THE INDUCTANCES OF A D.C. MACHINE UNDER TRANSIENT CONDITIONS

by

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ABSTRACT

The behaviour of inductance, self and mutual, is of considerable interest, especially in the case of d.c. machines operating in power and control systems, where the accurate prediction of the behaviour is of importance under both transient and steady-state conditions.

Existing methods of measuring the inductances of the coils of a compound-wound d.c. machine with interpoles are compared with a view to recommending simple methods that will be suitable for all the coils from the highly-inductive shunt-field to the comparatively non-inductive armature.

The thesis discusses the distinction between the transient, incremental and apparent inductances and surveys appropriate methods of measuring them. The experimental work considers and eliminates, in turn, the effects of the carbon brushes by making measurements with them in the circuit and then with direct connections to the commutator of a 5 KW machine. In all the inductance measurements the effects of direct-axis saturation and hysteresis are considered. Hay's bridge is considered to be the most convenient single method suitable for measuring all the machine self-inductances. The bridge is operated over a frequency range of 1.66 c/s to 60 c/s to observe the effect of eddy currents in the solid iron driving flux into leakage paths at the higher frequencies. Mutual inductances are measured by a simple ammeter-voltmeter method and the variations with rotor angle of mutual inductances between the stator and direct-axis coils and the armature coil, are investigated for the purposes of commutation analysis. The polar moment of inertia of the rotor is determined by both mechanical and electro-mechanical tests.

Theoretical transient curves are predicted for the machine operating both as a generator and as a motor and the curves are compared with those from actual tests. Due to the upredictable nature of hysteresis and brush-contact effects when the machine is in normal operation, the distinction between transient, incremental and apparent inductance is found to be practically unimportant for this machine.

ACKNOWLEDGEMENTS

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LIST OF PRINCIPAL SYMBOLS

Φ	Flux (weber)
B	Flux density (tesla)
λ	Flux linkages (weber turns)
$\mathfrak{F}$	Magnetomotive force (Ampere-turns)
σ	Current-density (Ampere/meter^2)
H	Magnetizing force (Ampere turns/meter)
L	Self-inductance (Henry)
M	Mutual-inductance (Henry)
r	Resistance (ohms)
R	Rotational loss torque per unit angular velocity (Newton-metre/radian/sec)
Z	Transient impedence matrix (ohms)
i	Instanteneous Current (Ampere)
I	Steady-state current (Ampere)
e	Instanteneous voltage (volts)
E	Steady-state voltage (volts)
t	Instant of time (seconds)
T	Time-constant (second)
T_c	Time of Commutation (second)
J	Moment of Inertia (Kilogram-meter^2)
ω	Angular velocity (radians/second)
θ	Rotor angle (Mechanical degrees)
p	Operator $\frac{d}{dt}$
δ	Interval of Commutation (Mechanical degrees)
μ_o	Permeability of air (Henry/meter)
L', M'	Speed coefficients which differ from unprimed inductances if the field form is not sinusoidal
M_{fs}	Mutual inductance between shunt field and series field (Henry)

M_{fa}	Mutual inductance between shunt field and armature (Henry)
M_{sa}	Mutual inductance between series field and armature (Henry)
$M_{(a+i)f}$	Mutual inductance between shunt field and armature + interpoles (Henry)
$M_{(a+i)s}$	Mutual inductance between series field and armature + interpoles (Henry)

Subscripts or superscript 'f' refer to shunt field

Subscripts or superscript 'a' refer to armature

Subscripts or superscript '(a + i)' refer to armature +interpoles

Subscripts or superscript 's' refer to series field

Subscripts or superscript 'b' refer to short-circuited turns.

CHAPTER I

INTRODUCTION

The importance of coil inductances in machine analysis is due to the fact that they, more than any other factor, influence the performance of the machine.

In a machine having one stator winding and one rotor winding the quadrature-axis e.m.f. generated in the rotor as the result of "flux-cutting" is proportional to the stator current producing the direct axis flux, the rotor speed and the mutual inductance existing between the stator and the rotor coils.

If a machine has no stator winding, a direct-axis rotor current will produce a flux and consequently a quadrature-axis generated e.m.f. proportional to the direct-axis current, the rotor speed and the self-inductance of the direct-axis rotor coil.

The inductances in the above two cases are frequently termed speed coefficients because the production of a generated e.m.f. is dependent on speed. The electromagnetic torque produced by a machine is intimately related to the speed coefficient of inductance. It is produced by a coil current acting on a flux that is produced by another coil, the two coils being in space-quadrature. The electromagnetic torque in a machine is given by the product of the currents in the two coils and the speed coefficient of inductance.

Due to the nonlinearity of the magnetization curve for machines, the speed coefficients may be determined from the open-circuit characteristics either by experiment or from design data and they may be plotted against exciting current so that an appropriate value may be chosen for the particular condition of saturation under investigation. However, if a load is suddenly applied, if a short-circuit suddenly occurs, or if the input voltage changes by a step, the speed coefficients are no longer appropriate for evaluating e.m.f. and torque, because sudden changes introduce eddy-currents in any solid machine parts resulting in transient magnetomotive forces that change the air-gap flux. Further, if a d.c. machine is driving a load with a fluctuating torque, e.g. a d.c. motor driving a reciprocating compressor, the torque fluctuations produce corresponding current pulsations in the armature circuit at a frequency proportional to the rotor speed and an alternating current is thereby superimposed on the direct current. With a compound machine, the direct-axis flux will undergo a corresponding fluctuation causing an alternating voltage to be generated by the armature in addition to the normal average or d.c., value of e.m.f. .

Thus in the d.c. machine there are three types of inductance that may be required for analysis in terms of circuit theory, namely the speed coefficients, the transient inductances and those incremental inductances that limit alternating currents of a definite frequency. All these inductances have their seats in iron-cored coils or in solid iron.

In analysing a.c. machines all the above inductances may require consideration, together with the steady-state apparent inductance of the coils producing a reactance as the result of symmetrical working on the B-H loop. Leakage inductances also require consideration but these are due to flux in non-magnetic materials with a relative permeability of unity and are consequently independent of current. However, alternating-current machines are beyond the scope of this thesis. Inductances of coils at slot ripple frequencies, usually higher than 1 kc/s are also ignored in this work because they have no appreciable effect on the performance of the machine.

The appropriate inductance to use in analysing transient phenomena cannot be explained in terms of the B-H loop because the eddy-current magnetomotive forces distort the hysteresis loop during the transient period. The general effect is to reduce the value of inductance below the steady-state value due to high-frequency components of current causing eddy current m.m.fs. which, by Lenz's law, oppose the initiating m.m.f. and drive the flux into leakage paths. During the transient period the inductance increases from the initial value to the final steady-state value.

Speed coefficients of inductance may be measured experimentally from the open-circuit characteristics. Incremental inductances may be measured by using a.c. bridge circuits where the frequency at which the measurements are made should be of the same order as that to be encountered by the machines under working conditions.

Various methods of measuring the inductances of a machine were investigated and the results are compared in subsequent chapters. It is considered pertinent to conclude this chapter with a discussion of the definition of inductance leading to concepts of inductance that are appropriate to the coils encountered in d.c. machines.

In text books, the concept of inductance is usually developed from Faraday's law, a typical definition[16] being:

"Whenever total net magnetic flux through a closed circuit varies there is induced in the circuit a voltage whose magnitude is proportional to the rate of the diminution of the flux through the circuit".

A single turn coil, in a non-magnetic medium, carrying a current has a flux through it simply due to the current. Then, the coefficient of self-inductance is defined as:

$$L = \frac{\Phi}{I}, \quad \text{or} \quad \frac{N\Phi}{I} \quad \text{for N turns} \tag{1}$$

According to Faraday's law, an e.m.f. is induced for a changing current:

$$e = L\frac{dI}{dt} \tag{2}$$

For a coil with an iron-core, equation (1) is still valid but equation (2) must be replaced by

$$e = \frac{d}{dt}(LI) = (L + I\frac{dL}{dI})\ \frac{dI}{dt} \qquad (3)$$

For practical purposes, an assumption that the flux is proportional to the current is often made. However, due to the presence of saturation, inductance is a variable quantity. For the purposes of this work equation (3) will always be considered as true and an approximation will be made for linearity when required.

Similar definitions exist for the coefficient of mutual inductance. When a current flows in a circuit, a flux is set up and if part of this flux is linked with a second circuit, there is said to be mutual inductance between the two circuits. Thus, mutual inductance between two circuits (1) and (2) due to current in circuit (2) can be defined as

$$M_{12} = \frac{N_1 \Phi_1}{I_2} \qquad (4)$$

If the roles of two circuits are interchanged,

$$M_{21} = \frac{N_2 \Phi_2}{I_1} \qquad (5)$$

In certain instances, difficulty arises in interpreting these results. One difficulty arises from the imperfection of the definition of flux-linkage as "the sum of fluxes through all the turns"[17] of a coil. Consider for example, an air-cored solenoid (figure I-1) with widely-spaced turns and consider the flux in terms of tubes rather than lines, the relation between tubes and lines being

$$d\Phi = B_n\ ds$$

It is not possible to count the number of tubes of flux which are linked with a particular turn in such a coil because the turn is not a closed loop. It might seem possible to resolve the difficulty by fixing attention on any closed tube of flux '$d\Phi$' and counting the number of times (N) that the winding conductor passes through the loop. If this is possible, the flux-linkage is '$Nd\Phi$'.

Consider now fig. I-2(a), which shows a simple single-loop circuit linked with a flux 'Φ'. In order to calculate the flux through the circuit, imagine a surface or a cap, of which the circuit is the boundary, figure I-2(b), and consider the normal component of flux-density $B_n(x, y, z, t)$ at any point on the surface. In general, B_n is a function of (x, y, z) as well as of 't' for it may be varying in any arbitrary manner. The instantaneous value of the total flux through the surface - and therefore through the boundary circuit is

$$\Phi = \iint B_n dS \qquad (6)$$

in which dS is the area of an element of the surface where the normal component of the flux density is B_n and the integral is to be taken over the entire surface of which the circuit is the boundary.

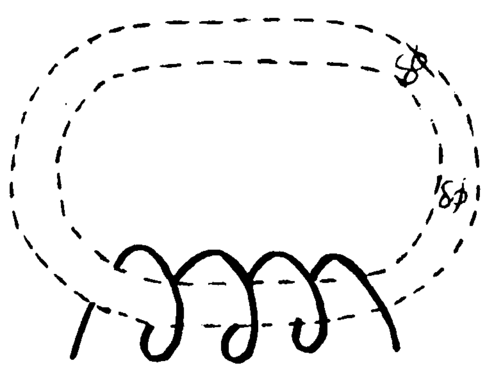


Fig. I-1

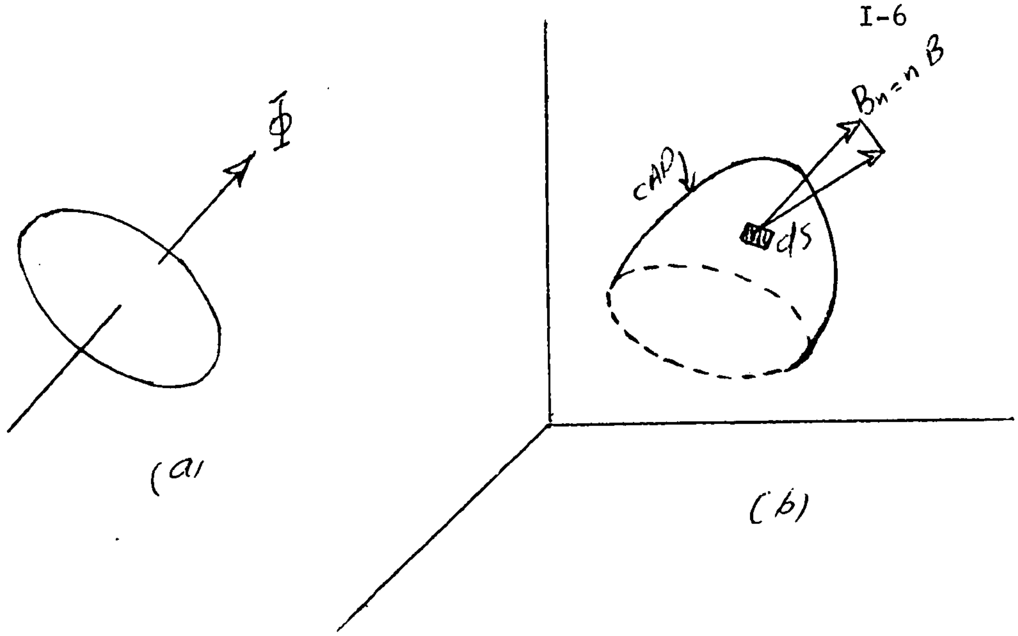


Fig. I-2.

It is to be noted that no restrictions are to be made as to the shape of the cap. In practice, it may become difficult to apply this interpretation to cases such as multi-turn and multi-layer windings in the field circuit of a d.c. machine but if flux-linkages are defined as "the product of the number of turns and the mean flux per turn" for an engineer's approximation, it is adequate for most purposes. However, the exact meaning of flux-linkage is "total flux across any surface which has its edge on the circuit"[16].

The distinction between the exact and the approximate definitions can be illustrated by some examples. Reference 16 and 17 present some of the situations under which the exact definition clears the misinterpretation that otherwise is created by the approximate definition.

An exact definition of flux-linkages applied to multi-layer coils is incomprehensible and some approximation is necessary to enable a working definition to be used. Many of the methods of measurement do not measure the inductance directly in terms of flux-linkages per ampere, but rather the e.m.f. induced, or the amount of energy stored, in an inductive circuit. Provided that the method of measurement simulates the actual working conditions for which the inductance value is required, the uncertainty in defining flux-linkages is of little practical consequence.

CHAPTER II

A SURVEY OF METHODS AVAILABLE FOR THE DETERMINATION OF COEFFICIENTS OF INDUCTANCE.

The methods available for the measurement of inductance can be divided into two broad groups:- those using (i) alternating current, and (ii) direct current, for the measurement. In the following, both a.c. and d.c. methods are described and the calculation of inductance from design data for an armature coil is also presented. Reference to a numerical method is given.

(I) A.C. Impedance Test:-

This is a simple test but gives results sufficiently accurate to discern a small difference between inductances measured at 60 c.p.s., and at 25 c.p.s. the latter being a little higher due to deeper flux penetration into the iron at lower frequencies. The test involves measurement of the a.c. impedance between the terminals of a stationary d.c. machine by impressing an alternating voltage on these terminals and measuring the resulting alternating current. Brush contact resistance can be eliminated by making solid connections on the commutator. The field winding may be left on open-circuit, if the unsaturated value of the inductance is desired. For obtaining saturated values of the inductance, the field winding should carry the normal field current. This test has been applied for machines ranging from 5 kw to 300 kw.

(II) From Design data:-

The inductance can be calculated from the design data. An outline of the procedure involved is as follows:-

(i) A magnetic circuit that affects one electric circuit is chosen and the total magnetomotive force existing in the magnetic circuit is determined.

(ii) The product of the magnetomotive force and the permeance of the magnetic circuit gives the total flux affecting the electric circuit under consideration.

(iii) The product of the total flux and the number of turns linked by the flux gives the total flux linkages in the circuit.

(iv) The total flux linkages divided by the current in the circuit gives the inductance. The above procedure is illustrated by calculating the inductances of an armature coil.

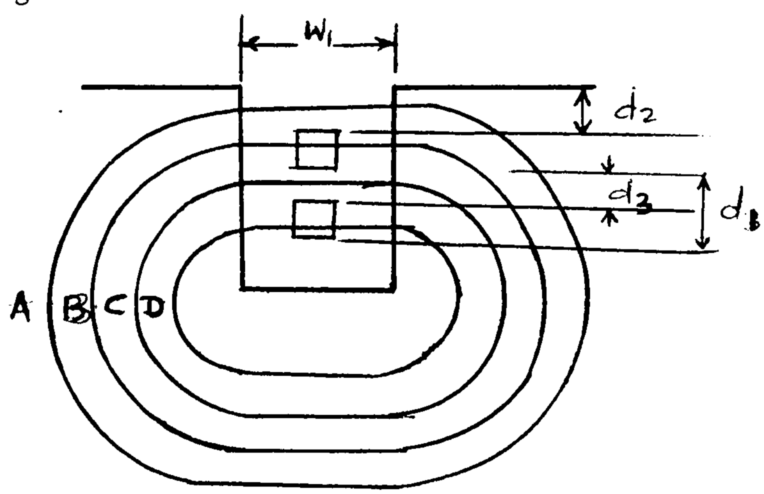


Fig. II-1

Consider the leakage reactance of the d. c. machine armature conductors embedded in a slot. The magnetomotive force around path A is:

Current in the slot = $\frac{I_a Z}{s}$

where I_a = armature current

Z = Total number of armature conductors on winding

s = Total number of armature slots

Assuming the permeance of the iron to be infinite:

Permeance around path A is $\mu_o l \frac{d_2}{w_1}$ where,

l = stacked length of rotor (meter)

d_2 = depth from top of the armature tooth to the top of copper (meter)

and w_1 = armature slot width (meter)

Therefore, flux per slot around path A

$$= \frac{Z}{s} \cdot I_a \cdot \mu_o \cdot 1 \frac{d_2}{w_1} \quad \text{(weber).}$$

This flux links all the conductors in the slot.

Therefore, linkage $\lambda_A = \mu_o \cdot (\frac{Z}{s})^2 \cdot \frac{I_a}{w_1} \cdot 1. d_2$

For path C, permeance is $\mu_o \cdot \frac{d_3 1}{w_1}$, where

d_3 = separation between top and bottom armature conductors (meter)

For path C, the flux is produced by half of the current. Therefore,

$$\Phi_c = \frac{Z}{s} \cdot \frac{I_a}{2} \cdot \mu_o \frac{d_3 1}{w_1} \quad \text{(weber)}$$

This flux links with half the conductors in the slot, and therefore,

$$\lambda_c = (\frac{Z}{s})^2 \frac{I_a}{4} \quad \mu_o \quad \frac{d_3 1}{w_1} .$$

In paths B and D, the mmf is linear and the linkages are parabolic. The average value will be one-third of the maximum linkages having a parabolic distribution along the slot. Then,

$$\lambda_{B,D} = \frac{\mu_o . Z^2 . 1. (d_1 - d_3). I_a}{3s^2 w_1}, \quad \text{where}$$

d_1 = total depth of the armature copper in slot (meter)

In one parallel circuit, there are s/a_a slots in series where a_a = number of parallel paths in the armature.

Hence, total flux-linkages per circuit is:-

$$\lambda_w = \frac{\mu_o . A^2 . 1. I_a}{s^2 . a_a . w_1} \quad [d_2 + \frac{d_3}{4} + \frac{d_1 - d_3}{3}] \quad \text{(weber-turns)}$$

Inductance of the circuit then, is $\frac{\lambda_w}{I_a}$, or the inductance of a_a parallel circuits is:-

$$L_a = \frac{\mu_o . z^2 . 1}{s^2 . a_a} \quad [d_2 + \frac{d_3}{4} + \frac{d_1 - d_3}{3}] \quad \text{(henry)}$$

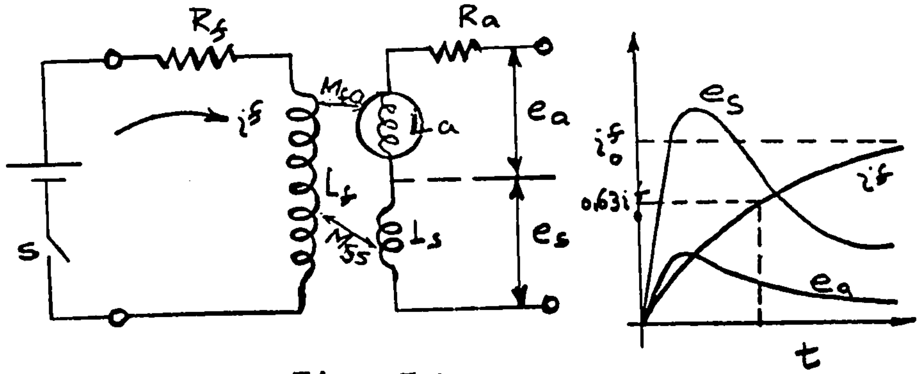
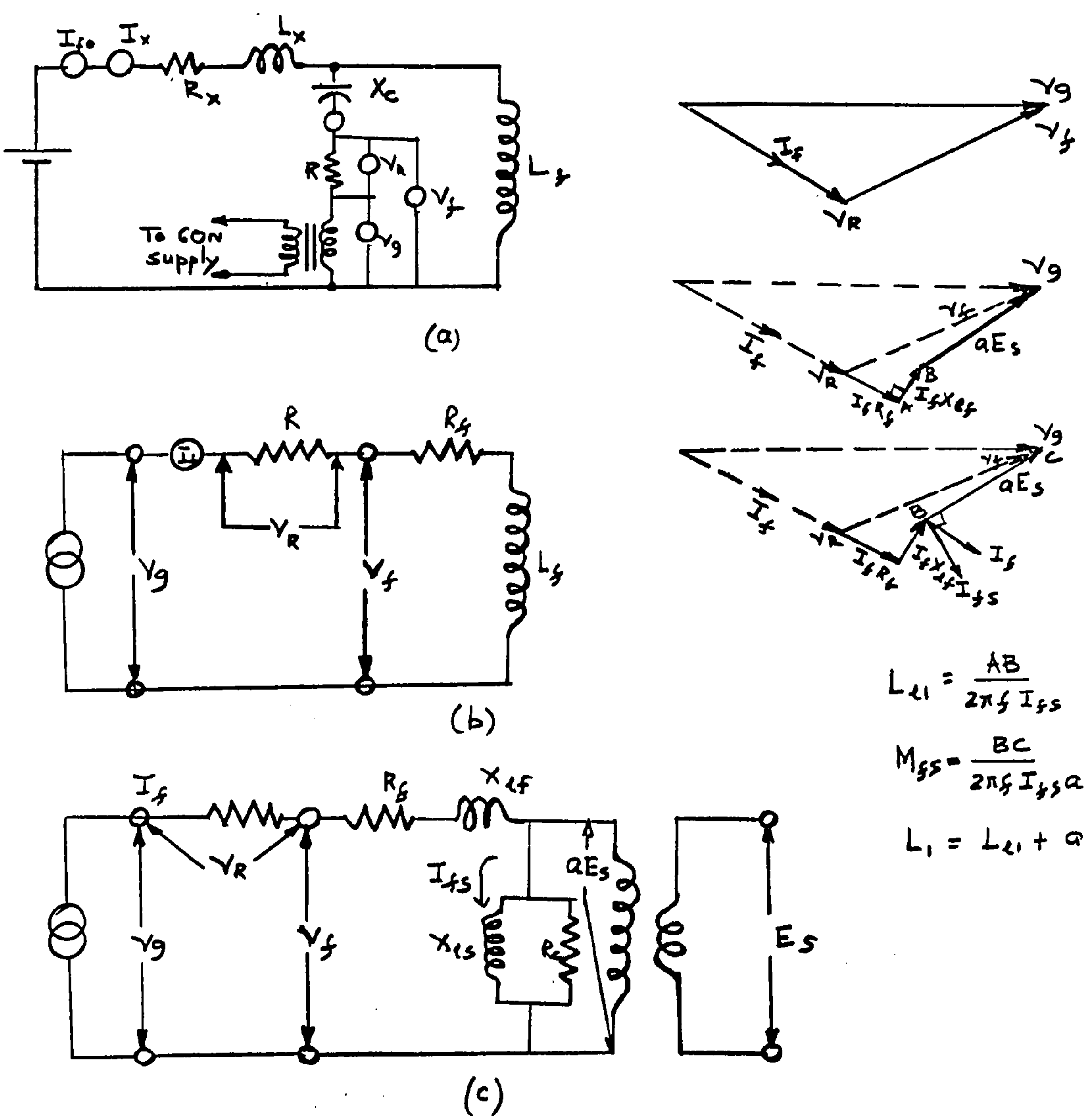


Fig II-2



$$L_{l1} = \frac{AB}{2\pi f I_{fs}}$$

$$M_{fs} = \frac{BC}{2\pi f I_{fs} a}$$

$$L_1 = L_{l1} + a M_{fs}$$

Fig II-3

The problem in the above method is greatly simplified by assuming infinite permeability in the iron path, uniform air gaps, linearity and the application of the theory of superposition. It is stated in reference (7), that the difference in the calculated and measured values is 12% maximum.

(III) Method suggested by Saunders[2]:-

For small currents Saunders has suggested a useful a.c. method, primarily based on transformer theory. A d.c. machine can be considered as a black-box with six terminals. Two methods are described by Saunders:

(i) transient method, originally suggested by Bowers

(ii) a.c. method suggested by Saunders

In the transient method, oscillograms are taken when switch s in fig. II-2 is closed. From the oscillograms, the following calculations can be made:

$$R_f = \frac{E_1}{I_{10}} \quad \text{and since} \quad \frac{L_f}{R_f} = T_f, \quad L_f \text{ can be calculated.}$$

The value M_{fs} can be calculated at any convenient point t_1 by taking the slope of the shunt-field current curve from which:

$$M_{fs} = \frac{e_s}{\left.\frac{d_i}{dt}\right|_{t_1}}$$

Similarly the test can be carried out on the series field and armature circuits. The transient method is laborious in that the interpretation of the oscillograms consumes a considerable time. Also, it is difficult to associate the value of the inductance found by this method with the degree of saturation because the flux during this period is produced not only by the actual coil magnetomotive force but by the resultant of this and eddy-current m.m.fs.

In the a.c. method, alternating current of either low or power frequencies instead of direct current (as employed by Bower) is used. Fig. (II-3) shows the development of the equivalent circuits and the respective vector diagrams. In circuit (a) the direct-current is superimposed while in (b) it is not. In the former, the effect of saturation is taken into account. It should be noted that in the development of vector diagrams, X_c is small compared to R and circuit (a) may be replaced by circuit (b).

Similar tests can be made for the series field and the armature circuit. It is recommended that when measurements are made, the armature is made to rotate at, say, 10% of the rated speed, so that some of the effects of commutation are included. The values obtained by this method may vary by 25% if the unsaturated values of the inductance are measured. However this seems to be the only a.c. method that can be conveniently used to measure all the inductances, self and mutual, of the d.c. machine.

Saunders' method measures the inductance offered to alternating current and he has used the method for servo-motors and motors used in instrumentation circuits.

(IV) <u>A.C. Bridges:-</u>

A.C. bridges can be conveniently used for the measurement of resistance, inductance or capacitance. Most of the bridges can be set up in the laboratory using a 60 c.p.s. supply. Hay's and Maxwell's bridges are especially useful for this purpose. If direct current is superimposed, the incremental inductance is measured. Selection of an appropriate bridge depends upon the amount of inductance involved in the circuit.

(V) <u>D.C. Methods:-</u>

Jones[4], Prescott and El-Kharashi[9] and Barton[8] have used this method employing direct current for the measurement.

Fundamentally, all three d.c. methods are similar: they differ in the device employed to measure the flux-linkages. The principle involved will be clear from the following: For two circuits (1) and (2), the e.m.f. induced in circuit (1) by a change in current i^2 in circuit (2) is given by:

$$e_1 = \frac{d}{dt}(M_{12}i^2)$$

(The index notation giving currents superscripts and voltages subscripts is used here). During a period from t_1 to t_2,

$$\int_{t_1}^{t_2} e_1 \, dt = (M_{12}i^{2''}) - (M_{12}i^{2''} - i^{2'}), \text{ where the}$$

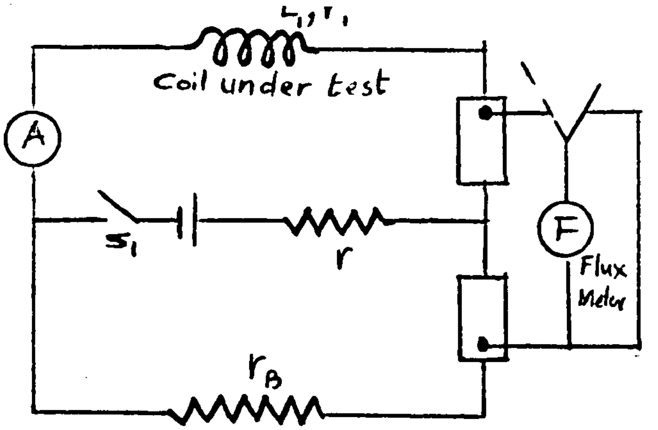
current changes from $i^{2'}$ to $i^{2''}$ during the period in question. If the current reverses from $-I^2$ to I^2, then

$$\int e_1 \, dt = 2M_{12}I^2$$

$$\text{i.e. } M_{12} = \frac{\int e_1 dt}{2I^2}$$

The integration can be carried out by a flux-meter, a ballistic galvanometer or an electronic integrator. The circuits are shown in Fig. II-4, 5 and 6.

These d.c. methods measure the inductance offered to direct current. They have been widely employed by Prescott, Barton and C.V. Jones for machines of almost all ratings. Prescott's method is particularly suitable for large inductances. In passing, it may be pointed out that, although the measurement of mutual inductance by these methods is straight-forward, that of self-inductance is involved due to the resistive voltage drop across the coil. The difficulty can be overcome by employing a bridge which should be balanced before the integration is performed. The theory of the flux-metric bridge can be found in Appendix I.

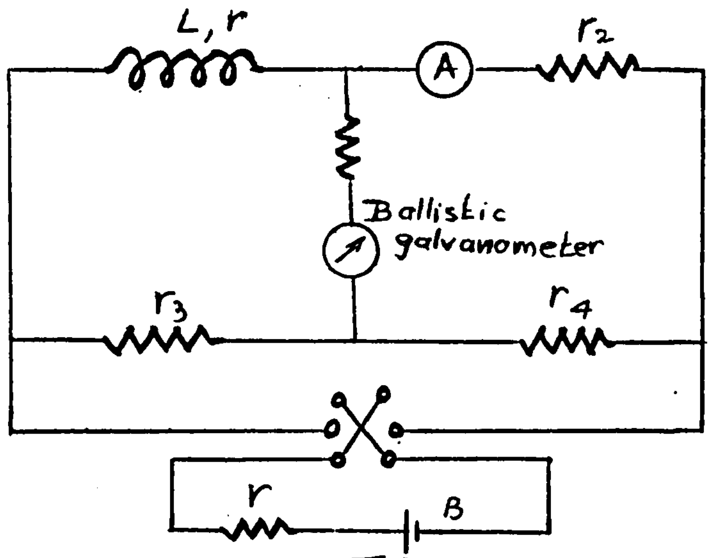


$$L_1 = \frac{\Delta \Phi}{\Delta I} = \frac{\Phi_0 - \Phi_1}{I_0 - I_1}$$

Φ_0 = Initial flux in coil
Φ_1 = Final " " "
I_1 = Current flowing in the coil 'A' at tir. t_1 after opening S_1

Fig II-4

Method used by Proscott and El-Kharashi



$$L = \frac{1}{2} \frac{\lambda}{I} \frac{r_3 + r_4}{r_4}$$

Fig. II-5

Methods used by C.V. Jones

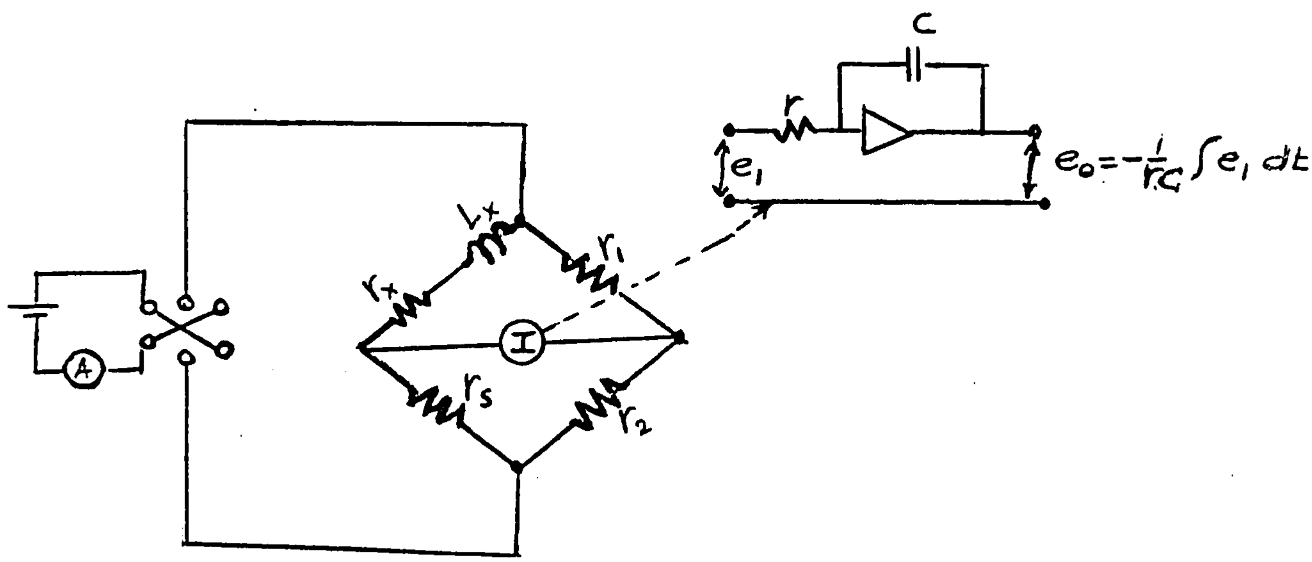


Fig. II-6

Method used by Barton

(VI) Numerical Method:-

This method uses Maxwell's equations and the approach is of a more theoretical nature and the method is practically inconvenient. Reference 6 discusses the method in detail.

(VII) Measurement of Impure Mutual Inductance:-

An impure mutual inductance is such that the voltages induced in the secondary are not in exact quadrature with the primary current. This may be due to capacitance and eddy-current effects. The method suggested by Hartshorn overcomes this difficulty and gives a measure of the impurity[19]. Most reference books on electrical measurements give details of the method.

Having made a general survey of the existing methods, the following comments are made: It seems that the a.c. bridge methods are most commonly used due to the simplicity of setting up the circuits and the good agreement of the results with those obtained by other methods. Most a.c. bridges are commercially available and if the accuracy of the results obtained using these bridges is tolerable, their use is certainly preferable to more sophisticated methods which require special equipment and also an experienced hand. Some parameters can be obtained by carrying out various tests on the machine; for instance, the direct-axis self-inductance of an alternator can be obtained from the short-circuit test. As observed by Lynn and Walshaw[11], the results obtained from this test are comparable with those obtained by more sophisticated methods such as the one employing the flux-metric bridge.

However, the simplest method seems to be the a.c. impedence test. References 7 and 10 include results obtained for different machines of different ratings using this method. If saturation is neglected during the test, the difference can be of the order of 25% as it was for Saunders' method. It is essential however, that direct-current should always be superimposed if conditions due to saturation are to be simulated.

D.C. methods suggested in references 1, 4 and 8 are very useful but require special equipment: the changes in flux must be measured accurately and the setting up of the bridge is laborious. The effects of inactive coupled circuits are avoided as the results depend only upon the initial and final steady-states.

Prescott[1] suggests that his method is to be used for large inductances carrying large currents. He utilises the method for the measurement of parameters of a large synchronous machine. C.V. Jones[4] utilises the method to measure the parameters of a d.c. machine while Barton[8] has used his method for the measurement of the parameters of small and medium-sized direct-current, synchronous and generalized machines.

Methods of measuring the inductance of a coil appropriate for analysing the conditions prevailing when a step voltage is applied to an unexcited coil, when alternating current is applied to an unexcited coil, or when a component of alternating-current is superimposed upon direct-current excitation, are typified by Bower's transient method, the a.c. impedance method and Saunders' method respectively. Alternative methods are available for each of the three conditions and a desirable conclusion to this work would have been to recommend one method for each of the three working conditions. Unfortunately this is impossible due to the wide range of inductance values encountered even in a given machine, the shunt-field inductance always being of a much higher order of value than the armature or interpole coils. The unpredicable hysteresis and brush-contact effects also introduce a source of error between analytical and experimental results.

