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A Study of the use of Wavelets in Cellular Wireless Communication Systems

By

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Abstract

In this thesis we review the fundamental concepts of wavelets and wireless cellular communication (chapters 2 & 3 respectively). We then explore the details of how wavelets can be used to multiplex users in a communication channel. Much of our exploration is devoted to gain insight into how this technique compares to existing multiple access schemes (chapter 4). A qualitative analysis of how wavelets support a multi-user scheme is provided (chapter 5). Performance issues related to the specific application of wavelets in a cellular communication environment are simulated, discussed, and compared with other multi-users schemes.

In this thesis we show how wavelets could potentially support a multi-user scheme that can provide a flexible transceiver structure of high capacity.

Acknowledgments

I would like to express my deep appreciation to Dr. T. Aboulnasr for her expert guidance and supervision throughout the course of this work. I am particularly grateful for her words of encouragement, and for taking the time to meet regularly in spite of her exceedingly demanding schedule as Dean of the Faculty of Engineering.

I would also like to thank Mr. Greg Carleton for sharing his extensive knowledge and experience in CDMA, Spread Spectrum system and Stochastic Analysis.

To Lise

Contents

List of Figures

List of Tables

List of Acronyms

A-CDMA	Asynchronous CDMA
AWGN	Additive White Gaussian Noise
BH-WDMA	Branch-Hopping WDMA
CDMA	Code Division Multiple Access
CDM	Code Division Multiplexing
DFT	Discrete Fourier Transform
DS-SS	Direct Sequence Spread Spectrum
FDMA	Frequency Division Multiple Access
FDM	Frequency Division Multiplexing
FFT	Fast Wavelet Transform
FH-SS	Frequency Hopping Spread Spectrum
FIR	Finite Impulse Response
LPI	Low Probability of Intercept
GSM	Global System Mobile
HDTV	High Definition TV
IFWT	Inverse Fast Wavelet Transform
IFFT	Inverse Fast Fourier Transform
OFDM	Orthogonal Frequency Division Multiplexing
PN	Pseudo-random Number
S-CDMA	Synchronous CDMA
SDR	Software Defined Radio
TDMA	Time Division Multiple Access
TDM	Time Division Multiplexing
WDMA	Wavelet Division Multiple Access
WDM	Wavelet Division Multiplexing
WPDM	Wavelet-Packet Division Multiplexing
FWT	Fast Wavelet Transform

Chapter 1

Introduction

1.1 Background and Problem Statement

During the recent past there has been a tremendous interest in the theory and application of Wavelets as a tool to perform analysis and decomposition of signals. By now just about anyone in the field of signal processing has at least heard the term *Wavelet* as one of the leading buzzwords that has captured the attention of researchers and practitioners alike. This growing interest has lead to a substantial amount of publications and research devoted to the exploration of wavelets for the purpose of solving contemporary problems. It requires only a quick literature survey to realize that most of the interest (and perhaps success **[Str1]**, **[Vet]**) of Wavelet applications has been in the fields of signal compression (particularly for images **[Man2]**) and signal analysis **[Str1]**. This is not a big surprise, it is perhaps even obvious to some, since the Wavelet transform allows us to decompose a signal into a set of constituent signals much like we do with the Fourier transform. Since the Wavelet transform has proven to be very efficient in providing flexible time-frequency localization properties, much research effort has been devoted to applications such as signal "de-noising", and detection of signal discontinuities. These applications appear to be a very "natural" use of wavelets for the time-frequency localization properties. There are however other applications that are not so obvious to many of us. One of them is the use of wavelets to efficiently multiplex signals in a communication channel. **This is the focal point of this thesis.** In fact the motivation behind this thesis is more specific than that: here we wish to *explore whether this method of multiplexing signals can in fact be a feasible option for a wireless cellular communication system.*

Existing commercial cellular networks are based on very mature and conventional techniques:

1. Time Division Multiple Access (TDMA). In this scheme each user is assigned a unique time slot but all users share the same frequency band.
2. Frequency Division Multiple Access (FDMA). In this scheme each user is assigned a unique frequency band but the transmission of different users overlaps in time.
3. Code Division Multiple Access (CDMA). Unlike the above schemes, each user is not assigned a given time slot or frequency band but a unique "code" (i.e. waveform).

It is only natural to ask:

- *Would a Wavelet-based scheme work for the requirements of a cellular wireless application?*
- *How does it differ from the traditional methods? Is this technique really "new" or simply another variation of the conventional techniques being used? Some researchers have referred to wavelets as means to implement spread spectrum systems* ***[Het],[Aka7]****. How does that work?*
- *What are the advantages and problems related to this multiple access method?*

Answering these and many other questions is the central goal of this investigation.

It is worth noting that Orthogonal Frequency Division Multiplexing (OFDM) has been considered for other applications such as Asymmetric Digital Subscriber Line (ADSL). However, we are not considering OFDM as part of the scope of our investigation because it has yet to be deployed in a cellular application.

1.2 Thesis Outline

In this section we provide a brief description of each chapter along with the intended flow of ideas.

Chapter 1: In this chapter we provide a brief introduction to the problem being analyzed as well as the motivation behind this work. A thesis outline (this section) is followed by a brief description of the expected contribution given by this investigation.

Chapter 2: Here we provide a basic description of the wavelet theory. The emphasis of this chapter is on an intuitive understanding to the answers to questions such as *what are wavelets? what is the wavelet transform and how to implement it?* In this chapter we also attempt to provide some insight into some of the characteristics of wavelets such as "regularity", "refinement", "order", and other terms.

Chapter 3: Before we explore the use of wavelets in cellular communications we first need to understand the application domain. Cellular wireless communications is an extremely vast topic, hence we need to focus on our area of interest and briefly review existing solutions to the cellular communication problem.

Chapter 4: Here we take the basic wavelet concepts described in Chapter 2 and extend them to formulate what could be regarded as a "universal" multiplexing structure. Here we can see how a structure known as a "transmultiplexer" can be combined with wavelets to form a very efficient multiplexing scheme. In this chapter we see how wavelets can provide an extremely versatile structure with very interesting characteristics. At the end of this chapter we begin to have a good understanding of how wavelets can be used to address the multi-user communication problem in a cellular system. We also show how the Noble Identities are used to bring the Wavelet structure from "multiplexing" to "multiple access".

Chapter 5: At this point we have defined a multiplexing scheme for a wireless cellular system, but how good is it? In this chapter we attempt to explore the performance of a Wavelet Division Multiple Access (WDMA) implementation, the suitability of this approach, its spectrum efficiency, its degradation in a fading channel, and other systems engineering related issues.

Chapter 6: In this chapter we summarize the results of our investigation and identify areas of future research.

Chapter 7: This final chapter contains a collection of references to some of the most relevant material consulted while carrying out this investigation. An effort has also been made to catalog the references according to their knowledge domain, i.e. wireless communication, wavelet theory and applications, etc.

The intended flow of ideas is illustrated in the Figure 1.2-1.

In this thesis we attempt to bridge two domains: wireless system engineering and wavelets. Hence the text has been written for an audience assumed to have a background in DSP or Wireless Engineering (not both). An effort has been made to avoid the assumed knowledge of very specialized topics in either field. Where possible, brief introductions are given for specific topics such as wireless standards, and mathematical concepts behind the theory of wavelets. In general, an attempt has been made to avoid mathematical detail to focus on intuitive understanding of concepts, focusing on the engineering aspects of a wavelet-based multi-user system.

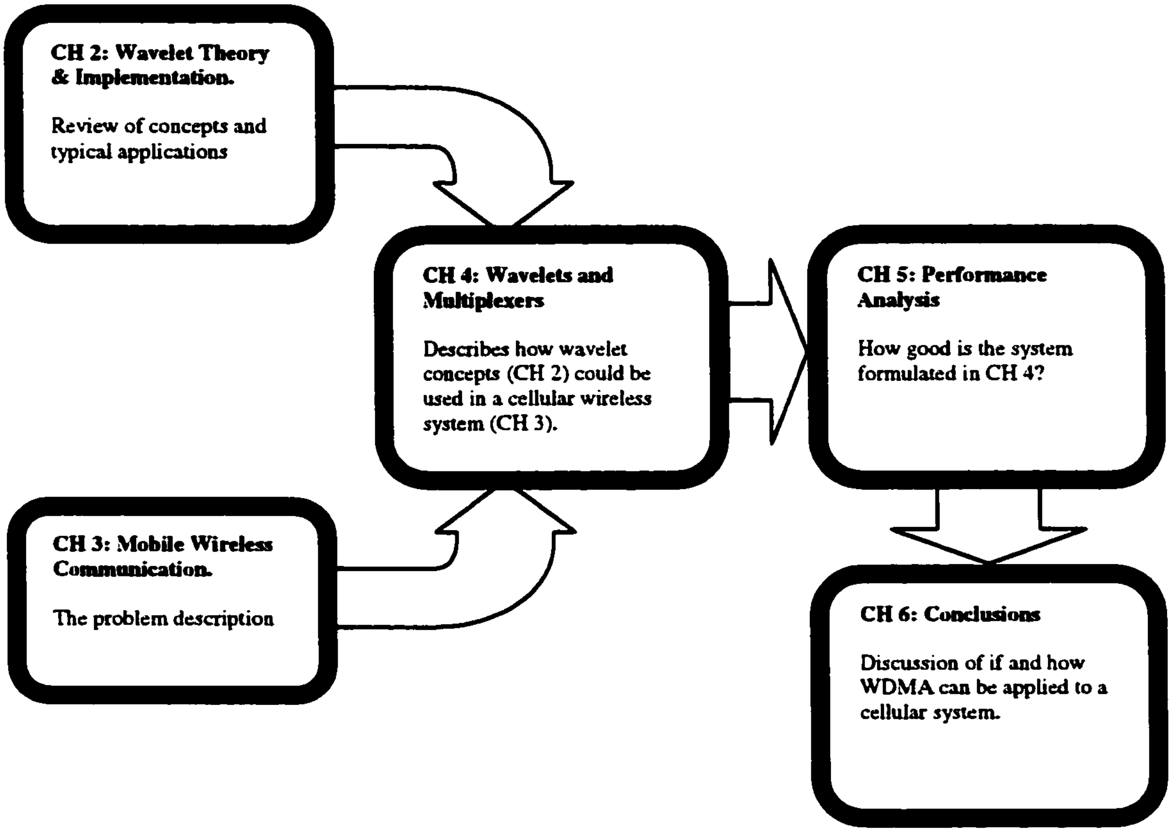


Figure 1.2-1 Visual Outline of Thesis.

1.3 Contribution

Some researchers (**[Aka]**, **[Dav]**, **[Wu]**, **[Orr]**) have recently suggested that wavelets can be used as a new technique for supporting a multi-user environment while providing higher capacity with "perhaps" a higher level of robustness to interference. However, beyond the basic description of this "first principle" we have seen very little detail concerning its implementation in a real life communication channel. In this thesis we attempt to provide exactly that, details about implementing a WDMA system (...and its associated problems). Throughout the course of this analysis we attempt to provide a detailed explanation of how

WDMA is built by connecting concepts from DSP, multi-rate systems, and functional analysis. Although this thesis has been written from a DSP point of view, an attempt has been made to show how details associated with DSP algorithms can actually transcend to influence system level issues. For instance, by the end of Chapter 4 we show how the multi-resolution details of the wavelet transform could be used to provide some of the features desired in a Third Generation (3G) wireless system.

Chapter 2

Wavelets: Theory and Implementation

2.1 The Concept of Wavelets

During the early 90's the concept of wavelets emerged as a new promising and exciting way of decomposing and analyzing signals. A problem for many signal processing engineers was simply to understand what this was all about. Given the fact that the wavelet concept originated in the field of mathematics, most of the existing publications were not easily exported to other disciplines. Although by now we have full textbooks dedicated to the subject of wavelet applications (**[Aka5]**, **[Str1]**, **[Vet1]**), the specific implementation details remain somewhat obscured by the rich mathematical language of functional analysis.

Without attempting to replace some of the excellent tutorials available today (**[PRC1]**, **[Vid]**, **[You]**), in this section we will attempt to provide a brief description of the wavelet concept so that we establish the necessary background.

One of the typical obstacles in understanding the notion of wavelets is that the subject is typically introduced as something "brand new" and loaded with a rich dosage of recursive equations. We will take an alternative approach and attempt to bridge the "known", Fourier Transform, to the "unknown", wavelets.

From basic functional analysis we know that the Fourier Transform (FT) is a tool that allows us to decompose a signal into a set of constituent signals. These constituent signals are a set of sines and cosines of various frequencies and amplitudes. Hence we can say that the FT is an instance of a more general principle that says that we can represent any signal by a linear combination of orthogonal set of signals. We refer to these constituent signals as the "basis" of the signal space. In the case of the FT we know that the *basis* is a set of waveforms that takes the shape of sines and cosines **[Oppl]**. The ultimate goal of this exercise is to obtain an *alternative* view of the same signal, hence we say that we have a "time-domain" view and a "frequency domain" view (i.e. the result of the transform). We say *frequency* domain since the concept of frequency is associated with the *period* of the various sines and cosines used to construct the desired signal.

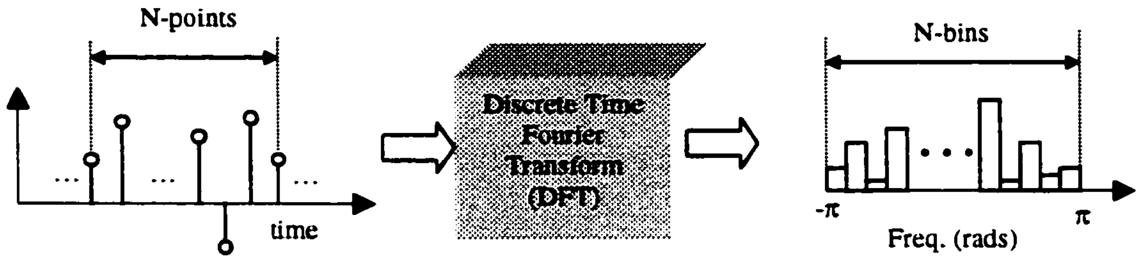


Figure 2.1-1 Basic Fourier Decomposition Process.

Figure 2.1-1 shows a discrete-time signal of N-points can be transformed into a set of N coefficients that describe the amplitude and phase of N sinusoidal waveforms. These sinusoidal waveforms vary in *equal* steps of frequency from the most negative to the most positive. Hence, the frequency "bins" shown above, are centered at each of N sinusoidal frequencies ranging from -π to π radians (here π is equal to ½ of the sampling frequency). For instance, if the input waveform would happen to be a sine wave, then output spectrum would have only one non-zero bin.

We also know this decomposition process is reversible, i.e. if we know the Fourier coefficients then we can reconstruct the signal in the time domain.

So how can we relate this to the Wavelet concept?

We can say just as the FT is attempting to decompose a signal in terms of sines and cosines, the Wavelet Transform (WT) is also attempting to do the same thing but in terms of other waveforms. Such waveforms are the so-called *wavelets*.

So what are wavelets anyway?

Unlike sines and cosines they are mostly of limited duration, for which reason they are termed wavelets (or small waves). It is this limited duration feature that allows us to not only localize signal characteristics in the frequency domain but also in the time domain.

Figure 2.1-2 Examples of Daubechies wavelets.

Figure 2.1-2 shows samples waveforms that are part of a family of wavelets named after the mathematician Ingrid Daubechies (we will later discuss the details behind these waveforms). Having seen these figures we then ask:

How does the discrete WT output differ from the discrete FT shown in Figure 2.1-1?

Perhaps the most well known difference is that the WT yields a frequency domain representation with "non-uniform" resolution. Unlike the result of the DFT in which we had N bins of *equal* bandwidth, with wavelets we can choose to have less than N bins with different bandwidths. This is perhaps better illustrated by answering the next question that typically comes to mind:

How does one obtain the WT of a signal?

Let's answer this question by implementing a very simple example based on the simplest of all wavelets: the Haar (also known as Daubechies of order 1 or 'db1' for short). Most of the basic literature typically first provides a definition of the "continuous" time version of the wavelet transform and only after, its discrete-time version is introduced **[Coh]**, **[Hes]**. Here we will not attempt to replicate such effort. Instead we will deal directly with the discrete-time WT, which is more appealing from a DSP point of view.

In order to obtain the wavelet coefficients, we follow the generic equation of an orthogonal transform:

$$\mathbf{c} = \mathbf{T}\mathbf{x}$$

where **c** is a matrix containing the wavelet coefficients, **T** is another matrix which represent the "operator" that will act on the signal **x** (which is also in a matrix form). Let's expand the above equation for our simple example in which we wish to perform a 4-point transform using the Haar wavelet **[Str1]**.

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ x_{n-3} \end{bmatrix}$$

Each of the rows in the transformation matrix **T** represents a "basis" waveform used to decompose the signal. In other words, knowing the coefficients **c** and the basis waveforms we can produce a linear combination that can reconstruct an approximation of the signal **x**. Just like in a 4-point DFT we would have 4 complex exponentials of different frequencies, here we have 4 wavelets of different "dilations" and "shifts". Just like in the FT we begin with two basic waveforms (sines and cosines) to produce all other basis needed, in the WT we also begin with two basic waveforms known as the "scaling" and "wavelet" functions. Once these two waveforms have been defined, all other basis functions are derived from here. Hence a family of wavelets can be formed as follows:

$$\varphi_{a,b}(t) = \frac{1}{\sqrt{a}}\varphi\left(\frac{t-b}{a}\right)$$

where **φ** is a fixed function known as the "mother wavelet". The factors **a** and **b** are the dilation and translation factors respectively. Note that the factor **1/a** is just to ensure that the functions $\boldsymbol{\varphi_{a,b}}$ have a constant norm.

So how can we associate this notation with the 4-point Haar example shown previously? First, we point out that the top row represents the "scaling" function, while the second row is the "wavelet" function. Furthermore, by adding restrictions to **a** and **b** we can obtain the multiresolution transform that leads to the matrix in our example. More specifically, we make the dilation factor be driven by a factor of $\mathbf{2^j}$, and the translation factor be a discrete interval **k** so that translations are only in steps equal to the wavelet duration **[Coh]**.

$$\varphi_{j,k}(t) = 2^{\frac{j}{2}}\varphi\left(2^{\frac{j}{2}}t - k\right)$$

With this equation we can see that the last two rows of the 4-point Haar matrix are derived from the second row, which is the mother wavelet (this is shown graphically in **Figure 2.1-3**). If we look at the matrix operation involved here, we can see that the first coefficient is obtained by multiplying each value of the data with each value of the basis function followed by a sum (i.e. apply a dot product). The operation to obtain this first wavelet coefficient $\mathbf{c_0}$ in fact matches the mathematical definition of correlation. Hence, $\mathbf{c_0}$ represents a measure of the correlation between the N-point input signal and the scaling function. The more the incoming signal looks like the scaling function, the larger the value of c_0.

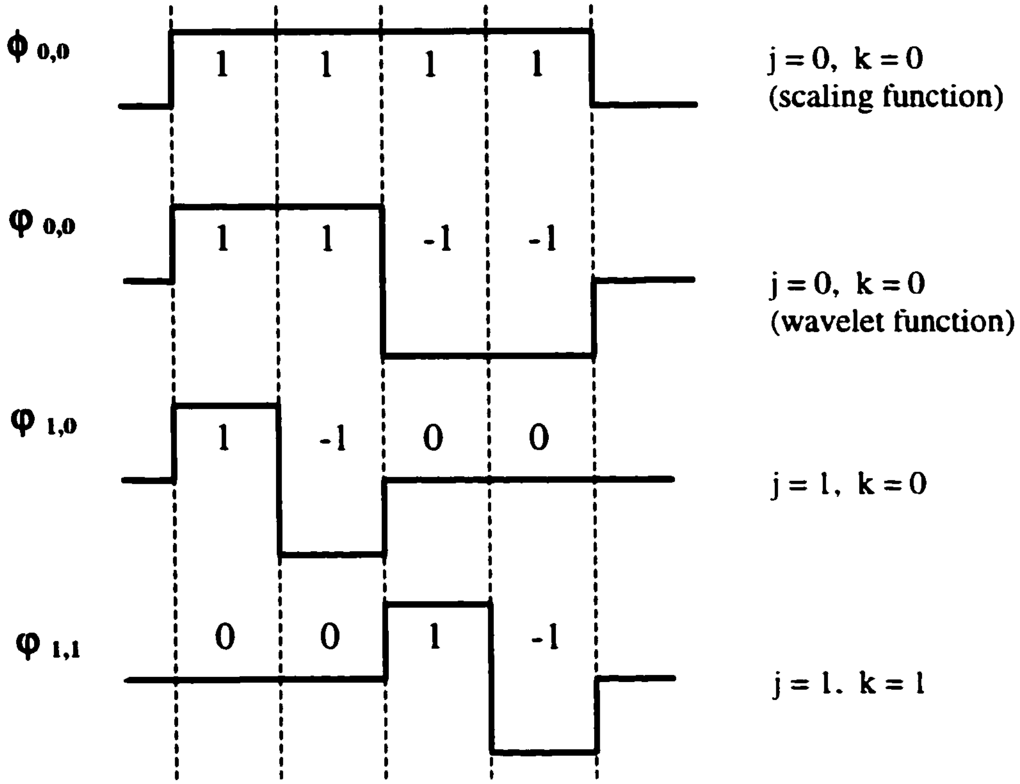


Figure 2.1-3 Basis functions for a 4-point Haar Wavelet Transform.

Let's note that for simplicity here we wrote the Haar coefficients with –1 and +1 values. In order to comply with the definition of orthogonality ($T^{-1}=T^T$, $T\,T^{-1}=I$), we need to multiply each row by a factor of $2^{j/2}$, which is the coefficient found in front of the wavelet definition. In the end, we arrive at the conventional reconstruction formula, which is a linear combination of the basis functions:

$$x(n) = c_0\phi_{00}+c_1\varphi_{00}+c_2\varphi_{10}+c_3\varphi_{11}$$

An important observation to make here is that although we end up with 4 coefficients, we do not have 4 frequency "bins" as in the case of the DFT. In fact we end up with only 3 (see **Figure 2.1-4** to visualize the correspondence between wavelet coefficients and frequency bands).

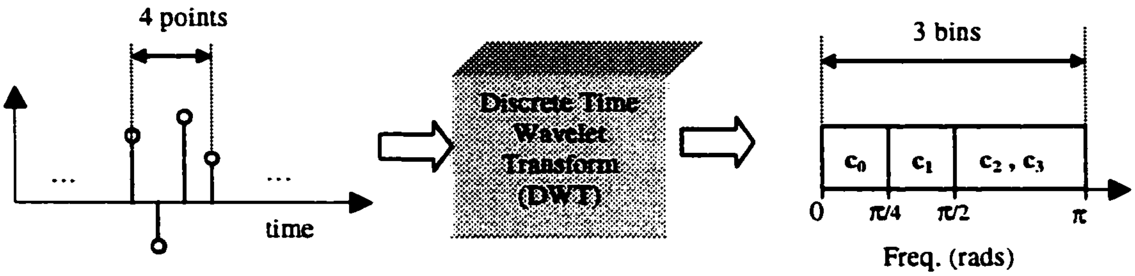


Figure 2.1-4 Basic 4-point Wavlet Decomposition Process.

There are two main observations we can make here:

1. The frequency bands are not equally spaced (we see that the third bin is twice as large as the first two). This brings about the notion of "multiresolution", i.e. we can see more details in some bands than in others. In this example, we oversimplify the transform by using only 3 bands but in practice we can have as many as we like and with different resolution arrangements. Note the coefficient associated with scaling function provides an indication of the "coarse" features of a signal. In Fourier analysis we would refer this coarse feature as the "lowpass" component of a signal. Complementary to that, we say that the coefficients associated with the wavelet function provide an indication of the "detail" features of a signal. Again, in Fourier term we would refer to this as the "highpass" components of the signal. The key difference is that in wavelet analysis we can *recursively* go from a coarse to a detail view, whereas in the Fourier transform we get a fixed view at once.
2. The second point is that we have two coefficients that describe the third band (i.e. from π/2 to π). The reason for this is that, if we recall, they come from basis which

do not overlap in time. Therefore, each of these coefficients represents the activity of the same frequency band but at different moments in time! This is what is meant by "time localization".

These observations are some of the most interesting features found in the Wavelet Transform, but absent in the Fourier Transform. In the past, researchers have attempted to achieve time-frequency localization with approaches like the Gabor Transform and other variants of the FT, but failed to obtain clean results due to the infinite time duration of the sinusoidal waveforms **[Hes]**.

In this very simple example, we dealt with the WT as an operation that takes N points and yields N coefficients. This means that if we were operating on a time series that represents a long data stream we would end up creating "blocks" of transformed data. Sometimes this causes a problem since the boundaries of such blocks produce a discontinuity. A common solution to this problem is to have "overlapping" sets of input data so that the so-called blocking effects is removed. This is the basis for the concept of Lapped Orthogonal Transforms (LOT) **[Str1]**. We will see that this is a very natural byproduct of an alternative implementation of the Wavelet transform, i.e. rather than using a matrix formulation we can use a "filter bank". This is the topic of the next section.

Aside: a bit of *(recent)* history

As mentioned previously, wavelets had their origin in the very abstract field of functional analysis. Hence it is not hard to imagine that the step from mathematics to engineering has not been a simple one. It is this step, among other things, that has been attributed to the French researcher Ingrid Daubechies who in the early 90's became the perceived creator of wavelets. It is worth noting that while Ingrid Daubechies was being credited for being the leading creator of the wavelet concept, she wrote a very humble article titled *Where Do Wavelets Comes From? – A personal point of view* **[Dau2]**. In this article Daubechies lists herself as a mere follower who simply studied the work of others, (a clear act of academic humility). This article is actually a very interesting chronicle of the recent development of the concept of wavelets.

2.2 The Fast Wavelet Transform

Just as in the case of the DFT where we have a "fast" version of the transform, i.e. the FFT, in wavelets we have the FWT (implemented in the form of a filter bank [**Mal**]). Since we are interested in the detail implementation issues, then the next question is *how do we exactly construct this wavelet-based filter bank?*

Figure 2.2-1 shows a typical dyadic structure that allows us to implement the same operation previously done with a matrix structure.

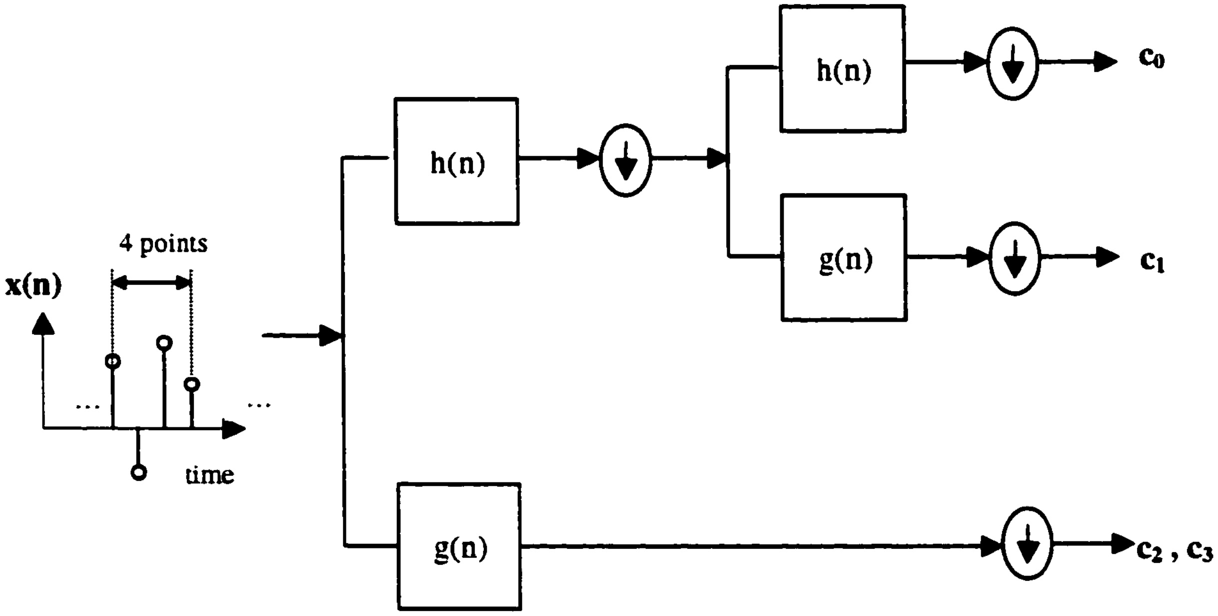


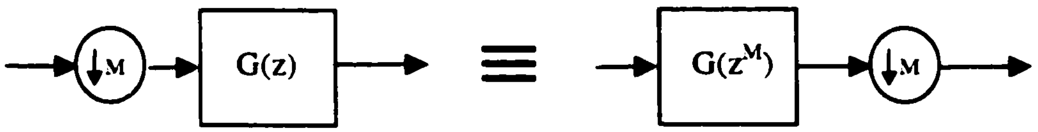
Figure 2.2-1 Dyadic tree structure for a 4-point WT (equivalent to that shown in Figure 2.1-4).

Although the structure above is only a "2-level" tree, the structure can be successively expanded to have as many levels as desired. Note that *the key element here is the filter pair: h(n) and g(n) which are related to the scaling and wavelet functions, respectively.*

The filters h(n) and g(n) have the basic characteristics of a “lowpass” and “highpass” behavior. In our example, the taps of filters h(n) and g(n) are [1 1] and [1 –1] respectively. Now the next question one would ask is: *how do we relate these filter taps to the basis functions found in our matrix formulation?*

The answer to this question can be found in the **Noble Identities [Vai]**. This concept is not native to wavelets but is actually taken from the extensive body of work already done in the field of Multirate Systems. These identities are reproduced here in **Figure 2.2-2**.

As we will see in subsequent chapters, the use of this identity is central to the focus of this thesis. For the time being we will see how the multi-stage structure shown in **Figure 2.2-1** can be reduced to a single stage structure with an “equivalent” filter (see **Figure 2.2-3**).



(a) Identity 1: Re-ordering of down-sampler and filter operations

(b) Identity 2: Re-ordering of up-sampler and filter operations

Figure 2.2-2 Noble Identities.

It is useful to recall that when we look at the Z-transform of a filter, $G(z^M)$ is simply G(z) with M-1 zeros inserted between successive taps!

If we again use our simple example of a 4-point Haar transform to gain insight into this concept we see that the "effective" impulse response of the first branch, $k_1(n)$, corresponds to the first row of the transform matrix shown on **page 2-5**.

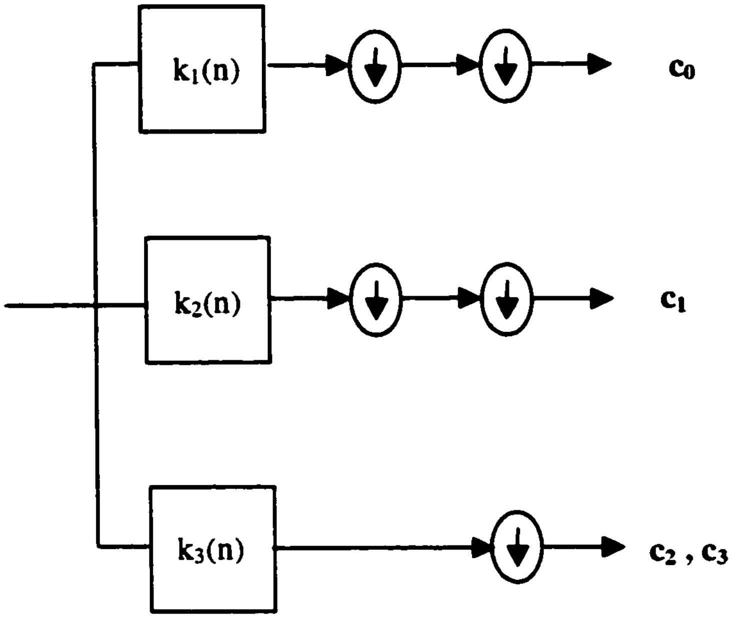


Figure 2.2-3 Dyadic tree structure after applying Noble Identity 1.

