

# Public Debt and GDP Growth in WAEMU Countries

by Junru Hou

(0300030859)

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Department of Economics of the University of Ottawa  
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Supervisor: Yazid Dissou

ECO 6999

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**Abstract:**

This paper tests the relationship between public debt and GDP per capita growth in WAEMU countries between 1980 and 2015 using panel data regression model. The influence on economic growth can be reflected by the changes in not only public debt but also other economic factors, such as the initial GDP per capita growth, openness, etc... There are three ways to solve panel data regression model: pooled OLS estimation, fixed effects estimation, and random effects estimation. Based on the assumptions and the results, dummy variables estimation, one of the methods of fixed effect estimation, is the preferred model. According to the results, a 1% increase in public debt will lead to a 0.014% slowdown in GDP per capita growth. Public debt can affect growth negatively with statistical insignificance; thus, public debt does not seem to negatively affect growth. Regarding to debt threshold, although debt thresholds vary across countries, there exists a common range of the debt thresholds in WAEMU countries: around 40%-60%, based on the two-way plots.

**Keywords:**

Public debt; Economic growth; Panel data regression.

## 1. Introduction

Nowadays, most countries have significant improvements in the economy. More and more individuals have better lives than before, and some even got rid of poverty. Public debt is one of the elements that can affect the economy.

Public debt is an essential method used by governments to collect resources for funding economic activities, like investment in public infrastructure that contributes to improving the standard of living of citizens. Besides, government incurs debt when the country is in a recession, in order to stimulate the economy.

Still, public debt can also cause some disadvantages. Since the government borrows funds from investors, investors must take financial risks. The higher is the ratio of public debt to GDP, the more risk investors undertake. Because of the growing risks, investors expect more returns so that the interest rates begin to increase. The governments must pay back more to investors, and then the government expenditure rises and hampers future spending on public infrastructure and growth. It thus appears that public debt can have negative on economic growth.

Recently, several researchers have been interested in the relationship between public debt and economic growth. Panizza and Presbitero (2014), Woo and Kumar (2014), Egert (2013), and Reinhart and Rogoff (2010) are some examples of studies on the relationship between public debt and economic growth in various countries. They obtained different results ranging from negative to no relationships between the two variables. While most of these studies relate to emerging and developed countries, there are not that many studies on low-income countries and especially African countries.

In this paper, we plan to fill this gap by analyzing the relationship between public debt and GDP growth in WAEMU (West African Economic Monetary Union) countries. This group comprises Benin, Burkina Faso, Cote d'Ivoire, Mali, Niger, Senegal, and Togo. We use panel regression methods to analyze the relationship between public debt and economic growth. In particular, we rely on the pooled estimation, fixed effect estimation, and random effect estimation in our approach.

The main finding is that there is a negative and non-significant relationship between public debt and GDP per capita growth in the WAEMU countries. A one percent increase in the ratio of public debt to GDP will cause a 0.014% reduction in GDP per capita growth. Overall, in the WAEMU countries the increase in public debt did not seem to hurt economic growth during the period of study

The remainder of the paper is organized as follows. The next section presents a review of the literature on the subject and the theoretical framework for analyzing the two variables. Section 3 discusses the stylized facts on debt and growth in the world and in the WAEMU countries in particular. In the fourth section, we present the data, the econometric model and discuss the results. We conclude in the last section.

## **2. Literature review and theoretical framework**

### **2.1 Literature review**

Several studies have been carried out to analyze the relationship between public debt and economic growth. A significant number of researchers have pointed out a negative relationship between public debt and growth. Some authors have raised the issue of the existence of the debt threshold from which an increase in the debt level has a negative impact on growth. This issue has led to heated discussions in the literature.

Panizza and Presbitero (2014) suggested that there is no evidence to show a causal relationship between public debt and growth. They used an instrumental variable method. Because an excellent instrument must have some effects on public debt, they set up an instrumental variable called valuation effect (VE), and the lagged value of VE is positively related to public debt with statistical significance. Also, they built up a baseline regression model. In their model, GDP growth is the dependent variable. The independent variables are public debt and some other economic variables as control variables. In the baseline specification that does not include the instrumental variable, a 10% increase in the debt to GDP ratio leads to an 0.18% slowdown of average growth. After adding the lagged value of VE to the model, they found a positive relationship between public debt and the

instrumental variable. Although the relationship between public debt and growth is positive, the coefficient of public debt becomes statistically insignificant, which means that there seems no causal relationship between public debt and growth.

Woo and Kumar (2014) suggested that the relationship between public debt and growth depends on the initial level of public debt. The regressors used in their model include GDP, population, investment, and government size. Overall, they found a negative relationship between initial government debt and subsequent growth of real GDP per capita. A 10% increase in initial debt-to-GDP is associated with a subsequent slowdown in GDP per capita growth by 0.24%. The subsequent growth of real GDP per capita with high initial debt is lower than that with low initial debt. This means that the higher public debt may lead to slower growth of GDP per capita.

Wani and Kabir (2016) focused on the relationship between public debt and economic growth in Afghanistan. They mentioned that government debt is one method of funding government operations, and governments can also create money to monetize their debts, thereby removing the need to pay interest. Domestic national debt may have little effect upon the economy because citizens, as potential taxpayers, are also indebted to themselves as holders of government debt. In their studies, domestic and external debt can represent public debt. Moreover, government stock and advances from commercial banks are two variables of domestic debt. They used the Harrod-Domar growth model, a multiple regression model, to solve their issues. In their model, the dependent variable is economic growth and the independent variables are government stocks, advances from commercial banks, and external debts. Based on their results, public debt lead to an inverse effect on economic growth.

Pescatori, Sandri, and Simon (2014) analyzed the relationship between public debt and growth over a long period and discussed whether 90% debt threshold is always held. The model focused on the relationship over a long period, and outcomes when the debt is above a certain level. Their model has four different periods: one year, five years, ten years, and fifteen years. Due to the outcomes of their model, there is no certain threshold. They also pointed out that debt's trajectory plays an essential role in affecting the relationship between public debt and economic growth. Moreover, they mentioned that at high levels

of debt, the relationship between public debt and GDP growth becomes relatively weak for any period except for the short term.

Egert (2013) used a formal econometric model to check whether the public debt thresholds used in the Reinhart-Rogoff dataset follows the Reinhart-Rogoff's results. Reinhart and Rogoff (2010) concluded that there are three debt thresholds: 30%, 60%, and 90%. But the 90% debt threshold is not supported in this study. Furthermore, public debt has a nonlinear effect on growth, and if public debt reaches 20% of GDP, there is a negative link between public debt and growth. However, considering individual countries, different countries have different debt thresholds.

Public debt seems to have a negative influence on growth when public debt reaches a high level. Thus, some researchers were interested in the issue of public debt above which level begins to affect growth negatively. Some studies used another model to check the debt threshold.

Constant (2016) focused on testing the nonlinearity and the effects of the threshold using the Panel Smooth Threshold Regression (PSTR) model. He collected data from 126 countries with different income-level groups averaged over 5-years periods from 1964-2014. At first, the linear model showed that public debt effects are adverse and significant. Then, the nonlinear testing showed a nonlinearity between public debt and economic growth. Moreover, under the threshold effect testing, it found a threshold as 94.47 percent. Testing the different income-level groups, respectively, the estimated threshold parameters are 95.12% for the advanced economies, 89.31% for the upper-middle-income, 84.75% for the lower-middle-income, and 77% for the low-income countries. The coefficients of the public debt variable below the threshold are positive and significant for all sub-samples; thus, public debt has a positive effect on growth when public debt is below threshold.

Ndoricimpa (2020) focused on the threshold effects of public debt on growth in some African countries. He mentioned that the increasing debt accumulation becomes unsustainable, causing repayment difficulties, a debt crisis, and a hindrance to economic growth. The model used in this research is also the Panel Smooth Threshold Regression (PSTR) model. He concluded that the debt threshold is between 62% and 66% for the whole sample. Low debt is growth-neutral, but higher public debt is detrimental to growth. For

middle-income and resource-intensive countries, the debt threshold is between 58% and 63%. As part of robustness checks, there is a dynamic panel threshold model used to deal with the endogeneity of debt, and a much higher debt threshold was estimated at 74.3%. While low public debt is either growth-neutral or growth-enhancing, high public debt is consistently detrimental to growth. There is no single debt threshold applicable to all African countries, and the debt threshold level is sensitive to modeling choices. Under the two-regime PSTR model, the first regime gives a public debt-to-GDP threshold of 65.9%, and the second is a debt threshold of 61.9%. A low regime does not have any significant effect on growth, but high public debt has a robust significant adverse effect on growth, at a 1% level. This suggests that high debt is detrimental to growth and low debt is growth-neutral. Each additional percentage point of debt-to-GDP beyond the threshold reduces annual growth by 0.024%.

Eberhardt and Presbitero (2015) discussed the relationship between public debt and economic growth in a large number of countries all over the world. Based on total public debt data, long-term debt coefficients differ across countries. Moreover, the countries with higher average debt-to-GDP ratios are more likely to harm long-term GDP growth. Some economic factors used in their study are GDP, population, capital stock, and total public debt stock. There are three models used in their study. The first model is a linear dynamic model, a log-linearized Cobb-Douglas production function augmented with a debt stock term. However, there exists a nonlinearity in the long-run relationship between debt and growth across countries. The asymmetric dynamic model, an asymmetric long-run regression model, is recommended to use under the nonlinearity conditions. There are two thresholds for the debt-to-GDP ratio: 90% and 60%. Their results showed no evidence for any systematic change in the relationship between debt and growth when countries shifted from a low to high debt regime. The last type of model is the static nonlinear model, which estimates highlight the heterogeneity in the country-specific results.

## **2.2 Theoretical framework**

First of all, several economic factors apart for public debt can also affect economic growth. The level of initial GDP per capita is a crucial factor that can affect growth. A higher level of initial GDP per capita is harder to reach a high level of growth. For example, in Table

1, most developed countries have high GDP per capita with relatively low growth rates. Their average GDP per capita growth was only 1.50 between 2015 and 2019. Furthermore, some countries even have negative growth in specific periods. Therefore, if the GDP per capita in some countries has already reached a relatively high level, these countries have difficulties to grow at a high speed.

