The Effect of Composite Action of Steel Beam

and Concrete Slab on the Analysis

Results of Steel Frames

by

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TABLE OF CONTENTS

TABLE OF CONTENTS

SYNOPSIS

When composite action is provided for members in a structural system, a greater saving in materials and also greater structural rigidity can be achieved. Composite construction for this reason is becoming increasing popular both in bridge structures and more recently in building frames.

This report presents a theoretical investigation of the effects of providing composite action in a building steel frame and its advantages; economic as well as structural rigidity when compared to a corresponding non-composite steel frame.

A comparison of the changes in moments and relative cost on a weight saving basis of a composite steel frame to that of a non-composite one is made with reference to a typical two-story, two-bay rigid frame(Fig.4-1). The span length is varied from twenty feet to fifty feet, and for each span length the live load to dead load ratio is varied from one to three. The composite and the non-composite frames are analysed using the STRUDL programs.

To realise the full advantages of composite action, the variation of the moment of inertia along the composite beam is taken into account in the analysis. Complete interaction is assumed between the two composite components: concrete and steel.

This investigation indicated there is a definite advantage

SYNOPSIS

in using composite construction — it has the effect of reducing the negative moments and increasing the positive moments. Since the negative moments are the controlling forces, lighter members can be used, while the increase in stress at the positive moment section can be taken care of by the composite section.

LIST OF FIGURES

LIST OF TABLES

LIST OF COMPUTER PROGRAMS

CHAPTER 1

INTRODUCTION

1.1 General

Composite construction as it is known today was first credited to H. Kahn in 1926, but the earliest investigations in this field were conducted by Truscon Steel Co. in the U.S. between 1921-23; by the National Physics Laboratory in England between 1922-23 and by the Dominion Bridge Co. of Canada in 1922. These investigations considered steel beams that were partially or completely encased in concrete and concerned mainly with the interaction resulting from natural bond between the two components. It was not until 1933 that mechanical shear connectors were employed to transfer shears in composite beams. These investigations began in Switzerland, and were followed by others in the United States and Europe (15).

However it was not until 1944 that a code governing the design and construction of composite beams was first published by American Association of State Highways Officials. Since then practically all investigations of composite action have utilized mechanical shear connectors. The development and adoption of a code governing building design followed somewhat later - it was in 1952 that the AISC incorporate a chapter on composite design of beams in it's specifications. Composite construction consisting of a concrete slab attached to a steel beam by mechanical shear connectors has since then become quite common and it is also widely accepted as indicated in a questionnaire conducted by the ACI (7). That composite construction has achieved wide recognition is also evidenced by the number of existing national codes governing its design (62, 56, 33, 5, 57).

A composite beam usually consists of a concrete slab attached to a steel beam or girder by mechanical shear connectors which forced the two components to act as an integral unit, when the beam is loaded. The shear connectors do this by performing the functions of transfering horizontal shears and preventing vertical separation between the slab and the beam. The shear connectors may be considered as the most important part of a composite beam.

To the present, a great many different types of shear connectors have been used sucessfully. Fig.1.1 shows the shear connectors in common use. Before the mid. 1950's, channels and spirals were by far the most popular (15). Research on composite construction using welded studs as shear connectors began in 1954. Due to the ease in fabrication and flexibility in design, they are currently the most popular. Other types of shear connectors such as bar and hoop, tee and hoop, angle, inclined stirrup and plate have been used with success, but they are not so popular as the welded studs.

The most common steel sections used in composite construction are the rolled wide flange beams and the built-up girders. Fig.2.1 shows some of the sections that have been used in composite construction.

The advantage of a composite beam lies in the utilization of the properties of the two different materials; compression in concrete and tension in the steel beam. The combined strength of the two components is very much greater than that, when the components are acting alone. With this in view, a properly designed composite beam can have the following advantages as compared to a non-composite one.

1) Savings in steel weight for a floor system.
2) Reduction in overall beam depth.
3) Longer spans are possible
4) Deflections are reduced.
5) Composite construction will result in a very stiff floor system which can withstand strong vibrations from machinery and other moving loads.

1.2 Object of Study

Before the advent of high-speed computers, it was usually considered impractical to carry out an 'exact' analysis for a continuous composite structure, particularly a building frame. Lacking such facilities, the result was to employ an approximate method; consequently the full advantages of composite action were not realised. The alternative is to use simple beam construction.

However with the availability and wide use of large capacity and high-speed computers nowadays, it is just as easy to analyse a rigid frame with non-uniform members as it is for a simple beam structure.

The object of this investigation is to determine the effect of composite action in a building frame by making an exact analysis, so that the full benefit afforded by composite action is realised.

The steel section used in this study will be a rolled section without cover plates at the negative moment section or at the bottom flange of the positive moment and composite section. The use of a rolled section is by no means the most economical section for a composite beam. A non-synmetrical section with cover plates at the bottom flange of the composite section, in combination with cover plates at the negative moment region, is more economical. In general, further reduction in steel weight can be achieved by transfering steel from the top flange to the bottom flange as it is done in built-up sections. However the prime concern of this report is to determine the effect of composite action based only on the rolled steel section alone - so that fabrication will also be limited to a minimum.

In addition, complete interaction between the steel and concrete will be assumed for the analysis.

1.3 Previous Research

The earliest studies on composite beams began in the early 1920's and they dealt primarily with interaction between the steel and concrete resulting from natural bond between the two components. Experiments were carried out in Canada, Britain, Europe and the U.S.A. at about the same time (15).

In Canada, the first tests were reported by McKay, Gillespie and Leluau in 1923. Only two specimens-rolled sections fully encased in concrete, were tested. McKay followed in 1927 with a series of tests on 13 specimens; 7 were simply supported, of which four were fully encased I-beams, two with only the top flange encased and one with bond absent from the top flange of a fully encased I-beam, and six fully encased beams were tested for the study of continuity. They reported good interaction between the steel beams and concrete. Bond failures occured in the two partially encased beams and on the basis of the tests he carried out, McKay recommended a working bond stress of 240 psi for fully encased beams.

In England, tests on fourteen floor specimens were carried out at the National Physics Laboratory between 1922 and 1923. A more comprehensive study was reported by Batho, Lash and Kirkham in 1939 on tests performed on twenty-seven composite simple beams. Eighteen of these beams were fully encased I-beams, while nine were partially encased. From these tests, the investigators concluded that the theory of reinforced concrete (assuming complete interaction between steel and concrete) is applicable to composite beams as long as bond between steel and concrete is not broken. They recommended an allowable bond stress of 60 psi for fully encased beams and 50 psi for partically encased beams.

In Europe, tests on composite beams without shear connectors were reported by Stüssi, Cumbernac and Baes. Stüssi reported that beams tested failed at between 80% and 100% of the theoretical fully plastic capacity and that preloading has no effect on the ultimate capacity. He suggested that composite beams can be designed on the basis of their ultimate moment capacity.

The first composite beams tested in the U.S.A. were slightly different, in that natural bond was angmented by some mechanical means. Tests in 1923 carried out at the University of Nebraska, Perduc University, Mass. Institute of Technology and at the Truscon Steel Co. has rolled beams with prongs attached to the edges of the top flanges. Good interaction between steel and concrete and high overload capacities were observed. R.A. Caughey in 1929 also recommended an allowable bond stress of 60 psi for fully encased beams, on the basis of tests on six composite T-beams.

Most of the previous tests investigated behavior under static loading only. In 1939, Greunig in Germany investigated the effect of vibrating load by testing four fully encased beams under an oscillatory load. More recently in 1951, Fuller performed a test on an actual bridge structure (3) with partially encased beam (only top flange encased in concrete). He reported both a definite presence and absence of composite action in different parts of the bridge slab and stringer structure.

Partically encased beams as indicated above cannot be fully relied on to provide the bond necessary for composite action when subjected to large fluctuating loads. Stüssi and Caughey advocated the use of mechanical shear connectors to carry the horizontal shear. The first systematic studies of beams with mechanical connectors was carried out by in 1933 in Switzerland. The mechanical connectors employed were spiral shear connectors and the specimens tested were mainly push-outs and a few T-beams. Further tests on both push-outs and T-beams with the same type of connectors followed in the U.S. These tests were conducted at Columbia University, Lehigh University and by Mains at Lehigh University, and by Viest for Nelson Stud Welding between 1937 and 1956 (15).

Experiments with other types of shear connectors, such as hooks, rolled shapes, and rectangular bars were also carried out at about the same time, mainly in Europe. On the other hand American investigators favored the use of flexible connectors such as channels and studs. Prior to the introduction of stud connectors, channels were very popular. Two extensive investigations of channel shear connectors were carried out at the University of Illinois by Siess, Viest, & Newmark and by

Viest, Siess, Appleton, & Newmark covering the period from 1942 to 1950. Experiments with stud shear connectors began in 1954 at the University of Illinois and at Lehigh University. Because of the ease of fabrication, stud connectors are currently the most popular. More recent investigations of stud connectors were conducted by Thurlimann, King, Slutter, & Driscoll, Toprac, and Slutter and Fisher. These papers were published in Highways Research Record (11, 31, 32, 34). These studies included the tests of small scale and full size specimens for both composite beams and push-out tests. These studies dealt with the general behavior of composite structures and mainly with the fatigue behavior of stud connectors.

Behavior of Shear Connectors

The functions of the shear connectors are to transfer horizontal shear between the slab and the beam and to prevent vertical uplift of the slab from the beam. These two functions are more clearly defined in some connectors such as the bar and hoop connector (Fig.1-1), than in others such as the stud, where the same element performs both functions. The root of the stud transmits the horizontal shear while the head holds the slab down. Failure of the shear connection can occur by the shear connector shearing or by the crushing of the concrete, or both.

It is evident that the properties of shear connectors can best be determined experimentally. The most satisfactory means of assessing the performance of a shear connector is to test a beam provided with the shear connectors under normal loading conditions. Such a procedure would be very costly as there are many variables to be considered; such as slab size and span length. A more economical method is to simulate the behavior of a shear connector in push-out test (Fig.1-2). This method has been widely used for evaluating the shear strength and load-slip characteristics of shear connectors. The advantage is that the loads to which the shear connectors are subjected, can be more easily evaluated.

The size of the slab will affect the behavior of the connector. High local stresses in the concrete in the immediate vicinity of the shear connector will cause premature failure of the concrete if the slab is too small. The crushing failure of concrete with bearing pressures as high as three times the cylinder strength of concrete were reported by Ros, Graf and at the State University of Iowa (15). This high state of pressure exerted by the connectors on the concrete was possible because of a triaxial state of stress.

Before failure, elastic deformation and slip will occur between the connector and the concrete. The capacity of the connector is evaluated on the basis of a load-slip curve, with the useful capacity arbitrarily defined as the load corresponding to a residual slip of 0.003 inch (13). A typical load slip curve is shown in Fig.1-3.

Analytical expressions for the useful capacity or critical loads for different types of shear connectors are given in Ref. 6, 9, & 10. The critical loads are given in terms of the concrete strength and the geometry of the connectors.

For a channel connector, the critical load is given by:

$$Q_{uc.} = 180 \ (h + 0.5t) \ W \sqrt{f'_c} \quad \ldots\ldots\ldots\ldots\ldots \quad (1.1)$$

where
- h = maximum thickness of channel flange
- t = thickness of web of channel
- w = length of channel measured in a transverse direction on the flange of the beam

The capacity of a stud connector may be caculated by the expressions:

For studs having $hs / d \geq 4.2$,

$$Q_{uc.} = 330\ d^2 \sqrt{f'_c} \quad \text{.........................} \quad (1.2)$$

For studs having $hs / d < 4.2$,

$$Q_{uc.} = 80\ hs\ ds \sqrt{f'_c} \quad \text{.........................} \quad (1.3)$$

And the capacity of one pitch of a spiral is given by:

$$Q_{uc.} = 3840\ d\ \sqrt[4]{f'_c} \quad \text{.........................} \quad (1.4)$$

In a more recent investigation, Slutter and Driscoll developed analytical expressions for the ultimate strength of shear connectors from push-out tests (14). The equations are similar to those for the useful capacity of shear connectors.

The ultimate strength of stud connector is given by:

$$Q_{u.} = 930\ d^2 \sqrt{f'_c} \quad \text{.........................} \quad (1.5)$$

The formula for the ultimate strength of one turn of a spiral is given as:

$$Q_{u.} = 8000\ d\ \sqrt[4]{f'_c} \quad \text{.........................} \quad (1.6)$$

And the ultimate strength of a channel connector is equal to:

$$Q_{u.} = 550\ (\ h + 0.5t\ )\ w \sqrt{f'_c} \quad \text{................} \quad (1.7)$$

The values of the allowable shear load of stud and channel connectors given in the AISC specification (62) may be obtained by dividing the ultimate capacity by a load factor of 2.5.

Although the push-out test is a useful way of evaluating and comparing the performance of different types of shear connectors, it cannot be expected to exhibit all the characteristic of a beam test. It should be noted that a number of difference exist between the conditions obtained in a push-out test and a beam test.

(1) The slab in a push-out test is not in overall compression as in the beam slab.

(2) The stress distribution over the slab depth is different from that in a beam.

(3) The uplift observed in a beam test is restricted in a push-out test.

In a recent investigation to determine the fatigue behavior of stud and channel connectors by means of push-out tests, Slutter and Fisher (34) found that the lower range of failure of an earlier beam tests (31) is about equal to the mean failure behavior of the push-out specimens. They concluded that the push-out tests therefore represent a lower bound for connector failure.

Behavior of Composite Beams

The concrete slab acts as a cover plate in the composite beam. This is made possible by the use of shear connectors which ensured that the steel beam and concrete act as a unit. If sufficient shear connectors are provided, the full plastic moment may be reached without the failure of the connectors.

When the moment on the positive composite section is increased beyond the working load, the bottom flange of the steel beam yields first and the neutral axis moves upwards causing tensile cracks at the underside. Cracking or spalling of the concrete at the compression face occurs when the strain reaches about 0.38 per cent. The ultimate strength of the section is then almost reached. As the curvature of the section increases, crushing of the slab extends downwards, while strain hardening begins in the bottom flange of the beam - the load being carried remains approximately constant. As the curvature is increased still further, the carrying capacity of the composite section is eventually reduced by extensive crushing of the concrete.

This general pattern of behavior is exhibited by beams subjected to positive bending and for the case where the neutral axis is near the top steel flange or in the slab. In cases where the number of connectors are limited to such an extent, the failure of the connectors occur before the maximum moment is developed. The ultimate moment capacity may also not develop if the plastic neutral axis is close to the bottom beam flange, as crushing at the surface of the concrete occurs before full plasticity can develop in the steel section.

When a composite beam is under load, slip between the slab and the beam will occur. This indicates the presence of incomplete interaction, which is due partly to deformation of the shear connectors and partly to the deformation of the concrete around the shear connectors.

Observations on composite beam tests (11), showed that for small loads and even for full live load, no significant slip occured. This is possible, because the shear at the interface is resisted by bond. The bond resistance develops only if sufficient connectors are provided to hold the beam and slab in close contact. In this respect, it is difficult to distinguish between pure adhesive bond resistance, and the resistance which results from the mechanical keying action of the normal roughness of the beam surface.

The first decrease in interaction takes place as a result of bond failure. It has been found that a composite beam loses interaction in a direct proportion to the loss of effective area of shear connectors (31). Bond failure usually starts at the end of a beam or at the point of inflection and progresses towards midspan. End shear connectors are therefore the first to undergo an increase in stress and consequently the first to fail.

Investigation of composite beams with incomplete interaction was conducted by Newmark, Siess & Viest (2). They showed that for properly designed beams with adequate shear connection, the slips between the beam and concrete were so small, that for all practical purposes, a design based on full composite action will give satisfactory results. The results of this tests are shown in Figs.1-4 and 1-5, which show the load vs deflection and load vs strain curves.

They also presented a theory to take into account the effect of imperfect interaction. The theory was based on the assumption of continuous imperfect connection between the two connecting elements and linear strains in the slab and the beam. The test results are generally in good agreement with the results computed from the theory.

Ultimate Strength and Shear Connectors Requirement

Slutter and Driscoll (26), in their investigation of the ultimate strength of composite concrete and steel beams reported that there was a definite relationship between the ultimate strength of the shear connectors and the ultimate flexural capacity of the beam.

They also showed that if the sum of the ultimate strength of all the shear connectors was adequate to satisfy the internal forces at ultimate load condition, then the theoretical ultimate moment could be attained Fig.(1-6).

A criterion for determining the minimum number of shear connectors required was established, based on the fulfilment of the equilibrium condition at ultimate load. Using this assumption, the ultimate flexural capacity of a beam can be determined for the case where the number of connectors provided were less than that required to develop the theoretical ultimate bending moment.

The study also showed that connectors need not be spaced according to the intensity of static shear to develop the ultimate strength. Due to the redistribution of load on the connectors, uniform spacing of connectors was satisfactory for most loading conditions if an adequate number of connectors was provided. Neither the ultimate strength nor deflections were appreciably influenced by the uniform spacing of shear connectors.

Chapman and Balakrishnan (22, 23) reported that the moment calculated from experimental stress distribution exceeded the theoretical plastic moment by as much as 23 to 25 per cent. This phenomenon was due to strain hardening in the steel beam.

It had also been shown by experiments conducted by Mains and at the University of Illinois (15) that shoring had little or no effect on the ultimate capacity of the composite beam. However in the working load range, the supported beam had about 15% greater capacity than the unsupported beam.

Elastic and Ultimate Strength Theory

The composite section is proportioned by the theory of transformed section - usually the concrete is transformed into an equivalent steel area. In addition the concrete is assumed incapable of resisting tensile stresses.

This theory assumes a straight-line stress distribution and complete interaction between the concrete and steel.

All experimental studies have shown that the transformed section theory is applicable, and also gives excellent approximation even if there is incomplete interaction which will be the case in practice.

For these reasons, the transformed section theory has been generally accepted for design purposes. This theory is also given in most standard structural design textbooks (52, 25, 17, 47).

The ultimate strength for composite concrete and steel beams may be classified into two categories: the first is based solely on the statical equilibrium of internal forces and the second takes into account the distribution of strains in addition to equilibrium of internal forces.

The second theory can theoretically account for the effect of strain hardening which can contribute as much as 25% to the carrying capacity of the beam, but it has not become popular as it is difficult to apply.

The first theory is based on the assumption that at the maximum flexural capacity, each element of the section has reached the plastic state of stress. This method has the advantage of simplicity and represents the lower limit of the bending capacity. Experiments conducted by Slutter and Driscoll (26) showed that the theoretical ultimate moment based on this theory can be reached as long as sufficient shear connectors are provided. Most of the recent revisions in the AISC codes governing composite construction were based on the results of these experiments (26).

Continuous Composite Beams

In connection with research into the behavior of continuous composite bridges, Viest, Fountain and Siess (10) discussed the results of a static test on two model continuous composite bridges: one with shear connectors throughout the length of the beam and one with shear connectors only in the positive moment section.

From these studies, it was concluded that, (i) in the negative moment regions only the slab reinforcement can act compositely with the steel beam; (ii) with shear connectors throughout the length of the beam, the slab reinforcement was fully effective; while with shear connectors in the positive moment regions, the slab reinforcement was only partly effective, and (iii) the behavior of both types of beam was about the same from comparison of strain distribution.

Based on the above considerations, it was concluded that the use of an elastic analysis is justified, and no special provisions are needed for the design of continuous composite beams.

In a report on '<u>Composite Design for Buildings</u>', Culver, Zarzeczny and Driscoll (19) tested a continuous composite beam to establish the feasibility of applying plastic design to a continuous composite beam. The results of the test indicated that plastic design of composite beams was feasible. They recommended that only the steel beam be considered as effective over the negative moment region, and expansion joints provided at this section to eliminate cracking of the slab.

In a later study, Slutter and Driscoll (26) also investigated the feasibility of applying the concept of plastic analysis together with ultimate strength theory to the design of continuous composite members. A two span continuous composite beam was tested statically to its ultimate load. The ultimate moment of the positive moment region was taken as that of the composite section, while the ultimate moment of the negative moment region was taken as that of the steel beam plus the slab reinforcement. It was noted that the theoretical plastic collapse load was exceeded in the test, even though the beam theoretically had inadequate shear connection throughout its length. Cracks were also observed to form in the negative moment region, and expansion joints or additional slab reinforcement was suggested as means to control the cracking.

They concluded that although the feasibility of plastic design of continuous composite members cannot be fully evaluated from the test of only one member, it appears that members in which the negative plastic hinge forms first could be designed by plastic analysis. It was also pointed out that plastic analysis of a member where the positive plastic hinge formed first, is also feasible, because the positive hinge undoubtedly has sufficient rotation capacity to permit formation of a negative hinge.

In a further investigation of the feasibility of applying the concept of plastic analysis and ultimate strength theory to the design of continuous composite beams Daniels and Fisher (40) tested four continuous composite beams. Each beam consisted of two equal spans of 25 feet. On the basis of these tests, they concluded that plastic analysis and ultimate strength can be used for the design of continuous composite beams. The longitudinal slab reinforcement can also be used in the design. However they recommended that further studies are required to establish the rotation capacity of the negative plastic hinges, because local flange and web buckling near the interior supports of continuous composite beams limit the ultimate load capacity of the beam.

Two recent investigations into this aspect were carried out by Johnson, Greenwood, and Dalen (55) on one hand and by Daniels, Kroll, and Fisher (59) on the other. In the latter, the conclusion reached was that there is sufficient rotation capacity of the composite beam in the vicinity of the joint to permit the use of plastic design theory in the analysis.

Application to Building Frames

It has been shown that the composite concrete and steel beam is a very strong structure with a large reserve strength and ductility beyond the load causing first yielding. In addition a smaller steel section can normally be used for composite construction.

The use of stud shear connectors also facilitate the fabrication of composite beams, and with the economy that is possible in this type of construction, the use of composite construction in buildings has become very popular and wide spread.

Numerous buildings have been built with this type of construction, but only the more notable ones were reported (13, 16, 20, 21, 29).

Hooper and Hotchkiss (13) reported that composite design saved 20% to 25% in steel tonnage, for a Federal court house and office building in Brooklyn, N.Y. The seven story court house and four story office building - sixteen million dollar project, was the largest building of composite construction, ever built up to 1960. A number of framing schemes were investigated. It was found that reinforced concrete was competitive on a cost basis, but composite concrete and steel was chosen because of a lighter total overall weight which required smaller columns and smaller footings. All the beams were treated as simply-support.

Three buildings of composite design were reported in Architectural Forum (16). The first was a two-story manufacturing plant with a floor space of around 600,000 sq.ft; the second one was a million dollar addition to Princeton University and the third was Detroit's Cobo Hall with an exhibition floor space of 400,000 sq.ft. For the first one, a steel saving of 25% was reported; for the second, there was a steel work cost saving of 20% (230 tons of steel was required compared to 350 tons for a conventional design); and for the third a 25% saving in girder beam weight was reported. It was also pointed out that a steel saving of less than 15% will usually not justify composite design strictly on a cost basis.

This is not necessary true in every case, Mayes (20) reported that for a 47,000 sq.ft. office building of composite design, there is an average saving of 13.8% in materials and a 12.6% on cost.

Leebrook (49) in a comparison between a conventional and a composite design, showed that there was as much as 43% savings in steel weight for the floor framing and an estimated cost reduction of 30% for floor framing.

Creasy (24) in a discussion on the relative cost of composite construction also made an analysis of a building frame. The analysis indicated a reduction of approximately 15% in the weight of the steel skebeton, and approximately 20% in the overall cost of the composite frame as compared with a conventional design.

As can be seen, a wide variation in the savings in steel weight and cost reduction in composite construction is possible. It ranges from a low of 12% to as high as 43%.

Plastic composite design had also been applied to buildings with success in a limited way. Most of these were built in England. Johnson, Finlinson, and Heyman (58) in the design of a five story building frame found that plastic composite design gave substantial reduction in cost when compared with the established elastic composite design method.

New Concepts in Composite Construction

A number of new concepts in composite construction have been reported in Ref.46. Prestressed planks, precast concrete slabs, and steel decking were used. Of these, the use of steel decking appeared to be the most attractive. It combines the advantage of rapid erection and also acts as a permenant formwork. Research into this type of construction by McDermott (39) and Robinson (54) indicated that it is feasible to design such composite floor by the conventional elastic method. The CSA standard (56) also recognises the use of this type of construction by including a section on such a system in its specification.

The most recent innovation in composite construction could well be the introduction of expoxy resin compound as a shear connector for composite beams. The extensive use of bonding plastic by the aircraft industry in bonding metal cores to metallic skins for forming sandwich panels, suggested that applications of expoxy compound as a shear connector are warranted. Research by Kriegh, Nordby and Endebrock (35) confirmed that expoxy compound can serve as a reliable and safe shear connector for composite T-beams. The test results also indicated that successful behavior of expoxy joined T-beams can be expected under either static or dynamic loading.

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1.5 NOTATION

a	=	Distance from the top of concrete slab to N.A. of composite section, when N.A. is in slab.
A_b	=	Area of bottom flange of plate girder
A_B	=	Area of rolled steel beam
A_c	=	Transformed effective area of concrete slab (bt/n)
A_p	=	Area of steel cover plate
A_r	=	Area of longitudinal slab reinforcement
A_s	=	Total area of steel section
A_t	=	Area of top flange of plate girder
A_w	=	Area of web of plate girder
b	=	Effective width of concrete slab
C	=	Compressive force of concrete slab
C.g.	=	Centre of gravity
C_L	=	M_L / WL^2
C_R	=	M_R / WL^2
d	=	Depth of steel section or diameter of stud connector
d_w	=	Depth of web
e, e', e''	=	Moment arms, Eqs. (3.15e) and (3.16d)
e_c	=	Distance from top of steel section to C.g. of concrete slab.
e_r	=	Distance from top of steel section to C.g. of longitudinal slab reinforcement
E_c	=	Modulus of elasticity of concrete

E_s = Modulus of elasticity of steel

f_b = Stress at bottom of steel section

f_c = Stress at top of concrete slab

f_s = Allowable steel stress

f_t = Stress at top of steel section

F_y = Yield strength of steel

f'_c = Compressive strength of concrete

F_s = Factor of safety

g = Distance of top layer of bars from c.g. of slab reinforcement

h = Column height or maximum thickness of channel flange

h_s = Stud connector height

H = Horizontal shear force

I = Moment of inertia

I_B = Moment of inertia of rolled steel section

I_c = Moment of inertia of composite section

I_r = Moment of inertia of slab reinforcement about own cenfroidal axis

I_s = Moment of inertia of steel section

I_w = Moment of inertia of web of plate girder

K = Numerical factor depending on type of loading
K = 1 for transient loads
K = 3 for sustained loads

K_c = $A_c / (A_c + A_s)$

K_r = $A_r / (A_s + A_r)$

K_s = A_p / A_s

L = Span length

L.L. = Live load on composite section

M = Moment

M_{DC} = Moment due to D.L. on composite section

M_{DS} = D.L. moment acting on steel section alone

M_L = Restraining moment at left support

M_{LL} = Live load moment

M_R = Restraining moment at right support

M_u = Ultimate moment

n = Modular ratio (E_s / E_c)

N = Number of shear connectors

N.A. = Neutral axis

q = Capacity of one shear connector

q_u = Ultimate strength of one shear connector

Q_c = Statical moment of transformed concrete section about N.A. of composite section

Q_u = Ultimate capacity of shear connector

Q_{uc} = Useful capacity of shear connector

S = Spacing of connectors along beam

S_{tr} = Section modulus of bottom steel fiber, composite section

S_s	=	Section modulus of bottom steel fiber, steel section
S_{cc}	=	Section modulus of top concrete fiber, composite section
S_{tc}	=	Section modulus of top steel fiber, composite section
S_{ts}	=	Section modulus of top steel fiber, steel section
S_r	=	Relative stiffness of column and beam (S_c / S_B)
S_B	=	Stiffness of beam (I_B / L_B)
S_c	=	Stiffness of column (I_c / L_c)
t	=	Slab thickness
t_b	=	Thickness of bottom flange
t_p	=	Thickness of cover plate
t_t	=	Thickness of top flange
T	=	Tensile force in steel section
v	=	Shear per unit length
V_c	=	Vertical shear acting on composite section
V_h	=	Total horizontal shear to be resisted by connectors
w	=	Uniform loading or length of channel measured in a transverse direction on the flange of the beam
Y_{bs}	=	Distance from N.A. to bottom steel fiber, steel section
Y_{cc}	=	Distance from N.A. of composite section to top of concrete fiber
Y_{ts}	=	Distance from N.A. to top of steel fiber, steel section

$\overline{Y}_s$	=	Shift in N.A. due to addition of cover plate
Z	=	Plastic section modulus
Z_{tc}	=	Distance from N.A. of composite section to top of steel beam at ultimate load
Δx	=	Deflection at distance x from left support
ϕ	=	Load reduction factor (= 0.85)

CHAPTER 2

THEORY FOR COMPOSITE MEMBERS

2.1 Assumptions

For design purposes, a composite-floor is assumed to be a series of T-beams, each made up of one steel beam and a part of the concrete slab. The properties of the composite section are usually calculated by the theory of transformed section. It is usually more convenient to transform the effective cross-sectional area of the concrete slab into an equivalent steel area. This is done by dividing the effective concrete area by an appropriate modular ratio n, where

$$n = \frac{E_s}{E_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.1)$$

Values of n from the AISC codes and CSA standard are used when available or it can be computed from Eq.(2.1), if the modulii of elasticity for different concrete strength are available. Values of n for different values of f'_c can also be obtain in the AASHO specifications (Sec.1.9.1).

