

**OPTIMUM MOUNTING OF ELECTRONIC
CIRCUIT BOARDS
FOR
COMPONENTS AND CIRCUITS
SURVIVABILITY**

**THESIS SUBMITTED
TO THE SCHOOL OF GRADUATE STUDIES
AS PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF APPLIED SCIENCE**

by

Ali Ghazizadeh

Department of Mechanical Engineering

UNIVERSITY OF OTTAWA

OTTAWA, ONTARIO, DECEMBER, 1993



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ISBN 0-612-15623-0

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ABSTRACT

Electronic circuit boards are being continually employed in new and increasingly demanding environments (e.g. satellite, aerospace, marine, automobile, etc.) where the board and mounted components are exposed to vibration of varying amplitude and frequencies. Optimum mounting and component placement with the aim of reducing the effect of vibration on the equipment has gained considerable attention in recent years. The goal of these efforts is to prolonging the service life of the system and reducing costly down time. Electronic boards are often mounted on four rigid support legs. Their vibration is a function of the location of the support legs, and board physical specifications such as length, width, weight, and placement of the components mounted on them.

In this study, an analytical method of plate vibration analysis is employed to find the board's free vibration. With the help of nonlinear optimization methods, optimum mounting of circuit boards are investigated. It will be shown that square board has a better performance than any other

board shapes. Furthermore for any board there is a particular set of support points that can reduce the board vibrations considerably. Results of the studies will be presented for a board having some relatively heavy components on it, as well as when the weight of the components are negligible. Finally a table of suitable support points will be introduced outlining optimum support points for eight rectangular shapes. For each of these shapes a graph of unsuitable regions is provided to help the designer to avoid placing delicate components over those regions of the plate. Although the optimization computer runs are lengthy it is well worth it, since it needs to be carried out only once for a new circuit board. Further more, a fair amount of insight can be obtained from the tabulated results, thus eliminating the need for optimization in non critical circuit boards. The tabulated results would also help in determining a good starting point for optimization, thus reducing the search time.

ACKNOWLEDGMENTS

First and foremost I would like to express my deep appreciation to Professor Atef Fahim for his guidance and supervision which has made this thesis possible. I also appreciate the guidance and helpful advice of Professor D.J. Gorman during the whole course of this work.

A sincere thanks to the staff of the Department of Mechanical Engineering, in particular Mr. Don Seaman and Ms. Solange Amyot, for their help that was provided when needed.

I am indebted to my many colleagues and friends at the University of Ottawa, especially Dr. Y. Alizadeh, Dr. P. Norazar, Mr. M. Kabkanian, Mr. S. Farahat and Ms. S. Fulford for their various contribution to this work.

Special thanks to my family, particularly my wife N. Roostaie, for their tolerance, encouragement, and moral support.

Last but not least, I would like to express my sincerest gratitude to my mother, Ms. T. Kheradmand for her love and encouragement throughout the whole time of my studies.

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NOMENCLATURE

SYMBOL	EXPRESSION
<i>a, b</i>	plate lateral dimensions
<i>C</i>	mass vector
<i>c</i>	dimensionless mass vector
<i>D</i>	$=Eh^3 / [12(1-\nu^2)]$, plate flexural rigidity
<i>d</i>	maximum displacement of the plate
<i>E</i>	Young's modulus of the plate material
<i>f</i>	plate vibration frequency (Hertz)
<i>h</i>	plate thickness
<i>L</i>	support point vector
<i>l</i>	dimensionless support point vector
<i>m</i>	mass of the component
<i>r</i>	radius of curvature
<i>t</i>	time (second)
<i>P</i>	plate vibration energy

SYMBOL	EXPRESSION
$W(\zeta, \eta, t)$	plate lateral displacement
$W(\zeta, \eta)$	amplitude of the plate lateral displacement
$\hat{W}(\zeta, \eta)$	normalized $W(\zeta, \eta)$ with respect to $W(\zeta, \eta)_{max}$
X	design vector
x, y	plate coordinates
α	reflection coefficient
β	contraction coefficient
γ	expansion coefficient
ζ	$=x/a$, dimensionless plate special coordinates
η	$=y/b$, dimensionless plate special coordinates
κ	position characteristic of a component
κ_c	cutoff position characteristic of a board
κ_{max}	maximum position characteristic of a board
λ^2	$=\omega^2 a(\rho / D)^{1/2}$, plate eigenvalue
λ'^2	$=\omega^2 b(\rho / D)^{1/2}$, alternative formulation of the eigenvalue
ν	poisson's ratio for the plate material
ρ	mass of the plate per unit area
ω	circular frequency of the plate
φ	$= b/a$, plate aspect ratio
φ'	$= a/b$, inverse of plate aspect ratio

CHAPTER 1

INTRODUCTION

This research work deals with the optimum mounting of electronic circuit boards for components and circuit survivability. Current advances in consumer electronics resulted in electronic circuits being present in an ever increasing number of goods and equipment operating in demanding environments. The complexity and density of the circuits have also increased considerably and resulted in them being susceptible to damage from board vibration. Electronic circuit boards are usually made of nonconductive materials in rectangular shapes and are supported on four rigid supports. These boards are generally subject to disturbances that cause them to vibrate. Even in cases of the extreme favourable condition where the boards are used in stationary applications, forced vibration due to cooling fans may arise. The vibration results in cyclic surface strain and fatigue to the printed circuits on the surface of these boards.

This research will attempt to address a number of points relating to the

vibration of electronic circuit boards. These points may be outlined as follow:

1. How to reduce circuit board (plate) vibration?
2. What are the physical characteristics of the circuit board that affect its vibration and how?
3. What method of plate vibration analysis is suitable for solving this problem?
4. What method of optimization is best suited for the optimization task?
5. How accurate and efficient is the problem solving method?

This research requires thorough familiarity with the both fields of optimization and plate vibration analysis.

The application of this research is to prolong the service life of electronic circuit boards and the delicate components mounted on them. Designing the circuit board to only withstand the static stresses proves to be inadequate, and the large cyclic displacements and stresses caused by load vibration must be considered. While it may be impossible to eliminate the vibration of electronic circuit boards, this work shows that by judicious choices of the support point locations the vibration over large regions of the board may be reduced to harmless values. The results of this work, when encapsulated with circuit design software, would allow circuit board designers to include vibration induced fatigue analysis of the printed circuit in the design process. While the

concepts developed in the course of this work is applicable to any board with different boundary conditions, it will be applied here to boards with free edges mounted using four interior rigid point supports.

Previous works which have been done on the subject matter of this work are reviewed in Chapter 2. They are mostly on plate vibration analysis, but the main purpose of most of them is to reduce the side effects of vibration of the plate. Kunz is perhaps the first one that has mentioned the importance of this subject from an optimization point of view [1]. In the latest work that has been done with his contribution [2], the frequency of the vibration of the board was maximized in order to optimize its vibration characteristic. A sequential search method was used to predict the objective function.

The plate vibration analysis that was used in this research is an analytical method that has been developed by Gorman [3]. The solution basically consists of finding the eigenvalue, and calculating the plate lateral displacement and its derivatives. Once the eigenvalue matrix has been established, the eigenvalue of the matrix is calculated through a trial and error procedure then the plate lateral displacement and its derivatives are calculated. Both the eigenvalue matrix and the displacement of the plate are functions of dimensionless coordinates of the support points, aspect ratio of the plate, plate rigidity, and dimensionless coordinates, and mass ratio of the attached masses. Chapter 3 is devoted to the theory behind this method of plate vibration analysis.

The optimization problem is a nonlinear unconstrained one, where the derivatives of the objective function are not readily available. Thus the more efficient gradient search techniques can not be used, and instead pattern search methods have to be employed. These are; Powell's method, Hooke and Jeeves, and the Simplex method. Test results show that the Simplex method is the most efficient in solving this type of optimization problem. Three main factors are considered to be design variables, these are: dimensionless coordinates of the point supports, aspect ratio of the board, and weight and location of an attached mass called dead weight. Dead weight is a small piece of the metal attached to the plate in a position such that it can reduce the plate vibration level. Chapter 4 contains a detail description of optimization procedure.

The research is primarily theoretical (plate vibration software was verified experimentally [4]). The calculation of the plate vibration analysis is coded in Fortran language. The optimization process is coded in C language. The C optimization program calls the Fortran software as part of the objective function. The Waterloo C compiler is employed to compile and link the two programs. The optimization procedure is carried out for two types of electronic circuit boards which are boards having light mounted components, and those having heavy ones. A table of optimum rigid point supports for plates with different aspect ratios is developed as well. The table is applicable to any set of plates including circuit boards with constant area and Poisson' ratio of 0.5

regardless of the size of the plates. This will be helpful when the designer can select a plate with a particular aspect ratio from the range of tabulated values or when it is not easy to employ the optimization procedure. Dead weights are applied as design variables in order to minimize the vibration level of two sample boards which are supported by a particular set of rigid point supports. These are presented in Chapter 5.

The plate vibration optimization developed in the current research is a novel approach. Experimental works have been carried out on the accuracy of the calculation of the plate eigenvalue (frequency of vibration) using the same method that is applied in this research [2,5]. Experimental work to verify the results in this thesis constitute phase 2 of the research. These issues will be discussed in Chapter 3.

CHAPTER 2

LITERATURE REVIEW

Thin plate vibration analysis is a well established field of study in which a large number of research articles have been published. Different approaches have been proposed during the last two decades to analyze the plate free vibration. Among them are; energy method, analytical method, finite differences and finite elements methods, the Lagrangian multiplier method, and Green's functions method. In addition, a large data base of experimental results has been collected. Although these efforts primarily focus on the analysis of plate vibration and determination of the vibration frequencies, the underlying aim is to reduce the unwanted vibrations.

Unfortunately there has been no substantial effort to reduce plate vibration by utilizing optimization techniques before 1980. For electronic circuit boards, Kunz [6] in 1989 discussed the importance of minimizing the level of plate vibration in order to prolong the service life of electronic circuit boards. The goal was to find the optimum support location of the circuit board

by maximizing the board natural frequency. Using a typical circuit board, he found the natural frequencies of various point support locations (screw attachments) by means of a sequential search technique. The experimental data were plotted and curve fitted. Finally the point support coordinates which results in a board having the highest natural frequency were considered as optimal support location. There is a more recent publication by Pitarresi and Kunz [5] that presents a simple techniques for the rapid estimation of the optimal point support locations of the vibrating plate. A relatively detailed description of their work will be presented in this review.

Other than Kunz's work which is very close to the present study, and directly aims at minimizing of thin plate vibration, there are a large number of publications dealing with optimization methods utilized for thick plate vibration. In this category two publications are noted here. These are from Inoue and his colleagues from Tohoku University [7,8]. They used a finite element shell analysis, a modal analysis, and a structural optimization method in order to minimize the vibration energy of thin plate structures. The design variable was the thickness of the plate and the objective function was plate energy.

An important issue is to determine the effect of the board deflection on the mounted components. Wong and his colleagues [9] simulated the surface mounted electronic assemblies under general mechanical loading condition using a hybrid analytical/experimental analysis approach. Their method

combines analytical techniques with experimentally determined load-deformation characteristic of the surface-mounted assemblies to predict their loading and deformation. The circuit (IC) package and the PC board are modeled as two separate structures connected by a series of nonlinear springs along the edges of the package. When the board is deflected subject to a particular static load, the displacement and rotation of the package is determined by using an optimization procedure. To do that, the resulting bending moments and forces in the package are calculated with respect to the displacement and rotation of the package. These terms are then squared and balanced by weighting factors and added up to form the objective function. The displacement and rotation of the package are design variables. Using dynamic forces, the found displacement and rotation of the package can be utilized for the determination of the harmful bending moments and stresses due to the natural vibration of the board that affect the package leads.

In the analysis of plate vibration with interior point support Gorman has many publications [10,11,12,13]. In his most recent work in cooperation with Singal [14] they developed an analytical-type solution based on the superposition method for developing the free-vibration frequencies and mode shapes of rectangular plates anchored using arbitrarily located rigid point supports. An extensive investigation on the comparison of experimental and analytical results were conducted as well. The testing was performed using an impact hammer and a faced accelerometer. The experimental results showed

very close agreement with the analytical results. They report also a similar investigation for plates having the same type of boundary conditions but with attached masses [3]. This is the most comprehensive approach in the free vibration analysis of rectangular plate with internal point support which has been done through analytical solutions to the governing plate vibration equation.

Finite element method was used by Venkateswara [15] and Raju and Amba-Rao [16] to fully analyze the rectangular plate problem with free edge boundary conditions and internal point supports positioned along the diagonals. Their experimental results agreed well with the analytical results. Their approach however does not lend itself to optimization methods due to the following drawbacks.

- 1) For each set of support point locations other than on the diagonals, the evaluation of the plate vibration requires a change in the boundary conditions, and hence a reformulation of the problem.
- 2) Each point support must be located at the node of an element mesh.
- 3) The analysis requires an excessively long run times.

The energy method has been used by Narita [17] for plates with internal supports along the diagonals. A similar approach has been used by Laura [18] to analyze rectangular plates with attached mass and internal point support. In these cases a polynomial coordinate function is coupled with the Rayleigh-

Schmidt approach. This technique is based on making the maximum strain energy of the vibrating system equal to maximum kinetic energy.

As mentioned earlier, one of the most related works to the topic of this research has been done by Pitarresi and Kunz [5]. Since it is the more recent publication on this subject, it is beneficial to expand on its review here. The objective function in this research is the natural frequency of the plate vibration. The support points are considered to be symmetrical with respect to the axes of coordinates, hence the design variables are the coordinates (X,Y) of one of the support point locations. They use a quadratic polynomial to represent the first natural frequency corresponding to the coordinates of any set of support points. A least square error analysis is carried out to obtain the constants of the polynomial. The optimum support point location is obtained by setting the gradient of the polynomial to zero. The optimum support point location for plates having different aspect ratios from 1 to 5 are then determined. The data of these cases are taken from Gorman's analytical method. The result shows that the optimum solution for square plates should lie along the diagonals at a location where

$$X_{opt}/a = Y_{opt}/b = \pm 0.466$$

The analysis concludes that for rectangular plates, the optimum location of the point supports tends to lie closer to the axis of the longer plate dimension as the aspect ratio increases. They have also verified the accuracy of calculation by using data taken from finite element method and experimental data as well.

CHAPTER 3

PLATE FREE VIBRATION

The free vibration analysis of plates [10] used in this study is not part of this research work, rather it is used here due to its many beneficial features. It is applicable to free edge rectangular plates having arbitrary rigid point supports, which is exactly the boundary condition of most of the electronic circuit boards. It is also possible to take into account the mass of components mounted at any arbitrary location using the above mentioned method. Furthermore, it has the advantages of suitability for inclusion in the objective function of optimization. For convenience of referencing and continuity of reasoning, a detailed description of this method of analysis will be presented in this chapter.

Generally in this method [1] the plate is built up with a number of building blocks. These blocks are simple cases which can be easily analyzed. To build up the plate, these building blocks will be superimposed on each other and the result must satisfy the boundary conditions of the main plate. Each

building block is a simple plate for which a fourth order governing differential equation is solved with a series form solution that is attribute to Levy, and further developed by Gorman. The fourth order differential equation describes the behaviour of the plate (building block) when it is excited. This solution will result in the plate lateral displacement function containing a number of constant driving coefficients which must be eliminated by considering the plate boundary conditions. To do that a relatively large number of linear homogenous equations must be solved simultaneously. The dimensionless frequency of vibration (eigenvalue) is determined numerically by setting the determinant of the set of the linear homogeneous equations to zero. The constant coefficients are then determined and substituted back in the plate lateral displacement equation. Finally the first and second derivatives of the plate displacement along the length and width of the plate are calculated.

3.1 General Method of Plate Vibration Analysis

The rectangular plates dealt with here are assumed to be thin with small deflection. The following is assumed [19]:

1. There is no extension in the mid-plane of the plate. This plane will remain neutral during the bending of the plate.
2. Those points of the plate lying initially on a normal to the mid plane of the plate remain on the normal to the mid plane after bending.

3. The normal stresses in the direction transverse to the plate can be ignored.

The general method of the plate vibration analysis utilized in this research consists of the following three intermediate calculations:

- a) Modification of a fourth order differential static equation governing the pure bending of the plate subject to the lateral forces to handle internal forces.
- b) Application of the solution proposed by Levy to the differential equation.
- c) Enforcement of the boundary conditions.

These steps will be reviewed here briefly.

3.1.1 Governing Differential Equation

The differential equation governing the pure bending of a plate subject to lateral static loading is given by Timoshenko [15] as follows:

$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2 \frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{q(x,y)}{D} \quad (3.1)$$

where q is the applied static load.

For dynamic loading Gorman [1] replaces q by the inertia forces as follows:

$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2 \frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{\omega^2 \rho}{D} W(x,y) \quad (3.2)$$

To simplify the analysis, dimensionless space variables ζ and η are utilized where $\zeta=x/a$, $\eta= y/b$. a and b are the plate dimensions as shown in Figure 3.1. In dimensionless form the governing differential equation is given by [1]:

$$\frac{\partial^4 W(\zeta,\eta)}{\partial \eta^4} + 2\varphi^2 \frac{\partial^4 W(\zeta,\eta)}{\partial \eta^2 \partial \zeta^2} + \varphi^4 \frac{\partial^4 W(\zeta,\eta)}{\partial \zeta^4} - \varphi^4 \lambda^4 W(\zeta,\eta) = 0 \quad (3.3)$$

where $\varphi= b/a$ is the plate aspect ratio and

$$\lambda^2 = \omega a^2 \sqrt{\frac{\rho}{D}} \quad (3.4)$$

is the eigenvalue (dimensionless frequency) of the plate.

For a particular set of boundary conditions, the solution process consists of determining the eigenvalue λ^2 , from which a specific plate natural frequency ω (rad/s) can be found.

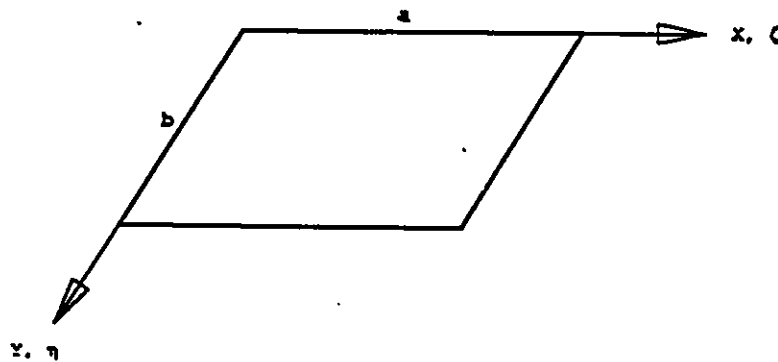


Figure 3.1 Rectangular plate with typical coordinates axes.

