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Abdul Salam YASSINE
AUTEUR DE LA THÈSE - AUTHOR OF THESIS

M. A. Sc. (Electrical Engineering)
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Department of Electrical Engineering
FACULTÉ, ÉCOLE, DÉPARTEMENT - FACULTY, SCHOOL, DEPARTMENT

TITRE DE LA THÈSE - TITLE OF THE THESIS

Performance Evaluation and Game Theoretic Pricing of Optimal Routing and
Flow Assignment in Optical Networks

D. Makrakis
DIRECTEUR DE LA THÈSE - THESIS SUPERVISOR

CO-DIRECTEUR DE LA THÈSE - THESIS CO-SUPERVISOR

EXAMINATEURS DE LA THÈSE - THESIS EXAMINERS

V. Groza

J. Talim

J.-M. De Koninck, Ph.D.

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Performance Evaluation and Game Theoretic Pricing of Optimal Routing and Flow Assignment in Optical Networks

By
Abdulsalam Yassine
B.s.c in Eng., Beirut Arab University, 1993

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Abstract

Transport service providers need control and optimization strategies for wavelength management, network provisioning, restoration, and protection allowing them to define and deploy new services offers. In this thesis, we apply microeconomic models to investigate the behavior of optical networks in different market environments, and the effect of different pricing schemes in the profitability of optical networks. The routing and wavelengths allocation were modeled using network flows optimization. Network pricing decisions were modeled by applying a game theoretic approach, where the game payoff is the profit of operating the network. We have developed a customer/supplier interaction protocol for network competition control. Two models of competition were introduced. The first model is called "active price competition", allows the customer (wavelength buyer) to interact in order to lower the prices. The second model, is called "passive price competition", assumes that the customer is passive and does not interact with the competitors; the customer accepts the lowest offered price or does not buy even from the lowest bidder if the price is above the buyer's maximum set cost value. In this model, the competitors monitor their own profit and the customer requests success rate to determine their pricing strategy. In both models, we examined different case studies, comparisons and performance evaluations. A mixed integer optimization is employed to determine the wavelengths allocation and flow assignment of the requested customer demand.

Acknowledgements

During the time I was working on my thesis, I accumulated various intellectual debts for which a mere acknowledgement is small repayment. I would like to express my genuine appreciation to my supervisor, Professor Dimitrios Makrakis, for his constant support and guidance.

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Above all, I cannot express my full gratitude to my mother and my father for their love and prayers. Lastly, I would like to thank my wife for the time she spent waiting me to come home.

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List of Acronyms and Symbols

Acronym	Definition
MC	Marginal Cost
TC	Total Cost
MR	Marginal Revenue
TR	Total Revenue
SLA	Service Level of Agreement
BB	Bandwidth Broker
ACA	Admission Control Agent
DWDM	Dense Wavelength Division Multiplexing
ADM	Add/Drop Multiplexers
OXC	Optical Cross Connects
WRON	Wavelength Routed Optical Networks
RWA	Routing and Wavelength Assignment
MIP	Mixed Integer Programming
IOTS	IntelliLight Optical Transport Service
MON	Multi-service Optical Network
TSP	Transport Service Providers
ISP	Internet Service Providers
CRTC	Canadian Radio-Television Commission
FCC	Federal Communication Commission
NAP	Network Access Points
VC	Virtual Connections

Chapter 1

1.1 Introduction

In today's telecommunication practice, the engineering and the economic aspects of the telecommunication networks are considered separate entities. Service providers strive with existing pricing schemes to provide better services in lower prices and attain maximum competitive advantages [1]. The main challenge facing service providers is to make available a universal business model, which represents the service quality and network performance, and can be used to derive optimal pricing schemes [1]. In practice, pricing depends on many parameters of the actual market - including carried traffic, network architecture, underlying technology, market regulations and the resulting cost of traffic transport. Recent technology advances, like the introduction of the fiber optic lines, the mobile communication networks, the transformation of cable networks to carry phone calls and the amazing growth of the Internet, combined with the deregulation of the telecommunication market and the proliferation of the Internet (ex. EU and USA), have created a highly competitive environment for transport service providers [2]. The intense competition on the telecommunication market drives the search for discovering new innovative approaches, capable of generating revenue. Several countries are planning for a competitive telecommunications industry and are passing new legislation in order to prepare the legal grounds for competition [2] [4] [5] [6]. As we mentioned above, the challenge that faces the communication networks, service providers and vendors, is providing a universal model that could be adopted globally. The complication will be taken to a higher level when different providers will own different parts of the network. Different networks owners will find themselves carrying traffic from other networks based on laws and contracts [4] [5] [7]. The need to deliver different service quality to different services and users will force network providers to act on a new pricing scheme, which will allow them to differentiate prices for users who are willing to pay extra for a higher quality. Such price discrimination will result in the providers' profit maximization and will keep them well positioned in a competitive market.

In this dissertation, we deploy microeconomic models and network flow optimization methods to investigate the behavior of optical networks in competitive market environments and the effect of price competition on the network profitability. In this respect, our work is a head-start; it provides the fundamental microeconomics aspects of the competition, which is applicable for optical networking and their future business models. It facilitates the strategic and long term network planning based on profit/loss analysis. It also determines a given network architecture's level of profit forecast based on competition.

1.2 Pricing in future Communication Networks

Internet is the most popular in a long progression of communication technologies. It is a growing area of concern to members of the technical, business, academic, and user communities. Due to the considerable growth in applications, as well as the huge increase in the number of users and connections, Internet has become an important medium for communication, information spreading, and electronic commerce [8] [9] [10]. The focus in this evolution is on the user perception, and in particular on how service has to be tailored to the current needs of the user in terms of prices. Also, this will create a potential for brokers to get involved heavily in price marketing, where they can influence users preferences to pay in return of a specific quality [8][9][10]. There is also a huge potential of bandwidth being traded as a commodity at the network provider, as opposed to the user, level. For example, Band- X is a bandwidth exchange that enables companies to offer their network and bandwidth resources to potential network providers [8] [9] [10]. In such market, competition among users as well as among service providers will naturally exist. Users will compete for different types of traffic (data, multimedia, virtual reality, voice, video, etc.) and have the willingness to pay more for better quality of service, therefore, map their expenses to pricing plans, which guarantee the desired quality of service. Service providers will compete to offer the best prices matched to different quality of service and hence use prices with service discrimination as valuable tools that can provide higher revenues and increase utilization efficiency of a network. Thus increase social welfare in general and stay competitive.

1.3 Previous work

Since mid 90's network economy and pricing have attracted the interest of researchers. Significant work has been done in resource allocation problems using pricing techniques, microeconomics and game theory [12] [13] [14] [15] [17] [18]. The objective is to formulate a resource allocation problem, and show that under certain conditions we can attain a satisfying network allocation under a given optimization criterion. Some investigate the effect of pricing incentives [17], others investigate flow/congestion control and routing [12], [13], [14], [15] from a pricing perspective. Some researches investigate the problem of selfish users competing for limited resources of a network until reaching equilibrium [18]. The novelty in this dissertation is the investigation of price competition among optical networks and its integration with routing and wavelength allocation. It results in the service provider's profit maximization while satisfying the customer bandwidth requests.

1.4 Objective

The main objective of this work is to investigate the role of pricing in optical communication networks in terms of bandwidth resource allocation, optimal network's resource utilization, improvement of the user's as well as the network service provider's business goals (minimal cost for a service, network profit maximization) and survivability under different market environments.

Specific objectives include:

1. To determine the behavior of optical communication networks in different market environments and the effect of pricing in their profitability and competitiveness.
2. To implement various microeconomic models and competition scenarios, which are applicable to optical networks (e.g.: monopoly, duopoly, game theory, auctioning, etc.).
3. To determine the requirements that price competition imposes on network planning based on profit/loss analysis.

1.5 Contribution

In this thesis we examined the role of pricing and the effect of market behaviour on the profitability and economic feasibility of optical networks. To the best of the authors' knowledge this work is the first to use microeconomic models and game theoretic pricing in optical networks. We have developed an interaction protocol between customers (wavelengths buyers) and the competing suppliers (optical networks). Another contribution is the use of profit/loss analysis as a criterion for network connectivity planning and for improving its competitiveness and market survivability.

Publications [65] and [70] deal with a game theoretic based model for pricing based on Bertrand duopoly competition. In both publications we proved that in a competitive environment a unique and stable Nash equilibrium exists. In the presented thesis these conclusions were extended by analysis and simulations, where the price for unit wavelength is dependent on the current state of the network, understood here as the current routing and wavelength assignment.

1.6 Thesis Organization

The remaining of the thesis is organized as follows. In Chapter 2, we provide a brief introduction to microeconomics models, market competition, game theory and the concept of equilibrium. In Chapter 3, we present the network management technology, charging systems, different bandwidth broker architecture, and pricing techniques. In Chapter 4, we present a case study of wavelength assignment and optimal routing in terms of profit maximization of a monopolistic optical network. In chapter 5, we present a game theoretic pricing approach of optimal routing and wavelength assignment in optical networks. Finally, the dissertation is concluded and some suggestions for future work are presented in Chapter 6.

Chapter 2

Introduction to Microeconomics and Game Theory

2.1 What is Microeconomics?

Economics is the study of how society allocates scarce resources to satisfy the needs of its members for goods and services [19]. Microeconomics is a part of economics, which mainly deals with firms and the relations of buyers and sellers in different kinds of markets [19]. It does not deal with the fluctuations of the economy in total or with specific problems such as unemployment, inflation or budgets. These are the topics of macroeconomics, the other major subject of economics. Microeconomics on the other hand is the study of how firms make decisions and how buyers and sellers interact to determine market prices. These prices reflect the way resources are allocated and their payback. For this reason, economists sometimes prefer to call microeconomics as “price theory” [19].

2.2 Basic Terminology of Microeconomics

2.2.1 Market

Market is defined as a physical place, geographical area or Internet where buyers and sellers interact to determine the price of a good [20]. The market is collective of suppliers and consumers, where competition takes different forms. Market relations could take two forms; competitive and noncompetitive. In a competitive market the buyers and the sellers cannot influence the price, an example being the agricultural market. In a noncompetitive market, the individual producer can influence the price. Example of non-competitive market is the gas and oil market, controlled by OPEC [20].

2.2.2 Commodity

Commodity is an actual product produced in factories, fields, mines, etc., usually agreed upon through regulation or specification, which can be commercially traded [21]. Also commodity could be any index (e.g. stocks), rate (e.g. mortgage interest rates) or service that is a price determinant of an agreement, originally between two parties, a buyer and a seller [21].

2.2.3 Demand

The demand for a commodity is defined to be the relationship that exists between the price of the commodity and the quantity requested in a given time period [20]. Any change in the price of the commodity affects the quantity's demand. There are several factors that affect the demand. These factors include: tastes and preferences, the prices of related goods, consumer's income, the number of consumers, customer loyalty, expectations of future prices, income etc. Figure 2.1 below shows the case of a linear demand curve of a consumer for a typical commodity. The curve has a downward slope, emphasizing the fact that as the price of a commodity becomes lower, people tend to buy more of it, and vice-versa [20].

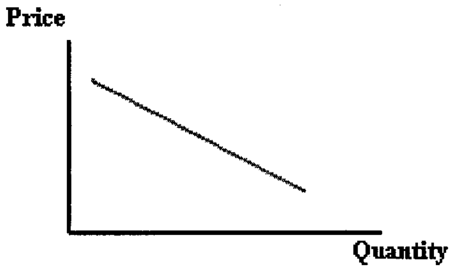

Figure 2.1: Demand Curve

Aggregate demand: The aggregate demand for a commodity is summation of the quantities demanded by each consumer in the market at each price level.

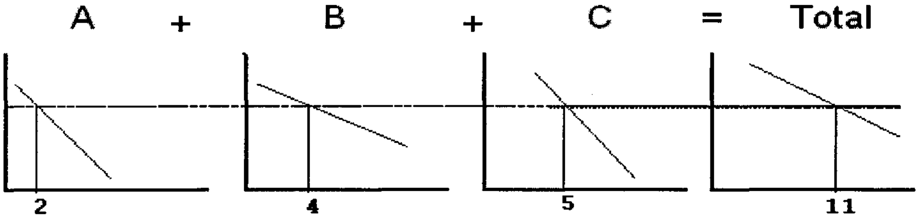

Figure 2.2: Aggregate demand

The situation is depicted graphically in figure 2.2. At a price level of 5. A demanded say 2 apples, B demanded 4, and C 5. The total aggregate demand is thus 11 apples at a price of 5. When the price changes this process is repeated for all consumers, thus the overall market demand curve is determined.

2.2.4 Supply

Supply is the relationship that exists between the price of a commodity and the quantity offered in a given time period. If the price of a resource is changed (e.g. increased) it will affect the profitability of producing the commodity, which results in change (e.g. reduction) of the quantity that suppliers are willing to offer for sale at certain each price. The factors that can affect the supply include: the prices of resources, technology, the forecasting of the market, the number of producers, and the prices of commodities and services, which are related [19] [20].

The supply curve shown in figure (2.3) is a typical linear supply curve applying to a firm, which decided to market a particular commodity. The supply curve slopes upwards, emphasizing the fact that as the market price increases, the firm is willing to produce greater value of the commodity [20].

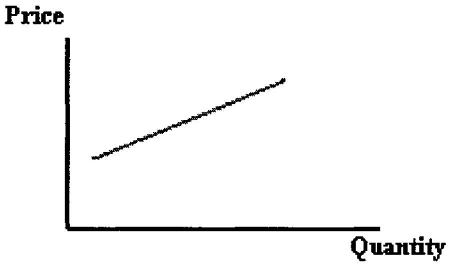

Figure 2.3: Supply curve

2.2.5 Elasticity of demand and supply

Elasticity is defined as an indicator to characterize the demand or supply [20]. The elasticity of demand equals the percentage change in demanded quantity with respect to a 1 percent change in price. Similarly, the elasticity of supply equals the percentage change of the supplied quantity in response to a 1 percent change in price. The elasticity of demand is usually negative and the elasticity of supply is positive. The most commonly used elasticity measure is the **price elasticity of demand** [20], defined as:

$$E = -\frac{\frac{\Delta d}{d}}{\frac{\Delta p}{p}}$$

where d is the demanded quantity and p is price.

The price elasticity of demand is a measure of the sensitivity of the quantity demanded to the change in the price of the commodity. There are three kinds of demand: *Inelastic demand***:** The percentage change in demanded quantity is less than the percentage change in price. That is, $E < 1$. Example: Gas commodity. *Elastic demand:* The percentage change in quantity is greater than the percentage change in price. That is, $E > 1$. Example: Gold commodity. *Unit elasticity***:** The percentage change in quantity equals the percentage change in price. That is, $E = 1$.Example: Meet commodity.

2.2.6 Total production cost

Total production cost or total cost (TC) consists of two parts: total fixed costs and total variable costs. Total fixed costs (TFC) are costs that are not subject to change as the output changes [20]. The level of total fixed costs is the same at all levels of output (even when output equals zero). Examples of such fixed costs include rent, annual fees, and monthly connection fees for utilities, which include only fixed monthly fees but not the portion of utility fees that varies with the level of use. Total variable costs (TVC) are costs that change whenever the output changes. Labor costs, material costs and energy costs are examples of variable costs [20]. Variable costs are equal to zero when no output is produced and increase when the output increases [20]. Table 2.1 below contains a hypothetical list of total fixed cost and total variable cost for a certain quantity Q. We can use the TFC and TVC lists to calculate the total cost list for this firm. For every output, $TC = TFC + TVC$. The total fixed costs are the same for each output and total variable costs are expected to change as the output changes.

Q	TFC	TVC	TC
0	5	0	5
5	5	30	35
10	5	40	45
15	5	50	55
20	5	60	65
25	5	90	95
30	5	100	105

Table 2.1: Total fixed costs and total variable cost

Figure 2.4 shown below illustrates the relationship between total cost, total variable costs and total fixed costs for certain quantity Q.

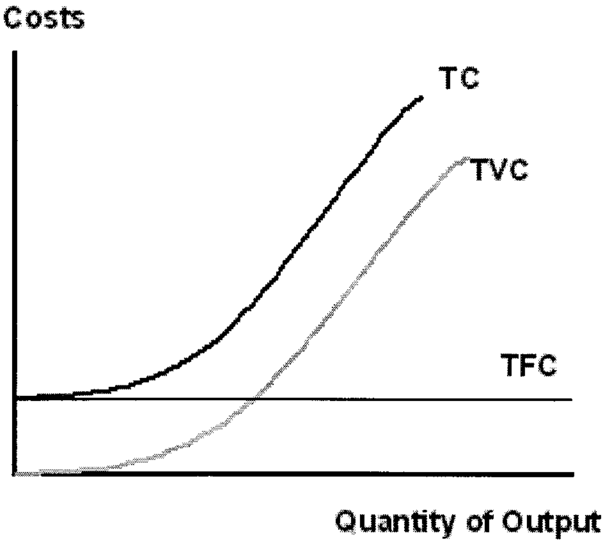

Figure 2.4: Relation between TFC, TVC and TC

2.2.7 Marginal production cost

The marginal production cost or the marginal cost **(MC)** can be seen as the cost of an additional unit of output [20]. Marginal cost can be measured as:

$$MC = \frac{\Delta TC}{\Delta Q}$$

Consider, for example, the interval between 10 and 15 units of output in Table 2.1. Since total cost increases by 10 (from 45 to 55) when 5 additional units of output are produced, we conclude that the marginal cost is 10/5 = 2.

2.2.8 Total revenue and marginal revenue

Total Revenue (TR) is defined as: *TR = p x Q, where p is the price and Q is the quantity* [20].

Suppose that a firm is facing a downward sloping demand curve for its product. When the price decreases, the quantity demanded by consumers increases. The revenue will decrease as the price of each unit of output decreases and vice-versa. On the other hand, the total revenue will increase even when the price falls if the quantity increases by a large percentage to offset the reduction in price per unit [20] [19].

Using the above discussion, we can conclude that a decrease in price will lead to:

- an increase in total revenue when the demand is elastic;
- no change in total revenue when the demand is unit elastic;
- a decrease in total revenue when the demand is inelastic.

In a similar manner, an increase in price will lead to:

- a reduction in total revenue when the demand is elastic;
- no change in total revenue when the demand is unit elastic;
- an increase in total revenue when the demand is inelastic.

In figure 2.5 we can see the relationship between total revenue and demand elasticity along a linear demand curve. The upper part of the demand curve is highly elastic (E > 1), while the bottom part is highly inelastic (E < 1). In between, elasticity becomes smaller as price decreases and quantity increases. At some point, demand changes from being elastic to inelastic. The point at which that occurs is the point at which demand is unit elastic (E =1) [20].

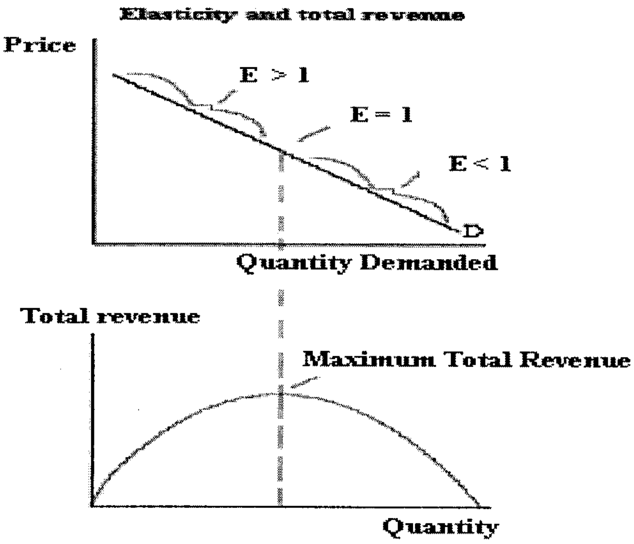

Figure 2.5: Elasticity and total revenue [20]

Total revenue increases as quantity increases (and price decreases) in the region where demand is unit elastic. Total revenue decreases as quantity increases (and price decreases), in the inelastic portion of the demand curve. Thus total revenue is maximized at the point at which demand is unit elastic [20].

Marginal Revenue (MR) is defined as the additional revenue, resulting from the sale of an additional unit of output [20].

$$MR = \frac{\Delta TR}{\Delta Q}$$

If a firm is facing an elastic demand curve (E > 1), the price of the commodity is the same at any given output. In this case, marginal revenue is simply equal to the market price. Suppose, for example, that a product sells for $2 per kilogram. The marginal revenue of this firm from the sale of an additional kilogram of the same product is simply $2. Suppose, however, that a firm is facing a downward sloping demand curve. In this case, it must decrease the price if it wishes to sell additional units of this commodity. In this case, marginal revenue is less than the price. Example: consider the situation where the price of a product is $8, the firm can sell 4 units of output while receiving a total revenue equal to $8 x 4 = $32. If it wishes to sell the 5th unit of output, it must lower the price to $5. Its total revenue in this case will be $37. The marginal revenue is $5 less than the price of $8 [20].

2.2.9 Profit maximization

Economists assume that firms select prices and output levels that maximize their profits [20] where profit is defined as Total revenue – Cost.

Let us consider a firm's decision about whether to produce more or less output. The first thing the firm should look at is if the marginal revenue is greater than the marginal cost. Then, in this case, the production of an additional unit of output will increase revenue more than the cost. Thus, the profit will increase as well in this case; the firm decision should be to increase the production. In the opposite case, if the marginal cost exceeds the marginal revenue; the decision should be to refrain from increasing the production. In this case, firms can increase their profits by producing less. A profit-maximizing firm will produce more output when MR > MC and less output when MR<MC. If MR = MC, the firm has no incentive to produce either more or less output where MR = MC.

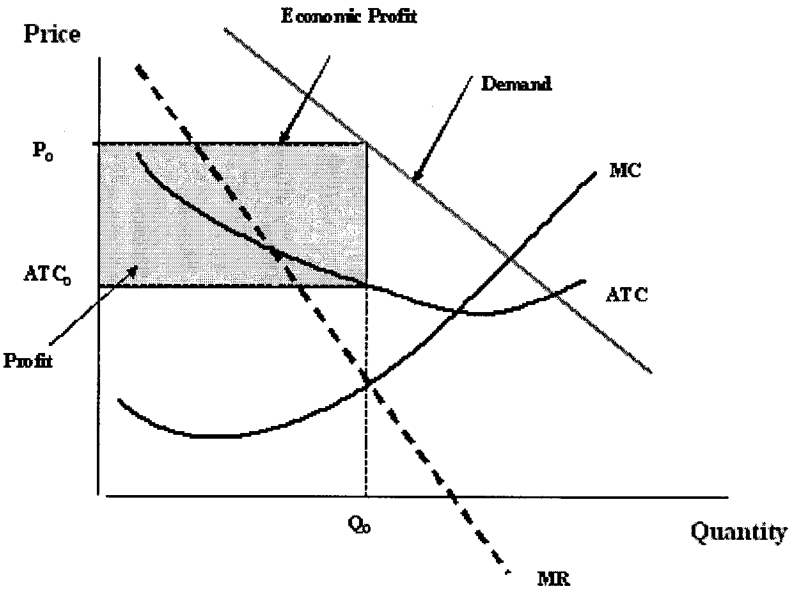

Figure 2.6: profit maximization [20]

Figure 2.6 shows the profit-maximizing levels of price and output for a firm facing a downward sloping demand curve. As we have mentioned above, the profit-maximizing level happens at the point where MR = MC. In the figure, this point is Q_o, the level of output at which the MR and MC curves intersect. The price that firms can charge to sell this much output is given by the demand curve. In this example, the price equals P_o. The size of the shaded area in the diagram above represents the size of economic profit made by this firm and is equal to [(P_o- ATC_o) Qo] [20]. Note that the height of this rectangle equals the difference between the price of the commodity and the average total cost. The base of the rectangle is equal to the quantity of output sold by the firm..

2.3 Types of market Structure

2.3.1 Monopoly market

A monopoly market can be described as follows. A single seller producing a product with no close substitutes, which means nobody else is producing a similar commodity or offering the same service. The firm is a price maker, because it faces a downward sloping demand curve for its product (in fact, this demand curve is the market demand curve) [24] [25].

2.3.2 Oligopoly market

An oligopoly market can be described as follows. A small number of firms produce most output. The product may be either standardized or differentiated, and each firm realizes that its profitability depends on the actions and reactions of rival firms. Furthermore, when making decisions concerning price or output, each firm has to take into account the expected reaction of its rival firms [24] [25]. Because there are few firms in an oligopoly industry, each firm's output is a large share of the market. Because of this, each firm's pricing and output decisions have a substantial effect on the profitability of the other firms. Because of this mutual interdependence, oligopoly firms often engage in strategic behavior, which occurs when the best outcome for one party is determined by the actions of other firms [24] [25].

2.3.3 Duopoly market

A duopoly market is a special type of oligopoly market structure that contains two large producers or sellers. The two firms are more or less equally powerful firms. An example of duopoly is the global wireline telephone switching market, which is dominated by Lucent and Nortel. Since the duopoly model is a special case of the oligopoly markets, it tends to inherit its tendency to compete, collude and form cartels [24] [25].

2.4 Game Theory

The idea of game theory was first introduced by John Von Neumann and Oscar Morgenstern in their famous 1944 book *"Theory of games and economical Behavior"*, which proposed that most economic questions could be analyzed as games [27]. Game theory is a formal way to analyze interaction among a group of rational players who behave strategically. Game theory concerns with how individuals make decisions when they are aware that their actions affect others and when each individual takes this into account. The elements of the Game theory, according to [27], are Game Strategy, Players and Payoffs. *Game Strategy* is the set of actions a player may take and is known as the space strategy profile $S=(s_1,\dots,s_n)$, where $s_1,s_2,..,s_n$ are the strategies of each player. *Players* are the decision makers of the game. *Payoffs* are the players' utility $U= (u_1,\dots,u_n)$ where $u_1,u_2,..,u_n$ are the utilities of each player for each profile in the space strategy [27].
To illustrate the game theory we will present an example of the famous Prisoner's Dilemma game. In this game, suppose that the police has arrested two people whom they know have committed an armed robbery together. Unfortunately, they lack enough admissible evidence to get a jury to convict. They do, however, have enough evidence to

send each prisoner away for two years for theft of the getaway car. The police inspector made the following offer to each prisoner: If you will confess to the robbery, implicating your partner, and he does not also confess, then you will go free and he will get ten years. If you both confess, you will each get 5 years. If neither of you confess, then you will each get two years for the auto theft. Our first step in modeling this situation as a game is to represent it in terms of utility functions. Both partners' utility functions are identical:

Go free = 4, 2 years =3, 5 years =2, 10 years =0

The numbers above are now used to express a partner's *payoff* in the various outcomes possible in the situation. We will refer to them as 'Player I' and 'Player II'. Now we can represent the entire situation on a matrix; this is the strategic form of the game:

		Player II	
		Confess	Refuse
Player I	Confess	2,2	4,0
	Refuse	0,4	3,3

Each cell of the matrix gives the payoffs to both players for each combination of actions. Player I's payoff appears as the first number of each pair, Player II's as the second. If both of them confess then each receives a payoff of 2 (5 years in prison each). This appears in the upper-left cell. If neither of them confesses, each receives a payoff of 3 (2 years in prison each). This appears as the lower-right cell. If one confesses and the other does not, then the one who confess receives a payoff of 4 (going free) while the other a payoff of 0 (ten years in prison). This appears in the upper-right cell. The reverse situation appears in the lower-left cell.

Each player evaluates two possible actions, by comparing his payoffs in each column, since this shows which of the actions is preferable for each possible action by the other player. Observe: If your partner confesses, then you get a payoff of 2 by confessing and a payoff of 0 by refusing. If your partner refuses, you get a payoff of 4 by confessing and a payoff of 3 by refusing. Therefore, you are better off confessing regardless of what the other player does. The other player, meanwhile, evaluates his actions by comparing his payoffs, and he comes to exactly the same conclusion with the first player. Whenever one action of a player is superior to his other actions for each possible action by the opponent, we say that the first action dominates the second one. In the Prisoner's Dilemma, then, confessing dominates refusing for both players. Both players know this about each other, thus entirely eliminating any temptation to depart from the dominated path. Thus both players will confess, and both will go to prison for 5 years. There is something disturbing about the outcome of the Prisoner's Dilemma. Had both players refused to confess, they would have arrived at the lower-right outcome in which each go to prison for only 2 years, therefore both earning higher utility than each receives when confessing. This is the most important fact about the Prisoner's Dilemma, and its significance for game theory is quite general. We will therefore return to it later when we discuss equilibrium concepts in game theory. When we represent the Prisoner's Dilemma as a strategic-form game, we implicitly assume that the prisoners can't attempt collusive agreement since they choose their actions simultaneously. In this case, agreement before the fact cannot help. If you are convinced that your partner will stick to the bargain then you can seize the opportunity to go free by confessing.