Country	2015	2016	2017	2018	2019
Australia	0.73	1.18	0.66	1.37	0.36
Belgium	1.44	0.97	1.51	1.03	0.90
Canada	-0.09	-0.14	1.95	0.60	0.22
France	0.75	0.83	2.08	1.64	1.37
Germany	0.86	1.41	2.08	1.22	0.29
Italy	0.88	1.47	1.82	0.99	0.51
Japan	1.33	0.64	2.34	0.53	0.86
Portugal	2.21	2.34	3.76	2.80	2.31
Spain	3.92	2.94	2.65	1.90	1.37
Singapore	1.77	1.91	4.24	2.95	-0.41
United States	2.13	0.84	1.57	2.65	1.85
United Kingdom	1.55	1.15	1.20	0.73	0.84
Data from database: World Development Indicators					

However, as for developing countries they can be divided into two different groups. Some developing countries have a steady economic and social environment, while other developing countries are still facing severe conditions like war, disaster, and recession. Those countries with a relatively steady environment have perfect chances to improve their economy. For example, based on Table 2, China and India have high GDP per capita growth, reaching over 6% recently. At the same time, there are also some developing countries with lower and even negative growth. Most of these countries are at the early stages level of development. However, nowadays, due to international trade, more and more countries become wealthy and develop rapidly.

Afghanistan	-1.62	-0.54	0.08	-0.56	0.55
Argentina	1.63	-3.11	1.61	-3.47	-3.13
Brazil	-4.35	-4.07	0.51	0.53	0.38
China	6.50	6.27	6.35	6.27	5.73
India	6.80	7.08	5.91	5.02	3.96
Indonesia	3.56	3.76	3.84	3.98	3.88
Korea, Rep.	2.27	2.54	2.87	2.18	1.83
Malaysia	3.69	3.04	4.31	3.33	2.95
Thailand	2.72	3.04	3.71	3.82	2.08
Data from database: World Development Indicators					

The second factor is the openness measured as the sum of exports and imports of goods and services. Because international trade plays a more important role nowadays, the volumes and values of exports and imports increased to a significant level in most countries worldwide. In the meantime, all countries can benefit from international trade. However, developed countries with high technology can gain more benefits than developing countries. For example, developed countries can import labor and raw material at a relatively low price from developing countries and export some products and techniques at a high price to developing countries. Thus, developed countries always play a dominant role in international trade.

Moreover, total population and population growth are also important factors. Total population represents the potential market in a country and determines the upper limit of economic development. Population growth can have a long-run effect on the economy.

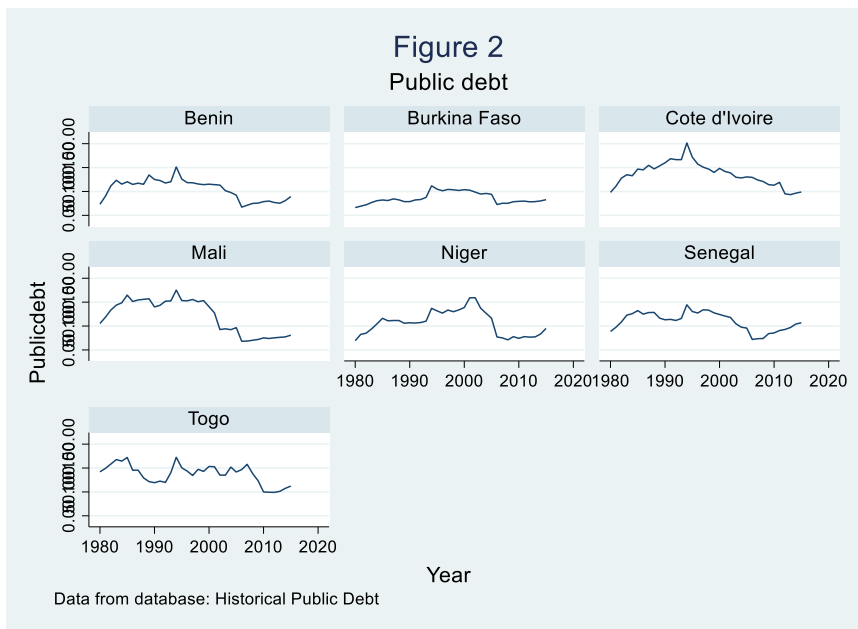
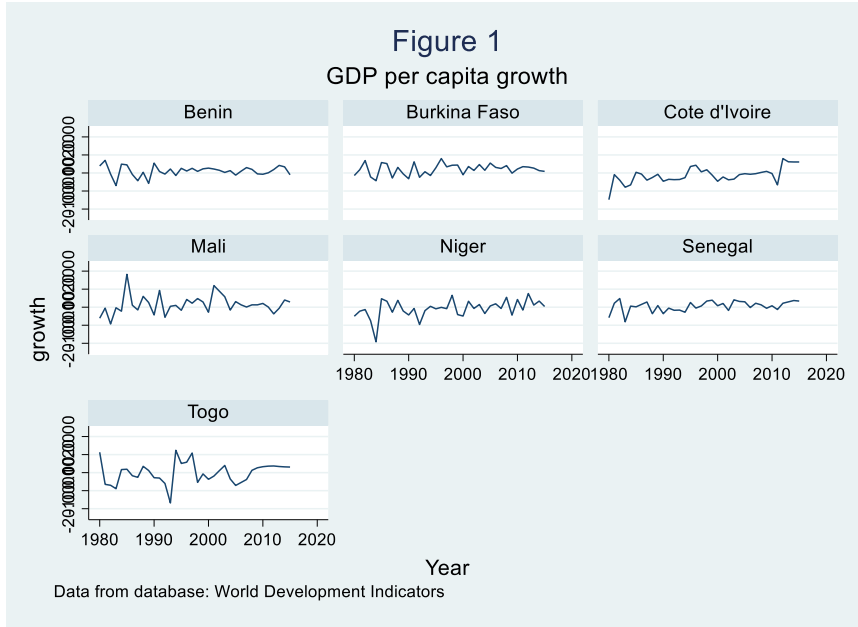
Other factors can also affect economic growth, such as inflation, government size, and domestic investment. First, the inflation rate can reflect the purchasing power of a country. With a certain level of price increases, inflation can lead to an increase in GDP. However, when the price level increases to extremely high levels, inflation can cause uncertainty and instability in society. Second, government expenditure can reflect government size. An increase in government expenditure can lead to a budget deficit. The budget deficit can increase the risks of investors and affect their confidence. Then, the

country's development also needs domestic investment. Domestic investment can provide jobs, improve infrastructure, and release the burden of the government budget.

Secondly, it is vital to identify the specific situations and developing trends of the WAEMU countries in recent decades. Moreno-Dodson and Bayraktar (2015) mention that one of the reasons why WAEMU countries have the lowest GDP per capita levels, and relatively low and irregular GDP per capita growth rates is the relatively high population growth. The data is collected from the World Bank and IMF database from 1980 to 2015. Based on Figure 1, which is the trend of GDP per capita growth for all WAEMU countries, GDP per capita growth fluctuates around zero. In most WAEMU countries, it is between -10% and 10%, such as Benin, Burkina Faso, Cote D'Ivoire, and Senegal. However, GDP per capita growth in Mali is between -10% and 20%. Moreover, GDP per capita growth in Niger and Togo is between -20% and around 10%.

As for public debt, shown in Figure 2, all WAEMU countries seem to have a similar trend of public debt from 1980-2015. From 1980 to the mid-1990s, all WAEMU countries except for Togo have an increasing trend. However, the increasing trends in Togo were before the mid-1980s, and between the 1990s and the mid-1990s. There was a decreasing trend in Togo during the mid-1980s and around the 1990s. Furthermore, from the mid-1990s to the mid-2020s, the trends in all WAEMU countries were decreasing. After that, all countries were increasing their public debt.

Also, a staff report of WAEMU countries (IMF African department, 2019) mentioned that the economy in WAEMU countries has a growing trend recently. Real GDP growth has reached 6 percent for the last seven years. Public debt grew to 52.5 percent of GDP in 2018 from 17.5 percent over the last five years. The cost related to debt increased to 33 percent of revenue in 2018 from 20.75 percent in 2015, and its servicing increased to 9.1 percent of revenue from 6.7 percent.



A panel data model can be an excellent method to analyze and test the relationship between growth and public debt. In this panel data model, GDP per capita growth is the dependent variable, and public debt is one of the independent variables. Other independent economic factors are the control variables. Also, in order to build up this model, there are some assumptions. Firstly, the model should be linear. This means that all variables, including public debt, are linearly related to growth. Secondly, all variables are independent from each other. Next, the error term should be uncorrelated with all

independent variables. Besides, there are three ways to deal with panel data models: pooled OLS estimation, fixed effects estimation, and random effects estimation. Within effect estimation and dummy variables estimation are two ways of fixed effects estimation. Based on the previous assumptions and the following results, dummy variables estimation seems to be the preferred model.

Some studies concluded that there is a negative relationship between public debt and growth, or there is no causal relationship because of statistical insignificance. Some studies also found a nonlinear relationship between growth and public debt using by the Panel Smooth Threshold Regression (PSTR) model. The PSTR model, which is nonlinear, uses a hypothesis test to check the effects of threshold and test whether there is a linear relationship between variables. As for the debt threshold, Reinhart and Rogoff (2010) suggested that there are three debt thresholds: 30%, 60%, and 90%, while several studies discussed Reinhart-Rogoff's conclusion and found that this conclusion does not seem to be always held. However, whether this conclusion is correct or not is still discussed by other researchers. This paper will also check whether the debt thresholds exist in each WAEMU country.

### **3. Data and model**

The data used in this study were collected from the World Bank Dataset and IMF Dataset between 1980 and 2015. They include GDP per capita growth, initial GDP per capita measured as Log of GDP per capita, public debt measured as the ratio of public debt to GDP, openness measured as the sum of exports and imports of goods and services to GDP, population growth, total population measured as the log of the total population, the inflation rate, government size measured as the ratio of government expenditures to GDP and the ratio of domestic investment to GDP. Table 3 presents some descriptive statistics of the model variables during the period of analysis.

