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Stochastic Analysis of Human-Machine Systems

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Stochastic Analysis of Human-Machine Systems

By

Aashish Satish Shah

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Abstract

Over the last few years, there has been a tremendous growth in the use of automation techniques in engineering systems. This has resulted in a corresponding increase in the use of electrical and/or electronic devices such as sensors, electrical switches and circuits, electronic diodes, etc. One of the specific properties of these devices is that they are three-state devices, which means that the device operates satisfactorily in its normal mode but fails in either of the two mutually exclusive failure modes called open-mode and short-mode (or closed-mode). In systems composed of such devices, the redundancy may either increase or decrease the system reliability. Despite the widespread use of automation, many engineering systems are interconnected by human links, and in most situations the human link is inevitable irrespective of the degree of automation. Thus far in the literature, it appears that very little work has been done to properly understand the behaviour of such systems, particularly under the influence of human errors and common-cause failures.

To address this issue, this study presents a stochastic analysis of three-state device systems subjected to critical human errors and common-cause failures. The systems analysed incorporate elements of commonly used redundant configurations such as parallel, series, k-out-of-n and standby. For the systems that are subjected to constant failure and repair rates, the *Markov State Space* technique is used to perform the reliability and availability analysis. Generalized and special-case expressions for the system reliability, the mean time to failure, the steady state availability and the time-dependent availability for each of these systems are developed. For the parallel system with non-constant failure and repair rates, the *Device of Stages* approach is used to formulate a modified Markov model with the addition of dummy states. The impact of human error, common-cause failure, failed system repair policy and elements of redundant configurations on the values of the system reliability, the mean time to failure, the time-dependent availability and the steady state availability is demonstrated by means of plots.

Upon comparison of the plots for each of these systems, the best system configuration is identified. When the open-mode failure rate of the device is higher than its short-mode failure rate, it is concluded that the k-out-of-n standby system has the highest values of system performance indices followed by the parallel, k-out-of-n and series systems in that order.

In short, the analysis presented in this study provides a better understanding of the behaviour of three-state device systems with human errors and common-cause failures. Ultimately, this can help engineers in their aim to design and operate a safe and a reliable system.

Chapter 1

Introduction

Humans play an increasingly important role in every phase of system life cycle: design, production and operation to a varying degree. Their performance is subject to error. The overall system reliability depends on the reliability of this 'human' link as well as that of its technical aspects. The reliability evaluation of these systems would be incomplete if the human error possibilities or probabilities are overlooked [81]. According to Issac and Kirwan [162], human error contributes either directly or indirectly to 90% or more air traffic management accidents, 70% to 90% nuclear accidents and almost 98% of medical accidents. Therefore, human reliability should be an important consideration in all stages of system design and management.

A *human error* is the failure to perform a given task that could result in the disruption of scheduled operations or damage to property and equipment. Human error can be either a critical or a non-critical human error. A critical human error is the one which can result in the failure of the total system whereas a non-critical human error leads to only non-total system or other minor failures. *Human reliability* is defined as "the probability that a job or task will be successfully completed by personnel at any required minimum time if the time requirement exists" [81]. Furthermore, *human reliability analysis* is defined as "any method or approach that can be used to estimate the quantitative or qualitative contribution of human performance to engineering system, reliability and safety" [81].

Sometimes, human errors may occur as *common-cause failures*. A common-cause failure may simply be defined as any instance where multiple units or components fail due to a single cause. Thus, the occurrence of common-cause failures leads to lower system reliability. Many factors lead to common-cause failures which include abnormal external environment, deficiency in design of equipment/s, operations and maintenance errors, functional deficiency, external catastrophe, etc.

Component redundancy is usually employed in order to ensure that the system reliability is higher than those of its sub-systems (or components). The term *redundant* means the presence of an alternate means which may be either identical or non-identical, for accomplishing a given task successfully. The commonly employed redundant configurations are parallel, k-out-of-n, and standby systems. A *parallel system* is one in which the system succeeds only if at least one of the components that make up the system succeeds. Therefore, such a system is called a *completely* or *totally redundant system*. A *k-out-of-n system* is made up of n identical and independent units in parallel of which at least k (where $k < n$) of the components should operate normally for the system to succeed. Hence, such a system is called a *partially redundant system*. When $k = 1$, the k-out-of-n system becomes a parallel system and when $k = n$, it becomes a *series* or a *non-redundant system*. In both parallel and k-out-of-n systems, all the units are active (are operating at any instant of time). Therefore, such kind of redundancy is termed as *hot redundancy*.

Standby redundancy is one in which there are one or more back-up unit/s called standby unit/s that does not operate until it is needed and is switched on only when the main unit or system fails. Standby redundancy is also known as *cold redundancy* as the standby unit/s is not active (or operating) when it is in standby mode. As a result, the standby unit/system cannot fail when it is in standby mode. But, normally this is not the case as standby unit/system is active (but not operating) and hence can fail when it is in standby mode. If the standby unit/s fails in standby mode, and if the associated failure rate of the standby system in standby mode is less than that of the failure rate of standby system in operating mode, then such a system is called *warm standby system*.

Usually, engineering systems are composed of components or devices having only two states, i.e., normal operating state and failure state. However, there are certain situations wherein we encounter systems that have three states. A device is said to have three states if it operates satisfactorily in its normal mode but fails in either of the two mutually exclusive failure modes. Two typical examples of a three-state device are a fluid flow valve and an electronic diode. The two failure modes pertaining to both these devices are open, close and open, short, respectively. In systems having such devices, the redundancy may either increase or decrease the system reliability. This depends on factors such as the dominant mode of device failures, the type of system configuration, and the number of redundant devices. A comprehensive bibliography and literature survey on three-state device systems is available in Refs. [82] and [203] respectively.

This study is concerned with the reliability and availability analysis of human-machine systems that are made up of three-state devices subjected to human errors and common-cause failures. The systems in this study incorporate elements of several commonly used redundant configurations: parallel, k-out-of-n, series and standby. Generalized and special-case expressions for the various system performance indices such as system reliability, system mean time to failure, time-dependent system availability and system steady state availability are presented.

1.1 Literature Review and Current Status of Human Reliability

Human reliability is a constantly evolving field. Over the years, there has been a significant rise in number of publications devoted to this area. This section presents a comprehensive survey and review of published literature on human reliability for the period 1993-2004. A review of publications prior to this period is available in Ref. [81]. The publications are classified into six broad categories:

1. Human error identification, classification and analysis.
2. Human error data.
3. Human error rate prediction, human reliability analysis (HRA) and probabilistic safety assessment (PSA) methodology.
4. Reliability evaluation of man-machine systems.
5. Literature survey and general review articles.
6. Application areas.

Table 1-1 presents these categories along with their respective references. Figures 1-1 and 1-2 show the distribution of publications for the period 1993-2004. Figure 1-3 depicts a percentage pie chart of publications for the Table 1-1 classifications and Figure 1-4 shows a percentage pie chart of publications specifically for the Table 1-1 application area category. Publications belonging to each category of Table 1-1 are reviewed in the following sections.

1.1.1 Human error identification, classification, and analysis

A large number of publications appeared on human error identification, classification and analysis during the period 1993-2004.

In 1993, Greenberg and Small [131] developed the concept of error monitoring, a form of human-centered support to analyze and minimize human error. In 1994, a total of seven publications appeared. Sudano and Marietta [327] asserted that human error is the most complex and least understood factor in the failures of complex systems, accounting for as much as 60% to 80% of complex system failures and as much as 96% of simple system failures. In addition, they discussed task breakdowns between the human element and software/hardware for improved design of human machine interface in complex systems.

Waters [356] argued that violation of standards during design is an important cause of human errors that can lead to both trivial and major consequences. Fields et al [109] discussed formal notations for describing and reasoning about the system behaviour and applying them for human error tolerant design. Simpson [301] identified potential human error audit as an important tool for targeting accident prevention strategies and thus to reduce the possibility of human error.

Dougherty [89] argued that the rule-based operating procedures are not flawless and stressed the importance of reframing them on the basis of cognitive human reliability assumptions. Skof and Molan [302] claimed that human errors are the consequences of cognitive and sensorimotor availability, the low levels of which are the root causes of knowledge and skill based errors. Bennet et al [29] argued that a cost-benefit analysis is just a single phase of a complete risk management program that would more than likely start with any probabilistic risk assessment.

Category	References
• Human Error Identification, Classification, and Analysis	4, 17, 19, 20, 21, 26, 27, 29, 31, 32, 33, 37, 38, 39, 42, 45, 49, 50, 60, 61, 65, 66, 69, 70, 71, 72, 75, 83, 85, 88, 89, 90, 92, 93, 98, 99, 103, 106, 108, 109, 112, 129, 131, 132, 141, 144, 145, 146, 154, 158, 159, 164, 179, 188, 189, 194, 195, 200, 205, 209, 213, 221, 233, 234, 235, 236, 241, 242, 247, 249, 251, 268, 272, 274, 281, 289, 292, 301, 302, 304, 313, 318, 327, 330, 341, 342, 343, 344, 345, 346, 356, 363, 364
• Human Error Data	2, 14, 16, 24, 30, 48, 52, 62, 114, 136, 160, 170, 180, 184, 187, 216, 229, 259, 260, 279, 296, 323, 324, 351
• Human Error Rate Prediction, HRA, and PSA Methods	10, 18, 23, 25, 28, 34, 35, 43, 44, 51, 53, 58, 63, 77, 86, 94, 101, 105, 110, 111, 113, 115, 116, 119, 126, 130, 134, 144, 147, 148, 149, 150, 161, 173, 175, 178, 181, 182, 183, 191, 192, 196, 206, 215, 225, 226, 227, 228, 230, 231, 244, 262, 269, 270, 271, 278, 280, 282, 283, 287, 288, 290, 295, 300, 305, 306, 314, 315, 316, 317, 320, 322, 326, 331, 332, 333, 336, 337, 338, 347, 348, 349, 350, 352, 361, 365, 368, 369, 382, 383, 385, 386
• Reliability Evaluation of Man-Machine Systems	5, 59, 74, 79, 80, 87, 95, 123, 125, 165, 166, 168, 171, 193, 210, 211, 218, 219, 239, 245, 246, 256, 257, 310, 311, 312, 325, 328, 329, 381
• Literature Survey, and General Review Articles	46, 81, 121, 135, 190, 198, 217, 232, 252, 261, 275, 276, 309, 321
• Applications Areas	
• Nuclear Power Plants	11, 36, 40, 67, 68, 100, 107, 117, 122, 140, 151, 155, 160, 167, 169, 172, 174, 176, 185, 202, 208, 214, 220, 223, 237, 238, 250, 254, 255, 263, 264, 273, 277, 293, 307, 308, 319, 334, 353, 354, 357, 358, 373, 374, 375, 376, 377, 378, 384
• Aerospace and Aviation	7, 54, 55, 56, 91, 102, 129, 143, 153, 162, 163, 199, 201, 204, 207, 240, 243, 253, 258, 284, 285, 286, 291, 298, 299, 335, 362, 366
• Manufacturing, Process and Maintenance	1, 15, 22, 41, 47, 132, 197, 212, 265, 266, 339, 340, 379, 380
• Structural Engineering	57, 96, 97, 127, 177, 248
• Computer and Information Systems	3, 6, 64, 73, 104, 139, 186, 297, 303
• Marine Engineering	8, 9, 12, 138, 152
• Others	13, 78, 118, 133, 157, 222, 355, 367, 370, 371, 372

Table 1-1: Classification of publications on human reliability

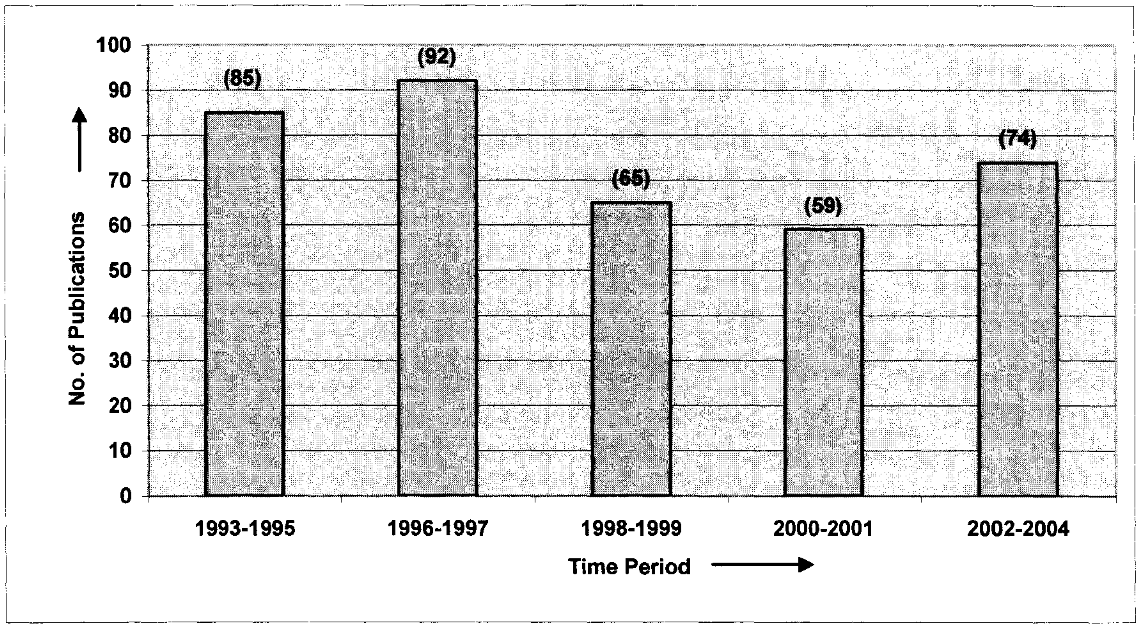

Figure 1-1: The distribution of publications on human reliability for the period 1993-2004

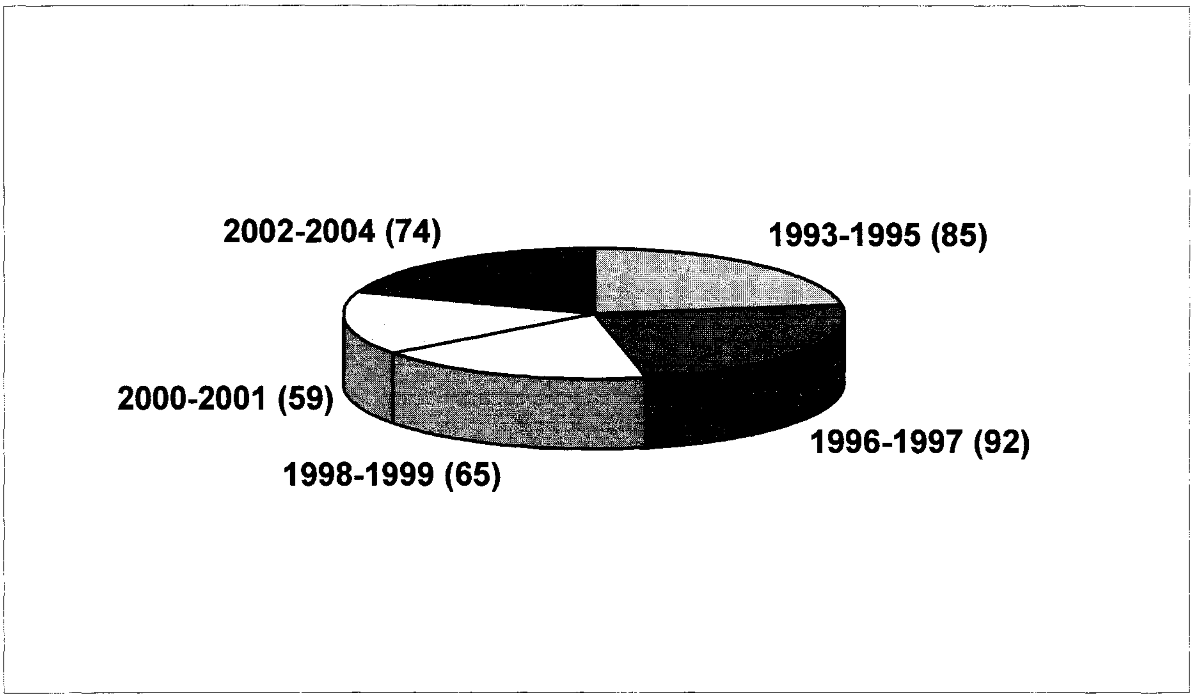

Figure 1-2: A pie-chart of publications on human reliability for the period 1993-2004. (The numerals in parentheses indicate the number of publications).

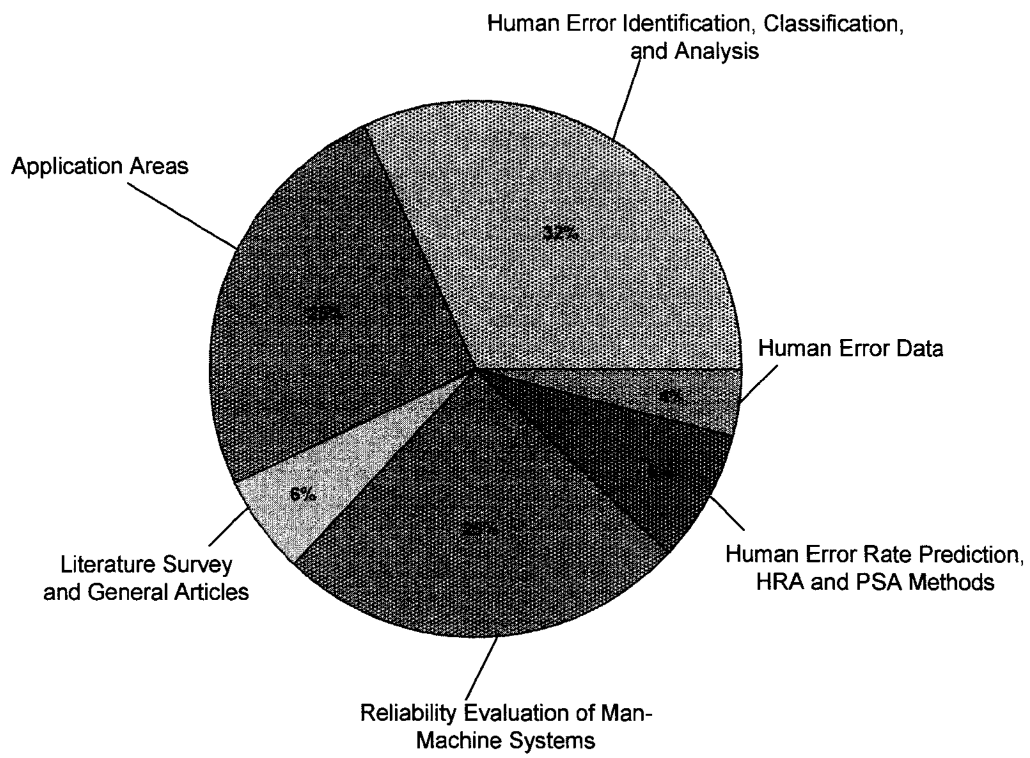

Figure 1-3: A percentage pie chart for Table 1-1 classifications

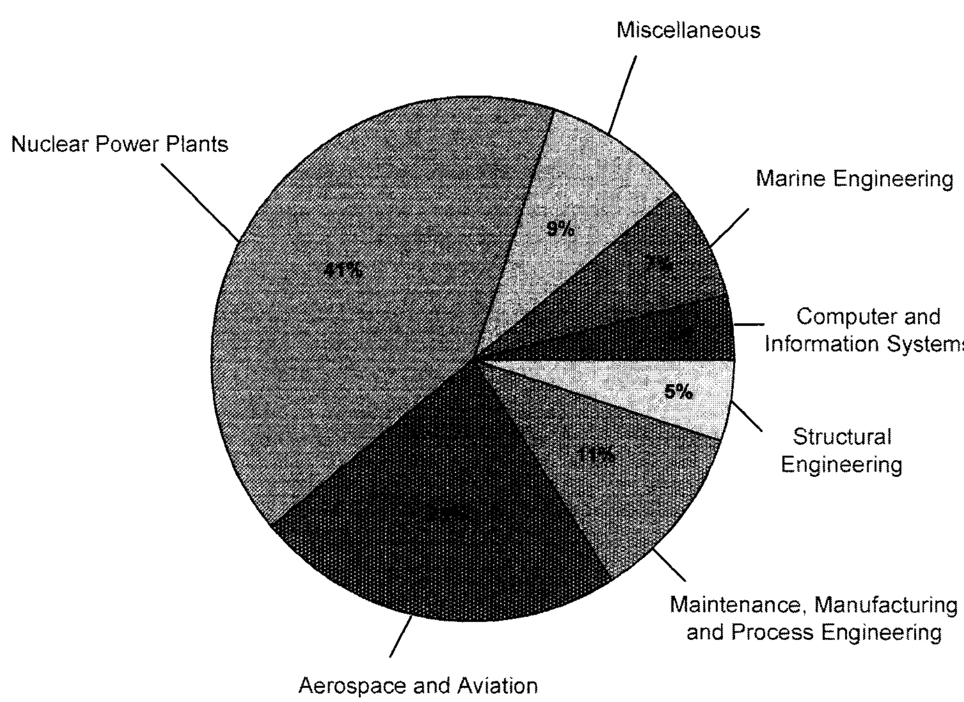

Figure 1-4: A percentage pie chart of publications for Table 1-1 category: application areas

In 1995, a total of sixteen publications appeared. Bradley [38] used fault tree methodology using a coded approach to analyze the causes of various types of human error in nine well documented disasters occurring between 1861 and 1987. In reference [39], he analyzed three additional disasters and claimed that in all types of human error, the operator error predominates. Van der Schaff [343] studied near miss reporting cases in Dutch chemical processing industry and proposed an incident causation model with human error recovery and classification of errors based on Rasmussen's Skill-Rule-Knowledge (SRK) model. Dougherty [85] in his study attempted to extend the classical slip-error taxonomy of human error with the inclusion of a separate category - 'violation'. Embrey [98] provided an excellent understanding of the nature of cognitive and conceptual errors, their causes, classification, prediction, and reduction using the SRK Model, Generic Error Modeling System (GEMS), and the Step Ladder Model (SLM). Smidts et al [304] developed a taxonomy of human cognitive errors based on the Information Decision Action (IDA) Model and discussed its benefits in human error data collection and analysis.

Van Vuuren and Van der Schaff [346] reviewed two research projects of modeling organizational factors responsible for human errors in human reliability analysis. Collins and Fragola [66] provided a background of the recent advancement in human error evaluation technique in which proper consideration is given to latent events. Sepanloo [292] et al discussed human error tolerant interfaces that help in analyzing and identifying human errors, without carrying tests or analysis on the floor. Hulet and Carroll [158] discussed various human failures that occur in safety management and provided guidelines on how to prevent them for a safer world. Other studies reported in 1995, related to human error identification, classification and analysis are presented in Refs. [72, 103, 111, 112, 194, 345].

In 1996, a total of eight publications appeared. Grabowski and Roberts [128] examined three large scale systems in the U.S. and claimed that by considering human error as the only cause of failure in these systems would be improper; the organizational and societal factors that contribute to human error must be considered. Stang [313] presented an analysis to evaluate the root causes of the Chernobyl disaster and claimed that the operator error was the root cause of the disaster. Ayers et al [17] explored the possible relationship between operator experience and risk of an accident; they concluded that the accident risk was not solely dependent on operator experience; other factors like mechanical or environmental could also affect the accident risk.

Dassonville et al [71] conducted a simulation experiment to evaluate "trust" and its relationship to "stress" as a contributing factor to human error in a man-machine system. Brookes [42] explored various causes of human error in engineering systems and the means by which they can be minimized. Bubb [45] described the use of system ergonomic rules to design the technical layout by examining the human conditions of information processing that enhance human reliability. Dearden and Harrison [75] demonstrated how the impact of human error can be assessed prior to, or in parallel with the design of the Human Machine Interface (HMI), and showed how impact assessments could be used to allow risk analysts to inform HMI designers about the relationship between operator actions and system hazards.

In 1997, a total of ten publications appeared. Koval and Floyd [195] classified human errors in industry as either human-based or situation-based. Each of these can then be of several types: design, assembly, operation, installation, inspection, etc. They concluded that while human-caused errors contributed 20% of all human errors in industry, 80% of them were the result of situation-based human errors. Edkins and Pollock [93] explained the Railway Accident Investigation Tool (RAIT) to perform retrospective analysis of 112 railway accidents, based on Reason's Generic Error Modeling System. Kohda et al [189] presented a prediction method using the classification scheme of human erroneous actions. Friedlander and Evans [108] described the correlation between organizational culture and the frequency, severity and nature of human errors. McCarthy et al [221] provided a framework to consider the organizational context of human error that aids in the identification and analysis of a number of contexts for vulnerability in high-consequence systems. Nelms [241] examined the identification and minimization issues of the latent causes of human error in industrial failures. Pyy [274] presented an experience based study aimed to identify and analyze various maintenance related failures prone to human error. Clarke [61] discussed the effect of human error on quality and reviewed how the human reliability concepts can be applied in total quality management. Other studies reported in 1997, related to human error identification, classification and analysis are presented in Refs. [236, 242].

In 1998, a total of four publications appeared. Neives and Sage [247] presented various taxonomies, causes and types of human and organizational error and explained how the results of their study can be used in process and system re-engineering design. Baxter and Bass [26] published an article on the role of 'Situation Awareness' (SA) in Human Error and Reliability. They identified nine lessons to highlight and refine the concept of SA. Mackieh [213] described a study aimed to identify various performance shaping factors (PSFs) that affect the performance of operators in industry.

In 1999, a total of nine publications appeared. Sasou and Reason [289] discussed taxonomy of human team errors and its relationship with the PSFs. Bes [31] presented a case study of human errors in dynamic environment consisting of air traffic controllers and an expert system. Stewart and Chase [318] examined quality issues in service industry and argued that human error is a direct cause of substantial number of service failures and developed a framework based on GEMS to understand the role of human error. Busse [50] asserted the importance of cognitive architecture as a framework to error modeling as cognitive processes are one of the most important factors contributing to operator/human error. Helfrich [140] investigated the concept of human error and reliability from a socio-psychological perspective and pointed out that "social" influence must be considered along with respective "psychological" influences. He proposed two approaches for studying and classifying errors: Firstly, according to their frequency of occurrence and secondly, according to their cause of occurrence.

Noerager et al [249] proposed Human Factors Analysis Software Tool (HFAST), a software tool that aids in identifying human error in design, operation, and maintenance. Sutcliffe et al [330] presented a model based approach to assess the implications of human error in system requirements. Chubb and Williamson [60] proposed a regulatory model to mitigate the occurrence of human error in performance-based safety regulations. Huang et al [154] discussed the success of operator error prevention system in Taiwan semi-conductor industry.

In 2000, a total of thirteen publications appeared. Droivoidsmo [90] discussed how human errors can be analyzed by means of training and simulation. Westfall-Lake [363] emphasized the need for understanding two different aspects: organizational and operational in order to develop any strategy for preventing human error in safety-sensitive operations. Linsenmaier and Straeter [209] stressed the importance of situation awareness in estimating human error. They attempted to remove the discrepancy between subjective and actual situation awareness in human factors event trees (HFET) and described it using a man-machine system framework. Boussouffara and Elzer [37] investigated the influence of human machine interfaces in technical systems. Besnard [32] stressed the importance of cognitive processes that influence expert error and the need to integrate them with system design for a more reliable operation.

Abdelhamid et al [4] proposed an accident root cause tracing model (ARCTM) for identifying the root causes of accidents. Forsythe et al [106] proposed an 'organic' model for providing a new perspective to approach human error in engineering systems. The model is based on the fact that systems take organic properties with human involvement and that the variability in systems may lead to errors. Izso [164] described a scheme for making a distinction between design errors and user errors on the basis of statistical data by using a simple binomial model. Hiramatsu and Obara [144] categorized different rear-end collision scenarios in automotive industry by the type of human error involved in them and validated them by using the Monte-Carlo Method. Gross et al [132] presented a study to identify latent causes of human error at electric utility generating stations.

Van der Schaaf and Kanse [342, 345] examined various facets of human error by emphasizing the positive role that the human operator plays in preventing small failures into becoming major system breakdowns. Other studies reported in 2000, related to human error identification, classification and analysis are presented in Refs. [27, 99].

In 2001, a total of six publications appeared. Hirotsu et al [145] presented multivariate analysis of human error incidents and concluded that 60% of human error occurred in maintenance and 20% in inspection. Baker [19] examined the main causes of accidents in Boilers and Pressure Vessels and claimed that human operator error and human maintenance error is often to blame.

Noyes [250] asserted that human error is detrimental to proper functioning of systems and discussed how human error concepts can be used in the context of safety-critical system design. Kletz [188] pointed out that most human errors are voluntary actions, that is, even if a human operator knows what to do, he/she decides not to do it which often results in a system breakdown. Pyy [272, 274] studied the plant specific maintenance data of nuclear power plants and presented an extensive analysis and classification of maintenance failures due to human errors. Resnick [281] presented an approach for task based evaluation of human error analysis and illustrated it by using a simple case study.

In 2002, a total of seven publications appeared. Cohen [65] suggested that "fatigue" is an important source of human error and hence accidents. He discussed various methods that could be used to reduce the risk of accidents by combating fatigue. In the same year, Dang and Reer [69] reviewed identification and quantification issues with respect to decision and commission errors. They concluded that though the proposed methods of errors of commission (EOCs) are good enough for identification, a combination of empirical work and expert judgment are the best way to deal with quantification issues of decision and commission errors. Mostia [234] presented a classification and cause analysis of human errors occurring in instrument design, construction, operation, and maintenance.

White [364] presented sensitivity analysis of operator error and Van der Linden et al [341] attempted to validate various exploration strategies based on error consequences and performance. Muschara [235] developed a dual human performance strategy based on both error management and design-at-depth approaches and Levy et al [205] applied a psychological theory of human cognition to the real world environment to improve system safety.

In 2003, a total of nine publications appeared. Daniel and Heising [70] studied the impact of organizational factors on maintenance and argued that their results would aid in optimizing human reliability. Mostia [233] asserted that proper understanding of the types and sources of error can be helpful in developing methods and models to mitigate human error. Balkey [20] pointed out that it is very important to go into the internals of an accident to analyze all its causes. Hobbs and Williamson [146] established the associations between various types of human errors and the contributing factors in safety-critical environments. Peterson [268] presented a detailed discussion of categories for human error and its causes, and culture and safety aspects related to human error. Kim and Jung [180] compared various PSF taxonomies used in the PSA and HRA studies in accident management. Based on this study, they developed taxonomy of PSFs for HRA in emergency situations.

Duffey and Saull [92] proposed Duffey-Saull Method [DSM] analysis using a combination of empirical and theoretical approach that can analyze and prevent human errors in technological systems. Busby et al [49] attempted to determine the distribution of people's problem solving, in the context of it being the source of failure in complex and hazardous systems.

In 2004, Doos et al [83] studied human errors on the basis of whether they are risk-creating errors or risk-triggering errors and claimed that risk-creating errors constituted almost 93% of all errors.

1.1.2 Human Error Data

Human error data collection and quantification is an important step for any meaningful quantitative human reliability analysis. This section presents the status of research on human error data, its collection techniques among other related aspects.

In 1994, Macwan et al [216] discussed the quantification of human error by using existing HRA methods with simple terms like success or failure instead of considering multiple error expressions. In the same year, Moo Sung and Chang Kue [229] presented quantification of human error probabilities (HEPs) in implementing accident management strategies on the basis of the quantified correlation of performance requirement and performance achievement.

In 1995, Bennett [30] argued that any Bayesian Human Reliability Analysis model needs proper data for a meaningful analysis and provided a scheme of an effective data collection and analysis. Burns and Turner [48] presented a method to estimate screening-level total failure probability of human error events. In the same year, Clarke and Wimpenny [62] described the automatic data logger developed at the Rolls-Royce and Associates Limited that can automatically quantify data. In another article [63] they presented various techniques for collecting human error data.

In 1996, a total of five publications appeared. Balkey [21] discussed the importance of integrating risk based inspection with human error for which plant or system specific human error data is vital. Park and Jung [259] presented an approach to modify the nominal human error probabilities based on the rationale that the human error likelihood depends upon the combined effects of PSF's. In the same year, they [260] also presented a method for estimating human error probabilities from paired comparison ratios and asserted that this rationale integrates many experts' opinions. Byungjoon and Bishu [52] used fuzzy regression analysis to estimate operator response times as a function of the performance shaping factors such as age, sex, experience, and education. Gao et al [114] proposed an analytical model to quantify human error probabilities by using a three level influence diagram.

In 1997, a total of four publications appeared. Shen et al [296] presented a method to conduct cognitive model-based analysis of human error data and Kirwan et al [184] discussed the development of 'CORE-DATA' - Computerized Operators Reliability and Error Database and its application in HRA. In the same year, Klein et al [187] pointed out that organizational databases have a significant rate of errors which need to be minimized or eliminated altogether in order to perform any meaningful HRA.

In 1998, Auflick et al [16] discussed the HRA centered knowledge based expert system that can be used to quantify human error data and probabilities. In the same year, Basra and Kirwan [24] discussed further development to 'CORE-DATA' and various data collection methods and provided a comprehensive comparison of these methods.

In 1999, Auflick [14] reviewed the HRA Expert Quantification System developed by the Los Alamos National Laboratory. Kirchsteiger et al [180] described a knowledge-based text retrieval method for MARS (Major Accident Reporting System) database and its application in determining the human contribution to accidents.

In 2000, Iliffe et al [160] discussed the features and benefits of the Active Database System developed at the Loughborough University. A year later, in 2001, Vaurio [351] developed general equations and numerical tables for the quantification of probabilities of sequentially-dependent repeatable human errors.

In 2004, a total of seven publications appeared. Straeter [323] discussed the importance of maintaining a continuum between qualitative and quantitative approaches of human reliability assessment and provided guidelines for any future data collection and quantification. In another publication that year, Straeter et al [324] summarized the state of data and potential solutions to assess the errors of commission in HRA/PSA. Fujita and Hollnagel [110] proposed a simple quantification method, which is simply an improvement of the basic screening method used in Cognitive Reliability and Error Analysis Method (CREAM). Hallbert et al [136] discussed empirical data sources and their utility in HRA. In another publication by Park et al [170] in that year examined the performance of the human performance database developed in a simulator studies during emergencies at a nuclear power plant (NPP). They suggested that these simulation studies can be a useful source of collecting data for performing HRA. Other studies reported in 2004, related to human error data are presented in Refs. [279, 180].

1.1.3 Human Error Rate Prediction and HRA/PSA Methods

Over the years, significant amount of work has been done in developing tools, methods and frameworks for predicting and measuring the reliability of humans and safety assessment. A review of literature belonging to this category for the period 1994-2004 is presented below.

In 1994, a total of ten publications appeared. Moieni et al [225, 226] reviewed the advances in human reliability methods in their work published in two parts. Montmayeul et al [227] enumerated three possible channels for the improvement of PSA as a tool in assisting in processing system safety problems. Reer [278] asserted the importance of risk-taking behavior as an important factor of accident causation and proposed incorporating it into any human reliability analysis. Wright et al [369] extended Systematic Human Action Reliability Procedure (SHARP) by providing a software engineering notation to derive human error tolerance requirements.

In the same year, Wilson et al [365] described the findings of a benchmarking exercise of a new automated technique developed as part of the U.S. Nuclear Regulatory Commission's Accident Sequence Evaluation Program (ASEP). Macwan and Mosleh [215] presented the Human Interaction TimeLINE (HITLINE) methodology and Nelson et al [244] explored the concept of applying human error analysis throughout the life cycle of complex systems by incorporating it into system design. Ryan and Siu [287] dealt with issues related to the extension of HRA methods to system design and management and provided certain insights in this aspect. Steinberg [317] presented human error models for performing structural reliability analysis.

In 1995, a total of nine publications appeared. Degen et al [77] discussed the development of the Technique for Human Error Rate Prediction (THERP) by using an expert system based on knowledge engineering environment. Ujita et al [338] elucidated an approach for the evaluation of expert's performance in emergency situations, based on both cognitive and behavioral viewpoints. Blackman and Byers [34] described the development and testing of a behavior-based human reliability analysis method. Richards et al [282] explored prospects and efficacy of applying "visualization" to enhance HRA processes, and the management and communication of HRA results. Moo Sung and Chang Kue [228] described a dynamic HRA method and its application in human error quantification. Gerdes [119] developed a selection matrix based on a goal and approach oriented classification of HRA techniques useful for selecting a particular HRA methodology for any specific industry. Heyes [142] presented criticality analysis of the current PRA techniques with respect to quantifying risks associated to human errors and Parry [262] proposed a set of requirements for improving HRA methods and their use in PSA.

In 1996, a total of thirteen publications appeared. Zhang et al [383] discussed the significance of human error factor identification in complex man-machine systems, the disadvantages of the existing human error factor identification (HEFI) methods, and proposed a set of criteria for a good HEFI method. Chen and Chen [58] pointed out that Human Error Assessment and Reduction Technique (HEART) needs some modifications in order to make it more accurate and consistent. Schade et al [290] presented a hybrid model based on Success Likelihood Index Method – Multi Attribute Utility Decomposition (SLIM-MAUD) and Human Cognitive Reliability (HCR) method for performing a more effective HRA. Stanton and Baber [314, 315] described the Task Analysis for Error Identification (TAFEI) procedure for a systematic approach to human error identification and introduced the concept of predicting 'designer error' based on HRA methods like Systematic Human Error Reduction and Prediction Approach (SHERPA) and TAFEI. Basra and Kirwan [23] presented Computerized Human Error Analysis Trees (CHEAT) to identify and analyze human errors for HRA/PSA. Hollywell [149] reviewed the state of research into incorporating human dependent failures in task and risk analysis techniques.

Hollnagel [147] reviewed a wide range of first and second generation HRA methods from the operator modeling perspective. He discussed the importance of cognitive operator modeling and the improvements needed in methods for a more realistic HRA. Stutzke and Doughtery [326] reviewed the development of 'A Technique for Human Event Analysis' (ATHEANA) and claimed it can be helpful in a HRA/PSA as it would consider the effects of the errors of commission. Boniface and Bea [35] discussed Human Error Risk Reduction

Operating System (HERROS), a modeling system to study human and organizational error in different mishaps. Mosleh et al [231] presented the IDA Cognitive model, a cause-oriented model that characterizes cognitive dynamics along with human factors and discussed its utility in HRA in emergency and safety-critical situations.

Julius and Jogerson [173] developed a procedure for the Probabilistic Risk Assessment to identify and analyze the impact of errors of commission that contribute significantly to risk during non-power operational mode of a nuclear power plant (NPP) . Hahn et al [134] demonstrated a prototype knowledge based system for estimating human error probabilities by means of various available HRA and PRA methods. Richei et al [288] presented Human Error Rate Assessment and Optimizing System (HEROS), a procedure for evaluating, analyzing, and optimizing the man-machine interface in PSA.

In 1997, Thompson et al [337] described the basic purpose, principles and process of ATHEANA, and Taylor et al [336] discussed its development and implementation issues. In the same year, Kirwan [181] developed the Human Reliability Management System (HRMS) and it's scaled down version Justification of Human Error Data Information (JHEDI), and in other two articles [182, 183], he attempted to validate various HRA techniques by deriving a common set of criteria.

In 1998, a total of seven publications appeared. Eisawy et al [94] compared THERP and Detailed Block Diagram Analysis (De-BDA) to quantify human error in an analysis concerning Inchass research reactor, Egypt and concluded that it is easier to simulate the intermediate steps and erect step blocks using De-BDA. Swanson [333] presented the Operational Human Performance Reliability Assessment (OHPRA), a method that integrates the objectives of HRA with both safety and productivity perspectives. Wen and Hwang [361] presented a graphical modeling and analysis tool for human fault diagnosis tasks based on both line-chart and Petri-nets. Stanton and Stevenage [316] attempted to validate the SHERPA approach and Brown and Haugene [43] presented a probabilistic risk model that takes into account management and organizational factors related to human error. Dougherty [86] examined and evaluated the ATHEANA approach to determine whether it is effective in performing HRA by considering errors of commission. Sharit [295] provided certain guidelines and a framework for applying formal HRA methods and principles in high-risk industries. He concluded that the written procedures are an important aspect for HRA in high risk industries.

In 1999, a total of six publications appeared. Holmberg et al [150] developed a HRA approach that integrates probabilistic and psychological approaches of human reliability. Hollnagel [148] et al described a human error analysis project in which human performance and error mode prediction are based on Cognitive Reliability and Error Analysis Method (CREAM). Galliers [113] presented a method for the assessment of the implications of human error on user interface design of safety-critical design.

Vanderhaegen [347] presented APRECIH, a French acronym for Preliminary Analysis of Consequences of Human Unreliability, a generic method to analyze the consequences of human unreliability and to generate a set of design recommendations to increase system safety. In another article, he [348] proposed a model to describe both human and machine dysfunctions and to guide the development of error prevention systems.

In 2000, a total of eight publications appeared. Seock [161] discussed the applicability of first and second generation HRA techniques, particularly ATHEANA and CREAM in the design review process of man-machine interfaces. Decision Trees were used by Baumont et al [25], in the HORRRAM (Human and Organizational Reliability Aspects in Accident Management) method, developed to integrate human aspects with the PSA. Pyy [271] described a decision reliability evaluation method for evaluating human reliability in decision making related to NPP's. Grams [130] addressed the implementation concerns of Decision Event Tree, a normative model of operator behavior. Byers et al [51] compared Simplified Plant Analysis Risk HRA methodology with other HRA methods. Kolarik [191] described a real-time conditional human reliability assessment model that predicts the likelihood of human error in a future time. Straeter [322] discussed the analysis of behavioral data and operator error modeling in human reliability analysis.

In 2001, a total of eight publications appeared. Ang et al [10] studied the development of integrated models for the assessment of severe accident management (SAMEM) strategies based on the Probabilistic Safety Assessment and the Risk Oriented Accident Analysis Methodology. They proposed and demonstrated three integrated accident management models. Pocock et al [270] described the Technique for Human Error Assessment (THEA) and its utility in early system design for reliable performance. Visser and Wieringa [352] presented a methodology to predict human error probabilities (PREHEP) in the early phase of the design process, as the basis of selecting functional process units. Pillay and Wang [269] described an approach to integrate human reliability assessment with decision making process using AHP-Analytical Hierarchy Process. Richei et al [283] studied HEROS, a procedure for evaluating and optimizing the man-machine interface in PSA based on fuzzy set theory expert system. Kim [178] examined the characteristics of the first and the second generation HRA methods with respect to their applicability in a man-machine interface design review. He claimed that the second generation HRA methods like ATHEANA and CREAM are much more promising in the design review process than their first generation HRA counterparts. Vanderhaegen [349] described ACIH, a French acronym for Analysis of Consequences of Human Unreliability, a non-probabilistic approach for identifying both tolerable and intolerable sets of human behavioral degradations that affect the safe operation of the system.

In 2002, Spurgin [306] presented a critical review of HRA methods and practices used in the context of PSA and Ayyub et al [18] reviewed different methods that can be used to develop risk-based standards for system safety. In the same year, Shu et al [300] proposed a team behavior network model that can simulate and analyze operator team errors in a dynamic environment and Gao and Zhang [116] discussed the HRA and human error data management system developed to provide a computer aided tool to analyze and manage human error events. In another article in the same year, Ferrante et al [101] presented Human Error Analysis Support Tool (HEAST) based on the paradigm of human error minimization. Other studies reported in 2002, related to human error rate prediction, and HRA/PSA methods are presented in Refs. [305, 386].

In 2003, Zhang et al [382] proposed the integration of THERP and HCR in a human factor event analysis model to create a new model that has the positive features of both the former methods and Bubb [44] discussed the development of Connectionism Assessment of Human Reliability (CAHR), a tool to evaluate events that lead to human and cognitive error and to develop strategies to reduce the probability of human error. In the same year, Bell and Scolaro [28] presented Team Interaction Analysis with Reusable Agents (TIARA), an approach to understand and evaluate interactions among team members and Gao et al [115] discussed the development of a computer-aided human reliability analysis method for integrating the HRA approach with computing concepts.

A year later, in 2004, Reer et al [280] described CESA (Commission Errors Search and Analysis), and demonstrated its utility in THERP based human reliability assessment with plant specific errors of commission. Forester et al [105] proposed an expert elicitation process of estimating human error probabilities by using the ATHEANA method. Human Error Risk Management in Engineering Systems (HERMES), a second generation HRA method that includes both cognitive and dynamic inclusions in its analysis was described by Cacciabue [53]. In the same year, Kolarik et al [192] presented a human performance reliability model based on multivariate forecasting and exponential smoothing, state-space modeling, and fuzzy logic.

1.1.4 Reliability Evaluation of Man-Machine Systems

In 1994, Jacob and Atri [165] presented stochastic analysis of a two-unit system with standby, common-cause failures, and critical human errors. In the same year, Jiang et al [168] developed two two-state and two multi-state mathematical models to evaluate human reliability with operation-inspection sequence.

In 1995, a total of four publications appeared. Dearden and Harrison [74] showed how mathematical models are useful in verifying the properties that contribute to human error early in design stage. Dhillon and Yang [80] developed a mathematical model to perform reliability and availability analyses of a general standby system with increasing human error rates, common-cause failure and arbitrary failed system repair rates. In the same year, they [246] presented stochastic analysis of a system with constant human error and common mode failure rates. In another article, they [245] developed a probabilistic model to perform availability analysis of a standby system with common cause failures and human error.

In 1996, a total of six articles appeared. Sur and Sarkar [329] developed a probabilistic model representing a redundant system with common cause failure, logic failure, and human error. As the number of states was high, the model was solved using standard numerical methods. Mahmoud and Esmail [218] conducted probabilistic analysis of a warm two-unit standby redundant system subject to hardware and human failures by using the regenerative point technique. Pandey et al [256, 257] presented stochastic modeling of a power-loom plant consisting of two units A and B connected in series. Unit B had 'n' number of components with a k/n configuration. The system was analyzed considering critical human errors and common cause failures. Sur [328] presented a probabilistic model for 'n' unit non-repairable system with 'k' units operating and (n-k) units kept in standby mode as partially energized units. Switching system failure, human error and common cause failures were considered in the model.

In 1997, a total of six publications appeared. Dougherty [87] investigated whether human failure is stochastic in nature or not. His study concluded that the uncertainty associated with human error does not make its assessment precise but rather probable or stochastic in nature. Goel et al [125] presented Markovian analysis of a two-unit redundant system with human error. Jacob et al [166] developed a probabilistic model of a two-unit deteriorating standby system with repairs, common cause failures and critical human errors.

Sridharan and Mahanavadivu [312] presented three models for performing reliability and availability analyses of two non-identical unit parallel systems with common cause failure and human error and El-Damcese [95] developed a Markov model representing a 2-unit multiplex system with common-cause failure and human error.

In 1998, Mahmoud and Esmail [219] performed stochastic analysis of a two-unit warm standby system with slow switch mechanism and human error using exponential time distributions. Liu and Wang [211] developed a continuous-time Markov Chain model with human and recovery factors to analyze the reliability of a man-machine system. In the same year, Strutt et al [325] developed a time dependent stochastic model for predicting human reliability based on probability performance requirement and resource consumption.

In 2000, Giuntini [123] proposed a mathematical methodology based on the Weibull hazard rate and showed how human reliability can be characterized and estimated on the lines of hardware and software reliability. He claimed that human hazard rate curve follows closely to the bath-tub hazard curve of hardware failures. Two years later, in 2002, Sridharan and Kalyani [310] presented stochastic analysis of a k-out-of-n system with common cause failures and human errors. In 2004, Chiodo et al [59] developed an analytical model based on conditional Weibull hazard rate with a random scale parameter, and by assuming system reliability as a function of technological, information, and human components.

1.1.5 Literature Survey, Review and General Articles

In 1994, Dhillon and Yang [81] presented a comprehensive literature review on the area of Human error and reliability. They reviewed over 380 publications that appeared in journals, conference proceedings, and other sources during the period 1979-1992.

In 1997, Ramey-smith et al [275] described the progress and objectives of the research initiative of the United States Nuclear Regulatory Commission (US NRC) with respect to human reliability and error. In 1998, LaSala [198] presented a historical review and developments in the field of human performance reliability (HPR). In this paper, he reviewed the development of HPR, compared hardware and human reliability, and provided certain future directions.

In 2002, Spurgin [309] presented a critical review of the current Human Reliability Methods and identified benefits and limitations of these methods. In the same year, Redmill [276] discussed the current state of research in human factors in risk analysis and suggested certain guidelines for future research. Also in 2002, Straeter and Kirwan [321] compared the state of

HRA developments with respect to nuclear power plants (NPP) and air traffic management (ATM) industries and identified what has been already accomplished and what scenarios need to be addressed in future.

In 2003, Bubb [46] discussed the new Association of German Engineers (VDI) code of practice on human reliability which contained the requirements and evaluation methods of human error. In a review on human error and reliability from an aging workforce perspective, in 2003, Haight [135] concluded that older people are at a disadvantage when it comes to overall task performance as they are more prone to making errors because of their decreased physical, mental, and decision making capacities.

1.1.6 Application Areas

Most of the earlier human reliability methods had been developed in the context of nuclear power plants, but lately areas like Aerospace and Aviation, Maintenance, Marine, Manufacturing, Medicine and many others, are increasingly concerned with human reliability and error. Developments in these areas are reviewed below.

i) Human Reliability Methods in Nuclear Industry

In 1994, a total of eight publications, on human reliability methods in nuclear industry, appeared. Converse [67] examined and evaluated the Computerized Procedures Manual (COPMA-II), a computer-based system to evaluate and mitigate the risk due to human error in nuclear power plants. Geeting and Gerrard [117] employed human error based fault tree analysis for determination of criticality frequency in nuclear/radioactive operations. Jacobsson and Svensen [167] discussed the self-reported human errors in nuclear power station control room working and Jin and Poong Hyun [169] presented optimal inspection period analysis with human error considerations.

Lin et al [208] presented the HRA conducted as part of a PSA level 1 of a nuclear power plant during low power and shut down operations to evaluate accident sequences. Ohtsuka et al [255] used training simulator studies to conduct operator performance analysis of a nuclear power plant. Soth [308] presented an extensive HRA, based on THERP, to quantify proceduralized operator actions to reflect a realistic treatment by integrating emergency operating procedures into accident sequences. Vo et al [353] developed a human reliability analysis for post–accident human actions with respect to potential seismic events at a NPP.

In 1995, John and Bindon [172] discussed the role of an operator in safety in nuclear industry and Bott [36] described the HRA methodology used in nuclear weapon disassembly process. In the same year, Reece and Hill [277] presented human performance analysis based on descriptive modeling of radiation exposure events in nuclear industry and Song-Hua et al [307] demonstrated how model-based human error taxonomy can be useful in the analysis of nuclear reactor operating events.

In 1996, a total of seven publications appeared. Antonov [11] studied the first generation Russian NPP's and provided the data on reliability and safety of these plants. Kirwan et al [185] presented a case study of human reliability analysis in a nuclear power plant and Mackieh and Cilingir [214] presented a study concerning human error events in the operation of an experimental nuclear reactor. Yoon et al [376] developed a new framework for error analysis based on an operator's decision making model. Wahlstrom [354] provided an account of safety precautions by emphasizing human and organizational error in high–risk nuclear power industry. Marko and Darula [220] presented techniques for the assessment of human error and common cause failures in a nuclear power plant. Other studies reported in 1997, related to human error events and HRA/PSA in nuclear power plants are presented in Refs. [107, 176].

In 1997, Bremner and Alsop [40] described a HRA screening method that can be used in the PSA studies and Patterson et al [264] presented a full scope simulator study of human and operator error in nuclear power plants, the means by which they can be prevented, and concepts of error detection and recovery. Also, in 1997, Nakatani et al [238] described Dynamic Interaction Analysis Support System (DIAS) consisting of a Distributed Simulation Unit and an Interaction Analyzer, which is useful in human interface evaluation of NPPs.

They showed how DIAS can be used in THERP. In the same year, Pvy and Andersson [273] reviewed the Integrated Sequence Analysis Program developed in the Nordic countries as a potential solution to HRA problems by creating realistic and more integrated ways of performing HRA. Other studies reported in 1997, related to human error events and HRA/PSA in nuclear power plants are presented in Refs. [196, 237, 358, and 378].

In 1998, a total of six publications appeared. Huang and Zhang [155] presented root cause analysis of human error events in operation and maintenance at Daya Bay NPP. Norros [250] presented the Contextual Analysis of Worker's Practices (CAWP), developed to assess and evaluate the operator's working practices in nuclear power plants. Shanley et al [293] described an approach for evaluating human actions for shutdown PSA of NPP's. Wei et al [357] discussed how human error probabilities can be deduced for PSA/HRA application using simulation studies. Yang and Yang [373] refined the human cognition reliability model and discussed the new model and its practical application to NPP. Zhao [384] explained how Human Performance Enhancement System (HPES) methodology can be applied during nuclear power plant event root cause analysis and discussed its practical implications.

In 1999, Mirabdobalqi [223] examined two power plants in Iran from the perspective of the operator and concluded that human factors principles along with enhanced control systems, and role of management in maintaining proper manpower resources, can help in minimizing human error in power plants. In the same year, Crawford [68] used HRA, PRA and THERP guidelines to determine the operator HEP's in nuclear reactors. Other studies reported in 1999, related to human error events and HRA/PSA in nuclear power plants are presented in Refs. [254, 377].

In 2000, Straeter [319] presented the analysis and assessment of the Errors of Commission (EOC) in NPP operations and Taeho and Sangman [334] described a human-machine interface study concerning control room operator action analysis. A year later, in 2001, Fang et al [100] conducted performance evaluation of a NPP with respect to human operator actions.

In 2002, Gertman et al [122] presented the results of a study concerning the contributions of human performance to risk in operating events at commercial nuclear power plants, based on Standardized Plant Analysis Risk (SPAR) models. In the same year, Jung et al [174] conducted a performance time evaluation study for human reliability analysis using full-scope simulation of nuclear power plants and Yokobayashi et al [374] discussed the modeling of human error events for a seismic PSA of nuclear power plant.

In 2003, Yong Suk et al [375] presented analysis of human error in trip events in Korean NPPs, by prioritizing human error causes according to their significance. In 2004, Lee et al [203] asserted that organizational deficiency is one of the main latent causes of human and hardware failures.

ii) Human Reliability in Aerospace and Aviation

In 1993, Graeber and Marx [129] summarized Maintenance Error Decision Aid (MEDA) to improve aircraft maintenance based on singular human events and trends. A year later, in 1994, Hibit and Marx [143] discussed how MEDA can be utilized effectively in aircraft maintenance operations.

In 1995, a total of five publications appeared. Allen and Rankin [7] further described MEDA in aircraft maintenance with respect to empirical human error data collection, analysis, and the performance shaping factors contributing to human errors. Also in 1995, Cacciabue and Cojazzi [54] conducted a case study to demonstrate the applicability of 'Human Error Risk Management in Engineering Systems' (HERMES) in the field of aviation safety using Simulation models and reliability approaches by taking into account both the dynamic and cognitive natures associated with human error. In another article in 1995, Romahn and Schafer [286] described an automated tool called 'SmartTranscript' for conducting the analysis and classification of pilot action protocols while operating the flight management system. A year later, in 1996, Noyes et al [253] discussed the design and development of flight-deck warning systems from user and human error perspectives.

In 1997, a total of five publications appeared. Schutte and Willshire [291] claimed that an effective design of the flight deck can prevent the occurrence of human errors. They presented the philosophy, status, and process of this design effort. Nelson [243] described HRA methods and frameworks specific to commercial aviation and asserted the importance of integrated design environment in an effective human performance. Drury et al [91] efforts dealt with the measuring aspect of human detection performance during aircraft visual detection. Other studies reported in 1997, related to human error events and HRA/PSA in aerospace and aviation are presented in Refs. [162, 204].

A year later, in 1998, Riley et al [284] discussed the concept of a pilot-centered autoflight system and its associated human factor issues. In the same year, Paries and Merritt [258] presented four models of human error and their potential contributions to aviation safety based on cultural and cognitive perspectives of error management.

In 1999, Cartmale and Forbes [55] demonstrated the application of Technique for Human Error Assessment to Air Traffic Control (THEA). They argued how effectively THEA can be used in procedural analysis to identify and analyze potential human errors in air traffic control (ATC). In the same year, Hopkin [153] discussed safety and human error concerns of automated air traffic management.

In 2000, Latorella and Prabhu [200] reviewed methods and approaches to identify, analyze, and prevent human errors in aircraft maintenance and inspection and Li et al [207] studied human errors from the cockpit design perspective.

In 2001, Shorrock et al [299] reported two case studies enumerating the various approaches of human error assessment and their application in U.K. air traffic management and Nechaev [240] asserted that work and rest planning is an important method of human error prevention and management.

In 2002, Shorrock and Kirwan [298] presented a human error identification technique called TRACEr-(technique for the retrospective and predictive analysis of cognitive errors) in ATC.

Other studies reported in 2002, related to human error events and HRA/PSA in aerospace and aviation are presented in Refs. [163 and 335].

iii) Human Reliability in Maintenance, Process and Manufacturing Engineering

In 1994, Bridges et al [41] presented a four step strategy to evaluate causes of human error and to improve process safety by incorporating human error occurrence in process hazard analysis. In 1995, Auflick and Auflick in [15] presented 'discriminant analysis' to determine if any relationship exists between human performance and a stuck pipe and concluded that improved situational awareness and human factors changes can lead to 70% reduction in cost associated with stuck pipes.

In 1997, Utkin et al [339] considered human behavior as fuzzy in nature, and presented mathematical analysis of computer integrated manufacturing systems.

In 1999, Yu et al [379, 380] presented HECA (human error criticality analysis), to identify potentially critical problems caused by human error in the human operated systems and Chadwell et al [56] conducted a detailed study and enumerated various causes and contributions of human error in petroleum refining industry. In the same year, an analytical framework HEDOMS (Human Error and Disturbance Occurrence in Manufacturing Systems) for assessing human reliability in manufacturing systems was developed by Barrosso and Wilson [22, 265].

In 2000, Kuo-Wei et al [197] discussed an expert system called Fault Recovery Management mechanism (FRMM) based on cognitive task analysis for maintenance tasks.

In 2002, Buehlmann and Thomas [47] studied the impact of human error on lumber yield by using computer based simulation.

In 2003, Luczak et al [212] discussed the development of an error compensating autonomous production cell to mitigate human error.

iv) Human Reliability in Structural and Construction Engineering

In 1995, El-Shahhat et al [97] proposed three approaches for addressing the issue of human errors in engineering design and construction. They, also, presented a mathematical model to investigate the effects of error, and a framework to reduce the occurrence of errors. In 1997, Gori and Muneratti [127] discussed deterministic and non deterministic aspects of the structural design process, and classified failures that occur in the structural design due to human errors.

In 2000, Chen and Zhang [57] explored the possibility of human error in estimating engineering structural reliability. The article concentrated on common human errors in knowing the distributed rule of random variables.

v) Human Reliability in Computer and Information Systems

In 1994, Cockram et al [64] discussed various faults that occur in a software generation process and the associated human error. In 1998, Abdel-Ghaffar [3] claimed that both substitution and transposition errors that are the most common errors in computing systems are caused primarily by human operators.

In 1995, Kitajma and Polson [186] presented a computational comprehension based model of skill use that provides an explanation of action slips.

In 1999, Smidts [303] established a relationship between human errors in software development and debugging activities and the software failure intensity function.

In 2000, Aitken and Melham [6] utilized interactive proof construction for identifying and classifying programming and other errors in human-computer interaction. They proposed a new taxonomy of errors for interactions and experimentally assessed its adequacy as usability metric.

vi) Human Reliability in Marine Engineering

In 1996, Archer et al [12] developed simulation models to demonstrate WinCrew, a human performance assessment tool developed by the Army Research Laboratory. In 1997, Anderson et al [9] emphasized the need for the integration of human engineering in Navy as it leads to improved performance, crew safety, and reduced workload. In 1998, Anderson et al [8] proposed reduction in manning as a possible means to improve the effectiveness and human reliability in the U. S. Navy. In the same year, Harrald et al [138] used simulation studies to model human error in a marine system. Their study was aimed to analyze and classify human and organizational error in a marine system.

vii) Other Applications

In 1996, Wall [355] discussed the inspection human error quantification and modeling to predict human reliability. In 1999, Asanov and Larichev [13] demonstrated that experienced persons are better off in coordination efforts than the non-experienced ones.

In 2003, Dhillon [78] published a book entitled "Human Reliability and Error in Medical System". This book presented various aspects including history, development of human error and reliability in the medical field as well as listed over 350 publications on human error and reliability in health care.

1.2 Motivation, Objective and Contribution of the Study

Over the years, there have been several publications on human error analysis of redundant systems [80, 95, 125, 165, 166, 218, 219, 245, 246, 256, 257, 310, 312, 328, and 329]. In all of these publications, the systems considered where assumed to have only two states. The work did not address adequately the negative impact of human errors and common-cause failures on the performance of human-machine systems that are made up of three-state devices. Also, most of the work was focussed primarily on the qualitative and quantitative treatments of human error prediction, human performance reliability and probabilistic safety assessment. This work is one of the first attempts in integrating human reliability and hardware reliability for systems made up of three-state devices.

Stochastic processes are extremely effective tools for the investigation of system reliability, in particular for the investigation of time behaviour of repairable systems. When system failure and repair rates are assumed to be exponentially distributed, it leads us to the *Markov* model. To analyse systems wherein the failure and repair rates are non-constant (non-exponential), the *Device of Stages* approach is used to formulate an approximate Markov model with the addition of *dummy* states.

The main objective of this study is to analyze the effect of human errors and common-cause failures on various redundant systems made up of three-state devices. This study investigates the performance of parallel, k-out-of-n, series, k-out-of-n-standby systems made up of three-state devices. Generalized expressions for various performance measures such as system reliability, availability, and mean time to failure are obtained. Further, the system reliability and availability are compared for all these systems to identify the best system configuration.

The main differences between this study and those of the previous researchers are as follows:

- The models that where proposed by previous researchers were for human-machine systems comprising of two-state devices only. In this study, three-state device human machine systems are investigated.

- The models in this study integrate hardware failures, critical human error, common-cause failures, and a repairable system.

- The Device of Stages – Markov method is used to investigate three-state device systems with non-constant critical human error and non-constant human error repair rates. Previous researchers used the Supplementary Variable – Markov method to study such systems.

1.3 Organization of Study

This thesis is divided into six Chapters as outlined below.

Chapter 1 gives a brief introduction to human reliability and presents a comprehensive literature review in this area for the period 1993-2004. It discusses the motivation, objective and the contribution of this study.

Chapter 2 presents a study of n-three-state device parallel system under four kinds of repair policies. The general and special case expressions for state probabilities, system reliability, the mean time to failure, the system availability and the steady state availability expressions are developed.

Chapter 3 extends the study presented in Chapter 2 to general three-state k-out-of-n and series systems. A comparison between the special cases of these models with special case models of Chapter 2 for the same values of model parameters is made.

In **Chapter 4**, investigates the effect of cold standby on the reliability of three-state device systems. A special kind of standby configuration called k-out-of-n standby system is presented. A comparison between the special cases of these models with those of the special case models of Chapters 2 and 3 for the same specified values of model parameters is made and the best system is identified.

Chapter 5 presents the study of two-unit parallel system which is subjected to critical human error and common-cause failure and repair rates that are not exponentially distributed. The *Device of Stages* approach is used to analyse the system.

Chapter 6 discusses the results of the analysis conducted in this study and presents conclusions and recommendations for further study.

Chapter 2
Three-State Device Parallel System

This Chapter presents a probabilistic analysis of generalized three-state device parallel system subjected to critical human error and common-cause failures. All the transition rates (i.e., open-mode failure rate, short-mode failure rate, critical human error rate, common-cause failure rate, and repair rates) are assumed constant. The system is analysed under four different situations: without repair, with Type I repair policy, with Type II repair policy and with Type III repair policy. Under each of these policies, two special case models are presented. The Markov method was used to develop the general and special case expressions for state probabilities, time-dependent system reliability, mean time to failure, time-dependent and steady state availability. Some plots are shown for specified values of the model parameters for the special case models.

2.1 Three-State Device Parallel System

The description of the system

The reliability block diagram of n-three state device parallel system with critical human error and common-cause failures is shown in Figure 2-1. This type of redundancy represents a system in which for its success, at least one device must operate normally, if all other devices have failed in open-mode. The system fails when a device fails in a short-mode, or a common-cause failure or a critical human error occurs or all devices fail in open-mode. All the units incorporated in this system are assumed to be identical and any operable unit is capable of keeping the system operating successfully for a given period of time.

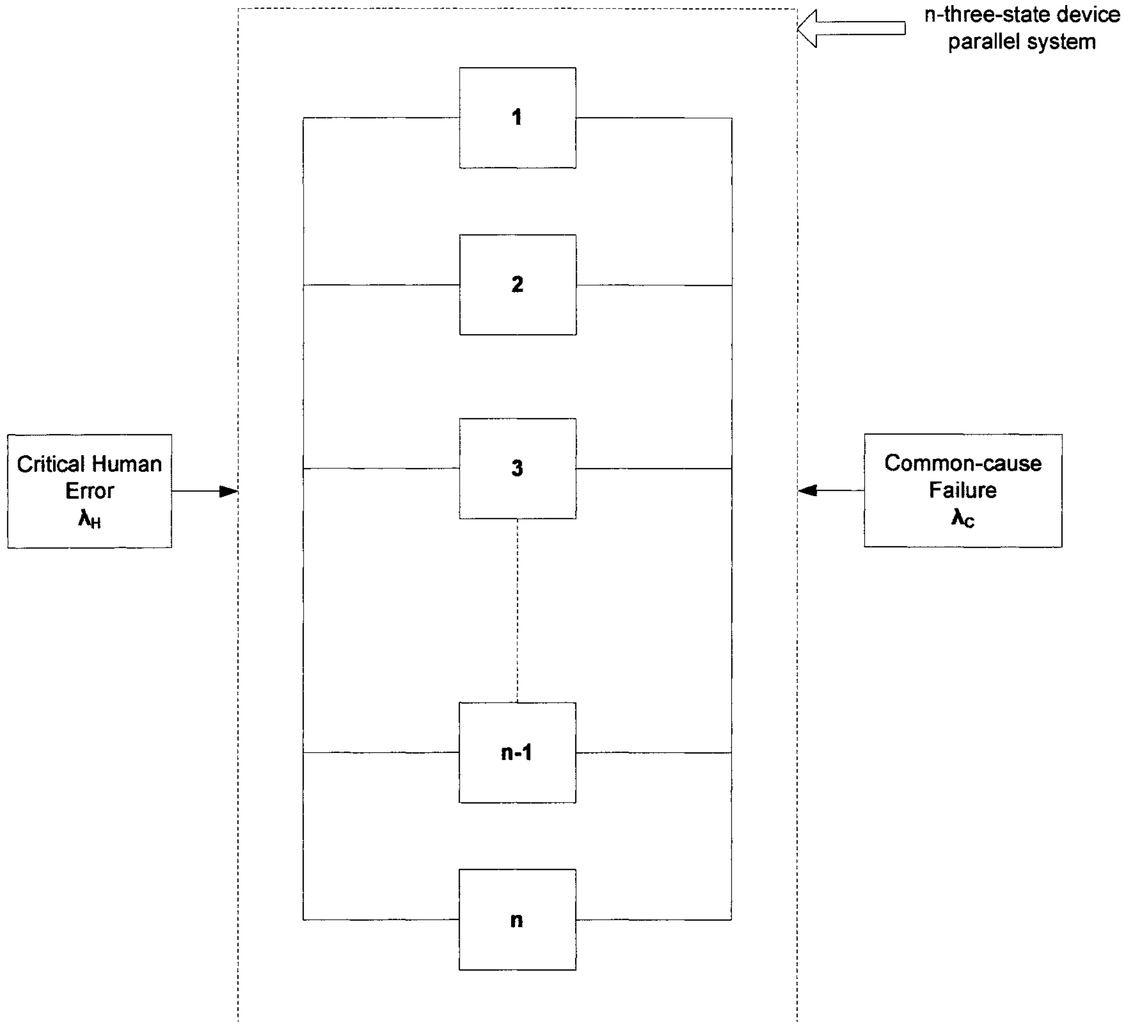

Figure 2-1: Block diagram of an n-three-state device parallel system

The system is analysed under four repair policies which are explained below.

1. **Without Repair:** This case deals with a situation in which the system can neither be repaired from a partially failed state nor from a completely failed state. Therefore, if the system fails, it cannot be restored back.

2. **Type I Repair Policy:** In this repair policy, only a partially failed system can be repaired. The repair starts when any of the system units fail and the system is restored

back to its previous state. But, if there is a complete system failure, the system cannot be restored back.

3. **Type II Repair Policy:** Unlike in Type I repair policy, this repair policy deals with the repair of a system only when the system fails completely either due to open-mode failures or a short-mode failure or a common-cause failure or a critical human error occurs. The failed system is restored back to its initial operating state (i.e., system state at time $t = 0$).

4. **Type III Repair Policy:** In this case, in addition to the partially failed system being repaired to its previous operating state, the completely failed system is also repaired back to its initial operating state (i.e., system state at time $t = 0$). Thus, Type I repair and Type II repair policies as discussed above are special cases of Type III repair policy.

Figure 2-2 shows the state transition diagram for Figure 2-1 under without repair policy. Similarly, Figure 2-3 shows the state transition diagram for Figure 2-1 under Type I repair policy, Figure 2-4 shows the state transition diagram for Figure 2-1 under Type II repair policy, and Figure 2-5 shows the state-transition diagram for Figure 2-1 under Type III repair policy. Numerals and single letters in boxes denote the system states. Other symbols used in the diagram are defined in the notation section.

Assumptions

The following assumptions are associated with the analyses in this study:

1. The system is composed of ‘n’ independent and identical devices in parallel.
2. Each unit (three-state device) can fail either in open-mode or in short-mode. These two failures are mutually exclusive of one another.
3. A critical human error or a common-cause failure can occur and trigger the whole system failure when two or more devices are operating.

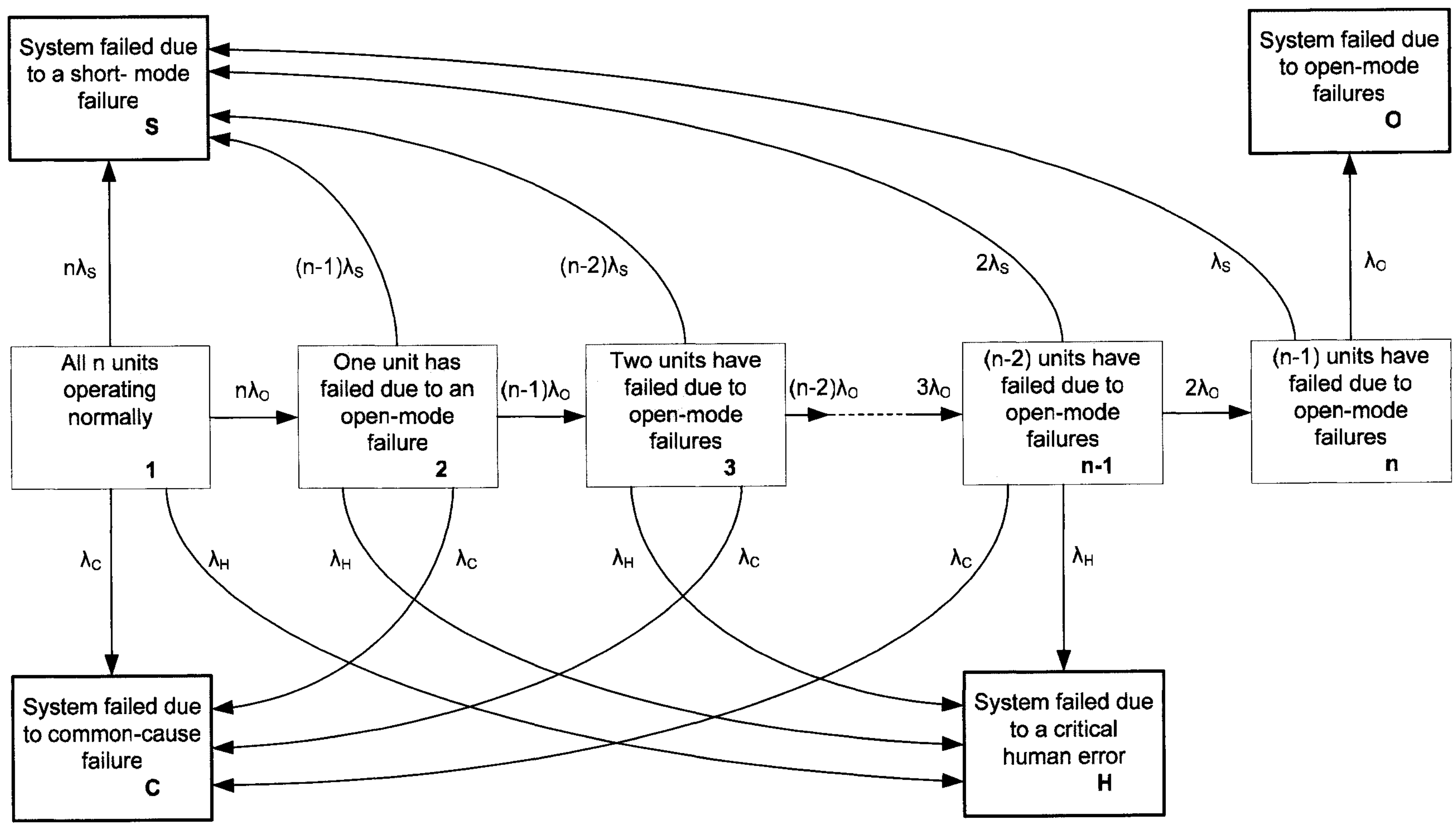

Figure 2-2: State transition diagram for an n-three-state device parallel system under without repair policy

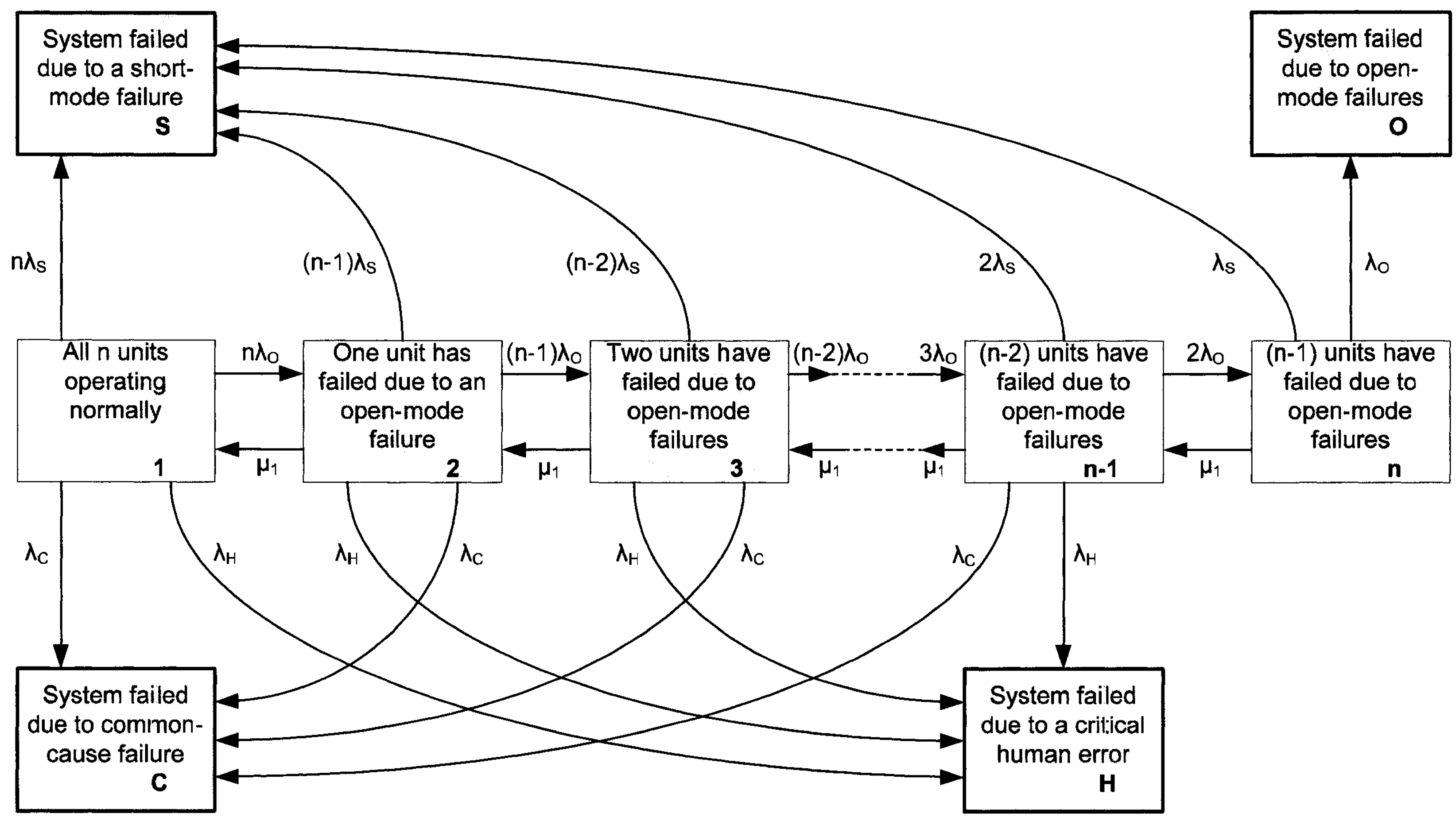

Figure 2-3: State transition diagram for an n-three-state device parallel system under Type I repair policy

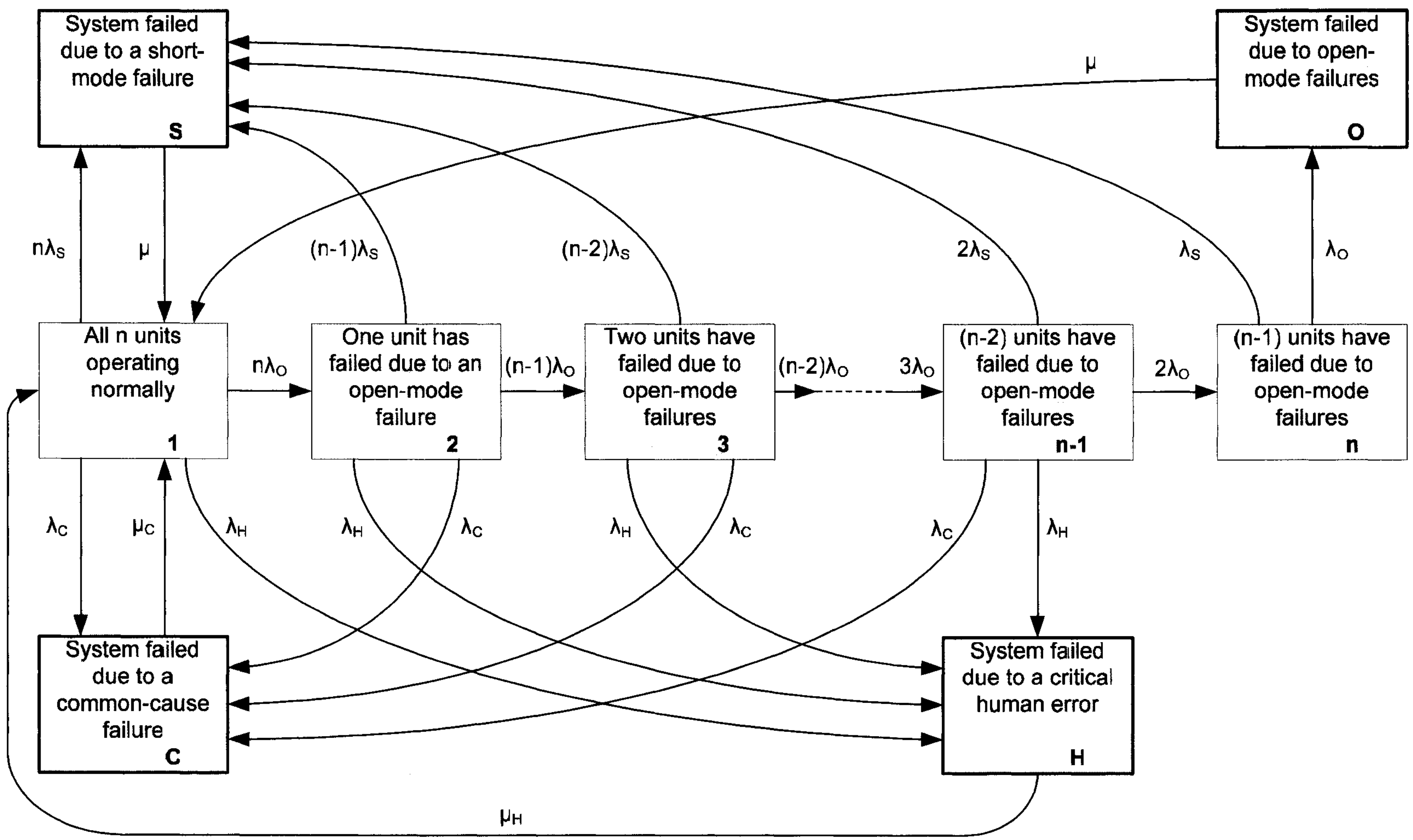

Figure 2-4: State transition diagram for an n-three-state device parallel system under Type II repair policy

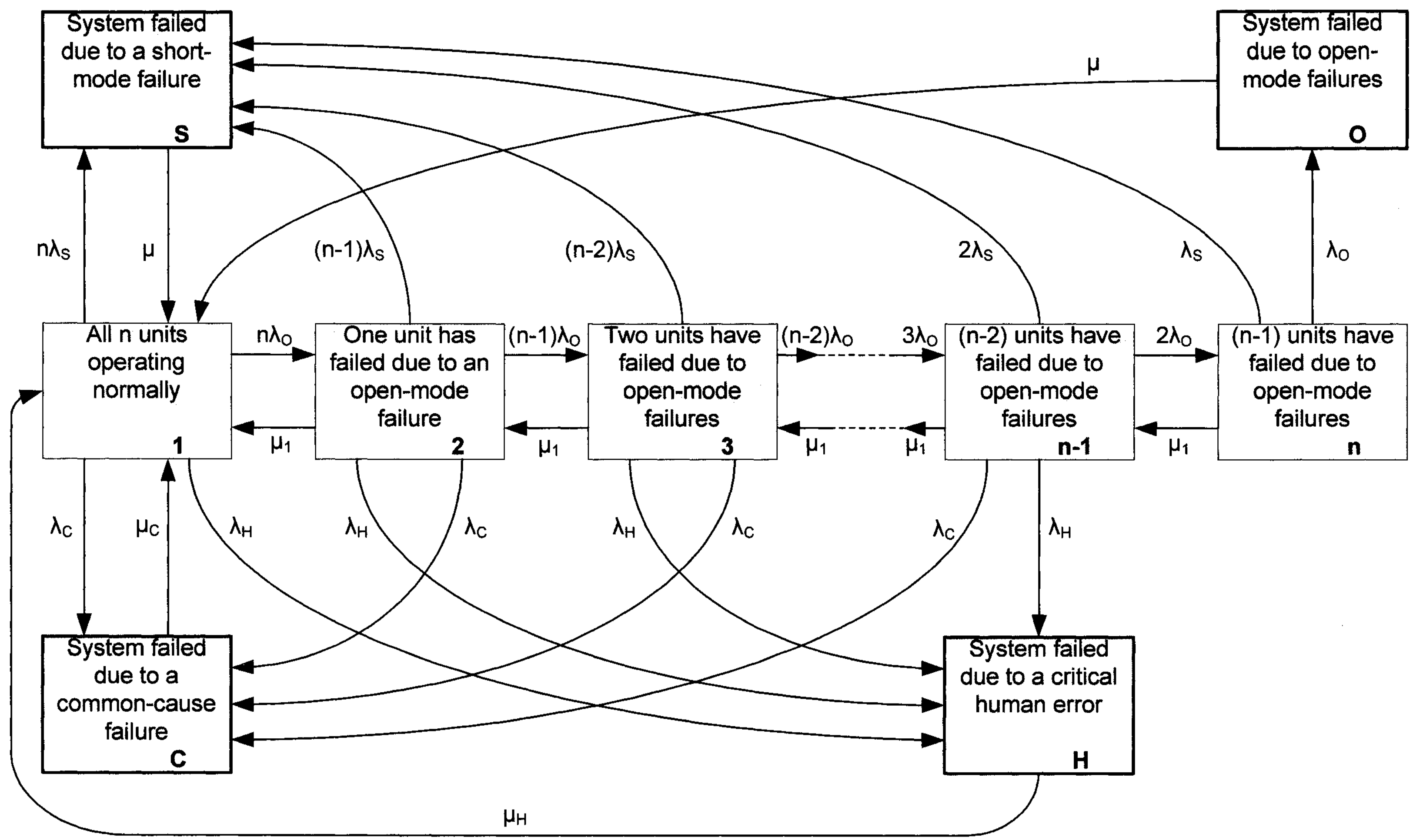

Figure 2-5: State transition diagram for an n-three-state device parallel system under Type III repair policy

4. All failures are statistically independent.
5. The failure rates for all the failures are constant.
6. The repaired unit/system is as good as new.
7. The repair rates for all types of failures are constant.
8. Repair rates are the same for completely failed system due to open-mode or short-mode failure.
9. All repair rates are the same for the partially failed system.
10. Common-cause failure rates are the same for the partially or fully operating system.
11. Critical human error rates are the same for the partially or fully operating system.

