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THREE ESSAYS ON THE ECONOMICS OF CLIMATE CHANGE

Thesis submitted to the School of Graduate and Postdoctoral Studies in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics

By

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ESSAY THREE: GLOBAL TRADING OF EMISSIONS PERMITS AND ECONOMIC GROWTH IN AN OVERLAPPING-GENERATIONS MODEL

ABSTRACT

This doctoral thesis assesses the impacts of the price policy that aims to set a price for carbon in Canada and analyzes the effects of an emissions cap on the welfare of the current and future generations and energy input substitution. In Essay One, a four-stage game is modeled to formulate the price policy and study its impacts. The main result is that through lobbying activities, the dirty-good sector induces the government to implement a price policy of a carbon that is lower than the expected price of carbon. The consequence of the policy is a transfer of income from the owners of the specific factor used in the production of the clean good and workers to the owners of specific factor used in the production of the dirty good.

In Essay Two, an overlapping-generations model is developed to study the impacts of the climate-change policy on the welfare of the current and future generations, the dynamic path of energy and permit prices, and resource substitution. The analysis shows that intergenerational equity issue arises when the oil input is large and when polluters are free to discharge greenhouse gases into the atmosphere. On the equity ground, this result justifies government intervention. The analysis also shows that the use of renewable energy can be introduced quickly if the government implements a stringent policy on climate change. In addition, the process of resource substitution – renewable energy for fossil fuels – is a gradual process and takes place in three stages. The economy will be completely sustained by renewable energy after the stock of fossil fuels is depleted.

The one-country model of Essay Two is extended into a two-country world, which is also subjected to a climate-change policy under the form of a global cap on greenhouse gases emissions. It is assumed that the emissions permits are allocated to each country according to its population and that the population of the South is higher than that of the North. The analysis shows that the North will import emissions permits and export the consumption good to pay for these permits.

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Brenda Tang

INTRODUCTION TO THE THESIS

From the "Project Green Plan" to "Made-in-Canada Plan" are we closer to Kyoto level of about 570 mega tonnes of carbon dioxide equivalent? It is the intention of the Conservative government to develop a new made-in-Canada plan to curb greenhouse gases emissions but at the same time cutting various air pollutants, taking an integrated approach to both global warming and clean air issues in Canada. Whether this new plan still provides flexibility mechanisms[1] should Canada continue to regulate greenhouse gases and how it addresses the consequences of not complying with Kyoto Protocol, to which Canada is a signatory are still not clear. Flexibility mechanisms are the commitments of the previous government – the Liberal - to support the large final emitters in complying with their obligations during the Kyoto period. However, mechanisms such as Price Assurance Mechanism (PAM) and Technology Investment Funds were not fully evaluated against efficiency and effectiveness of achieving the reduction targets. They could potentially reduce the liquidity of the carbon trading markets and distort the market price of carbon. For example, to fulfill the price commitment of $15 (should it be implemented), the government intends to create emissions units called the PAM units and sell these units at CA$15 a unit, making the government one of the competing suppliers of carbon units in Canada. Note that by buying PAM units from the government, companies are transferring their obligations to the government to reduce greenhouse gases emissions to comply with the Kyoto target.

[1] Flexibility mechanisms are the Price Assurance Mechanism, which guarantees the price of $15 per tonne of greenhouse gases and the Technology Investment Funds, where the large final emitters can make contribution to the funds at a rate of $15 per tonne and count it toward their legal obligations.

PAM units are rights to emit, and one PAM unit allows one tonne of carbon dioxide equivalent to be emitted. Given the uncertainty of the supply sources of carbon units and high transaction costs, it is speculated that the large final emitters would transfer their obligations to the government by buying PAM units. Because PAM units do not represent real greenhouse gases reductions, the government would have to purchase from other suppliers of carbon units, making the government one of the biggest buyers in the carbon trading markets. Depending on the carbon prices, the government could lose or gain from selling PAM units and buying carbon units to meet the Kyoto target. Therefore, the Price Assurance Mechanism could have significant implications on who bears the cost of reducing greenhouse gases in Canada and that those implications have not yet been formally analyzed. The objective of Essay One is to study the formulation of the price policy and its impacts on welfare.

In Essay One, a four-stage game of endogenous policy formation is constructed to rationalize the price policy and study the impacts of such policy on the wage rate, rent, and welfare. The main findings are as follows. First, the social welfare is maximized when the government does not implement any price policy to ease the burden of the industry in reducing their greenhouse gases emissions. Second, to reduce the compliance cost, the dirty sector offers political contribution to the government if the government implements the price policy that is set below the expected price of carbon. As a result, wage rate increases making the owners of specific sector in clean sector worse off as their labor cost increases. The workers are better off in terms of increased labor income but also worse off in terms of paying for greenhouse gases reductions. The net effect is not clear. The owners of specific factor in the dirty sector are better off as the expected payoff increases with the price policy. Third, the social welfare decreases by a small amount as part of the compliance cost shifts to the other two groups of the society.

Although Canada is a signatory to the Kyoto Protocol and that by the end of 2012 Canada, along with other Kyoto members must achieve their targets, Canada is not positioning itself to meet those obligations as those obligations can only be fulfilled at a very high cost. The only compliance option that seems feasible for Canada is to acquiring carbon units from the other Kyoto members. Note that Canada's greenhouse gases have increased by about 25% from the 1990 levels and that preparing itself to be at 6% below its 1990 levels by 2012 is almost an impossible task.[2] A significant amount of resources have to be allocated at the expense of other priorities and such large spending is often difficult to justify as the benefits are to be incurred in the future and that those benefits are very difficult to monetize. The timing of the realization of benefits depends on the availability of cleaner inputs, technologies, and productive resources, urban planning, the amount of research and development, etc.. Many researchers have examined the impacts of an environmental policy on economic growth, the well-being of present and future generations, and the environment under different frameworks, such as over-lapping generation models or applied general equilibrium models. Others have studied how the market allocates renewable or exhaustible resources and the associated price paths. Nevertheless, how environmental policy affects the timing of inputs used in the production, the welfare of current and future generations, and energy input substitutability have not yet been studied. The objective of Essay Two is to study such

[2] Information obtained from a contact person at Environment Canada.

effects in an overlapping-generations model when the government caps emissions and requires emitting industries to pay for their discharges. The contribution of Essay Two is further emphasized in Essay Two. In the model, there are three goods in the economy: a consumption good; oil, which is the main source of greenhouse gases, and a renewable energy. The consumption good is produced with the help of labor, capital, and energy. Energy input comes from two sources – oil and backstop. When using oil as energy input, firms emits greenhouse gases and therefore must pay for those emissions. The effective price of oil is the compliance cost plus the price of oil. The least cost energy input will be chosen to produce the consumption good. Emissions are endogenously determined and the processes of resource depletion and resource substitution are explicitly formulated in the model.

The main findings of Essay Two are as follows. First, if the initial stock of fossil fuels is large and if the government does not implement any policy on climate change, then the oil input will be large at beginning of time. Second, the excessive burning of fossil fuels at the beginning leads to a rise of greenhouse gases concentrations in the atmosphere, and the ensuing negative production externality causes output in the next period to fall, and this creates a serious problem of intergenerational equity. Third, the competitive equilibrium under non-intervention is not Pareto optimal: in any period when oil constitutes part of the energy input, its use can be cut slightly and the reduction transferred to the following period. This action can raise the lifetime utility of the young generation of the period if the current drop in output caused by a lower oil input falls completely on the young generation and the incremental output in the following period –

due to a higher oil input and a better environment – is allocated to the old generation of that period. Fourth, the government can induce the use of renewable energy quickly if it implements a stringent policy on climate change. Fifth, the process of resource substitution – renewable energy for fossil fuels – takes place in three stages. In the first stage, fossil fuels constitute the only source of energy input used in the production of the consumption good. Furthermore, as long as the emissions cap is binding and the stock of fossil fuels not yet depleted, the price of oil rises, but the price of emissions permits declines steadily, and the price of emissions permits drops to zero once the cap is not binding. In the second stage, the backstop is brought into use, and the two technologies co-exist during a certain number of periods. The stock of fossil fuels is depleted at the end of the second stage. When the economy enters the third stage, the economy is completely sustained by renewable energy in its convergence to sustainable development.

The one-country model of Essay Two is extended into a two-country world, which is also subjected to a climate-change policy under the form of a global cap on greenhouse gases emissions. The emissions permits are issued in each period and these permits will expire at the end of the period. The emissions permits are allocated to each country according to its population; that is, an individual in the North is allocated the same number of permits as an individual in the South. The major results of the essay are as follows. First, in the long run, after the world has exhausted its stock of fossil fuels, the world will be sustained by renewable energy. The capital stock in each country converges to its steady state level. Second, the share of energy input – in the world's energy input in each period – depends on its technological level and its population. If the population of the South is much larger than the population of the North, and if its population size dominates its technological disadvantage, then the South will have a larger stock of backstop capital than the North in the long run. On the other hand, if the population of South is much more impatient than the population in the North, then the South will have a lower capital stock in the long run and will import renewable energy from the North, and pay for the energy imports with part of the output of the consumption good that the South produces. Third, if neither country implements any climate-change policy, then under the competitive equilibrium the abundance of oil will induce excessive burning of this fossil fuel, resulting a high concentration of greenhouse gases in the atmospheres, with the ensuing consequence of lower consumption for future generations. The competitive equilibrium under the policy of non-intervention is also not Pareto optimal: by appropriately changing the intertemporal allocation of resources, we can raise the welfare of one generation without lowering the welfare of the other generations. Fourth, if the rate of capital depreciation is not too high, then the price of oil is rising, but the price of emissions permits is declining as long as the cap on global greenhouse gases emissions is biding. The rising price of oil signifies its scarcity, while the declining of the price of emissions permits reflects the fact that the stock of fossil fuels is being drawn down, and atmospheric pollution is becoming less and less of a problem. Once the cap is no longer binding, the price of emissions permits will fall to 0, and the threat of climate change caused by greenhouse gases no longer exists. The behavior of the price of oil and the price of emissions permits along the equilibrium path underlines the process of energy substitution – renewable energy for fossil fuels. Finally, the North will import emissions permits and export the consumption good to pay for these permits. Intuitively, one might suspect that the South with its high population will burn more fossil fuels to feed its population. However, the model assumes a lower technological level for the South and an allocation of emissions permits on an equitable per capita basis. The former assumption implies a lower energy input per worker, while the latter assumption implies an equal number of permits per worker, leading to the result that there is an excess demand for permits per worker in the North. This allocation mechanism allows the South to expropriate part of the output of the consumption good produced by the North during the initial periods.

Essay One

THE COST OF REDUCING GREENHOUSE GAS EMISSIONS FROM THE INDUSTRIAL SECTORS IN CANADA

1. INTRODUCTION

ON DECEMBER 2002, Canada ratified the Kyoto Protocol, committing herself – during the period between 2008 and 2012 – to reduce greenhouse gas emissions by 6 percent below the 1990 levels. This translates into an emission target of 571 mega tonnes of carbon dioxide equivalent, which to be achieved during the first commitment period.[1] To reach the target, an emission reduction of approximately 240 mega tonnes must be achieved given the projected greenhouse gas emissions in year 2010 of 810 mega tonnes under the "business as usual" case. To fulfill its international commitment, the Government of Canada has identified the sectors to be targeted and the reduction to be achieved by each sector. The large final emitters are the sectors that would be required to achieve a significant reduction during the first period of commitment. These sectors consist of thermal electricity; oil and gas, such as crude oil production, natural gas distribution and transmission; mining; and manufacturing, such as cement, pulp and paper, non-ferrous smelting and refining, aluminum, potash, nitrogen fertilizer, and lime. Together, they would be responsible for an annual reduction of 45 mega tonnes of carbon dioxide equivalent starting from 2008 through 2010.

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[1] Climate Change for Canada, 2002, p. 59.

Many sectors have raised their concerns about the impact of the climate-change policy on their own competitiveness. Sectors that compete on the international market, such as pulp and paper, argued that they will lose their market shares to their US competitors because the US producers are not subject to similar regulatory requirements. Added to the fear of increased competition is the uncertainty surrounding the cost of meeting these obligations. The large final emitters are made up of about 800 companies, and these companies have various compliance options. Some of the options involve changing the input mix – moving away from high-carbon content coal to low-carbon content coal; buying international project-based credits and Assigned Amount units, etc. Other options involve making changes, such as retrofitting equipment and installing abatement technology. As for greenhouse gas abatement technology, it is not always available to all sectors. On the other hand, retrofitting equipment or investing in abatement technology often entails significant capital expenditures, and the streams of benefits yielded by such investments are uncertain. The uncertainty arises from the ambiguity of these industries' post-2012 obligations, which are not resolved until the next rounds of the Kyoto negotiation. Therefore, it is difficult for firms to determine which solution is the most cost-effective.

Thus it is not surprising to witness the vociferous opposition to the ratification of the Kyoto Protocol by the industries that are the sources of heavy greenhouse gas emissions. In their efforts to lessen the impact on their own sectors of the government's policy on climate change, the owners of the sector-specific capital stocks in these industries have lobbied policy makers vigorously. As a result, flexibility mechanisms are to be put in place to help the industries to comply with their emission targets. One of the flexibility mechanisms that the Government of Canada provides to the large final emitters in achieving the reduction target is the Price Assurance Mechanism (PAM). The Price Assurance Mechanism assures that it will not cost the sectors involved more than $15 per tonne to meet their legal obligations during the Kyoto compliance period, 2008-2012. To implement this policy the government intends to issue PAM units and sell them at $15 per unit. One unit allows one tonne of greenhouse gases to be emitted. As discussed above, other compliance options available to large final emitters are retrofitting equipments, purchasing Assigned Amount units, and installing abatement equipments, etc.

The price policy of $15 per tonne was formulated to secure Canadian competitiveness through various debates between the industry stakeholders and the government. The government's evaluation of the overall impacts of the Canadian climate-change policy was based on two carbon prices: the best scenario price of $10 per tonne and the most pessimistic scenario price of $50 per tonne.[2] For example, at the sectoral level, it is expected that the climate-change policy would have a significant impact on sectors, such as coal-fired plants, cement, and lime. Based on the assumption of a permit price of $10 per tonne and 85 percent free permit allocation and hypothetical emissions intensities for the polluting sectors in year 2010, the climate-change policy would increase the unit cost of production by CA$1.18, CA$1.85, and CA$4.73 for the cement, lime, and aluminum

[2] Climate Change for Canada, 2002, p. 31.

sectors, respectively.[3] Note that the results are highly sensitive to the assumed greenhouse gas emissions reductions, the assumptions, and the methodologies employed.

To comply with the Kyoto Protocol target, the government implements a domestic policy – an environmental regulation that requires the polluting sector to remit carbon credits and/or PAM units (discussed further below) equal to the amount of greenhouse gases the polluting sector releases during the Kyoto compliance period, 2008-2012. In the model we formulate, it is assumed that control options such as internal reduction of greenhouse gases are not feasible for the dirty-good sector.

Carbon credits are issued and sold by the Kyoto community.[4] They are the only units that can be used for Kyoto compliance and are only available in the international carbon market for trading when the Kyoto compliance period begins. It is unclear as to how other member countries are meeting their targets, and hence the future prices of a carbon credit are highly uncertain.[5] To gain the acceptance from the polluting sector, the government is committed to reduce the uncertainty surrounding the compliance cost to the polluting sector by setting a price for one tonne of greenhouse gases emitted during the Kyoto compliance period. In trying to influence the policy being formulated, the polluting sector lobbies the government for favorable policies by offering political contributions.

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[3] Ibid, p. 31.
[4] The carbon trading market is exogenous to the model.
[5] Member countries will have to develop their own domestic policies to achieve their Kyoto targets.

Now the traditional view on the behavior of the State is that the government behaves as a benevolent dictator, carrying out economic policies to maximize social welfare. However, in reality to achieve political goals, governments often implement policies that distort the allocation of resources and affect the distribution of income. The new political economy of endogenous policy formation offers several explanations for these distorting policies. According to Hillman (1982), Long and Vousden (1991); the government carries out a distorting policy to maximize – not economic efficiency – but political support. Brock and Magee (1978) suggested that it is the electoral competition that the cause of distorting policies. Mayer (1984) pointed to the factor-ownership distribution cum voter participation costs under majority voting as the driving force behind the endogenous policy formation. Recently, Grossman and Helpman (1994, GH hereafter) applied the theoretical machinery of a first-price menu auction developed by Bernheim and Whinston (1986, BW hereafter), propounded the influence-buying approach of endogenous policy formation: special-interest groups offer payments to policy makers to obtain favorable policies.

In this essay, we formulate a model to explain how the price of a PAM unit is set by the government from the perspective of influence buying[6] in a small open economy. There are two production sectors in the economy, and each sector produces a freely traded good. The two traded goods are called the clean good and the dirty good. The clean good is produced from labor and capital. The dirty good is also produced from labor and

[6] Fredriksson and Sterner (2005) employed the influence-buying model developed by GH to study the impacts of lobbying on an environmental tax in an institution that has refunded emissions payment programs (REPs). Under these programs, the tax revenue is recycled to provide incentive to reduce greenhouse gas emissions.

capital. However, the production of the dirty good requires a third input – coal – which is the source of greenhouse gas emissions in this sector. Coal mining requires only labor, and the coal output is assumed to be non-traded. Following Samuelson (1971) and Jones (1971), we shall assume that capital is sector-specific. The production structure in our model is a simpler version of that found in Corden and Neary (1982), who drew on Jones, op cit., in formulating a model for analyzing the problem known as the Dutch disease.

In the model, the owners of the specific factor used in the production of the dirty good see their incomes tied to the ratification of the Kyoto Protocol have an incentive to form a special-interest group to lobby policy makers for policies that lessen the adverse impact of the Kyoto Protocol. Furthermore, because this group is highly concentrated, it is easier and less costly for its members to get organized as a special-interest group. It is the lobbying activities of the owners of the specific factor in the sector producing the dirty good that drive the results of our model.

The model we formulate is a four-stage game whose extensive form can be described as follows. In the first stage, the polluting sector communicates a contingent payment schedule to the government, with the payment depending on the price of a PAM unit set by the government. In the second stage, the government takes the contingent payment schedule as given, and sets a price of a PAM unit to maximize its objective function. The objective function of the government depends, of course, on the payment it will receive. Recognizing that the government is not without a conscience, the model assumes that social welfare – taken to be the sum of the net payoffs of all the groups in the economy –

is also a component of the objective function of the government. As is the tradition in the influence-buying approach, the preferences of consumers in our model are assumed to be quasi-linear. It is in the third stage that the government implements the price policy by issuing PAM units and sells them at the set price. However, by selling PAM units the government is assuming the responsibility of buying carbon credits to meet the Kyoto obligation, as PAM units are artificially created by the government, and do not represent real greenhouse gases emissions permits. Hence, the government will lose if the realized market price of a carbon credit is higher than that of a PAM unit, and gain in the opposite event. The model assumes that the government maintains a balanced budget; that is, if it loses, it will tax the citizens to pay for the shortfall, and if it gains it will distribute the profits to the citizens. The government is also assumed to impose a condition on the polluting sector that it must decide on the number of PAM units it wishes to purchase before the compliance period begins. That is, the purchasing plan of PAM units must be made and executed before the uncertainty on the price of a carbon credit is resolved. Under these restrictions, and given the price policy set in the second stage, the polluting sector purchases PAM units in the third stage of the game. In the model, it is assumed that the polluting sector is risk neutral, so that the number of PAM units purchased in the third stage maximizes the expected rent net of the cost of the PAM units purchased. In the fourth stage, the uncertainty on the price of carbon is resolved. All sectors carry out their production plans to maximize their profits. If the number of PAM units purchased in the third stage is not sufficient to cover the greenhouse gas emissions generated by the output of the dirty good, then the polluting sector must enter the international market for carbon to purchase the additional emissions permits needed. An additional unit of the dirty good beyond the level that can be covered by the number of PAM units already purchased thus means incremental cost in complying with the Kyoto Protocol and incremental costs in inputs – labor and coal – used. In the rent maximization problem faced by the polluting sector, the incremental cost in complying with the environmental regulation causes a discrete jump in the marginal cost curve of the polluting sector, and this jump is the source of considerable complications in finding the general equilibrium of the small open economy in the fourth stage.

The main contributions of the essay can be described as follows. Under the benchmark – when the polluting sector is unrestrained and can discharge any amount of greenhouse gas into the atmosphere without having to pay any cost – the output of the dirty good is at its highest level possible and GDP is maximized, but social welfare is not. If the countries that ratify the Kyoto Protocol as a group adheres to the cap imposed by this international environmental treaty, then the amount of greenhouse gas generated by the polluting sector under the benchmark will add to the cap, and thus raises the disutility of environmental degradation borne by the residents of the small open economy. This is the content of Proposition 1. If the government wants to adopt this policy, then it should take as given the cap and tries to find the level of the dirty good that maximizes GDP net of the disutility of environmental degradation, not the policy of allowing the sector producing the dirty good to pollute the atmosphere with impunity.

If the government adopts the policy of ratifying the Kyoto Protocol and lets the polluting sector pay the entire cost of complying with this international environmental treaty, then

GDP net of the country's cost of purchasing the needed carbon credits in the international market is maximized. That is, the policy of non-intervention after ratifying the Kyoto Protocol is socially optimal for the small open economy. This important result is stated as Proposition 3. There is thus no justification on purely economic grounds to help lessening the burden of compliance borne by the polluting sector. The policy of selling PAM units to the sector producing the dirty good can only be explained by the lobbying efforts of this sector.

The main results of the essay are stated in Proposition 6, which asserts that the price of a PAM unit set by the government is below the expected price of carbon in the international market. The putative justification for selling PAM units is to provide insurance – for the polluting sector – against the uncertainty in the price of carbon credits. This is certainly a valid argument if one assumes that this sector is risk averse. Then selling PAM units at the price equal to the expected price of carbon will certainly be welcomed by the polluting sector. However, when the polluting sector is risk neutral, as is assumed in our model, the polluting sector is only willing to purchase PAM units at a price below the expected price of carbon credits in the international market. The expected green fund is thus negative, and, a fortiori, the expected transfer to each individual in the small open economy. Through its political contributions, the polluting sector manages to induce the latter player to set the price of a PAM unit below the expected price of carbon in the international market. Relative to the policy under which the polluting sector must bear the entire cost of compliance, the owners of the specific factor used in the production of the clean good unambiguously lose. The loss comes from two sources.

First, for any realized price of carbon in the international market, the output of the dirty good is higher with PAM than without PAM, which in turn implies that the equilibrium wage rate is higher with PAM than without PAM. The higher wage rate will reduce the rent earned by the specific factor in the clean-good sector. Second, the negative expected transfer adds to the loss caused by the higher wage rate. As for the workers, the higher wage rate raises their income, but the negative expected transfer works in the opposite direction. Without a more detailed analysis, it is not possible to tell if the workers are better-off or worse-off with PAM, although we suspect that the workers will be worse-off. This is indeed the case for the numerical example presented in Section 11.

The essay is organized as follows. In Section 2, the structure of the economy – technology and preferences – are described. Section 3 presents the analysis for the scenario – called the benchmark – under which the government does not subscribe to the Kyoto Protocol, and allows the polluting sector to pollute the atmosphere unrestrained and with impunity. Section 4 presents the analysis under the scenario that the government ratifies the Kyoto Protocol, but does not carry out any policy to lessen the impact of this international environmental treaty. This is the worst-case scenario for the polluting sector because it has to bear the entire burden of complying with the Kyoto Protocol. The game of endogenous climate-change policy is presented in Section 5. The game formulated is a four-stage game, and its solution is presented in Sections 6 through 10. More specifically, the fourth-stage of the game is solved in Section 6. It is in this section that the impact of the climate-change policy on the general equilibrium allocation of resources and the impact on the environment is analyzed in detail. Section 7 contains the solution of the third stage of the game. It is in this section that the determination of the number of PAM units purchased by the polluting sector is analyzed. The equilibrium of the game is defined in Section 9. The determination of the price of PAM units set by the government in the second stage of the game is presented in Section 10. A numerical example to illustrate the main results of the game is presented in Section 11. Some concluding remarks are given in Section 12.

2. THE STRUCTURE OF THE SMALL OPEN ECONOMY

2.1. *The Production Sectors*

Consider a small open economy in which two goods – called good 0 and good 1 – are produced. Good 0, which is taken as the numéraire, is produced with the help of labor and capital. Good 1 is produced from labor, capital, and coal. It is the burning of coal to provide the energy input needed in the production of good 1 that is the source of greenhouse gases in the model. Coal mining is carried out by perfectly competitive firms, using labor as the only input. In what follows, we shall refer to good 0 as the clean good and good 1 as the dirty good. Both goods are assumed to be freely traded.

Labor is assumed to be homogenous and perfectly mobile. Its fixed supply is denoted by $\bar{L}$. Capital, on the other hand, is sector-specific, and the capital stock in sector i is denoted by $\bar{K}_i, i = 0,1$.

The output of good 0 is assumed to be given by the following production function:

(1) $Y_0 = \overline{K}_0^{\alpha_0} L_0^{1-\alpha_0}$,

where Y_0 is the output; L_0 is the labor input; and $\alpha_0, 0 < \alpha_0 < 1$, is a parameter. The technology used in the production of the dirty good is assumed to be given by the following production function:

(2) $Y_1 = \min\left[\overline{K}_1^{\alpha_1} L_1^{1-\alpha_1}, C\right]$

where Y_1 is the output; L_1 is the labor input; and C is the coal input. Also, $\alpha_1, 0 < \alpha_1 < 1$, is a parameter. As specified, the technology used to produce the dirty good is a nested production function with two levels. At the first level, labor and capital are combined according to a Cobb-Douglas production function to yield a composite factor. At the second level, the composite factor is combined with a fossil fuel in a fixed proportion to produce the dirty good.

As for coal mining, it is assumed that one unit of labor is able to mine a tonnes of coal, and that the coal mining sector is perfectly competitive. Thus if we let L_2 denote the labor input used in the coal mining sector, then the output of coal is given by

(3) $Y_2 = aL_2$.

To keep the exposition from becoming burdensome, we shall assume that coal is not traded. Furthermore, it is assumed that the burning of each tonne of coal discharges ε tonnes of greenhouse gases into the atmosphere. The amount of greenhouse gases generated by burning C tonnes of coal is thus equal to εC.

Let N be the size of the population of the small open economy. We shall assume that N is a continuum of measure 1. Each individual in the population is assumed to own only one type of input – labor or a specific input, but not both. We shall refer to the owners of the specific input used in the production of good 0 and the owners of the specific input used in the production of good 1, respectively, as group 0 and group 1. The workers – as a group – will be referred to as group 2. For each $i = 0,1,2$, let γ_i denote the size of group i relative to the total population. Thus group 0 and group 1 obtain their incomes through the ownership of sector-specific capital, while group 2 obtains its income by selling its labor. The aggregate labor supply in the small open economy is thus given by $\overline{L} = \gamma_2$. We expect that γ_2 is large relative to $\gamma_i, i = 0,1$.

2.2. *The Government*

The objective of the government is to choose an environmental policy – represented by the price of PAM units – that maximizes social welfare, taken to be the sum of the payoffs of all the groups in the small open economy. The government plans to issue PAM units and sell them at a predetermined price to the polluting industries. Let A and p_A denote, respectively, the number of PAM units that the government issues, based on the submissions of purchase plans[7] and the price of a PAM unit. One PAM unit entitles its owner to emit one tonne of greenhouse gases. We assume that if a firm in sector 1 chooses to buy PAM units, it is required to submit a purchase plan at the beginning of the compliance period. The purchase plan details the number of PAM units

[7] Hence, A is also the number of carbon credits that the Government would have to purchase to comply with the Kyoto agreement.

the firm wishes to purchase. Once submitted and approved, the firm must pay for those units, regardless of whether or not those units would be used for compliance. PAM units cannot be banked or traded. Consequently, the cost of PAM units could be considered as sunk cost, once committed by the firm.

To comply with the Kyoto Protocol at the end of the year, the government has to buy carbon credits from the carbon markets at a price, say p_{Ca}, which is probably higher than p_A, and this entails a loss for the government. However, if the price of a carbon credit is less than p_A, the government would make a profit from selling PAM units to the polluting sector at a price higher than the price it pays for these PAM units in the international carbon markets. If the price of a carbon, p_{Ca}, is the same as the price of a PAM unit, p_A, the government will break even, assuming that there is no administrative cost associated with issuing and selling PAM units. The price of carbon in the international carbon markets depends on the availability of and the demand for those compliance units, and is not known at the time the government issues PAM units. In what follows, we shall assume that denote p_{Ca} is a random variable with distribution function $F(p_{Ca})$. We shall also assume that the government is a price taker on the international markets of carbon credits.

The green fund – or profit – of the government is given by

(4) $$T = (p_A - p_{Ca})A.$$

If it turns out that $p_A < p_{Ca}$, then the green fund is negative, and the government – in its effort to pay for the loss involved in selling PAM units – will have to reduce its spending on some of the current programs or impose a green tax on the polluting good. For our purpose, we shall assume that the green fund – positive or negative – is redistributed equally to all the individuals in the economy, with the interpretation that a positive green fund involves a transfer, while a negative green fund suggests a tax. This assumption ensures that the public budget is always balanced.

2.3. *Preferences and Utility Maximization*

We suppose that individual preferences are identical and represented by the following quasi-linear utility function: $(x_0, x_1, E) \rightarrow x_0 + u_1(x_1) - u_2(\overline{E})$, where x_i is the consumption of good $i, i = 0,1$, and $\overline{E}$ is the cap on greenhouse gases imposed by the Kyoto Protocol. Also, u_1 is the sub-utility function associated with the consumption of the dirty good, and u_2 is the disutility function associated with the stock of greenhouse gases.[8] We shall assume that $\overline{E}$ is exogenous; that is, an increase in greenhouse gas emissions by Canada entails a reduction – of the same magnitude – in the greenhouse gas emissions of the rest of the world. Thus under the Kyoto Protocol, the disutility caused by

[8] Climate change could lead to loss of biodiversity, rise in sea level, damage to infrastructures, and loss of agricultural productivity. These are production externalities caused by an increase in the stock of greenhouse gases in the atmosphere. On the consumption side, climate change could result in the emergence or exacerbation of a large number of potential public health problems, such as heat-induced mortality and the spread of malaria and dengue fever. In this essay, we choose to focus on the consumption externalities of climate change. Production externalities are modeled in Essays Two and Three.

the stock of greenhouse gas emissions can be taken as exogenous. We assume that u_1 is continuously differentiable, strictly increasing, and strictly concave.

An individual with an income m at his disposal solves the following utility maximization problem:

(5) $\max_{x_1}\left[m - p_1 x_1 + u_1(x_1) - u_2(\bar{E})\right].$

Assuming that the above problem has an interior solution, we have the following expression for the demand for good 1:

(6) $x_1(p_1) = [u_1']^{-1}(p_1).$

The indirect utility function of an individual is then given by

(7) $v(p_1, m, E) = m + s_1(p_1) - u_2(\bar{E}),$

where

(8) $s_1(p_1) = u_1(x_1(p_1)) - p_1 x_1(p_1)$

is the consumer surplus enjoyed by this individual. Observe that the consumer surplus is constant because of free trade and because the price of the dirty good on the world market, namely p_1, is given. Hence the utility enjoyed by an individual will only change when her income, m, or the amount of greenhouse gases generated by the polluting sector, namely E, changes.

3. THE BENCHMARK: GENERAL EQUILIBRIUM UNDER THE ASSUMPTION THAT NO RESTRAINTS AND NO COSTS ARE IMPOSED ON THE POLLUTING SECTOR FOR ITS GREENHOUSE GAS EMISSIONS

In this sub-section, we find the competitive equilibrium under the scenario that the government refuses to sign the Kyoto Protocol. The sector producing the dirty good is then unrestrained in its greenhouse gas emissions and does not have to pay for its polluting activities.

3.1. *Profit Maximization in the Sector Producing the Clean Good*[9]

Let p_L denote the prevailing wage rate. The sector producing the clean good solves the following profit maximization problem:

(9) $$\max_{L_0}\left[\overline{K}_0^{\alpha_0} L_0^{1-\alpha_0} - p_L L_0\right] = \Pi_0(p_L).$$

The demand for labor by the sector producing the clean good is

(10) $$L_0(p_L) = \overline{K}_0\left(\frac{1-\alpha_0}{p_L}\right)^{1/\alpha_0}.$$

The output of the clean good is

(11) $$Y_0(p_L) = \overline{K}_0\left(\frac{1-\alpha_0}{p_L}\right)^{(1-\alpha_0)/\alpha_0}.$$

Using (9), (10), and (11), we obtain the following expression for the profit of the sector producing the clean good:

(12) $$\Pi_0(p_L) = \alpha_0\overline{K}_0\left(\frac{1-\alpha_0}{p_L}\right)^{(1-\alpha_0)/\alpha_0}.$$

3.2. *Profit Maximization by the Sector Producing the Dirty Good*

[9] Note that the profit maximization problem faced by the sector producing the clean good does not depend on the environmental policy adopted by the government.

If Y_1 is the output of the dirty good, then the labor input and the coal input needed to produce this output are given, respectively, by $L_1 = \left(Y_1/\overline{K}_1^{\alpha_1}\right)^{1/(1-\alpha_1)}$ and $C = Y_1$. The cost of inputs – labor and coal – needed to produce Y_1 units of the dirty good is given by

$$(13) \quad \Gamma(Y_1, p_L) = p_L \overline{K}_1^{-\alpha_1/(1-\alpha_1)} Y_1^{1/(1-\alpha_1)} + \frac{p_L}{a} Y_1.$$

Note that on the right side of (13), the first term represents the labor cost, while the second term represents the cost of the coal input. Furthermore, the cost of the coal input has been obtained by using the fact that p_L / a is the equilibrium price of coal, given the wage rate p_L.

When the polluting sector is free to emit greenhouse gases into the atmosphere, it solves the following profit maximization problem:

$$(14) \quad \max_{Y_1} \left[p_1 Y_1 - \Gamma(Y_1, p_L)\right] = \Pi_1(p_L).$$

The solution of the profit maximization problem (14) is given by

$$(15) \quad Y_1(p_L) = \overline{K}_1 \left[\left(\frac{p_1}{p_L} - \frac{1}{a}\right)(1-\alpha_1)\right]^{\frac{1-\alpha_1}{\alpha_1}}.$$

Also, the demand for labor by the polluting sector is given by

$$(16) \quad L_1(p_L) = (1-\alpha_1)^{\frac{1}{\alpha_1}} \overline{K}_1 \left(\frac{p_1}{p_L} - \frac{1}{a}\right)^{\frac{1}{\alpha_1}}.$$

Using (15) and (16), we obtain the following expression for the profit of the polluting sector under the scenario that the government does not ratify the Kyoto Protocol:

$$(17) \quad \Pi_1(p_L) = \overline{K}_1 p_L \left(\frac{\alpha_1}{1-\alpha_1}\right) \left\{(1-\alpha_1)\left(\frac{p_1}{p_L} - \frac{1}{a}\right)\right\}^{1/\alpha_1}.$$

From the profit maximization problem of coal producing sector, we obtain the following for the labor demand:

$$(18)\qquad L_2(p_L) = \frac{\overline{K}_1}{a}\left\{(1-\alpha_1)\left(\frac{p_1}{p_L} - \frac{1}{a}\right)\right\}^{(1-\alpha_1)/\alpha_1}.$$

The aggregate demand for labor – as a function of the wage rate – is then given by

$$L(p_L) = L_0(p_L) + L_1(p_L) + L_2(p_L)$$

$$(19)\qquad = \overline{K}_0\left(\frac{1-\alpha_0}{p_L}\right)^{1/\alpha_0} + (1-\alpha_1)^{\frac{1}{\alpha_1}}\overline{K}_1\left(\frac{p_1}{p_L} - \frac{1}{a}\right)^{\frac{1}{\alpha_1}} + \frac{\overline{K}_1}{a}\left\{(1-\alpha_1)\left(\frac{p_1}{p_L} - \frac{1}{a}\right)\right\}^{(1-\alpha_1)/\alpha_1}.$$

Observe that $L(p_L)$ is continuous and strictly decreasing from $+\infty$ to 0 as p_L rises from 0 to infinity. Hence there exists a unique value of p_L, say $\overline{p}_L$, such that

$$(20)\qquad L(\overline{p}_L) = \overline{K}_0\left(\frac{1-\alpha_0}{p_L}\right)^{1/\alpha_0} + (1-\alpha_1)^{\frac{1}{\alpha_1}}\overline{K}_1\left(\frac{p_1}{p_L} - \frac{1}{a}\right)^{\frac{1}{\alpha_1}} + \frac{\overline{K}_1}{a}\left\{(1-\alpha_1)\left(\frac{p_1}{p_L} - \frac{1}{a}\right)\right\}^{(1-\alpha_1)/\alpha_1} = \gamma_2.$$

The wage $\overline{p}_L$ is the equilibrium wage rate under the policy of non-intervention. In what follows, we shall let

$$(21)\qquad \overline{Y}_0 = Y_0(\overline{p}_L) = \overline{K}_0\left(\frac{1-\alpha_0}{\overline{p}_L}\right)^{(1-\alpha_0)/\alpha_0},$$

$$(22)\qquad \overline{Y}_1 = Y_1(\overline{p}_L) = \overline{K}_1\left[\left(\frac{p_1}{\overline{p}_L} - \frac{1}{a}\right)(1-\alpha_1)\right]^{\frac{1-\alpha_1}{\alpha_1}},$$

and

(23) $\overline{GDP} = \overline{Y}_0 + p_1\overline{Y}_1$

denote, respectively, the output of the clean good, the output of the dirty good, and the GDP of the economy under the assumption that the government refuses to sign the Kyoto Protocol.

The following proposition describes the implications of the decision not to sign the Kyoto protocol.

PROPOSITION 1: *GDP is maximized under the benchmark equilibrium. However, maximizing GDP by allowing the sector producing the dirty good a free reign in polluting the atmosphere leads to an excessive emission of greenhouse gases if the rest of the world does not deviate from the cap on greenhouse gas emissions imposed by the Kyoto Protocol. That is, in the first-order approximation, a slight reduction in the output of the dirty good has zero impact on GDP, but a positive on the quality of the global environment, and the end result is a net gain in social welfare.*

PROOF: To prove the first part of the proposition, we use reductio ad absurdum. If Proposition 1 is not true, then we can find a list of feasible outputs, say (Y_0, Y_1), such that

(24) $Y_0 + p_1Y_1 > \overline{Y}_0 + p_1\overline{Y}_1$.

