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INVESTIGATION OF PRACTICAL STRATEGIES
FOR REDUCING INTERMODULATION EFFECTS
IN SATELLITE COMMUNICATION SYSTEMS

by

F. NUR ÖZMIZRAK

A thesis submitted to the School of Graduate Studies and Research
of the University of Ottawa
in partial fulfillment of the requirements
of Master of Science in Systems Science

Ottawa, Ontario, 1986



F. Nur Özmizrak, Ottawa, Ontario, Canada, 1986

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ISBN 0-315-30968-7

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ABSTRACT

This thesis investigates methods for solving the (carrier) frequency assignment problem which is concerned with avoiding or minimizing third-order intermodulation noise in communication systems. This noise is due to several carriers using a common power amplifier which has nonlinear characteristics. The work presented in this study focuses on satellite communication systems but is valid for any communication system with several "equal-level" carriers sharing a common nonlinear repeater.

Unlike most of the previous studies of this problem which were concerned with the intermodulation free case, our study primarily examines heuristic methods for obtaining frequency assignments that yield minimum worst-case intermodulation. Two heuristics are formulated and their performance is compared with a recently proposed method that has appeared in the literature. The results indicate that both of the proposed strategies provide significantly better execution-time performance.

ACKNOWLEDGEMENTS

I would like to express my deep sense of gratitude to my supervisor, Professor Louis G. Birta, for his advice, encouragement and valuable guidance throughout the course of my study and research. He was kind and patient and spent considerable amount of his precious time not only directing the research but also during the preparation of this thesis document.

I would also like to express my sincere thanks to Dr. X.T. Vuong for suggesting the topic of this thesis. His many helpful comments, his useful suggestions concerning practical strategies for its solution, and his ever willingness to explain the concepts involved in this thesis topic are very much appreciated.

I would like to thank Professors M.D. Atkinson, J. Urrutia, and A. Warburton for stimulating discussion on this thesis topic. Professor A. Warburton's many useful comments and suggestions during the early stages of this work are especially acknowledged.

Sincere thanks are also due to Professor T.I. Ören for his advice and constant encouragement throughout the course of my study.

Financial support from National Science and Engineering Research Council of Canada through grant number A8109 is gratefully acknowledged.

I would like to thank the Computer Science Department of the University of Ottawa for providing the necessary facilities throughout the course of my studies.

I would also like to thank my colleague Dr. G.H. Masapati for his invaluable help and constant support during the process of writing, and typing some of the tables in this thesis.

I am also extremely grateful to my husband, Murat Özmizrak, for his unwavering support and constant encouragement without which it would have been impossible to finish this thesis project. I will always appreciate his incredible tolerance and help. I would also like to express my deep gratitude for his excellent typing of this thesis.

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Chapter 1

INTRODUCTION

The objective of a communication system is to transmit information signals (waveforms) from one point to another (or possibly many other). The information can be video, voice, data, or a combination of these.

The points between which the information is to be sent may be located in close proximity or may be continents or even planets apart. During the information transmission, various anomalies and interference effects enter the system operation. The common name for these effects is "noise". Noise can be classified into three:

- i) Thermal noise,
- ii) Intermodulation noise (IM-product),
- iii) Interference.

Thermal noise is due to the random motion of electrons. It is present in any active device and would still be present even after the electronic equipment has been perfectly insulated from external interference. It is an inescapable background to all electronic processes. The higher the

temperature, the faster the electrons move, and the higher the power of the thermal noise. The effect of thermal noise is reduced by employing a low (thermal) noise amplifier (LNA) at the first amplification stage of a receive system.

Intermodulation products (noise) are generated when two or more carriers are passed through a nonlinear device. Depending on whether the associated nonlinear device is active or passive, intermodulation is classified as active or passive respectively. Passive intermodulation products are generally small in power and very often can be ignored. Active intermodulation products are often of concern because active devices are much more nonlinear, particularly when they are operated near or at their saturation regions to nearly fully or fully utilize their available RF (radio frequency) powers. In satellite communication, the available RF power of a high power amplifier (HPA) is fixed and expensive. It is therefore desirable to operate as close to the saturation region as possible. An effective way to achieve this is to assign frequencies of the carriers properly so that the number of products falling into the carriers is minimized, as locations (frequencies) of the products depend on locations of the carriers.

Interference, the third type of noise, is generated when other systems in the environment are interfering with the system under consideration. In this case, it is desirable to construct a suitable system (low antenna sidelobe gains,

low signal sidelobe power spectra, and so on) in appropriate environment that accomplishes communication operation with minimal interference.

The quality of a received signal reflects the performance of the system. For a digital signal, it is specified in terms of the bit-error-rate (BER) or equivalently the probability that a received bit is erroneous (a one is received as a zero or vice versa), and for an analog signal, it is the signal-to-noise ratio (SNR). There is a one-to-one relationship between any of these two specifications to the ratio of the carrier power to the total noise power (CNR) at the input of the demodulator. The greater this carrier-to-noise ratio (CNR), the better is the quality of the signals that are received.

Let

- P_C : carrier power,
- P_T : thermal noise power,
- P_{IM} : intermodulation noise power,
- P_I : interference noise power, and
- P_N : the total noise power.

Then, CNR, carrier-to-noise ratio is given by:

$$\text{CNR} = \frac{P_C}{P_N} \quad \dots(1.1)$$

where

$$P_N = P_T + P_{IM} + P_I$$

In general P_T and P_I are fixed for a given communication system. Hence the term P_{IM} in the total noise P_N is the only quantity available for adjustment in trying to increase CNR. We note furthermore that when P_C is increased, P_{IM} is increased more rapidly than P_C . Then it is desirable to keep P_{IM} at a level as low as possible even when P_C is high. This can be achieved by properly arranging the carrier frequencies.

Intermodulation problem in various communication systems has been the subject of intensive research since 1950. Babcock [3] was the first to analyze this problem for radio systems. His work was concerned with the frequency of occurrence of intermodulation products. He obtained intermodulation-free (IM-free) operating channels by controlling IM-products through proper channel selection. His investigations provided optimum frequency assignments without third-order IM-product for operating channels up to 9 (i.e. $K \leq 9$) and suboptimum assignment for 10 operating channels ($K = 10$).

Robinson and Bernstein [29] approached the problem using the concept of difference sets (a good survey of the concept of difference sets is given by Marshall Hall [20]). Using the constructive procedure of perfect difference sets as explained by Singer [33], Robinson and Bernstein [29] generated the IM-free assignments for up to 24 channels (i.e. $K \leq 24$). However, these assignments have not been shown to be optimum.

Gardner [13] has presented the frequency assignment problem in terms of a series of interesting mathematical games.

It is desirable to analyze the intermodulation minimum (IM-minimum) assignment problem in terms of bandwidth available. For, it may not be cost-effective or possible to have tremendously large bandwidth to yield IM-free assignment.

In the IM-minimum problem, there are specified number of operating channels (K) and available channels (bandwidth N). That is, K frequency carriers are assigned to N channels in such a way that in the resulting assignment, the maximum number of IM-products falling on any operating channel will be minimum. Johansen and Paulsen [18] carried out an investigation on the behavior of the carrier-to-noise ratio by selectively spacing the carriers so as to avoid IM-products as much as possible.

Fang and Sandrin [9] also studied the problem of IM-free assignment with the help of difference triangles. Exhaustive enumeration method was used to obtain the optimum third-order IM-free assignments for 2 to 11 operating channels (i.e. $K \leq 11$). However, as the number of operating channels increases this method rapidly becomes computationally intractable.

Because of the impracticality of obtaining IM-free optimum assignments for large K , and because of the poor bandwidth utilization that such assignments yielded, recent

theoretical studies have been directed towards finding the lower and upper bound values on:

- a) the total IM-products, and
- b) the maximum number of products falling on any assigned channel (carrier)

for the optimum channel assignment for a given (K, N) pair. Maxemchuk and Schiff [23] and Hirata [15] have carried out theoretical analyses in this area. (The results from Hirata [15], however, cannot be of practical use as he used 2 instead of 4 as the power ratio between the two third-order IM-products; see page 29).

Eng and Stern [8] considered a slightly different problem (the passive intermodulation problem). They proposed a practical method for solving the minimum order and type of IM-products that fall in a specified frequency band. It involves forward and backward tree searches. These are also discussed extensively in [22, 24, 25].

Atkinson et al. [1,2] generated close upper and lower bounds on N for IM-free assignments using the concept of difference sets [29]. They have also given suboptimum IM-free solutions for $K \leq 101$. Previously suboptimum solutions were known only for $K \leq 24$.

Recently a practical and effective method has been proposed by Okinaka et al. [26] for the IM-minimum problem. Their approach provides a means for obtaining an approximate

solution for any given K and N . The solution generated by the proposed algorithm does however depend on the assumed initial channel allocation. The algorithm is basically made up of a series of insertion and deletion operations of carriers into the available channels. Therefore it is referred to as the Sequential Insertion and Deletion (SID) algorithm throughout the thesis.

In this thesis, formation of IM-products are briefly discussed. To avoid or minimize them, the frequency assignment problem (FAP) is studied in detail. A very efficient algorithm is given for solving the FAP. An improved version of the SID algorithm is also given.

The original objective of this thesis was to obtain optimum IM-free channel allocations for any K value. However, there are two problems with optimum IM-free investigation; namely:

- i) It is not possible to solve it for large values of K because of computation time,
- ii) Even if it is solved, the bandwidth utilization would probably be very poor and solution therefore has no practical value.

Because of the above stated problems, the goal was altered by fixing N to solve an IM-minimum frequency assignment problem. It turned out unfortunately that this problem is no less difficult than the problem of IM-free, in terms of time complexity.

It was, therefore, decided to direct the research effort towards ways of obtaining suboptimum assignments for any arbitrary values of K and N .

The proposed algorithm, DELIN, in this thesis uses improved criteria in channel selectivity when deletion and insertion operations are carried out. This results in better stopping criteria for the proposed algorithm, SID, by Okinaka et al. [26]. This thesis also includes the development of initial channel allocation method for the SID.

Okinaka et al. [26] concluded that the influence of the initial channel allocation on the result is insignificant. However, the same is not true for the computational time. In this thesis, it is shown that a "good" initial assignment decreases the computational time significantly.

A very practical and efficient heuristic algorithm, IN, for solving the IM-minimum FAP is proposed in this thesis. This has resulted in fairly good reduction in computational time (i.e. at least by a factor of 3 with respect to SID algorithm). The results are also satisfactory compared to lower and upper bound values, some of the known optimum values, and also to the assignments given in Okinaka et al. [26].

This thesis also includes some IM-minimum and/or IM-free optimum assignments which are not known previously. Those assignments are obtained for a given value of K , along with

N, where N starting from K, up to IM-free bandwidth value for that K. The largest (K,N) pairs experimented are (20,24) and (9,40) in this case.

The thesis also contains a discussion on the use of backtracking technique in obtaining the IM-minimum frequency assignments for successive values of K.

The principle of communication satellite is briefly described in Chapter 2, since IM-noise (IM-products) is an important factor in designing a satellite communication system.

Relevant characteristics of IM-products are described along with a few examples for some orders in Chapter 3.

Typical problems such as the optimum and suboptimum IM-free frequency assignment problem, and IM-minimum frequency assignment problem are explained in Chapter 4.

The solution procedures for the previously mentioned frequency assignment problems (FAP) are discussed, and practical and efficient heuristic algorithms (DELIN, IN, INU) are presented as solution techniques to the IM-minimum FAP in Chapter 5.

Computational results obtained from program EXHAUST, DELIN, IN, and INU are summarized in Chapter 6. The performance of the proposed heuristic algorithms and the comparative study with the Okinaka et al. [26] algorithm are presented in this Chapter.

Finally Chapter 7 contains the concluding remarks.

All the computational results are also presented in tabular form in the Appendix.

Chapter 2

SATELLITE COMMUNICATION

2.1. SATELLITE COMMUNICATION SYSTEMS

A Satellite Communication system has two major components; namely one or more earth stations and communication satellites. Such a system can have several different forms [11] as shown in Figure 2.1. System I is a ground-ground configuration which has an uplink from a ground-based earth station to a satellite, and a down-link from the satellite back to a ground station. System II is a ground-crosslink-ground system which has a satellite crosslink between two satellites prior to down-link transmission. System II does not yet exist. Worldwide communications between remote earth stations in different hemispheres can be carried out by such crosslinks. System III is a ground-user-relay system involving earth stations, near-earth users, and satellites. System III does not exist yet, but probably will exist in early 1990's.

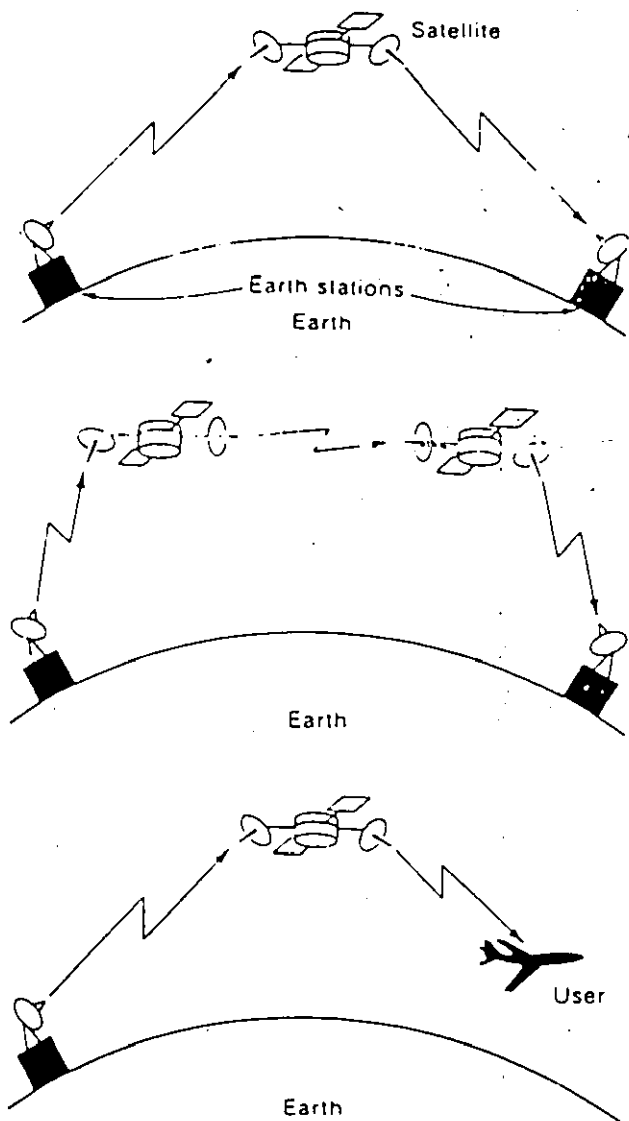


Figure 2 . 1. Satellite systems. I. Ground-ground, II. Ground-crosslink-ground, III. Ground-user relay. [11]

2.2. EARTH STATIONS

An earth station is a transmitting and/or receiving power station operating in conjunction with an antenna subsystem. Earth stations are usually classified as being either large or small depending upon the level of their radiated power, and their antenna sizes. Larger stations may use antenna dishes as large as 30 meters in diameter. Smaller stations may use antenna dishes as small as 0.6 meter in diameter.

A given earth station may be designed to operate as a transmitting station only, a receiving station only, or as both.

In a transmitting earth station, the baseband information signals (voice, television, and data) which provide the input to the system, originate from various possible sources. These signals are first multiplexed (MUX), encoded, and (frequency or phase) modulated onto intermediate frequency (IF) carriers. These are then translated to radio frequencies (RF) for high power amplification (HPA) and transmission. The transmit earth station antenna is finally used to direct the energy of the carriers in the form of electromagnetic waves to the direction of the satellite. For the uplink path to exist, the transmit earth station must locate within a region on Earth. This region is called the transmit area and its size (can be as large as 1/3 of the Earth) and shape are governed by the design of the satellite's receive antenna subsystem. The first component

of a receiving earth station, after a receive antenna, is a low noise amplifier (LNA) which is then followed by a frequency translator to IF. At the IF, the specific uplink carriers that are to be received are first separated, and then demodulated to baseband. The baseband is then decoded, demultiplexed (DEMUX) and transferred on to the destination. For the downlink path to exist, the receive earth station antenna must be pointed to the satellite and must locate within the receive area which is governed by the design of the satellite's transmit antenna subsystem. The transmit antenna and receive antenna are the same.

Figure 2.2 shows block diagrams for both transmitting and receiving earth stations [11].

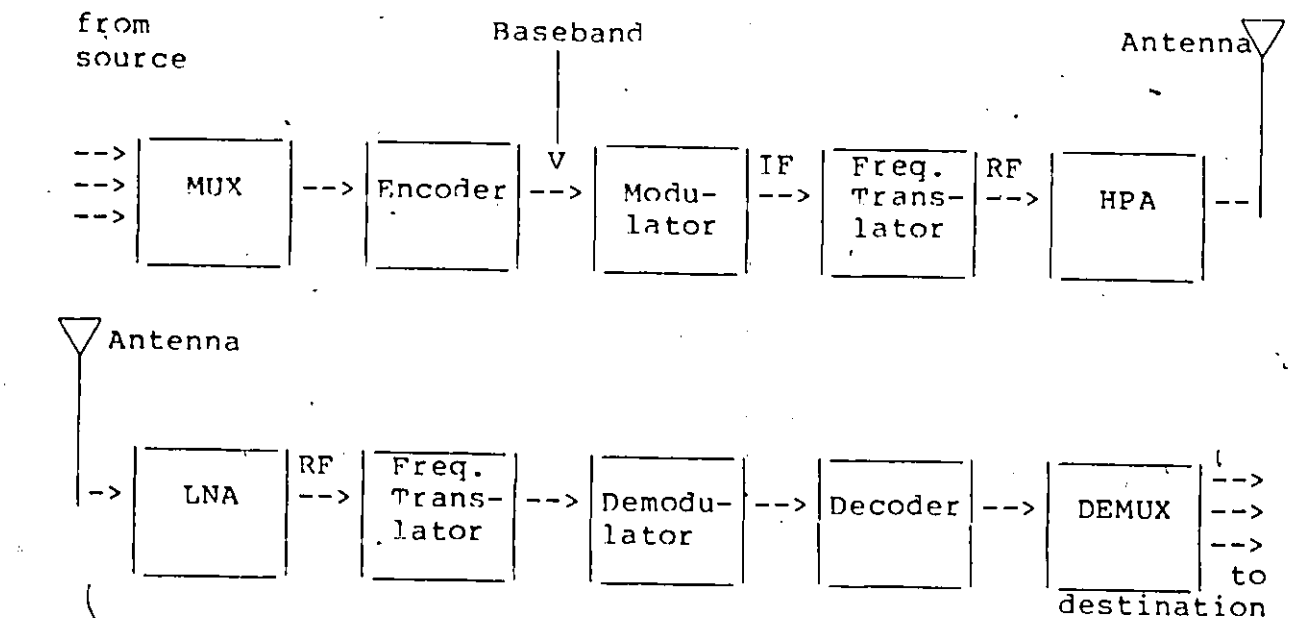


Figure 2.2. Earth Station Block Diagram

2.3. COMMUNICATION SATELLITES

A communication satellite typically costs about 20-50 million dollars to manufacture and about the same amount to place high up in the sky. At the present technology, it has a design lifetime of 10 years.

To avoid pointing adjustment (tracking) of earth station antennas and to be able to provide communication service for 24 hours/day, all communication satellites are stationary with respect to any point on the Earth. To be so without requiring any energy, the satellites must be located in the geostationary orbit which is on the equatorial plane and about 36,000 Km above the Earth.

A communication satellite acts like a radio relay in the sky. As shown in Figure 2.3, the receive antenna subsystem is used to receive carriers sent from the Earth. The low noise amplifier is used to perform amplification without adding too much (thermal) noise to the transmission. The carriers are then separated by the demultiplexer for power amplification. They are then combined together and beamed down to the receive area on the Earth.

A satellite transponder refers to satellite components that perform amplification and frequency translation of carriers associated with a certain frequency band, typically 36 MHz or 54 MHz wide. Frequency translation is necessary to avoid interference between uplink carriers and downlink carriers.

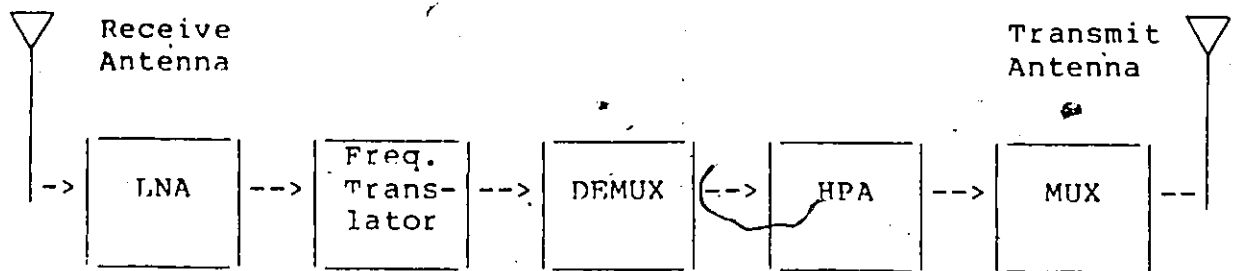


Figure 2.3. Satellite Block Diagram

The frequency spectrum used in a satellite link is governed by ITU, an organ of the United Nations. For the fixed satellite service, it consists of 4/6 GHz (C-band), 12/14 GHz (K_u -band), and 20/30 GHz (K_a -band). The first number in each case refers to the frequency of the downlink, and the second number refers to the frequency of the uplink. The bandwidth of the C-band and the K_u -band is 500 MHz and that of the K_a -band is 1000 MHz. Using the C-band is subject to terrestrial microwave interference as this band is also shared with terrestrial microwave transmission. One may encounter, using the K_u -band and particularly the K_a -band, outage or impairment due to rain. There is not yet any satellite operating at the K_a -band presently in North America.

Most satellites have more than one transponder, and bandwidth handled by a transponder is different from one satellite design to another (e.g. ANIK-C has transponders with a bandwidth of 54 MHz, ANIK-D has those with 36 MHz).

How this bandwidth is utilized depends on the earth-station equipment. Suppose 36 MHz bandwidth is available, and voice signals are desired to be transmitted. Earth voice signals typically have a bandwidth of 45 KHz. In this case, 800 (i.e. $36000/45$) channels (slots) will be available.

Assuming 800 channels are available and 100 carriers are to be assigned (i.e. $N=800$, $K=100$), the relation among the carrier, its frequency bandwidth, the channel, and the total number of available channels (bandwidth N) is shown in Figure 2.4.

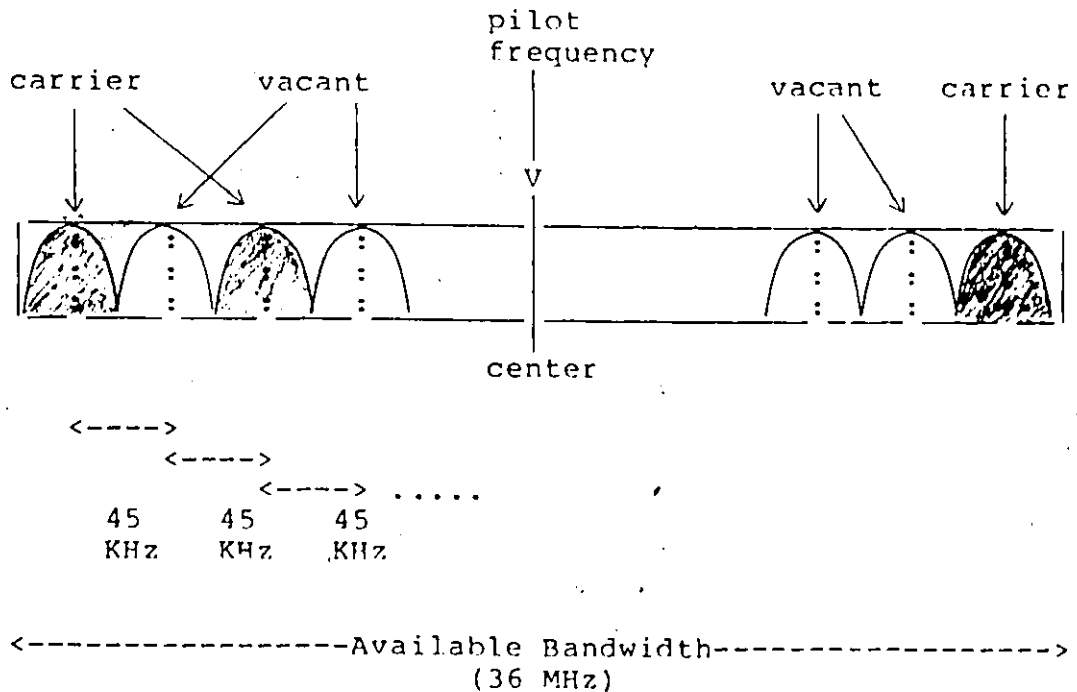


Figure 2.4. Bandwidth Representation of a Transponder with voice channels 45 KHz apart

($N = 800$ and $K = 100$)

For the purpose of illustration the frequency assignment presentation has been simplified as follows: Each slot has been given a number starting with 1. This is called "normalization". Each carrier is represented simply by an arrow (the fact that it has a "width" is suppressed). The 1-st and N-th slots are always assigned; the remaining 98 carriers (i.e. 100-2) are assigned in between. The resulting configuration is shown in Figure 2.5.

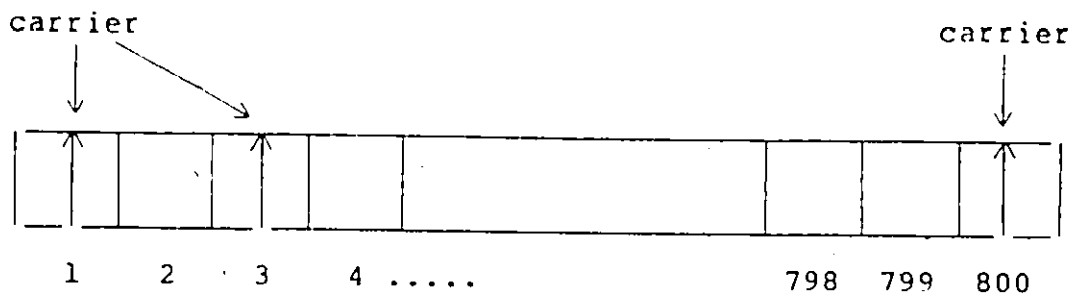


Figure 2.5. Simplified Bandwidth Representation of
a Transponder; $N = 800$ and $K = 100$

The considerations in this thesis are confined to the specialized case where all the carriers have the same power and the same bandwidth.

At the transponder, the carriers tend to modulate one another because of the nonlinear characteristics of the satellite's high power amplifier (HPA). In order to avoid the IM-products causing interference, the transmission may be operated below the full power so that no carrier

saturates the amplifier. This might not be desirable in terms of the efficiency of satellite capacity since it will decrease the carrier-to-noise ratio (Equation 1.1). Another solution is to arrange the carrier frequencies so that the resulting output will be IM-free. But this may require a large frequency bandwidth and therefore may not be cost effective. A modified approach could then be to arrange the carrier frequencies within the available bandwidth to minimize the IM-noise over the bandwidth.

The proper arrangements of carrier frequencies, namely the optimum/suboptimum channel allocation (frequency assignment) is the subject of investigation in this thesis and is studied in detail in the following chapters.

2.4. MODULATION, ENCODING, AND DECODING

As in any communication system the basic components of satellite systems are transmitter and receiver, other than the satellite. These will be briefly described in this section along with the figures of the complete system.

In communication links, information is transmitted by "modulating" it onto electromagnetic carriers. They are then propagated to the receiver. At the receiver, the information is demodulated (extracted) from the carrier to complete the link. In "analog" modulation systems, the information waveforms (voice, video, sometimes teletype,

etc) are modulated directly from the source onto the carrier. At transponders, the frequencies of the radio link (RF frequencies) are made to carry a lower intermediate frequency (IF) centered at 70 MHz. The higher frequency is said to be modulated by the information that is to carry. A demodulation process is then required to recover the data. The electronic circuit which does the modulation and demodulation is called a MODEM (a combination of the first letters of "MODulation" and "DEModulation" (Figure 2.6).

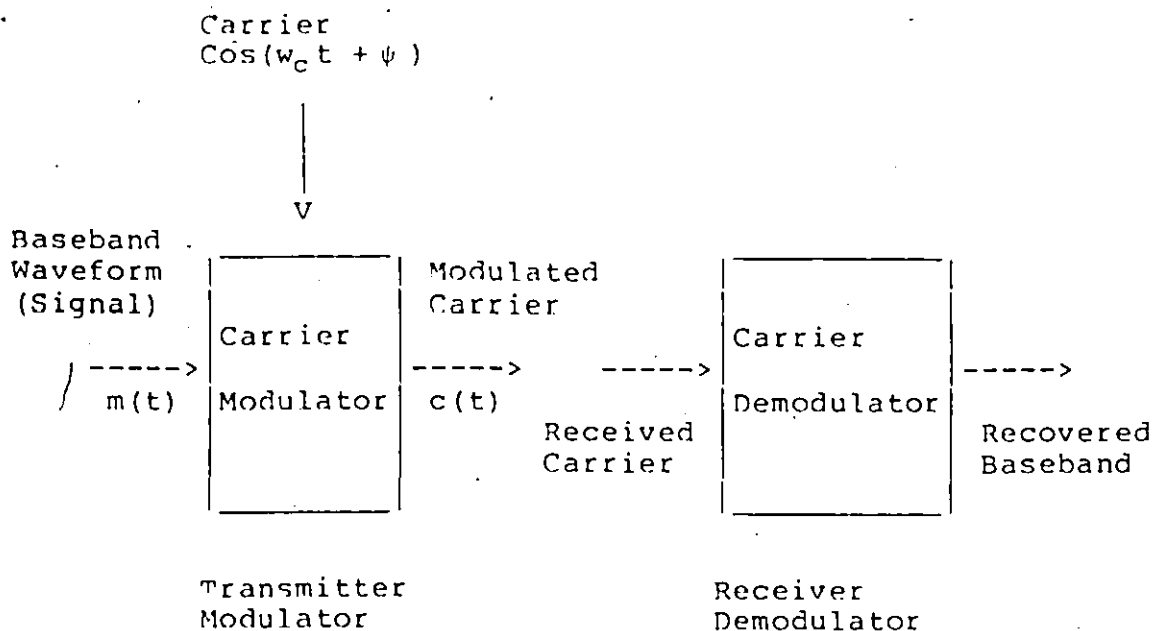


Figure 2.6. Analog Communication - Basic Components

In "digital" communication systems, source information is first converted into sequences of data bits (binary symbols), which are then "encoded" onto RF carriers for

carrier modulation. At the receiver, the demodulated waveform are "decoded" back into the desired data sequence, from which the source information is retrieved. The electronic circuit which does the conversion of analog signals into a digital form before transmission is called CODEC (a combination of the first letters of "COde" and "DECode" (Figure 2.7).

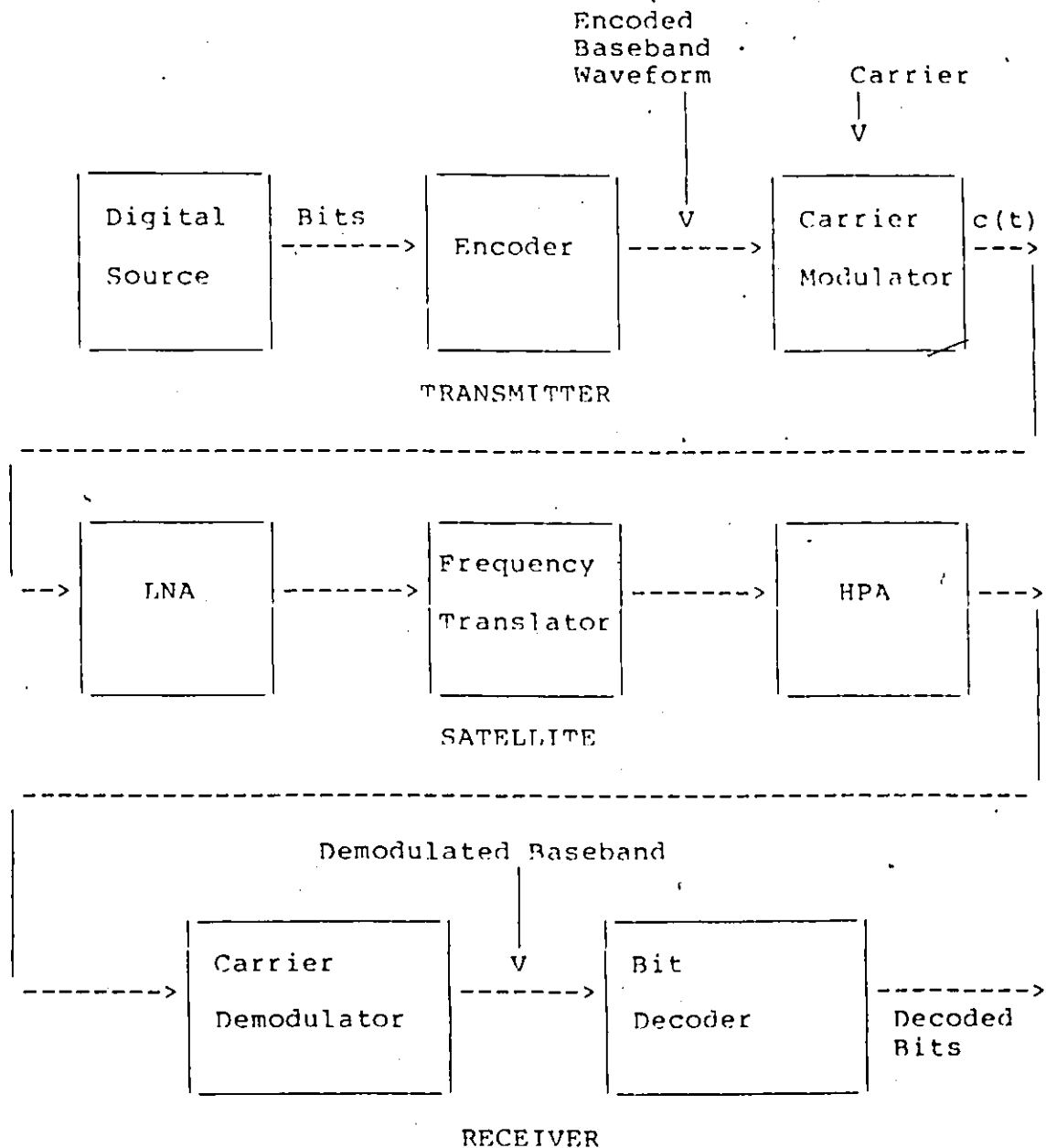


Figure 2.7. Digital Communication Block Diagram along with the Satellite

Chapter 3

INTERMODULATION PRODUCTS

Intermodulation products (IM-products) are produced when two or more carriers (spectral components) are passed through a nonlinear device. These products can be a significant source of communication impairment if their power levels are high (comparable with the carriers' power level) and fall into the carriers' channels (frequency bands occupied by the carriers). To avoid the impairment, one can take either or both of the following actions:

The first action is to reduce the power levels of the IM-products by operating the nonlinear device at its "linear" region (see Figure 3.1). That is, one does not make full use of the available (saturated) output power that the nonlinear device can supply. For a power amplifier such as a Traveling Wave Tube Amplifier (TWTA), a nonlinear device which performs the last amplification on carriers before the carriers radiate as electromagnetic waves in space, the available power is large and such an operation means a tremendous waste of power. For a power amplifier onboard a satellite, the available power is also very expensive, such

an operation is definitely not desirable.

The other action is to reduce the number of products falling into the carriers' channels by properly assigning frequencies (channels) to the carriers, as the frequencies of the products depend on the frequencies of the carriers constituting them. This action can be taken only if there are sufficient freedom of moving the carriers around. That is, if the nonlinear device's usable bandwidth is sufficiently larger than the carriers' combined bandwidths.

The thesis work is associated with the second action, and this chapter provides an introduction to intermodulation analysis for nonlinear devices, intermodulation properties and terminologies relevant to the thesis work.

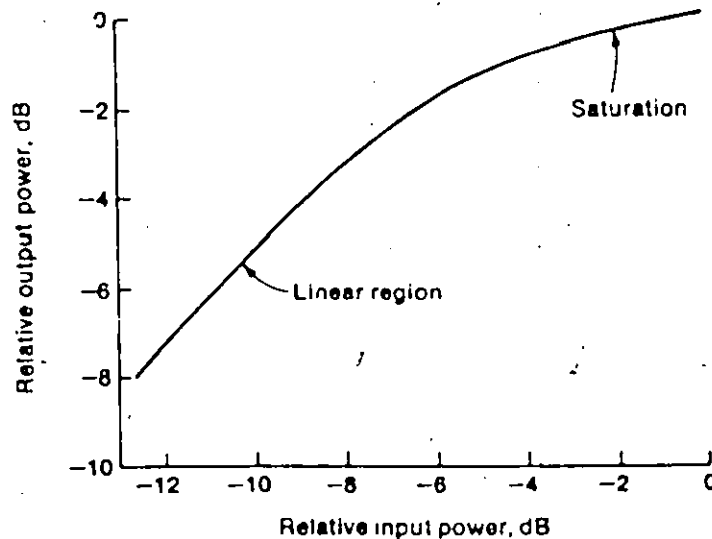


Figure 3.1. TWTA Input-Output Power Characteristic
(Normalized Power)

3.1. NONLINEAR DEVICES

Nonlinear devices are normally classified into three categories with respect to memory [37], namely:

- i) Memoryless nonlinear devices,
- ii) Effectively memoryless nonlinear devices,
- iii) Nonlinear devices with memory.

Memoryless nonlinear devices respond to input signals without delay. They can be characterized by

$$v(t) = f[u(t)] \quad \dots(3.1)$$

where $u(t)$ is the input voltage and $v(t)$ is the output voltage. If $u(t)$ consists of K narrowband carriers (whose bandwidth is much smaller than carrier frequency), then

$$u(t) = \sum_{i=1}^K A_i(t) \cos[w_i t + \psi_i(t)] \quad \dots(3.2)$$

which can be put into the following form:

$$u(t) = A(t) \cos[wt + \psi(t)] \quad \dots(3.3)$$

where $A(t)$ is the amplitude, w is an arbitrary frequency, and $\psi(t)$ is phase of the composite carrier.

Through the use of first order Chebyshev transform [4] and the bandpass assumption (the condition where the bandwidth of a nonlinear device is very small with respect to operating frequency; the nonlinear device only allows the fundamental components to pass through), $v(t)$ can then be

written as:

$$v(t) = G(A) \cos[\omega t + \psi(t)] \quad \dots(3.4)$$

where $G(A)$ is called the AM/AM characteristics of the memoryless nonlinear device.

Memoryless nonlinear devices do not exist in reality since, by definition, they must respond instantaneously to input signals and furthermore must introduce no phase shift at their outputs.

TWTAs are of effectively memoryless nonlinear type. These devices have both AM/AM and AM/PM transfer characteristics. That is, if the input $u(t)$ is given by Equation 3.2, then the output is:

$$v(t) = G(A) \cos[\omega t + \psi(t) + F(A)] \quad \dots(3.5)$$

where $F(A)$ is the AM/PM transfer characteristics which represents an amplitude-dependent phase shift in the output signal.

The third class of amplifiers, nonlinear with memory, are not of interest in this thesis.

3.2. ANALYSIS OF INTERMODULATION PRODUCTS

If input to a memoryless nonlinear device $u(t)$ consists of K narrowband carriers as described by Equation 3.2, and the device is characterized by Equation 3.1, then by expanding

$v(t)$ to a multiple Fourier series, the output of the device can be written in the following form:

$$v(t) = \sum_{\substack{k_1, k_2, \dots, k_K \geq -\infty \\ (k_1 + k_2 + \dots + k_K) = 1}}^{\infty} A(k_1, k_2, \dots, k_K) \cos \left[\sum_{i=1}^K k_i (\omega_i t + \psi_i(t)) \right] \quad \dots (3.6)$$

where $A(k_1, k_2, \dots, k_K)$ are the Fourier coefficients, and k_1, k_2, \dots, k_K are integers that must satisfy the constraint:

$$\sum_{i=1}^K k_i = 1$$

The output consists of original signals plus infinite IM-products.

IM-products are categorized by three properties:

- i) center frequency (location),
- ii) power spectrum, and
- iii) spectrum shape.

(i) The center frequency of each IM-product depends on the locations of the carriers by the following equation:

$$F(k_1, k_2, \dots, k_K) = \sum_{i=1}^K k_i f_i \quad \dots (3.7)$$

where K is the number of input frequencies (carriers), f_i is the center frequency of the i -th input signal. The coefficients k_i are positive/negative integers.

Power amplifiers are bandpass by nature and design: their

bandwidths are very small with respect to their operating frequencies. That is, they only let the fundamental products to pass through and filter out the other products. The fundamental products are those whose frequencies are in the operating frequency ranges of the bandpass devices, i.e. they are those satisfying the following constraint:

$$\sum_{i=1}^K k_i = 1 \quad \dots (3.8)$$

The order of each IM-product is defined by

$$\text{order} = \sum_{i=1}^K |k_i| \quad \dots (3.9)$$

(ii) The power of an IM-product is independent of the carrier frequencies but is a function of the carrier input power and the characteristics of the nonlinear device. IM-product level (power) decreases rapidly with increasing order. For example in most cases, fifth and higher order nonlinearities generate IM-products whose power is sufficiently small to be neglected [5]. In satellite communication systems, it is normally sufficient to consider only third-order IM-products [26]. Hence fifth and higher order effects are, for the most part, ignored throughout this thesis.

(iii) The spectrum shape of an IM-product depends on shapes of the carriers and is obtained by a process known as convolution. Details of this procedure are beyond the scope of this presentation.

3.3. EXAMPLE OF INTERMODULATION PRODUCT (3 CARRIERS)

Consider the case where input consists of three carriers:

$$u = A_1 \cos \theta_1 + A_2 \cos \theta_2 + A_3 \cos \theta_3$$

$$\text{with } \theta_i = \omega_i t + \psi_i(t) \quad i=1,2,3$$

and $\psi_i(t) = 0$ in the interest of simplicity.

A nonlinear device (memoryless and bandpass) with input u and output f can always be described by a power series [30]; hence

$$f(u) = c_1 u + c_2 u^2 + c_3 u^3 \quad \dots (3.10)$$

Here the real coefficients c_j are determined by the particular device and its operating environment.

Using the appropriate trigonometric identities (and eliminating terms which do not pass through the zonal filter), the output can be simplified to $v = Q + P$ where:

$$\begin{aligned} Q = & [c_1 A_1 + (3/2)c_3 (A_1^2 A_2^2 + A_1^2 A_3^2) + (3/4)c_3 A_1^3] \cos \theta_1 \\ & + [c_1 A_2 + (3/2)c_3 (A_1^2 A_2 + A_2^2 A_3^2) + (3/4)c_3 A_2^3] \cos \theta_2 \\ & + [c_1 A_3 + (3/2)c_3 (A_1^2 A_3 + A_2^2 A_3^2) + (3/4)c_3 A_3^3] \cos \theta_3 \end{aligned}$$

and

$$\begin{aligned} P = & (3/4)c_3 [A_1^2 A_2 \cos(2\theta_1 - \theta_2) + A_1 A_2^2 \cos(2\theta_2 - \theta_1) \\ & + A_1^2 A_3 \cos(2\theta_1 - \theta_3) + A_1 A_3^2 \cos(2\theta_3 - \theta_1) \\ & + A_2^2 A_3 \cos(2\theta_2 - \theta_3) + A_2 A_3^2 \cos(2\theta_3 - \theta_2)] \\ & + (3/2)c_3 A_1 A_2 A_3 [\cos(\theta_1 + \theta_2 - \theta_3) \\ & + \cos(\theta_1 + \theta_3 - \theta_2) \\ & + \cos(\theta_2 + \theta_3 - \theta_1)] \end{aligned}$$

Here Q denotes the wanted output signals (carriers), whereas P denotes the IM-products. Observe that in the first term of P (the one whose coefficient is $(3/4)c_3$) the terms are of the form $(2A-B)$, whereas in the second term of P (the one whose coefficient is $(3/2)c_3$) the terms are of the form $(A+B-C)$, where A , B , C represent one of the θ_j 's which are frequencies. These terms are referred to in the literature [36] as "third-order IM-products" and they are dominant. Moreover $(A+B-C)$ terms are called [23] "third-order triple IM-products", and $(2A-B)$ terms are called "third-order double IM-products". For convenience in the sequel, we shall frequently abbreviate these to simply "triple product" and "double product" respectively.