For a quick review of the existing methods to measure the coefficients of inductance, a matrix is shown in Table II-1.

TABLE II-1

Inductance	Shunt	Series	Armature	Interpoles	Mutual	Remarks
Transient Inductance						
(a) Bower's Method	X	X	X	X	X	Accurate but Laborious.
(b) D.C. Methods: (i) Prescott's (ii) Jones' (iii) Barton's	X	X	X	X	X	Require special equipment e.g. accurate ballistic galvanometer or an integrator. Balancing of bridge requires an experienced hand.
Incremental Inductance						
(a) Saunder's	X	X	X	X	X	Approximate. Only a.c. method available to measure all the inductances of a d.c. machine.
(b) Hay's Bridge	X	X	X			Simple but limited application. Can be easily simulated with common laboratory equipment
(c) Resonance method	X					Simple. Applicable to large inductance values only. No special equipment required.

CHAPTER III

Measurement of the Coefficients of Self and Mutual Inductances and the Moment of Inertia of a d.c. Machine.

Measurement of Coefficients of Inductance:-

Various methods of measuring the coefficients of self and mutual inductance have been described. The main alternatives were those methods using (i) an a.c. source and (ii) a d.c. source. Initially, the application of d.c. methods to obtain the values of inductance under steady-state and transient d.c. conditions was investigated. It was found that the d.c. methods with a ballistic galvanometer involved the use of a low-resistance in two of the four bridge arms. This increased the power requirements and needed a ballistic galvanometer of high accuracy. Barton's method of replacing the ballistic galvanometer by an electronic integrator was favoured but it was decided that a different approach would be taken.

By employing a.c. bridges for the measurements it was expected that the use of a 60 c.p.s. single-phase source would make the measured values approximate, but the main object was to make accurate measurements and to substitute measured values in the theoretical equations describing the transient response. The characteristics thus obtained by substituting these a.c. values were compared with those obtained under actual d.c. conditions. As can be seen later, the deviation between the two characteristics is quite small. Moreover, the values of inductance obtained using an a.c. bridge with superimposed direct current are the incremental inductances. In other words, these values would make the relationship $e = L \frac{di}{dt}$ true for small variations of currents about specified working conditions, thus making possible the solution of nonlinear equations by the theory of small perturbations. It is also emphasized that these 60 c.p.s. methods are simpler than the other methods.

The machine under test is a 4-pole, 3kW compound d.c. machine having a simplex wave-wound armature winding and being equipped with two interpoles. The rated full-load armature current is 21 A, the rated field current is 0.7A and the rated speed is 1750 r.p.m. There are 95 commutator segments, 48 armature slots, 2 conductors per slot with one dummy winding and 2 pairs of carbon brushes.

(1) Shunt-Field Inductance:-

The measurement of the self-inductance of the shunt-field is quite involved, because this coil is magnetically coupled to some armature coils and to the series field. Saturation in the main poles makes this quantity nonlinear in nature, making it necessary to measure the inductance as a function of field current.

An initial estimate of the value of inductance involved was made using the a.c. impedance method described by Sniveley and Robinson[7]. This involves the measurement of the a.c. impedance between the field terminals of the stationary d.c. machine by impressing an alternating voltage and measuring the resulting alternating current. Direct current is passed through the field winding to simulate the working conditions with saturation. The value obtained was in the region of 18H to 10H for a direct current range of 0 - 1.6A.

With this estimation of the shunt field inductance and knowing the value of the resistance of the shunt-field, a choice was made as to which a.c. bridge should be employed. Hay's bridge was chosen for two reasons: (1) the 'Q' of the circuit is high as also are the hysteresis losses. Hay's bridge is particularly suitable for the purpose[3,19,20,21], (2) Direct current can be superimposed conveniently upon alternating current for this bridge. The arrangement of the bridge is shown in fig. (III-1).

The a.c. source for the bridge was 110 volts 60 c.p.s. fed through a Variac and the value of alternating current was 3mA. The direct current was superimposed as shown. As can be seen its path is restricted to arms I and III. Hence R_3 must be of a sufficient rating. R_2 consists of two variable resistance, one a coarse and the other a fine resistance box which can be varied in steps of 10 x 10000 ohms: R_4 is an ordinary rheostat. Two capacitance boxes are used so that increments of 0.001μF can be obtained. The detector used is a cathode-ray oscilloscope and when the bridge is balanced, no 60 c/s alternating current shows. Due to the presence of harmonics, the balance is a little difficult but can easily be realized after a little experience. Final balance can be obtained by varying R_3 and C.

The theory of the bridge is given in any book on Measurements. (Ref: 19, 20, 21.). At balance, L_f the shunt field inductance, is given by

$$L_f = \frac{R_2 R_3 C}{1 + \omega^2 R_4^2 C^2}$$

The values of R_4, C and ω are such that the second term of the denominator was found to be negligible for all the observations. For example, with $R_4 = 341$, $C = 10\mu F$, $\omega = 377$, $(\omega^2 C^2 R_4^2) = .00162$ which is negligible. Thus,

$L_f = R_2 R_3 C$, and the final result is apparently independent of frequency which is an advantage of Hay's bridge.

To study the effect of hysteresis, the d.c. field current was decreased in steps after reaching a maximum value: the polarities were then reversed to get the negative loop.

It is desirable that the value of the transient inductance measured should correspond to the actual working conditions of the machine. Hence, the same circuit was used to measure the inductance as a function of frequency, a step voltage being equivalent to a series containing a large number of frequency components. The variable frequency supply was obtained by driving a wound-rotor induction machine by a d.c. motor. When the 3-phase balanced supply is fed to the stator of the machine, the rotor generates a variable frequency supply, depending upon the speed of the driving d.c. machine. The alternating current was maintained at 3mA and three different sets of readings for direct current were taken, (i) at zero current (ii) at 0.3A. (iii) at 0.6A. The operation of the bridge was the same as before. Care had to be taken for readings below 25 cycles per second due to mechanical oscillations of the machine set disturbing the output.

The phenomenon of resonance was also utilized to measure the self-inductance of the shunt field coil. This is especially suitable since the impedance of the shunt-field circuit is quite high but high resistance instruments must be employed. At resonance, $L = \frac{1}{\omega^2 C}$. Here w = 377; hence $L_f = 7.04/C$ where C is in μF. At resonance, the impedance of the circuit is purely resistive. This resistance is large in the case of the shunt field, viz. $Z_{resonance} = \frac{1}{CR} = \frac{16}{0.35 \times 10^6 \times 128} =$ 35.2 megohms. Instead of using a bridge circuit, the circuit shown in fig. (III-2) was used. This circuit is not to be balanced, but resonance is realized by observing the minimum reading of ammeter A_1. The direct current is superimposed to obtain the shunt field inductance as a function of field current. Due to the high impedance of the shunt field it was decided to use a 400-volt a.c.

supply at 60 c.p.s. For this purpose three 3KVA, 220/120: 120V wound-core transformers were used. As direct current would flow through the transformer secondary windings, it was necessary to avoid d.c. magnetisation of the core. To avoid this the transformer secondaries were connected in the manner shown in fig. (III-3) where it can be seen that the direct current flows in opposite directions in the two half-secondaries of each transformer thus producing a net effect of zero magnetisation.

However, care must be taken to ensure that the direct current flowing through R_1 and R_2 is the same. This is necessary to achieve balanced d-c ampere-turns in the two half-secondaries. The capacitor was of the type that can be varied in steps from 0.001 x 10 μF to 0.1 x 10μF. The direct current was increased to 1.6A and this required a source of 220 volts because the resistance of the shunt-field itself was 128 ohms.

With these precautions, this simpler method gives results comparable to those obtained using Hay's bridge. To the writer's knowledge, this method is used for the first time for this purpose and is strikingly simple.

The effect of hysteresis was again studied by reducing the field current in steps from the maximum value, the negative loop being obtained by reversing the polarity of the d-c supply.

(2) <u>Self Inductance of the Aramature and Interpole:-</u>

The a.c. impedance test, as described initially in the previous case, was again carried out to ascertain the approximate value of the inductance involved. As interpoles are always electrically connected in series with the main armature winding, the combined inductance of the two windings was measured. As is known, the resistance of the armature is nonlinear in nature due to the brush and contact drop. The value of the

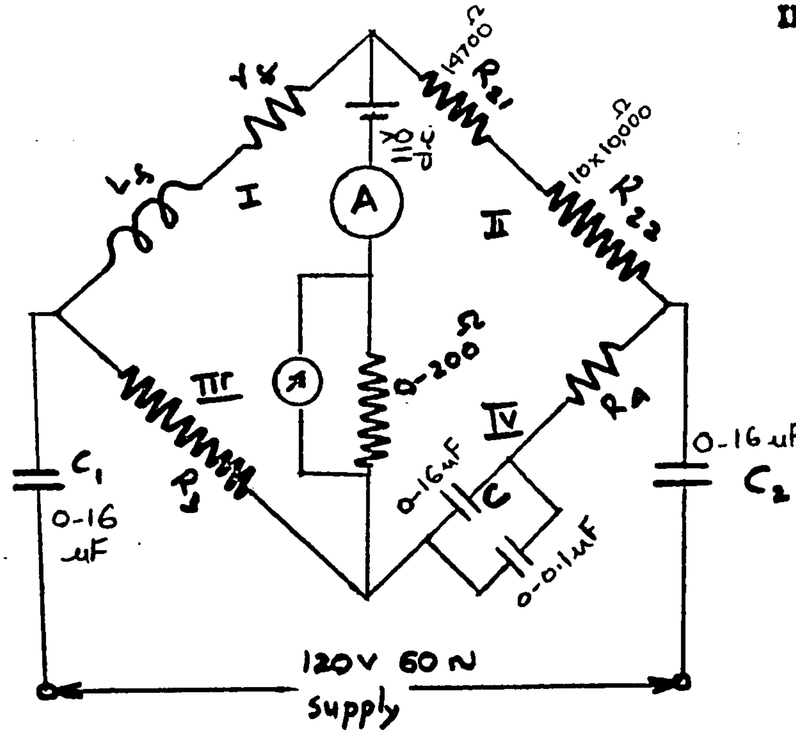


Fig. III-1

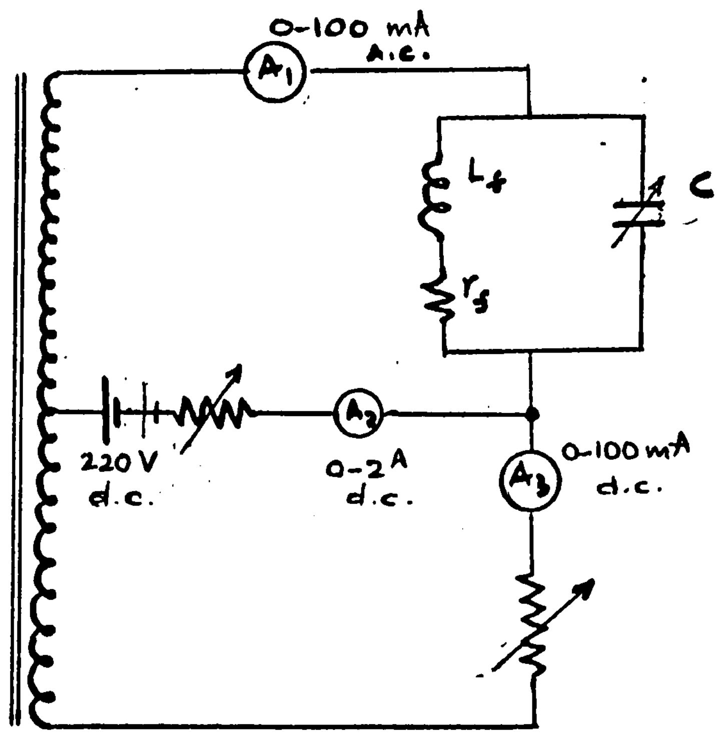


Fig. III-2

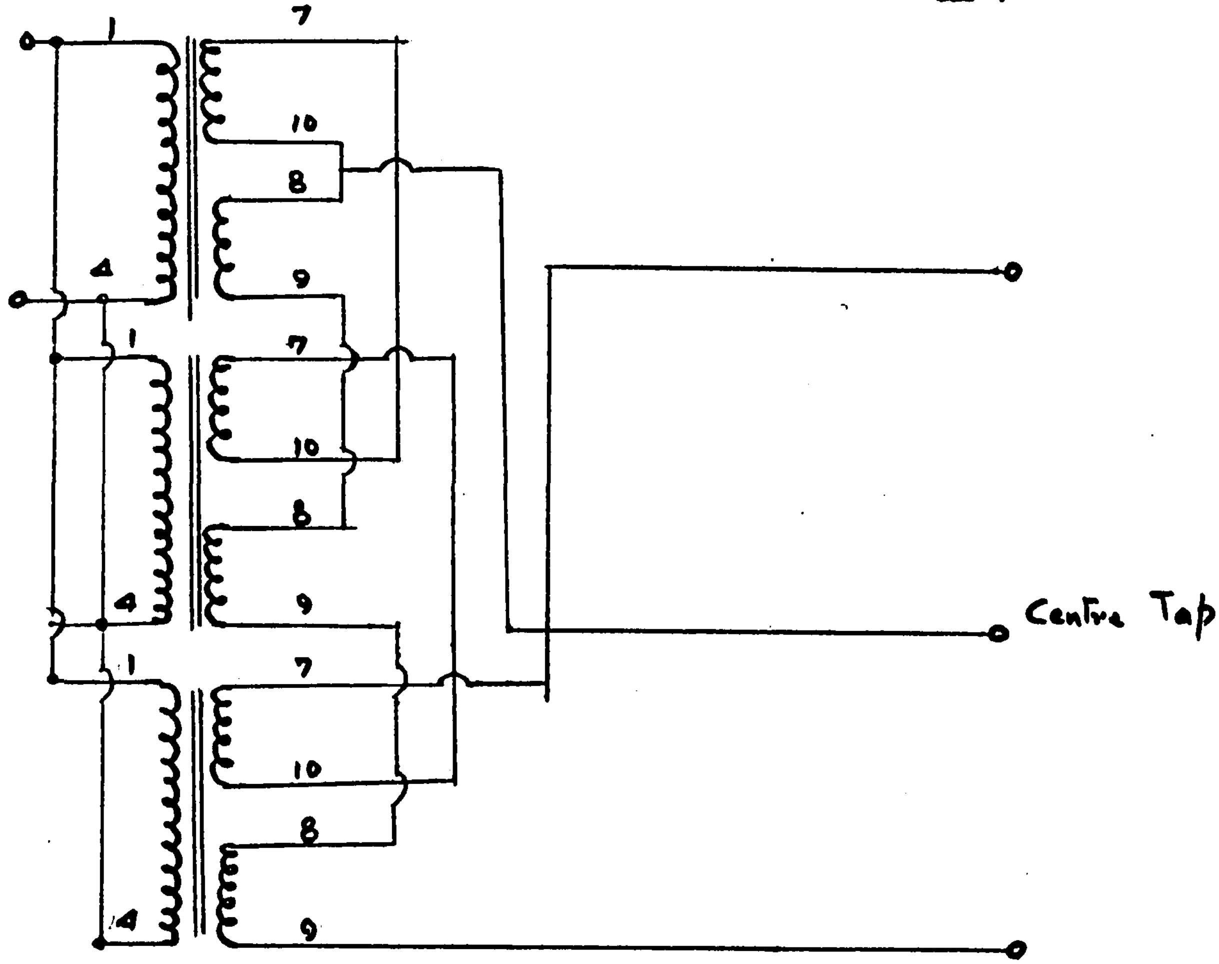


Fig. III-3

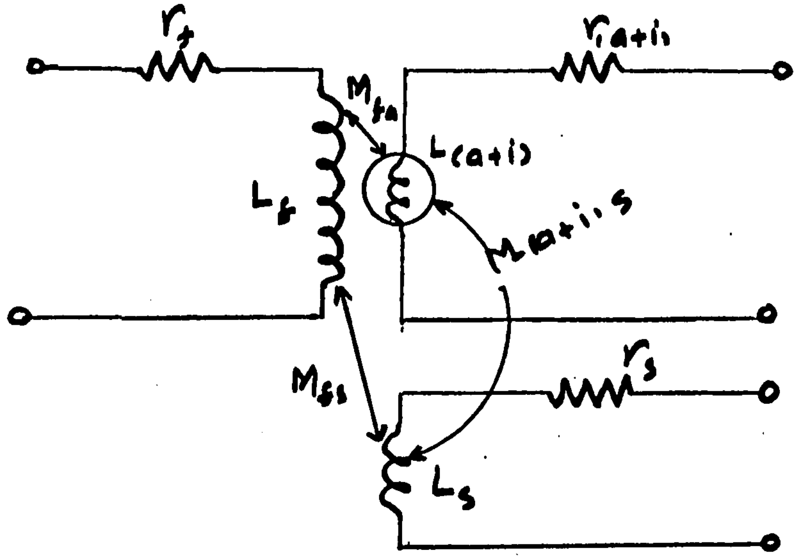


Fig. III-4

resistance substituted to evaluate the reactance from the impedance, is taken from the armature resistance current characteristics.

Both Maxwell's bridge and Hay's bridge were tried but Hay's Bridge was preferred because the Q of this circuit is of the order of 7 to 10. Unfortunately, the resonance method cannot be used here for it requires a large capacitance due to the low value of the armature inductance. With a typical value of armature inductance equal to 8.5 mH, $C = \dfrac{1 \times 10^3}{3.77 \times 8.5 \times 377} = 830 \mu F$ which is an awkwardly large capacitance.

Direct current is not superimposed in this case for several reasons. The armature self-inductance is independent of its own current; it consists largely of leakage inductance. Also, the current in the armature conductors is alternating and the wave-form is not sinusoidal, but not a very large error is introduced by assuming it to be so due to the fact of it being a distributed winding. In addition this inductance has a cyclic variation with rotor displacement, due to pole saliency.

One point with respect to these measurements is that as the value of the armature self-inductance was quite small, care had to be taken to account for stray inductances, as well as the inductances of the bridge components themselves. To minimize these effects, the values of the rheostats R_2 and R_3 used in opposite arms were always kept the same. It was to be expected that at 60 cycles per second, the effect of stray inductances would be negligible, as also would be the effect of capacitance to earth. The inductances of the rheostats and capacitor boxes used as the bridge components were separately measured and the highest value was found to be less than 100μH. This is neglected for the final calculation of the results. The term $\omega^2 C^2 R_4^2$ in the denominator of the expression for 'L_a' using Hay's Bridge was again found to be small and neglected. The results, therefore, are apparently independent of frequency.

The armature self-inductance is a function of the position of the rotor-axis and the variation is cyclic, approximating to a sine-series, Hay's bridge was used for the measurement and solid connections were made on the commutator to avoid the effects of brush and contact drop. There are 95 commutator segments: hence 1 commutator segment covers $\frac{360}{95}$ geometric degrees. The numbers on the x-axis of this characteristic (V-29) measure the displacement of the armature in terms of commutator segments.

An interesting point was to study the effect of the presence of shunt field current on the value of the armature quadrature-axis self-inductance, a phenomenon investigated by Jones[4]. For this purpose, the field current was increased from zero to 0.7A. for three different values of armature current, 1A, 3A and 5A. It seems that the effect of the presence of field current is to reduce the value of the armature self-inductance which confirms the findings of Jones.

(3) Measurement of Self-Inductance of the Series Field:-

The self-inductance of the series field is very low, as expected. An A.C. impedance test was first carried out. As the Q of this circuit is rather large $\frac{(3.5 \times 377 \times 10^3)}{.075} = 17.2$. Hay's Bridge was again preferred for this measurement. Stray inductances as well as the self-inductances of the bridge components again had to be considered but were found to be negligibly small. Actually, the series field inductance can be estimated from the value of the shunt field inductance by the relation: $\frac{L_f}{L_{series}} = \left(\frac{T_f}{T_{series}}\right)^2$

The actual tests on the machine were performed with a shunt connection but the series field inductance was measured to study its similarity in variation with that of the shunt field.

(4) Measurement of Mutual Inductance:-

As suggested by Saunders[2] the d.c. machine can be regarded as the equivalent circuit shown in fig. (III-4). The various magnetic couplings between the circuits can be taken to be represented by mutual inductances. Transformer theory can be applied to measure the values of these mutual inductances. In all cases, the armature is taken as the primary winding, mainly for the reason that currents from zero to 20A can flow through it. The value of mutual inductance can be determined from the fact that when the primary of a transformer is excited, the e.m.f. induced in the secondary is $M \frac{di}{dt}$, where M is the mutual inductance between the two circuits.

The mutual inductance between the shunt field and the armature and interpole winding is of particular interest. The quantity M'_{fa} in the impedance matrix [Appendix I equ. (1)] can also be determined from the open-circuit characteristics of the machine run as a generator from the relation: $E\text{ (generated)} = M'_d p\theta$. Where $p\theta$ = steady state speed, I^f = field current (steady state). If now, this mutual inductance is measured as a function of the field current, the two values differ due to the effect of non-sinusoidal flux distribution.

The quantity, M_{af}, is also a function of rotor position i.e. it is cyclic. This variation was investigated as in the case of the self-inductance of the armature and as expected, the value increased from zero to a maximum.

For the measurement of the mutual inductance, the effect of carbon brushes was studied. In one case, the armature current was supplied through the carbon brushes: in another, solid connections were made on the commutator by drilling holes in the commutator segments.

While doing the above experiments, readings were taken so that the mutual inductances between series and shunt fields and between series field and armature could be measured simultaneously. This was done by noting the voltmeter readings across the series and shunt windings. As the impedance of the shunt-field was quite high, high-resistance voltmeters were used for the voltage measurements.

(5) Measurement of Moment of Inertia:-

Three tests were made to estimate this constant. They were (a) the retardation test (b) measurement from losses (c) mechanical test.

(a) Retardation test:-

This test is especially suitable for large machines. It was found to be inaccurate and gave ambiguous results for the present machine, which is small and takes very little time to come to rest. For this test, a graph of speed against time was drawn for each of three different retardation conditions:-

(1) When the machine was running at rated speed (taken as 1800 rpm), both armature and field circuits were simultaneously opened. The machine was allowed to retard due to the mechanical losses only.

If the slope of the graph under this condition is $\frac{dn_1}{dt}$, then,

Mechanical losses = $1.67\, n\, J \frac{dn_1}{dt}$ watts where n = speed in r.p.s.

(2) The machine was allowed to retard after opening the armature circuit only. If the slope of this graph is $\frac{dn_2}{dt}$, (Mechanical + iron losses) = $1.67\, n\, J\, \frac{dn_2}{dt}$.

(3) The machine was allowed to retard after opening the armature circuit switch alone but the armature e.m.f. as used to supply a known load which was measured by means of a wattmeter. If the slope of this graph is dn_3/dt, then

Load power + (iron + mechanical + small armature copper) losses

$$= 1.67\, nJ \frac{dn_3}{dt}$$

Since the load power is measured by the wattmeter, (wattmeter reading + small armature copper losses) = $1.67\, nJ \left(\frac{dn_3}{dt} - \frac{dn_2}{dt} \right)$

$$\therefore \quad J = \frac{\text{Wattmeter reading + Armature Copper loss}}{1.67n\left(\frac{dn_3}{dt} - \frac{dn_2}{dt} \right)}$$

Instead of a wattmeter, a pen recorder was used with two channels to record voltage and current respectively.

The main difficulty is that the characteristics, particularly in the third case, are nonlinear making it difficult to estimate the value of J, which is a constant.

(b) By Measuring Iron losses:-

As described in case (2) above, a retardation curve was obtained. Also the iron losses were measured (by a separate test in which the motor was run at various speeds with constant excitation) using a pen-recorder and these were plotted against time. The kinetic energy $\frac{1}{2} J\omega^2$, where ω is the angular speed of the rotor is lost in mechanical and iron losses. If the area under the curve of iron loss versus time is calculated, the value of 'J' may be obtained. For different speeds 'J' was found to remain constant which was an improvement over the previous method.

(c) Mechanical test:-

This is considered to be the most accurate of the three methods and the value obtained by this method is taken for the final results.

For this test, a small peg was mounted on the rim of the flywheel attached to the shaft of the rotor. Over this peg a loop at the end of a length of string, to which a weight was attached, was hooked. The weight was dropped through a known height and the number of revolutions that the flywheel made after the string left the peg was measured, and also the time taken for the wheel to come to rest. The initial angular velocity was also measured. This is the velocity at the instant the string leaves the peg. This measurement must be very accurate and is necessary because the retardation is not uniform. Then, the moment of inertia can be calculated from the following relation:-

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} \omega^2 (1 + \frac{n_1}{n_2})$$

where m = mass of the weight,

h = height through which the weight is dropped,

ω = initial angular velocity,

n_1 = no. of turns required to wind up the weight to the starting point,

n_2 = revolutions made after the string leaves the peg.

The above equation can be derived as follows:-

The kinetic energy of the flywheel at the instant the string leaves the peg = $\frac{1}{2} J\omega^2$. All this energy is used to drive the rotor against windage and frictional forces so that n_2 revolutions are completed.

$$\therefore \quad \frac{1}{2} I\omega^2 = n_2 f$$

where f = work done against frictional force per revolution of rotor i.e. a constant average force is assumed to act during the period of retardation.

i.e. $f = \frac{1}{2} I \frac{\omega^2}{n_2}$

Equating potential energy to the kinetic energies of the weight and the rotor and to the energy lost in friction

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} J \omega^2 (1 + \frac{n_1}{n_2})$$

The moment of inertia obtained by this method was 3.73 lb. ft.^{2} or 0.1518 kg-m^2 as compared to 3.01 lb. ft.^{2} obtained by the second method.

CHAPTER IV

DISCUSSION OF THE MEASURED PARAMETERS AND RESULTS.

The three previous chapters have outlined the concept of inductance, the methods available and those employed for the measurement of the coefficients of inductance. In this chapter, the behaviour of the parameters as measured by the methods mentioned in the previous chapter, is discussed. Two other features are also presented:-

(i) Effect of carbon brushes on the value of the inductance.

(ii) Analysis of the process of commutation from the unified theory.

(1) Shunt-field inductance:-

The shunt-field inductance is a function of its own current. This is to be expected as the shunt field is wound on the main-poles. The magnetization curve gives an indirect indication that this inductance is a function of the exciting current. The variation of this inductance with the field current is shown on pages V-8, V-12 and V-13, and the magnetization curve is on page V-3.

The characteristics on page V-8 and V-12 show that the inductance rises initially as the field current is increased from zero to 0.3A, and then falls for the higher values of field current. Unfortunately, the law that this variation obeys cannot be derived easily as, due to the presence of iron, the flux is not proportional to the exciting current. However, this variation can be explained from the magnetization curve on page V-3, the incremental inductance being proportional to the slope of this curve which increases over the region from zero to 0.3A and then decreases gradually. Leakage flux is ignored for the purpose of this explanation.

Both Hay's bridge and the resonance method were employed; the resonance method is simpler, requires less equipment and the results are comparable to those obtained using Hay's bridge. The characteristics obtained by the two methods are plotted on page V-13.

The maximum difference in the two values is $3\frac{1}{2}\%$ of the maximum value of the shunt field inductance. No particular reasons can be given to account for the difference but the fact that Hay's bridge gives results independent of frequency while the resonance method depends upon frequency may be noted. However, the simplicity of the resonance method commends its use over Hay's bridge; but for circuits involving low inductance values, the resonance method cannot be conveniently applied at 60 c.p.s. .

The inductance thus measured is an incremental one. Two definitions of the incremental inductance are stated for the purpose of this work:-

(1) "When part of the magnetic circuit is in iron, the relationship between the flux and the exciting current is nonlinear. But it is possible to consider the relationship $e = L\frac{di}{dt}$ or $e = M\frac{di}{dt}$ to be true for small variations of the current about the specified values, in which case 'L' and 'M' are the incremental inductance with respect to these specified conditions"[18].

(2) "When a small a.c. magnetization is superimposed upon a large d.c. magnetization, the inductance offered to the small alternating current is called the incremental inductance"[26].

The incremental inductance depends upon the magnitude of both a.c. and d.c. magnetization and upon the magnetic history of the core. The effect is the presence of a minor hysteresis loop on the usual hysteresis loop, as shown in fig. IV-1[26]. A comment on the butterfly shape of the minor loop seems to be necessary. In general, in dealing with incremental inductance one must distinguish between the consequences of superimposing an incremental d.c. magnetization and an incremental a.c. magnetization. In the former case it is also necessary to distinguish between the effects of a positive polarity increment and of a negative polarity increment; each increment gives a loop but one loop slopes upward and the other downward, as in figure IV-1, in which the butterfly shape comes from the fact that both positive and negative increments of magnetization are illustrated. The loops, however, are not drawn to be mathematically perfect, but to illustrate the general idea of incremental inductance. The eight individual loops, however, are roughly elliptical in shape. The incremental permeability and hence the incremental inductance is proportional to the slope of the line joining the tips of this loop.

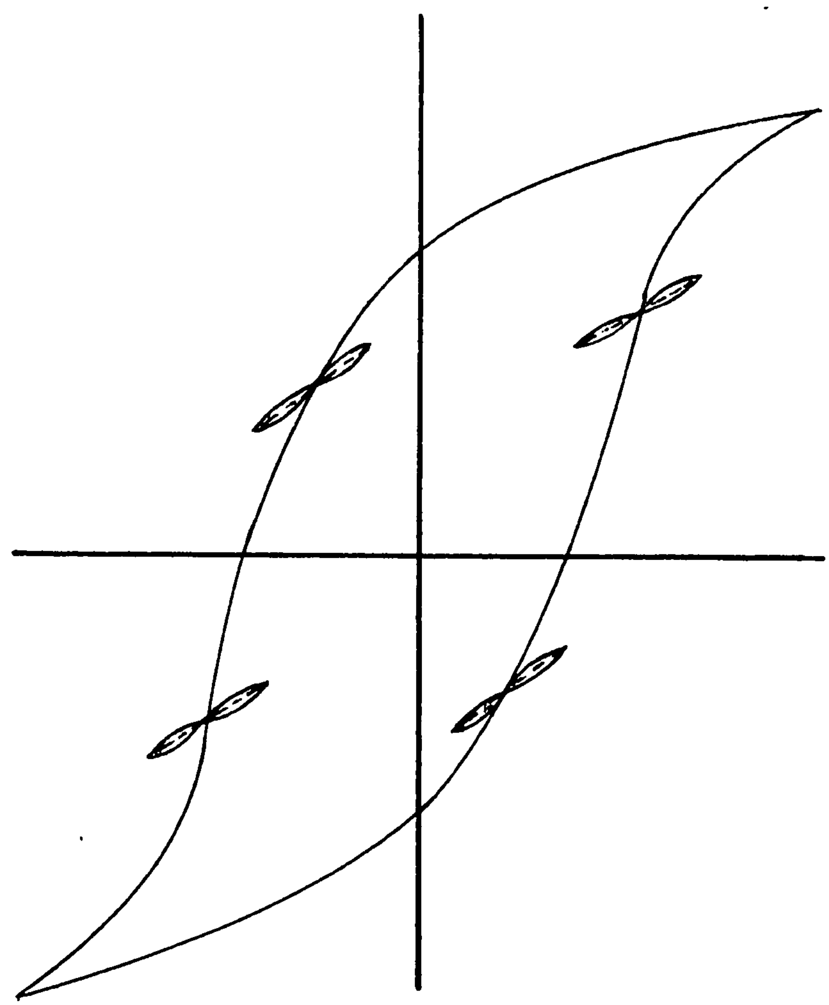


Figure IV-1.

An interesting point was to study the effect of hysteresis upon the nature and value of the shunt field inductance. The magnetization or ϕ - I curve, on page V-3 illustrates the effect of hysteresis. Since the inductance is proportional to the slope of the ϕ - I curve, it can be expected that the inductance would follow a curve derived from the hysteresis curve. Eight minor loops are shown in figure IV-1. The characteristics on pages V-8 and V-12 are obtained using Hay's bridge and the resonance method respectively and they illustrate both the effect of hysteresis and the dependence upon the slopes of the four minor loops of figure IV-1. A mean characteristic from pages V-8 and V-12 is plotted on page V-13. The value of the shunt field inductance for the final calculations is taken from this mean curve.

A point of discussion arises as to the substitution of a.c. values under actual d.c. conditions. The usual argument is that the value of the inductance would be lower at 60 c.p.s. A test, therefore, was made to measure this inductance as a function of frequency, down to $2\frac{1}{2}$ c.p.s. The curve was then extrapolated to zero c.p.s. The characteristics on page V-16 and V-17 show that the values at zero and 60 c.p.s. do not vary greatly.

(2) Series-field Inductance:-

The variation of series-field inductance is of the same nature as that of the shunt field because the two coils are wound on the same magnetic circuit. If leakage fluxex are ignored, the two self-inductances are related by: $\dfrac{L_f}{L_s} = \left(\dfrac{T_f}{T_s}\right)^2$.

Ignoring the leakage flux is a reasonable assumption although it would vary with the load. Moreover the leakage paths for the series coil are different from those of the shunt coil, the series turns being wound over the shunt turns on each pole. The characteristics are shown on page V-19 and V-21.

(3) Self-Inductance of the Armature and Interpoles:-

(a) Variation with armature current:

Since the armature and interpoles are always connected in series and as they appear as a single term in the final impedance matrix, their combined inductance was measured, using Hay's bridge. The characteristics are shown on page V-25 and V-27.

(b) Variation with rotor-angle:

In the generalized theory of electrical machines, one important feature is the variation of armature self-inductance with the rotor angle. To explain this statement, consider the following basic relationship:-

$$e_a = r_a i^a + p(L_a i^a) \qquad (1)$$

When the rotor rotates, the inductance is a function of time and 'p' operates both on L_a and i^a. The inductance has a cyclic spatial variation and it can therefore be represented by a Fourier series. The series for the self-inductance of the armature alone can be represented as:

$$L_a = L_{ao} + \sum_{n=1}^{\infty} L_{an} \operatorname{Cos}(2n\theta) \qquad (2)$$

where θ = angle between the armature winding and the direct-axis.

Due to the distribution of the winding the convergence of the series is fairly rapid and hence we neglect all the terms with coefficients of four and above. Then for a 2-pole machine, equation (1) becomes:-

$$L_a = L_{ao} + L_{ai} \operatorname{Cos} 2\theta.$$

For $\theta = 0$, $L_a = L_{ao} + L_{ai}$ (3)

For $\theta = 90^o$, $L_a = L_{ao} - L_{ai}$ (4)

In equations (3), and (4) L_a is the d-axis and q-axis inductance respectively. Hence, we can derive the following well-known relationship,

$$L_{ao} = \frac{1}{2}(L_d + L_q) \tag{5}$$

and $$L_{ai} = \frac{1}{2}(L_d - L_q) \tag{6}$$

where L_d and L_q are the d-axis and q-axis armature inductance respectively.

The characteristics on page V-29 show that when the mechanical degrees are converted to electrical degrees, the above equations are true. Thus, the armature self-inductance is a maximum when the rotor (qr) axis is in line with the main-pole axis and a minimum when the qr-axis is at 90^o to the main-pole axis, i.e. directly under the interpoles.

The curve illustrating the variation of armature self-inductance with rotor angle is not smooth for the following reasons:-

(i) the saliency of the main poles which is partially compensated for by the distribution of the winding.

(ii) tooth ripple causes fluctuations having a wavelength equal to the slot pitch i.e. 15^o elec.,

(iii) the effects of carbon brushes, and

(iv) experimental errors.

An interesting point during these measurements was to observe the effect of field current on the self-inductance of the armature and interpoles, a phenomenon usually ignored in the generalized theory. The characteristics on page V-32 indicate the decrease in this self-inductance due to the presence of the field current. The value also depends upon the magnitude of the armature current. The effect is attributed to cross-saturation and is in line with the similar observations made by Jones[4]. As he observes: "The wider applications of these are important, namely that it is unwise to assume (as is frequently done in synchronous machine theory) that d-axis parameters are unaffected by q-axis saturation and vice-versa".