Notation

The following notation is associated with models presented in this Chapter:

n	Number of units.
t	Time.
s	Laplace transform variable.
L^{-1}	Inverse Laplace transform.
λ_O	Constant open-mode failure rate of the operating unit (three-state device).
λ_S	Constant short-mode failure rate of the operating unit (three-state device).
λ_C	Constant common-cause failure rate of the system.
λ_H	Constant critical human error rate of the system.
μ	Constant repair rate of the system from open-mode or short-mode failure.
μ_1	Constant repair rate of the unit (three-state device) for a partially failed system.
μ_C	Constant repair rate of the system when it failed due to a common-cause failure.
μ_H	Constant repair rate of the system when it failed due to a critical human error.
$P_i(t)$	The probability that the system is in state i at time t; for $i = 1, 2, 3.....$n.

$P_j(t)$ The failed system state probability; for $j = C$ (due to a common-cause failure), $j = H$ (due to a critical human error), $j = S$ (due to a short-mode failure), $j = O$ (due to an open-mode failure).

$p_i(s)$ The Laplace transform of the state probability, for i=1, 2, 3…..n.

$p_j(s)$ The Laplace transform of the failed system state probability; for $j = C$ (due to a common-cause failure), $j = H$ (due to a critical human error), $j = S$ (due to a short-mode failure), $j = O$ (due to an open-mode failure).

P_i The steady state probability of the system being in state i, for i=1, 2, 3 …..n.

P_j The failed system steady state probability; for $j = C$ (due to a common-cause failure), $j = H$ (due to a critical human error), $j = S$ (due to a short-mode failure), $j = O$ (due to an open-mode failure).

$R(s)$ Laplace transform of the system reliability.

$R(t)$ System reliability at time t.

$MTTF$ System mean time to failure.

$A(s)$ Laplace transform of the system availability.

$A(t)$ System availability at time t.

AV_{ss} System steady state availability.

2.2 Three-State Device Parallel System without Repair

In this section, n-three-state device parallel system without repair and its two special cases: two-unit parallel system without repair and three-unit parallel system without repair are presented.

2.2.1 General parallel system without repair

The state transition diagram for the general model without repair is shown in Figure 2-2. Using the Markov method, the system of differential equations associated with Figure 2-2 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) \tag{2-1}$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H\right]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) \tag{2-2}$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -\left[\lambda_O + \lambda_S\right]P_n(t) + 2\lambda_O P_{n-1}(t) \tag{2-3}$$

$$\frac{dP_O(t)}{dt} = \lambda_O P_n(t) \tag{2-4}$$

$$\frac{dP_S(t)}{dt} = \lambda_S \sum_{i=1}^{n}(n-i+1)P_i(t) \tag{2-5}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-1} P_i(t) \tag{2-6}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-1} P_i(t) \tag{2-7}$$

At time t=0, $P_1(0)$= 1, and all other initial state probabilities are equal to zero. Taking Laplace transforms of Equations (2-1) – (2-7) and solving the resulting equations, we get the following set of equations:

$$p_1(s) = \frac{1}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{2-8}$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_O}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H}\right]p_{i-1}(s) \quad \text{for all } i = 2 \text{ to } (n-1) \tag{2-9}$$

$$p_n(s) = \left[\frac{2\lambda_O}{s + \lambda_O + \lambda_S}\right]p_{n-1}(s) \tag{2-10}$$

$$p_O(s) = \frac{\lambda_O}{s} p_n(s) \tag{2-11}$$

$$p_S(s) = \frac{\lambda_S}{s}\sum_{i=1}^{n}(n-i+1)p_i(s) \tag{2-12}$$

$$p_C(s)=\frac{\lambda_C}{s}\sum_{i=1}^{n-1}p_i(s) \tag{2-13}$$

$$p_H(s)=\frac{\lambda_H}{s}\sum_{i=1}^{n-1}p_i(s) \tag{2-14}$$

By taking the inverse Laplace transforms of Equations (2-8) – (2-14), the time-dependent state probabilities can be obtained. The time-dependent system reliability is given by

$$R(t)=\sum_{i=1}^{n}P_i(t)=L^{-1}\left[\sum_{i=1}^{n}p_i(s)\right] \tag{2-15}$$

2.2.2 Special case model 2-A: Two-unit parallel system without repair

The state transition diagram for a two-unit parallel system without repair is shown in Figure 2-6. By setting n=2 in Figure 2-2, from Equations (2-8) – (2-14), we get the following set of equations:

$$p_1(s)=\frac{1}{s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H} \tag{2-16}$$

$$p_2(s)=\frac{2\lambda_O}{(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{2-17}$$

$$p_O(s)=\frac{2\lambda_O^{\,2}}{s(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{2-18}$$

$$p_S(s)=\frac{2\lambda_S(s+2\lambda_O+\lambda_S)}{s(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{2-19}$$

$$p_C(s)=\frac{\lambda_C}{s(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{2-20}$$

$$p_H(s)=\frac{\lambda_H}{s(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{2-21}$$

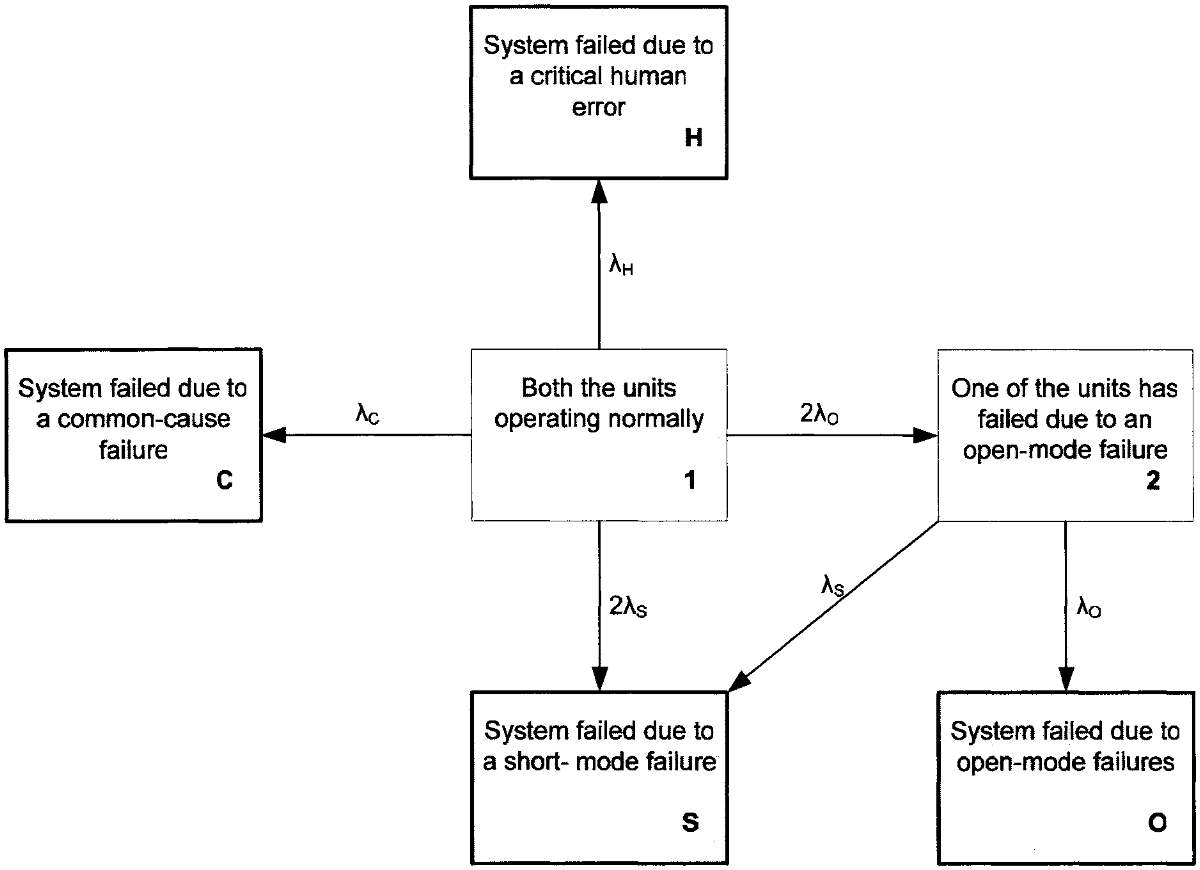

Figure 2-6: State transition diagram for a two-unit parallel system without repair

By taking inverse Laplace transforms of Equations (2-16) – (2-21), the time-dependent state probabilities can be obtained. By adding Equations (2-16) and (2-17) and taking inverse Laplace transforms, we get the following expression for the system reliability:

$$R(t)=\frac{1}{\lambda_O+\lambda_S+\lambda_C+\lambda_H}\left[\frac{\lambda_S+\lambda_C+\lambda_H-\lambda_O}{e^{(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)t}}+\frac{2\lambda_O}{e^{(\lambda_O+\lambda_S)t}}\right] \tag{2-22}$$

The system mean time to failure is given by

$$MTTF=\int_0^{\infty}R(t)\,dt=\frac{1}{\lambda_O+\lambda_S+\lambda_C+\lambda_H}\left[\frac{\lambda_S+\lambda_C+\lambda_H-\lambda_O}{2\lambda_O+2\lambda_S+\lambda_C+\lambda_H}+\frac{2\lambda_O}{\lambda_O+\lambda_S}\right] \tag{2-23}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 2-7. For the specified values of the model parameters, the plots of Equation (2-22) are shown in Figure 2-8. Similarly, for the specified values of the model parameters, the plots of Equation (2-23) are shown in Figure 2-9.

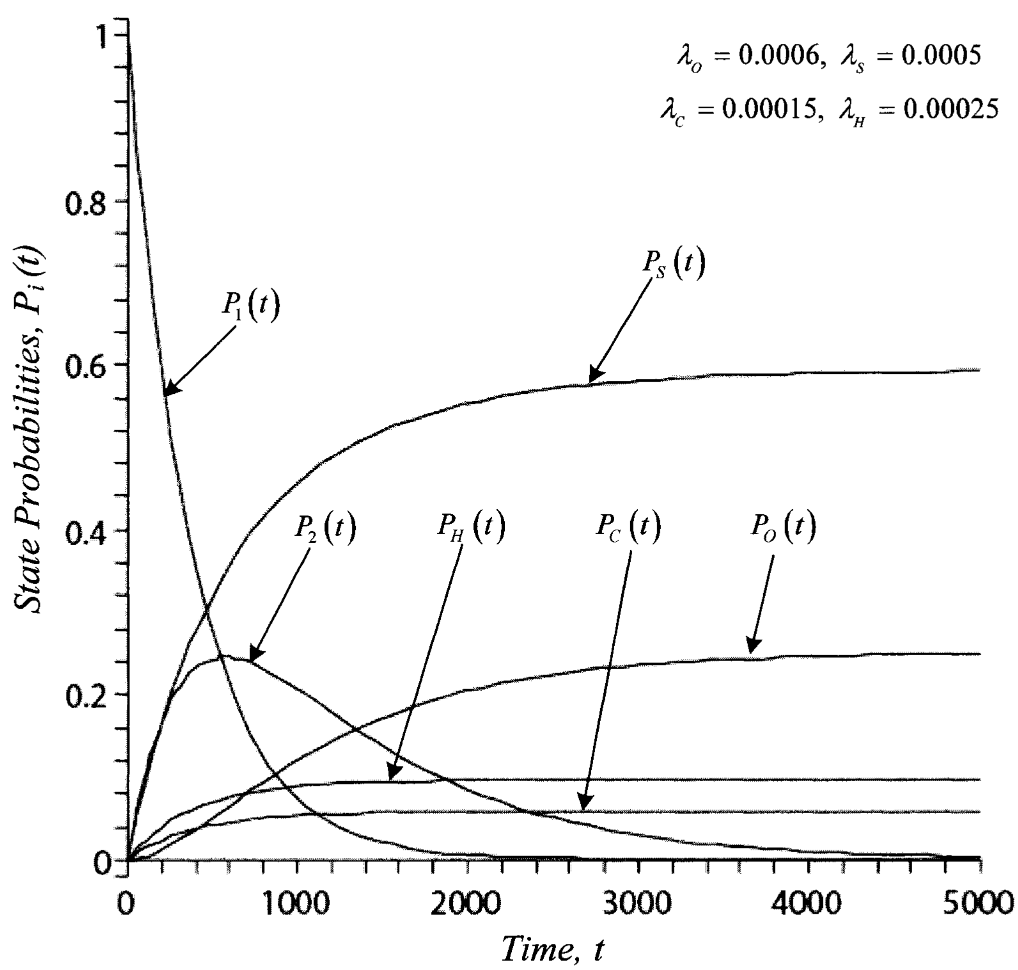

Figure 2-7: State probability plots of a two-unit parallel system without repair

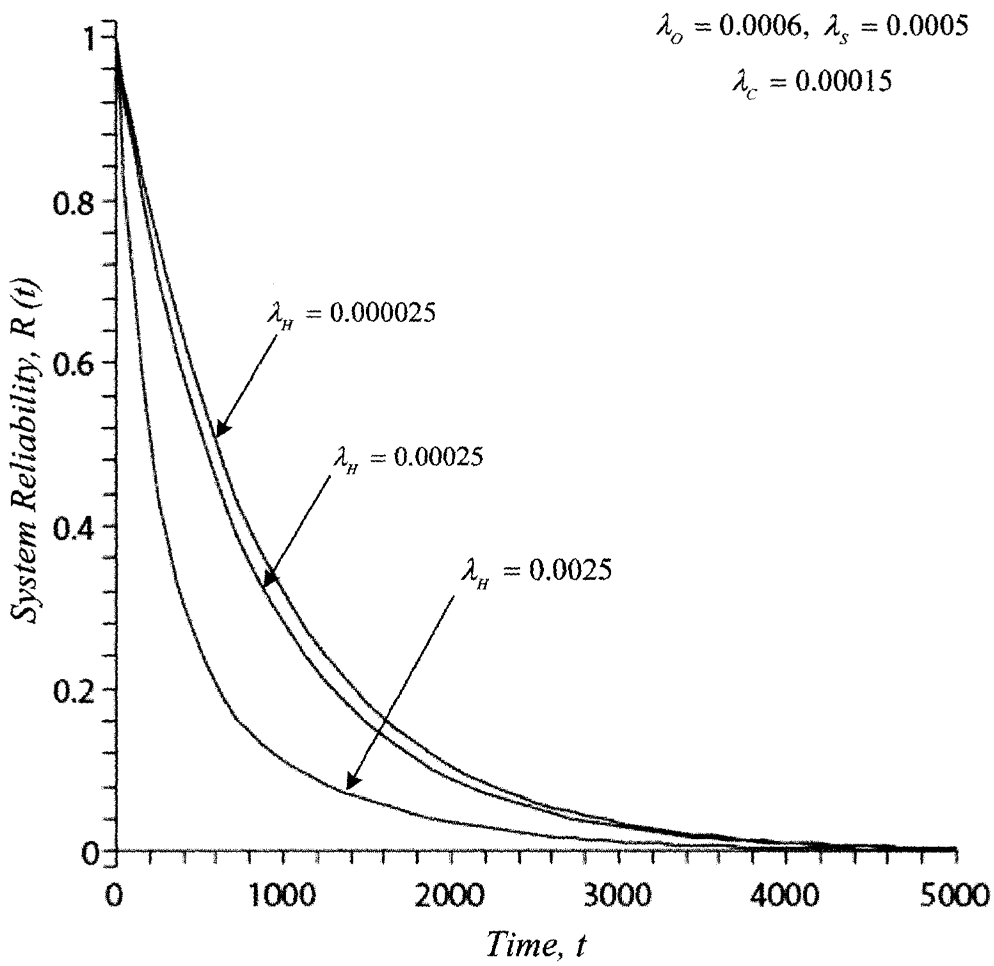

Figure 2-8: Reliability plots of a two-unit parallel system without repair

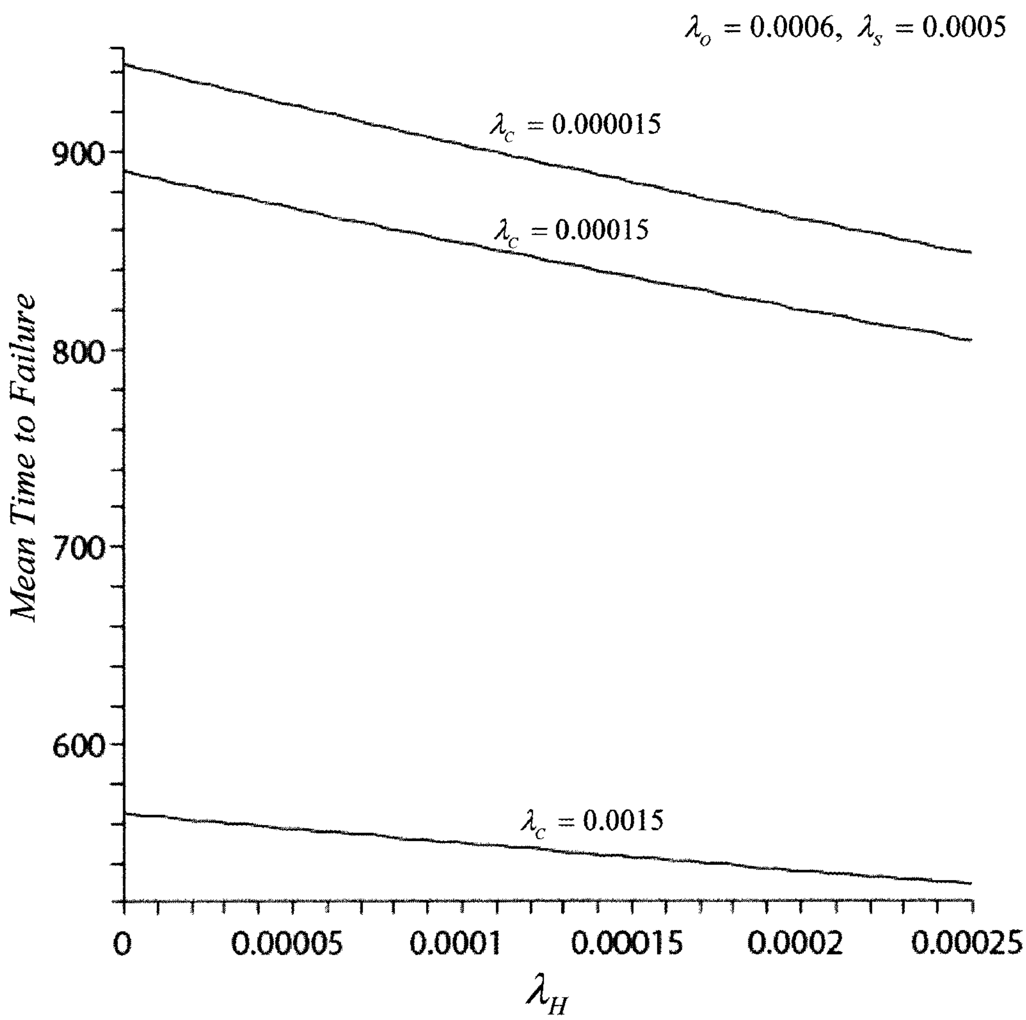

Figure 2-9: Mean time to failure plots of a two-unit parallel system without repair

2.2.3 Special case model 2-B: Three-unit parallel system without repair

The state transition diagram for a three-unit parallel system without repair is shown in Figure 2-10.

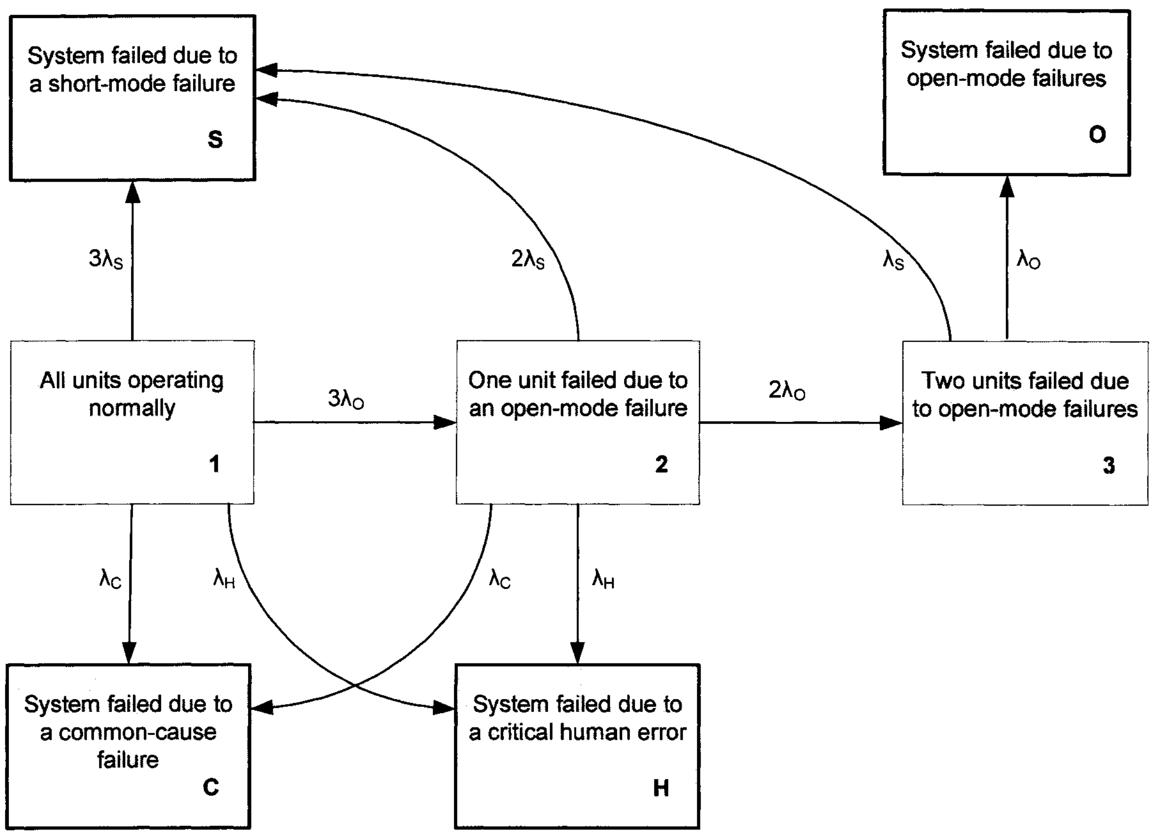

Figure 2-10: State transition diagram for a three-unit parallel system without repair

By substituting n=3 in Equations (2-8) – (2-14), we obtain the following set of equations:

$$p_1(s) = \frac{1}{s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H} \tag{2-24}$$

$$p_2(s) = \left[\frac{3\lambda_O}{s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H}\right]\left[\frac{1}{s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H}\right] \tag{2-25}$$

$$p_3(s)=\left[\frac{2\lambda_O}{s+\lambda_O+\lambda_S}\right]\left[\frac{3\lambda_O}{s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H}\right]\left[\frac{1}{s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H}\right] \tag{2-26}$$

$$p_O(s)=\frac{\lambda_O}{s}\left[\frac{2\lambda_O}{s+\lambda_O+\lambda_S}\right]\left[\frac{3\lambda_O}{s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H}\right]\left[\frac{1}{s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H}\right] \tag{2-27}$$

$$p_S(s)=\frac{3\lambda_S}{s}\left[\frac{(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+2\lambda_O(s+2\lambda_O+\lambda_S)}{(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)}\right] \tag{2-28}$$

$$p_C(s)=\frac{\lambda_C}{s}\left[\frac{s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)}\right] \tag{2-29}$$

$$p_H(s)=\frac{\lambda_H}{s}\left[\frac{s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)}\right] \tag{2-30}$$

By taking the inverse Laplace transforms of Equations (2-24) – (2-30), we obtain the expressions for the time-dependent state probabilities. By adding Equations (2-24) – (2-26) and then taking inverse Laplace transforms, the following expression for the system reliability results:

$$\begin{aligned}R(t)=L^{-1}\left[\sum_{i=1}^{3}p_i(s)\right]&=\frac{2\lambda_O(\lambda_O-\lambda_S-\lambda_C-\lambda_H)+\lambda_S(2\lambda_S+\lambda_C+\lambda_H)}{(\lambda_O+\lambda_S)(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)e^{(3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)t}}\\&+\frac{3\lambda_O(\lambda_S+\lambda_C+\lambda_H-\lambda_O)}{(\lambda_O+\lambda_S)(\lambda_O+\lambda_S+\lambda_C+\lambda_H)e^{(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)t}}\\&+\frac{6\lambda_O^{\,2}}{(\lambda_O+\lambda_S+\lambda_C+\lambda_H)(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)e^{(\lambda_O+\lambda_S)t}}\end{aligned} \tag{2-31}$$

The system mean time to failure is obtained by

$$MTTF=\int_0^\infty R(t)\,dt=\frac{(\lambda_O+\lambda_S)(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+3\lambda_O(3\lambda_O+\lambda_S)}{(\lambda_O+\lambda_S)(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)} \tag{2-32}$$

In order to make a consistent comparison with the two-unit parallel system and other models, same values of the model parameters have been taken for all the models in this study unless otherwise stated. For the specified values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 2-11. For the given values of the model parameters, the plots of Equation (2-31) are shown in Figure 2-12. Similarly, for the specified values of the model parameters, the plots of Equation (2-32) are shown in Figure 2-13.

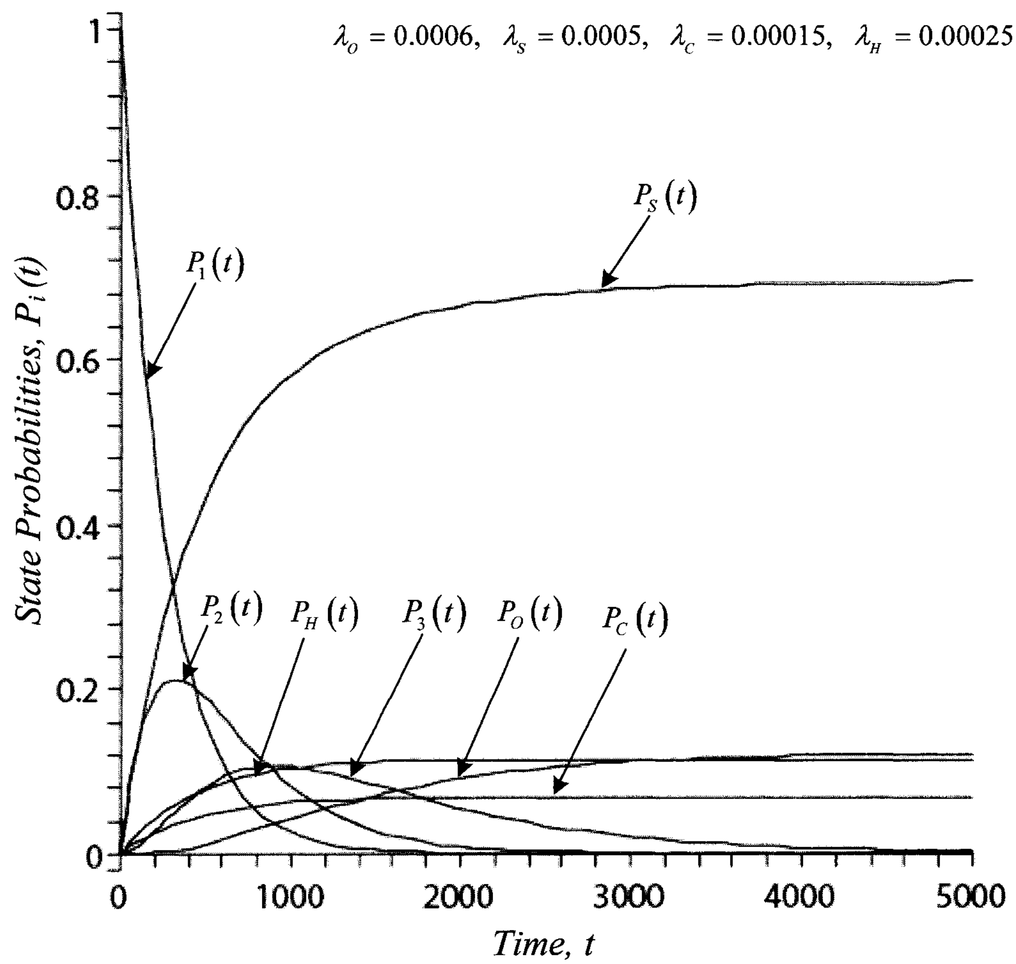

Figure 2-11: State probability plots of a three-unit parallel system without repair

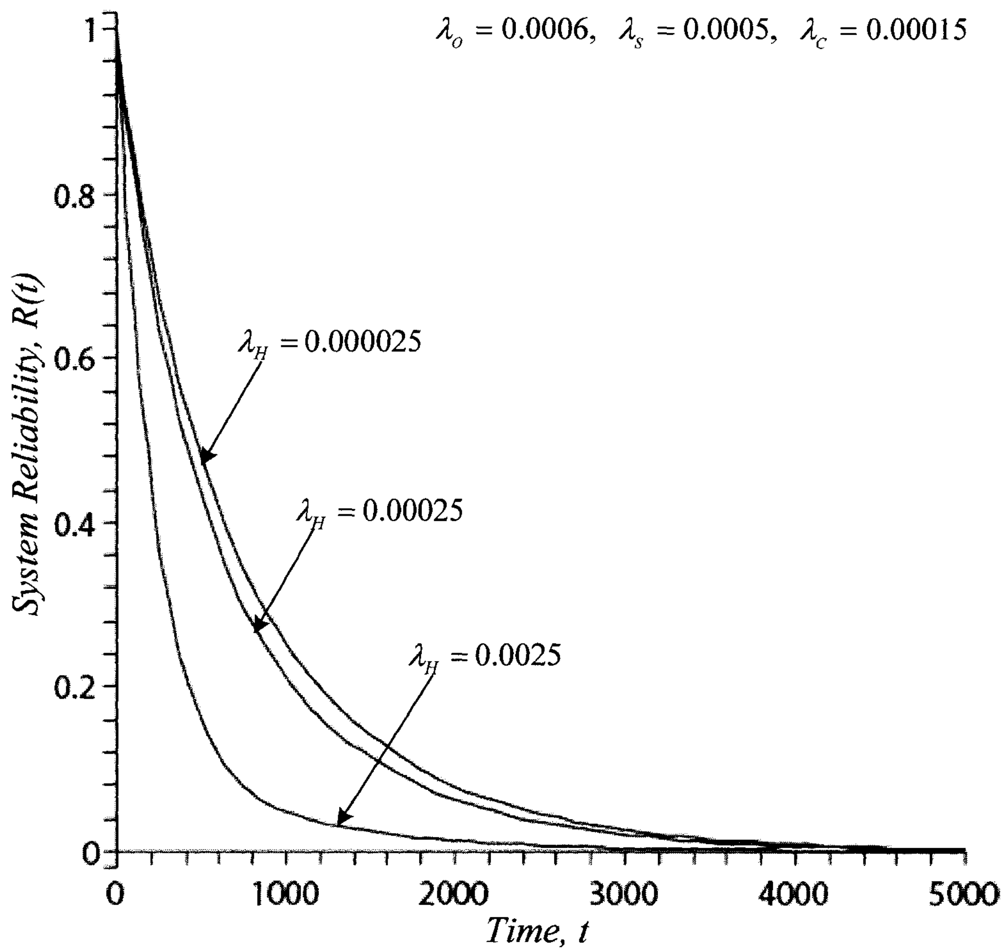

Figure 2-12: Reliability plots of a three-unit parallel system without repair

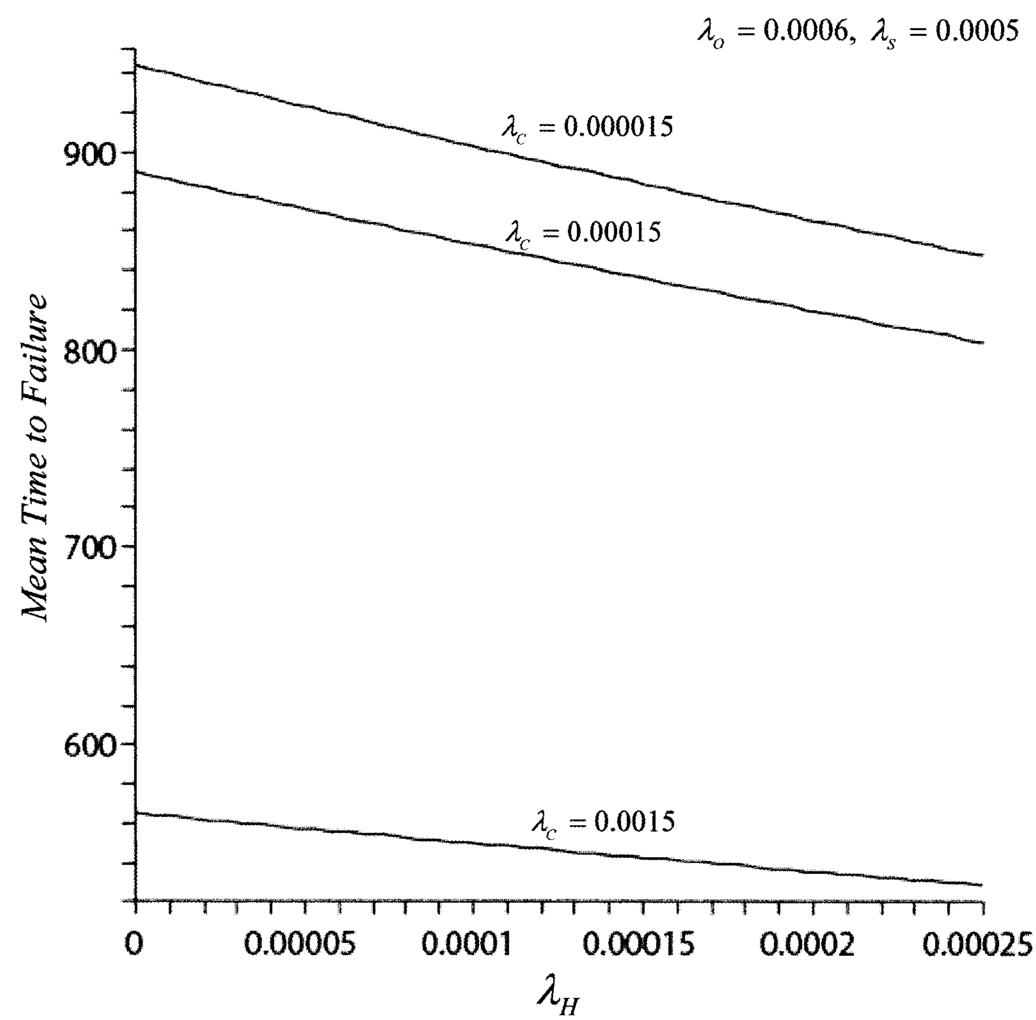

Figure 2-13: Mean time to failure plots of a three-unit parallel system without repair

2.3 Three-State Device Parallel System under Type I Repair Policy

In this section, the n-three-state device parallel system under Type I repair policy and its two special cases: two-unit parallel system with Type I repair policy and three-unit parallel system with Type I repair policy are presented. Under this repair policy, the partially failed system can be repaired back to its previous operating state. If the system fails completely, it cannot be restored. The system reliability and mean time to failure are compared for all these systems

2.3.1 General parallel system under Type I repair policy

Figure 2-3 shows the state transition diagram of n-three-state device parallel system under Type I repair policy. The following system of differential equations is associated with Figure 2-3:

$$\frac{dP_1(t)}{dt} = -[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H]P_1(t) + \mu_1 P_2(t) \tag{2-33}$$

$$\frac{dP_i(t)}{dt} = -[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) + \mu_1 P_{i+1}(t) \tag{2-34}$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -[\lambda_O + \lambda_S + \mu_1]P_n(t) + 2\lambda_O P_{n-1}(t) \tag{2-35}$$

$$\frac{dP_O(t)}{dt} = \lambda_O P_n(t) \tag{2-36}$$

$$\frac{dP_S(t)}{dt} = \lambda_S \sum_{i=1}^{n}(n-i+1)P_i(t) \tag{2-37}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-1} P_i(t) \tag{2-38}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-1} P_i(t) \tag{2-39}$$

At time t=0, $P_1(0)$ = 1, and all other initial state transition probabilities are equal to zero. By taking the Laplace transforms of Equations (2-33) – (2-39) and solving the resulting equations, following set of equations result:

$$p_1(s) = \frac{1+\mu_1 p_2(s)}{s+n\lambda_O+n\lambda_S+\lambda_C+\lambda_H} \tag{2-40}$$

$$p_i(s) = \frac{(n-i+2)\lambda_O p_{i-1}(s)+\mu_1 p_{i+1}(s)}{s+(n-i+1)\lambda_O+(n-i+1)\lambda_S+\lambda_C+\lambda_H+\mu_1} \quad \text{for all } i=2 \text{ to } (n-1) \tag{2-41}$$

$$p_n(s) = \frac{2\lambda_O p_{n-1}(s)}{s+\lambda_O+\lambda_S+\mu_1} \tag{2-42}$$

$$p_O(s) = \frac{\lambda_O p_n(s)}{s} \tag{2-43}$$

$$p_S(s) = \frac{\lambda_S}{s}\sum_{i=1}^{n}(n-i+1)p_i(s) \tag{2-44}$$

$$p_C(s) = \frac{\lambda_C}{s}\sum_{i=1}^{n-1}p_i(s) \tag{2-45}$$

$$p_H(s) = \frac{\lambda_H}{s}\sum_{i=1}^{n-1}p_i(s) \tag{2-46}$$

The time-dependent state probabilities can be obtained by computing the inverse Laplace transforms of Equations (2-40) – (2-46). The time-dependent system reliability is given by

$$R(t) = L^{-1}\left[\sum_{i=1}^{n}p_i(s)\right] \tag{2-47}$$

The system mean time to failure is obtained as

$$MTTF = \lim_{s\to 0}\left[\sum_{i=1}^{n}p_i(s)\right] \tag{2-48}$$

2.3.2 Special case model 2-C: Two-unit parallel system with Type I repair

The state transition diagram for a two-unit parallel system under Type I repair policy is shown in Figure 2-14.

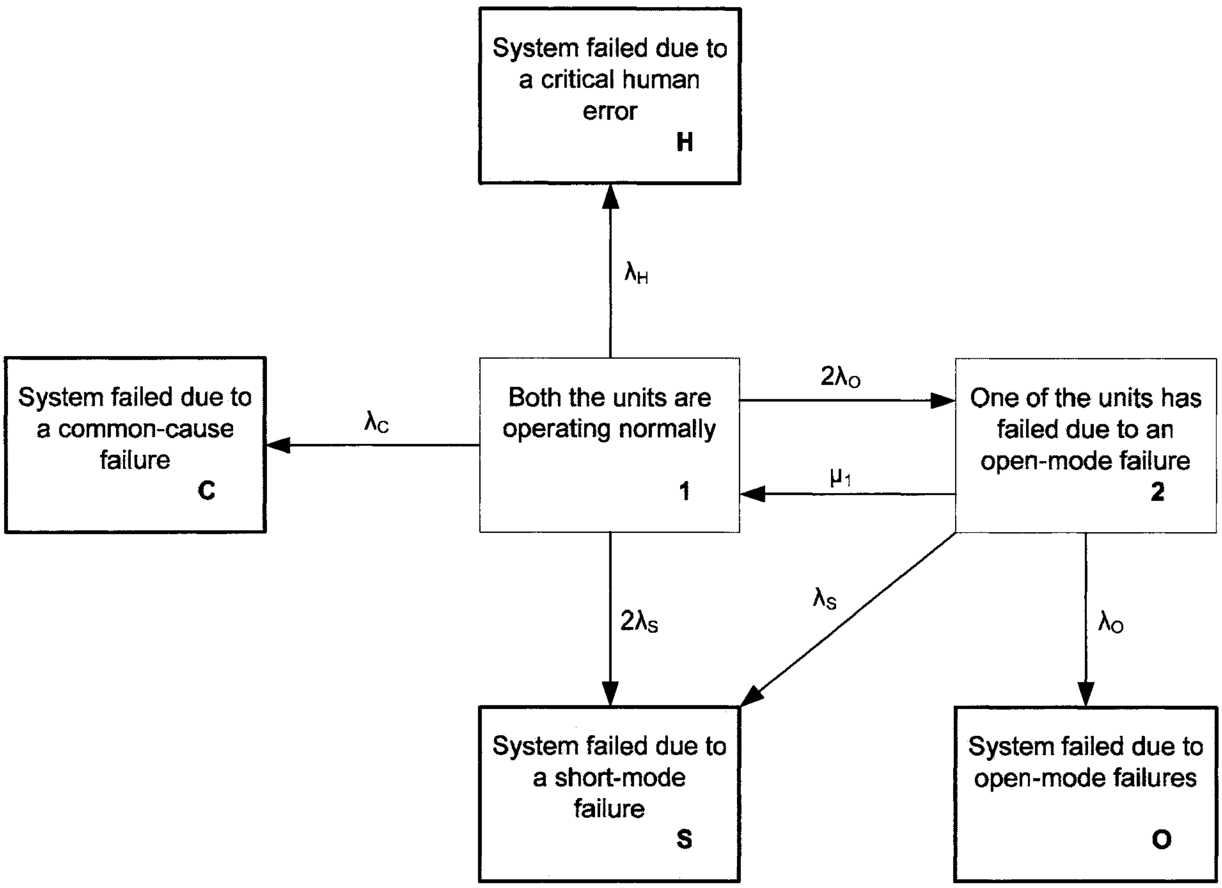

Figure 2-14: State transition diagram for a two-unit parallel system with Type I repair policy

For n=3 in Figure 2-3, from Equations (2-40) – (2-46), we get the following set of equations:

$$p_1(s) = \frac{s+\lambda_O+\lambda_S+\mu_1}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O} \quad (2\text{-}49)$$

$$p_2(s) = \frac{2\lambda_O}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O} \quad (2\text{-}50)$$

$$p_O(s) = \frac{2\lambda_O^2}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O\right]} \quad (2\text{-}51)$$

$$p_S(s) = \frac{2\lambda_S\left(s+2\lambda_O+\lambda_S+\mu_1\right)}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O\right]} \quad (2\text{-}52)$$

$$p_C(s) = \frac{\lambda_C\left(s+\lambda_O+\lambda_S+\mu_1\right)}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O\right]} \quad (2\text{-}53)$$

$$p_H(s) = \frac{\lambda_H\left(s+\lambda_O+\lambda_S+\mu_1\right)}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O\right]} \quad (2\text{-}54)$$

By taking the inverse Laplace transforms of Equations (2-49) – (2-54), the time-dependent transition state probabilities can be obtained. The time-dependent system reliability can be obtained by adding Equations (2-49) and (2-50) and then taking inverse Laplace transform which can be written as follows:

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\frac{s+3\lambda_O+\lambda_S+\mu_1}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O}\right] \quad (2\text{-}55)$$

The system mean time to failure is given by

$$MTTF = \lim_{s\to 0}\left[\frac{s+3\lambda_O+\lambda_S+\mu_1}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O}\right] \quad (2\text{-}56)$$

For the specified values of the model parameters, the plots of the state probabilities as a function of time, t, are shown in Figure 2-15, the plots of Equation (2-55) are shown in Figure 2-16 and the plots of Equation (2-56) are shown in Figure 2-17.

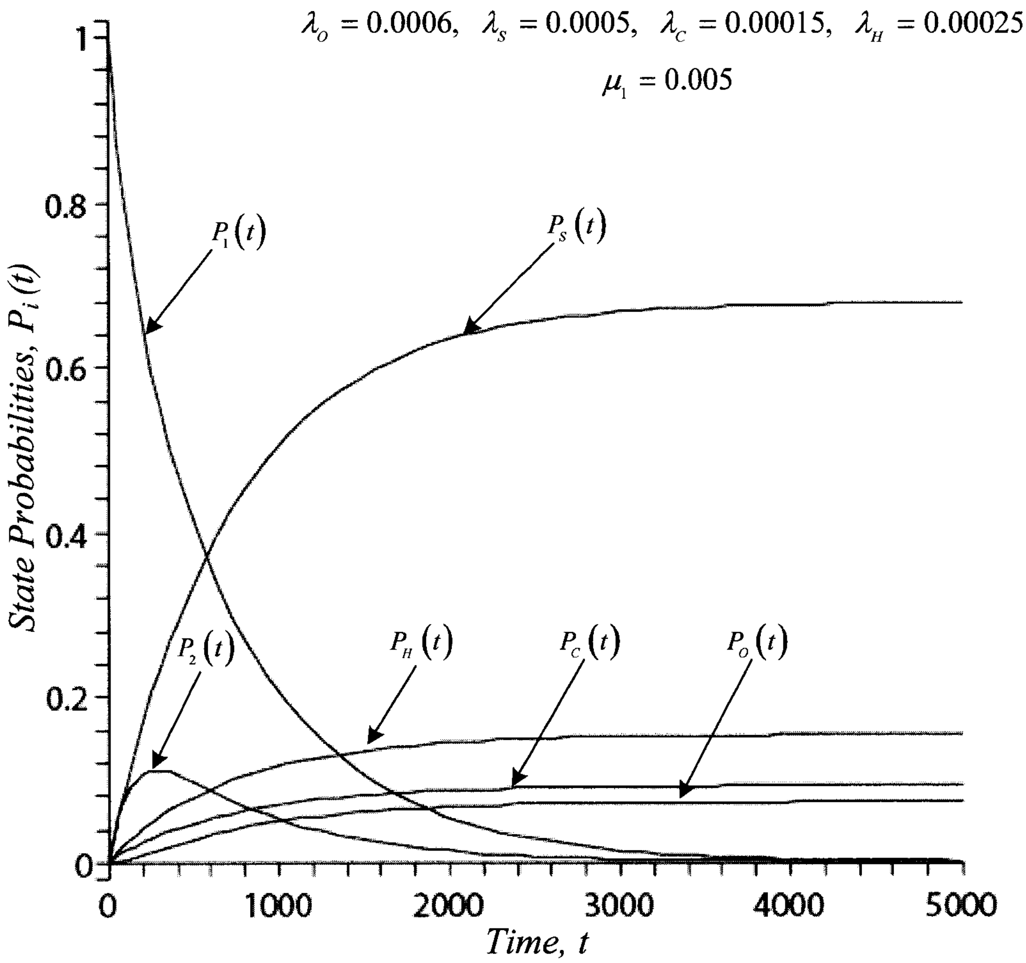

Figure 2-15: State probability plots of a two-unit parallel system with Type I repair policy

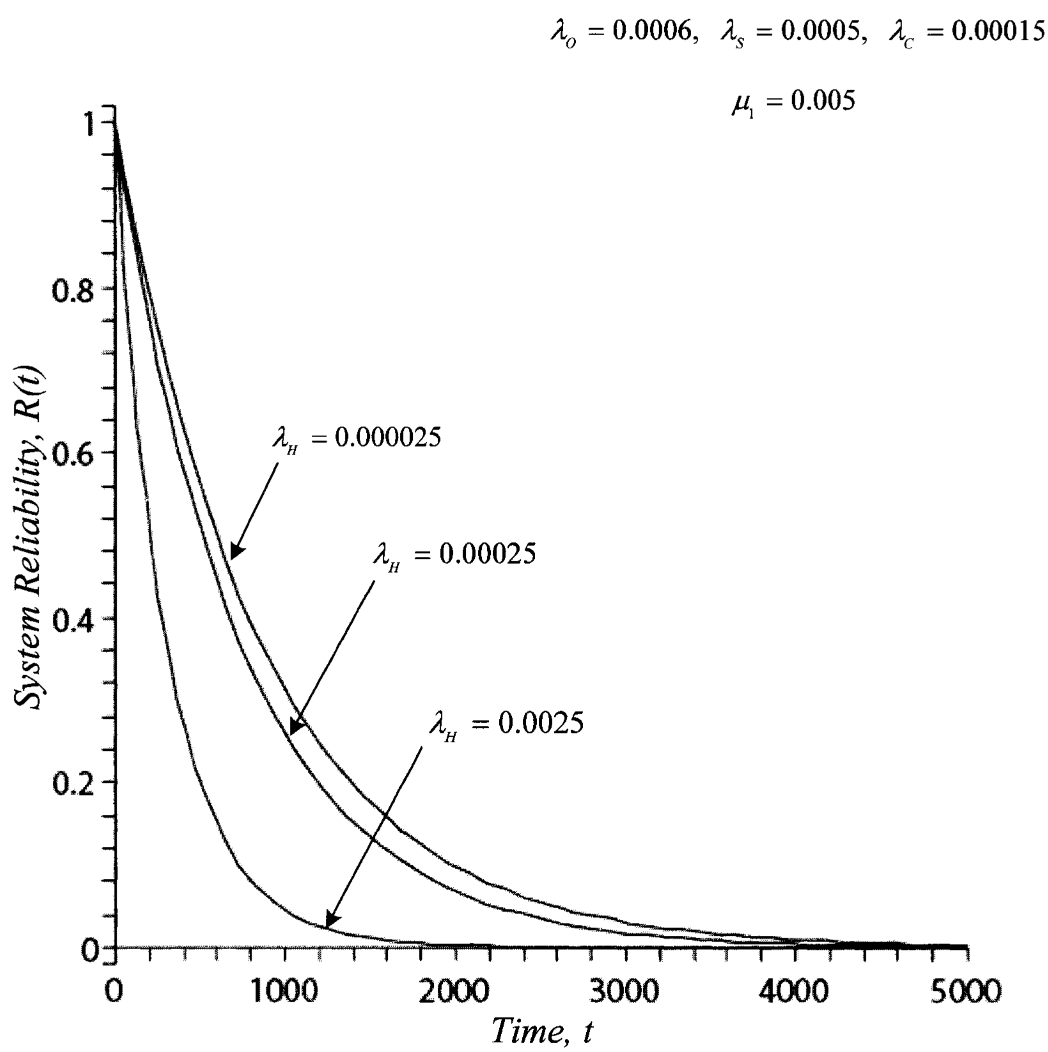

Figure 2-16: Reliability plots of a two-unit parallel system with Type I repair policy

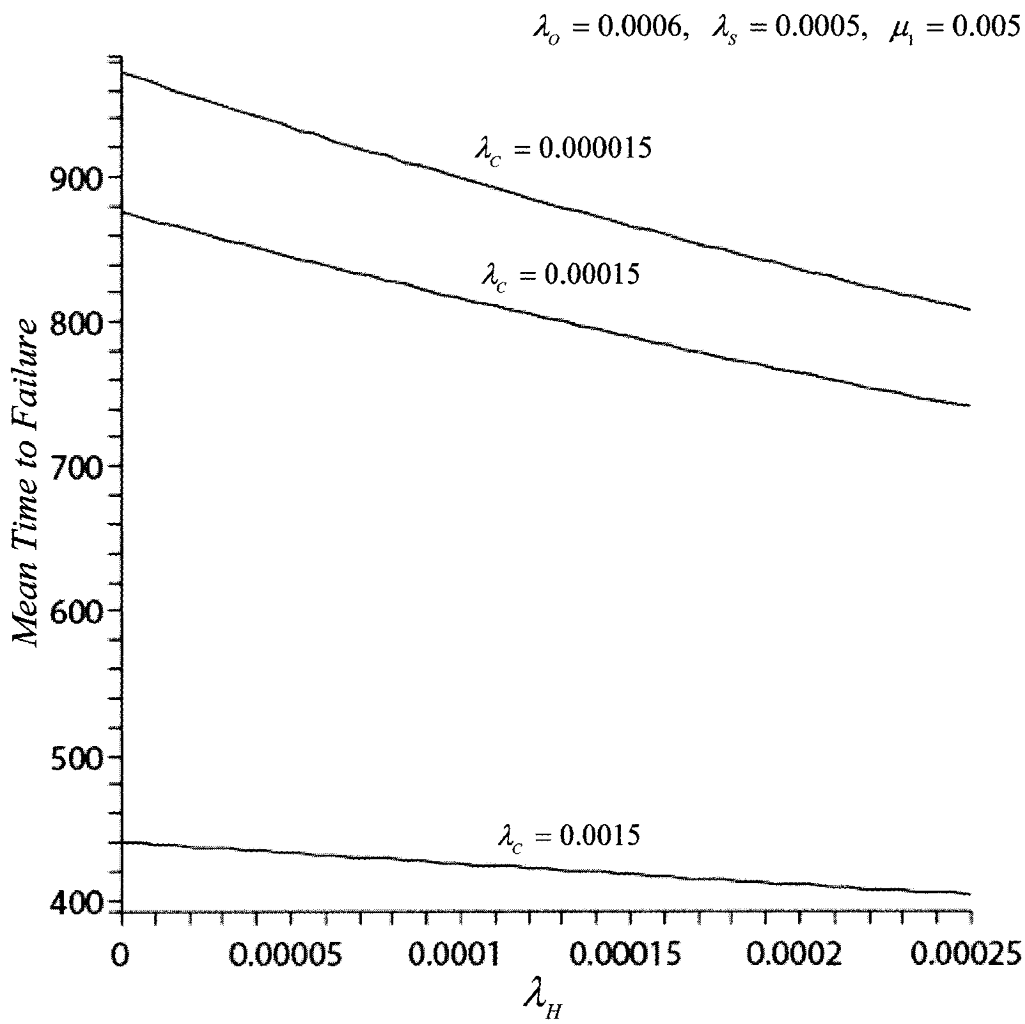

Figure 2-17: Mean time to failure plots of a two-unit parallel system with Type I repair policy

2.3.3 Special case model 2-D: Three-unit parallel system with Type I repair

Figure 2-18 depicts the state transition diagram for a three-unit parallel system under Type I repair policy.

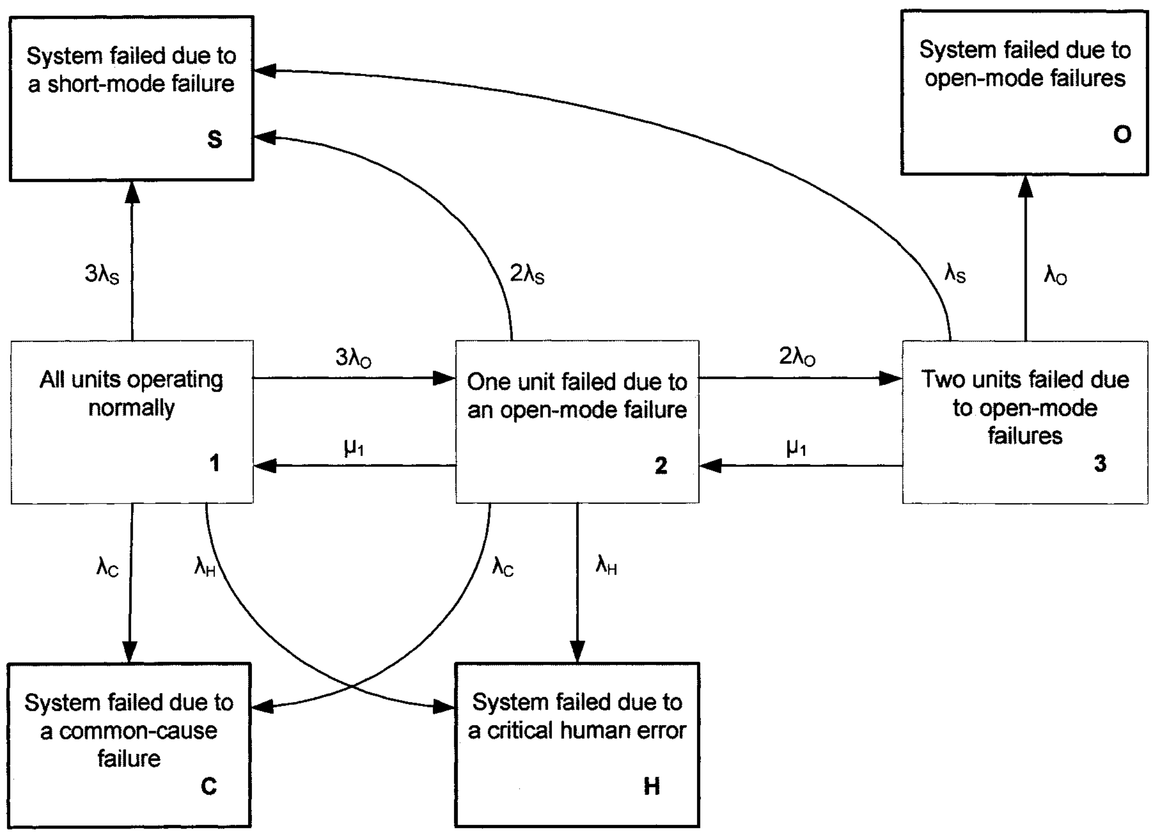

Figure 2-18: State transition diagram for a three-unit parallel system with Type I repair policy

By setting n=3 in Equations (2-40) – (2-46), we obtain the following set of equations associated with Figure 2-18:

$$p_1(s) = \frac{B(BC - 2\lambda_O \mu_1)}{A(2,1)} \tag{2-57}$$

$$p_2(s) = \frac{3\lambda_O BC}{A(2,1)} \tag{2-58}$$

$$p_3(s) = \frac{6\lambda_O{}^2 B}{A(2,1)} \qquad (2\text{-}59)$$

$$p_O(s) = \left(\frac{\lambda_O}{s}\right)\frac{6\lambda_O{}^2 B}{A(2,1)} \qquad (2\text{-}60)$$

$$p_S(s) = \left(\frac{3\lambda_S}{s}\right)\frac{(B+2\lambda_O)(BC-2\lambda_O\mu_1)+2\lambda_O{}^2(B+2\mu_1)}{A(2,1)} \qquad (2\text{-}61)$$

$$p_C(s) = \left(\frac{\lambda_C}{s}\right)\frac{(B+3\lambda_O)(BC-2\lambda_O\mu_1)+6\mu_1\lambda_O{}^2}{A(2,1)} \qquad (2\text{-}62)$$

$$p_H(s) = \left(\frac{\lambda_H}{s}\right)\frac{(B+3\lambda_O)(BC-2\lambda_O\mu_1)+6\mu_1\lambda_O{}^2}{A(2,1)} \qquad (2\text{-}63)$$

where:

$$A(2,1) = (AB-3\lambda_O\mu_1)(BC-2\lambda_O\mu_1)-6\mu_1{}^2\lambda_O{}^2$$
$$A = s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H$$
$$B = s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1$$
$$C = s+\lambda_O+\lambda_S+\mu_1$$

By taking the inverse Laplace transforms of equations (2-57) – (2-63), the time-dependent state probabilities can be obtained. By adding equations (2-57) – (2-59) and then taking the inverse Laplace transforms, we get the following expression for the time-dependent system reliability:

$$R(t) = L^{-1}[R(s)] = L^{-1}\left\{\frac{(BC-2\lambda_O\mu_1)(B+3\lambda_O)+6\lambda_O{}^2(B+1)}{A(2,1)}\right\} \qquad (2\text{-}64)$$

The system mean time to failure is given by

$$MTTF = \lim_{s\to 0}\left\{\frac{(BC-2\lambda_O\mu_1)(B+3\lambda_O)+6\lambda_O{}^2(B+1)}{A(2,1)}\right\} \qquad (2\text{-}65)$$

For the given values of the model parameters, the plots of the state probabilities as a function of time, t, are shown in Figure 2-19. For the specified values of the model parameters, the plots of Equation (2-64) are shown in Figure 2-20. Similarly, for the given values of model parameters, the plots of Equation (2-65) are shown in Figure 2-21.

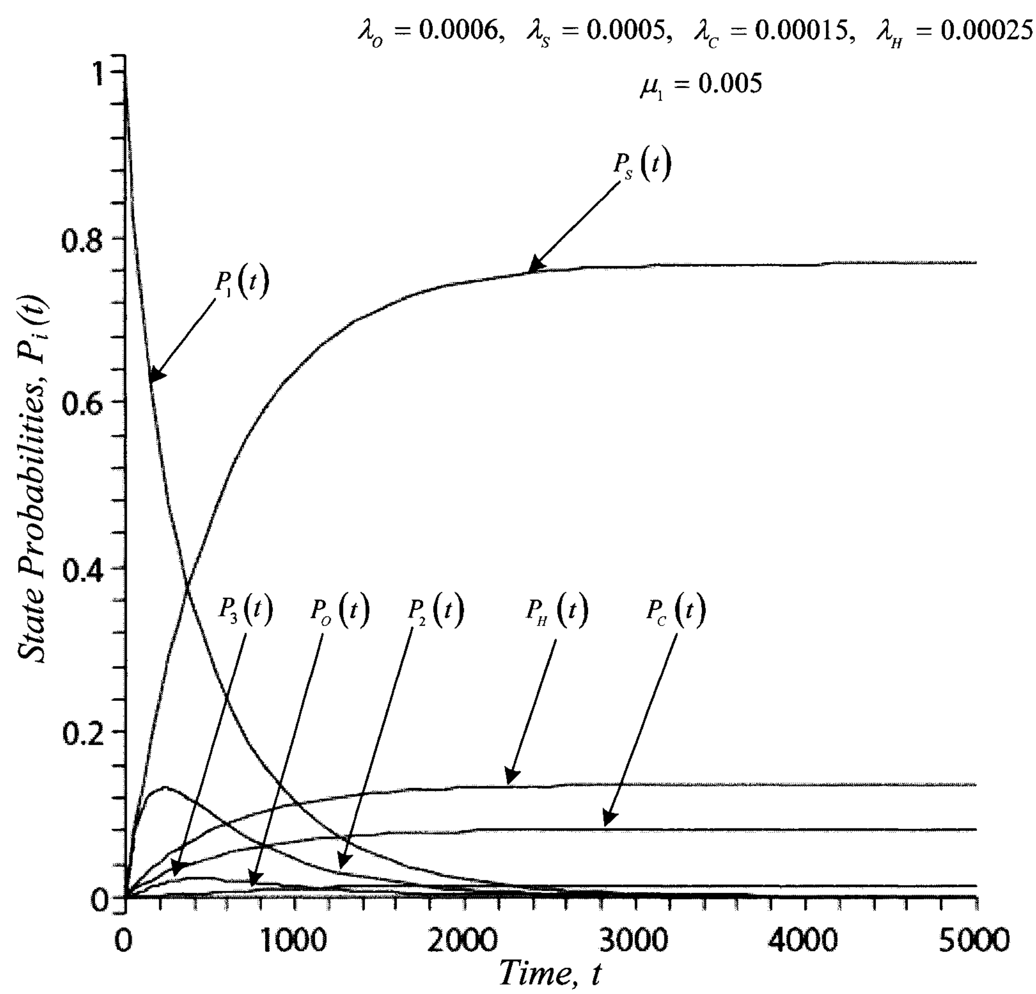

Figure 2-19: State probability plots of a three-unit parallel system with Type I repair policy

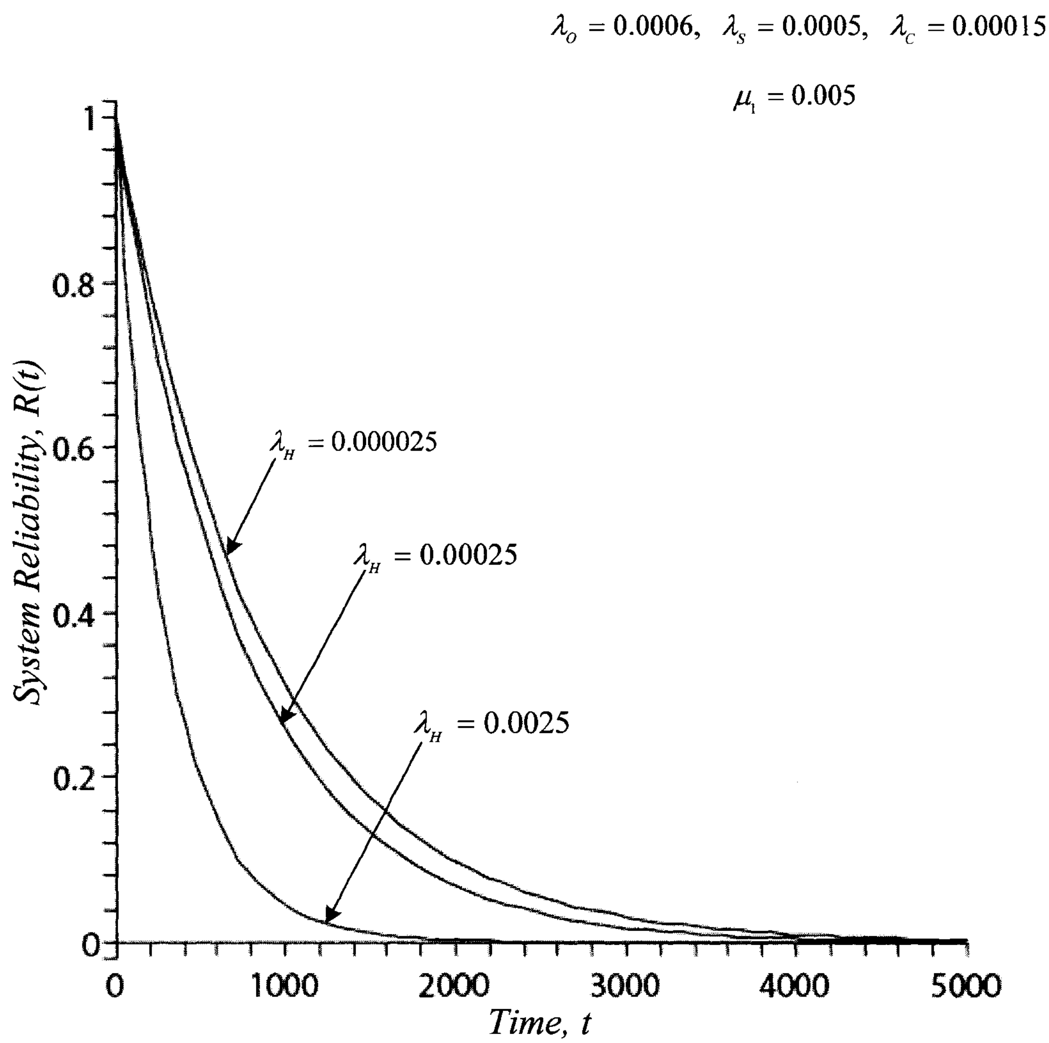

Figure 2-20: Reliability plots of three-unit parallel system with Type I repair policy

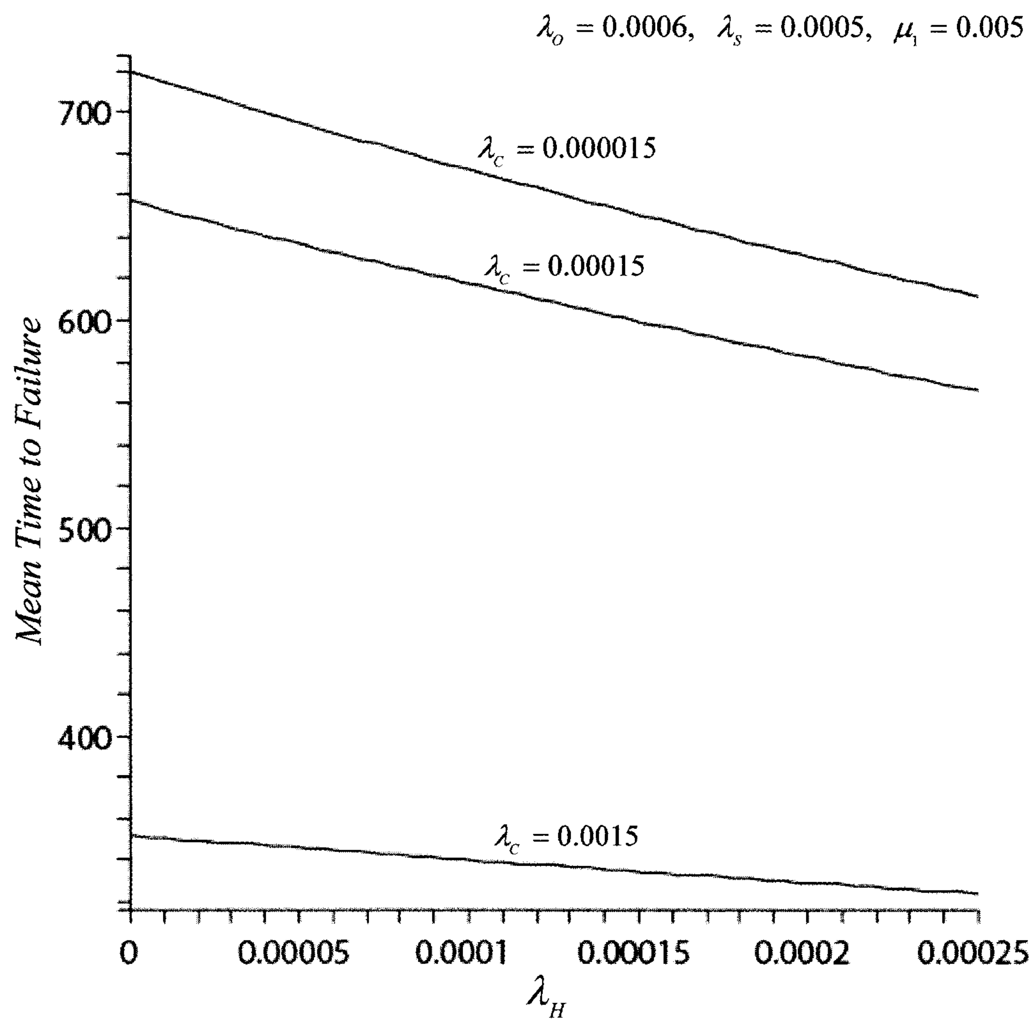

Figure 2-21: Mean time to failure plots of three-unit parallel system with Type I repair policy

2.4 Three-State Device Parallel System under Type II Repair Policy

In this section, the n-three-state device parallel system under Type II repair policy and its two special cases: two-unit parallel system with Type II repair and three-unit parallel system with Type II repair are discussed. Under this repair policy, the completely failed system is restored back to the initial operating state, (i.e. system state at time t =0).

2.4.1 General parallel system under Type II repair policy

The block diagram and the state transition diagram for the general model with Type II repair policy is shown in Figures 2-1 and 2-4 respectively. Using the Markov method, the system of differential equations associated with Figure 2-4 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) + \mu\left[P_O(t) + P_S(t)\right] + \mu_C P_C(t) + \mu_H P_H(t) \quad (2\text{-}66)$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H\right]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) \quad (2\text{-}67)$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -\left[\lambda_O + \lambda_S\right]P_n(t) + 2\lambda_O P_{n-1}(t) \quad (2\text{-}68)$$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + \lambda_O P_n(t) \quad (2\text{-}69)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S \sum_{i=1}^{n}(n-i+1)P_i(t) \quad (2\text{-}70)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-1} P_i(t) \quad (2\text{-}71)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-1} P_i(t) \quad (2\text{-}72)$$

At time t=0, $P_1(0)=1$, and all other initial state probabilities are equal to zero. Taking Laplace transforms of Equations (2-66) – (2-72) and solving the resulting equations, we get the following set of equations:

$$p_1(s)=\frac{1+\mu\left[p_O(s)+p_S(s)\right]+\mu_C p_C(s)+\mu_H p_H(s)}{s+n\lambda_O+n\lambda_S+\lambda_C+\lambda_H} \tag{2-73}$$

$$p_i(s)=\left[\frac{(n-i+2)\lambda_O}{s+(n-i+1)\lambda_O+(n-i+1)\lambda_S+\lambda_C+\lambda_H}\right]p_{i-1}(s) \quad \text{for all } i=2 \text{ to } (n-1) \tag{2-74}$$

$$p_n(s)=\left[\frac{2\lambda_O}{s+\lambda_O+\lambda_S}\right]p_{n-1}(s) \tag{2-75}$$

$$p_O(s)=\left[\frac{\lambda_O}{s+\mu}\right]p_n(s) \tag{2-76}$$

$$p_S(s)=\frac{\lambda_S}{s+\mu}\sum_{i=1}^{n}(n-i+1)p_i(s) \tag{2-77}$$

$$p_C(s)=\frac{\lambda_C}{s+\mu_C}\sum_{i=1}^{n-1}p_i(s) \tag{2-78}$$

$$p_H(s)=\frac{\lambda_H}{s+\mu_H}\sum_{i=1}^{n-1}p_i(s) \tag{2-79}$$

By taking the inverse Laplace transforms of Equations (2-73) – (2-79), the time-dependent state probabilities can be obtained. The time-dependent system availability is given by

$$A(t)=\sum_{i=1}^{n}P_i(t)=L^{-1}\left[\sum_{i=1}^{n}p_i(s)\right] \tag{2-80}$$

2.4.2 Special case model 2-E: Two-unit parallel system with Type II repair

The state transition diagram of a two unit parallel system with Type II repair policy is shown in Figure 2-22.

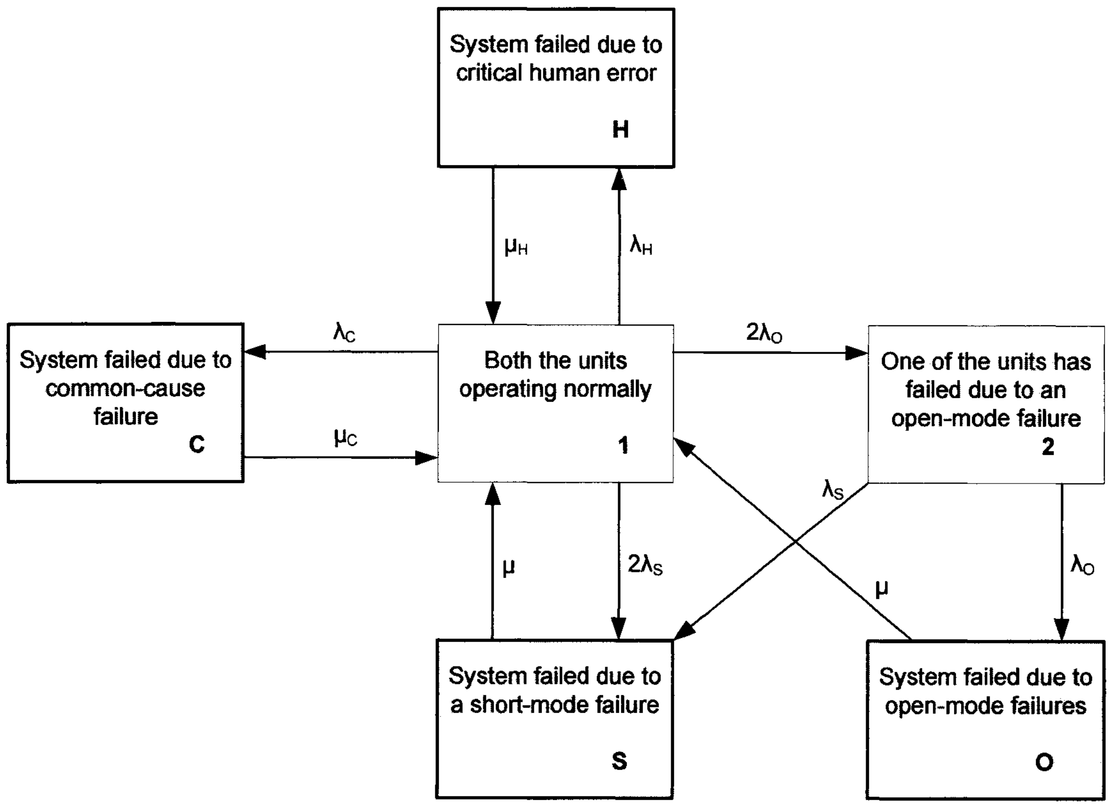

Figure 2-22: State transition diagram of a two unit parallel system with Type II repair policy

For n=2 in Figure 2-4 from Equations (2-73) – (2-79), we get the following set of equations associated with Figure 2-22:

$$p_1(s)=\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+\lambda_O+\lambda_S)}{A(2,2)} \tag{2-81}$$

$$p_2(s)=\frac{2\lambda_O(s+\mu)(s+\mu_C)(s+\mu_H)}{A(2,2)} \tag{2-82}$$

$$p_O(s)=\frac{2\lambda_O^{\,2}(s+\mu_C)(s+\mu_H)}{A(2,2)} \tag{2-83}$$

$$p_S(s)=\frac{2\lambda_S(s+2\lambda_O+\lambda_S)(s+\mu_C)(s+\mu_H)}{A(2,2)} \tag{2-84}$$

$$p_C(s)=\frac{\lambda_C(s+\mu)(s+\mu_H)(s+\lambda_O+\lambda_S)}{A(2,2)} \tag{2-85}$$

$$p_H(s) = \frac{\lambda_H (s+\mu)(s+\mu_C)(s+\lambda_O+\lambda_S)}{A(2,2)} \tag{2-86}$$

where

$$A(2,2) = (s+\mu_C)(s+\mu_H)\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\mu)(s+\lambda_O+\lambda_S)-2\mu\lambda_O^2-2\mu\lambda_S(s+2\lambda_O+\lambda_S)\right]$$
$$-(s+\mu)(s+\lambda_O+\lambda_S)\left[(s+\mu_H)\mu_C\lambda_C+(s+\mu_C)\mu_H\lambda_H\right]$$

By taking the inverse Laplace transforms of equations (2-81) – (2-86), the time-dependent state probabilities can be obtained. By adding Equations (2-81) and (2-82) and then taking inverse Laplace transforms, we get the following expression for the system time dependent availability:

$$A(t) = L^{-1}\left[\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+3\lambda_O+\lambda_S)}{A(2,2)}\right] \tag{2-87}$$

Steady State Conditions: We can compute the steady state conditions directly from Equations (2-66) – (2-72), (i.e., for n=2) by letting all derivatives equal to zero and solving the resultant equations by utilizing the total probability axiom: $\sum P_i = 1$. Thus, we get the following set of equations:

$$P_1 = \frac{\mu\mu_C\mu_H(\lambda_O+\lambda_S)}{\mu_C\mu_H\left[\lambda_O(2\lambda_O+4\lambda_S+3\mu)+\lambda_S(2\lambda_S+\mu)\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{2-88}$$

$$P_2 = \frac{2\lambda_O\mu\mu_C\mu_H}{\mu_C\mu_H\left[\lambda_O(2\lambda_O+4\lambda_S+3\mu)+\lambda_S(2\lambda_S+\mu)\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{2-89}$$

$$P_O = \frac{2\lambda_O^2\mu_C\mu_H}{\mu_C\mu_H\left[\lambda_O(2\lambda_O+4\lambda_S+3\mu)+\lambda_S(2\lambda_S+\mu)\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{2-90}$$

$$P_S = \frac{2\lambda_S(2\lambda_O+\lambda_S)\mu_C\mu_H}{\mu_C\mu_H\left[\lambda_O(2\lambda_O+4\lambda_S+3\mu)+\lambda_S(2\lambda_S+\mu)\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{2-91}$$

$$P_C = \frac{\lambda_C(\lambda_O+\lambda_S)\mu\mu_H}{\mu_C\mu_H\left[\lambda_O(2\lambda_O+4\lambda_S+3\mu)+\lambda_S(2\lambda_S+\mu)\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{2-92}$$

$$P_H = \frac{\lambda_H (\lambda_O + \lambda_S) \mu \mu_C}{\mu_C \mu_H [\lambda_O (2\lambda_O + 4\lambda_S + 3\mu) + \lambda_S (2\lambda_S + \mu)] + \mu (\lambda_O + \lambda_S)(\mu_C \lambda_H + \mu_H \lambda_C)} \quad (2\text{-}93)$$

By adding Equations (2-88) and (2-89), we get the following expression for the system steady state availability:

$$AV_{ss} = \frac{\mu \mu_C \mu_H (3\lambda_O + \lambda_S)}{\mu_C \mu_H [\lambda_O (2\lambda_O + 4\lambda_S + 3\mu) + \lambda_S (2\lambda_S + \mu)] + \mu (\lambda_O + \lambda_S)(\mu_C \lambda_H + \mu_H \lambda_C)} \quad (2\text{-}94)$$

For the same values of the model parameters as chosen earlier, the plots of the state probabilities as a function of time, t, are shown in Figure 2-23. For the given values of the model parameters, the plots of Equation (2-87) are shown in Figure 2-24. Similarly, for the specified values of the model parameters, the plots of Equation (2-94) are shown in Figure 2-25.

2.4.3 Special case model 2-F: Three-unit parallel system with Type II repair

The state transition diagram of a three unit parallel system with Type II repair policy is shown in Figure 2-26.