Let L_0, L_1, and L_2 denote, respectively, the labor inputs used in the production of the clean good, in the production of the dirty good, and in coal mining. Then the equilibrium condition in the labor market is

(25) $$L_0 + L_1 + L_2 = L_0(\overline{p}_L) + L_1(\overline{p}_L) + L_2(\overline{p}_L) \\ = \gamma_2.$$

Multiplying the first line of (25) by $\overline{p}_L$, then subtracting the result from (24), we obtain

(26) $$[Y_0 - \overline{p}_L L_0] + [p_1 Y_1 - \overline{p}_L L_1 - \frac{\overline{p}_L}{a} a L_2] \\ > [\overline{Y}_0 - \overline{p}_L L_0(\overline{p}_L)] + [p_1 \overline{Y}_1 - \overline{p}_L L_1(\overline{p}_L) - \frac{\overline{p}_L}{a} a L_2(\overline{p}_L)].$$

Inequality (26) can be rewritten as follows:

(27) $$[Y_0 - \overline{p}_L L_0] - [\overline{Y}_0 - \overline{p}_L L_0(\overline{p}_L)] \\ > [p_1 \overline{Y}_1 - \overline{p}_L L_1(\overline{p}_L) - \frac{\overline{p}_L}{a} a L_2(\overline{p}_L)] - [p_1 Y_1 - \overline{p}_L L_1 - \frac{\overline{p}_L}{a} a L_2].$$

Observe that when the equilibrium wage $\overline{p}_L$ prevails, the expression $[Y_0 - \overline{p}_L L_0]$ represents the rent earned by the specific factor used in the production of the clean good if the labor input in this sector is L_0. Furthermore, because L_0 is not necessarily the optimal labor input, this rent cannot exceed $[\overline{Y}_0 - \overline{p}_L L_0(\overline{p}_L)]$, the maximum rent that the specific factor earns under the competitive equilibrium. Hence the left side of inequality (27) is less than or equal to 0. In the same manner, observe that on the right side of inequality (27) the expression inside the first pair of square brackets represents the rent earned by the specific input used in the production of the dirty good under the competitive equilibrium, while the expression inside the second pair of square brackets represents the rent earned by this specific factor under the production plan that gives rise to Y_1. Hence the right side of inequality (27) must be greater than or equal to 0. The arguments just presented thus lead to the following chain of inequalities:

(28) $$0 \geq [Y_0 - \overline{p}_L L_0] - [\overline{Y}_0 - \overline{p}_L L_0(\overline{p}_L)] \\ > [p_1 \overline{Y}_1 - \overline{p}_L L_1(\overline{p}_L) - \frac{\overline{p}_L}{a} a L_2(\overline{p}_L)] - [p_1 Y_1 - \overline{p}_L L_1 - \frac{\overline{p}_L}{a} a L_2] \geq 0,$$

which are inconsistent. We have just proved that GDP is maximized under the benchmark equilibrium. To prove the statement on welfare, note that starting from the benchmark equilibrium a slight reduction, say $\Delta Y_1 < 0$, in the output of the dirty good has zero impact on GDP in the first order, but will reduce the global greenhouse gas emissions by the amount $\varepsilon \Delta Y_1$, leading to a variation in the disutility due to greenhouse gas emissions of $u_2'(\overline{E} + \varepsilon \overline{Y}_1)\varepsilon \Delta Y_1 < 0$. ■

Before ending this section, we derive some technical results – stated under the form of a lemma – that will prove useful later. Thus let Y_1 be a feasible level of output for the dirty good. Then the labor input that the polluting sector uses in producing Y_1 is given by $L_1 = \left(Y_1 / \overline{K}_1^{\alpha_1}\right)^{1/(1-\alpha_1)}$, and the labor input used in the mining sector to produce the coal input in producing Y_1 is Y_1 / a. The output of the clean good, given Y_1, is then equal to

(29) $$Y_0(Y_1) = \overline{K}_0^{\alpha_0}\left(\gamma_2 - [Y_1 / \overline{K}_1^{\alpha_1}]^{1/(1-\alpha_1)} - Y_1 / a\right)^{1-\alpha_0}.$$

Equation (29) represents the production frontier of the small open economy. Observe that this frontier is downward sloping and strictly concave, which leads to the following result:

LEMMA 1: *Let Y_1 be a feasible level for the output of the dirty good and $Y_0(Y_1)$, as defined by* (29)*, be the output of the clean good. Also, let*

(30) $$GDP(Y_1) = Y_0(Y_1) + p_1 Y_1$$

denote the gross domestic product associated with Y_1. Then

(a) $Y_0(Y_1)$ *is strictly decreasing and strictly concave in Y_1; and*

(b) *$GDP(Y_1)$ is strictly concave in* Y_1.

4. GENERAL EQUILIBRIUM UNDER THE KYOTO PROTOCOL: THE POLLUTING SECTOR BEARS THE ENTIRE COST OF EMISSIONS PERMITS

Suppose that the government signs the Kyoto protocol, but does not carry out any policy to help lessening the cost of compliance by the polluting sector. Under this scenario, the polluting sector solves the following profit maximization problem:

(31) $\max_{Y_1}\left[p_1 Y_1 - \Gamma(Y_1, p_L) - p_{Ca}\varepsilon Y_1\right] = \Pi_1(p_L, p_{Ca}),$

where p_{Ca} is the realized price of carbon in the international market.

In what follows, we shall assume that p_{Ca} is a random variable taking on values inside a compact interval $[\underline{p}_{Ca}, \overline{p}_{Ca}]$, with $0 < \underline{p}_{Ca} < \overline{p}_{Ca}$, of the real line. The distribution of p_{Ca} will be denoted by $F(p_{Ca})$. It is assumed that this distribution function has a density $f(p_{Ca})$, which is continuous and strictly positive on the interval $[\underline{p}_{Ca}, \overline{p}_{Ca}]$, In what follows, we shall denote the expected price of carbon in the international market by $E[p_{Ca}]$.

The solution of the profit maximization problem (31) is given by

(32) $$Y_1(p_L, p_{Ca}) = \left[\left(\frac{p_1 - \varepsilon p_{Ca}}{p_L} - \frac{1}{a}\right)(1-\alpha_1)\right]^{\frac{1-\alpha_1}{\alpha_1}} \overline{K}_1.$$

Also, the demand for labor by the polluting sector is given by

(33) $$L_1(p_L, p_{Ca}) = (1-\alpha_1)^{\frac{1}{\alpha_1}} \overline{K}_1 \left(\frac{p_1 - \varepsilon p_{Ca}}{p_L} - \frac{1}{a} \right)^{\frac{1}{\alpha_1}}.$$

Using (32) and (33), we obtain the following expression for the profit of the polluting sector under the scenario that the government does not ratify the Kyoto Protocol:

(34) $$\Pi_1(p_L, p_{Ca}) = \overline{K}_1 p_L \left(\frac{\alpha_1}{1-\alpha_1} \right) \left\{ (1-\alpha_1) \left(\frac{p_1 - \varepsilon p_{Ca}}{p_L} - \frac{1}{a} \right) \right\}^{1/\alpha_1}.$$

From the zero profit condition in the coal producing sector, we obtain the following expression for its labor demand:

(35) $$L_2(p_L, p_{Ca}) = \frac{\overline{K}_1}{a} \left\{ (1-\alpha_1) \left(\frac{p_1 - \varepsilon p_{Ca}}{p_L} - \frac{1}{a} \right) \right\}^{(1-\alpha_1)/\alpha_1}.$$

The aggregate demand for labor – as a function of the wage rate – is then given by

(36) $$\begin{aligned} L(p_L, p_{Ca}) &= L_0(p_L, p_{Ca}) + L_1(p_L, p_{Ca}) + L_2(p_L, p_{Ca}) \\ &= (1-\alpha_0)^{1/\alpha_0} \overline{K}_0 \left(\frac{1}{p_L} \right)^{1/\alpha_0} + (1-\alpha_1)^{\frac{1}{\alpha_1}} \overline{K}_1 \left(\frac{p_1 - \varepsilon p_{Ca}}{p_L} - \frac{1}{a} \right)^{\frac{1}{\alpha_1}} \\ &\quad + \frac{\overline{K}_1}{a} \left\{ (1-\alpha_1) \left(\frac{p_1 - \varepsilon p_{Ca}}{p_L} - \frac{1}{a} \right) \right\}^{(1-\alpha_1)/\alpha_1}. \end{aligned}$$

Observe that $p_L \to L(p_L, p_{Ca})$ is continuous and strictly decreasing from $+\infty$ to 0 as p_L rises from 0 to infinity. Hence there exists a unique value of p_L, say $p_L(p_{Ca})$, such that

(37) $$\begin{aligned} L(p_L(p_{Ca})) &= (1-\alpha_0)^{1/\alpha_0} \overline{K}_0 \left(\frac{1}{p_L(p_{Ca})} \right)^{1/\alpha_0} + (1-\alpha_1)^{\frac{1}{\alpha_1}} \overline{K}_1 \left(\frac{p_1 - \varepsilon p_{Ca}}{p_L(p_{Ca})} - \frac{1}{a} \right)^{\frac{1}{\alpha_1}} \\ &\quad + \frac{\overline{K}_1}{a} \left\{ (1-\alpha_1) \left(\frac{p_1 - \varepsilon p_{CA}}{p_L(p_{Ca})} - \frac{1}{a} \right) \right\}^{(1-\alpha_1)/\alpha_1} \\ &= \gamma_2. \end{aligned}$$

The wage $p_L(p_{Ca})$ is the equilibrium wage rate that prevails under the assumption that the polluting sector bears the full burden of purchasing the greenhouse gas emissions to comply with the Kyoto Protocol, given that the realized price of carbon in the international market is p_{Ca}. The following proposition describes the impact of the realized price of carbon on the small open economy when the cost of complying with the Kyoto Protocol falls entirely on the polluting sector.

PROPOSITION 2: *Suppose that the government signs the Kyoto Protocol, but does not carry out any program to lessen the compliance cost borne by the polluting sector. Then a rise in the realized price of carbon in the international market induces a fall in the equilibrium wage rate. The fall in the equilibrium wage rate causes labor to move out of the sector producing the dirty good and the coal mining sector in to the sector producing the clean good, causing the latter sector to expand and the former sectors to contract.*

PROOF: A rise in p_{Ca} shifts the aggregate demand for labor $p_L \to L(p_L, p_{Ca})$ downward, resulting in a fall of the equilibrium wage rate. A lower equilibrium wage rate will induce the sector producing the clean good to use more labor, causing this sector to expand. Given a fixed labor supply, the increase labor input in this sector implies a reduction in the labor inputs of the polluting sector and the coal mining sector, causing these two sectors to contract. ■

The following proposition gives a characterization of the equilibrium when the polluting sector has to bear the full cost of complying with the Kyoto Protocol.

PROPOSITION 3: *Suppose that the government signs the Kyoto Protocol, but is not willing to help alleviating the cost of complying with this international environmental treaty. Then for the small open economy GDP, net of the total costs of emissions permits purchased in the international market, is maximized under the competitive equilibrium. Furthermore, given quasi-linear preferences, social welfare is also maximized when GDP, net of total cost of emissions permits, is maximized.*

PROOF: To prove the first part of the proposition, we use reductio ad absurdum. If Proposition 1 is not true, then we can find a list of feasible outputs, say (Y_0, Y_1), such that

(38) $$Y_0 + p_1 Y_1 - p_{Ca}\varepsilon Y_1 > Y_0(p_L(p_{Ca})) + p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - p_{Ca}\varepsilon Y_1(p_L(p_{Ca}), p_{Ca}).$$

Let L_0, L_1, and L_2 denote, respectively, the labor inputs used in the production of the clean good, in the production of the dirty good, and in coal mining. Then the equilibrium condition in the labor market is

(39) $$\begin{aligned} L_0 + L_1 + L_2 &= L_0(p_L(p_{Ca})) + L_1(p_L(p_{Ca}), p_{Ca}) + L_2(p_L(p_{Ca}), p_{Ca}) \\ &= \gamma_2. \end{aligned}$$

Multiplying the first line of (39) by $p_L(p_{Ca})$, then subtracting the result from (38), we obtain

(40)

$$\begin{aligned} &[Y_0 - p_L(p_{Ca})L_0] + \left[p_1 Y_1 - p_L(p_{Ca})L_1 - p_{Ca}\varepsilon Y_1 - \frac{p_L(p_{Ca})}{a} aL_2 \right] \\ &> [Y_0(p_L(p_{Ca})) - p_L(p_{Ca})L_0(p_L(p_{Ca}))] + \left[\begin{array}{l} p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - p_L(p_{Ca})L_1(p_L(p_{Ca}), p_{Ca}) \\ - p_{Ca}\varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) \\ - \dfrac{p_L(p_{Ca})}{a} aL_2(p_L(p_{Ca}), p_{Ca}) \end{array} \right]. \end{aligned}$$

Inequality (40) can be rewritten as follows:

$$[Y_0 - p_L(p_{Ca})L_0] - [Y_0(p_L(p_{Ca})) - p_L(p_{Ca})L_0(p_L(p_{Ca}))]$$

$$(41) \qquad > \begin{bmatrix} p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - p_L(p_{Ca})L_1(p_L(p_{Ca}), p_{Ca}) \\ - p_{Ca}\varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) \\ - \dfrac{p_L(p_{Ca})}{a} aL_2(p_L(p_{Ca}), p_{Ca}) \end{bmatrix}$$

$$- \left[p_1 Y_1 - p_L(p_{Ca})L_1 - p_{Ca}\varepsilon Y_1 - \frac{p_L(p_{Ca})}{a} aL_2 \right].$$

Observe that when the equilibrium wage $p_L(p_{Ca})$ prevails, the expression $[Y_0 - p_L(p_{Ca})L_0]$ represents the rent earned by the specific factor used in the production of the clean good if the labor input in this sector is L_0. Furthermore, because L_0 is not necessarily the optimal labor input, this rent cannot exceed $[Y_0(p_L(p_{Ca})) - p_L(p_{Ca})L_0(p_L(p_{Ca}))]$, the maximum rent that the specific factor earns under the competitive equilibrium. Hence the left side of inequality (41) is less than or equal to 0. In the same manner, observe that on the right side of inequality (41) the expression inside the first pair of square brackets represents the rent earned by the specific input used in the production of the dirty good under the competitive equilibrium, while the expression inside the second pair of square brackets represents the rent earned by this specific factor under the production plan that gives rise to Y_1. Hence the right side of inequality (41) must be greater than or equal to 0. The arguments just presented thus lead to the following chain of inequalities:

$$
(42)\quad \begin{aligned}
&0 \geq [Y_0 - p_L(p_{Ca})L_0] - [Y_0(p_L(p_{Ca})) - p_L(p_{Ca})L_0(p_L(p_{Ca}))] \\
&> \left[\begin{aligned} &p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - p_L(p_{Ca})L_1(p_L(p_{Ca}), p_{Ca}) \\ &- p_{Ca}\varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) \\ &- \frac{p_L(p_{Ca})}{a} aL_2(p_L(p_{Ca}), p_{Ca}) \end{aligned}\right] \\
&- \left[p_1 Y_1 - p_L(p_{Ca})L_1 - p_{Ca}\varepsilon Y_1 - \frac{p_L(p_{Ca})}{a} aL_2 \right] \geq 0,
\end{aligned}
$$

which are inconsistent. We have just proved that GDP, net of the cost of emissions permits, is maximized under the competitive equilibrium.

To prove the statement on welfare, note that when the government subscribes to the Kyoto Protocol, the cap imposed by this international environmental treaty is implemented, and this means that the disutility – due to the stock of greenhouse gases in the atmosphere – suffered by each individual in the small open economy will be constant and equal to $u_2(\bar{E})$. Furthermore, because the consumer surplus yielded by the dirty good is constant, the utility enjoyed by an individual in the small open economy depends only on her income. Therefore, when GDP, net of compliance cost, is maximized, social welfare is also maximized. ■

The implication of Proposition 3 is clear. If preferences are quasi-linear, then from the purely economic efficiency point of view there is no justification for the government to help the polluting sector in its efforts to comply with the Kyoto Protocol. Thus when the government engages in selling PAM units to the polluting sector, it cannot justify this policy on economic grounds, unless it claims that the sector is risk averse and selling PAM units is tantamount to providing insurance to the polluting sector. Even if one is willing to accept this insurance policy argument, the price of a PAM unit should be set at its actuarially fair value, namely the expected price of carbon in the international market. However, the price of $CAN15 for a PAM unit set by the government is widely believed to be below the expected price of carbon in the international market. Hence motives other than those of a benevolent dictator must be found to explain the behavior of the government when it chooses to sell PAM units to the polluting sector at a price below the expected price of carbon in the international market, and this we propose to do in the rest of the essay.

Proposition 3 provides a convenient way for computing the competitive equilibrium under the scenario that the government ratifies the Kyoto Protocol, but refuses to help the polluting sector in its efforts to comply with this international environmental treaty. Indeed, if p_{Ca} is the realized price of carbon in the international market, then $Y_1(p_L, p_{Ca})$, the output of the dirty good under the competitive equilibrium, is the value of Y_1 that satisfies the following first-order condition:

(43) $$GDP'(Y_1) = Y_0^{'}(Y_1) + p_1 = \varepsilon p_{Ca} > 0.$$

Invoking (b) of Lemma 1, we can then assert that a rise in p_{Ca} will lead to a decline in the output of the dirty good as well as a decline in GDP, net of compliance cost. The decline in the output of the dirty good implies a rise in the output of the clean good and a fall in the equilibrium wage rate. It is also worth pointing out the output of the dirty good is always strictly lower under the scenario that the polluting sector bears the entire burden of the compliance cost than under the scenario that it is allow to emit greenhouse gases with impunity. This result follows directly from (43), the strict concavity of $GDP(Y_1)$, and the fact that $GDP'(\overline{Y}_1) = 0$ that is asserted by Proposition 1.

We shall let

(44) $\Omega(p_{Ca}) = Y_0(p_L(p_{Ca})) + p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - \varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) p_{Ca}$

denote GDP net of the cost of complying with the Kyoto Protocol, as a function of the realized price of carbon in the international market, and

(45) $\overline{\Omega} = \int \Omega(p_{Ca}) dF(p_{Ca})$

denote the expected social welfare under the scenario that the polluting sector bears the entire burden of complying with the Kyoto Protocol.

Now according to Proposition 2, the higher is the realized price of carbon in the international market, the lower is the equilibrium wage rate, and the lower the output of the dirty good. Thus when the realized price of carbon in the international market attains its highest value, the output of the dirty good will be at *its lower bound*

(46) $\underline{Y}_1 = Y(p_L(\overline{p}_{Ca}), \overline{p}_{Ca})$.

We shall assume that $\underline{Y}_1 > 0$, which ensures, according to Proposition 2, that

(47) $0 < \underline{Y}_1 < Y(p_L(p_{Ca}), p_{Ca})$,

for all $p_{Ca} < \overline{p}_{Ca}$.

5. THE GAME OF ENDOGENOUS CLIMATE-CHANGE POLICY DETERMINATION

The game of endogenous policy formation has four stages, and its extensive form is as follows.

In the first stage, group 1 communicates to the policy makers a contingent payment schedule $\varphi : p_A \to \varphi(p_A)$, where $\varphi(p_A)$ represents the payment that it is willing to make to the government if the policy p_A is implemented. The payment $\varphi(p_A)$ – also called the political contribution in the literature – is expressed in terms of the numéraire, and is valued by policy makers because it can be used as a political support in financing future election or simply put away for personal use.

In the second stage, the policy makers take as given the contingent payment schedule $\varphi(p_A)$ and implement a policy according to some criterion which depends on the payment as well as on some measure of social welfare. We would like to point out at this point that the game we formulate is that of a principal-agent problem – with group 1 as the principal and the policy makers as the agent – because we assume that neither group 0 nor group 2 is organized as a special-interest group. Indeed, if workers are represented by a strong labor union, and/or if owners of the specific factor used in the production of the clean good organize themselves as a special-interest group, then all these groups will compete in lobbying the policy makers. In this case, the game is one of many principals against a common agency, which could be analyzed with the help of the theoretical machinery developed by Bernheim and Whinston (1986).

In the third stage, group 1 takes as given the environmental policy implemented in the second stage by the government and decides on the number of PAM units they wish to buy. Of course, this decision is also influenced by their expectations about the price of carbon that will prevail later on the international carbon market.

In the fourth stage, the uncertainty about the price of carbon is resolved. Group 1 can now buy carbon credits – if they choose to do so – on international markets at the newly resolved price p_{Ca}. The PAM units that the polluting industries bought from the government in the third stage as well as the carbon credits bought on the international market can then be used to comply with the environmental regulation. The production sectors – sector 0, sector 1, and the coal mining sector – carry out their production plans to maximize profits. Note that the production plan carried out by group 1 must comply with the environmental regulation, and that at the end of the fourth stage group 1 will make the payment it promised the government in the first stage. As for the government, it has to go to the international market to purchase the carbon units that it issued as PAM units in the third stage of the game. Also, for consumers, they make their consumption decisions and these decisions depend on their income as well as the prices they face.

To solve the game of endogenous environmental regulation, we use backward induction and begin with the last stage.

6. THE FOURTH STAGE OF THE GAME: CLIMATE-CHANGE POLICY AND ITS IMPACT ON RESOURCE ALLOCATION

Suppose that A is the number of PAM units that the polluting sector bought from the government in the third stage of the game. Also, suppose that p_{Ca} is the realized price of carbon on the international market. We shall now determine the general equilibrium for the small open economy. There are three possibilities to consider: (i) $A = 0$, (ii) $A \geq \varepsilon\overline{Y}_1$, and (iii) $0 < A < \varepsilon\overline{Y}_1$.

Under possibility (i), the polluting sector did not buy any PAM units in the third stage of the game, possibly because the price of PAM units had been set too high. The competitive equilibrium in the fourth stage – after the price of carbon in the international market has been realized – is the one obtained under the scenario that the polluting sector bears the entire burden of compliance, and this scenario has been analyzed in Section 4.

Under possibility (ii), the number of PAM units bought in the third stage of the game is at least sufficient to cover the amount of greenhouse emissions generated under the benchmark equilibrium. Hence the competitive equilibrium under this possibility is that of the benchmark. We shall show later that possibility (ii) cannot arise in equilibrium.

It remains to analyze possibility (iii). Given A and p_{Ca}, the total costs – the costs of inputs plus the compliance costs – required for producing Y_1 units of the dirty good are then given by

$$\text{(48)} \quad \Gamma^{\#}(Y_1, p_L, p_{Ca}, A) = \begin{cases} \Gamma(Y_1, p_L), \text{ if } Y_1 \le A/\varepsilon, \\ \Gamma(Y_1, p_L) + p_{Ca}(\varepsilon Y_1 - A), \text{ if } Y_1 > A/\varepsilon. \end{cases}$$

Observe that the first line on the right side of (48) represents the total cost of producing Y_1 units of the dirty good, given that the PAM units purchased in the third stage are more than enough to cover the amount of greenhouse gases generated in the production of this level of output. The second line on the right side of (48) represents the total cost of producing Y_1 when the number of PAM units purchased fall short of the emission permits needed to cover the pollution generated by this level of output. In this case, the amount of greenhouse gases in excess of the amount that can be covered by the PAM units acquired in the third stage, namely $\varepsilon Y_1 - A$, must be paid for by purchasing carbon credits on the international market at the realized price p_{Ca}.

The properties of the marginal cost curve of the polluting sector are given in the following lemma:

LEMMA 2: *As Y_1 rises from 0 to $Y_1 \le A/\varepsilon$, the marginal cost of the polluting industry also rises with Y_1 and is given by*

$$\text{(49)} \quad D_1\Gamma^{\#}(Y_1, p_L, p_{Ca}, A) = \frac{p_L}{1-\alpha_1}\overline{K}_1^{-\alpha_1/(1-\alpha_1)}Y_1^{\alpha_1/(1-\alpha_1)} + \frac{p_L}{a}.$$

At $Y_1 = A/\varepsilon$, the marginal cost curve takes an upward jump of size $p_{Ca}\varepsilon$. For $Y_1 > A/\varepsilon$, the marginal cost also rises with Y_1 and is given by

$$\text{(50)} \quad D_1\Gamma^{\#}(Y_1, p_L, p_{Ca}, A) = \frac{p_L}{1-\alpha_1}\overline{K}_1^{-\alpha_1/(1-\alpha_1)}Y_1^{\alpha_1/(1-\alpha_1)} + \frac{p_L}{a} + p_{Ca}\varepsilon.$$

Furthermore, an increase in the wage rate or an increase in the realized price of carbon in the international market shifts the marginal cost curve $Y_1 \to D_1\Gamma^{\#}(Y_1, p_L, p_{Ca}, A)$ *upward, while an increase in the number of PAM units shifts this curved downward.*

Suppose that p_L is the wage rate that prevails in the fourth stage. The polluting sector solves the following profit maximization problem:

(51) $$\max_{Y_1}\left[p_1 Y_1 - \Gamma^{\#}(Y_1, p_L, p_{Ca}, A)\right] = \Pi_1^{\#}(p_L, p_{Ca}, A).$$

The preceding maximization has a unique solution, say $Y_1^{\#}(p_L, p_{Ca}, A)$, which might be to the left of the kink, at the kink, or to the right of the kink of $\Gamma^{\#}(Y_1, p_L, p_{Ca}, A)$ – depending on the values of $p_1, p_L, p_{Ca},$ and A. We shall denote by $Y_1^{\#}(p_L, p_{Ca}, A)$ the output of the dirty good in the fourth stage, after the uncertainty in the price of carbon in the international market has been resolved. Note that if $p_1 \le D_1\Gamma(A/\varepsilon, p_L)$, then

(52) $$Y_1^{\#}(p_L, p_{Ca}, A) = \left[\left(\frac{p_1}{p_L} - \frac{1}{a}\right)(1-\alpha_1)\right]^{\frac{1-\alpha_1}{\alpha_1}} \overline{K}_1 \le \frac{A}{\varepsilon}.$$

On the other hand, if $D_1\Gamma(A/\varepsilon, p_L) < p_1 \le D_1\Gamma(A/\varepsilon, p_L) + p_{Ca}\varepsilon$, then

(53) $$Y_1^{\#}(p_L, p_{Ca}, A) = \frac{A}{\varepsilon}.$$

Finally, if $p_1 > D_1\Gamma(A/\varepsilon, p_L) + p_{Ca}\varepsilon$, then

(54) $$Y_1^{\#}(p_L, p_{Ca}, A) = \overline{K}_1\left[\frac{(1-\alpha_1)}{p_L}\left(p_1 - \frac{p_L}{a} - p_{Ca}\varepsilon\right)\right]^{(1-\alpha_1)/\alpha_1}.$$

The determination of $Y_1^{\#}(p_L, p_{Ca}, A)$ – for the case $p_1 > D_1\Gamma(A/\varepsilon, p_L) + p_{Ca}\varepsilon$ – is depicted in Figure 1. Observe that a higher wage rate or a higher price of carbon in the international market leads to a lower output of the dirty good, while a higher value of A leads to a higher output of the dirty good.

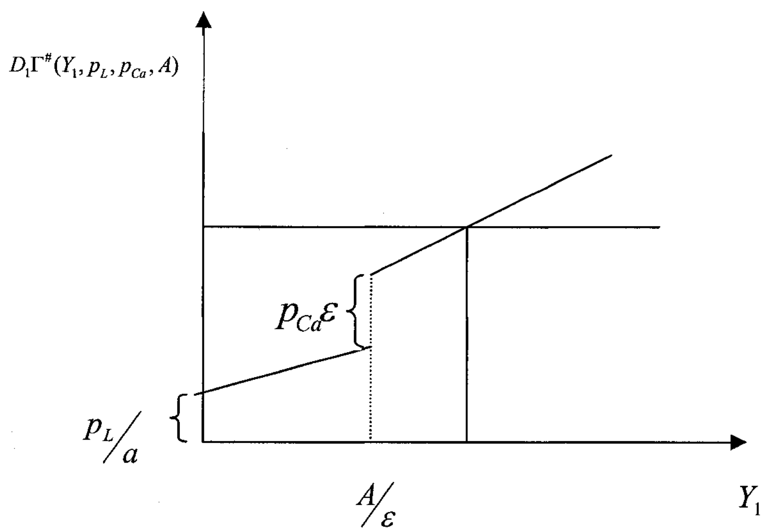

Figure 1.-- The determination of the output of the polluting sector in the fourth stage.

The demand for labor by the polluting sector can be computed from the output of the dirty good according to the following formula:

$$(55) \qquad L_1^{\#}(p_L, p_{Ca}, A) = \left[\frac{Y_1^{\#}(p_L, p_{Ca}, A)}{\overline{K}_1^{\alpha_1}}\right]^{\frac{1}{1-\alpha_1}}.$$

The labor input in the coal mining sector to provide the energy input needed in the production of $Y_1^{\#}(p_L, p_{Ca}, A)$ units of the dirty good is given by

$$(56) \qquad L_2^{\#}(p_L, p_{Ca}, A) = \frac{Y_1^{\#}(p_L, p_{Ca}, A)}{a}.$$

The aggregate demand for labor is then given by

(57) $$L^{\#}(p_L, p_{Ca}, A) = L_0(p_L) + L_1^{\#}(p_L, p_{Ca}, A) + L_2^{\#}(p_L, p_{Ca}, A).$$

Observe that the aggregate demand for labor $p_L \to L^{\#}(p_L, p_{Ca}, A)$ is downward-sloping. Furthermore, a rise in A, the number of PAM units purchased by the polluting sector, shifts it upward, but a rise in the realized price of carbon in the international market shifts it downward. Because $p_L \to L^{\#}(p_L, p_{Ca}, A)$ is continuous, and because $L^{\#}(p_L, p_{Ca}, A)$ tends to infinity (zero) when the wage rate tends to zero (infinity), there exists a unique value of p_L, say $p_L(p_{Ca}, A)$, such that $L^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) = \gamma_2$. The wage rate $p_L(p_{Ca}, A)$ is the equilibrium wage rate in the fourth stage of the game, given (p_{Ca}, A). The following lemma is immediate.

LEMMA 3: *A rise in A, the number of PAM units purchased by the polluting sector in the third stage of the game, ceteris paribus, raises* $p_L(p_{Ca}, A)$, *the equilibrium wage rate. On the other hand, a rise in the realized price of carbon in the international market, every other thing equal, induces a fall in the equilibrium wage rate.*

The determination of the equilibrium wage is depicted in Figure 2. Three excess aggregate demand curves for labor are shown in Figure 2. The one labeled *BB'* represents the excess aggregate demand for labor under the benchmark. The curve *CC'* represents the excess aggregate demand curve for labor for the case the government ratifies the Kyoto Protocol and the polluting sector bears the entire cost of compliance.

The thick one, labeled AA' is the excess aggregate demand curve for the case the government ratifies the Kyoto Protocol and engages in selling PAM units to the polluting sector. This curve has been drawn for a given value of A and a given value of p_{Ca}.

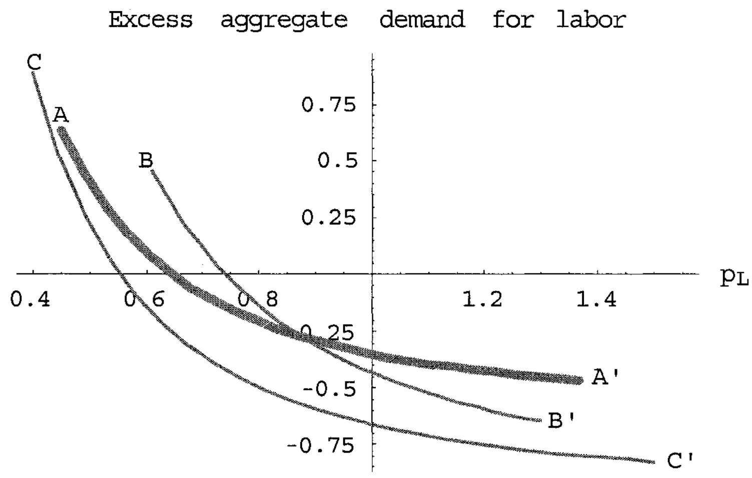

Figure 2.— The excess aggregate demand for labor: BB' (benchmark), CC' (the polluting sector pays the entire compliance cost), AA' (with PAM).

The following lemma asserts that unless the number of PAM units purchased in the third stage of the game is excessive, they will be fully utilized as part of the efforts expended by the polluting sector to comply with the Kyoto Protocol.

LEMMA 4: *Suppose that* $0 < A < \varepsilon \overline{Y}_1$. *Then* $\varepsilon Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \geq A$; *that is, the greenhouse gas emissions generated by the equilibrium output of the dirty good will be greater than or equal to* A, *which means that all the PAM units bought in the third stage of the game will be utilized to comply with the Kyoto Protocol.*

PROOF: To prove the lemma, suppose the contrary, say $\varepsilon Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) < A$. If this were the case, then the first-order condition for profit maximization in the polluting sector is

(58) $$p_1 = D_1\Gamma(Y_1^{\#}(p_L(p_{Ca}, A), P_{Ca}, A), p_L(p_{Ca}, A)).$$

Furthermore, because $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) < \overline{Y}_1$, we must have $p_L(p_{Ca}, A) > \overline{p}_L$. Using this last inequality, (54), and the first-order condition for profit maximization by the polluting sector under the benchmark, we obtain the following self-contradictory chain of inequalities.

(59) $$\begin{aligned} p_1 &= D_1\Gamma(\overline{Y}_1, \overline{p}_L) \\ &> D_1\Gamma(Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A), \overline{p}_L) \\ &> D_1\Gamma(Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A), p_L(p_{Ca}, A)) = p_1. \end{aligned}$$

■

Now according to Lemma 4, we must have $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \geq A/\varepsilon$ when $0 < A < \varepsilon\overline{Y}_1$. Next, note that if $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) = A/\varepsilon$, then the following first-order condition must hold:

(60) $$D_1\Gamma(A/\varepsilon, p_L(p_{Ca}, A)) < p_1 \leq D_1\Gamma(A/\varepsilon, p_L(p_{Ca}, A)) + p_{Ca}\varepsilon.$$

Furthermore, the total labor inputs used in the polluting and mining sectors are given by

(61) $$L_1^{\#}(p_L(p_{Ca}, A), A) + L_2^{\#}(p_L(p_{Ca}, A), A) = \left[\frac{A}{\varepsilon\overline{K}_1^{\alpha_1}}\right]^{\frac{1}{1-\alpha_1}} + \frac{A}{\varepsilon a},$$

which leads to the following expression for the equilibrium wage rate

(62) $$p_L(p_{Ca}, A) = (1-\alpha_0)\overline{K}_0^{\alpha_0}\left(\gamma_2 - \left[\frac{A}{\varepsilon \overline{K}_1^{\alpha_1}}\right]^{\frac{1}{1-\alpha_1}} - \frac{A}{\varepsilon a}\right)^{-\alpha_0}.$$

The following lemma is now immediate:

LEMMA 5: *Suppose that* $0 < A < \varepsilon \overline{Y}_1$. *Define*

(63) $$\omega(A) = (1-\alpha_0)\overline{K}_0^{\alpha_0}\left(\gamma_2 - \left[\frac{A}{\varepsilon \overline{K}_1^{\alpha_1}}\right]^{\frac{1}{1-\alpha_1}} - \frac{A}{\varepsilon a}\right)^{-\alpha_0}.$$

We have the following results:

(a) *The equilibrium output of the dirty good is equal to* A/ε, *i.e.,* $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) = A/\varepsilon$ *if and only if the following condition is satisfied:*

(64) $$D_1\Gamma(A/\varepsilon, \omega(A)) < p_1 \leq D_1\Gamma(A/\varepsilon, \omega(A)) + p_{Ca}\varepsilon.$$

(b) *If* $p_1 > D_1\Gamma(A/\varepsilon, \omega(A)) + p_{Ca}\varepsilon$, *then the number of PAM units purchased in the third stage of the game is not sufficient to cover the greenhouse gas emissions generated by the equilibrium output of the dirty good, and the polluting industry must also purchase carbon credits in the international market to comply with the Kyoto Protocol. In this case, the equilibrium output of the dirty good is the value of* Y_1 *that solves the following first-order condition*

(65) $$p_1 = D_1\Gamma(Y_1, p_L(p_{Ca})) + p_{Ca}\varepsilon,$$

where $p_L(p_{Ca})$, *we recall, is the wage rate that will prevail if the polluting sector has to bear the entire burden of compliance studied in Section* 4.

For any value $A < \varepsilon \overline{Y}_1$ and any value of p_{Ca}, let

$$(66)\quad \begin{aligned}\Omega^{\#}(p_{Ca},A) &= Y_0(p_L(p_{Ca},A)) + p_1 Y_1(p_L(p_{Ca},A), p_{Ca},A) \\ &\quad - p_{Ca}\varepsilon Y_1(p_L(p_{Ca},A), p_{Ca},A)\end{aligned}$$

denote the social welfare – GDP net of the country's cost of complying with the Kyoto Protocol – in the fourth period, as a function of the number of PAM units purchased by the polluting sector in the third stage of the game and the realized price of carbon in the international market. The expected social welfare, as a function of A, is then given by

$$(67)\quad \overline{\Omega}^{\#}(A) = \int \Omega^{\#}(p_{Ca},A)dF(p_{Ca}).$$

To complete this section, we state – for the sake of completeness – the following version of Proposition 2 that characterizes the competitive equilibrium when the government engages in selling PAM units to the polluting sector.

PROPOSITION 4: *A rise in the realized price of carbon on the international market will trigger a decline in the wage rate. Labor moves out of the polluting sector into the sector producing the clean good. The output of the clean good expands while the output of the dirty good contracts. The contraction of the polluting sector means that less greenhouse gases will be generated. In terms of rents, the owners of the specific input used in the production of the clean good are better-off while it is not clear whether they will rise or fall for the owners of the specific input in the polluting sector and the workers are worse-off.*

7. THE THIRD STAGE OF THE GAME: THE PAM PURCHASING PLAN

Let $E[p_{Ca}]$ denote the expected price of carbon in the international market. Suppose that we are at the beginning of the third stage of the game and that p_A is the price of a PAM unit set by the government in the second stage. We shall assume that $p_A \geq \underline{p}_A > 0$, where $\underline{p}_A$ represents the government's lower bound for the price of a PAM unit.

Let

(68) $\Phi_1(A) = \int \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) dF(p_{Ca})$.

As defined, $\Phi_1(A)$ represents the expected rent earned by the specific factor in the polluting sector, conditioned on the number A of PAM units purchased in the third stage of the game and before the political contributions are made. The problem of the polluting sector in this stage is to determine the number of PAM units that it wishes to purchase from the government. Assuming that the polluting sector is risk neutral, we can state its problem formally as follows:

(69) $\max_A \Phi_1(A) - p_A A$..

The following figure depicts the expected equilibrium rent earned by the specific factor in the polluting sector – for the numerical example presented in Section 11 – as a function of the number of PAM units that this sector purchased in the third stage of the game.

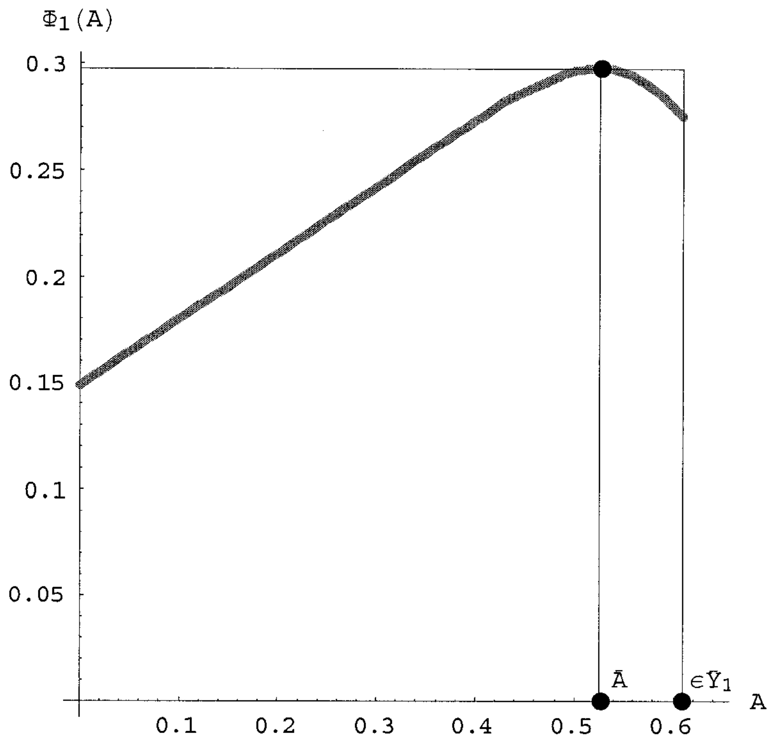

Figure 4.— The expected equilibrium rent earned by the specific factor in the polluting sector, as a function of the number of PAM units purchased.

As can be seen from Figure 4, the expected equilibrium rent rises steadily with A initially, reaches a maximum, then begins to decline steeply as A approaches the number of emissions permits needed to comply with the Kyoto Protocol if the output of the dirty good is at the benchmark level. The result that a higher value of A leads to a lower level of expected equilibrium rent might seem paradoxal, but not difficult to explain. The intuition is that when the output of the dirty good is at or close to its benchmark level, the first-order condition for profit maximization implies that price is equal to marginal cost, with marginal cost being evaluated at the benchmark output and the benchmark wage rate. A marginal decline in the output of the dirty good causes cost – evaluated at the benchmark wage rate – to fall by the price of dirty good, yielding no net gain in rent.

