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INDUSTRIAL AND TRADE POLICIES:  
A MULTISECTORAL MODEL  
WITH INCREASING RETURNS TO SCALE  
AND IMPERFECT COMPETITION

by

Sahala Benny Pasaribu

Thesis submitted to  
the School of Graduate Studies and Research  
in partial fulfilment of the requirements for the  
Ph.D. degree in Economics

Department of Economics  
Université d'Ottawa/ University of Ottawa  
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## ABSTRACT

The thesis develops a multisectoral model of the Johansen type intended for analyzing industrial and trade policies. The innovations brought by the thesis include a) increasing returns to scale and imperfect competition, b) technological progress via the learning curve. The theoretical structure of the mathematical model is developed step by step and in great detail, with a somewhat expanded discussion of the role of the government sector in industrial and trade policies. Using the data from the 19-sector IO Table of Indonesia for the year 1985, we calibrate the mathematical model. The result is a CGE model of Indonesia, coded in FORTRAN, that can be used for many varied policy analyses. With the help of this CGE model, some simulations are carried out to evaluate the quantitative effects of some industrial and trade policies applied on one imperfectly competitive industry. Simulations are also carried out to study the effects of technological change -- through the learning curve -- on the Indonesian economy for several years.

JEL Classification: C68, F13, L52.

Key Words: Multisectoral Growth Model, Imperfect Competition, Increasing Returns to Scale, Technological Progress, Industrial and Trade Policies.



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Sahala Benny Pasaribu

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## PART I

### INTRODUCTION TO THE THESIS

## CHAPTER 1

### INTRODUCTION TO THE THESIS

#### 1. Background

Mapping out a strategy to promote economic development has always been a crucial problem for the developing countries, especially the newly industrializing nations. A feasible solution to this problem is to adopt an industrial policy in which the government attempts to channel resources into sectors that it considers as important for future economic growth. Industrial policy seems to have played a major role in the economic development of Japan, and a number of newly industrializing countries have, in fact, adopted industrial policies quite similar to those pursued in Japan.

Industrial policy is defined as a policy that affects the economic welfare of a country by intervening in the allocation of resources between industries or sectors and within industrial organization of specific industries or sectors. In more concrete terms, industrial policies refer to

- (i) policies that affect the industrial structure of a country. These policies aim at protecting and nurturing upcoming industries or assisting the shift of resources away from declining industries. In carrying out these policies, the government often intervenes in transactions with foreign countries, such as trade and direct investment or adopts pecuniary measures, such as subsidies and taxes to assist the targeted industries.
- (ii) policies designed to correct market failures associated with technology development and imperfect information. These policies encourage resources to move to more

desirable areas either through subsidies and tax measures or by providing accurate information.

- (iii) policies designed to raise economic welfare through administrative measures that affect the organization of individual industries. These policies, such as depression cartels or investment cartels, intervene directly in either the competitive structure of industries or the allocation of resources.
- (iv) policies based on political demands rather than economic considerations. Some typical examples of these policies are voluntary export restraints and multilateral agreements, which are often adopted to deal with problems in trade friction.

To justify an active government program that encourages the shift of resources, it would be necessary to show that for some reason, the shift, if left to market forces, will take place too slowly; that is, there exists some market failure that provides a justification for government intervention. Economists studying industrial countries have identified two kinds of market failures that seem to be present and relevant in the industrial policies adopted by advanced economies. One of these involves the public good nature of research and development (R&D) activities. The other is the presence of monopoly profits in highly concentrated industries.

R&D has been a widely discussed topic. The main controversy revolves around whether private incentives, relative to the social optimum, result in excessive or too little R&D activity. On this point, the conventional wisdom concentrates on the public-good nature of technology and knowledge and concludes that R&D activities, given private incentives alone, tend to be socially too little, and some form of government assistance in R&D is necessary. The argument used to support this conclusion is as follows. An agent

who develops a new technology is always threatened by the possibility of imitation by other economic agents. This is known as the problem of appropriability. The benefits of a new technology tend to spill over to other firms due to its public good nature. The fruits of a new technology also spill over to consumers through lower prices and increased diversity in consumption.

The increased profits showered on other firms as well as the increase in consumer surplus due to the development of a new technology should be counted as part of the social benefits. However, for a private economic agent contemplating the development of a new technology, only private benefits --the increase in his profits due to the new technology minus development costs -- are taken into account when he decides whether to proceed with an R&D program or not. As a result, R&D activities tend to fall below the socially optimal level if left to market forces. Considered from this perspective, government assistance becomes necessary.

When markets are not perfectly competitive, government intervention can raise national welfare by shifting oligopoly rents from foreign to domestic firms. Krugman (1989, p. 1207) listed four main arguments in support of strategic trade policies, which can be used to

- (i) extract rents from foreign monopolists;
- (ii) shift rents from foreign to domestic firms;
- (iii) help domestic firms move down their cost curves;
- (iv) promote additional entry when this is desirable.

Governments can actively promote exports by means of specific export subsidies, export taxes, and also by procurement or by promoting R&D activities, which endogenously describes economies of scale. Under the assumption of increasing returns to scale, another way of promoting exports is by restricting imports, which may consist of specific import tariff or an import quota. Krugman (1984) demonstrated that all these measures help domestic firms to increase market shares both at home and abroad.

## **2. Objectives and Methodology**

The main objective of this thesis is to provide a framework suitable for analyzing and evaluating industrial and trade policies in an objective manner; that is, to find an optimal industrial and trade structure for the economy under consideration. The thesis also addresses the question concerning the appropriate measures to be adopted if an industrial or trade policy is carried out. The framework proposed is along the line of applied general equilibrium analysis, which contrasts sharply with most models in the literature used to analyze industrial policies. These models are often formulated under a partial equilibrium framework and usually abstract from factor markets. The results of the latter models often contradict each other, and their generality, as noted by Krugman (1989, p. 1181) often turns out to be illusory.

There are several arguments that can be advanced in support of adopting the general equilibrium framework. The most compelling argument involves the general equilibrium effects of industry linkages. Consider, for example, the construction of a large-

scale blast furnace to reduce costs of production. The heavy investment required in the construction of the blast furnace means that without a considerable demand for steel the project will be unprofitable. However, if the blast furnace is built and scale economies allow steel to be supplied at lower prices, then steel demand might increase substantially.

First, a drop in steel prices makes the existing industries, like ship building, more competitive on the international markets, raising derived demand for steel. The lower prices of steel might also induce new firms to enter the ship building industry. Second, declining steel prices can create entirely new industries, like the automobile industry, which were not economically viable before due to cost considerations. Thus the automobile industry provides a new source of derived demand for steel and such a derived demand rises further with the development of industries auxiliary to the production of automobiles. Third, declining steel prices can result in lower transport costs as industries like ship building, automobiles, and other related industries expand, thereby reducing the costs of raw materials for the steel industry itself. It is clear that these industry linkages can only be visualized and studied under the general equilibrium framework.

Another argument in favor of general equilibrium framework invokes the general equilibrium impact of strategic trade policies. If these policies help domestic firms to capture foreign markets, then the world demand for domestic primary factors of production will expand and the end result will be a rise in the prices of these primary factors. The rise in factor prices helps to improve the income position of the home country relative to that of its trading partners. This welfare gain spreads through the entire

economy and in the end the national gains are much larger than the private gains for the protected industry itself. The general equilibrium effects thus reinforce the rent shifting argument advanced by Krugman (1984). Although these general equilibrium effects have received little attention in the literature, they are the most important features of Japan's postwar industrial policy; see, for example, Itoh et al (1991).

Policy applications of general equilibrium modeling have undergone an explosive growth in the last decade and a half. The focus on economy-wide resource allocation that characterizes the general equilibrium approach has made this approach highly attractive for planning and policy analysis. See, for example, the book entitled "General Equilibrium Models for Development Policy," (A World Bank Research Publication) by Kemal Dervis et al (1982). A representative selection of the numerous and varied applications of general equilibrium modeling to economic policy analysis -- from the Common Agricultural Policy Analysis (CAP) of the European Union (EU) to Energy Policy Analysis in Sweden -- can be found in "General Equilibrium Modeling and Economic Policy Analysis," edited by Lars Bergman et al (1990).

Models of applied general equilibrium are commonly known as computable general equilibrium (CGE) models. Most CGE models are intended for some kind of numerical comparative static analysis of changes in exogenous conditions. Two reasons are often advanced to emphasize the usefulness of CGE models.

First, due to the size and complexity of the model, which is regarded as relevant and adequate in the analysis of a specific issue, analytical solutions are difficult or

impossible to obtain. A CGE model then might make it possible to gain useful insights into the economic problem being studied. Furthermore, in the case when analytical solutions are indeterminate, a CGE model can yield definite results in relevant special cases.

Second, when it is the order of magnitude of various effects that matters, a CGE model might prove useful. Thus a CGE model can be regarded as a tool for adding quantitative information to qualitative results.

Bergman (1990) has identified four approaches to applied general equilibrium modeling. The first approach has its origin in the doctoral dissertation of Johansen (1960), whose main purpose was to analyze the process of economic growth in Norway with the help of a numerical multisectoral model. The second approach is due largely to Arnold C. Harberger, Herberth C. Scarf, John B. Shoven, and John Whalley. This approach is represented by a number of CGE models aimed at elucidating efficiency and income effects of trade or tax and transfer policies; for an exposition of this approach, see Shoven and Whalley (1992). The third approach is essentially due to Dale W. Jorgenson (1984a, 1984b) and can be characterized as an econometric approach to CGE modeling. The fourth approach is due to Victor Ginsburgh and Jean Waelbroeck (1981, 1984) and can be characterized as an extension of activity analysis and linear programming modeling.

Because the CGE model we develop in this thesis belongs to the Johansen class, we shall now explain this approach in more detail. It is now commonly agreed that CGE modeling began with Johansen's doctoral dissertation "A Multisectoral Study of Economic Growth." Johansen's main purpose was to study the sectoral reallocation of

labor and capital, changes in sectoral terms of trade, etc., in the process of economic growth of Norway.

The basic tool in his analysis was a disaggregated numerical model, which later has become known as the multisectoral growth (MSG) model. Johansen's MSG model was essentially that of closed economy; the only endogenous foreign trade variables involve certain imported inputs that cannot be produced at home. The rest of Norway's foreign trade was treated as exogenously determined and was added to the demands for public consumption and net investments -- also assumed to be exogenously given. On the resource side, the aggregate supply of labor and capital as well as the rate of technical progress are also assumed to be exogenously determined.

In the original version of the MSG model, there were 20 real production sectors, each sector using other sectors' outputs as intermediate inputs in its own production process. Johansen's MSG model was intended to be a tool for long-term economic planning and forecasting.

The specification of Johansen's MSG model was entirely nonstochastic. Thus most parameters of the model were simply estimated on the basis of a single observation of the resource allocation across the sectors covered by the model. Johansen's method of parameter estimation has later been called the "reference equilibrium" method. The procedure adopted by Johansen for obtaining a numerical solution to his MSG model involves a log-linear approximation of the general equilibrium solution around the reference equilibrium. The MSG model then becomes a linear system in percentage

changes of the endogenous and exogenous variables, and the percentage changes of the endogenous variables can be easily obtained by a simple matrix inversion.

The advantage of this technique is that it is simple and relatively cheap to implement on a computer, which is of course important for large-scale models. The disadvantage is that if the variations in the exogenous variables are not small, the error involved in approximating the variations in the endogenous variables might be considerable.

Johansen's MSG model was taken over by the Norwegian Ministry of Finance in the 1970s and has become the primary framework for long-term macroeconomic planning and forecasting in Norway. The MSG model has been constantly extended and improved. The fourth and latest version, MSG-4, appeared in 1980; see Longva et al (1985).

Further development of Johansen's MSG model was carried out by Dixon et al (1982) in "ORANI: A Multisectoral Model of the Australian Economy." Compared with the original MSG model, ORANI represents a significant increase in size. Relative to other CGE models, ORANI treats 113 producing sectors, 115 domestically produced commodity categories and an equal number of imported commodity categories, nine types of labor, seven types of agricultural land and 113 types of industry-specific capital. Another pronounced difference between the original MSG model and ORANI is that the latter is a truly open economy. Finally, ORANI is numerically solved by an elaborated version of Johansen's original linearization technique. The numerical technique proposed in ORANI is the Euler method used to obtain the numerical solution of an ordinary

differential equation. A CGE model of the United States economy in the Johansen's style has been built by John V. Colias (1988, 1985). See also David Kendrick (1984), who has also developed a CGE model of the United States economy coded in the GAMS (General Algebraic Modeling System) language.

### **3. Contributions of the Thesis**

Most CGE models are developed using the hypothesis of perfect competition. Attempts have been made to introduce imperfect competition into CGE modeling. See, for example, Harris (1984), who introduces monopolistic competition a la Negishi (1961, 1972) and economies of scale -- under the form of a cost function with fixed cost and constant average variable cost. However, when it is about oligopolistic competition, no strategic interaction among the rival firms is modeled explicitly. Instead, ad hoc assumptions about the pricing behavior of oligopolistic firms are often made.

A main objective of the thesis is to fill part of this lacuna. Thus taking as the starting point the theoretical structure of ORANI, we introduce oligopoly into this multisectoral model. The multisectoral model we develop also treats the case of increasing returns to scale. This is because increasing returns to scale is presumably one reason for the existence of monopoly and monopolistic competition.

In the multisectoral model we develop, the production sector is divided into two groups: one group consists of the perfectly competitive industries, while the other group consists only of oligopolistic industries. All commodities are traded internationally. In each

oligopolistic industry, there are a small number of firms -- domestic and foreign -- who supply their products on two markets -- one at home and one abroad. The CGE model we develop can thus be characterized as an almost small open economy. Here almost small can be interpreted as most industries are perfectly competitive.

As already intimated, we attribute the existence of imperfect competition to increasing returns to scale, i.e., each oligopolistic firm operates according to a technology that exhibits increasing returns to scale. In our model, increasing returns to scale is modeled at the production function level. That is, if all inputs are doubled, output will be more than doubled. This is in contrast with Harris (op cit.), whose economies of scale derive from the fact that the same fixed cost is spread over many more units of output as the volume of production increases. Thus, given a constant average variable cost, the average cost will decline as output increases. Presumably, the fixed cost is a proxy for the size of the plant and, therefore, if one double the size of the plant as well as all other inputs, output will also double.

The introduction of increasing returns to scale and oligopolistic competition is made to accommodate highly concentrated industries which are often the focus of strategic industrial and trade policies pursued by national governments. For case studies of such strategic trade policies, we refer the reader to Laura D'Andrea Tyson and David B. Yoffie (1993), who discussed the roles played by the United States government and the Japanese government in fostering their own semiconductor industries on the world market; Benjamin Gomes-Casseres (1993), who discussed the liaisons between firms,

governments, and the global competition on the world computer market; Richard H. Vietor and David B. Yoffie (1993), who discussed how national governments use regulatory regimes on the markets for telecommunications to further the interests of domestic firms and shut out foreign competition.

The extensions that our multisectoral model offers require considerable efforts in handling oligopolistic interactions, increasing returns to scale, and even the nature and meaning of general equilibrium itself. The original contributions of the thesis in these areas will be found in Chapters 6 and 13.

In modeling general equilibrium under imperfect competition, there are two approaches: the objective approach and the subjective approach. According to the objective approach, oligopolists in each industry are assumed to know exactly the demand curve facing their industry. At the present time, a satisfactory objective theory of general equilibrium under imperfect competition is not yet available. Thus we opt for the subjective approach of Negishi (1961, 1972).

For each oligopolistic industry in our model, firms -- domestic as well as foreign -- operate on two markets, one at home and one abroad. While the foreign demand curve is assumed to be known, the domestic demand curve can only be subjectively perceived by the firms operating in this oligopolistic industry. More precisely, each oligopolistic firm, given the outputs chosen by its rivals, has a perceived residual demand curve for its product. Its strategy is to choose the output level that maximizes its perceived profit. The domestic market is in equilibrium if the market supply -- derived under the hypothesis of

maximizing perceived profits -- is equal to the market demand. A detailed treatment of oligopolistic behavior under the subjective approach of Negishi (op cit) is given in Chapter 6.

General equilibrium occurs when all markets clear. The definition of subjective general equilibrium under imperfect competition is given in Chapter 13. Such an equilibrium will take place when the subjective estimates of the domestic market demand curve for an oligopolistic industry -- by all the firms in this industry -- are mutually coherent and, therefore, is somewhat akin to the notion of rational expectation equilibrium.

Another major contribution of the thesis involves the modeling of technological progress along the hypothesis of learning by doing propounded by Arrow (1962). Here cumulative output is interpreted as the stock of knowledge acquired through the production process and exerts a positive effect on the parameters -- input-output coefficients, for example -- of the production function. The contribution of the thesis in this aspect is presented in Chapter 7 under the rubric "The Learning Curve." The learning curve can be the target of various industrial policies. In order to help an infant industry move down the learning curve, the government can raise a tariff wall against its foreign competitors, subsidize its exports, or carry out a preferential procurement policy in favor of this infant industry. These industrial policies constitute part of our simulation exercises. The government can also subsidize R&D activities. However, given the current state of empirical research on the production function of R&D, we feel that it is premature to

introduce this important subject into CGE modeling. We hope that this will be next on our research agenda.

The readers who are familiar with ORANI will recognize that Chapters 2-5, 8-12 are nothing but the theoretical structure of this large-scale multisectoral model of the Australian economy restated in our own notations and in our own chosen order. We have taken great care in showing how increasing returns to scale, imperfect competition, and endogenous technological progress can be introduced into the MSG model. Also, we have expanded ORANI's treatment of the government sector. In particular, the government budget constraint has been added, with various types of expenditures precisely identified to form the basis of trade and industrial policies. The result is a mathematical model with a flavor distinctively of general equilibrium. Our efforts, we believe, will make constructive criticism of the model and its extensions much easier.

Besides the introduction of imperfect competition, increasing returns to scale, and the learning curve into the theoretical structure of the multisectoral model, the thesis also seeks to contribute at the applied level. The result of our applied work is a CGE model of Indonesia that can be used for various simulation purposes.

The data for our CGE model comes from the Indonesian 19-sector IO Table for the year 1985, published in 1989. We merge the two sectors "public administration and defense" and "unspecified sector" of this IO Table into one sector called "unspecified sector" because the former sector contains mostly zeros in both its row and column. Our CGE model thus has 18 sectors that we reorder as follows.

01. Paddy
02. Other farm food and crops
03. Other crops
04. Livestock and its products
05. Forest products
06. Fishery
07. Mining and quarrying
08. Manufacture of other products not elsewhere classified
09. Electricity, gas, and water supply
10. Construction and building
11. Trade
12. Restaurant and hotel
13. Financial intermediaries, real estate, and business services
14. Other services
15. Unspecified sector
16. Transport and communication
17. Manufacture of food, beverages, and cigarettes
18. Petroleum refinery

Each industry is assumed to produce one commodity. Furthermore, the first 15 industries are perfectly competitive, while the last three, namely 16, 17, and 18, are oligopolistic.

Assuming that the original 19-sector IO Table represents an equilibrium of the Indonesian economy in 1985, i.e., the benchmark equilibrium, we use the data from this Table to compute a linear approximation -- in percentage change of the endogenous and exogenous variables -- around the benchmark equilibrium. Thus computer programs in FORTRAN are written to compute all the coefficients of the mathematical model

expressed in its linear form in Chapter 14. These coefficients depend on various elasticities and the endogenous as well as exogenous variables of the model. The values of the elasticities are either lifted from econometric studies or obtained from a process known as calibration. When a set of parameters are calibrated, we need a set of equations containing these parameters. In this set of equations, the parameters are treated as unknown while the values of the endogenous and exogenous variables are known from the IO Table. Solving this system of equations for the values of the parameters, we obtain the values of various elasticities -- assumed to be constant -- that characterize the CGE model.

After the mathematical model -- expressed as a linear in percentage changes -- has been calibrated, that is the benchmark equilibrium has been calculated, it can be used for policy analysis. The CGE model is designed to answer the following type of question. What will be the quantitative changes in the endogenous variables if some exogenous variables are allowed to change by a certain proportion? The procedure used to compute the new equilibrium associated with this comparative-static exercise is the Euler method used to obtain the solution of a system of ordinary differential equations. We have written a comprehensive computer program -- in FORTRAN -- to evaluate the results of such a comparative-static exercise. This computer program represents the CGE model we develop. It is made up of many subroutines that carry out the required computations at every step in the long march to the new equilibrium.

To illustrate how the CGE model can be used for analyzing industrial and trade policies, we carry out several simulation exercises. In the simulations, industry 17

(Manufacturing of food, beverages, and cigarettes) is the target of these policies. The reasons, why this industry is chosen are as follows. First, it is one of the three industries under imperfect competition. Second, it is highly concentrated and considered more difficult to develop since it requires a huge amount of capital and technology for both infrastructures (like roads to and from rural areas, electricity, and communication) and human resources, especially in rural areas. Third, this industry is one of the main industries that the Indonesian government chooses to promote. In terms of contributions, this industry, in 1985, contributed approximately 9.92 percent (Rp. 16,514 billion) to the national output (Rp. 166,423 billion), less than 4 percent to GDP, and 0.7 percent to total export. Regarding labor costs, this industry paid, in the same period, Rp. 746 billion or approximately 2.75 percent of total labor costs paid by all industries. Because of these reasons, industry 17 has received considerable attention in Five-Year Development Plans.

The following industrial and trade policies are targeted at industry 17: (1) a 20 percent increase in tariff applied on the import of this industry; (2) a 20 percent increase in the export subsidy imposed on the domestic oligopolistic firms of industry 17; (3) a 20 percent increase in the government procurement of the output of the domestic firms operating in industry 17. The detailed results and their analysis are presented in Chapter 17. Here we present and discuss the effects on macro variables of these industrial policies.

As can be seen from the Table 1.1, all the industrial policies applied on industry 17 have a positive impact on GDP, CPI, exports, balance of payments, and aggregate employment. For example, a 20 percent increase in tariff raises GDP by 0.0028 percent.

The numbers for a 20 percent increase in export subsidy and 20 percent increase in government procurement are 0.0169 percent and  $0.41 \times 10^{-9}$  percent, respectively. All these effects -- although positive -- are small. This might be explained by the relatively small size of industry 17 -- less than 4 percent of GDP -- in the Indonesian economy. In particular, because our data shows an extremely low level, close to zero, of government procurement, an increase of 20 percent in this policy variable has a very small impact on macro variables. It can also be discerned from the above table that export subsidy is much more efficient than tariff or government procurement in increasing GDP, export, the balance of payments, and aggregate employment or reducing the domestic price level.

Table 1.1. The Effects (in Percent) on Macro Variables of Policy Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

	<i>20% Tariff Increase</i>	<i>20% Export Subsidy Increase</i>	<i>20% Government Procure. Increase</i>
GDP	0.0028	0.0169	$0.41 \times 10^{-9}$
CPI	$-0.58 \times 10^{-6}$	$-0.39 \times 10^{-4}$	$-0.16 \times 10^{-9}$
Exports	0.0049	0.0805	$0.13 \times 10^{-8}$
BOP	0.0526	0.2646	$0.42 \times 10^{-8}$
Aggr. Employment	$0.62 \times 10^{-6}$	$0.42 \times 10^{-4}$	$0.17 \times 10^{-9}$

Note: Industry 17 is perfectly competitive.

We also carry out a simulation exercise to study the effects of technological change. The following table summarizes the results on macro variables of the increase in productivity via the learning curve of industry 17, all alone, i.e., technological change is assumed to take place only for domestic oligopolists operating in industry 17 while the technologies of the remaining industries are assumed to be unchanging.

In sharp contrast with the industrial policies discussed above, the effects on macro variables of the technological improvement in industry 17 are dramatic. GDP increases by 1.53 percent in the first year, 1.02 percent in the second and 0.08 percent in the fifth year. The fact that these rates decrease through time might be explained by the fact that learning takes place at higher rate at the beginning and tapers off as time goes on. Intuitively, we expect that newly established industries will have the most to learn and as they move down their learning curves, their contributions to the economy will be significant.

Table 1.2. The Effects (in Percent) on Macro Variables of Technological Change Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

	YEAR 1	YEAR 2	YEAR 5
GDP	1.5252	1.0240	0.0774
CPI	-0.0016	-0.0011	-0.0001
Export	4.5876	3.1879	0.2488
BOP	15.0730	11.7440	1.0411
Aggr. Employment	0.7012	0.4636	0.0346

Note: Industry 17 is imperfectly competitive.

Export also increases considerably: 4.59 percent in year 1, 3.19 percent in year 2, and 0.25 percent in year 5. The balance of payments improve dramatically: an increase of 15.07 percent in year 1, 11.74 percent in year 2, and 1.04 percent in year 5. Similar increases are also observed in aggregate employment, though at a smaller level. The above table also reveals that productivity gains also help to reduce the domestic price level, although the changes are quite small.

Several conclusions can be drawn from the simulation exercises. First, the numerical results indicate that industrial and trade policies can increase GDP, aggregate employment, export and improve the balance of payments. These policies also have a positive impact on the CPI. Furthermore, our simulations show that technological change can have a dramatic impact on these macro variables. The policy implication of the effects of technological change is that R&D subsidies might be an efficient tool for promoting economic growth.

#### **4. Organization of the Thesis**

The thesis consists of five parts. The first part contains only one chapter, namely Chapter 1, which serves as the introduction to the thesis. In Chapter 1, we explain the objectives of the thesis, its methodology, its contributions, its organization, and notations we adopt. Part II includes Chapter 2 - 14 in which the theoretical structure of the mathematical model is presented. The complete mathematical model -- expressed as a linear system of percentage changes of endogenous and exogenous variables -- is presented in Chapter 14. Part III contains a single chapter, namely Chapter 15. In Chapter 15, we explain how the solution of the model is computed according to the Euler method and the nature of the solution. Part IV contains two chapters, namely, Chapter 16 and 17. In Chapter 16, we explain the kind of raw data used and how this raw data is transformed into data files needed in the computations of solution of the CGE model we develop. In Chapter 17, the results and analysis of some simulation exercises -- for the purpose of

industrial and trade policies analysis -- are presented. Part V, and also the last part, contains only one chapter, namely Chapter 18. In Chapter 18, we draw our conclusions that emphasize our contributions, simulation results, and some reasonable future research areas.

## 5. Notations

Because a CGE model is intended for policy analysis that involves many independent sectors at a micro level, the number of variables -- endogenous as well as exogenous -- and the number of parameters are considerable. A judicious use of symbols is, therefore, a necessity in maintaining the precision and transparency of the economic concepts used in the model. The symbols chosen in this thesis represent our subjective trade-off between economic contents, economy of mathematical representation and the limits imposed by the computer software -- Microsoft Word -- used in typing the thesis.

The symbols used in the thesis can be seen as belong to one of the three categories. In the first category, one find the endogenous or exogenous variables. The symbols used to represent these variables are capital letters of the English alphabet. These symbols carry several subscripts and, sometimes, one superscript. For example,  $X_{ijs}$  denotes the amount of input  $j$  from source  $s$  (domestic or foreign) used by industry  $i$ , while  $C_{js}$  is the private consumption of commodity  $j$  from source  $s$ . Another example is  $Y_{ji}$ , which denotes the output of commodity  $j$  produced domestically. The price of one unit of  $X_{ijs}$  is denoted by the double symbol  $PX_{ijs}$ . Similarly, the price of 1 unit of good  $js$  for private consumption is

$PC_j$ , while the basic price (the price received by producers) of 1 unit commodity  $j$  produced domestically is  $PY_{j1}$ .

The second category of variables contains all the technological coefficients. Each technological coefficient is represented by a symbol that contains two or more capital letters and always begins with letter A. For example in equation (3.4),  $AX_{ij}$ , represents the technological coefficient associated with the variable  $X_{ij}$ , of the CES production in question.

The third category of variables contains the parameters (elasticities of various production functions, utility function) that are assumed to be constant in the model. These parameters can be recognized by the symbols that begin with a lower-case letter of the Greek alphabet. Again, in equation (3.4),  $\alpha X_{ij}$  is the parameter associated with  $X_{ij}$ , while  $\rho X_{ij}$  is the parameter associated with  $X_{ij}$ .

The notations are explained when they are first used in the thesis. A precise understanding of the symbols can only be obtained at this point in time, given the complexity of the mathematical model. We have taken great care in choosing and explaining these symbols.

For comparative static analysis, we have used the hat (^) calculus to express the results of all the comparative static exercises carried out. That is for a variable  $x$ , the percentage change in this variable, namely  $dx/x$ , will be denoted by  $\hat{x} := dx/x$ . In choosing this convention to denote the percentage change of a variable in question, we have been guided by the notations used in international trade to express results of comparative static analysis.

PART II

THE THEORETICAL STRUCTURE OF THE MODEL

## CHAPTER 2

### INTRODUCTION TO THE MODEL

Because the main objective of the thesis is to find an optimal industrial and trade structure, not to investigate its income distribution, we have chosen to model the production sector in great detail but adopt a more aggregate version of the consumer sector. In this way, we will be able to analyze the economic structures (for specific and aggregate sector under considerations) effects of industrial and trade policies.

The model contains  $jj$  produced commodities, where  $jj$  is a positive integer, and three primary factors of production. The produced commodities are indexed by  $j$ ,  $j=1, \dots, jj$ , and can either be consumed or used as intermediate inputs in the production process. These commodities are further differentiated by origin. Thus an additional index, say  $s$ , is needed to indicate whether the commodity is produced domestically, say  $s=1$ , or imported, say  $s=2$ . Among the produced commodities,  $jjc$  of them are bought and sold on perfectly competitive markets, while the rest are bought and sold on oligopolistic markets. Here  $jjc$  is a positive integer strictly less than  $jj$ . Without any loss of generality, we shall assume that commodities  $j=1, \dots, jjc$  are bought and sold on perfectly competitive markets and commodities  $j=jjc+1, \dots, jj$  are bought and sold on oligopolistic markets.

The three primary factors of production are labor, capital, and agricultural land. We shall use two indices to denote primary factors of production. More precisely  $(jj+1, 1)$ ,  $(jj+1, 2)$ , and  $(jj+1, 3)$ , indicate, respectively, that the primary factors of production are

labor, capital, and agricultural land. If we are not very particular about which primary factor is being considered, the generic symbol  $j_s$  will be used. Here of course  $j=j+1$  and  $s=1,2,3$ . In the model, labor is further disaggregated into  $t$  skill categories, where  $t$  is a positive integer. The triple index  $jst$ , with  $j=j+1$ ,  $s=1$ ,  $t=1,\dots,t$ , will be used to indicate that we are dealing with labor of type  $t$ .

In our model, domestic producers are grouped into industries. By an industry, we mean a group of firms that produce and sell the same number of commodities. We shall assume that a domestic firm belongs to only one industry and that all the domestic firms in a given industry have the same technology. In an industry where domestic and foreign firms compete, the production technology of the foreign firms might be different from that of the domestic firms.

Let  $i$  be a positive number that we interpret as the number of industries in our model. Industries are indexed by  $i$ . For each  $i=1,\dots,i$ , let  $L(i)$  be a subset of  $\{1,\dots,j\}$  that represents the commodities produced by industry  $i$ . For most industries,  $L(i)$  contains exactly one element, i.e., these industries produce only one commodity. An industry  $i$  for which  $L(i)$  has more than one element is a multiproduct industry. Let  $i$  be a positive number strictly less than  $i$ . We shall assume that

$$\bigcup_{i=1}^i L(i) = \{1, \dots, j\}$$

i.e., industries  $1,\dots,i$  constitute the perfectly competitive sector of the domestic economy. The remaining industries, namely  $i+1,\dots,i$ , are assumed to be oligopolistic. Furthermore, it is supposed that an oligopolistic industry produces only one commodity.

Hence the number of oligopolistic industries, namely  $ii-ii_c$ , is exactly equal to  $jj-jj_c$ , the number of commodities bought and sold on imperfectly competitive markets. Furthermore, the production technology of perfectly competitive industries exhibit constant returns to scale, while oligopolistic industries operate under increasing returns to scale.

For each  $j=1, \dots, jj_c$ , we assume that good  $j_1$  (good  $j$  produced domestically) and good  $j_2$  (good  $j$  produced abroad) are imperfect substitutes. A competitive firm that produces goods  $j_1$  can either sell its output at home or on the foreign markets. Goods  $j_2$ ,  $j=1, \dots, jj_c$ , are assumed to be brought into the domestic economy by competitive importers.

On the other hand, for each  $j=jj_c+1, \dots, jj$ , we shall assume that goods  $j_1$  and  $j_2$  are perfect substitutes. That is, we make the Armington assumption only for goods bought and sold under perfect competition. Furthermore, for each  $j=jj_c+1, \dots, jj$ , good  $j$  is bought and sold on two segregated markets -- one at home and one abroad. Domestic as well as foreign producers are assumed to sell their products on both of these markets. To model their strategic behavior, we adopt the Cournot framework, i.e., oligopolistic firms compete in quantity strategies.

In the production process, firms use as intermediate inputs the produced commodities, the three primary factors, and another input called "other cost" tickets. Other cost tickets are a useful device for handling production taxes, costs of holding liquidity, costs of holding inventories, and other miscellaneous production costs. Other cost tickets will be indexed by the subscript  $jj+2$ .

On the consumption side, we assume that households can be represented by a single consumer. This consumer maximizes his utility subject to a budget constraint, where his income comes from the sale of labor, capital rental, agricultural land rental, and the share in the profits of the domestic imperfectly competitive firms.

The following figure shows the main distinctive characteristics between perfect and imperfect competitive industries.

Figure 2.1. Distinctive Characteristics of Perfect and Imperfect Competitive Industries.

No.	Descriptions	Perfect Competitive Industry	Imperfect Competitive Industry
1.	Industry index	$i=1, \dots, iic$	$i=iic+1, \dots, ii$
2.	Commodity index	$j=1, \dots, jjc$	$j=jjc+1, \dots, jj$
3.	Firm index	none	Domestic firm: $f=1, \dots, F(i,1)$ Foreign firm: $f=1, \dots, F(i,2)$
4.	Production: a. Technology b. Output	Constant returns to scale Single or multiple commodities	Increasing returns to scale Single commodity
5.	Consumption: a. Source of income  b. Demand curve  c. Substitution Elasticity	Sale of labor, rent of capital and land  Linear expenditure system  Imperfect substitutes (Armington)	Sale of labor, rent of capital and land, and profit shares  Domestic demand: subjective demand a la Negishi  Perfect substitutes
6.	Basic prices in a. Domestic currency b. Foreign currency	$PY_{j1}$ may differ from $PY_{j2}$ $PE_{j1}$ is f.o.b. price of exported goods	$PY_{j1} = PY_{j2}$ $PQ_{j2}$ is the price prevailing in foreign markets

**CHAPTER 3**  
**TECHNOLOGY OF PRODUCTION**

Let

$$(3.1) \quad X_i = \left\{ \left( X_{ijs} \right)_{\substack{j=1, \dots, jj \\ s=1, 2}}, \left( X_{(i, jj+1, t)} \right)_{t=1, \dots, tt}, \left( X_{(i, jj+1, s)} \right)_{s=2, 3}, \left( X_{(i, jj+2)} \right) \right\}$$

be an input combination used by a firm in industry  $i$ ,  $i=1, \dots, ii$ , in the production process. Here  $X_{ij1}$  and  $X_{ij2}$ , for  $j=1, \dots, jj$ , denote, respectively, the amounts of the  $j^{\text{th}}$  commodity produced domestically and imported that are used as inputs by this firm. The variable  $X_{i, jj+1, t}$  denotes the labor input of type  $t$ ,  $t=1, \dots, tt$ . Capital inputs and agricultural land inputs are denoted, respectively, by  $X_{i, jj+1, 2}$  and  $X_{i, jj+1, 3}$ . The last variable in the above input combination, namely  $X_{i, jj+2}$ , denotes the input of other cost tickets.

The input combination  $X_i$  can be used in many different ways to produce various combinations of output. We shall let

$$(3.2) \quad Y_i = (Y_{ij1})_{j \in L(i)}$$

denote an output combination feasible under  $X_i$ . Here we recall that  $L(i)$  is the set of commodities produced by industry  $i$ .

The technology of this firm can be represented by a correspondence

$$(3.3) \quad \Gamma_i: X_i \rightarrow \Gamma_i(X_i),$$

that associates with each input combination  $X_i$  a production possibility frontier that is feasible under  $X_i$ , i.e.,  $\Gamma_i(X_i)$  is the set of output vectors  $Y_i$  that are feasible under  $X_i$ . We shall now describe  $\Gamma_i$  in more detail.

For each intermediate input  $j=1, \dots, jjc$ , we assume that there is imperfect substitutability between domestic and foreign sources. Thus if  $j$  denotes steel, then there is imperfect substitutability between  $X_{ij1}$  (domestic steel) and  $X_{ij2}$  (imported steel). To capture this idea of imperfect substitutability, we assume that  $X_{ij1}$  and  $X_{ij2}$  are combined to yield units of "effective steel input", say  $X_{ij}$ , according to the following equation.

$$*(3.4) \quad X_{ij} = CES \left\{ \left( \frac{X_{ijs}}{AX_{ijs}} \right)_{s=1,2}, (aX_{ijs})_{s=1,2}, \rho X_{ij} \right\}.$$

Here  $AX_{ijs}$ ,  $aX_{ijs}$ ,  $s=1,2$ , and  $\rho X_{ij}$  are technological coefficients, and CES is the constant-elasticity-of substitution production function, i.e.,

$$y = \left\{ (x_s)_{s=1,2}, (a_s)_{s=1,2}, \rho \right\} \text{ implies } y = (a_1 x_1^\rho + a_2 x_2^\rho)^{\frac{1}{\rho}}.$$

We also assume that there is imperfect substitutability of labor of different skills to yield units of "effective labor input", say  $X_{(i,jj+1,1)}$  according to the following formula

$$*(3.5) \quad X_{(i,jj+1,1)} = CRESH \left\{ \left( \frac{X_{(i,jj+1,1,t)}}{AX_{(i,jj+1,1,t)}} \right)_{t=1}^{tt}, \left( [\alpha X_{(i,jj+1,1,t)}] \cdot [\beta X_{(i,jj+1,1,t)}] \right)_{t=1}^{tt}, [\kappa X_{(i,jj+1,1)}] \right\}$$

Here  $AX_{(i,jj+1,1,t)}$ ,  $\alpha X_{(i,jj+1,1,t)}$ ,  $\beta X_{(i,jj+1,1,t)}$ ,  $t=1, \dots, tt$ , and  $\kappa X_{(i,jj+1,1)}$  are technological coefficients, and CRESH is the constant ratio of elasticity of substitution, homothetic

production function, i.e.,  $y = CRESH\left\{(x_k)_{k=1}^n, (\alpha_k, \beta_k)_{k=1}^n, \kappa\right\}$ , implies that  $y$  is

$$\text{defined implicitly by } \sum_{k=1}^n \frac{\beta_k}{\alpha_k} \left(\frac{x_k}{y}\right)^{\alpha_k} - \kappa = 0.$$

Finally we allow for substitution among labor, capital, and agricultural land to yield units of "effective primary factor inputs", say  $X_{(i,jj+1)}$ , according to the following formula

$$*(3.6) \quad X_{(i,jj+1)} = CRESH \left\{ \left( \frac{X_{(i,jj+1,s)}}{AX_{(i,jj+1,s)}} \right)_{s=1}^3, \left( [\alpha X_{(i,jj+1,s)}], [\beta X_{(i,jj+1,s)}] \right)_{s=1}^3, [\kappa X_{(i,jj+1)}] \right\}$$

where  $AX_{(i,jj+1,s)}$ ,  $\alpha X_{(i,jj+1,s)}$ ,  $\beta X_{(i,jj+1,s)}$ ,  $s=1,2,3$ , and  $\kappa X_{(i,jj+1)}$  are technological coefficients.

We do not allow substitution between different intermediate inputs (chemicals, steel, etc.) or between them and primary factors. We shall assume that the effective inputs  $X_{ij}$ ,  $j=1, \dots, jj+2$ , are combined according to the following Leontief technology to yield an activity level, say  $Z_i$ , for this firm

$$*(3.7) \quad [AXZ_i]Z_i = \text{Leontief} \left\{ \left( \frac{X_{ij}}{AX_{ij}} \right)_{j=1}^{jj+2} \right\} = \text{Min}_{j=1, \dots, jj+2} \left( \frac{X_{ij}}{AX_{ij}} \right)$$

Here  $AX_{ij}$ ,  $j=1, \dots, jj+2$ , and  $AXZ_i$  are technological coefficients.

We observe that the activity level  $Z_i$ , as defined by (3.7), that results from the input combination  $X_i$  is linear homogenous in  $X_i$ : a doubling of all the inputs will double the activity level. Technical progress for the production function (3.7) is reflected by a decrease in the values of the technological coefficients  $AX_{ij}$ ,  $j=1, \dots, jj+2$  and  $AXZ_i$ . The

technological coefficient  $AXZ_i$  is used for handling neutral technical progress, while the technological coefficients  $AX_{ij}$ 's are used to handle various types of biased technical progress associated with the use of various inputs. In the same manner, the coefficient  $AX_{ij}$  in (3.4),  $AX_{(i,jj+1,1,0)}$  in (3.5), and  $AX_{(i,jj+1,s)}$  in (3.6) can be used to deal with biased technical progress specific to each of the inputs used.

The technological coefficients can vary through time in an exogenous manner. In this case, the model will not try to explain technical progress but accepts it as given. On the other hand, firms can engage in R&D to actively influence the evolution of these technological coefficients. Of course, different R&D programs will affect different technological coefficients. Due to the spillover nature of technical progress and the public good aspects of R&D, there might even be a strong argument for governments to support R&D.

Having determined the activity level  $Z_i$  that is obtained from a given input combination  $X_i$ , we now determine the production frontier associated with  $Z_i$ . Recall that  $L(i)$  is the set of commodities produced by industry  $i$ . Now, there are certain groups of commodities made up of individual commodities that are always produced in fixed relative proportions. Such a group of commodities is known as a composite commodity. We shall let  $YY_{ij}$  denote the output of the  $j^{\text{th}}$  composite commodity,  $j=1, \dots, nn(i)$ , that industry  $i$  produces.

Observe that in the notation for composite commodities,  $YY$  indicates that we are dealing with a composite commodity. The first subscript, namely  $i$ , serves to identify the

industry; the second subscript, namely  $j$ , runs from 1 to  $nn(i)$  serves to identify the order of the composite commodity in question in the list of the composite commodities produced by industry  $i$ ; the last subscript, namely 1, indicates that this composite commodity is produced domestically.

Let  $(L(i, j))_{j=1}^{nn(i)}$  be a partition of  $L(i)$ . Here we identify  $L(i, j)$  as the  $j^{\text{th}}$  composite commodity produced by industry  $i$ , that is,  $L(i, j)$  consists exactly of the individual commodities  $j' \in L(i)$  that make up the  $j^{\text{th}}$  composite commodity produced by industry  $i$ . One unit of the  $j^{\text{th}}$  composite commodity produced by industry  $i$  is supposed to contain  $1/AY_{ij'1}^j$  units of the  $j'^{\text{th}}$  single commodity produced domestically,  $j' \in L(i)$ . Here  $AY_{ij'1}^j$ ,  $j' \in L(i)$  and  $j=1, \dots, nn(i)$ , are technological coefficients. Mathematically, one unit of the  $j^{\text{th}}$  composite commodity produced by industry  $i$  can be represented by  $(1/AY_{ij'1}^j)_{j' \in L(i, j)}$ .

The activity level  $Z_i$  will yield a production possibility frontier, say  $\Gamma_i(X_i)$ , which is a surface in the  $nn(i)$ -dimensional Euclidean space and which consists of nonnegative output vectors  $YY_i = (YY_{ij1})_{j=1}^{nn(i)}$  of composite commodities that satisfy the following relation:

$$*(3.8) \quad \frac{Z_i}{A_{YYZ_i}} = CRETH \left\{ \begin{array}{l} \left( [YY_{ij1}] [A_{YY_{ij1}}] \right)_{j=1}^{nn(i)}, \\ \left( [\alpha_{YY_{ij1}}], [\beta_{YY_{ij1}}] \right)_{j=1}^{nn(i)}, [\kappa_{YY_i}], h_i \end{array} \right\}$$

where  $A_{YYZ_i}$ ,  $A_{YY_{ij1}}$ ,  $\alpha_{YY_{ij1}}$ ,  $\beta_{YY_{ij1}}$ ,  $\kappa_{YY_i}$ , are technological coefficients, and

$h_i: y \rightarrow h_i(y)$  is a real-valued function defined for all  $y \geq 0$ , which is assumed to be strictly increasing, continuously differentiable, and satisfies  $h(0)=0$ .

Here  $y = CRETH\{(x_k)_{k=1}^n, (\alpha_k, \beta_k)_{k=1}^n, \kappa, h\}$  implies that  $y$  is defined implicitly by

$$*(3.9) \quad \sum_{k=1}^n \frac{\beta_k}{\alpha_k} \left( \frac{x_k}{h(y)} \right)^{\alpha_k} - \kappa = 0.$$

We observe from (3.9) that if  $h_i(y)=y$ , then an increase in the activity level  $Z_i$  in (3.8) will cause the production possibility frontier to shift outward in the same proportion. This is the case of constant returns to scale: an increase of all the inputs by a given proportion will expand the production possibility frontier by the same proportion.

If  $h_i(y)$  is convex, then the technology of production exhibits increasing returns to scale. On the other hand, if  $h_i(y)$  is concave, industry  $i$  operates under conditions of decreasing returns to scale. In our model, to capture the assumption of increasing returns to scale, we shall adopt the following functional form of  $h_i(y)$  for oligopolistic industries

$$*(3.10) \quad h_i(y) = y^{\theta_i}, \quad i=iic+1, \dots, ii$$

where  $\theta_i$  is a positive constant strictly greater than 1.

For an industry  $i$  that only produces single commodities (i.e., no composite commodities) we shall simplify notations and use the following version of (3.8) to describe its production frontier

$$*(3.11) \quad \frac{Z_i}{AYZ_i} = CRETH \left\{ \left( \left[ Y_{ij1} \right] \left[ AY_{ij1} \right] \right)_{j \in L(i)}, \left( \alpha_{Y_{ij1}}, \beta_{Y_{ij1}} \right)_{j \in L(i)}, \kappa_{Y_i}, h_i \right\}$$

## CHAPTER 4

### THE BASIC PRICE SYSTEM

We imagine that for each  $j=1, \dots, jj$ , there exists a geographical area, called the market for commodity  $j1$ , where producers and domestic buyers of this commodity come to trade. On such a market, the law of one price must apply and we shall let  $PY_{j1}$  be the market price of that commodity that prevails on this market. This is the price received by all producers that we call basic value. It excludes sales taxes, transport costs, margins for wholesales and retailers, and other margin costs involved in the transfer of good  $j1$  from producers to users. A buyer pays the basic value for a unit of commodity  $j1$  on this market. On top of this basic value the buyer also has to pay taxes, transport costs, and other margin costs to have this commodity delivered to the location he desires. If a producer has to incur transport and margin costs in transporting his output to the market, then these costs will be counted as costs of production and will not be included in the basic value  $PY_{j1}$ .

Clearly  $PY_{j1}$  should be expressed in domestic currency for  $j=1, \dots, jjc$ , i.e., for all the commodities produced domestically and sold on perfectly competitive markets at home. For  $j=jjc+1, \dots, jj$ , there are also foreign oligopolists who sell these commodities on the domestic markets. In this case,  $PY_{j1}$  is the price in domestic currency received by domestic as well as foreign oligopolists.

and other margins costs involved in the delivery of competitive exports to domestic ports, but it excludes transport costs from domestic ports to the final destination abroad, i.e.,  $PE_{j1}$  is the f.o.b. price in foreign currency. Thus if we visualize domestic ports as the market for competitive exports, then the law of one price implies that  $PE_{j1}$  is the price of commodity  $j1$  prevailing on this market.

For each  $j=j1c+1, \dots, j1$ , let  $PQ_{j2}$  be the price of one unit of commodity  $j$ , bought and sold under imperfectly competitive conditions, on the foreign market.  $PQ_{j2}$  is the basic value of good  $j$ , expressed in foreign currency on the foreign market. For a domestic producer who sells such a commodity on the foreign market,  $PQ_{j2}$  represents the price, in foreign currency, that he obtains for 1 unit of his product. This price, again, does not include sales taxes, transport, or other margin costs in delivering his output to the foreign market. These costs should be considered as part of the costs of production.

For each  $j=1, \dots, j1c$ , let  $PY_{j2}$  be the basic price in domestic currency, of imported good  $j$ . Again, if we visualize domestic ports as the market for competitive imports, then the law of one price must rule and  $PY_{j2}$  is the price received by importers that excludes transport and other margin costs involved in delivering the imports to final domestic users.

If we denote by  $PY_{j2}$  the basic price in domestic currency of imported good  $j$ ,  $j=j1c+1, \dots, j1$ , then assuming that foreign oligopolists transport their goods to the domestic market for good  $j$ , we must have  $PY_{j2} = PY_{j1}$ . The equality  $PY_{j2} = PY_{j1}$  comes from the assumption that commodities  $j1$  and  $j2$  are perfect substitutes.

## CHAPTER 5

### PROFIT MAXIMIZATION: THE PERFECTLY COMPETITIVE INDUSTRIES

#### 1. The Problem

Let  $X_i$  be an input combination chosen by a domestic firm in a perfectly competitive industry  $i$ ,  $i=1, \dots, iic$ , and  $Z_i$  be the activity level associated with  $X_i$ , as defined by equation (3.7). If  $Y_i$  is an output vector feasible under the activity level  $Z_i$  (see equation (3.8)), then the profit made by the firm under this production plan is

$$(5.1.1) \quad \begin{aligned} & \sum_{j \in L(i)} [PY_{j1}] [Y_{ij1}] - \sum_{j=1}^{jj} \sum_{s=1}^2 [PX_{ijs}] [X_{ijs}] \\ & - \sum_{t=1}^{tt} [PX_{(i,jj+1,1,t)}] [X_{(i,jj+1,1,t)}] - \sum_{s=2}^3 [PX_{(i,jj+1,s)}] [X_{(i,jj+1,s)}] \\ & - [PX_{(i,jj+2)}] [X_{(i,jj+2)}] \qquad \qquad \qquad i = 1, \dots, iic \end{aligned}$$

where  $PX_{ijs}$  is the price of intermediate input  $j$ ,  $j=1, \dots, jj$ , from source  $s$ ,  $s=1,2$ , that industry  $i$  has to pay;  $PX_{(i,jj+1,1,t)}$  is the price of labor of type  $t$ ,  $t=1, \dots, tt$ , paid by industry  $i$ ;  $PX_{(i,jj+1,2)}$  is the rental rate of capital in industry  $i$ ;  $PX_{(i,jj+1,3)}$  is the rental rate of agricultural land in industry  $i$ ;  $PX_{(i,jj+2)}$  is the price of "other cost" tickets that industry  $i$  has to pay in the production process, and  $PY_{j1}$  is the basic price industry  $i$  obtains from selling one unit of the  $j^{\text{th}}$  produced commodity. Because the producers in industry  $i$  are perfectly competitive, all these prices are given.

In our model, we assume that producers treat all factors of production as variable. In particular, they act as if they rent their fixed capital and agricultural land. Furthermore, both capital and agricultural land are assumed to be industry-specific, i.e., these two

factors of production are not mobile between industries. These assumptions mean, in effect, that there is a market for capital and a market for agricultural land for each industry. Moreover, producers in each industry treat the rental prices of capital and agricultural land specific to this industry as given. The rental rates adjust so that for each  $i=1, \dots, iic$ , the sum of demands from all the producers in industry  $i$  equals the available supplies of capital and agricultural land of type  $i$ .

In (5.1.1),  $PX_{ijs}$ ,  $j=1, \dots, jj$ ,  $s=1, 2$ , represents the "true cost" paid by industry  $i$  for a unit of good  $js$ . To obtain a unit of good  $js$ , a producer in this industry must first go to the market for  $js$ . On this market, he must pay the basic price  $PY_{j1}$  ( $PY_{j2}$ ) for one unit of the good if it is domestically produced (imported). The producer then has to pay taxes on this unit as well as transport and margin costs to bring this unit back to where the industry is located. These extra costs must be added to the basic price  $PY_{j1}$  ( $PY_{j2}$  if  $s=2$ ) to arrive at the true cost  $PX_{ijs}$ . Because of differences in taxes, transport costs, and other margin costs, different industries will pay different prices for the same commodity. Hence  $PX_{ijs}$ ,  $i=1, \dots, iic$ , will differ across industries.

The structure of the profit-maximization problem for industry  $i=1, \dots, iic$ , can now be described as follows:

- a. First, taking the input and output prices as given, the industry chooses an activity level  $Z_i$  and computes the minimum cost for sustaining this activity level and the maximum revenue obtained from this same activity level. The difference between the minimum

cost and maximum revenue is then the profit associated with the chosen activity level

$Z_i$ .

b. Next, choose the activity level that maximizes total industry profit.

## 2. Cost Minimization

Let  $Z_i$  be a given activity level for industry  $i$ . We want to find the input combination that yields this activity level at minimum cost. To this end, let

$$X_i = \left( \left( X_{ijs} \right)_{j=1, \dots, jj; s=1, 2}, \left( X_{(i, jj+1, t)} \right)_{t=1, \dots, n}, \left( X_{(i, jj+1, s)} \right)_{s=2, 3}, X_{(i, jj+2)} \right)$$

be an input combination that yields activity level  $Z_i$ . It follows from equations (3.7), (3.4), (3.6), and (3.5) that we must have

$$* (5.2.1) \quad [AXZ_i]Z_i = \text{Leontief} \left( \frac{X_{ij}}{AX_{ij}} \right)_{j=1}^{jj+2}, \quad i=1, \dots, iic$$

where

$$* (5.2.2) \quad X_{ij} = \text{CES} \left\{ \left( \frac{X_{ijs}}{AX_{ijs}} \right)_{s=1, 2}, \left( aX_{ijs} \right)_{s=1, 2}, \rho X_{ij} \right\}, \quad j=1, \dots, jj.$$

$$* (5.2.3) \quad X_{(i, jj+1)} = \text{CRESH} \left\{ \left( \frac{X_{(i, jj+1, s)}}{AX_{(i, jj+1, s)}} \right)_{s=1}^3, \left( [\alpha X_{(i, jj+1, s)}] [\beta X_{(i, jj+1, s)}] \right)_{s=1}^3, \kappa X_{(i, jj+1)} \right\}$$

$$* (5.2.4) \quad X_{(i, jj+1, 1)} = \text{CRESH} \left\{ \left( \frac{X_{(i, jj+1, 1, t)}}{AX_{(i, jj+1, 1, t)}} \right)_{t=1}^{n''}, \left( [\alpha X_{(i, jj+1, 1, t)}] [\beta X_{(i, jj+1, 1, t)}] \right)_{t=1}^{n''}, \kappa X_{(i, jj+1, 1)} \right\}$$

The total cost of the input combination  $X_i$  is given by

$$(5.2.5) \quad \sum_{j=1}^{jj} \sum_{s=1}^2 PX_{ijs} [X_{ijs}] + \sum_{t=1}^n PX_{(i,jj+1,t)} [X_{(i,jj+1,t)}] \\ + \sum_{s=2}^3 PX_{(i,jj+1,s)} [X_{(i,jj+1,s)}] + PX_{(i,jj+2)} [X_{(i,jj+2)}]$$

It follows directly from (5.2.1) that given the activity level  $Z_i$ , the effective inputs are completely determined, i.e.,

$$*(5.2.6) \quad [AX_{ij}][AXZ_i][Z_i] = X_{ij}, \text{ for } j=1, \dots, jj+2.$$

Hence for each  $j, j=1, \dots, jj+2$ , the inputs from various sources that make up the  $j^{\text{th}}$  effective input should be chosen to yield  $X_{ij}$  at minimum cost.

## 2.1. Cost Minimization: Intermediate Inputs

For each  $j=1, \dots, jj$ , industry  $i$  solves the following cost minimization problem

$$(5.2.1.1) \quad \text{Minimize } \sum_{s=1}^2 [PX_{ijs}] [X_{ijs}]$$

subject to

$$(5.2.1.2) \quad [AX_{ij}][AXZ_i][Z_i] = CES \left\{ \left( \frac{X_{ijs}}{AX_{ijs}} \right)_{s=1,2}, (aX_{ijs})_{s=1,2}, \rho X_{ij} \right\}$$

Note that equation (5.2.1.2) is obtained by combining equation (5.2.2) and (5.2.6).

The first order conditions for this minimization problem are

$$*(5.2.1.3) \quad PX_{ijs} - \frac{\lambda}{AX_{ijs}} D_s CES \left\{ \left( \frac{X_{ijs}}{AX_{ijs}} \right)_{s=1,2}, (aX_{ijs})_{s=1,2}, \rho X_{ij} \right\} = 0, \quad s=1,2,$$

$$*(5.2.1.4) \quad [AX_{ij}][AXZ_i][Z_i] - CES \left\{ \left( \frac{X_{ijs}}{AX_{ijs}} \right)_{s=1,2}, (aX_{ijs})_{s=1,2}, \rho X_{ijs} \right\} = 0.$$

In (5.2.1.3),  $\lambda$  is the Lagrange multiplier and  $D$  is differential operator, i.e.,  $D_s$  CES,  $s=1,2$ , is the partial derivative of the CES production function with respect to the  $s^{\text{th}}$  variable. The optimal values of the inputs  $X_{ijs}$ ,  $s=1,2$ , as defined by (5.2.1.3) and (5.2.1.4) depend on the exogenous variables  $Z_i$ ,  $PX_{ijs}$ ,  $s=1,2$ , and the technological coefficients  $AXZ_i$ ,  $AX_{ij}$ ,  $AX_{ijs}$ ,  $s=1,2$ . When these exogenous variables change, the optimal values for  $X_{ijs}$ ,  $s=1,2$ , also change and we have the following comparative static results

$$\begin{aligned} \text{^(5.2.1.5)} \quad \hat{X}_{ijs} - \hat{Z}_i + \sigma X_{ij} \left( \hat{P}X_{ijs} - \sum_{k=1}^2 SX_{ijk} [\hat{P}X_{ijk}] \right) - \hat{A}X_{ijs} - \hat{A}X_{ij} \\ - AXZ_i + \sigma X_{ij} \left( \hat{A}X_{ijs} - \sum_{k=1}^2 SX_{ijk} [\hat{A}X_{ijk}] \right) = 0, \\ i=1,\dots,ic, j=1,\dots,jj, s=1,2, \end{aligned}$$

where

$$\sigma X_{ij} := \frac{1}{1 - \rho X_{ij}}, \text{ and } SX_{ijs} := \frac{PX_{ijs} [X_{ijs}]}{\sum_{k=1}^2 PX_{ijk} [X_{ijk}]}$$

Observation:

$\sigma X_{ij}$  is the elasticity of substitution between the  $j^{\text{th}}$  intermediate input produced domestically and the  $j^{\text{th}}$  intermediate input imported by industry  $i$ ; and  $SX_{ijs}$  is the share of the cost of the  $j^{\text{th}}$  intermediate input from source  $s$  in the total cost of the  $j^{\text{th}}$  intermediate input from both sources - domestic and imported - in industry  $i$ .