By setting n=3 in Equations (2-73) - (2-79), we get:

$$p_1(s) = \frac{A(2,3)}{A(2,4)} \quad (2\text{-}95)$$

$$p_2(s) = \frac{3CDEF\lambda_O}{A(2,4)} \quad (2\text{-}96)$$

$$p_3(s) = \frac{6\lambda_O^2 DEF}{A(2,4)} \quad (2\text{-}97)$$

$$p_O(s) = \frac{6\lambda_O^3 EF}{A(2,4)} \quad (2\text{-}98)$$

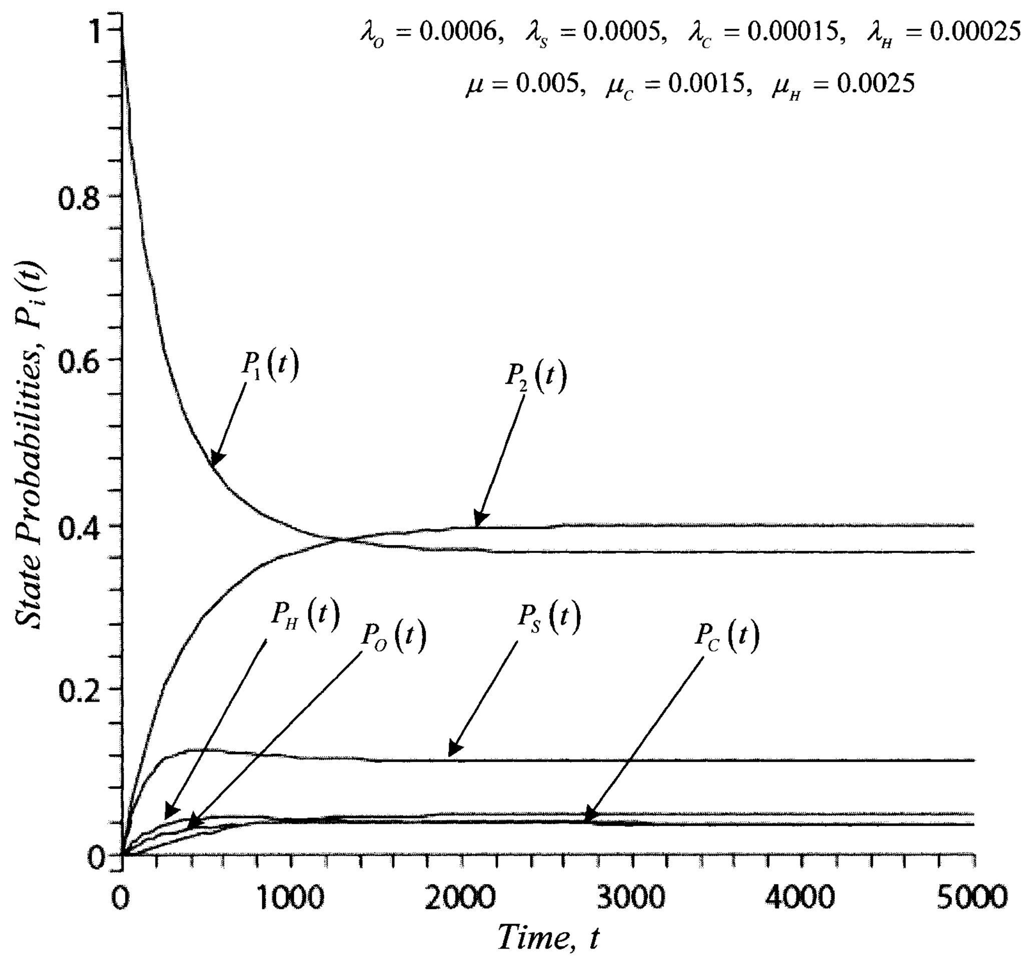

Figure 2-23: State probability plots of two-unit parallel system with Type II repair policy

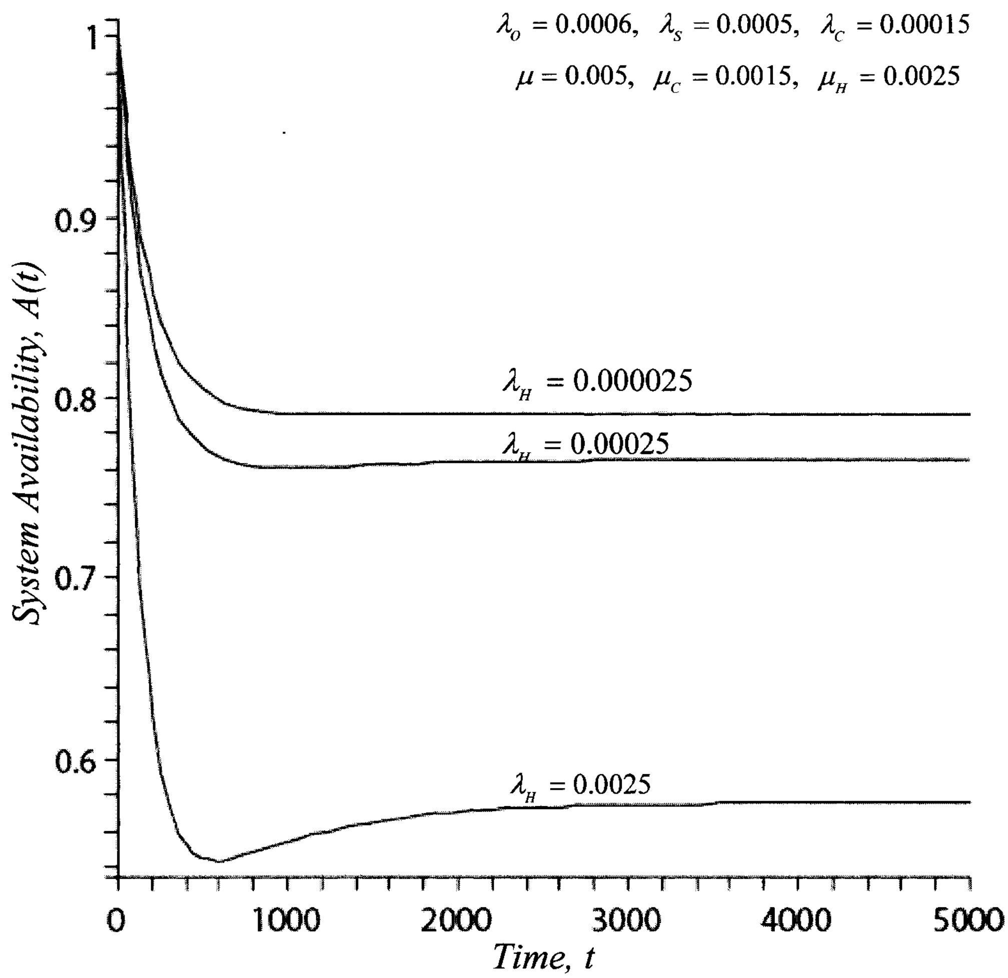

Figure 2-24: Availability plots of two-unit parallel system with Type II repair policy

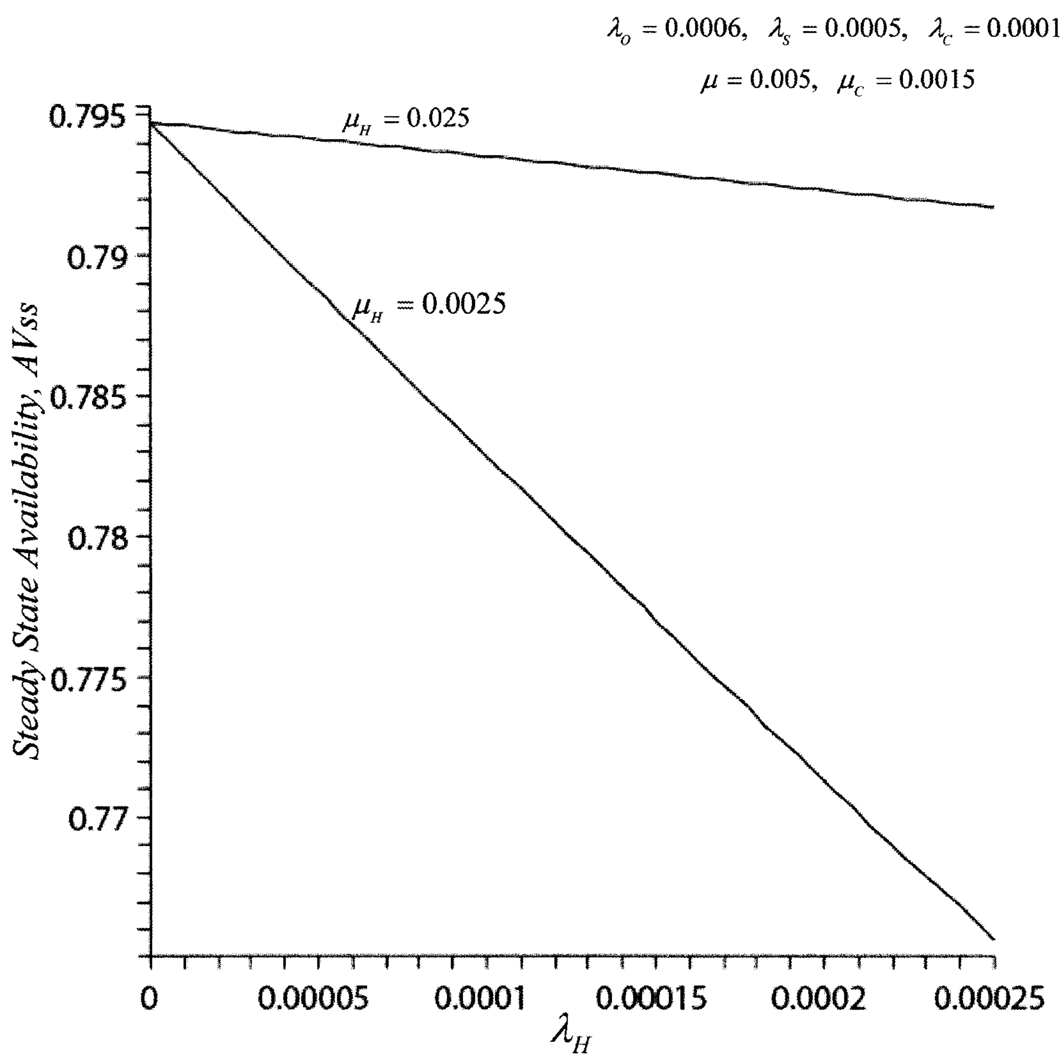

Figure 2-25: Steady state availability plots of two-unit parallel system with Type II repair policy

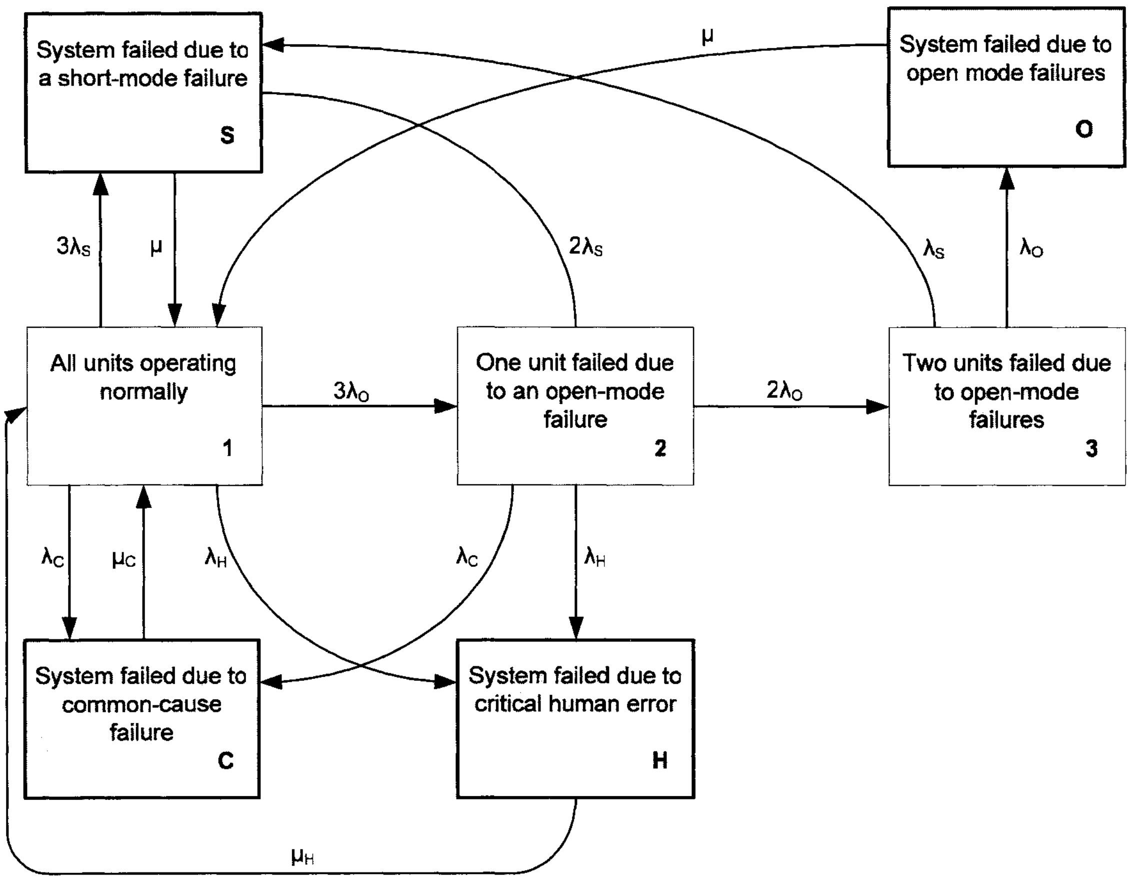

Figure 2-26: State transition diagram of a three-unit parallel system with Type II repair policy

$$p_S(s) = \frac{3EF\lambda_S\left(BC + 2C\lambda_O + 2\lambda_O{}^2\right)}{A(2,4)} \tag{2-99}$$

$$p_C(s) = \frac{CDF\lambda_C\left(B + 3\lambda_O\right)}{A(2,4)} \tag{2-100}$$

$$p_H(s) = \frac{CDE\lambda_H\left(B + 3\lambda_O\right)}{A(2,4)} \tag{2-101}$$

By adding Equations (2-95) - (2-97) and then taking inverse Laplace transforms, we get the following expression for the system time dependent availability:

$$A(t) = L^{-1}\left[A(s)\right] = \frac{DEF\left(BC + 3C\lambda_O + 6\lambda_O^2\right)}{A(2,4)} \tag{2-102}$$

where

$A(2,3) = BCDEF$

$A(2,4) = ABCDEF - 3\mu EF\left[C\lambda_S(B + 2\lambda_O) + 2\lambda_O^2(\lambda_O + \lambda_S)\right] - CD(B + 3\lambda_O)\left[F\lambda_C\mu_C + E\lambda_H\mu_H\right]$

$A = s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$

$B = s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H$

$C = s + \lambda_O + \lambda_S$

$D = s + \mu$

$E = s + \mu_C$

$F = s + \mu_H$

Steady State Conditions: The steady state conditions can be computed directly from Equations (2-66) - (2-72), for n=3, by setting all derivatives equal to zero and solving the resulting equations by utilizing the total probability axiom: $\sum P_i = 1$. Thus, we obtain the following set of equations:

$$P_1 = \frac{\mu\mu_C\mu_H(\lambda_O + \lambda_S)(2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)}{A(2,5)} \tag{2-103}$$

$$P_2 = \frac{3\lambda_O\mu\mu_C\mu_H(\lambda_O + \lambda_S)}{A(2,5)} \tag{2-104}$$

$$P_3 = \frac{6\lambda_O^2\mu\mu_C\mu_H}{A(2,5)} \tag{2-105}$$

$$P_O = \frac{6\lambda_O^3\mu_C\mu_H}{A(2,5)} \tag{2-106}$$

$$P_S = \frac{3\lambda_S\mu_C\mu_H\left[(\lambda_O + \lambda_S)(4\lambda_O + 2\lambda_S + \lambda_C + \lambda_H) + 2\lambda_O^2\right]}{A(2,5)} \tag{2-107}$$

$$P_C = \frac{\lambda_C\mu\mu_H(\lambda_O + \lambda_S)(5\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)}{A(2,5)} \tag{2-108}$$

$$P_H = \frac{\lambda_H \mu \mu_C (\lambda_O + \lambda_S)(5\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)}{A(2,5)} \tag{2-109}$$

where

$$A(2,5) = (\lambda_O + \lambda_S)(2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)\left[\mu\mu_C\mu_H + 3\mu_C\mu_H\lambda_S + \mu\mu_H\lambda_C + \mu\mu_C\lambda_H\right]$$
$$+3\lambda_O(\lambda_O + \lambda_S)\left[\mu\mu_C\mu_H + 2\mu_C\mu_H\lambda_S + \mu\mu_H\lambda_C + \mu\mu_C\lambda_H\right] + 6\lambda_O^2\mu_C\mu_H\left[\lambda_O + \lambda_S + \mu\right]$$

By adding Equations (2-103) - (2-105), we get the following expression for the system steady state availability:

$$AV_{SS} = \frac{\mu\mu_C\mu_H\left[(\lambda_O + \lambda_S)(5\lambda_O + 2\lambda_S + \lambda_C + \lambda_H) + 6\lambda_O^2\right]}{A(2,5)} \tag{2-110}$$

For the given values of the model parameters, the plots of the state probabilities as a function of time, t, are shown in Figure 2-27. For the specified values of the model parameters, the plots of Equation (2-102) are shown in Figure 2-28. Similarly, for the given values of the model parameters, the plots of Equation (2-110) are shown in Figure 2-29.

2.5 Three-State Device Parallel System under Type III Repair Policy

In this section, the general parallel system under Type III repair policy and its two special case models: two-unit parallel system with Type III repair policy and three-unit parallel system with Type III repair policy are discussed. In this case, both partially failed system and completely failed system is repaired. The partially failed system is repaired back to its previous operating state while the completely failed system is restored to its initial operating state (i.e., system state at time, t =0).

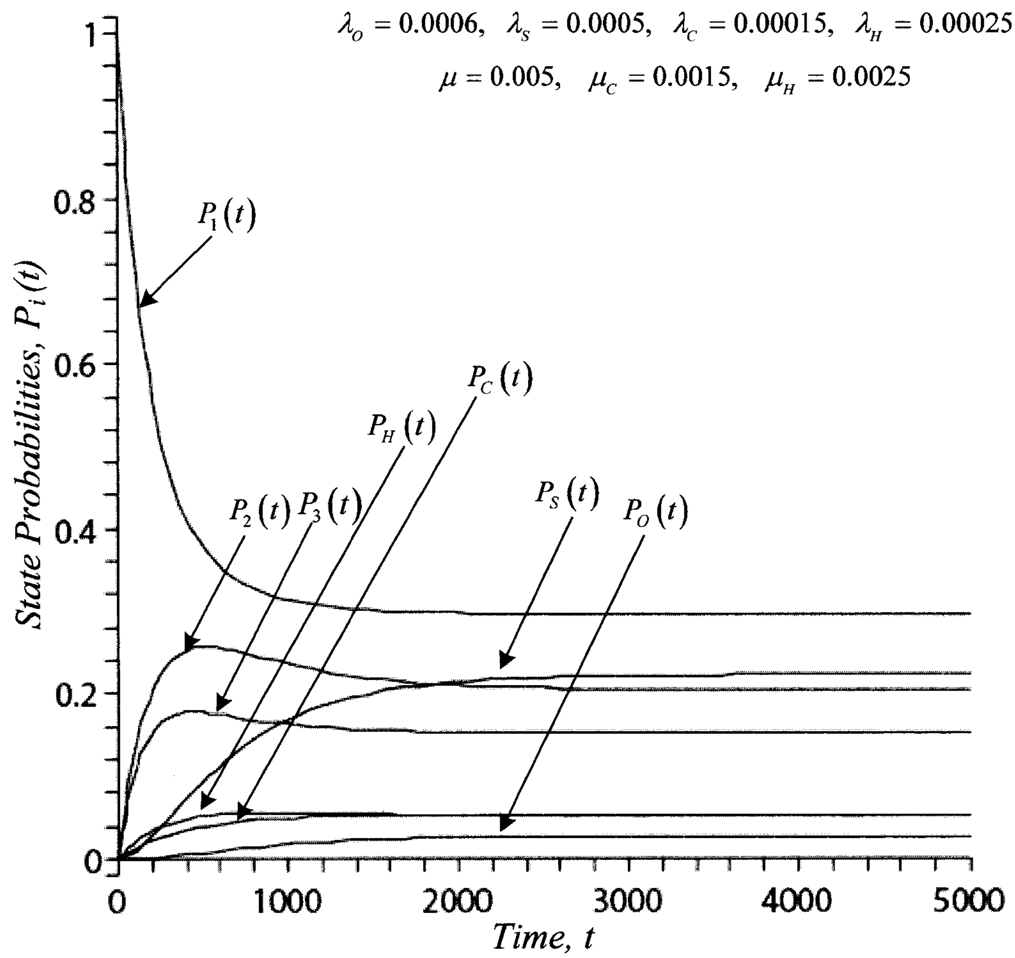

Figure 2-27: State probability plots of three-unit parallel system with Type II repair policy

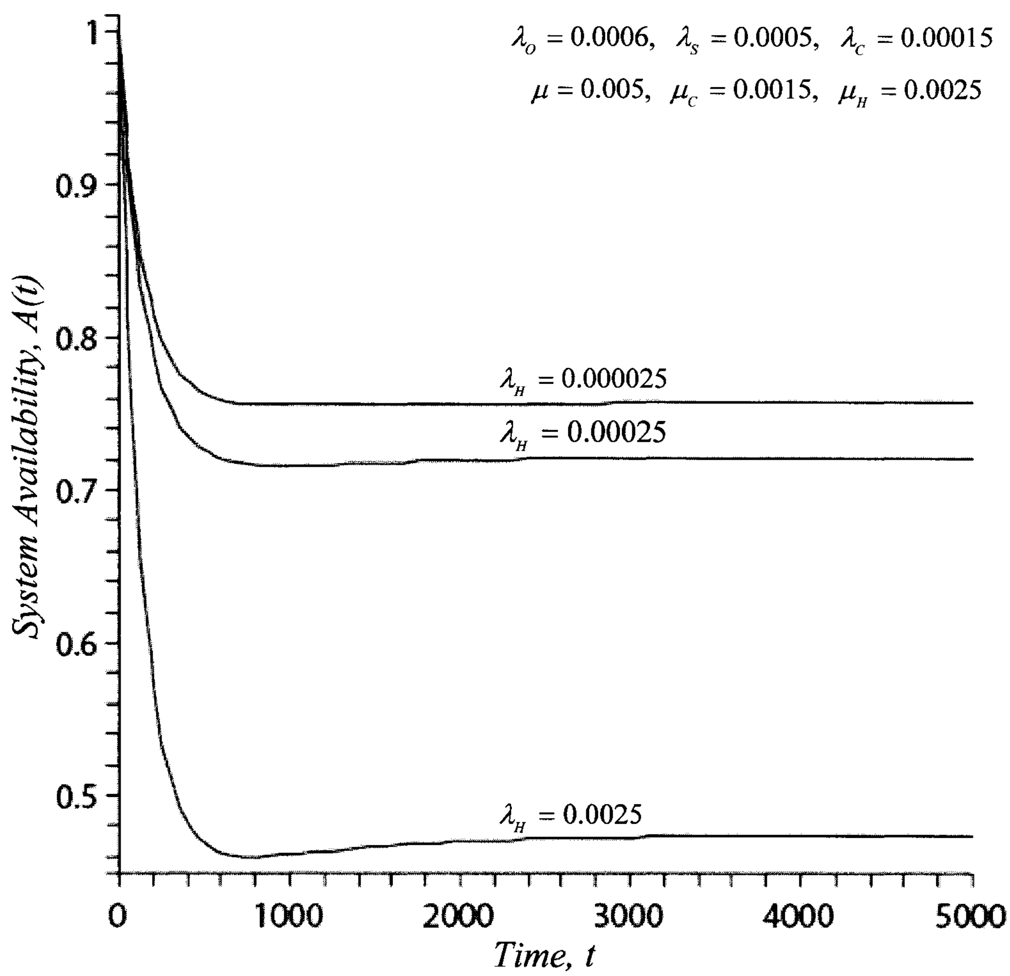

Figure 2-28: Availability plots of three-unit parallel system with Type II repair policy

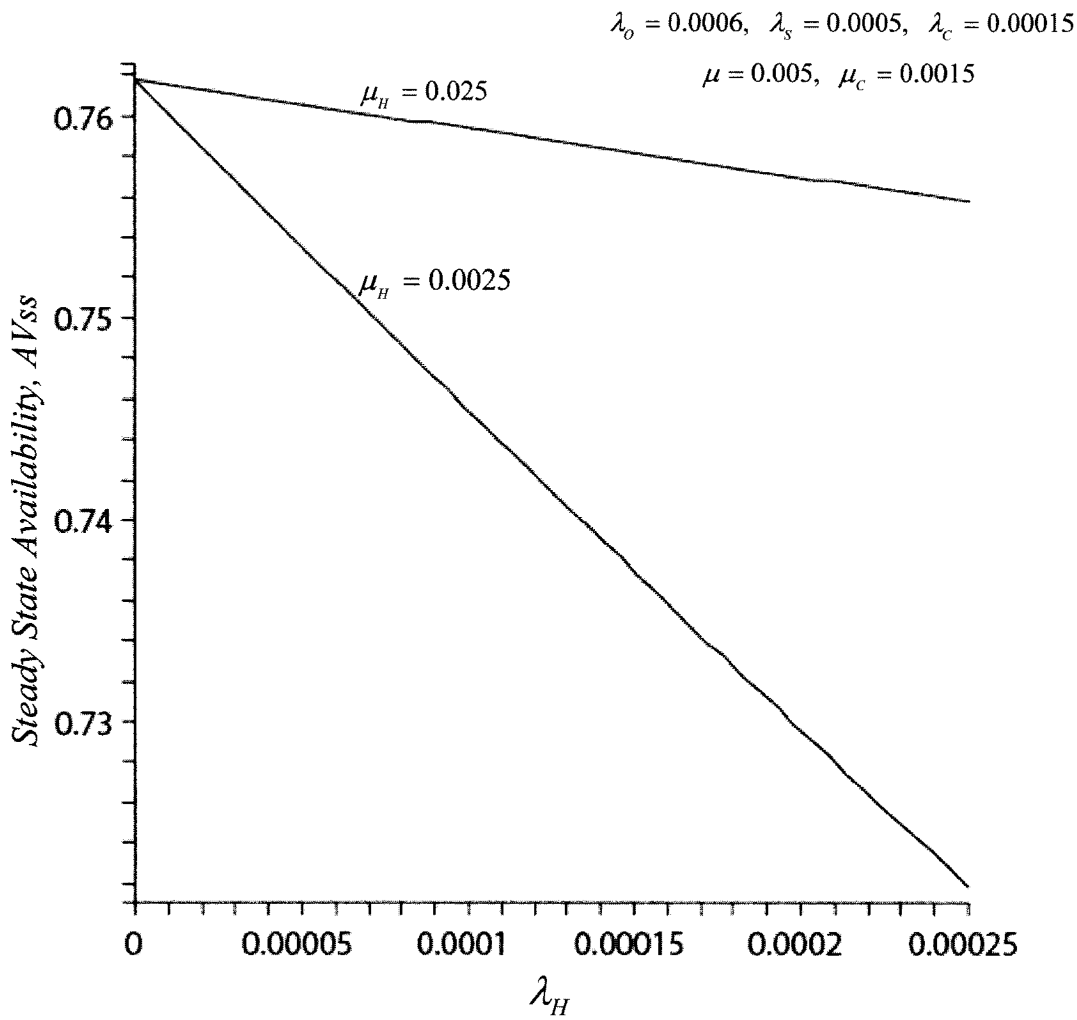

Figure 2-29: Steady state availability plots of three-unit parallel system with Type II repair policy

2.5.1 General parallel system under Type III repair policy

The block diagram and the state transition diagram for the general parallel system under Type III repair policy is shown in Figure 2-1 and Figure 2-5 respectively. Using the Markov method, the system of differential equations associated with Figure 2-5 is as follows:

$$\frac{dP_1(t)}{dt} = -[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H]P_1(t) + \mu_1 P_2(t) + \mu[P_O(t) + P_S(t)] + \mu_C P_C(t) + \mu_H P_H(t) \quad (2\text{-}111)$$

$$\frac{dP_i(t)}{dt} = -[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) + \mu_1 P_{i+1}(t) \quad (2\text{-}112)$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -[\lambda_O + \lambda_S + \mu_1]P_n(t) + 2\lambda_O P_{n-1}(t) \quad (2\text{-}113)$$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + \lambda_O P_n(t) \quad (2\text{-}114)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S \sum_{i=1}^{n} (n-i+1) P_i(t) \quad (2\text{-}115)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-1} P_i(t) \quad (2\text{-}116)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-1} P_i(t) \quad (2\text{-}117)$$

At time t=0, $P_1(0)$ = 1, and all other initial probabilities are equal to zero. Taking Laplace transforms of Equations (2-111) – (2-117) and solving the resulting equations, we get the following set of equations:

$$p_1(s) = \frac{1 + \mu[p_O(s) + p_S(s)] + \mu_1 p_2(s) + \mu_C p_C(s) + \mu_H p_H(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \quad (2\text{-}118)$$

$$p_i(s) = \frac{(n-i+2)\lambda_O p_{i-1}(s) + \mu_1 p_{i+1}(s)}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1} \quad \text{for all } i = 2 \text{ to } (n-1) \quad (2\text{-}119)$$

$$p_n(s) = \frac{2\lambda_O p_{n-1}(s)}{s+\lambda_O+\lambda_S+\mu_1} \tag{2-120}$$

$$p_O(s) = \frac{\lambda_O p_n(s)}{s+\mu} \tag{2-121}$$

$$p_S(s) = \frac{\lambda_S}{s+\mu}\sum_{i=1}^{n}(n-i+1)p_i(s) \tag{2-122}$$

$$p_C(s) = \frac{\lambda_C}{s+\mu_C}\sum_{i=1}^{n-1}p_i(s) \tag{2-123}$$

$$p_H(s) = \frac{\lambda_H}{s+\mu_H}\sum_{i=1}^{n-1}p_i(s) \tag{2-124}$$

By taking the inverse Laplace transforms of Equations (2-118) - (2-124), the time-dependent state probabilities can be obtained. The time-dependent system availability is given by

$$A(t) = L^{-1}\left[\sum_{i=1}^{n}p_i(s)\right] \tag{2-125}$$

2.5.2 Special case model 2-G: Two-unit parallel system with Type III repair

By setting n=2 in Figure 2-5, we obtain the state transition of a two-unit parallel system with Type III repair policy which is shown in Figure 2-30. The following set of equations is associated with Figure 2-30:

$$p_1(s) = \frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+\lambda_O+\lambda_S+\mu_1)}{A(2,6)} \tag{2-126}$$

$$p_2(s) = \frac{2\lambda_O(s+\mu)(s+\mu_C)(s+\mu_H)}{A(2,6)} \tag{2-127}$$

$$p_O(s) = \frac{2\lambda_O^{\,2}(s+\mu_C)(s+\mu_H)}{A(2,6)} \tag{2-128}$$

$$p_S(s) = \frac{2\lambda_S(s+\mu_C)(s+\mu_H)(s+2\lambda_O+\lambda_S+\mu_1)}{A(2,6)} \tag{2-129}$$

$$p_C(s) = \frac{\lambda_C(s+\mu)(s+\mu_H)(s+\lambda_O+\lambda_S+\mu_1)}{A(2,6)} \tag{2-130}$$

$$p_H(s) = \frac{\lambda_H(s+\mu)(s+\mu_C)(s+\lambda_O+\lambda_S+\mu_1)}{A(2,6)} \tag{2-131}$$

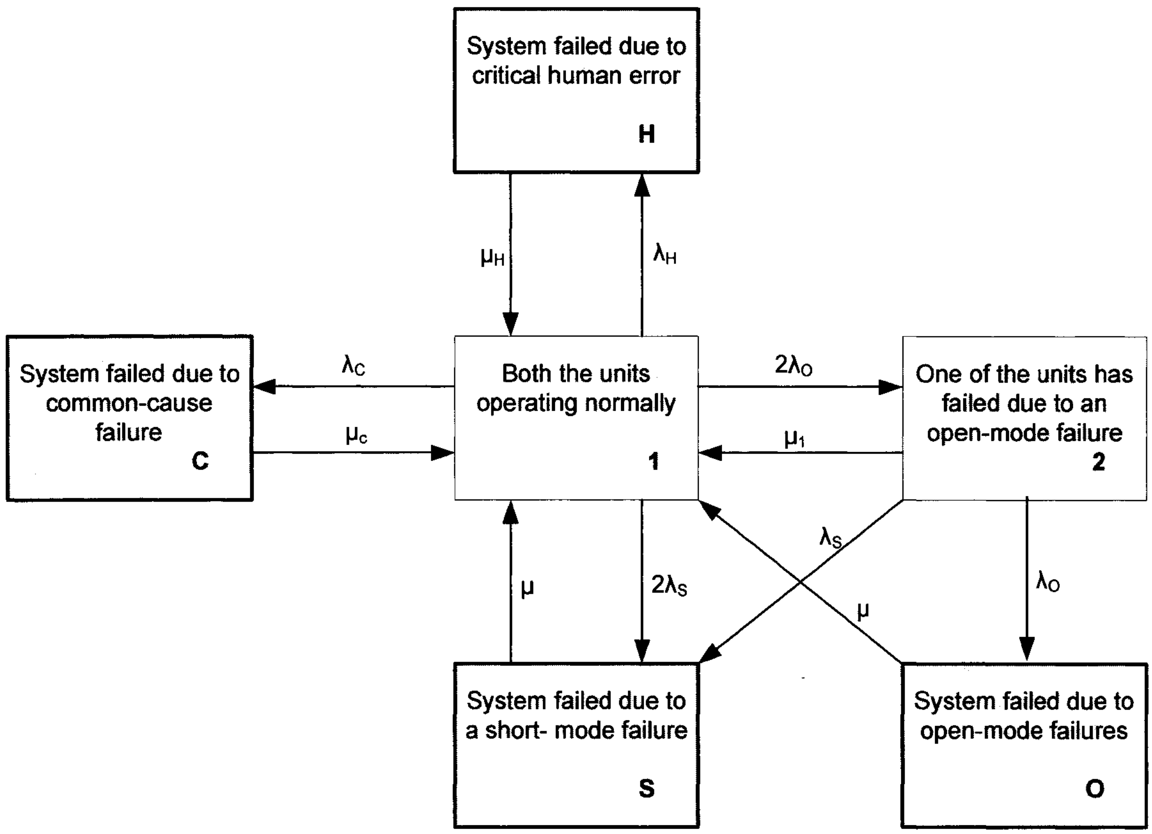

Figure 2-30: State transition of two-unit parallel system with Type III repair policy

By taking the inverse Laplace transforms of Equations (2-126) – (2-131), the time-dependent state probabilities can be obtained. By adding Equations (2-126) and (2-127) and then taking inverse Laplace transforms, we get the following expression for the system time-dependent availability:

$$A(t) = L^{-1}\left\{\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+3\lambda_O+\lambda_S+\mu_1)}{A(2,6)}\right\} \tag{2-132}$$

where

$$A(2,6) = (s+\mu)(s+\mu_C)(s+\mu_H)\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_O\right]$$
$$-2\mu(s+\mu_C)(s+\mu_H)\left[\lambda_O^2+\lambda_S(s+2\lambda_O+2\lambda_S+\mu_1)\right]$$
$$-(s+\lambda_O+\lambda_S+\mu_1)(s+\mu)\left[\mu_C\lambda_C(s+\mu_H)+\mu_H\lambda_H(s+\mu_C)\right]$$

Steady State Conditions: We can compute the steady state conditions directly from Equations (2-111) to (2-117) (i.e., for n=2), by putting all derivatives equal to zero and solving the resulting equations by utilizing the total probability axiom: $\sum P_i = 1$. Thus, we get the following equations:

$$P_1 = \frac{\mu\mu_C\mu_H(\lambda_O+\lambda_S+\mu_1)}{A(2,7)} \tag{2-133}$$

$$P_2 = \frac{2\lambda_O\mu\mu_C\mu_H}{A(2,7)} \tag{2-134}$$

$$P_O = \frac{2\lambda_O^2\mu_C\mu_H}{A(2,7)} \tag{2-135}$$

$$P_S = \frac{2\lambda_S\mu_C\mu_H(2\lambda_O+\lambda_S+\mu_1)}{A(2,7)} \tag{2-136}$$

$$P_C = \frac{\lambda_C\mu\mu_H(\lambda_O+\lambda_S+\mu_1)}{A(2,7)} \tag{2-137}$$

$$P_H = \frac{\lambda_H\mu\mu_C(\lambda_O+\lambda_S+\mu_1)}{A(2,7)} \tag{2-138}$$

By adding Equations (2-133) and (2-134), we get the following expression for the system steady state availability:

$$AV_{SS} = P_1 + P_2 = \frac{\mu\mu_C\mu_H(3\lambda_O+\lambda_S+\mu_1)}{A(2,7)} \tag{2-139}$$

where

$$A(2,7)=\mu_C\mu_H\left[2\lambda_O(\lambda_O+2\lambda_S)+2\lambda_S(\lambda_S+\mu_1)+\mu(3\lambda_O+\lambda_S+\mu_1)\right]+\mu(\lambda_O+\lambda_S+\mu_1)(\mu_C\lambda_H+\mu_H\lambda_C)$$

For the values of the model parameters as specified, the plots of the state probabilities as a function of time, t, are shown in Figure 2-31, the plots of Equation (2-132) are shown in Figure 2-32, the plots of Equation (2-139) are shown in Figure 2-33.

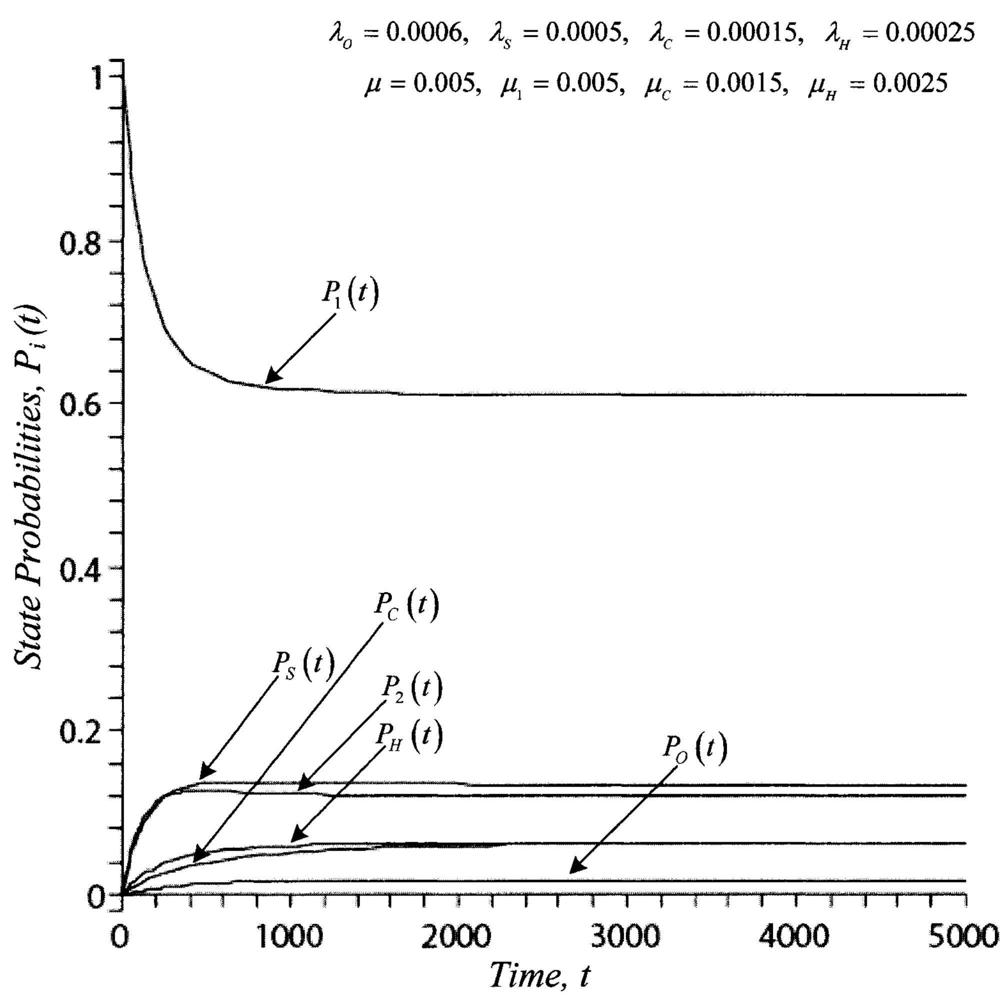

Figure 2-31: State probability plots of two-unit parallel system with Type III repair policy

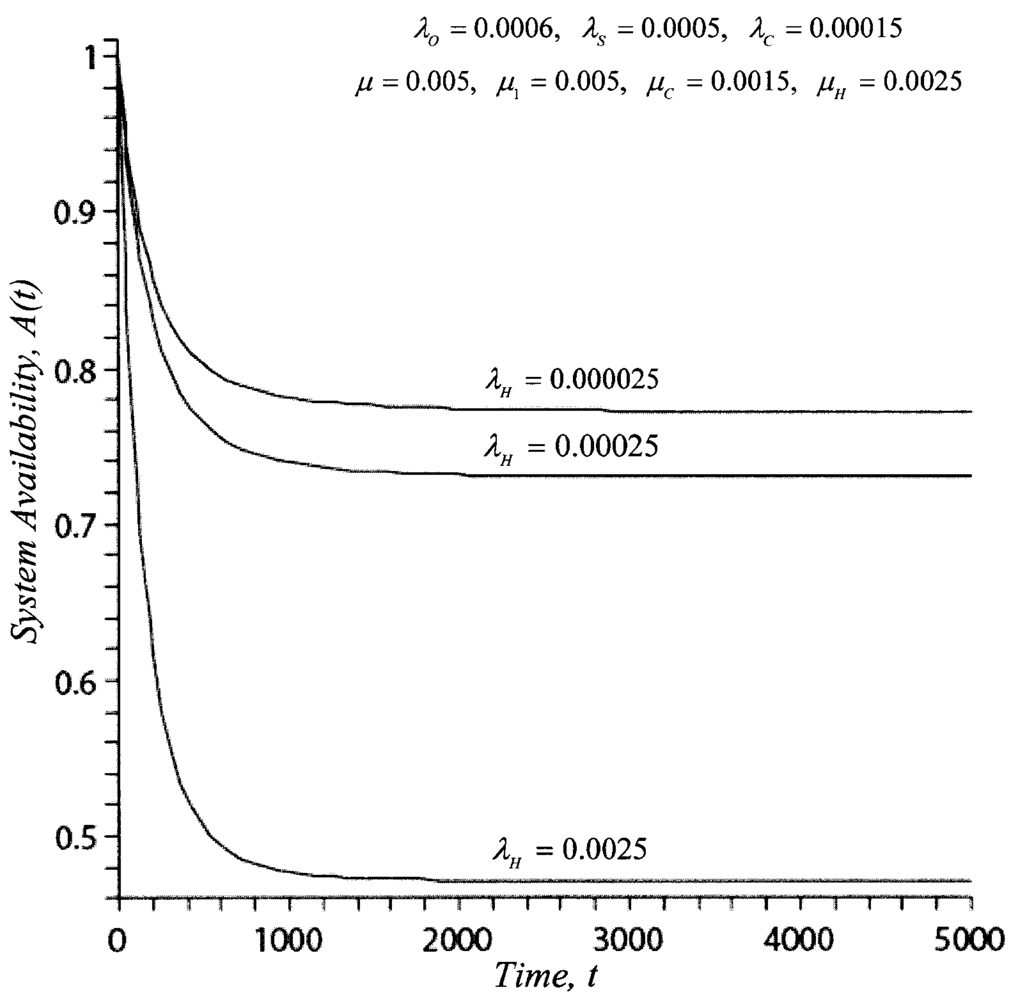

Figure 2-32: Availability plots of two-unit parallel system with Type III repair policy

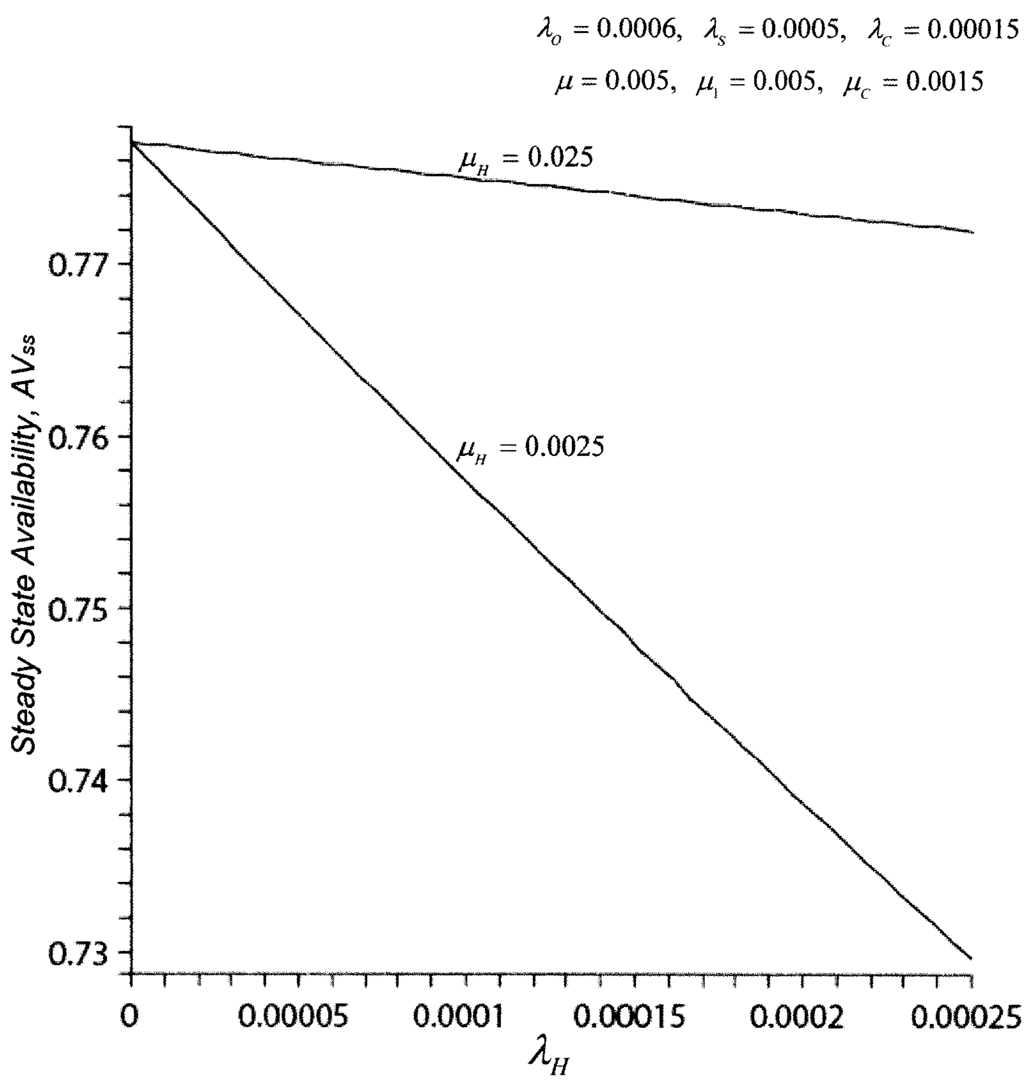

Figure 2-33: Steady state availability plots of two-unit parallel system with Type III repair policy

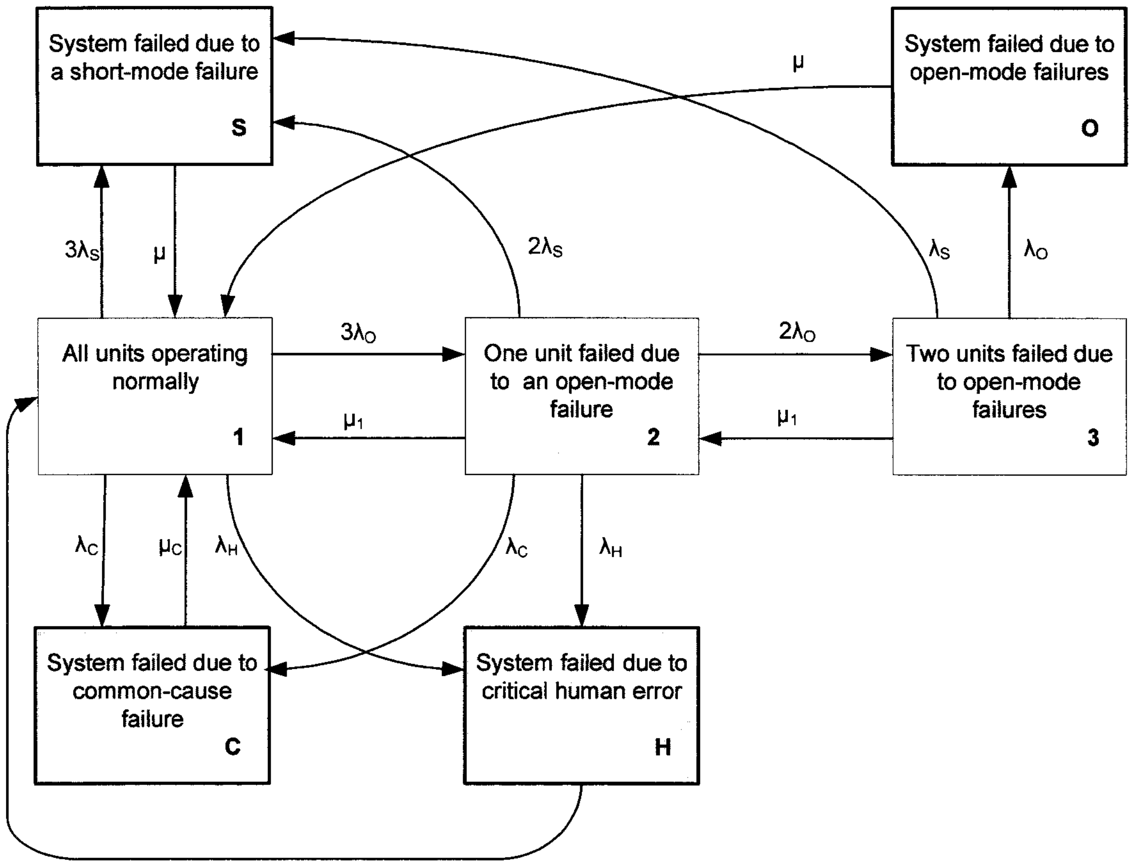

Figure 2-34: State transition diagram for three-unit parallel system with Type III repair policy

2.5.3 Special case model 2-H: Three-unit parallel system with Type III repair

Figure 2-34 shows the state transition diagram of a three-unit parallel system under Type III repair policy. By setting n=3 in Equations (2-118) – (2-124), the following set of equations are obtained:

$$p_1(s) = \frac{DEF(BC - 2\mu_1\lambda_O)}{A(2,8)} \tag{2-140}$$

$$p_2(s) = \frac{3CDEF\lambda_O}{A(2,8)} \tag{2-141}$$

$$p_3(s) = \frac{6DEF\lambda_O^2}{A(2,8)} \tag{2-142}$$

$$p_O(s) = \frac{6EF\lambda_O^3}{A(2,8)} \tag{2-143}$$

$$p_S(s) = \frac{3\lambda_S EF\left[BC - 2\mu_1\lambda_O + 2C\lambda_O + 2\lambda_O^2\right]}{A(2,8)} \tag{2-144}$$

$$p_C(s) = \frac{\lambda_C DF\left[BC - 2\mu_1\lambda_O + 3C\lambda_O\right]}{A(2,8)} \tag{2-145}$$

$$p_H(s) = \frac{\lambda_H DE\left[BC - 2\mu_1\lambda_O + 3C\lambda_O\right]}{A(2,8)} \tag{2-146}$$

By taking the inverse Laplace transforms of Equations (2-140) – (2-146), the time-dependent state probabilities can be obtained. By adding Equations (2-140) – (2-142) and then taking inverse Laplace transforms, we get the following expression for the system time dependent availability:

$$A(t) = L^{-1}\left[A(s)\right] = L^{-1}\left\{\frac{DEF(BC - 2\mu_1\lambda_O + 3C\lambda_O + 6\lambda_O^2)}{A(2,8)}\right\} \tag{2-147}$$

where

$$A(2,8) = ADEF(BC - 2\mu_1\lambda_O) - 3CDEF\mu_1\lambda_O - 3\mu EF\left[2\lambda_O^3 + 2\lambda_O\lambda_S(C + \lambda_O - \mu_1) + \lambda_S BC)\right]$$
$$-D(\mu_C\lambda_C F + \mu_H\lambda_H E)(BC + \lambda_O(3C + 2\mu_1))$$

$$A = s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$$

$$B = s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1$$

$$C = s + \lambda_O + \lambda_S + \mu_1$$

$$D = s + \mu$$

$$E = s + \mu_C$$

$$F = s + \mu_H$$

Steady State Conditions: The steady state conditions are computed by substituting n=3 in Equations (2-111) to (2-117), setting all derivatives equal to zero and solving the resulting equations by utilizing the total probability axiom: $\sum P_i = 1$. Thus, we get the following set of equations:

$$P_1 = \frac{\mu\mu_C\mu_H(BC - 2\mu_1\lambda_O)}{A(2,9)} \tag{2-148}$$

$$P_2 = \frac{3\mu\mu_C\mu_H C\lambda_O}{A(2,9)} \tag{2-149}$$

$$P_3 = \frac{6\mu\mu_C\mu_H\lambda_O^2}{A(2,9)} \tag{2-150}$$

$$P_O = \frac{6\mu_C\mu_H\lambda_O^3}{A(2,9)} \tag{2-151}$$

$$P_S = \frac{3\mu_C\mu_H\lambda_S\left[BC + 2\lambda_O(C + \lambda_O - \mu_1)\right]}{A(2,9)} \tag{2-152}$$

$$P_C = \frac{\mu\mu_H\lambda_C\left[BC + \lambda_O(3C - 2\mu_1)\right]}{A(2,9)} \tag{2-153}$$

$$P_H = \frac{\mu\mu_C\lambda_H\left[BC + \lambda_O(3C - 2\mu_1)\right]}{A(2,9)} \tag{2-154}$$

By adding Equations (2-148) – (2-150), we get the following expressions for the system steady state availability:

$$AV_{SS} = \frac{\mu\mu_C\mu_H\left[BC + \lambda_O\left(3C + 6\lambda_O - 2\mu_1\right)\right]}{A(2,9)} \tag{2-155}$$

where

$$A(2,9) = \mu\mu_C\mu_H(BC - 2\mu_1\lambda_O + 3C\lambda_O + 6\lambda_O^2) + 3\mu_C\mu_H(2\lambda_O^3 + 2C\lambda_S\lambda_O + \lambda_S(BC - 2\mu_1\lambda_O) + 2\lambda_S\lambda_O^2) + \mu(BC - 2\mu_1\lambda_O + 3C\lambda_O)(\mu_H\lambda_C + \mu_C\lambda_H)$$

$$B = 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1$$

$$C = \lambda_O + \lambda_S + \mu_1$$

For the specified values of the model parameters, the plots of the state probabilities as a function of time, *t*, are shown in Figure 2-35. For the given values of the model parameters, the plots of Equation (2-132) are shown in Figure 2-36. Similarly, for the specified values of the model parameters, the plots of Equation (2-139) are shown in Figure 2-37.

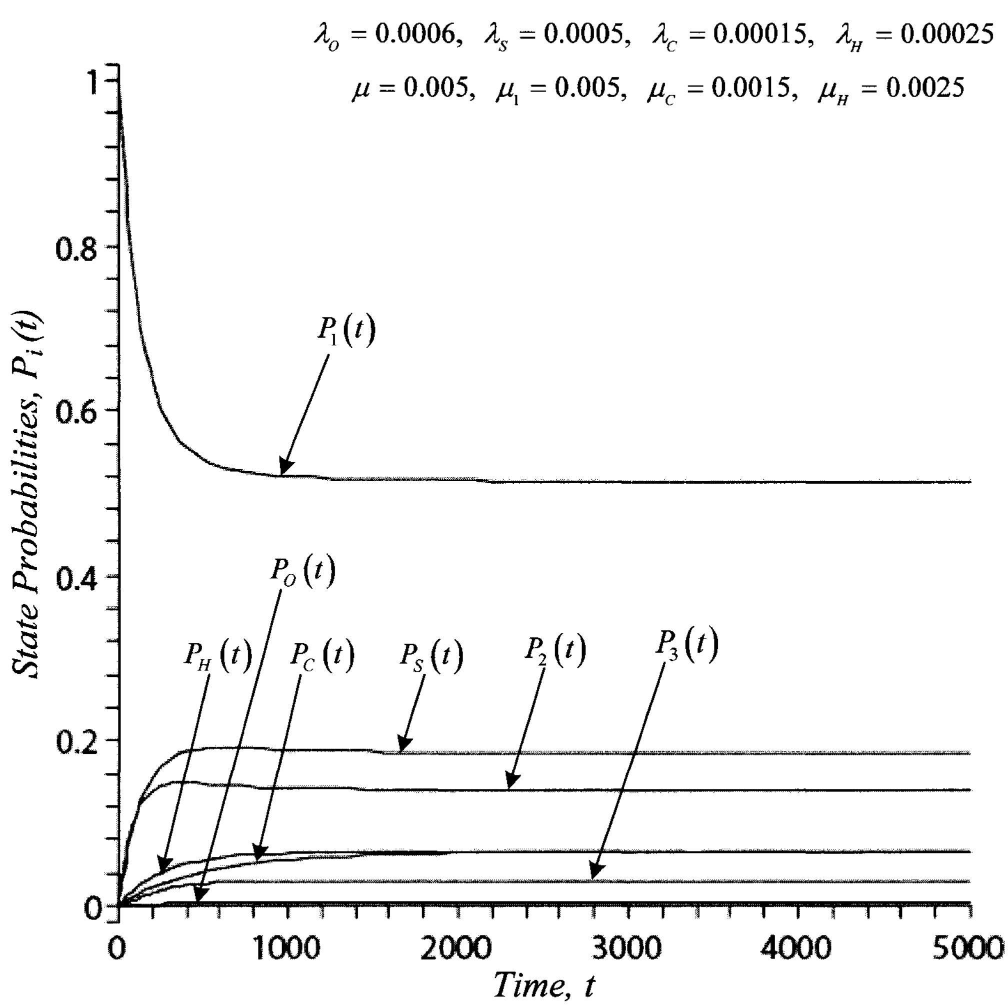

Figure 2-35: State probability plots of three-unit parallel system with Type III repair policy

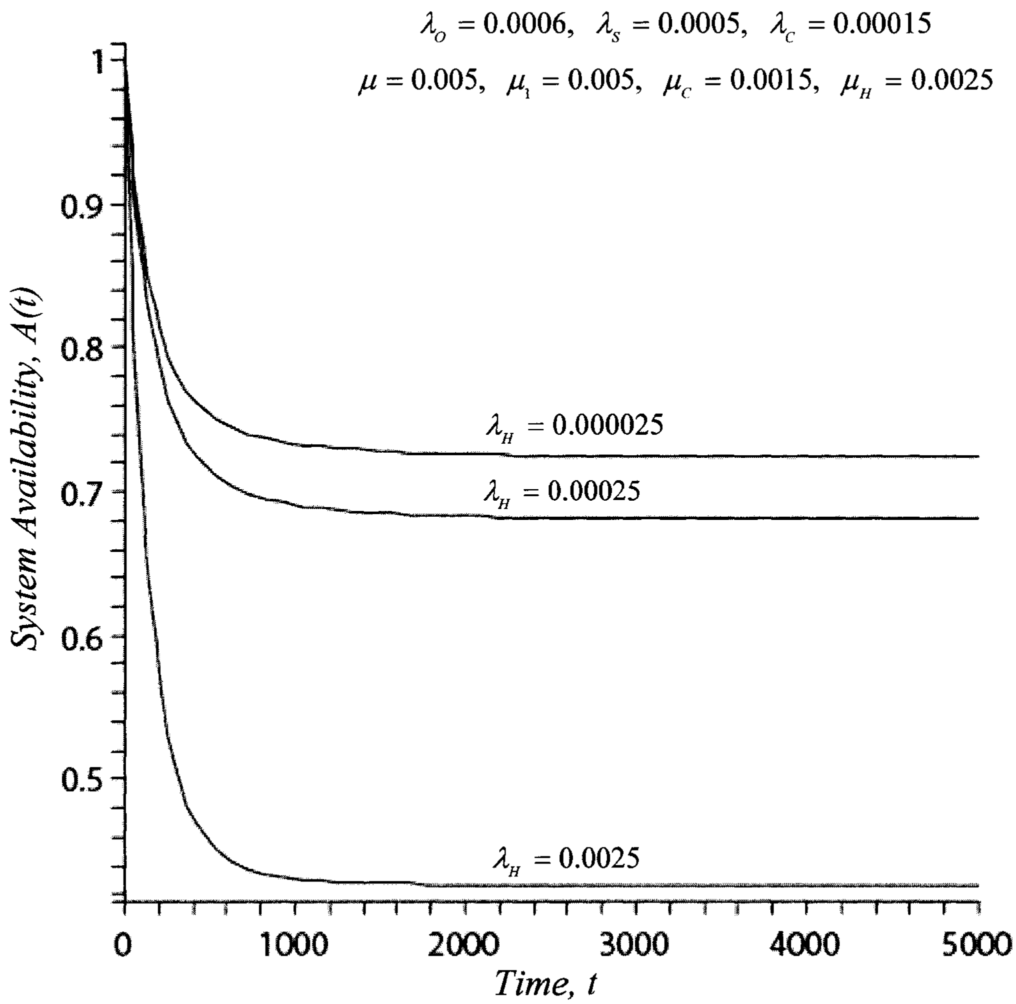

Figure 2-36: Availability plots of three-unit parallel system with Type III repair policy

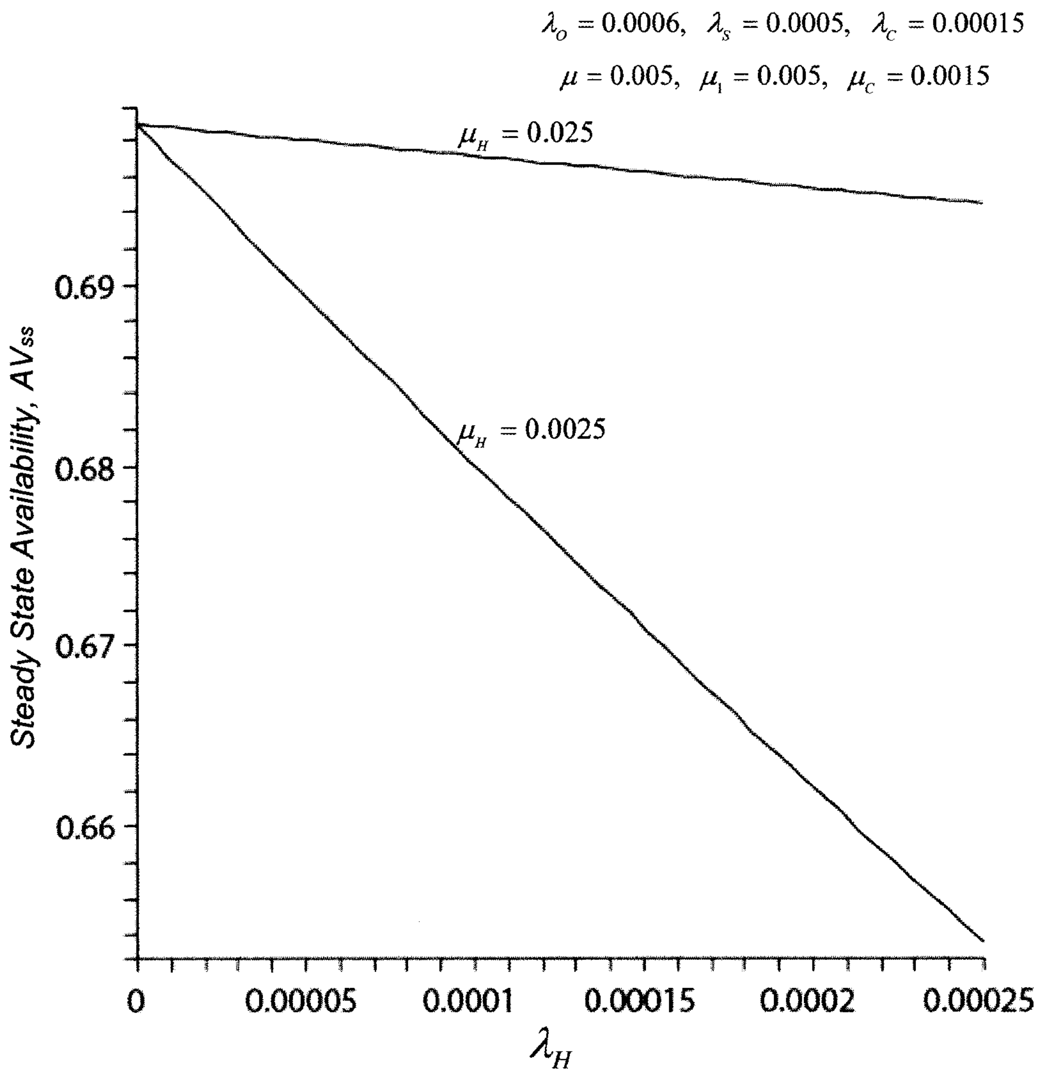

Figure 2-37: Steady state availability plots of three-unit parallel system with Type III repair policy

2.6 Summary

The analysis presented in this Chapter indicates that the values of the various system performance measures such as system reliability, mean time to failure, system time-dependent and steady state availability decreases with either increasing number of units or increasing value of the critical human error rate. Furthermore, it is observed that the system mean time to failure decreases with increasing common-cause failure rate and the system steady state availability increases with increasing human error repair rate. When the system is subjected to Type II and Type III repair policies, it is observed that the time-dependent system availability approaches to a constant value as time becomes very large$\left(i.e., \ t = \infty\right)$. This constant value is, in fact, the system steady state availability.

From the state probability plots, it is observed that the probability of short-mode failure (P_S) increases with increasing number of units while the probability of open-mode failure (P_O) decreases with increasing number of units. Also, it is evident that at any given time t, the sum of all the state probabilities equals to one$\left(i.e., \ \sum P_i = 1\right)$.

When examining the effect of Type I repair action on the system and comparing it with the system without repair, it is observed that the Type I repair action does not lead to a significant increase in the system reliability and mean time to failure. From the reliability plots, it is observed that for higher values of the critical human error rate, the reliability of the system with Type I repair is less than that for the system without repair. The same trend is observed in the mean time to failure plots: for higher values of the common-cause failure rate, the mean time to failure of the system with Type I repair policy is less than that for the system without repair. It is noted that when the system is subjected to Type I repair, the probability of open-mode failure decreases but the repair action does not help in reducing the probability of short-mode failure.

Similarly, when comparing the effect of Type II repair and Type III repair on the system, it is noted that Type III repair action leads to a reduction in the system time-dependent and steady state availabilities.

Chapter 3

Three-State Device K-out-of-N and Series Systems

In the previous Chapter, the parallel system was analysed in which for the system success at least one device must operate normally if all other devices have failed in open-mode. This Chapter extends the analyses of the Chapter 2 to k-out-of-n and series configurations. A comparison between the various performance measures (i.e., system reliability, mean time to failure, time-dependent availability and steady state availability) for these systems with those for the parallel system is made.

3.1 Three-State Device K-out-of-N System

The description of the system

This is another type of redundancy. The system consists of n identical devices of which, for the system success, at least k $(1 < k < n)$ devices must operate normally, if all other devices have failed in open-mode. Therefore, there are four kinds of system failures. The first is open-mode failure (**O**) which means that $(n\text{-}k)$ devices have failed due to open-mode failures. The second is short-mode failure (**S**) which means that any of the devices has failed due to a short-mode failure. Also, the system can fail due to common-cause failure (**C**) and critical human error (**H**) from any of the operable states as long as there are at least two operable units in the system.

The reliability block diagram of a general three state device k-out-of-n system with critical human error and common-cause failures can be represented as in Figure 3-1. The state transition diagram for this system under without repair policy is shown in Figure 3-2. Similarly, Figure 3-3 shows the state transition diagram for the general model under Type I repair policy, Figure 3-4 shows the state transition diagram for the general model under Type II repair policy, and Figure 3-5 shows the state-transition diagram for general model under Type III repair policy. Numerals and single letters in boxes denote the system states. Other symbols used in the diagram are defined in the notation section.

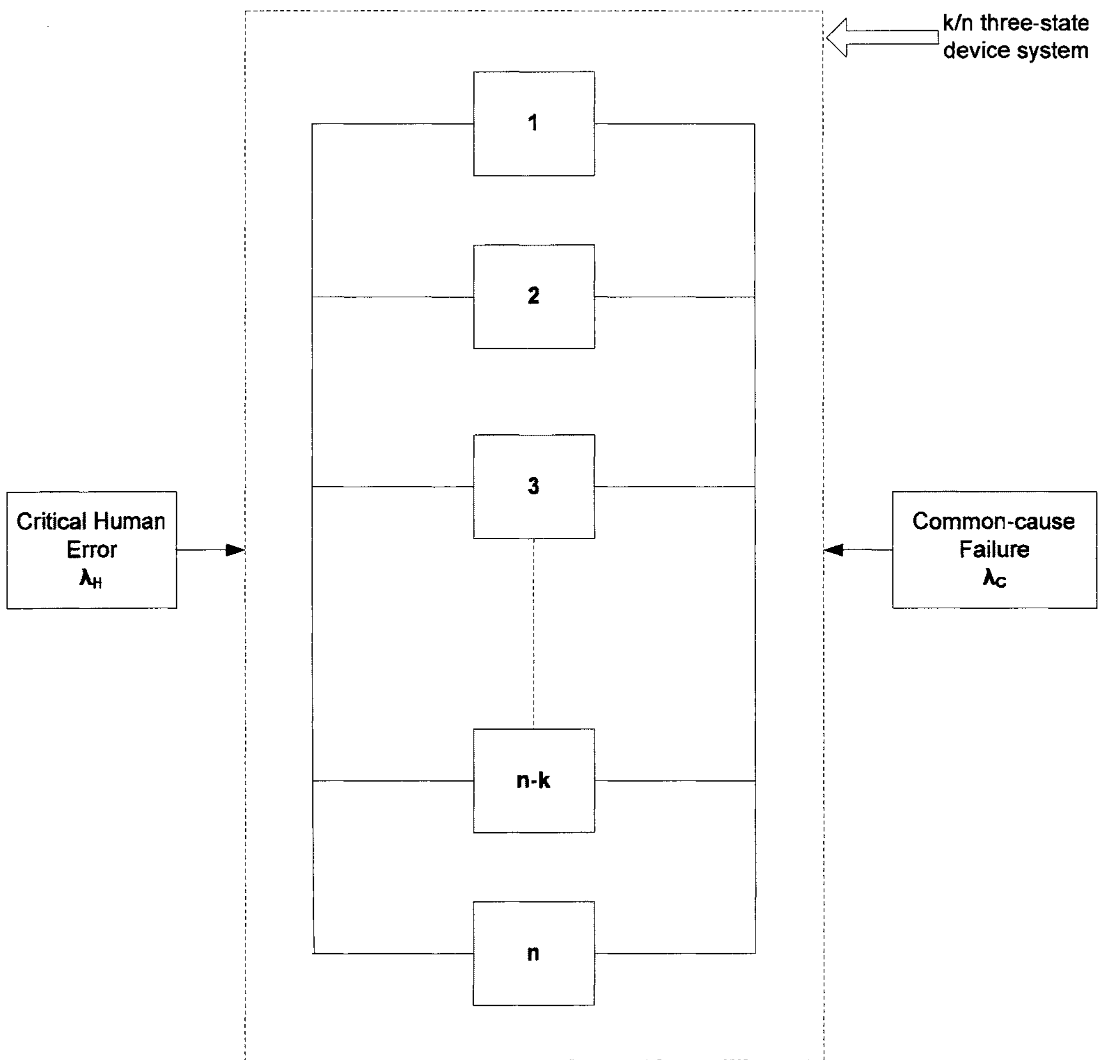

Figure 3-1: Block diagram of k-out-of-n three-state device system

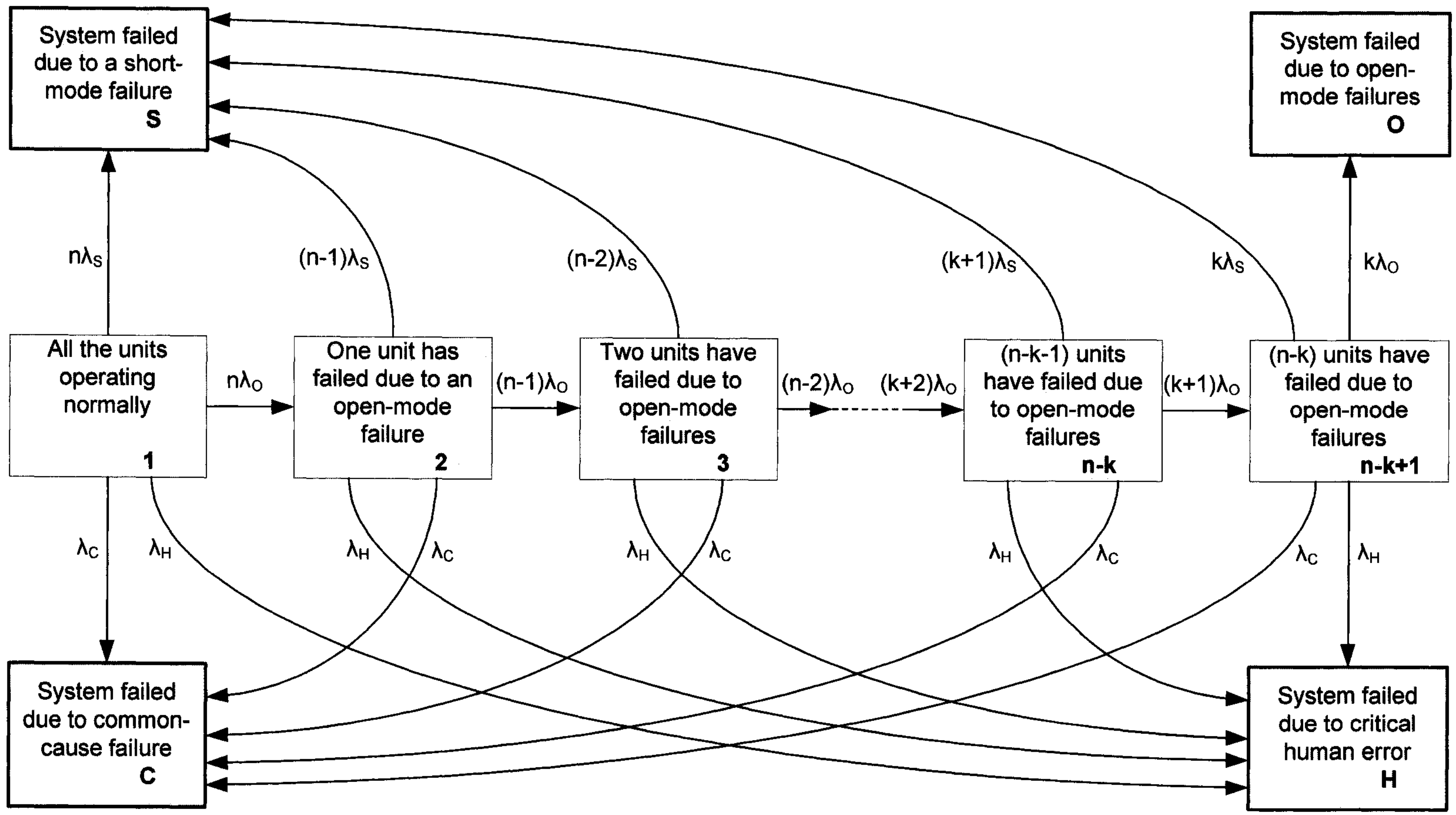

Figure 3-2: State transition diagram for three-state device k-out-of-n system under without repair policy

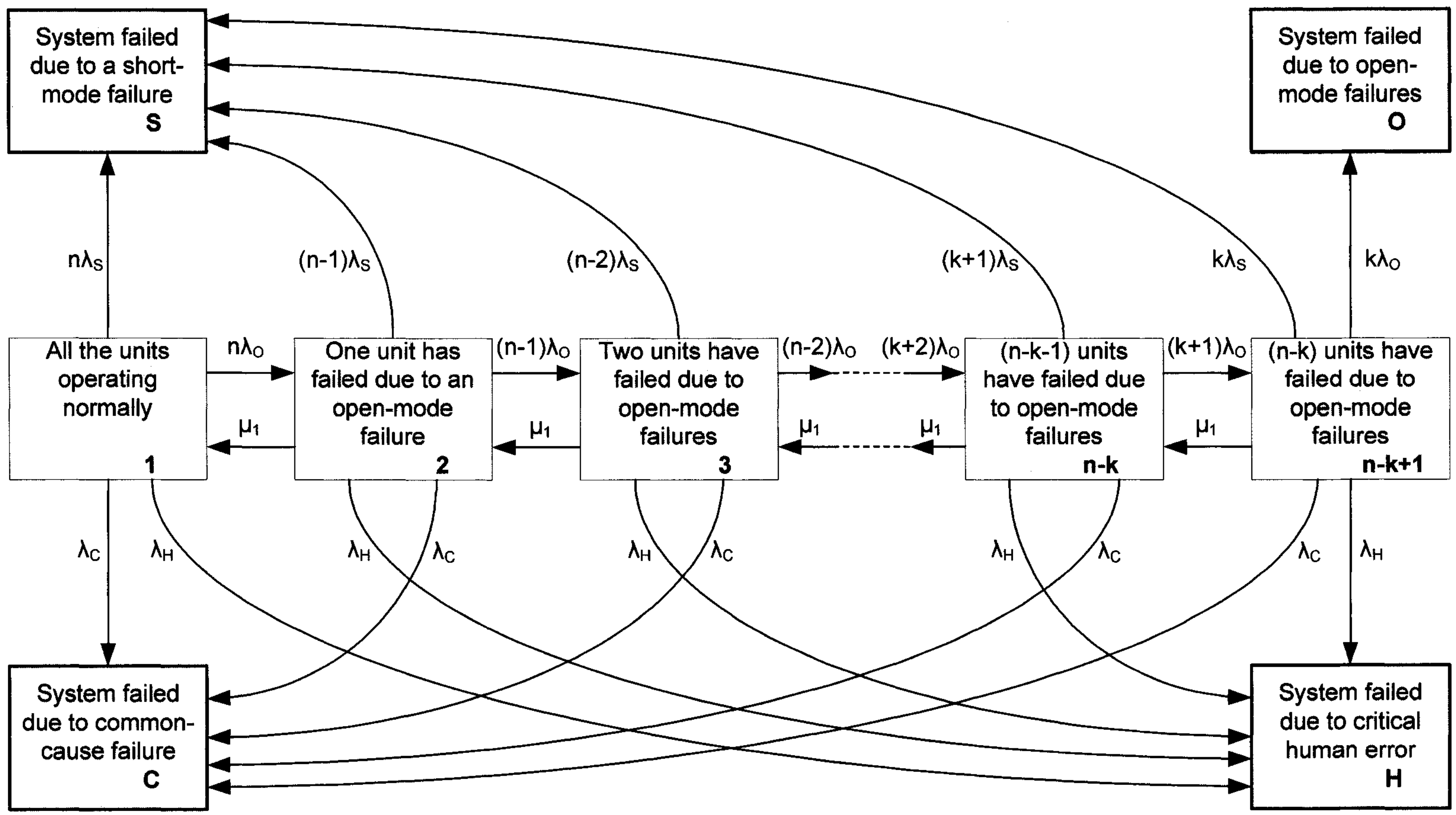

Figure 3-3: State transition diagram for three-state device k-out-of-n system under Type I repair policy

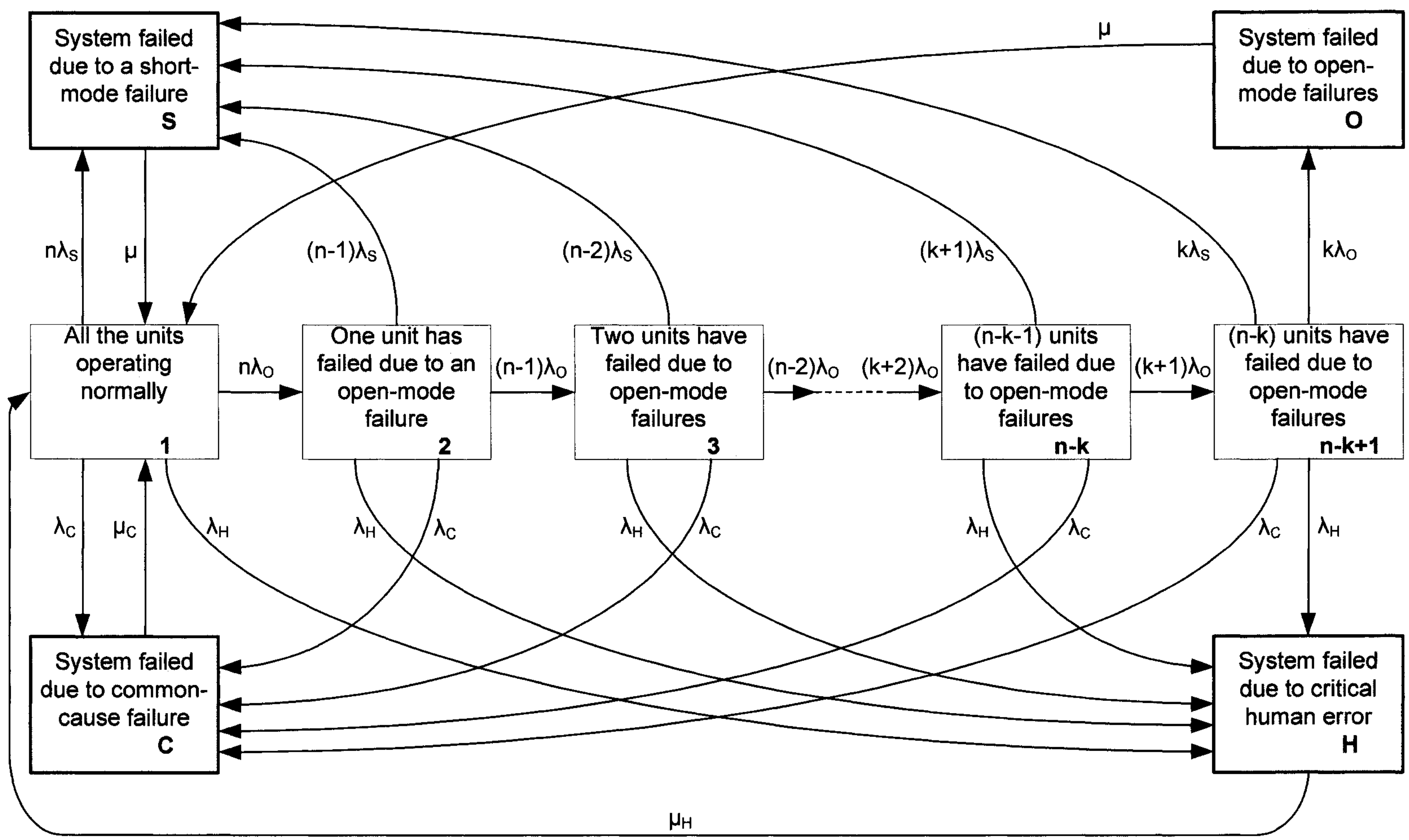

Figure 3-4: State transition diagram for three-state device k-out-of-n system under Type II repair policy

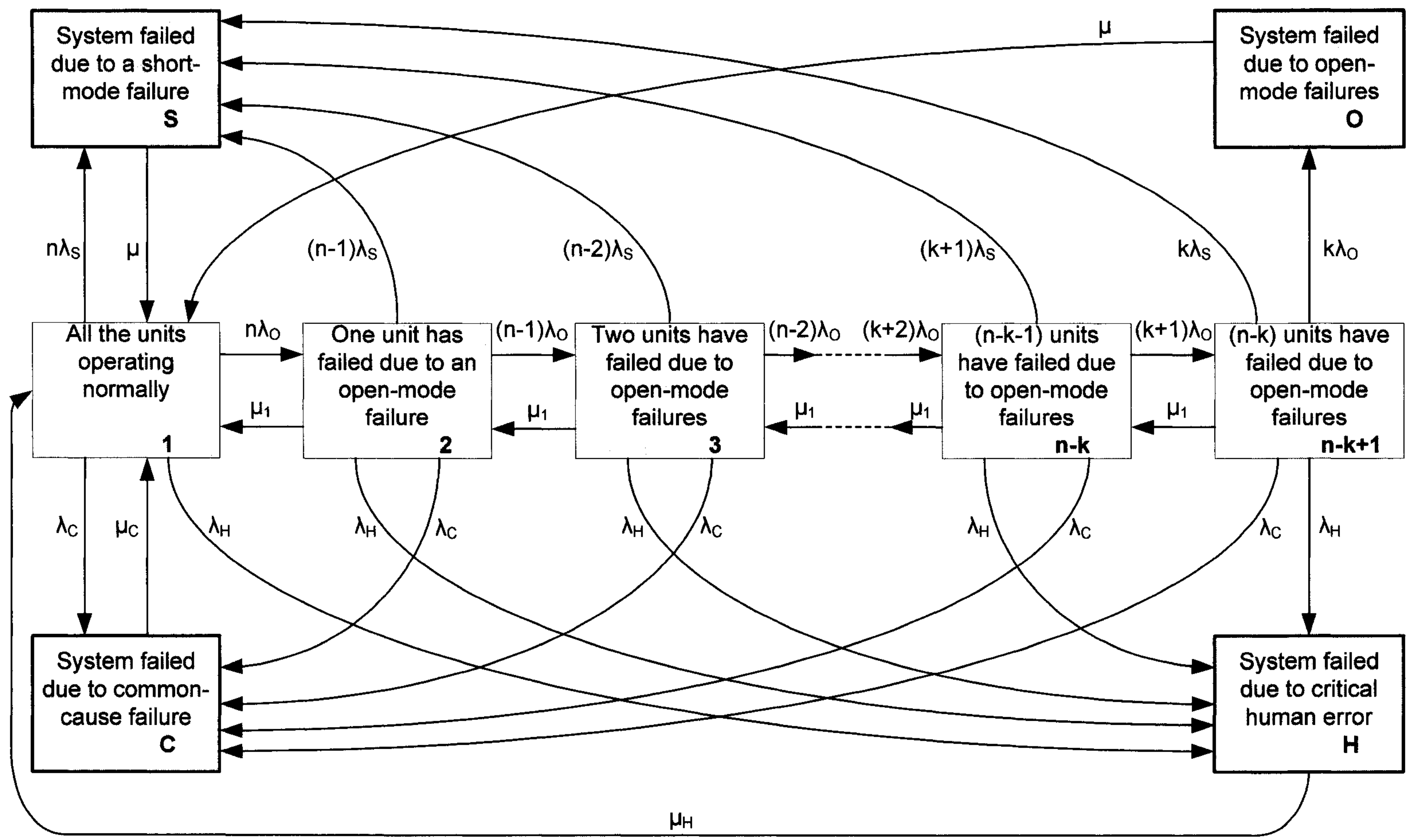

Figure 3-5: State transition diagram for three-state device k-out-of-n system under Type III repair policy

Assumptions

The following assumptions are associated with the analysis in this Chapter:

1. The system is composed of 'n' independent and identical devices in parallel.
2. Each unit (three-state device) can fail either in open-mode or in short-mode. These two failures are mutually exclusive of one another.
3. A critical human error or a common-cause failure can occur and trigger the whole system failure when two or more devices are operating.
4. All failures are statistically independent.
5. The failure rates for all the failures are constant.
6. The repaired unit/system is as good as new.
7. The repair rates for all types of failures are constant.
8. Repair rates are the same for completely failed system due to open-mode or short-mode failure.
9. All repair rates are the same for the partially failed system.
10. Common-cause failure rates are the same for the partially or fully operating system.
11. Critical human error rates are the same for the partially or fully operating system.

