USE OF INPUT-OUTPUT ANALYSIS FOR SENSITIVITY MEASUREMENT PURPOSES

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LIST OF ABBREVIATIONS

A.E.R. .... American Economic Review

A.S.A. .... American Statistical Association

C.J.E.P.S. .... The Canadian Journal of Economics and Political Science

c.i.f. .... Cost, insurance, and freight

D.B.S. .... Dominion Bureau of Statistics

Ectra............. Econometrica

E.J. .... Economic Journal

E.R. .... Economic Record

f.o.b. .... free on board

J.A.S.A. .... Journal of American Statistical Association

J.E.A. .... The Journal of Economic Abstracts

N.B.E.R. .... National Bureau of Economic Research

n.e.s. .... not elsewhere specified (or included)

O.E.E.C. .... Organization of the European Economic Community

Q.J.E. .... Quarterly Journal of Economics

R.E.S. .... Review of Economic Statistics

R.E.STUD. .... The Review of Economic Studies

U.N. .... United Nations

INTRODUCTION

The purpose of this thesis is two-fold; in the first place, to review the conceptual framework used in input-output analysis; secondly, to analyse the sensitivity of commodity prices to changes in the prices of given primary inputs, (wages, profits, and imports) of the Canadian economy. For the empirical work the Canadian Input-Output data for the year 1949 is used (at this time this is the only available data on interindustry studies for this country). Since, to the author's knowledge, there is no alternative method of testing the results reported in this thesis, the author has tried to explain his results rather than assert positive conclusions.

It is believed that if the actual prices were available for the year 1950 or 1951 for the Canadian economy and if any way was found to establish the price of each primary input, it would be interesting to compare the actual prices with the ones predicted by the model. Such work should be quite useful for economic stabilization and incomes policy.

To assist the worker in the interindustry area, the author has given an exhaustive bibliography, which is not only relevant to this particular problem, but also covers practically every piece of literature on this subject from

Leontief's first paper in 1936 up to 1963, and some works after 1963.

CHAPTER I

CONCEPTUAL FRAMEWORK

In the accounting sense, interindustry analysis may be defined as a convenient way of showing the values of the flow of goods and services per unit of time, usually a year, among the various producers and users in different industries and sectors of the economic system as a whole. The national accounts format also serves a similar purpose. One can therefore say that input-output analysis is an extension of the social accounts. If this is so, there must be a close relationship between input-output analysis and social accounting analysis. That is, the interindustry flow matrix may be derived from the national accounting scheme. Nevertheless, interindustry analysis differs from social accounting analysis in many important aspects.[1] One important difference between these two is that in order to make them operationally useful several dissimilar concepts and techniques are used.

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1. J. Siegel Stanley, "A Comparison of the Structures of Three Social Accounting Systems"; Herman I. Liebling, "Interindustry Economics and National Income Theory", in Input-Output Analysis: An Appraisal, Studies in Income and Wealth, Vol. 18, N.B.E.R., Princeton U. Press, Princeton, 1955.

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On the surface, input-output analysis is quite elusive, since it does not explain its own language and the accounting basis on which it is developed. To see what lies behind the interindustry flow table, one must understand these two aspects of the analysis. Consequently, the remaining portion of this chapter will be divided into two parts. Part one will deal with the various concepts used in input-output analysis and part two with the accounting framework of this analysis.

PART I

(A) The Concept of Activity or Process

In the economic analysis the terms activity and process are generally used interchangeably. Such usage will be maintained throughout in this work. Dorfman et al. define activity informally. According to them,

> "Intuitively, . . ., a process is some physical operation and it may be almost any physical operation, e.g., consuming something, storing something, selling something, throwing something away in a particular manner."[2]

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2. Robert Dorfman, Paul A. Samuelson, and Robert Solow, Linear Programming and Economic Analysis, McGraw-Hill, New York, 1958, p. 29.

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Dorfman, et al. also have given a formal definition of activity,[3] which is similar to the one given by Koopmans, and which is more suitable for input-output analysis. Koopmans says,

> "In our static model an activity consists of the combination of certain qualitatively defined commodities in fixed quantitative ratios as "inputs" to produce as "outputs" certain other commodities in fixed quantitative ratios to inputs".[4]

It should be noted that the performing of a service, say training a person or transporting something, is also considered as an activity.

So far, the concept of activity has been discussed in very general terms, as applied in activity analysis. It can be given more specific meaning when used in input-output analysis. In this context, the concept of activity implies an industry or a sector of production, transportation, storage, services and selling. In the Leontief system an activity is represented by a column of coefficients for each input and each output. The j-th

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3. Ibid., p. 132.

4. Tjalling C. Koopmans (ed.), Activity Analysis of Production and Allocation, Proceedings of a Conference, Wiley, New York, 1951, pp 35-36.

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activity can be written as a column vector,

1.1
$$A_j = \begin{bmatrix} a_{1j} \\ \vdots \\ a_{ij} \\ \vdots \\ a_{nj} \end{bmatrix}$$

which indicates the rate of flow per unit of time of each of n commodities involved in the unit level of that activity. We shall convene that negative coefficients of A_j indicate that the commodity involved is used up by the activity; positive coefficients, that the commodity is produced. A value $a_{ij} = 0$ indicates that the i-th commodity is not involved in the j-th activity.

All the activities represented by the input-output matrix can be written as,

1.2
$$A = (A_1 \cdot\cdot A_j \cdot\cdot A_n) = \begin{bmatrix} a_{11} \cdots a_{1j} \cdots a_{1n} \\ \vdots \qquad\qquad \vdots \\ a_{n1} \cdots a_{nj} \cdots a_{nn} \end{bmatrix}$$

assuming a square matrix. This is the familiar technology matrix. The extent to which an activity is utilized is defined by the gross output in the input-output system. If the level of activity j is represented by X_j, the total of each input used or output produced by the activity is then $x_{ij} = a_{ij} X_j$, with output positive and input negative.

(B) The Concept of Commodity

A layman generally thinks of a commodity as a physical good only. An economist conceives of a commodity in terms of a physical good as well as a service. Koopmans says that,

> "Commodities include primary factors of production, such as labor of various grades, including land giving access to mineral resources; intermediate products, such as coal, pig iron, steel; and final products".[5]

As in input-output analysis, he assumes each commodity to be homogeneous qualitatively and continuously divisible quantitatively.

In the interindustry model sense, commodities can be divided into two categories. The first group contains those commodities which are themselves consumed by the consumers; that is, those commodities which give utility. This group of commodities can further be divided into two classes. (a) Final commodities, which are not available in nature and which are produced from intermediate commodities and from commodities which are available in nature.

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5. Ibid., p. 35.

(b) Primary desired, those commodities which are available in nature and which satisfy consumers' desires without going through any processing.

The second group of commodities constitutes those commodities which are not desired in themselves but which are used to produce the desired commodities. These are called intermediate commodities.

There is another group of commodities which is not considered in input-output analysis. This group includes those commodities which are available in nature but which have no economic value (free goods), or which have no utility, or which have disutility.

The classification of commodities can be put in tabular form.

TABLE I[a]

COMMODITY CLASSIFICATION

Commodity	Desired in itself	Not desired in itself
Not available in nature	Final	Intermediate
Available in nature	Primary desired	Primary not desired

a. Ibid., p. 41, (Title Author's)

(C) The Concepts of Output and Input

The concept of output implies bringing of goods and services into existence by the use of various inputs. Thus output may refer to primary desired commodity or intermediate commodity. The D.B.S. defines output as,

> "The output of industries consists of new goods and services brought into existence through the transformation of materials, labor, capital, and other inputs".[6]

The input is a counterpart of the output. The input of one industry is the output of the other industry and vice versa. Generally an industry consumes several inputs from various industries and conversely it supplies its output to a number of industries as inputs. Broadly, the input may be defined as goods and services, human and non-human, which are consumed in the production of new goods and services. The input is divided into two main categories, primary and intermediate. Imports, labor, and anything else which is not produced in the production sector but which is consumed in this sector is called primary input. All the input which is drawn from the output of domestic industries is termed intermediate input,

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6. D.B.S., Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949, Catalog No. 13-513, p. 8.

in the sense that it is consumed in the production process.

For the economy as a whole intermediate output is equal to the domestic content of total intermediate input and total final output equals total primary input. The equality between total final output and total primary input is similar to the equality between gross national expenditure and gross national product.

(D) The Concepts of Firm, Establishment and Industry

For classification and aggregation purposes it is desirable to differentiate between the concepts of firm and establishment.

The firm is a legal entity which engages in the production of one or more (as generally is the case) commodities and which operates one or more establishments at one or more places. Normally, a firm produces several dissimilar kinds of commodities.

An establishment is an operating unit by itself. It may constitute a part of a firm or it may be controlled and operated independently of any other establishment. Such a producing unit is assumed to produce mainly one homogeneous commodity. In the Canadian input-output analysis, an establishment is considered as,

"For statistical purposes, the

> establishment is defined as the smallest unit which is a separate operating entity capable of reporting all elements of basic industrial statistics." [7]

If the industrial unit from which data are collected is defined in this way, this means that data are collected from the most homogeneous units that maintain accounts which permit them to report on their main elements of input and output. Typically the establishment is the mine in mining, the plant in manufacturing, the store in retail trade, and so on.