Many of the interactions in the business world could be modeled using the game theory methodology. Prisoner's dilemma is not the only game theory model, which can be used to model economic scenarios. Other models can be applied to different situations and, in many cases, can suggest the best outcome for all parties concerned. John Von Neumann and Oscar Morgenstern assumed that the key link between microeconomics and game theory is rationality [26] [27]. Microeconomics is based on the assumption that individuals are absolutely rational in their economic choices [26] [27]. Specifically, the assumption is that each person maximizes his or her profits, income, or subjective benefits in the circumstances that she or he faces. This hypothesis serves a double purpose in the study of the allocation of resources, because it narrows the range of possibilities somewhat and absolutely rational behavior is more predictable than irrational behavior [26]. Game theory was intended to confront economic problems: to provide a theory of economic and strategic behavior when people interact directly.

2.4.1 Cooperative and non-cooperative games

Oligopolistic games may be categorized according to the degree of cooperation among the players, the number of players, and the payoff structure. Oligopolistic players interact in two ways. In cooperative game, firms make binding agreements to cooperate or form a cartel. In non-cooperative game, firms cannot make binding agreements, so they act independently [26].

2.4.2 Nash Equilibrium

Equilibrium in game theory is determined by the condition that all players choose action from the space strategy profile $S= (s_1,\ldots,s_n)$, where $s_1,s_2,..,s_n$ are the strategies of each player, as the best response to the anticipated play of the opponent. In 1950, John Nash introduced the formulation of equilibrium in game theory, which later became known as "Nash Equilibrium" [26]. Nash equilibrium of a strategic game is an action profile (list of actions, one for each player) with the property that no player can increase his payoff by choosing a different action, *given* the other players' actions. Assume we have a set of players $I = (1,..,n)$. Each player *i's strategy space* belongs to the space strategy profile $S=(s_1,\ldots,s_n)$ of all players, we say a strategy profile s*, which is feasible and belongs to S is "Nash Equilibrium", such that "every player is playing a best response to the strategy choices of his opponents" more formally, we say that s* is a Nash Equilibrium if $(\forall i \in I)$ $(\forall s_i \in S)$ $u_i(s_i{}^*,s_{-i}{}^*) \geq u_i(s_i,\ s_{-i}{}^*)$ where u_i is the player's utility corresponding to his Nash equilibrium strategy $s_i{}^*$ and the other player's Nash equilibrium strategy represented by $s_{-i}{}^*$ [26].

A basic definition is this: If there is a set of strategies for a game with the property that no player can benefit by changing his strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute a Nash equilibrium.

Nash equilibrium in the prisoners' Dilemma example would be (2,2) , assuming that the prisoners cannot attempt collusive agreement since they choose their actions simultaneously. Otherwise, if you are convinced that the other player will stick to the bargain then you can seize the opportunity to go scot-free by confessing.

2.5 Models of competition

In this section we will discuss models of competition of market structure, where each firm tries to take a strategic market decision given its beliefs about how other firms behave. The three best known models are the Cournot, Bertrand and Stackelberg Models.

2.5.1 Cournot Model

The French mathematician Augstin Cournot developed the first model of non-coorporative oligopoly behavior in 1838 [24][25][26][27]. Cournot assumed that each firm acts independently and attempts to maximize its profit by choosing its output. A Cournot firm makes one of the simplest possible assumptions: other firms continue to produce the same level of output no matter how it behaves. That is, each firm assumes other firms are satisfied to continue selling their current quantity of output. The Cournot model can be used to study an industry with any number of firms. We will take an example of two firms; a duopoly case. The Cournot duopoly models a situation in which each firm chooses its output independently, and the market determines the price at which it is sold. Specifically, if firm 1 produces the output y_1 and firm 2 produces the output y_2 then the price at which each unit is sold is $P\ (y_1 + y_2)$, where P is the inverse demand function. Denote firm 1's total cost function by $TC_1\ (y)$ and firm 2's by $TC_2\ (y)$.

Then firm 1's total revenue for the pair of outputs (y_1, y_2) is $P\ (y_1 + y_2)\ y_1$, and its profit is given by $P\ (y_1 + y_2)\ y_1 - TC_1\ (y_1)$. Firm 2's revenue is $P\ (y_2 + y_2)\ y_2$, and hence its profit is given by $P\ (y_1 + y_2)\ y2 - TC_2\ (y_2)$. Firm 1's profit-maximizing output, when firm 2's output is y_2, is the output y_1 that maximizes firm 1's profit; that is, the value of y_1 that maximizes $P\ (y_1 + y_2)\ y1 - TC_1\ (y_1)$. In the Cournot model, the two firms only use strategies for choosing output levels. Obviously, the strategy of a firm's rival affects a firm's profits, since each firm's profits are a function of the output of the other firm. A set of outputs is said to be a Cournot (Nash) equilibrium if, holding the output levels (strategies) of all firms constant, no firm can obtain higher profits (payoff) by choosing a different output level (strategy) [24][25][26][27].

2.5.1 Bertrand Model

In 1883 Joseph Bertrand proposed a price competition model [24][25][26][27] based on the production marginal cost of unit production output, in which two or more firms compete with each other by undercutting each other's prices until they reach the non-profit point, where no one is able to compete further. The Bertrand model can be interpreted as a game in which the strategies of firms are the prices they choose. To illustrate the pricing competition in Bertrand model, we will consider the case discussed in the Cournot model and will assume that the two firms are firm 1 and firm 2 with output y_1 and y_2 respectively. In this case the buyers will buy from the firm with lower price. We also will assume the two firms offer prices simultaneously. Once a price is announced, the firms cannot change it and must sell to any customer who wants to buy at its offered price. If the two firms offer the same price, then they split the market equally. If firm 1 chooses a price that is greater than its marginal cost, then its interest is to change the price just slightly below the price of its rival. That way, it will capture the entire market and the firm will encounter additional profit on each of the new orders because the price exceeds the marginal cost. Had the firm chosen to charge a price higher than the price of its rival, it would have not sold anything and would have earned no profits. Because each firm knows that the other firm will be best served by charging a price just slightly lower than the rival's, the price will be bid down to the marginal cost. To determine the Nash equilibrium for the two firms when played in prices, we know that a Nash equilibrium is one in which each firm's expectation regarding the action of its rival is precisely the rival's best response to the strategy chosen by the firm in question, in anticipation of that response. There is, essentially, only one Nash Equilibrium for the Bertrand duopoly game which assumes that both firms have the same marginal cost c. It is the price pair $(p_1^*=c, p_2^*= c)$ where p_1^* and p_2^* are Nash equilibrium prices for firm 1 and firm 2 respectively. If firm 1 sets this price in the expectation that firm 2 will do so, and if firm 2 acts in precisely the same manner, neither will be disappointed. Hence, the outcome of the Bertrand duopoly game is that the market price equals marginal cost c [24] [25] [26] [27].

2.5.3 Stackelberg Model

In 1934 Heinrich von Stackelberg considered the same model as Cournot competition except he pointed out that the results would be quite different if the two firms chose quantities sequentially [26][27]. Stackelberg claimed that Cournot model is unrealistic because it is rare that firms would select quantities simultaneously, without any ability to watch the choices of their competitors. One firm gets to act before the other. For this reason, Stackelberg model is often called a leader follower model. The question here is which position is better - to go first or second? Would you prefer to have to select your quantity first, or would you prefer if your competitor selected his quantity first, and you, only upon observing this decision, had to make yours. Suppose firm 1 is allowed to choose its output first. Then firm 1 has to determine what the best response of firm 2 is, knowing the output level that firm 1 chooses. For firm 2 to determine its own output is not hard, because firm 2 will know firm 1's output level when firm 1 chooses its own. The most obvious is that firm 1 will pick the output level that maximizes its profit, given the best response of firm 2 to firm 1's choice. In this case, firm 1 is the Stackelberg-leader, and consequently firm 2's best response, which is supposed to maximize its profit will be the Stackelberg-follower's output level. Explicitly determining which firm moves first in this game eliminates much of the guessing to the other firm's best response in Stackelberg model. If firm 1 is able to *commit* to a quantity higher than the equilibrium in such game, then firm 1 is able to force firm 2 to cut back on its own production. This commitment plays a significant role in determining the market share of each firm. The first firm to commit will acquire a higher market share as compared to the market share it would have acquired, should both companies have made simultaneous move in the game.

2.6 Summary

This chapter introduced some principles of microeconomics and game theory. This included a brief definition of market, demand, supply, commodity and market elasticity of demand and supply. We have also presented the three different kinds of demand elasticity: *Inelastic demand, Elastic demand and Unit elasticity.* The difference between total production cost and marginal production, variable costs and fixed costs as well as the total revenue and marginal revenue were covered. Game theory and its elements, non-cooperative games and Nash equilibrium were explained to some extend in this chapter. We also have introduced the three underlying types of competition models; Cournot model, Bertrand model and Stackelberg model.

Chapter 3

Network Management technology, Charging Systems, Bandwidth Broker, and Pricing Techniques

3.1 Introduction

The telecommunications world of today offers many services, spreading over a wide range of fields (telephony, telex, fax, teleconferencing, cable TV, videoconferencing and multimedia document exchange). These services make use of different physical and logical network infrastructures. Issues of management, security, pricing, routing, billing, planning, performance and customer satisfaction are currently dealt in different management layers at the service provider's level [28].

The major goals of a network management system are to:

- Improve network availability and service.
- Reduce complexity.
- Reduce operational and maintenance costs.

The network management system can effectively reduce the cost and complexity of today's networks by providing a set of integrated tools that allow a network manager or support staff to quickly isolate and diagnose network issues. The ability to analyze and correct network problems from a remote location is critical to the management of both network and personnel resources [28]. The requirements of a network management system have been categorized as part of the OSI specification for systems management, which is used as a base line for the key functional areas of network management on any system [28].

3.2 OSI management system

The OSI system covers the basic network management areas, which includes five functions: Fault, Configuration, Accounting, Performance and Security Management (FCAPS) [30] [31] [32].

Configuration management activities include the configuration, maintenance, and updating of network components. Configuration management also includes notification to network users of pending and performing configuration changes [30] [31] [32].

Fault management is responsible for maintaining the entire network system in its nominal state of operation by carrying out corrective and maintenance tasks. It is also responsible of diagnosing network faults and repairing or re-configuring the relevant network elements [30] [31] [32].

Performance management is responsible for regulating the network to achieve the levels of performance agreed with the customers [30] [31] [32].

Security management is responsible for the protection of network resources and the integrity and privacy of information that is exchanged between various management network function blocks [30] [31] [32]. Security services include authentication, access control, integrity, etc.

Accounting management provides a set of activities for measuring the activation and use of the network services. Service and resources records are gathered from the network and are processed to generate customer invoices. Accounting management is also responsible for bidding procedures, cost determination, and price setting. This task also involves the exchange of information with the accounting management systems of other management domains. Information is also provided for network planning and for assessing how well business objectives have been achieved in terms of profitability and market survivability. In the figure below we can see how all management blocks combine together to serve the networks as a whole [30] [31] [32].

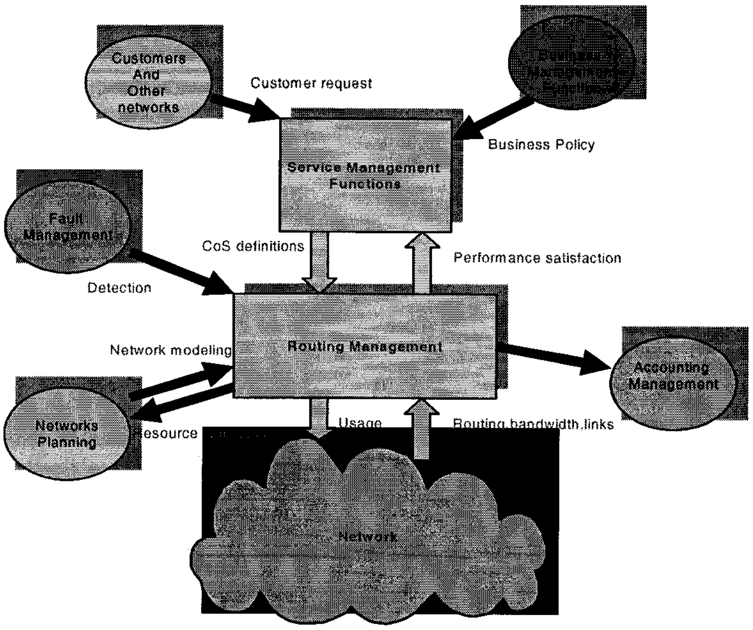

Figure 3.1: Network management blocks [31]

The following sections will mainly concentrate on the accounting management, since in our work we are dealing with billing and pricing.

3.3 Charging systems in communication networks

Charging systems in communication networks have broad functionalities, which include setting of prices, charge calculation, billing, and in some cases maintenance of service classes, user profiles, customer data, identities, and banking account data [31][32][33].

An example of architecture for a charging system is described below (Georg Carle, Felix Hartanto, Michael Smirnov, Tanja Zseby) [30]. The architecture model classifies charging, accounting, and describes their interaction (see figure 3.2). At the right, five layers are shown that describe processing for charging and accounting. A configuration plane allows for providing configuration parameters for the processing layers. Configuration parameters are derived from pricing policy, charging policy, accounting policy and metering policy. These policies can be provided by interaction of dedicated policy servers with the corresponding entities of the configuration plane.

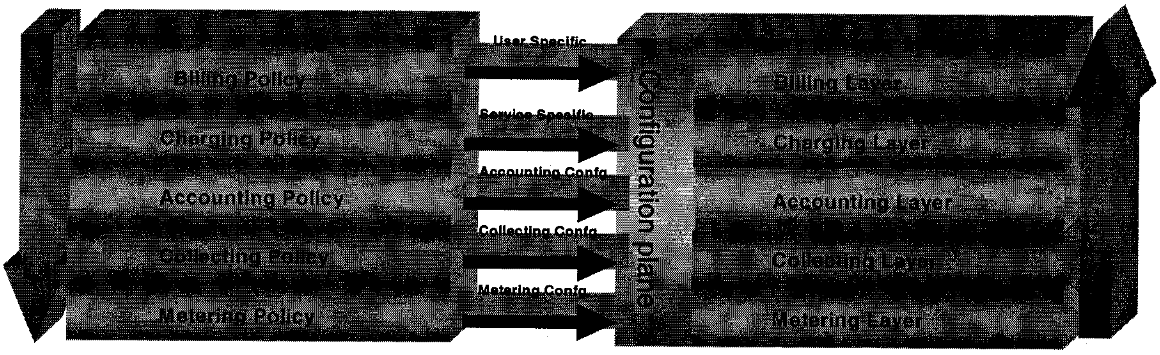

Figure 3.2: Charging and accounting framework [30]

The *metering layer* provides the functions to obtain reservation information in the network and to meter actual usage of network resources. Meters can be placed at the edge routers only or at multiple core routers points [30].

The *collecting layer* includes functional entities that access data provided by metering entities and then send it to the accounting layer for more processing. The collecting layer is also responsible for selecting appropriate placement. Additionally, this layer may distribute collected usage data to other domains in a multi-provider environment, in which more service providers interact together and may carry traffic from each other [30].

The accounting layer process collected usage data (from collectors within the same domain and other domains) and reservation data, consolidate them based on services parameters, and create accounting data sets (i.e., accounting records) which are passed to the charging layer for pricing assignment.

The *charging layer* is responsible of determining the costs based on the data it receives from the accounting layer. It is doing so by using service specific tariff parameters. Based on the policies and agreements between service providers and customers or service providers themselves the cost of usage could vary depending on usage of the resources. Based on the resource usage a detailed assessment will be provided for billing the customer or for internal analysis (auditing) by the service provider. A simple evaluation of current costs can be used for displaying an estimation of accumulated costs for the service user, or for control purposes by the customer organization or by the provider.

The *billing layer takes* the evaluation transferred from the charging layer into monetary units and generates a bill for a customer. This process may combine technical considerations with economic considerations, such as volume of resources used by the customers, and marketing methods, level of competition, other providers and discounts (e.g. customer appreciation).

The left part of figure 3.2 represents the policy plane, i.e. technical and commercial rules for setting parameters based on various factors such as, network configuration (important for metering policies) or the market conditions (important for charging and billing policies). The parameters are then carried to the appropriate layers through a configuration plan.

Based on the framework in figure 3.2, a sample architecture for implementing a charging and accounting service is presented in figure 3.3 [30]. As we can see from the architecture, if we need to support multicast charging, meters placement can take place at the edge routers as well as at the core routers. In order to charge, the collected information needs to be returned to the sender's domain (e.g. provider domain A), so that the provider can reconstruct the multicast tree structure in calculating the overall costs. Collecting entities at each domain collects data from the meters. The data structure from the collecting layer contains reserved and used resources for a traffic flow.

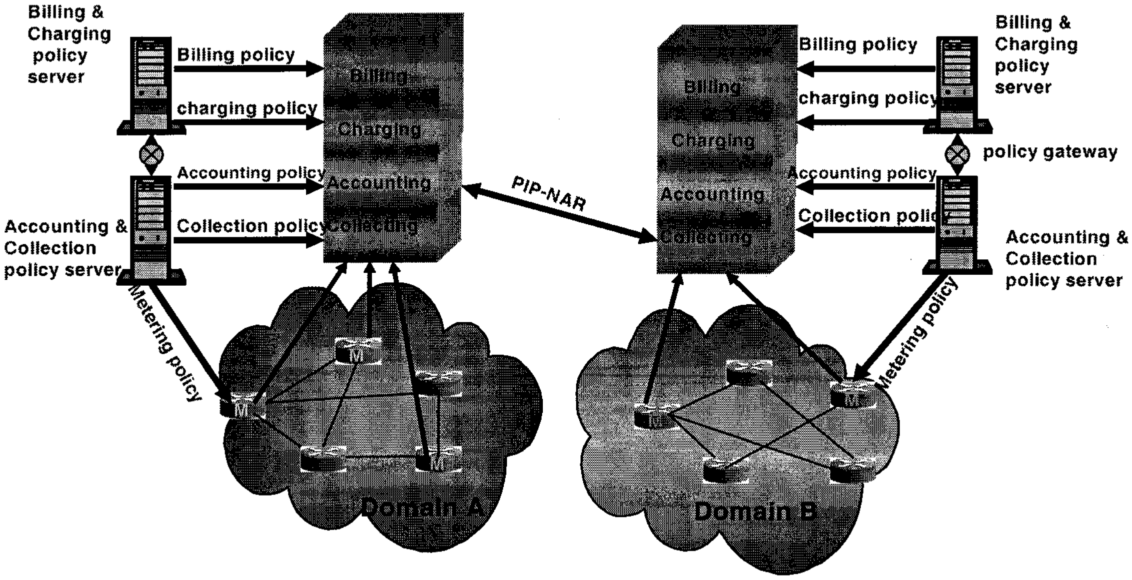

Figure 3.3: Architecture for multicast charging and accounting service [30]

3.4 Bandwidth Brokers

In today's growing telecommunication industry and the huge potential of electronic trade, the bandwidth broker became a major player in providing vast area of differentiated services, negotiation, assurance of service level of agreement and policies [35][37]. The bandwidth broker helps provide preferred service to users by allocating network bandwidth to them based on a set of policies and agreements between users and service providers. A typical bandwidth broker mostly insures its responsibilities of managing resources and policies in a single domain [35]. The main role of the bandwidth broker is to act on behalf of users on price negotiation with service providers, protect user preferences with respect to quality of service, resource reservation and pricing fluctuation through contract and mutual agreements. Bandwidth brokers will be an integral part to providing these preferred services [35].

3.4.1 High-Level Requirements and Design

A bandwidth broker can be looked at as an agent that is responsible for allocating bandwidth resources across a domain to end users and applications [37]. In order to do this, the bandwidth broker must be aware of the amount of resources that are already allocated within its domain and the amount of resources that are available. When a user or application wants to send data across the domain, they send a bandwidth request to the bandwidth broker. The bandwidth broker determines if sufficient resources are available to meet the request [37], and decides if the request should be accepted or rejected. Decisions depend on the current network resource loads and the amount of additional load the user's request would impose on those resources. It is also guided by a set of policies and rules that are pre-determined by the user and service provider through a Service Level Agreement (SLA). The bandwidth broker has to agree in garneting the service specified in the SLA by not offering service if there are insufficient resources [37].

3.4.2 Policy and Decision Making Requirements

A bandwidth broker must hold on to the policies and SLA that are in place between customers and service providers [37]. It must also monitor and manage the resources in his domain to make sure that no new traffic is accepted in the domain if it affects the resources promised to users already utilizing the domain. If the bandwidth broker has to communicate with bandwidth brokers in other domains, then they must adhere and agree to the SLAs in order for the integration to be realized effectively. These SLAs are bilateral, both parties agree to them [37].

3.4.3 Operational and Design Requirements

The main operational requirement of a bandwidth broker is to track resource allocation and SLAs. To ensure this process is performed reliably, the bandwidth broker must have the means to authenticate the user/application that is requesting service. Then the bandwidth broker must access the specifications in the user's SLA in order to decide if it should accept or reject the request. If the request to allocate bandwidth is granted, proper allocation should be made within network nodes in order to accommodate the user's needs. Implicit in these requirements, the bandwidth broker must also know its domain topology so that it can assign the most suitable path [37].

3.4.4 Design

There are several designs and implementations of bandwidth brokers under study at educational institutions and organizations [37]. The low-level workings of the bandwidth brokers are based on the organization's needs and available resources, but have similar functionality at a high level. The design of a bandwidth broker varies from implementation to implementation. However, the core components and the functions they serve are generally found in most bandwidth broker designs.

Following is an example presenting the bandwidth broker design as described by Sreekantan and Rao of the University of Kansas [38]. A bandwidth broker consists of a bandwidth broker database, a bandwidth broker server, two bandwidth broker command line interfaces (one for Service Level Agreements, and the other for bandwidth allocation requests), and a bandwidth broker router configuration client. Figure 3.4 shows the components of the bandwidth broker and how they interact.

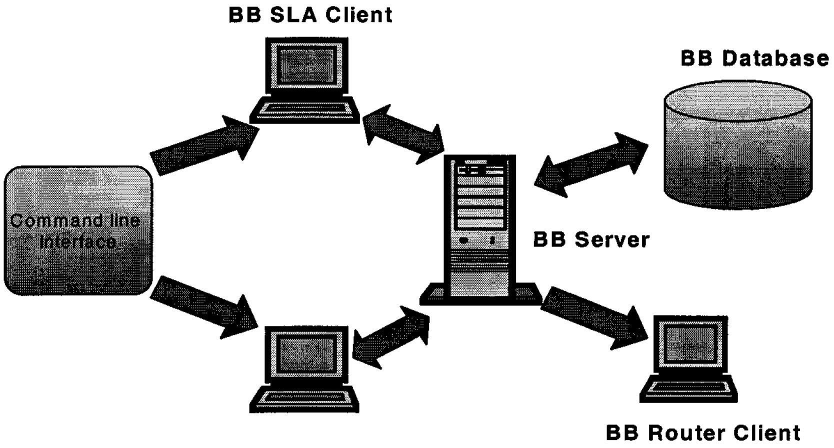

Figure 3.4: Bandwidth Broker (BB) architecture [38]

1) Bandwidth broker database (BB Database): The bandwidth broker database stores all data related to the bandwidth broker's domains, including the SLAs, policies and resource allocations of the current users. The database maintains complete information about the topology in its own domain, the routers within its domain, and the bandwidth brokers in the adjacent domains [37].

2) Bandwidth broker server (BB Server): The bandwidth broker server communicates with the bandwidth broker database, command line interfaces, and the router configuration clients. It is responsible for the verification of users/applications service requests, for updating the bandwidth broker database, and for forwarding configuration data to routers [37].

3) Bandwidth broker command line interface: The bandwidth broker command line interface is responsible for the operations of the bandwidth broker system through specific commands. There are commands for Service Level Agreements and bandwidth allocation requests [37].

4) Bandwidth broker router configuration client: Routers are configured by the bandwidth broker router configuration client in the domain based on data requested for a particular user or application. The configurations meant to provide the agreed upon service within the bandwidth broker's region. Resources are configured whenever a SLA or bandwidth allocation requests is verified [37].

3.5 Bandwidth Broker Implementation

There are several implementations of bandwidth brokers in the industry and in the academic arena. Below we will provide an example of bandwidth broker designed by Siemens.

3.5.1 Siemens Bandwidth Broker

The Siemens bandwidth broker is a centralized base structure, which controls the policy decision for differentiated services domains; it is used by applications on hosts and is capable of communicating with bandwidth brokers of other domains. Most platforms can take advantage of accessing it. It is compatible and accessible through many programming languages. Figure 3.5 shows the Siemens bandwidth broker implementation architecture. As we can see in the figure, the bandwidth broker can interact with the edge routers and provisions the core routers of the network. Siemens bandwidth brokers provide additional customized interfaces including World Wide Web for direct operation by human users and interaction with bandwidth brokers belonging to different service providers' domains. Figure 3.6 is a two-layer approach of implementing the Siemens bandwidth broker in which the resource management is performed at a higher layer and the admission control at a lower layer.

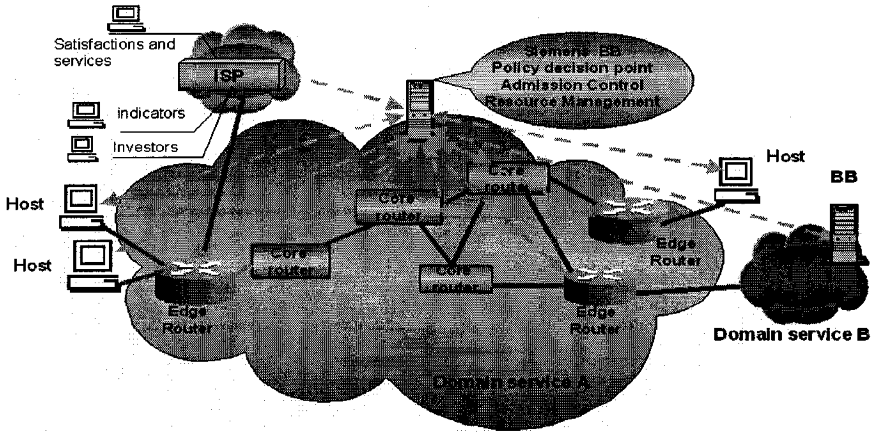

Figure 3.5: Siemens bandwidth broker implementation [35]

Assignments of the Admission Control Agent (ACA) is to handle the user QoS requests (e.g. via RSVP or http servlets), check available resources and admit or reject customer requests, setup edge routers with appropriate policies, request additional resources from the Resource Control Agent and release resources if no longer required.

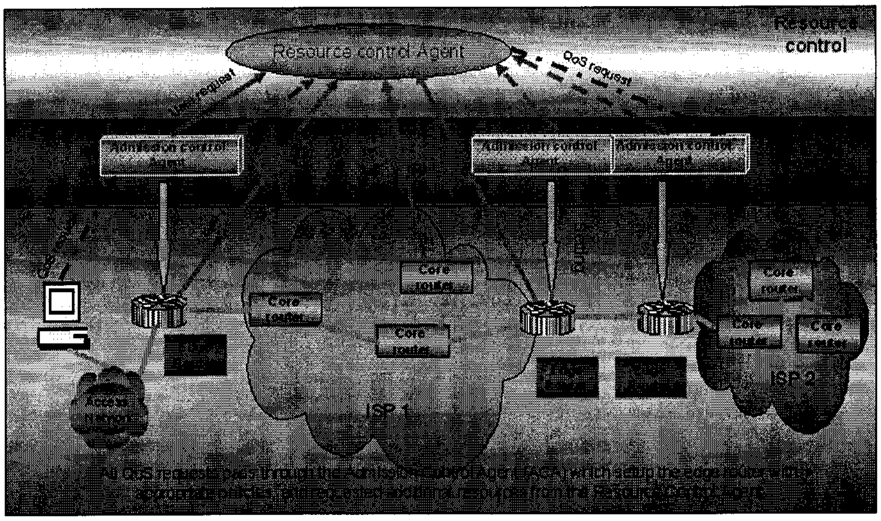

Figure 3.6: Siemens implementation following a two-layer approach [35]

3.6 Pricing techniques in communication networks

The literature is full of bidding methods, pricing techniques and market structures of communication networks. Examples can be found in [40][41][42][43]. In this section, we will present some of the basic pricing techniques in communication networks.

3.6.1 Auction Market

Figure 3.7 shows the auction method as described in [39]. In this method, users bid at a centralized broker for resources. Auction must close before resources are sold. The bidding is a time scale process. The wining user is allowed to transmit his traffic to the network. This is an effective centralized method in which the bandwidth broker controls all the agreements between the service providers and the users wishing to use the network.

