thick-wall cylinder analogy and assumed a uniform corrosion over the circumference of the rebar.

2.3.2.1 Bazant (1979)

Bazant was one of the first researchers who presented a model for corrosion-induced cracking in 1979. The following are some of the assumptions for this model:

1. The concrete is considered as a homogeneous linear elastic material.
2. There are two crack (failure) surfaces, which each makes a 45° angle with the cover surface (Figure 2.20).
3. The average tensile stress at the crack surfaces at failure is equal to the tensile strength of concrete (f'_t).
4. The corrosion rate is constant.

Unlike the second assumption, it has been observed in experimental tests that one main crack usually reaches the cover surface and widens over time (Andrade et al. 1993; Merino 2014). Since the tensile strains over the surfaces are not the same and concrete is not a plastic material, the third assumption is also not an accurate assumption. Bazant's model has been found to significantly underestimate the cracking time (Newhouse and Weyers 1996).

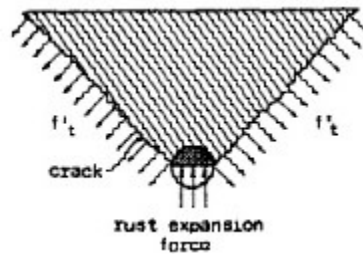


Figure 2.20: Failure surfaces (reproduced from Bazant, 1979)

2.3.2.2 Liu and Weyers (1998)

Liu and Weyers' model is very similar to Bazant's model. However, they proposed two reasons (assumptions) to explain why Bazant's model underestimates the cracking time:

1. The corrosion rate is not constant, and it decreases with time (see Section 2.2.1.1).
2. There is a void area around the rebar which must be filled before any pressure is applied to the concrete cover (see Section 2.2.2.3).

2.3.2.3 Wang and Liu (2004a)

Wang and Liu (2004a) employed a thick-walled cylinder approach, dividing the cylinder into an inner cracked and an outer un-cracked cylinder (Figure 2.21). A bilinear tension softening was considered for the cracked concrete. Although this model was proposed after Liu and Weyers' model, they did not consider the porous zone around the reinforcing steel bar. The authors incorporated their model to analyze the effect of corrosion on the bond-strength of the rebar.

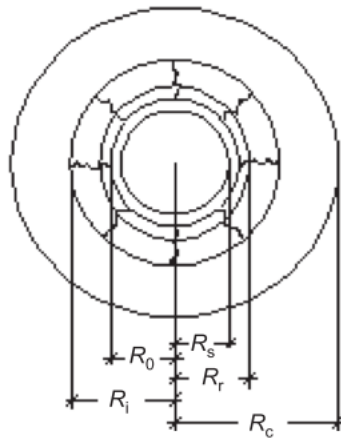


Figure 2.21: Divided cylinder (reproduced from Wang and Liu, 2004a)

Predicted results did not compare well with experimental data. Two rust expansion coefficients of $R_v = 2$ and 3 were used to find the critical attack penetration, providing a very broad range of results varying by a factor of 2. Nonetheless, not all the experimental results fall in this range, especially for C/d_b ratios larger than 4. Although the rust

expansion coefficient is not known for each set of experimental tests, it is likely similar for each of them, since they were all tested in the same environmental conditions.

2.3.2.4 Bhargava et al. (2006)

Bhargava et al. (2006) also used a thick-walled cylinder approach to model corrosion-induced concrete cracking, dividing the cylinder into an inner cracked cylinder and an outer un-cracked cylinder. They employed a bilinear tension softening relation for the cracked cylinder. The concrete was assumed to be a linear elastic, isotropic and homogenous material. Young's modulus was reduced for the inner cylinder to account for cracking. Unlike Wang and Liu's model, this model considered a porous area around the rebar as proposed by Liu and Weyers (1998). However, no rust products were considered to fill the radial cracks. Unlike most of the above models, the authors claimed that the tensile strength of the cover concrete had a major influence on the critical attack penetration and cover cracking time. The authors also used a nonlinear corrosion rate as proposed by Liu and Weyers (1998). A constant rust expansion coefficient of 3.39 was assumed.

2.3.2.5 Li et al. (2006)

Li et al. (2006) developed a new model in an effort to not only predict the critical attack penetration but also to model the behaviour of the concrete cover after cracking as the crack widens. They did not consider the effect of i_{cor} on the attack penetration needed to create a certain crack width, as mentioned in Section 2.2.1.1. The concrete cover was assumed to be a quasi-brittle material with an inner cracked and an outer un-cracked cylinder. Smeared cracking, a constant reduced Young's modulus, $\alpha E_{c,ef}$ (where $\alpha < 1$), and tension softening were assumed for the cracked cylinder. They provided a comparison for only a single experimental test.

2.3.2.6 Balafas and Burgoyne (2011)

A divided thick-walled cylinder analogy with an inner cracked cylinder and an outer un-cracked cylinder was used by the authors. A linear distribution of tangential strain in the radial direction of the cracked cylinder was assumed. Smeared cracking and a bilinear tension-softening curve was used. Fracture-mechanics was employed to define a cover's failure.

The compressibility of rust products was considered using a bulk modulus of 0.5 GPa. The rust expansion coefficient was assumed to be 2.96. The available space within the cracks was considered with the assumption that all the available space within the cracks is filled. A nonlinear production of rust products instead of Faraday's law was presumed.

2.3.2.7 Val and Chernin (2012)

Like other theoretical models, Val and Chernin (2012) also employed the thick-walled cylinder theory to model cover cracking. The thick-walled cylinder in their model consists of an inner cracked cylinder and an outer un-cracked cylinder. The un-cracked cylinder was idealized as a linear elastic ($E_{c,ef}$), homogenous material, whereas the cracked cylinder had a constant Young's modulus in the longitudinal (z) and radial directions ($E_{c,ef}$). Young's modulus of the cracked cylinder in the tangential direction varied as a function of cracking (Equation 2.22). Equation 2.23 ensures the continuity of both stresses and strains at the boundary between the two cylinders.

$$[2.21] \quad E_r = E_z = E_{c,ef} = \text{Constant}$$

$$[2.22] \quad E_\theta = E_{c,ef}F(r)$$

$$[2.23] \quad F(r) = \left(\frac{r}{r_c}\right)^n$$

where E_r , E_z , and E_θ are Young's modulus in the radial, z, and tangential directions, respectively, $E_{c,ef}$ is the long term effective Young's modulus of concrete, n is an empirical parameter calculated from Equation 2.24, r_c is the crack front, and r is the radial coordinate ($r < r_c$).

$$[2.24] \quad n = \begin{cases} 1.5, & f'_c \leq 38.4 \text{ MPa} \\ 0.081 \exp(0.076f'_c), & f'_c > 38.4 \text{ MPa} \end{cases}$$

The authors analyzed the cover assuming it was in a state of plane-strain instead of plane-stress. However, this assumption only slightly changes the results, and none of the two assumptions is quite accurate. The steel reinforcement and the cover around it are either in tension or compression depending on their location in the structural member.

Val and Chernin (2012) also proposed another approach to account for the porosity in the concrete cover and the porous zone within the concrete cover. The authors assumed that, instead of a void area which has to be filled before any pressure is imposed to the concrete cover, a constant portion of rust products always fills the void areas and the remainder imposes the internal pressure, which in turn generates more space to be occupied (Equation 2.25):

$$[2.25] \quad \Delta_{\text{tot}} = \Delta_{\text{cr}} + \Delta_{\text{diff}}$$

where Δ_{tot} is the total volume of rust products, Δ_{diff} is the portion of the rust products which fills the available pores, and Δ_{cr} is the portion of the rust products which imposes pressure and pushes the inner boundaries of the inner cylinder. They suggested a $\frac{\Delta_{\text{diff}}}{\Delta_{\text{tot}}}$ ratio of 0.68 for experimental tests and 0.73 for natural conditions. Although Δ_{diff} is constant and relatively large, there is no experimental explanation for it.

2.3.3 Numerical Models

Numerical models are usually used to model non-linear behaviours (e.g., the non-linear behaviour of concrete in tension or compression). The resulting equations stating the equilibrium should be solved numerically either by finite difference or finite element. Therefore, they need a computer to be run and it takes more time to solve them than analytical or empirical models.

2.3.3.1 Molina et al. (1993)

Molina et al. (1993) used a finite element analysis to simulate the cover cracking observed in four experimental tests they had reported in Andrade et al. (1993). The finite element model was based on smeared-cracking and linear tension softening. The corrosion load was modelled by imposing initial thermal strains and changing the elastic properties to simulate the uniform expansion and softening, respectively. The bulk modulus of water (2.0 GPa) was used to model compressibility of the corrosion products. Faraday's law was used to correlate time to attack penetration. A rust expansion coefficient of 2.0 was used.

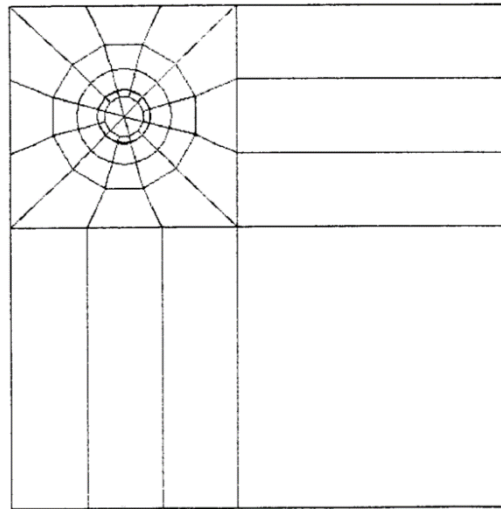


Figure 2.22: Mesh representing one of the specimens (reproduced from Molina et al., 1993)

The authors found the maximum internal pressures to induce cracking to be between 11 and 14 MPa. From the parametric studies, they found:

- Increasing the rust expansion coefficient from 2 to 3 doubles the rate of the cracking;
- Decreasing the tensile strength decreases the internal pressure;
- The maximum internal pressure does not change by increasing the fracture energy;
- Doubling the bulk modulus to 4.0 does not change the results.

2.3.3.2 Pantazopoulou and Papoulia (2001)

Pantazopoulou and Papoulia (2001) proposed an approach based on the thick-walled cylinder analogy where cracked concrete is assumed as an orthotropic material. The concrete's behaviour in compression was represented by Hognestad parabola. Smeared cracking and a bilinear tension softening was employed to model the post-crack behaviour of the concrete in tension. The problem was solved using finite differences and by discretizing the concrete cover along the radial direction (Figure 2.23).

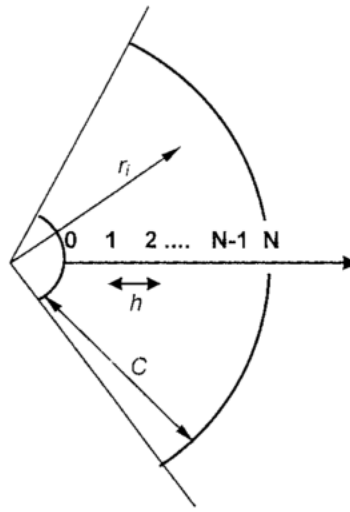


Figure 2.23: Discretizing concrete cover (reproduced from Pantazopoulou and Papoulia, 2001)

The authors did not provide a broad comparison of critical attack penetrations (or corresponding times) between their predicted results and the experimental results, providing a comparison for only a single experimental test.

The model was solved with two upper and lower limits, by considering that either all or none of the available space within the cracks was filled with corrosion products. The total crack width at the rebar/concrete interface was estimated using Equation 2.26:

$$[2.26] \quad w_{cr} = \varepsilon_{\theta} \times 2\pi \times r_b$$

where w_{cr} , and ϵ_θ are the total crack width and tangential strain at the concrete/rebar interface, and r_b is the rebar's radius.

The upper limit (based on the assumption of cracks full of corrosion products) of steel loss corresponding to the first crack appearance was found to be 5 times larger than the lower limit (based on the assumption of no corrosion products within the cracks).

The model was solved using Faraday's law and the non-linear rust production rate proposed by Liu and Weyers (1998). Compressibility of corrosion products was not considered in the model. The model also did not consider the porous zone around the reinforcement.

Based on this model, the authors believe that increasing the tensile strength of the concrete cover increases the time corresponding to cover-cracking.

2.3.3.3 Du et al. (2006)

Du et al. (2006) used two-dimensional finite element analysis to simulate concrete cover cracking induced by reinforcement corrosion. The finite element mesh consisted of plane-strain quadrilateral elements with 8 nodes. The concrete's behaviour in tension was modeled using a fixed smeared crack approach. In addition to uniform corrosion, two localized corrosion cases were modeled using elliptically imposed displacements (Figure 2.24).

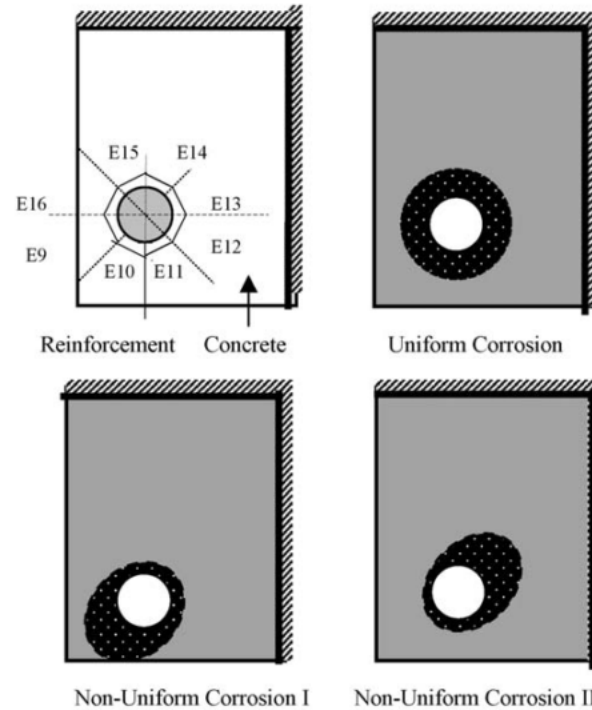


Figure 2.24: Non-uniform corrosion modeled by Du et al. (2006) (reproduced from Du et al., 2006)

The authors found four consecutive cracking stages in all cases: (i) internal cracking, (ii) external cracking, (iii) penetration cracking, and (iv) ultimate cracking.

There was a good agreement between their finite element results and the experimental results when they were compared based on the maximum internal pressure. However, the results for the critical attack penetrations were not in agreement with experimental results. For one case, where the experimental tests showed 1.3% and 1.58% of mass loss to cause cover-cracking for plain and ribbed bars, respectively, the authors' finite element model estimated mass losses between 0.03% to 0.22% to cause cracking of the concrete cover, using a rust expansion coefficient between 1.75 and 6.25. Du et al. (2006) did not consider the available space within the cracks as well as the porous zone around the rebar in their calculations.

2.3.4 Summary

Based on the literature review presented here, some of the identified gaps are as follows:

- 1 Not all of the models considered the porous zone around a corroding rebar (e.g., Wang and Liu (2004a), Pantazopoulou and Papoulia (2001), Du et al. (2006)).
- 2 Only Molina et al. (1993) and Balafas and Burgoyne (2011) considered the compressibility of corrosion products due to the internal pressure at the rebar/concrete interface.
- 3 In many cases the authors did not provide a broad comparison against experimental data. For example, Pantazopoulou and Papoulia (2001) and Li et al. (2006) only provided a comparison against a single experimental test.
- 4 The available space within the cracks is not considered to accommodate the corrosion products in most of the models. When considered, it is assumed that all of the available space is filled by corrosion products or an upper and lower limit are provided assuming that all or none of the available space is filled (e.g., Pantazopoulou and Papoulia (2001)). Since results based on an upper limit are many times larger than the ones obtained with a lower limit, the limits are not practically useful to estimate the cracking behaviour of the real structural element due to corrosion.
- 5 In some models, the authors provided a broad range of results using different rust expansion coefficients to fit the experimental results (e.g., Wang and Liu (2004a)). Although the exact value of the rust expansion coefficient is unknown for each experimental test, not providing a single suggested value makes the presented models impractical. For example, a sample with a rust expansion coefficient of 3.0 cracks two times faster than a similar sample with a rust expansion coefficient of 2.0. In addition, rust expansion coefficients are similar for different samples in an experimental set, as all of them were examined in a similar environment.

In this thesis, an attempt is made to address the gaps identified in the available literature. Chapter 3 provides a numerical approach and Chapter 4 provides two analytical approaches to model the corrosion-induced cracking considering the effects of the porous zone,

compressibility of corrosion products, and rust diffusion into the cracks. Chapter 5 provides a comprehensive comparison between the results based on the different modelling approaches and the experimental data reported in the literature. Constant input values (instead of a broad range) and experimental data provided by different researchers (instead of one experimental set) are used. It was also found that there is a great time-lapse between the time the first crack reaches the surface and when it becomes visible to the bare eyes of the examiner. This fact was neglected by the previous researchers. Consequently, a new interpretation of the experimental data has been established and the results were compared based on the new interpretation. Chapter 6 validates the use of thick-walled cylinder used in Chapters 3 and 4 by comparing the results based on FEM to the results found in the previous chapters.

Chapter 3

Numerical Model

3.1 Modelling Assumptions

Since corrosion-induced cracks tend to form along the longitudinal axis of the concrete element and parallel to the steel reinforcement, it is justifiable to simplify the problem as a two-dimensional one, assuming a state of plane stress. The numerical model proposed herein also assumes the concrete cover to act as a thick-walled cylinder subjected to the internal pressure caused by the expansion of corrosion products. The expansion pressure induced by corrosion at the reinforcement and concrete interface is mostly balanced by rings of tensile stresses around the rebar. This approach has previously been used by several researchers (Bhargava et al. 2006; Li et al. 2006; Pantazopoulou and Papoulia 2001; Val and Chernin 2012; Wang and Liu 2004a; Liu and Weyers 1998). The capacity of the thick-walled cylinder is limited by the tensile strength of concrete, and consequently failure of finite rings around the reinforcement occurs by concrete cracking.

Starting from the force equilibrium along the radial direction, and the compatibility of the deformations of any elements within the thick-walled cylinder (Figure 3.1), Equations 3.1 to 3.3 can be written:

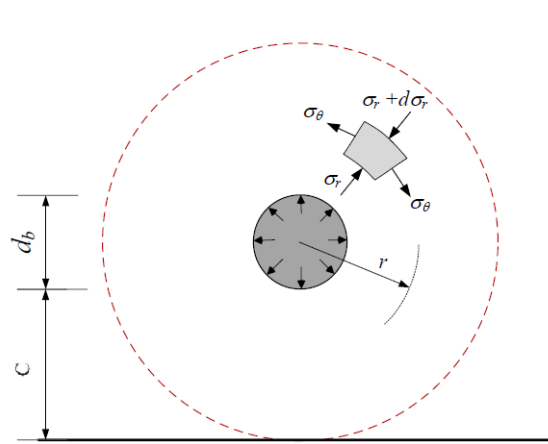


Figure 3.1: Force equilibrium and compatibility of an element within the thick-walled cylinder (adapted from Martin-Perez, 1999)

$$[3.1] \quad \sigma_r + \frac{d\sigma_r}{dr} r - \sigma_\theta = 0$$

$$[3.2] \quad \varepsilon_r = \frac{du}{dr}$$

$$[3.3] \quad \varepsilon_\theta = \frac{u}{r}$$

where ε_r and ε_θ are the radial and hoop strains, respectively, σ_r and σ_θ are the corresponding radial (compressive) and hoop (tensile) stresses, r is the radius of the concrete ring where stress or strain is being calculated, and u is the radial displacement at r . Variable r_b is the rebar's radius, and C is the cover depth. In Figure 3.1, d_b refers to the bar diameter.

If plane stress is assumed and concrete is treated as an orthotropic material, the stresses and strains in the concrete cover are related by Equation 3.4 and Equation 3.5:

$$[3.4] \quad \varepsilon_r = \frac{1}{E_r} (\sigma_r - \nu_{r\theta} \sigma_\theta)$$

$$[3.5] \quad \varepsilon_\theta = \frac{1}{E_\theta} (\sigma_\theta - \nu_{\theta r} \sigma_r)$$

where E_r and E_θ are the secant moduli of concrete along the radial and hoop directions, respectively, and $\nu_{r\theta}$ and $\nu_{\theta r}$ are the corresponding Poisson's ratios. Equations 3.4 and 3.5 can be rearranged as Equations 3.6 and 3.7:

$$[3.6] \quad \sigma_r = \frac{1}{1-\nu_{\theta r}\nu_{r\theta}} (E_r \varepsilon_r + \nu_{r\theta} E_\theta \varepsilon_\theta)$$

$$[3.7] \quad \sigma_\theta = \frac{1}{1-\nu_{\theta r}\nu_{r\theta}} (E_\theta \varepsilon_\theta + \nu_{\theta r} E_r \varepsilon_r)$$

Equation 3.8 can be derived by substituting Equation 3.6, Equation 3.7, Equation 3.2, and Equation 3.3 into Equation 3.1. Equation 3.8 can be rearranged as Equations 3.9 and 3.10:

$$[3.8] \quad E_r \frac{du}{dr} + \nu_{r\theta} E_\theta \frac{u}{r} + E_r \frac{d^2u}{dr^2} r + \nu_{r\theta} E_\theta \frac{du}{dr} - \nu_{r\theta} E_\theta \frac{u}{r} - E_\theta \frac{u}{r} - \nu_{\theta r} E_r \frac{du}{dr} = 0$$

$$[3.9] \quad E_r \frac{d^2u}{dr^2} r + (E_r - \nu_{\theta r} E_r + \nu_{r\theta} E_\theta) \frac{du}{dr} - E_\theta \frac{u}{r} = 0$$

$$[3.10] \quad \frac{d^2u}{dr^2} + \left(1 - \nu_{\theta r} + \nu_{r\theta} \frac{E_\theta}{E_r}\right) \frac{1}{r} \times \frac{du}{dr} - \frac{E_\theta}{E_r} \times \frac{u}{r^2} = 0$$

Since the concrete has been assumed to be an orthotropic material, Equation 3.11 should be satisfied:

$$[3.11] \quad \nu_{\theta r} E_r = \nu_{r\theta} E_\theta$$

Therefore, Equation 3.10 becomes:

$$[3.12] \quad \frac{d^2u}{dr^2} + \frac{1}{r} \times \frac{du}{dr} - \frac{E_\theta}{E_r} \times \frac{u}{r^2} = 0$$

Equation 3.12 is a second-order linear differential equation describing the radial deformation (u) where r is the radial coordinate. The boundary conditions are given by Equations 3.13 and 3.14:

$$[3.13] \quad u(r = r_b) = \Delta r_b$$

$$[3.14] \quad \sigma_r(r = r_b + C) = 0$$

where Δr_b is the uniform radial deformation at the reinforcement and concrete interface (inner boundary of the thick-walled cylinder) imposed by the corrosion expansion.

Equation 3.15 is derived by substituting Equations 3.2 and 3.3 into Equation 3.6, and enforcing Equation 3.14 as a boundary condition:

$$[3.15] \quad \sigma_r(r = r_b + C) = \frac{du}{dr} + \nu_{r\theta} \frac{E_\theta}{E_r} \times \frac{u}{r} = 0$$

The differential equation (Equation 3.12), along with the boundary conditions (Equations 3.13 and 3.15) can be solved numerically by discretizing the concrete cover into $n+1$ ($r_r, r_1, r_2, \dots, r_n$) at an equal spacing of h (Figure 3.2).

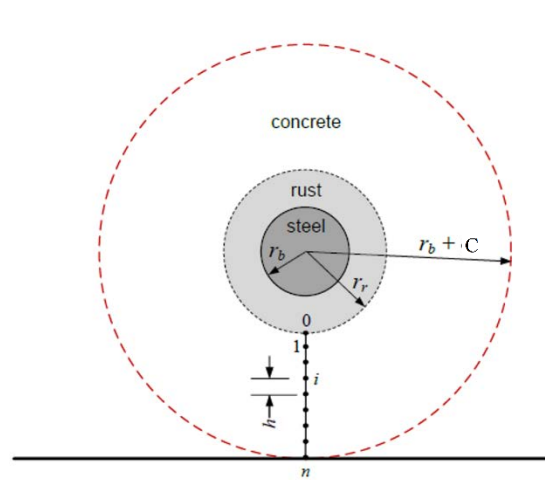


Figure 3.2: Numerical discretization of the concrete cover along the radial direction (adapted from Pantazopoulou and Papoulia, 2001)

The terms $\frac{d^2u}{dr^2}$ and $\frac{du}{dr}$ in Equation 3.12 can be approximated by the first central finite difference approximations (Equations 3.16 and 3.17) at each discrete point:

$$[3.16] \quad \frac{d^2u}{dr^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}$$

$$[3.17] \quad \frac{du}{dr} = \frac{u_{i+1} - u_{i-1}}{2h}$$

where the subscript i ($0, 1, 2, \dots, n$) denotes the location of the discrete point along the concrete cover.

Equation 3.18 can be derived by substituting Equations 3.16 and 3.17 into Equation 3.12:

$$[3.18] \quad \left(\frac{1}{h^2} - \frac{1}{2hr_i} \right) u_{i-1} - \left(\frac{2}{h^2} + \frac{E_{\theta,i}}{E_{r,i}} \times \frac{1}{r_i^2} \right) u_i + \left(\frac{1}{h^2} + \frac{1}{2hr_i} \right) u_{i+1} = 0$$

where i ranges from 0 to n , and h represents the spacing between discrete points.

Enforcing the boundary condition at the inner boundary of the thick-walled cylinder (Equation 3.13) into Equation 3.18 for $i=1$ results in Equation 3.19:

$$[3.19] \quad \left(\frac{2}{h^2} + \frac{E_{\theta,1}}{E_{r,1}} \times \frac{1}{r_1^2} \right) u_1 - \left(\frac{1}{h^2} + \frac{1}{2hr_1} \right) u_2 = \left(\frac{1}{h^2} - \frac{1}{2hr_1} \right) u_0 \quad \text{for } i=1$$

where $u_0 = \Delta r_b$.

Using the finite difference approximation (Equation 3.17) in the boundary condition expression given by Equation 3.15 results in Equation 3.20, which can be rearranged as Equation 3.21:

$$[3.20] \quad \frac{u_{n+1} - u_{n-1}}{2h} + v_{r\theta,n} \frac{E_{\theta,n}}{E_{r,n}} \times \frac{u_n}{r_n} = 0 \quad \rightarrow$$

$$[3.21] \quad u_{n+1} = u_{n-1} - \left(v_{r\theta,n} \frac{E_{\theta,n}}{E_{r,n}} \times \frac{2h}{r_n} \right) u_n$$

Substituting Equation 3.21 into Equation 3.18 for $i=n$ results in Equation 3.22:

$$[3.22] \quad \frac{2}{h^2} u_{n-1} - \left(\frac{2}{h^2} + \frac{E_{\theta,n}}{E_{r,n}} \times \frac{1}{r_n^2} \left(1 + v_{r\theta,n} \left(1 + \frac{2}{h} r_n \right) \right) \right) u_n = 0 \quad \text{for } i=n$$

Equations 3.18, 3.19, and 3.22 result in a system of n linear equations:

$$[3.23] \quad [K]\{U\} = \{F\}$$

where:

$$[3.24]$$

$$[K] = \begin{bmatrix} \frac{2}{h^2} + \frac{E_{\theta,1}}{E_{r,1}} \times \frac{1}{r_1^2} & -\frac{1}{h^2} - \frac{1}{2hr_1} & 0 & \dots & 0 & 0 & 0 \\ \frac{1}{h^2} - \frac{1}{2hr_2} & -\frac{2}{h^2} - \frac{E_{\theta,2}}{E_{r,2}} \times \frac{1}{r_2^2} & \frac{1}{h^2} + \frac{1}{2hr_2} & \dots & 0 & 0 & 0 \\ 0 & \frac{1}{h^2} - \frac{1}{2hr_3} & -\frac{2}{h^2} - \frac{E_{\theta,3}}{E_{r,3}} \times \frac{1}{r_3^2} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\frac{2}{h^2} - \frac{E_{\theta,n-2}}{E_{r,n-2}} \times \frac{1}{r_{n-2}^2} & \frac{1}{h^2} + \frac{1}{2hr_{n-2}} & 0 \\ 0 & 0 & 0 & \dots & \frac{1}{h^2} - \frac{1}{2hr_{n-1}} & -\frac{2}{h^2} - \frac{E_{\theta,n-1}}{E_{r,n-1}} \times \frac{1}{r_{n-1}^2} & \frac{1}{h^2} + \frac{1}{2hr_{n-1}} \\ 0 & 0 & 0 & \dots & 0 & \frac{2}{h^2} & -\frac{2}{h^2} - \frac{E_{\theta,n}}{E_{r,n}} \times \frac{1}{r_n^2} \times \left[1 + \nu_{r\theta,n} \times \left(1 + \frac{2}{h} r_n \right) \right] \end{bmatrix}$$

$$[3.25] \quad \{U\}^T = \{u_1 \quad u_2 \quad u_3 \quad \dots \quad u_{n-2} \quad u_{n-1} \quad u_n\}$$

$$[3.26] \quad \{F\}^T = \left\{ \left(\frac{1}{h^2} - \frac{1}{2hr_1} \right) \times \Delta r_b \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \right\}$$

The secant moduli $E_{\theta,i}$ and $E_{r,i}$ depend on the principal strains at nodal location i (Tastani and Pantazopoulou 2013).

Once the vector of nodal displacements $\{U\}$ is obtained by solving the system of equations given by Equation 3.23, the stresses σ_r and σ_θ along the concrete cover are obtained by substituting Equations 3.16, 3.17, 3.2, and 3.3 into Equations 3.6 and 3.7:

$$[3.27] \quad \sigma_{r,i} = \frac{1}{1-\nu_{\theta,r,i}\nu_{r\theta,i}} \left(E_{r,i} \frac{u_{i+1}-u_{i-1}}{2h} + \nu_{r\theta,i} E_{\theta,i} \frac{u_i}{r_i} \right)$$

$$[3.28] \quad \sigma_{\theta,i} = \frac{1}{1-\nu_{\theta,r,i}\nu_{r\theta,i}} \left(E_{\theta,i} \frac{u_i}{r_i} + \nu_{\theta,r,i} E_{r,i} \frac{u_{i+1}-u_{i-1}}{2h} \right)$$

where u_{-1} and u_{n+1} are obtained from Equation 3.18 for $r = r_b$ ($i = 0$) and $r = r_b + c$ ($i = n$), respectively:

$$[3.29] \quad u_{-1} = \left(\frac{1}{h^2} - \frac{1}{2hr_b} \right)^{-1} \times \left[\left(\frac{2}{h^2} + \frac{E_{\theta,0}}{E_{r,0}} \times \frac{1}{r_b^2} \right) u_0 - \left(\frac{1}{h^2} + \frac{1}{2hr_b} \right) u_1 \right]$$

$$[3.30] \quad u_{n+1} = \left(\frac{1}{h^2} + \frac{1}{2h(r_b+c)} \right)^{-1} \times \left[- \left(\frac{1}{h^2} - \frac{1}{2h(r_b+c)} \right) u_{n-1} + \left(\frac{2}{h^2} + \frac{E_{\theta,n}}{E_{r,n}} \times \frac{1}{(r_b+c)^2} \right) u_n \right]$$

3.2 Modelling the Concrete's Behaviour in Tension and Compression

3.2.1 Concrete in Tension

It has been observed in experimental tests (Evans and Marathe 1968) that the tensile stress in concrete at the location of a crack does not drop to zero immediately following crack initiation. Instead, the tensile stress decreases from its peak value (i.e., tensile strength) as the total tensile strain increases (Figure 3.3). This post-peak behaviour is known as tension softening (strain softening). Finally, the tensile stress reaches zero when the tensile strain is higher than the ultimate strain (ϵ_u).

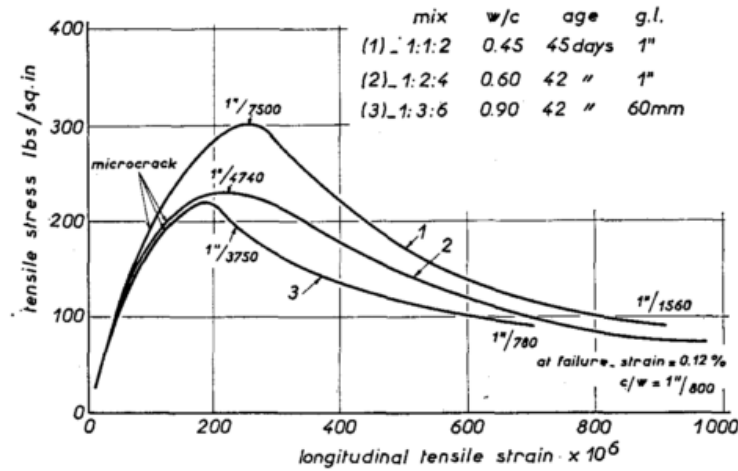


Figure 3.3: Experimental tensile stress-strain diagram for concrete (Evans and Marathe 1968)

The standard cohesive crack model was introduced by Hillerborg et al. (1976) to be employed in finite element analysis. In this model, the relation between strain and stress is assumed to be linear elastic (E_0) for un-cracked concrete, and concrete is assumed to be cracked when the principal tensile strain (ϵ_t) is greater than the crack strain: (ϵ_t) > (ϵ_{cr}), which is the strain corresponding to the tensile strength. Cracked concrete still bears some residual stress because the crack actually does not represent a single wide crack, but rather a combination of many micro-cracks in a region referred to as a crack zone. Therefore, there are some remaining bridges to transfer stress across the crack zone. When the tensile strain reaches the ultimate strain (ϵ_u), the tensile stress approaches zero; at this stage, the crack opening has become wide enough that the two faces of the concrete crack are completely separated.

The tensile stress-strain diagram for concrete in this work consists of two portions, an elastic portion and a strain-softening curve. The strain-softening behaviour is generally non-linear (Figure 3.3). However, it can be approximated by linear, bilinear or trilinear models as well as nonlinear models, as reported by Gettu et al. (2007). A linear, a bilinear, and a nonlinear model are all used here to model the post peak behaviour of concrete in tension. Regardless of the chosen strain-softening curve, the area under the tensile stress-strain diagram multiplied by the width of the crack band front (characteristic width of the advancing micro-crack zone) (w_c) is equal to the fracture energy of concrete (G_f). The

fracture energy of concrete is a material property and is defined as the dissipated energy caused by localized cracking per unit area of plane (Bažant and Oh 1983; Bažant 1986). The optimum value of the crack width of the band front has been found to be approximately three times the maximum aggregate size (d_a) (Bažant and Oh 1983).

Linear Tension Softening Model:

In this model, it is assumed that the residual stress after cracking decreases linearly with increasing total strain as shown in Figure 3.4.

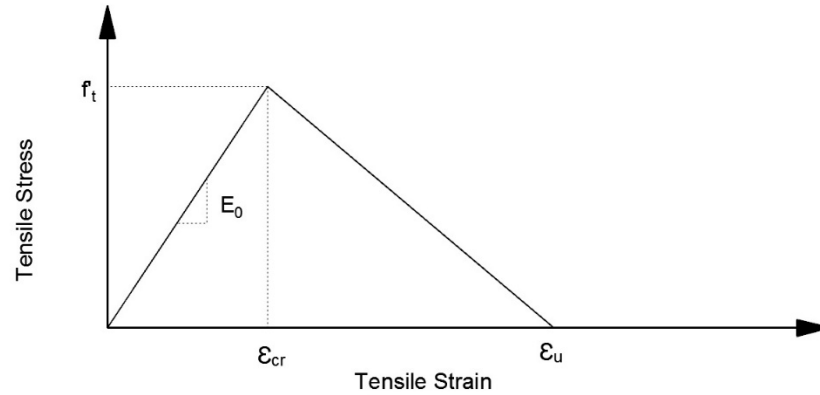


Figure 3.4: Tensile stress-strain diagram with a linear strain-softening curve

In the case of linear strain-softening, the ultimate tensile strain can be calculated from Equation 3.31:

$$[3.31] \quad \epsilon_u = \frac{2 \times G_f}{f_t' \times w_c}$$

Bilinear Tension Softening Model:

In the bilinear model, the softening (Figure 3.5) consists of two linear segments (CEB-FIP Model Code 2010), where ϵ_1 and ϵ_u can be calculated from Equations 3.32 and 3.33:

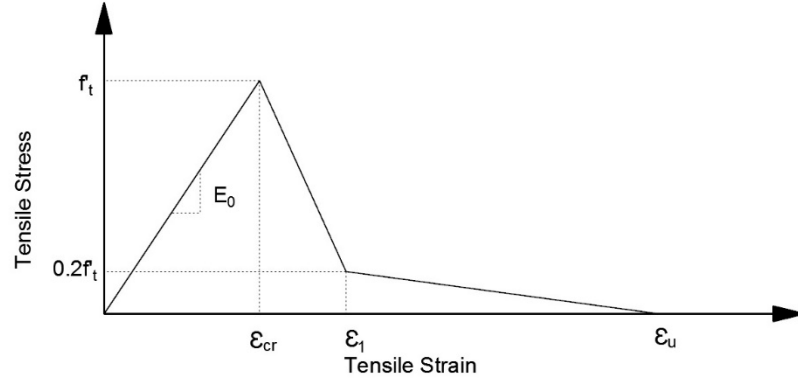


Figure 3.5: Tensile stress-strain diagram with a bilinear strain-softening curve

$$[3.32] \quad \varepsilon_1 = \frac{G_f}{f'_t \times w_c} + \varepsilon_{cr}$$

$$[3.33] \quad \varepsilon_u = \frac{5 \times G_f}{f'_t \times w_c} + \varepsilon_{cr}$$

Nonlinear Tension Softening Model:

A nonlinear strain-softening curve (Figure 3.6) was also employed to run the model proposed in Section 3.1. The nonlinear softening model used is that proposed by Hordijk (1991) and given by Equation 3.34.

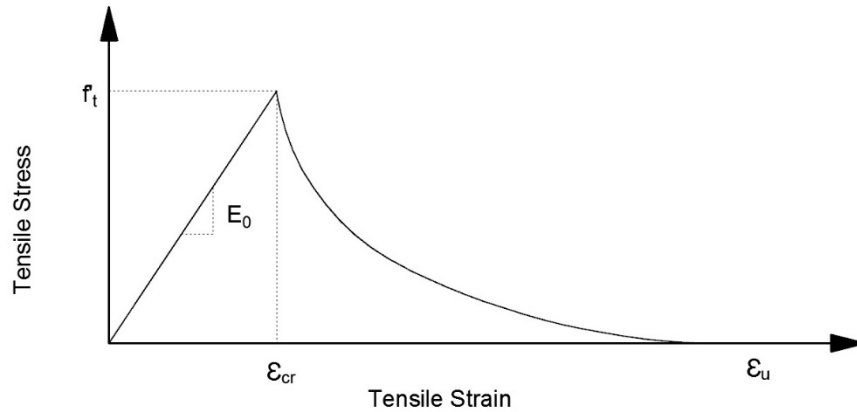


Figure 3.6: Tensile stress-strain diagram with a nonlinear strain-softening curve

$$[3.34] \quad \sigma_t = f'_t \times (1 + (c_1 \times \varepsilon_{ratio})^3) \times e^{-c_2 \times \varepsilon_{ratio}} - \varepsilon_{ratio} (1 + c_1^3) e^{-c_2}$$

where $c_1 = 3.0$, $c_2 = 6.93$. The strain ratio ϵ_{ratio} can be found using Equation 3.35:

$$[3.35] \quad \epsilon_{\text{ratio}} = \frac{(\epsilon - \epsilon_{\text{cr}})}{(\epsilon_{\text{u}} - \epsilon_{\text{cr}})}$$

where ϵ_{u} can be found using Equation 3.36:

$$[3.36] \quad \epsilon_{\text{u}} = \frac{5.136 \times G_f}{f'_t \times w_c} + \epsilon_{\text{cr}}$$

Figure 3.7 compares the three strain-softening models for a concrete with tensile strength $f'_t = 3.94$ MPa, elastic modulus $E_c = 35$ GPa, compressive strength $f'_c = 42.3$ MPa, and maximum aggregate size $d_a = 20$ mm:

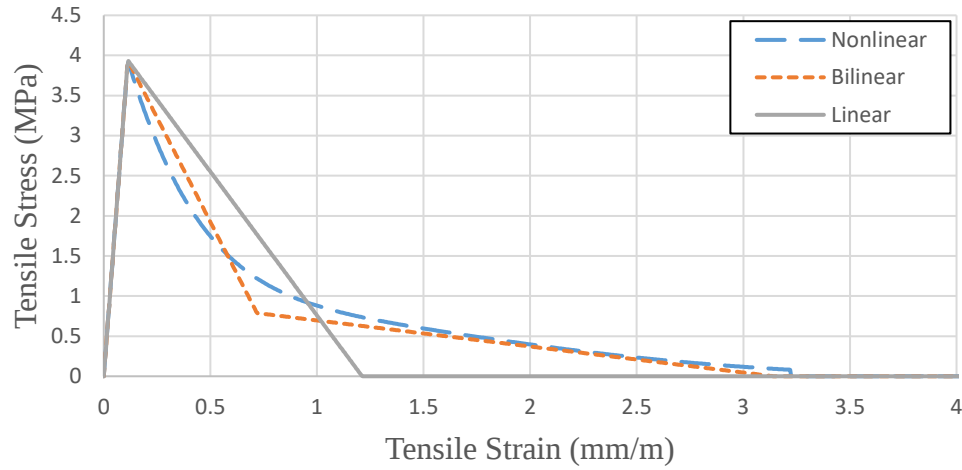


Figure 3.7: Comparison of different tensile strain-softening curves

The nonlinear curve is used as the default strain-softening model in the analyses.