Industry is defined in units of establishment rather than in units of firm because it is difficult to find a suitable scheme to aggregate the activities of a firm. To attempt to define each separate activity as the basic unit means, in most cases, adopting a unit of firm which is smaller than the unit used for accounting purposes. But most firms which produce commodities do not keep separate statistics for each activity of shipments, inventories, employment, payrolls, materials, process supplies, fuel and electricity used. In most cases, however, such statistics are available for each establishment (factory, farm, mill, mine, and so forth).

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7. D.B.S., Standard Industrial Classification Manual, Catalog No. 12-501, Queen's Printer, Ottawa, 1960, p. 8.

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In economic analysis an industry is defined as a combination of two or more firms which produce similar output. This definition of the industry is not suitable for the interindustry analysis (as will be discussed in aggregation). For this purpose establishment, and not firm, is considered as a basic unit of industry. The industry, then, is defined as,

> "... a group of establishments which have sufficient common characteristics that may be grouped together for analytical purposes. They may, for example, manufacture similar end products (the furniture industry) or use the same principal component material (the wood products industry). Sometimes it is the technological process which is similar. The term industry is used in the Canadian input-output study to include all productive activity carried on in the economy except for non-commercial work done in a household. It includes the public administration and defense industry as well as persons or associations (hospitals and churches) who hire employees or otherwise engage in industrial transactions."[8]

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8. D.B.S., Supplement to the Inter-Industry Flow of Goods and Services, Canada, Catalog No. 13-513, Queen's Printer, Ottawa, 1949, p.7.

(E) Industrial Classification

Classification of industries means finding some criterion on the basis of which the whole industrial sector can be divided into numerous industries. One may ask why there is need for such classification. The answer is that industrial classification is necessary for aggregation. Because of numerous interdependent relationships among the activities and because of similarity among the industries, it is clear that rigid classification of the industries is not possible. Now the question arises as to what sort of criterion should be used as a basis of this classification. Various criteria have been suggested in the literature.[9] But the most commonly used criterion for

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9. J.B. Balderston and T.M. Whitin, "Aggregation in the Input-Output Models," in Oskar Morgenstern (ed.), Economic Activity Analysis, Wiley, New York, 1954.

Tibor Barna, "Classification and Aggregation in Input-Output Analysis" in Tibor Barna (ed.), The Structural Interdependence of the Economy, Wiley, New York, 1956.

D.B.S., Standard Industrial Classification, Catalog No. 12-501, 1948 and 1959 (edd.), Queen's Printer, Ottawa.

Mathilda Holzman, "Problems of Classification and Aggregation" in Wassily Leontief (ed.), Studies in the Structure of the American Economy, Oxford University Press, New York, 1953.

W.W. Leontief, The Structure of the American Economy, 1919 - 1929, Harvard University Press, 1941, Part I

Richard Stone, Input-Output and National Accounts, O.E.E.C., Paris, 1961, Chaps. II and III.

classification is either similarity of input or similarity of output. Whatever the criterion for classification, it must be chosen in light of the following:

(a) The appropriateness of the scheme for achieving the purpose for which the input-output analysis is constructed. (b) The degree of homogeneity of classification. (c) The degree of stability of the unit of classification over time. (d) Availability of data and the cost involved.

It should be noticed that no single scheme of classification would satisfy each and every one of these conditions better than all other schemes. Hence, what should be sought is a basis of classification which is better, as a whole, than other alternatives.

For the full validity of the basis of classification for input-output studies, the following conditions would have to be satisfied.

(a) Commodities produced by the same industry should be perfectly homogeneous (perfectly substitutable for each other in their uses).

(b) In industrial uses there should not exist any possibility of substituting the product of one industry for the product of another.

(c) In each industry only a single technique should be possible which should obey the law of constant returns to scale.

(F) Aggregation of the Industries

In general, aggregation can be defined as reducing the number of unknowns and equations in the given system. As applied to the input-output table, aggregation means:

> "reducing the size of the table by collecting the industries into classes and combining the production accounts of industries to a smaller table ...".[10]

Broadly speaking, there are two types of aggregations, as used in the theory of production: vertical and horizontal. When the output of one industry is wholly absorbed by a second industry and the production function of the second industry permits no substitution of any other product for the output of the first industry, the two industries may be combined in a single industry and such aggregation is called vertical aggregation. Aggregation is horizontal when the products in the same stage of manufacture are grouped together in one industry.

It was mentioned above that aggregation is necessary to reduce the size of the model, without explaining why a reduced model was desired. When industries are

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10. R. Stone, loc cit, p.101.

classified on the basis of similarity of input or output, the number of industries will be so large that a very large input-output matrix will be needed to show the inter-industry flow of goods and services. Such a matrix will be more difficult to handle, compute and invert than a small matrix. Therefore, industries must be aggregated to get a matrix which is manageable.

There are two basic steps in aggregation; (a) classification of the production sector into establishments on a similar input or output basis, (b) aggregating establishments into industries, again on a similar input or output basis. Great attention is paid not to lose any important information during the process of aggregation.

Two important limitations of aggregation have been pointed out. (1) Since we are dealing with grossly aggregated figures, they might often conceal some important factual complexities. (2) Different methods of aggregation give different results and one hardly knows which is the best method of aggregation to achieve an optimal solution for the problem at hand.[11]

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11. W.W. Leontief, "Comparison of Input-Output Classification", (Appendix 2)", in The Structure of the American Economy, Oxford University Press, 1953, pp. 497-525.

W.W. Leontief, "Recent Developments in the Study of Inter-industrial Relationships", Papers and Proceedings of the A.E.R., Vol. 39, No. 3 (Annual Supplement), 1949, pp. 211-40.

(G) Problems Associated with Classification and Aggregation

So far classification and aggregation have been discussed without referring to problems to be encountered in such endeavor. The main problems to be faced in such a task will be discussed here.

Treatment of Subsidiary Products, and Joint Products

In the industrial classification it was assumed that each industry produces only one commodity which is homogeneous and which is not produced by any other industry. But that assumption was only to simplify the analysis. In reality some industries may produce commodities other than their main product in the form of subsidiary or joint products. How should these be incorporated in the analysis with minimum error?

Subsidiary Product

An industry may produce a certain product which is mainly produced in another industry. If the main product of industry A is the subsidiary product of industry B and the production function of industry A is similar to that of industry B, then it is useful to transfer subsidiary product (costs and output associated therewith) of B to that of A. This is what has been done in the Canadian interindustry flow table of 1949. It seems likely that the assumption of a similar input structure for a given product will frequently be a best approximation of reality. The assumption may not be justified where different parts of an economy operate at markedly different technological levels; for example, the case of synthetic rubber and natural rubber.

The practical difficulties of disentangling the costs associated with subsidiary production from those associated with principal production will in many cases be considerable, since overlapping mixtures of product may occur in many industries so that it may be difficult to find a suitable cost structure to apply to any particular subsidiary production.

It can be consolidated only if the assumption is made that the output of each industry is made up of the different products in fixed proportions.

Joint-Products

In some production processes several different products (which are not produced in any other industry) are produced simultaneously. For example, slaughtering produces both meat and hides; cattle farming produces both milk and animals for slaughtering. These industries are difficult to handle in a satisfactory way and it is desirable that different products and their allocation to different uses should be shown separately so that industries of this kind will have several product rows instead of one.

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Problems Arising from the Assumption of Homogeniety of Input or Output

This is a problem which, if ignored, can have disastrous consequences. Assumption of homogeneity of input implies perfect substitutability (from the standpoint of the user) between all the components of aggregates in every activity; but actually it is not this which leads to problems. Items that are substitutes in some activities may not be substitutes at all in other activities. Coal and oil, for example, may be considered substitutes from the standpoint of the household, but not from the standpoint of the iron and steel industry.

In such cases the potential or desired outputs derived by solving the input-output system may underestimate or overestimate the capabilities of the economic system, depending on the assumption of homogeneity. If complete substitutability between components of each aggregate is assumed (but in fact the substitutability is less than perfect), the solution might be thought feasible while in fact it is not practical. Conversely, if it is assumed that substitutability between the components of aggregates is less than perfect, (but in fact it is perfect), it will be indicated that the solution is not feasible, while in fact it is practical.

As the amount of aggregation increases, the former assumption is likely to play a more important part than the latter. Thus, highly aggregative systems are likely to overestimate the attainable levels of production and consumption. On the other hand, systems involving much less aggregation are likely to underestimate the potentialities of the economy.

If the industries were classified on the basis of similar production functions (similar output from the standpoint of producer, given technology) and industries with similar production functions were grouped together, the products of the economy are seen in aggregates and not individually. But in case of final demand industries, in dealing with problems of estimating consumption patterns, an adequate classification is one which identifies substitutable groups of consumption goods. Complementary goods can be grouped together with no loss of information. But if a group of complementary goods have different production functions, so that a change in production function, and, therefore, its cost structure is not accompanied by similar changes in the production function for the rest of the group, the classification is inadequate.