However, a marginal decline in output certainly causes the wage rate to fall below its benchmark level, and this lower wage rate applies to all the previous units of the dirty good. The net effect of a marginal decline in the output of the dirty good, when the output is already close to its benchmark level, is thus a rise in the rent earned by the specific factor in the polluting sector.

Now a value of $A \geq \varepsilon\overline{Y}_1$ will lead to the benchmark equilibrium, and thus does not raise the revenue of the polluting sector. Also, according to the argument given in the preceding paragraph, the expected equilibrium rent earned by the specific factor used in the production of the dirty good is strictly decreasing as A enters a left neighborhood of $\varepsilon\overline{Y}_1$. Hence the expected equilibrium rent in the polluting sector – as a function of A – achieves a maximum at a point, say $\overline{A}$, that lies in the interior of the interval $[0, \varepsilon\overline{Y}_1]$. The value $\overline{A}$ represents the upper bound on the number of PAM units that the polluting sector is willing to purchase. Even if the price of a PAM unit drops to 0, it is not optimal for the polluting sector to purchase more than $\overline{A}$ units of PAM. Thus, the possibility (ii) stated in Section 6 cannot arise in equilibrium. In general, when the price of PAM units is taken into consideration, the number of PAM units that the polluting sector is willing to purchase will be strictly below $\overline{A}$.

The following lemma describes the main properties of the slope of the map $\Phi_1(A)$, or, equivalently, the inverse demand curve for PAM units.

LEMMA 6: *The incremental rent generated by successive units of PAM, namely* $\Phi_1^{'}(A)$, *varies continuously with* A. *It is constant and equal to the expected price of carbon in the international market as* A *rises from* 0 *to* $\varepsilon\underline{Y}_1$. *It has a kink at* $A = \varepsilon\underline{Y}_1$, *and is strictly decreasing in a right neighborhood of* $A = \varepsilon\underline{Y}_1$. *It remains below the expected price of carbon in the international market as* A *continues to rise, and finally becomes negative when* A *approaches* $\varepsilon\overline{Y}_1$.

PROOF: See Appendix A.

Figure 5 depicts the inverse demand curve for PAM units that is associated with the curve of Figure 4.

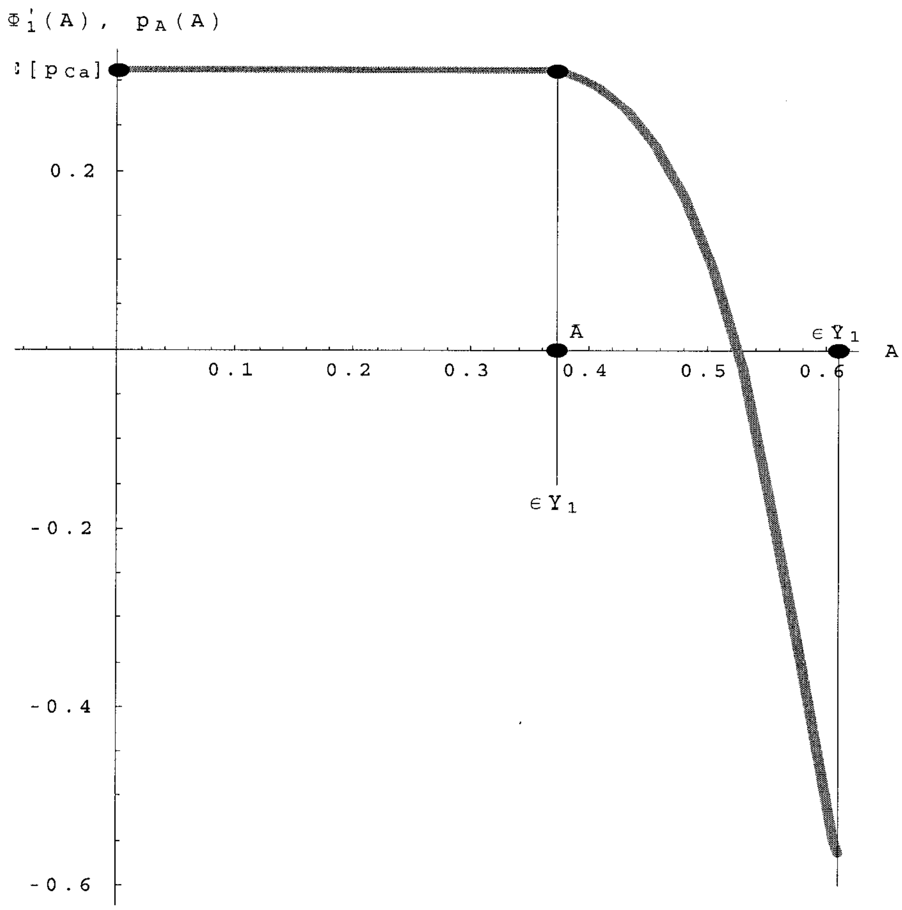

Figure 5.— The marginal product of PAM units (the incremental change in expected equilibrium rent in the polluting sector generated by successive units of PAM).

Observe that the inverse demand curve for PAM units begins with a horizontal line segment that ends when A reaches the critical value $\varepsilon\underline{Y}_1$, where, we recall, $\underline{Y}_1$, as defined by (46), is the lower bound of the output of the dirty good. The curve then begins its descent and crosses the horizontal axis. That is as A continues to rise to $\varepsilon\overline{Y}_1$, the marginal product curve continues to decline and finally becomes negative. In particular, note that the inverse demand curve for PAM units has a kink at $A = \varepsilon\underline{Y}_1$.

Now let $p_A \to A(p_A), p_A \geq \underline{p}_A$, be the map defined as follows:

$$
\begin{aligned}
A(p_A) &= 0 \text{ for } p_A > E[p_{Ca}] \\
&= \varepsilon \underline{Y}_1 \text{ for } p_A = E[p_{Ca}] \\
&= \text{the value of } A \text{ such that } \Phi_1'(A) = p_A \text{ for } \underline{p}_A \leq p_A < E[p_{Ca}].
\end{aligned}
\tag{70}
$$

The map represented by (70) is the demand for PAM units by the polluting sector. Note that the demand curve for PAM units experiences a discrete jump of size $\varepsilon \underline{Y}_1$ at $p_A = E[p_{Ca}]$. However, for $p_A < E[p_{Ca}]$, it is continuous and strictly decreasing.

8. THE SECOND STAGE OF THE GAME: THE IMPLEMENTATION OF CLIMATE-CHANGE POLICY

Suppose that we are at the beginning of the second stage of the game. At this stage of the game, the government has to decide on the environmental policy it wishes to implement, taking as given the contingent political contribution schedule φ communicated by the polluting sector.

Let p_A be the price of a PAM unit that the government charges the polluting sector. The best response – in the third stage of the game – of the polluting sector is to purchase $A(p_A)$ PAM units and obtains the following expected gross payoff before making the political contribution:

$$
\begin{aligned}
W_1(p_A) &= -p_A A(p_A) + \int \begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A(p_A)), p_{Ca}, A(p_A)) \\ + \gamma_1(p_A - p_{Ca})A(p_A) \end{bmatrix} dF(p_{Ca}) \\
&= \Phi_1(A(p_A)) - p_A A(p_A) + \gamma_1(p_A - E[p_{Ca}])A(p_A).
\end{aligned}
\tag{71}
$$

Observe that on the right side of the first equality in (71), the first expression under the integral sign represents the realized rents accruing to the fixed factor used in producing the dirty good; and the second expression represents the government transfer. For the small open economy, the prices of traded goods are given. Hence the consumer surplus $s(p_1)$ is constant. Also, under the Kyoto cap, the disutility caused by greenhouse gas emissions is $u_2(\overline{E})$, which is also constant if the government subscribes to the Kyoto Protocol. Because these two terms are constant, we have chosen to suppress them from the payoff of the owners of the specific factor used in the production of the dirty good. As defined, $W_1(p_A)$ represents the gross expected payoff – gross of political contribution – of group 1, as a function of the price of a PAM unit. The net expected payoff – net of political contributions – of group 1 is then given by $W_1(p_A) - \varphi(p_A)$.

The expected gross social welfare – before the political contributions are made – as a function of the environmental policy implemented is then given by

(72) $$W(p_A) = \overline{\Omega}^{\#}(A(p_A)),$$

and the expected net social welfare – net of political contributions – is given by $W(p_A) - \varphi(p_A)$. The expected payoff of the government is assumed to be given by

(73) $$\begin{aligned} G(p_A, \varphi) &= \varphi(p_A) + \lambda[W(p_A) - \varphi(p_A)] \\ &= (1-\lambda)\varphi(p_A) + \lambda W(p_A). \end{aligned}$$

In (73), λ represents the weight assigned to the expected net social welfare, with the political-contribution component receiving a weight equal to 1. In order for the government to accept political contributions, it is necessary that $\lambda < 1$. If $\lambda \geq 1$, a transfer from any group to the government will decrease the latter player's expected payoff.

Now let

(74) $\Re(\varphi) = \arg\max_{p_A} G(p_A, \varphi)$.

As defined, $\Re(\varphi)$ is the set of policies that are best against φ. The point-to-set map $\Re : \varphi \rightarrow \Re(\varphi)$ represents the best-response correspondence of the government. We are now ready for a formal definition of the equilibrium of the climate-change policy problem.

9. DEFINITION OF THE NASH EQUILIBRIUM

DEFINITION: Let $\varphi^* : p_A \rightarrow \varphi^*(p_A)$ be a feasible contingent payment schedule and p_A^* be the price for a PAM unit. The list (φ^*, p_A^*) is said to constitute a Nash equilibrium for the game of climate-change policy if the following conditions are satisfied:

(a) $p_A^* \in \Re(\varphi^*)$;

(b) For any feasible contingent payment schedule φ of group 1, we have

$$W_1(\varphi^*, p_A^*) - \varphi^*(\tau_A^*) \geq \sup_{p_A \in \Re(\varphi)} [W_1(p_A) - \varphi(p_A)].$$

Condition (a) asserts that p_A^* is a best response to φ^*, while condition (b) asserts that φ^* is a best strategy that the principal can adopt.

10. THE ENDOGENOUS DETERMINATION OF CLIMATE-CHANGE POLICY

According to Proposition 3 – stated and proved in Section 4 – letting the polluting sector bear the entire cost of complying with the Kyoto Protocol maximizes the country's GDP net of compliance cost. Hence if the objective of the government is to maximize social welfare, it should not subsidize this sector in any form. In the present context, it can set the price of a PAM unit equal to the expected price of carbon in the international market, and this policy will induce the polluting sector to comply with the Kyoto Protocol by purchasing all the emissions permits it needs in the international market for carbon. The maximum expected social welfare is then given by

$$(75) \qquad \Omega^{\#}(A(E[p_{Ca}])) = \Omega^{\#}(\varepsilon \underline{Y}_1) = \overline{\Omega}.$$

Now if $p_A < E[p_{Ca}]$ is a policy that group 1 wishes the government to implements, then the shortfall in the social welfare component of the government's payoff is $\overline{\Omega} - W(p_A)$. To induce the government into implementing this policy, group 1 must promise a payment of at least $\lambda[\overline{\Omega} - W(p_A)]/(1-\lambda)$. The net payoff of group 1 – after the payment has been made – is then equal to $W_1(p_A) - \lambda[\overline{\Omega} - W(p_A)]/(1-\lambda)$. Hence the policy that the owners of the specific input used in the production of good 1 wish the government to implement is the solution of the following maximization problem:

$$(76) \qquad \max_{\underline{p}_A \le p_A \le E[p_{Ca}]} \left[W_1(p_A) - \frac{\lambda}{1-\lambda}[\overline{\Omega} - W(p_A)] \right] = \mu_1.$$

We shall let p_A^* denote the value of p_A that solves (76). As defined, p_A^* is the price of a unit of PAM that the polluting sector wishes the government to implement. The environmental policy p_A^* maximizes the net payoff of group 1 while respecting the participating constraint of the government. Group 1 extracts the entire surplus generated by the participation of the government. To induce the government into setting p_A^* as the price of a PAM unit, the polluting sector could choose the following contingent political contribution schedule

(77) $\varphi^* : p_A \rightarrow \varphi^*(p_A) = \max[W_1(p_A) - \mu_1, 0]$

Bernheim and Whinston, op. cit., label such a contingent payment schedule a *truthful strategy.* In adopting the strategy represented by (77), group 1 only aims for a net payoff of μ_1. More precisely, if its gross expected payoff is less than μ_1, then it will not make any political contribution. On the other hand, any payoff in excess of μ_1 will be offered to the government as political contributions. For the government, a best response to φ^* is p_A^*. We have just established the following proposition:

PROPOSITION 5: *The pair* (φ^*, p_A^*) *constitutes a Nash equilibrium for the game of endogenous climat- change policy formation.*

Because the socially optimal number of PAM units is $\varepsilon\underline{Y}_1$, any value of $A > \varepsilon\underline{Y}_1$ will lower social welfare below the optimal level, i.e., $\Omega^{\#}(A) < \Omega^{\#}(\varepsilon\underline{Y}_1)$. The following lemma asserts that when A is slightly higher than the number of emissions permits needed to cover the air pollution generated at the lower bound of the output of the dirty good, the first-order loss in social welfare is 0. More precisely, we have

LEMMA 7: *We have* $D\Omega^{\#}(\varepsilon\underline{Y}_1) = 0$.

PROOF: See Appendix B.

Using (71) and (72) in (76), we obtain the following expression for the objective function of the maximization problem (76) in a left neighborhood of $p_A = E[p_{Ca}]$:

$$
\begin{aligned}
\Psi_1(p_A) &= W_1(p_A) - \lambda[\overline{\Omega} - W(p_A)] \\
&= \Phi_1(A(p_A)) - p_A A(p_A) + \gamma_1(p_A - E[p_{Ca}])A(p_A) \\
&\quad - \lambda[\overline{\Omega} - \overline{\Omega}^{\#}(A(p_A))],
\end{aligned} \tag{78}
$$

Now note that the left derivative of $\Psi_1(p_A)$ at $p_A = E[p_{Ca}]$ is

$$
\begin{aligned}
D^{-}\Psi_1(E[p_{Ca}]) &= D\Phi_1(A(E[p_{Ca}])D^{-}A(E[p_{Ca}])) - A(E[p_{Ca}]) \\
&\quad - E[p_{Ca}]D^{-}A(E[p_{Ca}])) \\
&\quad + \gamma_1 A(E[p_{Ca}]) + \gamma_1(E[p_{Ca}] - E[p_{Ca}])D^{-}A(E[p_{Ca}]) \\
&\quad + \lambda D\overline{\Omega}^{\#}(A(E[p_{Ca}]))D^{-}A(E[p_{Ca}]) \\
&= -A(E[p_{Ca}]) + \gamma_1 A(E[p_{Ca}]) \\
&= -(1-\gamma_1)\varepsilon\underline{Y}_1 < 0.
\end{aligned} \tag{79}
$$

Observe that the second equality has been obtained by using the fact that $D\Phi_1(A(E[p_{Ca}])) = E[p_{Ca}]$ and the third equality from the fact that $A(E[p_{Ca}]) = \varepsilon\underline{Y}_1$ and Lemma 7. The negative sign of the expression on the last line of (79) implies that the objective function in (76) is strictly decreasing in a left neighborhood of $E[p_{Ca}]$, which in turn implies that the price of a PAM unit that solves (76) is strictly lower than the expected price of carbon in the international market. We have just shown that $\underline{p}_A \leq p_A^{*} < E[p_{Ca}]$. In the numerical example presented in the next section, the price of a PAM unit set by the government actually reaches the lower bound $\underline{p}_A$.

The result $\underline{p}_A \leq p_A^* < E[p_{Ca}]$ has some important implications. First, the green fund is negative, and this means the transfer to each individual in the economy is negative. Second, the demand for PAM units by the polluting sector exceeds the numbers of emissions permits needed to cover the lower bound for the output of the dirty good, inducing an equilibrium wage rate that is higher than the equilibrium wage rate that will prevail if the polluting sector bears the entire cost of compliance. This last result implies that relative to the scenario under which the polluting sector has to bear the entire cost of compliance, the owners of the specific factor used in the production of the clean good lose. The loss comes from two sources: the higher wage rate, which reduces the rent in this sector, and the negative transfer. As for workers, the higher wage rate raises their welfare, but the negative transfer operates in the opposite direction. A priori, it is not clear whether the workers win or lose. However, we suspect that the workers also lose, and this is confirmed in the numerical example. We summarize the results just obtained in the following proposition, which constitutes the central result of the essay.

PROPOSITION 6: *The lobbying activities of the owners of the specific factor in the polluting sector induce the government to set the price of a PAM unit below the expected price of carbon in the international market. This policy results in a negative expected green fund, and leads to a negative transfer to each individual in the economy. Relative to the scenario under which the polluting sector bears the entire burden of the country's compliance with the Kyoto Protocol, the owners of the specific factor used in the production of the dirty good gains even after paying for the political contribution. This group extracts the entire surplus that results from the participation of the government in* *the endogenous policy formation game. For the owners of the specific factor used in the production of the clean good, they lose. Their loss comes from two sources: a higher wage rate that reduces the rent, and a negative transfer. As for workers, the higher wage rate raises their income, but the negative transfer operates in the opposite direction. Without more detailed analysis, it is not possible to state unambiguously whether they gain or lose, although we suspect that they also lose.*

11. A NUMERICAL EXAMPLE

In the numerical example, the following values for the parameters of the model are assumed.

Values for parameters

$K_0 = 1$	$\varepsilon = 1$	$\gamma_0 = 0.03$	$\underline{p}_{Ca} = 0.125$
$K_1 = 1$	$p_1 = 1.05$	$\gamma_1 = 0.02$	$\overline{p}_{Ca} = 0.625$
$\alpha_0 = \frac{1}{2}$	$a = 5$	$\gamma_2 = 0.95$	
$\alpha_1 = \frac{1}{2}$	$\lambda = 0.95$	$\underline{p}_A = 0.25$	

It is also assumed that p_{Ca} is a random variable and uniformly distributed on the interval $[0.125, 0.625]$.

Figure 6 depicts the joint payoff of the government and the dirty sector, as a function of the PAM units.

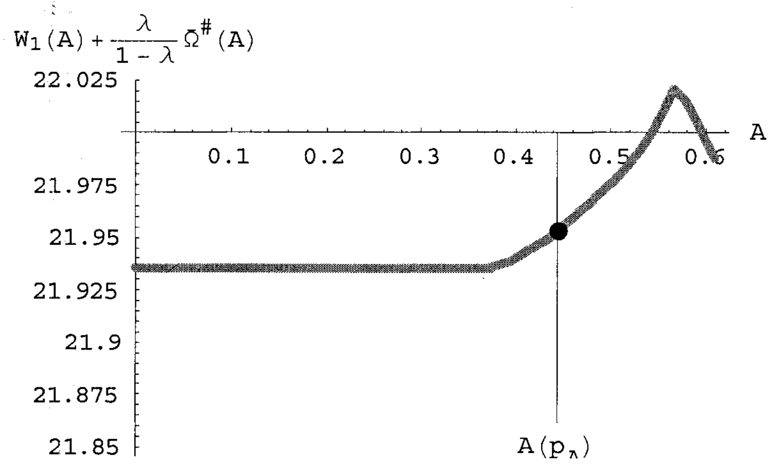

Figure 6.— The joint payoff of the government and the dirty sector as a function of the PAM units.

When the price of a PAM unit is equal to the expected price of carbon the joint payoff remains constant at $W_1(A)+\frac{\lambda}{1-\lambda}\overline{\Omega}^{\#}(A)=21.9355$ until A reaches $\varepsilon\underline{Y}_1$. The joint payoff starts to increase as A increases, that is, as the price of a PAM unit falls below the expected price of carbon, $p_A < E[p_{Ca}]$, and reaches its maximum value of 22.0208. Note that the price of a PAM unit can not fall below its floor price, $\underline{p}_A = 0.25$. Thus, given the joint payoff, the owners of specific factor in the dirty sector will induce the government to adopt the floor price of a PAM unit to maximize the joint payoff.

As a result, the expected wage rate increases by 0.00495, but the green fund decreases by 0.02773 leading to a net decrease of 0.02278 in the overall income of the workers. Hence workers are worse off. Although we do not compute the change in the expected rent earned by the owners of the specific factor in the clean-good sector, this group unambiguously lose for two reasons. First, the rise in the wage rate depresses rent. Second, the negative transfer reinforces the loss in rent. As for the owner of the specific factor in the dirty sector, the payoff, net of political contributions, is 0.16601, increasing from the payoff of 0.14848 this sector earns if it has to bear the entire burden of complying with the Kyoto Protocol. In terms of social welfare, the loss is insignificant, suggesting that the implementation of the price policy that is greater than $\underline{p}_A$ but less than $E[p_{Ca}]$ shifts the income of the owners of the specific factor in the clean-good sector and the income of the workers to the owners of specific factor in the dirty sector.

12. CONCLUDING REMARKS

The analysis of Section 4 supports the environmental policy – the Kyoto Protocol – by proving that social welfare is maximized if preferences are quasi-linear, and if the government does not intervene to implement a price policy that is favorable to the dirty-good sector in exchange for the political contribution from the owners of specific factor in this sector. The dirty-good sector in this case will bear the entire cost of compliance with the Kyoto Protocol. To lessen the impact of the environmental constraint on its own, the dirty-good sector, through lobbying activities, induces the government to implement a price policy of a PAM unit that is greater than its floor price but lower than the expected price of carbon. This leads to a small change in social welfare, but the consequence of the policy is a transfer of income from the owners of the specific factor used in the production of the clean good and workers to the owners of specific factor used in the production of the dirty good. The environmental policy benefits the owners of the specific factor in the dirty-good sector at the expense of the other groups, proving that the government is more receptive to external pressure because the industries involved contribute greatly to the economy in terms of employment and economic growth. The lobbying strength of an industry – and hence the size of the monetary contributions – depends on the economic stake of the industry and its contribution to the economy.

The model can be extended to include the non-traded good sector. If workers are allowed to organize themselves as a union, they might lobby for lax or vigorous environmental regulation; the result depends on the workers' trade-off between labor income and environmental quality. Furthermore, the competition among the various special-interest groups will allow the government to obtain a payoff above the reservation payoff.

APPENDIX A
PROOF OF LEMMA 6

Lemma 6 is proved through a series of claims.

CLAIM 1: *We have* $\Phi_1'(A) = E[p_{Ca}]$ *for* $0 < A < \varepsilon \underline{Y}_1$.

PROOF: Pick a value A that satisfies $0 < A < \varepsilon \underline{Y}_1$. Next, let $h > 0$ be a small number such that $0 < A - h < A + h < \varepsilon \underline{Y}_1$.

By definition, the right derivative of $\Phi_1(A)$ is given by

$$D^{+}\Phi_1(A) = \ell im_{h\downarrow 0}\frac{\Phi_1(A+h)-\Phi_1(A)}{h}$$

(A.1)
$$= \ell im_{h\downarrow 0}\frac{1}{h}\begin{bmatrix} \int \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h)dF(p_{Ca}) \\ -\int \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A)dF(p_{Ca}) \end{bmatrix}$$

$$= \int\left(\ell im_{h\downarrow 0}\frac{1}{h}\begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ -\Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix}\right)dF(p_{Ca}).$$

Now recall from (46) the definition of the lower bound $\underline{Y}_1$ for the output of the dirty good, which is the output chosen by the polluting sector when the realized price of carbon in the international market is at its highest level possible and when it has to bear the entire burden of compliance. The first-order condition that characterizes is

(A.2) $p_1 = D_1\Gamma(\underline{Y}_1, p_L(\overline{p}_{Ca}) + \varepsilon\overline{p}_{Ca}.$

Now let $p_{Ca} \leq \overline{p}_{Ca}$ be the realized price of carbon in the international market. If the polluting sector bearing the entire compliance cost, then the equilibrium wage rate will be $p_L(p_{Ca})$ and the equilibrium output of the dirty good will be $Y_1(p_L(p_{Ca}), p_{Ca})$. Furthermore, according to Proposition 2, we have $p_L(p_{Ca}) \geq p_L(\overline{p}_{Ca})$ and

(A.3) $Y_1(p_L(p_{Ca}), p_{Ca}). \geq Y_1(p_L(\overline{p}_{Ca}), \overline{p}_{Ca}) = \underline{Y}_1.$

Now note that at the same realized price of carbon p_{Ca} in the international market and given that the polluting sector purchased $A < \varepsilon\underline{Y}_1$ units of PAM in the third stage of the game, the wage $p_L(p_{Ca})$ is also the equilibrium wage rate with PAM and the output of the dirty good $Y_1(p_L(p_{Ca}), p_{Ca})$ is also the equilibrium output of the dirty good with PAM; that is, $p_L(p_{Ca}) = p_L(p_{Ca}, A)$ and $Y_1(p_L(p_{Ca}), p_{Ca}) = Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A)$. Thus under the equilibrium with PAM, the additional emissions permits – to be purchased in the international market for carbon – to comply with the Kyoto Protocol will be $\varepsilon Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) - A. = \varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) - A.$ The rent earned by the specific factor in the polluting sector is then given by

(A.4)
$$\begin{aligned} \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) &= p_1 Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \\ &\quad - \Gamma^{\#}\left(Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A), p_L(p_{Ca}, A), p_{Ca}, A\right) \\ &= p_1 Y_1(p_L(p_{Ca},), p_{Ca}) \\ &\quad - \Gamma\left({}_1 Y_1(p_L(p_{Ca},), p_{Ca}), p_L(p_{Ca}), p_{Ca}\right) \\ &\quad - [\varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) - A]p_{Ca}. \end{aligned}$$

Now if the polluting sector has bought $A+h<\varepsilon\underline{Y}_1$ units of PAM, then the argument used to obtain (A.4) for A units of PAM also holds for $A+h$ units of PAM, and the following version of (A.3) holds for $A+h$ units of PAM:

$$
\begin{aligned}
\Pi_1^{\#}(p_L(p_{Ca},A+h),p_{Ca},A+h) &= p_1Y_1^{\#}(p_L(p_{Ca},A+h),p_{Ca},A+h) \\
&\quad -\Gamma^{\#}\left(Y_1^{\#}(p_L(p_{Ca},A+h),p_{Ca},A+h),p_L(p_{Ca},A+h),p_{Ca},A+h\right) \\
\text{(A.5)} \qquad &= p_1Y_1(p_L(p_{Ca},),p_{Ca}) \\
&\quad -\Gamma\left({}_1Y_1(p_L(p_{Ca},),p_{Ca}),p_L(p_{Ca}),p_{Ca}\right) \\
&\quad -[\varepsilon Y_1(p_L(p_{Ca}),p_{Ca})-(A+h)]p_{Ca}.
\end{aligned}
$$

Subtracting (A.4) from (A.5), we obtain

(A.6) $\Pi_1^{\#}(p_L(p_{Ca},A+h),p_{Ca},A+h)-\Pi_1^{\#}(p_L(p_{Ca},A),p_{Ca},A)=hp_{Ca}.$

Using (A.6) in (A.1), we obtain

(A.7) $D^+\Phi_1(A)==\int p_{Ca}dF(p_{Ca}).=E[p_{Ca}].$

To complete the proOf of Lemma A1, note that the left derivative of $\Phi_1(A)$ at $A<\varepsilon\underline{Y}_1$ is given by

$$
\begin{aligned}
D^-\Phi_1(A) &= \ell im_{h\downarrow 0}\frac{\Phi_1(A)-\Phi_1(A-h)}{h} \\
\text{(A.8)} \qquad &= \ell im_{h\downarrow 0}\frac{1}{h}\begin{bmatrix}\int\Pi_1^{\#}(p_L(p_{Ca},A),p_{Ca},A+h)dF(p_{Ca}) \\ -\int\Pi_1^{\#}(p_L(p_{Ca},A-h),p_{Ca},A-h)dF(p_{Ca})\end{bmatrix} \\
&= \int\left(\ell im_{h\downarrow 0}\frac{1}{h}\begin{bmatrix}\Pi_1^{\#}(p_L(p_{Ca},A),p_{Ca},A) \\ -\Pi_1^{\#}(p_L(p_{Ca},A-h),p_{Ca},A-h)\end{bmatrix}\right)dF(p_{Ca}).
\end{aligned}
$$

The argument used to obtain (A.6) can be repeated to yield

(A.9) $\Pi_1^{\#}(p_L(p_{Ca},A),p_{Ca},A)-\Pi_1^{\#}(p_L(p_{Ca},A-h),p_{Ca},A-h)=hp_{Ca},$

The argument used to obtain (A.6) can be repeated to yield

(A.9) $\Pi_1^{\#}(p_L(p_{Ca},A),p_{Ca},A)-\Pi_1^{\#}(p_L(p_{Ca},A-h),p_{Ca},A-h)=hp_{Ca},$

■

CLAIM 2: *The derivative of* $\Phi_1(A)$ *at* $A=\varepsilon\underline{Y}_1$ *is* $\Phi_1'(\varepsilon\underline{Y}_1)=E[p_{Ca}]$.

PROOF: The proof that $D^-\Phi_1(A)=E[p_{Ca}]$, for $0<A<\varepsilon\underline{Y}_1$ can also be used to show that $D^-\Phi_1(\varepsilon\underline{Y}_1)=E[p_{Ca}]$. The right derivative of $\Phi_1(A)$ at $A=\varepsilon\underline{Y}_1$ is

$$D^{+}\Phi_1(\varepsilon \underline{Y}_1) = \ell im_{h\downarrow 0} \frac{\Phi_1(\varepsilon \underline{Y}_1 + h) - \Phi_1(\varepsilon \underline{Y}_1)}{h}$$

(A.11) $$= \ell im_{h\downarrow 0} \frac{1}{h}\left[\begin{array}{l} \int \Pi_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) dF(p_{Ca}) \\ - \int \Pi_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1), p_{Ca}, \varepsilon \underline{Y}_1) dF(p_{Ca}) \end{array}\right]$$

$$= \int \left(\ell im_{h\downarrow 0} \frac{1}{h}\left[\begin{array}{l} \Pi_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) \\ - \Pi_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1), p_{Ca}, \varepsilon \underline{Y}_1) \end{array}\right]\right) dF(p_{Ca}).$$

Now for any $p_{Ca} < \overline{p}_{Ca}$, we have $\underline{Y}_1 < Y_1(p_L(p_{Ca}), p_{Ca})$. Hence if $h > 0$ is sufficiently small, we will have

(A.12) $\varepsilon \underline{Y}_1 < \varepsilon \underline{Y}_1 + h < \varepsilon Y_1(p_L(p_{Ca}), p_{Ca})$.

Using (A.12), we can repeat the argument to obtain (A.6), we can then assert that

(A.13) $\Pi_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) - \Pi_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1), p_{Ca}, \varepsilon \underline{Y}_1) = hp_{Ca}$.

Because (A.13) holds for every $p_{Ca} < \overline{p}_{Ca}$, and because the density of the distribution function $F(p_{Ca})$ is assumed to be continuous, we must then have

(A.14) $D^{+}\Phi_1(\varepsilon \underline{Y}_1) = \int p_{Ca} dF(p_{Ca}) = E[p_{Ca}]$. ■

CLAIM 3: *For any given A satisfying $0 < A < \varepsilon \overline{Y}_1$ and any p_{Ca}, if*

(A.15) $p <\leq D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right). + \varepsilon p_{Ca}$,

then

$$\ell im_{h\to 0} \frac{\Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A)}{h}$$

(A.16) $$= \frac{1}{\varepsilon}\left[p_1 - D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right).\right] - D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A)$$

$$< p_{Ca}. - .D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A) < p_{Ca}.$$

PROOF: First, recall from Lemma 4 that when A satisfies $0 < A < \varepsilon \overline{Y}_1$, we have $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \geq A/\varepsilon$. Furthermore, when (A.15) holds, the polluting sector will not buy carbon credits in the international market, which then implies that $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) = A/\varepsilon$. Next, note that if h is a small number, then we stll have $p <\leq D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A+h)\right). + \varepsilon p_{Ca}$, and the preceding argument can be repeated to obtain $Y_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A+h) = (A+h)/\varepsilon$. Hence

$$\Pi_1^{\#}(p_L(p_{Ca}, A+\Delta A), p_{Ca}, A+h) - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A)$$

(A.17)
$$= p_1 \frac{A+h}{\varepsilon} - \Gamma\left(\frac{A+h}{\varepsilon}, p_L(p_{Ca}, A+h)\right) - p_1 \frac{A}{\varepsilon} + \Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)$$
$$= p_1 \frac{h}{\varepsilon} - \left[\Gamma\left(\frac{A+h}{\varepsilon}, p_L(p_{Ca}, A+h)\right) - \Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)\right].$$

Dividing (A.17) by h then letting $h \to 0$, we obtain

$$\ell im_{h\to 0} \frac{\Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A)}{h}$$

(A.18)
$$= \frac{p_1}{\varepsilon} - \left[D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)\frac{1}{\varepsilon} + D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)D_2 p_L(p_{Ca}, A)\right]$$
$$= \frac{1}{\varepsilon}\left[p_1 - D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)\right] - D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)D_2 p_L(p_{Ca}, A)$$
$$< p_{Ca} - D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)D_2 p_L(p_{Ca}, A) < p_{Ca}.$$

Note that the first inequality in (A.18) is due to (A.16), and the second inequality has been obtained by noting that $D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right)D_2 p_L(p_{Ca}, A) > 0.$ ■

Now under the scenario that the polluting sector bears the entire burden of complying with the Kyoto Protocol the equilibrium wage rate and the equilibrium output of the dirty good are both decreasing functions of the realized price of carbon. More precisely, when the realized price of carbon declines from $\overline{p}_{Ca}$ to $\underline{p}_{Ca}$, the equilibrium wage rate rises from $p_L(\overline{p}_{Ca})$ to $p_L(\underline{p}_{Ca})$ and the equilibrium output of the dirty good rises from $\underline{Y}_1$ to $Y_1(p_L(\underline{p}_{Ca}), \underline{p}_{Ca})$. Hence for each value of A that satisfies $\varepsilon\underline{Y}_1 \le A \le \varepsilon Y_1(p_L(\underline{p}_{Ca}), \underline{p}_{Ca})$, there exists a unique value of $p_{Ca} \in [\underline{p}_{Ca}, \overline{p}_{Ca}]$, say $q(A)$, such that $Y_1(p_L(q(A), q(A)) = A/\varepsilon$, and the first-order condition for profit maximization in the polluting sector is given by

(A.19) $p_1 = D_1\Gamma(A/\varepsilon, p_L(q(A))) + \varepsilon q(A)$.

Furthermore, $q: A \to q(A)$, $\varepsilon\underline{Y}_1 \le A \le \varepsilon Y_1(p_L(\underline{p}_{Ca}), \underline{p}_{Ca})$, is strictly decreasing.

CLAIM 4: *For any A satisfying $\varepsilon\underline{Y}_1 \le A < \varepsilon\overline{Y}_1$, we have $\Phi_1'(A) < E[p_{Ca}]$.*

PROOF: By definition,

$$\Phi_1'(A) = \ell im_{h\to 0} \frac{\Phi_1(A+h) - \Phi_1(A)}{h}$$

(A.20)
$$= \ell im_{h\to 0} \frac{1}{h}\begin{bmatrix} \int \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) dF(p_{Ca}) \\ - \int \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) dF(p_{Ca}) \end{bmatrix}$$

$$= \int \left(\ell im_{h\to 0} \frac{1}{h}\begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix}\right) dF(p_{Ca}).$$

Now if $\varepsilon \underline{Y}_1 \leq A \leq Y_1(p_L(\underline{p}_{Ca}), \underline{p}_{Ca})$, then $q(A)$ is well defined, and it is characterized by (A.19). In this case, the integral on the last line of (20) can be split into two integrals in the following manner:

$$\int \left(\ell im_{h\to 0} \frac{1}{h}\begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix}\right) dF(p_{Ca})$$

(A.21)
$$= \int_{\underline{p}_{Ca}}^{q(A)} \left(\ell im_{h\to 0} \frac{1}{h}\begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix}\right) f(p_{Ca}) dp_{Ca}$$

$$+ \int_{q(A)}^{\bar{p}_{Ca}} \left(\ell im_{h\to 0} \frac{1}{h}\begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix}\right) f(p_{Ca}) dp_{Ca}.$$

To evaluate the first integral on the right side of (A.21), note that for $p_{Ca} < q(A)$, (A.6) holds, which allows us to obtain

(A.22)
$$\int_{\underline{p}_{Ca}}^{q(A)} \left(\ell im_{h\to 0} \frac{1}{h}\begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ - \Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix}\right) f(p_{Ca}) dp_{Ca} = \int_{\underline{p}_{Ca}}^{q(A)} p_{Ca} f(p_{Ca}) dp_{Ca}.$$

As for the second integral on the right side of (A.21), note that for $p_{Ca} > q(A)$, (A.15) holds, which allows us to use (A.16) to obtain

$$\int_{q(A)}^{\bar{p}_{Ca}} \left(\ell im_{h\to 0} \frac{1}{h} \begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ -\Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix} \right) f(p_{Ca}) dp_{Ca}.$$

$$= \int_{q(A)}^{\bar{p}_{Ca}} \begin{pmatrix} \frac{1}{\varepsilon}\left[p_1 - D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right). \right] \\ -D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A) \end{pmatrix} f(p_{Ca}) dp_{Ca}.$$

(A.23)
$$= \int_{q(A)}^{\bar{p}_{Ca}} \left(\frac{1}{\varepsilon}\left[p_1 - D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right). \right] \right) f(p_{Ca}) dp_{Ca}.$$

$$- \int_{q(A)}^{\bar{p}_{Ca}} \left(D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A) \right) f(p_{Ca}) dp_{Ca}.$$

$$< \int_{q(A)}^{\bar{p}_{Ca}} p_{Ca} f(p_{Ca}) dp_{Ca}. - \int_{q(A)}^{\bar{p}_{Ca}} \left(D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A) \right) f(p_{Ca}) dp_{Ca}..$$

Using (A.22) and (A.23) in (A.21), we obtain

$$\int \left(\ell im_{h\to 0} \frac{1}{h} \begin{bmatrix} \Pi_1^{\#}(p_L(p_{Ca}, A+h), p_{Ca}, A+h) \\ -\Pi_1^{\#}(p_L(p_{Ca}, A), p_{Ca}, A) \end{bmatrix} \right) dF(p_{Ca})$$

(A.24)
$$< \int_{\underline{p}_{Ca}}^{q(A)} p_{Ca} f(p_{Ca}) dp_{Ca} + \int_{q(A)}^{\bar{p}_{Ca}} p_{Ca} f(p_{Ca}) dp_{Ca}.$$

$$- \int_{q(A)}^{\bar{p}_{Ca}} \left(D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A) \right) f(p_{Ca}) dp_{Ca}.$$

$$= E[p_{Ca}] - \int_{q(A)}^{\bar{p}_{Ca}} \left(D_2\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right) D_2 p_L(p_{Ca}, A) \right) f(p_{Ca}) dp_{Ca} < E[p_{Ca}].$$

The claim is now proved for the case $\varepsilon \underline{Y}_1 \le A \le Y_1(p_L(\underline{p}_{Ca}), \underline{p}_{Ca})$. Finally, for the case $A > Y_1(p_L(\underline{p}_{Ca}), \underline{p}_{Ca})$, we have

(A.25) $p <\le D_1\Gamma\left(\frac{A}{\varepsilon}, p_L(p_{Ca}, A)\right). + \varepsilon p_{Ca}$

for all $p_{Ca} \in [\underline{p}_{Ca}, \bar{p}_{Ca}]$, and applying Claim 3, we obtain

$$\int\left(\ell im_{h\to 0}\frac{1}{h}\begin{bmatrix}\Pi_1^{\#}(p_L(p_{Ca},A+h),p_{Ca},A+h)\\ -\Pi_1^{\#}(p_L(p_{Ca},A),p_{Ca},A)\end{bmatrix}\right)dF(p_{Ca})$$

(A.26)
$$<\int\left(p_{Ca}-D_2\Gamma\left(\frac{A}{\varepsilon},p_L(p_{Ca},A)\right)D_2p_L(p_{Ca},A)\right)dF(p_{Ca})$$

$$=E[p_{Ca}]-\int_{\underline{p}_{Ca}}^{\bar{p}_{Ca}}D_2\Gamma\left(\frac{A}{\varepsilon},p_L(p_{Ca},A)\right)D_2p_L(p_{Ca},A)f(p_{Ca})dp_{Ca}<E[p_{Ca}].$$

■

CEAIM 5: *We have* $D^-\Phi_1'(\varepsilon\underline{Y}_1)=0>D^+\Phi_1'(\varepsilon\underline{Y}_1)$; *that is, the inverse demand curve for PAM units has a kink at* $A=\varepsilon\underline{Y}_1$.