Notation

The following notation is associated with models presented in this Chapter:

n	Number of units.
t	Time.
s	Laplace transform variable.
L^{-1}	Inverse Laplace transform.
λ_O	Constant open-mode failure rate of the operating unit (three-state device).
λ_S	Constant short-mode failure rate of the operating unit (three-state device).
λ_C	Constant common-cause failure rate of the system.

λ_H	Constant critical human error rate of the system.
μ	Constant repair rate of the system from open-mode or short-mode failure.
μ_1	Constant repair rate of the unit (three-state device) for a partially failed system.
μ_C	Constant repair rate of the system when it failed due to a common-cause failure.
μ_H	Constant repair rate of the system when it failed due to a critical human error.
$P_i(t)$	The probability that the system is in state i at time t; for $i=1, 2, 3.....n\text{-}k+1$.
$P_j(t)$	The failed system state probability; for $j=C$ (due to a common-cause failure), $j=H$ (due to a critical human error), $j=S$ (due to a short-mode failure), $j=O$ (due to an open-mode failure).
$p_i(s)$	The Laplace transform of the state probability, for $i=1, 2, 3.....n\text{-}k+1$.
$p_j(s)$	The Laplace transform of the failed system state probability; for $j=C$ (due to a common-cause failure), $j=H$ (due to a critical human error), $j=S$ (due to a short-mode failure), $j=O$ (due to an open-mode failure).
P_i	The steady state probability of the system being in state i, for $i=1, 2, 3.....n\text{-}k+1$.
P_j	The failed system steady state probability; for $j=C$ (due to a common-cause failure), $j=H$ (due to a critical human error), $j=S$ (due to a short-mode failure), $j=O$ (due to an open-mode failure).
$R(s)$	Laplace transform of the system reliability.
$R(t)$	System reliability at time t.
$MTTF$	System mean time to failure.
$A(s)$	Laplace transform of the system availability.
$A(t)$	System availability at time t.
AV_{ss}	System steady state availability.

3.2 Three-State Device K-out-of-N System without Repair

This section presents the three-state device k-out-of-n system without repair and its two special cases: two-out-of-three system without repair and two-out-of-four system without repair. Generalized and special case expressions for system reliability and mean time to failure are developed. Some plots for the special case models are shown.

3.2.1 General k-out-of-n system without repair

The state transition diagram for the general model without repair is represented by Figure 3-2. Using the Markov technique, the system of differential equations associated with Figure 3-2 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) \qquad (3\text{-}1)$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H\right]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) \qquad (3\text{-}2)$$

for all $i = 2$ to $(n-k+1)$

$$\frac{dP_O(t)}{dt} = k\lambda_O P_{n-k+1}(t) \qquad \text{for all } k < n \qquad (3\text{-}3)$$

$$\frac{dP_S(t)}{dt} = \lambda_S \sum_{i=1}^{n-k+1} (n-i+1)P_i(t) \qquad \text{for all } k < n \qquad (3\text{-}4)$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \qquad (3\text{-}5)$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \qquad (3\text{-}6)$$

Initial Conditions: At time t=0, $P_1(0)=1$, and all other initial state transition probabilities are equal to zero.

By taking Laplace transforms of Equations (3-1) – (3-6) and solving the resulting equations, the following expressions for the Laplace transforms of the state probabilities results:

$$p_1(s) = \frac{1}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{3-7}$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_O}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H} \right] p_{i-1}(s) \tag{3-8}$$

for all $i = 2$ to $(n-k+1)$

$$p_O(s) = \frac{k\lambda_O}{s} p_{n-k+1}(s) \qquad \text{for all } k < n \tag{3-9}$$

$$p_S(s) = \frac{\lambda_S}{s} \sum_{i=1}^{n-k+1} (n-i+1) p_i(s) \qquad \text{for all } k < n \tag{3-10}$$

$$p_C(s) = \frac{\lambda_C}{s} \sum_{i=1}^{n-k+1} p_i(s) \qquad \text{for all } k \geq 2 \text{ and } k < n \tag{3-11}$$

$$p_H(s) = \frac{\lambda_H}{s} \sum_{i=1}^{n-k+1} p_i(s) \qquad \text{for all } k \geq 2 \text{ and } k < n \tag{3-12}$$

By taking the inverse Laplace transforms of Equations (3-7) – (3-12), the time-dependent transition state probabilities can be obtained. The time-dependent system reliability is given by

$$R(t) = \sum_{i=1}^{n} P_i(t) = L^{-1}\left[\sum_{i=1}^{n-k+1} p_i(s) \right] \tag{3-13}$$

The system mean time to failure is given by

$$MTTF = \lim_{s \to 0} \left[\sum_{i=1}^{n-k+1} p_i(s) \right] \tag{3-14}$$

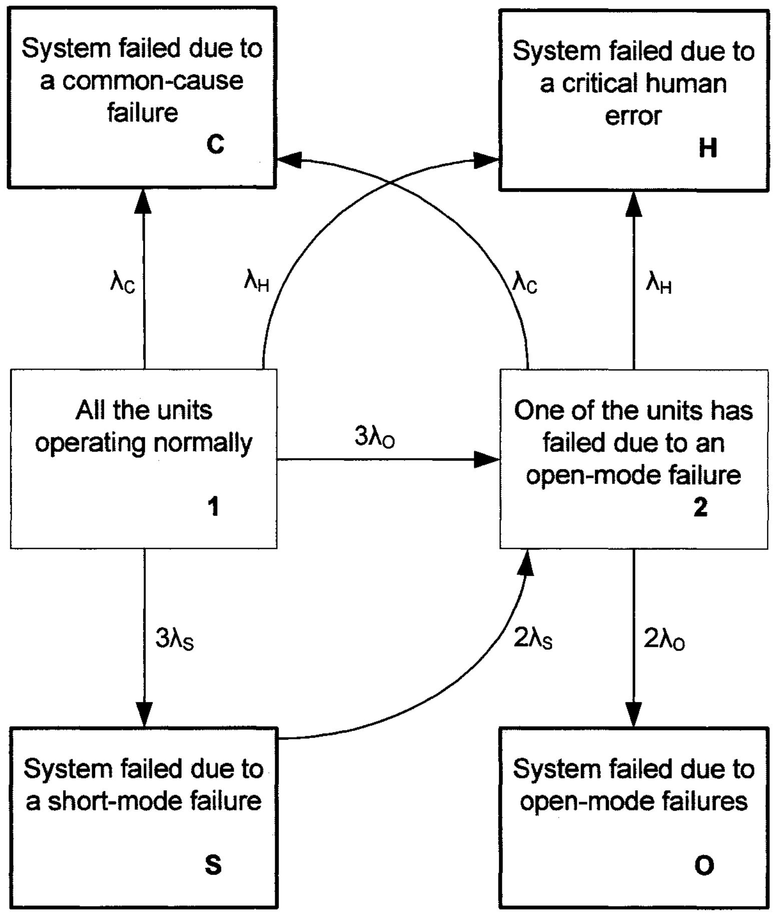

Figure 3-6: State transition diagram of two-out-of-three system with Type I repair

3.2.2 Special case model 3-A: Two-out-of-three system without repair

The state transition diagram for two-out-of-three system without repair is shown in Figure 3-6. For k=2 and n=3 in Figure 3-2, from Equations (3-7) – (3-12), we get the following set of equations:

$$p_1(s) = \frac{1}{s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H} \tag{3-15}$$

$$p_2(s) = \frac{3\lambda_O}{(s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H)(s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)} \tag{3-16}$$

$$p_O(s) = \frac{6\lambda_O^{\,2}}{s(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{3-17}$$

$$p_S(s) = \frac{3\lambda_S(s+4\lambda_O+2\lambda_S+\lambda_C+\lambda_H)}{s(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{3-18}$$

$$p_C(s) = \frac{\lambda_C(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H)}{s(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{3-19}$$

$$p_H(s) = \frac{\lambda_H(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H)}{s(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{3-20}$$

By taking inverse Laplace transforms of Equations (3-15) to (3-20), the time-dependent state probabilities can be obtained. By adding Equations (3-15) and (3-16) and then taking inverse Laplace transform, we obtain the following expression for the system reliability:

$$R(t) = \frac{1}{\lambda_O+\lambda_S}\left[\frac{\lambda_S-2\lambda_O}{e^{(3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)t}} + \frac{3\lambda_O}{e^{(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)t}}\right] \tag{3-21}$$

The system mean time to failure is given by

$$MTTF = \int_0^\infty R(t)\,dt = \frac{5\lambda_O+2\lambda_S+\lambda_C+\lambda_H}{(3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \tag{3-22}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-7. For the specified values of the model parameters, the plots of Equation (3-21) are shown in Figure 3-8. Similarly, for the specified values of the model parameters, the plots of Equation (3-22) are shown in Figure 3-9.

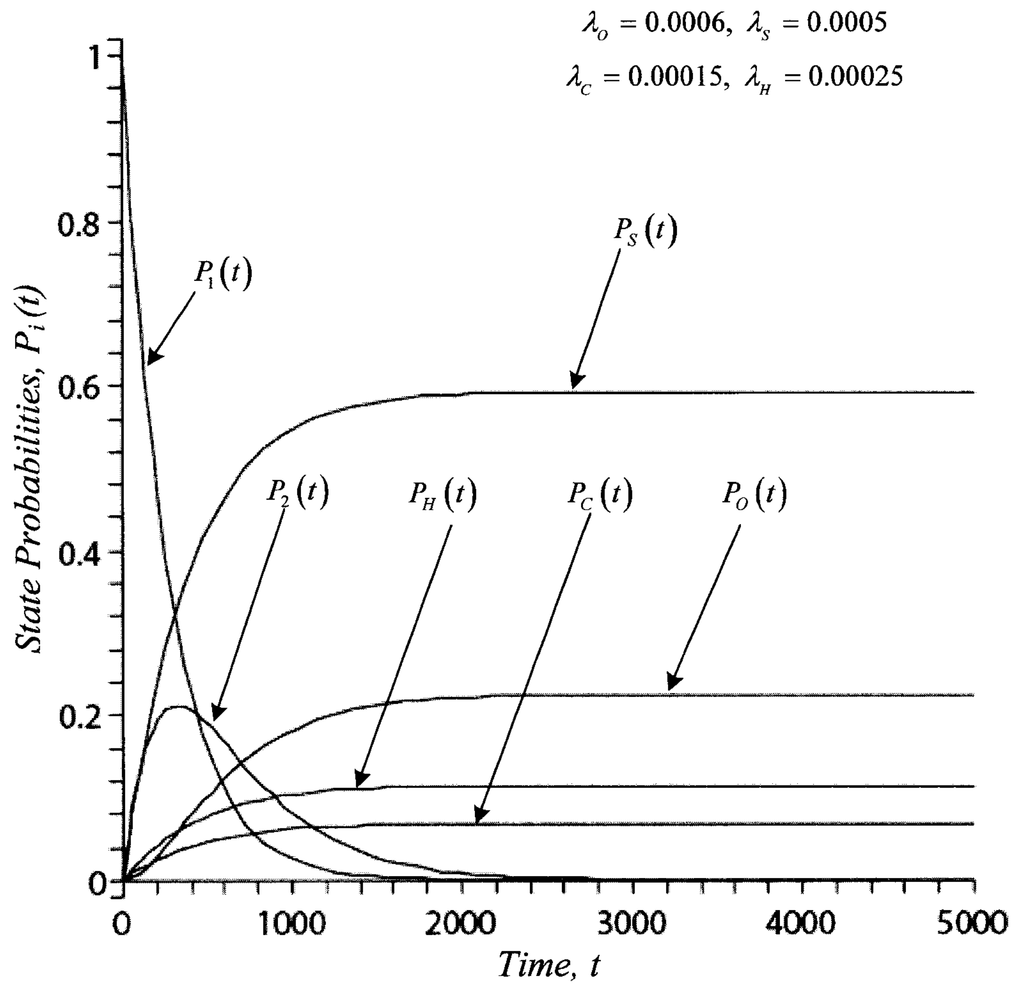

Figure 3-7: State probability plots of two-out-of-three system without repair

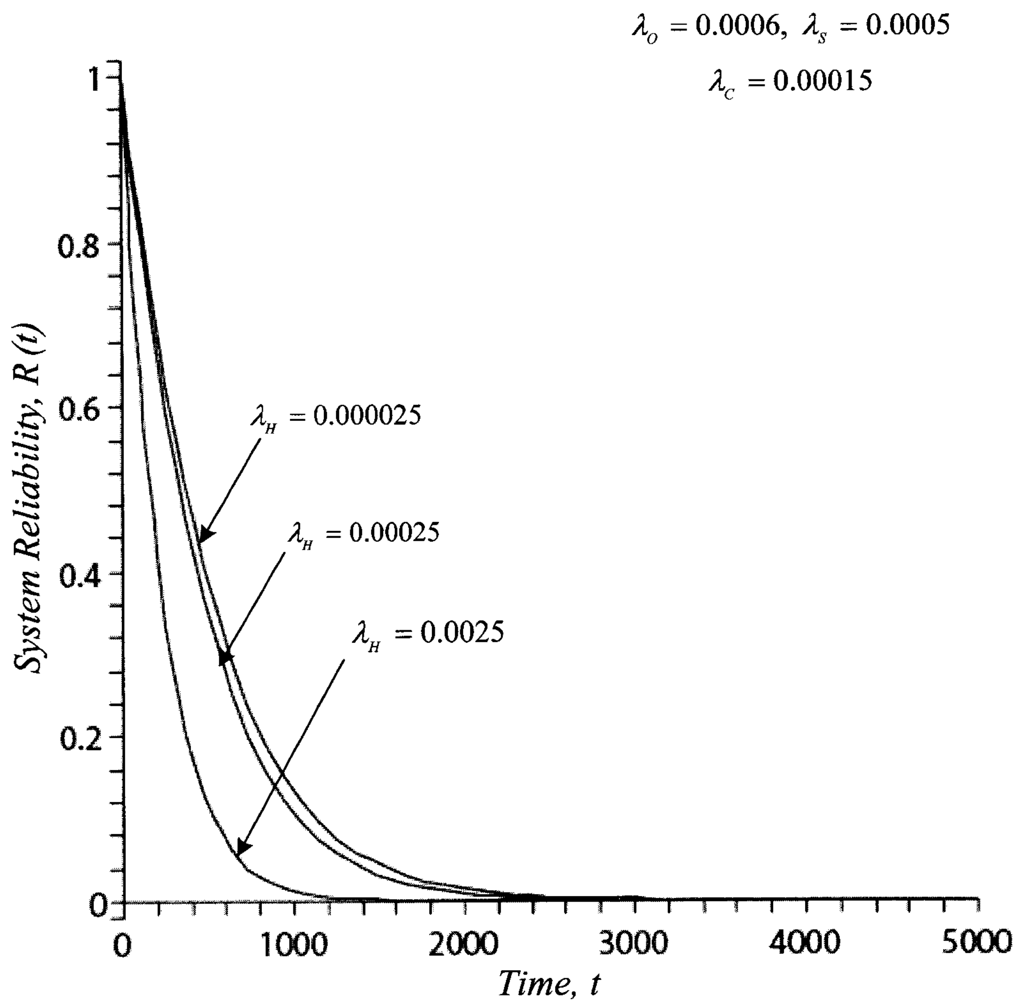

Figure 3-8: Reliability plots of two-out-of-three system without repair

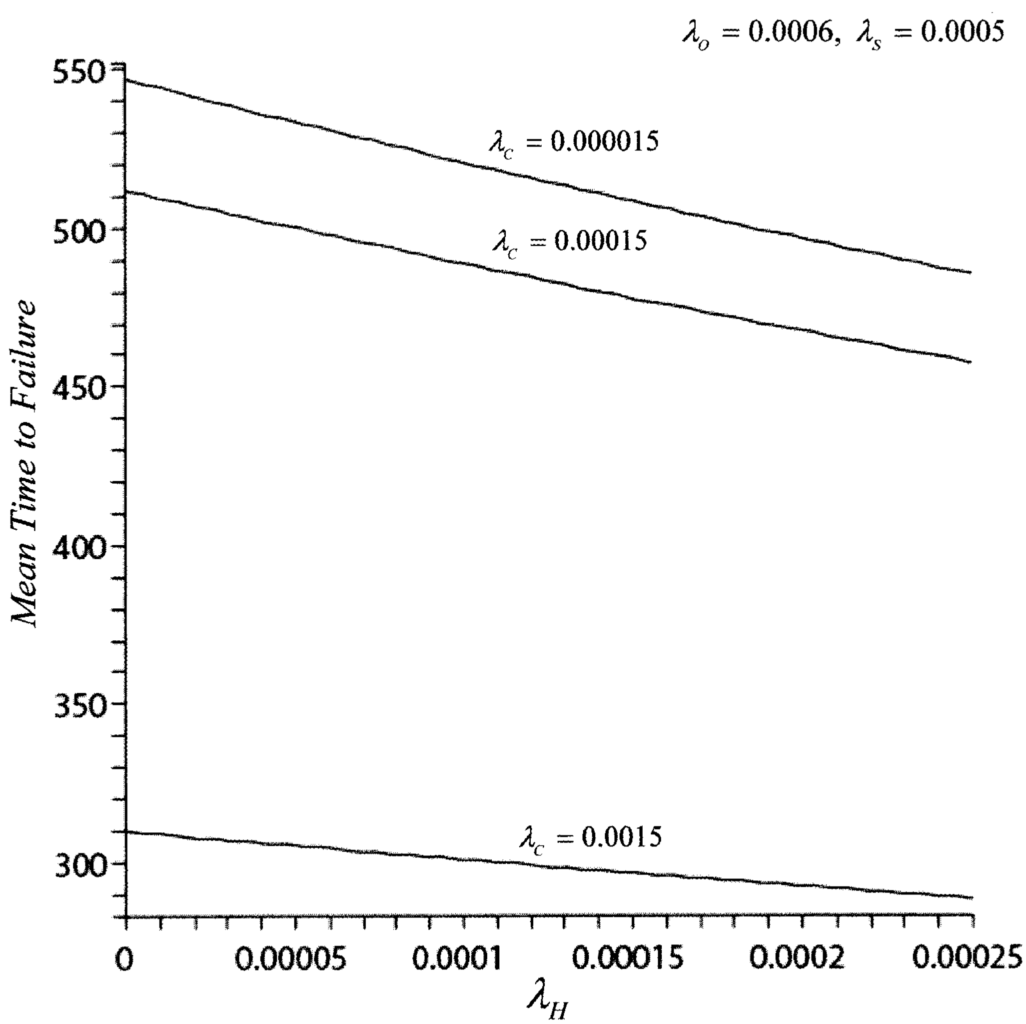

Figure 3-9: Mean time to failure plots of two-out-of-three system without repair

3.2.3 Special case model 3-B: Two-out-of four system without repair

The state transition diagram for two-out-of-four system without repair is shown in Figure 3-10.

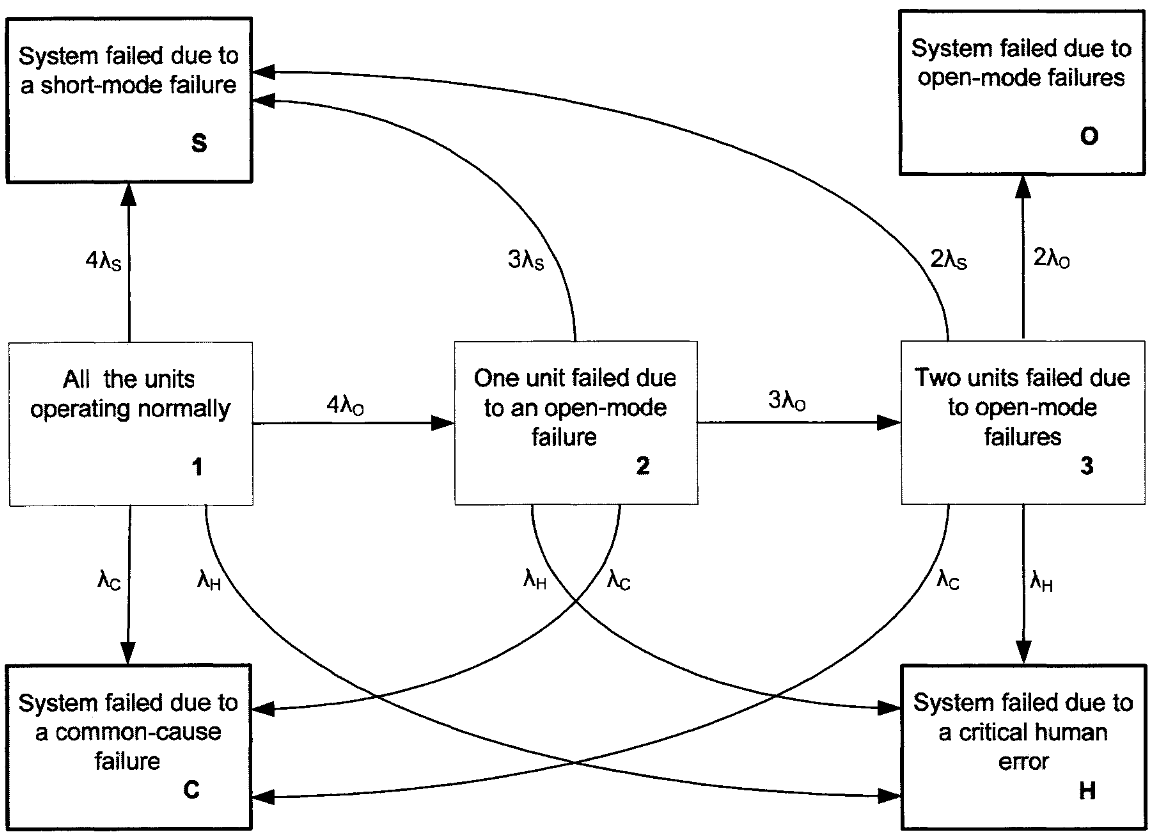

Figure 3-10: State transition diagram of two-out-of-four system without repair

By setting k=2 and n=4 in Equations (3-7) – (3-12), we obtain the following set of equations:

$$p_1(s) = \frac{1}{s + 4\lambda_O + 4\lambda_S + \lambda_C + \lambda_H} \tag{3-23}$$

$$p_2(s) = \left[\frac{4\lambda_O}{s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H}\right]\left[\frac{1}{s + 4\lambda_O + 4\lambda_S + \lambda_C + \lambda_H}\right] \tag{3-24}$$

$$p_3(s)=\left[\frac{3\lambda_O}{s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H}\right]\left[\frac{4\lambda_O}{s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H}\right]\left[\frac{1}{s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H}\right]$$

(3-25)

$$p_O(s)=\frac{2\lambda_O}{s}\left[\frac{3\lambda_O}{s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H}\right]\left[\frac{4\lambda_O}{s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H}\right]\left[\frac{1}{s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H}\right]$$

(3-26)

$$p_S(s)=\frac{4\lambda_S}{s}\left[\frac{(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+3\lambda_O(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+6\lambda_O^2}{(s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H)(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)}\right]$$

(3-27)

$$p_C(s)=\frac{\lambda_C}{s}\left[\frac{(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+4\lambda_O(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+12\lambda_O^2}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H)}\right]$$

(3-28)

$$p_H(s)=\frac{\lambda_H}{s}\left[\frac{(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+4\lambda_O(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)+12\lambda_O^2}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H)}\right]$$

(3-29)

The time-dependent state probabilities can be obtained by computing the inverse Laplace transforms of Equations (3-23) – (3-29). By adding Equations (3-23) – (3-25) and then taking the inverse Laplace transform, the following expression for the time-dependent system reliability is obtained:

$$R(t)=\sum_{i=1}^{3}P_i(t)=\frac{1}{(\lambda_O+\lambda_S)^2}\left\{\frac{3\lambda_O-2(\lambda_O^2-\lambda_S^2)}{2e^{(4\lambda_O+4\lambda_S+\lambda_C+\lambda_H)t}}+\frac{\lambda_O(4\lambda_O+4\lambda_S-3)}{e^{(3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)t}}+\frac{3\lambda_O}{2e^{(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)t}}\right\}$$

(3-30)

The system mean time to failure is given by

$$MTTF = \frac{(2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)(7\lambda_O + 3\lambda_S + \lambda_C + \lambda_H) + 12\lambda_O^2}{(2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)(3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H)(4\lambda_O + 4\lambda_S + \lambda_C + \lambda_H)} \quad (3\text{-}31)$$

For the specified values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-11, the plots of system reliability [i.e., Equation (3-30)] as a function of time, t, are shown in Figure 3-12 and the plots of mean time to failure [i.e., Equation (3-31)] are shown in Figure 3-13.

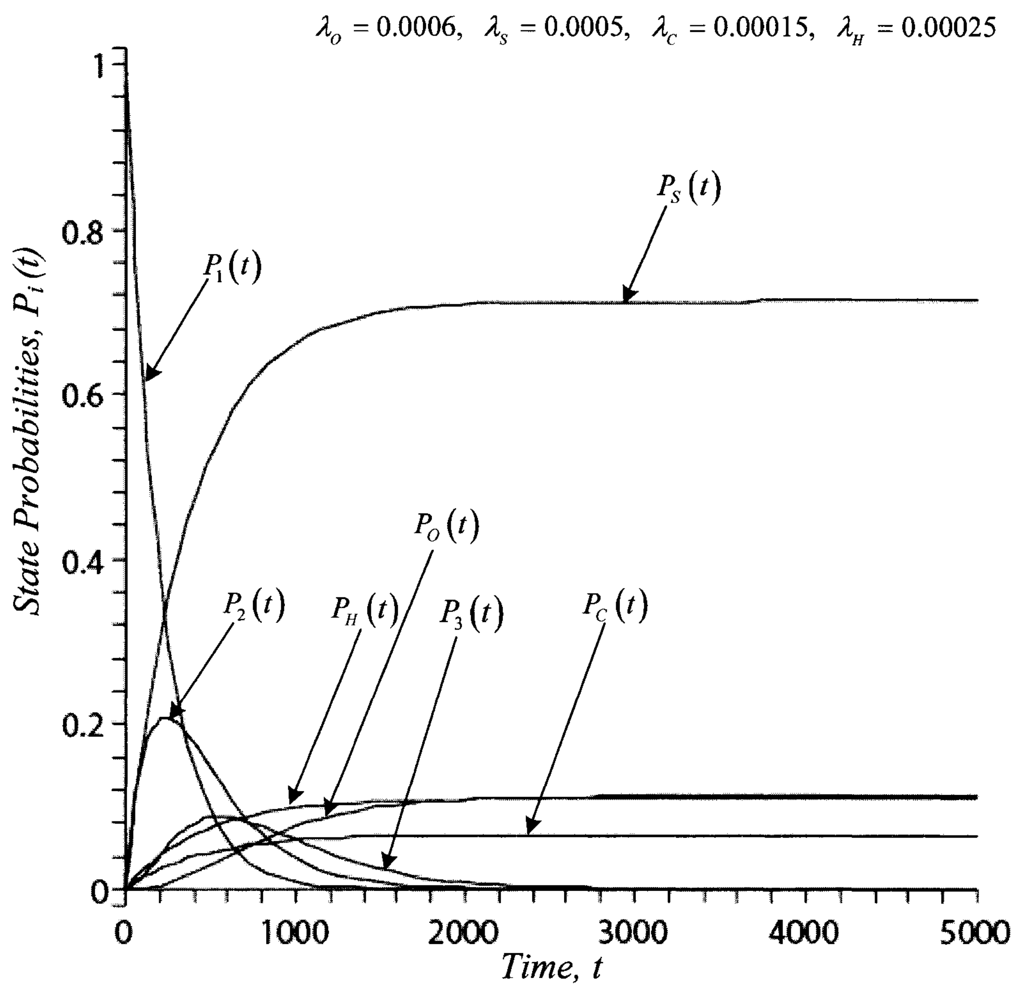

Figure 3-11: State probability plots of two-out-of-four system without repair

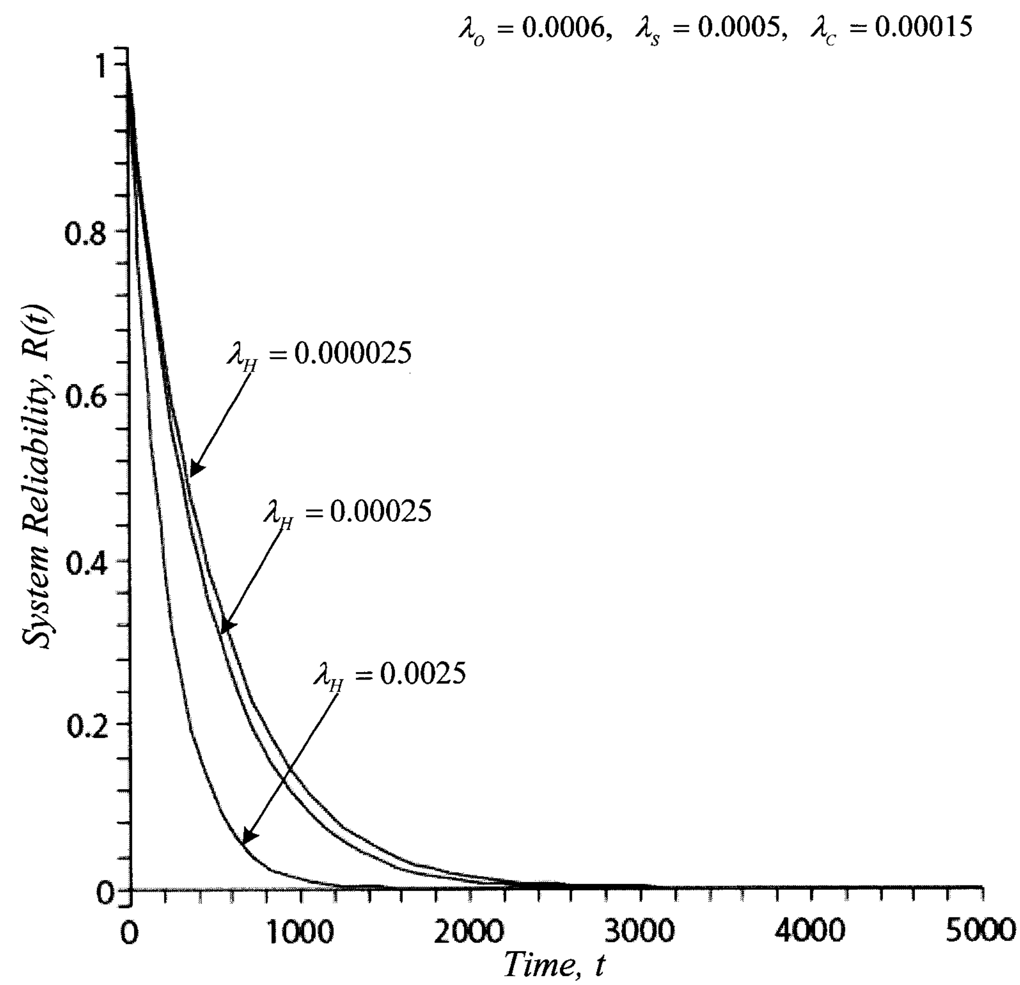

Figure 3-12: Reliability plots of two-out-of-four system without repair

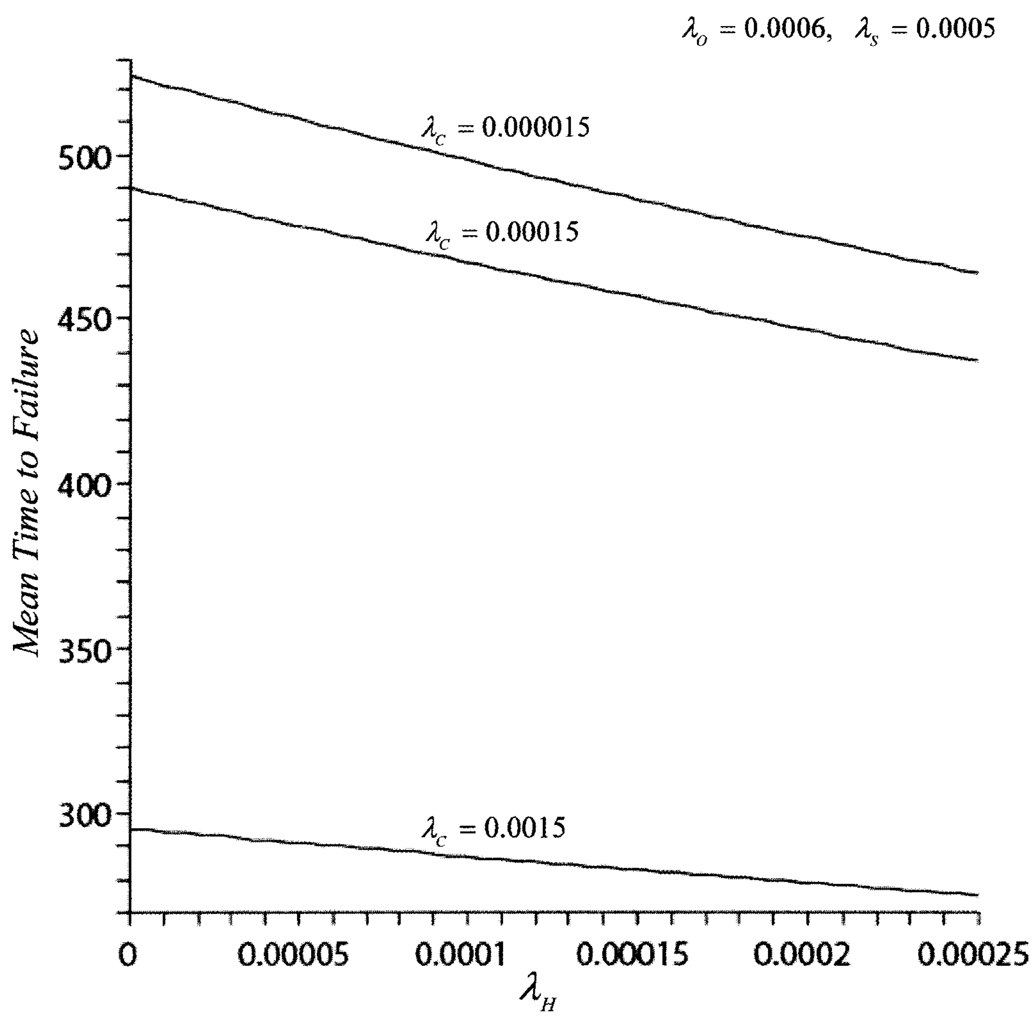

Figure 3-13: Mean time to failure plots of two-out-of-four system without repair

3.3 Three-State Device K-out-of-N System with Type I Repair Policy

In this section, three-state device k-out-of-n system with Type I repair policy and its two special cases: two-out-of-three system with Type I repair policy and two-out-of-four system with Type I repair policy are discussed. Some plots for the special case models are shown.

3.3.1 General k-out-of-n system with Type I repair

The block diagram and the state transition diagram for the general model with Type I repair policy is shown in Figures 3-1 and 3-3 respectively. Using the Markov method, the system of differential equations associated with Figure 3-3 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) + \mu_1 P_2(t) \tag{3-32}$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1\right]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) + \mu_1 P_{i+1}(t) \tag{3-33}$$

for all $i = 2$ to $(n-k)$

$$\frac{dP_{n-k+1}(t)}{dt} = -\left[k\lambda_O + k\lambda_S + \lambda_C + \lambda_H + \mu_1\right]P_{n-k+1}(t) + (k+1)\lambda_O P_{n-k}(t) \tag{3-34}$$

$$\frac{dP_O(t)}{dt} = k\lambda_O P_{n-k+1}(t) \qquad \text{for all } k < n \tag{3-35}$$

$$\frac{dP_S(t)}{dt} = \lambda_S \sum_{i=1}^{n-k+1} (n-i+1)P_i(t) \qquad \text{for all } k < n \tag{3-36}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \tag{3-37}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \tag{3-38}$$

At time t=0, $P_1(0)$ = 1, and all other initial state transition probabilities are equal to zero.

By taking Laplace transforms of Equations (3-32) – (3-38) and solving the resulting equations by applying the initial conditions, we get the following set of equations:

$$p_1(s) = \frac{1 + \mu_1 p_2(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{3-39}$$

$$p_i(s) = \frac{(n-i+2)\lambda_O p_{i-1}(s) + \mu_1 p_{i+1}(s)}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1} \quad \text{for all } i = 2 \text{ to } (n-k) \tag{3-40}$$

$$p_{n-k+1}(s) = \frac{(k+1)\lambda_O p_{n-k}(s)}{s + k\lambda_O + k\lambda_S + \lambda_C + \lambda_H + \mu_1} \tag{3-41}$$

$$p_O(s) = \frac{k\lambda_O p_{n-k+1}(s)}{s} \quad \text{for all } k < n \tag{3-42}$$

$$p_S(s) = \frac{\lambda_S}{s} \sum_{i=1}^{n-k+1} (n-i+1) p_i(s) \quad \text{for all } k < n \tag{3-43}$$

$$p_C(s) = \frac{\lambda_C}{s} \sum_{i=1}^{n-k+1} p_i(s) \quad \text{for all } k \geq 2 \text{ and } k < n \tag{3-44}$$

$$p_H(s) = \frac{\lambda_H}{s} \sum_{i=1}^{n-k+1} p_i(s) \quad \text{for all } k \geq 2 \text{ and } k < n \tag{3-45}$$

By taking the inverse Laplace transforms of Equations (3-39) – (3-45), the time-dependent state probabilities can be obtained. The time-dependent system reliability is obtained as follows:

$$R(t) = L^{-1}\left[\sum_{i=1}^{n-k+1} p_i(s)\right] \tag{3-46}$$

The system mean time to failure is calculated as:

$$MTTF = \lim_{s \to 0}\left[\sum_{i=1}^{n-k+1} p_i(s)\right] \tag{3-47}$$

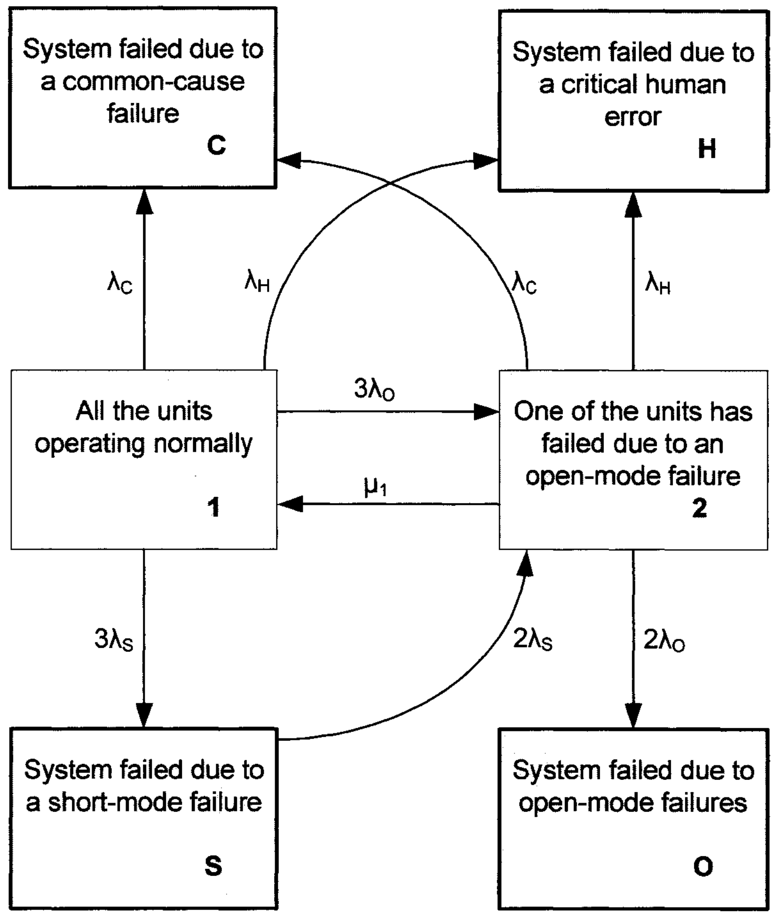

Figure 3-14: State transition diagram of two-out-of-three system with Type I repair policy

3.3.2 Special case model 3-C: Two-out-of-three system with Type I repair

The state transition diagram of two-out-of-three system with Type I repair policy is represented by Figure 3-14. By substituting for k=2 and n=3 in Equations (3-39) to (3-45), the following set of equations results:

$$p_1(s) = \frac{s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1}{(s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H)(s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1) - 3\mu_1\lambda_O} \tag{3-48}$$

$$p_2(s) = \frac{3\lambda_O}{(s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H)(s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1) - 3\mu_1\lambda_O} \tag{3-49}$$

$$p_O(s) = \frac{6\lambda_O^{\,2}}{s\left[(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)-3\mu_1\lambda_O\right]} \tag{3-50}$$

$$p_S(s) = \frac{3\lambda_S\left(s+4\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1\right)}{s\left[(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)-3\mu_1\lambda_O\right]} \tag{3-51}$$

$$p_C(s) = \frac{\lambda_C(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{s\left[(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)-3\mu_1\lambda_O\right]} \tag{3-52}$$

$$p_H(s) = \frac{\lambda_H(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{s\left[(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)-3\mu_1\lambda_O\right]} \tag{3-53}$$

By taking the inverse Laplace transforms of Equations (3-48) – (3-53), the time-dependent state probabilities can be obtained. Adding Equations (3-48) and (3-49) and then taking the inverse Laplace transform, we obtain the following expression for time-dependent system reliability:

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\frac{s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1}{(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)-3\mu_1\lambda_O}\right] \tag{3-54}$$

The system mean time to failure is given by:

$$MTTF = \lim_{s\to 0}\left[\frac{s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1}{(s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)-3\mu_1\lambda_O}\right] \tag{3-55}$$

For the specified values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-15, the plots of Equation (3-54) [i.e., system reliability as a function of time, t] are shown in Figure 3-16 and the plots of Equation (3-55) [i.e., mean time to failure] are shown in Figure 3-17.

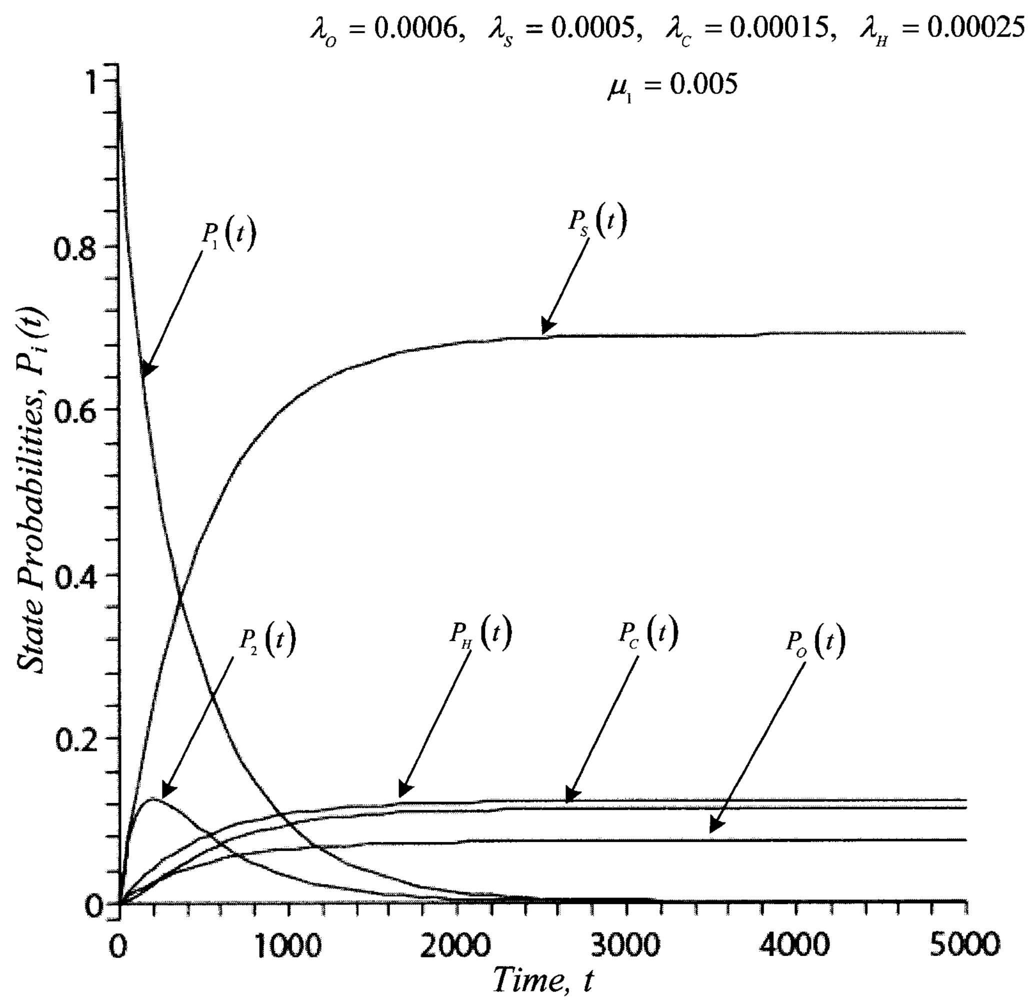

Figure 3-15: State probability plots of two-out-of-three system with Type I repair

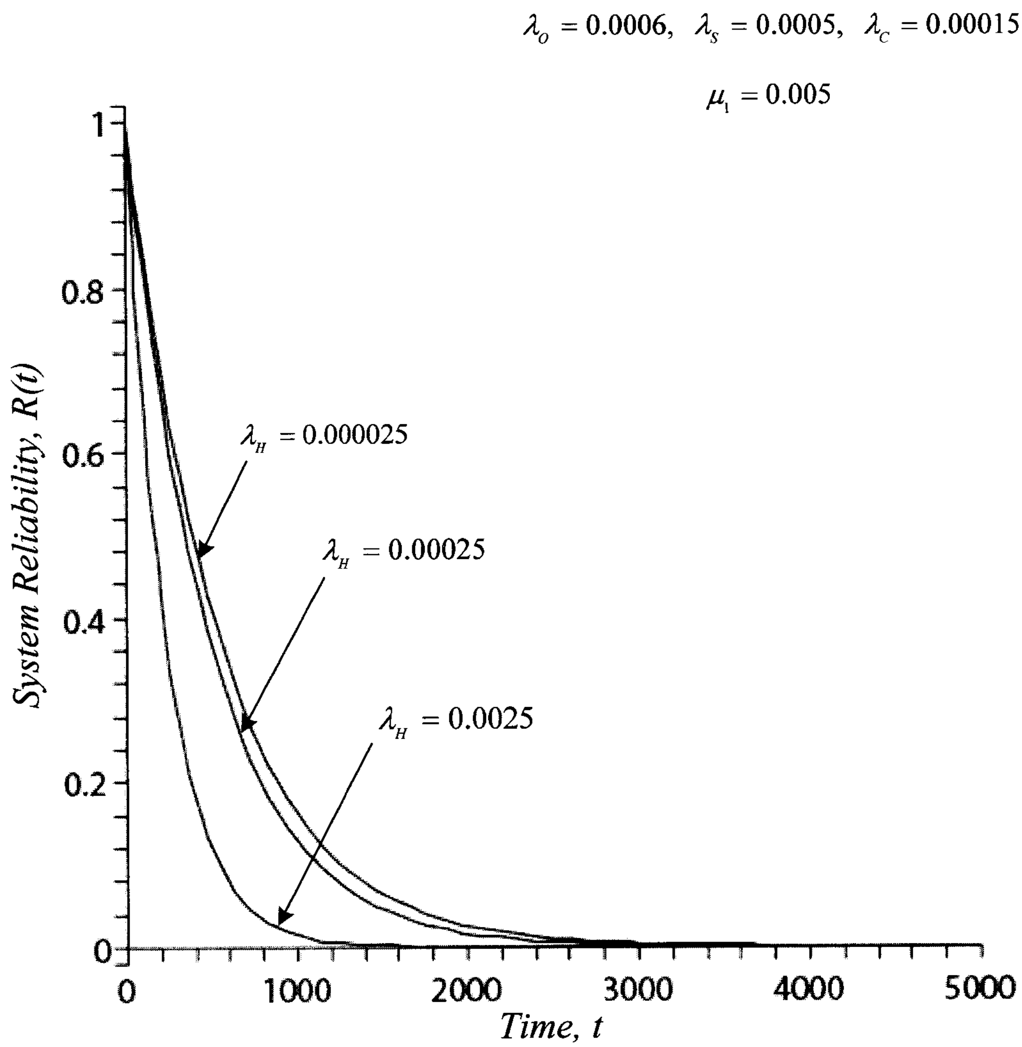

Figure 3-16: Reliability plots of two-out-of-three system with Type I repair

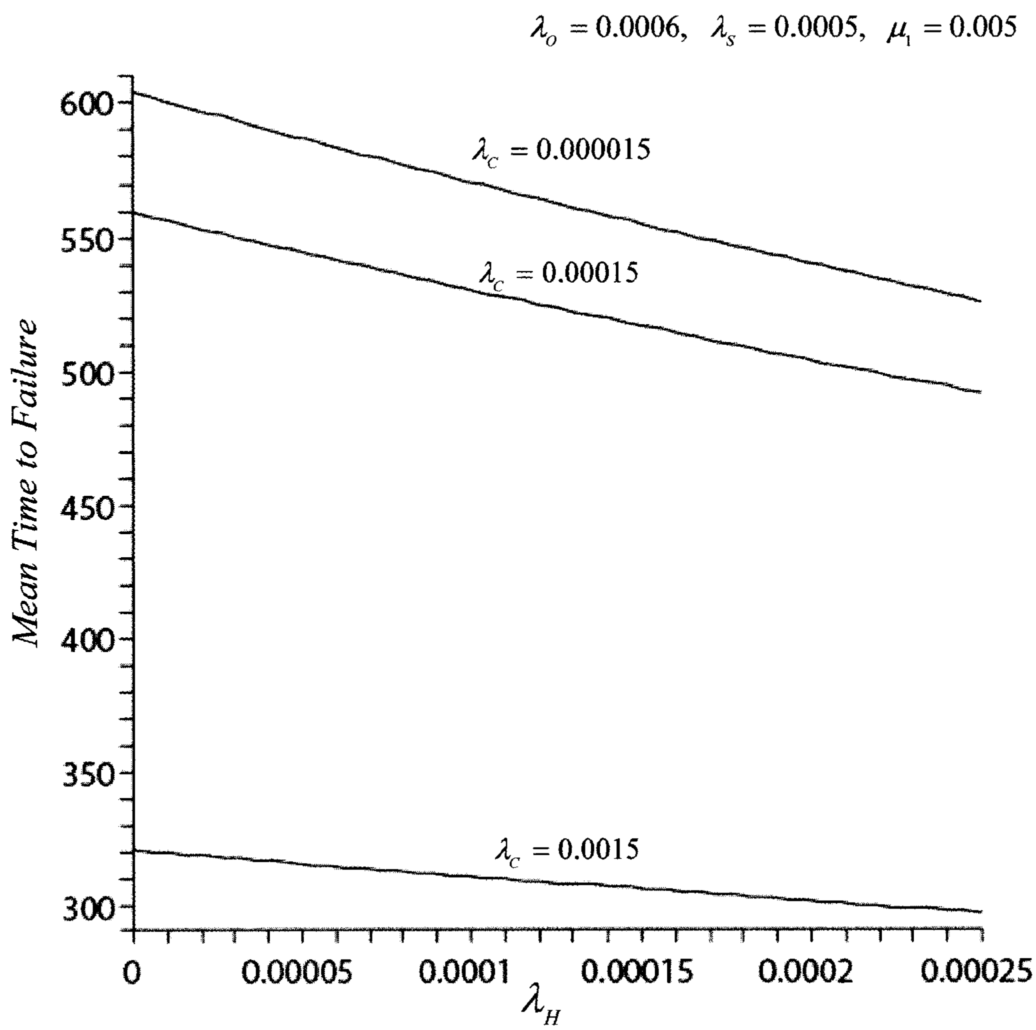

Figure 3-17: Mean time to failure plots of two-out-of-three system with Type I repair

3.3.3 Special case model 3-D: Two-out-of-four system with Type I repair

The state transition diagram of two-out-of-four system with Type I repair policy is depicted in Figure 3-18.

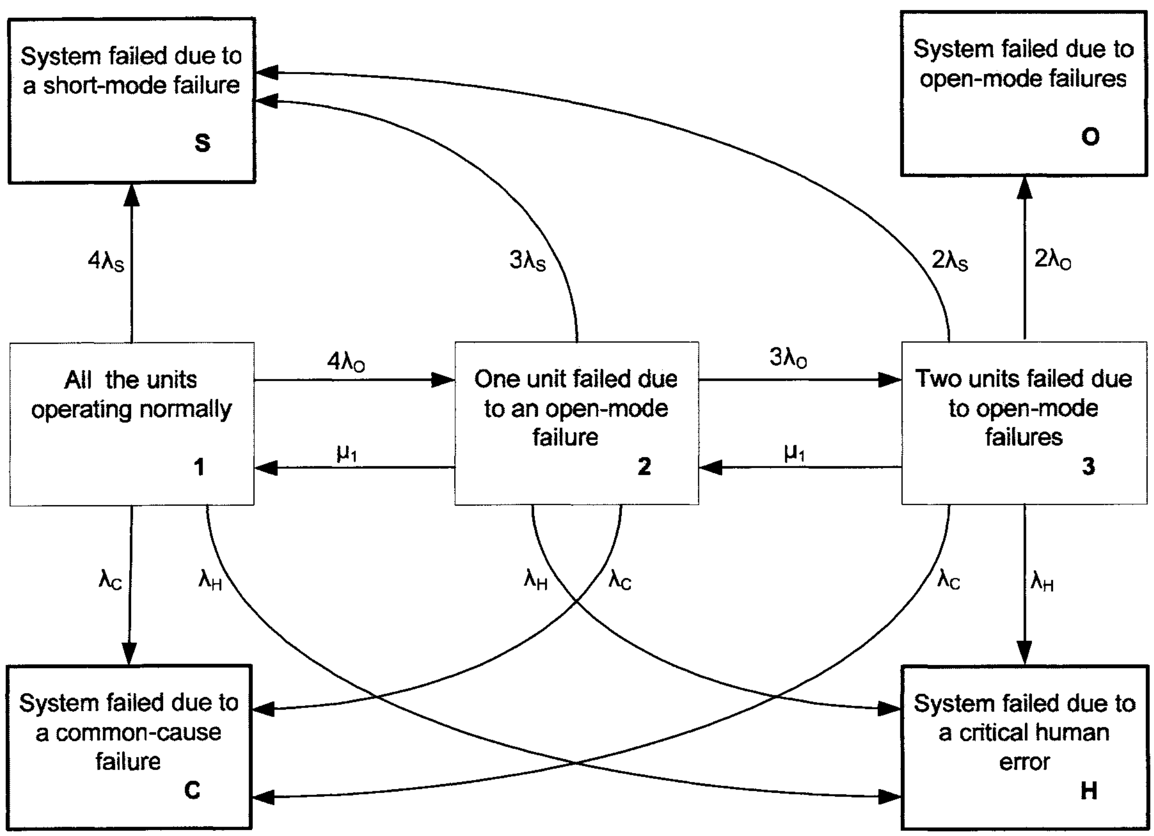

Figure 3-18: State transition diagram of two-out-of-four system with Type I repair policy

For k=2 and n=4 in Figure 3-3, from Equations (3-39) – (3-45), we get the following set of equations:

$$p_1(s) = \frac{BC - 3\mu_1\lambda_O}{ABC - \mu_1\lambda_O(3A + 4C)} \tag{3-56}$$

$$p_2(s) = \frac{4C\lambda_O}{ABC - \mu_1\lambda_O(3A + 4C)} \tag{3-57}$$

$$p_3(s) = \frac{12\lambda_O^2}{ABC - \mu_1\lambda_O(3A+4C)} \tag{3-58}$$

$$p_O(s) = \frac{24\lambda_O^3}{s\left[ABC - \mu_1\lambda_O(3A+4C)\right]} \tag{3-59}$$

$$p_S(s) = \left(\frac{4\lambda_S}{s}\right)\left[\frac{BC - 3\mu_1\lambda_O + 3C\lambda_O + 6\lambda_O^2}{ABC - \mu_1\lambda_O(3A+4C)}\right] \tag{3-60}$$

$$p_C(s) = \left(\frac{\lambda_C}{s}\right)\left[\frac{BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^2}{ABC - \mu_1\lambda_O(3A+4C)}\right] \tag{3-61}$$

$$p_H(s) = \left(\frac{\lambda_H}{s}\right)\left[\frac{BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^2}{ABC - \mu_1\lambda_O(3A+4C)}\right] \tag{3-62}$$

where:

$A = s + 4\lambda_O + 4\lambda_S + \lambda_C + \lambda_H$

$B = s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H + \mu_1$

$C = s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1$

By taking the inverse Laplace transforms of Equations (3-56) – (3-62), the time-dependent state probabilities can be obtained. By adding Equations (3-56) – (3-58) and then taking the inverse Laplace transform, we get the following expression for the system reliability:

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left\{\frac{BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^2}{ABC - \mu_1\lambda_O(3A+4C)}\right\} \tag{3-63}$$

The system mean time to failure is given by

$$MTTF = \lim_{s\to 0}\left[\frac{BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^2}{ABC - \mu_1\lambda_O(3A+4C)}\right] \tag{3-64}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-19. For the specified values of the model parameters, the plots of Equation (3-63) are shown in Figure 3-20. Similarly, for the specified values of the model parameters, the plots of Equation (3-64) are shown in Figure 3-21.

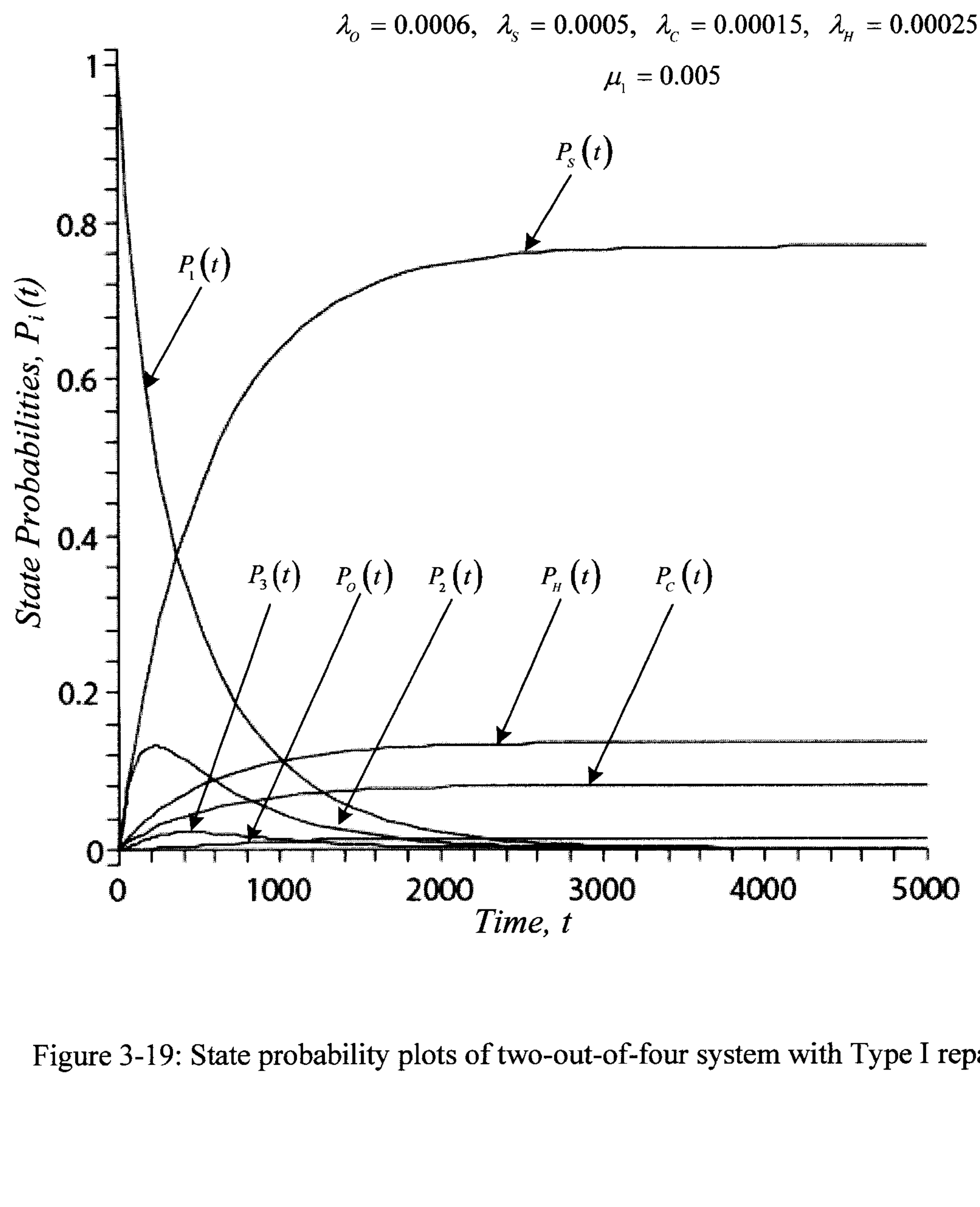

Figure 3-19: State probability plots of two-out-of-four system with Type I repair

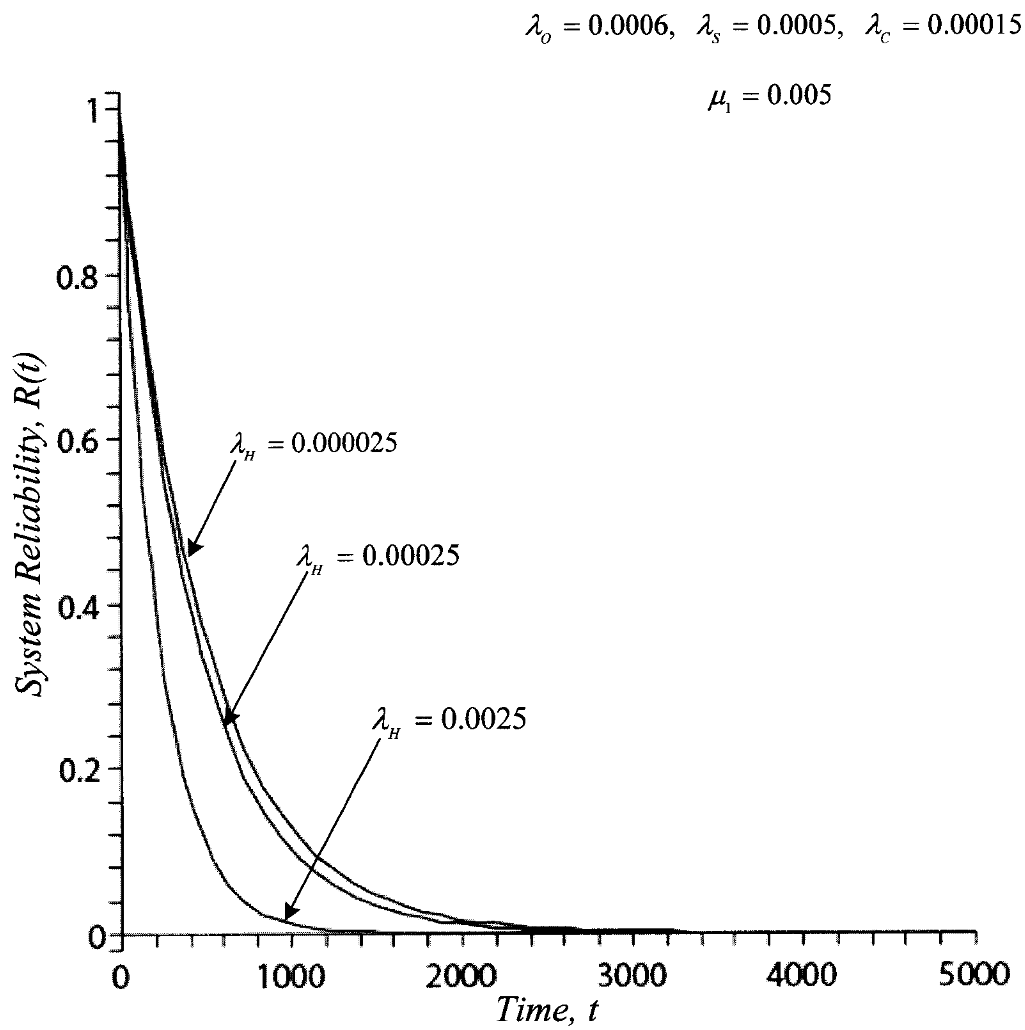

Figure 3-20: Reliability plots of two-out-of-four system with Type I repair

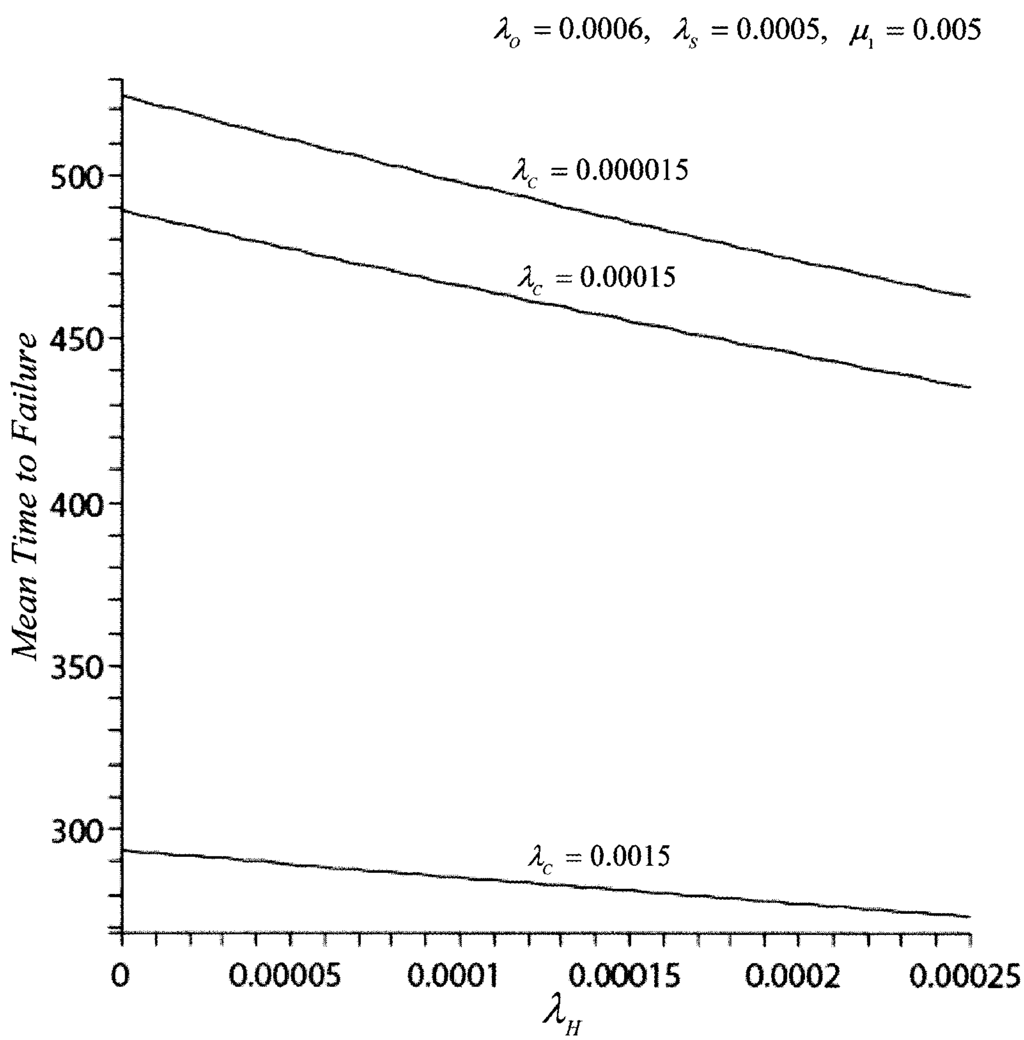

Figure 3-21: Mean time to failure plots of two-out-of-four system with Type I repair

3.4 Three-State Device K-out-of-N System with Type II Repair Policy

In this section, three-state device k-out-of-n system with Type II repair policy and its two special cases: two-out-of-three system with Type II repair policy and two-out-of-four system with Type II repair policy are presented.

3.4.1 General k-out-of-n system with Type II repair

The state transition diagram for the general k-out-of-n system under Type II repair policy is shown in Figure 3-4. Using the Markov state-space technique, the system of differential equations associated with Figure 3-4 is obtained as follows:

$$\frac{dP_1(t)}{dt} = -[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H]P_1(t) + \mu[P_O(t) + P_S(t)] + \mu_C P_C(t) + \mu_H P_H(t) \qquad (3\text{-}65)$$

$$\frac{dP_i(t)}{dt} = -[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) \qquad (3\text{-}66)$$

for all $i = 2$ to $(n-k+1)$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + k\lambda_O P_{n-k+1}(t) \qquad \text{for all } k < n \qquad (3\text{-}67)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S \sum_{i=1}^{n-k+1} (n-i+1)P_i(t) \qquad \text{for all } k < n \qquad (3\text{-}68)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \qquad (3\text{-}69)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \qquad (3\text{-}70)$$

At time t=0, $P_1(0) = 1$, and all other initial state probabilities are equal to zero. Taking Laplace transforms of Equations (3-65) – (3-70) and solving the resulting equations, we obtain the following set of equations:

$$p_1(s) = \frac{1 + \mu\left[p_O(s) + p_S(s)\right] + \mu_C p_C(s) + \mu_H p_H(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{3-71}$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_O}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H}\right] p_{i-1}(s) \tag{3-72}$$

for all $i = 2$ to $(n-k+1)$

$$p_O(s) = \left[\frac{k\lambda_O}{s+\mu}\right] p_{n-k+1}(s) \qquad \text{for all } k < n \tag{3-73}$$

$$p_S(s) = \frac{\lambda_S}{s+\mu} \sum_{i=1}^{n-k+1} (n-i+1) p_i(s) \qquad \text{for all } k < n \tag{3-74}$$

$$p_C(s) = \frac{\lambda_C}{s+\mu_C} \sum_{i=1}^{n-k+1} p_i(s) \qquad \text{for all } k \geq 2 \text{ and } k < n \tag{3-75}$$

$$p_H(s) = \frac{\lambda_H}{s+\mu_H} \sum_{i=1}^{n-k+1} p_i(s) \qquad \text{for all } k \geq 2 \text{ and } k < n \tag{3-76}$$

By taking the inverse Laplace transforms of equations (3-71) – (3-76), the time-dependent state probabilities can be obtained. The time-dependent system availability is given by:

$$A(t) = \sum_{i=1}^{n} P_i(t) = L^{-1}\left[\sum_{i=1}^{n-k+1} p_i(s)\right] \tag{3-77}$$

3.4.2 Special case model 3-E: Two-out-of-three system under Type II repair

Figure 3-22 shows the state transition diagram of two-out-of-three system under Type II repair policy. Setting k=2 and n=3 in Equations (3-71) – (3-76), the following set of equations results:

$$p_1(s) = \frac{BCDE}{A(3,1)} \tag{3-78}$$

$$p_2(s) = \frac{3\lambda_O CDE}{A(3,1)} \tag{3-79}$$

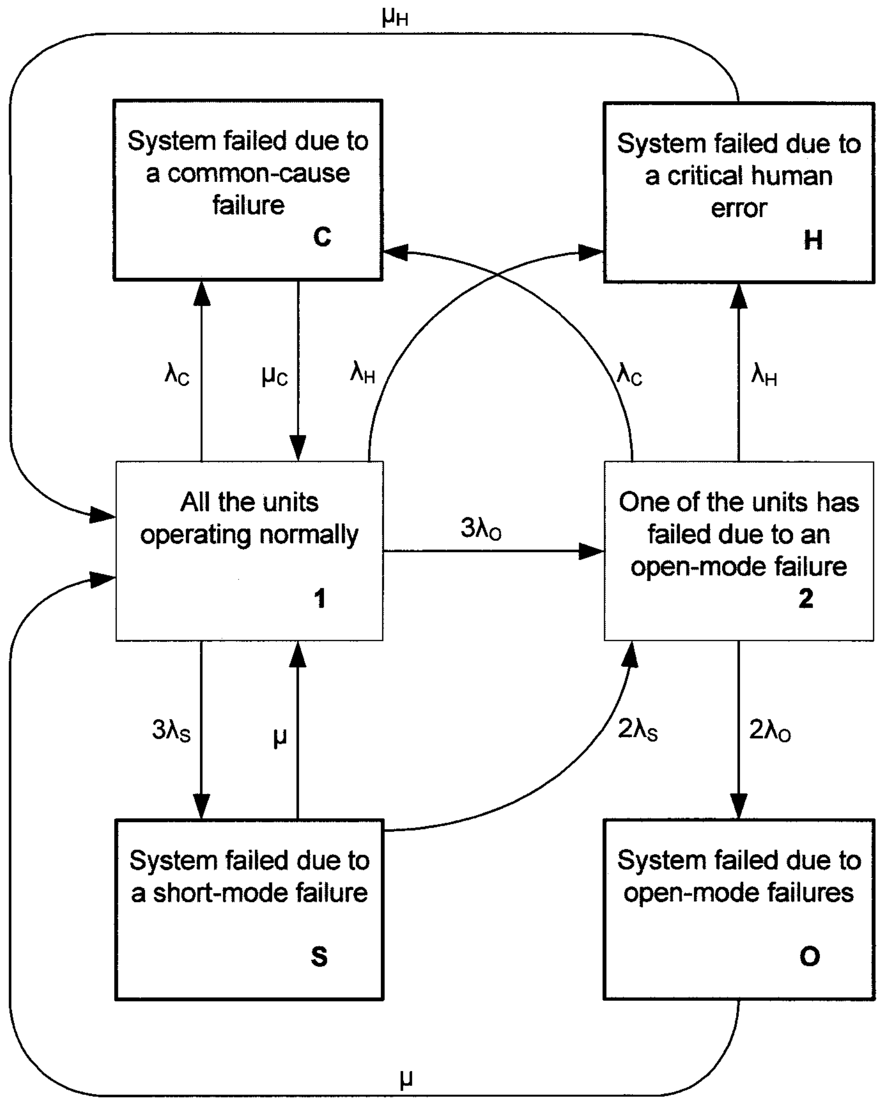

Figure 3-22: State transition diagram of two-out-of-three system with Type II repair

$$p_O(s)=\frac{6\lambda_O^{\,2}DE}{A(3,1)} \tag{3-80}$$

$$p_S(s)=\frac{3\lambda_S DE(B+2\lambda_O)}{A(3,1)} \tag{3-81}$$

$$p_C(s)=\frac{\lambda_C CE(B+3\lambda_O)}{A(3,1)} \tag{3-82}$$

$$p_H(s)=\frac{\lambda_H CD(B+3\lambda_O)}{A(3,1)} \tag{3-83}$$

where:

$$A(3,1)=ABCDE-3\mu DE[2\lambda_O(\lambda_O+\lambda_S)+B\lambda_S]-C(B+3\lambda_O)(\mu_C\lambda_C E+\mu_H\lambda_H D)$$
$$A=s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H$$
$$B=s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H$$
$$C=s+\mu$$
$$D=s+\mu_C$$
$$E=s+\mu_H$$

The time-dependent state probabilities can be obtained by taking the inverse Laplace transforms of Equations (3-78) – (3-83). By adding Equations (3-78) and (3-79) and then taking the inverse Laplace transform, we obtain the following expression for system availability:

$$A(t)=L^{-1}\left[\frac{CDE(B+3\lambda_O)}{A(3,1)}\right] \tag{3-84}$$

Steady State Conditions: We can compute the steady state conditions directly from equations (3-65) to (3-70), (i.e., for k=2 and n=3), by putting all derivatives equal to zero and solving the resultant equations utilizing the total probability axiom: $\sum P_i=1$. Thus, we get the following set of equations:

$$P_1=\frac{\mu\mu_C\mu_H B}{A(3,2)} \tag{3-85}$$

$$P_2=\frac{3\lambda_O\mu\mu_C\mu_H}{A(3,2)} \tag{3-86}$$

$$P_O=\frac{6\lambda_O{}^2\mu_C\mu_H}{A(3,2)} \tag{3-87}$$

$$P_S=\frac{3\lambda_S\mu_C\mu_H(B+2\lambda_O)}{A(3,2)} \tag{3-88}$$

$$P_C = \frac{\lambda_C \mu \mu_H (B+3\lambda_O)}{A(3,2)} \tag{3-89}$$

$$P_H = \frac{\lambda_H \mu \mu_C (B+3\lambda_O)}{A(3,2)} \tag{3-90}$$

where

$$A(3,2) = \mu(B+3\lambda_O)(\mu_C\mu_H + \mu_H\lambda_C + \mu_C\lambda_H) + 3\mu_C\mu_H[2\lambda_O(\lambda_O+\lambda_S) + B\lambda_S]$$

$$A = 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$$

$$B = 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H$$

System steady state availability can now be calculated as:

$$AV_{ss} = \frac{\mu\mu_C\mu_H (B+3\lambda_O)}{A(3,2)} \tag{3-91}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-23. For the specified values of the model parameters, the plots of Equation (3-84) are shown in Figure 3-24. Similarly, for the specified values of the model parameters, the plots of Equation (3-91) are shown in Figure 3-25.