The width of concrete slab that is assumed effective as the flange of the T-beam, is governed by specifications given in the AISC and the CSA codes for composite construction (56, 62).

For slabs extending on both sides of a beam, the effective width of the concrete flange must not exceed:

(1) one-fourth of the beam span;

(2) The flange width of the steel beam plus sixteen times the slab thickness;

(3) The distance from the centre of the steel beam to the centres of adjacent steel beams.

For slabs extending on only one side of a beam, the effective concrete flange width shall not exceed:

(1) one-twelfth of the beam span;

(2) six times the slab thickness;

(3) one-half the clear distance to an adjacent steel beam.

2.2 Loading Conditions

Normally in building designs, the following types of loading are considered: Dead loads, Live loads, Wind loads and Deformational loads such as creep and shrinkage.

The dead loads in building frames will include the beams, columns, the floor or concrete slab, the walls, partitions and ceiling plasters.

The live loads in building design are usually considered as uniformly distributed over the floor area. They may be placed in an arbitrary position to give a critical condition: e.g. loading two adjacent spans to give a maximum negative moment or loading alternate spans to give a maximum positive moment.

For structures subjected to vibrations or impact, such as the effects of machinery or moving loads, the usual way to account for such dynamic effects is to increase the live load by a certain factor as in highway bridge design (57).

2.3 Deformational Stresses

Concrete is a variable material, and may shrink or expand depending on its composition and atmospheric condition. Concrete when loaded continues to deform with time - a phenomena called creep. In a composite beam, deformation in the slab induces deformation in the beam which in turn canses deformational stresses in both the beam and the slab.

In the positive moment regions of the composite beams, the slab is subjected to permanent compressive stresses due to the dead load and live load of long duration. The resulting creep increases the compressive stresses in the top flange of the steel beam and the tensile stresses in the bottom flange, and decreases the compressive stresses in the slab. In some structures, such as warehouses, where the live loads are kept on the structures for a long time, a similar type of behavior can be expected. Theoretically, the magnitude of the creep stresses in any beam may be computed if the creep characteristics of the concrete are known. This analysis is complicated and, in view of the variability of the creep characteristics of concrete, of questionable design value. A simpler procedure to account for the effects of creep is to increase the value of the modular ratio n in the computation of the section properties of the composite beams (1). A factor of K = 3 is recommended in the AASHO specifications (57). The effect of increasing n is to reduce the section properties of the composite section, so that the steel section has to carry the redistributed stress.

After casting, during the first few months, the concrete slab tends to shrink. Shrinkage has a similar effect on the stress conditions in a composite beam as creep: it sets up tensile stresses in the slab and the bottom flange of the steel beam, and compressive stresses in the top flange. If the tensile stresses in the slab exceed the tensile strength of the concrete, cracks may form. In simple beams and in the positive moment regions of continuous beams, the dead and live load stresses counteract the shrinkage stresses to close the cracks and thus restore the effectiveness of the slab in resisting compressive stresses. In the negative moment regions of continuous composite beams, the loads tend to cause further opening of cracks, so that in these regions, shrinkage cracking makes the slab ineffective in resisting stresses. Accordingly, the effect of shrinkage on the slab stresses may ordinarily be neglected in the design. In practice however, consideration must be given to control slab cracking. Effective or not, a minimum amount of slab reinforcement is prescribed by sections 807 and 911 of the ACI 318-63, to control shrinkage and temperature cracks.

An approximate calculation of the steel stress caused by shrinkage can be made by assuming a shrinkage coefficient of 0.0002 in./in. for a reinforced concrete slab (10). The calculation of stresses may be made by assuming the shrinkage force to be an eccentric compressive force of 0.0002 $E_c bt$ applied at the centroid of the slab. In the case of the AASHO specifications, a twenty-five per cent overstress is permitted, when shrinkage stresses are considered.

Measurements of shrinkage stresses in an actual building structure (36) showed that they varied widely within the building, and this suggests that the use of complex design procedures based upon idealized shrinkage equations could well be unjustified in many cases.

2.4 Continuous Beams

Continuous spans are more advantageous than simple spans when foundation conditions and desired span lengths are favorable for this type of construction.

The main advantages to be gained from using continuous construction are:

(1) A stronger and a more rigid structure

(2) Longer span lengths are possible

(3) Less materials are required - consequently a more economic structure

(4) Shallower members can be used

These advantages are further enhanced when continuous composite construction is used. In addition, composite construction increases the stiffness and the ultimate carrying capacities of the structure. The increase in stiffness will lead to a decrease in live-load deflections and vibrations, while the increase in the ultimate capacity results in a higher factor of safety.

In computing the forces in a continuous composite beam, the stiffness in the positive moment regions will be taken as that of the composite section made up of the steel beam and the concrete slab; the stiffness in the negative moment regions as that of the steel beam alone or that of the steel beam plus the slab longitudinal reinforcement. The concrete is considered ineffective in resisting tension, and therefore is not considered in the negative moment section. In the negative moment regions, only the slab reinforcement can act compositely with the steel beam. For the slab reinforcement to be fully effective in the negative moment regions of a composite beam, shear connectors must be placed throughout the length of the beam. With shear connectors omitted from the negative moment regions, the slab reinforcement is only partly effective. Therefore in calculating the forces in a beam in such a case, only the steel section is considered effective in carrying the total negative moments.

The use of reinforceing steel in the slab may not increase the section sufficiently to resist the negative moment stresses in the bottom flange. It should be noted that the longitudinal steel does not share in resisting non-composite dead loads. The most efficient method of providing for the negative moment is to increase the main steel beam section. This can be done by the addition of equal cover plates to the top and bottom flanges. The necessary area of each cover plate for a rolled beam can be calculated from the following formula:

$$A_p = \frac{(M/f_s)\,(d + 2t_p)}{(d + t_p)^2} - 2I_B \quad \ldots\ldots\ldots\ldots \quad (2.2)$$

The flange areas for a symmetrical built-up girder can be evaluated by the formula:

$$A_b = \frac{M}{f_s\ d_w} - \frac{A_w}{6} \qquad \text{(2.3)}$$

If the design is based on the assumption that the negative moments are carried solely by the steel section, a symmetrical steel section is desired. Even though the concrete slab is not considered to resist the negative moment stresses, the minimum amount of reinforcement must still be placed in the slab to control cracking as prescribed by section 911 of the ACI Building Code.

2.5 Influence of Construction Methods

The method of construction will alter the stresses in the steel section and the concrete slab, depending on whether it is shored or unshored construction. These differences must be included in the design.

Temporary supports or shores are used commonly in composite construction for fuildings. Usually the steel beams carry their own weight, while the shores support the dead weight of the slab, so that the structure is partially shored.

To make the construction fully shored, it would be necessary to induce with temporary supports an upward deflection of the steel beams prior to casting the concrete slab, so as to eliminate dead load stresses caused by the beam weight. This is a delicate and complicated operation, which usually has little or no effect on the stress conditions and design of the beams. Use of full shoring is, therefore, not recommanded for building construction.

For partially shored construction, the shores usually are left in place until the concrete reaches 75 per cent of the 28-day compressive strength. Upon removal of the shores, the dead load together with the live load will be supported by the composite section. The section properties of the steel beam alone do not enter into the calculation of stresses - the only stress in the steel section acting alone will be that due to its own weight, which is negligible.

If, on the other hand, shoring is not used to support the dead load of the concrete slab, the dead load is then carried by the steel members alone. Stresses due to dead load are calculated based on the section properties of the steel member alone. The composite section will be effective only for the superimposed live loads.

From the point of view of the strength of the member in the elastic range, the second method, unshored construction is the least economical way of constructing a composite member. The first method partially shored construction usually adopted in use is better. An alternate method of construction is to bend the steel section upwards by jacks so it is prestressed, thus providing the most economical use of materials. However in practice, this may not be too feasible especially for a rigid frame building, but in a simple-support structure it can be considered.

Necessary provisions should be taken in the design and construction to prevent excessive dishing of the slab in beams built with shores and excessive thickening of the slab in beams built without shores. The determining factor in deciding whether or not to use shores is often the nagnitude of the dead load deflection.

The AISC specification considers all dead load and live load as applied to the composite section regardless of the method of construction. The method of construction does not alter the ultimate strength of the member, but does affect the actual steel stresses appreciably. For beams built without temporary supports, the actual stresses may be substantially in excess of the computed value, because the dead load is in reality resisted by the steel section alone. However, a maximum limit for bottom flange steel stress is fixed by AISC formula 17 (Sec.1.11.2) at approximately 0.82 Fy or the section modulus of the composite section used in computation should not exceed the value:

$$S_{bc} = (1.35 + 0.35 \frac{M_L}{M_D}) S_{bs} \quad \text{.................} \quad (2.4)$$

The section normally used in composite construction for building is a rolled section or a rolled section with cover plate on the bottom flange in the portion where composite action is effective. When the negative moment is assigned to the steel section alone, symmetrical cover plates can be used to strengthen the section. Typical sections that can be used are shown in Fig.(2-1).

CHAPTER 3

DESIGN OF COMPOSITE BEAMS

3.1 Formulas for DL and LL Stresses

Bending stresses at working loads are based on the elastic properties of the cross-sections. The calculation of actual stresses for unshored construction requires the definition of several sets of section properties.

The section properties are usually computed by the moment-of-inertia method and the theory of transformed area. The concrete is transformed into an equivalent steel area, so the concrete slab can be treated in the same manner as a steel cover plate.

The calculation of actual stresses is also complicated by the fact that dead load stresses do not remain constant with time because of the effect of creep and shrinkage of the concrete. With time, the effectiveness of the concrete slab decreases and the steel stresses due to dead load increase as a result. For this reason, stresses caused by dead load on the composite section are usually determined with section properties calculated using a value of modular ratio n equal to 3 times the elastic value of n.

If the steel section consists of a rolled beam with a steel cover plate on the bottom flange Fig.(3-1) the section properties can be calculated from the following formulas:

$$K_s = A_p \,/\, A_s \quad \ldots\ldots \quad (3.1a)$$

$$\overline{Y_s} = 0.5\ (\,d + t_p\,)\ K_s \quad \ldots\ldots \quad (3.1b)$$

$$I_s = 0.5\ (\,d + t_p\,)\ \overline{Y_s}\ A_B + I_B \quad \ldots\ldots \quad (3.1c)$$

$$S_{ts} = \frac{I_s}{0.5d + t_p - \overline{Y_s}} \quad \ldots\ldots \quad (3.1d)$$

$$S_s = \frac{I_s}{0.5d + \overline{Y_s}} \quad \ldots\ldots \quad (3.1e)$$

If the steel section is an unsymmetrical welded plate girder Fig.(3-2), the following formulas are used:

$$\overline{Y_s} = \frac{0.5\ (d_w + t_b)\ A_b - 0.5\ (d_w + t_t)\ A_t}{A_s} \quad \ldots\ldots \quad (3.2a)$$

$$I_s = 1/4\ (d_w + t_t)^2\ A_t + 1/4\ (d_w + t_b)^2\ A_b$$

$$+ I_w - A_s(\overline{Y_s})^2 \quad \ldots\ldots \quad (3.2b)$$

$$S_s = \frac{I_s}{\tfrac{1}{2}d_w + t_b - \overline{Y_s}} \quad \ldots\ldots \quad (3.2c)$$

$$S_{ts} = \frac{I_s}{\tfrac{1}{2}d_w + t_t + \overline{Y_s}} \quad \ldots\ldots \quad (3.2d)$$

For composite sections, the neutral axis may be located either below the slab in the steel section or in the slab itself.

If the neutral axis for a composite section is located below the slab Fig.(3-3), the full cross section of the slab is effective in resisting the stresses. The section properties can be computed from the following formulas:

$$K_c = A_c \,/\, (A_c + A_s) \quad \ldots\ldots\ldots\ldots \quad (3.3a)$$

$$\overline{Y_c} = (Y_{ts} + e_c) \, K_c \quad \ldots\ldots\ldots\ldots \quad (3.3b)$$

$$I_c = (Y_{ts} + e_c) \, \overline{Y_c} \, A_s + I_s + A_c \, t^2/12 \quad \ldots\ldots \quad (3.3c)$$

$$S_{tr} = I_c \,/\, (Y_{bs} + \overline{Y_c}) \quad \ldots\ldots\ldots\ldots \quad (3.3d)$$

$$S_{tc} = I_c \,/\, (Y_{ts} - \overline{Y_c}) \quad \ldots\ldots\ldots\ldots \quad (3.3e)$$

$$S_{cc} = I_c \,/\, (Y_{ts} - \overline{Y_c} + e_c + \tfrac{1}{2}t) \quad \ldots\ldots\ldots \quad (3.3f)$$

If the neutral axis is located in the slab Fig.(3-4) only the portion of the slab located above the neutral axis is considered effective in resisting stresses. However, equations (3.3) give sufficiently accurate results even if the neutral axis is located in the slab as long as the following condition is satisfied:

$$d/t \geq \frac{1}{3} \frac{A_c}{A_s}$$

Otherwise, section properties can be computed from the following formulas:

$$\overline{Y_c} = Y_{ts} + e_c + 0.5\ t - Y_{cc} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4a)$$

$$I_c = A_c \frac{Y_{cc}^{\ 3}}{3_t} + A_s \overline{Y_c}^2 + I_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4b)$$

$$S_{tr} = I_c \ / \ (\overline{Y_c} + Y_{bs}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4c)$$

$$S_{tc} = I_c \ / \ (\overline{Y_c} - Y_{ts}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4d)$$

$$S_{cc} = I_c \ / \ Y_{cc} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4e)$$

If steel reinforcement is provided in the slab in the negative moment section Fig.(3-5), the following formulas apply:

$$K_r = A_r \ / \ (A_s + A_r) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5a)$$

$$\overline{Y_c} = (Y_{ts} + e_r) \ K_r \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5b)$$

$$I_c = (Y_{ts} + e_r) \ \overline{Y_c} \ A_s + I_s + I_r \quad \ldots\ldots\ldots\ldots \quad (3.5c)$$

$$S_{rc} = I_c \ / \ (Y_{ts} - \overline{Y_c} + e_r + g) \quad \ldots\ldots\ldots\ldots \quad (3.5d)$$

To facilitate the calculation of stresses for both shored and unshored constructions, a summary of the section properties is given below together with the formulas for both steel and concrete stresses. M_D, M_L, and M_{DC} are respectively moment due to dead load on steel member, moment due to live load and moment due to dead load on the composite member. The stresses f_c, f_t, and f_b are the stresses at the top of the concrete slab, the top of the steel member, and bottom of the steel member, respectively.

Typical stress patterns for unshored construction, shored construction and prestressed steel section construction are shown in Figures 3-6, 3-7 and 3-8 respectively.

SECTION MODULUS FOR CALCULATION OF STRESSES

Member	Unshored Dead Load	Live Load	Dead Load on Composite Section	Location of Stress
No Cover plate	———	$S_c = I_c/Y_c$	$*S'_c = I'_c/Y'_c$	Top of Concrete
	$S = I_s/\overline{Y}$	$S_t = I_c/Y_t$	$S'_t = I'_c/Y'_t$	Top of Steel
	$S = I_s/\overline{Y}$	$S_b = I_c/Y_b$	$S'_b = I'_c/Y'_b$	Bottom of Steel
With Cover plate	———	$S_{cp} = I_{cp}/Y_{cp}$	$S'_{cp} = I'_{cp}/Y'_{cp}$	Top of Concrete
	$S_{st} = I_p/Y$	$S_{tp} = I_{cp}/Y_{cp}$	$S'_{tp} = I'_{cp}/Y'_{tp}$	Top of Steel
	$S_{sb} = I_p/Y$	$S_{bp} = I_{cp}/Y_{bp}$	$S'_{bp} = I'_{cp}/Y'_{bp}$	Bottom of Steel

* Letters with prime are for reduced section properties.

STRESSES IN CONCRETE AND STEEL

SHORED CONSTRUCTION

	No Cover Plate	With Cover Plate
f_c =	$M_{dc}/(n'S'_c) + M_L/(nS_c)$	$M_{dc}/(n'S'_{cp}) + M_L/(nS_{cp})$
f_t =	$M_{dc}/S'_t + M_L/S_t$	$M_{dc}/S'_{tp} + M_L/S_{tp}$
f_b =	$M_{dc}/S'_b + M_L/S_b$	$M_{dc}/S'_{bp} + M_L/S_{bp}$

UNSHORED CONSTRUCTION

	No Cover Plate	With Cover Plate
f_c =	$M_{dc}/(n'S'_c) + M_L/(nS_c)$	$M_{dc}/(n'S'_{cp}) + M_L/(nS_{cp})$
f_t =	$M_D/S_s + M_L/S_t + M_{dc}/S'_t$	$M_D/S_{st} + M_L/S_{tp} + M_{dc}/S'_{tp}$
f_b =	$M_D/S_s + M_L/S_b + M_{dc}/S'_b$	$M_D/S_{sb} + M_L/S_{bp} + M_{dc}/S'_{bp}$

3.2 Shear Connectors

To insure that the concrete and steel act as a single unit in composite construction, shear connectors must be provided at the junction between the steel flange and the concrete slab. These shear connectors must be capable of (i) transfering horizontal shear, (ii) resisting relative moments between the slab and the beams and (iii) resisting both horizontal and vertical movements.

Natural bond action between the concrete slab and the steel beam may provide some degree of composite action. However, this bond may be destroyed during the life of the structure, either by shrinkage of the concrete slab or by live load vibrations, and therefore cannot be depended upon for the transfer of shear. It will be assumed in design that all shears caused by forces acting on the composite structure are transmitted by the shear connectors.

Forces to be considered in shear connectors design are: dead loads and live loads applied after the concrete has attained 75 per cent of its 28-day strength, creep and shrinkage of concrete.

Dead load may be carried either by the steel beams alone or by the composite section. Shear connectors should be designed only for that portion of dead load carried by the composite section.

Since live load is applied only after the concrete has reached its specified strength, shear connectors should be designed for the full live loads plus impact if any.

Sustained loads can cause creep in the concrete. The effect of creep is to relieve stresses in the concrete and thus to decrease the forces transmitted by the shear connectors. Therefore the effect of creep is disregarded in the design of shear connectors.

Shrinkage in concrete is also resisted by the shear connectors. However in the positive moment region, the shrinkage force acts in a direction opposite the maximum horizontal shear force due to dead and live loads. Therefore, the effects of shrinkage may also be ignored in the design.

Other effects such as expansion and differential temperature changes need be considered only in exceptional cases. Normally it is desirable and safe to design shear connectors for horizontal forces caused by the full dead and live loads acting on the beam regardless of whether it is shored or unshored construction.

Design of Shear Connectors

Regardless of what method is used to design the shear connectors, it is imperative to unsure that shear connection should not be the cause of beam failure. The failure of the connectors will drastically reduce the carrying capacity of the beam and this can lead to a catastrophic failure.

It follows, that a satisfactory basis for design is one that provides sufficient shear connectors to resist the total horizontal shear force at the failure of the beam. Recent studies have shown that the ultimate flexural strength of composite beams can be achieved if sufficient connectors are provided to resist the maximum horizontal force in the slab or the beam in their plastic state of stress (26).

At the ultimate moment of a composite beam, two stress distributions are possible as shown in Fig.(3-9): for case I the plastic neutral axis is in the slab while for case II the plastic neutral axis in the beam. For any compsite beam, the maximum horixontal force to be resisted in the positive moment region between the points of inflection shall be taken as the smaller value obtained from the two expression:

$$V_h = 0.85\ f'_c bt \quad \text{.............................} \quad (3.6)$$

$$V_h = A_s\ F_y \quad \text{.................................} \quad (3.7)$$

In continuous composite beam, where slab reinforcement is considered to act compositely with the steel beam in the negative moment regions, sufficient shear connectors must be provided in these regions between adjacent points of contraflexure to resist the horizontal force, V_h, equal to the yield strength of the longitudinal reinforcement of the slab:

$$V_h = A_{sr} F_{yr} \quad \text{.............................} \quad (3.8)$$

The minimum number of shear connectors required in the positive or negative moment regions between the points of contraflexure is given by,

$$N = \frac{V_h}{q} \quad \text{.................................} \quad (3.9)$$

For the positive moment regions V_h from either Eq.(3.6) or (3.7) is used, while for the negative moment regions Eq.(3.8) applies. q is the allowable horizontal shear load per connector. Values of q for stud and channel connectors are given in Table 1.11.4 of the AISC specifications, and reproduced here as Table 3-1. The AISC specifications also give a simpler procedure to determine the number of shear connectors. This procedure makes use of the effective concrete flange area A_c, the weight per foot of the steel section, and the stud coefficient. This procedure is outlined on page 2-142 of the AISC specification.

Both the CSA standard and the AISC specifications permit the number of shear connectors determined by Eq.(3.9) or the stud coefficient method, to be spaced uniformly along the beam. Where concentrated load and uniform load occur together, the number of shear connectors required between the concentrated load and the nearest point of zero moment shall not be less than that determined by:

$$N_2 = \frac{N_1 \left(\frac{MB}{M_{max}} - 1 \right)}{B - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.10)$$

where N_1 = total number of connectors determined by Eq.(3.9)

M = moment at a concentrated load point

M_{max} = maximum positive moment

$B = S_{tr} / S_s$

It is necessary to decide the limiting load of the connector to ensure against failure. A theoretical minimum shear connector requirement is one in which the connectors fail when the beam reaches its fully plastic moment. This is a 100 per cent design, but before the fully plastic moment is reached, extensive slip can occur, and the stress distribution in the beam is considerably modified, so that the suggested rectangular stress block to calculate the maximum horizontal shear is no longer valid. It can be said with certainly that if sufficient shear connectors are provided to ensure that the fully plastic moment can be developed, then slip at working load will not significantly modify the stress distribution. The AISC allowable load for shear connectors provides a factor of safety of 2.5 according to Slutter and Driscoll (26).

Slutter and Fisher (34) proposed a more economical method of designing shear connectors. They recommended using a load reduction factor $\phi = 0.85$ for the ultimate shear strength of the shear connector. This would result in a lower factor of safety; approximately half of that provided in the AISC specification. The number of shear connectors required is given by:

$$N = \frac{V_h}{\phi\, q_u} \qquad (3.11)$$

3.3 Deflections

The deflection of a composite beam may be calculated by the same analytical methods used in calculating deflections of non-composite members. The methods of moment-areas, elastic weights, congugate-beam method and the method of least work give reasonably accurate results if proper values of the modular ratio are used. A comprehensive account of the different methods are given in Chapter 17 of Ref.48.

Composite beams are usually non-prismatic so that exact computations of deflections would have to account for the changes in the moment of inertia. However this can be ignored in initial deflection calculations. It is sufficiently accurate in many cases to base the calculation on the moment of inertia of the maximum positive moment section. The errors caused by this approximation are usually small, and if the deflections appear to be critical from these approximate calculations, then the calculations may be refined by considering the non-prismatic member.

Dead Load Deflections

A composite beam is usually subjected to both composite and non-composite dead loads. Thus, in calculating dead load deflections, it is necessary to consider two beam sections.

Dead load deflections for unshored members are calculated with the properties of the steel beam alone. Deflections caused by composite dead loads or shored construction are computed with the properties of the composite section. The calculation for shored dead loads and long term live loads should be calculated using a value of 3 times n the modular ratio for the composite section properties, to allow for the effect of creep in the concrete.

The deflection for uniform dead load can be computed at any point on the beam Fig.(3-10) by the following formula:

$$\Delta x = \frac{WX}{24E_sI}\left[X^3 - \left(2L + \frac{4M_l}{WL} - \frac{4M_r}{WL}\right)X^2 + \frac{12M_lX}{W} + L^3 - \frac{8M_lL}{W} - \frac{4M_rL}{W}\right] \quad \ldots\ldots\ldots (3.14)$$

W = uniform dead load (kip/in)

L = span length - in

E_s = modulus of elasticity - ksi

I = moment of inertia of steel section or of composite section - in^4

X = distance along beam (in.)

M_L = restraining moment at left end (k-in.)

M_r = restraining moment at right end (k-in.)

Live Load Deflections

Live loads are resisted by the composite section. For short-term live loads, the composite section properties for deflection calculations, should be calculated on the basis of the elastic modular ratio.

The live load deflections for the building beam can also be computed by equation (3.14), as live load is usually assumed uniform in building design.

The AASHO specification limity the live load deflection including the effect of impact to 1/800 of the span, while the AISC specifications limit it to 1/360 of the span. The AASHO specifications (Sec.1.6.10) also suggests two limiting slenderness ratios for composite beams, as does the AISC specifications. The aim is to prevent excessive vibrations.

The AISC specifications suggest that the overall depth of the composite section (concrete slab + steel beam) be at least 1/22 of the span for F_y = 36 ksi and 1/16 for F_y = 50 ksi. The depth / span ratio of the steel section alone should not be less than 1/20 for any grade of steel. The span length for simple beam is the distance between the centre of the bearings, and for that of continuous beam the distance between the dead load points of contraflexure.

3.4 Ultimate Strength of Composite Beams

The concept of ultimate strength design in reinforced concrete can also be applied to a composite steel-concrete member (26). The ultimate strength of a composite beam is calculated from the fully plastic distribution of stresses.

Normally, a composite beam fails due to the crushing of the concrete slab. At this instant a fully plastic state of stress is assumed for both concrete and steel. The assumed stress distribution is shown in Fig.3-9. The plastic state of stress is represented by stresses of $0.85f'_c$ in concrete and F_y in the steel. Case I applies when the neutral axis is in the slab or when the concrete slab is adequate to resist the total compressive force at ultimate load. Case II applies when the neutral axis is below the slab or the concrete slab is not large enough to resist the total compressive force.

Referring to Fig.3-9 the appropriate equations for computing the ultimate moments are:

Case I

$$C = 0.85f'_c b_a \quad (3.15a)$$

$$T = A_s F_y \quad (3.15b)$$

$$C = T \quad (3.15c)$$

$$a = A_s F_y / (0.85f'_c b) \quad (3.15d)$$

$$M_u = T_e = T \left(\frac{d}{2} + t - \frac{a}{2} \right) \quad (3.15e)$$

Case II $C = 0.85f'_c bt$ .... (3.16a)

$T = A_s F_y - C'$ .... (3.16b)

$T = C + C'$ .... (3.16c)

$M_u = Ce' + C'e''$ .... (3.16d)

The values of e' and e" must be determined from the stress distribution and geometry of the cross section.

For a composite beam, with the slab in tension in the negative moment region the ultimate moment can be determined by a similar consideration as the above two cases. When the slab reinforcement is taken into account Fig.(3-11) the following formulas apply:

$A'_s = 0.5 (A_s - A_{re})$ .... (3.17a)

$Z_{tc} = t_t + \frac{A'_s - A_t}{t_w}$ .... (3.17b)

$e = d - Y_{bs} + e_r$ .... (3.17c)

$e' = d - Y_{bs} - 0.5t_t$ .... (3.17d)

$e'' = d - Y_{bs} - Z_{tc} + 0.5 (Z_{tc} - t_t)$ ..... (3.17e)

$Z = A_{re}e + 2A_t e' + 2 (A'_s - A_t) e''$ ..... (3.17f)

The ultimate moment is computed from the plastic section modulus Z thus:

$$M_u = Z F_y \quad (3.18)$$

At ultimate load, all dead and live loads are carried by the composite section, irrespective of construction methods. Further more, deformational stresses such as creep may be assumed to have no effect on the ultimate strength of a composite beam.

CHAPTER 4

THEORETICAL INVESTIGATIONS OF THE EFFECTS OF COMPOSITE ACTION IN BUILDING FRAMES

4.1 Introduction

Generally the type of construction controls the economy of a structure and the method of analysis. For structural steel frames two framing schemes are available. The first type is usually refered to as simple framing. With this type of framing, the end moments are eliminated but it also results in the use of the largest possible beam or girder. However, elimination of the end moments makes possible the use of the simplest connections. There is also no problem as far as analysis is concerned, as only the simple beam moment is required.

The other type is rigid framing. As its name implies, the connection must have sufficient rigidity to hold its original direction virtually unchanged. The effect of rigid frame construction is to reduce the size of the steel beams or girders, but the expense of fabricating connections capable of fully rigid action may offset the savings in girder material.

For both simple and rigid framing, a more economical design might be obtained by using composite action in the positive moment regions of the beams by providing shear connectors between concrete and the steel section.

The purpose of this report is to investigate theoretically the advantage of providing composite action for beams in a rigid frame; the amount of steel that can be saved as compared to a non-composite design.

4.2 Theoretical Investigation

The typical frame chosen for the theoretical analysis is a two story- two bay frame with fixed foundations Fig.(4 1).