Industrial classification, moreover, is static in nature and therefore does not take into account changes in input and output which result in turn from changes in demand and technology.

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Problem of Unallocated Items

No matter what the basis of classification is, there are some unallocated items left, in the form of receipts or payments, which create problems.[12] It is possible to construct a basic input-output table with reasonably good estimates of receipts and payments for various industries. It is likely that in many industries it will not be possible to allocate all components of costs and receipts. To overcome this difficulty and to balance the accounts, what usually is done is to create for these unidentified residuals a dummy sector, called "Unallocated" and represented in the table by a separate row (for unidentified cost elements) and column (for shipments to unidentified destinations). One of the aims of classification and aggregation should be to make these residuals as small as possible, but they are difficult to eliminate completely.

If one wishes to eliminate these entries from the input-output table completely, what procedure should he follow? There are only two ways to accomplish this end; either ignore these items, with a loss in information, or find some method to incorporate these items in the table, with no loss of information. To incorporate the unallocated items in the table,

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12. Duane W. Evans, "Input-Output Computations", in Tabor Barna (ed.), op. cit.

it is suggested that it should be distributed among the establishment entries throughout the input-output table. Several rather mechanical approaches to the problem of distributing the unallocated entries within the body of the table have been suggested.

PART II

Accounting Framework of Input-Output Analysis

The accounting framework of input-output analysis is considered today to be a part of social accounting. In the literature, interindustry accounting has been developed from social accounting.[13] In what follows an

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13. H.B. Chenery and D.G. Clark, Interindustry Economics, Wiley, 1959, Chaps. II and III.

Harold C. Edey and A.T. Peacock, National Income and Social Accounting, Hutchinson's University Library, London, 1954.

Ragnar Frisch, "From National Accounts to Macro-economic Decision Models" in Milton Gilbert et al. (eds.), International Association for Research in Income and Wealth Series IV, Bowes and Bowes, London, 1955.

E. Fuerst, "The Matrix as a Tool in Macro-accounting", R.E.S., Vol. 37, 1955, pp. 35-45.

Eugene Grassberg, "Social Accounts and Input-Output Tables", Accounting Research, Vol. 6, No. 4, pp. 303-309.

W.W. Leontief, The Structure of the American Economy, 1919 - 1929, Harvard University Press, 1941, Part I.

Herman I. Liebling, op. cit.

Richard Stone, op. cit.

Richard Stone and G. Croft-Murry, Social Accounting and Economic Models, Bowes and Bowes, London, 1959.

input-output accounting framework will be developed as in the literature, and two important difficulties encountered will be discussed.

Consider a closed economy. All the annual transactions in this economy can be divided into two broad categories: those related to productive activity, bringing of goods and services into being, and all other forms of activity. Let X represent production accounts associated with the first activity and $\bar{X}$ with the second. If it is assumed that these accounts can be drawn up in consolidated form, they can be set up in the table as follows:

TABLE 2[a]

BROAD CATEGORIES OF ECONOMIC TRANSACTIONS

	X	$\bar{X}$
X	-	E
$\bar{X}$	G	-

Each row of this square table contains the incomings (or receivables) or a particular account while the

[a] Richard Stone, Input-Output and National Accounts, O.E.E.C., Paris, 1961, p.22.

corresponding column contains the outgoings (or payables) of that account. The two blank entries imply that there are no recorded incomings of X which are also outgoings of X and there are no recorded incomings of $\bar{X}$ which are also outgoings of $\bar{X}$. The entry labelled E represents the value of all kinds of output absorbed by $\bar{X}$ or what is usually termed as gross final expenditure or final product. The entry labelled G represents the gross value added in all forms of production, that is, the sum of the income accruing from productive activity plus provisions for depreciation. From the balance properties of accounts G is equal to E.

Now relax the assumption of a closed economy. More information can be acquired if X and $\bar{X}$ are disaggregated. Assume that there are n industries in the production sector and each industry uses as intermediate input the output of every other industry and each industry uses some primary input. Assume, also, that the final demand sector (all other forms of activities) is divided into investment, consumption, government expenditure, and exports and that each of these sectors consumes the output of every industry together with inputs from the primary sector. Represent the total output of the i-th industry by X_i and the amount of output of this industry going to the j-th industry, as input, by x_{ij}, and to the final consumers as

z_{ik}; where, i, j = 1, ..., n and k = 1, ..., 4. Designate the p-th primary input going to the j-th industry as f_{pj} the final consumers as g_{pk}, where p = 1, ..., m. With the foregoing discussion in mind the accounting framework of input-output analysis can be represented in the tabular form as follows:

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TABLE 3[a]

COMMODITY FLOW AMONG VARIOUS SECTORS

		Purchasing Sector				
		Intermediate Use		Final Use		Total Use = Total Supply
		Industries 1,...,j,...n	Total Intermediate Use	Investment, Consumption, Government, Exports	Total Final Use	
Producing Industries	1	$x_{11}\cdots x_{1j}\cdots x_{1n}$	W_1	$z_{11}\cdots z_{1k}\cdots z_{14}$	v_1	X_1
	⋮	⋮	⋮	⋮	⋮	⋮
	i	$x_{i1}\cdots x_{ij}\cdots x_{in}$	W_i	$z_{i1}\cdots z_{ik}\cdots z_{14}$	v_i	X_i
	⋮	⋮	⋮	⋮	⋮	⋮
	n	$x_{n1}\cdots x_{nj}\cdots x_{nn}$	W_n	$z_{n1}\cdots z_{nk}\cdots z_{n4}$	v_n	X_n
Total Produced Inputs		$U_1 \ldots U_j \ldots U_n$				
Primary Inputs	1	$f_{11}\cdots f_{1j}\cdots f_{1n}$		$g_{11}\cdots g_{1k}\cdots g_{14}$		Y_1
	⋮	⋮		⋮		⋮
	p	$f_{p1}\cdots f_{pj}\cdots f_{pn}$		$g_{p1}\cdots g_{pk}\cdots g_{p4}$		Y_p
	⋮	⋮		⋮		⋮
	m	$f_{m1}\cdots f_{mj}\cdots f_{mn}$		$g_{m1}\cdots g_{mk}\cdots g_{m4}$		Y_m
Total Primary Inputs (value added)		$f_1\cdots f_j\cdots f_n$		$g_1\cdots g_k\cdots g_4$		F
Total Production		$x_1\cdots x_j\cdots x_n$		$Z_1\cdots Z_k\cdots Z_4$		X

[a] This table is an extended version of Chenery's table: H.B. Chenery and P.G. Clark, Inderindustry Economics, Wiley, New York, 1959, p. 15.

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It should be noted that X (production sector) and $\bar{X}$ (final use sector) have been further divided. X has been divided into n industries and $\bar{X}$ into four final consuming sectors. All these accounts shown in Table 3 can be further disaggregated but it will not be attempted here.[14]

Each row of Table 3 shows outgoings of one industry and the corresponding column shows incomings of the same industry. The row-sum shows total production and the column-sum, total consumption, and these are equal to each other. For the industrial sector as a whole, intermediate input is equal to intermediate output. Input of any industry is equal to its output. If intermediate input is equal to intermediate output, primary input must be equal to final consumption.

It should be apparent that the same sort of table can be built by consolidating the accounts of establishments and consumers. The corresponding table of value flows be measured in two basically different ways: (1) At the values received by producers from the sale of their goods and services, usually referred to as "producers' values" or as value at "producers' prices"; (2) at the values paid by

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14. D.B.S. "The 1961 Input-Output Table", The Canadian Statistical Review, Oct., 1963, p.ii.
Ragnar Frich, op. cit., pp. 16 - 17.

the users in the purchase of goods and services, usually referred to as "purchasers' values" or as the value at "purchasers' prices". While in certain instances the purchasers' value is equal to the producer's value, it is often the case that the purchaser's value exceeds the producer's value, particularly where physical commodities are concerned. The difference is composed mainly of the charges of transportation establishments and the margins of trading establishments involved in conveying the goods from producer to user.

Two Main Problems Associated with Accounting Framework of Input-Output Analysis

1. Problem of Distributive Margins and Stocks

It is mentioned above that the commodity flows shown in the matrix may be valued at producer's prices or purchaser's prices and the value of commodity flow measured by these two methods cannot usually be equal, because between the two pricing methods there are such spread items as trade margins, sales taxes, and transport costs. A further reason why consumption values need not correspond to output values is the possibility of changes in the volume of stock held by producers or consumers. Each flow may show all these differences. Writers on input-output have attempted to incorporate these items in the body of the input-output matrix.[15]

One must ask how such a task should be done without introducing too much error in the analysis. The only way to display the exact relationships is to split each cell of the matrix into producer's value, trade margins, sales taxes, transport costs and stock changes - adding up to consumer's value, but this is difficult.

In most models, trade margins are supposed to move proportionally with the commodity flows themselves. To simplify the input-output model, generally all trade margins on the sales of a sector are lumped together, and supposed to move proportionally with total sales of that sector, or, alternatively, the same is done with all margins on inputs.