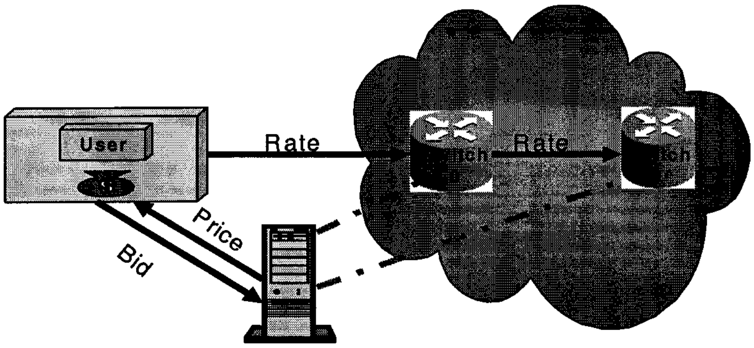

Figure 3.7: Auction market [39]

3.6.2 Smart Market

A method for pricing services in the Internet called the “smart market" was proposed by MacKie-Mason and Varian [44] (see figure 3.8). The user attaches a bid to the header of each sent packet. Routers in the network calculate a price based on the equilibrium price or the marginal cost of sending one more packet. Packets with bid amounts that exceed the current price p_0 for the link are transmitted. As a result, users have an incentive to quickly reveal the true value of each of their packets.

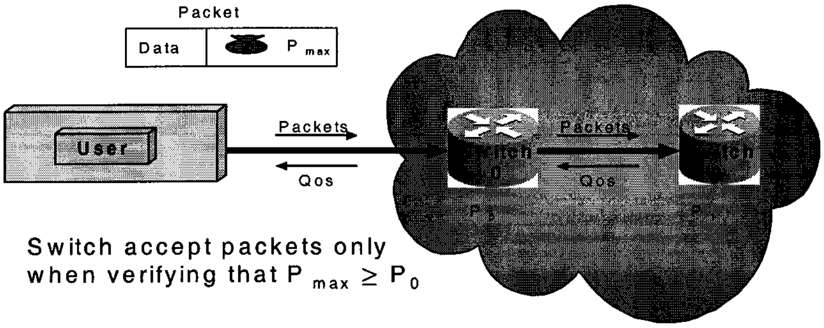

Figure 3.8: Smart market [39]

3.6.3 Spot Market

The pricing in a dynamic competitive market [45] or spot market is done to promote high utilization of resources and optimal allocations. The spot market has the unique ability to adjust to changing resource demands and provides users with immediate availability of resources. As seen in figure 3.9, the network economy consists of three entities: users (those who execute network applications), bandwidth brokers and switches. Users are consumers; switches are producers; bandwidth brokers exchange resources in the market. There are many different types of network resources desired by users. This market model considers link bandwidth; however, the techniques presented can be equally well applied to other network resources. Each link of the switch represents an independent market. Users compete for network resources where bandwidth brokers represent each user.

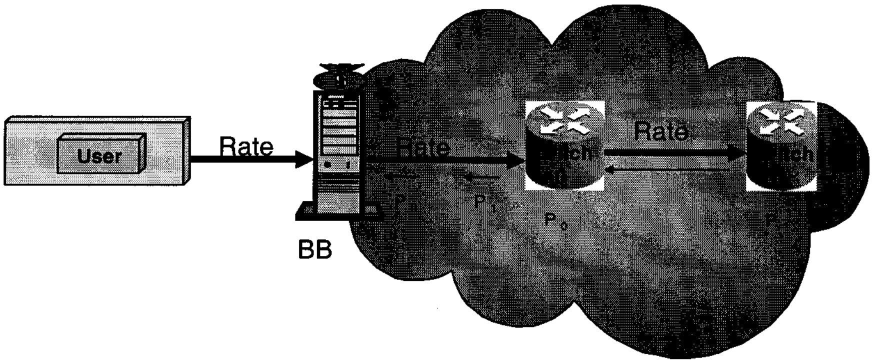

Figure 3.9: Spot market [39]

The Prices are updated at the switch based on supply and demand. Bandwidth is a non-storable resource. It has only immediate availability (no reservation overhead). Users are charged for usage (similar to residential electricity) and the bandwidth broker represents the user and is located at the network edge.

One disadvantage of the spot market approach is the absence of resource guarantees. It is possible that a user may enter the network during a period of time when prices are low, then at a later time find that prices have increased dramatically. These unpredictable price changes can cause the QoS of the user to suffer or even force the user to exit the network prematurely. For this reason, a method is needed to provide guarantees of resource availability (price stability). The multi-market approach addresses this issue and is described in the next section.

3.6.4 Multi Market

Similar to the spot market approach, a network is viewed as an economy consisting of three entities (users, bandwidth brokers and switches) and two different markets/resources (reserved and spot bandwidth) [39]. Switches own the bandwidth, which is sold in the reservation and spot markets. Reserved bandwidth has the advantage of ownership over a period of time, providing the user with some predictability of their expected traffic and quality of service. In contrast, spot bandwidth has the advantage of immediate availability without reservation overhead. Both market types are modeled as competitive markets; therefore efficient as well as optimal and fair allocations are possible. The bandwidth broker buys bandwidth to maximize the utility of the user. The multi-market approach uniquely integrates the benefits of the spot market optimality and equitable allocations with the price stability offered with the reservation market. The importance of the multi-market is underlined in the ability of users to dynamically change bandwidth quantities in response to market and source requirements. This is another unique feature of the multi-market approach [39].

3.7 Summary

This chapter described the network elements, which form the infrastructure of resource allocation, billing and call admission control as well as suitable network architectures.

We also described how pricing techniques are introduced and can be used in networks to trade resources (in most cases this is bandwidth). An important element is the bandwidth broker, which acts as the intermediary between the customers (users) and providers (networks and in finer scale, switches and routers). Users are represented in the economy by a bandwidth broker. The bandwidth broker buys bandwidth to maximize the utility of the user and considers the risks and benefits associated with the customer's requests. Architectures of bandwidth brokers have been described, along with several strategies in accepting user requests and allocate the resources. Such strategies correspond to various strategies, such as those described by the auction market, spot market and multi market.

In the spot market and multi market the computer network can be viewed as an economy consisting of three entities (users, bandwidth brokers and switches) and two different markets/resources. In the multi market, switches own the bandwidth, which is sold in the reservation and spot markets. Reserved bandwidth has the advantage of ownership over a period of time, providing the user with some predictability of their expected service. In contrast, spot bandwidth has the advantage of immediate availability without the reservation overhead.

Chapter 4

Performance Evaluation of Wavelength Allocation for Monopolistic Optical Networks under Demands Uncertainty

4.1 Introduction

The telecommunication service providers face ever increasing technological and economic challenges. They have to maintain their technological advantage and preserve profitability in a dynamic market. Employing Dense Wavelength Division Multiplexing (DWDM) is extremely cost effective for delivering high-speed transport services. DWDM uses multiple light wavelengths to transmit signals over a single optical fiber. It maximizes the use of installed fiber cable and allows new services to be quickly and easily provisioned over existing infrastructure. Flexible add/drop multiplexers (ADM) allow individual channels to be dropped and inserted along a route. Deployment of optical cross connects (OXC) interconnected by DWDM links, allow reconfigureability and scalability of the optical networks. The deployment of those components led to the deployment of Wavelength Routed Optical Networks (WRON) that have the potential to provide on-demand establishment of high-bandwidth connections, also called lightpaths. OXC's provide optical transport but they are only self-aware and there is a need for more intelligence for highly effective network provisioning. The ability to provide end-to-end lightpaths between end users is made possible through the deployment of intelligent routing algorithms. The lightpaths (known also as all-optical channels) require no processing or buffering at intermediate nodes. They provide a "circuit switched" interconnection between two end nodes, which may be located "far" from each other in the physical fiber network topology.

The establishment of those connections using appropriate wavelengths is known as Routing and Wavelength Assignment (RWA) [49].

The RWA can be cast as an optimization problem [49], and can be approached in a number of different ways, using various cost functions. In the related papers the approaches followed are: establishing all connections using a minimum number of wavelengths [68] [69]; establishing all connections using minimum path lengths [80]; maximizing the number of connections subject to a constraint on the number of wavelengths and/or the path lengths or their combinations [48][51].

In this chapter we are presenting a monopolistic case study in which we have only one transport service provider offering the wavelength sales. The main objective of this study is to investigate a profit/loss analysis of wavelength sales in such business models in different market environments under demand uncertainty. Our work in this area provides the network transport service providers with expectations on their cash flow based on market conditions, prices, profit and revenues.

4.2 Background

4.2.1 Retail Wavelength Sales

Wavelength sale services are currently available on all major long-haul routes [72] [73] [56]. The vast majority of sales remain wholesale [72]. Wavelengths are available at 1.2 GBps, 2.5 GBps or 10 GBps. Carriers have been the primary purchasers of wavelengths to date, but demand from other firms, especially financial services firms and service providers, such as Web-hosting firms, ISPs and content firms, is rapidly increasing. Small firms or users with limited bandwidth requirements usually avoid the expense of leasing or buying a wavelength [73]. Companies like

Sprint, SBC, Big Pipes, GO2Tel, Looking Glass, US Carrier, Bell South interconnection services, etc. offer wavelength retail services for large Internet and Application Service Providers (ISPs and ASPs), Carriers and Enterprise Organizations [73] [74].

4.2.2 Telecommunication Monopolistic market

The telecommunication industry has long been organized in most jurisdictions as a monopoly regulated by the state. Some countries like European Union and United States passed laws and acts to manage the transition from monopoly regulation of a dominant incumbent carrier to a competitive market [2]. In others like Latin America (World Bank study and London School of economics, see table 4.1) the state still owns the network infrastructure. Unregulated monopoly also exists in countries where the state owns the infrastructure and there is no independent regulatory system [5].

	Key reform law for telecoms	Year of first incumbent privatization action	Degree of privatization of incumbent operator	Period of exclusivity for initial new incumbent (begins-ends)	Level of competition (as of early 2000)			Effective creation of regulatory agency	Telecoms specific	Regulator financing	Regulator reports to:
					Local	Long distance	Digital cellular				
Argentina	1989	1990	Full	1990-2000	P	P	P	1990	Y	Regulatory fee	Ministry
Bahamas	1992	Delayed			M	M	M	1993	N	Government budget + auction revenue (license/spectrum) fee	
Bolivia	1995	1995	Partial	1995-2001	M	M	D	1995	Y	Reg. fee	Ministry
Brazil	1997	1998	Full	1998	P	P	P	1997	Y	Gvt + auction revenue	Ministry
Chile	1982	1987	Full	None	Full	Full	Full	1977	Y	Gvt	Ministry
Colombia	1991	Not yet	0	None	D	Full	D	1994	Y	Reg. fee	Ministry
Costa Rica	1996	Not yet	0		M	M	M	1996	N	Reg. Fee + gvt	Parliam.
Dom. Rep.	1998	1999	Full		Full	Full	Full	1998	Y	Auction revenue + gvt	Public controller
Ecuador	1995	Not yet	0	1995-2000	M	M	P	1995	Y	Auction revenue	Head of state
El Salvador	1995	1997/8	Partial	None	Full	Full	Full	1997	N	Auction revenue + gvt	Ministry
Guatemala	1995	1998	Full	None	Full	M	Full	1996	Y	Auction fee	Ministry
Honduras	1996	In process	0	1995-2005	Full	M	D	1996	Y	Gvt	Ministry
Jamaica		1989-91	Full		M	M	Full	1995	N	Reg. fee	Ministry
Mèxico	1990	1990-91	Full	1990-1996	Full	Full	Full	1996	Y	Gvt	Ministry
Nicaragua	1995	In process		1995-1999	M	M	Full	1995	Y	Auction fee	Head of State
Panama	1995	1997	Partial	1997-2002	M	M	P	1996	N	Auction fee + gvt	Head of state
Paraguay	1995	In process	0		M	M	Full	1995	Y	Auction fee	Ministry
Peru	1991	1994	Full	1994-1999	Full	Full	Full	1994	Y	Reg. fee	Ministry
Venezuela	1991	1991	Partial	1991-2000	M	M	Full	1991	Y	Reg. fee + gvt	Ministry
Uruguay	1992 referendum rejected liberalization	No	0		M	M	Full		Y	Gvt	Ministry

Level of competition: M – monopoly, D – duopoly, P – partial competition, F – full competition. These definitions are taken from ITU and refer to the number of licencees rather than the actual degree of competition.
Source: Authors' compilation from ITU, regulator's web sites and World Bank internal documents

Table 4.1: Summary of countries running monopolistic telecoms in Latin America [5]*

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* Up to the author's best knowledge this is the most recent publicly available study

4.2.3 Design Objective

The main design problem addressed in this chapter is to find the proper connectivity and traffic routing strategy within the optical network that maximizes profit and satisfies the customers' requests. The network traffic is given as a logical (virtual) topology that has to be mapped on the optical network. Transport network routing [55] procedures typically utilize explicit routing, where the path selection can be done either by an operator or by software-scheduling tools in resource management systems. This issue addresses mainly a backbone provider who runs a high capacity network and has connections to other backbone providers and Internet service providers. The design objective is to maximize the network service supplier profit and compute the actual set of flows through the network. In a switched optical network, end-to-end optical channel connections are requested with certain constraints. Path selection for a connection request should employ constrained routing based algorithms that balance multiple objectives, while conforming to physical constraints such as network topology and load balancing of network traffic in order to achieve the best utilization of network resources.

4.3 Economic Considerations

In order to achieve the design objective, a transport service provider, in addition to the technical aspects of the flow assignment and routing, addresses economic issues, such as service demand, profit maximization, revenue, market conditions etc. In this section, economic terms are introduced and explained, which are needed in the formulation of the allocation strategy.

4.3.1 Definition and modeling of demand for optical networks capacity

In section 2.2.5 of chapter 2 the definitions for demand elasticity and demand potential have been presented. For every commodity on the market there is a *demand* and a *price*. The *demand elasticity* E is defined within a time interval as the negative ratio of the change in demand over the relative change in price during that interval.

$$E = -\frac{\frac{\Delta D}{D}}{\frac{\Delta P}{P}} \tag{4.1}$$

Where Δd denotes the change in demand and Δp denotes the change in price. Considering the limit case where the time interval becomes infinitely small, we obtain [50]:

$$E = -\lim_{\Delta t \to 0} \frac{\frac{\left(\frac{\Delta D(t)}{D(t)}\right)}{\Delta t}}{\frac{\left(\frac{\Delta P(t)}{P(t)}\right)}{\Delta t}} = -\frac{\left(\frac{D'(t)}{D(t)}\right)}{\left(\frac{P'(t)}{P(t)}\right)} \tag{4.2}$$

Assuming a constant demand elasticity value E for every price and demand of interest, the price-demand relationship is defined as a demand function with constant elasticity [22], [50]. Assuming that E is independent of time, after solving the associated differential equation 4.2, the *demand function model* takes the following general form:

$$D(t) = AP(t)^{-E} \tag{4.3}$$

We will call this demand function model *deterministic demand function* (see figure 4.1). For simplicity we shall omit the dependence of the time t in the various functions. The scaling constant A is denoted as *demand potential.* Further, in order to avoid confusion in the notation we express the demand by "d" and the price by "p".

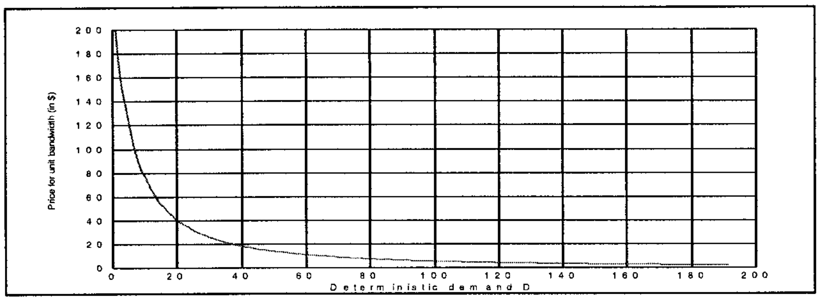


Figure 4.1: Deterministic demand model

As defined in chapter 2, section 2.2.8, the revenue denoted by "r", is the product of price and demand, henceforth :

$$r \;=\; p \cdot d \qquad (4.4)$$

4.3.2 Supplier's revenue

We assume that a network service provider is operating as *monopolist*, which means that he is the sole provider of services y. The supplier offers K different services, which are denoted as *vector of services* $\vec{y} = (y_1, \ldots, y_K)$ offered to the corresponding requests originating from different customers. A service is an end-to-end connectivity with certain number of wavelengths. As a monopolist, the supplier is free to set prices as he wishes; the only limitation is that as prices increase, customers are likely to buy lesser quantities of services. The supplier's profit is defined as the difference between the *supplier's revenue* $r(\vec{y})$ collected by selling services $\vec{y} = (y_1, \ldots, y_K)$ and the *production cost of* these *services* $c(\vec{y})$. The supplier's objective is to ***maximize his profit***:

$$\pi(y) = \max_{y \in Y} \; [r(\vec{y}) - c(\vec{y})] \qquad (4.5)$$

We assume that the price of the services occur in a linear manner, i.e. $r(\vec{y}) = p^T \cdot \vec{y}$, where p^T is a transpose price vector and $\vec{p} = (p_1, \ldots, p_K)$ is the *price vector* for charging customer requests. Based on the terms introduced, the *vector of services* is the set of *optical channels* or *lightpaths*, where every optical channel has fixed bandwidth. The cost of services can be controlled by the load balancing of the network traffic, which is applied with the objective to achieve the best utilization of network resources and leads to profit maximization.

We define $\vec{d} = (d_1, \ldots, d_K)$ as a *demand vector* of K service requests from customers, corresponding to the demand for wavelengths. Further, we define $\vec{A} = (A_1, .., A_K)$ as a *demand potential vector* (see equation 4.3) for these demands. It is assumed that the services $\vec{y}$, which the provider offers to the customers, meet the demand $\vec{d}$. The service is wavelength allocation over a requested optical connection. The *supplier's revenue* $r(\vec{y})$ has now the form $r(\vec{y}) = \sum_{i=1}^{K} p_i \cdot d_i$.

Taking into consideration the dependence between demand and pricing, described in equation 4.3, and assuming that the elasticity E of the optical network services is equal for all customer demands, we can derive the ***supplier's revenue*** as:

$$r(\vec{y}) = \sum_{k=1}^{K} p_k \cdot d_k = \sum_{k=1}^{K} \left(\frac{d_k}{A_k} \right)^{-\frac{1}{E}} \cdot d_k = \sum_{k=1}^{K} d_k^{\frac{E-1}{E}} \cdot A_k^{\frac{1}{E}} \qquad (4.6)$$

The *supplier's profit* or *payoff* is determined by the cost of operating the network and by the physical constraints associated with the network. The payoff is derived in the next section.

4.4 Routing and channel assignment in optical networks with Profit maximization

As mentioned earlier the formulation of the RWA as an optimization problem can be done in different ways, using different cost functions. In this section we define the profit function as an optimization function and we treat the RWA problem as a MIP (Mixed Integer Problem) optimization [49]. It is further assumed that the optical network consisting of OXCs does not have wavelength interchange capabilities. Figure 4.2 illustrates a simple logical topology mapped on the optical network. The network is modeled as a set of nodes, *Nodes=* $\{1,2,..,N\}$ and a set of optical links, $L=\{l_{xy}\}$ where l_{xy} denotes the bidirectional link from node x to node y. Every optical link l_{xy} is associated with a utilization cost p_{xy}. The set of available wavelengths is $\Lambda=\{1,2,..,W\}$. The network traffic is given as a set of connections has bandwidth demand d_k and is associated with wavelengths and a sequence of optical links. Based on this, we can now designate the network parameters and variables, used for the routing and wavelength allocation.

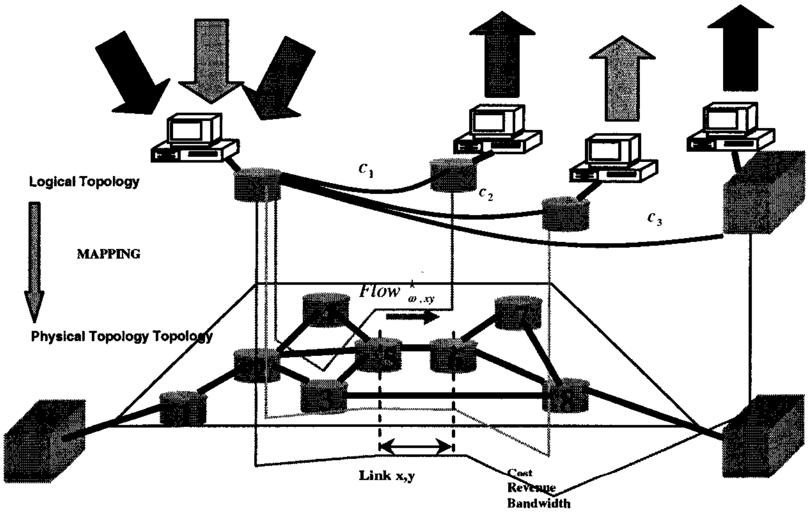


Figure 4.2: A network topology mapped on the optical network's infrastructure

The *network parameters* are:

v_k: the logical connection we want to realize

d_k: number of wavelengths demanded by customers for every logical channel

f_{xy}: the maximum number of wavelengths for the link l_{xy}

p_{xy}: the cost for allocation of a wavelength for the link l_{xy}

The *network variables* are:

$b^k_{w,xy}$: a flow binary variable, equal to one if a logical connection v_k is carried on link l_{xy} over fiber using wavelength w, and zero otherwise

Ω^k_w: a binary variable, equal to one if connection v_k is carried on wavelength w, and zero otherwise

Using these network parameters and variables, we define the *service production cost function* for operating the network as follows:

$$c(\vec{y}) = \sum_{k=1}^{K} \sum_{w=1}^{W} \sum_{l_{xy} \in L} p_{xy} \cdot b^{k}_{w,xy} \qquad (4.7)$$

Where W is the total number of wavelengths over the fiber.

Now, having the cost function $c(\vec{y})$ from equation 4.7, the supplier's revenue $r(\vec{y})$ from equation 4.6, and the general form for profit, we derive the *supplier's profit* for operating the optical network.

$$\pi(\vec{y}) = \sum_{k=1}^{K} d_k^{\frac{E-1}{E}} \cdot A_k^{\frac{1}{E}} - \sum_{k=1}^{K} \sum_{w=1}^{W} \sum_{l_{xy} \in L} p_{xy} \cdot b^{k}_{w,xy} \qquad (4.8)$$

The RWA can be formulated now as ***profit maximization***, subjected to ***constraints*** expressed by equations 4.8.1, 4.8.2, 4.8.3 and 4.8.4 given by the network topology:

- Flow conservation constraint: for every node x and neighboring nodes *j*:

$$\sum_{j \neq x} b^{k}_{w,xj} - \sum_{j \neq x} b^{k}_{w,jx} = \begin{cases} +\Omega^{k}_{w} & \text{if } x \text{ is source of } v_k \\ -\Omega^{k}_{w} & \text{if } x \text{ is destination of } v_k \\ 0 & \text{otherwise} \end{cases} \qquad (4.8.1)$$

Equation 4.8.1 is the flow conservation equation, which states that a connection v_k entering node x on wavelength *w* must leave the node on the same wavelength, thus ensuring wavelength continuity.

Capacity constraint

$$\sum_{w=1}^{W} \sum_{k=1}^{K} b^{k}_{w,xy} + \sum_{w=1}^{W} \sum_{k=1}^{K} b^{k}_{w,yx} \leq f_{xy} \qquad (4.8.2)$$

Equation (4.8.2) specifies the capacity limit of every optical link, where f_{xy} is the maximum number of wavelengths for link l_{xy} . The wavelength can be directed on the link from node *x* to node *y* or from node y to node x, but are not bi-directional simultaneously.

- Constraint for one traffic direction over single wavelength

$$b^{k}_{w,xy} + b^{j}_{w,yx} \leq 1 \qquad (4.8.3)$$

Equation 4.8.3 specifies that while the links (i.e. one fiber) are bi-directional on a single wavelength, the communication is only in one direction. As indicated earlier, $b^{k}_{w,xy}$ is a flow variable equal to one if a logical connection v_k is carried on link l_{xy} over fiber using wavelength *w*, and zero otherwise, $b^{k}_{w,xy} \in \{0,1\}$

- Traffic demand constraint

$$\sum_{w=1}^{W} \Omega^{k}_{w} = d_k \text{ , where } \Omega^{k}_{w} \in \{0,1\} \qquad (4.8.4)$$

Equation 4.8.4 ensures that the requested demand, interpreted as number of wavelengths for every optical connection, is actually allocated throughout the network.

This formulation can be cast as Mixed Integer Programming Formulation (MIP) [49]. In the sections bellow, we present a case study on how a transport provider can utilize the existing resources of a network; improve the network planning with regard to profitability and therefore stay in business.

4.5 RWA as Optimization Problem: Case Study

In this case study we present a realistic simulation of a virtual topology, realized on *CA*net3 Canada's National Optical Internet Project* [52] network. We assume that the analysis is based on a wavelength wholesale as described in section 4.2. The desired virtual topology consists of three optical channels, v_1, v_2 and v_3, with requested wavelength demands d_1, d_2 and d_3. Both, optical network topology and the desired virtual topology to be mapped are given in figure 4.3. According to [53], the price of one-meter optical cable per year is estimated at a cost of \$0.92. The distances $dist_{x,y}$ between the cities are known and this makes it possible to compute the utilization cost p_{xy} of every fiber link. In [56] the price for a 1.25 GB/s wavelength per month is \$140000. To obtain a realistic value for the demand potential A we need to apply equation 4.3. We assume that the fiber link capacity is 7 wavelengths and the bandwidth capacity of one wavelength is 2.5 GB/s. Our main objective from this case study is to examine the behavior of this particular network in different market environments and the effect of demand uncertainty in the profitability of the network. The RWA is carried out for the case where the service provider acts as *monopolist*. The solution of the optimization yields the wavelength allocation and the corresponding optimal profit, cost and pricing for the optical connections v_1, v_2 and v_3. Based on the optimization function expressed in equation 4.8 subjected the constraints 4.8.1, 4.8.2, 4.8.3 and 4.8.4, the simulation is carried out using the algebraic modeling language AMPL and the optimization tool CPLEX 7.0. We obtain the optimal wavelength demands d_1, d_2 and d_3 over the optical channels v_1, v_2 and v_3, given that the demand potentials A_1, A_2 and A_3 are known.

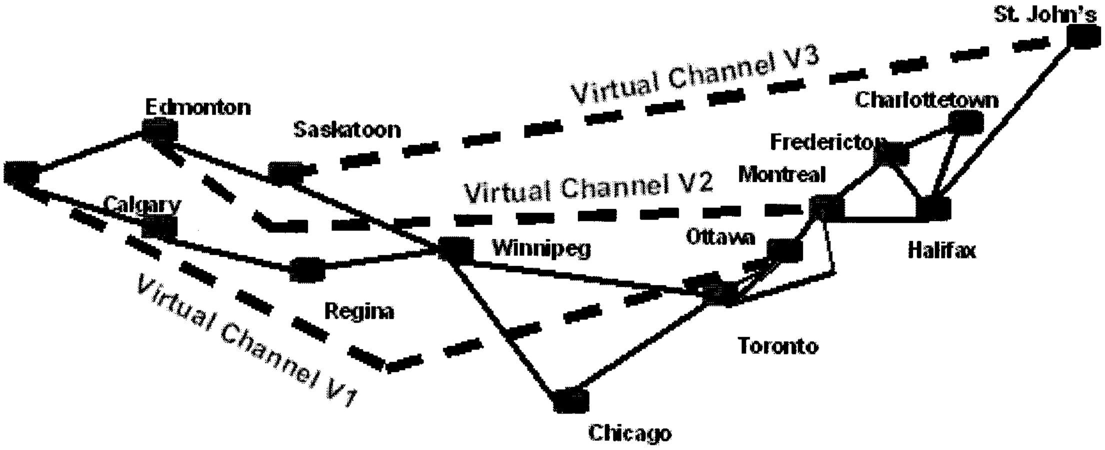


Figure 4.3: Optical network and the desired virtual topology

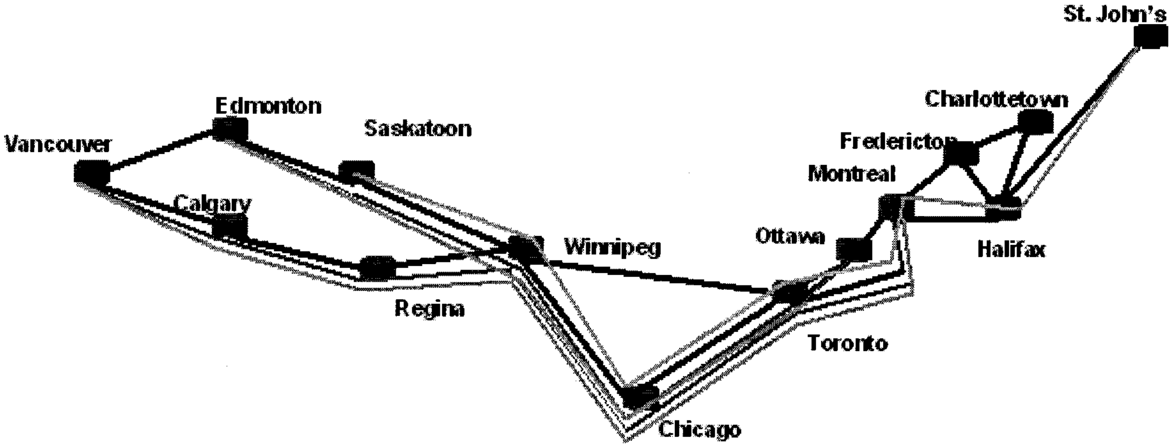


Figure 4.4: Resulting optimal allocation: $d_1=2$, $d_2=2$ and $d_3=1$

It is further assumed that the customer demands d_1, d_2 and d_3 are limited between 0 and 7 wavelengths. Any request from a customer higher than 7 wavelengths can not be allocated since it exceeds the capacity of a single link. The different demand market elasticities are taken between E=1.1 to E= 2.0 [81]. Figure 4.4 shows the resulting optimal wavelength allocation for demands $d_1=2$, $d_2=2$, *and* $d_3=1$ after optimization. Table 4.2 summarizes the results showing the demands at which the network attains maximum profit for each market elasticity. This simulation shows that this particular network can be operated profitably with modest load (maximum profit is achieved for demands $d1=2$, $d2=2$ *and* $d3=1$). For some demands (for example $d_1=6$, $d_2=6$ *and* $d_3=5$) there is no solution, because the demand exceeds the link capacity. Table 4.2 summarizes the results showing the demands at which the network attains minimum profit for each market elasticity.