(4) Short-circuit turns Inductance:

The measurements were made with solid connections on the commutator and the inductance was found to be independent of its own current but dependent upon the shunt field current. Measurements were therefore made with rated current in the field-winding. Because the short-circuited turns constitute a coil with a narrow spread, the variation of this self-inductance with rotor angle also has a trapezoidal waveform.

(5) Coefficients of Mutual Inductance :-

The coefficients of mutual inductances that are to be measured in a d.c. machine are those existing between:

(a) shunt field and armature
(b) shunt field and armature plus interpoles
(c) shunt field and interpoles
(d) shunt field and series field
(e) series field and armature
(f) series field and armature plus interpoles
(g) series field and interpoles.

Of these the first is the most important, it being the speed coefficient responsible for the generation of the normal e.m.f. and the production of normal torque in the d.c. machine. It is the quantity M'_{af} in the impedance matrix on page IV-23 and it differs from the quantity M_{af} if the field form is non-sinusoidal. Although the d.c. machine field form is approximately rectangular on no load, the effect of the distributed armature winding makes it possible to consider an equivalent lumped or concentrated coil and an approximately sinusoidal field form. If M'_{af} is obtained from the magnetization curve no approximations are necessary. The characteristics showing the variation of M'_{af} with field current is shown on page V-33.

As mentioned previously, mutual inductances, as distinct from speed coefficients are measured by applying simple transformer theory. One of the main objects of this work is to simplify the solutions of transient problems in a d. c. machine. Hence 60 c. p. s. methods are used and an attempt is made to find by what margin the solutions obtained from these measured values differ from the actual conditions. Although this method of measuring the mutual inductances between various magnetically coupled circuits is liable to criticism as to the accuracy of the results, its value lies in experimental ease and simplicity and Saunders[2] has utilized it to measure the inductance parameters of a d. c. machine.

In this project two distinct cases have been studied:-

(a) with brushes included in the armature circuit

(b) with solid connections to the commutator segments.

The brush short circuits three commutator segments and each brush is 3/8 in. thick and 3/4 in. wide.

The mutual inductance between the shunt field and the armature is a function of the field current, because the flux produced by the main-pole winding is nonlinearly related to the exciting current. As can be seen from the magnetization curve saturation does not seem to be appreciable below a field current of 0.6A.

This mutual inductance is a function of the rotor position. The precise law of the variation is unknown but it is cyclic and may be represented by the following Fourier series:

$$M_{af} = \sum_{n=1}^{\infty} M_{af} \text{Cos}\,(2_n - 1)\theta \quad (7)$$

Neglecting the terms with n equal to or greater than 2, for the 2-pole machine:-

$$M_{af} = M_{af1} \text{Cos}\,\theta \quad (8)$$

For $\theta = 0,\ M_{af} = M_{af1}$

$$\theta = 90^{\circ},\ M_{af} = 0 \quad (9)$$

It is evident that the maximum value would occur when the rotor - axis is in line with the main-pole axis, as this is the axis for maximum value of induced e.m.f. and zero speed e.m.f. .

(6) Effect of Carbon Brushes:-

As previously stated, two distinct cases were studied, one with brushes on the commutator and the other without. Before the effect of brushes can be discussed the influence of carbon brushes on the operation of the d.c. machine will be discussed.

The carbon brush contact-resistance characteristics depend upon the following factors in particular:-

(7) Resistivity:-

Although carbon-graphite mixtures are fairly good electric conductors among the nonmetals, their conductivities are low compared with those of metals. The lowest resistivity of electrographite is 300 microhm - in. The corresponding figure for copper is 0.7. Therefore, the resistivity of electrographite is 400 times, and of carbon is 3200 times, that of copper. Even so, the energy loss due to the contact resistance and friction is far larger than that due to the resistance of the body of the material. Thus, the resistivity of the brush material has a much smaller influence in determining the carrying capacity of a brush than the contact resistance, coefficient of friction and thermal conductivity.

(2) Thermal Conductivity:-

The thermal conductivity of the best electrographite is of the order of 0.3 calorie sec^{-1} cm^{-1} deg. C^{-1} and that of electrographite brush material is 0.15. Corresponding figures for metals are: (1) silver - 1.0 (2) Copper - 0.9 (3) iron - 0.11 The heat conductivity of graphite is thus comparable with that of the metals. The temperature at the contact surface will always be higher than that inside the body of the brushes and carbon starts to oxidise rapidly in air at temperature above 350°C. If, therefore, the thermal conductivity of the brush is inadequate for the particular duty, it is possible to get the temperature high enough to produce heavy wear due to burning.

(3) Density and Porosity:-

The density of pure graphite is 2.2. Actual densities of brushes range from 1.22 to 2.15.

(4) Elastic Properties:-

These have considerable influence on the smoothness of the "ride".

(5) Contact Properties:-

Contact properties such as friction and contact resistance cannot be defined without also specifying the ambient conditions which influence results. Among the condition which must be considered are:

(1) Nature of materials
(2) Conditions of surfaces e.g. clear, dirty, machined, polished.
(3) Pressure between surfaces.
(4) Intervening medium e.g. air, oil

(5) Movement - stationary, sliding, rotating and speed.

(6) Temperature.

(7) Current

(8) Factors influencing the intimacy of contact e.g. mechanical accuracy.

Some of these can be controlled and measured, some can be measured but not controlled. Sometimes variations different from those specified by carbon-brush manufacturers occur. This is most often due to ambient conditions changing.

The net area of true contact between the surfaces is extremely small, a typical value being of the order of 10^{-4} mm^2. Variation of load alters the area of contact. When a current is passed between a brush and metal, the crowding of current into extremely small areas of direct contact produces a "constriction" resistance at the contact face, which is many times greater than the resistance of the brush material.[24]

(6) Influence of pressure:-

This is one of the most important factors.

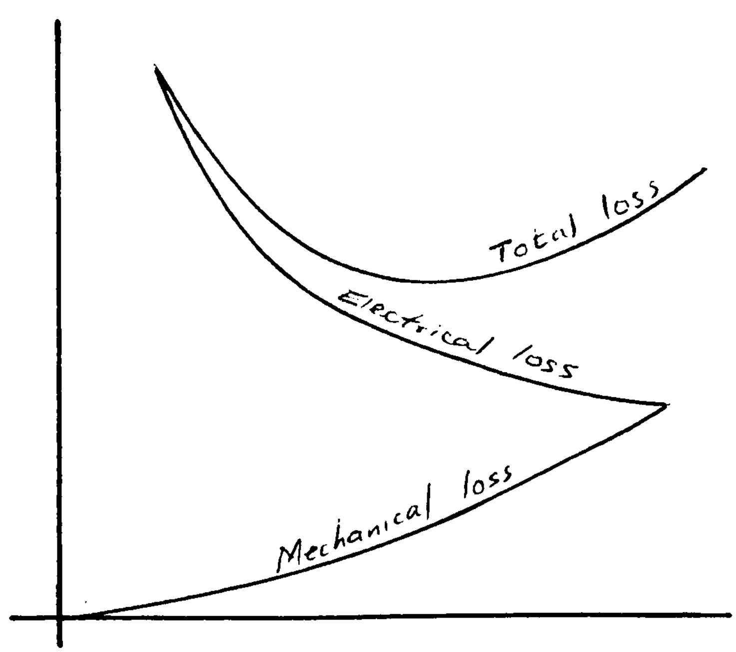


Fig. IV-2 Variation of brush surface losses with applied pressure.

The area of contact between two surfaces depends entirely upon the nature of the materials and the compressive forces between them. The coefficient of friction is the ratio that the force necessary to move one surface over another bears to the compressive force existing between them. The mechanical power loss due to friction is usually proportional to the brush pressure, the speed and the coefficient of friction but experience has shown that the coefficient of friction is not entirely independent of the brush-pressure. Generally, the coefficient of friction increases slightly at higher brush pressures. Doubling the brush pressure, therefore, more than doubles the friction loss. A typical curve showing power loss with brush pressure is given in fig. IV-2. Brush contact resistance and hence the electrical power loss is also very much dependent upon contact pressure.

The law connecting resistance with pressure for any electrical contact, moving or stationary, is of the form $R = K.P^{-n}$ where K = constant depending upon the specific conditions of the material, e.g. surfaces and n is a power usually between 0.5 and 1.0. For commutator machines at normal current densities $n = 0.5$ typically. Then R is inversely proportional to P.

(7) Influence of Current:-

The relationship between the potential difference or volt-contact drop at the contact surface and the value of current is not a simple one. If other conditions such as pressure, atmosphere etc. are assumed to be constant, a curve is obtained as shown in fig. 4-3[24]. It is noticed that the rate of increase in volt-drop gets progressively less as the current is increased. As can be seen, the curve is of the form: $V = KI^m$ where I = current density, K and m vary with brush grade and ambient conditions; 'm' is less than unity and is usually between 0.2 and 0.6. The value of 'm' varies not only with the brush grade, speed, pressure and so on, but also

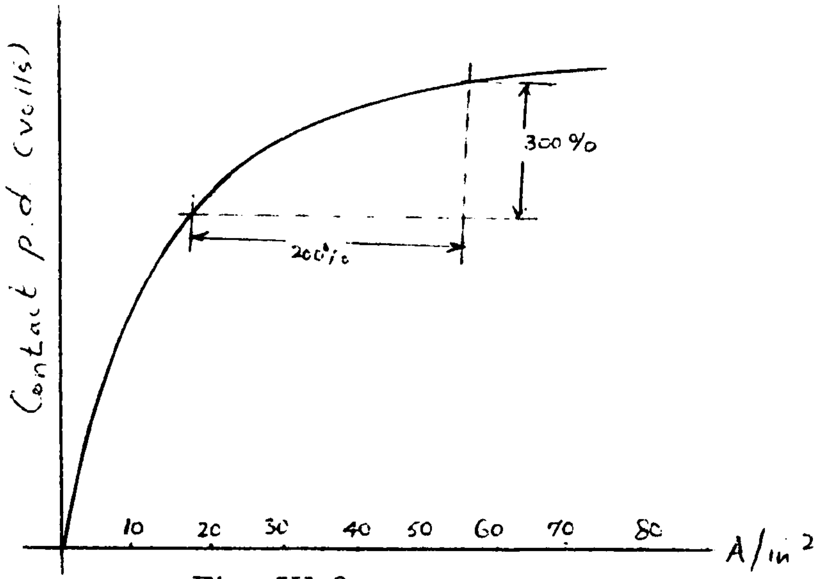


Fig. IV-3.

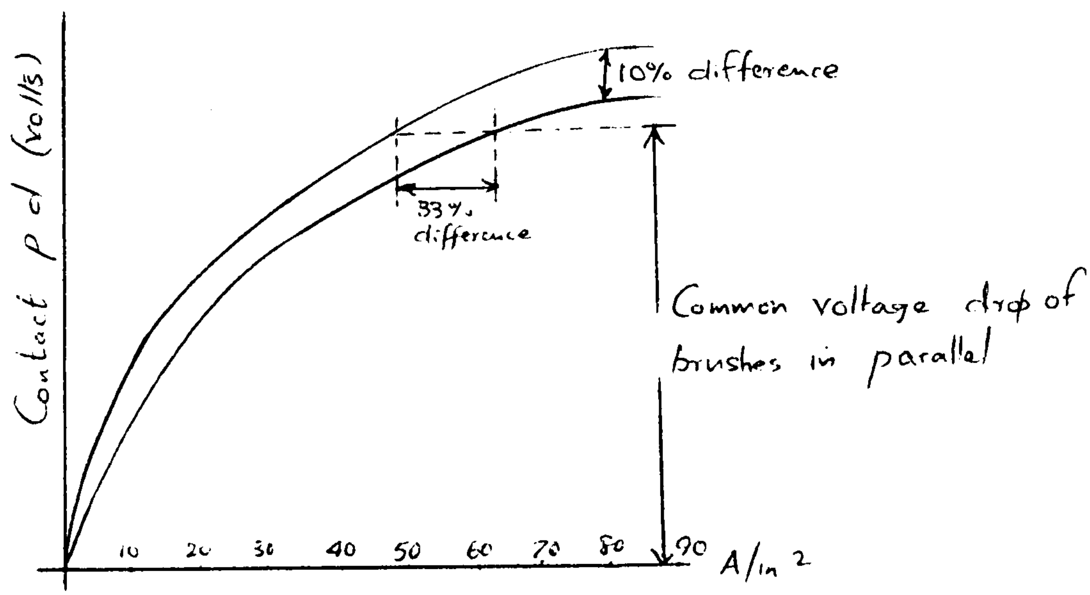


Fig. IV-4.

with the rate of change of current. Fig. IV-4[24] shows the contact drop/current density curves for two brushes with different applied pressures, the one being 20% higher than the other. If these two brushes were connected in parallel, the resultant difference in the current capacity would be 33% i.e. it is much more than the percentage difference in pressure applied to the brushes.

(8) Influence of Polarity:-

In general, the contact drop at the cathodic brush (motor negative or generator positive) is a little lower than that at the anodic brush (motor positive or generator negative). With electrographite brushes, this difference is smaller. With carbon and natural-grade graphite, the difference may be as much as 20%. With a copper-graphite brush, the contact drop at the motor positive brush may be two or three times higher than at the motor negative brush.

(9) Influence of Speed:-

Immediately a commutator or slip ring starts to move from rest, there is an increase in the contact drop of the order of 30%. At slow speeds, both the friction and contact drop are greater and the coefficient of friction is smaller than when running at faster speeds. Changes in contact drop are small but those in frcition are fairly marked. These effects are due to two causes:- (1) Mechanical (2) Aerodynamic. The former is due to the brush-holder mechanism and internal damping within the brush-material. The latter is due to the boundary layer of the air which helps the brush to act as a fluid-lubricated bearing.

Considering the influence of all these factors, it can be seen that carbon brushes do have a considerable effect on the behaviour of a machine. It is unfortunate that all the factors mentioned above are still not clearly understood as to their effects on the overall performance and that all the curves mentioned above are nonlinear.

Hence, a study is made of the effects of carbon brushes on the value of the inductances involved especially in the armature circuit. Examining the various characteristics of the mutual inductance, (pages V-33 to V-47) with and without the carbon brushes, it can be seen that the presence of carbon brushes is to decrease the mutual inductance. The difference is slight, the maximum being $2\frac{1}{2}\%$. However, the characteristics obtained without the brushes are smoother and more regular than those with brushes. This is attributed to the nonlinear effects brought in by the brushes and the erratic changes in contact surface. Thus, it may be advantageous to have measurements without brushes; that is by making solid connections on the commutator. The main reason for this being that it is impossible to analyse the effect of carbon brushes accurately. In the case of the measurement of the armature self-inductance, the brushes must be removed as the resistances in the four arms of the bridge have to be determined accurately. For mutual inductance measurements, however, the presence of brushes does not affect the numerical value nor the nature of the characteristics to any great extent. This conforms with the observations made by Barton[8]. This may be due to the contact resistance being higher than the resistance of the circuit under measurement.

However, two schools of thought exist in this respect. While some[4,7,8] insist that it is wise to remove brushes to eliminate their unknown behaviour, others[2] maintain that if brushes are kept in their position, the brush resistance and the inductive effects of the short-circuited coils will be reflected in the overall parameters; this in turn, is desirable in predicting the overall characteristics of the machine in which the brushes are always present under operating conditions.

(7) Commutation:-

Although the unified approach to the solution of electrical machine problems has advantages, it must be said that it was not quite comprehensive, because it ignored completely the process of commutation, which is so fundamental in d.c. machines. Some developments have been made in this direction but the only interesting papers on the subject are one by Jones and another by Jones and Barton.

It is interesting to investigate how the measured parameters can give a vivid picture of commutation in the present machine. Before this is done, it would be advantageous to see how the problem can be tackled from the Unified theory approach. The treatment that follows is based mainly on the paper by Jones[4].

The voltage generated in the windings of a normal d.c. commutator machine is alternating and the purpose of the commutator is to reverse the connections to the coils at the instant when the voltage generated in them changes sign. Twice in one cycle, the currents in the armature reverse and the time of reversal is very short. This process is completely repetitive in space. There is always some coil in which the reversal is taking place. The time of the operation is the time taken for the rotor to move through the thickness of the brush, minus one thickness of mica.

During commutation, the connections to the armature coil are not changed, so that during this small displacement the machine must behave as a normal slip ring machine. At the end of commutation, the connection is suddenly interrupted and then a similar cycle is repeated in the next coil. Hence, if the equations of a slipring primitive machine are known, the analysis of the commutator machine can be made on the same basis as that for a normal slipring primitive machine.

Consider the configuration in fig. IV-5. With the field form assumption made previously, the various coefficients of inductance may be written as a Fourier series:-

L_f and L_i are independent of θ

$$L_a = L_{ao} + \sum_{n=1}^{\infty} L_{an} \operatorname{Cos}(2n\theta) \tag{10}$$

$$M_{fi} = 0 \tag{11}$$

$$M_{fa} = \sum_{n=1}^{\infty} M_{fan} \operatorname{Sin}(2n-1)\theta \tag{12}$$

$$M_{fb} = \sum_{n=1}^{\infty} M_{fbn} \operatorname{Cos}(2n-1)\ \theta \tag{13}$$

$$L_b = L_{bo} + \sum_{n=1}^{\infty} L_{bn} \operatorname{Cos} 2n\theta \tag{14}$$

$$M_{ib} = \sum_{n=1}^{\infty} M_{ibn} \operatorname{Cos}(2n-1)\ \theta \tag{15}$$

$$M_{ia} = \sum_{n=1}^{\infty} M_{ian} \operatorname{Sin}(2n-1)\ \theta \tag{16}$$

$$M_{ab} = \sum_{n=1}^{\infty} M_{abn} \operatorname{Sin}(2n\theta) \tag{17}$$

Let δ = interval of commutation.

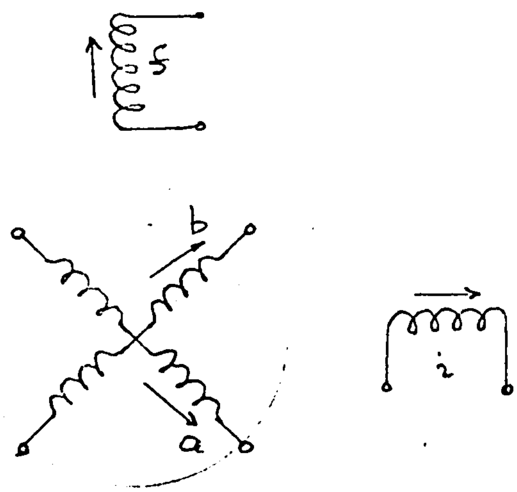
Since the axis of the coils undergoing commutation is the d-axis, the limits of θ during commutation is $-\frac{1}{2}\delta$ to $+\frac{1}{2}\delta$, $2\pi/\delta$ being the number of commutator segments per pair of poles. In practice, δ is usually small and it is a reasonable assumption to make that Sin $2\theta = 0$ and that Cos $2\theta = 1$. With this assumption, θ would disappear from the impedance matrix for a normal slipring primitive machine, and, on writing

$$p(MI) = MpI + M'I \text{ where } K' = \frac{dM}{d\theta} \text{ and 'p'}$$

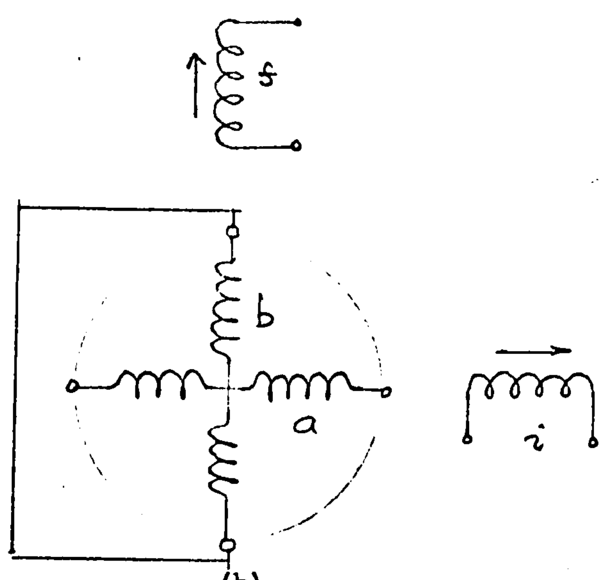
operates only on currents, and $p = \frac{d}{dt}$. The impedance matrix for a normal slipring primitive machine can be written as

$Z =$ (18)

	f	b	a	i
f	$r_f + pL_f$	pM_{fb}	pM_{fa}	pM_{fi}
b	pM_{fb}	$r_b + pL_b$	pM_{ba}	pM_{bi}
a	pM_{af}	pM_{ab}	$r_a + pL_a$	pM_{ai}
i	pM_{if}	pM_{ib}	pM_{ia}	$r_i + pL_i$



(a)



(b)

Fig. IV-5.

With the above assumption, impedance matrix (18) can be written as:-

$Z =$

	f	b	a	i
f	$r_f+L_f p$	$M_{fb}p$	$M_{fa}p$	0
b	$M_{fb}p$	$r_b+pL_b p$	M'_{bi}	$M_{ai}p$
a	M'_{fa}	M'_{ab}	$r_a+pL_a p$	$r_i+L_i p$
i	0	M'_{bi}	$M_{ai}p$	$r_i+L_i p$

(19)

The above impedance matrix applies to fig. IV-5 Note that (19) is symmetric.

The transformation tensor for fig. VIA-2 can be written as follows on inspection:-

$C =$

	f	b	a
f	1		
b		1	
a			1
i			-1

The armature and interpoles are connected in series but in a sense such that their mmfs are in opposition. After transformation, the voltage equation in the stationary reference frame is:-

	V
f	V_f
a	V_a
b	V_b

=

f	$r_f+L_f p$	$\omega M'_{fa}$	$M_{fb}p$
a	$\omega M'_{fa}$	$(r_a+r_i)+(L_a+L_i-2M_{ai})p$	$-\omega(M'_{ib}-M'_{ai})$
b	$M_{fb}p$	$-\omega(M'_{ib}-M'_{ai})$	$r_b+L_b p$

	i
f	i^f
a	i^a
b	i^b

(20)

In equation (20), V_a is the voltage applied to the armature and interpoles in series and i^a is their common current.

In fact, there are no external connections to the short-circuited turns; there are only two physical connections, one to the field and another to the armature. In tensor analysis it is convenient to consider as many equations as there are physical connections. Hence, 'b' row and column can be eliminated by utilizing matrix-algebra methods; however, for this particular case, it is easier to adopt another way; this consists in utilizing the equation for 'V_b' to analyze the process of commutation and later on, it can be dropped from equation (20), considering that it does not serve to analyze the overall performance of the machine.

Consider then, the short-circuited turns equation. From the voltage equation (20),

$$V_b = M_{fb} p i^f - \omega(M'_{bi} - M'_{ai})\, i^a + (r_b + L_b p)\, i^b \qquad (21)$$

clearly $V_b = 0$, since there is no externally applied voltage. Let us consider the first term of the equation (21). $M_{fb} p i^f$ is the induced emf in the turns due to the changing field current. In a d.c. machine, it is usually constant; this term, therefore vanishes. Then, equation (21) reduces to:

$$(r_b + L_b p) - \omega(M'_{bi} - M'_{ai}) i^a = 0 \qquad (22)$$

The significance of each term in equation (22) is as follows:-
$(r_b + L_b p) i^b$ are the resistive and self-inductive drops across the short circuited turns including the contact and brush resistance. The exact value of this is very difficult to establish. $\omega M'_{ai} i^a$ is the voltage generated in the turns of the coil 'b' by their rotation in the q-axis field, that would be produced by the main armature current alone.
M'_{bi} is the voltage generated in coil 'b' by their rotation in the q-axis field produced by the interpole current alone.

Equation (22) expresses the familiar fact that the turns on the interpoles must first counteract armature reaction and then set up a flux sufficient to produce a voltage equal and opposite to the resistive and self-inductive voltages in the turns undergoing commutation. From the design point of view, equation (22) may be solved assuming constant coefficients. It is a linear differential equation of the first degree with constant coefficients. The solution:-

$$i^b = Ae^{\frac{-r_b}{L_b}} + (M'_{bi} - M'_{ai}) \frac{\omega i^a}{r_b} \tag{23}$$

Constant 'A' can be evaluated from the following considerations; At time $t = \frac{1}{2} T_c$, $i^b = -i^a$. Then,

$$i^b = (M'_{bi} - M'_{ai}) \frac{\omega i^a}{r_b} - [1 + (M'_{bi} - M'_{ai}) \frac{\omega}{r_b}] i^a e^{-\frac{r_b}{L_b}(t + \frac{1}{2} T_c)} \tag{24}$$

For ideal commutation, $i^b = + i^a$; at $t = \frac{1}{2} T_c$; making this substitution and solving for M'_{bi} gives:

$$M'_{bi} = M'_{ai} + \frac{r_b}{\omega} (I + e^{\frac{-r_b}{L_b} T_c}) / (I - e^{\frac{-r_b}{L_b} T_c}) \tag{25}$$

Equation (25) can be looked upon as an alternative formula for the interpoles.

If the term $\frac{r_b}{L_b} T_c$ is assumed to be small, equation (25) becomes:-

$$M'_{bi} = M'_{ai} + \frac{2L_b}{\omega T_c} \text{ and putting } r_b = \frac{L_b}{T_c},$$

$$M'_{bi} = M'_{ai} + \frac{2L_b}{\delta} \quad \text{where } \delta = \omega T_c \tag{26}$$

If equation (26) is substituted in equation (24), linear commutation is achieved and the expression for i^b is:

$$i^b = \frac{t}{\frac{1}{2} T_c} \quad i^a \tag{27}$$

For a well-designed machine with linear commutation, the voltage equation (21) for the short-circuited turns simply gives an explanation of the process of commutation, and gives no further information about the performance of the machine.

From measurements on the machine:

Brush thickness = 0.375"

Mica thickness = 0.01"

Diameter of Commutator = 4.5"

Angular velocity at 1300 rpm = 188.5 rad/sec.

Peripheral speed at commutator = 2.25 x 188.5 in/sec.

$\therefore$ Time of commutation = $\frac{0.365}{424} \times 10^3$

i.e. T_c = 0.861 millisecond

$\delta = \omega T_c$

$= 0.162$ radian.

From equation 26, if idealized conditions are assumed,

$$M'_{bi} - M'_{ai} = \frac{2L_b}{\delta} = 1.71 \text{ mH.}$$

From equation (24),

$$\frac{i^b}{i^a} = 1.71 \times 10^{-3} \frac{\omega}{r_b} - (1+1.71 \times 10^{-3}) \frac{\omega}{r_b} e^{-\frac{r_b}{L_b}(t+\frac{1}{2} T_c)}$$

The value of the resistance of the short-circuited turns alone is 0.06Ω measured by Wheatstone bridge; carbon and brush resistance must be added to this value. Hence, two values will be assumed for r_b. (a) 0.06Ω (b) 0.1Ω. The first value is lower while the second may be higher than the actual value.

With these values of r_b, the following equations describe the conditions of commutation:

$$\frac{i^b}{i^a} = 5.35 - 6.35\ e^{0.463(t/T_c + 1/2)} \qquad (28)$$

with $r_b = 0.06\Omega$

$$\frac{i^b}{i^a} = 3.21 - 4.21\ e^{-0.77(t/T_c + 1/2)} \qquad (29)$$

The curves describing equations (28) and (29) are shown on page V-25. Very little difference can be noted for two different values of r_b. It can be seen that the machine is slightly under commutated at 1800 r.p.m. 1850 r.p.m. is found to be the ideal speed for the machine. The under commutation given by equations (28) and (29) may be due to the measurement of L_b at 60 c.p.s. If L_b had been of a little higher value, ideal commutation would result.

It can be claimed that a.c. measurements give fairly accurate results, and Jones' unified approach to the commutation problem is valid. The above approach also provides an alternative method for the design calculation of the interpole coil ampere turns.

(8) Moment of Inertia:-

Although the retardation test gives fairly accurate results for larger machines, no satisfactory electrical method seems to be available for smaller machines. However, the retardation test was carried out. The results showed that the speed/time characteristics were linear and the power/speed curve was nonlinear. The value of the slope of power/speed curve is required and as can be seen on page V-57, the slope changes at each point. For smaller machines, it is also difficult to read the wattmeter between two values of the speed; this was overcome by using a two-channel pen recorder. It is concluded, therefore, that the retardation test fails to give an accurate value for the moment of inertia for insertion in the differential equations.

A second test was performed which consisted of obtaining speed/time characteristics for the case when iron plus mechanical losses are measured. Then a power/speed curve was obtained using a pen recorder for the purpose. From the above two, a curve of power/time was obtained (page V-58). The area under this curve must be the kinetic energy lost by the machine in overcoming the losses. Knowing the speed and the losses, the value of the moment of inertia can be calculated. The moment of inertia as found by this method is almost constant at all speeds. However, the value differs from that found by the Mechanical test. The difference is attributed to the fact that the losses cannot be measured very accurately. It is recommended that the Mechanical method be used for smaller machines due to its simplicity; it is also less time consuming, involving only two tests as against four in the retardation test.

The third method is simple and direct. The only awkward feature being the measurement of the instantaneous initial speed of the flywheel. The value given by this test has been used in the subsequent transient calculations.

CHAPTER V

TABULATED OBSERVATIONS

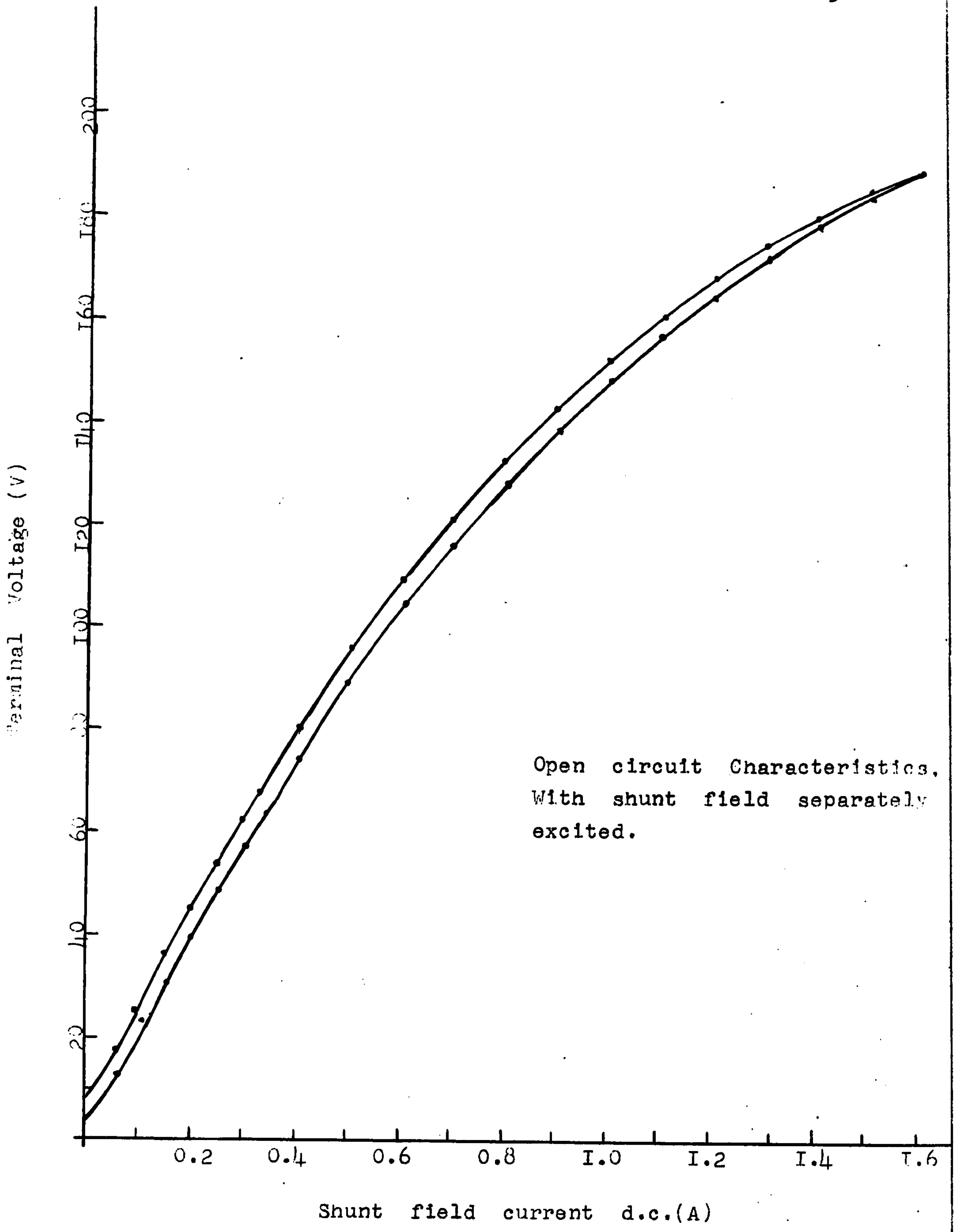
&

CHARACTERISTICS

O.C. Test on the d.c. m/c. Shunt field separately excited.

Speed = 1750 RPM

Field Current d.c. (A)	Terminal volts d.c.	Voltage volts d.c.
0.0	5	6.8
0.05	12.2	16.2
0.10	23.5	24.5
0.15	30.0	36.0
0.20	39.6	45.0
0.25	48.0	53.0
0.3	57.0	63.0
0.4	74.0	80.0
0.5	88.3	95.8
0.6	104.0	109.0
0.7	115.1	121.0
0.8	127.2	132.0
0.9	138.0	142.0
1.0	148.0	152.0
1.1	155.2	160.0
1.2	163.8	168.0
1.3	171.0	174.0
7.4	177.0	179.0
1.5	183.0	184.0
1.6	188.0	139.0


200
180
160
140
120
100
80
60
40
20
Terminal Voltage (V)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Shunt field current d.c.(A)
Open circuit Characteristics, With shunt field separately excited.