3.4.3 Special case model 3-F: Two-out-of-four system with Type II repair

The state space diagram of two-out-of-four system with Type II repair can be represented as in Figure 3-26. For k=2 and n=4 in Figure 3-4, from Equations (3-71) – (3-76), we get the following set of equations:

$$p_1(s) = \frac{BCDEF}{A(3,3)} \tag{3-92}$$

$$p_2(s) = \frac{4CDEF\lambda_O}{A(3,3)} \tag{3-93}$$

$$p_3(s) = \frac{12\lambda_O{}^2 DEF}{A(3,3)} \tag{3-94}$$

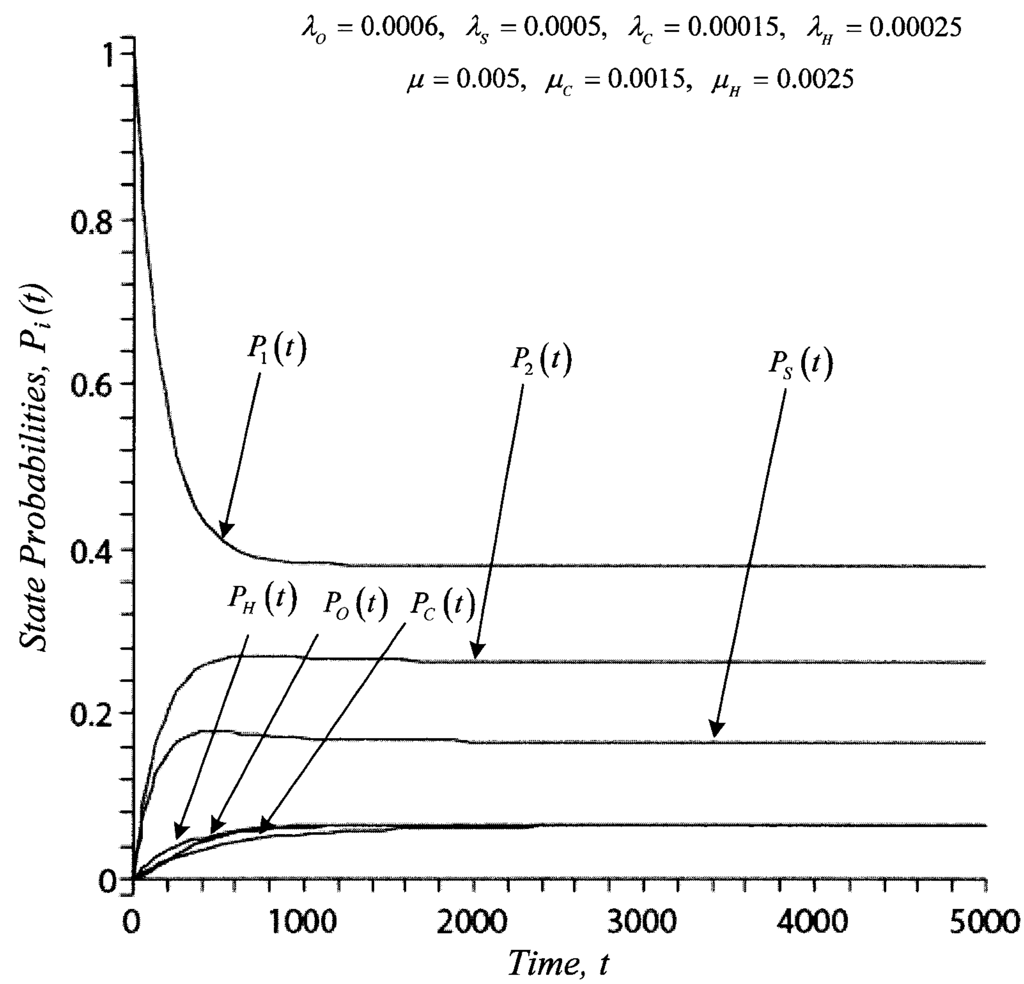

Figure 3-23: State probability plots of two-out-of-three system with Type II repair

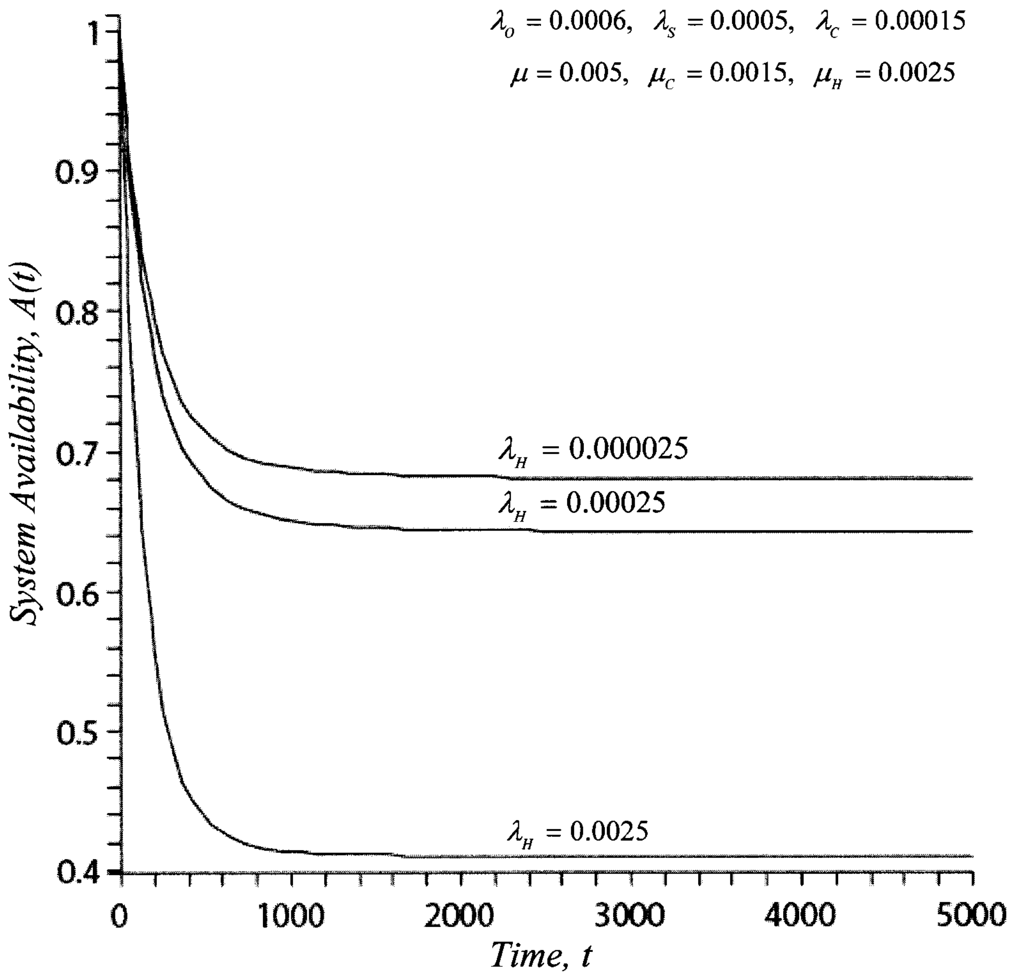

Figure 3-24: Availability plots of two-out-of-three system with Type II repair

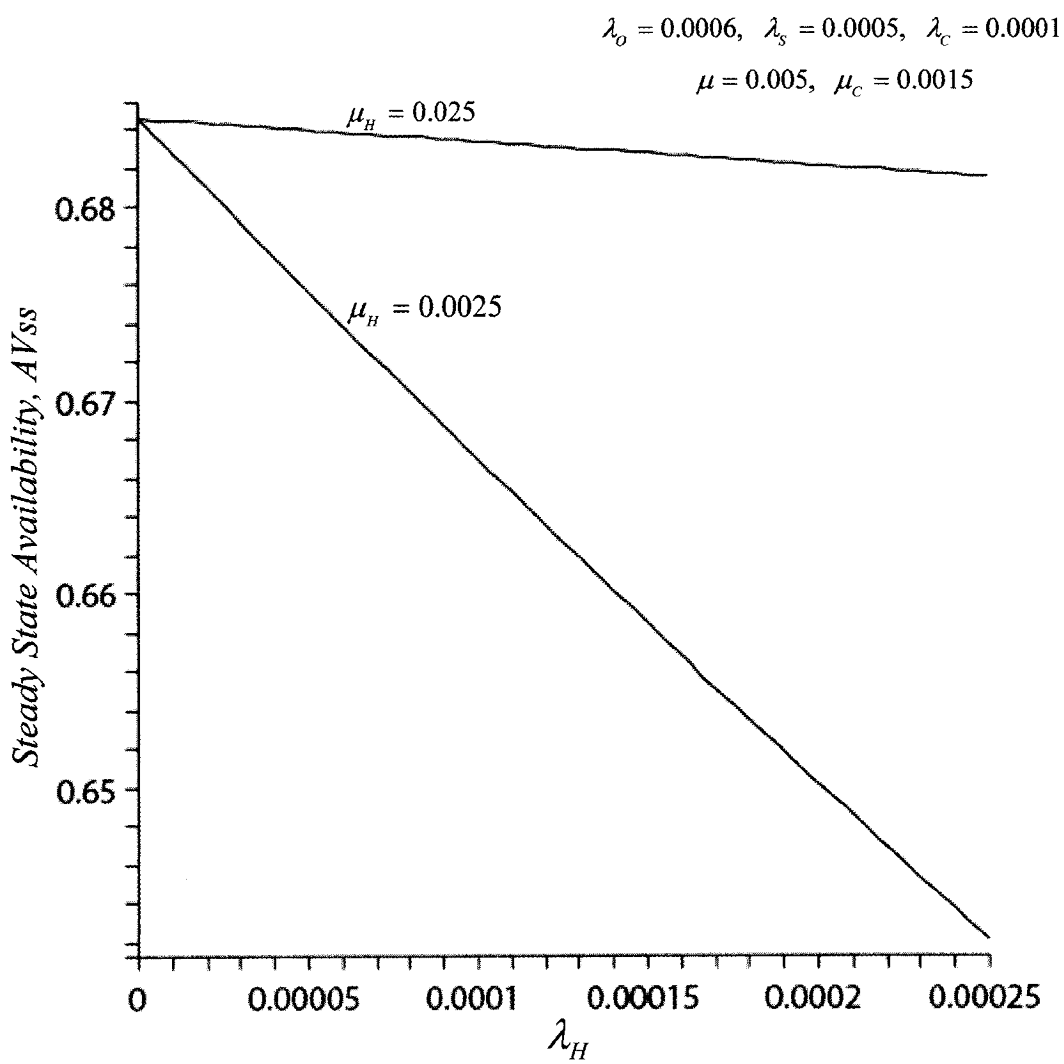

Figure 3-25: Steady state availability plots of two-out-of-three system with Type II repair

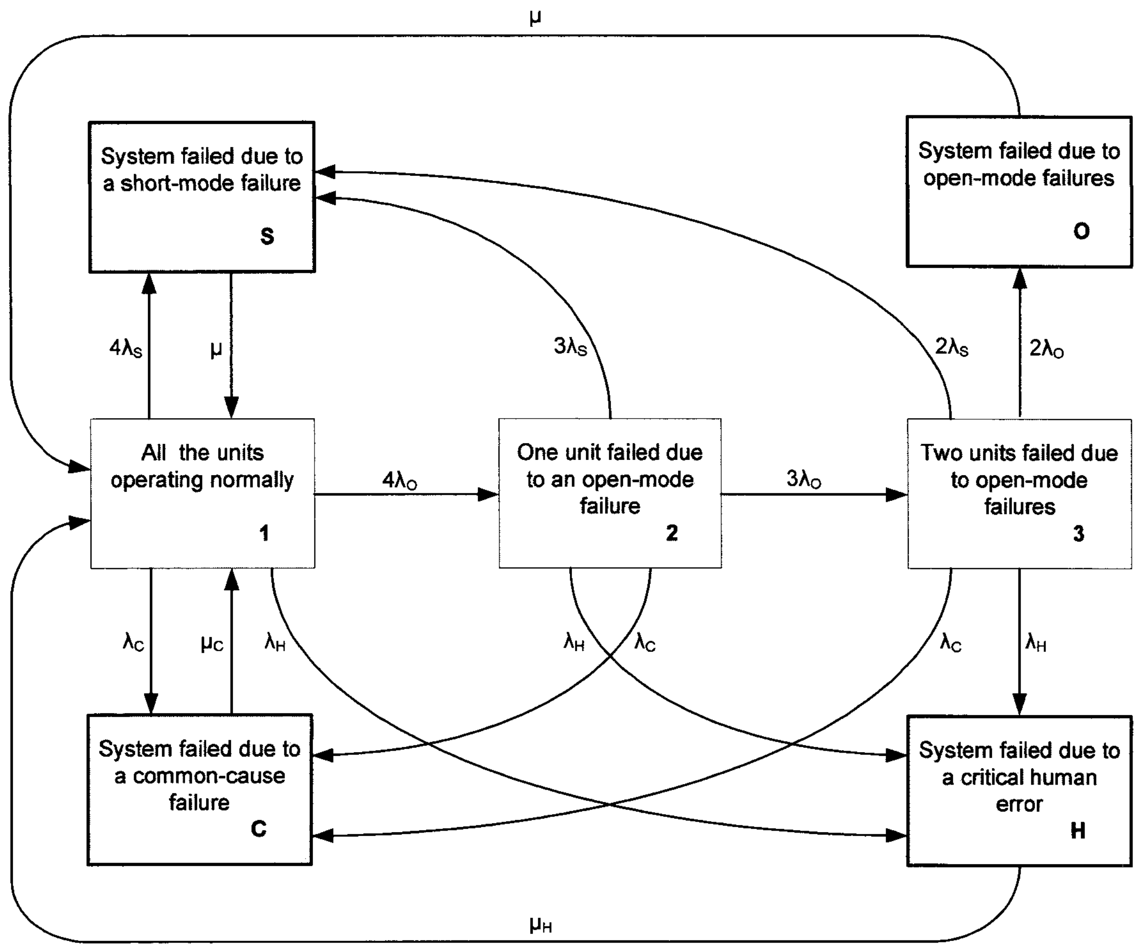

Figure 3-26: State transition diagram of two-out-of-four system with Type II repair

$$p_O(s) = \frac{24\lambda_O^{\,3} EF}{A(3,3)} \tag{3-95}$$

$$p_S(s) = \frac{4EF\lambda_S\left(BC + 3C\lambda_O + 6\lambda_O^{\,2}\right)}{A(3,3)} \tag{3-96}$$

$$p_C(s) = \frac{\lambda_C DF\left(BC + 4C\lambda_O + 12\lambda_O^{\,2}\right)}{A(3,3)} \tag{3-97}$$

$$p_H(s) = \frac{\lambda_H DE\left(BC + 4C\lambda_O + 12\lambda_O^{\,2}\right)}{A(3,3)} \tag{3-98}$$

where:

$$A(3,3)=ABCDEF-4\mu EF\left[C\lambda_S(B+3\lambda_O)+6\lambda_O^{\ 2}(\lambda_O+\lambda_S)\right]$$
$$-D(BC+4C\lambda_O+12\lambda_O^{\ 2})\left[F\lambda_C\mu_C+E\lambda_H\mu_H\right]$$

$$A=s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H$$
$$B=s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H$$
$$C=s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H$$
$$D=s+\mu$$
$$E=s+\mu_C$$
$$F=s+\mu_H$$

By taking the inverse Laplace transforms of equations (3-92) – (3-98), the time-dependent transition state probabilities can be obtained. By adding Equations (3-92) – (3-94) and then taking inverse Laplace transform, we obtain the following expression for the system time-dependent availability:

$$A(t)=L^{-1}\left[A(s)\right]=L^{-1}\left[\frac{DEF\left(BC+4C\lambda_O+12\lambda_O^{\ 2}\right)}{A(3,3)}\right] \tag{3-93}$$

Steady State Conditions: The steady state conditions can be calculated directly by setting all derivatives in Equations (3-65) to (3-70), (i.e., for k=2 and n=4) to zero, and solving the resultant equations by utilizing the total probability axiom: $\sum P_i=1$. Thus, we get the following set of equations:

$$P_1=\frac{\mu\mu_C\mu_H BC}{A(3,4)} \tag{3-94}$$

$$P_2=\frac{4\lambda_O\mu\mu_C\mu_H C}{A(3,4)} \tag{3-95}$$

$$P_3=\frac{12\lambda_O^{\ 2}\mu\mu_C\mu_H}{A(3,4)} \tag{3-96}$$

$$P_O=\frac{24\lambda_O^{\ 3}\mu_C\mu_H}{A(3,4)} \tag{3-97}$$

$$P_S = \frac{4\lambda_S \mu_C \mu_H \left[BC + 3C\lambda_O + 6\lambda_O^2\right]}{A(3,4)} \tag{3-98}$$

$$P_C = \frac{\lambda_C \mu \mu_H \left[BC + 4C\lambda_O + 12\lambda_O^2\right]}{A(3,4)} \tag{3-99}$$

$$P_H = \frac{\lambda_H \mu \mu_C \left[BC + 4C\lambda_O + 12\lambda_O^2\right]}{A(3,4)} \tag{3-100}$$

where

$$A(3,4) = \mu(BC + 4C\lambda_O + 12\lambda_O^2)\left[\mu_C \mu_H + \mu_H \lambda_C + \mu_C \lambda_H\right] + 4\mu_C \mu_H \left[6\lambda_O^2(\lambda_O + \lambda_S) + C\lambda_S(B + 3\lambda_O)\right]$$

$$B = 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$$

$$C = 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H$$

System steady state availability can now be calculated as:

$$AV_{SS} = \frac{\mu \mu_C \mu_H \left[BC + 4C\lambda_O + 12\lambda_O^2\right]}{A(3,4)} \tag{3-101}$$

For the specified values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-27, the plots of Equation (3-93) are shown in Figure 3-28 and the plots of Equation (3-101) are shown in Figure 3-29.

3.5 Three-State Device K-out-of-N System with Type III Repair Policy

In this section, three-state device k-out-of-n system with Type III repair policy and its two special cases: two-out-of-three system with Type III repair policy and two-out-of-four system with Type III repair policy are discussed.

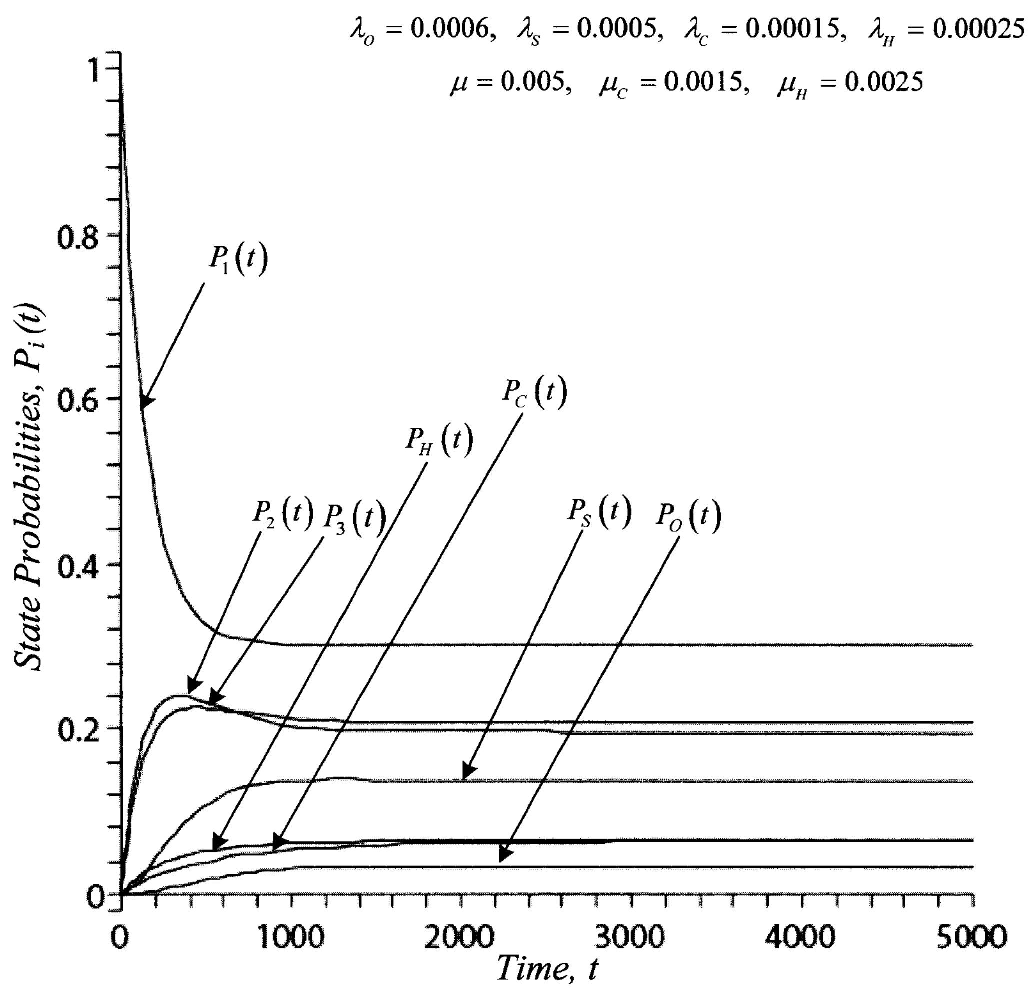

Figure 3-27: State probability plots of two-out-of-four system with Type II repair

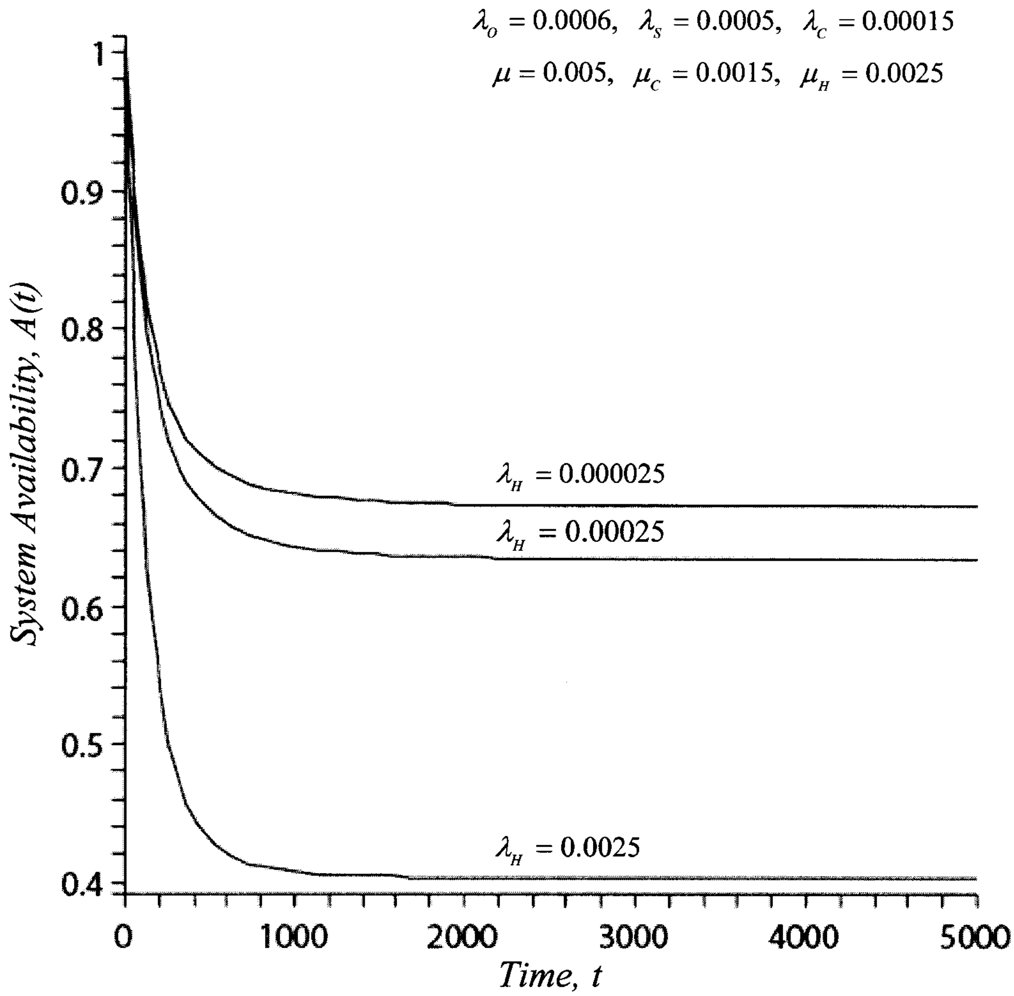

Figure 3-28: Availability plots of two-out-of-four system with Type II repair

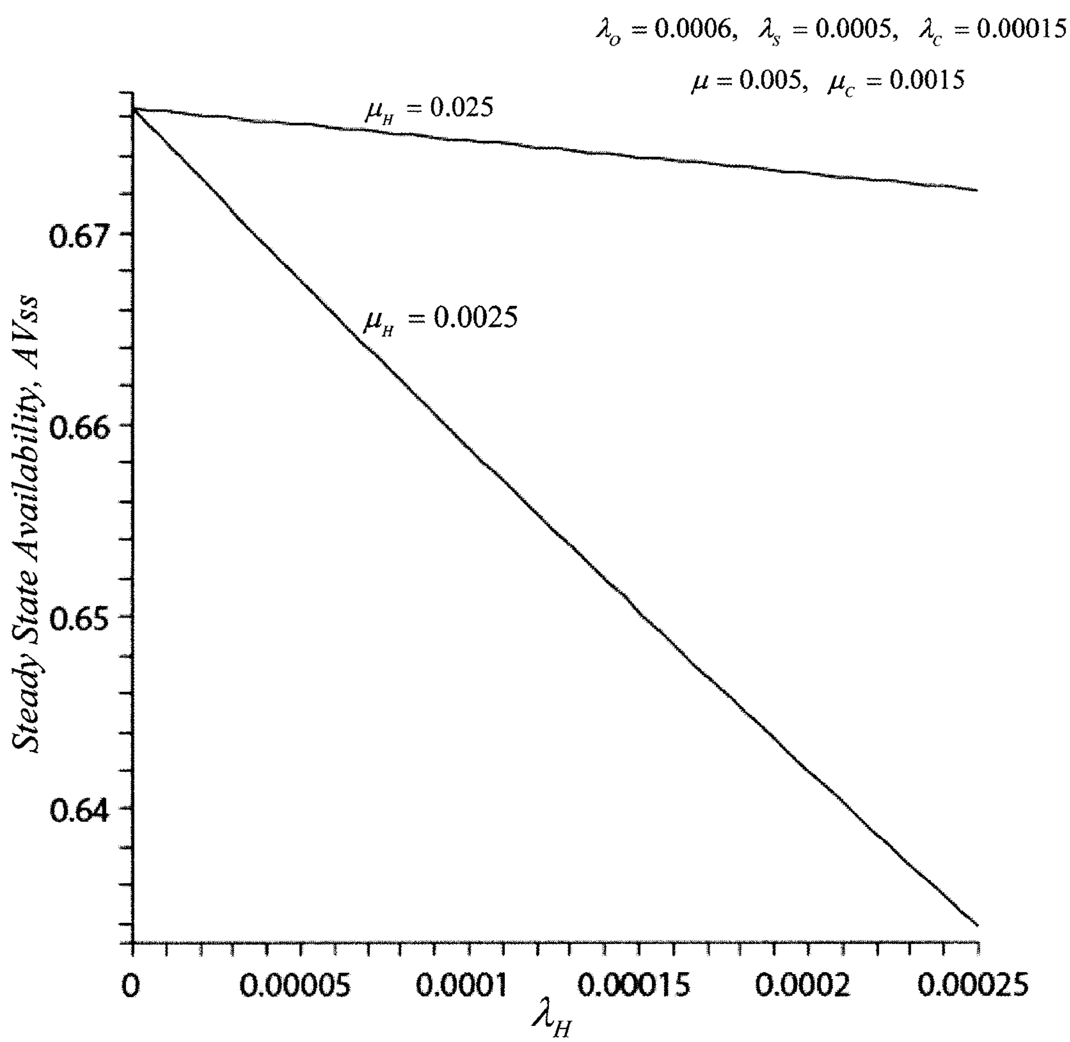

Figure 3-29: Steady state availability plots of two-out-of-four system with Type II repair

3.5.1 General k-out-of-n system with Type III repair

The block diagram and the state transition diagram for the general model under Type III repair policy is shown in Figures 3-1 and 3-5 respectively. Using the Markov method, the system of differential equations associated with Figure 3-5 is as follows:

$$\frac{dP_1(t)}{dt} = -[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H]P_1(t) + \mu_1 P_2(t) + \mu[P_O(t) + P_S(t)] + \mu_C P_C(t) + \mu_H P_H(t) \quad (3\text{-}102)$$

$$\frac{dP_i(t)}{dt} = -[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) + \mu_1 P_{i+1}(t) \quad (3\text{-}103)$$

for all $i = 2$ to $(n-k)$

$$\frac{dP_{n-k+1}(t)}{dt} = -[k\lambda_O + k\lambda_S + \lambda_C + \lambda_H + \mu_1]P_{n-k+1}(t) + (k+1)\lambda_O P_{n-k}(t) \quad (3\text{-}104)$$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + k\lambda_O P_{n-k+1}(t) \qquad \text{for all } k < n \quad (3\text{-}105)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S \sum_{i=1}^{n-k+1} (n-i+1)P_i(t) \qquad \text{for all } k < n \quad (3\text{-}106)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \quad (3\text{-}107)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-k+1} P_i(t) \qquad \text{for all } k \geq 2 \text{ and } k < n \quad (3\text{-}108)$$

At time t=0, $P_1(0) = 1$, and all other initial state transition probabilities are equal to zero. Taking Laplace transforms of Equations (3-102) – (3-108) and solving the resultant equations, we get the following set of equations:

$$p_1(s) = \frac{1 + \mu[p_O(s) + p_S(s)] + \mu_1 p_2(s) + \mu_C p_C(s) + \mu_H p_H(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \quad (3\text{-}109)$$

$$p_i(s) = \frac{(n-i+2)\lambda_O p_{i-1}(s) + \mu_1 p_{i+1}(s)}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1} \qquad \text{for all } i = 2 \text{ to } (n-k) \quad (3\text{-}110)$$

$$p_{n-k+1}(s)=\frac{(k+1)\lambda_O p_{n-k}(s)}{s+k\lambda_O+k\lambda_S+\lambda_C+\lambda_H+\mu_1} \qquad (3\text{-}111)$$

$$p_O(s)=\frac{k\lambda_O p_{n-k+1}(s)}{s+\mu} \qquad \text{for all } k<n \qquad (3\text{-}112)$$

$$p_S(s)=\frac{\lambda_S}{s+\mu}\sum_{i=1}^{n-k+1}(n-i+1)p_i(s) \qquad \text{for all } k<n \qquad (3\text{-}113)$$

$$p_C(s)=\frac{\lambda_C}{s+\mu_C}\sum_{i=1}^{n-i+1}p_i(s) \qquad \text{for all } k\geq 2 \text{ and } k<n \qquad (3\text{-}114)$$

$$p_H(s)=\frac{\lambda_H}{s+\mu_H}\sum_{i=1}^{n-i+1}p_i(s) \qquad \text{for all } k\geq 2 \text{ and } k<n \qquad (3\text{-}115)$$

By taking the inverse Laplace transforms of Equations (3-109) – (3-115), the time-dependent state probabilities can be obtained. The time-dependent system availability is given by:

$$A(t)=L^{-1}\left[\sum_{i=1}^{n-k+1}p_i(s)\right] \qquad (3\text{-}116)$$

3.5.2 Special case model 3-G: Two-out-of-three system with Type III repair

The state transition diagram of two-out-of-three system with type III repair can be represented as in Figure 3-30. By substituting k=2 and n=3 in Equations (3-109) – (3-115), we obtain the following set of equations:

$$p_1(s)=\frac{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)(s+\mu)(s+\mu_C)(s+\mu_H)}{A(3,5)} \qquad (3\text{-}117)$$

$$p_2(s)=\frac{3\lambda_O(s+\mu)(s+\mu_C)(s+\mu_H)}{A(3,5)} \qquad (3\text{-}118)$$

$$p_O(s)=\frac{6\lambda_O{}^2(s+\mu_C)(s+\mu_H)}{A(3,5)} \qquad (3\text{-}119)$$

$$p_S(s)=\frac{3\lambda_S(s+\mu_C)(s+\mu_H)(s+4\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{A(3,5)} \qquad (3\text{-}120)$$

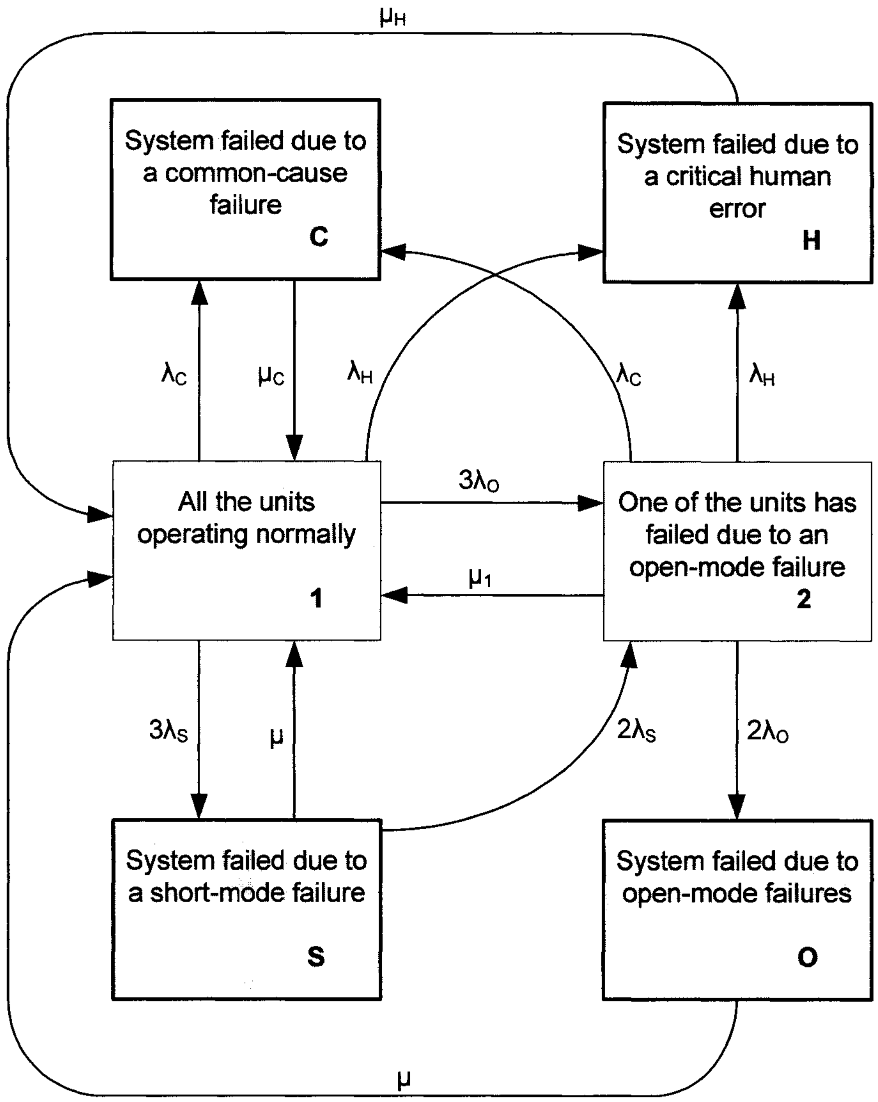

Figure 3-30: State transition diagram of two-out-of-three system with Type III repair

$$p_C(s) = \frac{\lambda_C(s+\mu)(s+\mu_H)\left(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1\right)}{A(3,5)} \tag{3-121}$$

$$p_H(s) = \frac{\lambda_H(s+\mu)(s+\mu_C)\left(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1\right)}{A(3,5)} \tag{3-122}$$

where:

$$A(3,5) = (s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)(s+\mu)(s+\mu_C)(s+\mu_H)$$
$$-3(s+\mu_C)(s+\mu_H)\{\lambda_O[\mu_1(s+\mu)+2\mu(\lambda_O+\lambda_S)]+\mu\lambda_S(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)\}$$
$$-(s+\mu)(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)[\mu_C\lambda_C(s+\mu_H)+\mu_H\lambda_H(s+\mu_C)]$$

By taking the inverse Laplace transforms of Equations (3-117) – (3-122), the time-dependent state probabilities can be obtained. By adding Equations (3-117) and (3-118) and then taking the inverse Laplace transforms, we obtain the following expression for system availability:

$$A(t) = L^{-1}\left\{\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{A(3,5)}\right\} \tag{3-123}$$

Steady State Conditions: We can compute the steady state conditions directly from Equations (3-102) – (3-108), (i.e., for k=2 and n=3) by putting all derivatives equal to zero and solving the resultant equations by utilizing the total probability axiom: $\sum P_i = 1$. Thus, we get the following set of equations:

$$P_1 = \frac{\mu\mu_C\mu_H(2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{A(3,6)} \tag{3-124}$$

$$P_2 = \frac{2\lambda_O\mu\mu_C\mu_H}{A(3,6)} \tag{3-125}$$

$$P_O = \frac{6\lambda_O^2\mu_C\mu_H}{A(3,6)} \tag{3-126}$$

$$P_S = \frac{3\lambda_S\mu_C\mu_H(4\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{A(3,6)} \tag{3-127}$$

$$P_C = \frac{\lambda_C\mu\mu_H(5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{A(3,6)} \tag{3-128}$$

$$P_H = \frac{\lambda_H\mu\mu_C(5\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1)}{A(3,6)} \tag{3-129}$$

where

$$A(3,6) = \mu(5\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1)(\mu_C\mu_H + \mu_C\lambda_H + \mu_H\lambda_C)$$
$$+ 3\mu_C\mu_H\left[2\lambda_O(\lambda_O + \lambda_S) + \lambda_S(2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1)\right]$$

By adding Equations (3-124) and (3-125), we get the following set the following expression for the system steady state availability:

$$AV_{SS} = P_1(t) + P_2(t) = \frac{\mu\mu_C\mu_H(5\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1)}{A(3,6)} \tag{3-130}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 3-31. For the specified values of the model parameters, the plots of Equation (3-123) are shown in Figure 3-32. Similarly, for the specified values of the model parameters, the plots of Equation (3-130) are shown in Figure 3-33.

3.5.3 Special case model 3-H: Two-out-of-four system with Type III repair

The state transition diagram of two-out-of-four system with Type III repair is shown in Figure 3-34. By substituting k=2 and n=4 in Equations (3-109) – (3-115), we get the following set of equations:

$$p_1(s) = \frac{DEF(BC - 3\mu_1\lambda_O)}{A(3,7)} \tag{3-131}$$

$$p_2(s) = \frac{4CDEF\lambda_O}{A(3,7)} \tag{3-132}$$

$$p_3(s) = \frac{12DEF\lambda_O^2}{A(3,7)} \tag{3-133}$$

$$p_O(s) = \frac{24EF\lambda_O^3}{A(3,7)} \tag{3-134}$$

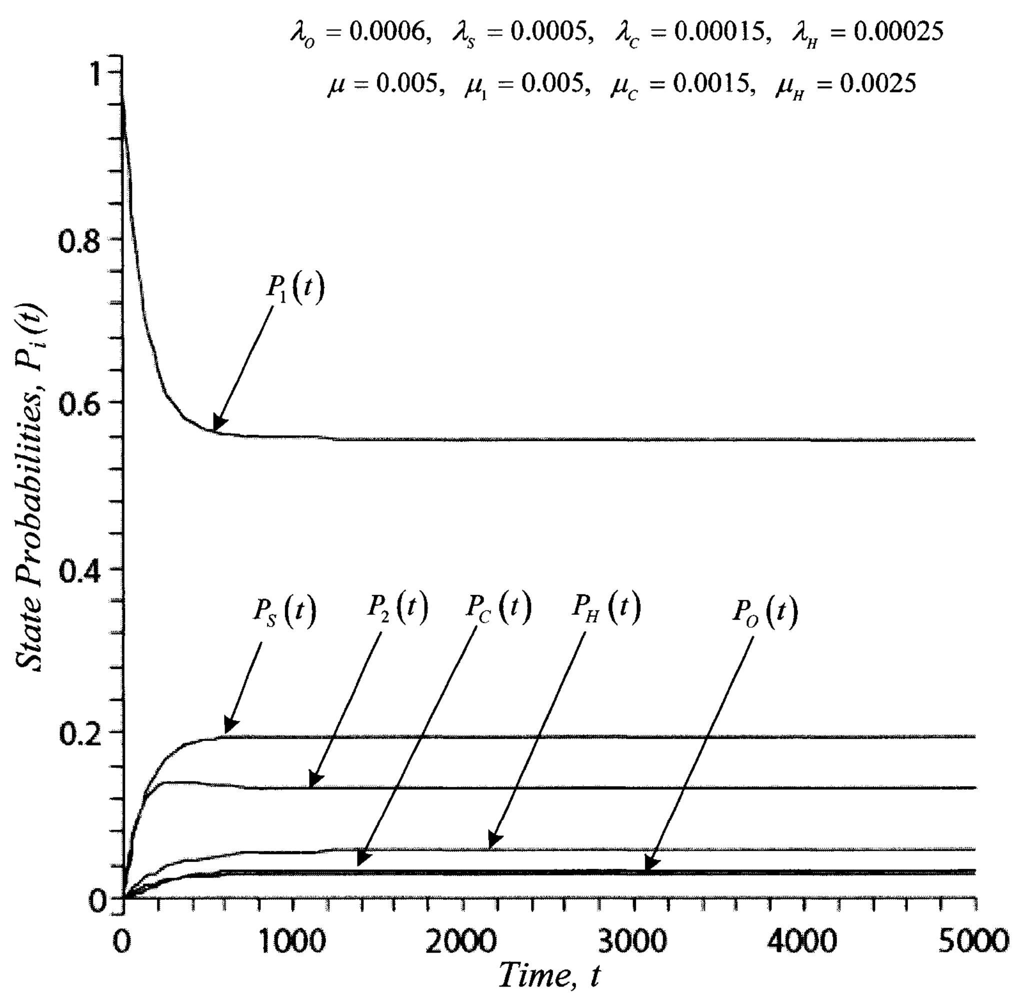

Figure 3-31: State probability plots of two-out-of-three system with Type III repair

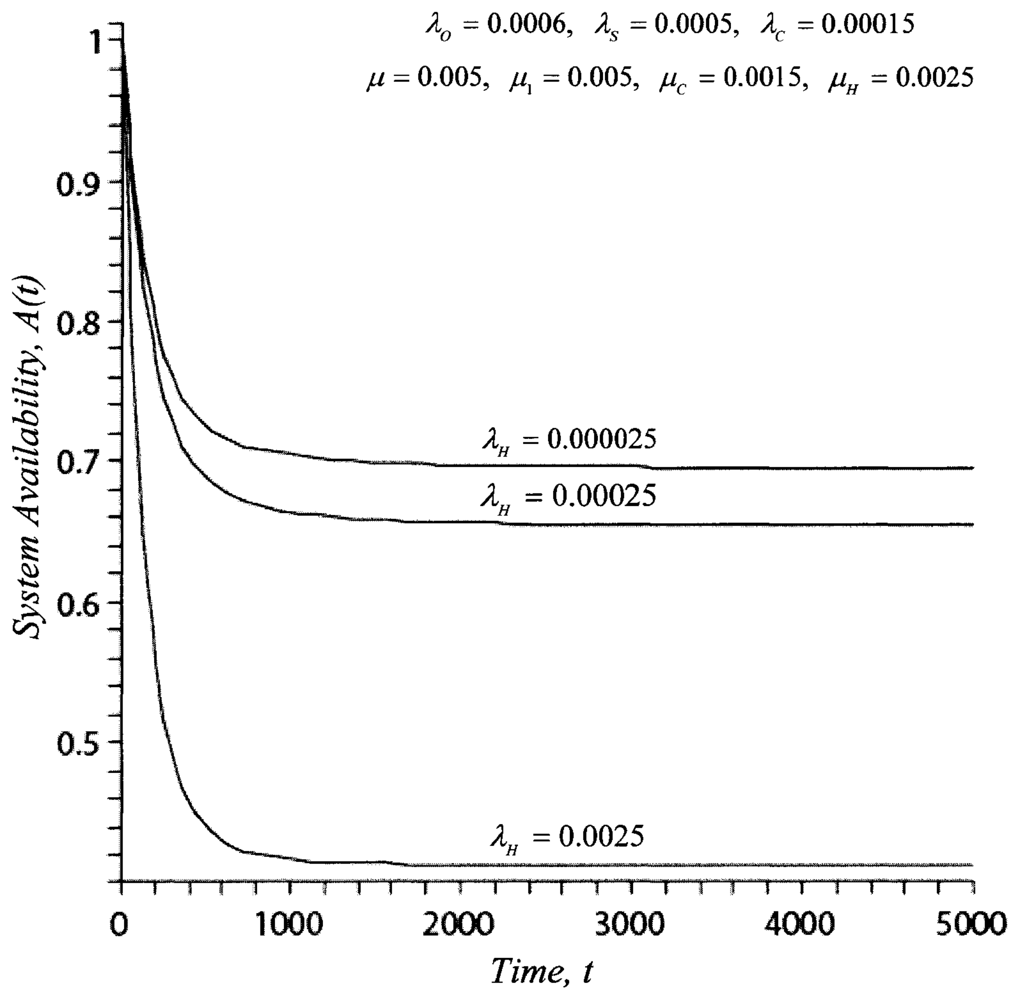

Figure 3-32: Availability plots of two-out-of-three system with Type III repair

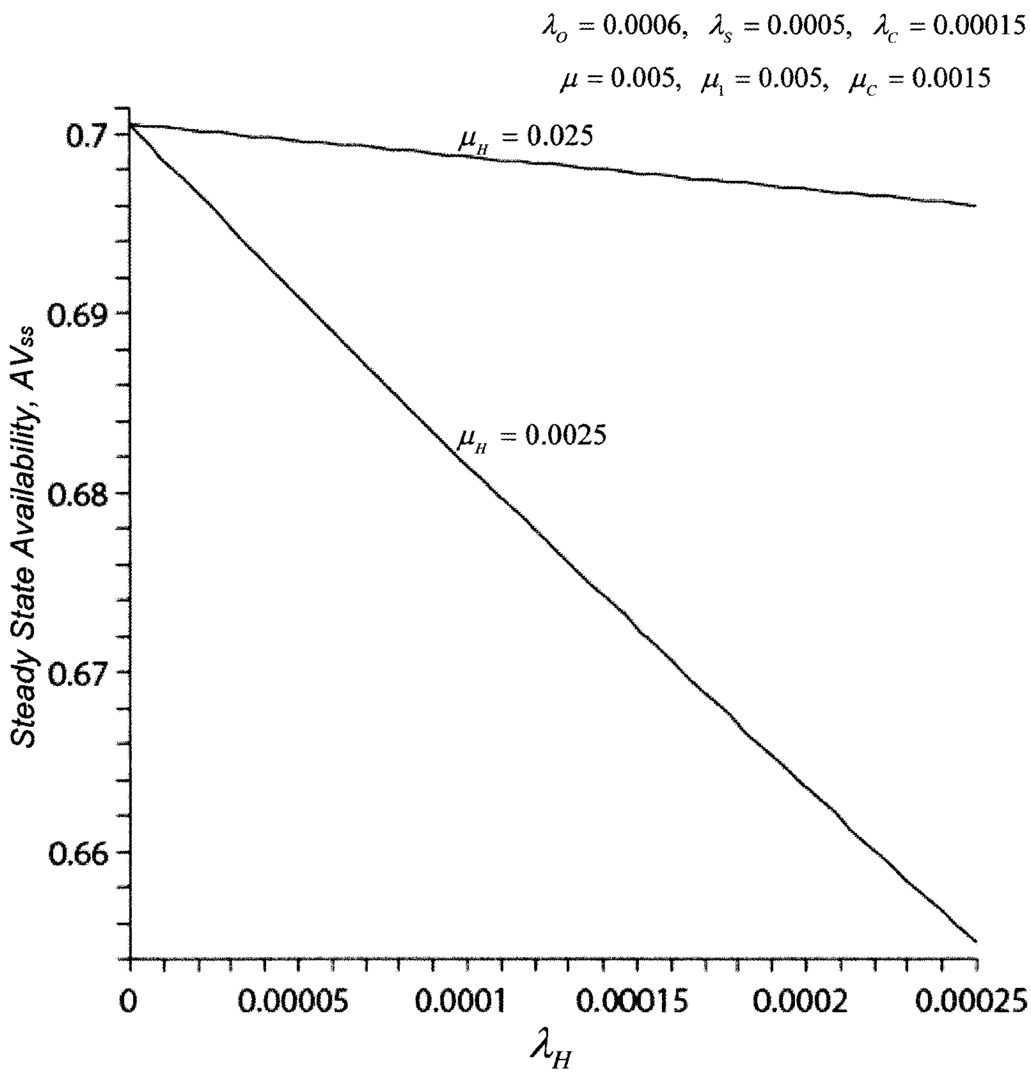

Figure 3-33: Steady state availability plots of two-out-of-three system with Type III repair

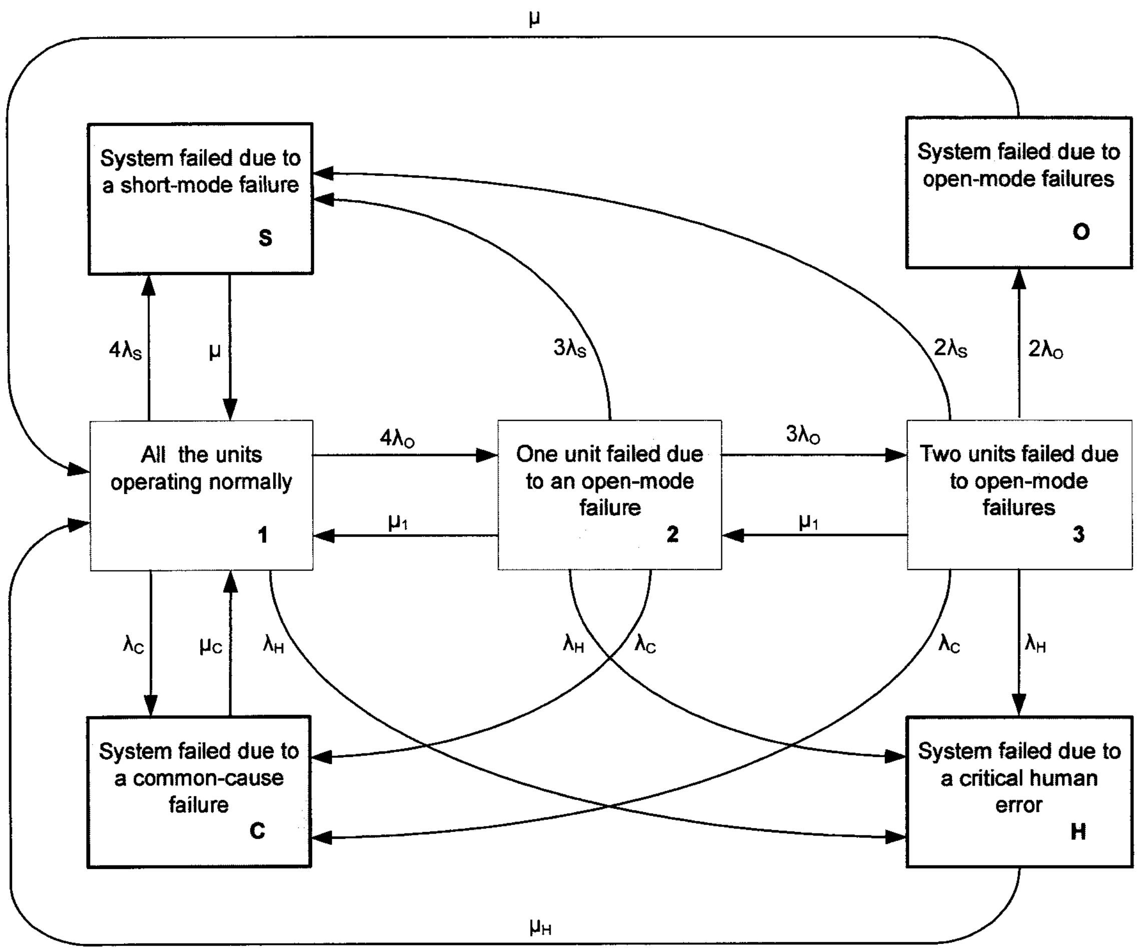

Figure 3-34: State transition diagram of two-out-of-four system with Type III repair

$$p_S(s) = \frac{4\lambda_S EF\left[BC - 3\mu_1\lambda_O + 3C\lambda_O + 6\lambda_O^{\ 2}\right]}{A(3,7)} \tag{3-135}$$

$$p_C(s) = \frac{\lambda_C DF\left[BC - 3\mu_1\lambda_O + 4C\lambda_O + 6\lambda_O^{\ 2}\right]}{A(3,7)} \tag{3-136}$$

$$p_H(s) = \frac{\lambda_H DE\left[BC - 3\mu_1\lambda_O + 4C\lambda_O + 6\lambda_O^{\ 2}\right]}{A(3,7)} \tag{3-137}$$

where:

$$A(3,7)=DEF\left[ABC-\mu_1\lambda_O(3A+4C)\right]-\mu EF\left[24\lambda_O^{\,2}(\lambda_O+\lambda_S)+4\lambda_S(BC+3C\lambda_O-3\mu_1\lambda_O)\right]$$
$$-D(\mu_C\lambda_C F+\mu_H\lambda_H E)\left[BC+4C\lambda_O-3\mu_1\lambda_O+12\lambda_O^{\,2}\right]$$

$$A=s+4\lambda_O+4\lambda_S+\lambda_C+\lambda_H$$

$$B=s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H+\mu_1$$

$$C=s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1$$

$$D=s+\mu$$

$$E=s+\mu_C$$

$$F=s+\mu_H$$

By taking the inverse Laplace transforms of Equations (3-131) – (3-137), the time-dependent state probabilities can be obtained. By adding Equations (3-131) – (3-133) and then taking the inverse Laplace transform, we obtain the following expression for system availability:

$$A(t)=L^{-1}\left[A(s)\right]=L^{-1}\left\{\frac{DEF(BC-3\mu_1\lambda_O+4C\lambda_O+12\lambda_O^{\,2})}{A(3,7)}\right\} \tag{3-138}$$

Steady State Conditions: We can compute the steady state conditions directly from Equations (3-102) – (3-108), (i.e., for k=2 and n=4) by putting all derivatives equal to zero and solving the resultant equations by utilizing the total probability axiom: $\sum P_i=1$. Thus, we get the following set of equations:

$$P_1=\frac{\mu\mu_C\mu_H(BC-3\mu_1\lambda_O)}{A(3,8)} \tag{3-139}$$

$$P_2=\frac{4\mu\mu_C\mu_H C\lambda_O}{A(3,8)} \tag{3-140}$$

$$P_3=\frac{12\mu\mu_C\mu_H\lambda_O^{\,2}}{A(3,8)} \tag{3-141}$$

$$P_O = \frac{24\mu_C\mu_H\lambda_O^{\ 3}}{A(3,8)} \tag{3-142}$$

$$P_S = \frac{4\mu_C\mu_H\lambda_S\left[BC - 3\mu_1\lambda_O + 3C\lambda_O + 6\lambda_O^{\ 2}\right]}{A(3,8)} \tag{3-143}$$

$$P_C = \frac{\mu\mu_H\lambda_C\left[BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^{\ 2}\right]}{A(3,8)} \tag{3-144}$$

$$P_H = \frac{\mu\mu_C\lambda_H\left[BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^{\ 2}\right]}{A(3,8)} \tag{3-145}$$

where:

$$A(3,8) = \mu\mu_C\mu_H(BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^{\ 2}) + 4\mu_C\mu_H\left[6\lambda_O^{\ 3} + \lambda_S(BC - 3\mu_1\lambda_O + 3C\lambda_O + 6\lambda_O^{\ 2})\right]$$
$$+ \mu(BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^{\ 2})(\mu_H\lambda_C + \mu_C\lambda_H)$$

$$B = 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H + \mu_1$$

$$C = 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1$$

By adding Equations (3-139) – (3-141), we get the following expression for the system steady state availability:

$$AV_{SS} = \frac{\mu\mu_C\mu_H\left[BC - 3\mu_1\lambda_O + 4C\lambda_O + 12\lambda_O^{\ 2}\right]}{A(3,8)} \tag{3-146}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, *t*, are shown in Figure 3-35. For the specified values of the model parameters, the plots of Equation (3-138) are shown in Figure 3-36. Similarly, for the specified values of the model parameters, the plots of Equation (3-146) are shown in Figure 3-37.

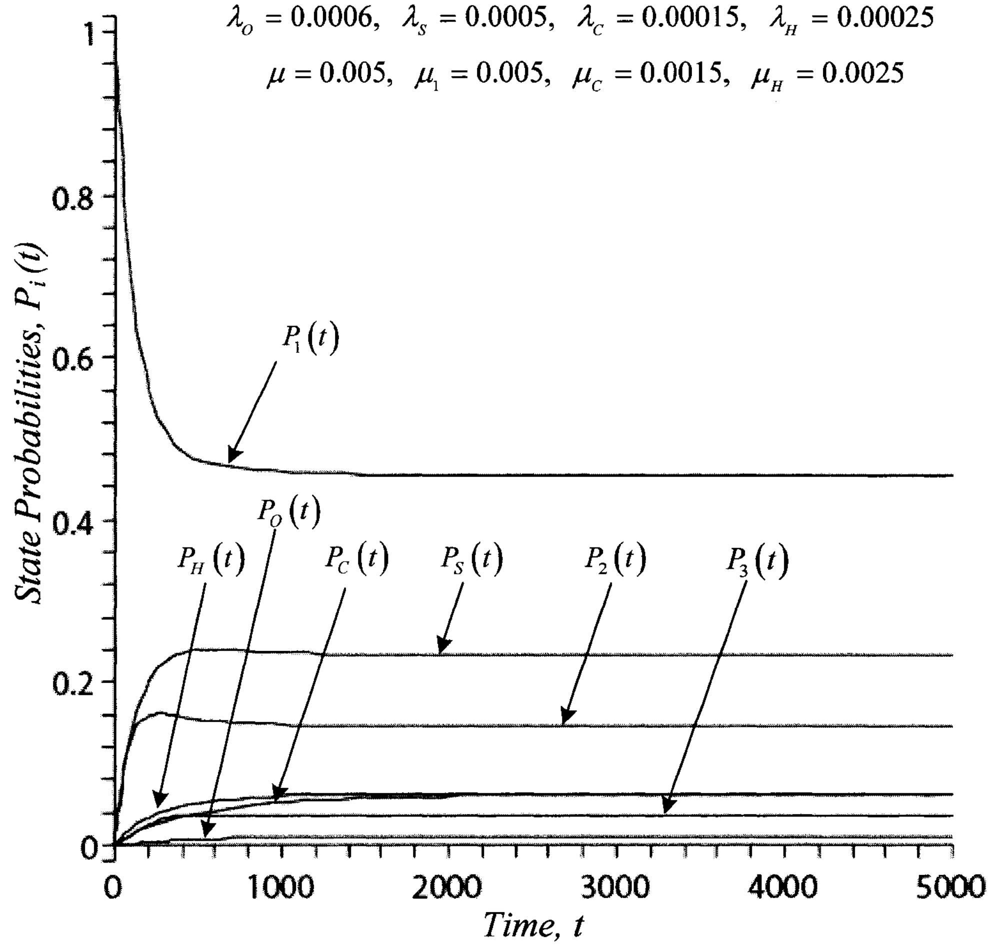

Figure 3-35: State probability plots of two-out-of-four system with Type III repair

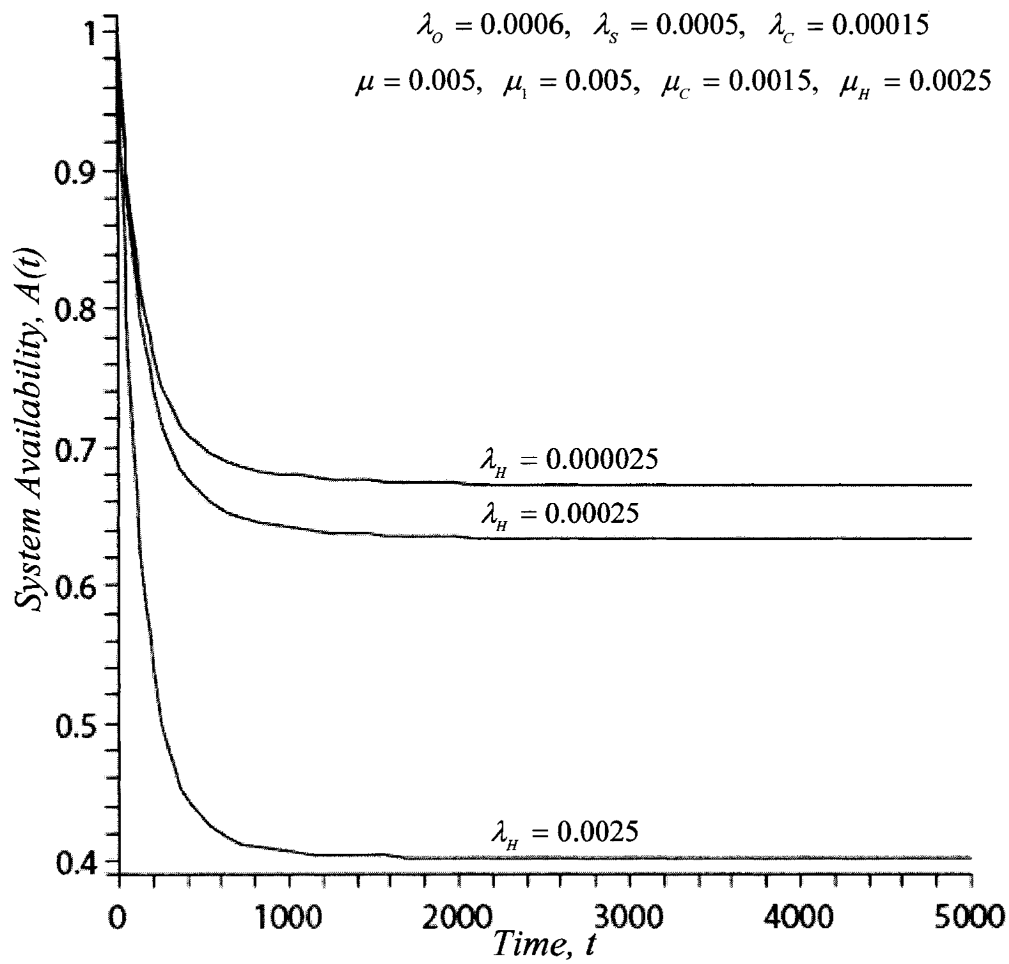

Figure 3-36: Availability plots of two-out-of-four system with Type III repair

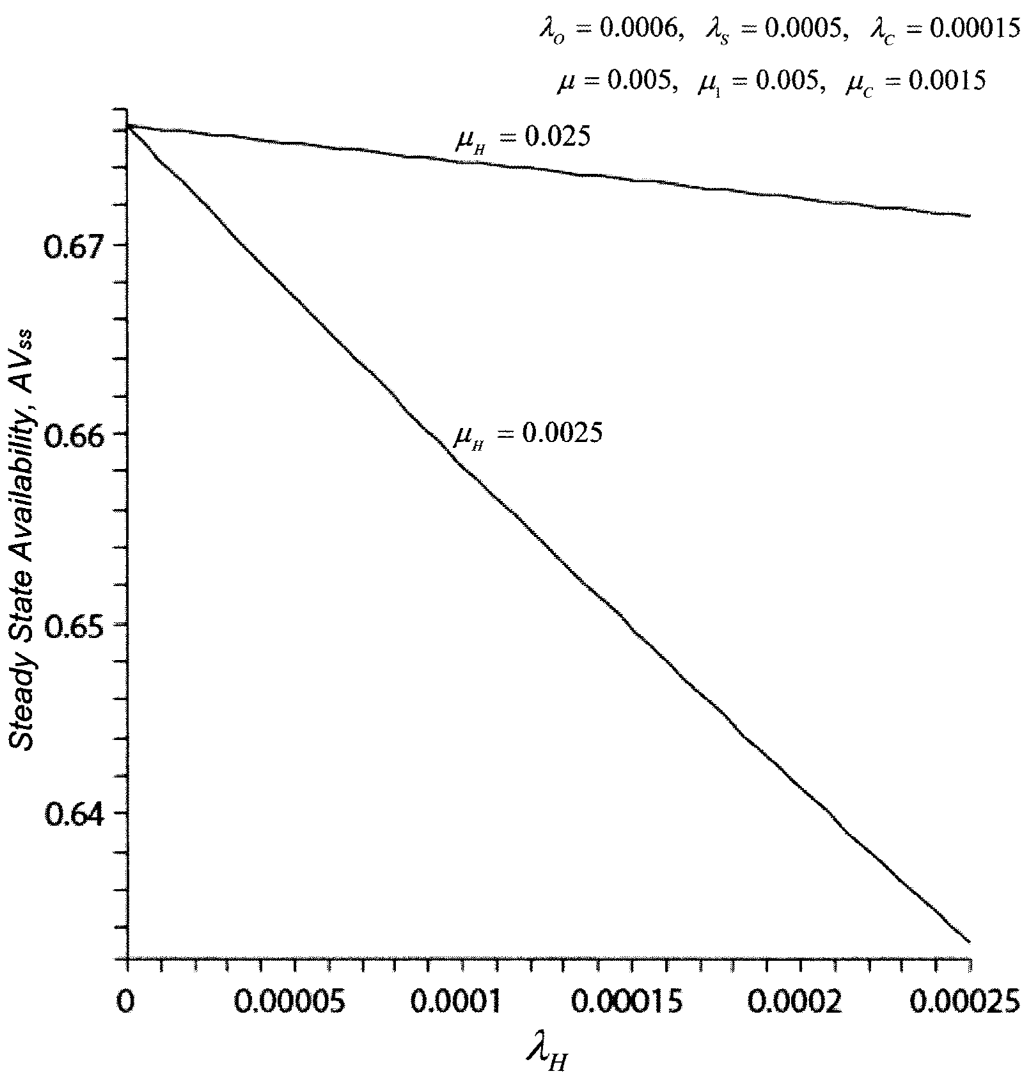

Figure 3-37: Steady state availability plots of two-out-of-four system with Type III repair

Setting:

$t = 100$ hours

$\lambda_O = 0.0006$ per hour, $\lambda_S = 0.0005$ per hour, $\lambda_C = 0.00015$ per hour

$\mu = 0.005$ per hour, $\mu_C = 0.0015$ per hour, $\mu_H = 0.0025$ per hour

For the values of the model parameters as specified above, Table 3-1 shows the comparison of system reliability and mean time to failure for different values of critical human error rates (i.e., λ_H) for the three-unit parallel system and two-out-of-three system without repair.

Similarly, for the given values of the model parameters, Table 3-2 shows the comparison of system time-dependent and steady state availabilities for different values of critical human error rate for the three-unit parallel system and two-out-of-three system under Type II repair policy.

The following notation associated with Tables 3-1 and 3-2:

$R_{Parallel}$	Reliability of three-unit parallel system without repair.
$R_{k/n}$	Reliability of two-out-of-three system without repair.
$MTTF_{Parallel}$	Mean time to failure of three-unit parallel system without repair.
$MTTF_{k/n}$	Mean time to failure of two-out-of-three system without repair.
$AV_{Parallel}$	Availability of three-unit parallel system with Type II repair.
$AV_{k/n}$	Availability of two-out-of-three system with Type II repair.
$AVSS_{Parallel}$	Steady state availability of three-unit parallel system with Type II repair.
$AVSS_{k/n}$	Steady state availability of two-out-of-three system with Type II repair.

The results of this analysis are discussed in Section 3.7.

λ_H (per hour)	$R_{Parallel}$ (%)	$R_{k/n}$ (%)	$MTTF_{Parallel}$ (hours)	$MTTF_{k/n}$ (hours)
0.0001	84.31226283	83.45879623	714.4163685	488.6461629
0.000125	84.10247808	83.25040983	705.0800441	483.1532105
0.00015	83.89321610	83.04254375	695.9595960	477.7777776
0.000175	83.68447587	82.83519668	687.0480151	472.5162170
0.0002	83.47625586	82.62836734	678.3385833	467.3650282
0.000225	83.26855484	82.42205441	669.8248586	462.3208505
0.00025	83.06137158	82.21625663	661.5006616	457.3804573
0.000275	82.85470472	82.01097271	653.3600627	452.5407478
0.0003	82.64855297	81.80620135	645.3973698	447.7987423

Table 3-1: The comparison of the system performance indices vs. critical human error rates for three-unit parallel system and two-out-of-three system without repair

λ_H (per hour)	$AV_{Parallel}$ (%)	$AV_{k/n}$ (%)	$AVSS_{Parallel}$ (hours)	$AVSS_{k/n}$ (hours)
0.0001	87.31042624	86.57979109	74.54303537	66.69805399
0.000125	87.12140116	86.39189479	74.14249943	66.27136379
0.00015	86.93283049	86.20443528	73.74504978	65.84992344
0.000175	86.74470101	86.01744031	73.35067027	65.43363969
0.0002	86.55702680	85.83087695	72.95934446	65.02242153
0.000225	86.36979234	85.64476529	72.57105534	64.61617988
0.00025	86.18300420	85.45908956	72.18578558	64.21482779
0.000275	85.99665641	85.27385539	71.80351735	63.81828016
0.0003	85.81075349	85.08906620	71.42423260	63.42645381

Table 3-2: The comparison of the system performance indices vs. critical human error rate for three-unit parallel system and two-out-of-three system under Type II repair policy

3.6 Three-State Device Series System

The description of the system

The block diagram of n-three state device series system with critical human error and common-cause failures is shown in Figure 3-38. This type of redundancy represents a system in which for its success, at least one device must operate normally, if all other devices have failed in short-mode. The system fails when a device fails in open-mode or a common-cause failure or a critical human error occurs or all the devices fail in short-mode. All the units incorporated in this system are assumed to be identical.

The series system and the parallel system (discussed in Chapter 2) are special cases of the k-out-of-n system. For k=1, the k-out-of-n system becomes a parallel system. For k=n, the system becomes a series system (an exception applies in the case of three-state device systems: open-mode failure parameters need to be interchanged with short-mode failure parameters and vice versa).

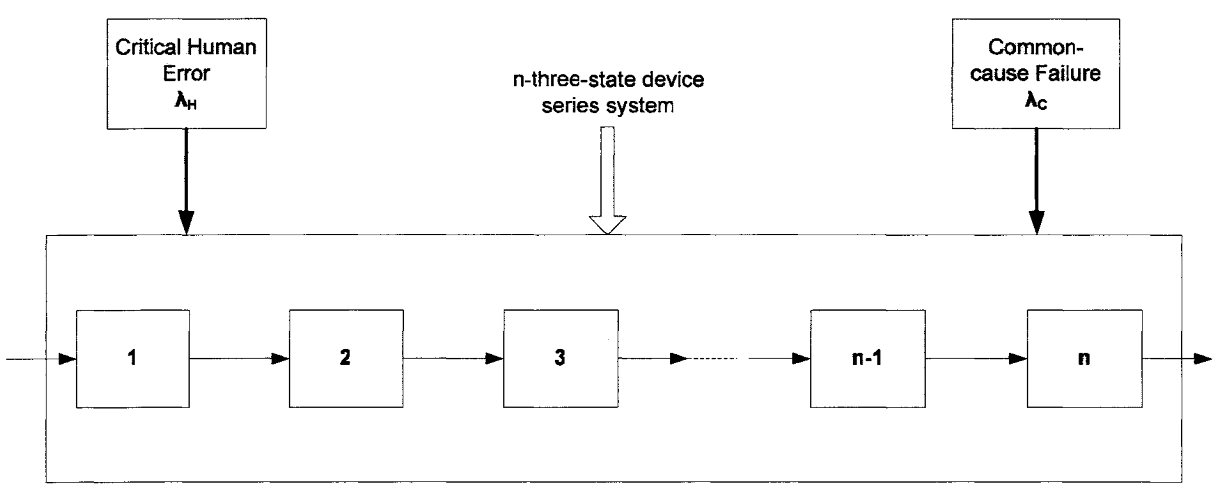

Figure 3-38: Block diagram of n-three-state device series system

Figure 3-39 shows the state transition diagram for Figure 3-38 under without repair policy. Similarly, Figure 3-40 shows the state transition diagram for Figure 3-38 under Type I repair policy, Figure 3-41 shows the state transition diagram for Figure 3-38 under Type II repair policy, and Figure 3-42 shows the state-transition diagram for Figure 3-38 under Type III repair policy. Numerals and single letters in boxes denote the system states. Other symbols used in the diagram are defined in the notation section as listed previously for the parallel system in Chapter 2.

Assumptions

The following assumptions are associated with the analyses in this study:

1. The system is composed of 'n' independent and identical devices in series.
2. Each unit (three-state device) can fail either in open-mode or in short-mode. These two failures are mutually exclusive of one another.
3. A critical human error or a common-cause failure can occur and trigger the whole system failure when two or more devices are operating.
4. All failures are statistically independent.
5. The failure rates for all the failures are constant.
6. The repaired unit/system is as good as new.
7. The repair rates for all types of failures are constant.
8. Repair rates are the same for completely failed system due to open-mode or short-mode failure.
9. All repair rates are the same for the partially failed system.
10. Common-cause failure rates are the same for the partially or fully operating system.
11. Critical human error rates are the same for the partially or fully operating system.

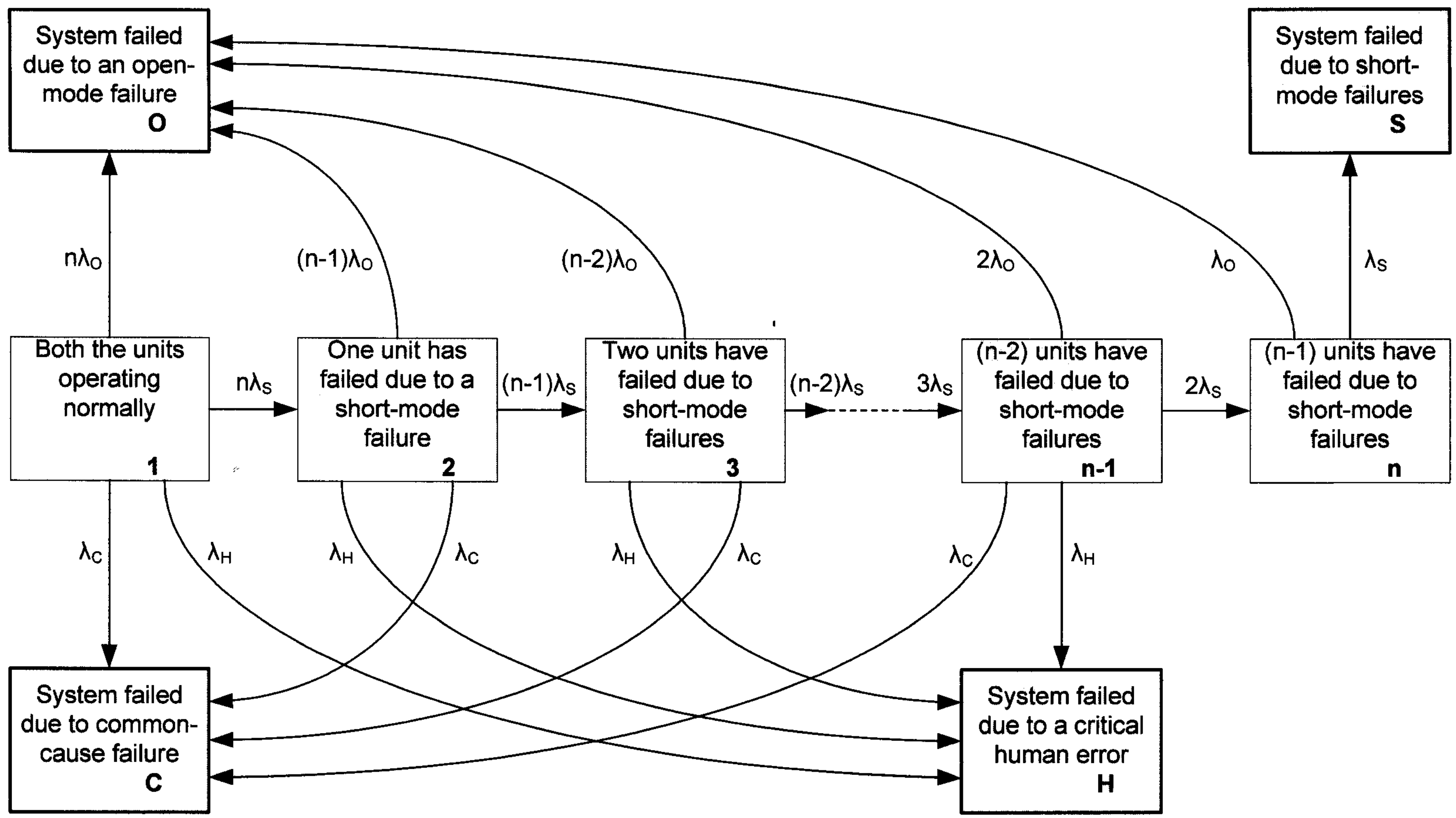

Figure 3-39: State transition diagram for n-three-state device series system under without repair policy

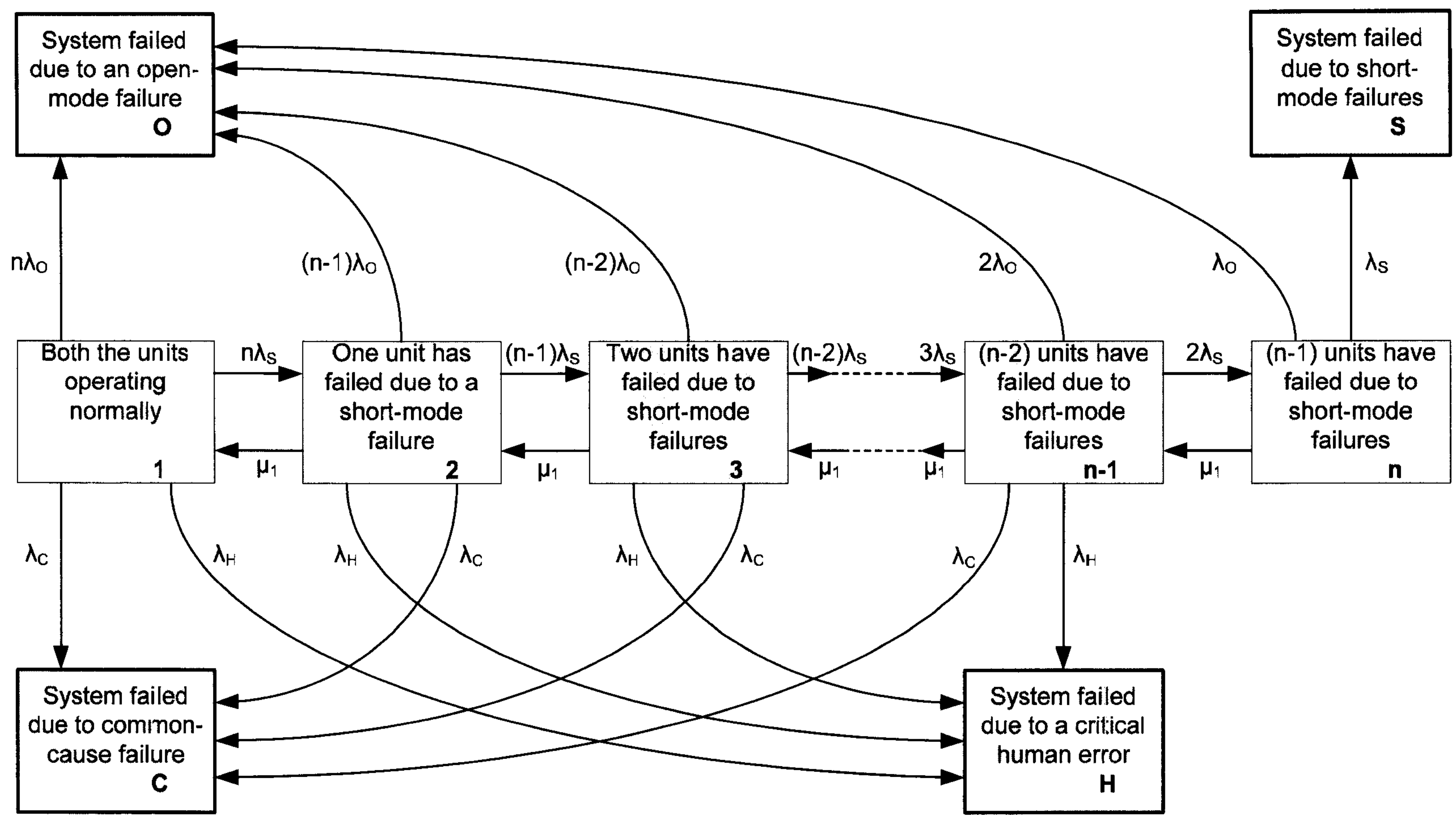

Figure 3-40: State transition diagram for n-three-state device series system under Type I repair policy

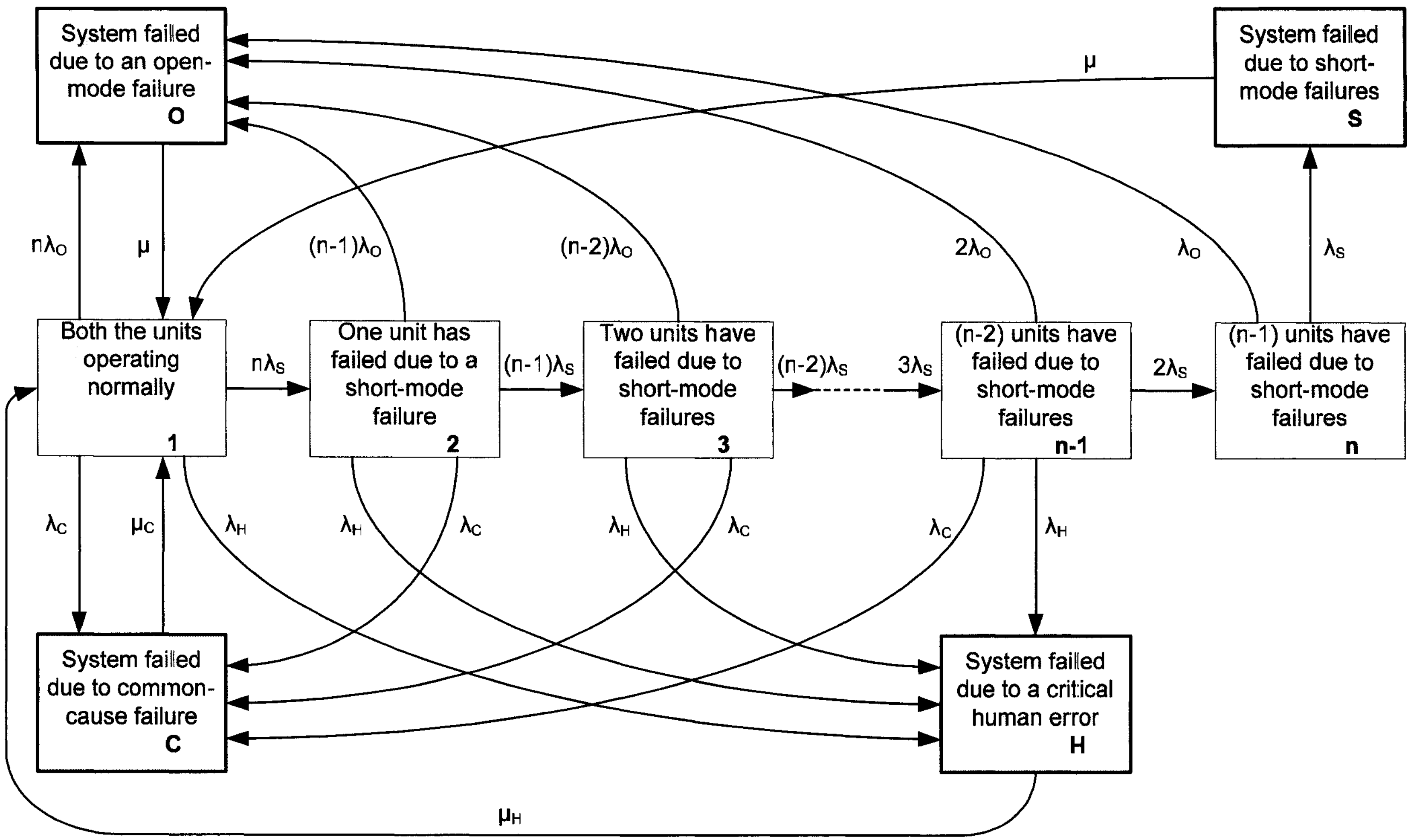

Figure 3-41: State transition diagram for n-three-state device series system under Type II repair policy

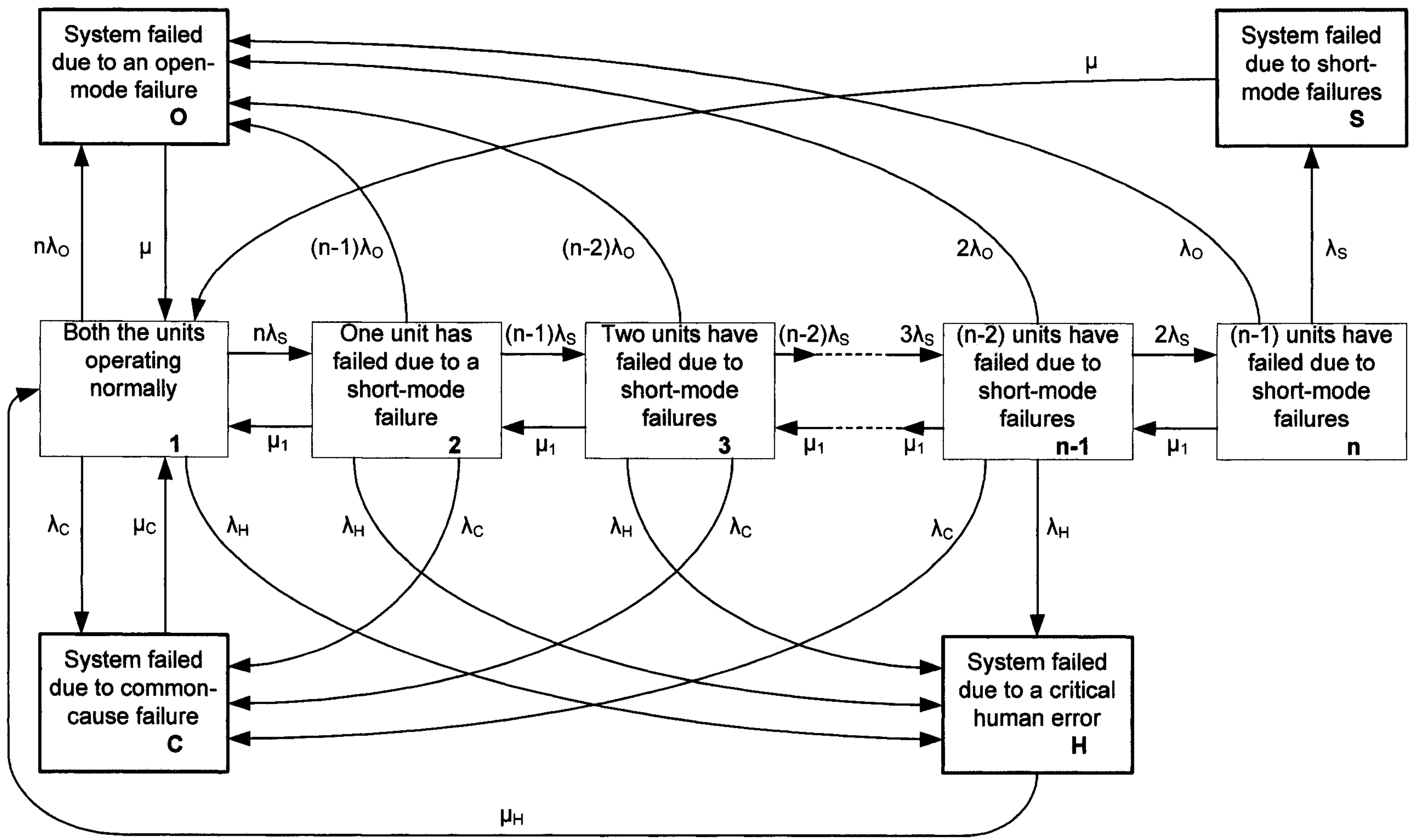

Figure 3-42: State transition diagram for n-three-state device series system under Type III repair policy

3.6.1 General series system without repair

The block diagram and the state transition diagram for the general model without repair is shown in Figures 3-38 and 3-39 respectively. Using the Markov state-space method, the system of differential equations associated with Figure 3-39 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) \tag{3-147}$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H\right]P_i(t) + (n-i+2)\lambda_S P_{i-1}(t) \tag{3-148}$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -\left[\lambda_O + \lambda_S\right]P_n(t) + 2\lambda_S P_{n-1}(t) \tag{3-149}$$

$$\frac{dP_S(t)}{dt} = \lambda_S P_n(t) \tag{3-150}$$

$$\frac{dP_O(t)}{dt} = \lambda_O \sum_{i=1}^{n}(n-i+1)P_i(t) \tag{3-151}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-1} P_i(t) \tag{3-152}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-1} P_i(t) \tag{3-153}$$

At time t=0, $P_1(0) = 1$, and all other initial state probabilities are equal to zero. Taking Laplace transforms of Equations (3-147) – (3-153) and solving the resulting equations, we get the following set of equations:

$$p_1(s) = \frac{1}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{3-154}$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_S}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H}\right]p_{i-1}(s) \quad \text{for all } i = 2 \text{ to } (n-1) \tag{3-155}$$

$$p_n(s) = \left[\frac{2\lambda_S}{s + \lambda_O + \lambda_S}\right] p_{n-1}(s) \tag{3-156}$$

$$p_S(s) = \frac{\lambda_S}{s} p_n(s) \tag{3-157}$$

$$p_O(s) = \frac{\lambda_O}{s} \sum_{i=1}^{n} (n - i + 1) p_i(s) \tag{3-158}$$

$$p_C(s) = \frac{\lambda_C}{s} \sum_{i=1}^{n-1} p_i(s) \tag{3-159}$$

$$p_H(s) = \frac{\lambda_H}{s} \sum_{i=1}^{n-1} p_i(s) \tag{3-160}$$

The time-dependent state probabilities can be obtained by taking the inverse Laplace transforms of Equations (3-154) – (3-160). The time-dependent system reliability is obtained by

$$R(t) = \sum_{i=1}^{n} P_i(t) = L^{-1}\left[\sum_{i=1}^{n} p_i(s)\right] \tag{3-161}$$

The system mean time to failure can be expressed as

$$MTTF = \lim_{s \to 0}\left[\sum_{i=1}^{n} p_i(s)\right] \tag{3-162}$$

3.6.2 Special case model 3-I: Two-unit series system without repair

The state transition diagram for a two-unit series system without repair can be represented as in Figure 3-43. By setting n=2 in Equations (3-154) – (3-160), we obtain the following set of equations associated with Figure 3-43:

$$p_1(s) = \frac{1}{s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H} \tag{3-163}$$

$$p_2(s) = \frac{2\lambda_S}{(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \quad (3\text{-}164)$$

$$p_S(s) = \frac{2\lambda_S^{\,2}}{s(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \quad (3\text{-}165)$$

$$p_O(s) = \frac{2\lambda_O(s+\lambda_O+2\lambda_S)}{s(s+\lambda_O+\lambda_S)(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \quad (3\text{-}166)$$

$$p_C(s) = \frac{\lambda_C}{s(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \quad (3\text{-}167)$$

$$p_H(s) = \frac{\lambda_H}{s(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)} \quad (3\text{-}168)$$

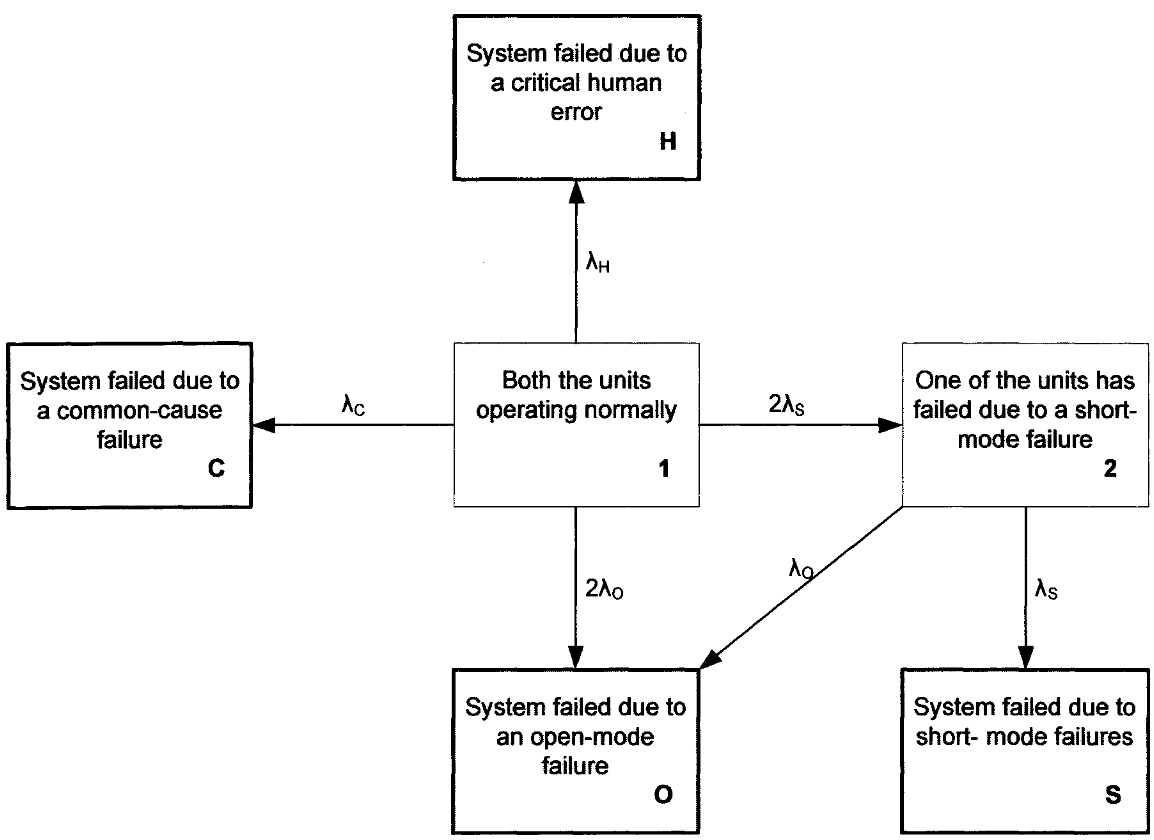

Figure 3-43: State transition diagram for a two-unit series system without repair

By taking inverse Laplace transforms of Equations (3-163) – (3-168), the time-dependent state probabilities can be obtained. By adding Equations (3-163) and (3-164) and taking inverse Laplace transforms, we following expression for the system reliability results:

$$R(t) = \frac{1}{\lambda_O + \lambda_S + \lambda_C + \lambda_H}\left[\frac{\lambda_O + \lambda_C + \lambda_H - \lambda_S}{e^{(2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H)t}} + \frac{2\lambda_S}{e^{(\lambda_O + \lambda_S)t}}\right] \quad (3\text{-}169)$$

The system mean time to failure is given by:

$$MTTF = \int_0^{\infty} R(t)\,dt = \frac{1}{\lambda_O + \lambda_S + \lambda_C + \lambda_H}\left[\frac{\lambda_O + \lambda_C + \lambda_H - \lambda_S}{2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H} + \frac{2\lambda_S}{\lambda_O + \lambda_S}\right] \quad (3\text{-}170)$$

For the specified values of the model parameters, the plots of state probabilities as a function of time, *t*, are shown in Figure 3-44, the plots of system reliability vs. time are shown in Figure 3-45 and the plots of *MTTF* vs. λ_H are shown in Figure 3-46.