3.1.2 Boundary Condition and Edge Reaction

A detailed description of the theoretical development of the boundary conditions is given by Timoshenko [15] and in terms of dimensionless coordinates in Reference [1]. In this work the plate is considered to have free edges. However for development of the solution, an understanding of the Levy type solution and hence of simply supported edge plates is necessary. Description of these two types of boundary condition follows:

Free Edge Boundary Condition

For a free edge plate no bending moments along the edges exist hence, for edges $\zeta=1$, and $\eta =1$ respectively:

$$\begin{aligned} \left(\frac{\partial^2 W(\zeta, \eta)}{\partial \zeta^2} + \frac{\nu}{\phi^2} \frac{\partial^2 W(\zeta, \eta)}{\partial \eta^2} \right)_{\zeta=1} &= 0 \\ \left(\frac{\partial^2 W(\zeta, \eta)}{\partial \eta^2} + \nu \phi^2 \frac{\partial^2 W(\zeta, \eta)}{\partial \zeta^2} \right)_{\eta=1} &= 0 \end{aligned} \quad (3.5)$$

also for no vertical edge reaction, at edges $\zeta=1$, and $\eta =1$ respectively :

$$\begin{aligned} \left(\frac{\partial^3 W(\zeta, \eta)}{\partial \zeta^3} + \frac{\nu^*}{\phi^2} \frac{\partial^3 W(\zeta, \eta)}{\partial \zeta \partial \eta^2} \right)_{\zeta=1} &= 0 \\ \left(\frac{\partial^3 W(\zeta, \eta)}{\partial \eta^3} + \nu^* \phi^2 \frac{\partial^3 W(\zeta, \eta)}{\partial \eta \partial \zeta^2} \right)_{\eta=1} &= 0 \end{aligned} \quad (3.6)$$

where $\nu^* = 2-\nu$.

Simply Supported Boundary Condition

Displacement and bending moment along the simply supported edges are zero, hence; for edge $\zeta=1$:

$$(W(\zeta,\eta))_{\zeta=1} = \left(\frac{\partial^2 W(\zeta,\eta)}{\partial \zeta^2} \right)_{\zeta=1} = 0$$

and for edge $\eta=1$:

$$(W(\zeta,\eta))_{\eta=1} = \left(\frac{\partial^2 W(\zeta,\eta)}{\partial \eta^2} \right)_{\eta=1} = 0$$

Distributed Bending Moment and Edge Reaction

Reference [1] gives the mathematical formulation of the distributed bending moments along the edges of rectangular plates in dimensionless coordinates as:

$$\begin{aligned} \frac{M_{\zeta} a}{D} &= - \left[\frac{\partial^2 W(\zeta,\eta)}{\partial \zeta^2} + \frac{\nu}{\phi^2} \frac{\partial^2 W(\zeta,\eta)}{\partial \eta^2} \right] \\ \frac{M_{\eta} b}{D} &= - \left[\frac{\partial^2 W(\zeta,\eta)}{\partial \eta^2} + \nu \phi^2 \frac{\partial^2 W(\zeta,\eta)}{\partial \zeta^2} \right] \end{aligned} \quad (3.9)$$

and gives the distributed vertical edge reactions along the two coordinates as:

$$\begin{aligned} \frac{V_{\zeta} a^2}{D} &= - \left[\frac{\partial^3 W(\zeta,\eta)}{\partial \zeta^3} + \frac{\nu}{\phi^2} \frac{\partial^3 W(\zeta,\eta)}{\partial \zeta \partial \eta^2} \right] \\ \frac{V_{\eta} \phi b^2}{D} &= - \left[\frac{\partial^3 W(\zeta,\eta)}{\partial \eta^3} + \nu \phi^2 \frac{\partial^3 W(\zeta,\eta)}{\partial \eta \partial \zeta^2} \right] \end{aligned} \quad (3.10)$$

3.1.3 Levy Type Solution

In the analysis of free vibration of rectangular plates with two opposite edges simply supported (edges $\xi=0$, and 1), the solution to Equation 3.3 can be cast in a series form attributed to M. Levy and further developed by Gorman [1] as follows:

$$W(\zeta, \eta) = \lim_{k \rightarrow \infty} \sum_{m=1}^k Y_m(\eta) \sin(m\pi\zeta) \quad (3.11)$$

As required each term in Equation 3.11 fully satisfies the boundary conditions at $\zeta=0$, and $\zeta=1$ (zero displacement, and zero bending moment). Substituting Equation 3.11 into the governing Equation 3.3 yields:

$$\frac{d^4 Y_m(\eta)}{d\eta^4} - 2\phi^2(m\pi)^2 \frac{d^2 Y_m(\eta)}{d\eta^2} + \phi^4 [(m\pi)^4 - \lambda^4] Y_m(\eta) = 0 \quad (3.12)$$

For each value of m , Equation 3.12 is an ordinary fourth-order homogeneous differential equation with constant coefficients. Its solution is given as follows: for $\lambda^2 > (m\pi)^2$

$$Y_m(\eta) = A_m \cosh \beta_m \eta + B_m \sinh \beta_m \eta + C_m \sin \gamma_m \eta + D_m \cos \gamma_m \eta \quad (3.13)$$

and, for $\lambda^2 < (m\pi)^2$

$$Y_m(\eta) = A_m \cosh \beta_m \eta + B_m \sinh \beta_m \eta + C_m \sinh \gamma_m \eta + D_m \cosh \gamma_m \eta \quad (3.14)$$

where:

$$\beta_m = \varphi \sqrt{\lambda^2 + (m\pi)^2} \quad (3.15)$$

and

$$\gamma_m = \varphi \sqrt{\lambda^2 - (m\pi)^2} \quad \text{or} \quad \gamma_m = \varphi \sqrt{(m\pi)^2 - \lambda^2} \quad (3.16)$$

whichever is real.

Equations 3.11, 3.13, and 3.14, represent the full solution to the free vibration of rectangular plates with two opposite edges simply supported. The final step in the solution consists of determining the coefficients of A_m , B_m , C_m , and D_m subject to the appropriate constraints of the boundary conditions on the remaining two opposite edges.

3.1.4 Concentrated Force Acting on the Plate

The effect of the point support is modeled by a harmonic concentrated force acting on the plate at the point of support. A mass m_i attached to the plate at coordinates u_i and v_i exerts a harmonic force on the plate at its point of attachment. The amplitude of this force is given by:

$$P = m_i \omega^2 a W(u_i, v_i) \quad (3.17)$$

or in the form of dimensionless variables [2]:

$$P^* = -2\varphi^4 M_{r_i} \lambda^4 W(u_i, v_i) \quad (3.18)$$

where $M_{r_i} = m_i/M$ is the mass ratio of the attached mass. Each of these two

types of concentrated forces is represented by a Dirac Delta function [10]. A detailed description of the Dirac Delta function is given in Reference [20] where the final form is given as:

$$q(\zeta) = 2P \sum_{m=1}^{\infty} \sin(m\pi u) \sin(m\pi \zeta) \quad (3.19)$$

Equation 3.19 is a series representation of the concentrated force P at coordinates u , and v which is expanded along the line, $\eta=v$.

3.2 Vibration of the Electronic Circuit Board

Electronic circuit boards are usually built in a rectangular shape with four rigid point supports. A large number of electronic components are mounted on these boards with electric connection between the components in the form of etched copper leads on the board surface. These boards can be considered as thin free edge rectangular plates with interior rigid point support and attached mass. Details of the solution to this sort of plate vibration problem are given in Reference [4]. In this work the authors utilize a set of four similar building blocks for analyzing the free plate itself and an additional one for every simple support point. They consider four such simple support points located at an appropriate distance from the centre of the support to represent a rigid point support as shown in Figure 3.2. Hence, in the present case for the four rigid supports, 16 building blocks in addition to the four required for the free plate itself reconstitute the plate. For every attached

mass an additional building block exactly similar to that used to model a simple support is considered as well. As an example, a plate having five concentrated mass requires a total of 25 building blocks in its analysis.

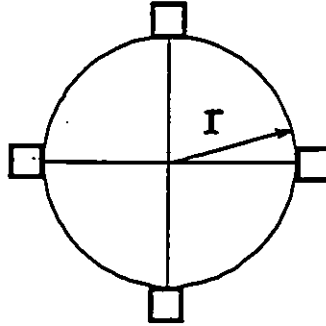


Figure 3.2 Location of four simple support points considered to represent a rigid point support of radius r .

3.3 Solution to the Building Blocks of the Plate

The building blocks required to solve for a plate with just one interior point support are shown in Figure 3.3. With reference to the figure, two small circle on an edge of a building block indicate slip shear condition. By definition, slip shear conditions at a particular edge of the plate implies a condition where the plate experiences zero vertical edge reaction and that the slope of the plate taken normal to the edge is equal to zero [1]. Each of the first four building blocks has one edge which is driven by a distributed harmonic bending moment or edge rotation of frequency ω . The fifth building block models an internal point support which is located at the coordinates $\xi=u$

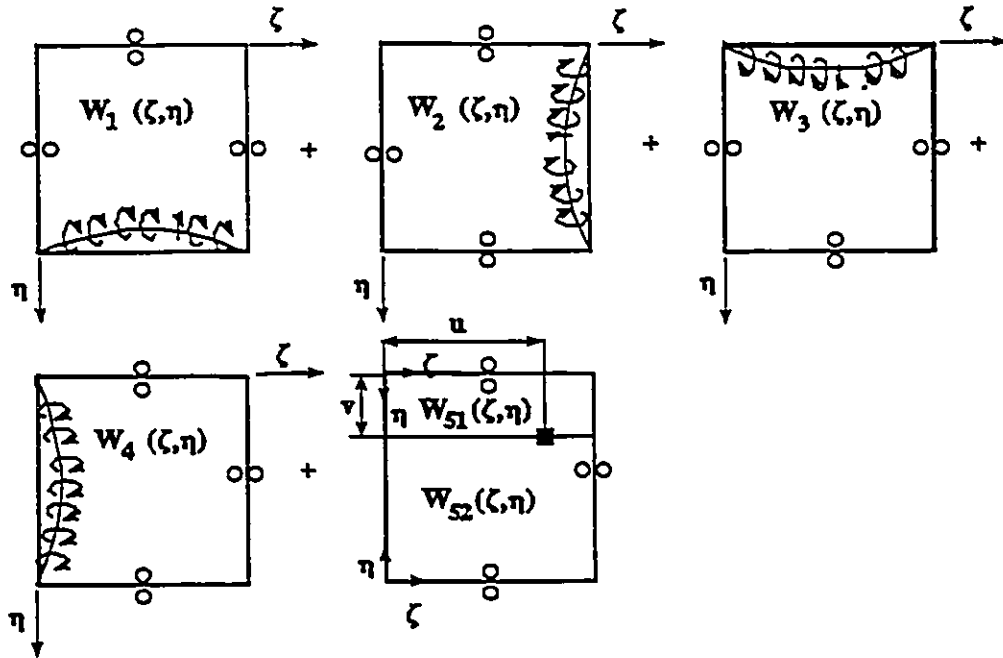


Figure 3.3 Building blocks used in analysis of a free edge plate with one simple point support .

This building block is divided into two segments with the dividing line drawn through the point support parallel to the ξ axis at $\eta=v$. The solution to the first building block is recalled here from Reference [4]. The lateral displacement function of this building block can be expressed as a Levy type solution (Equation 3.11):

$$W_1(\zeta, \eta) = \lim_{k \rightarrow \infty} \sum_{m=0}^k Y_m(\eta) \cos(m\pi\zeta) \quad (3.20)$$

This equation satisfies the boundary conditions at $\zeta=0$, and 1. Substituting for W , into equation 3.3 gives:

$$\frac{d^4 Y_m(\eta)}{d\eta^4} - 2\phi^2(m\pi)^2 \frac{d^2 Y_m(\eta)}{d\eta^2} + \phi^4 [(m\pi)^4 - \lambda^4] Y_m(\eta) = 0 \quad (3.21)$$

This is an ordinary fourth order homogeneous differential equation and can be solved for $Y_m(\eta)$ by enforcing the boundary conditions along the edges $\eta=0$ and $\eta=1$. We expand the bending moment, $M,(\zeta)$, in series form as follows:

$$\frac{b^2 M_1(\zeta)}{aD} = \sum_{m=0,1}^{\infty} E_m(\eta) \cos m\pi\zeta \quad (3.22)$$

The result, after substituting for $Y_m(\eta)$ into Equation 3.20 will be as follow:

$$W_1(\zeta, \eta) = \sum_{m=0,1}^{k^*} E_m \left(\theta_{11m} \frac{\cosh \beta_m \eta}{\sinh \beta_m} + \theta_{13m} \frac{\cos \gamma_m \eta}{\sin \gamma_m} \right) \cos m\pi\zeta + \sum_{k^*}^{\infty} E_m \left(\theta_{22m} \frac{\cosh \beta_m \eta}{\sinh \beta_m} + \theta_{23m} \frac{\cosh \gamma_m \eta}{\sinh \gamma_m} \right) \cos m\pi\zeta \quad (3.23)$$

where the first summation includes terms for which $\lambda^2 > (m\pi)^2$ only (β and γ have the same definition as before). The quantities θ_{11m} , θ_{13m} , θ_{22m} and θ_{23m} are functions of ν , ϕ , and λ .

The solutions to the other three building blocks are obtained from $W_1(\zeta, \eta)$ by appropriate rotation and/or reflection of the coordinate axes ζ , and η . For example for the case of $W_2(\zeta, \eta)$ the coordinate variables ζ , and η must be interchanged (reflection with respect to the diagonal) thus ϕ must be replaced by ϕ_1 and λ^2 by $\lambda^2 \phi^2$ as well.

The fifth building block models the point support by a dimensionless harmonic concentrated force P^* . The solution for each segment is considered separately. A Levy type solution can be developed for each segment in the cosine form. Using a second subscript to indicate the segments of the building block, this solution is expressed as follows:

$$W_{51}(\zeta, \eta) = \sum_{m=0,1}^{k^*} (A_m \cosh \beta_m \eta + B_m \cos \gamma_m \eta) \cos m\pi \zeta + \sum_{m=k^*}^{\infty} (A_m \cosh \beta_m \eta + B_m \cos \gamma_m \eta) \cos m\pi \zeta \quad (3.24)$$

and

$$W_{52}(\zeta, \eta) = \sum_{m=0,1}^{k^*} (C_m \cosh \beta_m \eta + D_m \cos \gamma_m \eta) \cos m\pi \zeta + \sum_{m=k^*}^{\infty} (C_m \cosh \beta_m \eta + D_m \cos \gamma_m \eta) \cos m\pi \zeta \quad (3.25)$$

In these two equations, A_m, B_m, C_m, D_m are four sets of constants which are determined by enforcing the boundary conditions prevailing along the boundary condition between two segments, namely:

1. The plate displacement must be continuous across the boundary between the two segments.
2. The slope of the plate taken in a direction normal to the boundary condition must also be continuous across the boundary.
3. Bending moment M_η must be continuous across the boundary between the plate segments.

4. There must be an equilibrium force balance between the applied force and the vertical edge reactions taken along the edges of the plates segment at the inter segment boundary.

The amplitude of the concentrated driving force is represented by a Dirac function using the cosine series:

$$P(\zeta) = \sum_{m=0,1} P \cdot \frac{\cos m\pi u}{\delta_m} \cos m\pi \zeta \quad (3.26)$$

where $\delta_m = 2$ for $m=0$ and 1 for $m \neq 0$ and $P^* = -2Pb^3/Da^2$. Finally by enforcing these boundary conditions, following two sets of equations are obtained.

For $\lambda^2 > (m\pi)^2$,

$$\begin{aligned} A_m \cosh \beta_m v + B_m \cos \gamma_m v - C_m \cosh \beta_m v^* - D_m \cos \gamma_m v^* &= 0 \\ A_m \beta_m \sinh \beta_m v - B_m \gamma_m \sin \gamma_m v + C_m \beta_m \sinh \beta_m v^* - D_m \gamma_m \sin \gamma_m v^* &= 0 \\ A_m \beta_m^2 \cosh \beta_m v - B_m \gamma_m^2 \sin \gamma_m v + C_m \beta_m^2 \cosh \beta_m v^* - D_m \gamma_m^2 \cos \gamma_m v^* &= 0 \quad (3.27) \\ A_m \beta_m^3 \sinh \beta_m v - B_m \gamma_m^3 \sin \gamma_m v + C_m \beta_m^3 \sinh \beta_m v^* - D_m \gamma_m^3 \sin \gamma_m v^* \\ &= (P^*/\delta_m) \cos m\pi u \end{aligned}$$

and for $\lambda^2 < (m\pi)^2$,

$$\begin{aligned} A_m \cosh \beta_m v + B_m \cosh \gamma_m v - C_m \cosh \beta_m v^* - D_m \cosh \gamma_m v^* &= 0 \\ A_m \beta_m \sinh \beta_m v - B_m \gamma_m \sinh \gamma_m v + C_m \beta_m \sinh \beta_m v^* - D_m \gamma_m \sinh \gamma_m v^* &= 0 \\ A_m \beta_m^2 \cosh \beta_m v - B_m \gamma_m^2 \sin \gamma_m v + C_m \beta_m^2 \cosh \beta_m v^* - D_m \gamma_m^2 \cosh \gamma_m v^* &= 0 \quad (3.28) \\ A_m \beta_m^3 \sinh \beta_m v - B_m \gamma_m^3 \sinh \gamma_m v + C_m \beta_m^3 \sinh \beta_m v^* - D_m \gamma_m^3 \sinh \gamma_m v^* \\ &= (P^*/\delta_m) \cos m\pi u \end{aligned}$$

where $u^* = 1-u$, and $v^* = 1-v$. These sets of equations can be solved as follow:

for $\lambda^2 > (m \pi)^2$,

$$\begin{aligned}
 A_m &= \frac{p^* \cos m\pi u \cosh \beta_m v^*}{\delta_m (\beta_m^2 + \gamma_m^2) \beta_m \sinh \beta_m} \\
 B_m &= \frac{p^* \cos m\pi u \cos \gamma_m v^*}{\delta_m (\beta_m^2 + \gamma_m^2) \gamma_m \sin \gamma_m} \\
 C_m &= \frac{p^* \cos m\pi u \cosh \beta_m v}{\delta_m (\beta_m^2 + \gamma_m^2) \beta_m \sinh \beta_m} \\
 D_m &= \frac{p^* \cos m\pi u \cos \gamma_m v}{\delta_m (\beta_m^2 + \gamma_m^2) \gamma_m \sin \gamma_m}
 \end{aligned} \tag{3.29}$$

and for $\lambda^2 < (m \pi)^2$,

$$\begin{aligned}
 A_m &= \frac{p^* \cosh m\pi u \cosh \beta_m v^*}{\delta_m (\beta_m^2 - \gamma_m^2) \beta_m \sinh \beta_m} \\
 B_m &= \frac{-p^* \cos m\pi u \cosh \gamma_m v^*}{\delta_m (\beta_m^2 - \gamma_m^2) \beta_m \sinh \gamma_m} \\
 C_m &= \frac{p^* \cos m\pi u \cosh \beta_m v}{\delta_m (\beta_m^2 - \gamma_m^2) \beta_m \sinh \beta_m} \\
 D_m &= \frac{-p^* \cos m\pi u \cosh \gamma_m v}{\delta_m (\beta_m^2 - \gamma_m^2) \gamma_m \sinh \gamma_m}
 \end{aligned} \tag{3.32}$$

The only unknown variable i.e., P^* can be obtained from the eigenvalue matrix calculation.

