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Mathematical Modeling and Parameter Identification for Unsteady Flow in Compound Channels.

By

Habib Abida

A thesis
presented to the University of Ottawa
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in
Civil Engineering

Department of Civil Engineering
University of Ottawa



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ISBN 0-315-85806-0

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UNIVERSITÉ D'OTTAWA
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To my Parents
and
my Wife

Abstract

This numerical study is divided into two main parts: In the first part a model for routing unsteady flows in compound channels, called RUFICC, is developed. [In this context the term "compound" implies that the channel cross-section includes a main (deep) channel with either one or two adjoining (shallow) flood plain zones]. The model accounts for flood plain contribution to system conveyance and also for lateral momentum transfer (LMT) between adjacent deep and shallow zones of compound flow fields. In the modeling approach composite channel sections are divided into representative deep and shallow zones. Continuity and momentum equations, developed separately for the aforementioned sub-sections, are then summed to yield the compound channel equations. The resulting one-dimensional model equations, which are a modified form of the St. Venant equations, are solved using a four-point implicit finite difference scheme.

Different procedures to account for LMT in the modeling process were considered. These included empirical models that were developed based on field and laboratory measurements under steady flow conditions. The effect of LMT was found to be insignificant in most of the applications considered. For a hypothetical channel with wide and rough flood plains LMT was found to be strongest at small flood plain depths and resulted in attenuation of the discharge hydrographs.

In the simulation exercises included in this work RUFICC's performance was compared to those of two frequently-used conventional models, namely: the Off-Channel Storage Model (OCSM) and the Separate Channel Model (SCM). For the applications considered a better agreement was achieved between RUFICC's simulated hydrographs and the observed data. Applying the two conventional models resulted in a marked delay in the fall of the recession curve of the simulated hydrographs. The OCSM also underestimated stages and discharges by more than 25%, especially for fairly high flood plain flows.

Flood routing exercises, which involve the solution of the St. Venant

equations, require that the geometric and hydraulic properties of the river reach under study be known. This includes the cross-sectional area of flow as well as the channel boundary roughness coefficients for different flow depths. Obtaining these data usually involves extensive field surveys, which can make the numerical river model an expensive design aid. Furthermore, roughness coefficients are generally estimated using arbitrary techniques, such as relying upon past experience or tables. Nevertheless, if suspect data are used the advantage in accuracy of using the complete St. Venant equations will be lost.

An alternative approach, that would significantly reduce the quantity of necessary data, would employ optimization techniques. Basically this means that an objective function, which represents the difference between simulated and observed values of discharge and/or flow depth, is minimized to yield the model parameters.

The second phase of this study concerned the testing of different optimization techniques to determine the most suitable optimization algorithm(s) in the estimation of flood routing data. The algorithms considered include Powell's and Rosenbrock's methods, and the Nelder and Meade Simplex algorithm. Regular channel data as well as compound channel data were used in these exercises.

The solution of the unsteady flow equations requires only cross-sectional area (A) and conveyance (K) as functions of flow depth (y). Thus, instead of following the conceptual approach of optimizing upon a channel's geometric and hydraulic parameters, optimization was performed upon abstract parameters in assumed $A(y)$ and $K(y)$ relationships. These types of relationships in the so-called 'Black-Box' approach would obviously speed up the computations involved in solving the unsteady flow equations. Furthermore, the relative simplicity of such relationships resulted in decreased computer times and reduced amounts of required computer storage.

Estimated data using the Rosenbrock and Simplex methods were then applied to route different flood events. Simulated peak stages and discharges were in good agreement with those estimated using actual routing data.

Acknowledgements

Special appreciation is expressed to Dr. R. D. Townsend for his assistance, guidance, and supervision of this research.

Special thanks are extended to W.R.C. Myers from the University of Ulster in Ireland and A. Treske from Germany for providing data used in the calibration and validation of the author's model.

Sincere appreciation is also expressed to the Tunisian Government and the National Science and Engineering Research Council of Canada (Grant # OGP0007443) for providing financial support throughout the author's graduate programme.

The author also wishes to thank his parents and his wife for their understanding, patience, and ever-lasting encouragement and support.

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Notations

A	= cross-sectional area of flow;
C_q	= a parameter accounting for the lateral momentum transfer mechanism;
C_r	= Courant number;
D	= coefficient of determination
Dv	= difference in flow velocity between main channel and flood plain;
E	= extensive property;
E_h	= error in peak water surface elevation;
E_q	= error in peak discharge;
f	= friction coefficient for the flood plain zone;
F_f	= friction force;
F_g	= gravity force;
F_p	= unbalanced pressure force;
F_r	= Froude Number;
g	= acceleration due to gravity;
H	= water surface elevation;
H_s	= simulated water surface elevation;
H_o	= observed water surface elevation;
I	= ratio of the flood plain to the composite section friction slopes;
K	= conveyance;
m	= mass;
M_e	= mean of the residuals;
M_f	= momentum correction factor for the composite section;
MR_{so}	= modified correlation coefficient;
N	= total number of nodes;
n	= Manning's roughness coefficient;
n_a	= apparent roughness coefficient;
n_e	= effective roughness coefficient;
$O(.)$	= observed value of stage or discharge;
p_i	= net momentum flux produced by lateral discharge in the x-direction :
P	= wetted perimeter;
Q	= actual discharge in the compound channel;
$\dot{Q}_i$	= isolated discharge in a sub-section of the compound channel;
Q_o	= observed discharge;
Q_s	= simulated discharge;
q	= lateral discharge;

R	=	hydraulic radius;
r	=	ratio of the flood plain to the main channel hydraulic radii;
$R_{,o}$	=	correlation coefficient;
$S(.)$	=	simulated value of stage or discharge;
$\bar{S}$	=	control surface;
S_f	=	friction slope;
S_0	=	channel bed slope;
t	=	time;
T_r	=	flood wave travel time;
u	=	lateral discharge velocity in the x-direction;
v	=	flow velocity;
V	=	volume;
V_w	=	flood wave velocity;
$\bar{V}$	=	control volume;
W_c	=	main channel width;
W_f	=	flood plain width;
x	=	distance along the channel;
y	=	flow depth;
y_r	=	main channel to flood plain depths ratio;
y_b	=	bank-full depth;
Z	=	channel bed elevation;
β	=	momentum correction factor for individual sub-section;
Δt	=	time step;
Δx	=	incremental distance along the channel;
ϵ	=	intensive property;
λ	=	coefficient to account for the lateral momentum transfer mechanism;
ρ	=	water density;
τ_a	=	apparent shear stress;
τ_f	=	flood plain boundary shear stress;
τ_0	=	main channel boundary shear stress.
θ	=	computational weighting factor;
ζ_i	=	net momentum flux produced by lateral discharge in the x-direction;

Abbreviations

ARM	=	apparent roughness model;
LMT	=	lateral momentum transfer;
MCI	=	i^{th} flood event in meandering channel;
OCSM	=	off-channel storage model;
PCI	=	i^{th} flood event in prismatic channel;
PEE	=	proportional error of estimate;
REE	=	reduced error of estimate;
RUFICC	=	mathematical model for routing unsteady flows in compound channels;
SAAD	=	sum of absolute area of divergence;
SAR	=	sum of absolute residuals;
SCM	=	separate channel model;
SD	=	standard deviation of the residuals;
SEE	=	standard error of estimate;
SSR	=	sum of squared residuals;

Chapter 1

INTRODUCTION

1.1 Preamble

In a surface water system, runoff, floods, droughts, and stream water quality interact very closely. Excessive rainfall from extreme storms results in flooding along rivers, while drought occurrence due to shortage of rainfall causes minimal streamflow, reduced water supply, restricted navigation and poor water quality.

To predict the interrelationships between rainfall, runoff, floods, droughts and stream quality, mathematical models are required to simulate flow characteristics in a stream resulting from rainfall. These models are crucial in predicting the characteristics of a flood wave and its change with time. The

characteristics include: (i) maximum water surface elevation and its rate of rise or fall (considered to be an important factor in the planning and design of structures across or along streams and rivers) (ii) peak discharge, which is required in the design of spillways, culverts, bridges and channels sections and (iii) total volume of water resulting from a design flood to assist in the design of storage facilities for flood control, irrigation and water supply.

Essentially the numerical modeling process proceeds by describing the physical system with a set of numbers and simulating the laws acting upon it with sets of operations on these numbers. In hydraulics the numbers generally used are measures of water depths, water surface elevations, flow velocities, and discharges. In most cases the sets of operations transform the description of the system at one time to a description at a later time, so that the numerical model traces the evolution in time of the set of numbers that describes the physical system. Accordingly, the physical evolution corresponding to various proposed engineering works and operations can be followed.

The development of practical numerical modeling techniques naturally brings with it a very thorough reformulation of hydraulics to suit the possibilities and requirements of the discrete, sequential and recursive processes of digital computation. The hydraulics that is reformulated to suit digital machine processes in this way is called computational hydraulics. The role of this particular area in hydraulics is to give formulations which allow for algorithmic representation of natural phenomena. These representation

must not only be acceptable and physically sound but must also yield reliable results within certain error estimates. In this overall context, numerical modeling provides a tool by means of which man can study and gain an understanding of hydraulic flow phenomena, select and design sound engineering projects, and predict extreme situations so as to be able to provide advance warning of their occurrence and importance.

Computational methods have been most widely applied to the area of 'free-surface flows'. Flows of this type occur in systems of channels, canals, rivers, lakes, estuaries, coastal areas and seas. Free-surface flows are associated with such phenomena as short-period waves on beaches and man-made structures, seiche actions in bays and harbors, tides and storm surges, flood waves, flows in sewer systems and much else besides.

1.2 Study Needs

Drinking water distributions, storm drainage systems, irrigation networks, flood forecasting and pollution control are becoming increasingly important as water resources become scarcer. To design, plan, and run these systems modeling is a necessary tool during the system design phase as well as at the simulation-optimization stage. The modeling technique must be extremely robust, easy to use and the information supplied must be of acceptable quality. As water flow in natural channels is generally unsteady, i.e. it varies continuously with time, unsteady open channel flow modeling

is required. The activity of mathematically modeling the progress of a flood wave (or hydrograph) is known as "flood routing". It is an integral component in any hydrologic model and is the most important activity in predicting flood stages and discharges as functions of time at a site along a river. Flood routing is employed in practice for the solution of a wide variety of problems associated with water use. Some of these include:

- predicting flood hydrographs for given or assumed initial conditions:
- determining hydrographs modified by reservoir storage:
- evaluating past floods for which records are incomplete:
- studying the effects of water resources development on the downstream flow conditions.

Flood routing is particularly important in compound channels, particularly when relatively shallow sections (flood plains) adjoin a deep (main) section. A flood plain, which is the flat land area bordering the main channel, is likely to be inundated during times of high floods. This can be quite catastrophic as flood plains of rivers are considered to be the most productive agricultural lands as well as attractive locations for dwellings and other structures. To alleviate the problem of perennial flooding various flood control measures need to be introduced. These include the construction of storage reservoirs to reduce peak flows, and flood embankments (dikes) to confine flows within main channels. The design of such flood control structures depends primarily on flood routing techniques and their reliability

in predicting, with reasonable accuracy, flows in the resulting compound sections.

While much work has been carried out in the past on unsteady flow in channels of simple cross-section, there has been a lack of research on the subject of routing floods through compound channels. The extension of unsteady flow theory to compound channels generally presents complications and uncertainties (Lai, 1986). This is due to the different phenomena and processes associated with this type of flow, which are not yet well understood. They consist mainly of lateral momentum transfer (occurring between the main channel and flood plain zones) and the question of flood plain conveyance (i.e. whether or not the flood plain zones contribute to overall system conveyance).

The generally-adopted procedure is to assume the river's composite section to be divided into two parts: a deep section, which serves for both storing and conveying water, and adjacent flood plain(s), believed to play a storage role only. However, this assumption can be justified only under special conditions, i.e. where the flow depths on the flood plain(s) are relatively small compared to that in the main channel. Flood plain flow has been shown to depend on channel geometry and the flood return period. For the less frequent (extreme flood) events the flood plain flow can be as large as the main channel flow and in these circumstances the composite section can be treated as a single unit (Bhowmik and Demissie, 1982).

In the case where flood plains are believed to contribute to stream conveyance, but not to the extent to treat the composite section as a single unit, the generally-adopted procedure is to increase the main channel top width to compensate for the flood plain flow contribution (Liggett, 1975). This technique, though simple and easy to use, is very arbitrary and its general use is therefore questionable (the procedure might be appropriate in the case of shallow flood plains however it should not be recommended when relatively high flood plain depths are encountered). In fact, based on an analysis of field data of flood flows of several streams, Bhowmik and Demissie (1982) observed that the discharge carrying capacity of flood plains reached as much as 80% of the total flow in the system.

As mentioned beforehand the other important consideration in modeling unsteady flow through compound channels is accounting for the interaction between the fast flowing water in the main channel and the slower moving flow of the flood plain. The difference in velocities between the two regions results in a deceleration of flow in the deep section and an acceleration in the flood plain zones. This lateral momentum transfer (LMT) mechanism is too complex to be modeled analytically, however many empirical procedures are available in the literature to account for it (Yen and Overton 1973, Wormleaton et al 1982, Prinos and Townsend 1983, Dracos and Hardegger 1987, and Wormleaton and Merret 1990 among others). It is worth mentioning that excluding the momentum transfer mechanism in the simulation process may lead to significant inaccuracies in river stage and discharge calculations, numerical modeling, and sediment transport

and erosion computations (Wormleaton and Merrett, 1990).

Good estimates of stage and discharge require not only a reliable unsteady flow model but also good data for the flood routing exercise. These data consist mainly of the river cross-section geometry and appropriate boundary roughness coefficients. Obtaining such data usually implies extensive field surveys, which can make the numerical modeling exercise very expensive. On the other hand the advantage in accuracy of using the complete St. Venant equations would be lost if suspect data were used.

1.3 Study Objectives

This study had two main objectives:

The first was to develop and validate a new one-dimensional mathematical model for routing unsteady flows in compound channels. The significance of the new model lies in the fact that it incorporates both the dynamics of the flood-plain flows and LMT effects in the numerical formulations. A one-dimensional model representation was selected because the characteristics of flood waves in natural channels are considered to be largely influenced by wave translation in the main flow direction (Fread 1976). LMT between adjacent shallow and deep zones has the effect of reducing velocities (and hence discharge) in the upper regions of the main channel flow. The model accounts for the resulting deficiency in main-channel conveyance through different empirical relationships, which relate the actual flow rate in the

compound channel sub-section to that for an equivalent single channel.

The second objective was to use optimization techniques to estimate the data required for routing floods through compound channels. Obtaining these data usually implies extensive field surveys, which can make a numerical modeling exercise very expensive. On the other hand the accuracy gained by using the complete St.Venant equations would suffer if suspect data were used. An objective function representing the differences between simulated and observed values of stage and/or discharge is minimized, thereby yielding the model parameters. (the flood routing data). Three optimization algorithms: the Nelder and Meade Simplex Method, Rosenbrock's method, and Powell's algorithm, were investigated in this phase of the study.

1.4 Thesis Description

Chapter II deals with the major research contributions of past years presented in a thematic rather than a chronological order (this format was considered more appropriate given the nature of the subject matter).

In Chapter III the unsteady flow model equations are developed. The empirical procedures adopted to account for the LMT mechanism are also described in this chapter.

Chapter IV deals with the numerical analysis of the unsteady flow model. The partial differential equations presented in chapter III are written in

difference form and a solution method is described.

Chapter V is devoted to the optimization analysis. The optimization algorithms adopted in the study are described, an objective function is written in terms of stage and discharge, and the optimization variables are identified.

In chapter VI several unsteady flow applications are considered to show the proposed model validity and reliability in predicting stages and discharges. Chapter VII deals with the calibration and validation of the proposed unsteady flow model. The chapter mainly presents graphical displays and statistics used to test the agreement between observed and simulated stage and discharge hydrographs.

In chapter VIII applications of the optimization techniques to estimate flood routing data are presented. A simple triangular channel is first considered. Then the application is extended to a prismatic compound channel.

Finally, chapter IX presents conclusions and recommendations for future research.

Chapter 2

LITERATURE REVIEW

2.1 Unsteady Flow Equations

The study of unsteady open channel flow started more than two centuries ago with the works of the French mathematicians Laplace (1775) and Lagrange (1781). However, it wasn't until 1871 that an advanced mathematical treatment of unsteady flow in open channels was introduced. Barré De Saint Venant, a man of exceptional originality, developed two partial differential equations (continuity and momentum) to describe the flow processes in rivers and channels (De Saint Venant, 1871)

In their original form (for unsteady flow in a prismatic channel) these

two equations are:

$$\frac{\partial A}{\partial t} + \frac{\partial(Av)}{\partial x} = 0 \quad (2.1)$$

and

$$\frac{\partial H}{\partial x} = \frac{1}{g} \frac{\partial v}{\partial t} + \frac{v}{g} \frac{\partial v}{\partial x} + \frac{P}{A} \frac{F}{\rho g} \quad (2.2)$$

where A = channel cross-sectional area; v = mean flow velocity; H = water surface elevation; $\frac{PF}{A\rho g}$ = friction slope; P = wetted perimeter; ρ = water density; g = gravitational acceleration; x = length along the assumed rectangular prismatic channel; and t = time.

The fundamental assumptions underlying the development of the St. Venant equations are:

1. the flow is one-dimensional, implying a constant velocity and a horizontal water surface across any section perpendicular to the longitudinal axis;
2. the variation of flow variables in the direction of flow is gradual, i.e. the streamline curvature is small and the vertical accelerations are negligible, which make the pressure distribution hydrostatic;
3. friction losses in unsteady flow can be approximated by those for steady flow;
4. the average channel bed slope is so small that $\sin \alpha$ can be replaced by $\tan \alpha$ and $\cos \alpha$ by unity, where α is the angle made by the channel bottom with the horizontal;

5. The bed of the channel is fixed, i.e. no erosion or deposition is assumed to occur.

The St. Venant equations were tested and verified experimentally through a study sponsored by the U.S. Federal Highway Administration (1970). This study, performed at Colorado State University, showed that the water stages and velocities computed by the basic St. Venant equations, using numerical integration schemes, were very close to those observed in the long circular conduit employed in the study. This proved the validity and reliability of the St. Venant equations in describing unsteady flows.

In practice, the St. Venant equations are considered as tools for flood routing problems. Here flood routing is defined as the procedure whereby the characteristics of a flood wave at one location along a channel are determined from known (or assumed) data at an upstream location. From this definition it can be seen that boundary and initial conditions are required for the exercise. The different types of possible boundary conditions include a hydrograph, normal or critical depth, or a stage-discharge relationship. The location of the boundary condition depends upon the flow regime. When the flow is subcritical the stream velocity is smaller than the celerity of the shallow water waves, which travel in both the upstream and downstream directions. Thus, boundary conditions must be placed on both the upstream and downstream ends of the channel. For the case of supercritical flows, however, waves travel only in the downstream direction and therefore both boundary conditions have to be located at the upstream

end of the system.

In application to irregular channels, the use of discharge rather than velocity in the unsteady flow equations is preferred because the former is a much smoother function of time and space. (Kabir,1984). So, using discharge Q as the dependent variable, the equations of unsteady, gradually-varied open channel flow are written as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (2.3)$$

$$\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial (\frac{Q^2}{A})}{\partial x} + g \frac{\partial y}{\partial x} = g(S_0 - S_f) \quad (2.4)$$

where Q is the flow discharge; A is the cross-sectional area; y is the water depth; x is the distance along the channel; S_0 is the channel bed slope; and S_f is the friction slope (see Fig. 2.1).

2.2 Solutions of the Unsteady Flow Equations

The two unsteady flow equations constitute a system of non-linear hyperbolic partial differential equations. Analytical solution to this type of problem does not exist because of:

- non-linearity of the hyperbolic partial differential equations:

- non-linearity of the friction slope term, which is proportional to discharge squared:
- the great diversity in shape and roughness of natural channels and the complexity of pattern of the lateral inflows and outflows generally complicate the analytical expressions to an extent that a closed-form integration of the partial differential equations is no longer feasible.

Accordingly the St. Venant equations can only be solved under simplifying assumptions, such as the use of one equation alone (generally the continuity equation) or by dropping those terms in the momentum equation considered to be negligible compared to others. The simplifications that were adopted to solve the St. Venant equations can be summarized as follows (Miller and Cunge, 1975):

1. use of the continuity equation alone, which is generally known as hydrologic routing;
2. use of the momentum equation alone;
3. simplification of the momentum equation by (a) neglecting the resistance term, (b) linearizing the resistance term, (c) neglecting the local acceleration term $\frac{\partial Q}{\partial t}$, (d) neglecting the convective acceleration term $\frac{\partial Q^2}{\partial x}$, or (e) some combination of the above;
4. relying on statistical information only, i.e. without directly using the continuity and momentum equations.

These simplification schemes are extensively used in practice for the purpose of flood routing, however special care is required to justify the use of a certain simplification for the particular problem being considered.

Among the above simplification schemes, two are often encountered in flood routing practice. These are:

- kinematic routing, where all of the terms of the momentum equation are dropped except for two, namely the channel bed slope S_0 and the friction slope S_f . The resulting equation is

$$S_0 = S_f \quad (2.5)$$

The kinematic model is generally accurate for steep slopes with supercritical velocities and for a very slowly rising flood hydrograph on mild channel slopes; it is mostly applicable to overland flow.

- In diffusive routing, the local and convective acceleration terms are neglected and the momentum equation takes the following form:

$$\frac{\partial y}{\partial x} = S_0 - S_f \quad (2.6)$$

The diffusion model was shown to be applicable for a wider range of bed slopes and wave periods than the kinematic model (Ponce et al, 1978).

With the advent of high speed digital computers there is no need for simplifying assumptions as the full St. Venant equations can be solved

numerically. However, the price is generally a complex solution procedure. Thus, in many instances "simplified" methods may still be preferred over "complete" methods.

2.3 Numerical Methods

The St. Venant equations were developed over a century ago, however it is only recently that significant research efforts have been devoted to solving these equations using numerical techniques. It is important to note that the equations are not exact, rather they are based on many simplifying assumptions. Nevertheless for most engineering applications these equations are considered to be sufficiently reliable (Strelkoff 1969, Yen 1973).

The first numerical study of unsteady open channel flow was undertaken by Stoker (1957), who carried out numerical computations for flood waves on the Ohio and Mississippi Rivers. The numerical methods generally used in flood routing problems are (1) the method of characteristics, (2) the finite difference method and (3) the finite element method.

2.3.1 Method of Characteristics (MC):

In this method, the two original partial differential equations are transformed into four simpler ordinary differential equations. Next solutions are

obtained by numerically integrating those equations using a finite difference scheme. The two most widely-used schemes adopt 'characteristics' or 'rectangular' grids. In the former a grid formed by the two sets of characteristic lines is used (Fig. 2.2). The problem is to find the location (and the values of v and y) of a new point, such as point 3 in the aforementioned figure, from known values at the two adjacent points 1 and 2. The second scheme of integration employs a regular grid (Fig. 2.3). In this instance solutions at grid points for $(t + \Delta t)$, such as point 3, are determined from known values of v and y at grid points for time t (points a, b, c, etc.) The method has been used extensively in practice and is thoroughly discussed in the literature (Abbot 1966 and 1975, Cunge et al. 1980, among others).

While the MC method has been shown to be computationally unattractive for application to (irregular) natural channels, it is nevertheless considered an effective tool for locating necessary boundary conditions, (Zoppou and O'Neill, 1981).

2.3.2 Finite Difference (FD) Methods:

In the FD method the derivatives in the partial differential equations are replaced by divided differences defined on a finite number of grid points within the considered domain. FD schemes can be classified as either "explicit" or "implicit". Numerical integration grids for both schemes are shown in Fig. 2.4.

- Explicit FD schemes:

The FD are written for each point on the forward time level in terms of the known values of the previous time level. Thus two linear algebraic equations, which can be solved explicitly for the two unknowns, are formed at each grid point in the x,t plane.

all explicit schemes are restricted in computational time step by the Courant condition:

$$r = \frac{\Delta t}{\Delta x} \leq \left| \frac{1}{v \pm \sqrt{gy}} \right| \quad (2.7)$$

where r is the Courant number, Δt is the time step, Δx is the incremental distance along the channel, and v is the flow velocity.

- Implicit FD schemes:

The implicit FD method was developed to overcome the limitations imposed on the size of the time step required for numerical stability of explicit schemes. The method, first suggested by Von Neumann, was first applied by O'Brien et al (1951) and later applied to unsteady flows in open channels by several investigators (Preissmann, 1961; Cunge and Wegner, 1964; Lai, 1965; and Amein, 1968).

Unlike the explicit method, solutions of the St. Venant equations are advanced from a previous time line to a future time line simultaneously for all points along the distance line. This results in a system of algebraic equations solved simultaneously for all dependent variables at all grid points. This method was shown to be suitable for irregular natural channels as it can handle varying channel geometry, even

where changes from section to section and in the bottom slope are significant (Amein and Fang, 1970). The implicit method is also considered advantageous for flood routing through compound channels. In fact the algebraic equations describing the flow between the flood plain and the main channel, being implicit in the stage, are solved at the same time as the overall solution of the flow (Price, 1974).

2.3.3 Finite Element (FE) Method:

The FE method is considered to be quite useful for two-dimensional flows in wide rivers or estuaries, where finite elements such as triangles can be successfully fitted to irregular boundaries (Lai, 1986). However, open channel flows are primarily one-dimensional and thus the FE method is not particularly advantageous in this instance. For this reason, application of the FE method to unsteady open channel flow problems has been limited with only a few cases reported in the literature, notably those of Cooley and Moin (1976) and Smith and Cheng (1976)

2.4 Unsteady Flow in Compound Channels

Flood routing through compound channels is lightly covered in the literature. There is no general solution to the unsteady flow equations for compound channels because of the complexity of the interaction phenomenon

between the main channel and flood plain flows. The generally-used procedure is to assume that the channel is divided into two parts: a deeper section, serving for both conveying and storing water, and a shallow flood plain section, considered to play a "storage" role only (Fig. 2.5). Therefore the momentum equation remains the same as for in-bank flow, while the continuity equation is slightly modified. The top width of the conveyance channel in the storage term ($\frac{\partial A}{\partial t}$) is substituted by the top width of the total channel. This conventional method for routing floods through compound channels is known as the "Off-Channel Storage" Method.

A different approach, known as the "single channel" method, is to average the flow across the composite cross-sectional area. This is based on the assumption that the total force resisting the flow is equal to the sum of the resisting forces in the different sub-sections of the compound channel. In this approach an equivalent roughness coefficient, which is a weighted average of the roughness coefficients of the flood plain and main channel sub-sections, is assumed. [An example formulation assumes that the total discharge in the compound channel is equal to the summation of the discharges of its sub-sections].

In practice, because of changing flow conditions, it is not always clear whether a flood plain zone acts as a storage reservoir or carries a significant portion of the flood flow. Moreover, the interaction between the fast moving flow in the main channel and the relatively slower flow of the flood plain is not thoroughly understood. These important considerations make

unsteady flow analysis in compound channels a very complicated exercise (Barishnikov et al. 1971; Zheleznyakov, 1971 and Yen and Overton, 1973).

It is likely impossible to develop a universal flood routing model capable of handling all practical situations. However, the choice of any model would depend to a large extent on whether to account for the flood plain contribution to system conveyance or assume this zone to be just a storage reservoir. Bhowmik and Demissie (1982) showed that the flood plain carrying capacity depends mainly on the nature of the flood plain, the main channel, and the flood frequency. To test whether a flood plain and a main channel act as a single unit with uniform characteristics, Bhowmik and Demissie (1982) developed Fig. 2.6, based on field data analysis for several streams. Fig. 2.6 shows the variation of the ratio of flood plain area to total cross-sectional area with the flood plain discharge to total discharge ratio for floods with different return periods. If the characteristics of the whole flow area had been uniform, then all the points should have plotted on the 45 deg line. However, it is apparent that floods of low return periods correspond to small flood plain discharges but fairly large flood plain cross-sectional areas. This indicates that a part of the flood plain area during floods of low return periods act as a storage reservoir. It is also clear from the same figure that the points converge to the 45 deg line as the return period increases, implying a proportionality between cross-sectional area and discharge for floods of return periods exceeding 40 years. Under these conditions the main channel and flood plains can be treated as a single unit.

As was mentioned earlier, the conventional "open channel" approach for the estimation of discharge in a compound channel is to separate the channel cross-section into a "conveyance" part and a "storage" part. This method, however, neglects the effect of the flood plain flow in retarding or decelerating the flow in the main channel. This deceleration is explained by the lateral momentum transfer (LMT) mechanism, found to be critical for shallow flood plain depths (Wormleaton et al. 1982 and Prinos and Townsend, 1983); the condition for which the use of this technique is recommended. This technique generally assumes (imaginary) vertical interface planes, where the LMT effect is observed to be strongest (Wormleaton, 1982). As an alternative, Wormleaton recommended the use of horizontal or diagonal interface planes to artificially separate, for computational purposes, the different flow sections. Indeed, momentum transfer for these types of interface plane was not found to be as important as the case for vertical interface planes.

Decreased flow velocities (resulting from LMT) in the main channel of compound sections at low flood plain depths have been observed by many investigators. Based on laboratory tests Rajaratnam and Ahmadi (1979) in their experimental investigation showed that, for small flood plain depths, there is a sharp discontinuity of velocity and bed shear at the sides of the main channel due to strong LMT effects. However, as flood plain depth increased, the LMT effect gradually diminished, resulting in a smoother transition of velocity and bed shear from the deep section of the channel to the flood plains. The same phenomenon was also observed by Bhowmik

and Demissie (1982) while analyzing field data from the Sangamon River near Oakford and from the Salt Creek near Greenview, Illinois. The same authors also found that the average velocity of the compound section decreased until a minimum was reached at an average flood plain depth of about 35% of the average depth in the main channel. In a similar study, Karasev (1969) reported that the minimum average velocity for the composite section of a laboratory channel was reached at a stage where average depth on the flood plain was about 30 to 40% of the average depth in the main channel.

As depth on flood plains increase beyond this limit, mean flood plain velocities increase substantially. In these circumstances the average velocity in the flood plain zone was observed to approach a value close to the average velocity in the compound section (Bhowmik and Demissie, 1982). Accordingly, the flow can be considered effectively homogeneous throughout the cross-sectional area of flow, assumed to be a single unit. It should be noted, however, that this is true only when depths on the flood plain exceed 35 % of the average depth in the main channel. This is not universal but rather expected to vary from a reach to another as the carrying capacity of flood plains depends upon the size, shape, width, depth, and nature of the flood plains and main channel and on the frequency of the flood event.

Based largely on experimental studies many researchers (Wormleaton, 1982; Yen and Overton, 1973; and Prinos and Townsend, 1983 among others) have developed empirical relationships to account for the aforemen-

tioned LMT mechanism when flood plain depths are shallow (i.e. less than 35 % of the average depth in the main channel). However, these studies invariably assumed steady flow conditions and hence their usefulness with regard to modeling unsteady flows in compound channels is limited.

2.5 Lateral Momentum Transfer (LMT)

During periods of severe flooding over-bank flows occur in streams and rivers resulting in radically different flow regions, namely: a main channel flanked by one or two flood plains. The former is characterized by relatively large depths and high velocities while the latter has generally small depths and low or negligible velocities. The significant velocity difference between these flow regions causes an interaction phenomenon known as the "kinematic effect". This is manifested as banks of vortices (vortex "sheets") along the main channel-flood plain interface regions.

Sellin (1964) presented photographic evidence of these complex flow features by dusting aluminium powder on the water surface of his experimental compound channel. The interaction between the fast main channel flow and the slower flood plain flows results in a tangential force, which amounts to a drag force on the main channel flow and a propulsive force on the over-bank flow. Many research efforts, aimed at determining this tangential force, have been reported in the literature (Chow, 1959 and 1964; Delleur, 1967; Posey, 1967; Wright and Carstens, 1970; Yen and Overton,

1973; Wormleaton et al. 1982; Prinos and Townsend, 1983 among others). Because of the complexity of the LMT phenomenon (and its associated turbulence) most of these past studies were experimental.

In their experiments dealing with flow in two joined rectangular channels, Wright and Carstens (1970) observed a linear momentum transfer from the major to the minor channel. They also demonstrated that the apparent shear stress at the junction interface plane was of the same order of magnitude as the boundary shear stress in the major channel. Therefore, to properly account for the 'drag' effect on the major channel flow by the slower moving minor channel flow, the authors recommended that the (imaginary) junction plane be included in the major channel's wetted perimeter calculation.

Rajaratnam and Ahmadi (1979) also confirmed, through experiments, the transport of longitudinal momentum from a main channel to its flood plain. The bed shear in the flood plain near the junction interface with their compound channel's main channel was observed to increase considerably, while a decrease occurred in the main channel bed shear because of LMT effect. A method to estimate the loss in flow capacity of their compound channel, due to LMT, was also recommended.

Yen and Overton (1973) approached the problem in a different way. They considered that it was impossible to determine shear stresses on arbitrary division lines (that divide the composite section into subsections) and instead determined the plane of zero shear stress. In their experimental

study the conveyance of their compound channel's main channel was shown to significantly increase once the water surface exceeded the bank-full stage. This would imply that the main channel portion of a compound channel conveys water more effectively than the equivalent rectangular channel. Based on this finding, an empirical relationship was suggested for estimating discharge in the main portion of compound channels. Another equation for estimating the flood plain discharge, which was simply a modified version of Manning's equation, was also developed. Discharges computed using this procedure were compared to observed flows and reasonably good agreement was obtained.

Generally, methods for discharge assessment in compound channels assume vertical interface planes between the deep and shallow sections. Some of these methods, however, include the interface plane in the wetted perimeter computations (assuming the fluid shear stress at the junction is the same as the average boundary shear stress) and others exclude it (assuming zero apparent shear stress). Wormleaton et al (1982) proposed a parameter they called the "apparent shear stress ratio" (λ), defined as the ratio of apparent shear stress to the average boundary shear stress, to compare between the two stresses. This parameter will obviously be zero in the absence of apparent shear stress and will approach unity when apparent and average boundary shear stresses are approximately equal. The average boundary shear stress can be determined by equating the total shear force (product of the average shear stress and total area of boundaries including interface planes) and the fluid weight component. The apparent shear stress can be

evaluated by equating the forces over a unit length of the deeper section of the compound channel. The acting forces are:

1. The gravity force: $F_{gc} = \gamma A_{ci} S_0$, where γ is the fluid unit weight; A_{ci} is the cross-sectional area of the main channel; and S_0 is the channel bed slope.
2. The boundary shear force: $F_{bc} = \tau_m P_c$, where τ_m is the main channel boundary shear stress and P_c is its wetted perimeter.
3. The apparent shear force: $F_a = \tau_{ai} P_{ai}$, where τ_{ai} is the apparent shear stress for a specified interface (i) and P_{ai} is the length of the interface plane.

Equating the gravity force with the summation of the boundary and apparent shear forces yields the following expression for the apparent shear stress along any specified interface (i):

$$\tau_{ai} = \frac{1}{P_{ai}}(\gamma A_{ci} S - \tau_c P_c) \quad (2.8)$$

Wormleaton et al (1982) wrote this equation for the vertical interface and divided τ_{ai} by the average boundary shear stress to get the apparent shear stress ratio λ_v for vertical planes. Fig. 2.7 shows the variation of λ_v with the flood plain to the main channel depths ratio for different flood plain roughness coefficients. Curve A was developed using smooth concrete flood plains while Manning's roughness coefficient for curves B, C, and D

were 0.014, 0.017, and 0.021 respectively. λ_v was observed to increase as the flood plain depth decreased or the flood plain roughness increased. This implies that it is inadvisable to assume apparent and average boundary shear stresses to be equal for relatively low flood plain depths and high flood plain friction coefficients, where λ_v becomes much greater than unity. The same observation, i.e. that LMT's effect is strongest at low flood plain depths, was also made by Myers (1978) and Prinos and Townsend (1983).

It has been established that, in the case of vertical interface planes, the apparent shear stress is large at low flood plain depths (Wormleaton et al. 1982). Therefore, it would be useful to examine other possible division planes (where the apparent shear stress is lower) and hence can be ignored or equated to boundary shear stress. Indeed, the diagonal and horizontal interface planes (Figs. 2.8 and 2.9 respectively) did not show the dramatic increase of the apparent shear stress ratio values obtained for vertical interface planes at low flood plain depths. Consequently, horizontal or diagonal rather than vertical interface planes are recommended for discharge assessment in compound channels.

2.6 Improved Mathematical Modeling of Unsteady Flow in Compound Channels

One of the very few studies that considered the dynamic effect of flood plain or "berm" flow when modeling river stage and discharge is that of Tingsanchali and Ackermann (1976). First, a momentum correction factor (M) was introduced to relate momentum flux through a composite section with the summation of momentum flux through each individual subsection. This momentum correction factor is near unity for in-bank flow (no flood plain flow) and greater than unity for overbank flow during flood periods. Under conditions where the flood plain is considered to play only a storage role (no conveyance), the factor M becomes very large and can be approximated by the ratio of the compound channel cross-sectional area to that of the main channel flow area.

Because of the different flow characteristics in the different flow sections, momentum equations are written for each sub-section and consider both "dynamics" and "storage" effects. The momentum equation for the whole cross-section is obtained by adding those for the sub-sections.

In this study the effect of river meander is also considered. In nature, a river's main channel generally tends to meander at a higher degree than that of its flood plain(s). The length of the flood plain meander is made identical to that of the main channel through the use of a schematized system, where berm geometry and roughness coefficients are adjusted to be

compatible with the main channel properties. Since the volume of water in the actual and schematized systems are equal, the cross-sectional area of the berm portion in the schematized system is determined. Similarly, assuming no difference in water depths between the schematized and actual systems (and assuming the same discharge for the flood plain sub-section(s) in both systems), top widths and roughness coefficients are also adjusted.

Tingsanchali and Ackermann (1976) applied their model to determine flow conditions of the Bicol River in the Philippines for a flood that occurred in 1970. When compared to observed data, their model performed better than the "off-channel storage" model, which overestimated peak stages and underestimated peak discharges.

Fread (1976) proposed another one-dimensional model for routing floods through meandering channels. His model equations are a modified form of the complete St. Venant equations. Separate continuity and momentum equations are written for the flood plain and main channel sub-sections. On the other hand, an approximate ratio of the flow in the flood plain to that in the main channel is obtained using Manning's equation. In this approximation the friction slope is assumed equal to the ratio of the difference in water surface elevation to the distance along the channel or the flood plain. It should be noted, however, that this approximation neglects the contribution of inertia effects in evaluating friction loss, and is considered valid only for the case of gradually-varied unsteady flow. Taking into account the computed ratio of flows, the separate equations are then com-

bined and a weighted four point implicit finite difference scheme is used in the numerical solution. Fread compared the performance of his model with the "Separate Channel" and the "Off-Channel Storage" models and found that the former produced appropriately smaller flood wave attenuation and travel time, especially when channel meander is a factor.

Based on Laboratory tests, Nicollet and Uan (1979) developed an empirical relationship between discharge in the main channel of a composite section to the discharge in an equivalent regular section. This empirical model was used to account for LMT between the flood plain and main channel flows in their proposed gradually-varied unsteady flow equations. A momentum equation for the composite section, in terms of the flow parameters in both main channel and flood plain sub-sections was developed. Another momentum equation, describing motion in the "equivalent" main channel is also established. The continuity equation simply states that summation of discharges in the sub-sections is equal to the total flow through the composite section.

Nicollet and Uan developed another procedure for describing average flow conditions in a compound channel. The related momentum equation is coupled with expressions for equivalent conveyance of the composite section, an average composite momentum coefficient, and the distribution of flow between main channel and flood plain to describe unsteady gradually-varied flows in compound channels.

Kolovopoulos (1990) proposed a one-dimensional unsteady flow model

for the simulation of flows in looped and branched open channels. The author evaluated different steady-state models that account for LMT and concluded that the Prinos-Townsend (1984) equation gives accurate results for apparent shear stresses at the main channel flood plain interface. Accordingly, this equation was incorporated in Kolovopoulos' unsteady flow model to account for LMT effects. Applying this model showed that LMT results in: (1) an attenuation of discharge hydrographs at low depths, (2) a delay in the falling of water stages, (3) a shift in the loop rating curve, and (4) an increase in flood plain flow accompanied by a decrease in main channel carrying capacity. Kolovopoulos stated that, for the most part, the phenomenon of main channel-flood plain interaction can safely be ignored and any conventional flood routing models can be applied.

Kolovopoulos (1990) applied his model to eight test cases and concluded that it performed well for a wide range of unsteady flows, tidal flows, and regulated flows frequently encountered in practice. The author also analyzed other unsteady flow models now in use. He compared the performance of the "Single Channel" and the "Off-Channel Storage" models in regards to computed hydrograph peaks and attenuation times. The "Storage" Model was found to underestimate discharges and overestimate peak water surface elevations.

2.7 Justification for the Use of Optimization Methods

The solution of unsteady open channel flow (St. Venant) equations requires estimation of both the hydraulic characteristics (especially Manning's n in the friction slope term) and the geometrical properties of the river. Obtaining these data usually requires extensive surveys, which implies high associated costs. Moreover, these data have to be reasonably accurate, otherwise much of the advantage in accuracy gained through the complete numerical solution of the St. Venant equations is lost.

The traditional approach for estimating Manning's (n) is based on a trial and error procedure, where the unsteady flow equations are repeatedly solved for different assumed values of n . The n -value that finally gives the closest agreement between the solutions of the governing equations and the field observations is the required value. However, because of the non-linearity of the partial differential equations describing unsteady flow in open channels, this procedure is tedious and difficult.

An alternative way of estimating Manning's n (as well as the other required data) is through optimization techniques, where errors between simulated and observed hydrographs from previous flood events are minimized to yield the model parameters. This procedure is known as "Identification of parameters", and is the mathematical process whereby the parameters appearing in a differential equation defining a system are determined from

observations of system input and output (Beker and Yeh, 1972). Optimization involves a comparison between observed and simulated hydrographs to minimize the difference between the two. However part of this difference will be due to the finite difference error, which should be kept to a minimum so as not to interfere with the optimization process. Price (1974) compared the performances of four of the most widely used finite difference schemes and concluded that the four-point implicit method of Amein and Fang (1970) yielded the most accurate results. This conclusion was later confirmed by Wormleaton and Karmegam (1984). For this reason this particular finite difference scheme was adopted in the present study.

The generally-used optimization techniques for the 1-D unsteady flow model are the "influence coefficient algorithm" (Becker and Yeh, 1972a and 1972b), and the "least squares gradient method" (Marquardt, 1964). Wormleaton and Karmegam (1984) applied both techniques to a river reach extending from the Erwood to Belmont gaging stations on the River Wye in Great Britain. They concluded that the least squares gradient method was more robust and distinctly faster than the linear influence coefficient algorithm. They also concluded that since each evaluation involves a time consuming routing of the flood the favored optimization technique in any application is the one that requires the least number of objective function evaluations.

With regard to their application of the numerical river model to the River Wye, the authors analysed five flood events and concluded that the

results were sufficiently consistent to view optimization as a technique with distinct possibilities in the identification of numerical river modeling data. This does not imply that the optimization procedure should replace the traditional collection of survey data. However, it could be an attractive approach when considering fairly long, featureless and relatively inaccessible river reaches.

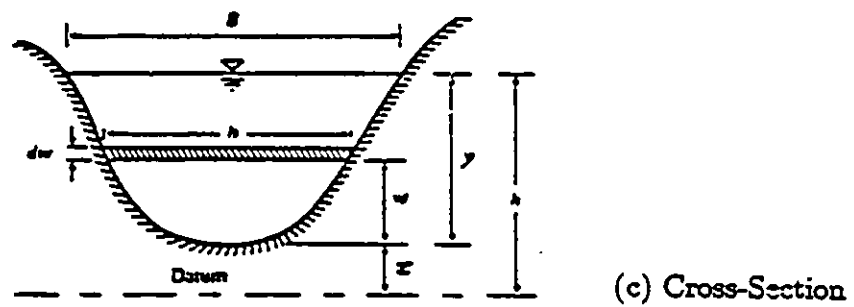
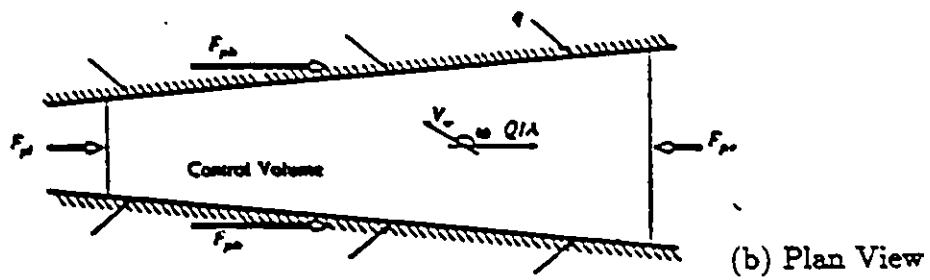
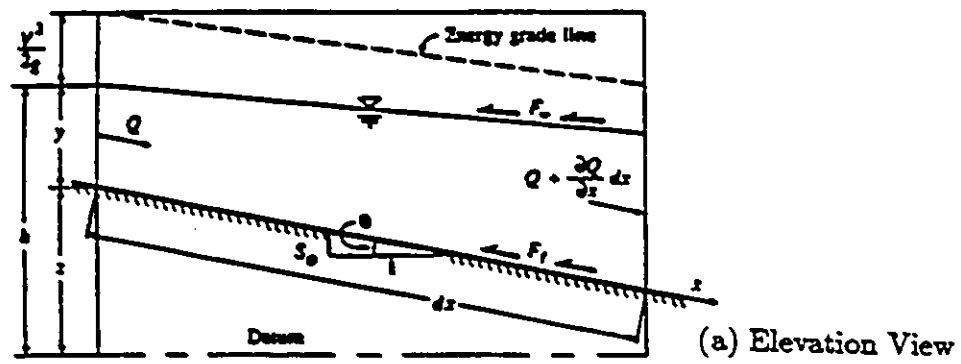


Figure 2.1: Definition Sketch of a Channel Reach for the Derivation of the St. Venant Equations.

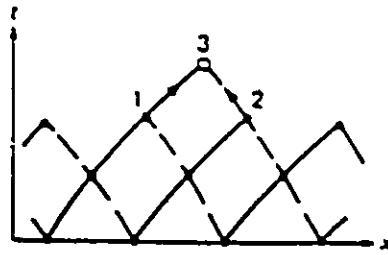


Figure 2.2: Characteristics Grid.

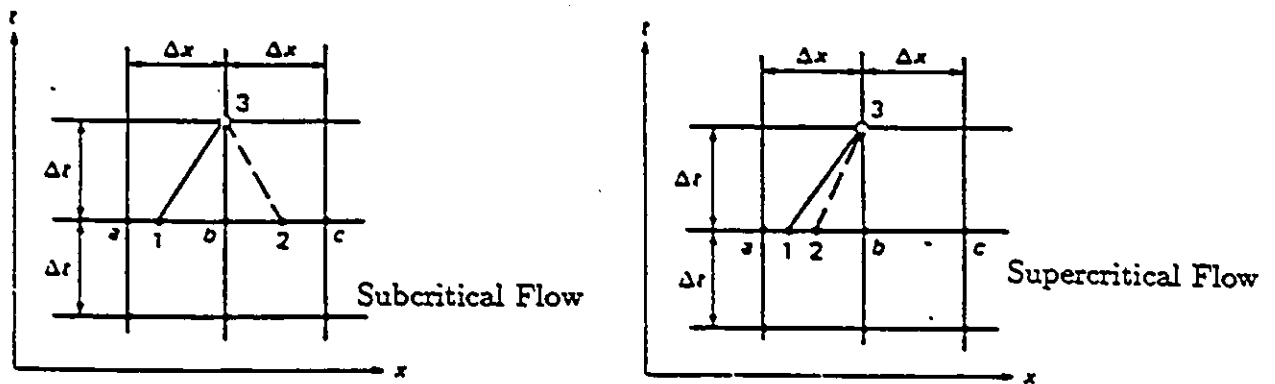


Figure 2.3: Rectangular Grid.

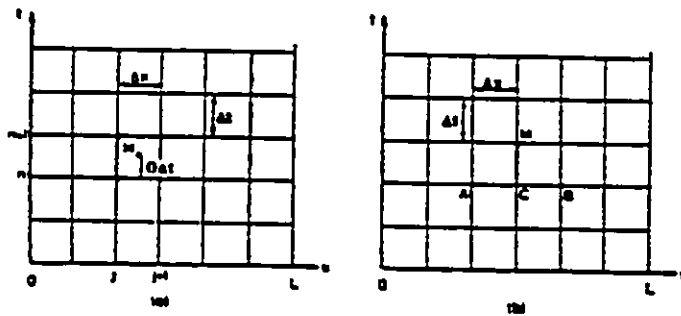


Figure 2.4: Grid of Points for (a) the Implicit and (b) the Explicit Schemes.

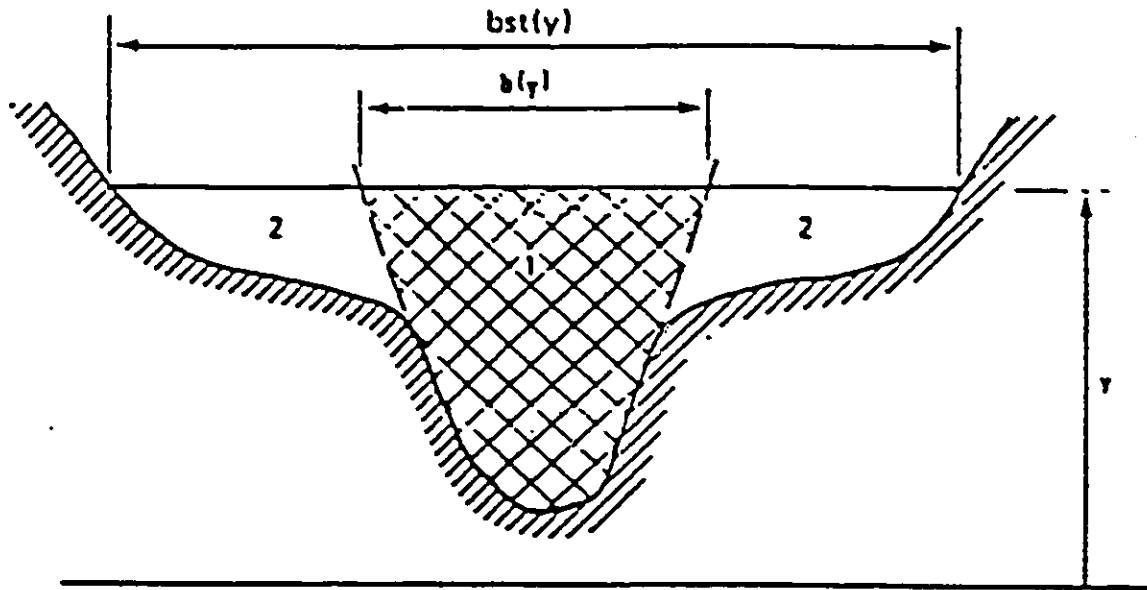


Figure 2.5: Illustration of 'Live' and 'Storage' Cross-Sectional Areas.

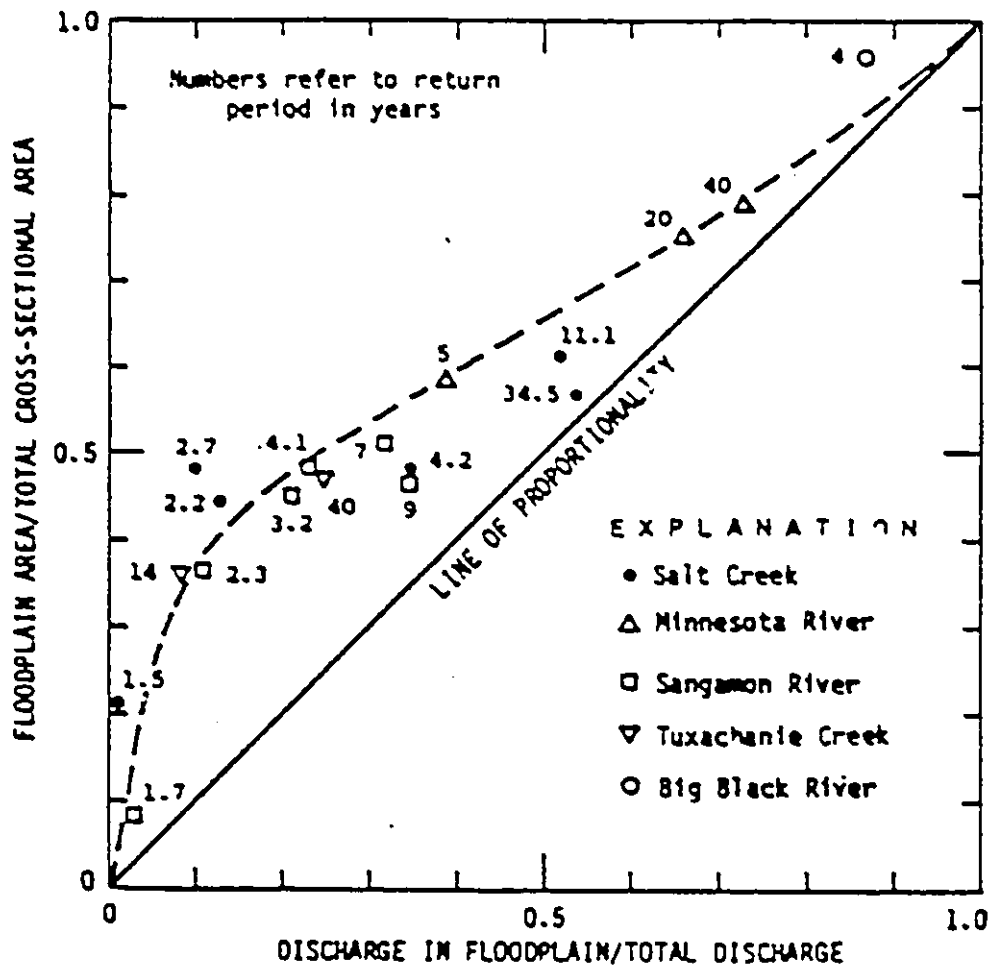


Figure 2.6: Relationship Between Ratio of Flood Plain Area to Total Cross-Sectional Area and Ratio of Discharge in the Flood Plain to Total Discharge (After Bhowmik and Demissie 1982).

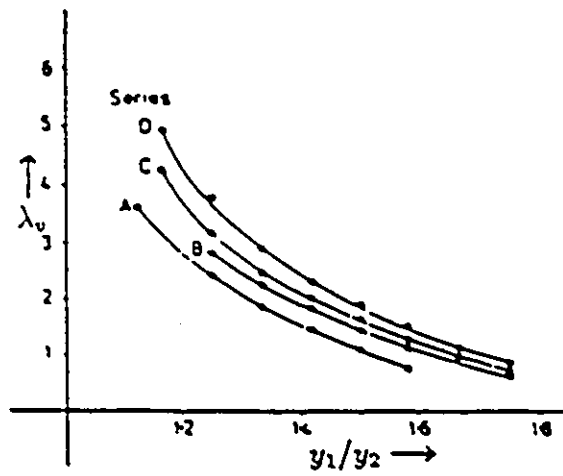


Figure 2.7: λ_v vs. Depth Ratio for Vertical Interfaces (After Wormleaton et al, 1982).

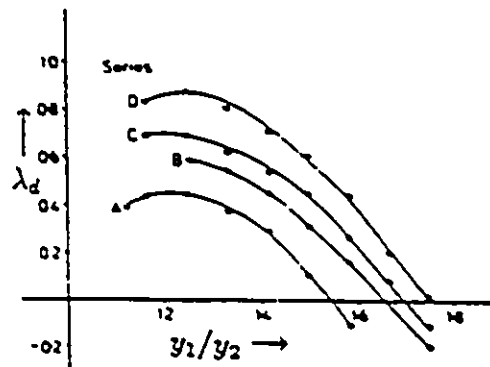


Figure 2.8: λ_d vs. Depth Ratio for Diagonal Interfaces (After Wormleaton et al, 1982).

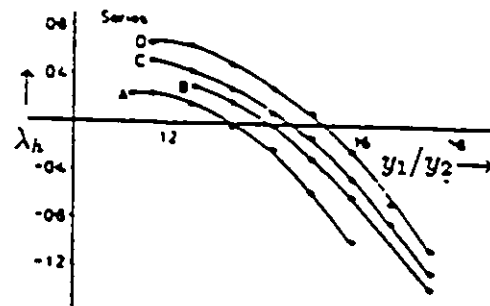


Figure 2.9: λ_h vs. Depth Ratio for Horizontal Interfaces (After Wormleaton et al, 1982).