3.6.3 General series system under Type I repair policy

The state space diagram for the general model under Type I repair policy is shown in Figures 3-40. Using the Markov technique, the corresponding system of differential equations associated with Figure 3-40 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) + \mu_1 P_2(t) \quad (3\text{-}171)$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1\right]P_i(t) + (n-i+2)\lambda_S P_{i-1}(t) + \mu_1 P_{i+1}(t) \quad (3\text{-}172)$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -\left[\lambda_O + \lambda_S + \mu_1\right]P_n(t) + 2\lambda_S P_{n-1}(t) \quad (3\text{-}173)$$

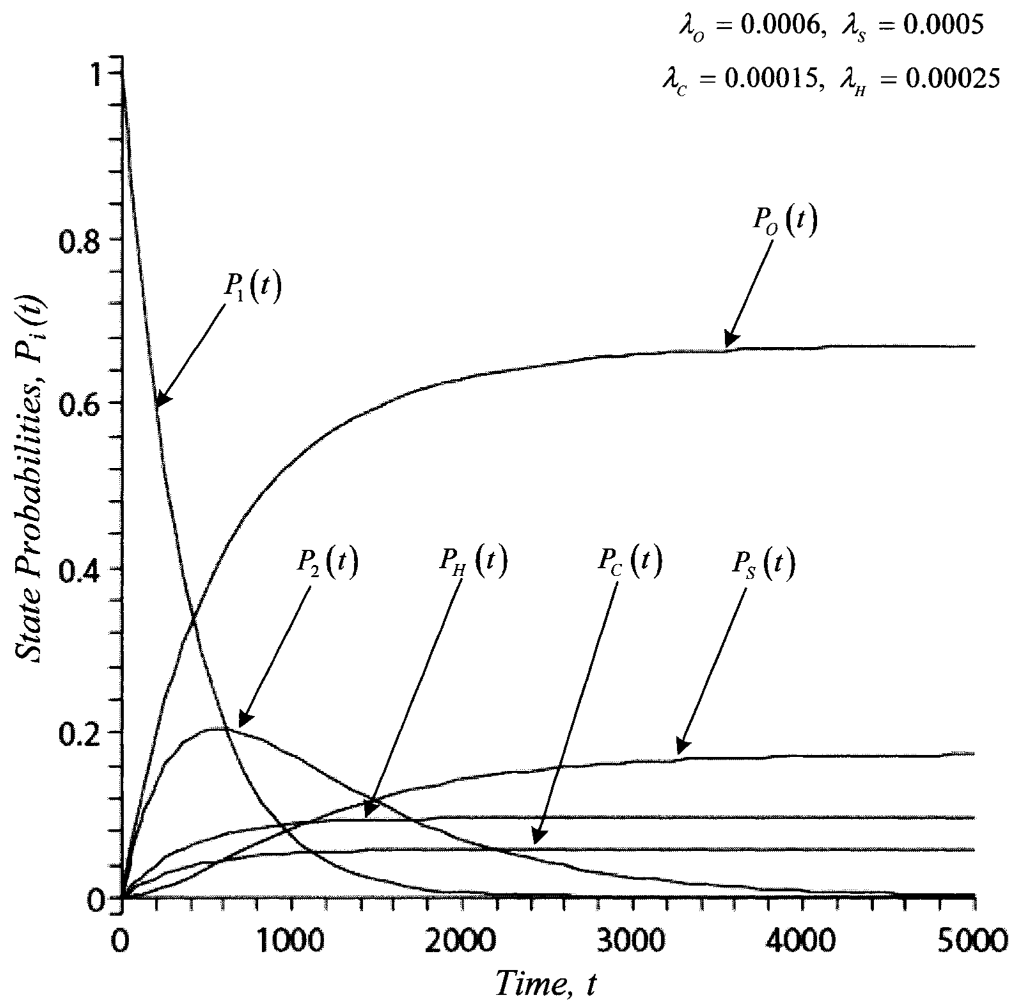

Figure 3-44: State probability plots of two-unit series system without repair

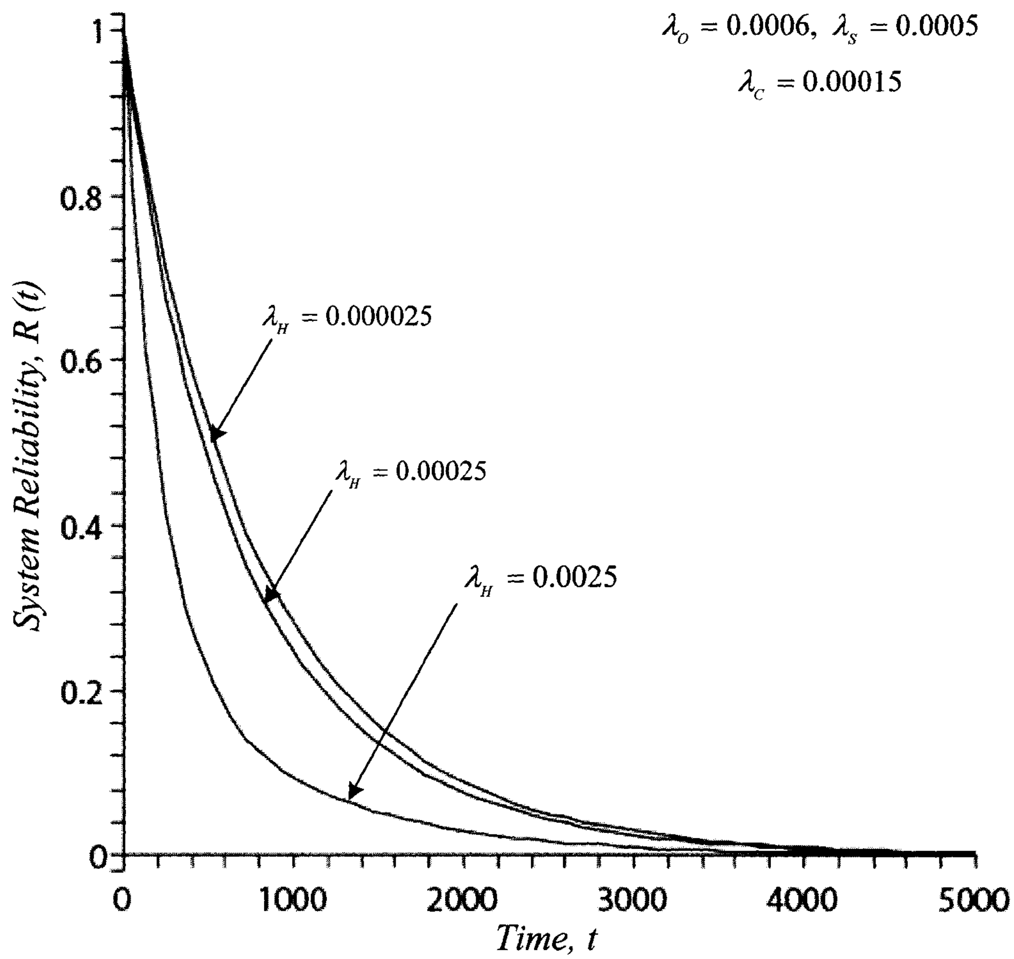

Figure 3-45: Reliability plots of two-unit series system without repair

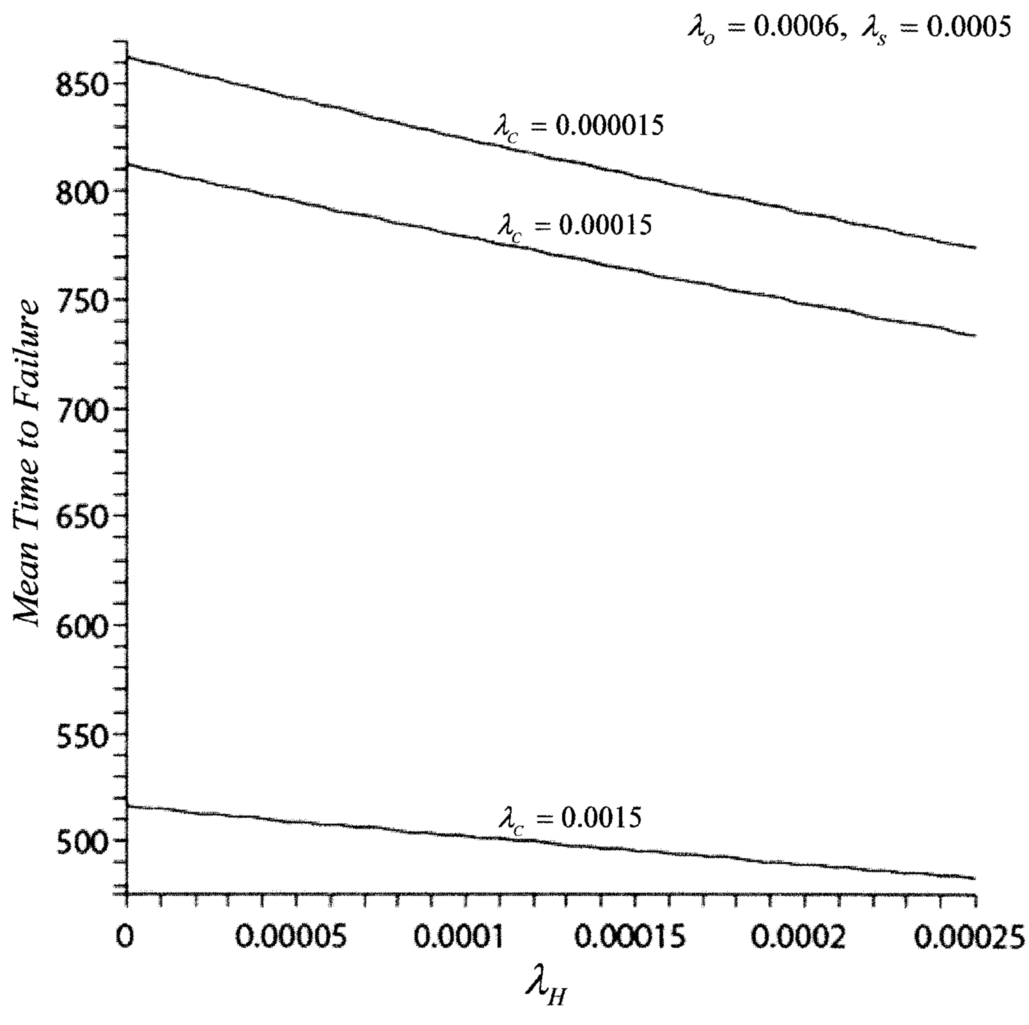

Figure 3-46: Mean time to failure plots of two-unit series system without repair

$$\frac{dP_S(t)}{dt} = \lambda_S P_n(t) \tag{3-174}$$

$$\frac{dP_O(t)}{dt} = \lambda_O \sum_{i=1}^{n} (n-i+1) P_i(t) \tag{3-175}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-1} P_i(t) \tag{3-176}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-1} P_i(t) \tag{3-177}$$

At time t=0, $P_1(0)$ = 1, and all other initial state transition probabilities are equal to zero. By taking Laplace transforms of Equations (3-171) – (3-177) and solving the resulting equations by utilizing the initial conditions, we get the following set of equations:

$$p_1(s) = \frac{1+\mu_1 p_2(s)}{s+n\lambda_O+n\lambda_S+\lambda_C+\lambda_H} \tag{3-178}$$

$$p_i(s) = \frac{(n-i+2)\lambda_S p_{i-1}(s)+\mu_1 p_{i+1}(s)}{s+(n-i+1)\lambda_O+(n-i+1)\lambda_S+\lambda_C+\lambda_H+\mu_1} \quad \text{for all } i = 2 \text{ to } (n-1) \tag{3-179}$$

$$p_n(s) = \frac{2\lambda_S p_{n-1}(s)}{s+\lambda_O+\lambda_S+\mu_1} \tag{3-180}$$

$$p_S(s) = \frac{\lambda_S p_n(s)}{s} \tag{3-181}$$

$$p_O(s) = \frac{\lambda_O}{s} \sum_{i=1}^{n} (n-i+1) p_i(s) \tag{3-182}$$

$$p_C(s) = \frac{\lambda_C}{s} \sum_{i=1}^{n-1} p_i(s) \tag{3-183}$$

$$p_H(s) = \frac{\lambda_H}{s} \sum_{i=1}^{n-1} p_i(s) \tag{3-184}$$

By taking the inverse Laplace transforms of Equations (3-178) – (3-184), we can obtain the time-dependent state probabilities. The time-dependent system reliability can be obtained using the following relation:

$$R(t) = L^{-1}\left[\sum_{i=1}^{n} p_i(s)\right] \tag{3-185}$$

The system mean time to failure can be found by

$$MTTF = \lim_{s\to 0}\left[\sum_{i=1}^{n} p_i(s)\right] \tag{3-186}$$

3.6.4 Special case model 3-J: Two-unit series system with Type I repair

The state transition diagram for a two-unit series system with Type I repair policy is shown in Figure 3-47. By substituting n=2 in Equations (3-178) – (3-184), the following set of equations results:

$$p_1(s) = \frac{s+\lambda_O+\lambda_S+\mu_1}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S} \tag{3-187}$$

$$p_2(s) = \frac{2\lambda_S}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S} \tag{3-188}$$

$$p_S(s) = \frac{2\lambda_S^{2}}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S\right]} \tag{3-189}$$

$$p_O(s) = \frac{2\lambda_O\left(s+\lambda_O+2\lambda_S+\mu_1\right)}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S\right]} \tag{3-190}$$

$$p_C(s) = \frac{\lambda_C\left(s+\lambda_O+\lambda_S+\mu_1\right)}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S\right]} \tag{3-191}$$

$$p_H(s) = \frac{\lambda_H\left(s+\lambda_O+\lambda_S+\mu_1\right)}{s\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S\right]} \tag{3-192}$$

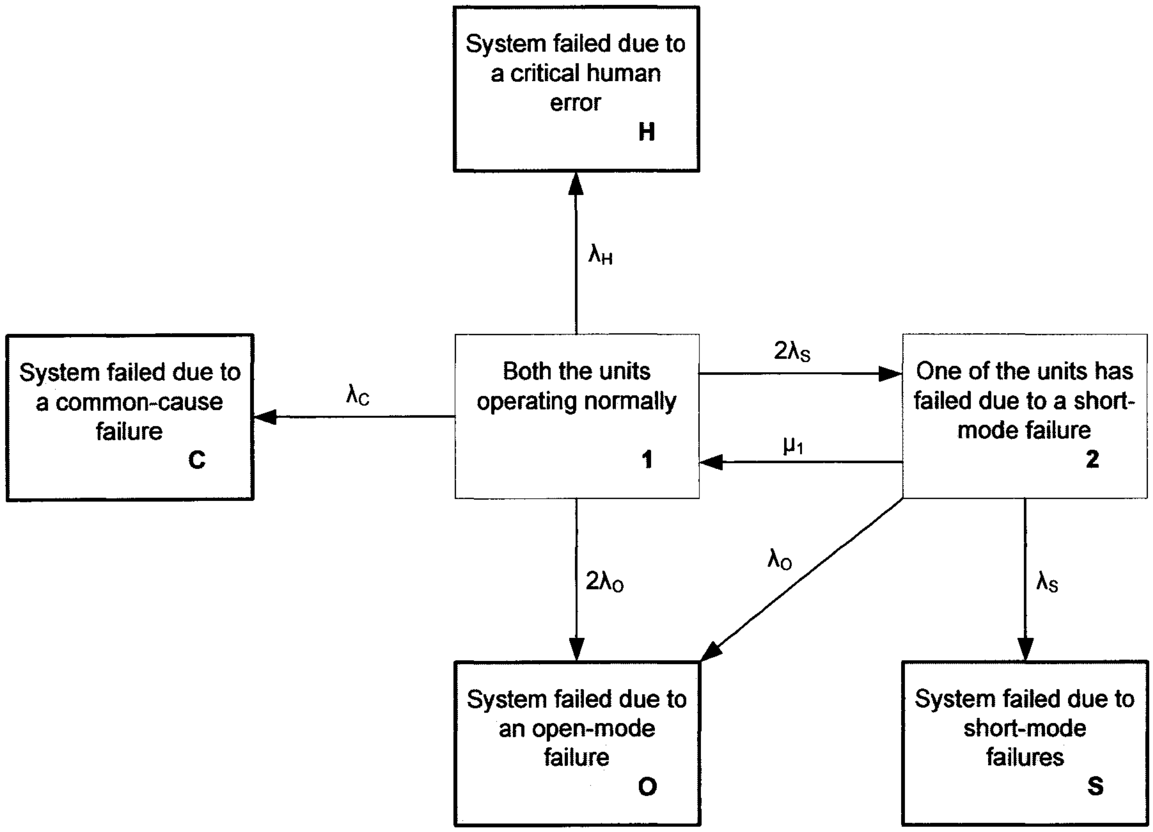

Figure 3-47: State transition diagram for a two-unit series system with Type I repair policy

By taking the inverse Laplace transforms of Equations (3-187) – (3-192), the time-dependent transition state probabilities can be obtained. The system reliability can be computed by adding Equations (3-187) and (3-188) and then taking the inverse Laplace transform, which can be expressed mathematically as follows:

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\frac{s+\lambda_O+3\lambda_S+\mu_1}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S}\right] \tag{3-193}$$

The system mean time to failure is given by

$$MTTF = \lim_{s\to 0}\left[\frac{s+\lambda_O+3\lambda_S+\mu_1}{(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S}\right] \tag{3-194}$$

For the given values of the model parameters, the plots of the state probabilities as a function of time, t, are shown in Figure 3-48. For the specified values of the model parameters, the plots of Equation (3-193) are shown in Figure 3-49. Similarly, for the specified values of the model parameters, the plots of Equation (3-194) are shown in Figure 3-50.

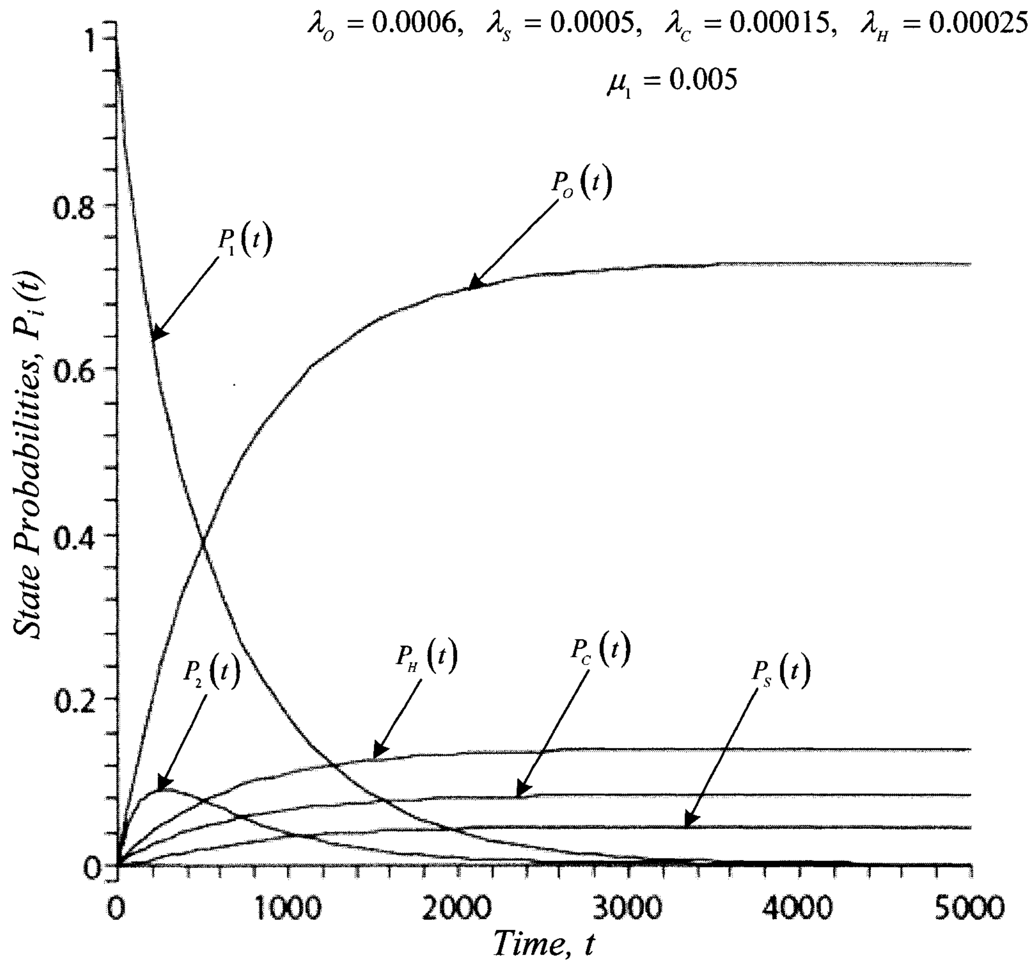

Figure 3-48: State probability plots of two-unit series system with Type I repair

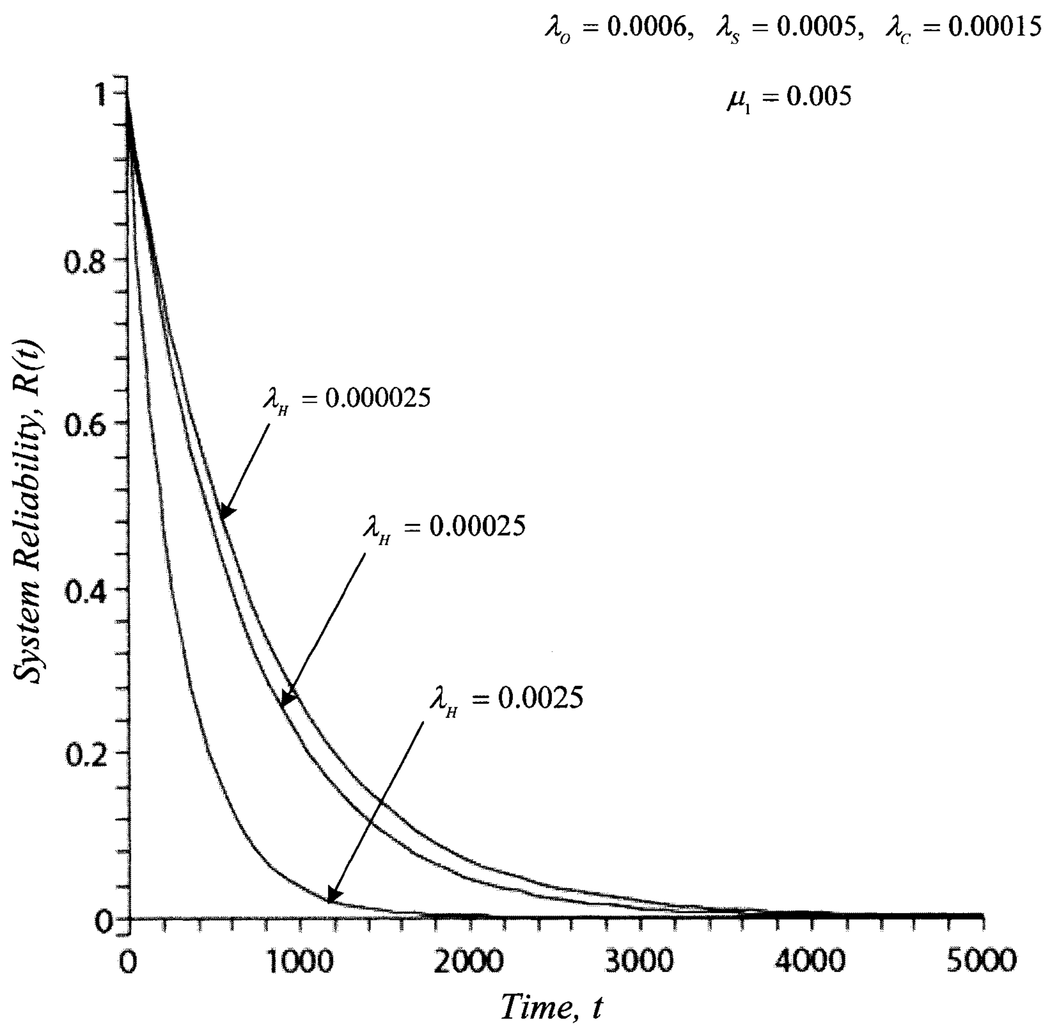

Figure 3-49: Reliability plots of two-unit series system with Type I repair

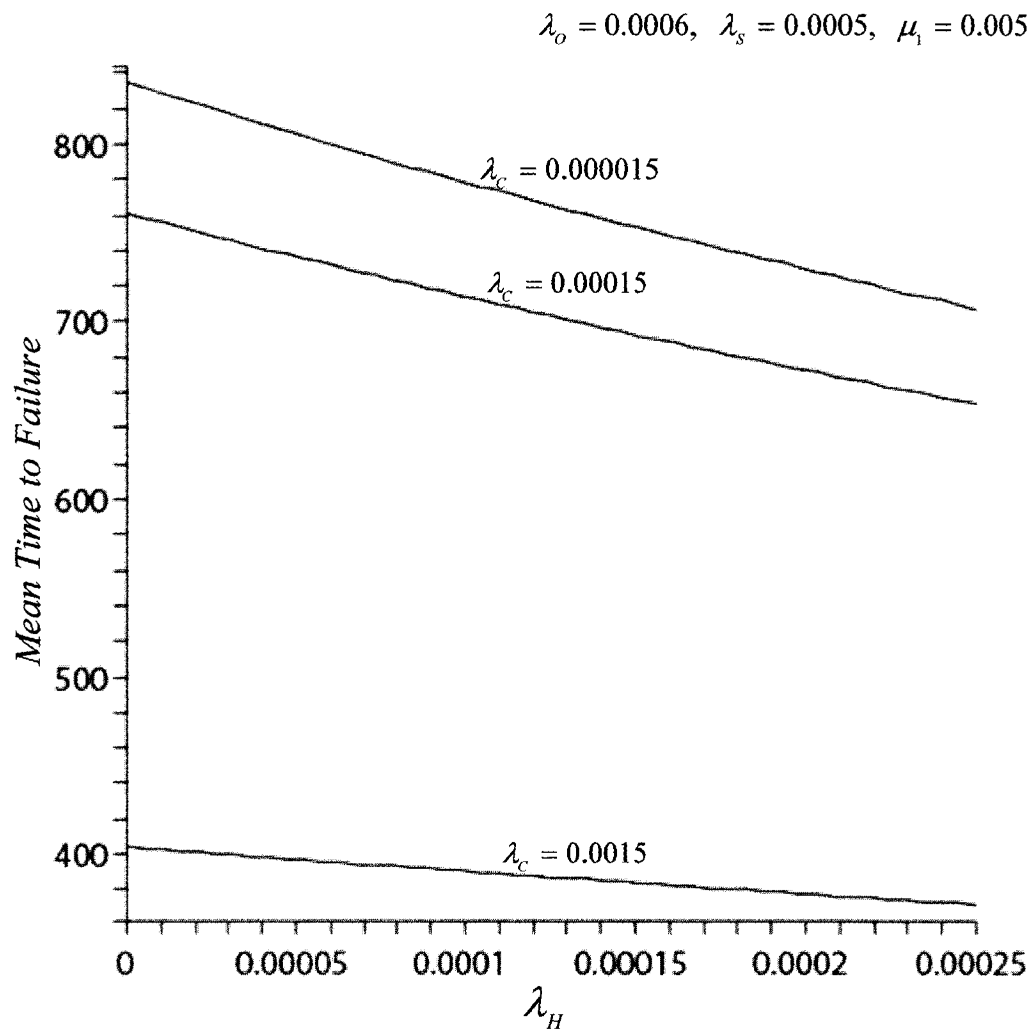

Figure 3-50: Mean time to failure plots of two-unit series system with Type I repair

3.6.4 General series system under Type II repair policy

The block diagram and the state transition diagram for the general model with Type II repair policy is shown in Figures 3-38 and 3-41 respectively. Using the Markov method, the system of differential equations associated with Figure 3-41 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) + \mu\left[P_O(t) + P_S(t)\right] + \mu_C P_C(t) + \mu_H P_H(t) \quad (3\text{-}195)$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H\right]P_i(t) + (n-i+2)\lambda_S P_{i-1}(t) \quad (3\text{-}196)$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -\left[\lambda_O + \lambda_S\right]P_n(t) + 2\lambda_S P_{n-1}(t) \quad (3\text{-}197)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S P_n(t) \quad (3\text{-}198)$$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + \lambda_O \sum_{i=1}^{n}(n-i+1)P_i(t) \quad (3\text{-}199)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-1} P_i(t) \quad (3\text{-}200)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-1} P_i(t) \quad (3\text{-}201)$$

At time t=0, $P_1(0) = 1$, and all other initial state probabilities are equal to zero. Taking Laplace transforms of Equations (3-195) – (3-201) and solving the resulting equations, we get the following set of equations:

$$p_1(s) = \frac{1 + \mu\left[p_O(s) + p_S(s)\right] + \mu_C p_C(s) + \mu_H p_H(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \quad (3\text{-}202)$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_S}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H}\right]p_{i-1}(s) \quad \text{for all } i = 2 \text{ to } (n-1) \quad (3\text{-}203)$$

$$p_n(s)=\left[\frac{2\lambda_S}{s+\lambda_O+\lambda_S}\right]p_{n-1}(s) \tag{3-204}$$

$$p_S(s)=\left[\frac{\lambda_S}{s+\mu}\right]p_n(s) \tag{3-205}$$

$$p_O(s)=\frac{\lambda_O}{s+\mu}\sum_{i=1}^{n}(n-i+1)p_i(s) \tag{3-206}$$

$$p_C(s)=\frac{\lambda_C}{s+\mu_C}\sum_{i=1}^{n-1}p_i(s) \tag{3-207}$$

$$p_H(s)=\frac{\lambda_H}{s+\mu_H}\sum_{i=1}^{n-1}p_i(s) \tag{3-208}$$

By taking the inverse Laplace transforms of Equations (3-202) – (3-208), the time-dependent state probabilities can be obtained. The time-dependent system availability is given by

$$A(t)=\sum_{i=1}^{n}P_i(t)=L^{-1}\left[\sum_{i=1}^{n}p_i(s)\right] \tag{3-209}$$

3.6.6 Special case model 3-K: Two-unit series system with Type II repair

The state transition diagram of a two-unit series system with Type II repair policy is shown in Figure 3-51. For n=2 in Figure 3-41 from Equations (3-202) – (3-208), we get the following set of equations:

$$p_1(s)=\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+\lambda_O+\lambda_S)}{A(3,9)} \tag{3-210}$$

$$p_2(s)=\frac{2\lambda_S(s+\mu)(s+\mu_C)(s+\mu_H)}{A(3,9)} \tag{3-211}$$

$$p_S(s)=\frac{2\lambda_S^{\,2}(s+\mu_C)(s+\mu_H)}{A(3,9)} \tag{3-212}$$

$$p_O(s)=\frac{2\lambda_O\left(s+\lambda_O+2\lambda_S\right)\left(s+\mu_C\right)\left(s+\mu_H\right)}{A(3,9)} \tag{3-213}$$

$$p_C(s)=\frac{\lambda_C\left(s+\mu\right)\left(s+\mu_H\right)\left(s+\lambda_O+\lambda_S\right)}{A(3,9)} \tag{3-214}$$

$$p_H(s)=\frac{\lambda_H\left(s+\mu\right)\left(s+\mu_C\right)\left(s+\lambda_O+\lambda_S\right)}{A(3,9)} \tag{3-215}$$

where

$$A(3,9)=\left(s+\mu_C\right)\left(s+\mu_H\right)\left[\left(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H\right)\left(s+\mu\right)\left(s+\lambda_O+\lambda_S\right)-2\mu\lambda_S^2-2\mu\lambda_O\left(s+\lambda_O+2\lambda_S\right)\right]$$
$$-\left(s+\mu\right)\left(s+\lambda_O+\lambda_S\right)\left[\left(s+\mu_H\right)\mu_C\lambda_C+\left(s+\mu_C\right)\mu_H\lambda_H\right]$$

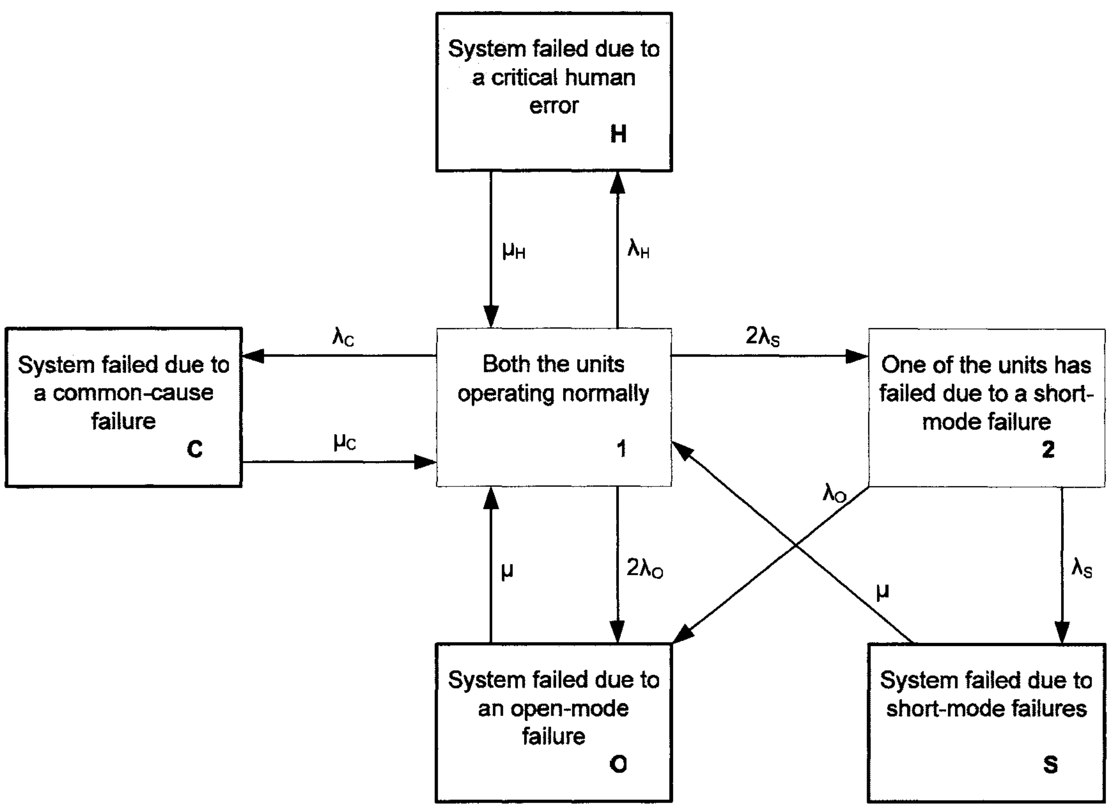

Figure 3-51: State transition diagram of a two unit series system with Type II repair policy

By taking the inverse Laplace transforms of equations (3-210) – (3-215), the time-dependent state probabilities can be obtained. By adding Equations (3-210) and (3-211) and then taking inverse Laplace transforms, we get the following expression for the system time dependent availability:

$$A(t) = L^{-1}\left[\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+\lambda_O+3\lambda_S)}{A(3,9)}\right] \tag{3-216}$$

Steady State Conditions: We can compute the steady state conditions directly from Equations (3-195) – (3-201), (i.e., for n=2) by putting all derivatives equal to zero and solving the resultant equations by utilizing the total probability axiom: $\sum P_i = 1$. Thus, we get the following set of equations:

$$P_1 = \frac{\mu\mu_C\mu_H(\lambda_O+\lambda_S)}{\mu_C\mu_H\left[\mu(\lambda_O+3\lambda_S)+2(\lambda_O+\lambda_S)^2\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{3-217}$$

$$P_2 = \frac{2\lambda_S\mu\mu_C\mu_H}{\mu_C\mu_H\left[\mu(\lambda_O+3\lambda_S)+2(\lambda_O+\lambda_S)^2\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{3-218}$$

$$P_S = \frac{2\lambda_S^2\mu_C\mu_H}{\mu_C\mu_H\left[\mu(\lambda_O+3\lambda_S)+2(\lambda_O+\lambda_S)^2\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{3-219}$$

$$P_O = \frac{2\lambda_O(\lambda_O+2\lambda_S)\mu_C\mu_H}{\mu_C\mu_H\left[\mu(\lambda_O+3\lambda_S)+2(\lambda_O+\lambda_S)^2\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{3-220}$$

$$P_C = \frac{\lambda_C(\lambda_O+\lambda_S)\mu\mu_H}{\mu_C\mu_H\left[\mu(\lambda_O+3\lambda_S)+2(\lambda_O+\lambda_S)^2\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{3-221}$$

$$P_H = \frac{\lambda_H(\lambda_O+\lambda_S)\mu\mu_C}{\mu_C\mu_H\left[\mu(\lambda_O+3\lambda_S)+2(\lambda_O+\lambda_S)^2\right]+\mu(\lambda_O+\lambda_S)(\mu_C\lambda_H+\mu_H\lambda_C)} \tag{3-222}$$

By adding Equations (3-217) and (3-218), we get the following expression for the system steady state availability:

$$AV_{ss} = \frac{\mu\mu_C\mu_H(3\lambda_O + \lambda_S)}{\mu_C\mu_H\left[\mu(\lambda_O + 3\lambda_S) + 2(\lambda_O + \lambda_S)^2\right] + \mu(\lambda_O + \lambda_S)(\mu_C\lambda_H + \mu_H\lambda_C)} \tag{3-223}$$

For the specified values of the model parameters, the plots of the state probabilities as a function of time, t, are shown in Figure 3-52, the plots of system availability as a function of time, t, are shown in Figure 3-53 and the plots of steady state availability given by Equation (3-223) are shown in Figure 3-54.

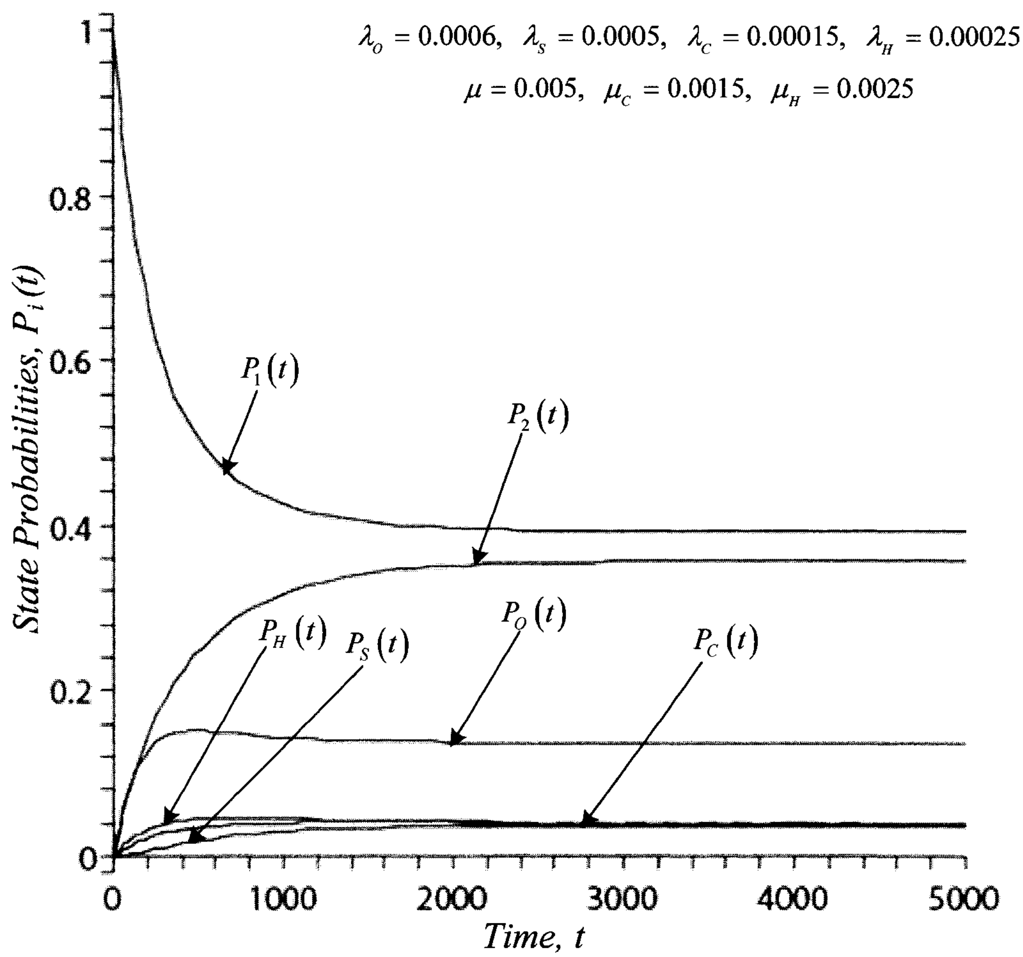

Figure: 3-52: State probability plots of two-unit series system with Type II repair

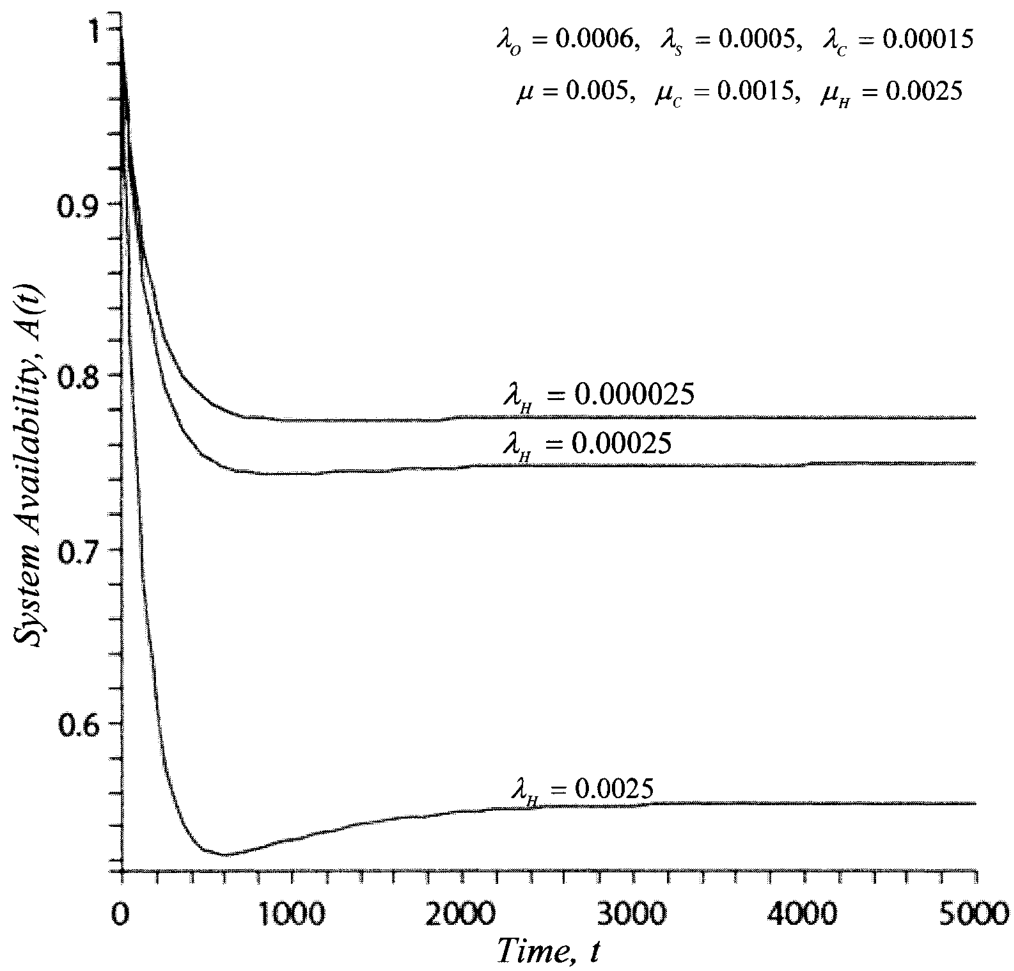

Figure 3-53: Availability plots of two-unit series system under Type III repair policy

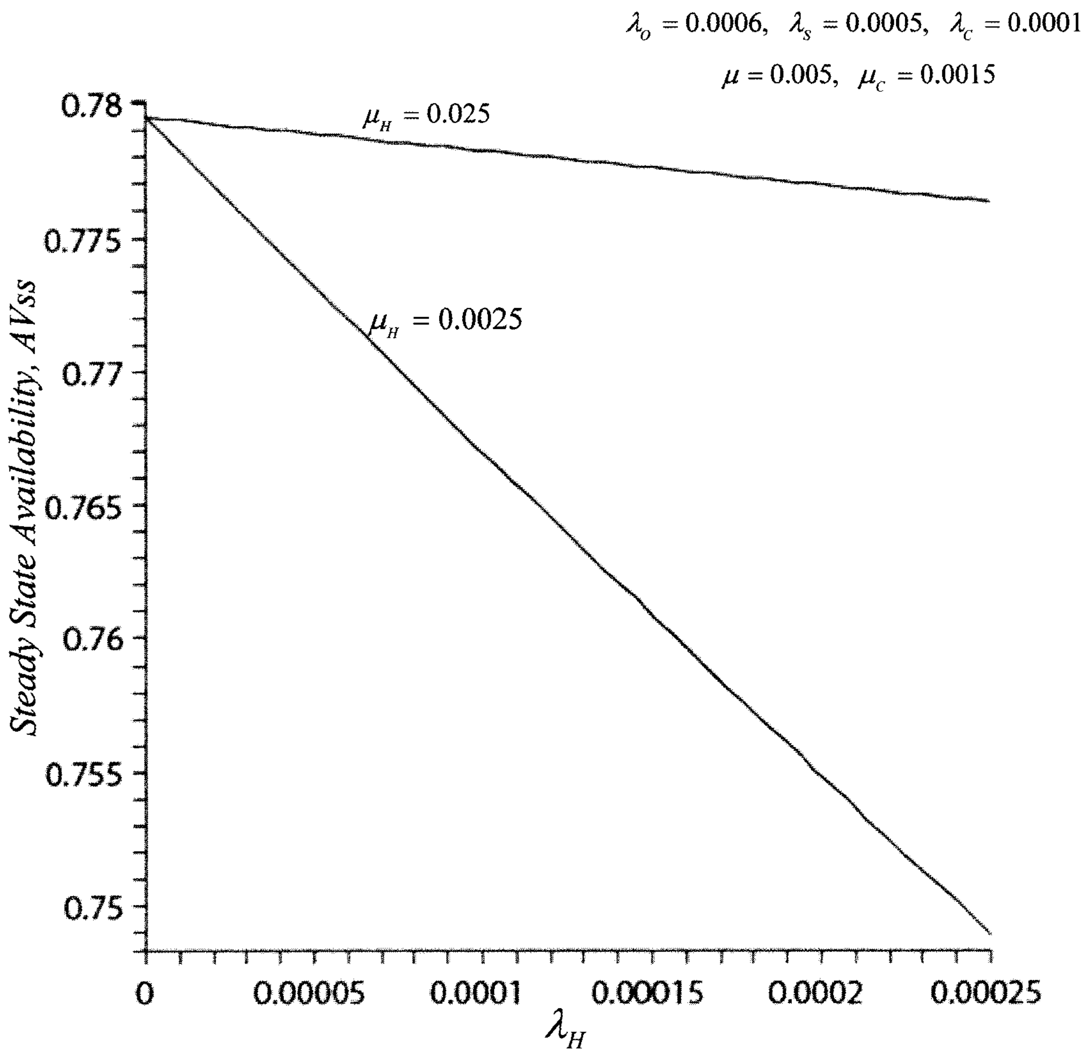

Figure 3-54: Steady state availability plots of two-unit series system under Type II repair policy

3.6.7 General series system under Type III repair policy

The state transition diagram for the general series system under Type III repair policy can be represented as in Figure 3-42. Using the Markov state-space technique, we obtain the following system of differential equations associated with Figure 3-42:

$$\frac{dP_1(t)}{dt} = -[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H]P_1(t) + \mu_1 P_2(t) + \mu[P_O(t) + P_S(t)] + \mu_C P_C(t) + \mu_H P_H(t) \quad (3\text{-}224)$$

$$\frac{dP_i(t)}{dt} = -[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1]P_i(t) + (n-i+2)\lambda_S P_{i-1}(t) + \mu_1 P_{i+1}(t) \quad (3\text{-}225)$$

for all $i = 2$ to $(n-1)$

$$\frac{dP_n(t)}{dt} = -[\lambda_O + \lambda_S + \mu_1]P_n(t) + 2\lambda_S P_{n-1}(t) \quad (3\text{-}226)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S P_n(t) \quad (3\text{-}227)$$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + \lambda_O \sum_{i=1}^{n} (n-i+1)P_i(t) \quad (3\text{-}228)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-1} P_i(t) \quad (3\text{-}229)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-1} P_i(t) \quad (3\text{-}230)$$

At time $t=0$, $P_1(0) = 1$, and all other initial probabilities are equal to zero. By taking Laplace transforms of Equations (3-224) – (3-230) and solving the resulting equations, the following set of equations results:

$$p_1(s) = \frac{1 + \mu[p_O(s) + p_S(s)] + \mu_1 p_2(s) + \mu_C p_C(s) + \mu_H p_H(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \quad (3\text{-}231)$$

$$p_i(s) = \frac{(n-i+2)\lambda_S p_{i-1}(s) + \mu_1 p_{i+1}(s)}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1} \quad \text{for all } i = 2 \text{ to } (n-1) \quad (3\text{-}232)$$

$$p_n(s) = \frac{2\lambda_S p_{n-1}(s)}{s + \lambda_O + \lambda_S + \mu_1} \tag{3-233}$$

$$p_S(s) = \frac{\lambda_S p_n(s)}{s + \mu} \tag{3-234}$$

$$p_O(s) = \frac{\lambda_O}{s + \mu} \sum_{i=1}^{n} (n - i + 1) p_i(s) \tag{3-235}$$

$$p_C(s) = \frac{\lambda_C}{s + \mu_C} \sum_{i=1}^{n-1} p_i(s) \tag{3-236}$$

$$p_H(s) = \frac{\lambda_H}{s + \mu_H} \sum_{i=1}^{n-1} p_i(s) \tag{3-237}$$

By taking the inverse Laplace transforms of Equations (3-231) – (3-237), the time-dependent state probabilities can be obtained. The time-dependent system availability can be computed using the following expression:

$$A(t) = L^{-1}\left[\sum_{i=1}^{n} p_i(s)\right] \tag{3-238}$$

3.6.8 Special case model 3-L: Two-unit series system with Type III repair

The state transition of a two-unit series system with Type III repair policy is shown in Figure 3-55. By setting n=2 in Equations (3-231) – (3-237), we get the following set of equations:

$$p_1(s) = \frac{(s + \mu)(s + \mu_C)(s + \mu_H)(s + \lambda_O + \lambda_S + \mu_1)}{A(3,10)} \tag{3-239}$$

$$p_2(s) = \frac{2\lambda_S (s + \mu)(s + \mu_C)(s + \mu_H)}{A(3,10)} \tag{3-240}$$

$$p_S(s) = \frac{2\lambda_S^{\,2} (s + \mu_C)(s + \mu_H)}{A(3,10)} \tag{3-241}$$

$$p_O(s) = \frac{2\lambda_O (s + \mu_C)(s + \mu_H)(s + \lambda_O + 2\lambda_S + \mu_1)}{A(3,10)} \tag{3-242}$$

$$p_C(s) = \frac{\lambda_C(s+\mu)(s+\mu_H)(s+\lambda_O+\lambda_S+\mu_1)}{A(3,10)} \tag{3-243}$$

$$p_H(s) = \frac{\lambda_H(s+\mu)(s+\mu_C)(s+\lambda_O+\lambda_S+\mu_1)}{A(3,10)} \tag{3-244}$$

where

$$A(3,10) = (s+\mu)(s+\mu_C)(s+\mu_H)\left[(s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H)(s+\lambda_O+\lambda_S+\mu_1)-2\mu_1\lambda_S\right]$$
$$-2\mu(s+\mu_C)(s+\mu_H)\left[\lambda_S^{\,2}+\lambda_O(s+\lambda_O+2\lambda_S+\mu_1)\right]$$
$$-(s+\lambda_O+\lambda_S+\mu_1)(s+\mu)\left[\mu_C\lambda_C(s+\mu_H)+\mu_H\lambda_H(s+\mu_C)\right]$$

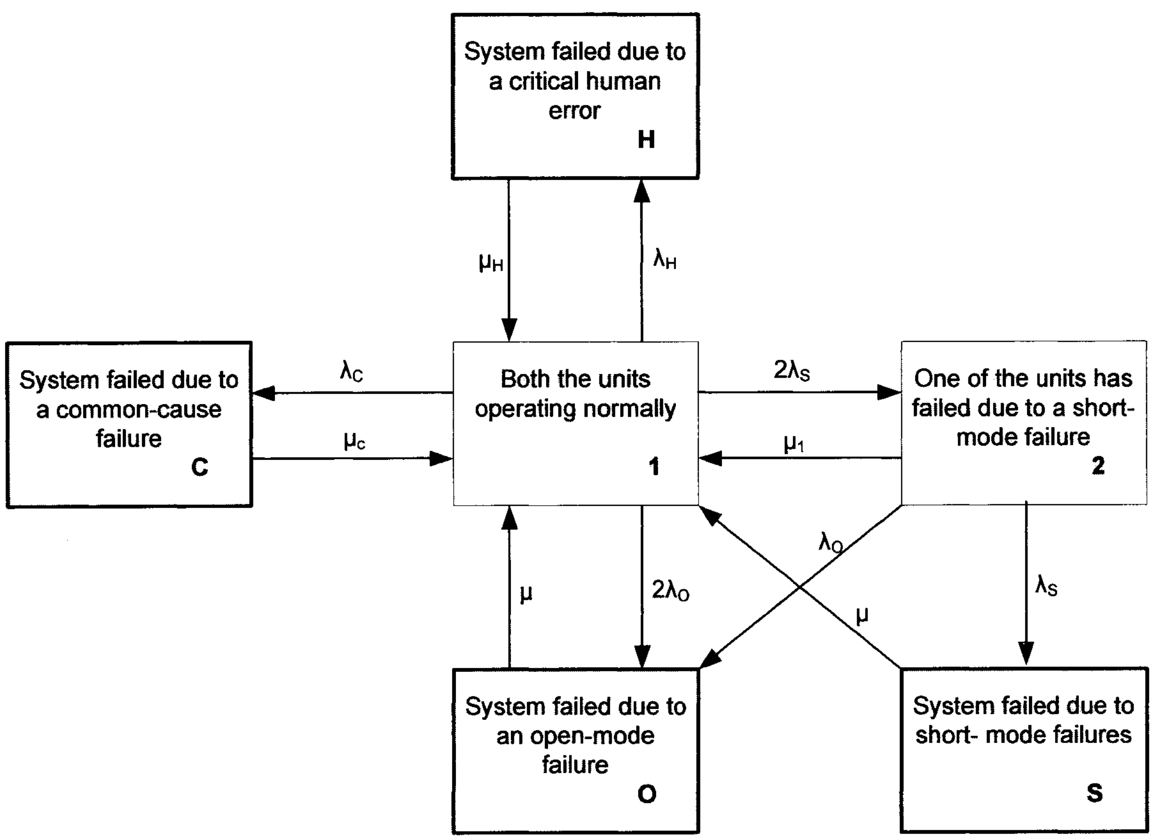

Figure 3-55: State transition of two-unit series system with Type III repair policy

By taking the inverse Laplace transforms of Equations (3-239) – (3-244), the time-dependent state probabilities can be obtained. By adding Equations (3-239) and (3-240) and then taking inverse Laplace transforms, we get the following expression for the system time dependent availability:

$$A(t) = L^{-1}\left\{\frac{(s+\mu)(s+\mu_C)(s+\mu_H)(s+\lambda_O+3\lambda_S+\mu_1)}{A(3,10)}\right\} \tag{3-245}$$

Steady State Conditions: The steady state conditions can be found directly from Equations (3-224) to (3-230), by substituting n=2 and equating all derivatives equal to zero. By solving the resulting equations utilizing the total probability axiom: $\sum P_i = 1$, the following set of equations results:

$$P_1 = \frac{\mu\mu_C\mu_H(\lambda_O+\lambda_S+\mu_1)}{A(3,11)} \tag{3-246}$$

$$P_2 = \frac{2\lambda_S\mu\mu_C\mu_H}{A(3,11)} \tag{3-247}$$

$$P_S = \frac{2\lambda_S^2\mu_C\mu_H}{A(3,11)} \tag{3-248}$$

$$P_O = \frac{2\lambda_O\mu_C\mu_H\left(\lambda_O+2\lambda_S+\mu_1\right)}{A(3,11)} \tag{3-249}$$

$$P_C = \frac{\lambda_C\mu\mu_H\left(\lambda_O+\lambda_S+\mu_1\right)}{A(3,11)} \tag{3-250}$$

$$P_H = \frac{\lambda_H\mu\mu_C\left(\lambda_O+\lambda_S+\mu_1\right)}{A(3,11)} \tag{3-251}$$

By adding Equations (3-246) and (3-247), we get the following expression for the system steady state availability:

$$AV_{SS} = P_1 + P_2 = \frac{\mu\mu_C\mu_H\left(\lambda_O+2\lambda_S+\mu_1\right)}{A(3,11)} \tag{3-252}$$

where

$$A(3,11) = \mu_C \mu_H \left[\mu(\lambda_O + 3\lambda_S + \mu_1) + 2\lambda_S^2 + 2\lambda_O(\lambda_O + 2\lambda_S + \mu_1) \right]$$
$$+ \mu(\lambda_O + \lambda_S + \mu_1)(\mu_C \lambda_H + \mu_H \lambda_C)$$

For the specified values of the model parameters, the plots of the state probabilities as a function of time, *t*, are shown in Figure 3-56. For the given values of the model parameters, the plots of Equation (3-245) are shown in Figure 3-57. Similarly, for the specified values of the model parameters, the plots of Equation (3-252) are shown in Figure 3-58.

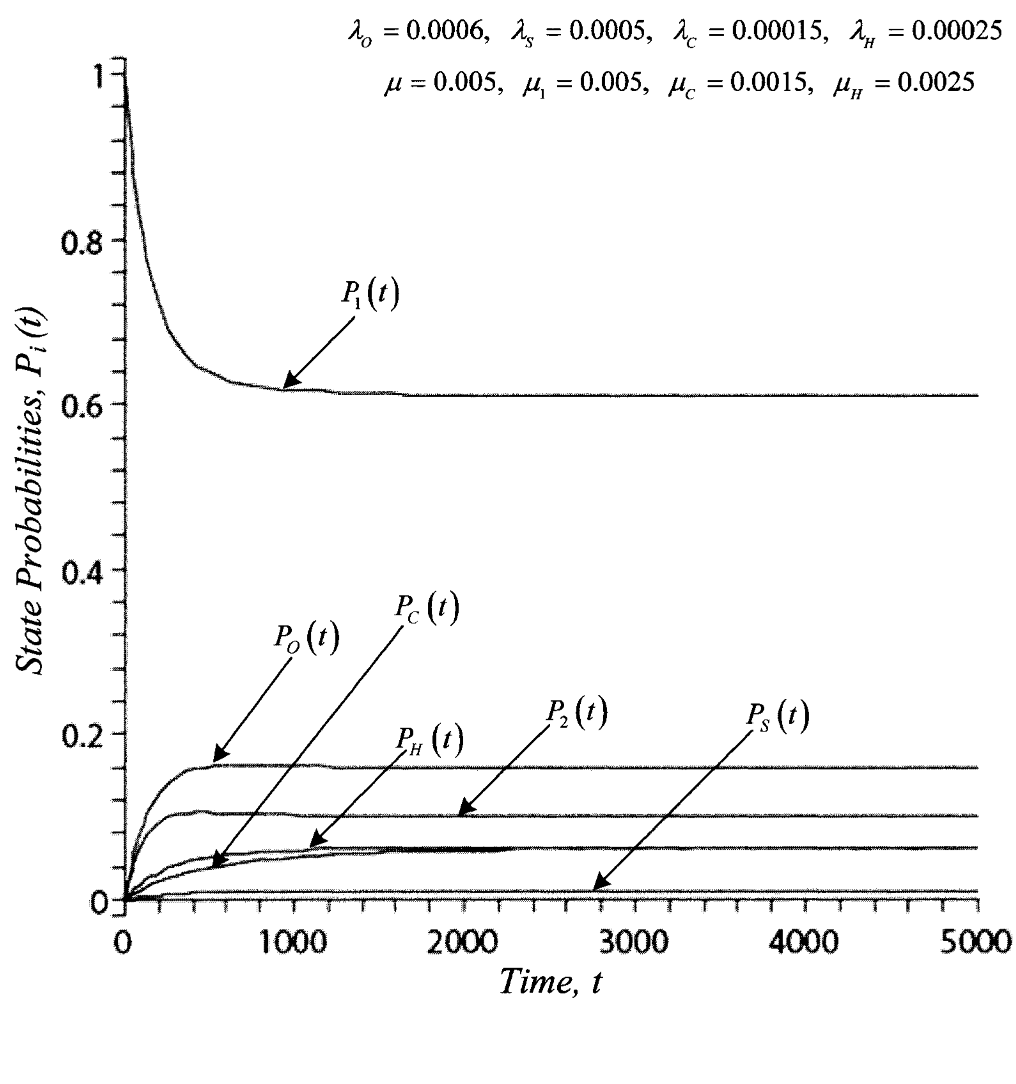

Figure 3-56: State probability plots of two-unit series system under Type III repair policy

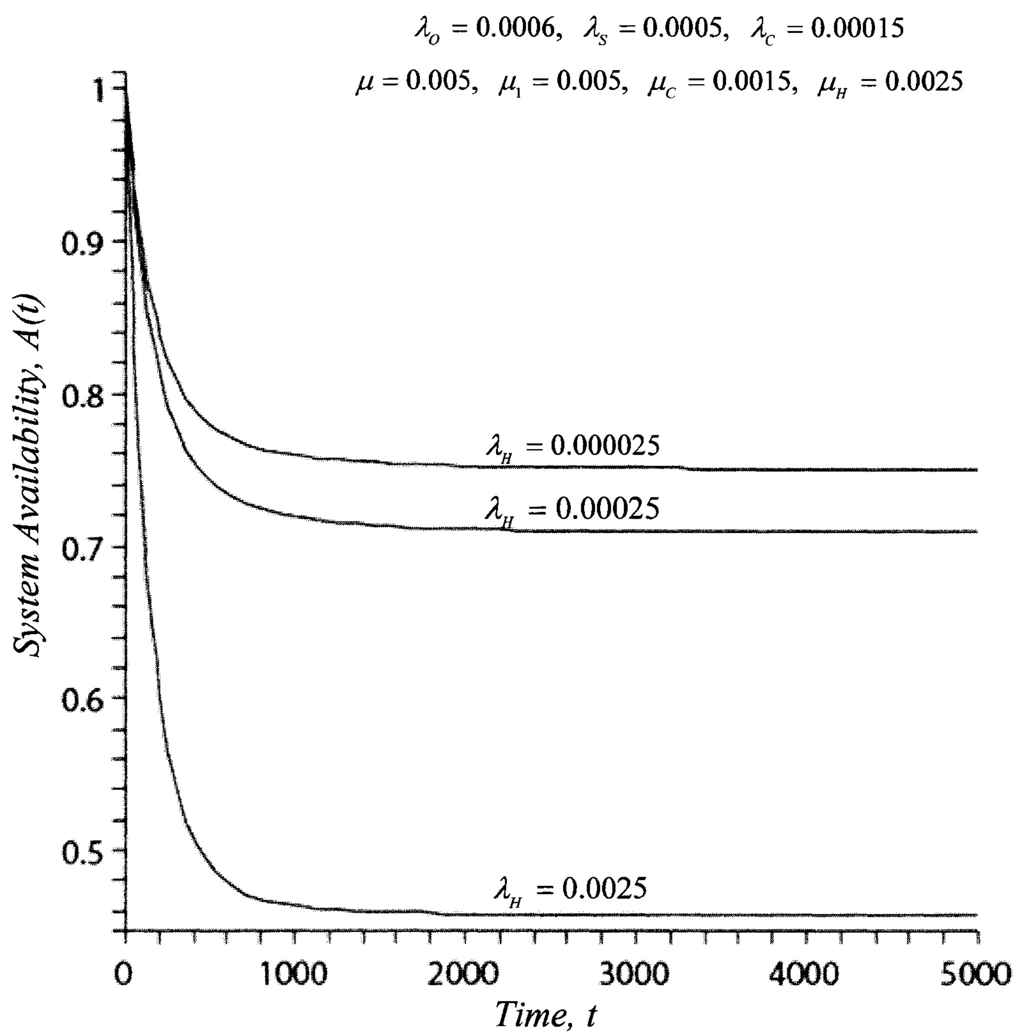

Figure 3-57: Availability plots of two-unit series system under Type III repair policy

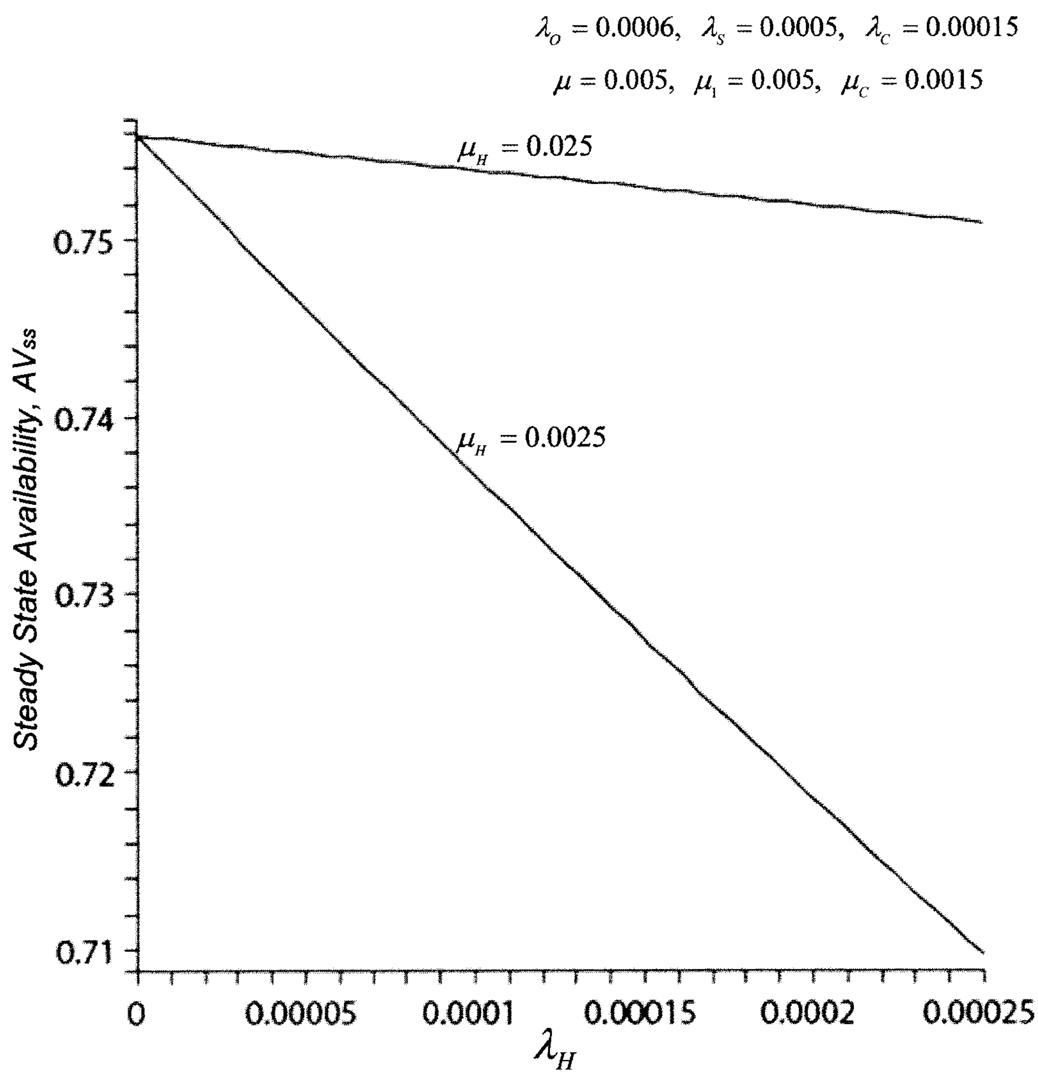

Figure: 3-58: Steady state availability plots of two-unit series system under Type III repair policy

Setting:

$t = 100$ hours

$\lambda_O = 0.0006$ per hour, $\lambda_S = 0.0005$ per hour, $\lambda_C = 0.00015$ per hour

$\mu = 0.005$ per hour, $\mu_C = 0.0015$ per hour, $\mu_H = 0.0025$ per hour

For the values of the model parameters as specified above, Table 3-3 shows the comparison of system reliability and mean time to failure for different values of critical human error rates (i.e., λ_H) for the two-unit parallel system and two-unit series system without repair.

Similarly, Table 3-4 shows the comparison of system time-dependent and steady state availabilities for different values of critical human error rate for the two-unit parallel system and two-unit series system under Type II repair policy.

The notation associated with Tables 3-3 and 3-4 is as follows:

$R_{Parallel}$	Reliability of two-unit parallel system without repair.
R_{Series}	Reliability of two-unit series system without repair.
$MTTF_{Parallel}$	Mean time to failure of two-unit parallel system without repair.
$MTTF_{Series}$	Mean time to failure of two-unit series system without repair.
$AV_{Parallel}$	Availability of two-unit parallel system with Type II repair.
AV_{Series}	Availability of two-unit series system with Type II repair.
$AVSS_{Parallel}$	Steady state availability of two-unit parallel system with Type II repair.
$AVSS_{Series}$	Steady state availability of two-unit series system with Type II repair.

The result of the analysis is presented in the next section.

λ_O (per hour)	λ_S (per hour)	$R_{Parallel}$ (%)	R_{Series} (%)	$MTTF_{Parallel}$ (hours)	$MTTF_{Series}$ (hours)
0.0009	0.0001	93.8611577	80.3514940	1166.666666	500.0000000
0.0008	0.0002	92.1724497	82.0402020	1083.333334	583.3333333
0.0007	0.0003	90.4837417	83.7289099	1000.000000	666.6666667
0.0006	0.0004	88.7950338	85.4176179	916.6666664	750.0000000
0.0005	0.0005	87.1063258	87.1063258	833.3333333	833.3333333
0.0004	0.0006	85.4176179	88.7950338	750.0000000	916.6666664
0.0003	0.0007	83.7289099	90.4837417	666.6666667	1000.000000
0.0002	0.0008	82.0402020	92.1724497	583.3333333	1083.333334
0.0001	0.0009	80.3514940	93.8611577	500.0000000	1166.666666

Table 3-3: The comparison of the system performance measures for two-unit parallel system and two-unit series system without repair

λ_O (per hour)	λ_S (per hour)	$AV_{Parallel}$ (%)	AV_{Series} (%)	$AVSS_{Parallel}$ (%)	$AVSS_{Series}$ (%)
0.0009	0.0001	94.72393753	84.00709663	82.35294119	66.66666667
0.0008	0.0002	93.39976118	85.36217513	81.25000000	70.00000001
0.0007	0.0003	92.07118766	86.71281954	79.99999999	72.72727272
0.0006	0.0004	90.73821416	88.05903686	78.57142858	74.99999999
0.0005	0.0005	89.40083224	89.40083224	76.92307694	76.92307694
0.0004	0.0006	88.05903686	90.73821416	74.99999999	78.57142858
0.0003	0.0007	86.71281954	92.07118766	72.72727272	79.99999999
0.0002	0.0008	85.36217513	93.39976118	70.00000001	81.25000000
0.0001	0.0009	84.00709663	94.72393753	66.66666667	82.35294119

Table 3-4: The comparison of system performance measures for two-unit parallel system and two-unit series system under Type II repair policy

3.7 Summary

The results of the analysis in this Chapter basically demonstrates the same trend as observed for the parallel system: system reliability (or system time-dependent availability in case of systems subjected to either Type II or Type III repair policies) decreases with increasing number of units and increasing critical human error rate. Similarly, the mean time to failure (or steady state availability in case of systems subjected to either Type II or Type III repair policies) decreases with increasing number of units, increasing critical human error rate and increasing common-cause failure rate.

From Tables 3-1 and 3-2, it can be observed that for the same values of the model parameters and the total number of devices *n*, the system reliability and availability of the parallel system is higher than that for the k-out-of-n system. Further, it can be noted that for a fixed value of *n*, an increase in the value of *k*, the least number of units needed for system operation would result in lower values of the system performance measures.

When comparing the series and the parallel systems (i.e. Tables 3-3 and 3-4), it is observed that when the value of the short-mode failure rate is higher than the value of open-mode failure rate, given the same values of other model parameters, the reliability of the series system is higher than that of the parallel system. On the contrary, when the value of open-mode failure rate is higher than the value of short-mode failure rate, for the same values of other model parameters, the parallel system is more reliable than the series system.

Chapter 4

Three-State Device K-out-of-N Standby System

In this Chapter, the effect of cold standby on the reliability and availability analysis of three-state device systems is examined. A new kind of standby system called the k-out-of-n standby system is introduced. The system is analysed under the same situations and assumptions as before. Generalized and special-case expressions for the various performance indices are obtained and some plots for the special-case models are shown. A comparison between the performance indices for this system with those for the parallel, series and k-out-of-n systems is made.

4.1 Three-State Device K-out-of-N Standby System

Redundancy is a concept in which alternate paths are provided so that the system continues to operate normally even when one or more components fail. The parallel, series (in the case of systems composed of three-state devices only) and k-out-of-n systems are all types of redundant systems. In these systems, all the components that make up the system are active (or on-line). Hence, such kind of redundancy is termed as 'hot' redundancy. Instead of having all the units on-line, it is possible that one or more units be kept on-line and one or more units can be kept off-line as back-up unit/s. When the on-line unit/s fails, the back-up unit/s can be switched into operation either by an automatic or by a manual switching mechanism. Such a kind of redundant configuration is called a standby system or a 'cold' standby redundancy.

The description of the system

Figure 4-1 represents the reliability block diagram of a three-state device k-out-of-n standby system. The system consists of n units in parallel (of which at least k units ($k<n$) must operate) and one unit on standby. As long as any one of the n units fails in short-mode or all n-k+1 units fail in open-mode, the standby unit is switched into operation. The system fails

when both the k-out-of-n system and the standby unit fail. Further, the system can fail due to common-cause failure and critical human error from any of the operable states as long as there are at least two operable units in the system. All the units incorporated in this system are assumed to be identical and the standby unit is capable of keeping the system operating successfully for a given period of time. The switching mechanism for switching on the standby unit when the k-out-of-n sub-system fails is assumed to be perfect.