3.4 Eigenvalue Matrix

In the previous section, the building blocks used in analysis of the plate were discussed and the lateral displacement function for each of them (Equations 3.23, 3.24, and 3.25) was obtained. From the first four building blocks E_m , E_n , E_p , and E_q are unknown coefficients that must be determined. m , n , p , and q are subscripts of the terms in the series of trigonometric functions that were used in the Levy type solution for these building blocks respectively. A number of building blocks of the fifth type are required to represent the point supports of the plates, the subscript coefficient of P thus will be indexed for each simple support point. For a case of four simple support points, these will be P_1 , P_2 , P_3 , and P_4 . The number of unknown coefficients for this case with four primary terms of trigonometric function used in the Levy type solution is 20 ("4k" unknowns for primary blocks and one for each simple point supports (where k is the number of the terms in the series of trigonometric functions)). Therefore 20 equations are required to solve this problem. These equations arise from the plate boundary conditions and support point displacements [14]. All of the equations are linear functions in these coefficients. They can be represented in matrix form as illustrated in abstraction in Figure 3.4. This is called the eigenvalue matrix.

The coefficients that form the first four rows of the eigenvalue matrix of Figure 3.4 are produced by expanding in cosine series the net contribution of all building blocks to moment along the edge, $\zeta=1$. After this expansion is

completed, each coefficient in the expansion (k terms) is set equivalent to zero. This gives rise to a homogenous equation [14]. The last four rows are obtained by the super position of the displacement at the point supports and requiring net displacement to vanish.

For a concentrated mass attached on the plate a single column and row exactly similar to those required for a point support is added to the matrix [5]. The building block associated with the mass is driven by the force,

$$P^* = -2\phi^4 M_r \lambda^4 W(u,v)$$

where M_r is mass ratio of the attached mass.

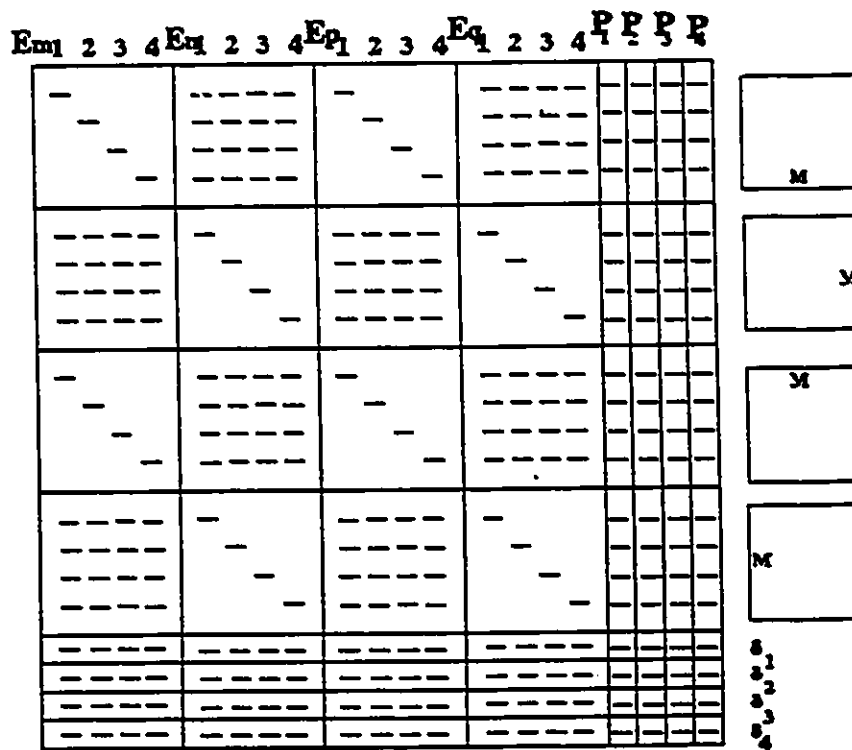


Figure 3.4 The eigenvalue matrix generated for a free edge plate with four simple interior point supports and by applying four primary terms in each building block.

Thus the eigenvalue matrix is a function of the lateral displacement of all of the building blocks that were considered already (Equations 3.23, 3.24, and 3.25) as well as their derivatives. A study of this matrix reveals that the plate's eigenvalue, and dimensionless lateral displacement function and its derivatives are functions of the support point coordinates (in dimensionless form), Poisson's ratio, the coordinates and mass ratio of the components on the plate, and the aspect ratio of the plate.

CHAPTER 4

OPTIMIZATION PROCEDURE

In the previous chapter the vibration of an electronic circuit board having four interior point supports and mass was discussed in detail, and the board lateral displacement function and its second derivatives with respect to the coordinates ζ and η were determined. However the aim of this optimization research is to eliminate -or at least- reduce the effects of board vibrations on the delicate components mounted on its lateral surface as well as the etched electronic circuits. The first step toward this end is to have a good understanding of the harmful effects of vibration and to design an objective function containing required design variables. The second step is to find a suitable optimization method which can minimize the objective function efficiently. This chapter will start by defining the terms required for developing the objective function. This is followed by the actual development

of the objective function. Two cases will be studied; a board having heavy mounted components on its lateral surfaces and one with light ones. Four objective functions to deal with the cases will be introduced. In the third section design variables are discussed in detail, both for a board with and without heavy components on it. In section four, different applications of the proposed objective functions and their relative design variables will be discussed. Section 5 is devoted to the type of the optimization problem and evaluation of the three nonlinear methods of optimization Simplex, Hooke and Jeeves, and Powell's methods and four proposed objective functions with respect to their suitability for the current research work.

4.1 Definitions

Support Point Vector L : It is a vector that contains the coordinates of the board support points. Generally for a four point support board, it might have eight members. But for simplicity and because of symmetric shape of the rectangular boards it is assumed that the support points are located at the corners of a smaller rectangle. In this case the coordinates of two opposite corners of the support rectangle will define the point support coordinates of the board. Figure 4.1 shows an example of a board whose supports is described by vector:

$$L = \{x_A, y_A, x_B, y_B\} \quad (4.1)$$

Dimensionless form of the support point vector is denoted by l , and all of its members are non-dimensionalized with respect to the board dimensions.

Mass Vector C: The mass vector of a component is defined as a vector of the coordinates of the component's position x , and y and its mass m . In Figure 4.1 the component M has a mass vector:

$$C = (x_M \ y_M \ m) \quad (4.2)$$

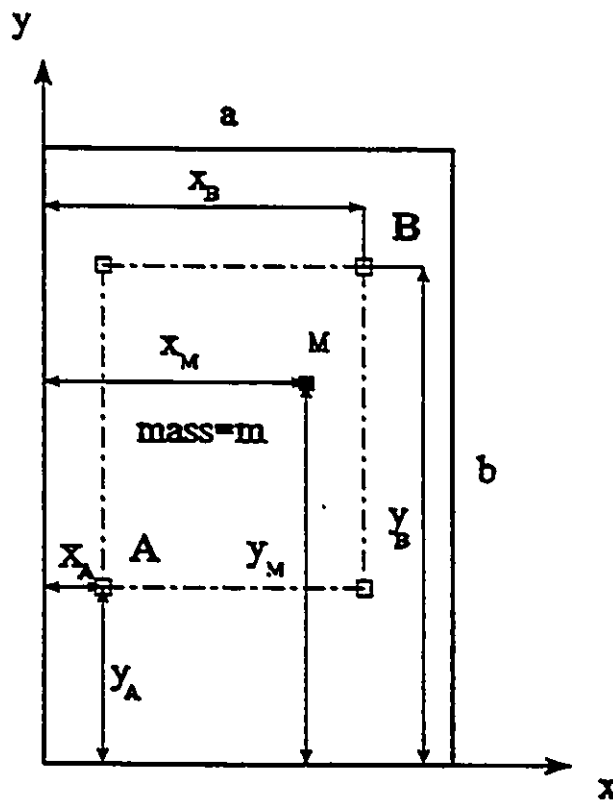


Figure 4.1 A board with support point vector $L = (x_A \ y_A \ x_B \ y_B)$ and a component of mass vector $C = (x_M \ y_M \ m)$.

In dimensionless form, the mass vector is denoted by small letter c . In this case x , and y are non-dimensionalized with respect to the board dimensions, and m with respect to the mass of the board M .

Position Characteristic κ : The position characteristic κ is a term that is composed of the requirements which are essential for a particular component to be safe when it is mounted at a location on a board exposed to vibration. For a component mounted on a vibrating board (Figure 4.2), the bending moment due to the board flexure and the inertia force due to the acceleration of the mass of the component are two main factors that result in a cyclic force that may dislodge the component from its anchoring point. The flexure of the board alone may cause stretching and damage to the circuit copper wiring imprinted on the board.

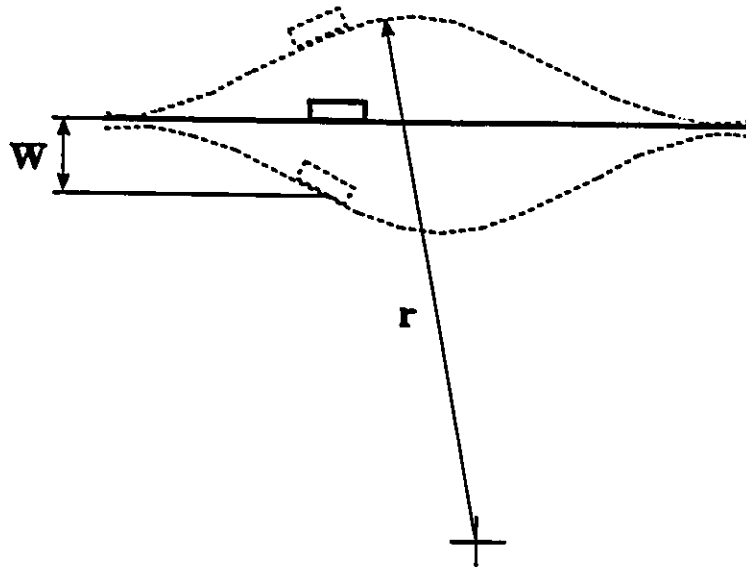


Figure 4.2 Flexure and displacement of the board affect the delicate components mounted on it.

The position characteristic for a component is defined as a linear function of the inertia force of a unit mass of the component $\omega^2 W$ (W is the lateral displacement of the component's position) and the radius of curvature r of the position on which it is mounted, hence:

$$\kappa = C_1 \omega^2 W + \frac{C_2}{r} \quad (4.3)$$

where C_1 , and C_2 , are two weighting factors that determine the relative importance of the inertia force, and bending moment in the definition of the position characteristic. κ_c is the cutoff position characteristic corresponding to the mounted sensitive component, or to the printed circuit on the board.

Suitable Region: A suitable region is a portion or some distinct portions of the board lateral surface which are suitable for a particular set of components specified by their maximum allowed position characteristic. In order to find the suitable region of a board, it is approximated by dividing it into a large number of rectangular elements in a way that the position characteristic of each of them can be considered constant and equal to the position characteristic of the centre point of that element. Thus the suitable region includes those elements that have a position characteristic less than the cutoff position characteristic.

4.2 Objective Function

This section will start with a look at the nature of the board vibration and what makes a board suitable for holding sensitive components. The excitation energy will then be considered in order to calculate the board vibration levels. Finally a number of different objective functions will be introduced and compared with each other.

4.2.1 Nature of a Suitable Board

In the search for a board specification that would prolong the service life of the electronic circuits printed on it, and that of the delicate mounted components, a major problem is overcome when the natural frequency of the board is known. In this case large cyclic displacements and stresses will be controlled by making sure that the natural frequency of the board is beyond the frequency range at which harmonic excitation is anticipated. In the majority of practical applications this is not possible and the system experience an extended range of different excitation frequencies or may even be subject to impact type excitation. In these cases, the electronic circuit board inevitably gets excited and it may undergo relatively large amplitude displacements and stresses. Theoretically the curvature of the board due to the flexure is more harmful to the circuit board and mounted delicate components than amplitude and frequency of displacement. Furthermore large components experience

more damage than small ones.

The curvature of a board at a particular location may be approximated by the second derivative of the board lateral displacement when the squared derivative of the lateral displacement \bar{W}' is ignored:

$$\frac{1}{r} = \frac{\bar{W}''}{\sqrt{1+\bar{W}'^2}} \approx \bar{W}'' \quad (4.4)$$

Substituting the curvature from Equation 4.4 into Equation 4.3 gives:

$$\kappa = c_1 \omega^2 \bar{W} + c_2 \bar{W}'' \quad (4.5)$$

The second derivative of lateral displacement of the board at a point may be evaluated along the coordinate axes parallel to the edge of the board and the larger one used as an approximation of the second derivative of the board at that particular location, hence:

$$\bar{W}'' = \text{Max}(\bar{W}''_x, \bar{W}''_y) \quad (4.6)$$

This approximation is justified since printed circuits and components are always aligned parallel to the board edge.

4.2.2 Board Vibration Energy

The amplitude of a board vibration is a function of the excitation energy. Vibration can be expressed as the cyclic conversion of kinetic and potential energies from one to the other. The kinetic energy P is given by:

$$P = \int_S \frac{1}{2} v^2 dm \quad (4.7)$$

where dm is the mass of an element of the board, and v is its maximum velocity of lateral displacement. The integration is taken over the surface of the board S . The maximum velocity of the board lateral displacement is given by:

$$v = W \times \omega \quad (4.8)$$

where W , and ω are the board lateral displacement and frequency of vibration respectively. Substituting for v in Equation 4.7 gives:

$$P = \int_S \frac{1}{2} W^2 \omega^2 dm \quad (4.9)$$

A single board can be excited when it is supported with different support vectors whereby the same amount of energy is absorbed. In this case the board experiences different mode shapes, vibration frequencies, and amplitudes of lateral displacement.

4.2.3 Definition of Objective Function

The optimization problem is expressed in terms of the position characteristic coefficient κ , suitable region, and board vibration energy P as follows:

The maximization of the number of elements approximating the board

for which the position characteristic κ_i is smaller than a given cutoff position characteristic κ_c . It is to be mentioned that the number of these elements should be sufficiently large so as to reflect the true vibration characteristics of the board. Alternatively the optimization problem can be cast in the form of a minimization problem as follows:

Minimize the number of unsuitable elements N_u , defined as those having position characteristic κ_i higher than κ_c from a grid of $n \times n$ elements on the board lateral surface.

This preceding objective function is dependent on the value of κ_c . To render the result of the optimization more valuable, this limitation should be removed. The generalization of the result will be explained through an example in the following paragraph.

Consider a study case where an electronic circuit board of 30x45 cms is divided into 441 elements the position characteristics of the elements are computed. The minimum and maximum position characteristic of these elements are κ_{min} and κ_{max} . Depending on the cutoff position characteristic κ_c , the number of unsuitable elements N_u will vary from 441 to zero. Figure 4.3 shows a plot of the number of unsuitable elements N_u for six cases of board anchoring points versus κ_c . The six different support point vectors are as follow:

$$L_1 = \{9.47 \ 1.19 \ 34.41 \ 31\}$$

$$L_2 = \{3.23 \ 10.95 \ 42.0 \ 26.57\}$$

$$L_3 = \{5.74 \ 7.34 \ 35.15 \ 36.0\}$$

$$L_4 = \{3.7 \ 5.13 \ 24.46 \ 39.67\}$$

$$L_5 = \{10.0 \ 5.46 \ 27.46 \ 38.45\}$$

$$L_6 = \{5.23 \ 12.8 \ 31.0 \ 35.46\}$$

Irrespective of κ_c , the optimum support point vector of such a board is the one which has the smallest area under the curve of the figure.

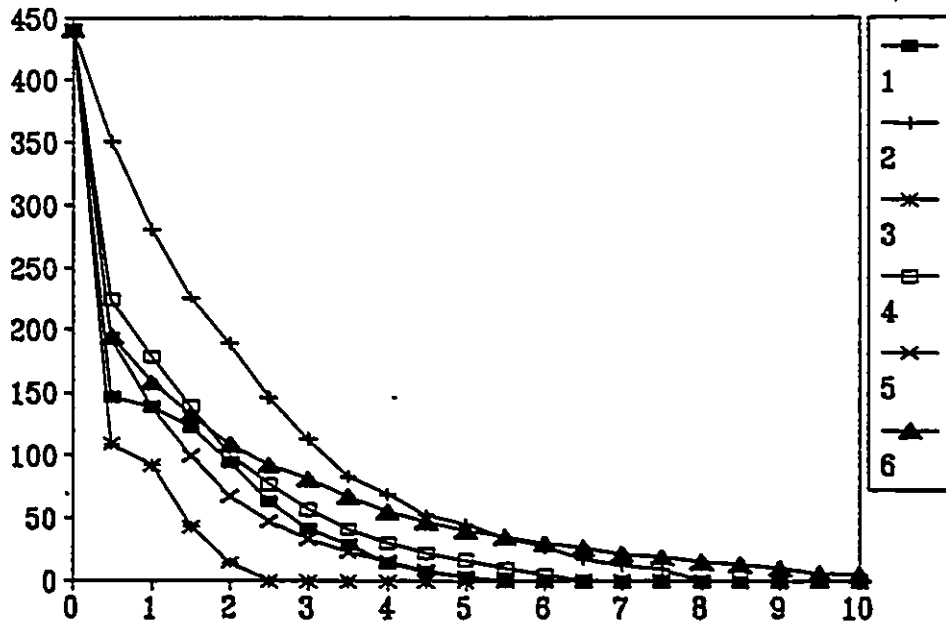


Figure 4.3 Graph of the number of the unsuitable elements N_u versus cutoff position characteristic κ_c for a board supported by six different support vectors.

This will correspond to the curve on the graph which has the sharpest sustained decline in number of unsuitable elements as κ_c is varied from 441 to zero (curve No. 3). This generalized form of objective function can be stated

as follows:

To minimize the area below the curve of the number of unsuitable elements N_u versus cutoff position characteristic κ_c as κ_c is varied from κ_{\min} to κ_{\max} .

Since the data of this graph are discrete, the area under each curve can be calculated as follows (Figure 4.4):

$$A_u = \frac{(\kappa_1 + \kappa_2)}{2} + \frac{(\kappa_2 + \kappa_3)}{2} + \dots + \frac{(\kappa_{n-1} + \kappa_n)}{2} \quad (4.5)$$
$$= \sum_{j=1}^n \kappa_j - \left(\frac{\kappa_1 + \kappa_n}{2}\right)$$

where κ_j is the position characteristic of the board for different elements sorted from minimum κ_1 to maximum κ_n .

4.2.4 Alternative Objective Functions

Other objective functions have been investigated in the course of this work. They are simpler in nature than the preceding one, however they can be used for particular applications. Their introduction here serves the purpose of alerting the reader to the fact that simpler objective functions than the one introduced earlier would not be as efficient in finding the optimum support

points for a board as will be seen later.