Most input-output accounts list imports at their c.i.f. value, even if carried in national vessels. If the shipping industry depends to a large extent on the volume of domestic imports (or exports), this ought to be reflected in the model. In general, however, the shipping industries depend much more on the world freight situation than on domestic developments, and the c.i.f. accounting method is an efficient way out of an awkward problem. If an industry performs trading and transport services for part of

15. C.P. Modlin and G. Rosenbluth, "The Treatment of Foreign and Domestic Trade and Transportation Charges in the Leontief Input-Output Table", in Oskar Morgenstern (ed.), op. cit. D.B.S., loc. cit.

its output, these can be transferred to the trade and transport industries, respectively.

The problem of stocks arises when more goods are made during the accounting period than are sold, and there will be an increase in output stocks. This problem is handled by transferring the increase in stocks to the accounts of next period. On the other hand, if more raw materials are bought in the period than are used in it there will be an increase of input stocks which also is transferred to the accounts of next period.

2. Treatment of Imports

Imports are generally divided into two classes, competitive and non-competitive. The former include commodities and services which are similar in nature to, or highly substitutable for, home production; they are an alternative source to domestic supply. Non-competitive imports comprise those foreign produced goods and services for which there are no similar or closely substitutable commodities produced in the domestic economy. In addition, non-competitives usually include net private and governmental unilateral payments abroad and personal governmental, and business expenditures in foreign countries. Non-competitive imports, by definition, are not an addition to the pool of home-produced commodities since there is no counterpart home production. Non-competitive imports are shown as direct purchases by the sectors which use the imported products. Competitive imports are shown as transferred to the sectors producing comparable products. They are implicitly distributed to domestic users through these channels, and hence are treated in a manner analogous to secondary products.

CHAPTER II

THE THEORETICAL FRAMEWORK

Because of the general interdependence of numerous economic phenomena, an economist faces difficulties in developing an economic model. From countless cause and effect relationships, he tries to extract the relationships which he thinks are most important for the model which he is interested in constructing. In doing so he is forced to introduce some simplifications in the form of basic assumptions. The severity of these assumptions varies from one model to another. In certain models the assumptions are grossly untrue and often they even contradict certain elements of economic theory. The very introduction of the assumptions of many models raises controversial methodological problems which are discussed in the economic literature.[1]

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1. M. Friedman, "The Methodology of Positive Economics", in Essays in Positive Economics, University of Chicago Press, Chicago, 1953.

R.F. Harrod, "Scope and Method of Economics", E.J., 1938.

J.N. Keynes, Scope and Method of Political Economy, Macmillan, London, 1891.

K. Klappholz, and J. Agassi, "Methodological Prescriptions in Economics", Economica, 1959.

T.C. Koopmans, "Measurement without Theory", R.E.S. Aug., 1957.

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Like most other economic models, the input-output model is based on certain strong assumptions and therefore subject to several limitations. Although there are a few assumptions which are common to all input-output models, the assumptions of a given model depend on the purpose for which the model is to be used.

This chapter will be divided into three parts. In the first part the assumptions of the input-output model (which is to be used in Chapter III) will be discussed; in the second part the model will be developed; and in the third part, the limitations of the model will be pointed out.

PART I

The Assumptions of the Model

Many writers on input-output analysis have discussed the assumptions of this subject.[2]

Lionel Robbins, An Essay on the Nature and Significance of Economic Science, Macmillan, London, 1932.

William S. Vickrey, Microstatics, Harcourt, Brace, and World, New York, 1964, Chaps. 1 and 2.

2. J. Balderston, "Models of General Economic Equilibrium", in Oskar Morgenstern (ed.), Economic Activity Analysis, Wiley, 1954.

H.B. Chenery, and P.G. Clark, Interindustry Economics, pp. 33-34, 39-42, 87-88.

Carl F. Christ, "A Review of Input-Output Analysis, Input-Output Analysis: An Appraisal", Studies in Income and Wealth, Vol. 18, N.B.E.R., 1955.

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Of all these assumptions, the following apply to the ideal version of the particular model which is to be developed in the next part.

(1) A Static Economy in Equilibrium and an Open Model

A static economy in equilibrium implies that only the current flows of inputs and outputs are considered and that the current inputs of an individual industry, as well as of the industrial sector as a whole, is equal to their corresponding current outputs. Dorfman defines an open model as,

> "The open model, then, does not yield the levels of economic activity consequent upon any government program or assumptions about final demand but, rather, only equilibrium levels in the sense that each sector produces just enough to meet its demands without either drawing down or adding to inventories or capital equipment".[3]

R. Dorfman, "The Nature and Significance of Input-Output", R.E.S., Vol. 36, 1954, pp. 121-123.

O. Eckstein, "The Input-Output System - Its Nature and Use", in Oskar Morgenstern, (ed.), op. cit.

N. Georgescu-Roegen, "Leontief System in the Light of Recent Results", R.E.S., 1950, pp. 214-222.

Leonid Hurwicz, "Input-Output Analysis and Economic Structure (a review article)", A.E.R., Vol. 45, 1955, pp. 626-636.

3. R. Dorfman, op. cit., p. 127.

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(2) Homogeneous Aggregation

It is assumed that the method of classification is such as to produce perfectly homogeneous aggregation. If the method of classification produces perfectly homogeneous aggregation, the statistics collected for the input-output table must also be considered appropriate for the calculations of the coefficients of production.

(3) Fixed Technical Coefficients and Production Functions, Homogeneous of Degree One

The technical coefficients for each industry are assumed to be fixed. This implies constant returns to scale, ruling out economies and diseconomies of scale.

If the structure of one year (or any variable which depends on the structure, say, price level) is to be compared with that of other years, an additional assumption must be made. That is, coefficients of production of one year in which the table is constructed are good approximations for the actual coefficients of the year of application; i.e. the coefficients are stable in the interval between model construction and model application.

The production functions are supposed to be homogeneous of degree one because the data available for the input-output table lends itself to the evaluation of such functions. However, nonlinear functions, relating to inputs and outputs, could be derived by using the transactions of more than one period, or by studying the technology of industries from technical information.

(4) No Substitution Among Inputs is Possible in the production of Goods and Services

This assumption stems from the assumption of constant coefficients of production. There is a corollary of this assumption. There is only one process used for the production of each output. Therefore, it must be possible to form the productive sector in such a way that a single production function can be assumed for each of them. If the output of one industry is the input of the others, the total of inputs equals the total of outputs.

PART II

The Development of the Model

Suppose that the producing sector is divided into n industries, each of which produces only one commodity which is homogeneous. Also assume that the output of each industry is the input of every other industry (excluding intra-industry consumption) and that each industry also supplies the final demand sector. Suppose further that there are m different kinds of primary inputs, each of which is consumed by every industry and by the final use sector. Suppose that the first industry produces P_1X_1 (where P_1 is the producer's unit price and X_1 is the total number of units produced by this industry) dollars worth of a certain commodity, of which it distributes P_1x_{1i} to the ith industry, and so on up to the nth industry to which it supplies P_1x_{1n} dollars worth. The balance, say, P_1b_1 dollars worth of its product goes to the final demand sector. The first letter in the subscript indicates the number of the industry which supplies the commodity and the second letter indicates the number of the industry which consumes the product.

It has been already mentioned in the first chapter that the total net output (total output minus the amount of its product absorbed within the same industry) of the first industry must be equal to the total intermediate sales to the remaining (n - 1) industries plus sales to the final demand sector. This accounting identity can be expressed as a compact equation, as follows:

$$P_1X_1 = P_1x_{1i} + \dots + P_1x_{1n} + P_1b_1 \text{ or,}$$

2.1 $$P_1X_1 - \dots - P_1x_{1i} - \dots - P_1x_{1n} = P_1b_1$$

Similarly, if for the remaining (n - 1) industries,

..., P_i, ..., P_n be the respective unit prices; ..., X_i, ..., X_n be the corresponding outputs and ..., b_i, ..., b_n respectively be the units of output going to the final demand sector, one equation can be written for each of these (n - 1) industries.