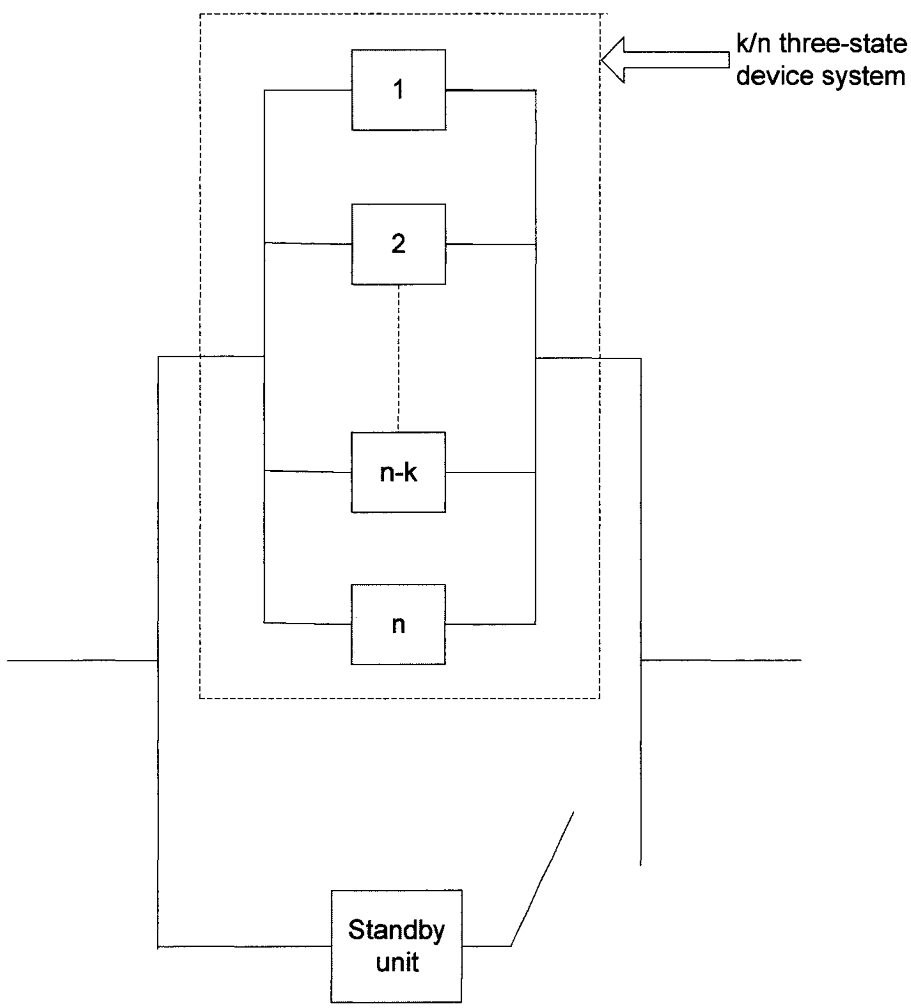

Figure 4-1: Block diagram of three-state device k-out-of-n system with one-unit on standby

In Figure 4-1, for k=n=1, the system becomes a pure cold-standby system. For k=1, it becomes a general parallel-standby system while for k=n, it becomes a general series-standby system. For all other values of k and n (k<n), it is a k-out-of-n standby system.

Figure 4-2 depicts the state transition diagram of the general system shown in Figure 4-1 under without repair policy. Similarly, Figure 4-3 shows the state transition diagram for the general model under Type I repair policy, Figure 4-4 shows the state transition diagram for the general model under Type II repair policy, and Figure 4-5 shows the state-transition diagram for general model under Type III repair policy. Numerals and single letters in boxes denote the system states. Other symbols used in the diagram are defined in the notation section.

Assumptions

The following assumptions are associated with the analyses in this section:

1. The system is composed of *n* units in parallel and one unit on standby.
2. Each unit, including the standby unit (three-state device) can fail either in open-mode or in short-mode. These two failures are mutually exclusive of one another.
3. A critical human error or a common-cause failure can occur and trigger the whole system failure when two or more devices are operating.
4. All failures are statistically independent.
5. The failure rates for all the failures are constant.
6. The repaired unit/system is as good as new.
7. The repair rates for all types of failures are constant.
8. Repair rates are the same for completely failed system due to open-mode or short-mode failure.
9. Repair rates are the same for partially failed k-out-of-n sub-system when the standby unit is in the standby (switched off/non-operating) mode.
10. Repair rates are the same for the partially failed k-out-of-n sub-system when the standby unit is in the operating mode.
11. Common-cause failure rates are the same for the partially or fully operating system.
12. Critical human error rates are the same for the partially or fully operating system.

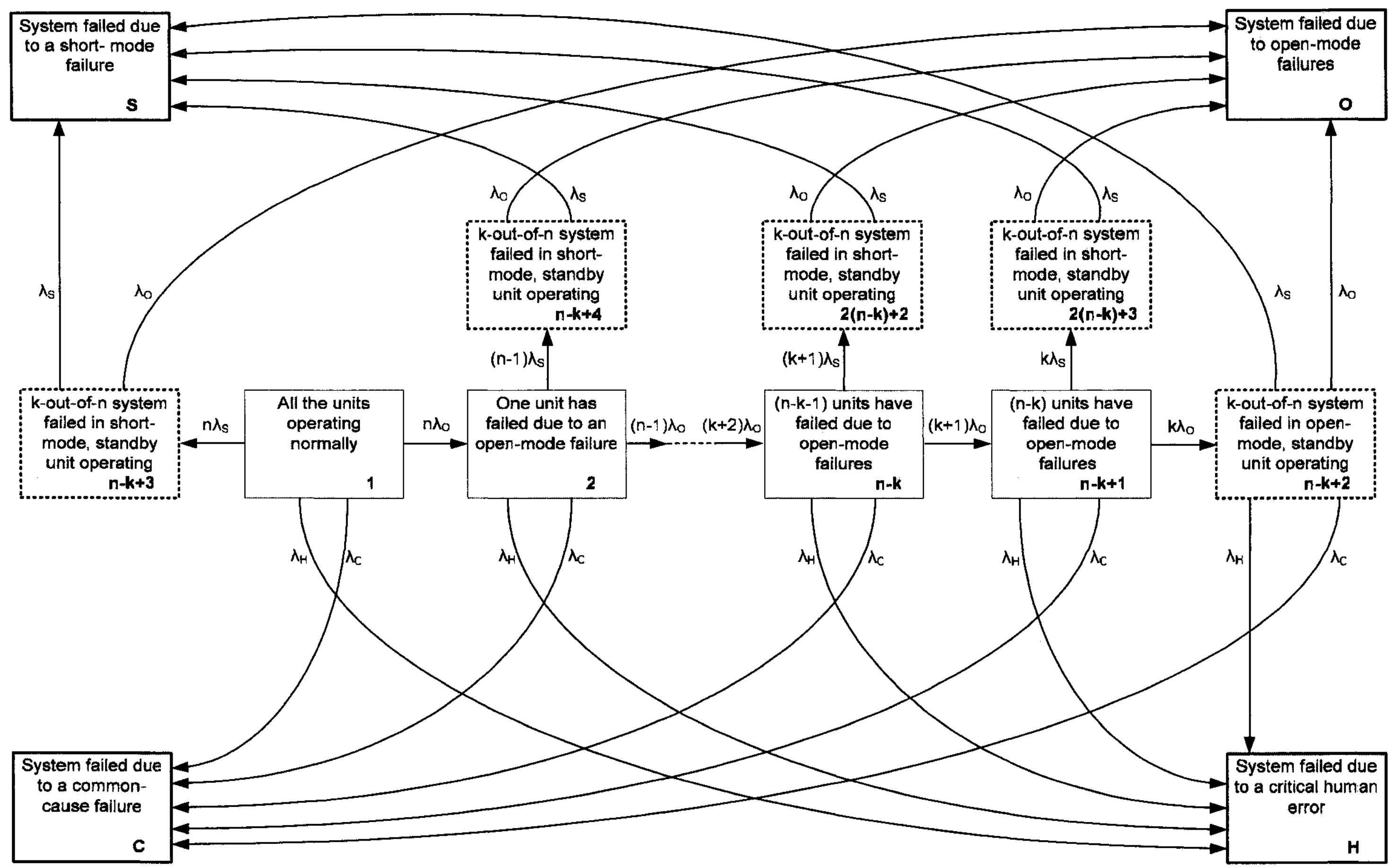

Figure 4-2: State transition diagram for three-state device k-out-of-n standby system under without repair policy

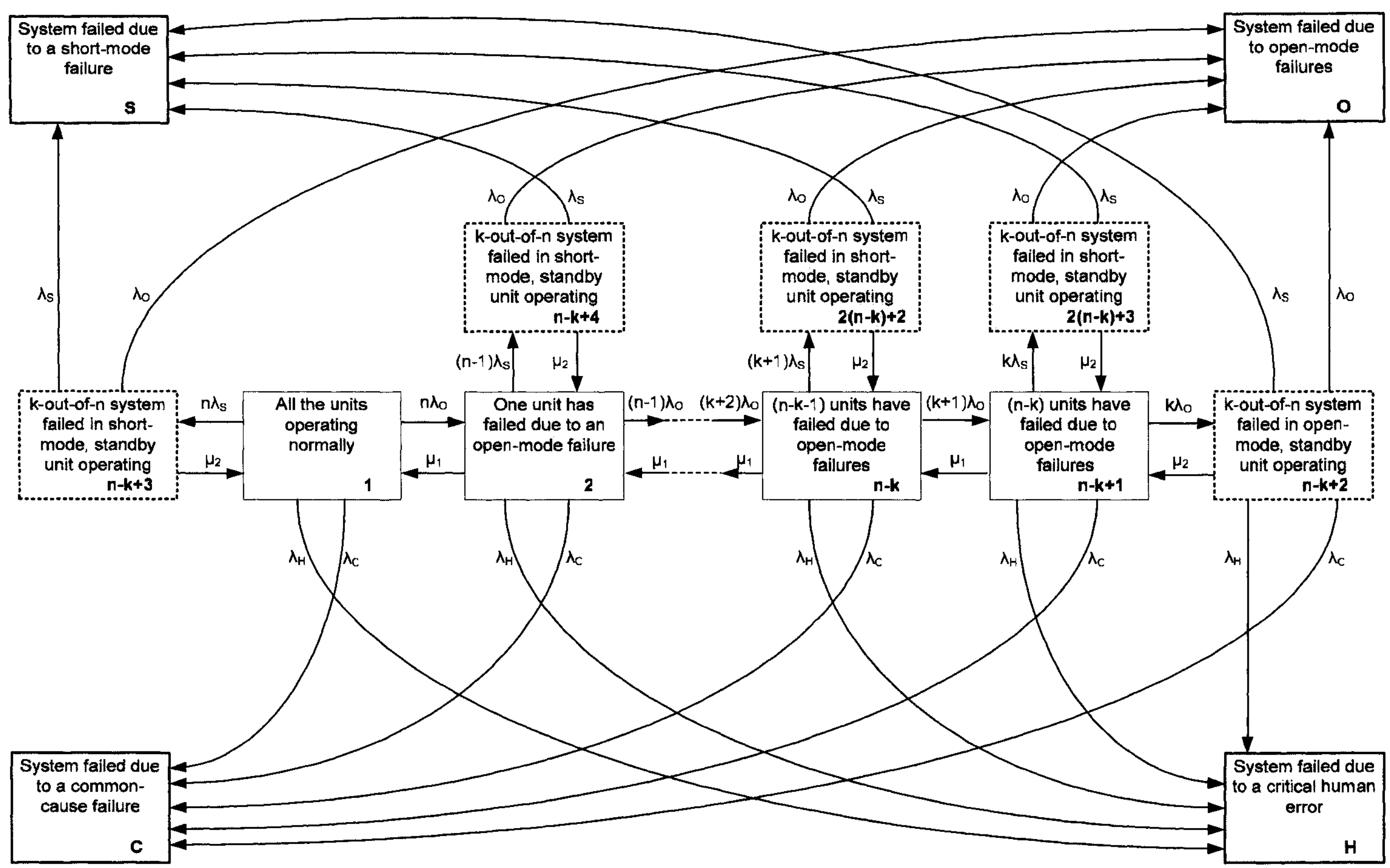

Figure 4-3: State transition diagram for three-state device k-out-of-n standby system under Type I repair policy

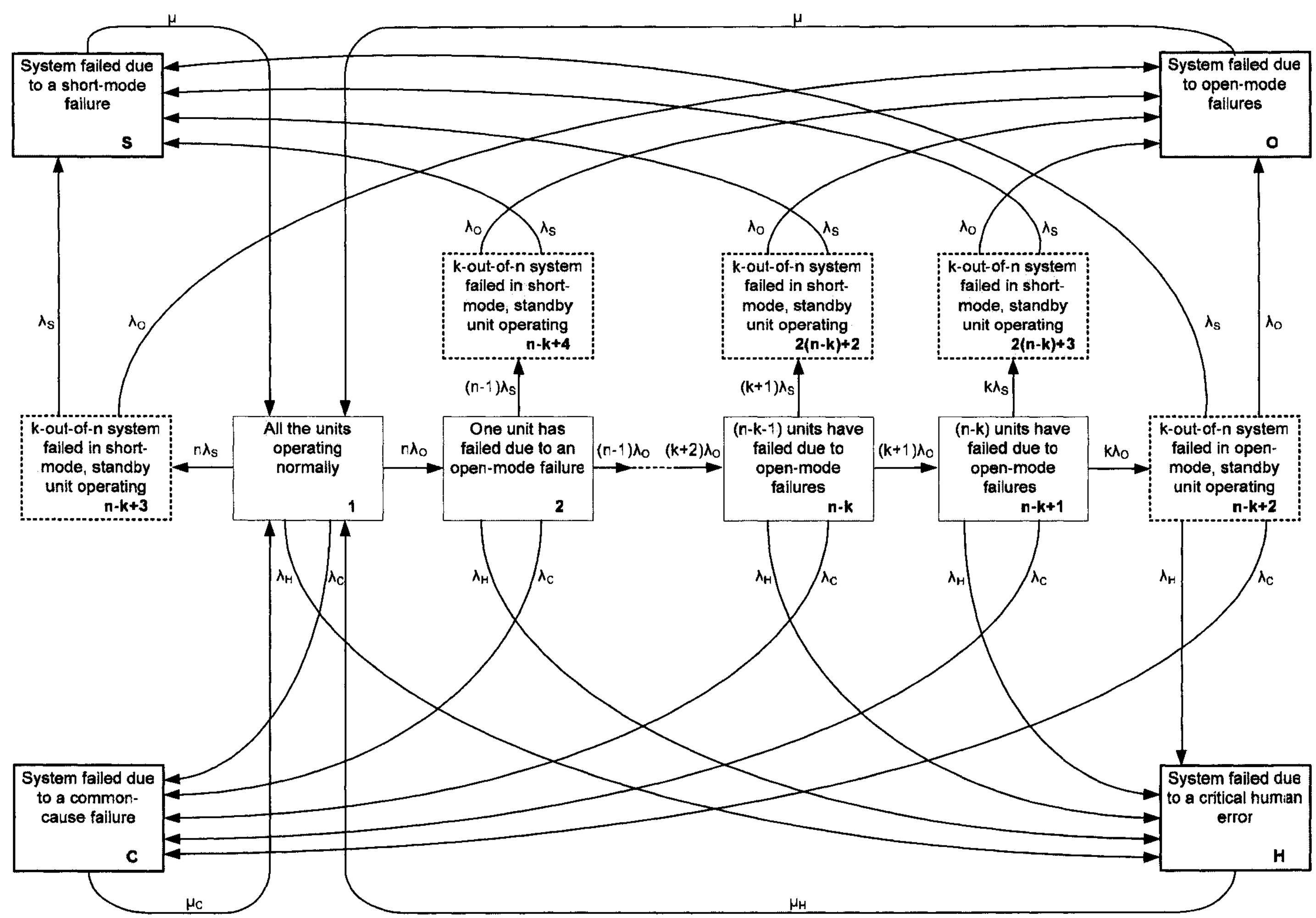

Figure 4-4: State transition diagram for three-state device k-out-of-n standby system under Type II repair policy

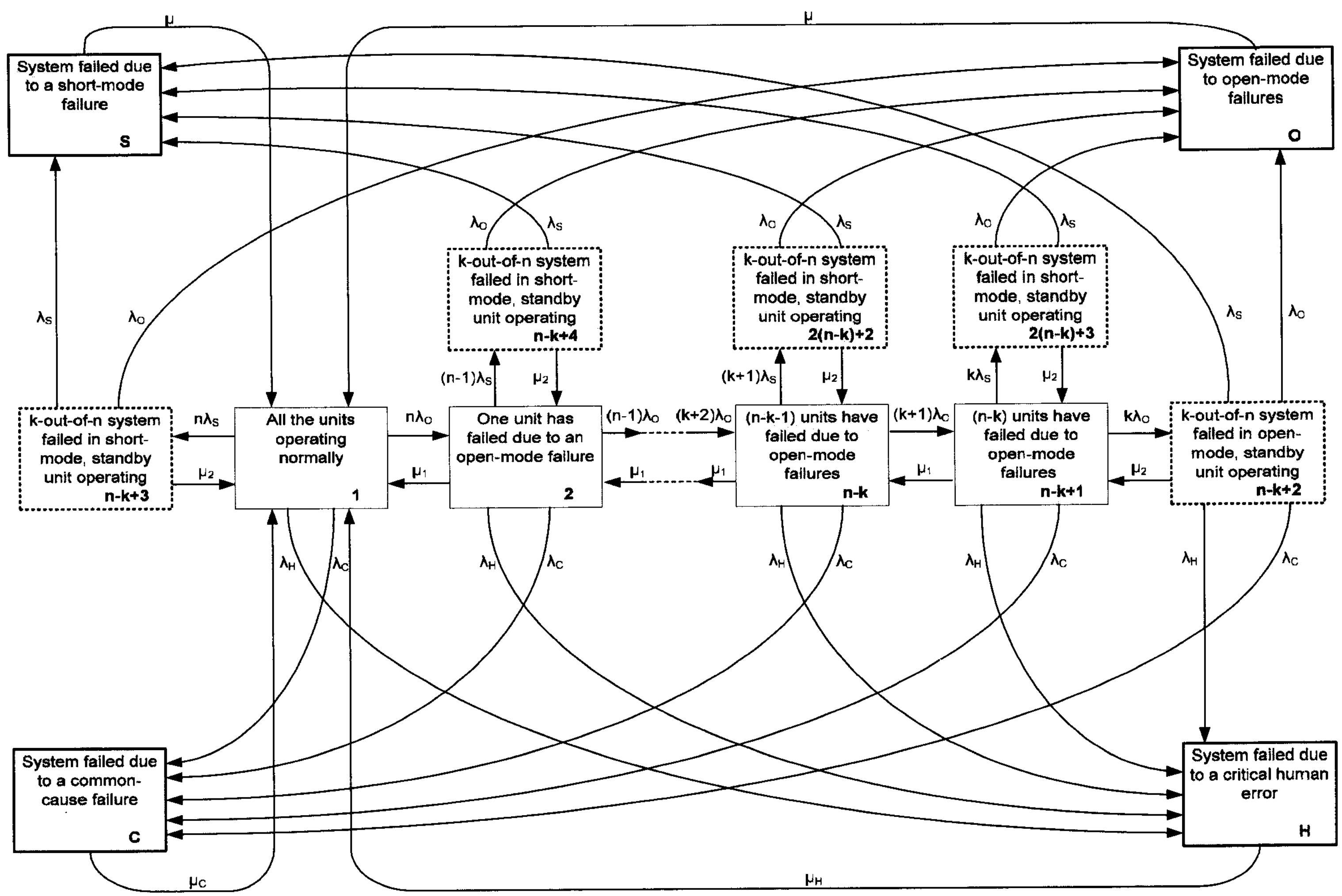

Figure 4-5: State transition diagram for three-state device k-out-of-n standby system under Type III repair policy

Notation

The following notation is associated with models presented in this section:

n	Number of units.
t	Time.
s	Laplace transform variable.
L^{-1}	Inverse Laplace transform.
λ_O	Constant open-mode failure rate of the operating unit (three-state device).
λ_S	Constant short-mode failure rate of the operating unit (three-state device).
λ_C	Constant common-cause failure rate of the system.
λ_H	Constant critical human error rate of the system.
μ	Constant repair rate of the system from open-mode or short-mode failure.
μ_1	Constant repair rate of the unit (three-state device) for a partially failed k-out-of-n sub-system when standby unit is in standby mode.
μ_2	Constant repair rate of the unit (three-state device) for a partially failed k-out-of-n sub-system when standby unit is operating.
μ_C	Constant repair rate of the system when it failed due to a common-cause failure.
μ_H	Constant repair rate of the system when it failed due to a critical human error.
$P_i(t)$	The probability that the system is in state i at time t; for i=1, 2, 3…..2(n-k)+3.
$P_j(t)$	The failed system state probability; for $j=C$ (due to a common-cause failure), $j=H$ (due to a critical human error), $j=S$ (due to a short-mode failure), $j=O$ (due to an open-mode failure).
$p_i(s)$	The Laplace transform of the state probability, for i=1, 2, 3…..2(n-k)+3.
$p_j(s)$	The Laplace transform of the failed system state probability; for $j=C$ (due to a common-cause failure), $j=H$ (due to a critical human error), $j=S$ (due to a short-mode failure), $j=O$ (due to an open-mode failure).
P_i	The steady state probability of the system being in state i, for i=1, 2…2(n-k)+3.

P_j The failed system steady state probability; for $j=C$ (due to a common-cause failure), $j=H$ (due to a critical human error), $j=S$ (due to a short-mode failure), $j=O$ (due to an open-mode failure).

$R(s)$ Laplace transform of the system reliability.

$R(t)$ System reliability at time t.

$MTTF$ System mean time to failure.

$A(s)$ Laplace transform of the system availability.

$A(t)$ System availability at time t.

AV_{ss} System steady state availability.

4.2 Three-State Device K-out-of-N Standby System under without Repair Policy

This section presents the analysis of three-state device k-out-of-n standby system under without repair policy. Generalized and special case expressions for system reliability and mean time to failure are developed. Some plots for the special case model are shown.

4.2.1 General k-out-of-n standby system under without repair policy

The state transition diagram for the general model without repair can be represented as in Figure 4-2. With the aid of the Markov method, the system of differential equations corresponding to Figure 4-2 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) \tag{4-1}$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H\right]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) \tag{4-2}$$

for $i = 2$ to $(n-k+1)$

$$\frac{dP_{n-k+2}(t)}{dt} = -\left[\lambda_O + \lambda_S + \lambda_C + \lambda_H\right]P_{n-k+2}(t) + k\lambda_O P_{n-k+1}(t) \tag{4-3}$$

$$\frac{dP_j(t)}{dt} = -[\lambda_O + \lambda_S]P_j(t) + (2n - j - k + 3)\lambda_S P_{j-n+k-2}(t) \tag{4-4}$$

for $j = (n-k+3)$ to $(2n-2k+3)$

$$\frac{dP_O(t)}{dt} = \lambda_O \sum_{i=n-k+2}^{2(n-k)+3} P_i(t) \tag{4-5}$$

$$\frac{dP_S(t)}{dt} = \lambda_S \sum_{i=n-k+2}^{2(n-k)+3} P_i(t) \tag{4-6}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-k+2} P_i(t) \tag{4-7}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-k+2} P_i(t) \tag{4-8}$$

Initial State Conditions: At time t=0, $P_1(0) = 1$, and all other initial state transition probabilities are equal to zero.

By taking Laplace transforms of Equations (4-1) – (4-8) and solving the resulting equations, the following expressions for the Laplace transform of the state probabilities result:

$$p_1(s) = \frac{1}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{4-9}$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_O}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H}\right] p_{i-1}(s) \quad \text{for } i = 2 \text{ to } (n-k+1) \tag{4-10}$$

$$p_{n-k+2}(s) = \left[\frac{k\lambda_O}{s + \lambda_O + \lambda_S + \lambda_C + \lambda_H}\right] p_{n-k+1}(s) \tag{4-11}$$

$$p_j(s) = \left[\frac{(2n-j-k+3)\lambda_S}{s + \lambda_O + \lambda_S}\right] p_{j-n+k-2}(s) \quad \text{for } j = (n-k+3) \text{ to } (2n-2k+3) \tag{4-12}$$

$$p_O(s) = \lambda_O \sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-13}$$

$$p_S(s) = \lambda_S \sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-14}$$

$$p_C(s) = \lambda_C \sum_{i=1}^{n-k+2} p_i(s) \tag{4-15}$$

$$p_H(s) = \lambda_H \sum_{i=1}^{n-k+2} p_i(s) \tag{4-16}$$

By taking the inverse Laplace transforms of Equations (4-9) – (4-16), the time-dependent transition state probabilities can be obtained.

The time-dependent system reliability is given by

$$R(t) = L^{-1}[R(s)] = L^{-1}\left[\sum_{i=1}^{2(n-k)+3} p_i(s)\right] \tag{4-17}$$

The system mean time to failure is given by

$$MTTF = \lim_{s \to 0}\left[\sum_{i=1}^{2(n-k)+3} p_i(s)\right] \tag{4-18}$$

4.2.2 Special case model 4-A: Two-out-of-three standby system under without repair

The state transition diagram for two-out-of-three standby system without repair is shown in Figure 4-6. For k=2 and n=3 in Figure 4-2, from Equations (4-9) – (4-16), we get the following set of equations:

$$p_1(s) = \frac{1}{A} \tag{4-19}$$

$$p_2(s) = \left[\frac{3\lambda_O}{B}\right]\left[\frac{1}{A}\right] \tag{4-20}$$

$$p_3(s) = \left[\frac{2\lambda_O}{C}\right]\left[\frac{3\lambda_O}{B}\right]\left[\frac{1}{A}\right] \tag{4-21}$$

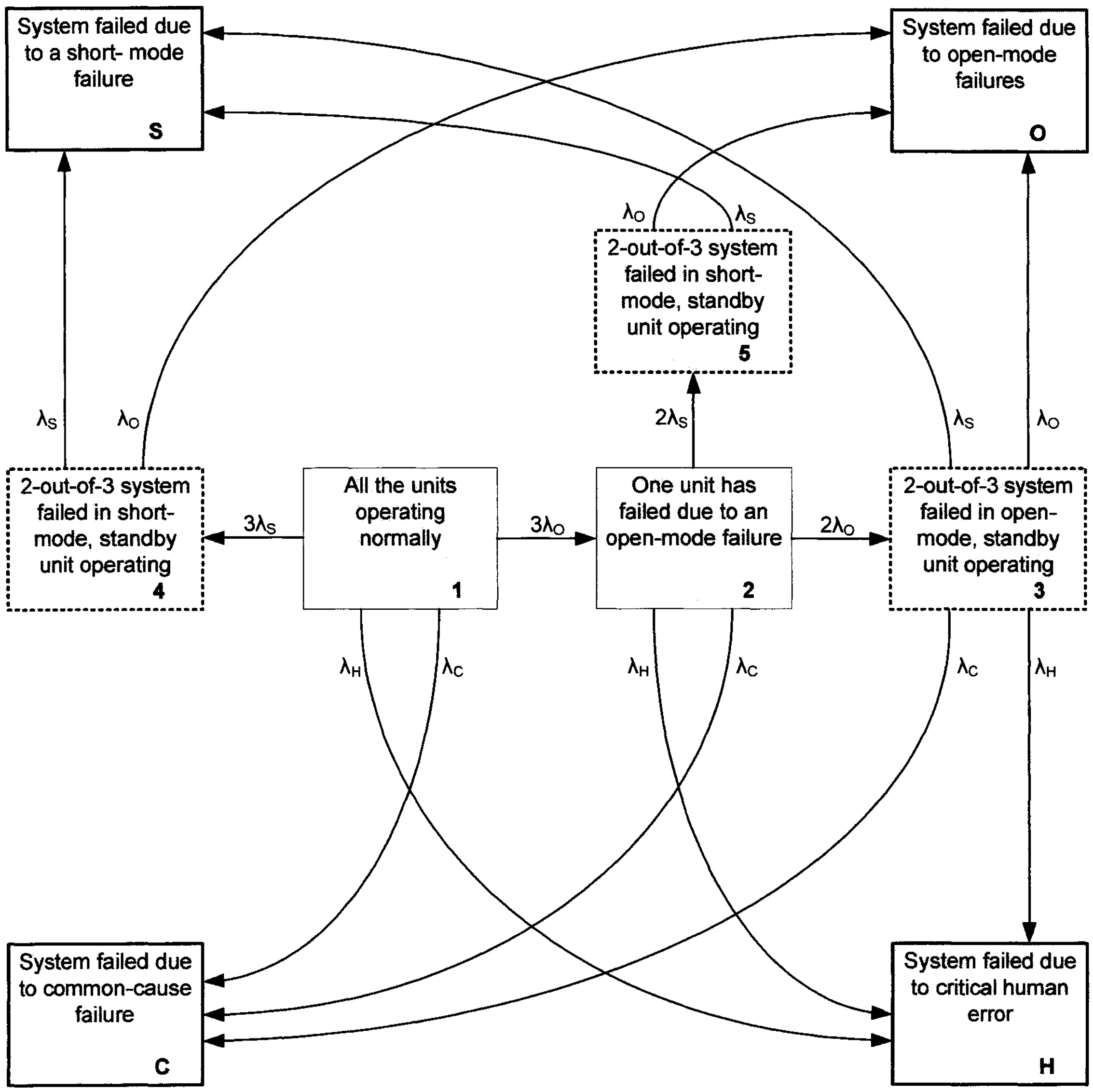

Figure 4-6: State transition diagram of two-out-of-three standby system without repair

$$p_4(s) = \left[\frac{3\lambda_S}{D}\right]\left[\frac{1}{A}\right] \tag{4-22}$$

$$p_5(s) = \left[\frac{2\lambda_S}{D}\right]\left[\frac{3\lambda_O}{B}\right]\left[\frac{1}{A}\right] \tag{4-23}$$

$$p_O(s) = \frac{3\lambda_O\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]}{sABCD} \quad (4\text{-}24)$$

$$p_S(s) = \frac{3\lambda_S\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]}{sABCD} \quad (4\text{-}25)$$

$$p_C(s) = \frac{\lambda_C\left[BC + 3C\lambda_O + 6\lambda_O^{\,2}\right]}{sABC} \quad (4\text{-}26)$$

$$p_H(s) = \frac{\lambda_H\left[BC + 3C\lambda_O + 6\lambda_O^{\,2}\right]}{sABC} \quad (4\text{-}27)$$

where

$A = s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$
$B = s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H$
$C = s + \lambda_O + \lambda_S + \lambda_C + \lambda_H$
$D = s + \lambda_O + \lambda_S$

By taking inverse Laplace transforms of Equations (4-19) to (4-27), the time-dependent state probabilities can be obtained. By adding Equations (4-19) – (4-23) and then taking inverse Laplace transform, we obtain the following expression for the system reliability:

$$R(t) = \sum_{i=1}^{5} P_i(t) = L^{-1}\left[\sum_{i=1}^{5} p_i(s)\right] \quad (4\text{-}28)$$

The system mean time to failure is given by

$$MTTF = \lim_{s\to 0}\left[\sum_{i=1}^{5} p_i(s)\right] \quad (4\text{-}29)$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 4-7. For the specified values of the model parameters, the plots of Equation (4-28) are shown in Figure 4-8. Similarly, for the specified values of the model parameters, the plots of Equation (4-29) are shown in Figure 4-9.

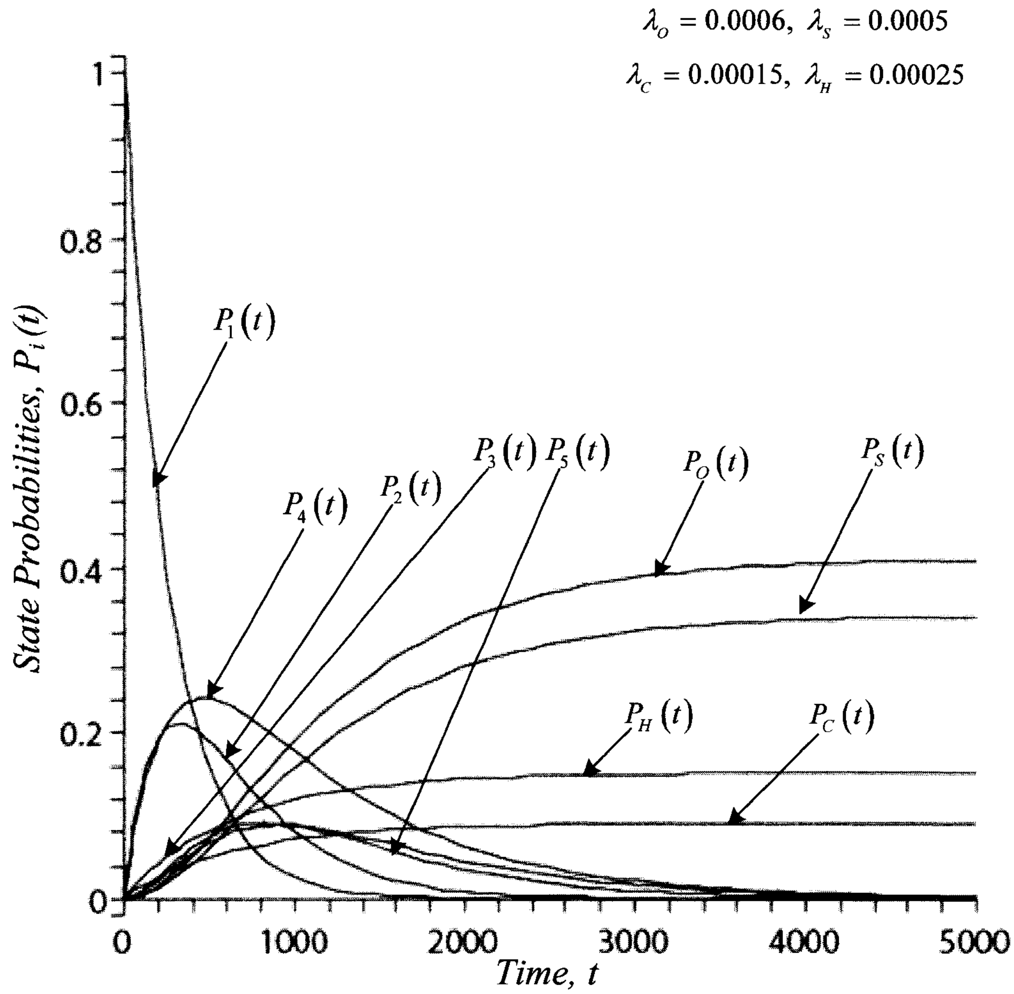

Figure 4-7: State probability plots of two-out-of-three standby system without repair

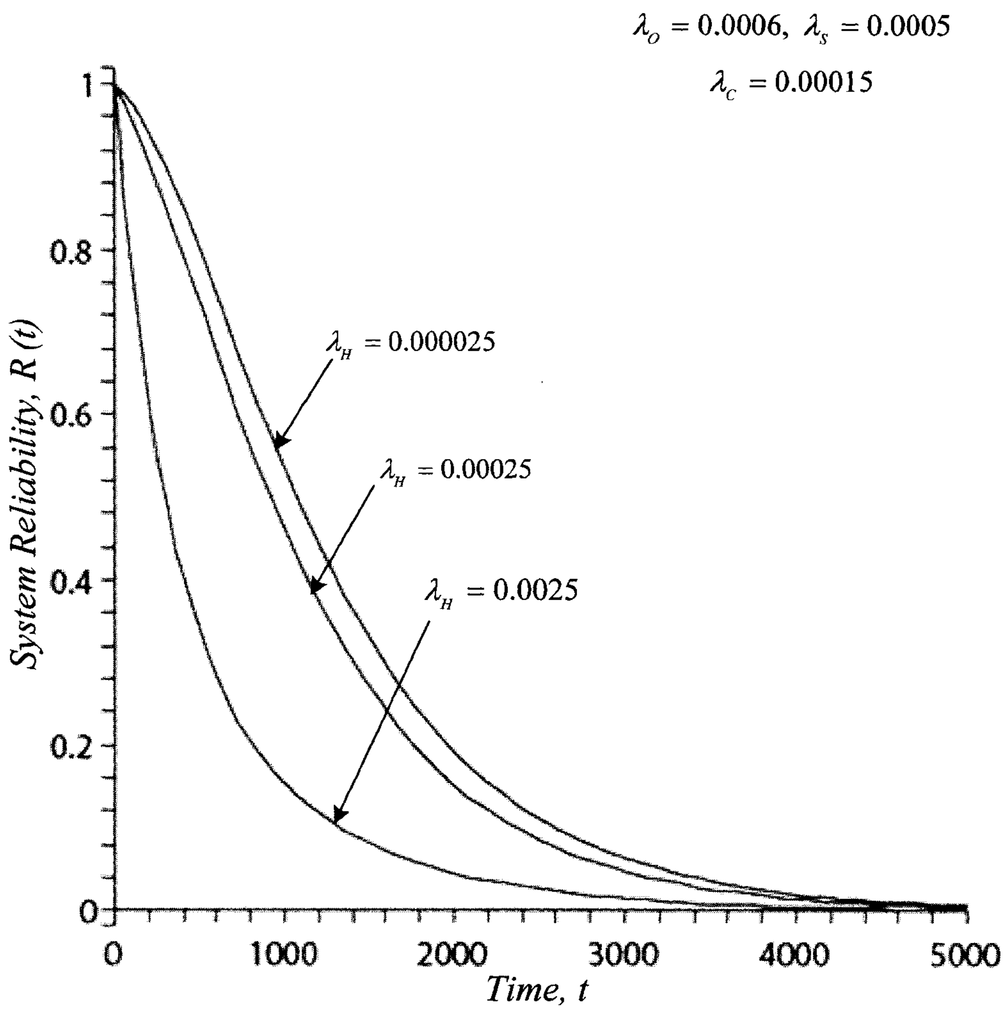

Figure 4-8: Reliability plots of two-out-of-three standby system without repair

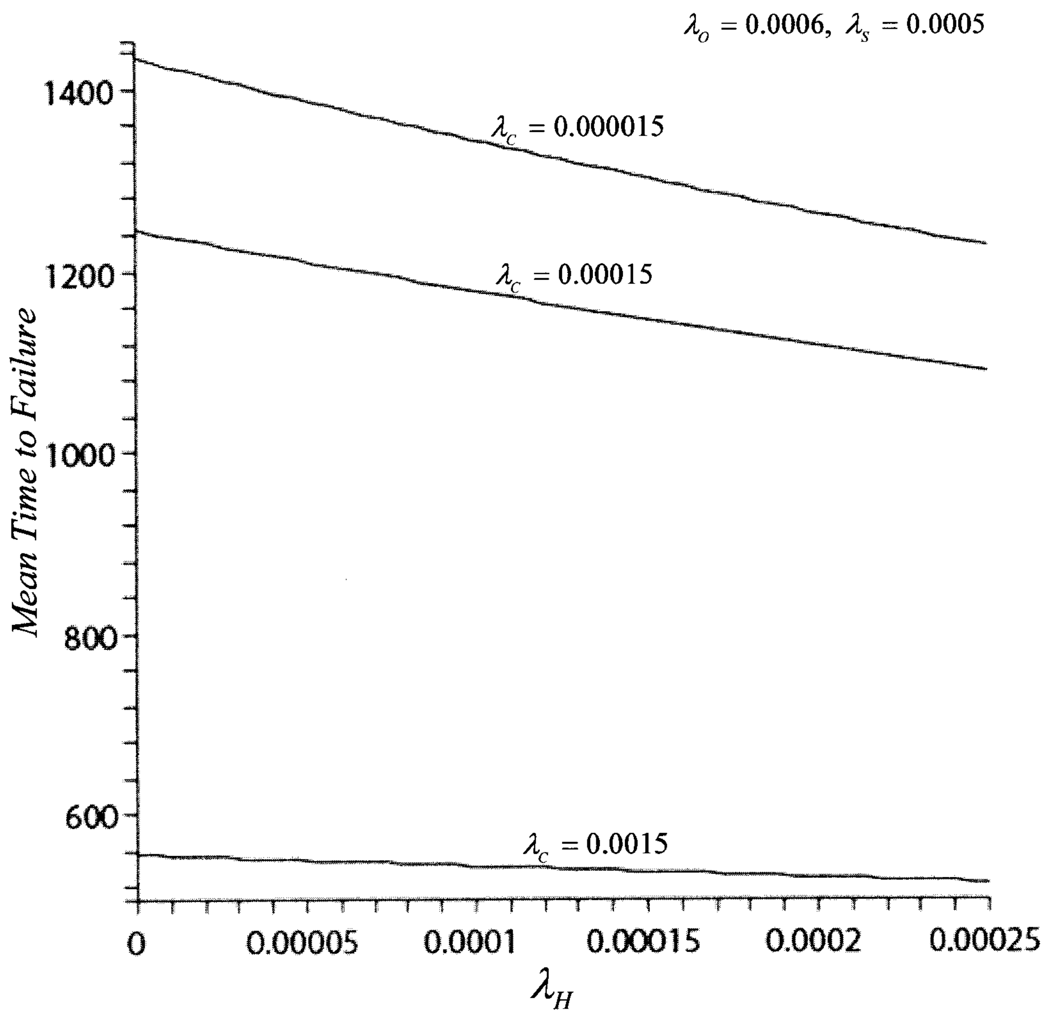

Figure 4-9: Mean time to failure plots of two-out-of-three standby system without repair

4.3 Three-State Device K-out-of-N Standby System under Type I Repair Policy

In this section, the analysis of the general model under Type I repair policy is presented. Generalized and special case expressions for system reliability and mean time to failure are obtained. Some plots for the special case model are shown.

4.3.1 General k-out-of-n standby system with Type I repair

The block diagram and the state transition diagram for the general model with Type I repair policy is shown in Figures 4-1 and 4-3 respectively. With the aid of Markov state-space technique, the system of differential equations corresponding to Figure 4-3 is as follows:

$$\frac{dP_1(t)}{dt} = -\left[n\lambda_O + n\lambda_S + \lambda_C + \lambda_H\right]P_1(t) + \mu_1 P_2(t) + \mu_2 P_{n-k+3}(t) \tag{4-30}$$

$$\frac{dP_i(t)}{dt} = -\left[(n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1\right]P_i(t) + (n-i+2)\lambda_O P_{i-1}(t) + \mu_1 P_{i+1}(t) + \mu_2 P_{i+n-k+2}(t)$$

$$\text{for } i = 2 \text{ to } (n-k) \tag{4-31}$$

$$\frac{dP_{n-k+1}(t)}{dt} = -\left[k\lambda_O + k\lambda_S + \lambda_C + \lambda_H + \mu_1\right]P_{n-k+1}(t) + (k+1)\lambda_O P_{n-k}(t) + \mu_2\left[P_{n-k+2}(t) + P_{2(n-k)+3}(t)\right] \tag{4-32}$$

$$\frac{dP_{n-k+2}(t)}{dt} = -\left[\lambda_O + \lambda_S + \lambda_C + \lambda_H + \mu_2\right]P_{n-k+2}(t) + k\lambda_O P_{n-k+1}(t) \tag{4-33}$$

$$\frac{dP_j(t)}{dt} = -\left[\lambda_O + \lambda_S + \mu_2\right]P_j(t) + (2n-j-k+3)\lambda_S P_{j-n+k-2}(t) \tag{4-34}$$

$$\text{for } j = (n-k+3) \text{ to } (2n-2k+3)$$

$$\frac{dP_O(t)}{dt} = \lambda_O \sum_{i=n-k+2}^{2(n-k)+3} P_i(t) \tag{4-35}$$

$$\frac{dP_S(t)}{dt} = \lambda_S \sum_{i=n-k+2}^{2(n-k)+3} P_i(t) \tag{4-36}$$

$$\frac{dP_C(t)}{dt} = \lambda_C \sum_{i=1}^{n-k+2} P_i(t) \tag{4-37}$$

$$\frac{dP_H(t)}{dt} = \lambda_H \sum_{i=1}^{n-k+2} P_i(t) \tag{4-38}$$

At time t=0, $P_1(0) = 1$, and all other initial state transition probabilities are equal to zero. By taking Laplace transforms of Equations (4-30) – (4-38) and solving the resulting equations by applying the initial conditions, we get the following set of equations:

$$p_1(s) = \frac{1 + \mu_1 p_2(s) + \mu_2 p_{n-k+3}(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{4-39}$$

$$p_i(s) = \frac{(n-i+2)\lambda_O p_{i-1}(s) + \mu_1 p_{i+1}(s) + \mu_2 p_{i+n-k+2}(s)}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1} \quad \text{for } i = 2 \text{ to } (n\text{-}k) \tag{4-40}$$

$$p_{n-k+1}(s) = \frac{(k+1)\lambda_O p_{n-k}(s) + \mu_2 \left[p_{n-k+2}(s) + p_{2(n-k)+3}(s) \right]}{s + k\lambda_O + k\lambda_S + \lambda_C + \lambda_H + \mu_1} \tag{4-41}$$

$$p_{n-k+2}(s) = \frac{k\lambda_O p_{n-k+1}(s)}{s + \lambda_O + \lambda_S + \lambda_C + \lambda_H + \mu_2} \tag{4-42}$$

$$p_j(s) = \frac{(2n-j-k+3)\lambda_S p_{j-n+k-2}(s)}{s + \lambda_O + \lambda_S + \mu_2} \quad \text{for } j = (n-k+3) \text{ to } (2n\text{-}2k+3) \tag{4-43}$$

$$p_O(s) = \lambda_O \sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-44}$$

$$p_S(s) = \lambda_S \sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-45}$$

$$p_C(s) = \lambda_C \sum_{i=1}^{n-k+2} p_i(s) \tag{4-46}$$

$$p_H(s) = \lambda_H \sum_{i=1}^{n-k+2} p_i(s) \tag{4-47}$$

By taking the inverse Laplace transforms of Equations (4-39) – (4-47), the time-dependent state probabilities can be obtained. The time-dependent system reliability is obtained as follows:

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\sum_{i=1}^{2(n-k)+3} p_i(s)\right] \quad (4\text{-}48)$$

The system mean time to failure is calculated as:

$$MTTF = \lim_{s\to 0}\left[\sum_{i=1}^{2(n-k)+3} p_i(s)\right] \quad (4\text{-}49)$$

4.3.2 Special case model 4-B: Two-out-of-three standby system under Type I repair

The state transition diagram of two-out-of-three standby system with Type I repair is represented by Figure 4-10. By substituting for k=2 and n=3 in Equations (4-39) to (4-47), the following set of equations results:

$$p_1(s) = \frac{D\left[BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right)\right]}{A(4,1)} \quad (4\text{-}50)$$

$$p_2(s) = \frac{3CD^2\lambda_O}{A(4,1)} \quad (4\text{-}51)$$

$$p_3(s) = \frac{6D^2\lambda_O{}^2}{A(4,1)} \quad (4\text{-}52)$$

$$p_4(s) = \frac{3\lambda_S\left[BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right)\right]}{A(4,1)} \quad (4\text{-}53)$$

$$p_5(s) = \frac{6CD\lambda_O\lambda_S}{A(4,1)} \quad (4\text{-}54)$$

$$p_O(s) = \frac{3\lambda_O\{2D\lambda_O[D\lambda_O + C\lambda_S] + \lambda_S[BCD - 2\mu_2(D\lambda_O + C\lambda_S)]\}}{sA(4,1)} \tag{4-55}$$

$$p_S(s) = \frac{3\lambda_S\{2D\lambda_O[D\lambda_O + C\lambda_S] + \lambda_S[BCD - 2\mu_2(D\lambda_O + C\lambda_S)]\}}{sA(4,1)} \tag{4-56}$$

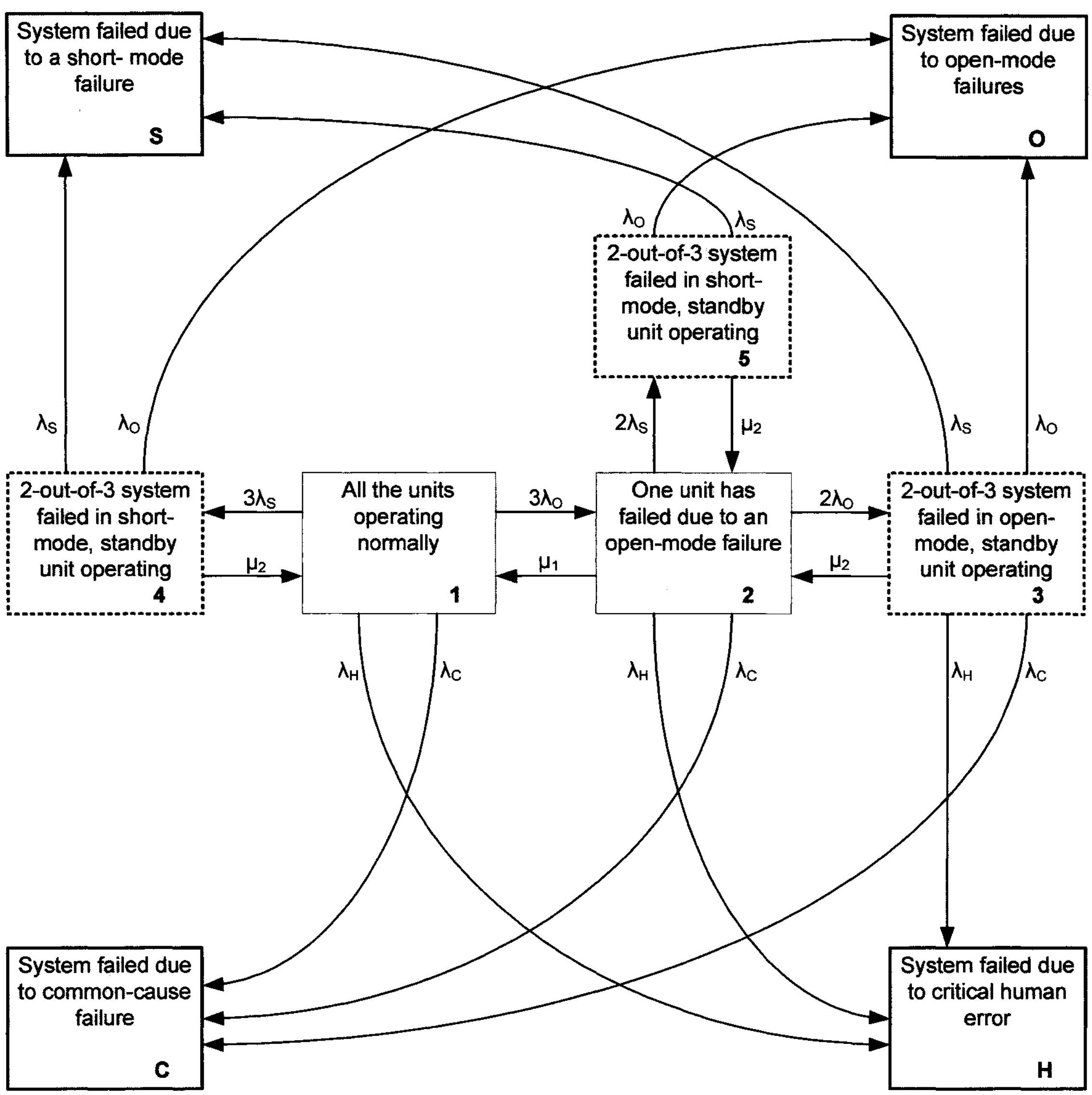

Figure 4-10: State transition diagram of two-out-of-three standby system with Type I repair

$$p_C(s) = \frac{D\lambda_C\left\{3CD\lambda_O + 6D\lambda_O{}^2 + BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right)\right\}}{sA(4,1)} \tag{4-57}$$

$$p_H(s) = \frac{D\lambda_H\left\{3CD\lambda_O + 6D\lambda_O{}^2 + BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right)\right\}}{sA(4,1)} \tag{4-58}$$

where

$$A(4,1) = \left[AD - 3\mu_2\lambda_S\right]\left[BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right) - 3\mu_1 CD^2\lambda_O\right]$$
$$A = s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$$
$$B = s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1$$
$$C = s + \lambda_O + \lambda_S + \lambda_C + \lambda_H + \mu_2$$
$$D = s + \lambda_O + \lambda_S + \mu_2$$

By taking the inverse Laplace transforms of Equations (4-50) – (4-58), the time-dependent state probabilities can be obtained. Adding Equations (4-50) and (4-54) and then taking the inverse Laplace transform, we obtain the following expression for time-dependent system reliability:

$$R(t) = L^{-1}\left\{\frac{3D\lambda_O\left[BC + 2\lambda_O + 2\lambda_S\right] + \left[D + 3\lambda_S\right]\left[BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right)\right]}{A(4,1)}\right\} \tag{4-59}$$

The system mean time to failure is given by:

$$MTTF = \lim_{s\to 0}\left\{\frac{3D\lambda_O\left[BC + 2\lambda_O + 2\lambda_S\right] + \left[D + 3\lambda_S\right]\left[BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right)\right]}{A(4,1)}\right\} \tag{4-60}$$

For the specified values of the model parameters, the plots of state probabilities as a function of time, *t*, are shown in Figure 4-11, the plots of Equation (4-59) [i.e., system reliability as a function of time, *t*] are shown in Figure 4-12 and the plots of Equation (4-60) [i.e., mean time to failure] are shown in Figure 4-13.

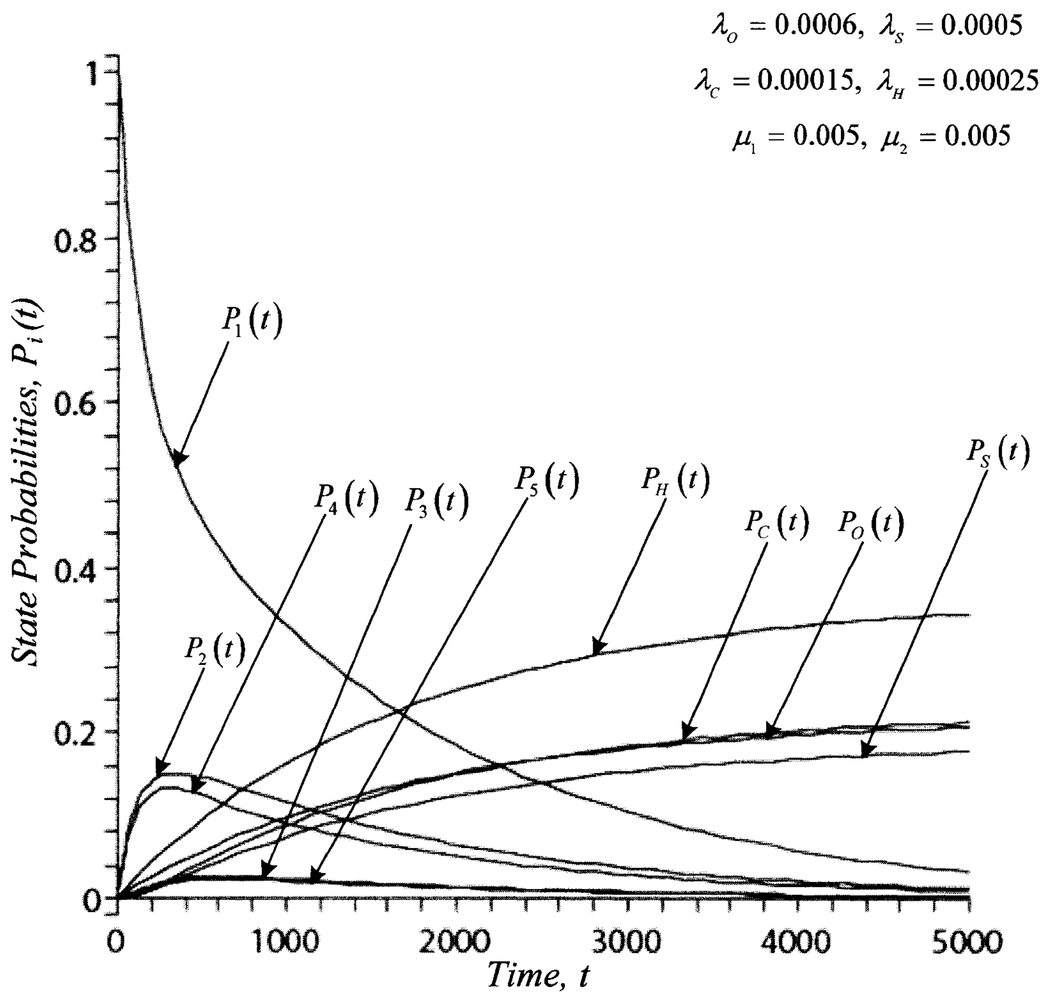

Figure 4-11: State probability plots of two-out-of-three standby system with Type I repair

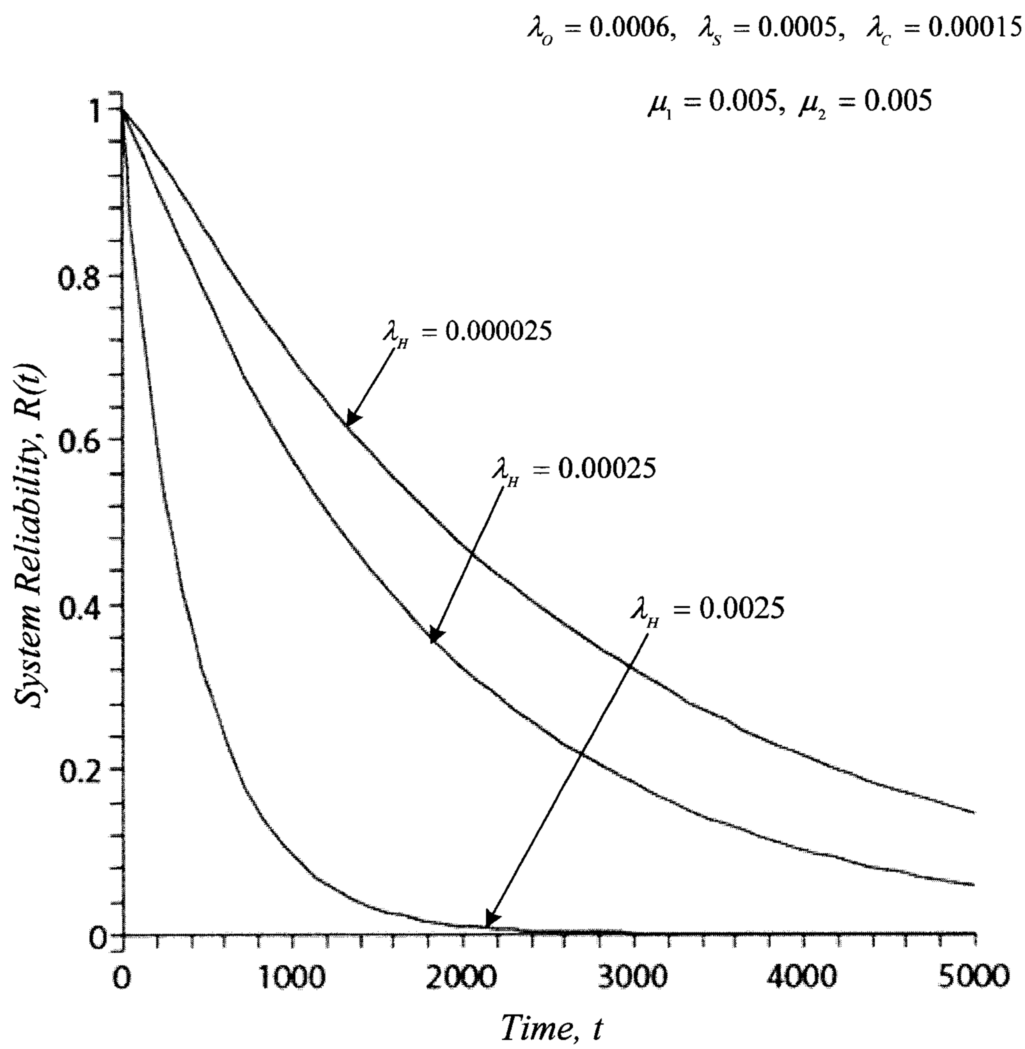

Figure 4-12: Reliability plots of two-out-of-three standby system with Type I repair

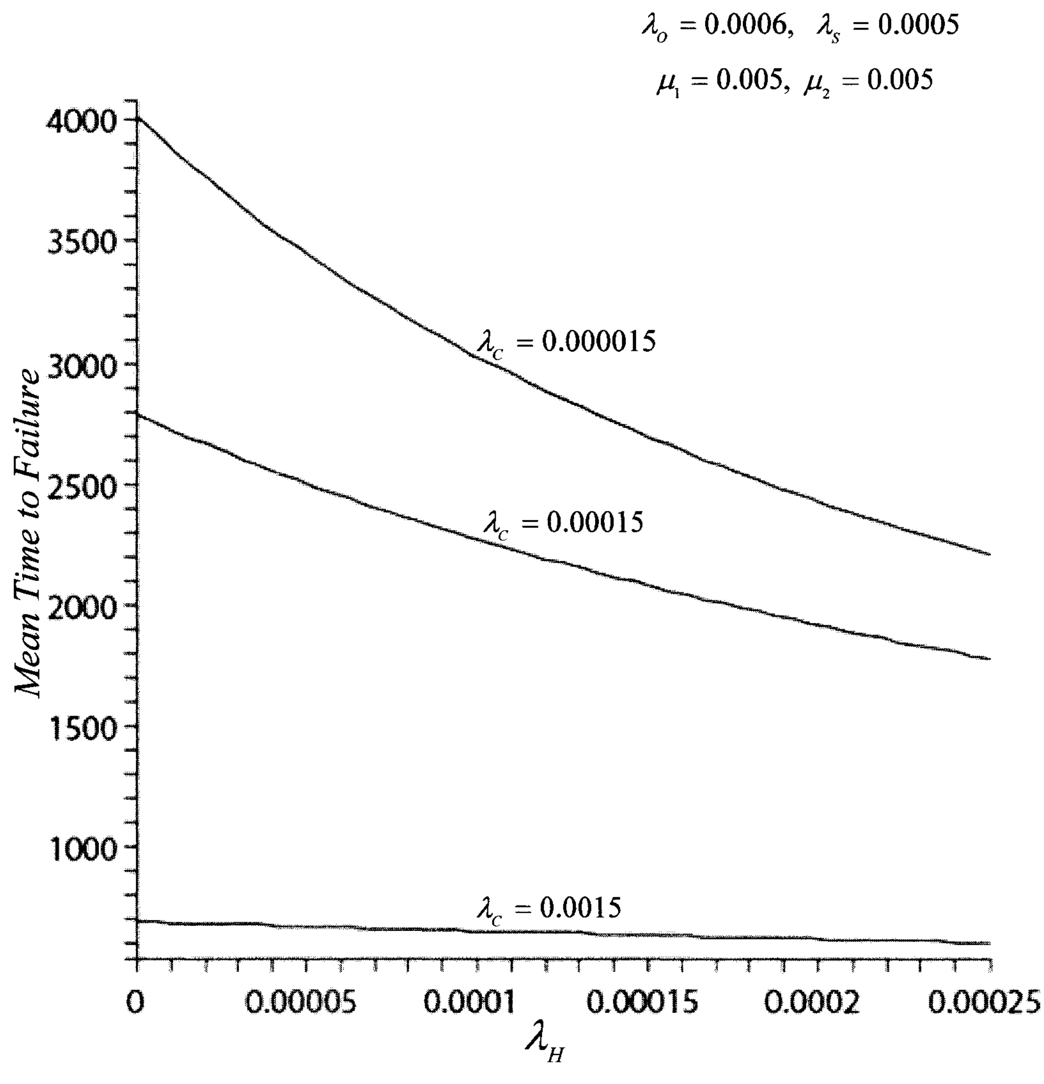

Figure 4-13: Mean time to failure plots of two-out-of-three standby system with Type I repair

4.4 Three-State Device K-out-of-N Standby System under Type II Repair Policy

In this section, the availability analysis of three-state device k-out-of-n standby system and its special case: two-out-of-three standby system under Type II repair policy is presented. The expressions for the time-dependent and steady state availabilities are obtained and some plots for the special case model are shown.

4.4.1 General k-out-of-n standby system under Type II repair policy

The state transition diagram for the general k-out-of-n standby system under Type II repair policy is shown in Figure 4-4. Using the Markov state-space technique, the system of differential equations associated with Figure 4-4 is obtained as follows:

$$\frac{dP_1(t)}{dt}=-\left[n\lambda_O+n\lambda_S+\lambda_C+\lambda_H\right]P_1(t)+\mu\left[P_O(t)+P_S(t)\right]+\mu_C P_C(t)+\mu_H P_H(t) \quad (4\text{-}61)$$

$$\frac{dP_i(t)}{dt}=-\left[(n-i+1)\lambda_O+(n-i+1)\lambda_S+\lambda_C+\lambda_H\right]P_i(t)+(n-i+2)\lambda_O P_{i-1}(t) \quad (4\text{-}62)$$

for $i=2$ to $(n-k+1)$

$$\frac{dP_{n-k+2}(t)}{dt}=-\left[\lambda_O+\lambda_S+\lambda_C+\lambda_H\right]P_{n-k+2}(t)+k\lambda_O P_{n-k+1}(t) \quad (4\text{-}63)$$

$$\frac{dP_j(t)}{dt}=-\left[\lambda_O+\lambda_S\right]P_j(t)+(2n-j-k+3)\lambda_S P_{j-n+k-2}(t) \quad (4\text{-}64)$$

for $j=(n-k+3)$ to $(2n-2k+3)$

$$\frac{dP_O(t)}{dt}=-\mu P_O(t)+\lambda_O\sum_{i=n-k+2}^{2(n-k)+3}P_i(t) \quad (4\text{-}65)$$

$$\frac{dP_S(t)}{dt}=-\mu P_S(t)+\lambda_S\sum_{i=n-k+2}^{2(n-k)+3}P_i(t) \quad (4\text{-}66)$$

$$\frac{dP_C(t)}{dt}=-\mu_C P_C(t)+\lambda_C\sum_{i=1}^{n-k+2}P_i(t) \quad (4\text{-}67)$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-k+2} P_i(t) \tag{4-68}$$

At time t=0, $P_1(0)$= 1, and all other initial state probabilities are equal to zero. Taking Laplace transforms of Equations (4-61) – (4-68) and solving the resulting equations, we obtain the following set of equations:

$$p_1(s) = \frac{1+\mu\left[p_O(s)+p_S(s)\right]+\mu_C p_C(s)+\mu_H p_H(s)}{s+n\lambda_O+n\lambda_S+\lambda_C+\lambda_H} \tag{4-69}$$

$$p_i(s) = \left[\frac{(n-i+2)\lambda_O}{s+(n-i+1)\lambda_O+(n-i+1)\lambda_S+\lambda_C+\lambda_H}\right]p_{i-1}(s) \quad \text{for } i=2 \text{ to } (n-k+1) \tag{4-70}$$

$$p_{n-k+2}(s) = \left[\frac{k\lambda_O}{s+\lambda_O+\lambda_S+\lambda_C+\lambda_H}\right]p_{n-k+1}(s) \tag{4-71}$$

$$p_j(s) = \left[\frac{(2n-j-k+3)\lambda_S}{s+\lambda_O+\lambda_S}\right]p_{j-n+k-2}(s) \qquad \text{for } j=(n-k+3) \text{ to } (2n\text{-}2k+3) \tag{4-72}$$

$$p_O(s) = \frac{\lambda_O}{s+\mu}\sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-73}$$

$$p_S(s) = \frac{\lambda_S}{s+\mu}\sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-74}$$

$$p_C(s) = \frac{\lambda_C}{s+\mu_C}\sum_{i=1}^{n-k+2} p_i(s) \tag{4-75}$$

$$p_H(s) = \frac{\lambda_H}{s+\mu_H}\sum_{i=1}^{n-k+2} p_i(s) \tag{4-76}$$

By taking the inverse Laplace transforms of equations (4-69) – (4-76), the time-dependent state probabilities can be obtained. The time-dependent system availability is given by:

$$A(t) = L^{-1}\left[A(s)\right] = L^{-1}\left[\sum_{i=1}^{2(n-k)+3} p_i(s)\right] \tag{4-77}$$

4.4.2 Special case model 4-C: Two-out-of-three standby system under Type II repair

Figure 4-14 shows the state transition diagram of two-out-of-three standby system under Type II repair policy.

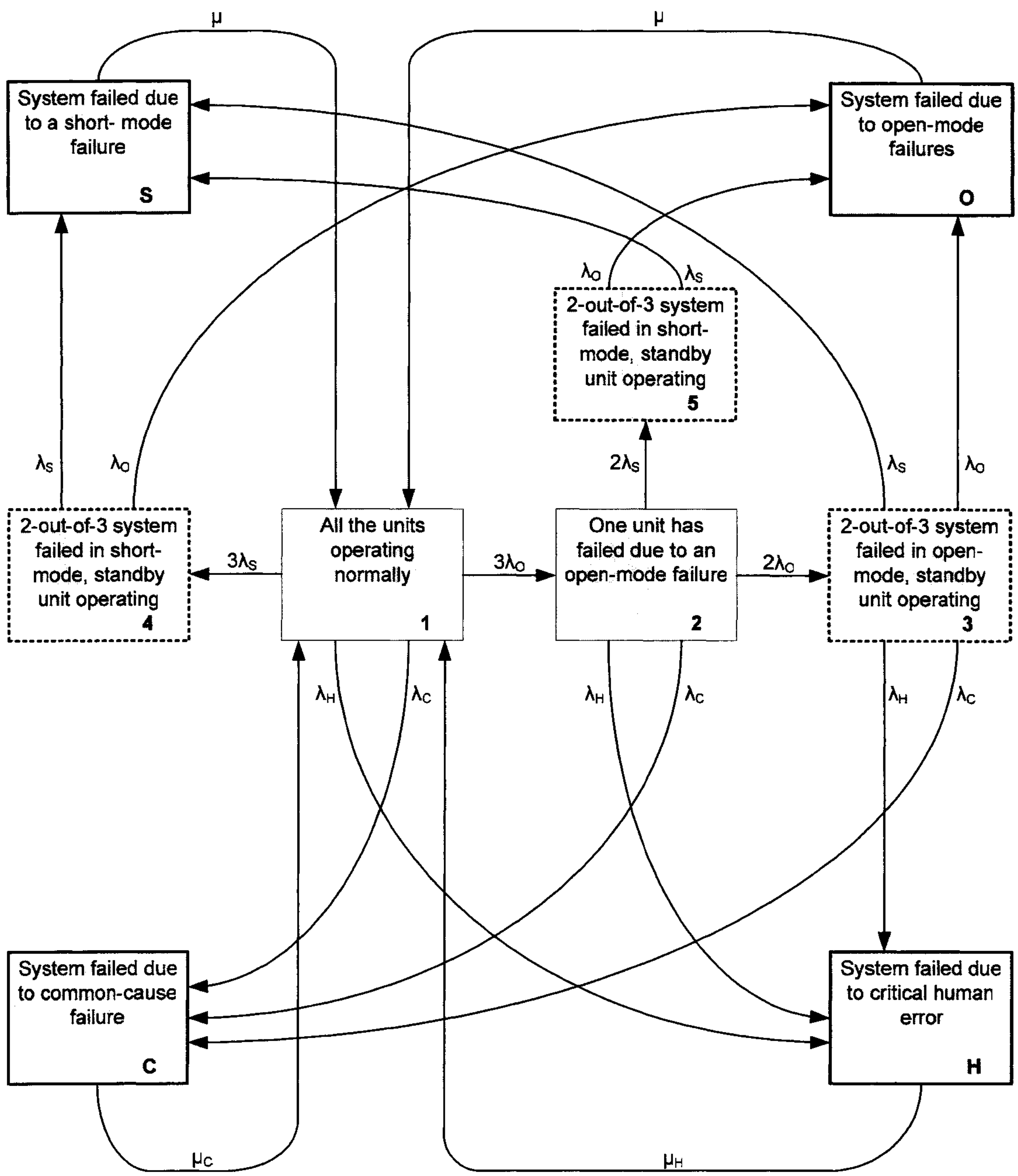

Figure 4-14: State transition diagram of two-out-of-three standby system with Type II repair

Setting k=2 and n=3 in Equations (4-69) – (4-76), the following set of equations results:

$$p_1(s) = \frac{BCDEFG}{A(4,2)} \tag{4-78}$$

$$p_2(s) = \frac{3\lambda_O CDEFG}{A(4,2)} \tag{4-79}$$

$$p_3(s) = \frac{6\lambda_O^2 DEFG}{A(4,2)} \tag{4-80}$$

$$p_4(s) = \frac{3\lambda_S BCEFG}{A(4,2)} \tag{4-81}$$

$$p_5(s) = \frac{6\lambda_O \lambda_S CEFG}{A(4,2)} \tag{4-82}$$

$$p_O(s) = \frac{3\lambda_O\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]FG}{A(4,2)} \tag{4-83}$$

$$p_S(s) = \frac{3\lambda_S\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]FG}{A(4,2)} \tag{4-84}$$

$$p_C(s) = \frac{\lambda_C\left[BC + 3C\lambda_O + 6\lambda_O^2\right]DEG}{A(4,2)} \tag{4-85}$$

$$p_H(s) = \frac{\lambda_H\left[BC + 3C\lambda_O + 6\lambda_O^2\right]DEF}{A(4,2)} \tag{4-86}$$

where:

$$A(4,2) = ABCDEFG - 3\mu FG\left(\lambda_O + \lambda_S\right)\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]$$
$$- DE\left[BC + 3C\lambda_O + 6\lambda_O^2\right]\left[\mu_C\lambda_C G + \mu_H\lambda_H F\right]$$

$$A = s + 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$$
$$B = s + 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H$$
$$C = s + \lambda_O + \lambda_S + \lambda_C + \lambda_H$$
$$D = s + \lambda_O + \lambda_S$$
$$E = s + \mu$$
$$F = s + \mu_C$$
$$G = s + \mu_H$$

The time-dependent state probabilities can be obtained by taking the inverse Laplace transforms of Equations (4-78) – (4-86). By adding Equations (4-78) – (4-82) and then taking the inverse Laplace transform, we obtain the following expression for system availability:

$$A(t) = L^{-1}\left\{\frac{EFG\left[BCD + 3CD\lambda_O + 6D\lambda_O^{\ 2} + 3BC\lambda_S + 6C\lambda_O\lambda_S\right]}{A(4,2)}\right\} \tag{4-87}$$

Steady State Conditions: We can compute the steady state conditions directly from equations (4-61) to (4-68), (i.e., for k=2 and n=3), by putting all derivatives equal to zero and solving the resultant equations utilizing the total probability axiom: $\sum P_i = 1$. Thus, we get the following set of equations:

$$P_1 = \frac{\mu\mu_C\mu_H BCD}{A(4,3)} \tag{4-88}$$

$$P_2 = \frac{3\mu\mu_C\mu_H CD\lambda_O}{A(4,3)} \tag{4-89}$$

$$P_3 = \frac{6\mu\mu_C\mu_H D\lambda_O^{\ 2}}{A(4,3)} \tag{4-90}$$

$$P_4 = \frac{3\mu\mu_C\mu_H BC\lambda_S}{A(4,3)} \tag{4-91}$$

$$P_5 = \frac{6\mu\mu_C\mu_H C\lambda_O\lambda_S}{A(4,3)} \tag{4-92}$$

$$P_O = \frac{3\mu_C\mu_H\lambda_O\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]}{A(4,3)} \tag{4-93}$$

$$P_S = \frac{3\mu_C\mu_H\lambda_S\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]}{A(4,3)} \tag{4-94}$$

$$P_C = \frac{\mu\mu_H D\lambda_C\left[BC + 3C\lambda_O + 6\lambda_O^{\ 2}\right]}{A(4,3)} \tag{4-95}$$

$$P_H = \frac{\mu\mu_C D\lambda_H\left[BC + 3C\lambda_O + 6\lambda_O^2\right]}{A(4,3)} \tag{4-96}$$

System steady state availability can now be calculated as:

$$AV_{SS} = \frac{\mu\mu_C\mu_H\left[BCD + 3CD\lambda_O + 6D\lambda_O^2 + 3BC\lambda_S + 6C\lambda_O\lambda_S\right]}{A(4,3)} \tag{4-97}$$

where:

$$\begin{aligned}A(4,3) = {} & \mu\mu_C\mu_H\left[D\left(BC + 3C\lambda_O + 6\lambda_O^2\right) + 3C\lambda_S\left(B + 2\lambda_O\right)\right]\\ & + 3\mu_C\mu_H\left(\lambda_O + \lambda_S\right)\left[2\lambda_O\left(D\lambda_O + C\lambda_S\right) + BC\lambda_S\right]\\ & + \mu D\left[\mu_H\lambda_C + \mu_C\lambda_H\right]\left[BC + 3C\lambda_O + 6\lambda_O^2\right]\end{aligned}$$

$$A = 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H$$

$$B = 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H$$

$$C = \lambda_O + \lambda_S + \lambda_C + \lambda_H$$

$$D = \lambda_O + \lambda_S$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 4-15. For the specified values of the model parameters, the plots of Equation (4-87) are shown in Figure 4-16. Similarly, for the specified values of the model parameters, the plots of Equation (4-97) are shown in Figure 4-17.

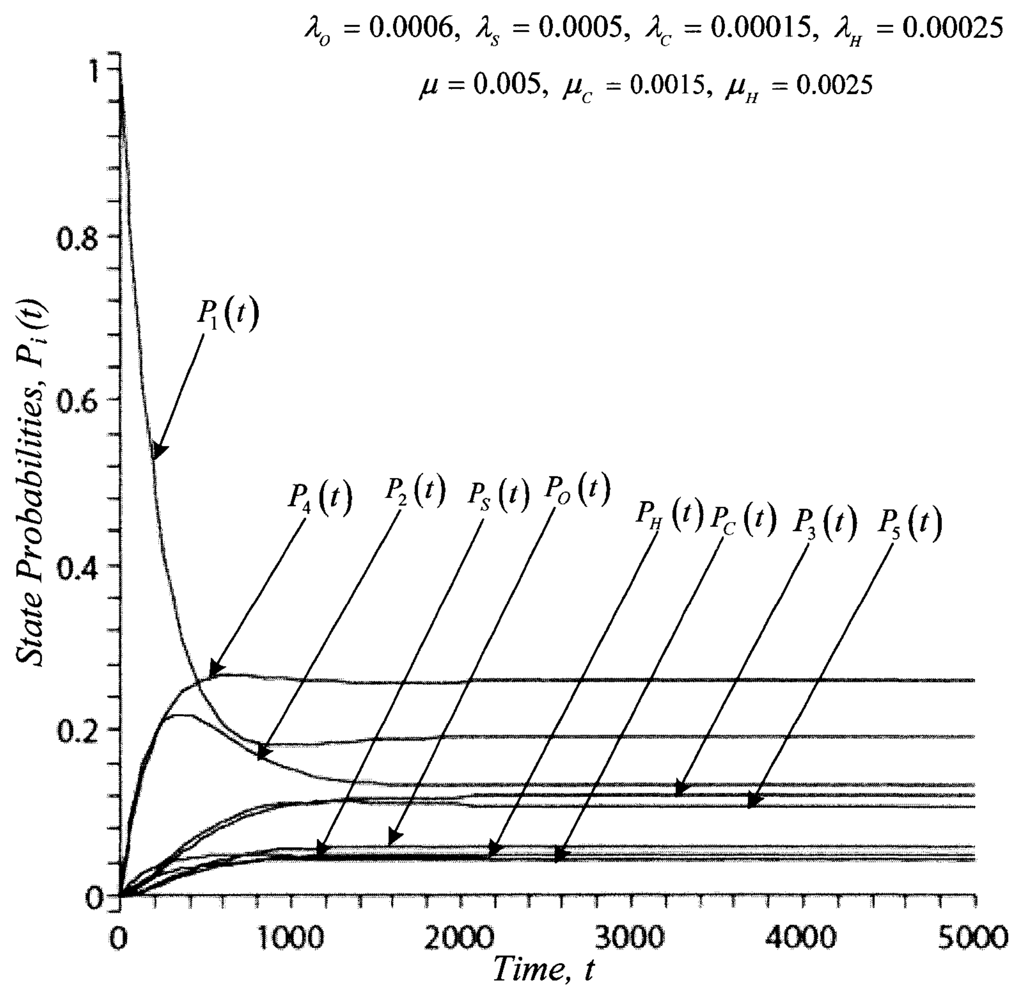

Figure 4-15: State probability plots of two-out-of-three standby system with Type II repair

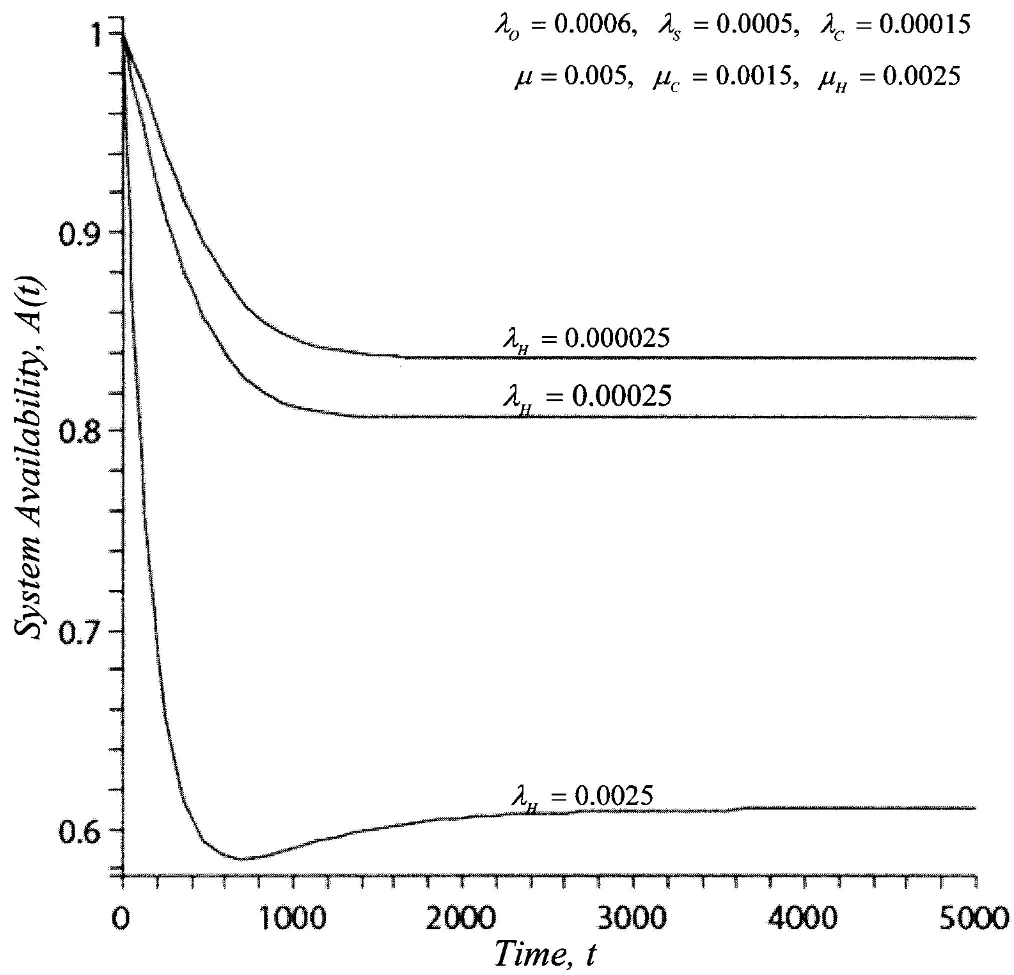

Figure 4-16: Availability plots of two-out-of-three standby system with Type II repair

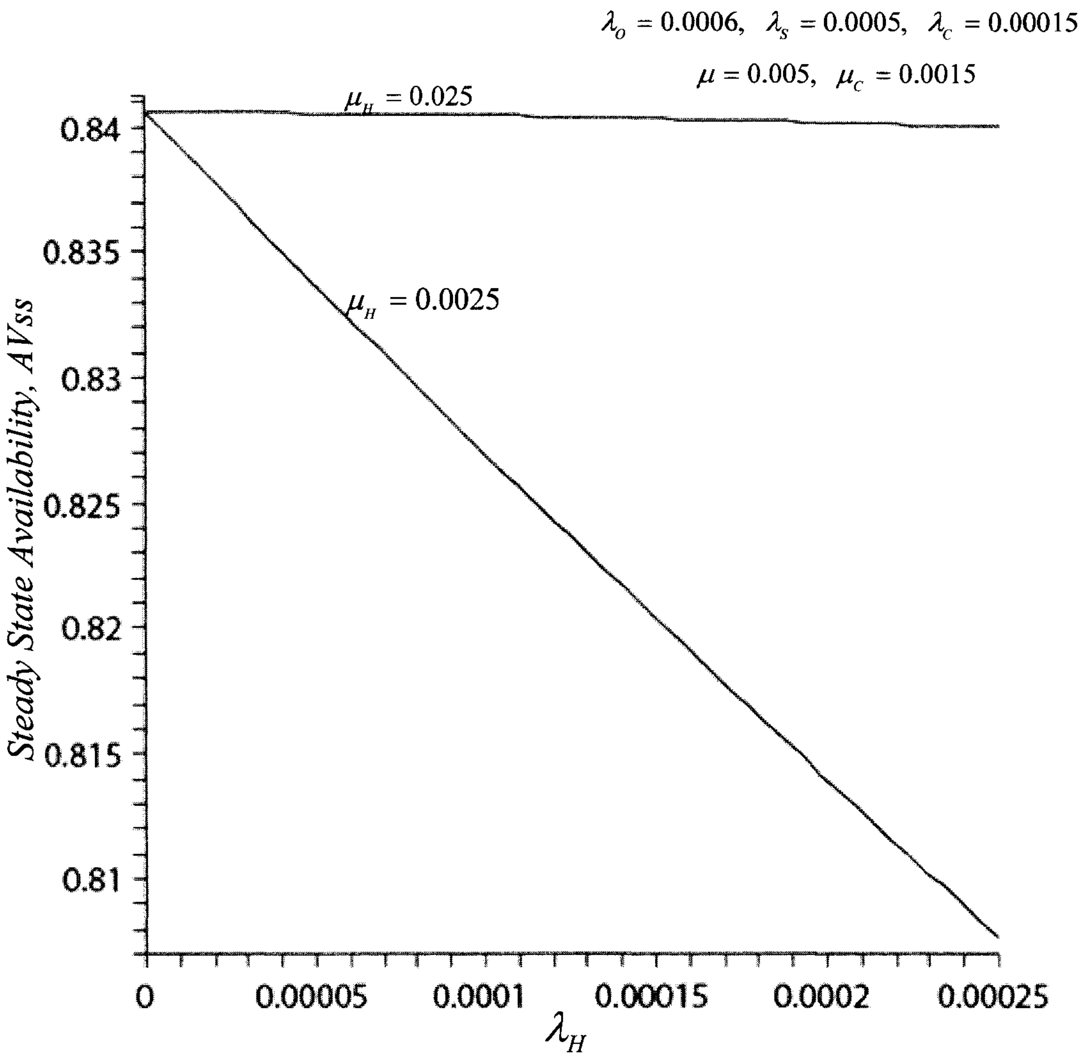

Figure 4-17: Steady state availability plots of two-out-of-three standby system with Type II repair

4.5 Three-State Device K-out-of-N Standby System under Type III Repair Policy

This section presents the availability analysis of three-state device k-out-of-n standby system and its special case: two-out-of-three standby system under Type III repair policy. The expressions for time-dependent and steady state system availabilities are obtained. For specified values of model parameters, some plots for the special case model are shown.