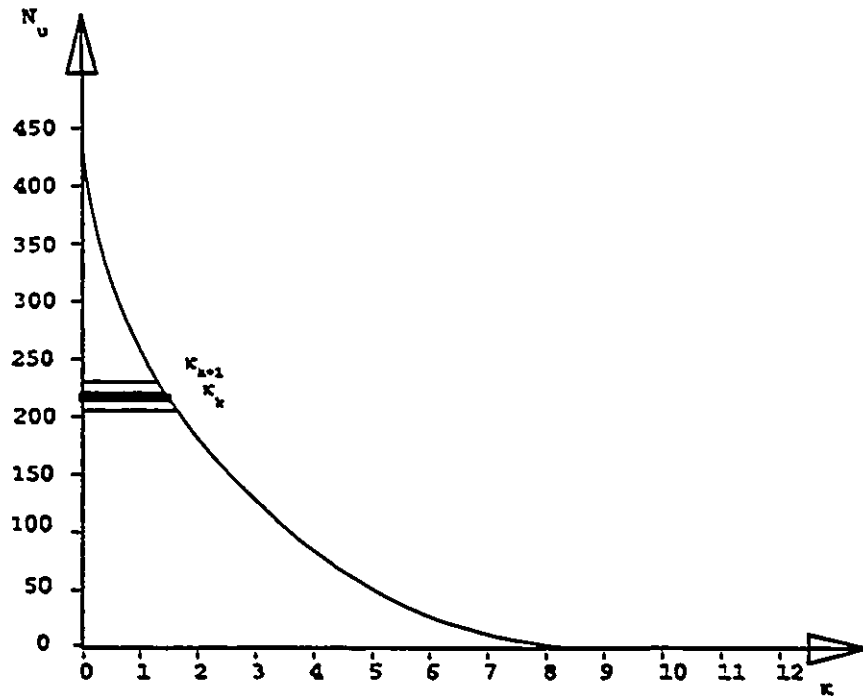


Figure 4.4 Area below curve of the number of unsuitable element N_u versus κ_c is divided into a large number of small trapeziums with unit height.

A relatively simple model would consist of choosing a board anchoring which has a minimum of the maximum position characteristic κ_{\max} . This is a typical min max optimization problem. Hence, to some extent we may expect to have a relatively large suitable region on the board. This objective function is expressed as :

**To find a support point vector which
minimizes the maximum of the board
position characteristic κ_{\max}**

Another objective function is designed so as to allow choosing a region of the board as the usable area and minimizing the position characteristic κ_c in that region at the expense of the rest of the board. This is accomplished by sorting the elements with respect to their position characteristics, electing the number of elements that reflect the desired area, and minimizing κ_c for that number of elements on of the board with a better cutoff position characteristic κ_c . The number of suitable elements is equal to the number of all of the elements on the board, minus the number of the unsuitable ones N_u . The objective function can then be defined as follows:

To find a support point vector which minimizes the cutoff position characteristic of the board κ_c for a particular number of elements on the board lateral surface .

In the former objective functions, one may consider that the position characteristic κ be a function of displacement, and frequency of vibration, or the second derivative of the board, by choosing C_1 and C_2 equal to zero respectively.

The last objective function to be introduced here is expressed in terms of the frequency of vibration. This idea was already considered as an objective

function by Kunz Reference [1]. While Kunz et al. did not consider the importance of curvature in reducing the service life of electronic circuit boards. It is generally accepted that a board with a higher vibration frequency has a smaller amplitude of displacement, and consequently smaller curvature in comparison with the case when the same board has a lower frequency where both of them are excited with a constant excitation energy. This objective function is relatively simple and it needs less CPU time to evaluate the objective function, because it does not require the calculation of lateral displacement and its second derivatives. Since the eigenvalue is proportional to the vibration frequency (Equation 3.4), it is proposed that the optimization problem be cast as follows:

**To find a board specification which maximizes
the fundamental eigenvalue λ^2 .**

It should be noted that this objective function is not applicable to the dimensionless problems which will be introduced later.

4.3 Design Variables

In this section, a complete discussion of the requirements for the calculation of a board vibration will be given for the cases when there is no heavy component on the board as well as when a group of heavy components are mounted on the board.

Recall from Chapter 3, the first step in evaluation of a board vibrations is to find the eigenvalue λ^2 , dimensionless lateral displacement function $W(\zeta, \eta)$, and its derivatives with respect to ζ and η where ζ , and η are dimensionless coordinates. In order to calculate these terms for a rectangular board supported by a particular support point vector, the following set of parameter is required:

1. Aspect ratio ϕ
2. Poisson's ratio ν
3. Dimensionless support point vector l
4. Radius of fasteners over length of the board r/a
5. Dimensionless mass vectors c_i for $i=1$ to k (where k is the number of the concentrated masses)

If we assume that the fasteners have a radius equal to a constant fraction of the board's width a , then r/a will be a constant and could be omitted from the list of the unknown variables. Another variable is Poisson's ratio ν . Fortunately it is almost equal to 0.333 for most of the engineering materials. And it is equal to 0.5 for polymeric materials, from which electronic circuit boards are normally built. Therefore it also can be assumed as a constant and omitted. Hence, the eigenvalue λ^2 , board lateral displacement $W(\zeta, \eta)$ and its derivatives are functions of ϕ , l , and c_i only as follows:

$$\lambda^2 = f(\varphi, l, c_i) \quad (4.11)$$

$$W(\zeta, \eta) = G(\varphi, l, c_i)$$

for $i=1$ to k where k is the number of masses attached to the board.

The next step is to calculate $W(x,y)$ and its derivatives with respect to the x and y axes. As indicated earlier $W(\zeta, \eta)$ is non-dimensionalized form of board lateral displacement $W(x,y)$ with respect to the dimensions of the board a , and b . Thus $W(\zeta, \eta)$ and its derivatives are given by:

$$\begin{aligned} W(\zeta, \eta) &= \frac{W(x,y)}{a} \\ \frac{\partial^2 W(\zeta, \eta)}{\partial \zeta^2} &= \frac{\partial^2 \left(\frac{W(x,y)}{a} \right)}{\partial \left(\frac{x}{a} \right)^2} = a \frac{\partial^2 W(x,y)}{\partial x^2} \\ \frac{\partial^2 W(\zeta, \eta)}{\partial \eta^2} &= \frac{\partial^2 \left(\frac{W(x,y)}{a} \right)}{\partial \left(\frac{y}{b} \right)^2} = a \varphi^2 \frac{\partial^2 W(x,y)}{\partial y^2} \end{aligned} \quad (4.12)$$

The lateral displacement and its derivatives are usually calculated in the normalized form with respect to the maximum lateral displacement. Thus for every point on a board with aspect ratio equal to φ and subject to a particular excitation which causes a maximum lateral displacement $W(\zeta, \eta)_{max}$ equal to d , $W(x,y)$ and its derivatives can be expressed as:

$$W(x,y) = ad \hat{W}(\zeta, \eta)$$

$$\frac{\partial^2 W(x,y)}{\partial x^2} = \frac{d}{a} \frac{\partial^2 \hat{W}(\zeta, \eta)}{\partial \zeta^2} \quad (4.13)$$

$$\frac{\partial^2 W(x,y)}{\partial y^2} = \frac{d}{a\phi^2} \frac{\partial^2 \hat{W}(\zeta, \eta)}{\partial \eta^2}$$

Where \hat{W} is the normalized form of the non-dimensional lateral displacement of the board. For simplicity the maximum second derivative of the board for each point $\bar{W}''(x,y)$ is defined as:

$$\bar{W}''(x,y) = \max \left\{ \frac{\partial^2 W(x,y)}{\partial x^2}, \frac{\partial^2 W(x,y)}{\partial y^2} \right\} \quad (4.14)$$

Substituting the right hand side of this equation from Equation 4.13 gives:

$$\bar{W}''(x,y) = \frac{d}{a} \max \left\{ \frac{\partial^2 \hat{W}(\zeta, \eta)}{\partial \zeta^2}, \frac{\partial^2 \hat{W}(\zeta, \eta)}{\phi^2 \partial \eta^2} \right\} \quad (4.15)$$

The dimensionless maximum lateral displacement of the board d can be calculated from board energy. Assuming that P is the absorbed energy of the board when it is excited and that the board is considered as a grid of $n \times n$ concentrated mass m_i where:

$$m_i = \begin{cases} M/n^2 & \text{for nodes located inside the board.} \\ M/2n^2 & \text{for nodes located at the edges.} \\ M/4n^2 & \text{for nodes located at the corners.} \end{cases}$$

then with reference to Equation 4.9 P is given by:

$$P = \frac{1}{2} \omega^2 \left\{ \sum_{i=1}^{(n+1)^2} m_i W^2(x,y) + \sum_{j=1}^l m_j W^2(x,y) \right\} \quad (4.17)$$

In this equation the first summation represents the absorbed energy by the board itself, and the second one represents the total energy absorbed by the mounted components. m_j is the mass of the components mounted on the board and l is the number of the components. Substituting for m_i from Equation 4.16, and m_j by Mm_{rj} and $W(x,y)$ from Equation 4.13 gives:

$$P = \frac{\omega^2 d^2 a^2 M}{2} \left\{ \sum_{i=1}^{(n+1)^2} \frac{k_k}{n^2} \hat{W}_i^2(\zeta, \eta) + \sum_{j=1}^l m_{rj} \hat{W}_j^2(\zeta, \eta) \right\} \quad (4.18)$$

where k_k is equal to 1, 1/2, or 1/4, and m_{rj} is the mass ratio of the components. Substituting ρ by M/A and a^2 by A/ϕ in the Equation 4.3 the frequency of vibration ω is given by:

$$\omega = \phi \lambda^2 \sqrt{\frac{D}{MA}} \quad (4.19)$$

Substituting for ω in Equation 4.18 and rearranging gives d as:

$$d = \left(\frac{2P}{\phi \lambda^4 D \left(\sum_{i=1}^{(n+1)^2} \frac{k_k}{n^2} \hat{W}_i^2(\zeta, \eta) + \sum_{j=1}^l m_{rj} \hat{W}_j^2(\zeta, \eta) \right)} \right)^{1/2} \quad (4.20)$$

Substituting d from Equation 4.20 into Equations 4.15, and 4.13 gives:

$$\bar{W}(x,y) = \left(\frac{2P}{A\lambda^4 D \left(\sum_{i=1}^{(n-1)^2} \frac{k_k}{n^2} \hat{W}_i^2(\zeta,\eta) + \sum_{j=1}^k m_{r_j} \hat{W}_j^2(\zeta,\eta) \right)} \right)^{1/2} \times \max \left\{ \frac{\partial^2 \hat{W}(\zeta,\eta)}{\partial \zeta^2}, \frac{\partial^2 \hat{W}(\zeta,\eta)}{\phi^2 \partial \eta^2} \right\} \quad (4.21)$$

and

$$W(x,y) = \frac{\hat{W}(\zeta,\eta)}{\phi \lambda^2} \times \left(\frac{2PA}{D \left(\sum_{i=1}^{(n-1)^2} \frac{k_k}{n^2} \hat{W}_i^2(\zeta,\eta) + \sum_{j=1}^k m_{r_j} \hat{W}_j^2(\zeta,\eta) \right)} \right)^{1/2} \quad (4.22)$$

Finally substituting ω , $W''(x,y)$, and $W(x,y)$ from Equations 4.19, 4.21, and 4.22 into Equation 4.5 the position characteristic is found as follow:

$$\kappa = \left\{ \frac{2P}{A \left\{ \sum_{i=1}^{(n-1)^2} \frac{k_k}{n^2} \hat{W}_i^2(\zeta,\eta) + \sum_{j=1}^k m_{r_j} \hat{W}_j^2(\zeta,\eta) \right\}} \right\}^{1/2} \times \left(C_1 \sqrt{\frac{\phi^2 \lambda^4 D}{M^2}} \hat{W}(\zeta,\eta) + \frac{C_2}{\sqrt{\lambda^4 D}} \max \left(\frac{\partial^2 \hat{W}(\zeta,\eta)}{\partial \zeta^2}, \frac{\partial^2 \hat{W}(\zeta,\eta)}{\phi^2 \partial \eta^2} \right) \right) \quad (4.23)$$

where:

n : Number of nodes on the board.

P : Board excitation energy.

λ^2 : Eigenvalue.

φ : Aspect ratio.

m_r : Mass ratio

A : Area of the board

In this equation λ^2 , $\hat{W}(\xi, \eta)$ and its derivatives are functions of φ , v , l , and c_1, c_2, \dots, c_n (Equation 4.11). The board physical specifications such as flexural rigidity, area and Poisson's ratio, and its excitation energy P are constant. The mass vector of the components that are located at predefined positions on the board are also constant parameters. The remainder of the parameters on the right hand side of the equation i.e. aspect ratio φ , and dimensionless support point vector l can be considered design variables. Furthermore dead weights may be attached to the board to affect its vibration characteristics. The mass vector of the dead weights can then be added to the list of the design variables. Aspect ratio is the other parameter that can be a design variable when the weight of the mounted components are negligible

4.4 Application of Objective Functions and Design Variables

Normally, vibration characteristic optimization is performed for an electronic circuit board having constant mass, area, aspect ratio, and flexural rigidity, and for a number of components mounted on the board. In this case

the support point vector is the design variable. This vector may have 2 members for a fully symmetric support points with respect to two axes of coordinates, 4 members for the case where the support points are located at the corners of a rectangle, and 8 members for the case with four scattered point supports. Furthermore, dead weights may be attached to the surface of the board to affect its vibration characteristics. This is applicable to those boards which have been supported by a set of definite support points, and the designer cannot change them. In this case the mass vectors of the dead weights can be considered design variables. In general each dead weight serves as three design variables. If the weights are constant, then each of them would contribute two design variables. Each of the proposed objective functions can be applied with one of the above mentioned design variables. When the position characteristic is used in the objective function, the relative importance of the inertia force and curvature is preserved by choosing a suitable value of the weighting factors C_1 and C_2 .

When the weight of the mounted components is negligible as compared to that of the board the weighting factor of the inertia force C_1 is set very small and can be considered equal zero. This simplification would result in the mass of the board dropping out from the objective function, and the normalized position characteristic can then be defined as follows:

$$\hat{k} = \kappa \sqrt{AD}^{1/2} = \left\{ \frac{2P}{\sum_{i=1}^{(n-1)^2} \frac{k_k \hat{W}_i^2(\zeta, \eta)}{n^2}} \right\}^{1/2} \times \left(\frac{1}{\lambda} \max \left(\frac{\partial^2 \hat{W}(\zeta, \eta)}{\partial \zeta^2}, \frac{\partial^2 \hat{W}(\zeta, \eta)}{\phi^2 \partial \eta^2} \right) \right) \quad (4.24)$$

In this case the objective function is a function of the board's Poisson's ratio, support point vector, and aspect ratio. The result of minimization can be applied for any set of the boards having the same Poisson's ratio and constant area and flexural rigidity.

4.5 Type of Optimization Problem

The objective function, given in Equation 4.5, is a nonlinear and multi-variable one. The partial derivatives of the objective function with respect to the design variables are not readily available. Their numerical approximation required a lengthy computerized calculation. Thus analytical method of optimization as well as gradient search ones are not suitable. Only direct search methods like Simplex and the pattern search methods, Powell's and Hook & Jeeves are considered. The required routines are programmed in C and the methods are borrowed from Reference [21].

4.5.1 Evaluation of Optimization Methods

In this section three methods of Simplex, Powell's, and Hook & Jeeves are evaluated in terms of finding the optimum support points of a sample board. The board has the following specifications:

A: 600 cm²

ϕ : 1

D:1838.59 N cm

ν : 0.5

Mass of the components are negligible.

The objective function is the area under the curve of the unsuitable elements of the board surface versus cutoff position characteristic κ_c , and the design variables are the members of the support point vector l . A set of eight support point vectors are produced randomly. Each of Powell's and Hooke and Jeeves methods are applied with four vectors from the set of the random support point vectors as initial design variables separately. This is done to increase the chance of getting the global minimum. Results of running the Hooke and Jeeves, and Powell's methods are given in Tables 4.1, and 4.2 respectively.

For Hooke and Jeeves method the step size of the design variables is equal to 0.1 and in its pattern search the cubic interpolation search is used with $\lambda=0.05$. The convergency criteria is set to stop the program when the progress is less than 0.01 after one complete iteration. It takes 66800 seconds to accomplish its task. Table 4.1 shows that this method has found four local minima using 554 function evaluations (113+224+130+77=544). The smallest minimum objective function, found by this method, belongs to the following support point vector:

$$l_{opt} = \{0.2684 \ 0.296 \ 0.611 \ 0.7206\}, \text{ with } F(l_{opt}) = 68.8$$

Table 4.1 Initial, and optimum support point vectors and their objective function evaluation, plus number of the function evaluation taken from Hooke and Jeeves method.

l_{int}	$F(l)$	n	l_{opt}	$F(l_{opt})$
{0.110 0.392 0.994 0.507}	110.4	113	{0.493 0.492 0.957 0.507}	83.18
{0.222 0.490 0.611 0.898}	110.1	224	{0.268 0.296 0.611 0.721}	68.8
{0.437 0.188 0.579 0.936}	121.6	130	{0.225 0.177 0.729 0.849}	77.85
{0.066 0.332 0.906 0.529}	109.7	77	{0.376 0.332 0.737 0.631}	81.44

For Powell's method the cubic interpolation is used in every pattern search with $\lambda=0.05$, and the convergency criteria is set to stop the program in a manner similar to the previous method. It takes 77560 seconds to accomplish the job. Table 4.2 shows that this method has found four local minima using 621 function evaluations ($180+164+149+128=621$). The smallest local minimum belongs to the following support point vector:

$$l_{opt} = \{0.2240 \ 0.267 \ 0.837 \ 0.9110\}, \text{ with } F(l_{opt}) = 64.6$$

Table 4.2 Initial, and optimum support point vectors and their objective function evaluation, plus number of the function evaluation taken from Powell's method.

l_{int}	$F(l)$	n	l_{opt}	$F(l_{opt})$
{0.437 0.188 0.579 0.963}	121.6	180	{0.232 0.188 0.589 0.926}	83.18
{0.118 0.301 0.586 0.758}	108.1	164	{0.138 0.311 0.615 0.7780}	68.8
{0.224 0.267 0.837 0.911}	124.6	149	{0.224 0.267 0.712 0.723}	64.6
{0.081 0.136 0.921 0.941}	155.9	128	{0.305 0.156 0.890 0.941}	93.25

The Simplex method requires five initial support point vectors to start with, hence a set of 15 random support point vectors is produced including eight vectors utilized to run the previous methods. For this method the required coefficients are set as follow:

Reflection coefficient=1

Contraction coefficient= 0.2

Expansion coefficient= 2

The convergency criteria is set to stop the program whenever the standard deviation of the objective function at the $n+1$ vertices of the current simplex is smaller than 0.01. Using the set of initial support vectors the Simplex method is applied to the same board, and results are given in Table 4.3.

With reference to the Table 4.3, in comparison with the previous methods the Simplex requires a large number of (more than 300) function evaluations for every run to optimize the objective function. But the result of each run is better than the results got from all of the four runs of each the previous methods.

It should be mentioned that the step size and initial design variables are two important factors that affect the result and efficiency of numerical search methods. The same method with different step sizes and initial design variables may find different local minima or the same one but utilizing different number of function evaluations. Thus it is difficult to ascertain a particular advantage for using Simplex method over the other two in dealing

with the problem from the limited number of sample run tested. However, it was still felt that the Simplex method is more efficient and easier to work with in the minimization of the board vibration characteristics.

Table 4.3 Initial, and optimum support point vectors and their objective function evaluation, plus number of the function evaluation taken from Simplex method.