3.2.2 Concrete in Compression

Hognestad's parabola (Collins and Mitchell 1997) has been used to define the general behaviour of concrete in compression, which is applicable for concrete with a 28-day compressive strength up to 40 MPa (Wong et al. 2013) and is given by Equation 3.37:

$$[3.37] \quad \sigma_c = f'_c \left[2 \left(\frac{\varepsilon_c}{\varepsilon_0} \right) - \left(\frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right]$$

where σ_c is the compressive stress, f'_c is the axial compressive strength of concrete, ε_c is the compressive strain, and ε_0 is the compressive strain corresponding to axial compressive strength, assumed to be 0.002.

The principal compressive strength of concrete decreases in the presence of transverse tensile strains. Hence, concrete may crush at a lower compressive stress than f'_c in a biaxial loading condition, which is the case of the thick-wall cylinder subjected to internal pressure. This phenomenon has been considered by using a compression softening model. Vecchio and Collins (1986) introduced a modification factor (β) for f'_c to take into account this softening effect (Equation 3.38):

$$[3.38] \quad \beta = \frac{1}{0.8 - 0.34 \left(\frac{\varepsilon_t}{\varepsilon_0} \right)} \leq 1.0$$

Equation 3.39 results from combining Equations 3.37 and 3.38 (Figure 3.8):

$$[3.39] \quad \sigma_c = \beta \times f'_c \left[2 \left(\frac{\varepsilon_c}{\varepsilon_0} \right) - \left(\frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right]$$

Figure 3.8 demonstrates the effects of different parameters such as compressive strain in the radial direction (ε_r), the axial compressive strength of concrete (f'_c), the compressive strain corresponding to axial compressive strength of concrete (ε_0), the modification factor (β) on Hognestad's parabola, and the secant modulus of the concrete cover in the radial direction (E_r).

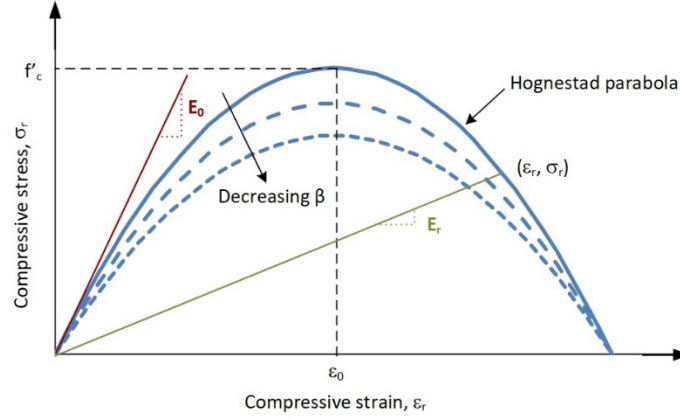


Figure 3.8: Compressive stress-strain diagram

3.3 Modelling the Compressibility of Materials

The numerical model accounts for the compressibility of corrosion products by considering their bulk modulus (Equation 3.40):

$$[3.40] \quad \Delta V_p = -\frac{V_{rr} \times P}{B_r}$$

where V_{rr} is the expected volume of corrosion products if they could expand freely, ΔV_p is the change in volume of the corrosion products due to the applied pressure, B_r is the bulk modulus of corrosion products, and P is the pressure at the reinforcement/concrete interface due to expansion of corrosion products.

The model also considers the compressibility of steel reinforcement. The change in the rebar's radius due to the pressure P is calculated considering the rebar as a linear elastic solid cylinder subjected to an external pressure (Figure 3.9) (Timoshenko 1956) (Equation 3.41).

$$[3.41] \quad \Delta r_{bp} = \frac{1-\nu_s}{E_s} \times P(r_b - x)$$

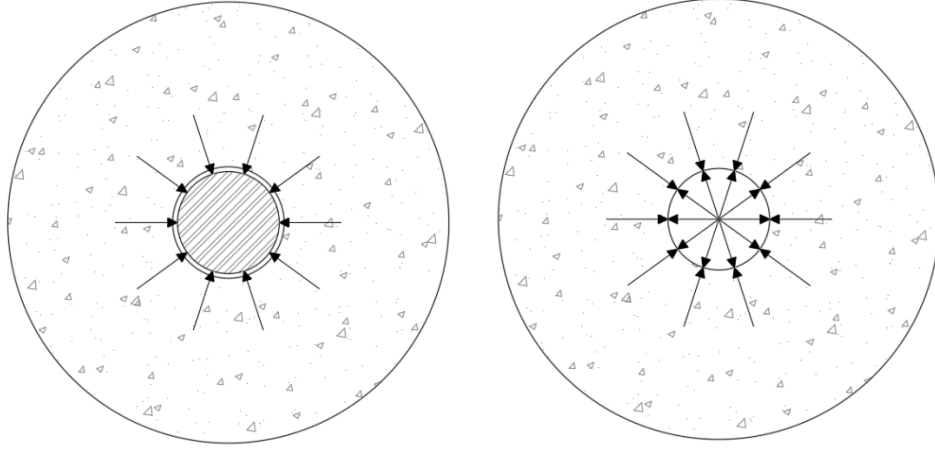


Figure 3.9: Steel reinforcement under uniform radial pressure

where ν_s and E_s are the Poisson's ratio and the Young's modulus of steel, respectively. P is the pressure at the interface, r_b is the original radius of the rebar and x is the attack penetration. The attack penetration is the radius of the original rebar minus the radius of the corroded rebar.

The pressure (P) at the rebar/concrete interface is equal to the radial stress in the concrete cover at the interface ($i = 0$). However, the stresses in the concrete cover cannot be accurately calculated unless Δr_b is known, which is also related to the compressibility of the materials and consequently to the pressure P . The calculations start with the assumption of free expansion (rigid corrosion products) to estimate the radial stress in the concrete cover at the interface. The program then runs the analyses for a second time considering the compressibility of the materials according to the calculated pressure in the previous step. The program iterates the analysis until the pressure converges based on Equation 3.42.

$$[3.42] \quad P_2 - P_1 = \max\left(0.1, \frac{P_2}{10}\right)$$

where P_2 and P_1 are the recent and previous calculated pressures, respectively.

3.4 Accounting for the Volume of the Cracks

As the concrete cover cracks due to corrosion-induced pressure, the cracks also provide some space within them to be filled by the corrosion products. This space is in addition to the space provided by the displaced inner boundary of the thick-walled cylinder. Assuming that the cracks are triangular, the volume of the corrosion products accommodated within the corrosion-induced cracks per unit length of the rebar, V_{cr} , can be estimated from Equation 3.43 (Figure 3.10):

$$[3.43] \quad V_{cr} = \frac{1}{2} C_v \times w_{cr} \times (r_c - r_r)$$

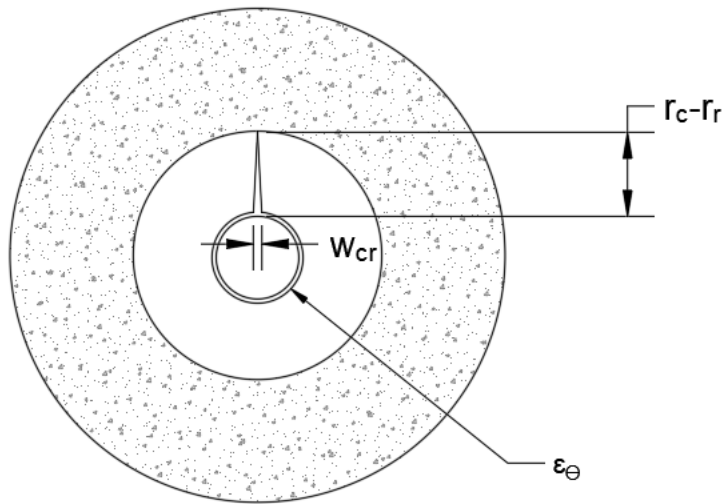


Figure 3.10: Accounting for the volume of the cracks

where r_c and r_r are the crack and rust fronts, respectively, w_{cr} is the total crack width at the reinforcement/concrete interface, and C_v is a crack volume coefficient. The crack volume coefficient accounts for the fact that not all of the available space within the cracks is filled by the corrosion products, as observed by Merino (2014).

According to Molina et al. (1993), the crack width, w_{cr} , can be estimated from Equation 3.44, where smeared cracking has been assumed:

$$[3.44] \quad w_{cr} = 2\pi r(\varepsilon_{\theta} - \varepsilon_{cr})$$

where r is the radius where ε_{θ} is calculated, and ε_{cr} can be calculated from Equation 3.45:

$$[3.45] \quad \varepsilon_{cr} = \frac{f'_t}{E_0}$$

where f'_t and E_0 are the tensile strength and the initial Young's modulus of the concrete, respectively.

3.5 Solution Procedure

The following flowchart summarizes the calculation procedure in the numerical model (Figure 3.11). Note that the procedure is simplified by removing some of the minor steps. The model considers the porous zone around the corroding rebar where it finds Δr_b .

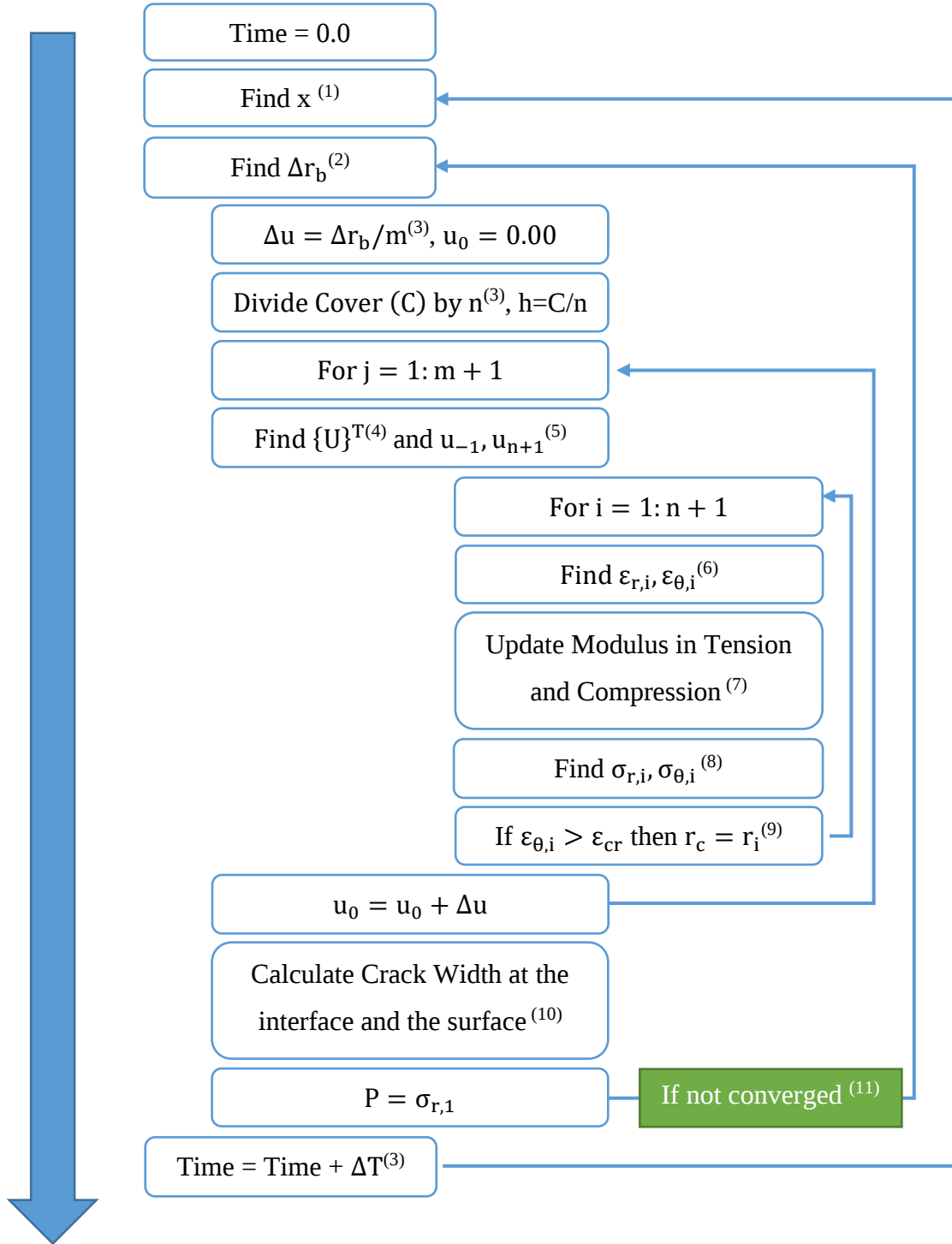


Figure 3.11: Solution procedure of proposed numerical model

(1) Faraday's law:

$$[3.46] \quad x = 0.0116 \times i_{\text{cor}} \times t$$

where x is the attack penetration in mm, t is the time after the start of the corrosion in years, and i_{corr} is the corrosion current density in $\frac{\mu\text{A}}{\text{cm}^2}$ ($100 \frac{\mu\text{A}}{\text{cm}^2}$ is used in the default calculation).

(2) Δr_b is calculated with Equation 3.47.

$$[3.47] \quad \Delta r_b = \sqrt{\frac{V_{\text{uct}}}{\pi}} - \Delta r_{\text{bp}} - r_b - t_p \geq 0$$

where t_p is the thickness of porous zone, V_{total} and Δr_{bp} are calculated with Equations 3.48 and 3.41, respectively. P is assumed zero in the first run. The crack front (r_c) is based on the crack front of the previous time step.

$$[3.48] \quad V_{\text{total}} = (\pi \times (r_b - x)^2) + V_{\text{rr}} - \Delta V_p - V_{\text{cr}}$$

where V_{rr} , V_{cr} , and ΔV_p are calculated with Equations 3.49, 3.43, and 3.40, respectively. V_{uct} is the uncompressed total volume of the corroded rebar and the rust around it (it does not consider the compressibility and does not include the volume of the corrosion products within the cracks).

$$[3.49] \quad V_{\text{rr}} = R_v \times \pi \times (r_b^2 - (r_b - x)^2)$$

(3) m is 100, n is 40, and ΔT is 0.1 day in the default calculations.

(4) Calculated with Equations 3.24-3.26.

(5) Calculated with Equations 3.29 and 3.30.

(6) Calculated with Equations 3.50 and 3.51:

$$[3.50] \quad \varepsilon_{r,i} = \frac{u_{i+1} - u_{i-1}}{2h}$$

$$[3.51] \quad \varepsilon_{\theta,i} = \frac{u_i}{h}$$

where $\varepsilon_{r,i}$ and $\varepsilon_{\theta,i}$ are, respectively, the radial and hoop strains at radial coordinate of r_i , where r_i is defined by Equation 3.52.

(7) Equations 3.27 and 3.28.

(8) $\sigma_{r,i}$ and $\sigma_{\theta,i}$ are calculated based on Section 3.2, $\varepsilon_{r,i}$, and $\varepsilon_{\theta,i}$. Consequently, secant moduli are calculated with Equation 3.53.

$$[3.52] \quad E_{c,i} = \frac{\sigma_{r,i}}{\varepsilon_{r,i}}, E_{t,i} = \frac{\sigma_{\theta,i}}{\varepsilon_{\theta,i}}$$

(9) r_i is defined by Equation 3.52:

$$[3.53] \quad r_i = r_b + (i - 1) \times h$$

(10) Equation 3.44 using u_1 and u_{n+1} for the crack width at interface and surface, respectively.

(11) Based on Equation 3.42.

Chapter 4

Analytical Model

4.1 Modelling Assumptions

The aim of this chapter is to develop a closed-form solution to calculate the internal pressures and deformations in the concrete cover as a result of reinforcement corrosion expansion (Figure 4.1). To achieve this objective, several assumptions have been made in addition to those adopted in the numerical model discussed in the previous chapter. These idealizations are summarized below.

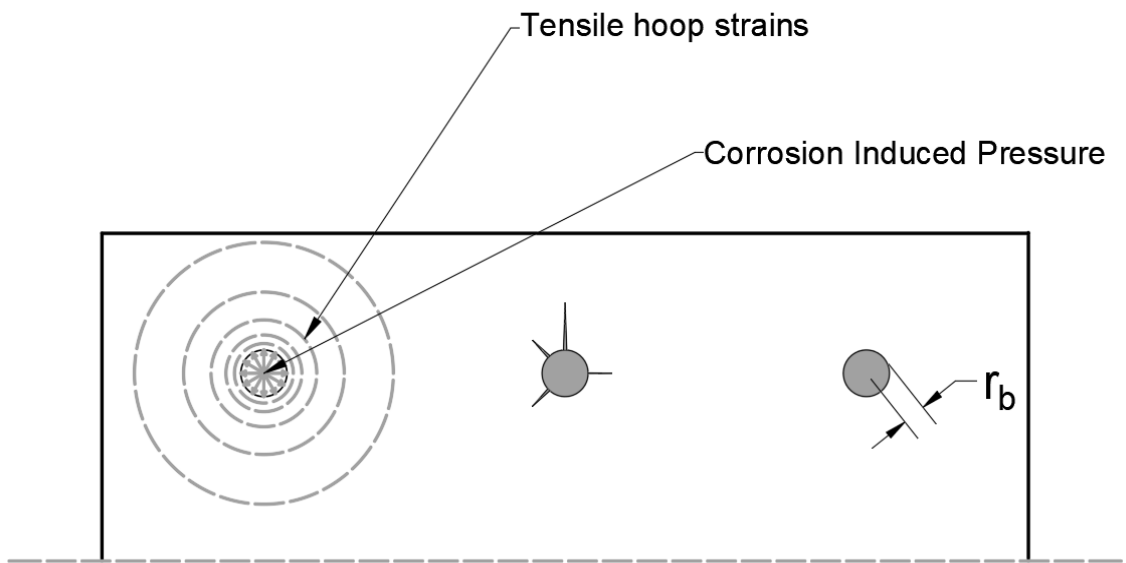


Figure 4.12: Partially cracked cover of RC due to reinforcement corrosion.

First Idealization (Thick-Walled Cylinder):

Like the numerical model, it is assumed that the concrete cover can be modelled as a thick-walled cylinder (Figure 4.2).

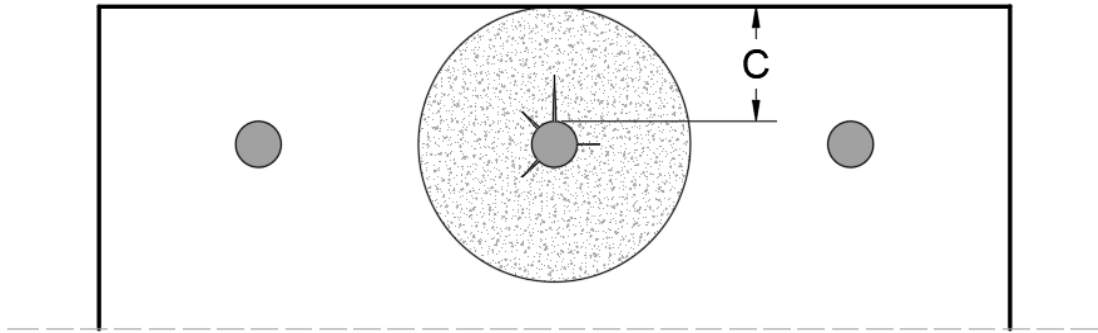


Figure 4.13: The thick-walled cylinder idealization of the concrete cover

Second Idealization (Dividing the Thick-Walled Cylinder):

The thick-walled cylinder is further divided into two concentric cylinders: an outer un-cracked cylinder and an inner cracked cylinder (Figure 4.3).

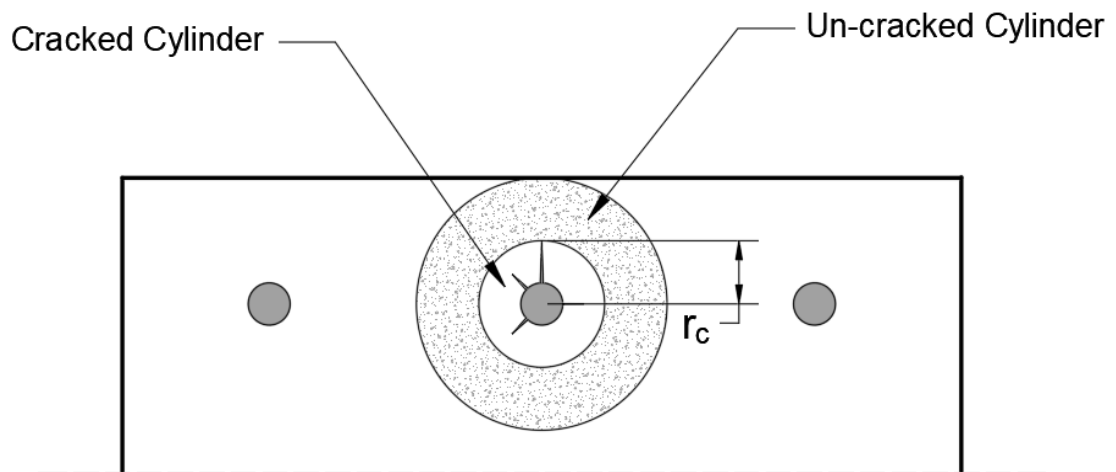


Figure 4.14: An outer un-cracked cylinder and an inner cracked cylinder

Third idealization (Smeared-Cracking):

It is assumed that there is no transfer of shear stresses at the interface between the two cylinders, as if there were rollers between them. Alternatively, it is assumed that radial cracks are equally smeared in the tangential direction. Any of the two assumptions cancels out shear stresses within both cylinders.

From the first three assumptions, Equation 4.1 can be concluded:

$$[4.54] \quad \begin{cases} \varepsilon_{\theta} > \varepsilon_{cr}, & r < r_c & (a) \\ \varepsilon_{\theta} < \varepsilon_{cr}, & r > r_c & (b) \\ \varepsilon_{\theta} = \varepsilon_{cr}, & r = r_c & (c) \end{cases}$$

where ε_{θ} is the hoop strain, r is the radial coordinate, r_c is the crack front, and ε_{cr} is the cracking strain, which can be calculated using Equation 4.2:

$$[4.55] \quad \varepsilon_{cr} = \frac{f'_t}{E_c}$$

where f'_t and E_c are the tensile strength and Young's modulus of the concrete cover, respectively.

The radial deformation at the crack front, u_b , can be found using Equation 4.3 (Timoshenko 1956):

$$[4.56] \quad \varepsilon_{cr} = \frac{u_b}{r_c} \quad \rightarrow \quad u_b = r_c \times \varepsilon_{cr}$$

where u_b is the radial deformation at the crack front (Figure 4.4).

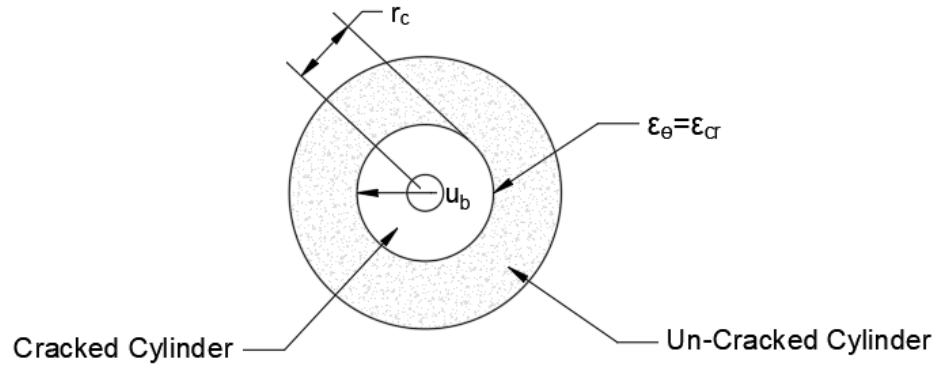


Figure 4.15: The outer un-cracked cylinder

Fourth idealization (Un-Cracked Cylinder):

Concrete is assumed to behave as a linear elastic material in both tension and compression. Consequently, Equation 4.4 can be written for the un-cracked thick-walled cylinder subjected to internal and external pressures (Timoshenko 1956) :

$$[4.57] \quad u_b = \frac{1-\nu_c}{E_c} \times \frac{(r_c^2 P_c - (C+r_b)^2 P_e) r_c}{(C+r_b)^2 - r_c^2} + \frac{1+\nu_c}{E_c} \times \frac{(P_c - P_e) r_c (C+r_b)^2}{(C+r_b)^2 - r_c^2}$$

where P_e and P_c are the outer and inner pressures applied to the un-cracked cylinder, respectively (Figure 4.5a), ν_c is the Poisson's ratio of concrete, C is the cover depth, and r_b is the rebar radius.

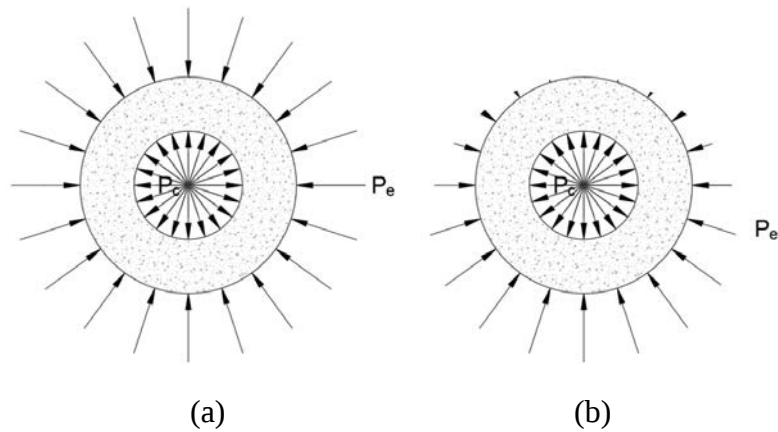


Figure 4.16: Applied pressures to inner and outer boundaries of un-cracked cylinder,

(a) Idealized uniform outer pressure, (b) More realistic non-uniform outer pressure

However, the outer pressure imposed by the concrete around the cylinder is not uniform (Figure 4.5.b) (it is zero at the outer concrete surface and higher in the regions with higher concrete depth). In this study, it is assumed to be zero on all sides (Equation 4.5):

$$[4.58] \quad P_e = 0.0$$

By using Equation 4.5, Equation 4.4 is simplified to Equation 4.6:

$$[4.59] \quad u_b = \frac{P_c r_c}{E_c((C+r_b)^2 - r_c^2)} \times [(1 - \nu_c)r_c^2 + (1 + \nu_c)(C + r_b)^2]$$

By noting that the radial compressive stress at the interface of the two cylinders σ_b is equal to the internal pressure P_c , Equation 4.6 can be rearranged as Equation 4.7:

$$[4.60] \quad \sigma_b = P_c = \frac{u_b E_c ((C+r_b)^2 - r_c^2)}{r_c [(1-\nu_c)r_c^2 + (1+\nu_c)(C+r_b)^2]}$$

Fifth idealization (Cracked Cylinder):

It is assumed that tensile hoop stresses are constant in the cracked cylinder. A lower limit and an upper limit are defined assuming concrete as (i) a brittle elastic material (concrete tension softening is ignored) and (ii) an elastoplastic material (Figure 4.6). The two sets of equations derived from the two assumptions (brittle elastic and elastoplastic) are combined in Equations 4.8, 4.10, 4.11, 4.12, and 4.24, in which the first terms represent the linear elastic behavior (brittle elastic model) and the second terms represent the plastic behavior (setting the latter one to zero reduces the equation to the brittle elastic model).

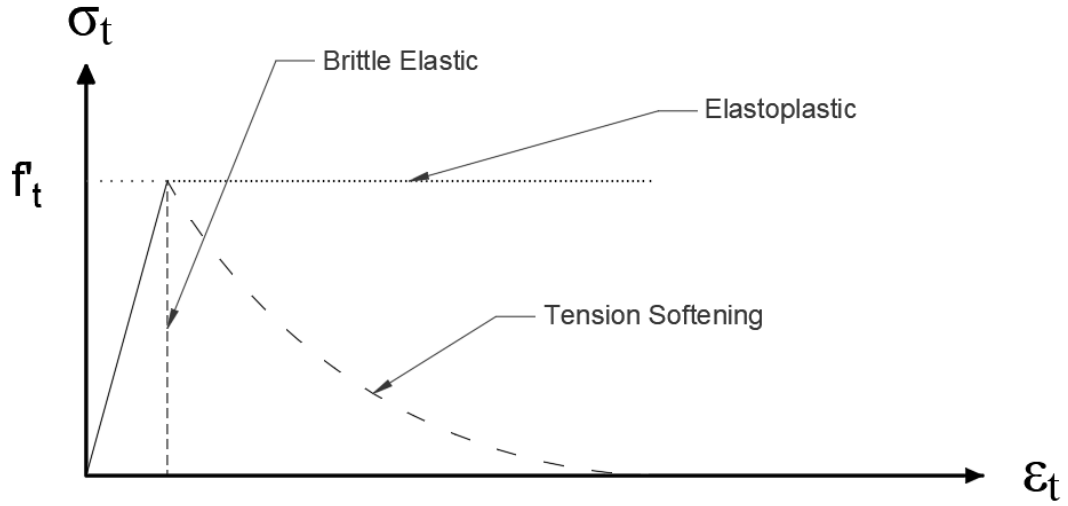


Figure 4.17: Idealized behaviour of concrete in tension

Consequently, Equation 4.8 is written using equilibrium in the radial direction (Figure 4.7):

$$[4.61] \quad P = \underbrace{\sigma_b \times \frac{r_c}{r_i}}_{\text{Brittle Elastic Term}} + \underbrace{\left[\frac{1}{r_i} \times \int_{r_i}^{r_c} \sigma_{\theta}(r) dr \right]}_{\text{Additional Elastoplastic Term}} = \sigma_b \times \frac{r_c}{r_i} + \left[\frac{1}{r_i} \times (f'_t \times (r_c - r_i)) \right]$$

where P is the corrosion-induced pressure, r_i is the inner radius of the cracked cylinder, and σ_{θ} is the tensile hoop stress at radial coordinate r . The inner radius r_i is obtained from:

$$[4.62] \quad r_i = r_b + t_p$$

where t_p is the thickness of the concrete porous zone around the rebar.

Radial compressive stresses and strains in the cracked cylinder (Figure 4.7) can be found using Equations 4.10 and 4.11:

$$[4.63] \quad \sigma_r = \underbrace{\sigma_b \times \frac{r_c}{r}}_{\text{Brittle Elastic Term}} + \underbrace{\left[\frac{1}{r} \times \int_r^{r_c} \sigma_\theta(r) dr \right]}_{\text{Additional Elastoplastic Term}} = \sigma_b \times \frac{r_c}{r} + \left[\frac{1}{r} \times (f'_t \times (r_c - r)) \right]$$

$$[4.64] \quad \epsilon_r = \frac{\sigma_r}{E_c} = \underbrace{\frac{r_c \times \sigma_b}{r \times E_c}}_{\text{Brittle Elastic Term}} + \underbrace{\left[\left(\frac{1}{r} \times \left(\frac{f'_t}{E_c} \times (r_c - r) \right) \right) + \left(v_c \times \frac{f'_t}{E_c} \right) \right]}_{\text{Additional Elastoplastic Term}}$$

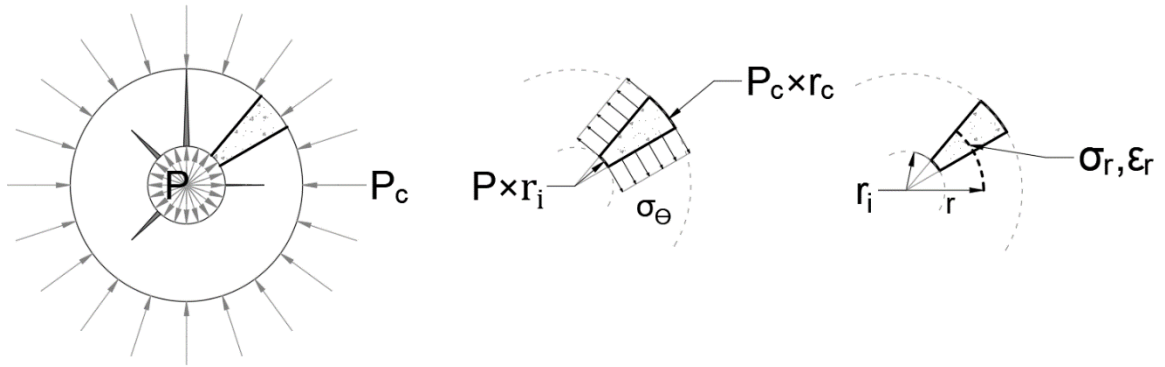


Figure 4.18: The cracked cylinder

where r is the radial coordinate ($r_b < r < r_c$). Consequently, the radial compressive deformation of the inner cracked cylinder is provided by Equation 4.12:

$$\begin{aligned}
& \text{Brittle Elastic Term} \\
[4.65] \quad u_c &= \int_{r_i}^{r_c} \varepsilon_r dr \rightarrow u_c = \underbrace{\frac{r_c \times \sigma_b}{E_c} \times \int_{r_i}^{r_c} \frac{1}{r} dr}_{\text{Brittle Elastic Term}} \\
& + \underbrace{\left[\frac{f'_t}{E_c} \times \left(r_c \times \int_{r_i}^{r_c} \frac{1}{r} dr - (1 - \nu_c) \int_{r_i}^{r_c} dr \right) \right]}_{\text{Additional Elastoplastic Term}} \\
\rightarrow u_c &= \underbrace{\frac{\sigma_b \times r_c}{E_c} \times \ln \frac{r_c}{r_i}}_{\text{Brittle Elastic Term}} + \underbrace{\left[\frac{f'_t \times r_c}{E_c} \ln \frac{r_c}{r_i} + \frac{(\nu_c - 1) \times f'_t \times (r_c - r_i)}{E_c} \right]}_{\text{Additional Elastoplastic Term}}
\end{aligned}$$

where u_c is the total radial compressive deformation of the cracked cylinder.

4.2 Computing Available Space for Corrosion Products

The radial deformation at the internal boundary of the cracked cylinder can be calculated using Equation 4.13 (Figure 4.8):

$$[4.66] \quad \Delta r_b = u_b + u_c$$

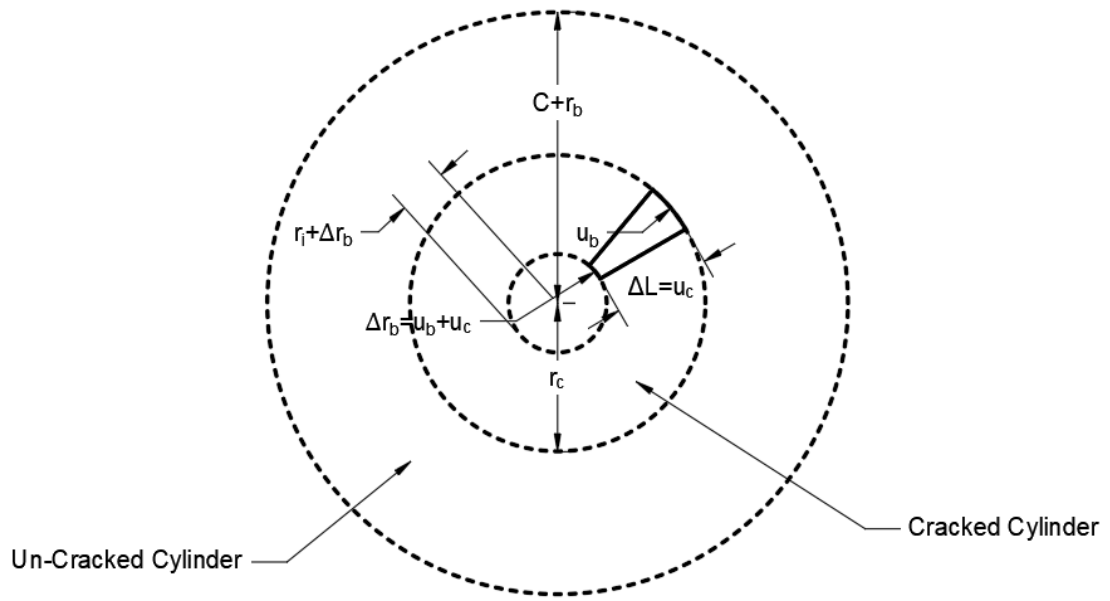


Figure 4.19: Radial deformation at the internal boundary of the cracked cylinder

The total available space to be occupied by the corrosion products consists of the available space around the rebar due to corrosion penetration and the available space within the corrosion-induced cracks (Equation 4.14) (Figure 4.9).

$$[4.67] \quad V_t = V_r + V_{cr}$$

where V_t is the total available volume per length of rebar, V_r and V_{cr} are the available spaces around the corroding rebar and within the cracks per length of rebar, respectively.

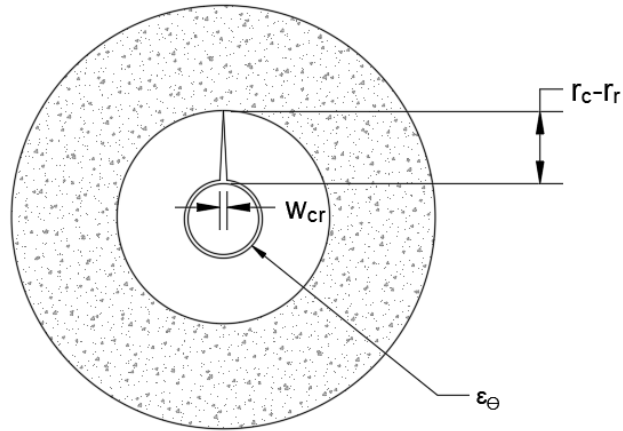


Figure 4.20: Crack width in inner cracked cylinder

The radial coordinate of the cracked cylinder's inner boundary after the deformation can be expressed by Equation 4.15 (Figure 4.10):

$$[4.68] \quad r_r = r_i + \Delta r_b$$

where r_r is the radial coordinate of the cracked cylinder's inner boundary after deformation has taken place (rust front).

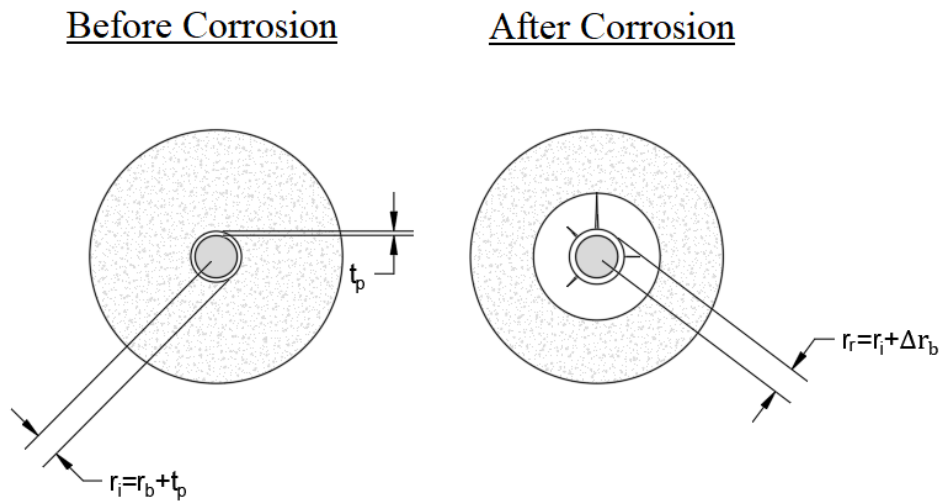


Figure 4.21: Definition of r_i and r_r

The radius of the corroded rebar can be found using Equation 4.16:

$$[4.69] \quad r_{rb} = \left(r_b^2 - \frac{\Delta V_s}{\pi} \right)^{\frac{1}{2}}$$

where r_{rb} is the radius of the corroded rebar, and ΔV_s denotes the volume of consumed steel due to corrosion.

If corrosion products are assumed to be rigid, the radial coordinate of the inner boundary of the cracked cylinder can be found using Equation 4.17:

$$[4.70] \quad r_{rg} = \left(r_{rb}^2 + \frac{\Delta V_r}{\pi} \right)^{\frac{1}{2}}$$

where r_{rg} is the radial coordinate of the inner boundary of the cracked cylinder if the corrosion products were rigid.

4.3 Considering the Compressibility of Rust

The analytical model can also account for the compressibility of corrosion products by considering the bulk modulus of the rust (Equation 4.18):

$$[4.71] \quad B_r = -V \left(\frac{dP}{dV} \right) = V_r \left(\frac{P}{\Delta V_p} \right)$$

where ΔV_p is the change in volume of the corrosion products due to the applied pressure, P is the corrosion-induced pressure, and B_r denotes the bulk modulus of the corrosion products. Therefore, the change in volume of the corrosion products due to the applied pressure can be calculated using Equation 4.19:

$$[4.72] \quad \Delta V_p = \frac{V_r \times P}{B_r}$$

The radial coordinate of the inner boundary of the cracked cylinder can be found using Equation 4.20:

$$[4.73] \quad r_r = \left(r_{rg}^2 - \frac{\Delta V_p}{\pi} \right)^{\frac{1}{2}} = r_i + \Delta r_b$$

where r_r is the radial coordinate of the inner boundary of the cracked cylinder.