Table 3 Descriptive Statistics						
Variables		Mean	Std. Dev.	Min	Max	Observations
Growth	Overall	0.41	4.43	-19.2	18.2	N = 248
	Between		1.19	-1.28	2.05	N = 7
	Within		4.29	-17.97	17.28	T-bar = 35.43
Initial GDP	Overall	2.65	0.25	2.20	3.19	N = 252
	Between		.212	2.42	2.96	n = 7
	Within		0.15	2.32	3.05	T = 36
Public debt	Overall	64.62	28.97	16.30	151.42	N = 250
	Between		18.28	36.22	87.35	n = 7
	Within		23.45	12.03	129.82	T-bar = 35.71
Openness	Overall	60.06	17.92	28.30	118.10	N = 252
	Between		16.01	39.53	85.44	n = 7
	Within		10.04	26.02	92.72	T-bar = 36
Population growth	Overall	2.91	0.52	1.30	4.40	N = 252
	Between		.285	2.55	3.43	n = 7
	Within		0.44	1.66	4.25	T-bar = 36
Total Population	Overall	6.95	0.21	6.43	7.37	N = 252
	Between		0.16	6.66	7.16	n = 7
	Within		0.13	6.69	7.23	T-bar = 36
Inflation	Overall	3.98	6.75	-7.80	39.20	N = 229
	Between		0.78	2.92	4.93	n = 7
	Within		6.72	-7.00	38.61	T-bar = 32.71
Government Expenditure	Overall	15.32	3.65	7.30	26.10	N = 239
	Between		2.33	13.17	20.20	n = 7
	Within		2.90	4.32	24.54	T-bar = 34.14
Domestic Investment	Overall	18.45	6.30	3.10	40.00	N = 247
	Between		2.88	13.34	21.85	n = 7
	Within		5.69	2.79	39.69	T-bar = 35.29

The model sets up a panel regression as follows:

$$y_{i,t} = \beta \frac{\text{Public debt}}{\text{GDP}}_{i,t} + z'_{i,t}\gamma + c_i + \varepsilon_{i,t}, \text{ eq. (1)}$$

where  $c_i$ : country fixed effects and  $z'_{i,t}$ : a matrix of control variables. There are seven countries over 36 years,  $i = 1, 2, \dots, 7$  and  $t = 1, 2, \dots, 36$ .

Three methods can deal with a panel regression: pooled estimation, fixed effect estimation (within effect and dummy variables), and random effect estimation.

First, the Pooled OLS model is a multiple linear regression without any country or time fixed effects. Let  $x_{i,t}$  be a matrix of all variables. The model is as follows:

$$y_{i,t} = x'_{i,t}\beta + \varepsilon_{i,t}, \text{ eq. (2)}$$

There are several important assumptions regarding the pooled OLS model. The first assumption is the linearity assumed at the beginning of this paper. The second is that the expected value of the errors is zero, and there is no relationship between errors and independent variables. The last assumption is that all error terms have the same variance and are not related to each other. Due to the country fixed effects, the independent variables may correlate to the error term. Thus, the estimators in this model may be biased.

Second, the within effect estimation, one of the methods to deal with fixed effect model, uses the differences from within effect mean. In that method, the country fixed effects can be dealt with as the intercept. The panel regression will be as follows:

$$y_{i,t} = x'_{i,t}\beta + c_i + \varepsilon_{i,t}, \text{ eq. (3) and } x_i = \begin{bmatrix} x'_{i,1} \\ \vdots \\ x'_{i,T} \end{bmatrix}$$

$$\text{Time averages: } \bar{y}_i = \frac{1}{T} \sum_{t=1}^T y_{i,t} = \frac{1}{T} \sum_{t=1}^T (x'_{i,t}\beta + c_i + \lambda_t + \varepsilon_{i,t}) = \bar{x}_i\beta + c_i + \bar{\varepsilon}_i, \text{ eq. (4)}$$

$$\text{where } \bar{x}_i = \frac{1}{T} \sum x_{i,t}, \text{ and } \bar{\varepsilon}_i = \frac{1}{T} \sum \varepsilon_{i,t}$$

Using differences from the mean, the panel regression can be rewritten as:

$$y'_{i,t} = y_{i,t} - \bar{y}_i = (x'_{i,t} - \bar{x}_i)\beta + (\varepsilon'_{i,t} - \bar{\varepsilon}_i) = x'_{i,t} \beta + \varepsilon'_{i,t}, \text{ eq. (5)}$$

which is a multiple regression.

The estimated coefficient is:

$$b_{fe} = \left( \sum_{i=1}^N \sum_{t=1}^T x'_{i,t} x_{i,t} \right)^{-1} \left( \sum_{i=1}^N \sum_{t=1}^T x'_{i,t} y'_{i,t} \right), \text{ eq. (6)}$$

Another method uses dummy variables estimation, which is also called Least square dummy variables (LSDV). This method uses a set of dummy variables. The model sets up is as follows:

$$y_{i,t} = x'_{i,t}\beta + d'_i c + \varepsilon_{i,t}, \text{ eq. (7) where } d_i \text{ are country dummies, which have } (N-1) \text{ elements.}$$

$$D_i = [d_{2i} \quad \dots \quad d_{Ni}]', \text{ where } d_{ni} = \begin{cases} 1 & \text{if } i = n \\ 0 & \text{otherwise} \end{cases}$$

$$x'_{i,t} = [1 \quad x_{1,it} \quad \dots \quad x_{k-1,it}]', \text{ which is the matrix of variables.}$$

The country fixed effects are:

$$c = [(c_2 - c_1) \quad \dots \quad (c_N - c_1)]'$$

$$\text{Moreover, the coefficients of the independent variables are } \beta = \begin{pmatrix} c_1 \\ \beta_1 \\ \vdots \end{pmatrix}.$$

Thus, this model can be rewritten as:

$$y = x\beta + Dc + \varepsilon = \tilde{x}\tilde{\beta} + \varepsilon, \text{ eq. (8)}$$

where  $\tilde{x} = [x \quad D]$ .

$$\text{The estimated coefficients of variables are } b_{dr} = (\tilde{x}'\tilde{x})^{-1}(\tilde{x}'y). \text{ eq. (9)}$$

Although the models of within effect and LSDV use different methods, their results are identical. The LSDV method is preferred and better than the within effect model. Park (2011) mentioned that the R-squared in the within-effect model is not correct. Because of correct estimation, goodness-of-fit, and group/time-specific intercepts, LSDV is the best estimation method compared with the other methods.

Finally, the Random effect model is also an important method to deal with these issues. One of the important differences between the fixed effect model and the random effect model is that the country or time effects do not affect any independent variables in the random effect model. In the random effect model, these country effects measure as error terms. The model of random effect is as follows:

$$y_{i,t} = x'_{i,t}\beta + (c_i + \varepsilon_{i,t}) = x'_{i,t}\beta + v_{i,t}. \text{ eq. (10)}$$

Due to the previous assumptions, the model's error terms do not have any relationship with independent variables. Since in the random effect model, these country effects measured as the error terms should not be correlated with independent variables.

## 4. Results

### 4.1 Pooled OLS regression

Table 4 shows the results of the pooled OLS regression. First, it is essential to check whether this model fits the data. Because the p-value of its F-test is close to 0, this model fits the dataset at a 5% significance level. Then, the coefficient of Public debt to GDP is 0.0025, which is positive but extremely small and close to 0. In this model, an increase in public debt to GDP can slightly increase GDP per capita. However, the p-value of public debt reaches 0.837, which is much larger than the 0.05 significance level. This means that we should not reject the null hypothesis  $H_0: \beta_i = 0$ . Thus, public debt is not statistically significant at the 0.05 level.

As for the initial GDP per capita, its coefficient is -3.515; thus, it harms GDP per capita growth. Because its p-value is 0.026, smaller than 5%, the null hypothesis  $H_0: \beta_i =$

0 should be rejected at the 0.05 significance level. Thus, initial GDP per capita is negatively related to growth and statistically significant.

The coefficient of openness is 0.00763, which suggests a positive relationship with GDP per capita growth. However, openness is not statistically significant at the 0.05.

Regarding population growth and total population, the coefficients are -2.1368 and 4.8, respectively. There is a negative relationship between population growth and GDP per capita growth, while total population can affect growth positively. Furthermore, their p-values are smaller than 0.05, which means that the null hypothesis is rejected.

As for the inflation rate, its coefficient is 0.0273; thus, there is a positive relationship between inflation and growth. Nevertheless, its p-value is 0.505, which is larger than the 0.05 significance level. The inflation rate is not statistically significant.

The coefficient of government expenditure is -0.0622, which means a slightly negative effect on growth. Moreover, its p-value is 0.452, which is also higher than the 0.05 significance level. Thus, government expenditure is not statistically significant.

As for investment, its coefficient is 0.238, so there is a positive relationship between investment and growth. Moreover, its p-value is close to 0, which is smaller than the 0.05 significance level.

Variables	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Initial GDP	-3.52	1.57	-2.24	0.03	-6.61	-0.42
Public debt	0.0025	0.012	0.21	0.834	-.021	.026
Openness	0.0077	0.021	0.36	0.72	-0.03	0.05
Pop. growth	-2.14	0.57	-3.76	0	-3.26	-1.01
Total Pop.	4.80	1.66	2.89	0.004	1.53	8.07
Inflation	0.03	0.04	0.67	0.505	-0.05	0.11
Gov. Exp	-.062	0.08	-0.75	0.45	-0.22	0.10
Domestic Inv	0.239	0.052	4.57	0	0.14	0.34
Constant	-21.65	10.60	-2.04	0.04	-42.55	0.76
R-squared	0.202					
F-test	6.58					
Prob > F	0					

In the pooled OLS model, GDP per capita growth can be affected positively by public debt, exports and imports, total population, inflation, and domestic investment. However,

their effects on growth are relatively small. Moreover, due to the statistical insignificance of economic factors' coefficients, there is no evidence that there are causal effects among these economic factors and growth. This means that these variables can improve economic growth, but they may improve or slow down economic growth in other periods. Total population and domestic investment can have positive effects on GDP per capita growth with statistical significance. In this model, when population and domestic investment increase, there is a high possibility that these factors can lead to a growing economy.