## 2.2. Cost Minimization: Primary Inputs

The problem is to choose  $X_{(i,jj+1,1,t)}$ ,  $t=1,\dots,tt$ ;  $X_{(i,jj+1,s)}$ ,  $s=2,3$ , to

$$(5.2.2.1) \quad \min \sum_{t=1}^{\mu} [PX_{(i,jj+1,1,t)}] [X_{(i,jj+1,1,t)}] + \sum_{s=2}^3 [PX_{(i,jj+1,s)}] [X_{(i,jj+1,s)}]$$

subject to

$$(5.2.2.2) \quad [AX_{(i,jj+1)}] [AXZ_t] [Z_t] = CRESH \left\{ \begin{array}{l} \left( \frac{X_{(i,jj+1,s)}}{AX_{(i,jj+1,s)}} \right)_{s=1}^3, \\ \left( [\alpha X_{(i,jj+1,s)}], [\beta X_{(i,jj+1,s)}] \right)_{s=1}^3, [\kappa X_{(i,jj+1)}] \end{array} \right\}$$

$$(5.2.2.3) \quad X_{(i,jj+1,1)} = CRESH \left\{ \begin{array}{l} \left( \frac{X_{(i,jj+1,1,t)}}{AX_{(i,jj+1,1,t)}} \right)_{t=1}^{\mu}, \\ \left( [\alpha X_{(i,jj+1,1,t)}], [\beta X_{(i,jj+1,1,t)}] \right)_{t=1}^{\mu}, [\kappa X_{(i,jj+1,1)}] \end{array} \right\}$$

We have the following first-order conditions

$$*(5.2.2.4) \quad [PX_{(i,jj+1,1,t)}] [AX_{(i,jj+1,1)}] [AX_{(i,jj+1,1,t)}] - \frac{\mu}{[AX_{(i,jj+1,1,t)}]} D_t CRESH \left\{ \begin{array}{l} \left( \frac{X_{(i,jj+1,1,t)}}{AX_{(i,jj+1,1,t)}} \right)_{t=1}^{\mu}, \\ \left( [\alpha X_{(i,jj+1,1,t)}], [\beta X_{(i,jj+1,1,t)}] \right)_{t=1}^{\mu}, [\kappa X_{(i,jj+1,1)}] \end{array} \right\} = 0$$

t=1,...,tt,

$$*(5.2.2.5) \quad X_{(i,jj+1,1)} - CRESH \left\{ \begin{array}{l} \left( \frac{X_{(i,jj+1,1,t)}}{AX_{(i,jj+1,1,t)}} \right)_{t=1}^{\mu}, \\ \left( [\alpha X_{(i,jj+1,1,t)}], [\beta X_{(i,jj+1,1,t)}] \right)_{t=1}^{\mu}, [\kappa X_{(i,jj+1,1)}] \end{array} \right\} = 0$$

$$*(5.2.2.6) \quad [PX_{(i,jj+1,s)}] [AX_{(i,jj+1,s)}] - \frac{\lambda}{AX_{(i,jj+1,s)}} D_s CRESH \left\{ \begin{array}{l} \left( \frac{X_{(i,jj+1,s)}}{AX_{(i,jj+1,s)}} \right)_{s=1}^3, \\ \left( [\alpha X_{(i,jj+1,s)}], [\beta X_{(i,jj+1,s)}] \right)_{s=1}^3, [\kappa X_{(i,jj+1)}] \end{array} \right\} = 0$$

s=2,3,

$$*(5.2.2.7) \mu - \lambda D_1 CRESH \left\{ \left( \frac{X_{(i,jj+1,s)}}{AX_{(i,jj+1,s)}} \right)_{s=1}^3, \left( \left[ \alpha X_{(i,jj+1,s)} \right], \left[ \beta X_{(i,jj+1,s)} \right] \right)_{s=1}^3, \left[ \kappa X_{(i,jj+1)} \right] \right\} = 0$$

$$*(5.2.2.8) \left[ AX_{i,jj+1} \right] \left[ AXZ_i \right] \left[ Z_i \right] - CRESH \left\{ \left( \frac{X_{(i,jj+1,s)}}{AX_{(i,jj+1,s)}} \right)_{s=1}^3, \left( \left[ \alpha X_{(i,jj+1,s)} \right], \left[ \beta X_{(i,jj+1,s)} \right] \right)_{s=1}^3, \left[ \kappa X_{(i,jj+1)} \right] \right\} = 0$$

In (5.2.2.4),  $\mu$  is the Lagrange multiplier associated with the constraint (5.2.2.3) and  $D$  is the differential operator, i.e.,  $D_t CRESH$ ,  $t=1, \dots, tt$ , is the partial derivative of CRESH with respect to the  $t^{\text{th}}$  variable. In (5.2.2.6),  $\lambda$  is the Lagrange multiplier associated with the constraint (5.2.2.2), and  $D_s CRESH$ ,  $s=1, 2, 3$ , is the partial derivative of CRESH with respect to the  $s^{\text{th}}$  variable.

Here, we obtained the following comparative static results concerning the demand for labor inputs of various skill types:

$$\begin{aligned} \hat{X}_{(i,jj+1,t)} &= \hat{X}_{(i,jj+1)} + \hat{A}X_{(i,jj+1,t)} \\ &- \left[ \sigma X_{(i,jj+1,t)} \right] \left\{ \hat{P}X_{(i,jj+1,t)} - \sum_{k=1}^u \left[ \tilde{S}X_{(i,jj+1,k)} \right] \left[ \hat{P}X_{(i,jj+1,k)} \right] \right\} \\ &- \left[ \sigma X_{(i,jj+1,t)} \right] \left\{ \hat{A}X_{(i,jj+1,t)} - \sum_{k=1}^u \left[ \tilde{S}X_{(i,jj+1,k)} \right] \left[ \hat{A}X_{(i,jj+1,k)} \right] \right\} \end{aligned}$$

for  $t=1, \dots, tt$ ,  $i=1, \dots, iic$ ,

where

$$\begin{aligned} \sigma X_{(i,jj+1,t)} &= \frac{1}{1 - \alpha X_{(i,jj+1,t)}} \\ \tilde{S}X_{(i,jj+1,t)} &= \frac{\left[ \sigma X_{(i,jj+1,t)} \right] \left[ SX_{(i,jj+1,t)} \right]}{\sum_{k=1}^u \left[ \sigma X_{(i,jj+1,k)} \right] \left[ SX_{(i,jj+1,k)} \right]} \end{aligned}$$

$$SX_{(i,jj+1,t)} = \frac{[PX_{(i,jj+1,t)}][X_{(i,jj+1,t)}]}{\sum_{k=1}^n [PX_{(i,jj+1,k)}][X_{(i,jj+1,k)}]}$$

The comparative static results for effective labor inputs, capital, and agricultural land are given by

$$\begin{aligned} \hat{X}_{(i,jj+1,s)} &= \hat{A}X_{(i,jj+1,s)} + \hat{A}X_{(i,jj+1)} + A\hat{X}Z_i + \hat{Z}_i \\ &- \sigma X_{(i,jj+1,s)} \left( [\hat{P}X_{(i,jj+1,s)}] - \sum_{k=1}^3 [\tilde{S}X_{(i,jj+1,k)}][\hat{P}X_{(i,jj+1,k)}] \right) \\ &\quad - \left( [\hat{A}X_{(i,jj+1,s)}] - \sum_{k=1}^3 [\tilde{S}X_{(i,jj+1,k)}][\hat{A}X_{(i,jj+1,k)}] \right) \end{aligned}$$

for  $s=1,2,3$  and  $i=1,\dots,ic$ ,

where

$$\begin{aligned} \sigma X_{(i,jj+1,s)} &= \frac{1}{1 - \alpha X_{(i,jj+1,s)}} \\ \tilde{S}X_{(i,jj+1,s)} &= \frac{[\sigma X_{(i,jj+1,s)}][SX_{(i,jj+1,s)}]}{\sum_{k=1}^3 [\sigma X_{(i,jj+1,k)}][SX_{(i,jj+1,k)}]} \\ SX_{(i,jj+1,s)} &= \frac{[PX_{(i,jj+1,s)}][X_{(i,jj+1,s)}]}{\sum_{k=1}^3 [SX_{(i,jj+1,k)}][X_{(i,jj+1,k)}]} \\ PX_{(i,jj+1,1)} &= \frac{\left( \sum_{t=1}^n [PX_{(i,jj+1,t)}][X_{(i,jj+1,t)}] \right)}{X_{(i,jj+1,1)}} \end{aligned}$$

Here, we obtain the following comparative static result for the price of one unit of effective labor inputs:

$$\begin{aligned} \hat{P}X_{(i,jj+1,1)} &= \sum_{t=1}^n [SX_{(i,jj+1,t)}][\hat{P}X_{(i,jj+1,t)}] \\ &\quad + \sum_{t=1}^n [SX_{(i,jj+1,t)}][\hat{A}X_{(i,jj+1,t)}] \end{aligned}$$

### 2.3. Cost Minimization: Other Cost Tickets

Let  $Z_i$  be the activity level for industry  $i$ . From equation (5.2.1), we have

$$*(5.2.3.1) \quad [AXZ_i][Z_i] = \frac{X_{i,jj+2}}{AX_{i,jj+2}},$$

which gives us immediately the following comparative static result

$$^{\wedge}(5.2.3.2) \quad \hat{X}_{(i,jj+2)} = \hat{Z}_i + A\hat{X}Z_i + \hat{A}X_{(i,jj+2)}, \quad i=1,\dots,ic.$$

### 3. Commodity Supply: Revenue Maximization

Let  $Z_i$  be a given activity level for industry  $i$  and  $(Y_{ij1})_{j \in L(i)}$ , be an output combination feasible under  $Z_i$ . The total revenue obtained from this output combination of produced commodities is

$$(5.3.1) \quad \sum_{j \in L(i)} [PY_{j1}][Y_{ij1}]$$

If we let  $(YY_{ij1})_{j=1}^{nn(i)}$  be the vector of composite commodities associated with

$(Y_{ij1})_{j \in L(i)}$ , then we must have

$$*(5.3.2) \quad Y_{ij'1} = \frac{YY_{ij1}}{AY_{ij'1}^j}, \quad j' \in L(i, j), j = 1, \dots, nn(i).$$

It follows directly from (5.3.2) that the revenue generated by one unit of the  $j^{\text{th}}$  composite commodity produced by industry  $i$  is given by

$$*(5.3.3) \quad PYY_{ij1} = \sum_{j' \in L(i, j)} PY_{j'1} / AY_{ij'1}^j.$$

Therefore in terms of composite commodities, (5.3.1) can also be expressed as

$$(5.3.4) \quad \sum_{j \in L(i)} [PY_{j1}] [Y_{ij1}] = \sum_{j=1}^{nn(i)} [PYY_{ij1}] [YY_{ij1}].$$

Given  $Z_i$ , the decision problem of industry  $i$  is to choose a combination of composite commodity  $(YY_{ij1})_{j=1}^{nn(i)}$  to maximize (5.3.4) subject to (3.8). We have the following first-order conditions

$$*(5.3.5) \quad PYY_{ij1} - \lambda D_j CRETH \left\{ \left( [YY_{ij1}] [AYY_{ij1}] \right)_{j=1}^{nn(i)}, \left( [\alpha YY_{ij1}], [\beta YY_{ij1}] \right)_{j=1}^{nn(i)}, [\kappa YY_i], h_i \right\} = 0$$

j=1, ..., nn(i),

$$*(5.3.6) \quad \frac{Z_i}{AYYZ_i} - CRETH \left\{ \left( [YY_{ij1}] [AYY_{ij1}] \right)_{j=1}^{nn(i)}, \left( [\alpha YY_{ij1}], [\beta YY_{ij1}] \right)_{j=1}^{nn(i)}, [\kappa YY_i], h_i \right\} = 0$$

In (5.3.5),  $\lambda$  is the Lagrange multiplier and  $D_j CRETH$  denotes the partial derivative of  $CRETH$  with respect to the  $j^{\text{th}}$  variable. The following comparative static results are derived from (5.3.2) and (5.3.3)

$$\begin{aligned} \hat{Y}_{ij1} &= \hat{Z}_i - A\hat{Y}_{ij1} - A\hat{Y}YZ_i \\ &+ [\sigma YY_{ij1}] \left( [P\hat{Y}_{ij1}] - \sum_{j'=1}^{nn(i)} [S\tilde{Y}_{ij'1}] [P\hat{Y}_{ij'1}] \right) \\ &- [\sigma YY_{ij1}] \left( [A\hat{Y}_{ij1}] - \sum_{j'=1}^{nn(i)} [S\tilde{Y}_{ij'1}] [A\hat{Y}_{ij'1}] \right) \end{aligned}$$

for  $i=1, \dots, iic$ , and  $j=1, \dots, nn(i)$ .

$$\hat{Y}_{ij'1} = \hat{Y}_{ij1} - \hat{A}Y_{ij'1}^j, \quad \text{for } j' \in L(i, j), j=1, \dots, nn(i).$$

$$P\hat{Y}_{ij1} = \sum_{j' \in L(i, j)} [SSY_{ij'1}^j] [P\hat{Y}_{j'1}] - \sum_{j' \in L(i, j)} [SSY_{ij'1}^j] [\hat{A}Y_{ij'1}^j]$$

for  $i=1, \dots, iic$ , and  $j=1, \dots, nn(i)$ ,

where

$$(5.3.10) \quad \sigma_{YY_{ij1}} = \frac{1}{\alpha_{YY_{ij1}} - 1}$$

$$(5.3.11) \quad \tilde{S}Y_{ij1} = \frac{[\sigma_{YY_{ij1}}][SYY_{ij1}]}{\sum_{j'=1}^{nn(i)} [\sigma_{YY_{ij'1}}][SYY_{ij'1}]}$$

$$(5.3.12) \quad SYY_{ij1} = \frac{[PYY_{ij1}][YY_{ij1}]}{\sum_{j'=1}^{nn(i)} [PYY_{ij'1}][YY_{ij'1}]}$$

$$(5.3.13) \quad SSY_{ij'1}^j = \frac{PY_{j'1} / AY_{ij'1}^j}{\sum_{k \in L(i,j)} PY_{k1} / AY_{ik1}^j} \quad \text{for } j=1, \dots, nn(i), j' \in L(i,j).$$

#### 4. Demands for Margins

Recall that  $X_{ijs}$  denotes the input demand for good  $j$  from source  $s$  by industry  $i$ . We suppose that the delivery of  $X_{ijs}$  to industry  $i$  requires various quantities of the commodities produced domestically and is described by the following equations

$$*(5.4.1) \quad MX_{(ijs,j'1)} = AMX_{(ijs,j'1)}[X_{ijs}],$$

$$\text{for } i=1, \dots, iic, j'=1, \dots, jj, j=1, \dots, jj, \text{ and } s=1,2.$$

Here,  $MX_{(ijs,j'1)}$  represents the total amount of good  $j'$  produced domestically needed in the delivery of  $X_{ijs}$  to industry  $i$  and  $AMX_{(ijs,j'1)}$  is a technological coefficient.

The above equation yields the following comparative static result

$$^{\wedge}(5.4.2) \quad \hat{M}X_{ijs,j'1} = \hat{A}M X_{ijs,j'1} + \hat{X}_{ijs}, \text{ for } i = 1, \dots, iic, j, j' = 1, \dots, jj, s = 1, 2.$$

For industries whose production involves the produced commodities, we have the following relationship between the basic values and the prices they actually pay for intermediate inputs

$$*(5.4.3) \quad PX_{ijs} = PY_{js} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMX_{ijs,j'1}] + HX_{ijs},$$

for  $i=1, \dots, iic$ ,  $j=1, \dots, jj$ , and  $s=1,2$ .

The left side of (5.4.3) is the price of 1 unit of good  $js$  used as an input by industry  $i$ . The first term on the right side of (5.4.3) is the basic value of 1 unit of good  $js$  on the market; the second term gives the margins required for delivering 1 unit of good  $js$  from the market place to industry  $i$ 's location. The third term,  $HX_{ijs}$ , represents the tax (or subsidy) that industry  $i$  has to pay on the market when it purchases 1 unit of good  $js$ . This tax is a sales tax, not a production tax.

Tax Equations: We assume that the sales taxes paid by domestic producers for intermediate inputs  $js$  are described by the following equation

$$*(5.4.4) \quad HX_{ijs} = \left( [\bar{HX}_{ijs}] [CPI] \right)^{[\alpha 1HX_{ijs}]} \left( [TX_{ijs}] [PY_{js}] \right)^{[\alpha 2HX_{ijs}]} (VX_{ijs})^{[\alpha 3HX_{ijs}]}$$

for  $i = 1, \dots, iic$ ,  $j = 1, \dots, jj$ ,  $s = 1,2$ .

where

$\bar{HX}_{ijs}$ ,  $\alpha 1HX_{ijs}$ ,  $\alpha 2HX_{ijs}$ ,  $\alpha 3HX_{ijs}$  are parameters;  $TX_{ijs}$  and  $VX_{ijs}$  are ad valorem and specific tax rates, respectively. CPI is the consumer price index.

Equation (5.4.4) yields the following comparative static result

$$^*(5.4.5) \quad \hat{HX}_{ijs} = [\alpha 1HX_{ijs}] [\hat{CPI}] + [\alpha 2HX_{ijs}] (\hat{TX}_{ijs} + \hat{PY}_{js}) + [\alpha 3HX_{ijs}] \hat{VX}_{ijs}, \quad i=1, \dots, iic, j=1, \dots, jj, s=1,2.$$

Equation (5.4.3) gives us the following comparative static result

$$\begin{aligned}
\hat{(5.4.6)} \quad [\hat{P}X_{ijs}] &= [W1PX_{ijs}] [\hat{P}Y_{js}] + [W2PX_{ijs}] [\hat{H}X_{ijs}] \\
&+ [W3PX_{ijs}] \left( \sum_{j'=1}^{jj} [W3PX_{(ijs,j'1)}] [PY_{j'1}] \right) \\
&+ [W3PX_{ijs}] \left( \sum_{j'=1}^{jj} [W3PX_{ijs,j'1}] [AMX_{(ijs,j'1)}] \right) \\
& \qquad \qquad \qquad i=1,\dots,ic, j=1,\dots,ij, s=1,2.
\end{aligned}$$

where the weights are defined as follows:

$$\begin{aligned}
[W1PX_{ijs}] &= \frac{PY_{js}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMX_{(ijs,j'1)}]} \\
[W2PX_{ijs}] &= \frac{HX_{ijs}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMX_{(ijs,j'1)}]} \\
[W3PX_{ijs}] &= 1 - [W1PX_{ijs}] - [W2PX_{ijs}] \\
[W3PX_{(ijs,j'1)}] &= \frac{[PY_{j'1}] [AMX_{(ijs,j'1)}]}{\sum_{k=1}^{jj} [PY_{k1}] [AMX_{(ijs,k1)}]}
\end{aligned}$$

## CHAPTER 6

### PROFIT MAXIMIZATION: THE IMPERFECTLY COMPETITIVE INDUSTRIES

#### 1. Introduction

The theory of competitive general equilibrium was fully developed and systematized in the 1950's; see, for example Arrow and Debreu (1954), Debreu (1959). The first attempt to introduce imperfect competition into general equilibrium was made by Negishi (1961). Since then a large number of contributions have appeared, but a satisfactory theory of general equilibrium under imperfect competition is yet to be formulated. For a review of the literature, we refer the reader to Hart (1985), Gary-Bobo (1989), and Giacomo Bonanno (1990).

Two main approaches have been taken in the study of imperfect competition in general equilibrium. One may be called the subjective (or perceived) demand approach; the other, for purpose of contrast, is referred to as the objective approach.

The objective approach begins with the hypothesis that the interdependence of prices and quantities in the economy are objectively given and that the imperfectly competitive firms have perfect information concerning the demands for their products. However, it is not a trivial matter to formalize and derive, in a general equilibrium context where everything depends on everything else, the functions that reflect the objective interdependencies and write down the true demand functions faced by a monopolist or a group of oligopolists. Among the research carried out under the objective approach, we

might mention Gabszewicz and Vial (1972), Fitzroy (1974), Laffont and Laroque (1973, 1976), Marschak and Selten (1974,1977), Nikaido (1975), Roberts and Sonnenschein (1977), Hart, op cit, Benassy (1975, 1988), Dierker and Grodal (1986), Heller (1986).

The subjective approach was developed in a seminal paper by Negishi, op cit; see also Negishi (1972). Refinements to Negishi's approach have been made by researchers, such as Arrow and Hahn (1971), Benassy (1976, 1989), Bonanno (1990), Silvestre (1977a, 1977b, 1978). The subjective approach rejects the idea that imperfectly competitive firms have perfect knowledge about the demands for their products. This approach rests upon the hypothesis that these firms have a subjective perception, perhaps erroneous, of the demands curves they face. These firms might attempt to estimate econometrically their demand curves from available data or historical observations. The estimated functions then represent more or less a good approximation of the real phenomena in the economy, and the partial ignorance of these phenomena might cause these firms to adopt behavioral patterns that do not appear in an objective universe. Subject to these perceived demand curves, imperfectly competitive firms will choose the production levels that maximize their perceived profits. The subjective theories held by these firms are not allowed to be completely arbitrary. The perceived demand curves must satisfy some minimal consistency requirements. First, they must be free from money illusion. Second, they must not be contradicted by observed signals, namely the objective phenomena in the economy.

Due to the theoretical difficulties as well as the computational complexities that must be surmounted to compute an equilibrium according to the objective approach, the subjective approach will be adopted in this thesis.

## 2. Demand Curves

Recall that the industries indexed  $i=[iic]+1, \dots, ii$  are oligopolistic and that each of them produces only one commodity. Hence without any loss in generality, we can suppose that the industries  $i, i=[iic]+1, \dots, ii$ , produce, respectively, the commodities  $j=[jic]+1, \dots, jj$ .

Now for each  $i=[iic]+1, \dots, ii$ , let  $F(i,1)$  and  $F(i,2)$  denote, respectively, the set of domestic producers and foreign producers that constitute this oligopolistic industry. The union of  $F(i,1)$  and  $F(i,2)$ , say  $F(i)$ , thus represents all the producers in this industry. We shall use the superscript  $f$  to index a producer in a particular industry. The oligopolistic industry that firm  $f$  belongs to can be identified, as usual, by the subscript  $i$ . We shall assume that producer in an oligopolistic industry competes in two segregated markets -- one at home and one abroad. Also, in modeling imperfect competition on markets for commodities  $j=[jic]+1, \dots, jj$ , we shall adopt the Cournot approach. Hence we shall assume that goods  $j_1$  and  $j_2$  are perfect substitutes and the firms selling these goods compete by using quantity strategies. We would like to emphasize that the imperfectly competitive producers in our model only have influence over the prices of their outputs, not over the prices of the inputs they buy. That is, all the firms are price takers when buying inputs needed for their production process. The model thus rules out monopsony.

For each  $j=jc+1, \dots, jj$ , let

$$*(6.2.1) \quad PQ_{j2} = \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}}$$

be the inverse demand curve for commodity  $j$  on the foreign market, assumed to be objectively known by all the firms – domestic or foreign – that supply this commodity on this market. Here  $\Psi_{j2}$  is a positive constant,  $Q_{j2}$  is the quantity demanded, and  $\varepsilon_{j2}$  is a positive constant strictly less than 1. Also,  $PQ_{j2}$  is the unit price expressed in foreign currency.

On the other hand, the firms only have conjectures about the price of good  $j$  on the domestic market. We assume that each firm  $f \in F(i)$  has a subjective theory about the relationship between price and quantity on this market that is captured under the form of a perceived inverse demand curve.

More precisely, we assume that the inverse demand curve perceived by a domestic firm  $f$  is given by

$$*(6.2.2) \quad PY_{j1}^f = \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f}, \quad f \in F(i, 1).$$

Here  $PY_{j1}^f$  is the price that the firm thinks will prevail on the market and  $Q_{j1}$  is the quantity demanded at this price. The subjective beliefs of the firm are captured by  $\Psi_{j1}^f$ , the position parameter, and  $\varepsilon_{j1}^f$ , the inverse of the elasticity of demand. For each possible value of  $(\Psi_{j1}^f, \varepsilon_{j1}^f)$ ,  $0 \leq \Psi_{j1}^f < \infty$ ,  $0 \leq \varepsilon_{j1}^f < 1$ , equation (6.2.2) gives a possible perceived demand curve for firm  $f$ . Thus when  $\Psi_{j1}^f$  varies inside the interval  $[0, \infty)$  and  $\varepsilon_{j1}^f$  varies inside the interval  $[0, 1)$ , equation (6.2.2) yields all the possible perceived demand curves for this firm.

The specification represented by (6.2.2) is based upon Gary-Bobo, op cit, (Chapter 4) and Silvestre (1977a), who have extended the seminal contributions of Negishi (1961,1972). Given (6.2.1) and (6.2.2), the domestic firm will choose a production plan that maximizes its total profits over the two markets, taking into consideration the production plans of the other oligopolists. In general equilibrium, the perceived value of  $(\Psi_{j1}^f, \varepsilon_{j1}^f)$ , say  $([\Psi_{j1}^f]^*, [\varepsilon_{j1}^f]^*)$ , must be such that the production plans chosen by this domestic firm together with the actions of all the other economic agents are mutually consistent in the sense that there is equality between supply and demand on all markets.

The perceived demand curve depicted in (6.2.2) is parametrized by two parameters  $\Psi_{j1}^f$  and  $\varepsilon_{j1}^f$ . A much simpler parametrization of this demand curve can be obtained by supposing that  $\varepsilon_{j1}^f$  is given a priori. In this case, the perceived demand curve is parametrized by only the position parameter, and it is this simple form of parametrization we adopt in this thesis. We observe that when  $\varepsilon_{j1}^f=0$ , the perceived demand curve is perfectly horizontal and we have the special case where the firm takes the price of its product as given, i.e., it is a perfectly competitive firm on the domestic market.

We shall assume that a version of (6.2.2) also holds for each foreign firm, i.e.,

$$*(6.2.3) \quad PY_{j1}^f = \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f}, \quad f \in F(i,2).$$

### 3. Profit Maximization: Domestic Imperfectly Competitive Firms

Let us now consider a particular commodity  $j$  that is bought and sold under imperfectly competitive conditions. The industry producing this commodity is

$i=j-[jic]+[iic]$ . For each  $f \in F(i,1)$ , let  $X_i^f$  be an input combination used by this firm. Due to the assumption that each imperfectly competitive firm produces only one commodity, the production possibility frontier feasible under  $X_i^f$ , namely  $\Gamma_i(X_i^f)$ , is reduced to a single element, i.e.,  $\Gamma_i(X_i^f) = Y_{ij1}^f$ , the amount of good  $j$  that this firm produces. More precisely, we have the following relation that defines  $Y_{ij1}^f$

$$\frac{Z_i^f}{AYZ_i} = CRETH \left\{ \left( [Y_{ij1}^f] [AY_{ij1}] \right)_{j \in L(i)}, \left( [\alpha Y_{ij1}] \cdot [\beta Y_{ij1}] \right)_{j \in L(i)}, [\kappa Y_i], h_i \right\}$$

and  $Z_i^f$  is the activity level corresponding to the input combination  $X_i^f$ . As explained in Chapter 3, we assume that  $h_i(y) = y^{\theta_i}$ , where  $\theta_i$  is a constant greater than 1. Also, because  $L(i)$  consists exactly of one element, namely commodity  $j1$ , the equation above will assume the following simpler form

$$\frac{\beta Y_{ij1}}{\alpha Y_{ij1}} \left( \frac{[AY_{ij1}] [Y_{ij1}^f]}{(Z_i^f / [AYZ_i])^{\theta_i}} \right)^{[\alpha Y_{ij1}]} - \kappa Y_i = 0,$$

which yields

$$(6.3.1) \quad Y_{ij1}^f = \frac{\zeta_{ij1} [Z_i^f]^{\theta_i}}{AY_{ij1} [AYZ_i]^{\theta_i}}$$

where we define

$$(6.3.2) \quad \zeta_{ij1} = \left( [\kappa Y_i] [\alpha Y_{ij1}] / \beta Y_{ij1} \right)^{\frac{1}{\alpha Y_{ij1}}}.$$

Let  $Y_{ij1,1}^f$  and  $Y_{ij1,2}^f$  be the amounts of good  $j$  that firm  $f$  sells on the domestic and foreign markets, respectively. Also, let  $GG_{ij1}^f$  be the amount procured by the government, then we must have the following relation

$$*(6.3.3) \quad Y_{ij1}^f = Y_{ij1,1}^f + Y_{ij1,2}^f + GG_{ij1}^f$$

Firm  $f$  will choose  $X_i^f$ ,  $Y_{ij1,1}^f$ ,  $Y_{ij1,2}^f$  to maximize its profit subject to its perception of the conditions on the domestic and foreign markets for good  $j$ . More precisely, the decision problem of this firm can be stated as follows:

Find  $Z_i^f$ ,  $Y_{ij1}^f$ ,  $Y_{ij1,1}^f$ ,  $Y_{ij1,2}^f$  to maximize the following total profits

$$(6.3.4) \quad Y_{ij1,1}^f \left( \Psi_{j1}^f \left( \sum_{f' \in F(i,1)} Y_{ij1,1}^{f'} + \sum_{f' \in F(i,2)} Y_{ij2,1}^{f'} \right)^{-\epsilon_{j1}^f} - HY_{j1,1}^1 \right) \\ + \Phi \left[ Y_{ij1,2}^f \right] \left( \Psi_{j2}^f \left( \sum_{f' \in F(i,1)} Y_{ij1,2}^{f'} + \sum_{f' \in F(i,2)} Y_{ij2,2}^{f'} \right)^{-\epsilon_{j2}^f} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) \\ + PGGY_{j1} \left[ GG_{ij1}^f \right] - Z_i^f \left[ UCZ_i \right] - \sum_{j'=1}^j \left[ AMY_{(ij1,1,j')} \right] \left[ PY_{j'1} \right] \left[ Y_{ij1,1}^f \right] \\ - \sum_{j'=1}^j \left[ AMY_{(ij1,2,j')} \right] \left[ PY_{j'1} \right] \left[ Y_{ij1,2}^f \right]$$

subject to (6.3.1) and (6.3.2).

In the above expression for firm  $f$ 's profits, the first term represents its revenues on the domestic markets. Here  $Y_{ij2,1}^{f'}$  represents the sale of a foreign firm  $f'$  on the domestic market. Also,  $HY_{j1,1}^1$  is the specific tax levied by the home government on 1 unit of the  $j^{\text{th}}$  commodity produced at home and sold on the domestic market. Following usual conventions, we allow  $HY_{j1,1}^1$  to be positive or negative, with the interpretation that a negative tax is really a subsidy.

The second term represents the firm's revenues - net of subsidies and tariffs - on the foreign market. Here  $Y_{ij2,2}^f$  represents the sale of a foreign firm  $f$  on the foreign market and  $HY_{j1,2}^1$  and  $HY_{j1,2}^2$  represent, respectively, the taxes levied by the home and foreign government on 1 unit of the  $j^{\text{th}}$  good produced domestically and sold on the foreign market. Usually, it is the case that  $HY_{j1,2}^1 \leq 0$  and  $HY_{j1,2}^2 \geq 0$ , i.e., the home country subsidizes exports while the foreign country imposes tariff on imports. Both  $HY_{j1,2}^1$  and  $HY_{j1,2}^2$  are expressed in foreign currency. Also,  $\Phi$  is exchange rate (the amount of domestic currency needed to buy one unit of foreign currency) used to convert earnings in foreign currency into domestic currency.

The third term represents the revenue obtained from the government procurement. Here  $GG_{ij1}^f$  is the quantity the government wants to purchase from the firm and  $PGGY_{j1}$  is the unit price the government is willing to pay. Both  $GG_{ij1}^f$  and  $PGGY_{j1}$  are fixed by the government, and, therefore, they are considered as exogenous variables by the firm. These variables often constitute the basis of an industrial policy to help a domestic industry move down the learning curve.

The fourth term represents the cost of inputs needed to sustain the activity level  $Z_i^f$ . Here  $UCZ_i$  is the minimum cost of sustaining 1 unit of activity level. Because the technology linking inputs and activity levels exhibits constant returns to scale, and also because all producers are assumed to be price takers on the input markets,  $UCZ_i$  is completely determined once input prices are given. The problem of cost minimization has been fully dealt with in Chapter 5.

The fifth term represents the margin costs involved in transporting the output  $Y_{ij1,1}^f$  to the domestic market, while the last term represents the margin costs involved in transporting the output  $Y_{ij1,2}^f$  to the foreign market. Here  $AMY_{(ij1,1,j'1)}$  is a technological coefficient representing the amount of good  $j'1$ ,  $j'=1, \dots, jj$ , needed in transporting 1 unit of good  $j$  from the producers of industry  $i$  to the domestic market for good  $j$ . The technological coefficient  $AMY_{(ij1,2,j'1)}$  represents the amount of good  $j'1$  needed by a producer of industry  $i$  in transporting a unit of good  $j$  to the foreign market.

Now define

$$*(6.3.5) \quad Q_{j1} = \sum_{f \in F(i,1)} Y_{ij1,1}^f + \sum_{f \in F(i,2)} Y_{ij2,1}^f$$

$$*(6.3.6) \quad Q_{j2} = \sum_{f \in F(i,1)} Y_{ij1,2}^f + \sum_{f \in F(i,2)} Y_{ij2,2}^f$$

Letting  $\lambda$  and  $\mu$  be the Lagrange multipliers associated with (6.3.1) and (6.3.3), we have the following first order conditions for an interior solution to (6.3.4).

$$*(6.3.7) \quad \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j1,1}^1 + Y_{ij1,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{(-\varepsilon_{j1}^f - 1)} - \sum_{j'=1}^{jj} AMY_{(ij1,1,j'1)} [PY_{j'1}] - \mu = 0,$$

$$*(6.3.8) \quad \Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 + Y_{ij1,2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{(-\varepsilon_{j2} - 1)} - \sum_{j'=1}^{jj} AMY_{(ij1,2,j'1)} [PY_{j'1}] - \mu = 0, \right.$$

$$*(6.3.9) \quad -\lambda + \mu = 0,$$

$$*(6.3.10) \quad -UCZ_i + \frac{\lambda [\zeta_{ij1}] [\theta_i [Z_i^f]]^{(\theta_i - 1)}}{AY_{ij1} [AYZ_i]^{\theta_i}} = 0,$$

$$*(6.3.11) \quad \frac{[\zeta_{ij1}][Z_i^f]^{\theta_i}}{AY_{ij1}[AYZ_i]^{\theta_i}} - Y_{ij1}^f = 0,$$

$$*(6.3.12) \quad Y_{ij1}^f - Y_{ij1,1}^f - Y_{ij1,2}^f - GG_{ij1}^f = 0.$$

Multiplying (6.3.10) by  $Z_i^f$ , we obtain the following equation

$$-UCZ_i[Z_i^f] + \frac{\lambda[\zeta_{ij1}][\theta_i][Z_i^f]^{\theta_i}}{AY_{ij1}[AYZ_i]^{\theta_i}} = 0, \quad \text{i.e.,}$$

$$*(6.3.13) \quad \sum_{j=1}^{jj} \sum_{s=1}^2 PX_{ijs} [X_{ijs}^f] + \sum_{t=1}^n PX_{(i,jj+1,1,t)} [X_{(i,jj+1,1,t)}^f] \\ + \sum_{s=2}^3 PX_{(i,jj+1,s)} [X_{(i,jj+1,s)}^f] + PX_{(i,jj+2)} [X_{(i,jj+2)}^f] \\ - \frac{\lambda[\zeta_{ij1}][\theta_i][Z_i^f]^{\theta_i}}{AY_{ij1}[AYZ_i]^{\theta_i}} = 0.$$

In (6.3.13), we have used

$$\left( \left( X_{ijs}^f \right)_{\substack{j=1, \dots, jj \\ s=1, 2}}, \left( X_{(i,jj+1,1,t)}^f \right)_{t=1, \dots, n}, \left( X_{(i,jj+1,s)}^f \right)_{s=2, 3}, \left( X_{(i,jj+2)}^f \right) \right)$$

to denote the cost-minimizing input combination that sustains the activity level  $Z_i^f$ .

#### 4. Comparative Static: Domestic Firms

Equation (6.3.7) gives us immediately the following comparative static result

$$(6.4.1) \quad W1Y_{ij1,1}^f \left( \hat{\Psi}_{j1}^f - \varepsilon_{j1}^f [\hat{Q}_{j1}] \right) - W2Y_{ij1,1}^f [\hat{H}Y_{j1,1}^1] \\ - W3Y_{ij1,1}^f \left( \hat{Y}_{ij1,1}^f + \hat{\Psi}_{j1}^f - (\varepsilon_{j1}^f + 1) [\hat{Q}_{j1}] \right) \\ - W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} [W4Y_{ij1,1,j'}^f] \left( [AM\hat{Y}_{(ij1,1,j')}] + [\hat{P}Y_{j1}] \right) \right) - \hat{\mu} = 0,$$

where we have let

$$(6.4.2) \quad W1Y_{ij,1}^f = \frac{\Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f}}{\left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j1,1}^1 + Y_{ij,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{jj} AMY_{(ij,1,j')} [PY_{j'1}] \right)}$$

$$(6.4.3) \quad W2Y_{ij,1}^f = \frac{HY_{j1,1}^1}{\left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j1,1}^1 + Y_{ij,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{jj} AMY_{(ij,1,j')} [PY_{j'1}] \right)}$$

$$(6.4.4) \quad W3Y_{ij,1}^f = \frac{Y_{ij,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1}]}{\left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j1,1}^1 + Y_{ij,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{jj} AMY_{(ij,1,j')} [PY_{j'1}] \right)}$$

$$(6.4.5) \quad W4Y_{ij,1}^f = \frac{\sum_{j'=1}^{jj} AMY_{(ij,1,j')} [PY_{j'1}]}{\left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j1,1}^1 + Y_{ij,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{jj} AMY_{(ij,1,j')} [PY_{j'1}] \right)}$$

$$(6.4.6) \quad W4Y_{ij,1,j'}^f = \frac{AMY_{(ij,1,j')} [PY_{j'1}]}{\sum_{k=1}^{jj} AMY_{(ij,1,k)} [PY_{k1}]}$$

Equation (6.3.8) gives us immediately the following comparative static result

$$(6.4.7) \quad W1Y_{ij,2}^f (\hat{\Phi} + \hat{\Psi}_{j2} - \varepsilon_{j2} [\hat{Q}_{j2}]) - W2Y_{ij,2}^f (\hat{\Phi} + [\hat{HY}_{j1,2}^1]) \\ - W3Y_{ij,2}^f (\hat{\Phi} + [\hat{HY}_{j1,2}^2]) - W4Y_{ij,2}^f (\hat{Y}_{ij,2}^f + \hat{\Psi}_{j2} - (\varepsilon_{j2} + 1) [\hat{Q}_{j2}] + \hat{\Phi}) \\ - W5Y_{ij,2}^f \left( \sum_{j'=1}^{jj} [W5Y_{ij,2,j'}^f] \left( [ \hat{AMY}_{(ij,2,j')} ] + [ \hat{PY}_{j'1} ] \right) \right) - \hat{\mu} = 0,$$

where we have let

$$(6.4.8) \quad W1Y_{ij1,2}^f = \frac{\Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} \right)}{\left( \Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) \right. \\ \left. + Y_{ij1,2}^f (-\varepsilon_{j2}) \left[ \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}-1} \right] \right) \\ \left. - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'1}] \right) \right)$$

$$(6.4.9) \quad W2Y_{ij1,2}^f = \frac{\Phi [HY_{j1,2}^1]}{\left( \Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) \right. \\ \left. + Y_{ij1,2}^f (-\varepsilon_{j2}) \left[ \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}-1} \right] \right) \\ \left. - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'1}] \right) \right)$$

$$(6.4.10) \quad W3Y_{ij1,2}^f = \frac{\Phi [HY_{j1,2}^2]}{\left( \Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) \right. \\ \left. + Y_{ij1,2}^f (-\varepsilon_{j2}) \left[ \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}-1} \right] \right) \\ \left. - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'1}] \right) \right)$$

$$(6.4.11) \quad W4Y_{ij1,2}^f = \frac{\Phi \left( Y_{ij1,2}^f (\varepsilon_{j2}) \left[ \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}-1} \right] \right)}{\left( \Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) \right. \\ \left. + Y_{ij1,2}^f (-\varepsilon_{j2}) \left[ \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}-1} \right] \right) \\ \left. - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'1}] \right) \right)$$

$$(6.4.12) \quad W5Y_{ij1,2}^f = \frac{\sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'1}]}{\left( \Phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) \right. \\ \left. + Y_{ij1,2}^f (-\varepsilon_{j2}) \left[ \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}-1} \right] \right) \\ \left. - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'1}] \right) \right)$$

$$(6.4.13) \quad W5Y_{ij1,2,j'1}^f = \frac{AMY_{(ij1,2,j'1)}[PY_{j'1}]}{\sum_{k=1}^{jj} AMY_{(ij1,2,k1)}[PY_{k1}]}$$

Equation (6.4.1) can be rearranged as follows

$$(6.4.14) \quad \begin{aligned} & \left( W1Y_{ij1,1}^f - W3Y_{ij1,1}^f \right) \hat{\Psi}_{j1}^f - \left( W1Y_{ij1,1}^f [\varepsilon_{j1}^f] - W3Y_{ij1,1}^f (\varepsilon_{j1}^f - 1) \right) \hat{Q}_{j1} \\ & - W2Y_{ij1,1}^f [\hat{H}Y_{j1,1}^1] - W3Y_{ij1,1}^f [\hat{Y}_{ij1,1}^f] \\ & - W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} \left[ W4Y_{ij1,1,j'1}^f \left[ \hat{A}MY_{(ij1,1,j'1)} \right] \right] \right) \\ & - W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} \left[ W4Y_{ij1,1,j'1}^f \left[ \hat{P}Y_{j'1} \right] \right] \right) - \hat{\mu} = 0, \end{aligned}$$

Equation (6.4.7) can be rearranged as follows

$$(6.4.15) \quad \begin{aligned} & \left( W1Y_{ij1,2}^f - W4Y_{ij1,2}^f \right) \hat{\Psi}_{j2} - \left( W1Y_{ij1,2}^f [\varepsilon_{j2}] - W4Y_{ij1,2}^f (\varepsilon_{j2} + 1) \right) \hat{Q}_{j2} \\ & - W2Y_{ij1,2}^f [\hat{H}Y_{j1,2}^1] - W3Y_{ij1,2}^f [\hat{H}Y_{j1,2}^2] - W4Y_{ij1,2}^f [\hat{Y}_{ij1,2}^f] \\ & + \left( W1Y_{ij1,2}^f - W2Y_{ij1,2}^f - W3Y_{ij1,2}^f - W4Y_{ij1,2}^f \right) [\hat{\Phi}] \\ & - W5Y_{ij1,2}^f \left( \sum_{j'=1}^{jj} \left[ W5Y_{ij1,2,j'1}^f \left[ \hat{A}MY_{(ij1,2,j'1)} \right] \right] \right) \\ & - W5Y_{ij1,2}^f \left( \sum_{j'=1}^{jj} \left[ W5Y_{ij1,2,j'1}^f \left[ \hat{P}Y_{j'1} \right] \right] \right) - \hat{\mu} = 0, \end{aligned}$$

Equation (6.3.13) gives us the following comparative static result

$$(6.4.16) \quad \begin{aligned} & \sum_{j=1}^{jj} \sum_{s=1}^2 WX_{ijs}^f \left( \hat{P}X_{ijs} + \hat{X}_{ijs}^f \right) \\ & + \sum_{t=1}^n WX_{(i,jj+1,1,t)}^f \left( \hat{P}X_{(i,jj+1,1,t)} + \hat{X}_{(i,jj+1,1,t)}^f \right) \\ & + \sum_{s=2}^3 WX_{(i,jj+1,s)}^f \left( \hat{P}X_{(i,jj+1,s)} + \hat{X}_{(i,jj+1,s)}^f \right) \\ & + WX_{(i,jj+2)}^f \left( \hat{P}X_{(i,jj+2)} + \hat{X}_{(i,jj+2)}^f \right) - \hat{\lambda} - \theta_i [\hat{Z}_i^f] \\ & + \hat{A}Y_{ij1} + \theta_i [A\hat{Y}Z_i] = 0, \end{aligned}$$

where

$$(6.4.17) \quad WX_{ijs}^f = \frac{PX_{ijs} \left[ X_{ijs}^f \right]}{\left( \sum_{j=1}^{ij} \sum_{s=1}^2 PX_{ijs} \left[ X_{ijs}^f \right] + \sum_{t=1}^n PX_{(i,jj+1,t)} \left[ X_{(i,jj+1,t)}^f \right] \right) + \sum_{s=2}^3 PX_{(i,jj+1,s)} \left[ X_{(i,jj+1,s)}^f \right] + PX_{(i,jj+2)} \left[ X_{(i,jj+2)}^f \right]}$$

$$(6.4.18) \quad WX_{(i,jj+1,t)}^f = \frac{PX_{(i,jj+1,t)} \left[ X_{(i,jj+1,t)}^f \right]}{\left( \sum_{j=1}^{ij} \sum_{s=1}^2 PX_{ijs} \left[ X_{ijs}^f \right] + \sum_{t=1}^n PX_{(i,jj+1,t)} \left[ X_{(i,jj+1,t)}^f \right] \right) + \sum_{s=2}^3 PX_{(i,jj+1,s)} \left[ X_{(i,jj+1,s)}^f \right] + PX_{(i,jj+2)} \left[ X_{(i,jj+2)}^f \right]}$$

$$(6.4.19) \quad WX_{(i,jj+1,s)}^f = \frac{PX_{(i,jj+1,s)} \left[ X_{(i,jj+1,s)}^f \right]}{\left( \sum_{j=1}^{ij} \sum_{s=1}^2 PX_{ijs} \left[ X_{ijs}^f \right] + \sum_{t=1}^n PX_{(i,jj+1,t)} \left[ X_{(i,jj+1,t)}^f \right] \right) + \sum_{s=2}^3 PX_{(i,jj+1,s)} \left[ X_{(i,jj+1,s)}^f \right] + PX_{(i,jj+2)} \left[ X_{(i,jj+2)}^f \right]}$$

$$(6.4.20) \quad WX_{(i,jj+2)}^f = \frac{PX_{(i,jj+2)} \left[ X_{(i,jj+2)}^f \right]}{\left( \sum_{j=1}^{ij} \sum_{s=1}^2 PX_{ijs} \left[ X_{ijs}^f \right] + \sum_{t=1}^n PX_{(i,jj+1,t)} \left[ X_{(i,jj+1,t)}^f \right] \right) + \sum_{s=2}^3 PX_{(i,jj+1,s)} \left[ X_{(i,jj+1,s)}^f \right] + PX_{(i,jj+2)} \left[ X_{(i,jj+2)}^f \right]}$$

Now from (6.3.9), we have  $\hat{\lambda} = \hat{\mu}$ . Using this result and (6.4.14), we can rewrite

(6.4.16) as follows

$$\begin{aligned} \wedge(6.4.21) \quad & \sum_{j=1}^{ij} \sum_{s=1}^2 WX_{ijs}^f \left[ \hat{X}_{ijs}^f \right] + \sum_{t=1}^n WX_{(i,jj+1,t)}^f \left[ \hat{X}_{(i,jj+1,t)}^f \right] \\ & + \sum_{s=2}^3 WX_{(i,jj+1,s)}^f \left[ \hat{X}_{(i,jj+1,s)}^f \right] + WX_{(i,jj+2)}^f \left[ \hat{X}_{(i,jj+2)}^f \right] \\ & + \sum_{j=1}^{ij} \sum_{s=1}^2 WX_{ijs}^f \left[ \hat{P}X_{ijs} \right] + \sum_{t=1}^n WX_{(i,jj+1,t)}^f \left[ \hat{P}X_{(i,jj+1,t)} \right] \\ & + \sum_{s=2}^3 WX_{(i,jj+1,s)}^f \left[ \hat{P}X_{(i,jj+1,s)} \right] + WX_{(i,jj+2)}^f \left[ \hat{P}X_{(i,jj+2)} \right] \\ & = \left( W1Y_{ij,1}^f - W3Y_{ij,1}^f \right) \hat{\Psi}_{j1}^f - \left( W1Y_{ij,1}^f \left[ \varepsilon_{j1}^f \right] - W3Y_{ij,1}^f \left( \varepsilon_{j1}^f + 1 \right) \right) \hat{Q}_{j1} \\ & - W2Y_{ij,1}^f \left[ \hat{H}Y_{j1,1}^1 \right] - W3Y_{ij,1}^f \left[ \hat{Y}_{ij,1}^f \right] \end{aligned}$$

$$\begin{aligned}
& - W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} \left[ W4Y_{ij1,1,j'}^f \left[ \hat{A}M\hat{Y}_{(ij1,1,j')} \right] \right] \right) \\
& - W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} \left[ W4Y_{ij1,1,j'}^f \left[ \hat{P}Y_{j'} \right] \right] \right) + \theta_i \left[ \hat{Z}_i^f \right] - \hat{A}Y_{ij1} - \theta_i \left[ A\hat{Y}Z_i \right]
\end{aligned}$$

Similarly, using  $\hat{\lambda} = \hat{\mu}$  and (6.4.15), we can also rewrite (6.4.16) as follows:

$$\begin{aligned}
\wedge(6.4.22) \quad & \sum_{j=1}^{jj} \sum_{s=1}^2 WX_{ijs}^f \left[ \hat{X}_{ijs}^f \right] + \sum_{t=1}^u WX_{(i,jj+1,1,t)}^f \left[ \hat{X}_{(i,jj+1,1,t)}^f \right] \\
& + \sum_{s=2}^3 WX_{(i,jj+1,s)}^f \left[ \hat{X}_{(i,jj+1,s)}^f \right] + WX_{(i,jj+2)}^f \left[ \hat{X}_{(i,jj+2)}^f \right] \\
& + \sum_{j=1}^{jj} \sum_{s=1}^2 WX_{ijs}^f \left[ \hat{P}X_{ijs} \right] + \sum_{t=1}^u WX_{(i,jj+1,1,t)}^f \left[ \hat{P}X_{(i,jj+1,1,t)} \right] \\
& + \sum_{s=2}^3 WX_{(i,jj+1,s)}^f \left[ \hat{P}X_{(i,jj+1,s)} \right] + WX_{(i,jj+2)}^f \left[ \hat{P}X_{(i,jj+2)} \right] \\
& = \left( W1Y_{ij1,2}^f - W4Y_{ij1,2}^f \right) \hat{\Psi}_{j2} - \left( W1Y_{ij1,2}^f \left[ \varepsilon_{j2} \right] - W4Y_{ij1,2}^f \left( \varepsilon_{j2} + 1 \right) \right) \hat{Q}_{j2} \\
& - W2Y_{ij1,2}^f \left[ \hat{H}Y_{j1,2}^1 \right] - W3Y_{ij1,2}^f \left[ \hat{H}Y_{j1,2}^2 \right] - W4Y_{ij1,2}^f \left[ \hat{Y}_{ij1,2}^f \right] \\
& + \left( W1Y_{ij1,2}^f - W2Y_{ij1,2}^f - W3Y_{ij1,2}^f - W4Y_{ij1,2}^f \right) \left[ \hat{\Phi} \right] \\
& - W5Y_{ij1,2}^f \left( \sum_{j'=1}^{jj} \left[ W5Y_{ij1,2,j'}^f \left[ \hat{A}M\hat{Y}_{(ij1,2,j')} \right] \right] \right) \\
& - W5Y_{ij1,2}^f \left( \sum_{j'=1}^{jj} \left[ W5Y_{ij1,2,j'}^f \left[ \hat{P}Y_{j'} \right] \right] \right) + \theta_i \left[ \hat{Z}_i^f \right] - \hat{A}Y_{ij1} - \theta_i \left[ A\hat{Y}Z_i \right].
\end{aligned}$$

Equation (6.3.11) gives us the following comparative static result:

$$\wedge(6.4.23) \quad \theta_i \left[ \hat{Z}_i^f \right] - \hat{A}Y_{ij1} - \hat{Y}_{ij1}^f - \theta_i \left[ A\hat{Y}Z_i \right] = 0.$$

Equation (6.3.12) gives us the following comparative static result:

$$\wedge(6.4.24) \quad \hat{Y}_{ij1}^f - SY_{ij1,1}^f \left[ \hat{Y}_{ij1,1}^f \right] - SY_{ij1,2}^f \left[ \hat{Y}_{ij1,2}^f \right] - SGG_{ij1}^f \left[ \hat{G}G_{ij1}^f \right] = 0,$$

where we have let

$$(6.4.25) \quad SY_{ij1,1}^f = Y_{ij1,1}^f / Y_{ij1}^f$$

$$(6.4.26) \quad SY_{ij1,2}^f = Y_{ij1,2}^f / Y_{ij1}^f$$

$$(6.4.27) \quad SGG_{ij1}^f = GG_{ij1}^f / Y_{ij1}^f$$

Finally, equations (6.3.5) and (6.3.6) give us the following comparative results:

$$\hat{Q}_{j1} - \sum_{f \in F(i,1)} SQY_{ij1,1}^f [\hat{Y}_{ij1,1}^f] - \sum_{f \in F(i,2)} SQY_{ij2,1}^f [\hat{Y}_{ij2,1}^f] = 0,$$

$$\hat{Q}_{j2} - \sum_{f \in F(i,1)} SQY_{ij1,2}^f [\hat{Y}_{ij1,2}^f] - \sum_{f \in F(i,2)} SQY_{ij2,2}^f [\hat{Y}_{ij2,2}^f] = 0,$$

where we have let

$$(6.4.30) \quad SQY_{ij1,1}^f = Y_{ij1,1}^f / Q_{j1} \quad \text{and} \quad SQY_{ij2,1}^f = Y_{ij2,1}^f / Q_{j1}, \quad f \in F(i,1),$$

$$(6.4.31) \quad SQY_{ij1,2}^f = Y_{ij1,2}^f / Q_{j2} \quad \text{and} \quad SQY_{ij2,2}^f = Y_{ij2,2}^f / Q_{j2}, \quad f \in F(i,2).$$

The linear equations in percentage changes (6.4.21) - (6.4.22) summarize the comparative static results concerning the variables associated with the output side, namely  $Z_i^f$ ,  $Y_{ij1}^f$ ,  $Y_{ij1,1}^f$ ,  $Y_{ij1,2}^f$ , and  $G_{ij1}^f$ . The linear equations in percentage changes (6.4.28) and (6.4.29) capture the relationship between the sales of the individual firms and the total supply on the domestic and foreign markets, respectively.

To obtain the complete set of linear equations in percentage changes that characterize the profit-maximizing behavior of firm  $f$ , we must also include those equations involving the input variables, namely  $Z_i^f$  and the input combination  $X_i^f$ . When these exogenous variables change, the optimal values for  $X_{ij}^f$ , also change and we have the following comparative static results

$$\begin{aligned} \hat{X}_{ijs}^f &= \hat{Z}_i^f + \hat{A}X_{ijs} + \hat{A}X_{ij} + \hat{A}\hat{X}Z_i - [\sigma X_{ij}] \left( [\hat{P}X_{ijs}] - \sum_{k=1}^2 [SX_{ijk}^f] [\hat{P}X_{ijk}] \right) \\ &\quad - [\sigma X_{ij}] \left\{ \hat{A}X_{ijs} - \sum_{k=1}^2 [SX_{ijk}^f] [\hat{A}X_{ijk}] \right\} \\ &\quad \text{for } i=ic+1, \dots, ii, \quad j=1, \dots, jj, \quad s=1,2, \quad f \in F(i,1), \end{aligned}$$

where

$$\sigma X_{ij} = \frac{1}{1 - \rho X_{ij}}, \text{ and } SX_{ijs}^f = \frac{[PX_{ijs}][X_{ijs}^f]}{\sum_{k=1}^2 [PX_{ijk}][X_{ijk}^f]}$$

We have obtained the following comparative static results concerning the demand for labor inputs of various skill types:

$$\begin{aligned} \wedge(6.4.33) \quad \hat{X}_{(i,jj+1,t)}^f &= \hat{X}_{(i,jj+1,t)}^f + \hat{A}X_{(i,jj+1,t)} \\ &- \left[ \sigma X_{(i,jj+1,t)} \right] \left\{ \left[ \hat{P}X_{(i,jj+1,t)} \right] - \sum_{k=1}^n \left[ \tilde{S}X_{(i,jj+1,k)}^f \right] \left[ \hat{P}X_{(i,jj+1,k)} \right] \right\} \\ &- \left[ \sigma X_{(i,jj+1,t)} \right] \left\{ \left[ \hat{A}X_{(i,jj+1,t)} \right] - \sum_{k=1}^n \left[ \tilde{S}X_{(i,jj+1,k)}^f \right] \left[ \hat{A}X_{(i,jj+1,k)} \right] \right\} \end{aligned}$$

for  $t=1, \dots, tt$ ,  $i=iic+1, \dots, ii$ ,

where

$$\begin{aligned} \sigma X_{(i,jj+1,t)} &= \frac{1}{1 - \alpha X_{(i,jj+1,t)}} \\ \tilde{S}X_{(i,jj+1,t)}^f &= \frac{[\sigma X_{(i,jj+1,t)}][SX_{(i,jj+1,t)}^f]}{\sum_{k=1}^n [\sigma X_{(i,jj+1,k)}][SX_{(i,jj+1,k)}^f]} \\ SX_{(i,jj+1,t)}^f &= \frac{[PX_{(i,jj+1,t)}][X_{(i,jj+1,t)}^f]}{\sum_{k=1}^n [PX_{(i,jj+1,k)}][X_{(i,jj+1,k)}^f]} \end{aligned}$$

The comparative static results for effective labor inputs, capital, and agricultural land are given by

$$\begin{aligned} \wedge(6.4.34) \quad \hat{X}_{(i,jj+1,s)}^f &= [\hat{A}X_{(i,jj+1,s)}] + [\hat{A}X_{(i,jj+1)}] + [A\hat{X}Z_i] + \hat{Z}_i^f \\ &- \sigma X_{(i,jj+1,s)} \left( \begin{aligned} &\left[ \hat{P}X_{(i,jj+1,s)} \right] - \sum_{k=1}^3 \left[ \tilde{S}X_{(i,jj+1,k)}^f \right] \left[ \hat{P}X_{(i,jj+1,k)} \right] \\ &\left[ \hat{A}X_{(i,jj+1,s)} \right] - \sum_{k=1}^3 \left[ \tilde{S}X_{(i,jj+1,k)}^f \right] \left[ \hat{A}X_{(i,jj+1,k)} \right] \end{aligned} \right) \end{aligned}$$

for  $s=1,2,3$  and  $i=iic+1, \dots, ii$

where

$$\begin{aligned}\sigma X_{(i,jj+1,s)} &= \frac{1}{1 - \alpha X_{(i,jj+1,s)}} \\ \tilde{S}X_{(i,jj+1,s)}^f &= \frac{\left[ \sigma X_{(i,jj+1,s)} \right] \left[ SX_{(i,jj+1,s)}^f \right]}{\sum_{k=1}^3 \left[ \sigma X_{(i,jj+1,k)} \right] \left[ SX_{(i,jj+1,k)}^f \right]} \\ SX_{(i,jj+1,s)}^f &= \frac{\left[ PX_{(i,jj+1,s)} \right] \left[ X_{(i,jj+1,s)}^f \right]}{\sum_{k=1}^3 \left[ SX_{(i,jj+1,k)}^f \right] \left[ X_{(i,jj+1,k)}^f \right]} \\ PX_{(i,jj+1,1)} &= \frac{\sum_{t=1}^n \left[ PX_{(i,jj+1,1,t)} \right] X_{(i,jj+1,1,t)}^f}{X_{(i,jj+1,1,t)}^f}\end{aligned}$$

Also, we obtain the following comparative static result for the price of one unit of effective labor inputs:

$$\begin{aligned}{}^{\wedge}(6.4.35) \quad \hat{P}X_{(i,jj+1,1)} &= \sum_{t=1}^n \left[ SX_{(i,jj+1,1,t)}^f \right] \left[ \hat{P}X_{(i,jj+1,1,t)} \right] \\ &\quad + \sum_{t=1}^n \left[ SX_{(i,jj+1,1,t)}^f \right] \left[ \hat{A}X_{(i,jj+1,1,t)} \right]\end{aligned}$$

Finally, we have the following comparative static result for other cost tickets

$${}^{\wedge}(6.4.36) \quad \hat{X}_{i,jj+2}^f = \hat{Z}_i^f + A\hat{X}Z_i + \hat{A}X_{(i,jj+2)}.$$

If the domestic firm  $f$  chooses to operate only on the domestic market, then  $Y_{ij1,1}^f > 0$  and  $Y_{ij1,2}^f = 0$ . In this case  $Y_{ij1,2}^f$  does not appear in (6.4.29). Furthermore, (6.4.22), which characterize the first-order condition for an interior solution on the foreign market, must also be removed from the set of linear equation in percentage changes that characterize the optimal solution of this firm. Similarly, if firm  $f$  chooses to operate only on the foreign market, then  $Y_{ij1,1}^f = 0$  and  $Y_{ij1,2}^f > 0$ . In this case,  $Y_{ij1,1}^f$  does not appear in

(6.4.28) and (6.4.21) must be removed from the set of linear equations in percentage changes that characterize the optimal solution of the firm.

### 5. Demands for Inputs and Demands for Margins at the Industry Level: Domestic Oligopolistic Industries

Let  $X_{ijs}$  be the total demand for intermediate input  $js$  by all the domestic imperfectly competitive firms in industry  $i$ , then we have

$$*(6.5.1) \quad X_{ijs} = \sum_{f \in F(i,1)} X_{ijs}^f \quad i=ii+1, \dots, ii, \quad j=1, \dots, jj, \quad \text{and } s=1,2.$$

Similarly, their demands for labor, capital, and agricultural land are given, respectively, by

$$*(6.5.2) \quad X_{(i,jj+1,1,t)} = \sum_{f \in F(i,1)} X_{(i,jj+1,1,t)}^f \quad i=ii+1, \dots, ii \quad \text{and } t=1, \dots, tt,$$

$$*(6.5.3) \quad X_{(i,jj+1,s)} = \sum_{f \in F(i,1)} X_{(i,jj+1,s)}^f \quad i=ii+1, \dots, ii \quad \text{and } s=2,3.$$

Also their demand for other cost tickets is given by

$$*(6.5.4) \quad X_{(i,jj+2)} = \sum_{f \in F(i,1)} X_{(i,jj+2)}^f \quad i=ii+1, \dots, ii.$$

The four preceding equations give us the following comparative static results, respectively

$$^{\wedge}(6.5.5) \quad \hat{X}_{ijs} = \sum_{f \in F(i,1)} SQX_{ijs}^f \left[ \hat{X}_{ijs}^f \right] \quad i=ii+1, \dots, ii, \quad j=1, \dots, jj, \quad \text{and } s=1,2.$$

where

$$(6.5.6) \quad SQX_{ijs}^f = \frac{X_{ijs}^f}{X_{ijs}} \quad i=ii+1, \dots, ii, \quad j=1, \dots, jj, \quad s=1,2, \quad \text{and } f \in F(i,1).$$

$$\hat{X}_{(i,jj+1,1,t)} = \sum_{f \in F(i,1)} SQX_{(i,jj+1,1,t)}^f \left[ \hat{X}_{(i,jj+1,1,t)}^f \right], \quad i=iic+1, \dots, ii, t=1, \dots, tt,$$

where

$$SQX_{(i,jj+1,1,t)}^f = \frac{X_{(i,jj+1,1,t)}^f}{X_{(i,jj+1,1,t)}} \quad i=iic+1, \dots, ii, j=1, \dots, jj, s=1,2, \text{ and } f \in F(i,1).$$

$$\hat{X}_{(i,jj+1,s)} = \sum_{f \in F(i,1)} SQX_{(i,jj+1,s)}^f \left[ \hat{X}_{(i,jj+1,s)}^f \right] \quad i=iic+1, \dots, ii \text{ and } s=2,3,$$

where

$$SQX_{(i,jj+1,s)}^f = \frac{X_{(i,jj+1,s)}^f}{X_{(i,jj+1,s)}} \quad i=iic+1, \dots, ii, s=2,3, \text{ and } f \in F(i,1).$$

$$\hat{X}_{(i,jj+2)} = \sum_{f \in F(i,1)} SQX_{(i,jj+2)}^f \left[ \hat{X}_{(i,jj+2)}^f \right] \quad i=iic+1, \dots, ii,$$

where

$$SQX_{(i,jj+2)}^f = \frac{X_{(i,jj+2)}^f}{X_{(i,jj+2)}} \quad i=iic+1, \dots, ii \text{ and } f \in F(i,1).$$

Recall that  $X_{ijs}$  denotes the input demand for good  $j$  from source  $s$  by industry  $i$ .