Equations 4.16, 4.17 and 4.20 can be combined into Equation 4.21:

$$[4.74] \quad r_r = \left(r_b^2 - \frac{\Delta V_s}{\pi} + \frac{V_r}{\pi} - \frac{\Delta V_p}{\pi} \right)^{\frac{1}{2}}$$

By substituting Equation 4.19 into Equation 4.21, Equation 4.22 is derived:

$$[4.75] \quad r_r = \left(r_b^2 - \frac{\Delta V_s - \Delta V_r \left(1 - \frac{P}{B_r} \right)}{\pi} \right)^{\frac{1}{2}}$$

The corrosion products within the corrosion-induced cracks can be calculated using Equation 4.23 (Figure 4.9) by assuming that the cracks are triangular in shape:

$$[4.76] \quad V_{cr} = \frac{1}{2} \times C_v \times w_{cr} \times (r_c - r_r)$$

where C_v is a crack volume coefficient, and w_{cr} can be calculated using Equation 4.24:

$$[4.77] \quad \begin{cases} w_{cr} = 0.0 \\ w_{cr} = 2\pi r(\varepsilon_\theta - \varepsilon_{cr}) \end{cases} = \underbrace{2\pi r(\varepsilon_\theta)}_{\text{Brittle Elastic Term}} + \underbrace{[-2\pi r(\varepsilon_{cr})]}_{\text{Additional Elastoplastic Term}}, \quad \begin{matrix} \varepsilon_\theta \leq \varepsilon_{cr} \\ \varepsilon_\theta > \varepsilon_{cr} \end{matrix}$$

4.4 Computing Attack Penetration and Corresponding Time

The rust expansion coefficient (R_v) is equal to the volumetric ratio of the corrosion products to the corroded parent steel (Equation 4.25):

$$[4.78] \quad R_v = \frac{V_r + V_{cr}}{\Delta V_s}$$

From Equations 4.22 and 4.25, the volume of consumed steel due to corrosion, ΔV_s , can be found from Equation 4.26:

$$[4.79] \quad \Delta V_s = \frac{\pi(r_r^2 - r_b^2) + V_{cr}}{R_v \left(1 - \frac{P}{B_r}\right) - 1}$$

The attack penetration producing ΔV_s is obtained from Equation 4.27. Note that the attack penetration was defined as the radius of the original rebar minus the radius of the corroded rebar:

$$[4.80] \quad x = r_b - \left(r_b^2 - \frac{\Delta V_s}{\pi}\right)^{\frac{1}{2}}$$

where x is the corresponding attack penetration.

The corresponding time to achieve the attack penetration (x) assuming uniform corrosion can be found using Faraday's law (Equation 4.28):

$$[4.81] \quad t = \frac{x}{0.0116 \times i_{cor}}$$

where t is the time after the start of the corrosion in years, x is given in mm, and i_{cor} is the corrosion current density in $\frac{\mu A}{cm^2}$.

4.5 Solution Procedure

The following flowchart summarizes the solution procedure for the analytical model (Figure 4.10). The analytical model solves the problem backwards compared to the numerical model, starting with an assumption of the location of the crack front instead of proceeding forward in time.

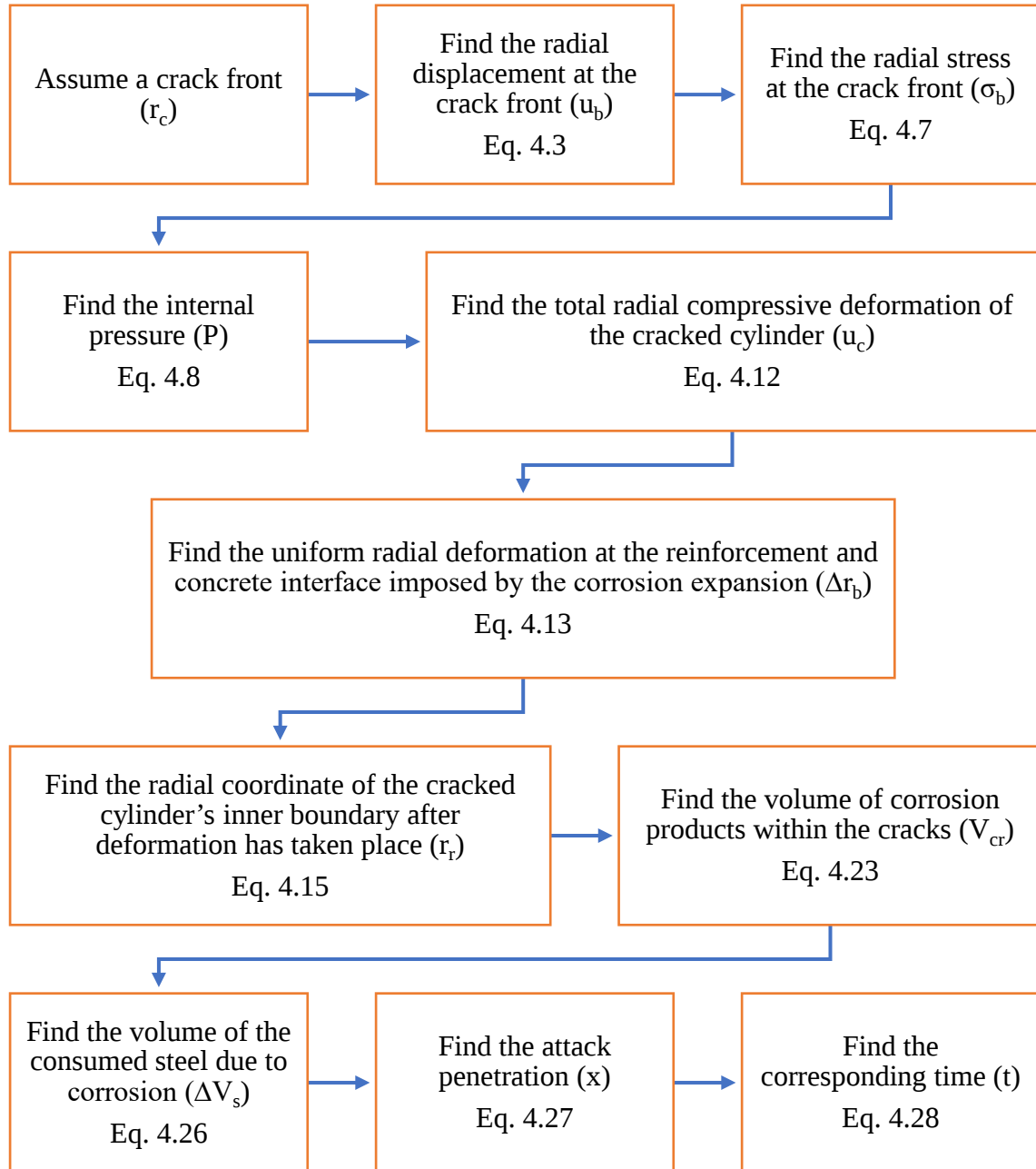


Figure 4.22: Solution procedure for the proposed analytical model

Chapter 5

Results and Comparison

The purpose of this chapter is to examine and validate the models presented in Chapters 3 and 4 by comparing them to the reported experimental data found in the literature as well as to each other. First, it is discussed how the input parameters were selected. Second, the results are compared against each other as well as the reported experimental data. Third, a parametric study is performed to demonstrate the effect of different input parameters on the results.

5.1 Reference Cases and Parameter Selection

Several experimental studies reported in the literature on the time to cover cracking due to reinforcement corrosion were used to benchmark the results of the models in Chapters 3 and 4 (Al-Sulaimani et al. 1990; Al-Saadoun and Al-Gahtani 1992; Andrade et al. 1993; Alonso et al. 1998; Liu and Weyers 1998; Torres-Acosta and Sagues 2004; Vu et al. 2005). In particular, the experimental results were used to calibrate the (i) rust expansion coefficient (R_v), (ii) porous zone thickness (t_p), and (iii) crack volume coefficient (C_v). These three parameters were calibrated to find a single value for each to best fit all of the experimental data used in this work. This approach is different to that of some other researchers (e.g., Wang and Liu (2004)), since only a single average value was sought instead of a very broad range of values. This approach ensures that the models are practical. The best fit was found using the numerical model presented in Chapter 3. Although the set

of values found using the numerical model does not provide the best fit for the analytical models, the same set of parameters was used for all the models for comparison purposes.

Experimental values reported by the researchers were directly used as input parameters in the models. However, in cases where input parameters were not reported in the original studies, they were assumed as described in the following subsections 5.1.1-5.1.4.

5.1.1 Corrosion Rate and Localization

As explained in Section 2.2.1.1, there is a lack of agreement among researchers on how to relate time (t) to corrosion attack penetration (x), let alone the fact that the nominal and actual current densities are not always the same (Alonso et al. 1998). Therefore, the models were verified by comparing the calculated critical attack penetration against the reported experimental values if the critical attack penetrations (or mass losses) were reported by the researchers. Note that the critical attack penetration refers to the attack penetration corresponding to through cracking of the concrete cover. This verification approach ensures that the validation is independent of the relationship between time and attack penetration. Faraday's law was used to estimate the experimental critical attack penetration if measured values of the critical attack penetrations were not reported.

In all analyses, it was assumed that corrosion was uniformly distributed around the perimeter and length of the reinforcing bar.

5.1.2 Steel and Concrete Properties

The modulus of elasticity and Poisson's ratio of steel were assumed to be 200 GPa and 0.3, respectively (CEB-FIP Model Code 2010). Poisson's ratio for concrete was assumed to be 0.2 (CEB-FIP Model Code 2010).

Where the researchers did not measure or report the concrete tensile strength (f'_t) or the initial elastic modulus (E_o), these values were estimated using Equations 5.1 and 5.2 as

given by the CEB-FIP Model Code (2010). When compressive strength (f'_c) was not reported and only tensile strength (f'_t) was given, the compressive strength (f'_c) was back calculated from Equation 5.1. Tensile strength, compressive strength, and Young's modulus of the studied tests ranged from 2.4 to 4.55 MPa, 20 to 55 MPa, and 19.6 to 37.4 GPa, respectively (Table 5.2).

$$[5.1] \quad f'_t = 0.3 (f'_c)^{\frac{2}{3}}$$

Equation 5.1 is valid for concretes with a 28-day compressive strength f'_c of less than or equal to 50 MPa. It estimates the mean value of the concrete tensile strength. The lower and upper bounds are 0.7 and 1.3 times the mean value, respectively. Although experimental values for f'_t go up to 5 MPa, the use of Equation 5.1 is not expected to make any practical difference for the purpose of this study.

$$[5.2] \quad E_o = E_{co} \times \alpha_E \times \left(\frac{f'_c}{10} \right)^{\frac{1}{3}}$$

where E_{co} is equal to 21.5 GPa, f'_c is the 28-day compressive strength of concrete, and α_E is a parameter related to the aggregate type, as listed in Table 5.1. In cases where the aggregate type is unknown, α_E was assumed to be 1.0.

Table 5.1: Aggregate type factor α_E (CEB-FIP Model Code 2010)

Type of Aggregate	α_E
Basalt, dense limestone aggregates	1.2
Quartzite aggregates	1.0
Limestone aggregates	0.9
Sandstone aggregates	0.7

The fracture energy (G_f) of concrete can be measured by experimental tests. However, since the experimental studies used for the validation of the models proposed herein did

not report the fracture energy of the concrete used, Equation 5.3 taken from the CEB-FIP Model Code (2010) was used to estimate the fracture energy of the concrete.

$$[5.3] \quad G_f = 73 \times f'_c{}^{0.18}$$

In cases where the researcher did not report the maximum aggregate size d_a , a value of 16 mm was assumed in lieu of more accurate values.

5.1.3 Geometric Dimensions

The experimental results include a broad range of reinforcing bar diameters and cover depths. The reinforcing bar diameters, cover depths, and cover-to-diameter ratios ranged from 8 to 38.1 mm, 20 to 76.2 mm, and 0.76 to 7.5, respectively.

The space between adjacent reinforcing bars (S) was assumed to be large enough so that the preferential crack propagation path is towards the concrete surface (Section 2.2.3.2). This assumption is consistent with the experimental data used in the validation.

5.1.4 Rust Properties

A bulk modulus of 0.5 GPa was used to consider the compressibility of corrosion products (Section 2.2.4.1).

5.2 Comparison of Models

5.2.1 Results against Time

The numerical and analytical models presented in Chapters 3 and 4, respectively, were used to simulate the experimental results of specimen SII_1 reported by Vu et al. (2005), wherein $d_b = 16$ mm, $C = 50$ mm, $f'_t = 3.94$ MPa, $f'_c = 42.25$ MPa, $E_c = 34.7$ GPa, $d_a = 20$ mm, $\nu_c = 0.2$, $E_s = 200$ GPa, $\nu_s = 0.3$. The attack penetration (x) was related to time through Faraday's law, where the corrosion current density (i_{cor}) was taken as 100

$\mu\text{A}/\text{cm}^2$. The thickness of the porous zone (t_p), the rust expansion coefficient (R_v), and the crack volume coefficient (C_v) were assumed to be $10\text{ }\mu\text{m}$, 2.75, and 0.5, respectively.

Figure 5.1 plots the resulting rust front (r_r) and radius of the corroded rebar ($r_b - x$) against time. The radius of the corroded rebar is independent of the chosen model (i.e., numerical or analytical). The results of rust front based on the three models nearly coincide with each other as the rate of the rust production is the same in all three models with the given assumptions; Faraday's law, the same corrosion current density and the same rust expansion coefficient are used in all three models. The very slight differences (around 0.2%) are due to the fact that the internal pressures calculated by each model are different from one another and thus cause different levels of compression of the corrosion products. The initial difference between the radius of the corroded rebar and the rust front is due to the thickness of the porous zone.

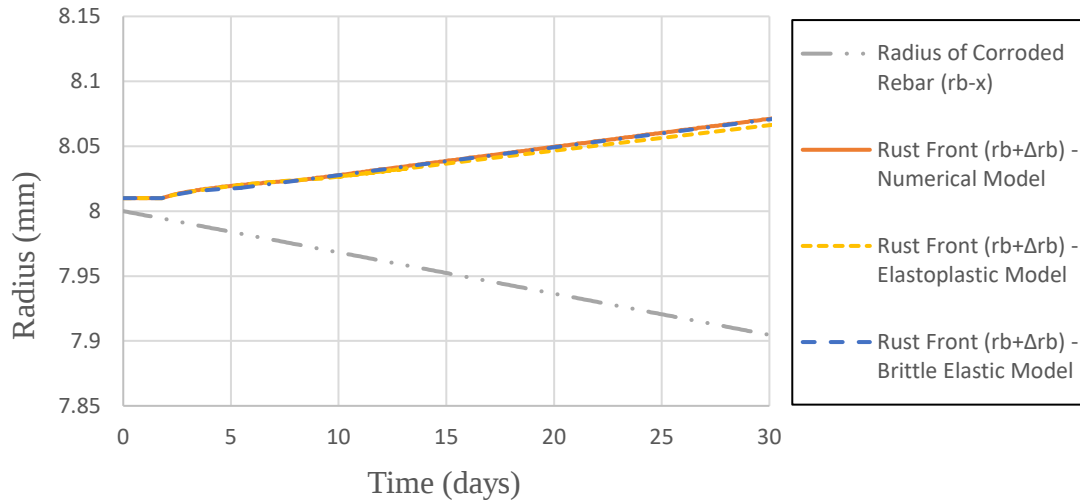


Figure 5.1: Rust front and the radius of corroded rebar against time

Figure 5.2 plots the crack front r_c against time based on the three proposed models: numerical, brittle elastic and elasto-plastic. The brittle elastic cylinder failed in a sudden manner when the crack front reached 53 mm. At a time of approximately 5 days, the crack-width suddenly increases from 53 mm to 58 mm. The chosen failure criterion and the brittle failure of the brittle elastic cylinder will be discussed further in this section. The two

analytical models provide an upper and a lower limit for the numerical results, as illustrated in Figure 5.2. The analytical models diverge as the crack front r_c increases. The first crack reaches the surface in 4.9, 8.0 and 9.2 days based on the brittle elastic cylinder, the numerical model and the elastoplastic cylinder, respectively.

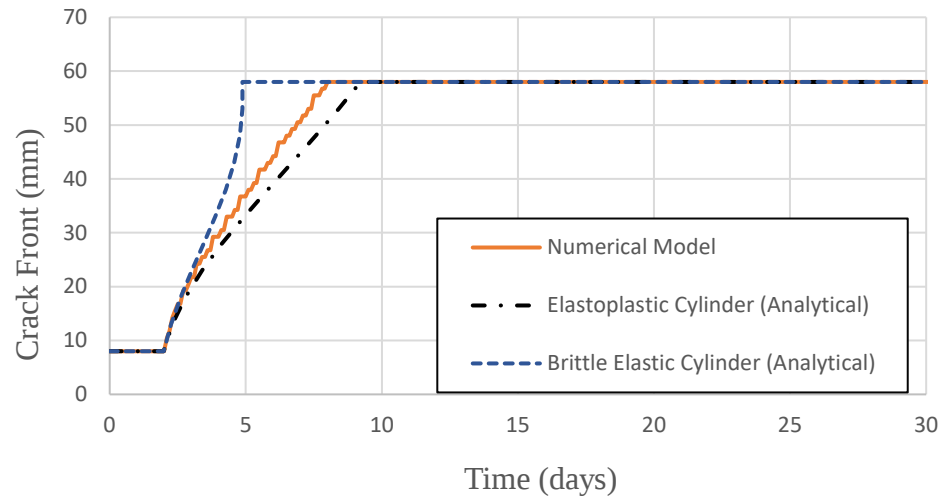


Figure 5.2: Crack front against time

Figure 5.3 plots the internal pressure generated as a result of the expansion of corrosion products against time based on the three different approaches. The results of the internal pressure buildup as well as the maximum internal pressure based on the numerical model lie between the results from the two analytical models. The results given by the analytical models diverge as time passes. The maximum internal pressure reached during the crack propagation process is 7.6, 18 and 24.6 MPa based on the brittle elastic cylinder, the numerical model and the elastoplastic cylinder, respectively.

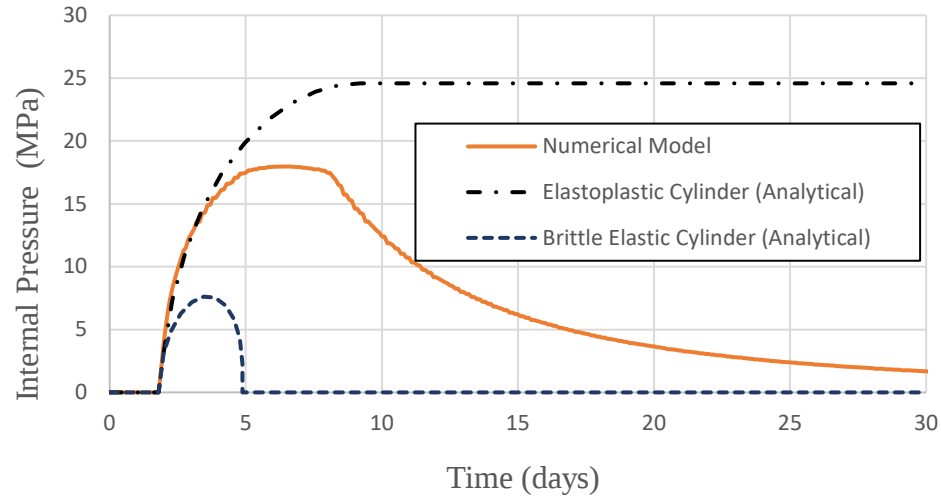


Figure 5.3: Internal pressure against time

Figure 5.4 plots the crack width against time based on the numerical model and the analytical models. The brittle behaviour of the elastic cylinder is also recognizable in this plot. At a time of approximately 5 days, the crack-width suddenly increases from zero to slightly over 0.05 mm. The first crack becomes visible to bare eyes in 5.5, 10.7 and 13.2 days based on the brittle elastic cylinder, the numerical model and the elastoplastic cylinder, respectively.

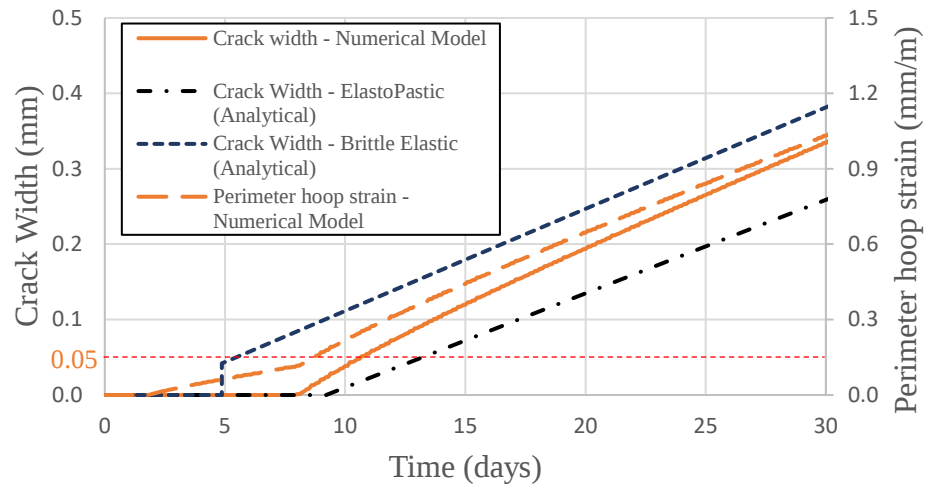


Figure 5.4: Crack width and perimeter hoop strain

The crack widths are calculated based on Equation 5.4:

[5.4]

$$\begin{cases} u_1(C + r_b) = \frac{w_{cr}}{2 \times \pi} + (C + r_b) \times \varepsilon_{cr} & \text{Elastoplastic or Numerical} \quad (a) \\ \Delta r_b = u_1(C + r_b) = \frac{w_{cr}}{2 \times \pi} & \text{Brittle Elastic Cylinder} \quad (b) \end{cases}$$

where variables are as defined in Chapter 3 and Chapter 4. Similar to previous diagrams, the two analytical models provide an upper and lower bound for the numerical results. The numerical results tend to be closer to the results based on the elastoplastic cylinder at the beginning of the cracking process and get closer to the results based on the brittle elastic cylinder as the cylinder loses its tensile capacity. The 0.05 mm line marks the first crack width which is visible to bare eyes. Figure 5.4 also plots the perimeter hoop strain ε_θ against time based on the numerical model.

Failure of Brittle Elastic Cylinder:

The brittle elastic cylinder fails either when the crack reaches the surface or when the crack propagation (increase of crack front) does not further increase the available space for the corrosion products to accommodate.

As explained in Chapter 4, Δr_b consists of two components, deformation u_b and u_c . Whereas u_b increases with the crack front, u_c initially increases to its maximum value and then decreases to zero as the internal pressure is dissipated. Consequently, the maximum value of Δr_b does not always occur when the crack front is at the cover's surface. Figure 5.5 plots u_b , u_c , and Δr_b corresponding to different crack fronts for the case discussed in this section. In this example, Δr_b is maximum at a crack front percentage of approximately 80%. The crack front percentage is defined by Equation 5.5.

$$[5.5] \quad \text{Crack front percentage} = \frac{\text{Crack front} - \text{Rebar's radius}}{\text{Cover depth}}$$

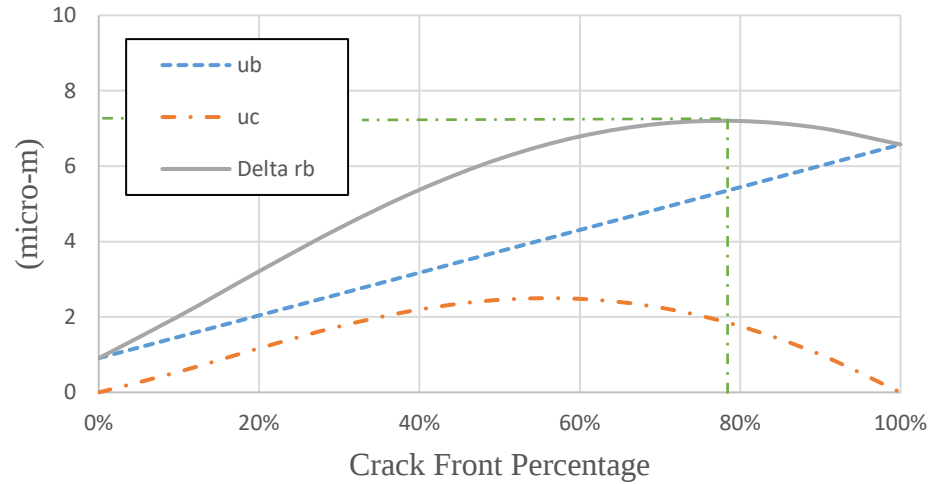


Figure 5.5: Calculated u_b , u_c , and Δr_b for different crack fronts based on the brittle elastic cylinder

Figure 5.6 plots the calculated Δr_b based on different crack fronts or different crack widths. The maximum calculated Δr_b based on different crack fronts corresponds to a crack front of 48 mm.

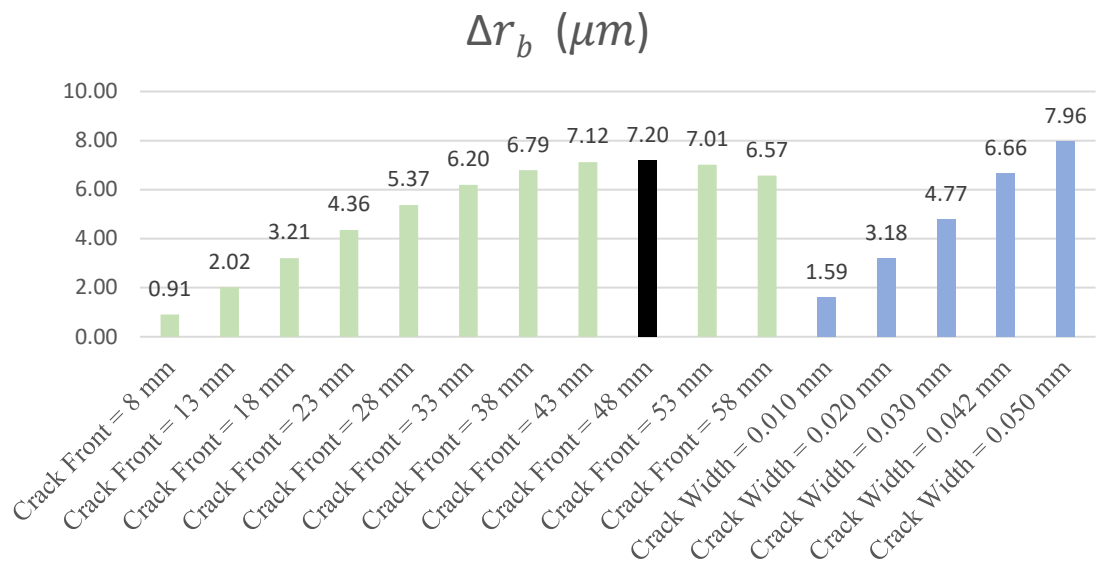


Figure 5.6: Calculated Δr_b for different crack fronts and crack widths

Figure 5.7 plots the calculated corresponding volume of consumed steel due to corrosion based on different crack fronts and different crack widths. Note that if Δr_b is known, the corresponding volume of consumed steel ΔV_s can be found from Equations 4.15, 4.23, and 4.26 (Figure 4.11). Figure 5.8 plots the corresponding uncompressed rust volume ($V_{rr} = R_v \Delta V_s$). According to Figure 5.8, when the crack front reaches 53 mm (90%), the crack propagation does not provide more space to the corrosion products anymore. Although Δr_b at a crack front percentage of 80% was maximum according to Figure 5.5, the maximum available space for corrosion products occurs at a crack front percentage of 90%. This is because the available space within the cracks V_{cr} always increases with the crack front. The behaviour of the elastic cylinder after the failure can be explained by Equation 5.4 (b). Based on this equation and Equations 4.15, 4.23, 4.26 (Figure 4.11), the failed cylinder must have at least a crack width of 0.042 mm to fit the same amount of corrosion products as it could fit just before its failure.

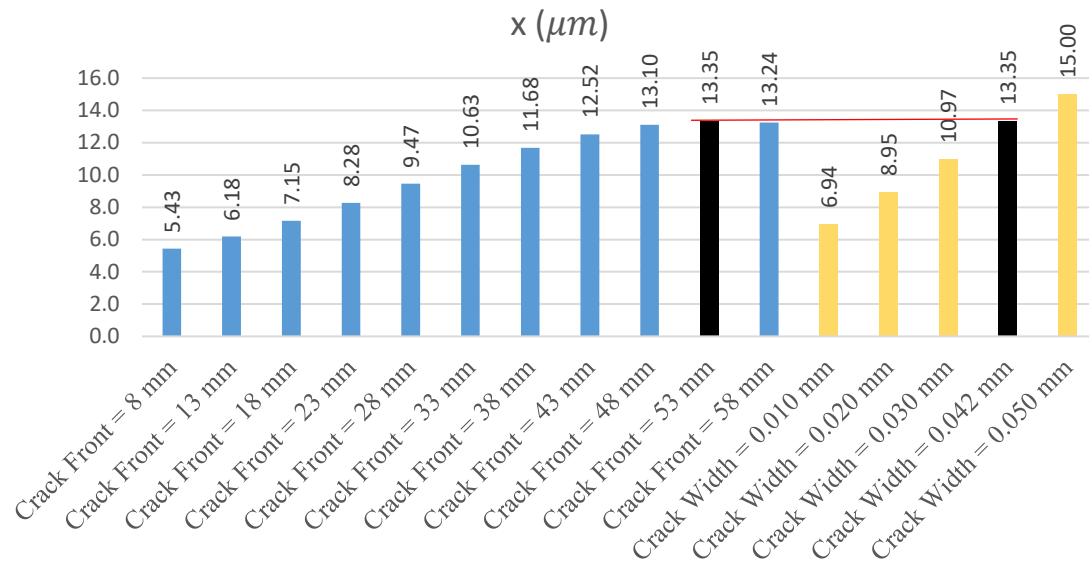


Figure 5.7: Calculated attack penetrations for different crack fronts and crack widths

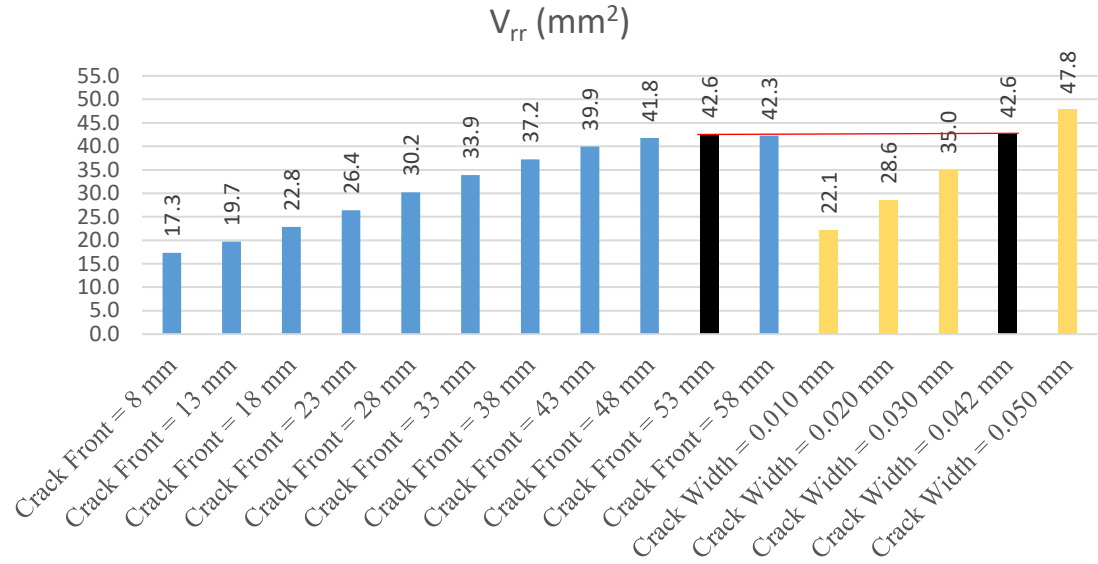


Figure 5.8: Calculated uncompressed rust volume based on different crack fronts and crack widths

5.2.2 Critical Attack Penetration

The critical attack penetration (x_{cr}) refers to the attack penetration corresponding to the time when a crack reaches the surface. However, experimental tests usually report the time or the attack penetration corresponding to the first crack which is visible to bare eyes (around 0.05 mm width) (Figure 5.4). This attack penetration is called the visual critical attack penetration (x_{vcr}) here. Based on the tests performed by Merino (2014), the visual critical attack penetration is about 2-3 times larger than the critical attack penetration for a rebar diameter and cover depth of 20 mm each. It is important to note that other researchers have compared the visual critical attack penetrations obtained from experimental tests to the critical attack penetrations calculated from their analytical or numerical models (e.g., Wang and Liu (2004a), Liu and Weyers (1998), Bhargava et al. (2006)), when in fact these two values correspond to different time frames.

5.2.2.1 Visual Critical Attack Penetration

Although the rust expansion coefficient (R_v), the thickness of the porous zone (t_p) around the rebar, and the crack volume coefficient (C_v) vary for each experimental test, a constant

porous zone thickness of 10 μm and crack volume coefficient of 0.5 were found to provide the best fit to the experimental data on average (based on the numerical approach). These values are within the acceptable range discussed in Section 2.2. The best fit was found by qualitative assessment based on trial and error.

Table 5.2 states the input data, some experimental results reported in the literature, and the results from the numerical and elastoplastic models for the visual critical attack penetrations. The numerical and analytical results are the attack penetrations corresponding to the first visual crack (crack width of 0.05 mm).

Figure 5.9 plots the ratio of the numerical results to the experimental results against the cover-to-diameter ratio. The average ratio and COV are 0.97 and 0.86, respectively. The numerical model can capture the overall trend observed in the experimental tests, although considerable scatter is observed in the results. The variability can be explained by the following:

- The input values for parameters R_v , C_v , t_p are chosen as constants in lieu of experimental values for each experimental test (which are not reported). However, they vary for different experimental tests.
- Although the value of 0.05 mm is chosen for the width of the first visual crack, this value can be slightly different based on the researcher.
- While the effect of w/c ratio on the porosity of the concrete is neglected, increasing w/c ratio increases the porosity in concrete. More porosity in concrete provides more space to accommodate the corrosion products, which consequently slows down the crack propagation. This effect can be incorporated in t_p and C_v parameters, although sufficient experimental data in the literature is currently lacking.
- Perfect uniform buildup of corrosion products around and along the reinforcement is assumed, which is not always the case.

Table 5.2: Comparison of visual critical attack penetrations between experimental results and proposed models

Experimental Test	d_b (mm)	C (mm)	f'_t (MPa)	f'_c (MPa)	E_o (MPa)	$d_a^{(1)}$ (mm)	Exp. x_{vcr} (μm)	Num. x_{vcr} (μm)	Anali. (E.P. ⁽²⁾) $x_{vcr}(\mu m)$	Num. /Exp.	E.P. /Exp.	E.P. /Num.
Liu and Weyers (1998)	16	25.4	3.30	31.5	<u>22062</u> ⁽³⁾	<u>16.0</u>	31	21	23	0.66	0.75	1.14
	16	50.8	3.30	31.5	<u>22062</u>	<u>16.0</u>	51	39	50	0.77	0.97	1.26
	12.7	50.8	3.30	31.5	<u>22062</u>	<u>16.0</u>	50	46	60	0.92	1.21	1.31
	16	76.2	3.30	31.5	<u>22062</u>	<u>16.0</u>	74	69	93	0.93	1.27	1.36
Andrade et al. (1993)	16	20	3.55	<u>39.6</u>	<u>23804</u>	<u>16.0</u>	17	18	20	1.06	1.18	1.11
	16	30	3.55	<u>39.6</u>	<u>23804</u>	<u>16.0</u>	18	23	27	1.26	1.50	1.19
Al-Saadoun and Al-Gahtani (1992)	8	36.8	<u>3.29</u>	35.4	<u>29491</u>	19.0	56	37	47	0.66	0.84	1.28
	9.5	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	42	28	34	0.68	0.83	1.22
	12.7	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	34	24	29	0.72	0.84	1.17
	19	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	23	21	23	0.9	1.00	1.11
	25.4	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	15	19	21	1.33	1.43	1.08
	38.1	29.1	<u>3.29</u>	35.4	<u>29490</u>	19.0	13	17	18	1.29	1.35	1.05
Alonso et al. (1998)	16	20	2.40	<u>22.1</u>	<u>19591</u>	12.0	15	18	18	1.19	1.23	1.04
	10	70	2.80	<u>27.8</u>	<u>21153</u>	12.0	68	87	116	1.28	1.71	1.33
	12	50	3.85	<u>44.7</u>	<u>24785</u>	12.0	55	46	64	0.84	1.16	1.39
	16	30	3.85	<u>44.7</u>	<u>24785</u>	12.0	25	24	28	0.94	1.11	1.18
	16	50	2.40	<u>22.1</u>	<u>19591</u>	12.0	31	38	42	1.23	1.37	1.11

	16	70	3.00	<u>30.8</u>	<u>21892</u>	12.0	51	59	75	1.17	1.48	1.27
Vu et al. (2005)	16	25	3.06	20.0	<u>27088</u>	20.0	21	20	21	0.97	1.02	1.05
	16	50	3.06	20.0	<u>27088</u>	20.0	26	39	41	1.5	1.60	1.06
	16	25	4.16	43.0	<u>34962</u>	20.0	15	19	22	1.24	1.40	1.13
	16	50	4.16	43.0	<u>34962</u>	20.0	21	35	43	1.68	2.10	1.25
	16	50	3.94	42.3	<u>34757</u>	20.0	53	34	42	0.64	0.79	1.23
	16	25	3.94	42.3	<u>34757</u>	20.0	18	19	21	1.04	1.17	1.13
	16	50	4.55	52.7	<u>37415</u>	20.0	65	34	44	0.52	0.68	1.31
	16	25	4.55	52.7	<u>37415</u>	20.0	30	19	22	0.65	0.74	1.14
Torres-Acosta and Sagues (2004)	13	39	<u>4.43</u>	55.0	<u>34156</u>	19.0	34	30	39	0.87	1.14	1.31
Al-Sulaimani et al. (1990)	10	75	<u>2.95</u>	30.0	<u>31008</u>	<u>16.0</u>	107	87	104	0.81	0.98	1.20
	14	75	<u>2.95</u>	30.0	<u>31008</u>	<u>16.0</u>	96	65	74	0.68	0.77	1.13
	20	75	<u>2.95</u>	30.0	<u>31008</u>	<u>16.0</u>	84	50	54	0.59	0.65	1.09
Average:							41.3	36.2	43.8	0.97	1.14	1.19
Correlation with experimental data:								0.8572	0.8323			
Correlation between the results of the numerical and the elastoplastic cylinder models:										0.9900		

⁽¹⁾ Maximum aggregate size, only used for the numerical model

⁽²⁾ Elastoplastic cylinder model

⁽³⁾ Italic-Underlined values are not reported by the researcher and they were assumed or calculated based on Section 5.1

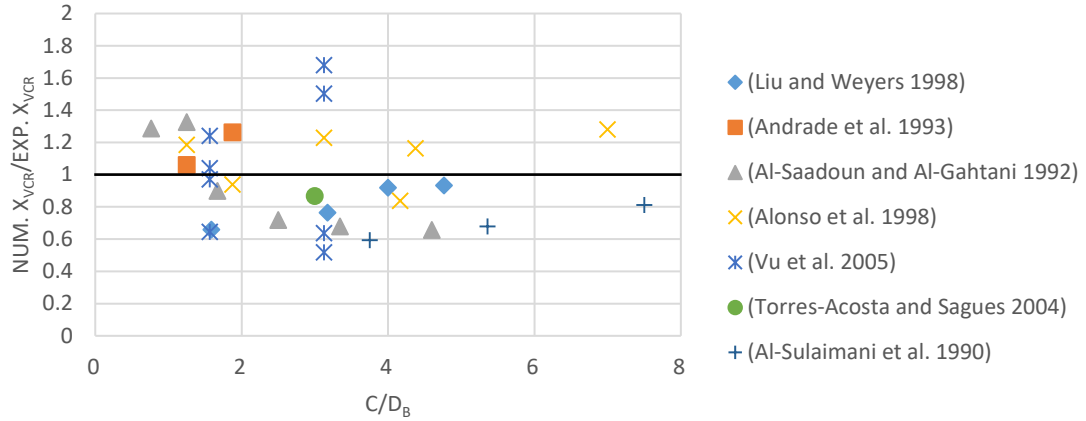


Figure 5.9: Ratio of the numerical to the experimental visual critical attack penetration

Figure 5.10 and Figure 5.11 plot the visual critical attack penetration against cover depth and cover-to-diameter ratio, respectively, for both the numerical and experimental results. As cover-to-rebar diameter ratio or cover depth increases, the visual critical attack penetration also increases according to the numerical model and the experimental data. These figures also demonstrate the variability in the experimental results reported in the literature if they were only compared based on one parameter (cover depth or C/d_b).