Furthermore, there are negative relationships between GDP per capita growth and the initial GDP per capita, population growth, and government expenditure. Both the initial GDP per capita and population growth can impede the growth with a high possibility. Government expenditure can lead to a negative effect on growth. However, because of the statistical insignificance, there is no evidence that there is a casualty effect between government expenditure and economic growth.

However, due to the countries' fixed effects, this model's results are inaccurate.

#### **4.2 Within effect model**

Based on Table 5, this model fits the dataset since the F-statistics is 7.03. Furthermore, the p-value is smaller than the 0.05 significance level. The coefficient of public debt to GDP is -0.13788, which shows a negative effect on GDP per capita growth but extremely small and close to 0. In this model, an increase in public debt to GDP can slightly decrease GDP per capita. However, its p-value reaches 0.431, which is much larger than the 0.05 significance level. This means that we cannot reject the null hypothesis  $H_0: \beta_1 = 0$ . Public debt is not statistically significant at the 0.05 level.

As for the initial GDP per capita, its coefficient is -10.872; thus, there is a negative effect on GDP per capita growth. Because its p-value is close to 0, the null hypothesis  $H_0: \beta_1 = 0$  should be rejected, so initial GDP per capita is statistically significant.

The coefficient of openness is 0.0049, so there is a positive relationship between openness and GDP per capita growth. However, it is not statistically significant at the 0.05 level. Regarding population growth and total population, the coefficients are -1.012 and 12, respectively. Population growth can affect growth positively, while total population is negatively related to GDP per capita growth. Moreover, the p-value of total population is

close to 0, which leads to a rejection of the null hypothesis. However, because the p-value of population growth is 0.148, we cannot reject the null hypothesis. Thus, total population is statistically significant at the 0.05 level, but the population growth is statistically insignificant.

As for the inflation rate, its coefficient is 0.034, so there is a positive relationship between inflation and growth. However, its p-value is 0.411, which is larger than the 0.05 level. The inflation rate is not statistically significant.

The coefficient of government expenditure is -0.073; thus, it has a slightly negative effect on growth. However, because its p-value is 0.458, government expenditure is not statistically significant.

As for domestic investment, its coefficient is 0.203, so there is a positive relationship between investment and growth. Also, its p-value is close to 0, leading to a rejection of the null hypothesis; thus, investment is statistically significant.

<b>Table 5 Fixed effect (within effect) model</b>						
Variables	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Initial GDP	-10.87	3.06	-3.55	0.000	-16.91	-4.83
Public debt	-0.013	0.02	-0.79	0.43	-0.05	0.02
Openness	0.005	0.03	0.16	0.87	-0.05	0.06
Pop. growth	-1.01	0.70	-1.45	0.15	-2.39	0.369
Total Pop.	12.00	2.77	4.33	0	6.53	17.47
Inflation	0.034	0.042	0.82	0.41	-0.05	0.12
Gov. Exp	-0.07	0.10	-0.74	0.46	-0.27	0.12
Domestic Inv	0.203	0.061	3.31	0.001	0.08	0.32
Constant	-53.54	18.04	-2.97	0.003	-89.11	-17.97
R-squared	0.2186					
F-test	7.03					
Prob > F	0					

GDP per capita growth can be affected positively by exports and imports, total population, inflation rate, and domestic investment. The values of exports and imports and

the inflation rate can improve economic growth slightly. However, due to the statistical insignificance of coefficients, there is no evidence to show that openness and inflation have any effect on growth. This means that these variables could have slowed down economic growth in some periods and could have improved the economy in other periods. However, total population and domestic investment positively affect GDP per capita growth with statistical significance.

Furthermore, initial GDP per capita, public debt, population growth, and government expenditure can affect GDP per capita growth negatively. Only initial GDP per capita can impede growth with a high possibility since there is statistical significance. Public debt, population growth, and government expenditure can have a negative effect on growth. However, because of the statistical insignificance, there is no evidence that there is a casualty effect.

#### **4.3 Dummy variables model**

As for the dummy variables estimation, its results, shown in Table 6, are almost the same as the results of within estimation. Every coefficient, standard error, t-statistics, and p-value are the same. Because there is a rejection for the null hypothesis of the F-test, both within effect model and dummy variables model fit better than the pooled OLS model. However, there also exist several differences.

Firstly, the F-statistic in the dummy variables model is 5.31, while in within effect estimation it is 7.03. Secondly, the R-squared in the dummy variables model is 0.2836, but the R-squared in the within effect model is 0.2186. A slightly higher R-squared in the dummy variable estimation method implies that the dummy variable model fits better than the within effect estimation model. Thus, the preference is to choose the dummy variables model.

<b>Table 6 Dummy Variable Model</b>						
Variables	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Initial GDP	-10.87	3.06	-3.55	0	-16.91	-4.83
Public debt	-0.013	0.02	-0.79	0.43	-0.05	0.02
Openness	0.005	0.03	0.16	0.87	-0.05	0.06
Pop. growth	-1.01	0.70	-1.45	0.15	-2.39	0.369
Total Pop.	12.00	2.77	4.33	0	6.53	17.47
Inflation	0.034	0.042	0.82	0.41	-0.05	0.12
Gov. Exp	-0.07	0.10	-0.74	0.46	-0.27	0.12
Domestic Inv	0.203	0.061	3.31	0.001	0.08	0.32
C1	-50.89	17.84	-2.85	0.005	-86.06	-15.72
C2	-55.12	18.03	-3.06	0.003	-90.67	-19.57
C3	-52.95	18.84	-2.81	0.005	-90.10	-15.80
C4	-54.50	18.20	-2.99	0.003	-90.39	-18.60
C5	-57.66	18.19	-3.17	0.002	-93.53	-21.79
C6	-50.90	17.92	-2.84	0.005	-86.24	-15.56
C7	-51.31	17.40	-2.95	0.004	-85.61	-17.01
R-squared	0.2836					
F-test	5.31					
Prob > F	0					

#### 4.4 Random effect model

Table 7 presents the results of the random effect model. The main difference between the random effect model and the fixed effect model is that there is no correlation between the specific effects and the random effect model's independent variables.

The coefficient of Public debt to GDP is 0.0025, which shows a slightly positive effect on GDP per capita growth. Nevertheless, its p-value reaches 0.837, which is much larger than the 0.05 significance level. This means that we cannot reject the null hypothesis  $H_0: \beta_i = 0$ ; thus, public debt is not statistically significant.

As for initial GDP per capita, its coefficient is -3.515; thus, there is a negative effect of initial GDP per capita on growth. Because its p-value is 0.025, which is smaller than 5%, the null hypothesis  $H_0: \beta_i = 0$  should be rejected. Therefore, initial GDP per capita is statistically significant.

The coefficient of openness is 0.0077. Openness has a positive effect on GDP per capita growth with statistical insignificance.

Regarding the population growth rate and total population, the coefficients are -2.1368 and 4.8, respectively. Population growth is negatively related to GDP per capita growth, while there is a positive relationship between total population and growth. Moreover, their p-values are both smaller than 0.05.

As for the inflation rate, its coefficient is 0.027 and its p-value is 0.504; thus, there is a positive relationship between inflation and growth with statistical insignificance.

The coefficient of government expenditure is -0.062; thus, government expenditure has a slightly negative effect on growth. Moreover, its p-value is 0.458, means that government expenditure is not statistically significant.

As for the investment, its coefficient is 0.239, so there is a positive relationship between investment and growth. Also, its p-value is close to 0. Investment is statistically significant.

**Table 7 Random Effect Model**

Variables	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Initial GDP	-3.52	1.57	-2.24	0.025	-6.59	-0.44
Public debt	0.0025	0.012	0.21	0.84	-0.021	0.026
Openness	0.0077	0.021	0.36	0.72	-0.034	0.049
Pop. growth	-2.14	0.57	-3.76	0	-3.25	-1.02
Total Pop.	4.80	1.66	2.89	0.004	1.55	8.05
Inflation	0.027	0.041	0.67	0.504	-0.053	0.11
Gov. Exp	-0.062	0.083	-0.75	0.45	-0.22	0.10
Domestic Inv	0.239	0.052	4.57	0	0.14	0.34
Constant	-21.65	10.60	-2.04	0.041	-42.43	-0.88
R-squared	0.1771					
Wald chi2(8)	52.68					
Prob > chi2	0					

In the random effect model, GDP per capita growth can be affected positively by public debt, openness, total population, inflation, and domestic investment. In this model, the increases in public debt to GDP, the values of exports and imports, and the inflation rate can slightly improve the economy. However, due to the statistical insignificance, there is no evidence that there are causal links between these factors and growth. This means that these variables may not always positively affect the economy, but they may be harmful to

the economy. Total population and domestic investment can have positive effects on GDP per capita growth with statistical significance. In this model, when population and domestic investment increase, there exists a high chance that these factors lead to a growing economy.

Furthermore, there are negative relationships between GDP per capita growth and initial GDP per capita, population growth, and government expenditure. Initial GDP per capita and population growth can impede the economy's growth with a high possibility. Government expenditure can lead to a negative effect on growth. However, because of the statistical insignificance, there is no evidence that there is a causal effect between government expenditure and economic growth.