Measurement of the self-inductance of the shunt-field using Hay's Bridge:-

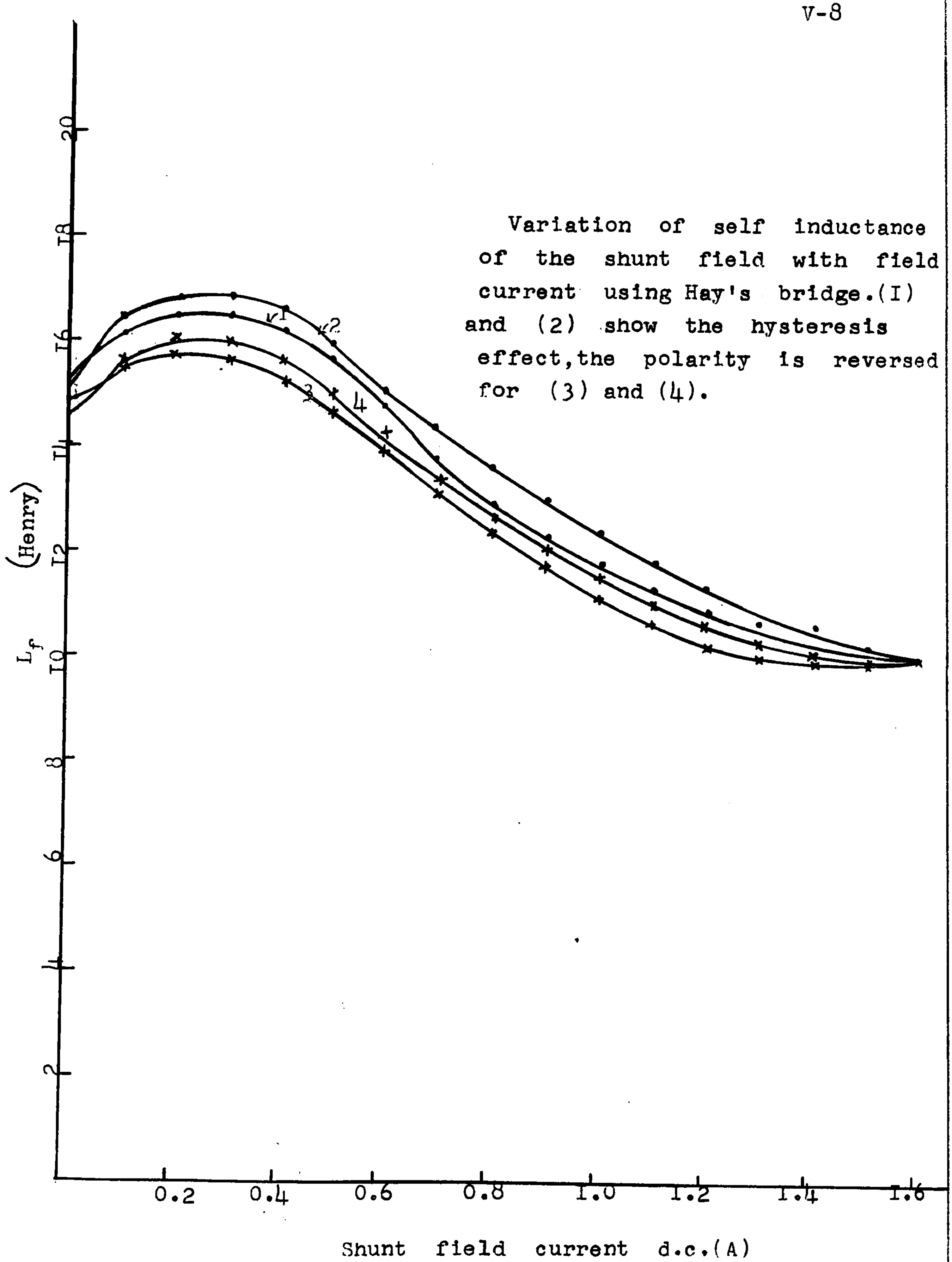
I_f(a.c) (Unit) mA	C μF	I_f(d.c) (A)	R_2 Ohms	R_3 Ohms	R_4 Ohms	L_f Henries
3	10	0	54000	28.2	340	15.2
"	10	0.1	56750	"	"	16.2
"	15	0.75	38500	"	"	16.3
"	15	0.2	39000	"	"	16.5
"	15	0.3	38900	"	"	16.45
"	15	0.4	38500	"	"	16.3
"	10	0.5	55400	"	"	15.6
"	10	0.6	52500	"	"	14.8
"	10	0.7	47900	"	"	13.5
"	10	0.8	45150	"	"	12.9
"	10	0.9	43600	"	"	12.3
"	10	1.0	41800	"	"	11.8
"	10	1.1	40000	"	"	11.3
"	10	1.2	38300	"	"	10.8
"	10	1.3	36900	"	"	10.4
"	10	1.4	35800	"	"	10.2
"	10	1.5	35600	"	"	10.05
"	10	1.6	35600	"	"	10.05
3	10	1.5	36200	28.2	340	10.2
"	10	1.4	37250	"	"	10.5
"	10	1.3	37600	"	"	10.6
"	10	1.2	40100	"	"	11.3
"	10	1.1	41900	"	"	11.8

$I_{a.c.}$ ma	C μF	$I_{d.c.}$ A	R_2 Ohms	R_3 Ohms	R_4 Ohms	L_f Henries
3	10	1.0	45100	28.2	340	12.4
"	10	0.9	44000	"	"	13.0
"	10	0.8	48700	"	"	13.6
"	10	0.7	51100	"	"	14.4
"	10	0.6	53250	"	"	15.0
"	10	0.5	56750	"	"	16.0
"	15	0.4	39200	"	"	16.6
"	15	0.3	39700	"	"	16.8
"	10	0.2	39500	"	"	16.7
"	10	0.1	58100	"	"	16.4
"	10	0.0	53250	"	"	15.0

(II) Hay's Bridge (Polarities Reversed)

I_f(a.c.) mA	C F	I_f(d.c.) (A)	R_2 Ohms	R_3 Ohms	R_4 Ohms	L_f Henries
3	10	0.0	52500	28.2	340	14.8
"	10	0.1	54600	"	"	15.4
"	15	0.2	37400	"	"	15.8
"	15	0.3	37100	"	"	15.7
"	15	0.4	35950	"	"	15.2
"	10	0.5	52100	"	"	14.7
"	10	0.6	49400	"	"	13.9
"	10	0.7	46500	"	"	13.1
"	10	0.8	44000	"	"	12.4
"	10	0.9	41500	"	"	11.7
"	10	1.0	39750	"	"	11.2
"	10	1.1	37600	"	"	10.6
"	10	1.2	36900	"	"	10.4
"	10	1.3	35750	"	"	10.1
"	10	1.4	35450	"	"	10.0
"	10	1.5	35450	"	"	10.0
"	10	1.6	35450	"	"	10.0
"	10	1.5	35450	"	"	10.0
"	10	1.4	35450	"	"	10.1
"	10	1.3	35750	"	"	10.3
"	10	1.2	36500	"	"	10.6
"	10	1.1	37600	"	"	11.0
"	10	1.0	39000	"	"	11.6
"	10	0.9	41100	"	"	12.0
"	10	0.8	42500	"	"	12.6
"	10	0.7	44700	"	"	13.5

I f(a.c.) mA	C F	I_f(d.c.) (A)	R_2 Ohms	R_3 Ohms	R_4 Ohms	L_f Henries
3	10	0.6	47900	28.2	340	14.3
"	10	0.5	50700	"	"	15.0
"	15	0.4	53200	"	"	15.6
"	15	0.3	55400	"	"	15.9
"	15	0.2	56750	"	"	16.0
"	10	0.1	55400	"	"	15.6
"	10	0.0	51750	"	"	14.6



Variation of self inductance of the shunt field with field current using Hay's bridge.(I) and (2) show the hysteresis effect,the polarity is reversed for (3) and (4).

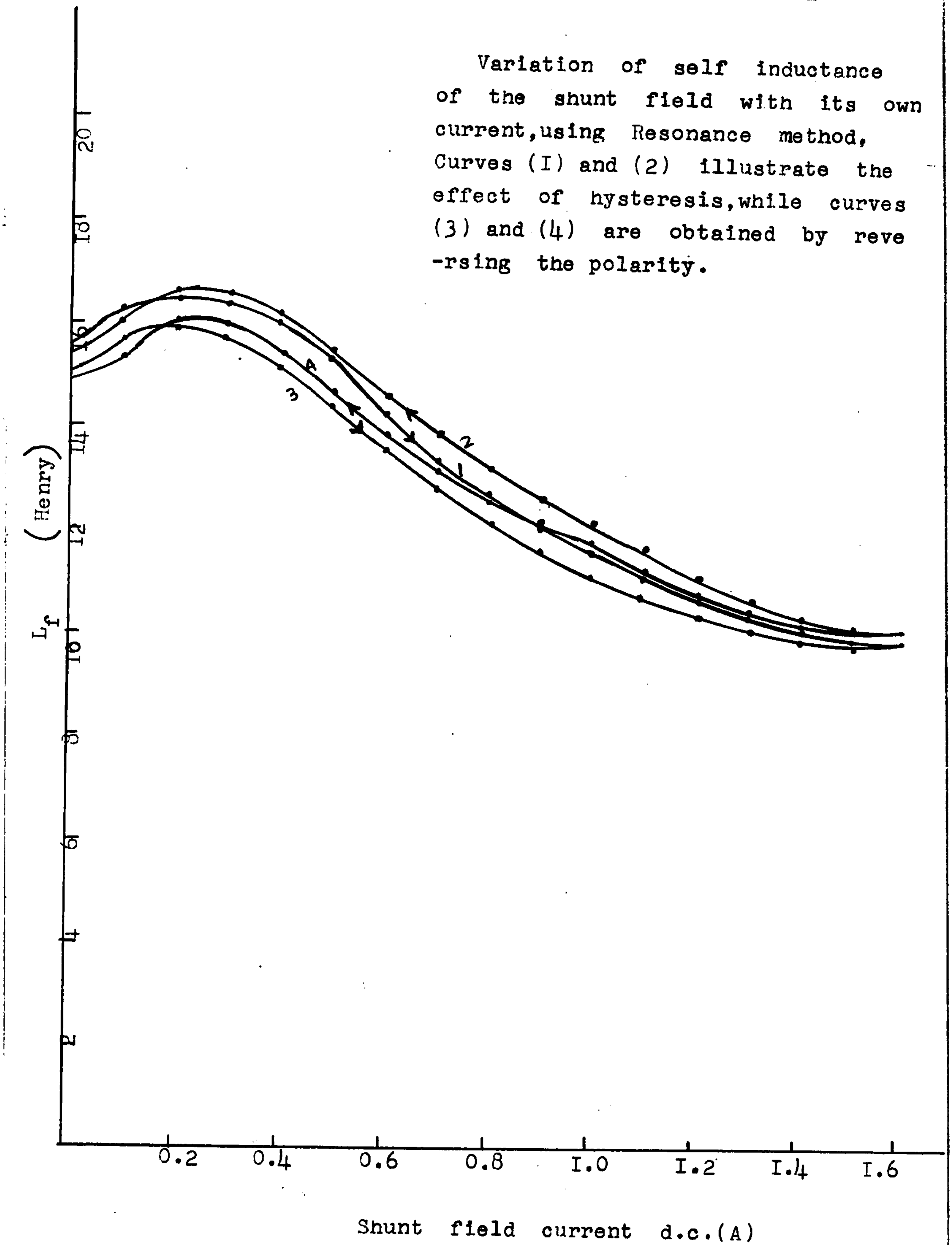
(I) Resonance Method for the measurement of shunt field Inductance

$\omega = 2\pi f$	Ia.c. for resonance mA	I_f(a.c.) mA	C μF	I_f(d.c.) (A)	L_f Henries
377	35	66	0.454	0.0	15.5
"	31.2	59	0.436	0.1	16.2
"	29.0	52	0.430	0.2	16.35
"	28.5	50	0.432	0.3	16.3
"	26.0	49	0.44	0.4	16.0
"	25.0	48	0.46	0.5	15.3
"	24.0	97.5	0.50	0.6	14.05
"	23.0	46.5	0.53	0.7	13.3
"	23.0	44.5	0.555	0.8	12.7
"	23.0	45.0	0.58	0.9	12.1
"	24.0	46.0	0.61	1.0	11.6
"	24.5	46.0	0.635	1.1	11.1
"	25.1	46.5	0.664	1.2	10.6
"	26.0	46.0	0.69	1.3	10.2
"	21.0	47.0	0.70	1.4	10.01
"	28.0	47.5	0.70	1.5	10.01
"	29.0	48.0	0.71	1.6	9.95
"	28.0	49.0	0.70	1.5	10.01
"	27.2	47.5	0.69	1.4	10.2
"	26.1	47.0	0.67	1.3	10.5
"	25.0	46.0		1.2	11.0
"	24.5	46.5	0.61	1.1	11.6
"	24.0	46.0	0.58	1.0	12.1

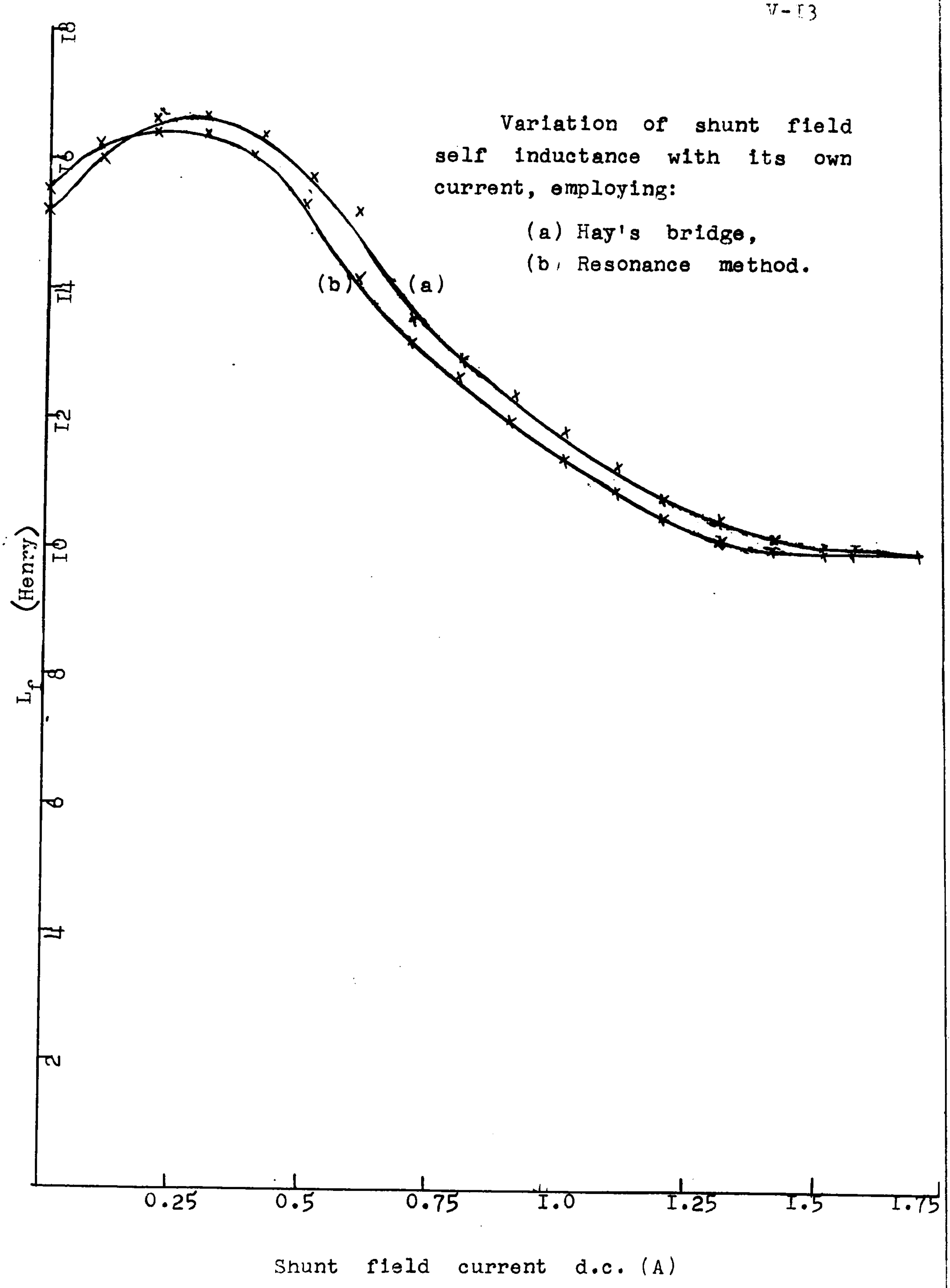
$\omega = 2\pi f$	Ia.c. for resonance mA	I_f(a.c.) mA	C μF	I_f(d.c.) (A)	L_f Henries
377	23.0	46.0	0.565	0.9	12.45
"	23.0	45.0	0.533	0.8	13.2
"	22.8	44.5	0.51	0.7	13.8
"	24.0	46.5	0.485	0.6	14.5
"	25.0	47.5	0.453	0.5	15.5
"	26.0	48.0	0.425	0.4	16.2
"	28.0	49.0	0.429	0.3	16.4
"	29.0	50.0	0.425	0.2	16.45
"	31.0	49.0	0.44	0.1	16.0
"	34.5	50.0	0.456	0.0	15.4

ω = 2πf f = 60 c.p.s.	Ia.c. (for Resonance) mA	Ia.c. (thro' field) mA	C μF	Id.c. (thro' field) (A)	Shunt field Inductance L Henries
377	35.0	65.0	0.47	0.0	15.0
"	31.0	58.0	0.45	0.1	15.6
"	29.0	51.0	0.445	0.2	15.8
"	28.0	50.0	0.451	0.3	15.65
"	26.0	49.0	0.466	0.4	15.1
"	25.0	48.0	0.489	0.5	14.4
"	24.0	47.5	0.52	0.6	13.5
"	23.5	46.0	0.55	0.7	12.8
"	23.0	46.0	0.582	0.8	12.1
"	23.0	45.0	0.606	0.9	11.6
"	24.5	44.5	0.628	1.0	11.2
"	25.0	45.0	0.65	1.1	10.8
"	25.5	46.0	0.69	1.2	10.2
"	26.0	46.5	0.704	1.3	10.0
"	27.1	47.0	0.715	1.4	9.85
"	28.2	47.5	0.719	1.5	9.8
"	29.0	48.0	0.719	1.6	9.8
"	28.1	49.0	0.715	1.5	9.85
"	27.5	48.0	0.703	1.4	9.95
"	26.4	47.4		1.3	10.4
"	25.0	47.0	0.65	1.2	10.8
"	24.4	46.0	0.628	1.1	11.2
"	23.0	46.4	0.634	1.1	11.7
"	23.0	46.2	0.582	0.9	12.1

$\omega = 2\pi f$ f = 60 c.p.s.	Ia.c.(for resonance) mA	Ia.c.(thro' field) mA	C μF	Id.c.(thro' field) (A)	Shunt fld. Inductance L Henries
377	22.7	45.5	0.563	0.8	12.5
"	25.0	45.0	0.535	0.7	13.15
"	25.7	46.0	0.51	0.6	13.8
"	26.5	46.5	0.477	0.5	14.75
"	27.5	47.0	0.464	0.4	15.2
"	28.1	47.5	0.442	0.3	15.9
"	29.0	48.0	0.44	0.2	16.0
"	30.5	49.0	0.46	0.1	15.3
"	32.0	49.5	0.472	0.0	14.9



Variation of self inductance of the shunt field with its own current, using Resonance method, Curves (I) and (2) illustrate the effect of hysteresis, while curves (3) and (4) are obtained by reve-rsing the polarity.


Variation of shunt field self inductance with its own current, employing:
(a) Hay's bridge,
(b) Resonance method.
(a)
(b)
L_f (Henry)
18
16
14
12
10
8
6
4
2
0.25
0.5
0.75
1.0
1.25
1.5
1.75
Shunt field current d.c. (A)

Variation of shunt field inductance with frequency:-

using Hay's Bridge

(I) d.c. through the field = 0 A A.C. thro' the field = 3 mA

Speed rpm	f cps.	R_2 Ohms	R_3 Ohms	C μF	L_f Henry
1800	60	54000	28.2	10	15.2
1550	51.66	"	28.5	"	15.35
1250	41.66	"	28.25	"	15.25
1000	31.33	"	28.1	"	15.15
800	26.66	"	28.5	"	15.35
500	14.66	"	28.25	"	15.25
350	11.66	"	28.3	"	15.2
180	6.03	"	28.3	"	15.2
65	1.66	"	28.35	"	15.29

(II) d.c. through the field = 0.3 Amp.

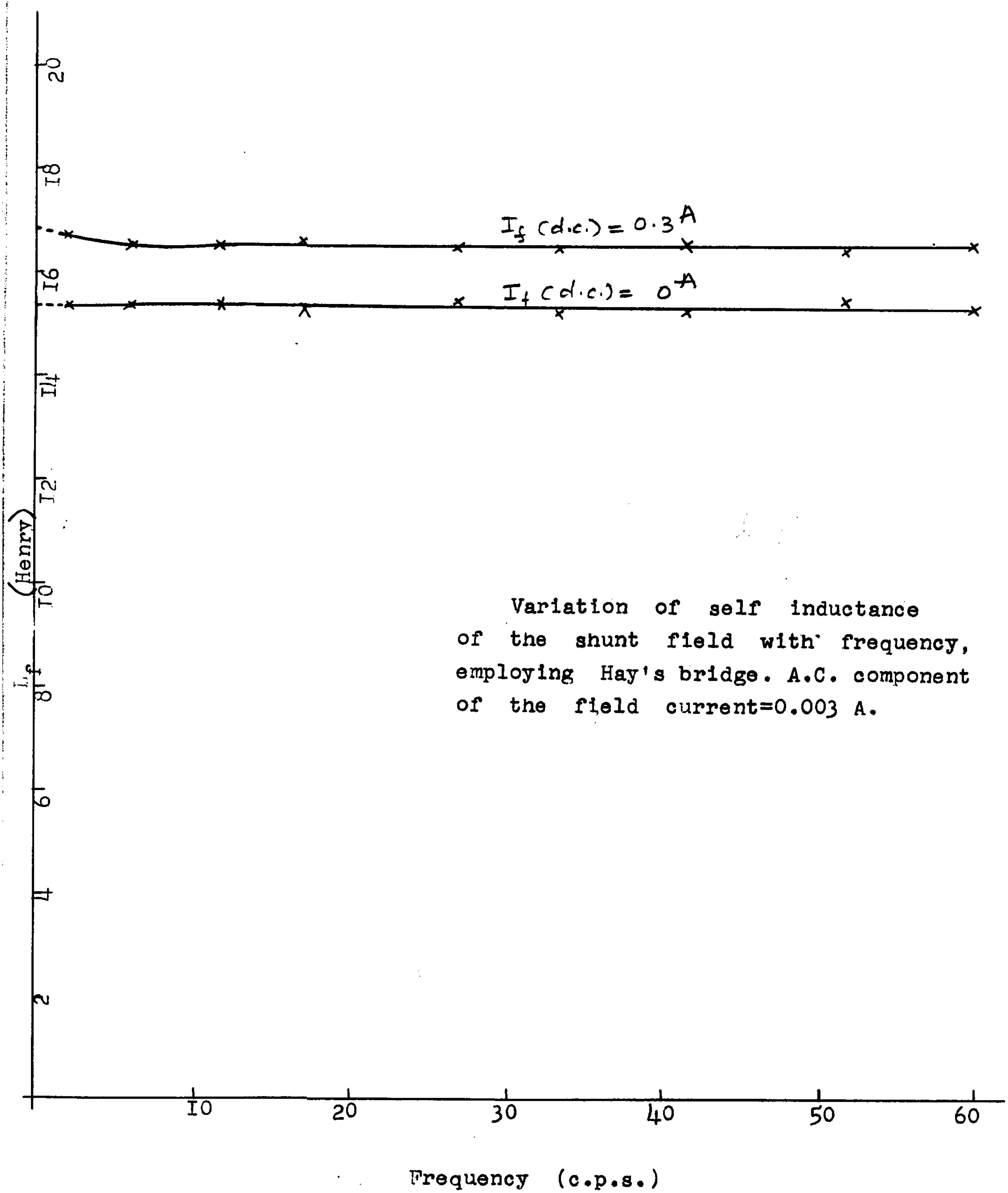
A.C. " " " = 0.003 A

Speed rpm	f cps.	R_2 Ohms	R_3 Ohms	C F	L_f Henry
1800	60	39000	42.3	10	16.5
1550	51.66	"	41.75	"	16.3
1250	41.66	"	42.3	"	16.5
1000	31.33	"	42.5	"	16.55
800	26.66	"	42.5	"	16.55
500	16.66	"	42.55	"	16.6
350	11.66	"	42.55	"	16.6
180	6.0	"	42.8	"	16.7
65	1.66	"	43.0	"	16.75

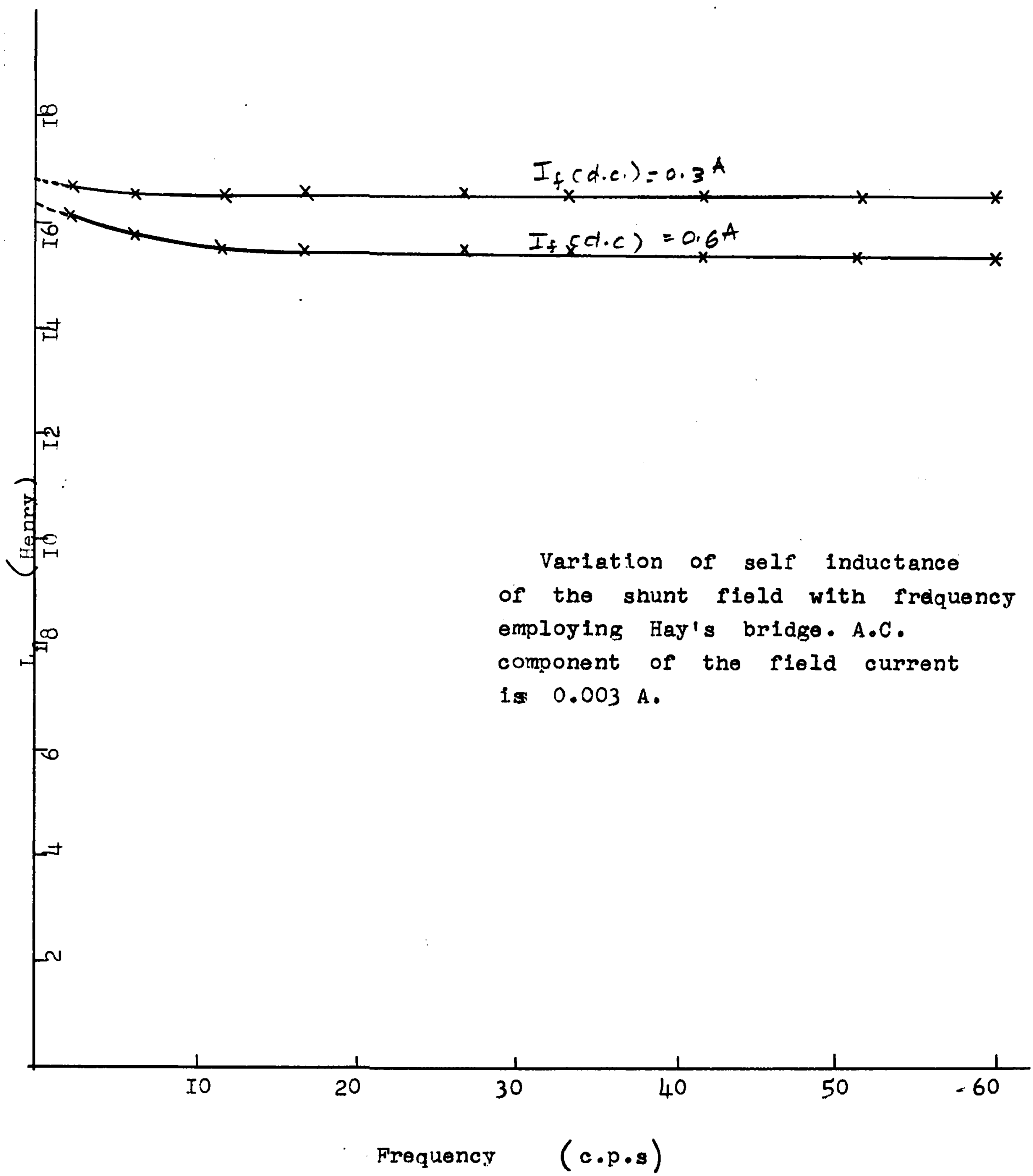
(III) D.C. Through the field = 0.6 Amp.

A.C. Through the field = 0.003 mA.

Speed rpm.	f cps.	R_2 Ohms	R_3 Ohms	C μF	L_f Henry
1800	60	41000	37.3	10	15.3
1550	51.66	"	37.6	"	15.4
1250	41.66	"	37.3	"	15.3
1000	31.33	"	37.7	"	15.45
800	26.66	"	37.7	"	15.45
600	16.66	"	37.8	"	15.45
350	11.66	"	38.0	"	15.6
180	6.0	"	38.6	"	15.8
65	1.66	"	39.2	"	16.1



Variation of self inductance of the shunt field with frequency, employing Hay's bridge. A.C. component of the field current=0.003 A.

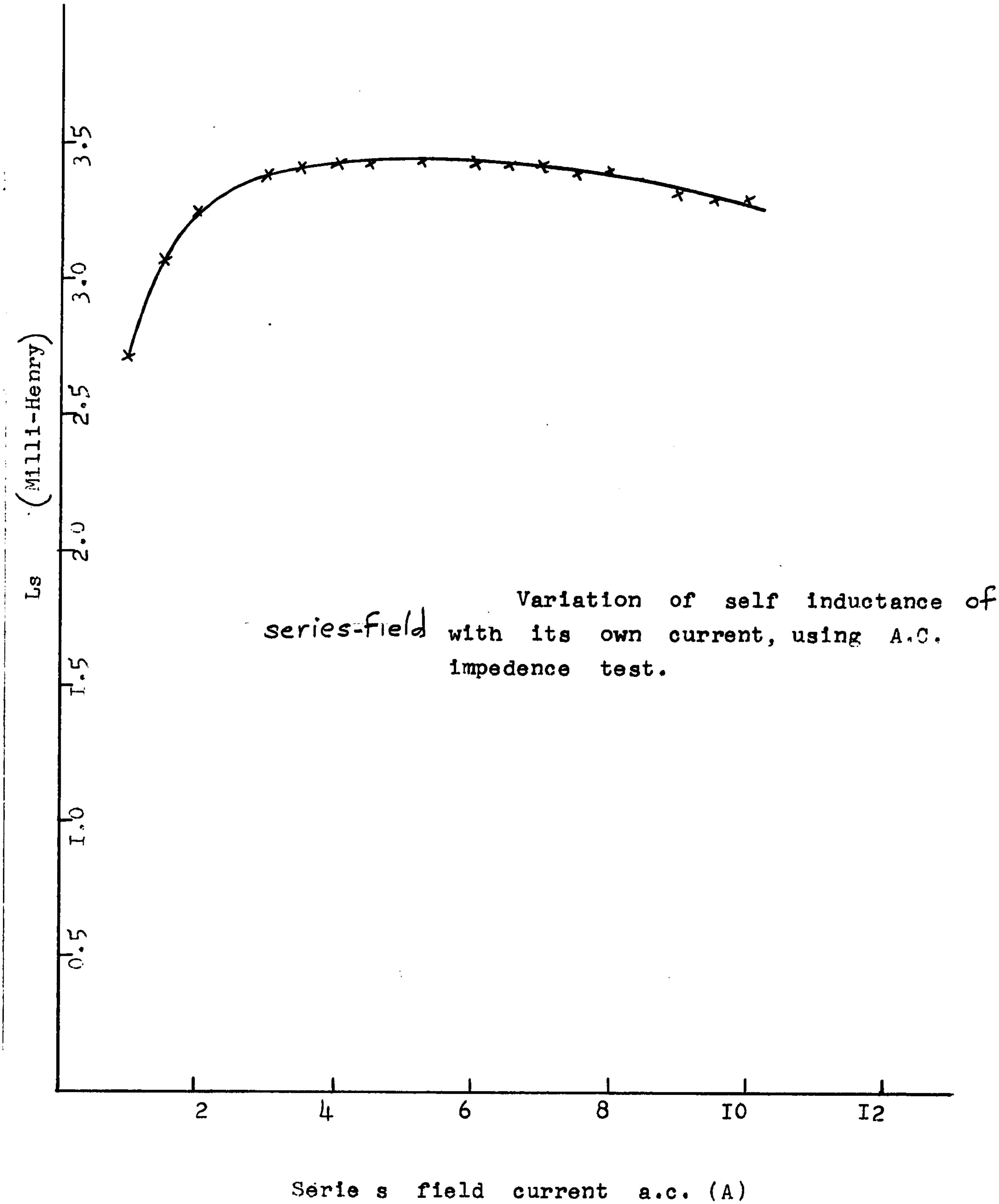


Variation of self inductance of the shunt field with frequency employing Hay's bridge. A.C. component of the field current is 0.003 A.

A.C. Impedance test for the measurement of self inductance of series field:-

V series Volts	V_f Volts a.c.	Is (A) a.c.	Z_s Ohms.	X series Ohms	L_s Millihenries
0.025	2	64 ma	-	-	-
0.05	1.3	0.1 Amp.	0.5	0.1483	3.94
0.14	9	0.2	0.7	0.216	5.74
0.40	24	0.5	0.8	0.248	6.54
1.025	52.5	1.0	1.025	1.015	26.90
1.70	87	1.5	1.142	1.133	30.5
2.46	132	2.0	1.230	1.22	32.4
3.85	225	3.0	1.280	1.27	33.7
4.40	260	3.5	1.275	1.278	33.0
5.15	300	4.0	1.290	1.283	34.0
5.81	340	4.5	1.290	1.283	34.0
6.75	400	5.2	1.395	1.296	34.1
7.80	460	6.0	1.30	1.290	34.2
8.40	500	6.5	1.29	1.283	34.0
9.05	540	1.0	1.29	1.283	34.0
9.60	570	1.5	1.280	1.227	33.7
10.10	610	8.0	1.295	1.286	34.1
11.25	680	9.0	1.25	1.242	33.0
11.75	715	9.5	1.24	1.245	32.8
12.4	750	10.0	1.24	1.245	32.8

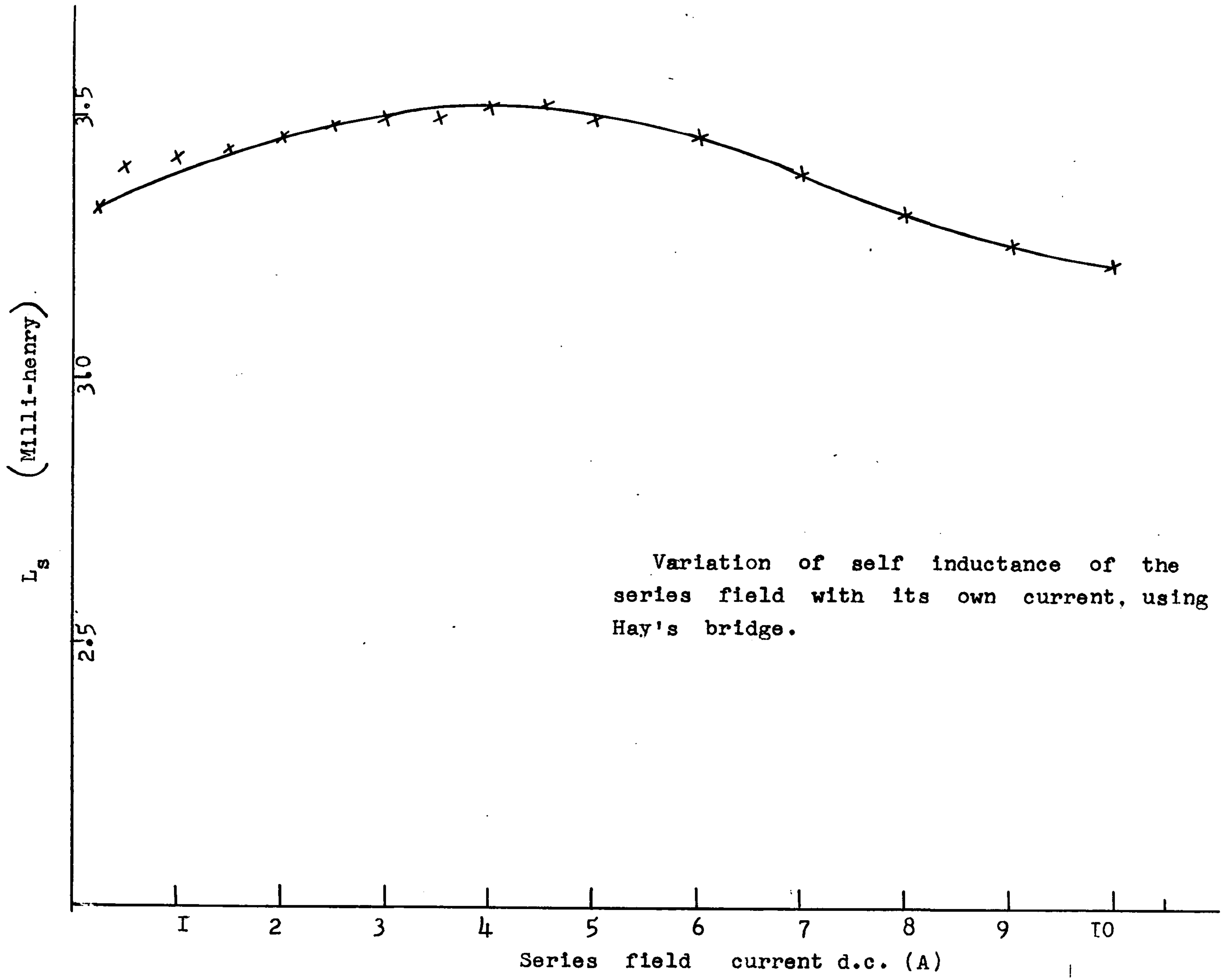
R series = 0.095 Ω


Ls (Milli-Henry)
3.5
3.0
2.5
2.0
I.5
I.0
0.5
Variation of self inductance of series-field with its own current, using A.C. impedence test.
2
4
6
8
IO
I2
Serie s field current a.c. (A)

Measurement of series - field self inductance using Hay's Bridge:-

A.C. component = 5 ma.