Chapter 3

DEVELOPMENT OF THE UNSTEADY FLOW MODEL

In this study new features, related to unsteady flow modeling in compound channels, are introduced. First, since past research has shown LMT effects in steady compound channel flows to be at times significant, it was considered important to account for LMT in the development of the proposed model (RUFICC). This has been achieved through the introduction in the unsteady flow equations of parameters that account for LMT-induced variations in main channel and flood plain flows. Secondly, similar studies on unsteady flow modeling in compound channels either treated flood plains as 'storage' components or considered the composite section as a single unit. However, in practice

flood plains may contribute to system conveyance and under these circumstances the flood plain friction slope should not be assumed zero. On the other hand, flow velocities in the flood plain section are generally smaller than those in the main channel and therefore the flood plain friction slope should not be made equal to that of the main channel. In this study a cosine interpolating function, relating main channel to flood plain friction slopes based on their relative flow depths, was proposed.

Accounting for LMT effects, for flood plain contribution to system conveyance, and introducing the aforementioned cosine interpolating function resulted in a modified set of unsteady flow equations.

3.1 Model Equations

The modified St. Venant equations for compound flow fields were developed using Reynolds' Transport Theorem. In this exercise all important flow processes associated with compound flows were included to make the mathematical model as representative of the real situation as possible. The model equations were one-dimensional because the characteristics of a flood wave were considered to be influenced predominantly by the one-dimensional wave motion along the longitudinal axis of the physical system. This also avoided the use of complex and computationally time-consuming two-dimensional flow models.

In developing this improved model for Routing Unsteady Flows In Compound Channels, RUFICC, a hypothetical composite channel section was divided into three parts: a left flood plain (sub-section 1), a main channel (sub-section 2) and a right flood plain (sub-section 3), see Fig. 3.1. Because of the different flow characteristics in these different sub-sections, Reynolds' Transport Theorem was applied separately to each sub-section to yield corresponding continuity and momentum relationships. The resulting equations were then summed to give the equations of flow for the composite section.

Derivation of the unsteady flow equations was based on Reynolds' Transport Theorem. The theorem states that: *"the total rate of change of an extensive property of a fluid is equal to the rate of change of extensive property stored in a control volume plus the net outflow of extensive property through the control surface."*, (Chow et al. 1988). The theorem can be expressed mathematically as:

$$\frac{dE}{dt} = \frac{d}{dt} \int_{\tilde{V}} \epsilon \rho dV + \int_{\tilde{S}} \epsilon \rho v \cdot d\vec{A} \quad (3.1)$$

where ρ = fluid density, v = flow velocity, V and A are the volume and cross-sectional area of the control volume respectively, E = extensive property of the fluid (dependent on the mass present), ϵ = intensive property of the fluid (independent of mass), t = time, and $\tilde{V}$ and $\tilde{S}$ refer to control volume and control section respectively. E and ϵ can be either scalar or vector quantities depending on the property being considered and are related through $\epsilon = \frac{dE}{dm}$, where m = mass.

3.1.1 Continuity equation

The extensive property $E = m$ and the intensive property $\epsilon (= \frac{dm}{dm}) = 1$. Since mass can be neither created nor destroyed, $\frac{dE}{dt} = \frac{dm}{dt} = 0$. Substituting these relationships into equation (3.1) yields:

$$\frac{dm}{dt} = \frac{d}{dt} \int_{\tilde{V}} \rho dV + \int_{\tilde{S}} \rho v \cdot dA \quad (3.2)$$

Now consider the main channel (sub-section 2). The first integral of equation (3.2) can be written as:

$$\frac{d}{dt} \int_{\tilde{V}} \rho dV = \frac{\partial(\rho A_2 dx)}{\partial t} \quad (3.3)$$

where A_2 = the main channel cross-sectional area of flow and dx = distance along the channel in the direction of flow. Integrating the last term of equation (3.2) over the control surface inlet yields:

$$\int_{inlet} \rho v \cdot dA = -\rho \dot{Q}_2 \quad (3.4)$$

where $\dot{Q}_2$ = isolated discharge corresponding to the main channel sub-section.

Integration of the same term of equation (3.2) over the control volume outlet gives:

$$\int_{outlet} \rho v \cdot dA = \rho \left(\dot{Q}_2 + \frac{\partial(\dot{Q}_2)}{\partial x} dx \right) \quad (3.5)$$

note that in applying Reynolds' Transport Theorem, inflows are negative while outflows are positive.

Substituting equations (3.3), (3.4), and (3.5) in equation (3.2) yields:

$$\frac{\partial(\rho A_2 dx)}{\partial t} - \rho \dot{Q}_2 + \rho \left(\dot{Q}_2 + \frac{\partial(\dot{Q}_2)}{\partial x} dx \right) = 0 \quad (3.6)$$

Simplifying and dividing throughout by ρdx results in:

$$\frac{\partial A_2}{\partial t} + \frac{\partial(\dot{Q}_2)}{\partial x} = 0 \quad (3.7)$$

which is the continuity equation for sub-section 2. Continuity equations for sub-sections 1 and 3 are also written in the same way, except that lateral discharges (q_{01} and q_{03}) have to be included. The equations are written as:

$$\frac{\partial A_1}{\partial t} + \frac{\partial \dot{Q}_1}{\partial x} - q_{01} = 0 \quad (3.8)$$

and

$$\frac{\partial A_3}{\partial t} + \frac{\partial \dot{Q}_3}{\partial x} - q_{03} = 0 \quad (3.9)$$

Equations (3.7), (3.8), and (3.9) are now summed to yield the continuity equation for the composite section. Summation of the first three terms gives $\frac{\partial(A_1 + A_2 + A_3)}{\partial t} = \frac{\partial A}{\partial t}$. However, the sum $\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3$ does not yield the actual flow rate in the composite section as isolated discharge is not equal to the actual flow rate in a sub-section of the compound channel. This is due to the aforementioned lateral momentum transfer between adjacent deep and shallow zones of compound flow fields. Therefore, the summation of the second three terms of Eqs. (3.7), (3.8) and (3.9) requires the introduction of the following parameter:

$$C_7 = \frac{1}{Q} (\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3) \quad (3.10)$$

where Q = the actual flow rate in the compound channel. On the other hand, $\dot{Q}_i = K_i S_{fi}^{1/2}$, where K_i = system conveyance and S_{fi} = friction slope for sub-section i . Friction slope for each sub-section (S_{fi}) is assumed to be equal to the product of a coefficient I_i , (that will be discussed further in section 3.2) and the composite section friction slope S_f . Accordingly isolated discharge in each sub-section is written as:

$$\dot{Q}_i = K_i (I_i S_f)^{1/2} \quad (3.11)$$

Now the actual flow rate in the composite section is: $Q = \sum_i^3 \dot{Q}_i$ where $\dot{Q}_i$ = the actual discharge in sub-section i , which is assumed equal to the product of another parameter, λ_i , and isolated discharge $\dot{Q}_i$. The parameter λ , which provides a means of accounting for LMT, is estimated using different empirical relationships (to be discussed in section 3.3). Including coefficients I_i and λ_i in equation (3.10), C_q is given by:

$$C_q = \frac{\sum_{i=1}^3 \dot{Q}_i}{\sum_{j=1}^3 \lambda_j \dot{Q}_j} = \frac{\sum_{i=1}^3 K_i I_i^{1/2}}{\sum_{j=1}^3 \lambda_j K_j I_j^{1/2}} \quad (3.12)$$

The resulting compound channel continuity equation is therefore written as:

$$\frac{\partial A}{\partial t} + \frac{\partial (C_q Q)}{\partial x} - q_{01} - q_{30} = 0 \quad (3.13)$$

Developing the second term yields:

$$\frac{\partial A}{\partial t} + C_q \frac{\partial Q}{\partial x} + Q \frac{\partial C_q}{\partial x} - q_{01} - q_{30} = 0 \quad (3.14)$$

3.1.2 Momentum equation

In this instance $E = mv$ and thus $\epsilon = \frac{dE}{dm} = v$. Also, according to Newton's second law, the change in momentum is equal to the summation of external forces:

$$\frac{dE}{dt} = \sum F \quad (3.15)$$

Substituting for E , ϵ , and $\frac{dE}{dt}$ in equation (3.1) yields:

$$\sum F = \frac{d}{dt} \int_V v \rho dV + \int_S v \rho v \cdot dA \quad (3.16)$$

Once again, in the case of the main channel, the summation of external forces is:

$$\sum F_2 = F_{g2} + F_{f2} + F_{p2} \quad (3.17)$$

where the forces are defined and evaluated as:

- Gravity force:

$$F_{g2} = \rho g A_2 S_0 dx \quad (3.18)$$

where S_0 = channel bed slope and g = acceleration due to gravity.

- Friction force:

$$F_{f2} = -\tau_0 P dx = -(\rho g R S_f) P dx = -\rho g A_2 S_{f2} dx \quad (3.19)$$

where τ_0 = boundary shear stress, P = wetted perimeter, R = hydraulic radius, and S_f = friction slope.

- Unbalanced pressure force F_{p2} (the resultant of the hydrostatic force on the left and right sides of the control volume and the pressure force exerted by the banks). This force is expressed as (Chow et al. 1988):

$$F_{p2} = -\rho g A_2 \frac{\partial y_2}{\partial x} \quad (3.20)$$

where y_2 = main channel flow depth.

Now, evaluate the right hand side terms of equation (3.16). When the continuity equation was developed the mass inflow rate to the control volume was found to be $-\rho \dot{Q}_2$. Thus the momentum inflow rate at inlet of the control volume is:

$$\int_{inlet} \int v \rho v . dA = -\rho \beta_2 v_2 \dot{Q}_2 - \zeta_2 dx \quad (3.21)$$

where ζ_2 = the net momentum flux produced by the lateral discharges in the x-direction. $\zeta_2 = \rho q_{12} u_{12} + \rho q_{23} u_{23}$ where q_{12} and q_{23} are lateral discharges to the main channel and u_{12} and u_{23} are their corresponding velocities in the x-direction.

$$\int_{outlet} \int v \rho v . dA = \rho \left(\beta_2 v_2 \dot{Q}_2 + \frac{\partial(\beta_2 v_2 \dot{Q}_2)}{\partial x} dx \right) \quad (3.22)$$

where β_2 is the Boussinnesq or momentum correction factor included to account for the non-uniformity of velocity distributions. Summing equations (3.21) and (3.22) yields the momentum inflow rate for the control section:

$$\int_S \int v \rho v . dA = \rho \frac{\partial(\beta_2 v_2 \dot{Q}_2)}{\partial x} dx - \zeta_2 dx \quad (3.23)$$

Finally, the momentum storage term is:

$$\frac{d}{dt} \int_V \int \int v \rho dV = \frac{d}{dt} (v_2 \rho A_2 dx) = \rho \frac{\partial \dot{Q}_2}{\partial t} dx \quad (3.24)$$

Substituting equations (3.18), (3.19), (3.20), (3.23) and (3.24) into equation (3.16) results in:

$$\begin{aligned} \rho g A_2 S_0 dx - \rho g A_2 S_{f2} dx - \rho g A_2 \frac{\partial y_2}{\partial x} dx - \rho \frac{\partial (\beta_2 v_2 \dot{Q}_2)}{\partial x} dx \\ - \rho \frac{\partial (\dot{Q}_2)}{\partial t} dx + \zeta_2 dx = 0 \end{aligned} \quad (3.25)$$

Dividing through by ρdx and replacing v_2 with $\frac{Q_2}{A_2}$ yields:

$$\frac{\partial (\beta_2 \frac{\dot{Q}_2^2}{A_2})}{\partial x} + \frac{\partial (\dot{Q}_2)}{\partial t} + g A_2 \left(\frac{\partial y_2}{\partial x} - S_0 + S_{f2} \right) - q_{12} u_{12} - q_{23} u_{23} = 0 \quad (3.26)$$

Assuming H to be the water surface elevation with respect to the main channel bottom we have:

$$H = y_2 = y_1 + y_{bl} = y_3 + y_{br} \quad (3.27)$$

where y_{bl} and y_{br} are bank-full depths at left and right banks respectively.

Thus:

$$\frac{\partial H}{\partial x} = \frac{\partial y_1}{\partial x} = \frac{\partial y_2}{\partial x} = \frac{\partial y_3}{\partial x} \quad (3.28)$$

and equation (3.26) can be written as:

$$\frac{\partial (\beta_2 \frac{\dot{Q}_2^2}{A_2})}{\partial x} + \frac{\partial (\dot{Q}_2)}{\partial t} + g A_2 \left(\frac{\partial H_2}{\partial x} + S_{f2} \right) - q_{12} u_{12} - q_{23} u_{23} = 0 \quad (3.29)$$

Following the same procedure, the momentum equations for sub-sections 1 and 3 are obtained:

$$\frac{\partial (\beta_1 \frac{\dot{Q}_1^2}{A_1})}{\partial x} + \frac{\partial (\dot{Q}_1)}{\partial t} + g A_1 \left(\frac{\partial H_1}{\partial x} + S_{f1} \right) - q_{01} u_{01} + q_{12} u_{12} = 0 \quad (3.30)$$

$$\text{and } \frac{\partial(\beta_1 \frac{\dot{Q}_1^2}{A_1})}{\partial x} + \frac{\partial(\dot{Q}_3)}{\partial t} + gA_3 \left(\frac{\partial H_3}{\partial x} + S_{f3} \right) + q_{23}u_{23} - q_{30}u_{30} = 0 \quad (3.31)$$

In order to sum the three momentum equations, a momentum correction factor M_f must be introduced to group the first terms of equations (3.29) to (3.31). M_f should relate the sum: $\beta_1 \frac{\dot{Q}_1^2}{A_1} + \beta_2 \frac{\dot{Q}_2^2}{A_2} + \beta_3 \frac{\dot{Q}_3^2}{A_3}$ with $\frac{Q^2}{A}$, which represents average flow conditions in the composite section, thus:

$$M_f = \frac{A}{Q^2} \left(\beta_1 \frac{\dot{Q}_1^2}{A_1} + \beta_2 \frac{\dot{Q}_2^2}{A_2} + \beta_3 \frac{\dot{Q}_3^2}{A_3} \right) \quad (3.32)$$

or

$$M_f = \frac{A}{Q^2} \left(\beta_1 \frac{K_1^2 S_{f1}}{A_1} + \beta_2 \frac{K_2^2 S_{f2}}{A_2} + \beta_3 \frac{K_3^2 S_{f3}}{A_3} \right) \quad (3.33)$$

where K refers to section conveyance.

Substituting $Q = \frac{1}{C_q}(\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3) = \sum_{i=1}^3 \lambda_i K_i I_i^{1/2} S_f^{1/2}$ and rearranging yields:

$$M_f = \frac{A}{\left(\sum_{i=1}^3 \lambda_i K_i I_i^{1/2} \right)^2} \sum_{j=1}^3 \beta_j I_j \frac{K_j^2}{A_j} \quad (3.34)$$

Adding the second terms of equations (3.29) to (3.31) requires the introduction of the parameter C_q . The compound channel momentum equation is therefore written as:

$$\frac{\partial(C_q Q)}{\partial t} + \frac{\partial(M_f Q^2/A)}{\partial x} + gA \frac{\partial H}{\partial x} + gFQ^2 - q_{01}u_{01} - q_{30}u_{30} = 0 \quad (3.35)$$

The term gFQ^2 comes from the summation of the terms: $gA_1 S_{f1}$, $gA_2 S_{f2}$ and $gA_3 S_{f3}$, which results in: $g \left[\sum_{i=1}^3 A_i I_i \right] S_f$. The total flow rate was written earlier as:

$$Q = \sum_{i=1}^3 \lambda_i K_i I_i^{1/2} S_f^{1/2} \quad (3.36)$$

Thus:

$$S_f = \frac{Q^2}{\left[\sum_{i=1}^3 \lambda_i K_i I_i^{1/2}\right]^2} \quad (3.37)$$

This yields the following:

$$g \left[\sum_{i=1}^3 A_i I_i \right] S_f = g \frac{\left[\sum_{i=1}^3 A_i I_i \right]}{\left[\sum_{j=1}^3 \lambda_j K_j I_j^{1/2} \right]^2} Q^2 = g F Q^2 \quad (3.38)$$

in which

$$F = \frac{\sum_{i=1}^3 A_i I_i}{\left(\sum_{j=1}^3 \lambda_j K_j I_j^{1/2} \right)^2} \quad (3.39)$$

Developing the first and second terms in equation (3.35) yields:

$$\begin{aligned} C_q \frac{\partial Q}{\partial t} + Q \frac{\partial C_q}{\partial t} + 2M_f \frac{Q}{A} \frac{\partial Q}{\partial x} - M_f \frac{Q^2}{A^2} \frac{\partial A}{\partial x} + \frac{Q^2}{A} \frac{\partial M_f}{\partial x} \\ + g A \frac{\partial H}{\partial x} + g F Q^2 - q_{01} u_{01} - q_{30} u_{30} = 0 \end{aligned} \quad (3.40)$$

In summary, routing floods through compound channels can be accomplished through the numerical solution of (3.14), (3.34), (3.39) and (3.40).

Once total flow rate through the compound channel is determined discharges through the flood plain and main channel sub-sections can also be evaluated separately. An equation expressing the main channel discharge as a fraction of the total compound channel discharge is developed. According to equation (3.36):

$$Q = Q_1 + Q_2 + Q_3 \quad (3.41)$$

where:

$$Q_i = \lambda_i K_i i_i^{1/2} S_f^{1/2} \quad i = 1, 3 \quad (3.42)$$

Writing equation (3.42) for sub-sections 2 and 3 respectively and combining both equations results in an expression of Q_1 in terms of Q_2 . Similarly Q_3 is

also expressed as a function of Q_2 . Substituting the resulting relationships in equation (3.41) yields the following relationship:

$$Q_2 = \frac{\lambda_2 K_2}{\sum_{i=1}^3 \lambda_i K_i I_i^{1/2}} Q \quad (3.43)$$

which gives the main channel discharge in terms of the total discharge through the compound channel. The flood plain discharge is then determined as the difference between these two discharges.

3.2 Subsection Friction Slopes

Past studies on the hydraulics of compound channels usually assume the same friction slope in the main channel and flood plain zones (Tingsanchali and Ackermann 1976). However, since streamwise mean flow velocity in a vertical clearly varies across a composite section of small flood plain depth, the slope of the energy grade line must also vary if the free surface at any channel cross-section is to remain horizontal, (Fig. 3.2). Assuming a constant friction slope throughout implies a non-horizontal cross-stream free surface, which clearly violates the one-dimensional flow hypothesis. This inconsistency was also noted by Cunge et al (1980). Thus, it can be concluded that assuming constant friction slopes is justified only for uniform horizontal velocity distributions, the condition at which the composite section is treated as a single unit.

Based on the analysis of field data Bhowmik and Demissie (1982) show

that a composite channel can be treated as a single unit if the flood plain depth exceeds 35% of the main channel depth. Therefore, the assumption of equal friction slopes cannot be justified for $\frac{y_r}{y_m} < 0.35$ since, under these circumstances, flow velocities in the main channel are much higher than those elsewhere. On the other hand, the conventional approach that assumes the flood plain to be a storage reservoir can be justified only for very small flood plain depths. So, we have two approaches to the problem that treat extreme cases, namely: very small flood plain depths and flood plain depths exceeding 35% of the main channel depth.

In this study a cosine interpolation function, which satisfies both of the above extremes in flood plain conveyance is suggested. It is assumed that $I_2 = S_{f2}/S_f = 1.0$, and I_1 and I_3 are given as:

$$I_1 = \frac{S_{f1}}{S_f} = \frac{1}{2} \left(1 - \cos \frac{\pi y_{r1}}{0.35} \right) \quad (3.44)$$

and

$$I_3 = \frac{S_{f3}}{S_f} = \frac{1}{2} \left(1 - \cos \frac{\pi y_{r3}}{0.35} \right) \quad (3.45)$$

where y_r = the ratio of flood plain to main channel depth. These equations imply that friction slope in the main channel is assumed equal to that of the composite section, while those of the flood plains (S_{f1} and S_{f3}) are close to zero for small flood plain depths and equal to S_f only when $y_r \geq 0.35$.

3.3 Modeling LMT Effects

The LMT's effect in retarding the main channel flow and accelerating that of the flood plain(s) is accounted for through the parameter λ , which is the ratio of isolated to actual flow rate in any flow sub-section. Several methods to evaluate the LMT coefficients λ_i , ($i = 1, 3$) are considered. All of these procedures are empirical and assume steady flow conditions apply.

Although the proposed empirical relationships to account for LMT were developed based on steady flow conditions, their application in unsteady flow modeling in compound channels is justified for two reasons: (i) No model capable of describing the complex interaction between 'main channel' and 'flood plain' flows under unsteady flow conditions presently exists, and (ii) including LMT by means of an empirical relationship developed for steady flows is obviously better than completely ignoring the process altogether. It has also to be kept in mind that unsteady flow equations are still being solved using steady flow formulae, such as Chezy or Manning, to evaluate the friction slope term, simply because no other alternative is available.

3.3.1 Nicollet and Uan's (1979) method

The first empirical procedure investigated in this study is one proposed by Nicollet and Uan (1979), in which the ratio of actual discharge in the main

channel sub-section to that of an equivalent simple rectangular section is given as:

$$\frac{Q_2}{\bar{Q}_2} = \lambda_0 = 0.9 \left(\frac{n_{13}}{n_2} \right)^{-1/6} \quad (3.46)$$

where n_{13} = an average Manning's roughness coefficient for both flood plain sub-sections. This equation accounts for the reduction in flow in the main channel (caused by the flood plain flow) by considering the drag force on the main channel flow produced by the slower flow water in the flood plain. [Note that only the main channel discharge is adjusted; the flow rate in the flood plain remains unchanged]. This assumption, however, should not affect the outcome since LMT's effect is strongest at the smaller flood plain depths. (the condition at which almost no conveyance occurs in these zones).

If r is any geometrical parameter characterizing over-bank flow (such as water depth, wetted perimeter, cross-sectional area, or hydraulic radius) clearly when $r = 0$ (no flood plain flow), $\frac{Q_2}{\bar{Q}_2} = 1$. On the other hand, it is important to note that the proposed empirical equation reaches an upper limiting value, r_s . Thus, we have: $\frac{Q_2}{\bar{Q}_2} = 1$ when $r = 0$ and $\frac{Q_c}{\bar{Q}_c} = N_0 = 0.9 \left(\frac{n_p}{n_c} \right)^{-1/6}$ when $r = r_s$. Nicollet and Uan (1979) suggested a cosine function that satisfies both boundary conditions to describe situations between these two extreme cases of flood plain conveyance. For r between 0 and r_s :

$$\frac{Q_2}{\bar{Q}_2} = \lambda = \frac{1}{2} \left[(1 - \lambda_0) \cos \frac{\pi r}{r_s} + (1 + \lambda_0) \right] \quad (3.47)$$

if r_s is assumed to be the ratio of flood plain to main channel hydraulic radii,

the authors suggest that it should be around 0.3. In other words equation (3.46) is valid when the flood plain hydraulic radius is approximately one third of that of the main channel.

3.3.2 Dracos and Hardegger's (1987) method

Most of the procedures developed to account for LMT are based on separate determination of the stage-discharge relations for the different sub-sections of a compound channel flow field, taking into consideration appropriate compatibility conditions along the separation interface (Nicollet and Uan 1979; Wormleaton et al 1982; Prinos and Townsend 1984; among others). Dracos and Hardegger (1987), however, proposed a single-channel method for the computation of uniform flow in channels with flood plains. They suggest to keep the definition of the hydraulic radius unchanged and examine the variation of the roughness coefficient when all measured quantities are introduced in one of the standard uniform flow equations, such as Manning's. The resulting roughness coefficient, called apparent roughness coefficient (n_a), was found to be smallest when bank-full stage just exceeded and then increases with increasing depth on the flood plain until it reaches a constant value that does not further increase with increasing depth (n_0). This implies that at this stage the compound channel starts to behave like a single unit with constant roughness coefficient.

The change in the ratio n_a/n_0 , which is a measure of the momentum

exchange between the flow in the main channel and the flood plain sub-sections, was related to the composite section hydraulic radius, flow depth and another parameter representing the increase in wetted perimeter when bank-full stage is just exceeded. Based on this correlation and the experimental data of James and Brown (1977), empirical relationships for n_a/n_0 as a function of flow depth were developed. Once $n_a = f(y)$ is known, the discharge for any flow depth y can be determined using Manning's equation.

When roughness coefficients in the flood plain and main channel are different, which is generally the case for natural rivers, an effective roughness coefficient representing average conditions in the compound channel should first be evaluated because Manning's equation as well as the proposed empirical relationships have been developed for channels with uniform roughness. Different procedures giving the effective roughness coefficient in terms of individual roughness coefficients and geometric properties of the different sub-sections of the compound channel are available in the literature. The three main methods that were suggested are the following:

$$n_e = \left(\sum_{i=1}^3 \frac{P_i n_i^{3/2}}{P} \right)^{2/3} \quad (3.48)$$

$$n_e = \left(\sum_{i=1}^3 \frac{P_i^{1/2} n_i^2}{P^{1/2}} \right)^{1/2} \quad (3.49)$$

$$n_e = \left(\frac{P R^{5/3}}{\sum_{i=1}^3 \frac{P_i R_i^{5/3}}{n_i}} \right) \quad (3.50)$$

where P and R are the composite section wetted perimeter and hydraulic radius respectively. Equation (3.48) assumes constant mean velocity for

all sub-sections while equation (3.49) is based on the assumption that the total resistance is equal to the resistance summed over the three zones of the compound channel. Finally, equation (3.50) treats the total discharge in the compound channel as the summation of discharges of its sub-sections.

3.3.3 Method based on Myers and Brennan's (1990) data

The data of Myers and Brennan (1990) was analyzed in this study to produce the following equation relating actual main channel discharge to that of an equivalent regular section.

$$\frac{Q_2}{Q_2} = \lambda = 0.917 \left(\frac{y_{13}}{y_b} \right)^{-0.0159} \left(\frac{W_t}{W_2} \right)^{-0.0675} \quad (3.51)$$

where W_t = total width of the composite section, y_{13} = average flood plain depth, and y_b = bank-full depth.

3.3.4 Φ -Indices method

An alternative method for estimating LMT coefficients, λ_i , is to relate the latter to the so-called " Φ -indices" (Radojkovic 1976).

A main channel of a composite section is separated from the flood plain sub-sections by imaginary vertical interface planes. The forces acting upon each sub-section (Fig. 3.3) are:

- F_{g1}, F_{g2}, F_{g3} = weight components in the direction of flow.
- F_{b1}, F_{b2}, F_{b3} = boundary shear forces.
- F_{a1}, F_{a3} = apparent shear forces.

A steady flow condition is assumed. Therefore, the summation of forces is equal to zero. For the main channel this yields:

$$F_{g2} = F_{b2} + F_{a1} + F_{a3} \quad (3.52)$$

The gravity force can be written as:

$$F_{g2} = \gamma A_2 S \quad (3.53)$$

where γ = the fluid specific weight, A_2 = the main channel cross-sectional area, and S = the channel bed slope.

The boundary shear force is expressed as:

$$F_{b2} = \tau_{b2} P_2 \quad (3.54)$$

where τ_{b2} = the boundary shear stress and P_2 = the main channel wetted perimeter. Finally, the apparent shear forces are

$$F_{a1} = \tau_{a1} y_1 \text{ and } F_{a3} = \tau_{a3} y_3 \quad (3.55)$$

Substituting equations (3.53) to (3.55) into equation (3.52) yields:

$$\gamma A_2 S = \tau_{b2} P_2 + \tau_{a1} y_1 + \tau_{a3} y_3 \quad (3.56)$$

Rearranging the terms of (3.56) results in the following relationship for boundary shear stress:

$$\tau_{b2} = \gamma R_2 S - \frac{\tau_{a1} y_1 + \tau_{a3} y_3}{P_2} \quad (3.57)$$

where R_2 = main channel hydraulic radius. On the other hand, the average boundary shear stress can also be expressed in terms of main channel velocity V_m , water density ρ , and a friction coefficient f .

$$\tau_{b2} = f \rho \frac{V_2^2}{2} \quad (3.58)$$

Combining equations (3.57) and (3.58) results in the following expression for main channel velocity.

$$V_2 = \sqrt{\frac{2g}{f} \left(R_2 S - \frac{\tau_{a1} y_1 + \tau_{a3} y_3}{\gamma P_2} \right)} \quad (3.59)$$

The relationship between the friction coefficient, C_f , and the roughness coefficients of Chezy, C , and Manning, n , is:

$$f = \frac{2g}{C^2} = 2g \left(\frac{n^2}{R_2^{1/3}} \right) \quad (3.60)$$

Combining equations (3.59) and (3.60) yields:

$$V_2 = \dot{V}_2 \sqrt{(\gamma A_2 S - \tau_{a1} y_1 - \tau_{a3} y_3) / \gamma A_2 S} \quad (3.61)$$

where $\dot{V}_2$ = velocity in the main channel given by Manning's formula (in which the interface is ignored in determining the wetted perimeter). Combining equations (3.56) and (3.61) yields:

$$V_2 = \dot{V}_2 \left(\frac{\tau_{b2} P_2}{\gamma A_2 S} \right)^{1/2} = \dot{V}_2 \left(\frac{F_{b2}}{F_{g2}} \right)^{1/2} \quad (3.62)$$

The ratio F_{b2}/F_{g2} is equal to Φ_2 (the main channel Φ index) which characterizes the degree of interaction between the main channel and flood plain sub-sections. Writing equation (3.62) in terms of discharge gives:

$$Q_2 = \dot{Q}_2 \Phi_2^{1/2} \quad (3.63)$$

A similar analysis can be carried out for the flood plain sub-sections in terms of their Radojkovic Φ -indices. This gives: $\Phi_1 = F_{b1}/F_{g1}$ and $\Phi_3 = F_{b3}/F_{g3}$. Thus, the compound channel discharge is written as:

$$Q = Q_1 + Q_2 + Q_3 = \dot{Q}_1 \Phi_1^{1/2} + \dot{Q}_2 \Phi_2^{1/2} + \dot{Q}_3 \Phi_3^{1/2} \quad (3.64)$$

where $\dot{Q}_1$, $\dot{Q}_2$, and $\dot{Q}_3$ are the "isolated" discharges as given by Manning's formula.

3.3.5 Estimation of Φ -Values from Channel Characteristics

According to equation (3.64), in order to estimate a compound channel discharge, Q , one first needs estimates of the flood plain and main channel Φ indices. These indices are related to the vertical interface apparent shear stress for which several empirical relationships are available in the literature.

First consider the steady flow condition for the left flood plain (sub-section 1). Here we may write:

$$F_{g1} + F_{a1} = F_{b1} \quad (3.65)$$

LMT is assumed to act as an accelerating force for the left flood plain flow and therefore is added to the gravity force. Dividing equation (3.65) by F_{g1} and rearranging yields:

$$\Phi_1 = \frac{F_{b1}}{F_{g1}} = 1 + \frac{F_{a1}}{F_{g1}} = 1 + \frac{\tau_{av1}y_1}{\gamma A_1 S} \quad (3.66)$$

Similarly the Φ index for the right flood plain can be written as:

$$\Phi_3 = 1 + \frac{\tau_{av3}y_3}{\gamma A_3 S} \quad (3.67)$$

where τ_{av} = the vertical interface apparent shear stress.

The steady flow condition for the main channel (sub-section 2) is written as:

$$F_{g2} = F_{b2} + F_{a1} + F_{a3} \quad (3.68)$$

Here apparent forces F_{a1} and F_{a3} act as retarding forces upon the main channel flow. Dividing equation (3.68) by F_{g2} and rearranging results in the following relationship for the main channel Φ index:

$$\Phi_2 = 1 - \frac{\tau_{av1}y_1 + \tau_{av3}y_3}{\gamma A_2 S} \quad (3.69)$$

As was mentioned earlier, several empirical relationships for estimating apparent shear stress τ_{av} are available in the literature. The following empirical equations were used in this study to estimate τ_{av} and hence the Φ -indices for the three sub-sections:

1. Wormleaton et al (1982):

$$\tau_{avi} = 13.84 \Delta v_i^{0.882} \left(\frac{W_i}{W_2} \right)^{-0.727} \left(\frac{y_i}{y_2 - y_i} \right)^{-3.123} \quad (3.70)$$

2. Prinos and Townsend (1984):

$$\tau_{avi} = 0.874 \Delta v_i^{0.92} \left(\frac{W_i}{W_2} \right)^{-0.514} \left(\frac{y_i}{y_2} \right)^{-1.129} \quad (3.71)$$

3.4 Channel Meander

In a compound channel the main channel generally tends to meander at a higher degree than its flood plain along the watercourse, resulting in different paths of the main channel and flood plain flows. Therefore, in order to apply equations (3.14), (3.34), (3.39), and (3.40) to a meandering channel, the berm distances along the watercourse have to be adjusted so that they are equal to those of the main channel. In this exercise distances along the left and right flood plains are assumed to be equal. Thus:

$$x_1 = x_3 = \frac{x_2}{L_r} \quad (3.72)$$

where L_r = a flow path length ratio defined as the ratio of the main channel to the flood plain lengths between the upstream and downstream boundaries. Introducing these changes to the continuity equation developed earlier, the parameter C_q is now written as:

$$C_q = \frac{1}{Q} (L_r \dot{Q}_1 + \dot{Q}_2 + L_r \dot{Q}_3) = \frac{\sum_{i=1}^3 L_{ri} K_i I_i^{1/2}}{\sum_{j=1}^3 \lambda_j K_j I_j^{1/2}} \quad (3.73)$$

where $L_{r1} = L_{r3} = L_r$, and $L_{r2} = 1.0$.

Similarly, the momentum correction factor M_f in the equation of motion

is also adjusted to account for channel meander. It is now written as:

$$M_f = \frac{A}{\left(\sum_{i=1}^3 \lambda_i K_i I_i^{1/2}\right)^2} \sum_{j=1}^3 \beta_j I_j L_{rj} \frac{K_j^2}{A_j} \quad (3.74)$$

Note that parameters C_q and F in the equation of motion remain unchanged.

3.5 Model Equations in a Non-Dimensional Form

For a general representation the dimensional model equations should be presented in a non-dimensional form. The characteristic variables used in the non-dimensionalization process are:

- characteristic depth (y_c), defined as the initial steady state uniform depth;
- characteristic velocity (v_c), defined as the uniform flow velocity;
- characteristic time (t_c), defined as the duration of the inflow flood hydrograph;
- characteristic distance (x_c), defined as $v_c t_c$;
- characteristic area (A_c), defined as y_c^2 ;
- characteristic discharge (Q_c), defined as $v_c y_c^2$; and

- characteristic slope (S_c), defined as y_c/x_c .

According to equation (3.12) C_q is written as a ratio of conveyances, making it already dimensionless. Similarly, M_f is also non-dimensional. On the other hand the parameter F is given by:

$$F = \frac{\sum_{i=1}^3 A_i I_i}{(\sum_{j=1}^3 \lambda_j K_j I_j^{1/2})^2} \quad (3.75)$$

implying a corresponding characteristic coefficient $F_c = A_c/K_c^2$, where K_c = a characteristic conveyance. Further development of F_c results in:

$$F_c = \frac{y_c^2 S_c}{Q_c^2} = \frac{1}{v_c^3 y_c t_c} \quad (3.76)$$

Finally, the characteristic variables for lateral inflows are introduced. Lateral inflows are defined as discharges for a given length along the channel. Thus the characteristic inflow rate is:

$$q_c = \frac{Q_c}{x_c} = \frac{y_c^2}{t_c} \quad (3.77)$$

Substituting dimensionless quantities for their corresponding dimensional ones in the model equations results in non-dimensional continuity and momentum equations, written respectively as:

$$\frac{\partial A}{\partial t} + C_q \frac{\partial Q}{\partial x} + Q \frac{\partial C_q}{\partial x} - q_{01} - q_{30} = 0 \quad (3.78)$$

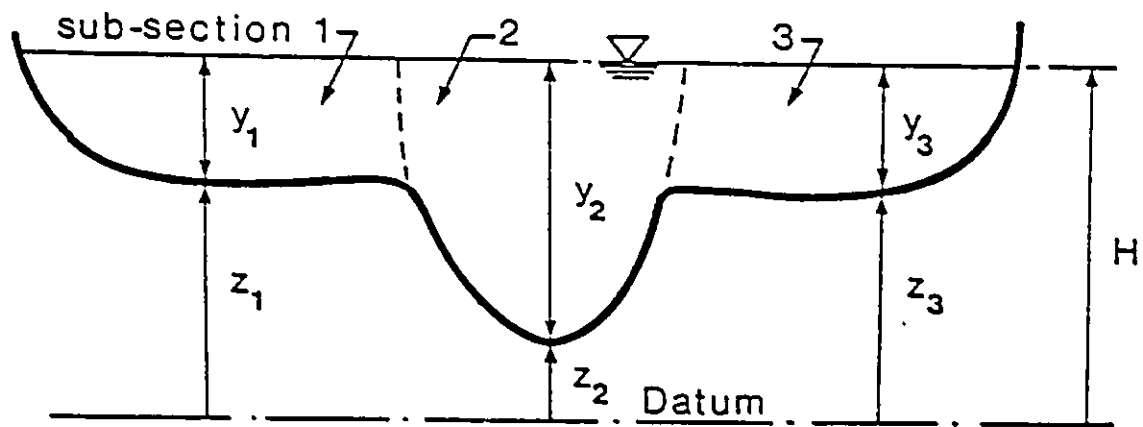
and

$$\begin{aligned} C_q \frac{\partial Q}{\partial t} + Q \frac{\partial C_q}{\partial t} + 2M_f \frac{Q}{A} \frac{\partial Q}{\partial x} - M_f \frac{Q^2}{A^2} \frac{\partial A}{\partial x} + \frac{Q^2}{A} \frac{\partial M_f}{\partial x} + gA \frac{\partial H}{\partial x} \\ + gFQ^2 - q_{01}u_{01} - q_{30}u_{30} = 0 \end{aligned} \quad (3.79)$$

3.6 Summary of the Model Assumptions

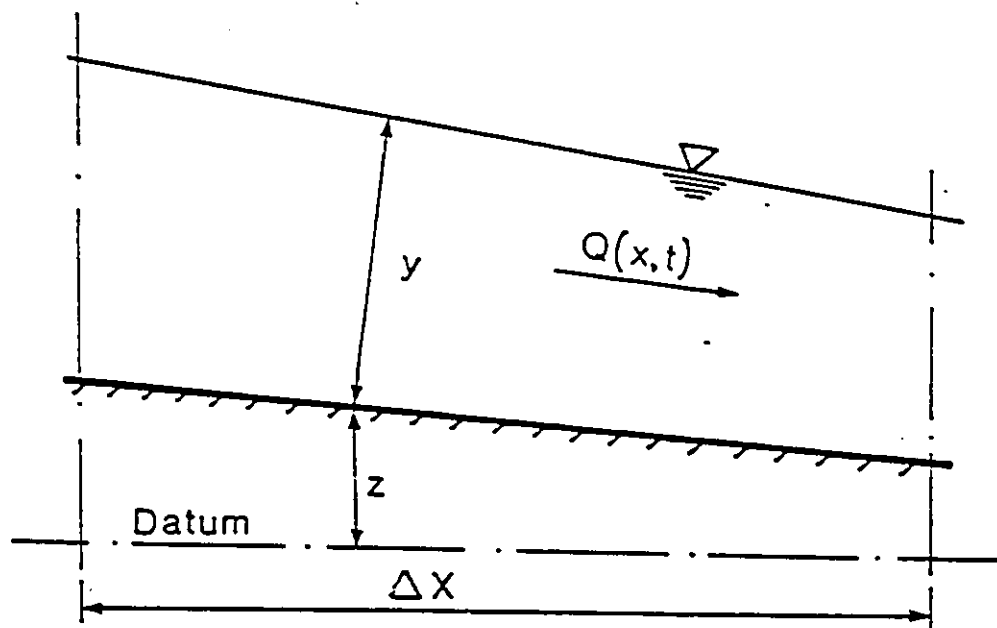
Besides the general assumptions underlying the development of the St. Venant equations, RUFICC also assumes the following:

1. empirical relationships that quantify LMT, even though developed for steady flow conditions, are assumed to be applicable to unsteady gradually-varied flows;
2. flood plains are not considered as storage reservoirs, but rather are assumed to contribute to system conveyance;
3. main channel and composite channel friction slopes are assumed equal;
4. flood plain friction slope is assumed equal to that for the composite section if the ratio of flood plain to main channel depths y_r equals or exceeds 0.35;
5. a cosine interpolation function gives the ratio of flood plain to composite section friction slopes in the interval $0 \leq y_r \leq 0.35$.



(a)

Channel Cross-Section



(b)

Incremental Reach

Figure 3.1: Definition Sketch of a Hypothetical Compound Channel Reach.

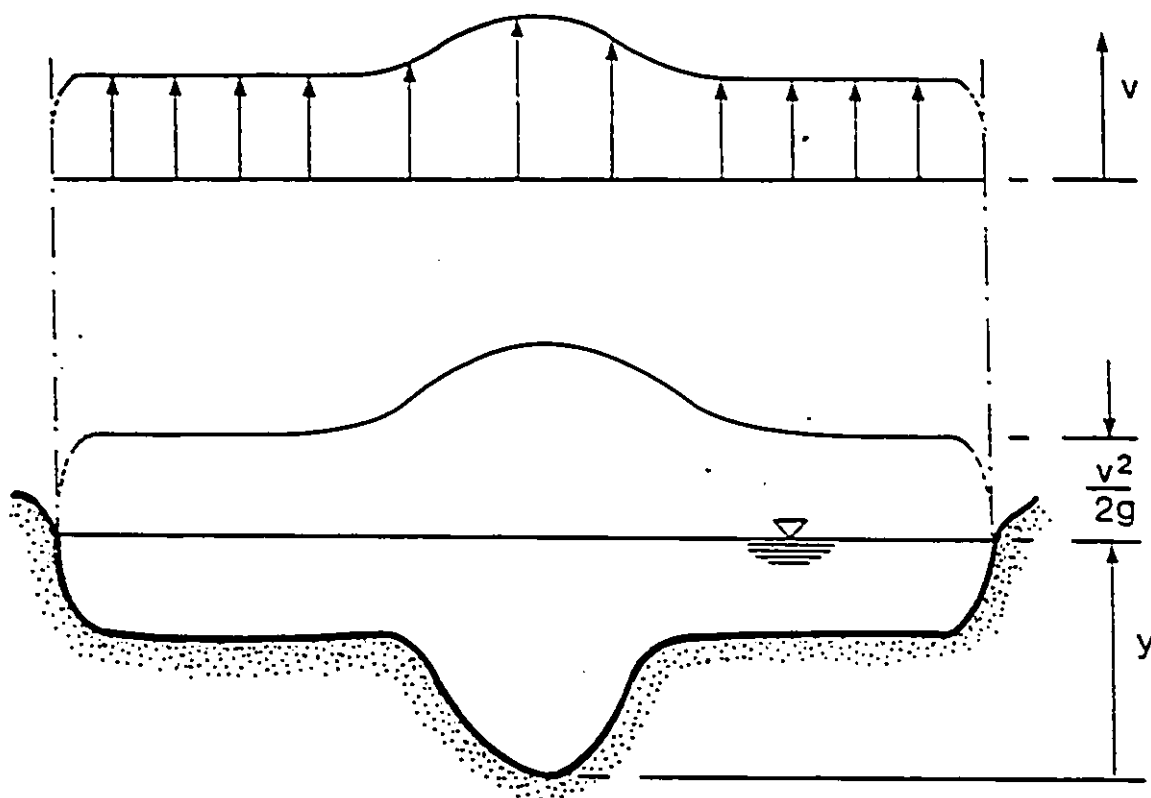


Figure 3.2: The Effect of a Horizontal Transverse Water Surface and a Non-Uniform Cross-Stream Velocity Distribution on the Total Energy Line at any Cross-Section.

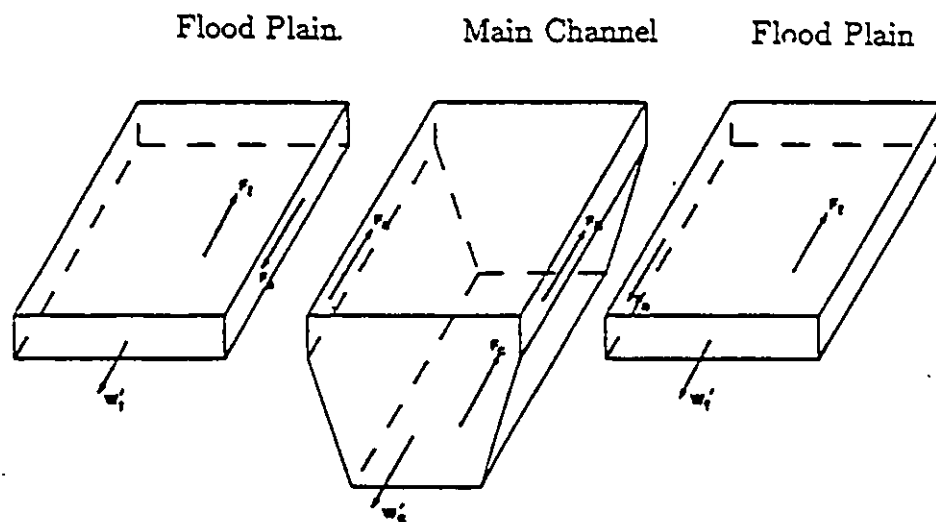


Figure 3.3: Diagram Showing Forces Acting in Compound Channel Sub-Sections.

Chapter 4

NUMERICAL ANALYSIS

The partial differential equations developed in the preceding chapter are modified forms of the St. Venant equations that are required to be solved numerically. The weighted four-point implicit finite difference (FD) scheme (Amein and Fang 1970) was selected for this exercise. This was done mainly to overcome the limitations imposed on the size of the time step required for the numerical stability of explicit schemes. The chosen scheme is also particularly well suited to applications involving natural channels. This is because it can handle varying channel geometry, even where changes from section to section and in the bottom slope are significant (Amein and Fang 1970).

4.1 Difference Equations

In the FD scheme the continuous time-space region, in which solutions of H and Q are to be obtained, is represented by a regular net of discrete points (Fig. 4.1). The discrete points are determined by the intersection of lines drawn parallel to the time (t) and space (x) axes. Lines parallel to the x-axis represent time lines and have a spacing Δt ; lines parallel to the t-axis represent space lines and have a spacing Δx . Each point in the rectangular network is identified by a subscript (i), which designates its space position and a superscript (j) which designates the time line. Time derivatives are written as:

$$\frac{\partial L}{\partial t} = \frac{(L_{i+1}^{j+1} + L_i^{j+1}) - (L_{i+1}^j + L_i^j)}{2\Delta t^j} \quad (4.1)$$

where L represents any dependent variable. Space derivatives and non-derivative terms are approximated by:

$$\frac{\partial L}{\partial x} = \frac{\theta(L_{i+1}^{j+1} - L_i^{j+1}) + (1 - \theta)(L_{i+1}^j - L_i^j)}{\Delta x^i} \quad (4.2)$$

and:

$$L = \frac{\theta(L_{i+1}^{j+1} + L_i^{j+1}) + (1 - \theta)(L_{i+1}^j + L_i^j)}{2} \quad (4.3)$$

in which θ is a weighting factor for position between two adjacent time lines. This factor is known to affect both numerical stability and accuracy of the different numerical schemes (Chaudhry and Contractor, 1973 and Fread, 1974 among others). The value of θ varies between 0 and 1. When $\theta = 0$, the numerical scheme reduces to the explicit method. A numerical scheme is unconditionally stable in the range $0.5 \leq \theta \leq 1.0$, however, accuracy

decreases as θ departs from 0.5 and approaches 1.0. Therefore, to avoid the weakly stable condition associated with $\theta = 0.5$ (and also minimize the loss in accuracy), θ can be increased slightly. In the present work a weighting factor $\theta = 0.6$ was used.

When the FD operators defined by equations (4.1) to (4.3) are used to replace the derivatives and other variables in equations (3.14) and (3.38), the following weighted four-point implicit FD equations are obtained:

- Continuity:

$$\begin{aligned} & \frac{(A_{i+1}^{j+1} + A_i^{j+1}) - (A_{i+1}^j + A_i^j)}{2\Delta t^j} \\ & + C_q \frac{\theta(Q_{i+1}^{j+1} - Q_i^{j+1}) + (1-\theta)(Q_{i+1}^j - Q_i^j)}{\Delta x^i} \\ & + Q \frac{\theta(C_{q,i+1}^{j+1} - C_{q,i}^{j+1}) + (1-\theta)(C_{q,i+1}^j - C_{q,i}^j)}{\Delta x^i} = 0 \quad (4.4) \end{aligned}$$

- Momentum:

$$\begin{aligned} & C_q \frac{(Q_{i+1}^{j+1} + Q_i^{j+1}) - (Q_{i+1}^j + Q_i^j)}{2\Delta t^j} + gFQ^2 \\ & + Q \frac{(C_{q,i+1}^{j+1} + C_{q,i}^{j+1}) - (C_{q,i+1}^j + C_{q,i}^j)}{2\Delta t^j} \\ & + 2M_f \frac{Q}{A} \frac{\theta(Q_{i+1}^{j+1} - Q_i^{j+1}) + (1-\theta)(Q_{i+1}^j - Q_i^j)}{\Delta x^i} \\ & - M_f \frac{Q^2}{A^2} \frac{\theta(A_{i+1}^{j+1} - A_i^{j+1}) + (1-\theta)(A_{i+1}^j - A_i^j)}{\Delta x^i} \\ & + \frac{Q^2}{A} \frac{\theta(M_{f,i+1}^{j+1} - M_{f,i}^{j+1}) + (1-\theta)(M_{f,i+1}^j - M_{f,i}^j)}{\Delta x^i} \\ & + gA \frac{\theta(H_{i+1}^{j+1} - H_i^{j+1}) + (1-\theta)(H_{i+1}^j - H_i^j)}{\Delta x^i} = 0 \quad (4.5) \end{aligned}$$

For the sake of simplicity, non-derivative terms in the above equations were not replaced by their FD approximations and terms representing lateral discharge were not included.

Variables with superscripts j are known from either the initial condition or previous computations. Variables with superscripts $j + 1$ are the unknowns that require solution. However, all the unknowns are not independent. The continuity and momentum equations contain four independent unknowns, namely discharge (Q) and stage (H) at grid points $(i, j + 1)$ and $(i + 1, j + 1)$. Two unknowns are common to any two neighboring cells. If N denotes the number of nodes, the total number of rectangular cells is $N - 1$. Applying equations (4.4) and (4.5) to all the cells results in $2(N - 1)$ equations with $2N$ unknowns. Two additional equations can be supplied by the prescribed boundary conditions. The resulting system of $2N$ non-linear equations needs to be solved for every time step. The Newton-Raphson iteration method is first used to reduce the non-linear system to a successive system of linear equations. The resulting linear equations are then solved using the Double-Sweep Solution Method.

4.2 Solutions of the Model Equations

Computations in the "equation reduction" procedure are started by assigning trial values to the $2N$ unknowns. When these trial values are substituted into the set of $2N$ equations, the right sides of the equations may

not vanish but acquire values known as residuals. The Newton-Raphson method provides a means for correcting the trial values until the residuals are reduced to a suitable tolerance level. This is usually accomplished in a small number of iterations.

The residual of the continuity equation after n iterations can be written as:

$$R_{ci}^n = \frac{(A_{i+1}^{j+1} + A_i^{j+1}) - (A_{i+1}^j + A_i^j)}{2\Delta t^j} + C_q \frac{\theta(Q_{i+1}^{j+1} - Q_i^{j+1}) + (1 - \theta)(Q_{i+1}^j - Q_i^j)}{\Delta x^i} + Q \frac{\theta(C_{qi+1}^{j+1} - C_{qi}^{j+1}) + (1 - \theta)(C_{qi+1}^j - C_{qi}^j)}{\Delta x^i} \quad (4.6)$$

This equation is a function of four unknown variables H_i^{j+1} , Q_i^{j+1} , H_{i+1}^{j+1} and Q_{i+1}^{j+1} . In the Newton-Raphson Method:

$$R_{ci}^{n+1} = R_{ci}^n + \frac{\partial R_{ci}^n}{\partial H_i^{j+1}} \Delta H_i^{j+1} + \frac{\partial R_{ci}^n}{\partial Q_i^{j+1}} \Delta Q_i^{j+1} + \frac{\partial R_{ci}^n}{\partial H_{i+1}^{j+1}} \Delta H_{i+1}^{j+1} + \frac{\partial R_{ci}^n}{\partial Q_{i+1}^{j+1}} \Delta Q_{i+1}^{j+1} = 0 \quad (4.7)$$

This can be rearranged as:

$$-\frac{\partial R_{ci}^n}{\partial H_{i+1}^{j+1}} \Delta H_{i+1}^{j+1} - \frac{\partial R_{ci}^n}{\partial Q_{i+1}^{j+1}} \Delta Q_{i+1}^{j+1} = \frac{\partial R_{ci}^n}{\partial H_i^{j+1}} \Delta H_i^{j+1} + \frac{\partial R_{ci}^n}{\partial Q_i^{j+1}} \Delta Q_i^{j+1} + R_{ci}^n \quad (4.8)$$

This equation is now written in a form that is suitable for application of the Double-Sweep method:

$$A_{ci} \Delta H_{i+1}^{j+1} + B_{ci} \Delta Q_{i+1}^{j+1} = C_{ci} \Delta H_i^{j+1} + D_{ci} \Delta Q_i^{j+1} + G_{ci} \quad (4.9)$$

where:

$$A_{ci} = -\frac{\partial R_{ci}^n}{\partial H_{i+1}^{j+1}} \quad (4.10)$$

$$B_{ci} = -\frac{\partial R_{ci}^n}{\partial Q_{i+1}^{j+1}} \quad (4.11)$$

$$C_{ci} = \frac{\partial R_{ci}^n}{\partial H_i^{j+1}} \quad (4.12)$$

$$D_{ci} = \frac{\partial R_{ci}^n}{\partial Q_i^{j+1}} \quad (4.13)$$

$$\text{and } G_{ci} = R_{ci}^n \quad (4.14)$$

Now, according to the residual equation (4.6) the derivatives can be expressed as:

$$\frac{\partial R_{ci}^n}{\partial H_{i+1}^{j+1}} = \frac{\Delta x_i}{2\Delta t^j} \frac{\partial A_{i+1}^{j+1}}{\partial H_{i+1}^{j+1}} \quad (4.15)$$

$$\frac{\partial R_{ci}^n}{\partial Q_{i+1}^{j+1}} = \theta \quad (4.16)$$

$$\frac{\partial R_{ci}^n}{\partial H_i^{j+1}} = \frac{\Delta x_i}{2\Delta t^j} \frac{\partial A_i^{j+1}}{\partial H_i^{j+1}} \quad (4.17)$$

$$\text{and } \frac{\partial R_{ci}^n}{\partial Q_i^{j+1}} = -\theta \quad (4.18)$$

Similarly, the residual momentum equation can be written as:

$$\begin{aligned} R_{mi}^n = & C_q \frac{(Q_{i+1}^{j+1} + Q_i^{j+1}) - (Q_{i+1}^j + Q_i^j)}{2\Delta t^j} + gFQ^2 \\ & + Q \frac{(C_{q_{i+1}}^{j+1} + C_{q_i}^{j+1}) - (C_{q_{i+1}}^j + C_{q_i}^j)}{2\Delta t^j} \\ & + 2M_f \frac{Q}{A} \frac{\theta(Q_{i+1}^{j+1} - Q_i^{j+1}) + (1-\theta)(Q_{i+1}^j - Q_i^j)}{\Delta x^i} \\ & - M_f \frac{Q^2}{A^2} \frac{\theta(A_{i+1}^{j+1} - A_i^{j+1}) + (1-\theta)(A_{i+1}^j - A_i^j)}{\Delta x^i} \end{aligned}$$

$$\begin{aligned}
& + \frac{Q^2}{A} \frac{\theta(M_{f_{i+1}}^{j+1} - M_{f_i}^{j+1}) + (1 - \theta)(M_{f_{i+1}}^j - M_{f_i}^j)}{\Delta x^i} \\
& + gA \frac{\theta(H_{i+1}^{j+1} - H_i^{j+1}) + (1 - \theta)(H_{i+1}^j - H_i^j)}{\Delta x^i} \quad (4.19)
\end{aligned}$$

Once again, to avoid further complexity, non-derivative terms were not replaced by their difference approximations. Following the same procedure adopted for the reduction of the continuity equation the following equation was obtained:

$$A_{mi}\Delta H_{i+1}^{j+1} + B_{mi}\Delta Q_{i+1}^{j+1} = C_{mi}\Delta H_i^{j+1} + D_{mi}\Delta Q_i^{j+1} + G_{mi} \quad (4.20)$$

where:

$$A_{mi} = -\frac{\partial R_{mi}^n}{\partial H_{i+1}^{j+1}} \quad (4.21)$$

$$B_{mi} = -\frac{\partial R_{mi}^n}{\partial Q_{i+1}^{j+1}} \quad (4.22)$$

$$C_{mi} = \frac{\partial R_{mi}^n}{\partial H_i^{j+1}} \quad (4.23)$$

$$D_{mi} = \frac{\partial R_{mi}^n}{\partial Q_i^{j+1}} \quad (4.24)$$

$$\text{and } G_{mi} = R_{mi}^n \quad (4.25)$$

The derivative terms appearing in equations (4.21) to (4.25) were evaluated based on equation (4.19); however they were not shown here for the sake of simplicity and convenience.