We suppose that the delivery of  $X_{ijs}$  to industry  $i$  requires various quantities of the commodities produced domestically and is described by the following equations

$$MX_{(ijs,j'1)} = AMX_{(ijs,j'1)} [X_{ijs}] \quad \text{for } j'=1, \dots, jj, j=1, \dots, jj, s=1,2, \text{ and } i=iic+1, \dots, ii.$$

Here,  $MX_{(ijs,j'1)}$  represents the total amount of good  $j'$  produced domestically needed in the delivery of  $X_{ijs}$  to industry  $i$  and  $[AMX_{(ijs,j'1)}]$  is a technological coefficient. The above equation yields the following comparative static result

$$\hat{M}X_{ijs,j'1} = \left[ \hat{A}M X_{ijs,j'1} \right] + \hat{X}_{ijs},$$

for  $i=ic+1, \dots, ii$ ,  $j,j' = 1, \dots, jj$ , and  $s=1,2$ .

For domestic oligopolists, we have the following relationship between the basic values and the prices they actually pay for intermediate inputs

$$PX_{ijs} = PY_{js} + \sum_{j'=1}^{jj} \left[ PY_{j'1} \right] \left[ \hat{A}M X_{ijs,j'1} \right] + HX_{ijs},$$

for  $i=ic+1, \dots, ii$ ,  $j=1, \dots, jj$ , and  $s=1,2$ .

The left side of (6.5.15) is the price of 1 unit of good  $js$  used as an input by industry  $i$ . The first term on the right side of (6.5.15) is the basic value of 1 unit of good  $js$  on the market; the second term gives the margins required for delivering 1 unit of good  $js$  from the market place to industry  $i$ 's location. The third term,  $HX_{ijs}$ , represents the tax (or subsidy) that industry  $i$  has to pay on the market when it purchases 1 unit of good  $js$ . This tax is a sales tax, not a production tax.

Tax Equations: We assume that the sales taxes paid by domestic producers for intermediate inputs  $js$  are described by the following equation

$$HX_{ijs} = \left( \left[ \bar{H}X_{ijs} \right] [CPI] \right)^{\alpha 1HX_{ijs}} \left( \left[ TX_{ijs} \right] \left[ PY_{js} \right] \right)^{\alpha 2HX_{ijs}} \left( VX_{ijs} \right)^{\alpha 3HX_{ijs}}$$

where  $\left[ \bar{H}X_{ijs} \right]$ ,  $\left[ \alpha 1HX_{ijs} \right]$ ,  $\left[ \alpha 2HX_{ijs} \right]$ ,  $\left[ \alpha 3HX_{ijs} \right]$  are parameters;  $TX_{ijs}$  and  $VX_{ijs}$  are ad valorem and specific tax rates, respectively. CPI is the consumer price index.

Equation (6.5.16) yields the following comparative static result

$$\hat{H}X_{ijs} = \left[ \alpha 1HX_{ijs} \right] \left[ \hat{C}PI \right] + \left[ \alpha 2HX_{ijs} \right] \left( \hat{T}X_{ijs} + \hat{P}Y_{js} \right) + \left[ \alpha 3HX_{ijs} \right] \hat{V}X_{ijs}$$

Equation (6.5.15) gives us the following comparative static result

$$\begin{aligned}
 \hat{P}X_{ijs} &= [W1PX_{ijs}] [\hat{P}Y_{js}] + [W2PX_{ijs}] [\hat{H}X_{ijs}] \\
 &+ [W3PX_{ijs}] \left( \sum_{j'=1}^{jj} [W3PX_{(ijs,j'1)}] [\hat{P}Y_{j'1}] \right) \\
 &+ [W3PX_{ijs}] \left( \sum_{j'=1}^{jj} [W3PX_{ijs,j'1}] [\hat{A}MX_{(ijs,j'1)}] \right)
 \end{aligned}$$

$i=ic+1, \dots, ii, j=1, \dots, jj, s=1, 2,$

where the weights are defined as follows:

$$\begin{aligned}
 [W1PX_{ijs}] &= \frac{PY_{js}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMX_{(ijs,j'1)}]} \\
 [W2PX_{ijs}] &= \frac{HX_{ijs}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMX_{(ijs,j'1)}]} \\
 [W3PX_{ijs}] &= 1 - [W1PX_{ijs}] - [W2PX_{ijs}] \\
 [W3PX_{(ijs,j'1)}] &= \frac{[PY_{j'1}] [AMX_{(ijs,j'1)}]}{\sum_{k=1}^{jj} [PY_{k1}] [AMX_{(ijs,k1)}]}
 \end{aligned}$$

The amount of good  $j$  supplied on the domestic market by domestic oligopolists is

$$*(6.5.19) \quad Y_{ij1,1} = \sum_{f \in F(i,1)} Y_{ij1,1}^f, \quad j=jjc+1, \dots, jj, i=j-jjc+ic.$$

The amount of good  $j'1$  required as margins to transport  $Y_{ij1,1}$  to the domestic market is

$$*(6.5.20) \quad MY_{(ij1,1,j'1)} = AMY_{(ij1,1,j'1)} [Y_{ij1,1}], \quad j=jjc+1, \dots, jj, i=j-jjc+ic, j'=1, \dots, jj.$$

The amount of good  $j$  supplied on the foreign market by domestic oligopolists is

$$*(6.5.21) \quad Y_{ij1,2} = \sum_{f \in F(i,1)} Y_{ij1,2}^f, \quad j=jjc+1, \dots, jj, i=j-jjc+ic.$$

The amount of good  $j$ '1 required as margins to transport  $Y_{ij1,2}$  to the foreign market is

$$*(6.5.22) \quad MY_{(ij1,2,j'1)} = AMY_{(ij1,2,j'1)} \left[ Y_{ij1,2} \right], \quad j=jc+1, \dots, jj, \quad i=j-jc+iic, \quad j'=1, \dots, jj.$$

Equations (6.5.19) through (6.5.22) give us the following comparative static results, respectively.

$$^{\wedge}(6.5.23) \quad \hat{Y}_{ij1,1} = \sum_{f \in F(i,1)} WY_{ij1,1}^f \left[ \hat{Y}_{ij1,1}^f \right], \quad j=jc+1, \dots, jj, \quad i=j-jc+iic,$$

where

$$WY_{ij1,1}^f = Y_{ij1,1}^f / Y_{ij1,1}, \quad f \in F(i,1), \quad i=iic+1, \dots, ii, \quad j=i-iic+jjc$$

$$^{\wedge}(6.5.24) \quad \hat{MY}_{(ij1,1,j'1)} = A\hat{MY}_{(ij1,1,j'1)} + \hat{Y}_{ij1,1}, \quad j=jc+1, \dots, jj, \quad i=j-jc+iic, \quad j'=1, \dots, jj.$$

$$^{\wedge}(6.5.25) \quad \hat{Y}_{ij1,2} = \sum_{f \in F(i,1)} WY_{ij1,2}^f \left[ \hat{Y}_{ij1,2}^f \right], \quad j=jc+1, \dots, jj, \quad i=j-jc+iic,$$

where

$$WY_{ij1,2}^f = Y_{ij1,2}^f / Y_{ij1,2}, \quad f \in F(i,1), \quad i=iic+1, \dots, ii, \quad j=i-iic+jjc$$

$$^{\wedge}(6.5.26) \quad \hat{MY}_{(ij1,2,j'1)} = A\hat{MY}_{(ij1,2,j'1)} + \hat{Y}_{ij1,2}, \quad i=iic+1, \dots, ii, \quad j=i-iic+jjc, \quad j'=1, \dots, jj.$$

## 6. Profit Maximization: Foreign Imperfectly Competitive Firms

We shall assume that all the foreign oligopolists in an imperfectly competitive industry possess the same technology of production. This technology is supposed to belong to the same class as that of domestic oligopolists. Unlike the case involving

domestic oligopolists, we shall not consider explicitly the input decisions of foreign oligopolists; our attention will be focused on their output decisions.

Let us consider a foreign oligopolist  $f \in F(i,2)$  and denote by  $Z_i^f$  and  $Y_{ij2}^f$  its chosen activity level and output, respectively. We assume that the following version of (6.3.1) exists between  $Z_i^f$  and  $Y_{ij2}^f$ :

$$(6.6.1) \quad Y_{ij2}^f = \frac{\zeta_{ij2} [Z_i^f]^{\theta_{i2}}}{AY_{ij2} [AYZ_{i2}]^{\theta_{i2}}},$$

where  $\zeta_{ij2}$ ,  $AY_{ij2}$ ,  $AYZ_{i2}$ , and  $\theta_{i2}$  are the counterparts of  $\zeta_{ij1}$ ,  $AY_{ij1}$ ,  $AYZ_i$ , and  $\theta_i$ , respectively.

Let  $Y_{ij2,1}^f$  and  $Y_{ij2,2}^f$  denote the sales of this firm on the domestic and foreign markets, respectively. Also, let  $GG_{ij2,2}^f$  be the part of output procured by the foreign government. Then the following relation must hold:

$$(6.6.2) \quad Y_{ij2}^f - Y_{ij2,1}^f - Y_{ij2,2}^f - GG_{ij2,2}^f = 0.$$

The problem of the foreign oligopolist  $f \in F(i,2)$  is to choose  $Z_i^f$ ,  $Y_{ij2}^f$ ,  $Y_{ij2,1}^f$ ,  $Y_{ij2,2}^f$  to maximize the following total profits, expressed in foreign currency.

$$(6.6.3) \quad \begin{aligned} & Y_{ij2,1}^f \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2 \right) / \Phi \\ & + PGGY_{j2,2} [GG_{ij2,2}^f] + [Y_{ij2,2}^f] \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j2,2}^2 \right) \\ & - Z_i^f [UCZ_{i2}] - \sum_{j'=1}^{jj} [AMY_{(ij2,1,j'2)}] [PY_{j'2}] [Y_{ij2,1}^f] \\ & - \sum_{j'=1}^{jj} [AMY_{(ij2,2,j'2)}] [PY_{j'2}] [Y_{ij2,2}^f] \end{aligned}$$

subject to (6.6.1) and (6.6.2).

In (6.6.3), the first term represents the revenues on the domestic markets. Here  $HY^1_{j2,1}$  and  $HY^2_{j2,1}$  are the specific taxes levied by the home and foreign governments, respectively. Also, the exchange rate is used to convert this earning into foreign currency. The second term represents the revenue made from the foreign government's procurement  $GG^f_{ij2,2}$ . Here the price offered by the foreign government is denoted by  $PGGY_{j2,2}$ , an exogenous variable.

The third term represents the revenue made on the foreign market. Here  $HY^2_{j2,2}$  is the specific tax levied by the foreign government on this firm. The fourth term represents the total cost required to sustain the activity level  $Z^f_i$ . Here  $UCZ_{i2}$  denotes the cost of sustaining 1 unit of activity level. The fifth term represents the margin costs involved in transporting the output  $Y^f_{ij2,1}$  to the domestic market, while the last term represents the margin costs involved in transporting the output  $Y^f_{ij2,2}$  to the foreign market. Observe that  $AMY_{(ij2,1,j'2)}$  and  $AMY_{(ij2,2,j'2)}$  are technological coefficients that represent, respectively, the amount of good  $j'2$ ,  $j'=1, \dots, jj$ , needed to transport 1 unit of good  $j2$  from the foreign producers of industry  $i$  to the domestic and foreign markets.

Letting  $\lambda$  and  $\mu$  be the Lagrange multipliers associated with the constraints (6.6.1) and (6.6.2), we have the following first-order conditions for an interior solution:

$$* (6.6.4) \quad \frac{1}{\Phi} \left( \Psi^f_{j1} [Q_{j1}]^{-\varepsilon^f_{j1}} - HY^1_{j2,1} - HY^2_{j2,1} + Y^f_{ij2,1} (-\varepsilon^f_{j1}) [\Psi^f_{j1}][Q_{j1}]^{-\varepsilon^f_{j1}-1} \right) - \sum_{j'=1}^{jj} AMY_{(ij2,1,j'2)} [PY_{j'2}] - \mu = 0,$$

$$* (6.6.5) \quad \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY^2_{j2,2} + Y^f_{ij2,2} \left( (-\varepsilon_{j2}) [\Psi_{j2}][Q_{j2}]^{-\varepsilon_{j2}-1} \right) - \sum_{j'=1}^{jj} AMY_{(ij2,2,j'2)} [PY_{j'2}] - \mu = 0,$$

$$*(6.6.6) \quad -\lambda + \mu = 0,$$

$$*(6.6.7) \quad -UCZ_{i2} + \frac{\lambda [\zeta_{ij2}] [\theta_{i2}] [Z_i^f]^{(\theta_{i2}-1)}}{AY_{ij2} [AYZ_{i2}]^{\theta_{i2}}} = 0,$$

$$*(6.6.8) \quad \frac{[\zeta_{ij2}] [Z_i^f]^{\theta_{i2}}}{AY_{ij2} [AYZ_{i2}]^{\theta_{i2}}} - Y_{ij2}^f = 0,$$

$$*(6.6.9) \quad Y_{ij2}^f - Y_{ij2,1}^f - Y_{ij2,2}^f - GG_{ij2,2}^f = 0.$$

## 7. Comparative Static: Foreign Firms

Equation (6.6.7) gives us the following comparative static result:

$$(6.7.1) \quad -\hat{UCZ}_{j2} + (\theta_{i2} - 1) [\hat{Z}_i^f] - \hat{AY}_{ij2} - \theta_{i2} [\hat{AYZ}_{i2}] + \hat{\lambda} = 0$$

Now equation (6.6.4) yields immediately the following comparative static result:

$$(6.7.2) \quad W1Y_{ij2,1}^f (\hat{\Psi}_{j1}^f - \varepsilon_{j1}^f [\hat{Q}_{j1}] - \hat{\Phi}) - W2Y_{ij2,1}^f (\hat{HY}_{j2,1}^1 - \hat{\Phi}) \\ - W3Y_{ij2,1}^f (\hat{HY}_{j2,1}^2 - \hat{\Phi}) - W4Y_{ij2,1}^f (\hat{Y}_{ij2,1}^f + \hat{\Psi}_{j1}^f - (\varepsilon_{j1}^f + 1) [\hat{Q}_{j1}] - \hat{\Phi}) \\ - W5Y_{ij2,1}^f \left( \sum_{j'=1}^{jj} [W5Y_{ij2,1,j'}^f] \left( [AM_{Y_{(ij2,1,j'2)}}] + [\hat{PY}_{j'2}] \right) \right) - \hat{\mu} = 0,$$

where we have let

$$(6.7.3) \quad W1Y_{ij2,1}^f = \frac{\left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} \right) / \Phi}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2 \right) + Y_{ij2,1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right) - \sum_{j'=1}^{jj} AM_{Y_{(ij2,1,j'2)}} [PY_{j'2}]}$$

$$(6.7.4) \quad W2Y_{ij2,1}^f = \frac{HY_{j2,1}^1 / \Phi}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2 \right) + Y_{ij2,1}^f (-\varepsilon_{j1}^f) \left[ \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right] - \sum_{j'=1}^{ij} AMY_{(ij2,1,j'2)} [PY_{j'2}] \right)}$$

$$(6.7.5) \quad W3Y_{ij2,1}^f = \frac{HY_{j2,1}^2 / \Phi}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2 \right) + Y_{ij2,1}^f (-\varepsilon_{j1}^f) \left[ \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right] - \sum_{j'=1}^{ij} AMY_{(ij2,1,j'2)} [PY_{j'2}] \right)}$$

$$(6.7.6) \quad W4Y_{ij2,1}^f = \frac{\left( Y_{ij2,1}^f (-\varepsilon_{j1}^f) \left[ \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right] \right)}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2 \right) + Y_{ij2,1}^f (-\varepsilon_{j1}^f) \left[ \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right] - \sum_{j'=1}^{ij} AMY_{(ij2,1,j'2)} [PY_{j'2}] \right)}$$

$$(6.7.7) \quad W5Y_{ij2,1}^f = \frac{\left( \sum_{j'=1}^{ij} AMY_{(ij2,1,j'2)} [PY_{j'2}] \right)}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2 \right) + Y_{ij2,1}^f (-\varepsilon_{j1}^f) \left[ \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right] - \sum_{j'=1}^{ij} AMY_{(ij2,1,j'2)} [PY_{j'2}] \right)}$$

$$(6.7.8) \quad W5Y_{ij2,1,j'2}^f = \frac{\left( AMY_{(ij2,1,j'2)} [PY_{j'2}] \right)}{\left( \sum_{k=1}^{ij} AMY_{(ij2,1,k2)} [PY_{k2}] \right)}$$

Now (6.6.6) implies  $\hat{\lambda} = \hat{\mu}$ , which together with (6.7.2) allow us to rewrite (6.7.1)

as

$$\begin{aligned}
(6.7.9) \quad U\hat{C}Z_{i2} = & \left( W1Y_{ij2,1}^f - W4Y_{ij2,1}^f \right) \hat{\Psi}_{j1}^f - \left( W1Y_{ij2,1}^f \left[ \varepsilon_{j1}^f \right] - W4Y_{ij2,1}^f \left( \varepsilon_{j1}^f + 1 \right) \right) \hat{Q}_{j1} \\
& - W2Y_{ij2,1}^f \left[ \hat{H}Y_{j2,1}^1 \right] - W3Y_{ij2,1}^f \left[ \hat{H}Y_{j2,1}^2 \right] - W4Y_{ij2,1}^f \left[ \hat{Y}_{ij2,1}^f \right] + \left( \theta_{i2} - 1 \right) \hat{Z}_i^f \\
& - \left( W1Y_{ij2,1}^f - W2Y_{ij2,1}^f - W3Y_{ij2,1}^f - W4Y_{ij2,1}^f \right) \hat{\Phi} \\
& - W5Y_{ij2,1}^f \left( \sum_{j'=1}^{jj} \left[ W5Y_{ij2,1,j'}^f \right] \left[ \hat{A}MY_{(ij2,1,j')} \right] \right) \\
& - W5Y_{ij2,1}^f \left( \sum_{j'=1}^{jj} \left[ W5Y_{ij2,1,j'}^f \right] \left[ \hat{P}Y_{j'} \right] \right) - \hat{A}Y_{ij2} - \theta_{i2} \left[ \hat{A}YZ_{i2} \right]
\end{aligned}$$

Equation (6.6.5) gives us immediately the following comparative static result

$$\begin{aligned}
(6.7.10) \quad W1Y_{ij2,2}^f \left( \hat{\Psi}_{j2} - \varepsilon_{j2} \left[ \hat{Q}_{j2} \right] \right) - W2Y_{ij2,2}^f \left[ \hat{H}Y_{j2,2}^2 \right] \\
- W3Y_{ij2,2}^f \left( \hat{Y}_{ij2,2}^f + \hat{\Psi}_{j2} - \left( \varepsilon_{j2}^f + 1 \right) \left[ \hat{Q}_{j2} \right] \right) \\
- W4Y_{ij2,2}^f \left( \sum_{j'=1}^{jj} \left[ W4Y_{ij2,2,j'}^f \right] \left( \left[ \hat{A}MY_{(ij2,2,j')} \right] + \left[ \hat{P}Y_{j'} \right] \right) \right) - \hat{\mu} = 0,
\end{aligned}$$

where we have let

$$(6.7.11) \quad W1Y_{ij2,2}^f = \frac{\left( \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}} \right)}{\left( \left( \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}} + Y_{ij2,2}^f \left( -\varepsilon_{j2} \right) \left[ \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}-1} \right] \right) \right.} \\
\left. - \left( \sum_{j'=1}^{jj} \left[ AMY_{(ij2,2,j')} \right] \left[ PY_{j'} \right] \right) - HY_{j2,2}^2 \right)$$

$$(6.7.12) \quad W2Y_{ij2,2}^f = \frac{-HY_{j2,2}^2}{\left( \left( \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}} + Y_{ij2,2}^f \left( -\varepsilon_{j2} \right) \left[ \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}-1} \right] \right) \right.} \\
\left. - \left( \sum_{j'=1}^{jj} \left[ AMY_{(ij2,2,j')} \right] \left[ PY_{j'} \right] \right) - HY_{j2,2}^2 \right)$$

$$(6.7.13) \quad W3Y_{ij2,2}^f = \frac{Y_{ij2,2}^f \left( -\varepsilon_{j2} \left[ \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}-1} \right] \right)}{\left( \left( \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}} + Y_{ij2,2}^f \left( -\varepsilon_{j2} \right) \left[ \Psi_{j2} \left[ Q_{j2} \right]^{-\varepsilon_{j2}-1} \right] \right) \right.} \\
\left. - \left( \sum_{j'=1}^{jj} \left[ AMY_{(ij2,2,j')} \right] \left[ PY_{j'} \right] \right) - HY_{j2,2}^2 \right)$$

$$(6.7.14) \quad W4Y_{ij2.2}^f = \frac{\left( \sum_{j'=1}^{jj} AMY_{(ij2.2,j'1)} [PY_{j'2}] \right)}{\left( \left( \Psi_{j2} [Q_{j2}] \right)^{-\varepsilon_{j2}} + Y_{ij2.2}^f (-\varepsilon_{j2}) \left[ \Psi_{j2} \left[ [Q_{j2}] \right]^{-\varepsilon_{j2}-1} \right] \right) - \left( \sum_{j'=1}^{jj} AMY_{(ij2.2,j'2)} [PY_{j'2}] \right) - HY_{j2.2}^2}$$

$$(6.7.15) \quad W4Y_{ij2.2,j'2}^f = \frac{AMY_{(ij2.2,j'2)} [PY_{j'2}]}{\sum_{k=1}^{jj} AMY_{(ij2.2,k2)} [PY_{k2}]}$$

Using (6.7.10) and  $\hat{\lambda} = \hat{\mu}$ , we can rewrite (6.7.1) as

$$\begin{aligned} \wedge(6.7.16) \quad \hat{UCZ}_{i2} &= \begin{pmatrix} W1Y_{ij2.2}^f \\ -W3Y_{ij2.2}^f \end{pmatrix} \hat{\Psi}_{j2} - \begin{pmatrix} W1Y_{ij2.2}^f [\varepsilon_{j2}] \\ -W3Y_{ij2.2}^f (\varepsilon_{j2} + 1) \end{pmatrix} \hat{Q}_{j2} \\ &- W2Y_{ij2.2}^f [\hat{HY}_{j2.2}^2] - W3Y_{ij2.2}^f [\hat{Y}_{ij2.2}^f] + (\theta_{i2} - 1) \hat{Z}_i^f \\ &- W4Y_{ij2.2}^f \left( \sum_{j'=1}^{jj} [W4Y_{ij2.2,j'2}^f] [AM\hat{Y}_{(ij2.2,j'2)}] \right) \\ &- W4Y_{ij2.2}^f \left( \sum_{j'=1}^{jj} [W4Y_{ij2.2,j'2}^f] [\hat{PY}_{j'2}] \right) - \hat{AY}_{ij2} - \theta_{i2} [A\hat{YZ}_{i2}] \end{aligned}$$

Equation (6.6.8) gives us immediately the following comparative static result:

$$\wedge(6.7.17) \quad \theta_{i2} [\hat{Z}_i^f] - A\hat{YZ}_{i2} - \hat{Y}_{ij2}^f - \theta_{i2} [A\hat{YZ}_{i2}] = 0.$$

Equation (6.6.9) gives us immediately the following comparative static result:

$$\wedge(6.7.18) \quad \hat{Y}_{ij2}^f - SY_{ij2.1}^f [\hat{Y}_{ij2.1}^f] - SY_{ij2.2}^f [\hat{Y}_{ij2.2}^f] - SGG_{ij2.2}^f [\hat{GG}_{ij2.2}^f] = 0,$$

where we have let

$$(6.7.19) \quad SY_{ij2.1}^f = Y_{ij2.1}^f / Y_{ij2}^f$$

$$(6.7.20) \quad SY_{ij2.2}^f = Y_{ij2.2}^f / Y_{ij2}^f$$

$$(6.7.21) \quad SGG_{ij2.2}^f = GG_{ij2.2}^f / Y_{ij2}^f.$$

Let  $Y_{j2}$  be the supply of good  $j$  on the domestic market by foreign imperfectly competitive firms. Then we have

$$*(6.7.22) \quad Y_{j2} = \sum_{f \in F(j-jjc+iic,2)} Y_{ij2,1}^f, \quad j=jjc+1, \dots, jj, \quad i=j-jjc+iic.$$

Equation (6.7.22) gives us the following comparative static result

$$^{\wedge}(6.7.23) \quad \hat{Y}_{j2} = \sum_{f \in F(j-jjc+iic,2)} [WY_{ij2,1}^f] \hat{Y}_{ij2,1}^f, \quad j=jjc+1, \dots, jj, \quad i=j-jjc+iic.$$

where

$$(6.7.24) \quad WY_{ij2,1}^f = Y_{ij2,1}^f / Y_{j2}, \quad j=jjc+1, \dots, jj, \quad i=j-jjc+iic, \quad f \in F(i,2).$$

## CHAPTER 7

### TECHNOLOGICAL CHANGE: THE LEARNING CURVE

The process of economic growth is driven by three fundamental forces: capital accumulation, a rising labor force, and technological progress. Traditional economic theory, while acknowledging the contribution of technological change, focused mainly on labor and capital -- the capital-labor ratio, to be more precise -- in explaining the rise in income per capita. The empirical studies of Abramovitz (1956) and Solow (1957) have made the overwhelming importance of technological change relative to labor and capital an incontrovertible fact in explaining the process of economic growth.

In this chapter, we introduce technological progress -- under the form of the learning curve -- into applied general equilibrium analysis. Given the difficulties in defining and measuring technological change, and given the present state of empirical research in science and technology, our attempt must necessarily be modest. To put our attempt of introducing technological change into applied general equilibrium in a proper perspective, we provide a general discussion of the process of technological progress in Section 1 of this chapter. In Section 2, we provide our specification of the learning curve. Section 3 explains how the learning curve provides a basis for carrying out industrial policies, especially strategic trade policies under conditions of increasing returns to scale and imperfect competition.

## **1. The Process of Technological Change**

In discussing technological change, economists often distinguish between invention and innovation. Invention, by itself, is not economic in nature; it is the discovery of a principle that enriches knowledge, but the discovery might stay confined in the domain of science without yielding any practical application. Furthermore, invention is characterized by extreme uncertainty and, therefore, is almost impossible to predict *ex ante*. Because of these reasons, invention is often excluded from the agenda of economic research.

Innovation – or more precisely, industrial innovation – on the other hand, is a purely economic application the main objective of which is to create a new production process by using resources in a new fashion. It is important to point out that innovation includes not only techniques of production, but also products, markets, and organizations.

In the recently flourishing field of Industrial Organization, much attention has been focused on the area of industrial innovation. Firms devote resources to R&D activities either to find a new product (product innovation) or a new process (process innovation) that can reduce the cost of producing a known commodity. The search for a new product or a new process is not always successful because the results of innovation R&D activities, like invention, are also fraught with uncertainty, although to a much lesser extent. In deciding whether to embark on an R&D program, a firm must weigh the potential monopoly profits made if the R&D program is successful against its costs.

A successful R&D program yields two distinct types of output. The first type of output involves some product-specific information that enables the firm to manufacture a

particular new good or produce an old good by a cheaper process. The second type of output involves some general technical information that might facilitate subsequent innovations. The product-specific information can be appropriated by the firm either through the protection of a patent or because the product produced from this blueprint is difficult to imitate. The general technical information, on the other hand, cannot be appropriated by the firm simply because intellectual property right is hard to define or enforce. The general technical information thus represents spillovers to the rest of the economy.

Because R&D is an activity that produces knowledge, a public good, imitation by rival firms might erode the potential earnings of an inventor. As a result, it is sometimes difficult to recover the costs of undertaking R&D activities through the functioning of the market mechanism. Therefore, private incentives for R&D have a tendency to result in a socially suboptimal level of R&D. The implication following from this viewpoint is that R&D activities should be subsidized.

On the other hand, there are other factors that tend to make R&D activities socially excessive. Now a firm can reap the fruit of R&D only if it beats its rivals in the development process. Under the rule of winner-takes-all given by the protection of a patent, there are no rewards in coming in second or third. Even when there is no legal protection involved, the rewards of a firm that comes in second are still substantially lower than those enjoyed by the winning firm. In a rush to beat each other in the R&D competition race, each firm will tend to invest more than its rivals. The result is a

duplication of R&D activities, i.e., private incentives cause the level of R&D to be socially excessive.

Furthermore, market structure also plays an important role in determining R&D activities. The incentives for undertaking R&D activities depend on whether the industry in question is a monopoly or competitive and whether the firm is a new entrant or has already established itself in the industry. For a succinct discussion of all the above R&D issues and their implications for industrial policy, we refer the reader to Itoh et al. (1991).

R&D activities lie at the heart of a theory of endogenous technological change formulated by Romer (1986). According to this researcher, a firm produces its output using four factors of production: labor, capital, its own technological knowledge and the economy's aggregate stock of technological knowledge. The economy's aggregate stock of technological knowledge can be considered as the sum of what all the firms and workers in the economy know.

Each firm can engage in R&D activities to accumulate knowledge, just as it can accumulate capital. Knowledge, like capital and labor, is subject to diminishing returns. So firms choose the right amount of knowledge in the same way they choose the right amount of labor and capital to maximize profits. When a firm embarks on an R&D program to accumulate technological knowledge, it generates externalities to the rest of the economy under the form of general technical information. These externalities represent an increase in the economy's aggregate stock of knowledge.

Because the economy's aggregate stock of knowledge enters the production functions of all the firms in the economy, aggregate output will increase proportionately more than the increase in this firm's stock of knowledge. For the economy as a whole, there is increasing returns to knowledge. It is this process of knowledge accumulation that generates endogenously the productivity gains that sustain growth in the long run.

To assess quantitatively the contribution of research to productivity growth, most empirical studies in the literature use a framework that extends the model of Solow (1957). A firm produces an output from three factors of production: labor, capital, and its stock of technological knowledge. For estimation purposes, the firm's production function is often specialized to be Cobb-Douglas, say

$$*(7.1.1) \quad Y_t = Ae^{\lambda t} \left( \sum_i \omega_i R_{t-i} \right)^\alpha K_t^\beta L_t^{1-\beta},$$

where  $Y_t$  is the firm's output in period  $t$ ;  $A$  is a constant;  $\lambda$  is the rate of neutral disembodied technical change;  $R_t$  is the real gross investment in research made in period  $t$ ;  $\omega_i$ ,  $i=0,1,\dots$ , are the coefficients used to capture the distributed-lag effects of past investments in research on the stock of research capital;  $K_t$  and  $L_t$  are the capital and labor inputs used in period  $t$ , respectively; also,  $\alpha$  and  $\beta$  are output elasticities.

Observe that in the above production function, the unobservable stock of technological knowledge is indexed by  $\sum_i \omega_i R_{t-i}$ . Also, constant returns to scale is assumed only with respect to the two traditional factors of production -- labor and capital. For an exposition of this framework and a detailed discussion of its shortcomings, we refer the reader to Griliches (1973). The above production function can easily be extended to

incorporate the externalities generated on this firm by other firms in the economy. All that needs to be done is to incorporate a multiplicative term that represents the economy's aggregate technological knowledge stock into the firm's production function. The assumption of constant returns to scale now applies to the firm's own stock of technological knowledge and the two traditional factors -- labor and capital.

Equations like (7.1.1) have been estimated for agriculture by Griliches (1964) and Evenson (1968); for chemical firms by Minasian (1969); and for selected manufacturing industries by Mansfield (1965).

## **2. The Learning Curve**

It has been observed repeatedly that in manufacturing input requirements per unit of output decreases as cumulative output increases. In a study of the effects of a new technique in steelmaking – continuous casting – Rosegger (1986, p. 87) found that man-hour per ton of output 12 months after commercial start-up were approximately half of what they had been at the outset. In the manufacturing of computer chips, most of the chips produced at the beginning did not work. Initially, usable chips accounted for about 7 to 10 percent of the total number of chips manufactured. This was because in some subtle way the conditions for production were not quite right although the design and the manufacturing process are largely a matter of experimenting with details over time. As the details are worked out, the proportions of usable chips rise sharply. The steadily increasing

performance in steel making and computer chip manufacturing can only be imputed to learning from experience.

The process of learning from experience -- or learning by doing -- is not a recently observed phenomenon. During World War II, it was found that a doubling of the cumulative output of a certain type of airframe resulted in a 15 percent reduction in labor inputs per airframe. A remarkably precise empirical relationship exists between  $N$ , the cumulative output and labor requirements: the labor requirement per airframe is proportional to  $N^{-1/3}$ ; for a full survey, see Asher (1956). During a period of twenty years (1932-52), the Horndall steel mill in Sweden produced steel continuously. Absolutely no changes were made to the plant or equipment; yet the output increased at a steady rate of 2 percent per year. The steady increase in productivity can only be attributed to the cumulative effects of a long and slow learning process.

Although the results of learning by doing are often documented under the form of productivity gain per man-hour, the intermediate input requirements are also observed to follow a decreasing course: there is a reduction in waste with increasing experience. This is clearly demonstrated in the process of computer chip manufacturing where the percentages of defective chips -- as high as 95 percent at the beginning -- decline significantly as experience accumulates. For intermediate inputs, though, the rate of improvement is often much lower than the rate of labor saving that can be realized within the same period of time.

Arrow (1962) advanced the hypothesis that technical change is a vast and prolonged process of learning about the environment in which we operate. Furthermore, learning can only take place through the attempt to solve a problem, and it is the very activity of production that creates problems for which solutions must be worked out. Because learning is a product of experience, the question is to choose the economic variable that represents experience. The economic examples advanced earlier suggest that cumulative output might be a good candidate for representing experience. Arrow, however, argued that cumulative gross investment is a better index of experience and assumed that technical change is completely embodied in new capital goods. Capital goods built at any moment incorporate all the knowledge then available, but once built their productive efficiency cannot be altered by subsequent learning.

Given the structure of the model in this thesis, technological change can be described by the evolution of the technological coefficients through time. One way of introducing endogenous technological change is to introduce R&D activities into the model. Extending the framework used by Griliches (1973), we can specify a functional relationship between investment in research and the technological coefficients. We have chosen to model technological change through the learning curve. Our choice is dictated both by the existence of numerous documentations of learning curves for various industries and the relatively underdeveloped empirical literature on the contribution of R&D for growth accounting.

Now consider a particular industry  $i$ . Let  $SIGMAZ_i(\tau)$  be the cumulative activity level up to period  $\tau$ , i.e., the sum of past activity levels up to time  $\tau$ . If the industry produces a single good, then there is a one-to-one correspondence in each period between output and activity level, and therefore,  $SIGMAZ_i(\tau)$  can be used as an index for cumulative output up to time  $\tau$ . On the other hand, if the industry produces several commodities, a problem might arise concerning how to measure its cumulative output. In this case  $SIGMAZ_i(\tau)$  provides a possible solution to this problem.

For each  $t=1, \dots, tt$ , the technological coefficient associated with labor input of type  $t$  is assumed to evolve through time according to the following equation:

$$*(7.2.1) \quad AX(\tau + 1)_{(i,jj+1,1,t)} = AX(0)_{(i,jj+1,1,t)} \left( SIGMAZ_i(\tau) \right)^{-\eta^X_{(i,jj+1,1,t)}}$$

where

$$SIGMAZ_i(\tau) = SIGMAZ_i(\tau-1) + Z_i(\tau).$$

In percentage change form

$$^{\wedge}(7.2.2) \quad \hat{AX}(\tau + 1)_{(i,jj+1,1,t)} = \hat{AX}(0)_{(i,jj+1,1,t)} - \eta^X_{(i,jj+1,1,t)} \left( \hat{SIGMAZ}_i(\tau) \right), \text{ where}$$

$$^{\wedge}(7.2.3) \quad \hat{SIGMAZ}_i(\tau) = \left[ \frac{SIGMAZ_i(\tau - 1)}{SIGMAZ_i(\tau)} \right] \left[ \hat{SIGMAZ}_i(\tau - 1) \right] \\ + \left[ \frac{Z_i(\tau)}{SIGMAZ_i(\tau)} \right] \left[ \hat{Z}_i(\tau) \right]$$

$\eta^X_{(i,jj+1,1,t)}$  is a parameter called the experience factor (see Fandel (1991), p.207) that is specific to the learning process of labor input of type  $t$  used in industry  $i$ .

For capital and land, we assume the following relation

$$*(7.2.4) \quad AX(\tau + 1)_{(i,jj+1,s)} = AX(0)_{(i,jj+1,s)} \left( SIGMAZ_i(\tau) \right)^{-\eta^X_{(i,jj+1,s)}}, \quad s=2,3.$$

In percentage change form

$$\hat{A}X(\tau + 1)_{(i,jj+1,s)} = \hat{A}X(0)_{(i,jj+1,s)} - \eta X_{(i,jj+1,s)}(\hat{SIGMAZ}_i(\tau)).$$

Here  $\eta X_{(i,jj+1,s)}$ ,  $s=2,3$ , denote, respectively, the experience factors specific to the learning process involving the use of capital and agricultural land in industry  $i$ .

We have decided not to model the technological change involved in the substitution among labor, capital, and land. We have also ignored the technological change involved in combining inputs from various sources to arrive at the effective inputs.

For intermediate input requirements, we assume that learning by doing takes place through the reduction of effective inputs per unit of activity level. More precisely, we assume

$$AX_{ij}(\tau + 1) = AX_{ij}(0)(SIGMAZ_i(\tau))^{-\eta X_{ij}}, \quad j=1, \dots, jj.$$

In percentage change form

$$\hat{A}X(\tau + 1)_{(ij)} = \hat{A}X(0)_{(ij)} - \eta X_{(ij)}(\hat{SIGMAZ}_i(\tau))$$

where  $\eta X_{ij}$  denotes the experience factor associated with effective intermediate input  $j$  used in industry  $i$ .

That the learning functions indeed follow such a course have been documented for aircraft construction by Wright (1936), Asher (1956), Alchian (1963); for shipbuilding by Searle (1945); for machine-tool building by Hirsch (1952,1956); for electronic and electro-mechanical products by Cole (1958), Conway and Schultz (1959); for steel, glass, and paper as well as electrical appliances by Baloff (1966, 1971).

### 3. The Learning Curve and Industrial Policies

Learning by doing -- or dynamic economies of scale -- constitutes the theoretical basis for the infant-industry argument. It is argued that an infant domestic industry,

because of high initial production costs, cannot compete against imports from foreign firms. However, if some forms of protection, such as tariffs, subsidies, or quotas are provided, the infant industry might be able to start production. As time goes on and experience accumulates, production will become more efficient. When the industry has moved down the learning curve sufficiently, the domestic industry might be able to compete without any protection from its own government. When such a time comes, all protective barriers can be dismantled. At this stage the domestic firms might even enter and compete on the international market. In helping the domestic industry move down the learning curve, the home government can use other policy tools, such as procurement or R&D subsidies. If the industry in question exhibits increasing returns to scale and its product is sold on imperfectly competitive markets, helping the domestic industry down its learning curve can be used as a strategic trade policy for shifting monopoly rents from foreign to domestic firms; see, for example, Krugman (1984), Spencer and Brander (1983).

In this thesis, the learning curves specified by (7.2.1), (7.2.2), and (7.2.3) will be used in conjunction with the model of imperfect competition formulated in Chapter 6. The incorporation of the learning curve into Chapter 6 may provide a better simulation result regarding the effects of some of the strategic trade policies mentioned above.

## CHAPTER 8

### CAPITAL INVESTMENT

#### 1. Inputs for Capital Investment

Let us consider capital investment in industry  $i$ . In this industry capital is created by using produced commodities, imported and produced domestically, according to the following technology

$$*(8.1.1) \quad [AI_i][I_i] = Leontief \left\{ \left( \frac{I_{ij}}{AI_{ij}} \right)_{j=1}^{jj} \right\},$$

where  $AI_j$ ,  $AI_{ij}$ ,  $j = 1, \dots, jj$ , are technological coefficients;  $I_i$  is the number of units of capital invested in industry  $i$ ;  $I_{ij}$  is the effective input of the  $j^{\text{th}}$  produced commodity used in capital production.

This effective input is obtained by combining the  $j^{\text{th}}$  produced commodity from both sources -- domestic and imported -- according to the following production function

$$*(8.1.2) \quad I_{ij} = CES \left\{ \left( \frac{I_{ijs}}{AI_{ijs}} \right)_{s=1,2} \left( aI_{ijs} \right)_{s=1,2}, \rho I_{ij} \right\},$$

where  $I_{ijs}$  is the amount of the  $j^{\text{th}}$  produced commodity from source  $s$ ,  $s = 1, 2$ , used by industry  $i$  to create its capital; and  $AI_{ijs}$ ,  $aI_{ijs}$ ,  $\rho I_{ij}$ ,  $j = 1, \dots, jj$ ,  $s = 1, 2$ , are technological coefficients.

For any given level of capital investment  $I_i$ , the producers of capital for industry  $i$  choose a combination of produced commodities ( $I_{ijs}$ ),  $j = 1, \dots, \bar{j}$ ,  $s = 1, 2$ , as inputs into the creation of capital to

$$(8.1.3) \quad \min \sum_{j=1}^{\bar{j}} \sum_{s=1}^2 [PI_{ijs}] [I_{ijs}]$$

subject to (8.1.1) and (8.1.2). Here  $PI_{ijs}$  is the price of intermediate input  $j$  from source  $s$  that producers who create capital for industry  $i$  have to pay.

Formally, problem (8.1.3) has the same structure as that of cost minimization for intermediate inputs that has already been solved in Section 2.1 Chapter 5 and we can immediately write down the comparative static results for (8.1.3) as follows.

$$\begin{aligned} \hat{I}_{ijs} = & \hat{I}_i + [\hat{A}I_{ijs}] + [\hat{A}I_{ij}] + [\hat{A}I_i] \\ & - [\sigma I_{ij}] \left( \left[ \hat{P}I_{ijs} \right] - \sum_{k=1}^2 [SI_{ijk}] \left[ \hat{P}I_{ijk} \right] \right) \\ & \left( \left[ \hat{A}I_{ijs} \right] - \sum_{k=1}^2 [SI_{ijk}] \left[ \hat{A}I_{ijk} \right] \right), \end{aligned}$$

$i=1, \dots, \bar{i}, j=1, \dots, \bar{j}, s=1, 2$

where

$$\sigma I_{ij} = \frac{1}{1 - \rho I_{ij}}, \quad \text{and} \quad SI_{ijs} = \frac{PI_{ijs} [I_{ijs}]}{\sum_{k=1}^2 PI_{ijk} [I_{ijk}]}$$

We also define

$$*(8.1.5) \quad PI_i = \frac{\sum_{j=1}^{\bar{j}} \sum_{s=1}^2 [PI_{ijs}] [I_{ijs}]}{I_i}.$$

As defined,  $PI_i$ , an endogenous variable, can be interpreted as the price of one unit of new capital in industry  $i$ , i.e., the price of one unit of capital currently produced.

## 2. Allocation of Investment Across Industries

In the preceding section, we have determined the optimal input requirements for creating a given number of units of capital, say  $I_i$ , in industry  $i$ . In this section, we explain how private investment is allocated across industries. We do not attempt to explain aggregate private investment. Our view is that aggregate private investment is best explained in a macromodel which captures the effects of monetary phenomena and macroeconomic policies. Consequently, without an appended macromodel, this multisectoral model results must be interpreted as showing the effects of a change in "tariff", etc., under the assumption that macroeconomic policies lead to a given (i.e., exogenous) change in the aggregate level of investment. This model does not reveal what macroeconomic policy leads to the assumed exogenous change in aggregate private investment. The implied assumption is that such a policy does exist.

We divide the  $ii$  industries into two groups. Let us assume that the first group consists of  $ii1$  industries, while the second group consists of  $ii-ii1$  industries. It is the investment in the first group that is treated as endogenous. Without any loss of generality, we shall assume that industries  $1, 2, \dots, ii1$  form the first group whose investment pattern we wish to explain. We shall denote by  $I_1$  the exogenous aggregate investment for the first group and see how  $I_1$ , that we also call total private investment expenditure, is allocated among the first  $ii1$  industries.

Let  $I_i$  be the level of capital investment in industry  $i$ , and recall that  $PI_i$ , defined in Section 1, is the price of one unit of capital currently produced for industry  $i$ . Then we have the following constraint

$$*(8.2.1) \quad I1 = \sum_{i=1}^{ii1} [PI_i][I_i]$$

Equation (8.2.1) describes the investment budget constraint for all the industries whose investment behavior we want to explain. This equation simply states that the sum of investment expenditures across those industries  $i \leq ii1$  is equal to the exogenous aggregate private investment.

For each  $i = 1, \dots, ii$ , let  $K_i$  be the current stock of capital in industry  $i$ . The stock of capital in this industry at the beginning of the next period is given by

$$(8.2.2) \quad K1_i = K_i(1 - d_i) + I_i,$$

where we have denoted by  $K1_i$  the capital stock of industry  $i$  one period from now. Also, we let  $d_i$  be the rate of depreciation for this industry.

Define

$$*(8.2.3) \quad R_i = \frac{PX_{(i,jj+1,2)}}{PI_i} - d_i$$

As defined  $R_i$  is the current net rate of return on capital.

The expected rate of return after one period is assumed to be given by

$$R1_i = [R_i] \left( \frac{K1_i}{K_i} \right)^{-[\alpha K_i]}, \text{ where } \alpha K_i \text{ is a positive parameter.}$$

We assume that the total aggregate private investment  $I_1$  is allocated among the first  $ii_1$  industries so that their rates of return expected in the next period are equalized, i.e.,

$$* (8.2.4) \quad R_1 = [R_i] \left( \frac{K_1}{K_i} \right)^{-[\alpha K_i]}, \quad i = 1, \dots, ii_1,$$

where  $R_1$  is their common expected rate of return in the next period.

In those industries for which the above rate of return theory is not appropriate, we use the following equations:

$$* (8.2.5) \quad I_i = [I_1 R]^{[\alpha I_i]} [SHFTI_i], \quad i > ii_1,$$

$$* (8.2.6) \quad I_1 R = \frac{I_1}{KPI},$$

where  $\alpha I_i$  is a positive parameter,  $SHFTI_i$  is a shift variable,  $I_1 R$  is the real level of aggregate private investment treated endogenously, and  $KPI$  is the capital-goods price index.

From equation (8.2.1), we have

$$(8.2.7) \quad I_1 + \left( \sum_{i=1}^{ii_1} [PI_i][I_i] - I_1 \right) = \left( \sum_{i=1}^{ii_1} [PI_i][I_i] \right) + \left( \sum_{i=1}^{ii_1} [PI_i][I_i] - I_1 \right) \\ = \sum_{i=1}^{ii_1} [PI_i][I_i]$$

Then if we let  $I_2 = \sum_{i=1}^{ii_1} [PI_i][I_i] - I_1$  denote the total investment of all the industries  $i$

for which  $i > ii_1$  then we can rewrite (8.2.7) as

$$* (8.2.8) \quad I_1 + I_2 = \sum_{i=1}^{ii_1} [PI_i][I_i] + I_2 = \sum_{i=1}^{ii_1} [PI_i][I_i],$$

total investment for all the industries in the economy. Using (8.2.8) we obtain the following comparative static result

$$(8.2.9) \quad \left(\frac{I1}{I1 + I2}\right)\hat{I}1 + \left(\frac{I2}{I1 + I2}\right)\hat{I}2 = \sum_{i=1}^{ii1} [SI_i][\hat{P}I_i] + \sum_{i=1}^{ii1} [SI_i][\hat{I}_i] + \left(\frac{I2}{I1 + I2}\right)\hat{I}2$$

where

$$(8.2.10) \quad SI_i = \frac{[PI_i][I_i]}{\sum_{k=1}^{ii} [PI_k][I_k]} = \text{the share of industry } i\text{'s investment in the total}$$

investment. It is clear that  $\left(\frac{I1}{I1 + I2}\right) = \sum_{i=1}^{ii1} [S_i]$ . Hence (8.2.9) can be rewritten as

$$(8.2.11) \quad \left(\sum_{i=1}^{ii1} [SI_i]\right)\hat{I}1 = \sum_{i=1}^{ii1} [SI_i] \left([\hat{P}I_i] + [\hat{I}_i]\right)$$

Equation (8.2.2) through (8.2.6) yield, respectively, the following comparative static results

$$(8.2.12) \quad \hat{K}1_i = \hat{K}_i \left(1 - \frac{I_i}{K1_i}\right) + \left(\frac{I_i}{K1_i}\right)\hat{I}_i, \quad \text{for } i = 1, \dots, ii.$$

$$(8.2.13) \quad \left(\frac{R_i}{R_i + d_i}\right)\hat{R}_i = \hat{P}X_{(i,jj+1,2)} - \hat{P}I_i, \quad \text{for } i = 1, \dots, ii.$$

$$(8.2.14) \quad \hat{R}_i - [\alpha K_i] (\hat{K}1_i - \hat{K}_i) = \hat{R}1$$

$$(8.2.15) \quad \hat{I}_i = [\alpha I_i][\hat{I}1R] + SH\hat{F}T I_i, \quad \text{for } i > ii1$$

$$(8.2.16) \quad \hat{I}1R = \hat{I} - K\hat{P}I$$

### 3. Demand for Margins

For the producers of capital in industry  $i$ , we assume the following demands for margins

$$(8.3.1) \quad MI_{(ijs,j'1)} = AMI_{(ijs,j'1)} [I_{ijs}],$$

where  $I_{ijs}$  is the demand for commodity  $j$  from source  $s$  to be used for capital investment in industry  $i$ ;  $MI_{(ijs,j'1)}$  is the amount of commodity  $j'$  produced domestically required in the delivery of  $I_{ijs}$  to the capital producers for industry  $i$ ; and  $AMI_{(ijs,j'1)}$  is a technological coefficient. We can obtain the following comparative static result from (8.3.1)

$$^*(8.3.2) \quad \hat{M}I_{(ijs,j'1)} = \hat{A}MI_{(ijs,j'1)} + \hat{I}_{ijs}, \quad i = 1, \dots, \bar{i}, j, j' = 1, \dots, \bar{j}, s = 1, 2.$$

For producers of capital goods for industry  $i$ , we have the following equation

$$^*(8.3.3) \quad PI_{ijs} = PY_{js} + HI_{ijs} + \sum_{j'=1}^{\bar{j}} [PY_{j'1}] [AMI_{(ijs,j'1)}],$$

for  $i=1, \dots, \bar{i}$ ,  $j=1, \dots, \bar{j}$ , and  $s = 1, 2$ .

In (8.3.3) the term on the left side represents the actual cost of 1 unit of good  $js$  to be used for capital formation in industry  $i$ . The first term on the right side represents, as in (5.4.3), the basic price on the market of 1 unit of good  $js$ ; the second term represents the taxes per unit of good  $js$  that has to be paid on the market at the time it is bought. The last term represents the cost of transporting 1 unit of good  $js$  back to the location of the producers of capital for industry  $i$ .

For producers of capital goods, we have the following sales tax equation:

$$^*(8.3.4) \quad HS_{ijs} = \left( [\bar{H}S_{ijs}] [CPS] \right)^{[\alpha 1 HI_{ijs}]} \left( [TS_{ijs}] [PY_{js}] \right)^{[\alpha 2 HI_{ijs}]} (VS_{ijs})^{[\alpha 3 HI_{ijs}]},$$

where  $\bar{H}I_{ijs}$ ,  $\alpha 1 HI_{ijs}$ ,  $\alpha 2 HI_{ijs}$ , and  $\alpha 3 HI_{ijs}$ , are parameters;  $TI_{ijs}$  and  $VI_{ijs}$  are ad valorem and specific tax rates, respectively. Also we recall, CPI is the consumer price index.

Equation (8.3.4) yields the following comparative static result

$$\hat{(8.3.5)} \quad \hat{HI}_{ijs} = [\alpha_1 HI_{ijs}] [C\hat{PI}] + [\alpha_2 HI_{ijs}] (\hat{TI}_{ijs} + \hat{PY}_{js}) + [\alpha_3 HI_{ijs}] [\hat{VI}_{ijs}]$$

Equation (8.3.3) yields the following comparative static result

$$\hat{(8.3.6)} \quad \begin{aligned} [\hat{PI}_{ijs}] &= [W_1 PI_{ijs}] [\hat{PY}_{js}] + [W_2 PI_{ijs}] [\hat{HI}_{ijs}] \\ &+ [W_3 PI_{ijs}] \left( \sum_{j'=1}^{jj} [W_3 PI_{(ijs,j'1)}] [\hat{PY}_{j'1}] \right) \\ &+ [W_3 PI_{ijs}] \left( \sum_{j'=1}^{jj} [W_3 PI_{ijs,j'1}] [\hat{AMI}_{(ijs,j'1)}] \right) \end{aligned}$$

where the weights are defined as follows:

$$\begin{aligned} [W_1 PI_{ijs}] &= \frac{PY_{js}}{PY_{js} + HI_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMI_{(ijs,j'1)}]} \\ [W_2 PI_{ijs}] &= \frac{HY_{ijs}}{PY_{js} + HI_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMI_{(ijs,j'1)}]} \\ [W_3 PI_{ijs}] &= 1 - [W_1 PI_{ijs}] - [W_2 PI_{ijs}] \\ [W_3 PI_{(ijs,j'1)}] &= \frac{[PY_{j'1}] [AMI_{(ijs,j'1)}]}{\sum_{k=1}^{jj} [PY_{k1}] [AMI_{(ijs,k1)}]} \end{aligned}$$

## CHAPTER 9

### CONSUMER DEMAND

#### 1. Problem Statement

Let  $C_{js}$ ,  $j=1, \dots, jj$ ,  $s=1,2$ , be the aggregate consumer demand for commodity  $js$  and  $PC_j$  be the price paid by domestic consumers for one unit of this commodity. Let  $M$  denote aggregate expenditure. Then the following aggregate budget constraint must hold:

$$(9.1.1) \quad \sum_{j=1}^{jj} \sum_{s=1,2} [PC_{js}] [C_{js}] = M$$

Because we are not interested in income distribution, we shall assume that domestic consumers behave as if they were a single consumer who chooses a consumption bundle  $(C_{js})_{j=1, \dots, jj, s=1,2}$  to maximize an aggregate utility function subject to the aggregate budget constraint (9.1.1). More precisely, given an aggregate expenditure level  $M$ , we must choose a consumption bundle  $(C_{js})_{j=1, \dots, jj, s=1,2}$  to

$$(9.1.2) \quad \max U \left\{ \left( \frac{C_j}{AC_j} \right)_{j=1}^{jj} \right\}$$

subject to (9.1.1) and (9.1.3), where

$$(9.1.3) \quad C_j = CES \left\{ \left( \frac{C_{js}}{AC_{js}} \right)_{s=1,2}, (aC_{js})_{s=1,2}, \rho C_j \right\} \quad j=1, \dots, jj.$$

Equation (9.1.3) shows how  $C_{js}$ ,  $s=1,2$ , are combined to yield  $C_j$ , the effective consumption of good  $j$ . Here  $AC_{js}$ ,  $aC_{js}$ ,  $\rho C_j$ ,  $j=1, \dots, jj$ ,  $s=1,2$ , are positive coefficients.

These positive coefficients together with the positive coefficients  $AC_j$ ,  $j=1, \dots, jj$ , that appear in (9.1.2) are introduced to allow for changes in tastes.

To solve the utility-maximization defined by (9.1.2), (9.1.1), and (9.1.3), we proceed as follows. Let

$$c_j = \frac{C_j}{AC_j}, \quad j=1, \dots, jj$$

$$c_{js} = \frac{C_{js}}{[AC_{js}][AC_j]}, \quad j=1, \dots, jj, \quad s=1, 2$$

$$p_{js} = [PC_{js}][AC_{js}][AC_j], \quad j=1, \dots, jj, \quad s=1, 2$$

$$a_{js} = aC_{js}, \quad j=1, \dots, jj, \quad s=1, 2$$

$$\rho_j = \rho C_j, \quad j=1, \dots, jj.$$

Then (9.1.2), (9.1.1), and (9.1.3) become

$$(9.1.4) \quad \max U \left\{ \left( c_j \right)_{j=1}^{jj} \right\}$$

subject to

$$(9.1.5) \quad \sum_{j=1}^{jj} \sum_{s=1, 2} [p_{js}][c_{js}] = M$$

$$(9.1.6) \quad c_j = CES \left\{ \left( c_{js} \right)_{s=1, 2}, \left( a_{js} \right)_{s=1, 2}, \rho_j \right\} \quad j=1, \dots, jj.$$

## 2. Solution of the Utility-Maximization Problem

In this section, we solve the utility-maximization problem defined by (9.1.4), (9.1.5), and (9.1.6). Letting  $\lambda$  be the Lagrange multiplier associated with the budget

constraint (9.1.5) and  $\mu_j, j=1, \dots, \bar{j}$ , be the multiplier associated with the constraint (9.1.6),

we have the following first-order conditions:

$$*(9.2.1) \quad D_j U \left\{ \left( c_j \right)_{j=1}^{\bar{j}} \right\} - \mu_j = 0, \quad j=1, \dots, \bar{j},$$

$$*(9.2.2) \quad -\lambda \left[ p_{js} \right] + \mu_j \left( D_s CES \left\{ \left( c_{js} \right)_{s=1,2}, \left( a_{js} \right)_{s=1,2}, \rho_j \right\} \right) = 0, \quad j=1, \dots, \bar{j}, \quad s=1,2$$

$$*(9.2.3) \quad M - \sum_{j=1}^{\bar{j}} \sum_{s=1}^2 \left[ p_{js} \right] \left[ c_{js} \right] = 0$$

$$*(9.2.4) \quad CES \left\{ \left( c_{js} \right)_{s=1,2}, \left( a_{js} \right)_{s=1,2}, \rho_j \right\} - c_j = 0, \quad j=1, \dots, \bar{j}.$$

Now let  $v_j = \mu_j / \lambda$ , then rewrite (9.2.2) as follows

$$*(9.2.5) \quad p_{js} + v_j \left( D_s CES \left\{ \left( c_{js} \right)_{s=1,2} \right\} \right) = 0, \quad j=1, \dots, \bar{j}, \quad s=1,2.$$

Observe that in (9.2.5) we have suppressed the parameters of the CES production function. It is clear from (9.2.5) that  $v_j > 0$ . Now multiplying (9.2.5) by  $c_{js}$ , then summing the results over  $s=1,2$ , we obtain

$$*(9.2.6) \quad \sum_{s=1}^2 p_{js} c_{js} + v_j \sum_{s=1}^2 D_s CES \left\{ \left( c_{js} \right)_{s=1,2} \right\} = v_j c_j, \quad j=1, \dots, \bar{j}.$$

Note that the second equality in (9.2.6) is due to Euler theorem applied to  $CES((c_{js})_{s=1,2})$ .

Summing (9.2.6) over  $j=1, \dots, \bar{j}$ , and using the budget constraint (9.1.5), we obtain

$$*(9.2.7) \quad M = \sum_{j=1}^{\bar{j}} \sum_{s=1}^2 \left[ p_{js} \right] \left[ c_{js} \right] = \sum_{j=1}^{\bar{j}} v_j c_j.$$

Now we observe that (9.2.1) and (9.2.7) together imply that the optimal solution of (9.1.4), (9.1.5), and (9.1.6) is also the solution of the following utility-maximization problem: Choose a combination  $(c_j)_{j=1, \dots, j\bar{j}}$  to

$$(9.2.8) \quad \max U\left\{\left(c_j\right)_{j=1, \dots, j\bar{j}}\right\}$$

subject to the following budget constraint

$$*(9.2.9) \quad M = \sum_{j=1}^{j\bar{j}} v_j c_j.$$

Note that for the maximization problem defined by (9.2.8) and (9.2.9),  $v_j$  can be interpreted as the consumption price of one effective unit of good  $j$ . We shall now exploit the properties of the solution of the problem defined by (9.2.8) and (9.2.9) to characterize the solution of the utility-maximization problem stated in Section 1.