More specifically we recall that h(n) and g(n) are [1 1] and [1 –1] respectively, hence:

$k_1(n) = Z^{-1}\{ H(z)\, H(z^2) \} = [1\ 1] * [1\ 0\ 1\ 0] = [1\ 1\ 1\ 1]$

$k_2(n) = Z^{-1}\{ H(z)\, G(z^2) \}\ [1\ 1] * [1\ 0\ {-1}\ 0] = [1\ 1\ {-1}\ {-1}]$

$k_3(n) = g(n) = [1\ {-1}]$ (since we did not need to apply the identity).

We can then see that the first two filters correspond to the scaling and wavelet functions while the filter $k_3(n)$ is the mother wavelet compressed by a factor of 2. The fact that coefficient c_2 will be obtained before c_3 is actually equivalent to the insertion of zeros in the last two rows of the matrix shown on **page 2-5**. Note that for every sample obtained from the top two branches we obtain two samples in the bottom branch. This means that we get an indicator of the same frequency band at two different moments in time, hence allowing us to perform time localization.

This elegant concept leads to a signal processing implementation that is very efficient and more native to DSP implementations. Just about every DSP processor has been optimized to process Finite Impulse Response (FIR) filters. An interesting observation we can make after this exercise is that ***the "effective" impulse response in each of the tree branches (i.e.*** **$k_i(n)$** ***), represents an approximation of the basis functions***. For instance, in this example we see that **$k_1(n)$** approximates $\phi_{0,0}$ (shown on **page 2-5**), **$k_2(n)$** approximates $\varphi_{0,0}$, and **$k_3(n)$** approximates $\varphi_{1,0}$. Note that $\varphi_{1,1}$ is just a time-shifted version of $\varphi_{1,0}$.

This observation leads to the concept of "refinement", which although somewhat theoretical, is central to the concept of wavelets. This is the subject or our next topic.

2.3 Refinement and Regularity

We can recall that if we perform an N-point DFT with N approaching a very large number, we are then approximating the *Continuous* Fourier Transform **[Opp1]**. In the same manner, if we create a filter structure with a very large number of stages we then approximate the *Continuous* Wavelet Transform.

Hence, as we increase the stages of signal decomposition (i.e. more branches), we say that the next equivalent impulse response of each branch has a higher level of "refinement". Therefore we can say that as we increase the level of *refinement* for the wavelets used, we are actually obtaining a more "detailed" decomposition of the signal being transformed.

It is important to note that we cannot achieve this desired refinement with just any filter! For instance, if h(n) is a lowpass filter designed as an "equiripple" filter, then the iteration process will diverge **[Str5]**. This is one of the reasons why the max-flat filters from the Daubechies family of filters are important for their property to be infinitely iterated, much like a fractal! **[Wor]**.

Another concept that is central to Wavelets is "regularity". The mathematical definition of this concept is somewhat technical **[Coh]**, but we can simply state that this property provides an indication of "differentiability". In visual terms, the smoother the waveform is, the higher the regularity. Let's reproduce **Figure 2.1-2** to illustrate this point.

The regularity of a certain wavelet is known, so for instance the Daubechies wavelets have a regularity of 0, 0.5, 0.91, and 2.15 (for db1, db2, db3, and db7 respectively). We have an asymptotic relation linking the size of the support (i.e. length = 2N) of the

Daubechies wavelets dbN and their regularity. When N becomes very large the regularity figure approaches N/5 **[Mat2]**.

Figure 2.3-1 Daubechies wavelets db1 (Haar), db2, db3, and db7.

The interesting point here is that unlike in Fourier analysis where we only have sines and cosines to choose from, in wavelet analysis we may choose a wavelet that "matches" the signal being decomposed. Thus, if we wish to analyze a waveform that contains many abrupt steps, we would choose the Haar wavelet, whereas in another case we could choose a wavelet with higher regularity. Deciding on the wavelet to use in a given case is certainly a topic of active research **[Str3]**.

It is important to emphasize that Daubechies wavelets are just one "family" of wavelets. We can also choose from other wavelet families such as Morlet, Meyer, Coiffman, and others [Str1], [Mat2]. In the interest of focus, we have chosen to restrict the scope of this thesis to deal with Daubechies wavelets since they are widely studied and well understood. We should remember that the design of new wavelet "families" is an area of active research with promising prospects (**[Aka7]**,**[Wor2]**,**[Tsa]**,**[Mon2]**). Extending the scope of this work to include other wavelet families is a subject of further research.

2.4 Wavelet Packets

In the previous sections we addressed the most popular notions related to wavelets: multiresolution, time-frequency localization and so on. These notions revolve around the use of the "dyadic" structure shown in **Figure 2.2-1**. In this structure we do not split every branch of the tree, hence we end up with output branches of different output rate. If on the other hand, we decide to split every branch in a uniform fashion then we obtain the structure shown in **Figure 2.4-1**.

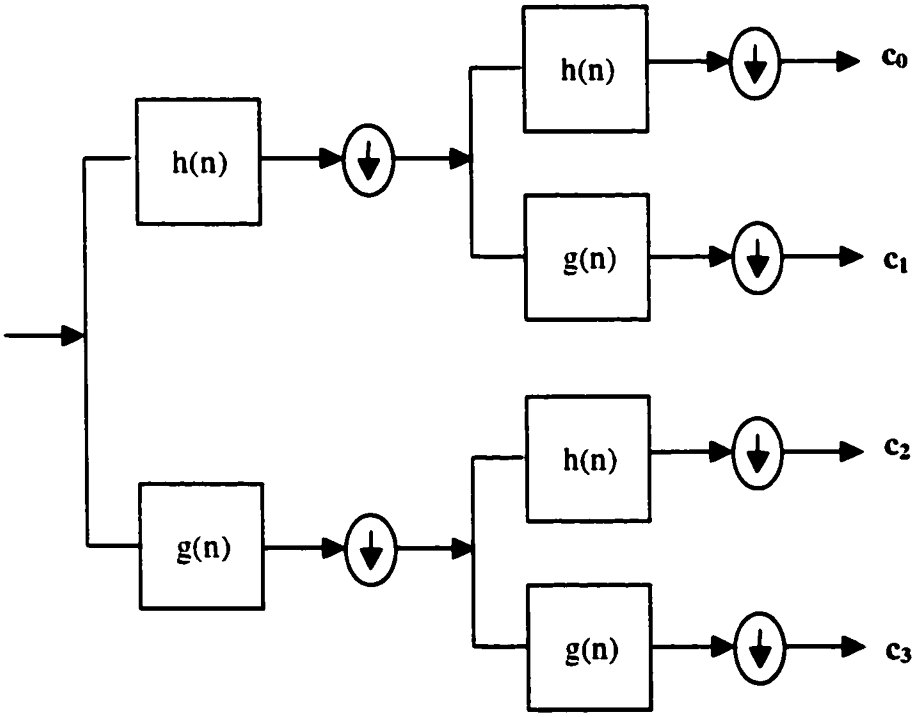


Figure 2.4-1 Wavelet Packet structure of two stages.

As we can see in this figure, all branches will produce outputs of equal rate. Although this essentially deprives us of the multi-resolution and time-frequency localization properties found in the dyadic structure, we will see that this structure will be very useful in other applications such as transmultiplexers (a concept central to this thesis).

Chapter 3

Mobile Wireless Communication

3.1 Introduction

Given the popularity of this application, it is not difficult to imagine that there is a large number of publications and textbooks on the subject. Hence in this chapter we do not attempt to add to such body of knowledge but rather to synthesize some relevant concepts that will allow us to establish a framework for the thesis. For instance, cellular networks consist of many complex systems that cover several knowledge bases. However we will only concern ourselves with the "air interface" of these networks. In this chapter, we will attempt to describe the "big picture" and gradually focus on the wireless air interface which is the focus of our attention.

3.2 Background

Although telephones have been with us since the beginning of the century, the notion of a "mobile" telephone was not available until the mid 40's when AT&T introduced the first system to interconnect mobile users (usually in automobiles). These systems were based on the concept of having a transmission tower located in a high point to offer "line-of-sight" link with all its mobile users (see **Figure 3.2-1**). Typical coverage was about 80 Km and naturally limited by the need to transmit high power radio waves which was more of a problem for the mobile unit than for the base station.

This system used the most intuitive form of user multiplexing: Frequency Division Multiple Access (FDMA). After all, we are very familiar with this concept. For instance even today when we want to listen to a particular radio broadcast station we simply "tune" our radio to catch our favorite program.

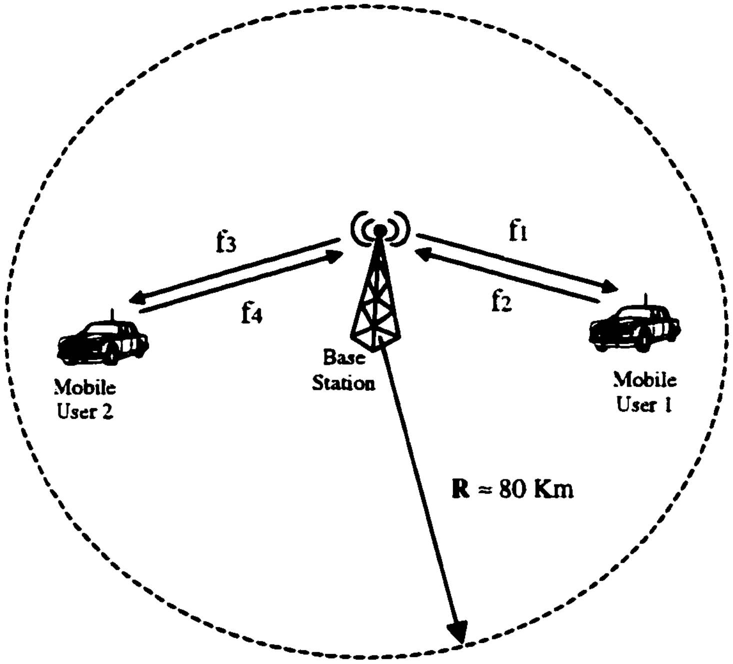


Figure 3.2-1 Pre-cellular Mobile Telephone System

This meant that each user would make use of different frequencies to avoid mutual interference. Since a telephone conversation naturally requires a full-duplex channel, these systems achieved two-way communication by assigning one frequency for the Mobile Station (MS) to Base Station (BS) link and another from the BS to the MS link. The former is known as the *reverse channel* while the latter is known as the *forward channel*. Even as of today this concept has not changed! Each frequency link occupied 120 KHz bandwidth to provide an effective 3 KHz channel using a form of Frequency Modulation (FM) very much like the one used today for public radio broadcasting.

We can easily see that there were a few of problems with this setup:

1. **Capacity**: Given the fact that each user would take up to 240 KHz of Radio Frequency (RF) bandwidth, it did not take many of them to use up all the available bandwidth allocated for this type of service.

2. **Robustness**: As the users would move further away from the base station it was harder to maintain LOS communication and hence the link quality became atrocious.
3. **Power**: Since the mobile station needed to reach the base station from anywhere in the city, its capacity to radiate power had to be large and hence made the transceiver heavy, and very costly.

This limited system would not evolve very much for the next 30 years. It has been said that in 1970 the Bell System in New York City could only support 12 simultaneous mobile conversations. The 13th user was blocked **[Gar]**. It was at this time that the Bell system was restructured to introduce the cellular concept.

3.3 The Cellular Concept

In simple terms, the cellular concept is based on the old philosophy of "divide and conquer". When dealing with a single 80 Km radius of RF coverage, we are limited by the fact that these waves attenuate at a rate inversely proportional to the square of the cell radius, $\mathbf{R}^2$ **[Che]**. Hence the farther we are, the harder it is to maintain a healthy radio link. It is this very weakness that allows us to build a cellular network. If we use many low powered base stations we can then allow multiple base stations to use the same frequency provided they are geographically separated by a distance larger than **R**. We can see that if we could handle M users in the old system based on a single area of coverage, then by having N cells then we can theoretically multiply our capacity by a factor of N.

In summary, the cellular concept is based on the following elements:

- Multiple Base Stations with Frequency Reuse
- Hand-Off and Central Control
- Cell splitting

The above elements are the focus of the following sections.

3.4 Multiple Base Stations with Frequency Reuse

Rather than using a single powerful base station, the cellular system is based on many interconnected "low-power" base stations that cover very small areas (typically from 2 to 12 Km as opposed to 70 to 80 Km). Since these low power transmitters can only cover a small area, then we can use the same frequency in another cell provided such cell is located far enough to avoid co-channel interference. The proper design of this aspect of a cellular system is know as "frequency planning". Here we divide the large area into a number of cells which are grouped in "clusters" for which we have a given reuse pattern as shown in **Figure 3.4-1**. Note that in a cluster of 7 cells (i.e. N=7) a frequency is only used every 4 adjacent cells. Recent advances in RF engineering allow systems to have N=4 which has more aggressive reuse patterns providing higher capacity (at the cost of very careful RF planning). Note that the hexagonal shape for these cells is fictitious and chosen purely for

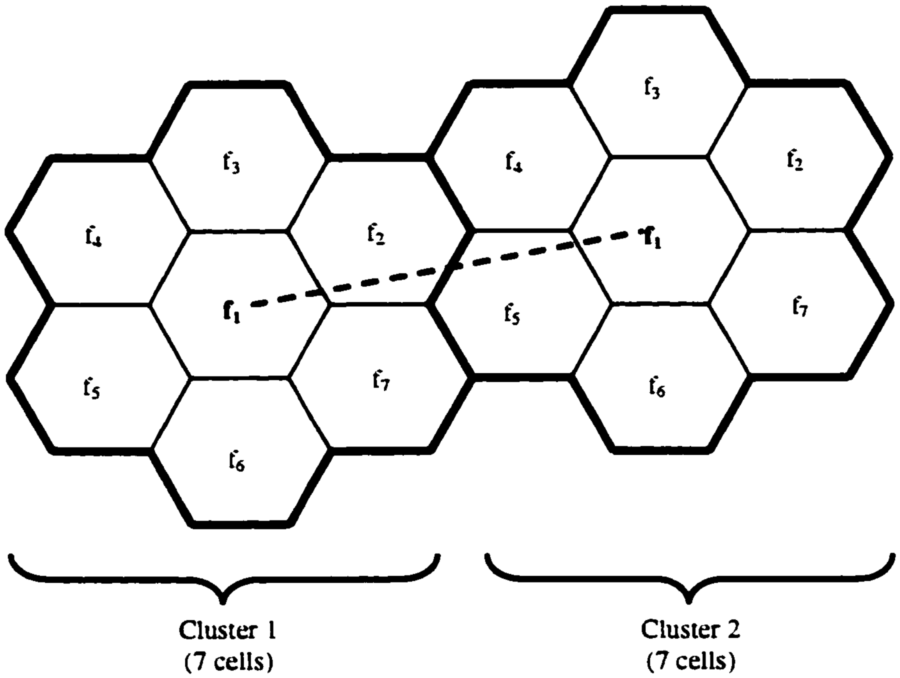


Figure 3.4-1 Frequency reuse concept in a N=7 cellular network

convenience [Haf]. In practice these cells do not have the ideal circular shape, rather they have extremely irregular shapes.

3.5 Hand-off and Central Control

All the benefits attained from replacing a single base station with many low-powered transmitters come at the cost of introducing a complex network to interconnect all the base stations in one system (See **Figure 3.5-1**).

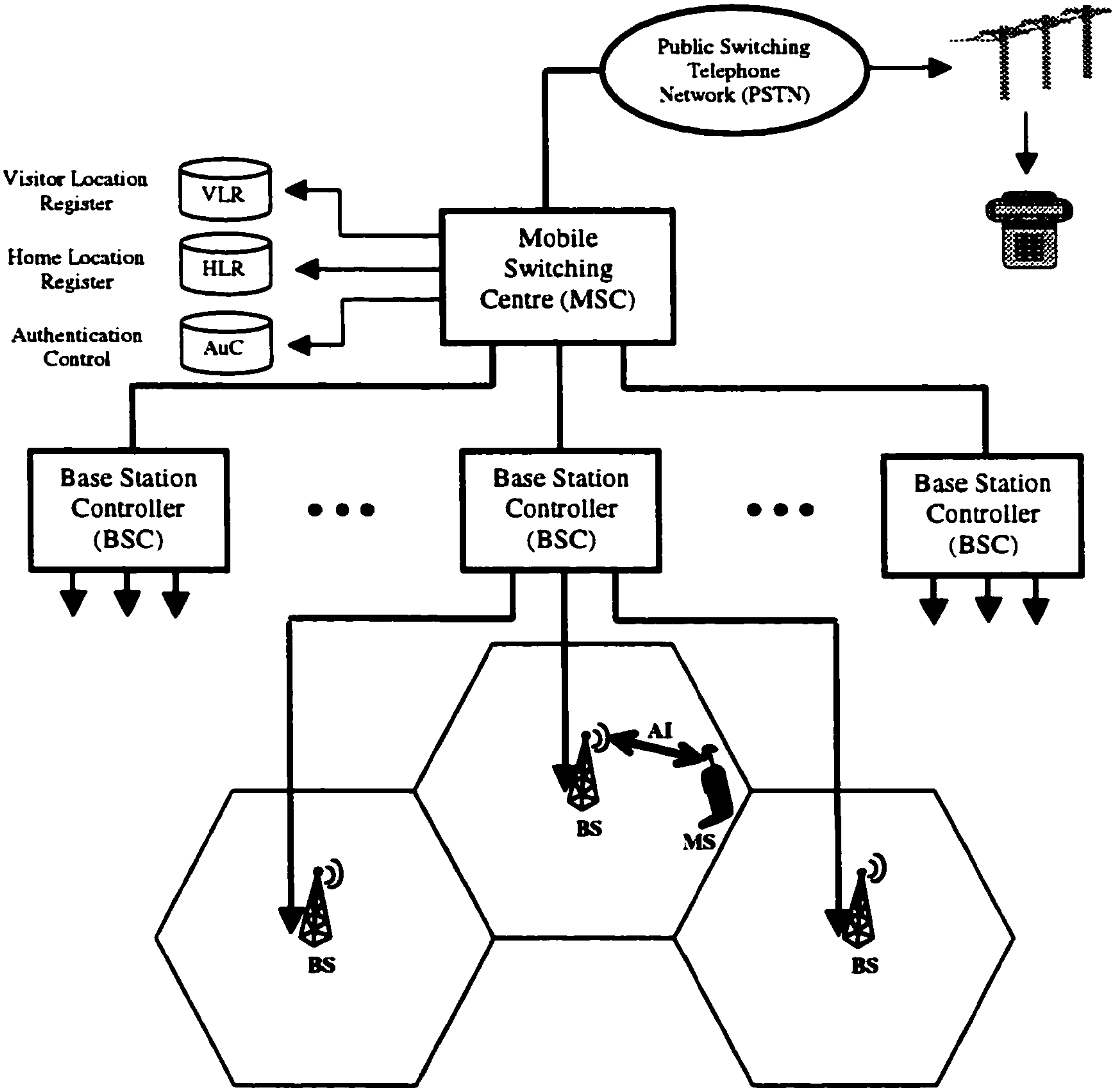


Figure 3.5-1 Wireless cellular network

When a user moves from one cell to another during a telephone call such user must be "handed-off" from one base station to another. Base stations therefore must be under some kind of hierarchical control to associate mobile users with cells. It is this distributed network concept that allows a user to move around (and be tracked) by the system in a seamless manner. Note that it is only the "Air Interface" (AI) portion of the system that is based on a radio link, i.e. the only "wireless" portion of the network. All other links above the BS hierarchy are typically T1 trunks, dedicated optical networks, PSTN, and other dedicated infrastructure. One can easily see that the so-called "wireless" networks are really full of wires!

At this point, it is extremely important to emphasize that the "Air Interface (AI)" is the main subject of concern to us. Here we present a broad system description not only to provide a context for our work but also to lay the foundation for the analysis of how the choice of a given multiple access scheme affects many system level issues such as hand-off, frequency planning and network growth.

3.6 Cell splitting

One can imagine that it is not easy to divide a given area (like a city) and create a number of "cells" at once. This would be tremendously costly and inefficient. Instead, when a given cell becomes too crowded, the cell is then split into two or more cells so that the number of users per cell is lowered. The challenge in this approach is that we have to re-arrange the way frequencies are allocated to each cell (i.e. frequency planning). In practice this is not an easy task!

Now that we have established the basic concept of interest for our work, we can focus on the main topic of our study: the air interface layer of the wireless network.

3.7 The Air Interface (AI)

The air interface is the part of the system where many users have to share a single medium, the RF channel. As we mentioned before, in the mid 40s mobile phone systems began with Frequency Modulation (FM) based technology in which users were multiplexed in frequency. When the cellular concept was introduced, the system architecture was radically different but the data modulation remained FM-based (about 30 KHz bandwidth per user) with a multiplexing scheme still based on FDMA. This technology represented the so-called **first generation** systems known as the Advanced Mobile Phone System (AMPS). By the time this technology had penetrated the market (around 1979) the maturing of micro controllers and digital signal processors allowed a new generation of cellular systems that came to be known as "digital" cellular. The term digital here referred to the fact that the user's speech was digitized, compressed, and then encoded using a digital data modulation technique such as Phase Shift Keying (PSK). This formed the basis for what we know as the **second generation** cellular systems.

When evolving from the first to the second generation system, not only did the data modulation change from analog (FM) to digital (PSK, MSK, etc), but the multiplexing of the air interface changed to one based on Time Division Multiple Access (TDMA). This technology penetrated the market by the late 80's and early 90's with tremendous success due to its increased capacity. In this system, 3 or more users would share the same frequency channel! It is not hard to see how the cellular network operators where able to place more users in the same RF bandwidth.

Some time around the mid 90's a new air interface mechanism was introduced. This multiplexing scheme was not based on TDMA nor FDMA but on using Code Division

Multiple Access (CDMA). Although this air interface was distinct and caused vast changes in the system, this technology was still considered a second generation cellular setup. Note that although the third generation of wireless has not yet been defined, in all likelihood it will be based on CDMA.

At this point, it is very important to highlight the relevant characteristics of each of these AI concepts. This can be illustrated by reviewing the three most widely used wireless standards deployed in North America: AMPS, IS-54, and IS-95. Although these standards are not the only ones in use throughout the world, they are representative of the three multiple access techniques of interest: FDMA, TDMA, and CDMA.

3.8 Advanced Mobile Phone System (AMPS)

Even though this technology is no longer considered "advanced", there still exists a large number of users relying on AMPS. In fact, this is still the default backup system for all other digital phones (TDMA, GSM, and CDMA)!

Mobile phones based on the AMPS technology are often referred to as *analog* phones because the voltage levels from the voice waveform $\lambda(t)$ is used to directly modulate the frequency of a data carrier (i.e. baseband sinusoidal waveform). This basedband signal is then used to modulate an RF carrier ω_{c1} that up-converts the signal to the 800 MHz band allocated for cellular communication. In **Figure 3.8-1** we also see that each user occupies a 30 KHz band centered at a given carrier located somewhere in the 800 MHz band. Although each user only requires a 25 KHz bandwidth, a 5 KHz ***guard band*** is given to avoid inter-user interference. This is a known drawback for all FDMA systems.

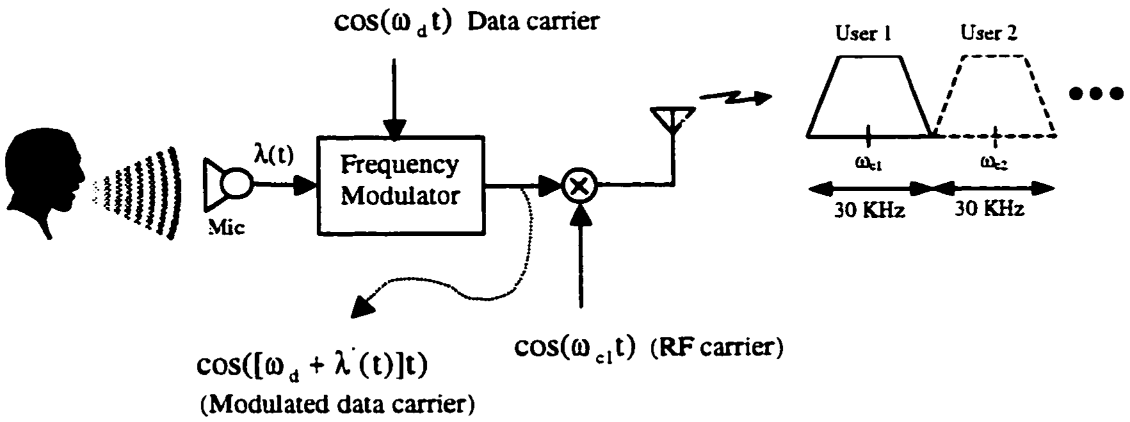


Figure 3.8-1 Simplified AMPS system (FDM Access)

Note that in this system we have carefully designed the frequency domain characteristics of the transmitted signal so that users will be essentially orthogonal in the frequency domain. However, no restrictions have been placed on the time domain signal. In sum, users overlap in time but not in frequency.

3.9 North American TDMA (IS-54)

When the digital technology of microprocessors, DSPs, and ASICs was mature enough to make its debut in the wireless world, there already existed a large number of AMPS users in North America. In order to make a graceful evolution from analog to digital systems the market adopted a standard that would be "RF compatible" with AMPS. Today such standard is known IS-54 or also known as North American TDMA. Meanwhile in Europe, the low population of mobile users allowed the creation of a standard that did not have the RF compatibility restriction. This standard is known as Global System for Mobile (GSM) communication. Given that both GSM and IS-54 are based on a TDM access technology, we will focus only on IS-54 to illustrate the TDMA issues of interest. For an excellent review of GSM and wireless standards evolution see **[Gar]** and **[Lee]**.

This digital system (or second generation wireless), is based on a number of new elements that interact very closely:

1. Use of a vocoder
2. Error correction
3. Interleaving
4. Digital data modulation
5. Shaping filters

All of the above elements allowed cellular systems to evolve from AMPS – FDMA to Digital TDMA. One of the major implications introduced by the above elements is that unlike AMPS where the signal is processed on a "sample-by-sample" basis, we now have to use a "block" processing philosophy. This is very much reflected in the TDMA air interface itself.

When we looked at AMPS we said that the user signal was restricted in the frequency domain with basically unlimited support in the time domain, TDMA on the other hand, places restrictions on the time domain. The time-domain signal formatting imposed by TDMA is shown in **Figure 3.9-1**:

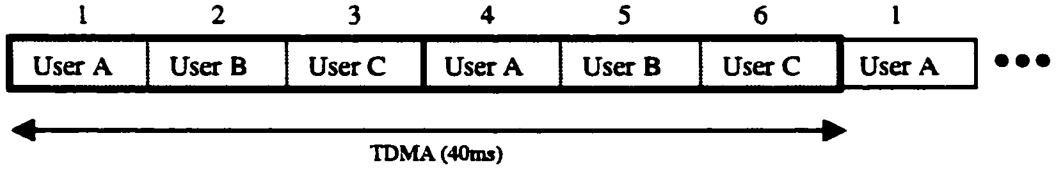


Figure 3.9-1 TDMA slot format

Here we can see that the timeline has been divided in *frames* of 40 ms, which are further subdivided into *slots* of 6.667 ms. **[IS-54]**. In theory this scheme can support 6 users (one per slot), however vocoder technology has not advanced enough to make this a reality. As of today, it takes two TDMA slots to transmit 40 ms of speech.

At this point it is useful to briefly view a simplified DSP block diagram of this system to appreciate the signal processing implications of this multiplexing scheme (**Figure 3.9-2**):

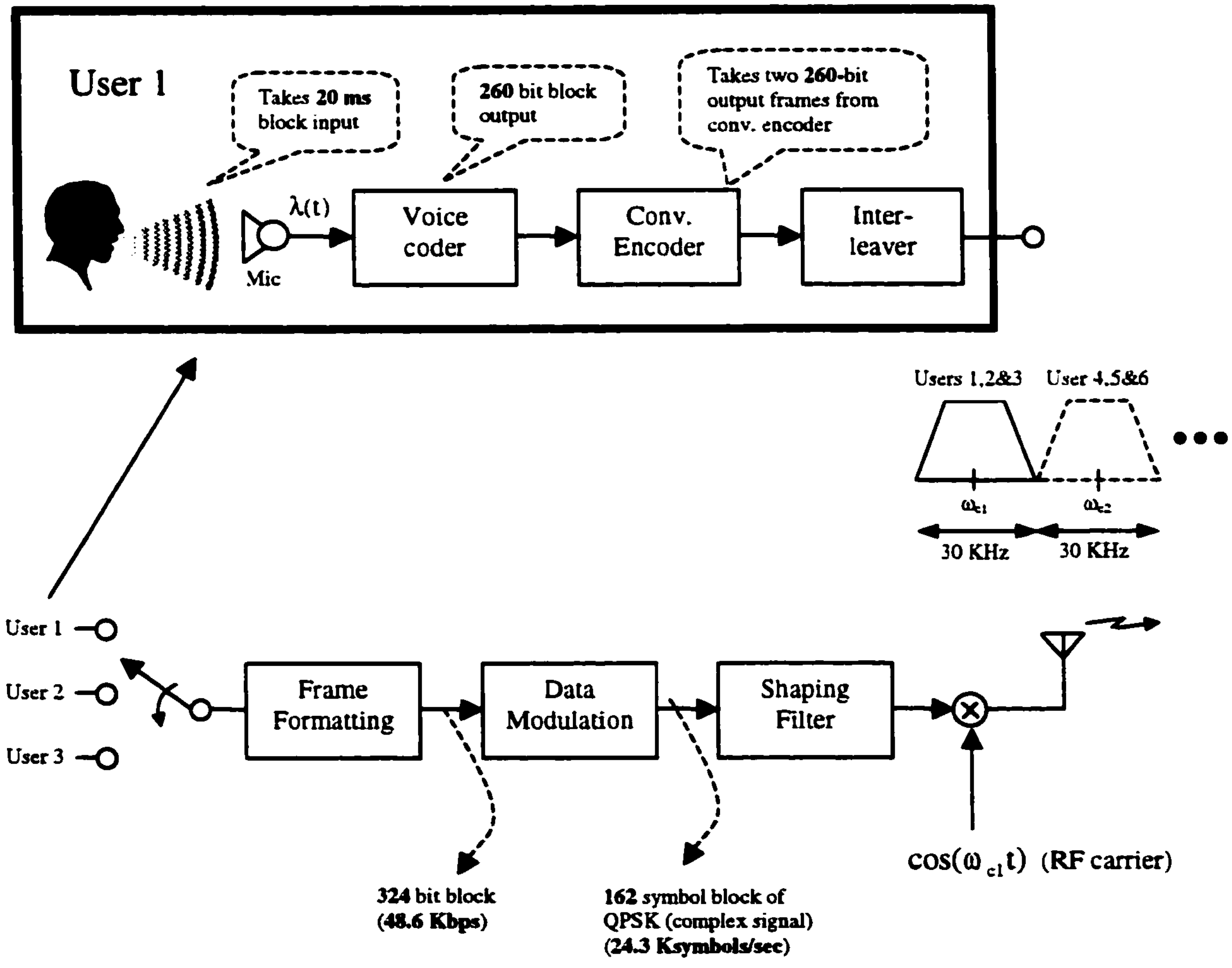


Figure 3.9-2 Simplified IS-54 system

The are several observations we can make about the system shown above:

1. More complexity than AMPS.

2. The data modulation is based on a bit stream which has no correlation to the user speech. While the signal $\lambda(t)$ has time variant spectrum with very non-stationary characteristics, the resulting bit stream used to modulate the phase of the data carrier has spectral characteristics (which are quite stationary). *The relevance of this observation is that for our simulation purposes we can simply replace all blocks prior to the data modulation block with a pseudorandom binary generator.*
3. This system has created the need to introduce ***time guards*** and synchronization sequences in the bit stream to maintain the slot sharing requirements. Note how the data rate is sharply increased from 260 to 324 bits after the Frame Formatting block has been applied (about 25% increase). This is mostly due to the introduction of TDMA.
4. While this system is generally referred to as TDMA, is really a combination of TDMA and FDMA. Since we can place no more than 3 users in one frequency carrier, this clearly does not provide adequate supply for the capacity requirement needs of a cell or even a sector. Even though a frequency channel is shared by 3 users via TDMA, the entire ensemble of users in a cell is also sharing the system via FDMA. **Figure 3.9-3** illustrates the difference between AMPS (analog FDMA) and

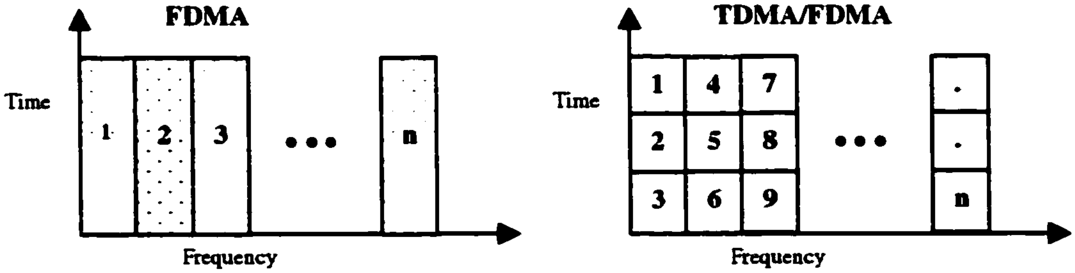


Figure 3.9-3 Time and frequency multiplexing schemes

IS-54 (digital – TDMA/FDMA). The relevance of this observation is that we not only have to introduce "guard time" to keep the 3 users from causing mutual interference in time but we must also have "guard bands" to prevent mutual interference in frequency.