Equations (4.9) and (4.20), when written for all the nodes, provide a system of linear equations which need to be solved for the nodal corrections of the unknowns, namely ΔH_i^{j+1} , ΔQ_i^{j+1} , ΔH_{i+1}^{j+1} , ΔQ_{i+1}^{j+1} . The system

of linear equations produces a banded matrix of the coefficients, which is solved using the "Double-Sweep" technique.

4.3 The Double-Sweep Solution Method

To solve equations (4.9) and (4.20) first the coefficients are evaluated in terms of either the initial or trial values of depth and discharge. Knowing the upstream boundary conditions the evaluation of the coefficients begins at the upstream end and proceeds until the downstream end of the reach under study. This completes the first sweep. Beginning next with the downstream boundary condition, the procedure then solves for the nodal corrections of the unknowns proceeding in the upstream direction to the first node. This completes the second sweep. The following analysis is taken from the work of Kabir (1984).

The procedures assume a linear relationship of the type

$$\Delta Q_i = E_i \Delta H_i + F_i \quad (4.26)$$

for point i on the time line $j + 1$. A similar relationship also exists for the next point $i + 1$. It will be shown below that:

$$\Delta Q_{i+1} = E_{i+1} \Delta H_{i+1} + F_{i+1} \quad (4.27)$$

Substituting equation (4.26) into equations (4.9) and (4.20) yields:

$$A_{ci} \Delta H_{i+1}^{j+1} + B_{ci} \Delta Q_{i+1}^{j+1} = (C_{ci} + D_{ci} E_i) \Delta H_i^{j+1} + D_{ci} F_i + G_{ci} \quad (4.28)$$

$$A_{mi}\Delta H_{i+1}^{j+1} + B_{mi}\Delta Q_{i+1}^{j+1} = (C_{mi} + D_{mi}E_i)\Delta H_i^{j+1} + D_{mi}F_i + G_{ci} \quad (4.29)$$

Equation (4.28) can also be written as:

$$\Delta H_i^{j+1} = \frac{A_{ci}}{C_{ci} + D_{ci}E_i}\Delta H_{i+1}^{j+1} + \frac{B_{ci}}{C_{ci} + D_{ci}E_i}\Delta Q_{i+1}^{j+1} - \frac{G_{ci} + D_{ci}F_i}{C_{ci} + D_{ci}E_i} \quad (4.30)$$

or, in the general form:

$$\Delta H_i^{j+1} = L_i\Delta H_{i+1}^{j+1} + M_i\Delta Q_{i+1}^{j+1} + N_i \quad (4.31)$$

$$\text{where: } L_i = \frac{A_{ci}}{C_{ci} + D_{ci}E_i}; M_i = \frac{B_{ci}}{C_{ci} + D_{ci}E_i}; N_i = -\frac{G_{ci} + D_{ci}F_i}{C_{ci} + D_{ci}E_i}.$$

Eliminating ΔH_i^{j+1} between equations (4.28) and (4.29), and then solving for ΔQ_{i+1}^{j+1} as a function of ΔH_{i+1}^{j+1} results in an equation of the same form as equation (4.27) where:

$$E_{i+1} = \frac{A_{ci}(C_{mi} + D_{mi}E_i) - A_{mi}(C_{ci} + D_{ci}E_i)}{B_{mi}(C_{ci} + D_{ci}E_i) - B_{ci}(C_{mi} + D_{mi}E_i)} \quad (4.32)$$

$$F_{i+1} = \frac{(G_{mi} + D_{mi}F_i)(C_{ci} + D_{ci}E_i) - (G_{ci} + D_{ci}F_i)(C_{mi} + D_{mi}E_i)}{B_{mi}(C_{ci} + D_{ci}E_i) - B_{ci}(C_{mi} + D_{mi}E_i)} \quad (4.33)$$

Thus, it is shown that if the linear relationship given by equation (4.22) exists for any point in the x-t plane, it also will exist for all the following points. Equations (4.32) and (4.33) show that there exists a recurrence relationship for the coefficients E and F.

Both coefficients E_{i+1} and F_{i+1} may be computed for any point $i + 1$ if the coefficients E_i and F_i for the preceding point i are known. The first values of E and F are given by the upstream boundary condition. The downstream boundary condition gives the values of ΔH^{j+1} for the last point. Equation (4.27) then gives the values of ΔQ^{j+1} for the last point

and equation (4.31) gives the value of ΔH^{j+1} for the preceding point. The computation proceeds in the upstream direction to the first node giving the correction values of ΔH^{j+1} and ΔQ^{j+1} for all the nodes. The trial values for the next iteration $n + 1$ for all the nodes are given by:

$$(H^{j+1})^{n+1} = (H^{j+1})^n + \Delta H^{j+1} \quad (4.34)$$

$$(Q^{j+1})^{n+1} = (Q^{j+1})^n + \Delta Q^{j+1} \quad (4.35)$$

The iteration procedure is repeated until the correction values of ΔH^{j+1} and ΔQ^{j+1} are reduced to a suitable tolerance level for that time step, thereby giving the desired solutions for this time step. The computations are then repeated for the next level, $j + 2$.

4.4 Boundary Conditions

The upstream and downstream boundary conditions are one of the following three possibilities: (1) $Q = Q(t)$; (2) $y = y(t)$; and (3) $Q = f(y)$; (all given). In this study the upstream boundary condition consists of either water surface elevation H or discharge Q , expressed as a function of time. The downstream boundary condition is either a rating curve provided by the user or a stage-discharge relationship obtained using one of the empirical procedures for estimating discharge in a compound channel (described in chapter III).

To use the Double-Sweep Method, the boundary conditions must be

expressed in the form:

$$\Delta Q_i = E_i \Delta H_i + F_i \quad (4.36)$$

so that at the upstream boundary, the coefficients E_1 and F_1 are known: and at the downstream boundary, the value of ΔH is known.

If the upstream boundary condition is given as an inflow discharge hydrograph

$$\Delta Q_1 = Q_1^{j+1} - Q_1^j \quad (4.37)$$

which is independent of ΔH_1 . Comparing equations (4.36) and (4.37) $E_1 = 0$ and $F_1 = Q_1^{j+1} - Q_1^j$. Then, whatever the computed value of ΔH_1 , ΔQ_1 will always be equal to the boundary condition value.

4.5 Check for Mass Conservation

The total change in volume is evaluated as the summation of all changes in incremental control volumes, each having a length Δx . Cross-sectional areas are evaluated at each node and the change in incremental control volume is estimated as:

$$V_k = \left[\frac{A_i^{j+1} + A_{i+1}^{j+1}}{2} - \frac{A_i^j + A_{i+1}^j}{2} \right] \Delta x \quad (4.38)$$

The total change in volume for the entire reach under study is determined as the summation of the elemental changes:

$$V_t = \sum_{k=1}^{N_1} V_k \quad (4.39)$$

where $N_1 = N - 1$, in which N = number of nodes.

The change in volume can also be evaluated as the product of discharge and the time increment Δt . So if Q_1 is the flow rate at time level j and Q_2 is the flow rate at time level $j + 1$, the change in volume would be:

$$\dot{V}_t = \Delta t(\theta(Q_{2u} - Q_{2d}) + (1 - \theta)(Q_{1u} - Q_{1d})) \quad (4.40)$$

where θ is the selected computational weighting factor and subscripts 'u' and 'd' refer respectively to upstream and downstream sections of the reach under study. If the difference between V_t and $\dot{V}_t$ is relatively small, continuity of mass is satisfied.

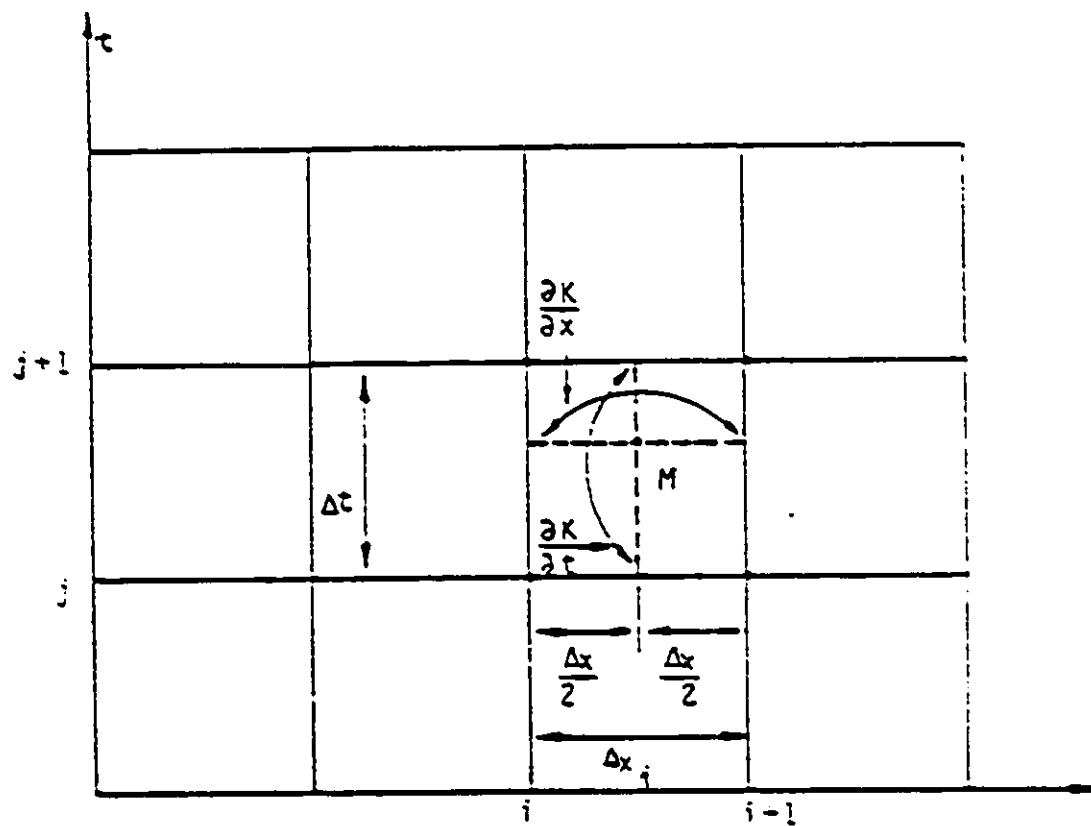


Figure 4.1: Weighted Four-Point Implicit Scheme.

Chapter 5

OPTIMIZATION MODEL

Optimization techniques have been successfully used by different investigators to identify flood routing data for the case of simple, regular-shaped channels. In this study the use of optimization techniques is extended to compound channels, where additional parameters associated with the flood plain zones are introduced.

The search method generally applied in the past to estimate the unsteady flow parameters is the so-called "Influence Coefficient" algorithm. In this study three search methods were employed and compared, not only in terms of their relative accuracy but also in terms of the computer time involved. These three optimization methods, namely: Powell's method, Rosenbrock's algorithm, and the Nelder and Meade Simplex method, were selected mainly because they do not require an estimate

of the objective function gradient. It is also important to note that **two different approaches were adopted in the evaluation of the objective function.** The first approach was the conceptual approach, where optimization was performed upon cross-sectional geometric and hydraulic parameters. The second formulation, termed the 'black box' approach, used abstract parameters in assumed conveyance and cross-sectional area relationships.

5.1 Description of the Optimization Algorithms

5.1.1 Powell's method

Powell's multivariable non-linear optimization method (Wilde and Beightler, 1967) is a "pattern search" method. This means that the method follows n unidirectional searches along the coordinate axes and then searches for the minimum along a pattern direction S_i defined as:

$$S_i = X_i - X_{i-m} \quad (5.1)$$

where X_i is the point obtained at the end of the m univariate steps and X_{i-m} is the starting point. In the next cycle one of the coordinate directions is dropped in favor of the pattern direction. For the minimization cycle, another coordinate direction is discarded in favor of the newly-generated

pattern direction. After all the initial coordinate directions are discarded, they are recovered and the procedure is repeated. This iterative procedure is continued until the desired minimum point is found. A flowchart describing this optimization method is shown in Fig. 5.1.

5.1.2 Rosenbrock's method

This "direct search" method (Wilde and Beightler, 1967) proceeds as follows:

1. A starting point and initial step sizes are chosen and the objective function is evaluated.
2. The first variable X_1 is stepped a distance S_1 parallel to the axis, and the function is evaluated. If the objective function value decreased, the move is termed a "success" and S_1 is increased by multiplying it by a factor γ ($\gamma \geq 1.0$); otherwise the move is termed a "failure" and S_1 is decreased by a factor α ($0 \leq \alpha \leq 1$), and the direction of movement is reversed.
3. Step 2 is repeated for all variables in consecutive sequences until a success and a failure have been encountered in all directions.
4. The axes are then related using the Gram-Schmidt orthogonalization process (Beaumont, 1965).
5. A search is made in each direction of the new coordinate axes.

6. The procedure terminates when the convergence criterion is satisfied.

A flowchart that summarizes all these steps is shown in Fig. 5.2.

5.1.3 Nelder and Meade simplex algorithm

The geometric figure formed by a set of $n + 1$ points in n -dimensional space is called a "simplex". The basic idea in the simplex method is to compare the values of the objective function in the $n + 1$ vertices of a general simplex and move this simplex gradually towards the optimum point during the iterative process (Wilde and Beightler, 1967). The movement of the simplex is achieved by using three operations known as reflection, contraction and expansion. If X_h is the vertex corresponding to the highest objective function value among the vertices of a simplex, we can expect the point X_r obtained by reflecting the point X_h in the opposite face to have the smallest value. If this is the case, then we can construct a new simplex by rejecting the point X_h from the simplex and including the new point X_r . Expansion of the simplex can also be adopted to speed up its movement towards the minimum value. On the other hand, in the case where the simplex rotates around one point, a contraction is carried out. These three operations are repeatedly used until convergence to the minimum occurs. All steps involved in this method are summarized in Fig. 5.3.

5.2 Objective Function

As in any optimization problem an objective function, which includes all parameters to be estimated, needs to be specified. The objective function should include the differences in depth and discharge between observed and simulated downstream hydrographs. The relative errors in stage and discharge at the j^{th} time increment can be defined as: $E_h^j = \frac{H_c^j - H_o^j}{H_o^j}$ and $E_q^j = \frac{Q_c^j - Q_o^j}{Q_o^j}$, where subscripts c and o refer to calculated and observed values respectively. Two broad classes of objective function F_o can be considered (Wormleaton and Narmegam, 1984):

$$\text{Minimize } F_o = \text{Max}(|E_h^j| + |E_q^j|) \quad (5.2)$$

or:

$$\text{Minimize } F_o = \sum_{j=1}^m (E_h^j + E_q^j)^2 \quad (5.3)$$

Equation (5.2) represents the "minimax" criterion, particularly suitable when the error at a critical time is of interest (e.g. where only peak discharge or depth is crucial). The least square criterion, represented by equation (5.3), is adopted in this study because it is considered more important that depth and discharge be modeled correctly over the entire hydrograph.

5.3 Model Parameters

5.3.1 Conceptual Approach

The optimization model parameters, i.e. the parameters to be estimated using optimization, are the roughness coefficients of the compound channel sub-sections and those parameters describing composite section geometry. To reduce model parameters to a manageable number only the main channel sub-section is assumed to be irregular (the flood-plain zones are approximated by rectangular sections). Therefore, assigning subscripts 1, 2 and 3 to the left flood plain, main channel, and right flood plain respectively, two of the model parameters are T_1 and T_3 , where T = sub-section top width.

In describing the main channel geometry the following relationship is assumed:

$$T_2 = cy_2^e \quad (5.4)$$

where y_2 = main channel flow depth and c and the e are constants to be estimated through the optimization exercise. The value of e varies with cross-sectional shape and is equal to zero for the rectangular, unity for the triangular, and takes any value between zero and one for the convex parabolic cross-sectional shapes.

The main channel cross-sectional area and wetted perimeter, which are both required for the flood routing exercise, can then be evaluated. The

cross-sectional area is given as:

$$A_2 = \int_0^{y_2} T dy = \int_0^{y_2} cy^e dy = \frac{c}{e+1} y_2^{e+1} \quad (5.5)$$

and the wetted perimeter is obtained from:

$$P_2 = 2 \int_0^{y_2} \left[1 + \left(\frac{eT}{2y} \right)^2 \right]^{1/2} dy \quad (5.6)$$

In the special case of a rectangular section $e = 0$ and equation (5.5) yields $P_2 = 2y_2$. Therefore, since the wetted perimeter includes the channel floor width, this component should be added the above solution. On the other hand, since both flood plains (sub-sections 1 and 3) are assumed rectangular, their flow areas and wetted perimeters are:

$$A_i = T_i y_i; \quad P_i = T_i + 2y_i; \quad \text{where } i = 1 \text{ or } 3 \quad (5.7)$$

Finally, based on Manning's equation, friction slopes for the three individual sub-sections of the compound channel are written as:

$$S_{fi} = \left[\frac{Q_i n_i}{A_i R_i^{2/3}} \right]^2; \quad \text{where } i = 1, 2, \text{ or } 3 \quad (5.8)$$

This introduces three new parameters to be estimated through optimization, namely n_1 , n_2 , and n_3 .

In summary, seven parameters (n_1 , n_2 , n_3 , T_1 , T_3 , c , and e), describing the hypothetical compound channel cross-section geometry and boundary roughness, are the optimization variables that need to be estimated in the optimization exercise.

5.3.2 'Black Box' Approach

The solution of the unsteady flow equations requires only cross-sectional area and conveyance as functions of flow depth. Thus, instead of following the conceptual approach of optimizing (i.e. upon cross-sectional geometric and hydraulic parameters), optimization can be performed upon abstract parameters in assumed $k(y)$ and $A(y)$ relationships, of the form:

$$A = ay^b \quad (5.9)$$

and

$$K = cy^d \quad (5.10)$$

where A = cross-sectional area of flow, K = channel conveyance, y = flow depth, and a , b , c , and d are real constants. This type of relationships would obviously speed up the computations involved in solving the unsteady flow equations. Furthermore, the relative simplicity of these relationships should lead to decreased computer times and reduced amounts of required computer storage. Finally this approach also avoids the use of empirical relationships, such as Manning's, considered to be valid only under "steady flow" conditions.

Since the proposed unsteady flow model routes floods in compound channels and accounts for flood plain contributions to system conveyance, two sets of equations (of forms 5.9 and 5.10) are required. Once again to reduce the number of coefficients to be estimated the flood plains are assumed to have rectangular cross-sections while the main channel can take

any convex parabolic shape ranging from a triangular to a rectangular cross-section. Thus, each flood plain cross-sectional area (A_p) is written as:

$$A_p = a_p y_p \quad (5.11)$$

where y_p = flood-plain depth and a_p = flood plain width (considered constant). Assuming Manning's equation applies, the flood plain conveyance (K_p) can be written as:

$$K_p = \frac{A_p}{n_p} R_p^{2/3} = \frac{A_p^{5/3}}{n_p W_p^{2/3}} \quad (5.12)$$

where n_p = flood plain Manning's roughness coefficient, R_p = section hydraulic radius and W_p = wetted perimeter. As the flood plain sections are assumed rectangular, the wetted perimeter can be written as:

$$W_p = a_p + y_p \quad (5.13)$$

or:

$$W_p = a_p + 2y_p \quad (5.14)$$

depending on whether one or two flood plain sections apply. Since flood plain depth is generally very much smaller than the section top width flood plain wetted perimeter can be approximated by the section top width (a_p):

$$W_p \approx a_p \quad (5.15)$$

Substituting equations (5.11) and (5.15) into equation (5.12) yields:

$$K_p = \frac{a_p}{n_p} y_p^{5/3} = c_p y_p^{5/3} \quad (5.16)$$

where c_p is another constant to be estimated.

In this study the exponent in equation (5.16) was fixed to keep the number of optimization variables small and thus speed up convergence and reduce computer time. The main channel cross-sectional area and conveyance are written respectively as:

$$A_m = a_m y_m^{b_m} \quad (5.17)$$

$$K_m = c_m y_m^{d_m} \quad (5.18)$$

where 'm' refers to the main channel cross-section and a_m , b_m , c_m , and d_m are four other coefficients to be estimated using optimization techniques. This brings the number of parameters to be estimated to six, namely a_p , c_p , a_m , b_m , c_m , and d_m .

In this second approach to the optimization problem the two flood-plain subsections are combined, i.e. they are represented by one cross-sectional area and one conveyance. Accordingly, our unsteady flow model which divides a composite section into three sub-sections, had to be slightly modified. The composite section continuity and momentum equations are not affected by this change; only the equations for parameters C_q , M_f and F are. The summation in these coefficients is now from $i = 1$ to $i = 2$ only.

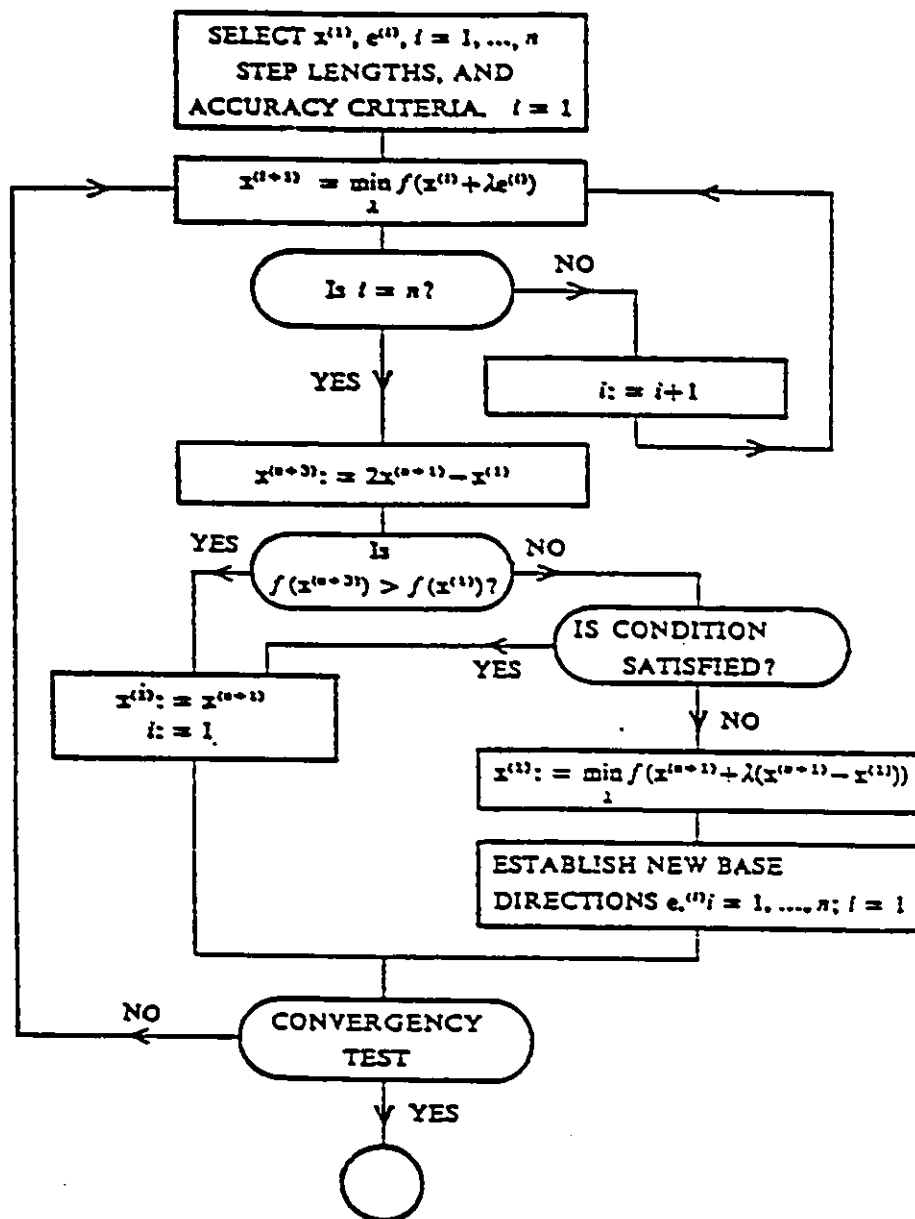


Figure 5.1: Flowchart for Powell's Method.

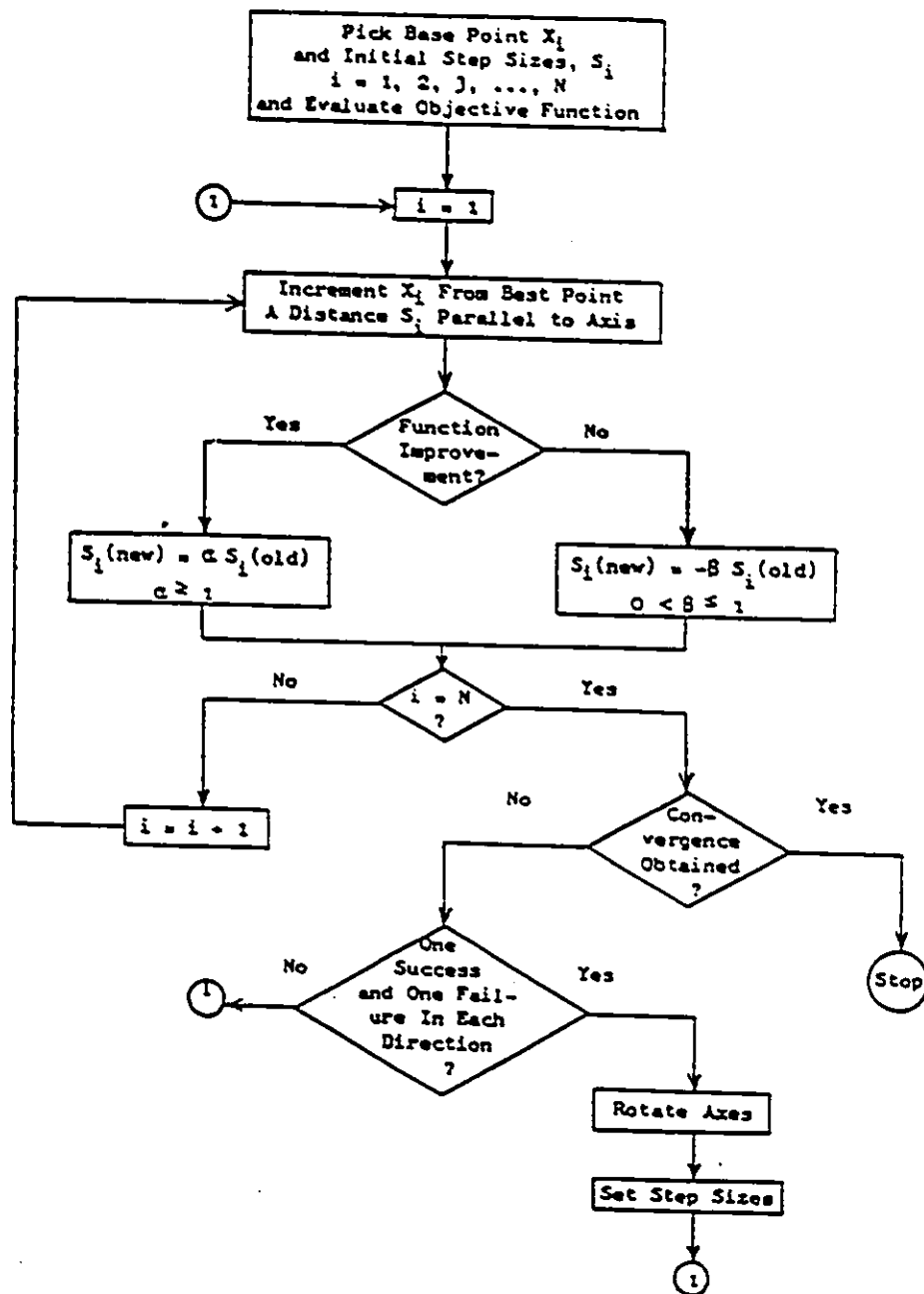


Figure 5.2: Flowchart for Rosenbrock's Method.

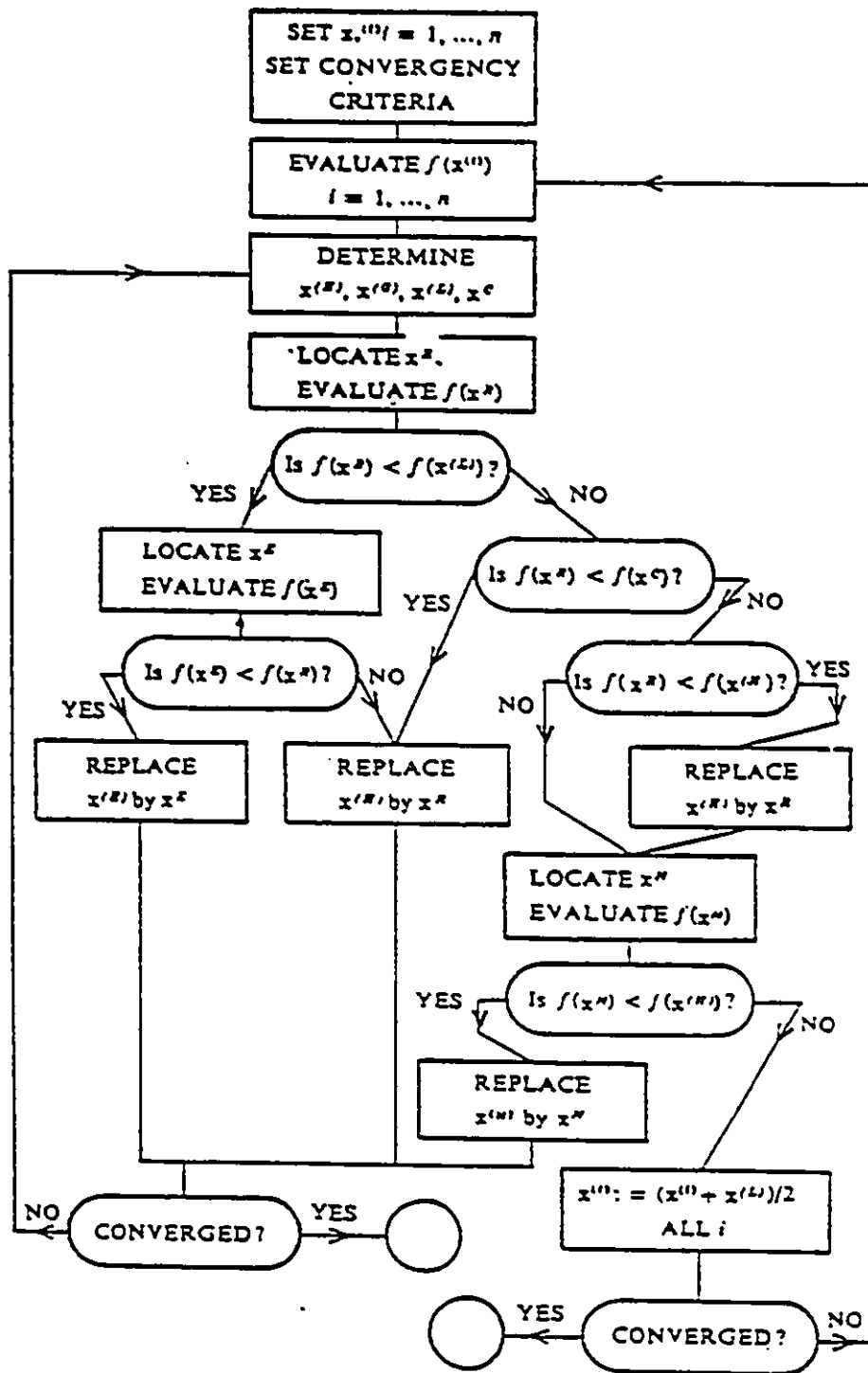


Figure 5.3: Flowchart for Nelder and Meade Simplex Method.

Chapter 6

UNSTEADY FLOW SIMULATIONS AND RESULTS

In this chapter, the performance and reliability of RUFICC in simulating stages and discharges for a wide range of unsteady flow problems is tested. First, the evolution with time of 10 flood waves generated in a prismatic compound laboratory channel were studied. Next, 8 flood events generated in a meandering laboratory channel were investigated. Following this exercise field data from the River Main in Northern Ireland were used to test the accuracy of the unsteady flow simulation. Finally, and to emphasize the importance of accounting for LMT when modeling unsteady flow in

compound channels, a hypothetical channel with overly wide and highly roughened flood plains was also considered.

6.1 Case I: Prismatic Channel Data

6.1.1 Data Used

The laboratory data set used was that of Treske (1980). Treske's experimental compound channel had two flood plains of different widths (Fig. 6.1). The working length of the channel was 210 m, the bed slope was 0.019 % and Manning's n for the channel bed was calculated to be 0.012. Depth and discharge, as functions of time, were measured at two stations that were the upstream and downstream extremities of the 210 m-long working section.

A total of ten model flood events in the prismatic channel were examined (Treske 1980). The characteristics of both inflow and outflow hydrographs for the 10 events are shown in Table 6.1. In this table, and in other subsequent tables for the prismatic channel application, reported stages are relative to the main channel bed elevation at the upstream station, which was 0.092 m. The corresponding maximum flood plain depths appear in Table 6.2, which indicates that events PC1 to PC4 correspond to in-bank flows, with no flood plain flow component. Measured discharge hydrographs were used as the upstream boundary condition in the simulation exercise.

The downstream boundary condition was determined from a preselected empirical procedure for estimating discharge in compound channels.

6.1.2 Results

RUFICC accounts for LMT by using any of the empirical procedures described in chapter III. The resulting peak stages and discharges are shown in Table 6.3. The same table also displays results when LMT was not accounted for in the simulation process. First, it is important to note that peak values of both stage and discharge are the same for flood events PC1 through PC4 no matter what method was used to account for LMT. This result was anticipated, as all of these flood events are in-bank floods and hence LMT between main channel and flood plain does not exist in this case. Nevertheless, Table 6.3 clearly shows that the results obtained using the different LMT procedures are comparable and all in good agreement with the observed data shown in Table 6.1.

Whether LMT was included or not in the analysis did not significantly affect the model outcome in terms of peak stages and discharges. Furthermore, according to Figs. 6.2 through 6.4, which show results for events PC5, PC7 and PC10 respectively, both hydrographs (i.e. with and without LMT) are almost identical. Nevertheless, for flood event PC5, including LMT in the analysis was observed to slightly overestimate stage and underestimate discharge. According to Table 6.1 event PC5 corresponds to

the smallest flood plain depth, the condition at which LMT is most important. This explains the slight difference in simulated stages and flow rates when LMT was accounted for in the analysis.

In order to compare results obtained using RUFICC with those of other conventional methods, simulations were performed using the off-channel storage method (OCSM), the separate channel model (SCM), as well as the apparent roughness model (ARM) proposed by Dracos and Hardegger (1987). The first of these three models assumes flood plain sub-sections to be storage reservoirs. In the second model, discharges for the different sub-sections are computed separately and then summed to yield the composite section discharge. The third model assumes the composite section to be a single unit, but modifies Manning's roughness coefficient to account for the lateral momentum transfer between main channel and flood plains. The three methods are described in more details in chapter II.

Simulated peak stages and discharges, using the aforementioned models as well as RUFICC, for flood events PC5 through PC10, are reported in Table 6.4. Computed discharge and stage hydrographs using RUFICC, OCSM and SCM, for flood events PC5, PC9 and PC10, are displayed in Figs. 6.5, 6.6 and 6.7 respectively. [In applying RUFICC the Prinos and Townsend (1984) empirical procedure was adopted to quantify LMT because this procedure was shown elsewhere to best describe the mechanism (Stephenson and Kolovopoulos 1990).] According to Table 6.4 there is a slight difference in the stages and flow rates obtained using the four dif-

ferent models. Furthermore, according to Figs. 6.5, 6.6 and 6.7, there is no marked difference in the shape of computed hydrographs, whether RUFICC or one of the conventional models was used. This is explained by the fact that the flood plain flow components for all flood events considered are very small. Hence the simulation result would be the same whether the flood plain is assumed to contribute to system conveyance or simply considered as a storage reservoir. It can be concluded therefore that conventional methods yield reasonably good results and can be considered reliable for the case of small flood plain flow components.

Compared to conventional methods, no better results were achieved in applying RUFICC to the prismatic channel data. This is because the flood plain flow component was very small for all events considered, the condition for which application of a conventional method like the OCSM and the SCM is justified. Since, for the case of small flood plain depths, no improvement in the performance of RUFICC could have been made, a different data set, for which the flood plain flow was significant was next selected.

6.2 Case II: Meandering Channel Data

6.2.1 Data Used

The second data set used in this study was also one generated by Treske (1980). Treske's meandering experimental channel had an elliptic main

channel and rectangular flood plains (Fig. 6.8). In the main channel sub-section the top width and the flow depth are related through the relationship $T_2 = 1.826y_2^{0.5}$. The maximum main channel width was 1.0 m and the bank-full depth was 0.3 m. The total flood plain width was 6.0 m while the width of each flood plain sub-section varied between 0.5 and 5.5 m because of the meander. The main channel bed slope was 0.0185 % and the total channel length was 245.56 m. A constant Manning's roughness coefficient of 0.015 was assigned to the whole composite section. The degree of meander was not the same for the entire channel. The channel was divided into two sub-reaches for which the main channel lengths were 113.85 and 131.71 m while the flood plain lengths were 105.0 and 113.0 m respectively. This results in a flood plain to main channel length ratio of 0.92 for the upstream sub-reach and 0.86 for the downstream one.

Treske (1980) studied a total of seventeen flood events in the meandering channel, for which measured stage and discharge hydrographs at an upstream and a downstream station were available. Since the first nine model flood events were all in-bank flows (i.e. they did not have any flood plain flow component), they were excluded from the present analysis. The flow characteristics of both inflow and outflow hydrographs, for the remaining eight flood events considered, are shown in Table 6.5. The corresponding maximum flood plain depths are shown in Table 6.6. According to Table 6.6 the maximum flood plain to main channel depths ratio varied between 8 and 33%. This represents a wide range of flood plain flow conditions that would conceivably arise in practice and also conditions where LMT

effects should not be ignored. It should also be noted at this stage that, for the meandering channel application, all water surface elevations are with respect to the main channel bed elevation, which is assumed to be zero for convenience. Thus, stage values reported in Table 6.5 and subsequent tables are actually flow depths in the main channel sub-section. The corresponding flood plain depths are the difference between these stage readings and the bank-full depth.

6.2.2 Results

As was the case for the prismatic channel application, simulations were performed using the different LMT procedures incorporated in RUFICC. Once again, using the different methods, peak stages and discharges were close to each other and therefore it is unnecessary to report all of these findings.

Simulated downstream peak stages and discharges using RUFICC and the two conventional models (OCSM and SCM) are displayed in Table 6.7. Compared to observed values (Table 6.5) stages and discharges obtained using both RUFICC and SCM are shown to be in a reasonably good agreement. The results obtained using OCSM, however, are unsatisfactory as stages and discharges are grossly underestimated, especially for relatively large flood plain flows. Consider flood event MC5, for instance: according to Table 6.7 simulated discharge using OCSM is $0.210m^3/s$ while the

observed value is $0.314\text{m}^3/\text{s}$, i.e. a difference of more than 30%.

The same observation was also made by Kolovopoulos (1990), who stated: "*The Off-Channel Storage Model underestimated the peak flow rate by 25 % while the effect of the momentum transfer mechanism in the calculated total flood discharge hydrograph was found to be minimal*". The gross underestimation of flood discharge by OCSM, especially for large flood plain flows, is due to the fact that the method neglects the flood plain contribution to system conveyance, which can be quite significant. On the other hand, as stated in earlier chapters, the effect of LMT becomes less significant as the flood plain depth increases, which explains the similarity in peak stages and discharges for high flood plain flows, whether LMT was included or not in the analysis.

Besides comparing simulated and observed peak stages and discharges a proper analysis also requires comparison of the corresponding hydrographs. Computed and observed hydrographs for flood events MC2 and MC3 (small flood plain depths) as well as for flood events MC7 and MCS (relatively large flood plain depths) are presented in Figs. 6.9 through 6.12 respectively. According to these figures the conclusion that OCSM significantly underestimates flow rates is confirmed. The same figures also show that, compared to observed data, the fall of the recession curve of computed hydrographs, using both conventional models, is delayed. On the other hand, the shape of the computed hydrographs using RUFICC is almost identical to that of observed hydrographs, especially for events MC7 and MCS.

where the flood plain flow component is fairly important (Figs. 6.11 and 6.12).

6.3 Case III: River Main Data

6.3.1 Data Used

In this field application a reach from the River Main in Northern Ireland is considered. The following data required for the unsteady flow simulation were provided by W.R.C. Myers of the University of Ulster at Jordanstown, Northern Ireland:

1. cross-sectional details at the upstream end, in the centre, and at the downstream end of the study reach. A typical cross-section of the river is shown in Fig. 6.13;
2. stage-discharge curves for both upstream and downstream stations;
3. time histories of depths at both upstream and downstream stations (measured using depth probes);
4. Manning's roughness coefficient n as a function of flow depth in the main channel and in the flood plain sub-sections (Fig. 6.14).

According to Fig. 6.14, Manning's n was provided separately for the main channel and flood plain sub-sections. This was achieved by taking measurements of velocity traverses that served in computing component discharges of the different sub-sections of the compound flow field. Fig. 6.14 shows an increase in the value of the main channel n for the "just beyond bank-full" condition. This increased flow resistance is due to momentum transfer to the flood plains (LMT), which becomes less significant with increasing flow depth.

The study reach length was 500 m and the average channel bed slope was 0.3 %. The more or less symmetrical compound section (Fig. 6.13) had two flood plains of almost the same width as the main channel. The in-bank depth varied between 0.95 and 1.05 m. The depth time series at the upstream station were used as the upstream boundary condition while the downstream boundary condition was determined from the stage-discharge curve provided. The selected time step and space increment were 1500 s and 400 m respectively. This choice of mesh size satisfies the criterion of Wormleaton and Karmegam (1984) for best accuracy of results when the four-point implicit FD scheme of Amein and Fang (1970) is adopted. This criterion is described in further detail later in this chapter (section 6.6).

6.3.2 Results

Fig. 6.15 shows the results of the unsteady flow simulation. Observed and simulated depths of flow are displayed as functions of time at the downstream end of the study reach. Comparing both time series shows reasonably good agreement between observed and simulated depths. However, it is also evident from the same figure that simulated flow depths are consistently slightly underestimated. This would imply that if different values of Manning's n were adopted a better agreement between observed and simulated stages could be achieved.

Manning's n is a non-physically measurable parameter and a great deal of uncertainty is usually associated with its estimation. For this particular application the values provided for Manning's n were determined experimentally for a uniform flow condition. Fread and Smith (1978) showed that the value of Manning's n depends not only on discharge and flow depth but also on the particular schematization used to describe the continuous channel geometry by a series of discrete representations along the reach of channel being modeled. This leads to the conclusion that n is best evaluated through calibration of the unsteady flow model, especially if reasonably accurate field data from past flood events is available.

The second hydrograph of the time series that occurred approximately at 200 hours from the beginning of the depth measurements was used in the calibration process. A single (average) value of Manning's n was assigned to

both the main channel and the flood plain. The value of Manning's n that resulted in the closest agreement between observed and simulated stages was determined by trial and error to be 0.036. This value was then used instead of the previously supplied relationships for Manning's n (Fig. 6.14) and the unsteady flow simulation was repeated. The results of this second simulation are shown in Fig. 6.16 which, compared to Fig. 6.15, shows a better agreement between observed and simulated depths. This exercise underscores the importance of calibration of unsteady flow models, especially in regards to the determination of Manning's n . Further discussion on this subject is included in chapter VIII.

6.4 Variation of the Model Parameters

Three important parameters are embedded in the proposed unsteady flow equations, developed in chapter III. They are C_q , M_f , and F , defined in equations (3.12), (3.34), and (3.39) respectively. When channel meander is a factor in the analysis, the definition of the parameter F remains unchanged while C_q and M_f are described by equations (3.73) and (3.74) respectively. According to the aforementioned equations, the model parameters are functions of λ (the parameter that accounts for LMT) as well as I [which refers to the proposed interpolating cosine function relating flood plain and main channel friction slopes]. They are also functions of other terms such as conveyance and cross-sectional area of flow.

Since λ , I , A , and K vary with y , it was felt necessary to examine the variation of the model parameters with flow depth and also the effect of including LMT on these parameters. The variation of C_q , F , and M_f with flow depth along the stage hydrograph is shown in Figs. 6.17 and 6.18 for flood events PC10 and MCS respectively. These figures show the variation of model parameters for the case where LMT was included as well as the case where it was neglected in the analysis. Examining first the variation of C_q it can be seen that, for the case where LMT was neglected, C_q takes the value of 1.0 for the prismatic channel (Fig. 6.17) and it is greater than 1.0 (reaching a maximum value at peak stage) for the meandering channel (Fig. 6.18). Including LMT in the analysis resulted in a further increase in the value of this parameter when over-bank flow just occurred. The difference, however, became less significant as the flood plain depth increased. This is consistent with the fact that the LMT effect is strongest when flood plain flow just occurs and then decreases with further increase in flood plain depth.

Examining now the parameter F , it is evident from both figures that it is usually very small (less than 0.06). This result is anticipated as F is the ratio of the summation of areas of the different sub-sections to the square of the summation of their conveyances, (see equation 3.39). The fact that this parameter is very small compared to the other two does not mean that it is less important as it is associated with the term Q^2 in the momentum equation. On the other hand, unlike C_q and M_f , which take the value of 1.0 for in-bank flows, the parameter F varies with flow depth even

though the flow is still confined within the banks. The maximum value for F is observed to occur for the smallest flow depth and then decreases as flow depth increases until it reaches a minimum at the peak and increases again along the recession curve of the hydrograph. For in-bank flow F is the ratio of the main channel cross-sectional area to the square of its conveyance. An increase in flow depth corresponds to an increase in both A and K^2 . However, it is obvious that the latter term increases at a much faster rate than the former, which explains the sharp decrease in F -values. When flood plain flow occurs, the parameter F continues to decrease very gradually with flow depth. It is also important to note that including or neglecting LMT in the analysis did not have any effect on the parameter F .

Finally, examining the parameter M_f shows that it takes the value 1.0 for in-bank flows (which is the value for Boussinesq coefficient) and then increases with increasing flow depth. Accounting for LMT results in a further increase of this parameter at small flood plain depths. The difference, however, becomes less significant as depth on the flood plain increases still further.

6.5 Effect of LMT in Unsteady Flow Modeling

In this study unsteady flow simulations were performed for a variety of flood hydrographs in an experimental prismatic channel, an experimental meandering channel, and finally in a reach of the River Main in Northern Ireland. In all of these applications the lateral momentum transfer was found to have an insignificant effect on the model output in terms of stages and discharges. It is known from the literature that LMT is strongest when there is a large difference between main channel and flood plain roughness coefficients, when depth on the flood plain is small compared to main channel depth, and when the flood plain section is wide compared to the main channel (Wormleaton et al 1982; Prinos and Townsend 1984; Wormleaton and Merrett 1990 among others). In fact, the larger is the flood plain roughness (compared to the main channel roughness), the lower the average flood plain velocity relative to that in the main channel, and therefore the higher the turbulence intensity is at a given flow depth. On the other hand, the wider the flood plain the smaller is the corresponding flow depth and therefore the stronger is the LMT mechanism.

To test the importance of LMT in unsteady flow simulations under the aforementioned conditions, a hypothetical compound channel was considered. A range of hydrographs was routed through the hypothetical channel, with and without LMT in the analysis, and the results were compared. The

channel considered had two flood plains of different widths (Fig. 6.19). A small main channel width was selected to keep the width to depth ratio small because the momentum exchange mechanism becomes insignificant if this ratio exceeds 5 (Kolovopoulos 1990). Manning's n was 0.03 for the main channel and 0.1 for the flood plain, implying a flood plain roughness more than three times that of the main channel. The reach length was 40 000 m and the average channel bed slope was 0.01 %. Initially the flow was uniform with a depth of 6.0 m, (the 'bank-full' depth was 6.2 m). The space increment Δx and the time step Δt were chosen to be 4000 m and 5000 s respectively. A synthetic inflow depth hydrograph was generated at the upstream end of the reach using the following relationship:

$$y = \frac{6.0}{const.} \left((const. + 1.0) - \cos \frac{2\pi t}{10^5} \right) \quad for \quad 0.0 \leq t \leq 10^5 s \quad (6.1)$$

where $const. = 10, 24,$ and 32 for hydrographs number I, II, and III respectively. Their respective peak stages were 7.2, 6.5, and 6.375 m, which shows that hydrograph I corresponds to high flood plain flows while hydrograph III is associated with low flood plain flows.

Three methods accounting for LMT were considered in this investigation. These methods, proposed by Nicollet and Uan (1979), Wormleaton et al (1982), and Prinos and Townsend (1984), are described in chapter V. It must be emphasized, once again, that all three procedures were developed and tested under 'steady state' conditions and for specific experimental flows and geometries. Therefore, using these procedures for the much more complex conditions of unsteady flow does not guarantee success. Nevertheless, it was felt that accounting for LMT in the unsteady flow analysis

through the use of these empirical steady-state procedures [being the only procedures presently available] is better than neglecting the mechanism altogether.

Simulated discharge and stage hydrographs, using the three empirical LMT procedures, are shown in Figs. 6.20 to 6.22 for the case of high, moderate, and low flood plain flows respectively. For comparison purposes simulated results obtained with excluding LMT effect, are also displayed in these figures. According to the figures, when flood plain flow first occurred, discharge and stage were maintained constant over an interval of time. This is due to the overly-wide flood plains which resulted in very small flood plain depths, that increased very slowly with time. The combined effect of small flood plain depths and large boundary roughness ($n = 0.1$) also resulted in very small discharges at the "just beyond bank-full" condition. Figs. 6.20 to 6.22 also show a very small difference between simulated stage hydrographs with and without LMT in the analysis. On the other hand, discharge hydrographs were attenuated when LMT was accounted for, especially for low flood plain flows (Fig. 6.22).

Comparing the performances of the different LMT procedures it can be seen that, in general, underestimation of discharge is greater using the Wormleaton et al (1982) method than using the Prinos and Townsend (1984) method. When the latter method was used the LMT effect was observed to be insignificant for hydrographs I and II, which correspond to high and moderate flood plain flow components respectively (Figs. 6.20

and 6.21). This result was anticipated as the significance of LMT decreases with increasing flood plain depth. Examining again Figs. 6.20 and 6.21 it can be seen that Nicollet and Uan (1979) empirical method did not perform well as it grossly underestimated discharge. On the other hand, while the other two methods also underestimate discharge for low flood plain flows [which is consistent with the observation that LMT is more significant for low flood plain flows] the opposite phenomenon was observed using Nicollet and Uan method. With this particular method the largest difference between simulated discharges (with and without LMT) occurred for the highest flood plain flow condition (Fig. 6.20). These results suggest that the use of Nicollet and Uan method to account for LMT in unsteady flow modeling is questionable. Their method is very simplistic as it quantifies LMT through an empirical relationship giving the main channel to its equivalent isolated discharge ratio as a function only of the ratio of flood plain roughness to that of the main channel (see chapter III). Consequently it neglects the effect of flood plain depth and width, both of which were shown to play important roles in quantifying LMT (Wormleaton et al 1982; Prinos and Townsend 1984 among others).

It can be concluded therefore that the presence of strong LMT effects generally results in attenuation of discharge hydrographs. However, no quantitative conclusion can be made as to the magnitude of this attenuation because of the lack of appropriate field and/or laboratory data against which simulation results can be compared. It must be emphasized again that LMT was found to be important only for this case of a hypothetical

channel with overly rough and wide flood plains. In most of the other applications treated in this chapter its effect was found to be insignificant. LMT can therefore be neglected for most applications and its small effect, if any, can be compensated for through the use of a marginally increased main channel roughness coefficient. In fact, since LMT acts to retard main channel velocity (and hence discharge) an increase in Manning's roughness coefficient, which is best obtained through calibration of the unsteady flow model, is the obvious solution to the problem.

6.6 Convergence and Stability

Flood routing is a numerical process in which the solution is highly dependent upon the choice of the space increment Δx and the time step Δt . These two variables have to be selected in a way to achieve convergence and stability of the numerical solution. Convergence is defined as the tendency to approach the exact solution as the computational grid is refined. [i.e. as Δx and Δt are both reduced]. This implies that a given scheme is convergent if an increasingly finer computational grid leads to a more accurate approximation of the unique solution. On the other hand, a numerical scheme is said to be stable if a small disturbance numerically introduced does not grow as the solution progresses in space and time.

Compared to explicit schemes and to the method of characteristics, the choice of Δt in the implicit method is not as restricted. Implicit schemes are

unconditionally stable (Price 1974), yet this does not guarantee convergence to a solution for any mesh size. Δt and Δx have to satisfy certain criteria to achieve convergence and stability of the numerical solution. Liggett and Woolhiser (1967) reported that if they increased the Δx to Δt ratio beyond that would be allowed for in an explicit FD scheme (Courant criterion), inaccuracy resulted and stability problems would sometimes occur. Therefore, they recommended the use of the Courant criterion as a good indicator when selecting the simulation time step, even though implicit schemes are theoretically not limited by such a stability criterion.

Williams (1969) stated that if the reach lengths are selected such that the travel time of the flow through the reach is kept smaller than one fifth of the time to peak of the hydrograph, the error induced is minor. Dealing specifically with the four-point implicit scheme of Amein and Fang (1970), which was adopted in this study, Price (1974) showed that the following criterion between the time step and space increment should be satisfied to minimize computational errors in the finite difference solution of the St. Venant equations:

$$\frac{\Delta x}{\Delta t} = V_w \quad (6.2)$$

where V_w = flood peak velocity. Later, to account for the variation of wave speed with flow depth in a natural river, Wormleaton and Karmegam (1984) showed that deviations from the aforementioned criterion should be such that $\Delta x/\Delta t \geq V_w$ to minimize finite difference error.

To test the validity of this latter criterion using RUFICC, 18 numer-

ical experiments with different mesh sizes were performed for the second flood event routed through the meandering channel (MC2). The results obtained are presented in Table 6.8, which shows the time step Δt , the space increment Δx , the time step to travel time ratio $\Delta t/T_r$, and the ratios of simulated to observed discharge and stage, Q_s/Q_o and H_s/H_o , respectively.

The flood peak velocity for flood event MC2 was determined to be 0.24 m/s. Comparing this value with $\Delta x/\Delta t$ ratios shown in Table 6.8 it can be seen that for runs 13-16, which do not satisfy the criterion suggested by Wormleaton and Karmegam (1984), the error in peak discharge is maximum. For other numerical experiments (5, 8, 9, and 12), even though the aforementioned criterion ($\Delta x/\Delta t V_w$) is satisfied, instability problems caused the termination of the simulation and no solution was obtained. Therefore, it can be concluded that the criterion suggested by Wormleaton and Karmegam yields accurate solutions but does not guarantee reaching one.

Numerical instability and program interruption was usually preceded by a sudden increase in mass conservation error, which shows the importance of continuity imbalance as a numerical criterion. In fact, the conservation of mass between the boundaries of a channel system has been accepted by many investigators as a good indicator of the accuracy of unsteady flow computational models (Gunaratnam and Perkins 1970; Theurer 1975; Wylie 1980 and others).

Table 6.8 also shows that, compared to simulated peak discharges, peak

stage values were less sensitive to changes in Δt and Δx . On the other hand, for each Δt chosen there is a particular Δx at which instability occurs. Instability and program interruption also occur if Δx is increased beyond this value. It is also evident from Table 6.8 that the larger Δt is the smaller is the corresponding critical Δx associated with program termination. The same table also shows that as Δx approaches the critical value that causes instability, the error in peak discharge increases. Consider, for instance, the case where $\Delta t = 20s$. Q_p/Q_o decreased from 1.009 to 0.979 when Δx was increased from 41.0 to 49.0 m. Eventually, instability and program interruption occurred when Δx was increased to 61.0 m. On the other hand, for a fixed Δt , if Δx is very small a result is obtained but the error in peak discharge becomes larger. This would suggest that there is an optimum value for Δx for which convergence to a solution is achieved and the error induced in peak discharge is minimum.

Based on this analysis the following concluding remarks can be made:

1. Δx and Δt should be chosen in a way to minimize conservation of mass error;
2. As recommended by Wormleaton and Karmegam (1984), $\Delta x/\Delta t$ should always be greater than the flood wave velocity to ensure accurate results;
3. for each selected time step there is an optimum Δx for which a most accurate solution is achieved:

4. the larger the time step the smaller the critical space increment at which instability and program interruption occurs:
5. the error induced in peak stage and discharge was generally small for all cases where convergence to a solution was achieved. Therefore, for a given choice of mesh size the model either converges to a solution with reasonable accuracy or, due to numerical instability, the computation process terminates before reaching a solution.

Table 6.1: Observed Flow Characteristics of Inflow and Outflow Flood Hydrographs for the Prismatic Channel.