Direct computation shows that  $D_{\text{CES}}((c_{js})_{s=1,2}) = a_{js} \left(\frac{c_{js}}{c_j}\right)^{\rho_j - 1}$ . Using this result

in (9.2.5), we obtain

$$*(9.2.10) \quad p_{js} - v_j a_{js} \left(\frac{c_{js}}{c_j}\right)^{\rho_j - 1} = 0, \quad j=1, \dots, j\bar{j}, \quad s=1,2$$

Equation (9.2.10) gives us immediately the following comparative static result:

$$(9.2.11) \quad \hat{p}_{js} - \hat{v}_j - (\rho_j - 1)(\hat{c}_{js} - \hat{c}_j) = 0.$$

Equation (9.2.4) gives us the following comparative static result:

$$\begin{aligned}
(9.2.12) \quad \hat{c}_j &= \sum_{s=1}^2 \left( D_s CES \left\{ (c_{js})_{s=1,2} \right\} \frac{c_{js}}{c_j} \right) \hat{c}_{js} \\
&= \sum_{s=1}^2 \left( a_{js} \left( \frac{c_{js}}{c_j} \right)^{\rho_j} \right) \hat{c}_{js} \\
&= \sum_{s=1}^2 \left( \frac{p_{js} c_{js}}{v_j c_j} \right) \hat{c}_{js}, \quad \text{by (9.2.10)} \\
&= \sum_{s=1}^2 [SC_{js}] \hat{c}_{js}
\end{aligned}$$

where we have let

$$(9.2.13) \quad SC_{js} = \frac{[PC_{js}][C_{js}]}{[PC_{j1}][C_{j1}] + [PC_{j2}][C_{j2}]} = \frac{p_{js} c_{js}}{p_{j1} c_{j1} + p_{j2} c_{j2}}$$

be the share of good  $j$  from source  $s$  in the total expenditure on good  $j$  from both sources  $s=1$  and  $s=2$ . In deriving the last equality in (9.2.12), we have also used (9.2.5).

If we multiply (9.2.11) by  $SC_{js}$ , then sum the results over  $s=1,2$ , we obtain

$$\sum_{s=1}^2 SC_{js} [\hat{p}_{js}] - \hat{v}_j - (\rho_j - 1) \left( \sum_{s=1}^2 SC_{js} [\hat{c}_{js}] - \hat{c}_j \right) = 0$$

The last term on the left side of this equation is zero by (9.2.12). Hence

$$(9.2.14) \quad \hat{v}_j = \sum_{s=1}^2 SC_{js} [\hat{p}_{js}]$$

Using (9.2.14) in (9.2.11), we obtain

$$\begin{aligned}
&\hat{p}_{js} - \sum_{s'=1}^2 SC_{js'} [\hat{p}_{js'}] - (\rho_j - 1) (\hat{c}_{js} - \hat{c}_j) = 0, \text{ i.e.,} \\
(9.2.15) \quad \hat{c}_{js} &= \hat{c}_j - \sigma_j \left( \hat{p}_{js} - \sum_{s'=1}^2 SC_{js'} [\hat{p}_{js'}] \right),
\end{aligned}$$

where we have let

$$(9.2.16) \quad \sigma_j = \frac{1}{1 - \rho_j}.$$

Using the definition of  $p_{js}$ ,  $c_{js}$ ,  $c_j$ , we have

$$(9.2.17) \quad \begin{aligned} \hat{p}_{js} &= \hat{P}C_{js} + \hat{A}C_{js} + \hat{A}C_j, \\ \hat{c}_{js} &= \hat{C}_{js} - \hat{A}C_{js} - \hat{A}C_j \\ \hat{c}_j &= \hat{C}_j - \hat{A}C_j, \end{aligned}$$

Substituting the values of  $\hat{p}_{js}$ ,  $\hat{c}_{js}$ , and  $\hat{c}_j$ , given by (9.2.17), into (9.2.15), we obtain

$$(9.2.18) \quad \begin{aligned} \hat{C}_{js} - \hat{C}_j - \hat{A}C_{js} + \sigma C_j \left( \hat{P}C_{js} - \sum_{s=1}^2 SC_{js'} \left[ \hat{P}C_{js'} \right] \right) \\ + \sigma C_j \left( \hat{A}C_{js} - \sum_{s=1}^2 SC_{js'} \left[ \hat{A}C_{js'} \right] \right) = 0 \end{aligned}$$

for  $j=1, \dots, \bar{j}$ ,  $s=1, 2$

where we have let

$$(9.2.19) \quad \sigma C_j = \frac{1}{1 - \rho C_j}, \quad j=1, \dots, \bar{j}.$$

Equation (9.2.18) gives the comparative static result for  $C_{js}$ ,  $j=1, \dots, \bar{j}$ ,  $s=1, 2$ . We shall now derive the comparative static result for  $C_j$ ,  $j=1, \dots, \bar{j}$ . To this end, first note that by using the values of  $\hat{p}_{js}$ , given by (9.2.17), in (9.2.14), we have

$$(9.2.20) \quad \begin{aligned} \hat{v}_j &= \sum_{s=1}^2 SC_{js} \left[ \hat{P}C_{js} \right] + \sum_{s=1}^2 SC_{js} \left[ \hat{A}C_{js} \right] + \hat{A}C_j \\ &= \hat{P}C_j + \sum_{s=1}^2 SC_{js} \left[ \hat{A}C_{js} \right] + \hat{A}C_j \end{aligned}$$

where we have let

$$(9.2.21) \quad \hat{P}C_j = \sum_{s=1}^2 SC_{js} \left[ \hat{P}C_{js} \right]$$

Now let us go back to the utility-maximization problem defined by (9.2.8) and (9.2.9). If we write  $c_j=c_j(v_1,\dots,v_{jj},M)$ ,  $j=1,\dots,jj$ , as the solution of this utility-maximization problem, then we obtain the following comparative static result immediately

$$(9.2.22) \quad \hat{c}_j = \left[ D_{jj+1} c_j(v_1, \dots, v_{jj}, M) \left( \frac{M}{c_j} \right) \right] \hat{M} \\ + \sum_{j'=1}^{jj} \left[ D_{j'j} c_j(v_1, \dots, v_{jj}, M) \left( \frac{v_{j'}}{c_j} \right) \right] \hat{v}_{j'} \\ = [\epsilon_{income}_j] \hat{M} + \sum_{j'=1}^{jj} [\epsilon_{price}_{jj'}] \hat{v}_{j'}$$

where we have let  $[\epsilon_{income}_j]$ ,  $j=1,\dots,jj$ , denote the income elasticity of effective good  $j$  and  $[\epsilon_{price}_{jj'}]$ ,  $j,j'=1,\dots,jj$ , denote the cross price elasticity of effective good  $j$  with respect to the price of effective good  $j'$ .

Substituting the value of  $\hat{c}_j$ , given by (9.2.17), and the value of  $\hat{v}_{j'}$ , given by (9.2.20), into (9.2.22), we obtain the following comparative static result for  $C_j$

$$(9.2.23) \quad \hat{C}_j - [\epsilon_{income}_j] [\hat{M}] - \sum_{j'=1}^{jj} [\epsilon_{price}_{jj'}] [\hat{P}_{C_{j'}}] \\ - \sum_{j'=1}^{jj} [\epsilon_{price}_{jj'}] \left( \sum_{s=1}^2 SC_{j's} [\hat{A}C_{j's}] + [\hat{A}C_{j'}] \right) - [\hat{A}C_j] = 0 \\ j=1,\dots,jj$$

### 3. Income and Price Elasticities

In our model, the aggregate utility function is assumed to be of the Klein-Rubin form, say

$$(9.3.1) \quad U(c_1, \dots, c_{jj}) = \sum_{j=1}^{jj} \delta_j \ln(c_j - \bar{c}_j)$$

where  $\delta_j, \bar{c}_j, j=1, \dots, jj$ , are positive parameters with  $\sum_{j=1}^{jj} \delta_j = 1$ . Here  $\bar{c}_j$  represents the subsistence consumption level for effective commodity  $j$ .

If we maximize (9.3.1) under the budget constraint

$$(9.3.2) \quad M - \sum_{j=1}^{jj} v_j c_j = 0,$$

then we obtain easily the following demand functions

$$(9.3.3) \quad c_j = \bar{c}_j + \frac{\delta_j}{v_j} \left( M - \sum_{j'=1}^{jj} v_{j'} \bar{c}_{j'} \right), \quad j=1, \dots, jj.$$

Using (9.3.3), we obtain immediately the following elasticity of income

$$(9.3.4) \quad \epsilon_{income_j} = \frac{\delta_j}{v_j c_j / M} = \frac{\delta_j}{Sc_j}, \quad j=1, \dots, jj,$$

where  $Sc_j$  is the share of good  $j$  in the total aggregate budget  $M$ . Similarly, the price elasticities can be computed directly from (9.3.3), and we have

$$(9.3.5) \quad \epsilon_{price_{jj'}} = -\delta_j \frac{v_{j'} \bar{c}_{j'} / M}{Sc_j}, \quad j, j' = 1, \dots, jj, \quad j \neq j'$$

$$\epsilon_{price_{jj}} = -\epsilon_{income_j} - \sum_{\substack{j'=1 \\ j' \neq j}}^{jj} \epsilon_{price_{jj'}}$$

#### 4. Demand for Margins

For households, we assume the following equations

$$*(9.4.1) \quad MC_{(js, j'1)} = AMC_{(js, j'1)} [C_{js}], \quad \text{for } jj' = 1, \dots, jj, \quad s=1, 2.$$

Here  $C_{js}$  is the amount of good  $j$  from source  $s$  consumed by households;  $MC_{(js,j'1)}$  is the amount of good  $j'$  produced domestically needed to deliver  $C_{js}$  to households; and  $AMC_{(js,j'1)}$  is a technological coefficient.

Equation (9.4.1) yields the following comparative static result

$$\hat{\Delta}(9.4.2) \quad \hat{MC}_{(js,j'1)} = \hat{AMC}_{(js,j'1)} + \hat{C}_{js}, \quad j,j' = 1, \dots, jj, s = 1, 2.$$

Finally, households pay  $PY_{js}$ ,  $j=1, \dots, jj$ ,  $s=1, 2$ , for 1 unit of good  $js$  on the market,  $HC_{js}$  as sales taxes on this unit, and transport this unit home at a cost of

$$\sum_{j'=1}^{jj} [PY_{j'1}] [AMC_{(js,j'1)}].$$

The actual price of 1 unit of good  $js$  for consumption purpose is

$$*(9.4.3) \quad PC_{js} = PY_{js} + HC_{js} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMC_{(js,j'1)}].$$

For consumers, we assume the following sales tax equation:

$$*(9.4.4) \quad HC_{js} = \left( [\bar{HC}_{js}] [CPI] \right)^{[\alpha 1 HC_{js}]} \left( [TC_{js}] [PY_{js}] \right)^{[\alpha 2 HC_{js}]} \left( VC_{js} \right)^{[\alpha 3 HC_{js}]}$$

where  $\bar{HC}_{js}$ ,  $\alpha 1 HC_{js}$ ,  $\alpha 2 HC_{js}$ , and  $\alpha 3 HC_{js}$ , are parameters;  $TC_{js}$  and  $VC_{js}$  are ad valorem and specific tax rates, respectively.

Equation (9.4.4) yields the following comparative static result

$$\hat{\Delta}(9.4.5) \quad \hat{HC}_{js} = [\alpha 1 HC_{js}] [\hat{CPI}] + [\alpha 2 HC_{js}] \left( \hat{TC}_{js} + \hat{PY}_{js} \right) + [\alpha 3 HC_{js}] [\hat{VC}_{js}]$$

Equation (9.4.3) yields the following comparative result

$$\begin{aligned} \hat{(9.4.6)} \quad [\hat{PC}_{js}] &= [W1PC_{js}] [\hat{PY}_{js}] + [W2PC_{js}] [\hat{HC}_{js}] \\ &+ [W3PC_{js}] \left( \sum_{j'=1}^{jj} [W3PC_{(js,j'1)}] [\hat{PY}_{j'1}] \right) \\ &+ [W3PC_{js}] \left( \sum_{j'=1}^{jj} [W3PC_{js,j'1}] [\hat{AMC}_{(js,j'1)}] \right) \end{aligned}$$

where

$$\begin{aligned} [W1PC_{js}] &= \frac{PY_{js}}{PY_{js} + HC_{js} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMC_{(js,j'1)}]} \\ [W2PC_{js}] &= \frac{HY_{ijs}}{PY_{js} + HC_{js} + \sum_{j'=1}^{jj} [PY_{j'1}] [AMC_{(js,j'1)}]} \\ [W3PC_{js}] &= 1 - [W1PC_{js}] - [W2PC_{js}] \\ [W3PC_{(js,j'1)}] &= \frac{[PY_{j'1}] [AMC_{(js,j'1)}]}{\sum_{k=1}^{jj} [PY_{k1}] [AMC_{(js,k1)}]} \end{aligned}$$

## CHAPTER 10

### COMPETITIVE EXPORTS AND IMPORTS

Let  $PE_{j1}$  be the export price, in foreign currency, obtained by exporting 1 unit of competitive commodity  $j$  produced domestically. Here  $PE_{j1}$ ,  $j=1, \dots, j1c$ , is f.o.b. price in foreign currency that includes payments for transport and other margins involved in the delivery of the commodity  $j$  to ports, but it excludes transport cost from the domestic ports to the final destination.

We assume that the foreign demand for the  $j^{\text{th}}$  commodity produced by the home country is given by

$$*(10.1) \quad PE_{j1} = F_{j1}(E_{j1})[SHFTE_{j1}], \quad j=1, \dots, j1c$$

where  $F_{j1}(E_{j1})$  is a non-increasing function of  $E_{j1}$ , the volume of exports of good  $j1$ ; and  $SHFTE_{j1}$  is a shift variable.

Equation (10.1) yields the following comparative static result

$$^{\wedge}(10.2) \quad \hat{P}E_{j1} = -\varepsilon_{j1} \hat{E}_{j1} + SH\hat{F}TE_{j1}, \quad j=1, \dots, j1c$$

where  $\varepsilon_{j1}$  is the reciprocal of the foreign elasticity of demand for the  $j^{\text{th}}$  commodity produced by the home country, i.e.,

$$\varepsilon_{j1} = -DF_{j1}(E_{j1}) \left( \frac{E_{j1}}{[F_{j1}(E_{j1})]} \right)$$

For exports, we assume the following equation for margin demands

$$*(10.3) \quad ME_{(j1,j'1)} = AME_{(j1,j'1)} [E_{j1}],$$

where  $E_{j1}$  is the quantity of good  $j1$  exported;  $ME_{(j1,j'1)}$  is the amount of good  $j'1$  produced domestically required to deliver  $E_{j1}$  from producers to the home country's port; and  $AME_{(j1,j'1)}$  is a technological coefficient. Then from (10.3) we obtain the following comparative static result

$$^(10.4) \quad \hat{ME}_{(j1,j'1)} = \hat{AME}_{(j1,j'1)} + \hat{E}_{j1}, \quad \text{for } j=1, \dots, jjc, \quad j'=1, \dots, jjc.$$

Competitive exporters make zero profits in equilibrium, as indicated by the following relation:

$$*(10.5) \quad [PE_{j1}] \Phi = PY_{j1} + \sum_{j'=1}^{jj} [PY_{j'1}] [AME_{(j1,j'1)}] + HE_{j1}, \quad \text{for } j=1, \dots, jjc.$$

The left side of (10.5) is the f.o.b. price paid by foreigners converted, via the exchange rate  $\Phi$ , to buy one unit of good  $j1$ . The right side represents the total cost of delivering one unit of good  $j1$  on board foreign ships. First, exporters have to buy 1 unit of good  $j1$  on the domestic market at the basic price  $PY_{j1}$  in domestic currency. Next, this 1 unit of good  $j1$  needs to be transported to a domestic port and loaded on a ship; the margins required are represented by the second term on the right side of (10.5). Finally, exporters have to pay export taxes, represented by the last term on the right side of (10.5).

The export tax is assumed to have the following form

$$*(10.6) \quad HE_{j1} = [sign(HE_{j1})] \left( [\bar{HE}_{j1}] [CPI] \right)^{[\alpha1HE_{j1}]} \\ \left( [TE_{j1}] [PE_{j1}] \Phi \right)^{[\alpha2HE_{j1}]} [VE_{j1}]^{[\alpha3HE_{j1}]},$$

Here  $\text{sign}(HE_{j1}) = +1$  in the case of a tax; and  $\text{sign}(HE_{j1}) = -1$  in the case of a subsidy.  $\bar{HE}_{j1}$ ,  $\alpha 1HE_{j1}$ ,  $\alpha 2HE_{j1}$ , and  $\alpha 3HE_{j1}$ , are all parameters;  $TE_{j1}$  is the ad valorem rate of protection; and  $E_{j1}$  is the specific export tax rate.

Importers who are perfectly competitive make zero profits in equilibrium. Let  $PY_{j2}$  be the price, in domestic currency, of one unit of good  $j2$  received by the importers at their domestic country's ports. We assume that

$$(10.7) \quad [PY_{j2}] = [PFY_{j2}] \Phi + [HY_{j2}^1], \quad \text{for } j = 1, \dots, j2c,$$

where  $PFY_{j2}$  is the price, in foreign currency, of one unit of commodity  $j$  produced abroad and delivered at the port (c.i.f.);  $\Phi$  is the exchange rate (the price, in domestic currency, of one unit of foreign currency); and  $HY_{j2}^1$  is the tariff in domestic currency imposed upon 1 unit of imported good  $j2$ .

Observe that the first term on the right side of (10.7) represents the price in domestic currency that importers have to pay foreigners to have one unit of good  $j2$  delivered to the domestic ports. This term plus the tariff importers have to pay, namely  $HY_{j2}^1$ , represent the total cost of importing one unit of good  $j2$ . The term on the left side of (10.7) namely  $PY_{j2}$  is the revenue importers obtain by selling 1 unit of good  $j2$  to the domestic demanders right at domestic ports.

We can model tariffs in a variety of ways and so we add the equation

$$*(10.8) \quad HY_{j2}^1 = \left( [\bar{HY}_{j2}] [CPI] \right)^{[\alpha 1HY_{j2}]} \\ \left( [TY_{j2}] [PFY_{j2}] \Phi \right)^{[\alpha 2HY_{j2}]} [VY_{j2}]^{[\alpha 3HY_{j2}]},$$

for  $j=1, \dots, jic$ ,

where  $\bar{HY}_{j2}$ ,  $\alpha 1HY_{j2}$ ,  $\alpha 2HY_{j2}$ , and  $\alpha 3HY_{j2}$ , are all parameters; CPI is the calculated consumer price index;  $TY_{j2}$  is the ad valorem rate of protection; and  $VY_{j2}$  is the specific rate of protection.

Equations (10.6) and (10.8) give us the following comparative static results, respectively

$$^{\wedge}(10.9) \quad \hat{HE}_{j1} = [\alpha 1HE_{j1}] [\hat{CPI}] + [\alpha 2HE_{j1}] (\hat{TE}_{j1} + \hat{PE}_{j1} + \hat{\Phi}) + [\alpha 3HE_{j1}] [\hat{VE}_{j1}]$$

$$^{\wedge}(10.10) \quad \hat{HY}_{j2}^1 = [\alpha 1HY_{j2}] [\hat{CPI}] + [\alpha 2HY_{j2}] (\hat{TY}_{j2} + \hat{PFY}_{j2} + \hat{\Phi}) + [\alpha 3HY_{j2}] [\hat{VY}_{j2}]$$

## CHAPTER 11

### GOVERNMENT SECTOR

#### 1. Introduction

In a modern mixed economy, state intervention is often called for to remedy the defects of the price system. The government might wish to reduce income and wealth inequality through taxation or cash transfers. Fiscal and monetary policies are conducted with the professed aim of maintaining a high and stable employment and a low level of inflation.

The government also engages in the provision of public goods, such as defense, law and order, which, if left to the market, will not be produced due to lack of private incentive. For roads and bridges, lighting and paving of public streets, which are not public goods, the rationale for collective provision resides in the fact that to charge a user of these facilities through the price mechanism would be disproportionately costly: it is more efficient to finance the construction of highways and bridges through some form of taxation and adopt the policy of no user-charge.

When an economic agent, through his production or consumption activities, generates negative spillover effects on other economic agents, social costs exceed private costs. To correct this form of market failure, the government might choose to regulate the activities of or to impose a tax on this agent. On the other hand, if the spillover effects are positive, the social benefits exceed private benefits of the economic agent. Because this

economic agent cannot reap the benefits of the externalities he showers on others, his activity level tends to be socially suboptimal.

The remedy for this form of market failure is to subsidize the activities that generate the positive externalities. Public expenditure on education and health care is often justified on the basis of the positive externalities these two activities generate for the whole economy.

According to the conventional theory of economic policy, the government is supposed to possess a social welfare function that depends on variables, such as income per capita, rate of economic growth, rate of inflation, distribution of income, etc.. Given a quantitative model of the economy and the behavior of its trading partners, this social welfare function is then maximized.

The solution of this constrained maximization problem yields the optimal intervention policy whose form is determined by the constraints identified in the quantitative model. In applied general equilibrium analysis, government expenditures are often broken down into transfers and real expenditures. The government is treated as an economic agent with its own utility function whose arguments consist exactly of the amounts of public goods and services it buys. Government real expenditures are then determined by maximizing this utility function.

The conventional theory of economic policy is hardly convincing in explaining how governments actually behave in reality. Much more useful as an explanatory tool is the public-choice approach, which is the application of economics to political science.

According to this approach, politicians have their own self interests, and behave in such a manner to further these interests instead of pursuing the objective of maximizing "the common good".

In the economy, there are various interest groups whose fortunes depend on the policies adopted by the government. These interest groups lobby the government by offering campaign contributions or the promise of delivering a block of votes. The utility function of politicians depend on both the number of votes they get and the contributions they can appropriate for their personal use. Politicians then behave as merchants of influence: the policies they adopt are the Nash equilibrium of a noncooperative game in which the players consist of the government and various interest groups. This approach has been proposed by Grossman and Helpman (1993a, 1993b) to explain protection and trade negotiations.

At the present time, modeling government behavior is not a top priority on the research agenda of applied general equilibrium analysis. Most research efforts are directed to more pressing and unresolved issues, such as the introduction of financial markets or explicit formulation of the dynamics of the economy. Because our main interests are in the area of industrial and trade policies, a full treatment of government behavior is beyond the scope of this thesis. In our model, the government's main objective is to choose industrial and trade policies to promote overall economic performance..

## 2. Government Expenditures

Now let  $G_{js}$  be the amount of good  $j$ ,  $j=1, \dots, jj$ , from source  $s$ ,  $s=1,2$ , bought by the government at the prices that prevail on these markets. Recall that in our model commodities  $j_1$  and  $j_2$ ,  $j=jjc+1, \dots, jj$ , are perfect substitutes. In this case  $G_{j_1}$  and  $G_{j_2}$  represent the amounts purchased from the domestic suppliers and the foreign suppliers, respectively.

When there are no domestic oligopolistic firms on the domestic market for good  $j$ ,  $j=jjc+1, \dots, jj$ , it is clear that  $G_{j_1}=0$ . We shall adopt the convention that the government, in filling its total demand  $G_{j_1}+G_{j_2}$ , will go first to the domestic oligopolistic firms. In case the total domestic supply falls short of  $G_{j_1}+G_{j_2}$ , the residual will be met from foreign sources.

In Chapter 6, we have also introduced direct procurement policies for which the government negotiates to buy an amount  $GG_{ij_1}^f$  of good  $j_1$  from domestic oligopolistic firm  $f$  in industry  $i$ ,  $i=iic+1, \dots, ii$ , at the exogenous (negotiated) price  $PGGY_{j_1}$ . Here recall that industry  $i$  produces a single output whose index satisfies the relation  $j-jjc=i-iic$ . If commodity  $j$  involves an industry that the government considers to be important, then a direct procurement program might be a good strategy for nurturing this targeted industry. The industry, with a guaranteed market for part of its output, can survive and reduce its production costs through time, through learning by doing. The economies of time thus realized might benefit the whole economy through interindustry linkages or help the industry to compete more effectively on the world market and thus shift oligopoly rents to the home economy.

The home government can also promote a targeted industry through a number of policies such as import tariff, export subsidies, and technological improvement. Our main task is to simulate these policies and to assess the economic implications of each policy.

Now let

$$*(11.2.1) \quad GG_{j1} = \sum_{f \in F(i,1)} GG_{ij1}^f, \quad j=jjc+1, \dots, jj, \quad i=j-jjc+iic,$$

be the total amount of good  $j$  that the government procures directly from the domestic oligopolists who produce this commodity. Total government expenditures,  $GEXP$ , is given by

$$*(11.2.2) \quad GEXP = \sum_{j=1}^{jj} \sum_{s=1}^2 PG_{js} [G_{js}] + \sum_{j=jjc+1}^{jj} PGG_{j1} [GG_{j1}] + GTRNSFR$$

On the right side of (11.2.2), the first term represents the total cost of purchasing the combination  $(G_{js})_{j=1, \dots, jj, s=1,2}$ ; the second term represents the total cost of the direct procurements  $(GG_{j1})_{j=jjc+1, \dots, jj}$ ; the last term,  $GTRNSFR$ , represents government transfers.

Furthermore,

$$*(11.2.3) \quad PG_{js} = PY_{js} + \sum_{j'=1}^{jj} AMG_{(js,j'1)} [PY_{j'1}], \quad j=1, \dots, jj, \quad s=1,2,$$

$$*(11.2.4) \quad PGG_{j1} = PGGY_{j1} + \sum_{j'=1}^{jj} AMG_{(j1,j'1)} [PY_{j'1}], \quad j=jjc+1, \dots, jj,$$

where  $AMG_{(js,j'1)}$  is a technological coefficient representing the amount of good  $j'1$  required in the delivery of 1 unit of good  $js$  to the government.

For our own use later, we define

$$*(11.2.5) \quad MG_{(js,j'1)} = AMG_{(js,j'1)} [G_{js}], \quad j=1, \dots, jj, \quad s=1,2, \quad j'=1, \dots, jj.$$

As defined,  $MG_{(j^1, j'1)}$  represents the amount of good  $j'1$  required in delivering  $G_j$  to the government. Similarly, we also define

$$*(11.2.6) \quad MGG_{(j^1, j'1)} = AMG_{(j^1, j'1)} \left[ GG_{j^1} \right], \quad j=j^1c+1, \dots, j^1j, j'=1, \dots, j^1j.$$

Equations (11.2.1) through (11.2.6) give us the following comparative static results

$$\wedge(11.2.7) \quad \hat{GG}_{j^1} = \sum_{f \in F(i,1)} WGG_{ij^1}^f \left[ \hat{GG}_{ij^1}^f \right], \quad j=j^1c+1, \dots, j^1j, i=j-j^1c+iic,$$

where

$$(11.2.8) \quad WGG_{ij^1}^f = GG_{ij^1}^f / GG_{j^1}, \quad f \in F(i,1), j=j^1c+1, \dots, j^1j, i=j-j^1c+iic,$$

$$\wedge(11.2.9) \quad \hat{GEXP} = \sum_{j=1}^{j^1j} \sum_{s=1}^2 WEXPG_{js} \left( \hat{PG}_{js} + \hat{G}_{js} \right) \\ + \sum_{j=j^1c+1}^{j^1j} WEXPGG_{j^1} \left( P\hat{GG}_{j^1} + \hat{GG}_{j^1} \right) \\ + [WGTRNSFR] [GTR\hat{NSFR}]$$

where

$$(11.2.10) \quad WEXPG_{js} = PG_{js} \left[ G_{js} \right] / GEXP, \quad j=1, \dots, j^1j, s=1,2$$

$$(11.2.11) \quad WEXPGG_{j^1} = PGG_{j^1} \left[ GG_{j^1} \right] / GEXP, \quad j=j^1c+1, \dots, j^1j$$

$$(11.2.12) \quad WGTRNSFR = GTRNSFR / GEXP$$

$$\wedge(11.2.13) \quad \hat{PG}_{js} = W1PG_{js} \left[ \hat{PY}_{js} \right] + \sum_{j'=1}^{j^1j} W2PG_{(js, j'1)} \left( \hat{AMG}_{(js, j'1)} + \hat{PY}_{j'1} \right) \\ j=1, \dots, j^1j, s=1,2$$

where

$$(11.2.14) \quad W1PG_{js} = PY_{js} / PG_{js}, \quad j=1, \dots, j^1j, s=1,2$$

$$(11.2.15) \quad W2PG_{(js, j'1)} = AMG_{(js, j'1)} \left[ PY_{j'1} \right] / PG_{js}, \quad j=1, \dots, j^1j, s=1,2, j'=1, \dots, j^1j$$

$$\wedge(11.2.16) \quad P\hat{G}G_{j1} = W1PGG_{j1} [P\hat{G}GY_{j1}] + \sum_{j'=1}^{jj} W2PGG_{(j1,j'1)} (A\hat{M}G_{(j1,j'1)} + \hat{P}Y_{j'1})$$

$j=jjc+1, \dots, jj$

where

$$(11.2.17) \quad W1PGG_{j1} = PGGY_{j1} / PGG_{j1}, \quad j=jjc+1, \dots, jj$$

$$(11.2.18) \quad W2PGG_{(j1,j'1)} = AMG_{(j1,j'1)} [PY_{j'1}] / PGG_{j1}, \quad j=jjc+1, \dots, jj, \quad j'=1, \dots, jj$$

$$\wedge(11.2.19) \quad \hat{M}G_{(js,j'1)} = A\hat{M}G_{(js,j'1)} + \hat{G}_{js}, \quad j=1, \dots, jj, \quad s=1, 2, \quad j'=1, \dots, jj$$

$$\wedge(11.2.20) \quad M\hat{G}G_{(j1,j'1)} = A\hat{M}G_{(j1,j'1)} + \hat{G}G_{j1}, \quad j=jjc+1, \dots, jj, \quad j'=1, \dots, jj$$

### 3. Government Revenues

Government revenues come from two sources: expenditure taxes and income taxes. We shall consider a subsidy as a negative tax. First, we deal with expenditure taxes.

Let

$$*(11.3.1) \quad GREV11 = \sum_{i=1}^{ii} \sum_{j=1}^{jj} \sum_{s=1}^2 HX_{ijs} [X_{ijs}]$$

$$+ \sum_{i=1}^{ii} \sum_{j=1}^{jj} \sum_{s=1}^2 HI_{ijs} [I_{ijs}] + \sum_{j=1}^{jj} \sum_{s=1}^2 HC_{js} [C_{js}]$$

$$*(11.3.2) \quad GREV12 = \sum_{j=1}^{jj} HY_{j2}^1 [Y_{j2}]$$

$$*(11.3.3) \quad GREV13 = \sum_{j=1}^{jjc} HE_{j1} [E_{j1}]$$

$$*(11.3.4) \quad GREV14 = \sum_{j=jjc+1}^{jj} HY_{j1,1}^1 [Y_{j1,1}] + \sum_{j=jjc+1}^{jj} HY_{j1,2}^1 [Y_{j1,2}]$$

Total expenditure taxes are then given by

$$*(11.3.5) \quad GREV1 = GREV11 + GREV12 + GREV13 + GREV14.$$

Now let GREV2 denote the total amount of income taxes collected. The precise equation used for calculating GREV2 is given in Chapter 12. Then total government revenues, GREV, is given by

$$*(11.3.6) \quad GREV = GREV1 + GREV2.$$

Equation (11.3.1) gives us the following comparative static result

$$\begin{aligned} \wedge(11.3.7) \quad GR\hat{E}V11 &= \sum_{i=1}^{ii} \sum_{j=1}^{jj} \sum_{s=1}^2 W1GREV11_{ijs} (\hat{H}X_{ijs} + \hat{X}_{ijs}) \\ &+ \sum_{i=1}^{ii} \sum_{j=1}^{jj} \sum_{s=1}^2 W2GREV11_{ijs} (\hat{H}I_{ijs} + \hat{I}_{ijs}) \\ &+ \sum_{j=1}^{jj} \sum_{s=1}^2 W3GREV11_{js} (\hat{H}C_{js} + \hat{C}_{js}) \end{aligned}$$

where

$$(11.3.8) \quad W1GREV11_{ijs} = HX_{ijs} [X_{ijs}] / GREV11$$

$$W2GREV11_{ijs} = HI_{ijs} [I_{ijs}] / GREV11$$

$$W3GREV11_{js} = HC_{js} [C_{js}] / GREV11$$

$$i=1, \dots, ii, \quad j=1, \dots, jj, \quad s=1, 2.$$

$$\wedge(11.3.9) \quad GR\hat{E}V12 = \sum_{j=1}^{jj} WGREV12_j (\hat{H}Y_{j2}^1 + \hat{Y}_{j2})$$

where

$$(11.3.10) \quad WGREV12_j = HY_{j2}^1 [Y_{j2}] / GREV12, \quad j=1, \dots, jj.$$

Equation (11.3.3) gives us the following comparative static result

$$\wedge(11.3.11) \quad GR\hat{E}V13 = \sum_{j=1}^{jj} WGREV13_j (\hat{H}E_{j1} + \hat{E}_{j1})$$

where

$$WGREV13_j = HE_{j1}[E_{j1}]/GREV13, \quad j=1, \dots, jjc.$$

Equation (11.3.4) gives us the following comparative static result

$$\begin{aligned} \wedge(11.3.12) \quad GR\hat{E}V14 &= \sum_{j=jjc+1}^{jj} W1GREV14_j \left( \hat{H}Y_{j1,1}^1 + \hat{Y}_{jj1,1} \right) \\ &+ \sum_{j=jjc+1}^{jj} W2GREV14_j \left( \hat{H}Y_{j1,2}^1 + \hat{Y}_{jj1,2} \right) \end{aligned}$$

where

$$\begin{aligned} (11.3.13) \quad W1GREV14_j &= HY_{j1,1}^1 [Y_{jj1,1}] / GREV14 \\ W2GREV14_j &= HY_{j1,2}^1 [Y_{jj1,2}] / GREV14, \quad j=jjc+1, \dots, jj. \end{aligned}$$

Equation (11.3.5) gives us the following comparative static result

$$\begin{aligned} \wedge(11.3.14) \quad GR\hat{E}V1 &= W1GREV1[GR\hat{E}V11] + W2GREV1[GR\hat{E}V12] \\ &+ W3GREV1[GR\hat{E}V13] + W4GREV1[GR\hat{E}V14] \end{aligned}$$

where

$$\begin{aligned} (11.3.15) \quad W1GREV1 &= GREV11/GREV1, \\ W2GREV1 &= GREV12/GREV1, \\ W3GREV1 &= GREV13/GREV1, \\ W4GREV1 &= GREV14/GREV1. \end{aligned}$$

Finally, equation (11.3.6) gives us the following comparative static result

$$\wedge(11.3.16) \quad GR\hat{E}V = W1GREV[GR\hat{E}V1] + W2GREV[GR\hat{E}V2]$$

where

$$\begin{aligned} (11.3.17) \quad W1GREV &= GREV1/GREV, \\ W2GREV &= GREV2/GREV. \end{aligned}$$

#### 4. The Government Budget Constraint and Economic Policies

Let  $GSAV$  denote government saving. Then we have

$$*(11.4.1) \quad GSAV = GREV - GEXP$$

When  $GSAV$  is positive (negative), we have a budget surplus (deficit). When  $GSAV=0$ , the government budget is balanced.

Given the structure of our model, a theoretically satisfactory treatment of government economic policies would require the use of intertemporal optimization. First, the government must fix a planning time horizon. Second, a time profile for each of the variables under government control, such as taxes, subsidies, expenditures on goods and services, must be chosen. Once the time profiles in these variables have been chosen, a static general equilibrium for each time period must be found. Third, compute the utility enjoyed by households under static general equilibrium for each time period to obtain the time profile of households' utilities for the entire planning time horizon. Fourth, use a discount rate to convert the time profile of households' utilities into discounted social welfare.

The objective of the government is to choose a time profile for each of the variables under its control to maximize discounted social welfare under the constraint of zero debt at the end of the time horizon. The government problem thus formulated is that of a Stackelberg differential game. At the present time such a model is technically unmanageable.

To focus our attention on industrial and trade policies, we shall adopt a more modest approach to economic policy. First, we impose a balanced-budget constraint on the government, i.e.,

$$*(11.4.2) \quad GREV - GEXP = 0.$$

Second, given the only constraint represented by (11.4.2) and the many instruments under its control, more definite results can only be obtained by endogenizing two control variables and forcing the rest to be exogenous. For example, to support an infant industry the government might impose a tariff on imports and distribute the tariff revenue thus obtained to the consumers. In this case, the tariff and government transfers are the only two endogenous policy variables. The government might choose  $G_{js}$  for some  $js$  as a policy variable. If  $G_{js}$  is exogenous, we suppose that it is given by

$$*(11.4.3) \quad G_{js} = [MR]^{\alpha G_{js}} [SHFTG_{js}], \quad j=1, \dots, j \text{ and } s=1, 2.$$

$$*(11.4.4) \quad MR = M/CPI$$

Here  $G_{js}$  = government demand for good  $j$  from source  $s$

$MR$  = real aggregate household expenditure

$M$  = aggregate household expenditure

$CPI$  = consumer price index

$\alpha G_{js}$  = a parameter

$SHFTG_{js}$  = a shift variable

Equations (11.4.3) and (11.4.4) give us immediately the following comparative static results

$$^{\wedge}(11.4.5) \quad \hat{G}_{js} = [\alpha G_{js}] [\hat{MR}] + SH\hat{FTG}_{js}, \quad j=1, \dots, j \text{ and } s=1, 2.$$

$$^{\wedge}(11.4.6) \quad \hat{MR} = \hat{M} - C\hat{PI}.$$

Finally, equation (11.4.2) gives us the following comparative static result

$$^{\wedge}(11.4.7) \quad \hat{GREV} - \hat{GEXP} = 0.$$

## CHAPTER 12

### NATIONAL INCOME ACCOUNTING AND MACROECONOMIC CLOSURES

#### 1. Introduction

Models in the Walrasian tradition are static in nature; they are not well suited for analyzing short-run phenomena, such as unemployment and inflation. The most satisfactory approach is to extend the Walrasian paradigm by introducing demand and supply functions for money and other financial assets. Unemployment might be analyzed according to the disequilibrium approach of Benassy (1982) in which economic agents face various rationing mechanisms on the markets they want to trade. The extended Walrasian paradigm would allow such important macroeconomics variables as the price level, the money supply, and the unemployment rate to be endogenized.

In applied general equilibrium analysis, the arrival date of the extended Walrasian paradigm is still in the distant future. In the meantime, in an attempt to deal with transient phenomena that take place one or two years in the future, researchers in applied general equilibrium analysis have incorporated some well-known features of conventional macroeconomics into their models. This chapter deals with national income accounting and macroeconomic closure rules used in the thesis.

## 2. Gross Domestic Product

Let GDP denote gross domestic product. using the value-added approach to national income accounting, we have

$$*(12.2.1) \quad GDP = \sum_{i=1}^{ii} VA_i,$$

where  $VA_i$ ,  $i=1, \dots, ii$ , is the value added generated by industry  $i$ . More precisely, for perfectly competitive industries, we have

$$*(12.2.2) \quad VA_i = \sum_{j \in L(i)} PY_{j1} [Y_{ij1}] - \sum_{j=1}^{jj} PY_{j1} [X_{ij1}] \\ - \Phi \sum_{j=1}^{jic} PFY_{j2} [X_{ij2}] - \sum_{j=jic+1}^{jj} (PY_{j2} - HY_{j2}^1) [X_{ij2}] \\ - \sum_{j=1}^{jj} \sum_{j'=1}^{jj} \sum_{s=1}^2 PY_{j'1} [MX_{(ijs, j'1)}], \quad i=1, \dots, iic.$$

For imperfectly competitive industries, we have

$$*(12.2.3) \quad VA_i = PY_{j1} [Y_{ij1,1}] + \left( (PQ_{j2} - HY_{j1,2}^1) Y_{ij1,2} \right) \Phi + PGGY_{j1} [GG_{j1}] \\ - \sum_{k=1}^{jj} PY_{k1} [X_{ik1}] - \Phi \sum_{k=1}^{jic} PFY_{k2} [X_{ik2}] \\ - \sum_{k=jic+1}^{jj} (PY_{k2} - HY_{k2}^1) [X_{ik2}] \\ - \sum_{k=1}^{jj} \sum_{j'=1}^{jj} \sum_{s=1}^2 PY_{j'1} [MX_{(iks, j'1)}] \\ - \sum_{j'=1}^{jj} \sum_{s=1}^2 PY_{j'1} [MY_{(ij1, s, j'1)}] \\ i=iic+1, \dots, ii, \quad j=i-iic+jic.$$

Equation (12.2.1) gives us the following comparative static result

$$\wedge(12.2.4) \quad G\hat{D}P = \sum_{i=1}^{ii} WVA_i [\hat{V}A_i],$$

where

$$WVA_i = VA_i / GDP, \quad i=1, \dots, ii$$

Equation (12.2.2) yields the following comparative static result

$$\begin{aligned}
 \wedge(12.2.5) \quad \hat{V}A_i &= \sum_{j \in L(i)} W1VA_{ij} (\hat{P}Y_{j1} + \hat{Y}_{ij1}) - \sum_{j=1}^{jj} W2VA_{ij} [\hat{X}_{ij1}] \\
 &\quad - \sum_{j=1}^{jic} W3VA_{ij} (P\hat{F}Y_{j2} + \hat{X}_{ij2} + \hat{\Phi}) \\
 &\quad - \sum_{j=jjc+1}^{jj} W4VA_{ij} \left( \left( \frac{PY_{j2}}{PY_{j2} - HY_{j2}^1} \right) \hat{P}Y_{j2} - \left( \frac{HY_{j2}^1}{PY_{j2} - HY_{j2}^1} \right) \hat{H}Y_{j2}^1 + \hat{X}_{ij2} \right) \\
 &\quad - \sum_{j=1}^{jj} \sum_{j'=1}^{jj} \sum_{s=1}^2 W5VA_{(ijs,j'1)} (\hat{P}Y_{j'1} + \hat{M}X_{(ijs,j'1)}), \quad i=1, \dots, iic
 \end{aligned}$$

where

$$\begin{aligned}
 (12.2.6) \quad W1VA_{ij} &= PY_{j1} [Y_{ij1}] / VA_i \\
 W2VA_{ij} &= PY_{j1} [X_{ij1}] / VA_i \\
 W3VA_{ij} &= \Phi [P\hat{F}Y_{j2}] [X_{ij2}] / VA_i \\
 W4VA_{ij} &= (PY_{j2} - HY_{j2}^1) X_{ij2} / VA_i \\
 W5VA_{(ijs,j'1)} &= PY_{j'1} [M\hat{X}_{(ijs,j'1)}] / VA_i.
 \end{aligned}$$

Equation (12.2.3) yields the following comparative static result

$$\begin{aligned}
 \wedge(12.2.7) \quad \hat{V}A_i &= W1VA_i (\hat{P}Y_{j1} + \hat{Y}_{ij1}) + W2VA_i \left( \left( \frac{PQ_{j2}}{PQ_{j2} - HY_{j1,2}^1} \right) \hat{P}Q_{j2} \right. \\
 &\quad \left. - \left( \frac{HY_{j1,2}^2}{PQ_{j2} - HY_{j1,2}^1} \right) \hat{H}Y_{j1,2}^1 \right. \\
 &\quad \left. + \hat{Y}_{ij1,2} + \hat{\Phi} \right) \\
 &\quad + W3VA_i (P\hat{G}G_{j1} + \hat{G}G_{j1}) - \sum_{k=1}^{jj} W4VA_{ik} (\hat{P}Y_{k1} + \hat{X}_{ik1}) \\
 &\quad - \sum_{k=1}^{jic} W5VA_{ik} (P\hat{F}Y_{k2} + \hat{X}_{ik2} + \hat{\Phi})
 \end{aligned}$$

$$\begin{aligned}
& -\sum_{k=ijc+1}^{ii} W6VA_{ik} \left( \begin{array}{c} \left( \frac{PY_{k2}}{PY_{k2} - HY_{k2}^1} \right) \hat{PY}_{k2} \\ - \left( \frac{HY_{k2}^1}{PY_{k2} - HY_{k2}^1} \right) \hat{HY}_{k2}^1 + \hat{X}_{ik2} \end{array} \right) \\
& -\sum_{k=1}^{ii} \sum_{j'=1}^{ii} \sum_{s=1}^2 W7VA_{(iks,j'1)} \left( \hat{PY}_{j'1} + \hat{MX}_{(iks,j'1)} \right) \\
& -\sum_{j'=1}^{ii} \sum_{s=1}^2 W8VA_{(ij1,s,j'1)} \left( \hat{PY}_{j'1} + \hat{MY}_{(ij1,s,j'1)} \right), \\
& \qquad \qquad \qquad i=ii+1, \dots, ii, \quad j=i-ii+ijc.
\end{aligned}$$

where

$$\begin{aligned}
(12.2.8) \quad W1VA_i &= PY_{j1} [Y_{ij1,1}] / VA_i \\
W2VA_i &= (PQ_{j2} - HY_{j1,2}^1) [Y_{ij1,2}] \Phi / VA_i \\
W3VA_i &= PGGY_{j1} [GG_{j1}] / VA_i \\
W4VA_{ik} &= PY_{k1} [X_{ik1}] / VA_i \\
W5VA_{ik} &= \Phi [PFY_{k2} [X_{ik2}]] / VA_i \\
W6VA_{ik} &= (PY_{k2} - HY_{k2}^1) X_{ik2} / VA_i \\
W7VA_{(iks,j'1)} &= PY_{j'1} [MX_{(iks,j'1)}] / VA_i \\
W8VA_{(ij1,s,j'1)} &= PY_{j'1} [MY_{(ij1,s,j'1)}] / VA_i
\end{aligned}$$

### 3. Aggregate Consumer Expenditure

Let *DEPR* denote capital depreciation for the whole economy. Then

$$*(12.3.1) \quad DEPR = \sum_{i=1}^{ii} [d_i [K_i] [PI_i]].$$

Households' income, say *INCOME*, is equal to gross domestic product minus capital depreciation.

$$*(12.3.2) \quad INCOME = GDP - DEPR$$

Letting  $aps$  denotes the average propensity to save and  $[\rho INCOME]$  the income tax rate, then aggregate consumer expenditure  $M$  is given by

$$*(12.3.3) \quad M = [INCOMEDISP](1 - aps)$$

where disposable income, say  $INCOMEDISP$ , is calculated by adding government transfers  $GTRNSFR$  to net income (after income taxes are paid).

$$*(12.3.4) \quad [INCOMEDISP] = (1 - \rho INCOME)[INCOME] + GTRNSFR$$

Equations (12.3.1) through (12.3.4) give us, respectively, the following comparative static results

$$\wedge(12.3.5) \quad D\hat{EPR} = \sum_{i=1}^n WDEPR_i (\hat{K}_i + \hat{PI}_i)$$

where

$$WDEPR_i = [d_i [K_i [PI_i]] / DEPR$$

$$\wedge(12.3.6) \quad IN\hat{COME} = \left( \frac{GDP}{INCOME} \right) G\hat{DP} - \left( \frac{DEPR}{INCOME} \right) D\hat{EPR}$$

$$\wedge(12.3.7) \quad \hat{M} = INCOM\hat{E}DISP$$

$$\wedge(12.3.8) \quad INCOM\hat{E}DISP = \left( \frac{(1 - \rho INCOME)}{INCOMEDISP} \right) [IN\hat{COME}] \\ + \left( \frac{GTRNSFR}{INCOMEDISP} \right) [G\hat{TR}NSFR]$$

#### 4. Balance of Payments

Let EXPORT and IMPORT denote the values of total exports and total imports, respectively. Then we have

$$*(12.4.1) \quad EXPORT = \Phi \left( \sum_{j=1}^{j^c} PE_{j1} [E_{j1}] + \sum_{j=j^c+1}^{jj} (PQ_{j2} - HY_{j1,2}^2) Y_{j1,2} \right)$$

$$*(12.4.2) \quad IMPORT = \Phi \left( \sum_{j=1}^{j^c} \left( \sum_{i=1}^{ii} X_{ij2} + C_{j2} + \sum_{i=1}^{ii} I_{ij2} + G_{j2} \right) [PFY_{j2}] \right) \\ + \sum_{j=j^c+1}^{jj} (PY_{j2} - HY_{j2,1}^1) Y_{j2}$$

In standard national income accounting, the current account is the sum of the balance of trade, the net interest payments received by domestic residents from the foreign country, and the net transfers, such as grant and foreign aid received by the home country. The sum of the current account and the capital account is then equal to the country's change in foreign reserves. Our model, like most models in applied general equilibrium analysis, excludes international capital flows and net transfers. Therefore, the balance of payments under a flexible exchange rate regime is given by

$$*(12.4.3) \quad EXPORT - IMPORT = 0.$$

Equation (12.4.1) yields the following comparative static result

$$\wedge(12.4.4) \quad \hat{EXPORT} = \hat{\Phi} + \sum_{j=1}^{j^c} W1EXPORT_j \left( \hat{PE}_{j1} + \hat{E}_{j1} \right) \\ + \sum_{j=j^c+1}^{jj} W2EXPORT_j \left( \left( \frac{PQ_{j2}}{PQ_{j2} - HY_{j1,2}^2} \right) \hat{P}Q_{j2} \right. \\ \left. - \left( \frac{HY_{j1,2}^2}{PQ_{j2} - HY_{j1,2}^2} \right) \hat{H}Y_{j1,2}^2 + \hat{Y}_{j1,2} \right)$$

where

$$W1EXPORT_j = \Phi [PE_{j1}] [E_{j1}] / EXPORT \\ W2EXPORT_j = \Phi (PQ_{j2} - HY_{j1,2}^2) [Y_{j1,2}] / EXPORT$$

Equation (12.4.2) yields the following comparative static result

$$\begin{aligned} \wedge(12.4.5) \quad \hat{I}MPORT = & \sum_{j=1}^{\hat{j}c} W1IIMPORT_j \left( \hat{\Phi} + P\hat{F}Y_{j2} + \hat{Y}_{j2} \right) \\ & + \sum_{j=jc+1}^{\hat{j}} W2IIMPORT_j \left( \begin{aligned} & \left( \frac{PY_{j2}}{PY_{j2} - HY_{j2,1}^1} \right) \hat{P}Y_{j2} \\ & - \left( \frac{HY_{j2,1}^1}{PY_{j2} - HY_{j2,1}^1} \right) \hat{H}Y_{j2}^1 + \hat{Y}_{j2} \end{aligned} \right) \end{aligned}$$

where

$$(12.4.6) \quad W1IIMPORT_j = \Phi \left[ Y_{j2} \right] \left[ PFY_{j2} \right] / IIMPORT$$

$$*(12.4.7) \quad Y_{j2} = \sum_{i=1}^{\hat{i}} X_{ij2} + C_{j2} + \sum_{i=1}^{\hat{i}} I_{ij2} + G_{j2}, \quad j=1, \dots, \hat{j}.$$

$$(12.4.8) \quad W2IIMPORT_j = \left( PY_{j2} - HY_{j2,1}^1 \right) Y_{j2} / IIMPORT$$

Recall that  $Y_{j2}$  is the total supply of imported good  $j$  on the domestic market.

Equation (12.4.7) yields the following comparative static result

$$\begin{aligned} \wedge(12.4.9) \quad \hat{Y}_{j2} = & \sum_{i=1}^{\hat{i}} W1Y_{j2,i} \left[ \hat{X}_{ij2} \right] + W2Y_{j2} \left[ \hat{C}_{j2} \right] \\ & + \sum_{i=1}^{\hat{i}} W3Y_{j2,i} \left[ \hat{I}_{ij2} \right] + W4Y_{j2} \left[ \hat{G}_{j2} \right], \quad j=1, \dots, \hat{j} \end{aligned}$$

where

$$W1Y_{j2,i} = X_{ij2} / Y_{j2}$$

$$W2Y_{j2} = C_{j2} / Y_{j2}$$

$$W3Y_{j2,i} = I_{ij2} / Y_{j2}$$

$$W4Y_{j2} = G_{j2} / Y_{j2}$$

The balance of payments equation (12.4.3) yields the following comparative static result

$$\wedge(12.4.10) \quad \hat{E}XPORT - \hat{I}MPORT = 0.$$

## 5. Macroeconomic Closures

We have the following national income identity

$$*(12.5.1) \quad GDP = M + I + GEXP$$

The macroeconomic closures used in our model include the national income identity (12.5.1), the aggregate consumer expenditure equation (12.3.3), the balance of payments equation (12.4.3), and the government balance budget constraint (11.4.2).

Equation (12.5.1) yields the following comparative static result

$$\wedge(12.5.2) \quad \hat{GDP} = \left(\frac{M}{GDP}\right)\hat{M} + \left(\frac{I}{GDP}\right)\hat{I} + \left(\frac{GEXP}{GDP}\right)\hat{GEXP}$$

Another closure rule, known as the Walrasian closure rule, is also used in our model. According to the Walrasian closure rule, labor markets are in equilibrium and wage rates are determined endogenously. To deal with unemployment, some computable general equilibrium models, such as Deardoff and Stern (1986) or Dixon et al (1982), use the Keynesian closure rule, which consists of dropping the requirement of full employment and using instead the assumption of a constant aggregate consumer expenditure.

The subject of macroeconomic closures remains a contentious issue concerning the role of traditional macroeconomics. See for example, Deardoff and Stern (1990) and Cooper et al (1985).

Finally, the exchange rate is endogenous, and one of the prices in the model is chosen as the numeraire.

## 6. Macro-Indices and Wage Indexation

The consumer price index is defined by the following equation

$$^{\wedge}(12.6.1) \quad C\hat{P}I = \sum_{j=1}^{jj} \sum_{s=1}^2 [WC_{js}] [\hat{P}C_{js}],$$

where the weights are given by

$$WC_{js} = \frac{[PC_{js}] [C_{js}]}{\sum_{j'=1}^{jj} \sum_{s'=1}^2 [PC_{j's'}] [C_{j's'}]}.$$

The capital goods price index is defined by

$$^{\wedge}(12.6.2) \quad K\hat{P}I = \sum_{i=1}^{ii1} [\tilde{S}I_i] [\hat{P}I_i]$$

where

$$\tilde{S}I_i = \frac{SI_i}{\sum_{i'=1}^{ii1} SI_{i'}}.$$

Aggregate employment is defined by

$$*(12.6.3) \quad L = \sum_{t=1}^n L_t$$

In percentage changes, (12.6.3) becomes

$$^{\wedge}(12.6.4) \quad \hat{L} = \sum_{t=1}^n [WL_t] \hat{L}_t,$$

where the weights are given by

$$WL_t = L_t / L, \quad t = 1, \dots, n.$$

Aggregate capital stock in the base-period value units

$$*(12.6.5) \quad K = \sum_{i=1}^{ii} K_i$$

In percentage changes, (12.6.5) becomes

$$\hat{K} = \sum_{i=1}^{ii} [WK_i] \hat{K}_i,$$

where the weights are given by

$$WK_i = K_i / K$$

Ratio of real private investment expenditure to real private consumption

$$* (12.6.7) \quad IMR = IIR / MR,$$

where IIR is already defined (see equation (8.2.6) and notice that  $MR = M/CPI$ ). In

percentage changes, (12.6.7) becomes

$$\hat{IMR} = \hat{IIR} - \hat{MR}$$

Aggregate agricultural land is defined by

$$* (12.6.9) \quad LAND = \sum_{i=1}^{ii} LAND_i.$$

In percentage changes, (12.6.9) becomes

$$\hat{LAND} = \sum_{i=1}^{ii} WLAND_i [\hat{LAND}_i]$$

where

$$WLAND_i = LAND_i / LAND$$

Wage indexation : In percentage change form, wage indexation is given by

$$\hat{P}X_{(i,jj+1,t)} = \left[ \omega PX_{(i,jj+1,t)} \right] [CPI] + \hat{F}PX_{(.,jj+1,t)} + \hat{F}PX_{(i,jj+1,t)} \\ + \hat{F}PX_{(.,jj+1,t)} + \hat{F}PX_{(i,jj+1,t)},$$

for  $i=1, \dots, ii$ , and  $t=1, \dots, tt$ ,

where  $\omega_{PX(i,jj+1,1,t)}$  is a parameter and the  $\hat{FPX}$ 's are variables. If  $\omega_{PX(i,jj+1,1,t)} = 1$ , for all  $t=1, \dots, tt$ , and the  $\hat{FPX}$ 's = 0 then we have full wage indexation.

"Other cost" tickets indexation is defined by

$$^{(12.6.12)} \hat{PX}_{(i,jj+2)} = \left[ \omega_{PX(i,jj+2)} \right] \left[ C\hat{PI} \right] + F\hat{PX}_{(i,jj+2)}$$

where  $\omega_{PX(i,jj+2)}$  is a parameter and  $\hat{FPX}_{(i,jj+2)}$  is a variable.

## CHAPTER 13

### GENERAL EQUILIBRIUM à LA NEGISHI

In Chapter 2 through 12, we have studied the optimizing behavior of individual economic agents -- firms, consumers, government. It is now time to bring together the decisions made by these economic agents and see if their individual choices are mutually consistent. Thus this chapter has two objectives. First, we define general equilibrium. Next, comparative static results for the market-clearing conditions are derived.

#### 1. States of the Economy

By a system of basic prices, we mean the array

$$P = \left( \left( PY_{js} \right)_{\substack{j=1, \dots, jj \\ s=1, 2}}, \left( PE_{j1} \right)_{j=1, \dots, jjc}, \left( PQ_{j2} \right)_{j=jjc+1, \dots, jj} \right).$$

Recall that  $\psi_{j1}^f$  is the value of the position parameter of the domestic inverse demand curve for good  $j1$ ,  $j=jjc+1, \dots, jj$ , perceived by an imperfectly competitive firm  $f$  – domestic or foreign – that produces the  $j^{\text{th}}$  commodity. The array

$$\Psi = \left( \Psi_{j1}^f \right)_{\substack{j=jjc+1, \dots, jj \\ f \in F(j-jjc+ic)}}$$

can be interpreted as a system of "pseudo-prices" in the Walrasian tradition. According to this interpretation, the Walrasian auctioneer will send to an imperfectly competitive firm  $f$  the value  $\psi_{j1}^f$  and the basic prices of the inputs this firm uses. Taking the pseudo-price  $\psi_{j1}^f$

and the prices of inputs as given, firm  $f$  then plays a Cournot game in quantity strategies against its rivals.

For each competitive industry  $i$ , let

$$\Omega_i = \left( X_i, Z_i, \left( Y_{ij1} \right)_{j \in L(i)} \right), \quad i=1, \dots, iic$$

be a production plan that specifies its input combination  $X_i$ , activity level  $Z_i$ , and output vector  $(Y_{ij1})_{j \in L(i)}$ .

Let  $\Omega_i^f$  be the production plan of an imperfectly competitive firm  $f$  in industry  $i$ , i.e.,

$$\Omega_i^f = \left( X_i^f, Z_i^f, Y_{ijs,1}^f, Y_{ijs,2}^f \right), \quad s=1,2, f \in F(i,s), i=iic+1, \dots, ii, j=i-iic+jjc.$$

Here we recall that  $Y_{ij}^f$  denotes this firm output and  $Y_{ij,1}^f, Y_{ij,2}^f$  its sales on the domestic and foreign markets, respectively. Also,  $s=1$  indicates that this is a domestic firm, while  $s=2$  refers to a foreign firm.

**Definition:** A state of the economy is a list

$$(13.1.1) \quad \left( P, \Psi, \Phi, \left( \Omega_i \right)_{i=1, \dots, iic}, \left( \Omega_i^f \right)_{\substack{i=iic+1, \dots, ii \\ f \in F(i)}}, \left( I_{ijs} \right)_{\substack{i=1, \dots, ii \\ j=1, \dots, jj \\ s=1,2}}, \right. \\ \left. \left( C_{js} \right)_{\substack{j=1, \dots, jj \\ s=1,2}}, \left( E_{j1} \right)_{j=1, \dots, jjc}, \left( Y_{j2} \right)_{j=1, \dots, jjc} \right)$$

We recall that in this list  $\Phi$  is the exchange rate;  $I_{ij}$  is the demand for intermediate input  $js$  to produce capital goods for industry  $i$ ;  $C_{js}$  is households' demand for good  $js$ ;  $E_{j1}$  is the volume of exports of good  $j1$ ;  $Y_{j2}$  is the volume of imports of good  $j2$ . The list defining a state of the economy contains only the most basic information about the

economy. From this list, one can compute  $PX_{ij}$ , (equation (5.4.3)),  $PI_{ij}$ , (equation (8.3.3))  $PC_j$ , (equation (9.2...)),  $PG_j$ , (equation (11.2.3)),  $PGG_{j1}$  (equation (11.2.4)) and the demands for margins. One can also compute GDP, national income,  $M$  (the aggregate consumer expenditure).

## 2. General Equilibrium: Definition

Definition: A state of the economy

$$\left( P, \Psi, \Phi, (\Omega_i)_{i=1, \dots, iic}, (\Omega_i^f)_{\substack{i=iic+1, \dots, ii \\ f \in F(i)}}, (I_{ijs})_{\substack{j=1, \dots, jj \\ s=1, 2}}, \right. \\ \left. (C_{js})_{\substack{j=1, \dots, jj \\ s=1, 2}}, (E_{j1})_{j=1, \dots, jic}, (Y_{j2})_{j=1, \dots, jic} \right)$$

is an equilibrium if the following conditions are satisfied.