PROOF: According to Lemmas A1 and A2, we have $D^-\Phi_1'(\varepsilon\underline{Y}_1)=0$. To compute the right derivative, let $h>0$ be a small number so that (A.24) applies. We have

$$\frac{\Phi_1'(\varepsilon\underline{Y}_1+h)-\Phi_1'(\varepsilon\underline{Y}_1)}{h}$$

$$<-\frac{1}{h}\int_{q(\varepsilon\underline{Y}_1+h)}^{\bar{p}_{Ca}}\left(D_2\Gamma\left(\frac{\varepsilon\underline{Y}_1+h}{\varepsilon},p_L(p_{Ca},\varepsilon\underline{Y}_1+h)\right)D_2p_L(p_{Ca},\varepsilon\underline{Y}_1+h)\right)f(p_{Ca})dp_{Ca}$$

$$=\left[-\frac{\bar{p}_{Ca}-q(\varepsilon\underline{Y}_1+h}{h}\right]\times$$

$$\left[\frac{1}{\bar{p}_{Ca}-q(\varepsilon\underline{Y}_1+h}\int_{q(\varepsilon\underline{Y}_1+h)}^{\bar{p}_{Ca}}\left(D_2\Gamma\left(\frac{\varepsilon\underline{Y}_1+h}{\varepsilon},p_L(p_{Ca},\varepsilon\underline{Y}_1+h)\right)D_2p_L(p_{Ca},\varepsilon\underline{Y}_1+h)\right)f(p_{Ca})dp_{Ca}\right]$$

When $h\downarrow 0$, the preceding expression becomes

(A.27)
$$D^+\Phi_1'(\varepsilon\underline{Y}_1)=\ell im_{h\downarrow 0}\frac{\Phi_1'(\varepsilon\underline{Y}_1+h)-\Phi_1'(\varepsilon\underline{Y}_1)}{h}$$
$$<-q'(\varepsilon\underline{Y}_1)(D_2\Gamma(\underline{Y}_1,p_L(p_{Ca},\varepsilon\underline{Y}_1)D_2p_L(p_{Ca},\varepsilon\underline{Y}_1))f(p_{Ca})<0.$$

■

Together, Claims 1, 2, 4, and 5 constitute the proof of Lemma 6.

APPENDIX B

PROOF OF LEMMA 7

Let $h > 0$ be sufficiently small, and suppose that the polluting sector purchased $\varepsilon \underline{Y}_1 + h$ units of PAM in the third stage of the game.

Next, recall the definition of the function $q(A)$, which was given immediately before the statement of Claim 4 of Appendix A. Using the definition of $q(\varepsilon \underline{Y}_1 + h)$, we obtain

(B.1) $Y_1(p_L(q(\varepsilon \underline{Y}_1 + h)), q(\varepsilon \underline{Y}_1 + h)) = (\varepsilon \underline{Y}_1 + h)/\varepsilon.$

Furthermore,

(B.2) $Y_1(p_L(p_{Ca}), p_{Ca}) < Y_1(p_L(q(\varepsilon \underline{Y}_1 + h)), q(\varepsilon \underline{Y}_1 + h)) = (\varepsilon \underline{Y}_1 + h)/\varepsilon,$

for $p_{Ca} > q(\varepsilon \underline{Y}_1 + h),$

and

(B.3) $Y_1(p_L(p_{Ca}), p_{Ca}) > Y_1(p_L(q(\varepsilon \underline{Y}_1 + h)), q(\varepsilon \underline{Y}_1 + h)) = (\varepsilon \underline{Y}_1 + h)/\varepsilon,$

for $p_{Ca} < q(\varepsilon \underline{Y}_1 + h).$

Now let us put ourselves under the scenario that the polluting sector has purchased $\varepsilon \underline{Y}_1 + h$ units of PAM from the government. It follows from (B.2) and (B.3) that as a function of the realized price of carbon in the international market, the equilibrium output of the dirty good under this scenario is given by

(B.4) $Y_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) = (\varepsilon \underline{Y}_1 + h)/\varepsilon,$

for $p_{Ca} > q(\varepsilon \underline{Y}_1 + h),$

and

(B.5) $Y_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) = Y_1(p_L(p_{Ca}), p_{Ca}),$

for $p_{Ca} < q(\varepsilon \underline{Y}_1 + h).$

That is, under the scenario with PAM units, the dirty good is only overproduced when the realized price of carbon in the international market exceeds $q(\varepsilon \underline{Y}_1 + h)$. Under such an event, the loss in GDP net of the country's cost of complying with the Kyoto Protocol is

(B.6)
$$\begin{bmatrix} Y_0\left(Y_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h)\right) + \\ p_1 Y_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) - \\ p_{Ca} \varepsilon Y_1^{\#}(p_L(p_{Ca}, \varepsilon \underline{Y}_1 + h), p_{Ca}, \varepsilon \underline{Y}_1 + h) \end{bmatrix} - \begin{bmatrix} Y_0\left(Y_1(p_L(p_{Ca},), p_{Ca},)\right) + \\ p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - \\ p_{Ca} \varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) \end{bmatrix}$$
$$= \begin{bmatrix} Y_0\left(\underline{Y}_1 + h/\varepsilon\right) + \\ p_1(\underline{Y}_1 + h/\varepsilon) - \\ p_{Ca} \varepsilon(\underline{Y}_1 + h/\varepsilon) \end{bmatrix} - \begin{bmatrix} Y_0\left(Y_1(p_L(p_{Ca},), p_{Ca},)\right) + \\ p_1 Y_1(p_L(p_{Ca}), p_{Ca}) - \\ p_{Ca} \varepsilon Y_1(p_L(p_{Ca}), p_{Ca}) \end{bmatrix}.$$

The loss in expected social welfare – when the polluting sector has purchased $\varepsilon \underline{Y}_1 + h$ units of PAM instead of $\varepsilon \underline{Y}_1 + h$ units of PAM – is thus given by

$$\Omega^{\#}(\varepsilon \underline{Y}_1 + h) - \Omega^{\#}(\varepsilon \underline{Y}_1) =$$

(B.7) $$\int_{q(\varepsilon \underline{Y}_1 + h)}^{\bar{p}_{Ca}} \begin{bmatrix} GDP(\underline{Y}_1 + h/\varepsilon) - GDP(Y_1(p_L(p_{Ca},), p_{Ca},)) \\ p_{Ca}\varepsilon((\underline{Y}_1 + h/\varepsilon) - Y_1(p_L(p_{Ca},), p_{Ca},)) \end{bmatrix} f(p_{Ca})dp_{Ca}.$$

It follows from (B.7) that

$$D\Omega^{\#}(\varepsilon \underline{Y}_1) =$$

(B.8) $$\ell\mathrm{im}_{h\downarrow 0} \frac{1}{h} \int_{q(\varepsilon \underline{Y}_1 + h)}^{\bar{p}_{Ca}} \begin{bmatrix} GDP(\underline{Y}_1 + h/\varepsilon) - GDP(Y_1(p_L(p_{Ca},), p_{Ca},)) \\ p_{Ca}\varepsilon((\underline{Y}_1 + h/\varepsilon) - Y_1(p_L(p_{Ca},), p_{Ca},)) \end{bmatrix} f(p_{Ca})dp_{Ca}.$$

To evaluate the limit of the expression on the second line of (B.8), we express ths expression as follows

(B.9) $$\left[\frac{\bar{p}_{Ca} - q(\varepsilon \underline{Y}_1 + h)}{h}\right] \times \left[\frac{1}{\bar{p}_{Ca} - q(\varepsilon \underline{Y}_1 + h)} \int_{q(\varepsilon \underline{Y}_1 + h)}^{\bar{p}_{Ca}} \begin{bmatrix} GDP(\underline{Y}_1 + h/\varepsilon) - GDP(Y_1(p_L(p_{Ca},), p_{Ca},)) \\ p_{Ca}\varepsilon((\underline{Y}_1 + h/\varepsilon) - Y_1(p_L(p_{Ca},), p_{Ca},)) \end{bmatrix} f(p_{Ca})dp_{Ca}\right].$$

Now note that

(B.10) $$\ell im_{h\downarrow 0} \frac{\bar{p}_{Ca} - q(\varepsilon \underline{Y}_1 + h)}{h} = -\ell im_{h\downarrow 0} \frac{q(\varepsilon \underline{Y}_1 + h) - q(\varepsilon \underline{Y}_1)}{h} = -q(\varepsilon \underline{Y}_1) > 0,$$

but

(B.11) $$\ell im_{h\downarrow 0} \frac{\int_{q(\varepsilon \underline{Y}_1 + h)}^{\bar{p}_{Ca}} \begin{bmatrix} GDP(\underline{Y}_1 + h/\varepsilon) - GDP(Y_1(p_L(p_{Ca},), p_{Ca},)) \\ p_{Ca}\varepsilon((\underline{Y}_1 + h/\varepsilon) - Y_1(p_L(p_{Ca},), p_{Ca},)) \end{bmatrix} f(p_{Ca})dp_{Ca}}{\bar{p}_{Ca} - q(\varepsilon \underline{Y}_1 + h)} = 0.$$

Hence $D\Omega^{\#}(\varepsilon \underline{Y}_1) = 0.$ ■

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Essay Two

CAPPING OF GREENHOUSE GAS EMISSIONS, ECONOMIC GROWTH, AND SUSTAINABLE DEVELOPMENT IN AN OVERLAPPING-GENERATIONS FRAMEWORK

1. INTRODUCTION

Global warming is currently a serious environmental threat faced by mankind. To stabilize the greenhouse gas concentration in the atmosphere, the United Nations Framework Convention on Climate Change sets out the objective of reducing global greenhouse gas emissions to a certain level. The target level should be achieved within a time frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened, and to enable economic development to proceed in a sustainable manner.[1] To that end, 162 countries have ratified the Kyoto Protocol, and by taking national measures, the parties listed in Annex 1 are obligated to achieve their reduction targets and fulfill their commitments. Not all largest emitters in the world are Annex 1 parties. Countries, such as China and India, have decided not to participate, arguing that global warming is due to the past greenhouse gas emissions of industrialized countries, not developing countries. The US also refuses to take action globally. Perhaps the most controversial issue about the Kyoto Protocol is that it is based on equity rather on efficiency grounds. This controversial issue was discussed by Nordhaus and Yang

[1] http://unfccc.int/essential_background/feeling_the_heat/items/2914.php.

(1996, NY hereafter) in RICE,[2] a computable general equilibrium model built by these researchers to study the environmental and economic impacts of various forms of international collaboration to fight global warming. NY found that the global emission reductions, the marginal costs of reducing carbon dioxide, and net benefits are higher when countries set their optimal policies to fight global warming cooperatively – a cooperative strategy – than when countries set their optimal policies without taking into account the effects of their emissions on the others. At the present time, it is this non-cooperative behavior that characterizes the strategies adopted by most nations. A global cooperative strategy on climate change requires China to implement a higher reduction rate than other countries, and that the reduction rate rises over time. Another interesting result is that not all parties gain from such cooperation. For example, the US will incur significant costs, but receive little benefits in return for its cooperation.

At the global level, to find the optimal climate-change policy, one can adopt the Ramsey framework and maximize a sum of discounted utilities; see, for example, the work of NY, discussed above. A charge often leveled against this approach is that it is not morally justifiable to discount the utilities of future generations. Another objection – and perhaps more serious – involves the lack of knowledge concerning the linkages between economic activities and climate change. The United Nations Framework Convention on Climate Change negotiated at the 1992 Rio Earth Summit adopts the precautionary principle and proposes that greenhouse gas emissions be capped at a rate that will prevent the greenhouse gas concentration from reaching a dangerous level that might cause

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[2] RICE is the acronym for Regional Integrated Model of Climate and the Economy.

irreversible and catastrophic change in climate. According to the simulation results of Howarth (2000), the precautionary principle may prescribe a greenhouse gas concentration of no more than twice the pre-industrial level, and allows for a long-run temperature change of no more than $2.9^{O}C$, with optimal emissions-abatement rates that rise from 31 percent to 85 percent between the years 2000 and 2105.

NY examined the impacts of international action – cooperatively or non-cooperatively – on the welfare of various regions in the world, but left the inter-generational equity issue untouched. Other researchers analyzed the impacts of a policy instrument on the economy, environment, and different generations under various frameworks. John, Pecchenino, Schimmelpfenning, and Schreft (1995) constructed an overlapping-generations model to study the impacts of a tax policy – implemented by a short-lived government – on the environment and capital accumulation. The myopia of a short-lived government leads it to maximizing the welfare of the current generation by setting a tax rate that ignores the impacts of such decision – via the bequest of environmental quality – on the future generations. These authors found that countries that better maintain their environment would enjoy higher environmental quality and capital accumulation in steady state. They also found that a better environmental maintenance technology will allow a country to accumulate more capital and have higher environmental quality. To internalize the impacts of a tax policy on future generations, these authors constructed and analyzed the problem faced by a central planner whose objective is to find the Golden Rule which maximizes the utility of a representative generation in the long run. They found that compared to the Golden Rule, the steady state under the competitive equilibrium might be inefficient – lower environmental quality coupled with over- or under-accumulation of capital – and that a tax transfer scheme could eliminate this efficiency.

The environmental effects of a tax policy were also examined by Bovenberg and Heijdra (1998) within an overlapping-generations framework. These authors studied the efficiency as well as the intergenerational distribution aspects of a tax policy. More specifically, they found that an environmental tax (equivalent to a tax on capital) improves efficiency by internalizing negative (pollution) externalities. They also found that implementing a tax policy would harm the old generations, but benefit the young and all future generations. Consequently, the old generation would not support such policy. In order to obtain the political support of the old generation, the government, according to these authors, could finance the expenditures associated with its environmental policy by issuing bonds to subsidize capital owners; that is, borrowing funds needed to carry out an environmental policy can be used to distribute the efficiency gains to all generations.

Gerlagh, Zwann, Hofkes, and Klaassen (2004) constructed a general equilibrium model called DEMETER (Decarbonisation Model with Endogenous Technologies for Emission Reductions), which includes the endogenous technological change in the energy sector and niche markets to study the implications of carbon taxes on emission levels. Niche markets are small markets with small numbers of customers, who discover advantages of using new carbon-free technologies, and these markets must exist in order for technology diffusion to take place and for new carbon-free technologies to mature in the energy sector. Also, the productions costs of carbon-free technologies will decline over time as a result of "learning by doing." Thus, it would be cheaper for firms to invest in new carbon-free technologies. Under these conditions and assumptions of GDP growth, population growth, and energy efficiency improvement, etc., the authors found that greenhouse gas emissions can be stabilized throughout the 21st century at a carbon tax of $50 per tonne, which is lower than the results offered in the literature.

Ono (2002) contributed to literature by examining the effects of lowering emissions quota on the environment and growth. The author constructed an overlapping-generations model based on the work of John and Pecchenio (1994) and John et al. (1995), and showed that lowering emission quota reduces the emission of harmful substances and thereby improves environmental quality. However, lowering emission quota also reduces labor income and income transfer from the government, which results in less resources being allocated to environmental maintenance. The author argued that the latter effect dominates the former effect. Therefore, reducing emission quota leads to environmental deterioration in the long run.

In this essay, we formulate a model of environmental policy, which takes the form of cap on greenhouse gases emissions, under the overlapping-generations framework. The cap on greenhouse gases emissions takes the form of a fixed number of emissions permits issued by the government in each period. There are three goods in the economy: a consumption good; oil – a generic term for fossil fuels, which is the main source of greenhouse gases – and a renewable energy. In the model, the consumption good is produced by competitive firms with the help of labor and energy, and the energy input comes from two sources – fossil fuels and a backstop. While fossil fuels can be extracted for use in the production of the consumption good, renewable energy can only be produced by accumulating backstop capital. The consumption good produced in any period can also be used as investment good to accumulate backstop capital. In our model, the greenhouse gas emissions are the direct results of the burning of fossil fuels, and the carbon dioxide emissions are endogenously determined by the amount of fossil fuels used, not the byproducts of output and emission intensity, an exogenous variable, as in the work of NY. Most theoretical studies on global warming do not attempt to model the burning of fossil fuels, which are the main sources of greenhouse gases, while the ones that explicitly link the input of fossil fuels to output often put no upper bound on the ultimate stock of these nonrenewable resources. Our model, in contrast, contains an explicit formalization of the process of resource depletion as well as the ultimate finite stock of fossil fuels that can be depleted.

In each period, five types of economic agents coexist in the economy: a young generation, an old generation, competitive firms producing the consumption good, competitive firms producing solar energy, and a government. These economic agents interact on five markets: the market for labor, the market for backstop capital, the market for oil, the market for solar energy, and the market for the consumption good. At the beginning of each period, a new generation is born. Each individual of the new generation lives for two periods. She works when she is young, and retires when she is old. A young individual has no assets, except for one unit of time that she offers in-elastically for sale on the labor market. Part of the wages the individual earns in her young age is spent on consumption; the remaining part is saved to provide for her old-age consumption. At the end of each period, the saving of the young individuals takes the form of oil and backstop capital that it purchases from the old individuals, who are the owners of the remaining oil stock and the depreciated capital. There is no population growth in the model, and in each period the size of a generation – young or old – is assumed to be a continuum of measure one. Also, there is no bequest in the model.

Climate change could lead to loss of biodiversity, rise in sea level, damage to infrastructures, loss of agricultural productivity... These are production externalities caused by an increase in the stock of greenhouse gases in the atmosphere. On the consumption side, climate change could result in the emergence or exacerbation of a large number of potential public health problems, such as heat-induced mortality and the spread of malaria and dengue fever. In Essay One, we choose to focus on the consumption externalities of climate change. In the present and following essays, we focus on production externalities, which take place in future periods.

The main contributions of the essay can be summarized as follows.

For an economy that has no oil to begin with or for an economy that has depleted its stock of fossil fuels, the model is reduced to the standard neoclassical growth model. The capital labor ratio converges to its long-run value monotonically, and in the steady state, the economy is sustained completely by renewable energy. These results are the contents of Proposition 1.

For an economy endowed with a stock of fossil fuels, the main results – when the government does not implement any climate-change policy – the competitive equilibrium is described by Propositions 3 and 4. According to Proposition 3, if the initial stock is large and if producers are allowed to discharge greenhouse gases into the atmosphere unrestrained and without paying any cost, then there will be excessive burning of fossil fuels in period 0, leading to a high concentration of greenhouse gases in the atmosphere in period 1, and the ensuing consequence of a low output of the consumption good in future periods. This proposition provides a powerful argument for intervention on equity grounds because the excessive burning of fossil fuels benefits the current generation at the expense of future generations. The competitive equilibrium also fails on efficiency grounds: it is not Pareto optimal according to Proposition 4. In any period oil is used, this energy input can be reduced slightly and the reduction transferred to the following period to increase the lifetime utility of the young generation without lowering the utility of the current old generation and the lifetime utility of future generations. The cut in current oil use, of course, reduces current output. However, the better environmental quality in the following period raises output in that period. If the fall in current output falls on the current generation and the rise in the next period's output is allocated to the current young generation- in its old age – then the lifetime utility of the current generation experiences a net gain, without any deterioration in the lifetime utility of any other generation. Proposition 4 thus reinforces Proposition 3 for intervention. Proposition

5 asserts that if the backstop is not used in period 0, then it will be brought into use in finite time. It is known that the oil stock is not necessarily depleted in finite time or even asymptotically; that is, part of the oil stock might be left in situ unexploited under the competitive equilibrium.[3] Proposition 6 asserts that this will not happen if the rate of capital depreciation is low.

For an economy with a large stock of fossil fuels, the climate-change policy will depress the price of oil in period 0. The cap on greenhouse gases emissions reduces the income of the old generation of period 0, making the generation worse-off. This is the content of Proposition 7. Capping greenhouse emissions thus has intergenerational equity implications: it benefits future generations at the expense of the current generations. A characterization of the competitive equilibrium under the cap is provided by Proposition 8. According to this proposition, if the backstop is not used in period 0, then it will be brought into use in finite time. Furthermore, a stringent cap will bring the backstop into operation sooner. As for the oil stock, if the rate of capital depreciation is low, then it will be depleted in finite time and the price of oil – as long as the oil stock has not been exhausted – will be rising through time. Also, in the time interval that does not stretch more that one period after the introduction of the backstop technology and as long as the cap is binding, the price of emissions permits is declining, reflecting the substitution of renewable energy for fossil fuels. In the long run, the economy will be completely sustained by renewable energy.

[3] See the numerical example in Hung and Quyen (2006).

The model we formulate also contributes to the modern literature in the economics of exhaustible resources. This literature, which can be said to began with Hotelling (1931), has mostly dealt with the depletion of a nonrenewable resource under the partial equilibrium framework. An exception is Olson and Knapp (1997, OK hereafter), who formulated an overlapping-generations model to analyze the depletion of an exhaustible resource. Unlike our model, the model of OK does not deal with pollution. Neither is there a substitute for the exhaustible resource in their model. Because the resource considered in their model is essential, the economy will collapse in the long run when it ultimately runs out of the resource. Agnani, Gutierrez, and Iza (2005) developed an overlapping-generations model which is similar to that of OK, but which includes natural capital and labor in the production process. These authors found that the economy can experience a positive steady-state growth rate if the labor share of output is high enough. In our model, when the fixed stock of fossil fuels dwindles, backstop capital will be accumulated, and a technology substitution – backstop for fossil fuels – will take place. The substitution of renewable energy for oil thus prevents our economy from collapsing – an unpleasant scenario in the model of OK – and allows our economy to evolve along a path of sustainable development.

This essay is organized as follows. Section 2 presents the model. Competitive equilibrium is defined in Section 3. Section 4 analyzes an economy without oil. The existence of competitive equilibrium for an economy with fossil fuels is discussed in Section 5. Section 6 analyzes the competitive equilibrium for an economy with oil resources under nonintervention. Section 7 analyzes the competitive equilibrium for an economy with oil resources, when the policy of climate change takes the form of emissions permits. Section 8 contains some concluding remarks.

2. THE MODEL

In the model, time is discrete and denoted by $t, t = 0,1,...$. There are three goods in the economy: a consumption good; oil – a generic term for fossil fuels, which is the main source of greenhouse gases – and a renewable energy. The consumption good is produced by competitive firms from two inputs – labor and energy. In each period, five types of economic agents coexist in the economy: a young generation, an old generation, competitive firms producing the consumption good, competitive firms producing solar energy, and a government. These economic agents interact on five markets: the market for labor, the market for backstop capital, the market for oil, the market for solar energy, and the market for the consumption good.

At the beginning of each period, a new generation is born. Each individual of the new generation lives for two periods. She works when she is young, and retires when she is old. A young individual has no assets, except for one unit of time that she offers in-elastically for sale on the labor market. Part of the wages the individual earns in her young age is spent on consumption; the remaining part is saved to provide for her old-age consumption. The individual does not care about her descendants and will leave no assets behind at the end of her life cycle. Although the individual does not care about her descendants, she cares deeply about the current as well as the future state of the environment, especially the quality of the air she breathes. There is no population growth in the model, and in each period the size of a generation – young or old – is assumed to be a continuum of measure one.

2.1. *The Evolution of the Stock of Greenhouse Gases*

Let H_t denote the stock of greenhouse gases in the atmosphere in period t. We shall assume that the stock of greenhouse gases decays naturally at rate γ where $0 < \gamma < 1$ is a parameter. The dynamics of this stock is governed by the following difference equation:

(1) $$H_{t+1} - H_t = E_t - \gamma H_t,$$

where E_t represents the emissions of greenhouse gases into the atmosphere in period t. The initial stock of greenhouse gases in the atmosphere, namely H_0, is given.

2.2. *Production Technologies*

The consumption good is produced by competitive firms from two inputs – labor and energy – and its form in period t is assumed to be given by the following Cobb-Douglas production function:

(2) $$Y = A\Omega(H)L^{\alpha}Z^{1-\alpha}.$$

In (2), A is the technology level; $\Omega(H)$ represents the output scaling due to the damage caused by climate change; L denotes the labor input; Z denotes the energy inputs; and

Y denotes the output of the consumption good. Also, $\alpha, 0 < \alpha < 1$, is the elasticity of output with respect to labor.

The output scaling factor $\Omega(H)$ is assumed to be a nonnegative, decreasing, and continuous function of the stock of greenhouse gases. Furthermore, there exists a threshold level $\underline{H}$ such that $\Omega(H) = 1$ for all $0 \le H \le \underline{H}$ and $\Omega(H)$ is strictly decreasing for all $H > \underline{H}$. Also, $\Omega(H) \to 0$ when $H \to +\infty$. A possible functional form for the output scaling factor due to environmental damage is

$$\text{(3)} \qquad \begin{aligned} \Omega(H) &= 1 \quad \text{for } 0 \le H \le \underline{H} \\ &= e^{-\lambda(H-\underline{H})}, H > \underline{H}. \end{aligned}$$

where $\underline{H}$ is a nonnegative constant and $\lambda > 0$ is a parameter.

The energy inputs come from two sources: oil and a backstop. Here oil is a generic term for fossil fuels – the main source of greenhouse gases – and the backstop can represent solar energy. We shall assume that oil can be extracted at negligible cost and that the burning of one unit of oil yields one British Thermal Unit[4] (Btu) and at the same time releases one unit of greenhouse gases into the atmosphere. While oil can be extracted at negligible cost, its stock is limited. The backstop on the other hand can provide an everlasting source of energy. However, harnessing the Sun's energy requires investments in backstop capital, say solar collectors. To keep the exposition as simple as possible, we shall assume that one unit of backstop capital produces one Btu. Also, backstop capital is assumed to depreciate at rate $\delta, 0 \le \delta \le 1$.

[4] One Bristish Thermal Unit (Btu) is the amount of heat required to raise the temperature by one Fahrenheit of one pound of water. http://www.webopedia.com/TERM/B/Btu.html.

In each period $t, t = 0,1,...,$ let L_t denote the labor input; Q_t denote the amount of oil – measured in Btus – extracted for use as part of the energy inputs used in the production of the consumption good; and K_t denote the stock of backstop capital. Because one unit of backstop capital produces one Btu, the total energy input in period t is $Q_t + K_t$, which, when combined with L_t units of labor, yields the following output of the consumption good: $Y_t = A\Omega(H_t)L_t^{\alpha}(Q_t + K_t)^{1-\alpha}$.

Observe that the production of the consumption involves only labor and energy, not capital, and that the backstop sector influences the production of the consumption good only indirectly through the amount of energy that the stock of backstop capital manages to produce and deliver to the consumption good sector. We shall assume that the consumption good can also be used as an investment good to augment the stock of backstop capital.

2.3. *The Government*

Suppose that the government puts in place an environmental policy – under the form of emissions permits – to cap the greenhouse gases emissions of firms. To comply with the environmental regulation, firms can either use clean energy from the backstop or buy emissions permits if they burn fossil fuels to provide part of the energy inputs used in the production of the consumption good. A certain number of emission permits are issued at the beginning of each period, and the polluting firms are required to purchase permits, if needed, to meet their obligations. Each permit allows 1 tonne of greenhouse gases emissions and will expire at the end of the period. Because each permit lasts only one period, banking permits is not possible.

For each $t = 0,1,...,$ let $E_t^{\#}$ be the number of emission permits issued by the government at the beginning of period t. By a policy on climate change, we mean an infinite sequence $\left(E_t^{\#}\right)_{t=0}^{\infty}$. Note that by controlling the number of permits the government is controlling the rate at which oil is extracted. The price of a permit in period t is denoted by $p_{E,t}$. Let E_t be the number of emissions permits demanded by the consumption good sector to comply with the environmental regulation. If $E_t \leq E_t^{\#}$, then the demand for emissions permits can be realized. On the other hand, if $E_t > E_t^{\#}$, then the price of emissions permits must rise to reduce its demand below $E_t^{\#}$. Thus after the price of emissions permits has adjusted so that the demand for emissions permits can be met, the revenues obtained from the sales of emissions permits is $p_{E,t}E_t$. We shall assume that the revenues raised from the emission permits are redistributed equally among all the members of the population. The transfer that each individual – young or old – receives in period t is thus given by

(4) $$m_t = \frac{p_{E,t}E_t}{2}.$$

2.4. *Profit Maximization*

In what follows, we shall choose the consumption good in each period as the numéraire. Also, we shall let $p_{L,t}, p_{X,t}, p_{B,t}$, and $p_{K,t}$ denote, respectively, the wage rate, the price of oil, the price of energy produced by the backstop, and the rental rate of backstop capital – all in period *t*.

2.4.1. *Profit Maximization in the Consumption Good Sector*

The representative firm in the consumption good sector solves the following profit maximization in period t:

$$\text{(5)} \qquad \max_{(L,Q,B)} A\Omega(H_t)L^{\alpha}(Q+B)^{1-\alpha} - p_{L,t}L - (p_{X,t}+p_{E,t})Q - p_{B,t}B.$$

Note that in (5) we have used B to denote the input of energy produced by the backstop. Let (L_t, Q_t, B_t) be the solution of this profit maximization problem. The following first-order condition characterizes the demand for labor:

$$\text{(6)} \qquad \alpha A\Omega(H_t)L_t^{\alpha-1}(Q_t+B_t)^{1-\alpha} - p_{L,t} = 0.$$

If $p_{X,t} + p_{E,t} < p_{B,t}$, then $Q_t > 0$ and $B_t = 0$, and the following first-order condition characterizes the demand for oil:

$$\text{(7)} \qquad (1-\alpha)A\Omega(H_t)L_t^{\alpha}Q_t^{-\alpha} - p_{X,t} - p_{E,t} = 0.$$

On the other hand, if $p_{X,t} + p_{E,t} > p_{B,t}$, then $Q_t = 0$ and $B_t > 0$, and the following first-order condition characterizes the demand for solar energy:

$$\text{(8)} \qquad (1-\alpha)A\Omega(H_t)L_t^{\alpha}B_t^{-\alpha} - p_{B,t} = 0.$$

When $p_{X,t} + p_{E,t} = p_{B,t}$, the oil input and the solar energy input are indeterminate, but their sum $Z_t = Q_t + B_t$ is uniquely determined and satisfies the following first-order condition:

$$\text{(9)} \qquad (1-\alpha)A\Omega(H_t)L_t^{\alpha}Z_t^{-\alpha} - p_{X,t} - p_{E,t} = 0.$$

As for the demand for emissions permits to comply with the environmental regulation, it is given by

(10) $E_t = Q_t$.

The output of the consumption good that emerges from the solution of the profit maximization problem (5) is then given by $Y_t = A\Omega(H_t)L_t^{\alpha}(Q_t + B_t)^{1-\alpha}$. We shall refer to the list $(Y_t, L_t, Q_t, B_t, E_t)$ as the optimal production plan in period t for the representative firm in the consumption good sector when it faces the price system $(p_{L,t}, p_{X,t}, p_{B,t}, p_{E,t})$ in that period.

2.4.2. *Profit Maximization in the Backstop Sector*

Because the technology used in the production of solar energy is linear, the solution of the profit maximization problem in the backstop sector is particularly simple. If $p_{B,t} > p_{K,t}$, then the output of solar energy is infinite. If $p_{B,t} < p_{K,t}$, then the backstop will shut down. When $p_{B,t} = p_{K,t}$, the output of solar energy is indeterminate and will adjust to its demand from the consumption good sector.

2.5. *Preferences and Lifetime Utility Maximization*

2.5.1. *Lifetime Utility Maximization of a Young Individual*

Consider an young individual of period *t*. Such an individual owns nothing except for one unit of time that she supplies in-elastically on the labor market at the wage rate $p_{L,t}$. She receives a transfer of m_t from the government, making her total income in period t equal to $p_{L,t} + m_t$. Out of this income, she spends a part on current consumption and invests the remaining part on the two real assets – capital and oil – to provide for her old-age consumption. Let c_t^0 be her current consumption and c_{t+1}^1 be her consumption in the next period, when she is old. Her lifetime utility is assumed to be given by the following logarithmic preferences:

(11) $Log(c_t^0) + \beta Log(c_{t+1}^1)$.

In (11), $\beta, 0 < \beta < 1$, is the factor the individual uses to discount future utilities. Let k_{t+1} and x_{t+1} denote, respectively, the amount of capital and the amount of oil she purchases and pays for from her saving. A lifetime plan for a young individual of period t is a list $\left(c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1}\right)$.

To find the optimal lifetime plan of a young individual of period t, first note that the rate of return to capital investment is $1 - \delta + p_{K,t+1}$ and the rate of return to oil investment is $p_{X,t+1} / p_{X,t}$. The rate of return to her saving obtained in period $t+1$ is then given by

(12) $$r_{t+1} = \max\left\{1 - \delta + p_{K,t+1}, \frac{p_{X,t+1}}{p_{X,t}}\right\}.$$

Note that if $1 - \delta + p_{K,t+1} > p_{X,t+1} / p_{X,t}$, then a young individual of period t will only invest in capital. On the other hand, if the inequality is reversed, then she will only invest in oil. When $1 - \delta + p_{K,t+1} = p_{X,t+1} / p_{X,t}$, the individual will be indifferent between the two real assets, and the investment mix is indeterminate. The lifetime utility maximization problem of a young individual of period t can be formally stated as follows:

(13) $$\max_{c_t^0} Log(c_t^0) + \beta Log[m_{t+1} + r_{t+1}(p_{L,t} + m_t - c_t^0)].$$

The following first-order condition characterizes the optimal current consumption:

(14) $$\frac{1}{c_t^0} - \beta \frac{r_{t+1}}{m_{t+1} + r_{t+1}(p_{L,t} + m_t - c_t^0)} = 0,$$

from which we obtain

(15) $$c_t^0 = \frac{1}{1+\beta}\left[p_{L,t} + m_t + \frac{m_{t+1}}{r_{t+1}}\right].$$

Using (15), we obtain the following expression for the saving made by a young individual of period t:

(16) $$\begin{aligned} s_t &= p_{L,t} + m_t - c_t^0 \\ &= \frac{1}{1+\beta}\left[\beta(p_{L,t} + m_t) - \frac{m_{t+1}}{r_{t+1}}\right]. \end{aligned}$$

As for the optimal investment portfolio of a young individual of period t, we have

(17) $$(k_{t+1}, x_{t+1}) = \begin{cases} (s_t, 0) \text{ if } 1-\delta + p_{K,t+1} > p_{X,t+1}/p_{X,t}, \\ \left(0, \dfrac{s_t}{p_{X,t}}\right) \text{ if } 1-\delta + p_{K,t+1} < p_{X,t+1}/p_{X,t}, \\ \left(k_{t+1}, \dfrac{s_t - k_{t+1}}{p_{X,t}}\right), 0 \le k_{t+1} \le s_t \text{ if } 1-\delta + p_{K,t+1} = p_{X,t+1}/p_{X,t}. \end{cases}$$

2.5.2. *Utility Maximization of an Old Individual*

The utility maximization problem of an old individual in period t is quite simple. Such an individual owns K_t units of capital and X_t units of oil. Because this is her last period, the individual will spend all her income and wealth on consumption. The consumption of an old individual in period t is thus given by

(18) $$c_t^1 = m_t + (1 - \delta + p_{K,t})K_t + p_{X,t}X_t.$$

3. DEFINITION OF COMPETITIVE EQUILIBRIUM

In the following definition of competitive equilibrium, we shall not consider explicitly the market for solar energy by considering only the price system in which the price of solar energy is equal to the rental rate of backstop capital in each period, i.e., $p_{B,t} = p_{K,t}$. Under such a price system, the market for solar energy is always in equilibrium, and the price of solar energy can be suppressed in the definition of an equilibrium price system.

Let $\left(E_t^\#\right)_{t=0}^{\infty}$ be a policy on climate change implemented by the government. Next, let $\mathcal{P} = (p_{L,t}, p_{X,t}, p_{K,t}, p_{E,t})_{t=0}^{\infty}$ be *a price system. An allocation induced by* $\mathcal{P}$ is an infinite sequence

$$\mathcal{A} = \left(c_0^1, (c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})_{t=0}^{\infty}, (Y_t, L_t, Q_t, B_t, E_t)_{t=0}^{\infty}, (K_t, X_t, H_t)_{t=0}^{\infty}\right)$$

with the following properties:

(i) $c_0^1 = m_0 + p_{X,0}X_0 + (1 - \delta + p_{K,0})K_0.$

(ii) $(c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})$ is the optimal lifetime plan for a young individual of period t, when the price system $\mathcal{P}$ prevails.

(iii) $(Y_t, L_t, Q_t, B_t, E_t)$ is an optimal production plan of the representative firm in the consumption good sector in period t, when the price system $\mathcal{P}$ prevails.

(iv) $(K_t, X_t) = (k_t, x_t)$, $\quad (t = 0,1,...)$.

(v) $H_{t+1} = (1-\gamma)H_t + Q_t$, $\quad (t = 0,1,...)$.

The pair $(\mathcal{P}, \mathcal{Q})$ is said to constitute *a competitive equilibrium induced by the climate-change policy* $\left(E_t^{\#}\right)_{t=0}^{\infty}$ if the following market-clearing conditions are satisfied for each $t = 0,1,...,$

(vi) $L_t = 1$,

(vii) $X_{t+1} + Q_t = X_t$,

(viii) $B_t = K_t$,

(ix) $E_t \leq E_t^{\#}$,

(x) $c_t^1 + c_t^0 + K_{t+1} = Y_t + (1-\delta)K_t$.

4. COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITHOUT OIL

Consider an economy that has exhausted its oil resources and that is now completely sustained by a backstop technology. Suppose that we are in period t and that K_t is the economy's capital stock in this period. After the oil stock has been depleted, no more greenhouse gases will be generated by production activities in the consumption good sector, and the evolution of the stock of greenhouse gasess in the atmosphere is governed by the following difference equation:

(19) $H_{t+1} = (1-\gamma)H_t.$

Furthermore, after oil exhaustion, all the energy needs of the economy will be met by the backstop. Under this scenario, we have $Q_t = 0, B_t = K_t,$ and the first-order condition (6) takes on the following form:

(20) $\alpha A\Omega(H_t)K_t^{1-\alpha} = p_{L,t}.$

Using (20) in (16), we obtain the following saving, which is also the capital labor ratio in the next period, of a young individual of period t:

$$(21)\quad \begin{aligned} K_{t+1} &= \frac{\beta}{1+\beta}p_{L,t} \\ &= \frac{\alpha\beta}{1+\beta}A\Omega(H_t)K_t^{1-\alpha}. \end{aligned}$$

The well-known difference equation (20) describes the transition of the capital labor ratio from one period to another. In the long run, the stock of greenhouse gases tends to 0, which implies that $\Omega(H_t) \to 1,$ and the capital labor ratio is given by

$$(22)\quad \overline{K} = \left[\frac{\alpha\beta A}{1+\beta}\right]^{1/\alpha},$$

and the convergence to the steady state capital labor ratio is monotone. We summarize the results just obtained in the following proposition:

PROPOSITION 1: *For an economy that has no oil or that has exhausted its oil stock and is now sustained completely by renewable energy, the backstop capital labor ratio converges monotonically to the steady state level* (22) *in the long run.*

5. EXISTENCE OF COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH FOSSIL FUELS

In the preceding section, we have studied an economy that is sustained completely by renewable energy. For such an economy, the competitive equilibrium can be computed recursively. For an economy with oil resources, it is not possible to find a competitive equilibrium in this recursive manner. The reason is that for an economy without oil the stock of capital is always fully utilized, which enables us to determine completely the price of energy in any period in terms of the capital stock. However, for an economy with oil resources, the amount of oil extracted in a period – and a fortiori the price of oil – is endogenous, and the oil stock might be exploited over many periods. The price of oil and the rental rate of capital cannot be computed directly from the stock of capital and the stock of oil. The existence of a competitive equilibrium for an economy with oil now becomes problematic. There is a literature on the existence of competitive equilibrium for overlapping-generations models, which began with Balasko and Shell (1980) and Balasko, Cass, and Shell (1980). In their efforts to generalize the finite-economy model of Arrow and Debreu to the overlapping-generations model, these two researchers have qualified the latter as one with double infinities: an infinity of consumers and an infinity of commodities. The infinity of consumers involve the infinite number of successive generations, while the infinity of commodities involve the infinite number of dated commodities – one commodity for each time period. These researchers established the existence of a competitive equilibrium for a simple overlapping-generation model of an exchange economy by showing that the competitive equilibrium for the overlapping-generations model is the limit in the product topology of a sequence of truncated economies. This proof technique is also used by Hung and Quyen (2006, HQ hereafter) in their work on growth, resource substitution, and endogenous fertility. In our model, there is added complications that arise from the temporal production externalities: an excessive burning of fossil fuels in one period raises the stock of greenhouse gases in the atmosphere, which shifts the production function of the consumption good downward. Although the existence of competitive equilibrium for the standard Arrow-Debreu model is well known, the question of existence when there are production externalities, according to McKenzie (1981), is not settled. Because the structure of production externalities in our model takes place in the direction of the arrow of time, the existence proof can be obtained by generalizing the existence proof given by Blad and Keiding (1990) for an exchange economy. The following proposition asserts the existence of a competitive equilibrium for an economy with oil resources and with a policy on climate change. Its existence proof – given in the appendix – is due to Quyen(2006).

PROPOSITION 2 (Quyen (2006)): *Suppose that the economy begins in period 0 with a stock of oil* $X_0 > 0$ *and a stock of backstop capital* $K_0 \geq 0$. *Also, the initial stock of greenhouse gases, namely* H_0*, is known. Let* $(E_t^{\#})_{t=0}^{\infty}$ *be a policy on climate change implemented by the government. There exists a competitive equilibrium, say* $(\mathcal{P}, \mathcal{Q})$*, with* $\mathcal{P} = (p_{L,t}, p_{X,t}, p_{K,t}, p_{E,t})_{t=0}^{\infty}$ *and*

$$\mathcal{Q} = \left(c_0^1, (c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})_{t=0}^{\infty}, (Y_t, L_t, Q_t, B_t, E_t)_{t=0}^{\infty}, (K_t, X_t, H_t)_{t=0}^{\infty}\right)$$

6. COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH OIL RESOURCES: THE CASE OF NO POLICY ON CLIMATE CHANGE

Let us first consider the case where the government does not have any environmental policy and the firms are free to discharge greenhouse into the atmosphere. This is the special case where the number of emissions permits issued in each period is so high, say $E_t^{\#} > X_0, t = 0,1,...,$ that the equilibrium price of emissions permits in each period is driven down to 0: the consumption good sector is free to discharge greenhouse gases into the atmosphere.

LEMMA 1: *Suppose that the government does not implement any policy on climate change. If the initial oil stock is large, then the oil input in period 0 will be large.*

PROOF: If the oil input in period 0 is not large when the initial oil stock is large, then there exists a number $M > 0$ such that $Q_0 < M$ when $X_0 \to +\infty$. Under such a scenario, the price of energy in period 0 will be bounded below by $(1-\alpha)A\Omega(H_0)(M + K_0)^{-\alpha}$ and the wage rate in period 0 bounded above by $\alpha A\Omega(H_0)(M + K_0)^{1-\alpha}$. Furthermore, the value of the oil investment of a young individual of period 0 must be less than her labor income, i.e.,

$$(23) \quad \begin{aligned} p_{X,0}(X_0 - Q_0) &< \alpha A\Omega(H_0)(Q_0 + K_0)^{1-\alpha} \\ &< \alpha A\Omega(H_0)(M + K_0)^{1-\alpha}. \end{aligned}$$

Because $p_{X,0} \geq (1-\alpha)A\Omega(H_0)(M+K_0)^{-\alpha}$, the left side of (23) will tend to infinity when $X_0 \to +\infty$, and this is not possible because the right side of the second inequality of (23) is finite. We have just proved that the oil input in period 0 will be high when the initial oil stock is large. ■

PROPOSITION 3: *Suppose that the government does not implement any policy on climate change. If the initial oil stock is large, then the output of the consumption good in period* 1 *will be small. More precisely,* $Y_1 \to 0$ *when* $X_0 \to +\infty$. *Under the competitive equilibrium, the abundance of oil resources induces an excessive burning of these fossil fuels, leading to a high concentration of greenhouse gases in the atmosphere, which in turn results in a low level of consumption for the young and old generations of period* 1.

PROOF: The competitive equilibrium wage rate in period 0 is given by

(24) $$p_{L,0} = \alpha A\Omega(H_0)(Q_0+K_0)^{1-\alpha},$$

and the saving of an individual of period 0 is given by

(25) $$s_0 = \frac{\beta}{1+\beta}p_{L,0} = \frac{\beta}{1+\beta}\alpha A\Omega(H_0)(Q_0+K_0)^{1-\alpha}.$$

Because the saving of a young individual of period 0 must be at least sufficient to purchase the oil stock remaining at the end of period 0, the following inequality must hold:

(26) $$\frac{\beta}{1+\beta}\alpha A\Omega(H_0)(Q_0+K_0)^{1-\alpha} \geq (1-\alpha)A\Omega(H_0)(Q_0+K_0)^{-\alpha}(X_0-Q_0),$$

which can be simplify to

(27) $$\frac{\beta}{1+\beta}\alpha(Q_0+K_0)\geq(1-\alpha)(X_0-Q_0),$$

or

(28) $$Q_0\geq\frac{(1-\alpha)(1+\beta)X_0-\alpha\beta K_0}{(1-\alpha)(1+\beta)+\alpha\beta}.$$

It follows from (28) that the remaining oil stock at the beginning of period 1 satisfies the following inequality

$$X_1=X_0-Q_0$$

$$\leq X_0-\frac{(1-\alpha)(1+\beta)X_0-\alpha\beta K_0}{(1-\alpha)(1+\beta)+\alpha\beta}$$

$$\leq\frac{[(1-\alpha)(1+\beta)+\alpha\beta]X_0-(1-\alpha)(1+\beta)X_0-\alpha\beta K_0}{(1-\alpha)(1+\beta)+\alpha\beta}$$

(29) $$\leq\frac{\alpha\beta}{(1-\alpha)(1+\beta)+\alpha\beta}(X_0-K_0).$$

The capital investment of a young individual of period 0 is given by

$$K_1=s_0-p_{X,0}X_1$$

$$=\frac{\beta}{1+\beta}\alpha A\Omega(H_0)(Q_0+K_0)^{1-\alpha}-(1-\alpha)A\Omega(H_0)(Q_0+K_0)^{-\alpha}(X_0-Q_0)$$

(30) $$=A\Omega(H_0)(Q_0+K_0)^{-\alpha}\left[\frac{\beta}{1+\beta}\alpha(Q_0+K_0)-(1-\alpha)(X_0-Q_0)\right]$$

$$=A\Omega(H_0)(Q_0+K_0)^{-\alpha}\left[\left(1-\alpha+\frac{\alpha\beta}{1+\beta}\right)Q_0+\frac{\alpha\beta}{1+\beta}K_0-(1-\alpha)X_0\right]$$

$$=A\Omega(H_0)(Q_0+K_0)^{-\alpha}\left[\frac{\alpha\beta+(1-\alpha)(1+\beta)}{1+\beta}Q_0+\frac{\alpha\beta}{1+\beta}K_0-(1-\alpha)X_0\right].$$

Now the output of the consumption good in period 1 is given by

$$Y_1 = A\Omega(H_1)(Q_1 + K_1)^{1-\alpha} \tag{31}$$
$$= Ae^{-\lambda[(1-\gamma)H_0+Q_0]}(Q_1 + K_1)^{1-\alpha},$$

assuming $\underline{H} = 0$.

Because $Q_1 \leq X_1$, the output of the consumption good in period 1 satisfies the following inequality:

$$Y_1 \leq Ae^{-\lambda[(1-\gamma)H_0+Q_0]} \times$$
$$\left(\begin{array}{l} \frac{\alpha\beta}{(1-\alpha)(1+\beta)+\alpha\beta}(X_0 - K_0) + \\ A\Omega(H_0)(Q_0 + K_0)^{-\alpha}\left[\frac{\alpha\beta+(1-\alpha)(1+\beta)}{1+\beta}Q_0 + \frac{\alpha\beta}{1+\beta}K_0 - (1-\alpha)X_0\right] \end{array} \right)^{1-\alpha}. \tag{32}$$

Because $Q_0 \leq X_0$, it follows from the preceding inequality that

$$Y_1 \leq Ae^{-\lambda[(1-\gamma)H_0+Q_0]} \times \left(\begin{array}{l} \frac{\alpha\beta}{(1-\alpha)(1+\beta)+\alpha\beta}(X_0 - K_0) \\ + \frac{\alpha\beta}{1+\beta}A\Omega(H_0)(Q_0 + K_0)^{1-\alpha} \end{array} \right)^{1-\alpha}. \tag{33}$$

Also, it follows from (28) that

$$X_0 \leq \frac{(1-\alpha)(1+\beta)+\alpha\beta}{(1-\alpha)(1+\beta)}Q_0 + \frac{\alpha\beta}{(1-\alpha)(1+\beta)}K_0. \tag{34}$$

Using (34) in (33), we can write

(35)

$$Y_1 \le Ae^{-\lambda[(1-\gamma)H_0+Q_0]} \times \left(\frac{\alpha\beta}{(1-\alpha)(1+\beta)+\alpha\beta} \left(\frac{(1-\alpha)(1+\beta)+\alpha\beta}{(1-\alpha)(1+\beta)} Q_0 + \frac{\alpha\beta}{(1-\alpha)(1+\beta)} K_0 - K_0 \right) + \frac{\alpha\beta}{1+\beta} A\Omega(H_0)(Q_0+K_0)^{1-\alpha} \right)^{1-\alpha}$$

$$= Ae^{-\lambda[(1-\gamma)H_0+Q_0]} \times \left(\frac{\alpha\beta}{(1-\alpha)(1+\beta)} Q_0 + \frac{\alpha\beta}{(1-\alpha)(1+\beta)+\alpha\beta} \frac{\alpha\beta-(1-\alpha)(1+\beta)}{(1-\alpha)(1+\beta)} K_0 + \frac{\alpha\beta}{1+\beta} A\Omega(H_0)(Q_0+K_0)^{1-\alpha} \right)^{1-\alpha}$$

Now when Q_0 is large, the exponential term $e^{-\lambda[(1-\gamma)H_0+Q_0]}$ will dominate the expression

(36)

$$\left(\frac{\alpha\beta}{(1-\alpha)(1+\beta)} Q_0 + \left[\frac{\alpha\beta}{(1-\alpha)(1+\beta)+\alpha\beta} \right] \left[\frac{\alpha\beta-(1-\alpha)(1+\beta)}{(1-\alpha)(1+\beta)} \right] K_0 + \frac{\alpha\beta}{1+\beta} A\Omega(H_0)(Q_0+K_0)^{1-\alpha} \right)^{1-\alpha}.$$

Thus under the scenario that the government does not implement any policy on climate change the output of the consumption good in period 1 under the competitive equilibrium will tend to 0 when the initial oil stock tends to infinity. ■

PROPOSITION 4: *Suppose that the government does not implement any policy on climate change. Consider a competitive equilibrium under which oil constitutes part of the energy input used in the production of the consumption good in at least one period. Then the competitive equilibrium is not Pareto optimal*

PROOF: Without loss of generality, suppose that $Q_t > 0$ for $t = 0$. If we cut back the oil input in period 0, say by ε and transfer it to period 1, we will be able to lower the stock of GHGs at the beginning of period 1. Of course, the output of the consumption good in period 0 will be lower. The variation in the output of the consumption good in period 0 is given by

(37) $$\Delta Y_0(\varepsilon) = A\Omega(H_0)(Q_0 - \varepsilon + K_0)^{1-\alpha} - A\Omega(H_0)(Q_0 + K_0)^{1-\alpha}.$$

If we maintain the same capital investment as the one made by the young generation of period and raises the oil input in period 1 above its competitive equilibrium, then the output of the consumption good in period 1 will rise by

(38) $$\begin{aligned}\Delta Y_1(\varepsilon) = & A\Omega((1-\gamma)H_0 + Q_0 - \varepsilon)(Q_1 + \varepsilon + K_1)^{1-\alpha} \\ & - A\Omega((1-\gamma)H_0 + Q_0)(Q_1 + K_1)^{1-\alpha}.\end{aligned}$$

The lifetime utility for a young individual of period 0 under this intervention is

(39) $$u(\varepsilon) = Log[c_0^0 + \Delta Y_0(\varepsilon)] + \beta Log[c_1^1 + \Delta Y_1(\varepsilon)].$$

Differentiating (39) with respect to ε, then evaluating the result at $\varepsilon = 0$, we obtain

(40) $$\begin{aligned} u'(0) = & -\frac{(1-\alpha)A\Omega(H_0)(Q_0 + K_0)^{-\alpha}}{c_0^0} + \\ & \beta\left[\frac{(1-\alpha)A\Omega((1-\gamma)H_0 + Q_0)(Q_1 + K_1)^{-\alpha} - A\Omega'((1-\gamma)H_0 + Q_0)(Q_1 + K_1)^{1-\alpha}}{c_1^1}\right] \\ = & -\frac{p_{X,0})}{c_0^0} + \beta\left[\frac{p_{X,1} + \lambda A\Omega((1-\gamma)H_0 + Q_0)(Q_1 + K_1)^{1-\alpha}}{\beta c_0^0 \frac{p_{X,1}}{p_{X,0}}}\right]\end{aligned}$$

$$= -\frac{p_{X,0}}{c_0^0} + \frac{p_{X,0}}{c_0^0}\left[1 + \frac{\lambda A\Omega((1-\gamma)H_0 + Q_0)(Q_1 + K_1)^{1-\alpha}}{p_{x,1}}\right]$$

$$= \frac{p_{X,0}}{c_0^0}\left[\frac{\lambda(Q_1 + K_1)}{1-\alpha}\right] > 0.$$

Because $u'(0) > 0$, the intervention just described raises the lifetime utility of the young generation of period 0 without lowering the old-age utility of the old generation of period 0 and the lifetime utilities of the generations to come. ■

PROPOSITION 5: *If* $X_0 > 0$ *and* $K_0 = 0$, *then there exists a positive integer, say* $T > 0$, *such that* $K_T > 0$. *That is, if the backstop is not brought into use in period* 0, *then it will be brought into use in finite time.*

PROOF: As long as oil is the only source of energy used in the production of the consumption good, then the wage rate and the price of oil are given, respectively, by $p_{L,t} = \alpha A\Omega(H_t)Q_t^{1-\alpha}$ and $p_{X,t} = (1-\alpha)A\Omega(H_t)Q_t^{-\alpha}$. Because all the saving of a young individual of period t is put in oil, we must have

$$s_t = \frac{\beta}{1+\beta} p_{L,t}$$

(41) $$= \frac{\beta}{1+\beta}\alpha A\Omega(H_t)Q_t^{1-\alpha}$$

$$= p_{X,t}X_{t+1} = (1-\alpha)A\Omega(H_t)Q_t^{-\alpha}(X_t - Q_t).$$

In the preceding expression, the equality between the expression on the right of the second equality in and the right side of the fourth equality allows us to write

(42) $Q_t = \eta X_t,$

where we have let

(43) $$\eta = \frac{(1-\alpha)(1+\beta)}{(1-\alpha)(1+\beta)+\alpha\beta}.$$

Equation (42) asserts that as long as the backstop has not been brought into use the oil consumption in each period is a constant fraction of the remaining stock. Using (42), we obtain the following difference equations, which describe the dynamics of the remaining oil stock and the consumption of oil, respectively, as long as the backstop has not been brought into use

(44) $X_{t+1} = (1-\eta)X_t$

and

(45) $Q_{t+1} = (1-\eta)Q_t.$

Also, as long as the backstop has not been brought into use the dynamics of the price of oil is governed by the following difference equation:

(46) $$\frac{p_{X,t+1}}{p_{X,t}} = \frac{\Omega(H_{t+1})}{\Omega(H_t)(1-\eta)^{\alpha}}.$$

Now as long as the backstop has not been brought into use, the rate of return to backstop capital investment cannot exceed the rate of return to oil investment and the price of oil cannot exceed the rental rate of backstop capital, i.e.,

(47) $$\frac{p_{X,t+1}}{p_{X,t}} = \frac{\Omega(H_{t+1})}{\Omega(H_t)(1-\eta)^{\alpha}} \geq 1-\delta+p_{K,t+1},$$

and

(48) $p_{K,t+1} \geq p_{X,t+1}.$

If the backstop is never brought into use, then as time increases indefinitely the stock of GHGs will be driven down to 0, which means that $\ell im_{t\to+\infty}[\Omega(H_{t+1})/\Omega(H_t)]=1$, and it follows from (46) that the price of oil will tend to infinity in the long run. This result and (48) then imply that the rental rate of backstop capital will also tend to infinity in the long run, contradicting the inequality in (47). Hence there exists an integer $T>0$ such that $K_T>0$, but $K_t=0, t<T$. ■

Whether $X_T>0$ or $X_T=0$ depends on the parameters of the model and the competitive equilibrium being considered. If $X_T=0$, then the economy from time T on is completely sustained by renewable energy, and its evolution through time is as described in Section 4. If $X_T>0$, then the two technologies might coexist during a certain time interval. Indeed, if both technologies are exploited in a period, say $t+1$, then both of the following conditions must hold:

(49) $$\frac{p_{X,t+1}}{p_{X,t}}=1-\delta+p_{K,t+1},$$

and

(50) $$p_{K,t+1}=p_{X,t+1}.$$

It then follows from (49) and (50) that

(51) $$p_{X,t+1}=\frac{(1-\delta)p_{X,t}}{1-p_{X,t}}$$

Observe that when (51) holds, we must have $p_{X,t}<1$, i.e., if both technologies are exploited during a period, then the price of oil in the preceding period cannot be too high.

Furthermore, (51) will continue to hold from the first until the last period that the two technologies are exploited simultaneously.

PROPOSITION 6: *If the rate of capital depreciation is low, then under a competitive equilibrium, the oil stock is exhausted in finite time. That is, there exists a positive integer* T *such that* $X_T = 0$.

PROOF: If the oil stock is not exhausted in finite time, then either it is exhausted asymptotically or it is never exhausted at all. In either case, each successive young generation invests in oil, and the following inequality must hold:

$$(52) \qquad \frac{p_{X,t+1}}{p_{X,t}} \geq 1 - \delta + p_{K,t+1}, \qquad (t = 1,2,...).$$

Now when δ is small, we have $\inf_{t\geq 1} p_{K,t} > \delta$, which means that the right side of the inequality (52) is strictly greater than 1. Hence the price of oil must rise through time to infinity in the long run.

We claim that for any time t, there exists a time $t' > t$ such that $K_{t'} > 0$. Indeed, if this is not true, then after a certain time, the backstop capital will never be used and all the energy needs of the economy are met by fossil fuels. The argument used in proving Proposition 5 can be repeated here to arrive at a contradiction. In period t', we must have

$$(53) \qquad \begin{aligned} \frac{p_{X,t'}}{p_{X,t'-1}} &= 1 - \delta + p_{K,t'} \\ &\leq 1 - \delta + p_{X,t'}, \end{aligned}$$

from which we obtain

(54) $\quad p_{X,t'} \le \dfrac{(1-\delta)p_{X,t'-1}}{1-p_{X,t'-1}}.$

Because $p_{X,t'-1}$ is large, the denominator of the expression on the right side of inequality (54) will be negative, while the price of oil in period t', namely $p_{X,t'}$, is positive, contradicting the premise of the reductio ad absurdum argument. ■

7. COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH OIL RESOURCES: THE CASE OF EMISSIONS PERMITS

To evaluate the impact on the pattern of resource allocation and welfare induced by a policy on climate change, we need a frame of reference, say a competitive equilibrium $(\overline{\mathcal{P}}, \overline{\mathcal{A}})$, that is assumed to prevail when there is no policy on climate change. Here $\overline{\mathcal{P}} = (\overline{p}_{L,t}, \overline{p}_{X,t}, \overline{p}_{K,t}, \overline{p}_{E,t})_{t=0}^{\infty}$ is the equilibrium price system and

$$\overline{\mathcal{A}} = \left(\overline{c}_0^1, (\overline{c}_t^0, \overline{c}_{t+1}^1, \overline{k}_{t+1}, \overline{x}_{t+1})_{t=0}^{\infty}, (\overline{Y}_t, \overline{L}_t, \overline{Q}_t, \overline{B}_t, \overline{E}_t)_{t=0}^{\infty}, (\overline{K}_t, \overline{X}_t, \overline{H}_t)_{t=0}^{\infty}\right)$$

is the equilibrium allocation induced by $\overline{\mathcal{P}}$. Next, let $(E_t^{\#})_{t=0}^{\infty}$ be a climate change policy implemented by the government and $(\mathcal{P}, \mathcal{A})$ be a competitive equilibrium induced by $(E_t^{\#})_{t=0}^{\infty}$, where $\mathcal{P} = (p_{L,t}, p_{X,t}, p_{K,t}, p_{E,t})_{t=0}^{\infty}$ is the equilibrium price system and

$$\mathcal{A} = \left(c_0^1, (c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})_{t=0}^{\infty}, (Y_t, L_t, Q_t, B_t, E_t)_{t=0}^{\infty}, (K_t, X_t, H_t)_{t=0}^{\infty}\right)$$

is the equilibrium allocation induced by $\mathcal{P}$.

Now in a period when both oil and solar energy are used in the production of the consumption good, then the following first-order condition must hold:

(55) $$(1-\alpha)A\Omega(H_t)(Q_t + K_t)^{-\alpha} = p_{X,t} + p_{E,t} = p_{K,t}.$$

If only solar energy is the only energy input used in the production of the consumption good, then the following condition must hold:

(56) $$\begin{aligned}(1-\alpha)A\Omega(H_t)K_t^{-\alpha} &= (1-\alpha)A\Omega(H_t)(Q_t + K_t)^{-\alpha} \\ &= p_{K,t} \le p_{X,t} + p_{E,t}.\end{aligned}$$

On the other hand, if only the stock of backstop capital is equal to 0, then oil is the only source of energy used in the production of the consumption good, and the following first-order condition must hold:

(57) $$\begin{aligned}(1-\alpha)A\Omega(H_t)Q_t^{-\alpha} &= (1-\alpha)A\Omega(H_t)(Q_t + K_t)^{-\alpha} \\ &= p_{X,t} + p_{E,t} \le p_{K,t}.\end{aligned}$$

In this case, we can assume that the inequality in (57) is an equality without disturbing the competitive equilibrium. Therefore, we can assume that the following conditions hold in each period:

(58) $$\begin{aligned}(1-\alpha)A\Omega(H_t)(Q_t + K_t)^{-\alpha} &= (1-\alpha)A\Omega(H_t)(E_t^{\#} + K_t)^{-\alpha} \\ &= p_{K,t} \le p_{X,t} + p_{E,t},\end{aligned} \qquad (t = 0,1,\ldots),$$

with the inequality becoming an equality if $Q_t > 0$. Note that the first equality in (58) is obtained by using the fact that $Q_t = E_t^{\#}$.

PROPOSITION 7: *Suppose that the initial oil stock is large. Then relative to the equilibrium under the policy of nonintervention, the climate-change policy depresses the price of oil in period* 0 *although the effective cost of using oil in this period is higher; that* *is,* $p_{X,0} < \bar{p}_{X,0}$, *but* $p_{X,0} + p_{E,0} > \bar{p}_{X,0}$. *The low price of oil under the climate-change policy reduces the income of the old generation and makes it worse off.*

PROOF: Under the climate-change policy $(E_t^{\#})_{t=0}^{\infty}$, the *effective unit cost of using oil* – the price of oil plus the cost of one emissions permit to comply with the climate-change policy – is given by

$$p_{X,0} + p_{E,0} = (1-\alpha)A\Omega(H_0)(E_0^{\#} + K_0)^{-\alpha}. \tag{59}$$

Furthermore, according to (16), the saving of a young individual of period 0 is given by

$$\begin{aligned} s_0 &= \frac{1}{1+\beta}\left[\beta(p_{L,0} + m_0) - \frac{m_1}{r_1}\right] \\ &\leq \frac{\beta}{1+\beta}(p_{L,0} + m_0) \\ &\leq \frac{\beta}{1+\beta}\left[\alpha A\Omega(H_0)(E_0^{\#} + K_0)^{1-\alpha} + (1-\alpha)A\Omega(H_0)(E_0^{\#} + K_0)^{-\alpha} E_0^{\#}\right] \\ &\leq \frac{\beta}{1+\beta} A\Omega(H_0)(E_0^{\#} + K_0)^{1-\alpha}. \end{aligned} \tag{60}$$

The saving of a young individual of period 0 is thus constrained to be bounded above by the expression on the last line of (60) no matter what the value of the initial oil stock. Thus when the oil stock is large, the price of oil must be low so that the young generation of period could buy $X_0 - E_0^{\#}$, the stock of oil that remains at the end of period 0. We have just shown that $p_{X,0} < \bar{p}_{X,0}$ and $p_{E,0} > 0$ when the initial oil stock is large. Thus

$$p_{X,0}(X_0 + K_0) + (1-\delta)K_0 < \bar{p}_{X,0}(X_0 + K_0) + (1-\delta)K_0. \tag{61}$$

Inequality (61) asserts that the income of an old individual is lower under the climate-change policy than under the policy of nonintervention. ■

Suppose that the objective of the climate-change policy is to stabilize the stock of greenhouse gases at a level, say $H^{\#}$, judged to be reasonable in the long run according to the precautionary principle. To stabilize the stock of greenhouse gases at the level $H^{\#}$, we shall assume a constant cap on greenhouse gases emissions given by $E_t^{\#} = E^{\#} = \gamma H^{\#}, t = 0,1,\ldots$ Also, we suppose that $K_0 = 0$; that is, the backstop is not yet active in period 0.

LEMMA 2: *If the initial oil stock is sufficiently large, then there exists a positive integer* $T^{\#}$ *such that the emissions cap is binding during the time interval between 0 and* $T^{\#}$, *with* $T^{\#}$ *included. Furthermore,* $T^{\#} \to +\infty$ *when* $X_0 \to +\infty$.

PROOF: First, we claim that the cap is binding in period 0. Indeed, if $Q_0 < E^{\#}$, then the price of emissions permits in period 0 will be 0, and this means that the price of oil in period 0 is given by

(62) $$p_{X,0} = (1-\alpha)A\Omega(H_0)(Q_0)^{-\alpha} > (1-\alpha)A\Omega(H_0)(E^{\#})^{-\alpha},$$

and the value of the oil investment of a young individual of period 0 is given by

(63) $$\begin{aligned} p_{X,0}(X_0 - E^{\#}) &= (1-\alpha)A\Omega(H_0)(Q_0)^{-\alpha}(X_0 - E^{\#}) \\ &> (1-\alpha)A\Omega(H_0)(E^{\#})^{-\alpha}(X_0 - E^{\#}). \end{aligned}$$

Because the oil investment of the young generation of period 0 constitutes part of the consumption of the old generation of that period, the left side of the equality in (63) is bounded above by the output of the consumption good in period 0. Using this result, we can rewrite (63) as follows:

$$A\Omega(H_0)(Q_0)^{1-\alpha} > p_{X,0}(X_0 - E^{\#}) \qquad (64)$$
$$> (1-\alpha)A\Omega(H_0)(E^{\#})^{-\alpha}(X_0 - E^{\#}).$$

When X_0 is large, the expression on the second line of (64) will be large, while the expression on the left side of the first inequality of (64) remains bounded above by $A\Omega(H_0)(E^{\#})^{1-\alpha}$, which is not possible. The argument just presented for period 0 can be repeated for any period $t > 0$ as long as the remaining oil stock at the beginning of that period is still large. The existence of $T^{\#}$ is now established. Note that $T^{\#} \leq X_0 / E^{\#}$. ∎

LEMMA 3: *Suppose that* $X_0 > 0$, *but* $K_0 = 0$. *If the emissions cap* $E^{\#}$ *is sufficiently low, then* $K_1 > 0$. *That is, if the backstop is not in operation in period* 0, *then it will be brought into use in period* 1 *by implementing a stringent climate-change policy.*

PROOF: To prove the lemma, suppose the contrary, say $K_1 = 0$, which means that a young individual of period 0 will only invest in oil, and the value of oil investment is then given by

$$p_{X,0}X_1 = s_0 = \frac{1}{1+\beta}\left[\beta(p_{L,0} + m_0) - \frac{m_1}{r_1}\right] \qquad (65)$$
$$= \frac{1}{1+\beta}\left[\beta\left(\alpha A\Omega(H_0)(E^{\#})^{1-\alpha} + \frac{1}{2}p_{E,0}E^{\#}\right) - \frac{1}{2}p_{E,1}E^{\#}\frac{p_{X,0}}{p_{X,1}}\right]$$

Also, the rate of return to oil investment for a young individual of period 0 is given by

$$r_1 = \frac{p_{X,1}}{p_{X,0}} \geq 1 - \delta + p_{K,1}$$
$$= 1 - \delta + p_{X,1} + p_{E,1} \qquad (66)$$
$$= 1 - \delta + (1-\alpha)A\Omega[(1-\gamma)H_0 + E^{\#}](E^{\#})^{-\alpha}.$$

Note that the expression on the last line of (66) will be large when the emissions cap $E^{\#}$ is low. That is, $(p_{X,1}/p_{X,0}) \to +\infty$ when $E^{\#} \to 0$.

Next, note that the value of the oil investment of a young individual of period 1 is given by

(67)
$$\begin{aligned} p_{X,1}X_2 = s_0 &= \frac{1}{1+\beta}\left[\beta(p_{L,1}+m_1) - \frac{m_2}{r_2}\right] \\ &= \frac{1}{1+\beta}\left[\beta\left(\alpha A\Omega(H_1)(E^{\#})^{1-\alpha} + \frac{1}{2}p_{E,1}E^{\#}\right) - \frac{1}{2}p_{E,2}E^{\#}\frac{p_{X,1}}{p_{X,2}}\right] \end{aligned}$$

It follows from (66) and (67) that

(68)
$$\frac{p_{X,1}X_2}{p_{X,0}X_1} = \frac{\beta\alpha A\Omega(H_1)(E^{\#})^{1-\alpha} + \frac{1}{2}E^{\#}\left(\beta p_{E,1} - p_{E,2}\frac{p_{X,1}}{p_{X,2}}\right)}{\beta\alpha A\Omega(H_0)(E^{\#})^{1-\alpha} + \frac{1}{2}E^{\#}\left(\beta p_{E,0} - p_{E,1}\frac{p_{X,0}}{p_{X,1}}\right)}$$

Now when $E^{\#} \to 0$, we have $X_2/X_1 \to 1$. Also, we have shown that $p_{X,1}/p_{X,0} \to +\infty$ if the lemma is not true. Hence the ratio on the left side of (68) will tend to infinity when $E^{\#} \to 0$. On the other hand, the expression on the last line of (68) tends to 1 when $E^{\#} \to 0$. Therefore, (66) cannot hold when $E^{\#} \to 0$, contradicting the premise of the reductio ad absurdum argument. ■

LEMMA 4: *If the rate of capital depreciation is low enough, then the price of oil will rise through time as long as the oil stock has not been depleted.*

PROOF: If $\delta = 0$, then $1-\delta + p_{K,t} = 1 + p_{K,t} > 1, t = 0,1,\ldots$ Using a continuity argument we can assert that if δ is small, then as long as the oil stock has not been depleted we will have

$$(69) \qquad \frac{p_{X,t+1}}{p_{X,t}} \geq 1-\delta + p_{K,t} > 1,$$

which implies that the price of oil will rise through time as long as the stock of oil has not been exhausted. ∎

Because the initial stock of backstop capital is equal to 0, the energy input in that period consists only of oil. Proposition 5, which holds for the case of nonintervention also holds for the case a climate-change policy is implemented; that is, the backstop will be brought into use in finite time. More precisely, there exists a time, say T^b, such that $K_{T^b} > 0$ and $K_t = 0, t < T^b$.

LEMMA 5: *If* $\hat{T} = \min\{T^b, T^\#\}$, *then during the time interval* $0 \leq t \leq \hat{T}$ *the emissions cap is binding and all the energy needs of the economy are met by oil. Furthermore, if the climate-change policy permits the stock of greenhouse gases to stabilize at a higher level than* H_0, *then the price of emissions permits is declining in the time interval* $0 \leq t \leq \hat{T}+1$.

PROOF: The first statement of the lemma follows directly from the definition of $\hat{T}$. To prove the second statement of the lemma, note that the effective unit cost of using oil during the time interval $0 \le t \le \hat{T}$ is given by

$$
\begin{aligned}
(70)\qquad p_{X,t} + p_{E,t} &= (1-\alpha)A\Omega(H_t)(Q_t)^{-\alpha} \\
&= (1-\alpha)A\Omega(H_t)(E^{\#})^{-\alpha}, \qquad (t = 0,\dots,T^b - 1),
\end{aligned}
$$

which is constant during the time interval $0 \le t \le \hat{T}$. We have already known that the price of oil is rising as long as the oil stock has not been depleted. Hence the price of emissions permits must be declining during the time interval $0 \le t \le \hat{T}$. Finally, in period $\hat{T}+1$, if the cap is not binding, then $p_{E,\hat{T}+1} = 0$. On the other hand, if the cap is binding, then the effective unit cost of using oil in period $\hat{T}+1$ is given by

$$
\begin{aligned}
(71)\qquad p_{X,\hat{T}+1} + p_{E,\hat{T}+1} &= (1-\alpha)A\Omega(H_{\hat{T}+1})(Q_{\hat{T}+1} + K_{\hat{T}+1})^{-\alpha} \\
&= (1-\alpha)A\Omega(H_{\hat{T}+1})(E^{\#} + K_{\hat{T}+1})^{-\alpha} \\
&\le (1-\alpha)A\Omega(H_{\hat{T}})(E^{\#})^{-\alpha} = p_{X,\hat{T}} + p_{E,\hat{T}}, \qquad (t = 0,\dots,T^b - 1).
\end{aligned}
$$

■

Combining Lemmas 2 through 5, we obtain the following proposition, which characterizes the competitive equilibrium induced by the climate-change policy that caps the greenhouse gases emissions at a constant level.

PROPOSITION 8: *Suppose that the initial stock of oil is large and that the climate-change policy involves capping the greenhouse gases emissions at a constant level. Under the competitive equilibrium, the process of technology substitution – renewable energy for fossil fuels – takes place in three stages. In the first stage, only oil is used. In the second stage, the energy needs of the economy are met by both oil and renewable energy. In the* *third stage – after oil exhaustion has occurred – the economy is completely sustained by renewable energy. In particular,*

(a) *If the backstop is not available in period* 0, *then it will be brought into use in finite time. Furthermore, if the cap is stringent enough, the backstop will be brought into use in period* 1.

(b) *If the rate of capital depreciation is low enough, then the oil stock is exhausted in finite time, and the price of oil will rise as long as oil exhaustion has not occurred.*

(c) *If the climate-change policy permits the stock of greenhouse gases to stabilize at a level higher than the initial level, then in the time interval that stretches from time* 0 *up to at most one period after the backstop is brought into use the price of emissions permits will be declining as long as the cap is binding.*

8. CONCLUSION

The model can be extended in several directions. First, abatement capital – as a means of reducing greenhouse gases and hence complying with the government policy – can be incorporated into the model. Second, R&D activities to find a greener technology can be introduced. Third, the model can be extended by considering various intergenerational transfer schemes to induce a more intergenerational distribution of income. Finally, because the model formulated in this essay is a global model which treats the world as a single country, it can be extended into a two-country model in which trade in oil, renewable energy, and emissions permits are allowed. Such a two-country model can be used to analyze cooperative as well as non-cooperative behavior of different nations in their strategies to deal with global warming. Furthermore, opening a country for trade in these commodities permits us to see how the strategies of various countries affect the terms of trade. The third essay offers such an extension of the current model.

APPENDIX

THE EXISTENCE OF A COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH OIL RESOURCES AND A CLIMATE-CHANGE POLICY

A. EXISTENCE OF COMPETITIVE EQUILIBRIUM FOR TRUNCATED ECONOMIES WITH FINITE TIME HORIZON

A.1. *Truncated Economies*

Suppose that the economy begins at time 0 in state (X_0, K_0, H_0), with $X_0 > 0, K_0 \geq 0, H_0 \geq 0$. Let T be a non-negative integer. If we truncate our economy at the end of period T, then we obtain an economy with a finite time horizon that we call the truncated economy with time horizon T. A price system for the truncated economy with time horizon T is a finite sequence $\wp^T = (p_{Y,t}, p_{L,t}, p_{X,t}, p_{B,t}, p_{K,t}, p_{E,t})_{t=0}^{T}$. Here $p_{Y,t}, p_{L,t}, p_{X,t}, p_{B,t}, p_{K,t},$ and $p_{E,t}$ denote, respectively, the price of the consumption good, the wage rate, the price of oil, the price of solar energy, the rental rate of backstop capital, and the price of emissions permits – all in period $t, t = 0, \ldots, T$. An *allocation induced by* $\wp^T$ is a list of finite sequences

$$\mathcal{Q}^T = \left(c_0^1, (c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})_{t=0}^{T-1}, (Y_t, L_t, Q_t, B_t, E_t)_{t=0}^{T}, (B_t^{\#}, K_t^{\#})_{t=0}^{T}, (K_t, X_t, H_t)_{t=0}^{T}, c_T^0\right)$$

with the following properties:

(i) $p_{Y,0} c_0^1 = m_0 + p_{X,0} X_0 + (1-\delta) p_{Y,0} K_0 + p_{K,0} K_0$.

(ii) $(c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})$ is an optimal lifetime plan for a young individual of period t, when the price system $\wp^T$ prevails.

(iii) $(Y_t, L_t, Q_t, B_t, E_t)$ is an optimal production plan of the representative firm in the consumption good sector in period t, when the price system $\mathcal{P}^T$ prevails.

(iv) $(B_t^{\#}, K_t^{\#})$ is an optimal production plan for the representative producer of solar energy in period t, when the price system $\mathcal{P}^T$ prevails.

(v) $(K_t, X_t, H_t) = (k_t, x_t, (1-\gamma)H_{t-1} + Q_t)$, $\quad (t = 1,2,\ldots,T)$.

(vi) $p_{Y,T}c_T^0 = m_T + p_{L,T}$.

Note that we have used $(B_t^{\#}, K_t^{\#})$ – with $B_t^{\#}$ as the output of solar energy and $K_t^{\#}$ as the input of backstop capital – to denote the production plan chosen by the representative firm in the backstop sector in period t. Also, observe that (vi) represents the consumption of a young individual in period T. Because the problem ends at the end of period T, a young individual of this period has no future to plan for and thus will neither save nor raise children; she will consume all her income (wages plus transfer) she earns.

The pair $(\mathcal{P}^T, \mathcal{Q}^T)$ is said to constitute *a competitive equilibrium for the truncated economy with time horizon T* if the following market-clearing conditions are satisfied:

(vii) $c_t^1 + c_t^0 + K_{t+1} = Y_t + (1-\delta)K_t$, $\quad (0 \le t \le T-1)$,

(viii) $c_T^1 + c_T^0 = Y_T + (1-\delta)K_T$.