Note that there are no fifth-order or higher order products and that c does not have any effect at the output. In general, the even terms in $f(u)$ do not have any effect on the odd products; and if the order of $f(u)$ is m , then there will not be any product whose order is greater than m .

When the amplitudes of the carriers are equal ($A_1 = A_2 = A_3$), the amplitude of triple products is twice the amplitude of double products, hence there is a four times factor in power level [40].

From Equation 3.6, the coefficients k_i for Example 3.3 can be stated as $k_1=1$, $k_2=1$, $k_3=-1$ for triple, and $k_1=2$, $k_2=-1$ for double products. Both types of IM-products satisfy the bandpass constraint (3.8). The order of triple and double products can be verified as three using the Equation 3.9.

3.4. NUMBER OF INTERMODULATION PRODUCTS IN FUNDAMENTAL ZONE
AND IN EACH CHANNEL.

The total number of fundamental IM-products for each order is independent of the locations (frequencies) of the K carriers. For some selected products, the numbers are given in Table 3.1.

Form	Order	Total Number of IM-product
2A-B	3	$K(K-1)$
A+B-C	3	$K(K-1)(K-2)/2$
3A-B-C	5	$K(K-1)(K-2)/2$
A+2B-2C	5	$K(K-1)(K-2)$
A+B+C-D-E	5	$K(K-1)(K-2)(K-3)(K-4)/12$

Table 3.1. Number of Possible IM-Products of Various Forms (assuming K input signals)

However the number of IM-products falling into channel of each carrier is directly related to the location (frequency) of the K carriers. This number can be determined by forming all possible combinations of IM-product of the specified form that is of interest, and checking to see how many are falling into one of the carrier channels.

To set the stage for the presentation in the remaining chapters of this thesis, we provide below a number of basic

definitions and related results. These are directly motivated by our previous discussions.

Definition 3.1. ((K,N) Assignment)

A (K,N) assignment, g , is a set of K distinct frequencies, (f_1, f_2, \dots, f_K) , selected from the set $\{1, 2, 3, \dots, N\}$ with the condition that 1 and N are always selected.

With regard to this definition, we note the following:

i) In reality, a frequency assignment involving K carriers has the form $F_i = F_1 + n_i \Delta F$ with $i=1, 2, \dots, K$ where ΔF is the smallest possible frequency increment and n_i are distinct integers in the range $[0, (N-1)]$ with $n_1=0$ and $n_K=(N-1)$. Here $(F_K - F_1)$ is the bandwidth. Let $f_i = [(F_i - F_1) / \Delta F] + 1 = n_i + 1$ (where F_i denotes a real frequency, and f_i is its integer representation), then the assignment can be characterized by the set of K f_i 's which are integers in the range $[1, N]$ and it is this representation that is commonly used in frequency assignments studies [8] (e.g. Figure 2.5 and in the basis for Definition 3.1).

ii) Our specific problem here is the case where all carrier frequencies are of the same type (i.e. bandwidth and power are the same).

Definition 3.2. (Third-order Triple Product IM-Source)

The ordered triplet of distinct frequencies (f_a, f_b, f_c) taken from the (K, N) assignment, q , is a third-order triple product IM-source (or simply a t-source) if

$$f_a + f_b - f_c = f_p$$

and $f_p \in q$. In this circumstance, q is said to contain a third-order triple IM-product on f_p .

Remarks: Two t-sources, which differ only in the relative positions of the first and second elements are considered to be identical. The third element of a t-source is referred to as its neg-constituent.

We note the following with respect to Definition 3.2:

- i) f_p cannot equal either f_a or f_b because otherwise, f_c would equal either f_a or f_b which violates the distinctness requirement;
- ii) If $f_p = f_c$ (i.e. the triple product on f_p is coincident with its neg-constituent), then the triple product on f_p is said to be unsymmetric;
- iii) If $f_p \neq f_c$ then the triple product on f_p is said to be symmetric.

Lemma 3.1

If the (K,N) assignment, g , contains a symmetric third-order triple product on f_p , i.e. $(f_a + f_b - f_c = f_p)$ with $f_p \neq f_c$, then g also contains symmetric third-order triple products on f_a , f_b , and f_c .

Proof

By assumption $f_a + f_b - f_c = f_p$ with all f_a, f_b, f_c, f_p distinct. Thus we can write:

$$\begin{aligned} f_a + f_b - f_p &= f_c \\ f_p + f_c - f_a &= f_b \\ f_p + f_c - f_b &= f_a \end{aligned}$$

from which the result follows from Definition 3.2.

O.E.D.

Definition 3.3. (Third-order Double Product IM-Source)

The ordered pair of distinct frequencies (f_b, f_c) taken from the (K,N) assignment, g , is a third-order double product IM-source (or simply a d-source) if

$$2f_b - f_c = f_p$$

and $f_p \in g$. In this circumstance, g is said to contain a third-order double IM-product on f_p .

We note the following with respect to Definition 3.3:

- i) f_p cannot equal either f_b or f_c because otherwise, f_c would equal f_b which violates the distinctness requirement;
- ii) Because f_c and f_p can be interchanged, the existence of a double product on f_p always implies the existence of a second double product (namely on f_c);
- iii) Because $f_p = 2f_b - f_c = f_b + f_b - f_c$ and rearrangement gives $f_p + f_c - f_b = f_b$, we observe the existence of an unsymmetric triple product on f_b .

Lemma 3.2

A (K, N) assignment, g , contains a third-order double IM-product if and only if it contains an unsymmetric third-order triple IM-product.

Proof

Suppose the double product $2f_b - f_c = f_p$ exists in g . Then we can write:

$$f_b + f_b - f_c = f_p$$

whose re-arrangement yields

$$f_p + f_c - f_b = f_b$$

where f_p, f_c , and f_b are distinct. Thus, by definition, g , contains an unsymmetric triple product on f_p .

Suppose g contains the unsymmetric triple product:

$$f_p + f_c - f_a = f_b \text{ for which } f_c \neq f_b.$$

Because unsymmetry implies $f_a = f_b$, we can write $2f_b - f_c = f_p$

Thus by definition, g contains a double product on f_p .

Q.E.D.

Corollary 3.2

If the (K,N) assignment g has no third-order triple IM-product, then it has no third-order double IM-product.

Our concern in this thesis is primarily with integrating the characteristics of (K,N) assignments. The characteristics are related to the distribution of third-order triple and double IM-products on frequencies within the assignment itself. Lemma 3.2 is especially valuable because it provides a means for establishing the location of third-order double IM-products once all third-order triple IM-products have been identified.

In particular we can proceed as follows:

$$\begin{aligned} \text{Let } S_3(g) &= \{(f_a, f_b, f_c) : (f_a, f_b, f_c) \text{ is a t-source for } g\} \\ S_2(g) &= \{(f_b, f_c) : (f_b, f_c) \text{ is a d-source for } g\} \end{aligned}$$

Assume now that $S_3(g)$ is available: The set $S_2(g)$ can, as a consequence of Lemma 3.2, be constructed by identifying each source $(f_a, f_b, f_c) \in S_3(g)$ which corresponds to an unsymmetric triple IM-product and placing into $S_2(g)$ the ordered pairs (f_c, f_b) and (f_c, f_a) .

Now let g be a (K,N) assignment containing the frequencies (f_1, f_2, \dots, f_K) . For $1 \leq i \leq K$, we use the notation $N_3(f_i; g)$ to denote the number of third-order triple products falling on f_i as a consequence of the (K,N) assignment g . We then define the worst triple intermodulation for g as:

$$WIM_3(g) = \max_{1 \leq i \leq K} \{N_3(f_i; g)\} \quad \dots(3.11)$$

Similarly, we define the total triple intermodulation for g as:

$$TIM_3(g) = \sum_{i=1}^K N_3(f_i; g) \quad \dots(3.12)$$

The quantities $N_2(f_i; g)$, $WIM_2(g)$, and $TIM_2(g)$ are defined in an analogous way within the context of third-order double IM-products. We note furthermore that, as a consequence of Lemma 3.2, the following is true:

$$\sum_{i=1}^K N_2(f_i; g) = TIM_2(g) = 2 N_u \quad \dots(3.13)$$

where N_u is the cardinality of the set $S_3(g)$ where

$$S_3(g) = \{(f_a, f_b, f_c) : (f_a, f_b, f_c) \text{ is an unsymmetric } t\text{-source for } g\}$$

The following example is presented to illustrate the calculation of the third-order triple and double IM-products of a given (K,N) assignment. We construct all possible products for both types, and then illustrate the use of Lemma 3.2 to determine the double products.

Example 3.2. (Calculation of Third-order Triple and Double IM-Products for a (K,N) Assignment)

Consider the (4,5) assignment (i.e. $K=4$, $N=5$) with carrier frequencies (1,3,4,5). From Table 3.1, the possible number of triple products is $K(K-1)(K-2)/2 = 12$ and the possible number of double products is $K(K-1) = 12$ in this case. These are given in Table 3.3.

Triple Product		Double Product	
Combinations (i.e. A+B-C)	Frequency (Eq'n 3.7)	Combinations (i.e. 2A-B)	Frequency (Eq'n 3.7)
1 + 3 - 4	0	2 x 1 - 3	-1
1 + 4 - 3	2	2 x 3 - 1 *	5
3 + 4 - 1	6	2 x 1 - 4	-2
1 + 3 - 5	-1	2 x 4 - 1	7
1 + 5 - 3 *	3	2 x 1 - 5	-3
3 + 5 - 1	7	2 x 5 - 1	9
1 + 4 - 5	0	2 x 3 - 4	2
1 + 5 - 4	2	2 x 4 - 3 *	5
4 + 5 - 1	8	2 x 3 - 5 *	1
3 + 4 - 5	2	2 x 5 - 3	7
3 + 5 - 4 *	4	2 x 4 - 5 *	3
4 + 5 - 3	6	2 x 5 - 4	6

Table 3.2. All Possible Third-order Triple and Double IM-Products for Example 3.2.

The number of third-order IM-products of each type falling onto carrier frequencies is given with Table 3.3.

Input Channel f_i	Number of IM-Products falling on f_i	
	Triple	Double
1	0	1
3	1	1
4	1	0
5	0	2

Table 3.3. Number of Third-order Triple and Double IM-Products falling on the Carrier Frequencies of Example 3.2.

We illustrate Lemma 3.2, by first identifying with a * the unsymmetric triple products in the first column of Table 3.2; i.e.:

$$i) \quad 1 + 5 - 3 = 3$$

$$\text{and } ii) \quad 3 + 5 - 4 = 4$$

These can be manipulated into:

$$i) \quad 2 \times 3 - 5 = 1 \quad \text{and} \quad 2 \times 3 - 1 = 5$$

$$ii) \quad 2 \times 4 - 5 = 3 \quad \text{and} \quad 2 \times 4 - 3 = 5$$

which are the four third-order double products that appear in the third column of Table 3.2.

The IM-products produced in a TWTA due to input/output amplitude nonlinearity and conversion effects (AM/AM and AM/PM) are predominantly due to third-order distortion for input powers up to saturation. If the number of third-order products falling onto each carrier frequency is known, the assignment can be evaluated with respect to the certain defined measures. One such evaluation measure of a given assignment in terms of baseband performance is called IM-advantage.

The IM-advantage of a (K,N) assignment, q , in terms of triple intermodulation is defined as:

$$IMA_3(q) = 10 \log_{10} [WIM_3(q^0) / WIM_3(q)] \quad \dots(3.14)$$

where q^0 represents a (K,K) assignment (which is unique).

The quantity $IMA_2(q)$ is defined in an analogous way within the the context of third-order double IM-products.

(K,K) assignments have been extensively studied [40] and in particular, it is known that the "center frequency" in such an assignment acquires the highest number of third-order IM-products.

As a consequence the following formulas can be used to determine the values of $WIM_3(q^0)$ [38] and $WIM_2(q^0)$:

$$\begin{aligned} \text{WIM}_3(g^0) &= (3K^2 - 10K + 8)/8 && K \text{ even} \dots (3.15) \\ &= [(3K^2 - 10K + 9) + 2(-1)^{(K+1)/2}]/8 && K \text{ odd} \end{aligned}$$

$$\begin{aligned} \text{WIM}_2(g^0) &= (K - 2)/2 && K \text{ even} \dots (3.16) \\ &= [(K - 2) - (-1)^{(K+1)/2}]/2 && K \text{ odd} \end{aligned}$$

Chapter 4

ANALYSIS OF THE FREQUENCY ASSIGNMENT PROBLEM (FAP)

The frequency assignment problem (FAP) is concerned with the effective use of the available bandwidth in communication systems in which the multiple carriers use a common power amplifier and hence give rise to IM-products. The amount of IM-noise falling onto carrier frequency slots can be reduced (and possibly eliminated) by properly arranging the carrier frequencies within the available bandwidth. The problem is therefore concerned with the best choice of the K frequencies which comprise a (K, N) assignment, g .

The FAP can be approached from either of two points of view, namely,

- a) Total elimination of IM-noise on the carrier slots,
- b) Toleration of some IM-noise on the carrier slots.

In the first case (the IM-free channel allocation) there is generally a very inefficient utilization of the available bandwidth, hence this approach has very limited practical relevance, particularly when there is a need to accommodate a reasonably large number of signal channels [15].

Solutions corresponding to the second approach are typically obtained by searching for a signal channel allocation for which the worst-case intermodulation is within a permissible level. Dramatic improvements in bandwidth utilization can usually be achieved without serious degradation.

In the discussion of this Chapter both approaches to the FAP are examined from the point of view of typical solution approaches.

Historically, most of the research efforts on the FAP has focused on obtaining IM-free channel allocation [2], [3], [9], [13], [23], [29], [32]. Any (K,N) assignment g for which $WIM_3(g)=0$, provides an IM-free solution to the FAP. If a (K,N) assignment g is IM-free and there exists no (\bar{K},N) assignment with $\bar{K} > K$ that is IM-free, then g is said to be optimal. An IM-free assignment that is not optimal is suboptimal. Typically the problem is switched by choosing a particular K and then attempting to establish a least N for which an IM-free assignment can be found.

Babcock [3] solved the optimum IM-free FAP for K in the range $3 \leq K \leq 10$, and also provided a suboptimum solution for $K = 11$. Later similar results were obtained by Fang and Sandrin [9]. These results along with the ones generated in this thesis can be found in Chapter 6 and Appendix.

Practical (i.e. efficient) methods for determining optimum IM-free solutions for arbitrary K has not yet been

developed. Exhaustive search remains as the only assured method, but it is computationally impractical (even the case of $K = 12$ has not yet been solved). However work with suboptimum approaches has progressed and in particular we note the recent work of Atkinson et al. [1, 2].

In the following sections, various techniques for investigating the IM-free FAP are given.

4.1. THE DIFFERENCE TRIANGLE FRAMEWORK

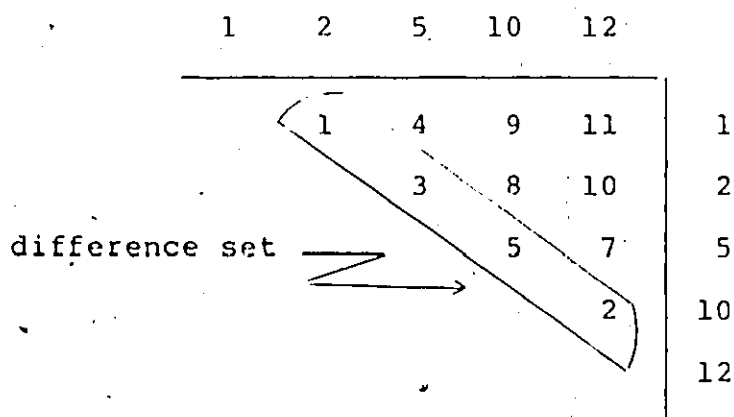
A difference triangle is usually represented by a difference set whose constructive procedure can be found in [33]. A procedure for constructing several difference triangles from a difference set is shown in detail in [29]. The concept of a difference set also provides a connection with related problems in coding [29] and graph theory [13].

The problem of finding a third-order IM-free frequency assignment can be shown to be equivalent to the search for a difference triangle with distinct integer elements [9]. If, in addition, such a difference triangle has the least possible largest element, then the corresponding frequency assignment is optimal [9].

The main purpose of studying difference triangles in this case is to recognize whether a given assignment is third-order IM-free or not. To verify that an assignment is third-order IM-free (not necessarily optimum), one can

construct the associated difference triangle and examine it for repeated entries in the triangle. If no repeated entries exist, then this assignment is IM-free. However, this procedure is not sufficient for determining optimality of the assignment, since a means for ensuring that the difference triangle has the least largest element has no known solution.

Shown below is a typical form of a difference triangle which is associated with the (5,12) assignment of (1,2,5,10,12). The constructive procedure for a difference triangle is as follows: the frequencies of the assignment are listed along the top and right edge of the triangle. The j -th entry in the i -th row with $j > i$ is obtained as $f_j - f_i$.



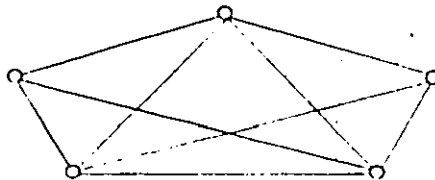
Notice that there are no repeated entries in this difference triangle; this confirms that there are no third-order IM-products. The diagonal elements of a difference triangle which has distinct entries, represent a difference set [9]. Hence the set (1,3,5,2), is a difference set. Note that it

is also possible to construct a difference triangle from a given difference set.

4.2. THE GRACEFUL GRAPH LABELLING FRAMEWORK —

The concept of graceful graphs provides an alternate framework for representing the IM-free FAP. The graceful labelling problem, however, is itself not an easy problem to solve [6]. Some interesting recent results that have direct relevance to the IM-free FAP can be found in [32]. In this section we illustrate the equivalence between the IM-free FAP and the problem of constructing a complete graph with graceful labelling.

A complete graph is a graph in which each pair of distinct vertices is joined by an edge; e.g.

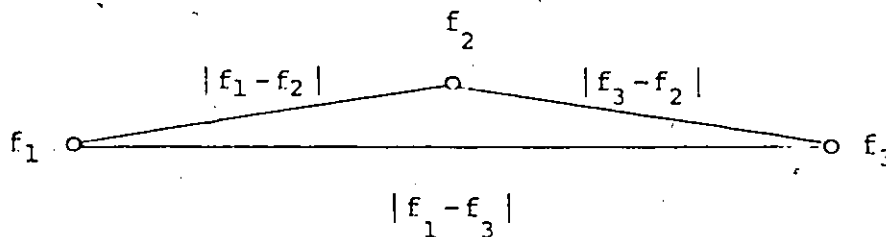


When it is possible to assign distinct non-negative integers to the vertices of a graph without loops so that:

- i) the largest such label is less than or equal to the number of edges e , and
- ii) the absolute differences between the pair of labels at the end points of each edge are all distinct,

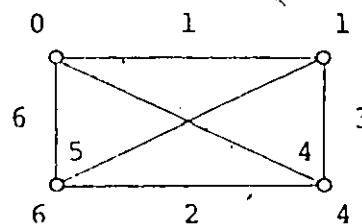
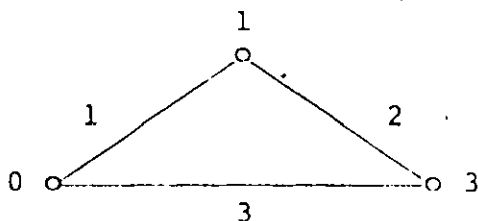
then such a graph is said to be graceful [14], [32].

A frequency assignment can be represented by a complete graph whose vertices denote the carrier frequencies. Consider, for example, the three-carrier case shown below:



If all differences are distinct, then the assignment is IM-free.

Numerical examples of complete graceful graphs for three and four vertices are given below:



The results of graceful labelling process yield the set of vertices $\{0, 1, 3\}$ and $\{0, 1, 4, 6\}$ respectively. To represent them as frequency assignments, we add one to each entry and obtain $(1, 2, 4)$ and $(1, 2, 5, 7)$ respectively. These assignments are known to be optimum IM-free. If we extend the discussion to more carriers, we can observe that graceful labelling is not possible. For example, for five carriers, it is known that the optimum IM-free assignment is $(1, 2, 5, 10, 12)$. By transforming this assignment into a

labelling, we obtain (0,1,4,9,11). Note that a complete graph with five vertices has ten edges (i.e. $e = 10$). Thus it is clear that graceful labelling is not possible for such a graph in which the smallest possible, largest label is eleven. This violates the condition that the largest label must be less than or equal to e for graceful labelling.

Golomb [14] showed that it is not possible to obtain a complete graceful graph for more than four vertices ($K > 4$). He posed at the same time the question of finding a labelling of a complete graph with K vertices, using the smallest possible labels so that all of the differences (i.e. edge labels) would be distinct [32]. These labellings are called Golomb's rulers. A Golomb ruler corresponds to the optimum IM-free FAP. However, solutions for this problem have been obtained only for small values of K by exhaustive search.

The IM-free FAP however is not of particular relevance to the objectives of this thesis, and therefore will not be pursued further.

4.3. THE IM-MINIMUM FREQUENCY ASSIGNMENT PROBLEM (FAP)

A (K,N) assignment g which provides an optimum IM-free solution to the FAP has the property that

$$WIM_3(g) = 0.$$

Such optimum IM-free solutions, however, have limited practical value. This is because the cost of such communication systems is directly related to the bandwidth requirement, N , and the value of N becomes unrealistically large as K increases ($N = O(K^2)$ [1,2]). As a result, practical solutions to the FAP are formulated in terms of permitting some amount of intermodulation effects. The problem then becomes one of finding (K,N) assignments that utilize the available bandwidth N in as efficient a way as possible without the constraint of totally eliminating intermodulation effects.

The common measure used for evaluating the quality of a (K,N) assignment, g , is based on worst-case effects. For this purpose, we define the maximum or worst-intermodulation (worst-IM) associated with the (K,N) assignment, g , as:

$$WIM(g) = \max_{1 \leq i \leq K} N(f_i; g) \quad \dots (4.1)$$

$$\text{where } N(f_i; g) = [a_3 N_{33}(f_i; g) + a_2 N_{22}(f_i; g)]$$

where a_3 and a_2 are parameters that provide a means for blending the effects of third-order triple and double IM-products. (Typically the values chosen for a_3 and a_2 are 1 and 0.25 respectively, since these reflect their relative contribution in terms of signal power.) The carrier frequency (or frequencies) which yield(s) the maximum value is (are) called the "worst frequency(ies)".

The IM-minimum frequency assignment problem (FAP) can then

be stated as follows: Among all possible (K,N) assignments (with K and N fixed) find an assignment g^* for which $WIM(g^*)$ has the least possible value. In general g^* is not unique.

Solving the IM-minimum FAP rapidly becomes computationally intractable as N increases because the only known solution methods rely on exhaustive search. Consequently, approximate solutions obtained via heuristic procedures provide the only practical alternative. The discussions in the remaining sections of this thesis are, in fact, devoted to this topic.

As an illustration of the nature of the computational task involved in an exhaustive search process, consider the case where $K=10$, $N=56$. The number of possible (K,N) assignments is in the order of 10^9 . For each such assignment, 360 third-order triple products need to be evaluated (i.e. sums of the form $(A+B-C)$) and 90 third-order double products need to be evaluated (i.e. sums of the form $(2A-B)$).

The best known reported work with heuristic procedures is that due to Okinaka et al. [26], and their approach is outlined in Chapter 5. The investigations in this thesis include experiments with their method as well as with some variations on it.

Related studies in the literature have been concerned with establishing Upper and Lower Bounds on $WIM(g^*)$ where g^* is a solution to the IM-minimum FAP [23]. Our discussions in the sequel examine the approximate solutions generated by

heuristic procedures with which we experiment within the context of these estimates. In Chapter 6, we will compare our results with known bounds on the optimum solution.

Chapter 5
SOLUTION PROCEDURES FOR THE
FREQUENCY ASSIGNMENT PROBLEM (FAP)

In this Chapter various procedures for obtaining both true and approximate solutions for the IM-minimum FAP are discussed. Included are a description of the Okinaka et al. heuristic [26] and two newly proposed alternate approaches whose solution speed is generally superior.

5.1. IM-MINIMUM FAP VIA EXHAUSTIVE SEARCH

True solutions to IM-minimum FAP can be obtained by exhaustive search in which all possible channel allocations are examined. Such an approach, however, becomes infeasible when the number of carriers and the bandwidth ratio become large because the number of possible channel allocations becomes unmanageable (as in optimum IM-free case).

One of the program packages (called EXHAUST) developed in the context of the research project described in this thesis is based on exhaustive evaluation for finding the true solutions for both the IM-minimum and IM-free FAPs. The

basic input to this program is the number of carriers K . The procedure in the program starts with a g^0 assignment and finds the true IM-minimum assignment(s) for the cases (K,N) with $N = K+1, K+2, \dots$ until an IM-free assignment is found. This sequential approach therefore generates, for a given K , the IM-minimum assignments for each bandwidth (i.e. N) up to the limiting case where an IM-free solution is possible.

For any particular (K,N) , there are, in general, multiple solutions to the IM-minimum FAP (i.e. multiple (K,N) assignments, g^* , which have the same smallest value for $WIM(g^*)$). In order to refine the reported results, a secondary criterion was therefore applied in the EXHAUST program among the equivalent IM-minimum solutions. This criterion was based on examining total intermodulation and choosing only those solution candidates that yielded a smallest value for $TIM(g^*)$. The use of total intermodulation as a secondary criterion in the search procedure is used extensively in other phases as outlined in the sequel.

Certain limited results have been obtained with the EXHAUST program and these correspond to selected (K,N) combinations with $K \leq 20$ and $K \leq N \leq 40$. In the literature [e.g. 5], optimum IM-free assignments are available for up to 11 carriers (i.e. $K \leq 11$), whereas IM-minimum assignments have never (to the author's knowledge) appeared in the literature. These results can be found in Chapter 6.

5.2. IM-MINIMUM FAP VIA BACKTRACKING

Backtracking has proved to be an efficient solution strategy for a variety of problems. Its applicability within the IM-minimum FAP context is briefly examined in this section.

Generally, in order to apply Backtracking, the desired solution to the problem must be expressible as an n -tuple such as (x_1, x_2, \dots, x_n) where x_i 's are chosen from some finite set. Often the problem calls for finding one or all solution vectors which satisfy an objective function [16]. In the case of the FAP, the desired solution is an assignment expressible as a K -tuple (f_1, f_2, \dots, f_K) where f_i 's are chosen from the finite integer set $(1, 2, \dots, N)$. Furthermore, solving the IM-minimum FAP calls for finding vectors which minimize the worst-intermodulation.

The underlying idea in Backtracking is to build up the solution vector (i.e. assignment) one component (i.e. carrier) at a time, and to use bounding values (i.e. worst-IM values for $K-i$, $i = 1, 2, \dots, (K-2)$, for fixed N) to test whether the vector being formed has any chance of success [16]. The objective is to identify partial solution vectors which cannot possibly lead to an optimum solution by avoiding fruitless computations that would otherwise be carried out in an exhaustive search.

Figures 5.1.(a), (b), and (c) illustrate the application of the backtracking method for solving the IM-minimum FAP for

each of the cases where $N = 7, 8,$ and 9 respectively. In each case, the value within a node is the worst-IM for the assignment that is comprised of the carriers that lie on branches of the path to that node. We assume that the tree is constructed from top to bottom, and left to right. The starting branch is always labelled $1,N$ because of the underlying constraint. The root node has no worst-IM value because none can be computed with only two carriers (hence the *).

There are basically two criteria which we could make use of when applying Backtracking to drop branches from further consideration. These are:

- i) All paths from a node can be pruned, if the worst-IM value associated with that node is greater than or equal to the smallest worst-IM value currently existing at the level where $K = N-1,$
- ii) A particular path from a node at level $K = j$ can be pruned, if it leads to a leaf node and the worst-IM value at the node is greater than or equal to an existing node value at level $K = j + 1.$

The likelihood of occurrence of (i) is very small (in the three examples considered, it never occurred). This is because of the characteristic of the FAP that as N becomes large, the worst-IM values differ significantly from level to level in such a way that as level increases, worst-IM values increase too.

LEVEL

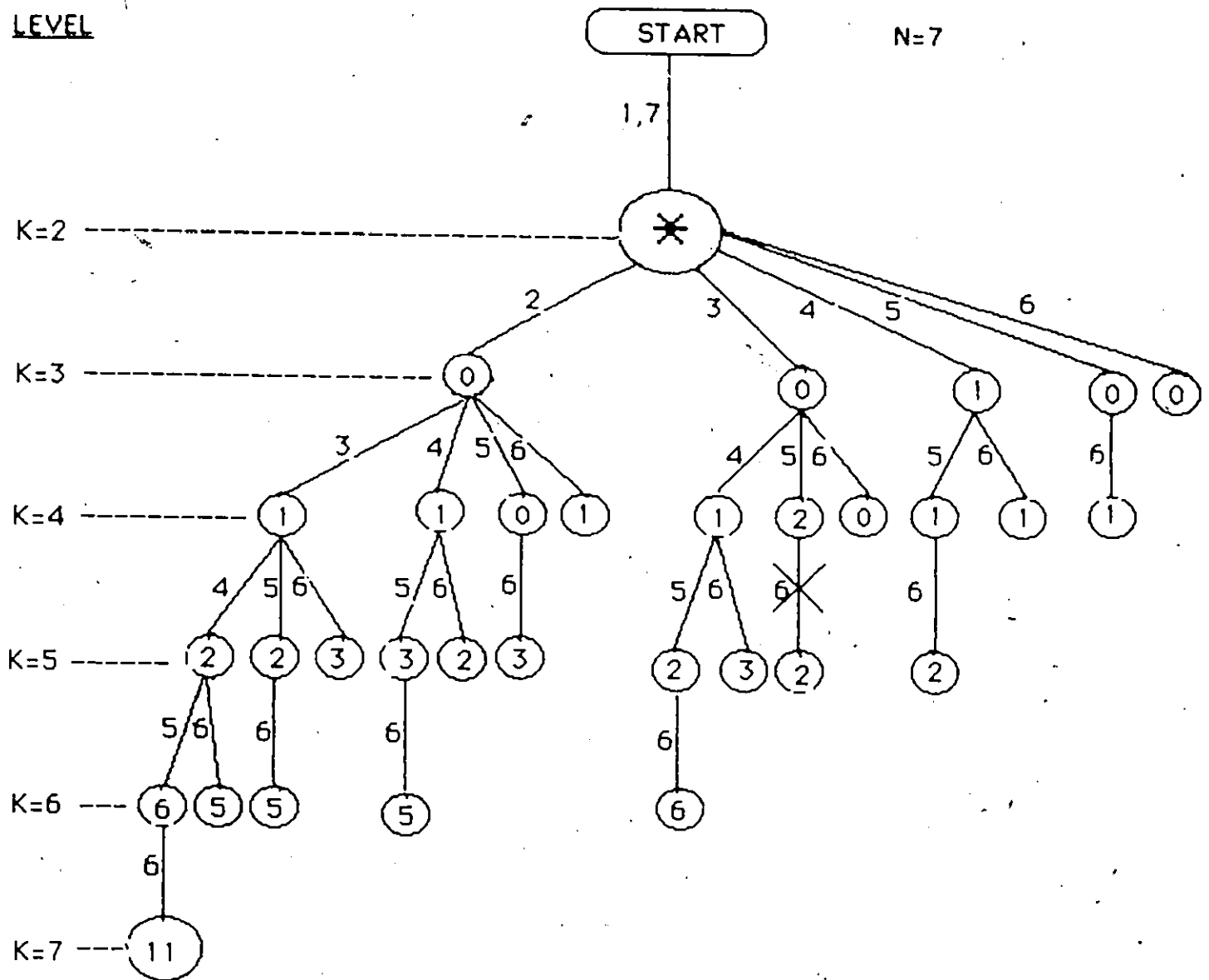


Figure 5.1(a). Backtracking Diagram For FAP With N = 7

LEVEL

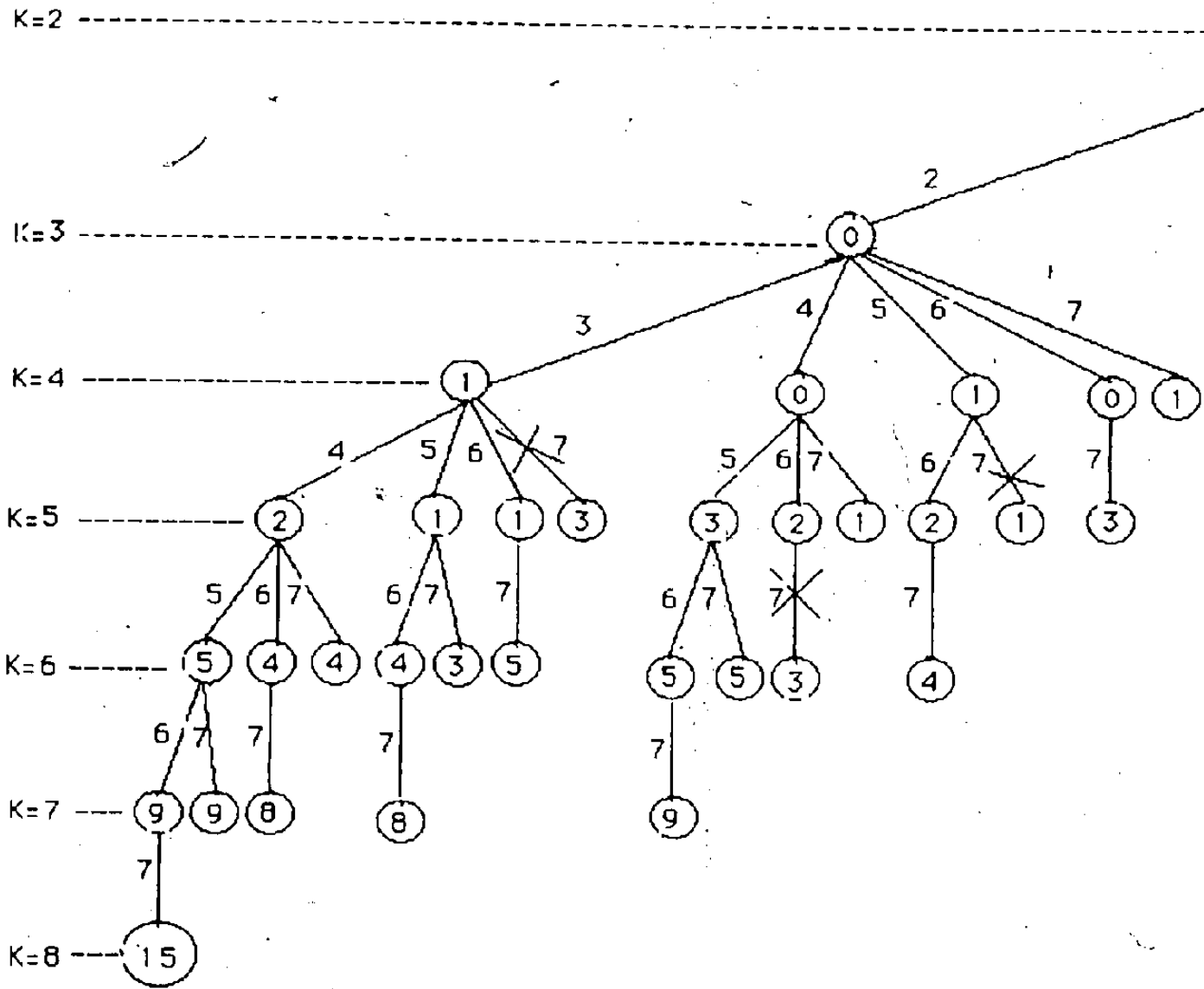
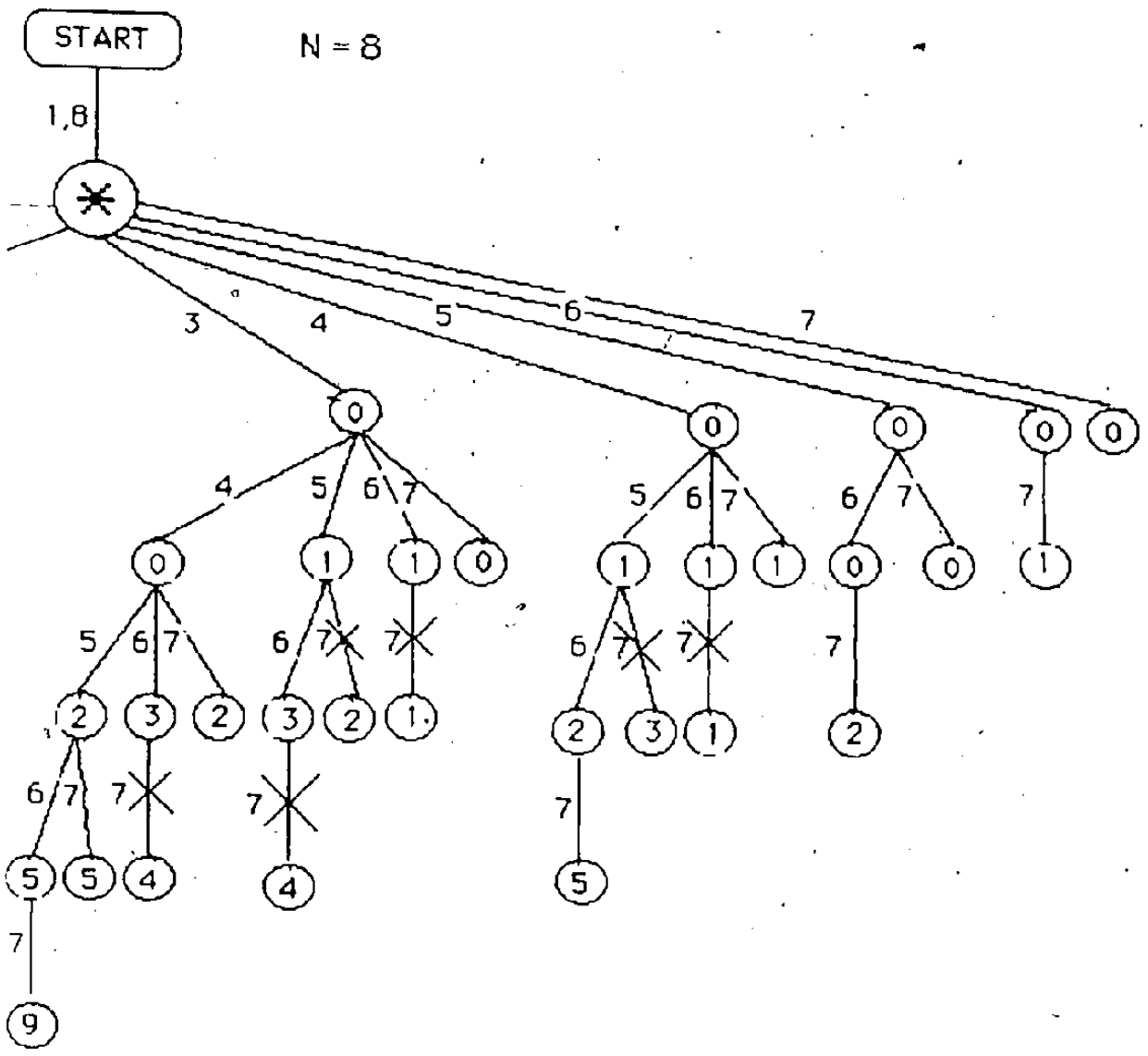


Figure 5.1(b). Backtracking Diagram For FAP With N = 8



LEVEL

K=2

K=3

K=4

K=5

K=6

K=7

K=8

K=9

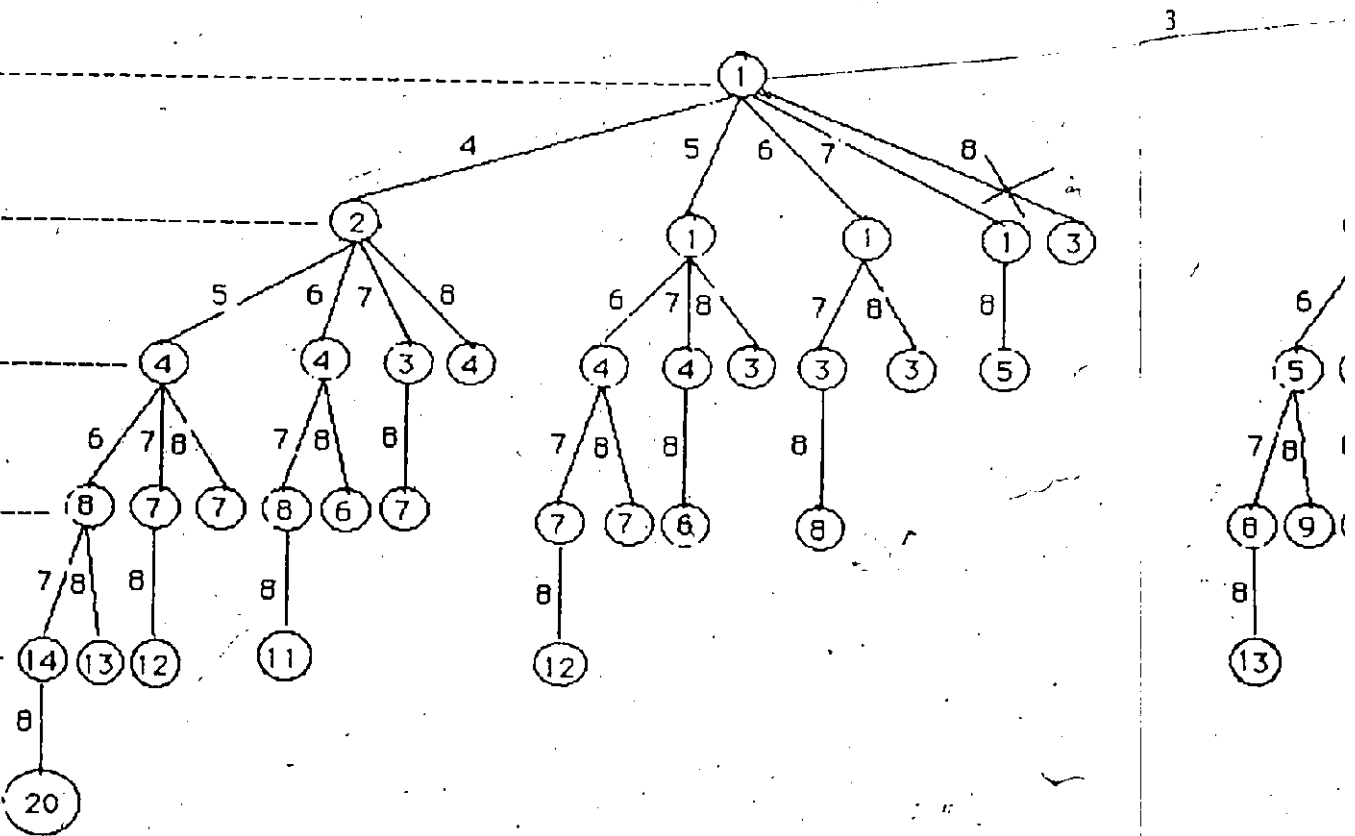
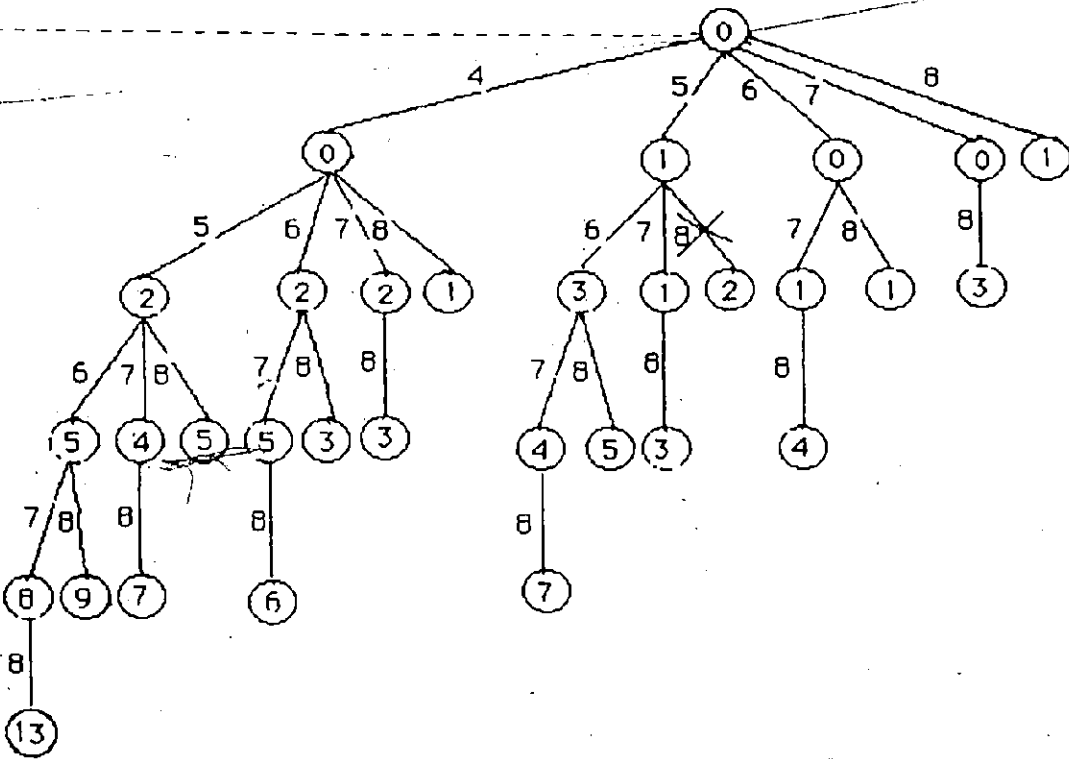