- a) For each  $i=1, \dots, iic$ , the production plan  $\Omega_i$  maximizes the profits of industry  $i$  under the price system  $P$ . Because such an industry is perfectly competitive and operates under conditions of constant return to scale, its profits must be equal to zero in equilibrium, i.e.,

$$\begin{aligned} *(13.2.1) \quad \sum_{j \in L(i)} [PY_{j1}] [Y_{ij1}] &= \sum_{j=1}^{jj} \sum_{s=1}^2 [PX_{ijs}] [X_{ijs}] \\ &+ \sum_{t=1}^{it} [PX_{(i, jj+1, 1, t)}] [X_{(i, jj+1, 1, t)}] \\ &+ \sum_{s=2}^3 [PX_{(i, jj+1, s)}] [X_{(i, jj+1, s)}] \\ &+ [PX_{(i, jj+2)}] [X_{(i, jj+2)}] \quad i=1, \dots, iic. \end{aligned}$$

- b) For each  $i=iic+1, \dots, ii$  and each  $f \in F(i)$ , the production plan  $\Omega_i^f$  maximizes the perceived profits of firm  $f$ , given

- b1) the prices of its inputs;
- b2) the production plans of its competitors, namely  $(\Omega_i^{f'})_{\substack{f' \in F(i) \\ f' \neq f}}$ ;
- b3) its perceived value of the position parameters  $\Omega_i^f$ , where  $j=i-iic+jjc$  is the index of the commodity it produces.

Furthermore, the perception of this firm must not be contradicted in equilibrium, i.e.,

$$*(13.2.2) \quad PY_{j1} = \left[ \Psi_{j1}^f \right] \left[ Q_{j1}^{-\varepsilon_{j1}^f} \right], \quad j=jjc+1, \dots, jj, \quad f \in F(j-jjc+iic),$$

where  $Q_{j1}$ , we recall, denote the total supply of good  $j$  on the domestic market by all the firms -- domestic or foreign.

- c) For each  $i=1, \dots, ii$ , the competitive producers of capital goods for industry  $i$  make zero profits in equilibrium, i.e.,

$$*(13.2.3) \quad [PI_i][I_i] = \sum_{j=1}^{jj} \sum_{s=1}^2 [PI_{ijs}][I_{ijs}], \quad i=1, \dots, ii.$$

- d) For competitive exporters, their profits are also zero in equilibrium, i.e.,

$$*(13.2.4) \quad [PE_i][\Phi] = PY_{j1} + \sum_{j=1}^{jj} [PY_{j1}][AME_{(j1, j'1)}] + HE_{j1} \quad j=1, \dots, jjc.$$

The left side of (13.2.4) is the f.o.b. price paid by foreigners converted, via the exchange rate  $\Phi$ , into domestic currency for 1 unit of good  $j1$  exported. The right side represents the total cost of delivering one unit of  $j1$  on board foreign ships. First, exporters have to buy one unit of  $j1$  on the domestic market at the basic price  $PY_{j1}$ . Next, this unit has to be transported to domestic ports and loaded on foreign ships; the

margins required are represented by the second term on the right side. Finally, export taxes, represented by the last term, must be paid.

e) In equilibrium, competitive importers also make no profits, i.e.,

$$*(13.2.5) \quad PY_{j2} = \left[ PFY_{j2} \right] \Phi + HY_{j2}^1, \quad j=1, \dots, jic.$$

Here  $PFY_{j2}$  is the price in foreign currency of 1 unit of commodity  $j$  produced abroad and delivered at domestic ports. This c.i.f. price is exogenous. Also  $HY_{j2}^1$  is the import tariff imposed by the home government.

f) The consumption bundle  $\left( C_{js} \right)_{\substack{j=1, \dots, j \\ s=1, 2}}$  maximizes households' utility, given that the

basic price system is  $P$  and that the aggregate consumer expenditure is  $M$ , with  $M$  computed according to equation (12.3.3)

g) There is equality of supply and demand on all markets.

The market-clearing conditions are as follows. For commodities bought and sold under perfectly competitive conditions, we have

$$*(13.2.6) \quad \sum_{i=1}^{iic} Y_{ij1} = \sum_{i=1}^{ii} X_{ij1} + \sum_{i=1}^{ii} I_{ij1} + C_{j1} + G_{j1} + E_{j1} \\ + \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MX_{(ij's, j1)} \right] \\ + \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MI_{(ij's, j1)} \right] + \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MC_{(j's, j1)} \right] \\ + \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MG_{(j's, j1)} \right] + \sum_{j'=1}^{jic} \left[ ME_{(j'1, j1)} \right] \\ + \sum_{i=iic+1}^{ii} \left[ MY_{(ij'1, 1, j1)} \right] + \sum_{i=iic+1}^{ii} \left[ MY_{(ij'1, 2, j1)} \right], \quad j=1, \dots, jic.$$

where the term on the left hand side denotes the total supply of good  $j$  by all the domestic competitive industries. On the right hand side, the first term represents the demand for

domestic input  $j$  by all the industries for current production; the second term is the demand for domestic input  $j$  by all the industries for capital investment; the third term is households' consumption of domestic good  $j$ ; the fourth term is government expenditures on domestic good  $j$ ; the fifth term is export of domestic good  $j$ ; the sixth term is the amount of domestic good  $j$  used as margins by industry  $i$  in obtaining  $X_{ijs}$  units of commodity  $j'$  from source  $s$  to be used as input in current production; the seventh term is the amount of domestic good  $j$  needed in delivering  $I_{ijs}$  units of commodity  $j'$ , from source  $s$  to industry  $i$  for capital construction; the eighth term is the amount of domestic good  $j$  needed in delivering  $C_{js}$  units of commodity  $j'$ , from source  $s$  to households; the ninth term is the amount of domestic good  $j$  needed in delivering  $G_{js}$  units of commodity  $j'$ , from source  $s$  to the government; the tenth term is the amount of domestic good  $j$  needed in delivering  $ME_{js}$  units of domestic commodity  $j'$  to domestic ports for exports. The last two terms represent, respectively, the amount of good  $j1$  required as margins by the domestic oligopolistic industries in delivering their outputs to the home and foreign markets.

The market-clearing conditions for commodity bought and sold under imperfectly competitive conditions are

$$\begin{aligned}
 *(13.2.7) \quad Q_{j1} &= \sum_{i=1}^i \sum_{s=1}^2 X_{ijs} + \sum_{i=1}^i \sum_{s=1}^2 I_{ijs} + \sum_{s=1}^2 C_{js} + \sum_{s=1}^2 G_{js} + GG_{j1} \\
 &+ \sum_{i=1}^i \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MX_{(ij's, j1)} \right] + \sum_{i=1}^i \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MI_{(ij's, j1)} \right] \\
 &+ \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MC_{(j's, j1)} \right] + \sum_{j'=1}^{j^c} \left[ ME_{(j'1, j1)} \right] \\
 &+ \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ MG_{(j's, j1)} \right] + \sum_{j'=j^c+1}^{jj} \left[ MGG_{(j'1, j1)} \right]
 \end{aligned}$$

$$+\sum_{i=\{iic\}+1}^{ii} \left[ MY_{(ij'1,1,j1)} \right] + \sum_{i=\{iic\}+1}^{ii} \left[ MY_{(ij'1,2,j1)} \right], \quad j=jjc+1, \dots, jj.$$

The term on the left hand side of (13.2.7) represent, respectively, the supply of commodity  $j$  by domestic and foreign producers. The right hand side is similar to the right side of (13.2.6) except for  $GG_{j1}$  that replaces  $E_{j1}$ .

For competitive exports, the price  $PE_{j1}$  must clear the market, i.e.,

$$*(13.2.8) \quad PE_{j1} = \Psi_{j1} F_{j1}(E_{j1}), \quad j=1, \dots, jjc.$$

For commodities bought and sold abroad under imperfectly competitive conditions, the market-clearing condition is

$$*(13.2.9) \quad PQ_{j2} = \left[ \Psi_{j2} \left[ Q_{j2}^{-\varepsilon_{j2}} \right] \right], \quad j=jjc+1, \dots, jj,$$

where we recall,  $Q_{j2}$  denotes the total supply on the foreign market for good  $j$  and  $PQ_{j2}$  denotes the price prevailing on this market.

For primary factor inputs, we have the following market-clearing conditions:

$$*(13.2.10) \quad L_t = \sum_{i=1}^{ii} X_{(i,jj+1,1,t)}, \quad t=1, \dots, tt$$

where  $L_t$  is the supply of labor of skill type  $t$ , and  $X_{(i,jj+1,1,t)}$  is the demand by industry  $i$  for labor of skill type  $t$ ;

$$*(13.2.11) \quad K_i = X_{(i,jj+1,2)}, \quad i=1, \dots, ii$$

where  $K_i$  is the supply of capital in industry  $i$ ; and  $X_{(i,jj+1,2)}$  is the demand for capital in industry  $i$ ;

$$*(13.2.12) \quad LAND_i = X_{(i,jj+1,3)}, \quad i=1,\dots,ii$$

where  $LAND_i$  is the supply of agricultural land in industry  $i$ ; and  $X_{(i,jj+1,3)}$  is the demand for agricultural land in industry  $i$ .

Observation: Equations (13.2.11) and (13.2.12) imply that both capital and land are

industry-specific: there is neither capital mobility nor land mobility across industries. On the other hand, there is labor mobility across industries, as indicated by equation (13.2.10).

Finally, under a flexible exchange rate regime, the balance of payments must be in equilibrium, i.e., equation (12.4.3) must hold.

### 3. Comparative Statics

Equation (13.2.1) gives us the following comparative static result

$$\begin{aligned} \wedge(13.3.1) \quad \sum_{j \in L(i)} \left[ \hat{P}Y_{j1} \right] \left[ SWY_{j1} \right] &= \sum_{j=1}^{jj} \sum_{s=1}^2 \left[ \hat{P}X_{ijs} \right] \left[ SWX_{ijs} \right] \\ &+ \sum_{t=1}^{tt} \left[ \hat{P}X_{(i,jj+1,1,t)} \right] \left[ SWX_{(i,jj+1,1,t)} \right] \\ &+ \sum_{s=2}^3 \left[ \hat{P}X_{(i,jj+1,s)} \right] \left[ SWX_{(i,jj+1,s)} \right] \\ &+ \left[ \hat{P}X_{(i,jj+2)} \right] \left[ SWX_{(i,jj+2)} \right] + \hat{A}_i \end{aligned}$$

where

$$\begin{aligned} \wedge(13.3.2) \quad \hat{A}_i &= \left[ A\hat{X}Z_i \right] + \left[ A\hat{Y}Z_i \right] + \sum_{j \in L(i)} \left[ A\hat{Y}Y_{j1} \right] \left[ SY Y_{j1} \right] \\ &+ \sum_{j=1}^{jj} \left[ \hat{A}Y_{j1} \right] \left[ SWY_{j1} \right] + \sum_{j=1}^{jj+2} \left[ \hat{A}X_{ij} \right] \left[ SWX_{ij} \right] \\ &+ \sum_{j=1}^{jj} \sum_{s=1}^2 \left[ \hat{A}X_{ijs} \right] \left[ SWX_{ijs} \right] \\ &+ \sum_{t=1}^{tt} \left[ \hat{A}X_{(i,jj+1,1,t)} \right] \left[ SWX_{(i,jj+1,1,t)} \right] \\ &+ \sum_{s=2}^3 \left[ \hat{A}X_{(i,jj+1,s)} \right] \left[ SWX_{(i,jj+1,s)} \right] \end{aligned}$$

where

$$SWX_{ijs} = PX_{ijs}[X_{ijs}] / TC_i,$$

and  $TC_i$  is the sum of all the terms on the right side of equation (13.2.1) which is the total cost of production in industry  $i$ .

$$\begin{aligned} SWX_{(i,jj+1,t)} &= [PX_{(i,jj+1,t)}][X_{(i,jj+1,t)}] / [TC_i] \\ SWX_{(i,jj+1,s)} &= [PX_{(i,jj+1,s)}][X_{(i,jj+1,s)}] / [TC_i] \\ SWX_{(i,jj+2)} &= [PX_{(i,jj+2)}][X_{(i,jj+2)}] / [TC_i] \end{aligned}$$

Equation (13.2.2) gives us the following comparative static result

$$\hat{P}Y_{j1} - \hat{\Psi}_{j1}^f + \varepsilon_{j1}^f [\hat{Q}_{j1}] = 0, \quad j=jj+1, \dots, jj, f \in F(j-jj+ic).$$

Equation (13.2.3) gives us the following comparative static result

$$\begin{aligned} (13.3.4) \quad [\hat{P}I_i] + \hat{I}_i &= \sum_{j=1}^{jj} \sum_{s=1}^2 [SWI_{ijs}] [\hat{I}_{ijs}] \\ &+ \sum_{j=1}^{jj} \sum_{s=1}^2 [SWI_{ijs}] [PI_{ijs}] \end{aligned}$$

where

$$[SWI_{ijs}] = \frac{[PI_i] I_i}{\sum_{j'=1}^{jj} \sum_{s'=1}^2 [PI_{ij's'}] I_{ij's'}}.$$

Using the expression for  $I_{ijs}$ , given by equation (8.1.4), we then have

$$\begin{aligned} (13.3.5) \quad \sum_{j=1}^{jj} \sum_{s=1}^2 [SWI_{ijs}] [\hat{I}_{ijs}] &= \hat{I}_i + \sum_{j=1}^{jj} \sum_{s=1}^2 [SWI_{ijs}] [\hat{A}I_{ijs}] \\ &+ \sum_{j=1}^{jj} \sum_{s=1}^2 [SWI_{ijs}] [\hat{A}I_{ij}] + [\hat{A}I_i] \\ &- \sum_{j=1}^{jj-1} [\sigma_{ij}^I] \left( \sum_{s=1}^2 [SWI_{ijs}] [\hat{P}I_{ijs}] \right. \\ &\quad \left. - \sum_{s=1}^2 [SWI_{ijs}] \left( \sum_{k=1}^2 [SI_{ijk}] [PI_{ijk}] \right) \right) \end{aligned}$$

$$- \sum_{j=1}^j [\sigma I_{ij}] \left( - \sum_{s=1}^2 [SWI_{ijs}] \left( \sum_{k=1}^2 [SI_{ijk}] [\hat{P}I_{ijk}] \right) \right)$$

We now show that the last two terms on the right side of equation (13.3.5) are equal to zero. Indeed, we have

$$(13.3.6) \quad \sum_{s=1}^2 [SWI_{ijs}] \left( \sum_{k=1}^2 [SI_{ijk}] [\hat{P}I_{ijk}] \right) = \sum_{k=1}^2 \left( [SI_{ijk}] \sum_{s=1}^2 [SWI_{ijs}] \right) [\hat{P}I_{ijk}]$$

where

$$[SI_{ijk}] = \frac{[PI_{ijk}] I_{ijk}}{\sum_{k'=1}^2 [PI_{ijk'}] I_{ijk'}}, \text{ and } \sum_{s=1}^2 [SWI_{ijs}] = \frac{\sum_{s=1}^2 [PI_{ijs}] I_{ijs}}{\sum_{j'=1}^j \sum_{s'=1}^2 [PI_{ij's'}] I_{ij's'}}$$

Hence

$$(13.3.7) \quad SI_{ijk} \left[ \sum_{s=1}^2 SWI_{ijs} \right] = SWI_{ijk}$$

Using (13.3.7), we can rewrite (13.3.6) as

$$(13.3.8) \quad \sum_{s=1}^2 [SWI_{ijs}] \left( \sum_{k=1}^2 [SI_{ijk}] [\hat{P}I_{ijk}] \right) = \sum_{k=1}^2 [SWI_{ijk}] [\hat{P}I_{ijk}]$$

It now follows from (13.3.8) that the fifth term on the right side of equation (13.3.5) is equal to zero. This argument can be repeated verbatim, with  $\hat{A}I_{ijs}$  in place of  $\hat{P}I_{ijs}$  to assert that the last (sixth) term on the right side of (13.3.5) is equal to zero. Now

substituting (13.3.5) into (13.3.4), we obtain the following end result

$$(13.3.9) \quad \hat{P}I_i = \sum_{j=1}^j \sum_{s=1}^2 [SWI_{ijs}] [\hat{P}I_{ijs}] + \sum_{j=1}^j \sum_{s=1}^2 [SWI_{ijs}] [\hat{A}I_{ijs}] + \sum_{j=1}^j [SWI_{ij}] [\hat{A}I_{ij}] + [\hat{A}I_i]$$

where

$$[SWI_{ij}] = \sum_{s=1}^2 [SWI_{ijs}].$$

Equation (13.2.4) gives us the following comparative static result

$$\begin{aligned} \wedge(13.3.10) \quad \hat{P}E_{j1} + \hat{\Phi} &= [W1PE_{j1}] [\hat{P}Y_{j1}] + [W2PE_{j1}] [\hat{H}E_{j1}] \\ &+ [W3PE_{j1}] \sum_{j'=1}^j [W3PE_{(j1,j'1)}] [\hat{P}Y_{j'1}] \\ &+ [W3PE_{j1}] \left( \sum_{j'=1}^j [W3PE_{(j1,j'1)}] [\hat{A}ME_{(j1,j'1)}] \right) \end{aligned}$$

where the weights are as follows:

$$\begin{aligned} W1PE_{j1} &= \frac{PY_{j1}}{[PY_{j1}] + [HE_{j1}] + \sum_{j'=1}^j [PY_{j'1}] [AME_{(j1,j'1)}]} \\ W2PE_{j1} &= \frac{HE_{j1}}{[PY_{j1}] + [HE_{j1}] + \sum_{j'=1}^j [PY_{j'1}] [AME_{(j1,j'1)}]} \\ W3PE_{j1} &= 1 - W1PE_{j1} - W2PE_{j1} \\ W3PE_{j1,j'1} &= \frac{[PY_{j'1}] [AME_{(j1,j'1)}]}{\sum_{k=1}^j [PY_{k1}] [AME_{(j1,k1)}]} \end{aligned}$$

From equation (13.2.5), we obtain the following comparative static result

$$\wedge(13.3.11) \quad [\hat{P}Y_{j2}] = [W1PY_{j2}] (P\hat{F}Y_{j2} + \hat{\Phi}) + [W2PY_{j2}] [\hat{H}Y_{j2}^1]$$

where the weights are given by

$$[W1PY_{j2}] = \frac{[PFY_{j2}] \Phi}{[PFY_{j2}] \Phi + HY_{j2}^1} \quad \text{and} \quad [W2PY_{j2}] = \frac{HY_{j2}^1}{[PFY_{j2}] \Phi + HY_{j2}^1}$$

Equation (13.2.6) gives us the following comparative static result

$$\begin{aligned} \wedge(13.3.12) \quad \sum_{i=1}^{ic} [WY_{ij1}] [\hat{Y}_{ij1}] &= \sum_{i=1}^i [WX_{ij1}] [\hat{X}_{ij1}] + \sum_{i=1}^i [WI_{ij1}] [\hat{I}_{ij1}] \\ &+ WC_{j1} [\hat{C}_{j1}] + WG_{j1} [\hat{G}_{j1}] + WE_{j1} [\hat{E}_{j1}] \\ &+ \sum_{i=1}^i \sum_{j'=1}^j \sum_{s=1}^2 [WMX_{(ij's,j1)}] [\hat{M}X_{(ij's,j1)}] \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMI_{(j's,j1)} \right] \left[ \hat{M}I_{(j's,j1)} \right] \\
& + \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMC_{(j's,j1)} \right] \left[ \hat{M}C_{(j's,j1)} \right] \\
& + \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMG_{(j's,j1)} \right] \left[ \hat{M}G_{(j's,j1)} \right] \\
& + \sum_{j'=1}^{jj} \left[ WME_{(j'1,j1)} \right] \left[ \hat{M}E_{(j'1,j1)} \right] \\
& + \sum_{i=ii c+1}^{ii} \left[ WMY_{(j'1.1,j1)} \right] \left[ \hat{M}Y_{(j'1.1,j1)} \right] \\
& + \sum_{i=ii c+1}^{ii} \left[ WMY_{(j'1.2,j1)} \right] \left[ \hat{M}Y_{(j'1.2,j1)} \right]
\end{aligned}$$

where

$$\begin{aligned}
WY_{j1} &= Y_{j1} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WX_{j1} &= X_{j1} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WI_{j1} &= I_{j1} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WC_{j1} &= C_{j1} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WG_{j1} &= G_{j1} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WE_{j1} &= E_{j1} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WMX_{(j's,j1)} &= MX_{(j's,j1)} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WMI_{(j's,j1)} &= MI_{(j's,j1)} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WMC_{(j's,j1)} &= MC_{(j's,j1)} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WMG_{(j's,j1)} &= MG_{(j's,j1)} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WME_{(j'1,j1)} &= ME_{(j'1,j1)} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WMY_{(j'1.1,j1)} &= MY_{(j'1.1,j1)} / \sum_{i'=1}^{iic} Y_{i'j1} \\
WMY_{(j'1.2,j1)} &= MY_{(j'1.2,j1)} / \sum_{i'=1}^{iic} Y_{i'j1}
\end{aligned}$$

Equation (13.2.7) gives us the following comparative static result

$$\begin{aligned}
(13.3.13) \quad \hat{Q}_{j1} = & \sum_{i=1}^{ii} \sum_{s=1}^2 [WX_{ijs}] [\hat{X}_{ijs}] + \sum_{i=1}^{ii} \sum_{s=1}^2 [WI_{ijs}] [\hat{I}_{ijs}] \\
& + \sum_{s=1}^2 [WC_{js}] [\hat{C}_{js}] + \sum_{s=1}^2 [WG_{js}] [\hat{G}_{js}] + [WGG_{j1}] [\hat{GG}_{j1}] \\
& + \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMX_{(ij's,j1)}] [\hat{MX}_{(ij's,j1)}] \\
& + \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMI_{(ij's,j1)}] [\hat{MI}_{(ij's,j1)}] \\
& + \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMC_{(j's,j1)}] [\hat{MC}_{(j's,j1)}] \\
& + \sum_{j'=1}^{jj} [WME_{(j'1,j1)}] [\hat{ME}_{(j'1,j1)}] \\
& + \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMG_{(j's,j1)}] [\hat{MG}_{(j's,j1)}] \\
& + \sum_{j'=j1c+1}^{jj} [WMGG_{(j'1,j1)}] [\hat{MGG}_{(j'1,j1)}] \\
& + \sum_{i=iic+1}^{ii} [WMY_{(ij'1,1,j1)}] [\hat{MY}_{(ij'1,1,j1)}] \\
& + \sum_{i=iic+1}^{ii} [WMY_{(ij'1,2,j1)}] [\hat{MY}_{(ij'1,2,j1)}], \quad j=j1c+1, \dots, jj.
\end{aligned}$$

Here the weights are defined as follows

$$\begin{aligned}
WX_{ijs} &= X_{ijs} / Q_{j1}, & WC_{js} &= C_{js} / Q_{j1} \\
WI_{ijs} &= I_{ijs} / Q_{j1}, & WG_{js} &= G_{js} / Q_{j1} \\
WI_{ijs} &= I_{ijs} / Q_{j1}, & WMX_{(ij's,j1)} &= MX_{(ij's,j1)} / Q_{j1} \\
WGG_{j1} &= GG_{j1} / Q_{j1}, & WMI_{(ij's,j1)} &= MI_{(ij's,j1)} / Q_{j1} \\
WMC_{(j's,j1)} &= MC_{(j's,j1)} / Q_{j1} \\
WMG_{(j's,j1)} &= MG_{(j's,j1)} / Q_{j1} \\
WME_{(j'1,j1)} &= ME_{(j'1,j1)} / Q_{j1} \\
WMGG_{(j'1,j1)} &= MGG_{(j'1,j1)} / Q_{j1} \\
WMY_{(ij'1,1,j1)} &= MY_{(ij'1,1,j1)} / Q_{j1} \\
WMY_{(ij'1,2,j1)} &= MY_{(ij'1,2,j1)} / Q_{j1}
\end{aligned}$$

The comparative static result of equation (13.2.8) is given by equation (10.2) of

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Equation (13.2.9) yields the following comparative static result

$$\hat{P}Q_{j2} = \hat{\Psi}_{j2} - \varepsilon_{j2} \hat{Q}_{j2}, \quad j=jjc+1, \dots, jj$$

Equation (13.2.10) gives us the following comparative static result

$$\hat{L}_i = \sum_{i=1}^i \left[ WX_{(i, jj+1, 1, t)} \right] \left[ \hat{X}_{(i, jj+1, 1, t)} \right],$$

where

$$\left[ WX_{(i, jj+1, 1, t)} \right] = X_{(i, jj+1, 1, t)} / \sum_{i=1}^i X_{(i', jj+1, 1, t)}$$

Equations (13.2.11) and (13.2.12) give us the following comparative static result

$$\hat{K}_i = \hat{X}_{(i, jj+1, 2)}$$

$$\hat{LAND}_i = \hat{X}_{(i, jj+1, 3)}$$

## CHAPTER 14

### THE COMPLETE MATHEMATICAL MODEL: A LINEAR SYSTEM IN PERCENTAGE CHANGES

In the thesis, the equations whose identifying numbers carry the star (\*) superscript constitute a statement of the mathematical model in its nonlinear form. The variables in the nonlinear form statement are all variables. Thus the nonlinear form statement of the model consists of equations, such as (6.3.7), (6.3.8), and (8.3.1),.... The linearized form -- in percentage changes -- of the mathematical model is represented by all the equations whose identifying numbers carry the hat (^) superscript.

Because our computational procedure is based upon the linearized form in percentage changes, we gather in this chapter all the equations that constitute the statement of the mathematical model under this linearized form.

Figure 14.1. The List of Equations, a Linear System in Percentage Changes

<u>No.</u>	<u>Equations</u>	<u>Subscript</u>	<u>Description</u>
5.2.1.5.	$\hat{X}_{ijs} - \hat{Z}_i + [\sigma X_{ij}] \left( [\hat{P}X_{ijs}] - \sum_{k=1}^2 [SX_{ijk}] [\hat{P}X_{ijk}] \right) - [\hat{A}X_{ijs}] - [\hat{A}X_{ij}] - [A\hat{X}Z_i] + [\sigma X_{ij}] \left( [\hat{A}X_{ijs}] - \sum_{k=1}^2 [SX_{ijk}] [\hat{A}X_{ijk}] \right) = 0$ <p>where</p> $\sigma X_{ij} = \frac{1}{1 - \alpha X_{ij}} \quad \text{and} \quad SX_{ijs} = \frac{[PX_{ijs}] X_{ijs}}{\sum_{k=1}^2 [PX_{ijk}] X_{ijk}}$	$i=1, \dots, ii,$ $j=1, \dots, jj,$ $s=1, 2$	Demands of perfectly competitive industry i for intermediate input j from source s: domestic and imports

$$\begin{aligned}
5.2.2.9 \quad & \hat{X}_{(i,jj+1,t)} - \hat{X}_{(i,jj+1)} - \hat{A}X_{(i,jj+1,t)} \\
& + \left[ \alpha X_{(i,jj+1,t)} \right] \left[ \hat{P}X_{(i,jj+1,t)} \right] \\
& - \left[ \alpha X_{(i,jj+1,t)} \right] \left[ \sum_{k=1}^n \left[ \tilde{S}X_{(i,jj+1,k)} \right] \left[ \hat{P}X_{(i,jj+1,k)} \right] \right] \\
& + \left[ \sigma X_{(i,jj+1,t)} \right] \left[ \hat{A}X_{(i,jj+1,t)} \right] \\
& - \left[ \alpha X_{(i,jj+1,t)} \right] \left[ \sum_{k=1}^n \left[ \tilde{S}X_{(i,jj+1,k)} \right] \left[ \hat{A}X_{(i,jj+1,k)} \right] \right] = 0
\end{aligned}$$

where

$$\begin{aligned}
\sigma X_{(i,jj+1,t)} &= \frac{1}{1 - \alpha X_{(i,jj+1,t)}} \\
SX_{(i,jj+1,t)} &= \frac{\left[ PX_{(i,jj+1,t)} \right] X_{(i,jj+1,t)}}{\sum_{k=1}^n \left[ PX_{(i,jj+1,k)} \right] X_{(i,jj+1,k)}} \\
\tilde{S}X_{(i,jj+1,t)} &= \frac{\left[ \sigma X_{(i,jj+1,t)} \right] \left[ SX_{(i,jj+1,t)} \right]}{\sum_{k=1}^2 \left[ \sigma X_{(i,jj+1,k)} \right] \left[ SX_{(i,jj+1,k)} \right]}
\end{aligned}$$

t=1,...,tt,  
i=1,...,iic  
Demands for  
labor per skill  
type by  
perfectly  
competitive  
industry i

$$\begin{aligned}
5.2.2.10 \quad & \hat{X}_{(i,jj+1,s)} - \hat{A}X_{(i,jj+1,s)} - \hat{A}X_{(i,jj+1)} - A\hat{X}Z_i + \hat{Z}_i \\
& + \alpha X_{(i,jj+1,s)} \left( \hat{P}X_{(i,jj+1,s)} - \sum_{k=1}^3 \left[ \tilde{S}X_{(i,jj+1,k)} \right] \left[ \hat{P}X_{(i,jj+1,k)} \right] \right) \\
& - \left( \hat{A}X_{(i,jj+1,s)} - \sum_{k=1}^3 \left[ \tilde{S}X_{(i,jj+1,k)} \right] \left[ \hat{A}X_{(i,jj+1,k)} \right] \right) = 0
\end{aligned}$$

s=1,2,3,  
i=1,...,iic  
Demands for  
primary factor s  
by perfectly  
competitive  
industry i

where

$$\begin{aligned}
\sigma X_{(i,jj+1,s)} &= \frac{1}{1 - \alpha X_{(i,jj+1,s)}} \\
SX_{(i,jj+1,s)} &= \frac{\left[ PX_{(i,jj+1,s)} \right] X_{(i,jj+1,s)}}{\sum_{k=1}^2 \left[ PX_{(i,jj+1,k)} \right] X_{(i,jj+1,k)}} \\
\tilde{S}X_{(i,jj+1,s)} &= \frac{\left[ \sigma X_{(i,jj+1,s)} \right] \left[ SX_{(i,jj+1,s)} \right]}{\sum_{k=1}^2 \left[ \sigma X_{(i,jj+1,k)} \right] \left[ SX_{(i,jj+1,k)} \right]} \\
PX_{(i,jj+1)} &= \frac{\sum_{t=1}^n \left( \left[ PX_{(i,jj+1,t)} \right] X_{(i,jj+1,t)} \right)}{X_{(i,jj+1)}}
\end{aligned}$$

$$\begin{aligned}
5.2.2.11 \quad & \hat{P}X_{(i,jj+1)} - \sum_{t=1}^n \left[ SX_{(i,jj+1,t)} \right] \left[ \hat{P}X_{(i,jj+1,t)} \right] \\
& - \sum_{t=1}^n \left[ SX_{(i,jj+1,t)} \right] \left[ \hat{A}X_{(i,jj+1,t)} \right] = 0
\end{aligned}$$

i=1,...,iic  
General price  
per unit of  
labor paid by  
perfectly  
competitive  
industry i

5.2.3.2	$\hat{X}_{i,jj+2} - \hat{Z}_i - A\hat{X}Z_i + \hat{A}X_{i,jj+2} = 0$	i=1,...,iic	Demands for "other costs" tickets by perfectly competitive industry i
5.3.7	$\hat{Y}_{ij1} - \hat{Z}_i + A\hat{Y}_{ij1} + A\hat{Y}Z_i$ $- [\sigma_{YY_{ij1}}] \left( P\hat{Y}_{ij1} - \sum_{j' \in L(i)} S\tilde{Y}_{ij'1} [P\hat{Y}_{ij'1}] \right)$ $+ [\sigma_{YY_{ij1}}] \left( A\hat{Y}_{ij1} - \sum_{j' \in L(i)} S\tilde{Y}_{ij'1} [A\hat{Y}_{ij'1}] \right) = 0$ <p>where</p> $\sigma_{YY_{ij1}} = \frac{1}{\alpha_{YY_{ij1}} - 1}$ $SSY_{ij1} = \frac{[PY_{ij1}] [YY_{ij1}]}{\sum_{j'=1}^{nn(i)} [PY_{ij'1}] [YY_{ij'1}]}$ $S\tilde{Y}_{ij1} = \frac{[\sigma_{YY_{ij1}}] SSY_{ij1}}{\sum_{j'=1}^{nn(i)} [\sigma_{YY_{ij'1}}] [SSY_{ij'1}]}$	j=1,...,nn(i) i=1,...,iic	Supplies of composite commodity j by perfectly competitive industry i
5.3.8	$\hat{Y}_{ij'1} - \hat{Y}_{ij1} + \hat{A}Y_{ij'1}^j = 0$	i=1,...,iic, j=1,...,nn(i) j' ∈ L(i,j)	Supplies of commodity j by perfectly competitive industry i
5.3.9	$P\hat{Y}_{ij1} - \sum_{j' \in L(i,j)} SSY_{ij'1}^j [P\hat{Y}_{ij'1}]$ $+ \sum_{j' \in L(i,j)} SSY_{ij'1}^j [\hat{A}Y_{ij'1}^j] = 0$ <p>where</p> $SSY_{ij'1}^j = \frac{[PY_{ij'1}] / [AY_{ij'1}^j]}{\sum_{k \in L(i,j)} ([PY_{ik1}] / [AY_{ik1}^j])}$	j=1,...,nn(i) i=1,...,iic	Price per unit of composite commodity j
5.4.2	$\hat{M}X_{(ij^s, j'1)} - A\hat{M}X_{(ij^s, j'1)} - \hat{X}_{ij^s} = 0$	jj'=1,...,jj i=1,...,iic s=1,2	Demands for margins to facilitate commodity flows to producers

$$5.4.5 \quad \hat{H}X_{ijs} - [\alpha_1 HX_{ijs}][C\hat{P}I] - [\alpha_2 HX_{ijs}](\hat{T}X_{ijs} + \hat{P}Y_{js}) - [\alpha_3 HX_{ijs}][\hat{V}X_{ijs}] = 0$$

$j=1, \dots, jj,$   
 $i=1, \dots, iic$   
 $s=1, 2$

Taxes  
(subsidies) paid  
by domestic  
producers for  
intermediate  
input  $js$  used  
for current  
production

$$5.4.6 \quad \hat{P}X_{ijs} - [W1PX_{ijs}][\hat{P}Y_{js}] - [W2PX_{ijs}][\hat{H}X_{ijs}] - [W3PX_{ijs}]\left(\sum_{j'=1}^{jj} [W3PX_{(ijs, j')}] [\hat{P}Y_{j'1}]\right) - [W3PX_{ijs}]\left(\sum_{j'=1}^{jj} [W3PX_{(ijs, j')}] [AMX_{(ijs, j')}] \right) = 0$$

$j=1, \dots, jj,$   
 $i=1, \dots, iic$   
 $s=1, 2$

Zero pure  
profits in the  
distribution of  
goods to  
domestic users

where

$$W1PX_{ijs} = \frac{PY_{js}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] AMX_{(ijs, j')}} AMX_{(ijs, j')}$$

$$W2PX_{ijs} = \frac{HX_{ijs}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}] AMX_{(ijs, j')}} AMX_{(ijs, j')}$$

$$W3PX_{ijs} = 1 - (W1PX_{ijs} + W2PX_{ijs})$$

$$W3PX_{(ijs, j'1)} = \frac{[PY_{j'1}] AMX_{(ijs, j'1)}}{\sum_{k=1}^{jj} ([PY_{k1}] AMX_{(ijs, k1)})}$$

$$6.4.21 \quad \sum_{j=1}^{jj} \sum_{s=1}^2 WX_{ijs}^f [\hat{X}_{ijs}^f] + \sum_{t=1}^n WX_{(i, jj+1, t)}^f [\hat{X}_{(i, jj+1, t)}^f] + \sum_{s=2}^3 WX_{(i, jj+1, s)}^f [\hat{X}_{(i, jj+1, s)}^f] + WX_{(i, jj+2)}^f [\hat{X}_{(i, jj+2)}^f] + \sum_{j=1}^{jj} \sum_{s=1}^2 WX_{ijs}^f [\hat{P}X_{ijs}] + \sum_{t=1}^n WX_{(i, jj+1, t)}^f [\hat{P}X_{(i, jj+1, t)}] + \sum_{s=2}^3 WX_{(i, jj+1, s)}^f [\hat{P}X_{(i, jj+1, s)}] + WX_{(i, jj+2)}^f [\hat{P}X_{(i, jj+2)}] - (W1Y_{ij1,1}^f - W3Y_{ij1,1}^f) \hat{\Psi}_{j1}^f + (W1Y_{ij1,1}^f [\varepsilon_{j1}^f] - W3Y_{ij1,1}^f (\varepsilon_{j1}^f + 1)) \hat{Q}_{j1} + W2Y_{ij1,1}^f [\hat{H}Y_{j1,1}^f] + W3Y_{ij1,1}^f [\hat{Y}_{ij1,1}^f] + W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} [W4Y_{ij1,1, j'1}^f] [AMY_{(ij1,1, j'1)}^f] \right) + W4Y_{ij1,1}^f \left( \sum_{j'=1}^{jj} [W4Y_{ij1,1, j'1}^f] [\hat{P}Y_{j'1}] \right) - \theta_i [\hat{Z}_i^f] + \hat{A}Y_{ij1} + \theta_i [A\hat{Y}Z_i] = 0$$

$f \in F(i, 1)$   
 $i = iic+1, \dots, ii$   
 $j = jjc+i-iic$

Profit-  
maximizing  
output of  
domestic  
oligopolists f

where

$$W1Y_{\bar{y}1,1}^f = \frac{\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f}}{\left( \Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j1,1}^1 + Y_{\bar{y}1,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{\bar{y}} AMY_{(\bar{y}1,1,j')} [PY_{j'1}] \right)}$$

$$W2Y_{\bar{y}1,1}^f = \frac{HY_{j1,1}^1}{\left( \Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j1,1}^1 + Y_{\bar{y}1,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{\bar{y}} AMY_{(\bar{y}1,1,j')} [PY_{j'1}] \right)}$$

$$W3Y_{\bar{y}1,1}^f = \frac{Y_{\bar{y}1,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f - 1}]}{\left( \Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j1,1}^1 + Y_{\bar{y}1,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{\bar{y}} AMY_{(\bar{y}1,1,j')} [PY_{j'1}] \right)}$$

$$W4Y_{\bar{y}1,1}^f = \frac{\sum_{j'=1}^{\bar{y}} AMY_{(\bar{y}1,1,j')} [PY_{j'1}]}{\left( \Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j1,1}^1 + Y_{\bar{y}1,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f - 1}] \right. \\ \left. - \sum_{j'=1}^{\bar{y}} AMY_{(\bar{y}1,1,j')} [PY_{j'1}] \right)}$$

$$W4Y_{\bar{y}1,1,j'1} = \frac{AMY_{(\bar{y}1,1,j'1)} [PY_{j'1}]}{\sum_{k=1}^{\bar{y}} AMY_{(\bar{y}1,1,k1)} [PY_{k1}]}$$

$$WX_{\bar{y}s}^f = \frac{PX_{\bar{y}s} [X_{\bar{y}s}^f]}{\left( \sum_{j=1}^{\bar{y}} \sum_{\sigma=1}^2 PX_{\bar{y}s} [X_{\bar{y}s}^f] + \sum_{\sigma=1}^{\alpha} PX_{(i,\bar{y}+11,\sigma)} [X_{(i,\bar{y}+11,\sigma)}^f] \right) \\ \left( + \sum_{\sigma=2}^3 PX_{(i,\bar{y}+1,s)} [X_{(i,\bar{y}+1,s)}^f] + PX_{(i,\bar{y}+2)} [X_{(i,\bar{y}+2)}^f] \right)}$$

$$WX_{(i,\bar{y}+11,t)}^f = \frac{PX_{(i,\bar{y}+11,t)} [X_{(i,\bar{y}+11,t)}^f]}{\left( \sum_{j=1}^{\bar{y}} \sum_{\sigma=1}^2 PX_{\bar{y}s} [X_{\bar{y}s}^f] + \sum_{\sigma=1}^{\alpha} PX_{(i,\bar{y}+11,\sigma)} [X_{(i,\bar{y}+11,\sigma)}^f] \right) \\ \left( + \sum_{\sigma=2}^3 PX_{(i,\bar{y}+1,s)} [X_{(i,\bar{y}+1,s)}^f] + PX_{(i,\bar{y}+2)} [X_{(i,\bar{y}+2)}^f] \right)}$$

$$WX_{(i,\bar{y}+1,s)}^f = \frac{PX_{(i,\bar{y}+1,s)} [X_{(i,\bar{y}+1,s)}^f]}{\left( \sum_{j=1}^{\bar{y}} \sum_{\sigma=1}^2 PX_{\bar{y}s} [X_{\bar{y}s}^f] + \sum_{\sigma=1}^{\alpha} PX_{(i,\bar{y}+11,\sigma)} [X_{(i,\bar{y}+11,\sigma)}^f] \right) \\ \left( + \sum_{\sigma=2}^3 PX_{(i,\bar{y}+1,s)} [X_{(i,\bar{y}+1,s)}^f] + PX_{(i,\bar{y}+2)} [X_{(i,\bar{y}+2)}^f] \right)}$$

$$WX_{(i,\bar{y}+2)}^f = \frac{PX_{(i,\bar{y}+2)} [X_{(i,\bar{y}+2)}^f]}{\left( \sum_{j=1}^{\bar{y}} \sum_{\sigma=1}^2 PX_{\bar{y}s} [X_{\bar{y}s}^f] + \sum_{\sigma=1}^{\alpha} PX_{(i,\bar{y}+11,\sigma)} [X_{(i,\bar{y}+11,\sigma)}^f] \right) \\ \left( + \sum_{\sigma=2}^3 PX_{(i,\bar{y}+1,s)} [X_{(i,\bar{y}+1,s)}^f] + PX_{(i,\bar{y}+2)} [X_{(i,\bar{y}+2)}^f] \right)}$$

6.4.22

$$\begin{aligned}
& \sum_{j=1}^j \sum_{s=1}^2 W X_{j s}^f \left[ \hat{X}_{j s}^f \right] + \sum_{t=1}^n W X_{(i, j+1, t)}^f \left[ \hat{X}_{(i, j+1, t)}^f \right] \\
& + \sum_{s=2}^3 W X_{(i, j+1, s)}^f \left[ \hat{X}_{(i, j+1, s)}^f \right] + W X_{(i, j+2)}^f \left[ \hat{X}_{(i, j+2)}^f \right] \\
& + \sum_{j=1}^j \sum_{s=1}^2 W X_{j s}^f \left[ \hat{P} X_{j s} \right] + \sum_{t=1}^n W X_{(i, j+1, t)}^f \left[ \hat{P} X_{(i, j+1, t)} \right] \\
& + \sum_{s=2}^3 W X_{(i, j+1, s)}^f \left[ \hat{P} X_{(i, j+1, s)} \right] + W X_{(i, j+2)}^f \left[ \hat{P} X_{(i, j+2)} \right] \\
& - \left( W 1 Y_{j 1, 2}^f - W 4 Y_{j 1, 2}^f \right) \hat{\Psi}_{j 2} \\
& + \left( W 1 Y_{j 1, 2}^f \left[ \varepsilon_{j 2} \right] - W 4 Y_{j 1, 2}^f \left( \varepsilon_{j 2} + 1 \right) \right) \hat{Q}_{j 2} \\
& + W 2 Y_{j 1, 2}^f \left[ \hat{H} Y_{j 1, 2}^1 \right] + W 3 Y_{j 1, 2}^f \left[ \hat{H} Y_{j 1, 2}^2 \right] + W 4 Y_{j 1, 2}^f \left[ \hat{Y}_{j 1, 2}^f \right] \\
& - \left( W 1 Y_{j 1, 2}^f - W 2 Y_{j 1, 2}^f - W 3 Y_{j 1, 2}^f - W 4 Y_{j 1, 2}^f \right) \left[ \hat{\Phi} \right] \\
& + W 5 Y_{j 1, 2}^f \left( \sum_{j'=1}^j \left[ W 5 Y_{j 1, 2, j'}^f \left[ \hat{A} M Y_{(j 1, 2, j')} \right] \right] \right) \\
& + W 5 Y_{j 1, 2}^f \left( \sum_{j'=1}^j \left[ W 5 Y_{j 1, 2, j'}^f \left[ \hat{P} Y_{j'} \right] \right] - \theta_i \left[ \hat{Z}_i^f \right] \right) \\
& + \hat{A} Y_{j 1} + \theta_i \left[ \hat{A} \hat{Y}_i \right] = 0
\end{aligned}$$

where

$$\begin{aligned}
W 1 Y_{j 1, 2}^f &= \frac{\Phi \left( \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}} \right)}{\left( \Phi \left( \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}} - H Y_{j 1, 2}^1 - H Y_{j 1, 2}^2 \right) \right. \\
& \left. + Y_{j 1, 2}^f \left( -\varepsilon_{j 2} \right) \left[ \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}-1} \right] \right) \\
& - \left( \sum_{j'=1}^j A M Y_{(j 1, 2, j')} \left[ P Y_{j'} \right] \right)} \\
W 2 Y_{j 1, 2}^f &= \frac{\Phi \left[ H Y_{j 1, 2}^1 \right]}{\left( \Phi \left( \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}} - H Y_{j 1, 2}^1 - H Y_{j 1, 2}^2 \right) \right. \\
& \left. + Y_{j 1, 2}^f \left( -\varepsilon_{j 2} \right) \left[ \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}-1} \right] \right) \\
& - \left( \sum_{j'=1}^j A M Y_{(j 1, 2, j')} \left[ P Y_{j'} \right] \right)} \\
W 3 Y_{j 1, 2}^f &= \frac{\Phi \left[ H Y_{j 1, 2}^2 \right]}{\left( \Phi \left( \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}} - H Y_{j 1, 2}^1 - H Y_{j 1, 2}^2 \right) \right. \\
& \left. + Y_{j 1, 2}^f \left( -\varepsilon_{j 2} \right) \left[ \Psi_{j 2} \left[ Q_{j 2} \right]^{-\varepsilon_{j 2}-1} \right] \right) \\
& - \left( \sum_{j'=1}^j A M Y_{(j 1, 2, j')} \left[ P Y_{j'} \right] \right)}
\end{aligned}$$

$f \in F(i, 1)$   
 $i = iic + 1, \dots, ii$   
 $j = jic + i - iic$

Profit-  
 maximizing  
 output of  
 domestic  
 oligopolists  $f$

$$W4Y_{ij1,2}^f = \frac{\phi \left( Y_{ij1,2}^f (\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2}^{-1}} \right)}{\left( \phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) + Y_{ij1,2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2}^{-1}} \right) - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'}] \right)}$$

$$W5Y_{ij1,2}^f = \frac{\sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'}]}{\left( \phi \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} - HY_{j1,2}^1 - HY_{j1,2}^2 \right) + Y_{ij1,2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2}^{-1}} \right) - \left( \sum_{j'=1}^j AMY_{(ij1,2,j')} [PY_{j'}] \right)}$$

$$W5Y_{ij1,2,j'}^f = \frac{AMY_{(ij1,2,j')} [PY_{j'}]}{\sum_{k=1}^j AMY_{(ij1,2,k')} [PY_{k}]}$$

- 6.4.23  $\theta_i [\hat{Z}_i^f] - \hat{A}Y_{ij1} - \hat{Y}_{ij1}^f - \theta_i [A\hat{Y}Z_i] = 0$  f ∈ F(i,1)  
i = iic+1, ..., ii  
j = jic+i-iic  
Output produced by domestic oligopolist firm f
- 6.4.24  $\hat{Y}_{ij1}^f - SY_{ij1,1}^f [\hat{Y}_{ij1,1}^f] - SY_{ij1,2}^f [\hat{Y}_{ij1,2}^f] - SGG_{ij1}^f [\hat{G}G_{ij1}^f] = 0$ ,  
where  
 $SY_{ij1,1}^f = Y_{ij1,1}^f / Y_{ij1}^f$ ,  $SY_{ij1,2}^f = Y_{ij1,2}^f / Y_{ij1}^f$ , and  
 $SGG_{ij1}^f = GG_{ij1}^f / Y_{ij1}^f$  f ∈ F(i,1)  
i = iic+1, ..., ii  
j = jic+i-iic  
Output supplied by domestic oligopolist firm f
- 6.4.28  $\hat{Q}_{j1} - \sum_{f \in F(i,1)} SQY_{ij1,1}^f [\hat{Y}_{ij1,1}^f] - \sum_{f \in F(i,2)} SQY_{ij2,1}^f [\hat{Y}_{ij2,1}^f] = 0$  j = jic+1, ..., jj  
Domestic sale of both oligopolists firms: domestic and foreign
- 6.4.29  $\hat{Q}_{j2} - \sum_{f \in F(i,1)} SQY_{ij1,2}^f [\hat{Y}_{ij1,2}^f] - \sum_{f \in F(i,2)} SQY_{ij2,2}^f [\hat{Y}_{ij2,2}^f] = 0$  j = jic+1, ..., jj
- where  
 $SQY_{ij1,1}^f = Y_{ij1,1}^f / Q_{j1}$  and  $SQY_{ij2,1}^f = Y_{ij2,1}^f / Q_{j1}$   
 $SQY_{ij1,2}^f = Y_{ij1,2}^f / Q_{j2}$  and  $SQY_{ij2,2}^f = Y_{ij2,2}^f / Q_{j2}$

$$\begin{aligned}
6.4.32 \quad & \hat{X}_{ys}^f - \hat{Z}_i^f - \hat{A}X_{ys} - \hat{A}X_{ij} - A\hat{X}Z_i \\
& + \left[ \alpha X_{ij} \right] \left\{ \hat{P}X_{ys} - \sum_{k=1}^2 \left[ SX_{jk}^f \right] \left[ \hat{P}X_{jk} \right] \right\} \\
& + \left[ \alpha X_{ij} \right] \left\{ \hat{A}X_{ys} - \sum_{k=1}^2 \left[ SX_{jk}^f \right] \left[ \hat{A}X_{jk} \right] \right\} = 0
\end{aligned}$$

$s=1,2$   
 $i=iic+1,\dots,ii$   
 $j=1,\dots,ij$   
 $f \in F(i,1)$

Demands for  
 intermediate  
 input  $js$  by  
 domestic  
 oligoplists firm  
 $f$

where

$$\begin{aligned}
\alpha X_{ij} &= \frac{1}{1 - \alpha X_{ij}} \\
SX_{ys}^f &= \frac{\left[ PX_{ys} \right] \left[ X_{ys}^f \right]}{\sum_{k=1}^n \left[ PX_{yk} \right] \left[ X_{yk}^f \right]}
\end{aligned}$$

$$\begin{aligned}
6.4.33 \quad & \hat{X}_{(i,jj+1,1,t)}^f - \hat{X}_{(i,jj+1,1)}^f - \hat{A}X_{(i,jj+1,1,t)} \\
& + \left[ \alpha X_{(i,jj+1,1,t)} \right] \left\{ \hat{P}X_{(i,jj+1,1,t)} - \sum_{k=1}^n \left[ \tilde{S}X_{(i,jj+1,1,k)}^f \right] \left[ \hat{P}X_{(i,jj+1,1,k)} \right] \right\} \\
& + \left[ \alpha X_{(i,jj+1,1,t)} \right] \left\{ \hat{A}X_{(i,jj+1,1,t)} - \sum_{k=1}^n \left[ \tilde{S}X_{(i,jj+1,1,k)}^f \right] \left[ \hat{A}X_{(i,jj+1,1,k)} \right] \right\} = 0
\end{aligned}$$

$t=1,\dots,tt$   
 $i=iic+1,\dots,ii$   
 $f \in F(i,1)$

Demands for  
 labor of skill  
 type  $t$  by  
 domestic  
 oligoplists firm  
 $f$

where

$$\begin{aligned}
\alpha X_{(i,jj+1,1,t)} &= \frac{1}{1 - \alpha X_{(i,jj+1,1,t)}} \\
SX_{(i,jj+1,1,t)}^f &= \frac{\left[ PX_{i,jj+1,1,t} \right] \left[ X_{(i,jj+1,1,t)}^f \right]}{\sum_{k=1}^n \left[ PX_{i,jj+1,1,k} \right] \left[ X_{(i,jj+1,1,k)}^f \right]} \\
\tilde{S}X_{(i,jj+1,1,t)}^f &= \frac{\left[ \alpha X_{i,jj+1,1,t} \right] \left[ SX_{(i,jj+1,1,t)}^f \right]}{\sum_{k=1}^n \left[ \alpha X_{i,jj+1,1,k} \right] \left[ SX_{(i,jj+1,1,k)}^f \right]}
\end{aligned}$$

$$\begin{aligned}
6.4.34 \quad & \hat{X}_{(i,jj+1,s)}^f - \hat{A}X_{(i,jj+1,s)} - \hat{A}X_{(i,jj+1)} - A\hat{X}Z_i - \hat{Z}_i^f \\
& + \alpha X_{(i,jj+1,s)} \left( \hat{P}X_{(i,jj+1,s)} - \sum_{k=1}^3 \left[ \tilde{S}X_{(i,jj+1,k)}^f \right] \left[ \hat{P}X_{(i,jj+1,k)} \right] \right. \\
& \left. - \hat{A}X_{(i,jj+1,s)} + \sum_{k=1}^3 \left[ \tilde{S}X_{(i,jj+1,k)}^f \right] \left[ \hat{A}X_{(i,jj+1,k)} \right] \right) = 0
\end{aligned}$$

$s=1,2,3$   
 $i=iic+1,\dots,ii$   
 $f \in F(i,1)$

Demands for  
 factor  $s$  by  
 domestic  
 oligoplist firm  $f$

where

	$\alpha X_{(i,jj+1,s)} = \frac{1}{1 - \alpha X_{(i,jj+1,s)}}$	$s=1,2,3$ $i=iic+1,\dots,ii$ $f \in F(i,1)$	Demands for factor s by domestic oligopolist firm f
	$SX_{(i,jj+1,s)}^f = \frac{[PX_{(i,jj+1,s)}][X_{(i,jj+1,s)}^f]}{\sum_{k=1}^3 [SX_{(i,jj+1,k)}^f][X_{(i,jj+1,k)}^f]}$		
	$\tilde{S}X_{(i,jj+1,s)}^f = \frac{[\sigma X_{(i,jj+1,s)}][SX_{(i,jj+1,s)}^f]}{\sum_{k=1}^3 [\sigma X_{(i,jj+1,k)}][SX_{(i,jj+1,k)}^f]}$		
6.4.35	$\hat{P}X_{(i,jj+1,1)} - \sum_{t=1}^n [SX_{(i,jj+1,1,t)}^f][\hat{P}X_{(i,jj+1,1,t)}] - \sum_{t=1}^n [SX_{(i,jj+1,1,t)}^f][\hat{A}X_{(i,jj+1,1,t)}] = 0$	$i=iic+1,\dots,ii$ $f \in F(i,1)$	General price of labor per unit paid by oligopolists
6.4.36	$\hat{X}_{(i,jj+2)}^f - \hat{Z}_i^f - A\hat{X}Z_i - \hat{A}X_{(i,jj+2)} = 0.$	$i=iic+1,\dots,ii$ $f \in F(i,1)$	Demands for "other costs" tickets
6.5.5	$\hat{X}_{ijs}^f - \sum_{f \in F(i,1)} SQX_{ijs}^f [\hat{X}_{ijs}^f] = 0$ <p>where</p> $SQX_{ijs}^f = \frac{X_{ijs}^f}{X_{ijs}}$	$i=iic+1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	Total demand for intermediate inputs js by domestic oligopolist industry i
6.5.7	$\hat{X}_{(i,jj+1,1,t)}^f - \sum_{f \in F(i,1)} SQX_{(i,jj+1,1,t)}^f [\hat{X}_{(i,jj+1,1,t)}^f] = 0$ <p>where</p> $SQX_{(i,jj+1,1,t)}^f = \frac{X_{(i,jj+1,1,t)}^f}{X_{(i,jj+1,1,t)}}$	$i=iic+1,\dots,ii$ $t=1,\dots,tt$	Total demand for labor type t by domestic oligopolist industry i
6.5.9	$\hat{X}_{(i,jj+1,s)}^f - \sum_{f \in F(i,1)} SQX_{(i,jj+1,s)}^f [\hat{X}_{(i,jj+1,s)}^f] = 0$ <p>where</p> $SQX_{(i,jj+1,s)}^f = \frac{X_{(i,jj+1,s)}^f}{X_{(i,jj+1,s)}}$	$i=iic+1,\dots,ii$ $s=1,2,3$	Total demand for factor by domestic oligopolist firm f in industry i
6.5.11	$\hat{X}_{(i,jj+2)}^f - \sum_{f \in F(i,1)} SQX_{(i,jj+2)}^f [\hat{X}_{(i,jj+2)}^f] = 0$ <p>where</p> $SQX_{(i,jj+2)}^f = \frac{X_{(i,jj+2)}^f}{X_{(i,jj+2)}}$	$i=iic+1,\dots,ii$	Total demand for other cost tickets by the domestic imperfectly competitive industry i

6.5.14	$\hat{M}X_{(ijs,j'1)} - \hat{A}M\hat{X}_{(ijs,j'1)} - \hat{X}_{ijs} = 0$	j,j'=1,...,jj i=iic+1,...,ii s=1,2	Demands for margins to facilitate commodity flows to producers
6.5.17	$\hat{H}X_{ijs} - [\alpha 1HX_{ijs}][C\hat{P}I] - [\alpha 2HX_{ijs}][\hat{T}X_{ijs} + \hat{P}Y_{js}] - [\alpha 3HX_{ijs}][\hat{V}X_{ijs}] = 0$	j=1,...,jj i=iic+1,...,ii s=1,2	Taxes (subsidies) paid by domestic producers for intermediate input <i>js</i> used for current production
6.5.18	$\hat{P}X_{ijs} - [W1PX_{ijs}][\hat{P}Y_{js}] - [W2PX_{ijs}][\hat{H}X_{ijs}] - [W3PX_{ijs}][\sum_{j'=1}^{jj} [W3PX_{(ijs,j'1)}][\hat{P}Y_{j'1}]] - [W3PX_{ijs}][\sum_{j'=1}^{jj} [W3PX_{(ijs,j'1)}][\hat{A}M\hat{X}_{(ijs,j'1)}]] = 0$	j=1,...,jj i=iic+1,...,ii s=1,2	Zero pure profits in the distribution of goods to domestic users
	where		
	$W1PX_{ijs} = \frac{PY_{js}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}][AM\hat{X}_{(ijs,j'1)}]}$		
	$W2PX_{ijs} = \frac{HX_{ijs}}{PY_{js} + HX_{ijs} + \sum_{j'=1}^{jj} [PY_{j'1}][AM\hat{X}_{(ijs,j'1)}]}$		
	$W3PX_{ijs} = 1 - (W1PX_{ijs} + W2PX_{ijs})$		
	$W3PX_{(ijs,j'1)} = \frac{[PY_{j'1}][AM\hat{X}_{(ijs,j'1)}]}{\sum_{k=1}^{jj} ([PY_{k1}][AM\hat{X}_{(ijs,k1)}])}$		
6.5.23	$\hat{Y}_{ij1,1} - \sum_{f \in F(i,1)} WY_{ij1,1}^f [\hat{Y}_{ij1,1}^f] = 0$	i=iic+1,...,ii j=i-iic+jjc	Output supplied at home by imperfectly competitive industry <i>i</i>
	where		
	$WY_{ij1,1}^f = Y_{ij1,1}^f / Y_{ij1,1}$		
6.5.24	$\hat{M}Y_{(ij1,1,j'1)} - \hat{A}M\hat{Y}_{(ij1,1,j'1)} - \hat{Y}_{ij1,1} = 0$	j=jjc+1,...,jj i=j-jjc+iic j'=1,...,jj	Margins needed to transfer domestic supply to domestic market

6.5.25  $\hat{Y}_{ij1,2} - \sum_{f \in F(i,1)} WY_{ij1,2}^f [\hat{Y}_{ij1,2}^f] = 0$  j=jjc+1,...,jj  
i=j-jjc+iic Output supplied in foreign market by imperfectly competitive industry i

where  $WY_{ij1,2}^f = Y_{ij1,2}^f / Y_{ij1,2}$

6.5.26  $\hat{M}Y_{(ij1,2,j'1)} - A\hat{M}Y_{(ij1,2,j'1)} - \hat{Y}_{ij1,2} = 0$  i=iic+1,...,ii  
j=i-iic+jjc,  
j'=1,...,jj Margins needed to transfer domestic supply to foreign market