4.5.1 General k-out-of-n standby system under Type III repair policy

The block diagram and the state transition diagram for the general model under Type III repair policy is shown in Figures 4-1 and 4-5 respectively. Using the Markov method, the system of differential equations associated with Figure 4-5 is as follows:

$$\frac{dP_1(t)}{dt}=-[n\lambda_O+n\lambda_S+\lambda_C+\lambda_H]P_1(t)+\mu_1P_2(t)+\mu_2P_{n-k+3}(t)+\mu\left[P_O(t)+P_S(t)\right]+\mu_CP_C(t)+\mu_HP_H(t) \quad (4\text{-}98)$$

$$\frac{dP_i(t)}{dt}=-\left[(n-i+1)\lambda_O+(n-i+1)\lambda_S+\lambda_C+\lambda_H+\mu_1\right]P_i(t)+(n-i+2)\lambda_OP_{i-1}(t)+\mu_1P_{i+1}(t)+\mu_2P_{i+n-k+2}(t)$$

$$\text{for } i=2 \text{ to } (n-k) \quad (4\text{-}99)$$

$$\frac{dP_{n-k+1}(t)}{dt}=-[k\lambda_O+k\lambda_S+\lambda_C+\lambda_H+\mu_1]P_{n-k+1}(t)+(k+1)\lambda_OP_{n-k}(t)+\mu_2\left[P_{n-k+2}(t)+P_{2(n-k)+3}(t)\right] \quad (4\text{-}100)$$

$$\frac{dP_{n-k+2}(t)}{dt}=-[\lambda_O+\lambda_S+\lambda_C+\lambda_H+\mu_2]P_{n-k+2}(t)+k\lambda_OP_{n-k+1}(t) \quad (4\text{-}101)$$

$$\frac{dP_j(t)}{dt}=-[\lambda_O+\lambda_S+\mu_2]P_j(t)+(2n-j-k+3)\lambda_SP_{j-n+k-2}(t) \quad (4\text{-}102)$$

$$\text{for } j=(n-k+3) \text{ to } (2n-2k+3) \quad (4\text{-}103)$$

$$\frac{dP_O(t)}{dt}=-\mu P_O(t)+\lambda_O\sum_{i=n-k+2}^{2(n-k)+3}P_i(t) \quad (4\text{-}104)$$

$$\frac{dP_S(t)}{dt}=-\mu P_S(t)+\lambda_S\sum_{i=n-k+2}^{2(n-k)+3}P_i(t) \quad (4\text{-}105)$$

$$\frac{dP_C(t)}{dt} = -\mu_C P_C(t) + \lambda_C \sum_{i=1}^{n-k+2} P_i(t) \tag{4-106}$$

$$\frac{dP_H(t)}{dt} = -\mu_H P_H(t) + \lambda_H \sum_{i=1}^{n-k+2} P_i(t) \tag{4-107}$$

At time t=0, $P_1(0) = 1$, and all other initial state transition probabilities are equal to zero. Taking Laplace transforms of Equations (4-98) – (4-107) and solving the resultant equations, we get the following set of equations:

$$p_1(s) = \frac{1 + \mu_1 p_2(s) + \mu_2 p_{n-k+3}(s) + \mu\left[p_O(s) + p_S(s)\right] + \mu_C p_C(s) + \mu_H p_H(s)}{s + n\lambda_O + n\lambda_S + \lambda_C + \lambda_H} \tag{4-108}$$

$$p_i(s) = \frac{(n-i+2)\lambda_O p_{i-1}(s) + \mu_1 p_{i+1}(s) + \mu_2 p_{i+n-k+2}(s)}{s + (n-i+1)\lambda_O + (n-i+1)\lambda_S + \lambda_C + \lambda_H + \mu_1} \tag{4-109}$$

$$p_{n-k+1}(s) = \frac{(k+1)\lambda_O p_{n-k}(s) + \mu_2\left[p_{n-k+2}(s) + p_{2(n-k)+3}(s)\right]}{s + k\lambda_O + k\lambda_S + \lambda_C + \lambda_H + \mu_1} \tag{4-110}$$

$$p_{n-k+2}(s) = \frac{k\lambda_O p_{n-k+1}(s)}{s + \lambda_O + \lambda_S + \lambda_C + \lambda_H + \mu_2} \tag{4-111}$$

$$p_j(s) = \frac{(2n-j-k+3)\lambda_S p_{j-n+k-2}(s)}{s + \lambda_O + \lambda_S + \mu_2} \tag{4-112}$$

$$p_O(s) = \frac{\lambda_O}{s+\mu} \sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-113}$$

$$p_S(s) = \frac{\lambda_S}{s+\mu} \sum_{i=n-k+2}^{2(n-k)+3} p_i(s) \tag{4-114}$$

$$p_C(s) = \frac{\lambda_C}{s+\mu_C} \sum_{i=1}^{n-k+2} p_i(s) \tag{4-115}$$

$$p_H(s) = \frac{\lambda_H}{s+\mu_H} \sum_{i=1}^{n-k+2} p_i(s) \tag{4-116}$$

By taking the inverse Laplace transforms of Equations (4-108) – (4-116), the time-dependent state probabilities can be obtained.

The time-dependent system availability is given by:

$$A(t) = L^{-1}\left[A(s)\right] = L^{-1}\left[\sum_{i=1}^{2(n-k)+3} p_i(s)\right] \tag{4-117}$$

4.5.2 Special case model 4-D: Two-out-of-three standby system under Type III repair

The state transition diagram of two-out-of-three standby system with Type III repair can be represented as in Figure 4-18. By substituting k=2 and n=3 in Equations (4-108) – (4-116), we obtain the following set of equations:

$$p_1(s) = \frac{DEFGH}{A(4,4)} \tag{4-118}$$

$$p_2(s) = \frac{3CD^2EFG\lambda_O}{A(4,4)} \tag{4-119}$$

$$p_3(s) = \frac{6D^2EFG\lambda_O^2}{A(4,4)} \tag{4-120}$$

$$p_4(s) = \frac{3\lambda_S EFGH}{A(4,4)} \tag{4-121}$$

$$p_5(s) = \frac{6CDEFG\lambda_O\lambda_S}{A(4,4)} \tag{4-122}$$

$$p_O(s) = \frac{3FG\lambda_O\left[2D\lambda_O\left(D\lambda_O + C\lambda_S\right) + H\lambda_S\right]}{A(4,4)} \tag{4-123}$$

$$p_S(s) = \frac{3FG\lambda_S\left[2D\lambda_O\left(D\lambda_O + C\lambda_S\right) + H\lambda_S\right]}{A(4,4)} \tag{4-124}$$

$$p_C(s) = \frac{DEG\lambda_C\left[H + 3CD\lambda_O + 6D\lambda_O^2\right]}{A(4,4)} \tag{4-125}$$

$$p_H(s) = \frac{DEF\lambda_H\left[H + 3CD\lambda_O + 6D\lambda_O^2\right]}{A(4,4)} \tag{4-126}$$

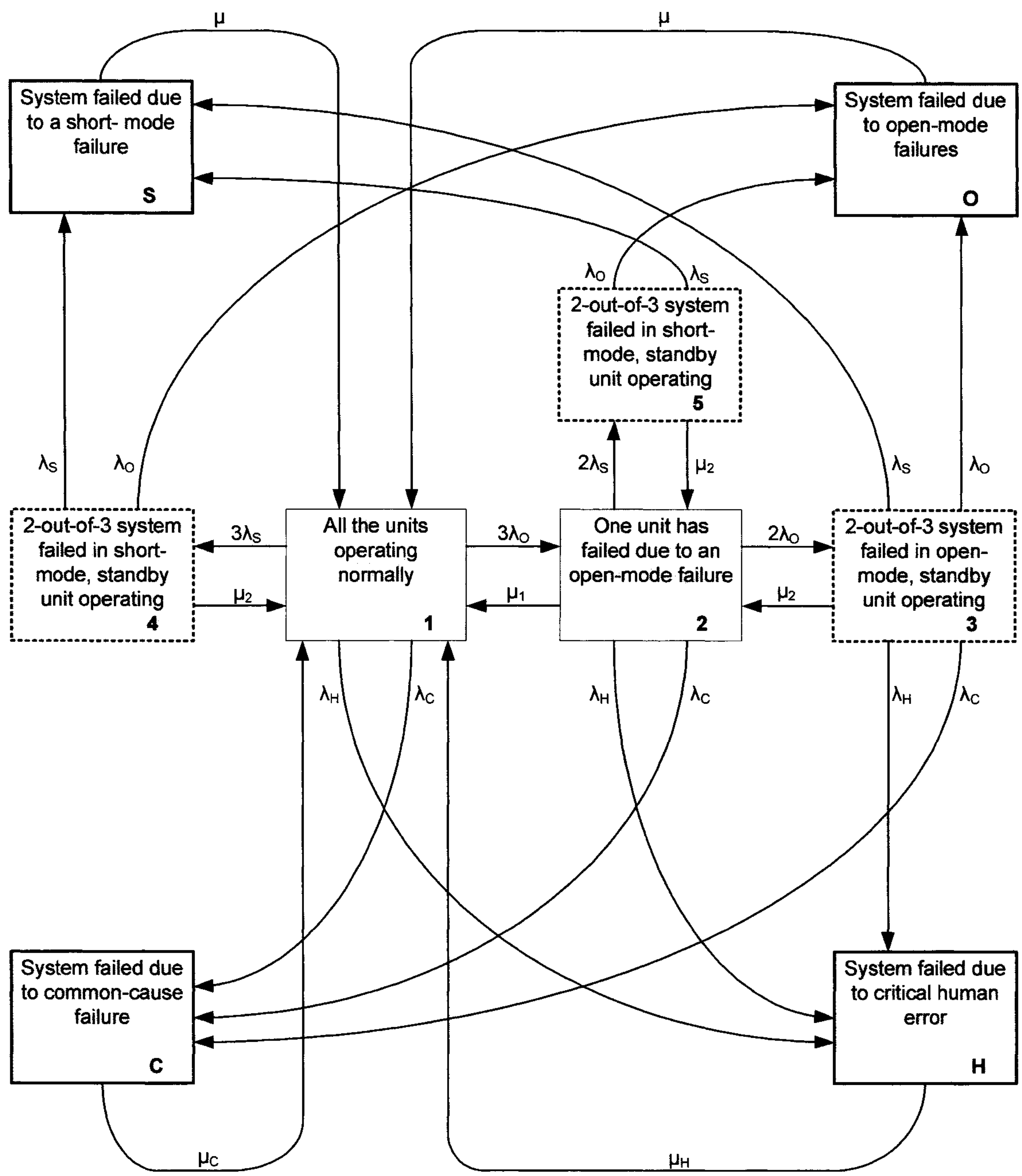

Figure 4-18: State transition diagram of two-out-of-three standby system with Type III repair

By taking the inverse Laplace transforms of Equations (4-118) – (4-126), the time-dependent state probabilities can be obtained. By adding Equations (4-118) – (4-122) and then taking the inverse Laplace transforms, we obtain the following expression for system availability:

$$A(t)=L^{-1}\left\{\frac{EFG\left[D\left(H+3CD\lambda_O+6D\lambda_O^2\right)+3\lambda_S\left(H+2CD\lambda_O\right)\right]}{A(4,4)}\right\} \tag{4-127}$$

where:

$$A(4,4)=EFG\left[ADH-3\left(CD^2\lambda_O\mu_1+\mu_2H\lambda_S\right)\right]-3\mu FG\left[2D\lambda_O\left(D\lambda_O+C\lambda_S\right)+H\lambda_S\right]\left(\lambda_O+\lambda_S\right)$$
$$-DE\left[\mu_C\lambda_C G+\mu_H\lambda_H F\right]\left[H+3CD\lambda_O+6D\lambda_O^2\right]$$
$$A=s+3\lambda_O+3\lambda_S+\lambda_C+\lambda_H$$
$$B=s+2\lambda_O+2\lambda_S+\lambda_C+\lambda_H+\mu_1$$
$$C=s+\lambda_O+\lambda_S+\lambda_C+\lambda_H+\mu_2$$
$$D=s+\lambda_O+\lambda_S+\mu_2$$
$$E=s+\mu$$
$$F=s+\mu_C$$
$$G=s+\mu_H$$
$$H=BCD-2\mu_2\left(D\lambda_O+C\lambda_S\right)$$

Steady State Conditions: We can compute the steady state conditions directly from Equations (4-108) – (4-116), (i.e., for k=2 and n=3) by putting all derivatives equal to zero and solving the resultant equations by utilizing the total probability axiom: $\sum P_i=1$. Thus, we get the following set of equations:

$$P_1=\frac{\mu\mu_C\mu_H DH}{A(4,5)} \tag{4-128}$$

$$P_2=\frac{3\mu\mu_C\mu_H CD^2\lambda_O}{A(4,5)} \tag{4-129}$$

$$P_3=\frac{6\mu\mu_C\mu_H D^2\lambda_O^2}{A(4,5)} \tag{4-130}$$

$$P_4=\frac{3\mu\mu_C\mu_H H\lambda_S}{A(4,5)} \tag{4-131}$$

$$P_5 = \frac{6\mu\mu_C\mu_H CD\lambda_O\lambda_S}{A(4,5)} \tag{4-132}$$

$$P_O = \frac{3\mu_C\mu_H\lambda_O\left[2D\lambda_O\left(D\lambda_O + C\lambda_S\right) + H\lambda_S\right]}{A(4,5)} \tag{4-133}$$

$$P_S = \frac{3\mu_C\mu_H\lambda_S\left[2D\lambda_O\left(D\lambda_O + C\lambda_S\right) + H\lambda_S\right]}{A(4,5)} \tag{4-134}$$

$$P_C = \frac{\mu\mu_H D\lambda_C\left[H + 3CD\lambda_O + 6D\lambda_O^{\ 2}\right]}{A(4,5)} \tag{4-135}$$

$$P_H = \frac{\mu\mu_C D\lambda_H\left[H + 3CD\lambda_O + 6D\lambda_O^{\ 2}\right]}{A(4,5)} \tag{4-136}$$

where:

$$\begin{aligned} A(4,5) = {} & \mu\mu_C\mu_H\left[DH + 3D\lambda_O\left[CD + 2D\lambda_O + 2C\lambda_S\right] + 3H\lambda_S\right] \\ & + 3\mu_C\mu_H\left(\lambda_O + \lambda_S\right)\left[2D\lambda_O\left(D\lambda_O + C\lambda_S\right) + H\lambda_S\right] \\ & + \mu D\left[\mu_H\lambda_C + \mu_C\lambda_H\right]\left[H + 3CD\lambda_O + 6D\lambda_O^{\ 2}\right] \end{aligned}$$

$$\begin{aligned} A &= 3\lambda_O + 3\lambda_S + \lambda_C + \lambda_H \\ B &= 2\lambda_O + 2\lambda_S + \lambda_C + \lambda_H + \mu_1 \\ C &= \lambda_O + \lambda_S + \lambda_C + \lambda_H + \mu_2 \\ D &= \lambda_O + \lambda_S + \mu_2 \\ H &= BCD - 2\mu_2\left(D\lambda_O + C\lambda_S\right) \end{aligned}$$

By adding Equations (3-124) and (3-125), we get the following set the following expression for the system steady state availability:

$$AV_{SS} = \frac{\mu\mu_C\mu_H\left[D\left(H + 3CD\lambda_O + 6D\lambda_O^{\ 2}\right) + 3\lambda_S\left(H + 2CD\lambda_O\right)\right]}{A(4,5)} \tag{4-137}$$

For the given values of the model parameters, the plots of state probabilities as a function of time, t, are shown in Figure 4-19. For the specified values of the model parameters, the plots of Equation (4-127) are shown in Figure 4-20. Similarly, for the specified values of the model parameters, the plots of Equation (4-137) are shown in Figure 4-21.

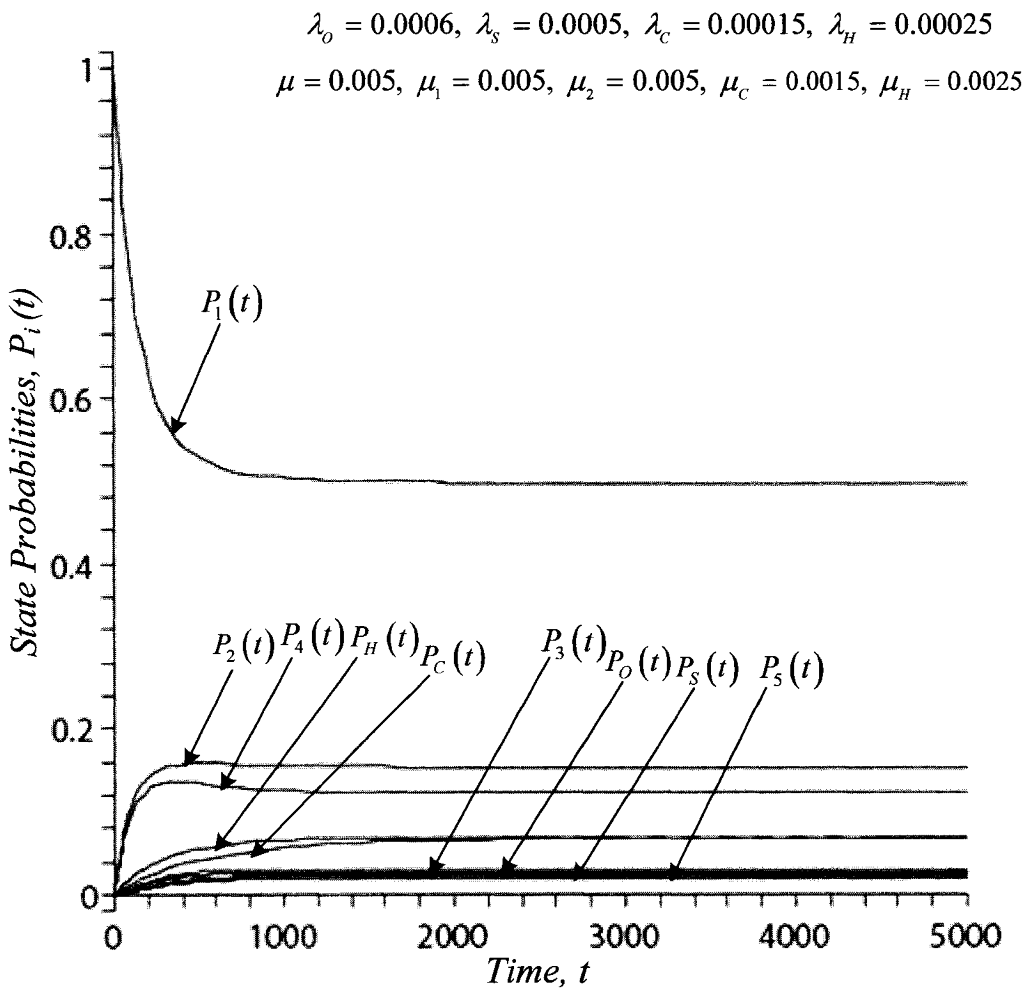

Figure 4-19: State probability plots of two-out-of-three system standby with Type III repair

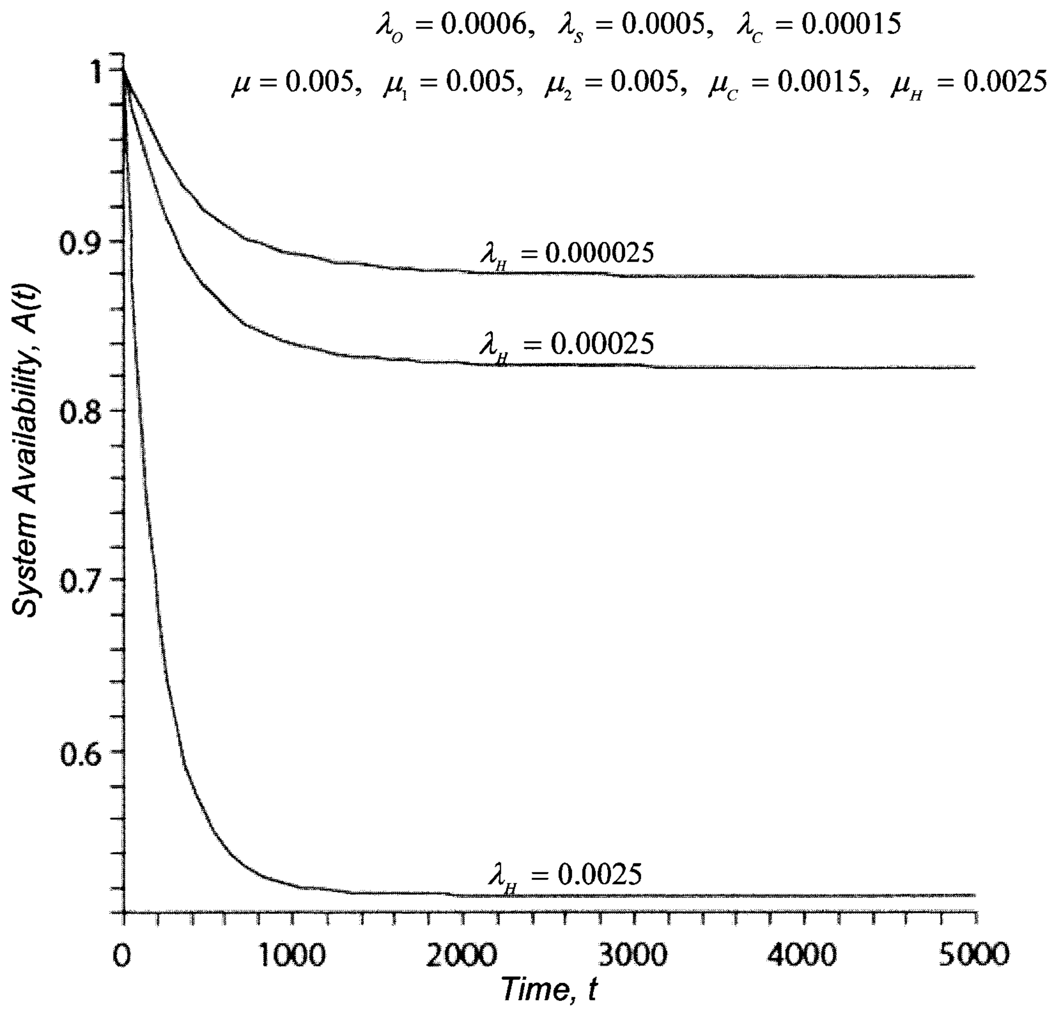

Figure 4-20: Availability plots of two-out-of-three standby system with Type III repair

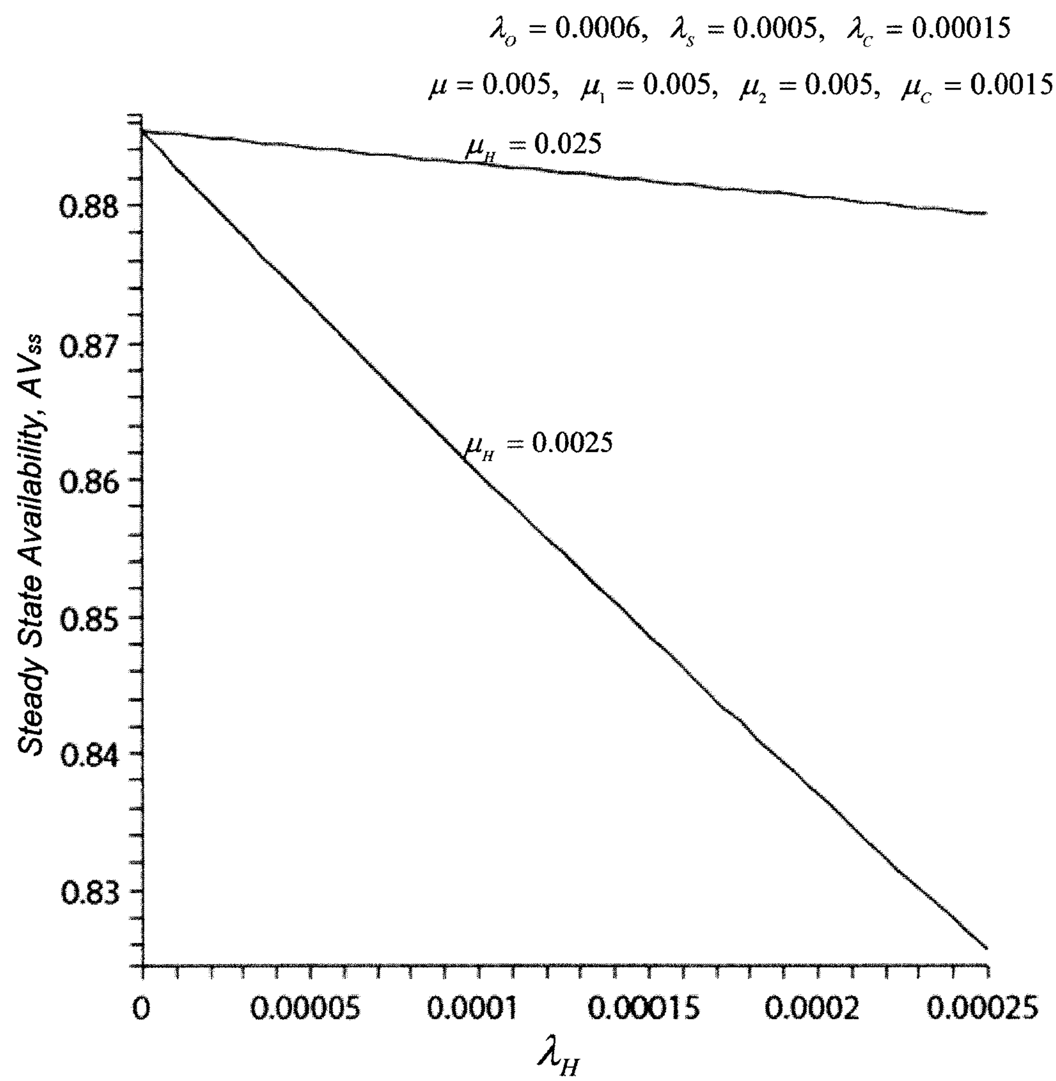

Figure 4-21: Steady state availability plots of two-out-of-three standby system with Type III repair

4.6 System Reliability and Availability Comparison

The expression for system reliability of a three-unit parallel system under without repair policy is given by Equation (2-31). Similarly, the expression for system reliability of a two-out-of-three system under without repair policy is given by Equation (3-21) and of a two-out-of-three system with one unit on standby under without repair policy is given by Equation (4-28). For the given values of model parameters, the plots of these Equations are shown in Figure 4-22.

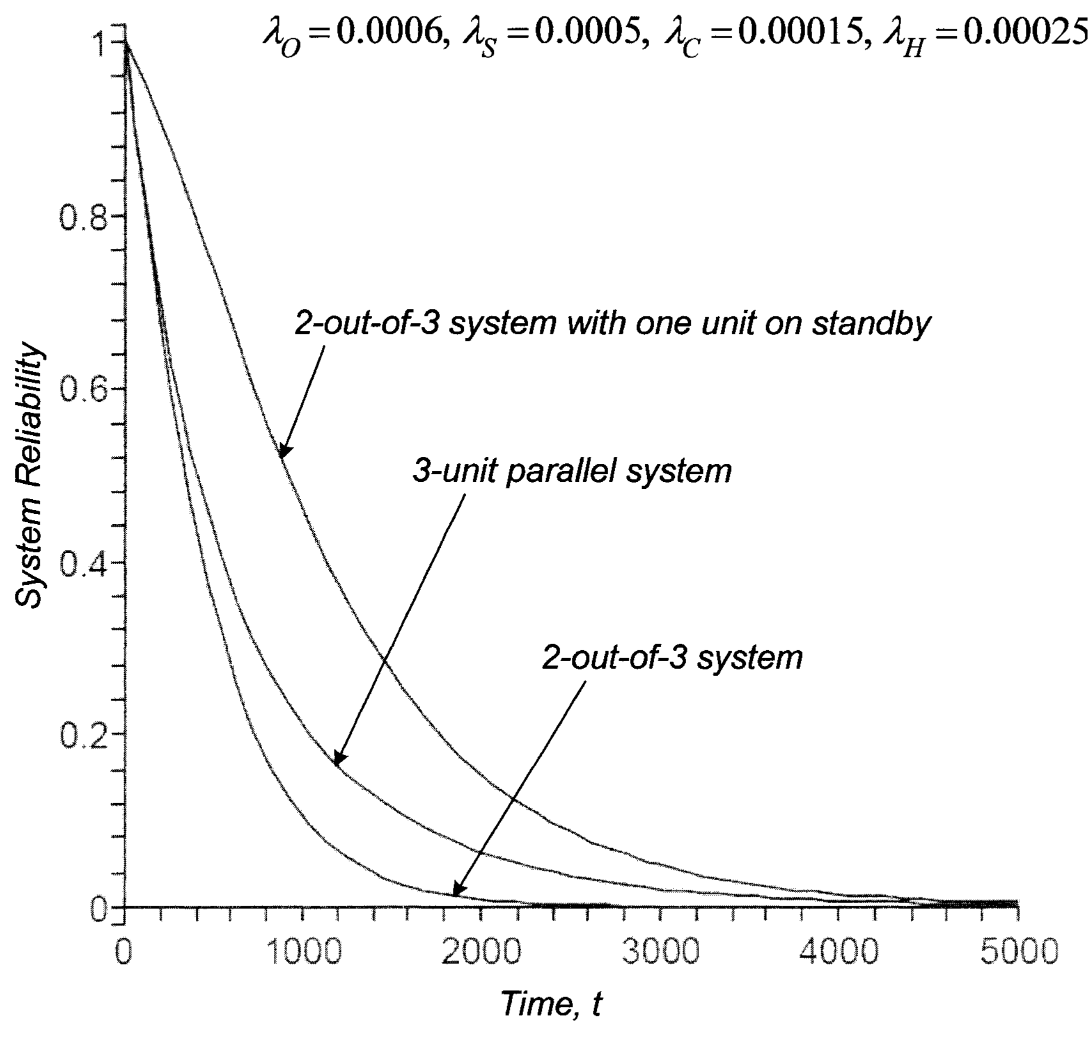

Figure 4-22: Reliability comparison plots for three-unit parallel system, two-out-of-three system and two-out-of-three system with one unit on standby

Similarly, for the specified values of the model parameters, the plots of system availability of a three-unit parallel system [i.e. Equation (2-102)], two-out-of-three system [i.e. Equation (3-84)] and two-out-of-three standby system [i.e. Equation (4-87)] under Type II repair policy are shown in Figure 4-23.

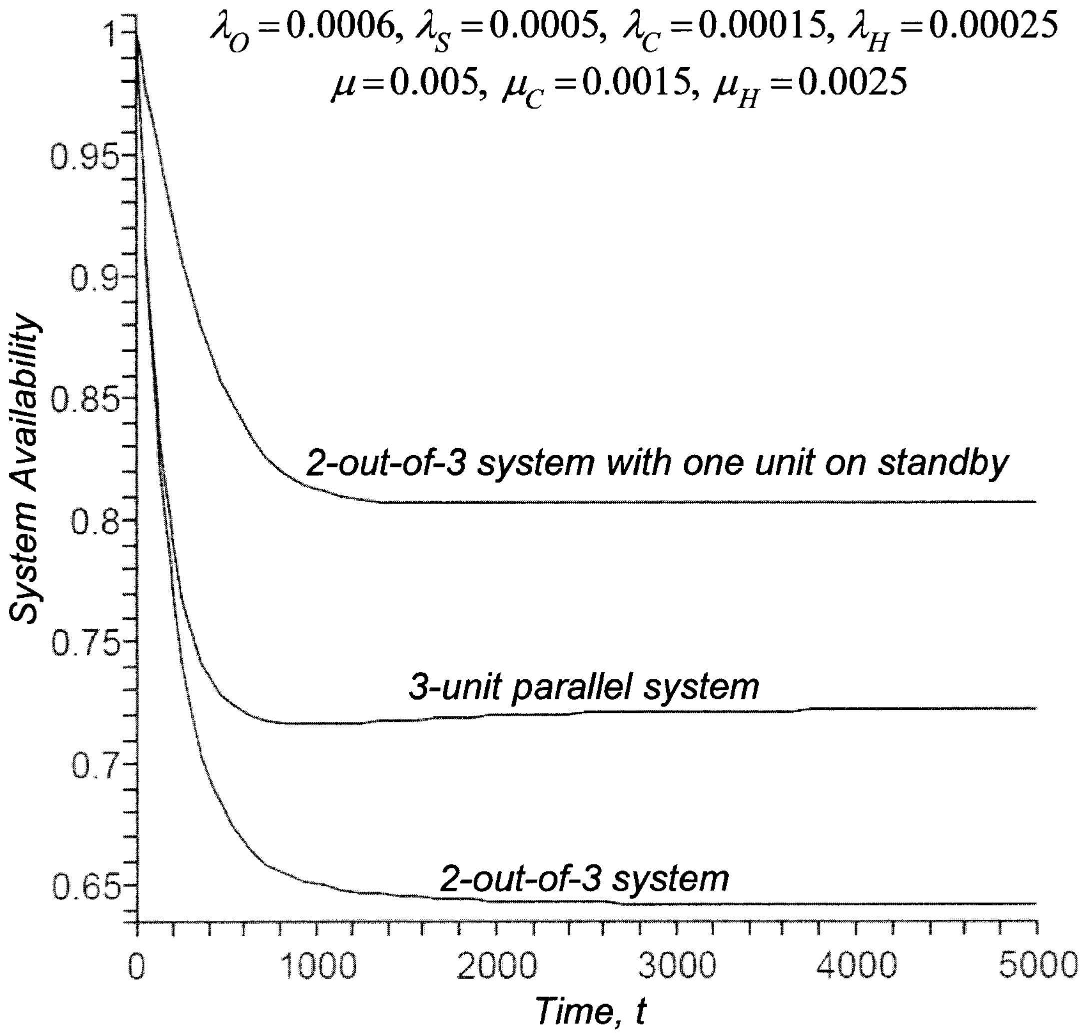

Figure 4-23: Availability comparison plots for three-unit parallel system, two-out-of-three system and two-out-of-three system with one unit on standby

4.7 Summary

This Chapter presents stochastic models for performing the reliability and availability analysis of three-state device k-out-of-n standby system subjected to critical human errors and common-cause failures. All failure and repair hazards for the system are assumed to be constant.

The analysis in this Chapter indicates that cold standby has a significant effect on the three-state device k-out-of-n system subjected to critical human errors and common-cause failures. Under the condition that open-mode failure rate of the unit is greater than its short-mode failure rate, it can be concluded that at any time t, the k-out-of-n standby system, under all the four maintenance policies (i.e., without repair, Type I repair, Type II repair and Type III repair), has a higher reliability and availability values than the parallel, series, and k-out-of-n systems.

Chapter 5

Three-State Device Parallel System with Non-Constant Critical Human Error and Common-Cause Failure and Repair Rates

In the previous Chapters, systems were analysed with an assumption that the critical human error rate, common-cause failure rate and the respective repair rates are exponentially distributed (i.e., constant). This Chapter presents the analysis of two-unit parallel system with non-constant critical human error and repair rates, non-constant common-cause failure and repair rates. When analysing systems with non-constant failure and repair rates, as the process is non-Markovian, the Markov state-space method cannot be used. Therefore, the *Device of Stages* approach is utilized to develop the expressions for various performance indices by formulating a modified markovian model with the addition of *dummy* states. The *Device of Stages* method is explained in detail in Appendix A.

5.1 Reliability Analysis of Parallel System with Non-Constant Critical Human Error and Common-Cause Failure Rates

In Section 2.2.2, the analysis of two-unit parallel system under without repair policy was presented. The system was analysed with constant critical human error and common-cause failure rates. This section presents the analysis of the same system when subjected to non-constant critical human error and common-cause failure rates. To facilitate the modeling of non-constant failure and/or repair rates, the *Device of Stages* approach is used, which aids in the formulation of a modified Markov model with the addition of dummy states. Figure 5-1 represents the state-space diagram for a two-unit parallel system under without repair policy with one dummy state each for a non-constant critical human error and common-cause failure rate respectively. The numerals and letters in the boxes denote the system states. All other symbols used in the diagram are defined in the notation section.

Notation

The symbols associated with the analyses in this Chapter are essentially the same as those defined previously in Chapter 2 except those mentioned below.

$\lambda_H(t)$ Time-dependent critical human error rate of the system.

α_1 Constant transition rate associated with critical human errors from initial operating state to dummy state.

α_2 Constant transition rate associated with critical human errors from dummy state to failed state.

α_H Steady state critical human error rate.

$\lambda_C(t)$ Time-dependent common-cause failure rate of the system.

α_3 Constant transition rate associated with common-cause failures from initial operating state to dummy state.

α_4 Constant transition rate associated with common-cause failures from dummy state to failure state.

α_4 Steady state common-cause failure rate.

$\mu_H(t)$ Time-dependent repair rate of the system when it failed due to critical human errors.

β_1 Constant transition rate associated with the system from initial operating state to dummy state when it failed due to critical human errors.

β_2 Constant transition rate associated with the system from dummy state to failed state when it failed due to critical human errors.

β_H Steady state repair rate of the system when it failed due to critical human errors.

$\mu_C(t)$ Time-dependent repair rate of the system when it failed due to common-cause failures.

β_3 Constant transition rate associated with the system from initial operating state to dummy state when it failed due to common-cause failures.

β_4 Constant transition rate associated with the system from dummy state to failed state when it failed due to common-cause failures.

β_4 Steady state repair rate of the system when it failed due to common-cause failures.

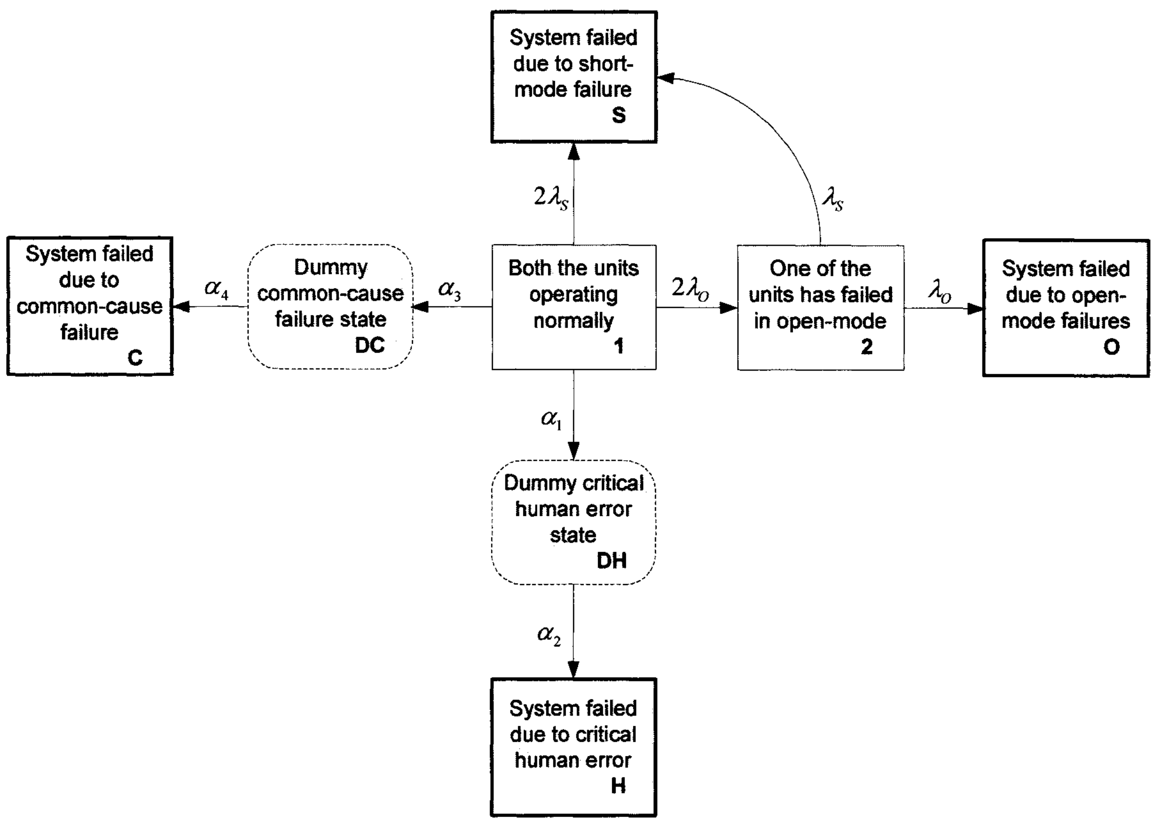

Figure 5-1: The state transition diagram of two-unit parallel system subjected to non-constant critical human error and common-cause failure rates

With the aid of Markov state-space technique, the system of differential equations associated with Figure 5-1 is as follows:

$$\frac{dP_1(t)}{dt} = -[2\lambda_O + 2\lambda_S + \alpha_1 + \alpha_3]P_1(t) \tag{5-1}$$

$$\frac{dP_2(t)}{dt} = -[\lambda_O + \lambda_S]P_2(t) + 2\lambda_O P_1(t) \tag{5-2}$$

$$\frac{dP_{DH}(t)}{dt} = -\alpha_2 P_{DH}(t) + \alpha_1 P_1(t) \tag{5-3}$$

$$\frac{dP_{DC}(t)}{dt} = -\alpha_4 P_{DC}(t) + \alpha_3 P_1(t) \tag{5-4}$$

$$\frac{dP_O(t)}{dt} = \lambda_O P_2(t) \tag{5-5}$$

$$\frac{dP_S(t)}{dt} = \lambda_S \left[2P_1(t) + P_2(t)\right] \tag{5-6}$$

$$\frac{dP_H(t)}{dt} = \alpha_2 P_{DH}(t) \tag{5-7}$$

$$\frac{dP_C(t)}{dt} = \alpha_4 P_{DC}(t) \tag{5-8}$$

Initial Conditions: At time $t = 0$, $P_1(t) = 1$, and all the other state probabilities equal to zero.

By substituting $\alpha_1 = \alpha_2 = \alpha_H$ and $\alpha_3 = \alpha_4 = \alpha_C$ in Equations (5-1) – (5-8) and solving the resulting equations by taking Laplace transforms, we obtain the following set of equations:

$$p_1(s) = \frac{1}{s + 2\lambda_O + 2\lambda_S + \alpha_H + \alpha_C} \tag{5-9}$$

$$p_2(s) = \left[\frac{2\lambda_O}{s + \lambda_O + \lambda_S}\right]\left[\frac{1}{s + 2\lambda_O + 2\lambda_S + \alpha_H + \alpha_C}\right] \tag{5-10}$$

$$p_O(s) = \left[\frac{\lambda_O}{s}\right]\left[\frac{2\lambda_O}{s + \lambda_O + \lambda_S}\right]\left[\frac{1}{s + 2\lambda_O + 2\lambda_S + \alpha_H + \alpha_C}\right] \tag{5-11}$$

$$p_S(s) = \frac{2\lambda_S\left(s + 2\lambda_O + \lambda_S\right)}{s\left(s + \lambda_O + \lambda_S\right)\left(s + 2\lambda_O + 2\lambda_S + \alpha_H + \alpha_C\right)} \tag{5-12}$$

$$p_{DH}(s) = \left[\frac{\alpha_H}{s + \alpha_H}\right]\left[\frac{1}{s + 2\lambda_O + 2\lambda_S + \alpha_H + \alpha_C}\right] \tag{5-13}$$

$$p_{DC}(s) = \left[\frac{\alpha_C}{s + \alpha_C}\right]\left[\frac{1}{s + 2\lambda_O + 2\lambda_S + \alpha_H + \alpha_C}\right] \tag{5-14}$$

$$p_H(s) = \frac{\alpha_H^{\ 2}}{s(s+\alpha_H)(s+2\lambda_O+2\lambda_S+\alpha_H+\alpha_C)} \tag{5-15}$$

$$p_C(s) = \frac{\alpha_C^{\ 2}}{s(s+\alpha_C)(s+2\lambda_O+2\lambda_S+\alpha_H+\alpha_C)} \tag{5-16}$$

Thus, we obtain the following expressions for system reliability and mean time to failure:

$$R(t) = P_1(t)+P_2(t)+P_{DH}(t)+P_{DC}(t) = L^{-1}\left[p_1(s)+p_2(s)+p_{DH}(s)+p_{DC}(s)\right]$$
$$= e^{-[2\lambda_O+2\lambda_S+\alpha_C+\alpha_H]t}\left\{1-\frac{2\lambda_O}{\lambda_O+\lambda_S+\alpha_C+\alpha_H}-\frac{\alpha_H}{2(\lambda_O+\lambda_S)+\alpha_C}-\frac{\alpha_C}{2(\lambda_O+\lambda_S)+\alpha_H}\right\}$$
$$+e^{-[\lambda_O+\lambda_S]t}\left\{\frac{2\lambda_O}{\lambda_O+\lambda_S+\alpha_C+\alpha_H}\right\}+e^{-\alpha_H t}\left\{\frac{\alpha_H}{2(\lambda_O+\lambda_S)+\alpha_C}\right\}+e^{-\alpha_C t}\left\{\frac{\alpha_C}{2(\lambda_O+\lambda_S)+\alpha_H}\right\}$$
(5-17)

$$MTTF = \lim_{s\to 0}\left[p_1(s)+p_2(s)+p_{DC}(s)+p_{DH}(s)\right] = \frac{5\lambda_O+3\lambda_S}{(\lambda_O+\lambda_S)(2\lambda_O+2\lambda_S+\alpha_C+\alpha_H)} \tag{5-18}$$

As per the discussion presented in Appendix A, it follows that the failure density function due to critical human errors and common-cause failures follow a gamma distribution. Thus, we get the following expressions for time-dependent failure rates:

$$\lambda_H(t) = \frac{\alpha_H^{\ 2} t}{\alpha_H t+1} \tag{5-19}$$

$$\lambda_C(t) = \frac{\alpha_C^{\ 2} t}{\alpha_C t+1} \tag{5-20}$$

For the specified values of the various parameters, the plots of Equation (5-17) are shown in Figure 5-2. Figure 5-3 shows the reliability comparison plots for a two-unit parallel system with constant critical human errors and common-cause failures and with non-constant critical human errors and common-cause failures. In this plot, it has been assumed that the steady state value of critical human error and common-cause failure rates is equal to constant critical human error and common-cause failure rates (i.e., $\alpha_H = \lambda_H$ and $\alpha_C = \lambda_C$).

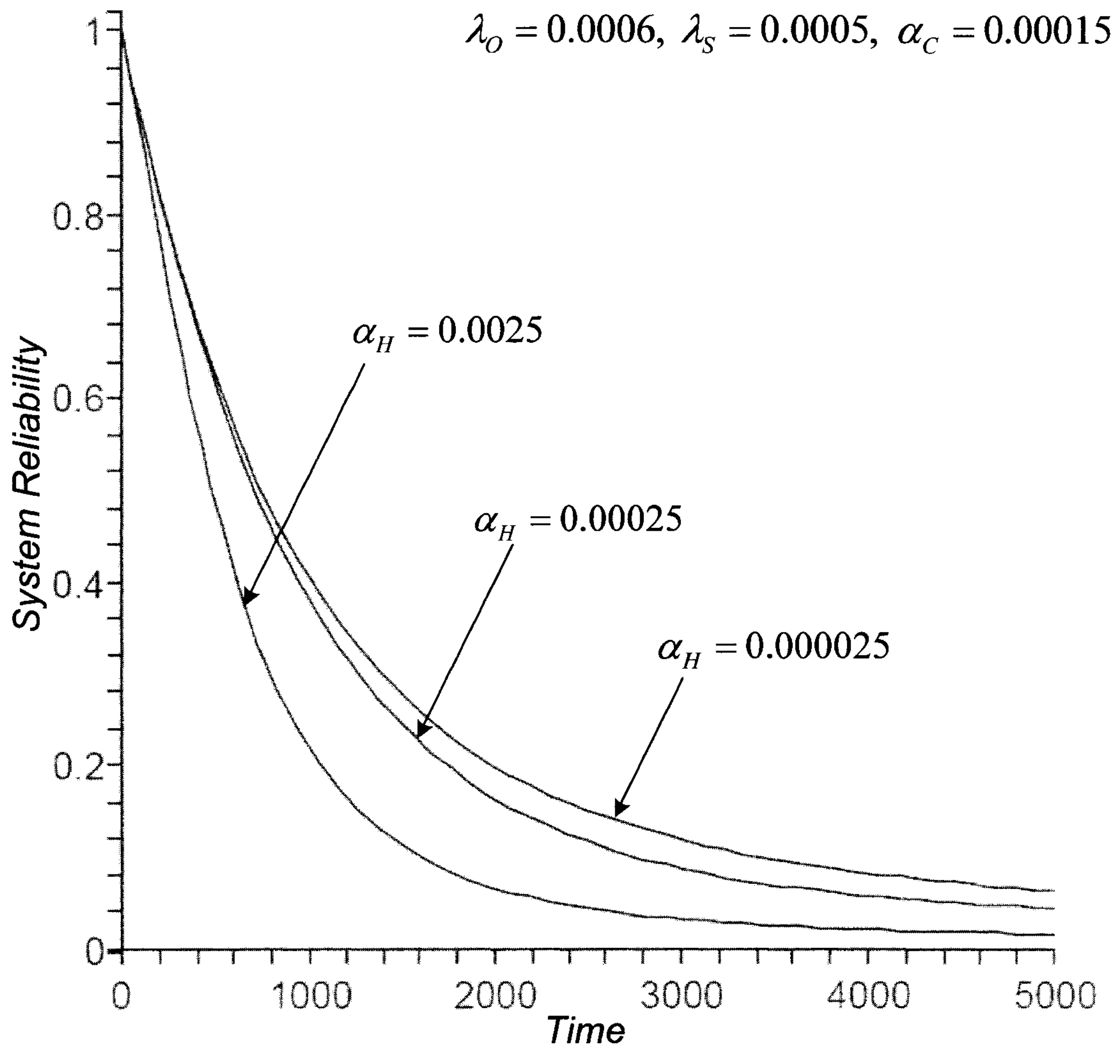

Figure 5-2: Reliability plots of two-unit parallel system with non-constant critical human error and common-cause failure rates

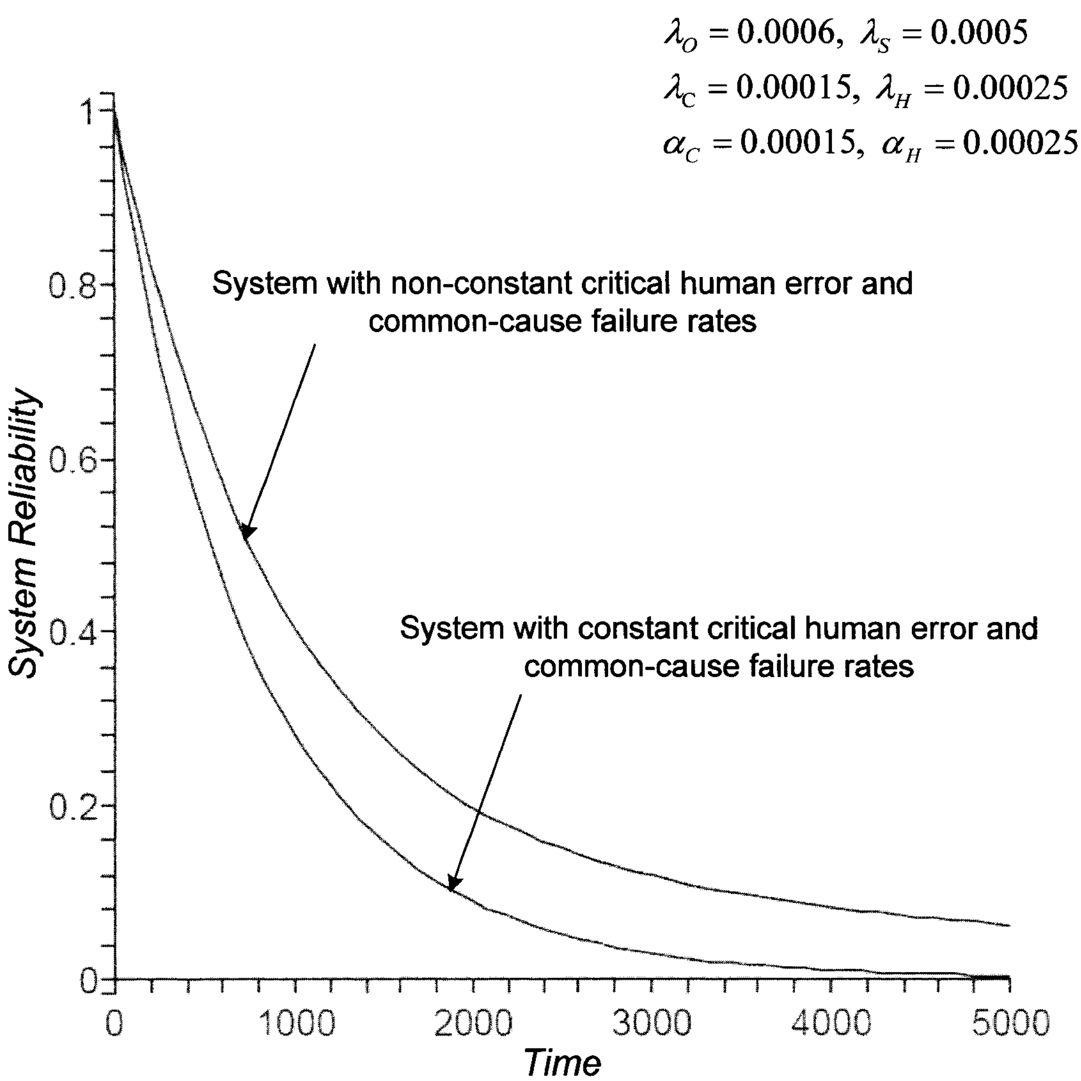

Figure 5-3: Reliability comparison plots of two-unit parallel system with constant and with non-constant critical human error and common-cause failure rates

5.2 Availability Analysis of Parallel System with Non-Constant Critical Human Error and Common-Cause Failure and Repair Rates subjected to Type II Repair Policy

Figure 5-4 represents the state transition diagram of a two-unit parallel system with non-constant critical human error, common-cause failure and repair rates.

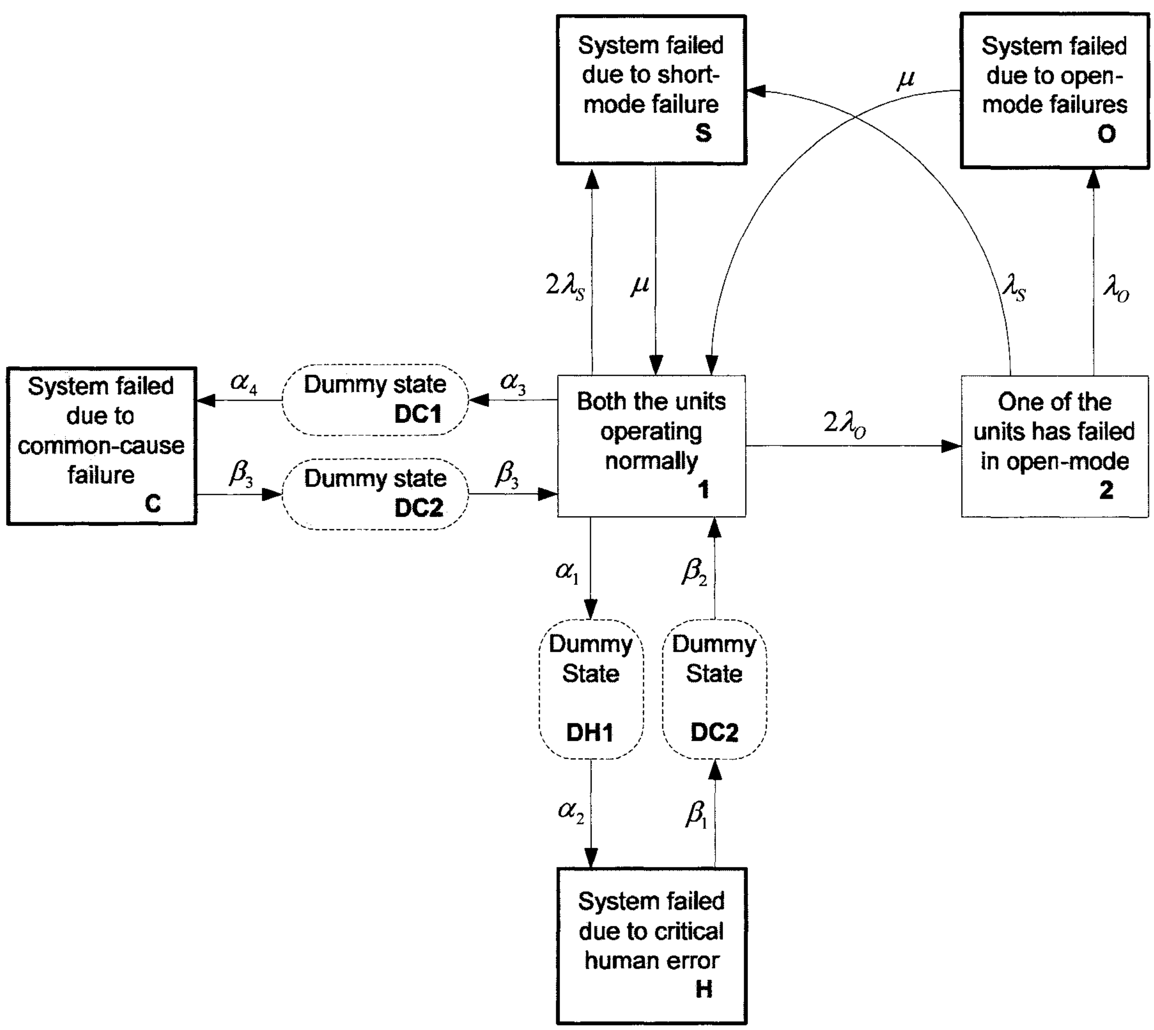

Figure 5-4: State space diagram of two-unit parallel system under Type II repair policy with non-constant critical human error, common-cause failure and repair rates

Using the same approach as before, we obtain the following system of differential equations corresponding to Figure 5-4:

$$\frac{dP_1(t)}{dt} = -[2\lambda_O + 2\lambda_S + \alpha_1 + \alpha_3]P_1(t) + \mu[P_O(t) + P_S(t)] + \beta_2 P_{DH2}(t) + \beta_4 P_{DC2}(t) \quad (5\text{-}21)$$

$$\frac{dP_2(t)}{dt} = -[\lambda_O + \lambda_S]P_2(t) + 2\lambda_O P_1(t) \quad (5\text{-}22)$$

$$\frac{dP_O(t)}{dt} = -\mu P_O(t) + \lambda_O P_2(t) \quad (5\text{-}23)$$

$$\frac{dP_S(t)}{dt} = -\mu P_S(t) + \lambda_S[2P_1(t) + P_2(t)] \quad (5\text{-}24)$$

$$\frac{dP_{DH1}(t)}{dt} = -\alpha_2 P_{DH1}(t) + \alpha_1 P_1(t) \quad (5\text{-}25)$$

$$\frac{dP_{DH2}(t)}{dt} = -\beta_2 P_{DH2}(t) + \beta_1 P_H(t) \quad (5\text{-}26)$$

$$\frac{dP_H(t)}{dt} = -\beta_1 P_H(t) + \alpha_2 P_{DH1}(t) \quad (5\text{-}27)$$

$$\frac{dP_{DC1}(t)}{dt} = -\alpha_4 P_{DC1}(t) + \alpha_3 P_1(t) \quad (5\text{-}28)$$

$$\frac{dP_{DC2}(t)}{dt} = -\beta_4 P_{DC2}(t) + \beta_3 P_C(t) \quad (5\text{-}29)$$

$$\frac{dP_C(t)}{dt} = -\beta_3 P_C(t) + \alpha_4 P_{DC1}(t) \quad (5\text{-}30)$$

By setting $\alpha_1 = \alpha_2 = \alpha_H$; $\alpha_3 = \alpha_4 = \alpha_C$; $\beta_1 = \beta_2 = \beta_H$; $\beta_3 = \beta_4 = \beta_C$ in Equations (5-21) – (5-30), equating the derivatives to zero and solving the resulting equations by making use of the total probability axiom: $\sum P_i = 1$ (where i represents all the states in the system), the following expressions for steady state probabilities are obtained:

$$P_1 = \frac{\mu\beta_H\beta_C(\lambda_O + \lambda_S)}{A(5,2)} \quad (5\text{-}31)$$

$$P_2 = \frac{2\mu\beta_H\beta_C\lambda_O}{A(5,2)} \tag{5-32}$$

$$P_O = \frac{2\lambda_O^{\,2}\beta_H\beta_C}{A(5,2)} \tag{5-33}$$

$$P_S = \frac{2\lambda_S(2\lambda_O + \lambda_S)\beta_H\beta_C}{A(5,2)} \tag{5-34}$$

$$P_{DH1} = \frac{\mu\beta_H\beta_C(\lambda_O + \lambda_S)}{A(5,2)} \tag{5-35}$$

$$P_{DH2} = \frac{\mu\alpha_H\beta_C(\lambda_O + \lambda_S)}{A(5,2)} \tag{5-36}$$

$$P_H = \frac{\mu\alpha_H\beta_C(\lambda_O + \lambda_S)}{A(5,2)} \tag{5-37}$$

$$P_{DC1} = \frac{\mu\beta_C\beta_H(\lambda_O + \lambda_S)}{A(5,2)} \tag{5-38}$$

$$P_{DC2} = \frac{\mu\alpha_C\beta_H(\lambda_O + \lambda_S)}{A(5,2)} \tag{5-39}$$

$$P_C = \frac{\mu\alpha_C\beta_H(\lambda_O + \lambda_S)}{A(5,2)} \tag{5-40}$$

Thus, the steady state availability of the two-unit parallel system is given by

$$\begin{aligned} AV_{SS} &= P_1 + P_2 + P_{DC1} + P_{DH1} + P_{DC2} + P_{DH2} \\ &= \frac{\mu\beta_H\beta_C(5\lambda_O + 3\lambda_S) + \mu(\lambda_O + \lambda_S)(\alpha_H\beta_C + \alpha_C\beta_H)}{A(5,2)} \end{aligned} \tag{5-41}$$

According to the discussion presented in Appendix A, when we model a non-constant hazard using one dummy state and setting $\alpha_1 = \alpha_2 = \alpha_H$; $\alpha_3 = \alpha_4 = \alpha_C$; $\beta_1 = \beta_2 = \beta_H$; $\beta_3 = \beta_4 = \beta_C$, we have that the failure density function of the non-constant hazard becomes a gamma distribution.

Thus, we obtain the following expressions for the time-dependent failure and repair rates:

$$\lambda_H(t) = \frac{\alpha_H{}^2 t}{\alpha_H t + 1} \tag{5-42}$$

$$\lambda_C(t) = \frac{\alpha_C{}^2 t}{\alpha_C t + 1} \tag{5-43}$$

$$\mu_C(t) = \frac{\beta_C{}^2 t}{\beta_C t + 1} \tag{5-44}$$

$$\mu_H(t) = \frac{\beta_H{}^2 t}{\beta_H t + 1} \tag{5-45}$$

For the given values of the model parameters, Figure 5-5 shows the steady state availability plots of two-unit parallel system subjected to non-constant critical human error and common-cause failure and repair rates.

5.3 Summary

This Chapter presents two mathematical models for performing the reliability and availability analysis of two-unit parallel system under without repair and Type II repair policies subjected to non-constant human error and common-cause failure and repair rates. The *Device of Stages – Markov Method* is used to develop expressions for various performance measures.

The analysis performed in this Chapter indicate that for the same values of model parameters, the system reliability, mean time to failure and steady state availability of the parallel system subjected to non-constant critical human errors and common-cause failures and the respective repair rates is higher than for the system subjected to constant critical human errors and common-cause failures and repair rates.

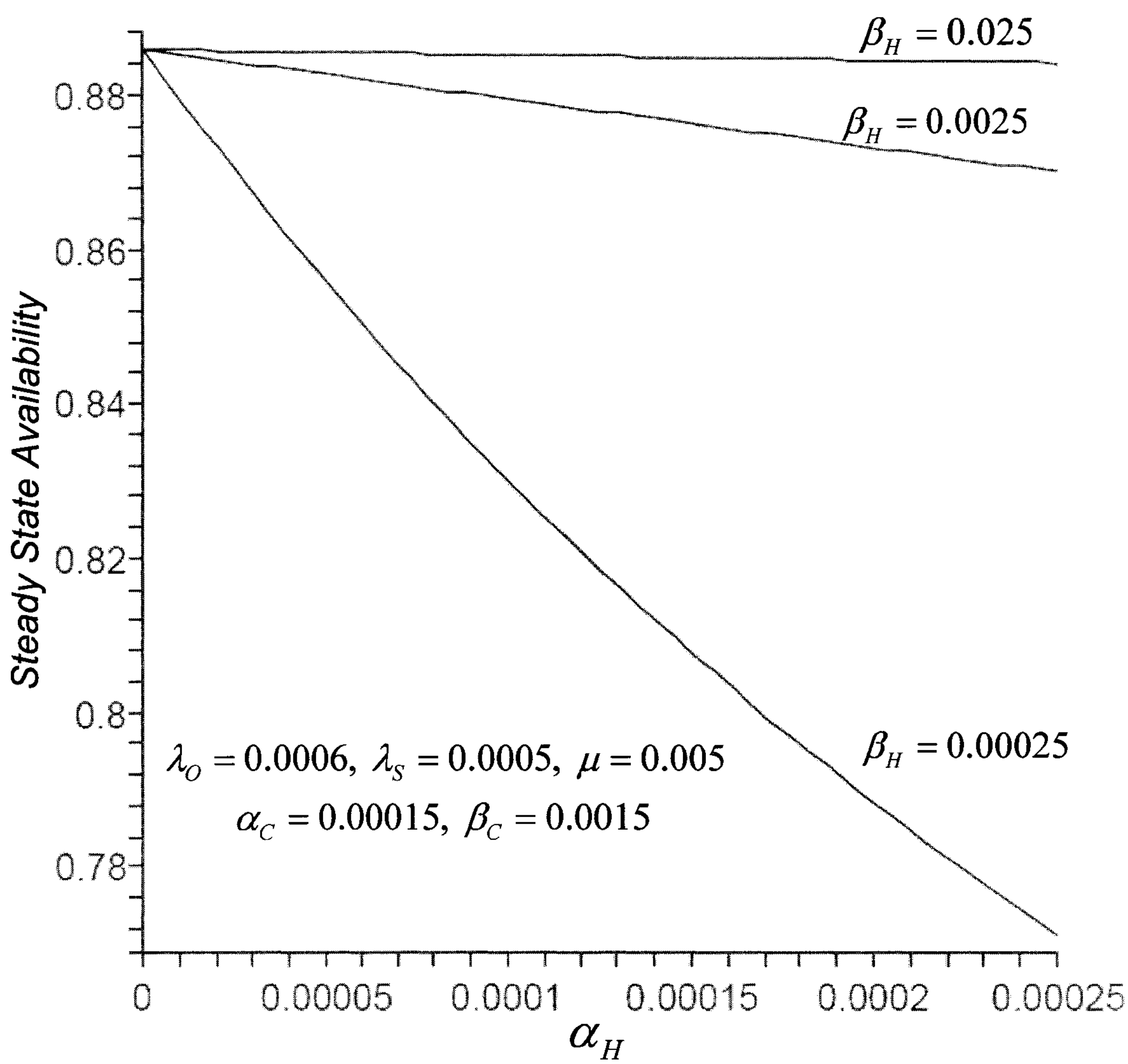

Figure 5-5: Steady state availability plots of a two-unit parallel system subjected to non-constant critical human errors and common-cause failures

Chapter 6

Discussion, Conclusions and Recommendations

6.1 Discussion

This study has presented a reliability and availability analysis of three-state device systems subjected to critical human errors and common-cause failures. Generalized and special-case expressions for system performance indices such as the system reliability, mean time to failure, time-dependent availability and steady state availability for various commonly used redundant configurations like parallel, series, k-out-of-n and k-out-of-n cold standby system are developed. The impact of critical human errors, common-cause failures and failed system repair policy on the system's performance is assessed by means of plots. The plots were plotted for the specified values of the model parameters using the mathematical software MAPLE v. 9.

With systems in which the failure and repair rates follow the exponential distribution, the transition rate from one state to another is constant and does not depend on how long the system spends in a given state nor does it depend on how it arrived at a particular state. Therefore, the stochastic process that governs this system is said to be 'memoryless'. This assumption leads us to the *Markovian Random Process*. Thus, the *Markov State Space* approach was used to develop expressions for system reliability, mean time to failure, time-dependent availability and steady state availability for systems with constant failure and repair hazard distributions.

When the systems are subjected to failure and/or repair hazard distribution which is non-exponential, then the process is said to be non-Markovian. In such cases, the *Device of Stages* approach is utilized to approximate a modified Markov model with the addition of *dummy* states.

6.2 Conclusions

The study clearly demonstrates that the values of system performance indices of three-state device systems depends upon factors such as type of redundant configuration, failed system repair policy, dominant mode of hardware failure, human error and common-cause failure. Therefore, it is concluded that the reliability and availability analysis of any three-state device system would be incomplete without giving due consideration to these factors.

The main results of the analysis presented in this study can be summarized as follows:

1. This study appears to be one of the first attempts to analyse a three-state device system under the influence of human errors and common-cause failures. It helps in a better understanding of the behaviour of such systems, which can aid engineers in their aim to design and operate a safe and reliable system.

2. The system reliability, mean time to failure, time-dependent and steady state system availability decreases as the number of devices in the system increases and as time becomes large. This is true regardless of which type of repair policy is used to analyse the system.

3. The values of the system performance indices decrease with increasing values of critical human error and common-cause failure rates. This is also true regardless of the fact that the system is repairable or not.

4. Cold standby has a significant effect on the performance of three-state device systems. Under the conditions that the open-mode failure rate of the device is higher than its short-mode failure rate, it is concluded that the k-out-of-n standby system has the highest values of system performance indices followed by the parallel, k-out-of-n and series systems in that order.

5. The parallel system subjected to non-constant critical human error and common-cause failure and repair rates has higher reliability and availability than the parallel system subjected to constant critical human error and common-cause failure and repair rates. This is true under the assumption that the steady state values of the non-constant failure/repair rates are equal to values of the constant failure/repair rates, respectively.

6.3 Recommendations

1. Commonly used three-state device redundant configurations have been studied in this research. The study can be extended further to complex three-state device networks like the bridge, series-parallel, parallel-series, etc.

2. The systems analysed in this study were composed of identical elements. Further research can be done to analyse systems made up of non-identical units.

3. Critical human errors which cause the total system failure were considered in this study. The models can be studied further considering both critical and non-critical human errors.

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Appendix A

Description of Device of Stages Approach

In a system composed of two or more units with constant failure rates, the system failure density function is not an exponential distribution even if those of individual units are exponential and vice-versa. Also, in most practical situations, the assumption of constant repair rates is not valid as arbitrary or increasing repair-hazard distributions are more common.

With systems in which the failure and repair rates are governed by the exponential distribution, the transition rate from one state to another is constant and does not depend on how long the system spends in a given state nor does it depend on how it arrived at a particular state. Hence, the stochastic process that governs this system is said to be "memory free". This assumption leads us to the *Markovian Random Process*. If the distribution is non-exponential, then the process is said to be non-Markovian. In such cases, the *Device of Stages* approach can be utilized to formulate a modified Markov model with the addition of *dummy* states.

Figure A-1 best illustrates the *Device of Stages* approach in modeling a non-constant failure rate by using one dummy state. In this example, the system can either be in the operating state or in the failed state. The non-constant failure rate of the system is given by $\lambda(t)$. The constant transition rate between the operating state and the dummy state and between the dummy state and the failed state is ϕ_1 and ϕ_2 respectively.

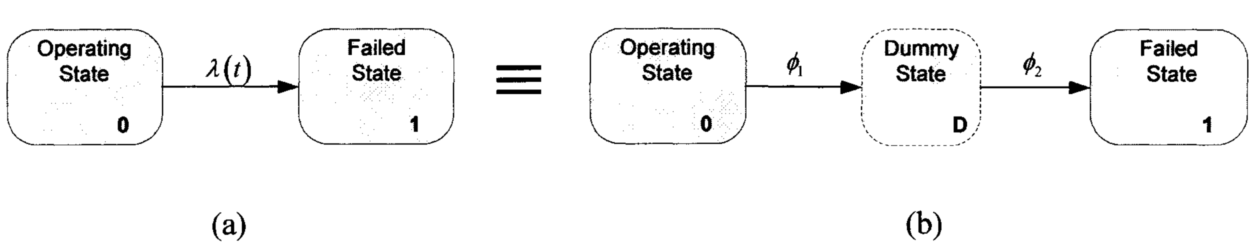

Figure A-1: Modeling a non-constant failure rate by the *Device of Stages* using one dummy state

Notation

t	time.
s	Laplace transform variable.
$\lambda(t)$	time-dependent failure rate of the system.
ϕ_1	constant failure rate from operating state to dummy state.
ϕ_2	constant failure rate from dummy state to failure state.
$P_0(t)$	Probability that the system is in operating state.
$P_D(t)$	Probability that the system is in dummy state.
$P_1(t)$	Probability that the system is in failed state.
$p_0(s)$	Laplace transform of the probability that the system is in operating state.
$p_D(s)$	Laplace transform of the probability that the system is in dummy state.
$p_1(s)$	Laplace transform of the probability that the system is in failed state.
$f(t)$	Probability density function of $\lambda(t)$.
$f_1(t)$	Probability density function of ϕ_1.
$f_2(t)$	Probability density function of ϕ_2.
$R(t)$	Reliability at time t.

With the aid of Markov state-space technique, we obtain the following system of differential equations corresponding to Figure 5-2 (b):

$$\frac{dP_0(t)}{dt} = -\phi_1 P_0(t) \quad \text{(A-1)}$$

$$\frac{dP_D(t)}{dt} = -\phi_2 P_D(t) + \phi_1 P_0(t) \quad \text{(A-2)}$$

$$\frac{dP_1(t)}{dt} = \phi_2 P_D(t) \quad \text{(A-3)}$$

At time $t = 0$, $P_0(t) = 1$ and $P_D(t) = P_1(t) = 0$. By taking the Laplace transforms of Equations (A-1) – (A-3) and solving the resulting equations, the following set of equations results:

$$p_0(s) = \frac{1}{s + \phi_1} \tag{A-4}$$

$$p_D(s) = \left(\frac{\phi_1}{s + \phi_1}\right)\left(\frac{1}{s + \phi_2}\right) \tag{A-5}$$

$$p_1(s) = \left(\frac{\phi_2}{s}\right)\left(\frac{\phi_1}{s + \phi_1}\right)\left(\frac{1}{s + \phi_2}\right) \tag{A-6}$$

The reliability is given by

$$R(t) = L^{-1}\left[p_0(s) + p_D(s)\right] = \frac{\phi_1 e^{-\phi_2 t} - \phi_2 e^{-\phi_1 t}}{\phi_1 - \phi_2} \qquad \{\phi_1 \neq \phi_2\} \tag{A-7}$$

By setting $\phi_1 = \phi_2 = \phi$, in Figure A-1 (b), the probability density function of non-constant failure rate $\lambda(t)$ becomes a gamma distribution with parameter ϕ. This can be proved by utilizing the convolution theorem.

Convolution Theorem: If $F(t) = L^{-1}\{f(s)\}$ and $G(t) = L^{-1}\{g(s)\}$ then it follows that

$$F * G = \int_0^t F(u)\, G(t-u)\, du = L^{-1}\{f(s)\, g(s)\}$$

where $F * G$ is called the convolution of two functions $F(t)$ and $G(t)$.

Hence, the probability density function of time-dependent failure rate $\lambda(t)$ can be obtained by the convolution of the probability density functions of constant transition rates ϕ_1 and ϕ_2.

Therefore, we have

$$f_1(t) = f_2(t) = \phi e^{-\phi t} \qquad (\because\ \phi_1 = \phi_2 = \phi = \text{ constant})$$

$$f(t) = L^{-1}\{f(s)\, g(s)\} = L^{-1}\left\{\frac{\phi}{s+\phi}\cdot\frac{\phi}{s+\phi}\right\} = \phi^2 t e^{-\phi t} \qquad \text{(A-8)}$$

which is the probability density function of gamma distribution with parameter ϕ.

Thus, we obtain the following expressions for the system reliability and time-dependent failure rate:

$$R(t) = 1 - \int_0^t f(u)\, du = (\phi t + 1)e^{-\phi t} \qquad \text{(A-9)}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\phi^2 t}{\phi t + 1} \qquad \text{(A-10)}$$

Figure A-2 shows the plots of time-dependent failure rate given by Equation (A-10). From the plots, it is evident that as time becomes very large, $\lambda(t)$ approaches to a constant value, i.e., $\lim_{t\to\infty} \lambda(t) = \phi$.

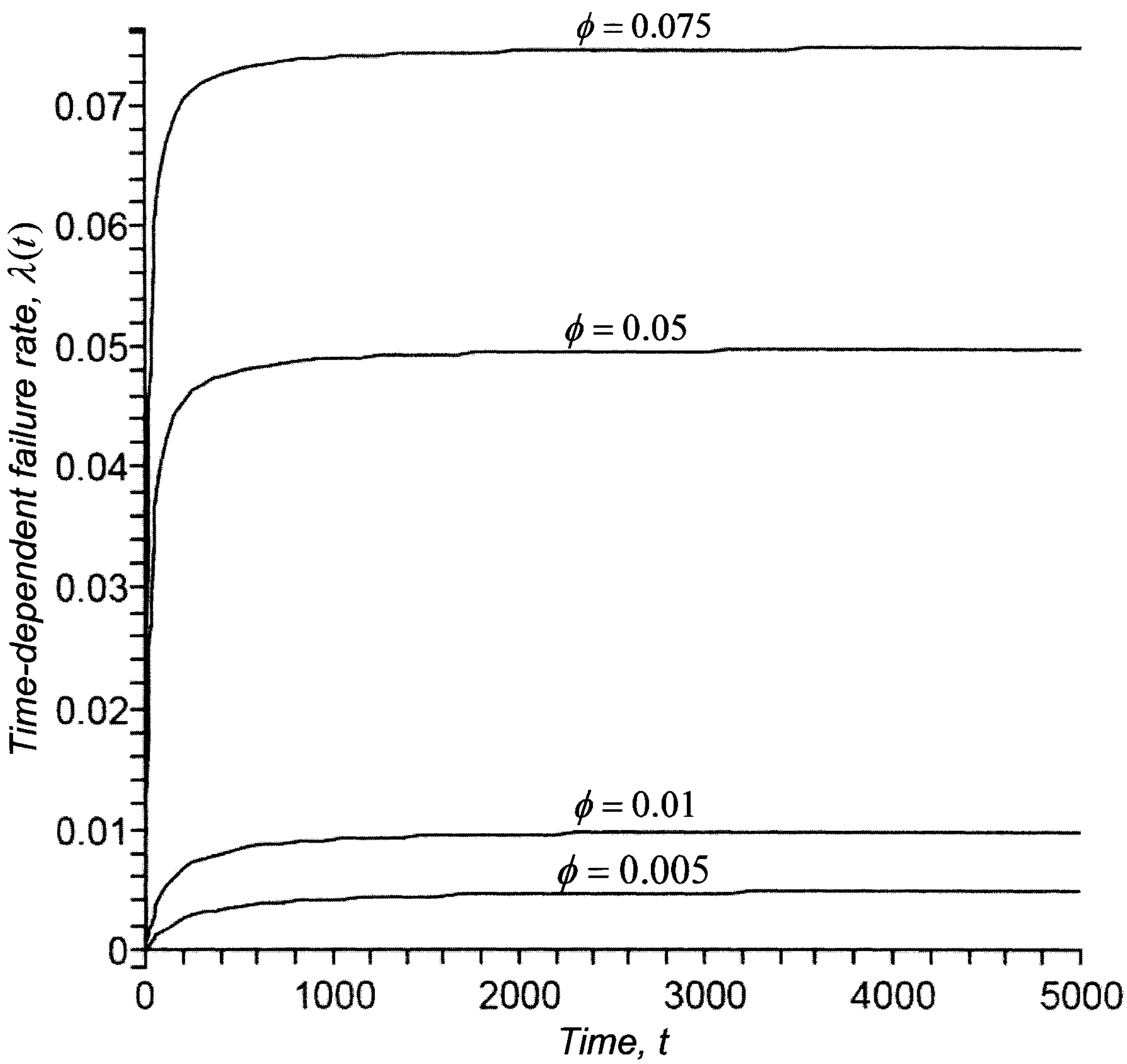

Figure A-2: Time-dependent failure rate $\lambda(t)$ plots for different values of the parameter ϕ