The object is to compare a composite design to a non-composite design by varying the span length L, the loading conditions and the column height h. The span length L is varied from 20 ft. to 50 ft. in increment of 10 ft. and the loading conditions are varied by having the live load to dead load ratio (LL/DL) equal to approximately 1, 2 and 3. The columns are varied for only one loading condition (LL/DL = 2) for the various spans, while the others are kept constant at 12 ft. The columns are varied from 12 ft. to 20 ft. for the bottom ones and to 15 ft. for the top ones.

In formulating this problem all the factors used are chosen within practical limits. A concrete slab of 5 inches is used in every case. This is an arbirtary choice and is adequate for most types of buildings. It also falls within the range commonly used in building construction (4 to 7 inch). Based on the slab thickness of 5 inch and a bent spacing of 15 feet, the concrete dead load is about 937 lbs per ft., and assuming the beam weight to be around 100 lbs, the total dead load (W) is assumed to be 1 kip per ft. Based on this load, the total uniform load (W_T) of 2 K/ft., 3 K/ft., and 4 Kip/ft. correspond to the LL/DL ratio of 1, 2, and 3 respectively. For wind loads a conservative value of 50 psf is used, compared to the 20-30 psf normally used in multi-story frames. This results in a 4.5 kip and 9.0 kip load at joint 1 and 4 respectively of Fig.4-1, for a column height of 12 feet.

It is pointed out that only the effect of composite action on the beam is investigated, the columns remain the same for both the non-composite and the composite design in every case studied.

4.3 Analysis

Many methods are available to solve the moments in a frame. Normally matrix methods of structural analysis are used, because it provides the simplest approach available for preparing the frame problem for solution by a computer.

The two methods commonly used are the Force method and the Displacement method. The Displacement method involves the formation of the static matrix (S) , the stiffness matrix (ST) , and the external load matrix (P). When these are available, the following matrix operation can be performed by a computer to obtain the force matrix (F), which represents the moments and the displacement matrix (X):

$$(X) = \left[(S)\ (ST)\ (S^T) \right]^{-1} (P) \quad \ldots\ldots\ldots\ldots \quad (4.1)$$

$$(F) = (ST)\ (S^T)\ (X) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

where (S^T) is the transpose of (S).

However the problem frequently encountered in a composite building is that the moment of inertia of the members is not constant. This brings up the question of what stiffness should be used for the composite member. If the member bends, so as to have positive moment throughout its entire length or if it is of simple framing, then the stiffness may be taken as I_c/L. For rigid framing, the situation is not so simple; negative bending occurs at both ends and positive bending at the centre of the span. Usually only the steel section is used to carry the negative moment, while composite action is provided for the positive moment section. The stiffness to be used must take into account the variation of moment of inertia along the beam; steel section at both ends and composite section at the centre. The method of determining the stiffness of such a member is by the method of Column Analogy - a good account of which is given in Ref.48. Chap.24.

This is not a very practical method expecially when the inflection point or the length of composite section is not known. With every change in composite section, a new stiffness calculation is required and consequently the formation of a new stiffness matrix. An exact solution will in every case require a number of iterations and this will be time consuming. With the number of problems that have to be solved, it is obviously not a very efficient method.

What is required is a method that allows the properties of the members to be changed, without any alteration to the other parts of the program. This is an iterative procedure. It would facilitate the computations greatly if only the areas, the moment of inertia of the steel beam and that of the composite section need be changed.

Fortunately a method that offers such capability and flexibility can be found in STRUDL (Structural Design Language), a problem oriented language. It is a large-scale computer system which permits the engineer to solve complex structures with a minimum of computer programming experience. Since STRUDL is a problem oriented language, it requires very little programming knowledge for its use or understanding. STRUDL is a subsystem of ICES (Integrated Civil Engineering System) developed by the Civil Engineering Systems Laboratory, Massachusetts Institute of Technology. ICES is an information processing system oriented toward the creation and use of engineering problem-solving capabilities. For further information on this computer program, refer to Appendix III and Refs. 37, and 43 to 45.

Effect of Composite Action on the Moments in a Frame

To determine the effects of providing composite action in a beam of a building frame, composite behaviour of 0.10L to 0.90L is assumed in the positive moment section equidistant from the ends, with variations in increments of 0.10L. For each length of composite section, the moments in the frame were determined by the STRUDL program. This is done for span lengths of 20 feet and 50 feet, and only one loading condition of LL/DL ratio equal to about 3 (W = 4K/ft.).

The results are shown in Figs. 4-2 and 4-3. The moments are plotted against the length of the composite section. The pattern of behavior is exactly the same for both spans. With 0.10L for the length of the composite section, the negative moments at joints 2 and 5, 1 and 4 are a maximum. There is a decrease with increasing length of the composite section. About 0.70L the moments at joints 2 and 5 begin to increase again, but at joints 1 and 4 the moments continued to decrease. The positive moments of beams B_1 and B_3 are a minimum for 0.1L of composite section length. There is an increase until to 0.70L when a decrease is noted. It is interesting to note that the positive moment curves are almost a mirror image of the negative moment curves of joints 2 and 5.

The negative moment controls the design for both composite and non-composite action. This can be seen in the curves and it occurs at joint 2 of the top beams. The moments for a section satisfying the non-composite design are shown by the symbols (I), superimposed on the moment curves which are established using a section satisfying the composite design. The exact solutions for the composite design (i.e. by providing the composite section between the inflection points) are shown by the symbols (⊕). It can be seen that they correspond closely to the minimum negative moments and the maximum positive moment obtained by arbirtarily placing the composite section at equal distances from both ends. The exact minimum negative moment for non-composite design for L = 20 ft. is 173 k ft. while for the composite design it is 123.0 K ft. - approximately 28.9% reduction. For L = 50 ft., the exact negative moment for non-composite design is 1051 K ft. while for composite design it is 860 K ft. - a reduction of approximately 18.2%.

From the above it can be seen that there is a decided advantage in using composite construction from the standpoint of structural stiffness and also savings in steel. It seems also that the amount of composite section rather than its location is more significant from the standpoint of analysis.

4.4 Empirical Relationship between Moments, Loading and Span Length

One of the most difficult problems in composite design is the determination of the correct beam size. A reasonable first trial section must be made in order for an exact analysis to be made without too many trials. This is desirable from the practical point of view.

A method of selecting the steel section for a trial composite design is given in Art. 403, p.104 of reference 47. It suggested that the trial section may be selected by increasing the allowable stress by a certain percentage for various values of modular ratio n. The percentage increase and the modular ratios given in reference 47 are :

n	$\Delta\sigma$ in %
6	37
8	33
10	28
12	24
15	20

For a modular ratio of 10 the allowable stress may be multiplied by a factor of 1.28, and this means that a composite member may use a steel section with a section modulus of about 78 percent of the comparable non-composite member. This method gives very good results, but in using this method, it implies that the non-composite member section must be known first, before a trial assumption can be made.

If it is desirable to provide composite action in a frame, it is more useful to be able to obtain a trial section based on the loading conditions and the span length. To do this, an empirical relationship between the moments in a composite frame, the span length and the loading condition is required. To obtain such a relationship, the typical frame of Fig. 4-1 is analysed for both non-composite and composite design for span lengths of 20, 30, 40 and 50 feet, and loading conditions of LL/DL ratios equal to 1, 2 and 3. In each of the above cases, the columns are kept constant at 12 feet height. The effects of varying the columns height(by changing to a 20 - 15 feet configuration) is also investigated for the different spans, but only for one loading condition of LL/DL = 2.

The results are plotted as curves of moments against span lengths for the various loading conditions in Figs. 4-4 to 4-8. The moments are given in terms of wL, where w is the applied uniform load(kip/ft) and L the span length(ft). The curves given in Figs. 4-4, 4-5, 4-6 and 4-7 are for the negative moments at joints 2, 5, 1 and 4 respectively. The controlling moments are at joint 2, as shown in Fig. 4-4. This is true in everyone of the cases investigated. The curves are more or less linear in relationship for moments at joints 2 and 5, but for joints 1 and 4, the points are more scattered for the case of the 30 feet span. Nevertheless for simplicity, a linear relationship has been assumed for the Moment vs. Span length curves, since these curves are to be used only as a guide for the first approximation. The controlling moments at joints 2 range in value from $0.073wL^2$ to $0.0775wL^2$ for w = 2 to 4 kips/ft respectively for L = 20 feet, while for L = 50 feet, the values range from $0.080wL^2$ to $0.086wL^2$ for the loading range. Values for span lengths between 20 and 50 feet fall within the values given by the above bounds. The results for the positive moment values are given in Fig. 4-8. The curves also indicate more or less a linear relationship between moment and span length, except in this case, the moment decreases with increasing span length; from a high of about $0.08wL^2$ for L = 20 feet, to a low of $0.07wL^2$ for L = 50 feet. In everyone of the cases investigated, the positive moment does not control the design.

The above curves can be used to estimate a trial steel section for composite design for a rigid frame. To use these curves (Figs. 4-4 to 4-8), the type of joint into which a member is framed must necessarily be taken into account. If a member is connected by a very rigid joint or if it is connected to a very stiff column, it can be assumed that it approaches the behavior of a fixed-end beam. For this purpose the composite design of a fixed-end beam is also investigated. The format of investigation is exactly the same as for the typical frame of Fig. 4-1, by varying the span lengths and loading conditions. The results are shown in Fig. 4-9. These curves can be used to determine trial sections for composite design if the members involved can be treated as a fixed-end beam.

In order to make a comparison, the non-composite design for the corresponding composite case is also carried out. The controlling moment is also the negative moment at joint 2. Wind load is not the controlling factor for this type of low building, only the gravity load is. To see the difference between the two types of design, the composite moment at joint 2 is expressed as a percentage of the non-composite moment at the same joint, and this is plotted against the span lengths for the various loading conditions. The curves obtained are shown in Fig. 4-10. The values for the composite moment range from 68% to 71.5% of the non-composite moment for the range of loading of w = 2 to 4 kips/ft for L = 20 feet. For L = 50 feet the range is 80% to 82% for the range of loading. It can be seen that approximately 20 to 30% reduction in moment or section modulus is possible - the larger reduction for the shorter span length of 20 feet and the smaller for the 50 feet span length. It can also be seen that the 22% possible reduction in section modulus suggested by the method of reference 47 falls within the above suggested values. The other possible reductions in moments for the composite beam for the other joints are as follows :

joints	%reduction
1	20.0
4	20.0
5	23.0

From an economic point of view, the savings in steel is usually the governing factor. The results of such an economic comparison is shown in Table (4-1). The weight of the non-composite beam and the composite beam is given and the difference in weight is expressed as a percentage of the former. It can be seen that in every case there is a saving in steel weight. The savings range from about 9% to 30% - with the higher percentages for the shorter spans and the lower percentages for the longer spans.

4.5 Location of Inflection Point

As has been previously determined in Section 4.3, the location of the inflection points is not significant from the standpoint of the analysis; the total length of the composite section provided is more significant. This is also shown by the fact that for every one of the composite cases investigated the length of composite section provided falls within the optimum range suggested by the curves of Figs. 4-2 and 4-3. The minimum length provided is 0.0705L and the maximum is 0.791L.

The location of the inflection points may not be important from an analytical point of view, but it is from the practical and the construction point of view. In order to be effective, the composite section must necessarily be placed in the positive moment region between the inflection points.

Reference 25 recommends a value of 0.10L from the ends as the location of the inflection points for design. This value is obtained from the redistributed moment for a fixed-end beam. In practice this is not true, even for rigid framing. For each of the cases investigated for the composite beam frame, there is strong evidence to indicate that the location of the inflection points depends on the relative stiffness between the beam and column, the type of joint the member is framed into,(whether it is a two, three or four member joint). Furthermore the location is also affected to a certain extent by the end conditions of the member - i.e. the inflection point at one end is affected by the end condition(type of joint) at the other end and vice versa.

The above must necessarily be true, since most members in a rigid frame have end conditions that are intermediate in behavior between a semi-rigid end in which a certain amount of rotation is possible and a fixed end where no rotation is possible. It would be more desirable to be able to predict a more accurate inflection point based on the member properties rather than the arbitrary value of 0.10L recommended in reference 25. Towards this end, the frame of Fig. 4-1 is further investigated by varying the sizes of the columns while keeping the composite beams constant. This is done for span lengths of 20, 30 and 50 feet, and only for one loading condition of w = 3 kips/ft. The column heights are kept constant at 12 feet in each case.

The format of the investigation is as follows : the interior columns are kept constant at the minimum allowable section while the exterior columns are varied in sizes. For each variation, the inflection point is changed, and the length of composite section is correspondingly reduced or increased to reflect this change.

One result that is obvious from this investigation is that with increasing column size, the inflection point is shifted towards the center portion of the beam, which means that the amount of composite section is reduced. To relate this change to the properties of the members, the distance of the inflection point from the end is plotted against the relative stiffness of the column and the beam. The stiffness of the beam and column is given by the following :

$$S_B = 4EI_B / L_B \quad \text{.......................} \quad (4.3)$$

$$S_C = 4EI_C / L_C \quad \text{.......................} \quad (4.4)$$

where I_B, L_B and I_C, L_C are the moment of inertia and length of the beam and column respectively. The relative stiffness of the column and beam is designated as ;

$$S_r = S_C / S_B \quad \text{.......................} \quad (4.5)$$

With the distance of inflection point from the end αL as ordinate and the relative stiffness Sr as abscissa, the results obtained are shown in Figs. 4-11 to 4-13 for the three cases. The behavior pattern is similar in the three cases. The change in inflection point at joints 1 and 4 is particularly sensitive to the change in relative stiffness S_r. When S_r is less than 1.0, the slope of the curve is rather steep, but when S_r is greater than 1.0, the slope gradually levels off until at S around 2.0 the curve more or less flattens out. To indicate this sensitivity, the increase in inflection point distance is calculated for two limits of S_r ; e.g. for the case of L = 20 feet, for S_r from around 0.2 to 2.6 the corresponding increase in αL for joint 1 goes from 0.054L to 0.1375L an increase of approximately 154%, while that for joints 2 and 5, the increase is from 0.1416L to 0.1665L - a 7.5% increase for a corresponding increase in S_r. This is the same for the other two cases, except the difference in the increase for joints 1 and 4 and joints 2 and 5 is not as large. For L = 30 feet, the increase for joints 1 and 4 is about 77% compared to 12% for joints 2 and 5, and for L = 50 feet, the increase is 61% and 6% for joints 1, 4 and joints 2 & 5 respectively. From the above, it can be seen that the change in inflection point at joints 2 and 5 are not sensitive at all to the variation in column size. For a relatively large variation in the relative stiffness ratio Sr the change in the inflection point is only 6 to 12%. For this reason no intermediate points are obtained and for simplicity the αL - S_r relationship for joints 2 and 5 is assumed to be a straight line relationship(Fig.4-15).

It is also apparent from the curves that the type of joint a member is framing into is also a factor in determining the inflection point. For the same relative stiffness ratio S_r, the inflection point distance αL for joint 4 is greater than that for joint 1. Also for the same S_r, the inflection point distance for joints 2 and 5 is much larger than that for joints 1 and 4. The conclusion drawn from the above findings is that the stiffer a joint is, the larger will be the inflection point distance αL.

Normally for joints with three or more members framing together such as joints 2 and 5 of the typical frame of Fig. 4-1, or when the stiffness of the column is very much larger than that of the beam, the inflection point distance αL can be approximated by using that of a fixed-end beam. For this purpose a fixed-end composite beam is investigated for the different spans and three loading conditions. The results obtained are shown in Fig. 4-14 with the inflection point distance αL as the ordinate and the span length as abscissa. Three curves are obtained for the three different loading conditions. The curves show that αL increases when the span length increases. From the standpoint of the length of composite section provided, the curves given, generally provide a conservative value.

By examing the curves of joints 1 and 4, for the three cases, it is seen that the shape of the curves are similar, and also the range in values are approximately the same. Since this is the case, the curves for the three cases are plotted together to form two curves for joints 1 and 4 for which the general case can be applied. The resulting curves are shown in Fig.4-15.

CHAPTER 5

Application and Discussion of Results

To determine the validity or applicability of the curves determined previously in Chapter 4, they are applied to the solution of two frames for which composite construction is to be used. The two frames to be analysed are shown in Figs. 5-1a and 5-1b. The frame for example No.1 is a 2 bay-3 story frame, with 36 ft. span length and 15 ft. columns. Actually the top story is a portal with a 72 ft. span, while the bottom left portion of the frame consists of a 30 ft. column. The second frame is a 3 bay-2 story frame with three equal spans of 30 ft. and 12 ft. columns. The joints of each frame are numbered in numerical order as are the beams for purpose of identification.

The frames for both examples are analysed first for non-composite design and the sections are as shown in Figs.5-2a and 5-4a together with the applied loadings. It is also assumed that the slab used for both frames is 5 inches in thickness. For example No. 1 a section of 33WF118 is required for beam B1, 21WF62 for beam B2 and B3, and for B4 a 21WF55 is adequate. The two top columns are 14WF150 sections, while the bottom columns are 14WF78 sections. All the designs are based on AISC specifications and all sections used are A36 steel, except for the two top columns of 14WF150 which are A242 steel. A large section and a higher strength steel is used for the two columns in order to resist the excessive bending that occurs there. For example

No. 2 with the same loading on each span, the same section is used for all the members and the required section is a 21WF68. The two interior columns are 12WF50 Sections, and the two exterior columns are 12WF58 Sections. It is pointed out that (wind+gravity) load is not the controlling factor in both cases, because of the 1/3 increase in allowable stress permitted by the AISC & CSA Specifications (Sec.1.5.6). The (DL +LL) moment diagram for non-composite design for the two examples are shown in Figs. 5-3a and 5-5a.

To determine the preliminary sections for composite design, the curves of Figs. 4-4 to 4-11 are used where applicable, if not the average value of percentage reduction of moment obtained in Section 4.4 of Chapter 4 is used. For beam No. 1 of example 1, the span length exceeds that given in the curves, therefore the percentage reduction given in Section 4.4 is used. Assuming the critical moment is at joint 1 or 2, the percentage reduction given for this type of joint is 20%, which would give a moment of 407 K-ft. from which the trial Section is determined. The required Section Modulus is 204 in^3 and the most economical section satisfying it is a 27WF94. For beams No. 2 and 3, the critical moment is assumed to be at joint 4 and the curves of Fig. 4-4 are used. The controlling moment for beam B4 is assumed to be at joints 6 or 7 and the curves of Fig. 4-7 apply. A summary of the moments derived from the given curves and the corresponding economical Sections are given below:

Beams	Moments (K-ft)		Economical Sections
	Assumed	Exact	
B1	407.0	427.0	27WF94
B2, B3	0.0769WL (199.0)	189.0	21WF55
B4	0.0465WL (121.0)	144.0	16WF40

From the above, it is seen that for beams 1 to 3 no revision is necessary compared to the exact sections shown in Fig. 5-2b. In comparing the assumed moments with the exact moments shown in Fig.5-3b, it is seen that the critical moments for beams B2 and B3 do not occur at joint 4, but at joint 3 and 5, and also there is a 16% between the assumed and the exact moment for beam B4, These discrepancies stem from the fact that the frame for example No.1 is quite different from the frame used in the previous investigations, where the end conditions of the members are quite different.

For example No.2, the frame resembles more closely the frame of Fig.4-1 used in the previous investigations. The controlling moments are assumed to be at joints 2 or 3 and at 6 and 7. For beams B1 and B3, the curves of Fig.4-4 are used, and for B4 and B6, the curves of Fig.4-5 are used. For beams B2 and B5, with 4 members at the joints, a fixed-end beam is assumed and the curves of Fig.4-9 apply. The derived moments, the exact moments as obtained from Fig.5-5b and the corresponding economical section are summarised below:

Beams	Moments		Economical Sections
	Assumed	Exact	
B1, B3	0.0787WL (212.0)	196.0	21WF55
B2, B5	0.0674WL (182.0)	187.0	21WF55
B4, B6	0.0758WL (204.0)	192.0	21WF55

Since all the members are assumed to be the same only B1 or B3 need be considered and with an assumed moment of 212.0 K-ft or a required section modulus of 106.0 in , the required section is a 21WF55.This section is the correct section as shown in Fig.5-4b. The assumed moment for the other beams also show very good agreement with the exact value.

After determining the preliminary steel sections, the next step is to determine the length of composite section to be provided or the location of the inflection points for each member. As a first approximation, the same amount of composite section located at equal distances from both ends may be assumed for each member or a more accurate approximation can be obtained by using the curves of Fig.4-11 to 4-15.

For example No.1, the inflection point distance αL for B1 is obtained by using the curve for joint 1 of Fig. 4-15. For beams B2 and B3, the curve for joint 4 of Fig. 4-15 is used for the ends at joints 3 and 5, and similarly for that of beam B4. For joint 4, the curve for joints 2 and 5 is used. The results as obtained by using the curves of Fig. 4-15 are summarised below togather with the exact values.

Beams	Joints	Sr	αL (inch) Assumed	Exact
B1	1 & 2	1.115	0.120L (104.0)	99.0
B2	3	0.827	0.139L (60.0)	69.0
	4	0.636	0.166L (71.0)	63.0
B3	4	0.636	0.166L (71.0)	72.0
	5	0.986	0.143L (62.0)	75.0
B4	6 & 7	0.96	0.142L (61.0)	54.0

It can be seen that there is quite a bit of difference between the assumed and the exact values especially for beams B2 and B3. Using these assumed values, the forces and moments calculated are practically the same as the values obtained when the exact values are used. These results are given in the computer output of Example No. 1.

Similarly the curves of Fig. 4-15 are used to determine the inflection point distance αL for the frame of example 2. The same general rule is applied in using the αL vs. Sr curves; curve for αL at joint 1 is applied to that of joint 1 and 4, curve for αL at joint 4 to that at joints 5 or 8. However in this case, beams B2 and B5 are treated as fixed-end beams and therefore inflection point distance αL curves for fixed-end beams of Fig. 4-14 are used instead. The results are summarised below.

Beams	Joints	Sr	αL (inch) Assumed	Exact
B1 or B3	1	0.371	0.078L (28.0)	30.0
	2	0.3075	0.1625L(58.5)	57.0
B4 or B6	5	0.371	0.110L (40.0)	39.0
	6	0.3075	0.1625L(58.5)	57.0
B2, B5	6 & 7	--	0.1545L(56.0)	57.0

It is seen that there is close agreement between the assumed and the exact values, which are shown in Fig. 5-4b togather with the exact length of Composite Section. The resulting moments based on the assumed values are also almost the same as those based on the exact values. (See Computer output of Ex.2)

It can be said that the curves of Fig. 4-15 can be used to determine the location of the inflection points with a good degree of accuracy as long as the configuratiion of the frame resemble the frame from which the given curves are obtained. For all other types of frame, the curves of Figs. 4-14 and 4-15 provide a reasonable basis for a first assumption.

From an economic standpoint, the composite designs for both examples are also more economical than that for the corresponding non-composite designs. Based on the weight of the steel beams alone, the savings in steel is 17.5% for the frame of example No. 1 and 19.1 % for that of example No. 2. If the saving is based on the total weight of the whole frame, then the saving is reduced to 10 % for frame 1 and 13.2 % for frame No. 2. The breakdown for the different components of both frames is shown in Table 5-1.

From the standpoint of safety, the composite beams also provide more than an adequate factor of safety. The ultimate strength of the composite beams for both frames are calculated. (See Appendix IV for calculations) For beam B1 of example No. 1 the ultimate moment is 15,930 kip-in while the total design moment is 7355 kip-in. The resulting factor of safety is 2.17. For beams of frame #2, the ultimate moment is 8150 kip-in while the maximum design moment is 2375 kip-in, with a resulting F. S. of 3.43 which is much higher than that for B1 of frame #1. For the steel section of B1 for frame #1 the F. S. is 1.95 and for that of frame #2 it is 1.93-this means in both cases the Composite Section is not likely to fail first.

CHAPTER 6

CONCLUSION

From the theoretical investigations performed on the typical frame of Fig. 4-1 and also from the application of the results from these investigations, the following conclusions can be drawn.

Providing composite action in a rigid frame has the effect of reducing the negative moments and increasing the positive moments in a beam.

The total length of the composite section provided is more significant from the analytical point of view than the exact location of the composite section along the beam. providing the optimum amount of composite section at equal distances from both ends of the beam will result in a moment that is almost the same as the exact value. The optimum length of the composite section is usually between 0.70L and 0.80L.

The curves developed in Chapter 4 for determining the preliminary steel section for composite design and also the curves for determining the location of the inflection points of a beam in a rigid frame give very good results, when the configuration of the frame is the same as, or resembles that used in the investigation. For other types of frame whose configuration differ from that of the given one such as that of example No. 1, the curves still provide a reasonable assumption which requires only one more iteration to arrive at the correct results. From a practical point of view, the first approximation with respect to the location of the inflection points or the length of composite section, is accurate enough since further refinements do not result in any further economy.

There is a definite advantage in using Composite Construction from the standpoint of either economy or structural stiffness.

The moment of inertia of a composite beam is usually 2.0 to 2.5 times as large as that of a comparable non-composite beam, and this means that the compsite beam is on the average 1.5 times as stiff as the non-composite section in a frame . A stiffer structure will also result in smaller deflections for the members. The ability of a composite structure to take overload is also much greater than a non-composite structure.

From the economic point of view less steel is required for the same loads and span when composite construction is used. This is confirmed in the investigations and also in the two examples presented. The percentage savings in steel weight are shown in Tables 4-1 and 5-1. The savings range from 10% to 30% which falls within the range reported in section 1.3. These savings are only for a single bent and for a large structure, a 10% saving can be quite substantial.

A further advantage of composite construction is the possibility of smaller over all floor depths. This aspect is of particular importance in tall buildings. With reduced beam depths, the height of the buildings will like wise be reduced. Consequently the column heights will also be shorter- and this will result in smaller costs for walls, partitions, plumbing, wiring, ducts for air conditioning, elevators shaft and to a certain extent the foundations. Fireproofing costs will also be reduced for a reduced beam depth.

A greater savings in steel weight and a greater reduction in beam depth is possible by providing cover plates at the non-composite negative moment section to balance the composite positive moment section. However the savings in steel in this case must be compared to the costs of fabrication. Usually the costs of fabrication exceed the cost reduction for short or lightly loaded spans.

In comparing the total overall economy, such factors as labour and material costs for the different structural elements must be taken into account. Whereever possible, fabrication details should be minimised to save shop time; duplication of parts such as field joints to reduce detailing time. The method of erection is also a factor affecting the overall economy, depending on whether it is shored or unshored construction. If shores are used which is usually the case - then rensable prefabricated shores should be used to save materials and more important to ensure efficiency and speed in erection.

It has been demonstrated that composite construction is more advantageous in almost every aspect than a non-composite one. A saving of from 10% to 30% is possible in the beam weight, and structurally a composite structure is also very much stronger. With modern techniques in welding, it is also possible to achieve a simpler and lighter joint capable of rigid frame action without having to resort to a balky type of bolted-bracket joint. With the above in view, whenever concrete or structural steel are considered as building components, continuous composite construction should be considered.

SUGGESTION FOR FUTURE STUDY

It should be noted that the advantage gained by providing composite action in a frame is based on the analysis of the frame acting independently of other structural elements such as the floor and the wall. In an actual structure, the floors and walls interact with the frame as a unit and can contribute substantially to the structural rigidity of the frame. In particular the composite floor is a very strong structure and can act as a very stiff plate. Further investigation both analytical and experimental is recommended to determine the degree of contribution by the composite floor.

Further investigation into the application of plastic composite design to rigid frames is also desirable. In this study, the moment capacity of the beam sections indicated that negative moment hinges in the frame is likely to form first. Model testing as carried out in Reference 60 can be carried out to determine the plastic behavoir of such a frame.

Further research into the use of epoxy compound as a shear connector is also recommended. The development of a reliable and economical epoxy compound can make composite construction for many light structures an economic proposition, which would not be otherwise.