Figure 5.1(c). Backtracking Diagram For FAP With N = 9



1 OF/DE

START

1,9



2

3



4



5



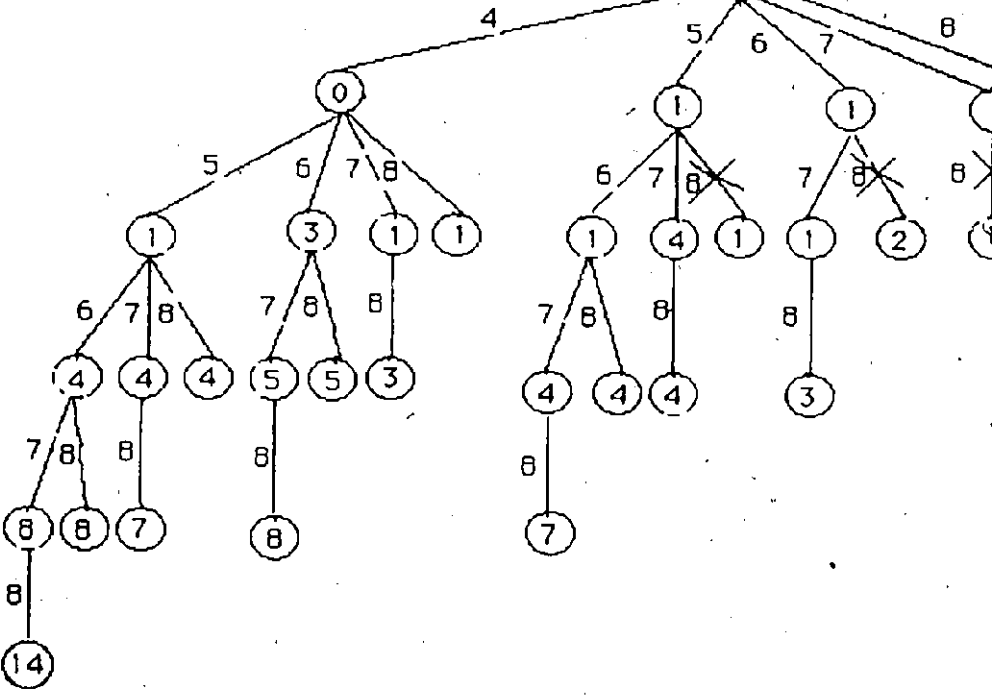
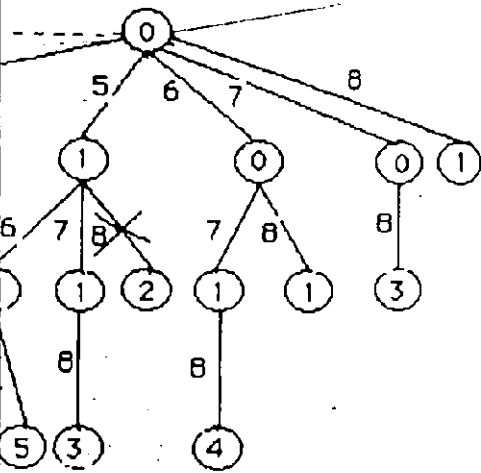
6



7

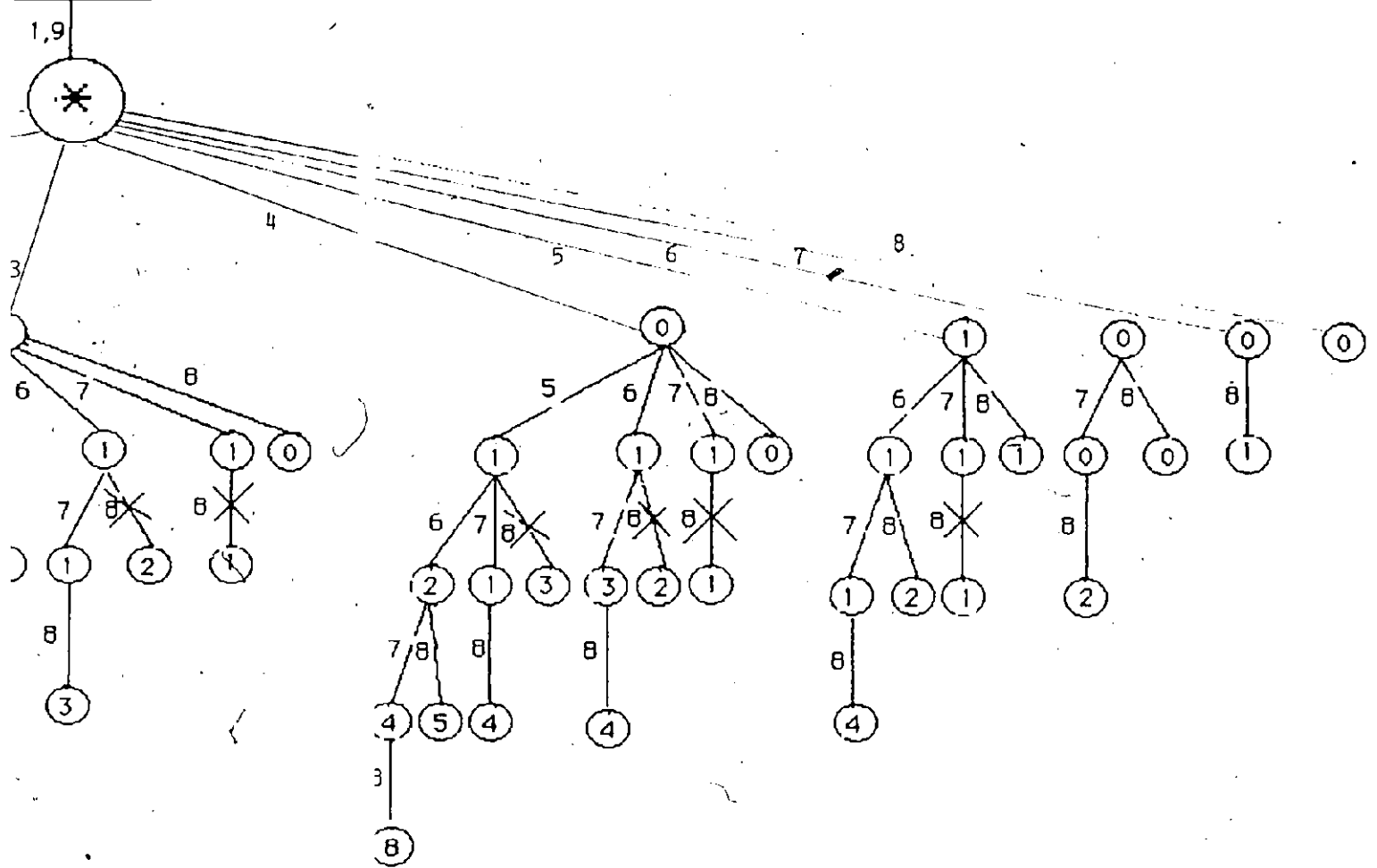


8



START

N = 9



2 OF/DE 2

Occurrences of (ii) are one out of 32 assignments in Figure 5.1.(a), nine out of 64 assignments in Figure 5.1.(b), and nine out of 128 assignments in Figure 5.1.(c). Those branches which could be pruned are indicated by an X. As N increases, the percentage of the assignments (2^{N-2} in total) that can be discarded does not increase but, unfortunately, decreases.

Therefore Backtracking (in the manner shown here) does not provide a particularly effective means for handling the IM-minimum FAP, but, as presented here, it theoretically makes possible the generation of true IM-minimum assignments for given N , and all $K \leq N$. IM-free assignments (i.e. those with zero worst-IM value) are also found. However these are not necessarily optimum. Determining whether or not they are optimum requires additional comparisons of these assignments with those obtained for the same K , but different N .

5.3. IM-MINIMUM FAP VIA THE OKINAKA HEURISTIC

Okinaka et al. [26] have presented a heuristic algorithm which gives an approximate solution to the IM-minimum FAP. For convenience, we refer to their approach as the "Sequential Insertion-Deletion" (SID) Method in the sequel. We now describe the method as presented in [26]. It will be noted that their procedure contains some arbitrariness. This defect is dealt with in the next section.

In the SID method, two types of operations called "deletion" and "insertion" are alternately carried out on some initially prescribed assignment.

The deletion operation transforms a (C,N) assignment into a $(C-1,N)$ assignment, while the insertion operation transforms a (C,N) assignment into a $(C+1,N)$ assignment.

In the deletion operation, one carrier is selected from the current assignment and removed from it. The specific carrier that is selected is the one which causes the most reduction in $WIM(g)$. The procedure is summarized in Figure 5.2 (the style of algorithm presentations follows that suggested by Ören [27]).

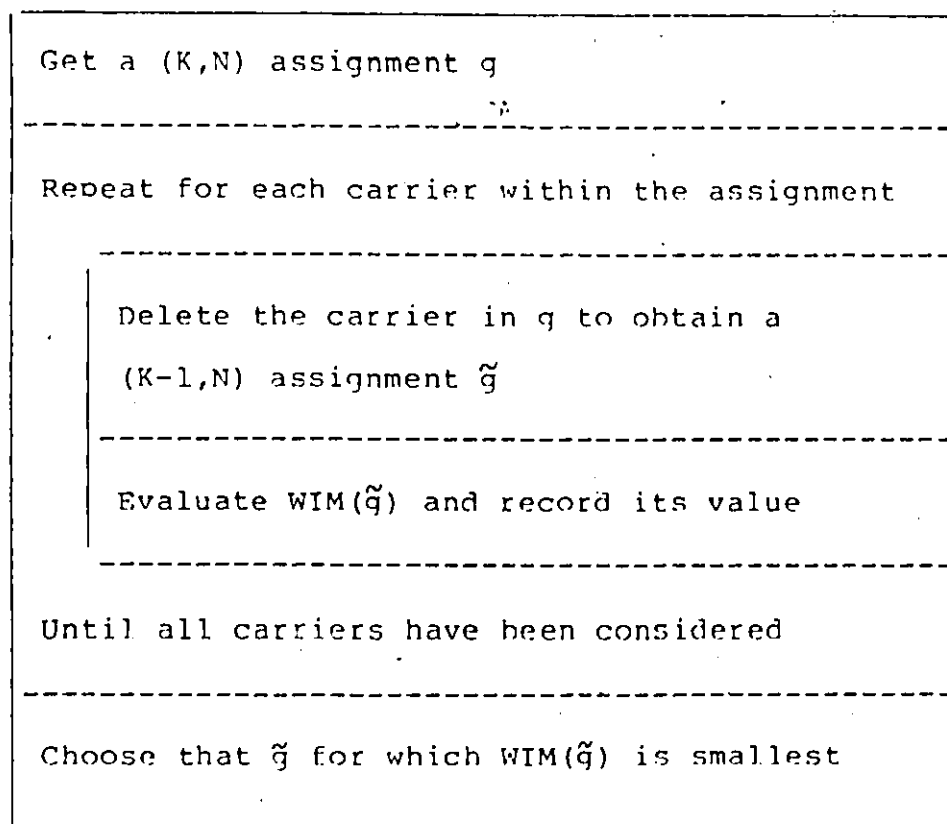


Figure 5.2. Deletion Procedure

In the insertion operation, an additional carrier is inserted into an unused slot in the current assignment. The slot selected is the one which causes the least increase in $WIM(g)$. The procedure is summarized in Figure 5.3.

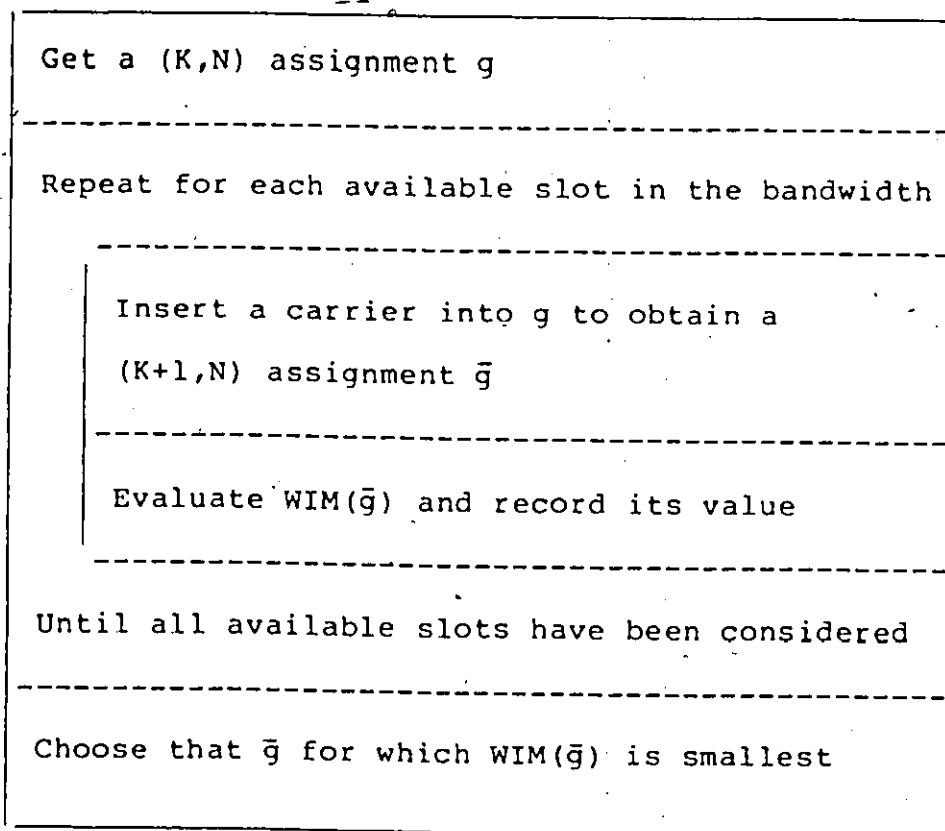


Figure 5.3. Insertion Procedure

The SID method begins with a (K,N) assignment and successively applies deletion-insertion and insertion-deletion operations. If the carrier slot deleted in the deletion operation is different from the slot selected in the next insertion operation (or vice versa), then this pair

of operations (i.e. deletion-insertion or insertion-deletion) yields an improved assignment (lower worst-IM) which then becomes the "current" assignment. This succession of deletion-insertion and insertion-deletion operations are repeated until no further improvement can be obtained, that is when the assignment is repeated. It should be emphasized that this process gives an approximate solution to IM-minimum FAP which is dependent on the initial frequency assignment that is used to initiate the procedure.

The SID procedure outlined in Figure 5.4 corresponds to the presentation as given in [26]. Unfortunately the algorithm contains a flaw that can lead to an unending loop. This can occur because the control criterion establishing completion is based on comparing an intermediate assignment (either \bar{g} or \tilde{g}) with the existing current assignment g (see * and ** in Figure 5.4). A situation which can, with reasonable likelihood, occur is one where these intermediate assignments are distinct from g , but have the same WIM value. The iteration therefore repeats but may yield an identical circumstance and this may continue endlessly.

This matter, as well as another shortcoming of the SID procedure are dealt with in the formulation of one of the alternate heuristics described in the following section.

```

Get an initial (K,N) assignment g
Initialize DI <-- 'SUCCESS'
Initialize ID <-- 'SUCCESS'
Initialize FIRST <-- 1
-----
While ( ID = 'SUCCESS' ) do
-----
Repeat
-----
    Invoke Deletion operation which yields a
    (K-1,N) assignment  $\tilde{g}$ 
-----
    Invoke Insertion operation which yields a
    (K,N) assignment  $\bar{g}$ 
-----
Until no reduction in WIM( $\bar{g}$ ) can be obtained
-----
If (  $\bar{g}$  is the same as g )
-----
    Then | set DI <-- 'FAILURE'
        | (i.e. no improvement in
        | deletion-insertion phase)
-----
    Else | replace g with  $\bar{g}$ 
-----
Endif
-----
If ( DI = 'SUCCESS' or FIRST = 1 )
-----
    Then | Repeat
        | -----
        | Invoke Insertion operation
        | which yields a (K+1,N) asg.  $\bar{g}$ 
        | -----
        | Invoke Deletion operation
        | which yields a (K,N) asg.  $\tilde{g}$ 
        | -----
        | Until no reduction in WIM( $\tilde{g}$ ) can
        | be obtained
    -----
Endif
-----
If (  $\tilde{g}$  is the same as g )
-----
    Then | set ID <-- 'FAILURE'
        | (i.e. no improvement in
        | insertion-deletion phase)
-----
    Else | replace g with  $\tilde{g}$ 
-----
Endif
-----
FIRST <-- 0
-----
Endwhile

```

..(*)

..(**)

Figure 5.4. SID Algorithm

5.4. PROPOSED ALTERNATE HEURISTIC METHODS FOR IM-MINIMUM FAP

In this section three alternate heuristic approaches to IM-minimum FAP are proposed. The first of these can be viewed as a refinement of the SID method described in the previous section, with the principal difference being in the strategy used for selecting carriers in the deletion and insertion phases. The other two alternatives are based on a different underlying concept. Comparative performance results for these three approaches are given in Chapter 6.

5.4.1. The DELIN Proposal

As presented in Figure 5.4, the SID algorithm contains a significant arbitrariness. This occurs in both the deletion and insertion activities contained in the procedure. In particular, it can be noted from Figure 5.2 and 5.3 that both these steps embody the implicit assumption that a unique assignment with a least worst-IM value is generated. This is, in fact, not true in general and consequently some refinement is necessary to more precisely define the procedure.

The DELIN proposed heuristic deals with this indefiniteness by incorporating a secondary criterion based on least total intermodulation and this criterion is used to refine (reduce) the set of equivalent assignments that have the same least value for worst intermodulation. The application

of this additional criterion does not guarantee a unique result but our experiments have demonstrated that it provides a substantial reduction in the number of equivalent results. When more than one assignment remain in this set, the one initially discovered is taken as the result. In Figure 5.5 and 5.6 we show the modified deletion and insertion procedures which incorporate the secondary criterion of least total intermodulation.

As a result of the use of the total-IM criterion, the DELIN procedure can conveniently circumvent the possible cycling that can occur in the SIQ procedure.

A way of circumventing the endless loop problem referred to earlier which could be considered is to replace the termination condition of (*) and (**) with:

IF ($WIM(\bar{g}) = WIM(g)$) then
and IF ($WIM(\tilde{g}_p) = WIM(g)$) then

respectively. However because there are generally many assignments that have the same minimum worst-IM, such a check would likely terminate the problem prematurely. In view of the incorporation within the deletion and insertion procedures of the secondary criterion based on total-IM, these conditions can be extended and made more useful in the following way:

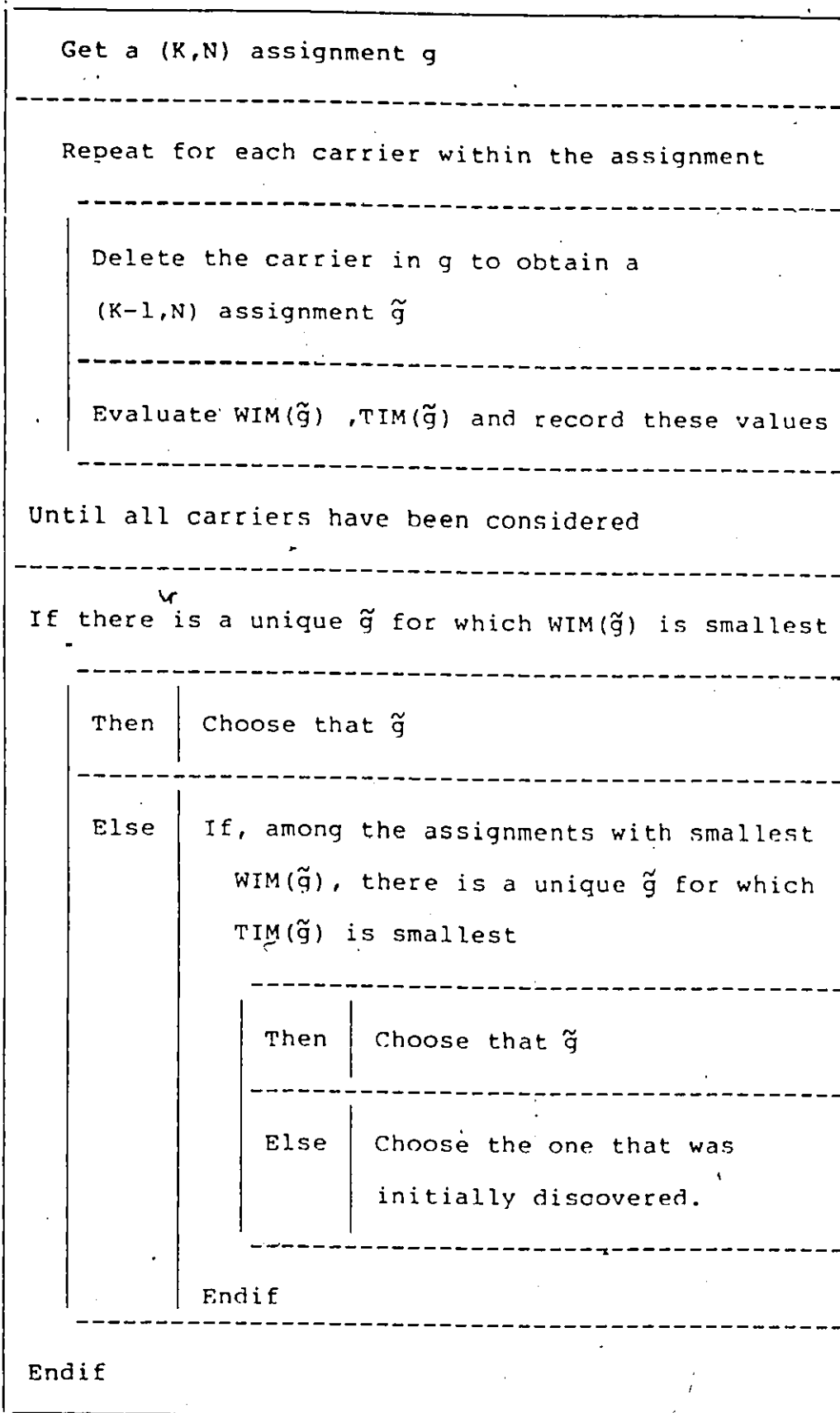


Figure 5.5. Deletion Procedure for DELIN

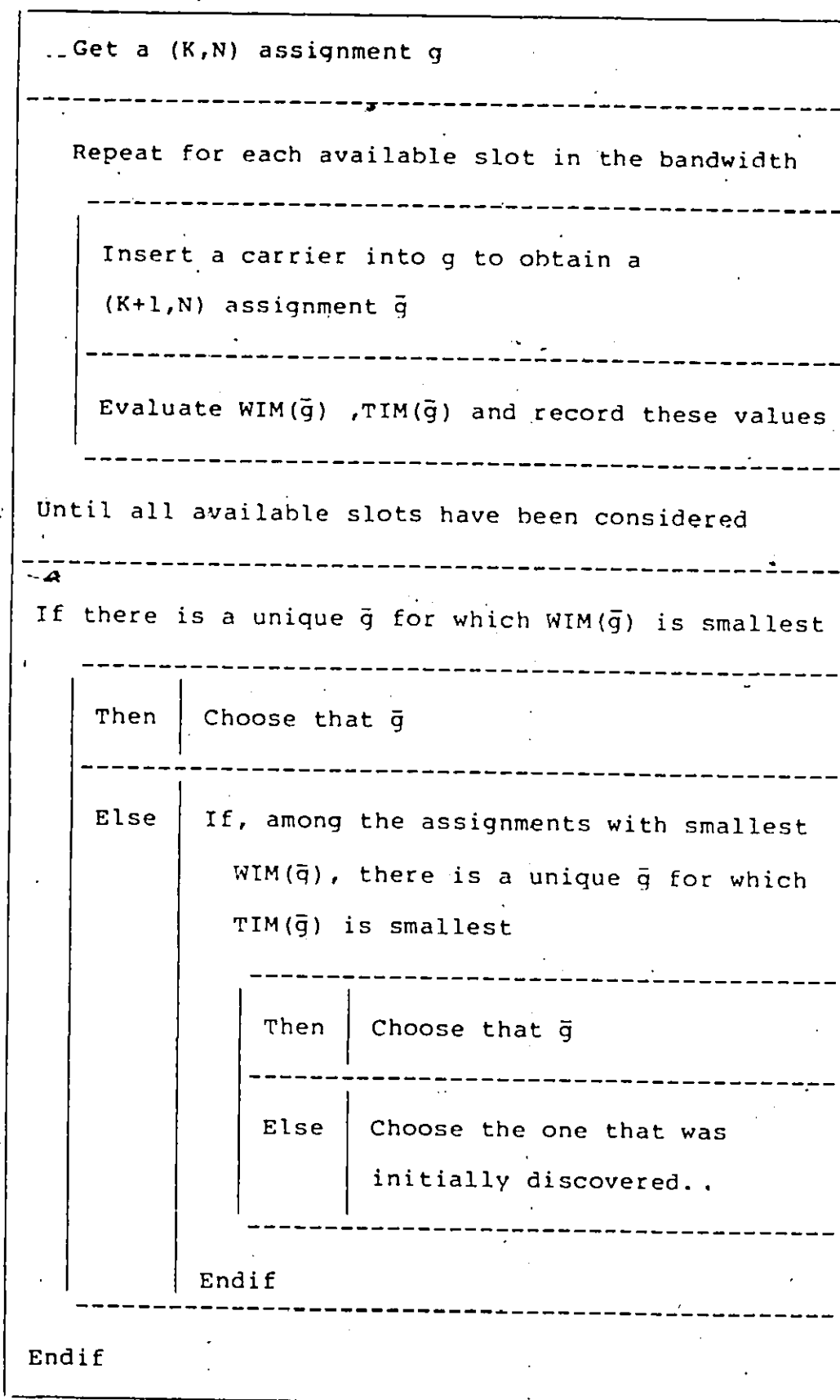


Figure 5.6. Insertion Procedure for DELIN and IN

IF ($WIM(\bar{g}) = WIM(g)$ and $TIM(\bar{g}) = TIM(g)$) then
 and IF ($WIM(\tilde{g}) = WIM(g)$ and $TIM(\tilde{g}) = TIM(g)$) then

The DELIN heuristic is, in fact, based on these conditions.

DELIN process as described above represents our implementation of the SID method. The assignments produced by this implementation usually differ from those reported by Okinaka et al. [26], but the WIM values are essentially the same (IM-advantage values differ by less than four percent).

5.4.2. The IN Proposal

The DELIN procedure (i.e. our implementation of the Okinaka et al. [26] heuristic) can be viewed as a perturbation process which successively refines an initially specified (K,N) assignment to generate an approximate solution to the IM-minimum FAP. An alternate approach that can be considered is one which is based on a constructive strategy. The IN and INU heuristics that are outlined in this section and in the one following, are formulated on this basis.

The Insert procedure as outlined in Figure 5.6 provides the underlying mechanism upon which the IN process is based. The procedure is executed (K-2) times in succession, creating from a basic (2,N) assignment, a sequence of (J,N) assignments, with $J = 3, 4, \dots, K$. The final (K,N) assignment in this sequence provides the approximate solution to the IM-minimum FAP.

Two noteworthy features of this process are:

- i) there is no requirement for an initial "priming" assignment to initiate the procedure; i.e., it is self-starting,
- ii) a by-product of the solution for the (K,N) case is a solution for each of the cases (J,N) with $J = 3, 4, \dots, (K-1)$ (this can be particularly significant and valuable in system design).

A Pseudo-code algorithm for the IN heuristic is given in Figure 5.7.

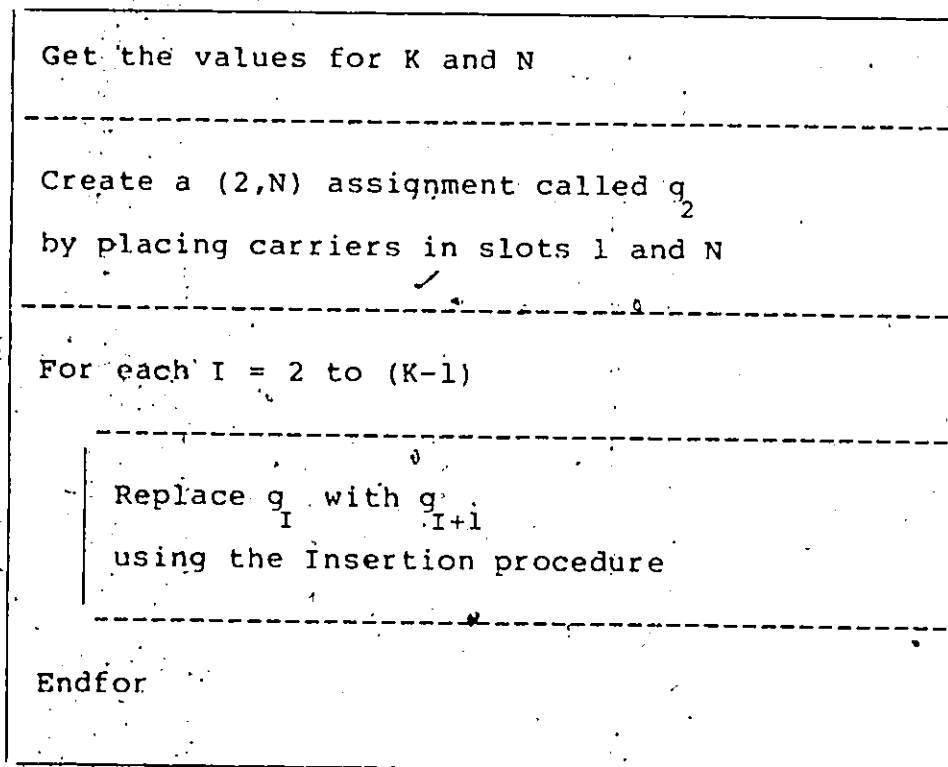


Figure 5.7. IN Algorithm

5.4.3. The INU Proposal

The INU procedure is a variation of the IN procedure in which an alternate strategy is used to select a (J,N) assignment from the set $E(J,N)$ which contains the assignments that minimize the worst-IM. Like the IN procedure, INU is a constructive process which successively generates a sequence of (J,N) assignments with $J = 3, 4, \dots, K$. A fundamental feature of each new assignment in this sequence is that a carrier is placed into a slot which was previously unassigned. Thus in anticipation of this insertion, it is relevant at the J -th step to take into account the intermodulation which falls on unassigned slots, since such slots will be utilized in subsequent steps.

More specifically, let $U(g_J)$ denote the unassigned slots associated with the (J,N) assignment g_J . The characteristic feature of the INU procedure is that the solution candidate g selected from $E(J,N)$ is the one for which $L[U(g_J)]$ is smallest where $L[U(g)]$ denotes the least intermodulation falling on a slot within $U(g)$. The case of non-uniqueness is handled by simply choosing the first candidate that was discovered by the procedure. In effect, the INU approach replaces the total intermodulation concept of the IN heuristic with the "unassigned slot" concept when making a selection from $E(J,N)$. A pseudo-code statement of the INU procedure is the same as given in Figure 5.7 with the fact that INU uses the modified insertion procedure which incorporates the concept of unassigned slot. In Figure 5.8 we show the modified insertion operation for procedure INU.

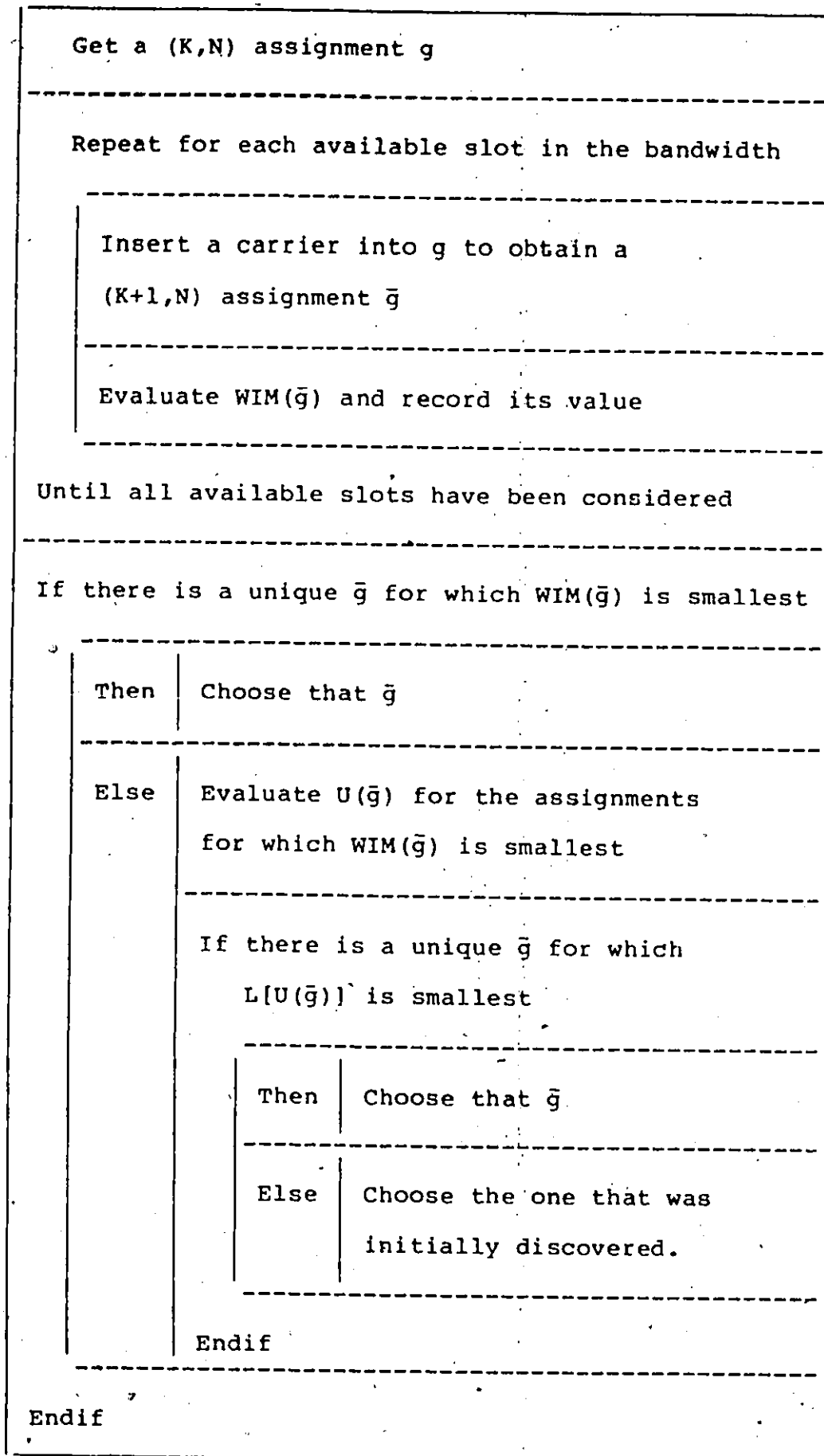


Figure 5.8. Insertion Procedure for INU

Chapter 6

COMPUTATIONAL RESULTS

This chapter summarizes the computational results obtained from experiments with the INTERMOD program. This program includes the FORTRAN code of the four FAP solution procedures, namely EXHAUST, DELIN, IN, and INU that are outlined in the preceding chapter. In response to an initial prompt, the user can specify the particular solution option that is desired. The INTERMOD program is comprised of approximately 1400 lines of FORTRAN code.

6.1. RESULTS WITH THE EXHAUST PROCEDURE

The IM-minimum results obtained by the EXHAUST program do not appear in the literature except for those whose results are in fact IM-free. Table 6.1 is the summary of the results for 168 different (K,N) pairs. For each pair, the following information is provided:

- i) the worst-IM value of the solution(s) obtained,
- ii) the number of solution(s) (assignments) that share this IM-worst value,

- iii) the minimum total-IM among the various solutions in (ii),
- iv) the number of solutions that share the minimum total-IM value given in (iii),
- v) the figure identification in the Appendix where the actual assignments corresponding to (iv) are explicitly given together with the intermodulation falling on each carrier.

Some of the results in Table 6.1 were obtained with the INTERMOD program running on an Amdahl mainframe while others were obtained using a DEC Rainbow microcomputer. In the latter case, substantial execution times were encountered; e.g. elapsed time for the case (K=9, N=24) was 6.5 hours and the elapsed time for (K=9, N=38) was 15 days.

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
4	4	2	1	6	1	A1.1
4	5	1	3	2	2	A1.2
4	6	1	6	1	4	A1.3
4	7	0	1	0	1	A1.4
5	5	4	1	16	1	A1.5
5	6	3	4	10	2	A1.6
5	7	2	6	6	1	A1.7
5	8	1	6	3	2	A1.8
5	9	1	18	2	4	A1.9
5	10	1	36	1	4	A1.10
5	11	1	60	1	14	A1.11
5	12	0	2	0	2	A1.12

Table 6.1. Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min Total-IM	Location of Assignments In Figure
6	6	7	1	34	1	A1.13
6	7	5	3	24	3	A1.14
6	8	3	2	16	2	A1.15
6	9	3	9	12	2	A1.16
6	10	2	2	10	2	A1.17
6	11	2	23	7	4	A1.18
6	12	1	8	6	8	A1.19
6	13	1	14	3	2	A1.20
6	14	1	70	2	4	A1.21
6	15	1	130	2	14	A1.22
6	16	1	304	2	52	A1.23
6	17	1	460	1	22	A1.24
6	18	0	4	0	4	A1.25

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained

By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments in Figure
7	7	11	1	61	1	A1.26
7	8	8	2	46	2	A1.27
7	9	6	3	35	1	A1.28
7	10	5	10	29	6	A1.29
7	11	4	10	24	6	A1.30
7	12	3	4	20	4	A1.31
7	13	3	29	16	8	A1.32
7	14	3	118	13	2	A1.33
7	15	2	22	11	4	A1.34
7	16	2	96	8	2	A1.35
7	17	1	2	7	2	A1.36
7	18	1	18	6	4	A1.37

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained

By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
7	19	1	34	4	4	A1.38
7	20	1	146	3	4	A1.39
7	21	1	268	2	2	A1.40
7	22	1	648	2	12	A1.41
7	23	1	122	2	56	A1.42
7	24	1	2314	1	20	A1.43
7	25	1	3310	1	46	A1.44
7	26	0	5	0	5	A1.45
8	8	15	1	100	1	A1.46
8	9	11	1	80	1	A1.47
8	10	9	2	64	2	A1.48
8	11	8	11	54	2	A1.49
8	12	6	2	46	2	A1.50

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
8	13	6	23	38	4	A1.51
8	14	5	20	33	4	A1.52
8	15	4	4	31	2	A1.53
8	16	4	68	25	10	A1.54
8	17	4	334	21	4	A1.55
8	18	3	24	20	4	A1.56
8	19	3	192	16	2	A1.57
8	20	3	663	15	6	A1.58
8	21	2	18	13	6	A1.59
8	22	2	148	10	2	A1.60
8	23	2	562	9	4	A1.61
8	24	1	2	7	2	A1.62
8	25	1	10	6	2	A1.63

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
8	26	1	36	5	2	A1.64
8	27	1	134	4	4	A1.65
8	28	1	344	4	22	A1.66
8	29	1	716	3	14	A1.67
8	30	1	1696	2	12	A1.68
8	31	1	3182	2	20	A1.69
8	32	1	6236	1	2	A1.70
8	33	1	10150	1	4	A1.71
8	34	1	N/A	N/A	N/A	N/A
8	35	0	1	0	1	[9]
9	9	20	1	152	1	A1.72
9	10	16	2	124	2	A1.73
9	11	13	3	104	2	A1.74

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained

By The Procedure EXHAUST

N/A : Not Available

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
9	12	11	4	89	2	A1.75
9	13	10	17	77	6	A1.76
9	14	9	40	66	2	A1.77
9	15	7	1	58	1	A1.78
9	16	7	74	52	4	A1.79
9	17	6	25	47	6	A1.80
9	18	6	260	42	6	A1.81
9	19	5	42	36	2	A1.82
9	20	4	8	32	2	A1.83
9	21	4	38	30	6	A1.84
9	22	4	232	27	4	A1.85
9	23	3	4	26	4	A1.86
9	24	3	26	21	2	A1.87
9	25	3	186	20	6	A1.88

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
9	26	3	888	18	2	A1.89
9	27	3	2620	17	42	A1.90
9	28	2	30	13	2	A1.91
9	29	2	94	13	4	A1.92
9	30	2	476	11	2	A1.93
9	31	2	1342	11	12	A1.94
9	32	1	4	9	4	A1.95
9	33	1	2	8	2	A1.96
9	34	1	14	8	4	A1.97
9	35	1	74	5	4	A1.98
9	36	1	198	5	2	A1.99
9	37	1	414	4	10	A1.100
9	38	1	1254	3	6	A1.101

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained

By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
9	39	1	2364	3	12	A1.102
9	40	1	5296	3	40	A1.103
9	41	1	N/A	N/A	N/A	N/A
9	42	1	N/A	N/A	N/A	N/A
9	43	1	N/A	N/A	N/A	N/A
9	44	1	N/A	N/A	N/A	N/A
9	45	0	1	0	1	[9]
10	10	26	1	220	1	A1.104
10	11	21	3	184	3	A1.105
10	12	18	8	158	5	A1.106
10	13	15	3	137	2	A1.107
10	14	13	2	122	2	A1.108
10	15	12	7	109	4	A1.109

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

N/A : Not Available

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
10	16	11	32	95	2	A1.110
10	17	10	44	84	2	A1.111
10	18	9	49	76	2	A1.112
11	11	33	1	305	1	A1.113
11	12	27	2	260	2	A1.114
11	13	23	5	226	2	A1.115
11	14	20	6	200	4	A1.116
11	15	18	4	177	2	A1.117
11	16	16	8	159	2	A1.118
11	17	15	50	145	2	A1.119
11	18	13	8	130	2	A1.120

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained

By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
12	12	40	1	410	1	A1.121
12	13	33	1	356	1	A1.122
12	14	29	4	314	3	A1.123
12	15	26	9	280	4	A1.124
12	16	23	6	251	2	A1.125
12	17	21	12	230	2	A1.126
12	18	19	9	210	2	A1.127
13	13	48	1	536	1	A1.128
13	14	41	2	470	2	A1.129
13	15	35	1	422	1	A1.130
13	16	32	6	377	2	A1.131
13	17	28	2	341	2	A1.132
13	18	26	4	314	2	A1.133

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K.	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
14	14	57	1	686	1	A1.134
14	15	49	3	608	3	A1.135
14	16	43	4	548	3	A1.136
14	17	39	5	497	2	A1.137
14	18	35	4	453	4	A1.138
14	19	33	48	416	2	A1.139
15	15	67	1	861	1	A1.140
15	16	58	2	770	2	A1.141
15	17	51	1	699	1	A1.142
15	18	47	8	637	2	A1.143
15	19	42	2	586	2	A1.144
15	20	39	8	543	2	A1.145

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
16	16	77	1	1064	1	A1.146
16	17	67	1	960	1	A1.147
16	18	60	4	876	3	A1.148
16	19	55	7	804	2	A1.149
17	17	88	1	1296	1	A1.150
17	18	78	2	1176	2	A1.151
17	19	69	1	1082	1	A1.152
17	20	64	6	994	2	A1.153
17	21	59	2	928	2	A1.154
18	18	100	1	1560	1	A1.155
18	19	89	3	1424	3	A1.156
18	20	80	4	1314	3	A1.157
18	21	74	9	1215	2	A1.158
18	22	68	2	1139	2	A1.159

Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

K	N	Worst-IM	No. of Cases With the Same Worst-IM	Minimum Total-IM	No. of Cases With the Same Worst-IM and Min. Total-IM	Location of Assignments In Figure
19	19	113	1	1857	1	A1.160
19	20	101	2	1704	2	A1.161
19	21	91	1	1579	1	A1.162
19	22	84	2	1468	2	A1.163
19	23	79	26	1374	2	A1.164
20	20	126	1	2190	1	A1.165
20	21	113	1	2020	1	A1.166
20	22	103	4	1878	3	A1.167
20	23	95	1	1762	1	A1.168

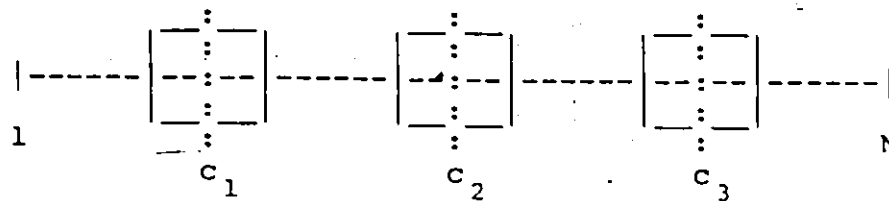
Table 6.1 (Continued)

Summary Of True IM-Minimum Results Obtained
By The Procedure EXHAUST

6.2. SENSITIVITY OF DELIN TO INITIAL STARTING ASSIGNMENT

The DELIN option in INTERMOD represents our program implementation of the Okinaka et al. heuristic [26], and our particular interest is with comparing the performance of our IN and INU heuristics with that of DELIN. However an intrinsic requirement of DELIN is an initial (K,N) assignment which serves to "prime" the DELIN solution process. In [26], the claim is made that this initial starting assignment has little consequence on the final result obtained. Because our performance comparisons are carried out in terms of solution time, it was felt that some experiments should be undertaken with DELIN to explore the sensitivity of its solution time to the initial assignment. Such experiments could provide the basis for ensuring that the chosen initial assignments in the tests with DELIN did not unreasonably impair its performance.