(ix) $L_t = 1$, $\quad (0 \le t \le T)$.

(x) $X_{t+1} + Q_t = X_t$, $\quad (0 \le t \le T-1)$.

(xi) $B_t = B_t^{\#}$, $\quad (0 \le t \le T)$.

(xii) $K_t^{\#} = K_t$, $\quad (0 \le t \le T)$.

(xiii) $E_t \le E_t^{\#}$, $\quad (0 \le t \le T)$.

Because in each period the government transfers all the revenues obtained from the sales of emissions permits equally to the consumers – young and old – we can assume without any loss of generality that the issued emissions permits are given equally to each consumer. Thus in each period the income of a young individual consists of her wages and the revenues she obtains by selling her endowment of emissions permits on the emissions permits market. As for an old individual, her income consists of the income yielded by her ownership of the oil stock and the stock of backstop capital as well as the sales of her endowment of emissions permits.

A.2. *Bounding Truncated Economies*

To establish the existence of a competitive equilibrium for a truncated economy, we first find a bound for it. To this end, let $(M_t)_{t=0}^T$ be a finite sequence chosen in the following manner. For $t = 0$, let M_0 be a positive number satisfying

$$M_0 > A\Omega(H_0)(X_0 + K_0)^{1-\alpha} + (1-\delta)K_0 + 1 + X_0 + E_0^{\#}.$$

For any $t, 0 < t < Y-1$,

$$M_{t+1} > A(X_0 + M_t)^{1-\alpha} + (1-\delta)M_t + 1 + X_0 + E_{t+1}^{\#}..$$

Next, let Ξ be the set consisting of the lists

$$\left(c_0^1, (c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})_{t=0}^{T-1}, (L_t, Q_t, B_t, E_t)_{t=0}^{T}, (B_t^{\#}, K_t^{\#})_{t=0}^{T}, c_T^0\right)$$

that satisfy the following conditions:

(A.2.1) $\quad c_0^1 \le M_0$;

(A.2.2) $\quad c_t^0 \le M_t, c_{t+1}^1 \le M_t, k_{t+1} \le M_t, x_{t+1} \le M_t, \quad (t = 0,...,T-1)$,

(A.2.3) $\quad L_t \le M_t, Q_t \le M_t, B_t \le M_t, E_t \le M_t, \quad (t = 0,...,T)$,

(A.2.4) $\quad B_t^{\#} \le M_t, K_t^{\#} \le M_t, \quad (t = 0,...,T)$,

(A.2.5) $\quad c_T^0 \le M_T$.

The set Ξ is a convex compact subset of the $(9T+6)$ – dimensional Euclidean space. An element of Ξ will be denoted by ξ. Also, let Δ be the $(6T+5)$ – dimensional unit simplex. The set $\Xi \times \Delta$ is convex and compact.

A.3. *Excess Demands*

Now for any element $(\xi, \vartheta^T) \in \Xi \times \Delta$, let

(A.3.1) $$\mathfrak{S}_{Y,t}(\xi) = c_t^1 + c_t^0 + k_{t+1} - A\Omega(H_t)L_t^{\alpha}(Q_t + B_t)^{1-\alpha} - (1-\delta)k_t, \quad (t = 0,...,T-1),$$

(A.3.2) $$\mathfrak{S}_{Y,T}(\xi) = c_T^1 + c_T^0 - A\Omega(H_T)L_T^{\alpha}(Q_T + B_T)^{1-\alpha} - (1-\delta)k_T.$$

Note that in (A.3.1) and (A.3.2), we have let

(A.3.3) $$H_{t+1} = (1-\gamma)H_t + Q_t, 0 < t < T-1,$$
H_0 is the initial stock of greenhouse gases.

Next, let

(A.3.4) $\quad \mathfrak{S}_{L,t}(\xi) = L_t - 1, \quad (t = 0,...,T)$,

(A.3.5) $\quad \mathfrak{S}_{X,t}(\xi) = Q_t + x_{t+1} - x_t, \quad (t = 0,...,T-1)$,

(A.3.6) $\quad \mathfrak{S}_{X,T}(\xi) = Q_T - x_T$,

(A.3.7) $\quad \mathfrak{S}_{B,t}(\xi) = B_t - B_t^{\#}, \quad (t = 0,...,T)$,

(A.3.8) $\quad \mathfrak{S}_{K,t}(\xi) = K_t - K_t^{\#}, \quad (t = 0,...,T)$,

(A.3.9) $\quad \mathfrak{S}_{E,t}(\xi) = E_t - E_t^{\#}, \quad (t = 0,...,T)$.

Let

(A.3.10) $$\mathfrak{S}(\xi) = \left(\mathfrak{S}_{Y,t}(\xi)\mathfrak{S}_{L,t}(\xi), \mathfrak{S}_{X,t}(\xi), \mathfrak{S}_{B,t}(\xi), \mathfrak{S}_{K,t}(\xi), \mathfrak{S}_{E,t}(\xi)\right)_{t=0}^{T}$$

be the matrix of excess demands associated with the allocation ξ.

A.4. *Existence of Competitive Equilibrium for a Truncated Economy Subject to Bounding Constraints*

CLAIM A1: *There exists an element* $(\bar{\xi},\bar{\wp}^T)\in\Xi\times\Delta$, *where*

$$\bar{\xi}=\left(\bar{c}_0^1,(\bar{c}_t^0,\bar{c}_{t+1}^1,\bar{k}_{t+1},\bar{x}_{t+1})_{t=0}^{T-1},(\bar{L}_t,\bar{Q}_t,\bar{B}_t,\bar{E}_t)_{t=0}^T,(\bar{B}_t^\#,\bar{K}_t^\#)_{t=0}^T,\bar{c}_T^0\right)$$

and

$$\bar{\wp}^T=(\bar{p}_{Y,t},\bar{p}_{L,t},\bar{p}_{X,t},\bar{p}_{B,t},\bar{p}_{K,t},\bar{p}_{E,t})_{t=0}^T.$$

which has the following properties:

First,

(A.4.1) $\bar{c}_0^1\in\arg\max c_0^1$

subject to the following budget constraint

(A.4.2) $\bar{p}_{Y,0}c_0^1\le(1-\delta)\bar{p}_{Y,0}K_0+\bar{p}_{K,0}K_0+\bar{p}_{X,0}X_0+\frac{1}{2}\bar{p}_{E,0}E_0^\#$,

and the bounding constraint (A.2.1).

Second, for each $t=0,...,T-1$,

(A.4.3) $(\bar{c}_t^0,\bar{c}_{t+1}^1,\bar{k}_{t+1},\bar{x}_{t+1})\in\arg\max Log[c_t^0]+\beta Log[c_{t+1}^1]$

subject to the following two budget constraints

(A.4.4) $\bar{p}_{Y,t}(c_t^0+k_{t+1})+\bar{p}_{X,t}x_{t+1}\le\bar{p}_{L,t}+\frac{1}{2}\bar{p}_{E,t}E_t^\#$,

(A.4.5) $\bar{p}_{Y,t+1}c_{t+1}^1\le(1-\delta)\bar{p}_{Y,t+1}k_{t+1}+\bar{p}_{K,t+1}k_{t+1}+\bar{p}_{X,t+1}x_{t+1}+\frac{1}{2}\bar{p}_{E,t+1}E_{t+1}^\#$,

and the bounding constraints (A.2.2).

Third, for each $t=0,...,T$,

(A.4.6) $(\bar{L}_t,\bar{Q}_t,\bar{B}_t,\bar{E}_t)\in\arg\max\begin{bmatrix}\bar{p}_{Y,t}A\Omega(\bar{H}_t)L_t^\alpha(Q_t+B_t)^{1-\alpha}-\bar{p}_{L,t}L_t\\-\bar{p}_{X,t}Q_t-\bar{p}_{B,t}B_t-p_{E,t}E_t\end{bmatrix}$

subject to

(A.4.7) $Q_t\le E_t$,

and the bounding constraints (A.2.3).*Note that in the objective function* (A.1.8), *we have defined*

(A.4.8) $\bar{H}_0=H_0$, $\bar{H}_{t+1}=(1-\gamma)\bar{H}_t+Q_t, 0<t<T-1$.

Fourth, for each $t=0,...,T$,

(A.4.9) $(\bar{B}_t^\#,\bar{K}_t^\#)\in\arg\max[\bar{p}_{B,t}B_t^\#-\bar{p}_{K,t}K_t^\#]$

subject to

(A.4.10) $\bar{B}_t^\#\le\bar{K}_t^\#$,

and the bounding constraints (A.2.4).

Fifth,

(A.4.11) $\bar{c}_T^0 = \arg\max c_T^0$

subject to the following budget constraint

(A.4.12) $\bar{p}_{Y,T} c_T^0 \le \bar{p}_{L,T} + \frac{1}{2} \bar{p}_{E,T} E_T^{\#}$.

and the bounding constraint (A.2.5).

The preceding conditions assert that the consumers maximize lifetime utility under budget constraints and bounding constraints. The last conditions involve market clearing.

(A.4.13) $\mathcal{E}_{Y,t}(\bar{\xi}) \le 0, \mathcal{E}_{L,t}(\bar{\xi}) \le 0, \mathcal{E}_{X,t}(\bar{\xi}) \le 0, \mathcal{E}_{B,t}(\bar{\xi}) \le 0, \mathcal{E}_{E,t}(\bar{\xi}) \le 0, \qquad (0 \le t \le T).$

A.5. *The First Stage of the Proof of the Existence of Competitive Equilibrium for a Truncated Economy Subject to Bounding Constraints*

A.5.1. *Some Preliminary Definitions*

For any matrix $a = (a_{ij})_{i=1,\dots,m, j=1,\dots,n}$, let $a^+ = (a_{ij}^+)_{i=1,\dots,m, j=1,\dots,n}$, be the matrix where $a_{ij}^+ = \max\{a_{ij}, 0\}$. Also, $|a| = \sum_{i,j} |a_{ij}|$ denotes the absolute norm of a.

For each $(\xi, \mathcal{P}^T) \in \Xi \times \Delta$, let

(A.5.1.1) $$f(\xi, \mathcal{P}^T) = \left((\xi, \frac{\mathcal{P}^T + [\mathcal{E}(\xi)]^+}{\left|\mathcal{P}^T + [\mathcal{E}(\xi)]^+\right|}) \right).$$

It is clear that $f(\xi, \mathcal{P}^T) \in \Xi \times \Delta$. The map $f : \Xi \times \Delta \to \Xi \times \Delta$, thus defined, is continuous. Furthermore, it is straightforward to show that $f(\xi, \mathcal{P}^T) = (\xi, \mathcal{P}^T)$ if and only if $\mathcal{E}^+(\xi) = \lambda \mathcal{P}^T$ for some positive number λ. Let

(A.5.1.2) $V_0 = \{(\xi, \mathcal{P}^T) \in \Xi \times \Delta \,|\, f(\xi, \mathcal{P}^T) \ne (\xi, \mathcal{P}^T)\}$

The set V_0 is open. The restriction of f to V_0 will be denoted by f_{V_0}.

To prove Claim A1, we use reductio ad absurdum. Suppose then that the claim is not true. Then for any element $(\xi, \mathcal{P}^T) \in \Xi \times \Delta$, one of the conditions (A.4.1) through (A.4.13) is violated. We shall now consider these violations one by one.

A.5.2. *The Budget Constraint of a Consumer is Violated*

Under this possibility, the budget constraint of one individual – young or old – of some period t is violated. Let us consider first the case he budget constraint for an old individual of period $t, 0 \le t \le T$, is violated, i.e., (A.5.2.1) $$p_{Y,t}c_t^1 > (1-\delta)p_{Y,t}k_t + p_{K,t}k_t + p_{X,t}x_t + \frac{1}{2}p_{E,t}E_t^{\#}.$$

We can then find a consumption level $\hat{c}_{t+1}^1$ such that

(A.5.2.2) $$p_{Y,t}c_t^1 > (1-\delta)p_{Y,t}k_t + p_{K,t}k_t + p_{X,t}x_t + \frac{1}{2}p_{E,t}E_t^{\#} \geq p_{Y,t}\hat{c}_t^1,$$

with the second inequality being strict if the right side of (A.5.2.1) is positive. If it happens that the right side of (A.5.2.1) is equal to 0, then $\hat{c}_t^1 = 0$. By continuity, we can then find a neighborhood of $(\xi, \wp^T)$, say $V(\xi, \wp^T)$, such that

(A.5.2.3) $$\widetilde{p}_{Y,t}\widetilde{c}_t^1 > (1-\delta)\widetilde{p}_{Y,t}\widetilde{k}_t + \widetilde{p}_{K,t}\widetilde{k}_t + \widetilde{p}_{X,t}\widetilde{x}_t + \frac{1}{2}\widetilde{p}_{E,t}E_t^{\#} \geq \widetilde{p}_{Y,t}\hat{c}_t^1$$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.2.4) $$f_{V(\xi,\wp^T)}. : V(\xi, \wp^T) \to \Xi \times \Delta$$

be the map defined by

(A.5.2.5) $$f_{V(\xi,\wp^T)}.(\widetilde{\xi}, \widetilde{\wp}^T). = (\widetilde{\xi}, \widetilde{\wp}^T) /. \widetilde{c}_t^1 \to \hat{c}_t^1,$$

where $(\widetilde{\xi}, \widetilde{\wp}^T) /. \widetilde{c}_t^1 \to \hat{c}_t^1$ denotes the result[5] obtained by replacing $\widetilde{c}_t^1$ in $(\widetilde{\xi}, \widetilde{\wp}^T)$ with $\hat{c}_t^1$. As defined, the map $f_{V(\xi,\wp^T)}$ changes the old-age consumption of an old individual of period t from $\widetilde{c}_t^1$ to $\hat{c}_t^1$, but keeping all the other components of the list $(\widetilde{\xi}, \widetilde{\wp}^T)$ unchanged.

Having considered the case the budget of an old individual is violated, we consider next the case the current budget of a young individual, namely (A.4.4), is violated. That is, for some $0 \leq t \leq T-1$,

(A.5.2.6) $$p_{Y,t}(c_t^0 + k_{t+1}) + p_{X,t}x_{t+1} > p_{L,t} + \frac{1}{2}p_{E,t}E_t^{\#}.$$

It follows from (A.5.2.6) that there exists a list $(\hat{c}_t^0, \hat{k}_{t+1}, \hat{x}_{t+1})$ such that

(A.5.2.7) $$p_{Y,t}(c_t^0 + k_{t+1}) + p_{X,t}x_{t+1} > p_{L,t} + \frac{1}{2}p_{E,t}E_t^{\#} \geq p_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + p_{X,t}\hat{x}_{t+1},$$

with the second inequality being strict if the right side of (A.5.2.6) is positive. By continuity, we can then find a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.2.8) $$\widetilde{p}_{Y,t}(\widetilde{c}_t^0 + \widetilde{k}_{t+1}) + \widetilde{p}_{X,t}\widetilde{x}_{t+1} > \widetilde{p}_{L,t} + \frac{1}{2}\widetilde{p}_{E,t}E_t^{\#} \geq \widetilde{p}_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + \widetilde{p}_{X,t}\hat{x}_{t+1},$$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.2.9) $$f_{V(\xi,\wp^T)}. : V(\xi, \wp^T) \to \Xi \times \Delta$$

be the map defined by

(A.5.2.10) $$f_{V(\xi,\wp^T)}.(\widetilde{\xi}, \widetilde{\wp}^T) = (\widetilde{\xi}, \widetilde{\wp}^T) /. \{\widetilde{c}_t^0 \to \hat{c}_t^0, \widetilde{k}_{t+1} \to \hat{k}_{t+1}, \widetilde{x}_{t+1} \to \hat{x}_{t+1}\}.$$

[5] This is the notation of the replacement rule of *Mathematica*.

For a young individual of period T, her budget constraint is violated if

(A.5.2.11) $$p_{Y,T}c_T^0 > p_{L,T} + \frac{1}{2}p_{E,T}E_T^{\#}.$$

In this case, we can find a consumption level $\hat{c}_T^0$ such that

(A.5.2.12) $$p_{Y,T}c_T^0 > p_{L,T} + \frac{1}{2}p_{E,T}E_T^{\#} \geq p_{Y,T}\hat{c}_T^0,$$

with the second inequality being strict if the right side of (A.1.39) is positive. By continuity, we can then find a neighborhood $V(\xi,\wp^T)$ of $(\xi,\wp^T)$ such that

(A.5.2.13) $$\tilde{p}_{Y,T}\tilde{c}_T^0 > \tilde{p}_{L,T} + \frac{1}{2}\tilde{p}_{E,T}E_T^{\#} \geq \tilde{p}_{Y,T}\hat{c}_T^0,$$

for all $(\tilde{\xi},\tilde{\wp}^T) \in V(\xi,\wp^T)$. Next, let

(A.5.2.14) $$f_{V(\xi,\wp^T)} : V(\xi,\wp^T) \to \Xi \times \Delta$$

be the map defined by

(A.5.2.15) $$f_{V(\xi,\wp^T)}(\tilde{\xi},\tilde{\wp}^T) = (\tilde{\xi},\tilde{\wp}^T)/.\tilde{c}_T^0 \to \hat{c}_T^0..$$

A.5.3. *All the Budget Constraints are Satisfied, but the Choices Made by some Individual are not Optimal.*

Suppose that the choices made by some individual, say a young individual of period $t, 0 \leq t \leq T-1$, satisfy the budget constraints, but do not maximize her lifetime utility. There are two possibilities to consider: both the current and the future budget constraints are binding and one of the budget constraints – current or future – is not binding.

If both budget constraints are binding, then there exists a lifetime plan $(\hat{c}_t^0, \hat{c}_{t+1}^1, \hat{k}_{t+1}, \hat{x}_{t+1})$ for that individual that satisfies the two budget constraints with strict inequalities but yields a higher lifetime utility that is,

(A.5.3.1) $$p_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + p_{X,t}\hat{x}_{t+1} < p_{L,t} + \frac{1}{2}p_{E,t}E_t^{\#},$$

(A.5.3.2) $$p_{Y,t+1}\hat{c}_{t+1}^1 < (1-\delta)p_{Y,t+1}\hat{k}_{t+1} + p_{K,t+1}\hat{k}_{t+1} + p_{X,t+1}\hat{x}_{t+1} + \frac{1}{2}p_{E,t+1}E_{t+1}^{\#},$$

and

(A.5.3.3) $$Log[\hat{c}_t^0] + \beta Log[\hat{c}_{t+1}^1] > Log[c_t^0] + \beta Log[c_{t+1}^1]$$

Hence there exists a neighborhood $V(\xi,\wp^T)$ of $(\xi,\wp^T)$ such that

(A.5.3.4) $$\tilde{p}_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + \tilde{p}_{X,t}\hat{x}_{t+1} < \tilde{p}_{L,t} + \frac{1}{2}\tilde{p}_{E,t}E_t^{\#},$$

(A.1.50) $$\tilde{p}_{Y,t+1}\hat{c}_{t+1}^1 < (1-\delta)\tilde{p}_{Y,t+1}\hat{k}_{t+1} + \tilde{p}_{K,t+1}\hat{k}_{t+1} + \tilde{p}_{X,t+1}\hat{x}_{t+1} + \frac{1}{2}\tilde{p}_{E,t+1}E_{t+1}^{\#},$$

and

(A.5.3.5) $$Log[\hat{c}_t^0] + \beta Log[\hat{c}_{t+1}^1] > Log[\tilde{c}_t^0] + \beta Log[\tilde{c}_{t+1}^1]$$

for all $(\tilde{\xi},\tilde{\wp}^T) \in V(\xi,\wp^T)$. Next, let

(A.5.3.6) $f_{V(\xi,\wp^T).} : V(\xi,\wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.3.7) $f_{V(\xi,\wp^T).}(\widetilde{\xi},\widetilde{\wp}^T). = (\widetilde{\xi},\widetilde{\wp}^T)/.\{\widetilde{c}_t^0 \to \hat{c}_t^0, \widetilde{c}_{t+1}^1 \to \hat{c}_{t+1}^1, \widetilde{k}_{t+1} \to \hat{k}_{t+1}, \widetilde{x}_{t+1} \to \hat{x}_{t+1}\}.$

If one of the budget constraints of the young individual is not binding, say the current budget constraint,

(A.5.3.8) $p_{Y,t}(c_t^0 + k_{t+1}) + p_{X,t}x_{t+1} < p_{L,t} + \frac{1}{2}p_{E,t}E_t^\#,$

then there exists a lifetime plan $(\hat{c}_t^0, \hat{c}_{t+1}^1, \hat{k}_{t+1}, \hat{x}_{t+1})$ for that individual that satisfies the following conditions:

(A.5.3.9)
$$p_{Y,t}(c_t^0 + k_{t+1}) + p_{X,t}x_{t+1} < p_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + p_{X,t}\hat{x}_{t+1} < p_{L,t} + \frac{1}{2}p_{E,t}E_t^\#,$$

(A.5.3.10) $p_{Y,t+1}\hat{c}_{t+1}^1 < (1-\delta)p_{Y,t+1}\hat{k}_{t+1} + p_{K,t+1}\hat{k}_{t+1} + p_{X,t+1}\hat{x}_{t+1} + \frac{1}{2}p_{E,t+1}E_{t+1}^\#,$

and that yields a higher lifetime utility. By continuity, there exists a neighborhood $V(\xi,\wp^T)$ of $(\xi,\wp^T)$ such that

(A.5.3.11)
$$\widetilde{p}_{Y,t}(\widetilde{c}_t^0 + \widetilde{k}_{t+1}) + \widetilde{p}_{X,t}\widetilde{x}_{t+1} < \widetilde{p}_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + \widetilde{p}_{X,t}\hat{x}_{t+1} < \widetilde{p}_{L,t} + \frac{1}{2}\widetilde{p}_{E,t}E_t^\#,$$

(A.5.3.12) $\widetilde{p}_{Y,t+1}\hat{c}_{t+1}^1 < (1-\delta)\widetilde{p}_{Y,t+1}\hat{k}_{t+1} + \widetilde{p}_{K,t+1}\hat{k}_{t+1} + \widetilde{p}_{X,t+1}\hat{x}_{t+1} + \frac{1}{2}\widetilde{p}_{E,t+1}E_{t+1}^\#,$

and

(A.5.3.13) $Log[\hat{c}_t^0] + \beta Log[\hat{c}_{t+1}^1] > Log[\widetilde{c}_t^0] + \beta Log[\widetilde{c}_{t+1}^1]$

for all $(\widetilde{\xi},\widetilde{\wp}^T) \in V(\xi,\wp^T)$. Next, let

(A.5.3.14) $f_{V(\xi,\wp^T).} : V(\xi,\wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.3.15) $f_{V(\xi,\wp^T).}(\widetilde{\xi},\widetilde{\wp}^T). = (\widetilde{\xi},\widetilde{\wp}^T)/.\{\widetilde{c}_t^0 \to \hat{c}_t^0, \widetilde{c}_{t+1}^1 \to \hat{c}_{t+1}^1, \widetilde{k}_{t+1} \to \hat{k}_{t+1}, \widetilde{x}_{t+1} \to \hat{x}_{t+1}\}.$

If the current budget constraint is binding, but the future budget constraint is not binding, say

(A.5.3.16) $p_{Y,t+1}c_{t+1}^1 < (1-\delta)p_{Y,t+1}k_{t+1} + p_{K,t+1}k_{t+1} + p_{X,t+1}x_{t+1} + \frac{1}{2}p_{E,t+1}E_{t+1}^\#,$

then there exists a lifetime plan $(\hat{c}_t^0, \hat{c}_{t+1}^1, \hat{k}_{t+1}, \hat{x}_{t+1})$ for that individual that satisfies the following conditions:

(A.5.3.17) $p_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + p_{X,t}\hat{x}_{t+1} < p_{L,t} + \frac{1}{2}p_{E,t}E_t^\#,$

(A.5.3.18) $$p_{Y,t+1}c_{t+1}^{1} < p_{Y,t+1}\hat{c}_{t+1}^{1} < (1-\delta)p_{Y,t+1}k_{t+1} + p_{K,t+1}k_{t+1} + p_{X,t+1}x_{t+1} + \frac{1}{2}p_{E,t+1}E_{t+1}^{\#},$$

and that yields a higher lifetime utility. By continuity, there exists a neighborhood $V(\xi,\wp^T)$ of $(\xi,\wp^T)$ such that

(A.5.3.19) $$\widetilde{p}_{Y,t}(\hat{c}_t^0 + \hat{k}_{t+1}) + \widetilde{p}_{X,t}\hat{x}_{t+1} < \widetilde{p}_{L,t} + \frac{1}{2}\widetilde{p}_{E,t}E_t^{\#},$$

(A.5.3.20) $$\widetilde{p}_{Y,t+1}\widetilde{c}_{t+1}^{1} < \widetilde{p}_{Y,t+1}\hat{c}_{t+1}^{1} < (1-\delta)\widetilde{p}_{Y,t+1}\widetilde{k}_{t+1} + \widetilde{p}_{K,t+1}\widetilde{k}_{t+1} + \widetilde{p}_{X,t+1}\widetilde{x}_{t+1} + \frac{1}{2}\widetilde{p}_{E,t+1}E_{t+1}^{\#},$$

and

(A.5.3.21) $$Log[\hat{c}_t^0] + \beta Log[\hat{c}_{t+1}^1] > Log[\widetilde{c}_t^0] + \beta Log[\widetilde{c}_{t+1}^1]$$

for all $(\widetilde{\xi},\widetilde{\wp}^T) \in V(\xi,\wp^T)$. Next, let

(A.5.3.22) $$f_{V(\xi,\wp^T).} : V(\xi,\wp^T). \to \Xi \times \Delta$$

be the map defined by

(A.5.3.23) $$f_{V(\xi,\wp^T).}(\widetilde{\xi},\widetilde{\wp}^T). = (\widetilde{\xi},\widetilde{\wp}^T)/.\{\widetilde{c}_t^0 \to \hat{c}_t^0, \widetilde{c}_{t+1}^1 \to \hat{c}_{t+1}^1, \widetilde{k}_{t+1} \to \hat{k}_{t+1}, \widetilde{x}_{t+1} \to \hat{x}_{t+1}\}.$$

For an old individual of period 0, if the choice she makes, namely c_0^1, satisfies the budget constraint but does not maximize her utility, then it can only mean that she does not use all the budget, i.e.,

(A.5.3.23) $$p_{Y,0}c_0^1 < p_{Y,0}(1-\delta)K_0 + p_{K,0}K_0 + p_{X,0}X_0 + \frac{1}{2}p_{E,0}E_0^{\#}.$$

Hence there exist a consumption level $\hat{c}_0^1 > c_0^1$ such that

(A.5.3.24) $$p_{Y,0}\hat{c}_0^1 < p_{Y,0}(1-\delta)K_0 + p_{K,0}K_0 + p_{X,0}X_0 + \frac{1}{2}p_{E,0}E_0^{\#}.$$

By continuity, there exists a neighborhood $V(\xi,\wp^T)$ of $(\xi,\wp^T)$ such that

(A.5.3.25) $$\widetilde{p}_{Y,0}\hat{c}_0^1 < \widetilde{p}_{Y,0}(1-\delta)K_0 + \widetilde{p}_{K,0}K_0 + \widetilde{p}_{X,0}X_0 + \frac{1}{2}\widetilde{p}_{E,0}E_0^{\#}.$$

(A.5.3.26) $$\hat{c}_0^1 > \widetilde{c}_0^1,$$

for all $(\widetilde{\xi},\widetilde{\wp}^T) \in V(\xi,\wp^T)$. Next, let

(A.5.3.27) $$f_{V(\xi,\wp^T).} : V(\xi,\wp^T). \to \Xi \times \Delta$$

be the map defined by

(A.5.3.28) $$f_{V(\xi,\wp^T).}(\widetilde{\xi},\widetilde{\wp}^T). = (\widetilde{\xi},\widetilde{\wp}^T)/.\widetilde{c}_0^1 \to \hat{c}_0^1.$$

Similarly, for a young individual of period T, if the choice she makes, namely c_T^0, satisfies the budget constraint, but does not maximize her utility, then it can only mean that she does not use all the budget, i.e.,

(A.5.3.29) $$p_{Y,T}c_T^0 < p_{L,T} + \frac{1}{2}p_{E,T}E_T^{\#}.$$

Hence there exists a consumption level $\hat{c}_T^0 > c_T^0$ such that

(A.5.3.30) $\quad p_{Y,T}\hat{c}_T^0 < p_{L,T} + \frac{1}{2} p_{E,T} E_T^{\#}.$

By continuity, there exist a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.3.31) $\quad \widetilde{p}_{Y,T}\hat{c}_T^0 < \widetilde{p}_{L,T} + \frac{1}{2} \widetilde{p}_{E,T} E_T^{\#}.$

(A.5.3.32) $\quad \hat{c}_T^0 > \widetilde{c}_T^0,$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.3.33) $\quad f_{V(\xi,\wp^T).} : V(\xi, \wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.3.34) $\quad f_{V(\xi,\wp^T).}(\widetilde{\xi}, \widetilde{\wp}^T). = (\widetilde{\xi}, \widetilde{\wp}^T) / .\widetilde{c}_T^0 \to \hat{c}_T^0.$

A.5.4. *All Consumers Maximize Lifetime Utility, but the Production Plan of a Producer of the Consumption Good does not Conform to the Environmental Regulation.*

Suppose that the choice made by each member of each generation maximizes her lifetime utility subject to the budget constraints, but the production plan chosen by the producers of the consumption good in a period $t, 0 \le t \le T,$ is either not conforming to the environmental regulation or not profit maximizing. This situation is formally the same as that of a consumer, with the environmental constraint playing the role of the budget constraint and the profit the role of utility.

Let us consider first the case the environmental constraint is violated, i.e.,

(A.5.4.1) $\quad Q_t > E_t.$

In this case, there exists an oil input $\hat{Q}_t$ such that

(A.5.4.2) $\quad Q_t > E_t \ge \hat{Q}_t,$

with the second inequality being strict if $E_t > 0$. By continuity, there exists a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.4.3) $\quad \widetilde{Q}_t > \widetilde{E}_t \ge \hat{Q}_t$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.4.4) $\quad f_{V(\xi,\wp^T).} : V(\xi, \wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.4.5) $\quad f_{V(\xi,\wp^T).}(\widetilde{\xi}, \widetilde{\wp}^T). = (\widetilde{\xi}, \widetilde{\wp}^T) / .\widetilde{Q}_t \to \hat{Q}_t.$

Next, suppose that the production plans of the producers of the consumption good all conform to the environmental regulation. However, for the producers in a certain period, say period $t, 0 \le t \le T,$ the input combination chosen does not maximize profit. As in the case of an individual, either the environmental constraint is binding or not binding.

If the environmental constraint is binding, i.e., if $Q_t = E_t$, then we can find an input combination $(\hat{L}_t, \hat{Q}_t, \hat{B}_t, \hat{E}_t)$ such that

(A.5.4.6) $\hat{Q}_t \leq \hat{E}_t, < E_t$

and

(A.5.4.7) $$\begin{aligned} & p_{Y,t} A\Omega(H_t) L_t^{\alpha}(Q_t + B_t)^{1-\alpha} - p_{L,t}L_t - p_{X,t}Q_t - p_{B,t}B_t - p_{E,t}E_t \\ & < p_{Y,t} A\Omega(H_t) \hat{L}_t^{\alpha}(\hat{Q}_t + \hat{B}_t)^{1-\alpha} - p_{L,t}\hat{L}_t - p_{X,t}\hat{Q}_t - p_{B,t}\hat{B}_t - p_{E,t}\hat{E}_t. \end{aligned}$$

By continuity, there exist a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.4.8) $$\begin{aligned} & \tilde{p}_{Y,t} A\Omega(H_t) \tilde{L}_t^{\alpha}(\tilde{Q}_t + \tilde{B}_t)^{1-\alpha} - \tilde{p}_{L,t}\tilde{L}_t - \tilde{p}_{X,t}\tilde{Q}_t - \tilde{p}_{B,t}\tilde{B}_t - \tilde{p}_{E,t}\tilde{E}_t \\ & < \tilde{p}_{Y,t} A\Omega(H_t) \hat{L}_t^{\alpha}(\hat{Q}_t + \hat{B}_t)^{1-\alpha} - \tilde{p}_{L,t}\hat{L}_t - \tilde{p}_{X,t}\hat{Q}_t - \tilde{p}_{B,t}\hat{B}_t - \tilde{p}_{E,t}\hat{E}_t. \end{aligned}$$

for all $(\tilde{\xi}, \tilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.4.9) $f_{V(\xi,\wp^T).} : V(\xi, \wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.4.10) $f_{V(\xi,\wp^T).}(\tilde{\xi}, \tilde{\wp}^T). = (\tilde{\xi}, \tilde{\wp}^T) / .\{\tilde{L}_t \to \hat{L}_t, \tilde{Q}_t \to \hat{Q}_t, \tilde{B}_t \to \hat{B}_t, \tilde{E} \to \hat{E}_t\}.$

If the environmental constraint is not binding, i.e., if $Q_t < E_t$, then we can find an input combination $(\hat{L}_t, \hat{Q}_t, \hat{B}_t, \hat{E}_t)$ such that

(A.5.4.11) $Q_t < \hat{Q}_t \leq \hat{E}_t, < E_t$

and

(A.5.4.12) $$\begin{aligned} & p_{Y,t} A\Omega(H_t) L_t^{\alpha}(Q_t + B_t)^{1-\alpha} - p_{L,t}L_t - p_{X,t}Q_t - p_{B,t}B_t - p_{E,t}E_t \\ & < p_{Y,t} A\Omega(H_t) \hat{L}_t^{\alpha}(\hat{Q}_t + \hat{B}_t)^{1-\alpha} - p_{L,t}\hat{L}_t - p_{X,t}\hat{Q}_t - p_{B,t}\hat{B}_t - p_{E,t}\hat{E}_t. \end{aligned}$$

By continuity, there exists a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.4.13) $\tilde{Q}_t < \hat{Q}_t \leq \hat{E}_t, < \tilde{E}_t$

and

(A.5.4.14) $$\begin{aligned} & \tilde{p}_{Y,t} A\Omega(H_t) \tilde{L}_t^{\alpha}(\tilde{Q}_t + \tilde{B}_t)^{1-\alpha} - \tilde{p}_{L,t}\tilde{L}_t - \tilde{p}_{X,t}\tilde{Q}_t - \tilde{p}_{B,t}\tilde{B}_t - \tilde{p}_{E,t}\tilde{E}_t \\ & < \tilde{p}_{Y,t} A\Omega(H_t) \hat{L}_t^{\alpha}(\hat{Q}_t + \hat{B}_t)^{1-\alpha} - \tilde{p}_{L,t}\hat{L}_t - \tilde{p}_{X,t}\hat{Q}_t - \tilde{p}_{B,t}\hat{B}_t - \tilde{p}_{E,t}\hat{E}_t. \end{aligned}$$

for all $(\tilde{\xi}, \tilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.4.15) $f_{V(\xi,\wp^T).} : V(\xi, \wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.4.16) $f_{V(\xi,\wp^T).}(\tilde{\xi}, \tilde{\wp}^T). = (\tilde{\xi}, \tilde{\wp}^T) / .\{\tilde{L}_t \to \hat{L}_t, \tilde{Q}_t \to \hat{Q}_t, \tilde{B}_t \to \hat{B}_t, \tilde{E} \to \hat{E}_t\}.$

A.5.5. *All Consumers Maximize Lifetime Utility Subject to the Budget Constraints, and all Producers of the Consumption Good Maximize Profit under the Environmental Constraint, but the Production Plan of a Producer in the Backstop Sector is either not Technologically Feasible or not Profit Maximizing.*

This case is formally the same as the case of the producers of the consumption good.Suppose now that all each member of each generation maximizes utility under the budget constraints and that the producers of the consumption good maximize profits in each period and also conform to the environmental regulation, but the production plan chosen by producers of renewable energy is either not technologically feasible or not profit maximizing. In the former case, we have

(A.5.5.1) $\quad K_t^\# - B_t^\# < 0$

for some $0 \le t \le T$. In this case, we can find a production plan $(\hat{B}_t^\#, \hat{K}_t^\#)$ and a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.5.2) $\quad \hat{B}_t^\# \le \hat{K}_t^\# \le \widetilde{K}_t^\# < \widetilde{B}_t^\#$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.5.3) $\quad f_{V(\xi,\wp^T).} : V(\xi, \wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.5.4) $\quad f_{V(\xi,\wp^T).}(\widetilde{\xi}, \widetilde{\wp}^T). = (\widetilde{\xi}, \widetilde{\wp}^T)/.\{\widetilde{B}_t^\# \to \hat{B}_t^\#, \widetilde{K}_t^\# \to \hat{K}_t^\#\}.$

In the latter case, there are two further possibilities to consider: the technological constraint holds with equality and the technological holds with strict inequality. If the technological holds with equality, then we can find a feasible production plan $(\hat{B}_t^\#, \hat{K}_t^\#)$ and a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.5.5) $\quad \hat{K}_t^\# - \hat{B}_t^\# < 0,$

(A.5.5.6) $\quad \widetilde{p}_{B,t}\hat{B}_t^\# - \widetilde{p}_{K,t}\hat{K}_t^\# > \widetilde{p}_{B,t}\widetilde{B}_t^\# - \widetilde{p}_{K,t}\widetilde{K}_t^\#$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.5.7) $\quad f_{V(\xi,\wp^T).} : V(\xi, \wp^T). \to \Xi \times \Delta$

be the map defined by

(A.5.5.8) $\quad f_{V(\xi,\wp^T).}(\widetilde{\xi}, \widetilde{\wp}^T). = (\widetilde{\xi}, \widetilde{\wp}^T)/.\{\widetilde{B}_t^\# \to \hat{B}_t^\#, \widetilde{K}_t^\# \to \hat{K}_t^\#\}.$

If the technological holds with strict inequality), say

(A.5.5.9) $\quad B_t^\# - K_t^\# < 0,$

then we can find a feasible production plan $(\hat{B}_t^\#, \hat{K}_t^\#)$ and a neighborhood $V(\xi, \wp^T)$ of $(\xi, \wp^T)$ such that

(A.5.5.10) $\quad \widetilde{B}_t^\# - \widetilde{K}_t^\# < \hat{B}_t^\# - \hat{K}_t^\# < 0,$

(A.5.5.6) $\quad \widetilde{p}_{B,t}\hat{B}_t^\# - \widetilde{p}_{K,t}\hat{K}_t^\# > \widetilde{p}_{B,t}\widetilde{B}_t^\# - \widetilde{p}_{K,t}\widetilde{K}_t^\#$

for all $(\widetilde{\xi}, \widetilde{\wp}^T) \in V(\xi, \wp^T)$. Next, let

(A.5.5.11) $\quad f_{V(\xi,\wp^T)} : V(\xi, \wp^T) \to \Xi \times \Delta$

be the map defined by

(A.5.5.12) $\quad f_{V(\xi,\wp^T).}(\widetilde{\xi}, \widetilde{\wp}^T) = (\widetilde{\xi}, \widetilde{\wp}^T)/.\{\widetilde{B}_t^\# \to \hat{B}_t^\#, \widetilde{K}_t^\# \to \hat{K}_t^\#\}.$

A.5.6. *Consumers Maximize Lifetime utility; Producers of the Consumption Good Maximize Profit Subject to the Technological and Environmental Constraints; Producers of Renewable Energy Maximize Profit Subject to the Technological Constraint; but One Market does not Clear.*

Suppose that each member of each generation maximizes utility subject to the budget constrains and all the producers – of the consumption good or of renewable energy – maximize profits while respecting the environmental regulation and the technological constraints. Then one of the market-clearing conditions must be violated if the claim is not true. In this case, "Walras law" must hold. Indeed, for each $t = 0,\dots,T-1$, we have

$$\begin{aligned}
&p_{Y,t}\mathcal{E}_{Y,t} + p_{L,t}\mathcal{E}_{L,t} + p_{X,t}\mathcal{E}_{X,t} + p_{B,t}\mathcal{E}_{B,t} + p_{K,t}\mathcal{E}_{K,t} + p_{E,t}\mathcal{E}_{E,t} \\
&\quad = p_{Y,t}\left(c_t^1 + c_t^0 + k_{t+1} - A\Omega(H_t)L_t^{\alpha}(Q_t + B_t)^{1-\alpha} - (1-\delta)k_t\right) \\
&\qquad + p_{L,t}(L_t - 1) + p_{X,t}(Q_t + x_{t+1} - x_t) + p_{B,t}(B_t - B_t^{\#}) \\
&\qquad + p_{K,t}(K_t^{\#} - k_t) + p_{E,t}(E_t - E_t^{\#}) \\
&\quad = p_{Y,t}c_t^1 - p_{Y,t}(1-\delta)k_t - p_{X,t}x_t - \frac{1}{2}p_{E,t}E_t^{\#} \\
&\qquad + p_{Y,t}(c_t^0 + k_{t+1}) + p_{X,t}x_{t+1} - p_{L,t} - \frac{1}{2}p_{E,t}E_t^{\#} \\
&\qquad - [p_{Y,t}A\Omega(H_t)L_t^{\alpha}(Q_t + B_t)^{1-\alpha} - p_{L,t}L_t - p_{X,t}Q_t - p_{B,t}B_t - p_{E,t}E_t] \\
&\qquad - [p_{X,t}B_t^{\#} - p_{K,t}K_t^{*}] \le 0.
\end{aligned} \tag{A.5.6.1}$$

For period T, we have

$$\begin{aligned}
&p_{Y,T}\mathcal{E}_{Y,T} + p_{L,T}\mathcal{E}_{L,T} + p_{X,T}\mathcal{E}_{X,T} + p_{B,T}\mathcal{E}_{B,T} + p_{K,T}\mathcal{E}_{K,T} + p_{E,T}\mathcal{E}_{E,T} \\
&\quad = p_{Y,T}\left(c_T^1 + c_T^0 - A\Omega(H_t)L_t^{\alpha}(Q_t + B_t)^{1-\alpha} - (1-\delta)k_T\right) \\
&\qquad + p_{L,T}(L_T - 1) + p_{X,T}(Q_T - x_T) + p_{B,T}(B_T - B_{tT}^{\#}) \\
&\qquad + p_{K,T}(K_T^{\#} - k_T) + p_{E,T}(E_T - E_{tT}^{\#}) \le 0.
\end{aligned} \tag{A.5.6.2}$$

Walras law now follows from (A.5.6.1) and A(5.6.2), i.e.,

$$\sum_{t=0}^{T}[p_{Y,t}\mathcal{E}_{Y,t} + p_{L,t}\mathcal{E}_{L,t} + p_{X,t}\mathcal{E}_{X,t} + p_{B,t}\mathcal{E}_{B,t} + p_{K,t}\mathcal{E}_{K,t} + p_{E,t}\mathcal{E}_{E,t}] \le 0. \tag{A.5.6.3}$$

Next, we show that when one of the market-clearing conditions is violated, it is not possible for $[\mathcal{E}(\xi)]^{+}$ to be proportional to $\mathcal{P}^T$, and thus $(\xi, \mathcal{P}^T) \in V_0$. Indeed, if $[\mathcal{E}(\xi)]^{+} = \lambda\mathcal{P}^T$ for some positive number λ, then the left side of (A.5.6.1) becomes

$$\sum_{t=0}^{T}[p_{Y,t}\mathcal{E}_{Y,t} + p_{L,t}\mathcal{E}_{L,t} + p_{X,t}\mathcal{E}_{X,t} + p_{B,t}\mathcal{E}_{B,t} + p_{K,t}\mathcal{E}_{K,t} + p_{E,t}\mathcal{E}_{E,t}] = \lambda > 0. \tag{A.5.6.4}$$

Clearly, (A.5.6.3) and A(5.6.4) are not mutually consistent.