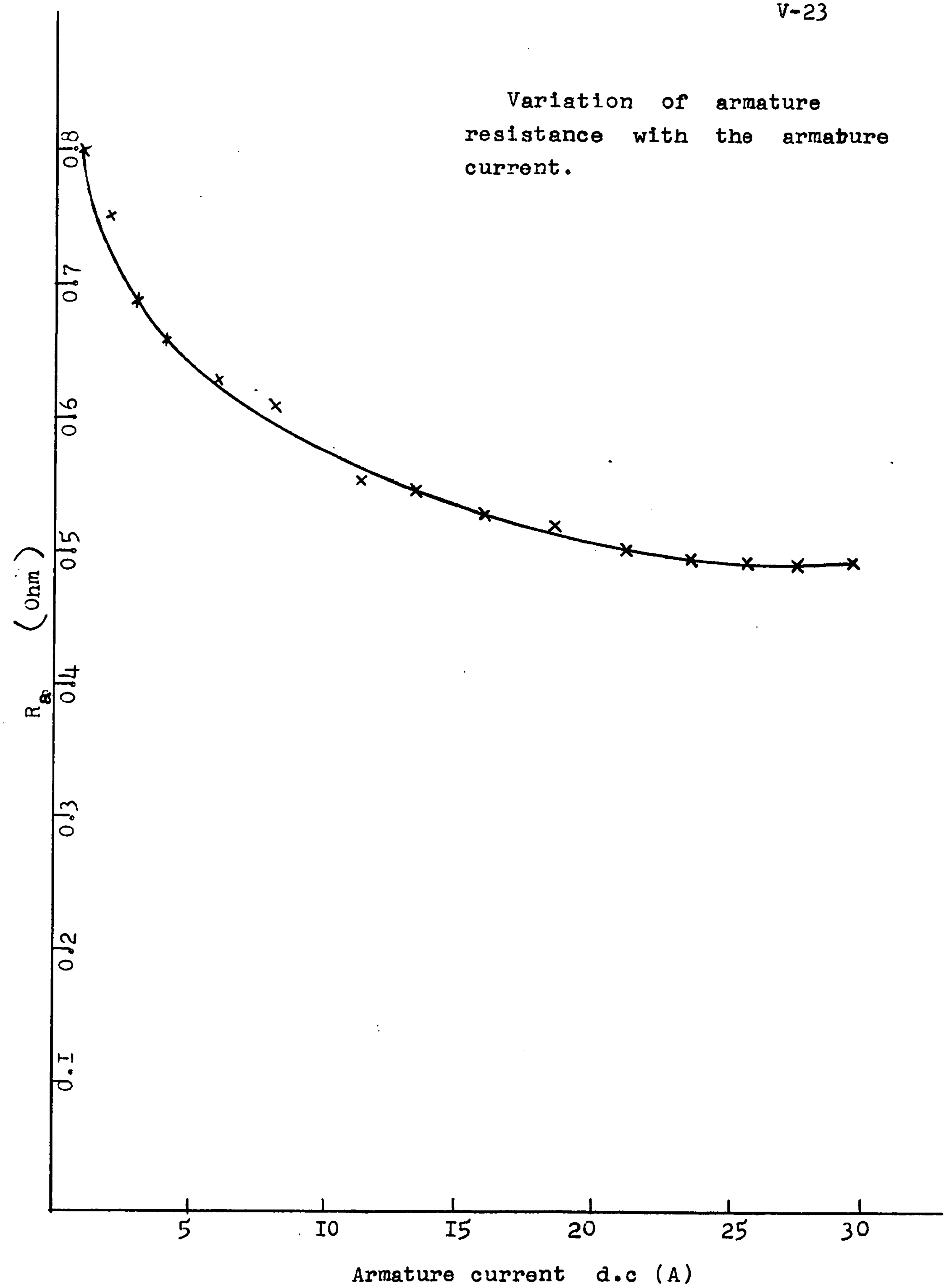
I_s(d.c.) (A)	R_2 Ohms	R_3 Ohms	C μF	L_s Millihenries
0.25	425	3.9	2.0	3.32
0.5	9.23	8.25	45.0	3.4
1.0	11.8	7.87	370.	3.42
1.5	24.75	3.255	43.0	3.44
2.0	104.8	4.71	7.0	3.46
2.5	102.0	3.8	9.0	3.48
3.0	19.45	4.6	29.0	3.49
3.5	105.0	3.33	10.0	3.5
4.0	105.0	3.34	10.0	3.51
4.5	105.0	3.35	10.0	3.52
5.0	105.0	3.33	10.0	3.5
6.0	104.8	4.71	7.0	3.46
7.0	9.23	8.25	45.0	3.4
8.0	425	3.9	2.0	3.32
9.0	425	3.85	2.0	3.27
10.0	425	3.67	2.0	3.22



Variation of self inductance of the series field with its own current, using Hay's bridge.

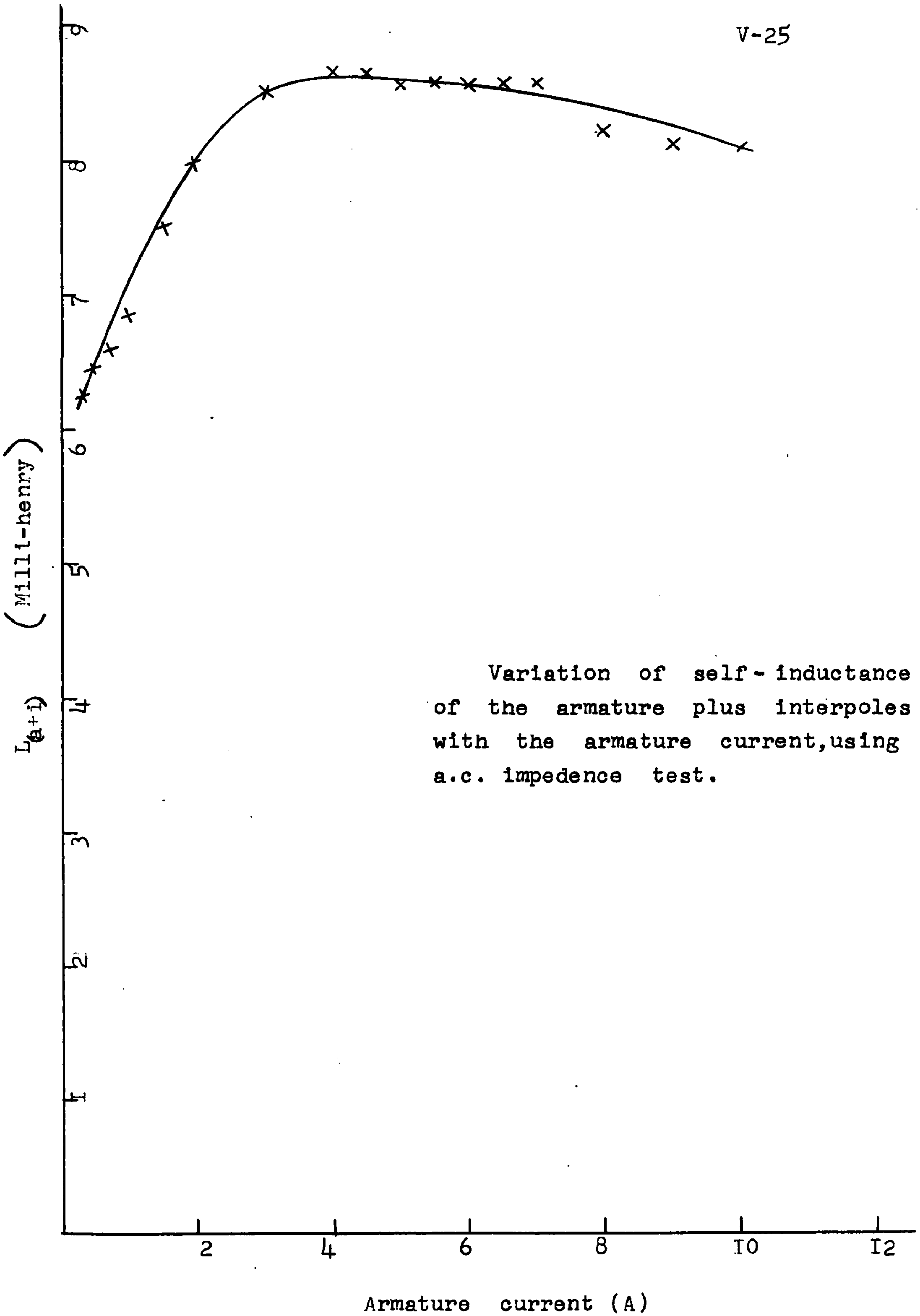
Measurement of Armature resistance:-

Va Volts d.c.	Ia (A) d.c.	Ra Ohms (d.c. value)
0.75	1.0	0.100
1.50	2.0	0.75
2.05	3.0	0.685
2.65	4.0	0.653
3.50	5.75	0.610
3.70	6.0	0.627
4.30	7.0	0.614
4.85	8.0	0.607
6.15	11.2	0.550
7.30	13.6	0.536
8.40	16.0	0.525
9.50	18.5	0.514
10.50	21.0	0.500
11.50	23.2	0.495
12.60	25.5	0.495
14.15	29.5	0.494


Variation of armature resistance with the armature current.
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
R_a (Ohm)
5
10
15
20
25
30
Armature current d.c (A)

A.C. Impedance Test - for the measurement of self inductance of armature plus interpoles:-

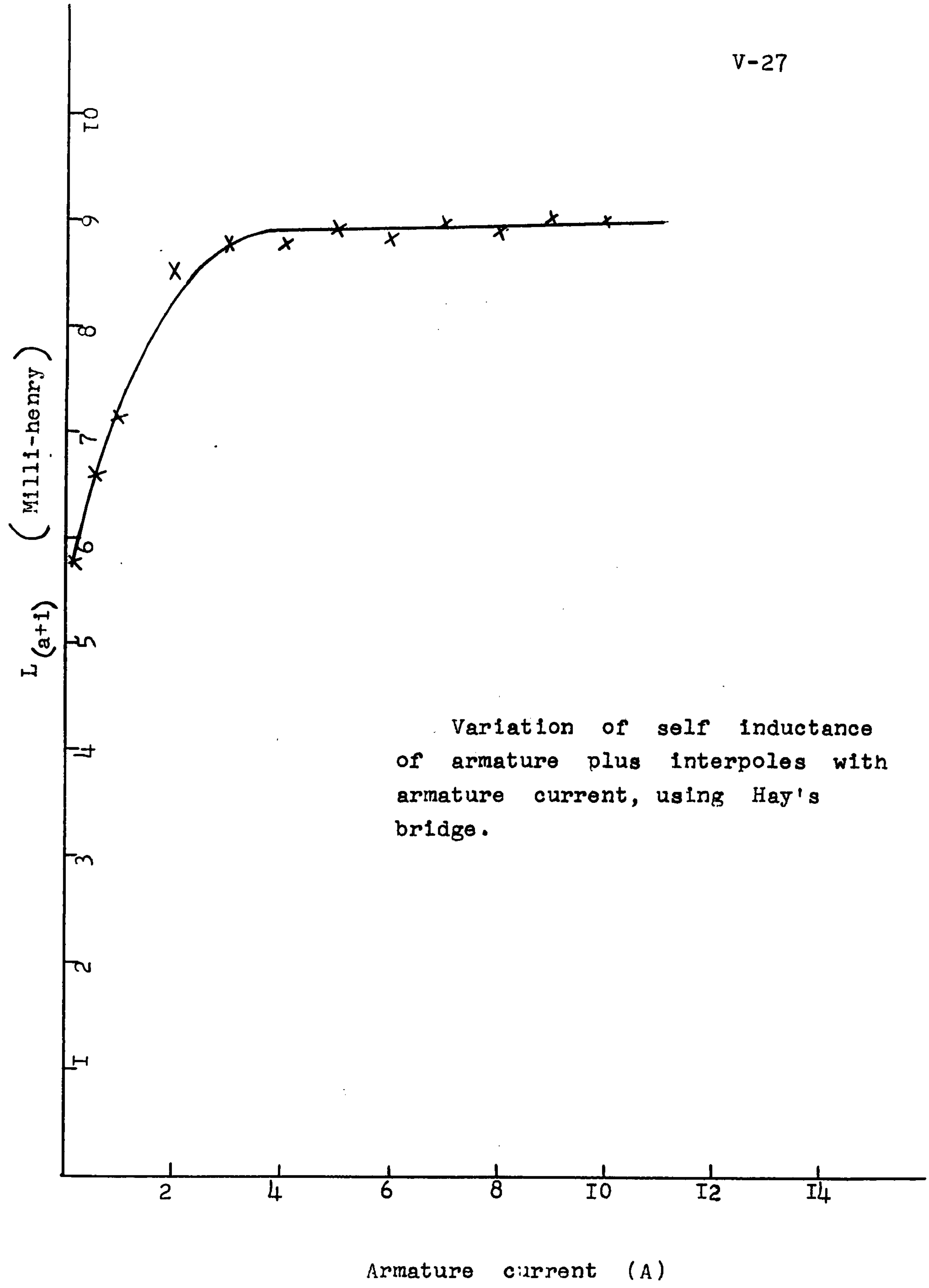
V_f Volts	Va Volts	Ia (A)	Za Ohm.	Ra Ohm.	Xa Ohm.	L(a+i) Millihenries
-	0.24	0.1	2.4	0.9	2.45	6.5[ma]
-	0.49	0.2	2.45	0.9	2.16	5.73
-	0.75	0.3	2.5	0.85	2.35	6.23
-	1.29	0.5	2.58	0.85	2.435	6.45
-	1.84	0.7	2.63	0.85	2.49	6.6
-	2.7	1.0	2.4	0.8	2.58	6.85
-	4.4	1.5	2.93	0.76	2.83	7.5
-	6.2	2.0	3.1	0.75	3.01	7.98
-	9.8	3.0	3.27	0.685	3.2	8.5
-	13.25	4.0	3.32	0.653	3.25	8.61
-	14.9	4.5	3.32	0.68	3.25	8.61
-	16.5	5.0	3.3	0.67	3.23	8.56
-	18.0	5.5	3.28	0.66	3.225	8.55
-	19.6	6.0	3.27	0.65	3.225	8.55
-	21.25	6.5	3.27	0.63	3.22	8.54
-	22.8	7.0	3.16	0.62	3.20	8.54
-	25.0	8.0	3.13	0.60	3.08	8.16
-	28.0	9.0	3.12	0.58	3.05	8.10
-	31.0	10.0	3.1	0.57	3.04	8.06



Variation of self-inductance of the armature plus interpoles with the armature current, using a.c. impedence test.

Measurement of Armature plus Interpole Self Inductance, using Hay's Bridge

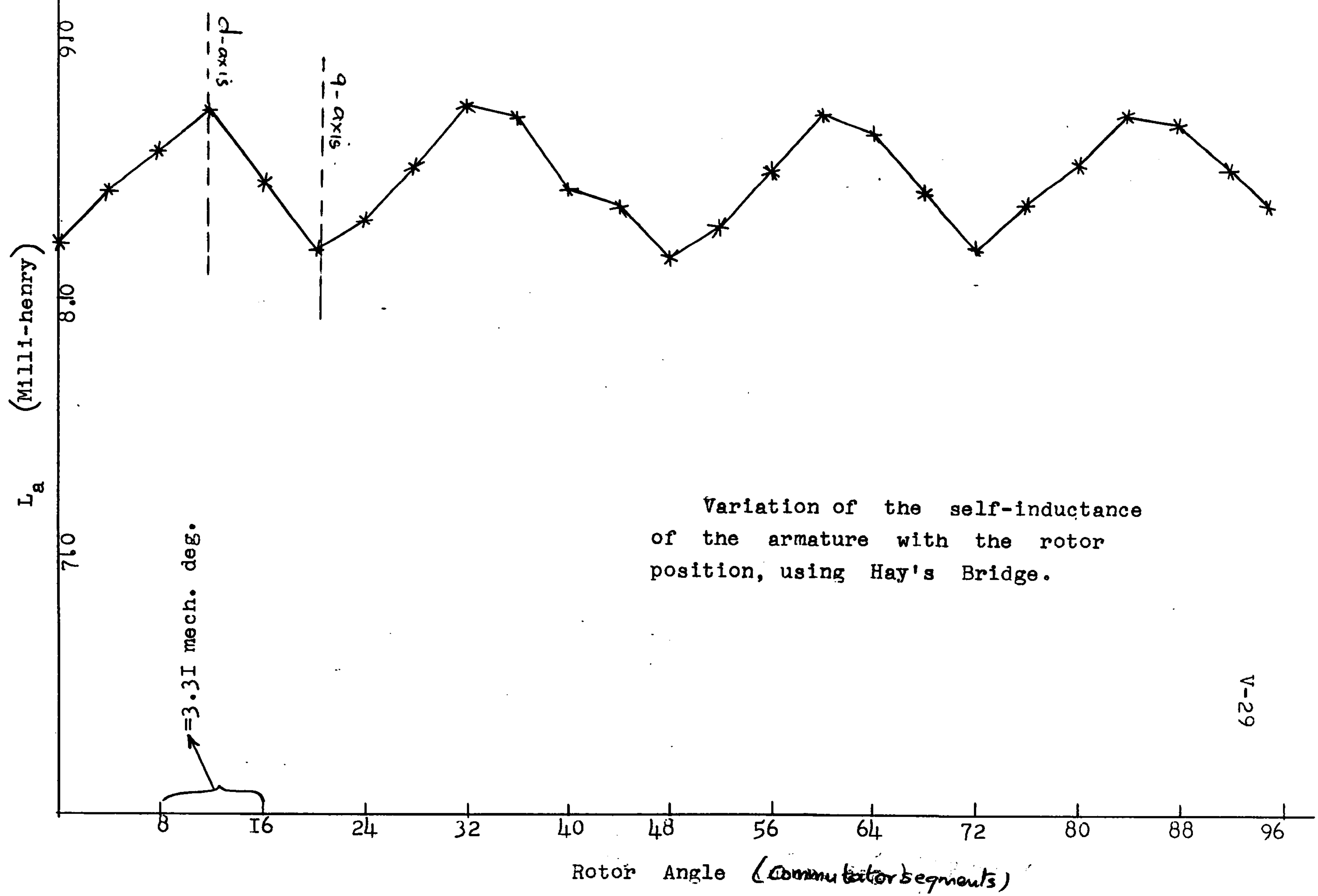
Arm. Current (A)	R_2 Ohms	R_3 Ohms	C F	$L_{(a+i)}$ Millihenries
0.08	1238	3.00	1.4	5.2
0.1	977	3.00	2.0	5.86
0.26	507	2.0	6.0	6.15
0.5	206	2.57	12.5	6.61
1.0	101.3	4.13	17.0	7.1
2.0	50.5	9.3	18.0	8.5
3.0	32.1	12.35	22.7	8.75
4.0	23.8	15.25	24.0	8.71
5.0	19.23	19.65	23.5	8.9
6.0	15.56	23.42	24.0	8.75
7.0	13.18	26.63	25.5	8.95
8.0	11.32	30.53	25.5	8.85
10.0	8.63	39.9	26.5	8.95



Variation of self inductance of armature plus interpoles with armature current, using Hay's bridge.

Variation of self inductance of the armature with the Rotor position using Hay's Bridge: Armature Current = 8^{A}. No carbon brushes.

R_2 Ohms	R_3 Ohms	C μF	Rotor Position	La. mH
35.32	13.65	17	0	8.2
"	14.0	"	4	8.4
"	14.27	"	8	8.56
"	14.51	"	12	8.71
"	14.04	"	16	8.44
"	13.12	"	20	8.18
"	13.6	"	24	8.29
"	14.15	"	28	8.5
"	14.55	"	32	8.74
"	14.5	"	36	8.70
"	14.0	"	40	8.41
"	13.95	"	44	8.36
"	13.59	"	48	8.15
"	13.79	"	52	8.27
"	14.15	"	56	8.5
"	14.51	"	60	8.71
"	14.38	"	64	8.64
"	14.0	"	68	8.41
"	13.6	"	72	8.18
"	13.95	"	76	8.36
"	14.2	"	80	8.52
"	14.5	"	84	8.70
"	14.45	"	88	8.67
"	14.15	"	92	8.5
"	13.6	"	95	8.18



Variation of the self-inductance of the armature with the rotor position, using Hay's Bridge.

Variation of Armature plus Interpoles self inductance with the field current (<u>using Hay's Bridge</u>)

Armature Current = 1 Ampere

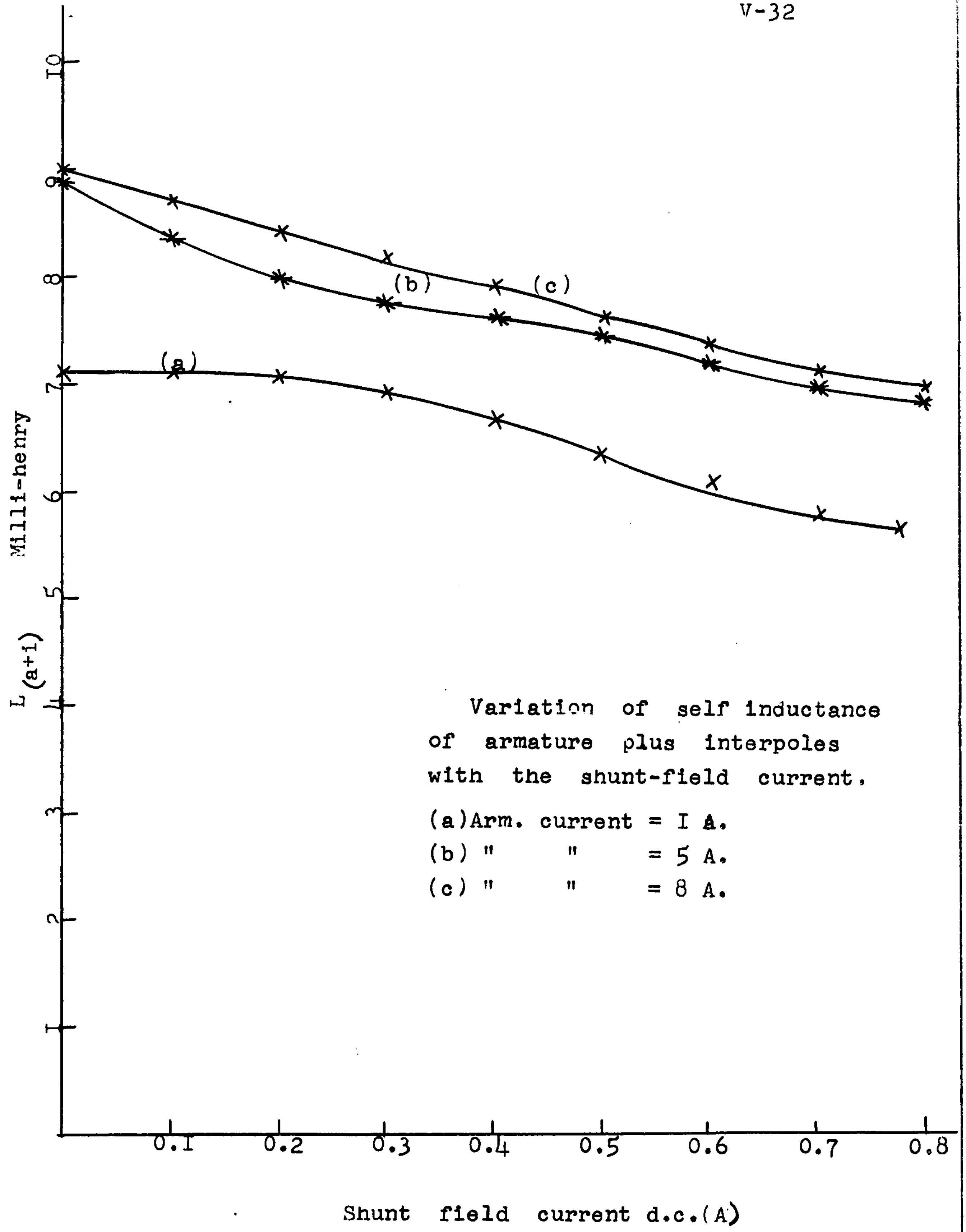
I_f (A)	R_2 (Ohms)	R_3 (Ohms)	R_4 (Ohms)	C (μF)	$L_{(a+i)}$ (Millihenries)
0	101.7	3.32	53	21	7.1
0.2	"	3.3	"	"	7.06
0.3	"	3.22	"	"	6.89
0.4	"	3.083	"	"	6.6
0.5	"	2.94	"	"	6.29
0.6	"	2.83	"	"	6.05
0.7	"	2.674	"	"	5.73
0.77	"	2.61	"	"	5.59

Armature Current = 5 Amperes

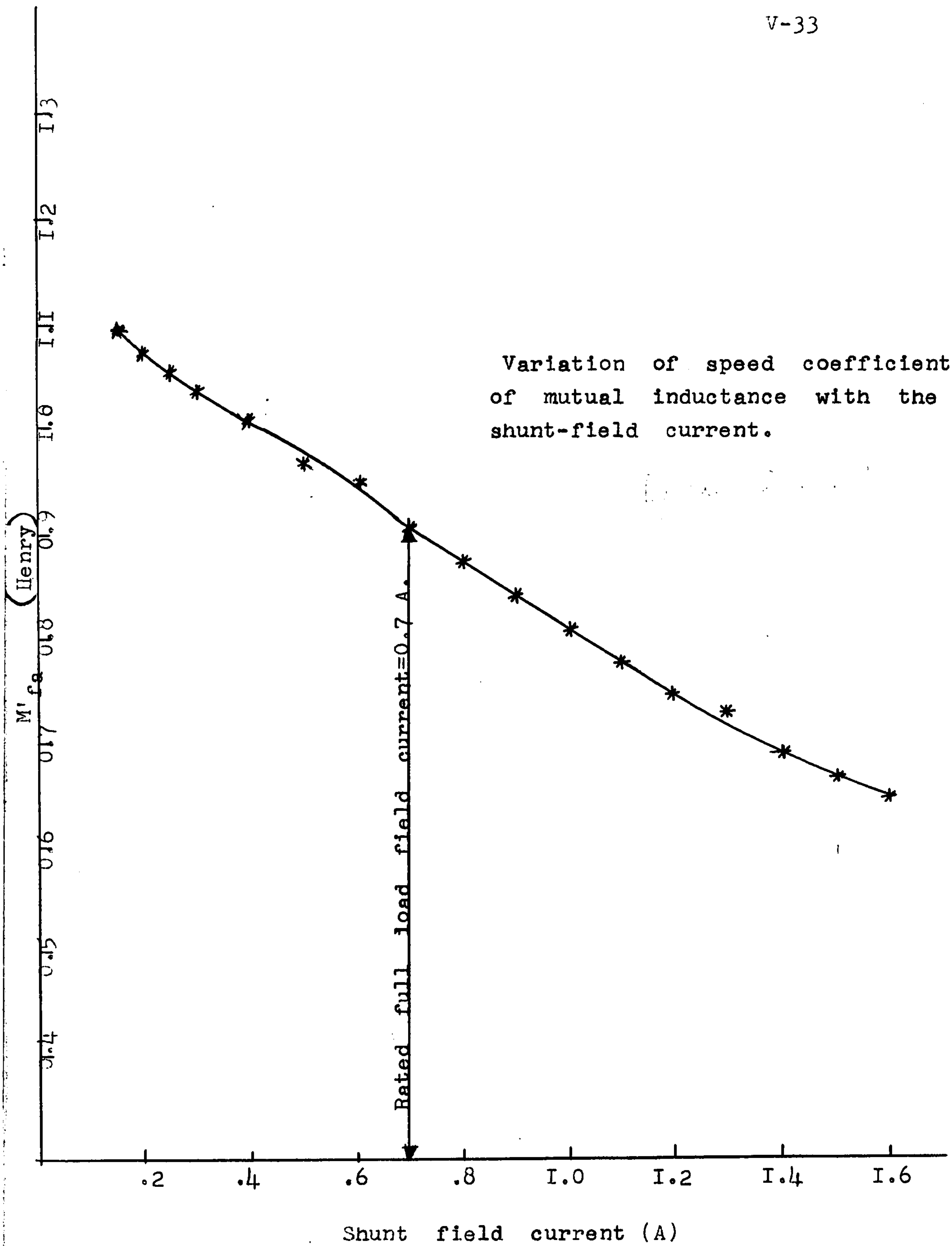
I_f (A)	R_2 (Ohms)	R_3 (Ohms)	R_4 (Ohms)	C (μF)	$L_{(a+i)}$ (Millihenries)
0	18.9	20.45	53	23	8.9
0.2	"	20.26	"	"	7.92
0.3	"	20.04	"	"	7.76
0.4	"	19.67	"	"	7.60
0.5	"	19.11	"	"	7.4
0.6	"	18.41	"	"	7.14
0.7	"	18.04	"	"	7.0
0.77	"	17.65	"	"	6.83

Armature Current = 10 Amperes

I_f (A)	R_2 (Ohms)	R_3 (Ohms)	R_4 (Ohms)	C (μF)	L(a+i) (Millihenries)
0	8.46	40.23	53	26.3	8.95
0.15	"	37.92	"	"	8.42
0.20	"	37.22	"	"	8.3
0.3	"	36.75	"	"	8.79
0.4	"	35.47	"	"	7.9
0.5	"	34.14	"	"	7.6
0.6	"	32.95	"	"	7.34
0.7	"	32.35	"	"	7.2
0.77	"	31.14	"	"	6.96



$L_{(a+i)}$ Milli-henry
10
9
8
7
6
5
4
3
2
1
(a)
(b)
(c)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Variation of self inductance of armature plus interpoles with the shunt-field current.
(a) Arm. current = I A.
(b) " " = 5 A.
(c) " " = 8 A.
Shunt field current d.c.(A)



Variation of speed coefficient of mutual inductance with the shunt-field current.
M'fa (Henry)
Rated full load field current=0.7 A.
.2
.4
.6
.8
I.0
I.2
I.4
I.6
Shunt field current (A)

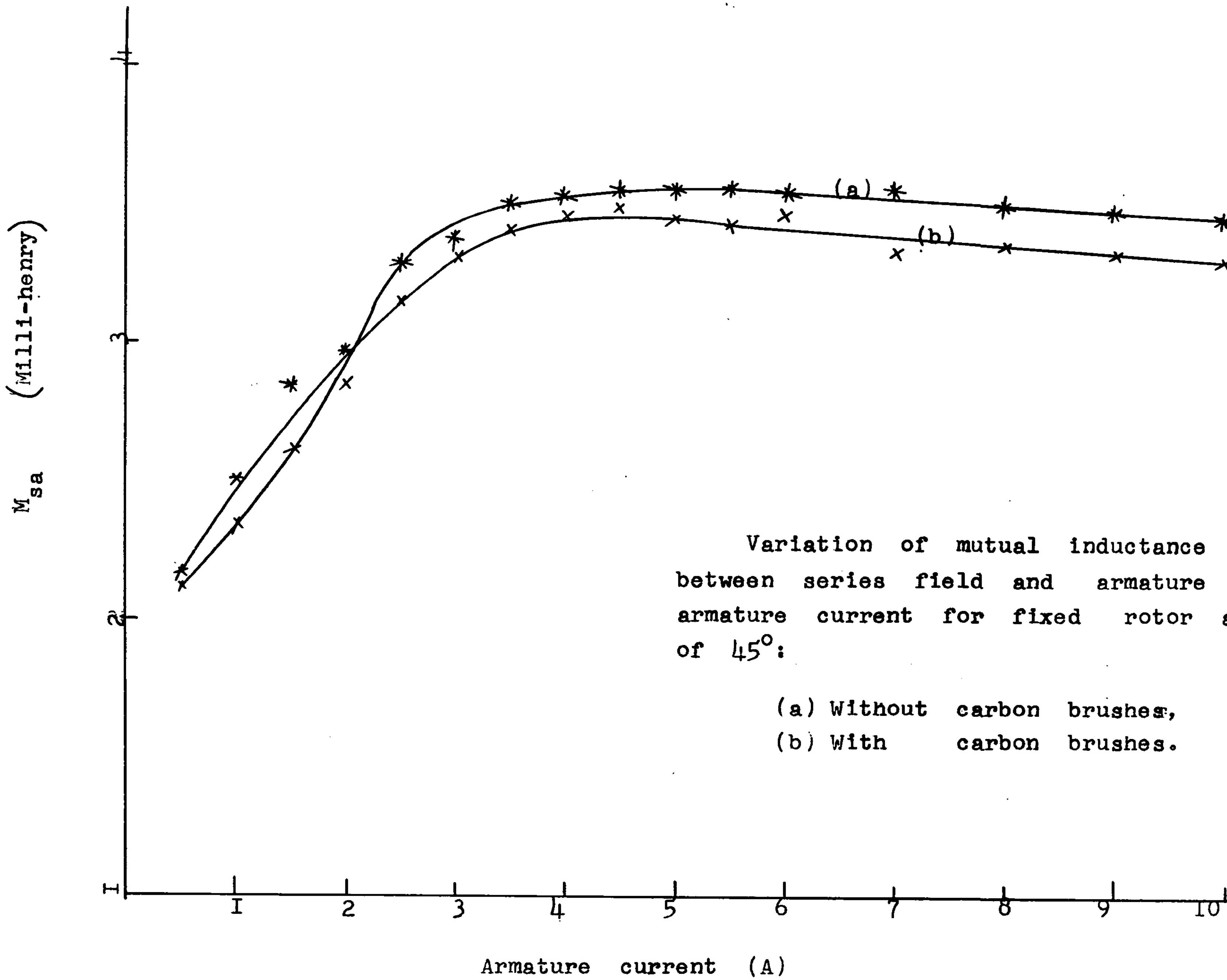
Variation of mutual inductance between armature and (i) shunt field (ii) series field with armature current, for fixed rotor angle of 45°, and without carbon brushes.