APPENDIX I

FIGURES

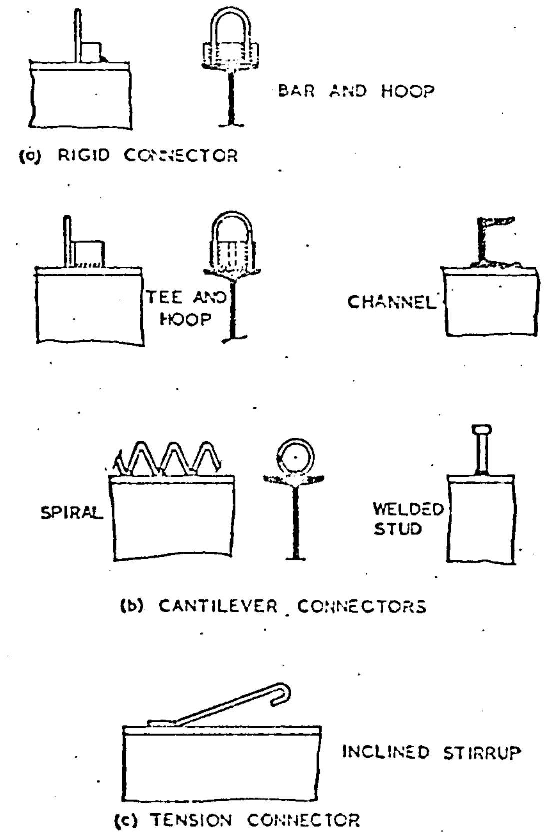


FIG. 1-1 TYPES OF SHEAR CONNECTORS

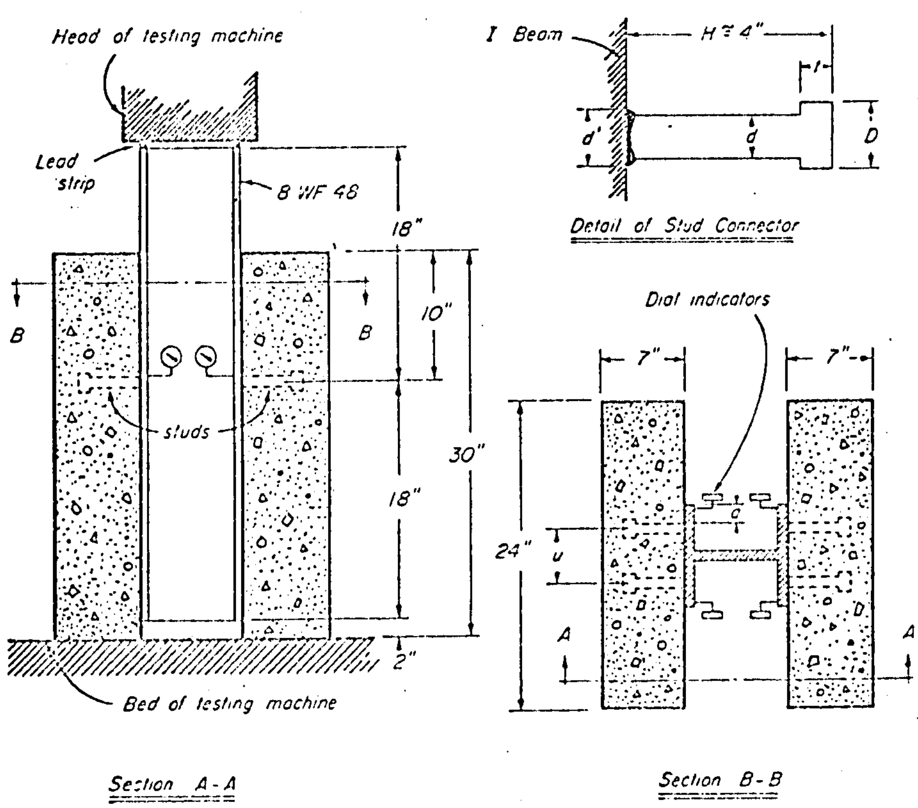


FIG. 1-2 DETAILS OF PUSH-OUT SPECIMENS TEST SET-UP

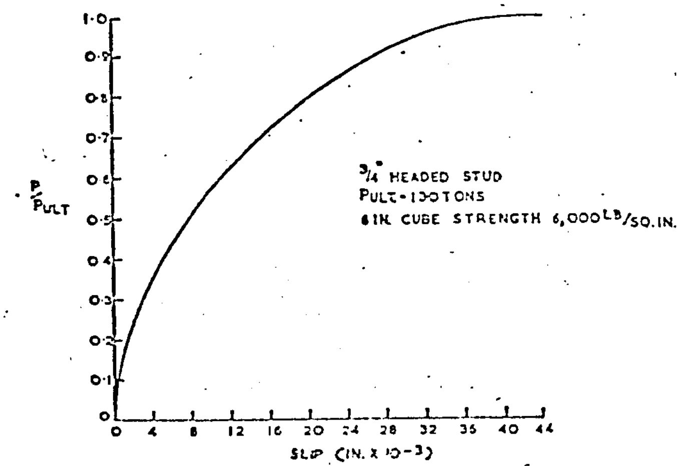


FIG. 1 - 3 LOAD-SLIP CURVE FROM PUSH-OUT TEST (22).

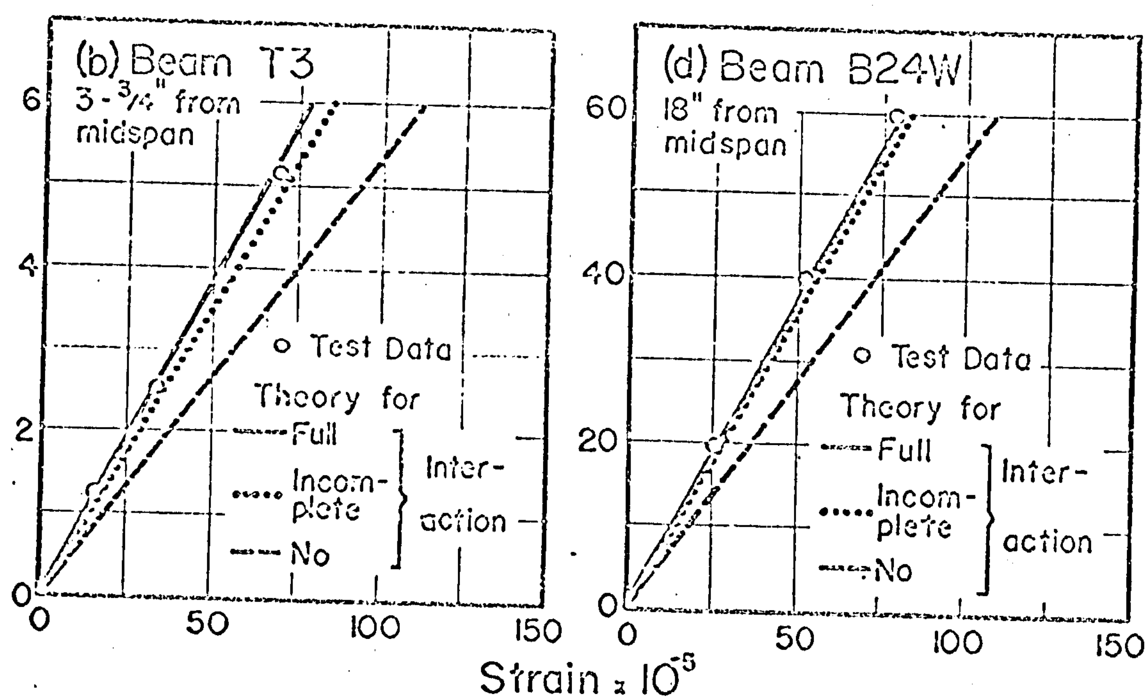


FIG. 1 - 4 LOAD-STRAIN CURVES (2)

Concentrated load at midspan

Strains on bottom flange of beam.

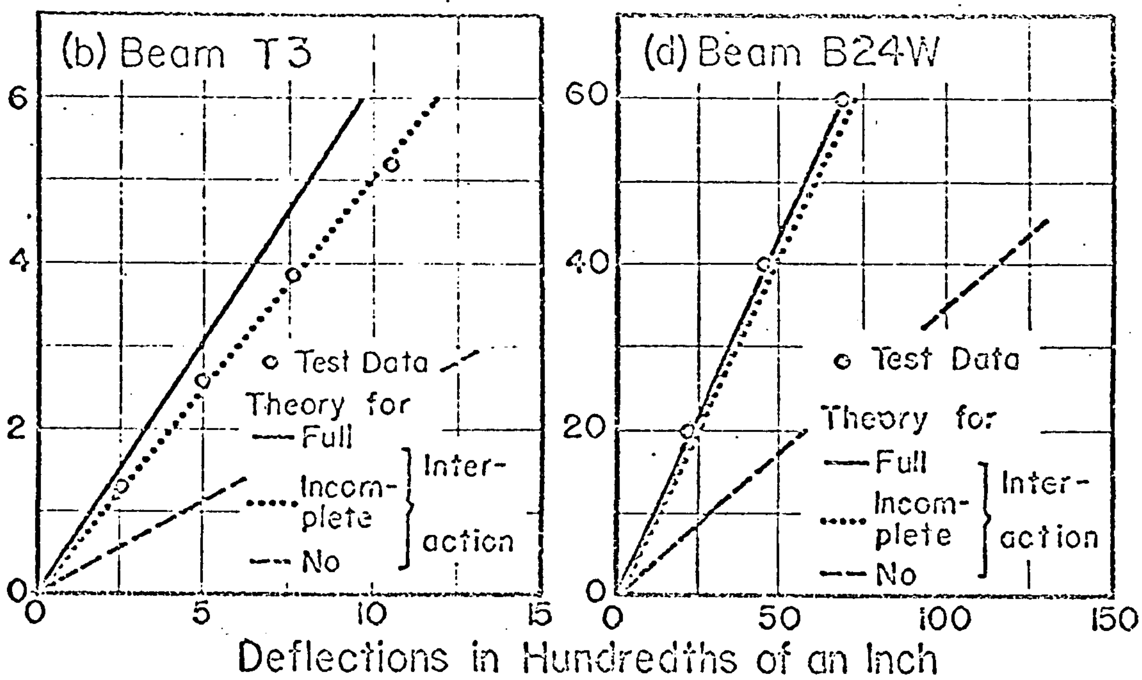


FIG. 1-5 LOAD-DEFLECTION CURVES(2)
Concentrated load at midspan
Deflections at midspan .
Theoret.values include shear deflections

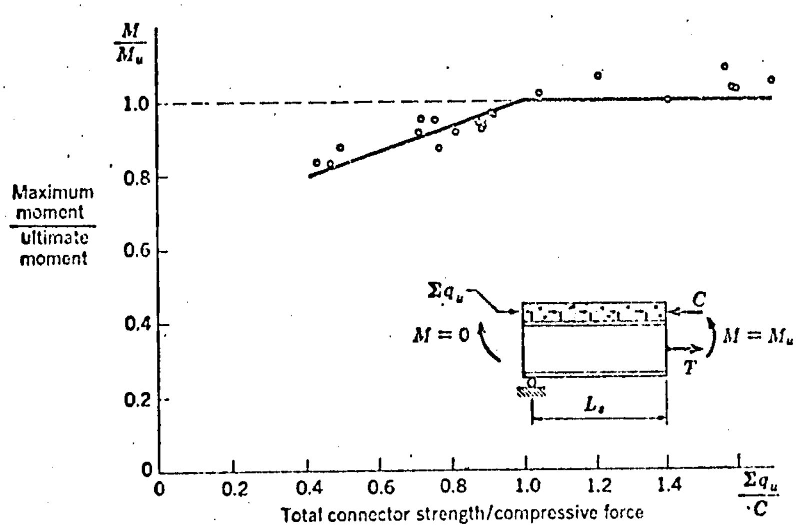


FIG. 1-6 RELATIONSHIP BETWEEN SHEAR CONNECTOR STRENGTH AND MOMENT CAPACITY(26).

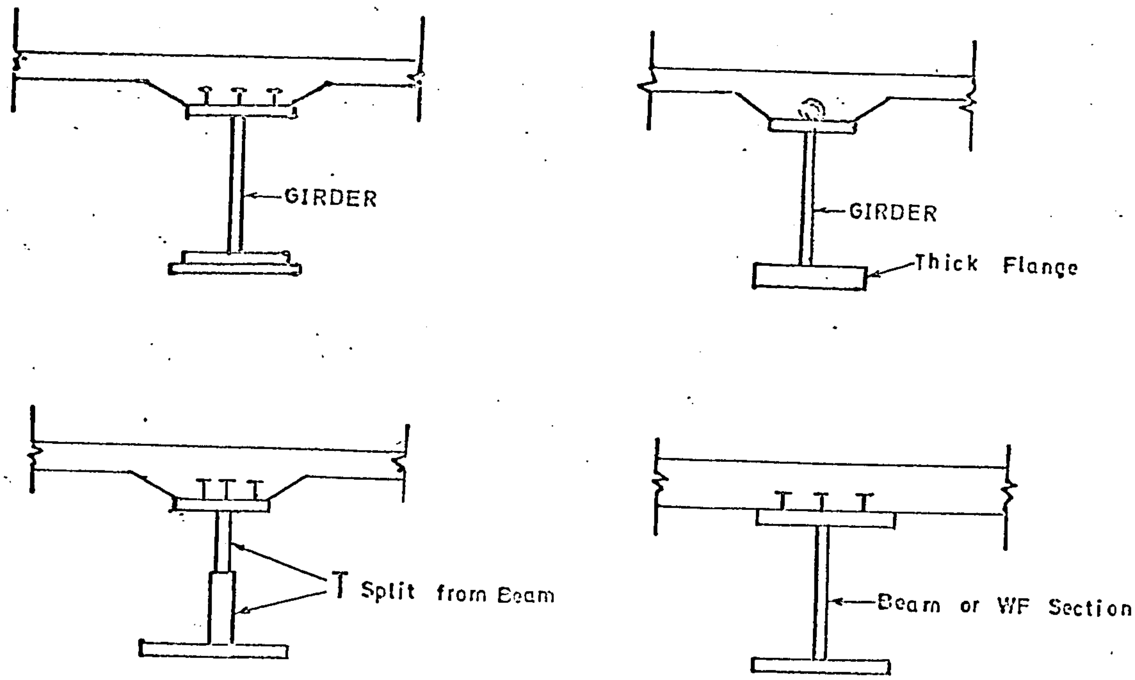


FIG. 2-1 TYPICAL SECTIONS USED IN COMPOSITE CONSTRUCTION FOR BUILDINGS

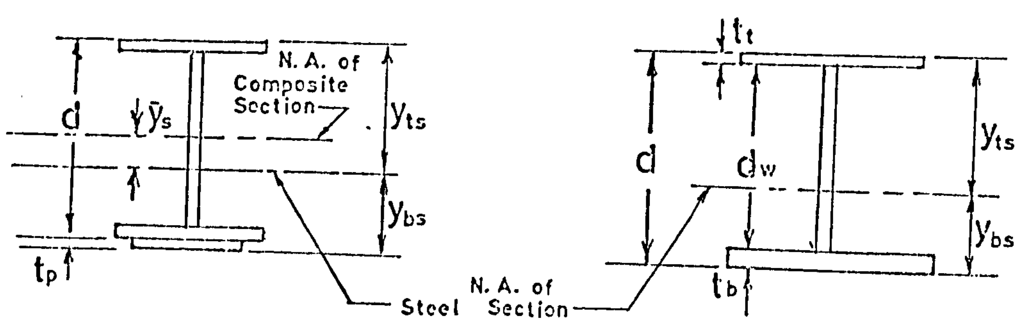


FIG. 3-1 ROLLED BEAM WITH COVER PLATE

FIG. 3-2 PLATE GIRDER

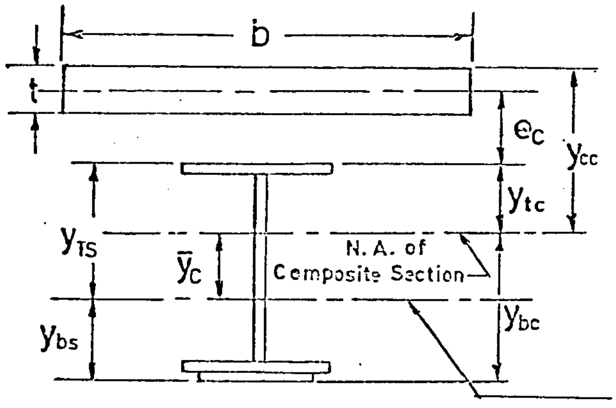


FIG. 3-3 COMPOSITE BEAM SECTION (N.A. BELOW SLAB)

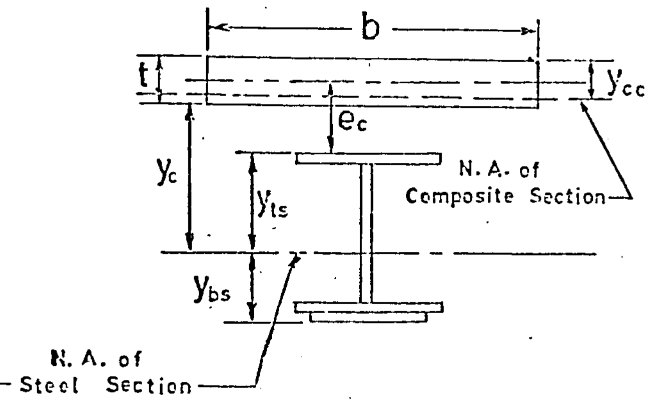


FIG. 3-4 COMPOSITE BEAM SECTION (N.A. IN SLAB)