In these experiments, both random and prespecified assignments were considered for the (K-2) carrier positions available in a (K,N) assignment (recall that carriers must always occupy positions 1 and N). In the random assignment, these (K-2) positions were allocated using a random number generator. In the non-random assignments, the remaining (K-2) carriers were arranged into groupings around a center frequency; e.g.,



The above assignment shows three groupings around the center frequencies of c_1 , c_2 , and c_3 . Within each grouping, the carriers occupy adjacent slots around the center frequency. In the non-random tests, several prespecified patterns were considered; namely:

- i) **Even (uniform):** In this case the spacing between carriers in the assignment is set equal or as equal as possible taking into account the constraints imposed by the values of K and N . It should be noted that this pattern is the starting pattern used by Okinaka et al. [26].
- ii) **Left Clustered:** In this case $(K-2)$ carriers are placed in $(K-2)$ adjacent slots immediately to the right of slot one.
- iii) **Right Clustered:** Here $(K-2)$ carriers are placed in $(K-2)$ adjacent slots immediately to the left of slot N .
- iv) **Middle Clustered:** In this case $(K-2)$ carriers are placed in $(K-2)$ adjacent slots centered at the $(N+1)/2$ (N odd) or $N/2$ (N even).
- v) **Edge Clustered:** In this case $(K-2)$ carriers are split into two equal groups which are then left clustered and right clustered.

vi) Triple Split Cluster: Here (K-2) carriers are divided into three equal groups which are then left, middle, and right clustered.

Extensive tests with these various initial assignment strategies were carried out. The general conclusion obtained from these tests is that the even initial assignment is significantly better in terms of solution time. Thus all the DELIN tests reported in the following were obtained using this choice for initial assignment. Exceptions to this general rule are cases where the results of IN or INU are used to initiate DELIN (see Table 6.3).

The results obtained also confirm that the quality of the final result (as measured by IM-advantage) produced by the Okinaka heuristic (as implemented in DELIN) is not sensitive to the initial starting assignment.

6.3. COMPARATIVE PERFORMANCE OF DELIN WITH IN AND INU

The main results of this thesis study are outlined in this section. These results are contained in Table 6.2; 6.3, and 6.4. Table 6.2 shows assignments obtained for sixteen different (K,N) pairs using the DELIN, IN, and INU procedures. In addition, results from Okinaka et al. [26] are given for comparison purposes. In each case the IM-advantage value is also provided. We note that for each (K,N) pair, the quality of the assignment in the four cases is essentially the same, although the assignments are different.

K	N	Frequency Assignment and IM-advantage Value (dB)			
		Okinaka et. al.	DELIN with Even	IN	INU
20	40	1 2 3 5 6 7 10 16 17 19 24 26 27 32 34 36 37 38 39 40 (4 45)	1 2 3 4 5 7 11 13 17 19 22 24 29 32 33 36 37 38 39 40 (4 57)	1 2 3 4 6 8 12 13 18 20 23 26 27 31 34 36 37 38 39 40 (4 47)	1 2 3 4 5 9 11 16 17 19 24 27 28 33 34 36 37 38 39 40 (4 57)
20	60	1 2 3 6 9 12 13 19 22 27 32 41 43 47 54 55 56 58 59 60 (6 73)	1 2 3 4 7 10 12 17 20 26 32 37 41 44 48 55 56 58 59 60 (6 69)	1 2 4 5 8 13 14 21 22 30 35 37 40 45 48 55 56 58 59 60 (6 53)	1 4 7 10 11 12 13 18 26 27 36 40 44 47 48 53 55 57 59 60 (6 23)
20	80	1 2 3 4 8 11 16 28 30 41 45 52 57 61 62 70 73 76 79 80 (8 18)	1 4 5 9 11 17 18 27 28 38 40 43 55 59 68 73 74 76 79 80 (7 99)	1 2 4 6 9 13 21 25 31 35 45 46 59 61 71 72 74 77 79 80 (8 22)	1 4 5 7 8 14 20 22 35 36 40 50 57 60 68 69 73 77 79 80 (8 22)
20	100	1 2 3 5 10 17 29 31 42 45 48 65 67 80 85 89 90 96 98 100 (9 54)	1 3 6 13 14 17 25 31 36 37 43 57 64 82 84 91 95 97 99 100 (9 54)	1 2 4 8 13 19 21 31 40 45 61 66 67 80 82 89 92 95 96 100 (9 24)	1 2 3 4 10 16 17 32 35 52 56 60 70 77 80 88 93 97 99 100 (9 54)

Table 6.2 Frequency Assignments With IM-advantage Values Obtained From Various Heuristics.

K	N	Frequency Assignment and IM-advantage Value (dB)																			
		Okinaka et. al.					DELIN with Even					IN					INU				
30	60	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	4	5	7
		6	7	10	13	15	7	8	9	10	17	7	8	13	14	15	8	10	11	12	13
		18	22	26	27	32	18	23	27	28	32	18	21	22	29	30	18	23	24	26	27
		34	36	40	42	45	34	36	40	45	45	34	35	37	40	45	32	36	40	41	44
		47	49	50	54	55	46	48	52	53	55	47	49	51	53	55	47	48	51	53	55
		56	57	58	59	60	56	57	58	59	60	56	57	58	59	60	56	57	58	59	60
		(4.30)					(4.33)					(4.25)					(4.22)				
30	90	1	2	3	4	6	1	2	3	4	7	1	2	4	5	7	1	2	3	4	6
		9	15	16	19	22	9	10	11	16	21	8	10	13	19	21	7	12	13	15	20
		28	31	36	39	42	26	29	32	38	43	26	31	38	41	45	22	29	31	37	46
		47	59	60	64	68	50	55	59	62	65	54	58	59	63	64	50	52	60	63	66
		70	78	79	81	83	69	75	78	80	82	72	74	79	80	84	70	75	78	82	83
		85	86	88	89	90	86	87	88	89	90	86	87	88	89	90	86	87	88	89	90
		(6.37)					(6.33)					(6.27)					(6.40)				
30	120	1	2	3	4	6	1	2	3	5	8	1	2	4	8	11	1	3	5	6	7
		9	13	17	19	32	9	13	15	16	23	12	13	18	21	31	9	10	16	22	26
		37	39	48	51	56	26	32	41	49	52	38	42	45	50	61	32	40	48	55	62
		62	69	76	77	86	58	67	71	83	87	63	66	76	81	92	73	76	81	88	90
		90	96	102	108	110	90	95	99	104	114	94	96	102	108	114	99	100	105	108	110
		113	117	118	119	120	115	117	118	119	120	116	117	118	119	120	113	117	118	119	120
		(7.85)					(7.88)					(7.63)					(7.71)				
30	150	1	2	3	5	10	1	3	4	5	6	1	2	4	6	8	1	3	5	8	10
		14	18	20	29	30	10	16	22	27	37	9	13	21	31	32	13	21	23	25	39
		40	45	52	66	70	40	56	66	68	69	33	41	45	54	66	55	64	70	71	85
		73	81	87	104	116	88	91	93	99	113	74	81	90	96	102	92	97	106	110	111
		117	126	135	140	143	119	126	127	135	136	112	128	130	133	136	119	120	130	138	141
		145	147	148	149	150	140	142	145	149	150	144	146	147	149	150	143	144	147	149	150
		(8.95)					(8.88)					(8.88)					(8.88)				

Table 6.2 (Continued)

K	N	Frequency Assignment and IM-advantage Value (dB)																			
		Okinaka et. al.					DELIN with Even					IN					INU				
40	80	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		6	7	8	10	13	6	7	8	9	10	6	7	8	9	12	6	7	8	10	11
		16	18	22	24	25	15	16	17	22	25	13	14	18	21	24	14	16	18	20	22
		27	28	32	35	39	26	31	33	37	39	25	28	31	35	37	28	29	33	35	36
		43	45	46	48	52	42	44	50	52	54	44	45	46	49	53	40	45	50	52	55
		61	62	64	65	66	56	59	62	64	65	58	59	61	63	67	57	59	60	65	66
		70	71	72	74	75	68	69	72	73	75	69	71	72	74	75	68	69	73	74	75
		76	77	78	79	80	76	77	78	79	80	76	77	78	79	80	76	77	78	79	80
		(422)					(423)					(421)					(421)				
40	120	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		6	7	8	12	15	8	11	12	13	21	8	11	12	13	17	6	7	9	10	16
		17	22	31	32	34	25	27	28	32	38	18	21	30	31	32	20	22	26	31	32
		39	45	55	57	65	41	42	50	53	55	38	42	45	50	54	40	43	48	55	56
		67	70	77	78	82	64	65	71	73	83	61	63	66	76	77	62	65	73	76	81
		88	91	96	97	100	87	89	99	100	102	81	88	92	94	96	88	90	92	99	100
		105	109	111	112	113	107	108	111	113	114	102	108	111	113	114	105	108	110	112	113
		116	117	118	119	120	115	117	118	119	120	116	117	118	119	120	116	117	118	119	120
		(625)					(621)					(621)					(624)				

Table 6.2 (Continued)

K	N	Frequency Assignment and IM-advantage Value (dB)															
		Okinaka et. al.					DELIN with Even	IN					INU				
40	160	1	2	3	5	7	N/A	1	2	3	4	6	1	2	3	4	7
		8	10	13	20	21		8	10	13	21	22	8	10	13	15	23
		25	30	31	37	40		30	31	32	42	45	24	25	39	41	46
		45	56	58	66	70		50	54	66	72	78	55	57	69	70	74
		71	72	79	92	104		89	91	97	105	106	82	95	96	101	105
		108	115	117	126	132		110	122	123	128	138	116	120	123	130	134
		136	145	148	151	152		144	145	148	149	152	140	147	148	150	153
		154	156	158	159	160		155	156	158	159	160	156	157	158	159	160
		(763)					(754)					(763)					
40	200	1	2	3	4	6	N/A	1	2	3	4	8	1	2	3	4	5
		9	10	13	20	26		9	13	15	21	22	10	16	18	20	26
		32	38	45	54	58		31	40	45	46	55	31	34	42	48	52
		71	75	83	85	86		62	66	81	86	94	64	70	73	77	104
		104	106	117	128	132		97	111	114	125	129	108	115	120	123	135
		144	146	161	164	175		137	142	153	163	167	143	156	160	163	169
		176	184	185	190	192		171	175	187	190	192	170	179	180	188	192
		194	195	198	199	200		196	197	198	199	200	193	197	198	199	200
		(874)					(866)					(872)					

Table 6.2 (Continued)

N/A : Assignment is not available because execution terminated due to excessive computational time.

K	N	Frequency Assignment and IM-advantage Value (dB)																			
		Okinaka et. al.					DELIN with Even					IN					INU				
50	100	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		6	7	8	9	10	6	7	9	10	11	6	7	8	9	11	6	7	8	10	11
		16	17	18	19	22	12	13	16	17	21	12	13	15	19	21	15	16	17	20	23
		24	25	28	31	36	23	24	27	31	37	23	27	28	31	34	25	28	31	32	35
		38	41	42	44	46	39	41	43	45	52	36	40	44	45	48	38	39	41	47	52
		53	56	58	62	64	54	56	60	62	63	51	58	61	62	66	54	56	58	60	66
		67	68	73	75	77	67	68	69	76	78	67	68	73	75	77	67	70	72	77	79
		81	85	86	88	89	81	82	86	87	88	80	82	85	86	87	80	81	86	88	89
		90	91	92	93	94	90	91	93	94	95	89	91	92	94	95	90	92	93	94	95
		96	97	98	99	100	96	97	98	99	100	96	97	98	99	100	96	97	98	99	100
		(413)					(410)					(416)					(414)				
50	150	1	2	3	4	5	N/A	1	2	3	4	5	1	2	3	4	5				
		6	7	8	9	12		6	8	9	10	13	6	8	10	11	13				
		20	21	24	29	33		16	21	23	31	32	16	21	23	25	27				
		36	38	43	46	48		33	41	45	46	54	33	39	40	45	48				
		58	60	64	66	76		57	58	65	66	74	55	56	64	70	71				
		77	85	90	96	98		80	81	90	92	96	81	85	86	92	97				
		99	105	109	113	118		102	107	109	112	119	99	106	110	111	119				
		123	128	131	134	135		123	126	128	130	133	120	128	130	133	137				
		138	139	141	142	144		136	139	141	142	144	138	141	143	144	145				
		146	147	148	149	150		146	147	148	149	150	146	147	148	149	150				
		(6.15)						(6.10)					(6.14)								

Table 6.2 (Continued)

N/A : Assignment is not available because execution terminated due to excessive computational time.

K	N	Frequency Assignment and IM-advantage Value (dB)															
		Okinaka et. al.					DELIN with Even	IN					INU				
50	200	1	2	3	4	5	N/A	1	2	3	4	6	1	2	3	4	5
		6	10	11	14	15		8	9	13	15	21	7	9	10	16	18
		20	27	33	35	40		22	30	31	39	40	20	26	31	34	42
		45	51	52	60	74		45	46	55	62	66	46	48	52	64	70
		76	78	81	92	98		75	81	86	94	97	73	77	82	89	94
		106	110	116	120	136		108	111	114	125	129	104	108	115	120	123
		140	142	143	145	157		137	142	149	153	163	135	140	143	146	156
		164	171	173	179	180		167	171	175	177	179	160	163	169	170	179
		184	185	188	192	195		187	189	190	192	194	180	188	189	192	193
		196	197	198	199	200		196	197	198	199	200	195	197	198	199	200
		(7.57)						(7.55)					(7.49)				
50	250	1	2	3	4	5	N/A	1	2	3	4	5	1	2	3	4	6
		6	10	11	18	20		7	8	11	13	21	7	13	14	15	18
		24	26	33	42	47		25	31	39	44	45	27	31	33	41	46
		57	61	72	73	77		51	58	66	69	81	47	52	61	65	77
		81	93	106	111	129		88	97	108	110	116	80	97	100	102	121
		131	139	150	153	156		123	135	149	151	162	128	131	138	148	154
		165	171	184	191	197		170	177	179	194	198	163	170	178	185	196
		203	208	214	216	226		202	210	211	223	225	203	206	212	220	226
		233	236	237	240	243		227	232	234	239	243	230	234	238	241	242
		245	247	248	249	250		244	245	248	249	250	243	247	248	249	250
		(8.65)						(8.56)					(8.56)				

Table 6.2 (Continued)

N/A : Assignment is not available because execution terminated due to excessive computational time.

The results given in Table 6.3 provide some insight into the possible usefulness of using the assignment obtained from either IN or INU to initiate DELIN. Table 6.3 considers the same sixteen cases used in Table 6.2, and results shown under the "Even" heading are a repetition of the results given in Table 6.2 under the "DELIN with Even" heading. In the remaining two columns are shown the assignments generated by DELIN when it is initiated with the assignment obtained from IN and INU respectively. The objective then is to compare the IM-advantage values in the last two columns of Table 6.3 with those in the last two columns of Table 6.2. It can be noted that the values in the former case are only infrequently larger and any difference is relatively minor. This suggests that there is little point in the approach of using DELIN to try to improve ("refine") the results obtained from IN or INU.

K	N	Frequency Assignment and IM-advantage Value (dB) Obtained from DELIN using the Initial Assignment														
		Even					output from IN					output from INU				
20	40	1	2	3	4	5	1	2	3	4	6	1	2	3	4	5
		7	11	13	17	19	8	12	13	16	20	9	11	16	17	19
		22	24	29	32	33	23	27	28	31	34	24	27	28	33	34
		36	37	38	39	40	36	37	38	39	40	36	37	38	39	40
		(4 5 7)					(4 5 7)					(4 5 7)				
20	60	1	2	3	4	7	1	2	4	5	8	1	2	4	7	11
		10	12	17	20	26	13	14	21	22	30	13	18	26	27	32
		32	37	41	44	48	35	37	40	45	49	36	40	47	48	53
		55	56	58	59	60	55	56	58	59	60	55	57	58	59	60
		(6 6 9)					(6 5 3)					(6 6 9)				
20	80	1	4	5	9	11	1	2	4	8	9	1	4	5	6	8
		17	18	27	28	38	13	21	25	31	35	14	20	22	35	36
		40	43	55	59	62	45	46	59	61	71	40	50	57	60	68
		73	74	76	79	80	72	74	77	79	80	69	73	77	79	80
		(7 9 9)					(8 2 2)					(8 2 2)				
20	100	1	3	6	13	14	1	2	4	8	13	1	2	3	4	10
		17	25	31	36	37	19	21	31	40	45	16	17	32	35	52
		43	57	64	82	84	61	66	67	80	82	56	60	70	77	80
		91	95	97	99	100	89	92	95	96	100	88	93	97	99	100
		(9 5 4)					(9 2 4)					(9 5 4)				

Table 6.3 Frequency Assignments With IM-advantage Values
Obtained From DELIN With Various Initial Assignments.

K	N	Frequency Assignment and IM-advantage Value(dB) Obtained from DELIN using the Initial Assignment														
		Even					output from IN					output from INU				
30	60	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		7	8	9	10	17	6	7	8	13	15	7	8	10	12	13
		18	23	27	28	32	18	21	22	29	30	18	23	24	26	31
		34	36	40	43	45	34	35	37	40	45	32	36	40	41	44
		46	48	52	53	55	47	49	51	53	55	47	48	51	53	55
		56	57	58	59	60	56	57	58	59	60	56	57	58	59	60
		(433)					(437)					(433)				
30	90	1	2	3	4	7	1	2	3	4	5	1	2	3	4	6
		9	10	11	16	21	7	10	13	19	21	7	12	13	15	20
		25	29	32	38	43	24	31	38	41	45	22	29	31	37	46
		50	55	59	62	65	54	58	59	63	64	50	52	60	63	66
		69	75	76	80	82	72	74	79	80	84	70	75	78	82	83
		86	87	88	89	90	86	87	88	89	90	86	87	88	89	90
		(633)					(640)					(640)				
30	120	1	2	3	5	8	1	2	4	8	11	1	3	5	6	7
		9	13	15	16	23	12	13	17	21	31	9	10	16	22	26
		26	32	41	49	52	38	42	45	50	61	32	40	48	55	56
		58	67	71	83	87	63	73	76	81	94	62	76	81	88	90
		90	95	99	104	114	96	102	103	108	114	99	100	105	108	113
		115	117	118	119	120	116	117	118	119	120	116	117	118	119	120
		(788)					(788)					(780)				
30	150	1	3	4	6	10	1	2	4	6	8	1	2	3	5	10
		10	16	22	27	37	9	13	21	31	32	13	21	23	29	39
		40	56	66	68	69	33	41	45	56	74	55	64	70	71	85
		88	91	93	99	113	81	90	96	102	112	92	97	106	110	115
		119	126	127	135	136	119	128	130	133	136	120	130	137	141	143
		140	142	145	149	150	144	146	147	149	150	144	145	147	149	150
		(888)					(899)					(899)				

Table 6.3 (Continued)

K	N	Frequency Assignment and IM-advantage Value (dB) Obtained from DELIN using the Initial Assignment														
		Even					output from IN					output from INU				
40	80	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		6	7	8	9	10	6	7	8	9	12	6	7	8	10	11
		15	16	17	22	25	13	14	18	21	24	14	16	18	20	22
		26	31	33	37	39	25	28	31	35	37	28	29	33	36	39
		42	44	50	52	54	44	45	46	49	53	40	45	50	52	55
		56	59	62	64	65	58	59	61	63	67	57	59	60	65	66
		68	69	72	73	75	69	71	72	74	75	68	72	73	74	75
		76	77	78	79	80	76	77	78	79	80	76	77	78	79	80
		(423)					(421)					(423)				
40	120	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		8	11	12	13	21	8	11	12	13	17	6	7	9	10	16
		25	27	28	32	38	18	21	30	31	32	20	22	26	31	32
		41	42	50	53	55	38	42	45	50	54	40	43	48	55	56
		64	65	71	73	83	61	63	66	76	77	62	65	73	76	81
		87	89	99	100	102	81	88	92	94	99	88	90	97	99	100
		107	108	111	113	114	102	108	111	113	114	105	108	110	112	113
		115	117	118	119	120	116	117	118	119	120	116	117	118	119	120
		(621)					(624)					(627)				

Table 6.3 (Continued)

K	N	Frequency Assignment and IM-advantage Value (dB) Obtained from DELIN using the Initial Assignment										
		Even	output from IN					output from INU				
40	160	N/A	1	2	3	4	6	1	2	3	4	7
			8	10	12	13	15	8	10	13	15	23
			22	30	31	42	45	24	25	39	41	46
			50	54	66	72	78	55	57	69	70	74
			89	91	97	105	106	82	95	96	101	105
			110	122	123	128	131	116	120	123	130	134
			138	144	145	148	152	140	147	148	150	153
			155	156	158	159	160	156	157	158	159	160
			(7 6 8)					(7 6 3)				
40	200	N/A	1	2	3	4	8	1	2	3	4	5
			9	13	15	21	22	10	16	18	20	25
			31	40	45	46	55	31	34	42	52	64
			62	66	81	86	94	69	73	77	104	108
			97	111	114	125	129	111	115	120	123	135
			137	142	153	163	167	143	156	160	163	169
			171	175	187	190	192	170	179	180	188	192
			196	197	198	199	200	193	197	198	199	200
			(8 6 6)					(8 7 2)				

Table 6.3 (Continued)

N/A : Assignment is not available because execution terminated due to excessive computational time

K	N	Frequency Assignment and IM-advantage Value (dB) Obtained from DELIN using the Initial Assignment														
		Even					output from IN					output from INU				
50	100	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
		6	7	9	10	11	6	7	8	9	11	6	7	8	10	11
		12	13	16	17	21	12	13	15	19	21	12	15	16	17	20
		23	24	27	31	37	23	27	28	31	34	23	28	31	32	35
		39	41	43	45	52	35	40	44	45	47	38	39	41	47	52
		54	56	60	62	63	48	51	61	62	66	54	56	58	60	66
		67	68	69	76	78	67	68	73	75	77	67	70	72	77	79
		81	82	86	87	88	80	82	85	86	87	80	81	86	88	89
		90	91	93	94	95	89	91	92	94	95	90	92	93	94	95
		96	97	98	99	100	96	97	98	99	100	96	97	98	99	100
		(410)					(417)					(416)				
50	150	N/A					1	2	3	4	5	1	2	3	4	5
							6	8	9	10	13	6	8	10	11	13
							16	21	23	31	32	16	21	23	25	27
							33	41	45	46	54	33	39	40	45	48
							57	58	66	74	80	55	56	64	70	71
							81	87	90	92	96	81	85	86	92	97
							102	107	109	112	119	99	106	110	111	119
							123	126	128	130	133	120	124	128	130	137
							136	139	141	142	144	138	141	143	144	145
							146	147	148	149	150	146	147	148	149	150
							(614)					(618)				

Table 6.3 (Continued)

N/A : Assignment is not available because execution terminated due to excessive computational time.

K	N	Frequency Assignment and IM-advantage Value (dB) Obtained from DELIN using the Initial Assignment																																																																																																						
		Even	output from IN					output from INU																																																																																																
50	200	N/A	1	2	3	4	6	1	2	3	4	5	8	9	13	15	21	7	9	10	16	18	22	30	31	39	40	20	26	31	34	42	45	46	55	62	66	46	48	52	64	70	75	81	86	94	97	73	77	82	89	94	108	111	114	125	129	104	108	115	120	123	137	142	149	153	163	135	140	143	146	160	167	171	175	177	179	163	169	170	175	179	187	189	190	192	194	180	188	189	190	192	196	197	198	199	200	193	197	198	199	200	(7 55)	(7 58)
50	250	N/A	1	2	3	4	5	1	2	3	4	6	7	8	11	13	21	7	13	14	15	18	25	31	39	45	51	27	31	33	41	46	58	69	71	81	88	47	52	65	77	80	97	100	108	110	116	97	100	102	121	128	123	135	149	151	162	131	138	148	154	163	170	177	179	194	198	170	178	181	185	189	202	210	211	223	225	203	206	212	220	226	227	232	234	239	243	230	234	238	241	242	244	245	248	249	250	243	247	248	249	250	(8 67)	(8 63)

Table 6.3 (Continued)

N/A : Assignment is not available because execution terminated due to excessive computational time.

The purpose of Table 6.4 is to provide solution time (CPU) for the three heuristic methods (for each of the sixteen cases considered previously). It should, in particular, be observed that the solution time of IN is always significantly less than that of both DELIN with "Even" initial assignment (by a factor of at least 3) and INU (by a factor of about 1.5). From the fourth and fifth columns in Table 6.4, it can be noted that the solution time of DELIN is substantially improved when its starting assignment is obtained from either IN or INU. This, of course, is not surprising.

K	N	* Execution Time (Seconds) for				
		DELIN using the Initial Assignment			IN	INU
		Even	output from IN	output from INU		
20	40	26	9	6	8	12
20	60	44	9	25	14	27
20	80	70	13	18	21	42
20	100	120	16	15	27	59
30	60	337	94	111	86	130
30	90	587	216	88	159	255
30	120	923	338	230	231	330
30	150	1162	246	530	304	488
40	80	2093	201	570	479	658
40	120	3507	757	430	885	1107
40	160	over 5000	1255	393	1285	1811
40	200	over 6000	518	1250	1684	2262
50	100	5625	1230	1170	1803	2356
50	150	over 10800	1387	1745	3333	4060
50	200	over 11000	1260	3590	4863	6299
50	250	over 12000	4677	3088	3266	8305

Table 6.4 Solution Times (In Seconds) For The Procedures
DELIN, IN and INU

* Rounded To The Nearest Integer

6.4. RESULTS FROM "IN" RELATIVE TO KNOWN VALUES AND BOUNDING FORMULAS

Table 6.5 provides the results from an interesting set of experiments carried out to examine the behavior of the IN heuristic for the cases where IM-free results are known. Ideally, for these cases one would like IN to generate an assignment for which the worst-IM value was zero. Table 6.5 gives the worst-IM values actually obtained and the corresponding solution times. In only two cases are the worst-IM values equal to zero. However in the remaining cases, the obtained value is small and, considering the relatively short solution times, these reflect well on the efficiency of IN heuristic. Note also that the worst-IM values shown are for the combined triple+double case, expressed in terms of triple.

K	N	Worst-IM resulting from IN	Execution Time (Seconds)
3	4	0	0.01
4	7	0	0.02
5	12	1	0.02
6	18	1	0.03
7	26	1	0.07
8	35	1	0.14
9	45	15	0.30
10	56	2	0.59
11	73	2	1.26
12	86	2	2.27
13	112	2	4.44
14	128	25	7.28
15	156	2	12.48
16	180	2	19.84
17	200	3	29.72

Table 6.5 Worst-IM Results and Solution Times. For The Procedure IN (with those K, N values for which IM-Free assignment is available [2], [9], [29]).

The objective in Figure 6.1 - 6.8 is to examine the results obtained from IN, relative to upper and lower bound formulas for IM-advantage for various bandwidth ratios, (N/K). The value for upper bound is obtained from the definition

$$\leftarrow \text{IMA}_3(g) = 10 \log_{10} [\text{WIM}_3(g^0) / \text{WIM}_3(g)]$$

by substituting the lower bound value for $\text{WIM}_3(g)$ that is given by Maxemchuk [23].

Each of the figures corresponds to a value of (N/K) in the range 2 to 9. In each case, the figure shows the upper and lower bound values, for N extending from 0 to 200. The points shown correspond to results obtained from IN and, where available, results obtained from EXHAUST. Note that in Figure 6.8 the upper bound value is not shown because the procedure from Maxemchuk yields a value of zero for the lower bound value on $\text{WIM}_3(g)$ in the range of N that is of interest.