Elasticity	Demand d1	Demand d2	Demand d3	($)price/wavelength	($)Revenue	($)Cost	($)MaxProfit
E=1.1	1	1	2	252865.8899	1011463.56	559080	452383.5595
E=1.2	1	1	2	257603.119	1030412.476	559080	471332.4759
E=1.3	1	1	2	261810.488	1047241.952	559080	488161.9518
E=1.4	1	1	2	265568.6265	1062274.506	559080	503194.5059
E=1.5	1	1	2	268943.4866	1075773.946	559080	516693.9465
E=1.6	1	1	2	271989.2633	1087957.053	559080	528877.053
E=1.7	1	1	2	274750.7448	1099002.979	559080	539922.9792
E=1.8	1	2	1	277265.175	1109060.7	515640	593420.7001
E=1.9	1	2	1	279563.7193	1118254.877	515640	602614.8771
E=2.0	2	2	1	252676.1902	1263380.951	652590	610790.9512

Table 4.2: Maximum profit, cost, and revenue for the optimal demand with different market elasticity

Elasticity	Demand d1	Demand d2	Demand d3	($)price/wavelength	($)Revenue	($)Cost	($)MaxProfit
E=1.1	7	0	7	56265.60518	787718.4725	2085720	-1298001.53
E=1.2	7	0	7	65202.69899	912837.7858	2085720	-1172882.21
E=1.3	7	0	7	73864.96978	1034109.577	2085720	-1051610.42
E=1.4	7	0	7	82200.1259	1150801.763	2085720	-934918.237
E=1.5	7	0	6	94685.92969	1230917.086	1924710	-693792.914
E=1.6	7	0	6	102362.5755	1330713.481	1924710	-593996.519
E=1.7	7	0	6	109652.2752	1425479.577	1924710	-499230.423
E=1.8	0	7	7	111947.6622	1567267.271	1986810	-419542.729
E=1.9	0	7	7	118501.9502	1659027.303	1986810	-327782.697
E=2.0	0	7	7	124728.2761	1746195.865	1986810	-240614.135

Table 4.3: Minimum profit, cost, and revenue for the optimal demand with different market elasticity

Showing the maximum and the minimum profit obtained from particular demands is very important to the transport service provider when analyzing the performance of the network. It helps determining the expectation of his cash flow and gives a wider perspective of the market and the demands should target the network's operation to stay profitable. The minimum profit case shows the worst case scenario for price setting. A very profitable network can turn to a loosing money network if the services were inappropriately priced.

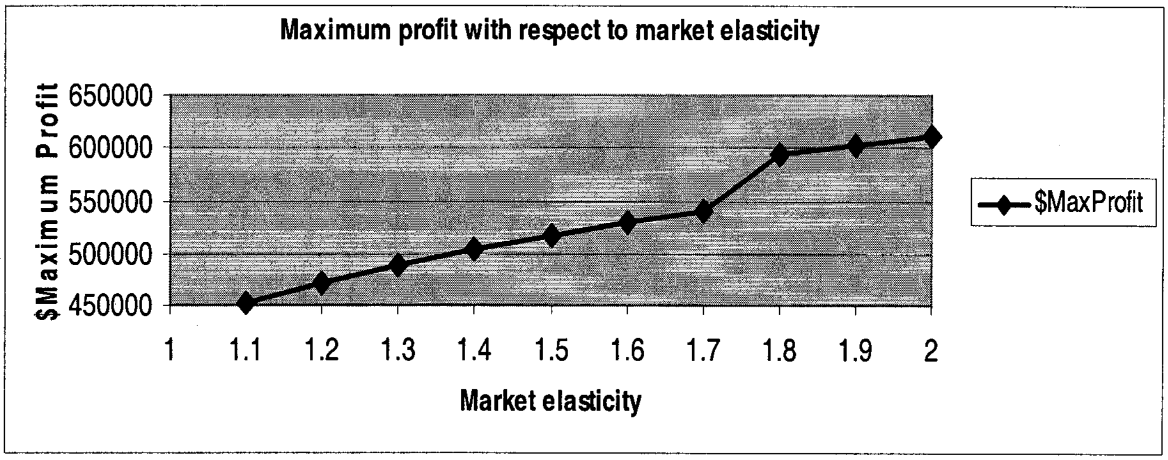


Figure 4.5: Maximum profit at different levels of market elasticity

In figure 4.5 we show the maximum profit possible when operating the network at different levels of market elasticity corresponding to optimal demands found in table 4.2. It is clear from figure 4.5 that as the elasticity increases, which means that the market becomes stronger, the profit from operating the network increases.

This simulation could represent a situation of operating an optical network in "economic boom" and "economic slowdown" market conditions in which transport service providers face profit/loss challenges. The best way to foresee them is a careful profit/loss analysis. We conclude that profitability of an optical network depends heavily on the market conditions. A very profitable optical network can turn into losing financial resources if the market environment turns from "hot" to "slow". Also these results lead to the following conclusions:

- full use of the available bandwidth resource does not provide maximum profit.
- not all demands can be satisfied.
- a call admission control, based on profit/loss consideration is necessary for a service provider in order to remain profitable.

In table 4.2, the transport network provider knows which profit should target for every market in order to obtain the maximum profit. The simulation assumes that the demand is deterministic. In reality, when facing customer demands, the service provider can control them by setting a suitable pricing policy. However as the decision to buy wavelength or not is in the hands of the customer, there is always a stochastic element which affects the customer's decision how many wavelengths to buy. The maximum profit business goal for a transport service provider could be formulated as follows: setting prices for a unit wavelength to achieve such utilization of the network resources that his profit of network operation is maximized. Let us assume that the administrator of the optical network targets certain profit to the corresponding deterministic demands $\boldsymbol{d_1}$, $\boldsymbol{d_2}$ and $\boldsymbol{d_3}$. For targeted profit $ 610790.9512 (see table 4.2) under market elasticity E=2.0, the corresponding deterministic demands are $d_1=2$, $d_2=2$, and $d_3=1$. The granularity of the fiber capacity is 1 wavelength and it is assumed to be 2.5 GB. This means a small stochastic change in demand can result in a new wavelength request. Also the cost of operating the network varies in a non-linear fashion since every link has a different cost and the routes used to satisfy the demand vary according to the availability of the capacity in each link along the route. Because of this uncertainty, we have assumed that after the transport network provider adjusted the price to target the demand that maximizes his profit, the demand could fluctuate, resulting in different profit expectations. In the simulation, we assume that the price-demand relation is a random process, where the demand value corresponding to a certain price is a Poisson random variable with mean corresponding to the deterministic value of the demand. In table 4.4 we have computed the profit resulting from 25, 50, 75, and 100 realizations of the random process. The corresponding average profit, standard deviation and the confidence interval are computed and presented in table 4.4. In figure 4.6 we see the decrease of profit compared to the maximum expected profit.

Random Process realizations	Average profit	Standard Deviation	95% Confidence Interval
25	$433970.50	$46396.72	$433711.51-$434230.26
50	$455791.52	$53980.07	$455602.15-$455973.42
75	$491502.15	$52503.41	$491340.23-$491652.58
100	$514102.56	$53500.35	$512701.24-$515511.56

Table 4.4: Profit of the network with stochastic demand

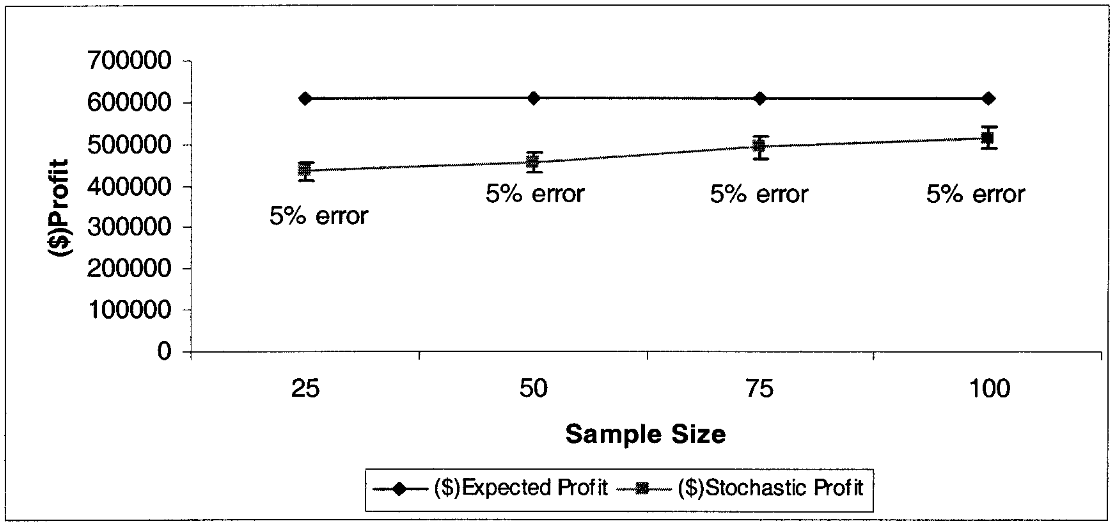


Figure 4.6: Comparison of stochastic profit with the expected profit

This study helps optical network service providers to define services strategy, leverage market characterization, quantify service revenue opportunities, and assess their business and the value of optical network investments in different market environments. It also answers many questions of the service providers such as the evaluation of trade-offs involving economics, risks and operational profit analysis. Also, this study assists service providers by demonstrating how demand will be realized in the network and guides them through their investment plans on optical networks based on different market environments.

4.6 Summary

In this chapter the Routing and Wavelength Allocation (RAW) algorithm in a Dense Wavelength Division Multiplexing (DWDM) optical network is formulated as a Mixed Integer Programming (MIP) problem. The presented work is based on an economic feasibility study for a service provider, acting as monopolist in the market, offering transport network services. An important conclusion of this case study is that full usage of the available resources does not necessarily provide maximum profit. Additionally, it shows how a very profitable network can turn in losing money only by a slight change in the market environment. It shows that a small change in customer demand results in big degradation of the expected profit. However, a monopolistic scenario is possible in heavily regulated environments and very remote areas where only one transport service provider is present. Therefore, it is necessary to consider the optimal resource allocation in a competitive network environment, which is the subject covered in the next chapter.

Chapter 5

Routing in Optical Networks based on Competitive Pricing Games

5.1 Optical networks in competitive markets

The research on competition between transport service providers becomes a very important issue especially in the light of worldwide globalization of trade and telecommunications. Our study aims at integrating business intelligence in the optical networks to increase the optical transport service provider's competitiveness and profitability in the telecommunication market. In [5], the World Bank and London School of Economics have provided an extensive study on governments and institutional networks operating as monopolists in a national market. The study has shown that monopolistic communication market cannot provide cost-effective services. In the majority of the industrialized world the competition is adopted as a solution (and to a lesser extent that technology has advanced to the point where competition is viable). The history of telecommunication industry has a distinct pattern of transformation, starting as unregulated monopoly, later to a fierce competition, then to regulated monopoly, and most recently to (de)regulated competition [2]. The current trend in optical networks is to open the entire wholesale market to competition [3]. As a result, we will see, instead of a single big market player, several service providers competing with each other to attract customer demand. Our study is a head-start and it is addressing a future competitive market. In this respect, it provides the fundamental microeconomics aspect of the competition and the possibilities of business opportunities. Our study guides transport service providers through their strategic and long term network planning based on profit/loss analysis. It also determines a given network architecture's level of profit forecast based on competition.

5.2 Wavelength trade and price competition among optical networks

In this section, we present the basic scenario for flow assignment in multi-wavelength optical networks based on competitive pricing. The following has to be taken into consideration:

(1) Currently, the bandwidth in the optical networks is offered / provided in the form of wavelengths wholesale to ISPs, Carriers Companies, Enterprise Organizations, Financial Firms, Application Service Providers (ASPs), Web hosting firms, health care centers, governments, etc. [72] [73]. The bandwidth of an optical light-path in the market over optical fiber is 2.5 Gbps and higher [61].

(2) Backbone long-haul optical network transport service providers (TSPs) are offering their services in terms of bandwidth measured in number of wavelengths.

(3) Customers (ISPs, Carriers Companies, Enterprise Organizations, Financial Firms, Application Service Providers (ASPs), Web hosting firms, health care centers, governments, etc.) request an end-to-end connectivity (virtual links), which forms a virtual network topology. The links of this virtual topology have to be mapped on the underlying optical networks.

Plenty of research studies were conducted in the past, concerning competitive routing in communication networks [47], [68], [62] [46] from the microeconomic point of view, where users act in a selfish manner and compete for network resources until they reach an equilibrium.

These studies were performed under the assumption that the user competes for scarce network resources in one network.

The novelty in our approach is the assumption that competitors now are the optical networks. They compete to allocate customers demand measured in number of wavelengths. The competition is assumed to be non-cooperative, which means, the competitors do not negotiate among each other. Up to the author's best knowledge, this approach has not been considered in the open scientific literature by anybody yet.

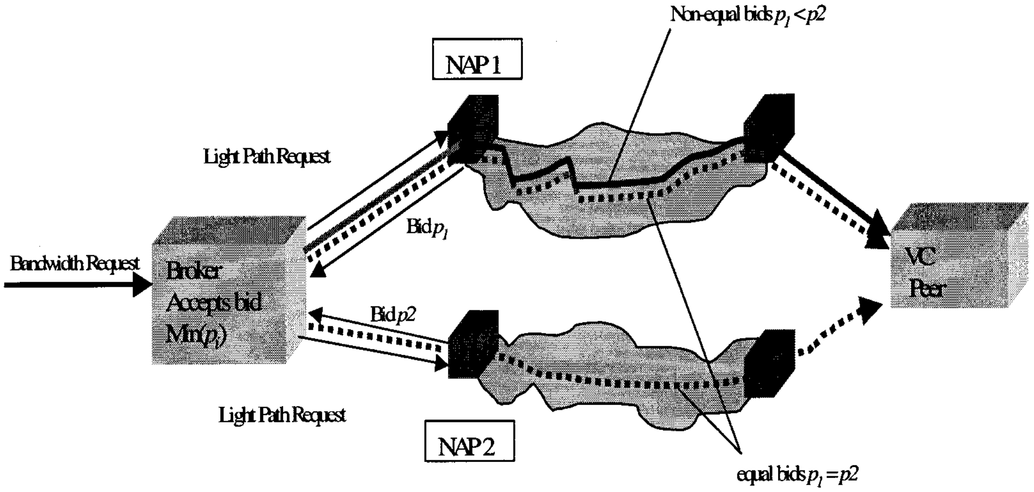


Figure 5.1: Price competition between TSPs

On figure 5.1 we present a typical scenario for network competition. We can identify a broker (broker acts on behalf of an institution, web host company, financial firm, etc.) who sends a request to the transport service providers to buy certain number of wavelengths. The transport service providers reply with their bids containing the price for the requested number of wavelengths. The broker accepts the bid with the lowest price and the winner network allocates the requested number of wavelengths.

5.3 Modeling the Price Competition

In this section we will consider a price competition game among non-cooperative optical networks. Non-cooperative means that the networks do not exchange information about their state topology, capacity, etc. In order to specify the process of competition we need to define it in terms of game theory:

Game theoretic formulation: The game between the network service providers for attracting customer demand can be seen as a non-coorporative game. The *competition* is offering competitive prices for allocating the requested wavelength demand. The *game strategy* is undercutting prices. *The players* are the networks. The *payoff* is the profit from allocating the requested demand.

As we have mentioned above, the game strategy is undercutting prices, which means a price setting game among competing networks. Each network is selfish and wants to maximize its profit independently and regardless from other networks. Many researches were conducted in the past considering the topic of price competition and price undercutting as we can see in [75], [76], [77], [78]. In [75] the authors assume that the buyers decide on the quantity and the price below what they are ready to buy for the entire quantity. The buyers post their quantity and the price they are willing to pay prior the competition. The sellers meeting the basic requirements of the buyers are qualified to compete. The seller with the lower price bid gets the deal. [76] models a price setting concept based on statistical gathering of historical data of market behavior. The model relies on marketing strategies to determine the most valuable customers, potential customers and below zero customers. This study does not require a competition in the market and the concept of game competition and players is not required. In [77] the authors assume that the broker is looking for the equilibrium price and then tries to make profit by charging more for buyers than they pay sellers. In this study, the broker looks for a matching equilibrium function, which satisfies the three parties. In [78] the author proposed a dynamic bid signaling and price negotiation protocol for service level of agreement trading between Internet service providers and consumers, called SLATP. The buyer sends his request asking for a price and the provider replies with his bid. The buyer replies back accepting or rejecting the bid and the provider confirm or reject the feedback. In our study, we present two models of competition; the first is called *active price competition* and the second is called *passive price competition.*

Active price competition:

This model is similar to the work in [78]. We assume that the suppliers (optical network service providers) are involved in a non-cooperative competitive game to allocate the customers demand of wavelengths and offer competitive prices. These prices are driven by the awareness of each supplier regarding the competition [26] [27]. The competition is initiated after the broker, who represents many customers, sends a request for logical link(s) connectivity (understood as virtual topology to be mapped on the physical networks) to the service providers. The service providers reply with price bids per unit wavelength. The broker replies to the service providers asking them to lower the bidding below the current minimum price without revealing the identity of the service provider who offered the minimum price. The interaction and the feedback between the broker and the service providers continue until the end of the competition, when only one service provider can offer the lowest price. The broker either accepts the lowest price offered, or rejects it, if he cannot afford. The decision about the number of wavelengths to be purchased is taken based on a price/demand curve, which determined by the broker based on a pool of individual customers he represents.

Passive price competition:

In this model, we assume there is no interaction between the broker and the suppliers (optical network service providers), except for the request of connectivity from the broker. This model simply applies incremental increases/decreases in price, and as long as the observed level of success rate (in terms of won/lost connectivity requests) increases, it continues to increase its price. If the success rate decreases, it decreases the price. In this model, the service provider is unaware of the customer's behavior, because of lack of feedback. The customer might have walked away either he can not afford the price or another service provider has won his request, but these facts are unknown to the service provider. The assumption in this model is that the service provider is only aware of the existence of the other service providers. This assumption is important to satisfy the requirements of the game, which requires defined players. This model is suitable for service providers newly entering the market.

In both models, active price competition and passive price competition, Nash equilibrium is a non-cooperative game solution [26] [27]. Our objective is to determine the performance of the optical network when reaching Nash equilibrium price. In such level of hard competition, we need to determine the profitability of the network and recommend where to upgrade the network to boost its competitive advantages.

In the subsection below we start by developing a high-level communication protocol supporting the first model, to explain the concept of price competition.

5.3.1 Active price competition : High-level communication protocol

In this model, the broker, representing many customers, decides the number of wavelengths to buy based on the cheapest price offer from the suppliers (optical network service providers) and the price-demand relationship *D (p)*. This price demand relationship is determined by the broker based on a pool of individual customers he represents. We develop a high-level communication protocol that supports an exchange of messages for competitive price setting based on the bids sent by the service providers and received by the broker. Before describing the protocol, we wish to point out the assumptions of the non-cooperative competition among suppliers.

a - The broker sends end-to-end connectivity requests to the networks asking for a price per unit wavelength.

b - Each network service provider first replies with reasonable price based on market awareness. Each network service provider is selfish and tries to increase his profit regardless of other networks.

c - After the initial bidding, the broker submits to the networks the minimum price offer without revealing the identity of the service provider who offered this price. The broker asks the networks to go lower than the current minimum price. If the minimum price is the same as the price previously offered, then the network does not have to go lower. In this case, the service provider can wait for another round.

d – If more than one service provider was able to answer with a lower price, then step c is repeated.

e- The competition ends when only one service provider was able to offer a lower price.

This approach of price setting is widely accepted business practice and happens daily in scale of hundreds of times. A real life business example would be a mortgage broker in a real estate company who acts on behalf of a property buyer. Assume that the broker is looking to find the best interest rate for a 1 million dollar mortgage from a financial institution; a bank for example. Assume there are five banks in the market. The broker first gets a quote from the five banks, and then he takes the cheapest interest rate and asks them again if they can go lower than the minimum interest rate he gets so far, without revealing the identity of the bank that offers this rate. If no one replies, then he accepts the current interest rate (the minimum he gets, if he can afford), otherwise, if two or three reply with a better offer than his current minimum interest rate, then he repeats his request again until only one bank can offer a better rate.

Direction	Messages	Comments
Customer to Supplier (Broker to Network Service provider)	***Reqc (X,Y)***	Initiate competition for the virtual link *X-Y*
	ocl (X,Y,P)	Offer a lower price than the current bid *P* for the virtual link *X,Y*
	Nack (X,Y)	The link request X-Y not granted
	Ack (X,Y,D)	The virtual link request X-Y is granted with D wavelengths to be accommodated
Supplier to Customer (Network Service Provider to Broker)	***offp(P,X,Y)***	The wavelength price offered for the virtual link X-Y is P dollars
Exceptions	***exc_1(D,P,X,Y)*** *(supplier to customer)*	The requested demand D over virtual link X-Y exceeds the available network capacity or No capacity to accommodate even a single wavelength of the link X-Y
	exc_2(X,Y,P) *(customer to supplier)*	The offered price results in 0 wavelengths demand from the customer, the customer can not afford

Table 5.1: Competition protocol messages

The messages exchange between brokers and service providers are shown in table 5.1. The requested service is the virtual connection between nodes X and Y. The networks compete by offering price *P* to win customer demand of wavelengths *D*. We have also developed exception messages for special cases concerning the competition. In the next section, we provide a detailed explanation regarding these messages and discuss the appropriate resolution scenarios.

In figure 5.2a, without loss of generality and for the sake of simplifying the protocol signaling, we will consider the case of two competing networks; Network A and Network B. It assumed that the duration of the customer connectivity request can take hours, days or weeks. The price competition is assumed to take milliseconds. Thus, the duration of the price competition is instantaneous compared to the duration of the request which might be hours, weeks or months. We present the sequence diagram of a general case of competition in which the broker accepts the final price (the price after finishing the competition) and the winner network can accommodate the requested number of wavelengths. From figure 5.2a, we can identify the following steps:

Step 1: The broker initiates the competition by sending his connection request to network A and network B.

Step 2: The suppliers (optical network service providers) respond with their price offerings.

Step 3: The broker responds to the offerings:

- The broker sends a request to the suppliers (optical networks) to undercut the minimum price of the offerings $P=min\ (P_1, P_2,\ldots)$ without revealing the identity of the network that offers the minimum price and then the suppliers repeat step 2.
- If only one competitor was able to lower the price while the other competitor cannot, then this competitor is a winner and the price is final.

Step 4: the broker notifies the winner competitor and sends his demand, measured in number of wavelengths.

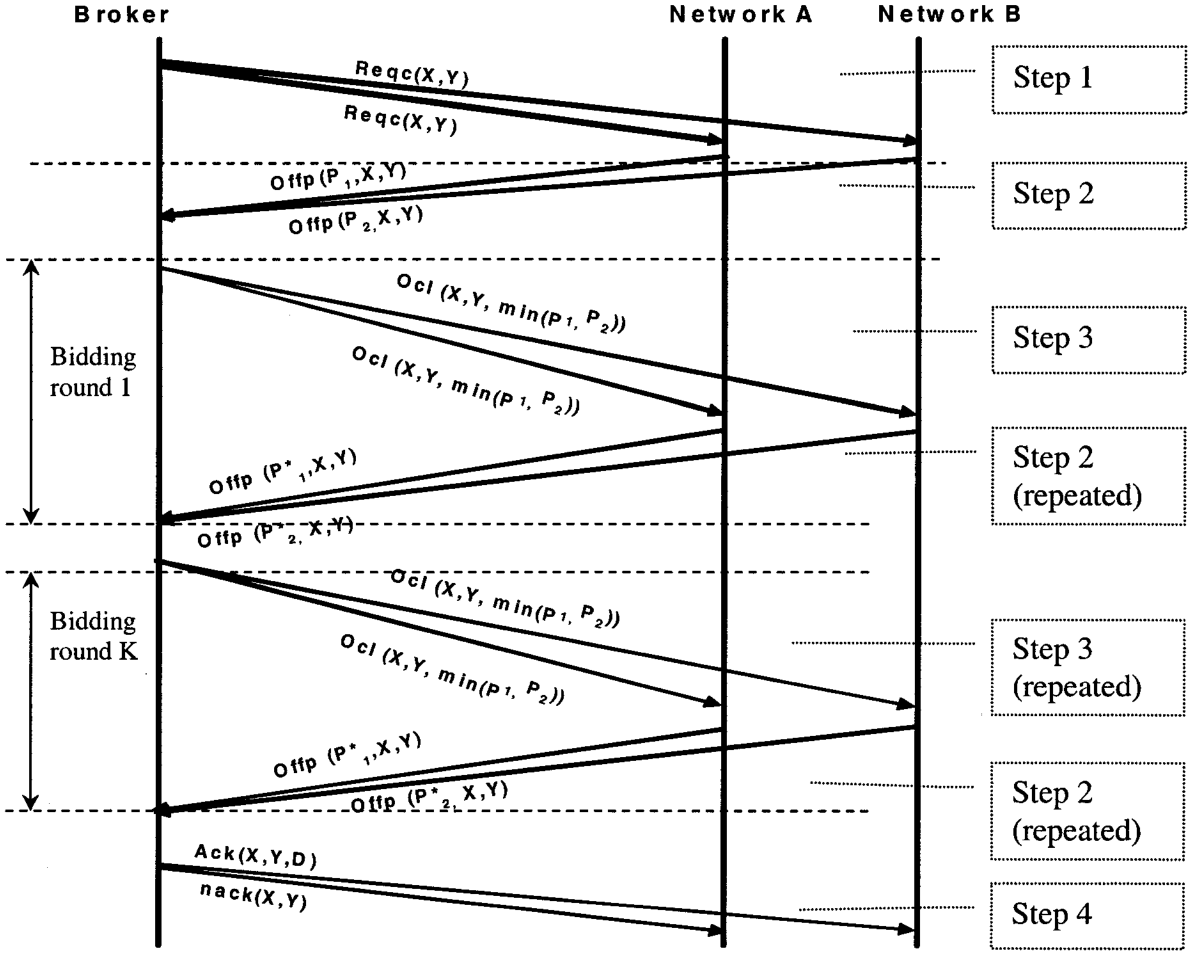


Figure 5.2a: Signaling of price competition

In case of having simultaneous arrivals of customer connections for the same channel, the network will try to accommodate both connections on the same route and offer the same price per wavelength. Otherwise, the network may use an alternative route. In this case, the requests will be charged different. The message exchange between the brokers and the competing networks is shown in figure 5.2b.