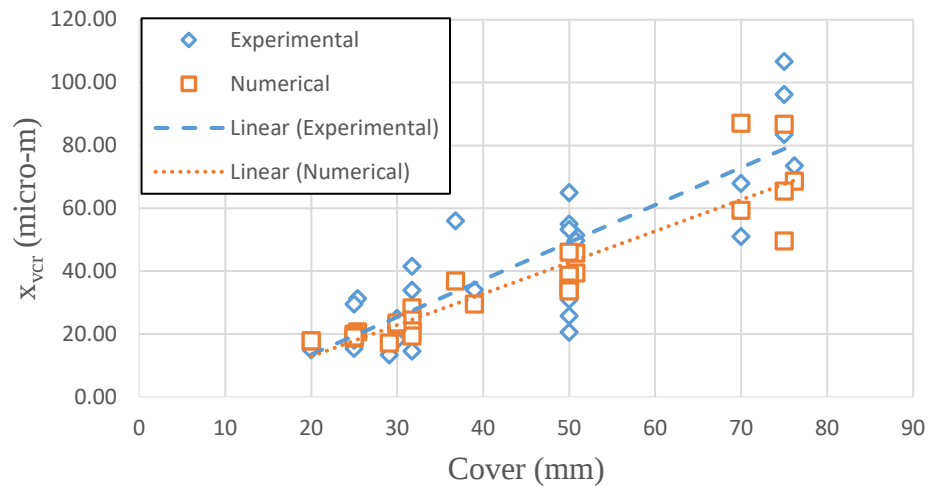


Figure 5.10: Numerical and experimental visual critical attack penetration against cover depth

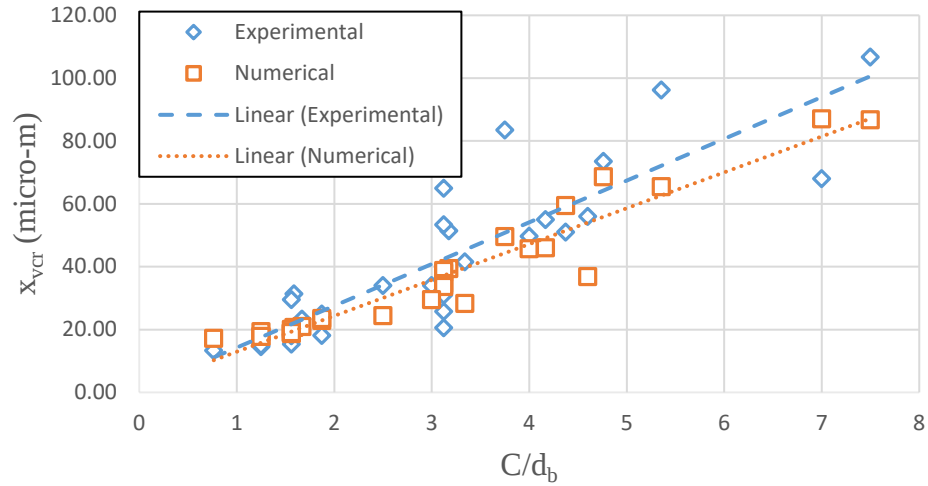


Figure 5.11: Numerical and experimental visual critical attack penetration against cover-to-diameter ratio

Figure 5.12 plots the ratio of the analytical results based on the elastoplastic cylinder to the numerical results for the visual critical attack penetrations. The ratio increases as the cover-to-diameter ratio increases. This model provides better predictions for C/d_b ratios lower than 3. The average ratio and COV are 1.19 and 0.99, respectively.

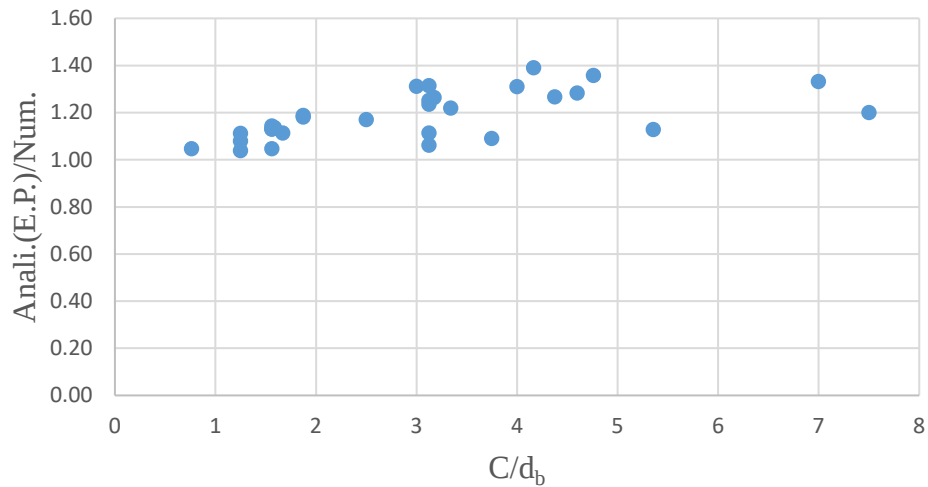


Figure 5.12: The ratio of the analytical results according to the elastoplastic cylinder to the numerical results for the visual critical attack penetrations

Since the experimental tests within a given set were exposed to a similar corrosive environment, it is very likely that the rust expansion coefficients are similar in all of them. Nonetheless, the rust expansion coefficients are likely different from one experimental set to another as they were performed in different environments. Table 5.3 lists the rust expansion coefficient which best fits the experimental data in each experimental set. Figure 5.13 plots the ratio of the numerical results to the experimental results against the cover-to-diameter ratio for the rust expansion coefficients stated in Table 5.3, instead of a constant rust expansion coefficient of 2.75. The values for the porous zone thickness ($10\mu\text{m}$) and the crack volume coefficient (0.5) were kept constant in all cases due to a lack of experimental data, although these are also likely to vary in reality. The correlation between the numerical results and the experimental data increases from 0.857 to 0.917 when the rust expansion coefficient is adjusted for each experimental study.

Table 5.3: Rust expansion coefficient for each experimental set.

Experimental Set	Rust Expansion Coefficient (R_v)
Liu and Weyers (1998)	2.435
Andrade et al. (1993)	3.030
Al-Saadoun and Al-Gahtani (1992)	2.628
Alonso et al. (1998)	2.940
Vu et al. (2005)	2.803
Torres-Acosta and Sagues (2004)	2.523
Al-Sulaimani et al. (1990)	2.213
Average:	2.653
Standard Deviation:	0.269
Coefficient of Variation:	0.101

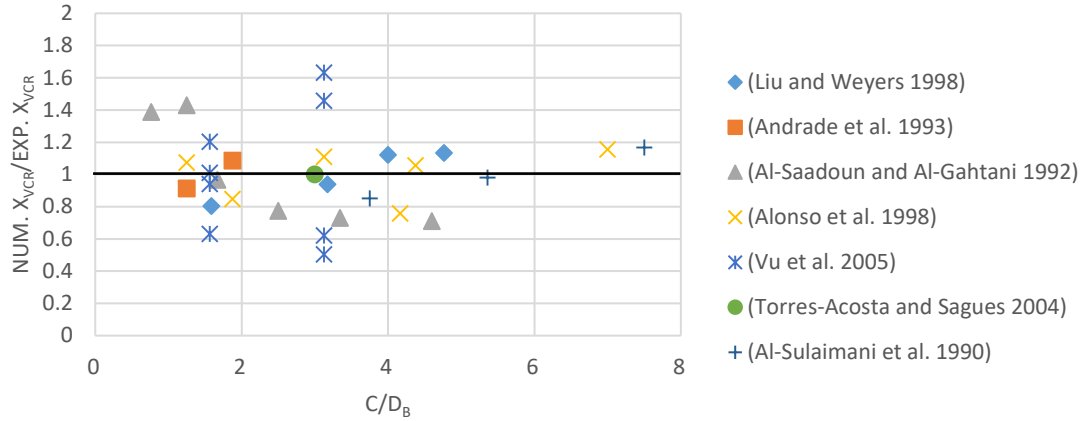


Figure 5.13: Ratio of the numerical to the experimental visual critical attack penetration if different rust expansion coefficients are used for each experimental set

5.2.2.2 Actual Critical Attack Penetration

Table 5.4 states the input data and the results for the numerical and analytical (based on both the elastoplastic cylinder and the brittle elastic cylinder) models for the actual critical attack penetrations. As explained previously, experimental results usually report the visual critical attack penetrations.

Figure 5.14 plots the ratio of the analytical results (based on the elastoplastic and brittle elastic cylinders) to the numerical results for the actual critical attack penetrations. The elastoplastic cylinder estimates the critical attack penetration larger than the numerical model, and the brittle elastic cylinder estimates it smaller than the numerical model. The results diverge as cover-to-diameter ratio increases, especially for the elastic cylinder. The ratio of the results based on the elastoplastic cylinder to the numerical model ranges between 0.91 to 1.41, which tends to increase with cover-to-rebar diameter ratio. The ratio of the results based on the brittle elastic cylinder to the numerical model ranges between 0.39 to 0.90, which tends to decrease with cover-to-rebar diameter ratio.

Table 5.4: Comparison of the actual critical attack penetrations from the three proposed models

Experimental Test	d_b (mm)	C (mm)	f'_t (MPa)	f'_c (MPa)	E_o (MPa)	$d_a^{(1)}$ (mm)	Num. x_{cr} (μm)	Anali. (E.P. ⁽²⁾) $x_{cr}(\mu m)$	Anali. (B.E. ⁽²⁾) $x_{cr}(\mu m)$	E.P. /Num.	B.E. /Num.	B.E. /E.P.
Liu and Weyers (1998)	16	25.4	3.30	31.5	<u>22062</u> ⁽³⁾	<u>16.0</u>	14	15	11	1.10	0.79	0.72
	16	50.8	3.30	31.5	<u>22062</u>	<u>16.0</u>	30	37	19	1.23	0.63	0.51
	12.7	50.8	3.30	31.5	<u>22062</u>	<u>16.0</u>	35	45	21	1.27	0.59	0.46
	16	76.2	3.30	31.5	<u>22062</u>	<u>16.0</u>	56	76	31	1.37	0.56	0.41
Andrade et al. (1993)	16	20	3.55	<u>39.6</u>	<u>23804</u>	<u>16.0</u>	11	12	10	1.09	0.86	0.79
	16	30	3.55	<u>39.6</u>	<u>23804</u>	<u>16.0</u>	15	18	12	1.20	0.80	0.67
Al-Saadoun and Al-Gahtani (1992)	8	36.8	<u>3.29</u>	35.4	<u>29491</u>	19.0	26	30	15	1.17	0.56	0.48
	9.5	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	18	21	12	1.16	0.65	0.57
	12.7	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	16	18	11	1.12	0.71	0.63
	19	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	14	15	11	1.07	0.77	0.72
	25.4	31.75	<u>3.29</u>	35.4	<u>29490</u>	19.0	13	13	10	1.05	0.81	0.78
	38.1	29.1	<u>3.29</u>	35.4	<u>29490</u>	19.0	11	12	10	1.04	0.90	0.86
Alonso et al. (1998)	16	20	2.40	<u>22.1</u>	<u>19591</u>	12.0	11	11	9	1.01	0.82	0.81
	10	70	2.80	<u>27.8</u>	<u>21153</u>	12.0	71	92	33	1.31	0.47	0.36
	12	50	3.85	<u>44.7</u>	<u>24785</u>	12.0	34	48	22	1.41	0.63	0.45
	16	30	3.85	<u>44.7</u>	<u>24785</u>	12.0	15	19	12	1.21	0.80	0.66
	16	50	2.40	<u>22.1</u>	<u>19591</u>	12.0	29	30	16	1.05	0.57	0.54

	16	70	3.00	<u>30.8</u>	<u>21892</u>	12.0	48	59	26	1.24	0.54	0.44
Vu et al. (2005)	16	25	3.06	20.0	<u>27088</u>	20.0	13	13	10	0.94	0.71	0.76
	16	50	3.06	20.0	<u>27088</u>	20.0	31	29	15	0.91	0.49	0.54
	16	25	4.16	43.0	<u>34962</u>	20.0	12	13	10	1.08	0.81	0.74
	16	50	4.16	43.0	<u>34962</u>	20.0	25	31	16	1.22	0.64	0.52
	16	50	3.94	42.3	<u>34757</u>	20.0	25	29	15	1.15	0.61	0.53
	16	25	3.94	42.3	<u>34757</u>	20.0	12	13	10	1.08	0.81	0.75
	16	50	4.55	52.7	<u>37415</u>	20.0	25	31	16	1.27	0.65	0.52
	16	25	4.55	52.7	<u>37415</u>	20.0	12	13	10	1.13	0.83	0.74
Torres-Acosta and Sagues (2004)	13	39	<u>4.43</u>	55.0	<u>34156</u>	19.0	20	26	14	1.29	0.70	0.54
Al-Sulaimani et al. (1990)	10	75	<u>2.95</u>	30.0	<u>31008</u>	<u>16.0</u>	71	79	28	1.11	0.39	0.35
	14	75	<u>2.95</u>	30.0	<u>31008</u>	<u>16.0</u>	51	55	23	1.08	0.45	0.41
	20	75	<u>2.95</u>	30.0	<u>31008</u>	<u>16.0</u>	39	40	19	1.02	0.49	0.48
Average:							26.8	31.4	15.9	1.15	0.67	0.59
Correlation with the numerical results:								0.9849	0.9691			
Correlation between the results of the elastoplastic cylinder and the brittle elastic cylinder:									0.9924			

⁽¹⁾ Maximum aggregate size, only used for the numerical model

⁽²⁾ E.P. denotes the elastoplastic cylinder, B.E. denotes the brittle elastic cylinder

⁽³⁾ Italic-Underlined values are not reported by the researcher and they were assumed or calculated based on Section 5.1.

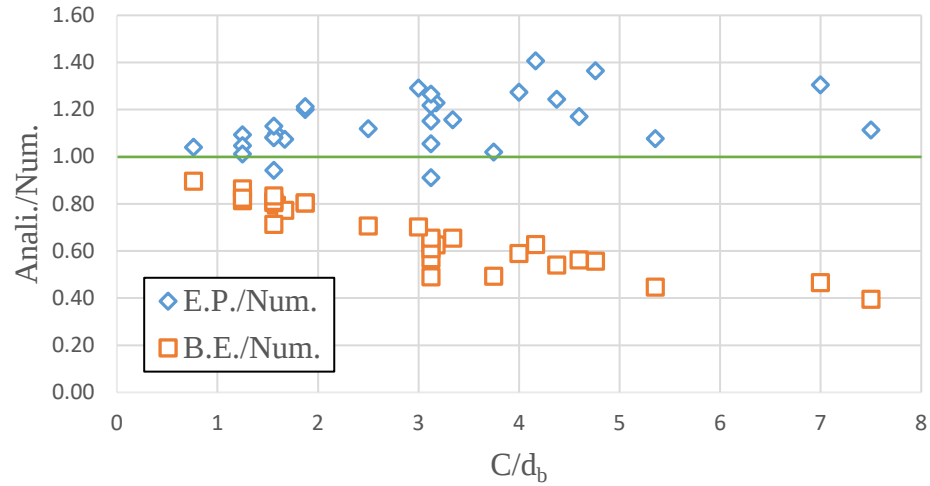


Figure 5.14: The ratio of the analytical results to the numerical results for the actual critical attack penetration

Figure 5.15 plots the ratio of the analytical results based on the brittle elastic cylinder to the analytical results based on the elastoplastic cylinder for the actual critical attack penetrations. The critical attack penetration estimated using the brittle elastic cylinder is always smaller than if using the elastoplastic cylinder. This is because the brittle elastic cylinder is weaker than the elastoplastic cylinder in tension. The ratio of the results based on the brittle elastic cylinder to the elastoplastic cylinder ranges between 0.35 to 0.86, which tends to decrease with C/d_b ratio.

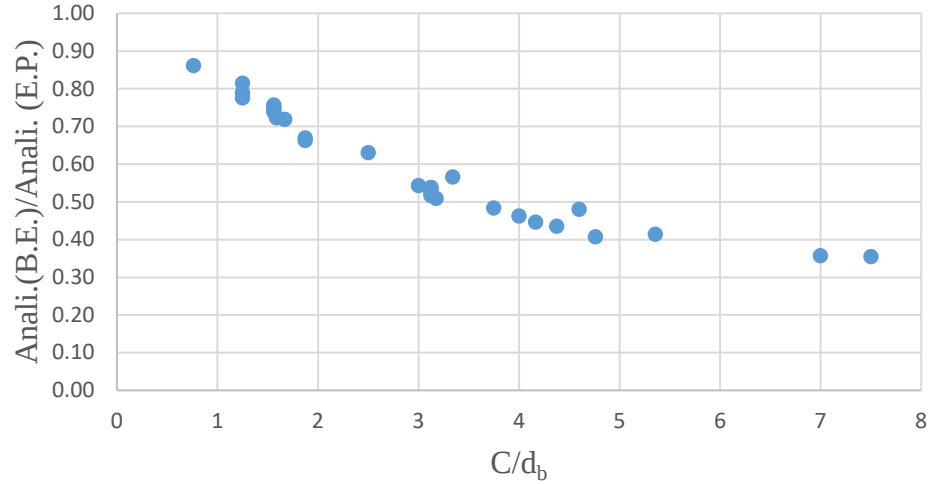


Figure 5.15: The ratio of the analytical results according to the brittle elastic cylinder to the elastoplastic cylinder for the actual critical attack penetration

5.2.3 Internal Pressure

5.2.3.1 Maximum Internal Pressure

Morinaga (1988) tested hollow cylindrical concrete specimens by applying an internal pressure to simulate the expansion of a corroding steel bar. Consequently, the author recorded the maximum internal pressures causing failure in the cylindrical specimens, which represent the concrete cover. Equation 5.6 was proposed to estimate the maximum internal pressure based on the experimental results.

$$[5.6] \quad P_{\max} = f'_t \left(1 + 2 \frac{C}{d_b} \right)^{0.85}$$

where $P_{\max}$ is the maximum internal pressure that can be applied before it causes failure in an experiment controlled by the applied load, in this case the internal pressure. In the case of a displacement-control problem (e.g., the expansion caused by the buildup of corrosion products around the reinforcing bar), it corresponds to the maximum internal pressure experienced at the concrete/rebar interface during the crack propagation.

Figure 5.16 plots the results based on the numerical and analytical models against Equation 5.6. The analytical results for the maximum internal pressure provide an upper and a lower limit to the numerical result. The maximum internal pressure increases with cover-to-rebar diameter ratio based on all models. The maximum internal pressure is also either proportional (i.e., the brittle elastic cylinder, the elastoplastic cylinder, Morinaga 1988) or slightly more than proportional (the numerical model) to the tensile strength of concrete cover. The numerical model, elastoplastic cylinder (analytical model) and Equation 5.6 provide similar results for cover-to-diameter ratios less than 3. As the cover-to-diameter ratio increases, the results start to diverge. The numerical results tend to get closer to the results based on the brittle elastic cylinder (analytical model) as cover-to-diameter ratio increases.

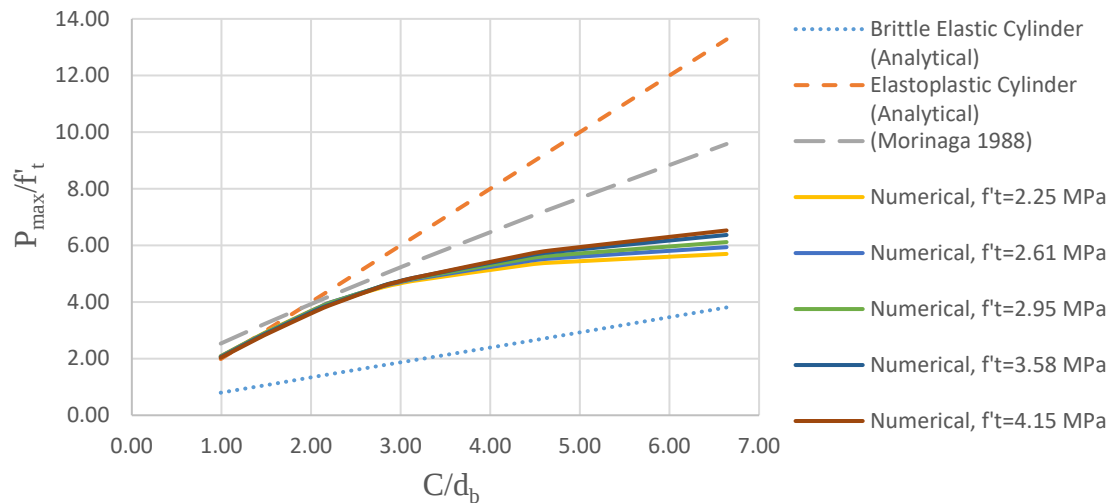


Figure 5.16: Comparison of the maximum internal pressure found by the numerical model, the analytical models, and that proposed by Morinaga (1988)

Williamson and Clark (2000) performed similar tests to Morinaga (1988) and reported that the tensile strength of the cover concrete (f'_t) had no effect on the maximum pressure (P_{max}), which is not in agreement with the findings reported herein.

5.2.3.2 Critical Crack Depth

The critical crack depth is defined as the crack front corresponding to the maximum internal pressure ($P_{\max}$). It has also been referred to as the optimum crack depth (Tepfers 1979). The ratio of the critical crack depth to the concrete cover depth is referred to herein as the critical crack depth ratio (D_{cr}). Figure 5.17 plots the critical crack depth ratio for different cover-to-diameter ratios based on the numerical and analytical models. The gray dots on Figure 5.17 are the numerical results based on different input parameters, and the red line is the trendline of the numerical results. The brittle elastic cylinder model discussed in Chapter 4 is a more advanced version of the cylinder discussed in Tepfers (1979), so it can be used to model the corrosion-induced cracking. Consequently, the brittle elastic cylinder provides the same results as Tepfers (1979) for the critical crack depth. As with the maximum internal pressure, the analytical models provide an upper and a lower limit to the numerical results for the critical crack depth. The results of the numerical model are closer to the results based on the elastoplastic cylinder for smaller cover-to-diameter ratios and approach the results based on the brittle elastic cylinder as the cover-to-rebar diameter ratio increases. This is because the smaller the cover-to-rebar diameter ratio, the smaller the hoop tensile strains at the cracked portion of the cylinder at the time corresponding to the maximum internal pressure. The smaller tensile strains mean the concrete is not softened much in tension, and it behaves more like an elastoplastic cylinder. Based on the elastoplastic cylinder, the maximum internal pressure always occurs when the crack reaches the surface ($D_{cr} = 100\%$). Conversely, the critical crack depth ratio decreases with cover-to-rebar diameter ratio in the numerical model (from values close to 100% down to 55%), and increases with cover-to-rebar diameter ratio in the brittle elastic cylinder (from values around 22% to 45%).

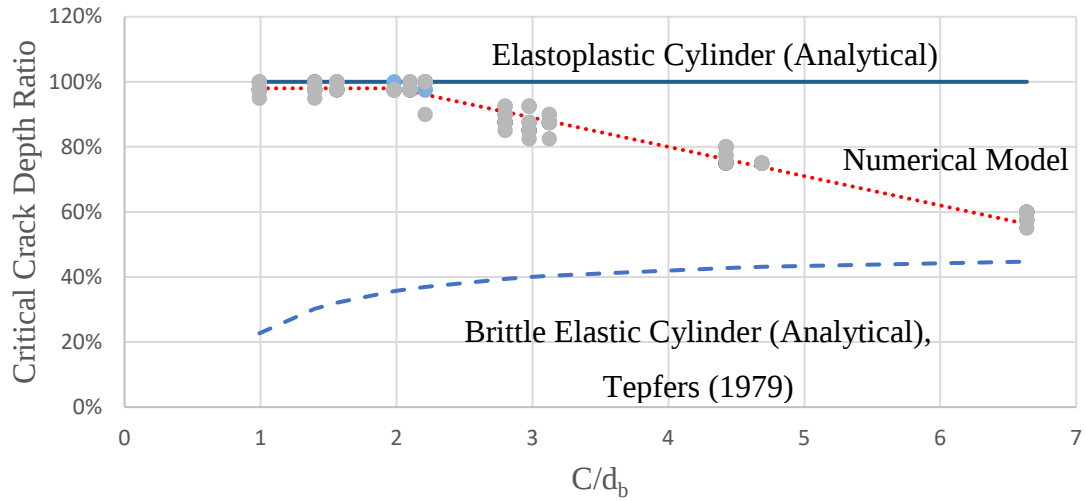


Figure 5.17: Comparison of the critical crack depth ratios found by the numerical model and the analytical models

5.3 Parametric Study

A parametric study using the proposed models was conducted on 12 geometric cases with different rebar diameters and concrete cover depths as listed in Table 5.5.

Table 5.5: Parametric analysis cases

Specimens	d_b (mm)	C (mm)	$\frac{C}{d_b}$
10M-25C	11.3	25	2.21
10M-50C	11.3	50	4.42
10M-75C	11.3	75	6.64
15M-25C	16	25	1.56
15M-50C	16	50	3.13
15M-75C	16	75	4.69
25M-25C	25.2	25	0.99
25M-50C	25.2	50	1.98
25M-75C	25.2	75	2.98
35M-50C	35.7	50	1.40

35M-75C	35.7	75	2.10
35M-100C	35.7	100	2.80

The parametric study looks into the effect of the different parameters stated in Table 5.6 on corrosion-induced cracking of the concrete cover. The parameters include concrete properties (i.e., Poisson's ratio, the tensile and the compressive strength, the creep coefficient, the maximum aggregate size, the porous zone thickness, and the crack volume coefficient), rust properties (i.e., rust expansion coefficient, bulk modulus), and the chosen tension-softening model.

Table 5.6: Range of values in the parametric study

Parameters	Values					
$\vartheta_c = \vartheta_{\theta r} = \vartheta_{r\theta}$	0.10	0.15	0.20⁽¹⁾	0.25		
f'_c (MPa) ⁽²⁾	20.0	25.0	30.0⁽¹⁾	40.0	50.0	
$\varnothing$ ⁽³⁾⁽⁴⁾	0.00⁽¹⁾	1.00	2.00	3.00	4.00	
d_a (mm) ⁽⁵⁾	12.0	16.0⁽¹⁾	20.0			
t_p (μ m)	0.00	5.00	7.50	10.0⁽¹⁾	12.5	15.0
C_v	0.00	0.40	0.50⁽¹⁾	0.60	1.00	
R_v	2.00	2.50	2.75⁽¹⁾	3.00	4.00	6.00
B_r (GPa)	0.20	0.50⁽¹⁾	2.00			
Tension Softening ⁽⁵⁾	Linear		Bi-linear		Non-linear⁽¹⁾	

⁽¹⁾ Default value, in bold font.

⁽²⁾ f'_t and E were consequently calculated using Equation 5.1 and Equation 5.2, resulting in values ranging from (2.25, 27,090) to (4.15, 36,760) MPa.

⁽³⁾ The Young's moduli were reduced using different creep coefficients.

⁽⁴⁾ Only used in the analytical models (elastoplastic and brittle elastic cylinders).

⁽⁵⁾ Only used in the numerical model.

5.3.1 Maximum Internal Pressure and Critical Crack Depth

5.3.1.1 Concrete Strength (f'_c, f'_t)

The maximum internal pressure was found to be linearly proportional to f'_t according to the analytical model and almost linearly proportional according to the numerical model. Note that f'_t and E were consequently calculated using Equations 5.1 and 5.2, respectively.

Figure 5.18 plots $(\frac{P_{\max}}{f'_t})$ against $(\frac{C}{d_b})$ for different tensile strengths of 2.25, 2.61, 2.95, 3.58, 4.15 MPa based on the numerical model. According to the numerical model, the effect of f'_t is slightly more than a proportional relation for $(\frac{C}{d_b})$ larger than 3.0.

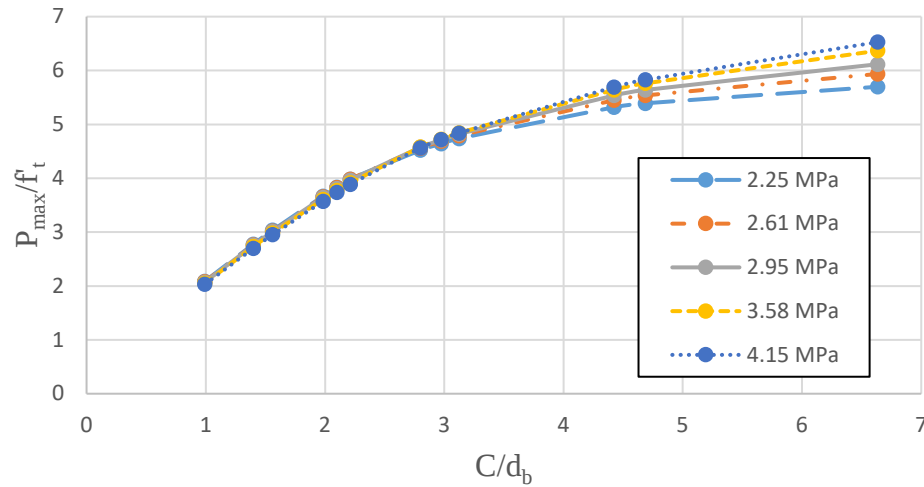


Figure 5.18: Effect of the tensile strength of the concrete on the maximum internal pressure according to the numerical model

The effect of the strength of the concrete was found to be negligible on the critical crack depth ratio (D_{cr}) based on the numerical model (Figure 5.19). Note that there is a slight variation in the numerical results due to numerical errors.

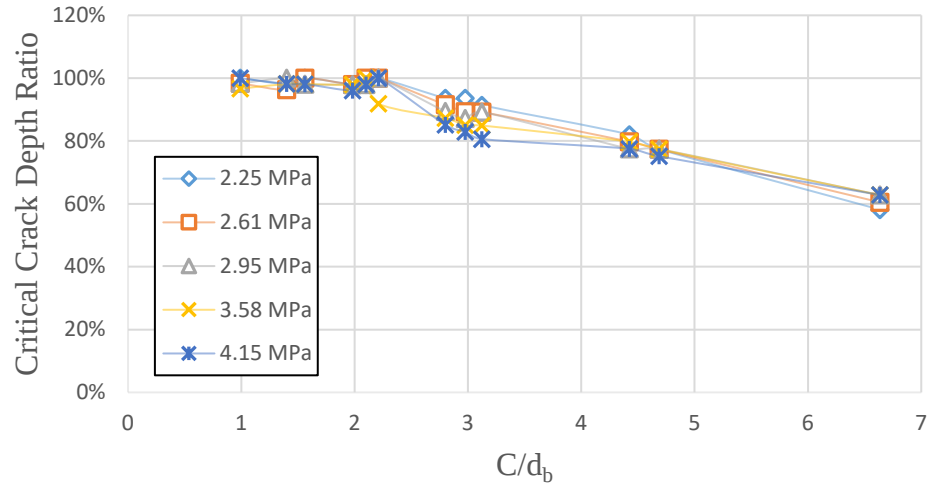


Figure 5.19: Effect of the tensile strength of the concrete on the critical crack depth ratio according to the numerical model

Figure 5.20 plots $(\frac{P_{\max}}{f'_t})$ against $(\frac{C}{d_b})$ for different tensile strengths of 2.25, 2.61, 2.95, 3.58, and 4.15 MPa according to both analytical models (elastoplastic and brittle elastic cylinders). Note that all curves of maximum internal pressure for different f'_t coincide if they are normalized by f'_t . The maximum internal pressure based on the brittle elastic cylinder is always smaller than the one computed by the elastoplastic cylinder. The difference increases with C/d_b ratio.

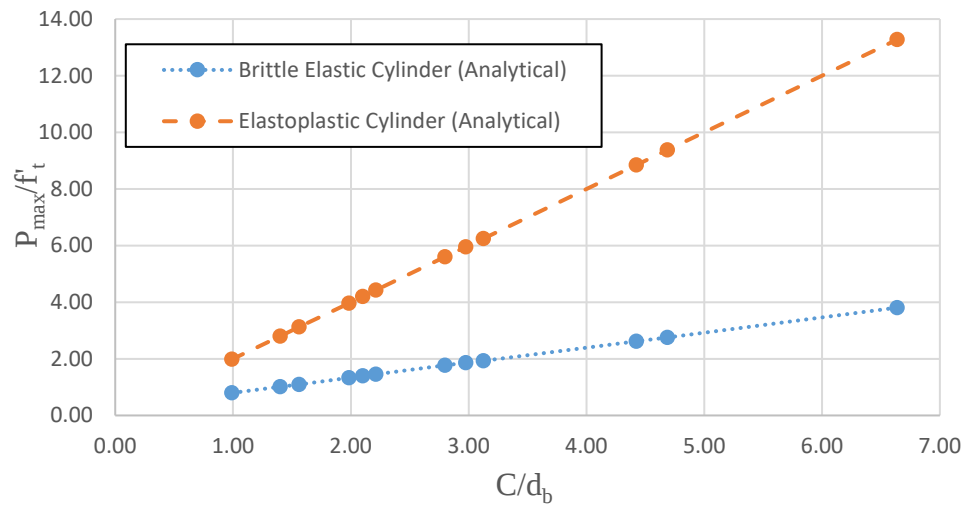


Figure 5.20: Effect of the tensile strength of concrete on the maximum internal pressure according to the analytical models

5.3.1.2 Poisson's ratio (ν_c):

The effect of Poisson's ratio was found to be negligible on the maximum internal pressure ($P_{\max}$) based on the numerical model and both analytical models. Figure 5.21 plots the maximum internal pressure ($P_{\max}$) for different Poisson's ratios based on the numerical model.

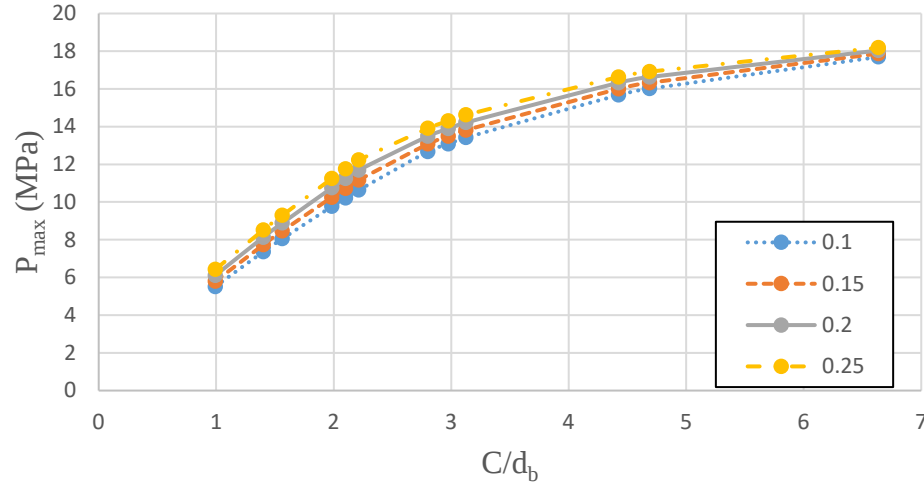


Figure 5.21: Effect of Poisson's ratio on the maximum internal pressure according to the numerical model

To better demonstrate the effect of Poisson's ratio, Figure 5.22 plots the maximum internal pressure divided by the maximum internal pressure if the Poisson's ratio was 0.2 according to the numerical model. The effect of Poisson's ratio on the maximum internal pressure was found to be less than 10% in all cases according to the numerical model. According to the numerical model, maximum internal pressure increases with increasing Poisson's ratio.

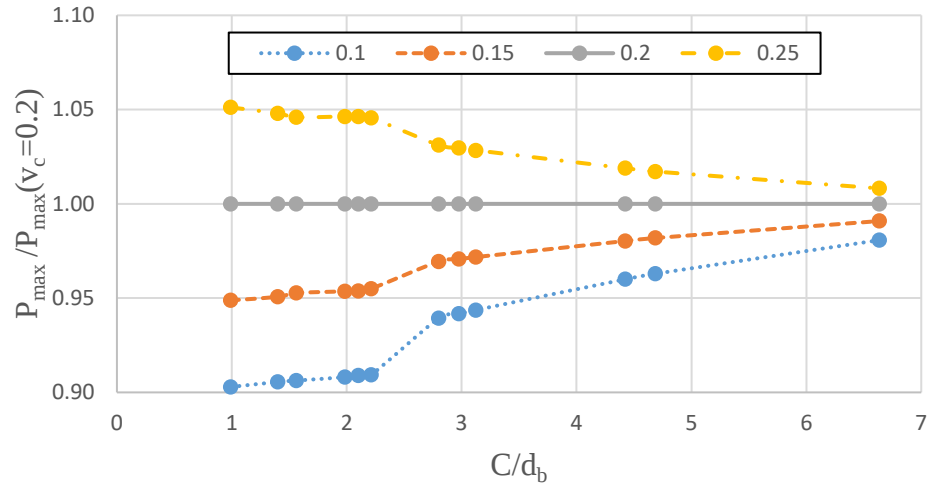


Figure 5.22: Relative effect of Poisson's ratio on the maximum internal pressure according to the numerical model

The effect of Poisson's ratio on the critical crack depth (D_{cr}) was also found to be negligible according to the numerical model (Figure 5.23). A maximum variation of 7% was observed; however, it is worth noting that the typical range for Poisson's ratio for concrete is between 0.18 to 0.2.

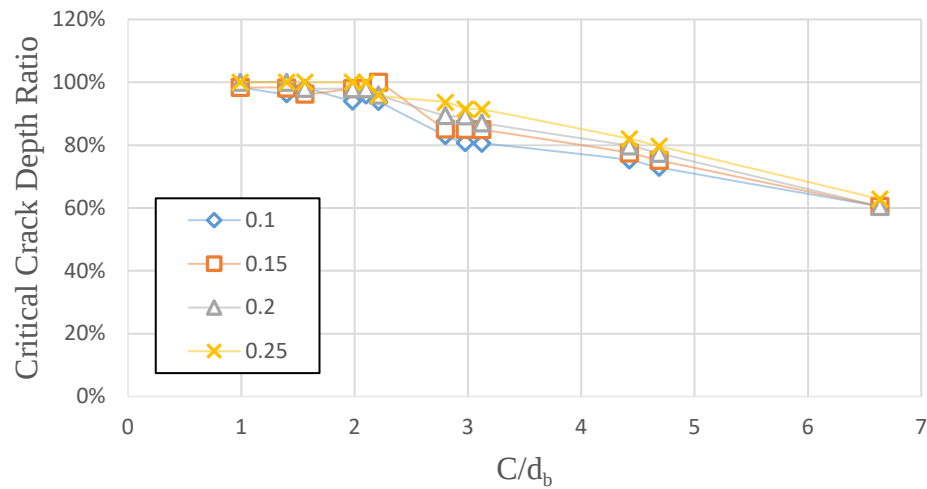


Figure 5.23: Effect of the Poisson's ratio on the critical crack depth ratio according to the numerical model

Figure 5.24 plots the maximum internal pressure divided by the maximum internal pressure if Poisson's ratio was 0.2 based on the brittle elastic cylinder. The effect of Poisson's ratio on the maximum internal pressure was found to be less than 6% in all cases according to the brittle elastic model. According to the brittle elastic model, maximum internal pressure decreases with increasing Poisson's ratio, which contradicts with the findings of the numerical model.

Poisson's ratio was found to have absolutely no effect on the maximum internal pressure based on the elastoplastic cylinder.

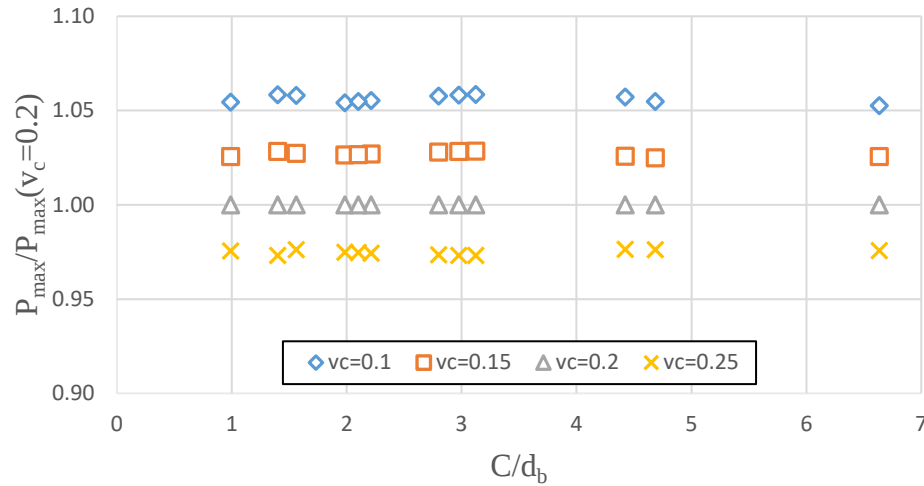


Figure 5.24: Relative effect of the Poisson's ratio on maximum internal pressure according to the brittle elastic cylinder model

5.3.1.3 Maximum Aggregate Size (d_a):

This parameter is only used in the numerical model, as it is used to calculate the fracture energy of concrete. The effect of the maximum aggregate size has been found to be negligible on the maximum internal pressure (P_{max}) and the critical crack depth (D_{cr}) (Figure 5.25, Figure 5.26, and Figure 5.27). Increasing the maximum aggregate size from 16 to 20 mm decreases the maximum internal pressure by less than 3%.

