Coupled motion control and estimation in a perimeter surveillance and area coverage problem

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Introduction

Area coverage with multi-agent systems

- **The problem of cooperative multi-agent control is to deploy autonomous agents over an environment to accomplish collective missions and tasks.**
- Advancements in this field have a wide range of **applications** such as:
    - search and rescue and **perimeter surveillance**
    - distributed sensing and monitoring for environmental monitoring and intervention
    - **optimal resources deployment and area coverage with multi-agent mobile robots**
- In area coverage, a team of mobile agents spatially configure themselves over an area of interest to **maximize a coverage metric that encodes agents' sensing performance and a risk density that weights points in the area.**

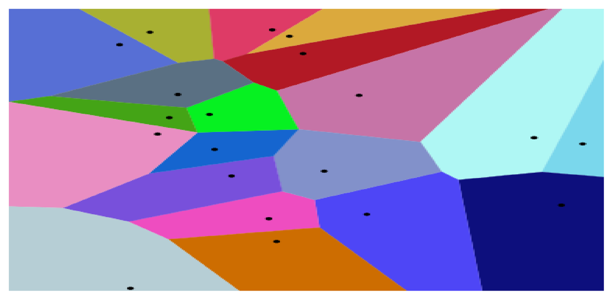

Figure 1: Voronoi diagram under Euclidean distance

- **Agents** with limited sensing range **need to work cooperatively** to **maximize the area coverage** and provide **reasonable estimations of randomly moving targets**.
- The probability of target states in the area is described by a time-varying risk density function. The **challenges** of this problem become:
    - **Resources allocation:** how do we best deploy a set of autonomous cooperating agents with respect to the task of area coverage?
    - **Optimal trajectories:** agents **react to uncontrolled events** accounting for spatial-temporal evolution of the risk.

Defining the objective: coverage metric

- **Objective function:** encodes the sensing performance and weighting regions of the 2D area.
- **Sensing performance:** assigns an area of influence to each agent **(partitioning of area).**
- **Risk field:** the relative importance of points in the area.

$$H(\boldsymbol{p},\mathcal{V},t)=\sum_{i=1}^{n}\int_{\mathcal{V}_i} f(r_i)\phi(\boldsymbol{q},t)d\Omega$$

How well does an agent sense a point in the area?

- **Sensing performance function** $f(r_i)=\kappa\exp\left({^{-r_i^2}}/{_{\sigma_i^2}}\right)$
- It is **measurable** and **differentiable** with respect to the argument $r_i=||p_i-q||$
- It is typically a strictly decreasing bell shaped function centered at the agent's state p_i.

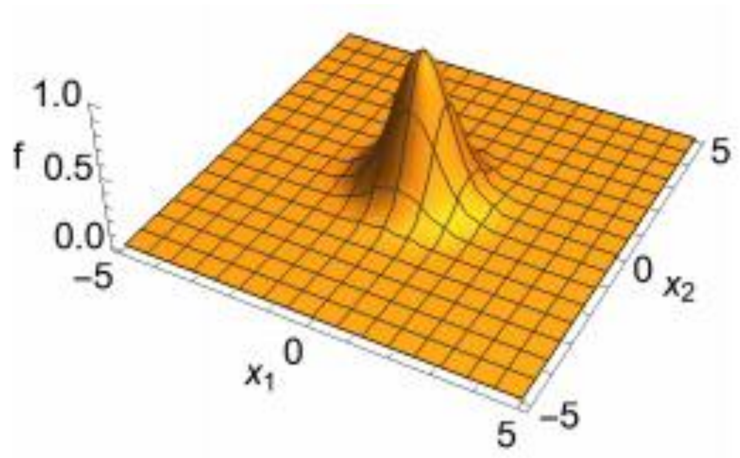

Figure 2: Sensor performance

How do we partition the area?

- **Voronoi diagram:** a partitioning of a plane into regions based on some metric to generators in a specific subset of the plane.
- **Agents ↔ Generators; Sensing Performance ↔ Metric**

$$\mathcal{V}_i=\{q\in\Omega: f(r_i)\geq f(r_j), \forall j\neq i\}$$

- **Centroidal Voronoi partition:** generators are regions' centroids.
- **Coverage is maximized when agents converge on centroids.**