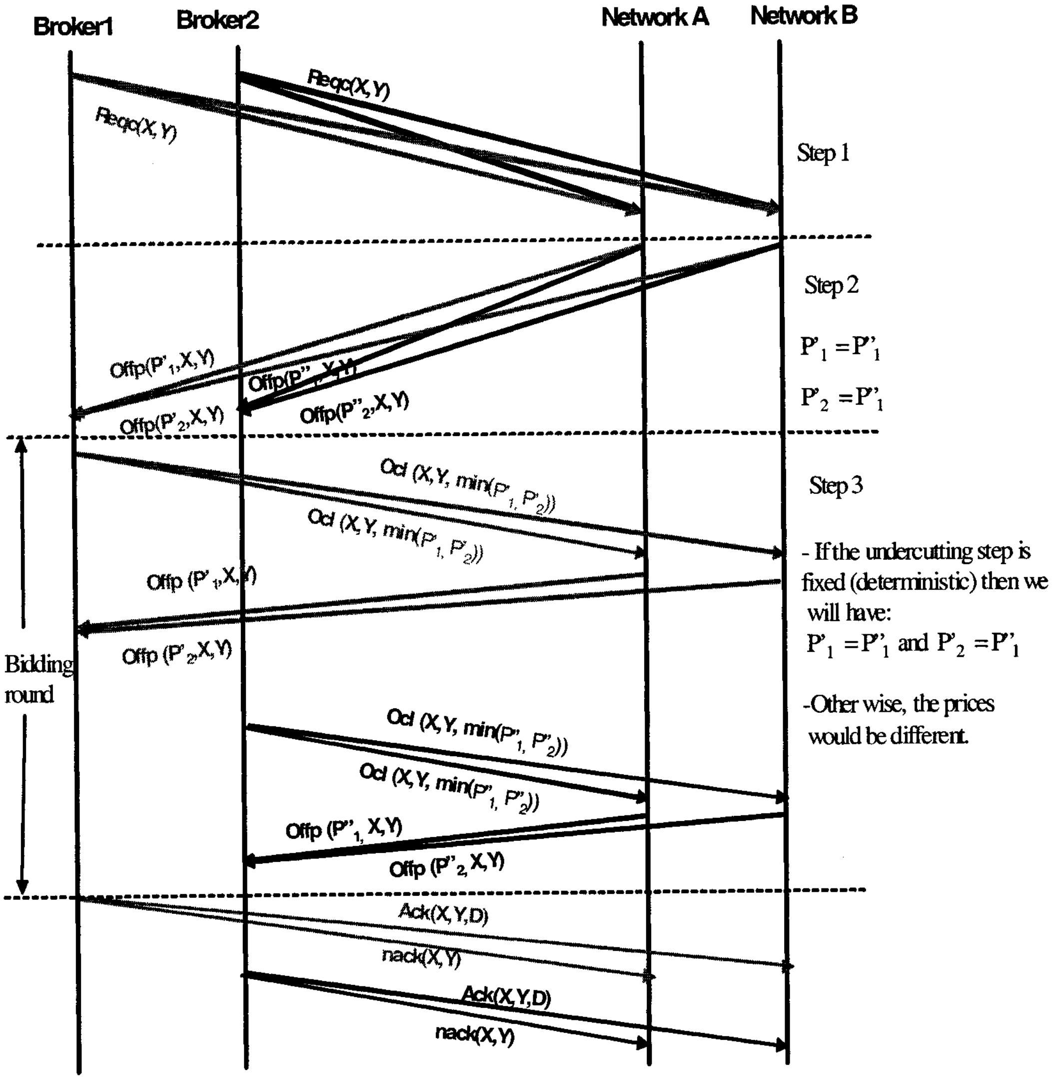


Figure 5.2b: Signaling of simultaneous connectivity requests

From this case, we conclude that although two simultaneous requests arrive at the same virtual channel and can be accommodated in the same route, they might end up with different prices per wavelength as a result of the competition. It happens based on the undercutting step, if the undercutting step is fixed and the offered prices were the same, then they will remain the same. If the undercutting step varies in a random fashion, then the prices might end up different.

5.3.2 Exception resolution

Exception 1

This exception, depicted in figure 5.3, represents the case when the capacity is not enough to accommodate the request for connection or the network does not have enough capacity to accommodate even a single wavelength, outlined as exception 1 in table 5.1. In this case, we assume the following:

(1) The "winning" network will allocate as many wavelengths as its capacity permits.

(2) Based on customer's budget and his willingness to pay, he buys as many wavelengths as his budget permits from the network that offers the second cheapest price.

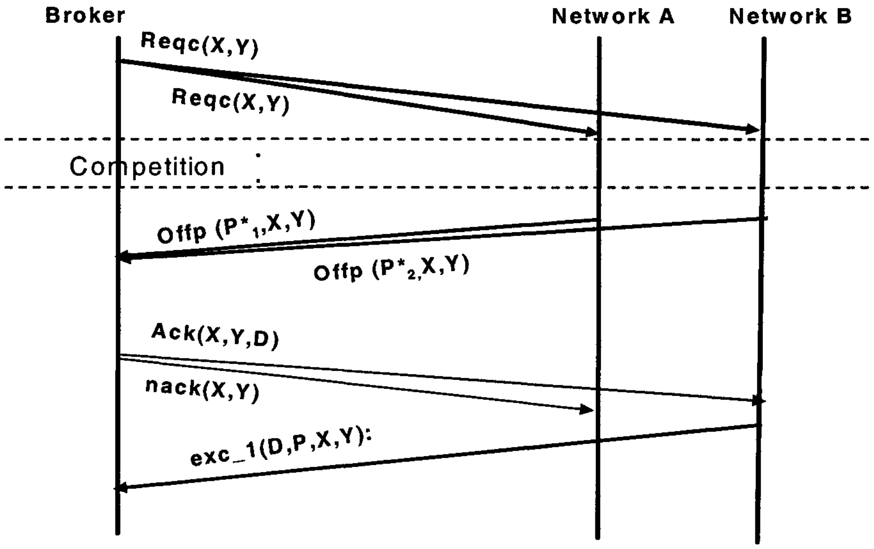


Figure 5.3: Signaling for exception 1

Exception 2

This exception, depicted in figure 5.4, represents the case when the final price of the winning network is much higher than what the customer can afford. This exception happens in the following scenarios:

1- The network management system has failed to estimate the market potential for driving a successful business in terms of offering affordable prices.

2- The cost of the network infrastructure is too high and has not been engineered economically.

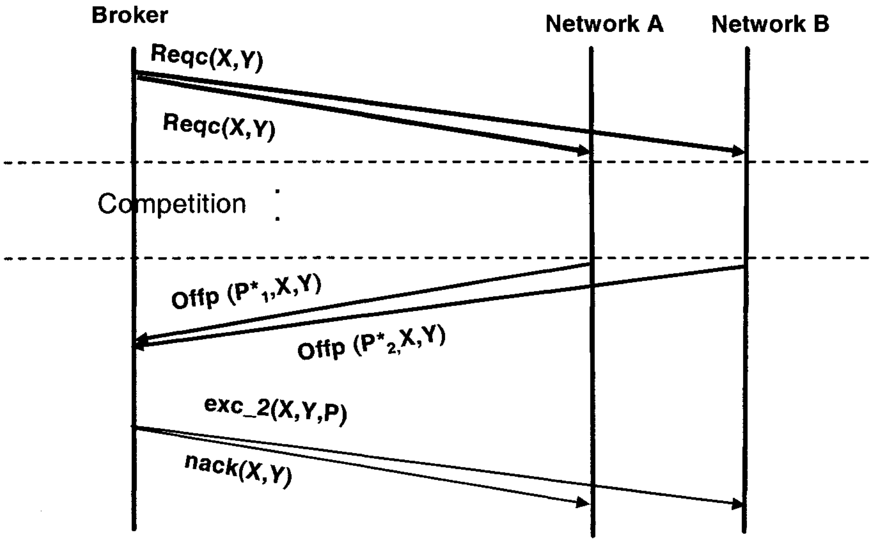


Figure 5.4: Signaling for exception 2

Exception 3:

This exception, depicted in figure 5.5, occurs when both networks end up offering the same price. The customer will choose one of the networks randomly with equal probability.

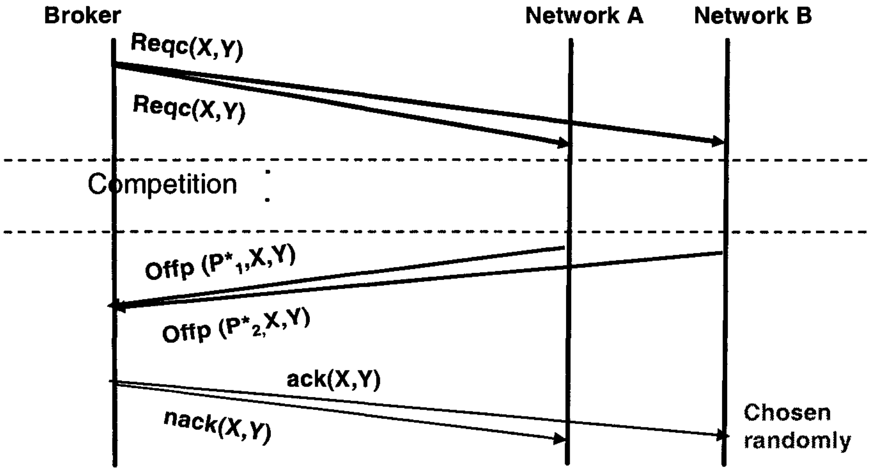


Figure 5.5: Signaling for exception 3

5.3.3 Elements of price competition in optical networks

In this section, we introduce two terms from microeconomics, which play a pivotal role in the price competition and are necessary for our analysis: the Marginal Cost and the Total Cost. Based on these two terms we can determine the competitiveness of the optical network.

- The *Total production cost (TC)* expresses the total expenses required for producing q items of a product and it is denoted as $TC(q)$ [please see chapter 2, section 2.2.6].
- The *Production Marginal Cost (MC)* is defined as the production cost per unit output [please see chapter 2, section 2.2.7]. It is defined as the change in the total production cost *(TC)* when producing an additional unit, i.e. $MC(q) = TC(q+1) - TC(q)$.

In the business case of wavelength allocation in optical networks the above terms are interpreted as follows:

- The production units are the wavelengths to be allocated.
- The unit production cost is the cost for allocating a single wavelength on a physical link. The cost per wavelength is determined by the need for wavelength regeneration in the optical switches and the optical fibers themselves. The power necessary for regeneration a wavelength can be as high as thousands of Watts [64]. This is the main reason why the total cost of using a fiber link depends on the flow (measured in number of wavelengths) over the physical link.
- The total cost is the sum of all physical link costs along the route for accommodating the requested number of wavelengths of the virtual link (point-to-point) connection.
- The marginal cost is the allocation cost of a single wavelength over a virtual link on the associated physical route of the network topology, given that the network has already

allocated a number of wavelengths in the same virtual channel. In other words, it is the cost associated with one additional unit of production (known here as unit wavelength).

5.3.4 Featuring the elements of price competition in optical networks

In order to understand and analyze the nature of price competition in optical networks we need to obtain the *total production cost (TC)* and the *marginal production cost (MC)* of wavelength allocation. In this section we will demonstrate that *TC* and *MC* for wavelength allocation of a virtual channel request are increasing in a piece-wise linear manner.

In the previous subsection *TC* was defined as the sum of all physical link costs along the route for accommodating the corresponding virtual channel (point-to-point) connection. In [48], we can find an extensive discussion and justification why the total cost is piece-wise linear. Here we provide a brief comment on how it affects our study. We present an example network (see figure 5.6) with the following assumptions:

- Customer sends a connection request for a virtual channel from Vancouver to Toronto
- We consider the following two alternative routes from Vancouver to Toronto:
    - A-B and C-D-E-F-G
- The capacities and prices of the physical links along these routes are shown in table 5.2.

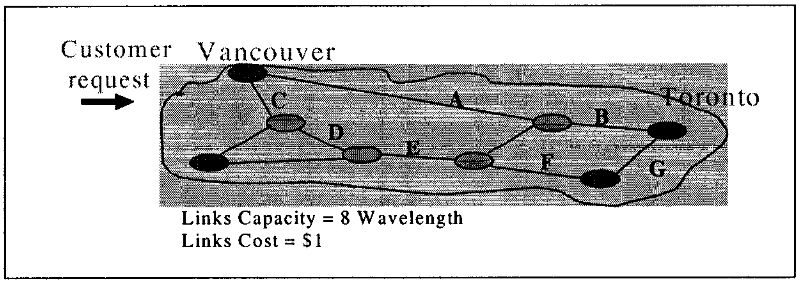


Figure 5.6: Example of an optical network

Physical Links:	Link Wavelength capacity	Cost for accommodating a wavelength ($) on this fiber
A	8	1
B	8	1
C	8	1
D	8	1
E	8	1
F	8	1
G	8	1

Table 5.2: Physical links in example network of figure 5.6

In table 5.3 we provide the total cost and the marginal cost associated with the requested number of wavelengths on virtual channel Vancouver - Toronto. The cheapest route of connection between Vancouver and Toronto is A-B. We assume that the allocation cost of an additional wavelength on a fiber is constant, as long the capacity of the fiber is not exceeded (see the definition of unit production cost in section 5.3.3). From the results, shown in table 5.3, it can be seen that for up to 8 requested wavelengths the whole demand can be accommodated on route A-B, because the capacity of the links along the route is not exceeded. For requests of 9 wavelengths and up, the additional wavelengths have to be allocated on the next cheapest route C-D-E-F-G. Requests over 17 wavelengths and more cannot be accommodated based on the fact that the capacity along these two routes is 8 wavelengths. The total cost *(TC)* and marginal cost *(MC)* are shown graphically on figure 5.7. Below are some observations based on figure 5.7:

- The total cost *(TC)* is monotonically increasing and piece-wise linear. The reason is that when the capacity of the cheapest route is exhausted, then the wavelengths are allocated on a more expensive route.
- The marginal cost *(MC)* is monotonically increasing and piece-wise constant. It is based on the fact that the allocation cost of a single wavelength along the route is constant, as

long the capacity of all links along the same route is not exceeded by the current accommodated requests (see the definition of unit production cost in Section 5.3.3).

- The marginal cost and the total cost are changing depending on the request, measured in number of wavelengths. It is very important to point out that the change in the marginal cost and the total cost affects the prices offered. This is the major difference between the price competition model we are presenting and the Bertrand duopoly price competition (chapter 2, section 2.5.1). Bertrand assumed that the marginal cost is constant and therefore the prices will reach equilibrium at this marginal cost.

Requested number of wavelengths	Routes	Total Cost ($)	Marginal Cost ($)
1	A-B: (accommodating 1 wavelength)	2	-
2	A-B: (accommodating 2 wavelengths)	4	2
3	A-B: (accommodating 3 wavelengths)	6	2
4	A-B: (accommodating 4 wavelengths)	8	2
5	A-B: (accommodating 5 wavelengths)	10	2
6	A-B: (accommodating 6 wavelengths)	12	2
7	A-B: (accommodating 7 wavelengths)	14	2
8	A-B: (accommodating 8 wavelengths)	16	2
9	A-B: (accommodating 8 wavelengths) C-D-E-F-G(accommodating 1 wavelength)	21	5
10	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating 2 wavelengths)	26	5
11	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating 3 wavelengths)	31	5
12	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating 4 wavelengths)	36	5
13	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating 5 wavelengths)	41	5
14	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating6 wavelengths)	46	5
15	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating 7 wavelengths)	51	5
16	A-B: (accommodating 8 wavelengths) C-D-E-F-G:(accommodating 8 wavelengths)	56	5
17	Can not accommodate this request, there is no capacity available	-	-

Table 5.3: TC and MC of wavelength allocation for Vancouver – Toronto virtual link

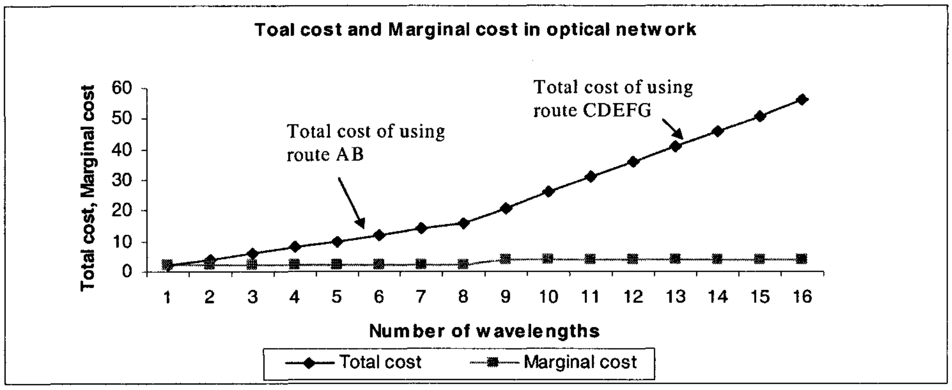


Figure 5.7: *TC* and *MC* of wavelength allocation on Vancouver – Toronto virtual link

5.3.5 Price undercutting and "winning" prices

In this section we will demonstrate different price undercutting approaches among competitors. As we have mentioned in sections 5.3, the customer decides, based on the cheapest offered price from the networks, how many wavelengths will request following a price-demand curve. The price competition in this case is determined by the marginal cost *(MC)* of unit wavelength. The *(MC)* depends on the route taken to allocate the wavelength, which entirely depends on the availability of the route when the request is submitted as we have shown in section 5.3.4. The network with lower marginal cost *(MC)* has a competitive advantage to win the competition by offering a cheaper price [26]. These results are driven entirely by the assumption of homogeneous products being offered in the market [26], which are in our case the wavelengths. Let us first present the price competition algorithm:

- The broker initiates a competition for a virtual channel (end-to-end) connection by sending the request to the competing networks.
- The competing networks respond with their initial price offers $B = (B_1, B_2, .., B_N)$, where N is the number of competing networks.

- After the initial bidding the broker submits to the networks the price of the minimum offer $P = \min (P_1, P_2, ..., P_N)$, without revealing the identity of the competitor who offered this price, and ask the competitors to undercut the current minimum price. $P_1, .. P_N$ are the prices offered by the competitors for a unit wavelength.
- The network that offered the lowest price has no incentive to undercut its own price immediately. It will let the round go in case nobody cuts lower.
- The competing network i undercuts the current bidding price P by an undercutting step value U_i as long as the corresponding marginal cost MC_i $(i=1..N)$ is not reached.
- The undercutting step value U_i defined by network i can be **deterministic** or **stochastic**. Each time the competitor undercuts the price, and the new price is $P^*_i = P - U_i$.

In subsections 5.3.5.1 and 5.3.5.2 we will explain by examples the deterministic and stochastic price competition mechanisms.

5.3.5.1 Deterministic price undercutting

Definition of deterministic price undercutting: It is a price undercutting in which the price undercut step U_i is constant for the entire duration of the competition for each network.

Deterministic price undercutting process between two competitors: In figure 5.8 we considered an example of two competing optical networks: *Network 1* and *Network 2*. The initial bids B_1 and B_2 start at a reasonable market price. Assume they start at $B = (350,150)$. The marginal MC_1 and MC_2 are assumed to be correspondingly $MC = (10, 50)$. The price undercuts U_1 and U_2 are $(U_1, U_2) = (3, 21)$. According to the competition algorithm, described in Section 5.3.5, the price undercutting develops as shown in table 5.4 below.

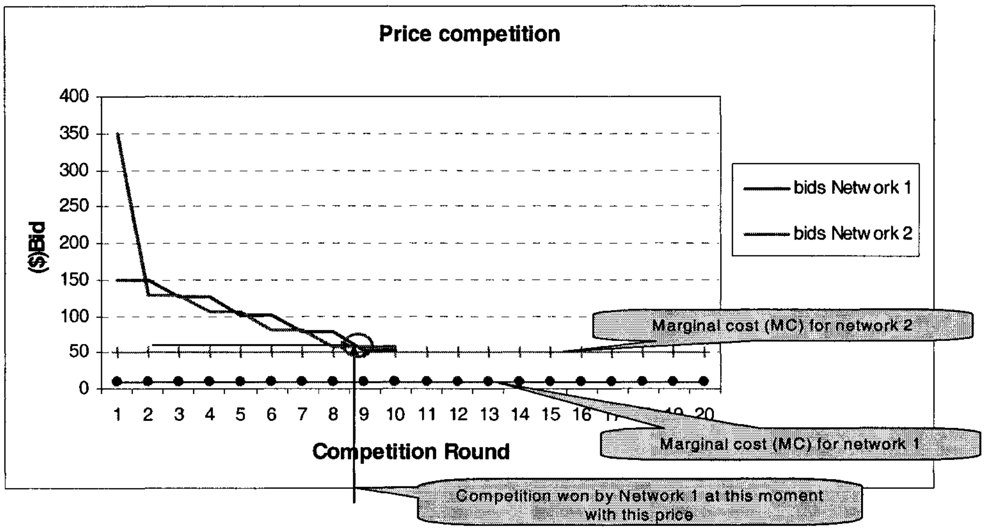


Figure 5.8: Deterministic price undercutting: 2 competitors

Round	Bid Network 1	Bid Network 2	Comment
1	150	350	Min (150$, 350$) =150$. Current bidding price is P=150$ New bid: $P_1^* = 150\$, P_2^*=P-U_2=150\$-21\$=129\$$
2	150	129	Min (150$, 129$) =129$. Current bidding price is P=129$ New bid: $P_1^*=P-U_1=129\$-3\$=126\$, P_2^*=129\$$
3	126	129	Min (126$, 129$) =126$. Current bidding price is P=126$ New bid: $P_1^* =126\$, P_2^*=P-U_2=126\$-21\$=105\$$
4	126	105	Min (126$, 105$) =105$. Current bidding price is P=105$ New bid: $P_1^*=P-U_1=105\$-3\$=102\$, P_2^*=105\$$
5	102	105	Min (102$, 105$) =102$. Current bidding price is P=102$ New bid: $P_1^*= 102\$, P_2^*=P-U_2=102\$-21\$=81\$$
6	102	81	Min (102$, 81$) =81$. Current bidding price is P=81$ New bids: $P_1^*=P-U_1=81\$-3\$=78\$, P_2^*=81\$$
7	78	81	Min (78$, 81$) =81$. Current bidding price is P=78$ New bids: $P_1^*=78\$, P_2^*=P-U_2=78\$-21\$=57\$$
8	78	57	Min (78$, 57$) =57$. Current bidding price is P=57$ New bids: $P_1^*=P-U_1=57\$-3\$= 54\$, P_2^* =57\$$
9	54	57	Min (54$, 57$) =54$. Current bidding price is P=54$ New bids: $P_1^*=54\$, P_2^* = 57\$$
10	54	57	Min (54$, 57$) =54. Current bidding price is P=54$ New bids: $P_1^*=54\$, P_2^* = 57\$$, Network 1 was the only supplier able to offer lower price than the current bidding price. Network 2 can not go below this price to a price value below its marginal cost

Table 5.4: Deterministic price undercutting for two competitors

The winning price is 54$/wavelength, and the "winner" is Network 1 after 10 rounds of bidding.

Deterministic price undercutting process among three competitors: Here we present deterministic examples for three competitors. The initial bids B_1, B_2 and B_3 and the marginal costs MC_1, MC_2 and MC_3 are chosen respectively $(B_1, B_2, B_3)=(150,200,100)$ and $(MC_1, MC_2, MC_3)=(15, 25, 55)$. The undercuts U_1, U_2 and U_3 for the deterministic cases are chosen as $(U_1, U_2, U_3)=(3, 21, 10)$.

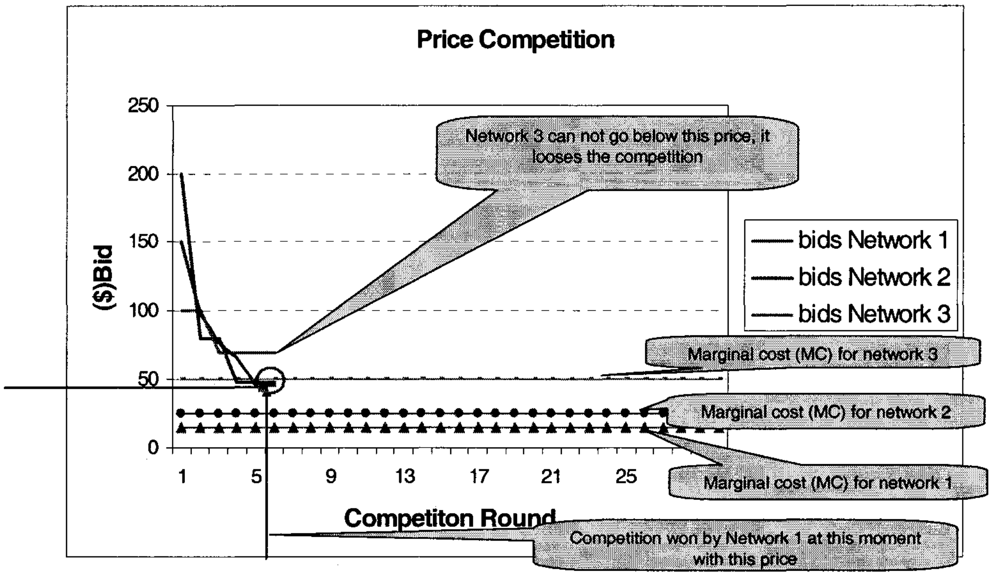


Figure 5.9: Deterministic price undercutting: 3 competitors

In the example of figure 5.9 the competition ends at a price of 45$ per unit wavelength won by network 1 after 6 rounds of bidding. The undercutting process is shown in table 5.5.

round	Bid for network			Comment
	1	2	3	
1	150	200	100	Min (150,200,100) =100. Current bidding price is P=100$ New bid P_1*= P -U_1= 97$; P_2*=P-U_2 =79$; P_3*=100$
2	97	79	100	Min (97, 79, 100) =79. Current bidding price is P=79$ New bid P_1*=P-U_1=76$; P_2*=79$; P_3*=69$
3	76	79	69	Min (76, 79, 69) =69. Current bidding price is P=69$ New bids: P_1*=P-U_1=66$; P_2*=P-U_2 =48$, P_3*=69$
4	66	48	69	Min (66, 48, 69) =48. Current bidding price is P=48$ New bid P_1*=P-U_1=45$; P_2*=48$; P_3*=69$;
5	45	48	69	Min (45, 48, 69) =45. Current bidding price is P=45$ New bid P_1*=P-U_1=45$; P_2*=48$; P_3*=69$;
6	45	48	69	network 1 was the only network able to offer lower price than the current bidding price.

Table 5.5: Deterministic price undercutting for three competitors

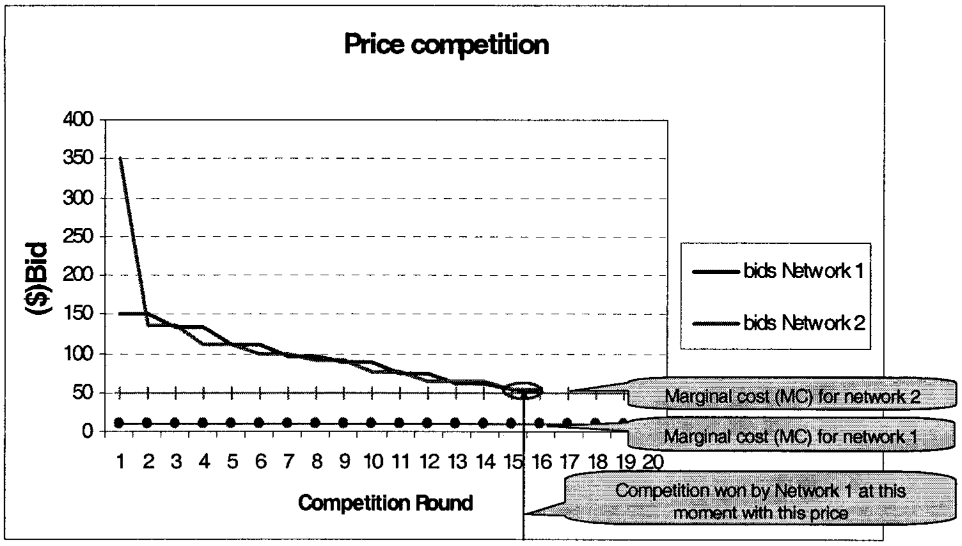
5.3.5.2 Stochastic price undercutting

In this section, we examine the case when the competitors have the choice of changing their undercutting steps during the competition in a stochastic fashion.

Definition of stochastic price undercutting: It is a price competition in which the price undercutting step U_i is chosen randomly in the interval $[L_i^{min}, L_i^{max}]$, where L_i^{min} and L_i^{max} are respectively the minimum and maximum possible undercutting steps for network i. The undercutting step U_i is a uniformly distributed random variable in the interval $[L_i^{min}, L_i^{max}]$.

Stochastic price undercutting between two competitors: In figure 5.10 we presented an example of the maximum and minimum undercutting steps, which corresponds to $L_1^{min} = 0.01$, $L_2^{min} = 0.01$ and $L_1^{max} = 3$, $L_2^{max} = 20$. The initial bids and marginal costs are identical to the deterministic example: $B = (350,150)$ and $MC = (10, 50)$. The competition ends at a price of 52.45$ per unit wavelength where Network 1 is a winner after 16 rounds of bidding.