Flood Event No.	Inflow Hydrograph				Outflow Hydrograph			
	t_p (min)	H_p (m)	t_p (min)	Q_p (m ³ /s)	t_p (min)	H_p (m)	t_p (min)	Q_p (m ³ /s)
PC1	14.0	0.388	13.0	0.186	15.0	0.351	15.0	0.158
PC2	24.0	0.408	21.0	0.190	24.0	0.368	24.0	0.171
PC3	39.0	0.425	35.0	0.192	38.0	0.383	38.0	0.183
PC4	72.0	0.435	68.0	0.197	72.0	0.392	70.0	0.190
PC5	16.0	0.501	14.0	0.387	19.0	0.458	19.0	0.255
PC6	24.0	0.528	21.0	0.397	27.0	0.481	27.0	0.291
PC7	33.0	0.541	29.0	0.403	35.0	0.495	35.0	0.315
PC8	43.0	0.555	38.0	0.409	43.0	0.512	43.0	0.344
PC9	75.0	0.573	68.0	0.422	75.0	0.533	75.0	0.380
PC10	102.0	0.580	99.0	0.412	102.0	0.540	102.0	0.394

Table 6.2: Maximum Flood Plain Flow Depths for the Flood Events Observed in the Prismatic Channel.

Flood Event	Inflow Hydrograph		Outflow Hydrograph	
	$y_1(m)$	$y_1/y_2(\%)$	$y_1(m)$	$y_1/y_2(\%)$
PC1	0.0	0.0	0.0	0.0
PC2	0.0	0.0	0.0	0.0
PC3	0.0	0.0	0.0	0.0
PC4	0.0	0.0	0.0	0.0
PC5	0.019	5.0	0.016	4.0
PC6	0.046	11.0	0.039	9.0
PC7	0.059	13.0	0.053	12.0
PC8	0.073	16.0	0.070	15.0
PC9	0.091	19.0	0.091	19.0
PC10	0.098	20.0	0.098	20.0

Table 6.3: Observed versus Simulated Stages and Discharges for the Case of Prismatic Channel [using RUFICC with Different LMT Methods].

Flood Event No.	Observed		LMT Method Used									
	Data		Nicollet and Uan		Wormleaton et al		Prinos and Townsend		Myers and Brennan		No LMT	
	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$
PC1	0.158	0.351	0.156	0.353	0.156	0.353	0.156	0.353	0.156	0.353	0.156	0.353
PC2	0.171	0.368	0.171	0.373	0.171	0.373	0.171	0.373	0.171	0.373	0.171	0.373
PC3	0.183	0.383	0.185	0.391	0.185	0.391	0.185	0.391	0.185	0.391	0.185	0.391
PC4	0.190	0.392	0.194	0.403	0.194	0.403	0.194	0.403	0.194	0.403	0.194	0.403
PC5	0.255	0.458	0.258	0.469	0.253	0.472	0.260	0.472	0.259	0.469	0.264	0.468
PC6	0.291	0.481	0.291	0.490	0.295	0.494	0.299	0.491	0.297	0.489	0.300	0.487
PC7	0.315	0.495	0.315	0.503	0.324	0.506	0.325	0.502	0.322	0.501	0.324	0.497
PC8	0.344	0.512	0.340	0.515	0.348	0.516	0.349	0.512	0.346	0.511	0.347	0.507
PC9	0.380	0.533	0.390	0.535	0.394	0.532	0.393	0.528	0.390	0.528	0.390	0.523
PC10	0.394	0.540	0.392	0.536	0.399	0.534	0.398	0.530	0.397	0.531	0.396	0.526

Table 6.4: Observed versus Simulated Stages and Discharges Using Different Routing Models (Prismatic Channel).

Flood Event No.	Observed Data		Conventional Models				Improved Models			
			OCSM		SCM		ARM		RUFICC	
	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$
PC5	0.255	0.458	0.261	0.466	0.263	0.468	0.274	0.474	0.260	0.472
PC6	0.201	0.481	0.286	0.487	0.299	0.486	0.300	0.486	0.299	0.491
PC7	0.315	0.495	0.313	0.497	0.320	0.500	0.324	0.497	0.325	0.502
PC8	0.344	0.512	0.337	0.507	0.343	0.510	0.346	0.507	0.349	0.512
PC9	0.380	0.533	385	0.526	0.395	0.530	0.390	0.523	0.393	0.528
PC10	0.394	0.540	0.404	0.528	0.412	0.531	0.394	0.525	0.398	0.530

Table 6.5: Observed Flow Characteristics of Inflow and Outflow Flood Hydrographs for the Meandering Channel.

Flood Event No.	Inflow Hydrograph				Outflow Hydrograph			
	T_p	H_p	T_p	Q_p	T_p	H_p	T_p	Q_p
	(min)	(m)	(min)	(m ³ /s)	(min)	(m)	(min)	(m ³ /s)
MC1	40.0	0.354	38.0	0.148	54.0	0.327	54.0	0.098
MC2	66.0	0.358	66.0	0.147	78.0	0.342	78.0	0.121
MC3	110.0	0.367	104.0	0.150	112.0	0.352	112.0	0.140
MC4	18.0	0.404	18.0	0.435	24.0	0.375	24.0	0.193
MC5	27.0	0.426	28.0	0.428	31.0	0.416	31.0	0.314
MC6	41.0	0.444	39.0	0.457	44.0	0.436	44.0	0.385
MC7	56.0	0.453	54.0	0.455	58.0	0.443	58.0	0.412
MC8	102.0	0.461	100.0	0.459	104.0	0.448	104.0	0.431

Table 6.6: Maximum Flood Plain Flow Depths for the Flood Events Observed in the Meandering Channel.

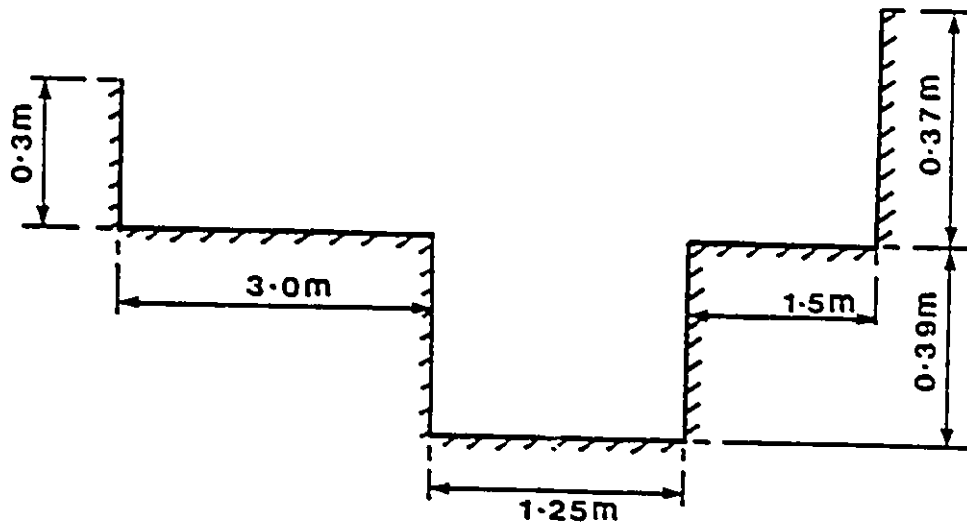
Flood Event	Inflow Hydrograph		Outflow Hydrograph	
	$y_1(m)$	$y_1/y_2(\%)$	$y_1(m)$	$y_1/y_2(\%)$
MC1	0.054	15.0	0.027	8.0
MC2	0.058	16.0	0.042	12.0
MC3	0.067	18.0	0.052	15.0
MC4	0.104	26.0	0.075	20.0
MC5	0.126	30.0	0.116	28.0
MC6	0.144	32.0	0.136	31.0
MC7	0.153	34.0	0.143	32.0
MC8	0.161	35.0	0.148	33.0

Table 6.7: Observed versus Simulated Stages and Discharges Using Different Routing Models (Meandering Channel).

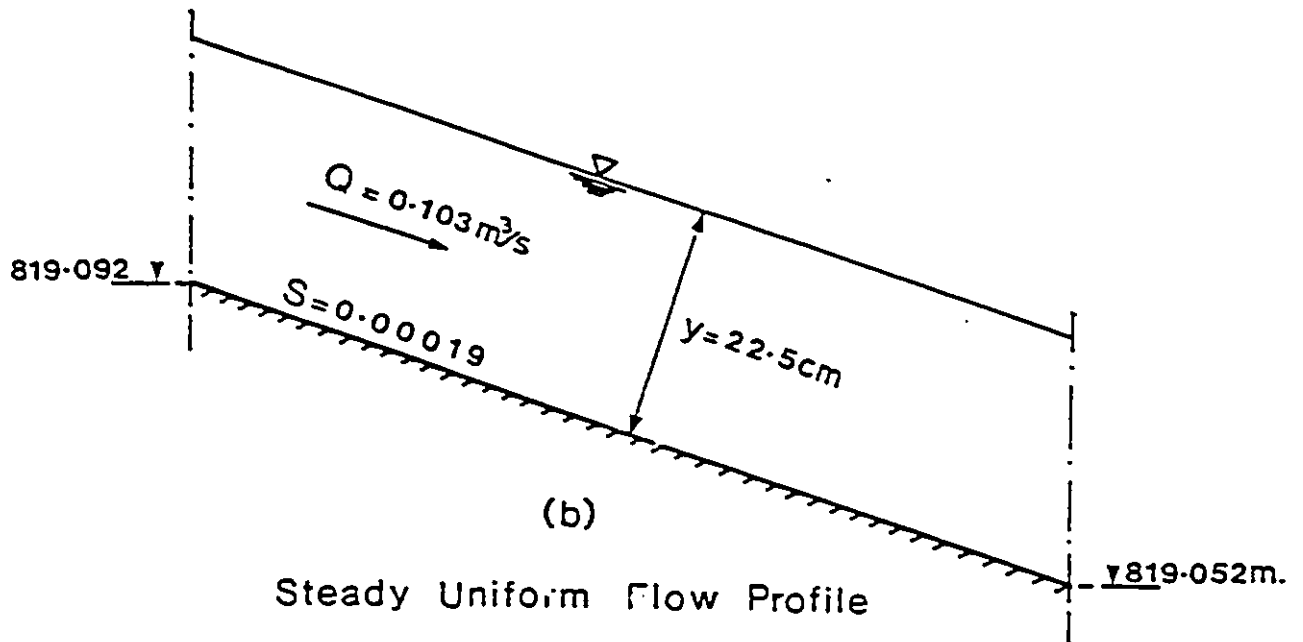
Flood Event No.	Observed Data		Conventional Models				Proposed Model	
			OCSM		SCM		RUFICC	
			$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$		
MC1	0.098	0.327	0.094	0.331	0.102	0.336	0.104	0.340
MC2	0.121	0.342	0.108	0.339	0.116	0.344	0.119	0.346
MC3	0.140	0.352	0.131	0.353	0.142	0.358	0.137	0.356
MC4	0.193	0.375	0.161	0.365	0.192	0.381	0.187	0.379
MC5	0.314	0.416	0.210	0.388	0.271	0.411	0.282	0.415
MC6	0.385	0.436	0.299	0.421	0.361	0.439	0.371	0.444
MC7	0.412	0.443	0.346	0.434	0.394	0.449	0.403	0.452
MC8	0.431	0.448	0.439	0.463	0.452	0.467	0.440	0.464

Table 6.8: Numerical Tests Using Different Time Steps and Space Increments (Flood Event MC2).

Run	$\Delta t(s)$	$\Delta x(m)$	$\frac{\Delta t}{T_c}$	$\frac{\Delta x}{\Delta t}$	$\frac{Q_s}{Q_o}$	$\frac{H_s}{H_o}$
1	20.0	22.5	0.31	1.13	0.995	1.012
2	20.0	34.0	0.19	1.70	1.002	1.015
3	20.0	41.0	0.17	2.05	1.009	1.015
4	20.0	49.0	0.14	2.49	0.979	1.009
5	20.0	61.0	0.11	3.05	*	*
6	40.0	22.5	0.62	0.56	0.994	1.012
7	40.0	28.4	0.50	0.71	0.979	1.009
8	40.0	30.7	0.44	0.77	*	*
9	40.0	34.0	0.39	0.85	*	*
10	60.0	5.20	4.00	0.09	0.979	1.009
11	60.0	22.5	0.92	0.38	0.994	1.012
12	60.0	28.4	0.75	0.47	*	*
13	120.0	5.20	8.00	0.04	0.978	1.009
14	120.0	12.3	3.40	0.10	0.978	1.009
15	120.0	13.5	3.00	0.11	0.978	1.009
16	120.0	18.9	2.20	0.16	0.990	1.012
17	120.0	22.5	1.85	0.19	0.993	1.012
18	120.0	28.4	1.50	0.24	*	*



(a)
Cross-Section



(b)
Steady Uniform Flow Profile

FIG. 6.1 Definition Sketch of Treske's Prismatic Channel.

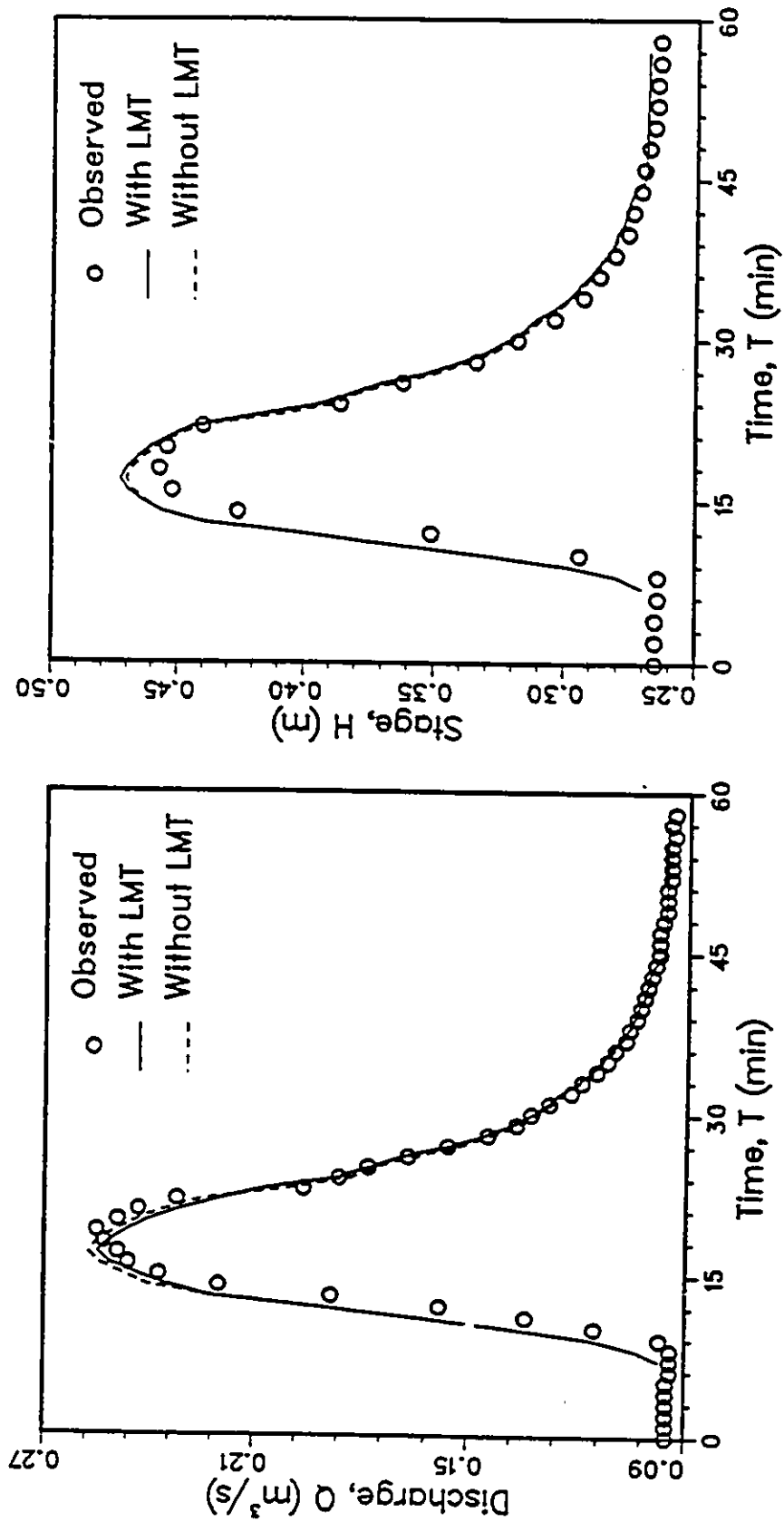


FIG. 6.2 Effect of LMT on Simulated Discharge and Stage Hydrographs, Prismatic Channel, Event PC5.

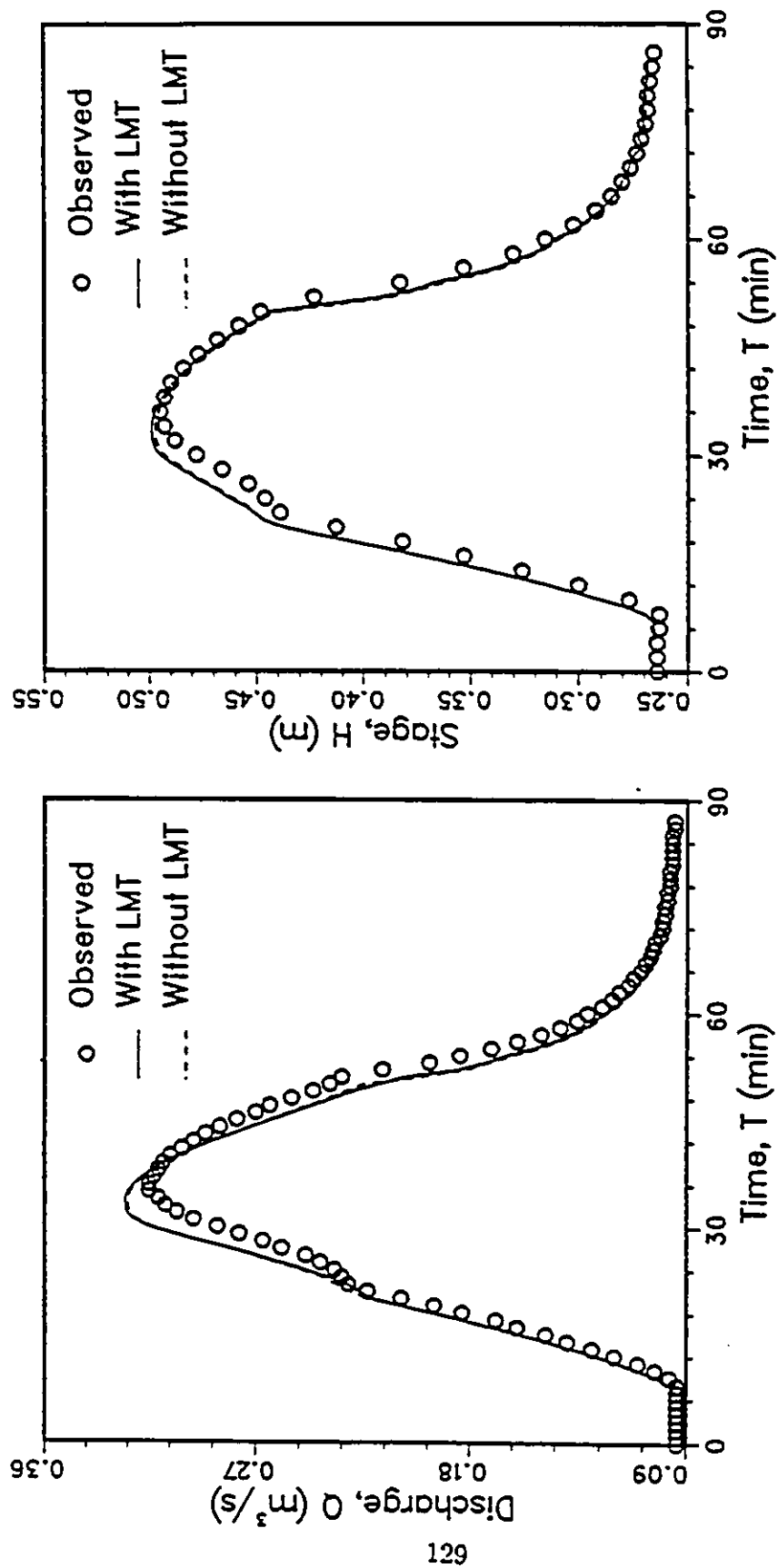


FIG. 6.3 Effect of LMT on Simulated Discharge and Stage Hydrographs, Prismatic Channel, Event PC7.

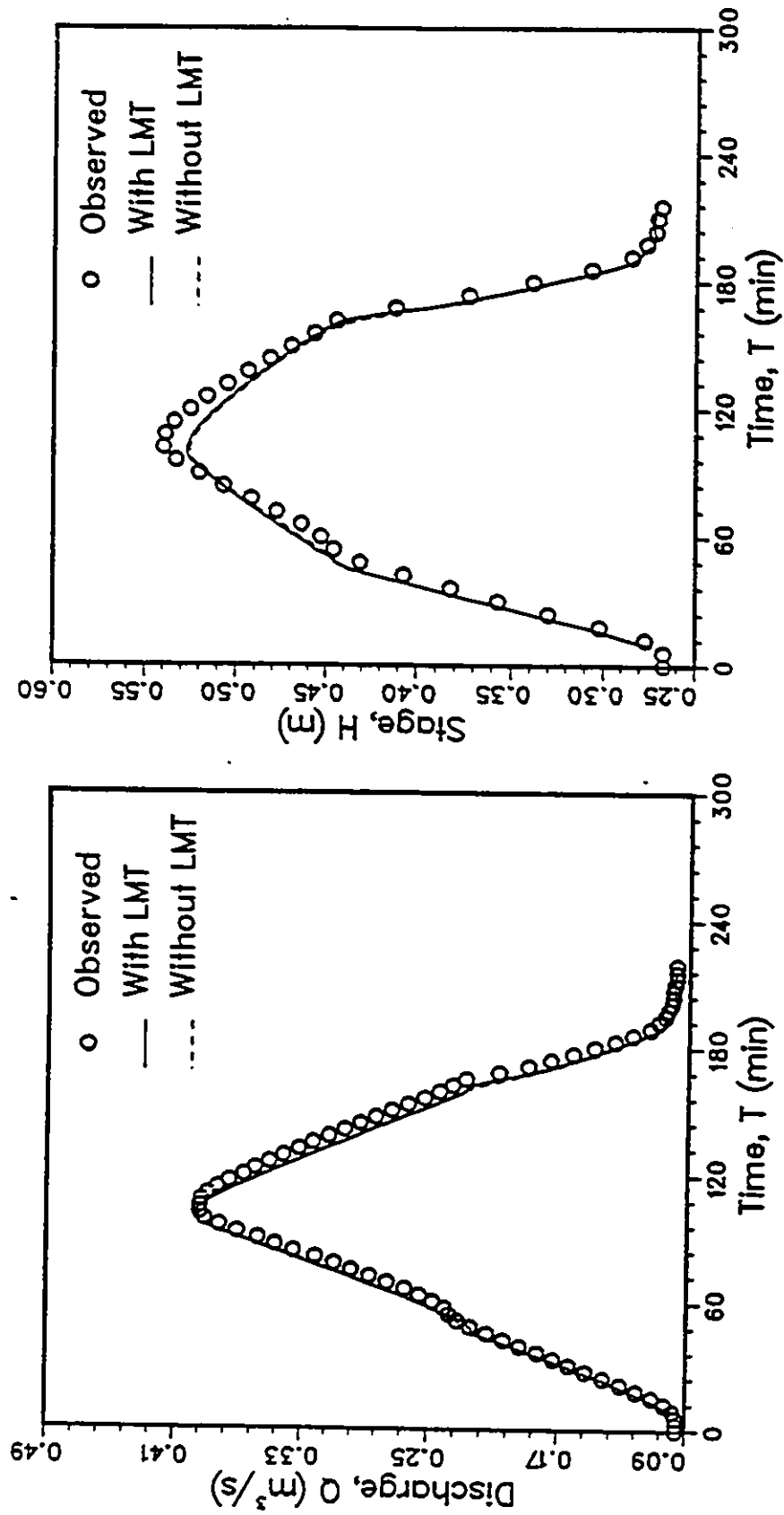


FIG. 6.4 Effect of LMT on Simulated Discharge and Stage Hydrographs, Prismatic Channel, Event PC10.

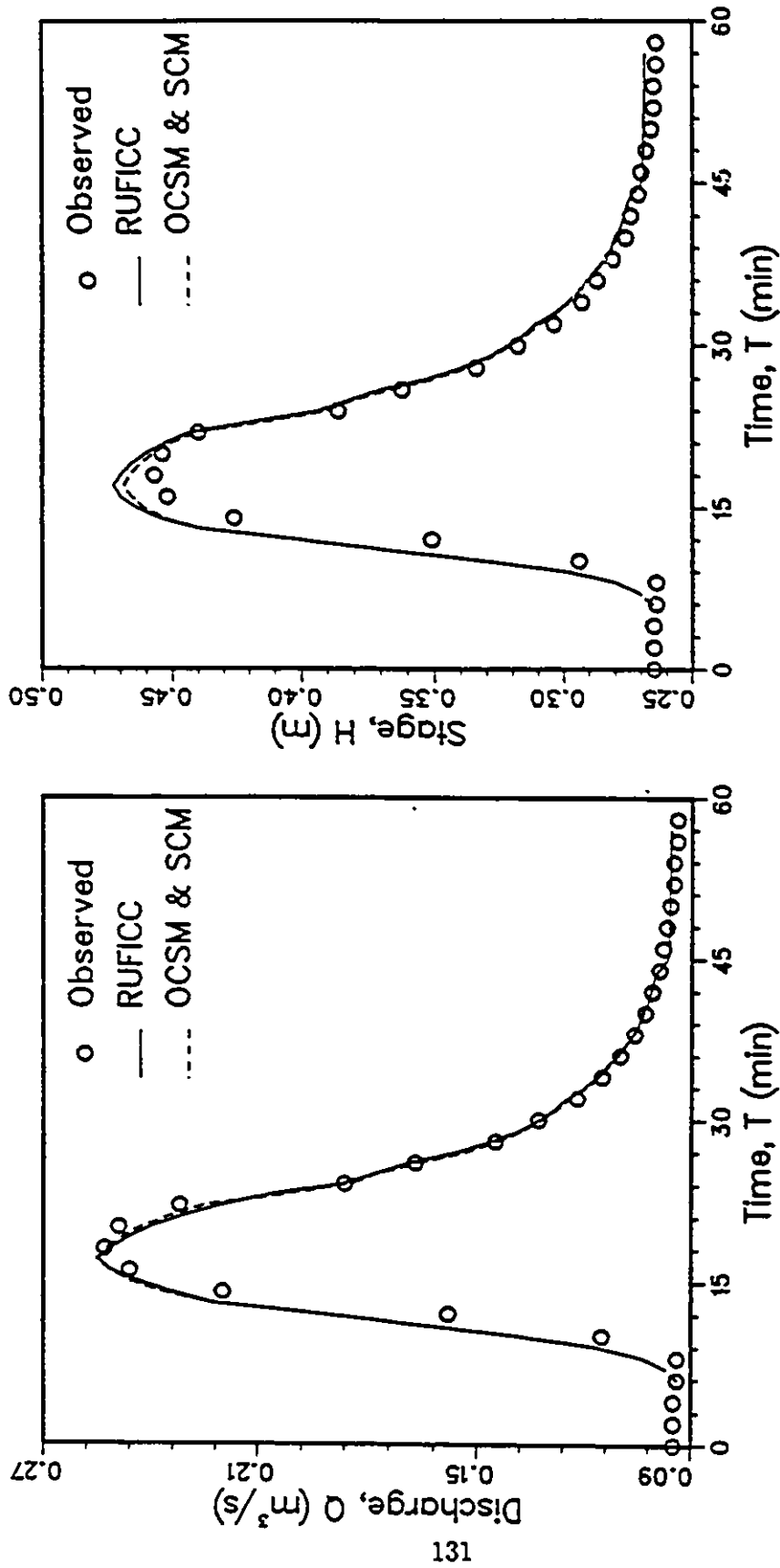


FIG. 6.5 Simulated versus Observed Discharge and Stage Hydrographs, Prismatic Channel, Event PC5.

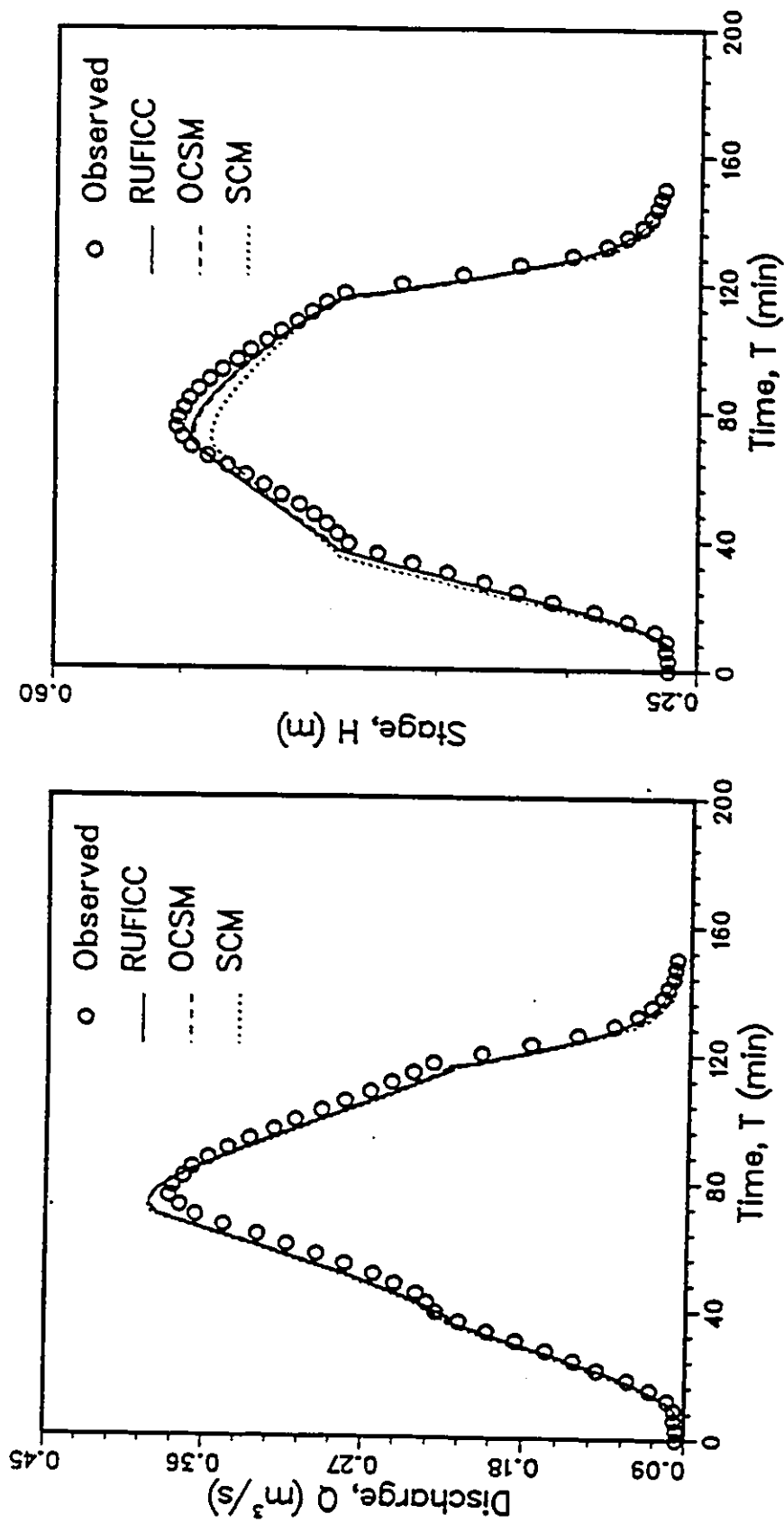


FIG. 6.6 Simulated versus Observed Discharge and Stage Hydrographs, Prismatic Channel, Event PC9.

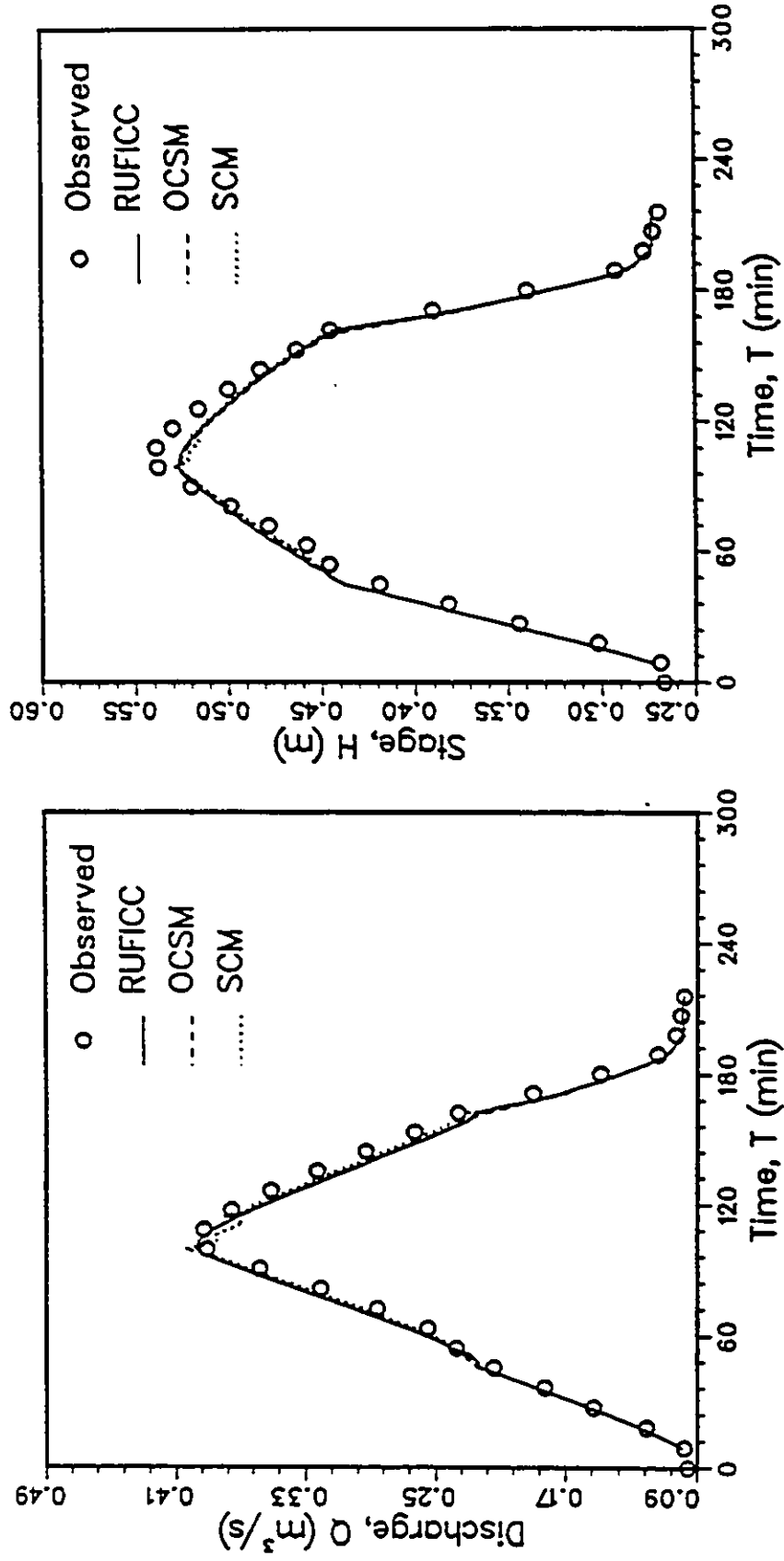
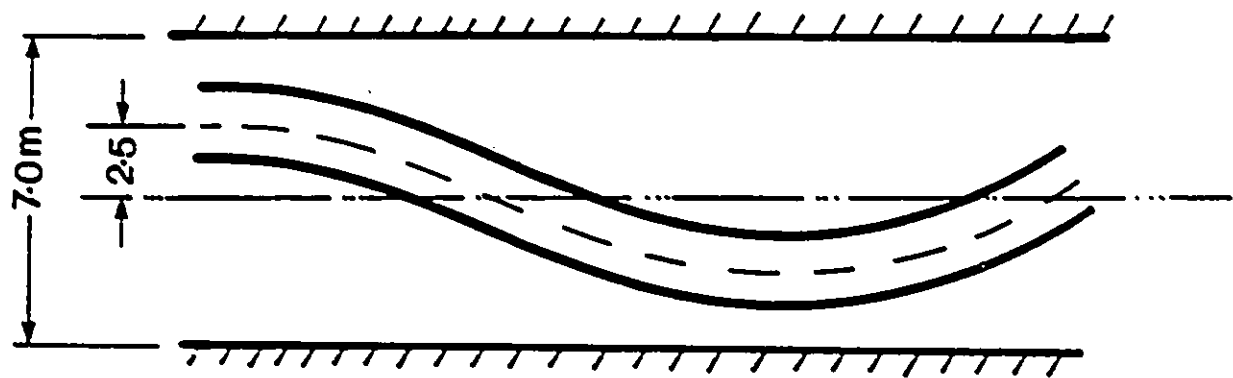
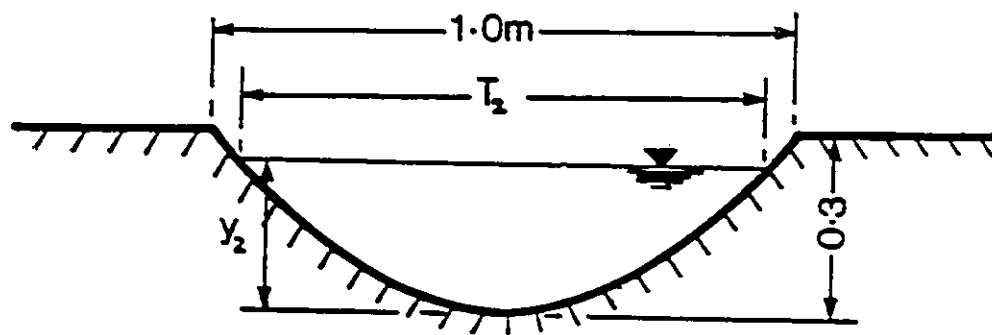


FIG. 6.7 Simulated versus Observed Discharge and Stage Hydrographs, Prismatic Channel, Event PC10.



PLAN VIEW



CROSS-SECTION

FIG. 6.8 Definition Sketch of Treske's Meandering Channel.

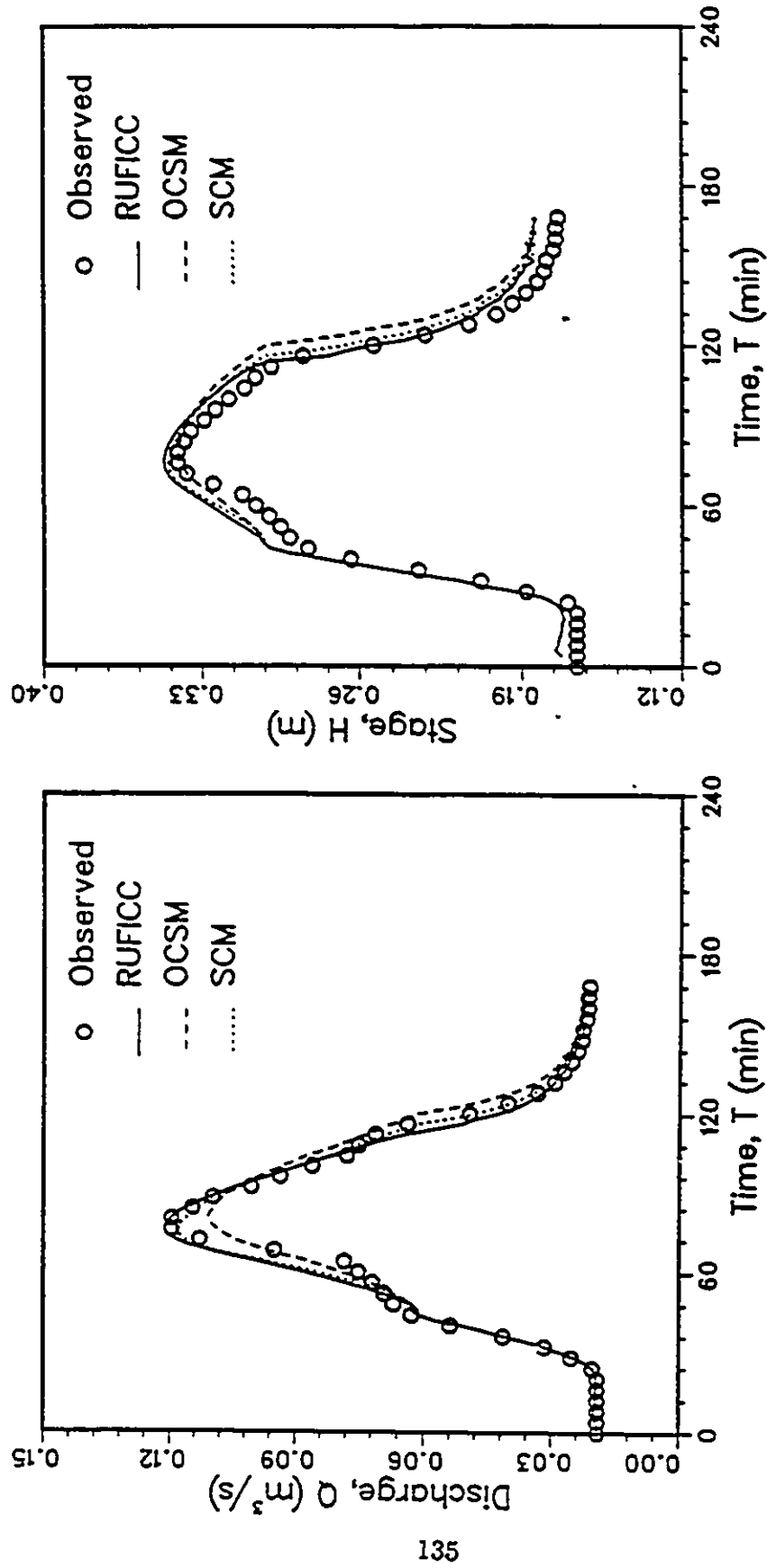


FIG. 6.9 Simulated versus Observed Discharge and Stage Hydrographs, Meandering Channel, Flood Event MC2.

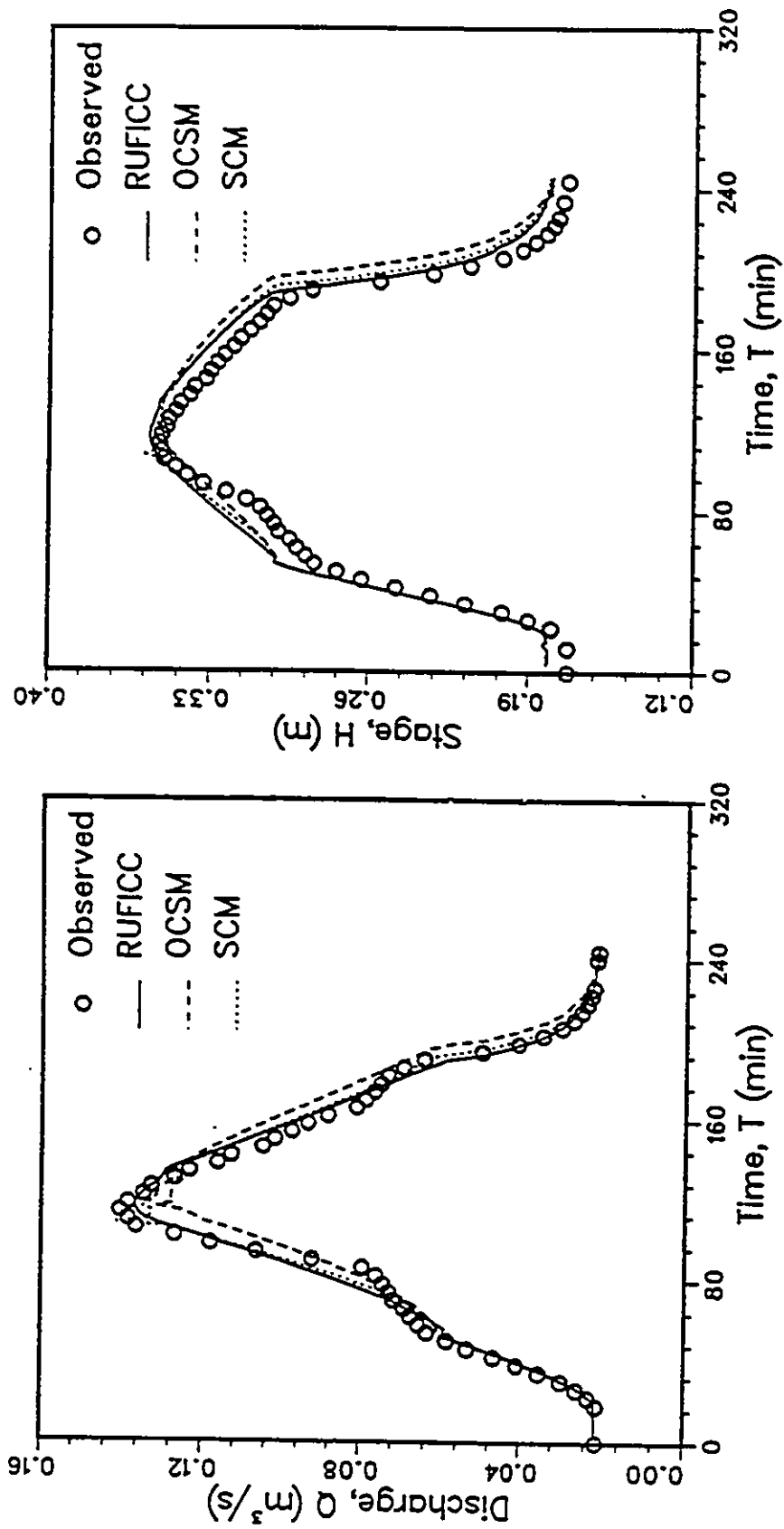


FIG. 6.10 Simulated versus Observed Discharge and Stage Hydrographs, Meandering Channel, Flood Event MC3.

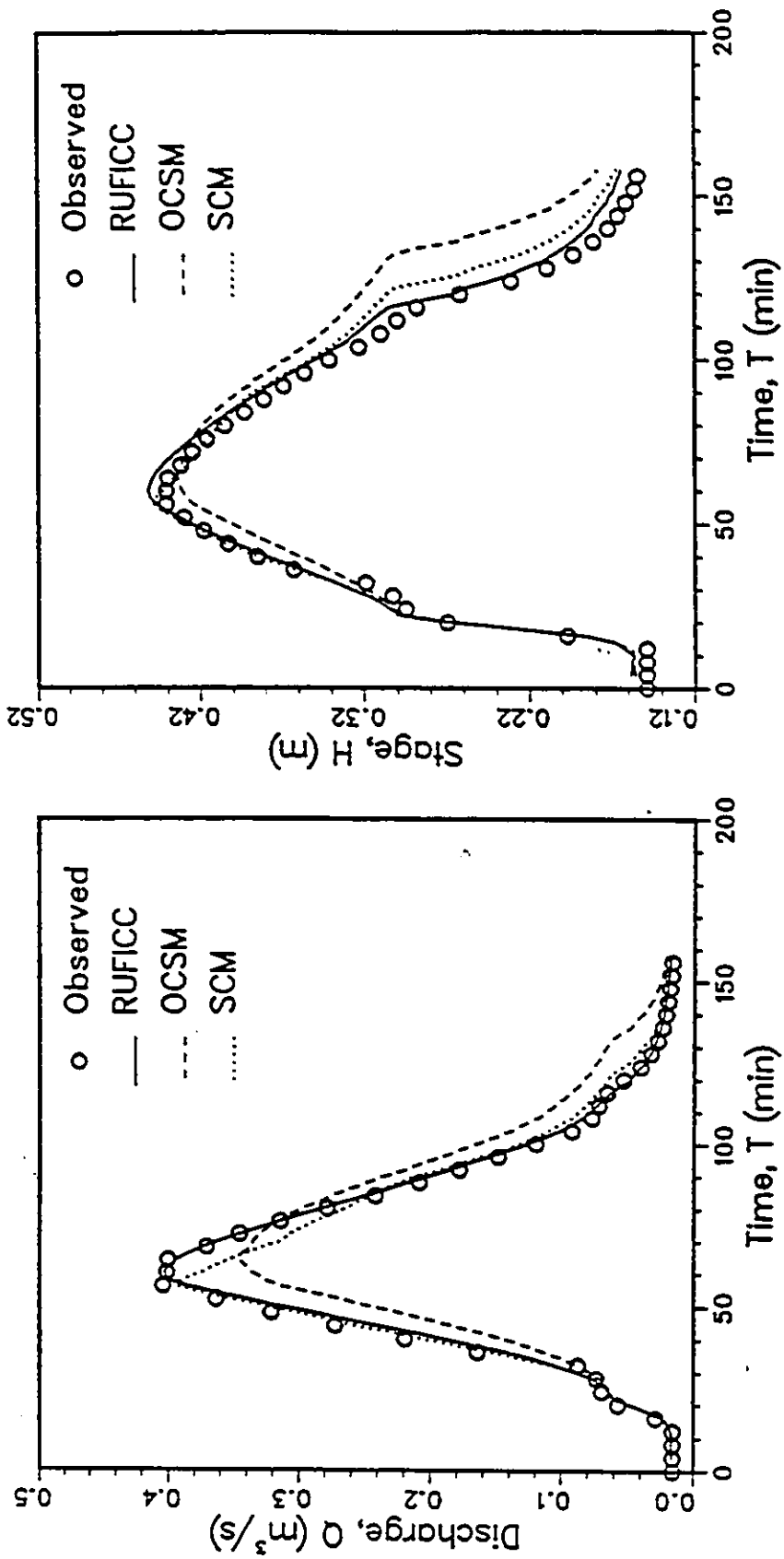


FIG. 6.11 Simulated versus Observed Discharge and Stage Hydrographs, Meandering Channel, Flood Event MC7.

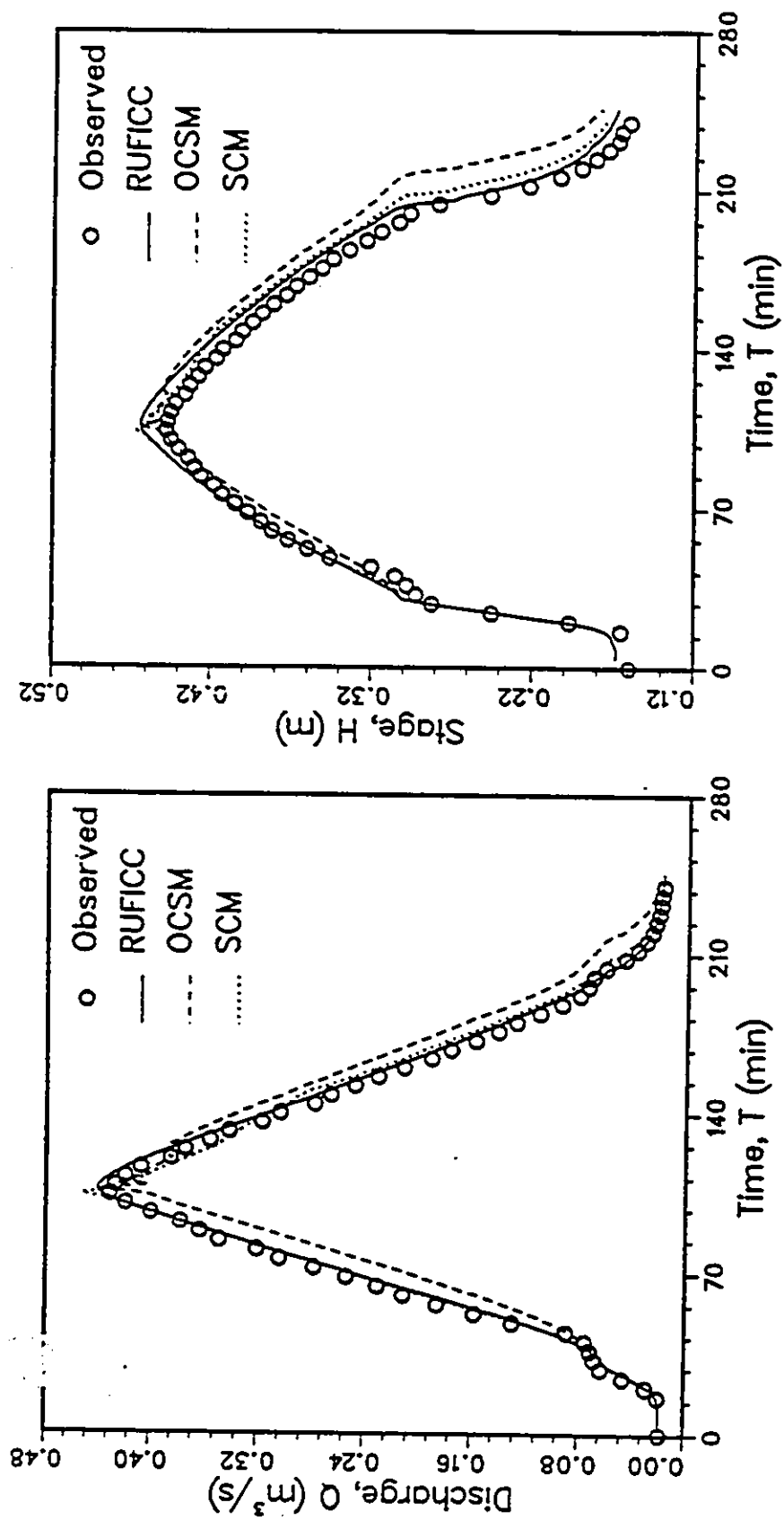


FIG. 6.12 Simulated versus Observed Discharge and Stage Hydrographs, Meandering Channel, Flood Event MC8.

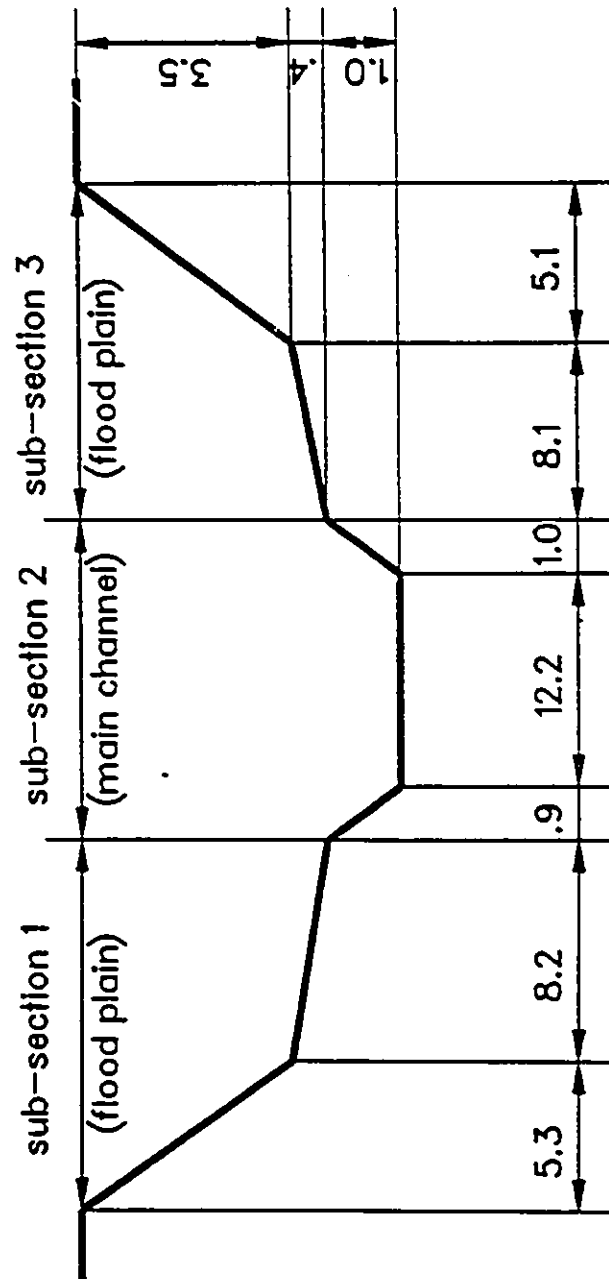


FIG. 6.13 Upstream Cross-Section of the River Main Study Reach, Northern Ireland
(not to Scale, all Dimensions in Meters).