To incorporate the primary input sector in the model, let Q_1Y_1, ..., Q_gY_g, ..., Q_mY_m (where Q's show the unit prices of the primary inputs and Y's the total physical amounts available, respectively), be the total value of m primary inputs. Assume that Q_1y_{11} dollars worth of the first primary output is consumed by the industry number one, Q_1y_{1i} by the industry number i, and so on up to Q_1y_{1n} dollars worth by the nth industry. The remainder, say Q_1d_1 dollars worth of primary output of this industry, goes to final consumption. As in the case of intermediate products, total primary output number one must be equal to the total primary input plus sales of this primary output to the final sector. This identity relationship can also be expressed as the following equation:

$$Q_1Y_1 = Q_1y_{11} + \dots + Q_1y_{1i} + \dots + Q_1y_{1n} + Q_1d_1, \text{ or,}$$

2.2 $$- Q_1y_{11} - \dots - Q_1y_{1i} - \dots - Q_1y_{1n} = Q_1d_1 - Q_1Y_1$$

Similarly, one equation can be written for each of the remaining (m - 1) primary inputs. The complete flow of goods and services among intermediate use, primary input, and final use sectors can be shown if n equations similar to 2.1 and m equations similar to 2.2 are written together as follows:

$$P_1X_1 - \ldots - P_1x_{1i} - \ldots - P_1x_{1n} = P_1b_1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$-P_ix_{i1} - \ldots + P_iX_i - \ldots - P_ix_{in} = P_ib_i$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$-P_nx_{n1} - \ldots - P_nx_{ni} - \ldots + P_nX_n = P_nb_n$$

2.3 $$-Q_1y_{11} - \ldots - Q_1y_{1i} - \ldots - Q_1y_{1n} = Q_1d_1 - Q_1Y_1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$-Q_gy_{g1} - \ldots - Q_gy_{gi} - \ldots - Q_gy_{gn} = Q_gd_g - Q_gY_g$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$-Q_mY_{m1} - \ldots - Q_my_{mi} - \ldots - Q_my_{mn} = Q_md_m - Q_mY_m$$

The part of the model 2.3 which is to the left of theequality signs shows the flow of intermediate and primary inputs among the n industries. The first n equations constitute the intermediate use sector. In this set of equations any row, say the ith, shows the distribution of the output of the ith industry among the remaining (n - 1) industries as input in order that these latter may be able to produce their own outputs. Any row, say the gth, in the remaining m equations shows the amount of the gth primary input which each of the n industries must combine with the intermediate inputs in order to produce the indicated outputs.

Any column, say the ith, in that part of model 2.3 which is to the left of the equality sign shows two items: (1) the total output of the ith industry; (2) the various inputs which it receives from the intermediate and the primary sectors. In total, the ith industry receives (n + m - 1) inputs [(n - 1) from the intermediate sector and m from the primary sector]. From industrial accounting, it is known that the output of each industry must be equal to its input. Therefore, if all the elements in the ith column are added together, the following equation is derived.

2.4 $$-P_1x_{1i} - \dots + P_iX_i - \dots - P_nx_{ni} - Q_1y_{1i} - \dots - Q_gy_{gi} - \dots - Q_my_{mi} = 0$$

One equation similar to 2.4 can be derived for the remaining (n - 1) industries. The complete model containing all n industries can be written as follows:

$$P_1X_1 - \dots - P_ix_{i1} - \dots - P_nx_{n1} - Q_1y_{11} - \dots - Q_gy_{g1} - \dots - Q_my_{m1} = 0$$

....

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2.5 $$-P_1x_{1i} - \ldots + P_iX_i - \ldots - P_nx_{ni} - Q_1y_{1i} - \ldots - Q_gy_{gi} - \ldots - Q_my_{mi} = 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$-P_1x_{in} - \ldots - P_ix_{in} - \ldots + P_nX_n - Q_1y_{1n} - \ldots - Q_gy_{gn} - \ldots - Q_my_{mn} = 0$$

It will be more useful if some relationship is established between input and output of the same industry. One convenient way to achieve this task is to express the system 2.5 in the form of input per unit of output. Since equations in this system show total output and also total input of various industries coming from intermediate and primary output sectors, if each equation is divided by the total output of the industry in the corresponding equation, the aim is achieved. Thus, by dividing the items in equations 1, ..., i, ..., n in 2.5 by X_1, ..., X_i, ..., X_n respectively, the following model is developed:

$$P_1\frac{X_1}{X_1} - \ldots - \frac{P_ix_{i1}}{X_1} - \ldots - P_n\frac{x_{n1}}{X_1} - Q_1\frac{y_{11}}{X\ 1} - \ldots - Q_g\frac{y_{g1}}{X_1} - \ldots - Q_m\frac{y_{m1}}{X_1} = 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

2.6 $$-P_1\frac{x_{1i}}{X_i} - \ldots + P_i\frac{X_i}{X_i} - \ldots - P_n\frac{x_{ni}}{X_i} - Q_1\frac{y_{1i}}{X_i} - \ldots - Q_g\frac{y_{gi}}{X_i}$$

$$-\dots- Q_m \frac{y_{mi}}{X_i} = 0$$

....

$$-P_1 \frac{x_{1n}}{X_n} -\dots- P_i \frac{x_{in}}{X_n} -\dots+ P_n \frac{X_n}{X_n} - Q_1 \frac{y_{1n}}{X_n} - Q_g \frac{y_{gn}}{X_n}$$
$$-\dots- Q_m \frac{y_{mn}}{X_n} = 0$$

The set of equations 2.6 can further be written as:

$$P_1 1 -\dots- P_i a_{i1} -\dots- P_n a_{n1} - Q_1 f_{11} -\dots- Q_g f_{g1} -\dots-$$
$$Q_m f_{m1} = 0$$

....

2.7 $$-P_1 a_{1i} -\dots+ P_i 1 -\dots- P_n x_{ni} - Q_1 f_{1i} -\dots- Q_g f_{gi} -\dots-$$
$$Q_m f_{mi} = 0$$

....

$$-P_1 a_{1n} -\dots- P_i a_{in} -\dots+ P_n 1 - Q_1 f_{1n} -\dots- Q_g f_{gn} -\dots-$$
$$Q_m f_{mn} = 0$$

The constants a's and f's are called coefficients of production and their sizes are determined from the technology of the period to which the sample pertains. Any row of coefficients in system 2.7 shows the production function for the corresponding industry. Therefore, all the rows taken as a whole describe the industrial production function. By using matrix algebra the model 2.7 can be written as follows:

2.8 $A'P - F'Q = 0$

Where A' is a transpose of A which is a matrix of order (n x n) of intermediate input coefficients and F' is the transpose of F which is a matrix of primary input coefficients of order (m x n). P is a column vector of order (n x 1) and Q is also a column vector, of order (m x 1).

Solving model 2.8 for P, one gets

$$P = A'^{-1} F'Q \quad \text{or,}$$

2.9 $$P' = Q' FA^{-1}$$

If the identity IQ = Q is combined with the equation 2.8, the price system can be written as follows:

2.10 $$(P' : Q') \begin{pmatrix} A & 0 \\ -F & I \end{pmatrix} = (0 : Q')$$

The rate of change of any component of P, say P_i, with respect of any component of Q, say Q_g, can be determined (see Appendix 1 for details) from system 2.9 as follows:

$$\text{let } (A^{-1}) = (T) = (t_{ij}) \quad (i,j = 1, \ldots, n).$$

2.11 Then $\dfrac{\partial P_i}{\partial Q_g} = f_{g1}t_{1i} + \ldots + f_{gi}t_{ii} + \ldots + f_{gn}t_{ni} = c_{gi}$

In connection with sensitivity analysis, there are two points which must be mentioned. In the first place one should note that the value of any P depends on every Q, every primary input coefficient, and every intermediate input coefficient (see Appendix 1). Since the P's are determined simultaneously, every P depends on every other P. Hence any P, say P_i, depends on every other element of structure in the whole system. Secondly, in deriving equation 2.11, everything else except the elements expressed in this equation are assumed to remain constant. Hence it is partial analysis.

PART III

Limitations of the Model

The elaborate model which has been developed in Part II is subject to a number of limitations. Almost all of them can be traced back to the assumptions discussed in Part I. This part will deal with limitations of the model which are due to these assumptions.

(1) A Static Economy in Equilibrium and an Open Model

It was pointed out in Part I, assumption I, that the assumption of a static economy in equilibrium implies that the total output of any industry in a given period is equal to its input within that period. Hence, there is no storage activity in such a model. Thus the problems of capital and capacity are ignored and they are left to be solved by dynamic models. Even when inventories are included in a static model, they are treated as a part of the bill of goods or are counted as one of the industries within the system for which fixed coefficients hold true.

The assumption of an open model omits the two important facets of economic activity, final consumption and investment. In such a model, consumption and investment are assumed given.

The assumption of a static economy can be defended in that, in the short-run, it holds virtually true. For the purpose at hand, it is not important to analyse investment and final consumption. Therefore, the assumption of an open model is a reasonable one.

(2) Homogeneous Aggregation

In Chapter I, it has been already pointed out that no method of aggregation is free from the errors due to classification; therefore, no aggregation is ideal. The problems of availability of data and the problems of subsidiary products cause errors in aggregation. However, one may assume that errors involved in aggregation do not render the model impracticable, and resulting simplifications are quite useful (c.f., Chapter I).

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(3) Fixed Technical Coefficients and Production Functions Homogeneous of Degree One

Leontief himself admits that the assumption of fixed coefficients implies rejection of the marginal productivity theory.[4] The marginal productivity of any single factor of production equals zero, because the output would not increase unless the inputs of all the other factors were also increased according to their respective coefficients.

The assumption of fixed input ratios, if held strictly, appears to imply the following general observations.