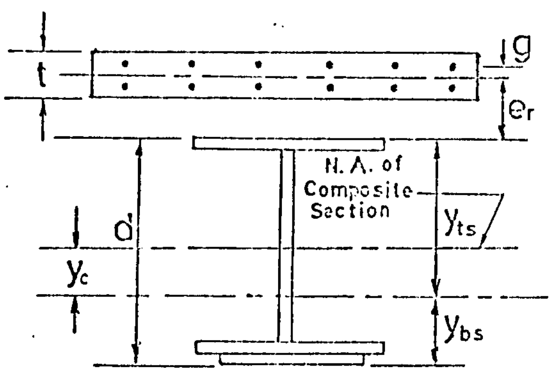


FIG. 3-5 COMPOSITE BEAM SECTION, SLAB IN TENSION WITH REINFORCEMENT

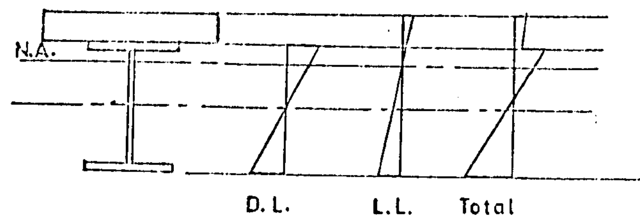


FIG. 3 - 6 STRESS DISTRIBUTION FOR UNSHORED CONSTRUCTION: D.L. TAKEN BY STEEL SECTION ALONE

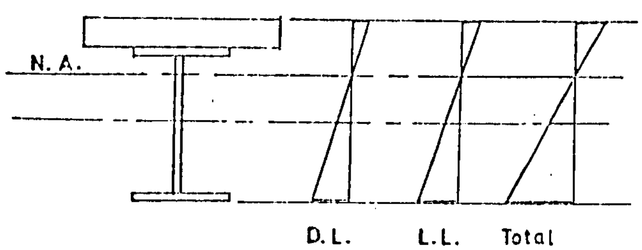


FIG. 3-7 STRESS DISTRIBUTION FOR SHORED CONSTRUCTION : D.L. TAKEN BY COMPOSITE SECTION

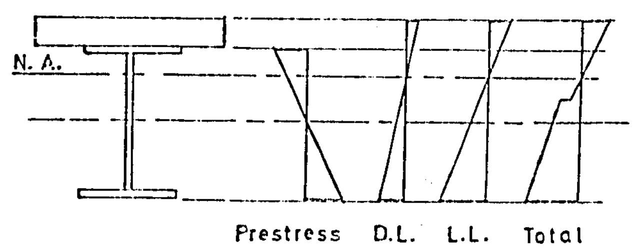


FIG. 3-8 STRESS DISTRIBUTION FOR THE CASE OF PRESTRESSED STEEL SECTION

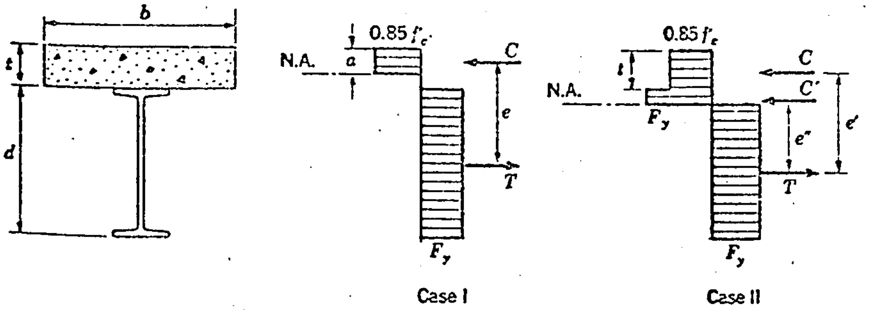


FIG. 3-9 STRESS DISTRIBUTION AT ULTIMATE MOMENT

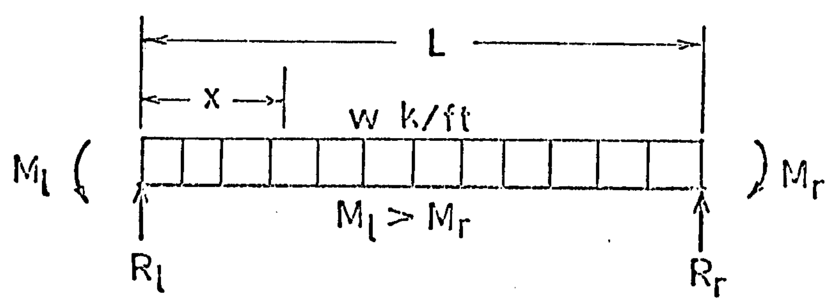


FIG. 3-10 BEAM UNDER UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS

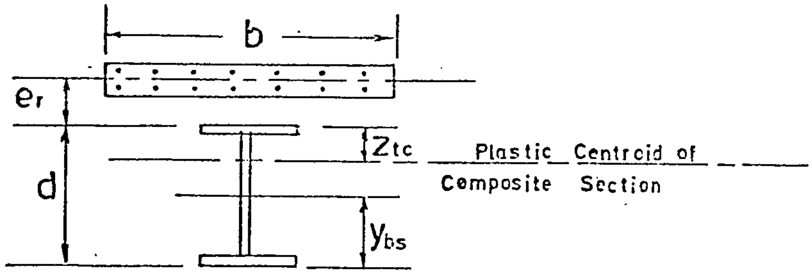


FIG. 3-11 COMPOSITE BEAM SECTION AT ULTIMATE LOAD CONDITION, WITH SLAB IN TENSION

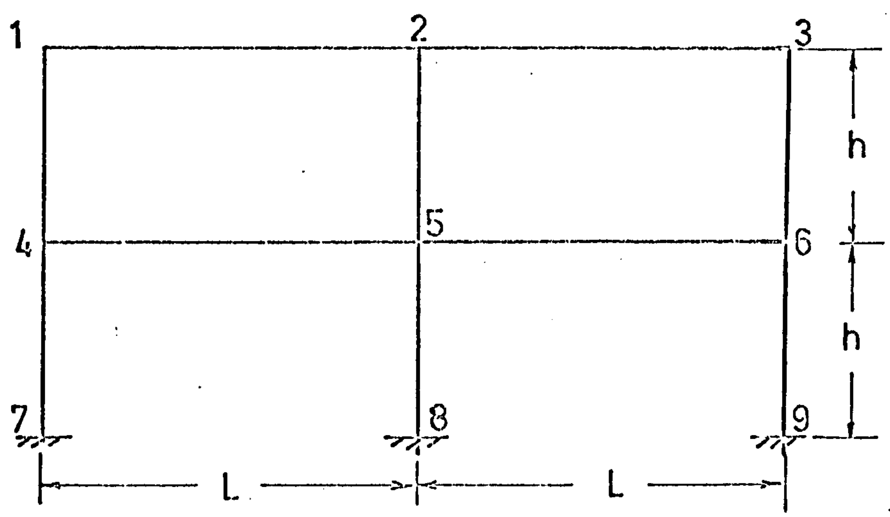


FIG. 4-1 TYPICAL TWO-STORY, TWO-BAY FRAME USED IN INVESTIGATION

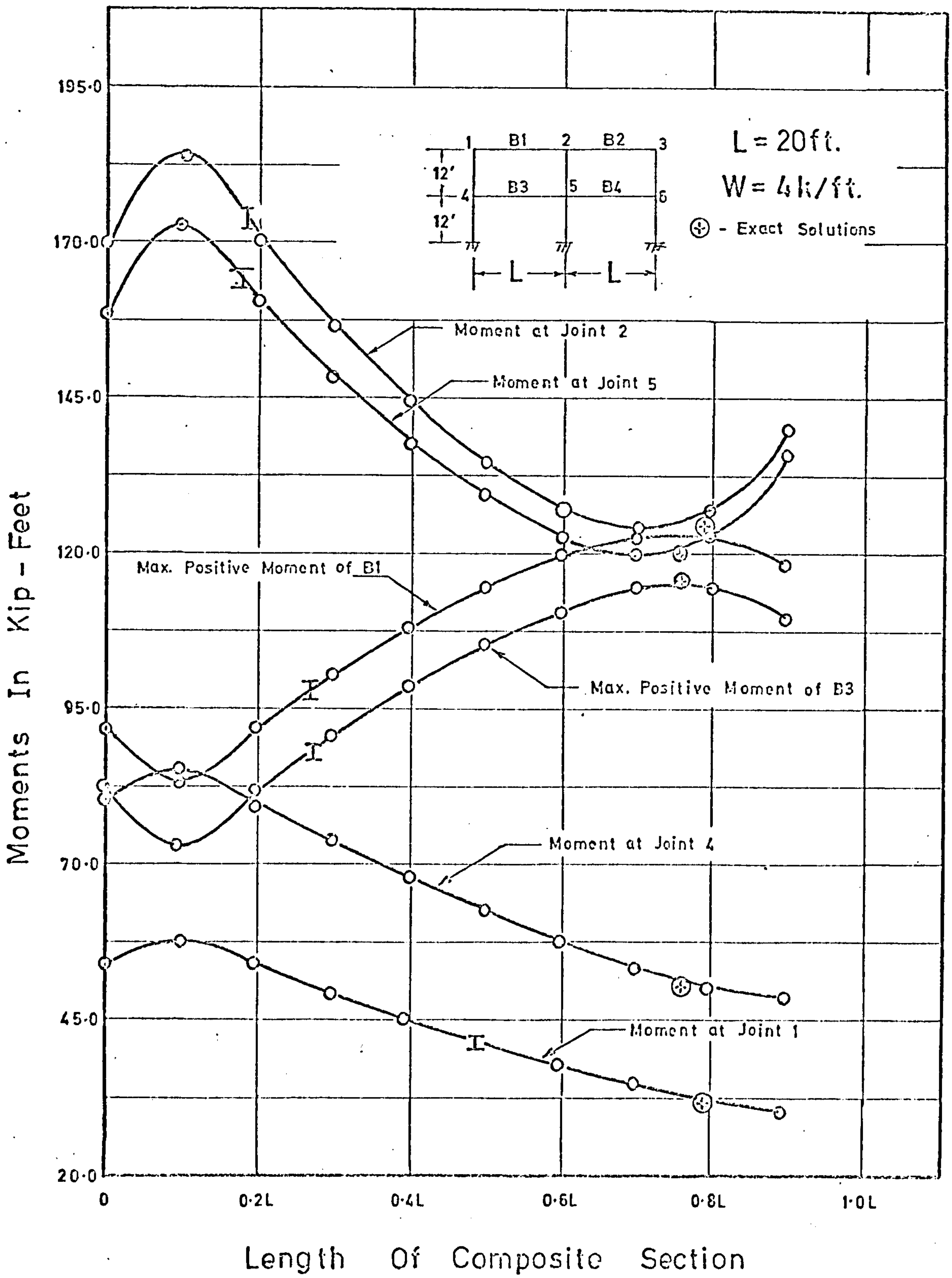


FIG. 4-2 EFFECT OF THE LENGTH OF COMPOSITE BEAM SECTION ON THE MOMENTS IN A TYPICAL FRAME

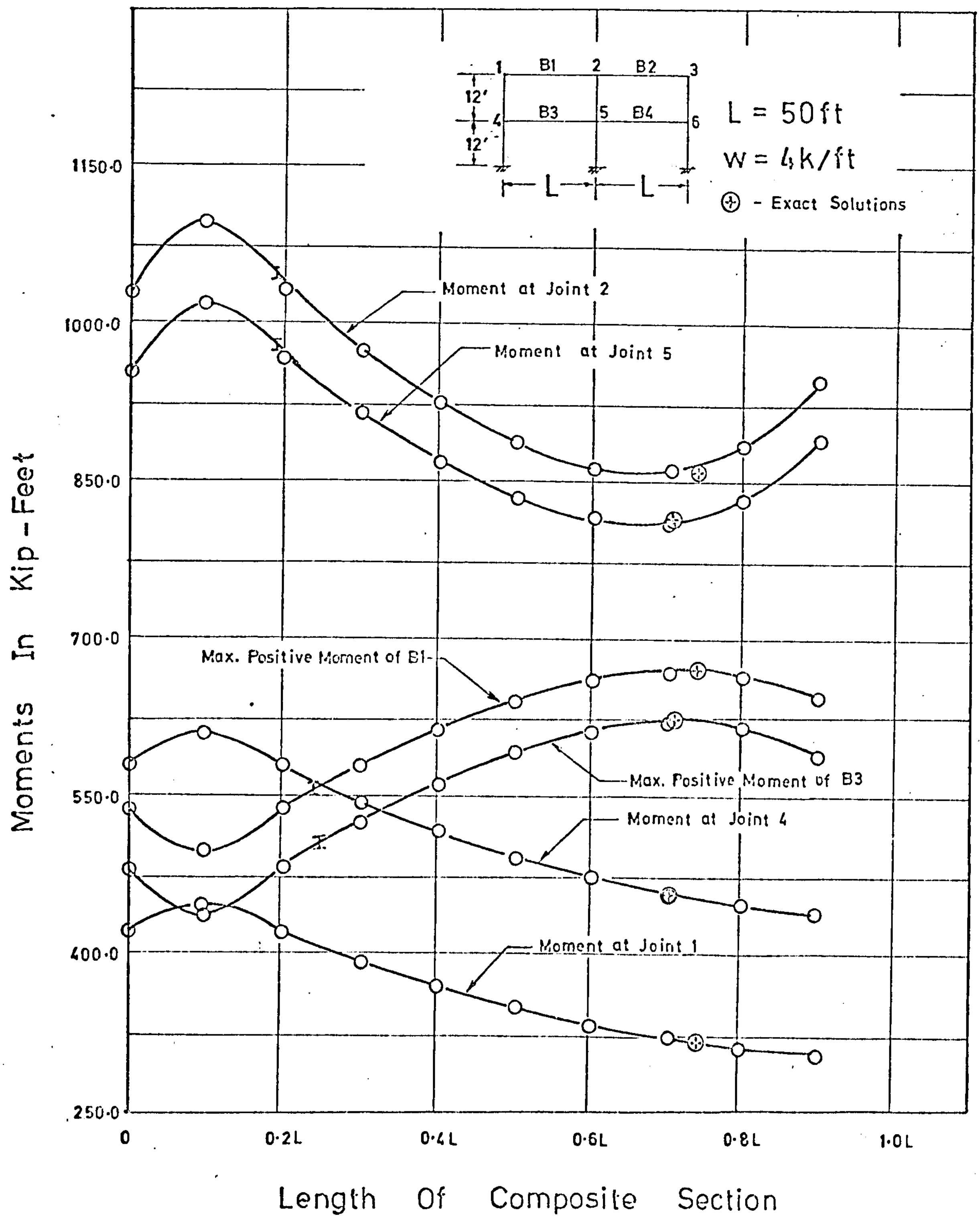


FIG. 4-3 EFFECT OF THE LENGTH OF COMPOSITE BEAM SECTION ON THE MOMENTS IN A TYPICAL FRAME

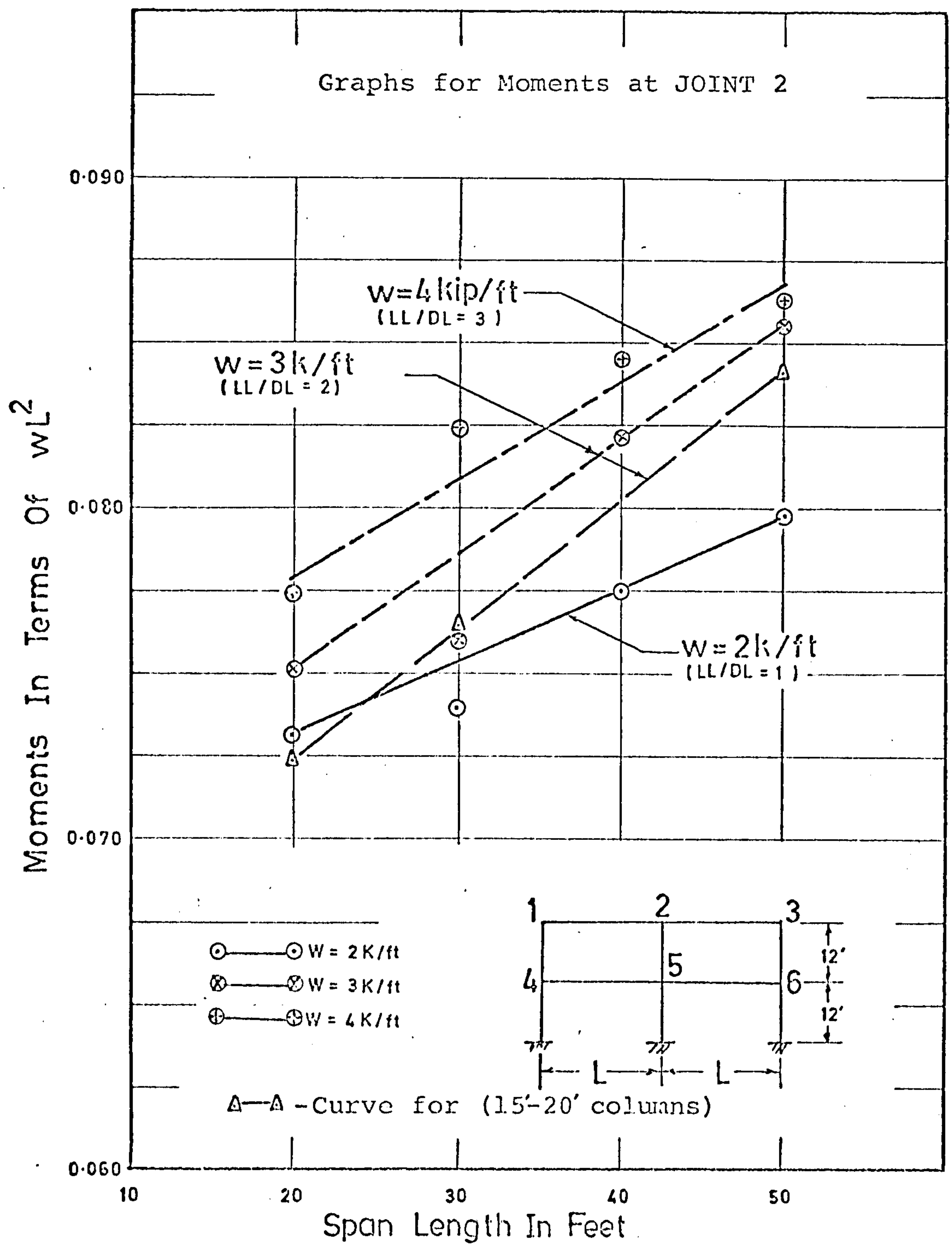


FIG. 4-4 EMPIRICAL RELATIONSHIP BETWEEN MOMENTS, LOADING, AND SPAN LENGTH FOR JOINT 2 OF TYPICAL FRAME.

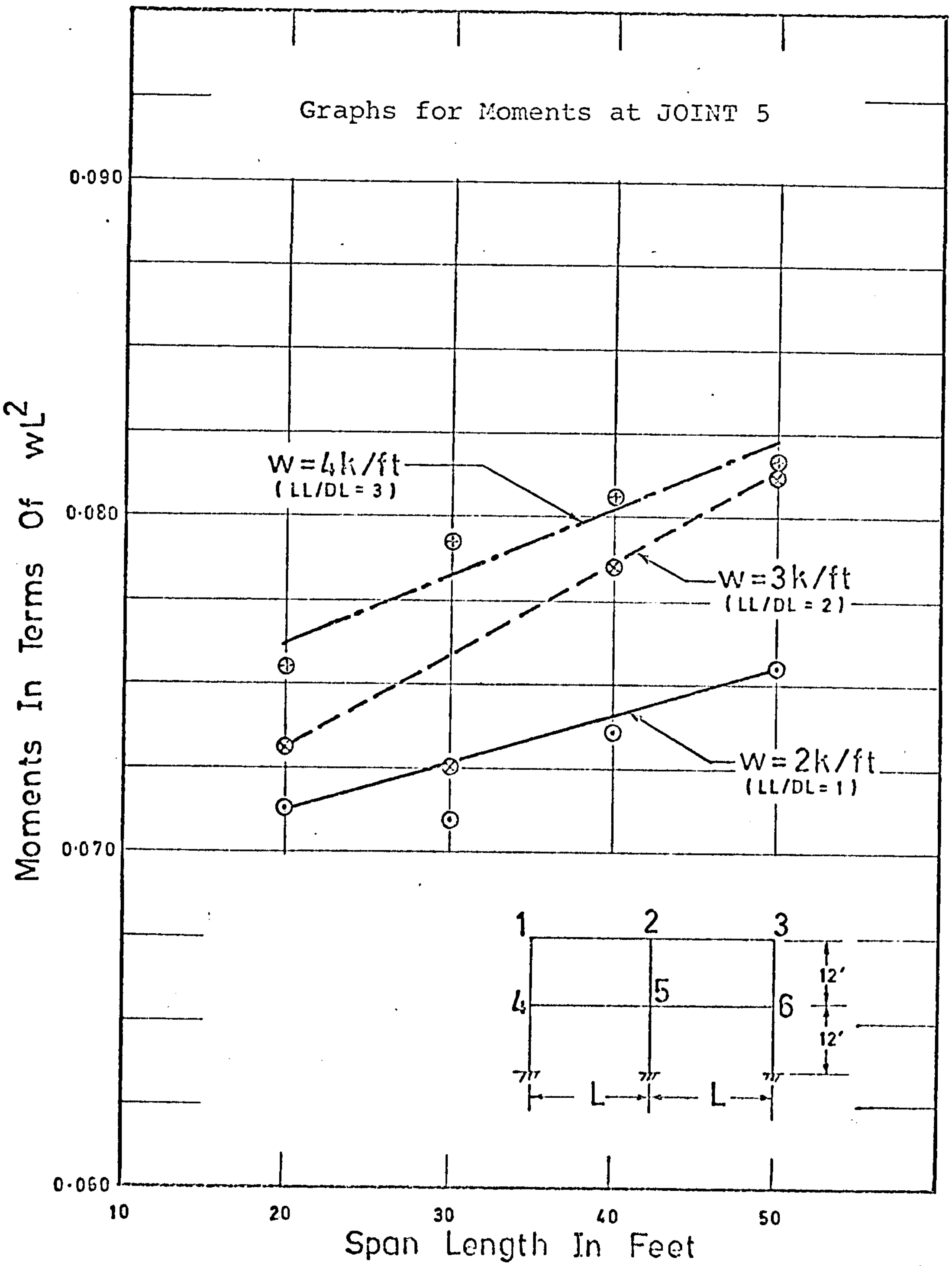


FIG. 4-5 EMPIRICAL RELATIONSHIP BETWEEN MOMENTS, LOADING, AND SPAN LENGTH FOR JOINT 5 OF TYPICAL FRAME.

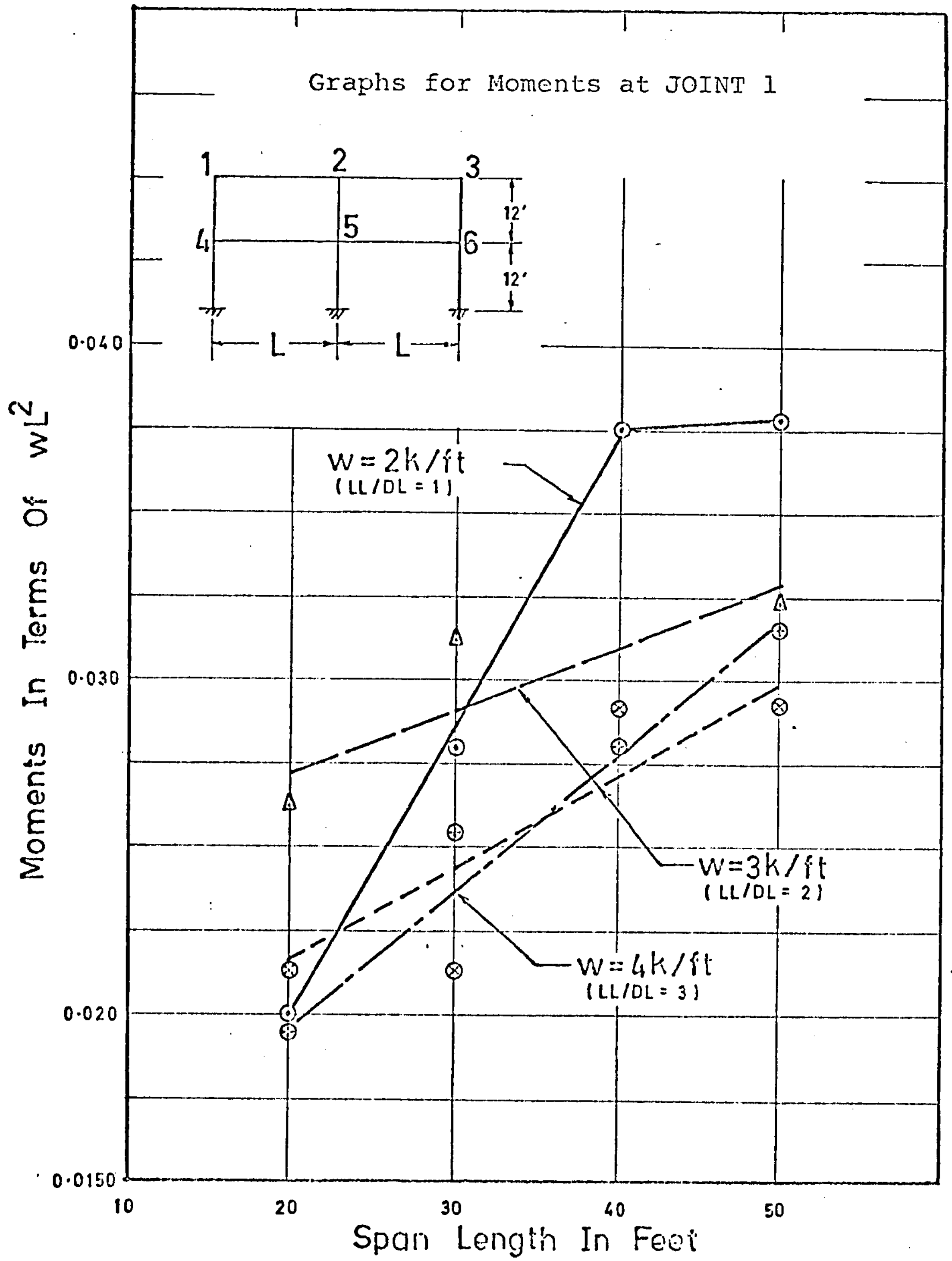


FIG. 4-6 EMPIRICAL RELATIONSHIP BETWEEN MOMENTS, LOADING, AND SPAN LENGTH FOR JOINT 1 OF TYPICAL FRAME.

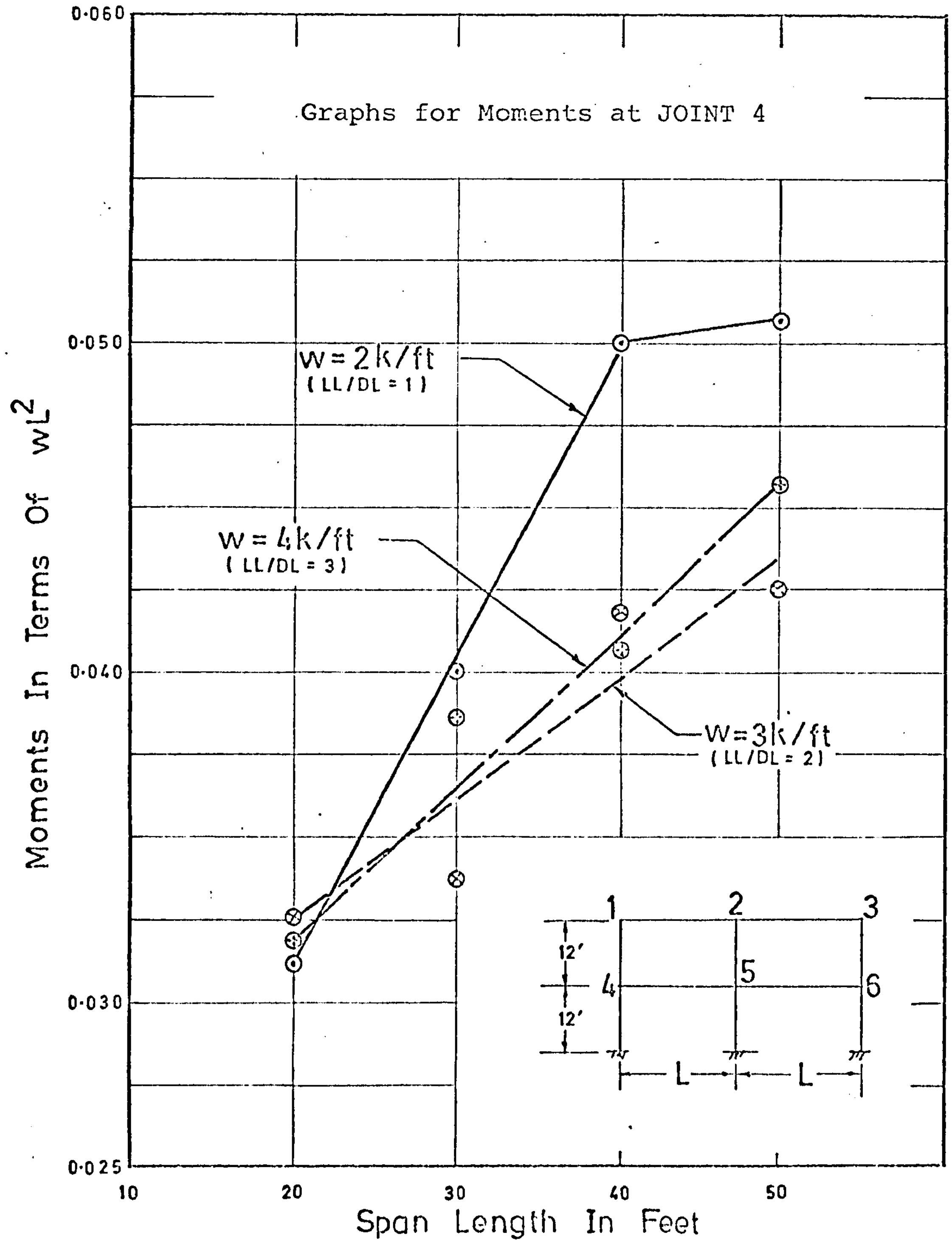


FIG. 4-7 EMPIRICAL RELATIONSHIP BETWEEN MOMENTS, LOADING, AND SPAN LENGTH FOR JOINT 4 OF TYPICAL FRAME.

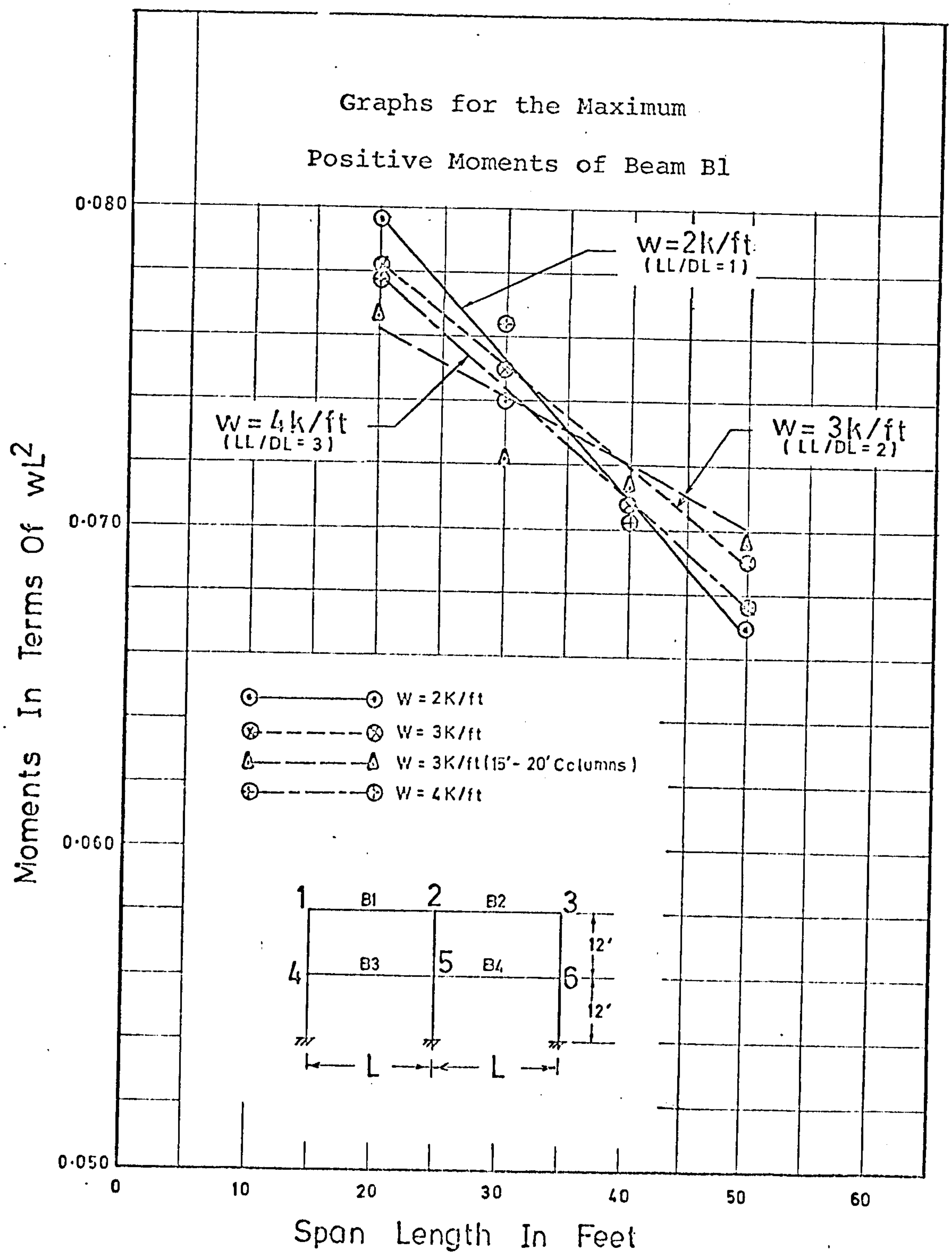


FIG. 4-8 EMPIRICAL RELATIONSHIP BETWEEN MOMENTS, LOADING AND SPAN LENGTH FOR THE MAXIMUM POSITIVE MOMENT OF BEAM B1 OF TYPICAL FRAME

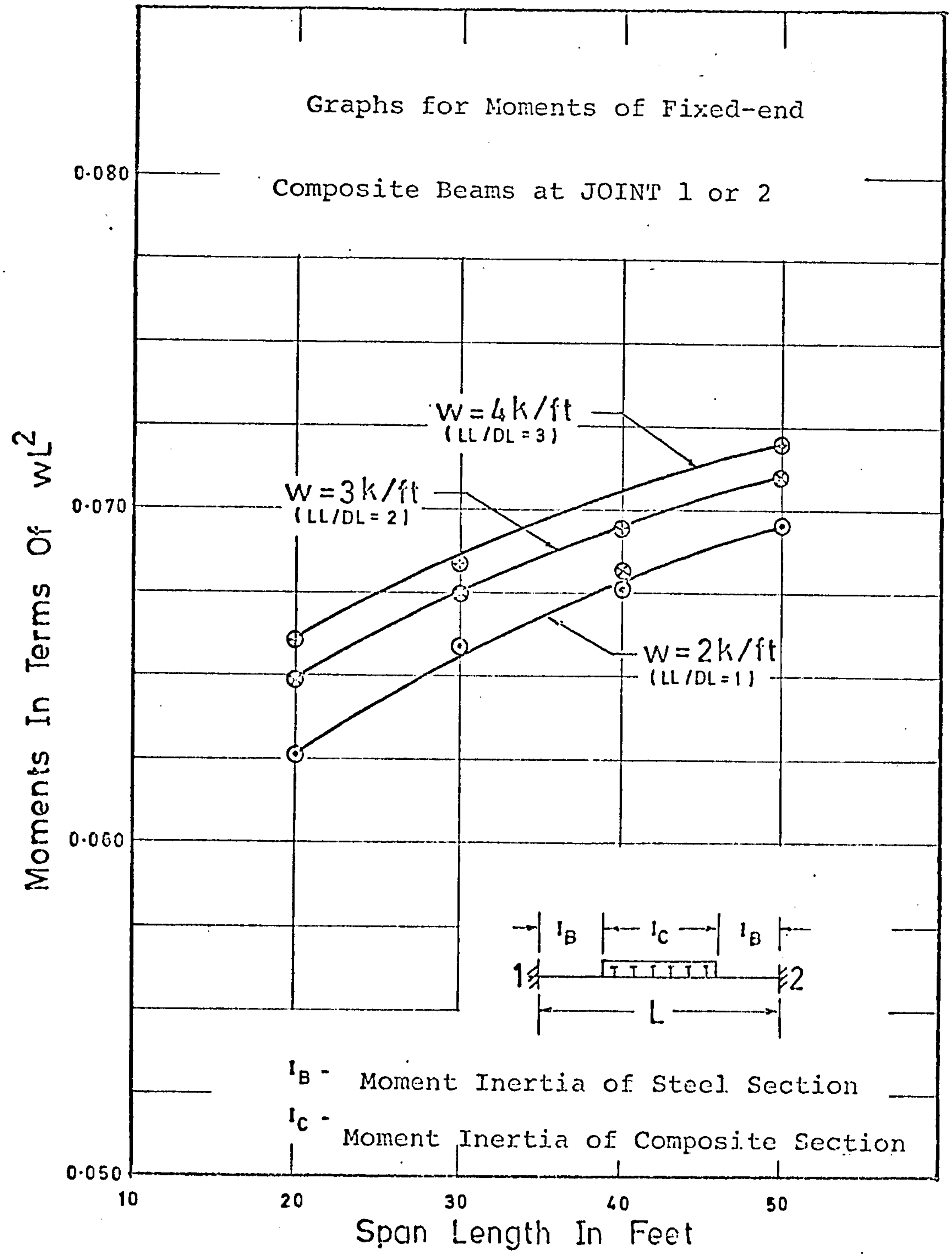


FIG. 4-9 EMPIRICAL RELATIONSHIP BETWEEN MOMENTS, LOADING, AND SPAN LENGTH FOR NEGATIVE MOMENTS OF FIXED-END COMPOSITE BEAMS.