6.7.9  $UCZ_{i2} - (W1Y_{ij2,1}^f - W4Y_{ij2,1}^f) \hat{\Psi}_{j1} + \left( \frac{W1Y_{ij2,1}^f [\epsilon_{j1}^f]}{-W4Y_{ij2,1}^f (\epsilon_{j1}^f + 1)} \right) \hat{Q}_{j1}$  i=iic+1,...,ii  
j=i-iic+jjc  
f \in F(i,2) Unit cost of output of domestic imperfectly competitive industry i

$$+ W2Y_{ij2,1}^f [\hat{H}Y_{j2,1}^1] + W3Y_{ij2,1}^f [\hat{H}Y_{j2,1}^2] + W4Y_{ij2,1}^f [\hat{Y}_{ij2,1}^f] - (\theta_{i2} - 1) \hat{Z}_i^f$$

$$- (W1Y_{ij2,1}^f - W2Y_{ij2,1}^f - W3Y_{ij2,1}^f - W4Y_{ij2,1}^f) \hat{\Phi}$$

$$+ W5Y_{ij2,1}^f \left( \sum_{j'=1}^{jj} [W5Y_{ij2,1,j'2}^f [A\hat{M}Y_{(ij2,1,j'2)}]] \right)$$

$$+ W5Y_{ij2,1}^f \left( \sum_{j'=1}^{jj} [W5Y_{ij2,1,j'2}^f [\hat{P}Y_{j'2}]] \right) + \hat{A}Y_{ij2} + \theta_{i2} [A\hat{Y}Z_{i2}] = 0$$

where

$$W1Y_{ij2,1}^f = \frac{(\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f}) / \Phi}{\frac{1}{\Phi} \left( \frac{\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2}{+Y_{ij2,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\epsilon_{j1}^f - 1}} \right) - \sum_{j'=1}^{jj} AMY_{(ij2,1,j'2)} [PY_{j'2}]}{HY_{j2,1}^1 / \Phi}$$

$$W2Y_{ij2,1}^f = \frac{\frac{1}{\Phi} \left( \frac{\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2}{+Y_{ij2,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\epsilon_{j1}^f - 1}} \right) - \sum_{j'=1}^{jj} AMY_{(ij2,1,j'2)} [PY_{j'2}]}{HY_{j2,1}^2 / \Phi}$$

$$W3Y_{ij2,1}^f = \frac{\frac{1}{\Phi} \left( \frac{\Psi_{j1}^f [Q_{j1}]^{-\epsilon_{j1}^f} - HY_{j2,1}^1 - HY_{j2,1}^2}{+Y_{ij2,1}^f (-\epsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\epsilon_{j1}^f - 1}} \right) - \sum_{j'=1}^{jj} AMY_{(ij2,1,j'2)} [PY_{j'2}]}{HY_{j2,1}^2 / \Phi}$$

$$W4Y_{ij2.1}^f = \frac{\left( Y_{ij2.1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right) / \Phi}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2.1}^1 - HY_{j2.1}^2 \right) + Y_{ij2.1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right) - \sum_{j'=1}^j AMY_{(ij2.1, j'2)} [PY_{j'2}]}$$

$$W5Y_{ij2.1}^f = \frac{\left( \sum_{j'=1}^j AMY_{(ij2.1, j'2)} [PY_{j'2}] \right)}{\left( \frac{1}{\Phi} \left( \Psi_{j1}^f [Q_{j1}]^{-\varepsilon_{j1}^f} - HY_{j2.1}^1 - HY_{j2.1}^2 \right) + Y_{ij2.1}^f (-\varepsilon_{j1}^f) [\Psi_{j1}^f] [Q_{j1}]^{-\varepsilon_{j1}^f - 1} \right) - \sum_{j'=1}^j AMY_{(ij2.1, j'2)} [PY_{j'2}]}$$

$$W5Y_{ij2.1, j'2}^f = \frac{\left( AMY_{(ij2.1, j'2)} [PY_{j'2}] \right)}{\left( \sum_{k=1}^j AMY_{(ij2.1, k2)} [PY_{k2}] \right)}$$

6.7.16

$$UCZ_{i2} - \left( \begin{array}{c} W1Y_{ij2.2}^f \\ -W3Y_{ij2.2}^f \end{array} \right) \hat{\Psi}_{j2} + \left( \begin{array}{c} W1Y_{ij2.2}^f [\varepsilon_{j2}] \\ -W3Y_{ij2.2}^f (\varepsilon_{j2} + 1) \end{array} \right) \hat{Q}_{j2}$$

$$+ W2Y_{ij2.2}^f [\hat{HY}_{j2.2}^2] + W3Y_{ij2.2}^f [\hat{Y}_{ij2.2}^f] - (\theta_{i2} - 1) \hat{Z}_i^f + W4Y_{ij2.2}^f \left( \sum_{j'=1}^j [W4Y_{ij2.2, j'2}^f] [AMY_{(ij2.2, j'2)}] \right) + W4Y_{ij2.2}^f \left( \sum_{j'=1}^j [W4Y_{ij2.2, j'2}^f] [\hat{PY}_{j'2}] \right) + \hat{AY}_{ij2} + \theta_{i2} [A\hat{YZ}_{i2}] = 0$$

where

$$W1Y_{ij2.2}^f = \frac{\left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} \right)}{\left( \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} + Y_{ij2.2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2} - 1} \right) - \left( \sum_{j'=1}^j AMY_{(ij2.2, j'2)} [PY_{j'2}] \right) - HY_{j2.2}^2 \right)}$$

$$W2Y_{ij2.2}^f = \frac{HY_{j2.2}^2}{\left( \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} + Y_{ij2.2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2} - 1} \right) - \left( \sum_{j'=1}^j AMY_{(ij2.2, j'2)} [PY_{j'2}] \right) - HY_{j2.2}^2 \right)}$$

$$W3Y_{ij2.2}^f = \frac{Y_{ij2.2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2} - 1}}{\left( \left( \Psi_{j2} [Q_{j2}]^{-\varepsilon_{j2}} + Y_{ij2.2}^f (-\varepsilon_{j2}) [\Psi_{j2}] [Q_{j2}]^{-\varepsilon_{j2} - 1} \right) - \left( \sum_{j'=1}^j AMY_{(ij2.2, j'2)} [PY_{j'2}] \right) - HY_{j2.2}^2 \right)}$$

i=ii+1,...,ii Unit cost of  
j=jjc+i-iiic output of  
f∈F(i,2) foreign  
imperfectly  
competitive  
industry i

$$W4Y_{j'2}^f = \frac{(\sum_{j'=1}^j AMY_{(j'2,2,j')} [PY_{j'2}])}{\left( (\Psi_{j'2} [Q_{j'2}]^{-\varepsilon_{j'2}} + Y_{j'2}^f (-\varepsilon_{j'2}) [\Psi_{j'2} [Q_{j'2}]^{-\varepsilon_{j'2}-1}) \right) - (\sum_{j'=1}^j AMY_{(j'2,2,j')} [PY_{j'2}] - HY_{j'2}^2)}$$

$$W4Y_{ij'2,2,j'2}^f = \frac{AMY_{(ij'2,2,j'2)} [PY_{j'2}]}{\sum_{k=1}^j AMY_{(ij'2,2,k2)} [PY_{k2}]}$$

$$6.7.17 \quad \theta_{i2} [\hat{Z}_i^f] - A\hat{Y}Z_{i2} - \hat{Y}_{ij'2}^f - \theta_{i2} [A\hat{Y}Z_{i2}] = 0 \quad \begin{array}{l} i=iic+1,\dots,i \\ i \\ j=i-iic+jjc \\ f \in F(i,2) \end{array} \quad \begin{array}{l} \text{Output produced by} \\ \text{foreign oligopolist} \\ \text{firm } f \end{array}$$

$$6.7.18 \quad \hat{Y}_{j'2}^f - SY_{j'2,1}^f [\hat{Y}_{j'2,1}^f] - SY_{j'2,2}^f [\hat{Y}_{j'2,2}^f] - SGG_{j'2,2}^f [\hat{GG}_{j'2,2}^f] = 0 \quad \begin{array}{l} i=iic+1,\dots,i \\ i \\ j=i-iic+jjc \\ f \in F(i,2) \end{array} \quad \begin{array}{l} \text{Output supplied by} \\ \text{foreign oligopolist} \\ \text{firm } f \end{array}$$

where

$$SY_{ij'2,1}^f = Y_{ij'2,1}^f / Y_{ij'2}^f$$

$$SY_{ij'2,2}^f = Y_{ij'2,2}^f / Y_{ij'2}^f$$

$$SGG_{ij'2,2}^f = GG_{ij'2,2}^f / Y_{ij'2}^f$$

$$6.7.23 \quad \hat{Y}_{j'2} - \sum_{f \in F(j-jjc+iic,2)} [WY_{ij'2,1}^f] \hat{Y}_{ij'2,1}^f = 0 \quad \begin{array}{l} j=jjc+1,\dots,j \\ j \\ i=j-jjc+iic \end{array}$$

where

$$WY_{ij'2,1}^f = Y_{ij'2,1}^f / Y_{j'2}^f$$

$$8.1.4 \quad \hat{I}_{ijs} - \hat{I}_i - \hat{A}I_{ijs} - \hat{A}I_{ij} - \hat{A}I_i + [\sigma I_{ij}] \left( \left[ \hat{P}I_{ijs} \right] - \sum_{k=1}^2 \left[ SI_{ijk} \right] \left[ \hat{P}I_{jk} \right] \right) - \sum_{k=1}^2 \left[ SI_{ijk} \right] \left[ \hat{A}I_{jk} \right] = 0 \quad \begin{array}{l} i=1,\dots,ii, \\ j=1,\dots,ij, \\ s=1,2 \end{array} \quad \begin{array}{l} \text{Demands for inputs to} \\ \text{capital creation} \end{array}$$

where

$$\sigma I_{ij} = \frac{1}{1 - \alpha I_{ij}} \quad \text{and} \quad SI_{ijs} = \frac{[PI_{ijs}] I_{ijs}}{\sum_{k=1}^2 ([PI_{ijk}] I_{ijk})}$$

$$8.2.11 \quad (\sum_{i=1}^{ii1} [SI_i]) \hat{I}1 - \sum_{i=1}^{ii1} ([SI_i] (\hat{P}I_i + \hat{I}_i)) = 0 \quad \text{Investment budget}$$

where

$$SI_i = \frac{[PI_i] I_i}{\sum_{k=1}^{ii1} ([PI_k] I_k)} \quad \text{and}$$

$$\sum_{i=1}^{ii1} SI_i = \frac{I1}{I1 + I2}$$

8.2.12	$\hat{K}1_i - \hat{K}_i \left(1 - \frac{I_i}{K1_i}\right) - \left(\frac{I_i}{K1_i}\right) \hat{I}_i = 0$	i=1,...,ii	Capital stock for the next period
8.2.13	$\left(\frac{R_i}{R_i + d_i}\right) \hat{R}_i - \hat{P}X_{(i,jj+1,2)} + \hat{P}I_i = 0$	i=1,...,ii	Rates of return on capital in each industry
8.2.14	$\hat{R}1_i - \hat{R}_i + [\alpha K_i] (\hat{K}1_i - \hat{K}_i) = 0$	i=1,...,ii1	Expected rates of return on capital
8.2.15	$\hat{I}_i - [\alpha I_i] \hat{I}1R - SH\hat{F}T I_i = 0$	i=ii1+1,...,ii	Equations for handling exogenous investment
8.2.16	$\hat{I}1R - \hat{I} + K\hat{P}I = 0$		Real private investment expenditure
8.3.2	$\hat{M}I_{(ijs,j'1)} - A\hat{M}I_{(ijs,j'1)} - \hat{I}_{ijs} = 0$	i=1,...,ii s=1,2, j',j=1,...,jj	Demands for margins to facilitate commodity flows to capital creators
8.3.5	$\hat{H}I_{ijs} - [\alpha 1HI_{ijs}] [C\hat{P}I] - [\alpha 2HI_{ijs}] (\hat{T}I_{ijs} + \hat{P}Y_{js}) - [\alpha 3HI_{ijs}] [\hat{V}I_{ijs}] = 0$	i=1,...,ii j=1,...,jj s=1,2	Taxes (subsidies) paid by domestic producers for intermediate input <i>js</i> used for capital creation
8.3.6	$\hat{P}I_{ijs} - W1PI_{ijs} [\hat{P}Y_{js}] - W2PI_{ijs} [\hat{H}I_{ijs}] - W3PI_{ijs} \left( \sum_{j'=1}^{jj} W3PI_{(ijs,j'1)} [\hat{P}Y_{j'1}] \right) - W3PI_{ijs} \left( \sum_{j'=1}^{jj} W3PI_{(ijs,j'1)} [A\hat{M}I_{(ijs,j'1)}] \right) = 0$	i=1,...,ii j=1,...,jj s=1,2	The actual cost of 1 unit good <i>js</i> to be used for capital formation in industry <i>i</i>
	where		
	$W1PI_{ijs} = \frac{PY_{js}}{PY_{js} + HI_{ijs} + \sum_{j'=1}^{jj} PY_{j'1} [A\hat{M}I_{(ijs,j'1)}]}$		
	$W2PI_{ijs} = \frac{HI_{ijs}}{PY_{js} + HI_{ijs} + \sum_{j'=1}^{jj} PY_{j'1} [A\hat{M}I_{(ijs,j'1)}]}$		
	$W3PI_{ijs} = 1 - W1PI_{ijs} - W2PI_{ijs}$		
9.2.18	$\hat{C}_{js} - \hat{C}_j - \hat{A}C_{js} + [\alpha C_j] \left( \hat{P}C_{js} - \sum_{k=1}^2 [SC_{jk}] [\hat{P}C_{js}] \right) + [\alpha C_j] \left( \hat{A}C_{js} - \sum_{k=1}^2 [SC_{jk}] [\hat{A}C_{js}] \right) = 0$	j=1,...,jj, s=1,2	Household demands for commodities classified by source

	where	$\alpha C_j = \frac{1}{1 - \alpha C_j} \text{ and } SC_{js} = \frac{[PC_{js}] C_{js}}{\sum_{k=1}^j ([PC_{jk}] C_{jk})}$	$j=1, \dots, j$ $s=1, 2$	
9.2.21	$\hat{P}C_j - \sum_{s=1}^2 [SC_{js}] [\hat{P}C_{js}] = 0$		$j=1, \dots, j$	General price of each commodity to households
9.2.23	$\hat{C}_j - \text{income}_j [\hat{M}] - \sum_{k=1}^j \text{price}_{jk} [\hat{P}C_k] - \hat{A}C_j - \sum_{k=1}^j (\text{price}_{jk} (\hat{A}C_k + \sum_{s=1}^2 ([SC_{ks}] \hat{A}C_{ks}))) = 0$		$j=1, \dots, j$	Household demands for commodities, undifferentiated by source
9.4.2	$\hat{M}C_{(js, j'1)} - \hat{A}M C_{(js, j'1)} - \hat{C}_{js} = 0$		$j, j'=1, \dots, j$ $s=1, 2$	Demands for margins to facilitate commodity flows to households
9.4.5	$\hat{H}C_{js} - [\alpha 1 HC_{js}] C \hat{P}I - [\alpha 2 HC_{js}] (\bar{T}C_{js} + \hat{P}Y_{js}) - [\alpha 3 HC_{js}] \hat{V}C_{js} = 0$		$j=1, \dots, j$ $s=1, 2$	Taxes (subsidies) paid by domestic consumption of goods $js$
9.4.6	$\hat{P}C_{js} - [W1 PC_{js}] [\hat{P}Y_{js}] - [W2 PC_{js}] [\hat{H}C_{js}] - [W3 PC_{js}] (\sum_{j'=1}^j [W3 PC_{(js, j'1)}] [\hat{P}Y_{j'1}]) - [W3 PC_{js}] (\sum_{j'=1}^j [W3 PC_{(js, j'1)}] [\hat{A}M C_{(js, j'1)}]) = 0$		$j=1, \dots, j$ $s=1, 2$	As mentioned above
	where	$W1 PC_{js} = \frac{PY_{js}}{PY_{js} + HC_{js} + \sum_{j'=1}^j [PY_{j'1}] AM C_{(js, j'1)}}$ $W2 PC_{js} = \frac{HC_{js}}{PY_{js} + HC_{js} + \sum_{j'=1}^j [PY_{j'1}] AM C_{(js, j'1)}}$ $W3 PC_{js} = 1 - (W1 PC_{js} + W2 PC_{js})$ $W3 PC_{(js, j'1)} = \frac{[PY_{j'1}] AM C_{(js, j'1)}}{\sum_{k=1}^j ([PY_{k1}] AM C_{(js, k1)})}$		
10.2	$\hat{P}E_{j1} + \varepsilon_{j1} [\hat{E}_{j1}] - SH \hat{F}TE_{j1} = 0$		$j=1, \dots, j$	Export demand functions
10.4	$\hat{M}E_{(j1, j'1)} - \hat{A}M E_{(j1, j'1)} - \hat{E}_{j1} = 0$		$j=1, \dots, j$ $j'=1, \dots, j$ $s=1, 2$	Demands for margins to facilitate commodity flows to domestic ports prior to export

- 10.9  $\hat{H}E_{j1} - [\alpha_1 H E_{j1}] [\hat{C}P I] - [\alpha_2 H E_{j1}] (\hat{T}E_{j1} + \hat{P}E_{j1} + \hat{\Phi}) - [\alpha_3 H E_{j1}] [\hat{V}E_{j1}] = 0$   $j=1, \dots, j_1c$  Export taxes on domestic good  $j$  under perfect competition
- 10.10  $\hat{H}Y_{j2}^1 - [\alpha_1 H Y_{j2}] [\hat{C}P I] - [\alpha_2 H Y_{j2}] (\hat{T}Y_{j2} + \hat{P}Y_{j2} + \hat{\Phi}) - [\alpha_3 H Y_{j2}] [\hat{V}Y_{j2}] = 0$   $j=1, \dots, j_1c$  Tariff per unit of imported good  $j$  under perfect competition
- 11.2.7  $\hat{G}G_{j1} - \sum_{f \in F(i,1)} [WGG_{ij1}^f] [\hat{G}G_{ij1}^f] = 0$   $j=j_1c+1, \dots, j_1j$   
 $i=j-j_1c+i$  Total amount of good  $j$  that government procures directly from the domestic oligopolists  
Where  
 $WGG_{ij1}^f = GG_{ij1}^f / GG_{j1}$
- 11.2.9  $\hat{G}E X P - \sum_{j=1}^{j_1j} \sum_{s=1}^2 W E X P G_{js} (\hat{P}G_{js} + \hat{G}_{js}) - \sum_{j=j_1c+1}^{j_1j} W E X P G G_{j1} (P \hat{G}G_{j1} + \hat{G}G_{j1}) - [W G T R N S F R] [G T R \hat{N} S F R] = 0$  Total government expenditure  
where  
 $W E X P G_{js} = P G_{js} [G_{js}] / G E X P$   
 $W E X P G G_{j1} = P G G_{j1} [G G_{j1}] / G E X P$   
 $W G T R N S F R = G T R N S F R / G E X P$
- 11.2.13  $\hat{P}G_{js} - W_1 P G_{js} [\hat{P}Y_{js}] - \sum_{j'=1}^{j_1j} W_2 P G_{(js,j'1)} (\hat{A}M G_{(js,j'1)} + \hat{P}Y_{j'1}) = 0$   $j=1, \dots, j_1j$   
 $s=1, 2$  Price per unit of good  $j$ s paid by government prevailing in the markets  
where  
 $W_1 P G_{js} = P Y_{js} / P G_{js}$   
 $W_2 P G_{(js,j'1)} = A M G_{(js,j'1)} [P Y_{j'1}] / P G_{js}$
- 11.2.16  $P \hat{G}G_{j1} - W_1 P G G_{j1} [P \hat{G}G Y_{j1}] - \sum_{j'=1}^{j_1j} W_2 P G G_{(j1,j'1)} (\hat{A}M G_{(j1,j'1)} + \hat{P}Y_{j'1}) = 0$   $j=j_1c+1, \dots, j_1j$  Price per unit of good  $j$  paid by government for its procurement  
where  
 $W_1 P G G_{j1} = P G G Y_{j1} / P G G_{j1}$   
 $W_2 P G G_{(j1,j'1)} = A M G_{(j1,j'1)} [P Y_{j'1}] / P G G_{j1}$

- 11.2.19  $\hat{M}G_{(js,j^1)} - \hat{A}M\hat{G}_{(js,j^1)} - \hat{G}_{js} = 0$   $j=1,\dots,jj,$   
 $s=1,2,$   
 $j^1=1,\dots,jj$  Demands for margins to facilitate commodity flows to government
- 11.2.20  $M\hat{G}G_{(j1,j^1)} - \hat{A}M\hat{G}_{(j1,j^1)} - \hat{G}G_{j1} = 0$   $j=jjc+1,\dots,jj$   
 $j^1=i,\dots,jj$  Demands for margins to facilitate commodity flows for government procurement
- 11.3.7  $GR\hat{E}V11 - \sum_{i=1}^{jj} \sum_{j=1}^{jj} \sum_{s=1}^2 W1GREV11_{ijs} (\hat{H}X_{ijs} + \hat{X}_{ijs})$   
 $-\sum_{i=1}^{jj} \sum_{j=1}^{jj} \sum_{s=1}^2 W2GREV11_{ijs} (\hat{H}I_{ijs} + \hat{I}_{ijs})$   
 $-\sum_{j=1}^{jj} \sum_{s=1}^2 W3GREV11_{js} (\hat{H}C_{js} + \hat{C}_{js}) = 0$   
 where  
 $W1GREV11_{ijs} = HX_{ijs} [X_{ijs}] / GREV11$   
 $W2GREV11_{ijs} = HI_{ijs} [I_{ijs}] / GREV11$   
 $W3GREV11_{js} = HC_{js} [C_{js}] / GREV11$
- 11.3.9  $GR\hat{E}V12 - \sum_{j=1}^{jj} WGREV12_j (\hat{H}Y_{j2}^1 + \hat{Y}_{j2}) = 0$   
 where  
 $WGREV12_j = HY_{j2}^1 [Y_{j2}] / GREV12$
- 11.3.11  $GR\hat{E}V13 - \sum_{j=1}^{jjc} WGREV13_j (\hat{H}E_{j1} + \hat{E}_{j1}) = 0$   
 where  
 $WGREV13_j = HE_{j1} [E_{j1}] / GREV13$
- 11.3.12  $GR\hat{E}V14 - \sum_{j=jjc+1}^{jj} W1GREV14_j (\hat{H}Y_{j1,1}^1 + \hat{Y}_{j1,1})$   
 $-\sum_{j=jjc+1}^{jj} W2GREV14_j (\hat{H}Y_{j1,2}^1 + \hat{Y}_{j1,2}) = 0$   
 where  
 $W1GREV14_j = HY_{j1,1}^1 [Y_{j1,1}] / GREV14$   
 $W2GREV14_j = HY_{j1,2}^1 [Y_{j1,2}] / GREV14$
- Government revenue from taxes on good js flowing to current production, capital creation and consumption
- Government revenue from tariff on imported good j
- Government revenue from export taxes on good j under perfect competition
- Government revenue from taxes on good j sold under imperfect competition

- 11.3.14  $GR\hat{E}V1 - W1GREV1[GR\hat{E}V11] - W2GREV1[GR\hat{E}V12] - W3GREV1[GR\hat{E}V13] - W4GREV1[GR\hat{E}V14] = 0$  Total government revenue from indirect taxes
- where  
 $W1GREV1 = GREV11/GREV1,$   
 $W2GREV1 = GREV12/GREV1,$   
 $W3GREV1 = GREV13/GREV1,$   
 $W4GREV1 = GREV14/GREV1.$
- 11.3.16  $GR\hat{E}V - W1GREV[GR\hat{E}V1] - W2GREV[GR\hat{E}V2] = 0$  Total government revenue
- where  
 $W1GREV = GREV1/GREV$   
 $W2GREV = GREV2/GREV$
- 11.4.5  $\hat{G}_{js} - \alpha G_{js}[\hat{M}R] - SH\hat{F}TG_{js} = 0$   $j=1,\dots,j$   
 $j,$  Government demands for commodities  
 $s=1,2$  classified by source
- 11.4.6  $\hat{M}R - \hat{M} + \hat{C}PI = 0$  Real household expenditure
- 11.4.7  $G\hat{R}EV - G\hat{E}XP - G\hat{S}AV = 0$  Balance budget constraint
- 11.4.8  $GR\hat{E}V2 - \rho INCOME[INC\hat{O}ME] = 0$  Total government revenue from income taxes
- 12.2.4  $G\hat{D}P - \sum_{i=1}^{ii} WVA_i[\hat{V}A_i] = 0$  Gross domestic product
- where  
 $WVA_i = VA_i/GDP$
- 12.2.5  $\hat{V}A_i - \sum_{j \in L(i)} WVA_{ij}(\hat{P}Y_{j1} + \hat{Y}_{j1}) + \sum_{j=1}^{jj} W2VA_{ij}[\hat{X}_{j1}]$   $i=1,\dots,ii$  Value added by industry under perfect competition  
 $+ \sum_{j=1}^{jjc} W3VA_{ij}(\hat{P}\hat{F}Y_{j2} + \hat{X}_{j2} + \hat{\Phi})$   
 $+ \sum_{j=jjc+1}^{jj} W4VA_{ij} \left( \left( \frac{PY_{j2}}{PY_{j2} - HY_{j2}^1} \right) \hat{P}Y_{j2} \right)$   
 $\left( - \left( \frac{HY_{j2}^1}{PY_{j2} - HY_{j2}^1} \right) \hat{H}Y_{j2}^1 + \hat{X}_{j2} \right)$   
 $+ \sum_{j=1}^{jj} \sum_{j'=1}^{jj} \sum_{s=1}^2 WSVA_{(ijs,j'1)}(\hat{P}Y_{j'1} + \hat{M}X_{(ijs,j'1)}) =$

where

$$\begin{aligned}
 W1VA_{ij} &= PY_{j1} [Y_{ij1}] / VA_i \\
 W2VA_{ij} &= PY_{j1} [X_{ij1}] / VA_i \\
 W3VA_{ij} &= \Phi [PFY_{j2}] [X_{ij2}] / VA_i \\
 W4VA_{ij} &= (PY_{j2} - HY_{j2}^1) X_{ij2} / VA_i \\
 W5VA_{(ij,s,j'1)} &= PY_{j'1} [MX_{(ij,s,j'1)}] / VA_i
 \end{aligned}$$

12.2.7

$$\begin{aligned}
 \hat{V}A_i - W1VA_i(\hat{P}Y_{j1} + \hat{Y}_{ij1}) - W2VA_i & \left( \left( \frac{PQ_{j2}}{PQ_{j2} - HY_{j,2}^2} \right) \hat{P}Q_{j2} \right. \\
 & \left. - \left( \frac{HY_{j,2}^2}{PQ_{j2} - HY_{j,2}^2} \right) \hat{H}Y_{j,2}^2 \right. \\
 & \left. + \hat{Y}_{ij,2} + \hat{\Phi} \right) \quad \begin{array}{l} j=i-ii+c+jc \\ i=ii+c+1, \dots \\ ,ii \end{array} \quad \begin{array}{l} \text{Value added by} \\ \text{industry under} \\ \text{imperfect} \\ \text{competition} \end{array} \\
 -W3VA_i(P\hat{G}G_{j1} + \hat{G}G_{j1}) + \sum_{k=1}^j W4VA_{ik}(\hat{P}Y_{k1} + \hat{X}_{ik1}) \\
 + \sum_{k=1}^{j^c} W5VA_{ik}(P\hat{F}Y_{k2} + \hat{X}_{ik2} + \hat{\Phi}) \\
 + \sum_{k=jj^c+1}^j W6VA_{ij} \left( \left( \frac{PY_{k2}}{PY_{k2} - HY_{k2}^1} \right) \hat{P}Y_{k2} \right. \\
 \left. - \left( \frac{HY_{k2}^1}{PY_{k2} - HY_{k2}^1} \right) \hat{H}Y_{k2}^1 + \hat{X}_{ik2} \right) \\
 + \sum_{k=1}^j \sum_{j'=1}^j \sum_{s=1}^2 W7VA_{(iks,j'1)} (\hat{P}Y_{j'1} + \hat{M}X_{(iks,j'1)}) \\
 + \sum_{j'=1}^j \sum_{s=1}^2 W8VA_{(ij1,s,j'1)} (\hat{P}Y_{j'1} + \hat{M}Y_{(ij1,s,j'1)}) = 0
 \end{aligned}$$

where

$$\begin{aligned}
 W1VA_i &= PY_{j1} [Y_{ij1}] / VA_i \\
 W2VA_i &= (PQ_{j2} - HY_{j,2}^2) [Y_{ij1,2}] \Phi / VA_i \\
 W3VA_i &= PGGY_{j1} [GG_{j1}] / VA_i \\
 W4VA_{ik} &= PY_{k1} [X_{ik1}] / VA_i \\
 W5VA_{ik} &= \Phi [PFY_{k2}] [X_{ik2}] / VA_i \\
 W6VA_{ik} &= (PY_{k2} - HY_{k2}^1) X_{ik2} / VA_i \\
 W7VA_{(iks,j'1)} &= PY_{j'1} [MX_{(iks,j'1)}] / VA_i
 \end{aligned}$$

$$W8VA_{(ij1,s,j'1)} = PY_{j'1} \left[ MY_{(ij1,s,j'1)} \right] / VA_i$$

$$12.3.5 \quad D\hat{E}PR - \sum_{i=1}^n WDEPR_i (\hat{K}_i + \hat{P}I_i) = 0$$

Depreciation

where

$$WDEPR_i = [d_i \mathbf{K}_i \mathbf{P}I_i] / DEPR$$

$$12.3.6 \quad IN\hat{C}OME - \left( \frac{GDP}{INCOME} \right) G\hat{D}P + \left( \frac{DEPR}{INCOME} \right) D\hat{E}PR = 0$$

Total national income

$$12.3.7 \quad \hat{M} - INCOM\hat{E}DISP = 0$$

Total income of households

$$12.3.8 \quad INCOM\hat{E}DISP - \left( \frac{(1 - \rho INCOME)}{INCOMEDISP} \right) [IN\hat{C}OME] \\ - \left( \frac{GTRNSFR}{INCOMEDISP} \right) [G\hat{T}RNSFR] = 0$$

Disposable income

$$12.4.4 \quad EX\hat{P}ORT - \hat{\Phi} - \sum_{j=1}^{j^c} W1EXPORT_j (\hat{P}E_{j1} + \hat{E}_{j1})$$

Total export

$$- \sum_{j=j^c+1}^j W2EXPORT_j \left( \begin{array}{l} \left( \frac{PQ_{j2}}{PQ_{j2} - HY_{j1,2}^2} \right) \hat{P}Q_{j2} \\ - \left( \frac{HY_{j1,2}^2}{PQ_{j2} - HY_{j1,2}^2} \right) \hat{H}Y_{j1,2}^2 + \hat{Y}_{j1,2} \end{array} \right) = 0$$

where

$$W1EXPORT_j = \Phi [PE_{j1} \mathbf{E}_{j1}] / EXPORT$$

$$W2EXPORT_j = \Phi (PQ_{j2} - HY_{j1,2}^2) [Y_{j1,2}] / EXPORT$$

$$12.4.5 \quad IMP\hat{O}RT - \sum_{j=1}^{j^c} W1IMPORT_j (\hat{\Phi} + P\hat{F}Y_{j2} + \hat{Y}_{j2})$$

Total import

$$- \sum_{j=j^c+1}^j W2IMPORT_j \left( \begin{array}{l} \left( \frac{PY_{j2}}{PY_{j2} - HY_{j2,1}^1} \right) \hat{P}Y_{j2} \\ - \left( \frac{HY_{j2,1}^1}{PY_{j2} - HY_{j2,1}^1} \right) \hat{H}Y_{j2}^1 + \hat{Y}_{j2} \end{array} \right) = 0$$

where

$$W1IMPORT_j = \Phi [Y_{j2} \mathbf{P}FY_{j2}] / IMPORT$$

$$W2IMPORT_j = (PY_{j2} - HY_{j2,1}^1) Y_{j2} / IMPORT$$

Total import

12.4.9	$\hat{Y}_{j2} - \sum_{i=1}^{ii} W1Y_{j2,i} [\hat{X}_{ij2}] - W2Y_{j2} [\hat{C}_{j2}]$ $- \sum_{i=1}^{ii} W3Y_{j2,i} [\hat{I}_{ij2}] - W4Y_{j2} [\hat{G}_{j2}] = 0$ <p>where</p> $W1Y_{j2,i} = X_{ij2}/Y_{j2}$ $W2Y_{j2} = C_{j2}/Y_{j2}$ $W3Y_{j2,i} = I_{ij2}/Y_{j2}$ $W4Y_{j2} = G_{j2}/Y_{j2}$	j=1,...,jj	Total volume of imported good j
12.4.10	$EXP\hat{O}RT - IMP\hat{O}RT = 0$		Balance of payments
12.5.2	$G\hat{D}P - \left(\frac{M}{GDP}\right)\hat{M} - \left(\frac{I}{GDP}\right)\hat{I} - \left(\frac{GEXP}{GDP}\right)G\hat{E}XP = 0$		Gross domestic product
12.6.1	$C\hat{P}I - \sum_{j=1}^{jj} \sum_{s=1}^2 [WC_{js}] [\hat{P}C_{js}] = 0$ <p>where</p> $WC_{js} = \frac{[PC_{js}] [C_{js}]}{\sum_{j'=1}^{jj} \sum_{s'=1}^2 [PC_{j's'}] [C_{j's'}]}$		Consumer price index
12.6.2	$K\hat{P}I - \sum_{i=1}^{ii1} [\tilde{S}I_i] [\hat{P}I_i] = 0$ <p>where <math>\tilde{S}I_i = \frac{SI_i}{\sum_{i'=1}^{ii1} SI_{i'}}</math></p>		Capital price index
12.6.4	$\hat{L} - \sum_{t=1}^n [WL_t] \hat{L}_t = 0$ <p>where</p> $WL_t = L_t/L$		Aggregate employment
12.6.6	$\hat{K} - \sum_{i=1}^{ii} [WK_i] \hat{K}_i = 0$ <p>where</p> $WK_i = K_i/K$		Total capital stock
12.6.8	$IM\hat{R} - \hat{I}1R + \hat{M}R = 0$		Ratio of real private investment to consumption spending

- 12.6.9  $\hat{P}X_{(i,jj+1,t)} - [\omega PX_{(i,jj+1,t)}][C\hat{P}I] - F\hat{P}X_{(i,jj+1,t)} - F\hat{P}X_{(i,jj+1,t)} - F\hat{P}X_{(i,jj+1,t)} = 0$   $i=1,\dots,ii$   
 $t=1,\dots,tt$  Wage indexation
- 12.6.10  $\hat{L}AND - \sum_{i=1}^{ii} [WLAND_i] \hat{L}AND_i = 0$  Total land  
Where  
 $WLAND_i = LAND_i / LAND$
- 12.6.11  $\hat{P}X_{(i,jj+2)} - [\omega PX_{(i,jj+2)}][C\hat{P}I] - F\hat{P}X_{(i,jj+2)} = 0$   $i=1,\dots,ii$  Price indexation  
for other cost
- 13.3.1  $\sum_{j \in L(i)} [\hat{P}Y_{j1}] [SWY_{j1}] - \sum_{j=1}^{jj} \sum_{s=1}^2 [\hat{P}X_{jys}] [SWX_{jys}] - \sum_{t=1}^{tt} [\hat{P}X_{(i,jj+1,t)}] [SWX_{(i,jj+1,t)}] - \hat{A}_i$   $i=1,\dots,ii$  Zero pure profits  
in production  
under competitive  
industry  
 $-\sum_{s=2}^3 [\hat{P}X_{(i,jj+1,s)}] [SWX_{(i,jj+1,s)}]$   
 $-\left[ \hat{P}X_{(i,jj+2)} \right] [SWX_{(i,jj+2)}] = 0$   
where  
 $SWX_{jys} = [PX_{jys}] X_{jys} / TC_i$   
 $SWX_{(i,jj+1,t)} = [PX_{(i,jj+1,t)}] X_{(i,jj+1,t)} / TC_i$   
 $SWX_{(i,jj+1,s)} = [PX_{(i,jj+1,s)}] X_{(i,jj+1,s)} / TC_i$   
 $SWX_{(i,jj+2)} = [PX_{(i,jj+2)}] X_{(i,jj+2)} / TC_i$
- 13.3.2  $\hat{A}_i - A\hat{X}Z_i - A\hat{Y}Z_i - \sum_{j=1}^{nn(i)} [A\hat{Y}_{j1}] [SY_{j1}] - \sum_{j \in L(i)} [\hat{A}Y_{j1}] [SWY_{j1}] - \sum_{j=1}^{jj+2} [\hat{A}X_{jy}] [SWX_{jy}] - \sum_{j=1}^{jj} \sum_{s=1}^2 [\hat{A}X_{jys}] [SWX_{jys}] - \sum_{t=1}^{tt} [\hat{A}X_{(i,jj+1,t)}] [SWX_{(i,jj+1,t)}] - \sum_{s=2}^3 [\hat{A}X_{(i,jj+1,s)}] [SWX_{(i,jj+1,s)}] = 0$   $i=1,\dots,ii$  Weighted sums of  
the technical-  
change terms  
affecting the  
production  
functions of each  
industry
- 13.3.3  $\hat{P}Y_{j1} - \hat{\psi}_{j1}^f + \varepsilon_{j1}^f [\hat{Q}_{j1}] = 0$   $j=jic+1,\dots,ji$   
 $feF(j-$   
 $jic+ic,1)$  Perceived  
domestic inverse  
demand for good j  
under imperfect  
competition

$$13.3.9 \quad \hat{P} I_i - \sum_{j=1}^j \sum_{s=1}^2 [SW I_{ijs}] [\hat{P} I_{ijs}] \quad i=1, \dots, ii \quad \text{Zero-profit prices for capital creation}$$

$$- \sum_{j=1}^j \sum_{s=1}^2 [SW I_{ijs}] [\hat{A} I_{ijs}]$$

$$- \sum_{j=1}^j [SW I_{ij}] [\hat{A} I_{ij}] - \hat{A} I_i = 0$$

where

$$SW I_{ijs} = \frac{[PI_i] I_i}{\sum_{j'=1}^j (\sum_{s'=1}^2 ([PI_{ij's'}] I_{ij's'}))}$$

$$\sum_{s=1}^2 SW I_{ijs} = \frac{\sum_{s=1}^2 ([PI_{ijs}] I_{ijs})}{\sum_{j'=1}^j (\sum_{s'=1}^2 ([PI_{ij's'}] I_{ij's'}))}$$

$$SW I_{ij} = \sum_{s=1}^2 SW I_{ijs}$$

$$13.3.10 \quad [\hat{P} E_{j1}] + \hat{\Phi} - [W1PE_{j1}] [\hat{P} Y_{j1}] - [W2PE_{j1}] [\hat{H} E_{j1}] \quad j=1, \dots, jjc \quad \text{Price per unit of exported good j under perfect competition}$$

$$- [W3PE_{j1}] \sum_{j'=1}^j [W3PE_{(j1,j'1)}] [\hat{P} Y_{j'1}]$$

$$- [W3PE_{j1}] \sum_{j'=1}^j [W3PE_{(j1,j'1)}] [\hat{A} M E_{(j1,j'1)}] =$$

where

$$[W1PE_{j1}] = \frac{PY_{j1}}{[PY_{j1}] + [HE_{j1}] + \sum_{j'=1}^j [PY_{j'1}] [AME_{(j1,j'1)}]}$$

$$[W2PE_{j1}] = \frac{HE_{j1}}{[PY_{j1}] + [HE_{j1}] + \sum_{j'=1}^j [PY_{j'1}] [AME_{(j1,j'1)}]}$$

$$[W3PE_{j1}] = 1 - [W1PE_{j1}] - [W2PE_{j1}]$$

$$[W3PE_{(j1,j'1)}] = \frac{[PY_{j'1}] [AME_{(j1,j'1)}]}{\sum_{k=1}^j [PY_{k1}] [AME_{(j1,k1)}]}$$

$$13.3.11 \quad \hat{P} Y_{j2} - [W1PY_{j2}] (P\hat{F}Y_{j2} + \hat{\Phi}) - [W2PY_{j2}] [\hat{H} Y_{j2}^1] = 0 \quad j=1, \dots, jjc \quad \text{Price per unit of imported good j under perfect competition}$$

where

$$[W1PY_{j2}] = \frac{[PFY_{j2}] \Phi}{[PFY_{j2}] \Phi + HY_{j2}}$$

$$[W2PY_{j2}] = \frac{HY_{j2}}{[PFY_{j2}] \Phi + HY_{j2}}$$

13.3.12

$$\begin{aligned}
& \sum_{i=1}^{ic} [WY_{j1}] [\hat{Y}_{j1}] - \sum_{i=1}^{ii} [WX_{j1}] [\hat{X}_{j1}] - \sum_{i=1}^{ii} [WI_{j1}] [\hat{I}_{j1}] \\
& - WC_{j1} [\hat{C}_{j1}] - WG_{j1} [\hat{G}_{j1}] - WE_{j1} [\hat{E}_{j1}] \\
& - \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMX_{(ij's,j1)}] [\hat{MX}_{(ij's,j1)}] \\
& - \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMI_{(ij's,j1)}] [\hat{MI}_{(ij's,j1)}] \\
& - \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMC_{(j's,j1)}] [\hat{MC}_{(j's,j1)}] \\
& - \sum_{j'=1}^{jj} \sum_{s=1}^2 [WMG_{(j's,j1)}] [\hat{MG}_{(j's,j1)}] \\
& - \sum_{j'=1}^{jj} [WME_{(j'1,j1)}] [\hat{ME}_{(j'1,j1)}] \\
& - \sum_{i=ii+1}^{ic} [WMY_{(ij'1,1,j1)}] [\hat{MY}_{(ij'1,1,j1)}] \\
& - \sum_{i=ii+1}^{ic} [WMY_{(ij'1,2,j1)}] [\hat{MY}_{(ij'1,2,j1)}] = 0
\end{aligned}$$

j=1,...,jjc

Market-clearing  
output under  
perfect competition

where

$$\begin{aligned}
WY_{j1} &= Y_{j1} / \sum_{i'=1}^{ic} Y_{i'j1}; \quad WX_{j1} = X_{j1} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WI_{j1} &= I_{j1} / \sum_{i'=1}^{ic} Y_{i'j1}; \quad WC_{j1} = C_{j1} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WG_{j1} &= G_{j1} / \sum_{i'=1}^{ic} Y_{i'j1}; \quad WE_{j1} = E_{j1} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WMX_{(ij's,j1)} &= MX_{(ij's,j1)} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WMI_{(ij's,j1)} &= MI_{(ij's,j1)} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WMC_{(j's,j1)} &= MC_{(j's,j1)} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WMG_{(j's,j1)} &= MG_{(j's,j1)} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WME_{(j'1,j1)} &= ME_{(j'1,j1)} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WMY_{(ij'1,1,j1)} &= MY_{(ij'1,1,j1)} / \sum_{i'=1}^{ic} Y_{i'j1} \\
WMY_{(ij'1,2,j1)} &= MY_{(ij'1,2,j1)} / \sum_{i'=1}^{ic} Y_{i'j1}
\end{aligned}$$

13.3.13

$$\begin{aligned}
& \hat{Q}_{j1} - \sum_{i=1}^{ii} \sum_{s=1}^2 \left[ WX_{ijs} \left[ \hat{X}_{ijs} \right] - \sum_{i=1}^{ii} \sum_{s=1}^2 \left[ WI_{ijs} \left[ \hat{I}_{ijs} \right] \right] \right. \\
& - \sum_{s=1}^2 \left[ WC_{js} \left[ \hat{C}_{js} \right] + \sum_{s=1}^2 \left[ WG_{js} \left[ \hat{G}_{js} \right] + \left[ WGG_{j1} \left[ \hat{GG}_{j1} \right] \right] \right. \\
& - \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMX_{(ij's,j1)} \left[ \hat{MX}_{(ij's,j1)} \right] \right. \\
& - \sum_{i=1}^{ii} \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMI_{(ij's,j1)} \left[ \hat{MI}_{(ij's,j1)} \right] \right. \\
& - \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMC_{(j's,j1)} \left[ \hat{MC}_{(j's,j1)} \right] \right. \\
& - \sum_{j'=1}^{jjc} \left[ WME_{(j'1,j1)} \left[ \hat{ME}_{(j'1,j1)} \right] \right. \\
& - \sum_{j'=1}^{jj} \sum_{s=1}^2 \left[ WMG_{(j's,j1)} \left[ \hat{MG}_{(j's,j1)} \right] \right. \\
& - \sum_{j'=jic+1}^{jj} \left[ WMGG_{(j'1,j1)} \left[ \hat{MGG}_{(j'1,j1)} \right] \right. \\
& - \sum_{i=iic+1}^{ii} \left[ WMY_{(ij'1,1,j1)} \left[ \hat{MY}_{(ij'1,1,j1)} \right] \right. \\
& \left. - \sum_{i=iic+1}^{ii} \left[ WMY_{(ij'1,2,j1)} \left[ \hat{MY}_{(ij'1,2,j1)} \right] \right] = 0
\end{aligned}$$

j=jic+1,...,jj

Quantity of  
good j sold  
in domestic  
market

where

$$WX_{ij1} = X_{ij1}/Q_{j1}; \quad WI_{ij1} = I_{ij1}/Q_{j1}$$

$$WC_{j1} = C_{j1}/Q_{j1}; \quad WG_{j1} = G_{j1}/Q_{j1}$$

$$WGG_{j1} = GG_{j1}/Q_{j1}$$

$$WMX_{(ij's,j1)} = MX_{(ij's,j1)}/Q_{j1}$$

$$WMI_{(ij's,j1)} = MI_{(ij's,j1)}/Q_{j1}$$

$$WME_{(j'1,j1)} = ME_{(j'1,j1)}/Q_{j1}$$

$$WMC_{(j's,j1)} = MC_{(j's,j1)}/Q_{j1}$$

$$WMG_{(j's,j1)} = MG_{(j's,j1)}/Q_{j1}$$

$$WMGG_{(j'1,j1)} = MGG_{(j'1,j1)}/Q_{j1}$$

$$WMY_{(ij'1,1,j1)} = MY_{(ij'1,1,j1)}/Q_{j1}$$

$$WMY_{(ij'1,2,j1)} = MY_{(ij'1,2,j1)}/Q_{j1}$$

13.3.14	$\hat{P}Q_{j2} - \hat{\Psi}_{j2} + \varepsilon_{j2}\hat{Q}_{j2} = 0$	$j=jjc+1,\dots,jj$	Foreign inverse demand for good j under imperfect competition
13.3.15	$\hat{L}_t - \sum_{i=1}^{ii} [WX_{(i,jj+1,1,t)}] [\hat{X}_{(i,jj+1,1,t)}] = 0$ where $WX_{(i,jj+1,1,t)} = X_{(i,jj+1,1,t)} / \sum_{i'=1}^{ii} X_{(i',jj+1,1,t)}$	$t=1,\dots,tt$	Labor demand per skill type
13.3.16	$\hat{K}_i - \hat{X}_{(i,jj+1,2)} = 0$	$i=1,\dots,ii$	Capital stock in industry i
13.3.17	$\hat{LAND}_i - \hat{X}_{(i,jj+1,3)} = 0$	$i=1,\dots,ii$	Agricultural land demanded by industry i

Figure 14.2. The List of Variables in Percentage Changes

<u>Variable</u>	<u>Subscript</u>	<u>Number</u>	<u>Description</u>
$Z_i$	$i=1,\dots,iic$	$iic$	Industry activity levels
$X_{ijs}$	$i=1,\dots,iic$ $j=1,\dots,jj$ $s=1,2$	$2iic \times jj$	Demands for inputs $j$ (domestic and imported) for current production of industry $i$
$X_{(i,jj+1,1,t)}$	$i=1,\dots,iic$ $t=1,\dots,tt$	$iic \times tt$	Demands for labor inputs by skill group for industry $i$
$X_{(i,jj+1,s)}$	$s=1,2,3$ $i=1,\dots,iic$	$3iic$	Industry $i$ 's demands for labor in general, capital and agricultural land
$X_{i,jj+2}$	$i=1,\dots,iic$	$iic$	Demands for "other cost" tickets
$PX_{ijs}$	$i=1,\dots,iic$ $j=1,\dots,jj$ $s=1,2$	$2iic \times jj$	Purchasers' prices for produced inputs $js$ for current production in industry $i$
$PX_{(i,jj+1,1,t)}$	$i=1,\dots,iic$ $t=1,\dots,tt$	$iic \times tt$	Prices paid by industry $i$ for units of labor of different skill types $t$
$PX_{(i,jj+1,s)}$	$i=1,\dots,iic$ $s=1,2,3$	$3iic$	Prices paid by industry $i$ for their labor in general, rental of capital and rental of land
$PX_{(i,jj+2)}$	$i=1,\dots,iic$	$iic$	Prices of "other cost" tickets to industry $i$
$AX_{ijs}$	$i=1,\dots,iic$ $j=1,\dots,jj$ $s=1,2$	$2iic \times jj$	Input- $js$ -augmenting Technological change for industry $i$
$AX_{ij}$	$i=1,\dots,iic$ $j=1,\dots,jj$	$iic \times jj$	Input- $j$ -augmenting Technological change for industry $i$
$AXZ_i$	$i=1,\dots,iic$	$iic$	Neutral-input-augmenting Technological change for industry $i$
$AX_{(i,jj+1,1,t)}$	$i=1,\dots,iic$ $t=1,\dots,tt$	$iic \times tt$	Specific-skill-augmenting Technological change for industry $i$

$AX_{(i,jj+1,s)}$	$i=1,\dots,iic$ $s=1,2,3$	$3iic$	Labor-, capital-, and agricultural land-augmenting Technological change for industry $i$
$AX_{(i,jj+1)}$	$i=1,\dots,iic$	$iic$	Primary factors-augmenting Technological change for industry $i$
$AX_{(i,jj+2)}$	$i=1,\dots,iic$	$iic$	"Other costs" ticket-augmenting Technological change for industry $i$
$A_i$	$i=1,\dots,iic$	$iic$	Weighted sums of the Technological change terms affecting the production functions for industry $i$
$MX_{(ijs,j'1)}$	$i=1,\dots,iic$ $j'=1,\dots,jj$ $j=1,\dots,jj$ $s=1,2$	$2iicxjj^2$	The amount of goods $j'1$ needed as margins to transport good $js$ to domestic industry $i$ for current production
$AMX_{(ijs,j'1)}$	$i=1,\dots,iic$ $j'=1,\dots,jj$ $j=1,\dots,jj$ $s=1,2$	$2iicxjj^2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ to domestic industry $i$ for current production
$YY_{ij1}$	$i=1,\dots,iic$ $j=1,\dots,nn(i)$	$iic \times nn(i)$	Supplies of domestic composite goods $j1$ by industry $i$
$Y_{ij'1}$	$i=1,\dots,iic$ $j' \in L(i,j)$	$iic \times L(i,j)$	Supplies of domestic goods $j'1$ by industry $i$
$Y_{ij1}$	$i=1,\dots,iic$ $j=1,\dots,nn(i)$	$iic \times nn(i)$	Supplies of domestic goods $j1$ by industry $i$
$YY_{j1}$	$j=1,\dots,jjc$	$jjc$	Total supplies of domestic commodities $j1$
$Y_{j2}$	$j=1,\dots,jjc$	$jjc$	Aggregate imports of commodity $j$ under perfectly competitive markets
$PYY_{ij1}$	$i=1,\dots,iic$ $j=1,\dots,jj$	$iic \times jj$	Basic prices of domestic produced good $j$ by industry $i$
$PY_{j1}$	$j=1,\dots,jjc$	$jjc$	Basic prices of commodities $j$ from source $s$
$PY_{j2}$	$j=1,\dots,jjc$	$jjc$	C.i.f. domestic currency import prices of commodity $j$
$PFY_{j2}$	$j=1,\dots,jjc$	$jjc$	C.i.f. foreign currency import prices of commodity $j$

$AY_{ij}^j$	$i=1,\dots,iic$ $j=1,\dots,nn(i)$ $j' \in L(i,j)$	$iic \times nn(i) \times$ $L(i,j)$	Technological coefficient
$AYY_{ij1}$	$i=1,\dots,iic$ $j=1,\dots,jj$	$iic \times jj$	Augmenting technological change with respect to domestic commodity output $j$
$AYYZ_i$	$i=1,\dots,iic$	$iic$	Neutral output-augmenting technological change for industry $i$
$HX_{ijs}$	$i=1,\dots,iic$ $j=1,\dots,jj$ $s=1,2$	$2iic \times jj$	Taxes per unit of sales of inputs $js$ for current production to industry $i$
$TX_{ijs}$	$i=1,\dots,iic$ $j=1,\dots,jj$ $s=1,2$	$2iic \times jj$	Variables that allow taxes on the sales of inputs $js$ to industries for current production to be modelled as ad valorem tax
$VX_{ijs}$	$i=1,\dots,iic$ $j=1,\dots,jj$ $s=1,2$	$2iic \times jj$	Variables that allow taxes on the sales of inputs $js$ to industry $i$ for current production to be modelled as specific tax
$HY_{j2}^1$	$j=1,\dots,jjc$	$jjc$	The tariffs per unit of imports under perfectly competitive industries
$TY_{j2}$	$j=1,\dots,jjc$	$jjc$	Variables that allow tariff per unit of imported goods $j$ to be modelled as ad valorem tax
$VY_{j2}$	$j=1,\dots,jjc$	$jjc$	Variables that allow tariff per unit of imports to be modeled as specific tax
$PY_{j1}$	$j=jjc+1,\dots,jj$	$jj-jjc$	Basic prices of commodity $j$ under imperfect competition
$PY_{j2}$	$j=jjc+1,\dots,jj$	$jj-jjc$	C.i.f. domestic currency import prices of commodity $j$ under imperfect competition
$Z_i^f$	$i=iic+1,\dots,ii$ $f \in F(i,1)$	$(ii-iic) \times F(i,1)$	Activity level of domestic firm $f$ in industry $i$ , under imperfect competition
$X_{ijs}^f$	$i=iic+1,\dots,ii$ $j=1,\dots,jj$ $f \in F(i,1)$	$(ii-iic) \times$ $jj \times F(i,1)$	Demands for intermediate inputs from source $s$ by domestic firm $f$ in industry $i$ , under imperfect competition

$X^f_{(i,jj+1,1,t)}$	$i=iic+1,\dots,ii$ $t=1,\dots,tt$ $f \in F(i,1)$	$(ii-iic) \times tt$ $x^F(i,1)$	Demands for labor inputs by skill group by domestic firm $f$ in industry $i$ , under imperfect competition
$X^f_{(i,jj+1,s)}$	$s=1,2,3$ $i=iic+1,\dots,ii$ $f \in F(i,1)$	$3(ii-iic)$ $x^F(i,1)$	Industry $i$ 's demands for labor in general, capital and agricultural land by domestic firm $f$ in industry $i$ , under imperfect competition
$X^f_{i,jj+2}$	$i=iic+1,\dots,ii$ $f \in F(i,1)$	$(ii-iic)$ $x^F(i,1)$	Demands for "other cost" tickets by domestic firm $f$ in industry $i$ , under imperfect competition
$AX_{ijs}$	$i=iic+1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2(ii-iic) \times jj$	Input- $j$ -augmenting technological change for domestic industry $i$ , under imperfect competition
$AX_{ij}$	$i=iic+1,\dots,ii$ $j=1,\dots,jj$	$(ii-iic) \times jj$	Input- $j$ -augmenting technological change for domestic industry $i$ , under imperfect competition
$AXZ_i$	$i=iic+1,\dots,ii$	$ii-iic$	Neutral-input-augmenting technological change for domestic industry $i$ , under imperfect competition
$AX_{(i,jj+1,1,t)}$	$i=iic+1,\dots,ii$ $t=1,\dots,tt$	$(ii-iic) \times tt$	Specific-skill-augmenting technological change for domestic industry $i$ , under imperfect competition
$AX_{(i,jj+1,s)}$	$i=iic+1,\dots,ii$ $s=1,2,3$	$3(ii-iic)$	Labor-, capital-, and agricultural land-augmenting technological change for domestic industry $i$ , under imperfect competition
$AX_{(i,jj+1)}$	$i=iic+1,\dots,ii$	$ii-iic$	Primary factors-augmenting technological change for domestic industry $i$ , under imperfect competition
$AX_{(i,jj+2)}$	$i=iic+1,\dots,ii$	$ii-iic$	"Other costs" ticket-augmenting technological change for domestic industry $i$ , under imperfect competition

$A_i$	$i=iic+1, \dots, ii$	$ii-iic$	Weighted sums of the technological change terms affecting the production functions for domestic industry $i$ , under imperfect competition
$MX_{(ijs,j'1)}$	$i=iic+1, \dots, ii$ $j'=1, \dots, jj$ $j=1, \dots, jj$ $s=1,2$	$2(ii-iic) \times jj^2$	The amount of goods $j'1$ needed as margins to transport good $js$ for current production of domestic imperfectly competitive industry $i$
$AMX_{(ijs,j'1)}$	$i=iic+1, \dots, ii$ $j'=1, \dots, jj$ $j=1, \dots, jj$ $s=1,2$	$2(ii-iic) \times jj^2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ for current production of domestic imperfectly competitive industry $i$
$\psi_{j1}^f$	$j=jjc+1, \dots, jj$ $f \in F(i,1)$	$(jj-jjc) \times F(i,1)$	A positive constant
$Q_{j1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Total amount of good $j$ supplied domestically by both domestic and foreign oligopolists
$Y_{j2}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Aggregate imports of commodity $j$ under imperfectly competitive markets
$Y_{ij1}^f$	$i=iic+1, \dots, ii$ $j=jjc+i-iic$ $f \in F(i,1)$	$(ii-iic) \times F(i,1)$	Total amount of good $j$ by domestic oligopolists $f$
$Y_{ij1,1}^f$	$i=iic+1, \dots, ii$ $j=jjc+i-iic$ $f \in F(i,1)$	$(ii-iic) \times F(i,1)$	Total amount of good $j$ supplied on the domestic markets by domestic oligopolists $f$
$HY_{j1,1}^1$	$j=jjc+1, \dots, jj$	$jj-jjc$	Specific taxes levied by domestic government on every sales of good $j1$ on domestic markets
$AYZ_i$	$i=iic+1, \dots, ii$	$(ii-iic)$	Technological change associated with activity level $Z_i$
$AY_{ij1}$	$i=iic+1, \dots, ii$ $j=jjc+i-iic$	$(ii-iic)$	Technological change associated with domestic output $Y_{ij1}$
$MY_{ij1,1,j'1}$	$i=iic+1, \dots, ii$ $j=jjc+i-iic$ $j'=1, \dots, jj$	$(ii-iic) \times jj$	The amount of good $j'1$ as margins needed to transport good $Y_{ij1}$ to the domestic markets

$AMY_{ij1,1j'1}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$ $j'=1,\dots,ij$	$(ii-iic) \times jj$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $j1$ from the domestic producers of industry $i$ to the domestic markets
$Y_{ij2}^f$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $f \in F(i,2)$	$(ii-iic) \times F(i,2)$	Total amount of good $j$ by foreign oligopolists $f$
$\Psi_{j2}$	$j=jjc+1,\dots,ij$	$ij-jjc$	A positive constant
$Q_{j2}$	$j=jjc+1,\dots,ij$	$ij-jjc$	Total amount of outputs supplied by domestic and foreign oligopolists on foreign markets
$HY_{j1,2}^1$	$j=jjc+1,\dots,ij$	$ij-jjc$	Export taxes in foreign currency levied by the home government on the sales of good $j1$ on foreign markets
$HY_{j1,2}^2$	$j=jjc+1,\dots,ij$	$ij-jjc$	Export taxes in foreign currency levied by the foreign government on the sales of good $j1$ on foreign markets
$Y_{ij1,2}^f$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$ $f \in F(i,1)$	$(ii-iic) \times F(i,1)$	Total amount of good $j$ supplied on the foreign markets by domestic oligopolists $f$
$MY_{ij1,2j'1}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$ $j'=1,\dots,ij$	$(ii-iic) \times jj$	The amount of good $j'1$ as margins needed to transport good $Y_{ij1}$ to the foreign markets
$AMY_{ij1,2j'1}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$ $j'=1,\dots,ij$	$(ii-iic) \times jj$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $j1$ from the domestic producers of industry $i$ to the foreign markets
$Y_{ij1,1}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$	$(ii-iic)$	Total amount of good $j$ supplied on the domestic markets by domestic oligopolists
$Y_{ij1,2}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$	$(ii-iic)$	Total amount of good $j$ supplied on the foreign markets by domestic oligopolists
$HY_{j2,1}^1$	$j=jjc+1,\dots,ij$	$ij-jjc$	Specific tariff levied by the domestic government on the sales of good $j2$ on domestic markets

$HY_{j2,1}^2$	$j=jc+1,\dots,jj$	$jj-jc$	Specific export taxes levied by the foreign government on the sales of good $j2$ on domestic markets
$Y_{ij2,1}^f$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$ $f \in F(i,2)$	$(ii-iic) \times F(i,2)$	The sales of good $j2$ by foreign oligopolist firm on the domestic markets
$AMY_{ij2,1,j2}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$ $j'=1,\dots,jj$	$(ii-iic) \times jj$	The amount of goods $j'2$ needed as margins to transport 1 unit of good $j2$ from the foreign producers of industry $i$ to the domestic markets
$UCZ_{i2}$	$i=iic+1,\dots,ii$	$(ii-iic)$	The cost of sustaining 1 unit of activity level of foreign oligopolistic industry
$AYZ_{i2}$	$i=iic+1,\dots,ii$	$(ii-iic)$	Technological coefficient
$AY_{ij2}$	$i=iic+1,\dots,ii$ $j=i-iic+jjc$	$(ii-iic)$	Technological coefficient
$HY_{j2,2}^2$	$j=jc+1,\dots,jj$	$jj-jc$	Specific taxes levied by foreign government on every unit sales of good $j2$ on foreign markets
$Y_{ij2,2}^f$	$i=iic+1,\dots,ii$ $j=1,\dots,jj$ $f \in F(i,2)$	$jj$	The sales of good $j2$ by foreign firm $f$ on the foreign markets
$AMY_{ij2,2,j'2}$	$i=iic+1,\dots,ii$ $j=jc+i-iic$ $j'=1,\dots,jj$	$(ii-iic) \times jj$	The amount of goods $j'2$ needed as margins to transport 1 unit of good $j2$ from the foreign producers of industry $i$ to the foreign markets
$I_{ijs}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2ii \times jj$	Demands for inputs (domestic and imported) for capital formation
$I_i$	$i=1,\dots,ii$	$ii$	Capital creation by using industry $i$
$II$		$1$	Total capital creation by using industry
$II_R$		$1$	Real private investment expenditure
$I$		$1$	Aggregate private investment
$K1_i$	$i=1,\dots,ii$	$ii$	Future capital stocks for industry $i$