Stochastic price undercutting among three competitors: Here we present an example of three competitors (see figure 5.11). The initial bids and marginal costs are chosen as in the deterministic example where $B = (150,200,100)$ and $MC = (15, 25, 55)$. The minimum and maximum undercutting values for the stochastic case are $L^{min} = (0.01,0.01,0.01)$ and $L^{max} = (3,21,10)$. In this example the competitors are Network 1, Network 2 and Network 3. The price competition ends at a price of 31.3 $ per unit wavelength where Network 1 is a winner after 11 rounds of bidding.

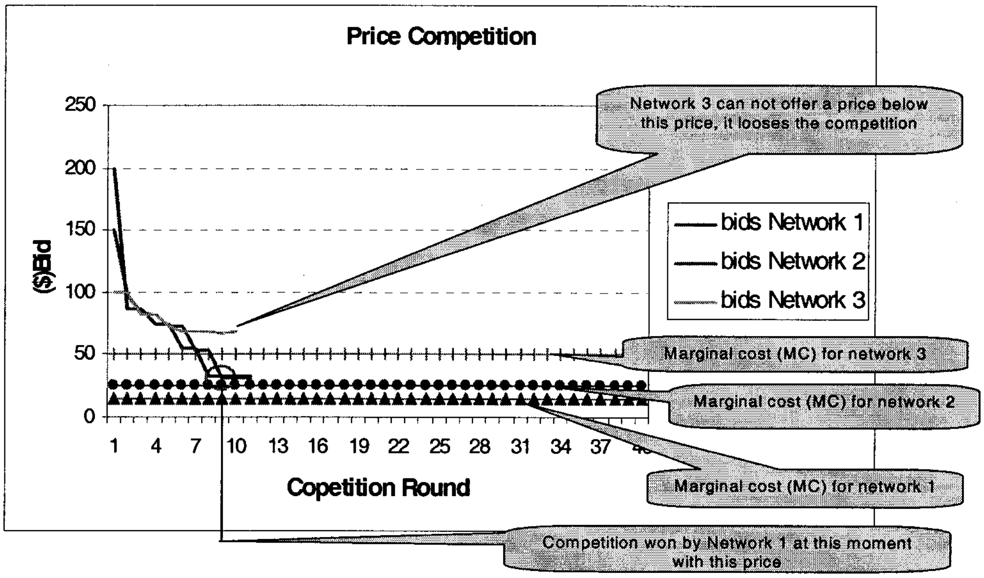


Marginal cost (MC) for network 2

Marginal cost (MC) for network 1

Competition won by Network 1 at this moment with this price

Figure 5.10: Stochastic price undercutting: 2 competitors



Network 3 can not offer a price below this price, it looses the competition

Marginal cost (MC) for network 3

Marginal cost (MC) for network 2

Marginal cost (MC) for network 1

Competition won by Network 1 at this moment with this price

Figure 5.11: Stochastic price undercutting: 3 competitors

In case of stochastic price competition of three competitors, Network 1 won the competition after 11 rounds while in deterministic case of three competitors Network 1 won the competition after 5 rounds. The reason is that Network 2 in the stochastic case has a choice of different undercutting steps which makes the network a hard competitor. Even though Network 1 won the competition, the wining price is less than the case of deterministic undercutting. The reason is that in the stochastic case the undercutting step is uniformly distributed and its corresponding average is half of the deterministic case.

5.3.5.3 Special cases of price competition

Based on what we have described in the previous related sections, the deterministic price undercutting and the stochastic price undercutting, there are two cases in which two competitors might reach a deadlock and none of them would have the incentive to lower the price. The first case is when both competitors start with the same price. For both of them this price is the current minimum price offered and therefore none of them sees the advantage to go lower. The second case can occur during the bidding process when both competitors might offer the same price. In this case, both of them have no incentive to lower the price. In deadlocks exceptions 2 and 3 might apply (see section 5.3.2).

5.3.6 Price equilibrium of homogenous product with different marginal cost

As discussed in section 5.3, the competition strategy is price undercutting and the payoff is the profit of operating the network. In this section we will present an analysis of the simplest case of competition, duopoly competition [26], and we will show that the process of price undercutting has a lower limit, which is the equilibrium price. The price of the competing networks depends on the marginal cost. The marginal cost changes according to the current state of the network as we have seen in section 5.3.4. Therefore, the *payoff* or the profit of the game also depends on the marginal cost, since the profit depends on the price. The equilibrium prices and the profit of 2 competing networks A and B will be as follows:

- if $MC_A > MC_B$ then equilibrium price of network B is $p_B = MC_A \pm e$

- if $MC_A<MC_B$ then equilibrium price of network A is $p_A= MC_B \pm e$
- if $MC_A= MC_B =MC$ then price $p_B= p_A =MC\pm e$, the user will choose Network A or Network B with equal probability.

Where e is bounded by the following inequality:

$min\ (U_A, U_B) \leq e \leq max(U_A, U_B)$ for deterministic price undercutting , and

$min\ (L_1^{min}, L_2^{min}) \leq e \leq max(L_1^{max}, L_2^{max})$ for stochastic price undercutting.

The outcome of the competition is decided by the marginal production cost *(MC)* of each competitor [27], [26]. The proof of the equilibrium can be found in the standard texts of industrial organization such as Shy (1995, 109-110) [79].

5.4 Routing and Wavelength Allocation and Marginal Cost Computation

In order to offer bids with minimal cost, a natural choice for routing in competitive optical networks appears to be the minimum cost routing [48],[51]. Therefore, the corresponding flow allocation has to be the minimum cost allocation, known also as min cost RWA. In chapter 4 section 4.4, we presented the *production cost function* wavelength allocation as follows:

$$c(y) = \sum_{k=1}^{k} \sum_{w=1}^{W} \sum_{l_{xy} \in L} p_{xy} \cdot b_{w,xy}^{k}$$

subjected to constraints expressed by equations 4.8.1, 4.8.2, 4.8.3 and 4.8.4 that are given by the network topology (please see chapter 4, section 4.4).

5.5 Business Model and Simulation results

In this section, we will describe the business model used for the simulations. In figure 5.12, we consider two service providers competing with each other to allocate the brokers' requests for end-to-end connectivity on virtual channels VC1, VC2, and VC3.

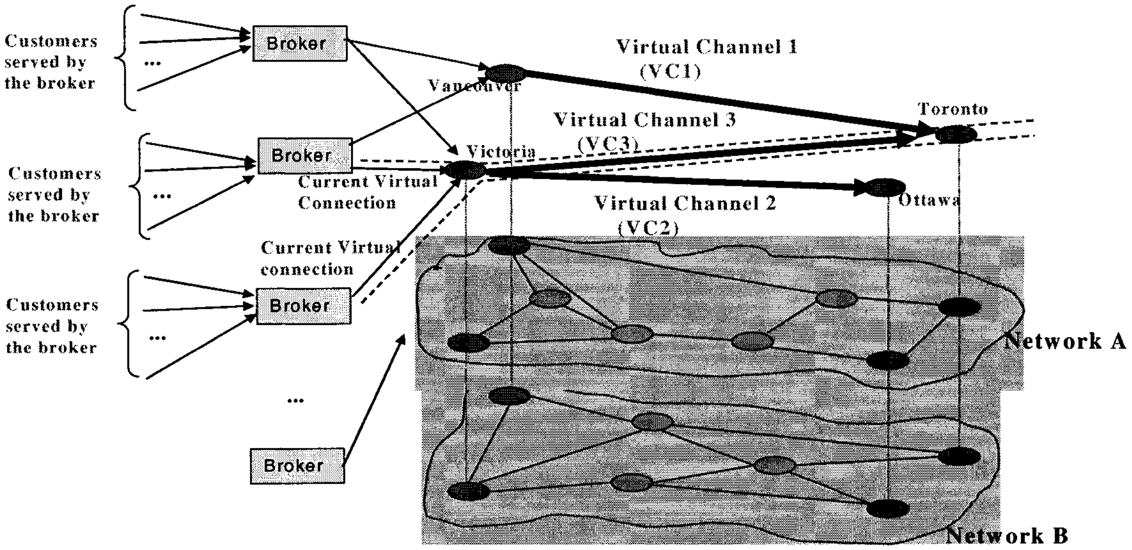


Figure 5.12: Competition business model

We make the following assumptions:

- The service provider offers virtual end-to-end connection as a service.
- The brokers can request end-to-end connection over any virtual channel.
- The broker decides the amount of wavelengths to be purchased based on his price-demand relationship.
- The brokers' requests arrivals are Poisson distributed, thus the inter arrival times are exponentially distributed. It is also assumed that the duration (holding time) of the requests is exponentially distributed. In figure 5.13, we present a case when multiple brokers' requests may overlap on the same virtual channel because of the holding time. In this case, we assume that the price offered does not change from the initial negotiated price for the duration period

of the request. In other words, the customer pays the same price for the duration of his request.

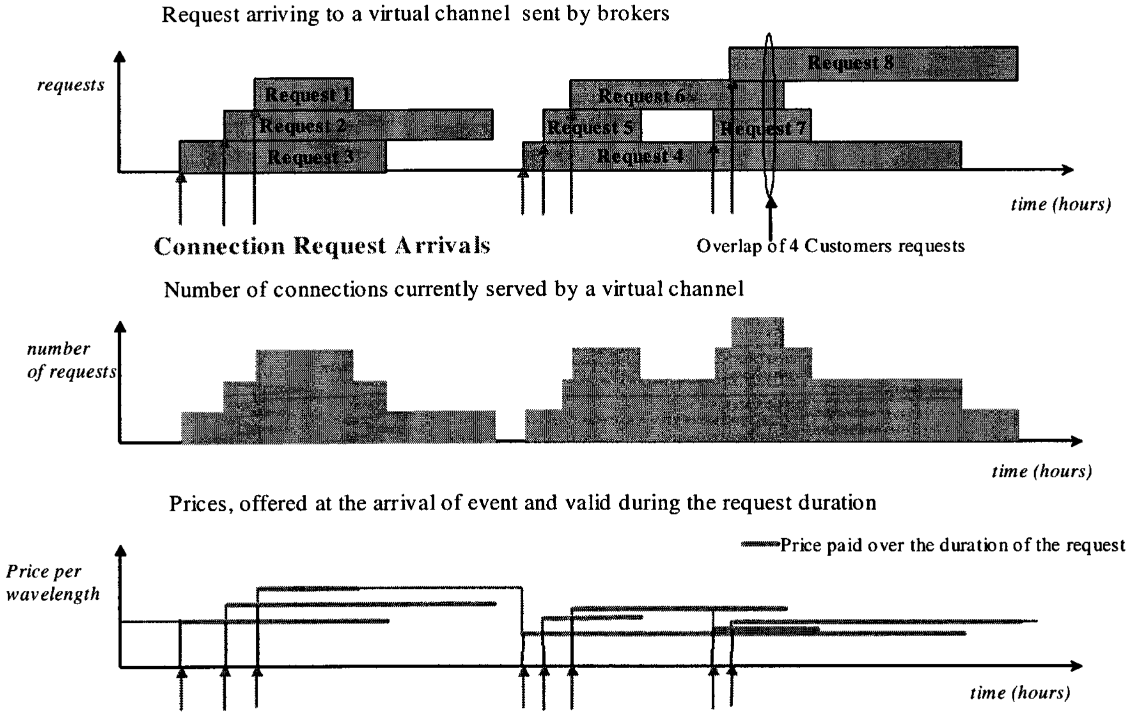


Figure 5.13: Requests overlapping on the same virtual channel

It also may occur that two or more requests arrive simultaneously in the same channel as we can see in figure 5.14. The analysis of this case is considered in section 5.3.1, figure 5.2b.

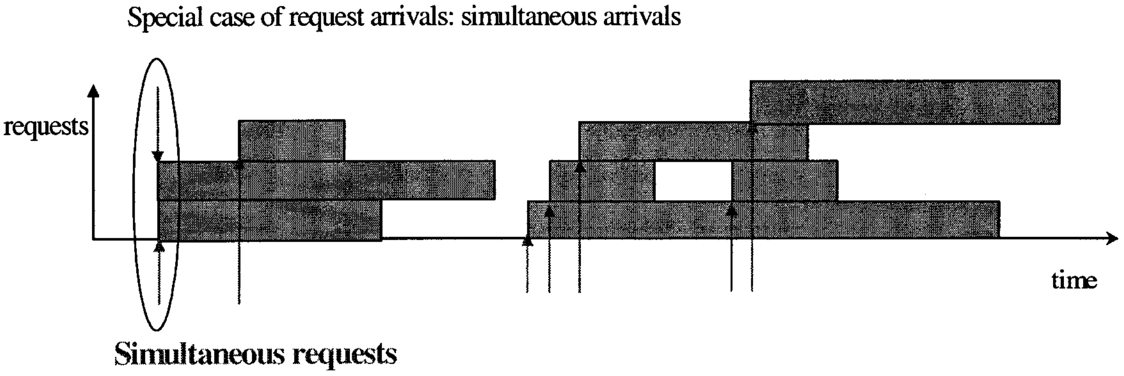


Figure 5.14: Arrival of simultaneous requests

5.5.1 Deterministic price-demand relationship and deterministic price undercutting

In this section, we consider the competition between transport service provider operating on Network A and transport service provider operating on Network B for the virtual channels VC1, VC2 and VC3 as shown on figure 5.12. We wish to point out here that the most widely used network in the open literature such as [82] and [83] is the NSF network, which has 16 nodes. Some researchers as those in [66], simulate their results using network size of 6 nodes, others in [84] use a network size of 8 nodes. In our simulations we used the AMPL modeling language software [58] with direct link to the solver CPLEX 7.0 [59], this software is limited to 300 variables, this is the reason we have only 8 nodes in the network. For this simulation, we assume that the competition and the price-demand relationships are deterministic. The undercutting steps for the competition are *U ($1, $1)*. The network parameters are assumed to be as follows: for all optical links in Networks A and Network B, the price for accommodating a wavelength is 1$ and the capacity of each link is 8 wavelengths. Furthermore, as mentioned in the above section, the arrival time of the requests for end-to-end connections is Poisson distributed. Figure 5.15 shows the number of active connections requests on each virtual channel. We also assumed that brokers have different price-demand functions, (see figure 5.16). The allocation of the wavelength requests is performed using the min-cost optimization routing, (see section 5.4). As mentioned earlier in section 5.3, our objective is to determine the profitability of the optical network when offering prices based on competition between the service providers. We need to determine the ability of the optical network to offer competitive prices to attract customer demand of wavelengths and to determine the survivability of the optical network in terms of total operational profitability. Furthermore, we need to provide a recommendation on network upgrade based on profit/loss analysis of the wavelengths sale over the virtual channels. In order to do this, we first need to determine the operational profit and the amount of wavelengths sale on virtual channels VC1, VC2, and VC3 for both networks. Then we provide the analysis based on these results.

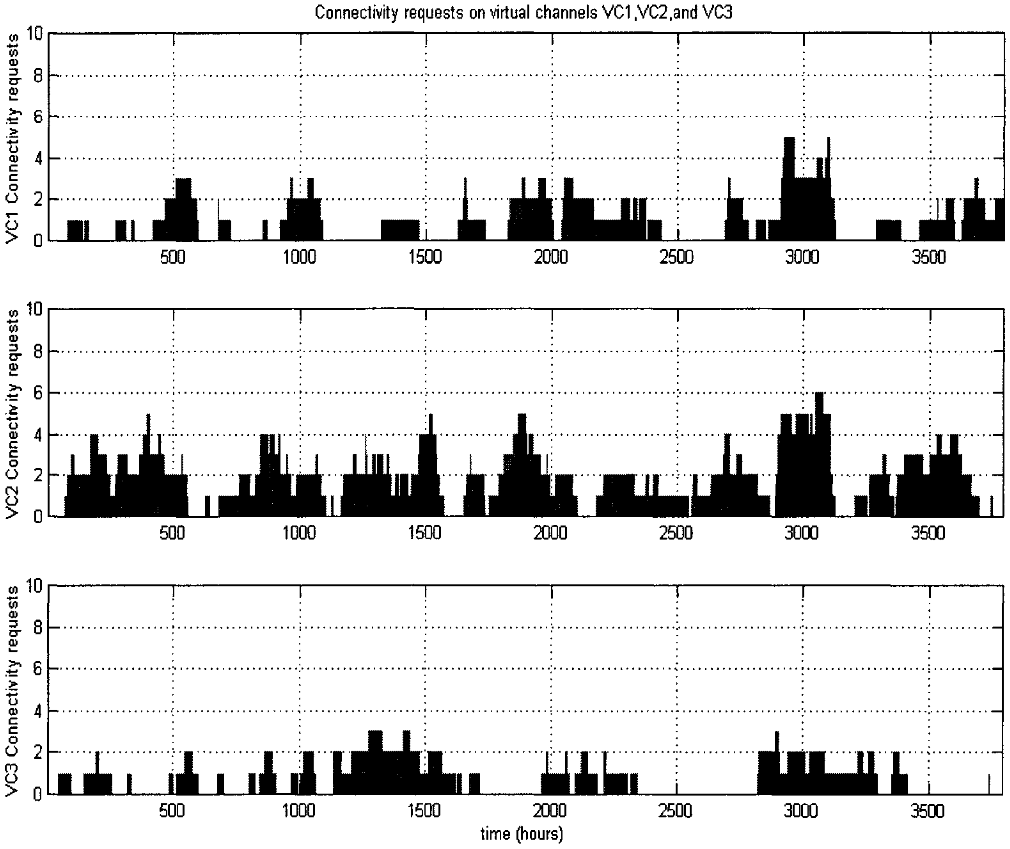


Figure 5.15: Number of connection requests on virtual channels VC1, VC2 and VC3

In figure 5.17 we show the number of active requests served on Network A and Network B. Finally, in figure 5.18 we show the resulting profit of Network A and Network B after the competition.

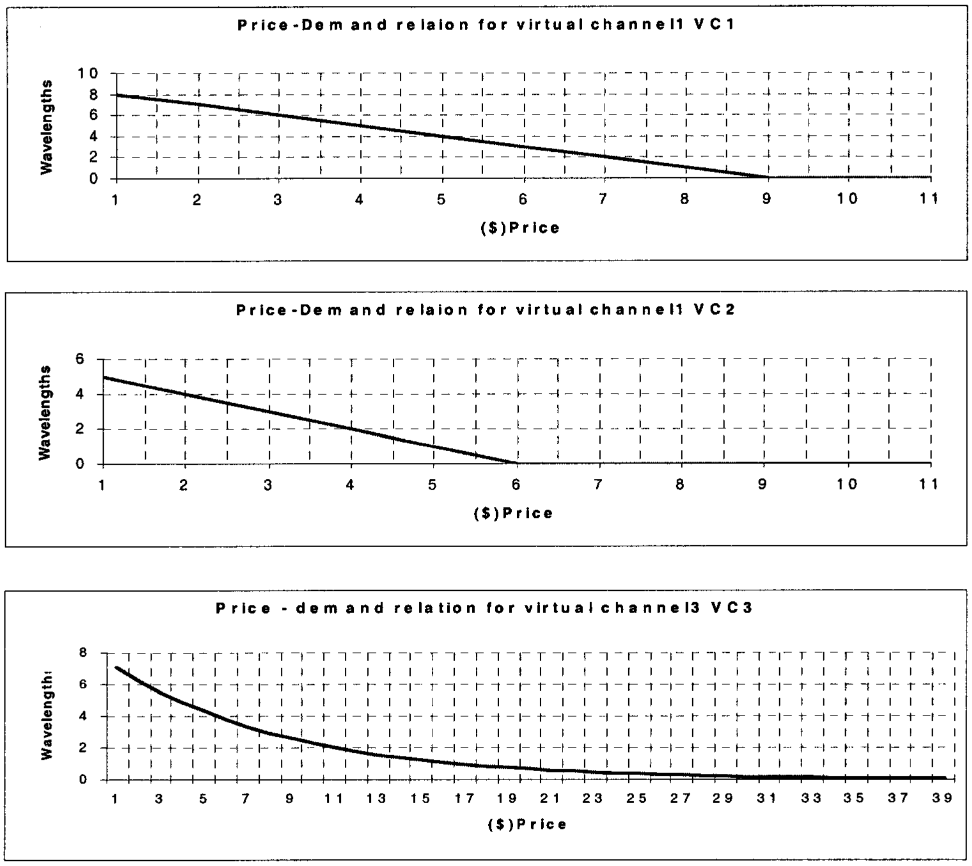


Figure 5.16: broker price-demand relationships

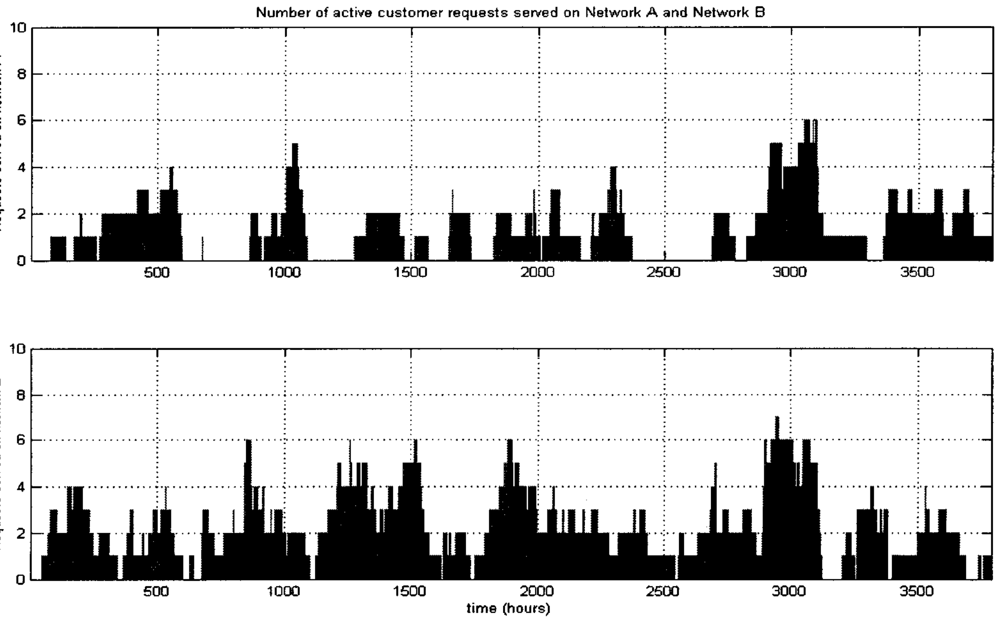


Figure 5.17: Number of active requests attracted and served by networks A and B

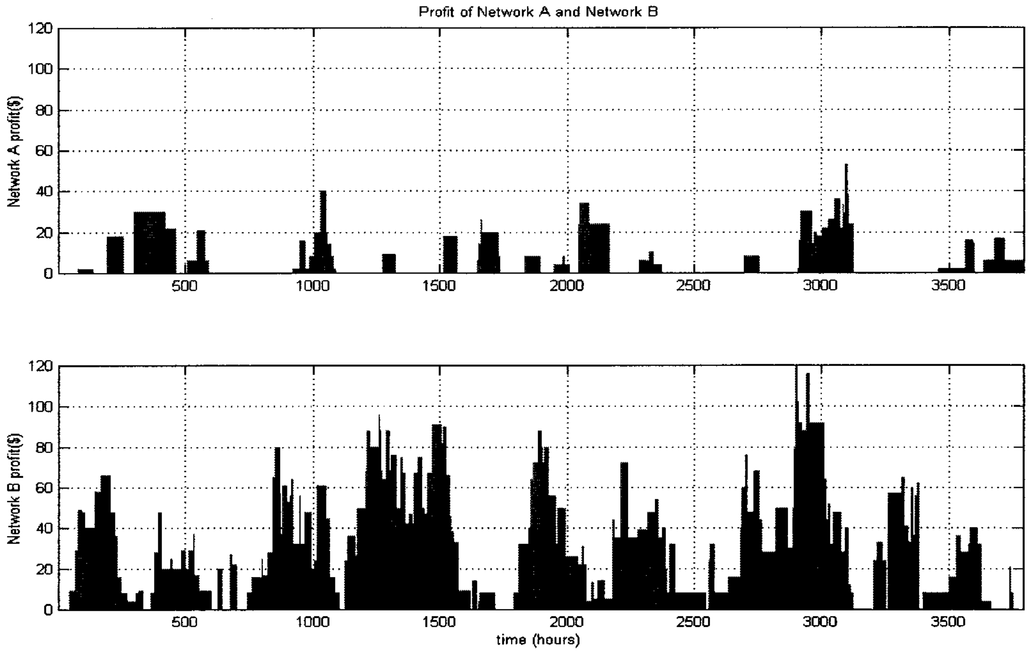


Figure 5.18: Resulting profit of operating the networks

Analysis of the simulation results

We have considered the case when the purchased wavelengths were deterministically computed based on the offered prices. The performance level of Network A in terms of profitability is much less when compared to Network B, (see figure 5.18). The average profit of network A for 50 simulation runs is $21962, while the average profit of network B is $187506. Figure 5.19 and figure 5.20, show the amount of wavelengths sold by Network A and Network B on the virtual channels VC1, VC2, and VC3. Figure 5.21 and figure 5.22, show the resulting profit of Network A and Network B from wavelengths sale. The profit from wavelengths sale as shown in figures 5.21 and 5.22 indicates that Network B was better off when competing for customer requests on virtual channel VC3.