A.6. *The Second Stage of the Proof of the Existence of Competitive Equilibrium for a Truncated Economy Subject to Bounding Constraints*

Let

(A.6.1) $$\mathcal{U} = \left\{ (\xi, \wp^T) \in \Xi \times \Delta \,\middle|\, \begin{array}{l} \text{one of the conditions A.5.2} \\ \text{through A.5.5 apllies to } (\xi, \wp^T) \end{array} \right\}.$$

Together V_0 and the family of neighborhoods $(V(\xi, \wp^T))_{(\xi,\wp^T)\in\mathcal{U}}$ cover the compact set $\Xi \times \Delta$. Hence $(V(\xi, \wp^T))_{(\xi,\wp^T)\in\mathcal{U}}$ has a finite subfamily $V(\xi(1), \wp^T(1)),..., V(\xi(n), \wp^T(n))$ such that $\{V_0, V(\xi(1), \wp^T(1)),..., V(\xi(n), \wp^T(n))\}$ covers $\Xi \times \Delta$, where $(\xi(i), \wp^T(i)) \in \mathcal{U}$ for each $i = 1,..., n$. To simplify the notations, we shall also write V_i for $V(\xi(i), \wp^T(i)), i = 1,..., n$.

Let $\varphi_0, \varphi_1,..., \varphi_n$ be a continuous partition of unity on $\Xi \times \Delta$ subordinate to $V_0, V_1,..., V_n$. Now for each $(\xi, \wp^T) \in \Xi \times \Delta$, let

(A.6.2) $$I(\xi, \wp^T) = \{i \in \{0,..., n\} | (\xi, \wp^T) \in V_i\},$$

and define

(A.6.3) $$F(\xi, \wp^T) = \sum_{i\in I(\xi,\wp^T)} \varphi_i(\xi, \wp^T) f_{V_i}(\xi, \wp^T).$$

It is clear that $F(\xi, \wp^T) \in \Xi \times \Delta$. The map $F : (\xi, \wp^T) \to F(\xi, \wp^T)$, thus defined, is a continuous map of the convex compact set $\Xi \times \Delta$ into itself.

Now for each $(\xi, \wp^T) \in \Xi \times \Delta$, let:

(A.6.4) $$(\underline{\xi}, \underline{\wp}^T) = F(\xi, \wp^T).$$

Next, let

(A.6.5) $$J(\xi, \wp^{T^T}) = \{i \in I(\xi, \wp^T) | \varphi_i(\xi, \wp^T) > 0\}.$$

If $J(\xi, \wp^T) = \{0\}$, then $F(\xi, \wp^T) = f_{V_0}(\xi, \wp^T) \neq (\xi, \wp^T)$, and this means that $(\xi, \wp^T)$ is not a fixed point of F. If $J(\underline{\xi}, \underline{\wp}^T) \neq \{0\}$, then

(A.6.6) $$J(\xi, \wp^T) - \{0\} = \{j_1,..., j_k\} \neq \phi.$$

Let us now look at the k neighborhoods $V(\xi(j_1), \wp^T(j_1)),..., V(\xi(j_k), \wp^T(j_k))$ to see if possibility A.5.2 applies to one of the points $(\xi(j_1), \wp^T(j_1)),..., (\xi(j_k), \wp^T(j_k))$. If it does, say A.5.2 applies to point $(\xi(j_1), \wp^T(j_1))$, then the budget constraint of one individual is violated at this point. As analyzed in A.5.2, the budget constraint might be violated for an old individual of period $t \leq T$, for a young individual of period $t < T$, or for a young individual of period T. Let us first consider the case the budget constraint of an old individual of period t is violated at point $(\xi(j_1), \wp^T(j_1))$. Then according to (A.5.2.5), the map $f_{V(\xi(j_1),\wp^T(j_1))}$ sends the component c_t^1 to $\hat{c}_t^1(j_1) < c_t^1$. Next, let us look at the map $f_{V(\xi(j_2),\wp^T(j_2))}$. If this map sends c_t^1 to a different level, then there are two possibilities to consider. Either the budget constraint for this old individual is also violated at the point

$(\xi(j_2), \wp^T(j_2))$, in which case $f_{V(\xi(j_2),\wp^T(j_2))}$ sends the component c_t^1 to $\hat{c}_t^1(j_2) < c_t^1$, or the budget constraint for the old individual is satisfied at $(\xi(j_2), \wp^T(j_2))$, but the old-age utility of the individual is not maximized. This last result can only mean that the value of her old-age consumption is strictly lower than her old-age income, and this continues to hold for any point in the neighborhood $V(\xi(j_2), \wp^T(j_2))$, which then implies that $(\xi, \wp^T) \notin V(\xi(j_2), \wp^T(j_2))$, contradicting the premise $(\xi, \wp^T) \notin V(\xi(j_2), \wp^T(j_2))$.

If the map $f_{V(\xi(j_2),\wp^T(j_2))}$ does not send c_t^1 to a different level, then look at $f_{V(\xi(j_3),\wp^T(j_3))}$ next, and repeat the process. We have just shown that each of the maps $f_{V_i}, i \in J(\xi, \wp^T)$, sends c_t^1 to itself or to some $\underline{c}_t^1 < c_t^1$, which then implies that $c_t^i \neq \sum_{i \in J(\xi,\wp^T)} \varphi_i(\xi, \wp^T) f_{V_i}(\xi, \wp^T)$, or the point $(\xi, \wp^T)$ is not a fixed point of F.

If A.5.2 does not apply to one of the points $(\xi(j_1), \wp^T(j_1)), \ldots, (\xi(j_k), \wp^T(j_k))$, then the budget constraint of each individual in each period is satisfied. We then check to see if A.5.3 applies to any one of them, i.e., if the choice made by an individual does not maximize utility subject to the budget constraint. If A.5.3 applies to one of these points, say $(\xi(j_1), \wp^T(j_1))$, then either the choice made by an old individual of period 0 or the choice made by a young individual of period $t \in \{0, \ldots, T\}$ does not maximize lifetime utility. For such an individual, say a young individual of period $t, 0 \le t < T$, the map $f_{V(\xi(j_1),\wp^T(j_1))}$ will send $(c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})$ to a lifetime plan $(\hat{c}_t^0, \hat{c}_{t+1}^1, \hat{k}_{t+1}, \hat{x}_{t+1})$ that satisfies the budget constraint and yields a utility level higher than that yielded by $(c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})$. For this same individual, the maps $f_{V(\xi(j_2),\wp^T(j_2))}, \ldots, f_{V(\xi(j_{k1}),\wp^T(j_k))}$ either map $(c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})$ to itself or to some better lifetime plan. The strict convexity of preferences then means that $c_t^i \neq \sum_{i \in J(\xi,\wp^T)} \varphi_i(\xi, \wp^T) f_{V_i}(\xi, \wp^T)$, or the point $(\xi, \wp^T)$ is not a fixed point of F.

The procedure just described for consumers can be repeated for producers of the consumption good as well as producers of renewable energy to show that if A.5.4 or A.5.5 applies to one of the points $(\xi(j_1), \wp^T(j_1)), \ldots, (\xi(j_k), \wp^T(j_k))$, the point $(\xi, \wp^T)$ is not a fixed point of F. According to (A.6.6), at least one of the conditions A.5.2 through A.5.5 must apply to one of the points $(\xi(j_1), \wp^T(j_1)), \ldots, (\xi(j_k), \wp^T(j_k))$, Hence F has no fixed point if Claim A1 is not true. However, according to the Brouwer fixed point theorem, F has a fixed point. Therefore, we conclude that Claim A1 is true.

CLAIM A2: *The bounding constraints are not binding for the utility and profit maximization problems stated in Claim A1. Furthermore, all the prices in* $\overline{\wp}^T$ *are positive* *and excess demand on each market in each period is equal to* 0. *The pair* $(\bar{\xi},\bar{\mathcal{P}}^T)$, *mentioned in Claim A1, is a competitive equilibrium of the truncated economy with time horizon T.*

Claim A2 is the last claim made by the standard proof method used in proving the existence of a competitive equilibrium for the Arrow-Debreu model. We shall not repeat the argument here, but refer the reader to Nikaido (1970).

B. EXISTENCE OF COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH OIL RESOURCES UNDER INFINITE TIME HORIZON

For each integer $T=1,2,...,$ let $(\mathcal{P}^T,\mathcal{Q}^T)$, where $\mathcal{P}^T=(p^T_{Y,t},p^T_{L,t},p^T_{X,t},p^T_{B,t},p^T_{K,t},p^T_{E,t})^T_{t=0}$ and

$$\mathcal{Q}^T=\begin{pmatrix} c_0^{1,T},(c_t^{0,T},c_{t+1}^{1,T},k^T_{t+1},x^T_{t+1})^{T-1}_{t=0},(Y^T_t,L^T_t,Q^T_t,B^T_t,E^T_t)^T_{t=0}, \\ (B_t^{\#,T},K_t^{\#,T})^T_{t=0},(K^T_t,X^T_t,H^T_t)^T_{t=0},c_T^{0,T} \end{pmatrix}$$

be a competitive equilibrium[6] for the truncated economy with time horizon *T*. We shall show that the sequence $(\mathcal{P}^T,\mathcal{Q}^T)^\infty_{T=1}$ has a subsequence that converges in the product topology of a denumerable family of Euclidean spaces and that the limit of this subsequence is a competitive equilibrium for the original economy. The existence proof is obtained at the end of a series of claims.

CLAIM B1: *The price systems in period* 0 *are uniformly bounded across the truncated economies with time horizons* $T=1,2,...$

PROOF: First, note that because the consumption good is taken to be the numéraire, its price is always equal to 1. Second, note that the wage rate in period 0 is uniformly bounded for all truncated economies because

$$\begin{aligned} p^T_{L,0} &= \alpha A\Omega(H_0)(L^T_0)^{\alpha-1}(Q^T_0+S^T_0)^{1-\alpha} \\ &\le \alpha A\Omega(H_0)(X_0+K_0)^{1-\alpha}. \end{aligned}$$

Third, to show that the price of oil and the price of solar energy are uniformly bounded across truncated economies, there are two cases to consider: $K_0>0$ and $K_0=0$.

If $K_0>0$, then

[6] Observe that we have used the superscript *T* to indicate the time horizon of the truncated economy in question. The superscript *T* is needed to distinguish one truncated economy from another in the sequence of truncated economies used in the proof of Proposition 5. Such a superscript is not needed in the proof – given in the appendix – of Proposition 4 because in that proof we consider only one truncated economy and thus no possibility for confusion might arise.

$$p^T_{B,0} = (1-\alpha)A\Omega(H_0)(L^T_0)^\alpha (Q^T_0 + B^T_0)^{-\alpha}$$
$$= (1-\alpha)A\Omega(H_0)(L^T_0)^\alpha (Q^T_0 + K_0)^{-\alpha}$$
$$\leq (1-\alpha)A\Omega(H_0)(K_0)^{-\alpha},$$
which means that the price of solar energy in period 0 is uniformly bounded across truncated economies. In this case we also have $p^T_{X,0} \geq p^T_{B,0}$. If $p^T_{X,0}$ is not uniformly bounded across truncated economies, then for any number $M > 0$ we can find a positive integer T such that $p^T_{X,0} > M > p^T_{B,o}$. Under this scenario, oil is only used for investment purposes, not for producing the consumption good. The value of the oil stock, namely $p^T_{X,0}X_0$ will tend to infinity when M tends to infinity. On the other hand, the output of the consumption good and a fortiori the wage rate of a young individual of period 0 is bounded above by $A\Omega(H_0)(K_0)^{1-\alpha}$, which implies that the young generation of period 0 does not earn enough wages to buy the oil stock. Hence the price of oil in period 0 must be uniformly bounded across truncated economies.

If $K_0 = 0$, then oil is the only source of energy used to produce the consumption good across truncated economies. If the price of oil is not uniformly bounded across truncated economies, then for any positive number M we can find a positive integer T such that $p^T_{X,0} > M$. Under this scenario, the amount of oil used in the production of the consumption good will be small, which implies that the output of the consumption good and a fortiori the wage rate of a young individual of period 0 will be small, and the young generation of period 0 will not earn enough wages to buy the oil stock. Hence the price of oil in period 0 must also be uniformly bounded across truncated economies in this case. As for the price of solar energy, we can set $p^T_{B,0} = p^T_{X,0}$, without disturbing the equilibrium of the truncated economy in question. Thus the prices of solar energy are also uniformly bounded across truncated economies.

Fourth, because the market for solar energy always clears, the price of solar energy must be equal to the rental rate of backstop capital in period 0. Hence the rental rates of backstop capital in period 0 are also uniformly bounded across truncated economies.

Finally, let us show that the prices of emissions permits in period 0 are uniformly bounded across truncated economies. To this end, let us consider a truncated economy with time horizon T and note that if $Q^T_0 = 0$, then there is an excess supply of emissions permits in period 0, which implies that the price of an emissions permit in this period is $p^T_{E,0} = 0$. Next, note that if the prices of emissions permits in period 0 are not uniformly bounded across truncated economies, then for any positive number M we can find a truncated economy such that $p^T_{E,0} > M > p^T_{B,0}$, which means that in this truncated economy oil is not used as part of the energy input in period 0 to produce the consumption good, i.e., $Q^T_0 = 0$ and $B^T_0 > 0$. However, a positive price for emissions permits means that the market for emissions permits in period 0 must clear, i.e., the demand for emissions permits in period 0 and a fortiori the demand for oil in period 0 by the representative firm in the consumption good sector are equal to $E_0^{\#} > 0$. We have just reached two contradictory results: the demand for oil in period 0 by the representative firm in the consumption good sector is both 0 and positive. ■

According to Claim B1, the sequence of five-dimensional vectors $(p_{L,0}^T, p_{X,0}^T, p_{B,0}^T, p_{K,0}^T, p_{E,0}^T)_{T=1}^{\infty}$ lies in a bounded set of the five-dimensional Euclidean space. Hence it contains a convergent subsequence that we denote by $\left(p_{L,0}^{\tau_0(n)}, p_{X,0}^{\tau_0(n)}, p_{B,0}^{\tau_0(n)}, p_{K,0}^{\tau_0(n)}, p_{E,0}^{\tau_0(n)}\right)_{n=1}^{\infty}$, where $\tau_0 : n \to \tau_0(n)$ is an increasing map from the set of positive integers into itself. Let

$$\left(p_{L,0}, p_{X,0}, p_{B,0}, p_{K,0}, p_{E,0}\right) = \ell im_{n\to+\infty}\left(p_{L,0}^{\tau_0(n)}, p_{X,0}^{\tau_0(n)}, p_{B,0}^{\tau_0(n)}, p_{K,0}^{\tau_0(n)}, p_{E,0}^{\tau_0(n)}\right)_{n=1}^{\infty}.$$

CLAIM B2: *The prices of energy in period* 1 *are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,\ldots$ *More precisely, we have*

$$\sup_{n>0}[\min\{p_{X,1}^{\tau_0(n)}, p_{B,1}^{\tau_0(n)}\}] < \infty.$$

PROOF: If the claim is not true, then for any number $M > 0$, we can find a positive integer n such that number $\min\{p_{X,1}^{\tau_0(n)}, p_{B,1}^{\tau_0(n)}\} > M$ for the truncated economy with time horizon $\tau_0(n)$. For this truncated economy, the condition that the marginal revenue product of energy is equal to each price implies

(B.1) $$\min\{p_{X,1}^{\tau_0(n)}, p_{B,1}^{\tau_0(n)}\} = (1-\alpha)(Q_1^{\tau_0(n)} + K_1^{\tau_0(n)})^{-\alpha} > M,$$

i.e.,

(B.2) $$Q_1^{\tau_0(n)} + K_1^{\tau_0(n)} < \left(\frac{1-\alpha}{M}\right)^{1/\alpha},$$

which implies that $K_1^{\tau_0(n)}$, the capital investment of a young individual of period 0, and $Q_1^{\tau_0(n)}$, the oil input in period 1 of the truncated economy being considered are both small when M is large. Now according to Claim 1, the total energy inputs in period 0 are uniformly bounded below across truncated economies, and this means that the wage rates in period 0 – and a fortiori the savings of a young individual of period 0 – are bounded below. Furthermore, the prices of oil in period 0 are also uniformly bounded across truncated economies. Hence most of the saving of a young individual of period 0 will be put into oil. Now using (B.2), we can then assert that the output of the consumption good in period 1 will be small when M is large. Also, using (B.1) and (B.2), we can assert that the value of the oil investment made by a young individual of period 1 will exceed the output of the consumption good when M is large, and this is not possible in equilibrium. ■

CLAIM B3: *The prices of oil, the prices of solar energy, and the prices of emissions permits in period* 1 *are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,\ldots$

PROOF: According to Claim B2, the prices of energy in periods 1 are uniformly bounded across truncated economies with time horizons $\tau_0(n), n = 1,2,\ldots.$ Hence there exists a number, say $M' > 0$, such that $\min\{p_{X,1}^{\tau_0(n)}, p_{B,1}^{\tau_0(n)}\} < M'$, for all $n = 1,2,\ldots.$

If the prices of oil are not uniformly bounded across truncated economies with time horizons $\tau_0(n), n = 1,2,\ldots,$ then for any number $M > 0$ sufficiently large there exists a positive integer n such that such that $p_{X,1}^{\tau_0(n)} > M > M' > p_{B,1}^{\tau_0(n)}$. Because M is large while M' is not, the rate of return to oil investment is much higher than the rate of return to backstop capital investment for a young individual of period 0, and this means that such an individual will only invest in oil, not capital. However, the strict inequality $p_{X,1}^{\tau_0(n)} > p_{B,1}^{\tau_0(n)}$ means that solar energy, not oil, will be demanded by the representative firm in the consumption good sector, contradicting the market-clearing condition for solar energy. Hence the prices of oil must be uniformly bounded across truncated economies with time horizons $\tau_0(n), n = 1,2,\ldots.$

If the prices of solar energy are not uniformly bounded across truncated economies with time horizons $\tau_0(n), n = 1,2,\ldots,$ then we can find a positive integer n such that $p_{B,1}^{\tau_0(n)}$ is arbitrarily large, and this means that a young individual of period 0 will only invest in capital. Because her wages are bounded below, her capital investment, namely $K_1^{\tau_0(n)}$, will not be small, which in turn implies that $p_{B,1}^{\tau_0(n)}$ will not be large, a contradiction to the reductio ad absurdum hypothesis.

Finally, the argument used to establish the result that the prices of emissions permits in period 0 are uniformly bounded across truncated economies with time horizons $\tau_0(n), n = 1,2,\ldots,$ can be repeated to assert that this result also holds in period 1. ■

CLAIM B4: *The states of the system in period* 1 *are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,\ldots$

PROOF: First, note that $X_1^{\tau_0(n)} \le X_0^{\tau_0(n)} = X_0$ because oil is an exhaustible resource. Second, note that $K_1^{\tau_0(n)} \le Y_0^{\tau_0(n)} + (1-\delta)K_0^{\tau_0(n)} \le (X_0 + K_0)^{1-\alpha} + (1-\delta)K_0$. Finally, note that

$$H_1^{\tau_0(n)} = Q_0^{\tau_0(n)} + (1-\gamma)H_0^{\tau_0(n)} \le E_0^{\#} + (1-\gamma)H_0^{\tau_0(n)}. \quad ■$$

CLAIM B5: *The price systems in period* 1 *are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,\ldots$

PROOF: According to Claim B4, the oil stock and the capital stocks are uniformly bounded across the truncated economies with time horizons $\tau_0(n), n = 1,2,\ldots$ Hence the wage rates in period 1 are also uniformly bounded across the truncated economies with time horizons $\tau_0(n), n = 1,2,...$ Claim B5 now follows from this result and Claim B3.

■

CLAIM B6: *The production plans in period* 0 *of the representative firm in the consumption good sector are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,...$

PROOF: We have

$$
\begin{aligned}
&(Y_0^{\tau_0(n)}, L_0^{\tau_0(n)}, Q_0^{\tau_0(n)}, B_0^{\tau_0(n)}, E_0^{\tau_0(n)}) \\
&= \left(A\Omega(H_0)(L_0^{\tau_0(n)})^{\alpha}(Q_0^{\tau_0(n)} + S_0^{\tau_0(n)})^{1-\alpha}, L_0^{\tau_0(n)}, Q_0^{\tau_0(n)}, B_0^{\tau_0(n)}, E_0^{\tau_0(n)})\right) \\
&= \left(A\Omega(H_0)(Q_0^{\tau_0(n)} + K_0^{\tau_0(n)})^{1-\alpha}, 1, Q_0^{\tau_0(n)}, K_0^{\tau_0(n)}, E_0^{\tau_0(n)})\right) \\
&\le \left(A\Omega(H_0)(X_0 + K_0)^{1-\alpha}, 1, X_0, K_0, E_0^{\#})\right).
\end{aligned}
\tag{B.3}
$$

Note that in (B.3), the second equality follows from the market-clearing conditions in the labor and solar energy markets, while the inequality is due to the oil stock constraint and the constraint imposed by the climate-change policy. ■

CLAIM B7: *The lifetime plans of a young individual of period* 0 *are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,...$

PROOF: First, note that for an individual of period 0 her current consumption, namely $c_0^{0,\tau_0(n)}$, is bounded above by $Y_0^{\tau_0(n)} \le A\Omega(H_0)(X_0 + K_0)^{1-\alpha}$, according to (A.5.2.5). Her old-age consumption is given by

$$c_1^{1,\tau_0(n)} = p_{X,1}^{\tau_0(n)} X_1^{\tau_0(n)} + (1 - \delta + p_{K,1}^{\tau_0(n)}) K_1^{\tau_0(n)} + p_{E,1}^{\tau_0(n)}) E_1^{\tau_0(n)}.$$

The right side of the preceding expression is uniformly bounded due to Claims 3 and 5 as well as the cap imposed on emissions in period. Hence $(c_0^{0,\tau_0(n)}, c_1^{1,\tau_0(n)}, k_1^{\tau_0(n)}, x_1^{\tau_0(n)}), n = 1,2,...,$ the lifetime plans of a young individual of period 0, are uniformly bounded across the truncated economies with time horizon s $\tau_0(n), n = 1,2,....$ ■

CLAIM B8: *The production plans in period* 1 *of the representative firm in the consumption good sector are uniformly bounded across the truncated economies with time horizons* $\tau_0(n), n = 1,2,...$

PROOF: The claim can be proved exactly as in Claim B6. ■

According to Claims B1 and B5, the price systems in period 0 and the price systems in period 1 are both uniformly bounded across the truncated economies with time horizons $\tau_0(n), n = 1,2,...$ Hence this sequence of truncated economies has a convergent subsequence that we denote by $\tau_1(\tau_0(n)), n = 1,2,...,$ where is an increasing map whose domain and range are the image of τ_0. According to Claims 6 and 8, the production plans in period 0 and the production plans in period 1 of the representative firm in the consumption good sector are uniformly bounded across the truncated economies with time horizons $\tau_0(n), n = 1,2,...$ Also, according to Claim B7, the lifetime plans of a young individual of period 0 are uniformly bounded across the truncated economies with time horizons $\tau_0(n), n = 1,2,...$ Hence the map τ_1 can be chosen so that the production plans in periods 0 and 1 of the truncated economies of the representative firm in the consumption good sector as well as the consumption of an old individual and the lifetime plans of a young individual of period 0 in the truncated economies with time horizons $\tau_1(\tau_0(n)), n = 1,2,...,$ converge. More precisely, the following limits exist:

(i) $\lim_{n\to+\infty}(p_{L,0}^{\tau_1(\tau_0(n))}, p_{X,0}^{\tau_1(\tau_0(n))}, p_{B,0}^{\tau_1(\tau_0(n))}, p_{K,0}^{\tau_1(\tau_0(n))}, p_{E,0}^{\tau_1(\tau_0(n))}) = (p_{L,0}, p_{X,0}, p_{B,0}, p_{K,0}, p_{E,0}),$

(ii) $\lim_{n\to+\infty}(p_{L,1}^{\tau_1(\tau_0(n))}, p_{X,1}^{\tau_1(\tau_0(n))}, p_{B,1}^{\tau_1(\tau_0(n))}, p_{K,1}^{\tau_1(\tau_0(n))}, p_{E,1}^{\tau_1(\tau_0(n))}) = (p_{L,1}, p_{X,1}, p_{B,1}, p_{K,1}, p_{E,1}),$

(iii) $\lim_{n\to+\infty}(Y_0^{\tau_1(\tau_0(n))}, L_0^{\tau_1(\tau_0(n))}, Q_0^{\tau_1(\tau_0(n))}, B_0^{\tau_1(\tau_0(n))}, E_0^{\tau_1(\tau_0(n))}) = (Y_0, 1, Q_0, B_0, E_0),$

(iv) $\lim_{n\to+\infty}(Y_1^{\tau_1(\tau_0(n))}, L_1^{\tau_1(\tau_0(n))}, Q_1^{\tau_1(\tau_0(n))}, B_1^{\tau_1(\tau_0(n))}, E_1^{\tau_1(\tau_0(n))}) = (Y_1, 1, Q_1, B_1, E_1),$

(v) $\lim_{n\to+\infty} c_0^{1,\tau_1(\tau_0(n))} = c_0^1,$

and

(vi) $\lim_{n\to+\infty}(c_0^{0,\tau_1(\tau_0(n))}, c_1^{1,\tau_1(\tau_0(n))}, k_1^{\tau_1(\tau_0(n))}, x_1^{\tau_1(\tau_0(n))}) = (c_0^0, c_1^1, k_1, x_1).$

The process just described can be repeated ad infinitum to obtain

(i) a sequence of increasing maps $\tau_0, \tau_1, \tau_2, ...$ with the domain and the range of one map – except τ_0 – being the image of the preceding map;

(ii) a price system $\mathcal{P} = (p_{Y,t}, p_{L,t}, p_{X,t}, p_{B,tt}, p_{K,t}, p_{E,t})_{t=0}^{\infty}$; and an allocation induced by $\mathcal{P}$, say

$$\mathcal{Q} = \begin{pmatrix} c_0^1, (c_t^0, c_{t+1}^1, k_{t+1}, x_{t+1})_{t=0}^{\infty}, (Y_t, Q_t, B_t, L_t, E_t)_{t=0}^{\infty}, (B_t^{\#}, K_t^{\#})_{t=0}^{\infty}, \\ (K_t, X_t, H_t)_{t=0}^{\infty} \end{pmatrix},$$

with the following properties:

(iii) for each $t = 0,1,...,$ we have

$$\lim_{n\to+\infty}\begin{pmatrix} p_{L,t'}^{\tau_t(...\tau_1(\tau_0(n)))}, p_{X,t'}^{\tau_t(...\tau_1(\tau_0(n)))}, p_{B,t'}^{\tau_t(...\tau_1(\tau_0(n)))}, \\ p_{K,t'}^{\tau_t(...\tau_1(\tau_0(n)))}, p_{E,t'}^{\tau_t(...\tau_1(\tau_0(n)))} \end{pmatrix}_{t'=0}^{t} = (p_{L,t'}, p_{X,t'}, p_{B,t'}, p_{E,t'})_{t'=0}^{t};$$

(iv) for each $t = 0,1,...,$ we have

$$\ell im_{n\to+\infty}\left(Y_{t'}^{\tau_t(...\tau_1(\tau_0(n)))}, L_{t'}^{\tau_t(...\tau_1(\tau_0(n)))}, Q_{t'}^{\tau_t(...\tau_1(\tau_0(n)))}, B_{t'}^{\tau_t(...\tau_1(\tau_0(n)))}, E_{t'}^{\tau_t(...\tau_1(\tau_0(n)))}\right)_{t'=0}^{t}$$

$$= \left(Y_{t'}, Q_{t'}, B_{t'}, L_{t'}, E_{t'}\right)_{t'=0}^{t};$$

(v) for each $t = 0,1,...,$ we have

$$\ell im_{n\to+\infty}\left(c_{t'}^{0,\tau_t(...\tau_1(\tau_0(n)))}, c_{t'+1}^{1,\tau_t(...\tau_1(\tau_0(n)))}, k_{t'+1}^{\tau_t(...\tau_1(\tau_0(n)))}, x_{t'+1}^{\tau_t(...\tau_1(\tau_0(n)))}\right)_{t'=0}^{t}$$

$$= \left(c_{t'}^{0}, c_{t'+1}^{1}, k_{t'+1}, x_{t'+1}\right)_{t'=0}^{t};$$

(vi) the pair $(\mathcal{P},\mathcal{Q})$ constitutes a competitive equilibrium for an economy with a stock of fossil fuels at the beginning.

The proof of the existence of a competitive equilibrium for an economy with oil resources under infinite time horizon is now complete. ∎

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Essay Three

GLOBAL TRADING OF EMISSIONS PERMITS AND ECONOMIC GROWTH IN AN OVERLAPPING-GENERATIONS MODEL

1. INTRODUCTION

The relationship between trade and environment has been long recognized and that the environmental consequences of trade liberalization have been examined extensively in the literature. Pethig (1979) was the first researcher to study this relationship using a general equilibrium model. The author considered an exogenous policy instrument – emission standards – and allowed the pollution discharge to be constrained by the emission standards. This researcher found that for two countries that are otherwise the same, but are different only in the emission standards, a country that permits higher levels of pollution emissions will export the pollution-intensive good – a factor endowment hypothesis to trade. Pethig concluded that unless developing countries impose environmental standards so that their comparative advantage is eliminated, there is a trade-off between gains from trade and environmental degradation. Although the paper recognized the environmental threats imposed by the lack of environmental standards, it focused mainly on the pattern of trade and what determines it. Antweiler, Copeland, and Taylor (2001) also provided some insights on environmental impacts of international trade in goods. These authors found that trade liberalization can increase welfare and lower pollution. Hence, freer trade is good for the environment. Bruneau (2005) investigated the impacts on welfare of the use of inefficient instruments to manage pollution and concluded that such instruments can lead to lower welfare under trade liberalization. Hence, trade will enhance welfare only if efficient instruments are used to set optimal pollution targets. The above mentioned studies together with others mentioned below do not consider input substitution as a possible means to comply with environmental policy. In combating global warming, especially in the energy sector, switching out of coal into, say natural gas, as an alternative is always feasible, but not always economical. In this paper, we construct an overlapping-generations model in which there are two possible sources of energy – renewable and non-renewable – to analyze the impacts of environmental policy on energy input substitution and, hence, on global pollution and trading.

Not all the models on trade and the environment assume that economic agents behave in a perfectly competitive manner. The relationship between trade and environment is also examined under the imperfect competition framework. For example, Conrad (1993) studied the relationship between environmental policy and firm's share of output in an international oligopoly market with two countries. This author found that if emissions are taxed, subsidies for pollution abatement could be granted to help a firm increasing its market share while maintaining environmental quality at the same time. However Conrad recognized that there is a redistribution of income from taxpayers to the firms' shareholders in using subsidies for pollution abatement. The model of Conrad captures the important fact that some industries, such as the pulp and paper industry and the steel industry, are made up of a small number of firms, and these firms compete with each other on the international markets.

Another important issue in the trade and environment literature concerns the coordination of environmental policies. Nordhaus and Yang (1996) analyzed the welfare implications on various regions of the globe when nations cooperate with each other or act in a non-cooperative manner in curbing greenhouse gases emissions. These authors found that cooperative behavior in curbing greenhouse gas emissions is an efficient one, but not all countries will gain from such an engagement. We also incorporate this feature into our model by assuming that transboundary pollution is being dealt with in a cooperative manner between the North and the South. There are no emission leakages and that permits are freely traded in the carbon market. We assume that emissions permits are allocated according to the scheme of one permit per head and that the Southern population is higher than that of the North. The implications of international collaboration in curbing global pollution were also examined in Conconi (2003), but with the presence of a green lobbying group.

There is also a large literature on how trade affects the use of natural resources and economic growth. Chichilnisky (1994) examined the impacts of trade on environmental resources under well- and ill-defined property rights. This researcher found that the South will over-export resource-intensive goods, and, as a result, the problem of the commons will be aggravated. Cabo, Escudero, Martin-Herran (2002) investigated the pattern of natural resources exploitation in response to technological transfer from the

North the South. Instead of examining the allocation of natural resources over time, Grimaud and Rouge (2005) investigated the problem of how a finite pollution stock, which is generated by the exploitation of non-renewable resources, is allocated among an infinite set of generations. One interesting result these researchers discovered is that at the optimum, an increase in the discount rate leads to a decrease in resource extraction. Like Grimaud and Rough, we analyze the impacts of environmental policy and hence global pollution on different generations.

The model we formalize in this essay is an extension of the one-country model of Essay Two into a two-country world, which is also subjected to a climate-change under the form of a global cap on greenhouse gases emissions. The global climate-change policy we analyze deals with the scientific uncertainty of climate change by adopting the precautionary principle and capping the global emissions of greenhouse gases at a level judged to be acceptable under the precautionary principle. The emissions permits are issued in each period and these permits will expire at the end of the period. The emissions permits are allocated to each country according to its population; that is, an individual in the North is allocated the same number of permits as an individual in the South.

In the model, there are two countries – called the North (N) and the South (S). There are three goods: a consumption good; oil – a generic term for fossil fuels, which is the main source of greenhouse gases – and a renewable energy. In each country, the consumption good is produced by competitive firms from two inputs – labor and energy, while renewable energy is produced by a backstop that uses only capital. The consumption good can also be used as investment to augment the stock of backstop capital. In each period, five types of economic agents coexist in each country: a young generation, an old generation, competitive firms producing the consumption good, competitive firms producing solar energy, and a government. These economic agents interact on five markets: the market for labor, the market for backstop capital, the market for oil, the market for solar energy, and the market for the consumption good. In the model, neither capital nor labor is mobile. On the other hand, oil, emissions permits, solar energy, and the consumption good are freely traded. At the beginning of each period, a new generation is born in each country. Each individual of the new generation lives for two periods. She works when she is young, and retires when she is old. A young individual has no assets, except for one unit of time that she offers in-elastically for sale on the labor market. Part of the wages the individual earns in her young age is spent on consumption; the remaining part is saved to provide for her old-age consumption. The individual does not care about her descendants and will leave no assets behind at the end of her life cycle. There is no population growth in the model, and in each period the size of a generation – young or old – in country i is assumed to be a continuum of measure $\overline{L}_i, i = N, S,$ with $\overline{L}_S > \overline{L}_N$.

The major results of the essay can be summarized as follows. In the long run, after the world has exhausted its stock of fossil fuels, the world will be sustained by renewable energy. The capital stock in each country converges to its steady state level. The share of energy input – in the world's energy input in each period – depends on its technological level and its population. If the population of the South is much larger than the population of the North, and if its population size dominates its technological disadvantage, then the South will have a larger stock of backstop capital than the North in the long run. On the other hand, if the population of South is much more impatient than the population in the North, then the South will have a lower capital stock in the long run and will import renewable energy from the North, and pay for the energy imports with part of the output of the consumption good that the South produces. These results are the contents of Proposition 1. If neither country implements any climate-change policy, then under the competitive equilibrium the abundance of oil will induce excessive burning of this fossil fuel, resulting a high concentration of greenhouse gases in the atmospheres, with the ensuing consequence of lower consumption for future generations. This result is asserted by Proposition 3, which is an extension of Proposition 3 of Essay Two into a two-country world. The competitive equilibrium under the policy of non-intervention is also not Pareto optimal: by appropriately changing the intertemporal allocation of resources, we can raise the welfare of one generation without lowering the welfare of the other generations. This is the message of Proposition 4. Proposition 3 provides an active global climate-change policy on equity grounds, while Proposition 4 a persuasive economic argument in favor of such a policy on efficiency ground. When a global policy on climate change – under the form of emissions permits – is implemented, the cap will be binding at the beginning if the stock of fossil fuels is large. Furthermore, if the rate of capital depreciation is not too high, then the price of oil is rising, but the price of emissions permits is declining as long as the cap on global greenhouse gases emissions is binding, and these results are the contents of Proposition 5. The rising price of oil signifies its scarcity, while the declining of the price of emissions permits reflects the fact that the stock of fossil fuels is being drawn down, and atmospheric pollution is becoming less and less of a problem. Once the cap is no longer binding, the price of emissions permits will fall to 0, and the threat of climate change caused by greenhouse gases no longer exists. The behavior of the price of oil and the price of emissions permits along the equilibrium path underlines the process of energy substitution – renewable energy for fossil fuels. Finally, Proposition 6 asserts that the North will import emissions permits and export the consumption good to pay for these permits. Intuitively, one might suspect that the South with its high population will burn more fossil fuels to feed its population. However, the model assumes a lower technological level for the South and an allocation of emissions permits on an equitable per capita basis. The former assumption implies a lower energy input per worker, while the latter assumption implies an equal number of permits per worker, leading to the result that there is an excess demand for permits per worker in the North. This allocation mechanism allows the South to expropriate part of the output of the consumption good produced by the North during the initial periods.