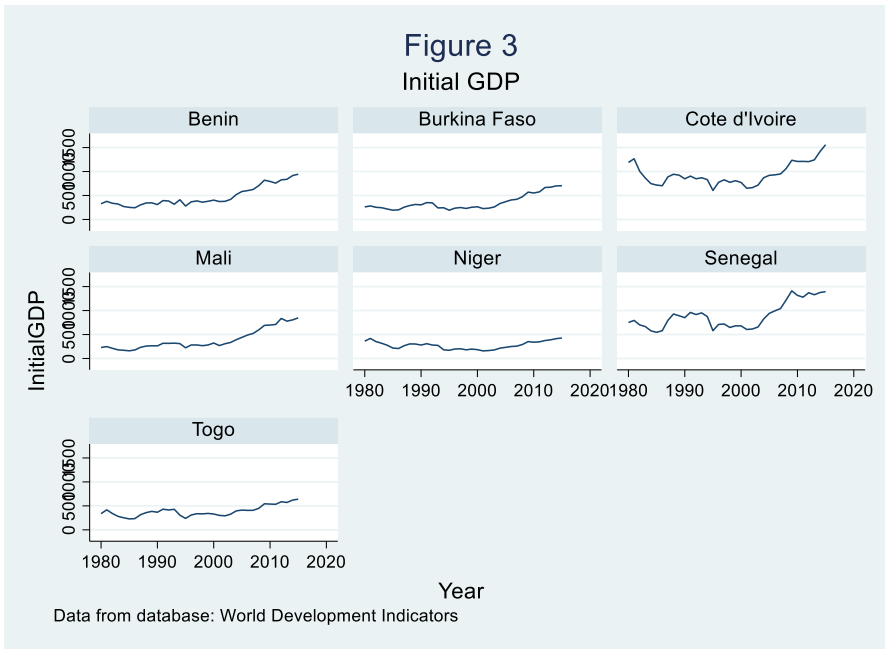
Based on the data, if the country effects measured as the error terms do not correlate with all independent variables, the random effect model results are almost the same as in the pooled OLS model. It seems like the country-specific effect can be precisely the error term.

#### **4.5 Conclusion and comparison of the four models**

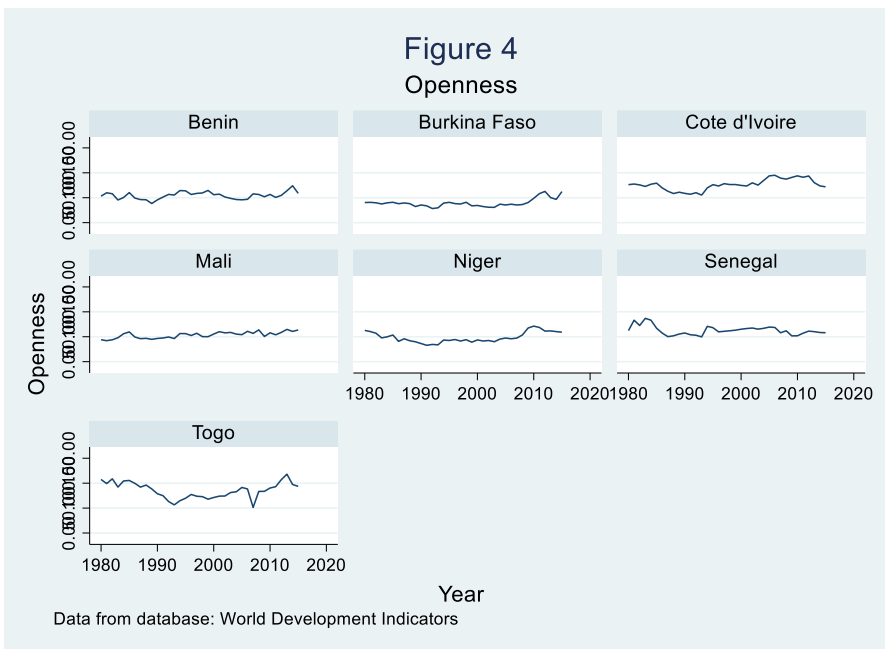
Table 8 compares the four models' results. At first, all models fit the data well; the p-values of the different specifications are all close to 0. The pooled OLS model and random effects have almost the same results, such as the same coefficient, standard deviation, and p-values of the estimated coefficients. Also, the fixed effects model and dummy variables model have the same results. Due to the higher R-squared, the dummy variables model is our preferred model. This also means that fixed effects play an essential role in this dataset. Secondly, based on the results of dummy variables, a 1% increase in public debt to GDP will cause a 0.014% reduction in GDP per capita growth. This constitutes a slight change in GDP per capita growth. Nevertheless, there is statistical insignificance between public debt to GDP and GDP per capita growth.

Variables	Pooled	Within effect	Dummy variable	Random effect
Initial GDP	-3.52 (1.57)	-10.87 (3.06)	-10.87 (3.06)	-3.52 (1.57)
Public debt	0.0025 (0.012)	-0.013 (0.02)	-0.013 (0.02)	0.0025 (0.012)
Openness	0.0077 (0.021)	0.005 (0.03)	0.005 (0.03)	0.0077 (0.021)
Population Growth	-2.14 (0.57)	-1.01 (0.70)	-1.01 (0.70)	-2.14 (0.57)
Total Population	4.80 (1.66)	12.00 (2.77)	12.00 (2.77)	4.80 (1.66)
Inflation	0.03 (0.04)	0.034 (0.042)	0.034 (0.042)	0.027 (0.041)
Government Expenditure	-0.062 (0.08)	-0.07 (0.10)	-0.07 (0.10)	-0.062 (0.083)
Domestic Investment	0.239 (0.052)	0.203 (0.061)	0.203 (0.061)	0.239 (0.052)
R-squared	0.202	0.2186	0.2836	0.1771

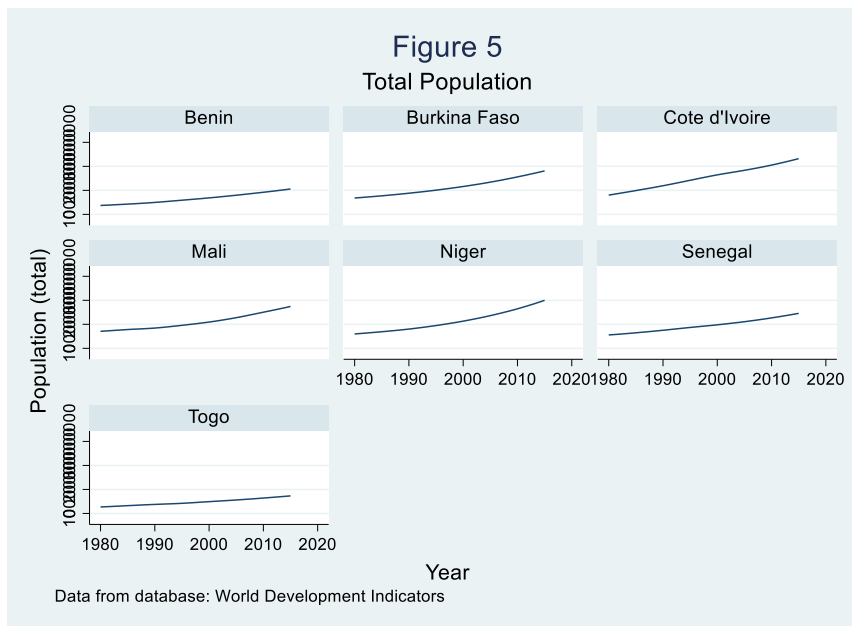
Initial GDP per capita affects GDP per capita growth negatively. This means that if there is a high GDP per capita in the previous year, it is hard to have a high-speed growth. Moreover, the relationship between initial GDP per capita and growth is statistically significant. The previous GDP per capita has a close link to growth. Based on Figure 3, almost all the WAEMU countries have increasing trends between 1980 and 2015.

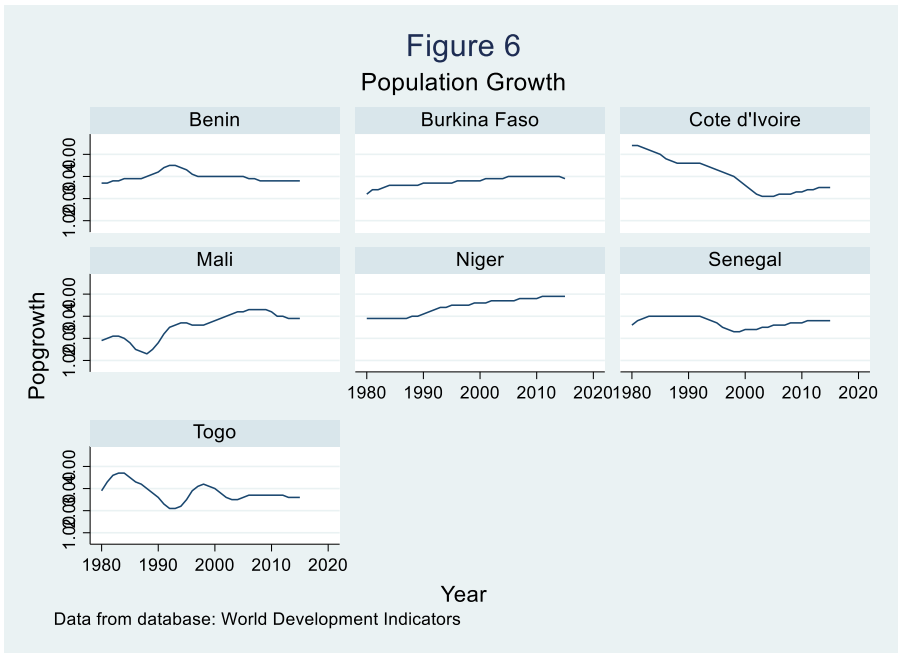


Openness is positively related to growth. As mentioned before, international trade can bring benefits to these countries and improve their economy. However, developed countries are the leaders of international trade. There is a slight growth in the economy as the values of exports and imports increase, but there is statistical insignificance. Moreover, in Figure 4, the exports and imports' values have changed only slightly between 1980 to 2015.