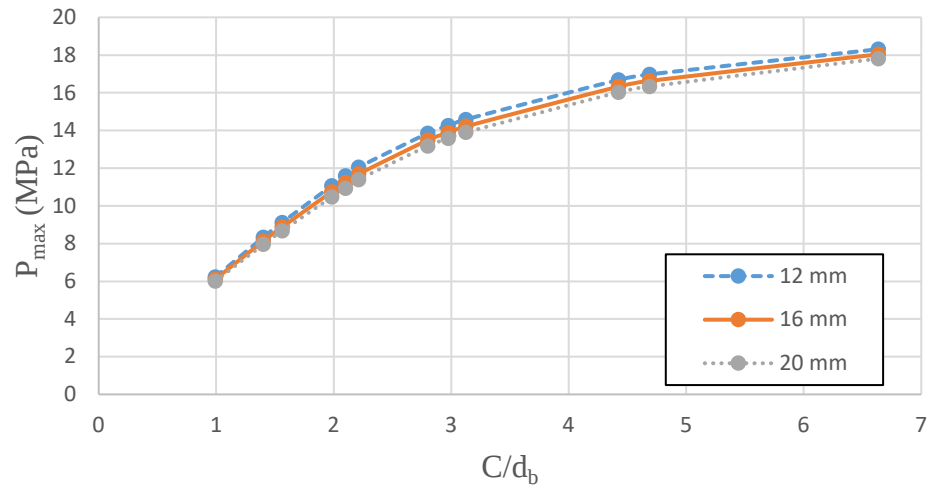


Figure 5.25: Effect of the maximum aggregate size on the maximum internal pressure

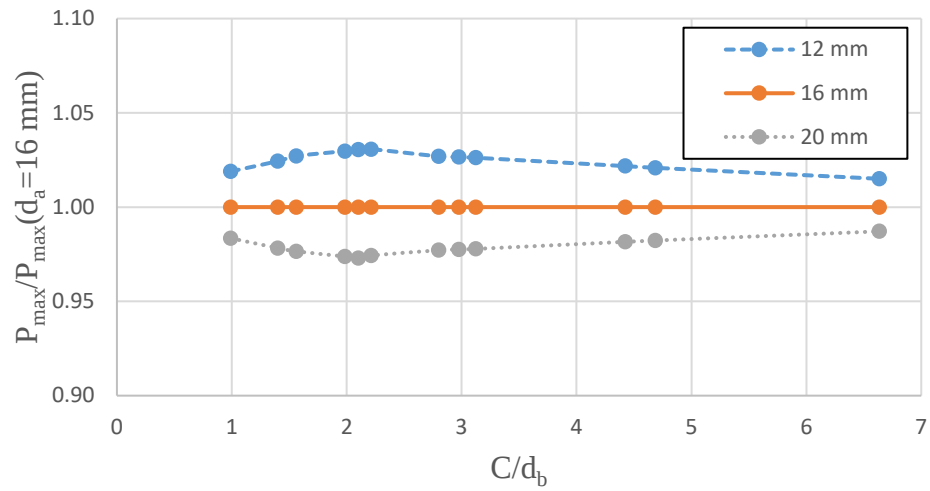


Figure 5.26: Relative effect of the maximum aggregate size on the maximum internal pressure

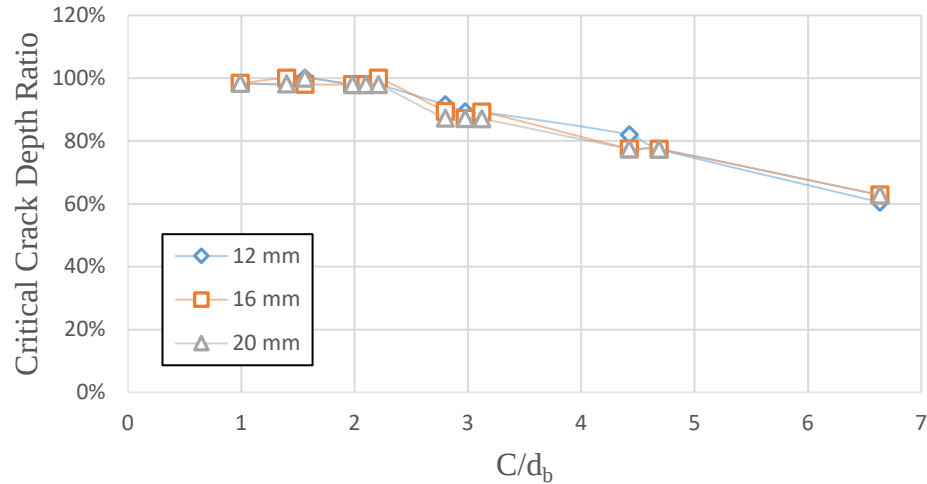


Figure 5.27: Effect of the maximum aggregate size on the critical crack depth

5.3.1.4 Tension Softening Model:

The tension softening model is only applicable to the numerical model. The effect of the chosen tension softening model was found to be negligible on the maximum internal pressure (P_{max}) and the critical crack depth (D_{cr}) (Figure 5.28, Figure 5.29, and Figure 5.30). The effect of the tension softening model on the maximum internal pressure can be explained by Figure 3.7. For the range of the studied cover-to-diameter ratios, when the maximum internal pressure occurs, the tensile hoop strains within the cylinder mostly lie at the beginning of the softening curve. The linear model provides higher tensile stresses than the bilinear model, and the bilinear model provides higher tensile stresses than the nonlinear model at the beginning of the softening curves. The chosen tension-softening model has less than 8% effect on the maximum internal pressure.

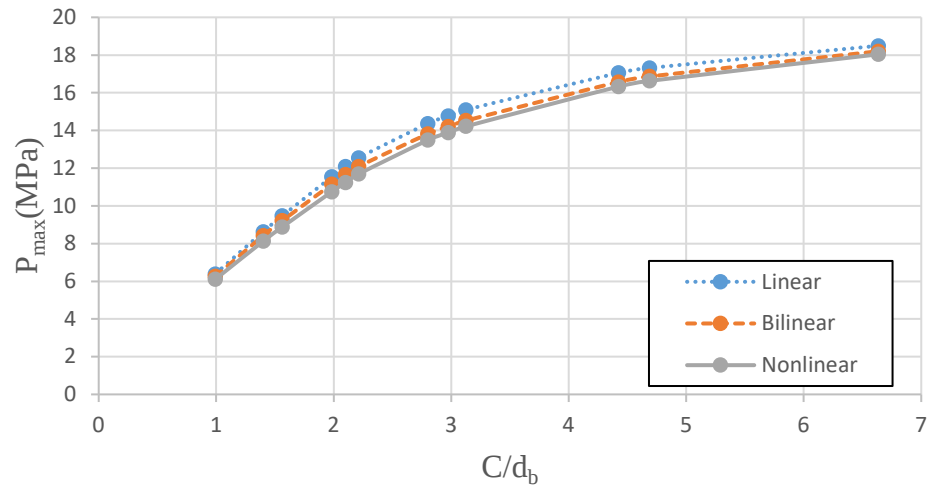


Figure 5.28: Effect of the chosen tension softening model on the maximum internal pressure

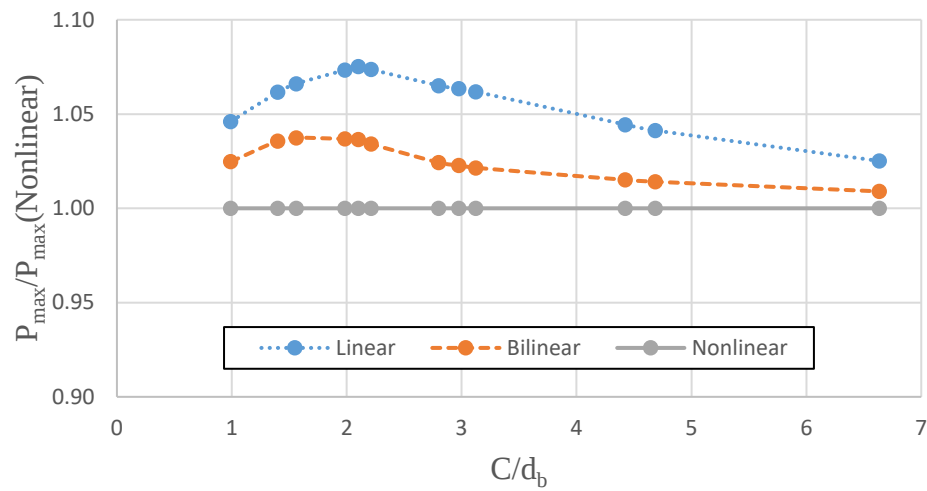


Figure 5.29: Relative effect of the chosen tension softening model on the maximum internal pressure

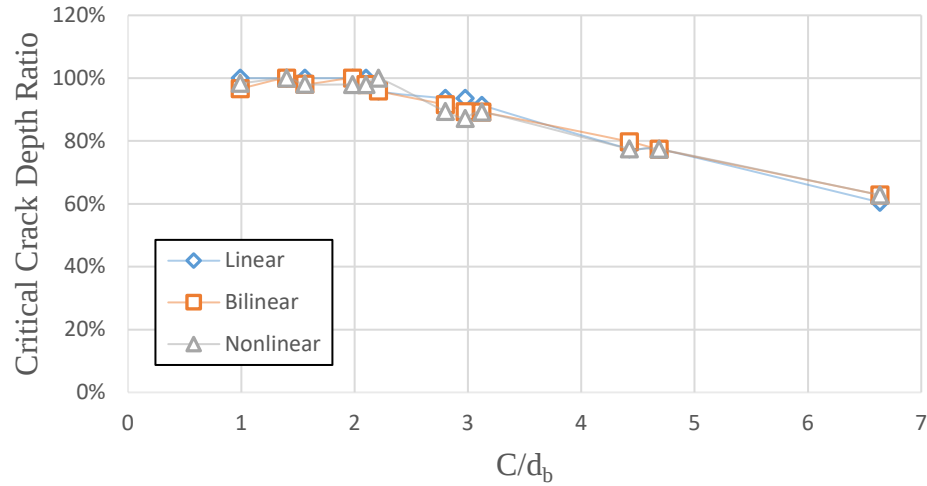


Figure 5.30: Effect of the chosen tension softening model on the critical crack depth ratio

5.3.1.5 Porous Zone Thickness (t_p), Rust Expansion Coefficient (R_v), Bulk Modulus of Corrosion Products (B_r), Crack Volume Coefficient (C_v), and Creep Coefficient (ϕ):

Sensitivity analyses based on the numerical and analytical models show absolutely no influence of t_p , R_v , B_r , C_v on the maximum internal pressure (P_{max}) and the optimum crack depth. Therefore, both the maximum internal pressure (P_{max}) and the critical crack depth (D_{cr}) are only functions of the cover dimensions and the concrete properties.

The creep coefficient (ϕ) was also found to have no effect on the maximum internal pressure (P_{max}) based on both analytical models, where E_c is found using Equation 5.7:

$$[5.7] \quad E_c = \frac{E_o}{1+\phi}$$

Note that f'_t was kept constant in the creep study.

5.3.2 Visual Critical Attack Penetration

As previously mentioned, the critical attack penetration refers to the attack penetration corresponding to the time when the first crack reaches the surface. The visual critical attack penetration is the attack penetration corresponding to the first crack width of 0.05 mm. In this section, the effect of different parameters on the visual critical attack penetration and actual critical attack penetration is investigated. Nonetheless, only the effect on the visual critical attack penetration is reported, as the effect of these parameters on the actual critical attack penetration were found to be similar.

5.3.2.1 Ratio of Visual Critical Attack Penetration (x_{vcr}) to Actual Critical Attack Penetration (x_{cr}):

Figure 5.31 plots the ratio of the visual critical attack penetration (x_{vcr}) to the actual critical attack penetration (x_{cr}) for different cover depths based on the numerical model and the elastoplastic cylinder model. The ratio decreases with increasing cover depth according to both the numerical and the elastoplastic cylinder models. The ratio based on both models are similar for different cover-to-rebar diameter ratios. According to the numerical and the elastoplastic models, the ratio of the visual critical attack penetration to the critical attack penetration lies between 1.7 to 1.2 for cover depths ranging from 25 to 100 mm. According to both models, the ratio decreases with increasing cover depth.

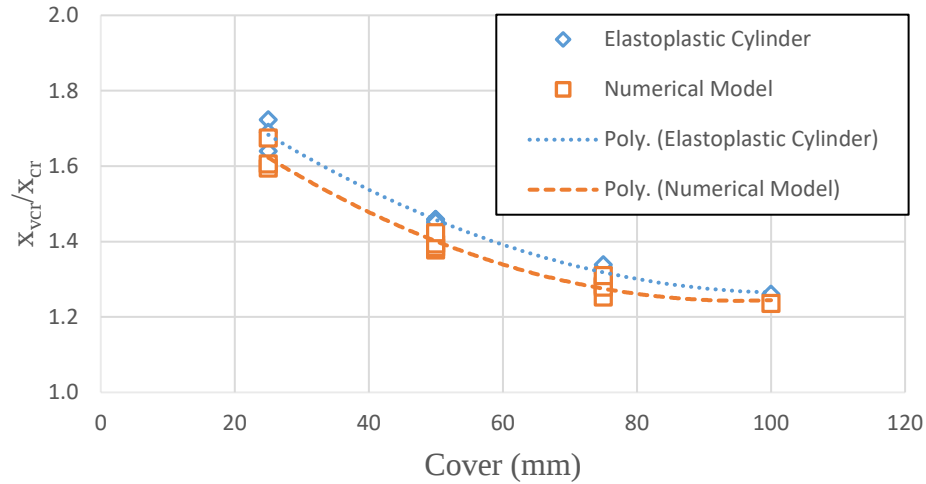


Figure 5.31: Ratio of the visual critical attack penetration to the actual critical attack penetration

5.3.2.2 Concrete Strength (f'_c, f'_t):

The effect of concrete strength on the visual critical attack penetration is less than 5% according to the results of the numerical model (Figure 5.32). This is due to the fact that although a higher tensile strength makes the thick-walled cylinder stronger in the hoop direction, it also makes the cylinder stiffer in the radial direction. Figure 5.33 plots the effect of concrete's tensile strength if the Young's modulus and the compressive strength are kept constant (based on the numerical model). It shows that the visual critical attack penetration would have increased with the tensile strength of concrete if the compressive strength and Young's modulus of concrete were not increased with the tensile strength of concrete. However, it is not usually the case.

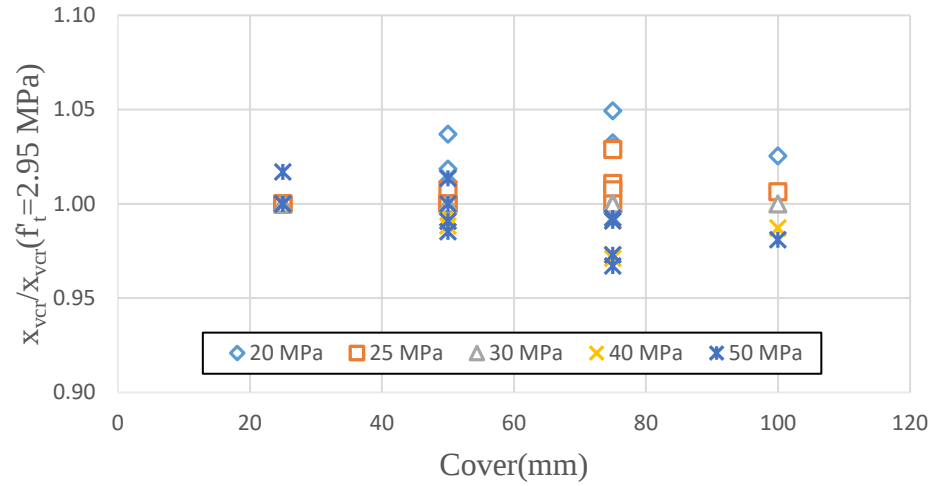


Figure 5.32: Effect of the concrete's strength on the visual critical attack penetration according to the numerical model

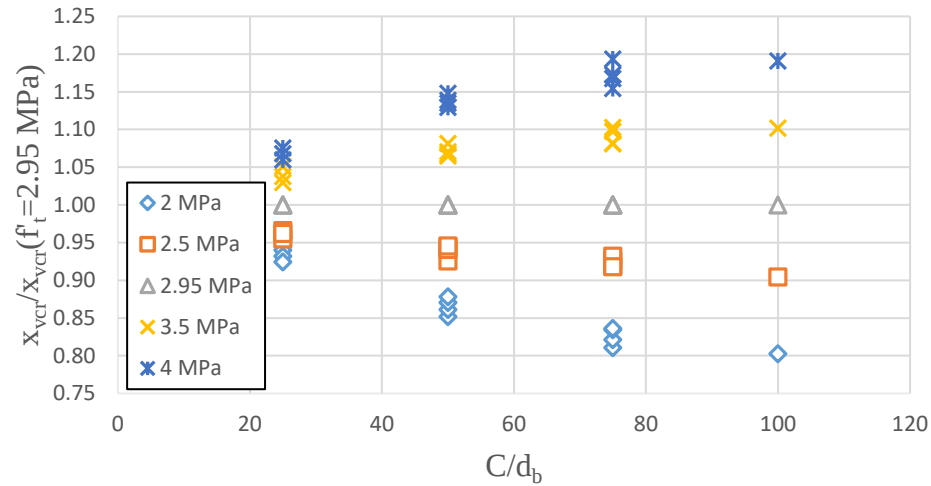


Figure 5.33: Effect of the concrete's tensile strength on the visual critical attack penetration according to the numerical model if the Young's modulus and the compressive strength were kept constant

Figure 5.34 plots the effect of the concrete's compressive strength on the visual critical attack penetration according to the elastoplastic cylinder model. Parameters f'_t and E were consequently calculated using Equations 5.1 and 5.2, respectively. Unlike the numerical

model, the elastoplastic model shows an increase in the visual critical attack penetration with increasing tensile strength.

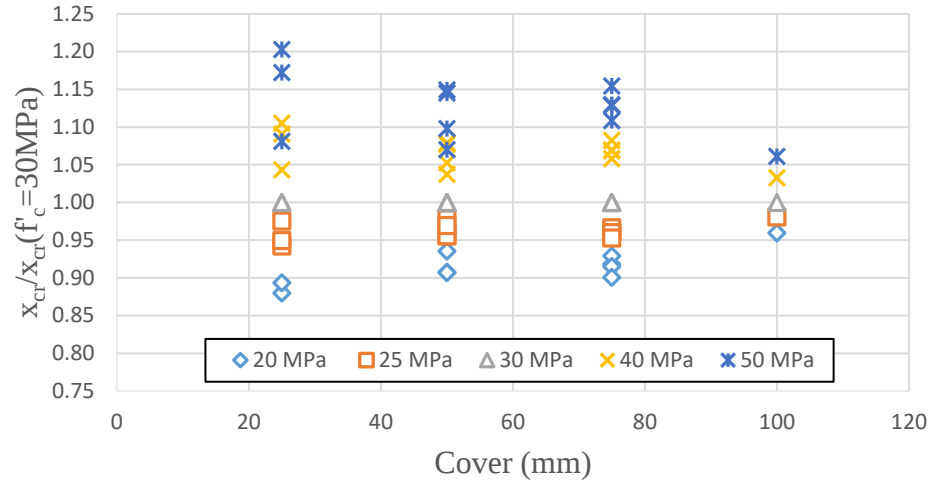


Figure 5.34: Effect of the concrete's compressive strength on the visual critical attack penetration according to the elastoplastic cylinder model

5.3.2.3 Poisson's ratio (ν_c):

The effect of Poisson's ratio on the visual critical attack penetration was found to be negligible according to both the numerical and the elastoplastic cylinder models, as respectively illustrated in Figure 5.35 and Figure 5.36. These figures plot the ratio of the visual critical attack penetration to the visual critical attack penetration if Poisson's ratio was 0.2. According to both models, increasing Poisson's ratio increases the visual critical attack penetration. However, Poisson's ratio is more influential according to the numerical model (less than 10%) than the elastoplastic cylinder model (less than 5%).

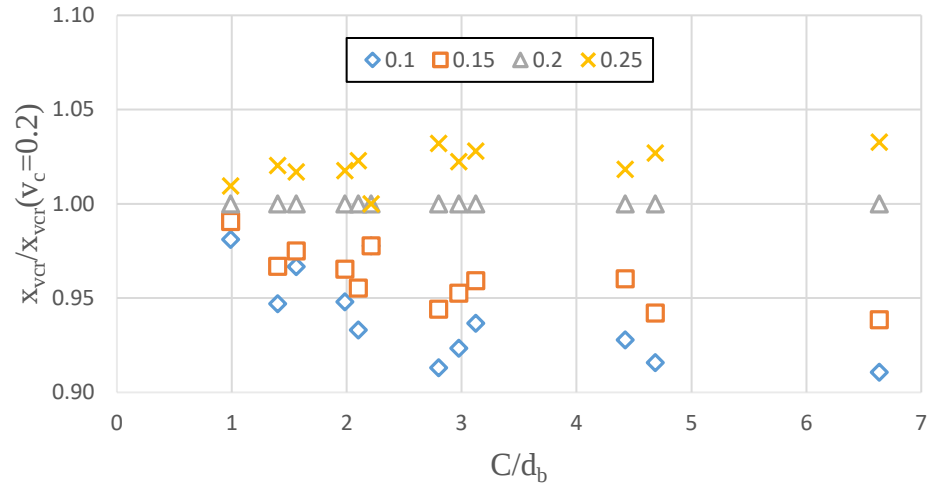


Figure 5.35: Effect of the Poisson's ratio on the visual critical attack penetration according to the numerical model

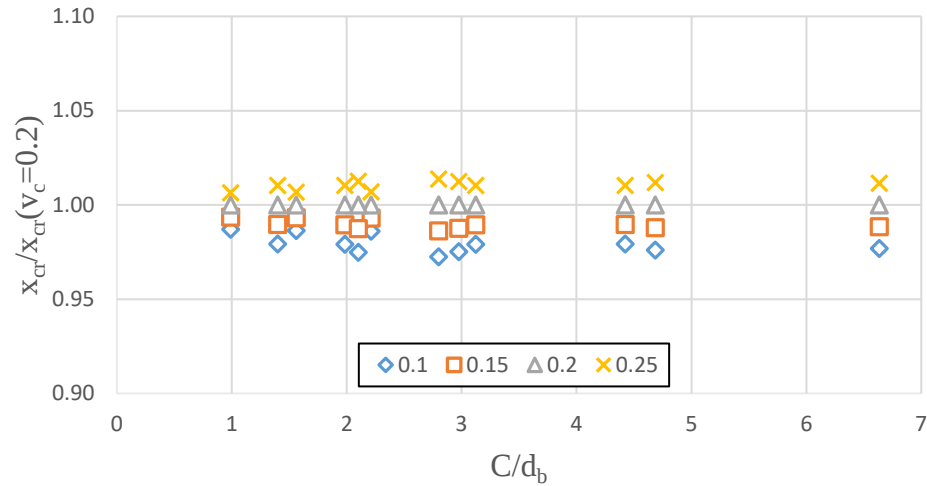


Figure 5.36: Effect of the Poisson's ratio on the visual critical attack penetration according to the elastoplastic cylinder model

5.3.2.4 Maximum Aggregate Size (d_a):

This parameter is only used in the numerical model. The effect of the maximum aggregate size was found negligible on the visual critical attack penetration. Figure 5.37 plots the ratio of the visual critical attack penetration to the visual critical attack penetration if the

maximum aggregate size was 16 mm. According to the figure, the differences obtained by using different aggregate sizes is small, within 5%.

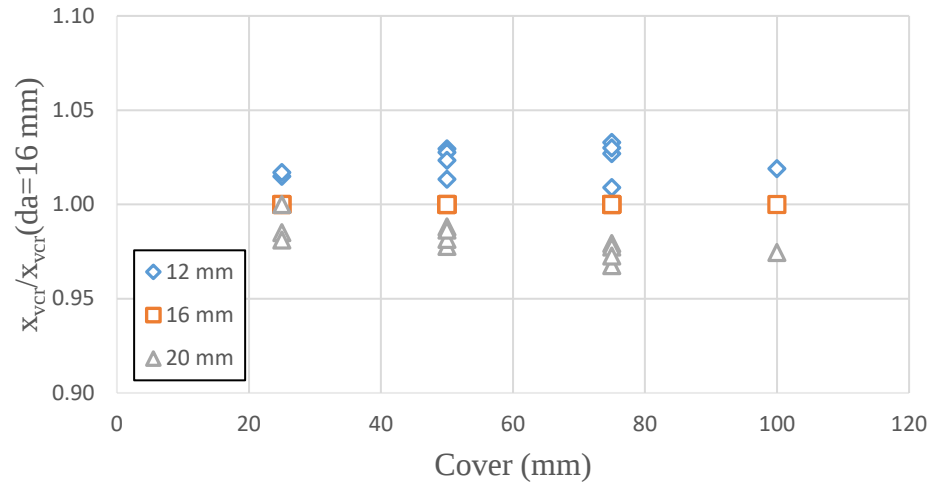


Figure 5.37: Effect of the maximum aggregate size on the visual critical attack penetration

5.3.2.5 Tension Softening Model:

The choice of a tension softening model for concrete is only applicable when using the numerical model. The effect of the used tension softening model was found to have less than 8% effect on the resulting visual critical attack penetration. Figure 5.38 plots the ratio of the visual critical attack penetration to the visual critical attack penetration if the nonlinear tension softening was used. As observed in the figure, the differences between the tension softening models increase as the cover-to-rebar diameter ratio increases.

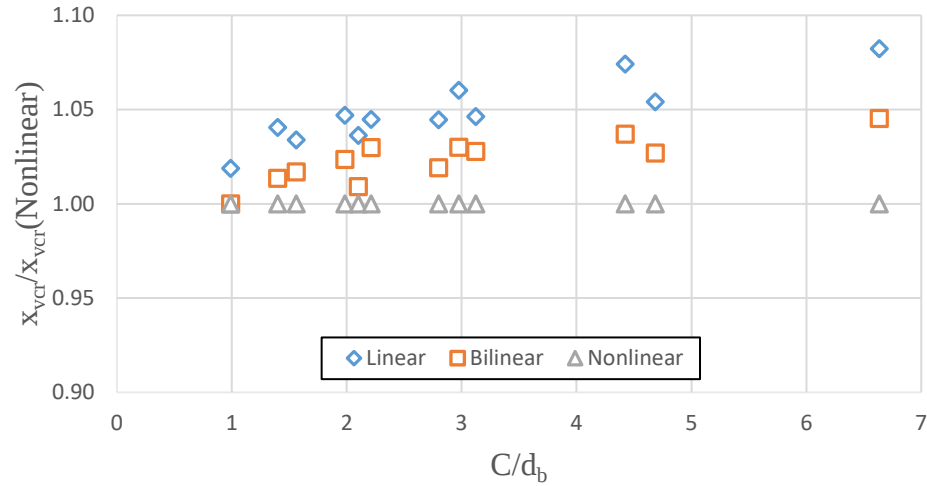


Figure 5.38: Effect of the tension softening model on the visual critical attack penetration

5.3.2.6 Porous Zone Thickness (t_p)

Figure 5.39 plots the ratio of the visual critical attack penetration to the visual critical attack penetration if a porous zone thickness of 10 μm was used. Both models (numerical and elastoplastic cylinder) provide the same results. The effect of accounting for a porous zone thickness in the calculations is larger for smaller cover depths, as the porous zone accounts for a higher percentage of all the available space where corrosion products are deposited (as high as 33% for a cover depth of 25 mm versus 11% for a cover depth of 100 mm).

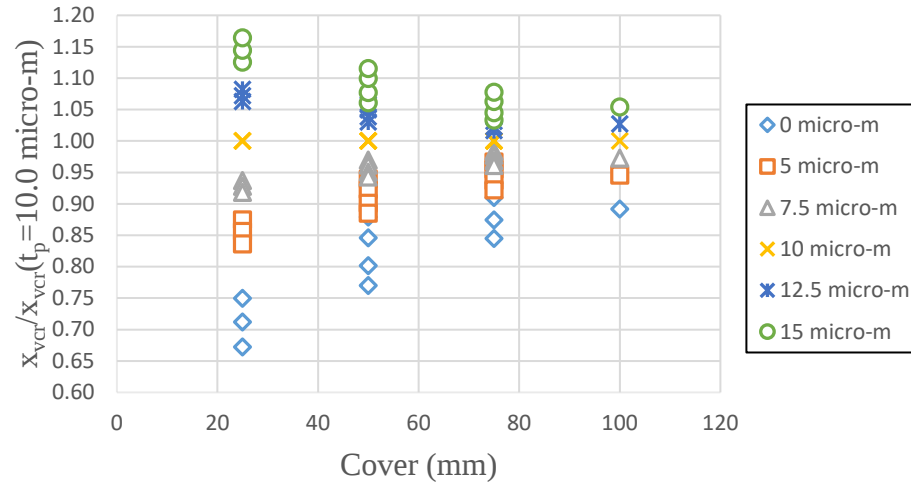


Figure 5.39: Effect of the porous zone thickness on the visual critical attack penetration

5.3.2.7 Rust Expansion Coefficient (R_v)

The effect of the rust expansion coefficient was found to be very significant according to both models (numerical model and elastoplastic cylinder). Figure 5.40 plots the ratio of the visual critical attack penetration to the visual critical attack penetration if a rust expansion coefficient of 2.75 was used. Both models (numerical and elastoplastic cylinder) provide the same results. As the value of R_v increases, the visual critical attack penetration decreases and it is independent of the C/d_b ratio. For example, if a rust expansion coefficient of 6.0 is used, the concrete cover cracks at 35% of the time calculated based on a rust expansion coefficient of 2.75.

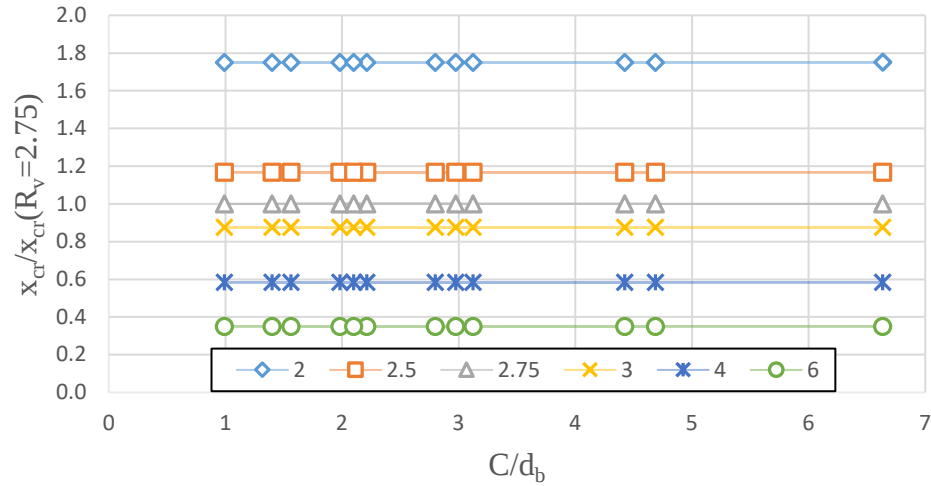


Figure 5.40: Effect of the rust expansion coefficient on the visual critical attack penetration

5.3.2.8 Bulk Modulus of Corrosion Products (B_r)

Figure 5.41 and Figure 5.42 plot the ratio of the visual critical attack penetration to the visual critical attack penetration if the bulk modulus was 0.5 GPa according to the numerical and elastoplastic cylinder models, respectively. The effect of the bulk modulus is more dominant in the elastoplastic cylinder model, since the internal pressure at the time corresponding to the visual critical attack penetration is larger than the one obtained with the numerical model. For example, using a bulk modulus of 0.2 GPa instead of 0.5 GPa for a cover-to-rebar diameter ratio of 6.64 increases the critical attack penetration by 8% or 27% based on the numerical model or the elastoplastic cylinder, respectively.

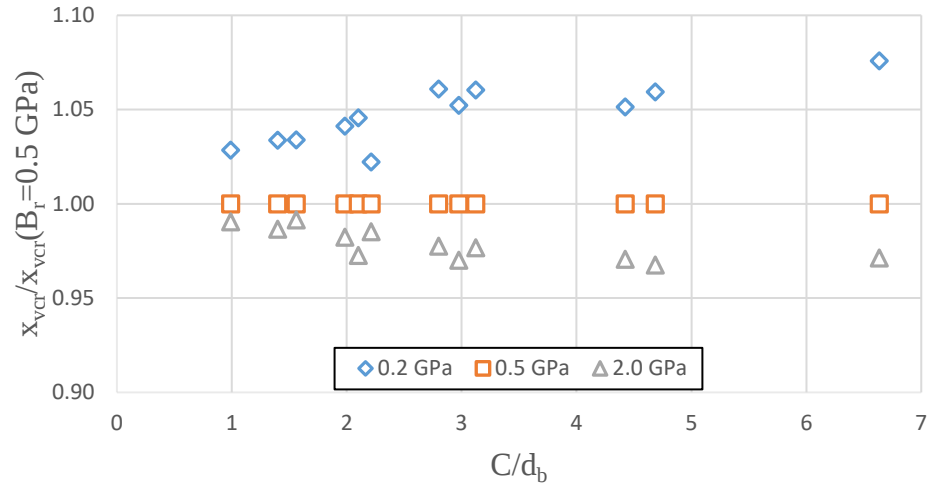


Figure 5.41: Effect of the bulk modulus on the visual critical attack penetration according to the numerical model

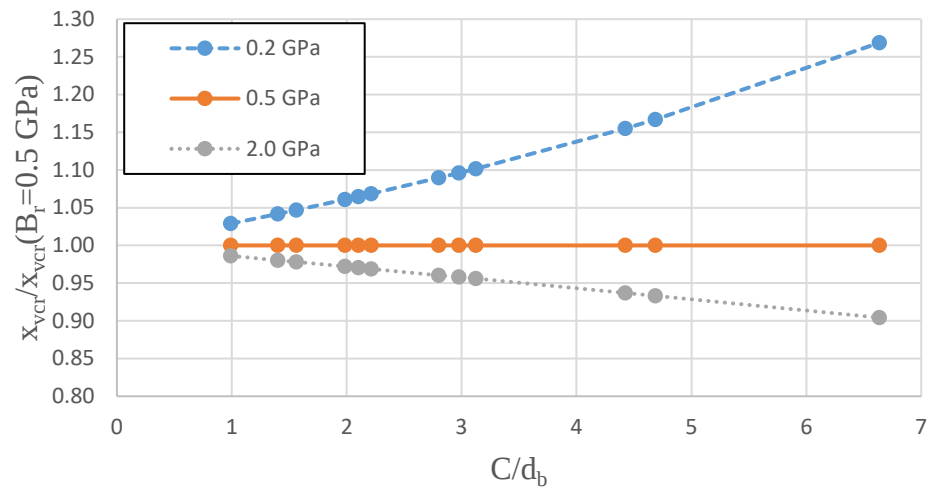


Figure 5.42: Effect of the bulk modulus on the visual critical attack penetration according to the elastoplastic cylinder model

5.3.2.9 Crack Volume Coefficient (C_v):

The effect of the crack volume coefficient was found to be very significant according to both models (the numerical and elastoplastic cylinder). The effect of the crack volume coefficient increases with increasing cover-to-rebar diameter ratio. Figure 5.43 and Figure 5.44 plot the ratio of the visual critical attack penetration to the visual critical attack penetration if the crack volume coefficient was zero according to the numerical and the

elastoplastic cylinder models, respectively. The impact of the percentage of crack volume occupied by corrosion products significantly increases as the concrete cover-to-rebar diameter ratio increases. For a cover-to-rebar diameter ratio of 6.64, the attack penetration corresponding to the first visual crack for a cylinder with cracks full of rust is approximately 6 times larger than the same cylinder with no rust within its cracks.

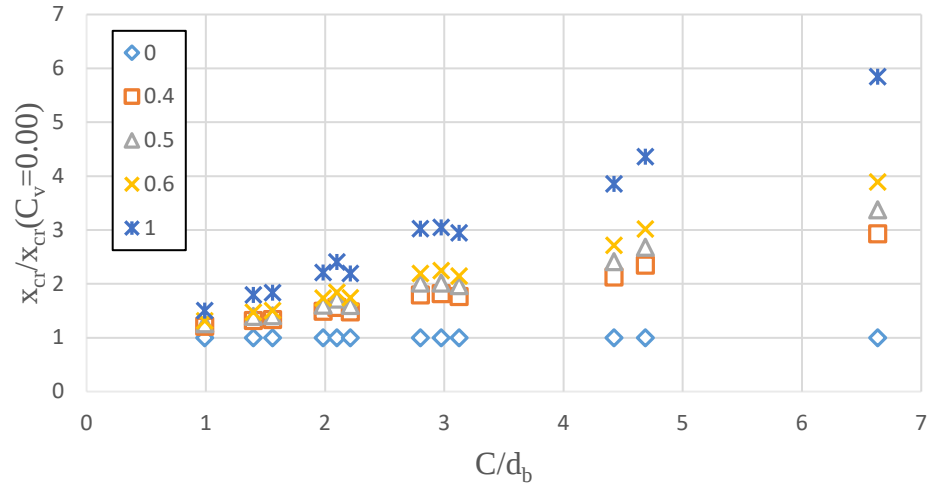


Figure 5.43: Ratio of the visual critical attack penetration to the visual critical attack penetration if the crack volume coefficient was zero according to the numerical approach

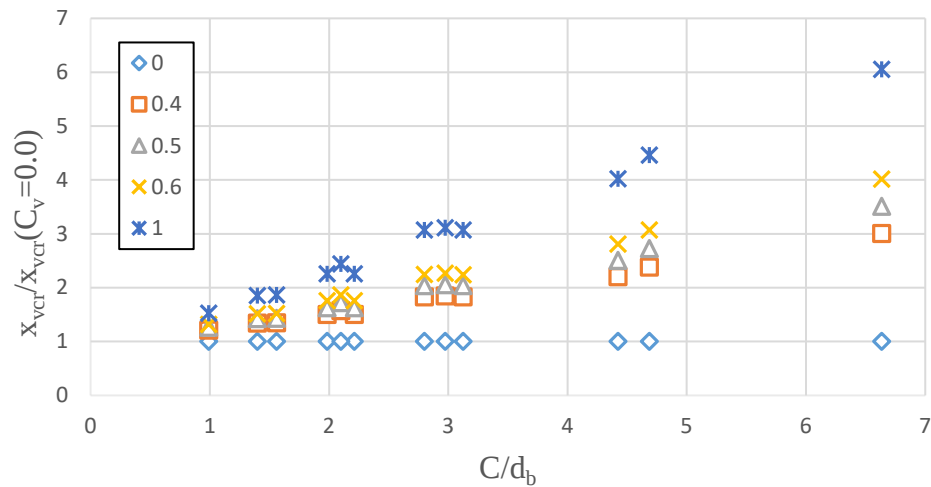


Figure 5.44: Ratio of the visual critical attack penetration to the visual critical attack penetration if the crack volume coefficient was zero according to the elastoplastic cylinder model

To better demonstrate the impact of the crack volume coefficient, the analysis was repeated for a case that ignores the presence of the porous zone around the reinforcing bar (i.e., $t_p = 0$). Figure 5.45 and Figure 5.46 plot the visual critical attack penetration against the concrete cover-to-rebar diameter ratio according to the numerical and elastoplastic cylinder models, respectively. If the porous zone is ignored, the graphs presenting the ratio of the results become almost linear with respect to C/d_b .

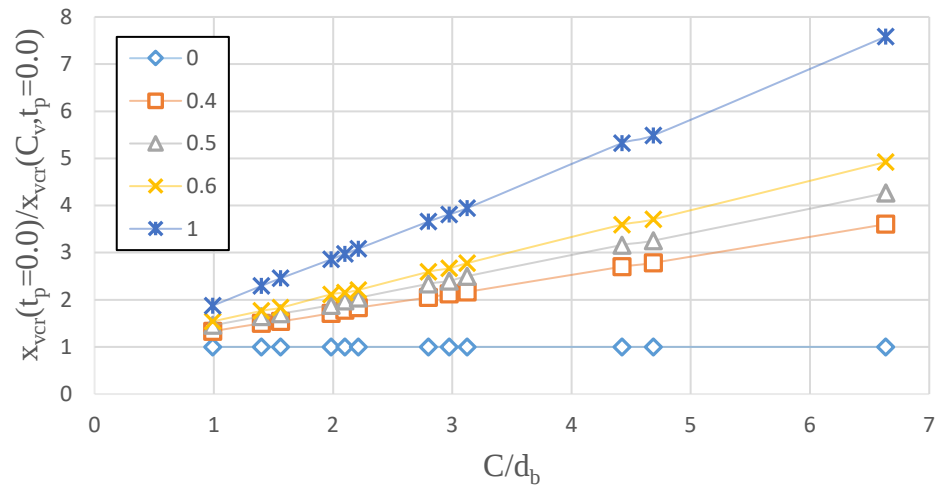


Figure 5.45: Ratio of the visual critical attack penetration without porous zone to the visual critical attack penetration without consideration of the cracks and porous zone according to the numerical model

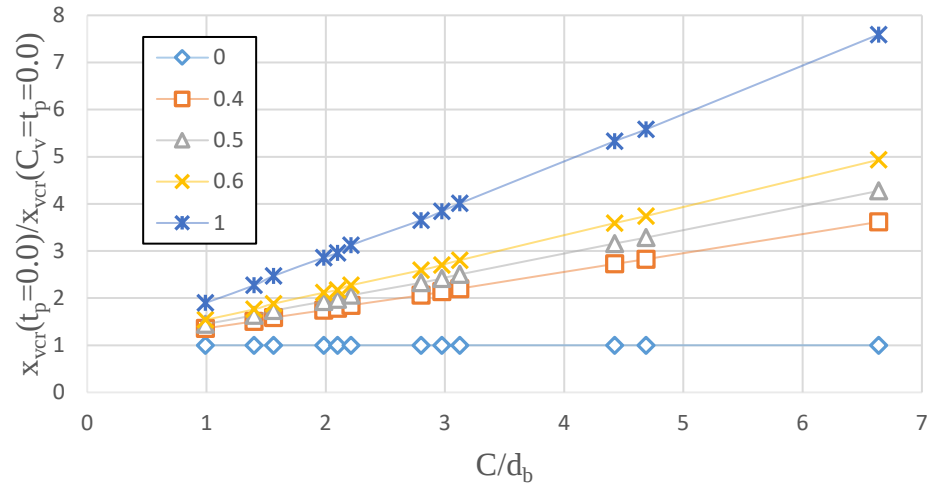


Figure 5.46: Ratio of the visual critical attack penetration without porous zone to the visual critical attack penetration without consideration of the cracks and porous zone according to the elastoplastic cylinder model

5.3.2.10 Creep Coefficient:

The effect of the creep coefficient is only investigated for the elastoplastic cylinder model. This is because the creep coefficient is applied to a constant Young's modulus (E), whereas the numerical model uses Hognestad's parabola to model the concrete's behaviour in compression. The results are based on two assumptions: (i) the creep in tension and compression is the same, and (ii) f'_t does not decrease with a sustained load.