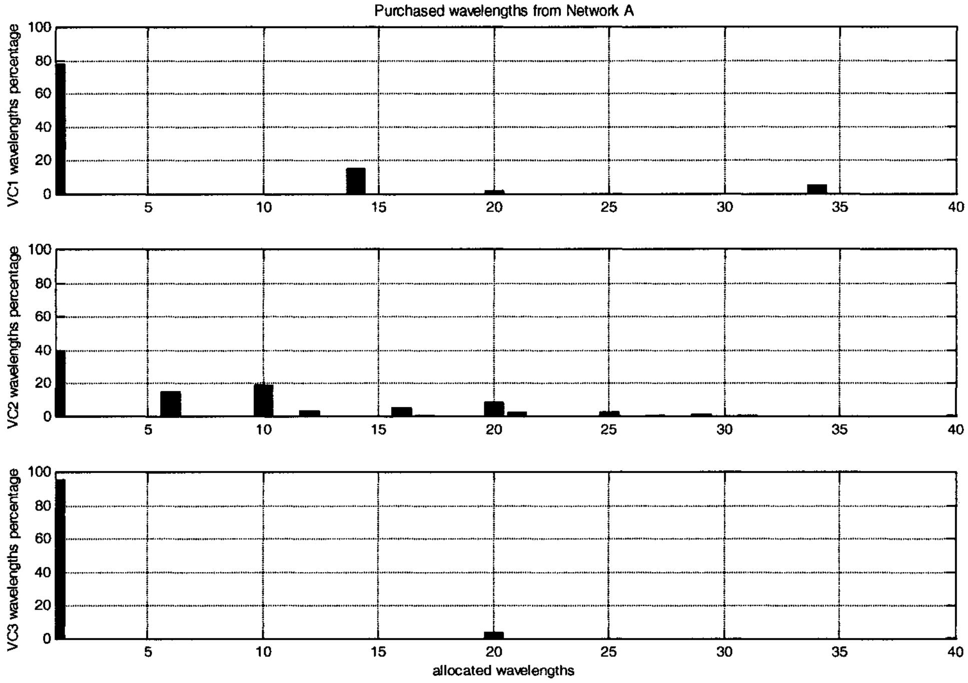


Figure 5.19: Wavelengths sold by Network A in percentage

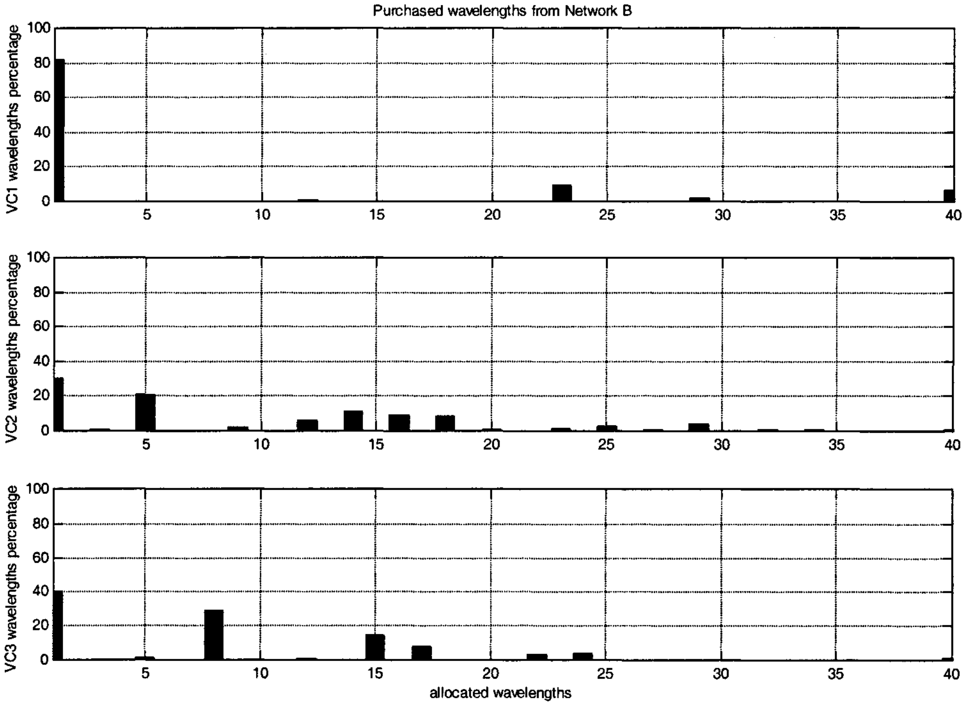


Figure 5.20: Wavelengths sold by Network B in percentage

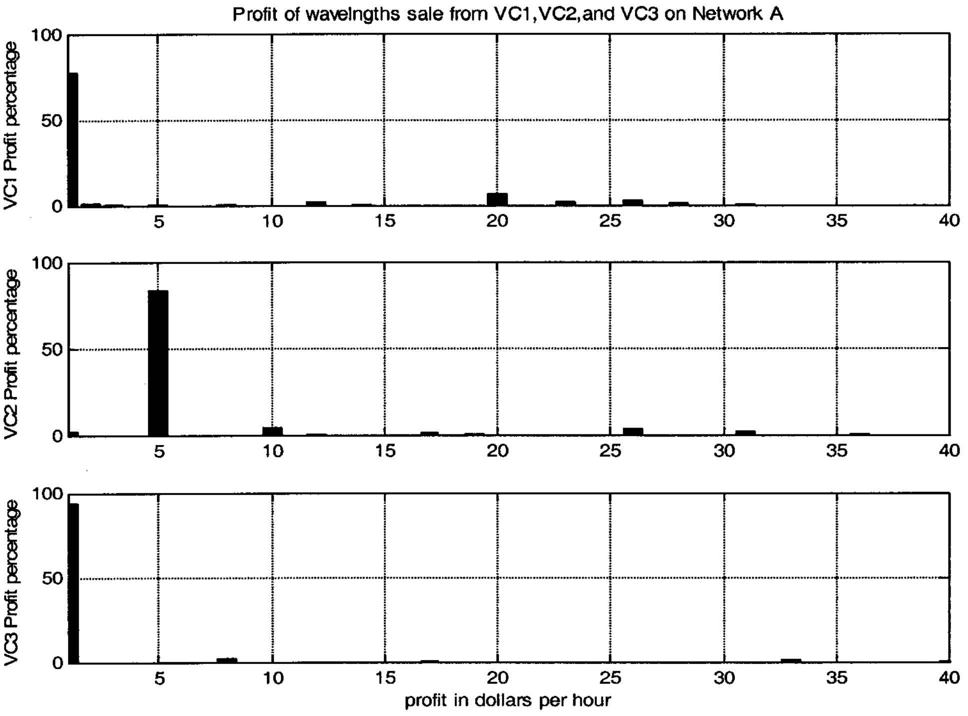


Figure 5.21: Profit percentage from wavelengths sale on Network A

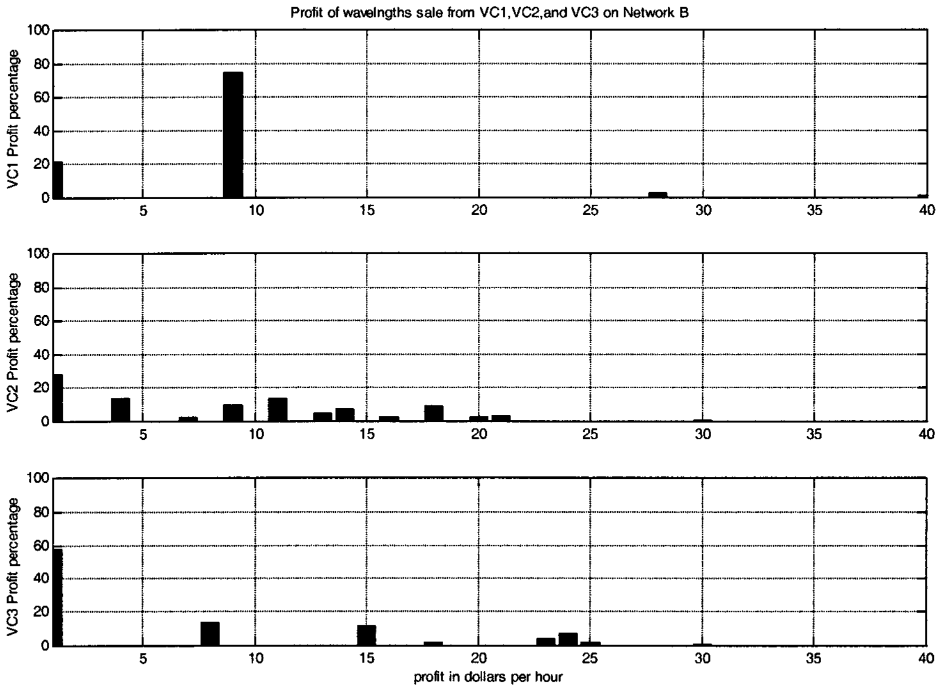


Figure 5.22: Profit percentage from wavelengths sale on Network B

In order to boost the competitive advantage of Network A, a physical connectivity upgrade along the routes that serve virtual channel VC3 needs to be considered. The decision where to place additional nodes and the corresponding appropriate links connections is a wide area of study, which is beyond the scope of this thesis. However, in this study, the profit/loss analysis can be used as a criterion, which helps network designers and network architectures in the upgrading decision of the optical network to increase its profitability. We will focus on analyzing the competition models and the resulting profit of operating the networks. In this respect, we are examining different scenarios of competition to determine the profitability of the optical networks. Next, we will compare the proposed dynamic pricing scheme with the static pricing.

<u>Comparison of dynamic and static pricing</u>

Now, let us consider the case when the most profitable network implements static pricing, which means it does not implement competitive dynamic pricing. It uses statically defined price per wavelength. Network A implements dynamic competitive pricing.

Static price network B ($)	Total Profit Network A (Dynamic pricing)	Total Profit Network B (Static pricing)
1	$0	$-13620
2	$228	$-802
3	$6020	$816
5	$14307	$945
10	$24147	$0

Table 5.6: Profit comparison when one network uses static pricing

The results in table 5.6 demonstrate that when the most profitable network uses static pricing, while the other network implements a dynamic pricing, the results are catastrophic.

If the "static pricing" network decides to offer low prices it operates with huge losses, if it decides to increase the price, then the other network can adjust its prices and offer better competitive prices. These results are interesting, because they show the sensitivity of the competitive market in terms of survivability. If one of the competitors in the market decides to compete by undercutting the prices, the rest of the players must react in order to capture customer demand of wavelength and stay in business; otherwise, the losses are unbearable.

In the next subsection, we will consider the stochastic behavior of the customer demand for wavelengths, while the price undercutting step remains deterministic.

5.5.2 Stochastic price-demand relationship and deterministic price undercutting

In this case, we assume that the competitors use undercutting steps, which are fixed (deterministic), but the price-demand relationship is stochastic. We modeled the stochastic behavior of the price-demand relationship as follows: the stochastic demand *D*(p)* for a price *p* is a Poisson random process with mean *D (p)*. Poisson random process provides non-negative values for the purchased number of wavelengths *D*(p)* for price *p*. In table 5.7, we perform 50 random process realizations to calculate the average profit of each network. We present the standard deviation and the confidence interval at 95%. In this example, the decrease in the resulting average profit was in the range of 5-8 % only. However, this example does not take into consideration the stochastic in the price undercutting step, which will be discussed in the next subsection.

Competing networks	Average Profit	Standard deviation	95% Confidence Interval	Comparison with the deterministic case
Network A	$20308	$2045	$20269-$20348	8.1 % less
Network B	$177688	$19635	$177570-$177810	5.5 % less

Table 5.7: Resulting profit when implementing stochastic price demand relationship

5.5.3 Deterministic price-demand relationship and stochastic price undercutting

In this case, we assume that the competitors implement stochastic price undercutting steps, while the price-demand relationship is deterministic. The price undercutting step U_i is chosen randomly in the interval $[L_i^{min}, L_i^{max}]$, where L_i^{min} and L_i^{max} are respectively the maximum and minimum possible undercutting steps for network *i*. In table 5.8, we perform 50 random process realizations to calculate the average profit of each network, the standard deviation and the confidence interval at 95%. We have performed the analysis for different undercutting intervals as shown in table 5.8. The performance of the networks compared to the deterministic case is lower. The reason is that in the stochastic case the undercutting step is uniformly distributed and its corresponding average is half of the deterministic case. However, this case does not take into consideration the variation on the price-demand relationship. In the next section, we examine a case when both the price-demand and the undercutting step behave randomly.

Undercutting step (uniformly distributed over the interval)	Competing networks	Average Profit	Standard deviation	95% Confidence Interval	Comparison with the deterministic case
[0.01; 1]	Network A	$19869	$2135	$19829-$19970	10.5 % less
	Network B	$176921	$20493	$176810-$177040	5.9 % less
[0.01; 2]	Network A	$19124	$2054	$19086-$19163	14.8 % less
	Network B	$174149	$20244	$174030-$174270	7.6 % less
[0.01; 3]	Network A	$18286	$1978	$18249-$18324	20 % less
	Network B	$172349	$20068	$172230-$172460	8.8 % less

Table 5.8: Resulting profit when implementing stochastic price undercutting

5.5.4 Stochastic price-demand relationship and stochastic price undercutting

In this simulation we assume that both, the price-demand relationship and the undercutting steps are stochastic. In order to examine the cross-influence of these parameters, we build the following simulation set. We run simulations for different undercutting steps and modeled the stochastic behavior of the price-demand relationship as Poisson random process. We performed 50 random process realizations to calculate the average profit of each network, the standard deviation and the confidence interval at 95%. This case is more realistic, because the undercutting steps are not fixed and the decision to buy wavelengths or not is in the hands of the customer. There is always a stochastic element which affects the customer's decision on how many wavelengths to buy. We observe that in this case, the combined effect of the undercutting and price-demand uncertainty causes a drop in the profit when increasing the interval of the undercutting step.

undercutting step (uniformly distributed over the interval)	Competing networks	Average Profit	Standard deviation	95% Confidence Interval	Comparison with the deterministic case
[0.01; 1]	Network A	$19310	$2139	$19272-$19349	13.7 % less
	Network B	$175061	$21180	$174950-$175180	7.1 % less
[0.01; 2]	Network A	$18190	$2330	$18153-$18228	20.7 % less
	Network B	$168501	$21975	$168390-$168620	11.2 % less
[0.01; 3]	Network A	$16840	$2425	$16804-$16876	30.1 % less
	Network B	$164202	$22186	$164090-$164310	14.2 % less

Table 5.9: Results of probabilistic price-demand relationship and undercutting steps

From the scenarios presented in the previous subsections, we have seen that Network A in general is performing worse than Network B in terms of profitability. As mentioned earlier, in order to improve the competitive advantage of Network A, physical connectivity upgrade along the routes serving the virtual channels where we experience low sale is needed.

In the Active price competition model, we presented different competition scenarios to analyze the networks' profitability. These analyses can be used as a criterion for network designers and network architectures to decide how and where to upgrade the network to improve its profitability.

In the next section, we present another model of competition between transport service provider operating on Network A and transport service provider operating on Network B.

5.6 Passive price competition model

In the passive price competition, we assume there is no negotiation between the broker and the suppliers (optical network service providers), except for the request of connectivity sent by the broker. The suppliers (optical network service provider), in this model, apply incremental increases/decreases in price per unit wavelength. The suppliers take their pricing decisions based on the history of "won" or "lost" customers' requests. The term "lost" of customer's request reflects the case when the customer might have walked away, either because he cannot afford the price or another service provider has won his request, but these facts remain unknown to the suppliers. As long as the observed rate of successfully allocated customers' requests increases, the supplier continues to change his price in the same direction. If the success rate of allocating customers' requests decreases, the supplier changes the direction of the price movement. Below we describe the algorithm used to increase/decrease the prices offered to the customers on the virtual channels.

We will first define the following notations:

- ***cmc***: current network marginal cost
- ***cp***: current network price
- ***tp***: current network profit
- ***coeff:*** price adjustment quotient. It determines the increase/decrease step of prices, assumed to be $0 \leq coeff \leq 1$.
- ***succ:*** success rate – the percentage of successfully allocated customers' requests by the network within a specific moving average window W.

The algorithm of price setting is as follows:

- The service provider follows the arriving customers' requests for end-to-end connections and the percentage of successfully allocated requests within a specific moving average window W.
- If the previous success rate ***succ*** (within the moving average window, see figure 5.23) is *below* the threshold ***thr1,*** (see figure 5.24), the price per unit wavelength is *decreased* as follows: if the current price ***cp*** per wavelength is higher than the current marginal cost ***cmc***, then the price is decreased with step ***(cp-cmc)*coeff***. If the current marginal cost ***cmc*** has changed, and the current price ***cp*** occurs to be lower than ***cmc***, then the current price is set to ***cmc***.
- If the previous success rate ***succ*** is *over* the threshold ***thr2,*** (see figure 5.24), the price per unit wavelength is *increased* as follows: if the current price ***cp*** per wavelength is higher than the current marginal cost ***cmc***, then the price is increased with step ***(cp-cmc)*coeff***. If the current marginal cost ***cmc*** has changed, and the current price ***cp*** occurs to be lower than ***cmc***, then the current price is equal to ***cmc***.

- If the success rate ***succ*** is <u>between</u> thresholds ***thr1*** and ***thr2,*** then the price per unit wavelength remains <u>*unchanged*</u>.
- The broker decides the number of wavelengths to be purchased based on his price-demand relationship (please see figure 5.16).

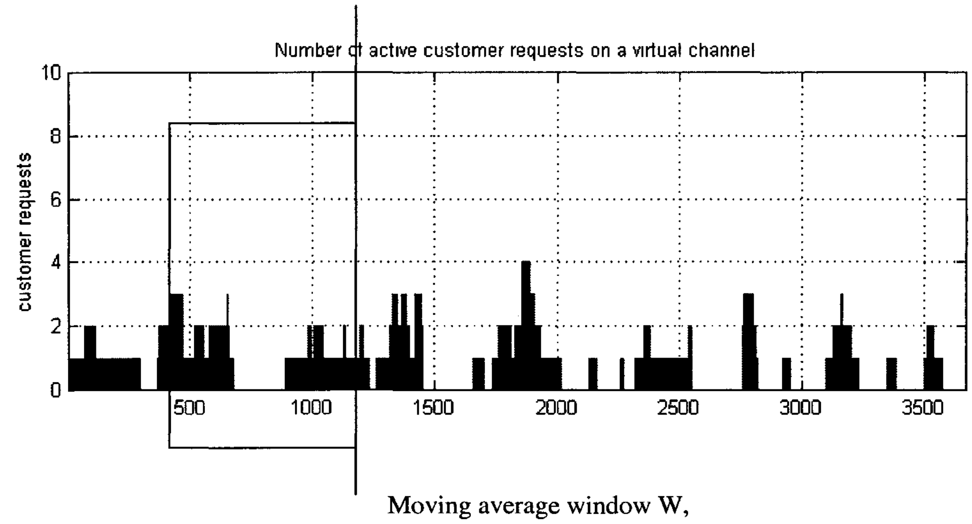


Figure 5.23: Passive competition – moving average window

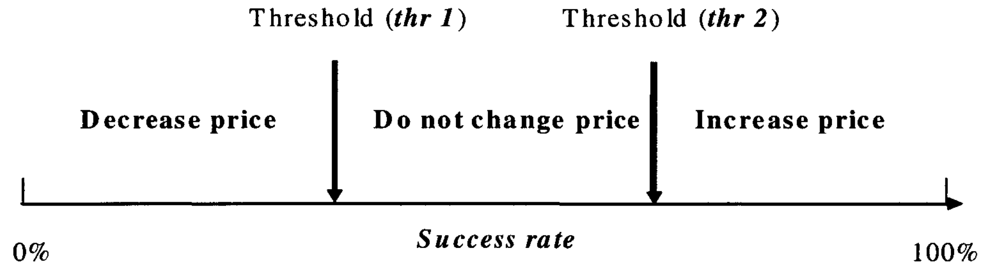


Figure 5.24: Threshold of price change

In figure 5.25, we are showing a signaling protocol describing the algorithm we have explained above. In this protocol, the broker, representing many customers, sends a request for end- to- end connection to the service providers.

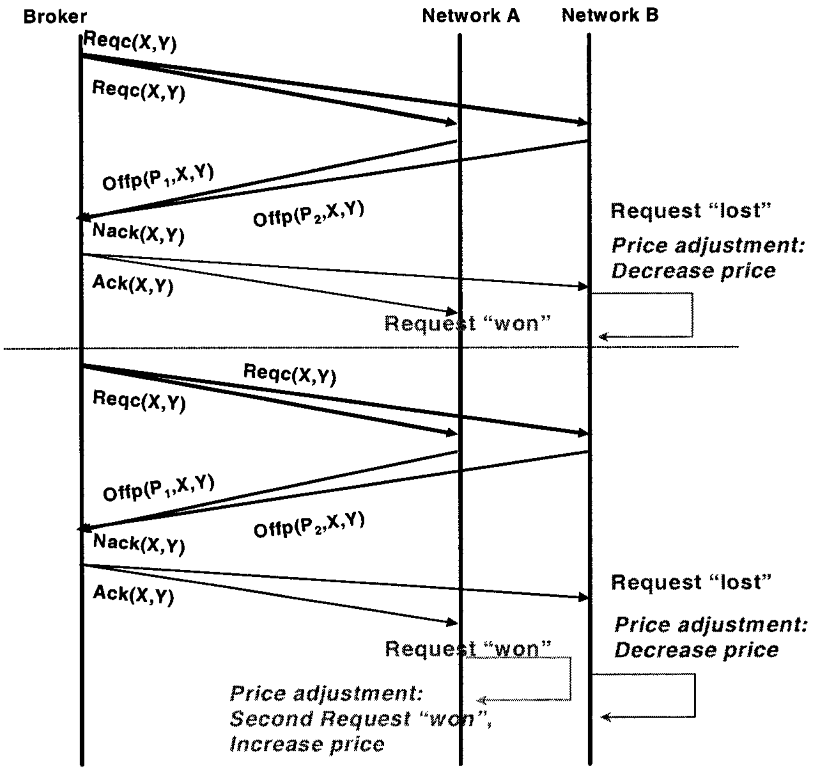


Figure 5.25: Signaling protocol of passive price competition

The service providers reply with their price offerings, the customer might walk away either he can not afford the price (based on his budget) or another service provider has won his request. These facts are unknown to the service providers. The suppliers perform their pricing decisions depending on the history of "won" or "lost" connection requests, for a specified moving average window.

Direction	Message	Comment
Customer to supplier (Broker to service provider)	***ReqC(X,Y)***	Request for a price send to the suppliers for a connection between nodes X and Y
supplier to customer (service provider to Broker)	***Offp(X,Y,P)***	Offered price per unit wavelength from the supplier
Customer to supplier (Broker to service provider)	***Ack(X,Y)***	Accepting the offered price for connection between nodes X and Y
Customer to supplier (Broker to service provider)	***Nack(X,Y)***	Rejecting the offered price for connection between nodes X and Y

Table 5.10: Messages between the customer and the suppliers

In table 5.10 above, we present the messages exchange between the brokers and the service providers.

In the next sections, we consider and present simulations for the following cases:

Case 1: Passive competition on one virtual channel with constant marginal cost. This is an ideal case when the fiber links have infinite capacity and can accommodate any request of the virtual connection along the same route. We present this case to demonstrate that the prices reach equilibrium when the marginal cost is constant.

Case 2: Passive competition on a single virtual channel with dynamic marginal cost. This case represents a scenario when the capacity of the fibers is limited and the marginal costs are changing depending on the route taken to accommodate the connection request. The prices vary depending on the marginal cost.

Case 3: Passive competition on 3 virtual channels with dynamic marginal cost and different moving average window sizes. This case represents a scenario in which the optical networks face competition. In this case, we show the prices offered to the customers, the profit of operating the networks and the resulting profit of the networks when implementing different moving average window sizes.

Case 4: Comparison between static price competition and dynamic price competition. Static price competition happens when one of the networks implements fixed prices, which means it does not implement competitive dynamic pricing or depend on historical information to increase or decrease the prices, but instead uses statically defined price per wavelength on all virtual connections.

5.7 Simulation results

In this section, we consider the competition between a transport service provider operating on Network A and a transport service provider operating on Network B as described in figure 5.12.

5.7.1 Passive competition on one virtual link with constant marginal cost

As mentioned in section 5.6, this represents a good approximation for the case where the links have very large capacity (ideally infinite capacity) and can accommodate any request of the virtual connection over the same route. Our aim is to demonstrate the existence of price equilibrium and how it is reached. Assume that the price adjustment quotient is 1 and the moving average window is 3. The thresholds *thr1* and *thr2* are assumed to be 0.4 and 0.6 respectively. The initial prices are set by the suppliers and assumed to be reasonable market prices. According to the algorithm described in section 5.6, we performed our simulation. As shown in figure 5.26, the arrival time of the incoming customers' requests is Poisson distributed. From figure 5.26 we can observe that the winning prices reach equilibrium as follows: for network A the wining prices equals to: $MC_B \pm e$, where MC_B is the marginal cost of Network B and e is bounded by *(cp-cmc)*coeff.* For network B the wining prices equals to: $MC_A \pm e$, where MC_A is the marginal cost of Network A and e is bounded by *(cp-cmc)*coeff.*

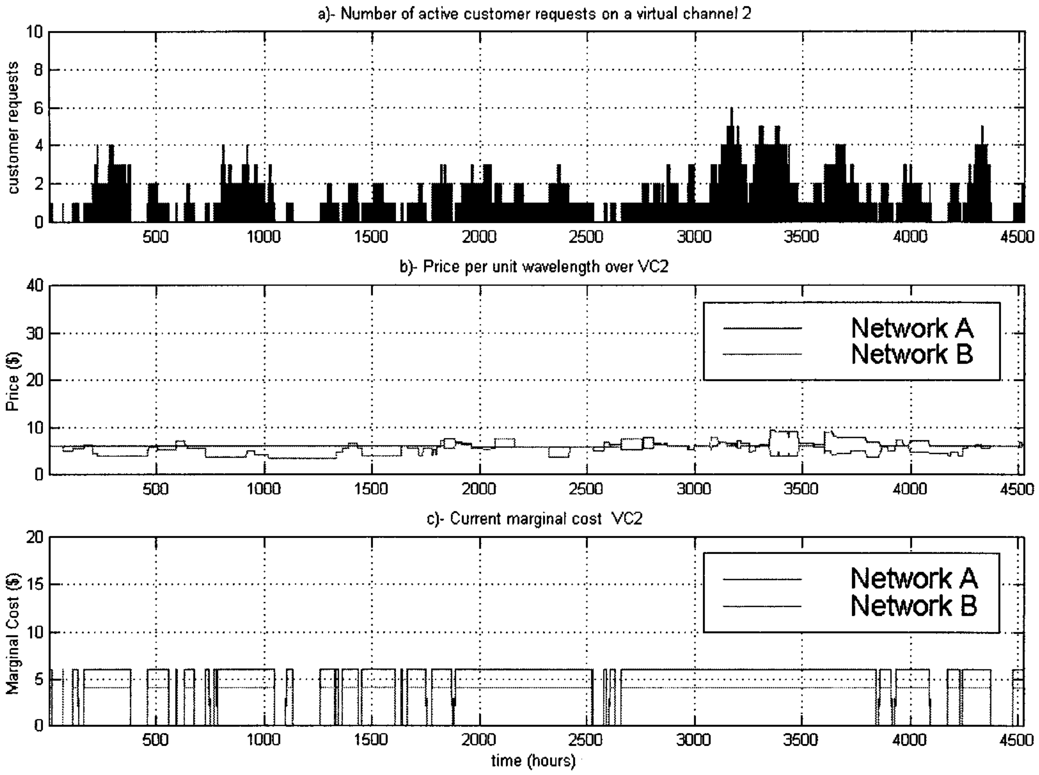


Figure 5.26: (a) Active requests (b) Prices offered (c) Marginal Cost

5.7.2 Passive competition on one virtual link with dynamic marginal cost

This case represents the scenario where the capacity of the fibers is limited and the marginal costs change depending on the route taken to accommodate the connection request. The prices vary depending on the marginal cost of both networks.. The price adjustment quotient is equal to 1 and the moving average window equals 3. The user behavior is described by the price-demand relationship (please see figure 5.16). In figure 5.27, we show the incoming customer requests, which as indicated earlier follow a Poisson distribution, the marginal cost of both networks, and the winning prices offered.

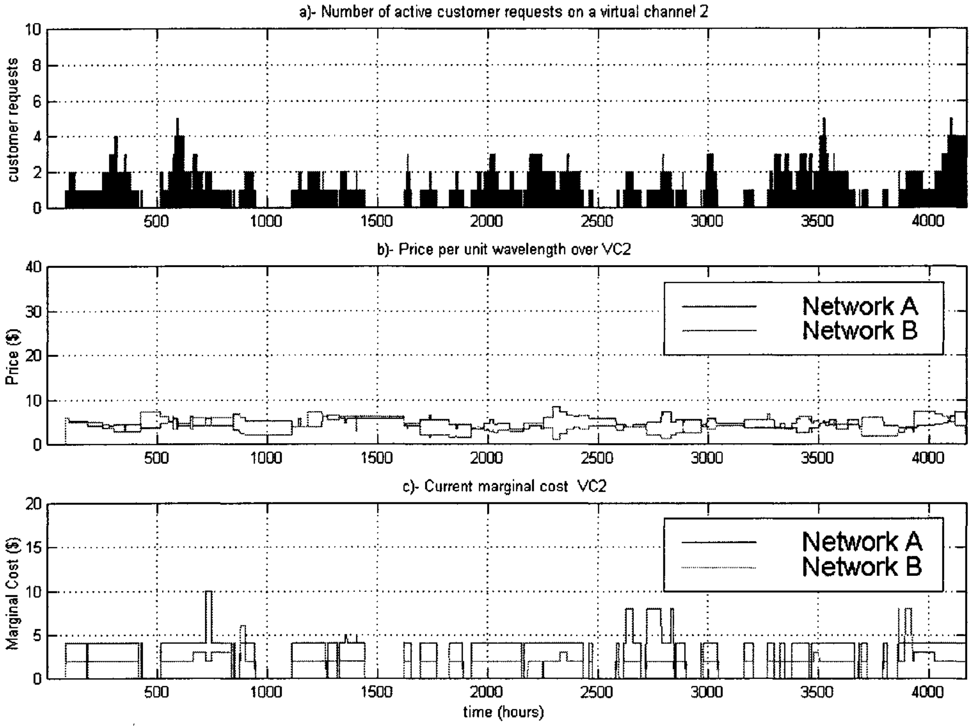


Figure 5.27: (a) Active requests (b) Prices offered (c) Marginal Cost

From figure 5.27, we can observe that the prices vary depending on the marginal cost. The prices are developing as follows: For Network A the wining price is equal to: $MC_B \pm e$, and for Network B the wining price is equal to: $MC_A \pm e$. The value of e depends on the current marginal cost and the value of the price adjustment quotient.

5.7.3 Passive competition on virtual channels VC1, VC2, and VC3 with dynamic marginal cost

In this case, we consider the competition between the transport service providers operating Networks A and B for the virtual connections VC1, VC2 and VC3 as shown on figure 5.12, section 5.5. The allocation of the wavelengths requests is performed using the min-cost optimization routing, as described in Section 5.4.

The competition in this scenario depends on the marginal cost of the network and the prices offered from its rival. As mentioned earlier in section 5.3, our objective is to determine the ability of the optical network to offer competitive prices to attract customer demand and also examine the survivability of the optical network in terms of profitability. Furthermore, we need to provide a recommendation on network upgrade based on profit/loss analysis of the wavelengths sale over the virtual channels. In order to do this, we first need to determine the operational profit and the amount of wavelengths sale on virtual channels VC1, VC2, and VC3 on both networks. Based on this, we have performed simulations for average moving windows equal to 1, 3, 6, 9, 12, 24, 34, and 50. The price adjustment quotient is chosen to be 1. The thresholds *thr1* and *thr2* are chosen to be 0.4 and 0.6 respectively. The influence of these parameters will be discussed later in the chapter. In figure 5.28, we show one possible realization of the incoming request for connection over virtual channels VC1, VC2, and VC3. In figures 5.29, 5.30, and 5.31, we show the prices offered and the profit of the wavelengths sale over virtual connections VC1, VC2, and VC3 for both networks in time. In figure 5.32, we show the profit percentage from wavelengths sale on the virtual connections VC1, VC2, and VC3 for Network A. In figure 5.33, we show the profit percentage from virtual connections VC1, VC2, and VC3 for Network B.