3.10 CDMA (IS-95)

The evolution from analog (FDMA) to digital (TDMA) wireless is almost a repeat of the evolution observed in the commercial wireline telephony systems. One can almost say that this evolution was expected. The same statement cannot be made about CDMA.

During the early 90's, a very distinct industry standard (IS) began to penetrate the wireless cellular industry: IS-95. The primary difference found in this standard was its air interface. Although this standard arrived after IS-54, it is still considered a "second generation" system. It had the same basic elements introduced by the digital philosophy: vocoder, error correction, interleaving, and digital modulation. However, there were at least two very unique aspects about this new standard:

1. In this system the signals corresponding to different users would overlap both in time and frequency. As we will see shortly, this is based on the principles of "orthogonal transformation" and spread spectrum communication. Signals from various users are "en*coded*" by making use of special waveforms (hence the term *Code* Division Multiple Access).
2. Another characteristic that makes IS-95 distinct from AMPS and IS-54 is the fact that the transmission is based on a *spread spectrum* scheme (the signal from one user spreads over 1.2288 MHz).

In **Figure 3.10-1** we illustrate the conceptual comparison between FDMA, TDMA, and CDMA multiplexing principles using the case of 4 users.

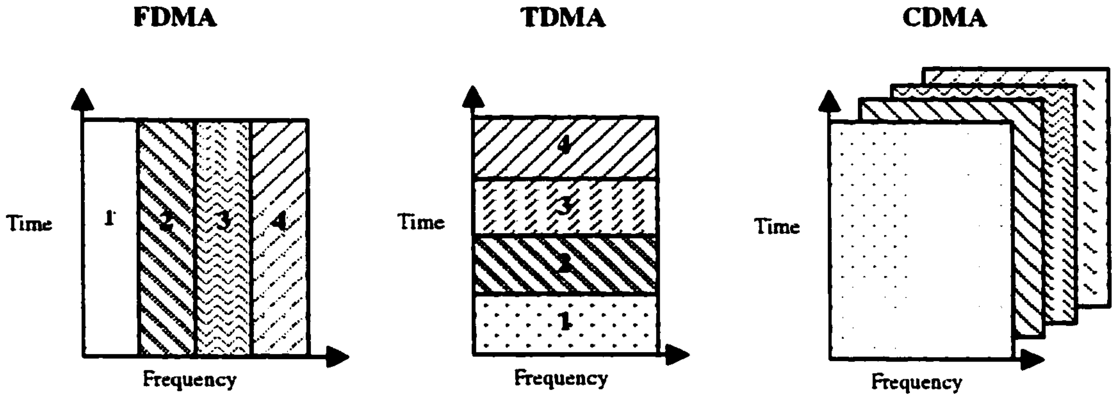


Figure 3.10-1 Conceptual illustrations of FDMA, TDMA, and CDMA.

If we think about time and frequency as dimensions or spaces, then we could say that in CDMA users are multiplexed in a third dimension.

Figure 3.10-2 shows a simplified signal processing diagram of the forward IS-95 channel. We may describe this block diagram by referring to the various points labeled **A** to **F**.

A) Here we can see that the vocoder follows the standard 20 ms block input. While the output data rate of the vocoder is variable, the subsequent encoding and interleaving ensures that the final output stream has a fixed 19.2 Kbps rate.

B) At this point, the bit stream has been scrambled with a PN sequence that is unique for each user (associated with the phone serial number). This PN sequence is typically known as the "long code" (long compared to the 15-bit PN code used for spreading). We can regard this process as some sort of encryption that leaves the data rate unaltered.

C) Here is where each bit coming from point B) is multiplied by a waveform which corresponds to one of 64 Walsh codes. Since each Walsh code has 64 bit periods we then raise the bit rate to 19.2K x 64 which results in the 1.2288 Mbps rate specified in the

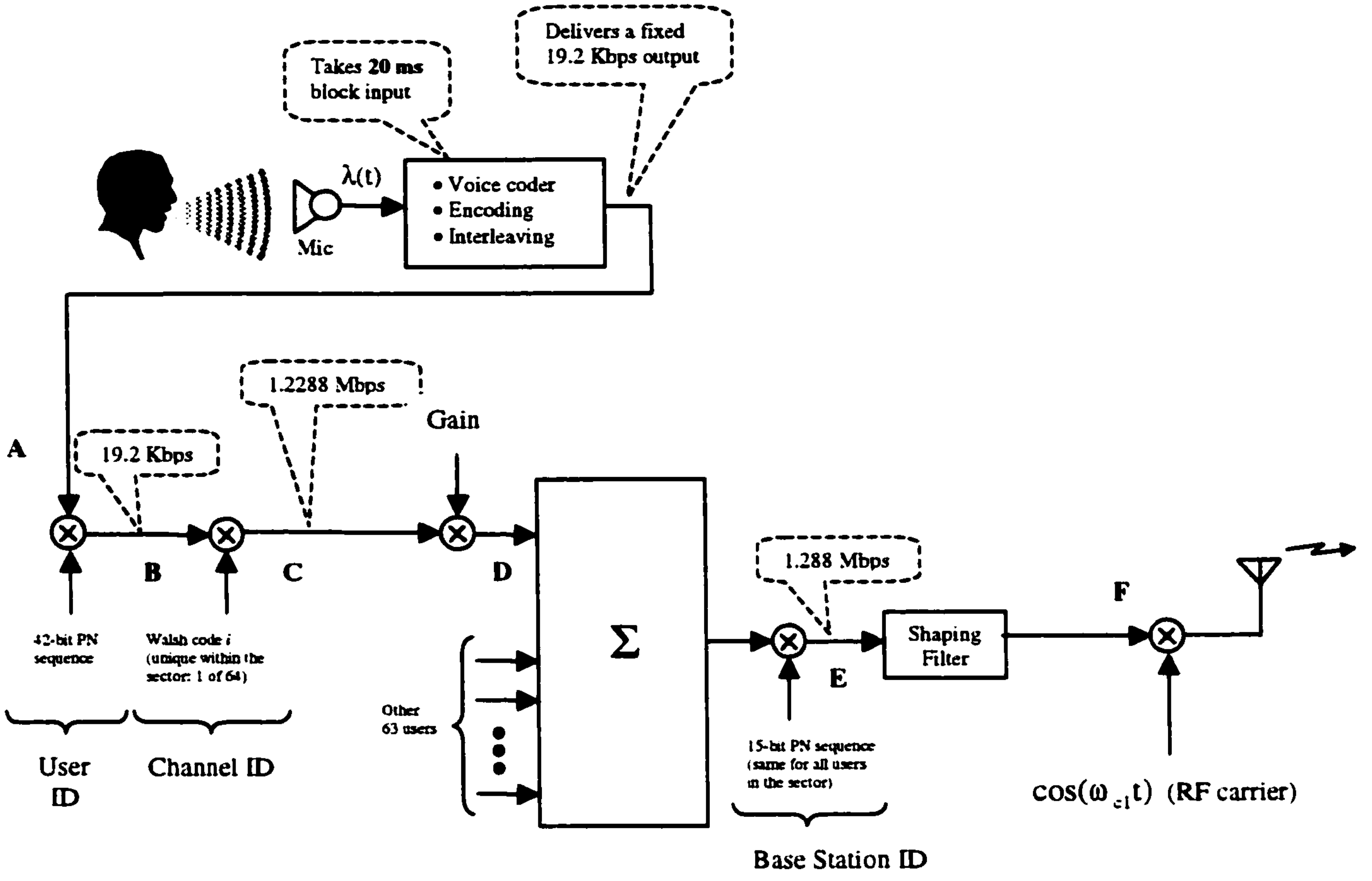


Figure 3.10-2 Simplified IS-95 Base Station Transmitter.

standard. It is customary to refer to the Walsh bits are "chips" to differentiate them from the actual data bits. Hence for each bit of information we send an entire waveform that belongs to an orthogonal set, **this is referred to as Synchronous CDMA [Sou]**. We'll expand on this topic in subsequent paragraphs.

D) We can see that the signal at point C) has a symbol rate that is much larger than that of the information rate (64 times larger). This does not appear to be very efficient. However, this signal can be summed with the other 63 signals because they all form an orthogonal set. By applying a gain factor to a given channel we are ensuring that this channel has more power and hence is easier to detect, i.e. has higher Energy-per-bit/noise ratio. This is part of the well-known power control scheme of IS-95. For instance, one of the 64 channels of each base station is dedicated to send a pilot signal which contains 25% of the total transmitted power to facilitate its detection. Once all the signals from all 64 channels have been summed, they can be sent over a single carrier. In theory we can have up to 55 users (the other 9 channels are for support), however, in practice only a fraction of that figure has been achieved **[Haf]**.

E) This is the point when the signal has been "spread" by multiplying the incoming signal with a 15-bit PN sequence of "minimum length" (typically known as M-sequence). This PN sequence is referred to as the "short code" or "spreading code". ***It is important to highlight that this PN sequence is shared by all users in the forward channel and hence does not play a role in separating users.*** However, it plays a role in making the transmission unique to that base station.

F) Given that we could not transmit binary signals (an extremely large bandwidth), we need to apply a "shaping" filter so that the transmitted pulse contains the minimum amount of excess bandwidth. This filter is very close to the popular Square Root Raised Cosine (SRRC) which is similar to a sinc(x) function.

One of the key differences between IS-95 and other systems is that the reverse channel is processed differently. For instance, Walsh codes are not used to separate users in the reverse channel (PN sequences are used instead), and the error protection is higher. **Figure 3.10-3** shows a simplified signal processing diagram of the reverse IS-95 transmission, i.e. from mobile to base station.

As we did with the forward channel we may also highlight key observations about this block diagram by referring to the various points labeled **A** to **D**.

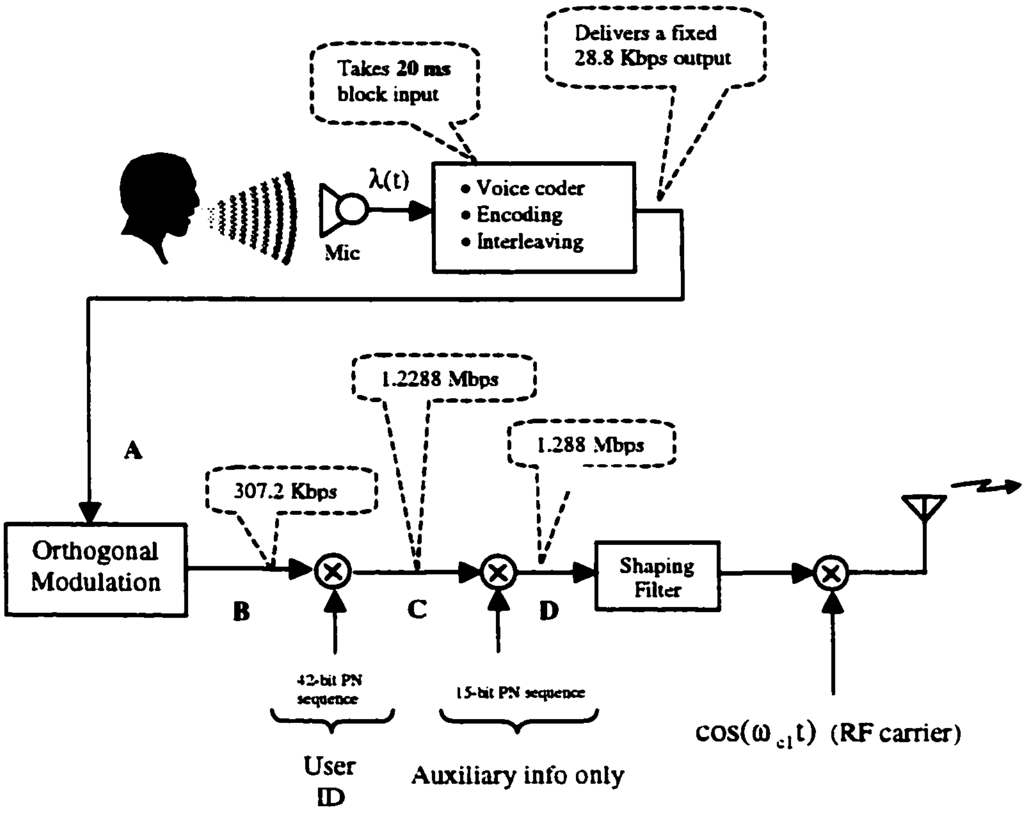


Figure 3.10-3 Simplified IS-95 Mobile Station Transmitter.

A) This is the same as in the transmitter but with enhanced channel coding of rate 1/3 to produce a fixed 28.8 Kbps rate.

B) A further form of signal encoding is applied to produce a signal with a higher rate. In this form of encoding we replace every 6 bits from the input with an entire row of the Walsh matrix (more details on this in section 4.6).

C) After the data stream has been encoded then a specific shift of the long PN code is used to spread the signal thereby raising the output rate to the standard 1.2288 Mbps. This is the part of the system that makes the signal "unique" not only in the cell but in general since the PN shift is based on the mobile user ID.

D) In this last section, the spreading code applied at the base station is also applied but with no offset. Thus all mobiles use the same code but only for auxiliary purposes **[Gar]**.

After our general review of the main characteristics of the IS-95 standard, it is worthwhile to focus on specific aspects of this system. We start with the generation of the Walsh codes and their use to multiplex users within a base station.

Walsh codes are rows or columns of the Hadamard matrix. These matrices can be recursively generated using the following formula:

$$A_n = \begin{bmatrix} A_{n-1} & A_{n-1} \\ A_{n-1} & \overline{A}_{n-1} \end{bmatrix} \quad \textit{where } \overline{A}_n \textit{ is the binary complement of } A_n$$

Let's generate the first few iterations using this formula:

$$[1] \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \ldots$$

Note however that in a communication system we do not use 1's and 0' but 1's and –1's to form a Non-Return to Zero (NRZ) sequence. Hence, every 0 in the above matrix is replaced with a "–1" entry. Now that we have a clear view of what the Walsh codes look like, let's examine the multiplexing process (see **Figure 3.10-4**):

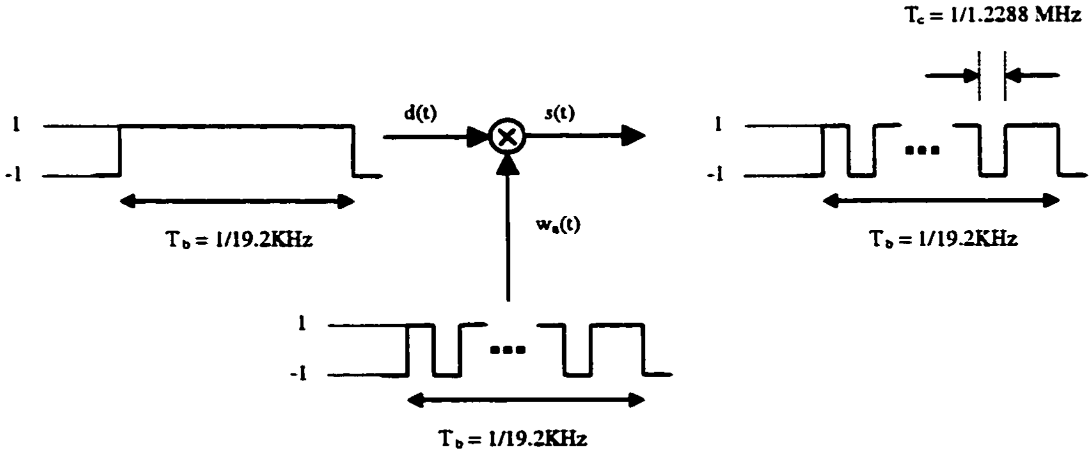


Figure 3.10-4 Multiplexing with Walsh codes.

We can see that since the signal *d(t)* can only take the values of 1 and –1 then the transmitted signal *s(t)* is either the original Walsh code, *w(t)*, or its inverted version *–w(t)*. We can see that the resulting signal for user *n* may look like the following:

$$s(t) = [\ w_n(t)\ \ w_n(t)\ \ -w_n(t)\ \ w_n(t)\ \ w_n(t)\ \ \ldots\ ]$$

So far we have only talked about the transmission aspect, the question is, ***how do we recover the data?*** As in many other communication systems, the chosen method to recover the data is a "correlation receiver".

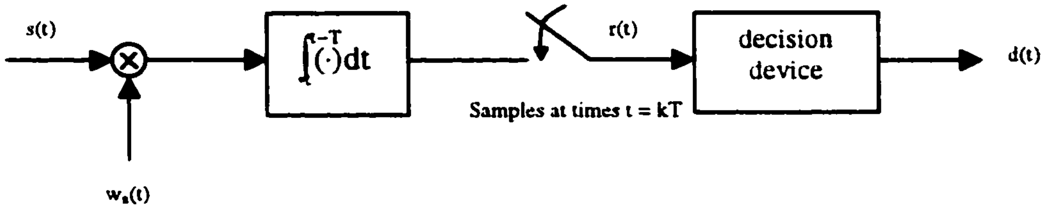


Figure 3.10-5 Correlation receiver structure.

Note that the signal before the sampler is really the dot product of s(t) and $w_n(t)$, i.e. $\langle s(t), w_n(t)\rangle$. Furthermore, we said before that s(t) was a collection of functions taking the value of $-w_n(t)$, or $w_n(t)$, hence the signal being sampled will be either $\langle w_n(t), w_n(t)\rangle$ or $\langle -w_n(t), w_n(t)\rangle$. Since this is the dot product of orthogonal functions, then the output of the sample would be either +1 or –1.

If the incoming signal s(t) contains a Walsh function which is not $w_n(t)$, but another row of the Hadamard matrix (i.e. another user), say $w_k(t)$, then the result of the dot product operation will be zero. This is how we detect the signal from a given user, even though it overlaps both in time and frequency with all other users. This entire process is referred to as *orthogonal spreading* **[IS-95]**.

Note that the reverse channel does not use Synchronous CDMA but ***Asynchronous CDMA [IS-95]*** *since the orthogonality conditions cannot be maintained due to channel degradation. This means that users are separated by the Pseudorandom Number (PN) sequence employed to encode them.* This is similar to the conventional military spread spectrum applications of the late 60's.

Aside:

It is interesting to point out that the popular *correlator receiver* can also be constructed with an alternative but equivalent structure known as the *matched filter*.

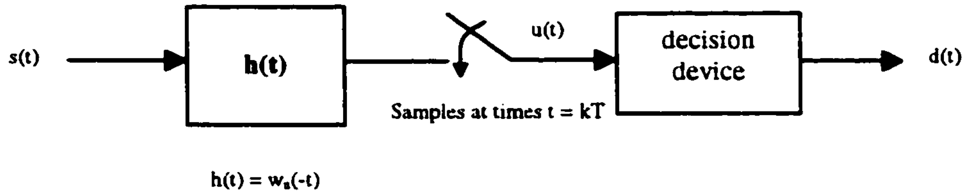


Figure 3.10-6 Matched filter receiver structure.

We can see that although the multiplication block appears to be removed, it has simply been moved into the filter. Let's consider the output of both structures, r(t) and u(t), from figures **3.10-5** and **3.10-6** respectively. The expression for r(t) can be formulated as follows:

$$\int_{t}^{t-T} s(t)w(t+\tau)d\tau \Rightarrow \pm \int_{t}^{t-T} w(t)w(t+\tau)d\tau$$

As we can see this expression coincides with the definition of the cross-correlation operation, hence the name correlation receiver. Given that in this case s(t) is really +/- w(t) the result looks more like an autocorrelation, however in practice the incoming signal s(t) is equal to w(t) with other components such as additive noise, multipath, and interference. Note that the switch or sampler has been modeled by taking the "definite" integral, while assuming synchronization to let us know when to begin integrating. This structure is also called "integrate and dump".

Similarly the expression for r(t) is simply the convolution of s(t) and h(t):

$$\int_{t}^{t-T} s(t)h(t-\tau)d\tau \Rightarrow \pm \int_{t}^{t-T} w(t)w(t+\tau)d\tau$$

However we know that s(t) is +/- w(t), and h(t) is the time-reversed version of w(t). Hence we can manipulate the signs and arrive at the same correlation expression.

This matched receiver structure formulation and this equivalence to the correlation receiver structure shows that in this case the "multiplication" operation can be replaced by the "filtering". This however is specific to our context, *the filter is the time-reversed version of the multiplicative function and the duration of the impulse response is exactly equal to the bit period.* This observation will become useful in subsequent chapters.

3.11 The Quest for Capacity

As we have seen in this chapter, much of the effort motivating the evolution of wireless systems is driven by the need to allow more users to share the same communication channel. We have seen that the spectrum efficiency is only one of several factors related to system capacity. In some cases the spectrum efficiency appears to be the single driver. For instance, we saw how second generation systems such as IS-54 (TDMA) allowed 3 users to fit in the 30 KHz spectrum of one AMPS user. However, in other cases it is not so clear. For instance, in GSM each user takes about 25 KHz (compared to the equivalent of 10 KHz in IS-54) yet the capacity factor is not 2.5 but 1.5. This is due to the fact that GSM uses a vocoder with higher data rate output and hence it is more resistant to interference, which in turn leads to more efficient frequency reuse patterns.

A very common benchmark for comparison is the system capacity based on the spectrum required to fit 6 AMPS users. **Table 3.11-1** provides such comparison **[Haf]**.

It is interesting to note that although IS-54 is the least sophisticated of all second generation systems, it is the one that provides the highest capacity. IS-95 was expected to provide outstanding capacity gains, however the systems deployed thus far have yet to approach such expectation.

Table 3.11-1 Number of users for various wireless systems

AMPS	6 users (180 KHz reference)
IS-54 (North American TDMA)	18 users
GSM (TDMA)	12 users
IS-95 (CDMA)	12 users

New issues are being raised as the industry struggles to define "third generation" systems. The next generation of air interface must provide support for more than just RF bandwidth efficiency. Here are some those desirable features:

- Data support: packet-based, as opposed to circuit-based switching. This includes media access control protocols to support concurrent sharing of a given channel.
- Asymmetric links: a high/low data rate configuration for the forward/reverse channel.
- Variable data rates: ability to change data rates during a given access session.
- Dynamic growth: easily re-configurable networks that can change size and density within days (not months).

At this point we have studied the most relevant wireless air interfaces being used in the cellular industry. Now that we have highlighted their most relevant characteristics, operation, and deficiencies, we proceed to analyze the implications of using wavelets to support a multi-user cellular air interface. In the following chapters we will describe how wavelets can be used to multiplex users in a channel and explore issues that are specific to the use of wavelets in a cellular wireless application.

Chapter 4

Wavelets and Multiplexers

4.1 What is the Connection Between Wavelets and Multiplexers?

In chapter 2 we saw how the wavelet transform was used as a tool to decompose a signal into a set of constituent waveforms. More specifically we saw how the DWT generated a set of coefficients that could then be used to reconstruct the signal originally decomposed or analyzed. It was not difficult to see how this tool could be applied to problems related to signal analysis, compression, and de-noising. However, it is not so obvious to see how we could use this tool to allow many signals to share a communication channel without creating mutual interference. It is only natural to ask: ***what do wavelets have to do with multiplexing users in a communication system?***

In the traditional application of the wavelet transform, we feed a signal into a decomposition or analysis block to obtain the wavelet coefficients (as shown in top of **Figure 4.1-1**). These values can then be used to reconstruct or synthesize the original signal.

It turns out that if we reverse the order of these blocks and replace the wavelet coefficients with signals from independent users, they can be combined into one signal that can be later decomposed to extract (or analyze) the signal from each user (see bottom of **Figure 4.1-1**). In broad terms, this implies that the Inverse Discrete Wavelet Transform (IDWT) is being used to allow many users to share the same channel and hence is acting as a *multiplexer*. Similarly, the DWT is used to decompose the received signal into the original set of constituent waveforms (one per user) thereby acting as a *de-multiplexer*. Some of the leading researchers (**[Aka]**, **[Het]**, **[Dav]**, among others) who are actively investigating this multiplexing method have proposed that the above system be implemented using filter banks.

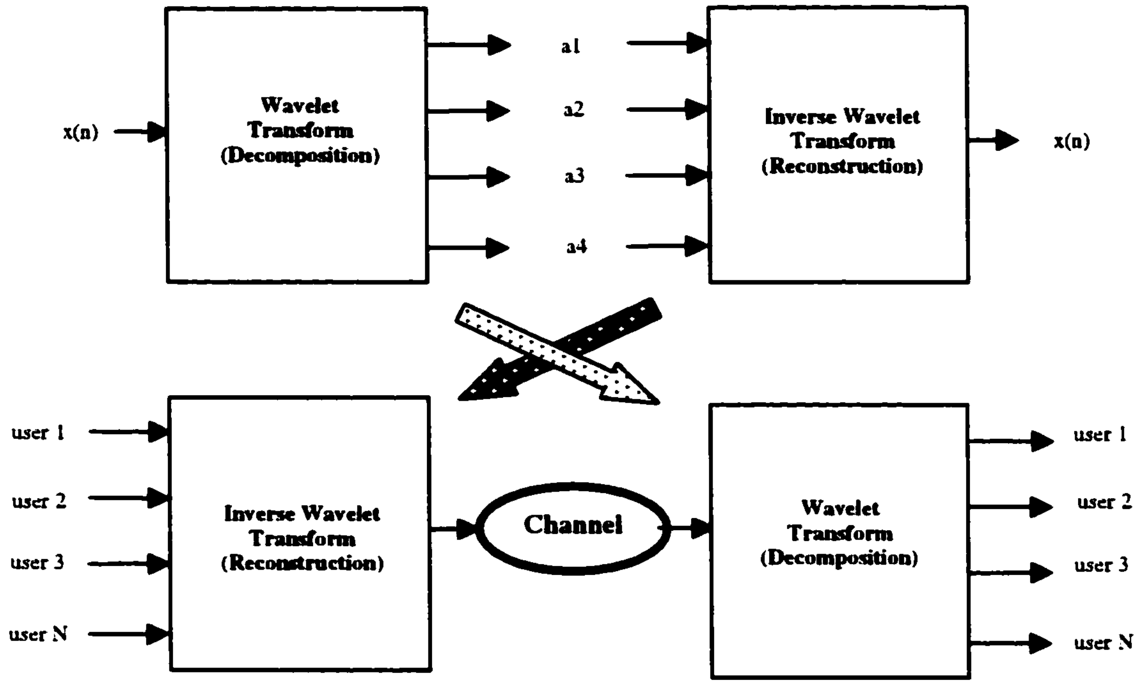


Figure 4.1-1 Traditional (top) and multiplexing wavelet application (bottom)

Recall that the filter bank structure has been proposed as a method of implementing the "Fast" Wavelet Transform (see Chapter 2). However, it should be noted that when the above structure is implemented with filter banks such structure is called a *transmultiplexer* ([Aka2], [Vet2], [Vai]).

At this point we have three independent concepts that are combined to produce the multiplexing scheme shown in **Figure 4.1-1**:

- **Wavelets**
- **PR-Filter Banks**
- **Transmultiplexers.**

The connection between the first two concepts has already been explored in Chapter 2, so the question now is *what are transmultiplexers and how do they relate to wavelets?*

4.2 Transmultiplexers

Prior to the development of digital circuits, telephone systems multiplexed users based on an FDM scheme **[Bel]**. Each user would "own" a 3 KHz channel that would be transmitted using Single Side Band (SSB) modulation. As the digital telephone switch began to appear, the multiplexing scheme was changed to TDM. In this case users would be multiplexed in time but with very demanding timing arrangements. This meant that signal processing was basically done on a sample by sample basis (that is still the case today). One can easily imagine that the transition from analog to digital switching technology would require many scenarios in which we would need to move signals from time multiplexing to frequency multiplexing and vice versa. The structure used to perform this conversion was the transmultiplexer **[Vai]**. One may speculate that the term ***transmultiplexer*** was chosen because this structure allowed a *trans*ition from time to frequency *multiplexers*.

Before we attempt to illustrate how a conventional transmultiplexer can convert TDM into FDM and vice versa, it is useful to examine first how a simple multi-rate structure can be used to realize a TDM system. **Figure 4.2-1** illustrates how 4 users represented by signals $\mathbf{x_1(n)}$, $\mathbf{x_2(n)}$, $\mathbf{x_3(n)}$, $\mathbf{x_4(n)}$, are being multiplexed in time to form one signal, i.e. $\mathbf{y(n)}$. There are a few key observations that we may want to highlight about the system depicted in **Figure 4.2-1.**

1. Each user signal is up-sampled by 4, hence only modulo 4 samples are non-zero. This means that we have increased the data rate by a factor of 4 and therefore have created 3 more mirror images of the spectrum (one for each inserted zero).

2. After up-sampling every signal we stagger the 4 signals by placing a delay of one sample relative to its neighboring user. This is in fact the block that prevent users from causing mutual interference.
3. The signal **y(n)** has a bandwidth which is 4 times that of each user.