Based on the assumptions stated above, the effects of different creep coefficients are plotted in Figure 5.47 to Figure 5.50, showing the ratio of the visual critical attack penetration to the visual critical attack penetration if the creep coefficient was zero. Creep increases the visual critical attack penetration as high as 290% for a creep coefficient of 4, cover depth of 75 mm and rebar diameter of 11.3 mm. The effect is more significant for larger cover depths and smaller rebars.

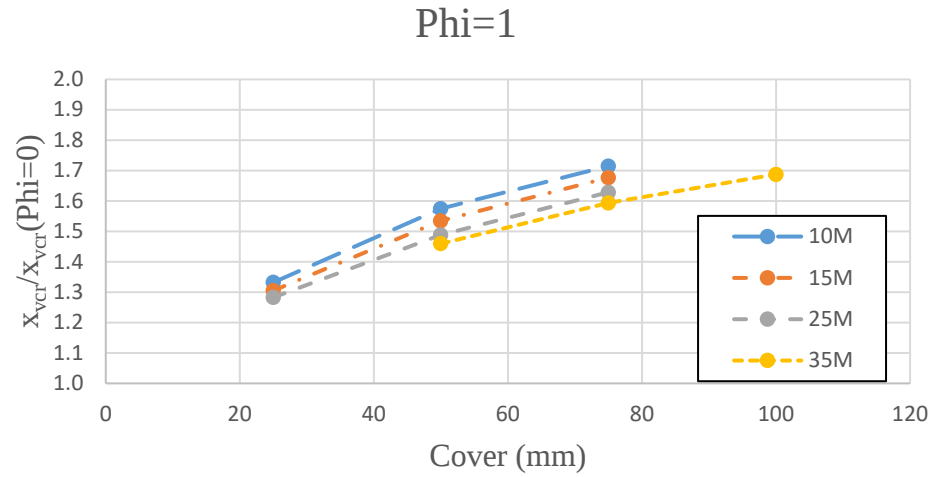


Figure 5.47: Effect of the creep coefficient $\phi = 1$ on the visual critical attack penetration

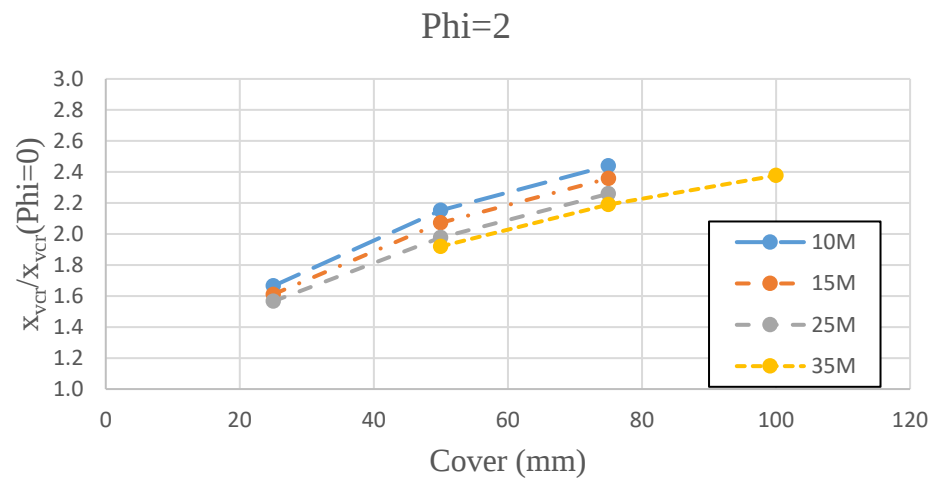


Figure 5.48: Effect of the creep coefficient $\phi = 2$ on the visual critical attack penetration

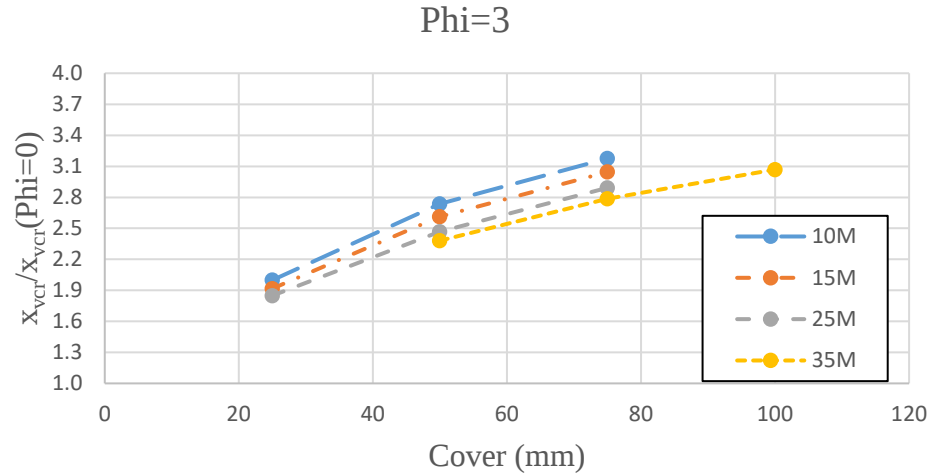


Figure 5.49: Effect of the creep coefficient $\phi = 3$ on the visual critical attack penetration

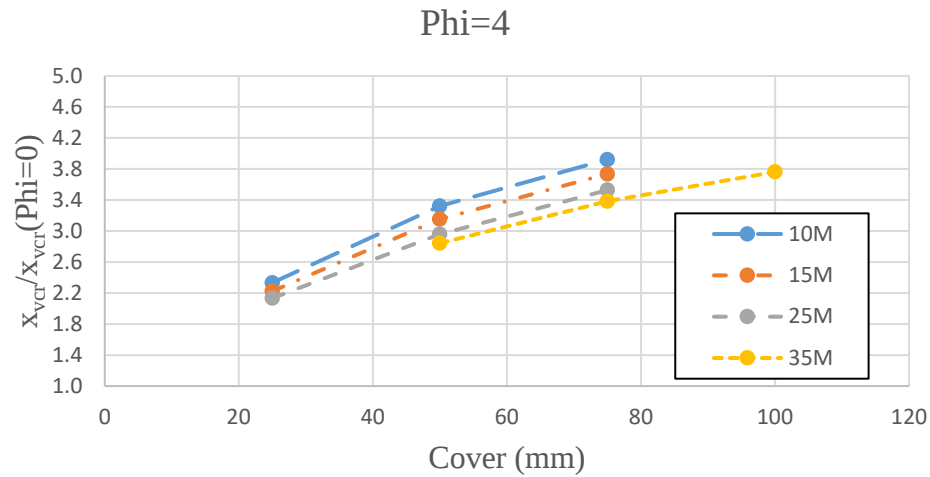


Figure 5.50: Effect of the creep coefficient $\phi = 4$ on the visual critical attack penetration

Note that creep is neglected in the validation of the models against the reported experimental data in the literature (Section 5.2.2.1). This is because there is a lack of agreement on the behaviour of concrete in tension under a sustained load among researchers. While some researchers believe the creep behaviour of concrete in tension is similar to its behaviour in compression, some believe the creep in tension is around one third of the creep in compression. In addition, the experimental works found in the

literature on the behaviour of concrete in tension under a sustained load only include specimens with an applied load lower than the tensile strength. Therefore, it is not known if the tensile strength of concrete decreases under a sustained load or not. The determining parameter in the cracking problem is the critical tensile strain (ϵ_{cr}), which is a combination of the tensile strength and Young's modulus.

5.3.2.11 Diameter and Cover Depth:

Figure 5.51 and Figure 5.52 respectively demonstrate the effect of different rebar diameters and cover depths on the critical attack penetration according to the numerical model. Figure 5.53 and Figure 5.54 demonstrate the effect of different rebar diameters and cover depths on the critical attack penetration according to the elastoplastic cylinder model. The results are based on the default values in Table 5.6. Increasing the cover depth while keeping the rebar diameter constant increases the visual critical attack penetration for both models (e.g, as much as 3.6 or 4.1 times for a rebar diameter of 11.3 mm if a cover depth of 75 mm is used instead of 25 mm based on the numerical model or the elastoplastic cylinder, respectively). Increasing the rebar's diameter while keeping the cover depth constant decreases the visual critical attack penetration for both models (e.g, as much as 2.2 or 2.6 times for a cover depth of 75 mm if a rebar diameter of 35.7 mm is used instead of 11.3 mm based on the numerical model or the elastoplastic cylinder, respectively).

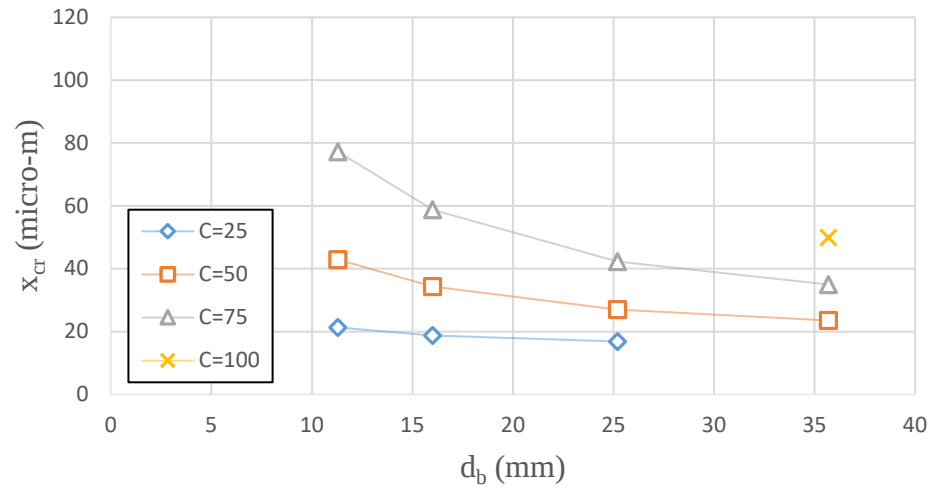


Figure 5.51: Effect of the rebar's diameter on the visual critical attack penetration according to the numerical approach

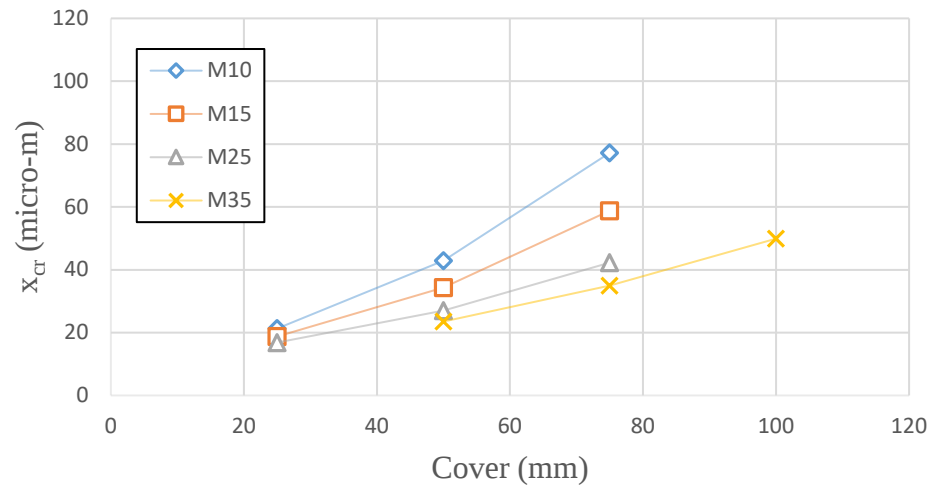


Figure 5.52: Effect of the cover depth on the visual critical attack penetration according to the numerical approach

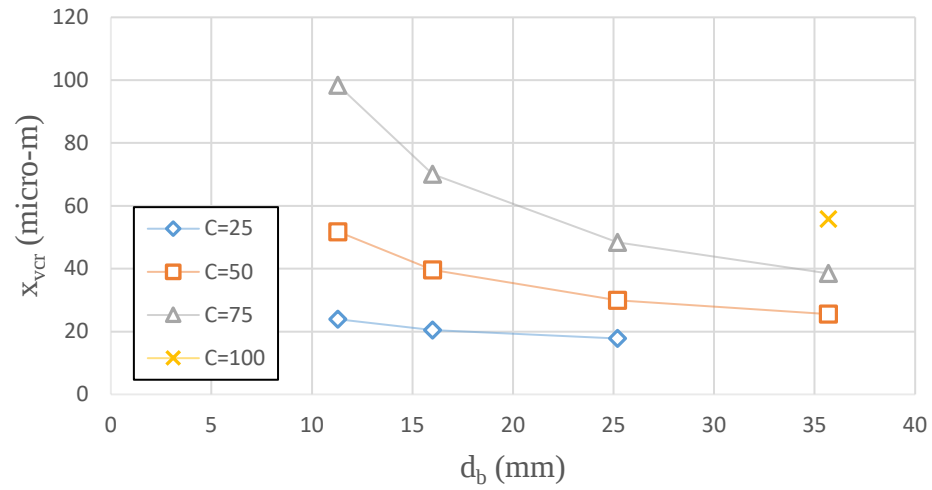


Figure 5.53: Effect of the rebar's diameter on the visual critical attack penetration according to the elastoplastic cylinder model

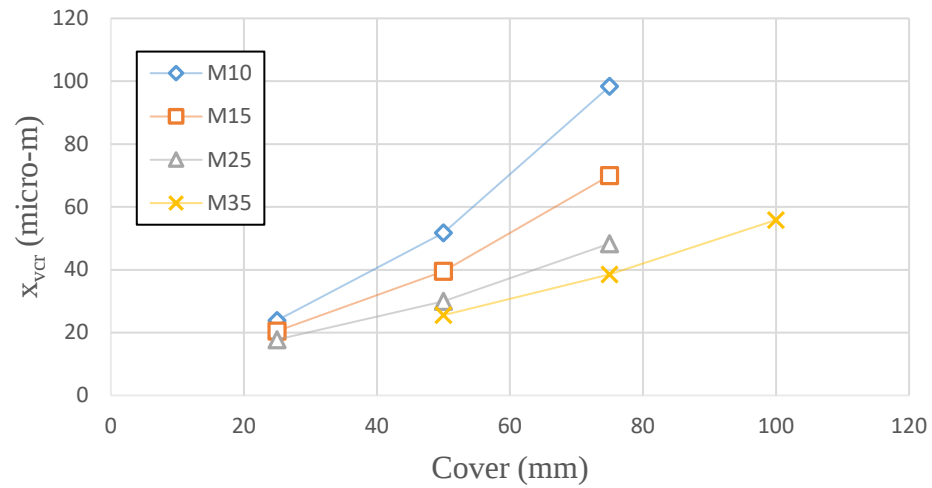


Figure 5.54: Effect of the cover depth on the visual critical attack penetration according to the elastoplastic cylinder model

5.4 Summary

Based on the comparison provided in Section 5.2:

- The brittle elastic cylinder results in a sudden brittle crack propagation to the concrete surface.
- The maximum internal pressure increases with cover-to-rebar diameter ratio based on all models. The maximum internal pressure is also found to be either proportional (i.e., the brittle elastic cylinder, the elastoplastic cylinder) or slightly more than proportional (the numerical model) to the tensile strength of the concrete cover.
- The results obtained by the numerical model are comparable to the experimental results reported in the literature; the average ratio of the numerical results to the experimental results was 0.97 with a COV of 0.86.
- Analytical models (elastoplastic and brittle elastic cylinders) provide upper and lower bounds to the results obtained by the numerical model (e.g., crack front, internal pressure, crack width). The results of the analytical models diverge as the cover-to-rebar diameter increases.
- The elastoplastic cylinder provides reasonably close results to those obtained by the numerical model for cover-to-rebar diameter ratios less than 3.

Based on the parametric study presented in Section 5.3:

- The ratio of the visual critical attack penetration to the critical attack penetration is between 1.7 to 1.2 for cover depths ranging from 25 to 100 mm. The ratio decreases as cover depth increases. Therefore, it is very important to compare the same time frames when it comes to comparing the experimental results to the results obtained by various models.
- Table 5.7 summarizes the effect of different parameters on the maximum internal pressure ($P_{\max}$) and the visual critical attack penetration (x_{vcr}) according to the numerical and elastoplastic cylinder models.

Table 5.7: Effect of inputs on results

Parameters		Effect		
Output		$P_{\max}$		x_{vcr}
Model	Num.	E.P.	Num.	E.P.
v_c	+ ⁽¹⁾ NEG ⁽²⁾	- ⁽³⁾ NEG	+NEG	+NEG
f'_t	+PRO ⁽⁴⁾	+PRO	-NEG	+MOD ⁽⁵⁾
$\emptyset$	N/A ⁽⁶⁾	NON ⁽⁷⁾	N/A	+SIG ⁽⁸⁾
d_a	-NEG	N/A	-NEG	N/A
t_p	NON	NON	+MOD	+MOD
C_v	NON	NON	+SIG	+SIG
R_v	NON	NON	-SIG	-SIG
B_r	NON	NON	-NEG	-MOD
Tension Softening	NEG	N/A	NEG	N/A
Model				

⁽¹⁾ “+” sign means increasing the input parameter increases the output result.

⁽²⁾ NEG: The effect is negligible, where a negligible effect is defined as an effect between 0% and 10%.

⁽³⁾ “-” sign means increasing the input parameter decreases the output result.

⁽⁴⁾ POR: The effect is Proportional.

⁽⁵⁾ MOD: The effect is moderate, where a moderate effect is defined as an effect between 10% to 100%.

⁽⁶⁾ N/A: Not Applicable or Not Available.

⁽⁷⁾ NON: There is no effect.

⁽⁸⁾ SIG: The effect is significant, where a significant effect is defined as an effect more than 100%.

Chapter 6

Finite Element Modelling

6.1 Finite Element Model

The main objective of this chapter is to investigate the effect of some parameters which are neglected in the previous models. These parameters might have influential effects on the cracking process of the concrete cover due to corrosion. The parameter investigations are as follows:

- i. Investigating the validity of the thick-walled cylinder analogy to model corrosion-induced cracking of RC structures (i.e., investigating the effect of the extra concrete around the hypothetical thick-walled cylinder);
- ii. Investigating the effect of an adjacent rebar;
- iii. Investigating the effect of the rebar's boundary conditions.

To achieve this objective, finite element software VecTor2 was used to model the cover cracking process of four different geometries (Figure 6.1). The finite element models (FEMs) are two-dimensional, and they consist of plane membrane, quadrilateral elements.

Since the auto-mesh function of VecTor2 was unable to mesh the model as desired, another commercial finite element software, ABAQUS, was used to mesh the models (with the exception of Geometry A in Figure 6.1). Consequently, node and element definitions were manually imported to the VecTor2 model.

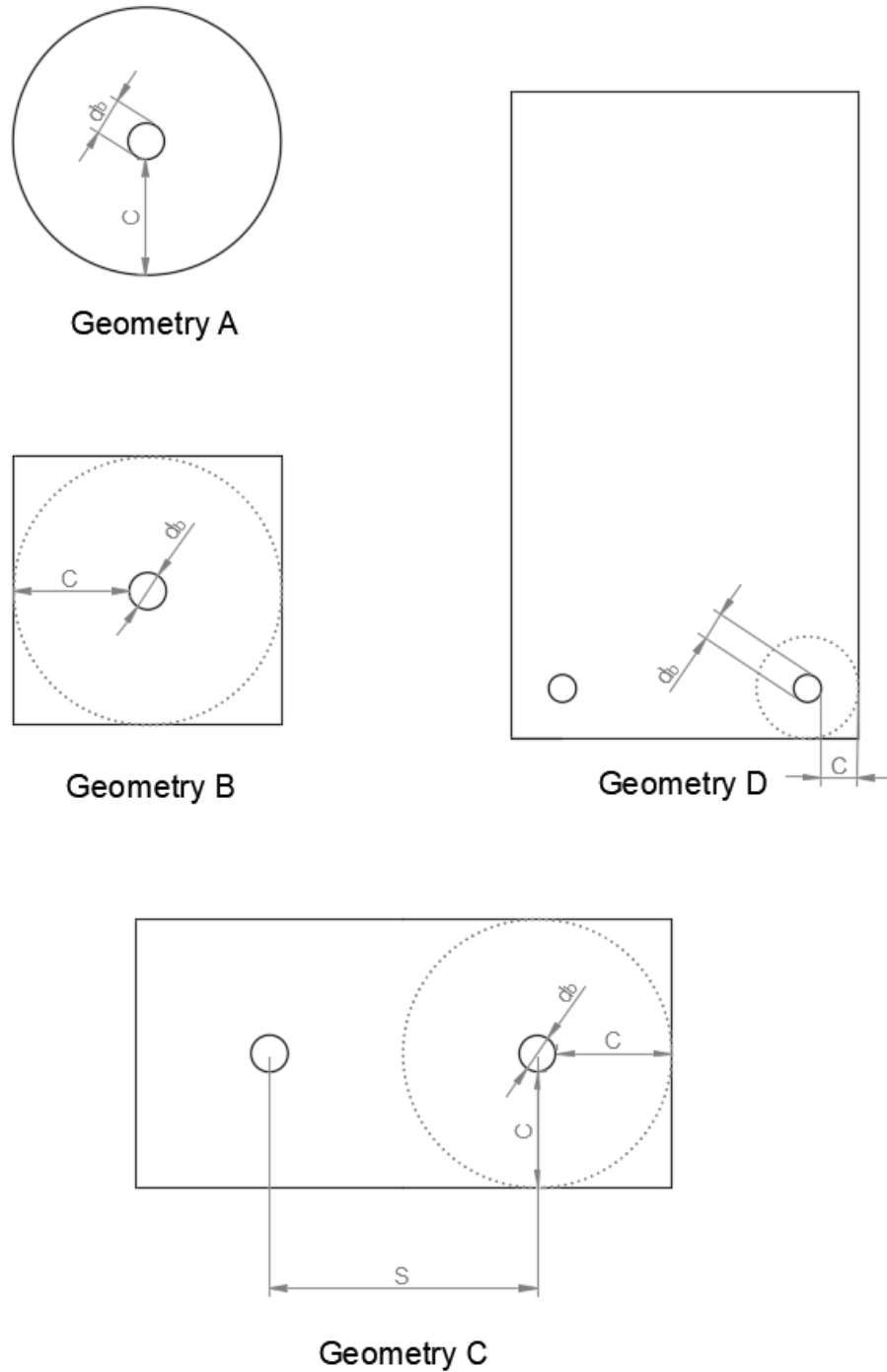


Figure 6.1: FEM geometries

Hognestad's parabola and the compression softening model proposed by Vecchio and Collins (1986) were used to model the concrete behaviour in compression. This is consistent with the numerical model discussed in Chapter 3.

Except for Geometry D (Section 6.5), all of the FE models use concrete with the same material properties as the cylinder which was investigated in Section 5.2.1:

- Tensile strength (f'_t) = 3.94 MPa
- Compressive Strength (f'_c) = 42.25 MPa
- Young's Modulus (E_c) = 34,700 MPa

6.1.1 Modelling the Corrosion-Induced Expansion

The corrosion-induced expansion at the steel/concrete interface is modelled using truss elements subjected to an equivalent thermal load (Equation 6.1).

$$[6.1] \quad \Delta r_b = L_t C_s \Delta T$$

where Δr_b is the applied radial displacement at the rebar/concrete interface, L_t is the original length of the truss elements (rebar's radius in this case), C_s is the truss elements' thermal expansion coefficient, and ΔT is the equivalent thermal load.

The Young's modulus and cross section of the truss elements were arbitrarily set to very large values so that the compression of the truss elements would be negligible. Nonetheless, when the analyses were run, the elongation of the truss elements were read and compared to the previous models to ensure that the results were independent of the chosen input values of C_s , ΔT , and Young's modulus of the truss elements.

Using truss elements instead of triangular or quadrilateral elements allowed a radial displacement to be applied without forcing any tangential displacement or applying any tangential stiffness (Figure 6.2).

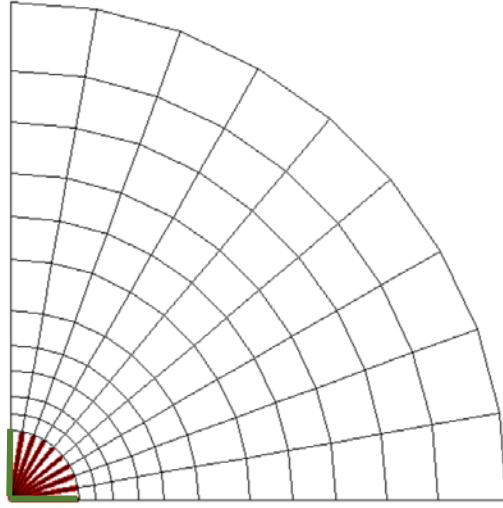


Figure 6.2: Truss elements (Geometry A)

6.1.2 Tension Softening and Mesh Size Sensitivity

According to Bazant and Oh (1983), finite element results are dependent on the chosen element size where tensile cracking occurs, and consequently, the fracture energy is selected as the failure criterion. In this situation, the element size should be chosen according to Equation 6.2:

$$[6.2] \quad h = w_c \times \cos \alpha$$

where w_c is the characteristic width, h is the element's width perpendicular to the crack, α is the angle between principal tensile strain and the element's side (if none of the sides is perpendicular to the crack) (Figure 6.3).

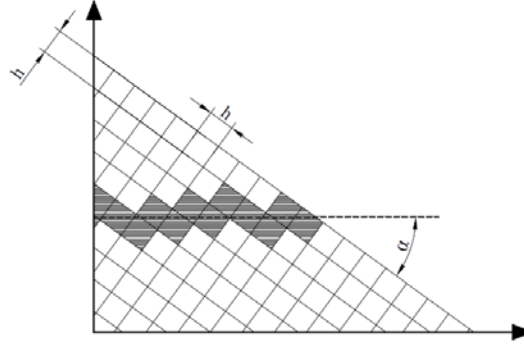


Figure 6.3: Definition of α (adapted from Bazant and Oh, 1983)

Alternatively, if another element size is chosen, the tensile strain-softening diagram must be modified in such a way that the area under the tensile stress-strain diagram is equal to Equation 6.3:

$$[6.3] \quad A = \frac{G_f}{h}$$

where G_f is the fracture energy of the concrete, and A is the area under the tensile stress-strain diagram. In other words, the product of the element's width (perpendicular to the crack) multiplied by the area under the tensile stress-strain diagram must be constant and equal to the fracture energy of the concrete.

VecTor2 allows users to define a single tensile stress-strain diagram manually or use one of the provided softening models. The first option can only be used if the element sizes are almost constant throughout the entire finite element mesh, as the software only allows the user to define one tensile stress-strain diagram for the whole model. The first option is used for all the models in this chapter, except for Geometry A, as the elements closer to the rebar are much smaller than the ones further away from it.

6.2 Mesh and Material Model Sensitivity (Geometry A)

In order to compare the finite element analysis (FEA) results with those found by the numerical (Chapter 3) and analytical (Chapter 4) models, the concrete cover was modelled as a slice of a quarter of a cylinder under plane stress conditions, as illustrated in Figure 6.1 (Geometry A). The rebar diameter (d_b) and the cover depth (C) were set as 16 mm and 50 mm, respectively. Due to the symmetry of the problem, symmetry boundary conditions were enforced along the edges. The center of the rebar was fixed along both Cartesian directions.

In order to apply a completely uniform expansion to the cylinder, the cross section of the truss elements at the model edges (green elements in Figure 6.2) were half that of the other truss elements in order to be compressed equally. The reaction forces (from the concrete) applied to the edge truss elements were also half of those applied to the inner truss elements.

In order to choose an appropriate tension-softening model, a sensitivity study was performed using different models offered by VecTor2. It was found that the results are dependent on the chosen tension-softening model, mesh size, and maximum crack spacing. Maximum crack spacings are defined in two directions: perpendicular to the x-reinforcement and perpendicular to the y-reinforcement. Maximum crack spacing is arbitrary in the geometries discussed in this chapter, as the quadrilateral elements represent plain concrete. However, the maximum crack spacing must be chosen reasonably as the results are dependent on the chosen values.

Figure 6.4 demonstrates the two mesh sizes used in the sensitivity analysis, as well as the boundary conditions. The boundaries are shown by small dots. In the coarse mesh, the geometry A is divided into 9 elements in the tangential direction, whereas the geometry is divided into 18 elements in the fine mesh. In both cases, they are divided in the radial direction in a way that the elements have an aspect ratio close to 1.

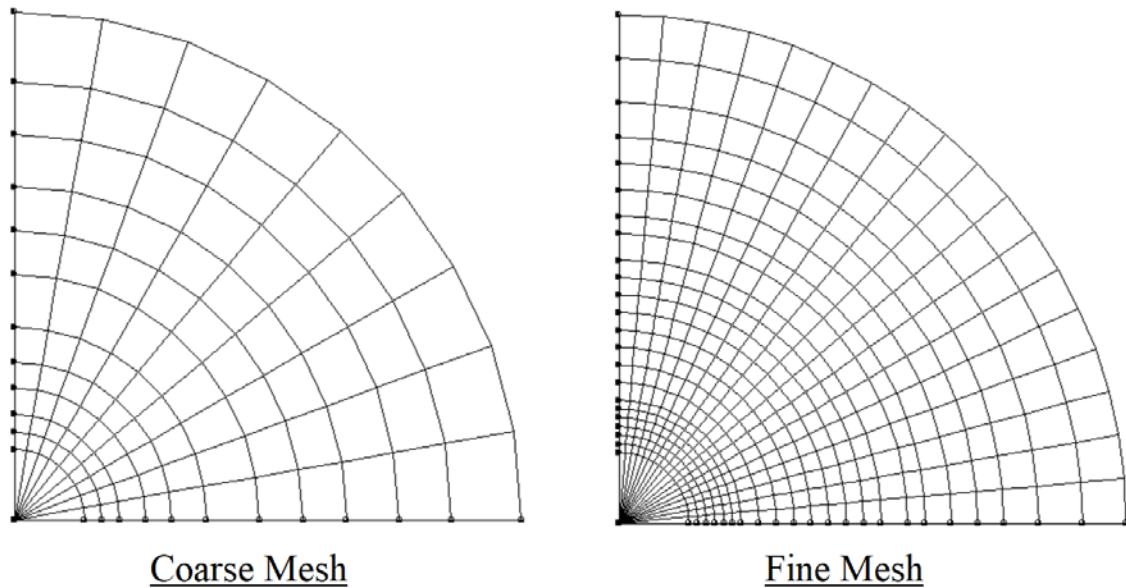


Figure 6.4: Cylinder with coarse and fine meshes

The FEA results along with the corresponding results of the numerical and analytical (both brittle elastic and elastoplastic) models presented in Chapters 3 and 4, respectively, are summarized in Table 6.1. The critical Δ_{rb} is the displacement of the internal boundary corresponding to the critical attack penetration (i.e., the corrosion-induced attack penetration that corresponds to a crack reaching the cover's surface). Tension softening models available in VecTor2 and used in the analyses are: linear, bilinear, and nonlinear (Hordijk, 1991). The VecTor2 results are reported to a precision of only two significant figures.

Table 6.1: Comparison of FEA with numerical and analytical models

Tension Softening	Mesh	Critical Δ_{rb} (μm)						
		Maximum Crack Spacing ⁽¹⁾ (mm)						
		2	5	10	20	60	120	Default
Linear	Coarse	12	- ⁽²⁾	-	-	-	12	12
	Fine	9	-	-	-	9	9	9
Bilinear	Coarse	14	-	13	13	-	13	13
	Fine	14	-	13	-	-	13	13
Nonlinear (Hordijk ,1991)	Coarse	14	-	13	11	9	9	7
	Fine	14	12	10	-	6	5	4
Numerical	13.54							
Brittle Elastic (Analytical)	6.66 (Crack depth ratio=100%) 7.29 (Crack depth ratio=80%) ⁽³⁾							
Elastoplastic (Analytical)	15.26							

⁽¹⁾ Perpendicular to x-reinforcement and y-reinforcement

⁽²⁾ The analysis is not run

⁽³⁾ The internal boundary displacement is maximum when the crack depth ratio is 80%.

It was found that if the nonlinear (Hordijk) tension softening model is used, the results are highly dependent on the maximum crack spacing. From the results, it is observed that the following conditions provide equivalent results:

$$\begin{aligned} &\text{Critical } \Delta_{rb} (\text{Mesh: Coarse, Maximum Crack Spacing} = 20 \text{ mm}) \\ &= \text{Critical } \Delta_{rb} (\text{Mesh: Fine, Maximum Crack Spacing} = 10 \text{ mm}) \end{aligned}$$

$$\begin{aligned} &\text{Critical } \Delta_{rb} (\text{Mesh: Coarse, Maximum Crack Spacing} = 10 \text{ mm}) \\ &= \text{Critical } \Delta_{rb} (\text{Mesh: Fine, Maximum Crack Spacing} = 5 \text{ mm}) \end{aligned}$$

The element's width in the coarse mesh is double that of the element's width in the fine mesh.

Another series of analyses was done using different crack spacings for different element sizes. The crack spacings in both directions were chosen equal to the elements' widths (perpendicular to the cracks). The analyses were done for two mesh sizes and two tension softening models (bilinear and nonlinear (Hordijk)). The results are tabulated in Table 6.2.

Table 6.2: Critical Δ_{rb} (μm) for different mesh size and tension softening models

	Bilinear	Nonlinear (Hordijk)
Coarse Mesh	13	13
Fine Mesh	13	12

The results are similar to each other and comparable to the numerical model's result (Chapter 3).

Based on the previous results, the finite element analyses presented in the following sections for representative cylinders (Geometry A) are run using the bilinear tension softening model available in VecTor2. The crack spacings are also chosen to be equal to the elements' widths.

The analysis results obtained from the coarse mesh with the bilinear tension softening model are used to plot the hoop and radial stresses along the two paths shown in Figure 6.5. They are plotted for different crack fronts approximately equal to 8 mm, 28 mm, and 58 mm. The results obtained from the numerical model (Chapter 3) are also included in the plots for comparison purposes (Figures 6.6 and 6.11).

The radial and hoop stresses along Path 1 are based on the principal stresses (f_1, f_2) found by VecTor2 on Path 1. The radial and hoop stresses along Path 2 are based on the stresses in the x and y direction (f_x, f_y) found by VecTor2 on Path 2.

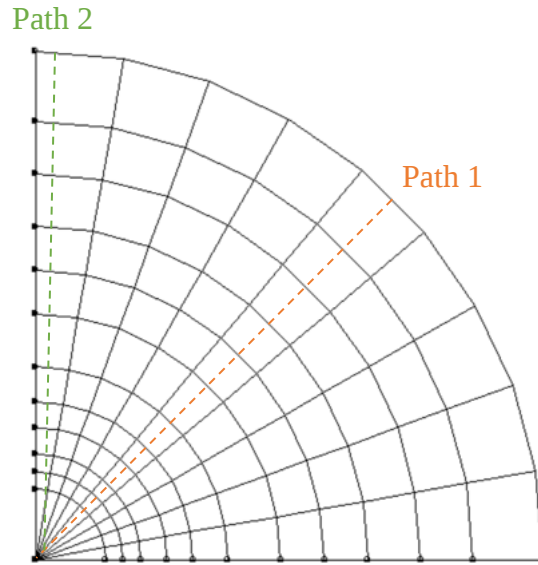


Figure 6.5: Two paths along which stresses are obtained

The results based on the numerical model and finite element analysis are similar. However, the finite element analysis provides higher tensile hoop stresses than the numerical model if the element is cracked. This is because the softening curve in the finite element analysis is stretched compared to the numerical model such that Equation 6.4 is satisfied.

$$[6.4] \quad A_1 \times h = A_2 \times 3d_a = G_f$$

where A_1 is the area under the tensile stress-strain curve in the finite element analysis, and A_2 is the area under the tensile stress-strain curve in the numerical model; A_2 is larger than A_1 as h is smaller than d_a here.