$R_i$	$i=1,\dots,ii$	$ii$	Current rates of return on fixed capital
$R1_i$	$i=1,\dots,ii$	$ii$	Economy-wide expected rate of return on capital for industry $i$
$PI_{ijs}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2ii \times jj$	Purchasers' prices for produced inputs $js$ for capital creation in industry $i$
$PI_i$	$i=1,\dots,ii$	$ii$	Costs of a unit of capital to industry $i$
$PI$		$1$	Costs of units of capital
$KPI$		$1$	Capital-goods price index
$SHFTI_i$	$i=1,\dots,ii$	$ii$	Shift variable for capital creation in industry $i$
$AI_{ijs}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2ii \times jj$	Input- $js$ -augmenting technological change with respect to capital creation
$AI_{ij}$	$i=1,\dots,ii$ $j=1,\dots,jj$	$ii \times jj$	Input- $j$ -augmenting technological change with respect to capital creation
$AI_i$	$i=1,\dots,ii$	$ii$	Neutral input-augmenting technological change with respect to capital creation in industry $i$
$MI_{(ijs,j'1)}$	$i=1,\dots,ii$ $j,j'=1,\dots,jj$ $s=1,2$	$2ii \times jj^2$	Demands for margins to facilitate input flows to industry $i$ for capital creation
$AMI_{(ijs,j'1)}$	$i=1,\dots,ii$ $j,j'=1,\dots,jj$ $s=1,2$	$2ii \times jj^2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ to domestic industry $i$ for capital creation
$HI_{ijs}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2ii \times jj$	Taxes per unit of sales of inputs $js$ for capital creation to industry $i$
$TI_{ijs}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2ii \times jj$	Variables that allow taxes on the sales of inputs $js$ to industry $i$ for capital formation to be modelled as ad valorem tax
$VI_{ijs}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$	$2ii \times jj$	Variables that allow taxes on the sales of inputs $js$ to industry $i$ for capital formation to be modelled as specific tax

$C_{js}$	$s=1,2$ $j=1,\dots,jj$	$2jj$	Household demands for good $j$ from source $s$
$C_j$	$j=1,\dots,jj$	$jj$	Household demands for good $j$
$PC_{js}$	$j=1,\dots,jj$ $s=1,2$	$2jj$	Purchasers' prices paid for commodities $js$ by households
$PC_j$	$j=1,\dots,jj$	$jj$	Purchasers' prices for consumer goods $j$
$AC_{js}$	$j=1,\dots,jj$ $s=1,2$	$jj$	Commodity- $js$ -augmenting change in household preferences
$AC_j$	$j=1,\dots,jj$	$jj$	Commodity- $j$ -augmenting change in household preferences
$MC_{(js,j'1)}$	$j,j'=1,\dots,jj$ $s=1,2$	$2jj^2$	Demands for margins to facilitate the flows of goods $js$ to households
$AMC_{(js,j'1)}$	$j,j'=1,\dots,jj$ $s=1,2$	$2jj^2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ to households
$HC_{js}$	$j=1,\dots,jj$ $s=1,2$	$2jj$	Taxes per unit of sales of commodity $js$ to households
$TC_{js}$	$j=1,\dots,jj$ $s=1,2$	$2jj$	Variables that allow taxes on the sales of commodity $js$ to households to be modelled as ad valorem tax
$VC_{js}$	$j=1,\dots,jj$ $s=1,2$	$2jj$	Variables that allow taxes on the sales of commodities $js$ to households to be modelled as specific tax
CPI		1	Consumer price index
$E_{j1}$	$j=1,\dots,jjc$	$jjc$	Export volumes of commodity $j1$ , that is sold under perfectly competitive markets
$PE_{j1}$	$j=1,\dots,jjc$	$jjc$	F.o.b. foreign currency export prices of commodity $j1$ , that is sold under perfectly competitive markets

$ME_{(j1,j'1)}$	$j'=1,\dots,j$ $j=1,\dots,jjc$	$jj \times jjc$	Demands for margins to facilitate the flows of exports $j$ from domestic perfectly competitive producers to the ports of exit
$AME_{(j1,j'1)}$	$j'=1,\dots,j$ $j=1,\dots,jjc$	$jj \times jjc$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $j1$ domestic perfectly competitive producers to the ports of exit for exports
$HE_{j1}$	$j=1,\dots,jjc$	$jjc$	Taxes per unit of exports $j1$ under perfect competition
$TE_{j1}$	$j=1,\dots,jjc$	$jjc$	Variables that allow taxes per unit of export good $j$ be modelled as ad valorem tax
$VE_{j1}$	$j=1,\dots,jjc$	$jjc$	Variables that allow taxes per unit of exports to be modelled as specific tax
$SHFTE_{j1}$	$j=1,\dots,jjc$	$jjc$	Shift variables for the demand for exports
$\Phi$		1	The exchange rate, domestic currency price of a unit foreign currency
$GEXP$		1	Government expenditures
$G_{js}$	$s=1,2$ $j=1,\dots,jjc$	$2jjc$	Government demands for good $j$ from source $s$ under perfectly competitive markets
$G_{js}$	$s=1,2$ $j=jjc+1,\dots,jj$	$2(jj-jjc)$	Government demands for good $j$ from source $s$ under imperfectly competitive markets
$GG_{j1}$	$j=jjc+1,\dots,jj$	$(jj-jjc)$	Government procurement of good $j$ from domestic oligopolists
$GG_{ij1}^f$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $f \in F(i,1)$	$(ii-iic)$ $\times F(i,1)$	Home government procurement of good $j1$ supplied by oligopolist firm $f$
$GG_{ij2,2}^f$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $f \in F(i,2)$	$(ii-iic)$ $\times F(i,2)$	Foreign government purchases of goods $j2$ from domestic firm $f$ in industry $i$

$PG_{js}$	$j=1, \dots, jjc$ $s=1,2$	$2jjc$	Price per unit of goods $js$ paid by government prevailed under perfect competitive markets
$PG_{js}$	$j=jjc+1, \dots, jj$ $s=1,2$	$2(jj-jjc)$	Price per unit of goods $js$ paid by government prevailed under imperfect competitive markets
$PGG_{j1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Price per unit of good $j1$ paid for direct government procurement under imperfect competitive markets
$PGGY_{j1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Basic price per unit of goods paid for direct government procurement to the domestic producer under imperfect competitive markets
$GTRNSFR$		1	Government transfer
$MG_{(js,j'1)}$	$j'=1, \dots, jj$ $j=1, \dots, jjc$ $s=1,2$	$2jj \times jjc$	Demands for margins to facilitate the flows of goods $js$ sold under perfectly competitive markets to government
$MG_{(js,j'1)}$	$j'=1, \dots, jj$ $j=jjc+1, \dots, jj$ $s=1,2$	$2jj \times (jj-jjc)$	Demands for margins to facilitate the flows of goods $js$ sold under imperfectly competitive markets to government
$AMG_{(js,j'1)}$	$j'=1, \dots, jj$ $j=1, \dots, jjc$ $s=1,2$	$2jj \times jjc$	The amount of good $j'1$ needed as margins to transport 1 unit of goods $js$ sold under perfectly competitive markets to government
$AMG_{(js,j'1)}$	$j'=1, \dots, jj$ $j=jjc+1, \dots, jj$ $s=1,2$	$2jj \times (jj-jjc)$	The amount of good $j'1$ needed as margins to transport 1 unit of goods $js$ to government
$MGG_{(j1,j'1)}$	$j'=1, \dots, jj$ $j=jjc+1, \dots, jj$	$jj \times (jj-jjc)$	Demands for margins to facilitate the flows of goods $js$ sold under perfectly competitive markets to government
$GREV11$		1	Government revenue from domestic sales of commodity $js$ for intermediate inputs, capital creations, and household consumption
$GREV12$		1	Government revenue from imported goods
$GREV13$		1	Government revenue from exported goods

GREV14		1	Government revenue from domestic and foreign sales paid by domestic firm under imperfectly competitive markets
GREV1		1	Total amount of expenditure taxes received by government
GREV2		1	Total amount of income taxes received by government
GREV		1	Total government revenue
SHFTG <sub>js</sub>	j=1,...,jj s=1,2	2jj	Shift variable
M		1	Money income of the households
MR		1	Real income
IMR		1	Ratio of real private investment expenditure to real private consumption spending
GDP		1	Gross domestic product
VA <sub>i</sub>	i=1,...,ii	ii	Value added accrues to industry i
DEPR		1	Depreciation
INCOME		1	National income
INCOMEDISP		1	Disposable income
EXPORT		1	Aggregate exports
IMPORT		1	Aggregate import
L		1	Aggregate employment
L <sub>t</sub>	t=1,...,tt	tt	Employment of labor by skill type t
K		1	Aggregate capital stock
K <sub>i</sub>	i=1,...,ii	ii	Capital stock used by industry i
LAND		1	Aggregate land used
LAND <sub>i</sub>	i=1,...,ii	ii	Land demanded by industry i
FPX <sub>(,jj+1,1)</sub>		1	
FPX <sub>(i,jj+1,1)</sub>	i=1,...,ii	ii	
FPX <sub>(,jj+1,1,t)</sub>	t=1,...,tt	tt	
FPX <sub>(i,jj+1,1,t)</sub>	i=1,...,ii t=1,...,tt	ii x tt	
FPX <sub>(i,jj+2)</sub>	i=1,...,ii	ii	

## PART III

### THE SOLUTION OF THE MODEL

## CHAPTER 15

### THE SOLUTION OF THE MODEL: ITS NATURE AND ITS COMPUTATION

The objective of this chapter is to explain the nature of the solution to our model as well as its computation. We also discuss how the model can be used for short-run analysis or long-run planning.

#### 1. Endogenous and Exogenous Variables

Our model contains a huge number of variables; we divide them into two groups. In the first group, we put those variables that are assumed constant by the models. These include the various elasticities that enter production functions, such as elasticities of substitution or transformation, price elasticities and elasticities of expenditure in the utility function of households, etc..

If possible, the values of these elasticities are taken from econometric studies or from similar source. Otherwise, they are calculated from the base-period data by a mathematical procedure known as calibration in the field of applied general equilibrium analysis; Shoven and Whalley (1992) contains a detailed discussion of this procedure. The remaining variables belong to the second group.

The variables in the second group are further divided into endogenous and exogenous variables. Clearly, an endogenous variable is one whose behavior we wish to

explain. An exogenous variable, however, is not explained in the model. There are several reasons why one chooses to make one variable exogenous.

First, in spite of its importance, this variable is determined outside the economic system. As an example, one might mention the weather as an important factor in influencing agricultural output and, therefore, farm income. Second, one might classify a variable as being exogenous due to lack of a theory to explain its behavior. This is the reason population growth and technical progress is often taken to be exogenous in multisectoral growth models. Sometimes, investments for capital accumulation are also considered to be exogenous.

In the multisectoral growth model MSG-4, the fourth and latest version of the Johansen model, used by Norwegian Ministry of Finances as a quantitative tool in macroeconomic planning, the production capacity of the economy is determined by the exogenous growth of the labor force, the exogenous technical change in each sector, and the exogenous accumulation of capital (see Longva et al (1985)). Precise numerical assumptions about their development are required to calculate growth paths five to thirty years into the future. Other variables, also often assumed to be exogenous, are tax rates, government expenditures, and so forth.

The choice of exogenous variables is also dictated by the time horizon under which one would like to conduct the analysis. In the short run, the capital stock should be exogenous while its rate of return should be determined within the system. In the long run when there is perfect capital mobility, the stock of capital should be endogenous while its

rate should be set exogenously equal to the world interest rate. If one wants to analyze unemployment in the short run, then the wage rates should be set exogenously; the unemployment level then becomes endogenous and is equal to the demand for labor.

## 2. Solution Computation

Suppose that the choice of endogenous and exogenous variables has been made, and the model has been calibrated to fit the base-period data. Let  $x^0 = (x_1^0, \dots, x_l^0)$  be the vector of exogenous variables and  $y^0 = (y_1^0, \dots, y_m^0)$  be the vector of endogenous variables obtained from the base-period data. Here  $l+m=n$ . The vector  $y^0$  is thus the solution of the model associated with the vector of exogenous variables  $x^0$ . The vector  $(x^0, y^0)$  satisfies the  $m$  relations that characterize an equilibrium.

Now, suppose that the vector of exogenous variables changes from  $x^0$  to  $x^0 + \Delta x$ . Let  $y^0 + \Delta y$  be the new vector of endogenous variables that is the solution of the model corresponding to  $x^0 + \Delta x$ . How should  $\Delta y$  be calculated? A possible answer to this question is as follows. First, let us write the  $m$  relations that define the equilibrium as

$$(15.2.1) \quad \begin{aligned} f_1(x^0, y^0) &= 0, \\ f_2(x^0, y^0) &= 0, \\ &\vdots \\ &\cdot \\ f_m(x^0, y^0) &= 0. \end{aligned}$$

Here  $f_1, \dots, f_m$  are real-valued functions of class  $C^2$  defined on an open subset of the  $n$ -dimensional Euclidean space.

Next, differentiate (15.2.1) at  $(x^0, y^0)$  to obtain a system of linear equations in percentage changes for the vectors  $x$  and  $y$ . In our model, the equations constituting this system are indicated by a superscript  $\hat{\phantom{x}}$  (hat) in the equation numbers that identify them. These equations represent the linearized form in percentage changes of the mathematical model and have been gathered in Chapter 14. For our purpose, we shall write this system more compactly under the following form

$$(15.2.2) \quad \left[ \mathfrak{I}(x^0, y^0) \right] \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = 0,$$

where  $\mathfrak{I}$  is an  $m \times n$  matrix whose entries can be computed directly from  $(x^0, y^0)$ ,

$$\hat{x} = \begin{bmatrix} dx_1/x_1^0 \\ \vdots \\ dx_l/x_l^0 \end{bmatrix}, \quad \hat{y} = \begin{bmatrix} dy_1/y_1^0 \\ \vdots \\ dy_m/y_m^0 \end{bmatrix}.$$

The linear system in percentage changes (15.2.2) can be rewritten as follows.

$$(15.2.3) \quad \left[ \mathfrak{I}1(x^0, y^0) \right] \hat{x} + \left[ \mathfrak{I}2(x^0, y^0) \right] \hat{y} = 0,$$

where  $\mathfrak{I}1(x^0, y^0)$  is the matrix made up of the first  $l$  columns of  $\mathfrak{I}(x^0, y^0)$  and  $\mathfrak{I}2(x^0, y^0)$  is the matrix made up of the last  $m$  columns of  $\mathfrak{I}(x^0, y^0)$ .

If  $\mathfrak{I}2(x^0, y^0)$  is invertible, then

$$(15.2.4) \quad \hat{y} = -\left[ \mathfrak{I}2(x^0, y^0) \right]^{-1} \left[ \mathfrak{I}1(x^0, y^0) \right] \hat{x}.$$

It follows directly from (15.2.4) that

$$\begin{aligned}
 (15.2.5) \quad \begin{bmatrix} dy_1 \\ \vdots \\ dy_m \end{bmatrix} &= - \begin{bmatrix} y_1^o & & 0 \\ & \dots & \\ 0 & & y_n^o \end{bmatrix} [\mathfrak{M}(x^o, y^o)]^{-1} [\mathfrak{M}(x^o, y^o)] \begin{bmatrix} 1/x_1^o & & 0 \\ \vdots & \dots & \vdots \\ 0 & & 1/x_l^o \end{bmatrix} \begin{bmatrix} dx_1 \\ \vdots \\ dx_l \end{bmatrix} \\
 &= - [\Lambda(x^o, y^o)] \begin{bmatrix} dx_1 \\ \vdots \\ dx_l \end{bmatrix}.
 \end{aligned}$$

Using (15.2.5), we can propose to approximate  $\Delta y$  by the following formula.

$$(15.2.6) \quad \begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_m \end{bmatrix} \approx - [\Lambda(x^o, y^o)] \begin{bmatrix} dx_1 \\ \vdots \\ dx_l \end{bmatrix}.$$

Expression similar to (15.2.6) was used by Johansen (1960) to approximate the change in the solution of his model associated with a change in the exogenous variables. This approach has also been used by Deardoff and Stern (1990) for approximating a new equilibrium for their world trade model.

Clearly (15.2.6) only yields good approximations when the changes in the exogenous variables are small enough; otherwise, the error might not be acceptable. To deal with this situation, Dixon et al (1982) have proposed to approximate the new solution by the Euler method used to obtain a numerical solution of a first-order ordinary differential equation. This technique can be explained as follows.

If  $D_2f(x^o, y^o)$ , the derivative of  $f$ ,  $f=(f_1, \dots, f_m)$ , with respect to  $y$  evaluated at  $(x^o, y^o)$ , is invertible, then by the implicit function theorem, there exists a unique function of class  $C^2$

$$(15.2.7) \quad g : N(x^0) \rightarrow N(y^0),$$

where  $N(x^0)$  is a neighborhood of  $x^0$  and  $N(y^0)$  is a neighborhood of  $y^0$ , such that  $g(x^0)=y^0$  and for all  $x$  in  $N(x^0)$ , we have  $g(x) \in N(y^0)$  and  $f(x, g(x)) = 0$ . The function  $g$  maps a vector of exogenous variables  $x$  to a solution  $y = g(x)$  of the model for  $x$  belonging to a neighborhood of  $x^0$ . Using (15.2.5), we can assert that derivative of  $g$  evaluated at  $(x^0, y^0)$  is  $Dg(x^0) = -\Lambda(x^0, y^0) = -\Lambda(x^0, g(x^0))$ . Therefore, for any  $x \in N(x^0)$ , we have

$$(15.2.8) \quad Dg(x) = -\Lambda(x, g(x)).$$

Now suppose that the vector of exogenous variables change from  $x^0$  to  $x^0 + \Delta x$  and  $x^0 + \Delta x$  belongs to some neighborhood  $N(x^0)$  for which the existence of a function  $g$ , defined by (15.2.7), is guaranteed. Then the variation in the solution of the model is

$$(15.2.9) \quad \Delta y = g(x^0 + \Delta x) - g(x^0).$$

To approximate (15.2.9) by the Euler method, let  $\tau$  be a variable,  $0 \leq \tau \leq 1$ , and define

$$(15.2.10) \quad \omega(\tau) = g(x^0 + \tau \Delta x), \quad 0 \leq \tau \leq 1.$$

We have  $\omega(0) = g(x^0) = y^0$ ,  $\omega(1) = g(x^0 + \Delta x)$ , and  $\Delta y = \omega(1) - \omega(0)$ . Differentiating (15.2.10) with respect to  $\tau$ , we obtain

$$(15.2.11) \quad \begin{aligned} \frac{d\omega}{d\tau} &= [Dg(x^0 + \tau \Delta x)] \Delta x \\ &= -[\Lambda(x^0 + \tau \Delta x, g(x^0 + \tau \Delta x))] \Delta x \\ &= -[\Lambda(x^0 + \tau \Delta x, \omega(\tau))] \Delta x \end{aligned}$$

In (15.2.11), the second equality is obtained using (15.2.8) and the last equality is obtained by using the definition of  $\omega(\tau)$ .

Now define

$$(15.2.12) \quad \mathfrak{G}(\tau, \omega) = -\Lambda(x^0 + \tau \Delta x, \omega)$$

where  $\omega = (\omega_1, \dots, \omega_m)$  is an  $m$ -vector. Then the differential equation (15.2.11) can be rewritten as

$$(15.2.13) \quad \frac{d\omega}{d\tau} = [\mathfrak{G}(\tau, \omega)]\Delta x, \quad \omega(0) = y^0.$$

Let  $\bar{N}(x^0)$  and  $\bar{N}(y^0)$  be the closures of  $N(x^0)$  and  $N(y^0)$ , respectively. Because both  $N(x^0)$  and  $N(y^0)$  can be taken to be bounded, their Cartesian product  $\bar{N}(x^0) \times \bar{N}(y^0)$  is a compact set in the  $n$ -Euclidean space. Furthermore, because  $f$  is of class  $C^2$ , the map  $\Lambda : (x, y) \in \bar{N}(x^0) \times \bar{N}(y^0) \rightarrow \Lambda(x, y)$  is of class  $C^1$ . Hence its derivative  $D\Lambda(x, y)$  is continuous on  $\bar{N}(x^0) \times \bar{N}(y^0)$ . In particular, the partial derivative of  $\Lambda$  with respect to  $y$  is bounded in  $\bar{N}(x^0) \times \bar{N}(y^0)$ , i.e., the norm of the linear map  $D_2\Lambda(x, y)$  is bounded

$$(15.2.14) \quad \max_{(x, y) \in \bar{N}(x^0) \times \bar{N}(y^0)} |D_2\Lambda(x, y)| < \lambda_{\max} < \infty,$$

where  $\lambda_{\max}$  is a positive number.

Using (15.2.14) and the definition of  $\mathfrak{G}$ , we can now assert that the norm of the partial derivative of  $\mathfrak{G}$  with respect to  $\omega$  satisfies the following relations

$$(15.2.15) \quad \max_{\substack{0 \leq \tau \leq 1 \\ \omega \in \bar{N}(y^0)}} |D_2\mathfrak{G}(\tau, \omega)| = \max_{\substack{0 \leq \tau \leq 1 \\ \omega \in \bar{N}(y^0)}} |D_2\Lambda(x^0 + \tau\Delta x, \omega)| < \lambda_{\max}.$$

Now it is well known (see Lang (1983, Corollary 4.3, p.107)) that the following mean value theorem holds

$$(15.2.16) \quad |\mathfrak{G}(\tau, \omega + \Delta\omega) - \mathfrak{G}(\tau, \omega)| \leq |\Delta\omega| \max_{0 \leq \xi \leq 1} |D_2 \mathfrak{G}(\tau, \omega + \xi \Delta\omega)|.$$

Here  $|\mathfrak{G}(\tau, \omega + \Delta\omega) - \mathfrak{G}(\tau, \omega)|$  and  $|\Delta\omega|$  denote, respectively, the Euclidean norms of the vectors  $\mathfrak{G}(\tau, \omega + \Delta\omega) - \mathfrak{G}(\tau, \omega)$  and  $\Delta\omega$ .

Together (15.2.16) and (15.2.15) imply that  $\mathfrak{G}(\tau, \omega)$  satisfies the following Lipschitz condition

$$(15.2.17) \quad |\mathfrak{G}(\tau, \omega) - \mathfrak{G}(\tau, \omega')| < \lambda_{\max} |\omega - \omega'| \text{ for all } \omega, \omega' \text{ in } N(y^0).$$

The Lipschitz condition (15.2.17) allows us to invoke the corollary of Theorem 3 on page 187 of Birkhoff and Rota (1969), which asserts that in the limit an approximate solution of (15.2.13) obtained by the Euler method converges uniformly to the exact solution of this differential equation. For our purposes an estimate of  $\omega(1)$  by the Euler method can be described as follows.

Let  $k$  be a positive integer and define a finite sequence  $(\tau^{k'}, \omega^{k'})_{k'=0}^k$  in the following manner

$$(15.2.18) \quad \begin{aligned} \text{a) } \tau^{k'} &= k'/k \\ \text{b) } \omega^0 &= y^0 \\ \omega^1 &= \omega^0 + \left[ \mathfrak{G}(\tau^0, \omega^0) \right] \left[ \frac{\Delta x}{k} \right] \\ &= \omega^0 - \left[ \Lambda(x^0 + \tau^0 \Delta x, \omega^0) \right] \left[ \frac{\Delta x}{k} \right] \\ \omega^2 &= \omega^1 + \left[ \mathfrak{G}(\tau^1, \omega^1) \right] \left[ \frac{\Delta x}{k} \right] \\ &= \omega^1 - \left[ \Lambda(x^0 + \tau^1 \Delta x, \omega^1) \right] \left[ \frac{\Delta x}{k} \right] \\ &\vdots \\ &\vdots \end{aligned}$$

$$\omega^{k'} = \omega^{k'-1} - \left[ A(x^o + \tau^{k'-1} \Delta x, \omega^{k'-1}) \right] \left[ \frac{\Delta x}{k} \right], \quad k'=1, \dots, k.$$

The Euler scheme defined by (15.2.18) can be used to approximate  $\omega(1)$  to any desired degree of accuracy. More precisely, we have

$$(15.2.19) \quad \lim_{k \rightarrow \infty} \omega^k = \omega(1) = g(x^o + \Delta x) = y^o + \Delta y.$$

### 3. Nature of the Computed Solution

The method proposed in the preceding section for computing the solution of the model gives an answer to the following question. Suppose that  $y^o$  is the solution of the model when vector of exogenous variables is  $x^o$ . What will be the solution of the model if the vector of exogenous variables is  $x^1$  instead? The Euler scheme, as represented by (15.2.18) gives us an answer to this purely comparative static exercise by providing an approximation of the exact solution  $y^1$  that is associated with the vector of exogenous variables  $x^1$ .

Although our model does not have any explicit dynamics, we would like to visualize the economy evolving from one time period to another. The driving forces behind this motion is the growth in the labor force, technical progress, and capital accumulation. The length of each period is assumed to be sufficiently short so that the labor force, the capital stocks, and the production technology can be considered as exogenous within each period.

However, there is also enough time in each period for capital investments and learning by doing to take place. The length of a time period is thus chosen so that the effects of capital investments and technical change only take place in the next period. In this fashion, the economy is assumed to be in static general equilibrium in each period, given the exogenous variables of this period. However, in the following period, the economy possesses a new stock of capital, a new stock of knowledge, and a new labor force. The static equilibrium for this period will not be the same as the static equilibrium of the last period. The Euler scheme in the preceding section can then be used to compute this equilibrium. If one links these static equilibria over a number of periods, one obtain a time path for the economic development of the economy under consideration. In this manner, our model can be used for economic planning or to assess the impact of industrial and trade policies after some time has elapsed.

## PART IV

# DATA SETS, DATA TRANSFORMATIONS, AND SIMULATIONS

## CHAPTER 16

### DATA SETS AND DATA TRANSFORMATION

#### 1. Presentation of Data Sets

We need two sets of data for computing general equilibrium: an input-output (IO) data set and an elasticity data set. Parameters, such as revenue, cost, and expenditure shares are calculated through a calibration process, using the IO data set. Other parameters, such as substitution or transformation elasticities, price elasticities, expenditure elasticities are assumed constant and may be taken from econometric studies or other literature. However, we may also calibrate price and expenditure elasticities following Frisch's formula (see Frisch, 1959).

The input-output (IO) data set is taken from an IO Table published by national bureaus of statistics in many countries. The IO Table presents data in matrix form. Each column shows intermediate and primary inputs used and obtained from its own and other industries in the production process. Each row shows how the output of each industry is distributed among intermediate and final demands. In addition, in its columns an IO table also lists trade margins, transport costs, and final demands such as private consumption, government expenditure, gross fixed capital formation, export, import, and changes in stocks. Its rows also list indirect taxes, subsidies, and components of primary factors such as the value of labor and capital uses, gross operating surplus, and depreciation.

For our computations, we need an IO data set that contains 4 (four) separate files: (a) one for domestic transactions for both current production, final demand, and capital formation; (b) one for import transactions for both current production, final demand, and capital formation; (c) one for trade margins and transport costs; and (d) one for indirect taxes and subsidies (net taxes). Domestic transactions will be free of imports, trade margins, transport costs, and net taxes. It is unlikely that an IO Table, presented in its usual format, is suited to our needs. Therefore, we shall transform the data from an ordinary IO Table into data files required by our CGE model. The method of transformation is explained in the next section.

## **2. Input-Output Data Set**

### **2.1. Indonesian Input-Output Table**

We intend to apply our model to Indonesia. First, The Bureau of Statistic of Indonesia (BSI) publishes an Input-output (IO) Table that is updated every five years. IO Table (Volume II) contains data for both 66 and 19 sectors. For our computations, we use the 19-sector IO Table for the year 1985, published in 1989. Moreover, we merge the sectors "public administration and defense" into a sector called the "unspecified sector" because the former sector contains mostly zeros in both its row and column. Therefore, we finally have 18 sectors in our application.

The 19 sector Indonesian IO Table consists of three basic tables: (a) transactions at

Table 16.1. Transactions at Purchasers Prices (Billion Rupiah)

	01	02	03	04	05	06	07	08	09	10
01	132.13	131.82	0	22.29	0	0	0	6873.69	.49	0
02	0	327.80	4.11	29.43	0	10.05	0	921.72	3.36	0
03	4.43	8.41	648.51	61.88	0	.06	0	2604.18	653.90	0
04	12.47	51.55	10.42	1649.23	0	.16	0	49.22	37.90	0
05	.56	1.57	11.21	7.64	9.67	13.48	1.76	8.34	1062.13	0
06	0	0	0	.69	0	200.03	0	182.29	.05	0
07	0	0	0	.03	0	8.38	640.67	8.07	769.40	5709.34
08	0	0	0	457.26	0	32.69	0	1316.19	81.34	0
09	554.37	347.24	407.62	67.18	65.99	84.34	295.26	452.54	9378.17	124.89
10	.34	5.94	42.27	40.07	43.57	88.24	168.03	111.51	553.16	30.66
11	0	0	7.25	11.26	3.04	1.49	4.80	35.53	194.20	41.92
12	5.74	9.37	94.18	15.42	16.03	5.87	105.86	17.95	51.01	44.22
13	0	0	0	0	0	0	0	0	0	0
14	0	.22	5.42	2.33	4.40	4.25	92.59	39.53	75.98	98.57
15	1.82	6.15	28.68	3.99	6.10	2.58	115.69	41.56	122.31	69.19
16	58.88	22.54	74.10	30.36	19.22	27.16	550.39	186.63	274.62	139.14
17	0	0	0	0	0	0	0	0	0	0
18	4.01	6.90	62.72	10.73	39.66	6.35	180.20	39.13	78.71	82.85
19	0	0	0	0	0	0	0	6.44	82.35	9.72
190	774.76	919.49	1356.51	2409.78	207.68	476.60	2155.25	12894.52	13419.09	6350.48
200	0	0	0	0	0	0	0	0	0	0
201	1324.35	949.07	1091.36	598.53	243.82	281.11	951.75	745.74	1941.15	294.19
202	4900.73	5526.07	2719.75	1795.26	1039.65	1296.72	12931.39	1704.10	5141.58	3579.57
203	83.12	34.26	127.27	39.77	109.37	68.05	667.88	492.68	739.37	409.03
204	57.79	40.20	22.84	30.18	11.59	10.46	20.94	677.33	306.40	4.55
205	0	0	0	0	0	0	0	0	-806.68	0
209	6365.99	6549.60	3961.22	2463.74	1404.43	1656.34	14571.96	3619.85	7321.82	4287.34
210	7140.75	7469.09	5317.72	4873.53	1612.11	2132.94	16727.21	16514.37	20740.90	10637.82

(Continued...)

(Continued...)

Table 16.1. Transactions at Purhasers Prices (Billion Rupiah)

	11	12	13	14	15	16	17	18	19	180
01	0	46.50	.03	0	0	0	0	.30	0	7207.25
02	0	0	.36	289.38	.84	0	0	38.96	0	1617.01
03	0	.93	.27	155.73	.23	0	0	4.08	0	4142.62
04	0	0	0	678.88	1.59	0	0	47.04	0	2538.46
05	0	556.69	.08	20.89	.10	0	0	2.64	0	1696.77
06	0	0	0	185.96	.76	0	0	6.27	0	576.05
07	104.45	1315.55	.05	.01	.06	0	0	0	0	8556.02
08	0	0	2.51	1108.12	14.87	2.49	0	61.90	0	3077.37
09	120.63	7218.57	217.49	165.13	287.68	179.60	0	2109.80	3.21	22080.21
10	791.28	1916.53	189.45	178.30	1484.86	29.21	0	102.50	3.23	5779.15
11	287.45	10.90	107.26	149.93	51.14	42.68	0	213.27	1.13	1163.25
12	50.29	31.39	88.40	64.99	179.65	436.63	0	111.53	0	0
13	0	0	0	0	0	0	0	0	0	1288.51
14	1.91	69.01	171.36	26.37	95.23	78.56	0	50.13	0	815.83
15	14.72	36.74	290.34	40.44	809.05	146.83	0	89.42	1.46	1827.05
16	14.13	409.76	642.36	213.75	358.31	608.07	0	312.58	0	3941.98
17	0	0	0	0	0	0	0	0	0	0
18	21.86	19.77	127.33	34.97	1169.90	167.60	0	276.81	.59	2350.09
19	0	.96	11.72	6.02	.01	.95	0	1.55	0	119.71
190	1406.72	11633.31	1848.98	3338.86	4454.29	1692.62	0	3428.76	9.63	68777.31
200	0	0	0	0	0	0	0	0	0	0
201	143.37	3232.80	2058.83	707.03	1741.17	1298.73	6071.4	3388.93	13.56	27076.92
202	197.86	2342.16	8542.66	1342.64	2755.99	4513.36	0	1966.87	28.86	62325.22
203	53.97	399.72	489.35	234.28	1166.90	429.18	303.57	365.93	.82	6214.54
204	.71	248.87	873.58	170.70	82.54	165.24	0	111.50	.49	2835.84
205	0	0	0	0	0	0	0	0	0	-806.68
209	395.91	6223.55	11964.43	2454.65	5746.59	6406.52	6375.0	5833.23	43.73	97645.88
210	1802.63	17856.86	13813.41	5793.51	10200.9	8099.14	6375.0	9261.99	53.35	166423.2

(Continued...)

(Continued...)

Table 16.1. Transactions at Purhasers Prices (Billion Rupiah)

	301	302	303	304	305	306	309	310	401	402
01	0	0	0	63.19	79.86	0	143.05	7350.30	0	0
02	8310.66	0	0	35.94	64.58	0	8411.18	10028.19	421.97	.20
03	1211.24	3.99	0	77.84	1473.52	0	2766.59	6909.21	372.06	.95
04	3107.60	.04	.54	40.66	34.03	0	3182.87	5721.32	14.75	.06
05	396.42	0	0	71.46	124.74	0	592.61	2289.38	3.40	.03
06	2366.96	0	0	12.32	272.71	0	2651.99	3228.04	1.17	.02
07	2.10	0	0	300.91	9799.36	0	10102.38	18658.39	1157.50	.73
08	15758.33	0	0	118.71	191.02	0	16068.06	19145.42	211.57	4.21
09	5875.43	718.20	5458.30	352.07	4105.39	49.26	16558.65	38638.86	9658.58	201.8
10	1652.19	181.71	0	-96.85	4983.54	0	6720.59	12499.74	476.64	4.19
11	522.24	117.13	0	0	0	0	639.37	1802.63	0	0
12	0	374.21	16194.1	0	0	0	16568.35	17856.86	0	0
13	0	0	0	0	0	0	0	0	0	0
14	4544.43	651.76	0	0	0	212.74	5408.92	6224.76	0	0
15	3913.94	330.23	0	0	0	571.56	4815.74	6642.79	0	0
16	4162.37	377.02	0	0	0	522.71	5062.09	9004.07	3.65	.04
17	0	6375.00	0	0	0	0	6375.00	6375.00	0	0
18	5395.18	2296.71	126.73	0	2.04	0	7820.66	10170.74	5.71	.14
19	-17.68	-25.90	0	0	35.47	0	-8.11	111.40	55.77	1.00
190	57201.40	11400.1	21779.7	976.23	21166.27	1356.27	113879.97	182657.29	12382.8	213.4
200	0	0	0	0	0	0	0	0	0	0

(Continued...)

(Continued...) Table 16.1. Transactions at Purchasers Prices (Billion Rupiah)

	403	404	409	501	502	503	509	600	700
01	0	0	0	186.37	0	23.17	209.55	7140.75	7350.30
02	6.72	0	428.89	903.91	872.38	353.93	2130.21	7469.09	10028.19
03	4.78	0	377.79	842.15	89.89	281.67	1213.70	5317.72	6909.21
04	.46	0	15.26	446.68	236.91	148.95	832.53	4873.53	5721.32
05	.18	0	3.61	494.80	48.88	129.99	673.67	1612.11	2289.38
06	.15	0	1.34	562.90	367.10	163.76	1093.76	2132.94	3228.04
07	8.81	0	1667.03	558.98	.89	204.30	764.16	16727.21	18658.39
08	13.66	0	229.43	1113.12	677.31	609.19	2401.63	16514.37	19145.42
09	507.30	207.37	10575.1	4865.39	580.52	1876.98	7322.89	20740.90	38638.86
10	3.24	0	484.07	691.51	271.74	414.60	1377.85	10637.82	12499.74
11	0	0	0	0	0	0	0	1802.63	1802.63
12	0	0	0	0	0	0	0	17856.86	17856.86
13	0	0	0	-10667.79	-3145.61	0	-13813.4	13813.41	0
14	0	431.24	431.24	0	0	0	0	5793.51	6224.76
15	0	648.43	648.43	0	0	-4206.52	-4206.52	10200.38	6642.79
16	.06	901.17	904.94	0	0	0	0	8099.14	9004.07
17	0	0	0	0	0	0	0	6375.00	6375.00
18	.44	902.46	908.75	0	0	0	0	9261.99	10170.74
19	1.47	0	58.25	0	0	0	0	53.35	111.60
190	547.27	3090.7	16234.1	0	0	0	0	166423.19	182657.29
200	0	0	0	0	0	0	0	0	0

Source: Indonesian Input-Output Table, 1985, Vol. 1, Central Bureau of Statistic, the Republic of Indonesia

Abbreviations:

01 = Paddy; 02 = Other farm food crops; 03 = Other crops; 04 = Livestock and its product; 05 = Forestry; 06 = Fishery; 07 = Mining and quarrying; 08 = Manufacture of food, beverage, and tobacco; 09 = Other manufacturing industries; 10 = Petroleum refinery; 11 = Electricity, gas, and water; 12 = Construction and building; 13 = Trade; 14 = Restaurant and hotel; 15 = Transportation and communication; 16 = Financial institution, real estate, and business services; 17 = Public administration and defense; 18 = Other services; 19 = Unspecified sector; 190 = Total intermediate input; 200 = Import; 201 = Wage and salary; 202 = Operating surplus; 203 = Depreciation; 204 = Indirect tax; 205 = Subsidy; 209 = Gross value added; 210 = Total input; 180 = Total intermediate demand; 301 = Private consumption (C); 302 = Government consumption (G); 303 = Gross fixed capital formation (GFCF); 304 = Change in stock; 305 = Export of goods; 306 = Export of services; 309 = Total final demands; 401 = Import of goods; 402 = Import sale tax; 403 = Import duty; 404 = Import of services; 409 = Total import; 501 = Wholesale trade margins; 502 = Retail trade margins; 503 = Transport costs; 509 = Total trade margins and transport costs; 600 = Total output; and 700 = Total supply.

Table 16.2. Transactions at Producers Prices (Billion Rupiah)

	01	02	03	04	05	06	07	08	09	10
01	130.15	123.25	0	21.71	0	0	0	6675.40	.48	0
02	0	302.42	3.25	24.54	0	.95	0	850.42	2.76	0
03	4.18	7.77	372.51	34.72	0	.05	0	2175.18	360.72	0
04	10.67	44.75	8.63	1414.90	0	.13	0	39.39	33.30	0
05	.39	1.12	8.01	3.39	8.40	9.48	1.28	6.47	737.79	0
06	0	0	0	.42	0	131.06	0	112.48	.03	0
07	0	0	0	.01	0	4.20	640.13	4.09	627.11	5708.74
08	0	0	0	378.83	0	26.16	0	1134.23	66.93	0
09	304.82	313.00	347.09	49.36	52.26	75.85	247.43	341.36	7994.52	103.19
10	.28	4.92	35.01	33.40	36.58	73.86	142.03	93.43	462.32	27.90
11	0	0	7.25	11.26	3.04	1.49	4.80	35.53	194.20	41.92
12	5.74	9.37	54.18	15.42	16.03	5.87	105.86	17.95	51.01	44.21
13	41.27	55.28	114.78	270.15	16.87	82.20	57.35	839.85	1559.43	19.22
14	0	.22	5.42	2.33	4.40	4.25	92.59	39.53	73.98	98.37
15	14.37	22.96	63.59	86.16	11.23	27.56	133.14	297.00	596.82	75.03
16	58.88	22.54	74.10	30.36	19.22	27.16	550.39	186.63	274.62	139.14
17	0	0	0	0	0	0	0	0	0	0
18	4.01	6.40	62.72	10.73	39.66	6.35	180.20	39.13	78.71	72.85
19	0	0	0	0	0	0	0	6.44	82.35	9.72
190	774.76	919.49	1356.51	2409.78	207.68	476.60	2155.25	12894.5	13419.1	6330.48
200	0	0	0	0	0	0	0	0	0	0
201	1324.35	949.07	1091.36	598.53	243.82	281.11	951.75	745.74	1941.15	294.19
202	4900.73	5326.07	2717.75	1795.26	1039.65	1296.72	12931.4	1704.10	5141.58	3579.37
203	83.12	34.26	127.27	39.77	109.37	68.05	667.88	492.68	739.37	409.03
204	57.79	40.20	22.84	30.18	11.59	10.46	20.94	677.33	306.40	4.55
205	0	0	0	0	0	0	0	0	-806.68	0
209	6365.99	6549.60	3961.22	2463.74	1404.43	1656.34	14571.96	3619.35	7321.82	4237.34
210	7140.75	7469.09	5317.72	4873.53	1612.11	2132.94	16727.2	16514.4	26740.9	10637.8

(Continued...)

(Continued...)

Table 16.2. Transactions at Producers Prices (Billion Rupiah)

	11	12	13	14	15	16	17	18	19	180
01	0	45.18	.03	0	0	0	0	.29	0	7001.49
02	0	0	.27	215.07	.61	0	0	29.73	0	1430.08
03	0	.87	.22	128.80	.20	0	0	3.41	0	3508.63
04	0	0	0	574.44	1.33	0	0	39.81	0	2167.34
05	0	390.17	.06	14.66	.07	0	0	1.86	0	1205.14
06	0	0	0	132.09	.49	0	0	4.78	0	381.36
07	100.82	727.11	.02	.01	.05	0	0	0	0	7812.34
08	0	0	2.05	918.78	11.38	1.77	0	54.09	0	2593.69
09	91.58	5299.27	164.84	127.67	238.48	152.15	0	1699.69	2.63	17825.22
10	680.43	1625.96	156.19	147.62	1254.72	24.53	0	85.76	2.79	4887.73
11	287.43	10.70	107.26	149.93	51.14	42.68	0	213.27	1.13	1163.25
12	50.29	31.39	88.40	64.99	179.65	436.63	0	111.53	0	1288.51
13	110.05	2274.45	66.36	401.63	202.18	25.17	0	348.17	.79	6485.21
14	1.91	69.01	171.36	26.37	95.23	78.56	0	50.13	0	815.83
15	48.18	728.50	310.32	162.59	870.55	134.49	0	195.31	1.70	3799.71
16	14.13	409.76	642.36	213.75	358.31	608.07	0	312.58	0	3941.98
17	0	0	0	0	0	0	0	0	0	0
18	21.84	19.77	127.33	54.97	1169.90	167.60	0	276.81	.59	2350.09
19	0	.94	11.72	6.02	.01	.95	0	1.55	0	119.71
190	1406.72	11633.31	1848.98	3338.86	4454.29	1692.62	0	3428.76	9.63	68777.31
200	0	0	0	0	0	0	0	0	0	0
201	143.37	3232.80	2058.83	707.03	1741.17	1298.73	6071.43	3388.93	13.56	27076.92
202	197.86	2342.16	8542.66	1342.64	2755.99	4513.36	0	1966.87	28.85	62325.22
203	53.98	399.72	489.35	234.28	1166.90	429.18	303.57	365.93	.82	6214.54
204	.71	248.37	873.58	170.70	82.53	165.24	0	111.50	.49	2835.89
205	0	0	0	0	0	0	0	0	0	-806.64
209	395.91	6223.55	11964.43	2454.65	5746.59	6406.52	6375.00	5833.23	43.73	97645.88
210	1802.63	17856.86	13813.41	5793.51	10200.88	8099.14	6375.00	9261.99	53.35	166423.19

(Continued...)

(Continued...)

Table 16.2. Transactions at Producers Prices (Billion Rupiah)

	301	302	303	304	305	306	309	310	401	402
01	0	0	0	61.68	77.58	0	139.26	7140.75	0	0
02	6281.12	0	0	34.92	51.86	0	6467.89	7897.98	421.97	.20
03	988.54	3.76	0	67.79	1126.81	0	2186.89	5695.52	372.06	.95
04	2655.21	.03	.47	35.57	30.16	0	2721.45	4888.79	14.75	.06
05	258.78	0	0	64.19	87.61	0	410.57	1615.71	3.40	.03
06	1566.66	0	0	9.49	176.77	0	1752.92	2134.28	1.17	.20
07	.21	0	0	299.67	9782.01	0	10081.90	17894.23	1157.50	.73
08	13882.99	0	0	106.90	160.21	0	14150.10	16743.80	211.57	4.21
09	4603.26	563.94	4634.56	313.90	3325.84	49.26	13490.75	31315.97	9658.52	201.82
10	1382.51	152.84	0	-114.23	4813.05	0	6234.16	11121.89	476.64	4.19
11	522.24	117.13	0	0	0	0	639.37	1802.63	0	0
12	0	374.21	16194.14	0	0	0	16568.35	17856.86	0	0
13	5334.21	140.63	631.68	73.89	1147.78	0	7328.20	13813.41	0	0
14	4544.43	651.76	0	0	0	212.74	5408.92	6224.76	0	0
15	5541.38	372.97	192.12	22.48	349.09	571.56	7049.60	10849.31	0	0
16	4162.37	377.02	0	0	0	522.71	5062.09	9004.07	3.65	.04
17	0	6375.00	0	0	0	0	6375.00	6375.00	0	0
18	5395.18	2296.71	126.73	0	2.04	0	7820.66	10170.74	5.71	.14
19	-17.68	-25.90	0	0	35.47	0	-8.11	111.60	55.77	1.00
190	57201.40	11400.10	21779.70	976.23	21166.27	1356.27	113879.97	182657.29	12382.77	213.36
200	0	0	0	0	0	0	0	0	0	0

(Continued...)

(Continued...) Table 16.2. Transactions at Producers Prices (Billion Rupiah)

	402	404	409	501	502	503	509	600	700
01	0	0	0	0	0	0	0	7140.75	7140.75
02	6.72	0	428.89	0	0	0	0	7469.09	7897.98
03	4.78	0	377.79	0	0	0	0	5317.72	5695.52
04	.46	0	15.26	0	0	0	0	4873.53	4888.79
05	.18	0	3.61	0	0	0	0	1612.11	1615.71
06	.15	0	1.34	0	0	0	0	2132.94	2134.28
07	8.81	0	1167.03	0	0	0	0	16727.21	17894.23
08	13.66	0	229.43	0	0	0	0	16514.37	16743.80
09	507.30	207.37	10575.07	0	0	0	0	20740.90	31315.97
10	3.24	0	484.07	0	0	0	0	10637.82	11121.89
11	0	0	0	0	0	0	0	1802.63	1802.63
12	0	0	0	0	0	0	0	17856.86	17856.86
13	0	0	0	0	0	0	0	13813.41	13813.41
14	0	431.24	431.24	0	0	0	0	5793.51	6224.76
15	0	648.43	648.43	0	0	0	0	10200.88	10849.31
16	.06	901.17	904.94	0	0	0	0	8099.14	9004.07
17	0	0	0	0	0	0	0	6375.00	6375.00
18	.44	902.46	908.75	0	0	0	0	9261.99	10170.74
19	1.47	0	58.25	0	0	0	0	53.35	111.60
190	547.27	3090.68	16234.1	0	0	0	0	166423.19	182657.29
200	0	0	0	0	0	0	0	0	0

Source: Indonesian Input-Output Table, 1985, Vol. 1, Central Bureau of Statistic, the Republic of Indonesia

Abreviations:

01 = Paddy; 02 = Other farm food crops; 03 = Other crops; 04 = Livestock and its product; 05 = Forestry; 06 = Fishery; 07 = Mining and quarrying; 08 = Manufacture of food, beverage, and tobacco; 09 = Other manufacturing industries; 10 = Petroleum refinery; 11 = Electricity, gas, and water; 12 = Construction and building; 13 = Trade; 14 = Restaurant and hotel; 15 = Transportation and communication; 16 = Financial institution, real estate, and business services; 17 = Public administration and defence; 18 = Other services; 19 = Unspecified sector; 190 = Total intermediate input; 200 = Import; 201 = Wage and salary; 202 = Operating surplus; 203 = Depreciation; 204 = Indirect tax; 205 = Subsidy; 209 = Gross value added; 210 = Total input; 180 = Total intermediate demand; 301 = Private consumption (C); 302 = Government consumption (G); 303 = Gross fixed capital formation (GFCF); 304 = Change in stock; 305 = Export of goods; 306 = Export of services; 309 = Total final demands; 401 = Import of goods; 402 = Import sale tax; 403 = Import duty; 404 = Import of services; 409 = Total import; 501 = Wholesale trade margins; 502 = Retail trade margins; 503 = Transport costs; 509 = Total trade margins and transport costs; 600 = Total output; and 700 = Total supply.

Table 16.3. Domestic Transactions at Producers Prices (Billion Rupiah)

	01	02	03	04	05	06	07	08	09	10
01	130.15	128.25	0	21.72	0	0	0	6675.40	.48	0
02	0	182087	3.23	24.64	0	.95	0	484.56	2.76	0
03	4.18	7.77	572.05	54.25	0	.05	0	2086.14	335.28	0
04	10.67	44.75	8.63	1410.92	0	.13	0	39.12	32.19	0
05	.39	1.12	8.01	5.39	8.40	9.49	1.28	6.47	754.30	0
06	0	0	0	.42	0	131.06	0	112.31	.02	0
07	0	0	0	.01	0	4.20	639.94	3.70	431.98	4749.16
08	0	0	0	340.90	0	25.97	0	1083.26	50.14	0
09	437.45	270.42	306.72	48.80	38.57	75.42	138.95	258.35	3695.63	64.35
10	.28	4.80	33.85	31.26	35.57	68.08	125.02	80.94	404.41	26.08
11	0	0	7.25	11.26	3.04	1.49	4.80	35.53	194.20	41.92
12	5.74	9.37	54.18	15.42	16.03	5.87	105.86	17.95	51.01	44.72
13	41.27	55.28	114.78	270.15	16.87	82.20	57.35	839.85	1559.45	19.22
14	0	.22	5.39	2.33	4.35	4.25	91.14	38.71	72.59	91.60
15	14.37	22.97	63.55	86.16	11.17	27.56	127.22	296.41	594.44	69.95
16	58.88	22.54	74.10	30.36	19.22	27.16	477.16	185.71	262.32	120.19
17	0	0	0	0	0	0	0	0	0	0
18	4.01	6.90	62.72	10.73	39.66	6.35	180.20	39.13	78.71	82.85
19	0	0	0	0	0	0	0	6.44	42.54	9.72
190	727.39	868.36	1314.45	2364.70	192.86	470.21	1948.92	12289.99	8562.43	5319.26
200	47.37	51.13	42.05	45.08	14.82	6.40	206.34	604.54	4856.65	1031.22
201	1324.35	949.07	1091.36	598.53	243.82	281.11	951.75	745.74	1941.15	294.19
202	4900.73	5526.07	2719.75	1795.26	1039.65	1296.72	12931.4	1704.10	5141.58	3579.57
203	83.12	34.26	127.27	39.77	109.37	68.05	667.88	492.68	739.37	409.03
204	57.79	40.20	22.84	30.18	11.59	10.46	20.94	677.33	306.40	4.55
205	0	0	0	0	0	0	0	0	0	0
209	6365.99	6549.60	3961.22	2463.74	1404.43	1656.34	14571.9	3619.85	7321.82	4287.34
210	7140.75	7469.09	5317.72	4873.53	1612.11	2132.94	16727.2	16514.37	20740.9	10637.8

(Continued...)

(Continued...)

Table 16.3. Domestic Transactions at Producers Prices (Billion Rupiah)

	11	12	13	14	15	16	17	18	19	180
01	0	45.18	.02	0	0	0	0	.29	0	7001.49
02	0	0	.27	212.28	.61	0	0	29.49	0	1052.76
03	0	.37	.22	124.82	.20	0	0	3.41	0	3189.23
04	0	0	0	574.09	1.33	0	0	39.69	0	2161.51
05	0	390.17	.06	14.65	.07	0	0	1.86	0	1201.65
06	0	0	0	131.74	.49	0	0	4.78	0	380.82
07	100.45	721.99	.02	.01	.05	0	0	0	0	6651.51
08	0	0	2.05	894.13	11.38	1.77	0	53.96	0	2463.56
09	73.16	3924.55	141.09	119.25	158.60	80.71	0	1205.48	2.63	11060.14
10	635.20	1512.95	144.21	136.88	1145.67	22.89	0	74.76	2.79	4485.61
11	287.45	10.90	107.26	149.93	51.14	42.68	0	213.27	1.13	1163.25
12	50.29	31.39	88.40	64.99	179.65	436.63	0	111.53	0	1288.51
13	110.05	2274.45	66.36	401.63	202.18	25.17	0	348.17	.79	6485.21
14	1.91	65.19	135.50	24.7	86.34	74.06	0	481.16	0	746.45
15	47.97	727.65	294.68	161.03	728.86	148.64	0	194.10	1.70	3618.42
16	13.65	321.98	467.78	185.25	286.00	410.92	0	303.23	0	3266.46
17	0	0	0	0	0	0	0	0	0	0
18	21.86	19.77	127.33	54.97	1169.90	167.60	0	169.89	.59	2243.16
19	0	.96	0	5.11	.01	.22	0	1.55	0	66.55
190	1341.99	10048.01	1575.24	3255.48	4022.46	1411.29	0	2803.63	9.63	58526.29
200	64.72	1585.30	273.74	83.38	431.83	281.33	0	625.13	0	10251.02
201	143.37	3232.78	2053.83	707.03	1741.17	1298.73	3071.4	3388.93	13.56	27076.92
202	197.86	2342.16	8542.66	1342.64	2755.99	4513.36	0	1966.87	28.36	62325.22
203	53.98	399.72	489.35	234.28	1166.90	429.18	303.57	265.93	.82	6214.54
204	.71	248.87	873.58	170.70	82.54	165.24	0	111.50	.49	2835.89
205	0	0	0	0	0	0	0	0	0	-806.63
209	395.91	6223.55	11964.4	2454.65	5746.59	6406.52	6375.0	5833.23	43.73	97645.88
210	1802.63	17856.36	13813.4	5793.51	10200.88	8099.14	6375.0	9261.99	53.35	166423.19

(Continued...)

(Continued...)

Table 16.3. Domestic Transactions at Producers Prices (Billion Rupiah)

	301	302	303	304	305	306	309	310	401	402
01	0	0	0	61.68	77.58	0	139.26	7140.75	0	0
02	6334.35	0	0	30.12	51.86	0	6416.33	7469.09	0	0
03	936.28	3.76	0	61.64	1126.81	0	2128.49	5317.72	0	0
04	2654.61	0	.33	26.91	30.16	0	2712.01	4873.53	0	0
05	258.66	0	0	64.19	87.61	0	410.46	1612.11	0	0
06	1565.86	0	0	9.49	176.77	0	1752.13	2132.94	0	0
07	.21	0	0	293.48	9782.01	0	10075.70	16727.21	0	0
08	13787.38	0	0	103.22	160.21	0	14050.81	16514.37	0	0
09	4057.89	387.53	1670.76	189.47	3325.84	49.26	9680.76	20740.90	0	0
10	1320.77	139.43	0	-121.04	4813.05	0	6152.21	10637.82	0	0
11	522.24	117.13	0	0	0	0	639.37	1802.63	0	0
12	0	374.21	16194.14	0	0	0	16568.35	17856.86	0	0
13	5334.21	140.63	631.68	73.89	1147.78	0	7328.20	13813.41	0	0
14	4302.06	532.28	0	0	0	212.74	5047.07	5793.51	0	0
15	5124.20	323.00	192.12	22.48	349.09	571.56	6582.45	10200.88	0	0
16	4002.29	307.68	0	0	0	522.71	4832.68	8099.14	0	0
17	0	6375.00	0	0	0	0	6375.00	6375.00	0	0
18	4593.35	2296.70	126.73	0	2.04	0	7018.82	9261.99	0	0
19	-22.77	-25.90	0	0	35.47	0	-13.20	53.35	0	0
190	54771.60	10971.47	18815.76	815.53	21166.27	1356.27	107896.90	166423.19	0	0
200	2429.81	428.63	2963.94	160.70	0	0	5985.08	16234.10	12382.77	213.38

(Continued...)

(Continued...) Table 16.3. Domestic Transactions at Producers Prices (Billion Rupiah)

	403	404	409	501	502	503	509	600	700
01	0	0	0	0	0	0	0	7140.75	7140.75
02	0	0	0	0	0	0	0	7469.09	7469.09
03	0	0	0	0	0	0	0	5317.72	5317.72
04	0	0	0	0	0	0	0	4873.53	4873.53
05	0	0	0	0	0	0	0	1612.11	1612.11
06	0	0	0	0	0	0	0	2132.94	2132.94
07	0	0	0	0	0	0	0	16727.21	16727.21
08	0	0	0	0	0	0	0	16514.37	16514.37
09	0	0	0	0	0	0	0	20740.90	20740.90
10	0	0	0	0	0	0	0	10637.82	10637.82
11	0	0	0	0	0	0	0	1802.63	1802.63
12	0	0	0	0	0	0	0	17856.86	17856.86
13	0	0	0	0	0	0	0	13813.41	13813.41
14	0	0	0	0	0	0	0	5793.51	5793.51
15	0	0	0	0	0	0	0	10200.88	10200.88
16	0	0	0	0	0	0	0	8099.14	8099.14
17	0	0	0	0	0	0	0	63745.00	63745.00
18	0	0	0	0	0	0	0	9261.99	9261.99
19	0	0	0	0	0	0	0	53.35	53.35
190	0	0	0	0	0	0	0	166423.19	166423.19
200	547.27	3090.68	16234.1	0	0	0	0	0	16234.10

Source: Indonesian Input-Output Table, 1985, Vol. 1, Central Bureau of Statistic, the Republic of Indonesia

Abbreviations:

01 = Paddy; 02 = Other farm food crops; 03 = Other crops; 04 = Livestock and its product; 05 = Forestry; 06 = Fishery; 07 = Mining and quarrying; 08 = Manufacture of food, beverage, and tobacco; 09 = Other manufacturing industries; 10 = Petroleum refinery; 11 = Electricity, gas, and water; 12 = Construction and building; 13 = Trade; 14 = Restaurant and hotel; 15 = Transportation and communication; 16 = Financial institution, real estate, and business services; 17 = Public administration and defense; 18 = Other services; 19 = Unspecified sector; 190 = Total intermediate input; 200 = Import; 201 = Wage and salary; 202 = Operating surplus; 203 = Depreciation; 204 = Indirect tax; 205 = Subsidy; 209 = Gross value added; 210 = Total input; 180 = Total intermediate demand; 301 = Private consumption (C); 302 = Government consumption (G); 303 = Gross fixed capital formation (GFCF); 304 = Change in stock; 305 = Export of goods; 306 = Export of services; 309 = Total final demands; 401 = Import of goods; 402 = Import sale tax; 403 = Import duty; 404 = Import of services; 409 = Total import; 501 = Wholesale trade margins; 502 = Retail trade margins; 503 = Transport costs; 509 = Total trade margins and transport costs; 600 = Total output; and 700 = Total supply.

purchasers' prices (Table 16.1); (b) transactions at producers' prices (Table 16.2); (c) domestic transactions at producers' prices (Table 16.3). Table 16.1 includes import transactions, trade margins, transport costs, and taxes. Table 16.2 includes domestic and import transactions at producers' prices. Table 16.3 contains data on domestic transactions at basic (producers) prices. Transaction for capital formation appears in only one column in all three tables. All three tables are far from meeting the requirements of our computations.

## **2.2. Data Transformation**

Prior to data transformation, each of the three tables taken from the Indonesian IO Table is reorganized so that the order of industries and commodities meets our needs. Industries and commodities are listed in the following order:

01. Paddy
02. Other farm food and crops
03. Other crops
04. Livestock and its products
05. Forest products
06. Fishery
07. Mining and quarrying
08. Manufacture of other products not elsewhere classified
09. Electricity, gas, and water supply
10. Construction and building
11. Trade
12. Restaurant and hotel
13. Financial intermediaries, real estate, and business services
14. Other services
15. Unspecified sector
16. Transport and communication
17. Manufacture of food, beverages, and cigarettes
18. Petroleum refinery

Basically the difference between the original list and our current list is only for industries or commodities 8, 10, 15, and 17. Considering the list of industries in Table 16.1 - 16.3, we observe that industries 8, 10, and 15 are now, respectively, called industries, 17, 18, and 16, whereas industry 17 is now assimilated into industry 15 which is originally named as industry 19. After moving these industries and reordering all the other industries, we finally arrive at the above list.