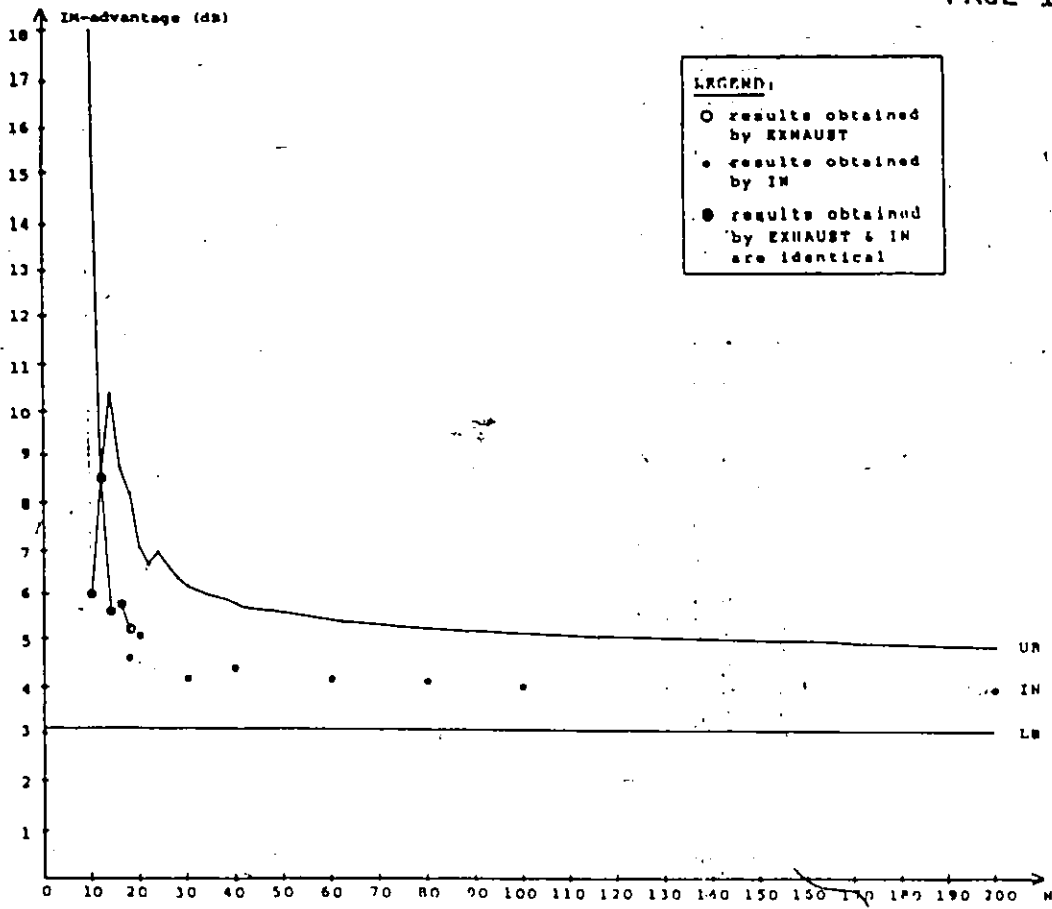


Figure 6.1. IM-advantage versus available bandwidth for $(N/K)-2$

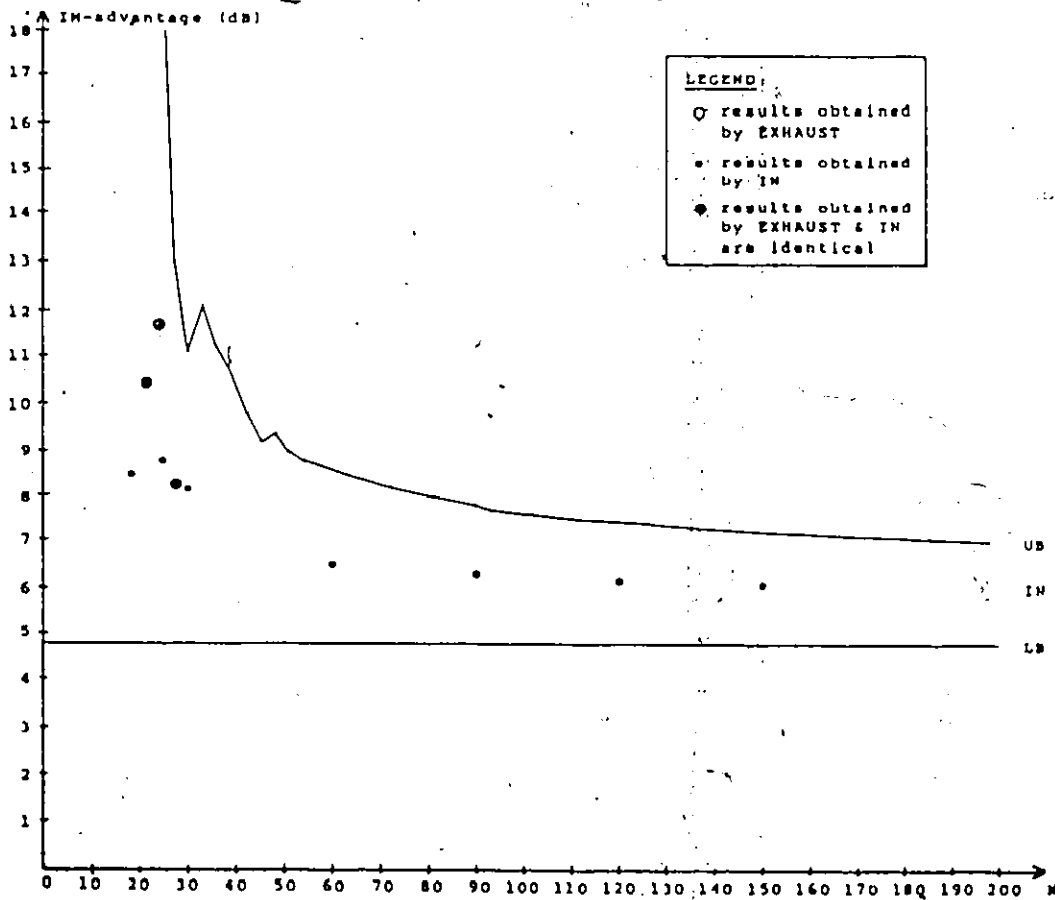


Figure 6.2. IM-advantage versus available bandwidth for $(N/K)-3$

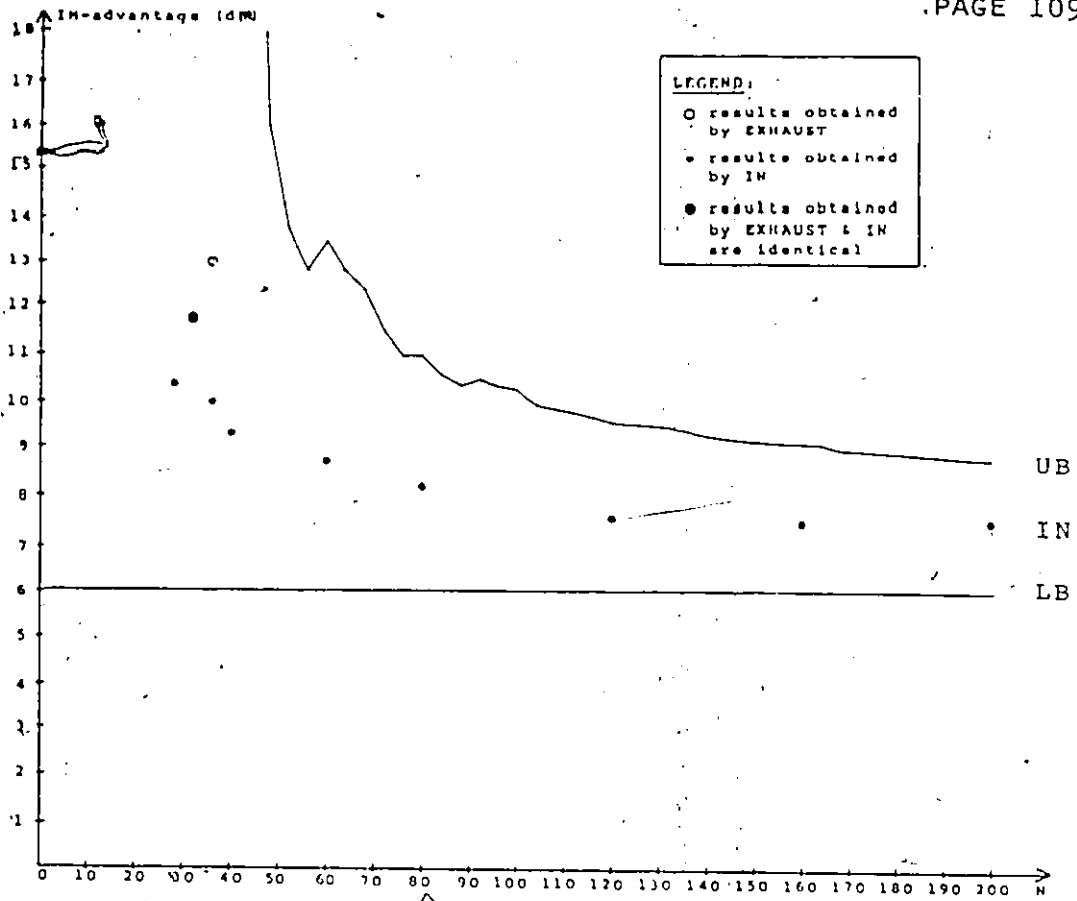


Figure 6.3. IM-advantage versus available bandwidth for $(N/K)=4$

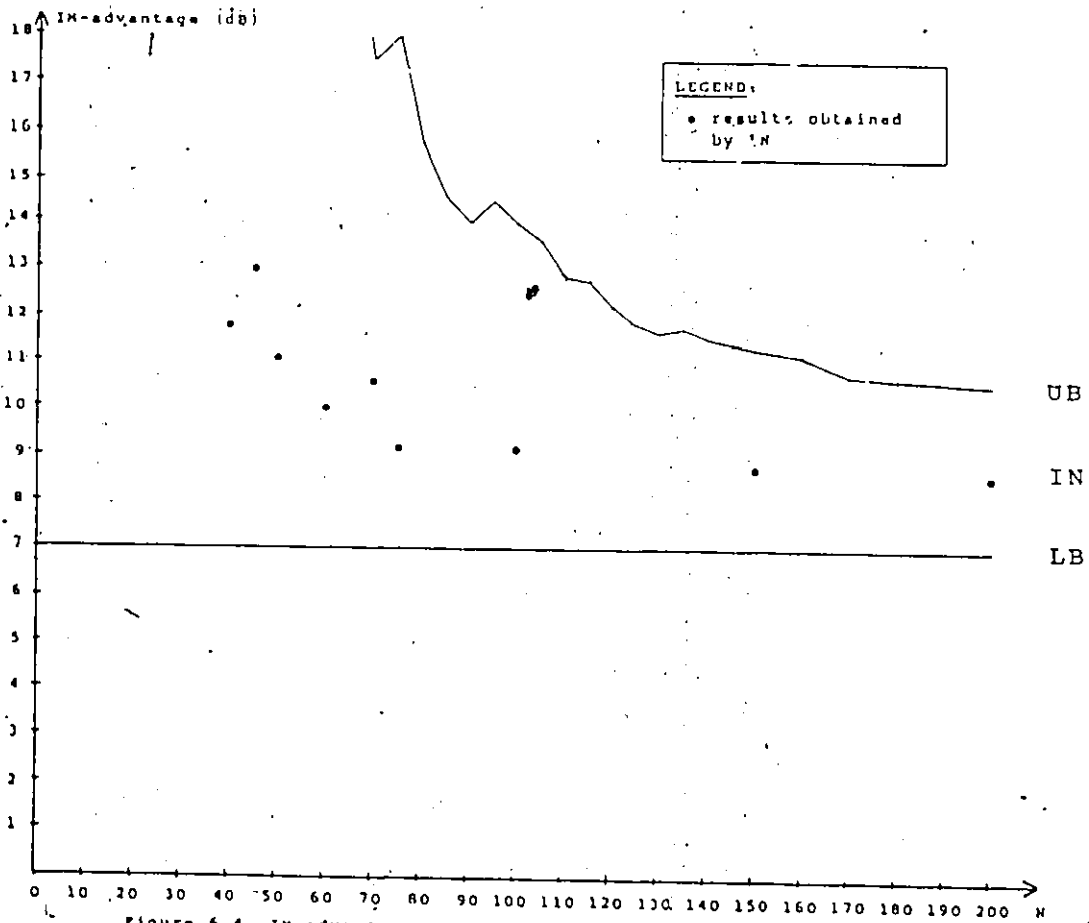


Figure 6.4. IM-advantage versus available bandwidth for $(N/K)=5$

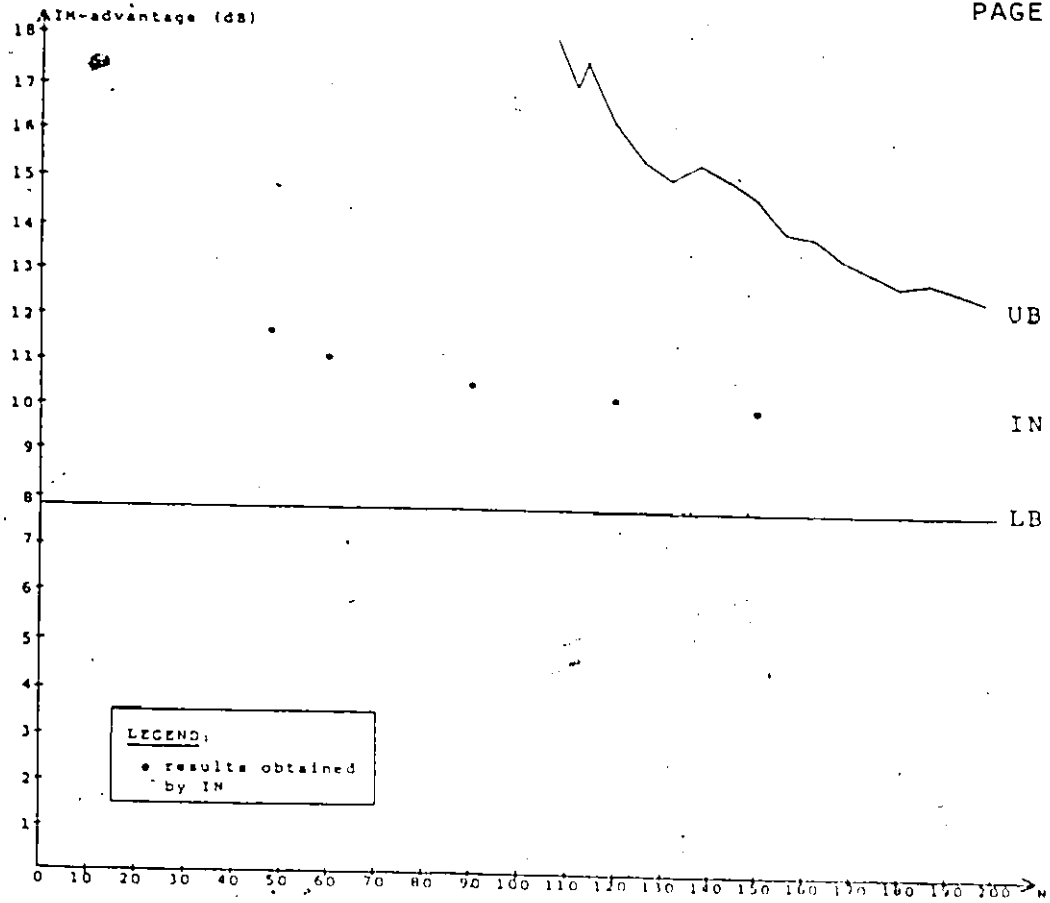


Figure 6.5. IM-advantage versus available bandwidth for $(W/K)-6$

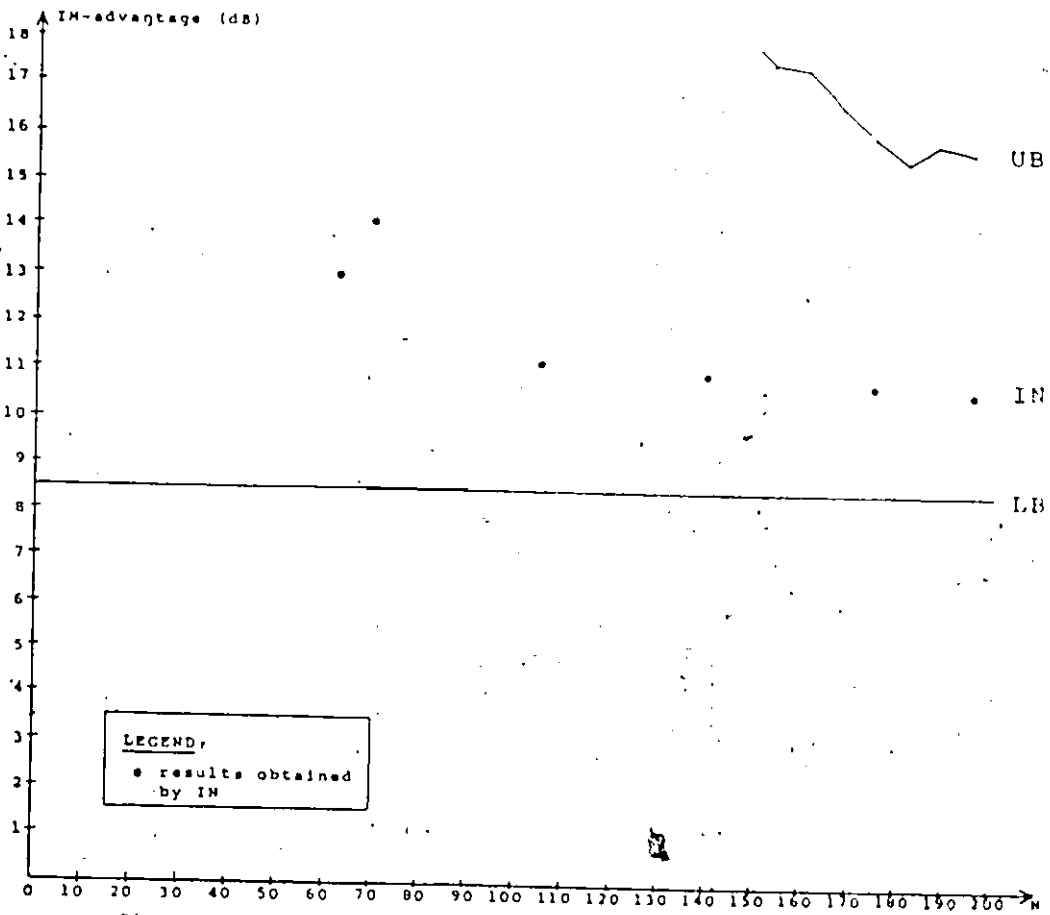


Figure 6.6. IM-advantage versus available bandwidth for $(W/K)-7$

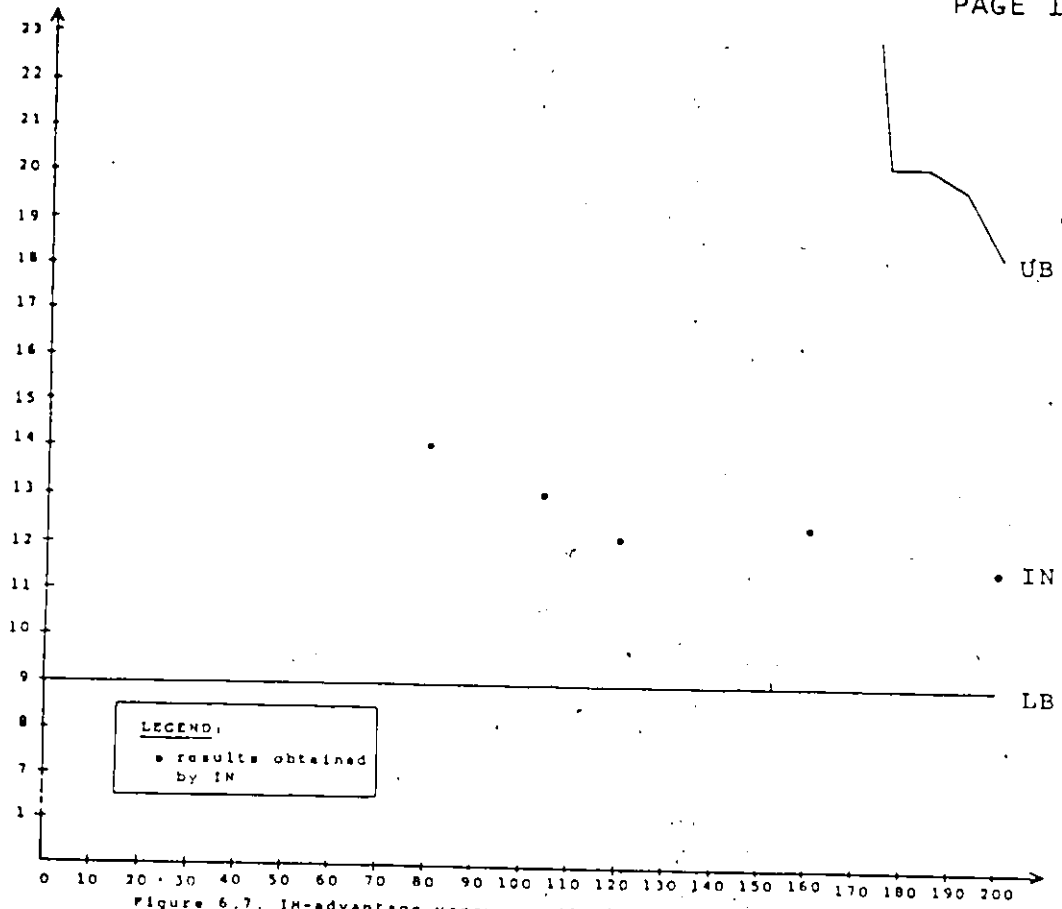


Figure 6.7. IM-advantage versus available bandwidth for $(N/K)=8$

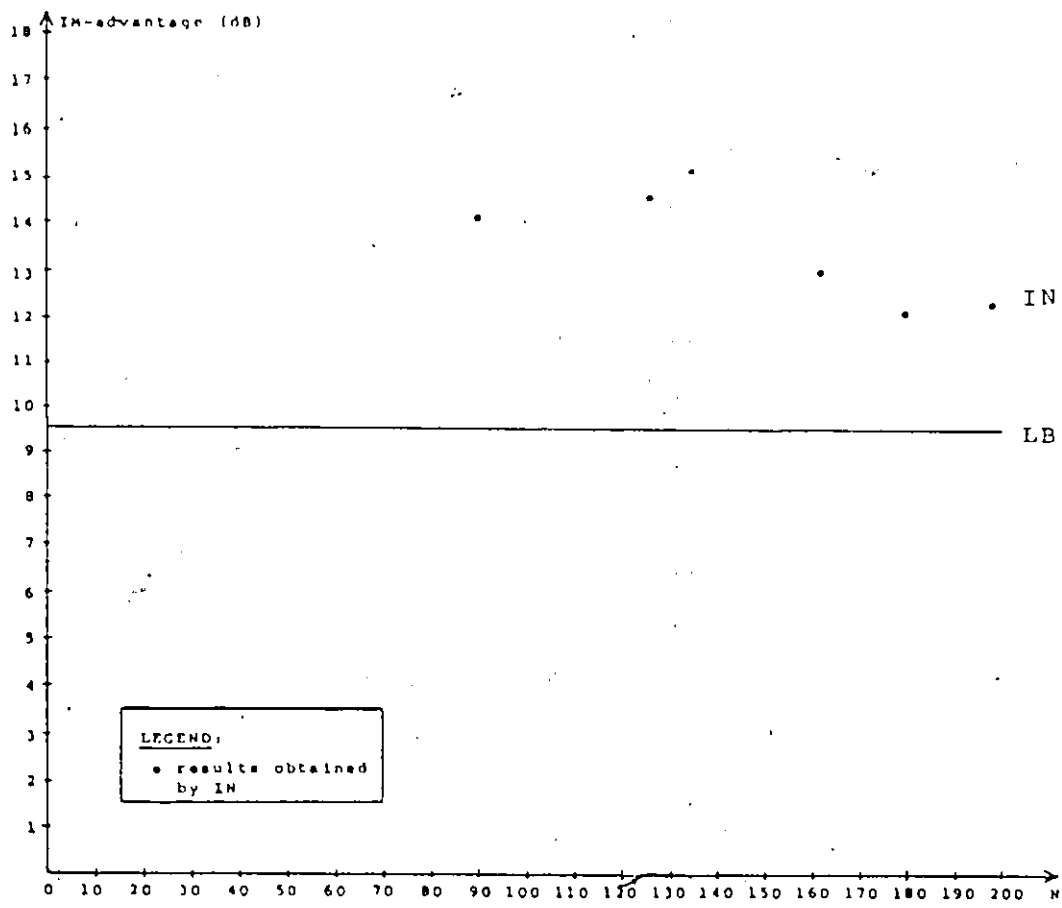


Figure 6.8. IM-advantage versus available bandwidth for $(N/K)=9$

As can be observed, the points all fall within the upper and lower bounds and furthermore the results of IN almost always coincide with the true results obtained from EXHAUST.

6.5. OVERHEAD OF OBTAINING BOTH DOUBLE AND TRIPLE PRODUCTS

Table 6.6 provides some insight into the solution time overhead incurred when double products are included in the analysis. From the data shown in this table, it can be seen that this overhead is in the order of 10%.

K	N	Execution Time (Seconds) for IN when determining	
		Triple Product	Triple + Double Product
20	40	7.70	8.57
20	60	14.12	15.63
20	80	20.64	22.67
20	100	27.16	29.82
30	60	86.22	93.22
30	90	158.83	171.28
30	120	231.18	249.44
30	150	304.23	327.47

Table 6.6 Solution Times For The Procedure IN When Determining Only Triple Products Or Triple + Double Products.

Chapter 7

CONCLUSIONS AND FURTHER STUDY

In this thesis various methods have been studied for solving the frequency assignment problem (FAP) with "equal-level" carriers in satellite communication systems. The study has exclusively considered third-order products (both double and triple) because these have dominant power.

Our initial effort in the study was to obtain solution to the IM-free FAP. However no method apart from exhaustive search could be uncovered and the inefficiency of this approach made it practically infeasible. Furthermore it became apparent that IM-free frequency assignments are inefficient in terms of bandwidth utilization, especially when large number of carriers must be accommodated. Therefore the IM-minimum frequency assignment problem was considered. Solutions to the IM-minimum frequency assignment problem yield significantly better bandwidth utilization [15], [23]. Unfortunately exhaustive search continues to be the only means for obtaining true solutions to this problem.

In this thesis two heuristic methods, namely IN and INU are developed to obtain approximate solutions to IM-minimum FAP.

The results with these heuristics are compared with the results obtained using heuristic recently proposed by Okinaka et al. [26]. An enhanced version of their proposal has been implemented in a program called DELIN (see section 5.4.1). The results obtained with the proposed heuristics are also compared with true solutions obtained using the program called EXHAUST. To our knowledge, the IM-minimum results from EXHAUST have never appeared in the literature (except those cases where an IM-free solution is available).

The results of our comparative studies are summarized below:

- i) The quality of the assignments in terms of IM-advantage obtained from DELIN, IN, and INU is almost identical.
- ii) The IN procedure generally runs at least three times faster than DELIN.
- iii) The execution time of INU is generally between that of IN and DELIN. This is reasonable since the execution time of INU is equal to that of IN plus the execution time for checking the unassigned channels in the case of a tie in (minimum) worst-IM.

Therefore the procedure IN is the best among three heuristics that were examined.

Other analyses of the results obtained with IN procedure have yielded the following observations:

- i) For the true solutions which are available from the EXHAUST program (for small values of K), the IM-advantage values of the approximate solutions obtained by IN are in close agreement (frequently identical).
- ii) The IM-advantage values of the approximate solutions produced by IN are always within the theoretical (lower and upper) bounds.
- iii) The difference between IM-advantage values obtained with IN and the upper bound becomes smaller as the bandwidth (and the number of carriers) increases for any particular bandwidth ratio (N/K). This suggests that the results produced by IN are also of good quality for large values of N , K (independent of N/K).

Another aspect of our study was an investigation of the sensitivity of the DELIN procedure (i.e. the Okinaka et al. heuristic [26]) to the initial assignment. Our experiments demonstrate that the "even" initial assignment is the most reasonable from the point of view of solution time. In particular it outperforms random assignment and other prespecified patterns.

It should also be noted that when determining double + triple products, all programs make use of Lemma 3.2. As a result, the overhead from the double products has a minor influence on the overall solution time.

As a possible area of further study, we recommend an examination of a generalization of the insertion (and deletion) procedure(s) that is (are) used in the DELIN, IN, and INU algorithms. In the current version, frequency carriers (slots) are inserted (deleted) one at a time; i.e., we use a "1-at-a-time" approach. The obvious generation is a "c-at-a-time" approach in which $c \leq K$ carriers (slots) are inserted (deleted) at each stage. The value of c could be a user supplied output parameter. When $c = K$, the procedure becomes an exhaustive search. Solution time would necessarily increase with increasing values of c , but better quality solutions could be anticipated. Similarly, other variants of neighbourhood search procedures could be used. For example, we might consider all assignments obtainable by exchanging c currently assigned slots with c currently unassigned slots.

A more extensive examination of branch and bound techniques could also be productive. There is a general feeling that the optimum FAP is NP-complete. It would be worthwhile to undertake a study of this question.

The interactive package, INTERMOD, which contains the program EXHAUST, and heuristics DELIN, IN, and INU is simple and user friendly. The package comprises about 1400 lines of FORTRAN code. The results presented in this thesis were obtained with the package running on an AMDAHL 470 V8.

APPENDIX: INTERMOD RESULTS

INTERMOD Output:

In this Appendix, we give the assignments corresponding to Table 6.1 and the complete INTERMOD output for the results given in Table 6.2 and Table 6.3.

For the former case the results are shown in Figures A1.1 through A1.168. A typical assignment has the following format:

```

F      1  2  3  5  8  9
I      1  3  2  2  2  2
    
```

This indicates a frequency assignment of (1,2,3,5,8,9) with one triple product falling on the first carrier frequency, three on the second, and two on each of the remaining frequencies. In those cases where the assignment is IM-free, a suitable message appears.

The results corresponding to Tables 6.2 and 6.3 are given in Figures A2.1-A2.3, and Figure A3.1-A3.2 respectively. Output produced by DELIN has the following form:

```

SCENARIO
*****
K=  3
N= 12

WITH TRIPLE
AND DOUBLE

FOR THE INSERT  INITIAL ASSIGNMENT:
  1  2  4  8  12
INITIAL WORST (D + T) = 1.00
INITIAL TOTAL (D + T) = 1.50

WORST : D + T = 1.00      TOTAL : D + T = 1.50
        D = 1           O = 2
        T = 1           T = 1

F      1      2      4      8      12
-----
IM  0.0    0.25    1.00    0.25    0.0
-----
D    0      1      0      1      0
-----
T    0      0      1      0      0
-----

IN-ADVANTAGE (DB) ==>  8.53
DATIMMED, JAN 01 1986  22113
ECPU= 0.03 SEC.
    
```

The above information has the following interpretation:

i) A (5,12) case is being considered with both triple (T) and double (D) products. The initial assignment \bar{g} is (1,2,4,6,12) which has been obtained as the output of IN or INU (indicated by the specification "INSERT INITIAL ASSIGNMENT").

ii) The initial assignment has

$$WIM(\bar{g}) = 1$$

$$TIM(\bar{g}) = 1.5$$

iii) The result generated by DELIN is the assignment, g , given by (1,2,4,6,12) (unchanged from \bar{g}):

iv) For this assignment:

$$WIM(g) = 1.0 \dots\dots\dots TIM(g) = 1.5$$

$$WIM_2(g) = 1 \dots\dots\dots TIM_2(g) = 2$$

$$WIM_3(g) = 1 \dots\dots\dots TIM_3(g) = 1$$

$$v) N(f_1;g) = 0.0 ; N(f_2;g) = 0.25 ; N(f_3;g) = 1.0 ;$$

$$N(f_4;g) = 0.25 ; N(f_5;g) = 0$$

$$N_2(f_1;g) = 0 ; N_2(f_2;g) = 1 ; N_2(f_3;g) = 0 ;$$

$$N_2(f_4;g) = 1 ; N_2(f_5;g) = 0$$

$$N_3(f_1;g) = 0 ; N_3(f_2;g) = 0 ; N_3(f_3;g) = 1 ;$$

$$N_3(f_4;g) = 0 ; N_3(f_5;g) = 0$$

$$vi) IMA(g) = 6.53 \text{ dB}$$

vii) The CPU time for the solution is 0.03 sec.

Output produced by either IN or INU has the same form, but does not contain any reference to an initial assignment.

In the following the results of the INTERMOD are given:

F=>	1	2	3	4
I=>	1	2	2	1

Figure Al.1

F=>	1	2	3	5	6
I=>	1	3	2	2	2
F=>	1	2	4	5	6
I=>	2	2	2	3	1

Figure Al.6

F=>	1	2	3	5
I=>	0	1	1	0
F=>	1	3	4	5
I=>	0	1	1	0

Figure Al.2

F=>	1	2	4	6	7
I=>	1	1	2	1	1

Figure Al.7

F=>	1	2	3	6
I=>	0	1	0	0
F=>	1	2	4	6
I=>	0	0	1	0
F=>	1	3	5	6
I=>	0	1	0	0
F=>	1	4	5	6
I=>	0	0	1	0

Figure Al.3

F=>	1	2	3	5	8
I=>	0	1	1	1	0
F=>	1	4	6	7	8
I=>	0	1	1	1	0

Figure Al.8

THE FOLLOWING ARE INTERMODULATION FREE

1	2	5	7
1	3	6	7

Figure Al.4

F=>	1	2	3	6	9
I=>	0	1	0	1	0
F=>	1	2	5	7	9
I=>	0	0	1	1	0
F=>	1	3	5	8	9
I=>	0	1	1	0	0
F=>	1	4	7	8	9
I=>	0	1	0	1	0

Figure Al.9

F=>	1	2	3	4	5
I=>	2	4	4	4	2

Figure Al.5

F=>	1	2	3	7	10
I=>	0	1	0	0	0
F=>	1	2	5	8	10
I=>	0	0	1	0	0
F=>	1	3	6	9	10
I=>	0	0	1	0	0
F=>	1	4	8	9	10
I=>	0	0	0	1	0

Figure Al.10

F=>	1	2	3	8	11
I=>	0	1	0	0	0
F=>	1	2	4	7	11
I=>	0	0	1	0	0
F=>	1	2	5	9	11
I=>	0	0	1	0	0
F=>	1	2	6	8	11
I=>	0	0	1	0	0
F=>	1	2	6	9	11
I=>	0	0	1	0	0
F=>	1	2	7	9	11
I=>	0	0	0	1	0
F=>	1	3	5	10	11
I=>	0	1	0	0	0
F=>	1	3	6	10	11
I=>	0	0	1	0	0
F=>	1	3	7	8	11
I=>	0	0	1	0	0
F=>	1	3	7	10	11
I=>	0	0	1	0	0
F=>	1	4	5	9	11
I=>	0	0	1	0	0
F=>	1	4	6	10	11
I=>	0	0	1	0	0
F=>	1	4	9	10	11
I=>	0	0	0	1	0
F=>	1	5	8	10	11
I=>	0	0	1	0	0

Figure Al.11

THE FOLLOWINGS ARE INTERMODULATION FREE

1	2	3	10	12
1	3	8	9	12
1	3	8	11	12
1	4	5	10	12

Figure Al.12

F=>	1	2	3	4	5	6
I=>	4	6	7	7	6	4

Figure Al.13

F=>	1	2	3	4	6	7
I=>	3	4	5	5	4	3
F=>	1	2	3	5	6	7
I=>	3	5	4	4	5	3
F=>	1	2	4	5	6	7
I=>	3	4	5	5	4	3

Figure Al.14

F=>	1	2	3	5	7	8
I=>	2	3	3	3	3	2
F=>	1	2	4	6	7	8
I=>	2	3	3	3	3	2

Figure Al.15

F=>	1	2	3	5	8	9
I=>	1	3	2	2	2	2
F=>	1	2	5	7	8	9
I=>	2	2	2	2	3	1

Figure Al.16

F=>	1	2	4	6	9	10
I=>	2	1	2	2	2	1
F=>	1	2	5	7	9	10
I=>	1	2	2	2	1	2

Figure Al.17

F=>	1	2	3	6	9	11
I=>	1	1	1	2	1	1
F=>	1	2	4	6	10	11
I=>	1	1	1	2	1	1
F=>	1	2	6	8	10	11
I=>	1	1	2	1	1	1
F=>	1	3	6	9	10	11
I=>	1	1	2	1	1	1

Figure Al.18

F=>	1	2	3	7	11	14
I=>	0	1	0	1	0	0
F=>	1	2	7	10	12	14
I=>	0	0	1	0	1	0
F=>	1	3	5	8	13	14
I=>	0	1	0	1	0	0
F=>	1	4	8	12	13	14
I=>	0	0	1	0	1	0

Figure Al.21

F=>	1	2	3	6	10	12
I=>	1	1	1	1	1	1
F=>	1	2	3	7	10	12
I=>	1	1	1	1	1	1
F=>	1	2	4	6	11	12
I=>	1	1	1	1	1	1
F=>	1	2	4	7	11	12
I=>	1	1	1	1	1	1
F=>	1	2	6	9	11	12
I=>	1	1	1	1	1	1
F=>	1	2	7	9	11	12
I=>	1	1	1	1	1	1
F=>	1	3	6	10	11	12
I=>	1	1	1	1	1	1
F=>	1	3	7	10	11	12
I=>	1	1	1	1	1	1

Figure Al.19

F=>	1	2	3	7	12	15
I=>	0	1	0	1	0	0
F=>	1	2	3	8	11	15
I=>	0	1	0	1	0	0
F=>	1	2	3	8	12	15
I=>	0	1	0	1	0	0
F=>	1	2	4	7	11	15
I=>	0	0	1	0	1	0
F=>	1	2	5	8	13	15
I=>	0	0	1	1	0	0
F=>	1	2	8	10	12	15
I=>	0	0	1	1	0	0
F=>	1	2	8	11	13	15
I=>	0	0	1	0	1	0
F=>	1	3	5	8	14	15
I=>	0	1	0	1	0	0
F=>	1	3	8	11	14	15
I=>	0	0	1	1	0	0
F=>	1	4	6	8	14	15
I=>	0	0	1	1	0	0
F=>	1	4	8	13	14	15
I=>	0	0	1	0	1	0
F=>	1	4	9	13	14	15
I=>	0	0	1	0	1	0
F=>	1	5	8	13	14	15
I=>	0	0	1	0	1	0
F=>	1	5	9	12	14	15
I=>	0	1	0	1	0	0

Figure Al.22

F=>	1	2	3	7	10	13
I=>	0	1	0	1	1	0
F=>	1	4	7	11	12	13
I=>	0	1	1	0	1	0

Figure Al.20

F=>	1	2	3	5	10	16
I=>	0	1	1	0	0	0
F=>	1	2	3	5	11	16
I=>	0	1	1	0	0	0
F=>	1	2	3	6	10	16
I=>	0	1	0	1	0	0
F=>	1	2	3	7	13	16
I=>	0	1	0	1	0	0
F=>	1	2	3	8	12	16
I=>	0	1	0	0	1	0
F=>	1	2	3	8	13	16
I=>	0	1	0	1	0	0
F=>	1	2	3	9	12	16
I=>	0	1	0	1	0	0
F=>	1	2	3	9	13	16
I=>	0	1	0	1	0	0
F=>	1	2	3	10	13	16
I=>	0	1	0	0	1	0
F=>	1	2	4	7	12	16
I=>	0	0	1	1	0	0
F=>	1	2	5	8	14	16
I=>	0	0	1	1	0	0
F=>	1	2	5	9	11	16
I=>	0	0	1	1	0	0
F=>	1	2	5	9	14	16
I=>	0	0	1	1	0	0
F=>	1	2	6	10	13	16
I=>	0	0	1	0	1	0
F=>	1	2	7	9	12	16
I=>	0	0	1	1	0	0
F=>	1	2	7	9	13	16
I=>	0	0	1	1	0	0
F=>	1	2	7	12	14	16
I=>	0	0	1	0	1	0
F=>	1	2	8	11	14	16
I=>	0	0	1	1	0	0
F=>	1	2	9	11	13	16
I=>	0	0	1	1	0	0
F=>	1	2	9	12	14	16
I=>	0	0	1	0	1	0

F=>	1	3	4	5	11	16
I=>	0	1	1	0	0	0
F=>	1	3	4	7	11	16
I=>	0	0	1	1	0	0
F=>	1	3	5	8	15	16
I=>	0	1	0	1	0	0
F=>	1	3	5	10	13	16
I=>	0	1	0	0	1	0
F=>	1	3	5	10	15	16
I=>	0	1	0	1	0	0
F=>	1	3	6	9	15	16
I=>	0	0	1	1	0	0
F=>	1	3	7	8	13	16
I=>	0	0	1	1	0	0
F=>	1	3	8	12	13	16
I=>	0	0	1	1	0	0
F=>	1	3	8	12	15	16
I=>	0	0	1	1	0	0
F=>	1	3	9	12	15	16
I=>	0	0	1	1	0	0
F=>	1	4	5	9	14	16
I=>	0	0	1	1	0	0
F=>	1	4	6	8	15	16
I=>	0	0	1	1	0	0
F=>	1	4	7	11	15	16
I=>	0	1	0	1	0	0
F=>	1	4	7	12	14	16
I=>	0	1	0	0	1	0
F=>	1	4	7	14	15	16
I=>	0	1	0	0	1	0
F=>	1	4	8	10	15	16
I=>	0	0	1	1	0	0
F=>	1	4	8	14	15	16
I=>	0	0	1	0	1	0
F=>	1	4	9	10	14	16
I=>	0	0	1	1	0	0
F=>	1	4	9	14	15	16
I=>	0	0	1	0	1	0
F=>	1	4	10	11	12	16
I=>	0	0	1	1	0	0

Figure A1.23

F=>	1	4	10	14	15	16
I=>	0	0	1	0	1	0
F=>	1	5	6	7	13	16
I=>	0	0	1	1	0	0
F=>	1	5	8	10	15	16
I=>	0	0	1	1	0	0
F=>	1	5	8	14	15	16
I=>	0	0	1	0	1	0
F=>	1	5	9	14	15	16
I=>	0	1	0	0	1	0
F=>	1	5	10	13	15	16
I=>	0	0	1	1	0	0
F=>	1	6	8	12	15	16
I=>	0	0	1	1	0	0
F=>	1	6	10	13	14	16
I=>	0	0	1	1	0	0
F=>	1	6	12	13	14	16
I=>	0	0	0	1	1	0
F=>	1	6	12	14	15	16
I=>	0	0	0	1	1	0
F=>	1	7	11	14	15	16
I=>	0	0	1	0	1	0
F=>	1	7	12	14	15	16
I=>	0	0	0	1	1	0

Figure Al.23 (cont.)

F=>	1	2	3	7	14	17
I=>	0	1	0	0	0	0
F=>	1	2	4	8	12	17
I=>	0	0	0	1	0	0
F=>	1	2	5	10	15	17
I=>	0	0	0	1	0	0
F=>	1	2	6	8	14	17
I=>	0	0	0	1	0	0
F=>	1	2	6	9	15	17
I=>	0	0	0	1	0	0
F=>	1	2	8	12	14	17
I=>	0	0	1	0	0	0

Figure Al.24

F=>	1	2	8	12	15	17
I=>	0	0	1	0	0	0
F=>	1	2	10	12	14	17
I=>	0	0	0	1	0	0
F=>	1	2	10	13	15	17
I=>	0	0	0	0	1	0
F=>	1	3	5	8	16	17
I=>	0	1	0	0	0	0
F=>	1	3	6	10	16	17
I=>	0	0	0	1	0	0
F=>	1	3	8	13	14	17
I=>	0	0	1	0	0	0
F=>	1	3	8	13	16	17
I=>	0	0	1	0	0	0
F=>	1	3	9	12	16	17
I=>	0	0	1	0	0	0
F=>	1	4	5	10	15	17
I=>	0	0	0	1	0	0
F=>	1	4	6	8	16	17
I=>	0	0	1	0	0	0
F=>	1	4	6	10	16	17
I=>	0	0	0	1	0	0
F=>	1	4	10	12	16	17
I=>	0	0	1	0	0	0
F=>	1	4	11	12	13	17
I=>	0	0	0	1	0	0
F=>	1	4	11	15	16	17
I=>	0	0	0	0	1	0
F=>	1	5	6	7	14	17
I=>	0	0	1	0	0	0
F=>	1	6	10	14	16	17
I=>	0	0	1	0	0	0

Figure Al.24 (cont.)

THE FOLLOWINGS ARE INTERMODULATION FREE

1	2	5	11	13	18
1	2	5	11	16	18
1	2	9	12	14	18
1	2	9	13	15	18
1	3	8	14	17	18
1	4	6	10	17	18
1	5	7	10	17	18
1	6	8	14	17	18

Figure Al.25

F=>	1	2	3	4	5	6	7
I=>	8	9	10	11	10	9	8

Figure Al.26

F=>	1	2	3	4	6	7	8
I=>	5	7	8	7	7	7	5
F=>	1	2	3	5	6	7	8
I=>	5	7	7	7	8	7	5

Figure Al.27

F=>	1	2	3	5	7	8	9
I=>	4	5	6	5	6	5	4

Figure Al.28

F=>	1	2	3	4	6	9	10
I=>	4	4	5	5	4	4	3
F=>	1	2	3	5	7	9	10
I=>	3	4	5	5	5	4	3
F=>	1	2	3	5	8	9	10
I=>	3	5	5	3	4	5	4
F=>	1	2	3	6	8	9	10
I=>	4	5	4	3	5	5	3
F=>	1	2	4	6	8	9	10
I=>	3	4	5	5	5	4	3
F=>	1	2	5	7	8	9	10
I=>	3	4	4	5	5	4	4

Figure Al.29

F=>	1	2	3	5	7	10	11
I=>	3	4	3	4	4	3	3

F=>	1	2	3	5	8	10	11
I=>	2	4	4	3	4	4	3

F=>	1	2	3	6	8	10	11
I=>	3	3	4	4	4	3	3

F=>	1	2	4	6	9	10	11
I=>	3	3	4	4	4	3	3

F=>	1	2	4	7	9	10	11
I=>	3	4	4	3	4	4	2

F=>	1	2	5	7	9	10	11
I=>	3	3	4	4	3	4	3

Figure Al.30

F=>	1	2	3	6	9	11	12
I=>	2	3	3	3	3	3	3

F=>	1	2	3	7	9	11	12
I=>	3	3	3	3	3	3	2

F=>	1	2	4	6	10	11	12
I=>	2	3	3	3	3	3	3

F=>	1	2	4	7	10	11	12
I=>	3	3	3	3	3	3	2

Figure Al.31

F=>	1	2	3	5	8	12	13
I=>	2	3	2	2	2	3	2

F=>	1	2	3	6	9	11	13
I=>	2	2	2	3	2	3	2

F=>	1	2	3	7	10	11	13
I=>	2	3	2	2	3	3	1

F=>	1	2	3	7	10	12	13
I=>	2	3	2	2	2	3	2

F=>	1	2	4	7	11	12	13
I=>	2	3	2	2	2	3	2

F=>	1	2	6	9	11	12	13
I=>	2	3	2	2	2	3	2

F=>	1	3	4	7	11	12	13
I=>	1	3	3	2	2	3	2

F=>	1	3	5	8	11	12	13
I=>	2	3	2	3	2	2	2

Figure Al.32

F=>	1	2	3	5	8	13	14
I=>	1	3	2	1	2	2	2
F=>	1	2	7	10	12	13	14
I=>	2	2	2	1	2	3	1

Figure Al.33

F=>	1	2	3	9	13	15	18
I=>	1	1	1	1	1	1	0
F=>	1	2	3	9	13	16	18
I=>	1	1	1	1	0	1	1
F=>	1	3	6	10	16	17	18
I=>	1	1	0	1	1	1	1
F=>	1	4	6	10	16	17	18
I=>	0	1	1	1	1	1	1

Figure Al.37

F=>	1	2	3	6	9	13	15
I=>	1	2	1	2	2	2	1
F=>	1	2	3	7	10	13	15
I=>	1	2	1	2	2	1	2
F=>	1	3	6	9	13	14	15
I=>	1	1	2	2	1	2	1
F=>	1	3	7	10	13	14	15
I=>	1	2	2	2	1	2	1

Figure Al.34

F=>		2	3	5	9	14	19
I=>	0	1	1	1	0	1	0
F=>	1	2	3	8	11	15	19
I=>	0	1	0	1	1	1	0
F=>	1	5	9	12	17	18	19
I=>	0	1	1	1	0	1	0
F=>	1	6	11	15	17	18	19
I=>	0	1	0	1	1	1	0

Figure Al.38

F=>	1	2	8	11	12	14	16
I=>	1	1	1	2	2	1	0
F=>	1	3	5	8	9	15	16
I=>	0	1	2	2	1	1	1

Figure Al.35

F=>	1	2	3	10	14	17	20
I=>	0	1	0	1	0	1	0
F=>	1	2	10	13	16	18	20
I=>	0	0	1	1	0	1	0
F=>	1	3	5	8	11	19	20
I=>	0	1	0	1	1	0	0
F=>	1	4	7	11	18	19	20
I=>	0	1	0	1	0	1	0

Figure Al.39

F=>	1	2	4	6	10	16	17
I=>	1	1	1	1	1	1	1
F=>	1	2	8	12	14	16	17
I=>	1	1	1	1	1	1	1

Figure Al.36

F=>	1	2	7	10	17	19	21
I=>	0	0	0	1	0	1	0
F=>	1	3	5	12	15	20	21
I=>	0	1	0	1	0	0	0

Figure Al.40

F=>	1	2	4	7	14	18	22
I=>	0	0	1	0	0	1	0
F=>	1	2	7	10	18	20	22
I=>	0	0	0	1	0	1	0
F=>	1	2	8	12	17	20	22
I=>	0	0	0	1	1	0	0
F=>	1	2	9	13	17	19	22
I=>	0	0	1	1	0	0	0
F=>	1	2	11	14	16	18	22
I=>	0	0	0	0	1	1	0
F=>	1	2	11	15	17	19	22
I=>	0	0	0	1	1	0	0
F=>	1	3	5	13	16	21	22
I=>	0	1	0	1	0	0	0
F=>	1	3	6	11	15	21	22
I=>	0	0	1	1	0	0	0
F=>	1	4	6	8	12	21	22
I=>	0	0	1	1	0	0	0
F=>	1	4	6	10	14	21	22
I=>	0	0	0	1	1	0	0
F=>	1	5	7	9	12	21	22
I=>	0	1	1	0	0	0	0
F=>	1	5	9	16	19	21	22
I=>	0	1	0	0	1	0	0

Figure Al.41

F=>	1	2	3	5	11	16	23
I=>	0	1	1	0	0	0	0
F=>	1	2	3	5	11	18	23
I=>	0	1	1	0	0	0	0
F=>	1	2	3	8	11	19	23
I=>	0	1	0	0	1	0	0
F=>	1	2	3	8	12	20	23
I=>	0	1	0	0	1	0	0
F=>	1	2	3	9	14	19	23
I=>	0	1	0	0	1	0	0
F=>	1	2	3	11	16	19	23
I=>	0	1	0	1	0	0	0

F=>	1	2	3	11	16	20	23
I=>	0	1	0	1	0	0	0
F=>	1	2	4	9	13	17	23
I=>	0	0	0	1	1	0	0
F=>	1	2	5	8	13	21	23
I=>	0	0	1	0	1	0	0
F=>	1	2	5	11	16	21	23
I=>	0	0	0	1	1	0	0
F=>	1	2	6	8	10	20	23
I=>	0	0	1	1	0	0	0
F=>	1	2	6	9	12	21	23
I=>	0	0	0	1	1	0	0
F=>	1	2	6	10	17	20	23
I=>	0	0	1	0	0	1	0
F=>	1	2	7	9	11	20	23
I=>	0	0	0	1	1	0	0
F=>	1	2	7	10	19	21	23
I=>	0	0	0	1	0	1	0
F=>	1	2	8	11	19	21	23
I=>	0	0	0	1	0	1	0
F=>	1	2	9	12	17	21	23
I=>	0	0	1	1	0	0	0
F=>	1	2	9	14	17	19	23
I=>	0	0	1	1	0	0	0
F=>	1	2	10	16	18	20	23
I=>	0	0	1	0	1	0	0
F=>	1	2	10	16	19	21	23
I=>	0	0	1	0	0	1	0
F=>	1	2	11	15	18	21	23
I=>	0	0	1	0	1	0	0
F=>	1	2	13	15	17	20	23
I=>	0	0	0	1	0	1	0
F=>	1	2	13	16	19	21	23
I=>	0	0	0	1	0	1	0
F=>	1	3	4	10	15	19	23
I=>	0	0	0	1	0	1	0
F=>	1	3	5	8	11	22	23
I=>	0	1	0	1	0	0	0
F=>	1	3	5	8	14	22	23
I=>	0	1	0	0	1	0	0

Figure Al.42

F=>	1	3	5	9	10	20	23
I=>	0	1	1	0	0	0	0
F=>	1	3	5	13	16	22	23
I=>	0	1	0	1	0	0	0
F=>	1	3	5	14	15	20	23
I=>	0	1	0	1	0	0	0
F=>	1	3	5	14	17	22	23
I=>	0	1	0	1	0	0	0
F=>	1	3	8	9	13	22	23
I=>	0	0	1	0	1	0	0
F=>	1	3	7	12	15	22	23
I=>	0	0	0	1	1	0	0
F=>	1	3	8	13	19	22	23
I=>	0	0	1	1	0	0	0
F=>	1	3	11	14	17	18	23
I=>	0	0	0	1	1	0	0
F=>	1	3	11	16	17	20	23
I=>	0	0	0	0	1	1	0
F=>	1	3	11	16	18	22	23
I=>	0	0	1	0	1	0	0
F=>	1	3	12	15	18	22	23
I=>	0	0	1	1	0	0	0
F=>	1	4	6	8	14	22	23
I=>	0	0	1	0	1	0	0
F=>	1	4	7	8	13	21	23
I=>	0	1	1	0	0	0	0
F=>	1	4	7	9	11	22	23
I=>	0	1	0	1	0	0	0
F=>	1	4	7	14	16	22	23
I=>	0	1	0	0	1	0	0
F=>	1	4	8	13	21	22	23
I=>	0	0	0	1	0	1	0
F=>	1	4	9	10	19	21	23
I=>	0	0	0	1	0	1	0
F=>	1	4	12	16	21	22	23
I=>	0	0	1	0	0	1	0
F=>	1	4	13	15	17	22	23
I=>	0	0	1	1	0	0	0
F=>	1	4	14	15	19	21	23
I=>	0	0	0	0	1	1	0

Figure Al.42 (cont.)

F=>	1	4	14	16	18	22	23
I=>	0	0	0	1	1	0	0
F=>	1	5	7	10	15	22	23
I=>	0	0	0	1	1	0	0
F=>	1	5	8	13	21	22	23
I=>	0	0	0	1	0	1	0
F=>	1	5	9	14	20	21	23
I=>	0	1	0	1	0	0	0
F=>	1	5	10	15	21	22	23
I=>	0	0	1	0	0	1	0
F=>	1	5	13	16	21	22	23
I=>	0	0	1	0	0	1	0
F=>	1	6	7	10	13	21	23
I=>	0	0	1	1	0	0	0
F=>	1	6	13	19	21	22	23
I=>	0	0	0	0	1	1	0
F=>	1	7	11	15	20	22	23
I=>	0	0	1	1	0	0	0
F=>	1	8	13	19	21	22	23
I=>	0	0	0	0	1	1	0

Figure Al.42 (cont.)