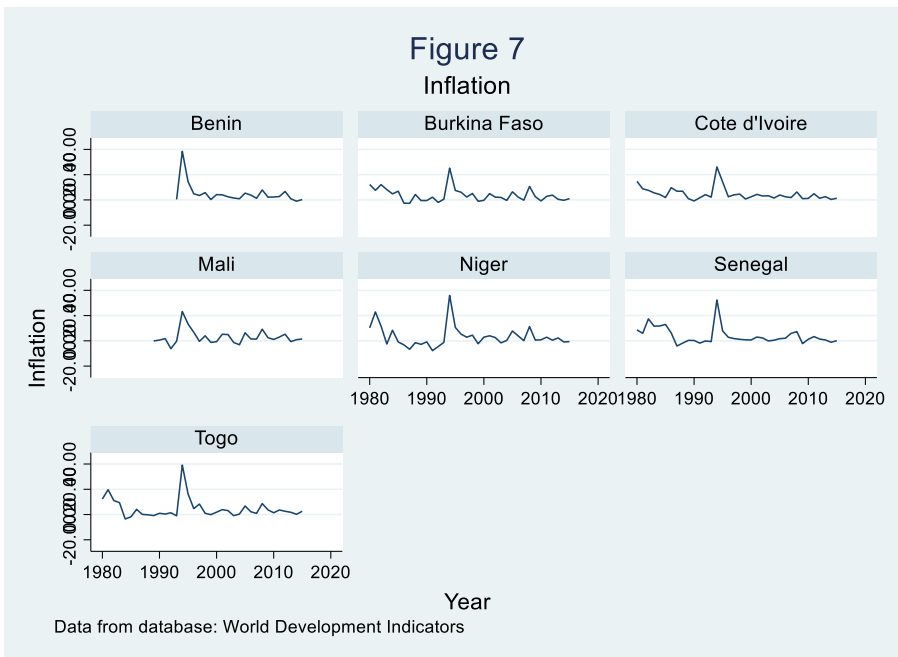


Total population has a positive effect on growth. A high level of population can represent plenty of consumers and workers and a larger potential market. These potential markets can create huge profits to society. An increase in total population can lead to an increase in economic growth. However, there is a negative relationship between population growth and GDP per capita growth. Population growth represents how many children are born in the current year. The government provides services to support citizens, and these welfare measures can increase the burden of the government. The countries will gain nothing from the newborn in the short run, but the new generation will contribute to the countries when they grow up. Therefore, an increase in population growth affects growth negatively in the current period, while it may lead to a positive effect on growth in the future. From Figure 5, all WAEMU countries have increasing trends of total population in recent decades. WAEMU countries, except for Benin and Togo, have almost the same levels of total population. The levels of population in Benin and Togo are lower than in other WAEMU countries. As for population growth shown in Figure 6, Benin, Niger, and Senegal have relatively flat trends between 1980 and 2015. The overall trends in Cote d'Ivoire and Togo is decreasing between 1980 and 2015, while Mali has an increasing trend. Moreover, the trends in all WAEMU countries become steadier between 2000 and 2015.

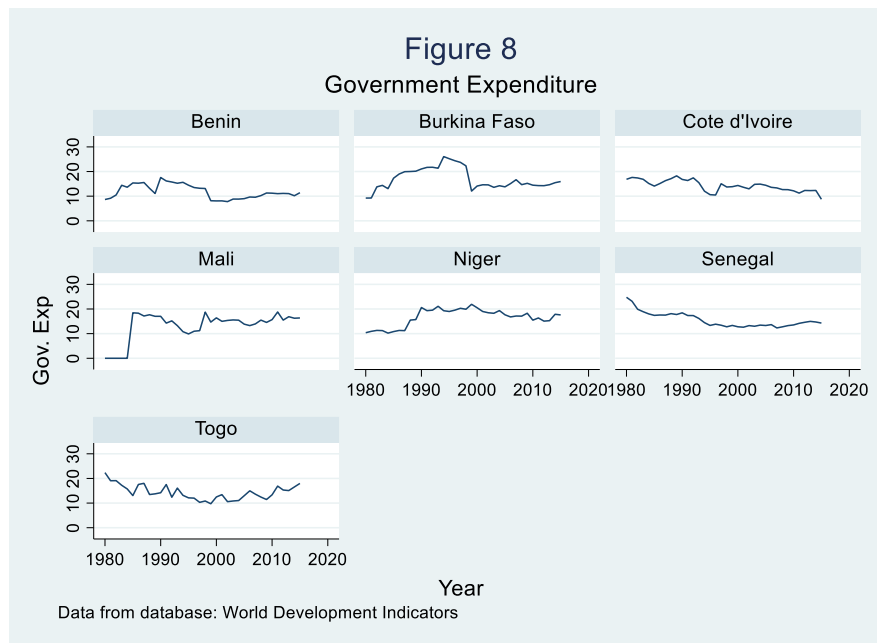




As for inflation, it has a positive effect on growth, but there is statistical insignificance. The inflation rate affects the purchasing power of the citizens. However, if the inflation rate becomes negative, deflation can induce a recession. Thus, it is essential to maintain the inflation rate at a certain level with slight fluctuations. In Figure 7, the inflation rate in WAEMU countries remained relatively low after 2000.

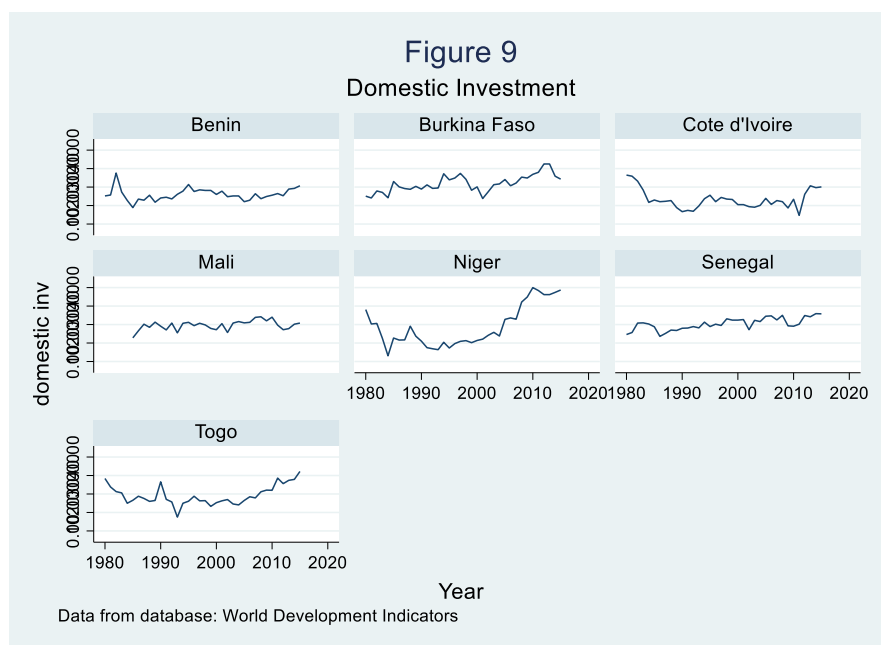


As for government expenditures, there is a negative relationship between government expenditure and GDP per capita growth rate. If there is a small government in the country, a small increase in government expenditure will easily cause government deficits. Government deficits will increase the risk of debt related to the government. Investors expect more returns because of the high risk, and the government pays more interest to the investors. Economic growth will be negatively affected. Based on Figure 8, every WAEMU country has a different trend in government expenditure. Some countries, such as Cote d'Ivoire and Senegal, have an overall decreasing trend. Benin and Burkina Faso both have an increasing trend at the beginning, a sudden decrease to a relatively low level, and then a steady trend until 2015. As for Niger, there is an overall increasing trend. In Togo, there is a decreasing trend until around 2000, and then government expenditure increases until 2015.



Domestic investment is positively correlated with growth with a statistical significance. Domestic investment is a direct way to enhance the growth of GDP. However, the wealth of citizens can play a significant role in domestic investment. The citizens who can invest in a large-scale project must have enough capital. Countries with low-income

levels cannot raise enough investments, and the levels of domestic investment in such countries must be at a low level. Figure 9 shows domestic investment in WAEMU countries. Some countries, such as Benin, Burkina Faso, Senegal, and Mali, have a relatively stable domestic investment. In such countries, the changes in domestic investment are not visible. Cote d'Ivoire, Niger, and Togo all decreased initially and then have an overall increasing trend. Domestic investments in Niger and Togo in 2015 are both higher than their domestic investments in 1980. However, domestic investments in Cote d'Ivoire in 2015 are lower than before. Overall, from 1980 to 2015, the levels of domestic investment in Niger and Togo increased, while these in Cote d'Ivoire decreased.



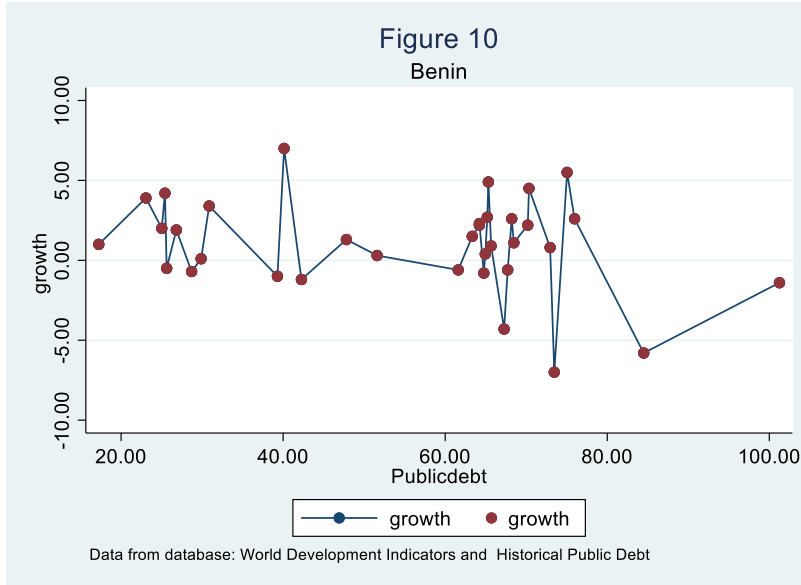
Public debt can affect economic growth negatively, but basically, there does not seem to be any causal links between public debt and growth. Public debt is still an essential way for the government to collect funds. At low levels of public debt, the government can have enough money to operate and invest, which is better for the economy. However, if the public debt to GDP is over a certain level, there seems to be a negative influence on society. This level of public debt can be called a threshold. Every country must consider the threshold effects of public debt.