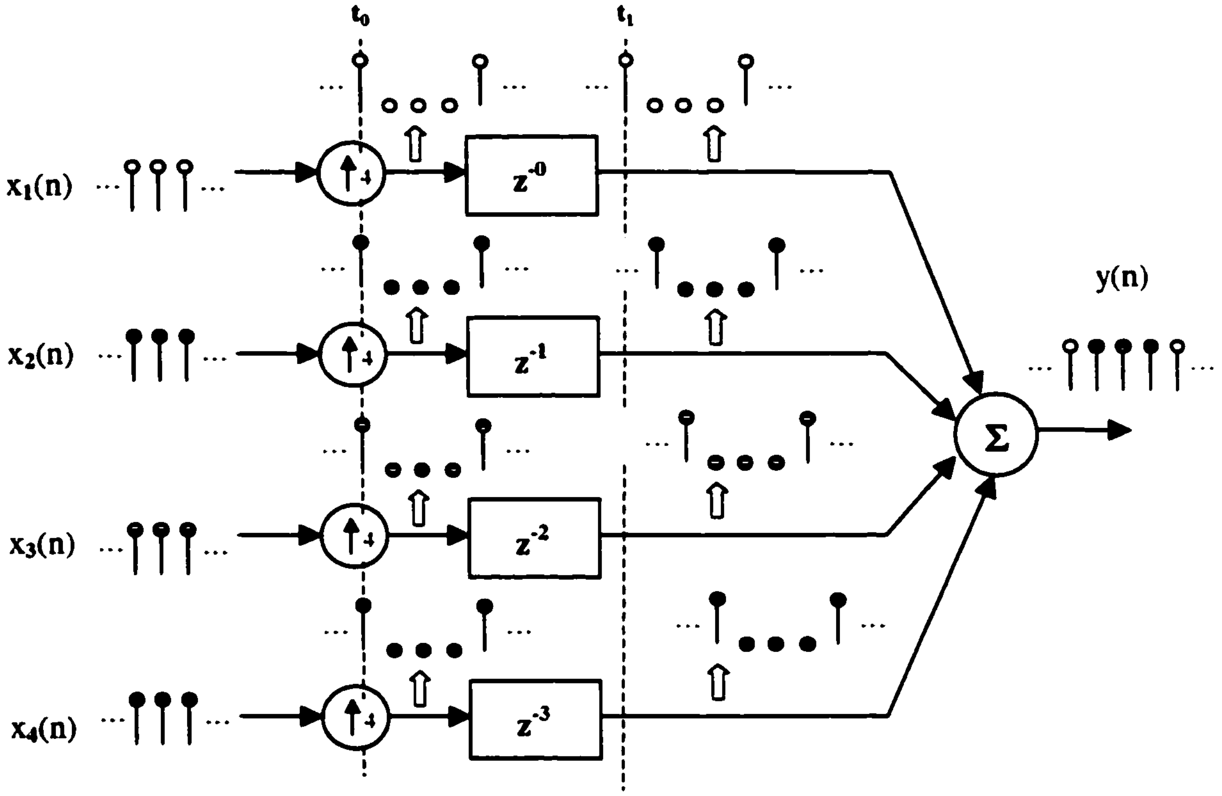


Figure 4.2-1 Simple multi-rate system to realize a 4-user TDM system.

4. The de-multiplexing system at the receiving end would simply be the reverse of the structure shown in **Figure 4.2-1**.

5. Note that all users **must** be synchronized. If such synchronization is lost there would be no way of separating each user without full or partial loss of information. **This is clearly a *time-variant* system.**

Now that we have reviewed the mechanics and basic characteristics of this system and its intermediate signals, let's consider a small variation of such system: *replacing the delay blocks with bandpass filters.* The lack of carefully placed delays would make all users overlap in time, however the filters introduced would separate the users in the frequency domain (hence creating an FDM system).

Let's consider a system whose input is a TDM signal such as **y(n)** (see **Figure 4.2-2**). In this block diagram we see that the up-sampling by 4 generates images of the frequency domain (note that: these are discrete signals). The use of ideal filters that are carefully allocated in the frequency domain allows us to select the spectral image we wish to retain. As shown in **Figure 4.2-2** the resulting signal **Z(ω)** contains the information of all 4 users clearly separated in the frequency domain. This shows how this structure can convert the incoming signal from TDM to FDM. It is only expected that the receiving end is the inverted structure of the multiplexer. In the de-multiplexer case we have a filter bank followed by "down-samplers".

Note however, that in our discussion we have assumed that the filters $\mathbf{H_1(z)}$ to $\mathbf{H_4(z)}$ are ideal. This was a necessary requirement to recover only the spectral image of the desired users. It is not hard to imagine that if such filters are non-ideal (see comparison in **Figure 4.2-3**), then some of the energy from a given user will leak into the neighboring frequency bands thereby causing interference (this is often referred to as *crosstalk*).

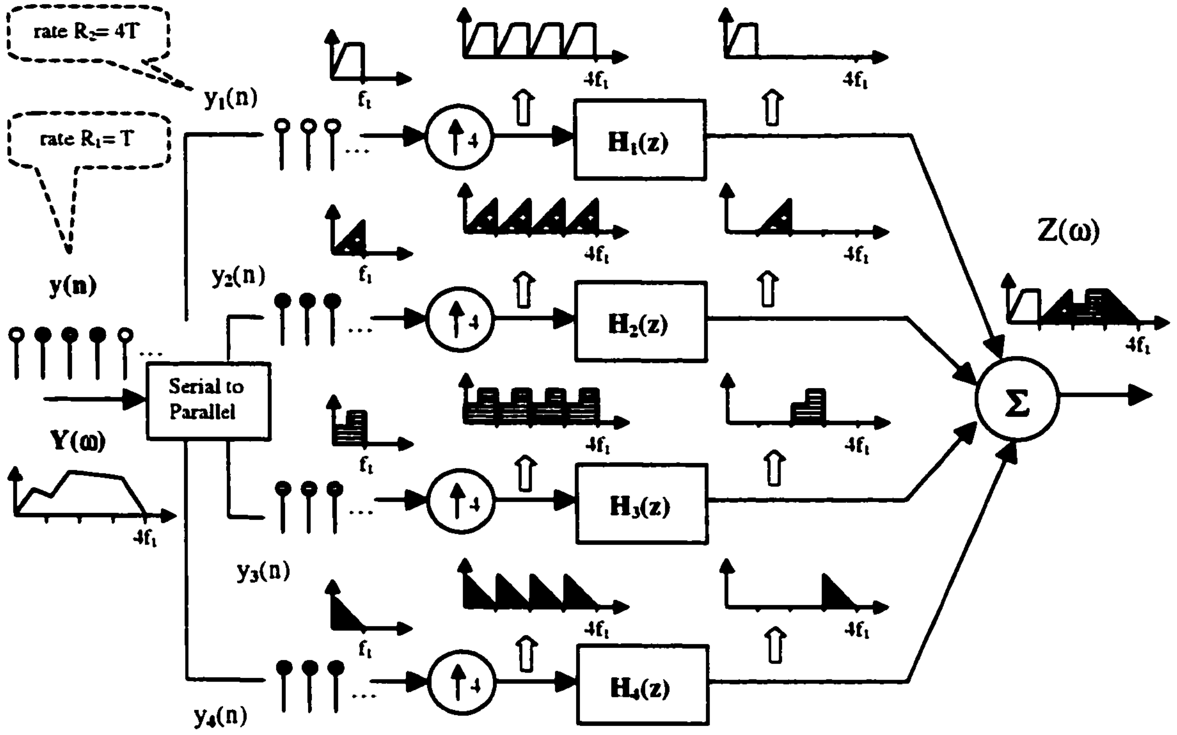
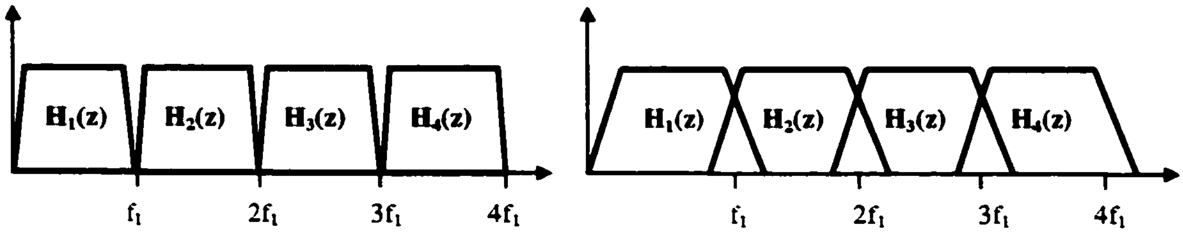


Figure 4.2-2 Simple multi-rate system to realize a 4-user FDM system.



(a) *(b)*

Figure 4.2-3 Ideal *(a)* and non-ideal *(b)* frequency response of filters $H_1(z)$ to $H_4(z)$.

A clear requirement to build adequate multiplexers was the design of very sharp filters with some separation between bands ("guard bands"). *The design effort was based on "minimizing" the amount of out of band energy for each user.* Naturally this would lead to smaller guard bands and hence a more efficient use of the spectrum.

During the late 80's, M. Vetterli made a remarkable contribution to the notion of transmultiplexers [Vet3]. This novel approach suggests that rather than attempting to suppress the out-of-band energy (i.e. making ideal filters), we could instead actually use non-ideal filters with significant overlap. This overlap however, would not be an arbitrary one. The suggestion was that by carefully designing the overlap among users, such overlap would be "cancelled" at the receiver. It turns out that the strategy to achieve this cancellation of interference is the same as that used to achieve "perfect reconstruction". In other words, M. Vetterli made the link between PR-filter banks and transmultiplexers.

The relevance of this new strategy is that rather than attempting to achieve suppression of out-of-band energy, we would instead achieve its cancellation. The benefits of this approach are:

1. We could use sub-optimal filters (i.e. fewer taps).
2. Save spectrum by avoiding the need for guard bands.
3. Avoid time guards by allowing signal to overlap in time.

Note that in this scheme the signals overlap both in time and in frequency and therefore such signals can only be de-multiplexed by using a third feature: the filter taps. Although we talk about filter taps, we are really referring to a collection of basis functions that form an orthogonal set. As we saw in Chapter 2, the filter taps for each branch are simply the discrete-time approximation of a continuous member function from an orthogonal set. This leads to the exploration of wavelets, as being the basis functions used to generate the filter taps of a transmultiplexer. In the next section we will see how wavelets can take transmultiplexers a step further by providing a template for a "universal" multiplexer that treats TDM, FDM & CDM-based multiplexers as special cases.

4.3 Wavelet-based Multiplexer

Now that we understand the concept of a transmultiplexer and how it can be used to convert TDM signals into FDM and vice versa, we can take this concept a step further. Let's consider a case in which the input is not one TDM signal that is distributed among many input branches of the transmultiplexer, but many independent signals in a multi-user environment. Furthermore, consider the case in which the filter coefficients are derived from wavelet basis. Making this suggestion raises many questions such as:

- How do we construct such a multiplexer?
- What is the output bandwidth?
- Does it matter which wavelet "order" we use?
- How do we relate this multiplexing scheme to those we already know?

Let's attempt to explore the above questions by building a simple 4-user system based on the Daubechies wavelets since they are perhaps the most mature and well studied.

Before we proceed to describe the wavelet multiplexer structure, let's bridge a few gaps by recalling a few concepts. For many of us, the first gap in understanding the use of wavelets for multiplexing comes from the fact some of the literature (**[Vet1]**, **[Wu1]**, **[Orr]**) tells us that *each user is projected on the basis vectors that span over the wavelet space and that such vectors are defined by:*

$$\varphi_{a,b}(t) = \frac{1}{\sqrt{a}}\varphi\left(\frac{t-b}{a}\right)$$

Although the above statement is correct, it does not begin to tell us how we would go about building such a system nor does it give us a sense of how to relate this to systems that we already know (as those described in Chapter 3).

This first problem with the given description is that "continuous-time" definitions do not facilitate the understanding of digital implementations, particularly when we are trying to realize this with DSP-based systems. Recall that:

1. Wavelets are simply waveforms of finite duration. Below we can see some examples of simple waveforms that are part of a family of Daubechies wavelets.

Figure 4.3-1 Daubechies wavelets of orders 1, 2, 3, and 7

2. In chapter 2 we formulated the link between the above continuous-time waveforms and a set of discrete-time coefficients.

$$\psi(t) \quad \Leftrightarrow \quad h(n) = [\, h_1,\ h_2,\ h_3,\ h_4 \ldots]$$

(continuous signal) (discrete signal)

3. These coefficients can be used to construct the filter bank structure that can continuously split a signal into a high and a low frequency component. In chapter 2 we showed that if we recursively convolve **h(n)** with an up-sampled version of itself then we would approximate the continuous-time signal **ψ(t)**. **Figure 4.3-2** depicts a simple 4-branch filter bank structure to implement the Forward and Reverse Fast Wavelet Transform.

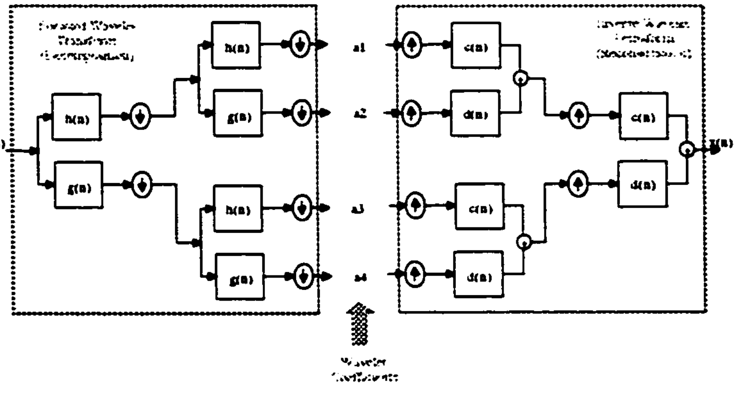


Figure 4.3-2 Forward and Inverse Fast Wavelet Transform (4 bands).

We can also recall that although we only know one set of coefficients, i.e. h(n), all other filter responses are easily derived. Consider the following example of a 4-tap filter.

$$h(n) = [\, h_1,\ h_2,\ h_3,\ h_4 \,] \Longrightarrow \begin{array}{l} g(n) = h_4, -h_3, h_2, -h_1 \\ c(n) = h_4, h_3, h_2, h_1 \\ d(n) = -h_1, h_2, -h_3, h_4 \end{array}$$

4. By inverting the sequence of the blocks shown in **Figure 4.3-2** we obtain the desired multiplexer and demultiplexer stages of our system. **Figure 4.3-3** depicts the block diagram for the multiplexer.

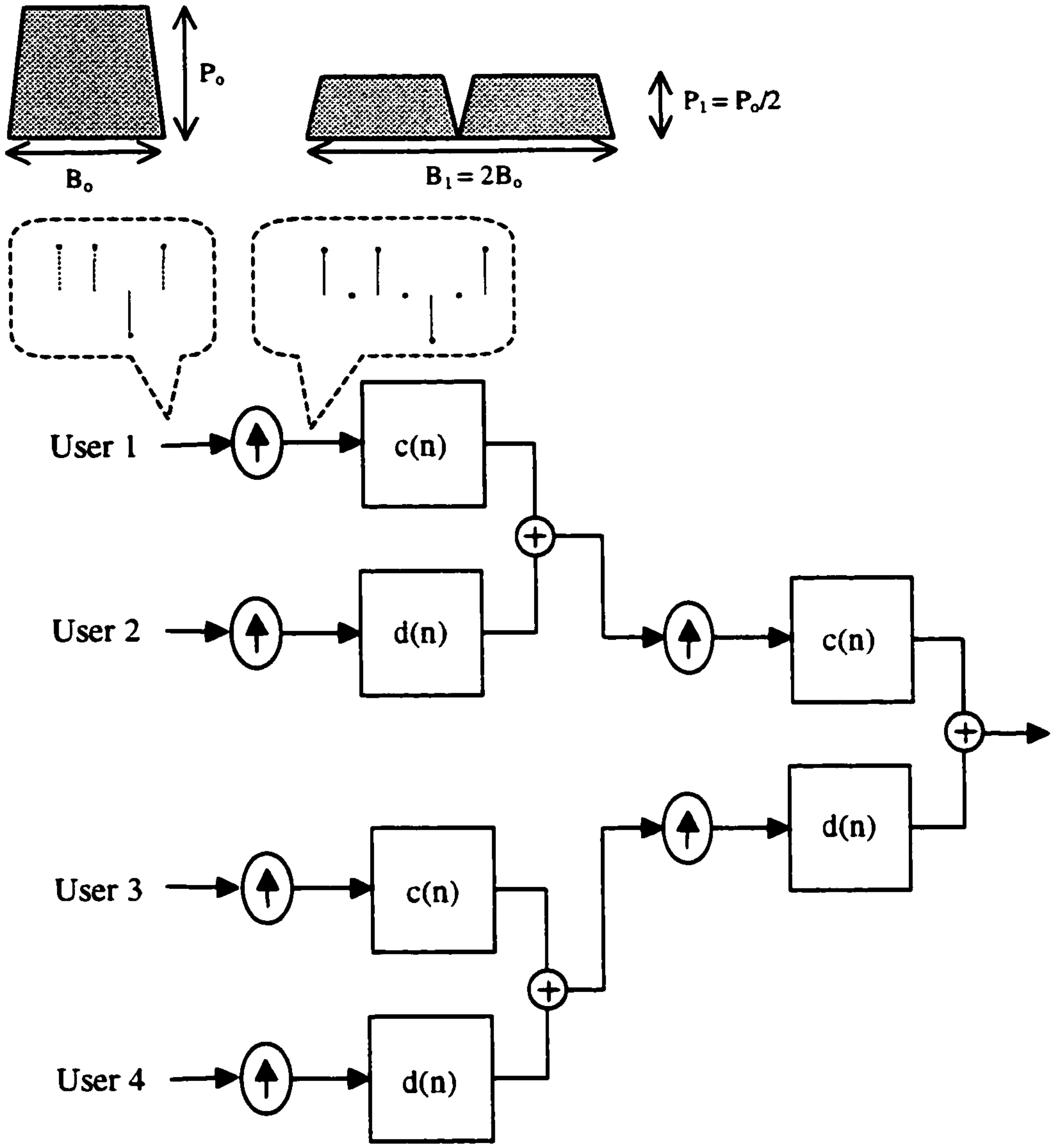


Figure 4.3-3 4-user wavelet-based multiplexer.

It is very important to recall that, as shown in **Figure 4.3-3**, every time we up-sample (i.e. insert a zero every other sample) we double the bandwidth and divide the power in half. As we spread the bandwidth with redundant copies of the original spectrum (via up-sampler) we then shape it with the subsequent filter (i.e. c(n) and d(n)).

Although we have shown how to construct a wavelet-based multiplexer, this structure does not lend itself to support a system in which users are located in different locations (such as cellular system). In other words, this structure can support Multiplexing, but not Multiple Access (MA).

So the question is *how do we move the structure depicted in **Figure 4.3-3** to the filter bank with individual processing entities?*

Thanks to the concept of "Noble Identities" **[Vai1]** described in Chapter 2 we can modify the given structure to obtain individual structures.

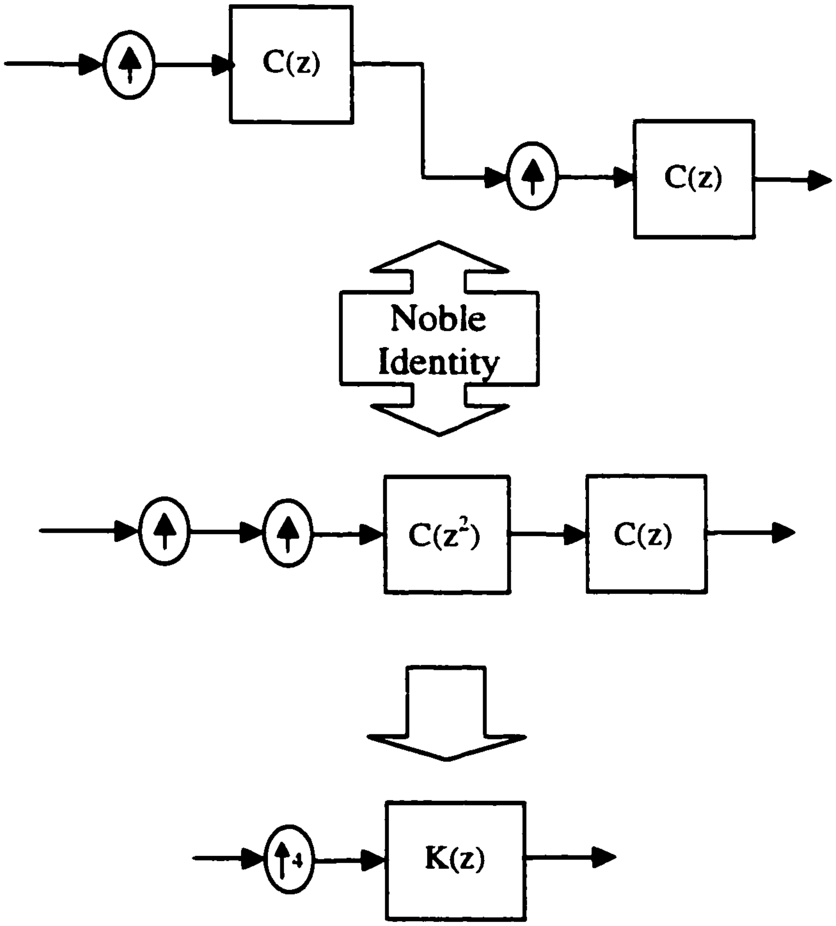


Figure 4.3-4 Use of noble identities to reduce branch for user 1.

Using the process shown in **Figure 4.3-4** we can then produce the final version of the wavelet-based multiplexer that matches the structure of the conventional transmultiplexer shown in **Figure 4.2-42. Figure 4.3-5** shows final structure with filters **$k_1(n)$** to **$k_4(n)$** for the equivalent filters corresponding to each user branch after applying the noble identities.

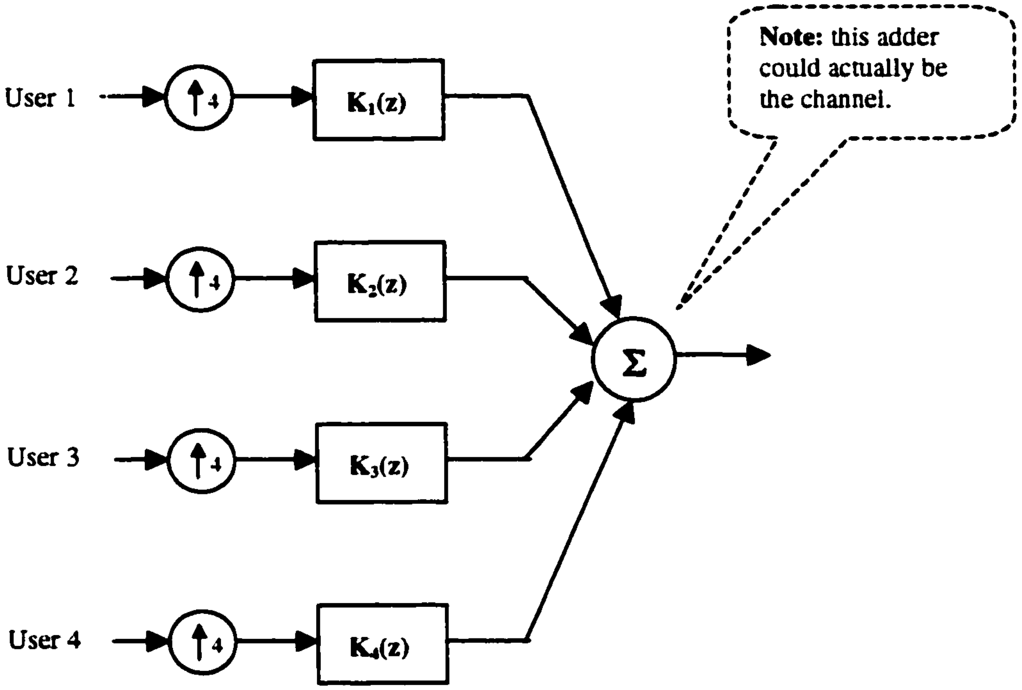


Figure 4.3-5 Four-user Wavelet-based multiplexer.

Note that if the filters **$k_1(n)$** to **$k_4(n)$** are designed to be very sharp filters we obtain a Frequency Division Multiplexing (FDM) system, similar to that shown in section 4.2 depicted in **Figure 4.2-42.** If on the other hand, these filters do not have a sharp response but are in fact spread over the entire band, then we obtain a system in which signals from different users overlap both in time and frequency. This represents a system with the characteristics of a CDM scheme. Finally, if the filters are simple time delays, we then obtain a TDM system.

We have now seen how wavelets extend the transmultiplexer functionality to provide a type of universal multiplexer, i.e. a system that can multiplex users as TDM, FDM, or CDM by simply changing the filter response of each branch! This is a very helpful understanding.

We began our analysis by thinking of wavelets as a "different" scheme to multiplex users, however, now we can view this concept as part of a more structured approach based on multi-rate signal processing. Subsequent sections will reveal connections that are very significant yet not obvious.

4.4 The Simple Case

In chapter 2 we discussed the "order" in the Daubechies wavelet family. We stated that wavelets with low order were basically very simple waveforms that would produce very slow cut-off filters. The simplest of all is the Daubechies of order 1, also known as the Haar **[Str1]**. The filter coefficients associated with this wavelet are shown below:

h(n) = [1 1]

It is not hard to see that this is the simplest of all lowpass filters (which is basically doing a moving average with a memory of two samples).

As stated in Chapter 2 and in section 4.3, we can obtain the other three set of filters to build our multiplexing structure:

g(n) = [-1 1]　　**c(n) = [1 1]**　　**d(n) = [1 –1]**

Furthermore, by applying the noble identities we can obtain our multiplexing filters coefficients for $\mathbf{k_1(n)}$ to $\mathbf{k_4(n)}$:

$\mathbf{k_1(n) = [1\ 1\ 1\ 1]}$　　$\mathbf{k_2(n) = [1\ 1\ -1\ -1]}$　　$\mathbf{k_3(n) = [1\ -1\ 1\ -1]}$　　$\mathbf{k_4(n) = [1\ -1\ -1\ 1]}$

Since the collective action of all these four filters is to actually perform the "inverse wavelet transform", it is useful to express these coefficients in the form of a matrix.

$$K = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

In order to gain some insight into this process let's have a detailed look at the signal processing associated with a given user. Consider for instance user 3:

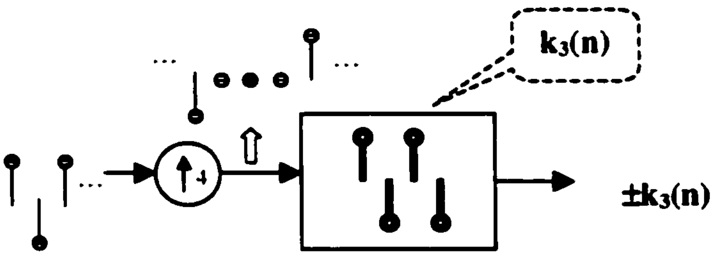


Figure 4.4-1 Detail processing for user 3 (given as an example).

Note that since the input signal is up-sampled just high enough to allow a single non-zero sample (of value ±1) to overlap with the filter response, hence we can consider such input as an impulse **$\delta(n)$** of positive or negative value (recall that the data source consists of a NRZ binary random sequence). From the convolution theorem we know that when we feed an impulse into a filter we obtain the "impulse response" of that filter. Hence for every data bit we feed into this system, a positive or negative version of the impulse response for the filter will be transmitted ***(note that this is true only when the impulse response length is equal to the up-sampling rate, in this case 4)***. Also note that if the given filter for that branch is not very sharp (as in this case), we have spread the input bandwidth by a factor of 4.

Although we have stated that the receiver structure for this system is simply a "mirror" image of the transmitter (note the phase differences), it is worthwhile to fully depict the multiplexing structure from start to end (**Figure 4.4-2**). Note that $\mathbf{K_1}^T$ denotes the transpose of $\mathbf{K_1}$.

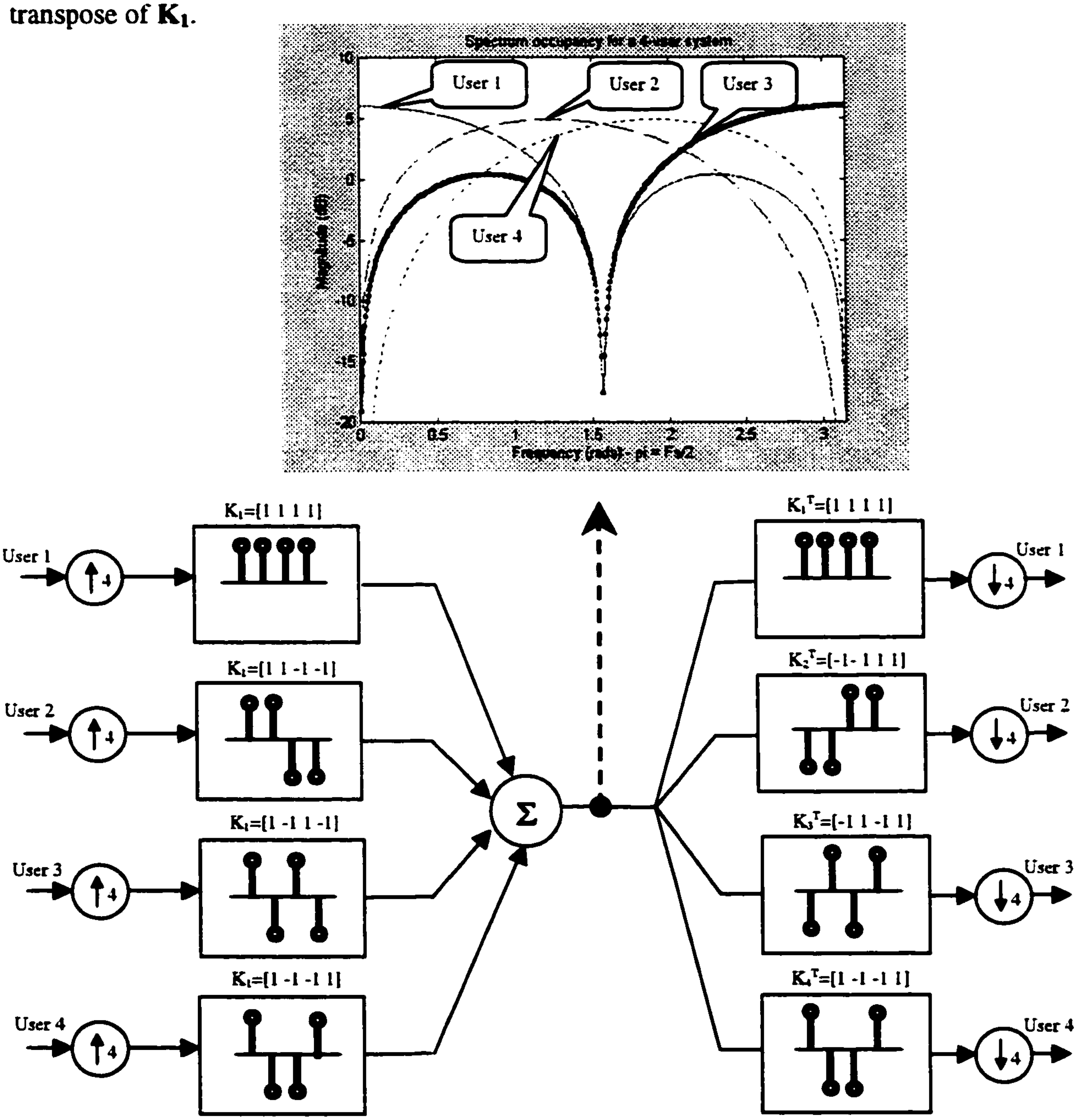


Figure 4.4-2 Details on a 4-user system and spectrum occupancy for each user.