> "First, it implies that all inputs are uniformally affected by a change in the scale of production, thus ignoring the time-honoured distinction between fixed and variable inputs and between short and long-run. Second, it assumes that industries can be classified sufficiently finely to eliminate multiproduct industries whose input structures would be affected by changes in the product-mix of their outputs. Third, it means that economizing substitutions among inputs due to changes in relative prices or availabilities are of negligible importance. Finally, it implies that technological changes in input structures are sufficiently rare and slow that they can be either discarded or adjusted for in simple fashion."[5]

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4. W. Leontief, The Structure of the American Economy, 1919-1929, 1941, pp. 37 - 40.

5. H. Chenery, et al., op. cit., pp. 157 - 158.

Furthermore, the production functions based on fixed technological relationships apply very well to a large part of the economy including manufacturing, mining and utilities.[5] But there are other areas, such as agriculture, foreign trade, government finance, and services where the concept does not apply well. But these sectors should also be included in the model in order to account for full output of the sectors with stable production functions.

The production functions are supposed to be homogeneous of degree one. This supposition neglects economies and diseconomies of scale.

Whatever the shortcomings of the assumption of fixed coefficients and production functions, homogeneous of degree one, it is argued that this is a first approximation to the more complex production functions. Non-proportional inputs, changes in product-mix, and technological changes all do occur and affect in reality production coefficients. The greater the time interval, the more likely will it be that the coefficients have been changed by technological innovations. But the empirical research suggests that in the short-run input coefficients are practically constant.[6]

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6. B. Cameron, "The Production Function in Leontief Models", R.E. STUD., Vol. 20, No. 1, 1952-1953, pp. 62-69.

This assumption is defended mainly on grounds of simplicity. One needs to observe a productive process just once, say at one point in time, to obtain estimates of all parameters of a simple proportion production function, and computations are simpler with this type than with nearly any other type of production function. The proponents of this assumption sometimes also argue that not enough is known to suggest what type of function should be used if the linear ones are discarded.

W. Leontief, "Structural Change", in W. Leontief (ed.), Studies in the Structure of the American Economy, Oxford University Press, 1953.

P. Serealson, "Norway", in T. Barna (ed.), The Structural Interdependence of the Economy, Wiley, New York, 1956.

(4) No Substitution Among Inputs

This assumption does not mean that changes in technology will not lead to changes in input coefficients. Dorfman[7] and others suggest that the assumption implies that with given technology there is one and only one optimal structure which will continue to be optimal no matter what the final demand happens to be. It does not imply that variations in relative prices will not induce changes in structure. (But it implies that relative prices cannot change; because relative prices of commodities will depend only on their direct and indirect labor content. Changes in wage rates will raise the prices of all commodities in a fixed proportion, leaving relative prices unchanged).

In the long-run, the degree of input substitution depends on two factors; in the first place, on the way in which industries are classified; secondly, on the circumstances associated with the system. Under the latter, the following four factors are responsible for changes in

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7. R, Dorfman, et al., Linear Programming and Economic Analysis, McGraw-Hill, New York, 1958, pp. 224 - 225, 248 - 252.

these input relations:

(1) Frequency of changes in the composition of demand for inputs.

(2) Frequency of changes in the relative prices of inputs.

(3) Frequency of changes in technology.

(4) Frequency of depletion of old and discovery of new resources.

Of these four, technology seems to have been the most important source of variation in input functions in the American Economy.[8] In the long-run, technological change is also responsible for most of the changes in relative prices that has taken place.

Leontief argues that the technology is such that no substitution is possible.[9] He suggests that a large proportion of what economists usually call substitution was due to large aggregates, such as consumption, in which a change in the proportion of automobiles and foodstuffs consumed, for example, would cause a change in the proportions of inputs of labour and capital used.

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8. H. Chenery et al., op. cit., p. 42.

9. W. Leontief, The Structure of the American Economy, 1919-1929, 1941, pp. 38-41.

This type of substitution, he says, is eliminated by the use of a finer sector breakdown. About substitution in a given productive process he suggested that there may be a high degree of complementarity among inputs, so that small changes in relative prices may not affect their proportions. If the factors of production are optimally allocated and the degree of complementarity is high, even violent variations in their relative prices affect proportions only slightly.

From the preceding discussion, one may conclude that in the short-run the scope for substitution among inputs is very limited. Therefore, substitution is not a problem in our model in the short-run. But in the long-run, substitution can change the structure of the model and will render the model operationally useless.

The Validity of the Model in General

As Friedman[10] and others argue, the validity of the model should not be judged by its assumptions but by its predictions. The assumptions of the model are beyond doubt unsatisfactory. The assumptions of the model can be sources of error. However, the analysis may be useful.

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10. M. Friedman, op.cit.

To evaluate its usefulness, the simplifying assumptions underlying the model should be checked from time to time and actual magnitudes of different variables should be compared with the corresponding variables predicted by the model.

CHAPTER III

PRICE SENSITIVITY, PRICE ELASTICITY, AND INFLATION

It has been argued that during normal times inflation in the United States and Canada has been more of the cost push type than the demand pull type. Using the 1939 structure of the United States, Leontief tried to estimate the extent of inflation that would have resulted from a ten percent increase in wage income and profit income.[1] In Hungary, Havas and Morva have studied price behavior of different industries,[2] using an input-output analysis similar to the one used by Leontief. Wonnacott used the input-output analysis to analyse the external aspect of inflation in the Canadian economy.[3] His study

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1. W. Leontief, "Wages, Profits and Prices", Q.J.E., Vol. 61, Nov., 1946, pp. 26 - 39.

2. P. Havas, "Examination of the Price System by Means of the Input-Output Table". T. Morva, "Some Experience of the Price Examination Performed by Means of the Chessboard Table of the Social Product", both in O. Lukas, Gy. Cukor, P. Havas, and Z. Roman, (eds.), Akademiac Kiado, Budapest, 1962, Translated into English by L. Varady.

3. R.J. Wonnacott, Canadian-American Dependence, North-Holland Publishing Co., Amsterdam, 1961.

PRICE SENSITIVITY, PRICE ELASTICITY, AND INFLATION is divided into two parts: (1) magnitude of inflation in each industry that resulted from imports into each industry from the United States only; (2) magnitude of inflation in each industry that resulted from total imports into each industry from the rest of the world. He used the 1949 structure and classification of the Canadian economy.

As can be seen from row 3, Table 4 (Appendix II), the results on price sensitivity which are reported in this thesis agree to a great extent with the conclusions of Wonnacott.[4]

In the remaining part of this chapter the cost push aspect of inflation that resulted in various industries due to increases in import prices, wage levels, and profits will be discussed. The model and aggregation of establishments used will be different than that used by Wonnacott.

The reader will recall that in Chapter II the price sensitivity coefficient of the ith industry with respect to a change in price of gth primary input was expressed as:

$$\frac{\partial P_i}{\partial Q_g} = c_{gi}$$

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4 Ibid. p. 140.

PRICE SENSITIVITY, PRICE ELASTICITY, AND INFLATION

The coefficient of price elasticity, E_{gi}, of the ith industry due to change in the price of the gth primary input is given by:

$$E_{gi} = \frac{\partial P_i}{\partial Q_g} \cdot \frac{Q_g}{P_i} = c_{gi} \cdot \frac{Q_g}{P_i}$$

Since in this work the structure of the base year 1949 is to be used, both Q_g and P_i are equal to one. Therefore, $E_{gi} = c_{gi}$.

The third row in Tables 4, 5, and 6 in Appendix II gives values of c_{gi} for three inputs (imports, labor, and capital), for the year 1949. These three tables and Table 7 are all self-explanatory except for one consideration. The elements in row 3 of each of the first three tables have been arranged in the order ofmagnitude of sensitivity coefficients. The numbers and names of the corresponding industries are written at the top of each column. Then in rows 1 and 2 were entered in their respective columns.

The magnitude of the direct impact of inflation on the economy which is caused by an increase in price of primary output of a given industry depends on:

(1) The value of the price sensitivity coefficient for a given industry. Other things being the same, the greater the value of the price sensitivity

coefficient for a given industry, the more inflationary that industry will be.

(2) The fraction of gross domestic product (G.D.P.) which is accounted for by the total output of the given industry. The larger the fraction of G.D.P. which is accounted for by the total output of a given industry, the more inflationary that industry will be.

To discuss any one of these two factors separately, in order to explain the magnitude of inflation in the economy caused by a given industry, does not seem to make as much sense as to discuss all these factors together. Moreover, to arrive at a definite value of the impact of inflation originating from a given industry, some sort of combination of the above-mentioned two factors must be sought by attaching to each factor a definite weight. Since, to the author's knowledge, no method of weighting has been as yet suggested, what the author plans to do is to discuss the inflationary aspect of each industry in rather general terms only.

Imports Absorbed by a Given Industry and Inflation

From Table 4 one notes that of all the industries, the products of the petroleum and coal industry (Column 34) had the highest sensitivity coefficient and a relatively high value of total output (3.7% of G.D.P.). Therefore, it seems that this industry would have been relatively more inflationary, from the point of view of the economy, if the prices of imports would have risen. One should note that the magnitude of inflation which would have resulted from this industry due to the increase of prices of imports would be much less than what would have been derived on the basis of elements in column 34, Table 4 only.