I	$F(I)$	n	I_{opt}	$F(I_{opt})$
{0.110 0.392 0.994 0.507}	110.4	378	{0.1914 0.2161 0.8085 0.6204}	64.1
{0.222 0.490 0.611 0.898}	110.1			
{0.437 0.188 0.579 0.936}	121.6			
{0.066 0.332 0.906 0.529}	109.7			
{0.118 0.301 0.586 0.758}	108.1			
{0.106 0.359 0.919 0.581}	107.5	320	{0.174 0.2828 0.6987 0.7096}	64.7
{0.188 0.241 0.769 0.811}	108.7			
{0.122 0.168 0.535 0.868}	108.3			
{0.330 0.179 0.584 0.841}	116.7			
{0.172 0.420 0.677 0.755}	108.2			
{0.164 0.480 0.713 0.525}	109.9	395	{0.1033 0.2293 0.8466 0.6495}	60.6
{0.031 0.037 0.927 0.514}	119.1			
{0.127 0.351 0.965 0.945}	112.5			
{0.112 0.206 0.596 0.817}	100.1			
{0.240 0.350 0.890 0.866}	104.4			

4.5.2 Evaluation Of Objective Functions

In section 4.1 four different objective functions were introduced. Generally speaking, in the current research it was shown that the best objective function is the one that can find a support point vector result in the largest region of the board being acceptable regardless of the cutoff position

characteristic (Section 4.2). Among the proposed objective functions, maximization of the eigenvalue (dimensionless frequency of vibration) is the one that has already been applied by Kunz [5]. In order to compare this objective function with the one suggested in this research, the optimization procedure is performed for the following board, applying each of the two objective functions separately:

A: 600 cm²

φ : 1.4

D:1838.59

ν : 0.5

Mass of the components mounted on the board is negligible.

Simplex method is used to come out the optimization and Table 4.4 summarizes the results.

Table 4.4 Comparison of maximization of the eigenvalue and minimization of the area under the curve of unsuitable elements versus cutoff position characteristic.

F(X)	Minimize the area under curve of unsuitable point versus κ_c	Maximize the eigenvalue
Optimum support point vector l_{opt}	(0.233 0.205 0.584 0.796)	(0.285 0.230 0.746 0.736)
Eigenvalue λ^2	19.57	24.31
radius of curvature cm	237	166
Objective Function	97.6	122.5
CPU (seconds)	46478	29866

The table shows that the maximum eigenvalue do not necessarily means the best situation in terms of the number of board's suitable elements with respect to the its flexure due to the vibration. However the eigenvalue maximization require less run-time in comparison with the proposed one. The reason is that the calculation of the mode shape and board position characteristic is not required in the eigenvalue evaluation. This amount of computer run-time consumption of the proposed objective function is not critical since the optimization needs to be performed once for every board to be deigned.

The minimization of the maximum position characteristic (Min Max κ) of the board is another proposed objective function. This is very useful when it is intended to mount delicate electronic components over the entire area of the board and the maximum position characteristic of the board must be as small as possible. The last objective function dealt with here entails minimizing the cutoff position characteristic of the board for a particular number of elements on the board lateral surface. To do that the position characteristic of the board elements is sorted from the minimum to the maximum. Then the n^{th} one will be minimized where κ_n is considered the cutoff position characteristic of the board. For this sample study n is equals to 400 out of the 441 elements on the board. This objective function would be used when some components are not sensitive to the vibration and can be located on an area of the board where high vibration would be tolerated in

order to reduce the vibration characteristics over the remainder of the board. These two objective functions are applied to the vibration characteristic minimization of the test board and the results are given in Table 4.5.

Table 4.5 Data from minimization of κ_{max} over the whole area of the board and over 90% of its elements.

Objective Function	Minimize κ_{max}	Minimize κ_{max} over 90% of the board element
l_{opt}	{0.166 0.389 0.895 0.611}	{0.288 0.183 0.507 0.883}
κ_{max}	0.109	0.214
Area under the curve of unsuitable points	478	390
CPU (seconds)	42797	36661

The graph of the number of unsuitable elements versus cutoff position characteristic κ_c are provided for the former objective functions in Figure 4.5. With reference to the figure the behaviour of the evaluated objective functions is as follow:

1. The curve belonging to the optimum support point vector found by minimization of the maximum position characteristic of the board has no unsuitable element at cutoff position characteristic equal to 0.12. This is a good record, but a large region of the board has a relatively high position characteristic in comparison with the other curves.

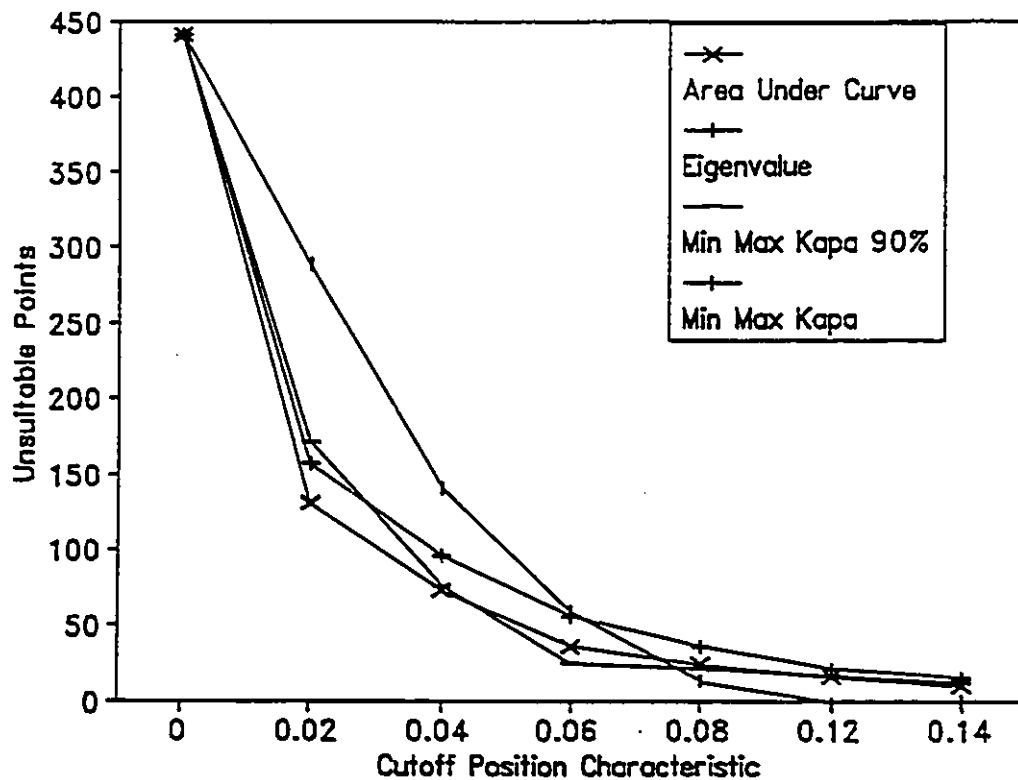


Figure 4.5 Graph of number of unsuitable point with respect to cutoff position characteristic for a board with different optimum support point vector.

2. The curves belonging to the eigenvalue maximization, and minimization of the area under the curve of unsuitable elements show a relatively similar behaviour over the following regions:

- a) For $\kappa_c < 0.2$ that is equivalent to having more than 170 unsuitable elements (38% of the board surface).
- b) For $\kappa_c > 0.14$ that is equivalent to having less than 20 unsuitable element (5% of the board surface).

This means that the first 38% of the board elements and the last 5% have almost similar position characteristic when it is supported by either of two mentioned optimum support point vectors, however, the middle part of curve that belongs to the objective function of the minimization of the area under the curve shows better vibration characteristics.

3. The curve belonging to the minimization of the maximum position characteristic over 90% of the board has the best behaviour around the point of 41 unsuitable elements that is equivalent to the 90% of the board.

CHAPTER 5

PRESENTATION OF RESULTS

In general, the solution to the minimization problems of the circuit board's vibration characteristics can be divided into two main groups. The first group consists of the solution to those boards on which the weight of the mounted components are negligible with respect to the weight of the board, or the components are distributed homogeneously over the entire regions of its surface. For this type of the boards, the vibration effect can be minimized by varying the support point vector and board aspect ratio. The point supports in this solution considered to be at the corners of a smaller rectangle because of homogeneous condition of the board surface. For simplicity they can also be considered as four fully symmetric points with respect to two coordinates axes. All of the proposed objective function are applicable to this type of boards. A table of suitable support point vector will also be introduced as a fast and ready to use solution to this type of boards. These approaches will be

discussed in sections 1, 2, and 3. Another group includes the solutions to the boards where the weight of their mounted components is not negligible. For these types of boards two solutions were proposed in the previous chapter, these are: the minimization with respect to the support point vector, and the dead weight approach. These types of solution and a study of the effect of the size and shape of the mounted components to the board vibration characteristics will be presented in sections 4, 5, and 6. In all of these solutions whenever the position characteristics is applied, the weighting factors of inertia force and bending moment C_1 , and C_2 must be determined prior to start minimization procedure. The experimental work is required to set a suitable value for them based on the sensitivity of the mounted components but for now their values are set by rule of thumbs.

5.1 Solution to Boards with Light Weight Components

For electronic circuit boards with light weight components the vibration characteristics are minimized using the support point vector as a design variable. The solution is exemplified by utilizing a sample board with the following specifications:

Area: $20 \times 30 = 600$ cms

Flexural rigidity: 1838.6 N cm

Poisson's ratio: 0.5

mass of the plate: 222 gr

Radius of fastener: 0.2 cm

Mass of the mounted components: negligible

The objective function for minimization is the area under the curve of the board unsuitable elements versus cutoff position characteristic. The weighting factor of the inertia force C_1 is set to 1 and that of the curvature C_2 is 50 (Equation 4.3). It is equivalent to considering the effect of a flexure with a radius of 200 cm equals to the effect of an inertia force of 0.25 Newton acting on a unit mass of the board due to the lateral displacement of the board.

Table 5.1 Data box from minimization of a sample board for minimum vibration characteristic with respect to the support point vector.

Support Point Vector		F	λ^2	κ_{\max}	κ_{\max} over 90%	
Initial support points	1	{0.110 0.392 0.994 0.507}	15405	6.69	20.4	8.34
	2	{0.222 0.490 0.611 0.898}	29025	5.05	416.8	11.41
	3	{0.437 0.188 0.579 0.936}	15251	10.6	32.4	6.95
	4	{0.313 0.363 0.868 0.758}	13540	4.97	25.9	7.84
	5	{0.118 0.301 0.586 0.758}	12757	11.9	23.9	7.50
Optimum		{0.223 0.286 0.661 0.770}	9515	17.8	23.4	5.55

The design variables are the members of the support point vector (in dimensionless form) located on the corners of a smaller rectangle. Results taken from utilizing the Simplex method are given in Table 5.1. The table

shows that the objective function evaluated for the board when it is supported by the optimum support point vector is considerably smaller than that when it is supported by any of the initial ones. The eigenvalue corresponding to the optimum support point vector is larger than that corresponding to any of the others.

Although the maximum position characteristic of the board supported by the optimum support point vector is not less than all of those corresponding to the initial support point vectors, the maximum position characteristic of 90% of the board elements is considerably smaller.

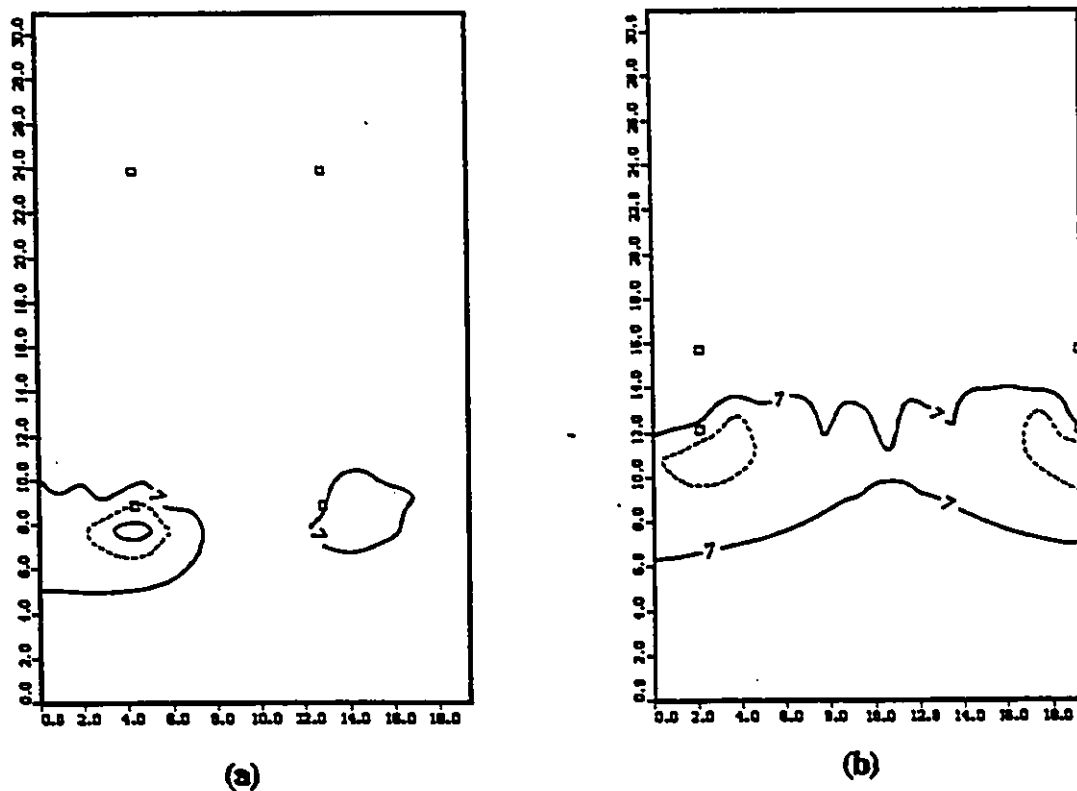


Figure 5.1 a) Contour plot of the board position characteristic when it is supported by optimum support point vector. b) Same plot for the same board when it is supported by a sample one.

A contour plot of the board position characteristic when it is supported by optimum support vector is provided in Figure 5.1 (a). Another plot for the same board when is supported by the initial support point (1) is also provided in Figure 5.1 (b).

In this figure the support points are indicated by small square symbols and the contour levels of the position characteristic are drawn to indicate the cutoff position characteristic. The board coordinate axes are scaled to show the size of the board and position of its support points. From now on, this type of contour plots will be used to show the vibration characteristics of the boards being investigated. Figure 5.1 shows that a relatively small region of the board is unsuitable when it is supported by the optimum support point vector, in comparison with the one where the board is supported by a randomly selected support point vector (initial support vector (1)). This approach gives a good understanding of the vibration characteristics for every single point of the board that is very useful. For example, a small portion of the unsuitable area of the board in Figure 5.1 (a) shows a very critical condition that must be taken into account.

5.2 Aspect Ratio as a Design Variable

In the early stage of designing of a circuit board, if the weight of the components are deemed to be negligible as compared to that of the board, or are going to be mounted homogeneously over the entire area of the board, the

aspect ratio can be added to the list of design variables. In this case the board area is considered as it is sufficient for mounting the required components and printing the related circuits regardless of their position on the board. The minimization procedure of the board vibration characteristic can then be used to determine the optimum shape of the board and the point support locations. In this case it would be beneficial to consider a larger board than required and then discard the region with more harmful vibration. A Simplex with five vertices is required to solve this problem. The solution is exemplified by utilizing a sample board with the following specifications:

Area: 600 cms

Flexural rigidity: 1838.6 N cm

Poisson's ratio: 0.5

mass of the plate: 222 gr

Radius of fastener: 0.2 cm

Mass of the mounted components: negligible

A set of 6 random design variable vectors are provided in order to utilize the Simplex method (Table 5.2). In the design variables column of this table the first four digits are members of the support point vector and the fifth is the aspect ratio of the board. The objective function for minimization is the area under the curve of the unsuitable elements versus cutoff position characteristic. The weighting factor of the inertia force C_1 is set to 1 and that of the curvature C_2 is 100 (Equation 4.3). It means that in comparison with

the previous case study, more weight is given to the bending moment force over inertia force due to the vibration of the board. The results of optimization is also given in Table 5.2. The optimum support point vector, and corresponding objective function evaluation, and aspect ratio as follow:

$$l_{opt} = \{0.295 \ 0.201 \ 0.673 \ 0.704\}$$

$$f(l_{opt}) = 8954$$

$$\varphi_{opt} = 1.03$$

It shows that the rectangles with an aspect ratio of 1 (square shape) is the optimum shape for the boards in terms of the vibration characteristics this result agree with the one reported by previous work on subject matter of this research [1].

Table 5.2 Data box from minimization of a sample board for minimum vibration characteristic with respect to the support point vector and aspect ratio.

Design Variables		F	λ^2	κ_{max}	κ_{max} over 90%	
Initial Design Variables	1	{0.110 0.392 0.994 0.673 3.33}	113842	1.69	152	85
	2	{0.407 0.222 0.590 0.821 1.27}	17099	17.05	20.1	6.4
	3	{0.331 0.198 0.737 0.928 2.54}	86055	5.6	93.8	37.6
	4	{0.188 0.479 0.936 0.765 4.11}	188751	4.94	200	137
	5	{0.226 0.332 0.806 0.541 1.86}	35236	4.28	99.5	25.5
	6	{0.032 0.467 0.659 0.611 2.86}	72238	2.47	113	55.7
Optimum		{0.295 0.201 0.673 0.704 1.08}	8954	24.6	23.5	5.84

5.3 Table of Optimum Support Point Vectors

Another approach is to use the normalized form of the position characteristic (Equation 4.24) by discarding the effect of the inertia force of the mounted components and hence setting the weighting factor of the inertia force $C_I=0$. This is justified because the board has only light weight components and their inertia forces hence are not considerable. In this case the result of the optimization procedure will be applicable to any set of the boards having the same Poisson ratio, but different constant flexural rigidity, and area. The results may be used as a good initial support point vector to minimize the board vibration characteristic in particular cases when the inertia force is considered as well. The optimization of the board vibration characteristic is performed for different aspect ratios from 1 to 2.5 in increments of 0.2. The optimum support point vectors and the values of the objective function, eigenvalue, and maximum position characteristic (in normalized form) are given in Table 5.3 with respect to different aspect ratios. This table shows that:

1. The square board (aspect ratio=1) has the smallest objective function. This indicate that such a board exhibits the best vibration characteristics for components and printed circuit survivability (same as what was given in previous section).
2. As the aspect ratio increases, so does the optimum objective function. This indicates that as the aspect ratio increases the

vibration characteristics of the board worsens.

3. An increase in the aspect ratio of the board results in a decrease in the frequency of vibration of the board.

4. The boards maximum position characteristic increases as the aspect ratio is increases.

Table 5.3 Table of suitable support point vectors for boards having Poisson ratio equals to 0.5.

No	φ	l	$F(l)$	λ	κ_{max}
1	1	{0.1740 0.2828 0.6987 0.7096}	10426	29.97	35.3
2	1.2	{0.1960 0.2220 0.6645 0.7299}	13313	23.80	57.4
3	1.4	{0.2852 0.2296 0.7462 0.7363}	15719	19.54	67.4
4	1.6	{0.2951 0.1147 0.5471 0.8350}	18927	17.13	72.2
5	1.8	{0.2422 0.2988 0.7002 0.7771}	27749	14.85	82.2
6	2	{0.2579 0.2681 0.7678 0.7246}	33683	14.85	91
7	2.2	{0.4043 0.1976 0.7211 0.7259}	37212	11.6	130
8	2.4	{0.1797 0.2652 0.5891 0.7929}	43949	11.26	134
9	2.5	{0.1595 0.2701 0.6719 0.7216}	69612	10.85	147

To make this table more useful it is accompanied by a set of 9 contour plots which indicate the unsuitable region of a sample board supported by these optimum support point vectors and having the relevant aspect ratios (Appendix 1). These contour plots are applicable to any set of boards having Poisson ratio $\nu=0.5$, and a constant area, and flexural rigidity. In order to use

this table the board designer must first decide on the aspect ratio of the board and choose it as small as possible. The support point vector can then be taken from the table and length of the board must be determined based on the suitable area of the sample board and the area required to locate all of the components.