The first 15 industries or commodities are presumed perfectly competitive while the last three industries or commodities are imperfectly competitive. Unlike industries under perfect competition, imperfectly competitive industries are considered more concentrated or perhaps face more institutional arrangements such as government regulations.

We begin the transformation of data from these three tables by listing the industries and final demands in the rows and the commodities, primary factors, and other costs in the columns (see Figure 16.1). The process of transformation then continues with the following steps.

First, we form two matrices on domestic transactions, called: (1) XCGE1, commodities used for current production ( $X_{ij1}$ ), consumption ( $C_{j1}$ ), government expenditure ( $G_{j1}$ ), and export ( $E_{j1}$ ); and (2) INV1, commodities used for capital formation ( $I_{ij1}$ ). Considering Table 16.3, we can easily generate matrix XCGE1, whose order is 21 x 18, by first taking out two columns on gross fixed capital formation and changes in stocks. We then take from the rows trade margins and transportation and communication the

values of the commodities that are not used as intermediate inputs, such as commodities that are used purely as trade margins and transport costs. In other words, matrix XCGE1 contains only the values of domestic transactions for intermediate inputs and some components of the final demands. In creating matrix INV1, we begin by taking the sum of the two columns gross fixed capital formation and change in stock and call the result  $I_j$ . Next we calculate the share of each commodity  $j$  used as intermediate inputs for current production ( $X_{ij}/X_{j1}$ , where  $X_{j1} = \sum_{i=1}^{ii} X_{ij}$ ) by every industry. This share is then multiplied by  $I_j$  to form the matrix INV1 whose size is of 18 x 18. For instance, for commodity 12 (construction and building), considering row 12 of Table 16.3,  $I_j = 16,194.14$ . This number is computed by summing gross fixed capital formation (column 303, that is 16,194.14) and changes in stocks (column 304, that is 0). The share of commodity 12 used as intermediate inputs by industry 17 is 0.0139, that is 17.95/1288.51 (the sum of intermediate use of commodity 12 (see column 180) is 1288.51 out of which, 17.95, is used by industry 8 (see column 8)). Therefore, the entry in the matrix INV1 (at row 17, column 10 after reordering the rows and columns) is 225.09, that is 0.0139 x 16,194.14. That means commodity 10 contributes 225.09 as inputs for capital formations in industry 17.

Figure 16.1: Input-output Data Set

	COMMODITIES		TRADE MARGINS FROM		TRANSPORT COSTS		NET TAXES		PRIMARY INPUTS				OTHER COST	TOTAL OUTPUT
	Domestic	Import	Domestic	Import	Domestic	Import	Domestic	Import	L	Lnd	Depr	II		
INDUSTRY (Current Production)	XCGE1	XCGE2	TMCP1	TMCP2	TCCP1	TCCP2	TAXCP1	TAXCP2	PRIMINP				OC	OUTPUT
PRIVATE CONS. (C)	XCGE1	XCGE2	TMCP1	TMCP2	TCCP1	TCCP2	TAXCP1	TAXCP2	0	0	0	0	0	0
GOV'NT CONS. (G)	XCGE1	XCGE2	TMCP1	TMCP2	TCCP1	TCCP2	TAXCP1	TAXCP2	0	0	0	0	0	0
EXPORT (E)	XCGE1	0	TMCP1	0	TCCP1	0	TAXCP1	0	0	0	0	0	0	0
INDUSTRY (Capital Formation) (I)	INV1	INV2	TMINV1	TMINV2	TCINV1	TCINV2	TAXINV1	TAXINV2	0	0	0	0	0	0
DUTY ON IMPORTS	0	DUTY	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL USAGE														

Second, we form two separate matrices for imports, namely: (1) XCG2, for transaction on imported intermediate inputs used for current production ( $X_{ij2}$ ), consumption ( $C_{j2}$ ), and government expenditure ( $G_{j2}$ ); and (2) INV2, for capital formation ( $I_{ij2}$ ). XCG2 can be formed by taking the difference between the second and the third IO tables and taking out both columns on: gross fixed capital formation and change in stock. Therefore the order of matrix XCG2 is 20 x 18. By the same process as for the domestic transactions, matrix INV2 can be formed by first computing the share of each commodity  $j2$  flowing as intermediate inputs for current production ( $X_{ij2}/X_{j2}$ ) by every industry. This share is then multiplied by  $I_j$  to form INV2. The order of INV2 is 18 x 18.

Third, we form eight matrices for margins: (1) one for trade margins from transactions of domestically produced commodities ( $j1$ ) used for current production by all industries, private consumption, government consumption, and export (TMCP1); (2) one for trade margins from transactions of imported commodities ( $j2$ ) used for current production by all industries, private consumption, government consumption, and export (TMCP2); (3) one for trade margins from transactions of domestically produced commodities ( $j1$ ) used as intermediate inputs for capital formation (TMINV1); (4) one for trade margins from transactions of imported commodities ( $j2$ ) used as intermediate inputs by all industries for capital formation (TMINV2); (5) one for transport costs margins needed to deliver domestically produced commodities ( $j1$ ) used for current production, consumption, government consumption, and export (TCCP1); (6) one for transport costs

margins needed to deliver imported commodity (j2) used for current production, consumption, government consumption, and export (TCCP2); (7) one for transport costs margins needed to deliver domestically produced commodities (j1) used as intermediate inputs for capital formation (TCINV1); and (8) one for transport cost margins needed to deliver imported commodities (j2) used as intermediate inputs for capital formation (TCINV2).

We begin by third step forming a matrix, called MT1 whose order is 20 x 18, for trade margins combined with transport costs. This matrix is calculated by taking the difference between Tables 16.1 and 16.2 of the Indonesian IO Table. Next, we need to divide the matrix MT1 into two matrices, one for trade margins called TM and one for transport costs called TC. Considering Table 16.3, we note that there is two columns for trade margins (columns 501 and 502) and one for transport costs (column 503). The row sums of these three columns are shown in column 509. We form matrix TM for trade margins by first taking the share of both trade margins (columns 501 plus 502) and transport costs (column 503) in the total trade margins and transport costs (column 509). Then, we multiply this share with each data entry in matrix MT1.

Subsequently, we can form matrix TC for transport costs by taking the share of each data entry in column 502 from row sum of MT1 and multiplying this share with each data entry in matrix MT1. Also, note that the uses of trade margins and transport costs by each component of final demands (including gross fixed capital formation and changes in stocks) appear in the last five columns of TM and TC, whose orders are 23 x 18. From

both TM and TC we shall set apart transactions of intermediate inputs flowing for current productions from those flowing for capital formations. The matrix for trade margins used for current production, called TMCP, is created by taking out from matrix TM the columns of gross fixed capital formation and changes in stocks. Following the same procedures we use to create TMCP, we can form the matrix for transport costs used for current production, called TCCP. Both TMCP and TCCP have order of  $21 \times 18$ .

Next, we need to form a matrix on trade margins used for capital formation, called TMINV. First, we need to sum both columns of gross fixed capital formations and changes in stocks that appear in TM. Then we follow the same procedures used to form matrix INV1 to generate matrix TMINV. Finally, Following the same procedures used to form TMINV, we can create a matrix on transport costs used for capital formation, called TCINV. Both TMINV and TCINV have the same orders of  $18 \times 18$ .

After forming matrices TMCP, TMINV, TCCP, and TCINV, we then create two matrices from each of the above matrices to distinguish import-related transactions from domestic transactions. First, we need to calculate the shares of domestic and imported commodities in total intermediate inputs, consumption, and government expenditure. These shares can be calculated by taking, respectively, the ratios of XCGE1 and XCG2 to the sum of XCGE1 and XCG2. For capital formation, we use, respectively, the ratios of INV1 and INV2 to the sum of INV1 and INV2. The calculated domestic shares are then multiplied by each of above matrices to form, respectively, TMCP1, TMINV1, TCCP1, and TCINV1. Accordingly, we multiply the import shares by each above matrix to form,

respectively, TMCP2, TMINV2, TCCP2, and TCINV2. The orders of TMCP1 and TCCP1 are 21 x 18, TMCP2 and TCCP2 are 20 x 18, and TMINV1, TMINV2, TCINV1, and TCINV2 are 18 x 18.

The last data file required is for indirect taxes and subsidies. This file contains net taxes for domestic and import flows in both current production and capital formation. From the IO Table we obtain rows for indirect taxes and subsidies and columns for taxes on imports. We, therefore, need to form 4 matrices for net taxes: (1) TAXCP1, net taxes on domestic flows for current production, consumption, government consumption, and export; (2) TAXINV1, net taxes on domestic flows for capital formation; (3) TAXCP2, net taxes on import flows for current production, consumption, government consumption, and export; (4) TAXINV2, net tax on import flows for capital formation.

Matrix TAXCP1 is formed by, first, taking the share of commodity j1 in total use of domestic intermediate inputs by industry i ( $X_{ij1}/X_i$ , where  $X_i = \sum_{j=1}^{jj} X_{ij1}$ ). Considering Table 16.3, this share is multiplied with the rows for the net taxes (rows 204 - 205). Matrix TAXINV1 can be formed by first taking the share ( $I_{ij1}/I_i$ , where  $I_i = \sum_{j=1}^{jj} I_{ij1}$ ) and then multiply this share with the net taxes. For import flow we can form TAXCP2 by first taking the share of commodity j2 from the total use of j2 by all industries ( $X_{ij2}/X_{j2}$ , where  $X_{j2} = \sum_{i=1}^{ii} X_{ij2}$ ) and then multiply this share with the column of import taxes in Table 16.3. Similar to the procedures used to create TAXINV1, we can form TAXINV2 by first calculating the share of commodity j2 for capital

formations ( $I_{ij2}/I_j$ , where  $I_j = \sum_{i=1}^i I_{ij2}$ ) and then this share is multiplied with the column for import taxes. Finally, we note that all these matrices now conform to the specification of Figure 16.1. That is, as mentioned in the beginning of this section, the original rows become columns and the original columns become rows, and the order of industries or commodities follows the above list.

### 3. Elasticities Data Set

Some elasticities, such as price and expenditure elasticities are calculated from IO data sets, using the Frisch formula. Other elasticities, such as elasticities of substitutions between import and domestic output, transformation elasticities, elasticities of substitution between primary factors, and export elasticities are fixed and adapted from other studies and research publications. It is unavoidable that these data sets may not be suitable for our studies. Therefore, policy recommendations, when it comes to the level of magnitudes, would have been more suitable as these data sets improve, especially with respect to quality. We then forewarn the readers to interpret cautiously the result of our simulations.

As for data on elasticity of substitution between import and domestic commodities ( $\sigma_{X_{ij}}$ ) it is fixed,  $\sigma_{X_{ij}} = 2$  for all commodities under perfect competition and  $\sigma_{X_{ij}} = 1$  for all commodities under imperfect competition. For Indonesia, this data is also used by Trewin et al (1993). Data on elasticities of substitution between primary factors ( $\sigma_{X_{(i,jj+1,s)}}$ ) is chosen to be  $\sigma_{X_{(i,jj+1,s)}} = 0.5$  for all factors and industries. In his detailed literature survey, Caddy (1976) pointed out that there is little basis for assigning different values to

different industries. The same value was also adopted by Trewin et al (op cit) for Indonesia. We also choose to fix the reciprocal export elasticity,  $\epsilon_{j1}$  at 0.05. This value is considered to be reasonable since Indonesia has not played a major role in the world exports.

#### **4. Indonesia's Economy and Sectoral Relative Importance**

Indonesia, with a population estimated at 186.00 million in 1992 (or 164.05 million in 1985), is the fourth most populous country in the world. Situated between the Asian and Australian continents, it extends over part of the world's largest archipelago with the area approximately of 1,92 million square kilometers comprising approximately 13,700 islands, of which 6000 are inhabited.

Agriculture, including forestry and fishing, is the dominant economic sector in 1985, accounting for 22.8 percent of GDP. Manufacturing sector is the second largest sector, accounting for 15.5% of GDP. While GDP grows by more than 6% per annum between 1985 and 1990, Manufacturing continues to be one of the most rapidly growing of Indonesia's major economic sector. Manufacturing becomes the dominant sector in 1990, accounting for 20.3% of GDP. Agriculture is thus the second largest sector, accounting for 19.8% of GDP.

In 1985, Gross Domestic Product (GDP) at current market prices amounted to Rp. 98,406.52 billion. Indonesia has been increasingly involving through trade with a numerous countries in the world. Net export -- export minus import -- is estimated to

amount to 7,049 billion rupiah in 1985 which then declined to 3,243 billion rupiah. The rate of inflation as measured by the Consumer Price Index (CPI) has been less than 10.0% in each year from 1984 to 1992. The CPI (1980=100) was approximately 159 in 1985 and then increased to 192 in 1990.

Table 16.4. Indonesia's Economic Performance in 1985 and 1990

Descriptions	1985	1990
Population (million persons)	164.05 <sup>(a)</sup>	186.00 <sup>(b)</sup>
Share in GDP (%) of <sup>(c)</sup>		
1. Agriculture	22.8	19.8
2. Mining and quarrying	14.8	12.2
3. Manufacturing	15.5	20.3
4. Others	53.1	47.7
GDP (billion rupiah) <sup>(c)</sup>	98,406.52	210,866.20
Net Export (billion rupiah) <sup>(c)</sup>	7,049.10	3,243.00
CPI (1980=100)	159	192 <sup>(d)</sup>

Source: Central Bureau of Statistics, Jakarta, Indonesia, except CPI which is taken from Statistical Year Book for Asia and the Pasific (1991), U.N. Publication.

Note:

(a) 1985 Intercensal population survey

(b) Data is for 1992 (based on population estimates)

(c) Data is taken from Indonesian Input-Output Table, (1990), volume I, p.68 and 70

(d) Data is for 1988.

In a more aggregate data, Table 16.4 also indicates the structural changes prevailing in the Indonesian economy throughout the second half of the 1980s. Where in 1985 the highest contribution to GDP accrues to agricultural sector (22.8 percent), in 1990 the highest share in GDP contributed by manufacturing sector (20.2 percent).

Furthermore, in a more elaborate manner, Table 16.5 shows the relative importance of every industry we study. This relative importance can be observed from the proportion of sectoral output used as intermediate inputs and sectoral contributions in GDP, export, import, and labor costs.

Column 2 of Table 16.5 indicate that the proportion of sectoral output flowing as intermediate inputs varies from 10.19 percent, the lowest, by industry 01 (Paddy) to 74.45 percent, the highest, by Industry 09 (Electricity, gas, and water). Regarding the sectoral shares in GDP (column 3 of Table 16.5), Industry 09 contributes the lowest, 0.4 percent, followed by Industry 05 (Forest products), 1.43 percent. The highest contribution comes from Industry 07 (Mining and quarrying), 14.82 percent, followed by Industry 11, 12.16 percent.

Indonesia has long evolved in trade with numerous countries. Among 18 sectors studied we find that both Industries 09 (Electricity, gas, and water) and 10 (Construction and building) do not have transactions in both export and import (see columns 4 and 5). Industry 14 (Services) contributes none to total export but 5.87 percent to total import. Other than those two industries whose shares in both export and import are zero, we also find three other industries whose imports are equal to zero; they are Industries 01 (Paddy), 06 (Fishery), and 11 (Trade). The highest contributions in both export and import come, respectively, from Industry 07 (43.43 percent) and Industry 08 (Other manufacturing), that is 63.76 percent.

Another economic indicator that shows the relative importance of the industry is the sectoral contribution in labor cost. Column 6 of Table 16.5 shows that among all industries, Industry 15 (Unspecified sectors) contributes 22.47 percent (the highest) to total employment and Industry 09 contributes only 0.53 percent (the lowest) to total employment.

Table 16.5. Sectoral Contributions in Intermediate Inputs, GDP, Export, Import, and Labor Costs (in Percent)

<i>No.</i>	<i>Industry</i>	<i>Output Used as Intermedi- ate Inputs</i>	<i>Shares in GDP</i>	<i>Shares in Export</i>	<i>Shares in Import</i>	<i>Shares in Labor Cost</i>
01.	Paddy	10.19	6.47	0.34	0	4.89
02.	Other farm food crops	11.63	6.66	0.23	2.73	3.51
03.	Other crops	24.71	4.03	5.00	2.40	4.03
04.	Poultry and its products	48.52	2.50	0.13	0.09	2.21
05.	Forest products	11.96	1.43	0.40	0.02	0.90
06.	Fishery	22.05	1.68	0.79	0	1.04
07.	Mining and Quarrying	11.65	14.82	43.43	7.48	3.51
08.	Other manufacturing	41.28	8.16	14.99	63.76	7.17
09.	Electricity, gas, water	74.45	0.40	0	0	0.53
10.	Construction and building	56.26	6.32	0	0	11.94
11.	Trade	11.41	12.16	5.09	0	7.60
12.	Restaurant and hotel	56.19	2.50	0.94	2.80	2.61
13.	Financial intermediaries, real estate and other services	17.43	6.51	2.32	5.85	4.80
14.	Other services	30.27	5.93	0	5.87	12.52
15.	Unspecified Sectors	18.04	6.53	0.16	0.36	22.47
16.	Transport and communication	39.43	5.84	4.10	4.19	6.43
17.	Manufacture of food, beverages, and cigarettes	74.42	3.70	0.71	1.37	2.75
18.	Petroleum refinery	50.00	4.36	21.37	3.08	1.09
	<i>Total</i>		<i>100</i>	<i>100</i>	<i>100</i>	<i>100</i>

Source: Calculated using Tables 16.1 through 16.3.

## CHAPTER 17

### SIMULATIONS

#### 1. Introduction

Our main purpose for carrying out this simulation is methodological. As a result, the simulation should be considered as an exercise to illustrate the solution method we use. It is worth noting, however, that relying on our data sets, we hope to observe whether our model can provide an alternative policy tool in analyzing the impact on such economic indicators as GDP, employment, inflation, of any change in such policy variables as tariffs, export subsidies, government procurement, and technological improvement.

As mentioned in Chapter 15, our simulation outcome is computed using the Euler method for 1 and 2 iterations, and then, by the extrapolation process, the values for an infinite number of iterations such that the linearization errors may be negligible, if at all. In every table of the simulation results, therefore, we present the values computed through extrapolation with infinitely many iterations.

We observe that all our policy simulations are targeted toward industry 17, which is manufacture of food, beverages, and cigarettes. This can be explained by two reasons. First, Indonesia, in its sixth Five-Year Development Plan (REPELITA VI) points out that the agroindustry constitutes the basis for transforming the economy from a predominantly agricultural economy into an industrialized one. The other reason is that this industry is one of the three industries under imperfect competition. It is highly concentrated and

considered more difficult to develop since it requires a huge amount of capital and technology for both infrastructures (like roads to and from rural areas, electricity, and communication) and human resources, especially in rural areas. The government has been promoting this industry, without neglecting other sectors. As we know, the agricultural sector contributes approximately 70 percent of employment in Indonesia. Furthermore, realizing that it has been dependent on the revenues from oil exports for too long, the government of Indonesia has also tried to promote non-oil exports through the promotion of the manufacturing and agricultural sectors.

In terms of contribution, this industry, in 1985, contributed approximately 9.92 percent (Rp. 16,514 billion) to the national output (Rp.166,423 billion), less than 4 percent to GDP, and 0.7 percent to total export. Regarding labor costs, in the same period, this industry paid Rp. 745.74 billion or approximately 2.75 percent of total labor costs paid by all industries. Therefore, given its economic potential and attention it receives in Five-Year Development Plans, the government expects to see some progress in its contributions to national development.

## **2. Selections of Endogenous and Exogenous Variables**

Before carrying out the simulations, we form the coefficient matrix  $\mathfrak{Z}$  (see equation (15.2.2)). The order of  $\mathfrak{Z}$  is  $m \times n$ , where  $m$  is the number of equations and  $n$  is the number of variables in our mathematical model -- expressed as a linear system in percentage changes in Chapter 14. We know that the size of  $\mathfrak{Z}$  increases as the number of

industries and commodities increases. For the 18 industries and commodities chosen,  $\mathfrak{J}$  contains approximately several tens of thousands of rows and more than a hundred thousand columns. Considering the limitations in computer power and the processing time required, we find it reasonable to condense the size of  $\mathfrak{J}$  through the substitution process to reduce the number of variables and equations.

After reducing the size of  $\mathfrak{J}$ , we finally have 211 equations and 6,706 variables. Out of 6706 variables, we then choose 211 as endogenous and 6,495 as exogenous. The list of endogenous and exogenous variables is shown in Figure 17.1. Figure 17.1 shows that rows 1 - 24 contain all the selected endogenous variables. Among these variables we include GDP, exports, the balance of payments, the consumer price index, industry employment and outputs, and so forth. Rows 25 - 74 contain exogenous variables, such as rental costs on per unit capital and agricultural land, all kinds of taxes, tariffs, exchange rates, government procurements, technological coefficients, and so forth. However, as we mentioned in Chapter 15, the order and the choice of endogenous and exogenous variables may change according to the needs of users.

Figure 17.1. The List of Variables after the Condensation

<u>No.</u>	<u>Variable</u>	<u>Subscript</u>	<u>Number</u>	<u>Description</u>
01.	$X_{(i,jj+1,s)}$	$i=1,\dots,ii$ $s=1,2,3$	3ii	Industry $i$ 's demands for labor in general capital and agricultural land
02.	$Y_{ij1}$	$i=1,\dots,ii$ $j=jjc+i-ii$	ii	Supplies of domestic composite goods $j1$ by industry $i$
03.	$Q_{j1}$	$j=jjc+1,\dots,jj$	$jj-jjc$	Total amount of good $j$ supplied domestically by both domestic and foreign oligopolists
04.	$Q_{j2}$	$j=jjc+1,\dots,jj$	$jj-jjc$	Total amount of outputs supplied by domestic and foreign oligopolists on foreign markets
05.	$PQ_{j2}$	$j=jjc+1,\dots,jj$	$jj-jjc$	Price of outputs supplied by domestic and foreign oligopolists on foreign markets
06.	$PY_{j1}$	$j=1,\dots,jjc$	$jjc$	Basic prices of domestic commodity $j$ under perfectly competitive markets
07.	$PY_{j2}$	$j=1,\dots,jjc$	$jjc$	Basic prices of imported commodity $j$ under perfectly competitive markets
08.	$PY_{j1}$	$j=jjc+1,\dots,jj$	$jj-jjc$	Price per unit of good $j$ on domestic markets under imperfectly competitive markets
09.	$PY_{j2}$	$j=jjc+1,\dots,jj$	$jj-jjc$	Price per unit of good $j$ on foreign markets under imperfectly competitive markets
10.	GDP		1	Gross domestic product
11.	CPI		1	Consumer price index
12.	EXPORT		1	Aggregate exports
13.	BOP		1	Balance of payments
14.	$R_i$	$i=1,\dots,ii$	ii	Current net rate of return on capital
15.	$R1$		1	Economy-wide expected rate of return on capital for industry $i$

16.	$PSI_{j1}^1$	$j=jjc+1, \dots, jj$	$jj-jjc$	A positive parameter in the domestic demand for good $j$ under imperfect competition
17.	$Y_{ii,1}^f$	$i=iic+1, \dots, ii$ $f \in F(i,1)$	$(ii-iic)$ $xF(i,1)$	Total amount of good $j$ supplied on the domestic markets by domestic oligopolists $f$
18.	$Y_{ii,2}^f$	$i=iic+1, \dots, ii$ $f \in F(i,1)$	$(ii-iic)$ $xF(i,1)$	Total amount of good $j$ supplied on the foreign markets by domestic oligopolists $f$
19.	$Y_{ii,2}^f$	$i=iic+1, \dots, ii$ $f \in F(i,2)$	$(ii-iic)$ $xF(i,2)$	The sales of good $j_2$ by foreign firm on the domestic markets
20.	$Y_{ii,2}^f$	$i=iic+1, \dots, ii$ $f \in F(i,2)$	$(ii-iic)$ $xF(i,2)$	The sales of good $j_2$ by foreign firm $f$ on the foreign markets
21.	IMR		1	Ratio of real private investment to real private consumption
22.	$K1_i$	$i=ii1+1, \dots, ii$	$ii-ii1$	Future capital stocks for industry $i$
23.	L		1	Total employment
24.	GSAV		1	Government saving
25.	$PX_{(i,jj+1,s)}$	$i=1, \dots, ii$ $s=1,2,3$	$3xii$	Prices paid by industry $i$ for labor, rental of capital, and land
26.	$X_{i,jj+2}$	$i=1, \dots, ii$	$ii$	Demands for "other cost" tickets
27.	$UCZ_{i2}$	$i=iic+1, \dots, ii$	$(ii-iic)$	The cost of sustaining 1 unit of activity level of foreign oligopolist industry
28.	$HX_{ijs}$	$i=1, \dots, ii$ $j=1, \dots, jj$ $s=1,2$	$2ii \times jj$	Taxes per unit of sales of inputs $js$ for current production to industry $i$
29.	$HI_{ijs}$	$i=1, \dots, ii$ $j=1, \dots, jj$ $s=1,2$	$2ii \times jj$	Taxes per unit of sales of inputs $js$ for capital creation to industry $i$
30.	$HC_{js}$	$j=1, \dots, jj$ $s=1,2$	$2jj$	Taxes per unit of sales of commodities $js$ to households
31.	$HE_{j1}$	$j=1, \dots, jjc$	$jjc$	Taxes per unit of exports

32.	$HY^1_{j1,1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Specific taxes levied by domestic government on every sales of good j1 on domestic markets
33.	$HY^1_{j1,2}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Specific taxes in foreign currency levied by the home government on the sales of good j1 on foreign markets
34.	$HY^2_{j1,2}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Specific taxes in foreign currency levied by the foreign government on the sales of good j1 on foreign markets
35.	$HY^1_{j2}$	$j=1, \dots, jj$	$jj$	Specific tariff levied by the domestic government on the sales of good j2 on domestic markets
36.	$HY^2_{j2,1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Specific taxes levied by the foreign government on the sales of good j2 on domestic markets
37.	$HY^2_{j2,2}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Specific taxes levied by foreign government on every unit sales of good j2 on foreign markets
38.	$\Phi$		1	The exchange rate, domestic currency price of 1 unit of foreign currency
39.	GTRNSFR		1	Government transfer
40.	$GG_{j1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Government procurement of good j1 from under imperfectly competitive markets
41.	$GG^f_{ij1}$	$i=iic+1, \dots, ii$ $j=i-iic+jjc$ $f \in F(i,1)$	$(ii-iic)$ $\times F(i,1)$	Government procurement of good j1 supplied by domestic oligopolist f in industry i
42.	$SHFTG_{js}$	$j=1, \dots, jj$ $s=1,2$	$2jj$	Shift variable in government demand for good js
43.	$PGGY_{j1}$	$j=jjc+1, \dots, jj$	$jj-jjc$	Basic price per unit of goods paid for direct government procurement under imperfect competitive markets
44.	$PFY_{j2}$	$j=1, \dots, jjc$	$jjc$	C.I.F. price per unit of imported good j2 in foreign currency

45.	$PSI_{j2}$	$j=jc+1, \dots, jj$	$jj-jc$	A positive parameter in foreign demands for good $j$ sold under imperfect competition
46.	$AX_{ijs}$	$i=1, \dots, ii$ $j=1, \dots, jj$ $s=1, 2$	$2ii \times jj$	Input- $js$ -augmenting technical change for industry $i$
47.	$AX_{ij}$	$i=1, \dots, ii$ $j=1, \dots, jj$	$ii \times jj$	Input- $j$ -augmenting technical change for industry $i$
48.	$AXZ_i$	$i=1, \dots, ii$	$ii$	Neutral-input-augmenting technical change for industry $i$
49.	$AX_{(i,jj+1,1,t)}$	$i=1, \dots, ii$ $t=1, \dots, tt$	$ii \times tt$	Specific-skill-augmenting technical change for industry $i$
50.	$AX_{(i,jj+1,s)}$	$i=1, \dots, ii$ $s=1, 2, 3$	$3xii$	Labor-, capital-, and agricultural land-augmenting technical change for industry $i$
51.	$AX_{(i,jj+1)}$	$i=1, \dots, ii$	$ii$	Primary factors-augmenting technical change for industry $i$
52.	$AX_{(i,jj+2)}$	$i=1, \dots, ii$	$ii$	"Other costs" ticket-augmenting technical change for industry $i$
53.	$AY_{ij1}$	$i=1, \dots, ii$ $j=i+iic-jjc$	$ii^2$	Technological coefficient
54.	$AY_{ij2}$	$i=iic+1, \dots, ii$ $j=i-iic+jjc$	$(ii-iic)$	Technological coefficient
55.	$AYZ_{i2}$	$i=iic+1, \dots, ii$	$(ii-iic)$	Technological coefficient
56.	$AI_{ijs}$	$i=1, \dots, ii$ $j=1, \dots, jj$ $s=1, 2$	$2ii \times jj$	Input- $js$ -augmenting technical change with respect to capital creation
57.	$AI_{ij}$	$i=1, \dots, ii$ $j=1, \dots, jj$	$ii \times jj$	Input- $j$ -augmenting technical change with respect to capital creation
58.	$AC_{js}$	$j=1, \dots, jj$ $s=1, 2$	$2 \times jj$	Commodity- $js$ -augmenting change in household preferences
59.	$AC_j$	$j=1, \dots, jj$	$jj$	Commodity- $j$ -augmenting change in household preferences

60.	$AMX_{(ijs,j'1)}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$ $j'=1,2$	$4ii \times jj$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ to domestic industry $i$ for current production
61.	$AMI_{(ijs,j'1)}$	$i=1,\dots,ii$ $j=1,\dots,jj$ $s=1,2$ $j'=1,2$	$4ii \times jj$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ to domestic industry $i$ for capital creation
62.	$AMC_{(js,j'1)}$	$j=1,\dots,jj$ $s=1,2$ $j'=1,2$	$4 \times jj$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $js$ to households
63.	$AME_{(j1,j'1)}$	$j=1,\dots,jjc$ $j'=1,2$	$jjc \times 2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $j1$ to the ports of exit for exports
64.	$AMG_{(js,j'1)}$	$j=1,\dots,jj$ $j'=1,2$	$jj \times 2$	Technical change associated with the use of services in facilitating the flows of goods $js$ to government
65.	$AMY_{ij1,1j'1}$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $j'=1,2$	$(ii-iic) \times 2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $j1$ from the domestic producers of industry $i$ to the domestic markets
66.	$AMY_{ij1,2j'1}$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $j'=1,2$	$(ii-iic) \times 2$	The amount of goods $j'1$ needed as margins to transport 1 unit of good $j1$ from the domestic producers of industry $i$ to the foreign markets
67.	$AMY_{ij2,1j'2}$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $j'=1,2$	$(ii-iic) \times 2$	The amount of goods $j'2$ needed as margins to transport 1 unit of good $j2$ from the foreign producers of industry $i$ to the domestic markets
68.	$AMY_{ij2,2j'2}$	$i=iic+1,\dots,ii$ $j=jjc+i-iic$ $j'=1,2$	$(ii-iic) \times 2$	The amount of goods $j'2$ needed as margins to transport 1 unit of good $j2$ from the foreign producers of industry $i$ to the foreign markets
69.	$SHFTI_i$	$i=1,\dots,ii$	$ii$	Shift variable for capital creation in industry $i$
70.	$SHFTE_{j1}$	$j=1,\dots,jjc$	$jjc$	Shift variable in foreign demand for export good $j1$ under perfect competition
71.	$K$		1	Total capital stock
72.	$LAND$		1	Total agricultural land

73.	I	1	Total investment
74.	II	1	Total aggregate private investment

### 3. Simulation Process

Prior to simulations, the mathematical model -- expressed as a linear system in percentage changes in Chapter 14 -- must be calibrated. The economy under consideration is assumed to be in equilibrium, and the equilibrium is commonly called the benchmark equilibrium. The parameters of the mathematical model are either chosen from econometric studies or computed from the known values of the endogenous and exogenous variables that represent the benchmark equilibrium.

In Chapter 15, we have used the compact symbol  $\mathfrak{J}(x^0, y^0)$  to denote the  $m \times n$  matrix (see equation (15.2.2)) that represents the mathematical model in its linearized form around the benchmark equilibrium. The matrix  $\mathfrak{J}(x^0, y^0)$  is partitioned into two matrices, say  $\mathfrak{J}_1(x^0, y^0)$  and  $\mathfrak{J}_2(x^0, y^0)$ , that correspond, respectively, to the vector of exogenous variables and the vector of endogenous variables. The matrix  $-\mathfrak{J}_2(x^0, y^0)^{-1} \mathfrak{J}_1(x^0, y^0)$ , (see equation (15.2.4)), represents the linear approximation of the percentage changes of the endogenous variables in terms of the percentage changes of the exogenous variables in a neighborhood of  $x^0$ . Next, from  $-\mathfrak{J}_2(x^0, y^0)^{-1} \mathfrak{J}_1(x^0, y^0)$ , we compute  $\Lambda(x^0, y^0)$ , (see equation (15.2.5)), which serves as the basis for computing the changes in the endogenous variables induced by the changes in the exogenous variables.

Having calibrated the model, we proceed with the computational programs for simulations. We choose to use the Euler process with 1- and 2-step iterations. Subsequently, applying the extrapolation process, we calculate the results for an infinite number of iterations. In these simulations, the object of industrial and trade policies is industry 17, namely manufacture of food, beverages, and cigarettes. More precisely, we calculate the effect on macro variables, industry employment, and industry output of hypothetical 20 percent tariff increase, 20 percent export taxes decrease, and 20 percent government procurement increase -- all applied in industry 17.

The last simulations are intended to evaluate the effects on macro variables, industry employment, and industry outputs of technological progress through learning curves -- directed specifically on technological progress through workers experiences (see equation (7.2.2) and (7.2.3) at industry 17). Here we assume that, the industry's cumulative output, called  $SIGMAZ_i(-1)$  is equal to zero. Moreover, at the beginning of the period, we assume that there is no technological progress, i.e., the percentage change of  $AX(0)_{(17,jj+1,1)}$  is set to zero. We also take the experience factor,  $\eta X_{(17,jj+1,1)}$ , as constant at 0.322 (see Fandel (1991), p.207). Finally, as the initial data we assume the percentage change in activity level of industry 17 is equal to 6 percent.

Based on these conditions, we calibrate the technological progress, in percentage change, for the following year, namely  $AX(1)_{(17,jj+1,1)}$ . After this calibration, we integrate the result into our mathematical model in order to evaluate the effects, for subsequent period of time, of this technological progress on macro variables, industry employment's,

and industry outputs. Furthermore, we also continue our simulations, following the same steps as mentioned above, for five years. In Tables 17.4, 17.5, and 17.6, we present the effects of technological progress through workers experiences on macro variables, industry employment's, and industry outputs for the subsequent years: year 1, year 2, and year 5.

#### 4. The effects of Policy Changes on Macro Variables

This section describes, given the economic environments as represented by the setting of exogenous and endogenous variables and elasticities, the effects on macro variables of such hypothetical government policies as 20 percent increase in specific tariff, 20 percent decrease in export tax, 20 percent increase in government procurement, all is targeted on industry 17; that is, manufacture of food, beverages, and cigarettes. We also try to explain the variations in macro variables across government

Table 17.1. The Effects (in Percent) on Macro Variables of Policy Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

	<i>20% Tariff Increase</i>	<i>20% Export Subsidy Increase</i>	<i>20% Gov't Procur. Increase</i>
GDP	0.0028	0.0169	$0.41 \times 10^{-9}$
CPI	$-0.58 \times 10^{-6}$	$-0.39 \times 10^{-4}$	$-0.16 \times 10^{-9}$
Export	0.0049	0.0805	$0.13 \times 10^{-8}$
BOP	0.0526	0.2646	$0.42 \times 10^{-8}$
Aggr. Employment	$0.62 \times 10^{-6}$	$0.42 \times 10^{-4}$	$0.17 \times 10^{-9}$

Note: Industry 17 is imperfectly competitive.

policies. Table 17.1 shows the effect of those government policies, respectively, in columns 2, 3, and 4.

In column 2 we observe that as we increase the tariff on imported manufacture of food, beverages, and cigarettes by 20 percent, GDP increases by 0.003 percent; domestic price decreases by an insignificant point ( $0.58 \times 10^{-6}$  percent); export increases by 0.005 percent; the balance of payment increases by 0.053 percent; and aggregate employment increases by an identical amount in percentage point as domestic price level.

Column 3 indicates the effects on macro variables of a hypothetical 20 percent increase in specific export subsidy on the output of this imperfectly competitive industry. Parallel to the tariff effects, we observe here that all concerned macro variables concerned but inflation go up. However, the magnitude, in absolute terms, of its effects is larger than that of tariff. GDP goes up by 0.017 percent; the domestic price level goes down by  $0.39 \times 10^{-4}$  percent; export rises up by 0.081 percent while balance of payment increases by 0.265 percent; and aggregate employment increases by  $0.42 \times 10^{-4}$  percent.

The last column shows the effects on macro variables of a hypothetical 20 percent increase in government procurement. In this case, we observe that GDP goes up by  $0.41 \times 10^{-9}$  percent, the domestic price level goes down by  $0.16 \times 10^{-9}$  percent. With quite small in magnitude, the increase is also experienced by export, the balance of payments, and aggregate employment. We also note that all these small changes happen because, due to lack of data for the initial year, we assign a very small value for government procurement

close to zero ( $0.1 \times 10^{-6}$ ). Moreover, we assume that the unit price set by government is unchanged.

Considering macro variables as general indications of economic performance, we may observe that the presence of a small specific tariff, export subsidy, or government procurement all helps to increase GDP. The CPI also declines under the three policies. Presumably, the increases in the outputs of the various sectors via industrial linkages help to reduce the general level of prices. Furthermore, beneficial effects of these policies on export, the balance of payments, and aggregate employment are also observed. As far as industry 17 is the targeted industry, and given the policy options available, it seems that export subsidy can provide a relatively higher economic performance.

##### **5. The effects of Policy Changes on Industry Employment**

Table 17.2 shows the effect on industry employment of such policy changes as mentioned above. Columns 2 - 4 contain the effects of, respectively, a tariff increase, an export tax decrease, and an increase in government procurement. In general, we observe that each of these policies, which is targeted at industry 17, does not significantly show variations across industries in both signs and magnitudes. Although manufacture of food, beverages, and cigarettes is the targeted industry, Industry 11 (Trade) and Industry 18 (Petroleum refinery) seem to benefit most from these policies.

Similar to the effect on aggregate employment, the variations of the effects of policy changes appear to be more significant across policies than across industries. Table

17.2 shows that the effects on industry employment of all three policy changes are all positive. The rate of increase, however, is higher when export subsidy is applied rather than other policies.

Table 17.2. The Effects (in Percent) on Industry Employment of Policy Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

No.	Industry	20% Tariff Increase	20% Export Subsidy - Increase	20% Gov't Procur. Increase
01.	Paddy	$0.46 \times 10^{-7}$	$0.31 \times 10^{-5}$	$0.12 \times 10^{-10}$
02.	Other farm food crops	$0.49 \times 10^{-7}$	$0.34 \times 10^{-5}$	$0.13 \times 10^{-10}$
03.	Other crops	$0.42 \times 10^{-7}$	$0.29 \times 10^{-5}$	$0.11 \times 10^{-10}$
04.	Poultry and its products	$0.44 \times 10^{-7}$	$0.29 \times 10^{-5}$	$0.12 \times 10^{-10}$
05.	Forest products	$0.48 \times 10^{-7}$	$0.33 \times 10^{-5}$	$0.13 \times 10^{-10}$
06.	Fishery	$0.48 \times 10^{-7}$	$0.33 \times 10^{-5}$	$0.13 \times 10^{-10}$
07.	Mining and Quarrying	$0.54 \times 10^{-7}$	$0.37 \times 10^{-5}$	$0.15 \times 10^{-10}$
08.	Other manufacturing	$0.44 \times 10^{-7}$	$0.29 \times 10^{-5}$	$0.12 \times 10^{-10}$
09.	Electricity, gas, water	$0.37 \times 10^{-7}$	$0.25 \times 10^{-5}$	$0.99 \times 10^{-11}$
10.	Construction and building	$0.27 \times 10^{-7}$	$0.18 \times 10^{-5}$	$0.71 \times 10^{-11}$
11.	Trade	$0.48 \times 10^{-7}$	$0.32 \times 10^{-5}$	$0.13 \times 10^{-10}$
12.	Restaurant and hotel	$0.40 \times 10^{-7}$	$0.27 \times 10^{-5}$	$0.11 \times 10^{-10}$
13.	Financial intermediaries, real estate and other services	$0.46 \times 10^{-7}$	$0.31 \times 10^{-5}$	$0.12 \times 10^{-10}$
14.	Other services	$0.24 \times 10^{-7}$	$0.16 \times 10^{-5}$	$0.63 \times 10^{-11}$
15.	Unspecified Sectors	$0.28 \times 10^{-9}$	$0.19 \times 10^{-7}$	$0.75 \times 10^{-13}$
16.	Transport and communication	$0.40 \times 10^{-7}$	$0.27 \times 10^{-5}$	$0.11 \times 10^{-10}$
17.	Manufacture of food, beverages, and cigarettes	$0.44 \times 10^{-7}$	$0.29 \times 10^{-5}$	$0.12 \times 10^{-10}$
18.	Petroleum refinery	$0.54 \times 10^{-7}$	$0.37 \times 10^{-5}$	$0.14 \times 10^{-10}$

Note: Industry 17 is imperfectly competitive.

## 6. The effects of Policy Changes on Industry Output

Table 17.3 shows that for the same policy changes we first observe that, given a policy change, the effects on output across industries in percentage are quite small. Secondly, similar to the effects on macro variables (Table 17.1) and industrial employment (Table 17.2), the effects on industry outputs appear to vary significantly across policies rather than across industries. In general we observe that all policy changes have a positive impact on industry outputs.

Considering the targeted industry 17 in Table 17.3, we observe that, given policy changes, part of its output that is sold in the domestic market increases by 0.006 percent as tariff increases, decreases by 0.011 percent as export subsidy increases, and increases by  $0.16 \times 10^{-7}$  percent as government procurement increases. Contrary to the sale at home, its foreign sale declines by 0.06 percent as tariff increases, rises by 6.53 percent as export subsidy increases, and increases by  $0.13 \times 10^{-5}$  as government procurement increases. Regarding supplies of foreign oligopolists in the domestic market, we observe a decline of 0.06 percent as tariff increase, a rise of 0.97 percent as export subsidy is raised, and a decline of  $0.10 \times 10^{-6}$  percent as government procurement increases. Similar changes of foreign oligopolists' sales in the domestic market -- except for the case of export subsidy increase -- are observed in other imperfectly competitive industries: industries 16 and 18, that is imports decline in industries 16 and 18, respectively, by 0.12 and 0.01 percents.

The numerical results confirm the intuition that under a tariff, domestic oligopolists will switch their sales to domestic markets and reduce sales in foreign markets.

Table 17.3. The Effects (in Percent) on Industry Output of Policy Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

<i>No.</i>	<i>Industry</i>	<i>20% Tariff Increase</i>	<i>20% Export Subsidy Increase</i>	<i>20% Gov't Procur. Increase</i>
01.	Paddy	$0.26 \times 10^{-7}$	$0.19 \times 10^{-5}$	$0.19 \times 10^{-11}$
02.	Other farm food crops	$0.39 \times 10^{-5}$	$0.54 \times 10^{-4}$	$0.65 \times 10^{-11}$
03.	Other crops	$0.31 \times 10^{-5}$	$0.18 \times 10^{-3}$	$0.40 \times 10^{-10}$
04.	Poultry and its products	$0.26 \times 10^{-5}$	$0.35 \times 10^{-4}$	$0.42 \times 10^{-10}$
05.	Forest products	$0.14 \times 10^{-5}$	$0.59 \times 10^{-4}$	$0.85 \times 10^{-11}$
06.	Fishery	$0.47 \times 10^{-5}$	$0.14 \times 10^{-3}$	$0.28 \times 10^{-10}$
07.	Mining and Quarrying	$0.73 \times 10^{-6}$	$0.52 \times 10^{-4}$	$0.40 \times 10^{-10}$
08.	Other manufacturing	$0.29 \times 10^{-3}$	$0.16 \times 10^{-3}$	$0.35 \times 10^{-10}$
09.	Electricity, gas, water	$0.16 \times 10^{-5}$	$0.17 \times 10^{-4}$	$0.12 \times 10^{-10}$
10.	Construction and building	$0.45 \times 10^{-5}$	$0.27 \times 10^{-4}$	$0.14 \times 10^{-10}$
11.	Trade	$0.15 \times 10^{-5}$	0.011	$0.26 \times 10^{-9}$
12.	Restaurant and hotel	$0.39 \times 10^{-5}$	$0.53 \times 10^{-4}$	$0.17 \times 10^{-9}$
13.	Financial intermediaries, real estate and other services	$0.25 \times 10^{-5}$	$0.32 \times 10^{-4}$	$0.21 \times 10^{-10}$
14.	Other services	$0.33 \times 10^{-5}$	$0.37 \times 10^{-4}$	$0.49 \times 10^{-10}$
15.	Unspecified Sectors	$0.47 \times 10^{-5}$	$0.64 \times 10^{-4}$	$0.16 \times 10^{-9}$
16.	Transport and communication			
	a. Sold in domestic market	0.0011	0.0067	$0.11 \times 10^{-9}$
	b. Sold in foreign markets	-0.1363	0.7895	$0.13 \times 10^{-7}$
	c. Import	-0.0213	-0.1234	$-0.21 \times 10^{-8}$
17.	Manufacture of food, beverages, cigarettes			
	a. Sold in domestic market	0.0006	-0.0109	$0.16 \times 10^{-7}$
	b. Sold in foreign markets	-0.0640	6.5280	$0.13 \times 10^{-5}$
	c. Import	-0.0550	0.9689	$-0.10 \times 10^{-6}$
18.	Petroleum refinery			
	a. Sold in domestic market	$0.39 \times 10^{-4}$	0.0004	$0.65 \times 10^{-11}$
	b. Sold in foreign markets	-0.0041	0.0219	$-0.61 \times 10^{-9}$
	c. Import	-0.0006	-0.0061	$-0.99 \times 10^{-10}$

Note: Industry 17 is imperfectly competitive.

The competitive edge yielded by the tariff also help domestic oligopolists to displace foreign sales at home market. On the other hand, when the home government subsidizes

exports, domestic oligopolists cut back sales in domestic markets and increase sales in foreign markets. With less attention in domestic markets, domestic oligopolists allow foreign imports to go up.

Regarding the variation across industries for a given policy, we may observe that unlike the other policy changes, export subsidy increase has a more dramatic effect on industry 11. The output of this industry increases by 0.1 percent, a rate much higher than that of any other industry except for that of industry 17. This happens because the increase of the sale in foreign market, especially by all imperfectly competitive industries, provides higher trade margins for wholesale and retailers.

#### 7. The effects of Technological Improvement on Macro Variables

Table 4 shows that all macro variables but the domestic price level increase at declining rates throughout the time periods specified. However, unlike the preceding simulations, here we observe that the magnitudes are quite large.

Table 17.4. The Effects (in Percent) on Macro Variables of Technological Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

	YEAR 1	YEAR 2	YEAR 5
GDP	1.5252	1.0240	0.0774
CPI	-0.0016	-0.0011	-0.0001
Export	4.5876	3.1879	0.2488
BOP	15.0730	11.7440	1.0411
Aggr. Employment	0.7012	0.4636	0.0346

Note: Industry 17 is imperfectly competitive.

Given other exogenous variables and all the specified parameters, GDP goes up by 1.53 percent in the first year, 1.02 percent in the second year, and 0.08 percent in the fifth year. Export and the balance of payments go up, respectively, by 4.59 and 15.07 percents in the first year, 3.19 and 11.7 percents in the second year, and 0.25 and 1.04 percent in the fifth year. Aggregate employment experiences a similar trend; that is, an increase of 0.7 percent in the first year, 0.46 percent in the second year, and 0.03 percent in the fifth year. The domestic price level, however, goes down by 0.002 percent in the first year, by 0.001 percent in the second year, and by 0.0001 percent in the fifth year.

#### **8. The effects of Technological Improvement on Industry Employment**

Similar to aggregate employment, industry employment shows a steady increase though at declining rates through time. Regarding the variation across industries, we may observe that employment in targeted industry 17 increases, as expected, at a larger rate than all other industries, whereas the smallest increase accrues to industry 15 since this industry does not use the output of industry 17 as its intermediate input.

Industry 17 shows a large increase in employment demanded, that is 1.23 percent for the first year, 0.81 percent for the second year, and another 0.06 percent in the fifth year. Industry 15 faces an increase in employment of  $0.80 \times 10^{-6}$  percent in the first year, an increase of  $0.19 \times 10^{-6}$  percent in the second year, and an increase of  $0.39 \times 10^{-7}$  percent in the fifth year.

Table 17.5. The Effects (in Percent) on Industry employment of Technological Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

No.	Industry	YEAR 1	YEAR 2	YEAR
01.	Paddy	$0.13 \times 10^{-3}$	$0.31 \times 10^{-4}$	$0.64 \times 10^{-5}$
02.	Other farm food crops	$0.14 \times 10^{-3}$	$0.34 \times 10^{-4}$	$0.69 \times 10^{-5}$
03.	Other crops	$0.12 \times 10^{-3}$	$0.29 \times 10^{-4}$	$0.58 \times 10^{-5}$
04.	Poultry and its products	$0.12 \times 10^{-3}$	$0.29 \times 10^{-4}$	$0.61 \times 10^{-5}$
05.	Forest products	$0.14 \times 10^{-3}$	$0.33 \times 10^{-4}$	$0.67 \times 10^{-5}$
06.	Fishery	$0.14 \times 10^{-3}$	$0.33 \times 10^{-4}$	$0.67 \times 10^{-5}$
07.	Mining and Quarrying	$0.15 \times 10^{-3}$	$0.37 \times 10^{-3}$	$0.76 \times 10^{-5}$
08.	Other manufacturing	$0.13 \times 10^{-3}$	$0.29 \times 10^{-4}$	$0.61 \times 10^{-5}$
09.	Electricity, gas, water	$0.11 \times 10^{-3}$	$0.25 \times 10^{-4}$	$0.52 \times 10^{-5}$
10.	Construction and building	$0.76 \times 10^{-4}$	$0.18 \times 10^{-4}$	$0.37 \times 10^{-5}$
11.	Trade	$0.13 \times 10^{-3}$	$0.32 \times 10^{-4}$	$0.66 \times 10^{-5}$
12.	Restaurant and hotel	$0.11 \times 10^{-3}$	$0.27 \times 10^{-4}$	$0.56 \times 10^{-5}$
13.	Financial intermediaries, real estate and other services	$0.13 \times 10^{-3}$	$0.31 \times 10^{-4}$	$0.64 \times 10^{-5}$
14.	Other services	$0.68 \times 10^{-4}$	$0.16 \times 10^{-4}$	$0.33 \times 10^{-5}$
15.	Unspecified Sectors	$0.80 \times 10^{-6}$	$0.19 \times 10^{-6}$	$0.39 \times 10^{-7}$
16.	Transport and communication	$0.11 \times 10^{-3}$	$0.27 \times 10^{-4}$	$0.56 \times 10^{-5}$
17.	Manufacture of food, beverages, and cigarettes	1.2348	0.8123	0.0605
18.	Petroleum refinery	$0.15 \times 10^{-3}$	$0.10 \times 10^{-3}$	$0.75 \times 10^{-5}$

Note: Industry 17 is imperfectly competitive.

### 9. The effects of Technological Improvement on Industry Output

In line with industry employment and GDP, industry outputs, presented in Table 17.6, also rise at decreasing rates over time. Again, the output of industry 17 goes up at a much higher rate than those of the other industries. Considering the distribution of its output, industry 17's sale on domestic market goes up by 2.5 percent in the first year, 1.69

percent in the second year, and 0.13 percent in the fifth year. Export from this industry also goes up by 263.48 percent in the first year, 107.94 percent in the second year, and by 3.27 percent in the fifth year. Imports in industry 17 decrease though at declining rates over time, that is a decline of 224.76 percent in the first year, 45.61 percent in the second year, and 2.09 percent in the fifth year. Similar trends and large in magnitudes are also observed in industries 16 and 18.

The productivity gain -- through learning by doing -- together with increasing returns to scale help industry 17 to grow at a very high rate. Its growth -- through interindustry linkages -- spills over into the other two oligopolistic industries, namely 16 (Transport and communication) and 18 (Petroleum refinery), which also exhibit increasing returns to scale. The results of the simulation also show high growth rates in these two industries.

Besides industry 17, industry 11 also experiences, for the same reason as when we increase export subsidy, a large output increase: 0.47 percent in the first year, 0.31 percent in the second year, and 0.023 percent in the fifth year. In short, we may conclude that with the exception of industry 11, all imperfectly competitive industries grow at faster rates than those of the other industries.

Table 17.6. The Effects (in Percent) on Industrial Output of Technological Changes Applied on Industry 17 (Manufacture of Food, Beverages, and Cigarettes)

No.	Industry	YEAR 1	YEAR 2	YEAR 5
01.	Paddy	$0.78 \times 10^{-4}$	$0.52 \times 10^{-4}$	$0.38 \times 10^{-5}$
02.	Other farm food crops	0.0033	0.0022	$0.16 \times 10^{-3}$
03.	Other crops	0.0076	0.0049	$0.37 \times 10^{-3}$
04.	Poultry and its products	0.0021	0.0014	$0.11 \times 10^{-3}$
05.	Forest products	0.0027	0.0018	$0.13 \times 10^{-3}$
06.	Fishery	0.0069	0.0045	$0.34 \times 10^{-3}$
07.	Mining and Quarrying	0.0022	0.0014	$0.11 \times 10^{-3}$
08.	Other manufacturing	0.0069	0.0045	$0.34 \times 10^{-3}$
09.	Electricity, gas, water	0.0011	0.0007	$0.56 \times 10^{-4}$
10.	Construction and building	0.0012	0.0008	$0.57 \times 10^{-4}$
11.	Trade	0.4737	0.3114	0.0231
12.	Restaurant and hotel	0.0032	0.0021	$0.16 \times 10^{-3}$
13.	Financial intermediaries, real estate and other services	0.0019	0.0013	$0.98 \times 10^{-4}$
14.	Other services	0.0025	0.0016	$0.12 \times 10^{-3}$
15.	Unspecified Sectors	0.0039	0.0026	$0.19 \times 10^{-3}$
16.	Transport and communication			
	a. Sold in domestic market	0.6216	0.4182	0.0316
	b. Sold in foreign markets	73.6980	178.6300	9.6855
	c. Import	-11.4990	-6.9074	-0.4751
17.	Manufacture of food, beverages, cigarettes			
	a. Sold in domestic market	2.5265	1.6965	0.1285
	b. Sold in foreign markets	263.4800	107.9400	3.2742
	c. Import	-224.7600	-45.6130	-2.0945
18.	Petroleum refinery			
	a. Sold in domestic market	0.02733	0.0182	0.0014
	b. Sold in foreign markets	2.1338	1.4611	0.1122
	c. Import	-0.4180	-0.2775	-0.0207

Note: Industry 17 is imperfectly competitive.

## 10. Conclusions

This chapter has demonstrated that, given its nature and the economic conditions represented by the data sets, the solutions to our model can be used for analyzing and evaluating industrial and trade policies. Our model is applied to Indonesia. For our computations, we use the 19-sector IO Table for the year 1985, published in 1989 by the Central Bureau of Statistic of the Republic of Indonesia. However, we merge the sector "public administration and defense" into the sector "unspecified sector" because the former sector contains mostly zeros in both its row and column. Therefore, we finally have 18 sectors in our application.

Prior to simulations, the mathematical model -- expressed as a linear system in percentage changes in Chapter 14 -- must be calibrated. The economy under consideration is assumed to be in equilibrium, and the equilibrium is commonly called the benchmark equilibrium. The parameters of the mathematical model are either chosen from econometric studies or computed from the known values of the endogenous and exogenous variables that represent the benchmark equilibrium.

Having calibrated the model, we proceed with the computational programs for simulations. We choose to use the Euler process with 1- and 2-step iterations. Subsequently, applying the extrapolation process, we calculate the results for an infinite number of iterations. In these simulations, the object of industrial and trade policies is industry 17, namely manufacture of food, beverages, and cigarettes.

Our simulations include two parts -- policy changes (tariff increase, export subsidy increase, and government procurement increase) and technological changes. More precisely, we calculate the effect on macro variables, industry employment, and industry output of a hypothetical 20 percent tariff increase, a hypothetical 20 percent export taxes decrease, and a hypothetical 20 percent government procurement increase -- all applied in industry 17. The last simulations are intended to evaluate the effects on macro variables, industry employment, and industry outputs of technological progress through learning curves -- directed specifically on technological progress through learning.

In the first part of the simulations (see tables 17.1 - 17.3), we observe that all these policy changes have positive impact on economic performances; GDP, export, the balance of payments, and aggregate employment increase whereas price level declines. Furthermore, variations in industry employment and outputs are quite small, except that industry 17 shows a higher growth, relative to other industries, in both its employment and outputs. This industry will sell more on the domestic market than in foreign markets when the government increases either tariff or procurement, and vice versa when the government provides more export subsidy.

In the second part of the simulations (see tables 17.4 - 17.6), we observe that, in sharp contrast to the policy changes discussed above, the effects on macro variables, industry employment, and industry outputs of the technological progress in industry 17 are dramatic. Technological improvement has a positive impact on macro variables, industry employment, and industry outputs.

The results of our simulation suggest that learning by doing has the most dramatic effect on GDP, export, the balance of payments, and aggregate employment. The policy implication that emerges from this numerical exercises is that R&D subsidies might be a good industrial policy. Also, compared to tariff or procurement, export subsidies seem to have a stronger impact on the economy. Intuitively, this result can be expected if the foreign oligopolistic markets are large, and, therefore, industrial or trade policies directed at these markets might give a much more positive effect on the domestic economy.

PART V

CONCLUDING REMARK

## CHAPTER 18

### CONCLUDING REMARK

In this thesis, we have built a multisectoral model intended for industrial and trade policy analysis. At the theoretical level, the innovations offered by the thesis consists of the introduction of increasing returns to scale and imperfect competition as well as the learning curve into the MSG model. Increasing returns to scale and imperfect competition is introduced to accommodate highly concentrated industries that are considered to be the target of strategic trade policies.

In conventional CGE models, technological progress is often assumed to be exogenous and takes the form of a Hicksian neutral shift of the production functions through time. In its treatment of technological progress, the thesis goes beyond the conventional assumption of exogenous Hicksian neutral technical progress and assumes that technical progress depends on the stock of knowledge accumulated in each industry. This is the hypothesis of learning by doing advanced by Arrow (1962). As a proxy in the stock of knowledge in each industry, we propose to use the cumulative output of this industry.

In our model, learning -- although endogenous -- is not consciously pursued by firms but is the byproduct of the production decisions of the producers in each industry. A more complete treatment would allow firm to learn actively. However, such a treatment must include several periods in the optimization problem solved by a firm, i.e., the CGE

model developed should be dynamic. Under the present state of the arts, such an extension is not yet available. Future extensions of the model might include endogenous investment, introduction of financial institutions, or linkages of several economies in a trade block.

Because the model is intended for industrial and trade policy analysis, we have presented a rather detailed modeling of the government sector. In particular, the government budget constraint is explicitly formulated. Also, a whole chapter is devoted to the macroclosure rules adopted in the model.

The theoretical structure is calibrated to Indonesian data obtained from the 1985 Indonesian IO Table. The numerical model thus obtained is then simulated under several policy scenarios: tariff against foreign firms, export subsidies given to domestic firms, and government procurement -- all targeted at industry 17, an oligopolistic industry. All these policies have a positive impact on the macrovariables, such as GDP, the CPI, the balance of payments, and the aggregate employment. The effects seem to be most favorable under the scenario of export subsidies. Presumably, this results follows from the fact that foreign markets are large, and thus an export subsidy gives the domestic oligopolists a more powerful incentive to compete on the foreign market. On the other hand, because the domestic market is relatively small, a tariff levied on foreign imports will have a lesser impact on the domestic economy.

We also simulate the model under the assumption that oligopolistic industry 17 (Manufacture of food, beverages, and cigarettes) experiences technological progress

through learning by doing. The numerical results show a dramatic impact of the technological progress in this industry on the economy.

The tentative conclusion -- with all the qualifications on the quality of the data used in calibrating the model -- that emerge from the simulation exercises suggests that R&D subsidies might have the most impact on the economy, with export subsidies occupying the middle ground and tariffs the least effective.

We would like to note in passing that our model does not incorporate any environmental dimension. In the simulation exercises, the growth induced on industry 16 (Transport and communication) and 18 (Petroleum and refinery) is quite high. These industries are rather pollution-intensive. Hence the gain in GDP should be balanced against the environmental degradation caused by the polluting industries. Again, this area perhaps is one of the extensions we hope to pursue in our future research agenda.

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