At the other extreme of the price sensitivity scale, the coal mining, crude petroleum, and natural gas industry (Column 5, Table 4) had lowest value of price sensitivity co-efficient. This does not mean that this industry would have contributed least to inflationary pressure, since this industry accounted for 1.2% of G.D.P. (Column 5, Table 7). The non-metal mining, quarrying and prospecting industry (Column 6, Table 4) seemed to have been least inflationary from the imports view point.

From Table 7 it can be seen that the greatest percentage of G.D.P. accounted for by the output of a single industry was 30%, which was due to the transportation, storage and trade industry (Column 38). But from Table 4 (Column 38) it can also be seen that this industry had a relatively low sensitivity coefficient. Therefore, one cannot conclude that since this industry's output accounted for a largest fraction of G.D.P., it would have caused the greatest inflation in the economy, if the price of imports would have increased.

Labor Input and Inflation

From Table 5 it is noted that the service (Column 42); and the transportation, storage and trade (Column 38) industries had significantly high price sensitivity coefficients. Moreover, total output of these industries accounted for a very high percentage (24% and 30% of G.D.P). One may safely conclude from this that these two industries would have been most inflationary.

On the other hand, it is also noted from Table 5 that the fishing, hunting and trapping (Column 3); and carbonated beverages (Column 13) industries had relatively low values of sensitivity coefficients. Moreover, as shown in Table 7 the former industry accounted for only .8% (Column 3) and the latter .6% (Column 13) of G.D.P. Therefore, these two industries seemed to have been least inflationary from the labor input point of view.

All the other industries seemingly fall between these two extremes in their contribution to inflation due to wage increases.

Investment Input and Inflation

In Table 6 it is indicated that of all the industries, the finance, insurance and real estate industry

(Column 41) had a relatively high price sensitivity coefficient. Furthermore, from Table 7 (Column 41) it can be seen that this industry accounted for a relatively large fraction of G.D.P. (14.2%). Therefore, it is likely that this industry was most inflationary from the capital input view point.

On the other hand, the price elasticity coefficient was relatively low for the fishing, hunting and trapping industry (Column 3, Table 6). Moreover, this industry accounted for only .8% of G.D.P. Hence, this industry seemed to be the least inflationary from the investment input view point.

All the other industries fell within the above two extremes in their contribution to inflation resulting from investment inputs.

Applications of the Analysis

The analysis which has been done in this chapter is rather general, but it may be useful for two important purposes. In the first place, the analysis concerning import input and inflation may be applied for better understanding of the external aspect of inflation. Secondly, the analysis concerning labor input and capital input may be applied in understanding the internal aspect of inflation and also for devising incomes policies, by using the economic structure of the year in which these policies are to be devised.

CHAPTER IV

SUMMARY AND CONCLUSIONS

In this thesis an attempt has been made to discuss the conceptual framework of interindustry studies, to develop Leontief's price model, and to apply this model to the Canadian economic structure of 1949.

The first chapter deals with the conceptual framework. In Part I of this chapter various concepts such as the concept of activity, the concept of input and output, the concepts of firm, establishment and industry, the concept of industrial classification, the concept of aggregation of establishments and industries, and the concept of subsidiary products, are discussed. Also in this part various conceptual problems associated with input-output analysis are discussed. Part II of Chapter I deals with the accounting framework of interindustry analysis and it also contains a discussion of the main problems associated with accounting framework.

The second chapter deals with Leontief's price model. In the first part of this chapter the assumptions of the model, such as static economy in equilibrium and an open model, homogenous aggregation, fixed technical coefficients and production functions homogenous of degree one, and no possibility of substitution among inputs in the production of goods and services, are discussed. In the second part of this chapter, the price model is developed. The third part of this chapter outlines limitations of the model. Almost all limitations are explained in terms of assumptions.

In Chapter III the model which is developed in Chapter II is illustrated with Canadian input-output data of 1949. The results are discussed in terms of price sensitivity of the output of a given industry with respect to price of its primary input, and direct inflation caused by an increase in price of input of a given industry. Only three primary inputs (imports, labor, and capital) are considered. It is argued that the direct impact of cost push inflation originating from a given industry depends on the price sensitivity coefficient and the relative output of that industry in the G.D.P.

The conclusions are tentative and can be confirmed only if industrial selling prices for the years of the early nineteen-fifties are available.

If the price of imports would have risen, relatively high inflation would have caused by the products of petroleum and coal; and transportation, storage, and trade industries; on the other hand, least inflation would have caused by the non-metal mining quarrying industry.

SUMMARY AND CONCLUSIONS

If the wage levelhad increased, the service and transportation, storage, and trade industries would have been most inflationary; on the other hand, the fishing, hunting and trapping; and carbonated beverages industries would have been least inflationary.

If the cost of capital would have increased, the finance, insurance, and real estate industry would have been most inflationary; on the other hand, the fishing, hunting and trapping industry would have been least inflationary.

In case of each of the three primary inputs, the rest of the industries fell within two extremes in their contribution to inflation; this can be readily seen in Appendix II.

APPENDIX I

The system 2.7 in Chapter II can be written as follows:

$$\begin{bmatrix} 1 \cdots -a_{i1} \cdots -a_{n1} \\ \cdots\cdots\cdots \\ -a_{1i} \cdots 1 \cdots -a_{ni} \\ \cdots\cdots\cdots \\ -a_{1n} \cdots -a_{in} \cdots 1 \end{bmatrix} \begin{bmatrix} P_1 \\ \cdot\cdot \\ P_i \\ \cdot\cdot \\ P_n \end{bmatrix} - \begin{bmatrix} f_{11} \cdots f_{g1} \cdots f_{m1} \\ \cdots\cdots\cdots \\ f_{1i} \cdots f_{gi} \cdots f_{mi} \\ \cdots\cdots\cdots \\ f_{1n} \cdots f_{gn} \cdots f_{mn} \end{bmatrix} \begin{bmatrix} Q_1 \\ \cdot\cdot \\ Q_g \\ \cdot\cdot \\ Q_m \end{bmatrix} = \begin{matrix} 0 \\ \cdot \\ 0 \\ \cdot \\ 0 \end{matrix}$$

$$A' P - F' Q = 0$$

$$P = A'^{-1} F' Q$$

$$P' = Q' F A^{-1}$$

Let $(A^{-1}) = (T)$, where T is a matrix of the order $(n \times n)$. Now,

$$P' = Q' FT = Q'C \quad (\text{where } C = FT)$$

Typical element c_{gi} of C can be represented as,

$$c_{gi} = \sum_{k=1}^{n} f_{gk} t_{ki}$$

$$P_i = \sum_{g=1}^{m} Q_g c_{gi}$$

$$= \sum_{g=1}^{m} Q_g \sum_{k=1}^{n} f_{gk} t_{ki}$$

APPENDIX I

$$= Q_1(f_{11}t_{1i}+\dots+f_{1i}t_{ii}+\dots+f_{1n}t_{ni})$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$+Q_g(f_{g1}t_{1i}+\dots+f_{gi}t_{ii}+\dots+f_{gn}t_{ni})$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$+Q_m(f_{m1}t_{1i}+\dots+f_{mi}t_{ii}+\dots+f_{mn}t_{ni})$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\frac{\partial P_i}{\partial Q_g} = f_{gi}t_{1i}+\dots+f_{gi}t_{ii}+\dots+f_{gn}t_{ni} = c_{gi}$$

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APPENDIX II

TABLE 4 VALUE OF IMPORTS INTO EACH INDUSTRY INPUT OF IMPORTS INTO EACH INDUSTRY PER DOLLAR OF OUTPUT OF THE INDUSTRY AND PRICE ELASTICITY COEFFICIENTS OF EACH INDUSTRY WITH RESPECT TO PRICES OF IMPORTS CANADA 1949

Industry Number	34	15	16	20	21	18	29	26	35	30	19	27	32	13
Name of Industry	Products of Petroleum and Coal	Confectionary and sugar refin ing	Miscellaneous Food Preparations	Textile Pro ducts (exc Clothing)	Clothing (textile and fur)	Rubber products	Transportation equipment	Primary Iron and Steel	Chemicals and allied Products	Jewellery and silverware (incl watch repair)	Leather products	Agr cultural implements	Electrical apparatus and supplies	Carbonated beverages
1 Imports of goods and services into each ndustry (millions of dollars	266 9	86 6	83 1	186 5	155 7	32 3	203 9	48 6	105 6	7 7	29 2	18 7	65 5	3 6
2 Inpu of imports into each industry per dollar of output of the ndustry	502	426	394	368	183	182	194	166	195	162	159	107	143	045
3 Price elasticity coefficients of each industry with respect to prices of imports	516	457	428	384	306	261	244	238	236	194	190	187	178	167