5.4 Boards With Heavy Mounted Components

When the weight of the mounted components are not negligible, the general method of board vibration minimization with respect to the support point vector is applicable. In this case the mass vector of those components which are relatively heavy have to be considered in the determination of the board vibration characteristics and position characteristic function. Each of the proposed objective function can be utilized. Because of the asymmetric form of the board in terms of the scattered mounted components, it is better to use four scattered support point vector with 8 members but it makes the optimization procedure too lengthy and time consuming. For the sample case minimization a four member support point vector is utilized. The board vibration optimization procedure in this case is exemplified by utilizing the same board used in the previous sections but with a set of five mounted components with mass vectors as follow:

$$c_1 = \{0.2 \ 0.2 \ 0.1\}$$

$$c_2 = \{0.8 \ 0.8 \ 0.1\}$$

$$c_3 = \{0.3 \ 0.9 \ 0.2\}$$

$$c_4 = \{0.1 \ 0.5 \ 0.15\}$$

$$c_5 = \{0.5 \ 0.1 \ 0.05\}$$

The result of minimization is given in Table 5.3.

Table 5.4 Data box from minimization of a sample board having five heavy components for minimum vibration characteristic with respect to the support point vector.

Support Point Vector		F	λ^2	K_{max}	K_{max} over 90%	
Initial support point vectors	1	{0.110 0.392 0.994 0.507}	2421	6.69	2.1	1.1
	2	{0.222 0.490 0.611 0.898}	3542	5.05	41	1.2
	3	{0.437 0.188 0.579 0.936}	3124	10.6	3.3	1.4
	4	{0.313 0.363 0.868 0.758}	2479	8.86	4.8	1.29
	5	{0.118 0.301 0.586 0.758}	2591	11.9	2.9	1.63
Optimum		{0.080 0.324 0.943 0.506}	1630	4.49	1.7	0.86

From comparison with the case of same board without mounted heavy components (Table 5.1) it is concluded that:

Mounting of the heavy components has increased the values of the board maximum position characteristic, and objective function. This also decreases the eigenvalue of the board vibration. In general, the board vibration characteristics is more unsuitable when a number of heavy components are positioned on the board.

A contour plot of the board vibration characteristic is provided in Fig 5.2.

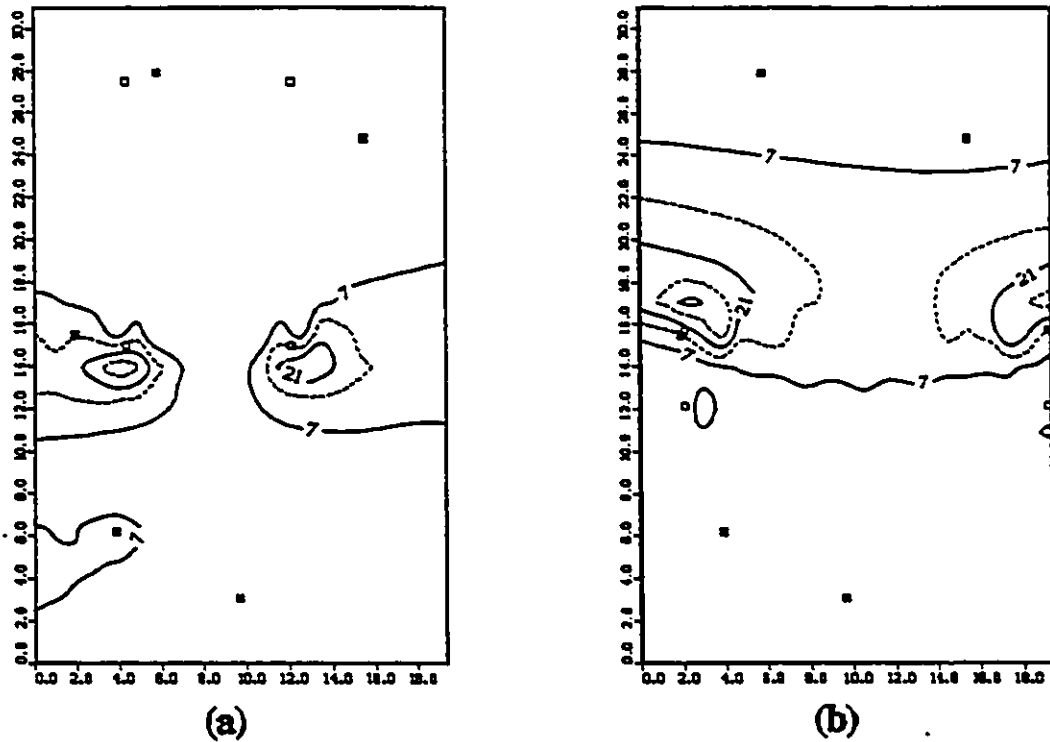


Figure 5.2 (a) Contour plot of the position characteristic for a board supported by optimum support point vector. (b) same plot when it is supported by a sample support vector.

In this figure the little black squares indicate the position of the heavy mounted components. It shows that after attachment of the heavy components, the board vibration characteristics are changed in comparison with the Figure 5.1 which shows the contour plots of the same board when there was no heavy components mounted. The position and extent of the unsuitable region are also not the same as in the previous case. Moreover the location of the optimum support points are not the same.

The extent of the unsuitable region of the board when it is supported by the optimum support vector is more than that of the same board when there is no heavy components on it, and is supported by its relevant optimum support point vector.

5.5 Effects of Size and Location of Components

To examine the effects of the size and location of the mounted components on the board vibration characteristics a sample board is considered with the following specifications:

Area: 150 cms

Aspect ratio=1.5

Flexural rigidity: 1838.6 N cm

Poisson's ratio: 0.5

Mass of the board: 55 gr

Radius of fastener: 0.3 cm

Support point vector: {0.05 0.95 8.45 8.82}

The weighting factors C_1 , and C_2 are set to zero and 1 respectively. The board vibration characteristics are determined and the contour plot of the board unsuitable region is provided in Figure 5.3 (a). A component having the following mass vector is then positioned on the board:

$$C=\{3 \ 3 \ 30\}$$

For simplicity the shape of the component and the way it is located on the board can be represented by different sets of small concentrated masses attached to the board.

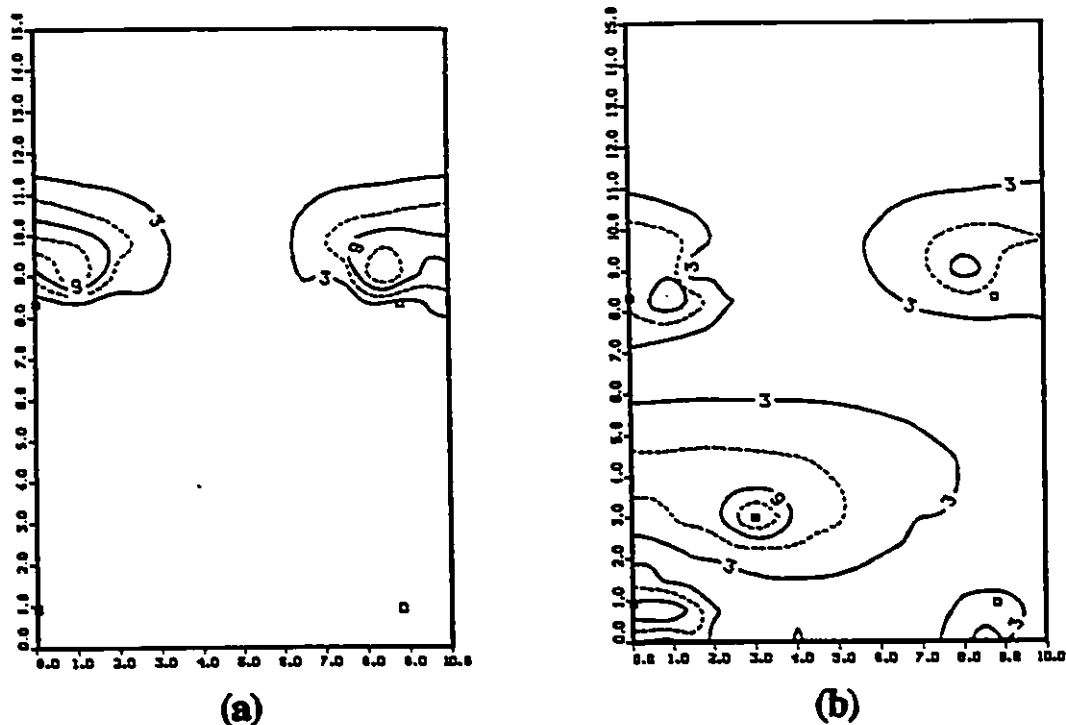


Figure 5.3 (a) Unsuitable region of a sample board. (b) Unsuitable region of the same board when a concentrated mass is located on it.

If the component is small it can be represented by a single concentrated mass having the same mass vector as the component itself. The vibration characteristics of a board with such a small heavy component mounted on it is shown in figure 5.3 (b). The figure shows that the attached mass produces a large unsuitable region around its position and two other unsuitable region at the corners of the board.

The effect of the mounted component when it is large and slender and is mounted across the length of the board can be investigated by representing it by a set of three small masses. It must be noted here that in this type of component representation, displacement of the substitute small masses is not constant and equal to the displacement of the component. But for simplicity it is considered as such, and more accurate determination is left for the future work. The mass vectors of this set are as follow:

$$C_1 = \{3 \ 1 \ 10\}, \quad C_2 = \{3 \ 3 \ 10\}, \quad C_3 = \{3 \ 5 \ 10\}$$

When it is mounted across the width of the board the set of three mass vector will be as follows:

$$C_1 = \{1 \ 3 \ 10\}, \quad C_2 = \{3 \ 3 \ 10\}, \quad C_3 = \{5 \ 3 \ 10\}$$

The unsuitable region of the board In these two cases are shown in Figure 5.4 (a, and b) respectively.

The unsuitable area in plot (a) of the figure is smaller than the other one. It shows that for this component it is better to locate it across the length of the board rather than width. In comparison with the previous case when the component was represented by a single concentrated mass (Figure 5.3 (b)) the unsuitable area of the both cases are smaller.

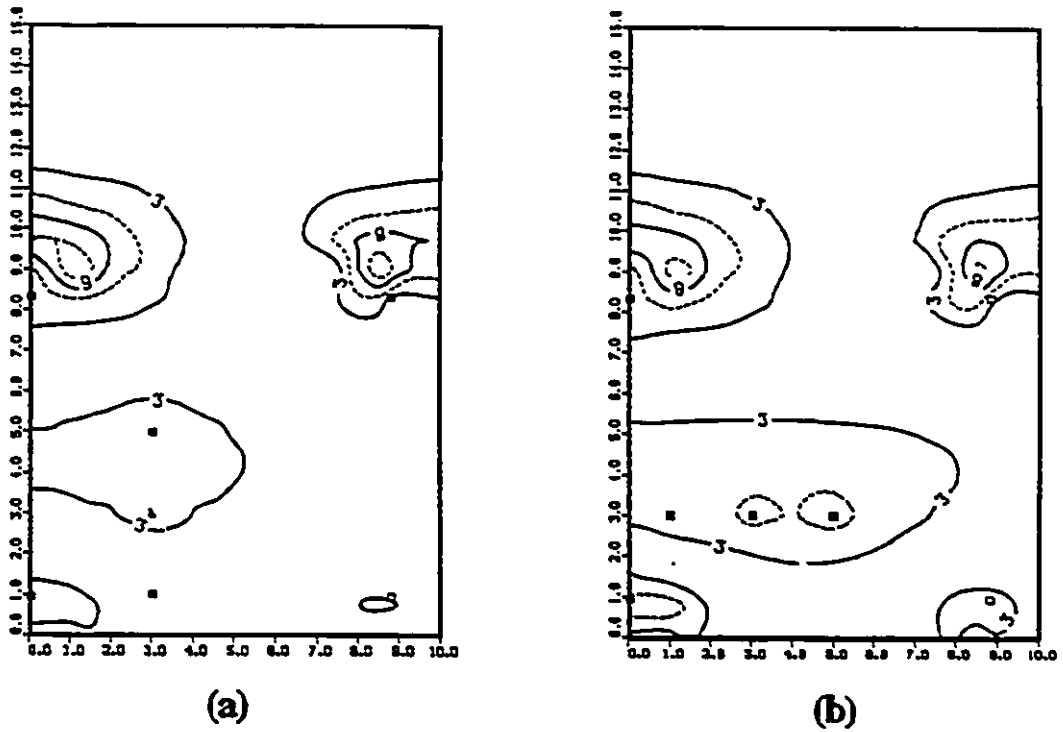


Figure 5.4 (a) Unsuitable region of a board having a distributed mass across the board's length. (b) Unsuitable region of the same board when the masses are distributed across its width.

The effects of the square components are investigated by dividing the concentrated mass into nine small masses distributed over a square area around the same point of attachment. For this purpose the following two sets of the mass vectors (sum of the weight of all small masses is equal to the one of the component):

set of small square:

$$C_1 = \{2 \ 2 \ 3.33\}, \quad C_2 = \{3 \ 2 \ 3.33\}, \quad C_3 = \{4 \ 2 \ 3.33\}$$

$$C_4 = \{2 \ 3 \ 3.33\}, \quad C_5 = \{3 \ 3 \ 3.33\}, \quad C_6 = \{4 \ 3 \ 3.33\}$$

$$C_7 = \{2 \ 4 \ 3.33\}, \quad C_8 = \{3 \ 4 \ 3.33\}, \quad C_9 = \{4 \ 4 \ 3.33\}$$

set of large square:

$$C_1=(1\ 2\ 3.33),\ C_2=(3\ 2\ 3.33),\ C_3=(5\ 2\ 3.33)$$

$$C_4=(1\ 3\ 3.33),\ C_5=(3\ 3\ 3.33),\ C_6=(5\ 3\ 3.33)$$

$$C_7=(1\ 4\ 3.33),\ C_8=(3\ 4\ 3.33),\ C_9=(5\ 4\ 3.33)$$

The unsuitable region of the board for both sets of the mass arrangements are shown in Figure 5.5 (a), and (b) respectively. The unsuitable region of the board is smaller in the case of having a smaller square components (a).

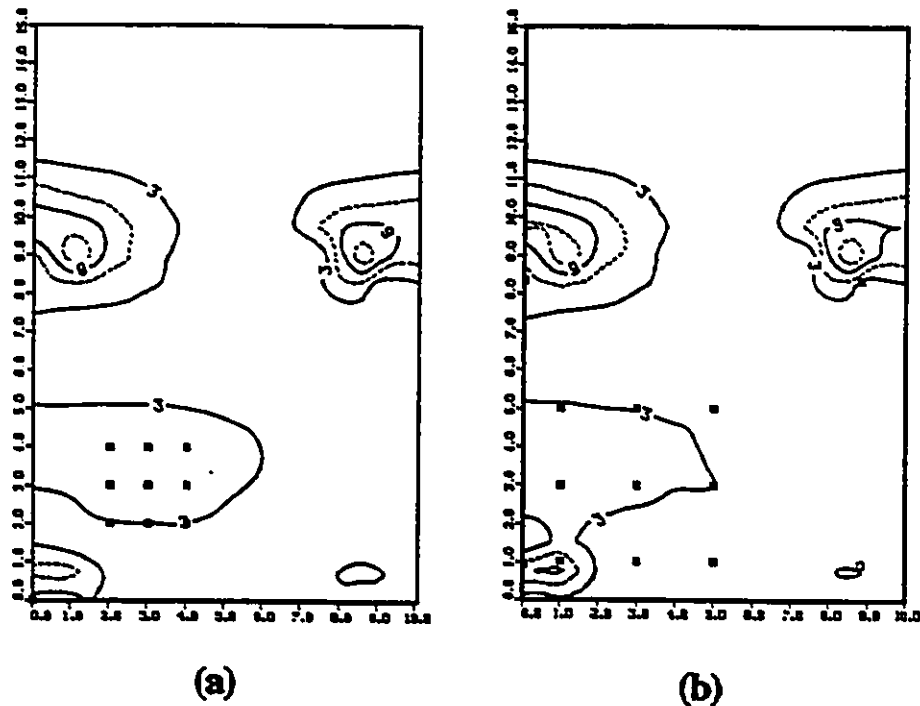


Figure 5.5 (a) Unsuitable region of a board having a distributed mass on a small square shape. (a) Unsuitable region of the same board when the masses are distributed over a larger square.

The objective function evaluation, maximum position characteristic, and maximum position characteristic over 90% of the board for each of the seven

mentioned type of loading is provided in Table 5.5.

Table 5.5 The objective function evaluation, and maximum position characteristic of the board and the one for 90% of its surface for a board with seven different distribution of attached mass.

Mass Distribution	$F(l_{opt})$	κ_{max}	κ_{max} over 90%
no mass attached	1138	30.24	6.97
one concentrated	2348	30.62	12.48
across the width	2190	26.40	11.10
across the length	1890	28.79	9.27
small square	1945	29.05	9.26
large square	1975	28.01	9.81

This table shows that any concentrated mass attached to the board can produce an unsuitable region on the board and the effect of distributed mass (large components) is less than concentrated mass (small but heavy components).

5.6 Vibration Characteristics Optimization Using Dead Weights

In the previous chapter it was shown that the objective function depends on the mass vectors of the components mounted on the board. In cases where the weights and locations of the components are predetermined as a result of the electronics design, they can be omitted from the list of the design variables.

Furthermore, when the shape, and size of the board, and anchoring points are predetermined, the vibration characteristics may be minimized by introducing a number of small weights attached to the board which are not part of the board's electronic system (dead weight). The dead weights and their coordinates can then be cast in the design variables for minimizing the objective function. If the board has a number of heavy components but the support point vector is not constant, it might be better to use support point vector to minimize the board vibration characteristics. This will also be investigated in this section.

A board with the following specification is exemplified:

A: 600 cms

φ : 1.6

Flexural rigidity: 1838.598 N cm

ν : 0.333

Mass of the board: 222 gr

Radius of fastener: 0.23 cm

Dimensionless support vector $l = \{0.172 \ 0.420 \ 0.677 \ 0.755\}$

Number of mounted component: 5

Dimensionless mass vectors of the components:

$c_1 = \{0.410 \ 0.792 \ 0.100\}$ $c_2 = \{0.107 \ 0.522 \ 0.290\}$

$c_3 = \{0.111 \ 0.398 \ 0.437\}$ $c_4 = \{0.688 \ 0.879 \ 0.236\}$

$c_5 = \{0.966 \ 0.132 \ 0.106\}$

A relatively higher weight is given to the bending force in comparison with the inertia force of the components by setting $C_1=1$, and $C_2=400$. Vibration characteristics of the board is given in Table 5.6 (first row). A contour plot of the board position characteristics is also provided in Figure 5.6 (a). The objective function in this case is the area under the curve of unsuitable elements versus cutoff position characteristic. The Simplex method is used to carry out the minimization. The minimization procedure is accomplished with utilizing one dead weight. Four starting points are needed for optimization. Results of the minimization of the board vibration characteristics is given in Table 5.6.