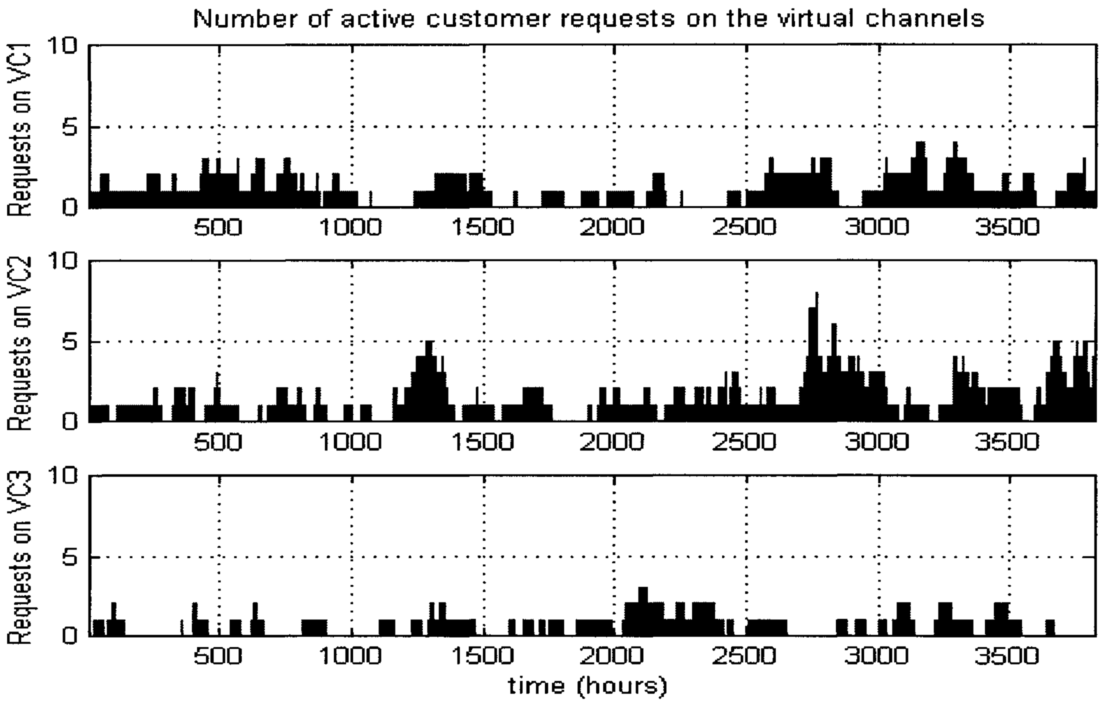


Figure 5.28: Incoming requests on virtual channels VC1, VC2 and VC3

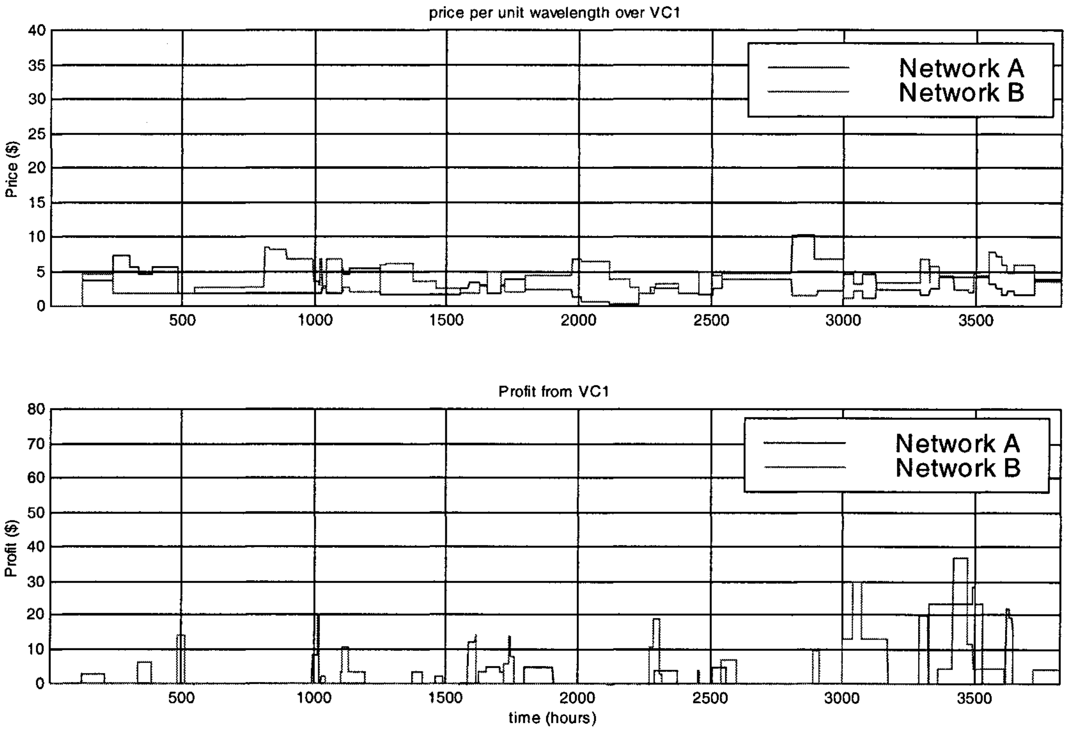


Figure 5.29: Prices offered for requests on VC 1 and the resulting profit

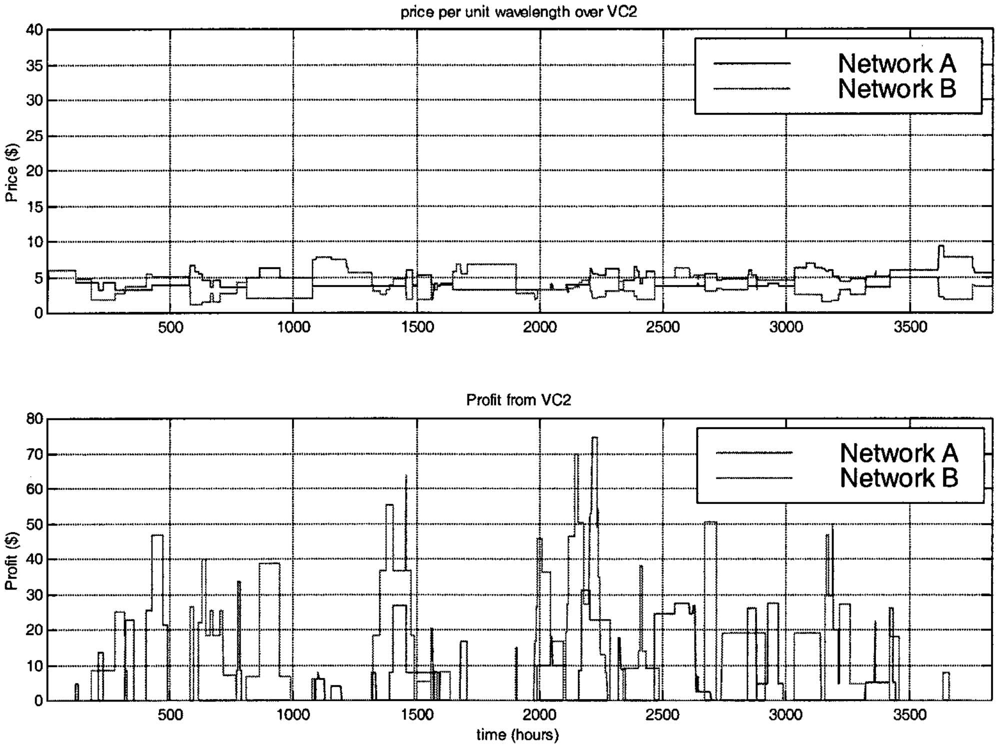


Figure 5.30: Prices offered for requests on VC 2 and the resulting profit

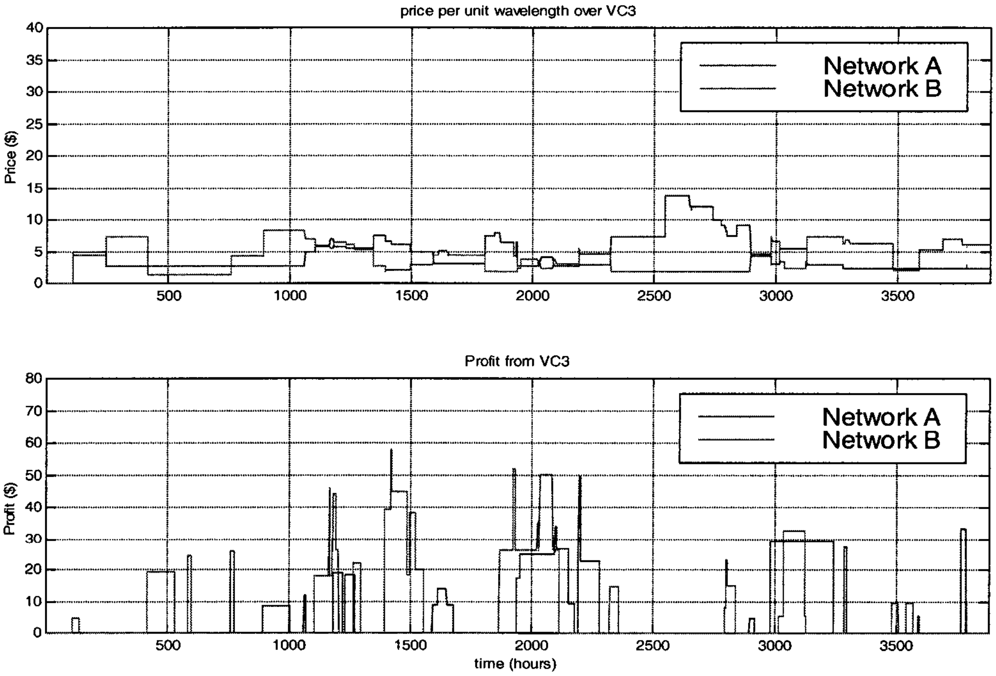


Figure 5.31: Prices offered for requests on VC 3 and the resulting profit

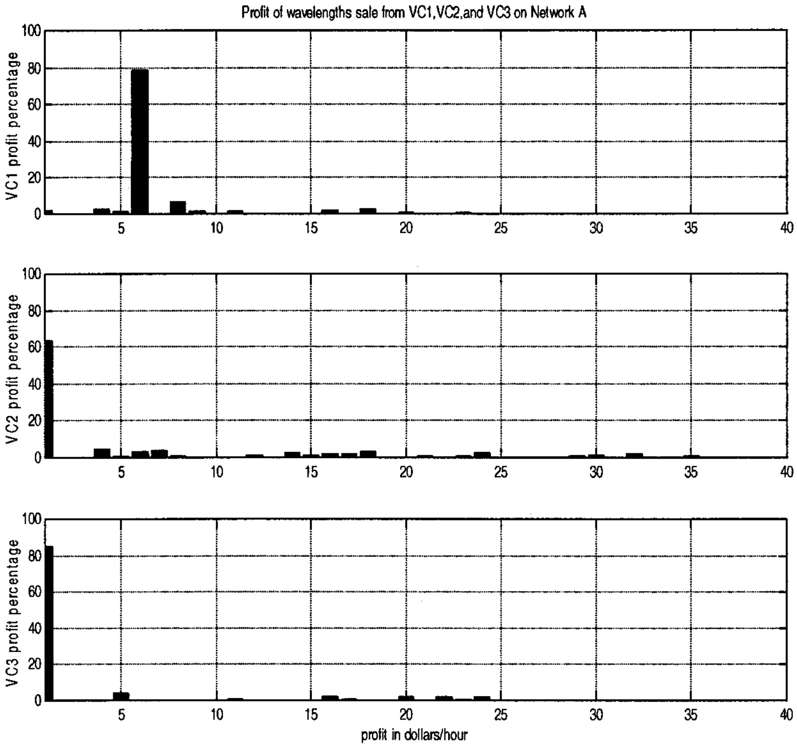


Figure 5.32: Profit percentage from wavelengths sale on Network A

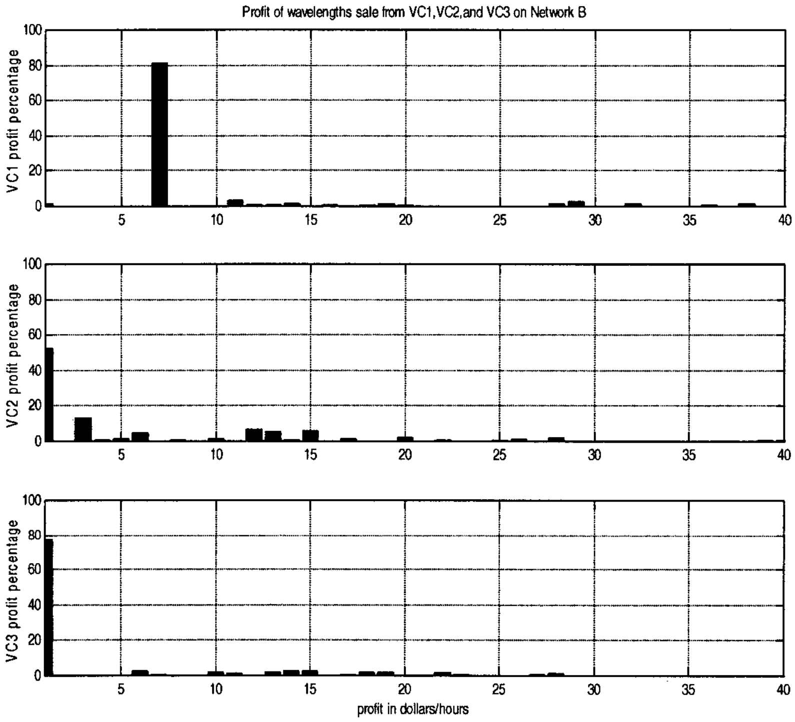


Figure 5.33: Profit percentage from wavelengths sale on Network B

Analysis of the simulation results

We have considered the case when the pricing decision of the two networks depends on a moving average window W, which measures the rate of successfully allocated requests for connection. The average profit of network A for 50 simulation runs is $56375, while the average profit of network B is $209883. The performance of Network A in terms of profitability is lower than Network B. The profit from wavelengths sale as shown in figure 5.32 indicates that Network B was better off when competing for customer requests on virtual channel VC3. In this simulation, we only considered a moving average window size equals to 3. In figure 5.34, we show the average profit of Network A and Network B for moving average windows equal to 1, 3, 6, 9, 12, 24, 34, and 50. From this figure we can see that the performance of Network A in terms of profitability is much lower from its rival Network B in all cases. To determine the weaknesses on Network A, we computed the average profit of wavelengths sale from virtual channels on both networks as shown in figure 5.35. From figure 5.35, we can see that VC3 of network A is performing the worst, which clearly indicates that physical connectivity upgrade needed to be considered in the routes serving this virtual channel.

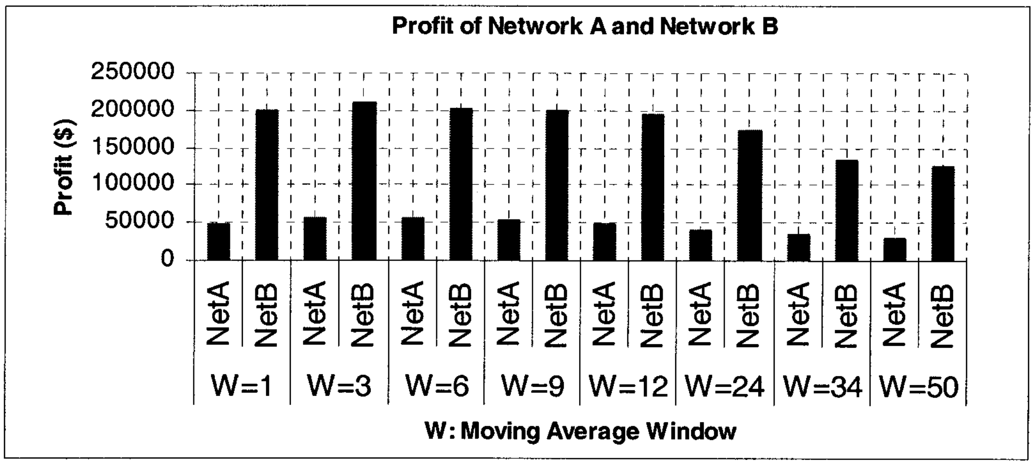


Figure 5.34: Profit of Network A and Network B for average windows

We also observe that increasing the moving average window results in profit decrease. This can be explained by the fact that the higher the size of the moving average window, the slower the network to react with the changes in the marginal cost, which we refer to as the state of the network.

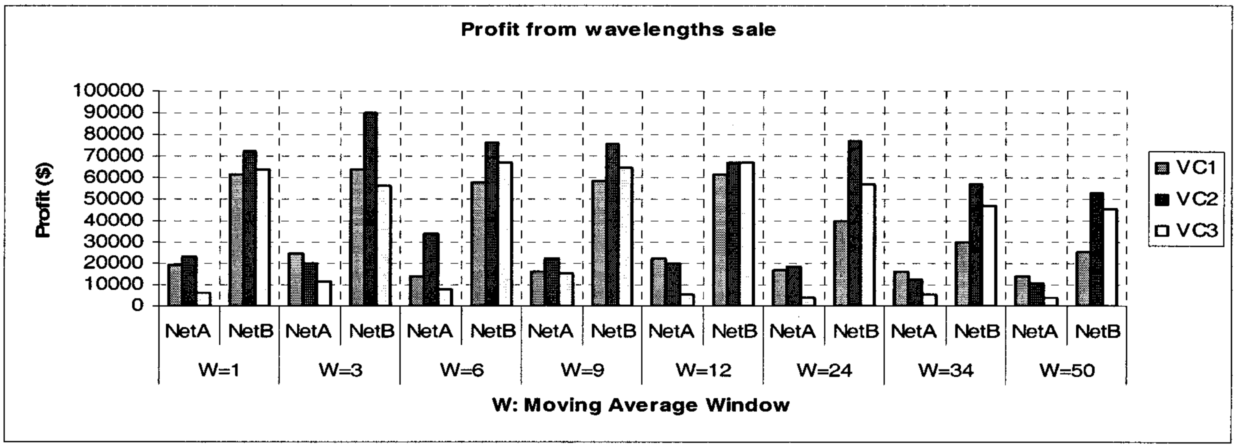


Figure 5.35: Profit of wavelengths sale on VC1, VC2, and VC3

In figure 5.36, we consider the influence of the price adjustment quotient on the network profitability. Assume that the moving average window is equal to 3 and the thresholds *thr1* and *thr2* are 0.4 and 0.6 respectively.

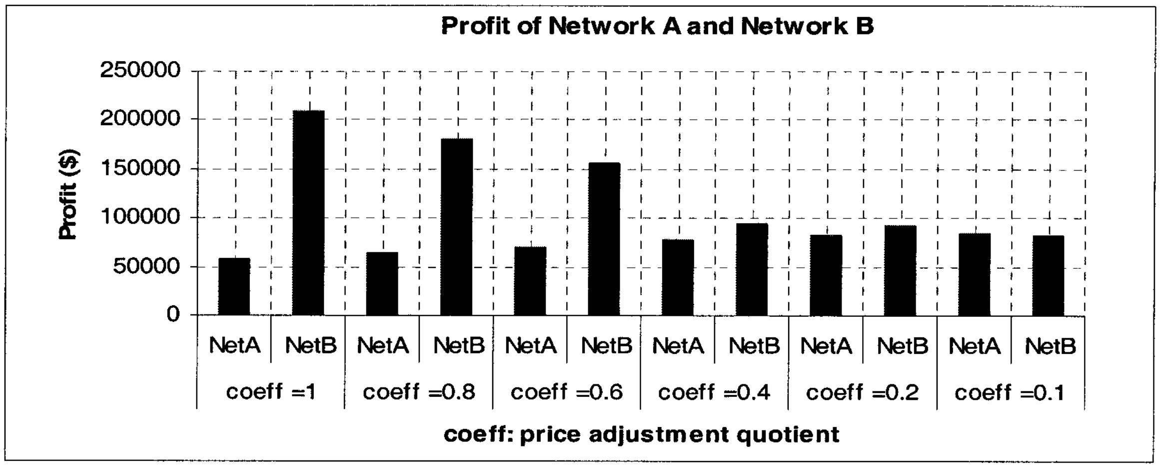


Figure 5.36: Profit of wavelengths sale for different price adjustment quotients

We observe that when the price adjustment quotient is very small, the profit of the two networks becomes less different, because the rate of price increase/decrease becomes very slow.

In table 5.11, we examine the case when on of the networks uses a small price adjustment quotient, while the other network uses a high price adjustment quotient.

Network A price adjustment *coeff*	Network B price adjustment *coeff*	Network A Profit	Network B Profit
1	0.1	$74,768	$56,098
0.1	1	$34,758	$81,967

Table 5.11: Profit of networks A and B for different price adjustment quotients

As we have mentioned above, when the network uses a small price adjustment quotient, its reaction to the changes in the network state becomes slower. This means that the rate of price increase/decrease is slow. The network implementing a small price adjustment quotient behaves almost statically, while the other network behaves dynamically.

In table 5.12, we examine different threshold values. We increased the average window size to 10 in order to cover more possible values of the success rate. Example of success values for window size 10 would be 0, 0.1, 0.2, 0.3, .., 0.9, 1. For window size 3 the corresponding values would be 0, 0.3, 0.66, and 1.

thr1	*thr2*	Network A Profit	Network B Profit
0.2	0.4	$54,768	$134,098
0.4	0.6	$64,758	$191,967
0.6	0.8	$52,542	$142,374

Table 5.12: Profit of network A and network B for different threshold values

From table 5.12, we can see that both networks are better off with moderate thresholds. If the thresholds are low, then the probability that the network will increase the prices is higher. On the other hand, if the thresholds are high, then the network is more likely to decrease the prices.

5.7.4 Comparison of dynamic pricing and static pricing

Let us assume that Network B (the most profitable network) implements static pricing, which means the prices are fixed and does not depend on the moving average window to increase/decrease the prices. Network A implements dynamic competitive pricing. The results in table 5.13 show that when the most profitable network decides to use static pricing, while the other network implements a dynamic pricing, the results are catastrophic. If the "static pricing" network decides to offer low prices it operates with losses, if it decides to increase the price, then the dynamic network can adjust its prices and offer better competitive prices and make high profits.

Network B: Static prices($)	**Total Profit Network A** *(dynamic pricing)*	**Total Profit Network B** *(static pricing)*
$1	$0	**-$3111**
$2	$216	$1795
$3	$10332	$2627
$4	$14656	$ 4486
$5	$19908	$ 1813
$6	$ 31026	$ 2495
$7	$30875	$2282
$8	$24720	$0
$9	$22631	$0
$10	$18484	$0

Table 5.13: Profit comparison when one network uses static pricing

Let us now consider the case when both networks implement static prices as shown in table 5.14. From table 5.13 and table 5.14, we can notice the difference in Network A's profit when implementing dynamic prices, which is considerably higher than the case when Network A implements static pricing schemes. The same conclusion applies for Network B as well.

Static prices network A; network B	**Total Profit Network A** *(static pricing)*	**Total Profit Network B** *(static pricing)*
$1	-$5398	-$7332
$2	-$3111	-$3746
$3	$4572	$7770
$4	$6047	$8054
$5	$8424	$15491
$6	$9702	$22042
$7	$6817	$13114
$8	$6444	$12849
$9	$3609	$11568
$10	$2686	$6168

Table 5.14: Profit comparison when both networks use static pricing

If one network decides to go below its rival price, then it will capture the market. The other network in this case will have the incentive to decrease its prices to make profit. These results are important, because they show the sensitivity of the competitive market in terms of survivability. If one of the players in the market decides to compete by dynamically change the prices, the rest of the players must react in order to capture the customer demand and stay in business; otherwise, the losses are catastrophic.

Another scenario would be if the fixed price is set to be higher from what the customer can afford. In this case, the network simply will make zero profit. This scenario happens when the network management system has failed to estimate the market potential for driving a successful business in terms of offering affordable prices. Also it could happen if the cost of the network infrastructure is too high and has not been engineered economically.

5.8 Comparison between active price competition and passive price competition

In table 5.15, we present a comparison between active and passive competition models. For this comparison, we present the average profit for each network and the average price per customer request served by networks A and B implementing both models.

	Active model	Passive model					
		W=1	W=3	W=6	W=9	W=24	W=50
	Average Profit	Average profit	Average profit	Average profit	Average profit	Average profit	Average profit
Deterministic active model , deterministic passive model							
Network A	$21962	$48576	$56375	$55356	$53870	$39456	$29456
Network B	$187506	$198363	$209883	$200837	$198454	$17319	$123719
Average price per request	$3.92	$5.44	$5.91	$5.63	$5.59	$5.01	$4.81
Stochastic active model, stochastic passive model							
Network A	$19310	$41569	$47732	$51785	$39860	$23759	$20175
Network B	$175061	$189438	$199648	$201845	$187210	$159743	$124951
Average price per request	$3.01	$5.26	$5.31	$5.86	$5.21	$5.22	$4.03

Table 5.15: Profit comparison between Active and Passive price competition

The deterministic active model is the case when the price undercutting step and the price-demand relationship are deterministic (see subsection 5.5.1). The stochastic active model is the case when both the undercutting step and the price-demand relationship are stochastic (see subsection 5.5.4). From table 5.15, we can draw the following conclusions about the active and passive price competition:

- In active price competition, the customer is a participant in the competition and price negotiation process. Therefore, the customer is able to obtain the lowest price possible.

As a result, the average price offered to the customer is lower compared to the average price obtained in the passive price competition model.

- On the other hand, in the passive price competition model, the pricing decisions do not require active involvement of the customer. As a result the prices remain higher, because the customer does not "actively press" the suppliers for lower prices. Implementing this model in optical communication networks can save the overhead complications of signaling between the customers and the suppliers, but requires the necessary intelligence to forecast and gather customer behavior and its modeling.
- As a conclusion, the active price competition is more beneficial to the customer, while the passive price competition is more beneficial to the supplier.

5.9 Summary

In this chapter we proposed a microeconomic approach for routing and wavelength allocation in competitive non-cooperative disjoint optical networks. We modeled the pricing decisions as a game between two players and developed a messaging protocol supporting the competition process. We have presented two models of price competitions, an active price competition and a passive price competition. We have also proposed new criterion for network upgrade based on profit/loss analysis, which helps transport service provider to leverage market characteristics and stay in business. In this regard, this study was a head-start in addressing a future competitive market of optical networks. It guides transport service providers through their strategic and long term network planning based on determining a given network architecture's level of profit forecast based on competition.

Chapter 6

6.1 Conclusion

Routing and Wavelength Allocation (RAW) in a Dense Wavelength Division Multiplexing (DWDM) optical network has been formulated as a Mixed Integer Programming (MIP) problem. With the immense bandwidth opportunities as trade commodity comes the challenge of cost-effective managing of the optical transport networks to increase market survivability and improve competitive advantages. The presented work is based on an economic feasibility study for a service provider, offering transport network services in a competitive market where a pricing decision has to be chosen carefully to acquire market shares and stay profitable. We have proposed a microeconomic approach for routing and wavelength allocation in competitive non-cooperative disjoint optical networks. We modeled the pricing decisions as a game between two players and developed a messaging protocol supporting the competition process. In our study, we presented two models of competition; the first is called *active price competition* and the second is called *passive price competition.*

In active price competition the suppliers allow customer negotiation, the prices offered on the active price competition are more customers oriented. In the passive price competition model, the interaction between customers and suppliers is not required. Therefore, implementing this model in optical communication networks can save the overhead complications of the signaling between the customers and the suppliers, but requires the necessary intelligence to forecast and gather customer behavior.

In both models, the active price competition and the passive price competition, we analyzed the profitability of the optical network and proposed a criterion based on profit/loss analysis to upgrade the network in order to boost its competitive advantages. Our study is a head-start in addressing a future competitive market of optical networks. It guides transport service providers through their strategic and long term network planning based on determining a given network architecture's level of profit forecast based on competition.

6.2 Future Work

The proposed Game Theoretic Pricing in Optical Networks can be further developed as a powerful tool for optical network planning and provisioning, thus increasing the effectiveness and competitiveness of an optical network service provider. A call admission control tool, based on profit/loss analysis, is necessary for a service provider in order to preserve the profitability and to future-proof his investments.

Following are some recommendations for future work:

- Implementation of heuristic (sub-optimal) routing strategies in the optical networks and their effects on the network competitiveness and profitability.
- Implementation of the proposed algorithm as extension to the existing interior and exterior protocols.
- Emulation of optical networks using open-source low cost test-beds and implementing a prototype of the proposed price competition models.
- Further study of customer behavior, modeling of customer preferences using Markov-Chains and customer welfare utility models.

- Implementation of Artificial Intelligence models (Neural Networks, Expert systems, Fuzzy Logic) to model and simulate customer behavior and preferences.

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