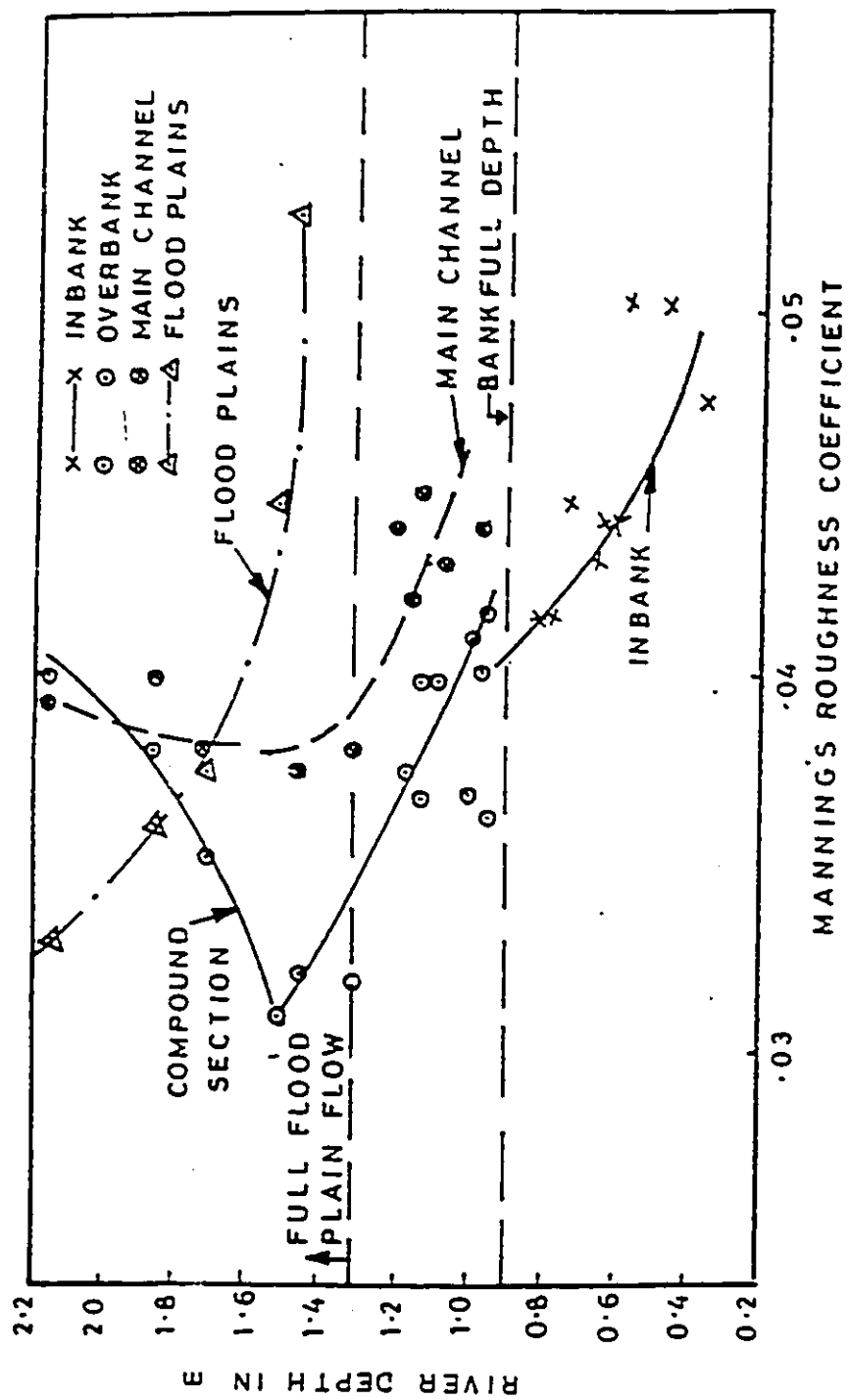


FIG. 6.14 Variation of Manning's Roughness Coefficient with Flow Depth, River Main Study Reach, (after Myers 1991).

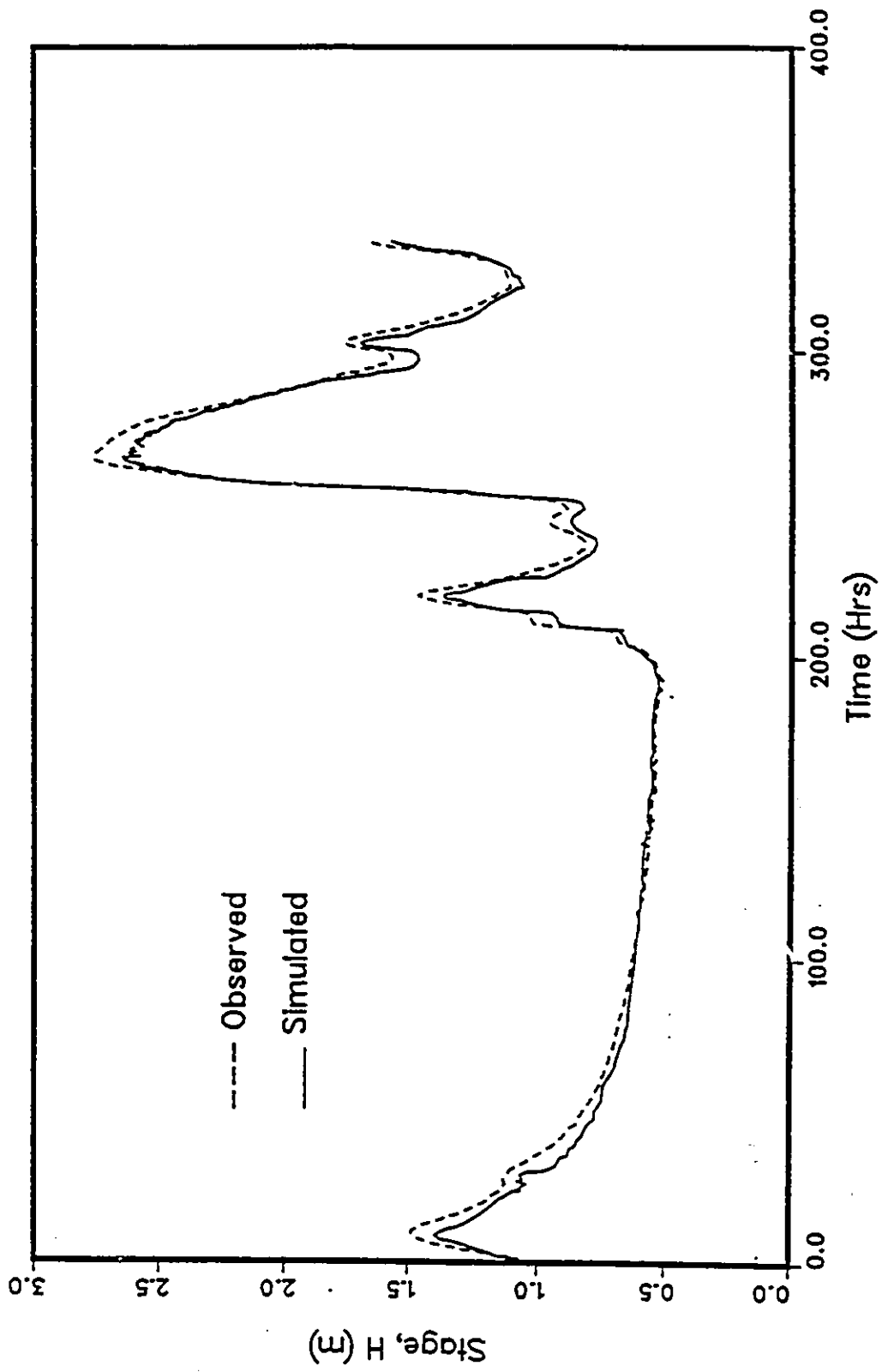


Fig. 6.15 Observed versus Simulated Stage Hydrographs for the River Main Reach, (Actual Data).

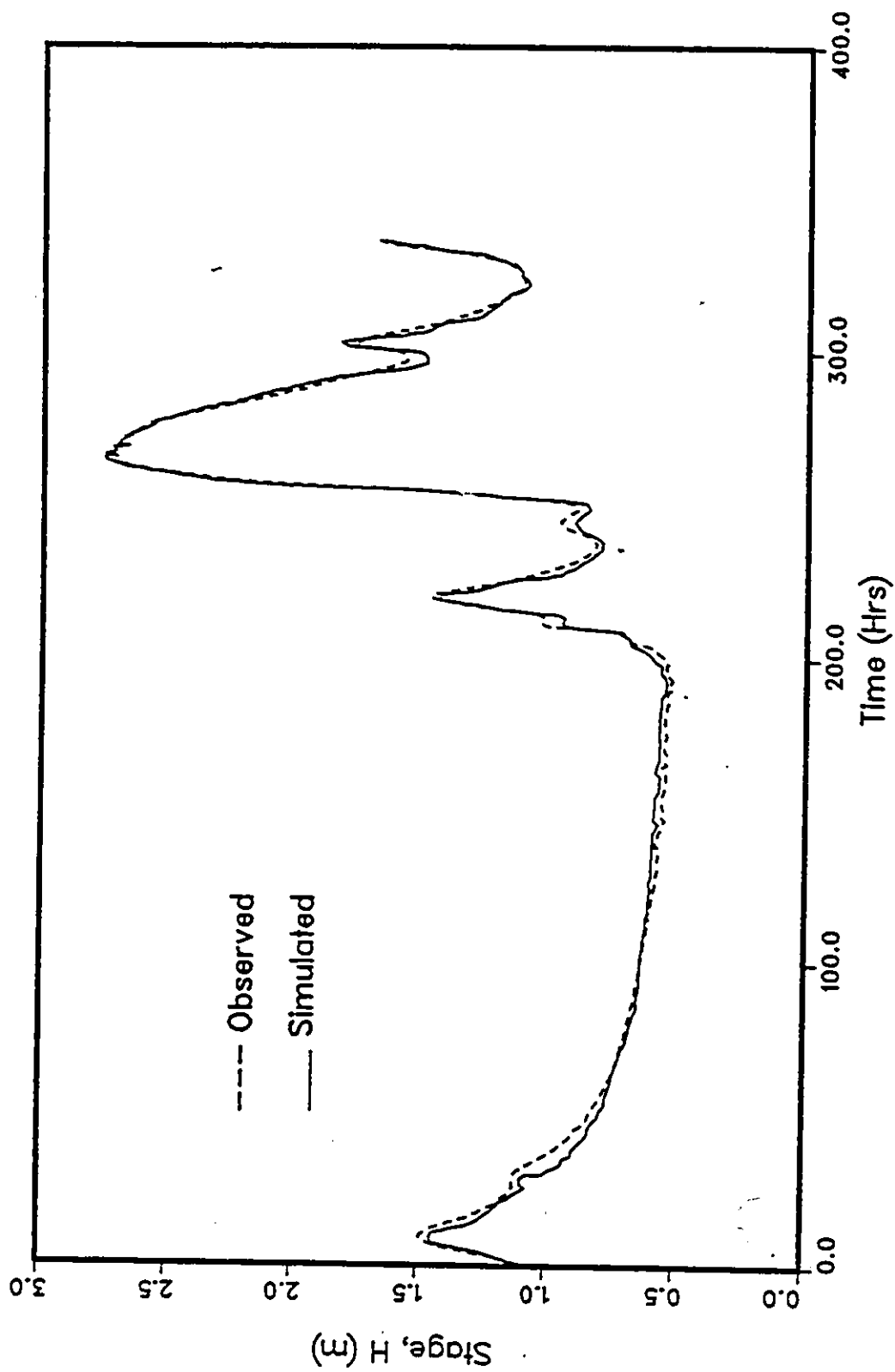


FIG. 6.16 Observed versus Simulated Stage Hydrographs for the River Main Reach,
(Precalibrated Manning's n).

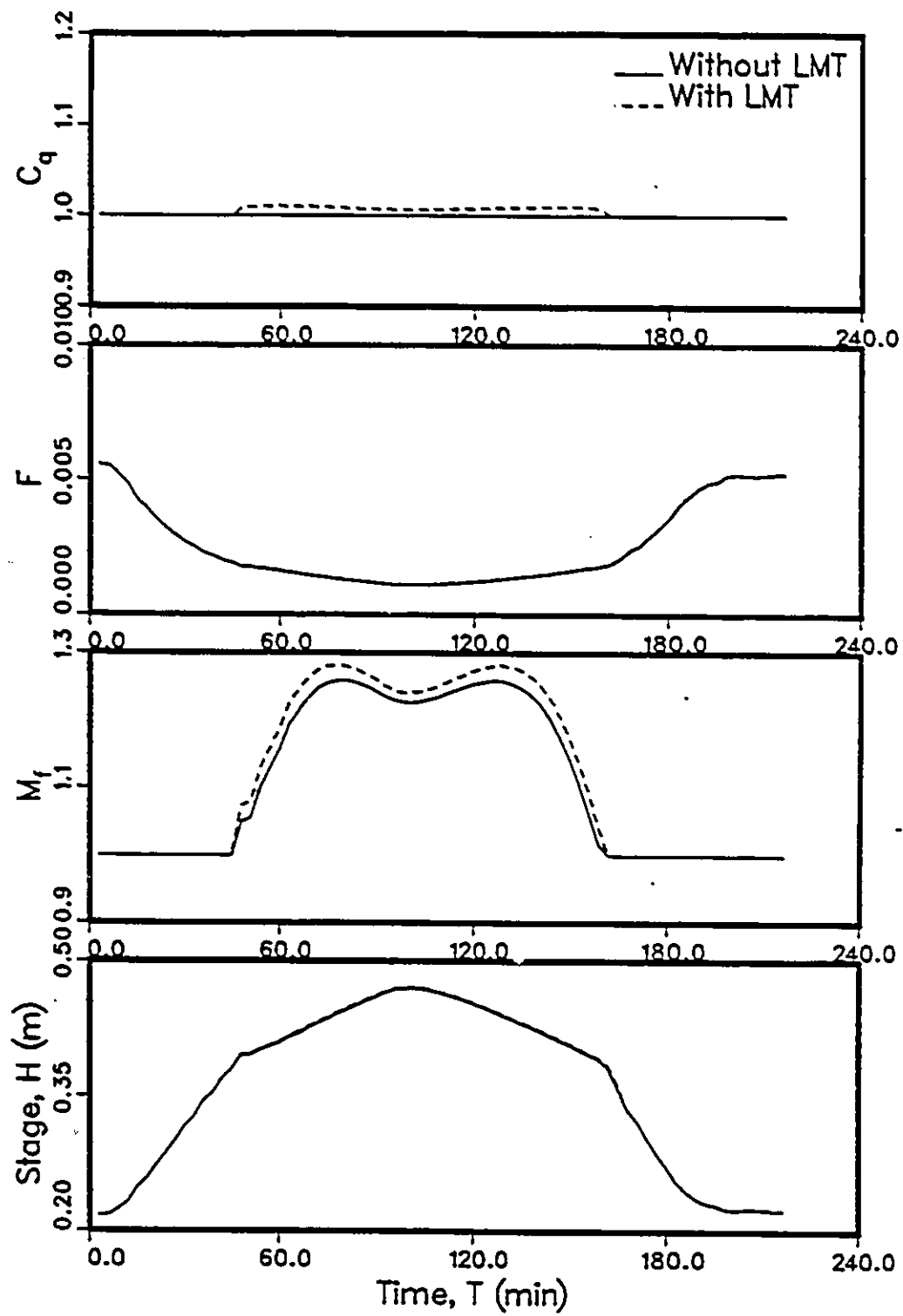


FIG. 6.17 Variation of Model Parameters M_f , F , and C_q with Depth of Flow, Flood Event PC10.

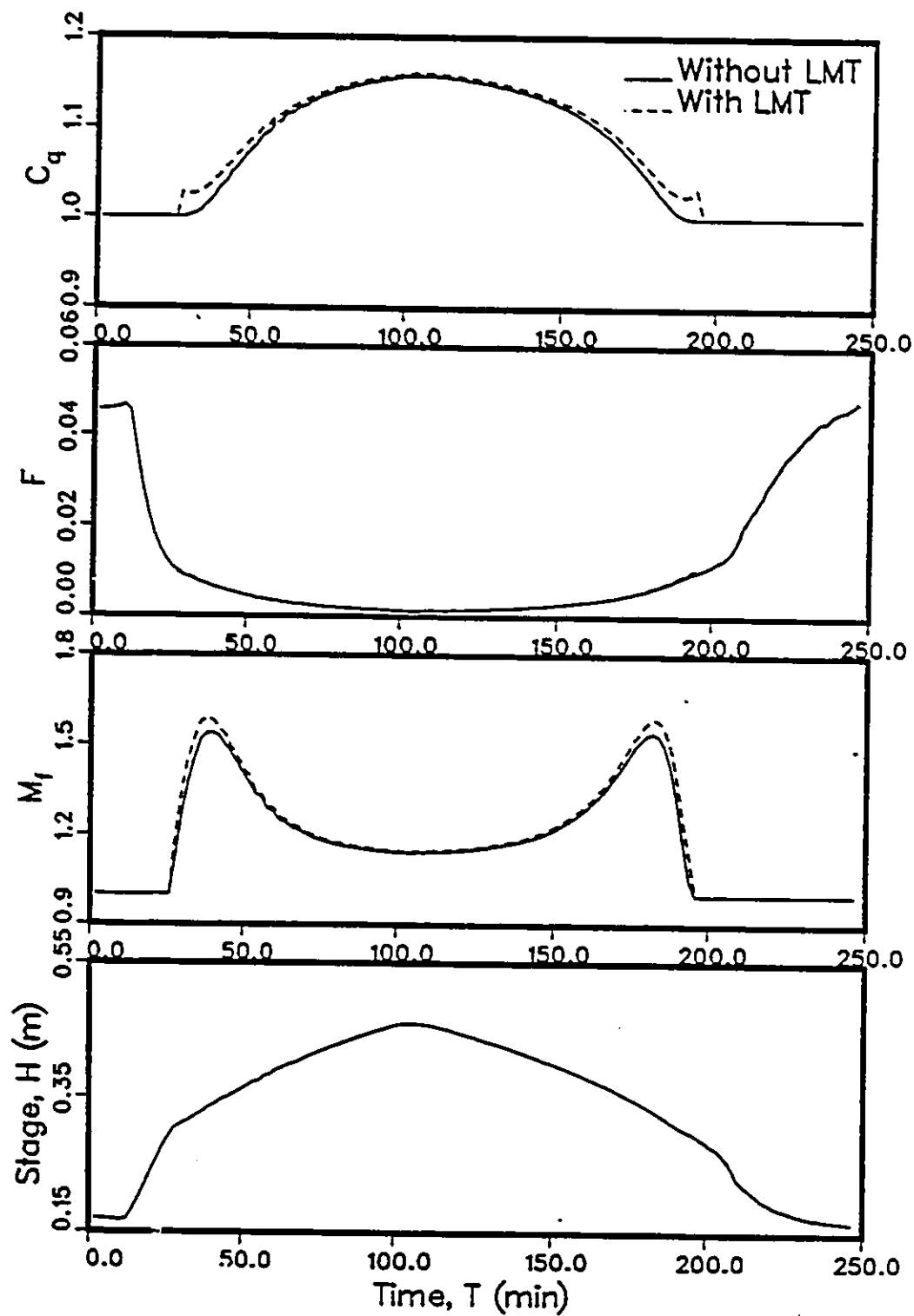


FIG. 6.18 Variation of Model Parameters M_f , F , and C_q with Depth of Flow, Flood Event MC8.

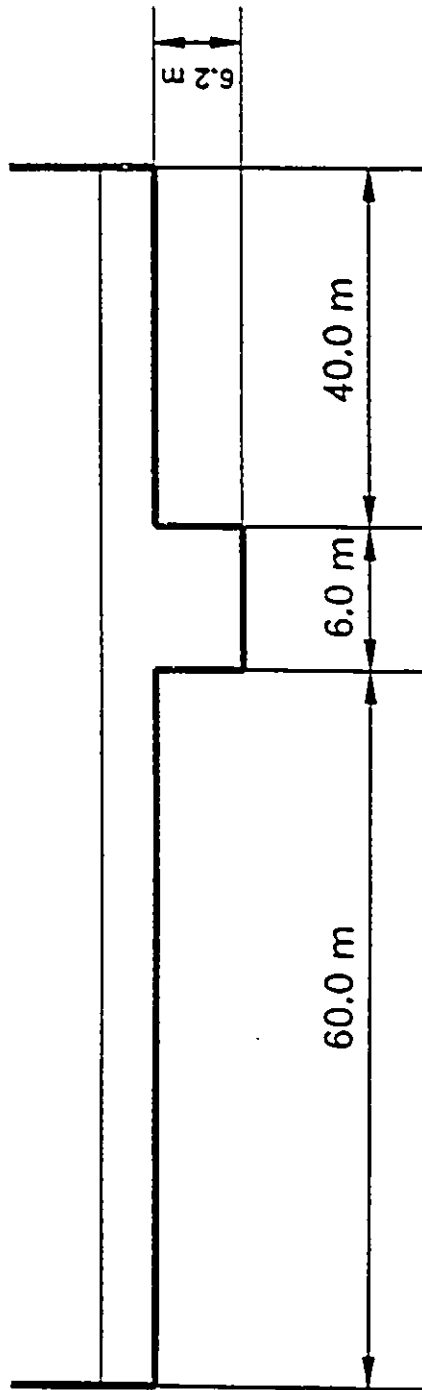


FIG. 6.19 Cross-Section of the Hypothetical Channel Considered.

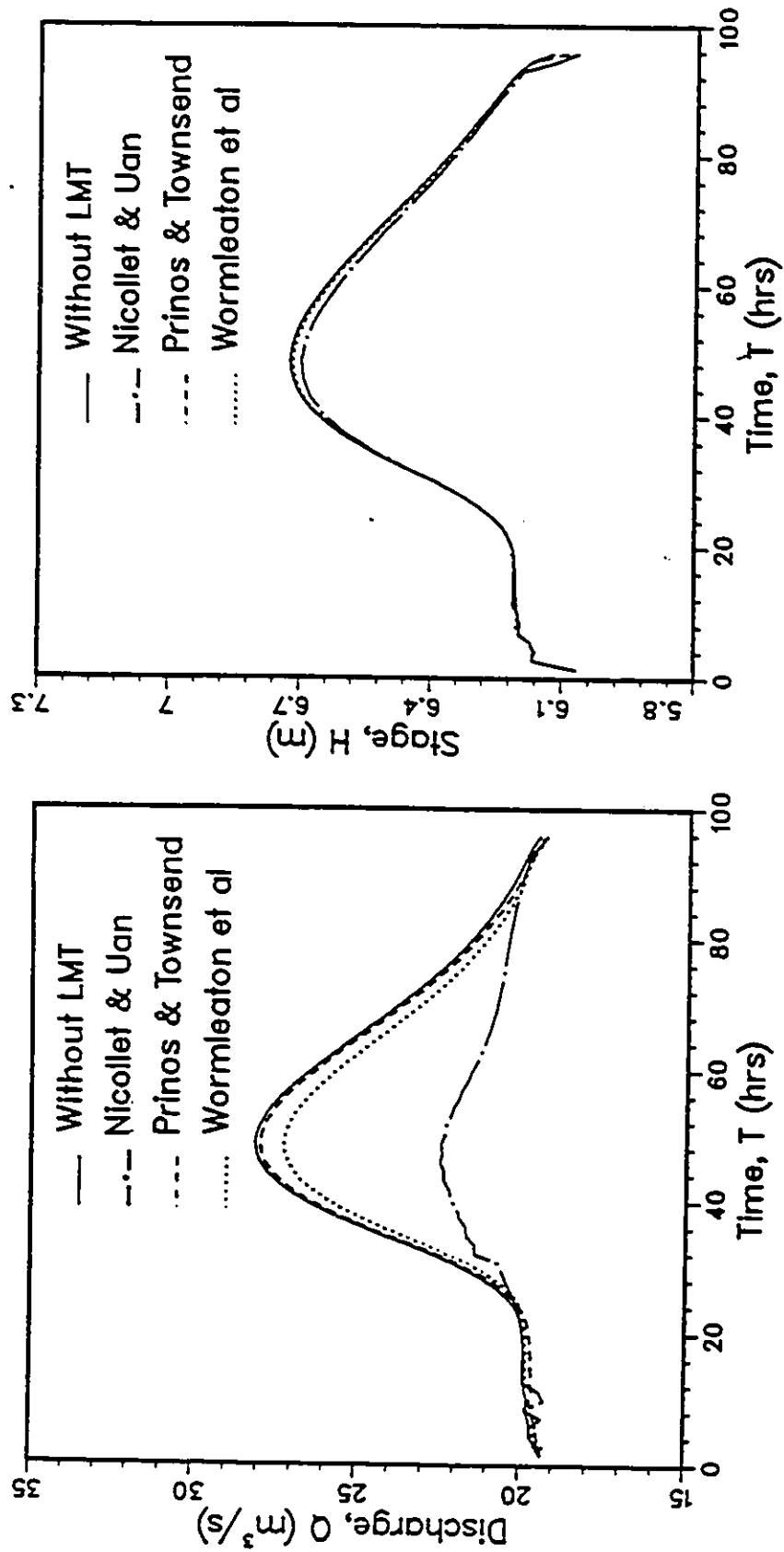


FIG. 6.20 Effect of LMT on Discharge and Stage Hydrographs, Hypothetical Channel Application, (Case I: High Flood Plain Flows).

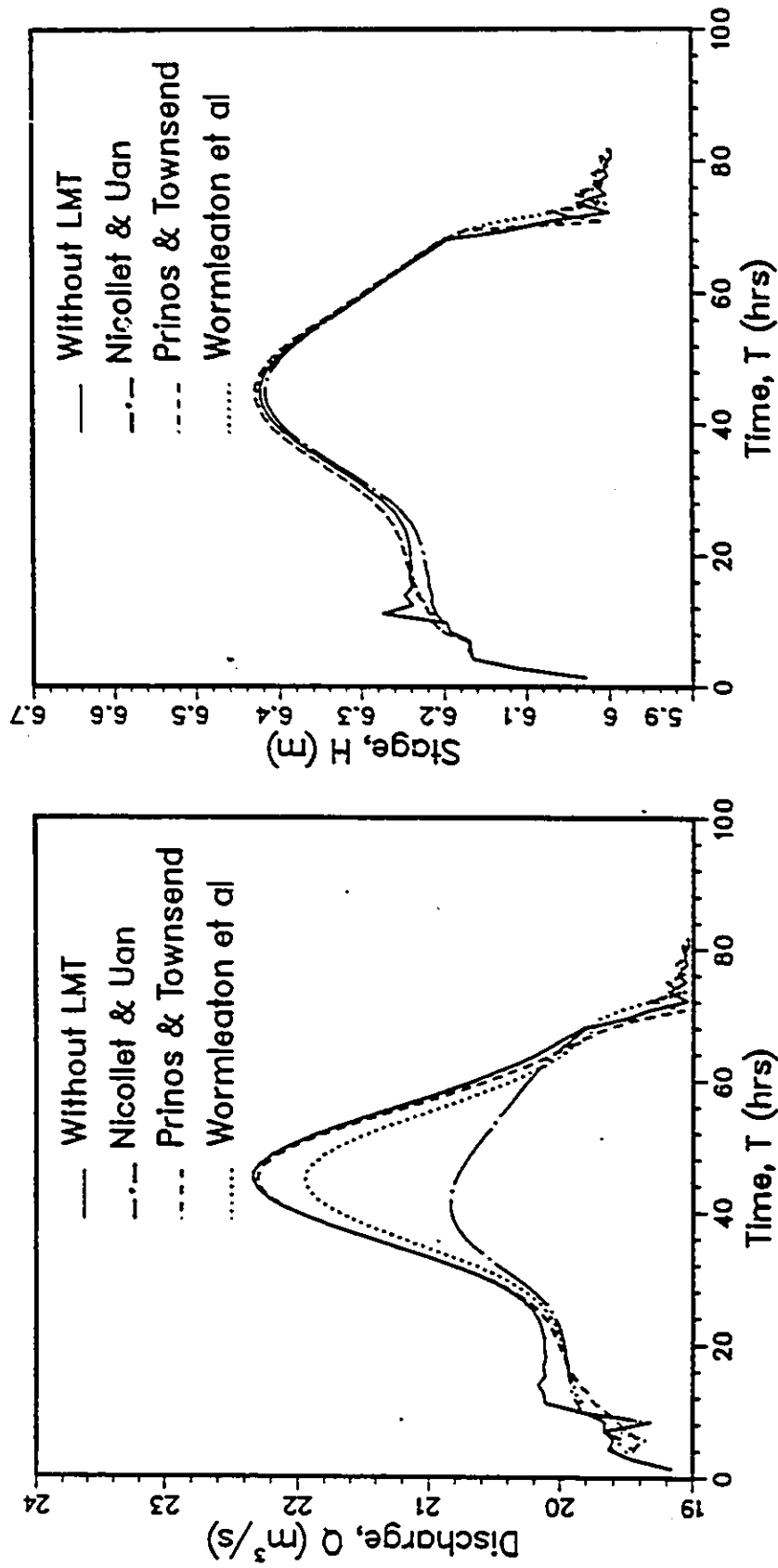


FIG. 6.21 Effect of LMT on Discharge and Stage Hydrographs, Hypothetical Channel Application, (Case II: Moderate Flood Plain Flows).

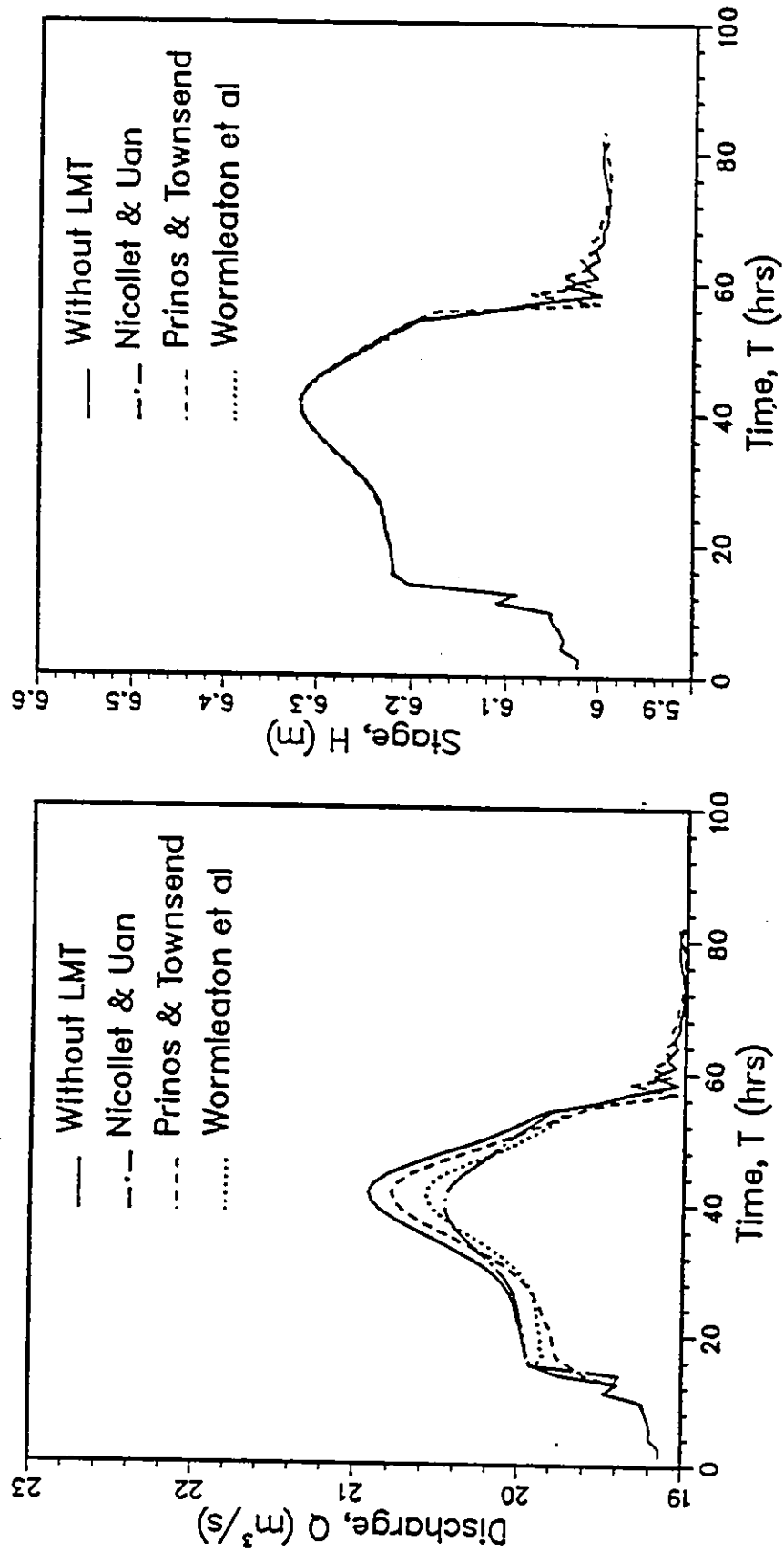


FIG. 6.22 Effect of LMT on Discharge and Stage Hydrographs, Hypothetical Channel Application, (Case III: Low Flood Plain Flows).

Chapter 7

CALIBRATION AND VALIDATION OF THE UNSTEADY FLOW MODEL

7.1 Model Calibration

Error generation is inherent in the mathematical and numerical description of physical systems and processes. In unsteady flow modeling errors result not only from representing the model partial differential equations by finite difference approximations (and the resulting truncation errors) but also from the simplifying assumptions underlying the unsteady flow model itself. Another major source of error is the fact that some of the parameters

embedded in the model equations cannot be measured directly. Examples of such parameters are the coefficient and exponent in the friction slope relationship, the variable that accounts for lateral momentum transfer between the main channel and flood plain sub-sections, and the ratio of their respective friction slopes.

While channel properties such as bed slope, reach length, and cross-sectional geometry can be directly measured, the other conceptual parameters mentioned earlier need to be estimated and then adjusted in a way to minimize errors between model output and the corresponding observed values of stage and discharge. This requires that the model be calibrated prior to its application.

The calibration process is mainly dependent on the use of proper values for the flow resistance coefficient. Adjustment of this parameter is necessitated by the subjectivity involved in its estimation based on field observations and the use of charts and tables. Evaluation of the roughness coefficient is complicated because of the energy-dissipation relationship, which is an approximation borrowed from steady uniform flow theory. Moreover, the flow resistance coefficient varies under changing flow conditions (i.e. it could be a function of discharge, flow depth, bed regime and turbulence among others). Nevertheless, Kolovopoulos (1990) concluded that *"for most river reaches the use of a constant roughness coefficient is adequate and the use of variable roughness coefficients should be considered only for secondary adjustments or for long simulation periods"*.

Other conceptual parameters that should be included in the calibration process are the LMT coefficient and the ratio of flood plain to main channel friction slopes. Different LMT methods, all based on steady flow conditions but under different testing scenarios, are incorporated in the model. In the calibration exercise the most suitable method for the particular system under investigation needs to be selected, with proper adjustments being made when necessary. The cosine interpolating function, relating the main channel and flood plain friction slopes, also needs to be adjusted to better describe the real situation and hence minimize errors in the model output.

Finally, as noted by Kolovopoulos (1990), only conceptual parameters in the unsteady flow model should be adjusted. The physical parameters, such as the channel geometry data, should generally remain unchanged as these data can be measured with reasonable accuracy. Furthermore, parameter values, whether physical or conceptual, must always remain rational and within reasonable limits under all circumstances.

7.2 Model Validation

The output of an unsteady flow model consists of stage and discharge hydrographs that need to be compared to observed data to check the model's performance. Lai et al (1978) argue that model verification using stages alone is insufficient to assure the validity of the results. However, since discharge is hard to measure during extreme flood events, accurate discharge

hydrographs are rarely available. Kolovopoulos (1990) suggests that measured discharges should be used as a supplementary check.

Model validation can range from being completely subjective, by relying strongly on visual impressions of the correspondence between the observed and simulated hydrographs, to a detailed statistical analysis that tests the agreement between the respective time series. Green (1985) suggests a number of statistics to be used in these model performance studies. These statistics, which were also employed by Kolovopoulos (1990), are used to test the performance of RUFICC.

7.2.1 Graphical displays

The following graphs are used to test the model performance and the agreement between the observed and simulated time series:

- **Continuous plot of simulated stage and discharge hydrographs superimposed on the recorded values:** A qualitative assessment is made by comparing peak values, hydrograph shape, and total flow volumes.
- **A scattergram of recorded data plotted against simulated values (stages or discharges):** This plot is particularly useful in regards to revealing the presence of systematic errors in the simulated hydrograph.

- **A scattergram of the error in simulated magnitude versus the observed magnitude:** This plot is similar to the last one, except that it emphasizes the relative error.

7.2.2 Statistics Used

The following statistics, which were described by Kolovopoulos (1990), were adopted in order to compare the observed with the simulated time series.

- Relative errors are first evaluated as:

$$E(i) = \frac{O(i) - S(i)}{O(i)} \quad (7.1)$$

where $O(i) = i^{th}$ observed value (stage or discharge), and $S(i) = i^{th}$ simulated value.

- **Sum of Squared Residuals (SSR):**

$$SSR = \sum_{i=1}^n (E(i))^2 \quad (7.2)$$

where n = the total number of data points used in the comparison.

- **Sum of Absolute Residuals (SAR):**

This statistic, proposed by Stephenson (1979), is written as:

$$SAR = \sum_{i=1}^n |E(i)| \quad (7.3)$$

Compared to SSR, using this statistic does not reduce the effect of the residuals with a value less than unity and does not emphasize the effect of large magnitude outliers (Green 1985).

- **Sum of Absolute Area of Divergence (SAAD):**

This statistic, which highlights the difference in shape between two hydrographs, is written as:

$$SAAD = \frac{1}{n} \sum_{i=1}^n \left| \frac{E(i) + E(i+1)}{2} \Delta t \right| \quad (7.4)$$

where Δt = time step.

- **Reduced Error of Estimate (REE):**

This statistic, proposed by Manley (1977), is given by the following relationship:

$$REE = \left(\frac{\sum_{i=1}^n (O(i) - s(i))^2}{\sum_{i=1}^n (O(i) - \bar{O})^2} \right)^{1/2} \quad (7.5)$$

where $\bar{O}$ = the mean of the observed values.

- **Proportional Error of Estimate (PEE):**

$$PEE = \left(\frac{\sum_{i=1}^n (O(i) - S(i))^2}{(\sum_{i=1}^n O(i))^2} \right)^{1/2} \quad (7.6)$$

This statistic, proposed by Manley (1978), avoids the major drawback of REE, namely that it emphasizes errors associated with large flows more than a sequence of errors at low discharges.

- **Mean of the Residuals (M_e):**

$$M_e = \frac{(\sum_{i=1}^n |E(i)|)}{n} \quad (7.7)$$

- **Standard Deviation of the Residuals (SD):**

$$SD = \left(\frac{\sum_{i=1}^n (E(i) - M_e)^2}{n} \right)^{1/2} \quad (7.8)$$

- **Standard Error of Estimate (SEE):**

This statistic, first introduced by Jewell et al (1978), is given by:

$$SEE = \left(\frac{\sum_{i=1}^n E(i)^2}{n-2} \right)^{1/2} \quad (7.9)$$

- **Correlation Coefficient (R_{so}):**

This is the most important and most frequently used criterion for measuring the agreement between two hydrographs (Sarma et al 1969; Snyder et al 1970). The closer the value of this coefficient is to 1.0 the better is the agreement. The correlation coefficient is defined as:

$$R_{so} = \frac{1}{n} \sum_{i=1}^n \left(\frac{O(i) - \bar{O}}{SD_o} \right) \left(\frac{S(i) - \bar{S}}{SD_s} \right) \quad (7.10)$$

where $\bar{O}$ and $\bar{S}$ are the mean values of the observed and simulated time series respectively and SD_o and SD_s are their respective standard deviations.

- **Modified Correlation Coefficient (MR_{so}):**

Because the correlation coefficient measures the agreement only if the two hydrographs being compared have nearly equal mathematical moments about a horizontal axis (Kolovopoulos 1990), MuCuen and Snyder (1975) proposed weighting it by the ratio of the standard deviations. Thus the Modified Correlation Coefficient is given by:

$$MR_{so} = \begin{cases} (SD_o/SD_s)R_{so} & \text{if } SD_o \leq SD_s, \\ (SD_s/SD_o)R_{so} & \text{if } SD_s \leq SD_o. \end{cases}$$

- **Coefficient of Determination D :**

This coefficient was proposed by Nash and Sutcliffe (1970).

$$D = 1 - \frac{\sum_{i=1}^n (O(i) - S(i))^2}{\sum_{i=1}^n (O(i) - \bar{O})^2} \quad (7.11)$$

The coefficient of determination is the square of the correlation coefficient and therefore will always be less than unity [and less than the correlation coefficient].

R_{so} , MR_{so} , and D are all measures of the degree of association between the observed and estimated values. They do not, however, reveal systematic errors if they exist.

7.3 Examples of Model Validation

Three examples were selected as sample applications to demonstrate the model validation test. The first example considers flood event 8 in the prismatic compound channel (PCS). Observed and simulated stage and discharge hydrographs for this particular flood event are shown in Fig. 7.1. Reasonably close agreement between the two sets of hydrographs was achieved. The corresponding scattergrams of observed and simulated stages and discharges (Fig. 7.2) indicate that the model accurately simulated both the high and low flows. Residuals in simulated stage and discharge are also observed to be fairly low, (see Fig. 7.3).

Values for the different statistics adopted in the validation process for both stage and discharge time series are shown in Tables 7.1 and

7.2 respectively. According to these tables the values of most of the coefficients are very close to unity and the correlation is excellent, which shows again the reasonably good agreement between simulated results and observed data.

The second example considered flood event S in the meandering channel (MCS). Observed versus simulated discharge and stage hydrographs for this particular flood event are shown in Fig. 7.4. It can be clearly seen from this figure that a reasonably good agreement between simulated and observed data was achieved, except that peak stage was slightly overestimated [the error was less than 4 %]. Scattergrams of observed and simulated stages and discharges for flood event MCS (Fig. 7.5) indicate that while low and high discharges are accurately estimated, a larger error was induced in simulating stages for low flows. Residuals in simulated magnitude shown in Fig. 7.6 confirm this conclusion. Examining now the values for the statistics considered, it is clear from Tables 7.3 and 7.4 that a good correlation was achieved since $R_{s,o}$, $MR_{s,o}$, and D are all close to unity. Error estimates are also fairly low.

Finally, results of the field application treated in both chapters VI and VIII were considered for statistical analysis. In this application of the River Main reach simulations were performed using actual data and then actual geometric data but a precalibrated Manning's n . Relying on visual inspection the latter simulation was shown to yield better agreement between simulated and observed hydrographs, (see

Figs. 6.15 and 6.16). Statistical parameters for both of these simulations are shown in Tables 7.5 and 7.6 respectively. Comparing both tables, it is clear that lower estimates of statistics like the sum of absolute area of divergence, reduced error of estimate, standard deviation of the residuals, and standard error of estimate were obtained for the second simulation (i.e. when a precalibrated Manning's n was adopted). On the other hand, even though the correlation coefficients were both close to unity, the modified correlation coefficient was shown to be higher for the case when a precalibrated Manning's n was used. A third simulation for the River Main reach was also performed using predetermined optimal data through a 'black box' optimization approach in chapter VIII, (Fig. 8.24). A summary of statistics for this latter application is shown in Table 7.7. Comparing Tables 7.5, 7.6, and 7.7 it is obvious that Table 7.7 (optimal data) shows the lowest estimates of error statistics with almost the same values for correlation coefficients as for Table 7.6 (precalibrated Manning's n).

Scattergrams of observed and simulated stages for this particular application using actual and optimal data are shown in Figs. 7.7 and 7.9 respectively. Comparing both figures it is clear that better results were obtained for the simulation where optimal data was used. In the case where actual data were adopted stages were observed to be consistently underestimated, (see Fig. 7.7). This conclusion is confirmed when Figs. 7.8 and 7.10, showing residuals for both simulations, are compared.

Table 7.1: Statistical Summary of the Stage Hydrograph for Event PCS.

Sum of Squared Residuals	=	0.093
Sum of Absolute Residuals	=	2.220
Sum of Absolute Area of Divergence	=	1.459
Reduced Error of Estimate	=	0.031
Proportional Error of Estimate	=	0.031
Standard Deviation of the Residuals	=	0.032
Standard Error of Estimate	=	0.033
Correlation Coefficient	=	0.9995
Modified Correlation Coefficient	=	0.9937
Coefficient of Determination	=	0.9990

Table 7.2: Statistical Summary of the Discharge Hydrograph for Event PCS.

Sum of Squared Residuals	=	3.056
Sum of Absolute Residuals	=	12.92
Sum of Absolute Area of Divergence	=	8.496
Reduced Error of Estimate	=	0.153
Proportional Error of Estimate	=	0.153
Standard Deviation of the Residuals	=	0.183
Standard Error of Estimate	=	0.185
Correlation Coefficient	=	0.9884
Modified Correlation Coefficient	=	0.9604
Coefficient of Determination	=	0.9769

Table 7.3: Statistical Summary of the Stage Hydrograph for Event MCS.

Sum of Squared Residuals	=	1.455
Sum of Absolute Residuals	=	7.507
Sum of Absolute Area of Divergence	=	7.600
Reduced Error of Estimate	=	0.072
Proportional Error of Estimate	=	0.072
Standard Deviation of the Residuals	=	0.111
Standard Error of Estimate	=	0.112
Correlation Coefficient	=	0.9979
Modified Correlation Coefficient	=	0.9686
Coefficient of Determination	=	0.9958

Table 7.4: Statistical Summary of the Discharge Hydrograph for Event MCS.

Sum of Squared Residuals	=	8.544
Sum of Absolute Residuals	=	21.94
Sum of Absolute Area of Divergence	=	22.27
Reduced Error of Estimate	=	0.105
Proportional Error of Estimate	=	0.105
Standard Deviation of the Residuals	=	0.269
Standard Error of Estimate	=	0.271
Correlation Coefficient	=	0.9946
Modified Correlation Coefficient	=	0.9842
Coefficient of Determination	=	0.9892

Table 7.5: Statistical Summary for the River Main Application (actual data).

Sum of Squared Residuals	=	2.704
Sum of Absolute Residuals	=	43.81
Sum of Absolute Area of Divergence	=	51.74
Reduced Error of Estimate	=	0.052
Proportional Error of Estimate	=	0.052
Standard Deviation of the Residuals	=	0.052
Standard Error of Estimate	=	0.052
Correlation Coefficient	=	0.9996
Modified Correlation Coefficient	=	0.9564
Coefficient of Determination	=	0.9992

Table 7.6: Statistical Summary for the River Main Application
(pre-calibrated Manning's n).

Sum of Squared Residuals	=	1.596
Sum of Absolute Residuals	=	32.19
Sum of Absolute Area of Divergence	=	38.06
Reduced Error of Estimate	=	0.032
Proportional Error of Estimate	=	0.032
Standard Deviation of the Residuals	=	0.040
Standard Error of Estimate	=	0.040
Correlation Coefficient	=	0.9995
Modified Correlation Coefficient	=	0.9919
Coefficient of Determination	=	0.9990

Table 7.7: Statistical Summary for the River Main Application (optimal data).

Sum of Squared Residuals	=	1.497
Sum of Absolute Residuals	=	24.71
Sum of Absolute Area of Divergence	=	36.52
Reduced Error of Estimate	=	0.049
Proportional Error of Estimate	=	0.049
Standard Deviation of the Residuals	=	0.043
Standard Error of Estimate	=	0.043
Correlation Coefficient	=	0.9988
Modified Correlation Coefficient	=	0.9891
Coefficient of Determination	=	0.9976
Coefficient of Efficiency	=	0.9987

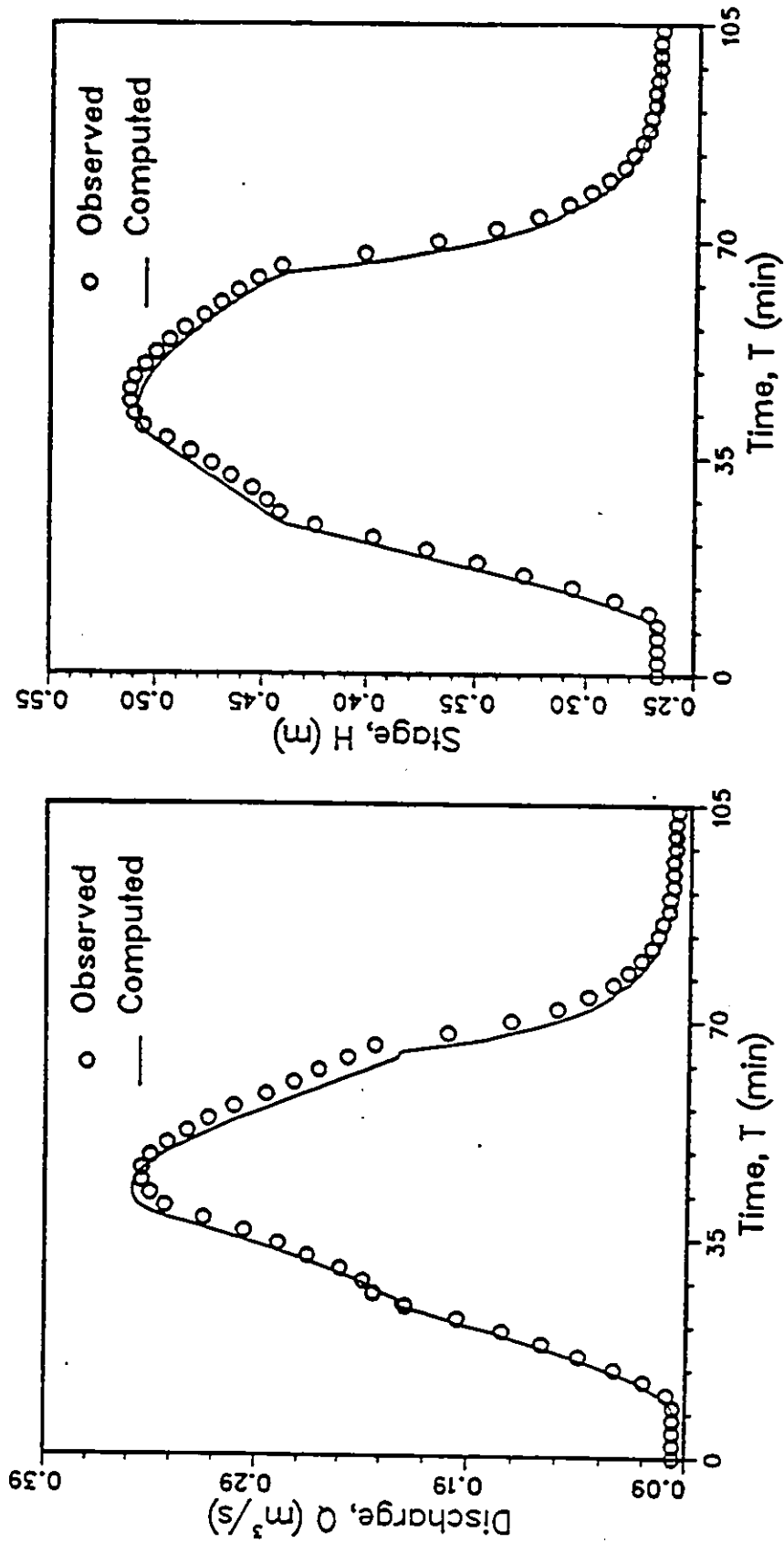


FIG. 7.1 Comparison of Observed and Simulated Discharge and Stage Hydrographs for Flood Event NO. 8 in Prismatic Channel (PC8).

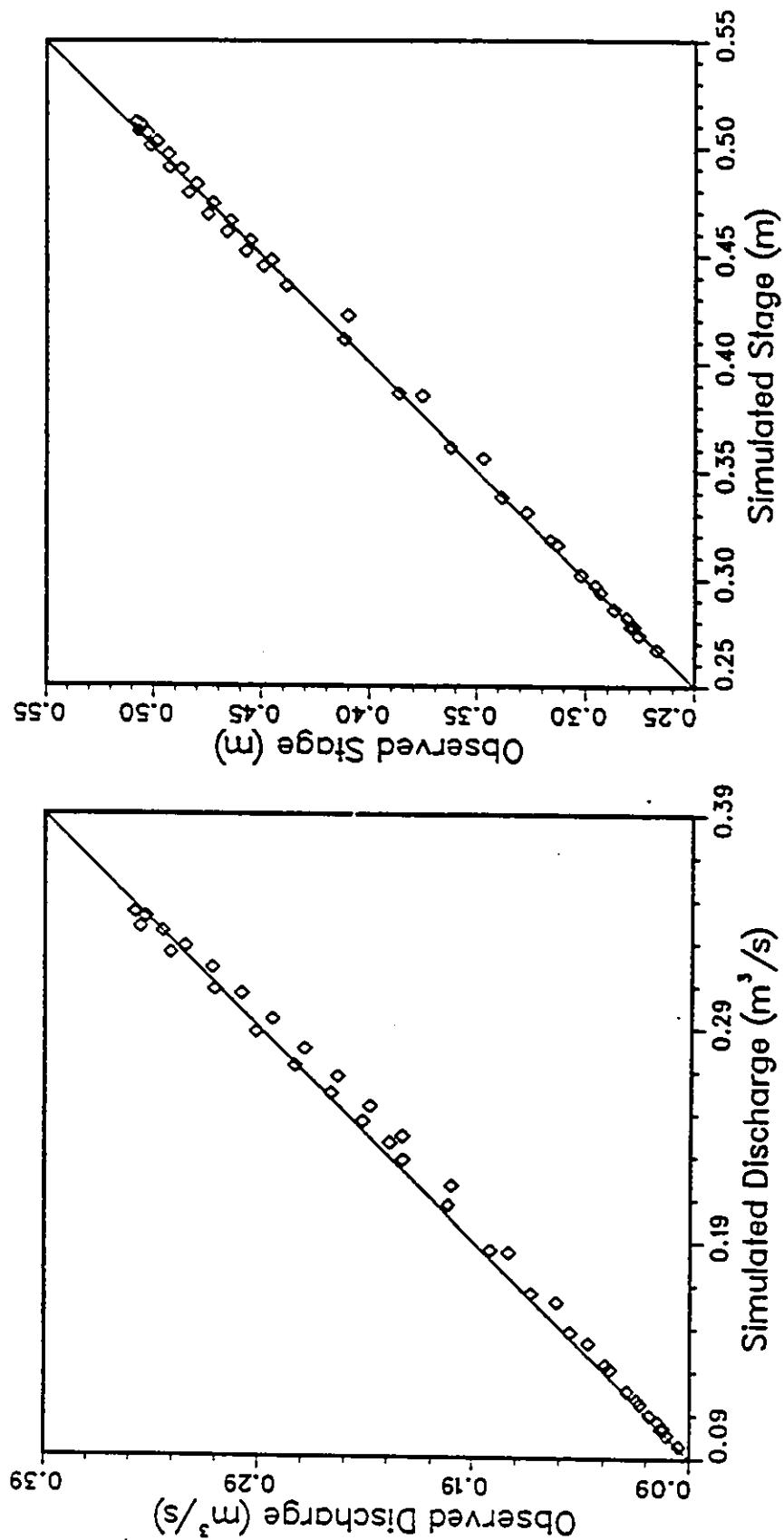


FIG. 7.2 Scattergram of Observed and Simulated Discharges and Stages for Flood Event No. 8 in Prismatic Channel (PC8).

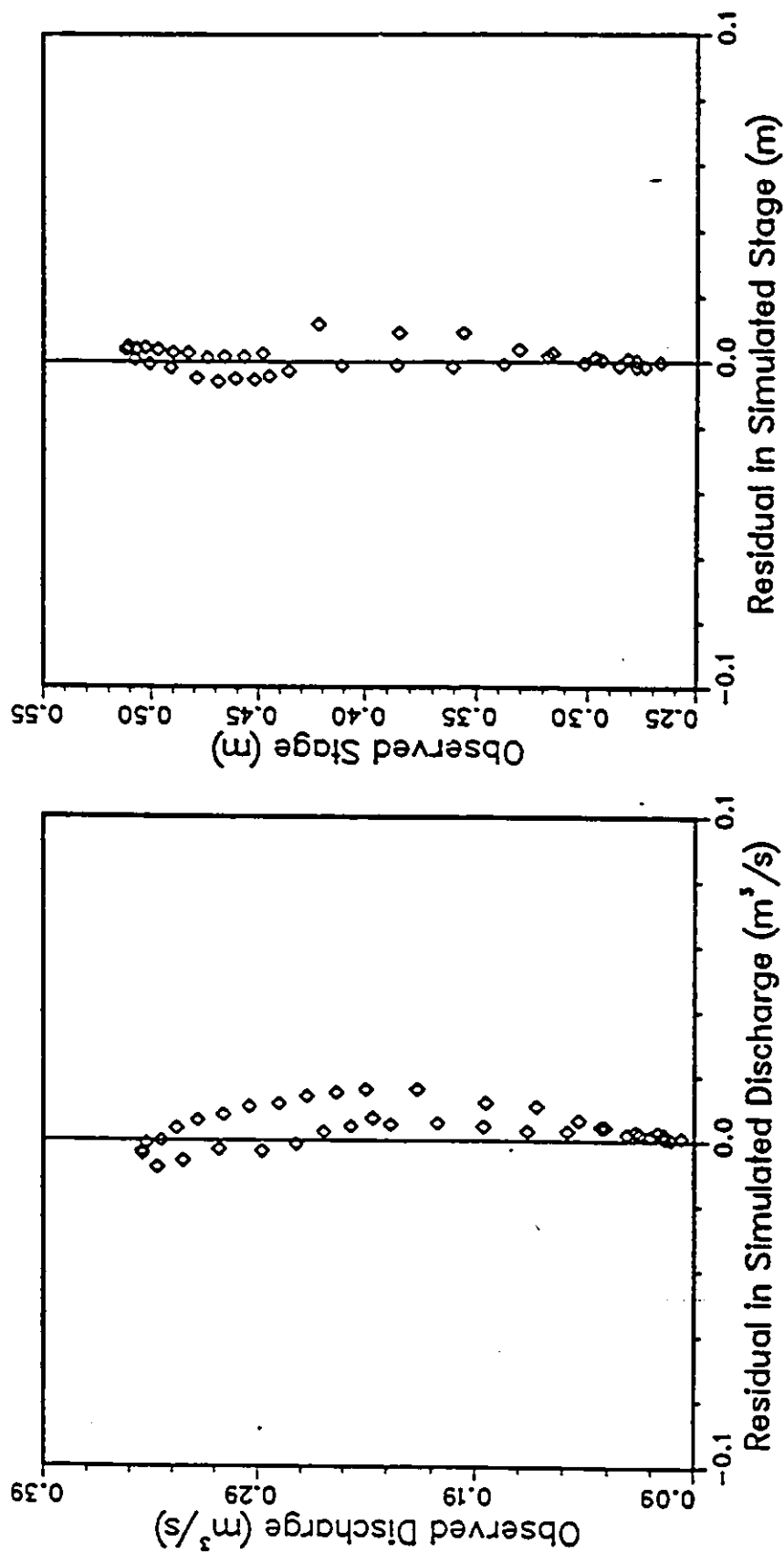


FIG. 7.3 Scattergram of the Error in Simulated Magnitude Against Observed Value for Flood Event No. 8 in Prismatic Channel (PC8).