Table 5.6 Data table taken from minimization of a sample board with a dead weight.

mass vector of dead weight		F(X)	λ^2	κ_{max}	κ_{max} over 90%
No dead weight		118319	5.45	378.73	78.89
Initial dead weights	{0.460 0.939 0.472}	119248	5.451	378.6	78.88
	{0.141 0.502 0.747}	118602	5.452	378.75	78.9
	{0.505 0.403 0.153}	118387	5.452	378.9	78.9
	{0.477 0.126 0.362}	77228	7.65	224.9	49.5
Optimum	{0.653 0.017 0.094}	67769	8.65	224.5	38

The board vibration characteristics for most of the initial conditions are worse than that of the board with no dead weight. When the optimum dead weight is attached to the board, the board vibration characteristics is

considerably improved. The Optimum dead weight has a weight of $(222 \times 0.094 = 20.9 \text{ gr})$ and must be attached at point P with the following coordinates:

$$P: (12.68, 0.53)$$

The contour plot of the board position characteristics when the board vibration characteristics are minimized by attachment of the dead weight is provided in Figure 5.6 (b).

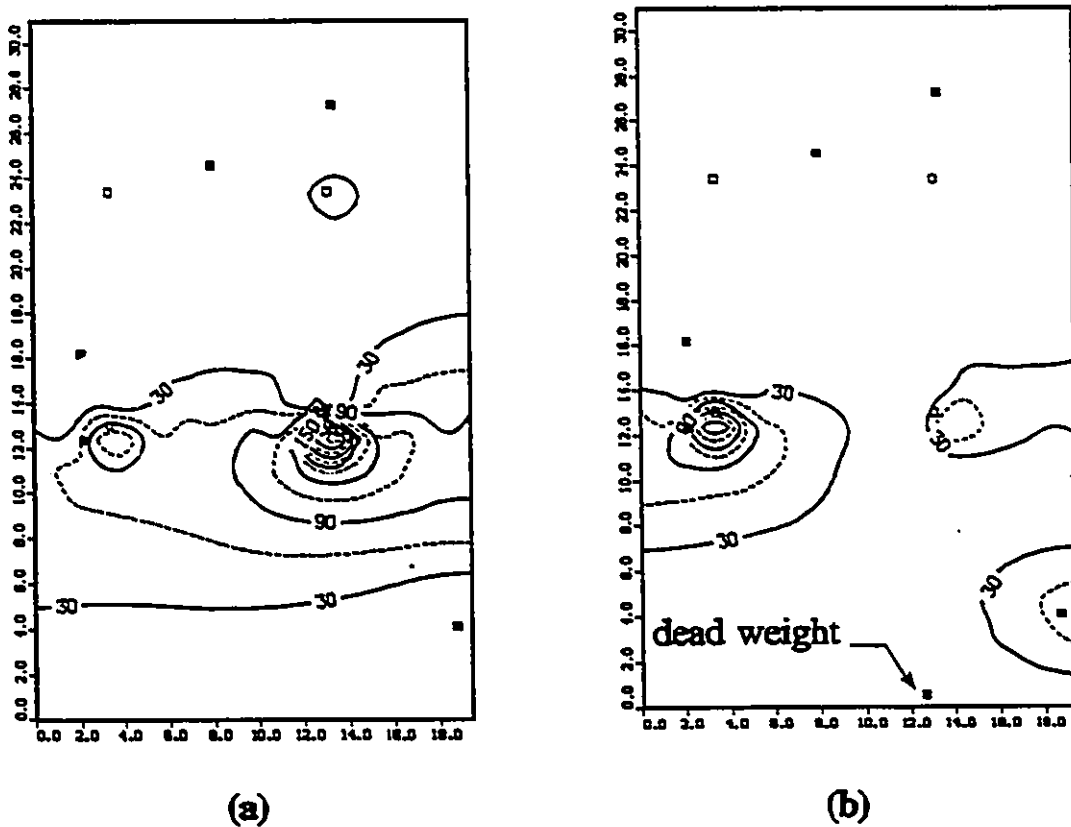


Figure 5.6 (a) The vibration characteristic of a sample board having five heavy components. (b) Same board when a dead weight of 20.9 gr is attached to the point (12.68, 0.53).

By attachment of the dead weight, the unsuitable region of the board is

reduced considerably. The location of the unsuitable region is different in comparison with the case with no dead weight. This also indicates that the dead weights can be used to relocate the unsuitable region of the board to other locations.

Another case study, presented here, consist of minimization of the board vibration characteristics utilizing two dead weights (6 design variable).

Table 5.7 Data table taken from minimization of a sample board with two dead weights.

Coordinates of dead weights		F(X)	λ^2	κ_{max}	κ_{max} over 90%
Initial dead weights	{0.586 0.623 0.438} {0.297 0.492 0.213}	118673	5.451	379	78.9
	{0.304 0.284 0.341} {0.118 0.384 0.239}	120983	5.35	382	80.6
	{0.529 0.118 0.545} {0.301 0.873 0.123}	191030	3.58	534.7	134
	{0.186 0.758 0.731} {0.104 0.342 0.052}	118365	5.452	378.6	79
	{0.443 0.819 0.187} {0.832 0.581 0.281}	123266	5.450	377.5	78.8
	{0.621 0.432 0.424} {0.780 0.615 0.730}	147370	5.450	373.5	78.7
	{0.855 0.385 0.351} {0.702 0.714 0.166}	316916	5.45	573.4	263
optimum	{0.659 0.149 0.227} {0.538 0.757 0.125}	62963	8.77	228	33.8
Optimum with respect to support point vector, no dead weight {0.1560 0.2234 0.5847 0.7329}		50019	9.45	65.8	20

A sample board similar to the one utilized in the previous case is considered and the optimization procedure is applied with the same condition as before but with two dead weights. The results of minimization are given in Table 5.7. The minimization of the board vibration with considering the support point vector as design variable is also carried out and the result is given in Table 5.7 (last row). The locations and weights of the optimum dead weights are determined as follow:

$$p_1 = (12.8, 4.6), m_1 = 61.5 \text{ gr}$$

and

$$p_2 = (10.4, 23.5), m_2 = 27.8 \text{ gr}$$

The table shows that in general, the attachment of each set of dead weights (initial dead weights) worsen the board vibration characteristics but in the optimum case the board vibration characteristics are improved substantially. In comparison with the case when the vibration characteristics of the same board was minimized with one dead weight, a relatively small improvement is achieved as a result of more flexibility with applying two dead weight offers.

The vibration characteristics of the board when supported by optimum support point vector is lower than those determined by attachment of dead weights. This shows that the dead weight solution might not be used in minimization of the board vibration characteristics when the support point vector can be changed. Contour plot of the board when its vibration characteristics are minimized by 2 dead weights and when it is supported by

optimum support point vector is provided in Figure 5.7 (a), and (b) respectively.

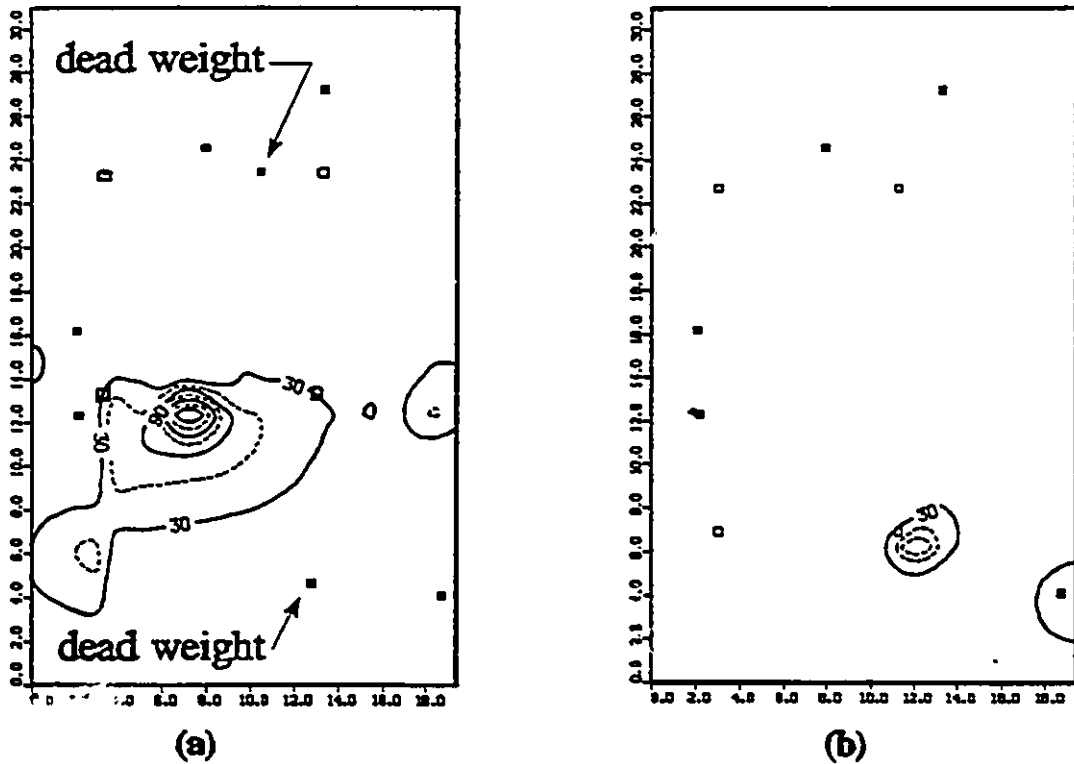


Figure 5.7 (a) Contour plot of a board when its vibration characteristic is minimized by 2 dead weights. (b) Same board when it is supported by optimum support point vector.

The solution to the boards vibration characteristics when a number of heavy components are mounted on the board can be summarized as follow:

For this type of the boards in general, the most effective way is to find the optimum support point vector. But when it is not allowed to change the support point vector, one or two dead weights can be helpful to reduce the

vibration characteristics. With 3 or more dead weights the number of design variables will increase to 9 and more. This make the Simplex method inefficient and more time consuming. Moreover the determination of the board vibration characteristic will be more lengthy as the number of attached mass increases. Utilising more than two dead weights is not thus recommended. In the previous section it was shown that for the boards having light components, the presence of a heavy component will produce an unsuitable region around its position and increase the extent of unsuitable region of the board considerably. The dead weights type of solution thus is not recommended for boards with no heavy mounted component.

CHAPTER 6

CONCLUSION AND FUTURE WORK

This research work consists of the following novel concepts which are aimed at reducing the board vibration characteristics of electronic circuit boards:

1. Consideration of the board flexure due to its vibration in addition to the vibration frequency in the objective function definition.
2. Development of the position characteristic of a board.
3. Consideration of the area under the curve of unsuitable elements of the board versus cutoff position characteristic as a generic objective function.
4. Development of the table of the suitable support point vectors

and contour plots of the unsuitable regions of the rectangular boards having the same Poisson's ratio.

5. Application of the dead weight in the minimization of the board vibration characteristics.

The effective use of optimization methods in order to reduce the board vibration characteristics and application of the proposed objective function in addition to the frequency of vibration are two distinguished feature of this research work. These issues and the recommendations for future work are discussed in this chapter.

6.1 Conclusion

This research work provides the theoretical means required to deal with the minimization of the vibration characteristics of electronic circuit boards in a considerably novel manner. The board flexure due to its vibration, and inertia force due to the mass of the board's elements, and mounted components were considered as the essential factors in evaluation of position characteristics of the board's elements. This lead to the definition of the cutoff position characteristic which is the minimum requirement for position characteristics of the board elements with respect to the expected service life of the board. This provide a useful way to deal with evaluation of the vibration characteristics of a board in the optimization procedure. Moreover the introduction of an objective function using the position characteristics of the

board but independent of cutoff position characteristics result in the minimization of the board vibration characteristics regardless of its cutoff position characteristic which is a more general approach. Development of the normalized position characteristics also gave a more generalization in the minimization procedure. With applying the normalized position characteristic, the objective function became independent of the aspect ratio and flexural rigidity of the board. By using this approach the table of the suitable support point vectors was introduced which paved the road to eliminate the accomplishment of the minimization procedure for a variety of electronic circuit boards which have light components or their components are distributed homogeneously over the board's lateral surface.

Different techniques for dealing with the problems of board vibration characteristics were introduced among them are: application of dead weight as a design variable when the board have a number of heavy components and the point supports of the board can not be changed, consideration of aspect ratio as a design variable at the early stage of designing of the electronic boards when the components and circuits are not mounted, and consideration of the shape and direction of the components which are mounted on the board in order to reduce their effect on of the board vibration characteristics.

To summarize the results, this research work shows that the vibration characteristics of any board with or without heavy mounted components, having constant dimensions or not can be minimized up to an admissible level

of the inertia and bending forces acting on the board circuits and its mounted components. This will result in a longer service life of the board.

In comparison with the previous work in which the frequency of the board free vibration was considered as objective function, it was shown that the proposed objective function is more powerful in terms of presenting a better efficiency and being able to investigate different area of the board. The functionality of this approach was verified by comparing the vibration characteristics of a number of sample boards supported by optimum support point vector to the ones of the same specifications but supported by the other support point vectors.

6.2 Recommendation

The reported research work was restricted to the investigation of free edge boards having four interior point supports. The objective function and the optimization procedure developed, however may be applied to other board boundary conditions and support or anchoring methods. The current work should be extended to investigate such other cases. Experimental and theoretical work is needed in order to corroborate the results of this research and develop it further:

To investigate the accuracy of the minimization of the board vibration characteristics through a set of the destructive tests which are performed by applying forced vibration with natural frequency of the board and measuring

the board service life. This kind of the tests should be carried out for a board which is supported by its optimum support point vector, and the result compared to the life expectancy of a similar board when it is supported by other support point vectors.

To investigate the relative effect of the flexure and inertia forces to the board's life time and set an appropriate value for weighting factors C_1 , and C_2 by applying the force vibration to a board having light component and comparing the results with those of another board having the same size but with heavy components mounted on it.

APPENDIXES

- Appendix 1: Table of Suitable Support Point Vectors and Suitable Region of the Boards Having Poisson's ratio of 0.5.
- Appendix 2: Provided program.

Appendix 1

Table of Suitable Support Point Vectors

and

Suitable Region of the Boards Having Poisson's Ratio of 0.5

For boards having Poisson's ratio equal to 0.5 the following table was developed in this research which contains the suitable support point vectors of the boards having different aspect ratios:

Table 1 Table of suitable support point vectors for boards having Poisson's ratio of 0.5.

No	ϕ	l	$F(l)$	λ^2	κ_{max}
1	1	{0.1740 0.2828 0.6987 0.7096}	10426	29.97	35.3
2	1.2	{0.1960 0.2220 0.6645 0.7299}	13313	23.80	57.4
3	1.4	{0.2852 0.2296 0.7462 0.7363}	15719	19.54	67.4
4	1.6	{0.2951 0.1147 0.5471 0.8350}	18927	17.13	72.2
5	1.8	{0.2422 0.2988 0.7002 0.7771}	27749	14.85	82.2
6	2	{0.2579 0.2681 0.7678 0.7246}	33683	14.85	91
7	2.2	{0.4043 0.1976 0.7211 0.7259}	37212	11.6	130
8	2.4	{0.1797 0.2652 0.5891 0.7929}	43949	11.26	134
9	2.5	{0.1595 0.2701 0.6719 0.7216}	69612	10.85	147

As a complimentary to this table, corresponding to every aspect ratio contour plot of the suitable region of a sample board or:

Area= 600 cms Flexural rigidity = 1838.6 N cm

Mass of the board: 222 g Radius of fastener/ width of the board: 0.012 is provided (plot (b) Figures 1 to 9). These graphs are served to help the designer to put the delicate components at the appropriate positions on the board lateral surface. A contour plot of the same board when supported by another support point vector is provided as well (plot (a) of the same figures). For this purpose the follows symmetrical support point vector is utilized:

$$l=\{0.2 \ 0.2 \ 0.8 \ 0.8\}$$

This is an ordinary support point vector that is more probable to be used in the mounting of the boards using rule of thumbs.

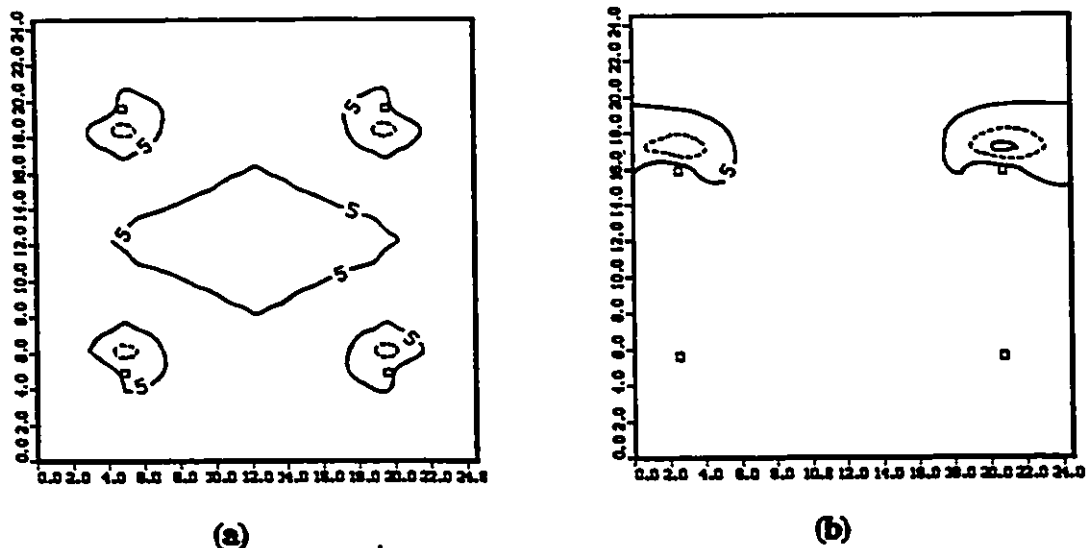
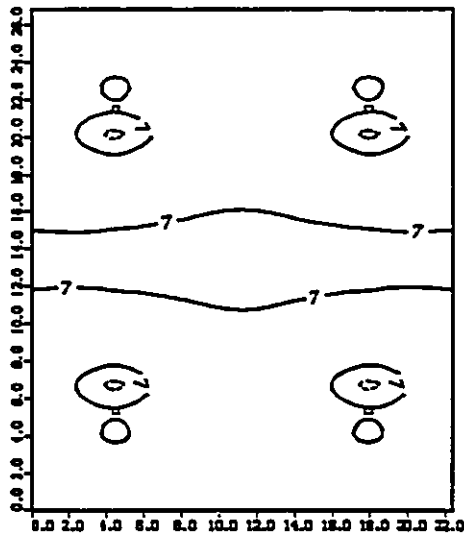
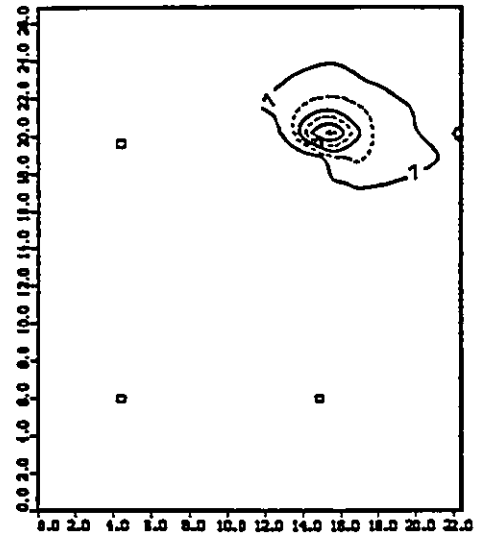


Figure 1 (a) Unsuitable region of a board of $\varphi=1$, supported by reference support vector. **(b)** Same board supported by $l=\{0.1740 \ 0.2828 \ 0.6987 \ 0.7096\}$.