## 5. Public debt thresholds

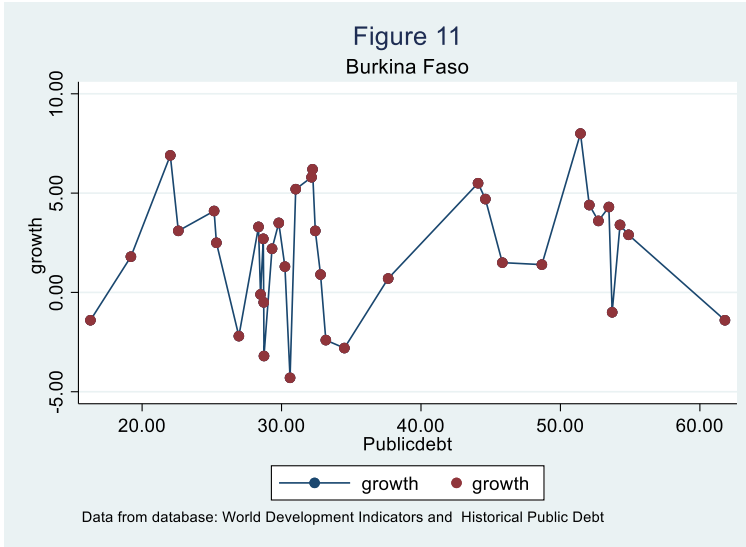
Based on the previous results, overall public debt seems to affect growth negatively. However, the influence of the changes in the level of public debt on growth is not apparent. When the public debt to GDP stays at a low level, it seems that there is a positive relationship between public debt and growth. More and more researchers point out that the high level of public debt will hurt growth due to the negative results. Thus, it is essential to check whether there exist any points where public debt begins to affect growth negatively in WAEMU countries.

Also, as mentioned in some studies, heterogeneity exists among countries; they are not all alike. Different countries seem to have different debt threshold levels due to the different conditions, such as economy, culture, and politics. We would like to analyze the debt threshold of WAEMU countries separately.

Figure 10 shows each level of public debt corresponding to the growth in Benin. The lowest level of the public debt is below 20%, and its corresponded growth rate is higher than 0. After that, the growth rate fluctuated around 0, until public debt reached 40% of GDP. Before public debt reaching 40%, the growth rate decreased from around 4% to negative when public debt increased from around 30% to 40%. Around 40% of GDP, the growth rate suddenly increased a lot to 5% and decreased to negative immediately. Later, the growth rate fluctuated around 0 again until public debt reached around 80% of GDP. The public debt at around 80% starts to hurt the growth continuously. The lowest growth rate will reach around -5% when public debt is between 80% and 100% of GDP. Overall, the debt threshold seems to be around 80% of GDP, leading to a continuous decrease in economic activity.



Based on Figure 11, public debt to GDP in Burkina Faso is relatively low compared to other WAEMU countries since the maximum level of public debt to GDP is 65%. The public debt starts at around 0, and its corresponding growth rate is lower than 0. With the increase in public debt until reaching around 25% of GDP, the growth rate begins to decrease from its peak point, around 7%. The growth rate trend is decreasing, but the growth rate fluctuated at around 0 when public debt is between 25% and 35%. When public debt is between 35% and 53% of GDP, the growth rate trend is increasing from negative to around 8%. After that, the growth rate decreases continuously. There seem to be several thresholds in Burkina Faso, 25%, 35%, and around 53%.



As for Cote d'Ivoire shown in Figure 12, its level of public debt is relatively high, between around 45% and 150%. When public debt reached around 47% of GDP, the growth rate reached its peak level, around 8.5%, and then the trend of growth is declining continuously, although there are some fluctuations around 0. Thus, there is a debt threshold of around 45% of GDP.

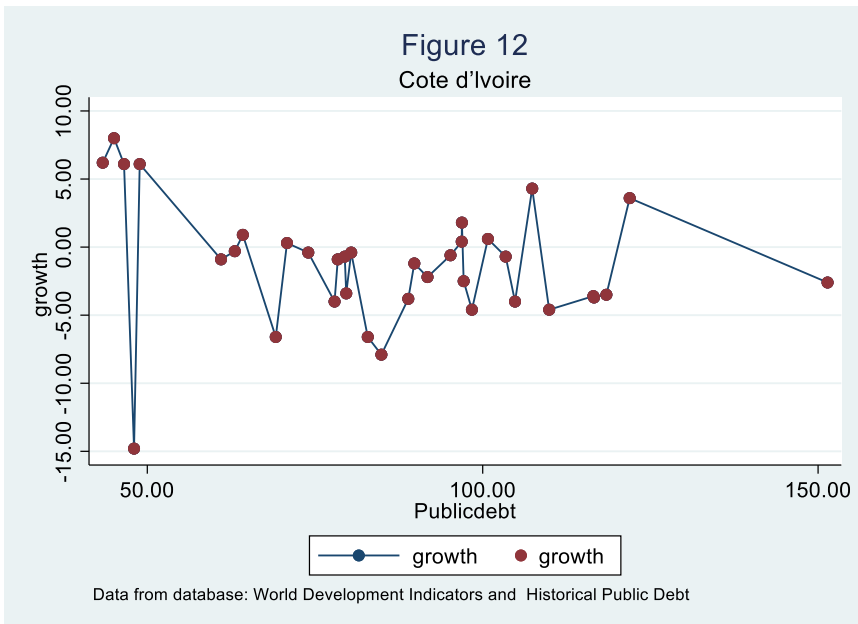
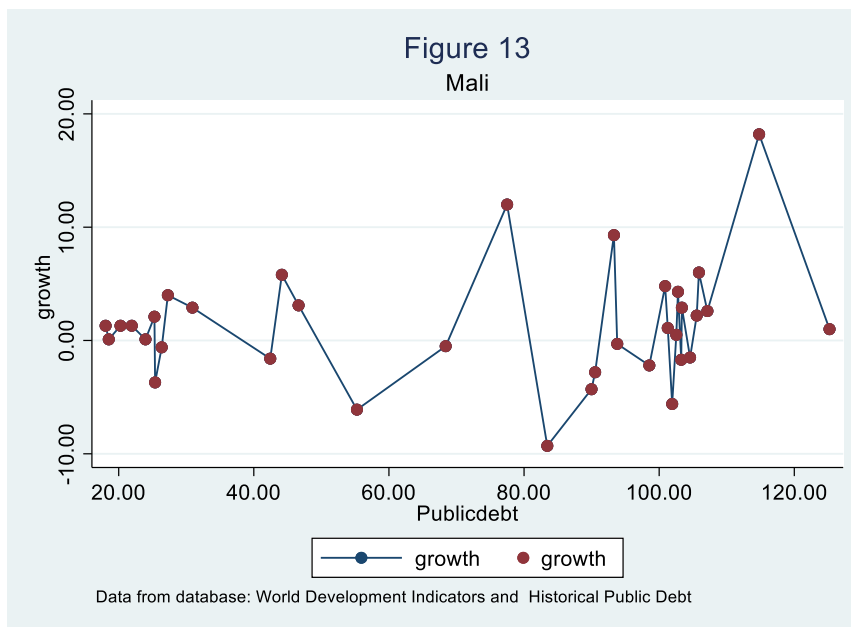
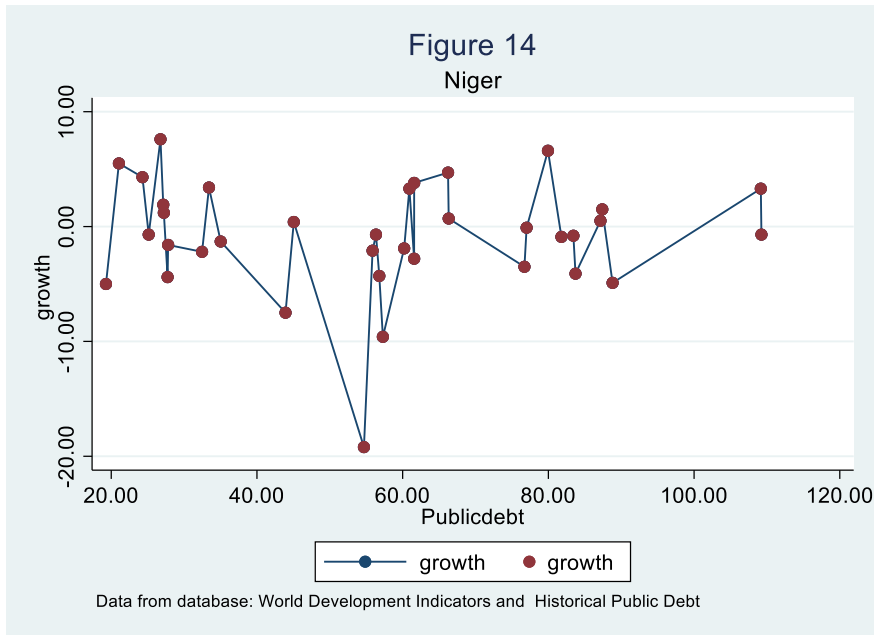


Figure 13 shows that several points in Mali cause significant changes in the growth rate. When the public debt is at a low level, the growth rate fluctuated around 0. Until the level of public debt reaches around 45%, the growth rate becomes negative. When public debt is between around 55% and 75% of GDP, the growth rate increased and reached a high level, over 10%. The growth rate decreases sharply to around -10% when the public debt reaches around 85% of GDP. When public debt is over 85%, there is an overall increasing growth rate trend, but the growth rate fluctuates around 0. There are no threshold points since there are no clear increasing or decreasing trends in growth rate between 0 and around 130% of public debt to GDP.



However, based on Figure 14, the growth rate in Niger is below or around 0 during the period of study. When the level of public debt is below 55% of GDP, the growth rate trend decreases to the lowest level, around -20%. After that, the growth rate continuously increases until around 80% of public debt to GDP. And then, the growth rate faces a continuously decreasing trend. Overall, there seem to be two thresholds, 55%, and 80%.



Based on Figure 15, overall, some points change the trends of growth rate in Senegal. Above 25% of GDP, the growth rate begins to decrease to around -6% until the public debt reaches 40%. And then, when the level of public debt is between 40% and 60%, there is an increasing trend in the growth rate. When the level of public debt is between 60% and around 85%, the growth rate fluctuates around 0. When public debt is above 85% of GDP, the growth rate starts to decrease. Thus, the thresholds in Senegal are 25%, 40%, and 85%.