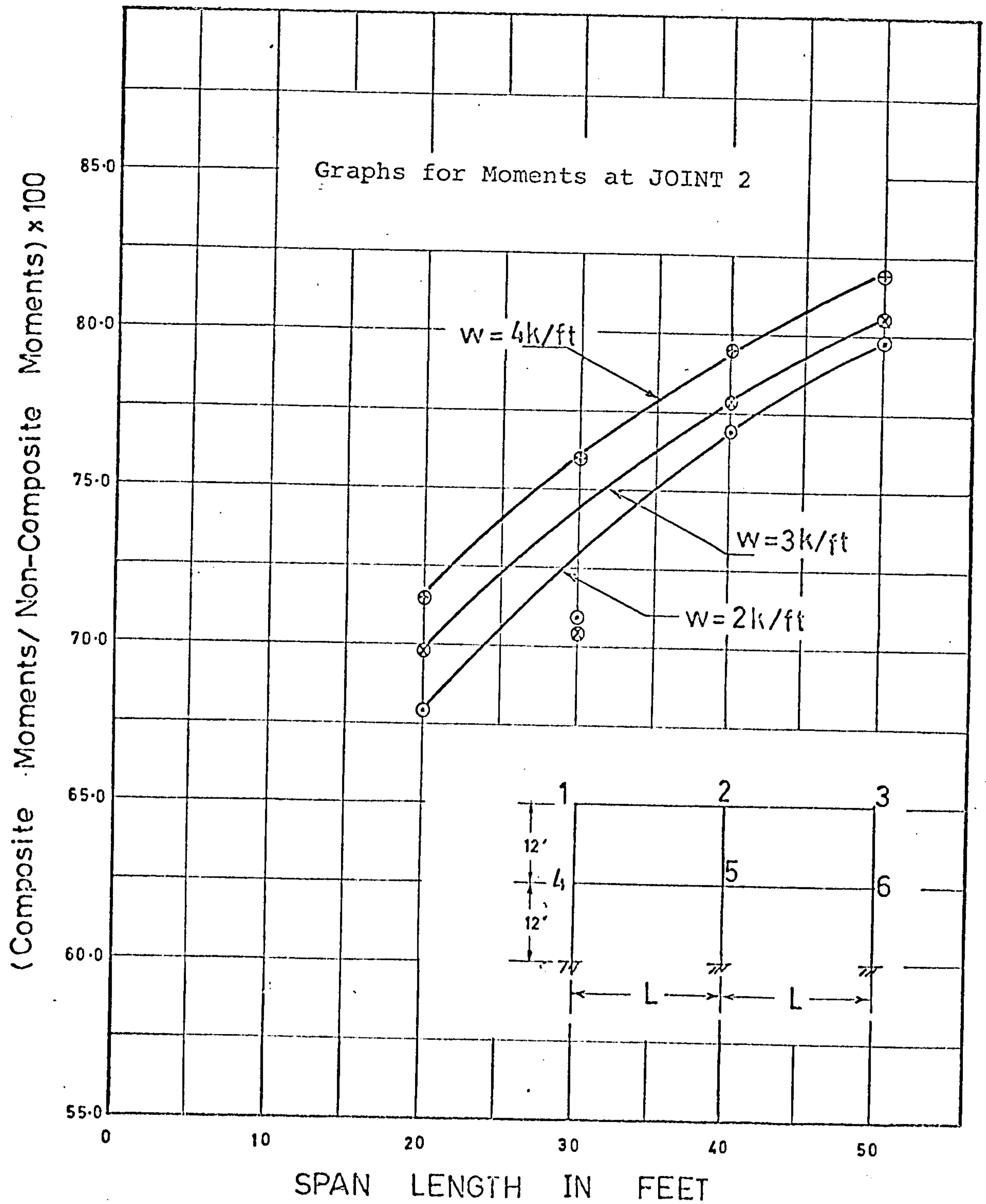


FIG. 4-10 EMPIRICAL RELATIONSHIP BETWEEN THE COMPOSITE MOMENTS EXPRESSED AS A PERCENTAGE OF THE NON-COMPOSITE MOMENTS, LOADINGS AND SPAN LENGTH FOR MOMENTS AT JOINT 2 OF TYPICAL FRAME

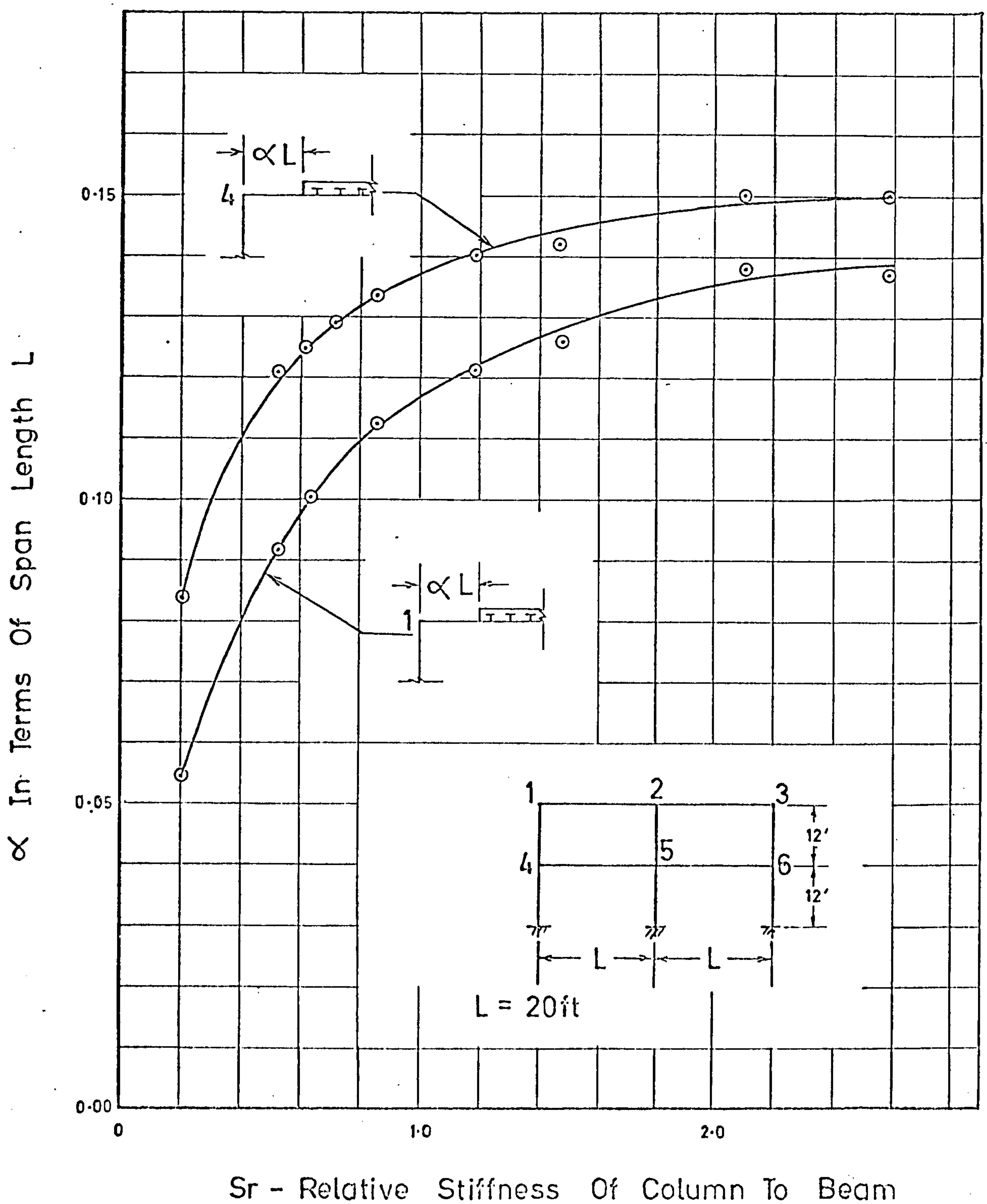


FIG. 4-11 INFLUENCE OF THE RELATIVE STIFFNESS OF COLUMN AND BEAM ON THE INFLECTION POINT DISTANCE FOR BEAMS OF TYPICAL FRAME

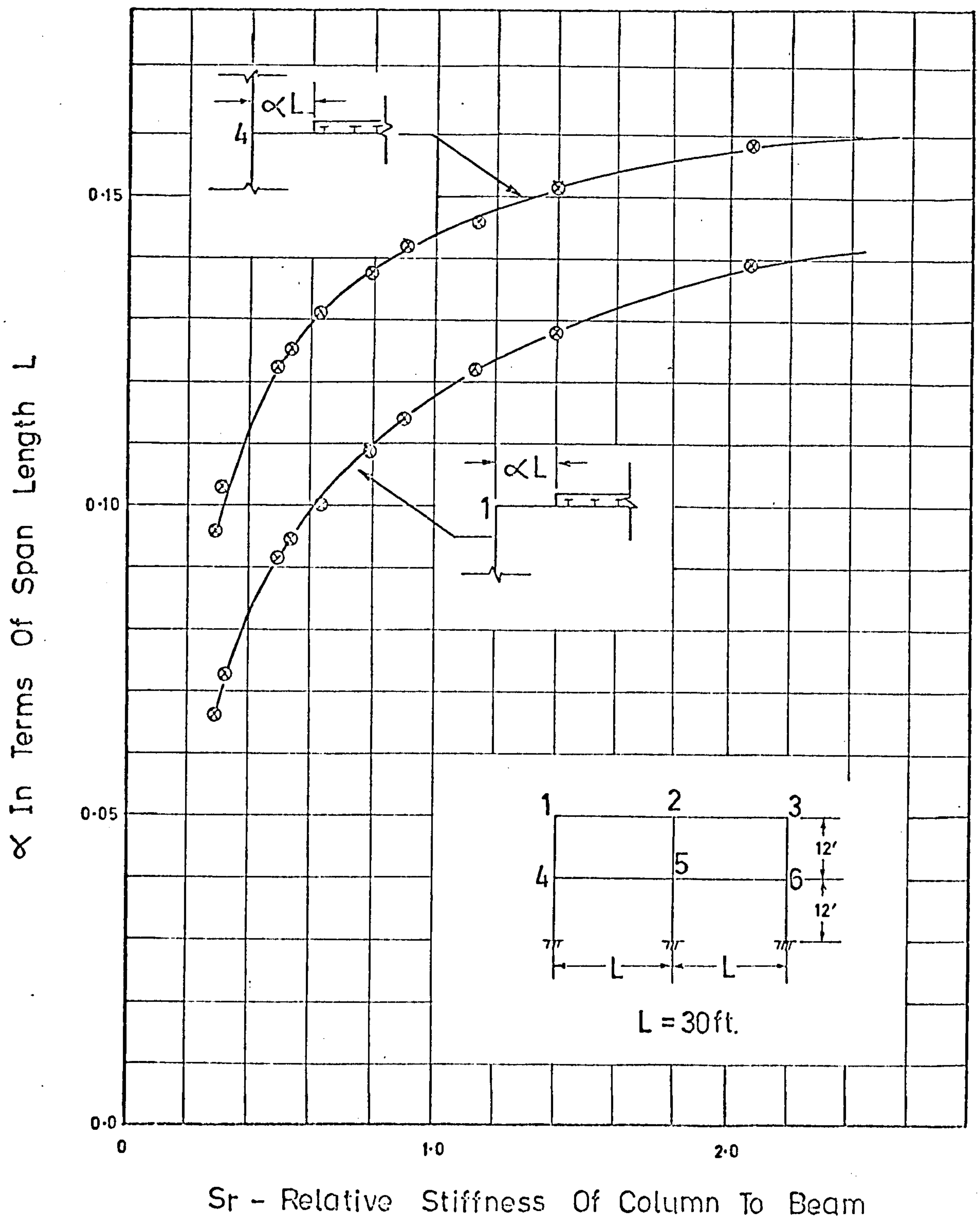


FIG. 4-12 INFLUENCE OF THE RELATIVE STIFFNESS OF COLUMN AND BEAM ON THE INFLECTION POINT DISTANCE FOR BEAMS OF TYPICAL FRAME

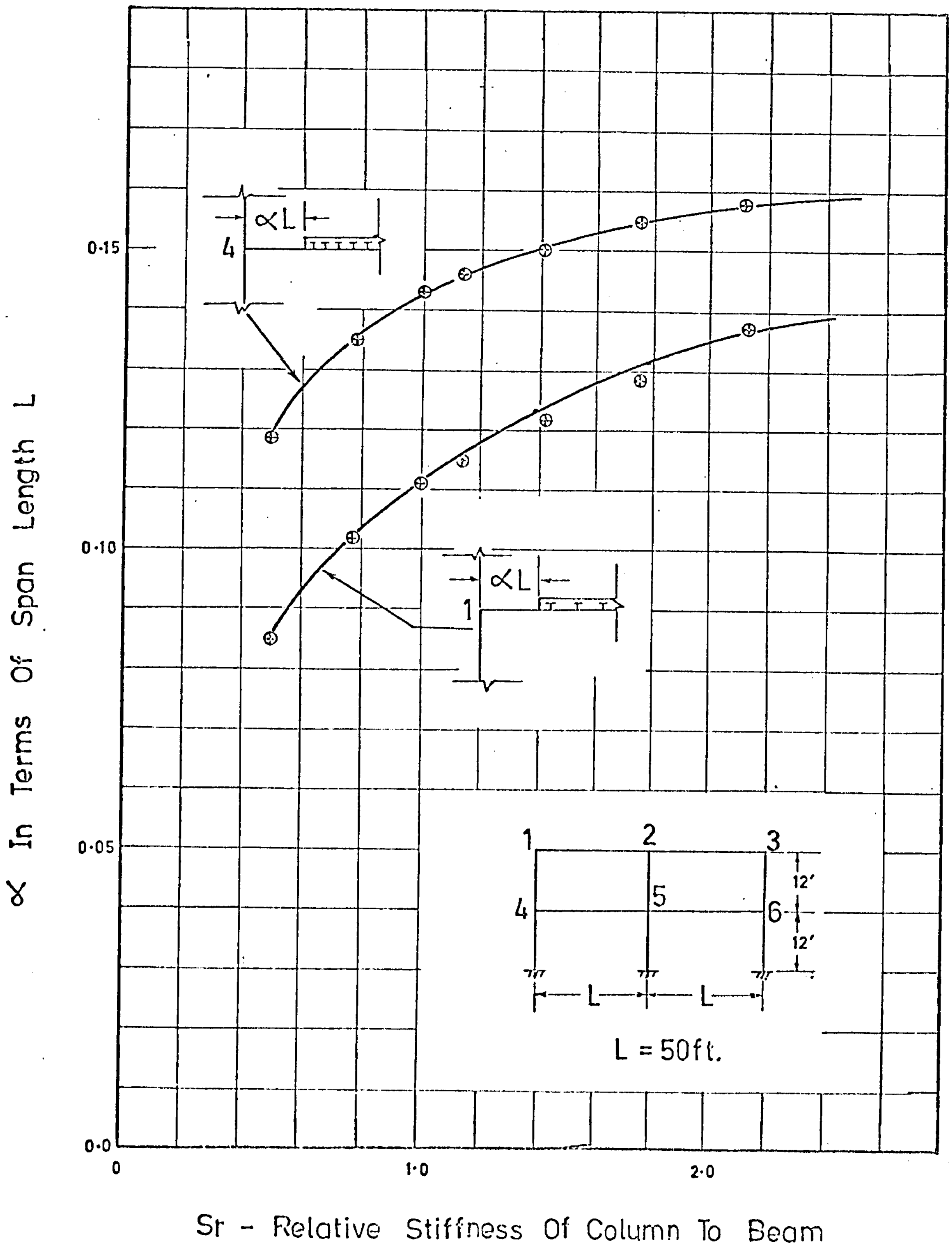


FIG. 4 - 13 INFLUENCE OF THE RELATIVE STIFFNESS OF COLUMN AND BEAM ON THE INFLECTION POINT DISTANCE FOR BEAMS OF TYPICAL FRAME

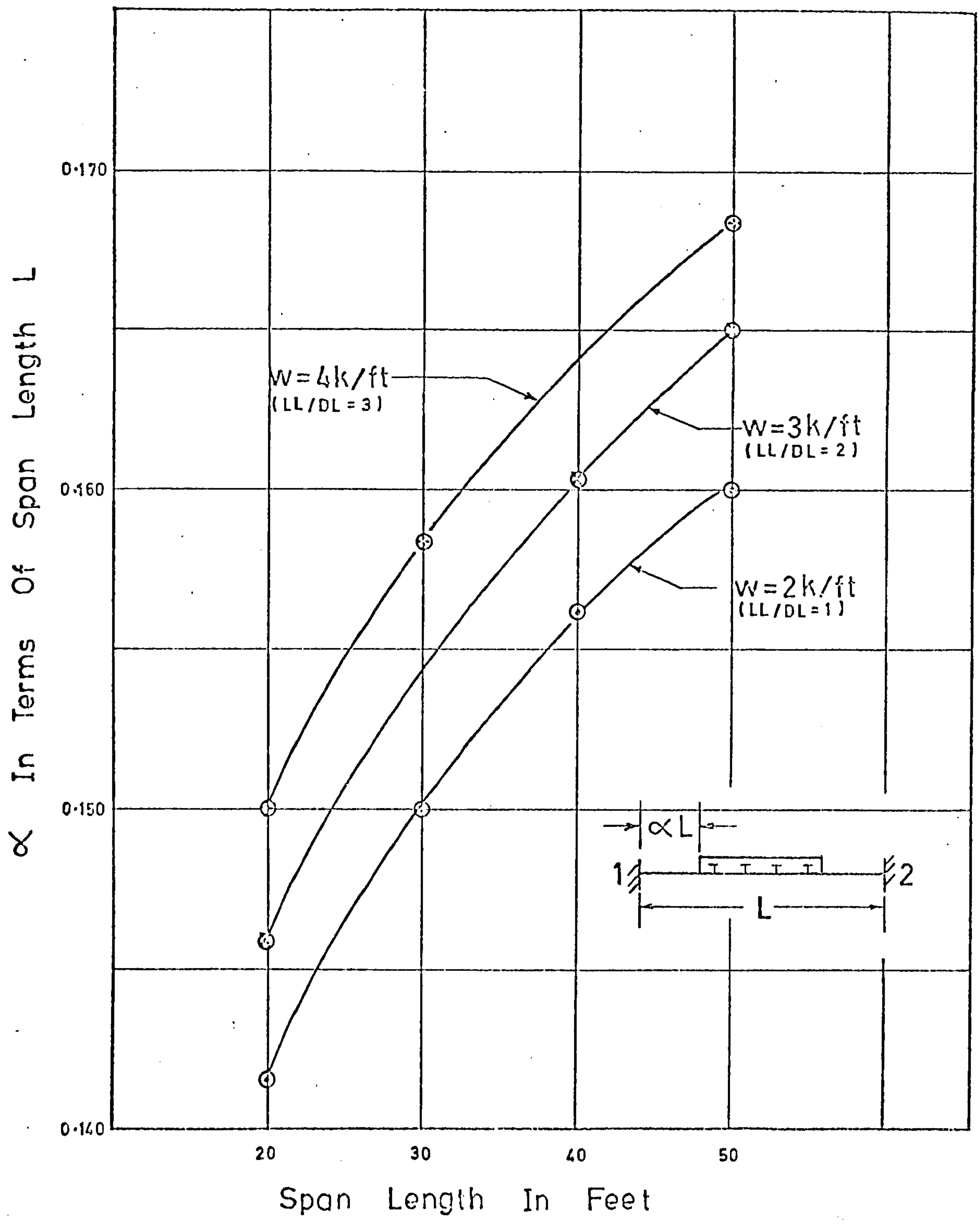


FIG. 4-14 EMPIRICAL RELATIONSHIP BETWEEN INFLECTION POINT DISTANCE AND SPAN LENGTH FOR FIXED-END COMPOSITE BEAMS

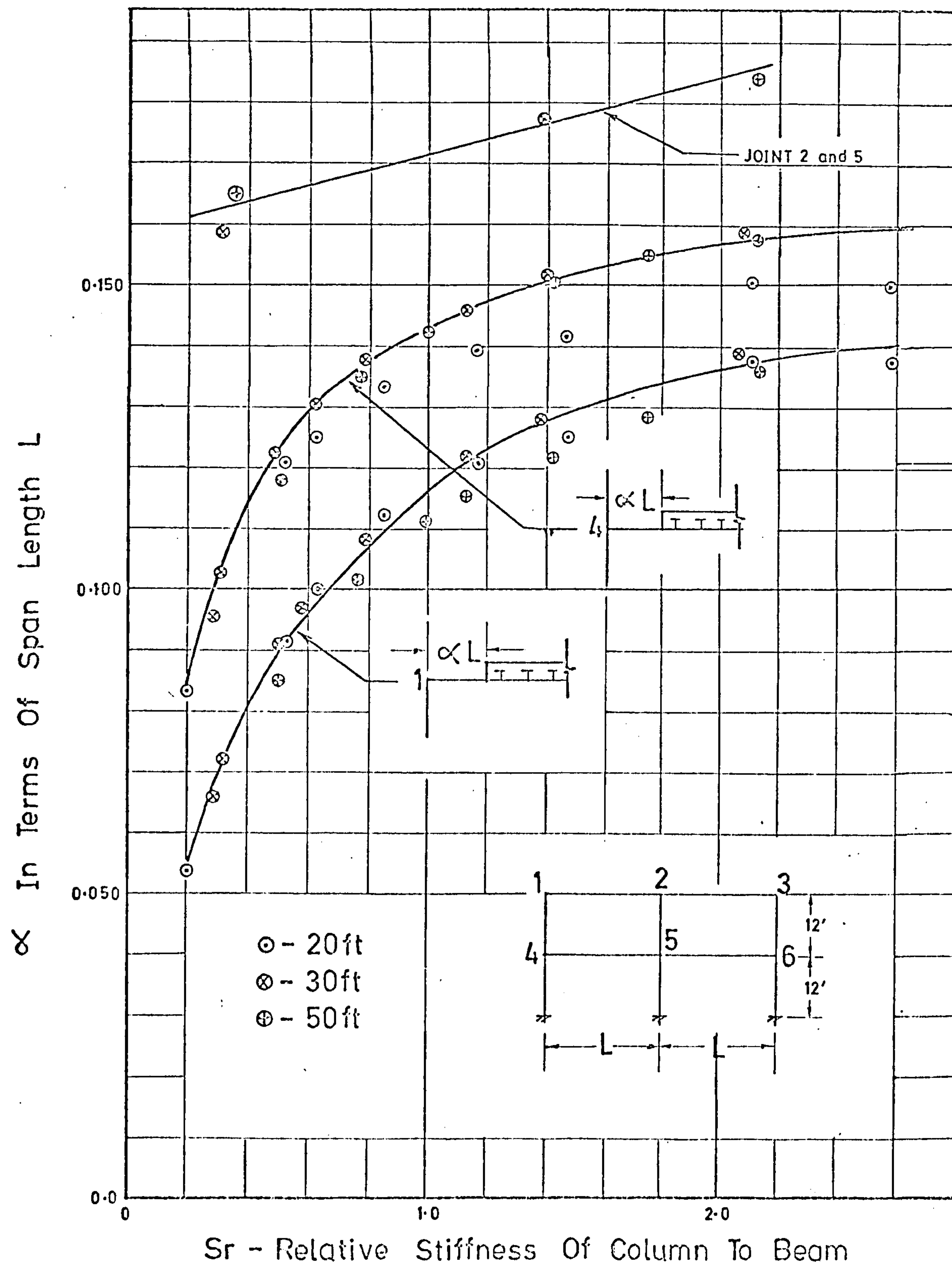


FIG. 4-15 EMPIRICAL RELATIONSHIP BETWEEN THE INFLECTION POINT DISTANCE AND THE RELATIVE STIFFNESS OF COLUMN AND BEAM FOR THE GENERAL CASE OF THE TYPICAL FRAME

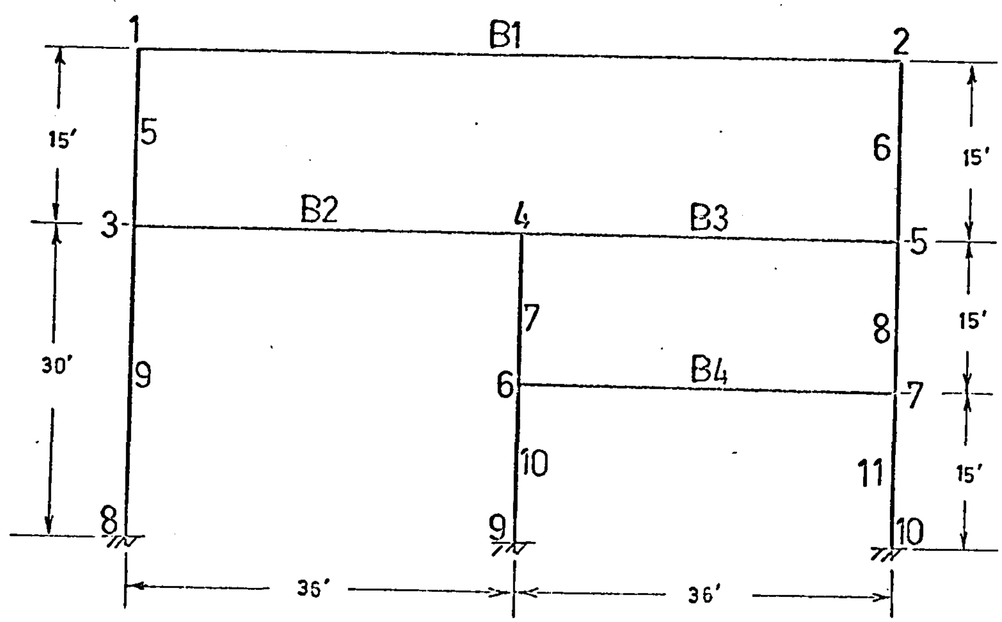


FIG. 5-1a FRAME FOR EXAMPLE No. 1

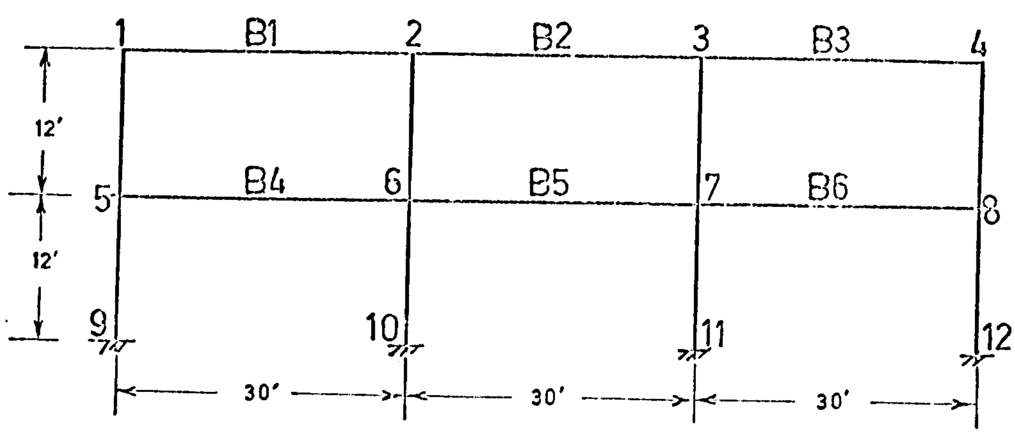


FIG. 5-1b FRAME FOR EXAMPLE No. 2

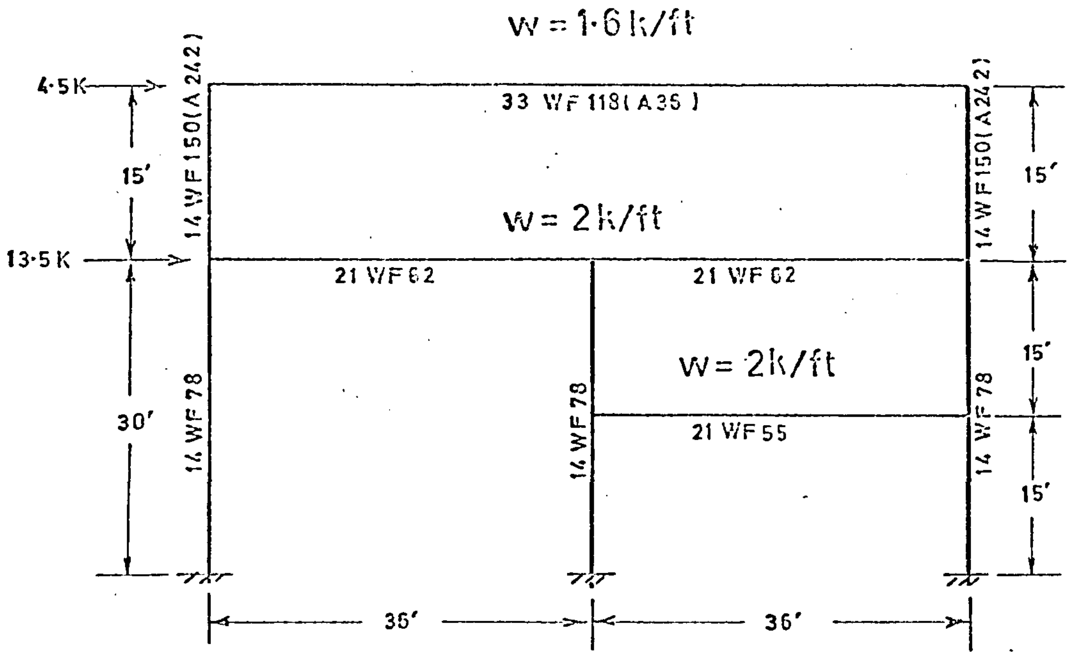


FIG. 5-2a FRAME FOR EXAMPLE No. 1 SHOWING SECTIONS FOR THE NON-COMPOSITE CASE

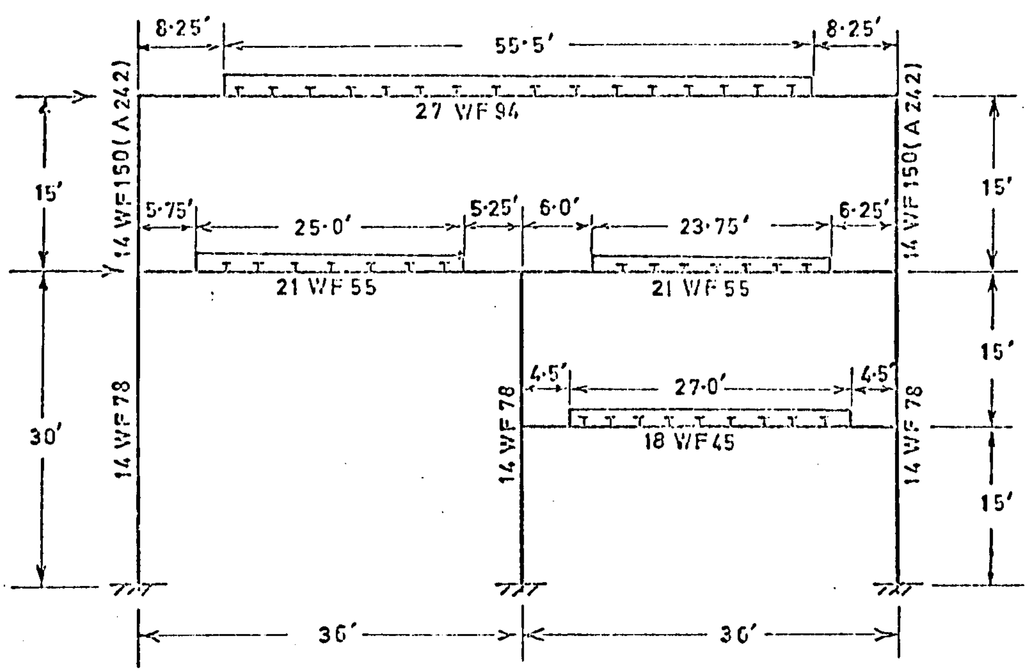


FIG. 5-2b FRAME FOR EXAMPLE No. 1 SHOWING SECTIONS FOR THE COMPOSITE CASE, TOGETHER WITH THE EXACT LENGTH OF COMPOSITE SECTION