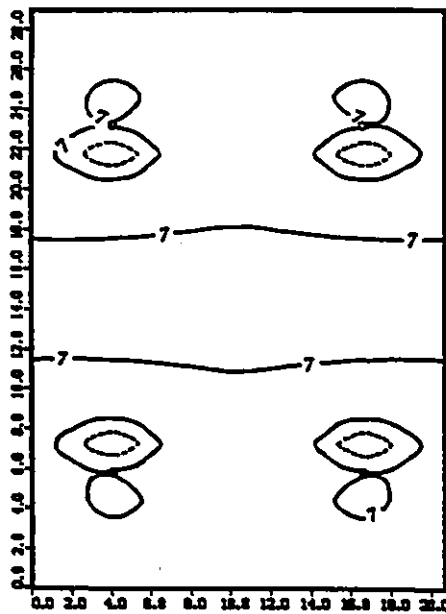


(a)

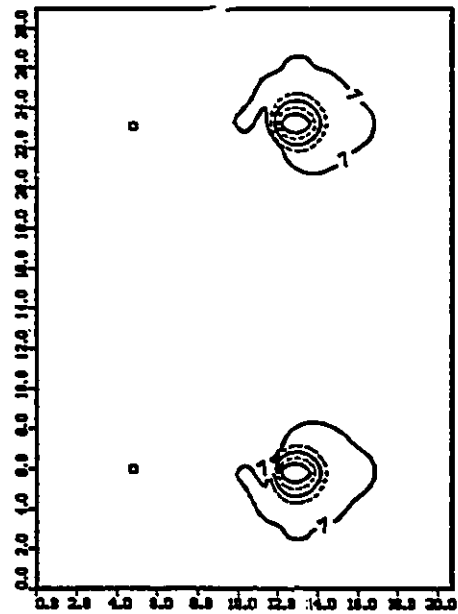


(b)

Figure 2 (a) Unsuitable region of a board of $\varphi=1.2$, supported by reference support vector. (b) Same board supported by $l=\{0.1960 \ 0.2220 \ 0.6645 \ 0.7299\}$.



(a)



(b)

Figure 3 (a) Unsuitable region of a board of $\varphi=1.4$ supported by reference support vector. (b) Same board supported by $l=\{0.2852 \ 0.2296 \ 0.7462 \ 0.7363\}$.

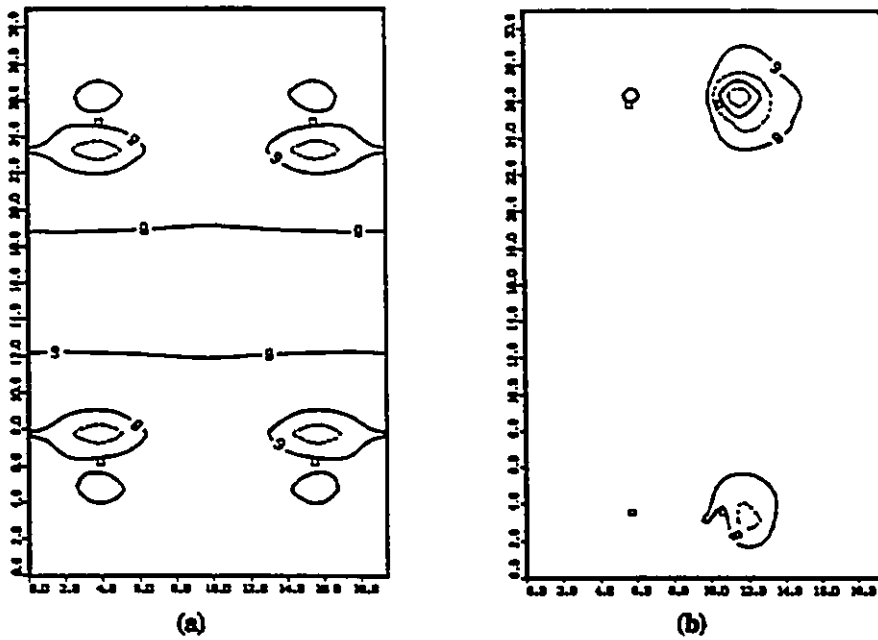


Figure 4 (a) Unsuitable region of a board of $\phi=1.6$ supported by reference support vector. (b) Same board supported by $l=\{0.2951 \ 0.1147 \ 0.5471 \ 0.8350\}$.

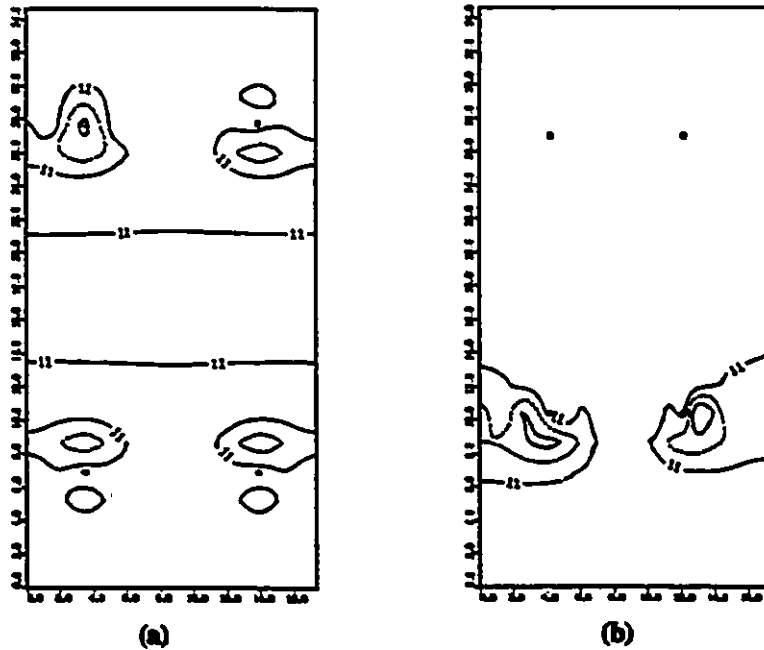


Figure 5 (a) Unsuitable region of a board of $\phi=1.8$ supported by reference support vector. (b) Same board supported by $l=\{0.2422 \ 0.2988 \ 0.7002 \ 0.7771\}$.

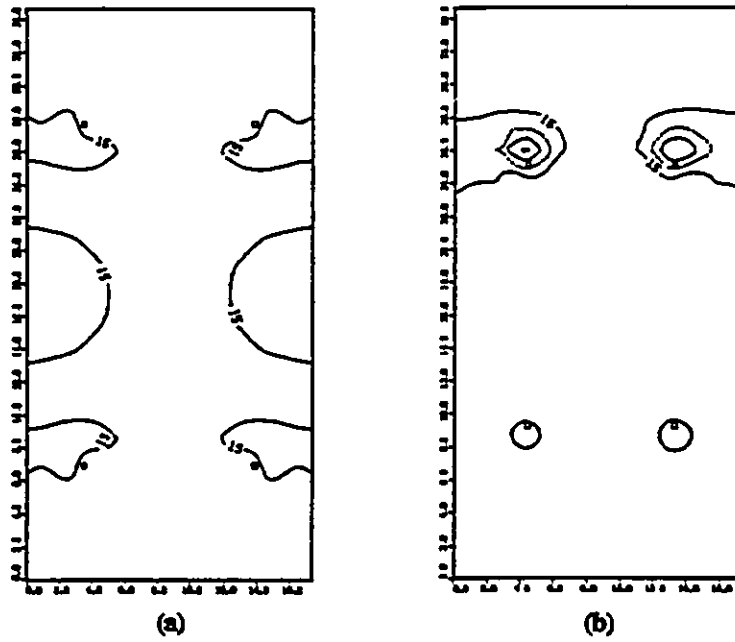


Figure 6 (a) Unsuitable region of a board of $\phi=2$ supported by reference support vector. (b) Same board supported by $l=\{0.2579\ 0.2681\ 0.7678\ 0.7246\}$.

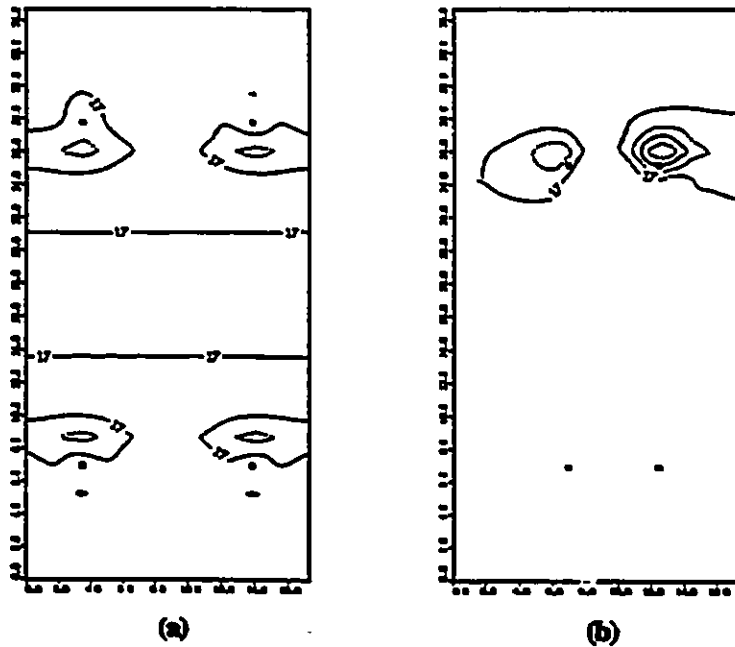


Figure 7 (a) Unsuitable region of a board of $\phi=2.2$ supported by reference support vector. (b) Same board supported by $l=\{0.4043\ 0.1976\ 0.7211\ 0.7259\}$.

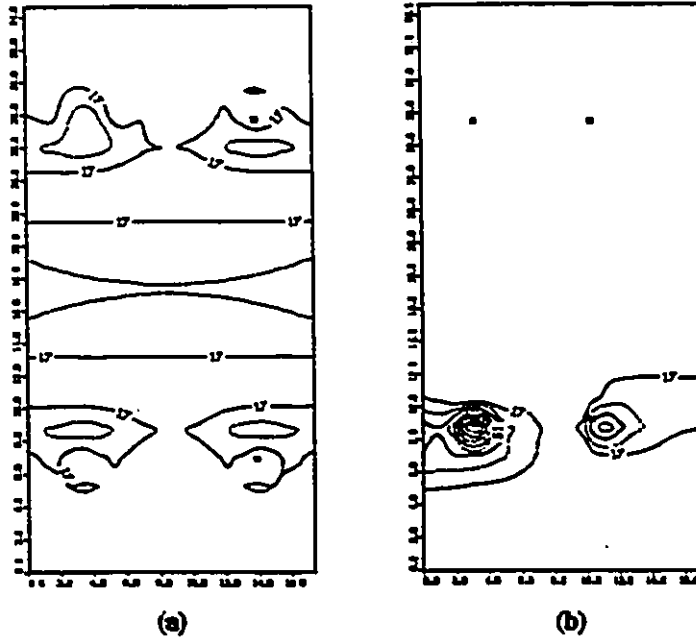


Figure 8 (a) Unsuitable region of a board of $\phi=2.4$ supported by reference support vector. (b) Same board supported by $l=\{0.1797\ 0.2652\ 0.5891\ 0.7929\}$.

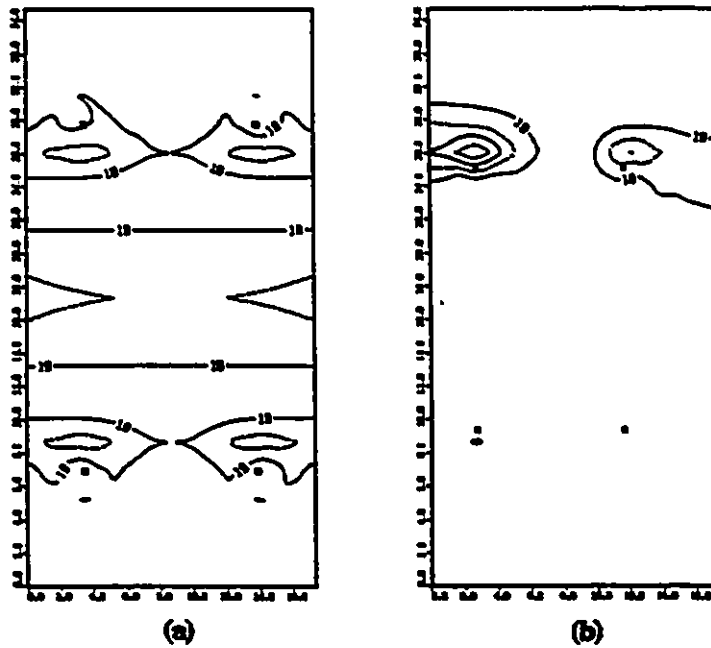


Figure 9 (a) Unsuitable region of a board of $\phi=2.5$ supported by reference support vector. (b) Same board supported by $l=\{0.1595\ 0.2701\ 0.6719\ 0.7216\}$.

Appendix 2

Provided Routine

A handy routine is provided to perform the minimization of the board vibration characteristics. It has all of the capabilities required to do the required minimization jobs utilizing different methods of optimization applied in this research i.e.:

- Simplex method
- Hooke and Jeeves method
- Powell's method

It is capable of handling different objective functions that are:

- Minimization of the area under the curve of unsuitable elements versus cutoff position characteristic
- Maximization of the board eigenvalue
- Minimization of the position characteristic of the board
- Minimization of the position characteristic of the board over 90% of its elements.

The design variables can be one of the following parameters:

- Support point vector in dimensionless form
- Dead weights
- Support point vector and aspect ratio

The program is capable of handling any kind of the board vibration

characteristics evaluation such as:

- Evaluation and production of the data required to draw the contour levels of the board's second derivative, position characteristic, and displacement
- Determination of the eigenvalue

The data required to run the program is received from an input file and result will be printed into an output file. The progress of the methods of optimization is reported after each iteration. The program consists of the following subroutines:

- Main program
- Input data
- Simplex method
- Hooke and Jeeves method
- Powell's method
- Objective function
- Data manipulator

The program is written in C and the software of the plate vibration analysis for rectangular plate having interior point support was written in Fortran and is used as a part of the objective function subroutine.

BIBLIOGRAPHY

1. Kunz, r. j., and Pitarresi, j. m., *"Analytical and Experimental Optimization of Support Locations for Vibrating Printed Boards"*, Ninth annual International Electronics Packaging Conference, San Diego, California, September, 1989, Pages 507-529.
2. Pitarresi, j. M. and Kunz, R. L., *A Simple Technique for Rapid Estimation of the Optimal Support Locations for a Vibrating plate*, Journal of Vibration and Acoustic, Vallum 114, Pages 112 -118, January 1992.
3. Gorman, D. J., *Free Vibration Analysis of Rectangular Plates*, Elsevier North Holeand, Inc., New York, New York, 1982.
4. Singal, R. K., Gorman, D. J., *A General Analytical Solution for Free Vibration of Rectangular Plates Resting on Fixed Supports and with Attached Masses*, Journal of Electronic Packaging, June 1992, Volume 114, Pages 239-245.
5. Singal, R. K., Gorman, D. J., *Effect of Attached Masses on Free Vibration of Rigid Point Supported Rectangular Plates*, AIAA Journal, March 1992, Volume 30, Number 3, Pages 853-855.
6. Kunz, R. J. and Pitarresi, J. m., *Analytical and Experimental Optimization of Support Locations for Vibrating Printed Circuit Board*, International Electronics Packaging Society Conference, 1989.
7. Inoue, Katsomi, Gakkai, Ronbunshue, and Chen, *Optimum Design of a Plate Based on Minimization of the Vibration Energy*, Japan Society of Mechanical Engineers, September 1990, Part C, Volume 57, Number 529,

Pages 2361-2366.

8. Inoue, Katsumi, Townsend, Dennis P., Coy, John J. , *Minimization of the Vibration Energy of Thin-Plate Structure*, ASME Design Engineering Division (publication), New York, NY, USA, 1992, volumn 44, Part 2, Pages 369-376.
9. Wong T-L, wang .J, and Stevens K. K., "Automated Hybrid Analysis of Surface Mounted Electronic Assemblies", Journal of Mechanical Design Transaction of ASME, 1993, Volume 115, Pages 435-440.
10. Gorman, D.j., "Free Vibration Analysis of Rectangular Plates with Symmetrically Distributed Point Supports Along the Edges, Journal of Sound and Vibration, 1980, Volume 73, Number 4, Pages 563-574.
11. Gorman, D.j., *An Analytical Solution for the Free Vibration Analysis of Rectangular Plates Resting on Symmetrically Distributed Point Support*, Journal of Sound and Vibration, 1981, Volume 71, Number 4, Pages 561-574.
12. Gorman, D. J., "A Note on the Free vibration of Rectangular Plates Resting on Symmetrically Distributed Point Supports", Journal of Sound and Vibration, Volume 131, Number 3, 1989, Pages 515-519.
13. Gorman, D. J., and Singal, R. K., "Analytical and Experimental Study of Vibrating Rectangular Plates on Rigid Point Supports", AIAA Journal, Volume 29, Number 5, 1991, Pages 838-844.
14. Gorman, D.J. and Singal, R.K., *Analytical and Experimental Study of Vibrating Rectangular Plates on Rigid Point Supports*, American Institute of Aeronautics and Astronautics Journal. May 1991, Volume 29, Number 5.
15. Venkateswara, G., *Vibration of Point Supported Plates*, Journal of Sound and Vibrations, 1973.-Volume 29, Number 3, Pages 387-391.
16. Raju, I.S. and Amba-Rao, C.L., *Free Vibration of a Square Plate Symmetrically Supported at Four Points on the Diagonals*, Journal of Sound and Vibration, 1983, Vallum 90, Number 2, Pages 291-297.
17. Narita, Y., *Free Vibration Analysis of Orthotropic Elliptical Plates Resting on Arbitrarily Distributed Point Supports*, Journal of Sound and Vibration, 1986, Volume 108, Number 1, Pages 1-10.
18. Laura, P.A.A and Cortinez, V.H., *Fundamental Frequency of Point-Supported Square Plates Carrying Concentrated Masses*, Journal of Sound and Vibrations, 1985, Volume 100, Number 3, Pages 456-458.

19. Timoshenko, S. and Woinowskt-Krieger, S., *Theory of plates and shell*, Second Edition, McGraw-Hill Book Company, Inc., U.S.A.,1987.
20. Gorman, D.J., *On Use of Dirac Delta Function for Representing Concentrated Forces and Moments in Structural Mechanics*, International Journal of mechanical Engineering Education, Volume 10, Number 3, Pages 177-183.
21. Rao S. S., "*Optimization Theory and Applications*", Wiley Eastern Limited, New Delhi, 1984.