Time varying/weight function

- **Area protection:** targets/external threats move inside the area; **weight function may evolve.**
- **Implication of evolving weight: time-varying centroids.**
- **Time varying risk determined by:**

$$\phi(\boldsymbol{q},t)=\bar{\phi}(\boldsymbol{q})+\sum_{j=1}^{m}\phi_j(\boldsymbol{q},t)$$

$$\phi_j(\boldsymbol{q},t)=\exp\left(-\frac{\left|\left|\boldsymbol{q}-\boldsymbol{s}^{[j]}\right|\right|^2}{2\sigma^2}\right)$$

The **control algorithms** proposed regulate the motions of the agents to react to external threats and maximize coverage **depend on knowledge of the threats' kinematics that is assumed to be known in previous work.** However, **in reality** this knowledge is obtained by **estimations through on-board sensory devices that are intrinsically affected by noise.**

Results

Estimation based on Kalman filtering

On-board sensory apparatuses take **noisy measurements** of threats' trajectories at discrete times, **approximated by:**

$$\boldsymbol{g}[k+1]=\begin{bmatrix}s_x[k+1]\\ s_y[k+1]\\ v_x[k+1]\\ v_y[k+1]\end{bmatrix}=\boldsymbol{\Phi}\boldsymbol{g}[k]+\boldsymbol{\Gamma}\boldsymbol{\alpha}[k]$$

Where $\boldsymbol{\Phi}=\begin{bmatrix}1&0&\gamma&0\\0&1&0&\gamma\\0&0&1&0\\0&0&0&1\end{bmatrix}$, $\boldsymbol{\Gamma}=\begin{bmatrix}\gamma^2/2&0\\0&\gamma^2/2\\\gamma&0\\0&\gamma\end{bmatrix}$

The state $\boldsymbol{g}[k]$ includes the **2D position and velocity** at time instant *k;* $\boldsymbol{\alpha}[k]=[\alpha_x[k],\alpha_y[k]]$ is the **2D acceleration input** at time instant *k*.

The **expected values of the threat's initial state** and its **error covariance** are given by:

$$E(\boldsymbol{g}[0])=\widehat{\boldsymbol{g}}[0],$$
$$E[(\boldsymbol{g}[0]-\widehat{\boldsymbol{g}}_0)(\boldsymbol{g}[0]-\widehat{\boldsymbol{g}}_0)^T]=\boldsymbol{P}[0].$$

Since the *i^th^* interceptor measures the line-of-sight distance between itself and the threat at time instant *k*, the **measurement model** can be expressed as:

$$\bar{\boldsymbol{z}}^{[i]}[k+1]=\boldsymbol{d}^{[i]}[k+1]+\boldsymbol{\xi}[k+1]$$

Where $\boldsymbol{d}^{[i]}[k]=[d_1^{[i]}[k],\ldots,d_N^{[i]}[k]]^T$ with $d_l^{[i]}[k]$ being **Euclidean distance** given by:

$$d_l^{[i]}[k+1]=\sqrt{(x^{[i]}[k+1]-x_l[k+1])^2+(y^{[i]}[k+1]-y_l[k+1])^2}$$

l = 1,…,N is the **2D position of the threat corresponding to the measured distance** $d_l^{[i]}$, and $\boldsymbol{\xi}[k]$ is the **measurement noise vector** with zero-mean and $\boldsymbol{\Sigma}$ measurement noise covariance matrix.

The state $\widehat{\boldsymbol{g}}[k]$ can be **estimated** using the **Extended Kalman Filter (EKF) algorithm.**