Crack front $\approx 58\text{mm}$:

Path1:

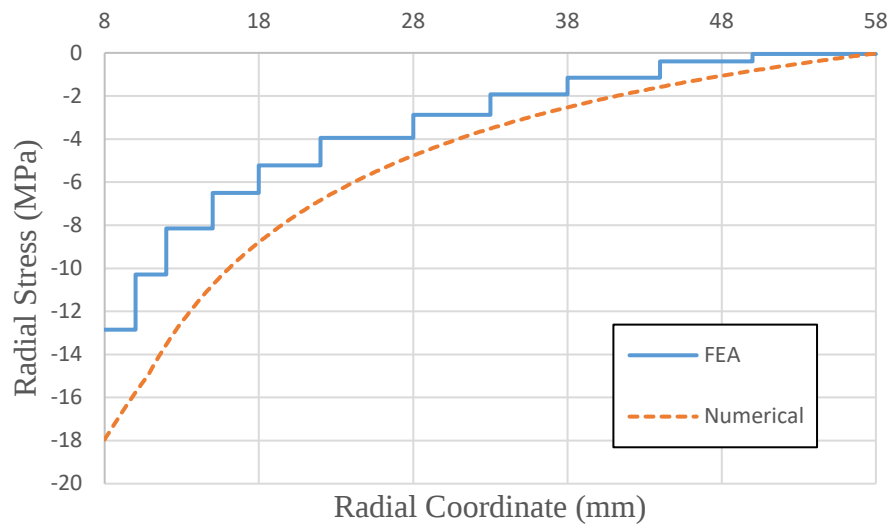
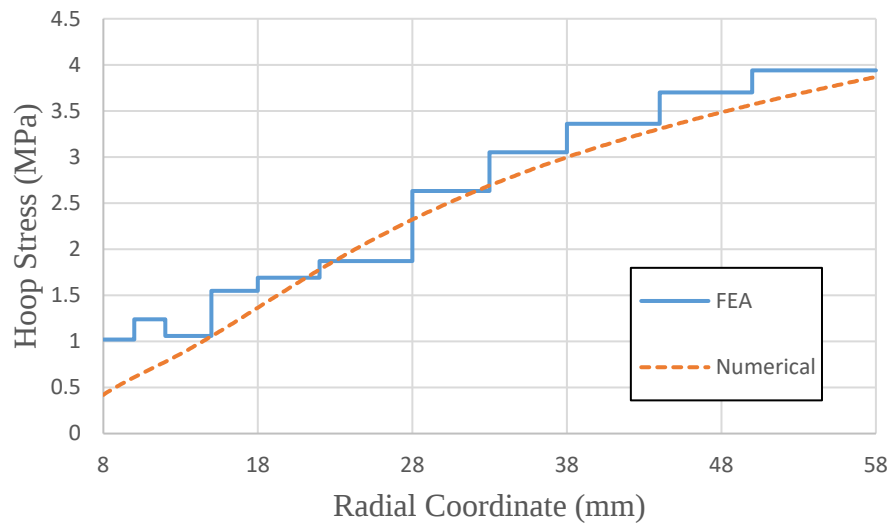


Figure 6.6: Hoop and radial stresses along Path 1 at a crack front of 58 mm

Path2:

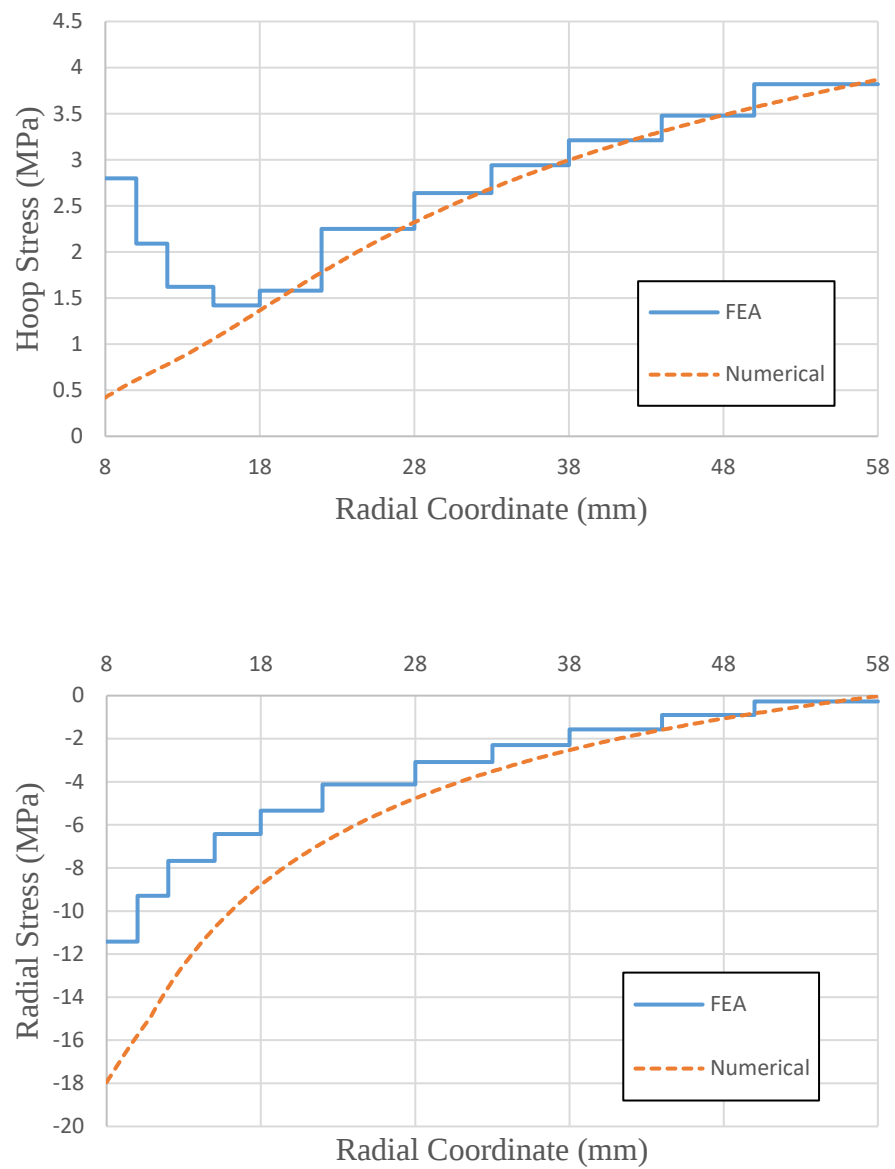


Figure 6.7: Hoop and radial stresses along Path 2 at a crack front of 58 mm

Crack front $\approx 28\text{mm}$:

Path1:

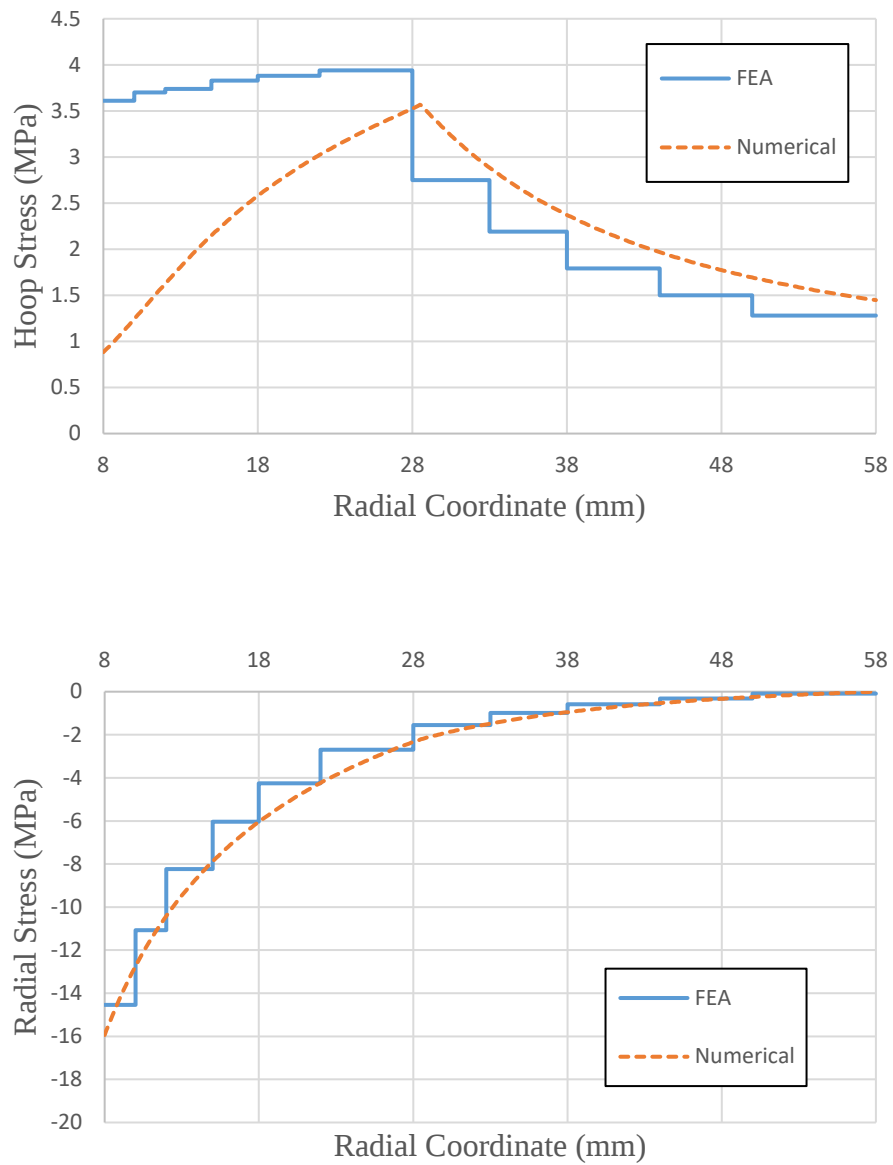


Figure 6.8: Hoop and radial stresses along Path 1 at a crack front of 28 mm

Path2:

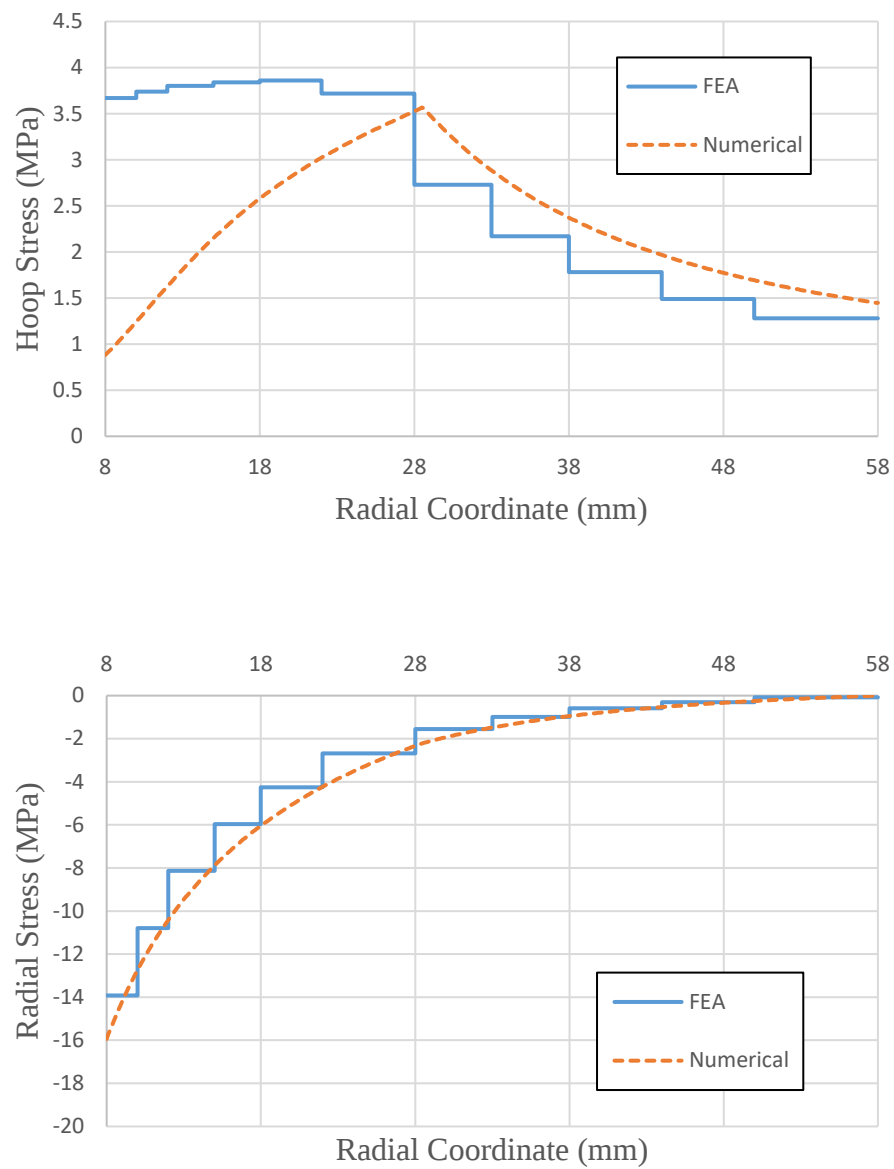


Figure 6.9: Hoop and radial stresses along Path 2 at a crack front of 28 mm

Crack front $\approx 8\text{mm}$:

Path1:

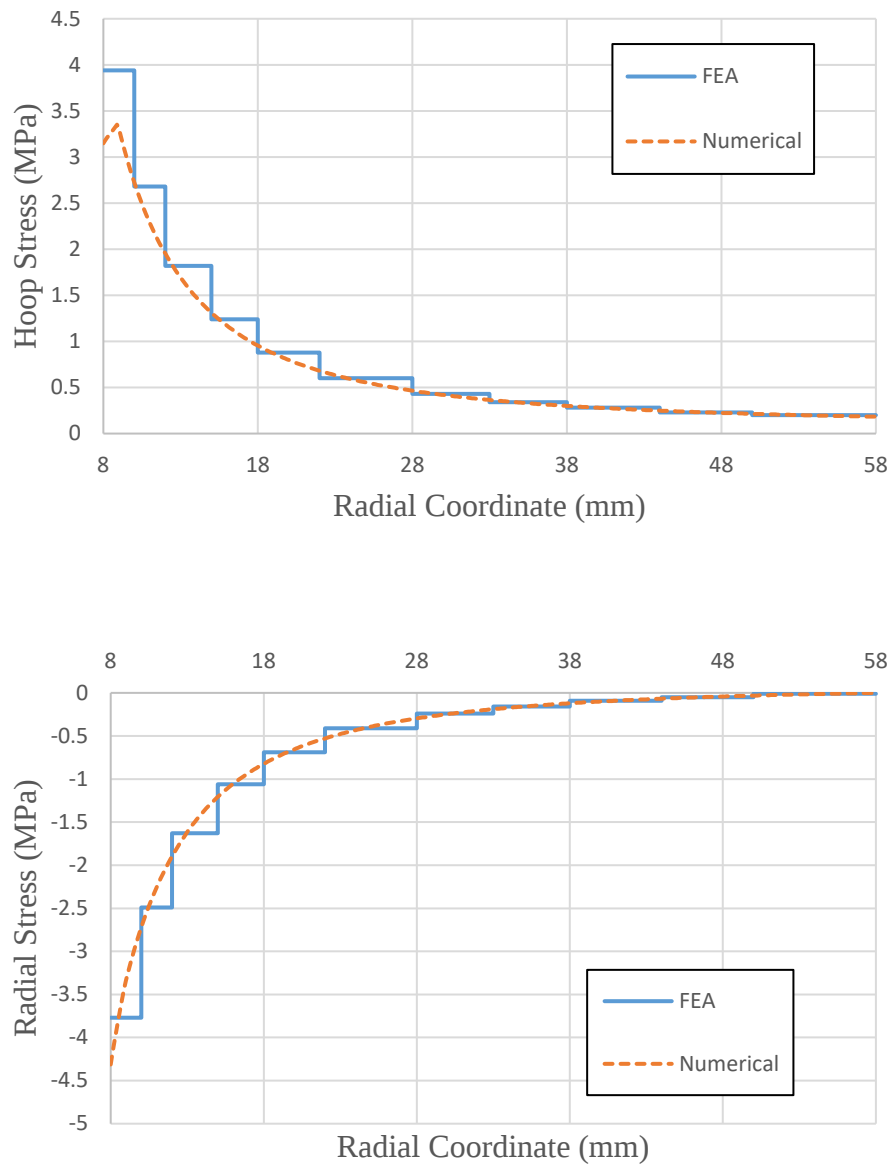


Figure 6.10: Hoop and radial stresses along Path 1 at a crack front of 8 mm

Path2:

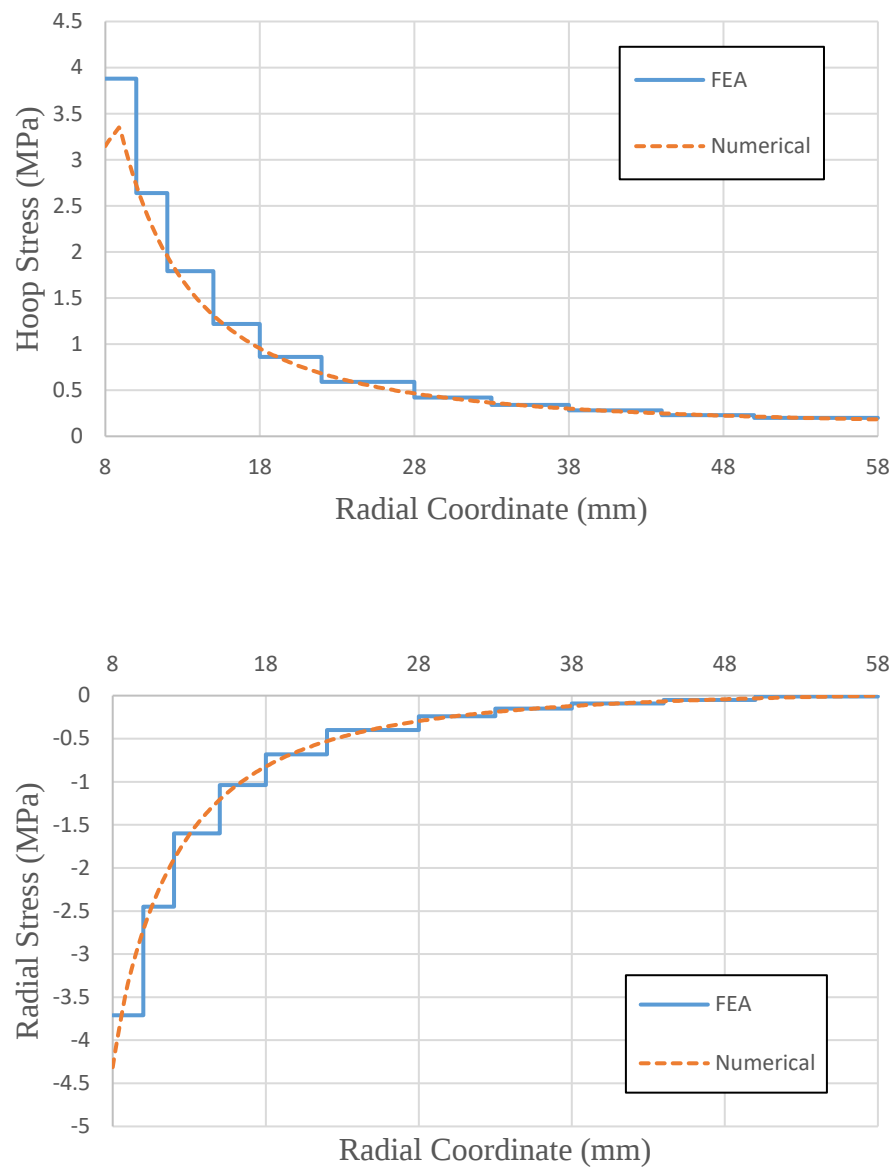


Figure 6.11: Hoop and radial stresses along Path 2 at a crack front of 8 mm

6.3 Geometry B

The corrosion-induced cover cracking models presented in Chapters 3 and 4 are based on the treatment of the concrete cover as a thick-walled cylinder subjected to the internal pressure resulting from the growth of corrosion products. To test this assumption, finite element analyses of the concrete cover were carried out using Geometry B shown in Figure 6.1. The rebar had a diameter of 16 mm, and the cover depth was 50 mm. One half of the geometry was modeled by using symmetry boundaries. The model was meshed with 3 different element sizes (Figure 6.12). The tensile stress-strain curves are defined manually based on the elements' sizes, as discussed in Section 6.1.2.

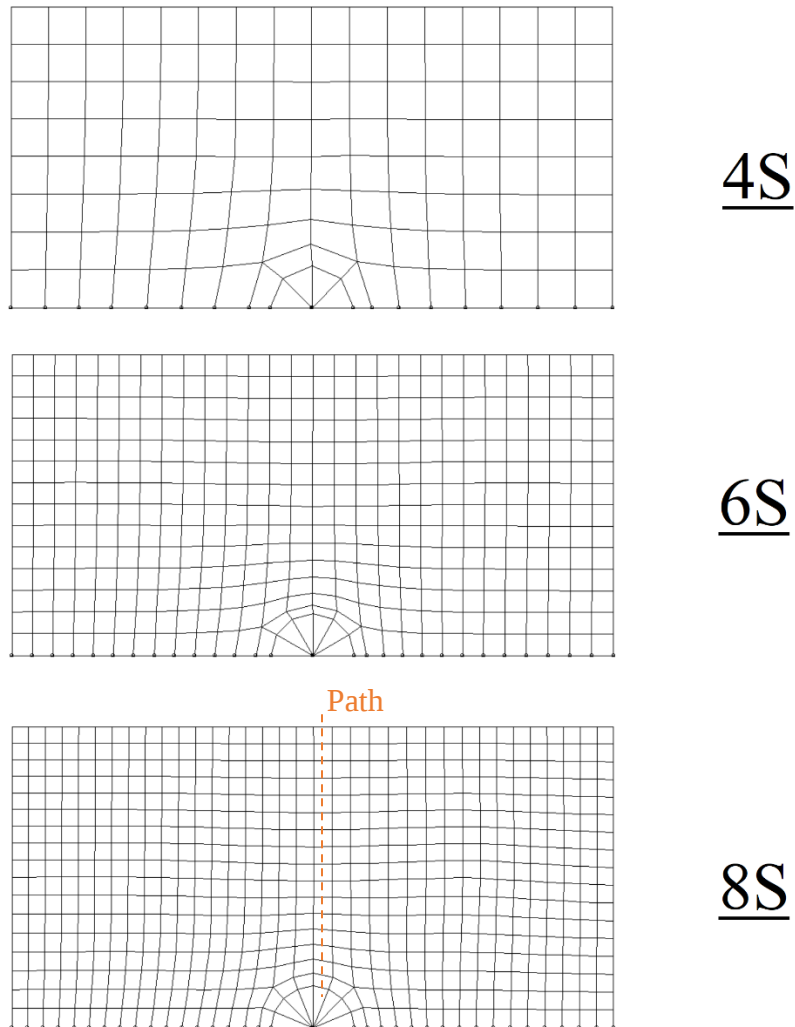


Figure 6.12: Different meshes for Geometry B

Similar to what is found by Du et al. (2006), the cracking process can be divided into 3 consecutive stages:

1. Internal Cracking: the first crack at the concrete/rebar interface.
2. External Cracking: the first crack at the cover's surface
3. Penetration Cracking: the first crack which penetrates from the cover's surface to the rebar's surface.

Figure 6.13, Figure 6.15, and Figure 6.17 plot the cracking pattern as well as the principal tensile stresses for stages 1, 2, and 3 (analyzed by 8S mesh), respectively. Figure 6.14, Figure 6.16, and Figure 6.18 plot the corresponding hoop and radial stresses based on f_x and f_y on the path shown on Figure 6.12. The first cracks appear at the concrete/rebar interface as the tensile strains are maximum (Stage 1, Figure 6.13 and 6.14). The expansion of corrosion products continues to push the internal boundary causing the internal cracks to propagate toward the surface. However, an external crack often appears at the cover's surface before any of the internal cracks reaches the surface. This is due to a stress concentration that is caused by a geometry that is not axisymmetric, so the stress distribution is not uniform in the hoop direction (Stage 2, Figure 6.15 and 6.16). The internal cracks stop propagating when the first external crack forms. The external cracks propagate towards the rebar until it reaches the internal cracks and forms the first penetration crack (Stage 3, Figure 6.17 and 6.18).

Table 6.3 provides the results found by the analyses with different mesh sizes. The internal cracking and the external cracking are more sensitive to the mesh size. This is due to the fact that whereas the maximum tensile stresses occur at a point, the 4-node isoparametric elements used have a constant tensile stress within the element.

Table 6.3: FE results for the attack penetration Δr_b for meshes 4S, 6S and 8S

Mesh	Average Elements' Width (mm)	Δr_b (μm)		
		Internal Cracking	External Cracking	Penetration Cracking
4S	7.25	2	14	14
6S	4.14	1	13	14
8S	3.41	1	12	14

The Δr_b corresponding to external cracking and penetration cracking are comparable to the critical Δr_b found by a representative cylinder (Geometry A, Section 6.2). A representative cylinder is a cylinder with the same rebar size and cover depth.

Stage 1: Internal Cracking

$$\Delta r_b = 1 \mu\text{m}$$

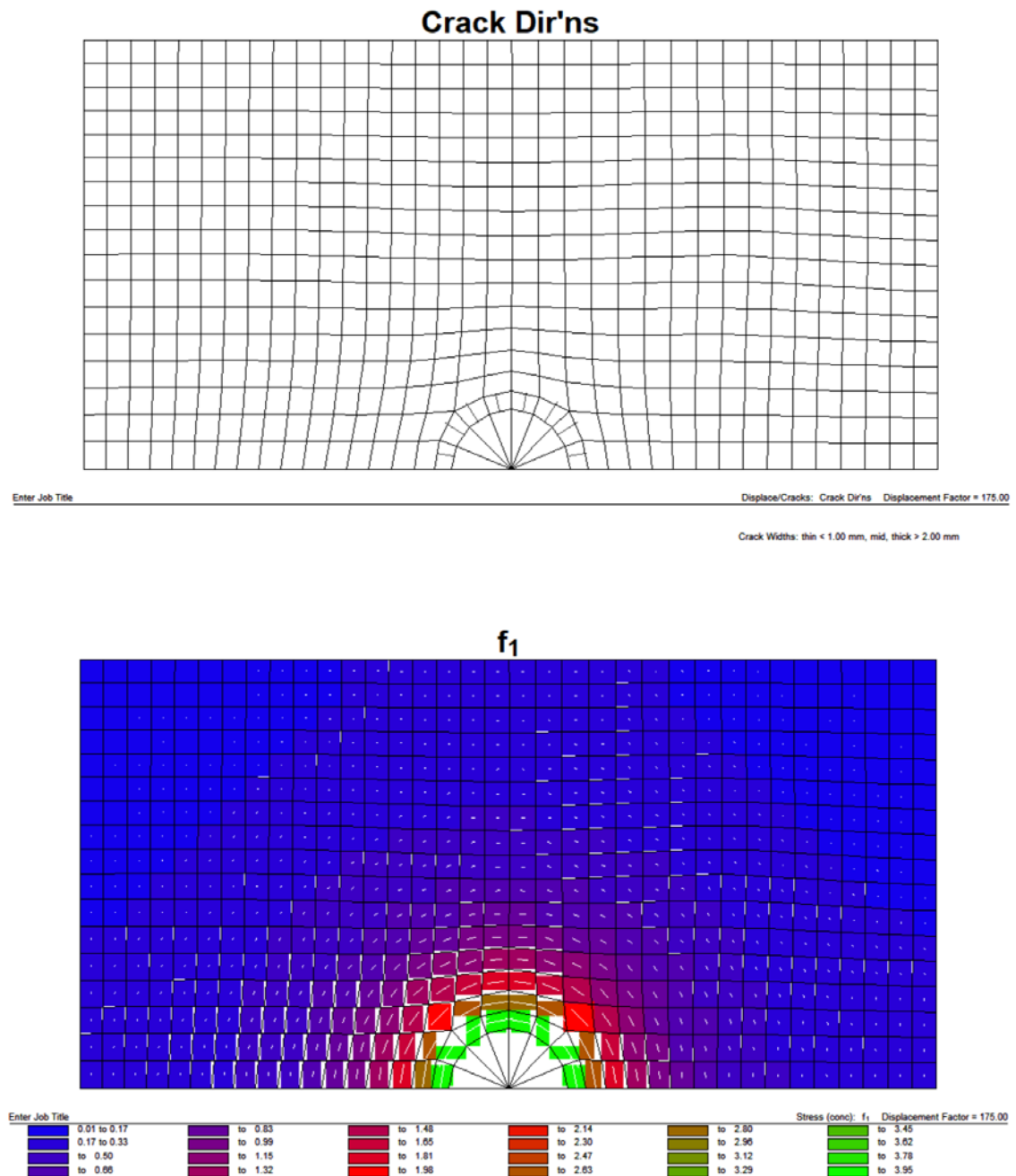


Figure 6.13: Stage1, Internal Cracking (legend values for f_1 are in MPa)

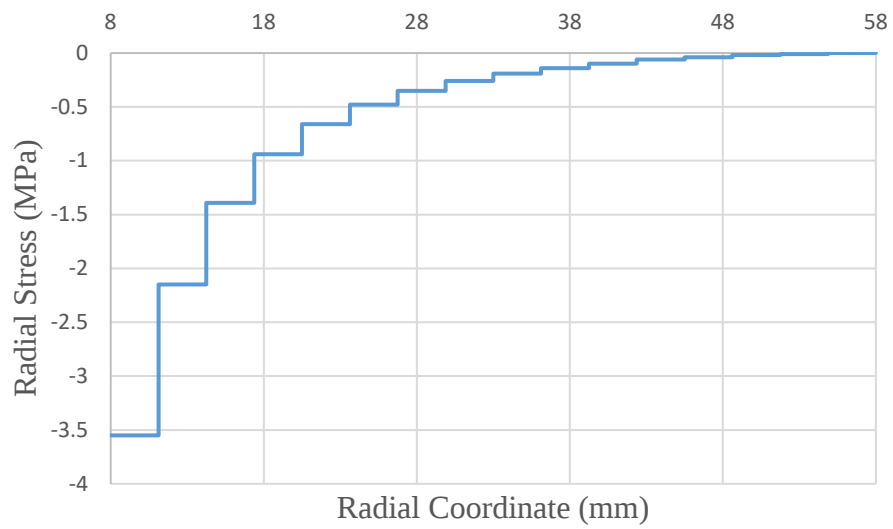
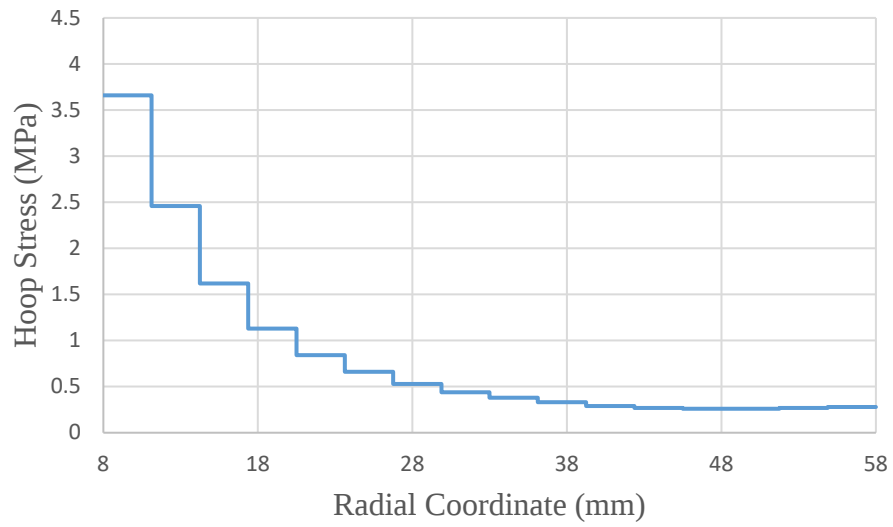


Figure 6.14: Stage 1, Radial and hoop stresses

Stage 2: External Cracking

$$\Delta r_b = 12 \mu\text{m}$$

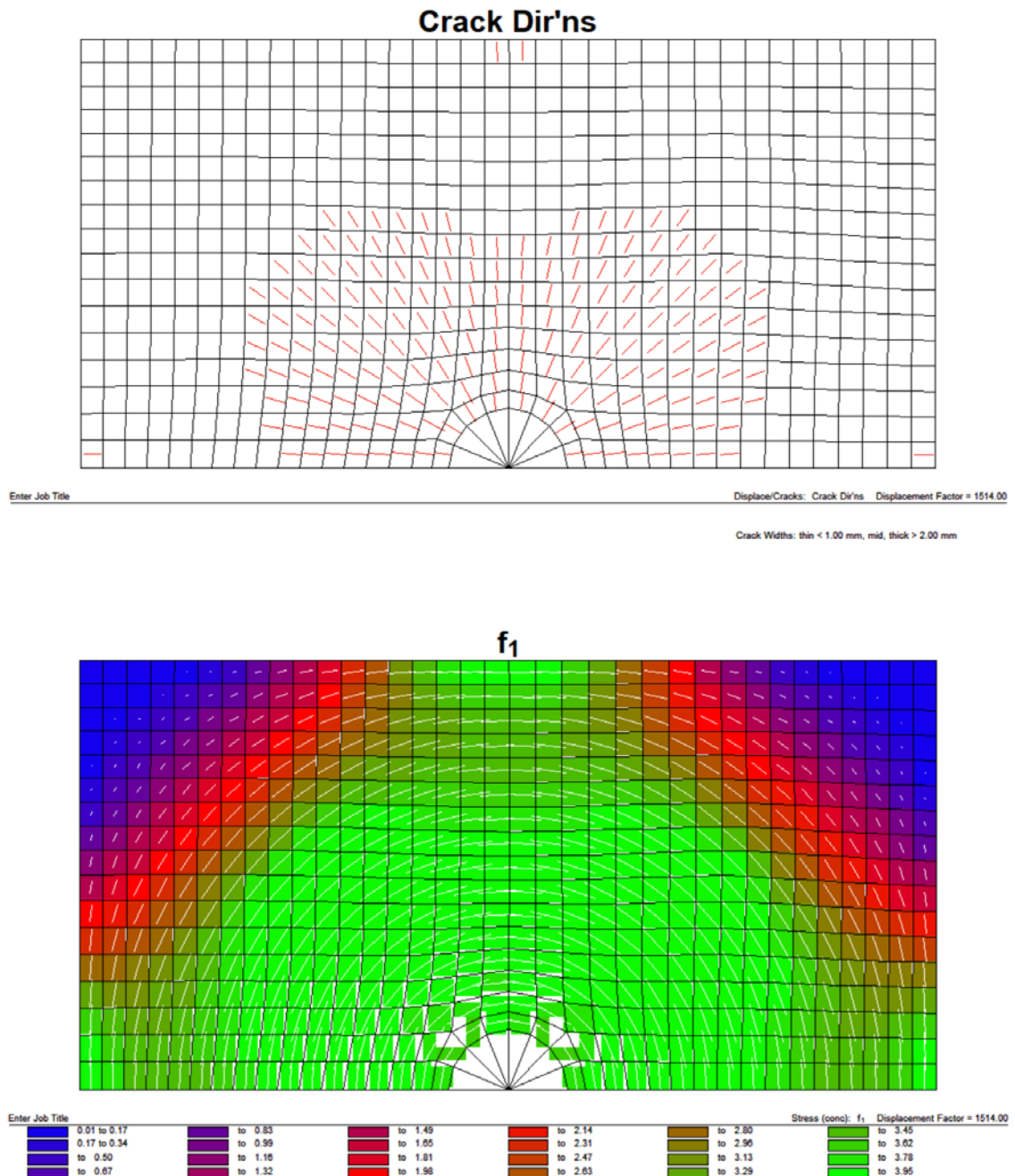


Figure 6.15: Stage 2, External Cracking (legend values for f_1 are in MPa)

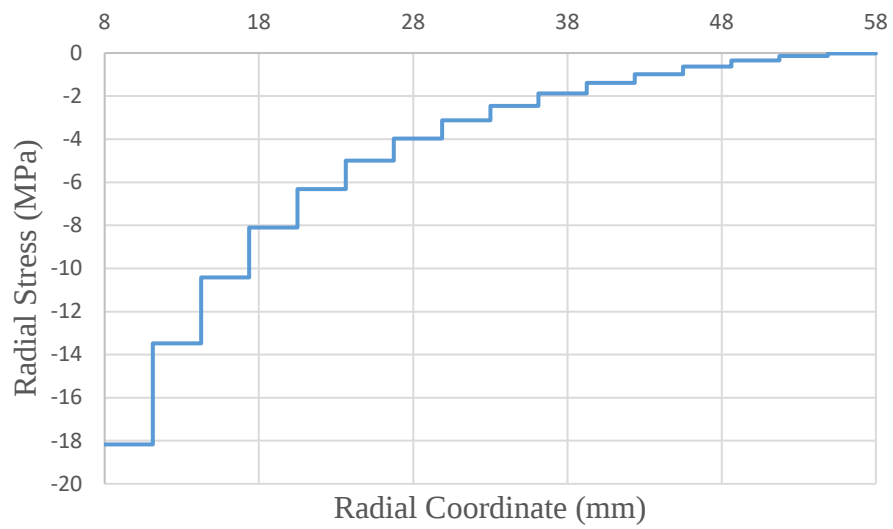
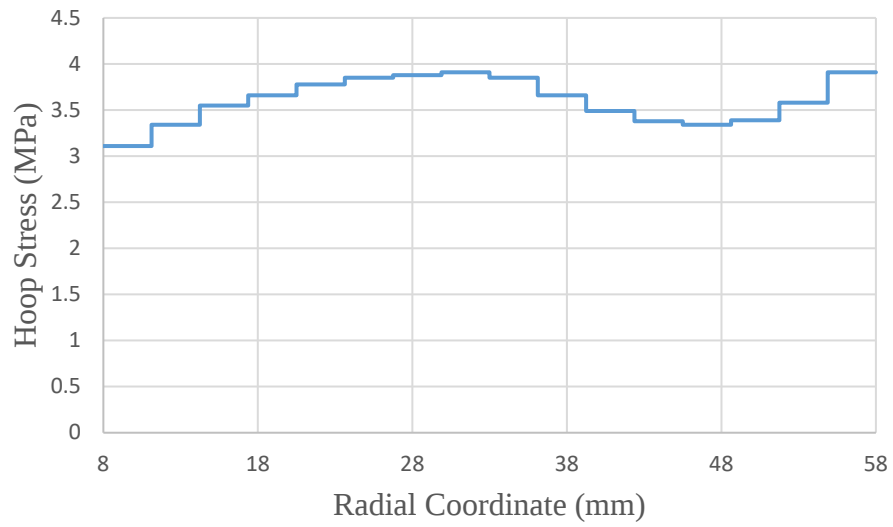


Figure 6.16: Stage 2, Radial and hoop stresses

Stage 3: Penetration Cracking

$$\Delta r_b = 14 \mu\text{m}$$

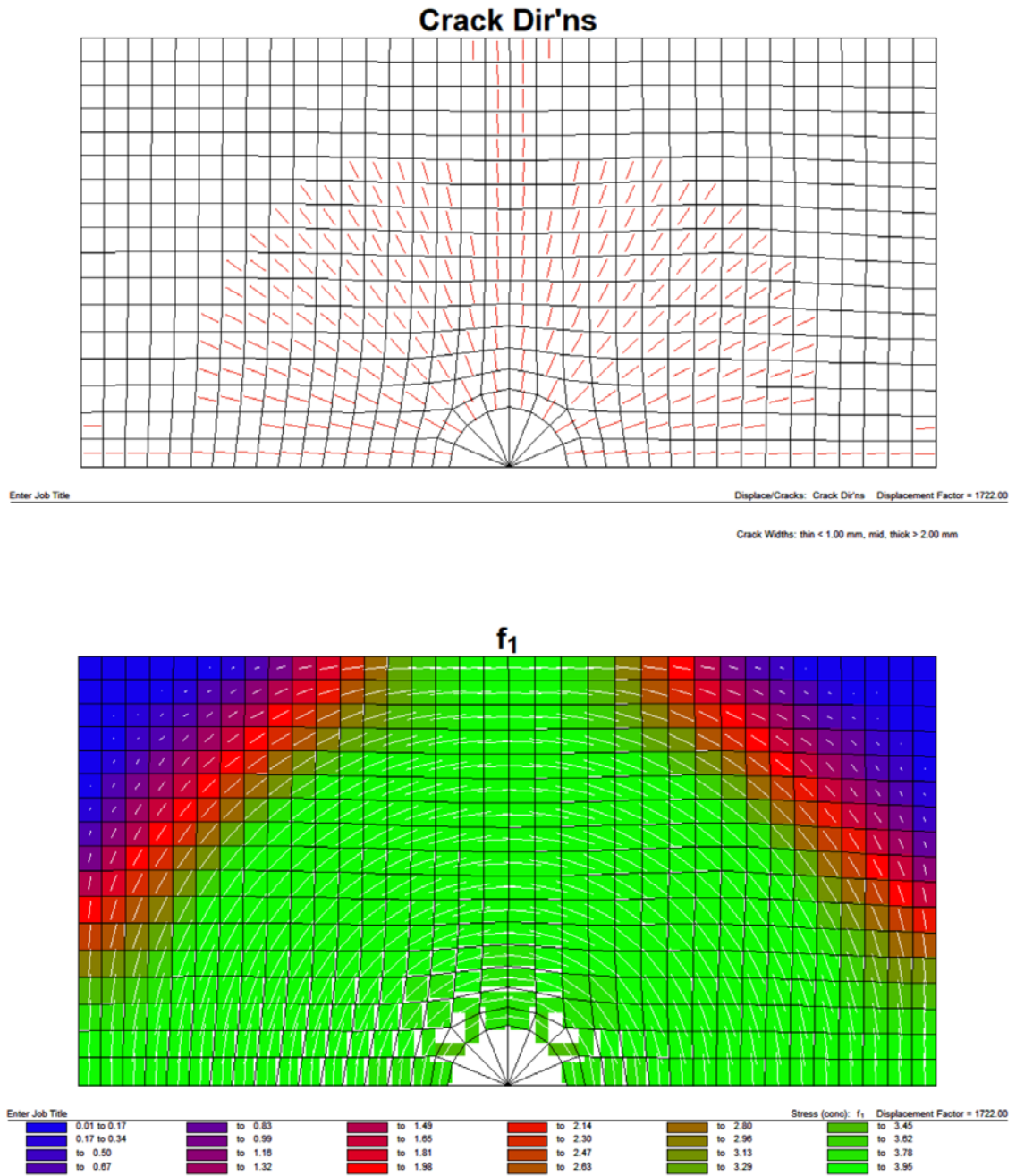


Figure 6.17: Stage 3, Penetration Cracking (legend values for f_1 are in MPa)

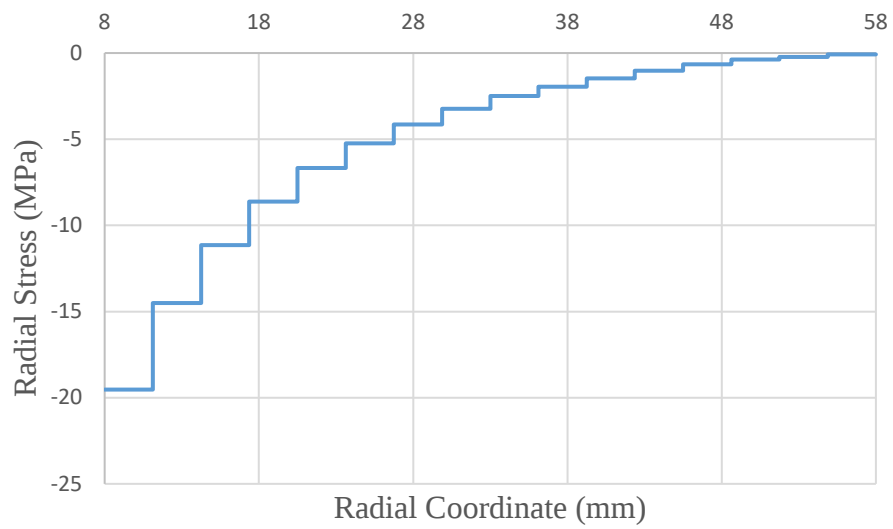
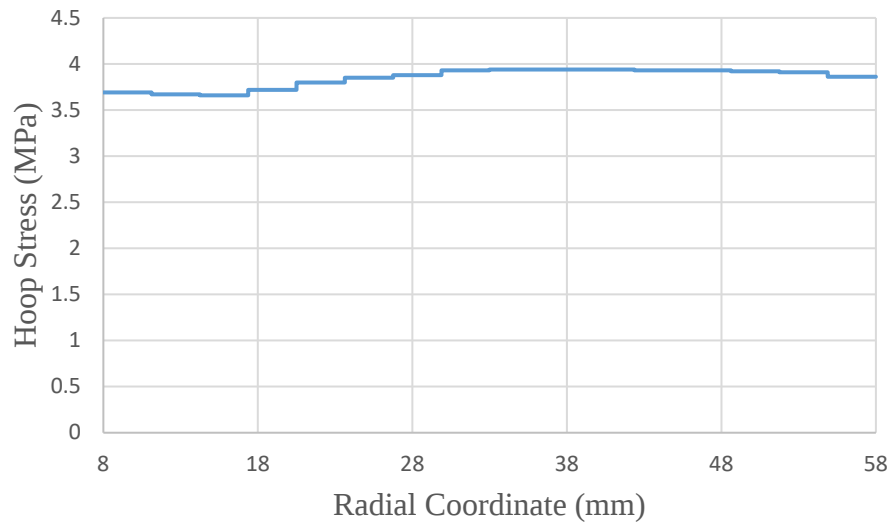


Figure 6.18: Stage 3, Radial and hoop stresses

6.4 Effect of an adjacent Rebar (Geometry C)

This section investigates the effect of an adjacent rebar by running a series of analyses with two rebars (Geometry C). The space between the two rebars (S) is the variable parameter (Figure 6.1). The center of the rebars are fixed in the plane, and the bottom side of the model is restrained in the y direction, as illustrated in Figure 6.19 by small dots.