F=>	1	2	3	12	16	19	24
I=>	0	1	0	0	0	0	0
F=>	1	2	3	12	17	20	24
I=>	0	1	0	0	0	0	0
F=>	1	2	5	11	17	19	24
I=>	0	0	0	1	0	0	0
F=>	1	2	5	11	17	22	24
I=>	0	0	0	1	0	0	0
F=>	1	2	8	13	16	22	24
I=>	0	0	0	1	0	0	0
F=>	1	2	7	10	20	22	24
I=>	0	0	0	0	0	1	0
F=>	1	2	9	13	19	22	24
I=>	0	0	0	1	0	0	0
F=>	1	2	9	15	19	21	24
I=>	0	0	0	1	0	0	0

Figure Al.43

F=>	1	3	5	15	16	21	24
I=>	0	1	0	0	0	0	0
F=>	1	3	5	15	18	23	24
I=>	0	1	0	0	0	0	0
F=>	1	3	8	12	15	23	24
I=>	0	0	0	1	0	0	0
F=>	1	3	8	14	20	23	24
I=>	0	0	0	1	0	0	0
F=>	1	3	9	12	19	23	24
I=>	0	0	0	1	0	0	0
F=>	1	3	10	16	20	21	24
I=>	0	0	0	0	1	0	0
F=>	1	4	5	9	15	22	24
I=>	0	0	1	0	0	0	0
F=>	1	4	6	10	16	23	24
I=>	0	0	0	1	0	0	0
F=>	1	4	9	10	20	22	24
I=>	0	0	0	0	0	1	0
F=>	1	5	8	13	22	23	24
I=>	0	0	0	0	0	1	0
F=>	1	6	8	14	20	23	24
I=>	0	0	0	1	0	0	0
F=>	1	6	9	13	22	23	24
I=>	0	0	0	0	0	1	0

Figure Al.43 (cont.)

F=>	1	2	3	7	15	18	25
I=>	0	1	0	0	0	0	0
F=>	1	2	3	7	15	22	25
I=>	0	1	0	0	0	0	0
F=>	1	2	3	10	15	21	25
I=>	0	1	0	0	0	0	0
F=>	1	2	4	9	15	21	25
I=>	0	0	0	0	1	0	0
F=>	1	2	5	11	16	23	25
I=>	0	0	0	0	1	0	0
F=>	1	2	5	12	17	19	25
I=>	0	0	0	1	0	0	0

F=>	1	2	5	12	17	23	25
I=>	0	0	0	1	0	0	0
F=>	1	2	6	14	18	22	25
I=>	0	0	0	1	0	0	0
F=>	1	2	7	8	11	22	25
I=>	0	0	0	1	0	0	0
F=>	1	2	7	9	12	21	25
I=>	0	0	1	0	0	0	0
F=>	1	2	7	10	21	23	25
I=>	0	0	0	0	0	1	0
F=>	1	2	8	12	20	22	25
I=>	0	0	0	1	0	0	0
F=>	1	2	8	12	20	23	25
I=>	0	0	0	1	0	0	0
F=>	1	2	9	14	19	23	25
I=>	0	0	0	1	0	0	0
F=>	1	2	9	16	19	21	25
I=>	0	0	1	0	0	0	0
F=>	1	2	9	16	20	22	25
I=>	0	0	1	0	0	0	0
F=>	1	2	11	14	19	21	25
I=>	0	0	1	0	0	0	0
F=>	1	3	5	10	11	22	25
I=>	0	1	0	0	0	0	0
F=>	1	3	5	16	17	22	25
I=>	0	1	0	0	0	0	0
F=>	1	3	5	16	19	24	25
I=>	0	1	0	0	0	0	0
F=>	1	3	6	14	15	21	25
I=>	0	0	0	1	0	0	0
F=>	1	3	6	14	18	24	25
I=>	0	0	0	1	0	0	0
F=>	1	3	7	12	17	24	25
I=>	0	0	0	1	0	0	0
F=>	1	3	8	15	16	19	25
I=>	0	0	1	0	0	0	0
F=>	1	3	8	15	21	24	25
I=>	0	0	1	0	0	0	0
F=>	1	3	9	14	21	24	25
I=>	0	0	0	1	0	0	0

Figure Al.44

F=>	1	3	12	15	19	20	25
I=>	0	0	0	0	0	1	0
F=>	1	3	12	17	18	22	25
I=>	0	0	0	1	0	0	0
F=>	1	4	6	10	17	24	25
I=>	0	0	0	0	1	0	0
F=>	1	4	8	14	18	24	25
I=>	0	0	0	1	0	0	0
F=>	1	4	8	9	14	23	25
I=>	0	0	0	1	0	0	0
F=>	1	4	9	10	21	23	25
I=>	0	0	0	0	0	1	0
F=>	1	4	10	12	20	24	25
I=>	0	0	0	1	0	0	0
F=>	1	4	11	19	23	24	25
I=>	0	0	0	0	0	1	0
F=>	1	4	15	16	21	23	25
I=>	0	0	0	0	0	1	0
F=>	1	4	15	17	19	24	25
I=>	0	0	0	1	0	0	0
F=>	1	5	7	10	17	24	25
I=>	0	0	0	0	1	0	0
F=>	1	5	7	12	15	24	25
I=>	0	0	0	0	1	0	0
F=>	1	5	11	12	20	23	25
I=>	0	0	0	1	0	0	0
F=>	1	5	11	16	23	24	25
I=>	0	0	0	0	0	1	0
F=>	1	5	11	17	22	24	25
I=>	0	0	1	0	0	0	0
F=>	1	5	14	17	19	24	25
I=>	0	0	0	0	1	0	0
F=>	1	6	7	11	14	23	25
I=>	0	1	0	0	0	0	0
F=>	1	7	9	14	21	24	25
I=>	0	0	0	1	0	0	0
F=>	1	7	10	11	18	23	25
I=>	0	0	0	0	1	0	0
F=>	1	8	11	19	23	24	25
I=>	0	0	0	0	0	1	0

Figure Al.44 (cont.)

THE FOLLOWINGS ARE INTERMODULATION FREE

1	2	5	11	19	24	26
1	2	8	12	21	24	26
1	2	12	17	20	24	26
1	3	4	11	17	22	26
1	3	6	15	19	25	26
1	3	7	10	15	25	26
1	3	8	14	22	23	26
1	3	8	16	22	25	26
1	4	5	13	19	24	26
1	5	10	16	23	24	26

Figure Al.45

F=>	1	2	3	4	5	6	7	8
I=>	9	12	14	15	15	14	12	9

Figure Al.46

F=>	1	2	3	4	6	7	8	9
I=>	8	10	11	11	11	11	10	8

Figure Al.47

F=>	1	2	3	4	6	8	9	10
I=>	7	8	9	9	8	9	8	6

F=>	1	2	3	5	7	8	9	10
I=>	6	8	9	8	9	9	8	7

Figure Al.48

F=>	1	2	3	5	7	8	10	11
I=>	5	7	6	8	8	8	7	5

F=>	1	2	4	5	7	9	10	11
I=>	5	7	8	8	8	6	7	5

Figure Al.49

F=>	1	2	3	5	7	10	11	12
I=>	5	6	6	8	6	6	6	5

F=>	1	2	3	6	8	10	11	12
I=>	5	6	6	8	6	6	6	5

Figure Al.50

F=>	1	2	3	5	7	10	12	13
I=>	3	5	5	6	5	6	5	3

F=>	1	2	3	5	8	11	12	13
I=>	4	6	5	4	4	5	6	4

F=>	1	2	3	6	9	11	12	13
I=>	6	5	4	4	5	6	6	4

F=>	1	2	4	7	9	11	12	13
I=>	3	5	6	5	6	5	5	3

Figure Al.51

F=>	1	2	3	5	7	10	13	14
I=>	5	4	5	4	5	3	4	
F=>	1	2	3	5	8	10	13	14
I=>	3	4	4	5	5	5	4	3
F=>	1	2	5	7	10	12	13	14
I=>	3	4	5	9	5	4	4	3
F=>	1	2	5	8	10	12	13	14
I=>	4	3	5	4	5	4	5	3

Figure Al.52

F=>	1	2	3	7	10	13	15	17
I=>	2	2	3	3	3	2	4	2
F=>	1	2	4	7	9	13	16	17
I=>	2	2	4	3	3	3	3	1
F=>	1	2	5	8	11	14	16	17
I=>	1	3	3	3	3	4	2	2
F=>	1	3	5	8	11	15	16	17
I=>	2	4	2	3	3	3	2	2

Figure Al.55

F=>	1	2	3	7	10	12	14	15
I=>	3	4	4	4	4	4	4	4
F=>	1	2	4	6	9	13	14	15
I=>	4	4	4	4	4	4	4	3

Figure Al.53

F=>	1	2	3	6	9	14	16	18
I=>	3	2	2	3	2	3	3	2
F=>	1	2	4	7	9	13	17	18
I=>	1	3	3	3	3	3	1	3
F=>	1	2	6	10	12	15	17	18
I=>	3	1	3	3	3	3	3	1
F=>	1	3	5	10	13	16	17	18
I=>	2	3	3	2	3	2	2	3

Figure Al.56

F=>	1	2	3	5	6	11	15	16
I=>	2	4	3	3	4	3	3	3
F=>	1	2	3	6	9	12	14	16
I=>	3	2	3	4	4	3	4	2
F=>	1	2	3	7	9	12	15	16
I=>	3	3	3	4	4	2	3	3
F=>	1	2	3	8	10	12	13	16
I=>	3	4	3	3	4	4	3	1
F=>	1	2	3	8	11	12	14	16
I=>	3	3	4	2	4	4	3	2
F=>	1	2	5	8	10	14	15	16
I=>	3	3	2	4	4	3	3	3
F=>	1	2	6	9	12	14	15	16
I=>	3	3	3	4	3	3	4	2
F=>	1	3	5	6	9	14	15	16
I=>	2	3	4	4	2	4	3	3
F=>	1	3	5	8	11	14	15	16
I=>	2	4	3	4	4	3	2	3
F=>	1	4	5	7	9	14	15	16
I=>	1	3	4	4	3	3	4	3

Figure Al.54

F=>	1	2	3	7	11	14	17	19
I=>	2	2	1	3	2	2	2	2
F=>	1	3	6	9	13	17	18	19
I=>	2	2	2	2	3	1	2	2

Figure Al.57

F=>	1	2	3	6	11	14	18	20
I=>	1	2	2	3	2	2	2	1
F=>	1	2	3	8	12	15	18	20
I=>	1	2	2	3	1	2	2	2
F=>	1	2	4	8	10	15	19	20
I=>	1	2	2	3	2	2	2	1
F=>	1	2	6	11	13	17	19	20
I=>	1	2	2	2	3	2	2	1
F=>	1	3	6	9	13	18	19	20
I=>	2	2	2	1	3	2	2	1
F=>	1	3	7	10	15	18	19	20
I=>	1	2	2	2	3	2	2	1

Figure Al.58

F=>	1	2	3	7	11	14	19	21
I=>	1	2	1	2	2	2	2	1
F=>	1	2	3	8	11	15	19	21
I=>	1	2	1	2	2	2	1	2
F=>	1	2	8	13	15	17	18	21
I=>	1	2	2	1	2	2	2	1
F=>	1	3	7	11	14	19	20	21
I=>	2	1	2	2	2	1	2	1
F=>	1	3	8	11	15	19	20	21
I=>	1	2	2	2	2	1	2	1
F=>	1	4	5	7	9	14	20	21
I=>	1	2	2	2	1	2	2	1

Figure Al.59

F=>	1	2	3	5	9	12	17	22
I=>	0	2	1	2	2	2	1	0
F=>	1	8	11	14	18	20	21	22
I=>	0	1	2	2	2	1	2	0

Figure Al.60

F=>	1	2	7	12	15	19	21	23
I=>	1	0	2	1	2	1	2	0
F=>	1	3	5	9	10	17	20	23
I=>	0	2	1	2	1	1	1	1
F=>	1	3	5	9	12	17	22	23
I=>	0	2	1	2	1	2	0	1
F=>	1	4	7	14	15	19	21	23
I=>	1	1	1	1	2	1	2	0

Figure Al.61

F=>	1	2	3	12	16	19	22	24
I=>	1	1	1	1	0	1	1	1
F=>	1	3	5	9	13	22	23	24
I=>	1	1	1	0	1	1	1	1

Figure Al.62

F=>	1	2	3	5	9	15	20	25
I=>	0	1	1	1	1	1	1	0
F=>	1	6	11	17	21	23	24	25
I=>	0	1	1	1	1	1	1	0

Figure Al.63

F=>	1	2	3	5	9	16	21	26
I=>	0	1	1	1	1	0	1	0
F=>	1	6	11	18	22	24	25	26
I=>	0	1	0	1	1	1	1	0

Figure Al.64

F=>	1	2	5	11	13	21	24	27
I=>	0	0	1	1	1	0	1	0
F=>	1	2	7	10	13	23	25	27
I=>	0	0	1	1	1	0	1	0
F=>	1	3	5	15	18	21	26	27
I=>	0	1	0	1	1	1	0	0
F=>	1	4	7	15	17	22	26	27
I=>	0	1	0	1	1	1	0	0

Figure Al.65

F=>	1	2	3	5	9	18	23	28
I=>	0	1	1	1	0	0	1	0
F=>	1	2	3	5	11	16	21	28
I=>	0	1	1	0	1	1	0	0
F=>	1	2	3	12	16	20	23	28
I=>	0	1	0	1	1	1	0	0
F=>	1	2	3	14	18	22	25	28
I=>	0	1	0	1	1	0	1	0
F=>	1	2	4	7	13	20	24	28
I=>	0	0	1	1	1	0	1	0
F=>	1	2	7	10	13	24	26	28
I=>	0	0	1	1	1	0	1	0
F=>	1	2	8	14	18	23	26	28
I=>	0	0	1	1	1	1	0	0
F=>	1	2	11	17	21	23	25	28
I=>	0	0	1	1	1	1	0	0

Figure Al.66

F=>	1	3	4	5	12	18	23	28
I=>	0	1	1	0	1	0	1	0
F=>	1	3	5	16	19	22	27	28
I=>	0	1	0	1	1	1	0	0
F=>	1	3	6	11	15	21	27	28
I=>	0	0	1	1	1	1	0	0
F=>	1	3	11	16	21	22	25	28
I=>	0	0	1	1	0	1	1	0
F=>	1	4	6	7	13	20	24	28
I=>	0	1	0	1	1	0	1	0
F=>	1	4	6	8	12	18	27	28
I=>	0	0	1	1	1	1	0	0
F=>	1	4	7	8	13	18	26	28
I=>	0	1	1	0	1	1	0	0
F=>	1	4	7	11	15	26	27	28
I=>	0	1	0	1	1	0	1	0
F=>	1	5	9	16	22	23	25	28
I=>	0	1	0	1	1	0	1	0
F=>	1	5	9	16	22	25	27	28
I=>	0	1	0	1	1	1	0	0
F=>	1	6	9	13	17	26	27	28
I=>	0	0	1	1	1	0	1	0
F=>	1	6	11	17	24	25	26	28
I=>	0	1	0	1	0	1	1	0
F=>	1	6	11	20	24	26	27	28
I=>	0	1	0	0	1	1	1	0
F=>	1	8	13	16	24	26	27	28
I=>	0	0	1	1	0	1	1	0

Figure Al.66 (cont.)

F=>	1	2	3	12	17	22	25	29
I=>	0	1	0	1	1	0	0	0
F=>	1	2	3	14	19	22	25	29
I=>	0	1	0	1	0	1	0	0
F=>	1	2	3	14	19	23	26	29
I=>	0	1	0	1	0	0	1	0
F=>	1	2	4	7	14	21	25	29
I=>	0	0	1	0	1	0	1	0
F=>	1	2	5	8	14	19	27	29
I=>	0	0	1	1	1	0	0	0
F=>	1	2	7	15	18	25	27	29
I=>	0	0	0	1	1	0	1	0
F=>	1	2	8	15	19	24	27	29
I=>	0	0	1	1	0	1	0	0
F=>	1	3	5	12	15	23	28	29
I=>	0	1	0	1	1	0	0	0
F=>	1	3	6	11	15	22	28	29
I=>	0	0	1	0	1	1	0	0
F=>	1	3	11	16	22	24	28	29
I=>	0	0	0	1	1	1	0	0
F=>	1	4	7	11	16	27	28	29
I=>	0	1	0	0	1	0	1	0
F=>	1	5	8	11	16	27	28	29
I=>	0	0	1	0	1	0	1	0
F=>	1	5	8	13	18	27	28	29
I=>	0	0	0	1	1	0	1	0
F=>	1	5	9	16	23	26	28	29
I=>	0	1	0	1	0	1	0	0

Figure Al.67

F=>	1	2	3	12	18	22	25	30
I=>	0	1	0	1	0	0	0	0
F=>	1	2	3	12	18	23	26	30
I=>	0	1	0	1	0	0	0	0
F=>	1	2	3	15	19	22	25	30
I=>	0	1	0	0	0	1	0	0
F=>	1	2	3	15	20	23	28	30
I=>	0	1	0	0	0	1	0	0
F=>	1	2	8	12	16	25	28	30
I=>	0	0	0	1	1	0	0	0
F=>	1	3	6	15	19	23	29	30
I=>	0	0	0	1	1	0	0	0
F=>	1	3	10	16	22	26	27	30
I=>	0	0	0	1	0	1	0	0
F=>	1	4	5	9	15	21	28	30
I=>	0	0	1	0	1	0	0	0
F=>	1	5	8	11	16	28	29	30
I=>	0	0	1	0	0	0	1	0
F=>	1	5	8	13	19	28	29	30
I=>	0	0	0	0	1	0	1	0
F=>	1	6	9	12	16	28	29	30
I=>	0	0	1	0	0	0	1	0
F=>	1	6	9	13	19	28	29	30
I=>	0	0	0	0	1	0	1	0

Figure Al.68

F=>	1	2	3	13	18	24	27	31
I=>	0	1	0	1	0	0	0	0
F=>	1	2	7	10	17	27	29	31
I=>	0	0	0	0	1	0	1	0
F=>	1	2	7	17	20	27	29	31
I=>	0	0	0	1	0	0	1	0
F=>	1	2	8	11	16	27	29	31
I=>	0	0	0	0	1	0	1	0
F=>	1	2	8	12	14	23	28	31
I=>	0	0	1	1	0	0	0	0
F=>	1	2	8	17	21	26	29	31
I=>	0	0	0	1	0	1	0	0

Figure Al.69

F=>	1	2	12	17	22	25	29	31
I=>	0	0	1	1	0	0	0	0
F=>	1	3	4	5	13	20	26	31
I=>	0	1	1	0	0	0	0	0
F=>	1	3	5	12	15	25	30	31
I=>	0	1	0	0	1	0	0	0
F=>	1	3	5	15	22	23	28	31
I=>	0	1	0	0	0	1	0	0
F=>	1	3	5	15	22	25	30	31
I=>	0	1	0	1	0	0	0	0
F=>	1	3	5	16	21	24	30	31
I=>	0	1	0	1	0	0	0	0
F=>	1	3	6	11	15	24	30	31
I=>	0	0	1	0	1	0	0	0
F=>	1	3	7	10	15	20	30	31
I=>	0	0	0	0	1	1	0	0
F=>	1	3	7	13	14	23	28	31
I=>	0	0	1	1	0	0	0	0
F=>	1	4	9	10	17	27	29	31
I=>	0	0	1	0	0	0	1	0
F=>	1	4	9	18	19	25	29	31
I=>	0	0	0	0	1	1	0	0
F=>	1	4	9	18	20	24	30	31
I=>	0	0	0	0	1	1	0	0
F=>	1	5	8	14	19	29	30	31
I=>	0	0	0	0	1	0	1	0
F=>	1	6	12	19	27	28	29	31
I=>	0	0	0	0	0	1	1	0

Figure Al.69 (cont.)

F=>	1	3	4	11	17	23	28	32
I=>	0	0	0	0	1	0	0	0
F=>	1	5	10	16	22	29	30	32
I=>	0	0	0	1	0	0	0	0

Figure Al.70

F=>	1	2	8	18	21	24	29	33
I=>	0	0	0	0	0	1	0	0
F=>	1	2	8	18	23	28	31	33
I=>	0	0	0	0	0	1	0	0
F=>	1	3	6	11	15	26	32	33
I=>	0	0	1	0	0	0	0	0
F=>	1	5	10	13	15	26	32	33
I=>	0	0	1	0	0	0	0	0

Figure Al.71

F=>	1	2	3	4	5	6	7	8	9
I=>	12	16	18	20	20	20	18	16	12

Figure Al.72

F=>	1	2	3	4	5	7	8	9	10
I=>	10	14	15	18	15	15	15	13	11
F=>	1	2	3	4	6	7	8	9	10
I=>	11	13	15	15	15	16	15	14	10

Figure Al.73

F=>	1	2	3	4	5	7	9	10	11
I=>	9	12	13	13	13	12	12	11	9
F=>	1	2	3	5	7	8	9	10	11
I=>	9	11	12	12	13	13	13	12	9

Figure Al.74

F=>	1	2	3	4	7	8	10	11	12
I=>	8	10	11	10	11	11	10	11	7
F=>	1	2	3	5	6	9	10	11	12
I=>	7	11	10	11	11	10	11	10	8

Figure Al.75

F=>	1	2	3	4	7	8	11	12	13
I=>	7	8	10	8	8	8	8	8	7
F=>	1	2	3	5	6	8	11	12	13
I=>	6	10	9	10	9	9	8	9	7
F=>	1	2	3	5	7	8	10	12	13
I=>	6	8	9	10	10	10	10	8	8
F=>	1	2	3	5	7	10	11	12	13
I=>	7	9	9	9	8	9	10	9	7
F=>	1	2	3	5	8	9	11	12	13
I=>	7	9	8	9	8	10	9	10	8
F=>	1	2	4	5	7	9	11	12	13
I=>	6	8	10	10	10	10	9	8	8

Figure Al.76

F=>	1	2	3	5	6	9	11	13	14
I=>	5	7	8	8	9	8	9	8	6
F=>	1	2	4	6	7	10	12	13	14
I=>	6	6	9	8	8	8	8	7	5

Figure Al.77

F=>	1	2	3	5	8	11	13	14	15
I=>	5	7	7	7	8	7	7	7	5

Figure Al.78

F=>	1	2	3	5	8	11	14	15	16
I=>	4	7	6	6	6	6	6	6	5
F=>	1	2	3	6	9	11	13	15	16
I=>	5	5	5	7	8	7	7	5	5
F=>	1	2	3	6	9	12	14	15	16
I=>	5	6	6	6	6	6	6	7	4
F=>	1	2	4	6	8	11	14	15	16
I=>	5	5	7	7	8	7	5	5	5

Figure Al.79

F=>	1	2	3	5	7	10	13	16	17
I=>	4	8	4	8	8	8	8	5	4
F=>	1	2	3	8	10	11	14	16	17
I=>	4	8	5	8	8	6	5	5	4
F=>	1	2	4	8	10	11	14	16	17
I=>	5	4	8	5	8	6	6	5	4
F=>	1	2	4	7	8	12	14	16	17
I=>	4	5	8	8	8	5	8	4	5
F=>	1	2	4	7	8	12	15	16	17
I=>	4	5	5	8	6	6	5	6	4
F=>	1	2	5	8	11	13	15	16	17
I=>	4	5	8	8	8	8	4	6	4

Figure Al.80

F=>	1	2	3	6	9	13	15	17	18
I=>	4	5	4	8	4	5	5	4	5
F=>	1	2	3	8	10	12	15	17	18
I=>	5	3	5	5	5	5	6	5	3
F=>	1	2	3	7	10	13	15	17	18
I=>	3	5	4	5	8	4	8	4	5
F=>	1	2	4	8	9	12	16	17	18
I=>	5	4	8	4	6	5	4	5	3
F=>	1	2	4	8	10	13	16	17	18
I=>	5	4	5	5	4	8	4	5	4
F=>	1	2	4	7	9	13	16	17	18
I=>	3	5	8	5	5	5	5	3	5

Figure Al.81

f=>	1	2	3	8	11	14	15	17	19
i=>	3	4	4	5	4	5	4	5	2
f=>	1	3	5	6	9	12	17	18	19
i=>	2	5	4	5	4	5	4	4	3

Figure Al.82

f=>	1	2	3	5	8	11	15	19	20
i=>	3	4	3	4	4	4	4	3	3
f=>	1	2	6	10	13	16	18	19	20
i=>	3	3	4	4	4	4	3	4	3

Figure Al.83

f=>	1	2	3	6	9	12	17	19	21
i=>	3	3	3	4	4	4	3	4	2
f=>	1	2	3	7	11	15	18	20	21
i=>	3	4	3	4	4	3	2	4	3
f=>	1	2	3	7	12	14	17	20	21
i=>	3	4	3	4	4	3	3	3	3
f=>	1	2	4	7	11	15	19	20	21
i=>	3	4	2	3	4	4	3	4	3
f=>	1	2	5	8	10	15	19	20	21
i=>	3	3	3	3	4	4	3	4	3
f=>	1	3	5	10	13	16	19	20	21
i=>	2	4	3	4	4	4	3	3	3

Figure Al.84

f==> 1 2 3 5 9 12 17 21 22

i==> 2 4 2 4 3 3 3 4 2

f==> 1 2 3 8 11 14 18 20 22

i==> 3 2 3 3 4 3 3 4 2

f==> 1 2 6 11 14 18 20 21 22

i==> 2 4 3 3 3 4 2 4 2

f==> 1 3 5 9 12 15 20 21 22

i==> 2 4 3 3 4 3 3 2 3

Figure Al.85

f==> 1 2 3 4 8 12 17 20 23

i==> 3 3 3 3 3 3 3 3 2

f==> 1 2 3 10 13 16 18 22 23

i==> 3 3 3 3 3 3 2 3 3

f==> 1 2 6 8 11 14 21 22 23

i==> 3 3 2 3 3 3 3 3 3

f==> 1 4 7 12 16 20 21 22 23

i==> 2 3 3 3 3 3 3 3 3

Figure Al.86

f==> 1 2 3 8 12 16 19 22 24

i==> 2 2 2 3 3 2 3 2 2

f==> 1 3 6 9 13 17 22 23 24

i==> 2 2 3 2 3 3 2 2 2

Figure Al.87

f==> 1 2 4 7 14 16 20 24 25

i==> 2 3 2 2 3 2 3 1 2

f==> 1 2 5 10 11 18 21 23 25

i==> 2 2 3 3 2 3 2 2 1

f==> 1 2 6 10 12 19 22 24 25

i==> 2 1 3 2 3 2 2 3 2

f==> 1 3 7 10 11 18 21 23 25

i==> 1 2 3 3 3 3 2 2 1

f==> 1 3 5 8 15 16 19 24 25

i==> 1 2 2 3 3 3 3 2 1

f==> 1 3 5 8 15 16 21 24 25

i==> 1 2 2 3 2 3 3 2 2

Figure Al.88

f==> 1 2 3 10 14 18 21 24 26

i==> 1 2 2 3 3 2 2 1 2

f==> 1 3 6 9 13 17 24 25 26

i==> 2 1 2 2 3 3 2 2 1

Figure Al.89

f==> 1 2 3 6 10 13 19 25 27
i==> 1 2 2 2 3 2 2 1 2

f==> 1 2 3 6 11 14 21 25 27
i==> 1 2 2 3 2 3 1 2 1

f==> 1 2 3 8 10 13 19 23 27
i==> 1 3 1 3 3 2 2 1 1

f==> 1 2 3 8 12 16 21 24 27
i==> 1 2 1 2 3 3 2 1 2

f==> 1 2 3 8 13 17 21 24 27
i==> 1 2 1 3 2 3 1 2 2

f==> 1 2 3 8 14 18 22 25 27
i==> 2 1 2 3 2 2 1 2 2

f==> 1 2 3 9 13 17 22 24 27
i==> 1 2 2 2 3 2 2 2 1

f==> 1 2 3 10 14 19 21 24 27
i==> 1 2 2 3 2 3 2 1 1

f==> 1 2 3 10 15 18 21 25 27
i==> 2 1 2 3 1 3 1 2 2

f==> 1 2 3 13 18 20 23 26 27
i==> 2 3 2 1 2 2 1 2 2

f==> 1 2 4 6 10 16 19 26 27
i==> 2 2 2 1 3 2 2 1 2

f==> 1 2 4 7 9 13 17 26 27
i==> 1 2 3 2 2 2 2 2 1

f==> 1 2 4 7 14 18 22 26 27
i==> 1 2 1 1 3 2 3 2 2

f==> 1 2 4 11 15 19 21 26 27
i==> 1 2 2 2 2 3 2 2 1

f==> 1 2 5 8 10 15 25 26 27
i==> 2 2 1 2 2 1 2 3 2

f==> 1 2 5 8 13 18 23 25 27
i==> 2 0 2 2 3 3 2 2 1

f==> 1 2 5 9 16 18 21 26 27
i==> 2 2 2 2 1 3 2 2 1

f==> 1 2 6 9 12 18 23 25 27
i==> 1 2 2 2 2 3 2 2 1

f==> 1 2 6 10 14 21 24 26 27
i==> 2 2 3 2 3 1 1 2 1

f==> 1 2 6 11 13 21 24 25 27
i==> 1 2 2 2 3 2 2 3 1

f==> 1 2 7 8 12 15 23 25 27
i==> 1 2 2 3 3 2 1 2 1

f==> 1 2 7 9 13 17 24 26 27
i==> 1 2 2 3 2 2 2 2 1

f==> 1 2 7 10 12 19 23 26 27
i==> 1 2 2 3 1 2 2 2 2

f==> 1 2 9 12 18 22 24 26 27
i==> 2 1 2 2 3 1 2 2 2

Figure A1.90

f==> 1 2 11 15 19 21 24 26 27
i==> 1 2 2 2 2 2 3 2 1

f==> 1 3 4 7 12 13 20 25 27
i==> 1 2 2 2 3 2 2 2 1

f==> 1 3 4 7 15 17 22 26 27
i==> 1 3 2 1 3 2 2 2 1

f==> 1 3 4 12 15 20 21 25 27
i==> 2 2 2 3 2 2 2 1 1

f==> 1 3 5 10 15 20 23 26 27
i==> 1 2 2 3 3 2 2 0 2

f==> 1 3 5 10 16 19 22 26 27
i==> 1 2 2 3 2 2 2 2 1

f==> 1 3 5 13 16 20 21 26 27
i==> 1 2 1 2 3 3 2 2 1

f==> 1 3 6 10 14 20 25 26 27
i==> 2 2 1 2 2 3 2 1 2

f==> 1 3 7 8 13 16 24 25 27
i==> 1 1 2 2 2 3 2 2 2

f==> 1 3 7 10 13 18 25 26 27
i==> 2 2 1 3 1 3 2 1 2

f==> 1 3 7 14 17 22 25 26 27
i==> 1 2 1 3 2 3 2 2 1

f==> 1 3 8 15 16 21 24 25 27
i==> 1 2 2 2 3 2 2 2 1

Figure A1.90(cont.)

f==> 1 3 9 15 18 22 25 26 27
i==> 2 1 2 2 3 2 2 2 1

f==> 1 4 6 11 15 19 25 26 27
i==> 1 2 2 2 3 2 2 2 1

f==> 1 4 7 9 14 18 25 26 27
i==> 1 1 2 3 2 3 2 2 1

f==> 1 4 7 11 15 20 25 26 27
i==> 2 2 1 3 2 3 1 2 1

f==> 1 4 7 12 16 20 25 26 27
i==> 2 1 2 3 3 2 1 2 1

f==> 1 5 9 15 18 20 25 26 27
i==> 1 1 2 2 3 3 1 3 1

Figure A1.90(cont.)

f==> 1 2 3 12 16 20 23 26 28
i==> 1 2 1 2 2 1 1 2 1

f==> 1 3 6 9 13 17 26 27 28
i==> 1 2 1 1 2 2 1 2 1

Figure A1.91

f=> 1 2 3 9 14 19 23 26 29
 i=> 0 2 1 2 2 2 1 2 1

f=> 1 2 3 9 15 19 24 26 29
 i=> 1 2 1 2 1 2 2 2 0

f=> 1 4 6 11 15 21 27 28 29
 i=> 0 2 2 2 1 2 1 2 1

f=> 1 4 7 11 16 21 27 28 29
 i=> 1 2 1 2 2 2 1 2 0

Figure Al.92

f=> 1 2 4 7 14 21 25 29 30
 i=> 1 2 1 1 1 0 2 1 2

f=> 1 2 6 10 17 24 27 29 30
 i=> 2 1 2 0 1 1 1 2 1

Figure Al.93

f=> 1 2 3 5 9 15 21 26 31
 i=> 0 1 1 1 1 2 2 2 1

f=> 1 2 3 6 11 18 25 29 31
 i=> 1 2 1 2 0 1 1 2 1

f=> 1 2 3 10 15 21 25 28 31
 i=> 0 1 1 1 2 2 1 2 1

f=> 1 2 7 8 16 19 27 29 31
 i=> 1 1 1 2 2 2 1 1 0

f=> 1 2 8 12 14 23 26 28 31
 i=> 0 0 1 2 2 1 2 2 1

Figure Al.94

f=> 1 2 12 15 22 23 27 29 31
 i=> 1 1 2 1 2 2 1 1 0

f=> 1 3 5 9 10 17 20 30 31
 i=> 0 1 1 2 2 1 2 1 1

f=> 1 3 5 13 16 24 25 30 31
 i=> 0 1 1 2 2 2 1 1 1

f=> 1 3 7 14 21 25 29 30 31
 i=> 1 2 1 1 0 2 1 2 1

f=> 1 4 6 9 18 20 24 30 31
 i=> 1 2 2 1 2 2 1 0 0

f=> 1 4 7 11 17 22 29 30 31
 i=> 1 2 1 2 2 1 1 1 0

f=> 1 6 11 16 23 27 29 30 31
 i=> 1 2 2 2 1 1 1 1 0

Figure Al.94(cont.)

f=> 1 2 3 16 20 24 27 30 32
 i=> 1 1 1 1 1 1 1 1 1

f=> 1 2 4 7 13 17 24 31 32
 i=> 1 1 1 1 1 1 1 1 1

f=> 1 2 9 16 20 26 29 31 32
 i=> 1 1 1 1 1 1 1 1 1

f=> 1 3 6 9 13 17 30 31 32
 i=> 1 1 1 1 1 1 1 1 1

Figure Al.95

f=> 1 2 3 16 21 24 27 31 33

i=> 1 1 1 1 0 1 1 1 1

f=> 1 3 7 10 13 18 31 32 33

i=> 1 1 1 1 0 1 1 1 1

Figure Al.96

f=> 1 2 3 14 18 24 27 32 34

i=> 1 1 1 1 1 1 0 1 1

f=> 1 2 3 17 22 25 28 32 34

i=> 1 1 1 1 0 1 1 1 1

f=> 1 3 7 10 13 18 32 33 34

i=> 1 1 1 1 0 1 1 1 1

f=> 1 3 8 11 17 21 32 33 34

i=> 1 1 0 1 1 1 1 1 1

Figure Al.97

f=> 1 2 3 8 14 17 27 31 35

i=> 0 1 0 1 1 1 0 1 0

f=> 1 2 3 15 20 25 28 31 35

i=> 0 1 0 1 1 1 1 0 0

f=> 1 5 8 11 16 21 33 34 35

i=> 0 0 1 1 1 1 0 1 0

f=> 1 5 9 19 22 28 33 34 35

i=> 0 1 0 1 1 1 0 1 0

Figure Al.98

f=> 1 2 3 16 20 24 27 30 36

i=> 0 1 0 1 1 0 1 1 0

f=> 1 7 10 13 17 21 34 35 36

i=> 0 1 1 0 1 1 0 1 0

Figure Al.99

f=> 1 2 3 16 21 27 30 33 37

i=> 0 1 0 1 0 1 1 0 0

f=> 1 2 3 18 23 27 31 34 37

i=> 0 1 0 1 0 1 0 1 0

f=> 1 2 7 10 13 20 33 35 37

i=> 0 0 1 1 0 1 0 1 0

f=> 1 2 8 11 14 19 33 35 37

i=> 0 0 1 1 0 1 0 1 0

f=> 1 2 8 19 23 27 32 35 37

i=> 0 0 0 1 1 1 1 0 0

f=> 1 3 5 18 25 28 31 36 37

i=> 0 1 0 1 0 1 1 0 0

f=> 1 3 5 19 24 27 30 36 37

i=> 0 1 0 1 0 1 1 0 0

f=> 1 3 6 11 15 19 30 36 37

i=> 0 0 1 1 1 1 0 0 0

f=> 1 4 7 11 15 20 35 36 37

i=> 0 1 0 1 0 1 0 1 0

f=> 1 5 8 11 17 22 35 36 37

i=> 0 0 1 1 0 1 0 1 0

Figure Al.100

f=> 1 2 3 12 18 26 31 34 38

i=> 0 1 0 0 1 1 0 0 0

f=> 1 3 4 5 13 20 27 33 38

i=> 0 1 1 0 0 1 0 0 0

f=> 1 3 5 16 21 28 29 35 38

i=> 0 1 0 1 0 1 0 0 0

f=> 1 4 10 11 18 23 34 36 38

i=> 0 0 0 1 0 1 0 1 0

f=> 1 5 8 13 21 27 36 37 38

i=> 0 0 0 1 1 0 0 1 0

f=> 1 6 12 19 26 34 35 36 38

i=> 0 0 0 1 0 0 1 1 0

Figure Al.101

f=> 1 2 3 13 18 26 32 35 39

i=> 0 1 0 0 1 1 0 0 0

f=> 1 2 3 15 24 28 31 34 39

i=> 0 1 0 1 0 0 1 0 0

f=> 1 2 3 15 24 29 32 35 39

i=> 0 1 0 1 0 0 1 0 0

f=> 1 2 4 7 12 19 26 35 39

i=> 0 0 1 1 0 1 0 0 0

f=> 1 2 7 10 18 25 35 37 39

i=> 0 0 0 1 1 0 0 1 0

Figure Al.102

f=> 1 2 12 15 18 30 35 37 39

i=> 0 0 0 1 1 0 0 1 0

f=> 1 3 5 10 22 25 28 38 39

i=> 0 1 0 0 1 1 0 0 0

f=> 1 3 5 15 22 30 33 38 39

i=> 0 1 0 0 1 1 0 0 0

f=> 1 5 8 11 16 25 37 38 39

i=> 0 0 1 0 0 1 0 1 0

f=> 1 5 8 14 22 27 37 38 39

i=> 0 0 0 1 1 0 0 1 0

f=> 1 5 14 21 28 33 36 38 39

i=> 0 0 0 1 0 1 1 0 0

f=> 1 6 9 12 16 25 37 38 39

i=> 0 0 1 0 0 1 0 1 0

Figure Al.102(cont.)

f=>	1 2 3 5 13 20 26 35 40	f=>	1 3 8 15 19 30 35 38 40
i=>	0 1 1 0 0 1 0 0 0	i=>	0 0 1 0 1 0 1 0 0
f=>	1 2 3 13 18 27 33 36 40	f=>	1 2 9 12 17 22 34 36 40
i=>	0 1 0 0 1 1 0 0 0	i=>	0 0 1 1 1 0 0 0 0
f=>	1 2 3 17 23 28 33 36 40	f=>	1 2 9 16 19 29 31 35 40
i=>	0 1 0 1 0 1 0 0 0	i=>	0 0 1 1 1 0 0 0 0
f=>	1 2 3 18 22 27 30 33 40	f=>	1 3 4 5 20 28 35 40
i=>	0 1 0 1 0 0 1 0 0	i=>	0 1 1 0 0 1 0 0 0
f=>	1 2 4 10 19 24 29 36 40	f=>	1 3 5 16 21 30 33 39 40
i=>	0 0 0 1 1 1 0 0 0	i=>	0 1 0 0 1 1 0 0 0
f=>	1 2 5 8 14 19 30 38 40	f=>	1 3 5 17 22 30 31 37 40
i=>	0 0 1 1 0 1 0 0 0	i=>	0 1 0 1 0 0 1 0 0
f=>	1 2 6 19 26 29 32 38 40	f=>	1 3 5 17 22 30 33 39 40
i=>	0 0 0 1 0 1 1 0 0	i=>	0 1 0 1 1 0 0 0 0
f=>	1 2 7 20 24 28 31 38 40	f=>	1 3 6 11 22 26 33 39 40
i=>	0 0 0 1 1 0 1 0 0	i=>	0 0 1 0 1 0 1 0 0
f=>	1 2 8 11 19 24 36 38 40	f=>	1 3 7 13 14 23 32 37 40
i=>	0 0 0 0 1 1 0 1 0	i=>	0 0 1 1 0 1 0 0 0
f=>	1 2 8 11 20 25 36 38 40	f=>	1 3 9 12 15 22 35 39 40
i=>	0 0 0 1 1 0 0 1 0	i=>	0 0 1 1 0 1 0 0 0
f=>	1 2 8 12 14 23 32 37 40	f=>	1 3 9 15 20 27 36 37 40
i=>	0 0 1 1 0 1 0 0 0	i=>	0 0 1 1 1 0 0 0 0
f=>	1 2 8 14 18 26 35 37 40	f=>	1 3 10 13 17 21 34 39 40
i=>	0 0 1 1 1 0 0 0 0	i=>	0 0 1 0 1 1 0 0 0

Figure A1.103

f==> 1 3 11 22 27 33 35 39 40

i==> 0 0 0 1 0 1 1 0 0

f==> 1 4 5 14 21 26 32 38 40

i==> 0 0 0 0 1 1 1 0 0

f==> 1 4 6 15 23 24 30 36 40

i==> 0 0 0 1 1 0 1 0 0

f==> 1 4 6 15 23 27 33 39 40

i==> 0 0 0 0 1 1 1 0 0

f==> 1 4 9 18 27 23 34 38 40

i==> 0 0 0 1 0 1 1 0 0

f==> 1 4 9 18 27 29 33 39 40

i==> 0 0 0 1 0 1 1 0 0

f==> 1 4 10 11 19 24 36 38 40

i==> 0 0 1 0 0 1 0 1 0

f==> 1 5 7 19 24 29 32 39 40

i==> 0 0 0 0 1 1 1 0 0

f==> 1 5 8 13 18 24 38 39 40

i==> 0 0 0 1 0 1 0 1 0

f==> 1 5 8 14 23 28 38 39 40

i==> 0 0 0 1 1 0 0 1 0

f==> 1 5 11 17 18 26 35 37 40

i==> 0 0 1 0 1 1 0 0 0

f==> 1 5 12 17 22 31 37 39 40

i==> 0 0 0 1 1 1 0 0 0

f==> 1 6 10 12 22 25 32 39 40

i==> 0 0 0 0 1 1 1 0 0

f==> 1 6 13 21 27 36 37 38 40

i==> 0 0 0 1 0 0 1 1 0

f==> 1 6 15 21 28 36 38 39 40

i==> 0 0 0 1 0 0 1 1 0

f==> 1 8 11 14 19 23 38 39 40

i==> 0 0 1 0 0 1 0 1 0

```
F=> 1 2 3 4 5 6 7 8 9 10
-----
I=> 16 20 23 25 26 26 25 23 20 16
```

Figure Al.104

```
F=> 1 2 3 4 6 9 10 12 13 14
-----
I=> 11 12 13 13 12 13 13 13 13 8
-----
F=> 1 2 3 5 6 9 11 12 13 14
-----
I=> 9 13 13 13 13 12 13 13 12 11
```

Figure Al .108

```
F=> 1 2 3 4 5 6 8 9 10 11
-----
I=> 14 17 20 21 21 21 20 19 17 14
-----
F=> 1 2 3 4 5 7 8 9 10 11
-----
I=> 14 18 19 21 20 20 21 19 18 14
-----
F=> 1 2 3 4 6 7 8 9 10 11
-----
I=> 14 17 19 20 21 21 21 20 17 14
```