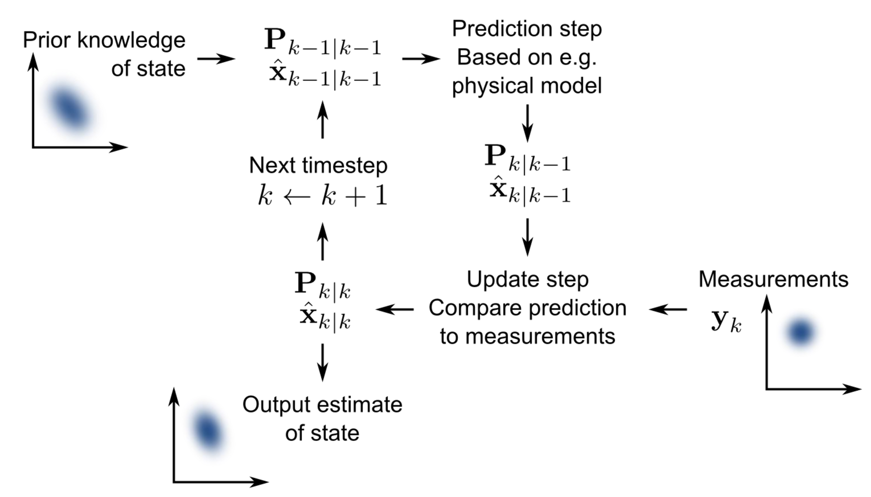

Figure 3: Basic concepts of Kalman filtering

Simulation results

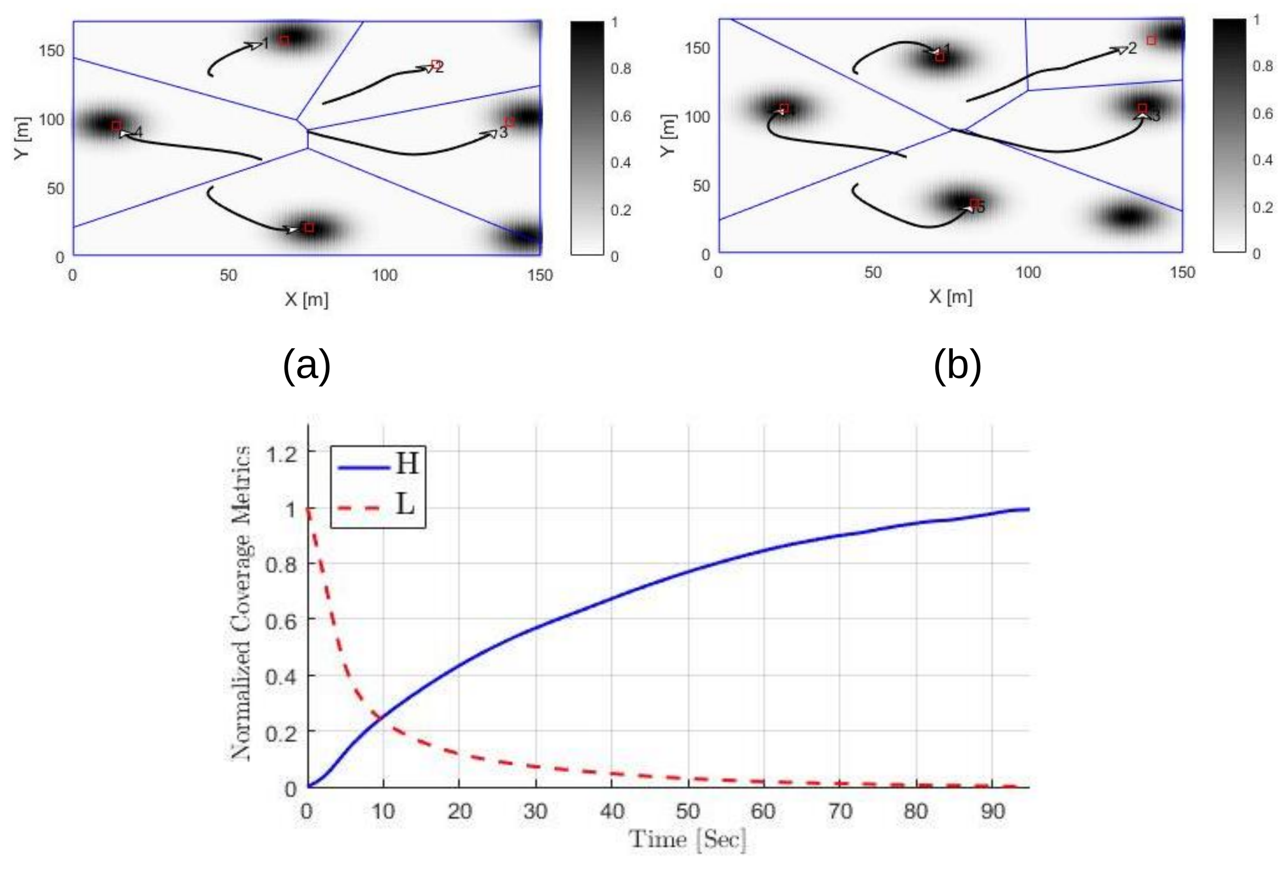

Figure 4: Coverage performance showing agents' optimal coverage at time (a) t = 60 s and (b) t = 100 s, and (c) proposed coverage performance

- The simulation considers a **convex 2D area Ω** with **6 mobile targets** and the deployment of **5 interceptors.**
- The **performance** of the proposed feedback control law in **maximizing the coverage metric** is summarized in Figure 4.
- The **coverage** (blue line) **increases as a result of the motion of the agents.**
- The vertical grayscale bars in Figure 4(a) and (b) represent the **level of density in Ω** with 0 (1) corresponding to lowest (highest) density.

References

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