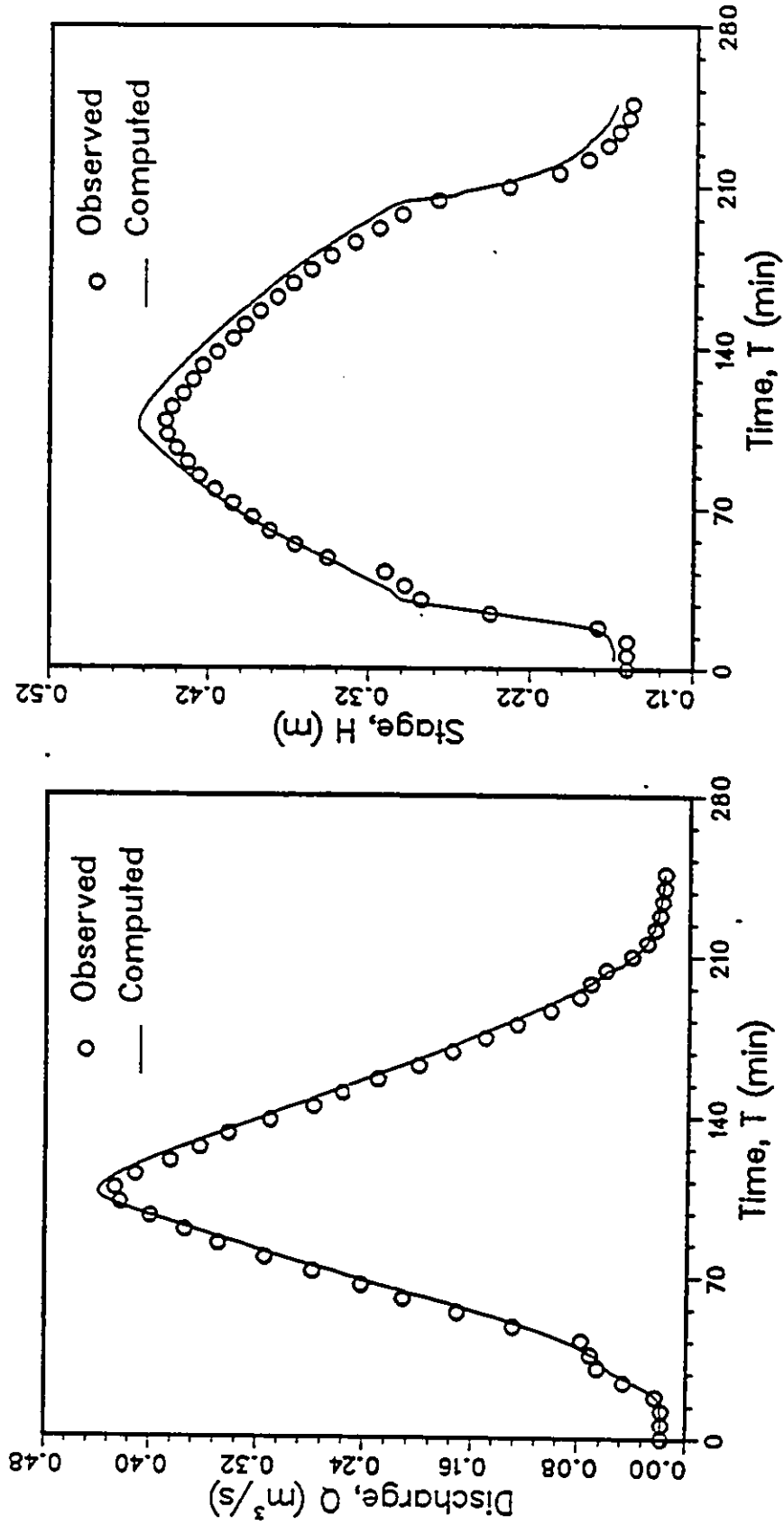


FIG. 7.4 Comparison of Observed and Simulated Discharge and Stage Hydrographs for Flood Event No. 8 in Meandering Channel (MC8).

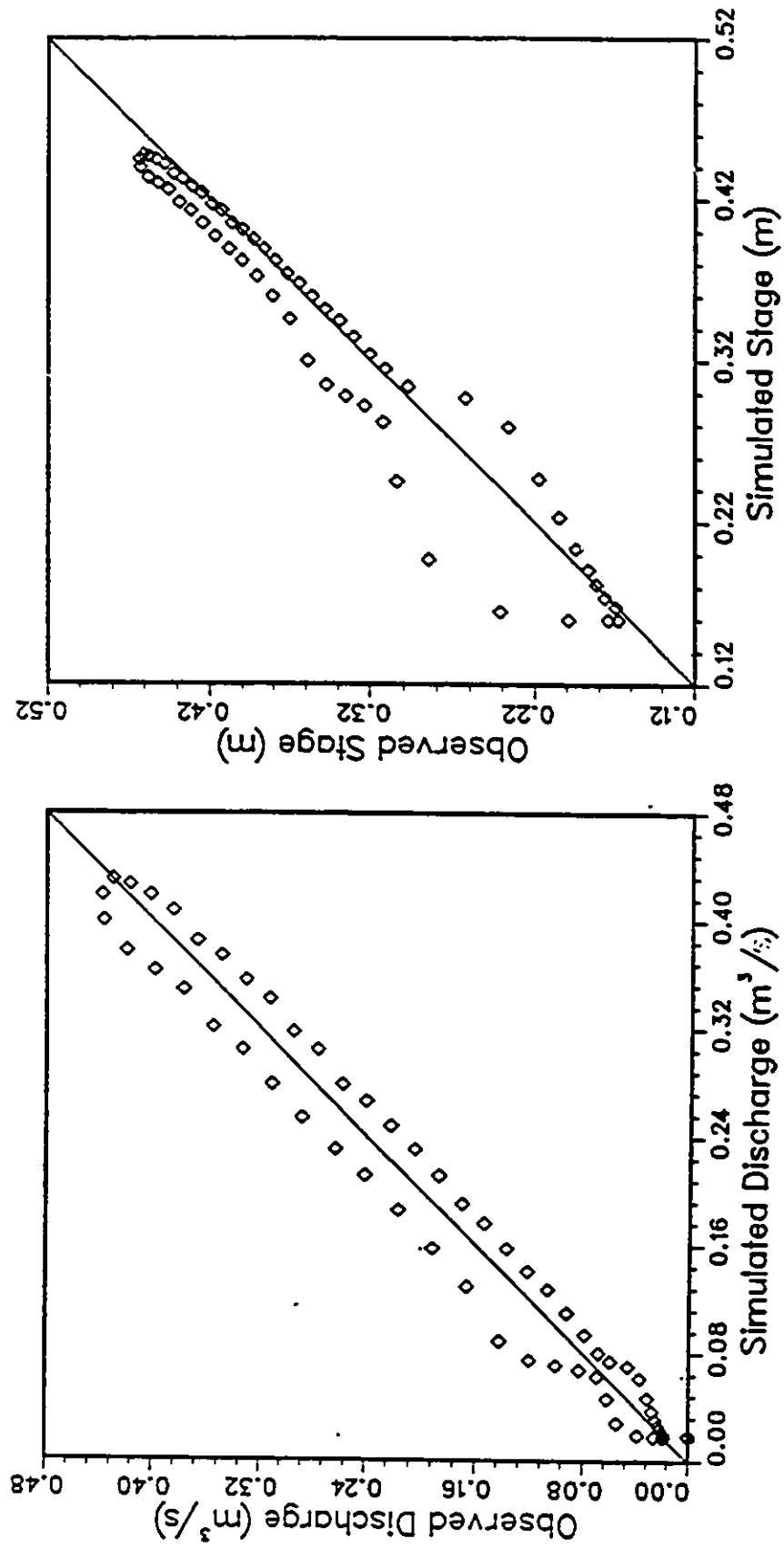


FIG. 7.5 Scattergram of Observed and Simulated Discharges and Stages for Flood Event No. 8 in Meandering Channel (MC8).

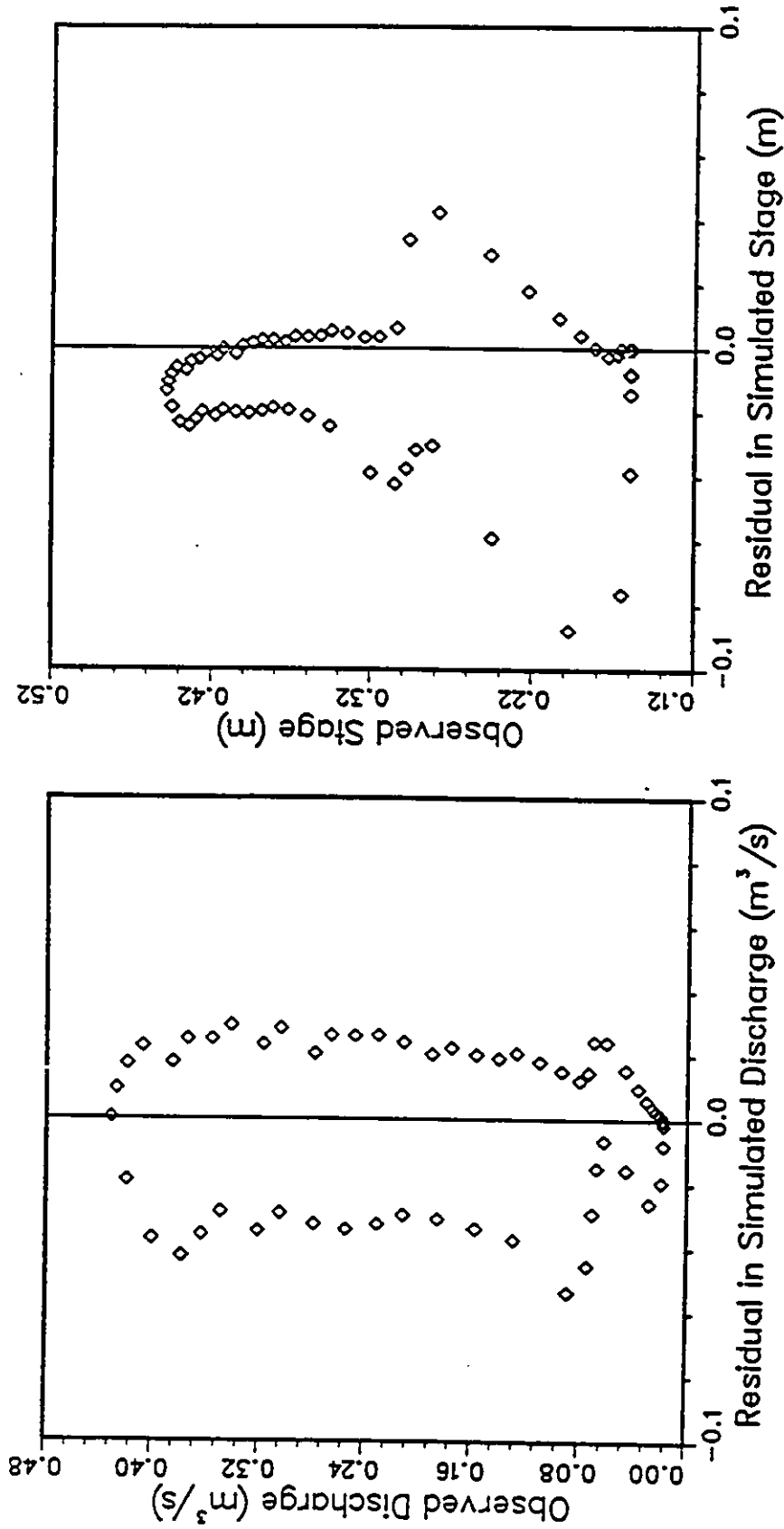


FIG. 7.6 Scattergram of the Error in Simulated Magnitude Against Observed Value for Flood Event No. 8 in Meandering Channel (MC8).

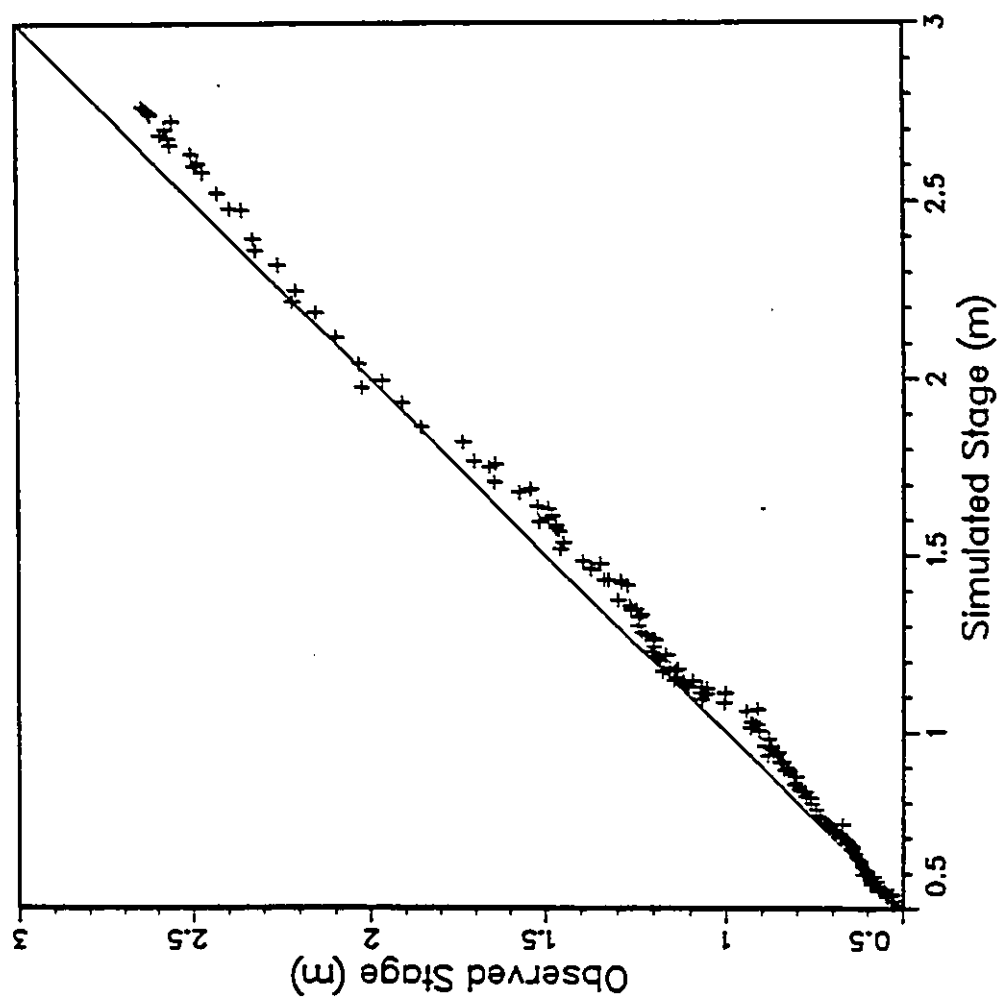


FIG. 7.7 Scattergram of Observed and Simulated Stages for the River Main Application (Actual Data).

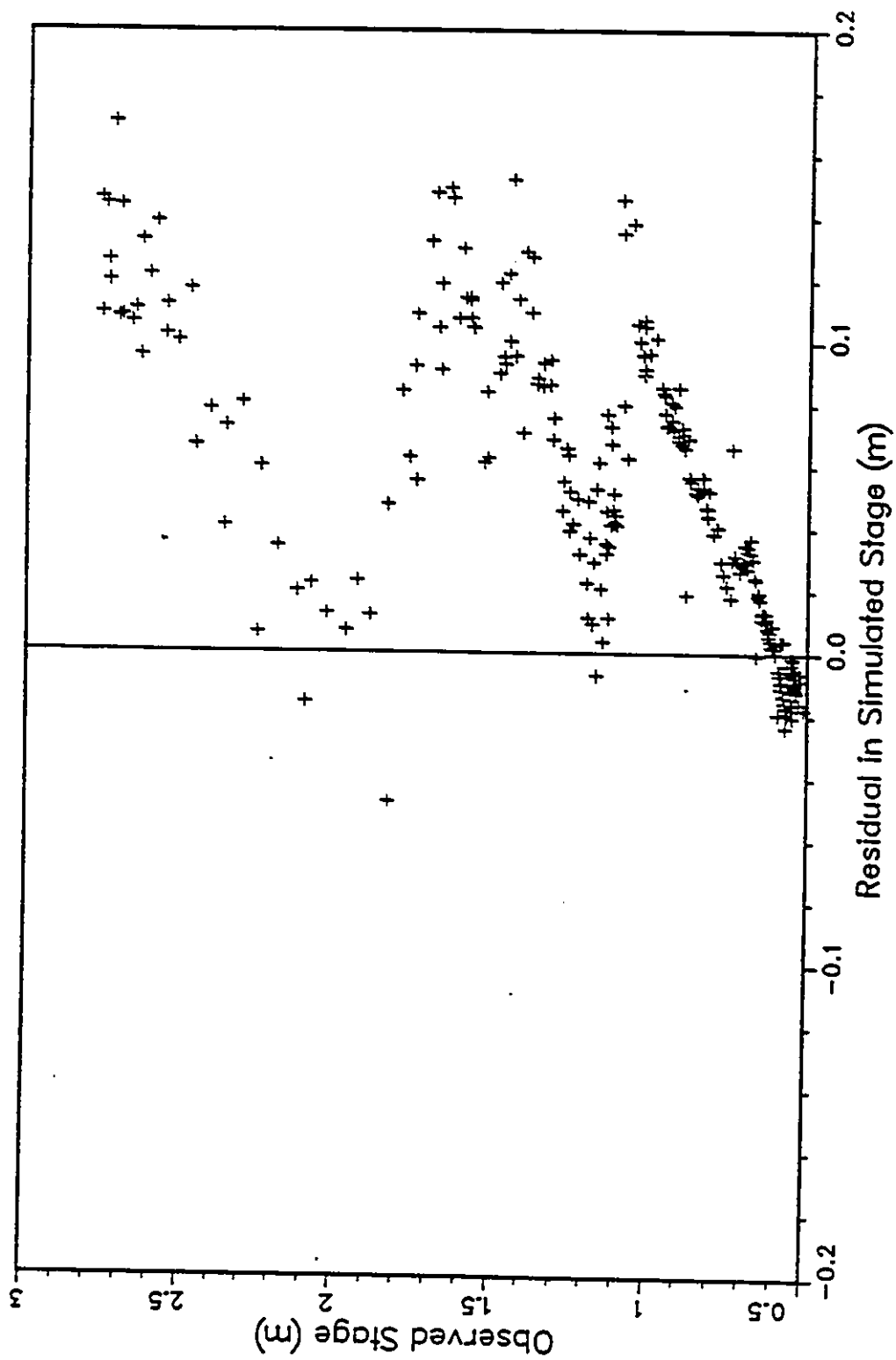


FIG. 7.8 Scattergram of the Error in Simulated Magnitude Against Observed Value for the River Main Application (Actual Data).

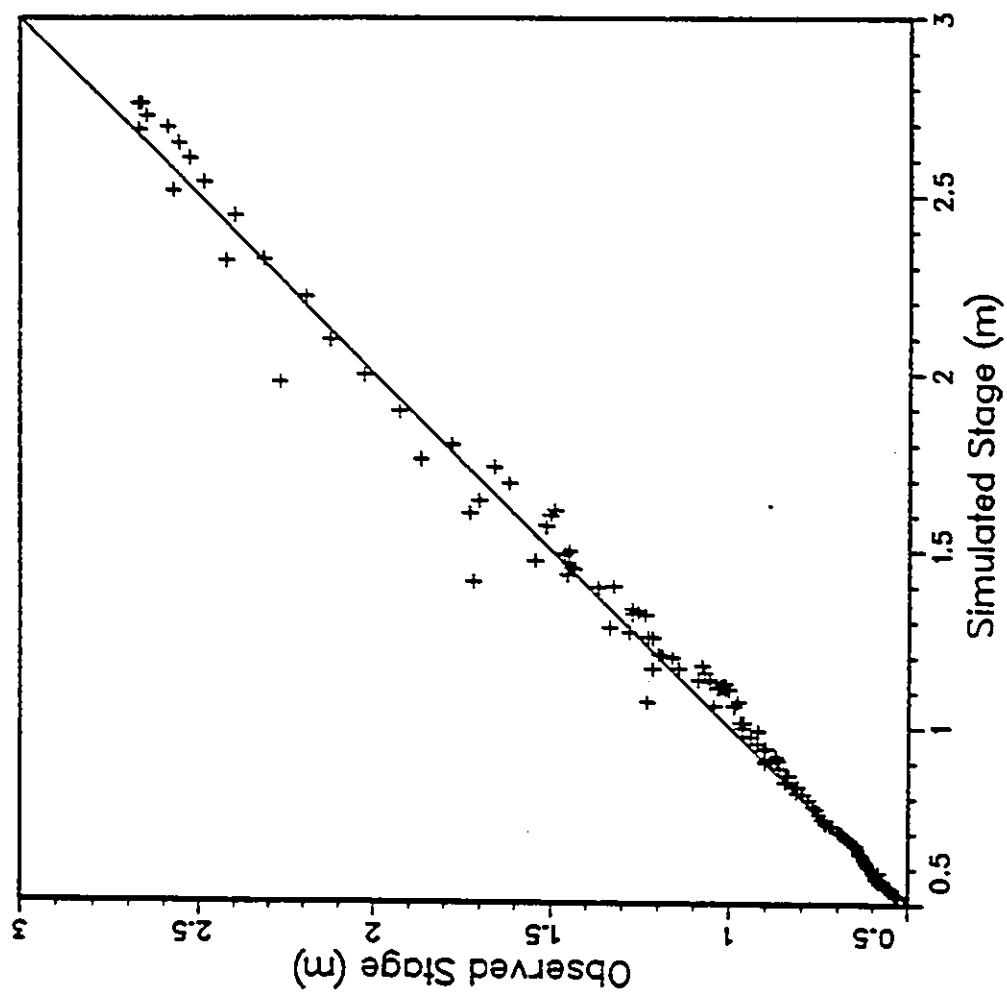


FIG. 7.9 Scattergram of Observed and Simulated Stages for the River Main Application (Optimal Data).

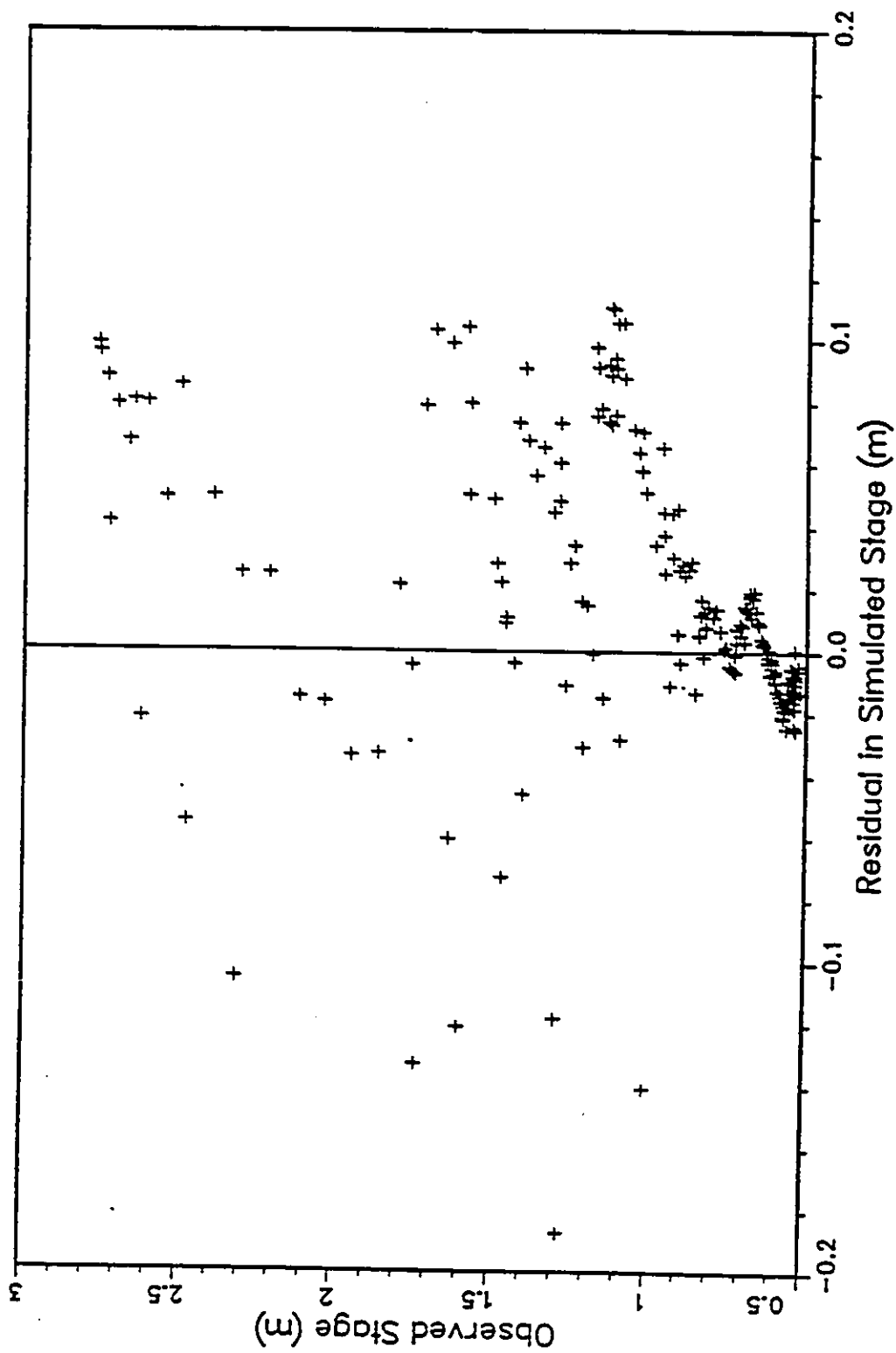


FIG. 7.10 Scattergram of the Error in Simulated Magnitude Against Observed Value for the River Main Application (Optimal Data).

Chapter 8

APPLICATION OF OPTIMIZATION METHODS

An optimization process involves calculating systematically better values for the model parameters until a minimum value for the objective function is reached, at which point the model parameters have optimum values. Therefore, the effectiveness of any optimization process depends on whether it converges to the right minimum and how fast the convergence is. Several examples were considered to test the performance of the three optimization techniques adopted in this study, (see chapter V).

A simple example was first considered to examine the feasibility of using the selected optimization techniques for the problem of identi-

fication of parameters for unsteady open channel flows. In this example, a prismatic channel of triangular section was chosen and a small computer program, solving the original St. Venant equations using the method of characteristics, was written. The program, considered as the objective function routine, was then coupled with one of the three optimization methods, and the optimization process started.

This exercise was quite successful as the data required for the flood routing exercise was estimated with reasonable accuracy compared to actual data. The optimization process was then extended to the case of compound channels, with RUFICC being used in the objective function evaluation. The results obtained for the conventional as well as the 'black box' formulations are presented later in this chapter.

8.1 Hypothetical Simple Channel Application

Before applying optimization methods to compound channels, where numerous parameters describing geometry and conveyance for the main channel and flood plain sub-sections are involved, it was felt important to validate the approach using a simple channel geometry. Accordingly, a single prismatic channel was first considered to examine the feasibility of optimization techniques and their reliability in estimating data required for the flood routing exercise.

The example is taken From "Fluid Mechanics in Water Resources

Engineering" by Li (1983). A channel of triangular cross-section is considered. The exact values of the constants c and e , which characterize the channel geometry in equation (5.4) are 6.0 and 1.0 respectively. Manning's roughness coefficient $n = 0.03$. The average channel bed slope is 8.1×10^{-5} . The initial state corresponds to a uniform flow condition with a flow depth $y_0 = 6.1m (= 20 \text{ ft})$ and a velocity $v_0 = 0.61m/s (= 2 \text{ ft/s})$.

The flow depth, y , at an upstream (origin) section ($x = 0$) then varies for a duration of 10^5 seconds, as follows:

$$y = \frac{y_0}{10} \left(11 - \cos \frac{2\pi t}{10^5} \right) \quad (8.1)$$

where $0 < t < 10^5$ seconds.

An outflow hydrograph at a station $91\,440m (= 3 \times 10^5 \text{ ft})$ from the origin was also given. This was considered as the observed hydrograph at the downstream station. Both inflow and outflow hydrographs provided are shown in Fig. 8.1.

The simulated hydrograph was obtained by integrating the St. Venant equations using the method of characteristics. Time and space steps were selected as $\Delta t = 5 \times 10^3$ seconds and $\Delta x = 45\,720m (= 1.5 \times 10^5 \text{ ft})$ respectively [these values were selected in a way to satisfy the Courant criterion required for numerical stability].

8.1.1 Results

Powell's method

Powell's multivariable non-linear optimization method was first applied to solve the problem and estimate the parameters. Convergence was very slow and the solution did not yield the correct minimum. Although it was supposed to be very close to zero, the final objective function value was 7.0065. The estimated parameters c , e , and n were 23.82, 0.93, and 0.035 respectively. This poor performance of Powell's algorithm is explained by its major drawback, namely the search directions tend to be linearly dependent on each other. This results in the collapse of the search directions, which limits the search space and therefore prevents convergence to the correct minimum.

Rosenbrock algorithm

The Rosenbrock algorithm was used with different initial guesses of the optimization variables and different step expansion and reduction values (ϵ and α respectively). First, it was observed that if ϵ is very small (close to 1.0) convergence is very slow. On the other hand, numerical instability and program interruption occurred for ϵ values greater than 1.7. Therefore, this value for the step expansion coefficient (ϵ) was considered to be the most suitable in this instance. In runs 1 to 5 (Table 8.1), the same initial guesses but different α values were considered. Based on the results obtained it can be seen

that values of the optimization variables (c , e , and n) varied very little with α , except when $\alpha = 0.55$ (when a really low estimate of Manning's n was obtained). On the other hand, the number of function evaluations was observed to increase considerably for α values exceeding 0.6. Very low α values ($\alpha = 0.25$) also resulted in a large number of function evaluations.

For the problem in hand, it should be noted that it is extremely important to keep the number of objective function evaluations to a minimum since every evaluation involves the numerical integration of the complete unsteady flow equations over a grid of space and time. Therefore, in order to minimize the number of objective function evaluations (and consequently computer time) $\alpha = 0.5$ was adopted in this study. In runs 7 to 10, α and ϵ were kept constant (0.5 and 1.7 respectively), but different initial guesses of the optimization variables (c , e , and n) were considered. Reasonably good results were obtained for e and n values, which were always close to the exact values (1.0 and 0.03 respectively). On the other hand, c was underestimated if the initial value is an underestimate of the exact value and overestimated if the initial value was an overestimate of the exact value.

8.1.2 Nelder and Meade's Simplex Algorithm

Four runs, with different initial guesses of the optimization variables, were considered (Table 8.2). The values of c , e , and n obtained were observed to be very close (or equal) to the exact values (6.0, 1.0, 0.03

respectively) for all initial guesses considered. Furthermore, the final objective function values were observed to be much lower than those obtained using the Rosenbrock algorithm. This would imply that the Simplex Method is the most suitable optimization technique for this particular example.

8.2 Laboratory Compound Channel Application

The composite prismatic channel that provided data for RUFICC calibration and validation exercises (chapter VI) was also adopted for this 'optimization' exercise. The data used was that for flood event 8 (Table 6.1). Measured discharge hydrographs at a designated upstream station were the model's upstream boundary condition. Observed hydrographs at a downstream station served to give the differences between observed and simulated stage and discharge values, required for the objective function evaluation.

8.2.1 Conceptual Formulation

In the earlier discussion of the conceptual formulation of the optimization model (chapter V) seven parameters (n_1 , n_2 , n_3 , T_1 , T_3 , c , and e) were considered the optimization variables. However, for this particular example Manning's roughness coefficients for the three dif-

ferent sub-sections are equal and hence the number of parameters is reduced to five.

Powell's multivariable non-linear optimization method was first applied to solve the problem and estimate the parameters. In this instance, convergence was very slow and the objective function never reduced to an acceptable value. This poor performance of Powell's algorithm was also noted while applying the optimization process to the previous example (simple triangular channel).

Two sets of initial guesses for the optimization parameters were considered for both the Rosenbrock and Simplex Methods. The values of the initial and final optimization parameters are shown in Table S.3. The first set of initial guesses yielded an initial objective function value of 38.05. This value was reduced to 0.72 using Rosenbrock's method and to 0.22 using the Nelder and Meade Simplex Algorithm. The required number of iterations to achieve convergence was 101 and 52 respectively. Using the second set of initial guesses, for which the initial objective function value was 3.82, much lower final objective function values (0.50 and 0.06 for the Rosenbrock and Simplex methods respectively) were obtained. The number of objective function evaluations was also reduced from 101 to 51 using Rosenbrock's method. This underscores the importance of good initial guesses, reparameter values, in the optimization exercise. In fact reasonable initial estimates speed up convergence and therefore save computer time. It is also worth mentioning that, compared to Rosenbrock's

method, the Nelder and Meade Simplex Algorithm resulted in much lower objective function values for both sets of initial guesses. The optimization parameters are also closer to the exact values.

The data obtained through the optimization exercise were used for the simulation of different flood events observed in Treske's (1980) compound prismatic channel. Simulated peak stages and discharges for the 10 flood events reported by Treske are shown in Table 8.4. Errors between observed and simulated stage and discharge values are shown in Table 8.5. Negative error values imply overestimation of discharge or stage. It should be noted that events 1 to 4 do not have any flood plain flow component even at the peak flow. Errors in stage and discharge (E_h and E_q respectively) are shown to be reasonably low and range from 0 to $\pm 4\%$ for run S_2 , for which the final objective function value was 0.06. It should be kept in mind that E_h and E_q are not due exclusively to the estimation of flood routing data but also to the unsteady flow model used and its reliability in simulating stages and discharges. In fact, the unsteady flow model adopted was known to slightly overestimate stages and underestimate discharges. As a result, E_h is generally shown to be negative while E_q is generally positive. Reported errors in stage and discharge using actual routing data were within $\pm 3\%$.

Comparing peak stages and discharges obtained using actual and optimal data is not considered sufficient and the simulated and observed hydrographs have also to be compared. Observed versus simulated

discharge and stage hydrographs (using actual and optimal data) for runs R_1 , S_1 , R_2 , and S_2 are shown in Figs. 8.2 through 8.5 respectively. In these figures flood event 10 was selected because it is associated with the highest flood plain depth and therefore the effect of the optimization parameters describing the flood plain sub-section can be emphasized. Furthermore, it would be impractical to show all simulated hydrographs for the 10 different flood events and 4 optimization experiments in graphical form.

Examining Figs. 8.2 to 8.5, it can be seen that generally a small difference between both simulated discharge hydrographs (using actual and optimal data) is observed. However, a larger error, especially near the peak, is generally induced in simulated stage hydrographs, (see Figs. 8.2 and 8.4). The largest difference between stage hydrographs obtained using optimal and actual data was associated with run R_1 , shown in Fig. 8.2. For this particular optimization experiment the final objective function value was 0.72, which is considered quite high. This was the result of poor initial estimates of the model parameters that yielded an initial objective function value of 38.05. This underscores again the importance of 'good' initial guesses before starting the optimization process. These results also emphasize that, for reliable estimates of the optimization model parameters, the corresponding optimum objective function value should be as low as possible (usually in the order of 0.1). Furthermore the optimization process should not be based on a single flood event but different flood events should be adopted and then the results, in terms of both the

optimization model parameters and their resulting simulated hydrographs for another flood event, should be compared.

8.2.2 'Black Box' Formulation

In this second formulation six parameters requiring estimation through the optimization process were generated (Chapter V). Initial as well as optimum values for these parameters are shown in Table S.6 for both Powell's and Rosenbrock's methods and in Table S.7 for the Nelder and Meade Simplex Algorithm.

Using Powell's Method the values of the optimization parameters did not change much from the original initial guesses, which proves again the method's limited performance. However, when the optimized parameters were used to route other flood events, the corresponding errors in stage and discharge were not significantly different from those using other methods. This is because good initial estimates were assigned to the model parameters prior to the optimization process.

After choosing a set of initial guesses that was close to true values, initial estimates were varied to examine the resulting impact on the optimization process. Using the Simplex Method in run S_1 initial estimates of a_m and c_p were changed to 1.0 and 1000.0 respectively, which are values considered to be far from actual values. Nevertheless, after optimization the final values for a_m and c_p were 1.44 and 467.0, which are close to the optimal parameters when good initial estimates were used, as in run S_2 . This shows that the method converges reasonably

well to the minimum even if initial estimates of some model parameters are not close to true values. Nevertheless, poor initial estimates translate into increased number of iterations and objective function evaluations. So, while good initial guesses do not significantly affect the optimization outcome, they reduce the number of objective function evaluations and therefore save computer time.

The optimization parameter values obtained using the different optimization techniques and the corresponding initial estimates were then used to route Treske's observed flood hydrographs. The resulting stages and discharges appear in Tables 8.8. and 8.9. The corresponding relative errors, compared to observed values, are listed in Table 8.10. Comparisons between observed and simulated discharge and stage hydrographs for flood event 10 and optimization runs P_1 , R_1 , P_2 , R_2 , S_1 and S_3 are shown in Figs. 8.6 through 8.11. Again these figures are representative of the results obtained, as it would be impractical to show all of the findings in graphical form. According to Figs. 8.6 through 8.11, stage and discharge hydrographs obtained using optimal parameters were shown to be in a good agreement with observed data and with stages and discharges generated using actual geometric and hydraulic properties. Near the peak, stages were shown to be overestimated using optimal data and underestimated using actual data. The error induced in absolute magnitude is generally about the same in both cases.

According to Table 8.10, using the different optimization experiments.

errors in stage and discharge are generally within $\pm 5\%$. This suggests that there is no significant difference in peak stages and discharges when different optimal parameters (obtained from different numerical experiments) are used. There is, for example, a difference of more than 20 % in the value of b_m between runs S_1 and S_4 . However, the corresponding values for both stage and discharge are almost the same for all 10 flood events (Table 8.9). This supports the conclusion that model parameters do not need to be the same for the different numerical experiments. In fact, what counts is their combined effect in the 'area' or 'conveyance' relationships required for the flood routing exercise.

Since the cross-sectional area, $A(y)$, and conveyance, $K(y)$, are operated upon in the optimization process, these values should be compared to check the consistency of solutions. Figs. 8.12 through 8.19 show functional relationships for the main channel cross-sectional area and conveyance in terms of flow depth, for all optimization experiments considered. It is clear from these figures that the 'conveyance' values show good agreement with actual data while the cross-sectional area variation does not show a similar consistency. This might be explained by looking at the relative values of the different terms of the unsteady flow equations.

Based on an 'order of magnitude' analysis, Henderson (1966) and Wormleaton and Karmegam (1984) showed that the terms in which the cross-sectional area are used are much smaller than those asso-

ciated with conveyance. This would imply that variations in $A(y)$ alone will have much less effect on the solution of the unsteady flow equations than variations in $K(y)$. Therefore the value of $K(y)$ will be much more constrained by the optimization process while larger variations in the value of $A(y)$ are to be expected.

Representative relationships for cross-sectional area and conveyance in terms of flow depth for the flood plain sub-section are shown in Figs. S.20 through S.22. These figures show that conveyance values as well as the values of cross-sectional area are in a reasonable agreement with observed data. The difference between the two, however, was shown to increase significantly with increasing flow depth, and specially when the depth of flow increases beyond the maximum stage associated with the flood event adopted in the optimization process. Compared to the conceptual formulation, the objective function evaluation using the 'black box' optimization model took much less computer time. In a particular application the time required for a single objective function evaluation on an AMDAHL 5680 mainframe computer was reduced from 52 to 16 seconds when the 'black box' formulation was adopted.

For both Rosenbrock's method and Powell's algorithm, the convergence criterion which has to be satisfied before terminating the optimization procedure was based on the difference in objective function value for two successive iterations. In this instance, the difference is compared to a given tolerance value [10^{-4} for this application] and

the procedure terminates if the former is less or equal to the latter. Using the Nelder and Meade Simplex method the objective function is evaluated at the vertices of the simplex and the variance of these objective function values is estimated. If it is less than or equal to a given critical value [10^{-10} for this application] the optimization procedure terminates and the desired minimum is reached. Two additional tests, using the Nelder and Meade Simplex Algorithm and the same set of initial estimates but different tolerance values (10^{-4} and 10^{-8} respectively), were performed. Table (8.11) lists the optimal parameters that were obtained as well as the required number of iterations and the final objective function values. In the second test the number of iterations was increased by more than 200 % and the objective function was decreased slightly. The optimal parameters resulting from both tests were used to route the 10 flood events generated in the experimental compound channel. Predicted stages and discharges are compared to observed data in Table 8.12. It is evident that the difference in the outcome of both tests is not significant.

Changing the final critical value of the variance from 10^{-4} to 10^{-8} produced a significant increase in the number of iterations with only a marginal improvement in the estimation of stages and discharges. Thus, the convergence criterion should be selected keeping in mind the degree of accuracy required in the modeling exercise and computer time allocation. Accordingly, the optimization process is stopped either when a particular value of the standard deviation of errors is reached, or when a number of iterations is completed.

8.3 Field Compound Channel Application

The 'black box' optimization approach was used in connection with the Nelder and Meade Simplex method to identify flood routing parameters for the River Main, considered earlier in the validation exercise for RUFICC (chapter IV). The study reach, 800 m long, was assumed prismatic and the flood plain was treated as a rectangular section even though a mild transverse slope was present according to the data provided.

Since only one time series of depths was available, it was divided into two segments. One part was used in the optimization process to evaluate the flood routing parameters while the other was used to check the accuracy of the optimum parameters obtained by comparing resulting downstream depths with observed data. Upstream and downstream stage hydrographs used in the optimization procedure are shown in Fig. 8.23. Based on an examination of data on the river geometry and boundary roughness coefficients a set of initial estimates for the optimization model parameters was selected. These initial guesses as well as the optimum parameters obtained are shown in Table 8.13. According to this table, the objective function value was reduced from 12.53 to 0.1105 in 23 iterations.

The optimum parameters were then used to route the complete time series of depths. Simulated and observed stages at the downstream

section of the study reach are shown in Fig. S.24, which shows reasonably good agreement between the two. On the other hand, no significant difference can be seen when the results obtained using optimal data (Fig. S.24) are compared to those obtained using complete actual geometry and roughness data (Fig. 6.15).

In this field application reasonably accurate depth measurements at both upstream and downstream ends of the study reach assisted in obtaining fairly accurate estimates of the optimal flood routing parameters. However, errors are generally induced in field data due to imprecise measurement procedures and small sampling rates. In these circumstances, the user should not rely on one single flood event in the optimization process, but several flood hydrographs should be considered.

Table 8.1: Numerical Experiments Using Rosenbrock Algorithm for the Hypothetical Channel Application.

Run	α	ϵ	c_o	e_o	n_o	c	e	n	F_o	No. It
1	0.25	1.70	5.3	0.8	0.02	5.60	0.979	0.0204	2×10^{-3}	97
2	0.50	1.70	5.3	0.8	0.02	5.70	1.000	0.0287	4×10^{-3}	82
3	0.60	1.70	5.3	0.8	0.02	5.65	1.000	0.0291	4×10^{-3}	88
4	0.75	1.70	5.3	0.8	0.02	5.64	1.000	0.0294	5×10^{-3}	181
5	0.85	1.70	5.3	0.8	0.02	5.58	0.936	0.0127	6×10^{-3}	184
6	0.50	1.70	5.3	0.9	0.02	5.59	1.000	0.0290	6×10^{-3}	134
7	0.50	1.70	5.3	1.0	0.03	6.00	1.000	0.0300	4×10^{-7}	15
8	0.50	1.70	5.3	1.0	0.03	6.39	1.000	0.0320	1×10^{-2}	38
9	0.50	1.70	5.3	1.0	0.02	6.95	0.995	0.0320	3×10^{-2}	89
10	0.50	1.70	5.3	1.0	0.035	6.40	0.997	0.0310	6×10^{-3}	59

Table 8.2: Numerical Experiments Using Simplex Method for the Hypothetical Channel Application.

Run	c_o	e_o	n_o	c	e	n	F_o	No. It
1	5.3	0.80	0.020	6.00	1.00	0.030	510^{-6}	60
2	4.5	0.80	0.020	5.94	1.00	0.031	310^{-4}	76
3	5.3	0.80	0.025	5.97	1.00	0.030	410^{-5}	70
4	5.3	0.95	0.020	6.00	1.00	0.030	510^{-6}	86

Table S.3: Summary of Model Parameters Using Rosenbrock (R) and Simplex (S) Methods (Conceptual Approach).

Variable and Symbol	Trial Value (1)	Optimal Values		Trial Value (2)	Optimal Values		Actual Value
		$R_1^\dagger$	$S_1^\dagger$		R_2	S_2	
Manning' s (n)	0.015	0.015	0.011	0.010	0.009	0.014	0.012
Top Width (T_1)	3.20	3.28	3.17	2.30	3.23	2.80	3.00
Top Width (T_3)	1.60	1.67	1.69	1.20	2.15	1.96	1.50
Coefficient (c)	1.00	1.14	1.18	0.90	1.01	1.44	1.25
exponent (e)	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$\dagger R$ for Rosenbrock Method

$\dagger S$ for Simplex Method

Table 8.4: Observed Versus Estimated Stages and Discharges (Conceptual Approach).

Event	Observed Values		Simulated Values Using Rosenbrock (R) and Simplex (S) Methods							
			R_1		S_1		R_2		S_2	
	No.	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$	$Q_p(m^3/s)$	$H_p(m)$
1	0.351	0.158	0.343	0.154	0.358	0.155	0.369	0.158	0.346	0.151
2	0.368	0.171	0.370	0.168	0.378	0.171	0.391	0.173	0.368	0.167
3	0.383	0.183	0.394	0.181	0.396	0.184	0.408	0.185	0.386	0.181
4	0.392	0.190	0.414	0.193	0.409	0.194	0.420	0.194	0.401	0.193
5	0.458	0.255	0.480	0.250	0.474	0.258	0.480	0.260	0.464	0.252
6	0.481	0.291	0.502	0.280	0.495	0.290	0.499	0.290	0.489	0.286
7	0.495	0.315	0.516	0.300	0.508	0.310	0.510	0.310	0.504	0.308
8	0.512	0.344	0.531	0.323	0.521	0.332	0.522	0.331	0.518	0.329
9	0.533	0.380	0.561	0.372	0.546	0.377	0.545	0.377	0.547	0.377
10	0.540	0.394	0.568	0.383	0.550	0.386	0.549	0.385	0.552	0.386

Table S.5: Percentage of Error in Stage and Discharge (Conceptual Approach).

Event No.	Run R_1		Run S_1		Run R_2		Run S_2	
	E_h	E_q	E_h	E_q	E_h	E_q	E_h	E_q
1	2.7	2.5	-2.3	1.9	-6.0	0.0	1.7	4.4
2	-0.6	1.8	-3.2	0.0	-7.3	-1.2	0.0	2.3
3	-3.3	1.1	-3.9	-0.5	-7.6	-1.1	-0.9	1.1
4	-6.5	-1.6	-5.0	-2.1	-8.2	-2.1	-2.6	-1.6
5	-5.4	2.0	-3.9	-1.2	-5.4	-2.0	-1.5	1.2
6	-4.9	3.8	-3.3	0.3	-4.2	0.3	-1.9	1.7
7	-4.7	4.8	-2.9	1.6	-3.4	1.6	-2.0	2.2
8	-4.1	6.1	-2.0	3.5	-2.2	3.8	-1.3	4.4
9	-5.8	2.1	-2.7	0.8	-2.5	0.8	-2.9	0.8
10	-5.7	2.8	-2.0	2.0	-1.8	2.3	-2.5	2.0

Table S.6: Optimal Parameters Using Powell's (P) and Rosenbrock's (R) Methods ('Black Box' Approach).

Optimization Parameter	Trial Value (1)	Optimal Values		Trial Value (2)	Optimal Values	
		P_1	R_1		P_2	R_2
a_m	1.25	1.41	1.45	1.25	1.50	1.41
b_m	1.00	0.88	0.70	0.80	0.50	1.01
c_m	60.0	68.2	67.5	60.0	64.2	66.5
d_m	1.50	1.49	1.50	1.50	1.47	1.47
a_p	4.50	4.50	5.25	4.50	4.50	4.68
c_p	400	380	475	400	338	469

Table 8.7: Optimal Parameters Using Nelder and Meade Simplex (S) Method ('Black Box' Approach).

Optimization Parameter	Run (S_1)		Run (S_2)		Run (S_3)		Run (S_4)	
	Initial	Optimal	Initial	Optimal	Initial	Optimal	Initial	Optimal
a_m	1.00	1.44	1.25	1.35	1.25	1.28	1.25	1.36
b_m	1.00	1.07	1.00	0.97	0.80	0.80	0.80	0.81
c_m	60.0	60.3	60.0	63.1	50.0	52.0	60.0	60.0
d_m	1.50	1.39	1.50	1.41	1.50	1.28	1.50	1.38
a_p	4.50	4.22	4.50	4.39	4.00	4.06	4.50	4.24
c_p	1000	467	400	426	500	515	400	427

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Event	Observed Values		Simulated Values Using Powell (P) and Rosenbrock (R) Methods								
			P ₁		R ₁		P ₂		R ₂		
			H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)	
No.											
1	0.351	0.158	0.344	0.150	0.346	0.148	0.349	0.148	0.345	0.151	
2	0.368	0.171	0.366	0.167	0.370	0.167	0.374	0.167	0.367	0.167	
3	0.383	0.183	0.384	0.182	0.388	0.181	0.393	0.182	0.384	0.181	
4	0.392	0.190	0.396	0.192	0.401	0.192	0.406	0.192	0.397	0.192	
5	0.458	0.255	0.464	0.255	0.462	0.255	0.467	0.253	0.465	0.255	
6	0.481	0.291	0.490	0.285	0.486	0.288	0.494	0.287	0.489	0.286	
7	0.495	0.315	0.506	0.304	0.500	0.308	0.511	0.308	0.504	0.306	
8	0.512	0.344	0.521	0.324	0.513	0.327	0.526	0.327	0.518	0.325	
9	0.533	0.380	0.555	0.366	0.542	0.368	0.559	0.369	0.551	0.366	
10	0.540	0.394	0.564	0.376	0.549	0.378	0.567	0.380	0.558	0.378	

Table 8.9: Observed Versus Estimated Stages and Discharges Using the Simplex Method ('Black Box' Approach).

Event	Observed Values		Simulated Values							
	Values		S ₁		S ₂		S ₃		S ₄	
	H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)	H _p (m)	Q _p (m ³ /s)
1	0.351	0.158	0.342	0.150	0.342	0.151	0.345	0.149	0.341	0.149
2	0.368	0.171	0.365	0.166	0.364	0.168	0.371	0.166	0.365	0.166
3	0.383	0.183	0.384	0.180	0.382	0.182	0.393	0.181	0.385	0.181
4	0.392	0.190	0.398	0.191	0.395	0.192	0.408	0.191	0.398	0.191
5	0.458	0.255	0.466	0.252	0.467	0.254	0.471	0.252	0.466	0.254
6	0.481	0.291	0.493	0.285	0.492	0.285	0.498	0.288	0.493	0.287
7	0.495	0.315	0.508	0.306	0.508	0.304	0.515	0.310	0.509	0.307
8	0.512	0.344	0.524	0.325	0.524	0.322	0.530	0.331	0.525	0.327
9	0.533	0.380	0.557	0.368	0.559	0.366	0.561	0.372	0.557	0.370
10	0.540	0.394	0.565	0.379	0.568	0.377	0.569	0.383	0.566	0.380

Table 8.10: Percentage of Error in Stage and Discharge ('Black Box' Approach).

Event No.	Run P_1		Run R_1		Run P_2		Run R_2		Run S_1		Run S_2		Run S_3		Run S_4	
	E_h	E_q	E_h	E_q	E_h	E_q	E_h	E_q	E_h	E_q	E_h	E_q	E_h	E_q	E_h	E_q
1	2.3	5.1	1.7	6.3	0.7	6.3	2.0	4.4	3.0	5.1	3.0	4.4	2.0	5.7	3.3	5.7
2	0.6	2.3	-0.6	2.3	-1.9	2.3	0.3	2.3	0.9	2.9	1.3	1.8	-0.9	2.9	0.9	2.9
3	-0.3	0.5	-1.5	1.1	-3.0	0.5	-0.3	1.1	-0.3	1.6	0.3	0.5	-3.0	1.1	-0.6	1.1
4	-1.2	-1.1	-2.6	-1.1	-4.1	-1.1	-1.5	-1.1	-1.8	-0.5	-0.9	-1.1	-4.7	-0.5	-1.8	-0.5
5	-1.5	0.0	-1.0	0.0	-2.2	0.8	-1.7	0.0	-2.0	1.2	-2.2	0.4	-3.2	1.2	-2.0	0.4
6	-2.1	2.1	-1.2	1.0	-3.0	1.4	-1.9	1.7	-2.8	2.1	-2.6	2.1	-4.0	1.0	-2.8	1.4
7	-2.5	3.5	-1.1	2.2	-3.6	2.2	-2.0	2.9	-2.9	2.9	-2.9	3.5	-4.5	1.6	-3.2	2.5
8	-2.0	5.8	-0.2	4.9	-3.0	4.9	-1.3	5.5	-2.6	5.5	-2.6	6.4	-3.9	3.8	-2.8	4.9
9	-4.6	3.7	-1.9	3.2	-5.4	2.9	-3.7	3.7	-5.0	3.2	-5.4	3.7	-5.8	1.8	-5.0	2.6
10	-4.9	4.6	-1.8	4.1	-5.5	3.6	-3.7	4.1	-5.1	3.8	-5.7	4.3	-5.9	2.8	-5.3	3.6

Table 8.11: Optimization Features for Tests with Different Convergence Criteria.

Optimization Parameter/Feature	Trial	Optimal Values	
	Value	I	II
Variance	-	10^{-4}	10^{-8}
F_o	-	0.0566	0.0377
No. of Iter.	-	34	101
a_m	1.25	1.68	1.81
b_m	1.00	0.84	1.25
c_m	60.0	57.6	59.9
d_m	1.50	1.35	1.37
a_p	4.5	4.32	6.00
c_p	400.0	471.0	575.0

Table 8.12: Estimated Stages and Discharges for the two Additional Tests with Different Convergence Criteria.

Flood	TEST I				TEST II			
Event	Q_p	H_p	E_q	E_h	Q_p	H_p	E_q	E_h
No.	(m^3/s)	(m)	(%)	(%)	(m^3/s)	(m)	(%)	(%)
PC1	0.144	0.333	8.9	5.1	0.147	0.336	7.0	4.3
PC2	0.161	0.358	5.8	2.7	0.163	0.358	4.7	2.7
PC3	0.178	0.381	2.7	0.5	0.178	0.379	2.7	1.0
PC4	0.190	0.397	0.0	-1.3	0.190	0.394	0.0	-0.5
PC5	0.244	0.457	4.3	0.2	0.243	0.457	4.7	0.2
PC6	0.284	0.486	2.4	-1.0	0.275	0.481	5.5	0.0
PC7	0.307	0.503	2.5	-1.6	0.295	0.495	6.3	0.0
PC8	0.328	0.517	4.7	-1.0	0.315	0.508	8.4	0.8
PC9	0.371	0.548	2.4	-2.8	0.359	0.538	5.5	-0.9
PC10	0.381	0.556	3.3	-3.0	0.372	0.547	5.6	-1.3

Table S.13: Optimization Features for River Main Reach.