Armature voltage = 110 volts

I_a (A)	V_f (Volts)	Vs (Volts)	M_{fa} (Henry)	M_{sa} (Millihenry)
0.5	27	0.4	0.143	2.12
1.0	61	0.9	0.162	2.39
1.5	100	1.49	0.1765	2.63
2.0	143	2.14	0.1895	2.84
2.5	207	3.1	0.2195	3.29
3.0	245	3.8	0.227	3.36
3.5	306	4.5	0.232	3.41
4.0	352	5.3	0.233	3.51
4.5	400	6.0	0.236	3.53
5.0	445	6.7	0.236	3.55
6.0	536	8.05	0.237	3.56
7.0	620	9.3	0.235	3.56
8.0	700	10.5	0.222	3.48
9.0	776	11.72	0.229	3.46
10.0	841	12.6	0.223	3.44

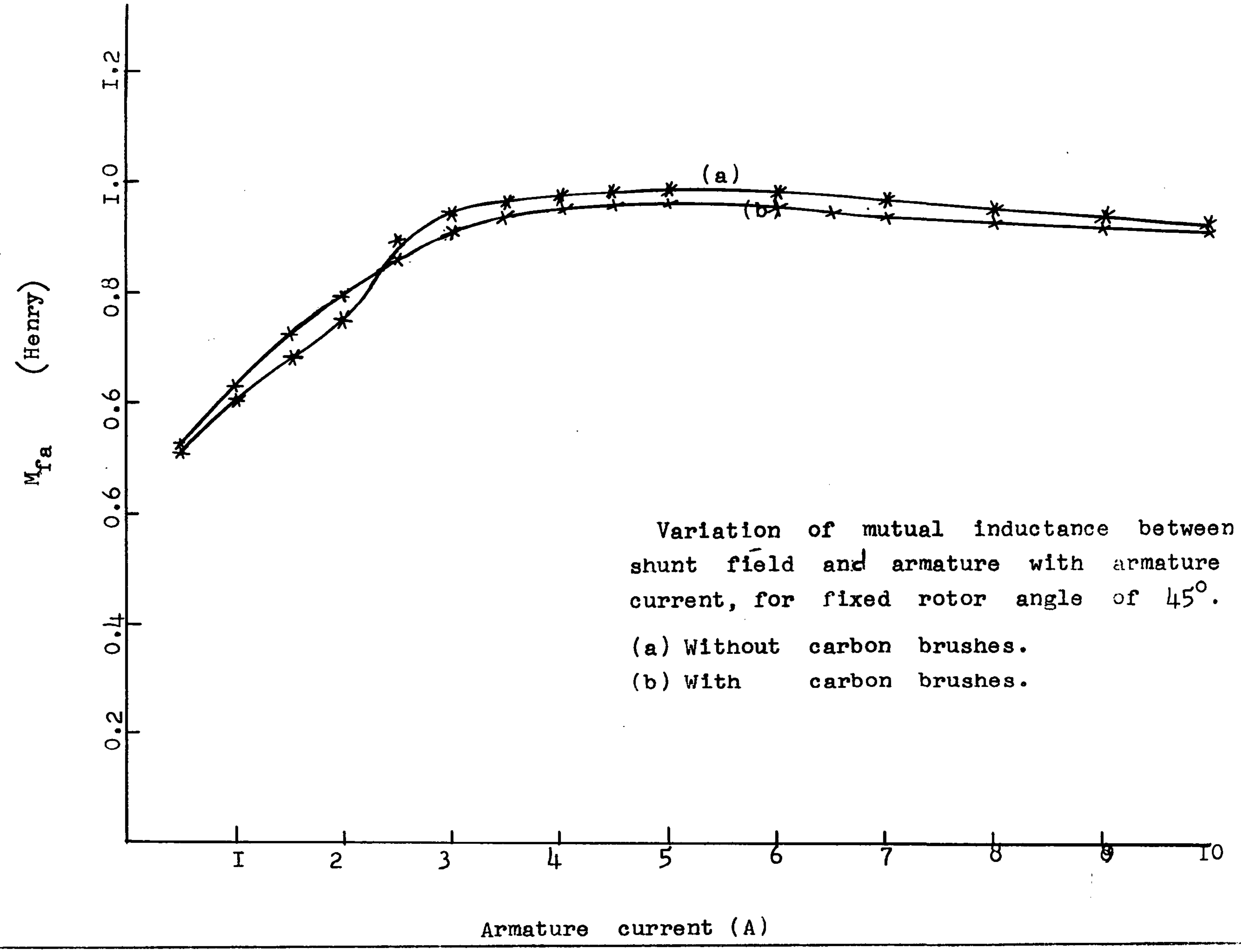
Variation of mutual inductance between armature and (i) shunt field (ii) series field with armature current, for fixed rotor angle of 45° and with carbon brushes.

I_a (A)	V_f (Volts)	V_s (Volts)	M_{fa} (Henry)	M_{sa} (Millihenry)
0.5	27.3	0.405	0.145	2.14
1.0	62.8	0.95	0.1659	2.52
1.5	104	1.56	0.185	2.86
1.0	150	2.24	0.198	3.09
2.5	205	2.95	0.218	3.13
3.0	250	3.75	0.221	3.32
3.5	298	4.5	0.226	3.41
4.0	350	5.37	0.232	3.56
4.5	392	5.95	0.231	3.51
5.0	440	6.65	0.2335	3.53
6.0	525	7.8	0.236	3.52
8.0	688	10.1	0.228	3.32
9.0	766	11.2	0.226	3.31
10.0	840	12.4	0.223	3.29



Variation of mutual inductance between series field and armature with armature current for fixed rotor angle of 45^{o}:

(a) Without carbon brushes,
(b) With carbon brushes.



Variation of mutual inductance between shunt field and armature with armature current, for fixed rotor angle of 45°.

(a) Without carbon brushes.

(b) With carbon brushes.

Variation of mutual inductance between (i) shunt field and armature (ii) series field and armature with the rotor position and without carbon brushes, for armature current = 2^A.

Comm. Segment	Vshunt Volts	V series Volts	Mfa Henry	Msa Millihenry	Arm. Voltage
0	160	2.4	0.212	3.18	110
1	158	2.36	0.2005	3.13	"
2	155	2.37	0.2060	3.06	"
3	148	2.2	0.196	2.92	"
4	136	2.04	0.1802	2.7	"
5	127	1.81	0.1605	2.4	"
6	100	1.5	0.1325	1.98	"
7	78.3	1.27	0.104	1.685	"
8	61.0	0.91	0.0809	1.205	"
9	44.5	0.66	0.059	0.875	"
10	31.0	0.46	0.0412	0.67	"
11	16.0	0.24	0.0212	0.318	"
12	6.04	0.02	-0.0053	- 0.0265	"
13	14.5	0.22	- 0.0192	- 0.31	"
14	30.0	0.45	- 0.0398	- 0.596	"
15	45.0	0.66	- 0.0596	-0.875	"
16	62	0.92	- 0.0823	-1.22	"
17	79	1.15	- 0.105	-1.525	"
18	100	1.5	- 0.1325	-1.99	"
19	120	1.8	- 0.159	-2.49	"
20	135	2.03	- 0.179	-2.69	"
21	146	2.2	- 0.1935	-2.92	"
22	155	2.31	- 0.206	-3.06	"
23	158	2.36	- 0.2095	-3.13	"
24	160	2.[illegible]2	- 0.2[illegible]	-3.27	"

Variation of mutual inductance between (i) shunt field and armature, (ii) series field and armature with the rotor position, for armature current = 5A and without carbon brushes.

Comm. Segments	V_f Volts	Vs Volts	M_{fa} Henry	Msa (Millihenry)	Arm. Va Volts
0	445	6.7	0.236	3.55	110V
1	443	6.5	0.235	3.45	110V
2	415	6.21	0.22	3.30	"
3	385	5.95	0.204	3.16	"
4	365	5.65	0.1935	3.00	"
5	330	5.20	0.175	2.76	"
6	280	4.40	0.1485	2.33	"
7	230	3.4	0.122	1.805	"
8	180	2.65	0.0955	1.305	"
9	131	1.96	0.0695	1.04	"
10	87	1.31	0.0461	0.695	"
11	41	0.6	0.02175	0.318	"
12	4	0.05	- 0.00212	- 0.0365	"
13	41	0.6	- 0.02175	- 0.318	"
14	86	1.3	- 0.0456	- 0.69	"
15	130	1.95	- 0.069	- 1.035	"
16	179	2.60	- 0.095	- 1.33	"
17	231	3.42	- 0.1225	- 1.812	"
18	281	4.41	- 0.1485	- 2.34	"
19	330	5.2	- 0.175	- 2.76	"
20	365	5.65	- 0.1935	- 3.00	"
21	385	5.95	- 0.204	- 3.16	"
22	415	6.21	- 0.22	- 3.3	"
23	443	6.5	- 0.235	- 3.45	"
24	445	6.7	- 0.236	- 3.55	"

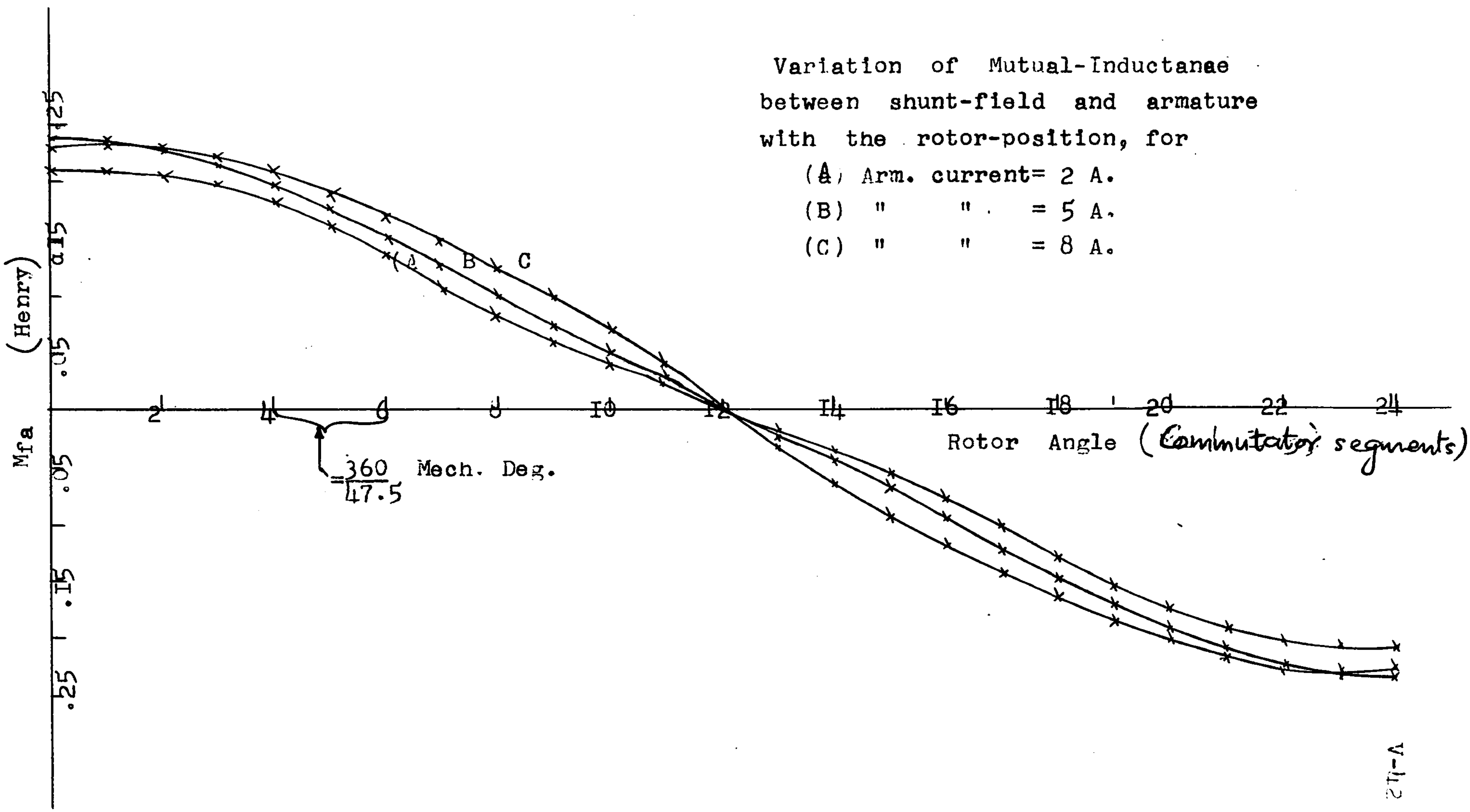
Note:- Column (1) represents the width of commutator segments.

They can be transformed into degrees as follows:-

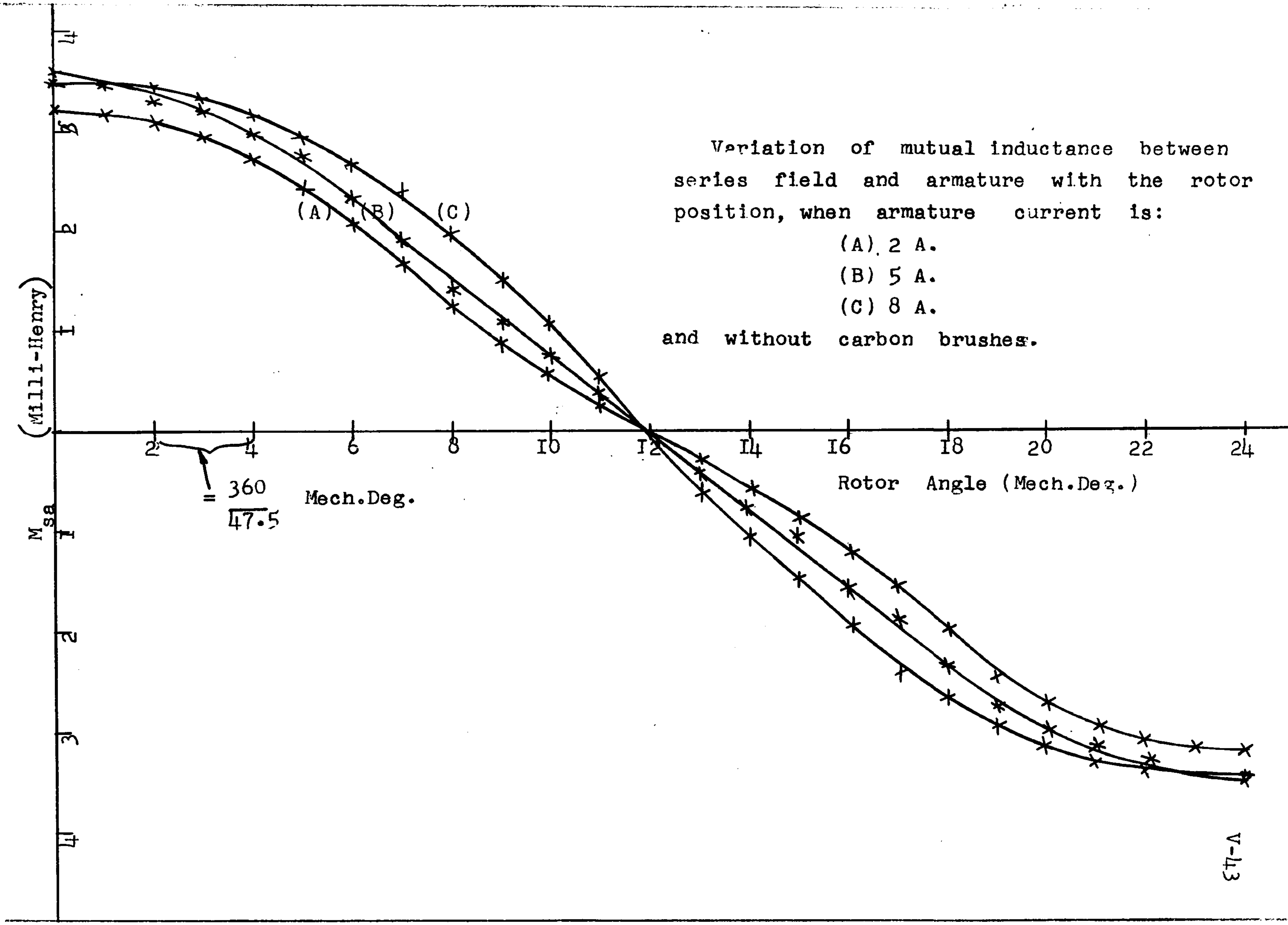
Since there are 95 commutator sigments, each one of them correspond to $\frac{360^\circ}{95}$. Thus, the difference between M at 1 and M at (2), will mean the variation of 'M' corres. to $\frac{360^\circ}{95}$.

Variation of mutual inductance between (i) shunt field and armature, (ii) series field and armature, with the rotor position, for armature current = 8^A and without carbon brushes.

	V_f (Volts)	V_s (Volts)	Mfa in (Henries)	Msa (Millihenries	Va Volts
0	700	10.5	0.232	3.48	110
1	690	10.4	0.229	3.45	"
2	681	10.3	0.226	3.415	"
3	665	10.0	0.217	3.32	"
4	614	9.5	0.203	3.15	"
5	564	8.9	0.187	2.95	"
6	498	8.7	0.165	2.65	"
7	437	7.25	0.145	2.4	"
8	356	6.0	0.118	1.99	"
9	233	4.5	0.094	1.491	"
10	196	3.2	0.065	1.061	"
11	99.5	2.0	0.033	0.664	"
12	- 18.1	- 1.0	- 0.006	- 0.0332	"
13	- 99.5	- 2.0	- 0.033	- 0.664	"
14	- 196	- 3.2	- 0.065	- 1.06	"
15	- 286	- 4.49	- 0.095	- 1.49	"
16	- 359	- 6.0	- 0.119	- 1.99	"
17	- 444	- 7.25	- 0.147	- 2.4	"
18	- 498	- 8.7	- 0.165	- 2.65	"
19	- 559	- 8.88	- 0.185	- 2.94	"
20	- 614	- 9.5	- 0.203	- 3.15	"
21	- 665	-10.0	- 0.214	- 3.32	"
22	-681	-70.3	- 0.226	- 3.415	"
23	- 690	-10.4	- 0.229	- 3.45	"
24	- 700	-10.5	- 0.232	- 3.48	"


Variation of Mutual-Inductanae
between shunt-field and armature
with the rotor-position, for
(A) Arm. current= 2 A.
(B) " " = 5 A.
(C) " " = 8 A.
A
B
C
M_{fa}
(Henry)
.25
.15
.05
.05
.15
.25
2
4
6
8
10
12
14
16
18
20
22
24
Rotor Angle (Commutator segments)
$=\frac{360}{47.5}$ Mech. Deg.

V-42



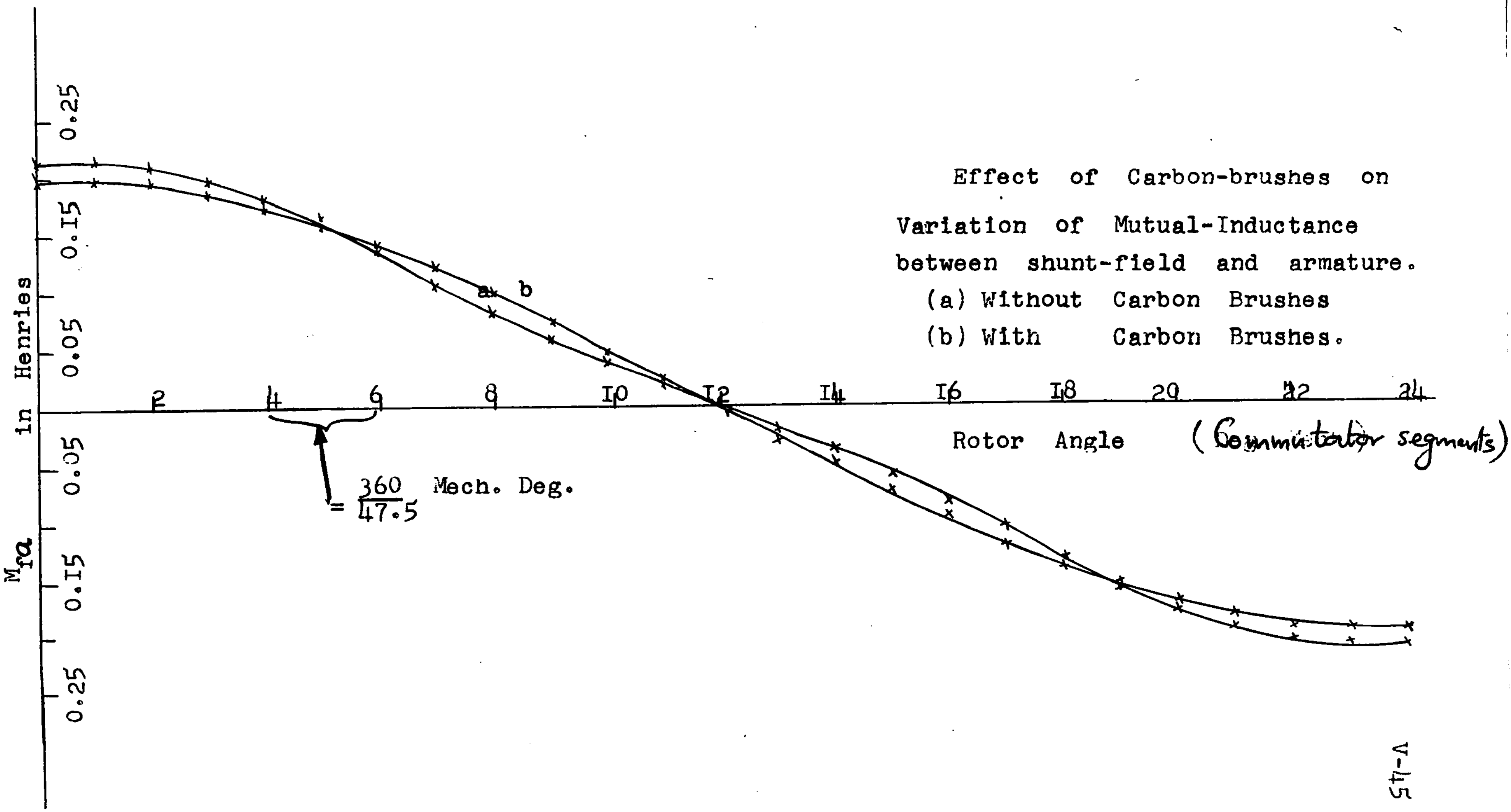
Variation of mutual inductance between series field and armature with the rotor position, when armature current is:

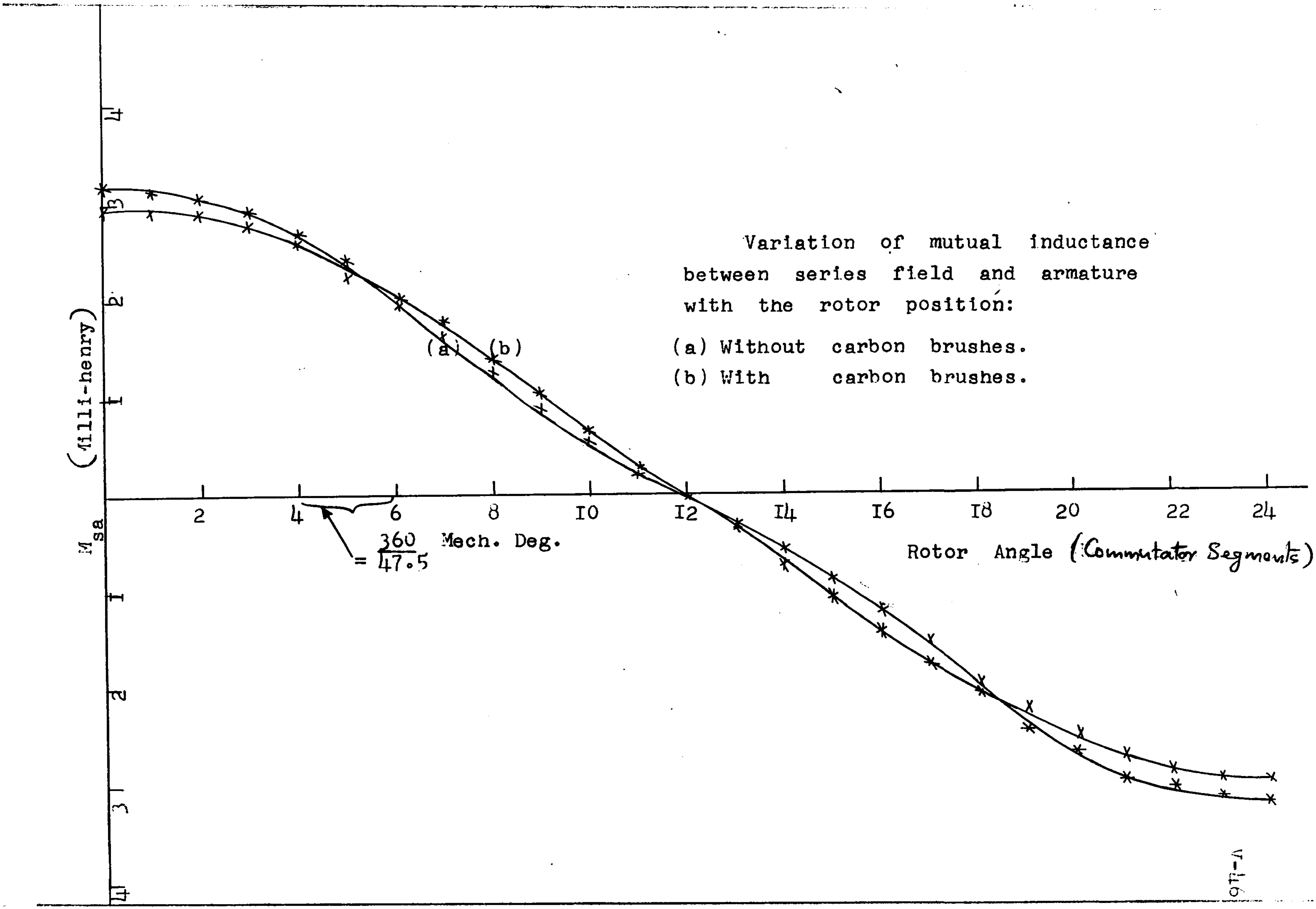
(A) 2 A.
(B) 5 A.
(C) 8 A.

and without carbon brushes.

Effect of Carbon brushes on mutual inductance between (i) shunt-field and armature, (ii) series-field and armature, for armature current = 2^A.

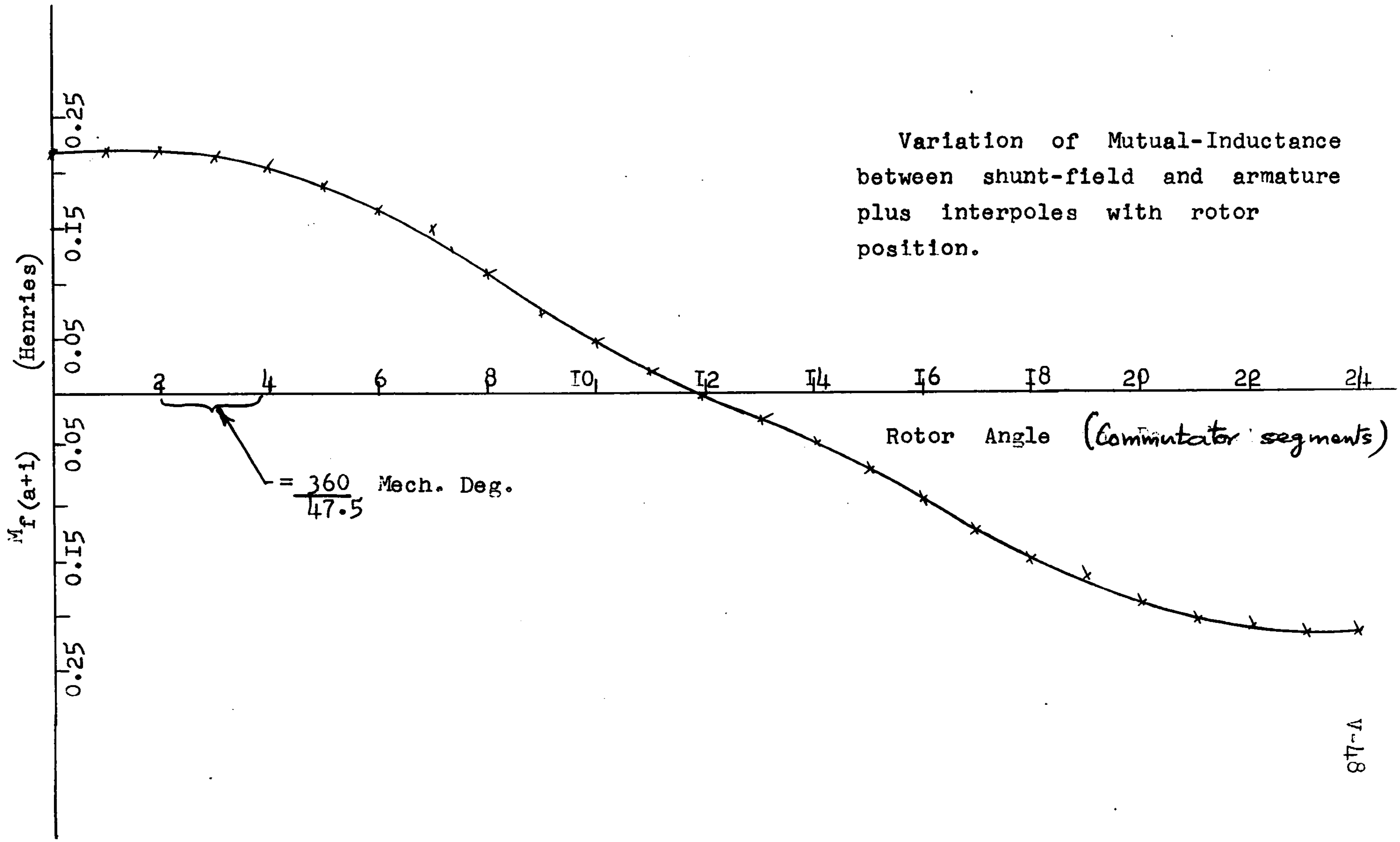
Comm. Segments	No carbon brushes	Mfa Henries With Carbon Brushes	No Carbon brushes	Msa Millihenry With carbon brushes
0	0.212	0.198	3.18	2.98
1	0.2095	0.196	3.13	2.95
2	0.206	0.1925	3.06	2.89
3	0.196	0.186	2.92	2.85
4	0.1802	0.171	2.7	2.60
5	0.1605	0.1565	2.4	2.25
6	0.1325	0.143	1.98	2.05
7	0.104	0.119	1.685	1.75
8	0.0809	0.0929	1.205	1.4
9	0.059	0.0705	0.875	1.05
10	0.0412	0.0465	0.61	0.7
11	0.0212	0.0194	0.318	0.3
12	- 0.0053	- 0.00259	-0.0265	- 0.0266
13	- 0.0192	- 0.0259	- 0.31	- 0.72
14	- 0.0398	- 0.046	- 0.596	- 0.7
15	- 0.0596	- 0.0705	- 0.875	- 1.05
16	- 0.0823	- 0.0929	- 1.22	- 1.4
17	- 0.105	- 0.119	- 1.525	- 1.75
18	- 0.1325	- 0.143	- 1.99	- 2.05
19	- 0.159	- 0.155	- 2.49	- 2.75
20	- 0.179	-0. 17	- 2.69	- 2.65
21	- 0.1935	-0.184	- 2.92	- 2.75
22	- 0.206	- 0.1925	- 3.06	- 2.85
23	- 0.2095	- 0.1975	- 3.13	- 2.95
24	- 0.212	- 0.199	- 3.21	- 2.9[illegible]


Effect of Carbon-brushes on
Variation of Mutual-Inductance
between shunt-field and armature.
(a) Without Carbon Brushes
(b) With Carbon Brushes.
a
b
Rotor Angle
(Commutator segments)
= 360/47.5 Mech. Deg.
M_fa in Henries
0.25
0.15
0.05
0.05
0.15
0.25
2
4
6
8
10
12
14
16
18
20
22
24

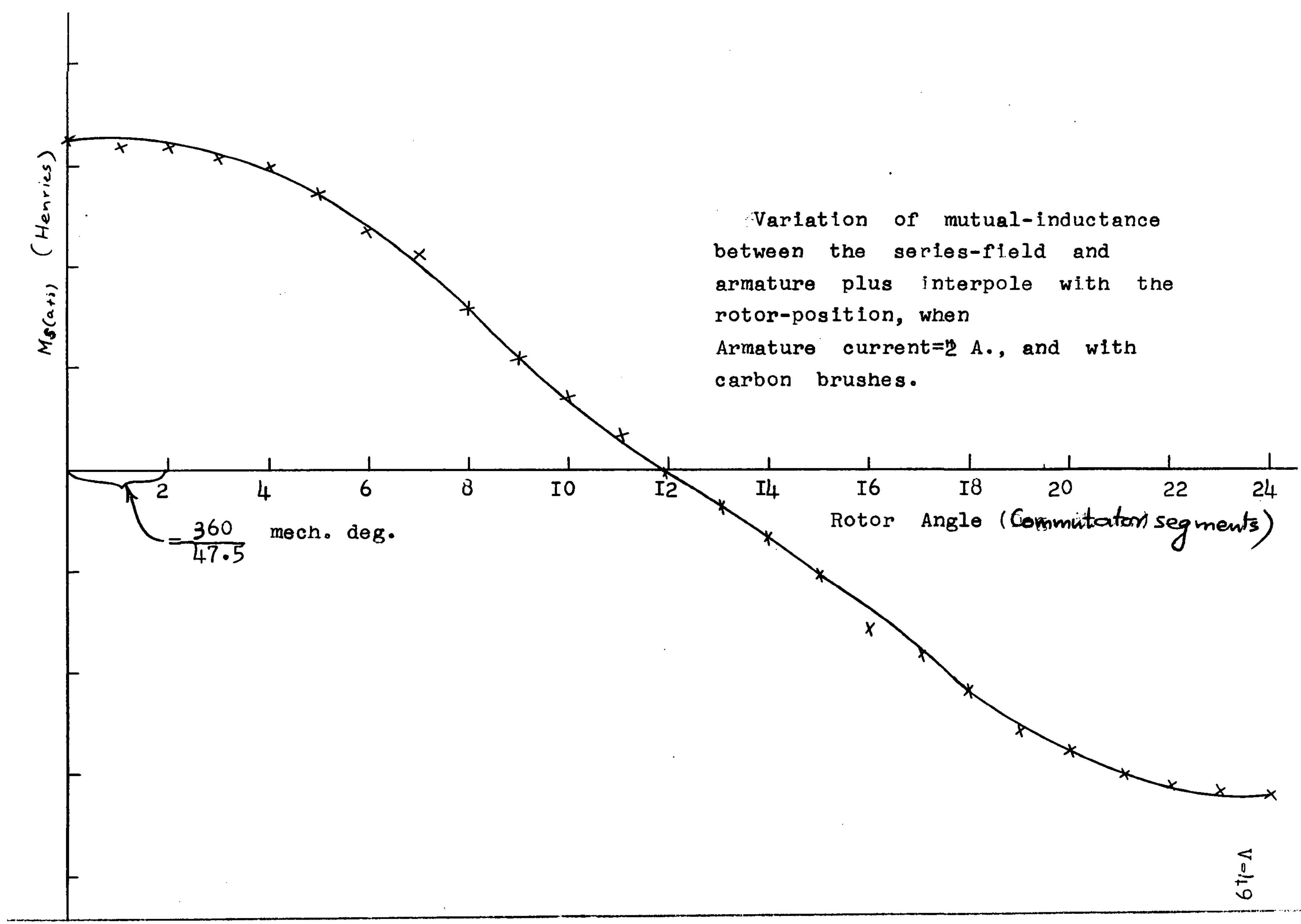

Variation of mutual inductance between series field and armature with the rotor position:
(a) Without carbon brushes.
(b) With carbon brushes.
(a)
(b)
M_{sa} (Milli-henry)
4
3
2
1
1
2
3
4
2
4
6
8
I0
I2
I4
I6
I8
20
22
24
= $\frac{360}{47.5}$ Mech. Deg.
Rotor Angle (Commutator Segments)

Variation of Mutual Inductance between (Armature + Interpoles) and (i) Shunt Field (ii) series field for Armature current = 2^A and with carbon brushes.