Figure Al.105

```
F=> 1 2 3 4 6 8 11 12 14 15
-----
I=> 9 10 12 12 12 11 12 12 10 9
-----
F=> 1 2 3 4 6 9 10 13 14 15
-----
I=> 10 11 11 11 11 12 12 11 11 9
-----
F=> 1 2 3 6 7 10 12 13 14 15
-----
I=> 9 11 11 12 12 11 11 11 11 10
-----
F=> 1 2 4 5 8 10 12 13 14 15
-----
I=> 9 10 12 12 11 12 12 12 10 9
```

Figure Al.109

```
F=> 1 2 3 4 5 7 8 10 11 12
-----
I=> 13 15 16 18 18 18 18 16 15 11
-----
F=> 1 2 3 4 5 7 9 10 11 12
-----
I=> 12 16 17 18 17 16 17 17 15 13
-----
F=> 1 2 3 4 6 7 9 10 11 12
-----
I=> 13 14 16 18 18 18 18 14 13
-----
F=> 1 2 3 4 6 8 9 10 11 12
-----
I=> 13 15 17 17 16 17 18 17 16 12
-----
F=> 1 2 3 5 6 8 9 10 11 12
-----
I=> 11 15 16 18 18 18 18 16 15 13
```

Figure Al.106

```
F=> 1 2 3 5 7 8 11 14 15 16
-----
I=> 8 10 9 10 11 11 10 9 9 8
-----
F=> 1 2 3 6 9 10 12 14 15 16
-----
I=> 8 9 9 10 11 11 10 9 10 8
```

Figure Al.110

```
F=> 1 2 3 4 6 7 10 11 12 13
-----
I=> 11 13 15 15 15 14 14 14 11
-----
F=> 1 2 3 4 7 8 10 11 12 13
-----
I=> 11 14 14 14 15 15 15 15 13 11
```

Figure Al.107

```
F=> 1 2 3 5 7 9 12 15 16 17
-----
I=> 7 8 9 10 10 9 8 8 8
-----
F=> 1 2 3 6 9 11 13 15 16 17
-----
I=> 7 8 8 8 9 10 10 9 8 7
```

Figure Al.111

```
F=> 1 2 3 5 8 9 13 15 17 18
-----
I=> 6 7 8 9 8 9 8 8 7
-----
F=> 1 2 4 6 10 11 14 16 17 18
-----
I=> 6 7 8 8 9 8 9 8 7
```

Figure Al.112

F=>	1	2	3	4	5	6	7	8	9	10	11
I=>	20	25	28	31	32	33	32	31	28	25	20

Figure Al.113

F=>	1	2	3	4	5	6	8	9	10	11	12
I=>	18	22	25	28	27	26	26	26	24	27	18
F=>	1	2	3	4	5	7	8	9	10	11	12
I=>	18	22	24	26	26	26	27	26	25	22	18

Figure Al.114

F=>	1	2	3	4	5	7	8	10	11	12	13
I=>	16	20	20	23	23	23	23	23	21	18	16
F=>	1	2	3	4	6	7	9	10	11	12	13
I=>	16	18	21	23	23	23	23	23	20	20	16

Figure Al.115

F=>	1	2	3	4	5	7	9	10	12	13	14
I=>	14	18	13	20	20	20	20	20	19	16	14
F=>	1	2	3	4	5	8	9	11	12	13	14
I=>	15	18	13	20	19	19	19	20	20	17	14
F=>	1	2	3	4	6	7	10	11	12	13	14
I=>	14	17	20	20	19	19	19	20	19	18	15
F=>	1	2	3	5	6	8	10	11	12	13	14
I=>	14	18	19	20	20	20	20	20	19	18	14

Figure Al.116

F=>	1	2	3	4	5	8	9	11	13	14	15
I=>	13	16	17	18	17	18	18	17	15	15	13
F=>	1	2	3	5	7	8	11	12	13	14	15
I=>	13	15	15	17	18	18	17	18	17	16	13

Figure Al.117

F=>	1	2	3	4	6	9	10	12	14	15	16
I=>	13	13	15	16	15	16	16	16	14	14	11
F=>	1	2	3	5	7	8	11	13	14	15	16
I=>	11	14	14	16	16	16	15	16	15	13	13

Figure Al.118

F=>	1	2	3	4	7	9	12	13	15	16	17
I=>	11	12	14	15	14	13	15	14	14	13	10
F=>	1	2	3	5	6	9	11	14	15	16	17
I=>	10	13	14	14	15	13	14	15	14	12	11

Figure Al.119

F=>	1	2	3	4	7	10	12	14	16	17	18
I=>	11	11	13	12	11	12	13	13	13	11	10
F=>	1	2	3	5	7	9	12	15	18	17	18
I=>	10	11	13	13	13	12	11	12	13	11	11

Figure Al.120

F=>	1	2	3	4	5	6	7	9	10	11	12	
I=>	25	30	34	37	38	40	40	39	37	34	30	25

Figure Al.121

F=>	1	2	3	4	5	6	8	9	10	11	12	13
I=>	23	27	30	32	33	33	33	33	32	30	27	23

Figure Al.122

F=>	1	2	3	4	5	6	8	10	11	12	13	14
I=>	21	24	28	28	28	28	27	28	28	27	25	20
F=>	1	2	3	4	5	7	8	10	11	12	13	14
I=>	21	24	25	28	28	28	28	28	28	25	24	21
F=>	1	2	3	4	5	7	9	10	11	12	13	14
I=>	20	25	27	26	28	27	28	29	29	28	24	21

Figure Al.123

F=>	1	2	3	4	5	7	8	10	12	13	14	15
I=>	18	22	24	25	26	26	26	24	23	21	19	
F=>	1	2	3	4	5	7	9	10	12	13	14	15
I=>	18	23	23	26	24	26	25	26	26	23	21	19
F=>	1	2	3	4	6	7	9	11	12	13	14	15
I=>	19	21	23	26	26	25	26	24	26	23	23	18
F=>	1	2	3	4	6	8	9	11	12	13	14	15
I=>	19	21	23	24	26	26	26	26	25	24	22	18

Figure Al.124

F=>	1	2	3	4	5	8	9	11	13	14	15	16
I=>	17	20	22	23	21	23	23	22	21	21	20	18
F=>	1	2	3	4	6	8	9	12	13	14	15	16
I=>	18	20	21	21	22	23	23	21	23	22	20	17

Figure Al.125

F=>	1	2	3	4	5	8	9	12	14	15	16	17
I=>	16	19	20	21	21	20	20	19	19	20	19	16
F=>	1	2	3	4	6	9	10	13	14	15	16	17
I=>	16	19	20	19	19	20	20	21	21	20	19	16

Figure Al.126

F=>	1	2	3	4	6	8	11	12	15	16	17	18
I=>	15	18	19	17	18	18	19	18	18	18	18	14
F=>	1	2	3	4	7	8	11	13	15	16	17	18
I=>	14	18	18	18	18	19	18	18	17	19	18	15

Figure Al.127

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13
I=>	30	38	40	44	46	47	48	48	46	44	40	36	30

Figure Al.128

F=>	1	2	3	4	5	6	7	9	10	11	12	13	14
I=>	27	33	36	39	40	41	40	40	40	38	36	32	28
F=>	1	2	3	4	5	6	8	9,10	11	12	13	14	
I=>	28	32	34	38	40	40	40	41	40	39	36	33	27

Figure Al.129

F=>	1	2	3	4	5	6	8	10	11	12	13	14	15
I=>	26	29	34	35	35	35	34	35	35	35	34	29	26

Figure Al.130

F=>	1	2	3	4	5	7	8	10	12	13	14	15	16
I=>	23	27	29	31	32	32	31	32	30	31	29	27	23
F=>	1	2	3	4	5	7	9	10	12	13	14	15	16
I=>	23	27	29	31	30	32	31	32	32	31	29	27	23

Figure Al.131

F=>	1	2	3	4	5	7	9	10	13	14	15	16	17
I=>	21	26	27	28	27	28	28	28	27	28	27	25	21
F=>	1	2	3	4	5	8	9	11	13	14	15	16	17
I=>	21	25	27	28	27	28	28	28	27	28	27	26	21

Figure Al.132

F=>	1	2	3	4	5	7	10	11	13	15	16	17	18
I=>	20	24	24	26	26	25	26	25	26	24	25	22	20
F=>	1	2	3	4	6	8	11	12	14	15	16	17	18
I=>	20	22	25	25	26	25	26	25	26	26	24	24	20

Figure Al.133

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14
I=>	38	42	47	51	54	56	57	57	56	54	52	47	42	36

Figure Al.134

F=>	1	2	3	4	5	6	7	8	10	11	12	13	14	15
I=>	33	38	43	46	48	49	49	49	48	47	45	42	38	33
F=>	1	2	3	4	5	6	7	9	10	11	12	13	14	15
I=>	33	39	42	46	47	49	48	48	49	47	46	42	38	33
F=>	1	2	3	4	5	6	8	9	10	11	12	13	14	15
I=>	33	38	42	45	47	46	45	49	49	48	46	43	38	33

Figure Al.135

F=>	1	2	3	4	5	6	7	9	11	12	13	14	15	16
I=>	30	36	39	43	43	43	42	41	42	42	41	40	35	31
F=>	1	2	3	4	5	6	8	9	11	12	13	14	15	16
I=>	31	35	38	41	43	43	43	43	43	43	41	38	35	31
F=>	1	2	3	4	5	6	8	10	11	12	13	14	15	16
I=>	31	35	40	41	42	42	41	42	43	43	43	39	36	30

Figure Al.136

F=>	1	2	3	4	5	6	8	10	11	13	14	15	16	17
I=>	29	32	36	39	38	38	39	38	38	39	37	35	32	28
F=>	1	2	3	4	5	7	8	10	12	13	14	15	16	17
I=>	28	32	35	37	39	38	38	39	38	38	38	36	32	29

Figure Al.137

F=>	1	2	3	4	5	6	8	10	11	14	15	16	17	18
I=>	27	30	34	35	35	34	35	34	34	33	34	32	30	26
F=>	1	2	3	4	5	6	9	10	12	14	15	16	17	18
I=>	27	30	33	35	35	34	34	34	34	34	34	32	31	26
F=>	1	2	3	4	5	7	9	10	13	14	15	16	17	18
I=>	26	31	32	34	34	34	34	34	34	35	35	33	30	27
F=>	1	2	3	4	5	8	9	11	13	14	15	16	17	18
I=>	26	30	32	34	33	34	34	35	34	35	35	34	30	27

Figure Al.138

F=>	1	2	3	4	5	7	9	10	13	14	16	17	18	19
I=>	24	28	29	32	32	32	31	33	32	32	31	29	27	24
F=>	1	2	3	4	6	7	10	11	13	15	16	17	18	19
I=>	24	27	29	31	32	32	33	31	32	32	32	29	25	24

Figure Al.139

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I=>	42	49	54	59	62	65	66	67	66	65	62	59	54	49	42

Figure Al.140

F=>	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16
I=>	39	45	50	53	56	57	58	57	57	55	53	49	45	39	
F=>	1	2	3	4	5	6	7	9	10	11	12	13	14	15	16
I=>	39	45	49	53	55	57	57	54	57	56	53	50	45	39	

Figure Al.141

F=>	1	2	3	4	5	6	7	9	11	12	13	14	15	16	17
I=>	36	42	46	49	50	51	50	49	50	51	50	50	46	42	36

Figure Al.142

F=>	1	2	3	4	5	6	8	10	11	12	14	15	16	17	18
I=>	34	38	42	44	45	45	47	47	47	47	45	44	41	39	33
F=>	1	2	3	4	5	7	8	9	11	13	14	15	16	17	18
I=>	33	36	41	44	45	47	47	47	47	45	45	44	42	38	34

Figure Al.143

F=>	1	2	3	4	5	6	8	10	11	14	15	16	17	18	19
I=>	32	36	40	41	42	42	41	41	40	41	42	41	39	36	32
F=>	1	2	3	4	5	6	9	10	12	14	15	16	17	18	19
I=>	32	36	39	41	42	41	40	41	41	42	42	41	40	36	32

Figure Al.144

F=>	1	2	3	4	5	7	8	11	12	14	16	17	18	19	20
I=>	31	33	35	37	38	39	39	39	37	39	38	36	35	33	29
F=>	1	2	3	4	5	7	9	10	13	14	16	17	18	19	20
I=>	29	33	36	38	38	35	37	39	39	39	38	36	35	33	29

Figure Al.145

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
I=>	49	56	62	67	71	74	76	77	77	76	74	71	67	62	56	49

Figure Al.146

F=>	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17
I=>	48	52	57	61	64	66	67	67	67	67	65	64	61	57	52	46

Figure Al.147

F=>	1	2	3	4	5	6	7	8	10	12	13	14	15	16	17	18
I=>	43	48	54	57	60	60	62	59	58	59	59	58	57	53	49	42
F=>	1	2	3	4	5	6	7	9	10	12	13	14	15	16	17	18
I=>	43	48	52	57	58	60	60	60	60	60	60	58	57	52	48	43
F=>	1	2	3	4	5	6	7	9	11	12	13	14	15	16	17	18
I=>	42	49	53	57	58	59	59	58	59	60	60	60	57	54	48	43

Figure Al.148

F=>	2	2	3	4	5	6	7	9	11	12	13	15	16	17	18	19
I=>	40	45	48	52	54	54	55	55	55	55	55	53	51	46	44	39
F=>	1	2	3	4	5	7	8	9	11	13	14	15	16	17	18	19
I=>	39	44	48	51	53	55	55	55	45	55	54	54	52	49	45	40

Figure Al.149

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
I=>	56	64	70	74	80	84	86	88	88	88	88	84	83	76	70	64	56

Figure Al.150

F=>	1	2	3	4	5	6	7	8	9	11	12	13	14	15	16	17	18
I=>	52	60	65	70	73	76	77	78	77	77	77	75	73	69	65	59	53
F=>	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18
I=>	51	59	65	69	73	75	77	77	77	78	77	76	73	70	65	60	52

Figure Al.151

F=>	1	2	3	4	5	6	7	8	10	12	13	14	15	16	17	18	19
I=>	50	55	62	65	68	69	69	69	69	69	69	69	69	65	62	55	50

Figure Al.152

F=>	1	2	3	4	5	6	7	9	11	12	13	15	16	17	18	19	20
I=>	46	52	56	60	62	62	63	64	64	64	64	63	62	59	51	51	46
F=>	1	2	3	4	5	6	8	9	10	12	14	15	16	17	18	19	20
I=>	46	51	59	59	62	63	64	64	64	64	63	62	60	58	52	46	

Figure Al.153

F=>	1	2	3	4	5	6	7	10	12	13	14	16	17	18	19	20	21
I=>	44	49	54	56	58	58	56	57	58	59	59	59	59	56	53	49	44
F=>	1	2	3	4	5	6	8	9	10	12	15	16	17	18	19	20	21
I=>	44	49	53	56	59	58	59	59	58	57	58	58	56	54	49	44	

Figure Al.154

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
I=>	64	72	79	85	90	94	97	99	100	100	99	97	94	90	85	79	72	64

Figure Al.155

F=>	1	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19
I=>	60	67	74	79	83	86	88	89	89	89	88	87	85	82	78	73	67	60
F=>	1	2	3	4	5	6	7	8	9	11	12	13	14	15	16	17	18	19
I=>	60	68	73	78	82	86	87	89	88	88	87	86	82	76	73	68	60	
F=>	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18	19
I=>	60	67	73	78	82	85	87	88	89	89	89	88	86	83	79	74	67	60

Figure Al.156

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I=>	56	64	59	75	77	80	80	80	79	78	79	79	78	77	73	70	63	57		
F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I=>	57	63	68	74	77	78	80	80	80	80	80	80	78	77	74	68	63	57		
F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I=>	57	63	70	73	77	78	79	78	79	80	80	80	77	75	69	64	56			

Figure Al.157

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
I=>	53	59	64	68	71	73	74	74	74	74	73	73	71	70	68	64	59	53			
F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
I=>	53	59	64	68	70	71	73	73	74	74	74	74	73	71	68	64	59	53			

Figure Al.158

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
I=>	51	57	51	66	67	68	68	67	65	66	68	68	64	68	64	61	56	50				
F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
I=>	50	56	61	64	68	68	68	68	65	65	67	68	64	67	66	61	57	51				

Figure Al.159

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
I=>	72	81	88	95	100	105	108	111	111	112	113	112	111	110	105	100	95	88	81	72

Figure Al.160

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I=>	68	76	83	88	92	96	99	100	101	101	100	100	100	95	96	92	88	82	76	68
F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I=>	68	76	82	88	92	96	98	100	100	100	100	100	99	96	93	88	83	76	68	

Figure Al.161

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
I=>	64	72	76	84	86	90	90	91	93	90	90	91	90	90	86	84	78	72	64		

Figure Al.162

F=>	1	2	3	4	5	6	7	8	10	12	13	14	16	17	18	19	20	21	22
I=>	62	67	73	77	81	82	83	84	84	84	84	84	84	83	80	77	72	67	60
F=>	1	2	3	4	5	6	7	9	10	11	13	15	16	17	18	19	20	21	22
I=>	60	67	72	77	80	83	84	84	84	84	84	84	83	82	81	77	73	67	62

Figure Al.163

F=>	1	2	3	4	5	6	7	8	11	12	13	15	17	18	19	20	21	22	23
I=>	58	65	69	72	75	78	78	78	77	79	78	76	78	77	74	72	68	65	57
F=>	1	2	3	4	5	6	7	9	11	12	13	16	17	18	19	20	21	22	23
I=>	57	65	68	72	74	77	78	78	78	79	77	78	78	78	75	72	69	65	58

Figure Al.164

F=>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I=>	81	90	98	103	111	116	120	123	125	126	126	125	123	120	116	111	105	97	90	81

Figure Al.165

F=>	1	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	21
I=>	77	85	92	98	103	107	110	112	114	113	113	113	112	110	107	103	98	92	85	77

Figure Al.166

F=>	1	2	3	4	5	6	7	8	9	10	12	14	15	16	17	18	19	20	21	22
I=>	73	80	88	93	98	100	103	103	103	102	101	102	101	100	96	93	87	81	72	
F=>	1	2	3	4	5	6	7	8	9	11	12	14	15	16	17	18	19	20	21	22
I=>	73	80	85	93	97	100	101	103	103	103	103	103	103	101	100	97	93	88	80	73
F=>	1	2	3	4	5	6	7	8	9	11	13	14	15	16	17	18	19	20	21	22
I=>	72	81	87	93	96	100	101	102	102	101	102	103	103	103	101	98	93	88	80	73

Figure Al.167

F=>	1	2	3	4	5	6	7	8	9	11	13	15	16	17	18	19	20	21	22	23
I=>	69	77	83	89	92	95	95	95	94	92	92	94	95	95	95	92	89	83	77	69

Figure Al.168

SCENARIO

 K= 20
 N= 40

FOR THE EVEN INITIAL ASSIGNMENT:

	1	3	5	7	9	11	13	15	17	19
	22	24	26	28	30	32	34	36	38	40

INITIAL WORST= 81
 INITIAL TOTAL= 1540

FINAL WORST= 44
 FINAL TOTAL= 427

F==>	1	2	3	4	5	7	11	13	17	19
IM==>	37	38	43	43	42	42	41	41	44	43
F==>	22	24	28	32	33	35	37	38	38	40
IM==>	44	40	38	46	41	43	41	44	41	37

IM-ADVANTAGE (DB) ==> 4.57

DAYTIM=TUE, AUG 20 1985 20148
 ECPU= 26.48 SEC.

SCENARIO

 K= 20
 N= 60

FOR THE EVEN INITIAL ASSIGNMENT:

	1	4	7	10	13	17	20	23	26	29
	32	35	38	41	44	48	51	54	57	60

INITIAL WORST= 81
 INITIAL TOTAL= 1420

FINAL WORST= 27
 FINAL TOTAL= 489

F==>	1	2	3	4	7	10	12	17	20	26
IM==>	24	24	25	23	25	26	23	26	23	25
F==>	32	37	41	44	48	55	56	58	59	60
IM==>	22	23	27	27	25	26	25	23	25	22

IM-ADVANTAGE (DB) ==> 6.89

DAYTIM=FRI, AUG 30 1985 16122
 ECPU= 43.58 SEC.

SCENARIO

 K= 20
 N= 80

FOR THE EVEN INITIAL ASSIGNMENT:

	1	5	8	13	18	22	26	30	34	38
	43	47	51	55	59	63	68	72	76	80

INITIAL WORST= 94
 INITIAL TOTAL= 1452

FINAL WORST= 20
 FINAL TOTAL= 348

F==>	1	4	5	9	11	17	18	27	28	38
IM==>	14	17	19	15	18	20	16	15	19	19
F==>	40	43	55	58	68	73	74	78	78	80
IM==>	18	20	18	17	17	16	19	17	16	17

IM-ADVANTAGE (DB) ==> 7.98

DAYTIM=FRI, AUG 23 1985 22120
 ECPU= 70.40 SEC.

Figure A2.1. DELIN Results corresponding to Table 6.2

SCENARIO

 K= 20
 N= 100

FOR THE EVEN INITIAL ASSIGNMENT:

1	8	11	17	22	27	32	37	43	48
53	58	64	69	74	79	84	90	95	100

INITIAL WORST= 94
 INITIAL TOTAL= 1428

FINAL WORST= 14
 FINAL TOTAL= 252

F==>	1	3	6	13	14	17	25	31	36	37
IM==>	13	12	13	13	13	13	14	14	13	13
F==>	43	57	64	82	84	91	95	97	98	100
IM==>	13	12	14	11	14	11	13	13	10	10

IM-ADVANTAGE (DB) ==> 9.54

DATTIM=FRI, AUG 23 1985 22:24
 ECPU= 119.84 SEC.

SCENARIO

 K= 30
 N= 60

FOR THE EVEN INITIAL ASSIGNMENT:

1	3	5	7	9	11	13	15	17	19
21	23	25	27	29	32	34	36	38	40
42	44	46	48	50	52	54	56	58	60

INITIAL WORST= 221
 INITIAL TOTAL= 5782

FINAL WORST= 111
 FINAL TOTAL= 3180

F==>	1	2	3	4	5	7	8	9	10	17
IM==>	81	87	103	106	109	108	108	107	107	104
F==>	18	23	27	28	32	34	38	40	43	45
IM==>	104	105	111	108	111	110	107	107	111	108
F==>	46	48	52	53	55	56	57	58	58	60
IM==>	107	108	110	111	108	107	107	104	102	94

IM-ADVANTAGE (DB) ==> 4.33

DATTIM=FRI, AUG 23 1985 22:36
 ECPU= 337.17 SEC.

SCENARIO

 K= 30
 N= 90

FOR THE EVEN INITIAL ASSIGNMENT:

1	4	7	10	13	16	19	22	26	29
32	35	38	41	44	47	50	53	56	59
62	65	68	72	75	78	81	84	87	90

INITIAL WORST= 230
 INITIAL TOTAL= 5334

FINAL WORST= 70
 FINAL TOTAL= 1985

F==>	1	2	3	4	7	9	10	11	16	21
IM==>	62	61	60	70	69	70	70	68	69	61
F==>	26	29	32	38	43	50	55	59	62	65
IM==>	69	69	69	68	61	66	65	70	70	70
F==>	69	75	78	80	82	86	87	88	89	90
IM==>	68	66	66	69	68	63	70	60	62	58

IM-ADVANTAGE (DB) ==> 6.33

DATTIM=FRI, AUG 23 1985 22:55
 ECPU= 586.73 SEC.

Figure A2.1. (continued)

SCENARIO

 K= 30
 N= 120

FOR THE EVEN INITIAL ASSIGNMENT:

1	5	9	13	17	22	28	30	34	38
42	48	50	54	58	63	67	71	75	79
83	87	91	95	99	104	108	112	116	120

INITIAL WORST= 215
 INITIAL TOTAL= 5142

FINAL WORST= 49
 FINAL TOTAL= 1384

F==>	1	2	3	5	8	9	13	15	16	23
IM==>	42	45	42	49	49	48	47	41	48	44
F==>	28	32	41	49	52	58	67	71	83	87
IM==>	47	48	43	44	48	43	48	48	44	48
F==>	80	95	99	104	114	115	117	118	119	120
IM==>	46	45	48	48	47	47	44	46	43	42

IN-ADVANTAGE (DB) ==> 7.88

DAYTIM=FRI, AUG 23 1985 23:21
 ECPU= 122.45 SEC.

SCENARIO

 K= 30
 N= 150

FOR THE EVEN INITIAL ASSIGNMENT:

1	6	11	16	22	27	32	37	42	47
52	58	63	68	73	78	83	88	93	99
104	109	114	119	124	129	135	140	145	150

INITIAL WORST= 219
 INITIAL TOTAL= 5128

FINAL WORST= 38
 FINAL TOTAL= 1082

F==>	1	3	4	5	6	10	16	22	27	37
IM==>	32	35	37	38	37	35	32	31	30	38
F==>	40	56	68	88	88	88	91	93	99	113
IM==>	35	35	38	38	39	38	39	38	35	38
F==>	119	126	127	135	138	140	142	145	149	150
IM==>	38	38	38	35	38	39	34	37	38	38

IN-ADVANTAGE (DB) ==> 8.88

DAYTIM=FRI, AUG 23 1985 23:49
 ECPU= 1182.81 SEC.

SCENARIO

 K= 40
 N= 80

FOR THE EVEN INITIAL ASSIGNMENT:

1	3	5	7	9	11	13	15	17	19
21	23	25	27	29	31	33	35	37	39
42	44	46	48	50	52	54	56	58	60
62	64	66	68	70	72	74	76	78	80

INITIAL WORST= 408
 INITIAL TOTAL= 14280

FINAL WORST= 208
 FINAL TOTAL= 7970

F==>	1	2	3	4	5	6	7	8	9	10
IM==>	178	184	190	193	198	198	200	203	204	202
F==>	15	18	17	22	25	28	31	33	37	38
IM==>	201	208	204	202	203	201	205	203	204	208
F==>	42	44	50	52	54	56	59	62	64	65
IM==>	204	202	204	206	208	207	202	208	208	208
F==>	68	68	72	73	75	78	77	78	78	80
IM==>	198	203	198	203	198	192	192	186	184	174

IN-ADVANTAGE (DB) ==> 4.23

DAYTIM=SAT, AUG 24 1985 01:12
 ECPU= 2082.97 SEC.

SCENARIO

 K= 40
 N= 120

FOR THE EVEN INITIAL ASSIGNMENT:

1	4	7	10	13	16	19	22	25	28
32	35	38	41	44	47	50	53	56	59
62	65	68	71	74	77	80	83	86	89
93	96	99	102	105	108	111	114	117	120

INITIAL WORST# 408
 INITIAL TOTAL# 12940

FINAL WORST# 132
 FINAL TOTAL# 5031

F#>	1	2	3	4	5	6	11	12	13	21
IM#>	115	120	118	125	119	121	124	129	128	127
F#>	25	27	28	32	38	41	42	50	53	55
IM#>	129	129	126	126	131	132	124	128	132	129
F#>	64	65	71	73	83	87	89	99	100	102
IM#>	125	130	129	126	127	125	128	130	130	129
F#>	107	108	111	113	114	115	117	114	119	120
IM#>	127	124	129	127	124	122	129	121	117	115

IM-ADVANTAGE (DB) ==> 6.21

DATTIM=SAT, AUG 24 1985 02:27
 ECPU# 3506.52 SEC.

SCENARIO

 K= 50
 N= 100

FOR THE EVEN INITIAL ASSIGNMENT:

1	3	5	7	9	11	13	15	17	19
21	23	25	27	29	31	33	35	37	39
41	43	45	47	49	52	54	56	58	60
62	64	66	68	70	72	74	76	78	80
82	84	86	88	90	92	94	96	98	100

INITIAL WORST# 648
 INITIAL TOTAL# 28512

FINAL WORST# 341
 FINAL TOTAL# 16253

F#>	1	2	3	4	5	6	7	9	10	11
IM#>	243	296	306	307	317	322	325	337	333	334
F#>	12	13	16	17	21	23	24	27	31	37
IM#>	336	340	328	334	326	327	323	317	325	331
F#>	38	41	43	45	52	54	56	60	62	63
IM#>	330	336	333	332	324	334	334	340	337	337
F#>	67	68	69	76	78	81	82	86	87	88
IM#>	334	326	325	326	329	317	341	334	332	337
F#>	90	91	93	94	95	96	97	98	99	100
IM#>	336	331	328	325	319	319	311	304	293	286

IM-ADVANTAGE (DB) ==> 4.10

DATTIM=SAT, AUG 24 1985 07:52
 ECPU# 5824.97 SEC.

Figure A2.1. (continued)

SCENARIO

 K# 20
 N# 40

VORST# 45
 TOTAL# 823

F#>	1	2	3	4	5	8	12	13	18	20
IM#>	38	34	33	41	41	40	42	42	42	45
F#>	23	26	27	31	34	35	37	38	39	40
IM#>	43	45	43	42	41	42	44	41	39	35

IN-ADVANTAGE (DB) ==> 4.47

DATTIM#FRI, AUG 30 1985 23:12
 ECP# 7.70 SEC.

SCENARIO

 K# 20
 N# 60

VORST# 28
 TOTAL# 482

F#>	1	2	4	5	8	13	14	21	22	30
IM#>	23	21	25	25	22	24	24	27	28	24
F#>	35	37	40	45	48	55	56	58	59	60
IM#>	24	27	28	24	28	25	24	24	28	19

IN-ADVANTAGE (DB) ==> 6.53

DATTIM#FRI, AUG 30 1985 23:11
 ECP# 14.12 SEC.

SCENARIO

 K# 20
 N# 80

VORST# 19
 TOTAL# 330

F#>	1	2	4	8	9	13	21	25	31	35
IM#>	17	16	15	17	14	18	18	16	14	19
F#>	45	46	59	61	71	72	74	77	78	40
IM#>	17	17	18	15	18	19	15	14	18	15

IN-ADVANTAGE (DB) ==> 8.22

DATTIM#SAT, AUG 31 1985 18:00
 ECP# 20.64 SEC.

SCENARIO

 K# 20
 N# 100

VORST# 85
 TOTAL# 248

F#>	1	2	4	8	13	18	21	31	40	45
IM#>	12	11	11	14	13	14	10	15	12	11
F#>	81	86	87	80	82	88	92	95	98	100
IM#>	14	13	11	15	13	12	9	13	12	11

IN-ADVANTAGE (DB) ==> 9.24

DATTIM#SAT, AUG 31 1985 18:01
 ECP# 27.16 SEC.

Figure A2.2. IN Results corresponding to Table 6.2

SCENARIO
 =====
 K= 30
 N= 80

WORST= 113
 TOTAL= 3185

F==>	1	2	3	4	5	7	8	13	14	15
IM==>	88	98	102	99	103	108	104	107	110	110
F==>	18	21	22	28	30	34	35	37	40	45
IM==>	111	107	108	112	111	113	110	111	108	111
F==>	47	49	51	53	55	56	57	58	58	60
IM==>	110	109	110	105	104	106	104	99	98	97

IM-ADVANTAGE (DB) ==> 4.25

DAYTIM=SAT, AUG 31 1985 18:03
 ECPU= 86.22 SEC.

SCENARIO
 =====
 K= 30
 N= 80

WORST= 71
 TOTAL= 1948

F==>	1	2	4	5	7	8	10	13	19	21
IM==>	58	57	62	71	64	66	68	68	60	64
F==>	26	31	38	41	45	54	58	59	63	64
IM==>	67	66	65	62	59	68	69	70	70	68
F==>	72	74	79	80	84	86	87	88	88	90
IM==>	66	68	69	65	71	69	64	60	64	59

IM-ADVANTAGE (DB) ==> 6.27

DAYTIM=SAT, AUG 31 1985 18:07
 ECPU= 158.83 SEC.

SCENARIO
 =====
 K= 30
 N= 120

WORST= 52
 TOTAL= 1375

F==>	1	2	4	6	11	12	13	18	21	31
IM==>	40	43	38	44	50	47	47	47	46	43
F==>	38	42	45	50	61	63	68	76	81	82
IM==>	45	49	46	43	49	51	52	51	46	50
F==>	94	96	102	108	114	116	117	118	119	120
IM==>	48	45	45	44	44	48	44	47	47	40

IM-ADVANTAGE (DB) ==> 7.63

DAYTIM=SAT, AUG 31 1985 18:12
 ECPU= 231.18 SEC.

SCENARIO
 =====
 K= 30
 N= 150

WORST= 39
 TOTAL= 1028

F==>	1	2	4	6	8	9	13	21	31	32
IM==>	36	29	34	33	33	38	34	37	32	36
F==>	33	41	45	54	66	74	81	90	98	102
IM==>	38	34	34	35	36	34	30	35	34	34
F==>	112	128	130	133	136	144	148	147	148	150
IM==>	34	34	32	35	36	34	35	33	36	31

IM-ADVANTAGE (DB) ==> 8.88

DAYTIM=SAT, AUG 31 1985 18:18
 ECPU= 304.23 SEC.

Figure A2.2. (continued)

SCENARIO

 K= 40
 N= 80

WORST= 209
 TOTAL= 7881

F==>	1	2	3	4	5	6	7	8	9	12
IM==>	177	185	193	198	206	203	205	207	209	208
F==>	13	14	18	21	24	25	28	31	35	37
IM==>	201	207	205	202	201	201	203	202	202	199
F==>	44	45	46	49	53	58	59	61	63	67
IM==>	188	200	200	199	200	204	202	199	202	203
F==>	69	71	72	74	75	78	77	78	79	80
IM==>	201	205	204	201	203	201	194	188	184	177

IM-ADVANTAGE (DB) ==> 4.21

DATTIM=SAT, AUG 31 1985 18:28
 ECPUM 479.30 SEC.

SCENARIO

 K= 40
 N= 120

WORST= 132
 TOTAL= 4885

F==>	1	2	3	4	5	8	11	12	13	17
IM==>	115	119	118	118	125	122	129	128	128	131
F==>	18	21	30	31	32	39	42	45	50	54
IM==>	127	123	132	127	131	132	130	125	129	129
F==>	61	63	66	76	77	81	88	92	94	96
IM==>	130	129	124	127	129	129	127	131	123	122
F==>	102	108	111	113	114	116	117	118	119	120
IM==>	122	125	119	121	122	117	120	119	118	110

IM-ADVANTAGE (DB) ==> 6.21

DATTIM=SAT, AUG 31 1985 18:47
 ECPUM 484.85 SEC.

SCENARIO

 K= 40
 N= 180

WORST= 97
 TOTAL= 3587

F==>	1	2	3	4	6	8	10	13	21	22
IM==>	80	86	90	88	88	84	92	91	95	94
F==>	30	31	32	42	45	50	54	66	72	78
IM==>	88	94	93	93	91	94	94	85	87	90
F==>	89	91	97	105	106	110	122	123	128	134
IM==>	92	85	82	84	92	84	88	88	93	93
F==>	144	145	148	149	152	155	158	158	159	160
IM==>	92	89	92	88	88	88	97	83	85	81

IM-ADVANTAGE (DB) ==> 7.94

DATTIM=SAT, AUG 31 1985 19:10
 ECPUM 1284.54 SEC.

Figure A2.2. (continued)

SCENARIO
 =====
 K= 40
 N= 200

WORST= 75
 TOTAL= 2743

F=>	1	2	3	4	5	6	7	8	9	10	11
IM=>	67	63	68	65	71	72	73	66	73	65	
F=>	31	40	45	46	55	62	66	61	66	54	
IM=>	67	67	67	74	72	67	73	66	75	75	
F=>	97	111	114	125	129	137	142	153	163	167	
IM=>	65	71	72	75	70	67	69	69	63	75	
F=>	171	175	187	190	192	196	197	198	199	200	
IM=>	68	65	68	66	65	69	62	70	67	62	

IM-ADVANTAGE (DB) ==> 8.66

DATTIM= SAT, AUG 31 1985 18:40
 ECPUM= 1683.62 SEC.

SCENARIO
 =====
 K= 50
 N= 100

WORST= 336
 TOTAL= 16171

F=>	1	2	3	4	5	6	7	8	9	11
IM=>	283	295	298	309	309	320	325	324	334	332
F=>	12	13	15	19	21	23	27	28	31	34
IM=>	334	334	333	331	332	328	331	329	332	331
F=>	36	40	44	45	46	51	58	61	62	68
IM=>	334	331	327	320	323	309	331	328	331	332
F=>	67	68	73	75	77	80	82	85	86	87
IM=>	336	335	336	336	335	330	332	334	330	333
F=>	88	91	92	94	95	96	97	98	99	100
IM=>	332	335	328	326	318	307	302	297	294	287

IM-ADVANTAGE (DB) ==> 4.16

DATTIM= SAT, AUG 31 1985 20:12
 ECPUM= 1802.92 SEC.

SCENARIO
 =====
 K= 50
 N= 150

WORST= 215
 TOTAL= 10171

F=>	1	2	3	4	5	6	7	8	9	10	13
IM=>	182	188	191	193	204	200	205	203	203	206	
F=>	18	21	23	31	32	33	41	45	46	54	
IM=>	207	205	204	207	204	202	202	203	204	214	
F=>	57	58	65	68	74	80	81	90	92	96	
IM=>	214	215	211	205	208	202	209	207	211	203	
F=>	102	107	109	112	119	123	126	128	130	133	
IM=>	202	209	208	206	208	211	206	205	204	208	
F=>	138	139	141	142	144	146	147	148	149	150	
IM=>	209	207	211	202	201	198	196	190	184	188	

IM-ADVANTAGE (DB) ==> 6.10

DATTIM= SAT, AUG 31 1985 21:32
 ECPUM= 3332.84 SEC.

Figure A2.2. (continued)

SCENARIO

 K= 50
 N= 200

WORST= 154
 TOTAL= 7337

FUN>	1	2	3	4	6	8	9	13	15	21
IMB>	137	135	140	144	146	146	142	150	144	151
FUN>	22	30	31	39	40	45	48	55	62	68
IMB>	145	152	152	153	147	146	147	147	144	152
FUN>	75	81	86	94	97	108	111	114	125	129
IMB>	145	142	150	146	142	144	149	149	151	145
FUN>	137	142	149	153	163	167	171	175	177	179
IMB>	144	141	148	151	148	152	152	148	152	154
FUN>	187	188	190	192	194	196	197	198	199	200
IMB>	147	154	149	148	149	149	142	148	140	138

IM-ADVANTAGE (DB) ==> 7.55
 DATTIM=SAT, AUG 31 1985 23:08
 ECPU= 4862.52 SEC.

SCENARIO

 K= 50
 N= 250

WORST= 122
 TOTAL= 5669

FUN>	1	2	3	4	5	7	8	11	13	21
IMB>	105	106	115	118	112	115	109	111	111	109
FUN>	29	31	38	44	45	51	58	66	69	81
IMB>	119	115	112	122	121	122	119	112	113	109
FUN>	88	97	108	110	118	123	135	149	151	162
IMB>	111	112	118	111	118	110	118	113	109	109
FUN>	170	177	179	194	198	202	210	211	223	225
IMB>	114	112	118	108	115	121	110	116	114	122
FUN>	227	232	234	239	243	244	245	248	249	250
IMB>	117	117	112	113	112	114	113	113	108	107

IM-ADVANTAGE (DB) ==> 8.56
 DATTIM=SUN, SEP 01 1985 00:59
 ECPU= 3266.22 SEC.

Figure A2.2. (continued)

SCENARIO

 K= 20
 N= 40

WITH TRIPLE

WORST# 44
 TOTAL# 834

F==>	1	2	3	4					17	18
IM==>	38	41	43	44	41	39	41	43	44	42
F==>	24	27	28	33	34	38	37	38	39	40
IM==>	43	43	41	41	44	44	42	42	42	38

IM-ADVANTAGE (DB) ==> 4.57

DATE= FRI, DEC 27 1985 20:31
 ECPU= 12.22 SEC.

SCENARIO

 K= 20
 N= 60

WORST# 30
 TOTAL# 927

F==>	1	4	7	10	11	12	13	18	26	27
IM==>	22	24	27	28	28	27	27	28	23	28
F==>	38	40	44	47	48	53	55	57	58	60
IM==>	24	27	30	30	28	29	25	25	24	22

IM-ADVANTAGE (DB) ==> 6.23

DATE= FRI, AUG 30 1985 16:07
 ECPU= 26.81 SEC.

SCENARIO

 K= 20
 N= 80

WITH TRIPLE

WORST# 19
 TOTAL# 343

F==>	1	4	5	7	8	14	20	22	35	34
IM==>	16	18	18	19	19	18	17	16	16	18
F==>	40	50	57	60	68	69	73	77	79	80
IM==>	18	18	18	17	15	17	19	16	15	15

IM-ADVANTAGE (DB) ==> 8.22

DATE= THU, JAN 02 1986 22:39
 ECPU= 41.60 SEC.

SCENARIO

 K= 20
 N= 100

WITH TRIPLE

WORST# 14
 TOTAL# 241

F==>	1	2	3	4	10	16	17	32	35	32
IM==>	10	12	13	12	13	12	14	14	12	13
F==>	58	60	70	77	80	88	93	97	98	100
IM==>	10	11	11	13	14	8	12	11	11	14

IM-ADVANTAGE (DB) ==> 3.54

DATE= FRI, DEC 27 1985 20:42
 ECPU= 59.40 SEC.

Figure A2.3. INU Results corresponding to Table 6.2

SCENARIO

 K= 30
 N= 60

WORST= 114
 TOTAL= 3234

F=>	1	2	4	5	7	8	10	11	12	13
IM=>	81	94	103	106	111	113	113	114	110	110
F=>	18	23	24	28	27	32	36	40	41	44
IM=>	108	113	113	112	113	109	108	111	110	114
F=>	47	48	51	53	55	56	57	58	58	60
IM=>	113	113	110	107	107	110	108	99	99	96

IM-ADVANTAGE (DB) => 4.22

DATTIM=THU, AUG 28 1985 18:38
 ECPU= 130.08 SEC.

SCENARIO

 K= 30
 N= 90

WORST= 69
 TOTAL= 1936

F=>	1	2	3	4	6	7	12	13	15	20
IM=>	59	59	65	65	67	68	68	68	68	68
F=>	22	29	31	37	46	50	52	60	63	66
IM=>	65	65	62	60	60	60	65	64	66	67
F=>	70	75	78	82	83	86	87	88	89	90
IM=>	64	68	67	65	66	65	66	62	63	60

IM-ADVANTAGE (DB) => 6.40

DATTIM=THU, AUG 28 1985 18:43
 ECPU= 255.48 SEC.

SCENARIO

 K= 30
 N= 120

WORST= 51
 TOTAL= 1373

F=>	1	3	5	6	7	9	10	16	22	26
IM=>	41	48	43	44	48	45	48	49	44	45
F=>	32	40	48	55	62	73	76	81	88	90
IM=>	43	45	40	44	39	46	42	48	47	47
F=>	98	100	105	108	110	113	117	118	119	120
IM=>	50	47	51	48	51	51	46	45	46	44

IM-ADVANTAGE (DB) => 7.71

DATTIM=THU, AUG 28 1985 18:50
 ECPU= 329.55 SEC.

SCENARIO

 K= 30
 N= 150

WITH TRIPLE

WORST= 39
 TOTAL= 1048

F=>	1	3	5	8	10	13	21	23	25	39
IM=>	30	32	34	35	34	33	33	33	35	33
F=>	55	64	70	71	85	92	97	108	110	111
IM=>	34	35	35	35	36	38	39	37	38	39
F=>	119	120	130	136	141	143	144	147	149	150
IM=>	37	33	34	38	37	37	38	34	32	33

IM-ADVANTAGE (DB) => 8.88

DATTIM=FRI, DEC 27 1985 21:06
 ECPU= 466.43 SEC.

Figure A2.3. (continued)

SCENARIO

K= 40
N= 80

WORST= 209
TOTAL= 7971

F==>	1	2	3	4	5	6	7	8	10	11
IN==>	180	180	192	191	200	204	201	206	207	208
F==>	14	16	18	20	22	28	29	33	35	36
IN==>	202	208	208	206	201	203	200	202	203	197
F==>	40	45	50	52	55	57	59	60	65	66
IN==>	195	196	204	201	205	207	209	204	205	203
F==>	68	69	73	74	75	76	77	78	79	80
IN==>	200	206	201	204	199	195	192	187	182	177

IN-ADVANTAGE (DB) ==> 4.21

DATTIM=PRI, AUG 09 1985 18:54
ECPU= 658.04 SEC.