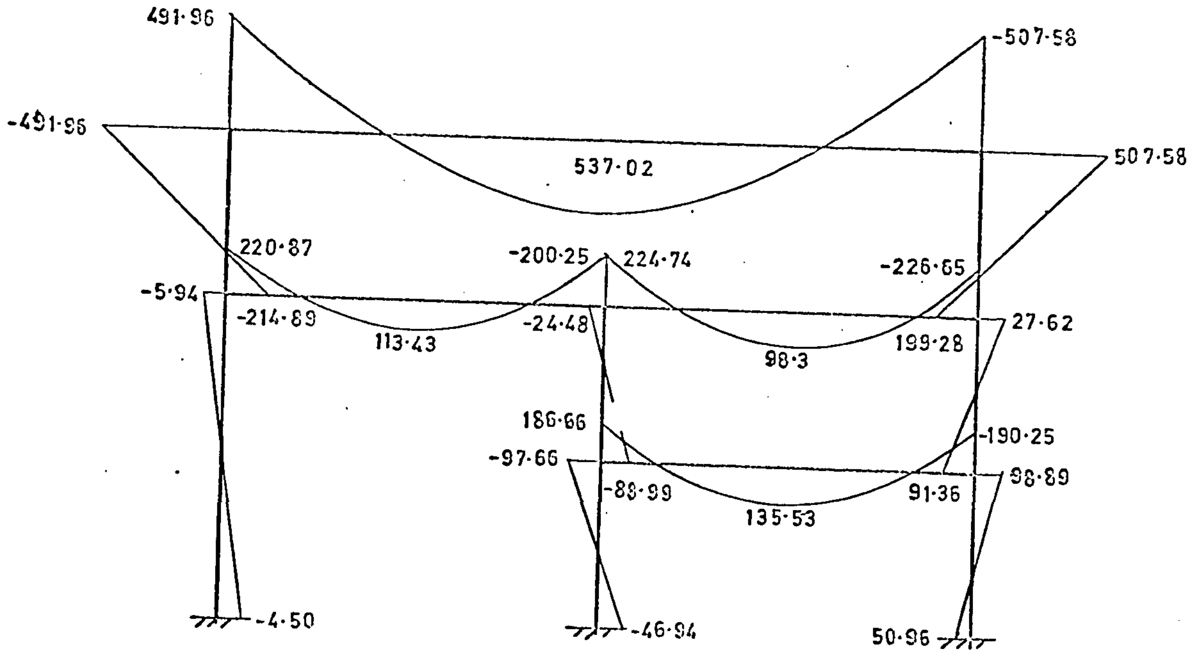


FIG. 5-3a D.L. + L.L. MOMENT DIAGRAM FOR THE FRAME OF EXAMPLE No. 1 FOR THE NON-COMPOSITE CASE

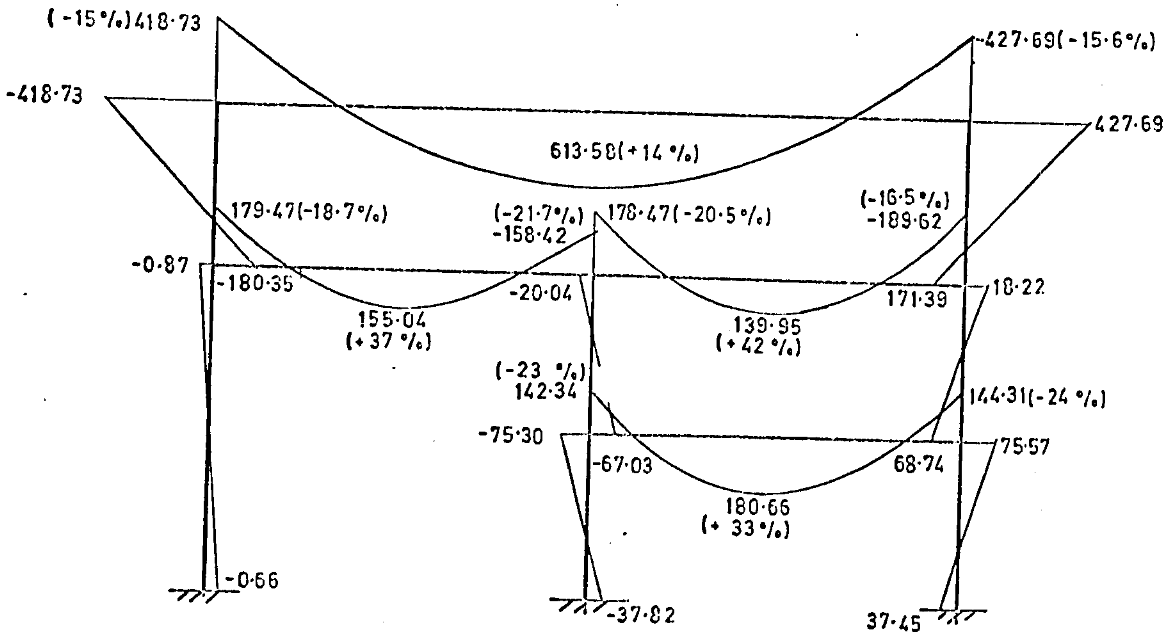


FIG. 5-3b D.L. + L.L. MOMENT DIAGRAM FOR THE FRAME OF EXAMPLE No. 1 FOR THE COMPOSITE CASE

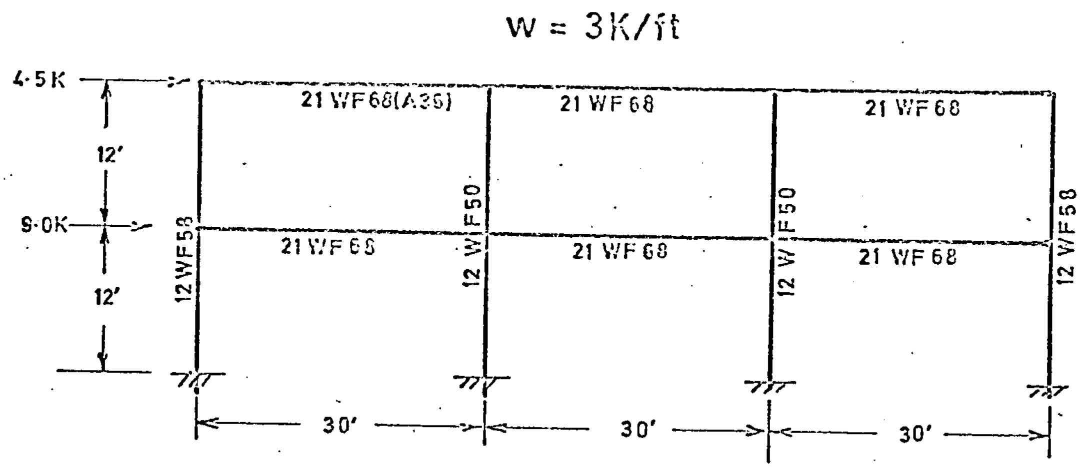


FIG. 5-4a FRAME FOR EXAMPLE No. 2 SHOWING SECTIONS FOR THE NON-COMPOSITE CASE

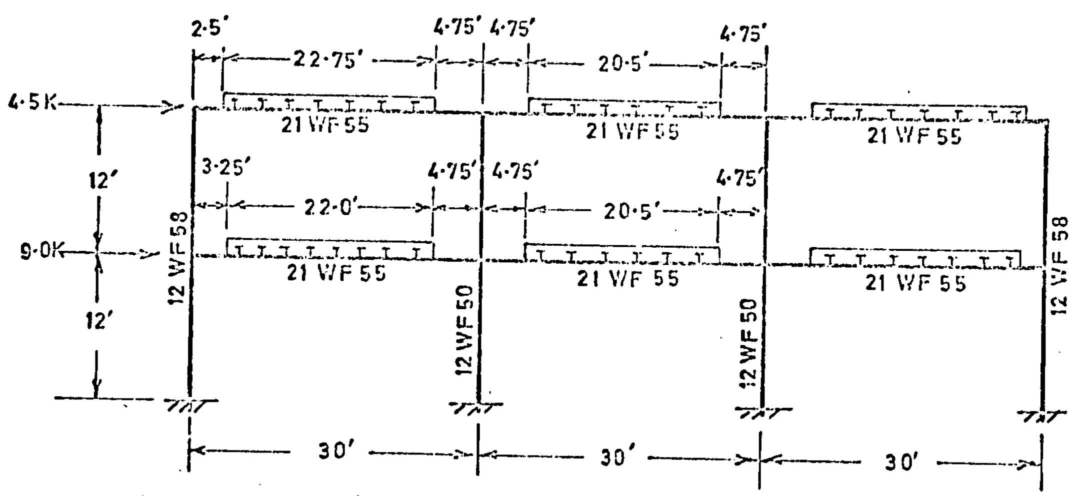


FIG. 5-4b FRAME FOR EXAMPLE No.2 SHOWING SECTIONS FOR THE COMPOSITE CASE, TOGETHER WITH THE EXACT LENGTH OF COMPOSITE SECTION

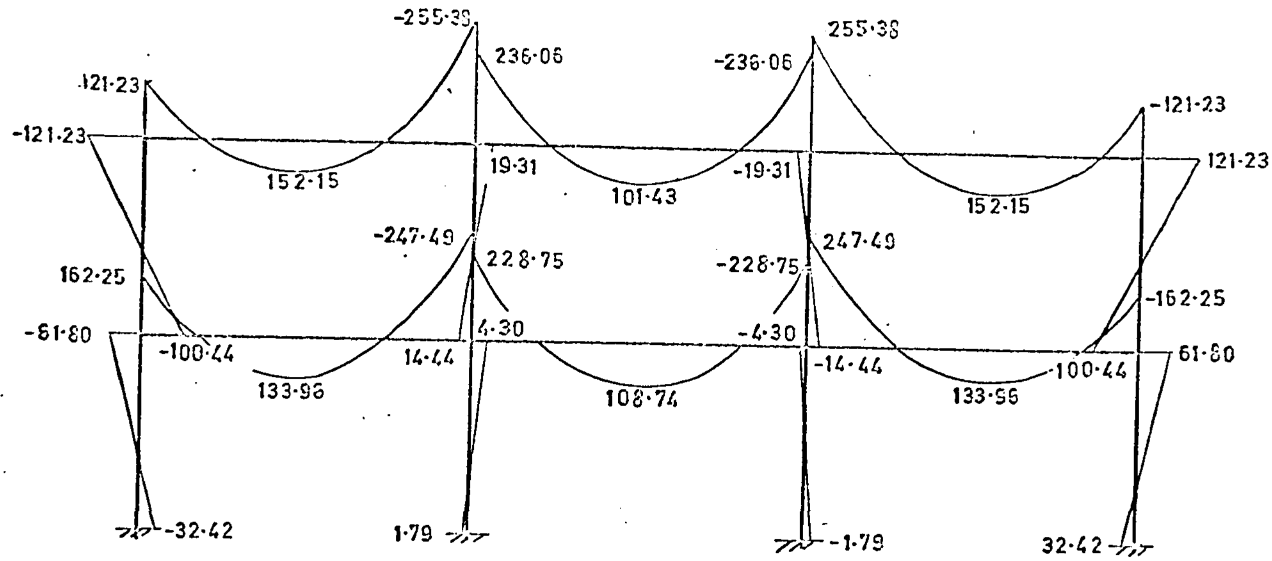


FIG. 5-5a D.L. + L.L. MOMENT DIAGRAM FOR THE FRAME OF EXAMPLE No. 2 FOR THE NON-COMPOSITE CASE

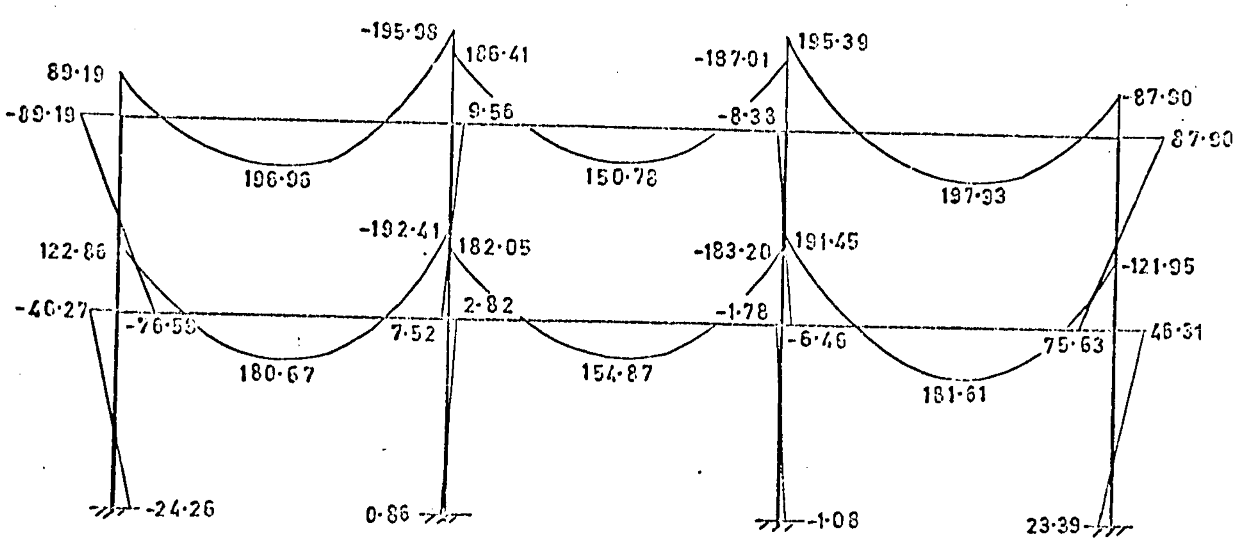


FIG. 5-5b D.L. + L.L. MOMENT DIAGRAM FOR THE FRAME OF EXAMPLE No. 2 FOR THE COMPOSITE CASE

APPENDIX II

TABLES

TABLE (3 - 1)

AISC ALLOWABLE LOAD PER CONNECTOR

Type of Connector	Allowable Horizontal Shear Load (q) (Kips) (Applicable only to concrete made with ASTM C33 aggregates)		
	f'c(kips per square inch)		
	3.0	3.5	4.0
½" diam. * 2" hooked or headed stud	5.1	5.5	5.9
5/8" diam. * 2½" hooked or headed stud	8.0	8.6	9.2
3/4" diam. * 3" hooked or headed stud	11.5	12.5	13.3
7/8" diam. * 3½" hooked or headed stud	15.6	16.8	18.0
3" channel, 4.1 lb.	4.3w	4.7w	5.0w
4" channel, 5.4 lb.	4.6w	5.0w	5.3w
5" channel, 6.7 lb.	4.9w	5.3w	5.6w

TABLE (4 - 1)

COMPARISON OF STEEL BEAM WEIGHT FOR THE CASE OF COMPOSITE AND NON - COMPOSITE DESIGNS

L	W	Non-composite Steel Weight	Composite Steel Wt.	% of Non-Composite Wt.	% of Savings
20	2	16B31	14B26	100 *	16.1
		620	520	83.90	
30	2	21WF55	16WF45	300	18.2
		1650	1350	81.75	
40	2	24WF76	21WF68	320	10.5
		3040	2720	89.50	
50	2	30WF108	27WF84	1200	22.2
		5400	4200	77.80	
20	3	16WF45	16B31	280	31.1
		900	620	68.90	
30	3	24WF68	21WF55	390	19.1
		2040	1650	80.90	
40	3	30WF99	27WF84	600	15.15
		3960	3360	84.90	
50	3	36WF135	33WF118	850	12.6
		6750	5900	87.50	
20	4	18WF50	16WF40	200	20.0
		1000	800	80.0	
30	4	27WF84	24WF68	480	19.0
		2520	2040	81.0	

TABLE (4 - 1) (cont'd)

L	W	Non-composite Steel Weight	Composite Steel Wt.	% of Non-Composite Wt.	% of Savings
40	4	33WF118	30WF108	400	8.5
		4720	4320	91.5	
50	4	36WF160	33WF141	950	11.90
		8000	7050	88.05	

* Top figure refers to the actual difference in lbs. of steel weight.

TABLE (5 - 1)

COMPARISON OF STEEL WEIGHT FOR COMPOSITE AND NON-COMPOSITE DESIGNS FOR EXAMPLES

Example 1

	Non-composite	Composite	Difference	% Saving
Beams	14,944	12,348	2596	17.4
Columns	11,520	11,520	---	
Total Wt.	26,464	23,868	2596	9.8

Example 2

	Non-composite	Composite	Difference	% Saving
Beams	12,240	9,900	2340	19.1
Columns	5,424	5,424		
Total Wt.	17,664	15,324	2340	13.25

APPENDIX III

DESCRIPTION OF COMPUTER PROGRAMS

APPENDIX III

DESCRIPTION OF COMPUTER PROGRAM

As pointed out previously in chapter 4, the special computer system called STRUDL (Structural Design Language) developed at M.I.T., is used to solve all the frames in the theoretical investgations. STRUDL is a command-structured language and a set of programmed procedures for processing the commands or instructions. The language is easily understandable to an engineer, as they consist of phrases in engineering terminology. Words and phrases used for inputing data are so structured that their meanings are self-evident to a structural engineer.

STRUDL is applicable to a wide range of structural problems. It is capable of carrying out calculations for a wide variety of elastic structures, including two and three dimensional bar and plate structures, rigid or pin-jointed frames, subjected to various types of loadings and constraints. Members in the structures may also be non-prismatic and have any orientation with respect to the frame.

Available analytical methods are :

Determinate analysis
Preliminary analysis
Stiffness analysis

The Determinate analysis is the solution by statics alone of a determinate structure - no member properties are required for this analysis. The Preliminary analysis is an approximate analysis procedure based on the user's supplied information such as the magnitude of a reaction or moment to render the problem statically determinate. The Stiffness analysis is a linear, elastic analysis procedure, in which the joint displacements are treated as unknowns and the member properties are required.

All input to STRUDL is made through problem - oriented commands. These are one or more English Keywords which in most cases is self-evident in meaning to the engineer. For input data, the user must specify the type of structure, the geometry of the structure, its topology or connectivety, the member's properties and the loading conditions. Geometry is specified by providing the coordinates of the joints, and the topology by specifying the incidences of each member. The properties of members may be given in a number of ways; by providing the section properties of a prismatic member or of segmented sections of a non-prismatic member. Loads may act on the members and joints and may have any arbitrary orientations. Wind loads are usually considered to act at the joints. Any number of loading conditions may be considered.

Many forms of output are available to the user. The analytic procedures provide member and forces (moments), reactions (shears), axial fprces, joint displacements and member distortions. Force envelopes are also available. Force or stress at a specified section along a member can also be calculated for different loading conditions to determine the maximum stress.

STRUDL is also structured in such a way as to be compatible with the logical sequence of design operations. The sequence of operations can be assumed to be as follows :

1) Data input, and modification
2) Analytic procedures
3) Data subsetting
4) Data output

The minimum specifications necessay for analysis and solutions by a computer would consists of the following :

STRUDL ' job id ' ' title '	(initialization)
TYPE ______	(structural type)
JOINT COORDINATES	(geometry)
MEMBER INCIDENCES	(topology)
MEMBER PROPERTIES ______	(physical properties)
LOADING 'name' 'title'	(loading description)
LOADING LIST _____	(pertinent loading)
______ ANALYSIS	(analysis)
LIST _______	(output)

An example of such a minimum specification for computer run is given below with the frame of example 1 as the example. (See Fig. 5-1a)

STRUDL ' job. id ' ' title '

All STRUDL programs must be preceded by this command. This command prepares the system to accept the input data for the problem. 'job. id ' is the problem or job identifier chosen by the user and is limited to 8 characters. ' title ' is an arbitrary problem title (limit to 64 characters) and is printed with the output for user's reference.

TYPE PLANE FRAME

The frame of example 1 and 2 is classified as slane frame, snch as plane Grid, plane Truss, Space Truss or Space Frame.

JOINT COORDINATES

This command specifies the geometry of the structure in a Cartosian Coordinate System. Lables for coordinate directions need not be given if the coordinate numbers are given in the same oder as x, y, z. Also if all the coords. are zero, they may be omitted entirely. The word SUPPORT may be abbreviated to S to describe the end condition. Thus for the frame of Fig. 5-1a, the joint coords. are :

1	0.0	45.	
2	72.0	45.	
....			
8	S		
9	36.	0.	S
10	72.	0.	S

MEMBER INCIDENCES

{ i } Starting joint end joint

Member incidences define the connectivity- each member in a structure is connected to 2 joints. { i } is the identifier for the member and it can be a integer or a alphanumeric. For convenience, the integer is used for example 1, thus the member incidences are :

1	1	2
2	3	4
3	4	5
4	6	7
....		
....		
11	7	10

MEMBER PROPERTIES (PRISMATIC or VARIABLE)

Depending on whether a member is uniform or non-uniform, the word prismatic or variable is used : prismatic for uniform and variable for non-uniform. A number of different format is available for specifying the member properties. For the problem of ex. 1 the following format is used :

For uniform member,

members' number AX ____ IZ ____ SZ ____

For non-uniform member,

List of member

SEGMENT ___ AX ___ IZ ___ L ___

LOADING ' name ' ' title '

Loading as its name implies, specifies the types of loading that are applied to the structure. 'name ' is an identifier and can be an integer or a alphanumeric, ' title ' is an arbitrary name which is printed with the output for more meaningful identification of results. As an example, the loading for exam. 1 can be specified as :

LOADING 2 ' DEAD LOAD + LIVE LOAD '

MEMBERS 2 TO 4 LOADS FORCE y UNFORM - 2.0

The above indicates that members 2 to 4 is under a uniform load of 2 kips per ft. The other terms are self-explainatory. For wind loading, it can be specified as,

LOADING 3 'WIND LOAD '

JOINT 1 LOADS FORCE X 4.5

The above indicates a load of 4.5 kip acting at joint 1.

LOADING LIST (n)

Usually there are more than one loading invalved, and the loading list can be used to select a particular loading for analysis.

For the analysis, 3 types are available, but for both example, only the Stiffness analysis is used.

For output the Command LIST will automatically print what is requested. An example of an output statement for the mements and shears in a frame can be written as,

LIST FORCES, REACTIONS

The format of all outputs in STRUDL is fixed, and the user has no contol over that,

For a detail treatment of this STRUDL program, References 43 to 45 should be consulted.

A sample STRUDL program on example 1 is given on the next page together with sample output.

All the ICES - STRUDL programs in this investigation are implemented on an IBM System/360, model 65 computer.

RESULTS OF COMPUTER PROGRAMS

FOR

EXAMPLE No. 1

NOTE : EXACT SOLUTIONS are given first, together with Moments for (DL+LL+WL), and also a force envelope for Beam B1

```
STRUDL ' M. ENG. _ EX. 1 '  ' 3 STORY _ 2 BAY _ 36 FT SPAN '

                    *******************************************
                    *                                         *
                    *              ICES STRUDL-I              *
                    *      THE STRUCTURAL DESIGN LANGUAGE     *
                    *                                         *
                    *   CIVIL ENGINEERING SYSTEMS LABORATORY  *
                    *  MASSACHUSETTS INSTITUTE OF TECHNOLOGY  *
                    *        CAMBRIDGE, MASSACHUSETTS         *
                    *           JUNE, 1968  MOD 2             *
                    *                                         *
                    *******************************************

TYPE PLANE FRAME

UNITS FEET KIPS

JOINT COORDS

  1  0.  45.

  2  72.0  45.

  3  0.  30.

  4  36.  30.

  5  72.  30.

  6  36.  15.

  7  72.  15.

  8  S

  9  36.  0.  S

  10  72.  0.  S
```

```
MEMBER INCIDENCES
 1   1   2
 2   3   4
 3   4   5
 4   6   7
 5   1   3
 6   2   5
 7   4   6
 8   5   7
 9   3   8
10   6   9
11   7  10
UNITS INCHES
MEMBER PROPERTIES PRISM
$   COLUMNS 7 TO 11   :  14WF78(A36), P=400K(15), P=236K(20)
  7 TO 11  AX 22.94  IZ 851.2
$   COLUMNS 5 & 6   :  14WF150(A242_46KSI), P = 1029K(15')
  5, 6  AX 44.08  IZ 1786.9
```

```
MEMBER PROPERTIES VARIABLE
$      3 STORY _ 2 BAY( 36FT. SPAN ),( 15' _ 20' COLS. )  W=2K/FT
$    BEAM FOR PRISM. SECTION   :  33WF118(A36),   S=358.3
$      SR=1.115
$    BEAM WITH  COMP. SEC.   :  27WF94(A36),   S=242.8
  1
SEG 1 AND 3  AX 27.65  IZ 3266.7  L 99.0
SEG 2  AX 72.15  IZ 7699.3  L 666.0
$    BEAM FOR PRISM. SECTION   :  21WF62(A36),  S=126.4
$    BEAMS WITH COMP. SEC.   :  21WF55(A36),   S=109.7
$      SR=0.827  &  SR3 = 0.986
  2
SEG 1  AX 16.18  IZ 1140.7  L 69.0
SEG 2  AX 60.68  IZ 3208.0  L 300.0
SEG 3  AX 16.18  IZ 1140.7  L 63.0
  3
SEG 1  AX 16.18  IZ 1140.7  L 72.0
SEG 2  AX 60.68  IZ 3208.0  L 285.0
SEG 3  AX 16.18  IZ 1140.7  L 75.0
$    BEAM FOR PRISM. SECTION   :  21WF55(A36),   S=109.7
$    BEAM # 4  :  18WF45(A36),   S=78.9
  4
SEG 1 AND 3  AX 13.24  IZ 704.5  L 54.0
SEG 2  AX 57.74  IZ 2130.4  L 324.0
CONSTANTS E 29000. ALL
```

```
UNITS FEET
$   SPECIFY LOADING CONDITIONS
LOADING 1 ' DEAD LOAD '
MEMBERS 1 TO 4  LOADS FORCE Y UNIFORM -1.0
LOADING 2  ' WIND LOAD '
  JOINT 1  LOADS FORCE X 4.5
  JOINT  3  LOADS FORCE X 13.5
LOADING 3  ' D.L. + L.L. '
  MEMBER 1  LOADS FORCE Y UNIFORM -1.6
  MEMBERS 2 TO 4  LOADS FORCE Y UNIFORM -2.0
LOADING 4  ' D.L. + L.L. + W.L. '
  MEMBER 1 LOADS FORCE Y UNIFORM -1.6
  MEMBERS 2 TO 4  LOADS FORCE Y UNIFORM -2.0
  JOINT 1  LOADS FORCE X 4.5
  JOINT  3  LOADS FORCE X 13.5
PRINT STRUCTURAL DATA ALL
```

```
***********************************************
*   PROBLEM DATA FROM INTERNAL STORAGE   *
***********************************************
```

JOB ID M. ENG. NAME - 3 STORY _ 2 BAY _ 36 FT SPAN

ACTIVE	UNITS	LENGTH	WEIGHT	ANGLE	TEMPERATURE	TIME
		FEET	KIP	RAD	DEGF	SEC

********** STRUCTURAL DATA **********

ACTIVE STRUCTURE TYPE - PLANE FRAME

ACTIVE COORDINATE AXES X Y

JOINT COORDINATES--/---STATUS---/

JOINT	X	Y	Z	CONDITION	STATUS
1	0.0	45.000	0.0		ACTIVE
2	72.000	45.000	0.0		ACTIVE
3	0.0	30.000	0.0		ACTIVE
4	36.000	30.000	0.0		ACTIVE
5	72.000	30.000	0.0		ACTIVE
6	36.000	15.000	0.0		ACTIVE
7	72.000	15.000	0.0		ACTIVE
8	0.0	0.0	0.0	SUPPORT	ACTIVE
9	36.000	0.0	0.0	SUPPORT	ACTIVE
10	72.000	0.0	0.0	SUPPORT	ACTIVE

MEMBER INCIDENCES-----------/ LENGTH------/ STATUS---/

MEMBER	START	END	LOCAL COORD.	STATUS
1	1	2	72.000	ACTIVE
2	3	4	36.000	ACTIVE
3	4	5	36.000	ACTIVE
4	6	7	36.000	ACTIVE
5	1	3	15.000	ACTIVE
6	2	5	15.000	ACTIVE
7	4	6	15.000	ACTIVE
8	5	7	15.000	ACTIVE
9	3	8	30.000	ACTIVE
10	6	9	15.000	ACTIVE
11	7	10	15.000	ACTIVE

MEMBER PROPERTIES

MEMBER	TYPE/SEG	SEG.L	COMP	AX	AY	AZ	IZ
1	VARIABLE						
	1	8.250	L	0.192	0.0	0.0	0.158
	2	55.500	L	0.501	0.0	0.0	0.371
	3	8.250	L	0.192	0.0	0.0	0.158
2	VARIABLE						
	1	5.750	L	0.112	0.0	0.0	0.055
	2	25.000	L	0.421	0.0	0.0	0.155
	3	5.250	L	0.112	0.0	0.0	0.055
3	VARIABLE						
	1	6.000	L	0.112	0.0	0.0	0.055
	2	23.750	L	0.421	0.0	0.0	0.155
	3	6.250	L	0.112	0.0	0.0	0.055
4	VARIABLE						
	1	4.500	L	0.092	0.0	0.0	0.034
	2	27.000	L	0.401	0.0	0.0	0.103
	3	4.500	L	0.092	0.0	0.0	0.034
5	PRISMATIC			0.306	0.0	0.0	0.086
6	PRISMATIC			0.306	0.0	0.0	0.086
7	PRISMATIC			0.159	0.0	0.0	0.041
8	PRISMATIC			0.159	0.0	0.0	0.041
9	PRISMATIC			0.159	0.0	0.0	0.041
10	PRISMATIC			0.159	0.0	0.0	0.041
11	PRISMATIC			0.159	0.0	0.0	0.041

MEMBER DEPTH

MEMBER	Y DEPTH	Z DEPTH	Y CENTROID	Z CENTROID

MEMBER CONSTANTS

CONSTANT	STANDARD VALUE	DOMAIN,	VALUE	MEMBER LIST
E	4176000.000000	ALL		
G	0.0	ALL		
DENSITY	1.728000	ALL		
CTE	1.000000	ALL		
BETA	0.0	ALL		

****
* END OF DATA FROM INTERNAL STORAGE *
****

LOADING LIST 3

STIFFNESS ANALYSIS

```
TIME FOR MODULE STCOMP FOR   11 MEMBERS=     0.73 SECONDS
TIME FOR MODULE STSTDP FOR   11 MEMBERS=     0.70 SECONDS
TIME FOR MODULE STMDDP FOR    4 LOADS=       2.83 SECONDS
TIME FOR MODULE STASDP FOR   11 MEMBERS=     0.55 SECONDS
TIME FOR MODULE STJPDP FOR   10 JOINTS=     0.05 SECONDS
TIME FOR MODULE STSLDP FOR NSOL=     7 IS    1.13 SECONDS
TIME FOR MODULE STD1BS FOR   10 JOINTS EQUALS     0.05 SECONDS
TIME FOR MODULE STD3BS FOR   10 JOINTS EQUALS     0.22 SECONDS
TIME FOR MODULE STBKSB FOR   11 MEMBERS=    12.35 SECONDS
```

LIST LOADS, FORCES, REACTIONS, DISPLACEMENTS

```
*****************************
*RESULTS OF LATEST ANALYSES*
*****************************
```

PROBLEM - M. ENG. TITLE - 3 STORY _ 2 BAY _ 36 FT SPAN

ACTIVE UNITS FEET KIP RAD DEGF SEC

ACTIVE STRUCTURE TYPE PLANE FRAME

ACTIVE COORDINATE AXES X Y

LOADING - 3 D.L. + L.L.