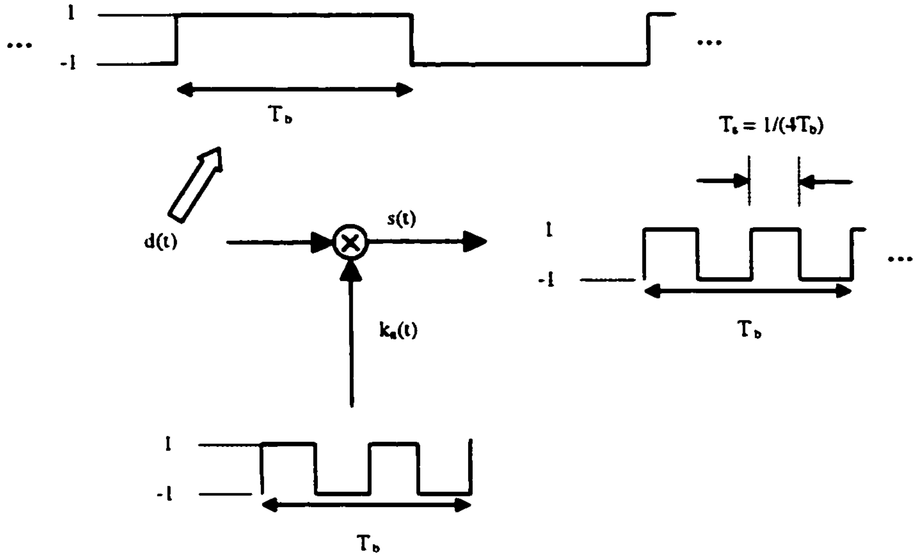


Figure 4.4-3 (a) Equivalent processing for system in Figure 4.4-1 (see "aside" in section 3.10 for details on this equivalence).

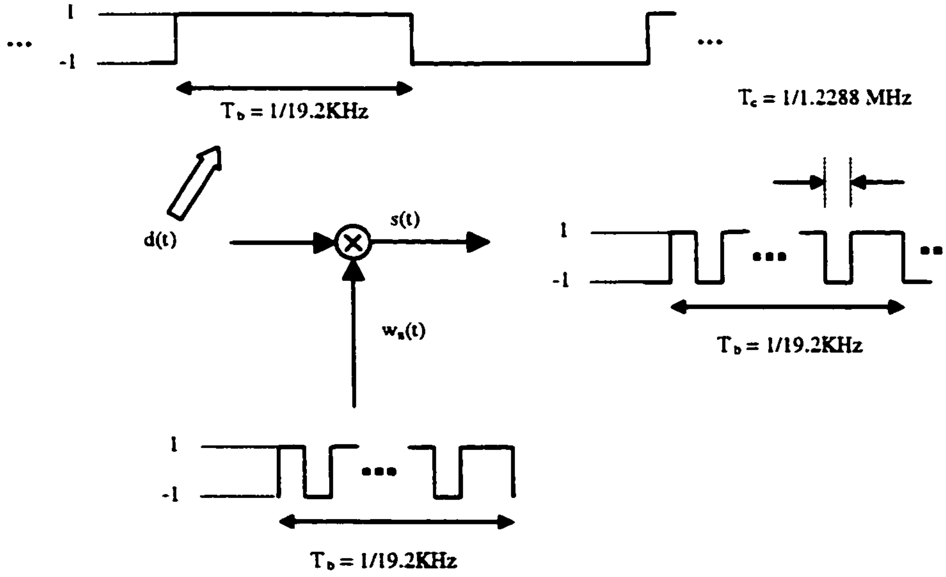


Figure 4.4-3 (b) Multiplexing with Walsh codes (reproduction of Figure 3.10-4).

It is interesting to note that the structure depicted in **Figure 4.4-1** is the equivalent to a non-multirate system in which a low frequency square wave is being multiplied by another square wave of higher frequency (see **Figure 4-124.4-3 (a)**). When describing this structure, one cannot help but to recognize the multiplexing mechanism used by IS-95 (Chapter 3). In such system users were multiplexed using Walsh codes. For convenience, we have reproduced **Figure 3.10-4** (see **Figure 4.4-3 (b)**). Recall that in section 3.10 we showed that the structures in Figure 4.4-3 can be implemented as an up-sampler followed by a filter.

In the interest of analysis let's consider a 4-user system being multiplexed with Walsh codes in the same manner as in IS-95. This will allow us to compare systems with the same input rate and the same number of basis functions.

Recall that each Walsh code corresponded to one of the rows of a Walsh-Hadamard matrix that can be built with the recursive process described in section 3-10. Comparing the wavelet matrix **K** applied to our simple 4-user system with the Non-Return-to-Zero (NRZ) Walsh-Hadamard matrix **W** we obtain the following:

$$K = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \qquad W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Note that these matrices are almost identical! All rows are the same but they are arranged in a slightly different order. If we use sub-indices to label each row we can see that all rows are identical but they differ in the order in which they appear. The first and last rows are the same while the second and third appear in reverse order. Since users are assigned almost randomly to each basis, then this small difference becomes irrelevant.

Here is how the Wavelet (Haar) matrix **K** rows compare to the Walsh-Haddamard **W**.

$$\mathbf{K}_1 = \mathbf{W}_1$$

$$\mathbf{K}_2 = \mathbf{W}_3$$

$$\mathbf{K}_3 = \mathbf{W}_2$$

$$\mathbf{K}_4 = \mathbf{W}_4$$

This indicates that the Synchronous CDMA method used in IS-95 actually uses the lowest order Daubechies wavelet: the Haar.

If we have shown that the forward channel of IS-95 is based on the simplest of all wavelets (Daubechies or order 1), then it is only natural to ask what are the issues associated with higher order wavelets? If we use a higher order wavelet, how does it differ from the case we have just analyzed?

4.5 Beyond the Simple Case

In the previous section we analyzed a case in which the wavelet basis consisted of a Daubechies of order 1 ("db1" for short notation). This means that the length of the FIR filter impulse response was 2, and the resulting function basis for each user was 4. In general, the length of each basis is equal to the number of users. We can recall that in IS-95 we have a matrix of 64 x 64 to support 64 users. In the Haar wavelet case, we obtained the same result, as shown in the previous section for 4 users (i.e. 2 levels for 2^2). If we want to expand to 64 users we would cascade 6 stages (i.e. 6 levels for 2^6)

If we are to maintain the same structure as that used in IS-95 (i.e. for every data bit we transmit either a positive or negative version of the basis function), then as we increase the filter length of the basis function we also increase the output symbol rate. This is clearly not acceptable since increasing this number would typically require more bandwidth.

Thus, applying the multiplexing structure used in IS-95 we are restricted to simple basis such as the Haar. However, if we do not restrict ourselves to use the structure in which we multiply every bit by a basis function (**Figure 4.4-3 (a)**), we can then take advantage of the more flexible features of the wavelet transform. More specifically we are referring to the ability to apply a **Lapped Orthogonal Transform (LOT) [Str1], [Vai1].**

Based on the notion of a LOT, we can then begin to think about transforms in which we change the length of the filters being used without affecting the input/output bandwidth ratio! This is a significant degree of freedom added to our design tools. This feature is extensively used in image processing to eliminate the sharp discontinuities associated with the blocking effects associated with block transforms **[Str1],[Aka5]**. As we described in Chapter 2 this operation essentially correlates the subsequent samples in the input stream.

So what does it all mean in our application?

Again, let's make use of our 4-user example to analyze the consequences of this concept. **Figure 4.5-1** depicts a system in which the filter taps were derived from a wavelet of order 3 (i.e. h(n) = [0.0352 -0.0854 -0.1350 0.4599 0.8069 0.3327]). As done previously, from h(n) we can then derive g(n), c(n), and d(n). From these values we then generate the necessary values for our filter bank coefficients, $k_1(n)$ to $k_4(n)$. **Figure 4.5-1** illustrates the overlapping effect caused by the increase in wavelet order.

The effect of this extension in the support of the basis function (i.e. longer duration), is that the output of the system shown in **Figure 4.5-1** has a more "compact" spectrum. From a DSP point of view this is actually very obvious since we are used to the idea that the sharpness of an FIR filter is improved by increasing the number of taps we are allowed to use. One would expect that as the order of the wavelet increases, so would the sharpness of the filter response. In terms of our multi-user application this means that users would overlap less and less in the frequency domain. In fact, if the wavelet order is made very high (i.e. very long filter length), then we would end up with a multiplexing system which would be close to a pure FDM. This is precisely the final observation made in **Section 4.3.** Let's compare the spectra associated with each user when using basis function derived from the wavelet 'db1' (that of IS-95), 'db3' and 'db7' (as examples of higher order wavelets). This comparison is illustrated in **Figure 4.5-2**. Here we can clearly appreciate that the 'db3' and 'db7' spectra depict the energy from each user in a more compact fashion than that of 'db1' (the later tends to 'spread' the signal energy). One could imagine that a possible advantage that comes from this localization is that the resulting waveform would have a sharper cross-correlation function (this would be useful in a case in which orthogonality is lost due to lack of inter-user synchronization). This is something that we explore in Chapter 5 where we analyze the performance associated with various wavelets.

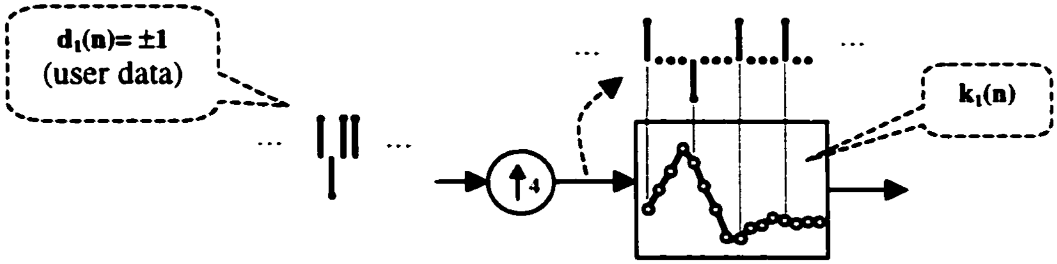


Figure 4.5-1 Detail processing for user 1 (given as an example).

It should be pointed out that the use of a basis function that is much longer than the bit duration is not entirely a wavelet-based concept but a more generic one that has been examined as "Signature Spread CDMA" **[Wor3]**.

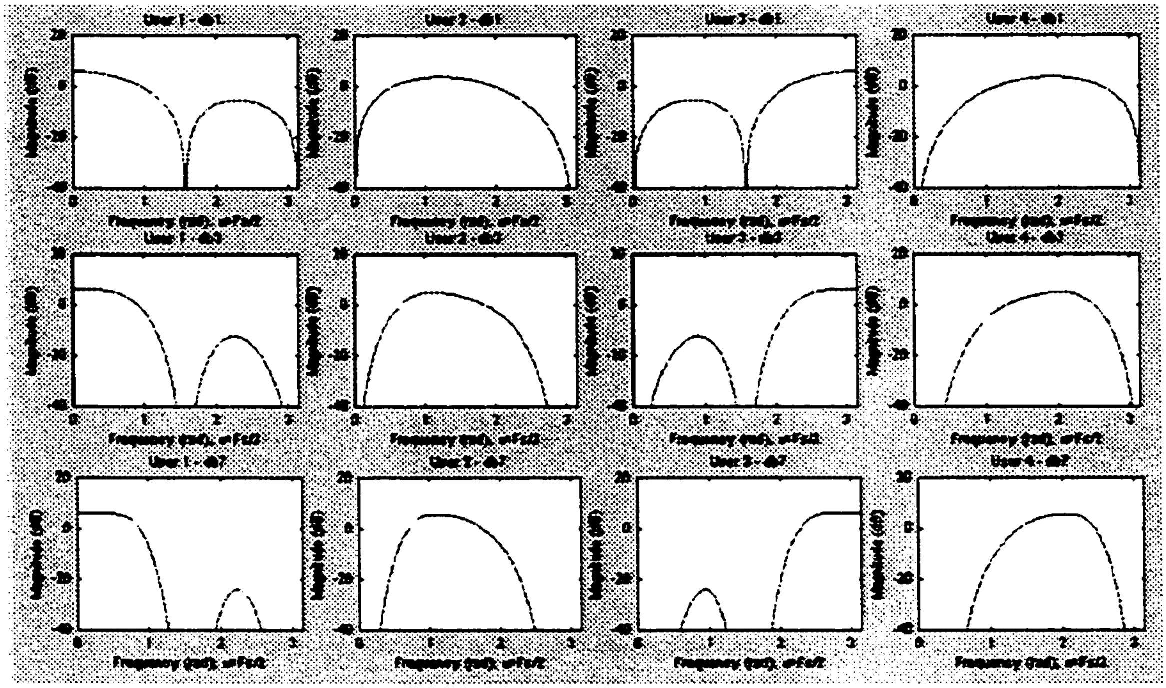


Figure 4.5-2 Comparison of user spectra for a 4-user system based on Daubechies wavelets of order 1, 3, and 7 (top to bottom, respectively).

An interesting feature:

So far we have been constantly discussing a wavelet-based multiplexer that is really based on the "wavelet packet" structure. As we discussed in Chapter 2, this is a tree structure in which all bands are split equally hence the data rates at all points of every branch are equal. However, in Chapter two we introduced the wavelet transform as a great tool to localize signal characteristics in both frequency and time. This localization was obtained through the used of a "dyadic" tree structure in which frequency bands were not split equally.

We can then apply the same structure to our multi-user system to provide channels with different data rates. For instance, consider how the basic 4-user packet structure can be modified into a system with only 3 users.

Figure 4.5-3 depicts such structures with their respective spectra. As we can see in this figure, by realizing this simple modification we can have a system in which User 3 can transmit at a data rate that is twice of that corresponding to User 1 and User 2. Since this modification is so trivial, it lends it self to providing a highly dynamic (i.e. reconfigurable) system. It is not hard to imagine that this feature is one of great importance in third generation wireless system in which the data traffic (as opposed to voice) is expected to be in high demand.

The second scenario in which this would also be of great interest is in the case of asymmetric channels, i.e. the forward channel is much wider than the reverse channel. Unlike existing voice channels in which the forward and the reverse link to the mobile have the same bandwidth requirement, in a data traffic environment such requirement would not be the same. For instance, if we are not dealing with a mobile wireless phone but with a mobile wireless "appliance" (e.g. internet browser), we would have a few key strokes being sent in the reverse channel while receiving large amount of data in the forward channel. This type of requirement would appear to be addressed by using a wavelet-based air interface. Another possible use for this feature would be unequal Quality of Service (QoS). If we allow some users to pay more in exchange for higher quality of service then we could assign extra bandwidth for such users. More bandwidth to transmit the same amount of information obviously means that we can use more error correction, etc.

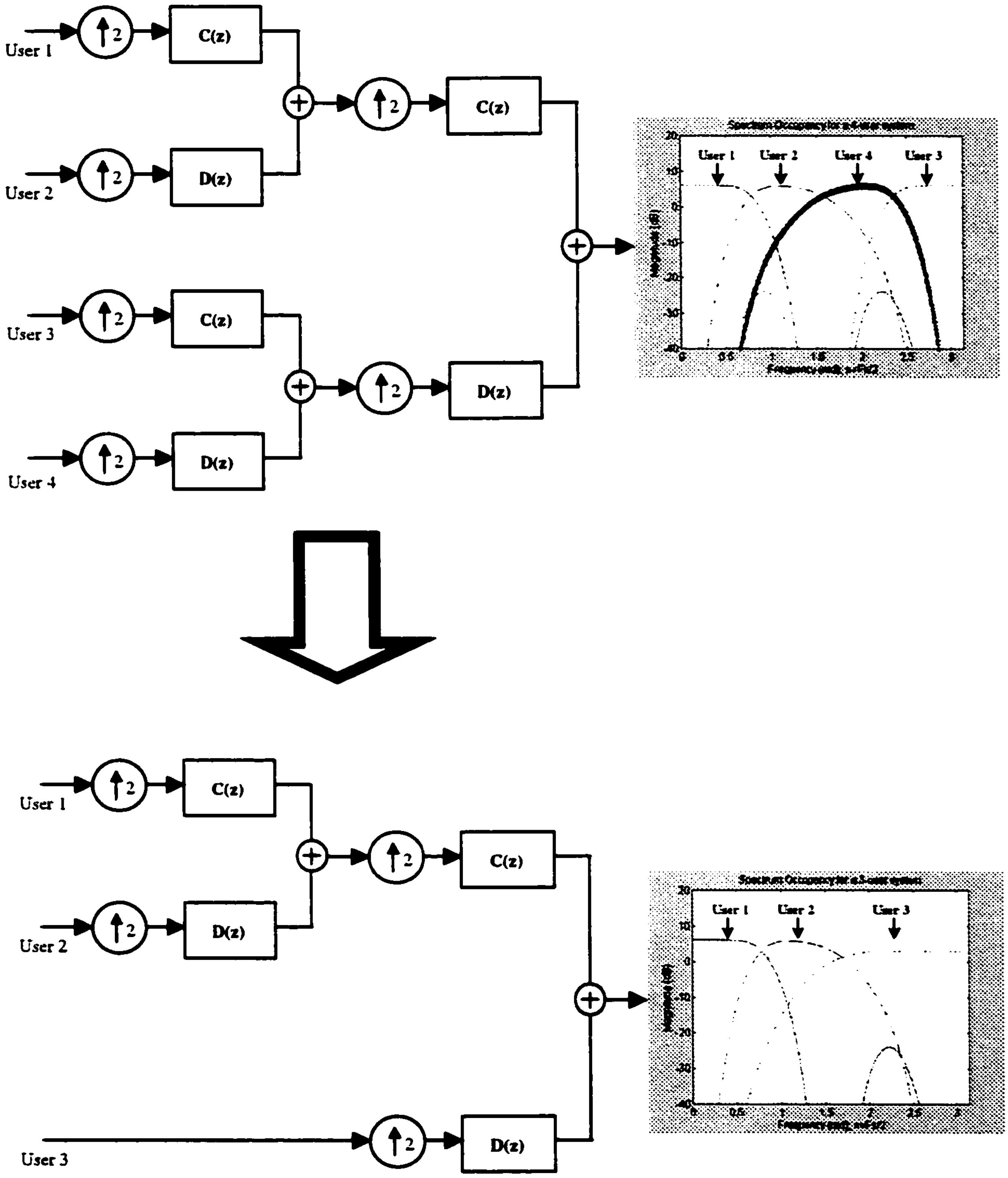


Figure 4.5-3 System modification to support unequal data rates (using 'db7')

4.6 Branch-Hopping Wavelet Multiplexing

In the previous section we described the makeup of a wavelet based multiplexing structure that allows many users to share the same communication channel. In such a structure, we examined how the transmitted signals for various users would overlap both in time and frequency. Hence we classified this scheme as CDMA as opposed to TDMA or FDMA. In the previous section we showed that a key characteristic of such system was that the transmitted bandwidth per user was much greater than their respective information bandwidth, hence "spreading" the signal bandwidth. This is why many researchers have introduced wavelets (and PR filters in general) as a form of spread spectrum CDMA (**[Aka6]**,**[Coc]**,**[Het]**). These researchers have compared wavelet-based systems to the traditional Direct Sequence (DS) spread spectrum implemented with Gold codes **[Aka1]** which are basically a subset of maximal length (ML) pseudo-random (PN) sequences.

As we know, spread spectrum systems are typically implemented as either DS or Frequency Hopping (FH) **[Pet]**,**[Fli]**. Unlike DS where the signal is spread over a large bandwidth with a unique PN code, the FH scheme is based on maintaining a narrowband signal but changing the frequency of the RF carrier. A significant challenge of such system is the fact that RF circuits have a settling time which limits fast hopping (i.e. changing RF frequencies more than once per bit period).

Hence it is no surprise that other researchers have also proposed wavelets-based schemes that extend the basic wavelet-based multiplexing structure to provide a form of "Wavelet Hopping" **[Dav]** **[Dan]**. *The main idea behind this concept is to dynamically change the user-to-wavelet mapping.* In **Section 4.5** we showed that although the user input signal would be spread over a larger bandwidth, the wavelet filter would "compact" much of that bandwidth in a visible region of the spectrum. The specific location of this concentration of energy is directly related to the wavelet branch used for each users (see **Figures 4.5-2** and **4.5-3**). If we think of dynamically changing the wavelet branch (or "signature") then the main frequency region associated with a given user would be changing constantly over time! This would be very similar to constantly changing the frequency carrier observed in an FH system. Let's illustrate this concept with the 4-user structure shown in **Figure 4.6-1** (suggested by **[Dav]**).

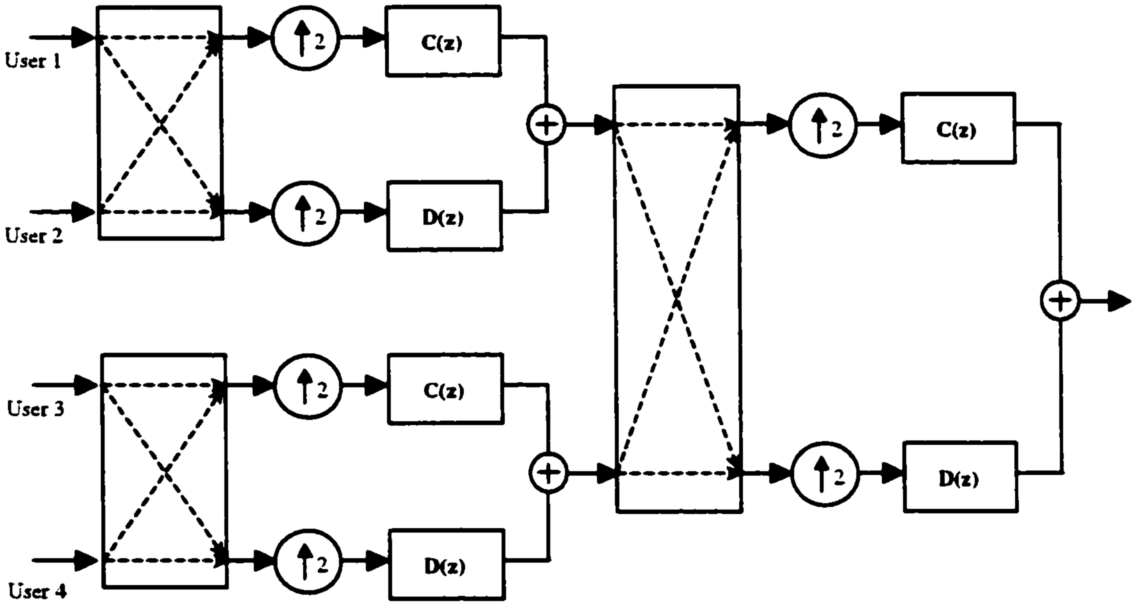


Figure 4.6-1 A 4-user Branch-Hopped Wavelet Division Multiplexing (BH-WMD) system

In previous systems we constructed our WDM system with a recursive arrangement of only two basic elements: a **sampler** (up/down), and a **filter** (lowpass/highpass). Here we see the addition of a **switch** as the third element. In **Figure 4.6-1** we can see how the position of the given switches can fully determine which wavelet basis is being used for transmission. This hopping strategy can provide the kind of performance averaging. In this manner a user does not remain in a bad channel for very long hence improving the performance over the case in which we use the fixed WDM scheme. In a multi-user environment this means that users would share any type of narrowband interference associated with a given frequency region.

There are a few important observations we should highlight about this concept:

- Synchronization is at the heart of this system.
- If we apply a form of 'Fast Hopping' there is a need to keep track of individual filter 'states' for each user. Particular care must be given to 'transient' zone for each filter when switching the input from one user to another (i.e. the "pipeline" issue). **This is an extremely difficult problem.**
- Just as speculated for FH-SS systems **[Fli]**, we can also expect that if the branch hopping rate is shorter than the delay spread in the channel, then multipath images could be significantly suppressed. Davison **[Dav]**, and Daneshgaran **[Dan]** have reported promising results.
- Naturally we expect to pay a higher price in terms of system complexity.
- When used in a single user system, this concept can provide a very promising form of "covert" or Low Probability of Intercept (LPI) communication **[San]**.
- Switching strategies are still areas of active research **[Dav]**,**[Dan]**.

Due to the multiple issues associated with this specific wavelet system, simulation of this design (chapter 5) is outside the scope of this thesis.

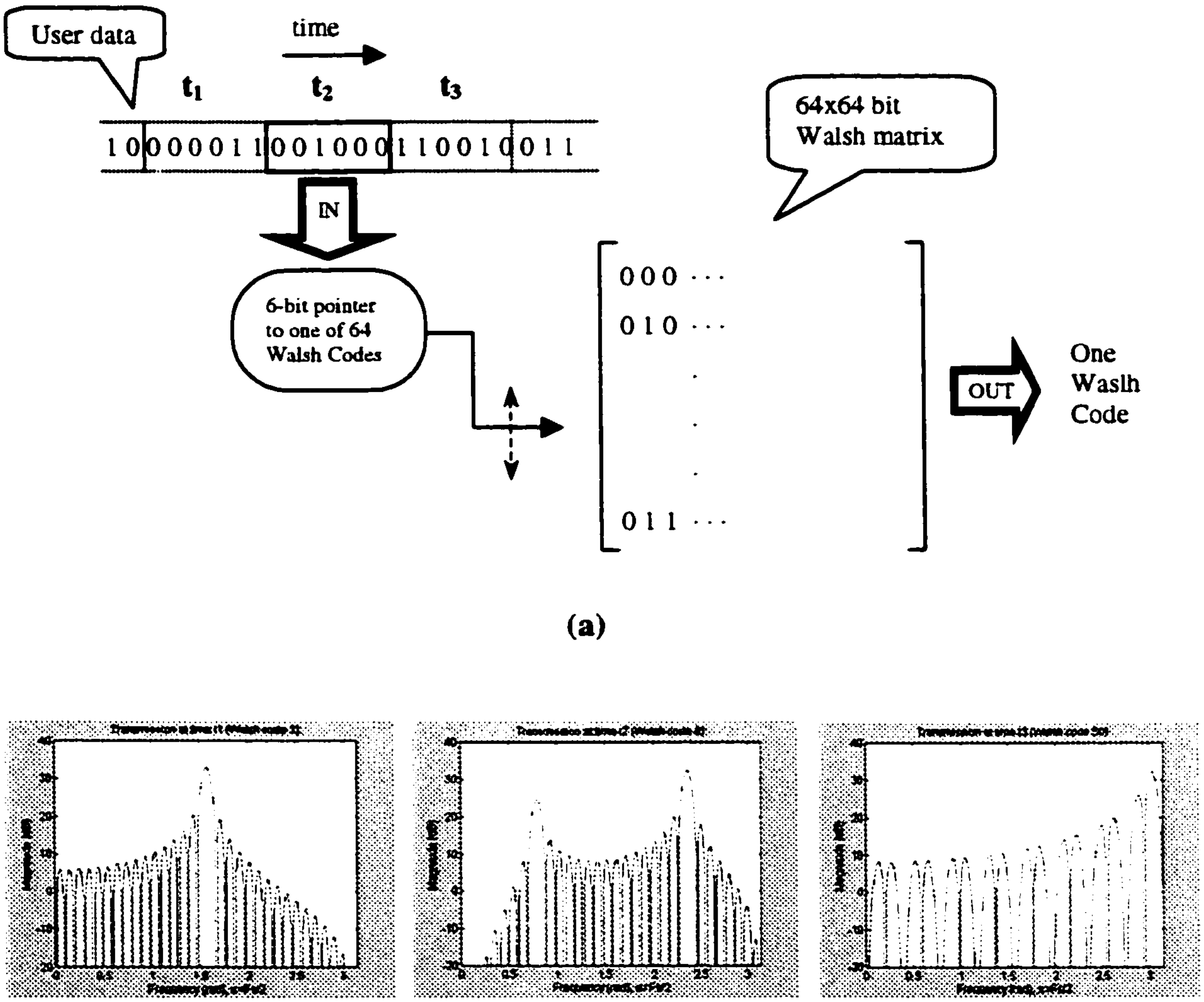
Is this Branch-Hopping a novel concept?

After very detailed analysis of a 'db1' wavelet-based transmultiplexer we showed that the equivalent system was being used in IS-95 as a means of multiplexing users in the forward link (**Section 4.4**). So we could then ask: is there a similar connection between orthogonal branch hopping and IS-95?

Before we provide a short answer to the question at hand, let's briefly review a few details about IS-95 again. As we have mentioned before, multiplexing of signals in the forward path (i.e. base station to mobile) is accomplished by assigning a Walsh code for each user (see details in Chapter 2). In this case the multiplexing process is based on the fact that Walsh codes form an orthogonal set (64x64 matrix), much like wavelets do. This orthogonality is highly dependent on the synchronized transmission of all users. However such condition is nearly impossible on the reverse channel (this is why the forward channel is said to employ "synchronous CDMA" as opposed to "asynchronous CDMA" for the reverse channel **[Sou]**, **[Haf]**). Since a mobile transmitter is unaware of other mobiles, and constantly changing location (hence changing propagation delays) it is difficult to synchronize all reverse path transmissions. So how does IS-95 address such problem?

Orthogonal Modulation

This term refers to the technique by which we map a bit sequence to a signal that is a member of an orthogonal set. This concept is perhaps best understood by looking at **Figure 4.6-2** where we depict the 64-ary Orthogonal Modulation employed by IS-95. Here we can see how a 6-bit input is converted into a 64-bit Walsh code, and more importantly, we see that over time, a single user would transmit all possible Walsh codes. Here we are doing the same code (or branch) hopping described earlier.



(a)

(b)

Figure 4.6-2 IS-95 orthogonal modulation (a) and transmitted signal (b) at times t1, t2 and t3.

If we were to realize this system with the equivalent Wavelet structure ('db1') we would see that ***IS-95 uses "branch hopping" with a hopping rate of 1 every 6 bits!*** (note that here the hopping depends on the data as opposed to some random sequence).

So is Branch-Hopping Wavelet Packet Multiplexing a new concept? Most likely we can say yes, however it is interesting to see that the same principle has been "implicitly" applied with an orthogonal transform implemented without using a recursive multi-rate structure (as that used in Wavelet Multiplexing).

It is fair to restate that the IS-95 structure is equivalent to the 'db1' wavelet implementation (which is the simplest) and hence lacks the high degree of freedom offered by higher order wavelets (as those shown in **Section 4.5**).

When we look at **Figure 4.6-2** we cannot help but ask: *if one user employs all 64 codes, what about the remaining users?* Well, since we have 64 possible codes to transmit, the probability of another mobile transmitting the same code is 1/64 or 1.5%. Although the orthogonality between users is not guaranteed due to asynchronous transmissions, such property still minimizes the cross-correlation among users. Some of the degradation due to the lack of orthogonality is compensated with added error correction (the reverse channel uses a convolutional error correction rate or 1/3 as opposed to 1/2 for the forward channel). However, the key element used to isolate users in the reverse channel is the use of a PN sequence (not shown in **Figure 4.6-2**), which is unique to each user in question. In this case we say that the reverse channel is based on Asynchronous CDMA.

Chapter 5

Performance Analysis

5.1 Simulation Strategy and Scope

In the previous chapter we examined how the wavelet transform could be employed to multiplex users in a communication channel. Furthermore, we highlighted some of the key characteristics of a wavelet-based transceiver and established some interesting connections to the use of Walsh codes currently found in one of the main wireless standards (IS-95). Thus far, we have described how the Wavelet Transform could be combined with the concepts of transmultiplexers and multirate signal processing to provide a multiplexing scheme with interesting properties. In essence, we answer the question *how does wavelet-based multiplexing work?* Naturally the next step is now to ask: *how good is it?* More specifically, how does the system degrade in the presence of noise and other non-ideal conditions such as multi-path. In addition to "channel-related" conditions, we must also examine the degradation effects from "systems-related" issues such as the unsynchronized transmissions from mobile units. The following are some channel and system characteristics identified as very relevant to a cellular wireless system:

- **Additive White Gaussian Noise**
- **Multipath**
- **Synchronization**
- **Narrowband Interference**
- **Signal Power**

The above list does not include one other problem that is typically found in a mobile wireless environment: Doppler Spread. This condition is related to the relative motion between the transmitter and receiver and determines the rate of change for the channel.