SCENARIO

K= 40
N= 120

WORST= 131
TOTAL= 4966

F==>	1	2	3	4	5	6	7	9	10	16
IN==>	116	122	126	127	127	126	124	127	125	120
F==>	20	22	26	31	32	40	43	48	55	56
IN==>	126	123	126	127	126	124	121	127	121	130
F==>	62	65	73	76	81	88	90	92	99	100
IN==>	123	126	127	119	118	125	122	129	123	127
F==>	105	108	110	112	113	116	117	118	119	120
IN==>	124	123	125	129	126	131	127	123	116	112

IN-ADVANTAGE (DB) ==> 6.24

DATTIM=PRI, AUG 09 1985 22:26
ECPU= 1106.55 SEC.

SCENARIO

K= 40
N= 180

WORST= 95
TOTAL= 3543

F==>	1	2	3	4	7	8	10	13	15	23
IN==>	84	86	92	89	87	91	90	93	93	88
F==>	24	25	39	41	48	55	57	61	70	74
IN==>	91	88	90	92	90	89	92	93	88	85
F==>	82	95	98	101	105	116	120	123	130	134
IN==>	81	92	95	88	88	88	89	86	82	92
F==>	140	147	148	150	153	155	157	158	159	160
IN==>	87	84	92	82	84	90	90	93	85	83

IN-ADVANTAGE (DB) ==> 7.63

DATTIM=SUN, AUG 11 1985 72144
ECPU= 1811.27 SEC.

Figure A2.3. (continued)

SCENARIO

 K= 40
 N= 207

WORST= 74
 TOTAL= 2745

F#>	1	2	3	4	5	10	18	18	20	26
IM#>	61	70	68	68	58	67	67	71	72	68
F#>	31	34	42	44	52	64	78	73	77	104
IM#>	70	74	71	73	68	70	72	64	67	61
F#>	108	115	120	123	135	143	156	160	163	169
IM#>	64	63	63	65	66	67	70	68	73	74
F#>	170	179	180	186	192	193	197	198	198	200
IM#>	74	72	73	68	73	68	61	67	65	65

IM-ADVANTAGE (DB) ==> 6.77

DATTIM=SAT, AUG 17 1985 05150
 ECPU= 2261.77 SEC.

SCENARIO

 K= 50
 N= 100

WORST= 336
 TOTAL= 18113

F#>	1	2	3	4	5	6	7	8	10	11
IM#>	284	288	288	303	308	310	318	317	321	323
F#>	15	16	17	20	23	25	28	31	32	35
IM#>	321	328	328	327	324	331	328	334	334	337
F#>	38	39	41	47	52	54	56	58	60	66
IM#>	338	334	329	328	325	328	332	336	331	334
F#>	67	78	72	77	79	80	81	86	88	88
IM#>	336	334	329	330	336	331	331	333	328	330
F#>	80	82	83	84	85	86	87	88	89	100
IM#>	332	327	325	324	328	314	305	300	292	281

IM-ADVANTAGE (DB) ==> 4.14

DATTIM=TUE, AUG 13 1985 22130
 ECPU= 2358.84 SEC.

SCENARIO

 K= 50
 N= 150

WORST= 213
 TOTAL= 10182

F#>	1	2	3	4	5	6	8	10	11	13
IM#>	188	193	202	203	204	210	211	208	213	206
F#>	18	21	23	25	27	33	38	40	45	48
IM#>	208	204	207	203	199	204	204	208	201	206
F#>	55	56	64	70	71	81	85	86	92	97
IM#>	208	205	200	204	198	201	205	207	198	202
F#>	99	108	110	111	119	120	128	130	133	137
IM#>	208	205	198	205	199	197	211	208	208	203
F#>	138	141	143	144	145	148	147	148	148	150
IM#>	208	207	212	209	212	202	198	203	194	196

IM-ADVANTAGE (DB) ==> 6.14

DATTIM=FRI, AUG 23 1985 02111
 ECPU= 4060.03 SEC.

Figure A2.3. (continued)

SCENARIO

 K= 50
 N= 200

WORST= 156
 TOTAL= 7304

F#>	1	2	3	4	5	7	9	10	16	18
IM#>	137	136	134	142	143	145	145	147	148	142
F#>	20	26	31	34	42	46	48	52	64	70
IM#>	145	146	148	149	150	155	149	148	149	148
F#>	73	77	82	88	94	104	108	115	120	123
IM#>	149	153	150	153	156	150	150	150	154	150
F#>	135	140	143	146	156	160	163	169	170	179
IM#>	144	151	148	149	152	142	153	148	145	141
F#>	180	188	189	192	193	195	197	198	199	200
IM#>	140	144	148	148	148	151	137	134	136	132

IM-ADVANTAGE (DB) ==> 7.48

DATTIM=FRI, AUG 23 1985 04:48
 CPU= 6299.35 SEC.

SCENARIO

 K= 50
 N= 250

WORST= 122
 TOTAL= 5707

F#>	1	2	3	4	6	7	13	14	15	18
IM#>	109	108	111	110	110	122	121	118	116	119
F#>	27	31	33	41	46	47	52	61	65	77
IM#>	116	120	121	120	121	118	115	115	118	112
F#>	80	97	100	102	121	128	131	138	148	154
IM#>	108	117	113	107	112	116	114	105	118	113
F#>	163	170	176	185	186	203	206	212	220	226
IM#>	111	110	118	108	118	113	116	115	113	117
F#>	230	234	239	241	242	243	247	248	249	250
IM#>	115	118	112	117	119	111	110	112	109	108

IM-ADVANTAGE (DB) ==> 8.56

DATTIM=FRI, AUG 23 1985 22:17
 CPU= 8305.40 SEC.

Figure A2.3. (continued)

SCENARIO
 =====
 K= 20
 N= 48

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	6	8	12	13	18	20
23	26	27	31	34	36	37	38	38	40

INITIAL WORST= 45
 INITIAL TOTAL= 823

FINAL WORST= 64
 FINAL TOTAL= 825

F==>	1	2	3	4	6	8	12	13	18	20
IM==>	38	39	41	41	40	40	44	43	42	43
F==>	23	27	28	31	34	34	37	34	38	40
IM==>	43	43	44	41	41	41	43	42	42	34

IM-ADVANTAGE (DB) ==> 4.57

DATTIM=MON, SEP 02 1985 18:45
 ECPU= 8.81 SEC.

SCENARIO
 =====
 K= 20
 N= 60

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	4	5	8	13	14	21	22	30
35	37	40	45	48	55	56	58	59	60

INITIAL WORST= 28
 INITIAL TOTAL= 492

FINAL WORST= 28
 FINAL TOTAL= 492

F==>	1	2	4	5	8	13	14	21	22	30
IM==>	23	21	23	25	22	26	24	27	28	24
F==>	35	37	40	45	49	55	56	58	59	60
IM==>	24	27	28	24	28	25	24	24	26	18

IM-ADVANTAGE (DB) ==> 6.53

DATTIM=MON, SEP 02 1985 18:48
 ECPU= 9.47 SEC.

SCENARIO
 =====
 K= 20
 N= 80

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	4	8	9	13	21	25	31	35
45	48	59	61	71	72	74	77	79	80

INITIAL WORST= 19
 INITIAL TOTAL= 330

FINAL WORST= 19
 FINAL TOTAL= 330

F==>	1	2	4	8	9	13	21	25	31	35
IM==>	17	16	15	17	18	18	18	16	14	19
F==>	45	48	59	61	71	72	74	77	79	80
IM==>	17	17	18	15	18	19	15	14	18	15

IM-ADVANTAGE (DB) ==> 8.22

DATTIM=MON, SEP 02 1985 18:48
 ECPU= 12.33 SEC.

Figure A3.1. DELIN Results corresponding to Table 6.3 (Initial Assignment obtained from IN)

SCENARIO

 K= 20
 N= 100

FOR THE INSERT INITIAL ASSIGNMENT:

1 2 4 8 13 19 21 31 40 49
 61 66 67 80 82 89 92 95 96 100

INITIAL WORST= 15
 INITIAL TOTAL= 248

FINAL WORST= 15
 FINAL TOTAL= 248

F==>	1	2	4	8	13	19	21	31	40	49
IM==>	12	11	11	14	13	14	10	15	12	11
F==>	61	66	67	80	82	89	92	95	96	100
IM==>	14	15	11	15	13	12	9	13	12	11

IM-ADVANTAGE (DB) ==> 9.24

DATTIM=MON, SEP 02 1985 18:54
 ECPU= 18.23 SEC.

SCENARIO

 K= 30
 N= 60

FOR THE INSERT INITIAL ASSIGNMENT:

1 2 3 4 5 7 8 13 14 15
 18 21 22 28 30 34 35 37 40 45
 47 48 51 53 55 56 57 58 59 60

INITIAL WORST= 113
 INITIAL TOTAL= 3185

FINAL WORST= 110
 FINAL TOTAL= 3164

F==>	1	2	3	4	5	7	8	13	14	15
IM==>	91	100	109	105	109	110	109	106	104	104
F==>	18	21	22	28	30	34	35	37	40	45
IM==>	107	104	104	109	109	110	110	110	104	107
F==>	47	48	51	53	55	56	57	58	59	60
IM==>	106	108	110	107	106	109	106	103	99	93

IM-ADVANTAGE (DB) ==> 4.37

DATTIM=MON, SEP 02 1985 18:56
 ECPU= 93.83 SEC.

SCENARIO

 K= 30
 N= 90

FOR THE INSERT INITIAL ASSIGNMENT:

1 2 4 5 7 8 10 13 18 21
 26 31 38 41 45 54 58 59 63 64
 72 74 78 80 84 86 87 88 89 90

INITIAL WORST= 71
 INITIAL TOTAL= 1948

FINAL WORST= 68
 FINAL TOTAL= 1956

F==>	1	2	3	4	5	7	8	10	13	18	21
IM==>	57	60	68	66	69	63	68	61	68	67	67
F==>	24	31	38	41	45	54	58	59	63	64	64
IM==>	66	65	65	61	63	68	69	67	69	68	68
F==>	72	74	78	80	84	86	87	88	89	90	90
IM==>	68	64	67	69	67	66	62	64	67	61	61

IM-ADVANTAGE (DB) ==> 6.40

DATTIM=MON, SEP 02 1985 19:11
 ECPU= 216.48 SEC.

Figure A3.1. (continued)

 K# 30
 N# 120

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	4	8	11	12	13	18	21	31
38	42	45	50	61	63	68	78	81	82
84	88	102	108	114	118	117	118	118	120

INITIAL WORST# 52
 INITIAL TOTAL# 1375

FINAL WORST# 49
 FINAL TOTAL# 1369

F##>	1	2	4	8	11	12	13	17	21	31
IM##>	43	42	44	45	47	47	47	47	44	44
F##>	38	42	45	50	61	63	73	76	81	84
IM##>	43	48	42	45	49	47	48	48	46	48
F##>	88	102	103	108	114	118	117	118	118	120
IM##>	45	48	48	48	48	48	48	43	48	40

IM-ADVANTAGE (DB) ==> 7.88

DATTIM=MON, SEP 02 1985 19:17
 ECPU# 337.75 SEC.

SCENARIO

 K# 30
 N# 150

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	4	8	9	13	21	31	32	
33	41	45	54	66	74	81	90	98	102
112	128	130	133	138	144	146	147	149	150

INITIAL WORST# 39
 INITIAL TOTAL# 1024

FINAL WORST# 38
 FINAL TOTAL# 1040

F##>	1	2	4	8	9	13	21	31	32	
IM##>	36	33	37	33	33	37	35	36	35	37
F##>	33	41	45	66	74	81	90	98	102	112
IM##>	38	32	31	34	33	28	34	32	32	36
F##>	119	128	130	133	138	144	146	147	149	150
IM##>	38	35	36	37	37	36	35	37	36	33

IM-ADVANTAGE (DB) ==> 8.95

DATTIM=MON, SEP 02 1985 19:22
 ECPU# 245.56 SEC.

SCENARIO

 K# 40
 N# 80

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	7	8	9	12
13	14	18	21	24	25	28	31	35	37
44	45	48	49	53	58	58	61	63	67
69	71	72	74	75	76	77	78	79	80

INITIAL WORST# 209
 INITIAL TOTAL# 7981

FINAL WORST# 208
 FINAL TOTAL# 7981

F##>	1	2	3	4	5	6	7	8	9	12
IM##>	177	185	193	198	208	203	205	203	208	208
F##>	13	14	18	21	24	25	28	31	35	37
IM##>	202	207	209	202	201	201	203	202	202	199
F##>	44	45	48	49	53	58	58	61	63	67
IM##>	198	200	200	199	200	204	202	199	202	203
F##>	69	71	72	74	75	76	77	78	79	80
IM##>	201	205	204	201	203	201	194	188	184	177

IM-ADVANTAGE (DB) ==> 4.21

DATTIM=MON, SEP 02 1985 19:28
 ECPU# 200.83 SEC.

Figure A3.1. (continued)

SCENARIO

 K= 40
 N= 120

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	8	11	12	13	17
18	21	30	31	32	38	42	45	50	54
81	83	88	78	77	81	88	92	94	98
102	108	111	113	114	116	117	118	119	120

INITIAL WORST= 132
 INITIAL TOTAL= 4885

FINAL WORST= 131
 FINAL TOTAL= 4882

F==>	1	2	3	4	5	8	11	12	13	17
IM==>	114	119	117	121	125	122	128	131	129	130
F==>	18	21	30	31	32	38	42	45	50	54
IM==>	129	124	130	129	128	129	128	128	130	124
F==>	81	83	88	78	77	81	88	92	94	98
IM==>	131	130	125	125	126	130	127	129	121	124
F==>	102	108	111	113	114	116	117	118	119	120
IM==>	125	127	122	124	121	115	120	120	120	111

IM-ADVANTAGE (DB) ==> 6.24

DATTIM=MON, SEP 02 1985 18:40
 ECPU= 757.04 SEC.

SCENARIO

 K= 40
 N= 180

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	6	8	10	13	21	22
30	31	32	42	45	50	54	66	72	78
89	81	97	105	108	110	122	123	128	138
144	145	148	149	152	155	158	154	159	160

INITIAL WORST= 97
 INITIAL TOTAL= 3587

FINAL WORST= 84
 FINAL TOTAL= 3587

F==>	1	2	3	4	6	8	10	12	13	15
IM==>	84	82	92	88	91	91	94	93	94	93
F==>	22	30	31	42	45	50	54	66	72	78
IM==>	92	88	89	93	93	93	92	88	86	90
F==>	89	81	97	105	108	110	122	123	128	138
IM==>	93	85	82	93	93	93	91	85	89	92
F==>	144	144	145	148	152	155	158	154	159	160
IM==>	91	92	88	89	93	88	94	84	84	82

IM-ADVANTAGE (DB) ==> 7.68

DATTIM=MON, SEP 02 1985 20:04
 ECPU= 1295.18 SEC.

Figure A3.1. (continued)

SCENARIO

 KM = 50
 NM = 100

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	7	8	9	11
12	13	15	19	21	23	27	28	31	34
38	40	44	45	48	51	58	61	62	68
67	68	73	75	77	80	82	85	86	87
89	91	92	94	95	96	97	98	99	100

INITIAL WORST= 336
 INITIAL TOTAL= 16171
 FINAL WORST= 335
 FINAL TOTAL= 16148

FMS>	1	2	3	4	5	6	7	8	9	11
IMS>	288	287	297	310	308	322	328	330	331	328
FMS>	12	13	15	19	21	23	27	28	31	34
IMS>	332	333	334	330	330	330	333	332	330	331
FMS>	38	40	44	45	47	48	51	61	62	68
IMS>	332	332	328	327	327	328	317	326	323	335
FMS>	67	68	73	75	77	80	82	85	86	87
IMS>	329	333	333	331	330	333	329	329	331	333
FMS>	89	91	92	94	95	96	97	98	99	100
IMS>	330	331	327	328	313	311	299	297	291	287

IM-ADVANTAGE (DB) ==> 4.17
 DATIM=MON, SEP 02 1985 20:38
 ECPU= 1230.45 SEC.

SCENARIO

 KM = 50
 NM = 150

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	8	8	10	13
16	21	23	31	32	33	41	45	46	54
57	58	65	66	74	80	81	90	92	96
102	107	108	112	119	123	126	128	130	133
138	139	141	142	144	146	147	148	149	150

INITIAL WORST= 215
 INITIAL TOTAL= 10171
 FINAL WORST= 213
 FINAL TOTAL= 10178

FMS>	1	2	3	4	5	6	8	9	10	13
IMS>	182	187	184	192	203	201	208	202	203	203
FMS>	16	21	23	31	32	33	41	45	46	54
IMS>	206	202	204	208	200	200	201	204	205	212
FMS>	57	58	66	74	80	81	87	90	92	96
IMS>	210	210	203	205	206	203	213	213	211	207
FMS>	102	107	108	112	119	123	126	128	130	133
IMS>	202	212	212	209	210	211	207	208	203	208
FMS>	138	139	141	142	144	146	147	148	149	150
IMS>	213	208	211	203	203	197	196	184	192	185

IM-ADVANTAGE (DB) ==> 6.14
 DATIM=MON, SEP 02 1985 21:04
 ECPU= 1387.45 SEC.

Figure A3.1. (continued)

SCENARIO

 N= 50
 N= 200

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	7	8	9	13	15	21
22	30	31	39	40	45	46	55	62	66		
75	81	86	94	97	108	111	114	125	129		
137	142	149	153	163	167	171	175	177	179		
187	189	190	192	194	196	197	198	199	200		

INITIAL WORST= 154
 INITIAL TOTAL= 7337

FINAL WORST= 154
 FINAL TOTAL= 7337

F=>	1	2	3	4	5	6	7	8	9	13	15	21
IM=>	137	135	140	144	148	146	142	150	144	151		
F=>	22	30	31	39	40	45	46	55	62	66		
IM=>	145	152	152	153	147	146	147	147	144	152		
F=>	75	81	86	94	97	108	111	114	125	129		
IM=>	145	142	150	148	142	148	149	149	151	145		
F=>	137	142	149	153	163	167	171	175	177	179		
IM=>	144	141	146	151	148	152	152	148	152	154		
F=>	187	189	190	192	194	196	197	198	199	200		
IM=>	147	154	149	146	149	149	142	148	140	136		

IM-ADVANTAGE (DB) => 7.55

DATTIM=MON, SEP 02 1985 21:27
 ECPUR 1260.09 SEC.

SCENARIO

 N= 50
 N= 250

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	7	8	11	13	21
25	31	39	44	45	51	58	66	69	81
88	97	104	110	116	123	135	149	151	162
170	177	179	194	198	202	210	211	223	225
227	232	234	239	243	244	245	248	249	250

INITIAL WORST= 122
 INITIAL TOTAL= 5688

FINAL WORST= 119
 FINAL TOTAL= 5688

F=>	1	2	3	4	5	7	8	11	13	21
IM=>	109	109	109	113	112	112	111	111	114	111
F=>	25	31	39	45	51	58	66	71	81	88
IM=>	115	114	111	112	119	115	116	119	114	117
F=>	87	100	108	110	116	123	135	149	151	162
IM=>	118	118	117	115	119	116	118	115	117	111
F=>	170	177	179	194	198	202	210	211	223	225
IM=>	114	115	117	111	115	117	107	117	110	118
F=>	227	232	234	239	243	244	245	248	249	250
IM=>	118	115	108	113	113	114	113	114	108	107

IM-ADVANTAGE (DB) => 8.67

DATTIM=MON, SEP 02 1985 22:49
 ECPUR 4876.55 SEC.

Figure A3.1. (continued)

SCENARIO

 N= 40
 N= 200

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	8	9	13	15	21	22
31	40	45	46	55	62	66	61	66	64
87	111	114	125	128	137	142	153	163	167
171	175	167	190	182	196	197	198	199	200

INITIAL WORST= 75
 INITIAL TOTAL= 2745

FINAL WORST= 75
 FINAL TOTAL= 2745

F##>	1	2	3	4	8	9	13	15	21	22
IM##>	67	63	69	65	71	72	73	66	73	65
F##>	31	40	45	46	55	62	66	61	66	64
IM##>	67	67	67	74	72	67	73	66	75	75
F##>	97	111	114	125	128	137	142	153	163	167
IM##>	69	71	72	75	70	67	68	69	63	75
F##>	171	175	167	190	182	196	197	198	199	200
IM##>	68	65	68	66	65	69	62	70	67	62

IM-ADVANTAGE (DB) ==> 0.66

DAYTIME=MON, SEP 02 1985 20:15.
 CPU= 518.38 SEC.

Figure A3.1. (continued)



SCENARIO
 =====
 K= 20
 N= 40

FOR THE INSERT INITIAL ASSIIONMENT:

	1	2	3	4	5	9	11	16	17	19
	24	27	28	33	34	35	37	38	39	40

INITIAL WORST= 44
 INITIAL TOTAL= 834

FINAL WORST= 44
 FINAL TOTAL= 834

F==>	1	2	3	4	5	9	11	16	17	19
IM==>	36	41	43	44	41	39	41	43	44	42
F==>	24	27	28	33	34	36	37	38	39	40
IM==>	43	43	41	41	44	44	42	42	42	38

IN-ADVANTAGE (OB) ==> 4.57
 DATTIM=THU, AUG 29 1985 18:52
 ECPU= 5.80 SEC.

SCENARIO
 =====
 K= 20
 N= 60

FOR THE INSERT INITIAL ASSIIONMENT:

	1	4	7	10	11	12	13	18	26	27
	36	40	44	47	48	53	55	57	58	60

INITIAL WORST= 30
 INITIAL TOTAL= 527

FINAL WORST= 27
 FINAL TOTAL= 495

F==>	1	2	4	7	11	13	18	26	27	32
IM==>	20	24	20	24	28	27	23	27	27	27
F==>	36	40	47	48	53	55	57	58	59	60
IM==>	26	22	26	27	27	28	28	24	24	22

IN-ADVANTAGE (DB) ==> 6.69
 DATTIM=FRI, AUG 30 1985 16:11
 ECPU= 25.14 SEC.

SCENARIO
 =====
 K= 20
 N= 80

FOR THE INSERT INITIAL ASSIIONMENT:

	1	4	5	7	8	14	20	22	35	36
	40	50	57	60	68	69	73	77	78	80

INITIAL WORST= 19
 INITIAL TOTAL= 343

FINAL WORST= 19
 FINAL TOTAL= 336

F==>	1	5	6	8	14	20	22	35	36	
IM==>	15	16	19	18	16	17	17	17	16	18
F==>	40	50	57	60	68	69	73	77	78	80
IM==>	17	18	17	18	18	18	18	17	15	14

IN-ADVANTAGE (OB) ==> 8.22
 DATTIM=THU, AUG 29 1985 18:52
 ECPU= 17.67 SEC.

Figure A3.2. DELIN Results corresponding to Table 6.3 (Initial Assignment obtained from INU)

SCENARIO

 K= 20
 N= 100

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	10	16	17	32	35	52
56	60	70	77	80	88	93	97	99	100

INITIAL WORST= 14
 INITIAL TOTAL= 241

FINAL WORST= 14
 FINAL TOTAL= 241

F==>	1	2	3	4	10	16	17	32	35	52
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	10	12	13	12	13	12	14	14	12	13
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
F==>	56	60	70	77	80	88	93	97	99	100
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	10	11	11	13	14	6	12	11	12	14

IM-ADVANTAGE (DB) ==> 9.54

DATTIM=THU, AUG 29 1985 18:52
 ECPU= 15.39 SEC.

SCENARIO

 K= 30
 N= 60

WITH TRIPLE

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	4	5	7	8	10	11	12	13
18	23	24	26	27	32	36	40	41	44
47	48	51	53	55	56	57	58	59	60

INITIAL WORST (T) = 114
 INITIAL TOTAL (T) = 3234

WORST= 111
 TOTAL= 3171

F==>	1	2	3	4	5	7	8	10	12	13
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	91	98	99	102	105	110	110	109	108	109
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
F==>	18	23	24	26	31	32	36	40	41	44
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	105	109	108	108	111	110	111	109	108	105
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
F==>	47	48	51	53	55	56	57	58	59	60
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	111	111	108	110	108	107	101	98	98	95

IM-ADVANTAGE (DB) ==> 4.33

DATTIM=FRI, DEC 27 1985 21:27
 ECPU= 110.78 SEC.

SCENARIO

 K= 30
 N= 90

WITH TRIPLE

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	6	7	12	13	15	20
22	29	31	37	46	50	52	60	63	66
70	75	78	82	83	86	87	88	89	90

INITIAL WORST (T) = 69
 INITIAL TOTAL (T) = 1936

WORST= 69
 TOTAL= 1936

F==>	1	2	3	4	6	7	12	13	15	20
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	58	59	65	65	67	66	69	68	68	68
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
F==>	22	29	31	37	46	50	52	60	63	66
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	65	65	62	60	60	60	65	64	66	67
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
F==>	70	75	78	82	83	86	87	88	89	90
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
IM==>	64	69	67	65	66	65	66	62	63	60

IM-ADVANTAGE (DB) ==> 6.40

DATTIM=FRI, DEC 27 1985 22:01
 ECPU= 88.45 SEC.

Figure A3.2. (continued)

 K= 30
 M= 120

WITH TRIPLE

FOR THE INSERT INITIAL ASSIGNMENT:

1 3 5 6 7 9 10 18 22 24
 32 40 48 55 62 73 76 81 88 90
 99 100 105 108 110 113 117 118 119 120

INITIAL WORST (T) = 51
 INITIAL TOTAL (T) = 1373

WORST= 50
 TOTAL= 1383

F==>	1	3	5	6	7	9	10	18	22	24
IM==>	41	47	48	48	49	47	48	45	44	47
F==>	32	40	48	55	54	62	76	81	88	90
IM==>	43	43	41	44	46	43	41	42	44	44
F==>	99	100	105	108	113	116	117	118	119	120
IM==>	48	47	48	44	50	49	48	45	47	45

IM-ADVANTAGE (DB) ==> 7.80

DATTIM=FRI, DEC 27 1985 22:13
 ZCPU= 229:08 SEC.

SCENARIO

 K= 30
 M= 150

FOR THE INSERT INITIAL ASSIGNMENT:

1 3 5 8 10 13 21 23 25 39
 55 64 70 71 85 92 97 106 110 111
 119 120 130 138 141 143 144 147 149 150

INITIAL WORST= 39
 INITIAL TOTAL= 1046

FINAL WORST= 38
 FINAL TOTAL= 1029

F==>	1	2	3	5	10	13	21	23	29	39
IM==>	29	32	34	36	33	32	32	34	36	31
F==>	55	64	70	71	85	92	97	106	110	115
IM==>	34	34	34	34	35	37	33	34	35	37
F==>	120	130	137	141	143	144	145	147	149	150
IM==>	33	31	35	38	33	36	37	37	33	35

IM-ADVANTAGE (DB) ==> 8.99

DATTIM=THU, AUG 08 1985 22:04
 ZCPU= 530.29 SEC.

SCENARIO

 K= 40
 M= 80

FOR THE INSERT INITIAL ASSIGNMENT:

1 2 3 4 5 6 7 8 10 11
 14 16 18 20 22 28 29 33 35 36
 40 45 50 52 55 57 59 60 65 66
 68 69 73 74 75 76 77 78 79 80

INITIAL WORST= 209
 INITIAL TOTAL= 7971

FINAL WORST= 208
 FINAL TOTAL= 7962

F==>	1	2	3	4	5	6	7	8	10	11
IM==>	181	183	191	193	199	201	204	208	206	204
F==>	14	16	18	20	22	28	29	33	36	39
IM==>	202	207	206	204	206	199	200	198	195	199
F==>	40	45	50	52	55	57	59	60	65	66
IM==>	200	195	196	198	204	208	206	202	202	204
F==>	68	72	73	74	75	76	77	78	79	80
IM==>	203	208	203	204	198	197	196	191	183	178

IM-ADVANTAGE (DB) ==> 4.23

DATTIM=FRI, AUG 09 1985 19:32
 ZCPU= 570.48 SEC.

Figure A3.2. (continued)

SCENARIO

 K= 40
 N= 120

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	7	8	9	10	11
20	22	28	31	32	40	43	48	55	56	
62	65	73	76	81	88	90	92	99	100	
105	108	110	112	113	116	117	118	119	120	

INITIAL WORST# 131
 INITIAL TOTAL# 4964

FINAL WORST# 130
 FINAL TOTAL# 4964

F=>	1	2	3	4	5	6	7	8	9	10	11
IM=>	118	121	125	125	125	124	128	128	128	127	121
F=>	20	22	26	31	32	40	43	48	55	56	
IM=>	126	123	127	125	123	124	123	126	122	127	
F=>	62	65	73	76	81	88	90	92	99	100	
IM=>	121	126	126	118	116	124	118	129	128	128	
F=>	105	108	110	112	113	116	117	118	119	120	
IM=>	123	124	126	129	128	130	124	122	119	113	

IM-ADVANTAGE (DB) => 8.77

DATTIM=SUN, AUG 11 1985 18159
 ECPU# 430.25 SEC.

SCENARIO

 K= 40
 N= 160

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	7	8	10	13	15	23
24	25	39	41	46	55	57	69	70	74
82	95	98	101	105	116	120	123	130	134
140	147	148	150	153	156	157	158	159	160

INITIAL WORST# 95
 INITIAL TOTAL# 3563

FINAL WORST# 95
 FINAL TOTAL# 3563

F=>	1	2	3	4	7	8	10	13	15	23
IM=>	84	88	92	89	87	91	90	93	93	88
F=>	24	25	39	41	46	55	57	69	70	74
IM=>	91	88	90	92	90	89	92	93	88	85
F=>	82	95	98	101	105	116	120	123	130	134
IM=>	81	92	95	90	90	88	85	86	82	92
F=>	140	147	148	150	153	156	157	158	159	160
IM=>	87	94	93	92	84	90	90	93	85	83

IM-ADVANTAGE (DB) => 7.63

DATTIM=MDN, AUG 12 1985 21100
 ECPU# 392.84 SEC.

Figure A3.2 (continued)

SCENARIO

 K= 40
 N= 200

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	10	15	18	20	25
31	34	42	48	52	64	70	73	77	104
108	115	120	123	135	143	158	160	183	189
178	179	180	188	192	193	197	198	199	200

INITIAL WORST= 74
 INITIAL TOTAL= 2745

FINAL WORST= 74
 FINAL TOTAL= 2737

F==>	1	2	3	4	5	10	15	18	20	25
IM==>	62	65	67	69	66	69	63	68	65	67
F==>	31	34	42	52	64	69	73	77	104	108
IM==>	68	73	66	65	68	74	69	69	74	72
F==>	111	115	120	123	135	143	158	160	183	189
IM==>	74	88	72	72	68	68	67	67	68	70
F==>	178	179	180	188	192	193	197	198	199	200
IM==>	73	72	72	68	73	67	64	65	62	67

IM-ADVANTAGE (DB) ==> 8.72

DATTIM=VED, AUG 21 1985 01:12
 ECPU= 1249.90 SEC.

SCENARIO

 K= 50
 N= 100

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	7	8	10	11
15	16	17	20	23	25	28	31	32	35
38	39	41	47	52	54	56	58	60	66
67	70	72	77	79	80	81	86	88	89
90	92	93	94	95	96	97	98	99	100

INITIAL WORST= 338
 INITIAL TOTAL= 16113

FINAL WORST= 336
 FINAL TOTAL= 16132

F==>	1	2	3	4	5	6	7	8	10	11
IM==>	288	290	308	307	315	318	325	327	330	331
F==>	12	15	16	17	20	23	26	31	32	35
IM==>	335	327	331	323	324	313	322	332	327	334
F==>	38	39	41	47	52	54	56	58	60	66
IM==>	329	331	324	324	322	327	328	333	329	329
F==>	67	70	72	77	79	80	81	86	88	89
IM==>	331	330	322	327	330	322	331	334	333	334
F==>	90	92	93	94	95	96	97	98	99	100
IM==>	338	333	330	326	323	319	318	304	293	285

IM-ADVANTAGE (DB) ==> 4.16

DATTIM=VED, AUG 14 1985 17:32
 ECPU= 1176.26 SEC.

Figure A3.2. (continued)

SCENARIO

 K= 50
 N= 150

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	8	10	11	13
18	21	23	25	27	33	39	40	45	48
55	58	64	70	71	81	85	88	92	97
99	108	110	111	119	120	128	130	133	137
138	141	143	144	145	146	147	148	149	150

INITIAL WORST= 213
 INITIAL TOTAL= 10182

FINAL WORST= 211
 FINAL TOTAL= 10178

F==>	1	2	3	4	5	6	8	10	11	13
IM==>	189	189	200	202	202	209	208	209	208	200
F==>	18	21	23	25	27	33	39	40	45	48
IM==>	200	205	207	206	208	205	207	206	203	206
F==>	55	58	64	70	71	81	85	88	92	97
IM==>	203	198	205	204	197	202	204	207	201	198
F==>	99	108	110	111	119	120	124	128	130	137
IM==>	209	210	201	203	205	202	208	209	207	200
F==>	138	141	143	144	145	146	147	148	149	150
IM==>	202	205	211	207	211	204	196	198	194	193

IM-ADVANTAGE (DB) ==> 8.18

DATTIM=TUE, AUG 27 1985 03:54
 ECPU= 1744.88 SEC.

SCENARIO

 K= 50
 N= 200

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	7	9	10	16	18
20	26	31	34	42	46	48	52	64	70
73	77	82	89	94	104	108	115	120	123
135	140	143	146	156	160	163	169	170	179
180	188	189	192	193	195	197	198	199	200

INITIAL WORST= 156
 INITIAL TOTAL= 7304

FINAL WORST= 153
 FINAL TOTAL= 7310

F==>	1	2	3	4	5	7	9	10	16	18
IM==>	138	137	133	143	139	143	144	145	146	144
F==>	20	26	31	34	42	46	48	52	64	70
IM==>	146	150	149	149	146	147	145	144	151	148
F==>	73	77	82	89	94	104	108	115	120	123
IM==>	150	153	147	150	153	151	146	150	152	152
F==>	135	140	143	146	160	163	169	170	175	179
IM==>	145	147	147	148	144	152	151	153	153	145
F==>	180	188	189	190	192	193	197	198	199	200
IM==>	146	148	150	149	149	149	139	135	136	133

IM-ADVANTAGE (DB) ==> 7.58

DATTIM=TUE, AUG 27 1985 19:55
 ECPU= 3589.77 SEC.

Figure A3.2. (continued)

SCENARIO
 =====
 K= 90
 N= 250

FOR THE INSERT INITIAL ASSIGNMENT:

1	2	3	4	5	6	7	13	14	15	18
27	31	33	41	48	47	52	51	55	77	
80	87	100	102	121	128	131	138	148	154	
183	170	178	185	188	203	208	212	220	228	
230	234	238	241	242	243	247	248	248	250	

INITIAL WORST= 122
 INITIAL TOTAL= 9707
 FINAL WORST= 120
 FINAL TOTAL= 9708

F==>	1	2	3	4	5	6	7	13	14	15	18
IM==>	108	112	106	110	113	120	115	117	117	118	118
F==>	27	31	33	41	48	47	52	55	77	80	
IM==>	119	113	118	120	116	109	113	115	118	111	111
F==>	87	100	102	121	128	131	138	148	154	163	
IM==>	118	110	113	110	116	116	106	112	114	115	115
F==>	170	178	181	185	189	203	208	212	220	226	
IM==>	113	117	120	116	120	116	115	118	118	118	118
F==>	230	234	238	241	242	243	247	248	248	250	
IM==>	115	118	117	114	114	113	113	110	110	103	103

IM-ADVANTAGE (DB) ==> 8.83
 DATIM=VED, AUG 28 1985 01:13
 ECPU= 3087.63 SEC.

Figure A3.2. (continued)

REFERENCES

- [1] Atkinson, M.D., Hassenklover, A. (1984), Sets of Integers with Distinct Differences, School of Computer Science, Carleton University, Ottawa, Ontario, Canada, October 1984.
- [2] Atkinson, M.D., Santoro, N., Urrutia, J. (1984), Integer Sets with Distinct Sums and Differences; Carrier Frequency Assignments for Non-linear Repeaters, School of Computer Science, Carleton University, Ottawa, Ontario, Canada, August 1984.
- [3] Babcock, W.C. (1953), Intermodulation Interference in Radio Systems: Frequency of Occurance and Control by Channel Selection, The Bell System Technical Journal, Vol. 32, No. 1, pp. 63-73.
- [4] Blachman, N.M. (1971), Detectors, Bandpass Nonlinearities, and their Optimization: Inversion of the Chebyshev Transform, IEEE Transactions on Information Theory, July 1971.
- [5] Bond, F.E., Meyer, H.F. (1970), Intermodulation Effects in Limiter Amplifier Repeaters, IEEE Transactions on Communication Technology, Vol. COM-18, No. 2, April 1970.

- [6] Bondy, J.A., Murty, U.S.R. (1980), Graph Theory with Applications, Department of Computer and Optimization, University of Waterloo, Ontario, Canada.
- [7] Chase, W.M., Rockway, J.W., Salisbury, G.C. (~~1975~~), A Method of Detecting Significant Sources of Intermodulation Interference, IEEE Transactions on Electromagnetic Compatibility, Vol. EMC-17, No. 2, May 1975.
- [8] Eng, K.Y., Stern, T.E. (1981), The Order-and-Type Prediction Problem Arising from Passive Intermodulation Interference in Communications Satellites, IEEE Transactions on Communications, Vol. COM-29, No. 5, May 1981, pp. 549-555.
- [9] Fang, R.J.F., Sandrin, W.A. (1977), Carrier Frequency Assignment for Nonlinear Repeaters, COMSAT Technical Review, Vol. 7, No. 1, Spring 1977, pp. 227-245.
- [10] Fuenzalida, J.C., Shimbo, O., Cook, W.L. (1973), Time-domain Analysis of Intermodulation Effects Caused by Nonlinear Amplifiers, COMSAT Technical Review, Vol. 3, No. 1, Spring 1973, pp. 90-141.
- [11] Gagliardi, R.M. (1978), Introduction to Communications Engineering, John Wiley & Sons Inc., USA.
- [12] Gagliardi, R.M. (1984), Satellite Communications, Lifetime Learning Publications, USA.

- [13] Gardner, M. (1972), Mathematical Games, Scientific American, No. 226, June 1972, pp: 108-112/114-118.
- [14] Golomb, S.W. (1972), How to Number a Graph, In: Graph Theory and Computing, R.C. Read (Ed.), Academic Press, NY, USA, pp.23-37.
- [15] Hirata, Y. (1978), A Bound on the Relationship Between Intermodulation Noise and Carrier Frequency Assignment, COMSAT Technical Review, Vol. 8, No. 1, Spring 1978, pp. 141-154.
- [16] Horowitz, E., Sahni, S. (1978), Fundamentals of Computer Algorithms, Computer Science Press Inc., USA, pp. 323-369.
- [17] Jay, F. (1984), IEEE Standard Dictionary of Electrical and Electronic Terms, IEEE.
- [18] Johannsen, K.G., Paulsen, F.L. (1976), Limiter Intermodulation Improvement Due to Selective Carrier Spacing, IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-12, No. 4, July 1976, pp. 451-458.
- [19] Lyons, R.G. (1975), Selecting Carrier Frequencies to Facilitate IM Products Measurements, IEEE Transactions on Communications, Vol. COM-23, No. 12, December 1975, pp. 1525-1526.
- [20] Marshall, Jr.H. (1956), A Survey of Difference Sets, Proceedings of American Mathematical Society, Vol. 7, December 1956, pp. 975-986.

- [21] Martin, J. (1978), Communications Satellite Systems, Prentice-Hall Inc., USA.
- [22] Mathur, K., Salkin, H.M., Nishimura, K., Morito, S. (1985), Applications of Integer Programming in Radio Frequency Management, Management Science, Vol. 31, No.7, July 1985.
- [23] Maxemchuk, N.F., Schiff, L. (1977), Third Order Intermodulation Interference - Bounds and Interference - Free Channel Assignment, IEEE Transactions on Communications, Vol. COM-25, No. 9, September 1977, pp. 1041-1046.
- [24] Morito, S., Salkin, H.M., Williams, D.E. (1980), Two Backtrack Algorithms for the Radio Frequency Intermodulation Problem, Applied Mathematics and Optimization, Vol. 6, pp. 221-240.
- [25] Morito, S., Salkin, H.M., Mathur, K. (1981), Computational Experience with a Dual Backtrack Algorithm for Identifying Frequencies Likely to Create Intermodulation Problems, IEEE Transactions on Electromagnetic Compatibility, Vol. EMC-23, No. 1, February 1981.
- [26] Okinaka, H., Yasuda, Y., Hirata, Y. (1984), Intermodulation Interference - Minimum Frequency Assignment for Satellite SPC Systems, IEEE Transactions on Communications, Vol. COM-32, No. 4, April 1984, pp. 462-468.

- [27] Ören, T.I. (1984), Graphic Representation of Pseudocodes and Computer Programs: A Unifying Technique and a Family of Documentation Programs. In: Proc. of EdCompCon-83 (First Educational Computing Conf. of IEEE Computer Society), D.C. Rine (Ed.), San Jose, CA, USA, October 18-20, 1983. IEEE Computer Society, New York, pp. 81-89.
- [28] Prentiss, S. (1983), Satellite Communications, TAB Books Inc., USA.
- [29] Robinson, J.P., Bernstein, A. (1967), A Class of Binary Recurrent Codes with Limited Error Propagation, IEEE Transactions on Information Theory, Vol. IT-13, No. 1, January 1967, pp. 106-113.
- [30] Sea, G.R. (1968), An Algebraic Formula for Amplitudes of Intermodulation Products Involving an Arbitrary Number of Frequencies, In: Proceedings of the IEEE, Vol. 56, No. 8, August 1968, pp. 1388-1399.
- [31] Shimbo, O. (1971), Effects of Intermodulation, AM/PM Conversion, and Additive Noise in Multicarrier TWT Systems, In: Proceedings of the IEEE, Vol. 59, No. 2, February 1971, pp. 230-238.
- [32] Simmons, G.J. (1985), Synch-Sets: A Variant of Difference Sets, In: Proceedings of the Fifth S-E Conference Combinatorics, Graph Theory and Computing, pp. 625-645.

- [33] Singer, J. (1938), A Theorem in Finite Projective Geometry and Some Applications to Number Theory, Transactions of American Mathematical Society, Vol. 43, May 1938, pp. 377-385.
- [34] Stremler, F.G. (1977), Introduction to Communication Systems, Addison-Wesley Publishing Company, USA.
- [35] Sunde, E.D. (1965), Intermodulation Distortion in Multicarrier FM Systems, Bell Telephone Laboratories Inc., Murray Hill, NJ, USA.
- [36] Sunde, E.D. (1969), Communication Systems, Engineering Theory, John Wiley & Sons Inc., USA.
- [37] Vuong, X.T., Guilbord, A.F. (1984), Modelling of Nonlinear Elements Exhibiting Frequency-dependent AM/AM and AM/PM Transfer Characteristics, Canadian Electrical Engineering Journal, Vol. 9, No.3.
- [38] Vuong, X.T., Birta, L.G. (1983), Optimal Characteristics of a Memoryless Amplifier System, Paper presented at the International Electrical, Electronics Conference and Exposition, Toronto, Ontario, Canada, September 26-28, 1983.
- [39] Vuong, X.T., Birta, L.G. (1984), Optimal Characteristics of a Memoryless SCPC System, Paper presented at the International Conference on Communications, Amsterdam, Netherlands, May 14-17, 1984.

[40] Wescott, R.J. (1967), Investigation of Multiple FM/FDM Carriers through a Satellite TWT Operation near Saturation, In: Proceedings of the IEE, Vol. 114, No.6, June 1967, pp. 726-740.