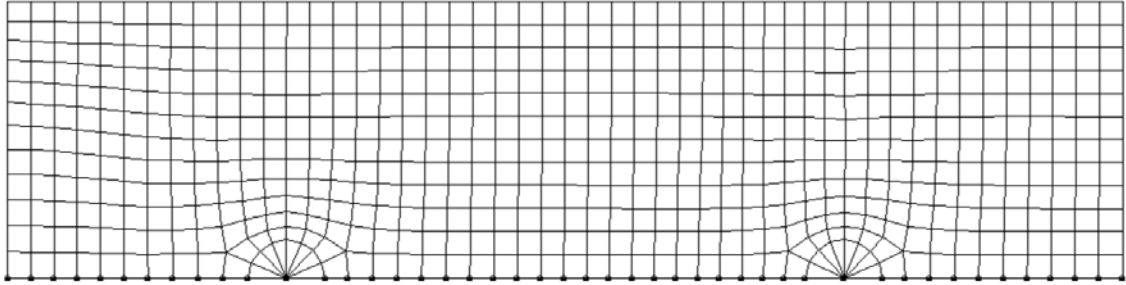


Figure 6.19: Boundary conditions of Geometry C

Parameter S' is defined as the clear space between the rebars:

$$[6.5] \quad S' = S - d_b$$

where S is the centre to centre space between the rebars, and d_b is the rebar diameter.

Table 6.4 provides the results found by the analyses with different rebar spacings. It also includes the results of Geometry A for different cover depths (30, 42, 50, and 60 mm) and the same rebar diameter (16 mm). It was found that if $S'/2$ is smaller than the clear concrete cover C , delamination is the first failure mode. Otherwise, the external cracking and penetration cracking occur before delamination. It was also found that Δr_b corresponding to the first failure mode (delamination if $S'/2 < C$, external cracking if $S'/2 > C$) is comparable to a representative cylinder (Geometry A) with the same rebar diameter and a cover depth equal to the smaller of $S'/2$ and C .

Table 6.4: Results from FEA of concrete cover with two reinforcing bars subjected to corrosion

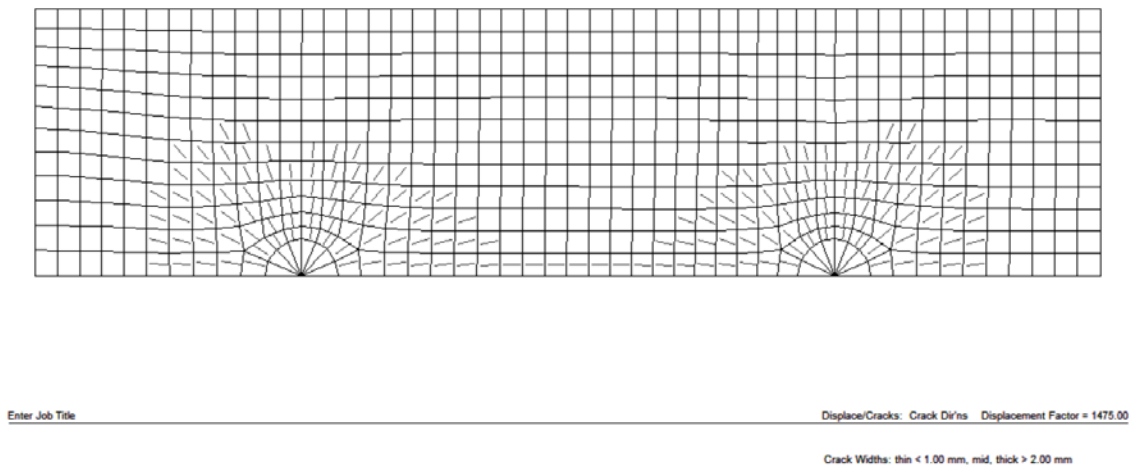
Specimens	d_b (mm)	C (mm)	S [S'] (mm)	Δ_{rb} (μm)	Failure Modes by Sequence
15M-30C	16	30	-	7	-
15M-60S'	16	50	76 [60]	6	Delamination
15M-42C	16	42	-	10	-
15M-84S'	16	50	100 [84]	9	Delamination
				14	External Cracking
				22	Penetration Cracking
15M-50C	16	50	-	13	-
15M-100S'	16	50	116 [100]	12	Delamination
				14	External Cracking
				18	Penetration Cracking
15M-60C	16	60	-	15	-
15M-120S'	16	50	136 [120]	13	External Cracking
				15	Penetration Cracking
				16	Delamination

Figure 6.20, Figure 6.21, and Figure 6.22 plot the cracking pattern as well as the principal tensile stresses for stages 1, 2, and 3 of specimen 15M-100S', respectively. Delamination occurs first followed by external and penetration cracking. The specimen shows a similar cracking behaviour due to corrosion as specimen 15M-50C (Section 6.3).

Stage 1: Delamination

$$\Delta r_b = 12 \mu\text{m}$$

Crack Dir'ns



f_1

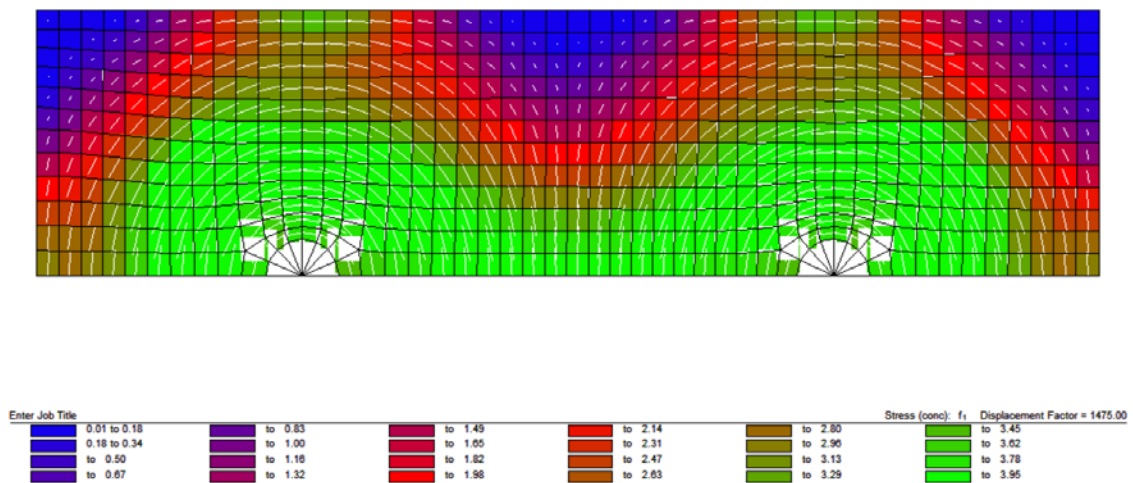
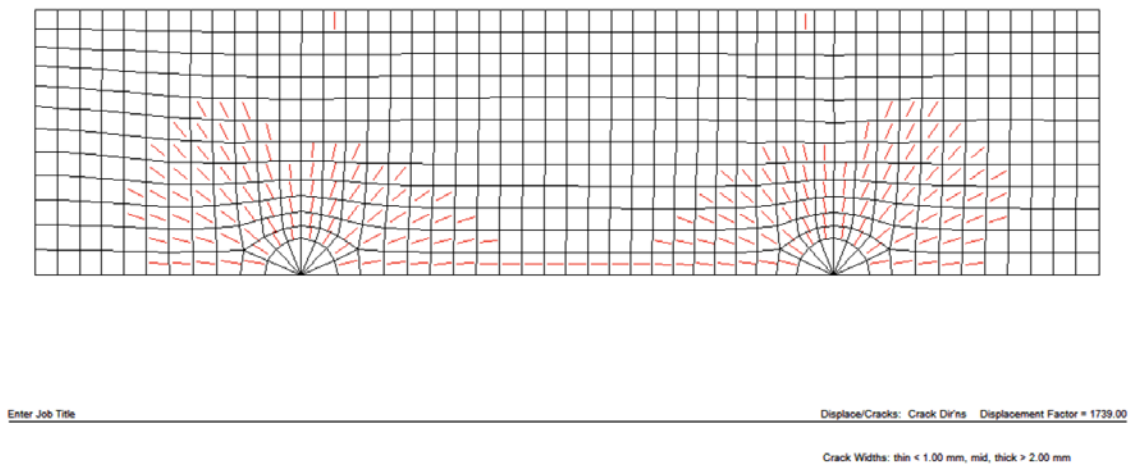


Figure 6.20: Stage 1, Delamination (legend values for f_1 are in MPa)

Stage 2: External Cracking

$$\Delta r_b = 14 \mu\text{m}$$

Crack Dir'ns



f_1

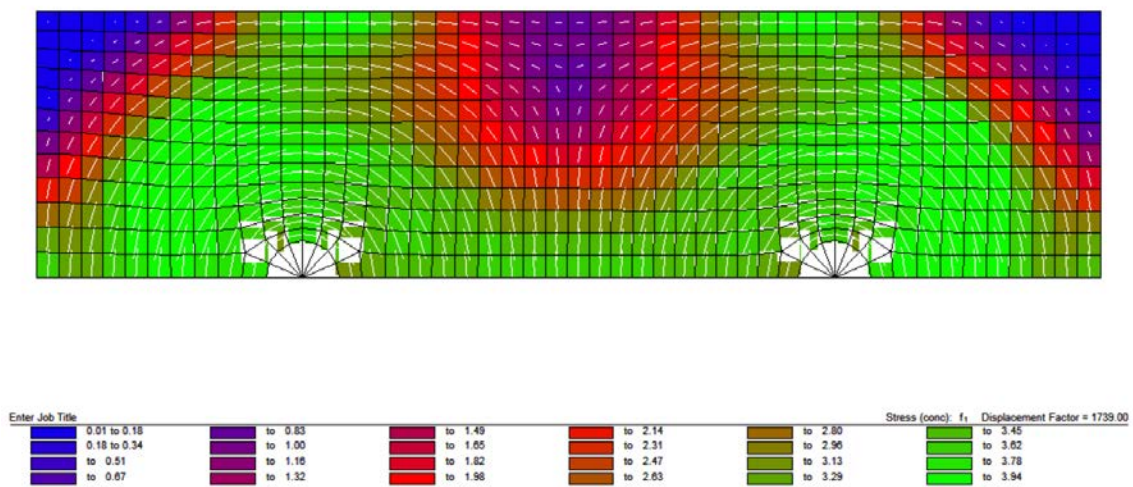
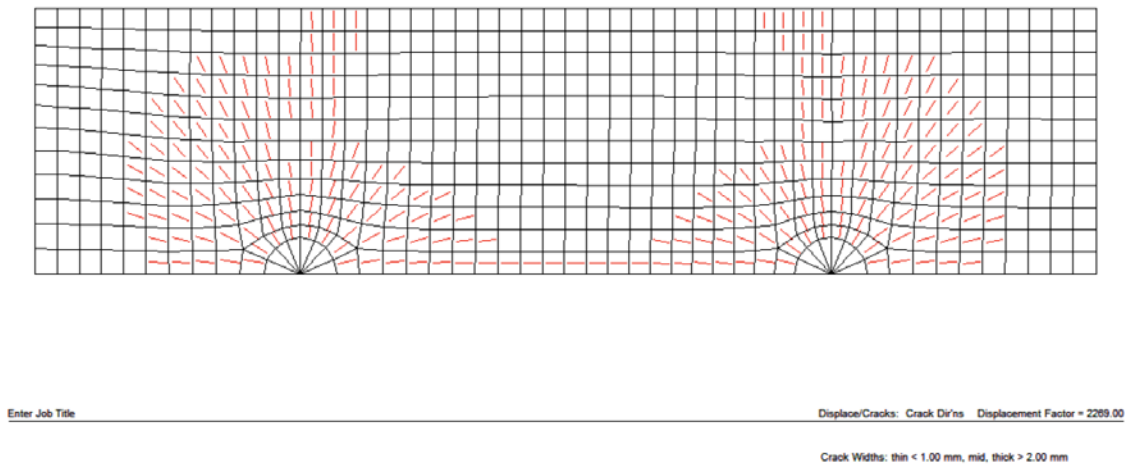


Figure 6.21: Stage 2, External Cracking (legend values for f_1 are in MPa)

Stage 3: Penetration Cracking

$$\Delta r_b = 18 \mu\text{m}$$

Crack Dir'ns



f_1

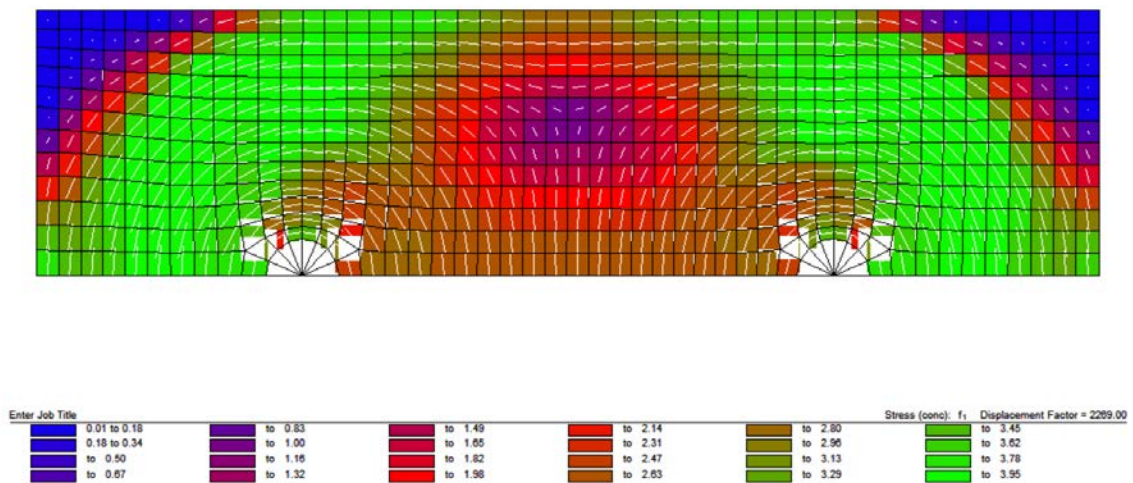


Figure 6.22: Stage 3, Penetration Cracking (legend values for f_1 are in MPa)

6.5 Effect of Boundary Conditions (Geometry D)

All the finite element analyses carried out up to this point have assumed the centre of the reinforcing steel bar to be fully restrained, either because it fell in a symmetry line (e.g., Section 6.36.4) or because it was manually enforced (e.g., results in Section 6.4). In reality, the centre of the corroding rebar might move in the plane if the condition around the rebar is not the same in all the directions. The purpose of this section is to investigate the effect of releasing this boundary condition at the centre of the corroding rebar. A released rebar is able to move in the plane as a result of corrosion.

To study this condition, a quarter (75 mm $\times$ 75 mm) of a beam cross section (Geometry D) is modeled, as illustrated in Figure 6.23. The rebar diameter and the cover depth are 12 mm and 16 mm, respectively. The left side is restrained in the x direction, and the upper side is restrained in the y direction (Figure 6.23). The material properties are:

- Tensile strength (f_t') = 4.70 MPa
- Compressive Strength (f_c') = 45.00 MPa
- Young's Modulus (E_c) = 32,000 MPa

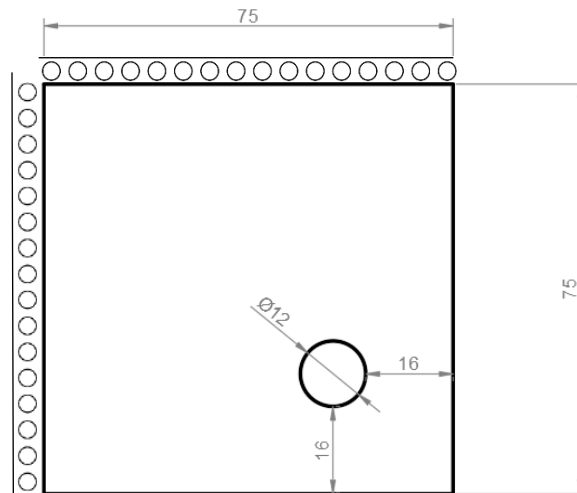


Figure 6.23: One quarter of a beam cross section, units in mm

It was found that the center of the corroding rebar tends to move toward the region that is less stiff (to the right and bottom in this example). This phenomenon causes non-uniform hoop stresses around the rebar, as seen in Figure 6.26 and Figure 6.27, as the center of the green circle does not lie on the center of the corroding rebar. This phenomenon accelerates the cracking process, and thus a lower Δr_b for crack penetration is obtained.

A quarter of a cylinder (Geometry A) with a rebar diameter of 12 mm and cover depth of 16 mm was also modeled to be compared to the results obtained from Geometry D. The bilinear tension softening model available in VecTor2 was used. The crack spacings were also chosen to be equal to the elements' widths. The critical Δr_b was found to be equal to 5 μm .

The critical Δr_b obtained based on Geometry D are 5 μm and 4 μm for a fixed and a released rebar, respectively. Therefore, the result based on the fixed rebar assumption corresponds to the representative cylinder (Geometry A), whereas the result based on the released rebar is smaller than the representative cylinder (Geometry A).

Fixed Rebar:

Stage 2: External Cracking

$\Delta r_b = 5 \mu\text{m}$, Time step: 764

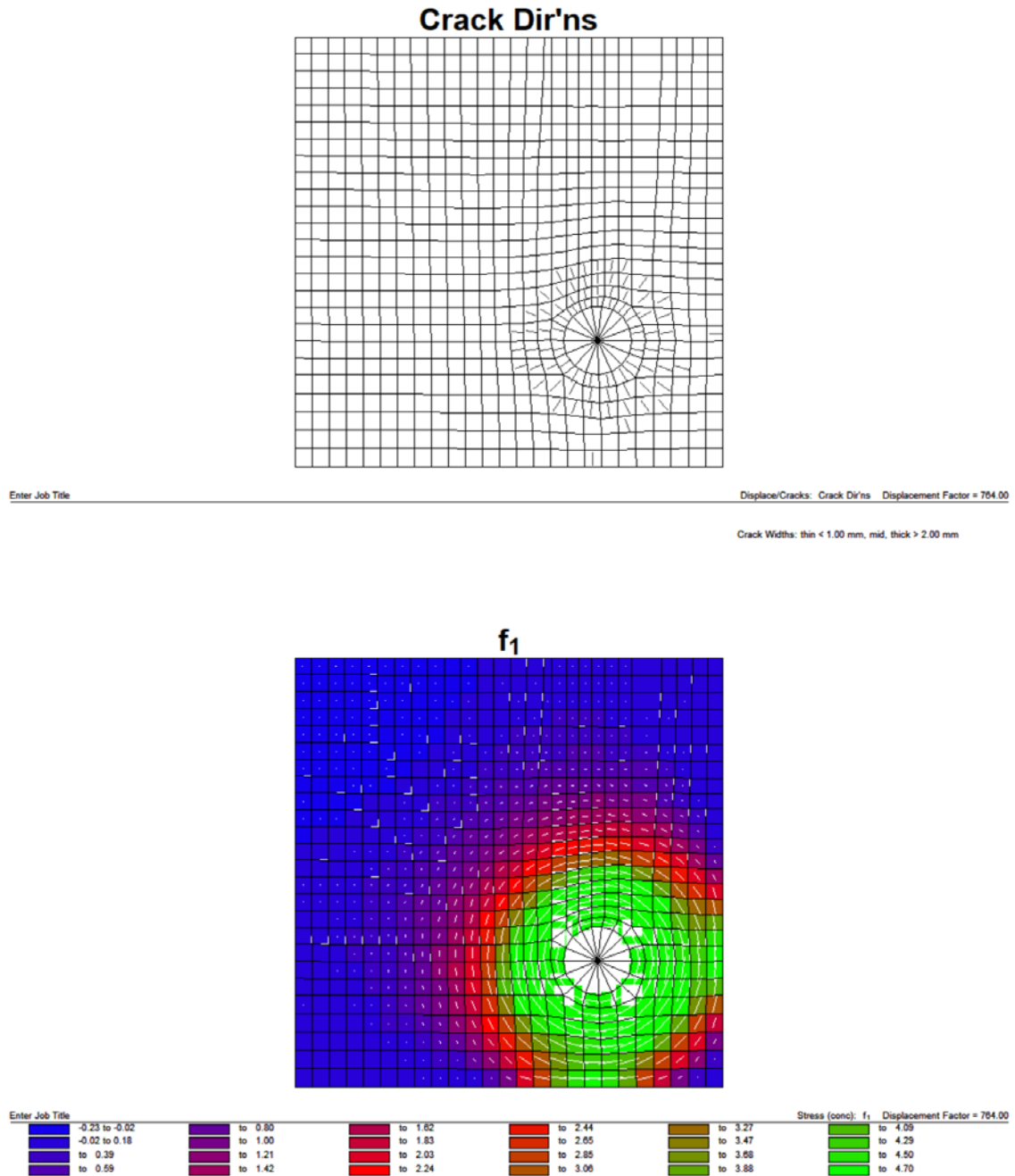


Figure 6.24: Fixed Rebar, Stage 2, External Cracking (legend values for f_1 are in MPa)

Stage 3: Penetration Cracking

$\Delta r_b = 5 \mu\text{m}$, Time step: 829

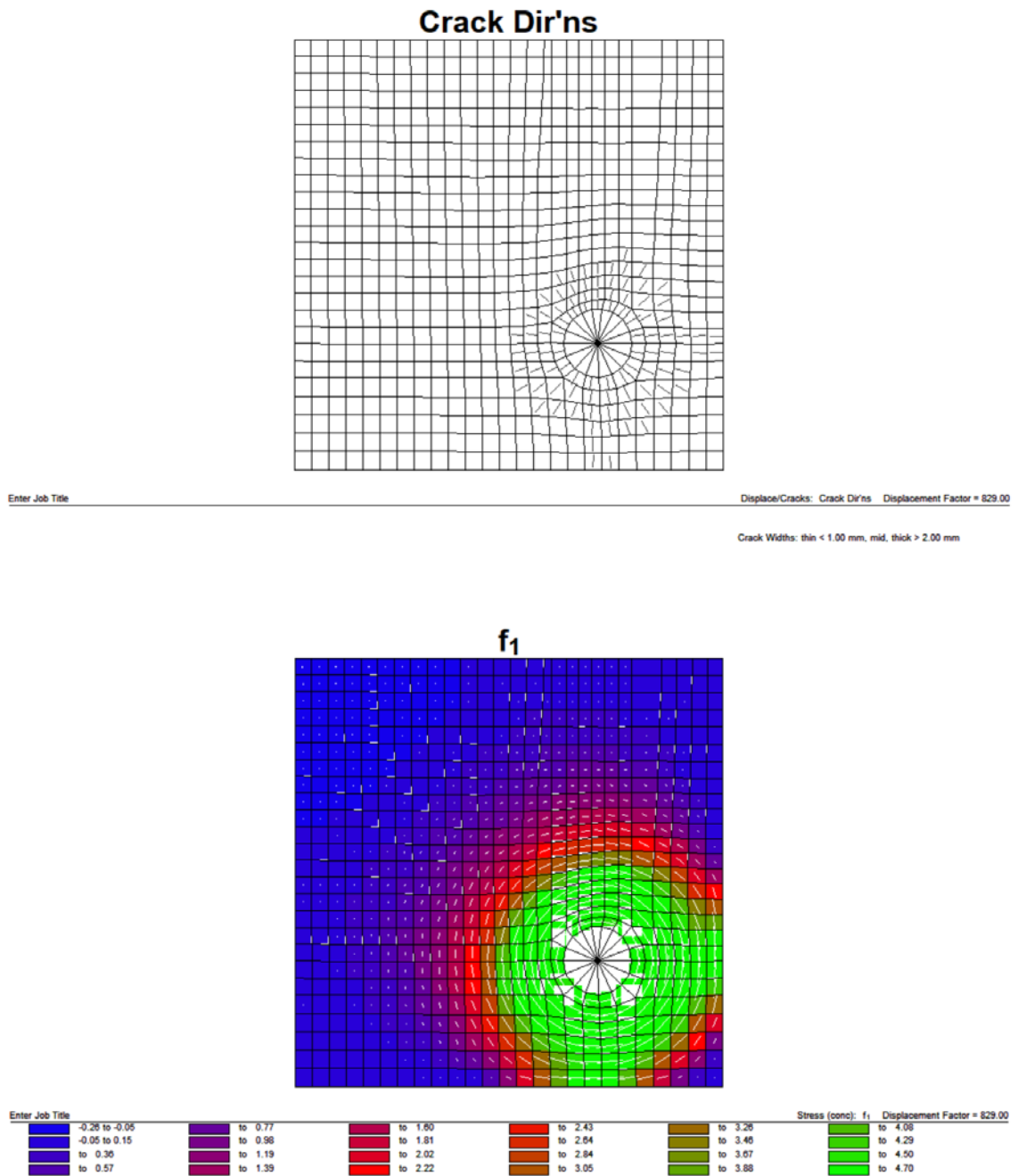


Figure 6.25: Fixed Rebar, Stage 3, Penetration Cracking (legend values for f_1 are in MPa)

Released Rebar:

Stage 2: External Cracking

$\Delta r_b = 4 \mu\text{m}$, Time step: 625

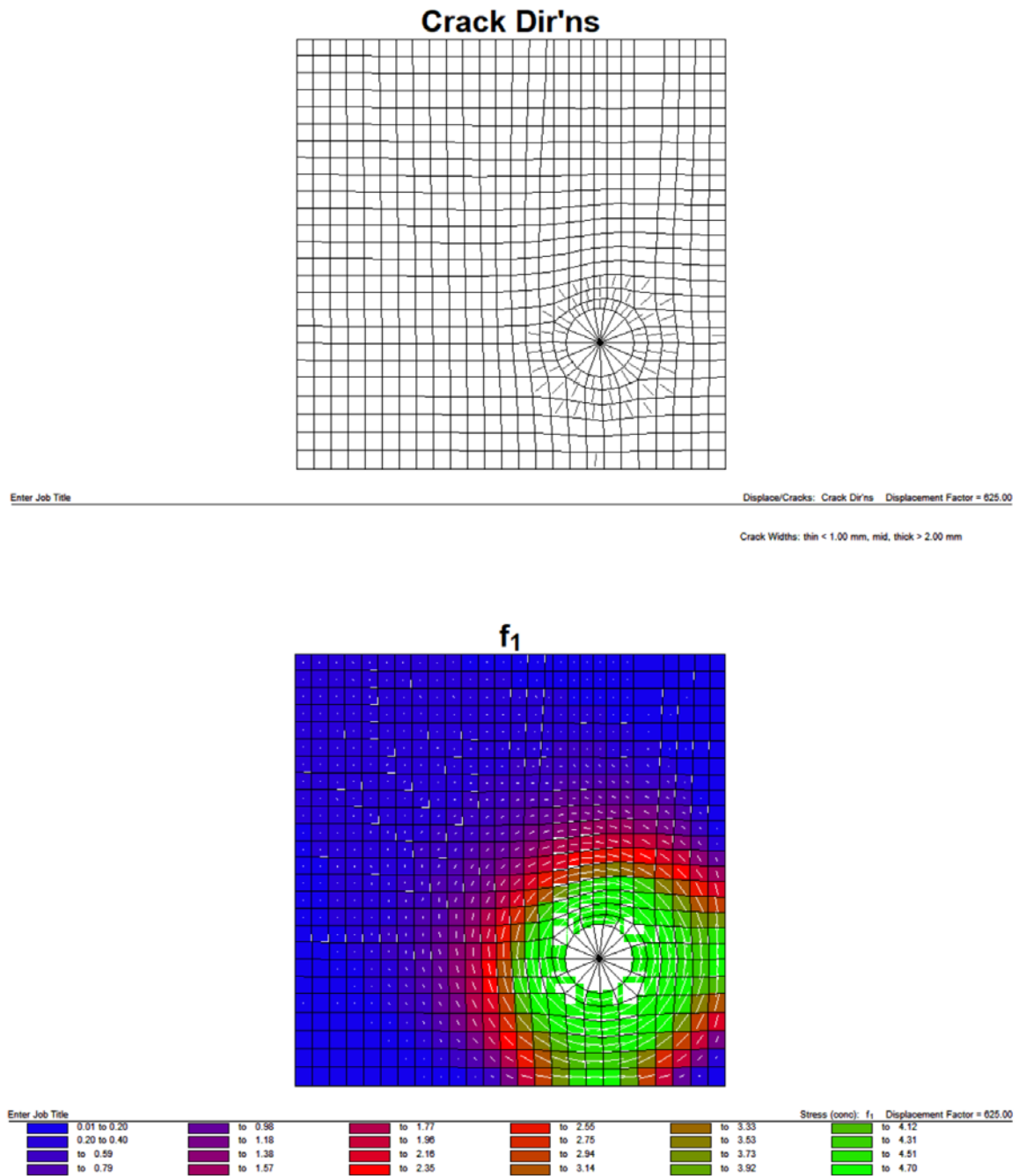


Figure 6.26: Released Rebar, Stage 2, External Cracking (legend values for f_1 are in MPa)

Stage 3: Penetration Cracking

$\Delta r_b = 4 \mu\text{m}$, Time step: 667

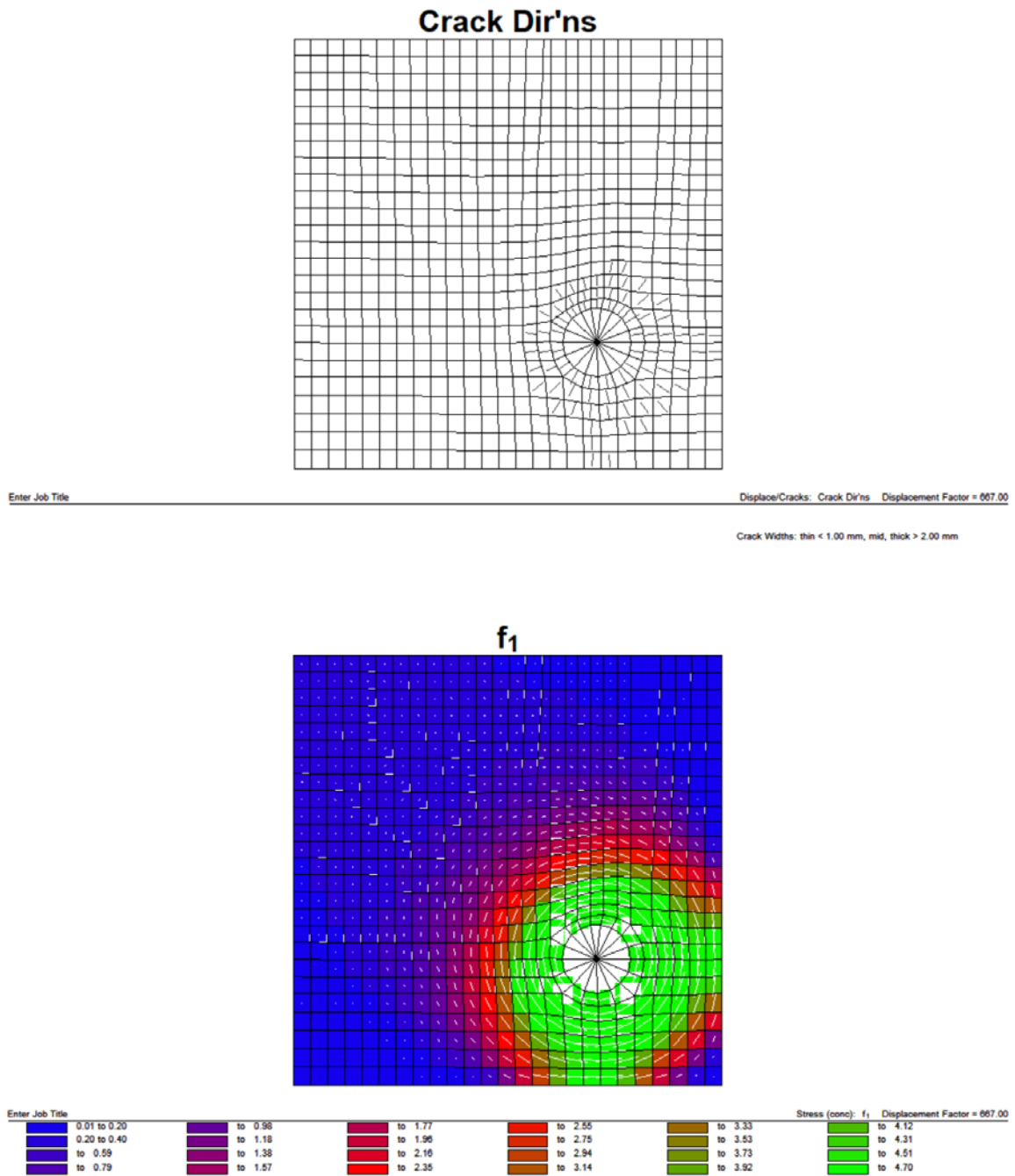


Figure 6.27: Released Rebar, Stage 3, Penetration Cracking (legend values for f_1 are in MPa)

6.6 Summary

Based on the work presented in this chapter, the thick-walled cylinder analogy is a reasonable assumption to model cover cracking due to corrosion-induced expansion in reinforcement concrete members. Consequently, the external radius of the thick-walled cylinder should be chosen as the minimum of the cover depth and half of the clear spacing between the rebars ($C = \min(\text{cover depth}, (S - d_b)/2)$). The internal radius must be chosen equal to the rebar's radius.

Whereas the crack propagation in Geometry A was found to start from the rebar/concrete interface and end at the concrete cover surface, the crack propagation of other geometries (Geometry B, C, and D) were found to have three stages: (i) internal cracking, (ii) external cracking, and (iii) penetration cracking. In other words, whereas the external cracking in Geometry A also marks the penetration cracking, external cracking occurs before penetration cracking in the other geometries (Geometries B, C, and D).

The boundary condition applied in the analysis of a corroding rebar affects the rate of crack propagation in a reinforced concrete member. Using a representative cylinder instead of modeling a more complex geometry is based on an assumption of uniform loading at the rebar/concrete interface. However, even in the case of uniform corrosion, it is not always an accurate assumption. It was found that if the centre of the reinforcing steel bar is not fully restrained or if it does not fall in a symmetry line, the center of the reinforcing steel bar would move in the plane towards the softer regions, causing non-uniform loading. It consequently increases the rate of crack propagation. More studies are needed to quantify this effect for different geometries.

Chapter 7

Concluding Remarks

7.1 Summary of Findings

A critical review of the reported experimental tests and the previously proposed models was conducted. Influential parameters and gaps in the previously proposed models were identified. Subsequently, a program was developed using MATLAB to solve corrosion-induced crack propagation numerically using finite differences. In addition, two analytical models were proposed, one based on a brittle elastic assumption and one based on an elastoplastic assumption. The results obtained from the numerical and analytical models were compared to the experimentally measured values found in the literature. Finally, finite element analysis was used to simulate the problem and the results were compared against the analytical and numerical results presented previously. The influence of some parameters, which were neglected in the models based on the thick-walled cylinder analogy, were also investigated to verify some of the simplifying assumptions used in this analogy.

Based on the work presented in this thesis, the main findings are as follows:

- Finite Element Analysis (Chapter 6) is more complex than the numerical model (Chapter 3) and the numerical model is more complex than the analytical models (Chapter 4). The number of simplifying assumptions reduces with the complexity

of the models. Therefore, more complex models better represent the reality. On the other hand, more computational effort/time is needed to solve them.

- Since the values of some of the most influential input parameters used in all of the models (e.g., rust expansion coefficient, porous zone thickness, crack volume coefficient, current density) are not well known, and due to high variability in the nature of the problem, any of the models provided here cannot serve as a deterministic model (unless more research is done on the selection of the input values). However, they may be suitable for probabilistic models if the probability distributions of the various parameters are known; this is outside the scope of the current study.
- The thick-walled cylinder analogy is a reasonable assumption to model cover cracking due to corrosion-induced expansion in reinforced concrete members. The external radius of the thick-walled cylinder should be chosen as the minimum of the cover depth and half of the clear spacing between the rebars ($C = \min(\text{cover depth}, (S - d_b)/2)$). The internal radius of the cylinder must be chosen equal to the rebar's radius.
- The analytical models (elastoplastic and brittle elastic cylinders) provide upper and lower limits to the results found with the numerical model (e.g., crack front, internal pressure, crack width). The results diverge as cover-to-rebar diameter increases, particularly for values of C/d_b greater than 3. The numerical results tend to be closer to the results based on the elastoplastic cylinder.
- The numerical and the elastoplastic models were found to be capable of predicting the visual critical attack penetration and the maximum internal pressure. The results

based on the two models are reasonably close to each other for cover-to-rebar diameter ratios smaller than 3.0.

- The ratio of the visual critical attack penetration to the critical attack penetration was found to range from 1.7 to 1.2 for cover depths from 25 to 100 mm. The ratio decreases as cover depth increases. Since the visual critical attack penetrations are significantly larger than the critical attack penetrations, it is very important to compare the same time frames when comparing experimental results to the results according to the numerical or analytical models. This point is mostly missed by previous researchers.
- While consideration of the diffusion of corrosion products into cracks was found to be essential in order to predict the visual critical attack penetration, considering all of the available space would over-estimate the visual critical attack penetration. The degree of over-estimation increases with the cover-to-rebar diameter ratio. Therefore, although the corrosion products diffuse into the cracks, they do not fill all of the available space.
- The rust expansion coefficient R_v , the crack volume coefficient C_v , and the porous zone thickness t_p were found to have significant effects on the crack propagation, the critical, and the visual critical attack penetrations. However, values of 2.75, 0.5, and 10 μm are recommended, respectively, to be used with the numerical model in order to estimate the visual and the critical attack penetrations. The maximum internal pressure resulting from the expansion of corrosion products is independent of these parameters.

- The boundary condition of a corroding rebar (fixed or released in the plane) affects the rate of crack propagation in a reinforced concrete member. Releasing the corroding rebar accelerates the crack propagation.

7.2 Recommendations for Future Work

Although the numerical and elastoplastic cylinder models were successful in modelling the crack propagation caused by steel reinforcement corrosion, several aspects were outside the scope of the current study. The following topics are recommended to be investigated further:

- While the rust expansion coefficient R_v , the crack volume coefficient C_v , and the porous zone thickness t_p have major effects on the cracking rate, there are few studies available to quantify them experimentally. In addition, since the values have significance influence on the results and they are not always constant, a guideline to select an appropriate value would be beneficial. The guideline should relate the values to other known parameters (e.g., relating the crack volume coefficient C_v and the porous zone thickness t_p to water-to-cement ratio, relating rust expansion coefficient R_v to environmental conditions such as temperature and humidity).
- While the corrosion products probably do not expand greatly after decompression (Konopka 2005), rust is modeled as an elastic material. If the numerical model or the brittle elastic model is used, this assumption accelerates the cracking process after the maximum internal pressure is reached. The internal pressure decreases after reaching its maximum value in these models. This assumption does not make any difference if the elastoplastic model is used, since the internal pressure does not decrease after reaching its maximum value. More advanced modelling is needed to consider the plastic behaviour of corrosion products in the numerical and the

brittle elastic models. Nonetheless, this assumption probably does not have a major impact on the results.

- While the analytical models are not as accurate as the numerical model in the modelling of crack propagation for a member cast with conventional concrete, they might be more successful in modelling other materials or novel concretes. For example, steel fiber-reinforced concrete has more plastic capacity in tension than conventional concrete. Consequently, it behaves more like an elastoplastic cylinder than conventional concrete. Unfortunately, due to lack of sufficient experimental data on the behaviour of steel fiber-reinforced concrete subjected to corrosion of an embedded rebar, a comparison could not be performed in this study.
- The effect of transverse reinforcements is not studied here. Transverse reinforcements might increase the internal pressure acting on the corrosion products, making the corrosion products more compressed compared to corrosion products in a similar member with no transverse reinforcement. Therefore, transverse reinforcement may increase the importance of considering the compressibility of corrosion products.
- While it was shown that the boundary conditions of the corroding rebar affects the rate of crack propagation (Section 6.5), more studies are needed to quantify this effect.
- More work is needed to determine the effect of a sustained load (creep) on the critical tensile strain of concrete (ϵ_{cr}). The critical tensile strain of concrete is one of the determining parameters in the cracking of RC structures (e.g., the results of the critical attack penetration, the visual critical attack penetration, and the crack width after cracking).

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