Using a typical case of 80 Km/h with an 880 MHz carrier, we expect that the channel would change every 15 ms (i.e. the "Coherence Time" of the channel). This does not become an issue during the processing of a single bit (or symbol) since the time window of concern is in the order of μs, hence this case has been excluded from the set of test cases. The problem of a changing channel is of paramount importance when building an equalizer for such a system (a topic that is extremely important but beyond the scope this thesis). In order to gain insight into the strengths and weaknesses of the wavelet-based transceiver we have deliberately omitted any "performance enhancing" elements typically found in real life systems (such as error correction, AGC, equalizers, etc.). Instead we are focusing on a "microscopic" view of the signal processing aspect of this multiple access scheme.

Now that we have established what the performance issues of interest are, we need to specify our frame of reference. In Chapter 3, we reviewed the basic operation of the most popular cellular systems and highlighted that existing systems are primarily based on two multiple access methods: **FDMA** (basis for AMPS, IS-54, and GSM) and **CDMA** (basis for IS-95). We recognized that while IS-54 is referred to as TDMA (since 3 users share the same carrier frequency), the predominant access method is FDMA. Some researchers have shown that wavelet-based transmission can offer a 30% improvement over FDMA systems **[Wu], [Dav2]** while providing superior performance to TDMA systems in the presence of impulse noise **[Wong], [Dav]**.

Hence, *for the purpose of our performance analysis we have chosen CDMA as our basic frame of reference*. It is worthwhile to note that in the case of CDMA we must recall that forward and reverse channel used slightly different schemes. The forward channel is based on Synchronous CDMA (S-CDMA) which is based on orthogonal spreading using Walsh codes while the reverse channel is based on Asynchronous CDMA (A-CDMA) which is based on the conventional spreading using PN sequences **[Sou2]**. Here we will examine both. Although it would be beyond the scope of this work to accurately simulate a very complex system such as IS-95, we can implement a simplified version of it, i.e. a system based on the same operating principles.

In summary, we want to explore two things:

1. Are some wavelets better than others, i.e. will a 'db7' provide better performance than a 'db2'?
2. How does a wavelet-based multiple access system compare with Walsh codes and PN sequences?

These performance issues were explored by building a MATLAB simulation. The block diagram for such simulation is shown in **Figure 5.1-1**. This model can be used to simulate aspects of both the forward and the reverse channel of a cellular environment.

This model is made up of three basic elements: Transmitter, Channel, and Receiver. Depending on which scenario we wish to explore, we change the TX/RX unit to be Walsh-based, PN-based, or Wavelet-based.

The Transmitter: this unit is replicated N-times (N being the total number of users) and contains the following modules:

Source: This block produces a random signal in the form of a binary Non-Return-to-Zero (NRZ) sequence.

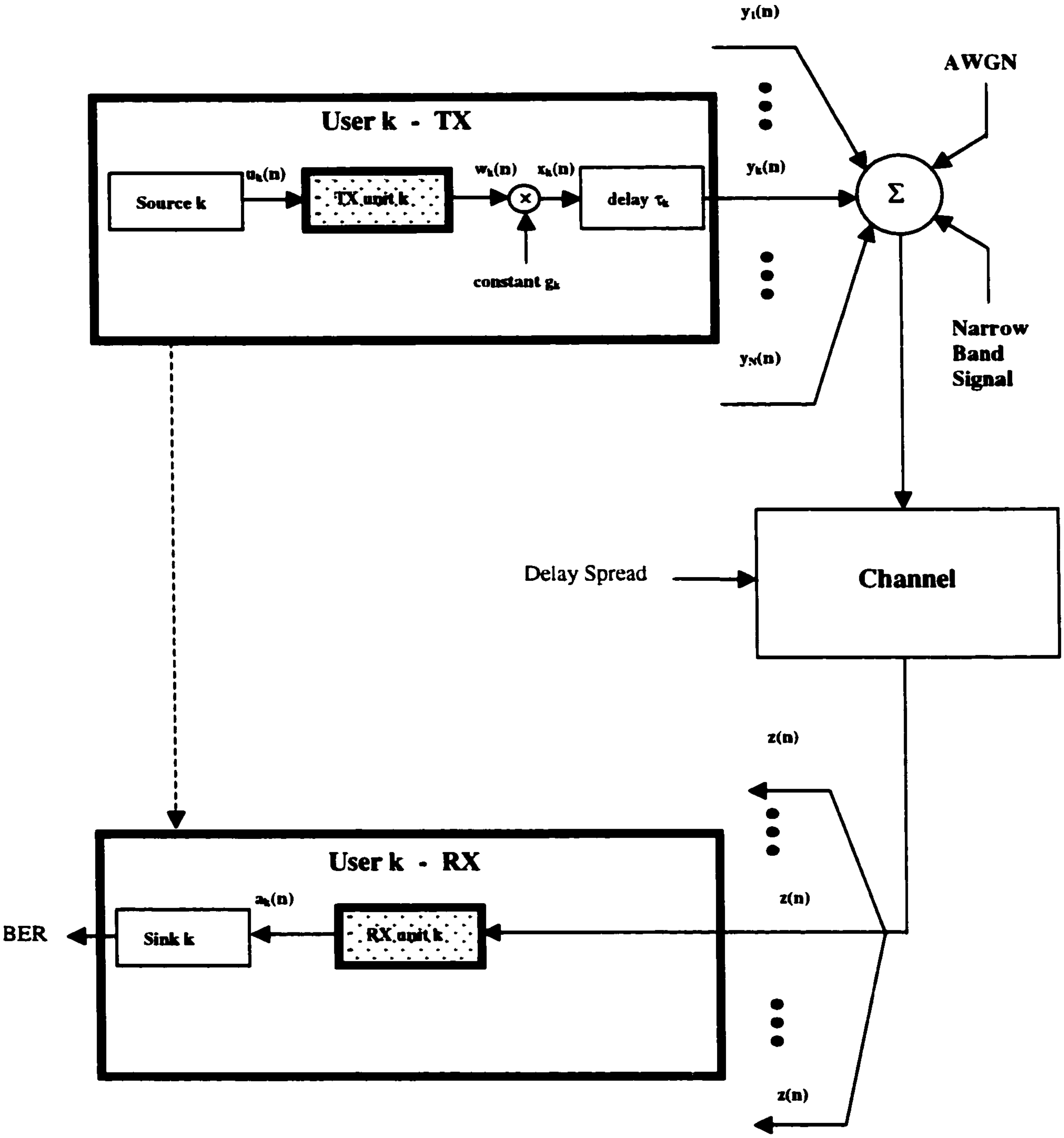


Figure 5.1-1 Simulation model for an N-user system

TX unit: This is the “kernel” of the transmitter to be simulated. **Figure 5.1-2** depicts the block diagram for both TX and RX units. Here we take advantage of the universal structure based on a transmultiplexer (see Chapter 4) and implement different multi-user schemes by just applying a different impulse response for $\mathbf{h_k(n)}$.

1. For WDMA $\mathbf{h_k(n)}$ is based on a wavelet filter, i.e. db1, db3, etc.
2. For CDMA $\mathbf{h_k(n)}$ consists of a segment of either a PN-sequence (N-samples long) for the asynchronous case (labels here as A-CDMA) or Walsh codes for the synchronous case (S-CDMA). Hence $\mathbf{h_k(n)}$ changes for every bit to be transmitted. Note that the typical representation of a CDMA system is based on a "multiplication" of the data stream and the PN sequence, while here we are using a "filtering" operation. For details on the equivalency of these two structures see sections 4.5 and 3.10.

Gain: In the transmitter block we can change the value of $\mathbf{g_k}$ to simulate different values of signal strength to reflect the proximity to the base station. If the transmitter block is being used to simulate the base station then this value can be set 1 for all users. This value is set to a constant during a given simulation run.

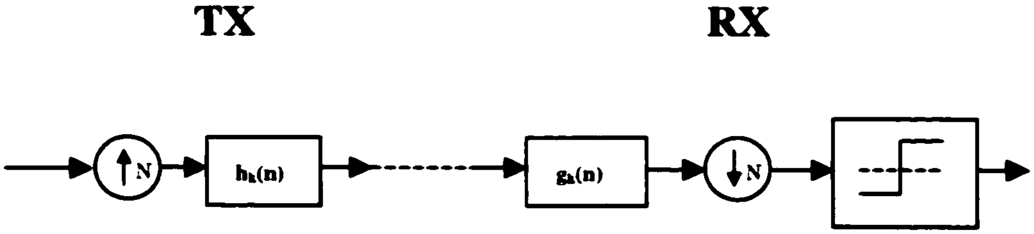


Figure 5-2 TX and RX units.

Delay: This block allows us to simulate the uncoordinated nature of the transmissions in the reverse channel due to propagation delays. Again, if the transmitter block is being used to simulate the forward path then this block is always set to provide zero delay for all users so that it reflects the implicit synchronization provided by the base station. This value is set to a constant during a given simulation run.

The Channel: The output of each transmitter is then corrupted with Additive White Gaussian Noise (AWGN) and a narrow band signal interferer. The result is then passed through a low-pass filter that consists of two taps (to reflect a two-ray model). These taps are spaced in time according to a given "delay spread". Typically these taps change randomly in time following a rate of change related to the "Doppler spread" (this reflects the speed of motion of the transmitter). This model however, does not introduce Doppler spread since we are evaluating a system without adaptive components.

The Receiver: similar to the transmitter, this unit is replicated N-times (N being the total number of users) and contains the following modules:

RX unit: This is the "kernel" of the receiver to be simulated. **Figure 5-2** depicts the RX unit as being a mirror image of the TX unit followed by a hard-decision decoding device. In this case all three receiving schemes are implemented by simply changing the value of $\mathbf{g_k(n)}$:

1. For WDMA $\mathbf{g_k(n)}$ is equal to $\mathbf{h_k(-n)}$.
2. For CDMA $\mathbf{g_k(n)}$ is equal to $\mathbf{h_k(-n)}$. Note that this changes for every bit when using PN codes .

Sink: This unit is basically an instrumentation device that determines if the received signal is a delayed version of the bit stream sent by the source block found in the transmitter. This block provides the actual performance measure of our model: Bit Error Rate (**BER**).

Note that the broken line that connects the transmitter and receiver blocks is shown to represent the hidden transfer of information to evaluate the system performance, eg. bit pattern sent, synchronization info, etc.

*It is important to highlight that the purpose of this performance analysis is not obtain a "performance specification" for WDMA (i.e. determining whether wavelets are "better" than Walsh codes or PN sequences), but rather to gain some **insight** into this emerging wavelet application.*

5.2 Additive White Gaussian Noise (AWGN)

No performance analysis would be complete without testing for the system degradation in the presence of AWGN. Hence, in this test case we examine how the BER declines as we add noise to the channel.

Setup:

In this test we measure the variance of the noise source (σ_n) and compare it with the variance of the sum of all transmitted signals (σ_s) to produce the ratio σ_s/σ_n. This ratio is then used to reflect the amount of noise in the system and allow us to make comparison on the relative performance of various systems.

Expectation:

Given that the noise source is flat across the spectrum we can expect all systems to degrade in a similar manner.

Results:

Before looking at specific results and comparisons between WDMA and other schemes it is worthwhile to look at some basic results such as the input/output signals for a system with and without noise (see Figure **5.2-1**).

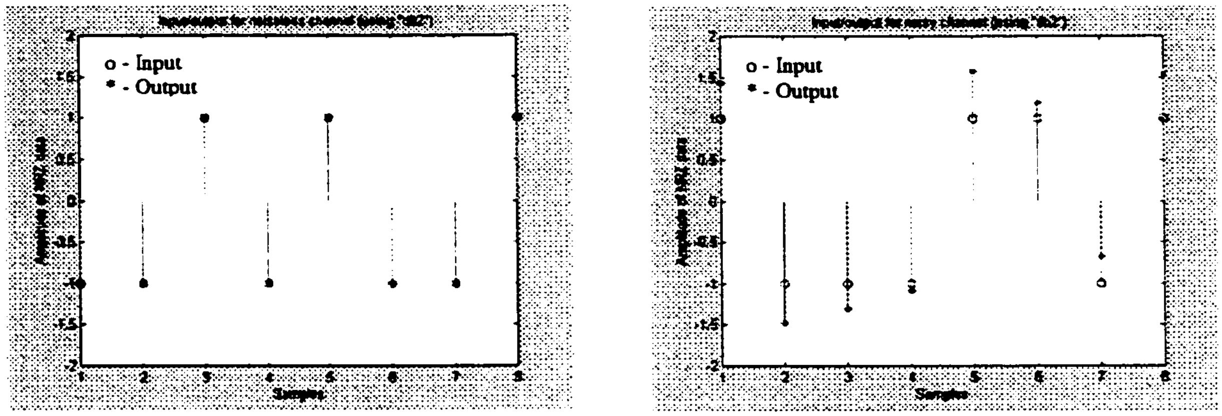


Figure 5.2-1 Input/Output comparison of a noiseless channel (left) and a noisy channel (right) using the Daubechies of order 2 (db2) as a basis (delay adjusted).

Now we can take a "microscopic" view of what is taking place at intermediate stages of our system (see Figure **5.2-2**). In this figure we can see how the input signal is up-sampled and then filtered. The output of this process is then added to the signal from other users after which is combined with the output of a noise source to illustrate how noise interferes with the perfect reconstruction process. In this figure we can also see that the impulse response for the receiver is simply a time-reversed version of the transmitter filter.

In this particular example we are only looking at a sequence of four bits (1,1,-1,1) which can be actually recovered from the output of the system (0.6708, 1.3299, -0.9773, 0.3704) by applying a simple decision device.

Naturally we are more interested in a more statistical measure of the BER figure for different noise levels and system configurations. **Figures 5.2-3** to **5.2-5** depict comparison for different wavelets (db1, db2, db..), number of users (4, 8, and 32), and different systems (A-CDMA, and S-CDMA), respectively.

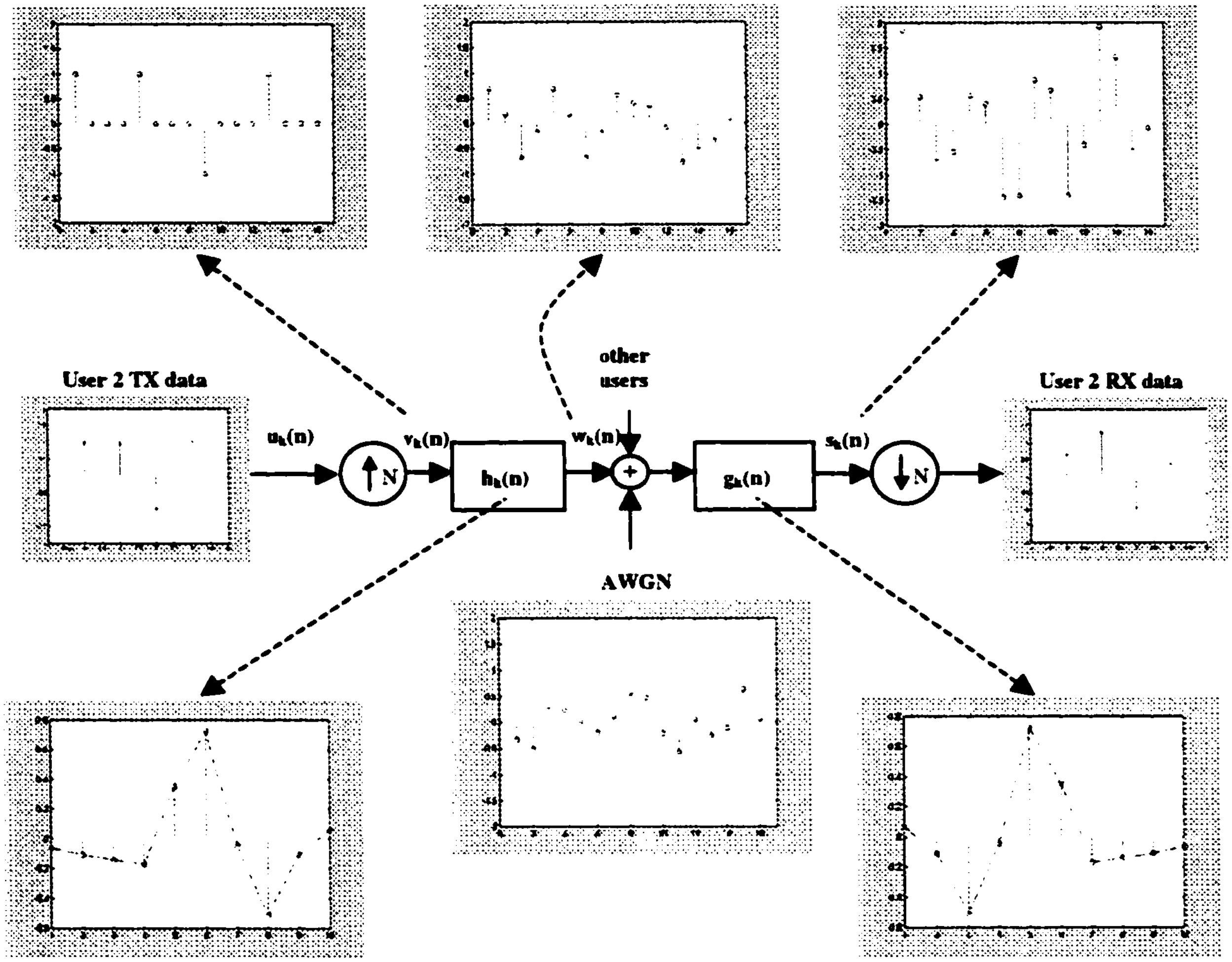


Figure 5.2-2 Detail view for User 2 in an 8-user WDMA system (using db2).

The following BER curves illustrate the relative performance for various systems/setups with respect to the SNR ratio (defined as σ_s / σ_n).

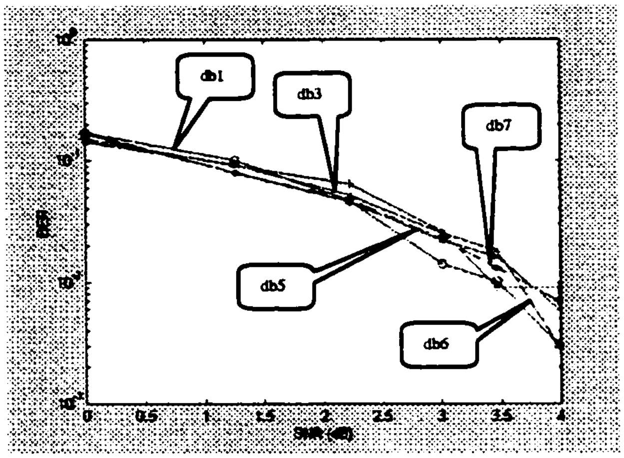


Figure 5.2-3 Comparing wavelets db1, db3, db5, db6, and db7 (operating in an 8-user system configuration).

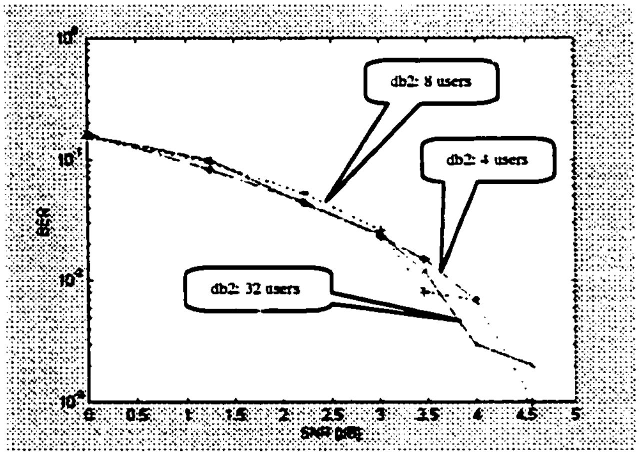


Figure 5.2-4 Comparing systems with 4, 8, and 32 users using db2.

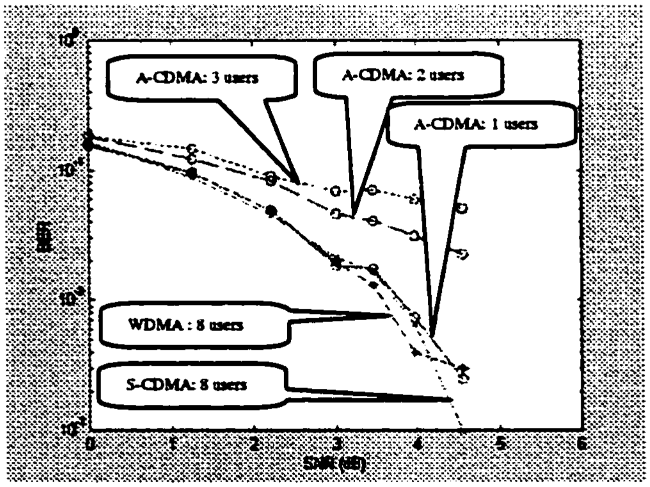


Figure 5.2-5 Comparing wavelets db7 with A-CDMA, and S-CDMA (operating in an 8-user system configuration).

Discussion:

So what do these graphs tell us?

1. **Figure 5.2-3** shows that the "order" of the wavelet does not have an effect in the system performance under AWGN.
2. **Figure 5.2-4** shows that whether we have a system supporting 4, 8, or 32 users there is no change in the system performance.

Figure 5.2-5 confirms that a system based on Walsh codes (S-CDMA) has the same basic performance as a Wavelet-based system **[Aka7]**. More interestingly, this graphs shows is that the PN-based system (A-CDMA) can only achieve similar performance when a single user is active, i.e. as more users become active the system begins to degrade. In sum, we see that under AWGN the WDMA, and S-CDMA systems do not suffer from "multi-user interference" whereas the A-CDMA does. We can say that as other users are added to the PN-based system, such users appear as additional AWGN interference – consistent with references **[Aka7]** and **[Sou2]**. An interesting observation about this fact is that if we were to build a system based on Orthogonal codes we would have a deterministic count for the number of users in the system (e.g. IS-95 has fixed number of 64 Walsh channel in the forward channel but no fixed constraint in the reverse channel where PN codes are used). However, in the case of the PN-based system there is not a deterministic limit (i.e. less noise will give more room for additional users thereby yielding a system with non-deterministic capacity).

5.3 Multipath

We know that in a cellular environment, particularly in urban sites, the transmitter and receiver communication is not restricted to line-of-sight. In the same way in which we perceive audio echoes in a room (due to reflected sound waves), we also have electromagnetic reflections in radio transmissions. The received signal arrives with one or more slightly delayed copies of itself (due to reflections of the same radio signal).

Setup:

A typical multipath scenario is depicted in **Figure 5.3-1**. It is natural to expect that as path **B** becomes longer the strength of the signal will decrease (radio waves attenuate proportionally to $1/d^2$ while traveling in free space, were **d** is the distance traveled **[Che]**).

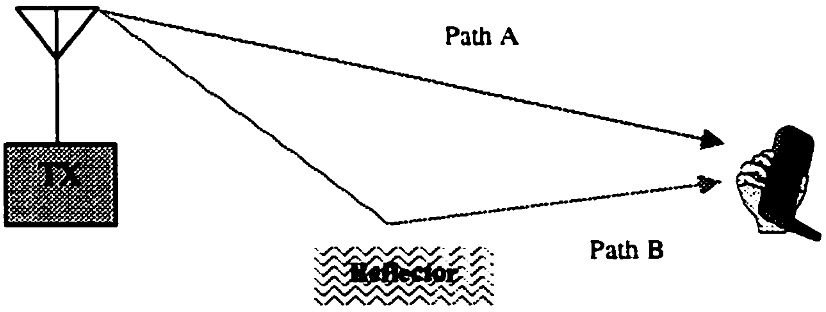


Figure 5.3-1 Multipath scenario with two-ray model

This means that the signal from path B should be weaker than that from path A. Here we consider the effect of having a single copy of the original signal (path A) plus a copy of itself (path B) with 75% of the transmitted power. This type of degradation is modeled as filter with two taps which are spaced by τ which represents the delay spread (see **Figure 5.3-2**). In this case τ takes discrete positive values to reflect the number of samples by

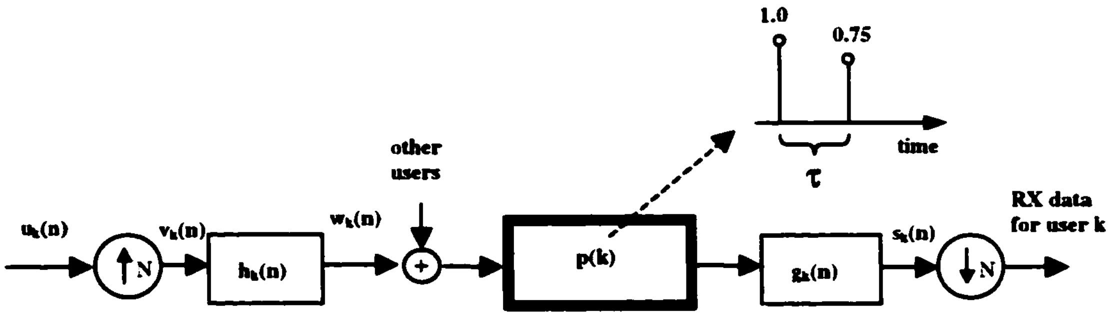


Figure 5.3-2 Setup for Multipath interference case.

which the path **B** signal is delayed. In this case one sample delay is based on the signal rate for the up-sampled signal **v(n)**. Note that this is only a fraction of the duration of one symbol, i.e. one sample of **u(n)**. For instance, in this experiment we use an 8-user system to examine the signal degradation when τ takes values of 1, 2, 3, 6 and 7. This value would represent a delay spread that ranges from 1/8 to 7/8 of a symbol.

Expectation:

Since A-CDMA is based on the transmission of PN sequences with very low cross-correlation and very sharp auto-correlation, we can expect this scheme to provide the best performance. S-CDMA (Walsh codes) and WDMA would be expected to provide similar performance (both poor) since they are based on the same principle of orthogonality. Any interference with such condition is expected to diminish our ability to achieve "perfect reconstruction" of the transmitted data.

Results:

Here we see the comparison between different systems (**Figure 5.3-3**), different Wavelets (**Figure 5.3-4**), and different users in a WDMA scheme (**Figure 5.3-5**).

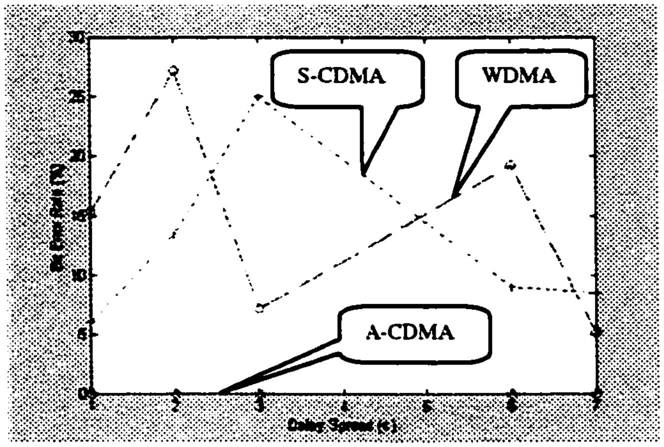


Figure 5.3-3 Effect of delay spread for A-CDMA, S-CDMA, and WDMA (db7). Viewing user 7 in each system.

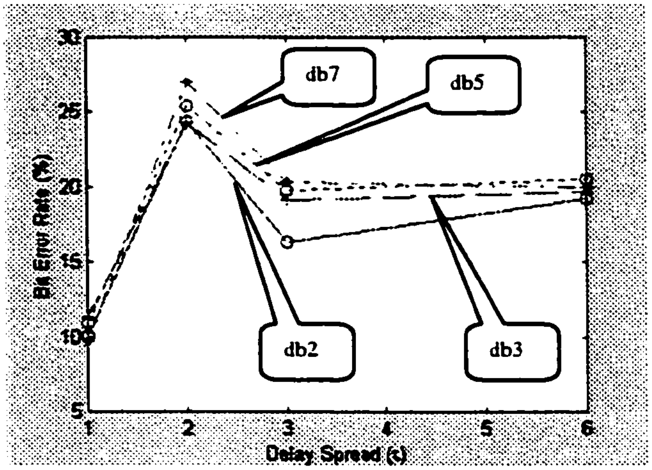


Figure 5.3-4 Effect of delay spread for different wavelet orders :db2,db3, db5, and db7 (user 3).

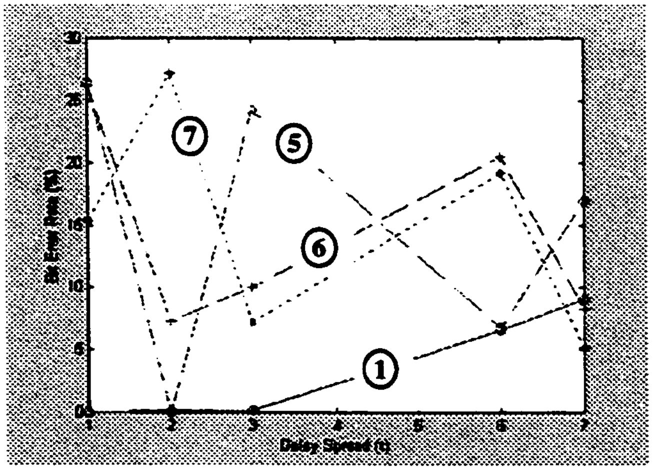


Figure 5.3-5 Effect of delay spread for different WDMA users: 1, 5, 6 and 7 system (using db7).

Each of these results highlights a different observation:

1. First we see that while A-CDMA (PN codes) is completely unaffected by the delay spread, the WDMA and S-CDMA schemes exhibit remarkable degradation for the same delay spread (**Figure 5.3-3**). Note that here we forced the A-CDMA system to support **one user only** so that we eliminate the "multi-user" interference which will appear as Gaussian Noise (and smear results). This allowed us to focus on the multipath problem only.
2. We also see that increasing the order of the wavelet used does not seem to have a significant effect in mitigating the effect of multipath different for Wavelets (**Figure 5.3-4**).
3. Finally in **Figure 5.3-5** we see a clear display of the disparity with which different users are affected by the same delay spread. In this case it was sufficient to randomly pick users 1, 5, 6, and 7 to display such variation.

Discussion:

While some of these results were expected, such as 1), others results were not so obvious, such as 3). Here are some questions worth asking:

1) *Why does A-CDMA significantly outperform the other schemes?*

It is no mystery that A-CDMA outperforms the other schemes since this system is based on PN sequences. We know that these sequences have very sharp autocorrelation functions (this also means very low cross-correlation with shifted version of itself). This is clearly illustrated by **Figure 5.3-6** where we see the comparison between the autocorrelation function for a PN-sequence and a wavelet ('db7'). We can see that the PN code has a very low "side lobes" so that any reflected signals that arrive with a delay greater than one sample will be easily rejected. In fact, the theoretical curve for this autocorrelation shows that for proper signal recovery we must be synchronized within a quarter of a sample **[Pet]**. This is the well-known property that makes Direct-Sequence Spread Spectrum (DS-SS) systems so resistant to multipath propagation **[Feh1]**.