Optimization Parameter/Feature	Trial Value	Optimal Value
Variance	-	0.3910^{-4}
F_o	12.53	0.1105
No. of Iter.	-	23
a_m	12.0	11.908
b_m	1.0	0.912
c_m	300.0	383.6
d_m	1.5	1.863
a_p	6.5	6.847
c_p	80.0	81.40

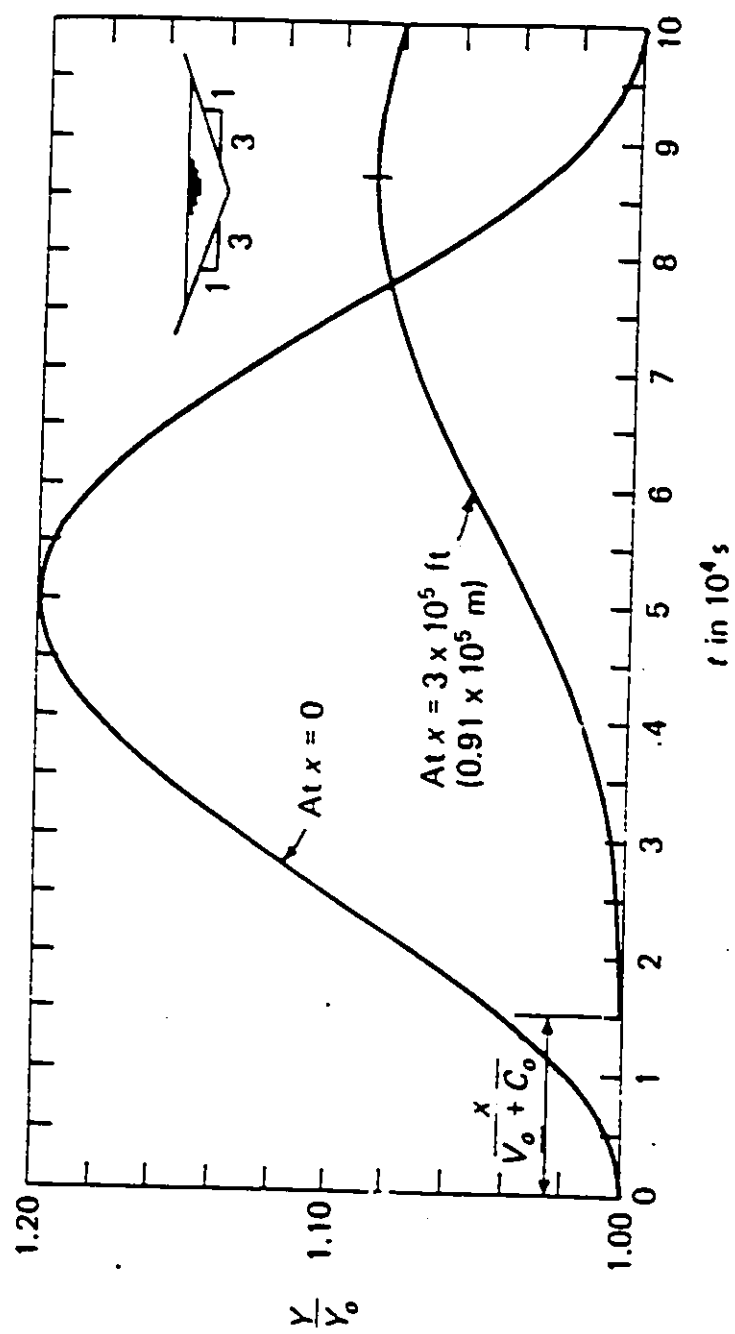


FIG. 8.1 Inflow and Outflow Stage Hydrographs for the Hypothetical Channel Application
(after LI 1983).

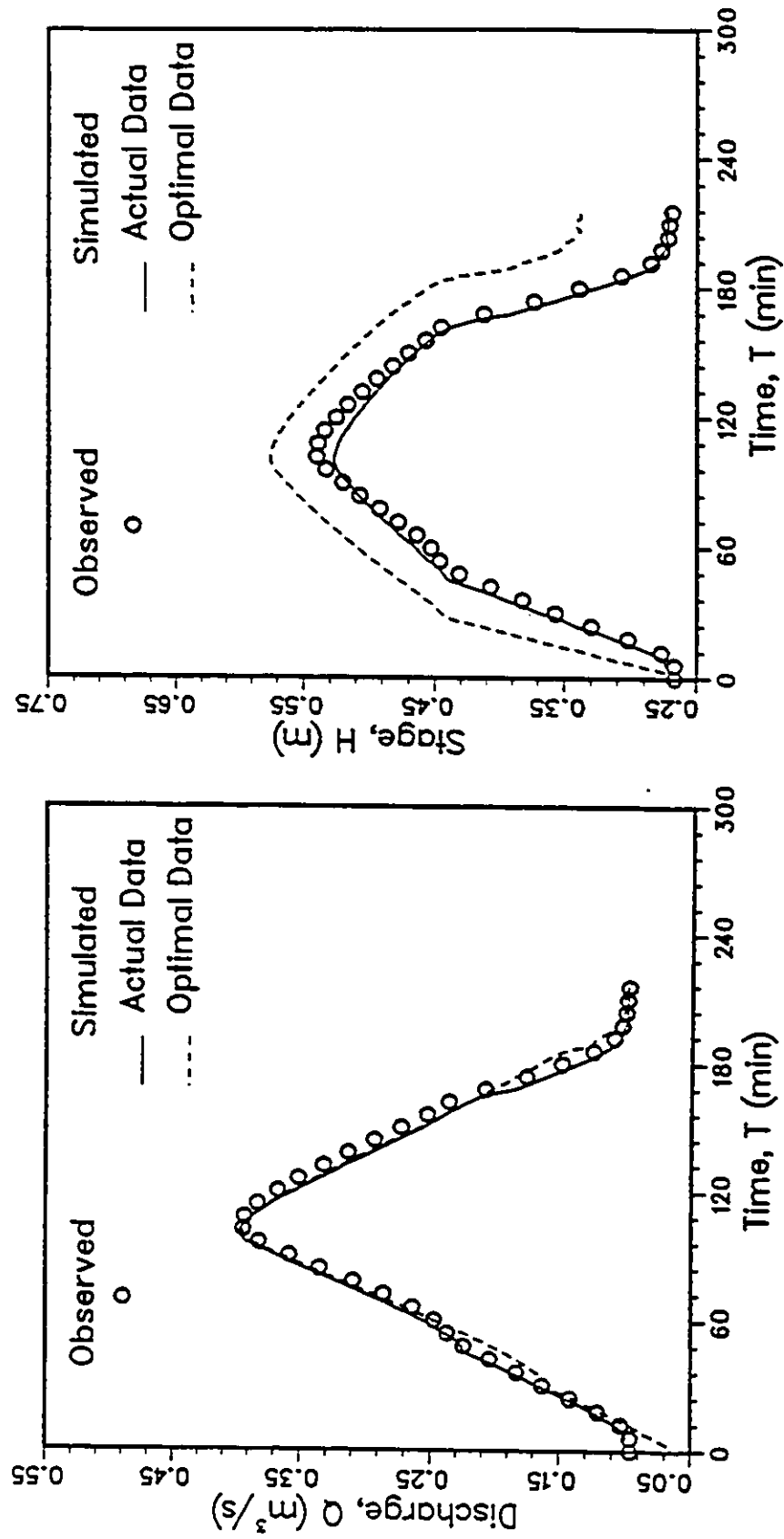


FIG. 8.2 Observed versus Simulated Discharge and Stage Hydrographs, Conceptual Approach, (Run R1, Event 10).

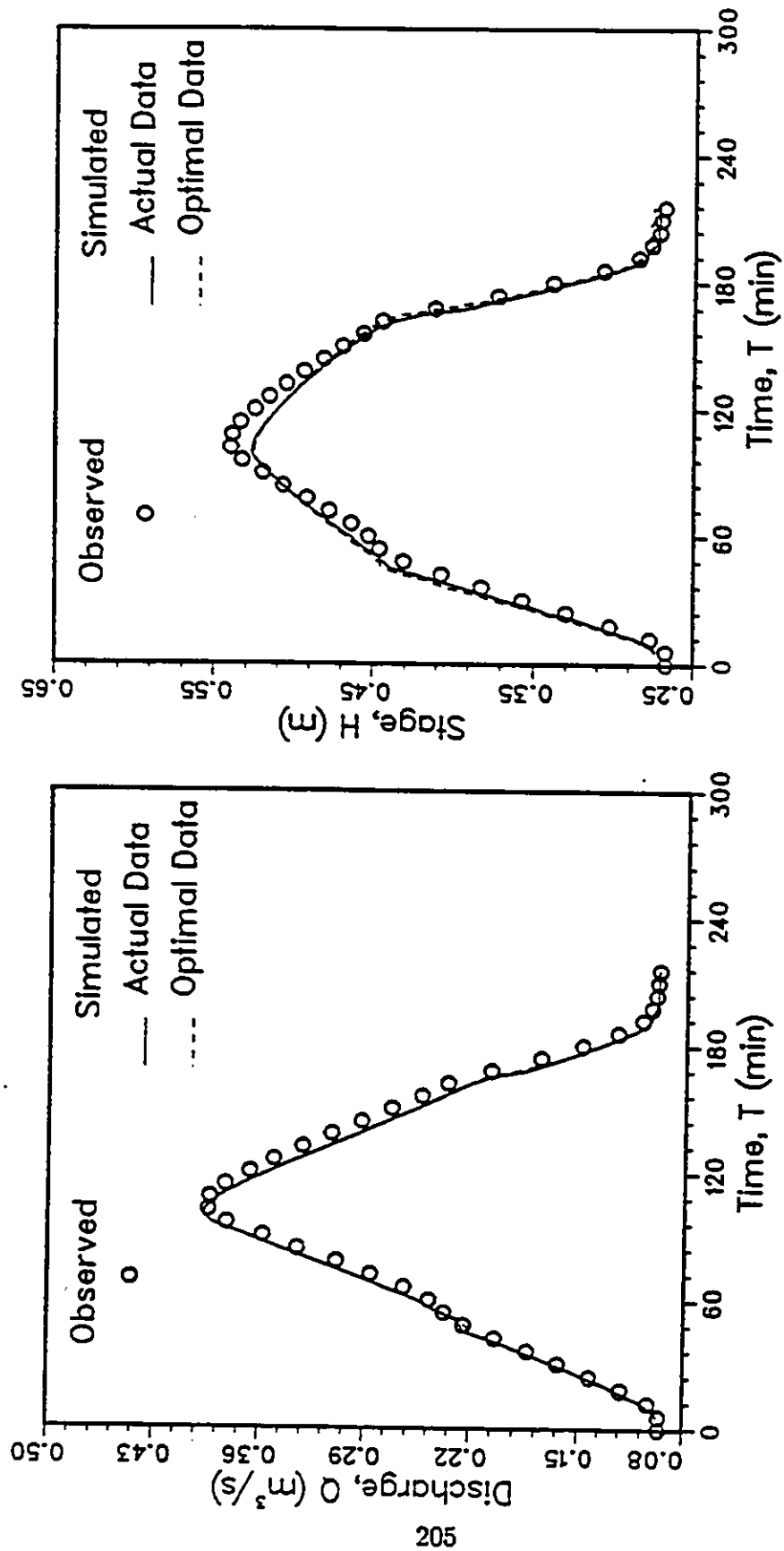


FIG. 8.3 Observed versus Simulated Discharge and Stage Hydrographs, Conceptual Approach, (Run S1, Event 10).

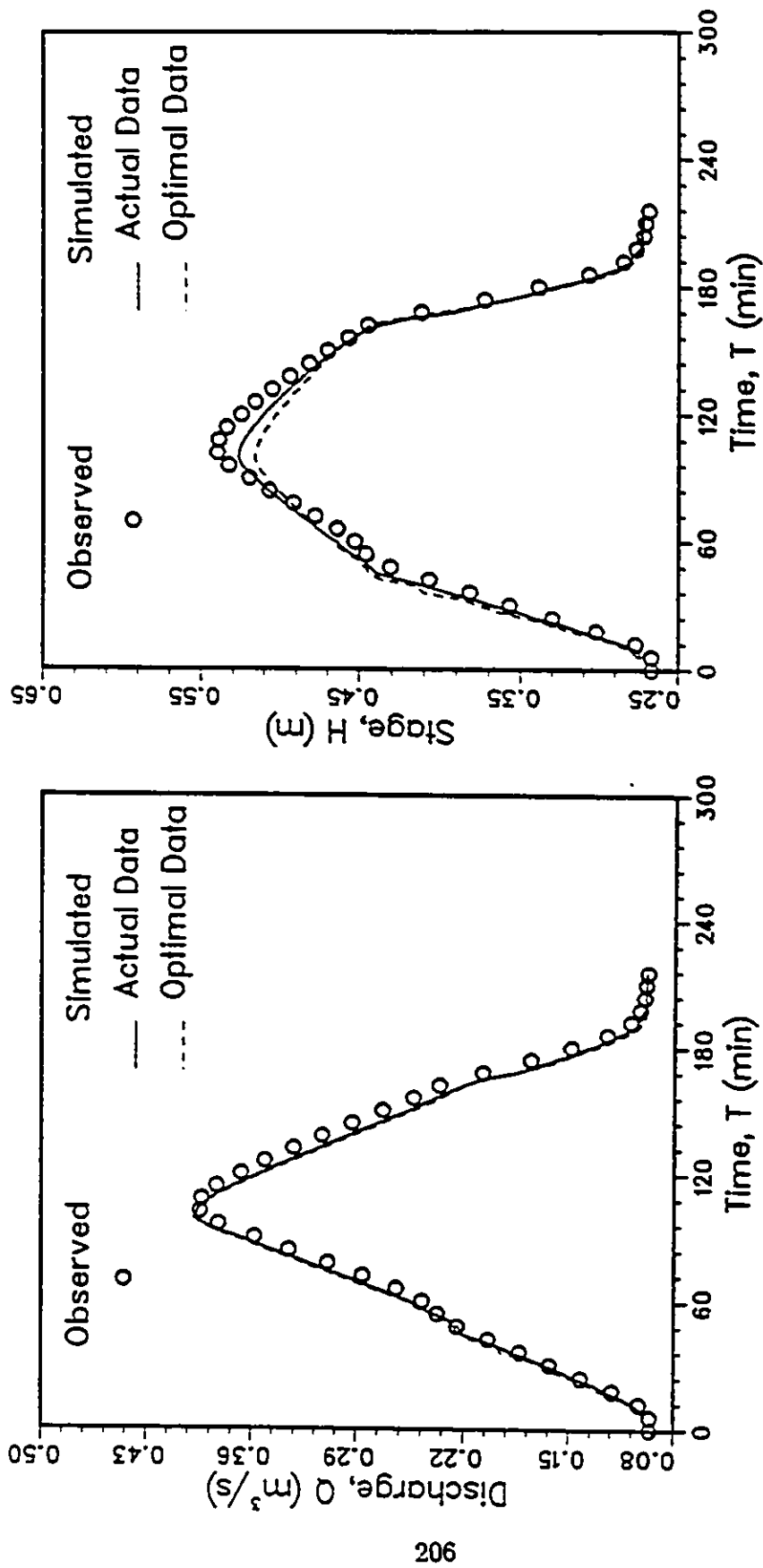


FIG. 8.4 Observed versus Simulated Discharge and Stage Hydrographs, Conceptual Approach, (Run R2, Event 10).

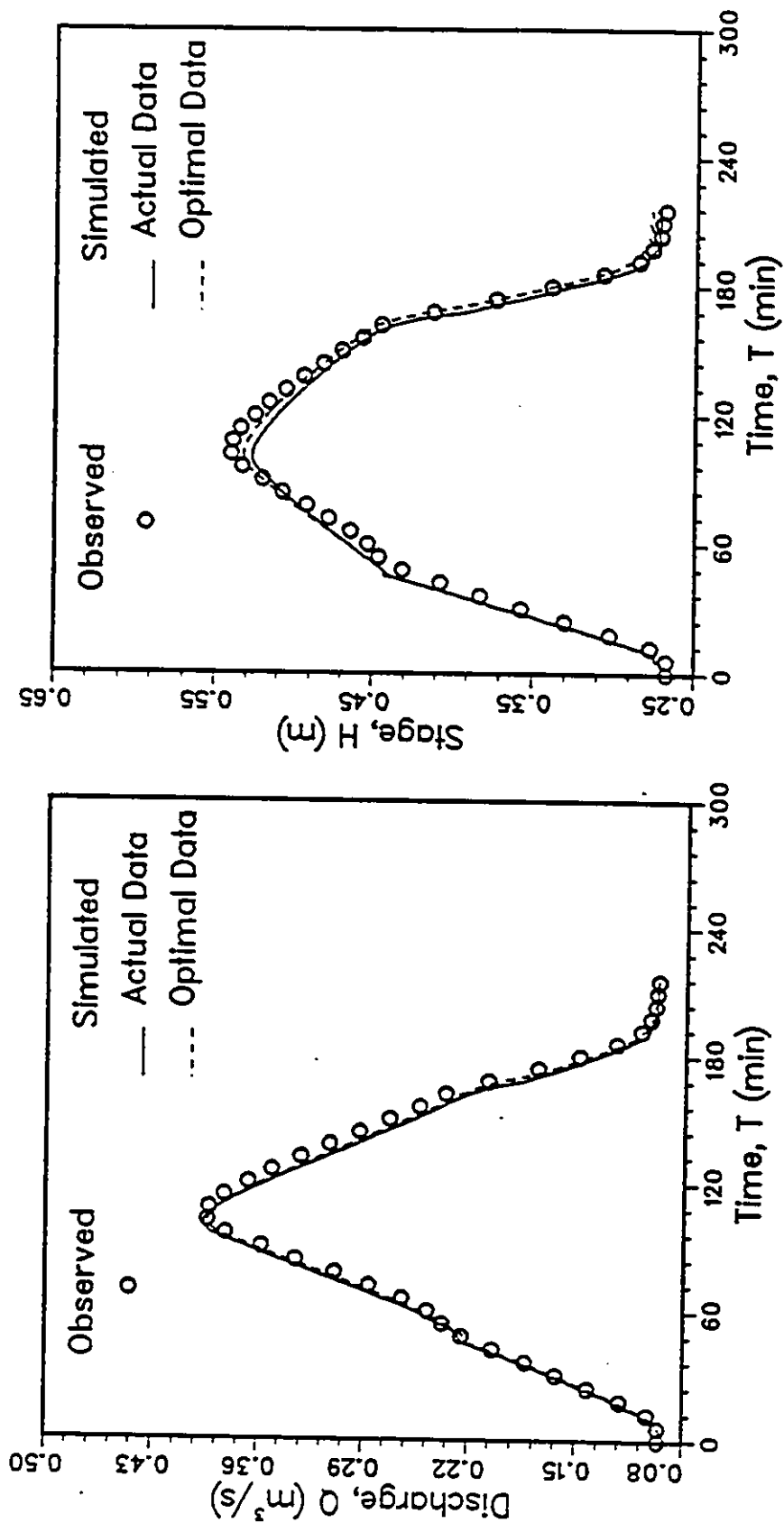


FIG. 8.5 Observed versus Simulated Discharge and Stage Hydrographs, Conceptual Approach, (Run S2, Event 10).

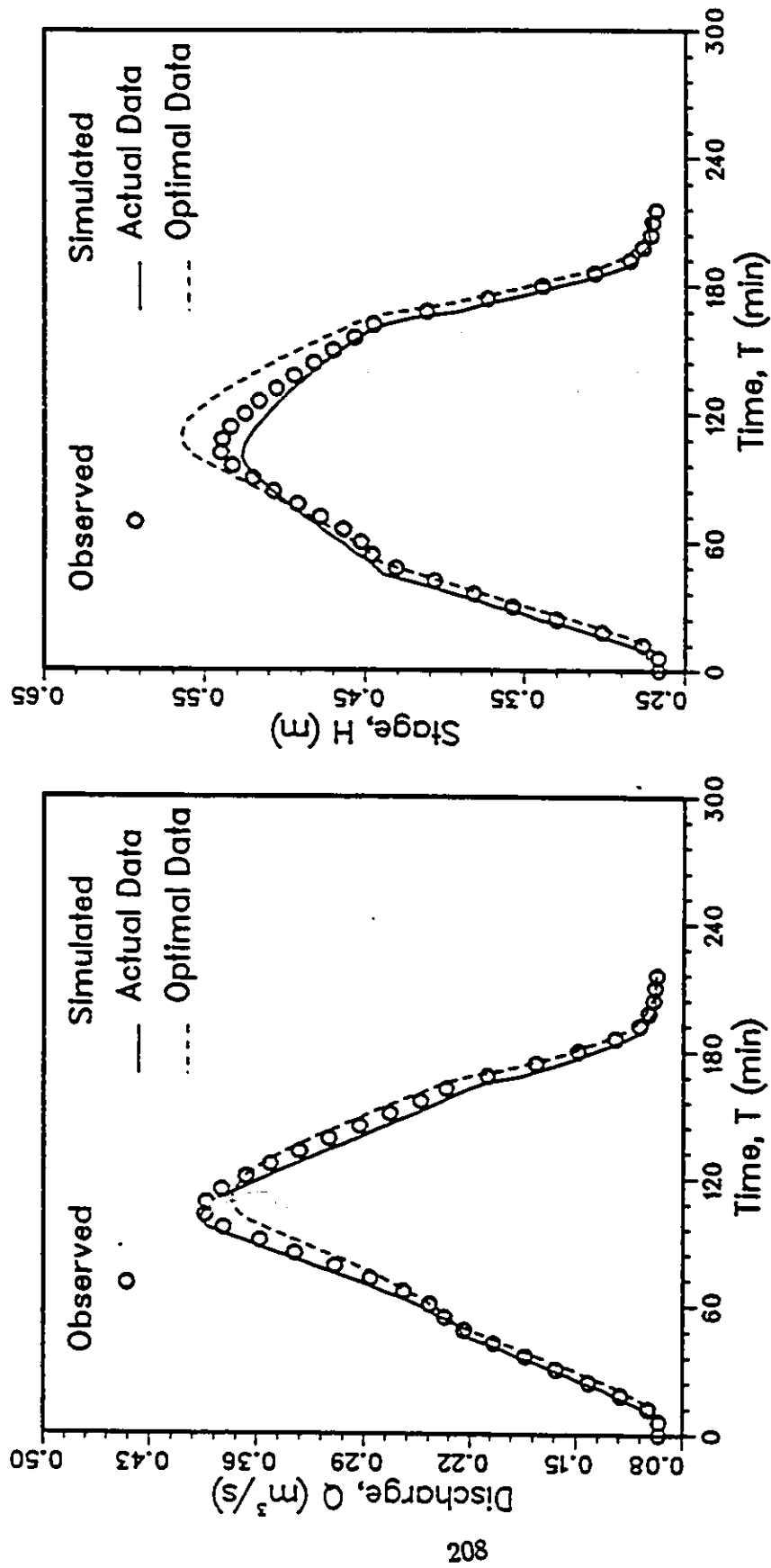


FIG. 8.6 Observed versus Simulated Discharge and Stage Hydrographs, Black Box Approach, (Run P1, Event 10).

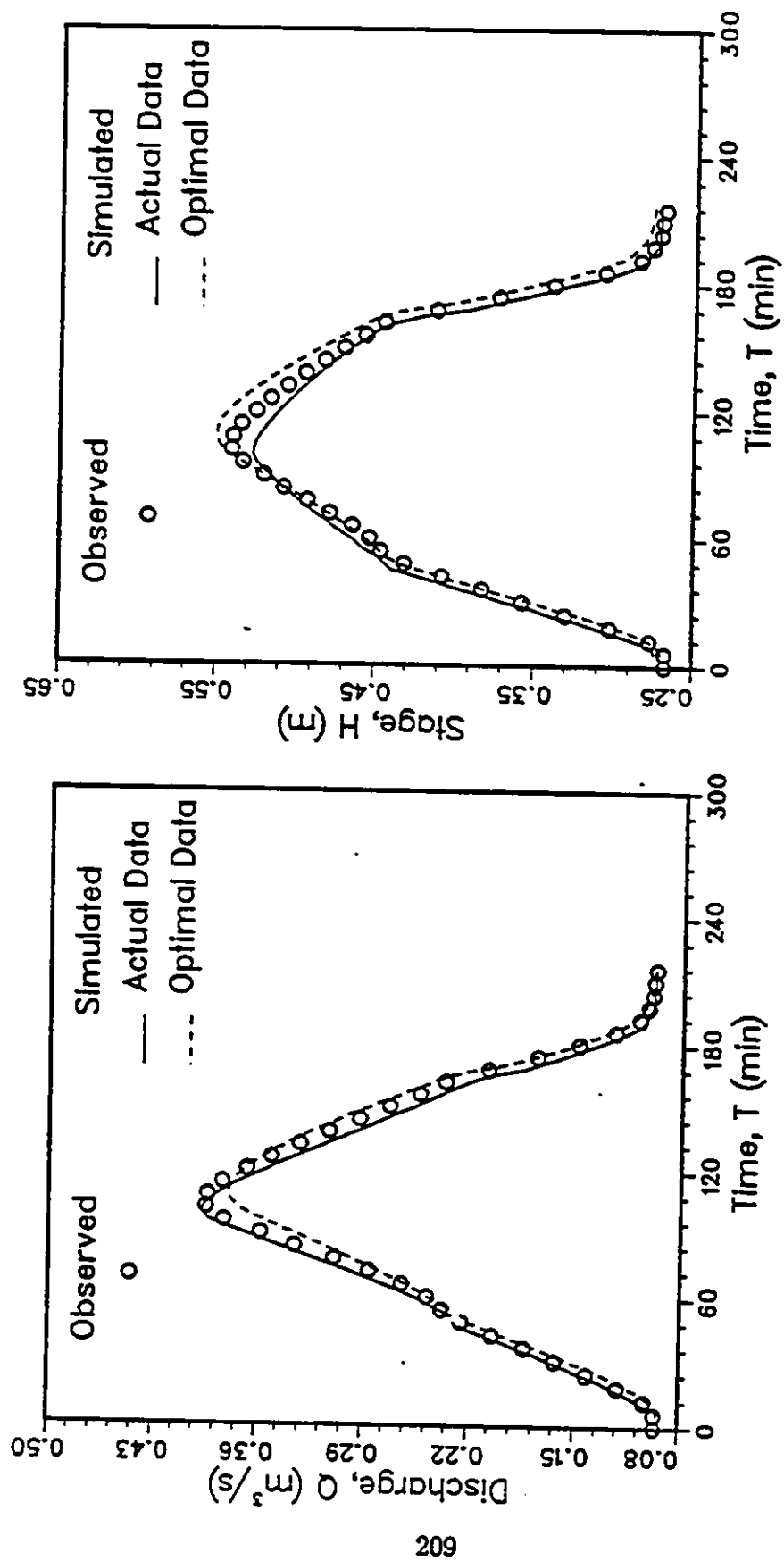


FIG. 8.7 Observed versus Simulated Discharge and Stage Hydrographs, Black Box Approach, (Run R1, Event 10).

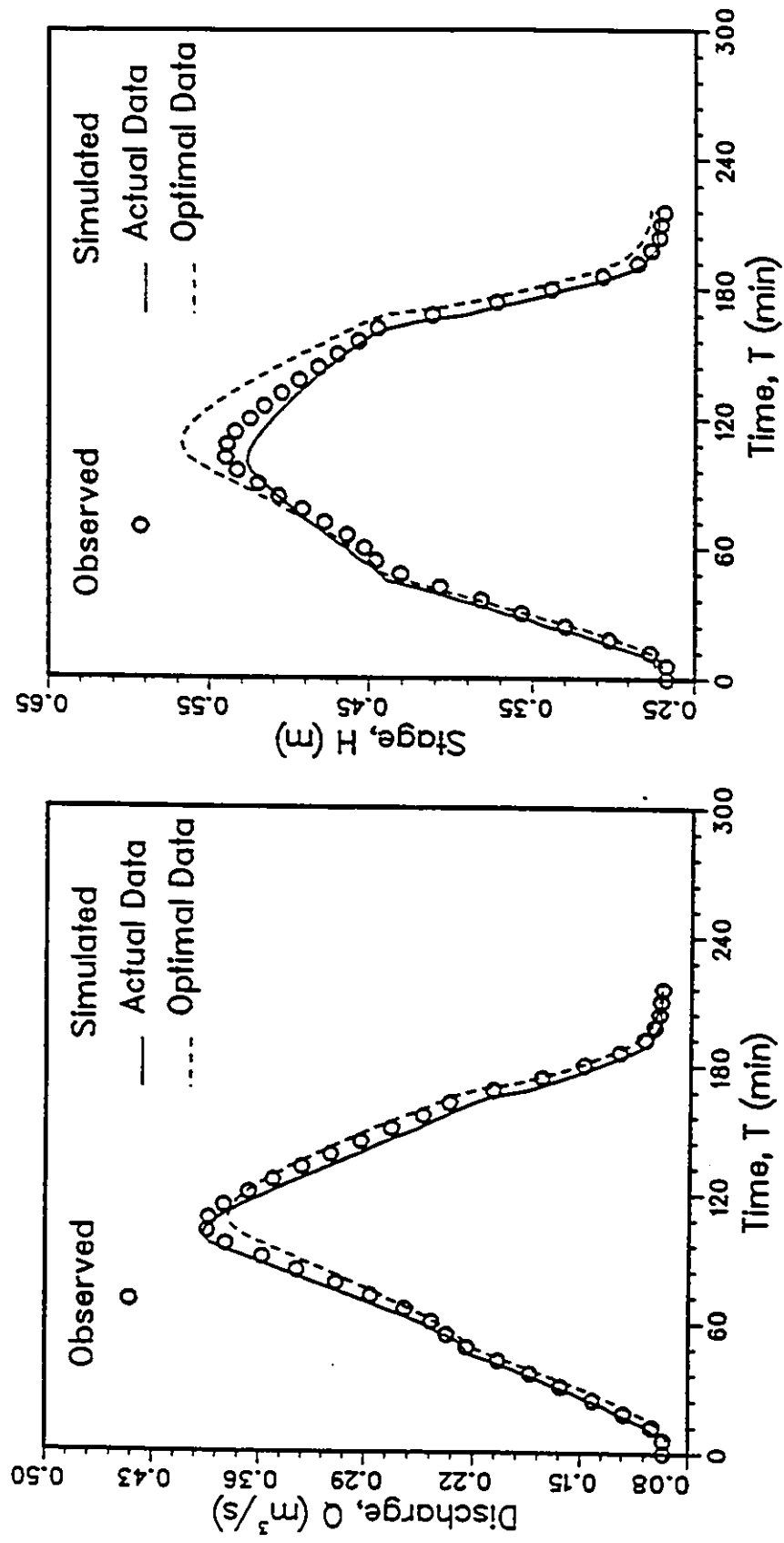


FIG. 8.8 Observed versus Simulated Discharge and Stage Hydrographs, Black Box Approach, (Run P2, Event 10).

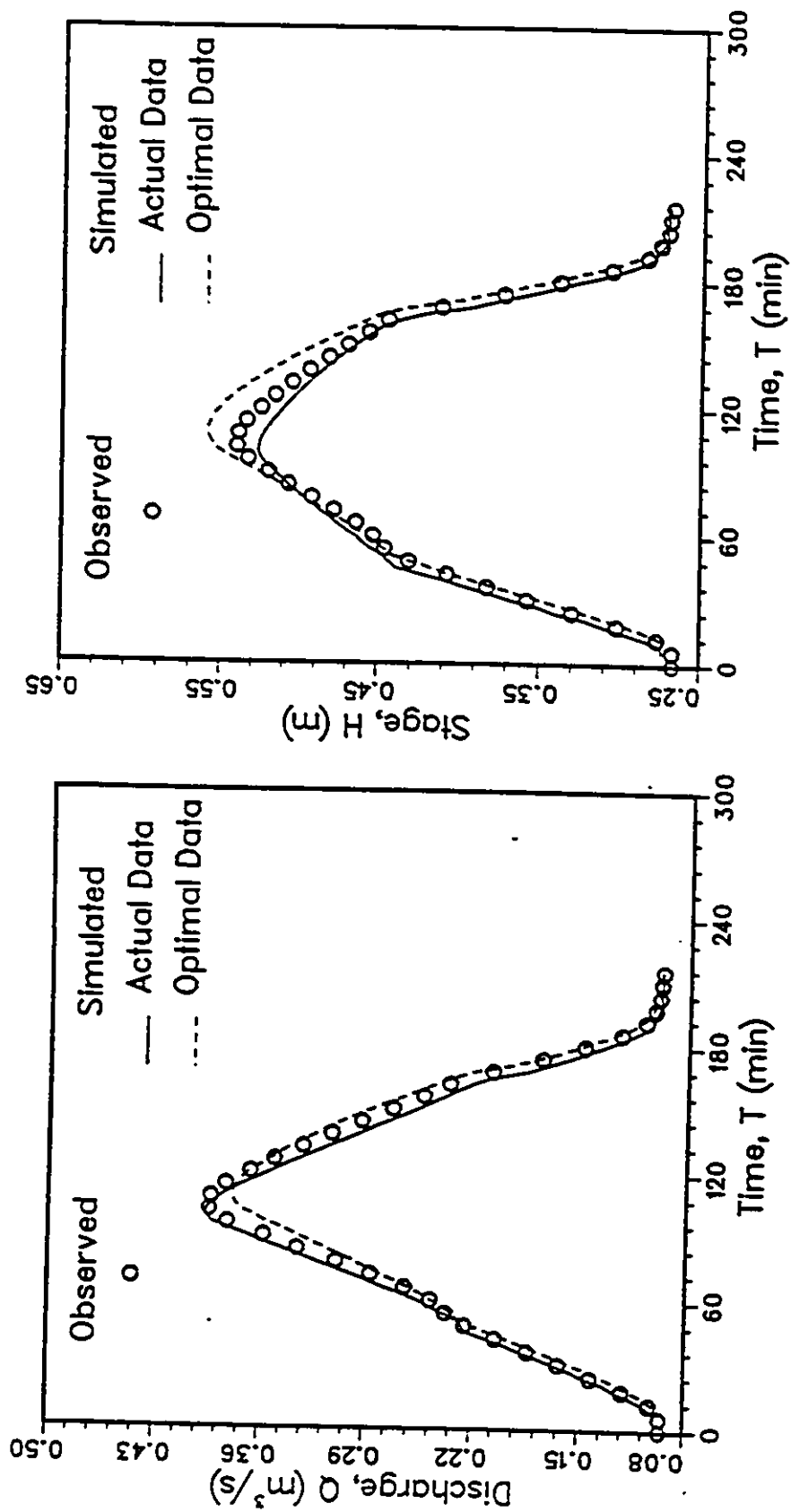


FIG. 8.9 Observed versus Simulated Discharge and Stage Hydrographs, Black Box Approach, (Run R2, Event 10).

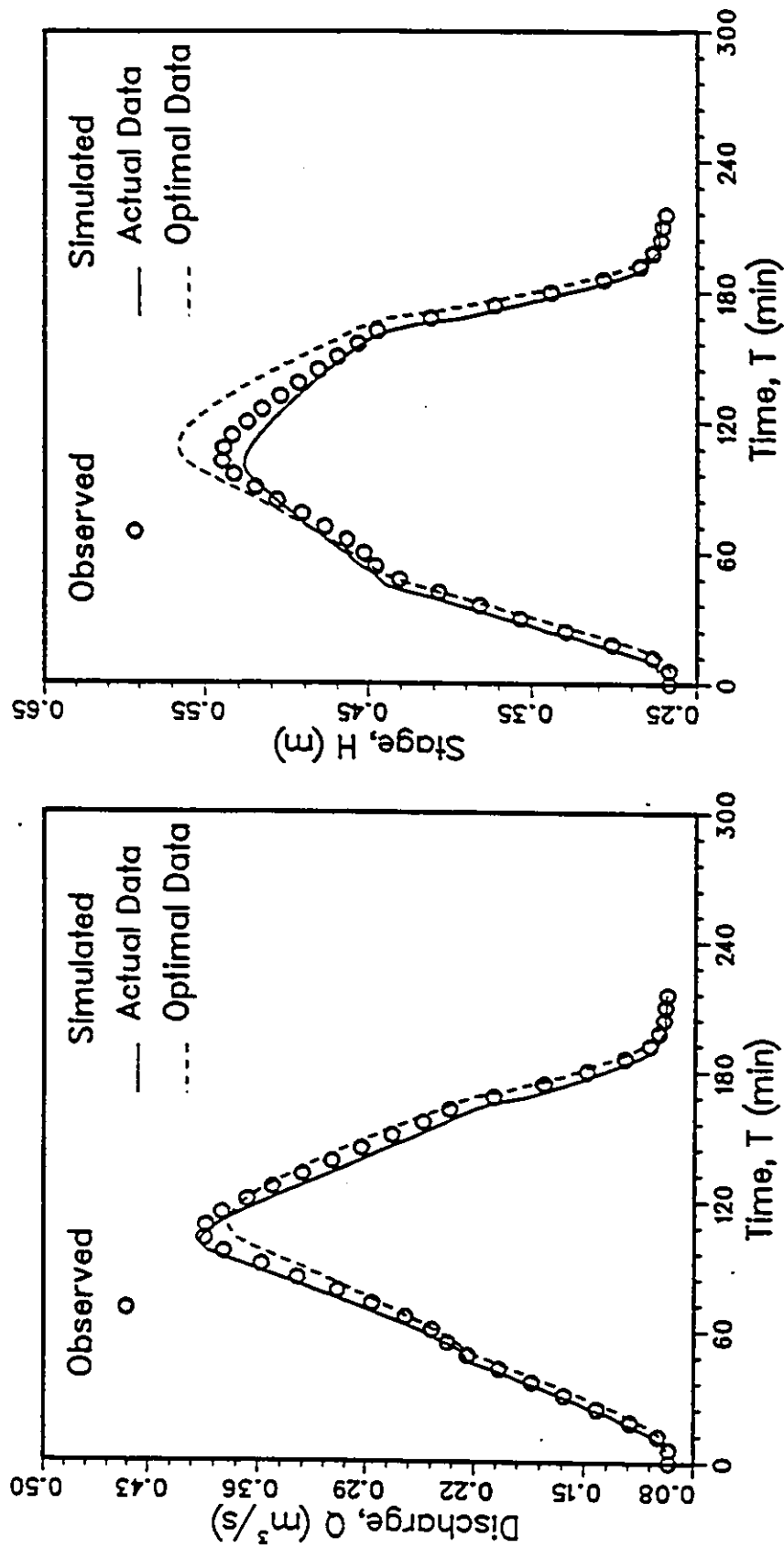


FIG. 8.10 Observed versus Simulated Discharge and Stage Hydrographs, Black Box Approach, (Run S1, Event 10).

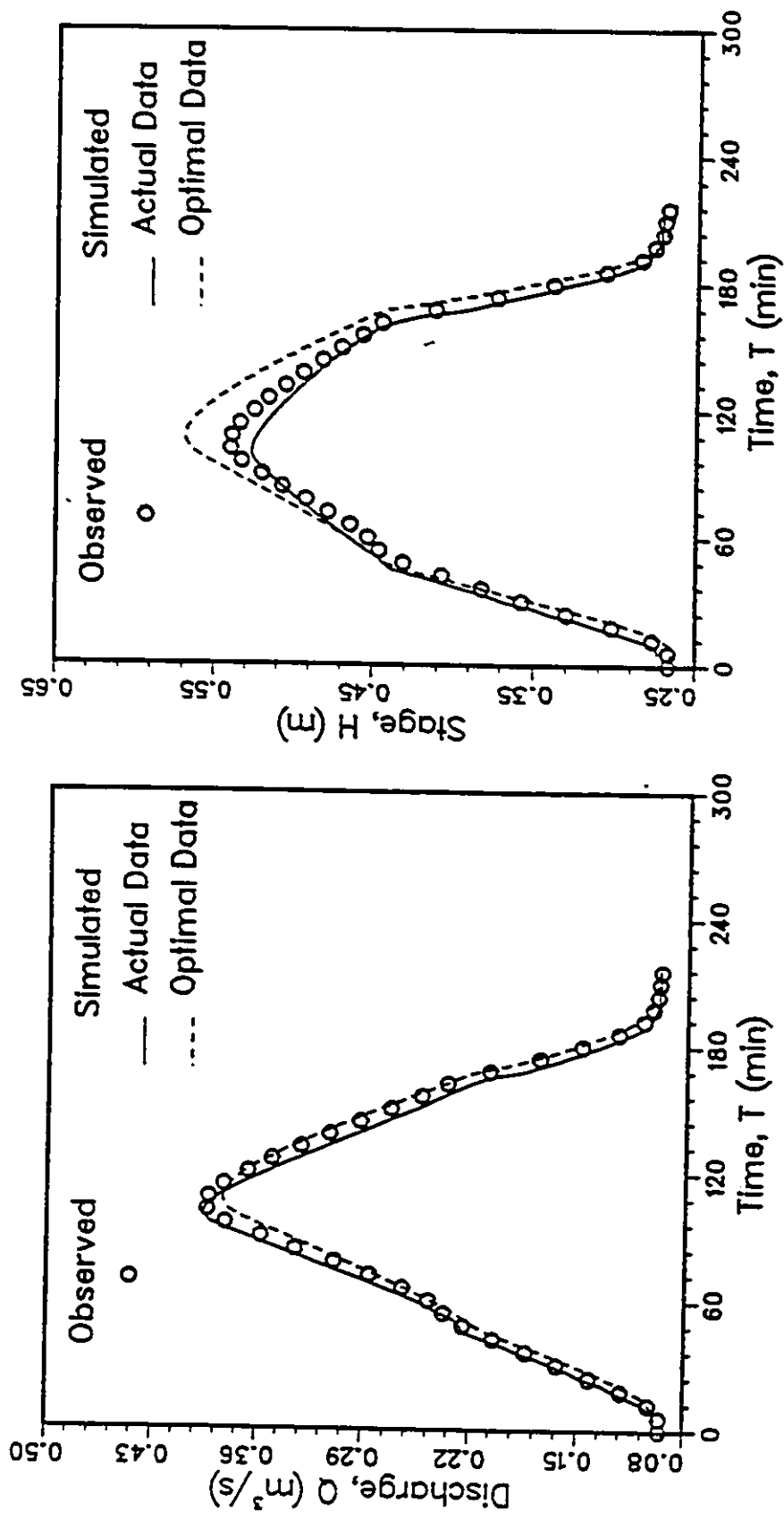


FIG. 8.11 Observed versus Simulated Discharge and Stage Hydrographs, Black Box Approach, (Run S3, Event 10).

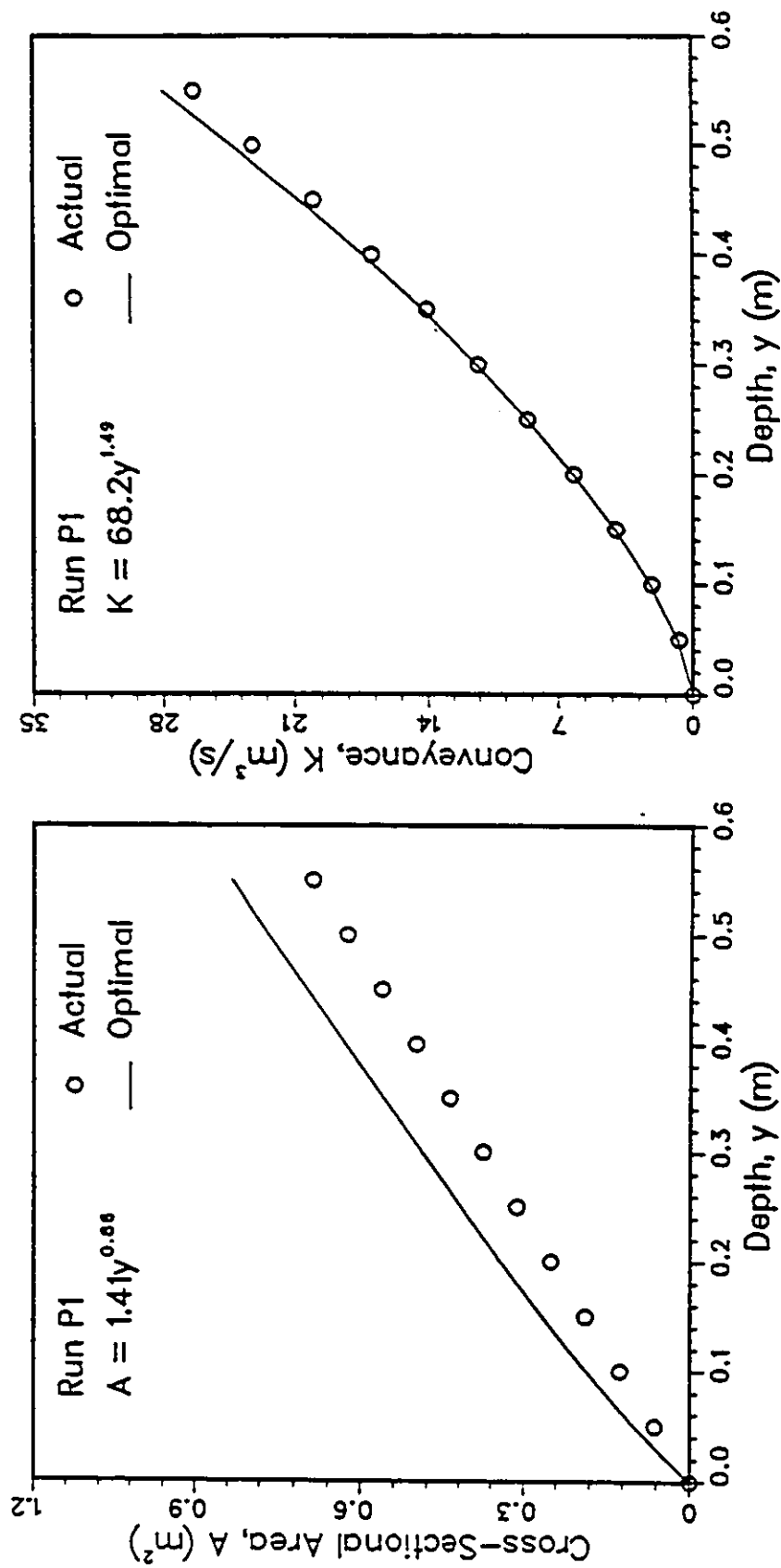


FIG. 8.12 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

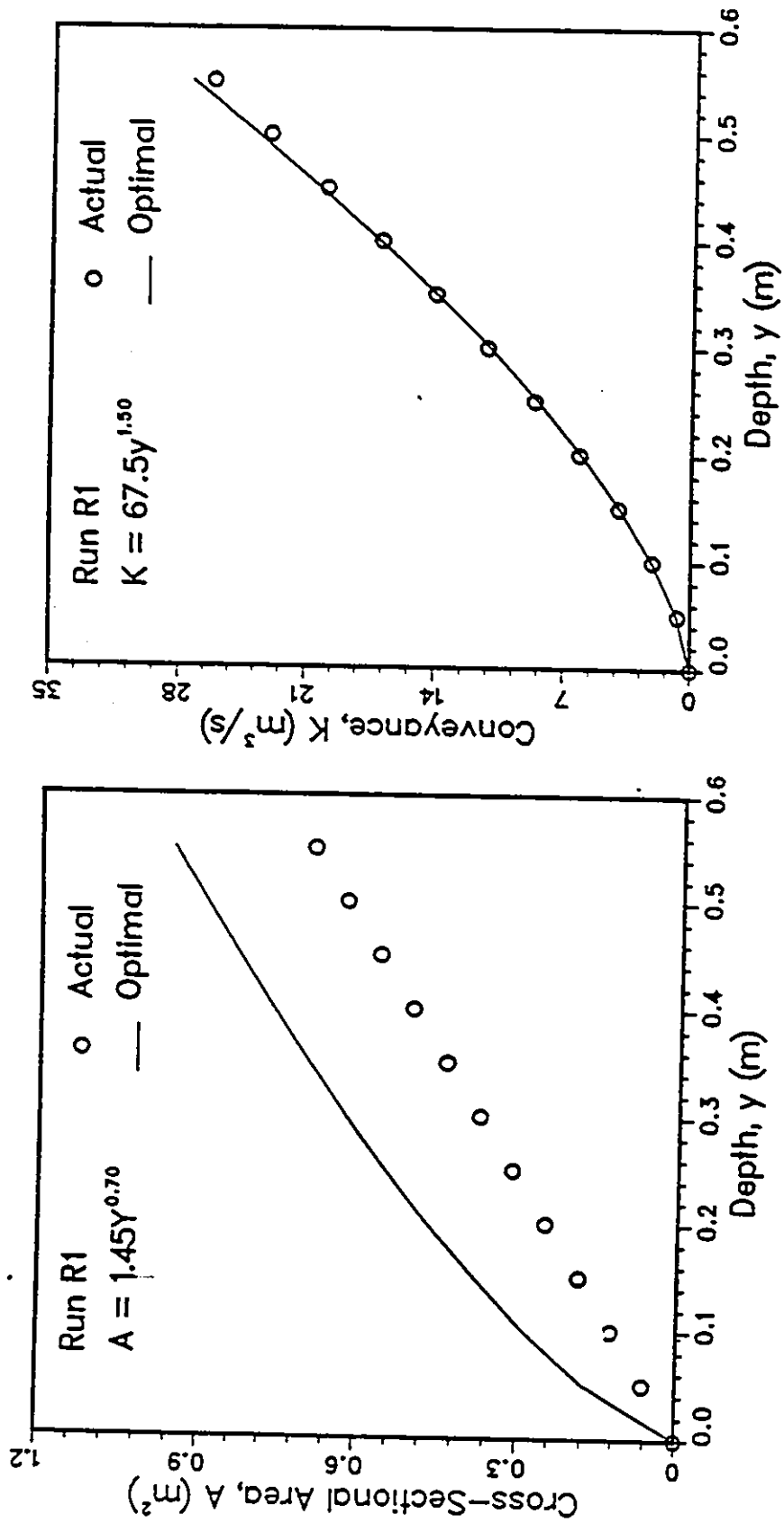


FIG. 8.13 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

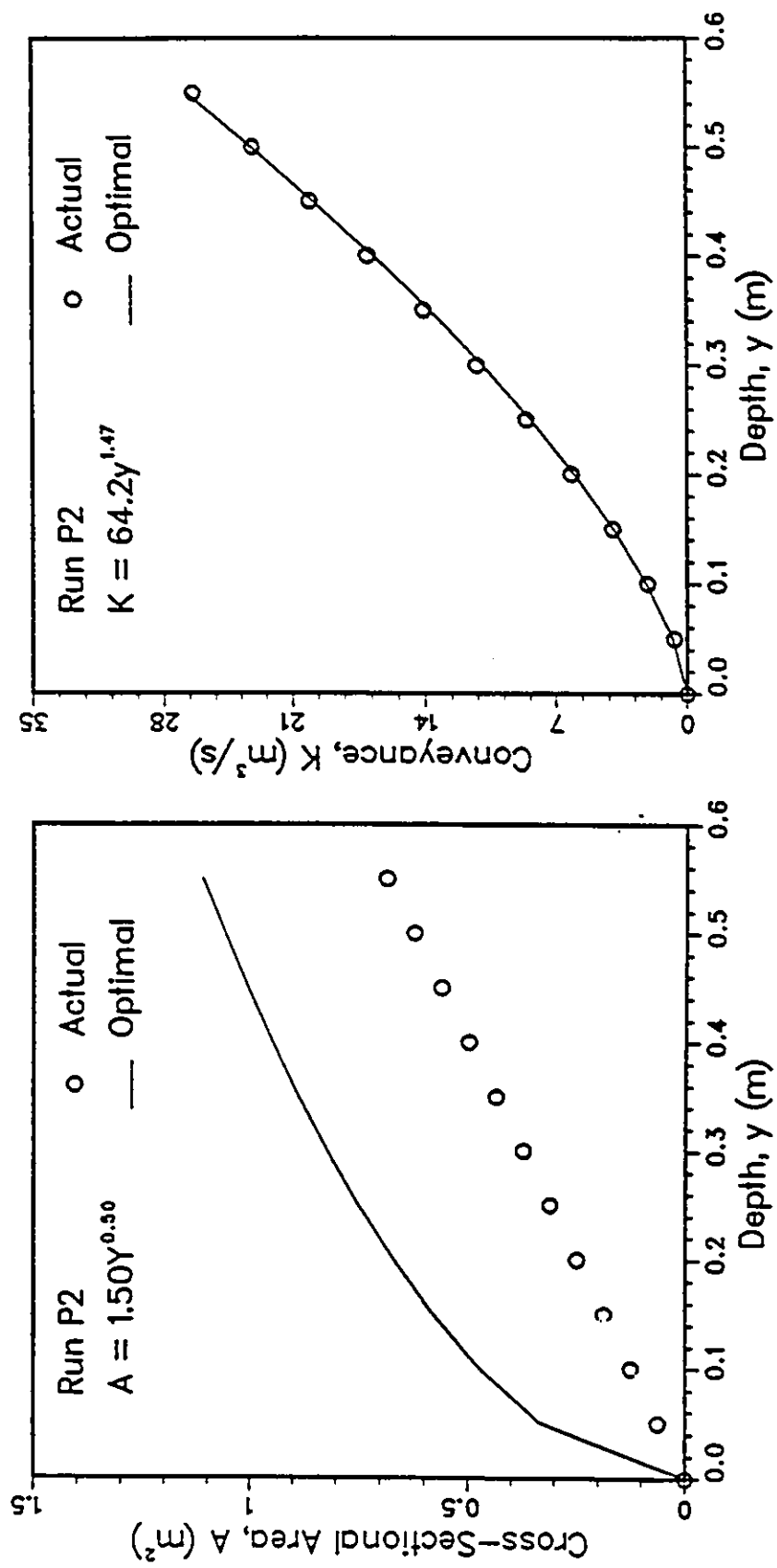


FIG. 8.14 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

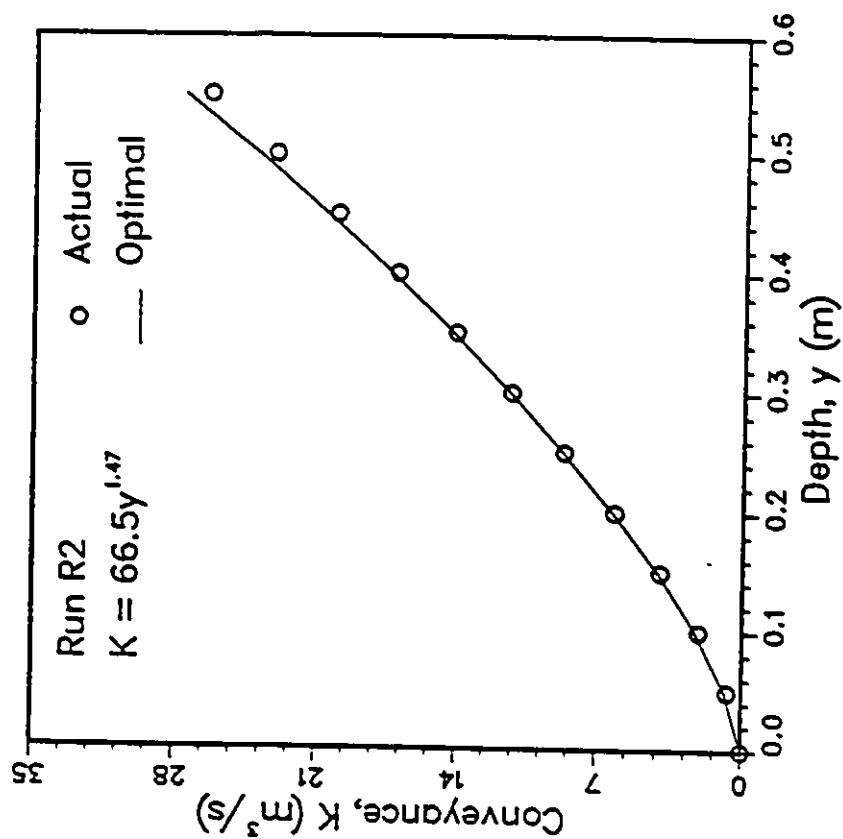
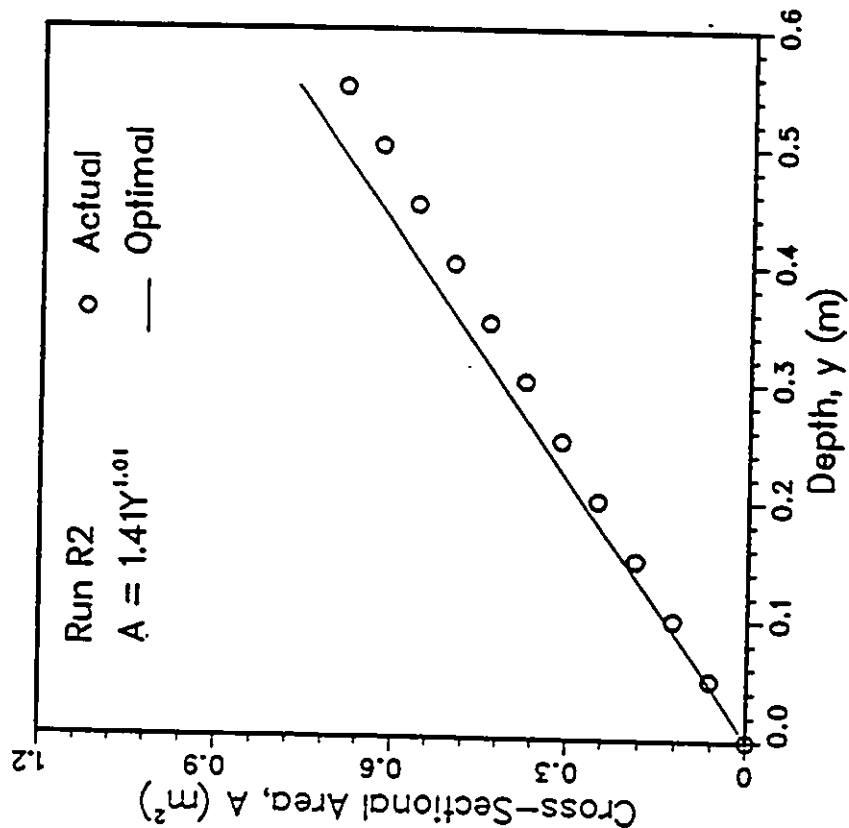