Industry Number	36	28	33	22	14	10	12	31	11	4	37	24	7	17
Name of Industry	Miscellaneous manufacturing industr es	Iron and steel products n e s	Non metallic mineral products	Furniture	Alcohol c beverages	Fruit and vegetable prepara ons	Bakery products	Non ferrous metal products n e s	Grain mill products	Metal refining and smelting and refining	Construction	Paper products	Meat products	Tobacco and tobacco products
1	23 2	95 0	26 9	1 9	14 0	5 9	8 4	11 1	14 5	45 5	111 9	47 5	17 4	4 8
2	137	107	113	073	063	041	032	057	036	0 1	038	049	026	037
3	166	157	157	135	128	123	113	104	099	098	098	086	083	076

Industry Number	9	8	3	1	23	25	38	42	40	6	2	39	41	5
Name of Industry	Fish process ng	Dairy products	Fishing hunting and trapping	Agriculture	Wood products (exc furnit re)	Printing publishing & allied in dustries	Transportat on storage and trade	Service industries	Electric power gas and water utilities	Non metal min ing quarrying & prospecting	Forestry	Commun cations	Finance insur ance & real estate	Coal min ng crude pe roleum and natural gas
1	1 9	3 7	1 8	26 1	13 5	7 9	110 6	64 4	4 1	8	5	3	6	6
2	015	008	015	011	023	.023	026	019	010	002	001	001	000	003
3	070	066	061	060	053	051	048	043	038	038	032	029	022	022

TABLE 5 TOTAL LABOR INPUT INTO EACH INDUSTRY LABOR INPUT INTO EACH INDUSTRY PER DOLLAR OF OUTPUT OF THE INDUSTRY AND PRICE ELASTICITY COEFFICIENTS OF EACH INDUSTRY WITH RESPECT TO WAGE LEVEL CANADA 1949

Industry Number	42	39	25	5	23	37	38	2	22	6	27	28	31	19
Name of Industry	Service Industries	Communication	Printing publishing and allied industr es	Coal min ng crude petroleum and natural gas	Wood products (exc furniture)	Construc ion	Transportation storage and trade	Forestry	Furniture	Non metal mining quarrying and prospecting	Agricultural implements	Iron and steel products n e s	Non ferrous metal products n e s	Leather products
1 Wages salaries and supplementary labour income (in millions of dollars)	1640	136	142	86	170	810	1 844	199	54	38	46	296	48	62
2 Labor input into each industry per dollar of output of the industry	478	456	407	491	284	272	430	461	329	386	262	333	244	341
3 Price elast city coefficients of each industry with respect to wage level	611	578	561	543	538	536	526	515	510	492	476	472	467	466

Industry Number	30	12	32	24	21	18	26	36	29	33	4	10	20	9
Name of Industry	Jewellery and silverware (incl watch repair	Bakery products	Electrical apparatus and supplies	Paper Products	Clothing (textile and fur)	Rubber products	Primary iron and steel	M scellaneous manufacturing industries	Transportat on equ pment	Non metallic mineral products	Metal refining and smelting and refin ng	Fruit and vege table prepara tions	Textile p oducts (exc clothing)	Fish processing
1	14	63	145	216	208	50	85	54	292	66	206	25	160	19
2	291	238	315	224	261	262	292	.318	278	277	321	171	313	154
3	454	451	448	441	438	433	423	423	418	414	41	400	377	371

Industry Number	11	17	35	40	13	14	8	7	41	15	16	3	34	1
Name of Industry	Grain mill products	Tobacco and tobacco pro ducts	Chemicals and allied products	Electric power gas and water utilities	Carbonated beverages	Alcoholic beverages	Dairy products	[illegible]	Finance insurance and real esta e	Confect onery and sugar refin ng	Miscellaneous food preparat ons	Fishing hunt ng and trapping	Products of petroleum & coal	Agricul ure
1	27	23	108	87	10	40	49	36	313	27	19	17	35	134
2	067	173	199	218	129	179	115	084	164	131	091	142	066	058
3	363	342	342	323	305	299	297	288	287	224	222	211	211	166

TABLE 6 TOTAL INVESTMENT INCOME IN EACH INDUSTRY INVESTMENT INPUT INTO EACH INDUSTRY PER DOLLAR OF OUTPUT OF THE INDUSTRY AND PROFIT ELASTICITY COEFFICIENT OF EACH INDUSTRY CANADA 949

Industry Number	40	4	14	41	13	24	27	33	28	17	6	26	35	31
Name of Industry	Electr c power gas and water utilities	Metal mining and smelting and refining	Alcoholic beverages	Finance insurance and real estate	Carbonated beverages	Paper products	Agricultural implements	Non metallic m neral products	Iron and steel products n e s	Tobacco and tobacco products	Non metal mining quarrying & prospecting	Primary iron and steel	Chemicals and allied products	Non ferrous metal products n e s
1 Investment Income (in m ons of dollars)	131	197	56	447	13	129	22	33	110	16	13	33	63	8
2 Investment Inpu into each industry per dollar of output of the industry	329	306	253	234	167	134	124	139	124	119	135	113	116	038
3 P of t elast c ty coefficients of each industry	360	349	305	262	234	.209	200	198	180	176	175	173	170	168

Industry Number	38	32	25	29	39	2	16	23	5	20	10	11	36	15
Name of Industry	Transportation storage & trade	Electrical apparatus and supplies	P nt ng pub lishing and all ed ndust ries	Transportation equ pment	Common cat ons	Forestry	Miscellaneous food preparations	Wood products (exc furni ture)	Coal min ng crude petro leum and natural gas	Textile pro ducts (exc clothing)	Fruit and vegetable p eparations	Grain mill products	Miscellaneous manufacturing industries	Confectionery and sugar refining
1	546	49	31	110	35	58	22	38	19	54	7	12	14	16
2	128	107	088	104	117	134	102	064	108	106	046	028	080	081
3	160	158	156	155	153	153	150	144	132	132	131	126	120	116

Industry Number	30	12	22	37	34	21	18	19	9	7	42	8	1	3
Name of Industry	Jewellery and s lverware (incl watch repair)	Bakery products	Furniture	Construction	Products of petroleum and coal	Clothing (tex t le and fur)	Rubber products	Leather products	Fish processing	Meat products	Service industries	Dairy products	Agriculture	F shing hun ing and trapping
1	1	9	8	5	24	26	5	6	3	9	106	4	36	1
2	027	033	050	017	045	036	028	042	026	014	031	009	015	009
3	116	112	110	109	092	090	088	084	084	083	076	073	058	035

TABLE 7 TOTAL OUTPUT OF EACH INDUSTRY EXPRESSED AS A PERCENTAGE OF GROSS DOMESTIC PRODUCT AT FACTOR COST

Industry Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Name of Industry	Agriculture	Forestry	Fishing hunt ing and trapp ing	Metal min ng and smelting and refining	Coal min ng crude petrol eum & natural gas	Non metal min ng quarrying and prospecting	Meat products	Dairy products	Fish processing	Fruit and vegetable preparations	Grain mill products	Bakery products	Carbonated beverages	Alcoholic beverages
Percent of G D P	16 1	2 9	8	4 3	1 2	7	4 7	3 0	8	1 0	2 9	1 8	6	1 5

Industry Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Name of Industry	Confectionery & sugar refining	M scellaneous food pre parations	Tobacco and obacco products	Rubber products	Leather products	Textile pro ducts (exc clothing)	Clothing (textile and fur)	Furniture	Wood products (exc furniture	Paper products	Printing pub lishing and allied industries	Primary iron and steel	Agricultural implements	Iron and steel products n e s
Percent of G D P	1 4	1 4	1 2	1 2	5	4 3	5 1	1 1	4 2	7 3	2 5	2 2	1 2	6 3

Industry Number	29	30	31	32	33	34	35	36	37	38	39	40	41	42
Name of Industry	Transportation equ pment	Jewellery and silverware incl watch repair)	Non-ferrous metal products n e s	Electrical apparatus and supplies	Non metallic mineral products	Products of petroleum and coal	Chemicals and all ed products	Miscellaneous manufacturing industries	Construction	Transportation storage and trade	Communicat on	Electric power gas and water utili ies	Finance in surance and real estate	Service industr es
Percent of G D P	7 6	4	1 4	3 3	1 7	3 7	4 0	1 2	20 0	30 1	2 2	3 3	14 2	23 8

Source Table 4 Row 1 conta ns elements of row 44, Table 1 Supplement to the Inter Industry Flow of Goods and Services Canada 1949
Row 2 contains elements of row 44 Table 2 Ibid
Row 3 contains elements of row 44 Table 3B Ibid

Table 5 Row 1 conta ns elements of row 48 Table 1, Supplement to the Inter Industry Flow of Goods and Services Canada 1949
Row 2 contains elements of row 48 Table 2 Ibid
Row 3 conta ns elements of row 48 Table 3B Ibid

Table 6 Row 1 con a ns elements of row 49 Table 1 Supplement to the Inter Industry Flow of Goods and Services Canada 1949
Row 2 conta ns elements of row 49 Table 2 Ibid
Row 3 conta ns elemen s of row 49 Table 3B Ibid

Table 7 Divide each element in column 52 (Table 1 Canada 1949) by G D P ($14 885 millions) and multiply the quotient by 100

N B Figures in Tables 4 5 and 6 are rounded to three digits from the original six digits