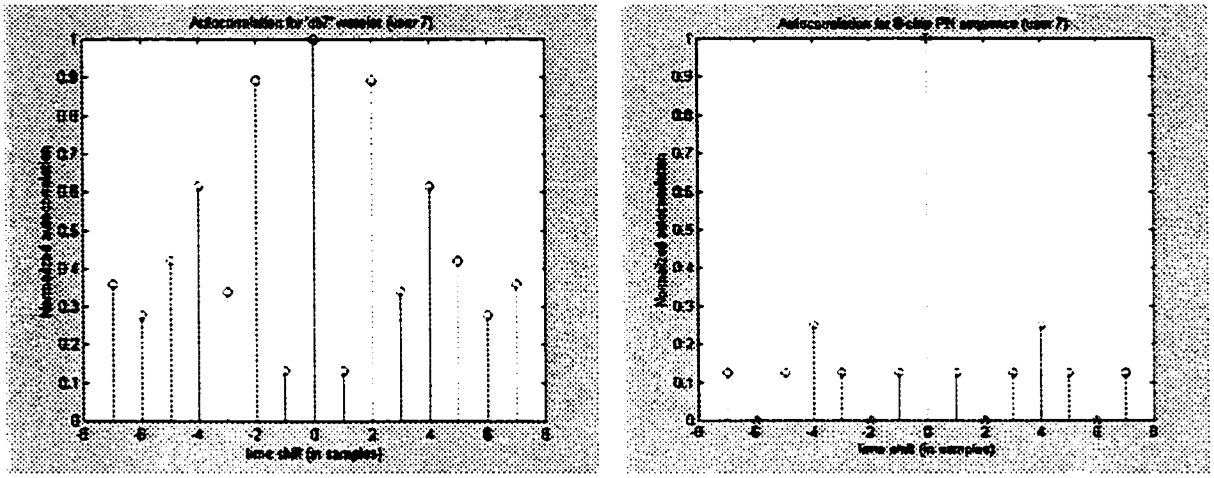


Figure 5.3-6 Autocorrelation function for a 'db7' wavelet (left) and PN sequence (right).

Another view that is worth looking at is the frequency domain. **Figure 5.3-7** (top) depicts the frequency response of the channel at different delay spread values. Here we can consider the frequency response of the channel for 4 values of delay spread and compare them to the frequency response for user 7 in all three schemes, see **Figure 5.3-7** (bottom). We can see that the channel distortion produces "nulls" that have a severe impact on the user whose energy in concentrated in that frequency band. For instance, we can see that user 7 of the WDMA scheme has most of its energy concentrated in the middle of the spectrum. Thus when τ is equal to 2 or 6 the most severe degradation is observed. The key observation here is that the spectrum for the same user in the A-CDMA system does not have any specific energy concentration. In fact its energy is "spread" over the full bandwidth thereby creating a form implicit frequency diversity that makes it resistant to frequency selective fading in the channel.

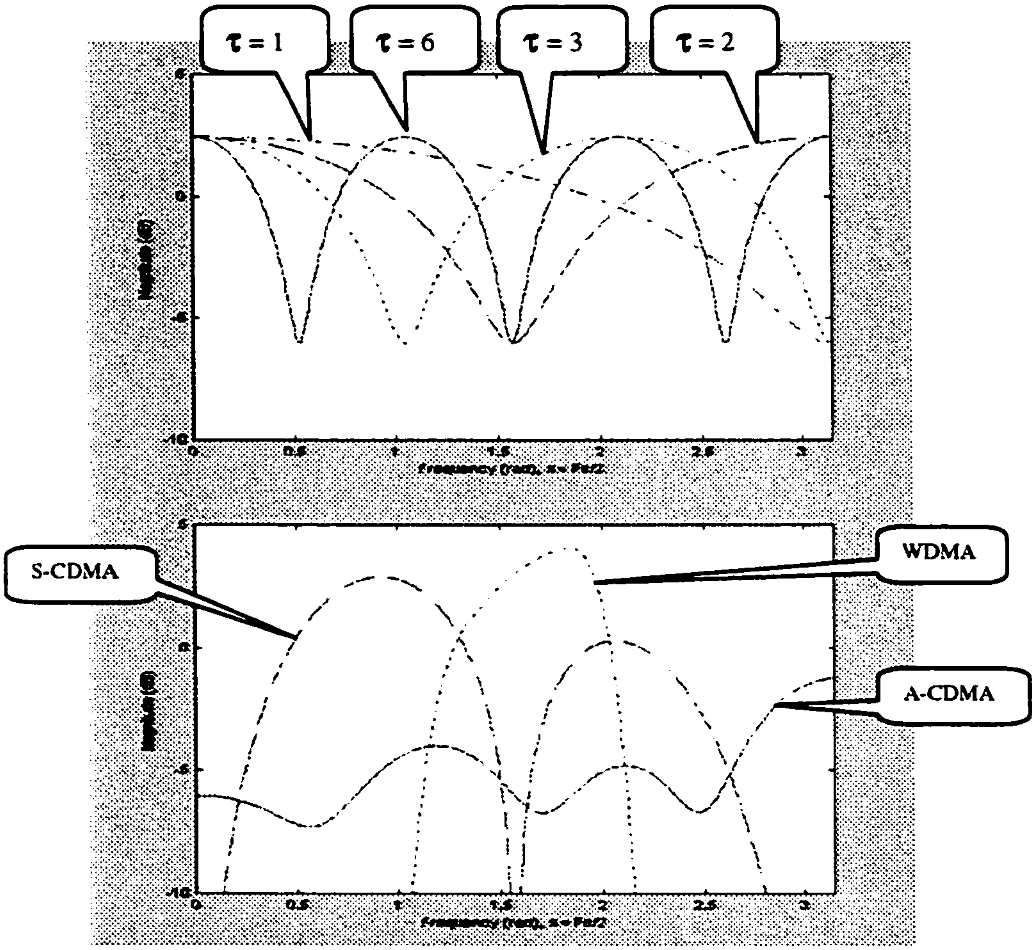


Figure 5.3-7 Frequency response for channel at different delay spread values (top) and frequency response for user 7 in different schemes (bottom).

Figure 5.3-7 (top) also shows that users in the S-CDMA scheme have less "concentration" of energy than in WDMA. This explains why in **Figure 5.3-4** we see that higher order wavelets perform slightly worse than those of lower order in reaction to multipath effects (we know that the higher the wavelet order, the more energy concentration we will expect).

2) *Figure 5.3-5 shows what appear to be erratic result, why is that?*

This figure shows the bit error rate for users 1, 5, 6, and 7 (chosen arbitrarily) as the delay spread changes from 1 to 7. We can see that different users are affected in a non-uniform fashion as the delay spread changes.

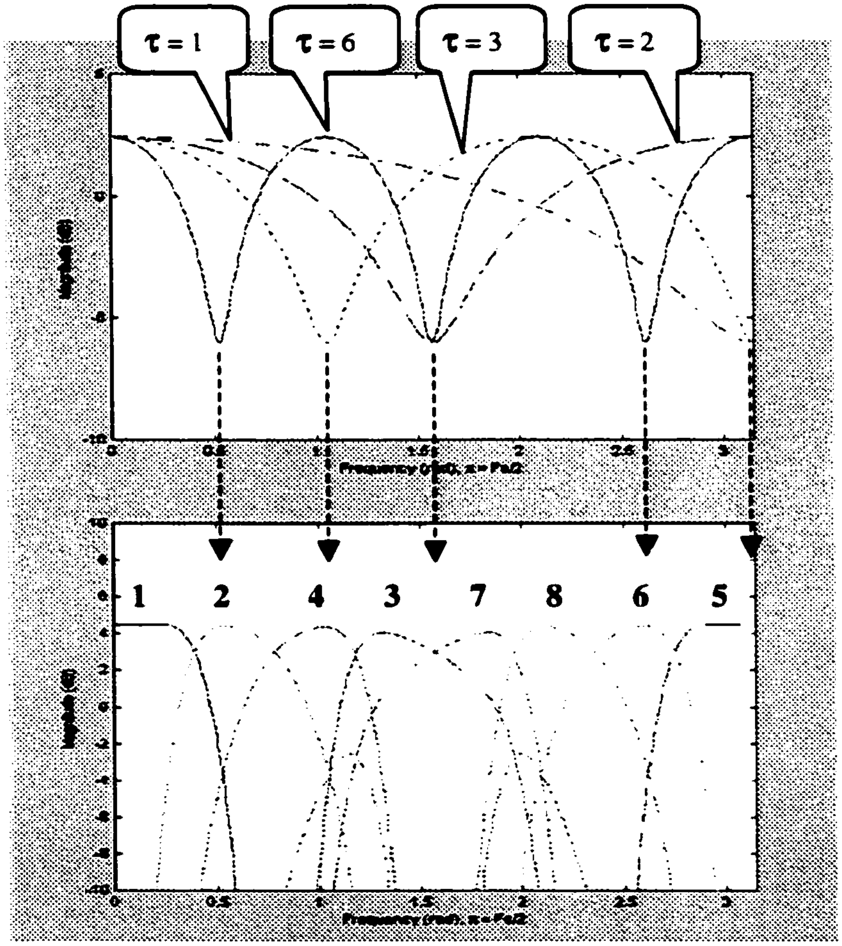


Figure 5.3-7 Frequency response for channel at different delay spread values (top) and frequency response users 1 to 8 in a WDMA scheme (bottom).

While this does not appear to have an obvious pattern, the explanation is clearer when we look at the frequency domain where we can see that as the delay spread changes, we create a channel response with nulls located at different locations in the spectrum. Since each user is concentrated in a relatively narrow portion of the spectrum, then we can expect that while some users are severely affected others do not show degradation. For instance, when the delay spread τ is 2 we see that the null in the channel will affect mostly user 7, then user 6, while users 1 and 5 are virtually unaffected. As we continue to correlate the location of the nulls with the user spectrum occupancy we find the results in **Figure 5.3-5** are actually very coherent.

While it was an interesting exercise to view the effect of specific values for the delay spread τ, in a real life scenario this value would be changing constantly. The speed with which it changes depends on the *coherence time* of the channel, which is related to the relative speed of motion of the receiver. Earlier we said that a mobile traveling at about 80 Km/hr would have a channel with a coherence time of about 15 ms. This means that every 15 ms we can expect the channel to change. This means on the average, we could expect that all users in the WDMA system will be affected by multipath.

Observation:

In our simulation we use the value of τ to represent the delay spread of the channel. To relate the values used for τ (1, 2, etc) to the actual delay spread of a typical wireless channel, recall that the delay spread is an attribute of the environment, not the system being used. Typical numbers range from 1 to 20 µs. Hence, if we had a narrowband system such as IS-54 where the symbol rate is about 41 µs so the multipath could be as much as ½ symbol, or $\tau = 4$ in our 8-user system.

5.4 Synchronization

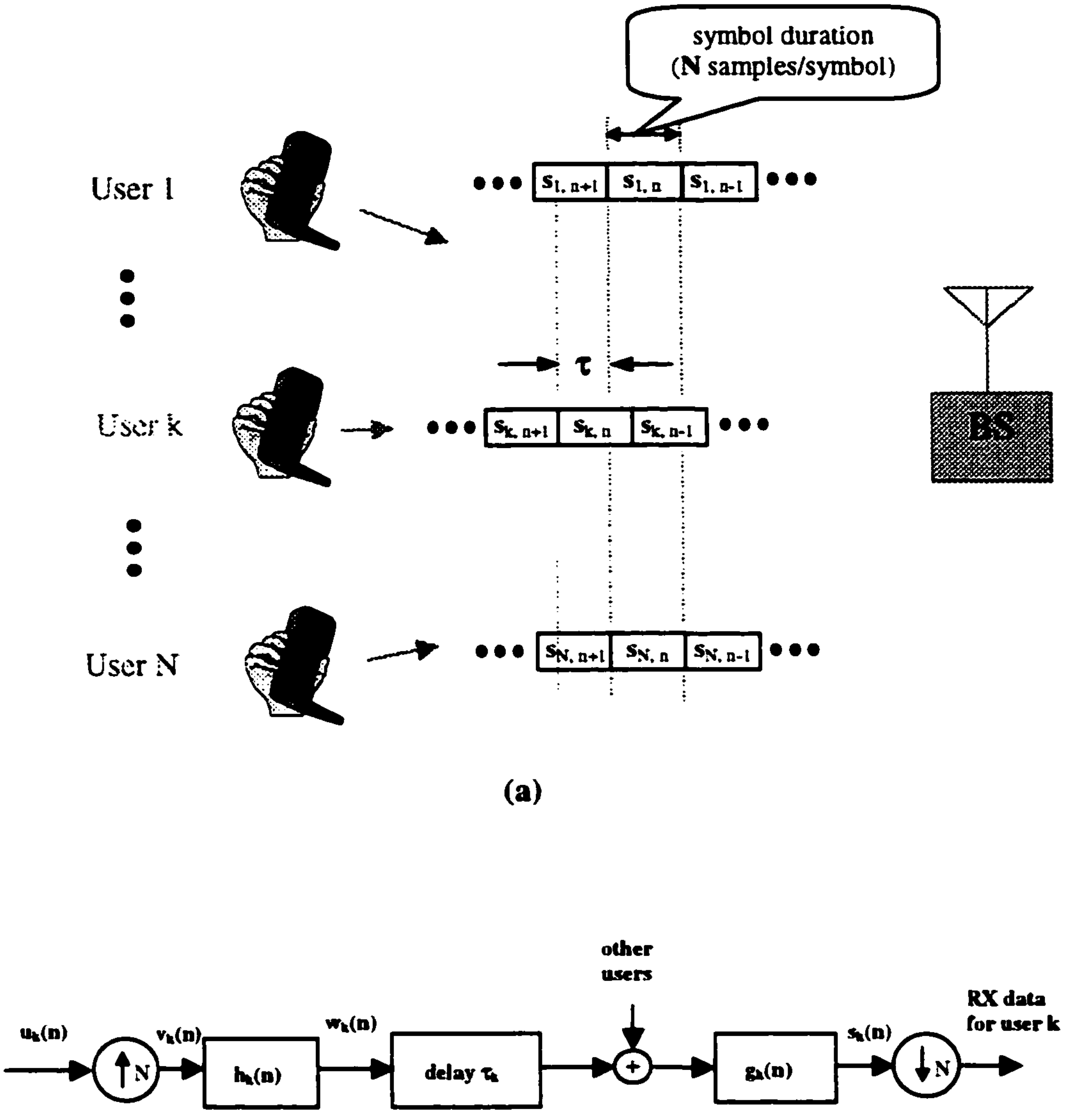
We can assume that in the forward path (i.e. base station to mobile) signals sent to the various mobiles are synchronized. Typically the mobile station monitors some kind of pilot channel to acquire sync (already done in IS-95/ CDMA, IS-54/TDMA, etc). However, in the reverse channel, the mobile units would be transmitting unaware of each other's presence (all signals would be effectively "summed" in the RF channel). By disrupting synchronization among users we are essentially disrupting orthogonality, hence we need to explore the effect of such disruption.

Setup:

Figure 5.4-1 shows a mobile to base station scenario along with the block diagram for this experiment. In this case we vary the value of τ_k for only one mobile (user *3*) and observe how other users are affected by this disruption. We set up a system with only 8 users so that we can observe if the degradation is a "two-way" problem, i.e. is the mobile out of sync affected as much as all others? Note that since we have an 8-user system we transmit 8-samples for every symbol (or bit) sent by the user. This means that when τ_k has a value of 2, we are essentially forcing user 3 to be out of sync by ¼ of a symbol (in A-CDMA this would mean "2 chips").

Expectation:

Here we expect WDMA and S-CDMA to be the least favorable performers since their operation is heavily based on orthogonality (basis for perfect reconstruction).



(a)

(b)

Figure 5.4-1 (a) Unsynchronized transmissions in the reverse path.
(b) Block diagram used in this simulation.

We know that the process by which we map bits to an orthogonal waveform is "time-variant" hence it is very clear that introducing a time delay for one user will create a lack of synchronization in the system. However it is hard to speculate on the severity of the degradation.

Since A-CDMA is based on PN sequences of very low cross-correlation it is natural to expect this setup to have no added degradation due to the lack of synchronization among users.

Results:

Tables 5.4-1 to 5.4-5 show the Bit Error Rate (BER) associated with db2, db3, db7, S-CDMA, and A-CDMA respectively.

Table 5.4-1 Effect of one unsynchronized transmitter (using Wavelet db2)

Delay τ for User 3	Bit Error Rate per user (%)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	0	0	0	0	0	0	0	0
1	0	0	19.5	0	0	0	12.2	0
2	0	0	1.4	0	0	0	0	0
3	0	0	16.2	0	0	0	10.9	0
4	0	0	4.9	3.2	0	0	0	0

The first row in the table above indicates that when user 3 is fully synchronized, i.e. τ is 0, then the BER figure is 0% for all users (this is what we expect since orthogonality has not been broken). The second row shows that when user 3 is out of sync by 1/8 of a symbol, i.e. τ is 1, then the BER figure is 19.5% for user 3, 12.2% for user 7 and 0% for all other users. The third row show the BER figures for $\tau = 1$, and so on. These results show that when user 3 is not in sync with other users the BER is significantly increased for user 3 and 7. However, this lack of synchronization causes high BER only for certain shifts. **Tables 5.4-2** and **5.4-3** show the system performance for the same test but with higher order wavelets, db3 and db7, respectively.

Table 5.4-2 Effect of one unsynchronized transmitter (using Wavelet db3)

Delay τ for User 3	Bit Error Rate per user (%)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	0	0	0	0	0	0	0	0
1	0	0	19.3	0	0	0	15.3	0
2	0	0	0	0	0	0	0	0
3	0	0	18.2	0	0	0	14.7	0
4	0	0	0	0	0	0	0	0

Table 5.4-3 Effect of one unsynchronized transmitter (using Wavelet db7)

Delay τ for User 3	Bit Error Rate per user (%)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	0	0	0	0	0	0	0	0
1	0	0	15.3	0	0	0	16.0	0
2	0	0	0	0	0	0	0	0
3	0	0	20.6	0	0	0	20.9	0
4	0	0	0	0	0	0	0	0

After observing the behavior of the Wavelet-based system, we then proceed to explore how the S-CDMA (Walsh codes), and A-CDMA (PN codes) perform under the same test (see results in **Tables 5.4-4** and **5.4-5**).

Table 5.4-4 Effect of one unsynchronized transmitter (using S-CDMA)

Delay τ for User 3	Bit Error Rate per user (%)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	0	0	0	0	0	0	0	0
1	0	0	21.2	6.0	0	0	0	0
2	0	0	7.7	0	0	0	0	0
3	0	0	17.5	7.1	0	0	0	0
4	0	0	9.1	0	0	0	8.0	0

Table 5.4-5 Effect of one unsynchronized transmitter (using A-CDMA)

Delay τ for User 3	Bit Error Rate per user (%)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	13.8	16.8	15.0	13.4	14.0	15.2	13.8	13.1
1	13.6	14.2	13.5	14.3	14.5	15.9	15.0	14.2
2	15.0	15.0	14.1	16.1	13.4	13.7	16.5	13.1
3	16.0	15.2	14.5	15.3	15.2	14.3	15.2	13.9
4	13.7	14.8	15.6	13.8	12.9	14.9	15.5	16.5

Here we can see that while the Walsh codes display a behavior similar to that of a wavelet-based system, the system based on PN codes yields different results. *The key observation here is that A-CDMA is not affected by the fact that user 3 is transmitting out of sync.*

Another interesting observation is that as the wavelet order is increased, the scope of the interference focuses on users 3 and 7 only (or 3 and 4 for Walsh codes). Also we note that for low order wavelets (e.g. db2) the BER figure for user 3 is higher than that for user 7, but this condition changes when using higher order wavelets (e.g. db7) the BER is the same for users 3 and 7.

Discussion:

It is important to keep in mind that since our simulation consisted of only a couple of thousand runs, the BER figures are likely to have a fair margin of error, i.e. we should not infer anything by observing a difference of $\pm 2\%$.

1) Why A-CDMA outperforms S-CDMA and WDMA?

This can be simply explained by the fact that A-CDMA is based on the use of PN sequences which do not rely on orthogonality. In contrast, the orthogonality condition required for wavelets and Walsh codes is clearly broken by disrupting the time-domain arrangement among users. This is one of the key challenges for applying an orthogonal-based multiple access technique in the reverse channel.

It is important to highlight that although the BER figures for the A-CDMA case (**Table 5.4-5**) appear to be high, the *relative* level of error did not change by disrupting the time synchronization among users. The already high BER figures are present with or without time alignment since they are due to "multi-user" interference. We know that A-CDMA is a system based on "interference management" since other users appear as AWGN. This problem was explored in section 4.2 where we saw that adding more users to the system is reflected in an increased noise level.

2) Why are users 3, 4, and 7 affected in S-CDMA?

The BER figures observed in **Table 5.4-4** appear to be somewhat erratic. We know that the frequency domain overlap between users 3 and 4 is higher than users 3 and 7, hence we expect some interference. However there seems to be a discrepancy between the BER figures for users 4 and 7, i.e. there does not seem to be a linear behavior that we can observe.

Since we know that shifting a digital filter in the time domain affects the frequency domain response, then we can expect that user 3 will not conform to the expected frequency response. We must recall that the basis functions applied for each user are in fact FIR filters! Hence, an answer to our question can be found by examining the "cross-correlation" between user 3 and all others. We can expect that if a shifted version of user 3 is highly correlated to another user, then mutual interference will follow. This can be illustrated by looking at **Table 5.4-6**, which is a copy of **Table 5.4-4** (results for S-CDMA) with the cross-correlation coefficient shown in brackets besides the error figure for each user. For example, we can see that in the second row of this table (i.e. user 3 is out of sync by 1/8 of a symbol), the BER figure for user 4 is 6% and the cross-correlation factor is -0.87. In this table we have used **bold** printing to highlight the non-zero BER figures to show the correlation between high BER and high cross-correlation factors. As we see these figures side by side we can then appreciate that the seemingly erratic results obtained for this test are in fact explained by the cross-correlation measure between the offending user (#3) and all other users.

Table 5.4-6 Effect of one unsynchronized transmitter (using S-CDMA) and its cross-correlation factors with all other users.

Delay τ	Bit Error Rate per user (%), and Cross-correlation factor (in brackets below)							
for User 3	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	0 (0.0)	0 (0.0)	0 -	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
1	0 (0.12)	0 (0.12)	**21.2** -	**6.0** **(-0.87)**	0 (0.12)	0 (0.12)	0 (0.12)	0 (0.12)
2	0 (0.25)	0 (0.25)	**7.7** -	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.25)	0 (0.0)
3	0 (0.12)	0 (0.12)	**17.5** -	**7.1** **(0.62)**	0 (-0.12)	0 (-0.12)	0 (0.12)	0 (-0.37)
4	0 (0.0)	0 (0.0)	**9.1** -	0 (0.0)	0 (0.0)	0 (0.0)	**8.0** **(-0.5)**	0 (0.0)

Another curious aspect about these results is that the "offending" user (#3) suffers more than the affected user (#4). We can attribute this difference to the fact user 4 has high correlation with user 3 only, but zero with all other users. However, that is not the case with user 3. We can see that all other users behave as *small sources of interference* that contribute to a higher BER figure.

3) Why do higher order wavelets limit the interference to two users only?

After looking at **Tables 5.4-1** (for db2) to **5.4-3** (for db7) we see that the interference begins to focus more on to users 3 and 7 with equal BER figures. This is due to the decrease in cross-correlation among users as the wavelet order is increased. This is clear when we superimpose the BER and cross-correlation figures (see **Table 5.4-7**).

Table 5.4-7 Effect of one unsynchronized transmitter (using 'db7' wavelets) and its cross-correlation factors with all other users.

Delay τ for User 3	Bit Error Rate per user (%), and Cross-correlation factor (in brackets below)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
0	0 (0.0)	0 (0.0)	0 -	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
1	0 (0.0)	0 (-0.01)	**15.3** -	0 (0.05)	0 (0.0)	0 (0.0)	**16.0** **(-0.50)**	0 (-0.01)
2	0 (0.0)	0 (-0.01)	0.0 -	0 (0.05)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
3	0 (0.0)	0 (0.0)	**20.6** -	0.0 (-0.03)	0 (0.0)	0 (0.0)	**20.9** **(0.70)**	0 (0.01)
4	0 (0.0)	0 (0.0)	0 -	0 (-0.08)	0 (0.0)	0 (0.0)	0.0 (0.0)	0 (0.0)

Observation:

In the interest of analytical work, we allowed only one user to break out of the collective synchronization, however in real life all users would potentially be out of sync. This means that without time alignment the high error rates would be present in all users. Time alignment among mobiles is already done in TDMA systems (at relatively low data rates), however it is hard to imagine how this could be achieved at very high data rate.

5.5 Narrowband Interference

It has been well documented that DS-SS systems consistently outperform narrowband systems (i.e. FDMA) when operating in the presence of a jammer [Pet]. Hence in this test case we are interested in exploring the performance of a wavelet-based system when adding a narrowband signal to the channel.

Setup:

As in other test cases we suppress all other forms of interference (AWGN, multipath, etc) to ensure that we isolate system performance related to narrowband interference. In this case the narrowband interference is modeled using a pure tone located at one half of the signal bandwidth ($f_i = f_s/4$). The power in this signal is set to be about 4 times higher than that of each user. The total number of users is set to 8 so that we can easily appreciate the effect on different users. Figure 5.5-1 shows the basic setup for this test case (N=8).

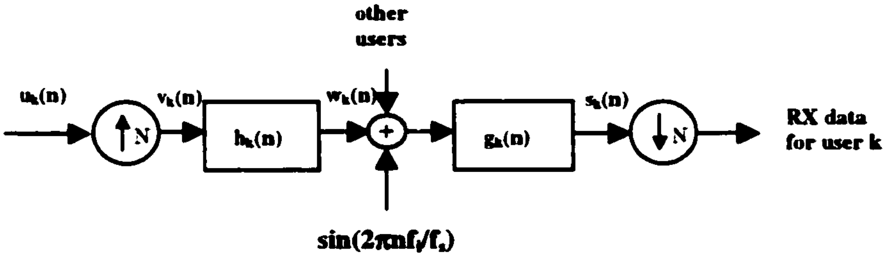


Figure 5.5-1 Setup for Narrowband interference case.

Expectation:

Given the processing gain found in the DS-SS system we expect to observe the least amount of interference in this case. In fact we expect that the A-CDMA (based on a PN sequence) will be completely unaffected by the presence of a tone since the jammer power will be spread while the desired signal is de-spread. In the case of an S-CDMA (based on Walsh codes) we expect that at least one user will be vulnerable to the presence of a tone since the signal is not totally spread, rather somewhat concentrated in a given frequency region. Since the wavelet scheme is similar to that of Walsh codes (i.e. based on orthogonality) we expect wavelets to have similar performance. One question here is: how many users are affected by this interference?

Results:

After creating an environment with a Signal-to-Interference (S/I) ratio of –6dB we obtained clear results about the effect of Narrowband interference on various users (see Table 5.5-1).

In this set of results we see that the A-CDMA (based on PN-sequences) is unaffected by the same interference level which had devastating effects for some users in the other schemes. For both wavelets and S-CDMA (based on Walsh codes), we note that two of the 8 users are severely affected by the presence a jammer.

Although not very noticeable, we can observe a minor difference in degradation levels for the affected users. Lower order wavelets appear to have a very slight advantage over the higher order ones. Walsh codes (equivalent to 'db1') have the lowest percentage of error.

Table 5.5-1 Effect of Narrowband Interference (8-user system, S/I = -6db)

System Type	Error per user (%)							
	User 1	User 2	User 3	User 4	User 5	User 6	User 7	User 8
db2	0	0	47.1	0	0	0	47.4	0
db3	0	0	48.1	0	0	0	48.8	0
db4	0	0	52.0	0	0	0	53.1	0
db8	0	0	56.2	0	0	0	55.7	0
A-CDMA (PN-seq.)	0	0	0	0	0	0	0	0
S-CDMA (Walsh codes)	0	0	45.6	46.2	0	0	0	0

Discussion:

The results shown in Table 5-5.1 raise some questions that are worthwhile exploring:

1) ***Why were users 3 and 7 so severely affected for wavelets?***

This question can be answered by looking at the spectral allocation of all 8 users and their relative location with respect to the interfering signal. In Figure 5.5-2 we can clearly see that the interference is directly located at the center of users 3 and 7 hence the explanation for their extreme degradation.

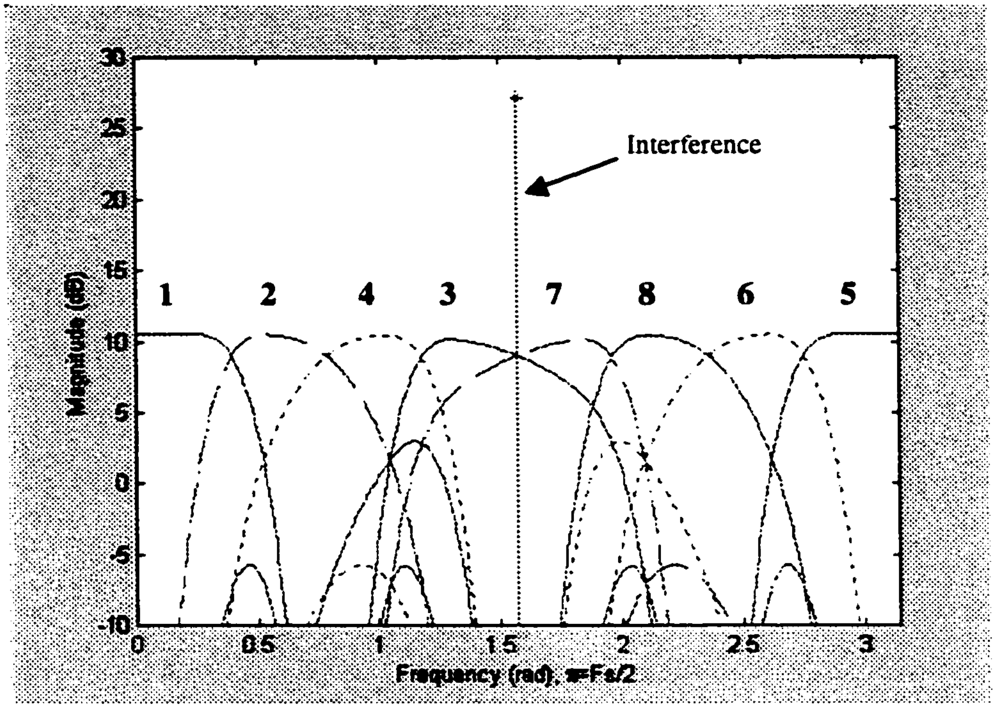


Figure 5.5-2 Narrowband interference with frequency $f_i = f_s/4$ in an 8-user system (using db8).

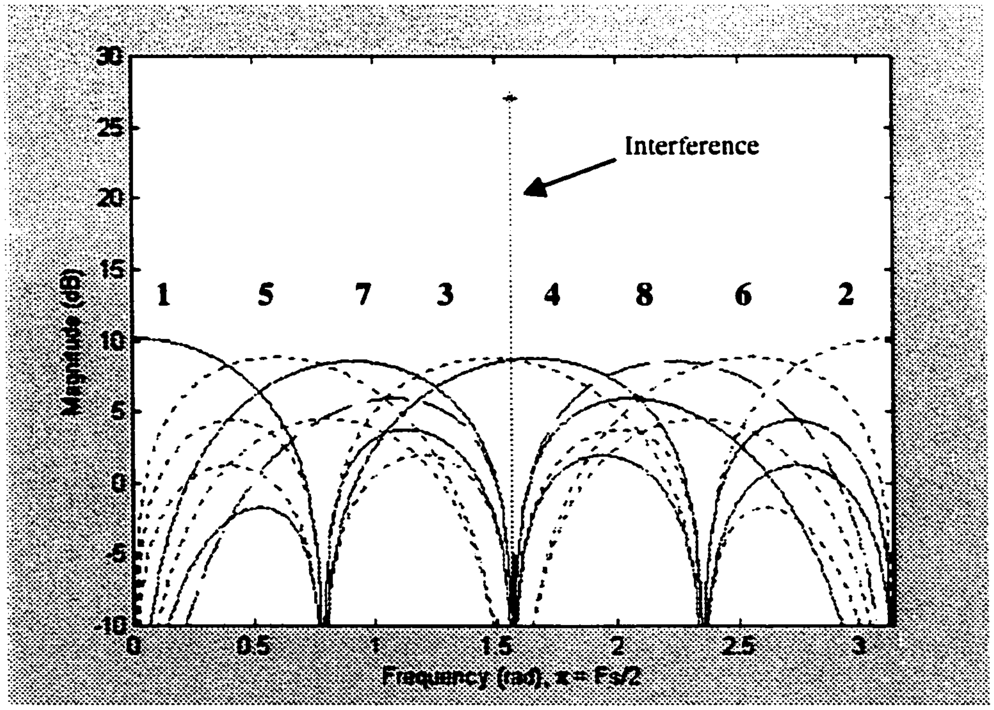


Figure 5.5-3 Narrowband interference with frequency $f_i = f_s/4$ in an 8-user system (using Walsh codes).