Comm Segments	V_f Volts	V_s Volts	M_f(ati) Henry	M_s(a+i) Millihenry
0	163	2.43	0.217	3.22
1	162.5	2.42	0.216	3.21
2	161	2.38	0.214	3.16
3	158	2.31	0.21	3.06
4	151	2.25	0.201	2.98
5	141	2.09	0.187	2.78
6	125	1.8	0.166	2.385
7	109	1.6	0.145	2.12
8	81	1.2	0.108	1.59
9	56	0.8	0.0745	1.06
10	35	0.53	0.0465	0.704
11	14.6	0.22	0.0194	0.292
12	1.5	0.09	- 0.00199	- 0.0119
13	19.5	0.25	- 0.0259	- 0.332
14	34.5	0.5	- 0.0458	- 0.665
15	53	0.78	- 0.0705	-1.035
16	73.5	1.19	- 0.0975	- 1.58
17	96	1.4	- 0.1275	- 1.86
18	114	1.65	- 0.1512	- 2.19
19	126	1.85	- 0.167	- 2.6
20	145	2.10	- 0.1925	- 2.785
21	152	2.25	- 0.202	- 2.98
22	158	2.32	- 0.210	- 3.08
23	160	2.37	- 0.212	- 3.145
24	162.5	2.42	- 0.216	- 3.21



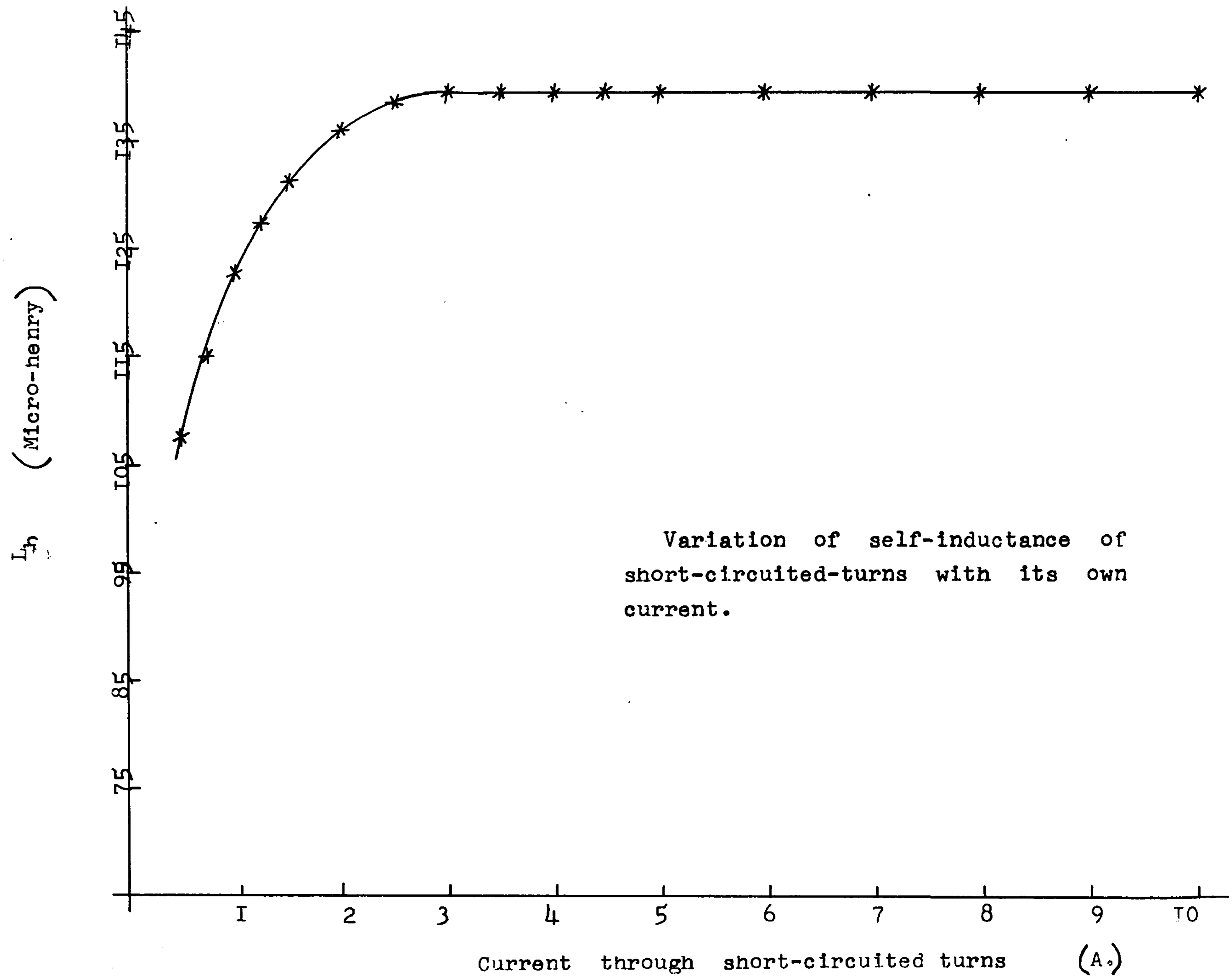
Variation of Mutual-Inductance between shunt-field and armature plus interpoles with rotor position.


Variation of mutual-inductance
between the series-field and
armature plus interpole with the
rotor-position, when
Armature current=2 A., and with
carbon brushes.
Ms(a+i) (Henries)
Rotor Angle (Commutator segments)
2
4
6
8
10
12
14
16
18
20
22
24
= 360/47.5 mech. deg.

Short-circuit turns self inductance using A.C. Impedance test:-

Field current = 0.7^A

$V_b = I_b Z_b$ Volts	I_b (A)	Z_b Ohms	R_b Ohms	L_b Microhenries
0.042	0.5	0.0835	0.06	107.5
0.086	1.0	0.086	"	122.5
0.13	1.5	0.088	"	131.5
0.18	2.0	0.089	"	135.1
0.225	2.5	0.090	"	139.0
0.276	3.0	0.092	"	140.0
0.322	3.5	"	"	"
0.368	4.0	"	"	"
0.414	4.5	"	"	"
0.450	5.0	"	"	"
0.552	6.0	"	"	"
0.644	7.0	"	"	"
0.735	8.0	"	"	"
0.828	9.0	"	"	"
0.92	10.0	"	"	"



Variation of self-inductance of short-circuited-turns with its own current.

Commutation Characteristics:-

Calculations for equs. (28) & (29):- Page:

Equ. (23) :-

$$\frac{i^b}{i^a} = 5.35 - 6.35\, e^{-0.463(\frac{1}{2}+t/T_c)} \quad \text{for } R_b = 0.06\ \Omega$$

With

$\frac{t}{T_c} = +0.05$; $\frac{i^b}{i^a} = 5.35 - 6.35\, e^{-0.463 \times 0.55} = +0.21$

$\frac{t}{T_c} = +0.1$; $i^b/i^a = 5.35 - 6.35\, e^{-0.463 \times 0.6} = +0.30$

$= +0.2$; $= 5.35 - 6.35\, e^{-0.463 \times 0.7} = +0.48$

$= +0.3$; $= 5.35 - 6.35\, e^{-0.463 \times 0.8} = +0.69$

$= +0.4$; $= 5.35 - 6.35\, e^{-0.463 \times 0.9} = +0.82$

$= +0.5$; $= 5.35 - 6.35\, e^{-0.463} = +0.97$

$\frac{t}{T_c} = -0.05$; $\frac{i^b}{i^a} = 5.35 - 6.35\, e^{-0.463 \times 0.45} = +0.05$

$= -0.1$; $= 5.35 - 6.35\, e^{-0.463 \times 0.4} = -0.04$

$= -0.2$; $= 5.35 - 6.35\, e^{-0.463 \times 0.3} = -0.22$

$= -0.3$; $= 5.35 - 6.35\, e^{-0.463 \times 0.2} = -0.44$

$= -0.4$; $= 5.35 - 6.35\, e^{-0.463 \times 0.1} = -0.65$

$= -0.5$; $= 5.35 - 6.35\, e^{0} = -1$

Equ. (29):-

$$\frac{i^b}{i^a} = 3.29 - 4.29\ e^{-0.77(\frac{1}{2} + \frac{t}{Tc})} \quad \text{for } R_b = 0.1$$

$t/_{Tc} = +0.05$ $\quad \frac{i^b}{i^a} = 3.29 - 4.29\ e^{-0.77 \times 0.55} = +0.18$

$= +0.1$ $\quad = 3.29 - 4.29\ e^{-0.77 \times 0.6} = +0.26$

$= +0.2$ $\quad = 3.29 - 4.29\ e^{-0.77 \times 0.7} = +0.45$

$= +0.3$ $\quad = 3.29 - 4.29\ e^{-0.77 \times 0.8} = +0.64$

$= +0.4$ $\quad = 3.29 - 4.29\ e^{-0.77 \times 0.9} = +0.795$

$= +0.5$ $\quad = 3.29 - 4.29\ e^{-0.77} = +0.92$

$\frac{t}{TC} = -0.05$ $\quad = 3.29 - 4.29\ e^{-0.45 \times 0.77} = +0.05$

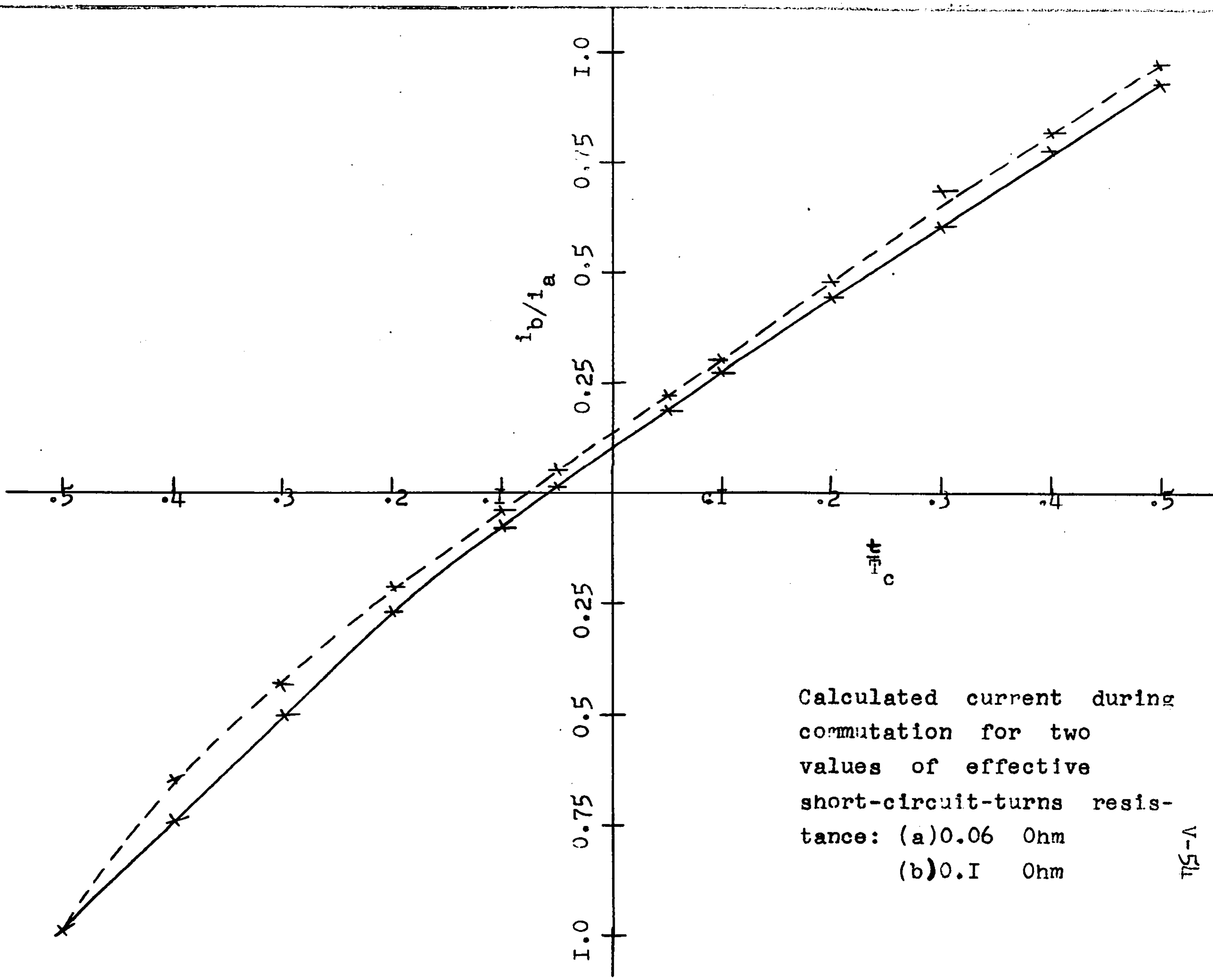
$= -0.1$ $\quad = 3.29 - 4.29\ e^{-0.4 \times 0.77} = -0.08$

$= -0.2$ $\quad = 3.29 - 4.29\ e^{-0.3 \times 0.77} = -0.27$

$= -0.3$ $\quad = 3.29 - 4.29\ e^{-0.2 \times 0.77} = -0.51$

$= -0.4$ $\quad = 3.29 - 4.29\ e^{-0.1 \times 0.77} = -0.73$

$= -0.5$ $\quad = 3.29 - 4.29\ e^{0} = -1$



Calculated current during commutation for two values of effective short-circuit-turns resistance: (a)0.06 Ohm
(b)0.I Ohm

Calculations for the moment of Inertia:-

(i) Using area method:-

At 1800 rpm, = 188.4 rad/sec.

Total Area under curve = $2043.6 \times \frac{550}{746}$ ft.lb.

$\therefore \frac{1}{2} J\omega^2 = 2043 \times \frac{550}{746}$

or J = 2.82 lb ft.^{2}

At 1200 rpm:- Area under curve = 884.3 W-Sec.

J = 2.7 lb ft.^{2}

At 600 rpm:- Area under curve = 217.7 W-Sec.

J = 2.68 lb ft.^{2}

(ii) Using mechanical test:-

m = 4.28 lb. h = 111.5"

n_1 = 44. revolutions; n_2 = 3.75 revolutions

$= \frac{2 n}{60} = \frac{2T \times 218}{60} = 22.82$ rad/sec.

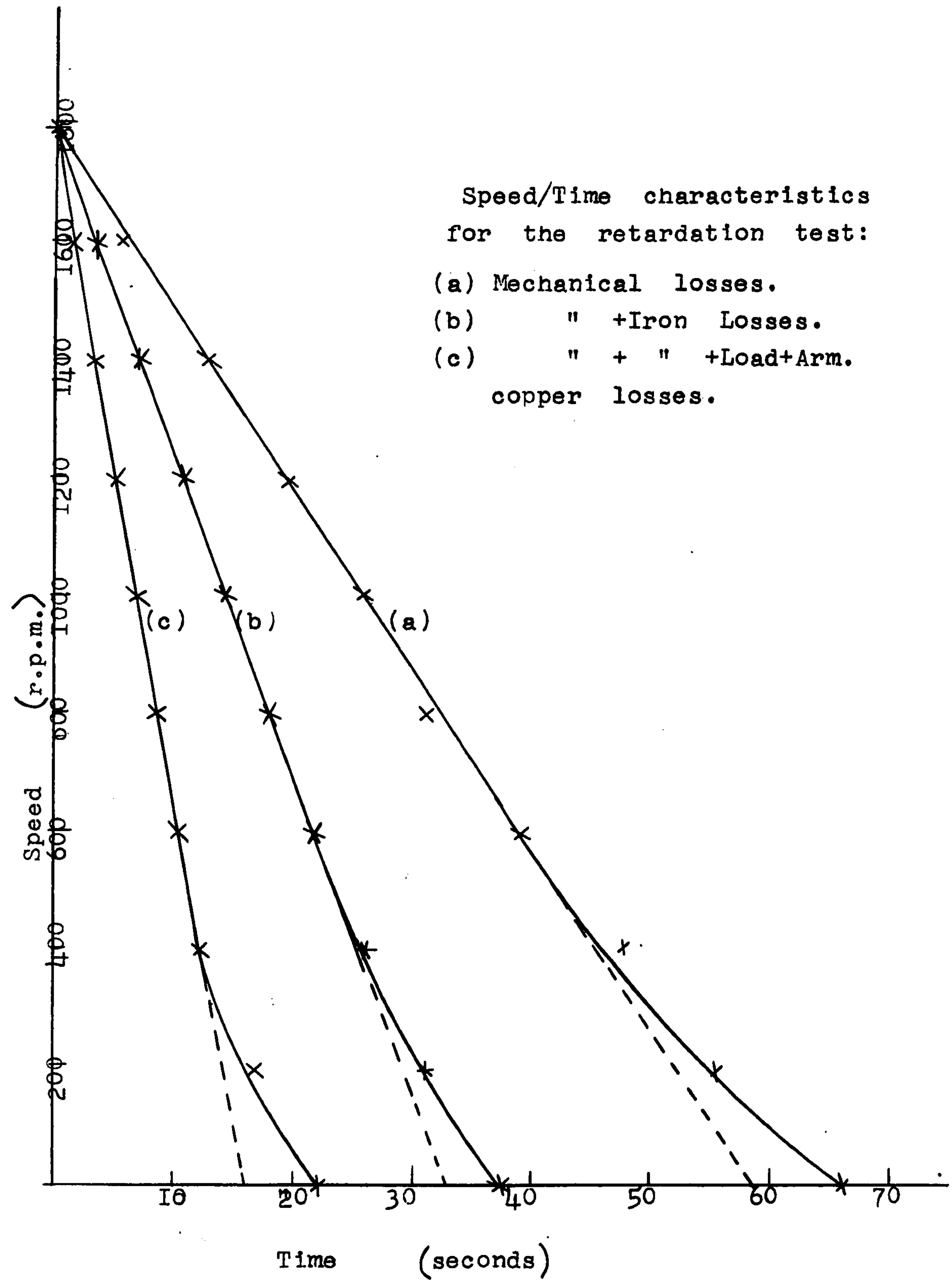
from equ: $mgh = \frac{1}{2} mV^2 + \frac{1}{2} Jw^2(1 + \frac{n_1}{n_2})$

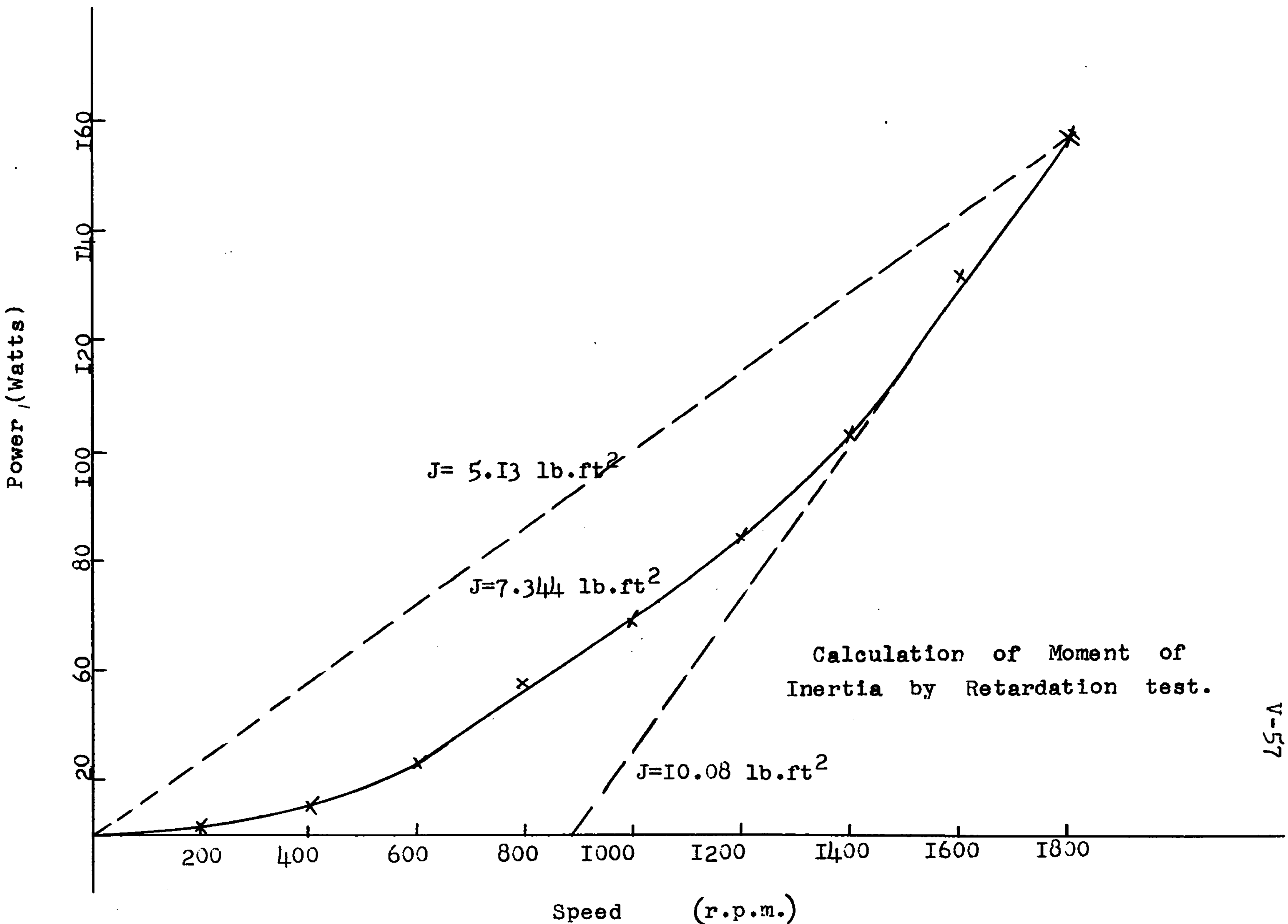
$\frac{1}{2} Jw^2 = 260.5$ J lb. ft.^{2}/sec.^{2}

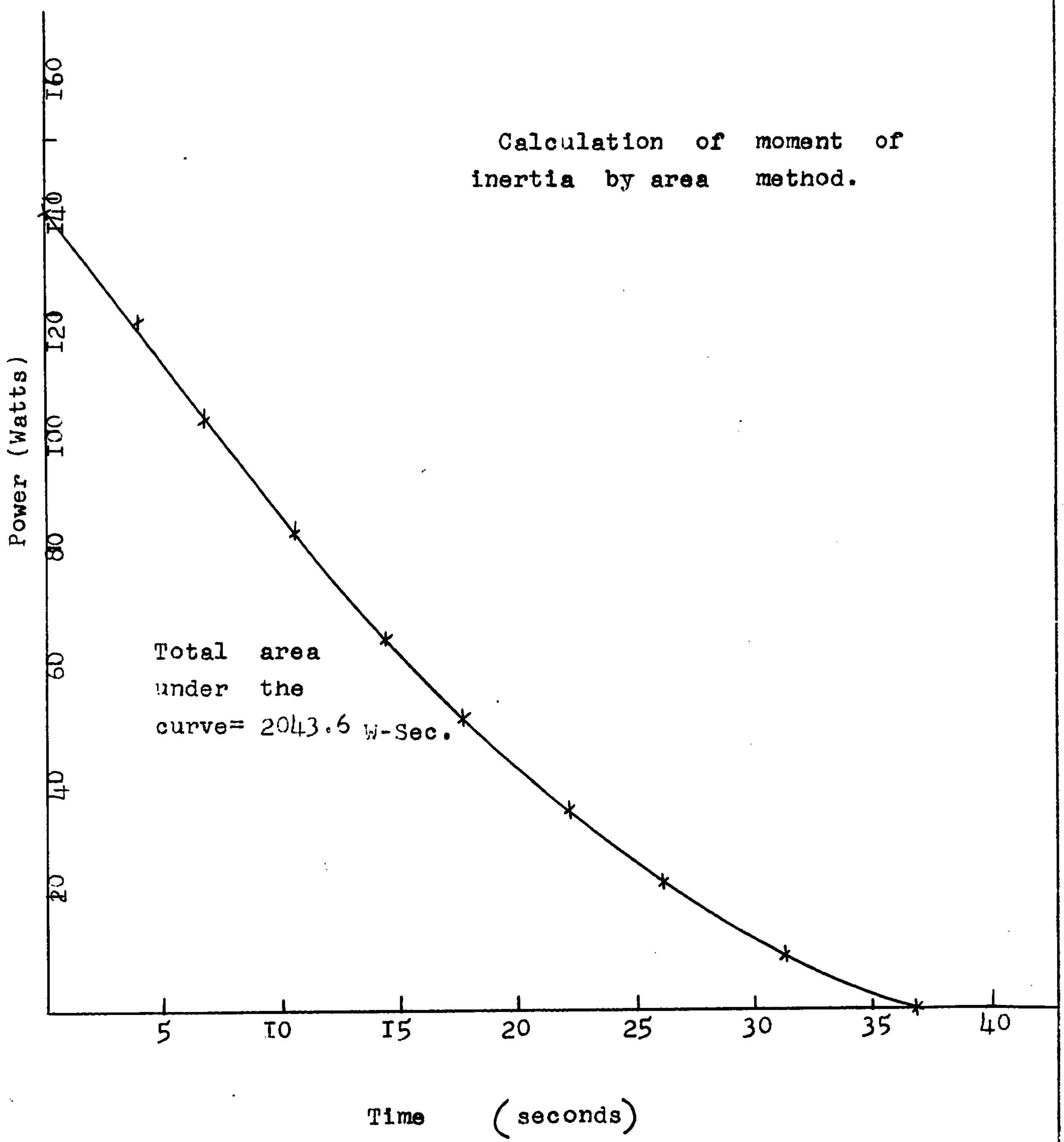
$(1 + n_1/n_2) = 1.094$

$\therefore$ J = 3.73 lb ft^2

or 0.1585 Kg-m^2


Speed/Time characteristics for the retardation test:
(a) Mechanical losses.
(b) " +Iron Losses.
(c) " + " +Load+Arm. copper losses.
1800
1600
1400
1200
1000
800
600
400
200
Speed (r.p.m.)
(a)
(b)
(c)
10
20
30
40
50
60
70
Time (seconds)


Calculation of Moment of
Inertia by Retardation test.
J= 5.I3 lb.ft2
J=7.344 lb.ft2
J=I0.08 lb.ft2
Power,(Watts)
20
60
80
I00
I20
I40
I60
200
400
600
800
I000
I200
I400
I600
I800
Speed (r.p.m.)


Calculation of moment of inertia by area method.
Total area under the curve= 2043.6 W-Sec.
Power (Watts)
160
140
120
100
80
60
40
20
5
10
15
20
25
30
35
40
Time (seconds)

CHAPTER VI

(1) Transients in the field and armature circuits of a d. c. separately-excited shunt generator.

(2) Tests on a separately-excited d. c. shunt motor.

The parameters of the d.c. machine under test have been obtained and, by using these, the problem of predicting transients can be solved. The transient analysis is made for the machine connected as a d.c. shunt generator and motor in turn. However, the method can readily be applied to a d.c. compound machine.

(1) Transients in the field and armature circuits of a d.c. generator:-

The machine was run as a d.c. shunt generator with brushes on the neutral axis for the test. The circuit employed is shown in fig. VI-1. If a step voltage is applied to the field circuit, transients occur in the field current, armature current and armature voltage. The step is applied by closing or opening the switch S_1 in fig. VI-1. The voltage across the field is a step function and is fed to one channel of a pen-recorder. Full-load field current is 0.7A. Two cases, in which the field current increases from (i) 0 to 0.3A (ii) 0.3A to 0.6A, are studied and the actual transients are recorded from experiment. Measured parameters are utilised to predict the rise or decay of the currents from initial to final steady-state conditions.

The transient equations for the field and armature current are developed in Appendix II and two equations employing sign conventions as explained in Appendix II, are as follows:-

$$e_f = -(r_f + L_f p)\, i^f \tag{1}$$

$$e_a = -(r_{a+i} + L_{a+i} p)\, i^a + M'_{fa} p\theta \; i^f \tag{2}$$

The solution of equation (1) is straight-forward, viz:-

$$i^f = -\frac{E_f}{r_f}(1 - e^{-t/T_f}) + I^f(o)\, e^{-t/T_f} \tag{3}$$

For case (i), mentioned above, $I^f(o) = 0$. E_f is the step-voltage in the field circuit.

For an exact solution it is necessary to regard the time constant T_f as a variable. Having determined values of L_f for various excitations, a family of simple exponential curves for the different exciting currents could be plotted and the time constant for each of the curves determined. It would then be possible to select a value of T_f, solve for i^f at a given 't' and, by means of a transcendental solution, to obtain a true value of i^f for the given 't'. Such a procedure would be extremely laborious and the justification is questionable when the inherent errors due to hysteresis are considered.

A compromise between the simple solution that assumes a constant self-inductance and the exact solution, may be achieved without undue labour by using a computer with the output recorded by an x-y plotter. Selected values of inductance, and hence the time constant T_f, were used with an analogue computer.

The values of L_f substituted in equation (3) were incremental values of the shunt-field inductance corresponding to the initial and final steady-state field currents. The values 0, 0.3A, 0.6A were chosen because the curve of L_f against I^f changes slope at 0.3A.

It can be seen, referring to the characteristics on pages VI-9 to VI-12, that the error between the predicted characteristics of the transients and the actual one, is by no means large. The difference is attributed to the following reasons:-

(i) Equation (2) is simplified for linear conditions,

(ii) The measurements are made employing alternating current,

(iii) Experimental error.

With the initial conditions of zero, the equations of transient field current are found to be as follows:-

$$i^f = -\frac{E_f}{128}(1 - e^{-0.125t}), \text{ for the predicted transient}$$

$$i^f = -\frac{E_f}{128}(1 - e^{-0.117t}), \text{ for the actually-recorded transient.}$$

These equations are for the case when the field current changes from 0 to 0. 3A. For the case when it changes from 0. 3A to 0. 6A, the equations are:-

$$i^f = -\frac{-E_f}{128}\left(1 - e^{-0.129t}\right), \text{ for the predicted transient}$$

$$i^f = -\frac{E_f}{128}\left(1 - e^{-0.1485t}\right), \text{ for the actually-recorded transient.}$$

Equation (2) can be solved similarly. The assumption here is made that the term($M'_{fa} p\theta$) remains constant. The speed is kept constant for this test but M'_{fa} is not a constant quantity. Two alternatives are: (1) to draw a line 0A as shown on the open circuit characteristic on page V-3 and to take the slope of this line as constant or, (2) from the characteristics on page V-3 the average value of M'_{fa} should be taken between initial and final steady-state field currents. The second alternative has been chosen here. With this, the effect of saturation is accounted for to some extent. Even with these assumptions, the solution of equation (2) is a lengthy one, as can be seen from equation (7) in appendix II. When initial conditions in both field and armature circuits are assumed to be zero, a simpler solution given by equation (9) is obtained. This is not strictly true as there is always some armature current flowing owing to the e. m. f. generated by the residual magnetism. Equation (8) in appendix II must therefore be employed. This equation was simulated on the analogue computer and the characteristics obtained are illustrated on pages VI-9 to VI-12 where the error between the predicted and recorded transients in the armature cirucit is not great.

It can be seen that if accurate results are required the value and the nature of the inductance play a major role in the prediction of the transients in a d. c. machine. Very accurate solutions of the transient problem are unjustifiable and it is sufficiently accurate to make assumptions that make the system linear.

(2) Tests on a d.c. motor.

The second test for the prediction of transients was made for the machine run as a nominal shunt motor, the field being excited from a separate source. The field current was maintained at a constant value of 0.7A which is the rated full-load field current. A step in the armature voltage was applied to produce transients in both the armature current and in the speed of the machine and both of these quantities were fed into a pen-recorder. The recording of transients in speed is particularly difficult because no entirely satisfactory method is available for the accurate measurement of speed. Most speed-indicating instruments introduce an error due to the friction in their internal rotating parts and part of the energy is lost to overcome this friction. Simple stroboscopic methods are useless because they do not enable the speed to be recorded. Two methods were adopted here:

(1) A tachometer-generator feeding its output voltage to the pen-recorder. The chart paper was then calibrated to give the speed-time curve. Friction is involved in this case.

(2) Employing a special method, making use of the pen-recorder In this method, a contact strip is placed so that it just touches the rim of the flywheel. An insulating tape is passed around the rim of the flywheel, leaving a small gap to make metallic contact with the strip so that each time contact is made, a blip is recorded on the pen-recorder time marker. Knowing the chart-speed, the machine speed can be calculated. Although this method is rather laborious, it gives satisfactorily accurate results. It can be noted that there is almost no friction. It was found that the curves obtained by the above two methods were almost identical indicating that the loading effect of the tachometer was negligible.

As shown in appendix II, the transient equations for the speed and the armature current can be obtained from the generalized theory of electrical machines. Equation (12) is of importance.

$$(p\theta) = \frac{e_a}{K_1 \quad 1 + T_m p(1+T_a p)} \tag{12}$$

Rigorous methods give the classical solution of the above equation but they involve long and tedious expressions, e. g. equations (12) and (14).* To obtain the curves from these expressions by substituting the values of measured parameters is so laborious that a simplification must be adopted. This is achieved by neglecting the armature time constant with respect to the mechanical time constant resulting in two simpler expressions:

$$(p\theta) = \frac{e_a}{K_1(1 + T_m p)(1+T_a p)} \tag{15}$$

$$(p\theta) = \frac{e_a}{K_1 \, (1 + T_m p)} \tag{16}$$

Solution of equation (16) is:-

$$(p\theta) = \frac{E_a}{K_1} (1 - e^{-t/T_m}) + p\theta(0) \, e^{-t/T_m} \tag{17}$$

Equation (17) can be plotted easily when the values of measured parameters are substituted.

A simple solution of equation (12) or (15) may be obtained by using an analogue computer. The only variable in these expressions is the resistance of armature plus brushes, since

$$T_m = \frac{Jr_a}{K_1 K_2} \quad \text{and} \quad T_a = \frac{L_a}{r_a}$$

The armature resistance is nonlinear; hence, equations containing this term are nonlinear and the exact solution is unknown.

* See Appendix VIII-13.

To obtain theoretical characteristics various values of armature resistance are taken from the armature resistance versus armature current characteristics on page V-15. These values are then substituted in equation (15) and the solution obtained from the analogue computer. Four curves, with corresponding values of armature resistance at 0.5, 0.6 0.7 and 0.8 ohm, are reproduced on pages VI-13 and VI-14. These curves are typical and the corresponding values of the armature current represent the entire working range of armature current of the machine i.e. from 0A to 30A.

It can be seen that the theoretical and experimental curves differ from each other. Most of the parameters, with the notable exception of the armature resistance, are constant. This is true because the moment of inertia is constant; the self-inductance of the armature is independent of its own current and the quantity M'_{fa} is a function of the field current which is constant. The armature resistance includes brush and contact resistance and as described in Chapter IV, this is nonlinear and its value cannot be estimated accurately. The friction due to the brushes is another factor that affects the speed transient appreciably.

The deviation between theoretical and experimental curves is then, attributed to the following reasons:

(1) Nonlinearity of the contact drop.

(2) Unknown effect of friction due to the brushes.

(3) Experimental error.

Another fact is mentioned in this connection. Under the actual test, the recorded transient showed that, although the speed reaches its steady-state speed exponentially, it takes a fairly long time to settle down to a definite steady-state value. On the other hand, the analogue computer curves show that the steady-state is reached in about 1.5 seconds. This is attributed to the friction due to the brushes and iron losses which are neglected by the analogue computer, only the value of armature resistance being considered.

If the step is applied both to the field as well as the armature circuits, the above analysis can be extended to cover this case. The method of approach for this problem is described at the end of Appendix III.