FIG. 8.15 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

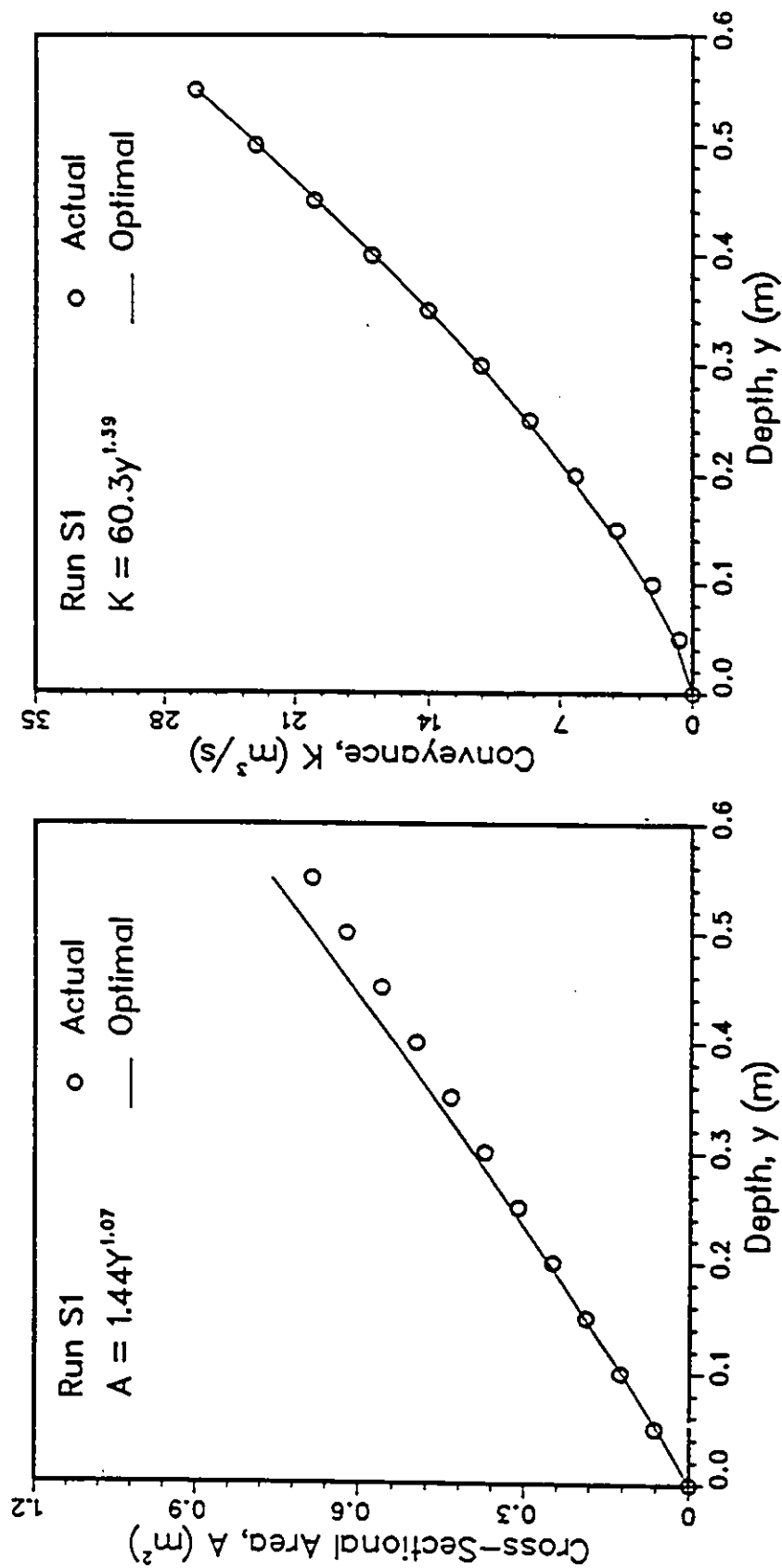


FIG. 8.16 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

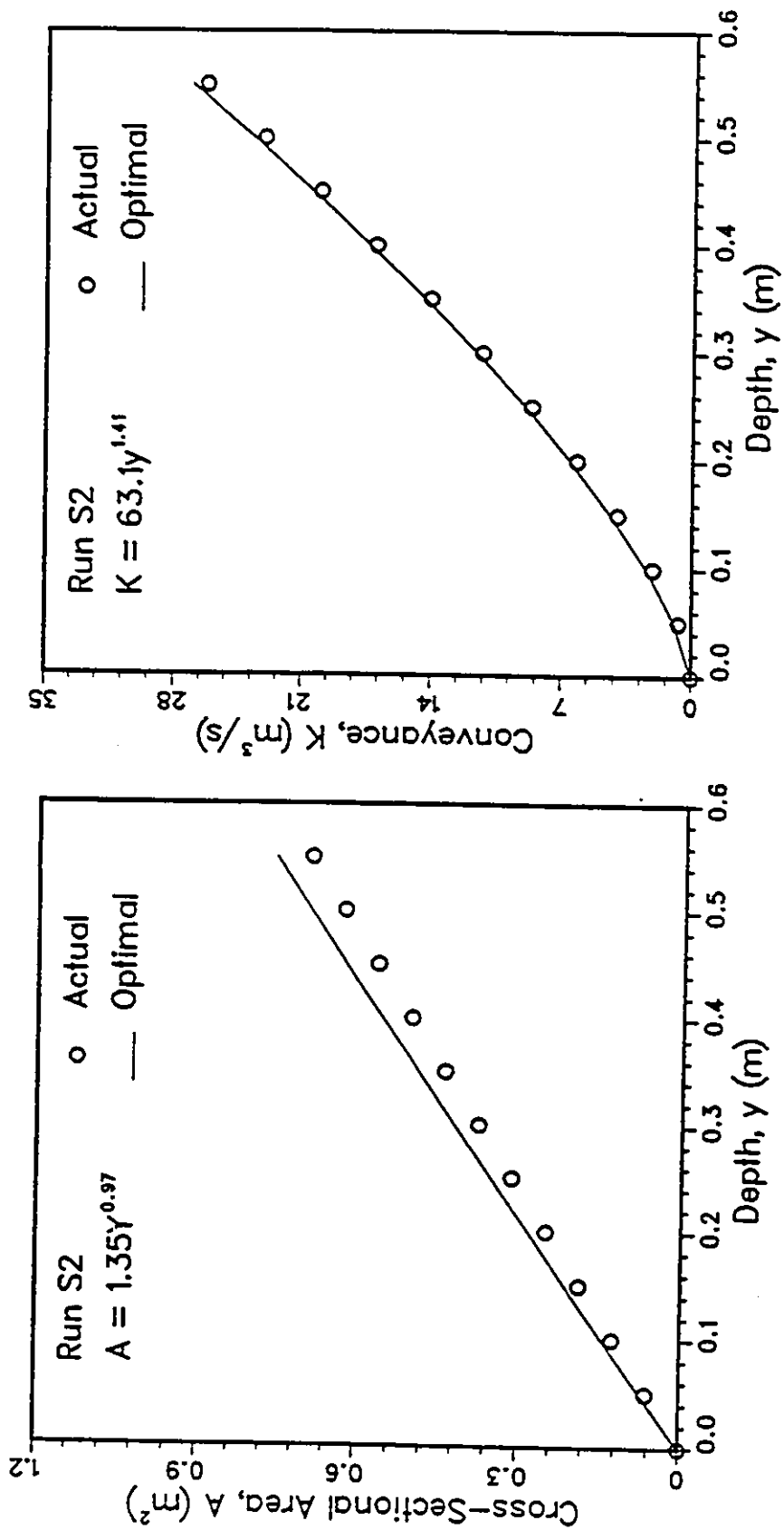


FIG. 8.17 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

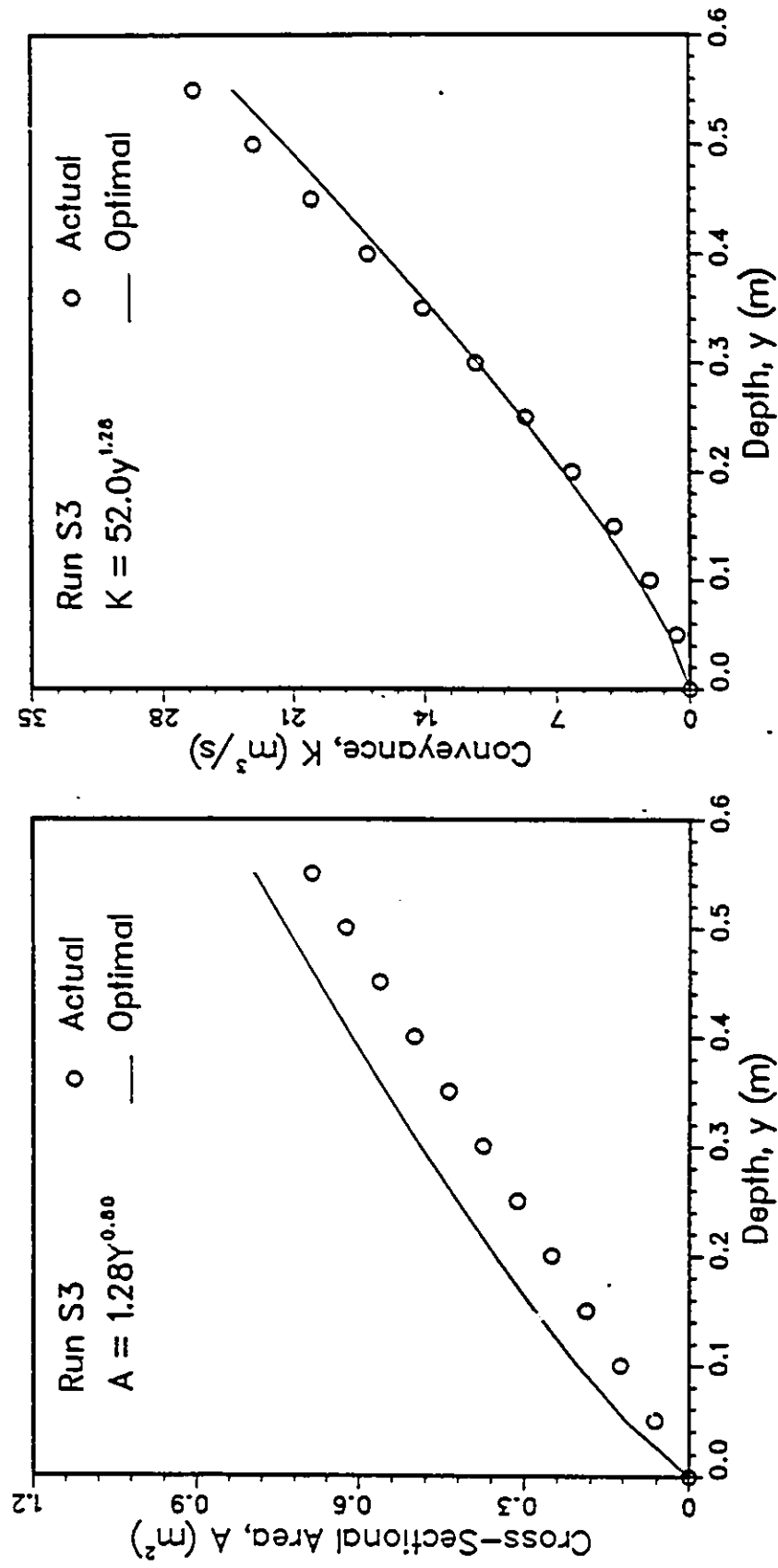


FIG. 8.18 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

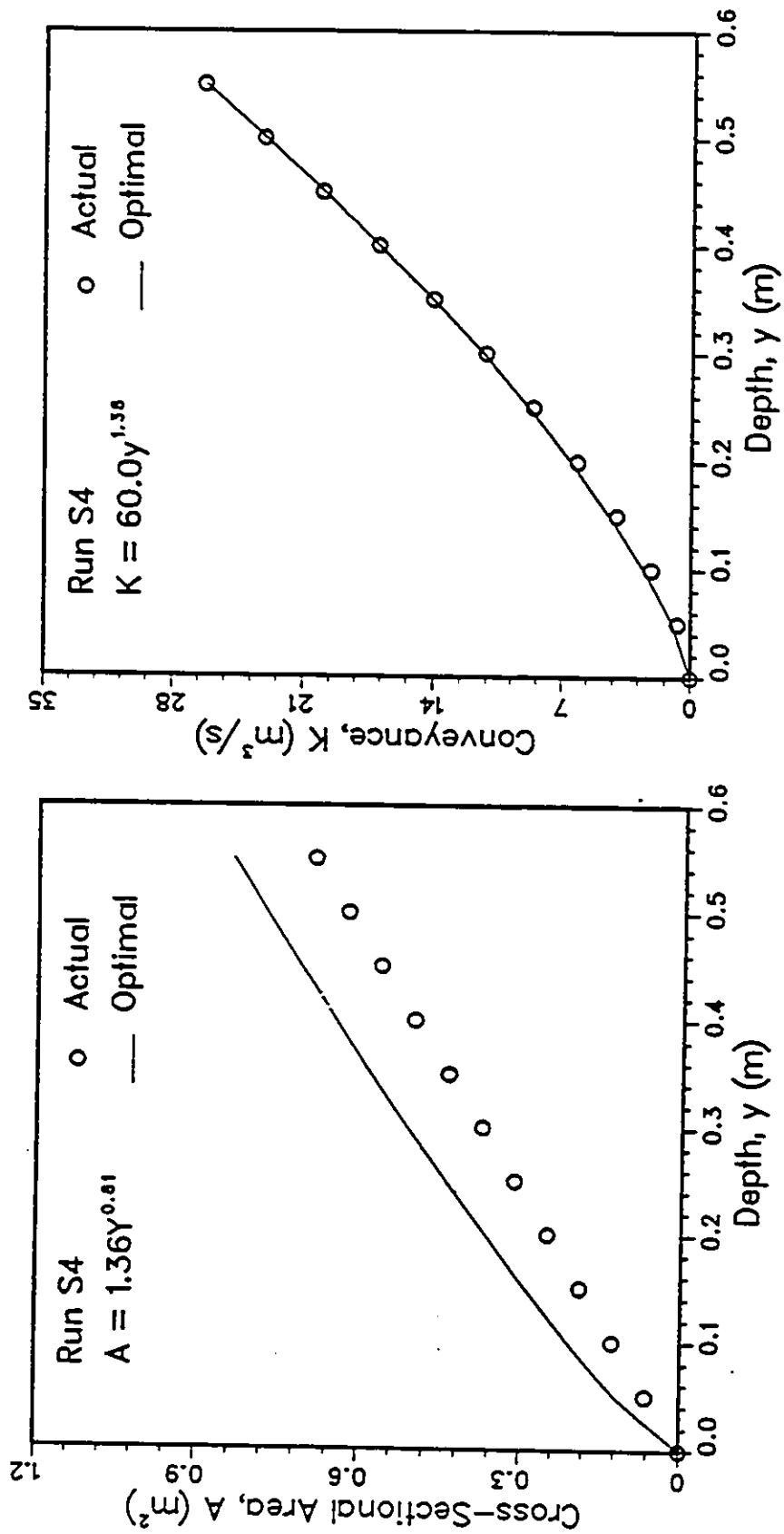


FIG. 8.19 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Main Channel).

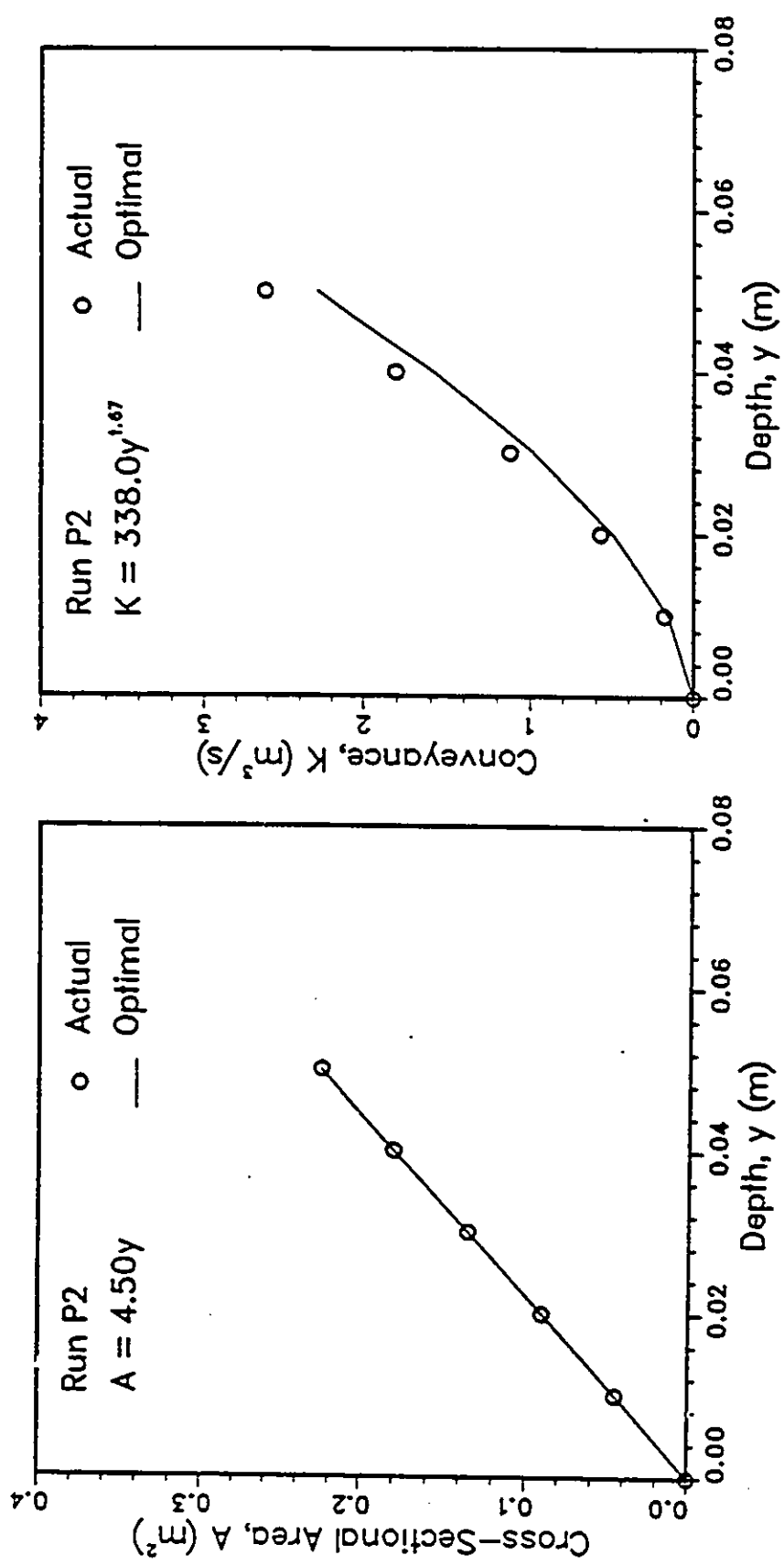


FIG. 8.20 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Flood Plain).

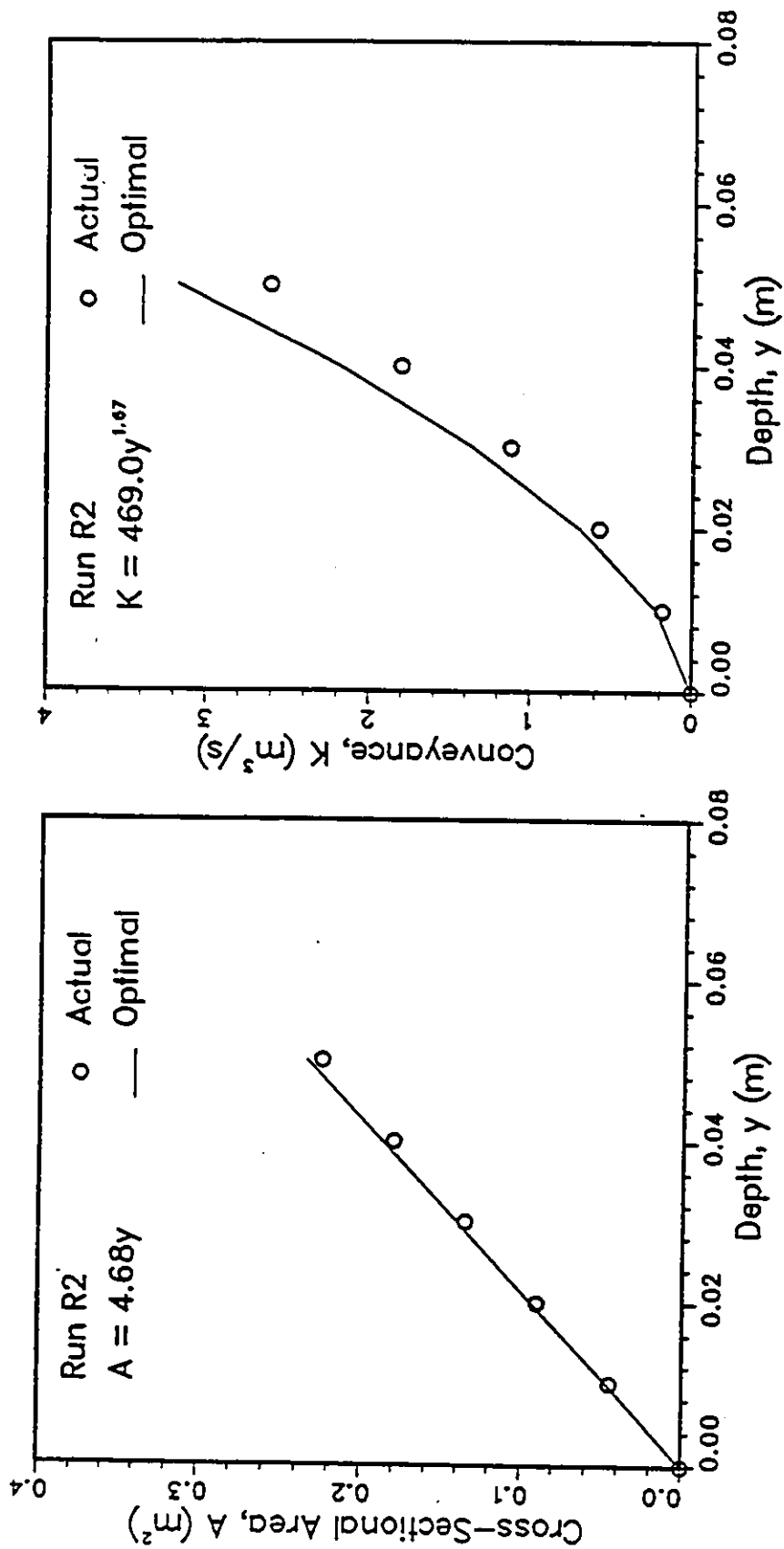


FIG. 8.21 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Flood Plain).

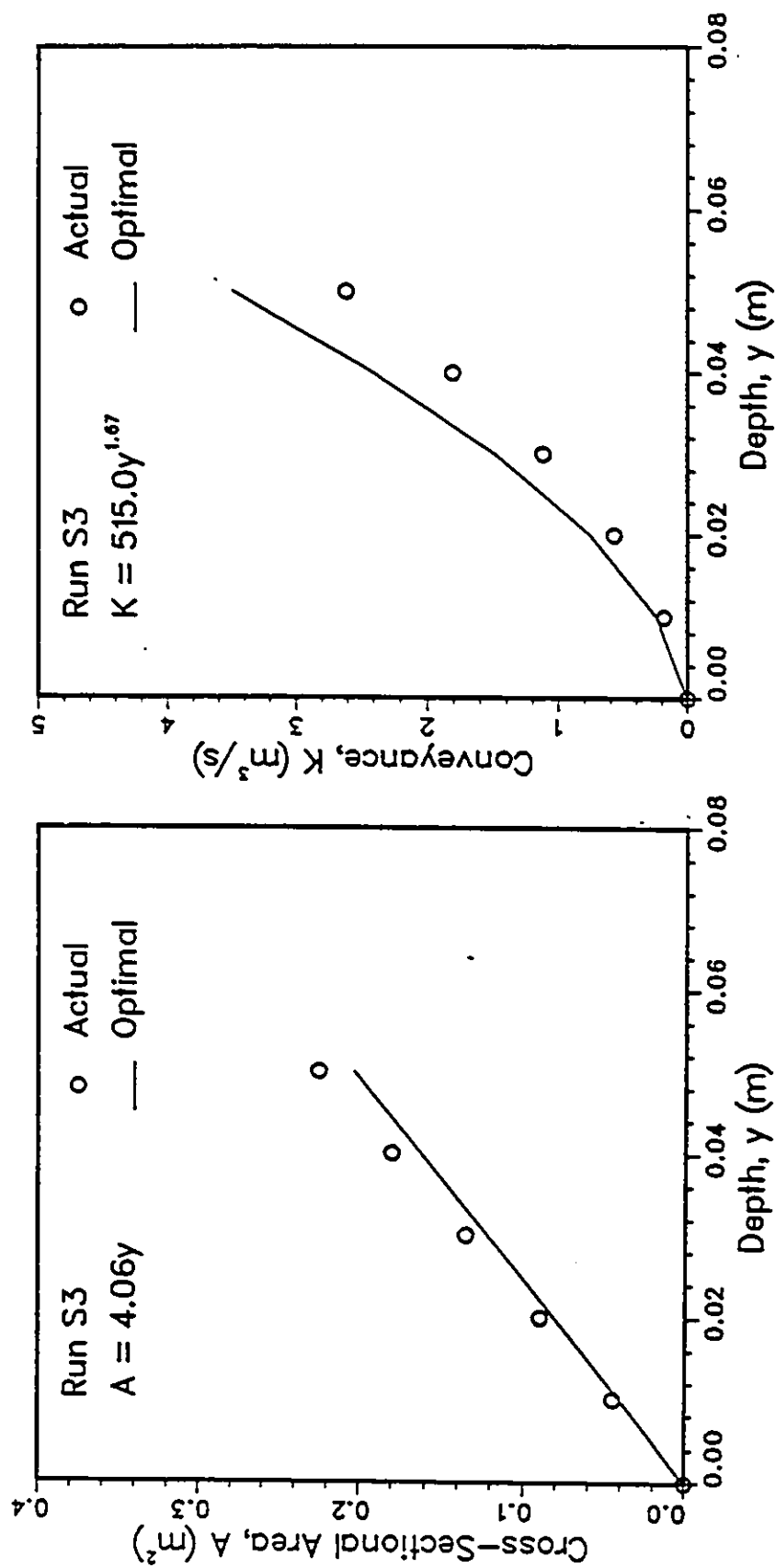


FIG. 8.22 Functional Relationships for Cross-Sectional Area and Conveyance in Terms of Flow Depth (Flood Plain).

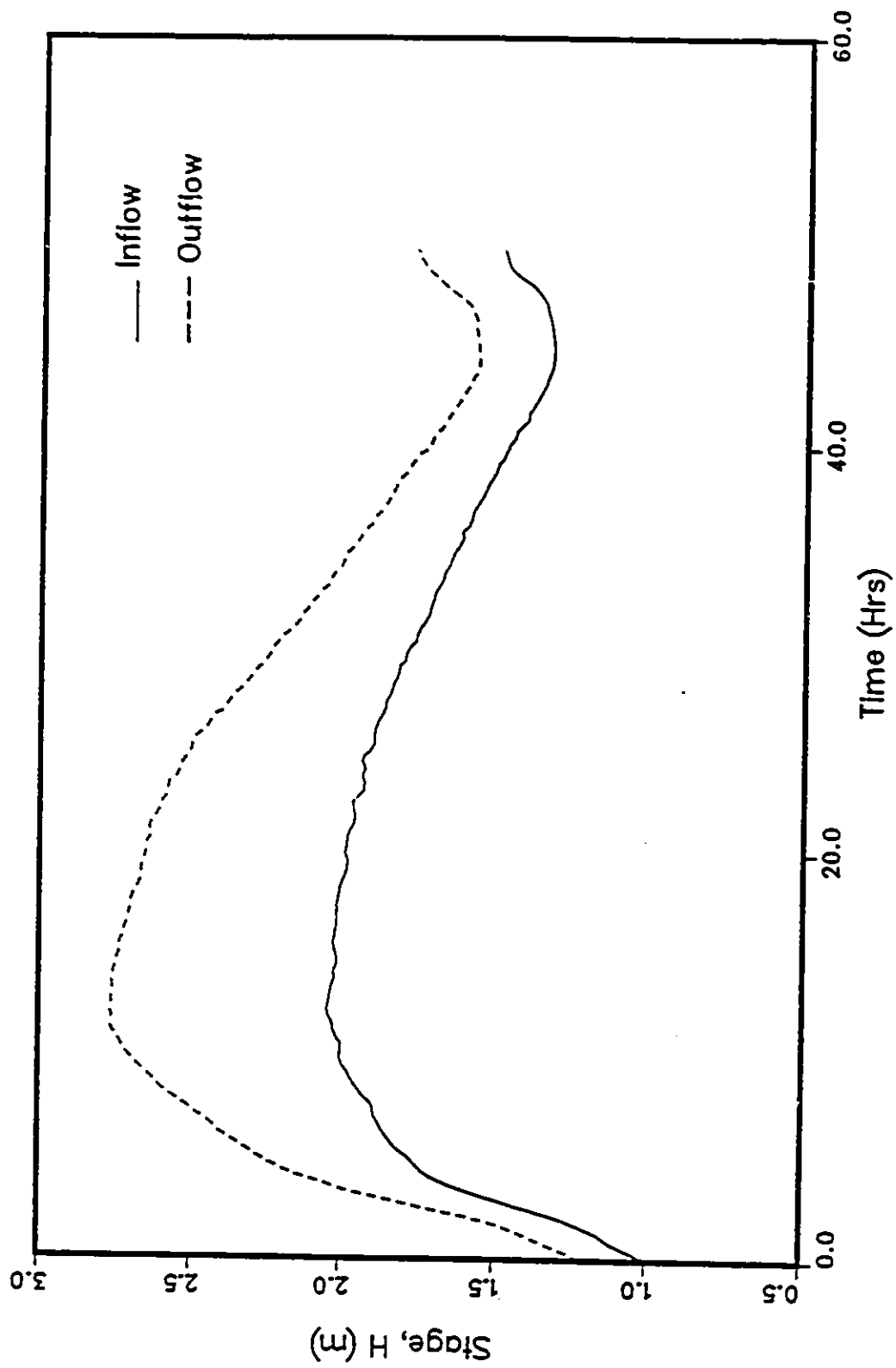


Fig. 8.23 Inflow and Outflow Stage Hydrographs Used in the Optimization Process, River Main Application.

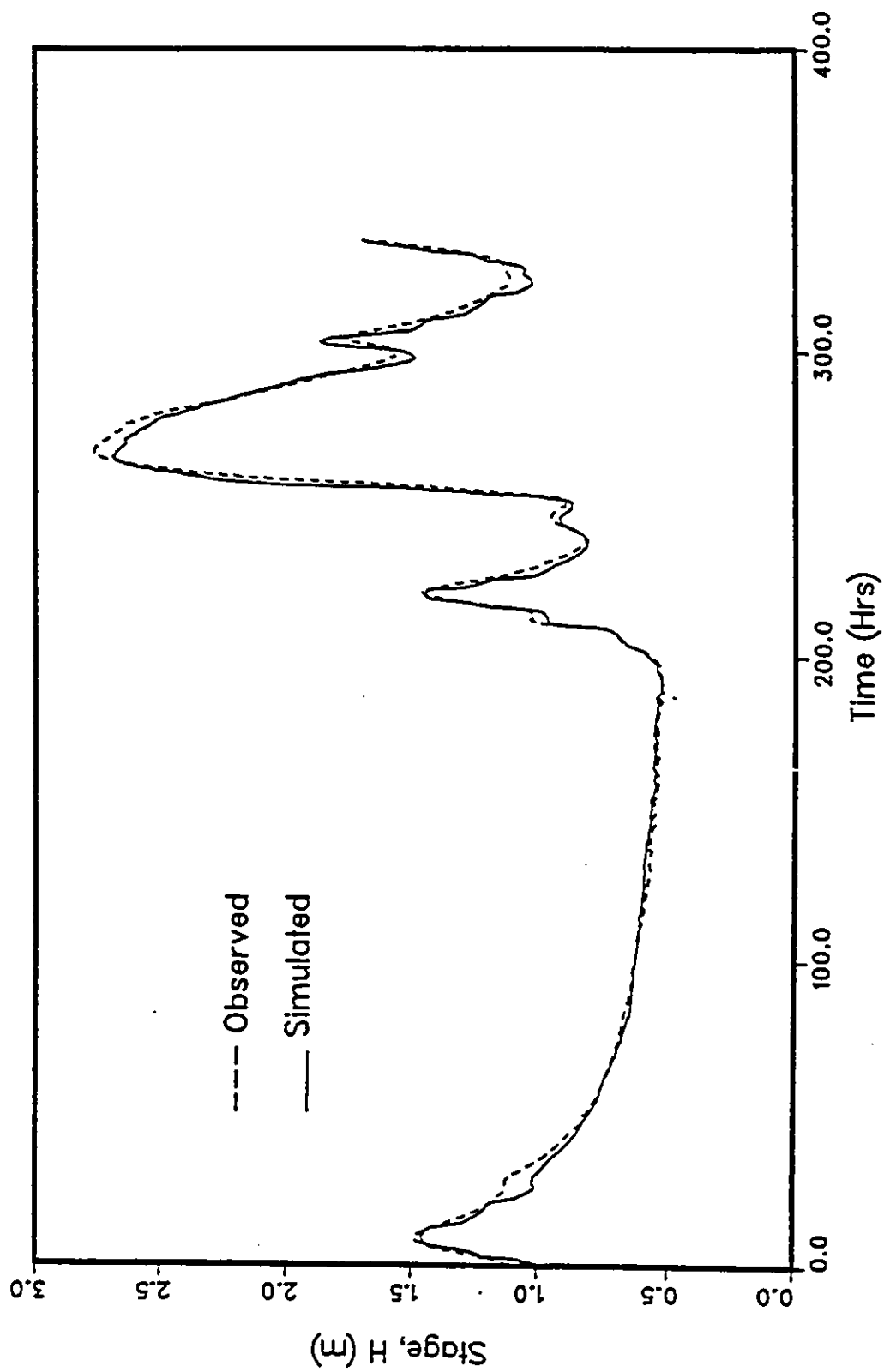


Fig. 8.24 Observed versus Simulated Stage Hydrographs for the River Main Reach, (Optimal Data).

Chapter 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 General Conclusions

The following are the principal conclusions relating to the first phase of the study, namely the development of the improved unsteady flow model for routing floods through compound channels:

1. the proposed model for routing unsteady flows in compound channels (RUFICC) yielded 'good' estimates of stage and discharge for a wide range of flood plain conditions. This underscores the importance of the new features in unsteady flow modeling (flood plain contribution to system conveyance, the cosine interpolating function relating flood plain and main channel fric-

tion slopes, and LMT) introduced in this study.

2. Compared to the Off-Channel Storage Model (OCSM) and the Separate Channel Model (SCM), the new model: RUFICC yielded best estimates of stage and discharge, especially when high flood plain flows were encountered.
3. Conventional methods (OCSM and SCM) yielded fairly good results for the case of low flood plain flows (i.e. flood plain to main channel depths ratios less than 10%).
4. The use of both conventional methods for a "high flood plain flow" condition resulted in a marked delay in the fall of the recession curve of simulated stage and discharge hydrographs.
5. The Off-Channel Storage Model (OCSM) significantly underestimated (by more than 25%) stages and especially discharges when applied to a "high flood plain flow" condition.
6. An LMT effect was found to be important only for the case of very wide and rough flood plains. Under these conditions the mechanism resulted in attenuation of discharge hydrographs, especially when low flood plain flows were encountered. However, no conclusion could be made with respect to the magnitude of this attenuation because of the lack of (i) a model that properly accounts for LMT under unsteady flow conditions, and (ii) appropriate field data.
7. Conservation of mass between the boundaries of a channel system can be considered as a good indicator of the accuracy of

unsteady flow computations using RUFICC.

8. As recommended by Wormleaton and Karmegam (1984), $\Delta x/\Delta t$ should always be greater than the flood wave velocity to ensure accurate results. However, this criterion does not always guarantee reaching a solution without program termination caused by instability problems.

The following conclusions refer to the second phase of the study, which dealt with the use of optimization methods for flood routing data estimation:

1. Minimization of the error between simulated and observed stages and discharges yielded reasonably good estimates of a channel's geometric and hydraulic properties required for the flood routing exercise.
2. Using optimized parameters to route different hydrographs in the same channel resulted in stages and discharges in good agreement with those obtained using actual data. Errors in stage and discharge were less than $\pm 5\%$ in most cases.
3. For certain conditions the individual optimized parameters were shown to be quite different for different optimization numerical tests. Nevertheless, the resulting values for the cross-sectional area and conveyance (the only requirement in the unsteady flow analysis), were shown to be in a reasonably good agreement with actual data.

4. The optimization process provides a rational basis for calculating hydraulic resistance parameters and therefore avoids the subjectivity associated with the estimation of Manning's roughness coefficients.
5. The 'black box' formulation avoids the use of steady flow formulae (such as Chezy's or Manning's) to evaluate the friction slope term in the unsteady open channel flow equations.
6. The 'black box' approach is especially advantageous for compound channels because of the uncertainty associated with the development of the composite section conveyance.
7. Compared to the conceptual formulation an objective function evaluation using the 'black box' approach took much less computer time.
8. The optimization process was very slow using Powell's Method. Objective function values at the minimum were also higher than those using the Rosenbrock or Simplex methods. This might be explained by the fact that the search directions in Powell's method tend to be linearly dependent on each other, which results in a reduction of the search space and therefore inhibits convergence.
9. Compared to Rosenbrock's method the Nelder and Meade Simplex algorithm generally resulted in much lower optimal objective function values and better estimates of the optimization parameters.

10. As each evaluation requires the complete numerical solution of the unsteady flow equations, any optimization algorithm selected for the estimation of flood routing parameters should involve the least number of objective function evaluations.
11. The closer the initial estimates of the optimization model parameters are to true values the faster is the convergence, which saves computer time. For initial guesses to be close to correct values, the former should be based on careful inspection of the study reach. It would also be good practice to evaluate the objective function for different sets of initial guesses before starting the optimization process and adopt the one that gives the smallest objective function value [a value less than 10, for example, is considered acceptable].
12. The final objective function value should be as low as possible (usually less than 0.1) in order to obtain accurate and reliable estimates of the optimization model parameters.
13. The convergence criterion used to stop the optimization process should depend not only on the accuracy required of the model outcome but also the allowable computer time. The optimization process should be stopped either when a particular value of the minimum is reached or when a number of iterations is completed.
14. To avoid an unnecessary number of objective function evaluations, the convergence criterion should not be overly restrictive.

15. The optimization process should not be based on a single flood event but different flood events should be adopted and then the results, in terms of both the optimization model parameters and their resulting simulated hydrographs for another flood event, should be compared. This is especially true for the case of field data, where errors are generally induced due to measurement procedures and low sampling rates.
16. Flood routing parameters estimation using optimization techniques should result in a saving of costs associated with detailed field surveys and also should improve the accuracy of numerical river models.
17. Optimization methods could not possibly replace completely the collection of survey data. However, their use could be advantageous in the case of fairly long, featureless and relatively inaccessible reaches of natural watercourses.

9.2 Recommendations for Future Research

1. Using a common data set, the performance of RUFICC should be compared to that of existing unsteady flow models like DWOPER, ONE-D, EXTRAN and others.
2. Since all available models that attempt to quantify LMT were developed for a steady state flow condition, the importance of this mechanism should be investigated experimentally under condi-

tions of unsteady flow.

3. In this study only vertical imaginary interface planes (separating shallow and deep regions of the flow field) were considered. Therefore, it would be appropriate to examine other possible alternatives such as horizontal and diagonal interface planes.
4. Through a detailed experimental program, utilizing different compound channel geometries and different flood events, develop an empirical model that relates 'flood plain' and 'main channel' friction slopes for unsteady flow conditions.
5. Generate both field and experimental data, consisting of stage and discharge time series, for channels where the LMT effect is believed to be significant. Such data would serve to compare the performance of the different LMT procedures by comparing their corresponding simulated hydrographs to the measured time series.
6. Account for the lateral mass transfer between the main channel and flood plains in the model formulation to determine its effect on unsteady flow simulations.
7. The performance, in terms of convergence and the corresponding number of iterations, of the optimization methods used should be compared to that of the gradient method and the influence coefficient algorithm.
8. In this study the 'least square' criterion was adopted for the objective function evaluation because it is believed to model stages

and discharges correctly over the whole hydrograph. It is now advisable to use the 'minimax' criterion for the same applications to examine whether there are any differences in corresponding peak stages and discharges.

9. Optimization procedures should be applied to other field data, where errors can be induced due to measurement procedures and low sampling rates. Under these circumstances the outcome of the optimization procedure is not expected to be as accurate. Also, the effect of induced errors in field data on the optimization results should be investigated.

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Appendix A

USER'S MANUAL

The computer model, RUFICC, is used for unsteady flow simulations in compound meandering channels. It uses a modified version of St. Venant equations in which the contribution of flood plain flow to system conveyance is considered. The model also accounts for the lateral momentum transfer mechanism (LMT) between the shallow and deep sub-sections of a compound channel. The LMT is accounted for through several empirical models available in the literature which were all developed under steady flow conditions. It is left to the user to apply any of the procedures available or not to account for LMT at all. It is also important to note that, unlike other models of a similar nature, this model does not assume the same friction slope for the different sub-sections of the compound channel. Rather it relates the flood plain friction slope to that of the main channel using a cosine interpolating function and based on the different flow conditions in the flood plain and the main channel and especially their relative depths of flow.

Besides solving the improved unsteady flow equations the model also includes, for comparison purposes, routines that perform flood routing using conventional methods ("Off-Channel Storage" and "Separate Channel" Methods).

The model uses the Four-Point Implicit Scheme to transform the partial differential equations into difference equations. The latter are then linearized and solved for using the Newton-Raphson Method and the Double-Sweep Technique. The model is written in FORTRAN and a separate data file is required for its execution. An output file, storing stages and discharges at all points of the computational grid, is then created.

The upstream boundary condition given to the model can either be a stage or a discharge hydrograph. The downstream boundary condition is a stage-discharge curve. If such a curve is not available to the user the model computes its own rating curve based on an empirical procedure for estimating steady-state discharge in compound channels. The procedure is analogous to Manning's equation for the case of prismatic regular channels.

The model handles the following two types of compound channel geometries:

1. Flood plains are approximated by rectangular sections while the main channel can be of any convex shape, for which a relationship between the flow depth and top width is supplied.
2. Any irregular composite section. If the geometric properties of the section change along the reach, they should be supplied at several stations and then the model estimates those for intermediate points using linear interpolation.

A.1 Description of Cards in the Data File

Variable	Description
Card I	Title Card
Name	A title to describe a particular run for later reference.
Card II	Indicators describing several input features
IND	Indicator for the flood routing method to be used =1, Improved Unsteady Flow Method; =2, "Separate Channel" Conventional Method; =3, "Off-Channel Storage" Conventional Method.
INDI	Indicator for the inflow hydrograph =1, discharge hydrograph; =2, depth hydrograph.
INDG	Indicator for the compound channel geometry =1, rectangular flood plains and a main channel of any convex shape are assumed; =2, a table of geometric properties (top width, wetted perimeter, cross-sectional area, etc.) as functions of flow depth is given for the different sub-sections of the compound channel. A separate program that performs all of these computations for any irregular compound channel is available.

INDM	Indicator for the Lateral Momentum Transfer Method used = -2, Dracos and Hardegger's (1987) Method; = -1, relationship developed by the author of this study based on Myers and Brennan (1990) data; =0, LMT is ignored; =1, Nicollet and Uan (1979); =2, Prinos and Townsend (1983); =3, Prinos and Townsend (1984); =4, Prinos (1985); =5, Wormleaton and Merrett (1990); =6, Wormleaton et al (1982).
INDB	Indicator for the downstream boundary condition =1, no stage-discharge curve is available; =2, a rating curve is to be provided as an input to the model.
INDX	Indicator to decide whether constant or variable space increments are to be selected =0, constant space increment; =1, the reach under study is divided into several sub-reaches and different space increments are to be adopted.
INDMN	Indicator for channel meander =0, no significant meander is present in the channel; =1, the channel is meandering.

INDL	Indicator for lateral discharge =0, no lateral inflow to the system is present; =1, lateral inflows are quite significant and have to be considered in the analysis.
INDR	Indicator for the relationship used in estimating a compound channel Manning's roughness coefficient.
Card III	Channel's Data
RL	reach length;
YI	initial steady state depth of flow;
S	channel bed slope;
DATUM	channel bed elevation for the upstream station.
Card IV	Time Parameters
TDF	duration of the inflow hydrograph;
DDDT	time step;
WF	computational weighting factor;
DTMAX	maximum time step;
ITRMAX	maximum number of iterations.
Card V	used if INDMN = 1
ALR	flood plain to main channel length ratio.

Card VI	used if $INDX = 0$
DDDX	space increment.
Card VII	used if $INDX = 1$
NSU	number of sub-reaches for which different space increments are assigned.
Card VIII	used if $INDX = 1$
	This card is repeated NSU times
AL(.)	length of a sub-reach;
DDX(.)	space increment assigned to that particular sub-reach.
Card IX	roughness coefficients
RN(1)	Manning's roughness coefficient for the left flood plain;
RN(2)	Manning's roughness coefficient for the main channel;
RN(3)	Manning's roughness coefficient for the right flood plain.
Card X	Boussinesq coefficients
BET(1)	Boussinesq coefficient for the left flood plain;
BET(2)	Boussinesq coefficient for the main channel;
BET(3)	Boussinesq coefficient for the right flood plain.

Card XI	used if $INDG = 2$
NBS	number of cross-sections along the study reach:
NPT	number of points in the geometric characteristics table for each cross-section.
Card XII	used if $INDG = 2$
	This card is repeated NBS times
BL(...)	bank-full depth at left bank;
BR(...)	bank-full depth at right bank.
Card XIII	used if $INDG = 2$
	This card, which gives the main channel geometric characteristics at each cross-section, is repeated NPT times.
HCU(...)	water surface elevation measured with respect to the main channel bed level;
ACU(...)	main channel cross-sectional area;
PCU(...)	main channel wetted perimeter;
TCU(...)	main channel top width;
DCU(...)	derivative of the wetted perimeter with respect to flow depth for the main channel sub-section.

Card XIV used if $INDG = 2$

This card, which gives geometric characteristics for the left flood plain at each cross-section, is repeated NPT times.

HCU(.,.) water surface elevation measured with respect to the main channel bed level;

ALU(.,.) cross-sectional area of the left flood plain sub-section;

PLU(.,.) wetted perimeter for the left flood plain sub-section;

TLU(.,.) top width for the left flood plain sub-section;

DLU(.,.) derivative of the wetted perimeter with respect to flow depth for the left flood plain sub-section.

Card XV used if $INDG = 2$

This card, which gives geometric characteristics for the right flood plain at each cross-section, is repeated NPT times.

HCU(.,.) water surface elevation measured with respect to the main channel bed level;

ARU(.,.) cross-sectional area of the right flood plain sub-section;

PRU(.,.) wetted perimeter for the right flood plain sub-section;

TRU(.,.) top width for the right flood plain sub-section;

DRU(.,.)	derivative of the wetted perimeter with respect to flow depth for the right flood plain sub-section.
Card XVI	used if INDB = 2
IU	number of points in the upstream stage-discharge curve:
ID	number of points in the downstream stage-discharge curve.
Card XVII	used if INDB = 2
	This card is repeated IU times
YUP(.)	depth at the upstream station above main channel bottom;
QUP(.)	corresponding discharge.
Card XVIII	used if INDB = 2
	This card is repeated ID times
YDO(.)	depth at the downstream station above main channel bottom;
QDO(.)	corresponding discharge.
Card IXX	used if INDL = 1
Q01	left flood plain lateral discharge;
U01	velocity component of the lateral inflow to the left

	flood plain in the longitudinal direction;
Q30	right flood plain lateral discharge;
U30	velocity component of the lateral inflow to the right flood plain in the longitudinal direction.
Card XX	used if $INDI = 1$
	This card is repeated NT times, where NT is equal to the duration of the inflow hydrograph divided by the time step.
QU(.)	ordinate of the discharge inflow hydrograph.
Card XXI	used if $INDI = 2$
	This card is repeated NT times, where NT is equal to the duration of the inflow hydrograph divided by the time step.
QU(.)	ordinate of the depth inflow hydrograph.

A.2 Sample Data and Output Files

A sample data file is shown on pages 261 and 262. The corresponding output file is shown on pages 263 to 265. After presenting channel geometry data as well as initial hydraulic conditions, the output file displays stages and discharges for all points of the computational grid. In the example considered 14 space increments and 73 time steps are encountered, which

translate into 1110 points in the computational mesh. As it would be impractical to report stages and discharges for all 1110 computational points in the output file, only stages and discharges that correspond to the first 'time' line of the computational mesh (for all points along the study reach) are presented herein.

The model also produces another output file, in which stages and discharges for the downstream station of the study reach are reported as functions of time. A sample of this particular file is shown on pages 266 and 267.

1.2.1 Sample Data File

THIS IS A SAMPLE EXAMPLE. FLOOD EVENT PC10 IS CONSIDERED.

1	1	1	2	1	1	0	0	0	1
210.0	0.217			0.00019	0.				
13140.0	180.0			0.6	180.	20			
15.0									
0.0115	0.0115			0.0115					
1.0	1.0			1.0					
0.0	0.39			0.39					
3.0	1.25			1.5					

.096
.095
.103
.114
.124
.135
.145
.156
.166
.177
.187
.198
.209
.220
.230
.241
.250
.261
.272
.282
.293
.304
.314
.325
.336
.346
.357
.368
.379
.389
.400
.411
.412
.402

.392
.382
.371
.361
.350
.340
.329
.318
.308
.297
.286
.276
.265
.255
.244
.233
.222
.212
.202
.193
.180
.170
.159
.148
.137
.126
.115
.104
.099
.099
.099
.098
.098
.098
.098
.098
.098
.098
.098

1.2.2 Sample Output File Showing Results for all Points of the Computational Mesh

THIS IS A SAMPLE EXAMPLE. FLOOD EVENT PC10 IS CONSIDERED.

INITIAL DATA

INITIAL DEPTH, YI= 0.2170M
CHANNEL BOTTOM SLOPE, SO= 0.000190
LEFT BANKFULL DEPTH, YBL= 0.3900M
RIGHT BANKFUL DEPTH, YBR 0.3900M

WEIGHTING FACTOR, THETA= 0.600
NO. OF STATIONS= 15 TOTAL LENGTH= 210.0000M
NO. OF TIME STEPS FOR CALCULATIONS, NT= 73
MANNING N FOR THE LEFT FLOOD PLAIN , N1= 0.01150
MANNING N FOR THE MAIN CHANNEL, N2= 0.01150
MANNING N FOR THE RIGHT FLOOD PLAIN , N3= 0.01150
INITIAL WIDTH, BI= 1.250 M
INITIAL X-SECTIONAL AREA, AI= 0.27125SQ.M
INITIAL STEADY STATE DISCHARGE, QI= 0.962E-01CMS
INITIAL FROUDE NUMBER F= 0.243

U/S INFLOW: DISCHARGE HYDROGRAPH, IND= 1

TIME IN SECONDS= 180.00 INFLOW= 0.9500E-01

NO. OF ITERATIONS 1

SECTION	PARAMETER N	PARAMETER I
1	1.000	0.000
2	1.000	1.000
3	1.000	0.000

FROUDE NO. AT U/S END, FN1= 0.2401

DELTA Q (CMS) AT U/S END, DQ1 = -0.00125

PARAMETER M =1.000 PARAMETER F=0.006

STAT NO.	DISTANCE (M)	DISCHARGE (CMS)	STAGE (M)
-------------	-----------------	--------------------	--------------

CHECK FOR MASS CONSERVATION

VOLLD, CU.M.=-0.2574E-01 VOLRD, CU.M.=-0.2574E-01

VOLUME DIFFERENCE, CU.M.=-0.3040E-05

TIME IN SECONDS= 360.00 INFLOW= 0.1030

NO. OF ITERATIONS 3

SECTION	PARAMETER N	PARAMETER I
---------	-------------	-------------

1	1.000	0.000
2	1.000	1.000
3	1.000	0.000

FROUDE NO. AT U/S END, FN1= 0.2588

DELTA Q (CMS) AT U/S END, DQ1 = 0.00800

PARAMETER M =1.000 PARAMETER F=0.006

STAT NO.	DISTANCE (M)	DISCHARGE (CMS)	STAGE (M)
-------------	-----------------	--------------------	--------------

1	0.000	0.103	0.218
2	15.000	0.103	0.218
3	30.000	0.103	0.218
4	45.000	0.103	0.218
5	60.000	0.102	0.218
6	75.000	0.102	0.218
7	90.000	0.102	0.218
8	105.000	0.102	0.218
9	120.000	0.102	0.218
10	135.000	0.102	0.218
11	150.000	0.101	0.218
12	165.000	0.101	0.217
13	180.000	0.101	0.218
14	195.000	0.101	0.220
15	210.000	0.099	0.222

CHECK FOR MASS CONSERVATION

VOLLD, CU.M.= 0.3036 VOLRD, CU.M.= 0.3651

VOLUME DIFFERENCE, CU.M.=-0.6148E-01

 TIME IN SECONDS= 540.00 INFLOW= 0.1140

1.2.3 Sample Output File Showing Results for the Downstream Station

THIS IS A SAMPLE EXAMPLE. FLOOD EVENT PC10 IS CONSIDERED.

TIME (SEC)	DISCHARGE (CMS)	STAGE (M)
360.0	0.099	0.222
540.0	0.102	0.226
720.0	0.107	0.233
900.0	0.114	0.244
1080.0	0.127	0.262
1260.0	0.134	0.272
1440.0	0.145	0.286
1620.0	0.157	0.302
1800.0	0.167	0.315
1980.0	0.179	0.331
2160.0	0.189	0.344
2340.0	0.198	0.356
2520.0	0.214	0.375
2700.0	0.224	0.390
2880.0	0.222	0.385
3060.0	0.232	0.397
3240.0	0.240	0.403
3420.0	0.248	0.408
3600.0	0.256	0.413
3780.0	0.267	0.419
3960.0	0.278	0.425
4140.0	0.289	0.430
4320.0	0.300	0.436
4500.0	0.312	0.441
4680.0	0.323	0.446
4860.0	0.335	0.451
5040.0	0.347	0.456
5220.0	0.358	0.460
5400.0	0.370	0.465
5580.0	0.382	0.469
5760.0	0.392	0.473

5940.0	0.396	0.474
6120.0	0.397	0.474
6300.0	0.393	0.473
6480.0	0.388	0.471
6660.0	0.380	0.468
6840.0	0.372	0.465
7020.0	0.363	0.462
7200.0	0.354	0.458
7380.0	0.344	0.455
7560.0	0.335	0.451
7740.0	0.325	0.447
7920.0	0.315	0.442
8100.0	0.305	0.438
8280.0	0.295	0.433
8460.0	0.286	0.429
8640.0	0.276	0.424
8820.0	0.266	0.419
9000.0	0.257	0.413
9180.0	0.247	0.408
9360.0	0.239	0.402
9540.0	0.230	0.396
9720.0	0.220	0.383
9900.0	0.210	0.370
10080.0	0.188	0.342
10260.0	0.173	0.323
10440.0	0.161	0.308
10620.0	0.149	0.291
10800.0	0.138	0.276
10980.0	0.127	0.262
11160.0	0.116	0.246
11340.0	0.108	0.234
11520.0	0.103	0.227
11700.0	0.102	0.225
11880.0	0.101	0.224
12060.0	0.101	0.224
12240.0	0.100	0.223
12420.0	0.098	0.220
12600.0	0.098	0.220
12780.0	0.098	0.220
12960.0	0.099	0.220