The paper is organized as follow. Section 2 presents the model. The definition of competitive equilibrium is provided in Section 3. Section 4 analyzes the competitive equilibrium for the world economy without oil. Section 5 presents the existence of competitive equilibrium for a world economy that is endowed with a stock of fossil fuels and that is subject to a cooperative policy on climate change under the form of tradable emissions permits. To establish the baseline for comparison, Section 6 presents the competitive equilibrium for a two-country world with oil resources under the policy of non-intervention. Section 7 analyzes the competitive equilibrium for a world economy which is endowed with oil resources and which is subject to a global policy on climate change under the form of tradable emissions permits. We then conclude in Section 8.

2. THE MODEL

In the model, time is discrete and denoted by $t, t = 0,1,...$. There are two countries – called the North (N) and the South (S) in the model. There are three goods: a consumption good; oil – a generic term for fossil fuels, which is the main source of GHGs – and a renewable energy. In each country, the consumption good is produced by competitive firms from two inputs – labor and energy, while renewable energy is produced by a backstop that uses only capital. The consumption good can also be used as investment to augment the stock of backstop capital.

In each period, five types of economic agents coexist in each country: a young generation, an old generation, competitive firms producing the consumption good, competitive firms producing solar energy, and a government. These economic agents interact on five markets: the market for labor, the market for backstop capital, the market for oil, the market for solar energy, and the market for the consumption good. In the model, neither capital nor labor is mobile. On the other hand, oil, emissions permits, solar energy, and the consumption good are freely traded.

At the beginning of each period, a new generation is born in each country. Each individual of the new generation lives for two periods. She works when she is young, and retires when she is old. A young individual has no assets, except for one unit of time that she offers in-elastically for sale on the labor market. Part of the wages the individual earns in her young age is spent on consumption; the remaining part is saved to provide for her old-age consumption. The individual does not care about her descendants and will leave no assets behind at the end of her life cycle. There is no population growth in the model, and in each period the size of a generation – young or old – in country i is assumed to be a continuum of measure $\overline{L}_i, i = N, S,$ with $\overline{L}_S > \overline{L}_N$.

2.1. *Production Technologies*

In country $i, i = N, S,$ and in period $t, t = 0,1,...,$ the consumption good is produced by competitive firms from two inputs – labor and energy – according to the following Cobb-Douglas production function:

$$(1) \qquad Y_{i,t} = A_i \Omega(H_t) L_{i,t}^{\alpha} Z_{i,t}^{1-\alpha},$$

where A_i denotes the level of technology; $\Omega(H_t)$ represents the output scaling factor due to environmental damage, which depends negatively on H_t, the stock of greenhouse gases in period t; $L_{i,t}$ denotes the labor input; $Z_{i,t}$ denotes the energy inputs; and $Y_{i,t}$ denotes the output of the consumption good. Also, $\alpha, 0 < \alpha < 1,$ is the elasticity of output with respect to labor, and this parameter is assumed to be the same for both countries. Thus the only parameter that differentiates the two countries – as far as the production of the consumption good is concerned – is their level of technology. We shall assume that $A_N > A_S$. The energy inputs come from two sources: oil and a backstop.

Here oil is a generic term for fossil fuels – the main source of greenhouse gases – and the backstop can represent solar energy.

We shall assume that oil can be extracted at negligible cost and that the burning of one unit of oil yields one Btu, and at the same time releases one unit of greenhouse gases into the atmosphere. While oil can be extracted at negligible cost, its stock is limited. The backstop on the other hand can provide an everlasting source of energy. However, harnessing the Sun's energy requires investments in backstop capital, say solar collectors. To keep the exposition as simple as possible, we shall assume that one unit of backstop capital produces one Btu. Also, backstop capital is assumed to depreciate at rate $\delta, 0 \le \delta \le 1$.

In each period $t, t = 0,1,...,$ in country i, let $L_{i,t}$, $Q_{i,t}$, and $B_{i,t}$ denote, respectively, the labor input, the oil input, and the renewable energy input used in the production of the consumption good. These inputs yield the following output of the consumption good:

(2) $$Y_{i,t} = A_i \Omega(H_t) L_{i,t}^{\alpha} (Q_{i,t} + B_{i,t})^{1-\alpha}.$$

Observe that the production of the consumption involves only labor and energy, not capital, and that the backstop sector influences the production of the consumption good only indirectly through the amount of energy that the stock of backstop capital manages to produce and deliver to the consumption good sector. We shall assume that the consumption good can also be used as an investment good to augment the stock of backstop capital.

2.2. *The Evolution of the Stock of Greenhouse Gases*

Let H_t denote the stock of greenhouse gases in the atmosphere in period t. We shall assume that the stock of greenhouse gases decays naturally at rate γ, where $0 < \gamma < 1$ is a parameter. The dynamics of this stock is governed by the following difference equation:

(3) $$H_{t+1} - H_t = E_t - \gamma H_t,$$

where E_t is the global emissions of greenhouse gases in period t. The initial condition H_0 is given.

2.3. *Global Policy on Climate Change*

Suppose that the governments in the two countries agree to implement a global policy on climate change – under the form of emissions permits – to cap the greenhouse gases emissions of firms. To comply with the environmental regulation, firms can either use clean energy from the backstop or buy emissions permits if they burn fossil fuels to provide part of the energy inputs used in the production of the consumption good. A certain number of emission permits are issued at the beginning of each period, and the polluting firms are required to purchase permits, if needed, to meet their obligations. Each permit allows 1 tonne of greenhouse gases emissions, and will expire at the end of the period. Because each permit lasts only one period, banking permits is not possible.

For each $t = 0,1,...,$ let $E^{\#}_{i,t}$ be the number of emission permits allocated to country i in period $t, i = N, S, t = 0,1,...,$ under some international agreement. We shall not attempt to explain how the allocation of emissions permits is arrived at through the negotiation between the two countries, but take the sequence of two-dimensional vector $\left(E^{\#}_{N,t}, E^{\#}_{S,t}\right)_{t=0}^{\infty}$ as given and call this sequence a global policy on climate change.

In any period, there is an international market for emissions permits. The price of a permit in period t is denoted by $p_{E,t}$. Let $E_{i,t}$ be the number of emissions permits demanded by the consumption good sector in country i in period t to comply with the environmental regulation. The global demand for emissions permits in period t is then given by $E_t = E^{\#}_{N,t} + E^{\#}_{S,t}$. If $E_t \leq E^{\#}_{N,t} + E^{\#}_{S,t}$ then the demand for emissions permits can be realized. On the other hand, if $E_t > E^{\#}_{N,t} + E^{\#}_{S,t}$, then the price of emissions permits must rise to reduce its demand below $E^{\#}_{N,t} + E^{\#}_{S,t}$. Thus after the price of emissions permits has adjusted so that the global demand for emissions permits can be met, the total revenues obtained from the sales of emissions permits is $p_{E,t}E_t$. We shall assume that the revenues raised from the emission permits in each country are redistributed equally among all the members of its population. In country i, the transfer that each individual – young or old – receives in period t is thus given by

$$(4) \qquad m_{i,t} = \frac{\sigma_{i,t} p_{E,t} E_t}{L_{i,t}},$$

where we have let

$$(5) \qquad \sigma_{i,t} = \frac{E^{\#}_{i,t}}{E^{\#}_{N,t} + E^{\#}_{S,t}}$$

denote the share of emissions permits allocated to country i in period t.

2.4. *Profit Maximization*

In what follows, we shall choose the consumption good in each period as the numéraire. Also, we shall let $p_{i,L,t}, p_{X,t}, p_{B,t},$ and $p_{i,K,t}$ denote, respectively, the wage rate, the price of oil, the price of renewable energy, and the rental rate of backstop capital – all in period t – that are faced by the economic agents of country $i, i = N, S$.

2.4.1. *Profit Maximization in the Consumption Good Sector*

In country i, the representative firm in the consumption good sector solves the following profit maximization in period t:

$$(6) \qquad \max_{(L_{i,t}, Q_{i,t}, B_{i,t})} A_i \Omega(H_t) L_{i,t}^{\alpha} (Q_{i,t} + B_{i,t})^{1-\alpha} - p_{i,L,t} L_{i,t} - (p_{X,t} + p_{E,t}) Q_{i,t} - p_{B,t} B_{i,t}.$$

Note that in (6) we have used B to denote the input of solar energy. Let $(L_{i,t}, Q_{i,t}, B_{i,t})$ be the solution of this profit maximization problem. The following first-order condition characterizes the demand for labor:

$$(7) \qquad \alpha A_i \Omega(H_t) L_{i,t}^{\alpha-1} (Q_{i,t} + B_{i,t})^{1-\alpha} - p_{i,L,t} = 0.$$

If $p_{X,t} + p_{E,t} < p_{B,t}$, then $Q_{i,t} > 0$, $B_{i,t} = 0$, and the following first-order condition characterizes the demand for oil:

$$(8) \qquad (1-\alpha) A_i \Omega(H_t) L_{i,t}^{\alpha} Q_{i,t}^{-\alpha} - p_{X,t} - p_{E,t} = 0.$$

On the other hand, if $p_{X,t} + p_{E,t} > p_{B,t}$, then $Q_{i,t} = 0$, $B_{i,t} > 0$, and the following first-order condition characterizes the demand for solar energy:

$$(9) \qquad (1-\alpha) A_i \Omega(H_t) L_{i,t}^{\alpha} B_{i,t}^{-\alpha} - p_{B,t} = 0.$$

When $p_{X,t} + p_{E,t} = p_{B,t}$, then oil input and the solar energy input are indeterminate, but their sum $Z_{i,t} = Q_{i,t} + B_{i,t}$ is uniquely determined and satisfies the following first-order condition:

(10) $(1-\alpha)A_i\Omega(H_t)L_{i,t}^{\alpha}Z_{i,t}^{-\alpha} - p_{X,t} - p_{E,t} = 0.$

As for the demand for emissions permits to comply with the environmental regulation, it is given by

(11) $E_{i,t} = Q_{i,t}.$

The output of the consumption good that emerges from the solution of the profit maximization problem (6) is then given by $Y_{i,t} = A_i\Omega(H_t)L_{i,t}^{\alpha}(Q_{i,t} + B_{i,t})^{1-\alpha}$. We shall refer to the list $(Y_{i,t}, L_{i,t}, Q_{i,t}, B_{i,t}, E_{i,t})$ as the optimal production plan in period t for the representative firm in the consumption good sector of country i, when this firm faces the price system $(p_{i,L,t}, p_{X,t}, p_{B,t}, p_{E,t})$ in that period.

2.4.2. *Profit Maximization in the Backstop Sector*

Because the technology used in the production of solar energy is linear, the solution of the profit maximization problem in the backstop sector in country i in any period is particularly simple. If $p_{B,t} > p_{i,K,t}$, then the output of solar energy in country i in period t is infinite. If $p_{B,t} < p_{i,K,t}$, then the backstop sector in country i will shut down in period t. When $p_{B,t} = p_{i,K,t}$, the output of solar energy in country i in period t is indeterminate, and it must adjust to global demand.

2.5. *Preferences and Lifetime Utility Maximization*

The state of the system at the beginning of period $t, t = 0,1,...,$ is represented by the list $\left((K_{i,t}, X_{i,t})_{i=N,S}, H_t\right)$, where $K_{i,t}$, $X_{i,t}$, and H_t represent, respectively, the capital stock, the remaining oil stock, and the stock of greenhouse gases in the atmosphere. We assume that the state of the global economy in period 0, namely $\left((K_{i,0}, X_{i,0})_{i=N,S}, H_0\right)$, is known.

2.5.1. *Lifetime Utility Maximization of a Young Individual*

Consider an young individual of period t in country i. Such an individual owns nothing except for one unit of time that she supplies in-elastically on the labor market at the wage rate $p_{i,L,t}$. She receives a transfer of $m_{i,t}$ from her own government, making her total income in period t equal to $p_{i,L,t} + m_{i,t}$. Out of this income, she spends a part on current consumption and invests the remaining part on the two real assets – capital and oil – to provide for her old-age consumption. Let $c_{i,t}^0$ be her current consumption and $c_{i,t+1}^1$ be her consumption in the next period, when she is old. Her lifetime utility is assumed to be given by

(12) $Log(c_{i,t}^0) + \beta_i Log(c_{i,t+1}^1)$,

where $\beta_i, 0 < \beta_i < 1$, is the factor the individual uses to discount future utilities. We shall assume that $\beta_N > \beta_S$; that is, consumers from the South are less patient than consumers in the North. Let $k_{i,t+1}$ and $x_{i,t+1}$ denote, respectively, the amount of capital and the amount of oil she purchases and pays for from her saving. A lifetime plan for a young individual of period t in country i is a list $\left(c_{i,t}^{0}, c_{i,t+1}^{1}, k_{i,t+1}, x_{i,t+1}\right)$.

To find the optimal lifetime plan of a young individual of period t in country i, first note that the rate of return to capital investment is $1-\delta+p_{i,K,t+1}$ and the rate of return to oil investment is $p_{X,t+1}/p_{X,t}$. The rate of return to her saving obtained in period $t+1$ is then given by

$$(13)\quad r_{i,t+1}=\max\{1-\delta+p_{i,K,t+1},\frac{p_{X,t+1}}{p_{X,t}}\}.$$

Note that if $1-\delta+p_{i,K,t+1}>p_{X,t+1}/p_{X,t}$, then a young individual of period t in country i will only invest in capital. On the other hand, if the inequality is reversed, then she will only invest in oil. When $1-\delta+p_{i,K,t+1}=p_{X,t+1}/p_{X,t}$, the individual will be indifferent between the two real assets, and the investment mix is indeterminate. The lifetime utility maximization problem of a young individual of period t can be formally stated as

$$(14)\quad \max_{c_{i,t}^{0}} Log(c_{i,t}^{0})+\beta_{i}Log[m_{i,t+1}+r_{i,t+1}(p_{i,L,t}+m_{i,t}-c_{i,t}^{0})].$$

$$(14)\quad \max_{c_{i,t}^{0}} Log(c_{i,t}^{0})+\beta_{i}Log[m_{i,t+1}+r_{i,t+1}(p_{i,L,t}+m_{i,t}-c_{i,t}^{0})].$$

The following first-order condition characterizes the optimal current consumption:

$$(15)\quad \frac{1}{c_{i,t}^{0}}-\beta_{i}\frac{r_{i,t+1}}{m_{i,t+1}+r_{i,t+1}(p_{i,L,t}+m_{i,t}-c_{i,t}^{0})}=0,$$

from which we obtain

$$(16)\quad c_{i,t}^{0}=\frac{1}{1+\beta_{i}}\left[p_{i,L,t}+m_{i,t}+\frac{m_{i,t+1}}{r_{i,t+1}}\right].$$

Using (16), we obtain the following expression for the saving made by a young individual of period t in country i:

$$(17)\qquad \begin{aligned} s_{i,t} &= p_{i,L,t} + m_{i,t} - c_{i,t}^{0} \\ &= \frac{1}{1+\beta_i}\left[\beta_i(p_{i,L,t} + m_{i,t}) - \frac{m_{i,t+1}}{r_{i,t+1}}\right]. \end{aligned}$$

As for the optimal investment portfolio of a young individual of period t in country i, we have

$$(18)\qquad (k_{i,t+1}, x_{i,t+1}) = \begin{cases} (s_{i,t}, 0) \text{ if } 1-\delta + p_{i,K,t+1} > p_{X,t+1}/p_{X,t}, \\ \left(0, \dfrac{s_{i,t}}{p_{X,t}}\right) \text{ if } 1-\delta + p_{i,K,t+1} < p_{X,t+1}/p_{X,t}, \\ \left(k_{i,t+1}, \dfrac{s_{i,t} - k_{i,t+1}}{p_{X,t}}\right), 0 \le k_{i,t+1} \le s_{i,t} \text{ if } 1-\delta + p_{i,K,t+1} = p_{X,t+1}/p_{X,t}. \end{cases}$$

2.5.2. *Utility Maximization of an Old Individual*

The utility maximization problem of an old individual in period t is quite simple. Such an individual owns $K_{i,t}$ units of capital and $X_{i,t}$ units of oil. Because this is her last period, the individual will spend all her income and wealth on consumption. The consumption of an old individual in period t is thus given by

$$(19)\qquad c_{i,t}^{1} = m_{i,t} + (1-\delta + p_{i,K,t})K_{i,t} + p_{X,t}X_{i,t}.$$

3. DEFINITION OF COMPETITIVE EQUILIBRIUM

In the following definition of competitive equilibrium, we shall not consider explicitly the market for solar energy by considering only the price system in which the price of solar energy is equal to the rental rate of backstop capital in each period, i.e., $p_{B,t} = p_{i,K,t}$. Under such a price system, the market for solar energy is always in equilibrium, and the price of solar energy can be suppressed in the definition of an equilibrium price system.

governments. Next, let $\mathcal{P} = \left((p_{i,L,t})_{i=N,S}, p_{X,t}, p_{B,,t}, (p_{i,K,t},)_{i=N,S}, p_{E,t}\right)_{t=0}^{\infty}$ be *a price system*.e two

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An allocation induced by $\mathcal{P}$ on country i is an infinite sequence

$$\mathcal{Q}_i = \left(c_{i,0}^1, (c_{i,t}^0, c_{i,t+1}^1, k_{i,t+1}, x_{i,t+1})_{t=0}^{\infty}, (Y_{i,t}, L_{i,t}, Q_{i,t}, B_{i,t}, E_{i,t})_{t=0}^{\infty}, (K_{i,t}, X_{i,t}, H_t)_{t=0}^{\infty}\right)$$

with the following properties:

(i) $c_{i,0}^1 = m_{i,0} + p_{X,0}X_{i,0} + (1 - \delta + p_{i,K,0})K_{i,0}$ is the consumption of an old individual of period 0 in country $i, i = N, S$.

(ii) $(c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})$ is the optimal lifetime plan for a young individual of period t in country $i, i = N, S, t = 0,1,...,$ when the price system $\mathcal{P}$ prevails.

(iii) $(Y_{i,t}, L_{i,t}, Q_{i,t}, B_{i,t}, E_{i,t})$ is an optimal production plan of the representative firm in the consumption good sector in period t in country $i, i = N, S, t = 0,1,...,$ when the price system $\mathcal{P}$ prevails.

(iv) $(K_{i,t}, X_{i,t}) = (L_{i,t}k_{i,t}, L_{i,t}x_{i,t}),$ $\qquad (t = 0,1,...).$

(v) $H_{t+1} = (1-\gamma)H_t + \sum_{i=N,S} Q_{i,t},$ $\qquad (t = 0,1,...).$

The list $(\mathcal{P},\mathcal{Q}_N,\mathcal{Q}_S)$ is said to constitute *a competitive equilibrium induced by the climate-change policy* $\left(E^{\#}_{N,t},E^{\#}_{S,t}\right)_{t=0}^{\infty}$ if the following market-clearing conditions are satisfied for each $t=0,1,\ldots,$

(vi) $\quad L_{i,t}=\overline{L}_i, \qquad (i=N,S),$

(vii) $\quad \sum_{i=N,S}\left(X_{i,t+1}+Q_{i,t}\right)=\sum_{i=N,S}X_{i,t},$

(viii) $\quad \sum_{i=N,S}B_{i,t}=\sum_{i=N,S}K_{i,t},$

(ix) $\quad \sum_{i=N,S}E_{i,t}\leq\sum_{i=N,S}E^{\#}_{i,t},$

(x) $\quad \sum_{i=N,S}\left(c^1_{i,t}+c^0_{i,t}+K_{i,t+1}\right)=\sum_{i=N,S}\left(Y_{i,t}+(1-\delta)K_{i,t}\right).$

4. COMPETITIVE EQUILIBRIUM FOR A GLOBAL ECONOMY WITHOUT OIL

Suppose that the global economy has exhausted its oil resources and that is now completely sustained by a backstop technology. After the oil stock has been depleted, no more greenhouse gases will be generated by production activities in the consumption good sectors, and the evolution of the stock of greenhouse gases in the atmosphere is governed by the following difference equation:

(20) $\quad H_{t+1}=(1-\gamma)H_t.$

Furthermore, after oil exhaustion, all the energy needs of the two economies will be met by the backstop. Under this scenario, we have $Q_{i,t}=0, B_{i,t}>0,$ and the first-order condition (7), which characterizes the demand for labor, takes on the following form:

(21) $\alpha A_i \Omega(H_t)(\overline{L}_i)^{\alpha-1} B_{i,t}{}^{1-\alpha} = p_{i,L,t}$.

Also, the first-order condition (9), which characterizes the demand for renewable energy, now assumes the following form:

(22) $(1-\alpha) A_i \Omega(H_t)(\overline{L}_i)^{\alpha} (B_{i,t})^{-\alpha} = p_{B,t}$.

In period t, the demand for solar energy by the consumption good sectors in the North and in the South satisfy the following system of equations:

(23)
$$\begin{aligned}&(1-\alpha) A_N \Omega(H_t)(\overline{L}_N)^{\alpha} (B_{N,t})^{-\alpha} \\ &\quad = (1-\alpha) A_S \Omega(H_t)(\overline{L}_S)^{\alpha} (B_{S,t})^{-\alpha} = p_{B,t},\end{aligned}$$

(24) $B_{N,t} + B_{S,t} = K_{N,t} + K_{S,t}$.

Together, (23) and (24) allow us to write

(25)
$$\begin{aligned}\frac{B_{N,t}}{(A_N)^{1/\alpha} \overline{L}_N} &= \frac{B_{S,t}}{(A_S)^{1/\alpha} \overline{L}_S} \\ &= \frac{B_{N,t} + B_{S,t}}{(A_N)^{1/\alpha} \overline{L}_N + (A_S)^{1/\alpha} \overline{L}_S} \\ &= \frac{K_{N,t} + K_{S,t}}{(A_N)^{1/\alpha} \overline{L}_N + (A_S)^{1/\alpha} \overline{L}_S}.\end{aligned}$$

It follows immediately from (25) that the demand for renewable energy in the North and the demand for renewable energy in the South are given by

(26) $B_{i,t} = \kappa_i (K_{N,t} + K_{S,t})$,

where we have let

(27) $$\kappa_i = \frac{(A_i)^{1/\alpha} \overline{L}_i}{(A_N)^{1/\alpha} \overline{L}_N + (A_S)^{1/\alpha} \overline{L}_S}, \qquad (i = N, S).$$

Note that (27) gives the share in global renewable energy use for country i in each period. Furthermore, theses shares – which depend only on the technology levels and the labor forces of the two countries – are constant through time. Also, the share of a country is an increasing function of its technology level and the size of its labor force. Hence whether the South demands more energy than the North depends on whether its larger population more than offsets its lower technology level. The following lemma is immediate.

LEMMA 1: *If the population in the South is much larger than the population in the North so that* $A_S(\overline{L}_S)^{\alpha} > A_N(\overline{L}_N)^{\alpha}$, *then* $\kappa_S > \kappa_N$, *i.e., the consumption good sector in the South uses more renewable energy than that in the North in each period.*

Using (21) and (26), we obtain the following expression for the saving of the young generation of period t in country i, which is also the capital stock of this country in the next period i:

$$
\begin{aligned}
K_{i,t+1} &= \overline{L}_i \frac{\beta_i}{1+\beta_i} p_{i,L,t} \\
(28) \qquad &= \overline{L}_i \frac{\beta_i}{1+\beta_i} \alpha A_i \Omega(H_t)(\overline{L}_i)^{\alpha-1}[\kappa_i(K_{N,t}+K_{S,t})]^{1-\alpha} \\
&= \frac{\alpha\beta_i A_i \kappa_i^{1-\alpha}\overline{L}_i^{\alpha}}{1+\beta_i}\Omega(H_t)(K_{N,t}+K_{S,t})^{1-\alpha}.
\end{aligned}
$$

Summing (28) over $i = N, S,$ we obtain the following difference equation that describes the evolution of the global stock of backstop capital:

$$
(29) \quad K_{N,t+1} + K_{S,t+1} = \alpha\Omega(H_t)\left[\sum_{i=N,S} \frac{\beta_i}{1+\beta_i} A_i \kappa_i^{1-\alpha}\overline{L}_i^{\alpha}\right](K_{N,t}+K_{S,t})^{1-\alpha}.
$$

If we let $K_{t+1} = K_{N,t+1} + K_{S,t+1}$, then the following difference equation governs the evolution of the global stock of backstop capital:

(30) $$K_{t+1} = \alpha\Omega(H_t)\left[\sum\nolimits_{i=N,S}\frac{\beta_i}{1+\beta_i}A_i\kappa_i^{1-\alpha}\overline{L}_i^{\alpha}\right]K_t^{1-\alpha}.$$

and in the long run the global stock of backstop capital converges globally to the following stationary level:

(31) $$\overline{K} = \ell im_{t\to+\infty}K_t = \alpha^{1/\alpha}\left[\sum\nolimits_{i=N,S}\frac{\beta_i}{1+\beta_i}A_i\kappa_i^{1-\alpha}\overline{L}_i^{\alpha}\right]^{1/\alpha}.$$

We summarize the result just obtained in the following lemma:

LEMMA 2: *In the long run – and after the oil stocks in the two countries have been exhausted – the global stock of backstop capital converges globally to the stationary level defined by* (31).

Now note that the share of country $i's$ in the global stock of backstop capital in period $t+1$ is given by

(31) $$\frac{K_{i,t+1}}{K_{N,t+1}+K_{S,t+1}} = \frac{\frac{\beta_i}{1+\beta_i}\alpha A_i\Omega(H_t)\kappa_i^{1-\alpha}\overline{L}_i^{\alpha}(K_{N,t}+K_{S,t})^{1-\alpha}}{\alpha\Omega(H_t)(K_{N,t}+K_{S,t})^{1-\alpha}\left[\sum_{j=N,S}\frac{\beta_j}{1+\beta_j}A_j\kappa_j^{1-\alpha}\overline{L}_j^{\alpha}\right]} = \frac{\frac{\beta_i}{1+\beta_i}A_i\kappa_i^{1-\alpha}\overline{L}_i^{\alpha}}{\left[\sum_{j=N,S}\frac{\beta_j}{1+\beta_j}A_j\kappa_j^{1-\alpha}\overline{L}_j^{\alpha}\right]}.$$

If the South is much more impatient than the North, i.e., if β_S is much smaller than β_N, then the ratio on the right side of (32) will be strictly less than $1/2$. The following lemma is immediate:

LEMMA 3: *If the population in the South is much more impatient than the population in the North, then the stock of backstop capital in the South will be lower than the stock of backstop capital in the North in each period* $t = 1,2,\ldots$.

Combining Lemmas 1 through 3, we obtain the following proposition:

PROPOSITION 1: *For a global economy that has no oil or that has exhausted its oil stock and is now sustained completely by renewable energy, the global stock of backstop capital converges globally to the stationary level* (31) *in the long run. Furthermore, if the population in the South is much larger and much more impatient than the population in the North, then along the equilibrium path the South will have a smaller stock of backstop capital and use more renewable energy than the North, i.e., in each period the South will import renewable energy from the North and pay for the energy imports with part of its output of the consumption good.*

5. EXISTENCE OF COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH FOSSIL FUELS

The proposition asserts the existence of a competitive equilibrium for an overlapping-generations model in a two-country world subject to a global policy on climate change. It is the two-country version of Proposition 2 of the preceding essay, and can be proved in exactly the same manner, with minor modifications for a world made up of two national economies.

PROPOSITION 2: *Let* $X_{i,0} > 0$ *and* $K_{i,0} \geq 0$ *denote, respectively, the initial stock of oil and the initial stock of backstop capital in country* $i, i = N, S$. *Let* $(E^{\#}_{N,t}, E^{\#}_{S,t})_{t=0}^{\infty}$ *be a global policy on climate change implemented by the two national governments. There exists a competitive equilibrium, say* $(\mathcal{P}, \mathcal{Q}_N, \mathcal{Q}_S)$, *with* $\mathcal{P} = (p_{N,L,t}, p_{S,L,t}, p_{X,t}, p_{N,K,t}, p_{S,K,t}, p_{E,t})_{t=0}^{\infty}$ *and*

$$\mathcal{Q}_i = \left(c^1_{i,0}, (c^0_{i,t}, c^1_{i,t+1}, k_{i,t+1}, x_{i,t+1})_{t=0}^{\infty}, (Y_{i,t}, L_{i,t}, Q_{i,t}, B_{i,t}, E_{i,t})_{t=0}^{\infty}, (K_{i,t}, X_{i,t}, H_t)_{t=0}^{\infty}\right)$$

6. COMPETITIVE EQUILIBRIUM FOR A TWO-COUNTRY WORLD WITH OIL RESOURCES: THE CASE OF NO POLICY ON CLIMATE CHANGE

Let us first consider the case where the two national governments do not have any environmental policy and the firms in the North as well as the firms in the South are free to discharge greenhouse gases into the atmosphere. This is a special case of the case where the number of emissions permits issued in each period is large enough, say $\sum_{i=N,S} E^{\#}_{i,t} > \sum_{i=N,S} E_{i,t}, t = 0,1,...,$ so that the equilibrium price of emissions permits in each period is driven down to 0: the consumption good sector is free to discharge greenhouse gases into the atmosphere. With appropriate modifications of their proofs, the results of the competitive equilibrium for the one-economy model under non-intervention in the preceding chapter extend naturally to the two-country world.

Proposition 3 of the preceding essay asserts, for a one-country world, that if the initial oil stock is large enough and if there is no policy on climate change, then there will be excessive burning of fossil fuels in period 0 under the competitive equilibrium, with the ensuing steep fall in consumption of the old and young generations of period 1. Proposition 3 of the present chapter extends Proposition 3 of the preceding chapter into the two-country world. It provides the rationale for an active global climate-change policy to counteract the negative production externalities generated by the excessive burning of fossil fuels in one period that spill over into the next period. Proposition 3 stated below asserts that without an interventionist climate-change policy, it is future generations that pay for the excessive current consumption of the current generations.

PROPOSITION 3: *Suppose that neither* government *implements any policy on climate change. If the initial oil stock is large, then the output of the consumption good in period* 1 in both countries *will be low. More precisely,* $[\sum_{i=N,S} Y_{i,1}] \to 0$ *when* $[\sum_{i=N,S} X_{i,0}] \to +\infty$. *Under the competitive equilibrium, the abundance of oil resources induces an excessive burning of these fossil fuels, leading to a steep rise in the concentration of greenhouse gases in the atmosphere, which in turn causes a drastic fall in the consumption of the young and old generations of period* 1 *in the North as well as in the South.*

PROOF: This proposition can be proved in the same manner as Proposition 3 of Essay Two, with only minor changes in notations to accommodate a two-country world.

Proposition 4 of Essay Two asserts for a one-country world that the competitive equilibrium under non-intervention is not Pareto optimal. Proposition 4 of the present essay – stated below – is an extension of this result into a two-country world. As in the one-country model, it is the production externalities generated by burning fossil fuels that is the source of inefficiency.

PROPOSITION 4: *Suppose that neither national government implements any policy on climate change. Consider a competitive equilibrium under which oil constitutes part of the energy input used in the production of the consumption in one country for at least one period. Then the competitive equilibrium is not Pareto optimal*

PROOF: This proposition can be proved in the same manner as Proposition 4 of Essay Two, with only minor changes in notations to accommodate a two-country world.

7. COMPETITIVE EQUILIBRIUM FOR AN ECONOMY WITH OIL RESOURCES: THE CASE OF EMISSIONS PERMITS

Suppose that the objective of the climate-change policy is to stabilize the stock of greenhouse gases at a level, say $H^{\#}$, judged to be reasonable in the long run according to the precautionary principle. To stabilize the stock of greenhouse gases at the level $H^{\#}$, we shall assume a constant cap on greenhouse gases emissions given by

$E_t^{\#} = E^{\#} = \gamma H^{\#}, t = 0,1,...$ Also, we suppose that $K_0 = 0$; that is, the backstop is not yet active in period 0.

In Essay Two, we showed that for the one-country world the backstop will be brought into use in finite time if the initial stock of backstop capital is 0.[1] Furthermore, when the initial oil stock is large, there exists a positive integer $T^{\#}$ such that the emissions cap is binding during the time interval between 0 and $T^{\#}$, with $T^{\#}$ included, and $T^{\#} \to +\infty$ when $X_0 \to +\infty$.[2] Also, if the rate of capital depreciation is low, then the price of oil will be rising along the equilibrium path as long as the oil stock has not been exhausted to induce the successive young generations into holding it.[3] These results continue to hold in a two-country world, and their proofs only need to be modified to accommodate the existence of a second country. We mention these results explicitly here because the will be used in the analysis that follows.

Because the initial stock of backstop capital is assumed to be equal to 0, the energy input in that period consists only of oil. Proposition 5 of Essay Two, which holds for the case of nonintervention in a one-country world also holds for the case of a two-country world subjected to a global policy on climate change; that is, the backstop will be brought into use in finite time. More precisely, there exists a time, say T^b, such that $K_{N,T^b} > 0$ or $K_{S,T^b} > 0$ and $K_{i,t} = 0$, for $i = N, S, t < T^b$.

[1] Proposition 5 , Essay Two.
[2] Lemma 2, Essay Two.
[3] Lemma 4, Essay Two.

Let $\hat{T} = \min\{T^b, T^\#\}$. then during the time interval $0 \le t \le \hat{T}$ the emissions cap is binding and all the energy needs of the economy are met by oil. Hence the effective unit cost of using oil during the time interval $0 \le t \le \hat{T}$ is given by

$$(34) \quad \begin{aligned} p_{X,t} + p_{E,t} &= (1-\alpha)A_N\Omega(H_t)(\bar{L}_N)^\alpha (Q_{N,t})^{-\alpha} \\ &= (1-\alpha)A_S\Omega(H_t)(\bar{L}_S)^\alpha (Q_{S,t})^{-\alpha}, \end{aligned}$$

$$(35) \quad Q_{N,t} + Q_{S,t} = E^\#.$$

It follows from (34) and (35) that

$$(36) \quad Q_{i,t} = \kappa_i E^\#, \qquad (0 \le t \le T).$$

where, we recall, κ_i is defined by (27). Using (36) in (34), we obtain

$$(37) \quad \begin{aligned} p_{X,t} + p_{E,t} &= (1-\alpha)A_N\Omega(H_t)(\bar{L}_N)^\alpha (\kappa_N E^\#)^{-\alpha} \\ &= (1-\alpha)A_S\Omega(H_t)(\bar{L}_S)^\alpha (\kappa_S E^\#)^{-\alpha}. \end{aligned}$$

It follows directly from (37) that during the time interval the global cap on greenhouse gases emissions is binding, the effective unit cost of burning oil, namely the purchase price of oil plus the cost of one emissions permit to comply with the Kyoto Protocol – is constant. Furthermore, when the rate of capital depreciation is low, the price of oil must be rising along the equilibrium path as long as the global world oil stock has not been depleted. Hence the price of emissions permits will be declining along the equilibrium path during the time interval the global cap on greenhouse gases emissions is binding if the rate of capital depreciation is low. This result – proved in Essay Two for a one-country world – is thus also true in a two-country world. We summarize the result just discussed in the following Proposition:

PROPOSITION 5: *Suppose that the rate of capital depreciation is low and that the backstop is not active in period 0. During the time interval the global cap on greenhouse emissions is binding and before backstop capital is accumulated, the price of oil is rising, but the price of emissions permits is declining along the equilibrium path.*

If the number of emissions permits allocated to each country is equal to its relative size in the world population, then the numbers of emissions permits allocated to the two countries in period t are given by

$$E_{i,t}^{\#} = \frac{\overline{L}_i}{\overline{L}_N + \overline{L}_S} E^{\#}, \qquad (i = N, S). \tag{38}$$

The excess demand for emissions permits in the North in period $t, 0 \le t \le \hat{T}$, is then given by

$$\begin{aligned} Q_{N,t} - E_{N,t}^{\#} &= \frac{(A_N)^{1/\alpha}\overline{L}_N}{(A_N)^{1/\alpha}\overline{L}_N + (A_S)^{1/\alpha}\overline{L}_S} E^{\#} - \frac{\overline{L}_{iN}}{\overline{L}_N + \overline{L}_S} E^{\#} \\ &= \left[\frac{(A_N)^{1/\alpha}\overline{L}_N}{(A_N)^{1/\alpha}\overline{L}_N + (A_S)^{1/\alpha}\overline{L}_S} - \frac{\overline{L}_N}{\overline{L}_N + \overline{L}_S}\right] E^{\#} \\ &= \frac{[(A_N)^{1/\alpha}\overline{L}_N][\overline{L}_N + \overline{L}_S] - \overline{L}_N[(A_N)^{1/\alpha}\overline{L}_N + (A_S)^{1/\alpha}\overline{L}_S]}{[(A_N)^{1/\alpha}\overline{L}_N + (A_S)^{1/\alpha}\overline{L}_S][\overline{L}_N + \overline{L}_S]} E^{\#} \\ &= \frac{[(A_N)^{1/\alpha} - (A_S)^{1/\alpha}]\overline{L}_N\overline{L}_S}{[(A_N)^{1/\alpha}\overline{L}_N + (A_S)^{1/\alpha}\overline{L}_S][\overline{L}_N + \overline{L}_S]} E^{\#} > 0. \end{aligned} \tag{39}$$

Note that the strict inequality on the last line of (39) follows from the assumption that the technological level of the North is higher than that of the South, namely $A_N > A_S$. The intuition behind this result is not hard to grasp. Because of its high technological level, the output per worker is higher in the North than in the South, which in turn implies that the marginal product of energy is higher in the North than in the South, inducing the North to use more oil per worker. On the other hand, the emissions permits are allocated on the basis of one permit per head. Therefore, the excess demand for emissions permits per worker in the North is higher than that in the South. Given the fixed number of emissions permits issued in each period, it can only mean that the North must import emissions permits from the South during the time interval the global cap on greenhouse emissions is binding. We summarize the result just obtained in the following proposition:

PROPOSITION 6: *Suppose that the rate of capital depreciation is low and that the backstop is not active in period 0. During the time interval the global cap on greenhouse emissions is binding and before backstop capital is accumulated, the North imports emissions permits from the South.*

We would like to note in passing that in any period, the share of energy input – oil or renewable energy or a combination of both – of a country in the total world's energy input is a constant fraction $\kappa_i, i = N, S,$ of this latter variable. Also, the energy input per worker in the North is higher than that in the South, and this result means that the along the equilibrium path, the wage rate is higher in the North than in the South.

8. CONCLUSION

The two-country overlapping-generations model we formulated and analyzed in this essay represents a first attempt to study the impact on the world economy of a global policy on climate change. The global number of emissions permits and its allocation between the two countries are taken as exogenous. The model is silent on how these numbers are arrived at. The next step is to endogenize these variables and explain how an international environmental treaty, such as the Kyoto Protocol, is negotiated. To be viable, an international environmental treaty must be self-enforcing; that is, each country must find it is in its own interest to subscribe to such an agreement. This is the greatest defect of the Kyoto Protocol because large countries like the US, China, and India do not find the international environmental treaty suits their interests. The model we formalize does not incorporate the scientific uncertainty concerning our knowledge of climate change and the potential catastrophic consequences of a high level of greenhouse gases in the atmosphere, either. Another direction for extending the model is to incorporate the uncertainty on the impact on humanity of climate change as well as the nonlinear behavior of the Earth's climate due to anthropomorphic action on the earth.

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