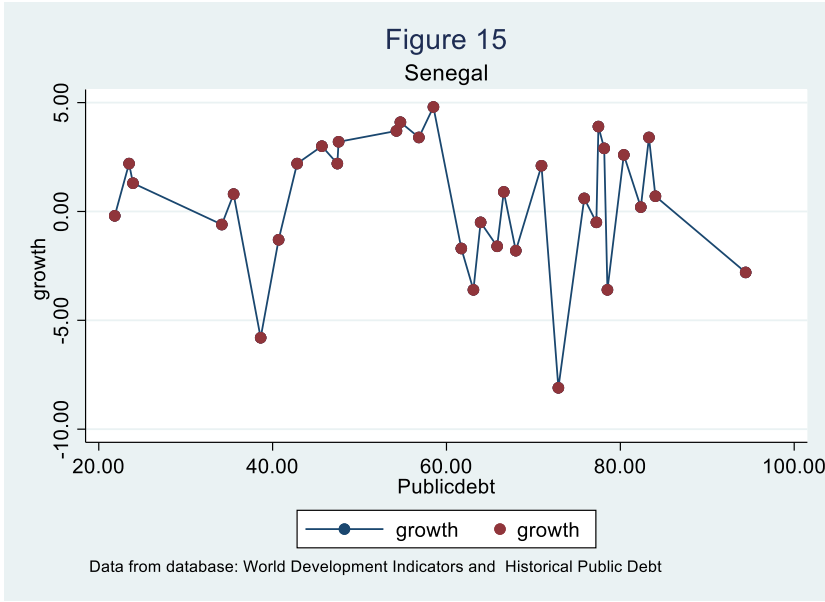
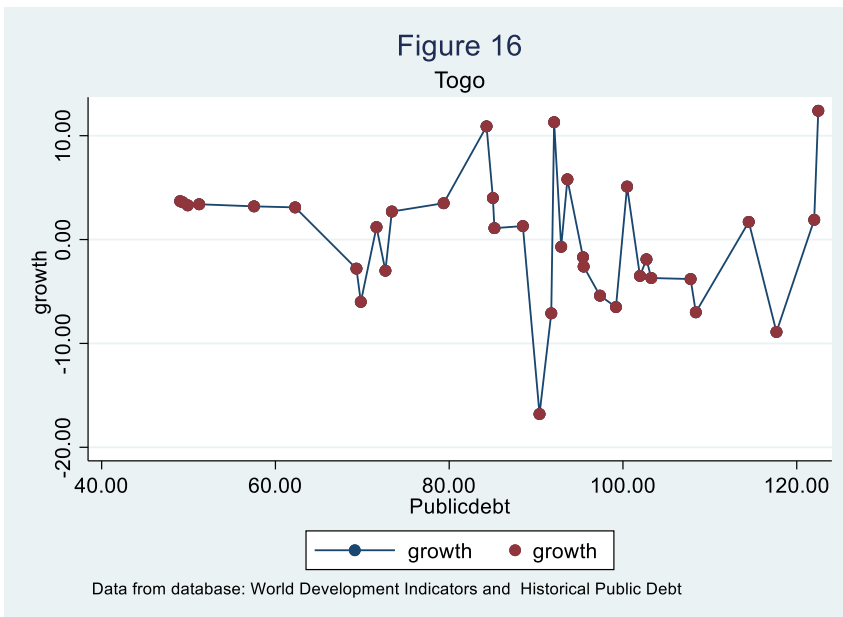


Figure 16 shows the relation between public debt and growth in Togo. When the level of public debt is beyond 85% of GDP, the growth rate begins to decrease with some fluctuations until the public debt reaches around 120% of GDP, which is exceptionally high. Moreover, when public debt is above 120%, there exists an increasing trend of growth. Thus, the debt thresholds in Togo seem to be 85% and 120%.



Thus, based on the two-way plots of growth and public debt, almost every WAEMU country has several thresholds. Also, in all WAEMU countries except for Togo, one of the thresholds is between around 40% and 55% of GDP. In Togo, when public debt is below around 60% of GDP, the economic growth is relatively stable. This analysis focused on the direct links between public debt and growth. However, because of the absence of other economic factors that may influence the growth rate at the same time, these debt thresholds may be inaccurate. As we can see in the plots, in each country, there always exists a situation such that a slight change in public debt could lead to a significant change in the growth rate. This situation happened because the changes in other economic factors may reflect the change in growth rate at the same time.

However, there is another model introduced in the literature, the Panel Smooth Transition model that is useful to test the threshold effects and linearity. Constant (2016) and Ndoricimpa (2020) used PSTR model in their studies. Moreover, Hurlin (2018) mentioned this model in his lecture notes.

We present the model in the following lines without estimating its parameters:

$$y_{it} = \alpha_i + \beta'_0 x_{it} + \beta'_1 x_{it} g(q_{it}, \gamma, c) + \varepsilon_{it}, \text{ eq. (11)}$$

where  $y_{it}$  is GDP per capita growth in country  $i$  at year  $t$ ,  $\alpha_i$  is the country fixed effect,  $x_{it}$  includes all independent variables, such as initial GDP per capita, public debt, openness, total population, population growth, inflation, government expenditure, and domestic investment.  $g(q_{it}, \gamma, c)$  is a logistic transition model, where  $q_{it}$  is the threshold variable public debt,  $c$  is a location parameter, and  $\gamma$  is a slope parameter.

The logistic transition function is shown as follows:

$$g(q_{it}, \gamma, c) = \left(1 + e^{-\gamma(q_{it}-c)}\right)^{-1}, \gamma > 0. \text{ eq. (12)}$$

There are two conditions for this logistic transition function. First, if  $\gamma$  is close to 0, the logistic function will be as follows:  $\lim_{\gamma \rightarrow 0} g(q_{it}, \gamma, c) = \frac{1}{2}$ . The panel smooth transition model

will be  $y_{it} = \alpha_i + \beta'_0 x_{it} + \beta'_1 x_{it} \frac{1}{2} + \varepsilon_{it} = \alpha_{it} + \left(\beta'_0 + \frac{\beta'_1}{2}\right) x_{it} + \varepsilon_{it}$ , eq. (13) which is a

linear regression model. Secondly, if  $\gamma$  is close to infinity, the logistic function will be as follows:  $\lim_{\gamma \rightarrow \infty} g(q_{it}, \gamma, c) = I_{(q_{it} > c)}$ . Thus, the panel smooth transition model will be  $y_{it} = \alpha_i + \beta'_0 x_{it} + \beta'_1 x_{it} I_{(q_{it} > c)} + \varepsilon_{it}$ , eq. (14) which is a panel transition model.

One of the essential applications of the PSTR model is to test the linearity of the model. Based on eq. (11), because the threshold variable must also be an element in the vector  $x_{it}$ , this model usually should be nonlinear. However, there are two conditions, such as  $\beta_0 = \beta_1$  or  $\gamma = 0$ , which can test the linearity of the model. For example, when  $\gamma = 0$ ,  $\beta'_1 x_{it} g(q_{it}, \gamma, c)$  is close to  $\frac{\beta'_1}{2} x_{it}$ , and the PSTR model is transformed to a linear model eq.(13). Whether the model is linear or not, we can use a hypothesis test to check. Let  $H_0: \gamma = 0$  or  $H_0: \beta_0 = \beta_1$ . If the null hypothesis is rejected, the model is nonlinear.

There is an assumption in this Panel Smooth Transition model that only two public debt groups include: high and low. However, suppose there are more than two groups,  $r + 1$ , the panel smooth transition model will be as follows:

$y_{it} = \alpha_i + \beta'_0 x_{it} + \sum_{j=1}^r \beta'_j x_{it} g_j(q_{it}, \gamma_j, c_j) + \varepsilon_{it}$ , eq. (16) where there are  $r$  transition functions  $g_j$ . How many regimes there are can also use hypothesis tests to solve. First, let  $H_0: r = r^*$  vs  $H_1: r = r^* + 1$ . Usually,  $r^*$  begins at 0. If there is no rejection of the null hypothesis,  $r^* + 1$  regimes are in this model. Otherwise, let  $H_0: r = r^* + 1$  vs  $H_1: r = r^* + 2$ . The hypothesis test will keep going until the first acceptance of  $H_0$ .

## 6. Conclusions

Public debt plays an important role in economic activities. Researchers are interested in how public debt affect economic growth. Panizza and Presbitero (2014) found that there is no evidence to show a causal relationship between public debt and growth. Woo and Kumar (2014) suggested a high level of public debt seems to affect growth negatively. Pescatori, Sandri, and Simon (2014) pointed out there is a relatively weak relationship between high levels of public debt and GDP growth over a long period, and debt's trajectory plays an essential role in affecting the relationship between two variables. Regarding to debt

threshold, Constant (2016) and Ndoricimpa (2020) found that debt thresholds vary across countries and 90% threshold is not always applied to every samples.

In this paper we analyzed the relationship between public debt and growth in WAEMU countries. As public debt is not the only factor that can affect the economy, we added to our analysis some critical control variables, such as the initial GDP per capita growth, openness, and government size, which might affect the economic growth rate. We used different specifications of the panel regression model to assess the relationship between the two variables.

The results of the dummy variables model that provided the highest R-square was our most preferred model. Its results suggest that there is a negative and non-significant relationship between public debt and GDP per capita growth in the WAEMU countries. A one percent increase in the ratio of public debt to GDP will cause a 0.014% reduction in GDP per capita growth. It follows that in the WAEMU countries the increase in public debt did not seem to hurt economic growth during the period of study. As for debt thresholds, WAEMU countries have different thresholds respectively, but there is a common range of the debt thresholds: around 40%-60%.

Moreover, the government has to implement some policies to control the level of public debt. Although the slowdown in economic growth does not seem to be caused by the increase in public debt, public debt still has a negative effect on growth with statistical insignificance. Also, some studies found that high level of public debt can negatively affect growth and the low-level leads to a growth-enhancing or growth-neutral. Overall, public debt should remain in a relatively low level. Combined with the range of thresholds, it is better for WAEMU countries to reach and remain a 40%-60% of public debt to GDP.

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