MEMBER FORCES

MEMBER	JOINT	/------ FORCE ------ AXIAL	SHEAR Y	------ MOMENT -----/ BENDING Z
1	1	39.9393158	57.4754639	418.7341309
1	2	-39.9393158	57.7243958	-427.6953125
2	3	-39.9464569	36.5848083	179.4794922
2	4	39.9464569	35.4151001	-158.4239960
3	4	-34.1408844	35.6901093	178.4707489
3	5	34.1408844	36.3097992	-189.6244965
4	6	1.7368813	35.9451599	142.3464966
4	7	-1.7368813	36.0547638	-144.3193207
5	1	57.4754639	-39.9393158	-418.7341309
5	3	-57.4754639	39.9393158	-180.3555756
6	2	57.7243958	39.9393158	427.6953125
6	5	-57.7243958	-39.9393158	171.3944855
7	4	71.1051788	-5.8055735	-20.0467682
7	6	-71.1051788	5.8055735	-67.0368500
8	5	94.0341797	5.7984333	18.2299957
8	7	-94.0341797	-5.7984333	68.7465210
9	3	94.0602417	0.0071403	0.8760908
9	8	-94.0602417	-0.0071403	-0.6618818
10	6	107.0503693	-7.5424566	-75.3096466
10	9	-107.0503693	7.5424566	-37.8271942
11	7	130.0889282	7.5353155	75.5728149
11	10	-130.0889282	-7.5353155	37.4569397

RESULTANT JOINT LOADS - SUPPORTS

JOINT	X FORCE	Y FORCE	Z MOMENT
	/---------- FORCE		----------/
8	-0.0071403	94.0602417	-0.6618818
9	7.5424566	107.0503693	-37.8271942
10	-7.5353155	130.0889282	37.4569397

RESULTANT JOINT LOADS - FREE JOINTS

JOINT	X FORCE	Y FORCE	Z MOMENT
	/---------- FORCE		----------/
1	0.0000000	0.0000000	0.0000000
2	-0.0000000	0.0000000	-0.0000000
3	-0.0000000	0.0000000	-0.0000000
4	-0.0000000	0.0000000	-0.0000000
5	0.0000000	-0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	-0.0000000

RESULTANT JOINT DISPLACEMENTS - SUPPORTS

JOINT	X DISP.	Y DISP.	Z ROT.
	/---------- DISPLACEMENT		----------/
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0

RESULTANT JOINT DISPLACEMENTS - FREE JOINTS

JOINT	X DISP.	Y DISP.	Z ROT.
	/---------- DISPLACEMENT		----------/
1	0.0021028	-0.0049151	-0.0048335
2	0.0002215	-0.0057308	0.0047991
3	-0.0019249	-0.0042417	0.0001346
4	-0.0004210	-0.0040179	0.0004160
5	0.0009311	-0.0050534	-0.0005425
6	-0.0000754	-0.0024137	-0.0016399
7	-0.0001431	-0.0029332	0.0016676

JOINT	X DISP.	Y DISP.	Z ROT.
1	0.0021028	-0.0049161	-0.0048335
2	0.0002215	-0.0057308	0.0047991
3	-0.0019249	-0.0042417	0.0001346
4	-0.0004210	-0.0040170	0.0004160
5	0.0009311	-0.0050534	-0.0005425
6	-0.0000754	-0.0024137	-0.0016399
7	-0.0001441	-0.0029332	0.0016676

LOADING - 4 D.L. + L.L. + W.L.

MEMBER FORCES

MEMBER	JOINT	AXIAL	SHEAR Y	BENDING Z
1	1	41.9665527	56.5171661	384.1315918
1	2	-41.9665527	58.6826935	-462.0895996
2	3	-27.5289154	34.1058807	130.1859131
2	4	27.5289154	37.8940430	-198.3720703
3	4	-29.9931030	33.1606903	139.0599060
3	5	29.9931030	38.8392334	-241.2729950
4	6	3.0630112	32.0295410	70.8450317
4	7	-3.0630112	39.9703674	-213.7796631
5	1	56.5171661	-37.4665527	-384.1315918
5	3	-56.5171661	37.4665527	-177.8668213
6	2	58.6826935	41.9665527	462.0895996
6	5	-58.6826935	-41.9665527	167.4085693
7	4	71.0546875	2.4641733	59.3121338
7	6	-71.0546875	-2.4641733	-22.3495178
8	5	97.5219269	11.9734564	73.8644409
8	7	-97.5219269	-11.9734564	105.7374115
9	3	90.6230621	3.5623655	47.6808319
9	8	-90.6230621	-3.5623655	59.1901398
10	6	103.0842438	-0.5988379	-48.4954987
10	9	-103.0842438	0.5988379	39.5129242
11	7	137.4923096	15.0364685	108.0422363
11	10	-137.4923096	-15.0364685	117.5047455

LIST FCRCE ENVELOPE MEMBER 1 SECTION DS 0. 2.0

****
RESULTS OF LATEST ANALYSES
****

PROBLEM - M. ENG. TITLE - 3 STORY _ 2 BAY _ 36 FT SPAN

ACTIVE UNITS FEET KIP RAD DEGF SEC

ACTIVE STRUCTURE TYPE PLANE FRAME

ACTIVE COORDINATE AXES X Y

INTERNAL MEMBER RESULTS

MEMBER FORCE ENVELOPE

MEMBER 1

DISTANCE FROM START	/----- FORCE ----- AXIAL	Y SHEAR	----- MOMENT ----- Z BENDING
0.0	-39.9393158	-57.4754639	-418.7329102
	-39.9393158	-57.4754639	-418.7329102
2.000	-39.9393158	-54.2754669	-306.9826660
	-39.9393158	-54.2754669	-306.9826660
4.000	-39.9393158	-51.0754700	-201.6306458
	-39.9393158	-51.0754700	-201.6306458
6.000	-39.9393158	-47.8754578	-102.6799927
	-39.9393158	-47.8754578	-102.6799927
8.000	-39.9393158	-44.6755219	-10.1293316
	-39.9393158	-44.6755219	-10.1293316
10.000	-39.9393158	-41.4755249	76.0213165
	-39.9393158	-41.4755249	76.0213165
12.000	-39.9393158	-38.2755280	155.7719879

	-39.9393158	-35.0755310	229.1239777
16.000	-39.9393158	-31.8755341	296.0744629
	-39.9393158	-31.8755341	296.0744629
18.000	-39.9393158	-28.6755219	356.6252441
	-39.9393158	-28.6755219	356.6252441
20.000	-39.9393158	-25.4755249	410.7756348
	-39.9393158	-25.4755249	410.7756348
22.000	-39.9393158	-22.2755280	458.5263672
	-39.9393158	-22.2755280	458.5263672
24.000	-39.9393158	-19.0755310	499.8771973
	-39.9393158	-19.0755310	499.8771973
26.000	-39.9393158	-15.8755350	534.8291016
	-39.9393158	-15.8755350	534.8291016
28.000	-39.9393158	-12.6755342	563.3798828
	-39.9393158	-12.6755342	563.3798828
30.000	-39.9393158	-9.4755344	585.5317383
	-39.9393158	-9.4755344	585.5317383
32.000	-39.9393158	-6.2755346	601.2805176
	-39.9393158	-6.2755346	601.2805176
34.000	-39.9393158	-3.0755348	610.6311035
	-39.9393158	-3.0755348	610.6311035
36.000	-39.9393158	0.1244648	613.5832520
	-39.9393158	0.1244648	613.5832520
38.000	-39.9393158	3.3244648	610.1337891
	-39.9393158	3.3244648	610.1337891
40.000	-39.9393158	6.5244331	600.2856445
	-39.9393158	6.5244331	600.2856445
42.000	-39.9393158	9.7244329	584.0371094
	-39.9393158	9.7244329	584.0371094
44.000	-39.9393158	12.9244328	561.3881836
	-39.9393158	12.9244328	561.3881836

46.000	-39.9393158	16.1244202	532.3391113
	-39.9393158	16.1244202	532.3391113
48.000	-39.9393158	19.3244324	496.8906250
	-39.9393158	19.3244324	496.8906250
50.000	-39.9393158	22.5244293	455.0415039
	-39.9393158	22.5244293	455.0415039
52.000	-39.9393158	25.7244110	406.7929687
	-39.9393158	25.7244110	406.7929687
54.000	-39.9393158	28.9244080	352.1437988
	-39.9393158	28.9244080	352.1437988
56.000	-39.9393158	32.1244049	291.0952148
	-39.9393158	32.1244049	291.0952148
58.000	-39.9393158	35.3244171	223.6464844
	-39.9393158	35.3244171	223.6464844
60.000	-39.9393158	38.5244141	149.7975616
	-39.9393158	38.5244141	149.7975616
62.000	-39.9393158	41.7244110	69.5487213
	-39.9393158	41.7244110	69.5487213
64.000	-39.9393158	44.9244080	-17.1000824
	-39.9393158	44.9244080	-17.1000824
66.000	-39.9393158	48.1244049	-110.1488190
	-39.9393158	48.1244049	-110.1488190
68.000	-39.9393158	51.3244019	-209.5975647
	-39.9393158	51.3244019	-209.5975647
70.000	-39.9393158	54.5243988	-315.4462891
	-39.9393158	54.5243988	-315.4462891
72.000	-39.9393158	57.7243958	-427.6953125
	-39.9393158	57.7243958	-427.6953125

EXAMPLE No. 1

Results of FIRST APPROXIMATION for (DL+LL) Moments

```
MEMBER PROPERTIES PRISM
$    COLUMNS 7 TO 11   :  14WF78(A36), P=400K(15), P=236K(20)
  7 TO 11  AX 22.94  IZ 851.2
$    COLUMNS 5 & 6   :  14WF150(A242_46KSI), P = 1029K(15')
  5, 6  AX 44.08  IZ 1786.9
MEMBER PROPERTIES VARIABLE
$       3 STORY _ 2 BAY( 36FT. SPAN ),( 15' _ 20' COLS. )  W=2K/FT
$    BEAM FOR PRISM. SECTION  :  33WF118(A36),  S=358.3
$       SR=1.115
$       FIRST APPROXIMATION
$    BEAM WITH COMP. SEC.   :  27WF94(A36),  S=242.8
  1
SEG 1 AND 3  AX 27.65  IZ 3266.7  L 104.0
SEG 2  AX 72.15  IZ 7699.3  L 656.0
$    BEAM FOR PRISM. SECTION  :  21WF62(A36),  S=126.4
$    BEAM WITH COMP. SEC.  :  21WF55(A36),  S=109.7
$       SR=0.827  &  SR3 = 0.986
  2
SEG 1  AX 16.18  IZ 1140.7  L 60.0
SEG 2  AX 60.68  IZ 3208.0  L 301.0
SEG 3  AX 16.18  IZ 1140.7  L 71.0
  3
SEG 1  AX 16.18  IZ 1140.7  L 71.0
SEG 2  AX 60.68  IZ 3208.0  L 299.0
SEG 3  AX 16.18  IZ 1140.7  L 62.0
$    BEAM FOR PRISM. SECTION  :  21WF55(A36),  S=109.7
$    BEAM B4 WITH COMP. SEC.  :  18WF45(A36),  S=78.9
  4
SEG 1 AND 3  AX 13.24  IZ 704.5  L 61.0
SEG 2  AX 57.74  IZ 2130.4  L 310.0
CONSTANTS E 29000. ALL
```

****
RESULTS OF LATEST ANALYSES
****

PROBLEM - M. ENG. TITLE - 3 STORY _ 2 BAY _ 36 FT SPAN

ACTIVE UNITS FEET KIP RAD DEGF SEC

ACTIVE STRUCTURE TYPE PLANE FRAME

ACTIVE COORDINATE AXES X Y

LOADING - 3 D.L. + L.L.

MEMBER FORCES

MEMBER	JOINT	/------------------- FORCE ------------------------/		
		AXIAL	SHEAR Y	BENDING Z
1	1	39.9487762	57.4665527	418.7080078
1	2	-39.9487762	57.7332916	-428.3103027
2	3	-39.9414368	36.5603790	179.8940735
2	4	39.9414368	35.4395447	-159.7184143
3	4	-34.1838694	35.7423553	179.1811523
3	5	34.1838694	36.2575684	-188.4543304
4	6	1.8298130	35.9260559	142.5598297
4	7	-1.8298130	36.0738525	-145.2194061
5	1	57.4665527	-39.9487762	-418.7080078
5	3	-57.4665527	39.9487762	-180.5234985
6	2	57.7332916	39.9487762	428.3103027
6	5	-57.7332916	-39.9487762	170.9213257
7	4	71.1818695	-5.7575741	-19.4627838
7	6	-71.1818695	5.7575741	-66.9008331
8	5	93.9908600	5.7649059	17.5330048
8	7	-93.9908600	-5.7649059	68.9405823
9	3	94.0269318	-0.0073315	0.6293717
9	8	-94.0269318	0.0073315	-0.8493174
10	6	107.1079254	-7.5873861	-75.6590424
10	9	-107.1079254	7.5873861	-38.1517792
11	7	130.0647430	7.5947180	76.2789001
11	10	-130.0647430	-7.5947180	37.6419067

RESULTS OF COMPUTER PROGRAMS

FOR

EXAMPLE No. 2

I Section Properties and Results for FIRST APPROXIMATION

II Section Properties and Results for EXACT SOLUTIONS

I

Section Properties and Results for

FIRST APPROXIMATION

```
$       FIRST APPROXIMATION
  1
SEG 1  AX 16.18  IZ 1140.7  L 28.0
SEG 2  AX 60.68  IZ 3208.0  L 273.5
SEG 3  AX 16.18  IZ 1140.7  L 58.5
  3
SEG 1  AX 16.18  IZ 1140.7  L 58.5
SEG 2  AX 60.68  IZ 3208.0  L 273.5
SEG 3  AX 16.18  IZ 1140.7  L 28.0
  4
SEG 1  AX 16.18  IZ 1140.7  L 40.0
SEG 2  AX 60.68  IZ 3208.0  L 261.5
SEG 3  AX 16.18  IZ 1140.7  L 58.5
  6
SEG 1  AX 16.18  IZ 1140.7  L 58.5
SEG 2  AX 60.68  IZ 3208.0  L 261.5
SEG 3  AX 16.18  IZ 1140.7  L 40.0
  2  5
SEG 1 AND 3  AX 16.18  IZ 1140.7  L 56.0
SEG 2  AX 60.68  IZ 3208.0  L 248.0
CONSTANTS E 29000. ALL
```

LOADING - 4 D.L. + L.L.

MEMBER FORCES

MEMBER	JOINT	FORCE AXIAL	FORCE SHEAR Y	FORCE BENDING Z
1	1	13.8136129	41.4298096	89.1765747
1	2	-13.8136129	48.5700531	-196.2788239
2	2	12.3733749	44.9912720	186.6124115
2	3	-12.3733749	45.0086517	-186.8726654
3	3	13.6249723	48.5834503	195.3469086
3	4	-13.6249723	41.4163971	-87.8398285
4	5	-7.9325304	42.6722717	122.8736572
4	6	7.9325304	47.3276367	-192.7039032
5	6	-6.8061323	44.9734802	182.2070770
5	7	6.8061323	45.0264435	-183.0010681
6	7	-7.8092337	47.3122101	191.4018250
6	8	7.8092337	42.6877136	-122.0344086
7	1	41.4298096	-13.8136129	-89.1765747
7	5	-41.4298096	13.8136129	-76.5868073
8	2	93.5613098	1.4402399	9.6664314
8	6	-93.5613098	-1.4402399	7.6164474
9	3	93.5920563	-1.2515974	-8.4742546
9	7	-93.5920563	1.2515974	-6.5449162
10	4	41.4163971	13.6249723	87.8398285
10	8	-41.4163971	-13.6249723	75.6597595
11	5	84.1020508	-5.8810778	-46.2868958
11	9	-84.1020508	5.8810778	-24.2860718
12	6	185.8624268	0.3138391	2.8804150
12	10	-185.8624268	-0.3138391	0.8856546
13	7	185.9307404	-0.2484941	-1.8558426
13	11	-185.9307404	0.2484941	-1.1260872
14	8	84.1041107	5.8157339	46.3746490
14	12	-84.1041107	-5.8157339	23.4141693

II

Section Properties and Results for EXACT SOLUTIONS

```
MEMBER PROPERTIES PRISM
$   COLUMNS 7, 10, 11, 14   :  12WF58(A36), P=302K(12'), BX=0.218, S=78.1
  7, 10, 11, 14  AX 17.06  IZ 476.1
$   COLUMNS 8, 9, 12, 13   :  12WF50(A36), P=236K(12'), BX=0.227,  S=64.7
  8, 9, 12, 13  AX 14.71  IZ 394.5
$     2 STORY _ 3 BAY ( 30' SPAN )( 12' _ 12' COLS )  W=3K/FT( LL/DL=2 )
$   BEAMS FOR PRISM. SECTIONS  :  21WF68(A36),   S=139.9
$   BEAMS WITH COMP. SECTION  :  21WF55(A36),   S=109.7
$    SR1  =  0.371
$   SR2  =  0.3075
MEMBER PROPERTIES VARIABLE
  1
SEG 1  AX 16.18  IZ 1140.7  L 30.0
SEG 2  AX 60.68  IZ 3208.0  L 273.0
SEG 3  AX 16.18  IZ 1140.7  L 57.0
  3
SEG 1  AX 16.18  IZ 1140.7  L 57.0
SEG 2  AX 60.68  IZ 3208.0  L 273.0
SEG 3  AX 16.18  IZ 1140.7  L 30.0
  2  5
SEG 1  AX 16.18  IZ 1140.7  L 57.0
SEG 2  AX 60.68  IZ 3208.0  L 246.0
SEG 3  AX 16.18  IZ 1140.7  L 57.0
  4
SEG 1  AX 16.18  IZ 1140.7  L 39.0
SEG 2  AX 60.68  IZ 3208.0  L 264.0
SEG 3  AX 16.18  IZ 1140.7  L 57.0
  6
SEG 1  AX 16.18  IZ 1140.7  L 57.0
SEG 2  AX 60.68  IZ 3208.0  L 264.0
SEG 3  AX 16.18  IZ 1140.7  L 39.0
CONSTANTS E 29000. ALL
```

LOADING - 4 D.L. + L.L.

MEMBER FORCES

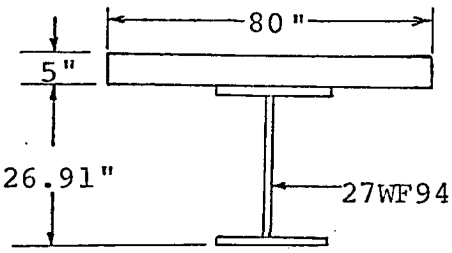
MEMBER	JOINT	FORCE AXIAL	FORCE SHEAR Y	FORCE BENDING Z
1	1	13.8160267	41.4401550	89.1950684
1	2	-13.8160267	48.5597076	-195.9873199
2	2	12.3913670	44.9801025	186.4191589
2	3	-12.3913670	45.0198059	-187.0143280
3	3	13.6287460	48.5828705	195.3967438
3	4	-13.6287460	41.4169922	-87.9073181
4	5	-7.9379492	42.6817780	122.8696594
4	6	7.9379492	47.3180847	-192.4134979
5	6	-6.8214445	44.9618073	182.0560760
5	7	6.8214445	45.0381165	-183.2004089
6	7	-7.8196249	47.3166656	191.4557495
6	8	7.8196249	42.6831818	-121.9528198
7	1	41.4401550	-13.8160267	-89.1950684
7	5	-41.4401550	13.8160267	-76.5971985
8	2	93.5398102	1.4246569	9.5681295
8	6	-93.5398102	-1.4246569	7.5277491
9	3	93.6026764	-1.2373762	-8.3823748
9	7	-93.6026764	1.2373762	-6.4661350
10	4	41.4169922	13.6287460	87.9073181
10	8	-41.4169922	-13.6287460	75.6375427
11	5	84.1219330	-5.8780775	-46.2724609
11	9	-84.1219330	5.8780775	-24.2644501
12	6	185.8196869	0.3081530	2.8296432
12	10	-185.8196869	-0.3081530	0.8681933
13	7	185.9574890	-0.2391963	-1.7892370
13	11	-185.9574890	0.2391963	-1.0811176
14	8	84.1001129	5.8091202	46.3153076
14	12	-84.1001129	-5.8091202	23.3941193

APPENDIX IV

SAMPLE CALCULATIONS

SAMPLE CALCULATION NO. I

SECTION PROPERTIES



f'c	n
3000psi	9

$A = 27.65 \text{ in}^2$

$I_o = 3266.7 \text{ in}^4$

$d = 26.91 \text{ in.}$

$S = 242.8 \text{ in}^3$

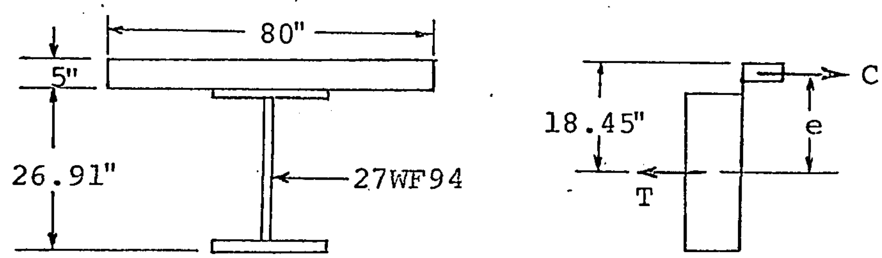
Element	Trans. Area	Moment Arm	Moment of Area	y	y2	Ay2	I_o
Slab	44.5	2.5	111.2	6.11	37.3	1660.0	92.6
WF	27.65	18.455	510.0	9.84	96.95	2680.0	3266.7
	72.15		621.2			4340.0	3359.3

$$Ic = I_O + Ay^2 = 3359.3 + 4340.0 = \underline{7699.3 \text{ in}^4}$$

SAMPLE CALCULATION NO. II

EXAMPLE 1

Ultimate Strength of Beam B1



Assume Fy = 36 ksi f'c = 3 ksi Steel Section: 27WF94

As = 27.65
d = 26.91
w = 0.490
b = 9.99
t = 0.747

$$AsFy = 27.65(36) = 995 \text{ kip}$$

$$a = \frac{AsFy}{0.85bf'c} = \frac{995}{0.85(80)3} = 4.875 \text{ in.}$$

This is Case I (See Sec. 3-4)

$$c = 0.85f'c(b)a = 0.85(3)80(4.875) = 995 \text{ kip}$$

Determine moment arm - e

$$e = 18.455 - 4.875/2 = 16.0175 \text{ in.}$$

Using Eq. (3.15e):

$$Mu = Te = 995(16.0175) = \underline{15{,}930 \text{ kip-in.}}$$

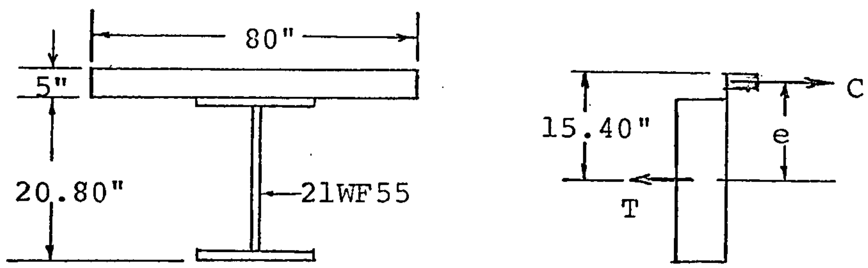
Total Design Moment = 613.58 kip-ft = 7355 kip-in.

Factor of Safety = 15,930/7355 = 2.17.

SAMPLE CALCULATION NO. III

EXAMPLE No. 2

Ultimate Strength of Beams B1 to B6



Assume $F_y = 36$ ksi $\quad f'_c = 3$ ksi $\quad$ Steel Section: 21WF55

$n = 9$ $\quad$ $A_s = 16.18 \text{ in.}^2$

$A_sF_y = 16.18(36) = 583$ kip $\quad$ $d = 20.80$ in.

$$a = \frac{A_sF_y}{0.85bf'_c} = \frac{583}{0.85(3)80} = 2.86 \text{ in.}$$

This is Case I (Sec. 3-4)

$$c = T = 0.85f'_c(a) = 583 \text{ kip}$$

Determine moment arm - e:

$$e = d/2 + t - a/2 = 10.40 + 5 - 1.43 = 13.97 \text{ in.}$$

Using Eq. (3.15e):

$$M_u = Te = 583(13.97) = 8150 \text{ kip-in.}$$

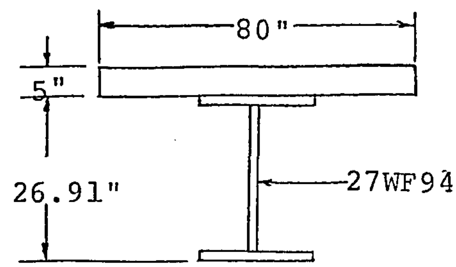
Total Design Moment = 197.93 kip-ft. = 2375 kip-in.

Factor of Safety = 8150/2375 = 3.43

SAMPLE CALCULATION NO. IV

EXAMPLE NO. 1

Design of Shear Connectors for Beam B1



Steel Section: 27WF94; As = 27.65 in.; Fy = 36 ksi

Concrete Strength f'c = 3 ksi

Using Eq. (3.6) & (3.7):

$$V_h = 0.85 f'_c A_c = 0.85(3)80(5) = 1020 \text{ kips}$$

$$V_h = A_s F_y = 27.65(36) = 995 \text{ kips}$$

the Steel Section controls.

Use 3/4 in. diameter stud connector.

Allowable Shear Load q = 11.5 kip per connector (Table 3-1)

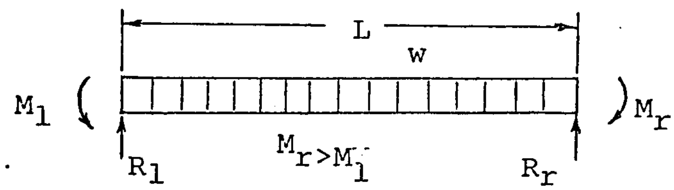
Using Eq. (3.9) the number of connectors required:

$$N = \frac{V_h}{q} = \frac{995}{11.5} = 86.5 \text{ connectors}$$

Use 87 3/4 diameter stud connectors.

SAMPLE CALCULATION NO. V

Deflection at mid-span of Beam B1 of Example No.2



w=3k/ft

L=30 ft.

Non-composite Deflection: Steel Section: 21WF68

M_l = 1453.0 kip-in. I_s = 1478.3 in.^{4}

M_r = 3065.0 kip-in. E = 30 x 10^6 psi

Using Eq. (3.14):

$$\Delta_x = \frac{w(x)}{24EI}\left[x^3 - (2L + \frac{4M_r}{wL} - \frac{4M_l}{wL})x^2 + \frac{12M_r x}{w} + L^3 - \frac{8M_r L}{w} - \frac{4M_l L}{w}\right]$$

Substituting in the given data,

Δ_x = 0.551 in.

Composite Deflection: Steel Section: 21WF55

M_l = 1070.0 kip-in. I_c = 3208.0 in.^{4}

M_r = 2345.0 kip-in. E = 30 x 10^6 psi

Substitute M_l, M_r, w,x,L, into Eq. 3.14;

Δ_x = 0.291 in.

APPENDIX V

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APPENDIX V

REFERENCES

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