2) *Why does the S-CDMA show different users being affected?*

This question again is something that can be answered by looking at the spectral allocation for each user. In the case of Walsh codes we can see that users are allocated in a slightly different arrangement than in wavelets (this was discussed in Chapter 4). In **Figure 5.5-3** we can see that it is users 3 and 4 that are located at the interfering frequency. Since Walsh codes are like 'db1' in the sense that they have very high side lobes, there is a certain amount of redundant energy outside the interference region. This may account for the very small advantage in error rate.

5.6 Signal Power

Although CDMA-DSSS has provided ample evidence of its robustness in the presence of multipath and narrowband interference (**[Sou]**, **[Gar]**, **[Vit1]**), one implementation problem in a multiuser environment is power control. Since each user transmission consists of a PN sequence whose spectral characteristics resembles that of white noise, then when we have many users 'stacked' on the same frequency band, increasing power for one user means increased interference for others! Hence CDMA based on DSSS is based on "interference management" so that all users get fair access to the channel. This management is accomplished by a very sophisticated scheme to regulate the transmission power from each mobile almost every millisecond **[IS-95]**. This also creates a problem of "admission control", i.e. how can we tell that there are too many users in the same spectral range? There is much active research on this subject **[Gar]**. So the question is: can we expect the same problem for a wavelet-based system?

Setup:

In this case we consider the so-called "near-far" effect that typically affects A-CDMA **[Aka6]**. **Figure 5.6-1** illustrates the case in which user 1 is closest to the base station (distance $\mathbf{d_1}$), user 2 and 4 are further (distance $\mathbf{d_2}$), and user 3 is at the furthest location (distance $\mathbf{d_3}$). In this case user 1 has twice the power of users 2 and 4 while user 3 only has half of it. In this case we injected a small amount of AWGN to show that users that are further away from the base station have higher BER than others that are closer.

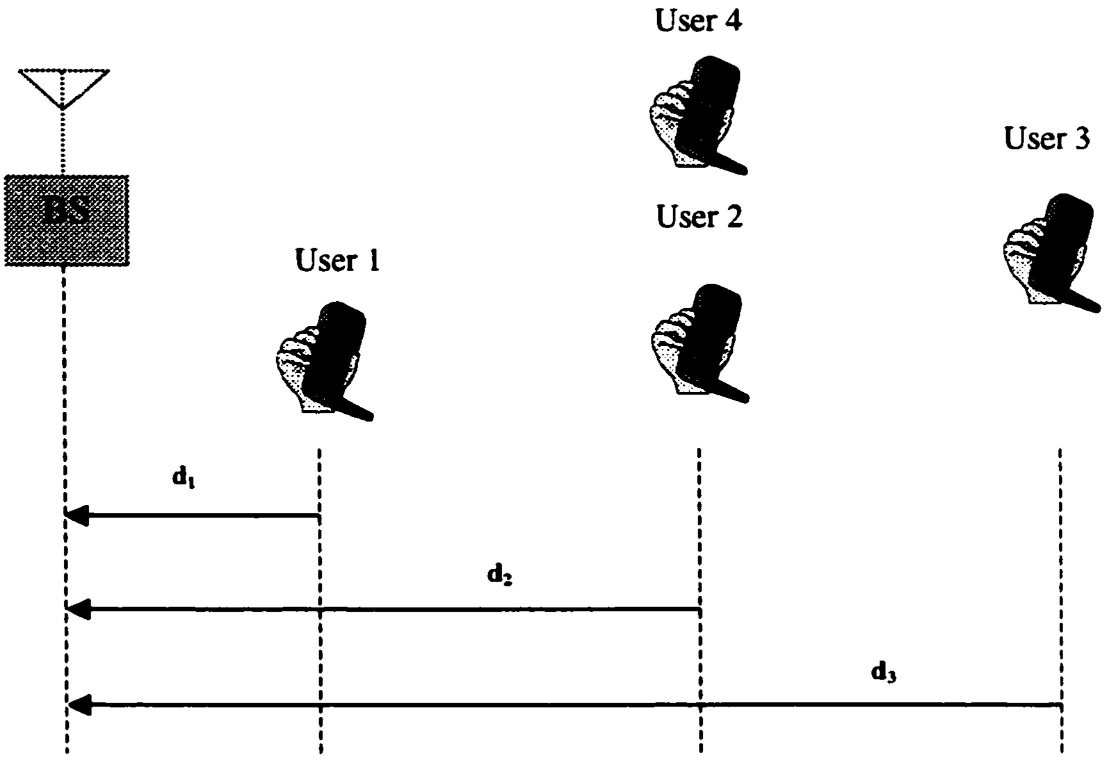


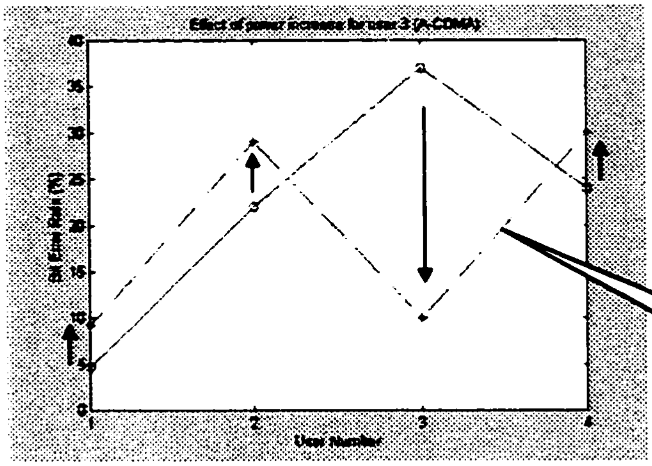
Figure 5.6-1 Scenario for power control test case.

Expectation:

As stated in the literature (**[Pet]**, **[Feh1]**, **[Aka6]**), we can expect the A-CDMA system to be the most sensitive to unequal power levels. We can expect the S-CDMA and wavelet based schemes to remain unaffected since we can freely overlap excess bandwidth thanks to the perfect reconstruction properties of such systems.

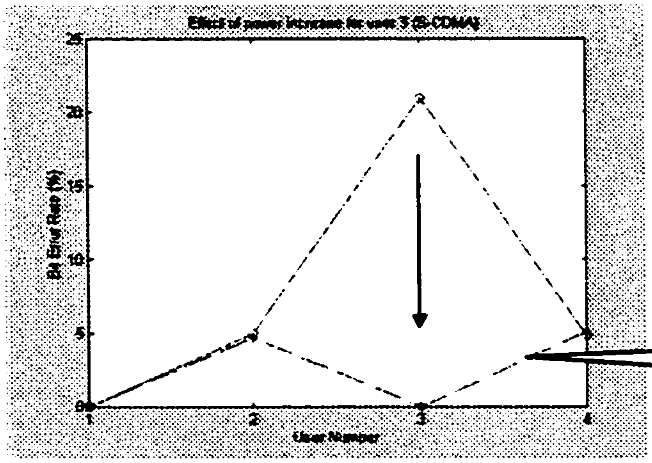
Results:

Figures 5.6-2 to 5.6-4 depict the effect of increasing the power for user 3 to compensate for its low level signal in schemes A-CDMA, S-CDMA, and WDMA, respectively.



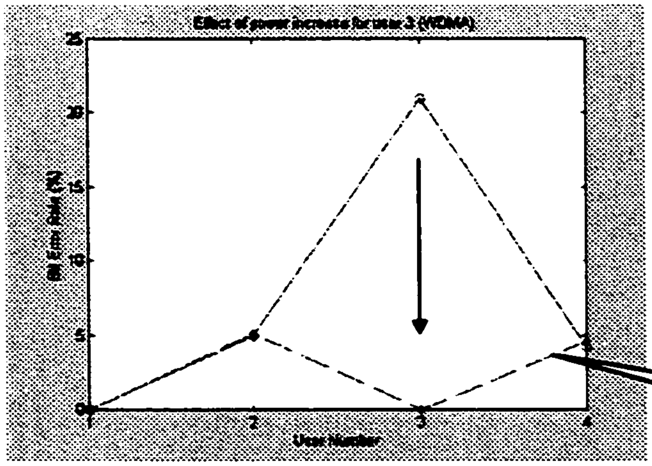
After power adjustment for user 3.

Figure 5.6-2 Effect of increasing power for user 3 by a factor of 4. A-CDMA case, i.e. PN sequences used.



After power adjustment for user 3.

Figure 5.6-3 Effect of increasing power for user 3 by a factor of 4. S-CDMA case, i.e. Walsh codes used.



After power adjustment for user 3.

Figure 5.6-4 Effect of increasing power for user 3 by a factor of 4. WDMA case, using db7.

Figure 5.6-2 shows that as we raise the power for user 3, its BER figure improves remarkably but at the expense of worsening the performance of other users. By contrast, we can see that in **Figures 5.6-3** and **5.3-4** the power increase for user 3 does not affect the performance of other users.

Discussion:

These results are expected since we know that S-CDMA and WDMA are based on orthogonality and hence do not suffer from multi-user interference. A-CDMA on the other hand is based on interference management, i.e. we know that other users behave as AWGN noise sources. Hence when one of those users becomes stronger, other users perceive it as a stronger noise source. These results illustrate the fact that in IS-95 systems we must maintain a very tight power control scheme so that all users have only the precise amount of power needed **[Feh]**.

Since both, S-CDMA and WDMA are based on orthogonal arrangements then the power control problem is not an issue. Changes in power for one user are "orthogonal" to other users.

Observation:

We must remind ourselves that in a practical scenario where we encounter multipath, synchronization problems, and other channel disturbances, we may not be able to ensure orthogonality among users. This means that as we lose the orthogonal arrangement among users we will then have to deal with the difficult issues of power control.

5.7 System Level Issues

In addition to the quantitative analysis described in previous sections, it is worthwhile to discuss other issues that may be more difficult to evaluate: system level issues. Although we cannot run a given simulation to obtain a figure of merit for each system, we can highlight some key characteristics of each system and provide some insight into how they relate to system as a whole. In this thesis we have turned our attention to the multiple access issue, which is a low level aspect of a wireless system. However, this low level aspect may have a tremendous influence in the way we architect the entire system.

When we read DSP textbooks and recent advances we often get a "microscopic" view of the algorithms and their performance, e.g. a vocoder with very low output data rate. However, this focused view must be extended to understand the macroscopic effect of a given DSP algorithm. For instance, having a vocoder that produces the lowest data rate would appear to the best choice since it would required less of the precious RF bandwidth. In reality this is not a clear choice since having a low rate vocoder would imply that the signal has less "redundancy" and therefore we would have to add redundancy (in the form of error correction) to resist a harsh RF channel. A clear example of that is GSM and TDMA (IS-54). The former uses a 13 Kbps vocoder while the later uses a 7.95 Kbps. However, TDMA does not allow for "twice" the number of users! The redundancy or apparent "waste" of bandwidth found in the GSM vocoder is in fact key to allowing a different "frequency planning" scheme. Similarly, in IS-95 the "variable" rate vocoder was not chosen arbitrarily but as a result of the power control issue. These are just examples of how the internal aspects of a given DSP block can influence larger system issues. The following is a brief discussion on how the choice of multiple access schemes can significantly impact system level issues.

5.7.1 Computational demand

When IS-95 was introduced there was a requirement for a different hardware platform in both, the base station and mobile units. Computational demand and a different front-end RF module were some of the factors behind this new requirement. A key factor in a spread spectrum multiple access scheme is that we need a RAKE receiver to combat frequency selective fading. In the IS-95 setup (4-finger RAKE receiver) this means that we essentially have four receivers working in parallel. This was a significant increase from the computational power required for conventional TDMA phones.

So what can we say about the computational requirement issue in a wavelet based system? Some key observations can be made by looking at a simple (4-user) system based on different schemes: Walsh codes, PN codes, and Wavelets.

In section 5.1 we illustrated how all of these schemes could be realized with a single structure by changing **h(n)** and **g(n)**. See **Figure 5.7.1-1**. where **N** is set to the number of users (equal to 4 in this case).

Let's consider the following cases:

1) Walsh codes: in this case **h(n)** is one of the rows of a 4x4 Walsh matrix. In this system we need to convolve **v(n)** with **h(n)** but we can take advantage of the fact that 3 out of every 4 samples in **v(n)** are zero (due to the upsampling by N=4). Hence for every required sample of **w(n)** we need to perform 1 multiplication and 0 additions. At the receiver we can also exploit the multi-rate structure. Although we do not have input samples of zero

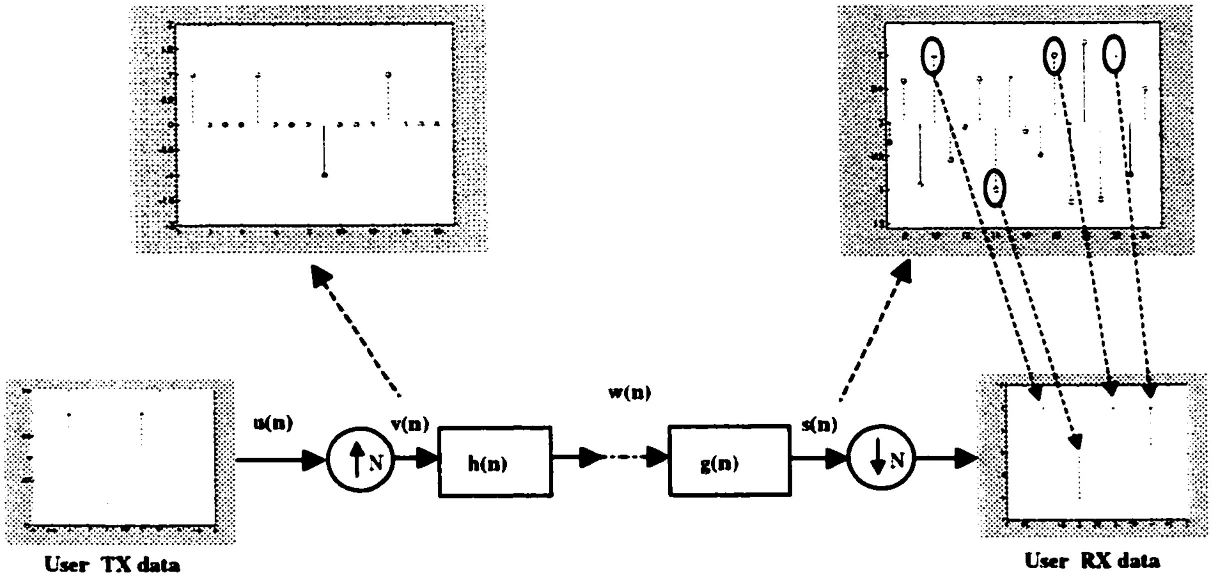


Figure 5.7.1-1 TX and RX kernels for a 4-user system showing some intermediate waveforms while transmitting a test sequence, u[n]=[1 −1 1 1].

value (as is the case for the transmitter block), we know that the downsampler will eliminate 3 out of every 4 samples. Therefore to obtain one sample of user data we need to perform 4 multiplications and one addition. This efficiency is just the standard benefit of using a multi-rate filter structure [Vai1].

2) PN codes: this case is identical to the previous one since the **h(n)** consists of a PN sequence of length 4. Hence our computational demand does not change.

3) Wavelets: in this case the computational demand will vary depending on the order of the wavelet being used. Unlike the previous cases, we have more freedom to select the "family" of values for **h(n)**. For instance, if we choose the 'db1' wavelet, then the length of **h(n)** will be 4 and therefore the computational demand will be exactly that of the previous cases. On the other hand, if we choose to use the 'db2' wavelet (which has longer support), then **h(n)** will be of length 10. In this case **h(n)** is longer and therefore will overlap with two non-zero samples of the input stream **v(n)**. This will lead to 2 multiplications and one addition required to produce one sample of **w(n)**. In general, the number of multiplications will be determined by the number of non-zero samples in **v(n)** that overlap with **h(n)**, i.e. **M** module **N** (where **M** is the length of the **h(n)** and **N** is the number of users). A similar argument applies to the receiver block.

In summary, we can see that wavelets do have a higher computational demand over Walsh or PN-based systems. As we increase the wavelet order, we also increase the computational requirements of the system.

Observation:

We are assuming that we know precisely which one of the N samples we need to keep at the receiver output! This means that we assume exact synchronization between the receiver and the transmitter (which is reasonable). However, as the mobile moves closer or further away from the base station we are going to experience propagation delays that will require us to "track" the precise down-sampling point. This means that in practice we need to perhaps compute some of the neighboring samples.

5.7.2 RF Management and Hand-off

One of the great advantages of using A-CDMA is that two base stations can make use of the same RF frequency. This is due to the fact that unlike if FDMA systems, the base stations are distinguished by phases of a PN sequence rather than RF carriers. In chapter 3 we saw that in IS-95 we have a combination of orthogonal multiplexing in the forward channel (acting as channelizer within the cell), but we also have PN sequence spreading to separate base stations. This combination of signal processing schemes allows a mobile to 'listen' to two or more base stations by using the same RF carrier. This feature is what allows IS-95 mobiles to execute "soft" hand-offs, i.e. make a new connection before breaking the old one. This feature removes a tremendous burden from the cellular deployment process in which one needs to assign RF carriers to specific cells while minimizing their co-channel interference.

If we were to accomplish a similar capability in a wavelet based system we would need to "stack" two wavelet schemes. In other words, would require one wavelet access scheme to separate users in a base station, and another higher-level scheme to separate base stations! This concept is not yet very clear and hence an interesting topic for further research.

Observation:

Current IS-95 systems still have to perform some kind of RF planning and frequency allocation since they remain, to some extent, FDMA systems. We must recall that one IS-95 carrier is only 1.25 MHz wide, which clearly insufficient to hold all the users in a cell. Hence, a cell makes use of 10 RF carriers to make full use of the 12.5 MHz officially allocated cellular band. It is interesting to see that in the larger scheme of things, all existing cellular standards are till FDMA based and therefore mobile units must remain capable of "re-tuning" RF circuitry.

5.7.1 Admission Control:

In previous sections we highlighted the fact that a drawback in PN-based systems was the fact that other users appeared as interference (hence the need for power control). A problem associated with this fact is that it becomes difficult to know the point at which the system has too many users. We can have a user yield the same BER under two very different scenarios. In one case, there are very few other users and high noise while in the other case there are many users and low noise. For this reason it has become very difficult to estimate the system "load" and hence subject of "multi-user" detection has become an extensive area of very active research **[Vit]**, **[Pet]**, **[Feh]**, **[Aka6]**. In essence, a PN-based system gives us a non-deterministic capacity.

A serious byproduct of this problem is that if we cannot easily determine when we have saturated the system, then it becomes difficult to provide a measurement for Quality of Service (QoS). One of the goals of third generation system is to provide flexible schemes in which some users can pay more than others in exchange for better service. As of today is not possible for a person to visit a cellular provider and request that he or she pay twice the regular phone rate to get better voice quality. AMPS, TDMA, and GSM are all systems based on a fixed bandwidth per user scheme. IS-95 has the potential to provide unequal QoS by allowing one user to use more power than others, however this cannot be easily done because of the difficulty in estimating the point at which the system has reached its capacity.

On the other hand, a wavelet-based system can provide unequal bandwidth (i.e. unequal QoS) while supporting a deterministic system capacity. We have seen that since the orthogonal multi-user scheme is not based on interference management but on a disciplined time-frequency arrangement, we can precisely determine the number of users (of possibly unequal bandwidth).

Chapter 6

Conclusions

In this thesis we reviewed the fundamentals concepts of wavelets and wireless cellular communication (chapters 2 & 3 respectively). We then explored the details of how wavelets can be used to multiplex users in a communication channel. Much of our exploration was devoted to gain insight into how this technique compared to existing multiple access schemes (chapter 4). After having done a qualitative analysis of wavelets to support a multi-user scheme, we then ran a series of simulations to gain insight into performance issues related to the cellular application (chapter 5).

At the start of this thesis, we began with the notion that wavelets can be used to support a multi-user communication environment (**[Het]**, **[Aka7]**, **[Wu]**). We then stated that we would focus our investigation on exploring whether this concept *can in fact be a feasible option for a wireless cellular communication system* (**Section 1.1**). In other words, ***Are Wavelet Division Multiple Access schemes a preferable choice to be deployed in place of existing systems?***

The answer appears more likely to be "no" (...for now).

The most significant problem we can highlight is that wavelet-based multiuser support is based on the principle of orthogonality, which imposes the requirement for "time alignment" among mobile users. This design constraint is very difficult to satisfy in the reverse channel of a cellular environment, particularly when we are dealing with very high data rates. This was demonstrated in section 5.4.

Since we are citing current technological limitations for the problems associated with a practical deployment of a WDMA system, then it is only fair to ask the question:

Could a Wavelet Division Multiple Access system be deployed in the future?

The answer appears to be "yes" (... with some qualifiers).

We have seen that wavelets present a very rich set of properties that could lead to a very efficient and flexible system. These attractive properties constitute a solid reason to pursue further research in the subject.

There are two technological developments that could make WDMA a feasible option in a cellular application. The first one is internal to the field of wavelet research, i.e. the development of new wavelet families that are less vulnerable to the problems we have mentioned before. Some developments include the design of wavelets that would be less susceptible to the loss of orthogonality, and so on. The second one is an external one, i.e. the development in wireless systems engineering and DSP technologies. Here we are referring to drastic changes in the way we architect cellular networks. For instance, as the cost of hardware reaches very low levels and the packet-based switching infrastructure is significantly extended, we could consider a cellular environment in which cells would have a radius in the order of meters and not kilometers. In this environment we would be dealing with mostly line-of-sight communication with very low propagation delays that would de-emphasize the time alignment problem. We are actually approaching a time when the notion of Software Defined Radios (SDR) is becoming possible. This is a case in which the DSP component is actually very close to the antenna, and base stations could be networked as part of an internet structure. We can even consider downloading different air interface protocols to fit the local wireless environment.

In **Chapter 4** we elaborated on how we could use the multi-rate structure to implement different access schemes by simply changing the basis waveform (i.e. filter coefficients). We specifically showed how the same multi-rate filter bank structure could be used to realize S-CDMA (Walsh codes), A-CDMA (PN codes), and WDMA by simply changing the filter coefficients. This particular feature makes this concept extremely appealing when thinking about SDR since we can conceive an "adaptive" system that can change multiple access schemes based on channel conditions. Furthermore, we talked about downloading protocols. We can think of a mobile entering a new cell and being downloaded a new "waveform" (i.e. basis function/filter taps) that has been optimized for that environment. This concept would go a long way towards alleviating the current lack of interoperability that leaves a GSM subscriber "out of service" although he is meters away from a CDMA base station.

In addition to the fundamental question about the feasibility of wavelets to support a multiple access scheme, we also began this investigation with the desire to answer other questions such as:

How does a wavelet-based system differ from the traditional methods?

After a very detailed exploration of a WDMA (Chapter 4), it became clear that this method was very different from TDMA but that it could behave anywhere between a FDMA and CDMA scheme. When the wavelet applied was a Daubechies of high order, then users would overlap in time but not in frequency (with the added benefit that "guard bands" were not required). On the other hand, if the wavelet order were lower, the signals would overlap both in frequency and in time (we showed that this is already being used in the forward channel of IS-95 in the form of Walsh codes but in conjunction with PN codes). In Chapter 4 we highlighted that Walsh codes were actually identical to a Daubechies wavelet of order one (i.e. the Haar). On this topic there is one important clarification to make, the scheme we are referring to is known as "synchronous" CDMA. This is different from the "asynchronous" CDMA that is implemented with PN sequences and has a more equally spread spectrum (more discussion on this later). When considering the literature of CDMA this terminology becomes very unclear and many times ambiguous. There are few but excellent references that clarify the CDMA terminology (**[Sou]**, **[Vet1]**, **[Aka4]**, **[IS-95]**). Having compared WDMA with the traditional multiple access methods we can now answer the next question:

Is this technique really "new" or simply another variation of the conventional techniques being used?

After having reviewed the existing multiple access schemes, i.e. TMDA, FDMA, and CDMA, we found that WDMA to be very different from the former but very similar to the later (Chapter 3 & 4). We compared WDMA, Synchronous CDMA (based on Walsh codes), and Asynchronous CDMA (based on PN codes) and established that WDMA is not a "brand new" concept for multiple access scheme, but rather one that is part of a more general model that also includes S-CDMA. We are referring to the notion of employing a family of orthogonal functions to multiplex users. We are saying that if we can associate the signal of a given user with a basis function of an orthogonal family, then we can later "detect" or "differentiate" such user from any other.

The concept of applying an inverse/forward transform as a multiplexer/demultiplexer is quite mature. In the early 90's we saw at least one High Definition TV (HDTV) proposal based on the Fourier Transform **[Led]**. In this case we used the "inverse" Fourier transform to multiplex many inputs into one stream and the Forward Fourier transform to reverse such process (this approach became known as Orthogonal Frequency Division Multiplexing or OFDM). We can say that WDM is part of the same family of "orthogonal-based" transceiver. In OFDM we are using sine and cosines as our family of basis functions whereas in WDM we are using wavelets. Comparing these two methods would be a great subject for further research.

Given that we have investigated the association between a wavelet-based system and CDMA (a term associated with spread spectrum systems), we can then answer the next question:

Some researchers have referred to wavelets as a means to implement spread spectrum systems [Het],[Aka7]. How does that work?

In Chapter 4 we showed that lower order wavelets produce users signals that would spread both in time and frequency, hence behaving similar to the traditional definition of a spread spectrum system **[Pat]**. However, as the wavelet order is increased, the signal power becomes more concentrated in a particular section of the transmission band (see diagrams in Chapter 4). The basic structure of the wavelet transmitter clearly showed that the input signal was subsequently up-sampled, in turn causing the output spectrum to be 2^L times wider (where L was the number of times we up-sampled). However, this expansion in the spectrum is "shaped" by the wavelet basis chosen for the system (see Chapters 4 & 5 for details). It is important to note that while the Daubechies wavelets provided a wide-band signal, the spectral energy was not "uniformly" distributed (as it is when we used the traditional PN-sequence method to spread the spectrum). Attempting to achieve this characteristic with other wavelet families is a subject of current research (**[Aka4]**, **[Orr]**, **[Mon1]**, **[Wor2]**). This particular characteristic is not a requirement for commercial cellular communication but highly desirable in Low Probability of Intercept (LPI) systems for covert applications.

After learning about the mechanisms and operating principle of WDMA we proceeded to examine specific performance issues related to a cellular environment. This effort was motivated by the desired to provide an answer to our last question, which is:

What are the advantages and problems related to this multiple access method?

Earlier in this chapter we referred to the problems associated with the multi-user time alignment requirement. On the positive side of things we highlighted issues related to flexibility, attractive features to support asymmetric links, unequal data rates among users, and lots of room to investigate new implementations.

In **Chapter 5** we ran a series of test cases that were not intended to provide a "performance specification", but to gain further insight into the operation of a wavelet based system. We found that such detection was not so simple in an environment in which the orthogonal arrangement was threatened. We noted that although PN sequences had problems related to power control and inter-user interference, they performed better where WDMA and S-CDMA failed. The reason is simple: *systems based on PN codes do not rely on orthogonality to perform their detection.* On the other hand we say that PN based systems had far less capacity than the orthogonal arrangement due to the fact that users behaved like noise sources to one another (this is consistent with previous research **[Sou]**, **[Aka2]**). This leads to another serious problem with PN based systems: admission control. Since the users appear as sources of noise, then it becomes difficult to determine if there is room for more users. In other words the capacity of the system is non-deterministic. This has proven to be a significant challenge where much active research is in progress under the subject of "multi-user detection" **[Haf]**. However, when we use an orthogonal systems such as the one studied here we have an a-priori knowledge that we can support as many users as basis function. For example, if we construct a system with a wavelet tree of 2 levels then we know that we can support a limit of 4 users (i.e. 2^L, where L is the number of levels).

A possible scenario to consider is to use an orthogonal-based system in the forward channel (where users can be time-aligned) and a PN-based scheme in the reverse channel.

Further Research

Here are some of the topics that need further study:

- *OFDM*: After having studied the details about wavelet-based multiplexing we can see that this is very similar to DFT-based OFDM. In fact we can even think of wavelets as being a form of OFDM (and less related to CDM). In retrospect we can see that it would have been very interesting to compare the use of wavelets and complex exponentials as orthogonal basis for the multiplexer. OFDM was not included initially since we limited ourselves to systems currently in use for wireless cellular communication.
- *Other wavelets*: In the interest of focus we restricted our analysis to Daubechies wavelets. However, there are many other families of wavelets that could be used in the same context. It would be of great interest to re-run the test cases described in **Chapter 5** and observe if we can say anything different about their performance (this could be extended to compare "the optimum" wavelet with OFDM).
- *New wavelets*: An interesting research topic would be to "design" new wavelets that are tailored for a particular channel. Is it possible that we can tailor a particular wavelet design to provide immunity to multipath? This would include the search for wavelets

that may have spectral characteristics that are "spread" rather than "compact" (as Daubechies wavelets are). This would be highly desirable for Low Probability of Intercept (LPI) systems.

- *Equalization*: In order to focus on specific behaviors of wavelets we restricted our simulation model to be static, i.e. the multipath was not changed with time (as it would due to the doppler spread). The question we would ask is: how do we go about designing a receiver that can remove the signal reflections introduced by non-stationary multipath reflections? In spread spectrum wide band systems we use the RAKE receiver to handle this problem. However, if we are using wavelets of higher order, we have a set of very compact spectra that coexist in a wide band. The approach to handle this problem would likely be different since we are not dealing with signals that have a very sharp autocorrelation function, as that of the PN sequences.
- *New Signal Envelops*: One of the characteristics associated with the output of a wavelet system is that the signal envelop has a very high peak-to-average ratio **[Wu]**. The problem with this is that we need a linear amplifier to avoid distorting the output signal. Linear amplifiers are now available but sufficiently expensive that we only find them in base stations but not in mobile units. The question would be: can we design a new wavelet prototype that can provide a low "peak-to-average" envelop?.
- *Base station synchronization*: One of the problems we highlighted here is that in the reverse channel we cannot easily synchronize users, hence we lose orthogonality (which in turn leads to poor performance). With the recent arrival of high power DSP processors, it is conceivable that we can use the base station as a type of "buffer" that

would collect the signals from all mobiles and synchronize them. This would remove the burden for the mobile to perform inter-user synchronization.

Chapter 7

References

7.1 Comments

This chapter contains a list of publications and sources consulted throughout the preparation of this thesis (**Section 7.2**). In addition to the conventional listing provided in **Section 7.2**, an effort has been made to catalog a subset of very useful (and essential) references (**Section 7.3**). For instance, useful publications on the general theory of wavelets have been separated from those that are intimately related to the application dealt within this thesis. It is hoped that such effort will provide a more useful set of references for those who wish to further investigate the topic presented in this thesis. The following categories have been identified:

1. General Wavelet Theory and Applications (other than multiplexers)
2. Wavelets for Multiplexers
3. Communications

Furthermore, specific references that were found to be exceptionally useful have been marked with an asterisk (e.g. **[Ari]***).

A note about the Internet

During the last decade we have seen a widespread use of the internet to disseminate technical information. While some of the content found in the internet is very useful and valuable, there is a larger amount of information which comes from sources with no academic (or equivalent) credentials. A strong effort has been made to ensure that the material used from the internet comes from sources with substantial academic credibility, eg. internet pages from professors at highly respected universities such as MIT, Duke University (these references are grouped separately).

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