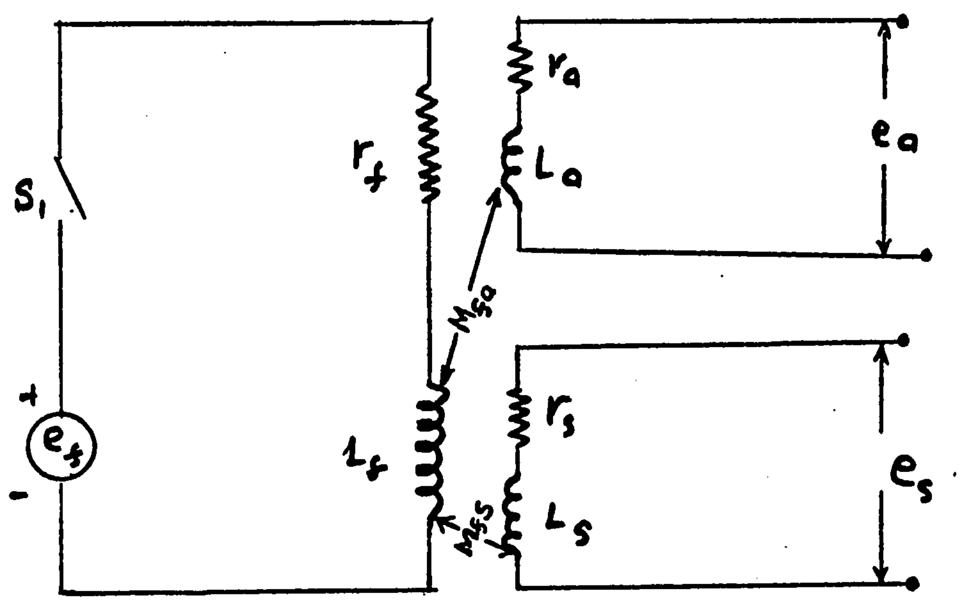
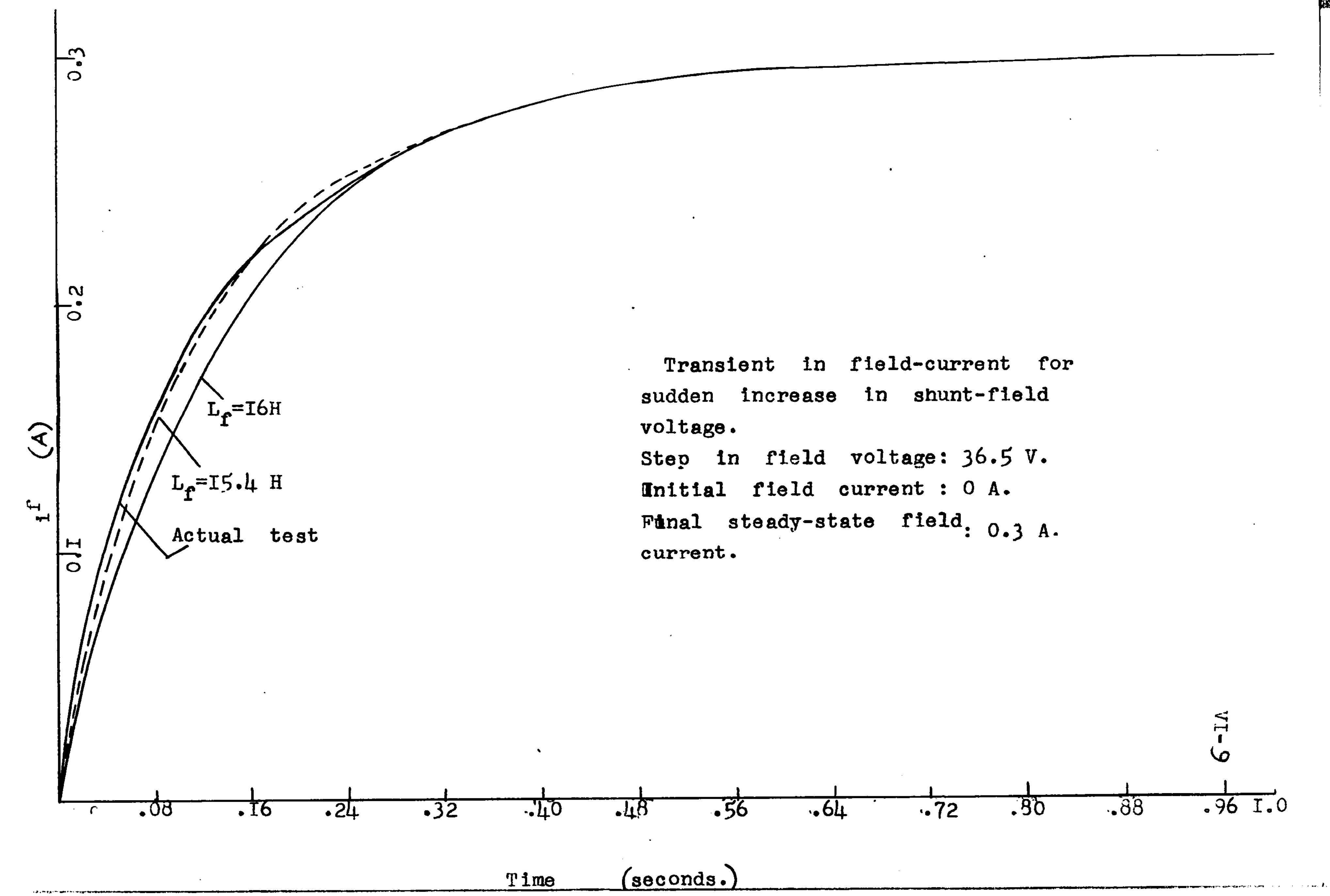
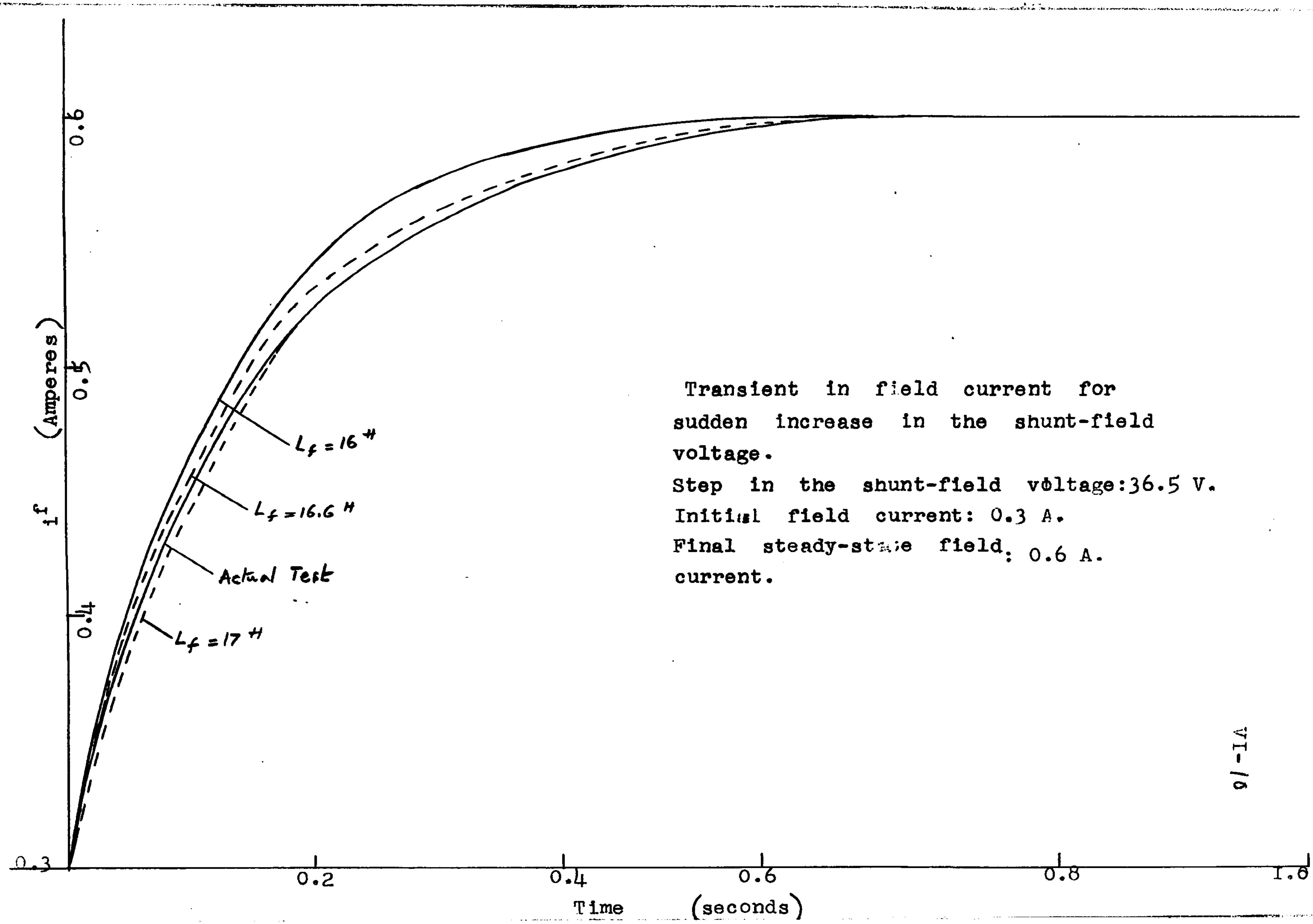
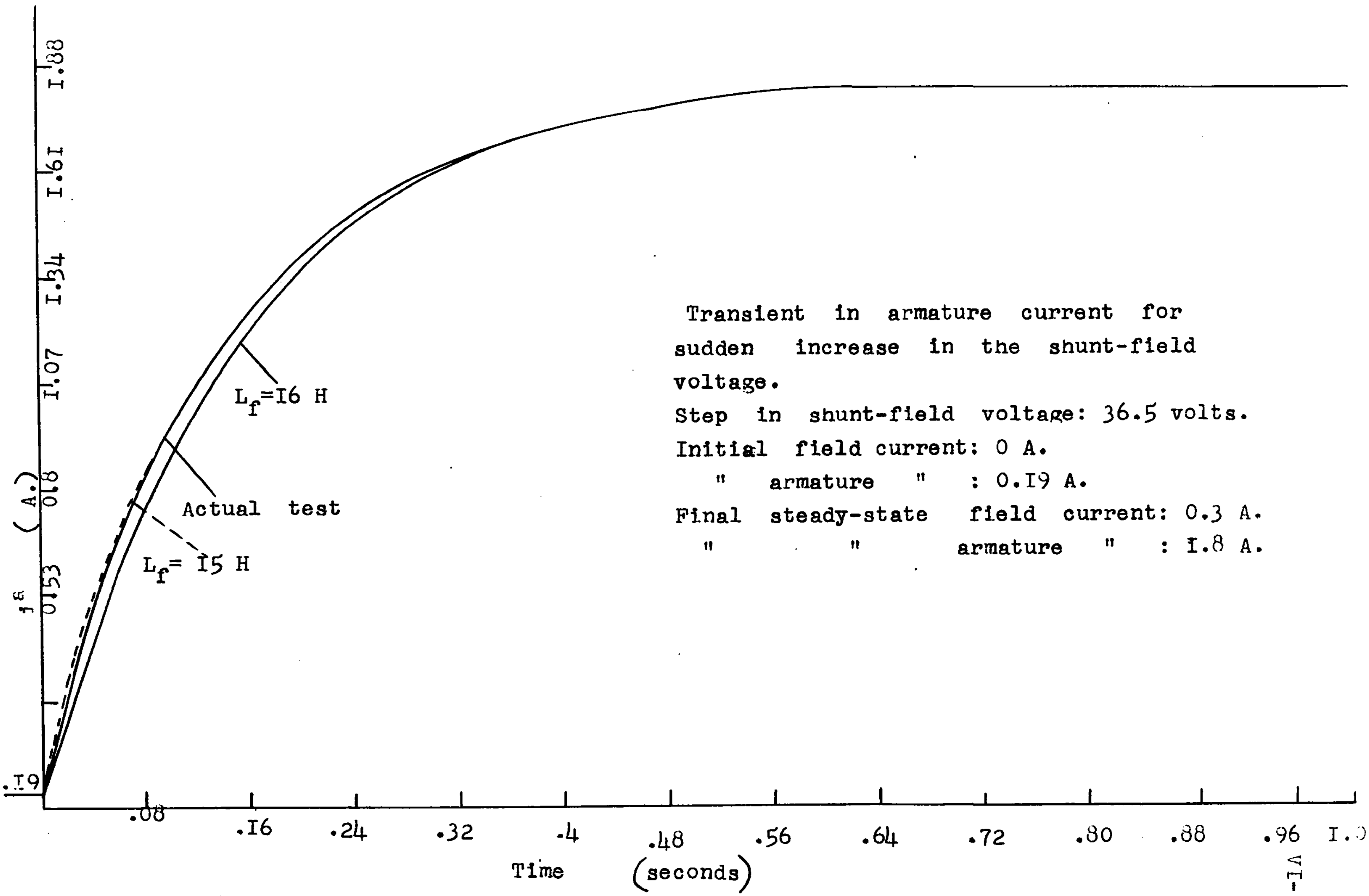


Figure VI-1

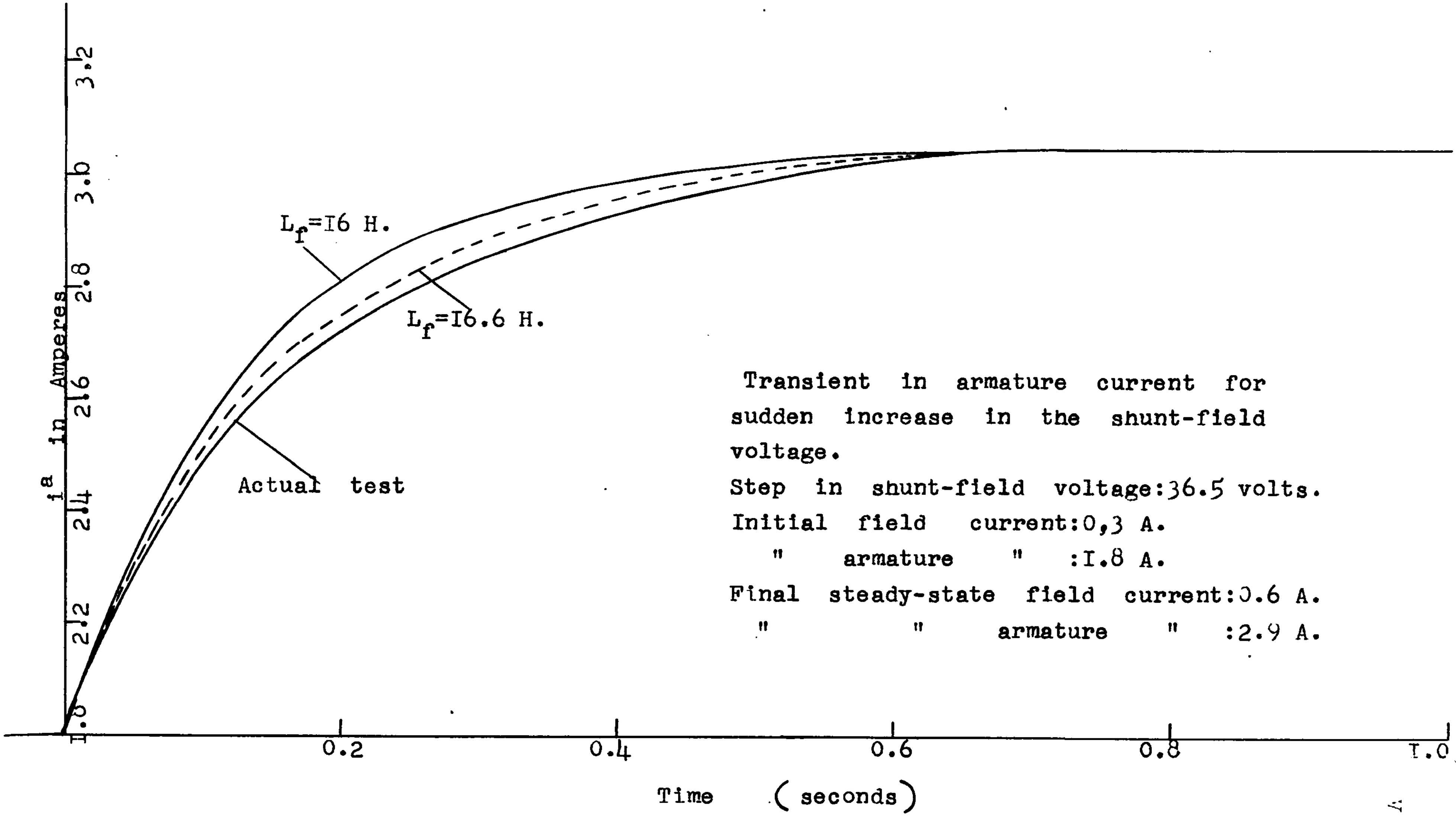

0.3
0.2
0.1
i_f (A)
L_f=I6H
L_f=I5.4 H
Actual test
Transient in field-current for sudden increase in shunt-field voltage.
Step in field voltage: 36.5 V.
Initial field current : 0 A.
Final steady-state field current: 0.3 A.
.08 .16 .24 .32 .40 .48 .56 .64 .72 .80 .88 .96 I.0
Time (seconds.)


Transient in field current for sudden increase in the shunt-field voltage.
Step in the shunt-field voltage: 36.5 V.
Initial field current: 0.3 A.
Final steady-state field current: 0.6 A.
$L_f = 16^H$
$L_f = 16.6^H$
Actual Test
$L_f = 17^H$
i^f (Amperes)
0.3
0.4
0.5
0.6
0.2
0.4
0.6
0.8
1.0
Time (seconds)



Transient in armature current for sudden increase in the shunt-field voltage.
Step in shunt-field voltage: 36.5 volts.
Initial field current: 0 A.
" armature " : 0.I9 A.
Final steady-state field current: 0.3 A.
" " armature " : I.8 A.
L_f=I6 H
Actual test
L_f= I5 H
(A.)
.I9
0.53
0.8
I.07
I.34
I.61
I.88
.08
.I6
.24
.32
.4
.48
.56
.64
.72
.80
.88
.96
Time (seconds)

VI-11



Transient in armature current for sudden increase in the shunt-field voltage.

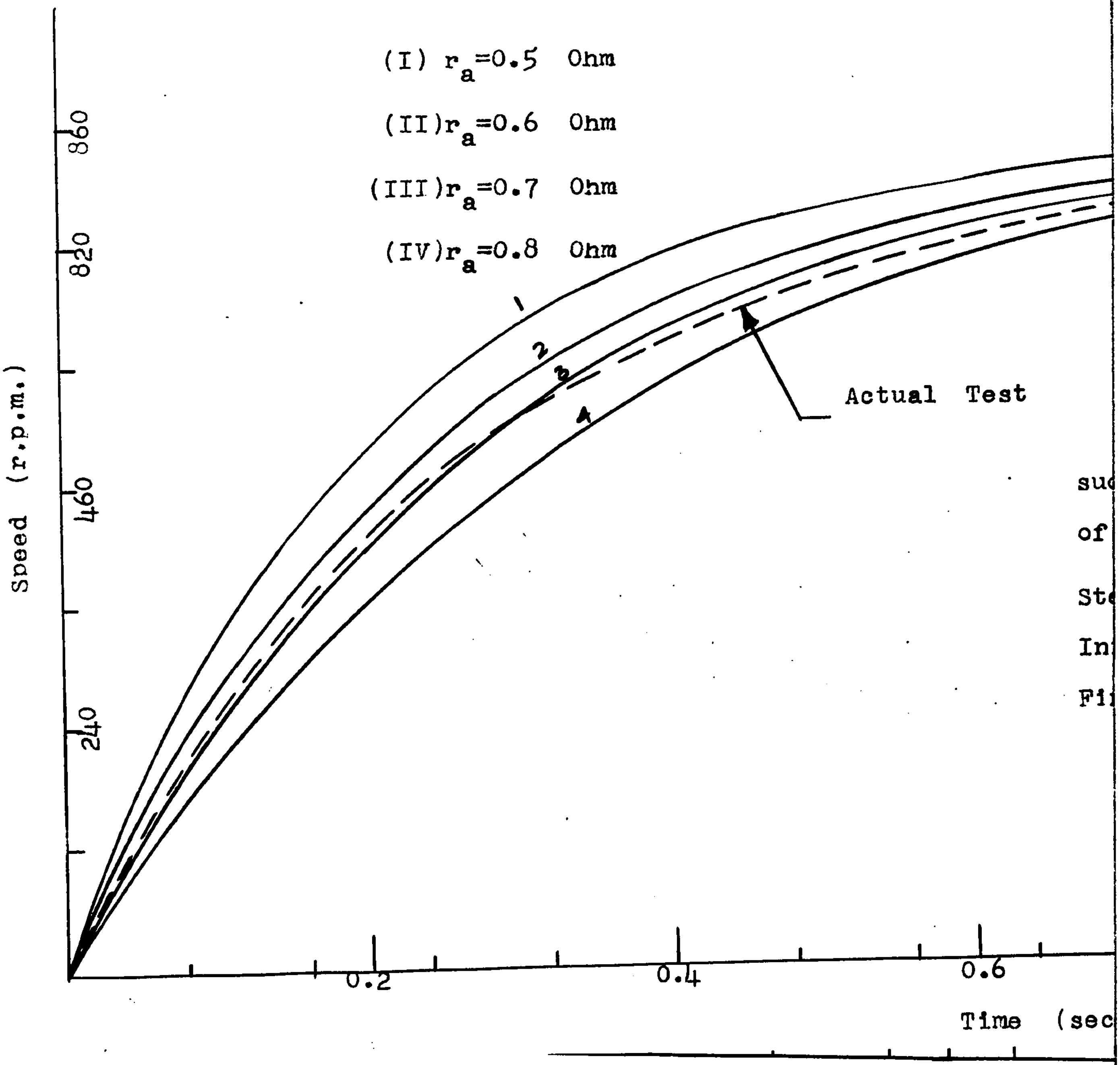
Step in shunt-field voltage:36.5 volts.

Initial field current:0,3 A.

" armature " :1.8 A.

Final steady-state field current:0.6 A.

" " armature " :2.9 A.



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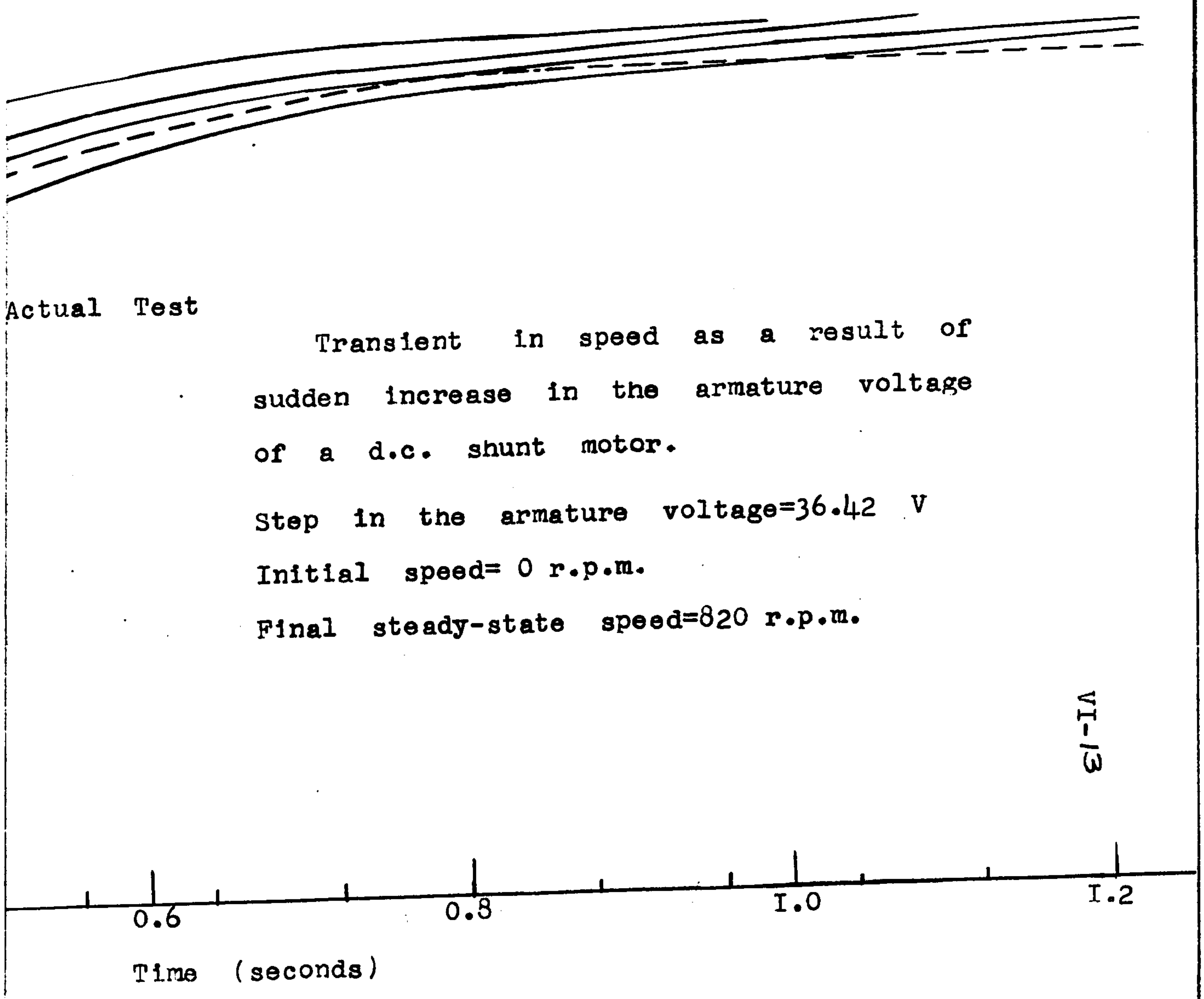
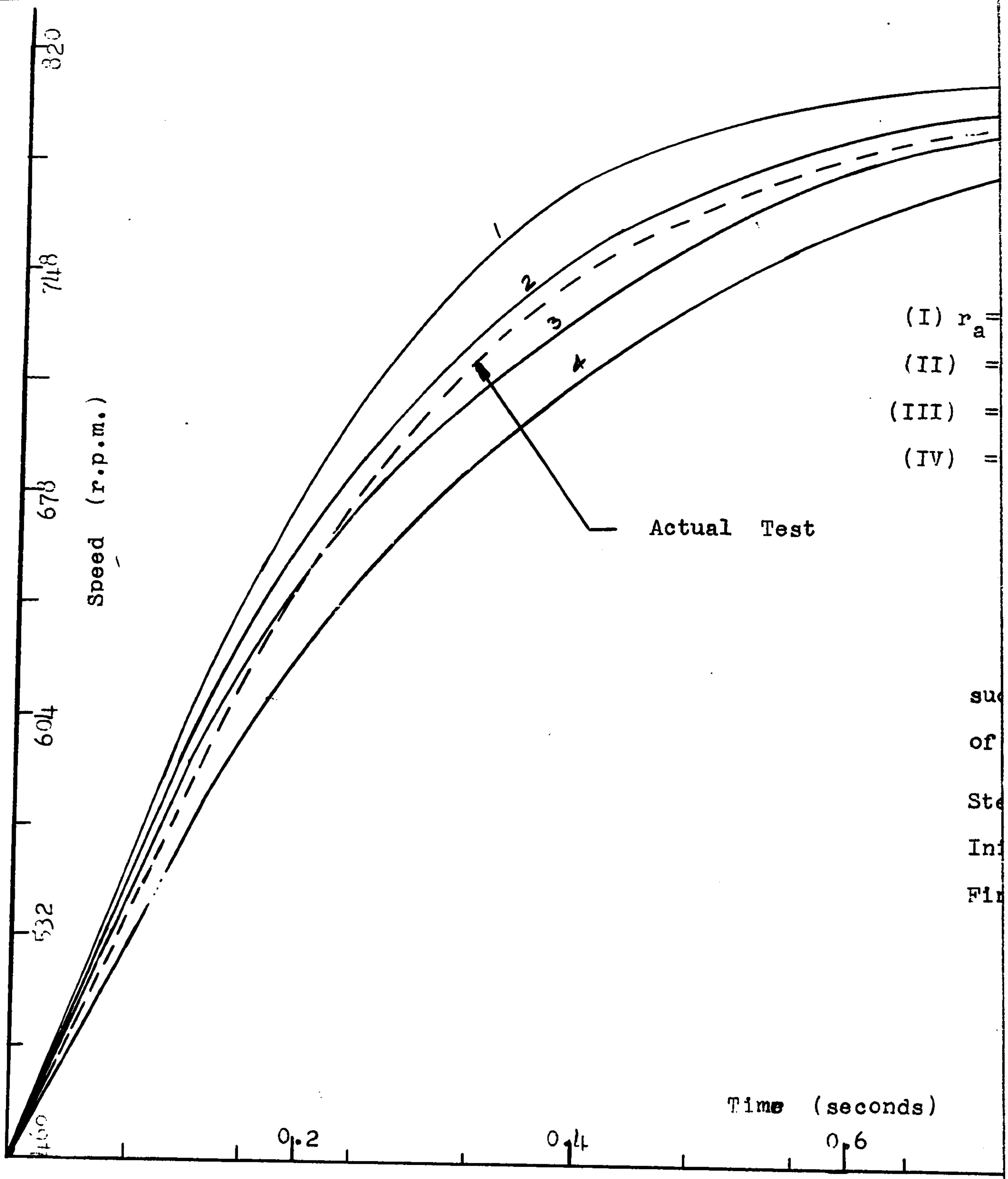


Transient in speed as a result of sudden increase in the armature voltage of a d.c. shunt motor.

Step in the armature voltage=36.42 V

Initial speed= 0 r.p.m.

Final steady-state speed=820 r.p.m.


820
748
678
604
532
460
Speed (r.p.m.)
1
2
3
4
(I) r_a=
(II) =
(III) =
(IV) =
Actual Test
Time (seconds)
0.2
0.4
0.6

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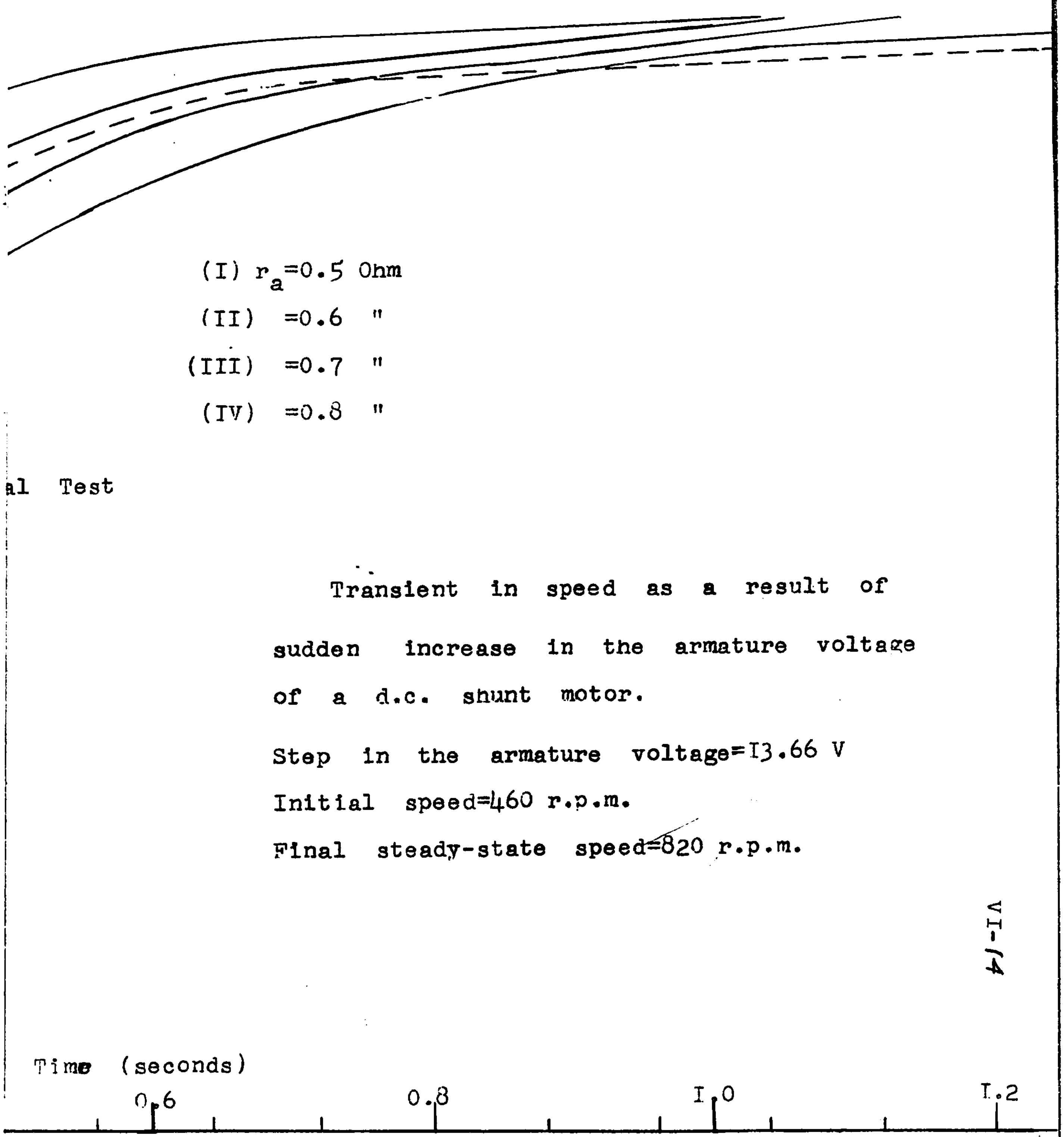


Transient in speed as a result of sudden increase in the armature voltage of a d.c. shunt motor.

Step in the armature voltage=I3.66 V

Initial speed=460 r.p.m.

Final steady-state speed=820 r.p.m.

CHAPTER VII

CONCLUSIONS

CONCLUSIONS

The definition of flux linkages as the total flux across any surface which has its edge on the circuit, is an exact one while the meaning of flux-linkage as the product of number of turns and the mean flux per turn is the engineer's approximation, accurate enough for most purposes.

The existing methods of measuring inductances using either alternating current or direct current are surveyed. Alternating current methods were used for the purposes of this work and their simplicity, compared to direct current methods which require special equipment and experience, is an important asset. All measurements were made with a single-phase, 60 c.p.s. supply which is an additional advantage. The simple resonance method is an addition to the existing methods and is found to be very useful.

Most of the results were expected. Hysteresis affects the shunt-field inductance and the field current affects the armature self-inductance. The latter is attributed to the effect of cross-saturation. The variation of self-inductance of the armature with the rotor-position agrees with the theoretical Fourier series.

The coefficient of mutual inductance has been measured using transformer theory, this being the simplest method that gives good results. The variation of mutual inductance with the rotor position also agrees with the theoretical Fourier series. Presence of brushes does not seem to affect the order of value of the mutual inductance; however, the curves obtained without the brushes are more regular and smooth.

The armature resistance is nonlinear due to the carbon-brush and contact resistances. This affects the measured value of the self-inductance of the armature and also the process of commutation. It is unfortunate that the exact degree to which the contact resistance affects the final results cannot be estimated accurately. With the development of brushless machines, the final results should be capable of more accurate prediction.

The measurement of the moment of inertia for small machines is yet another problem. For small machines, the method involving measurement of area to evaluate $\int P\,dt$ during retardation is found to be quicker, simpler and more accurate than the usual retardation test. If a mechanical test can be performed and if an accurate measurement of speed can be made, this is to be recommended.

Transients in the field and the armature circuits can be predicted with only a small error between predicted and recorded transients. The armature-circuit resistance, the friction due to the brushes and iron losses are factors of major importance when the transient in speed is to be predicted with accuracy.

With the measured parameters, other investigations can be made, in addition to those mentioned in this work. For example: a hunting analysis of a d.c. motor driving a load with a fluctuating torque or the extension of the method of analysis to d.c. compound and cross-field machines.

Some assumptions are always necessary to simplify the method of analysis. Although these assumptions limit the degree of accuracy of the final results, a good estimation of the machine behaviour can be obtained. In this connection, the following points are mentioned:

(1) An assumption has to be made for saturation. Saturation affects both the shunt-field inductance, as well as the speed coefficient of the mutual inductance between the shunt-field and armature plus interpoles. The methods mentioned in chapters IV and VI do take

saturation into account reasonably well.

(2) The leakage flux is assumed to be constant. On load, due to armature reaction, this is not true.

(3) The brushes are assumed to be on the neutral axis for the purpose of the transient analysis. On load, the magnetic axis shifts, thereby distorting the field form and changing inductance values.

(4) The nonlinearity of the armature resistance cannot be estimated with accuracy.

(5) The assumption that the field form is sinusoidal is not true but is justified because the rotor winding is distributed.

To conclude, the nonlinearity of some of the parameters presents the major difficulty in predicting the transients. The chief source of error is attributable to brush and contact resistances, which make it impossible to obtain an accurate estimation of commutation and armature-circuit currents. This necessarily limits the accuracy with which the performance of control systems may be analyzed. Development of brushless machines, with more predictable commutation and current-collection behaviour, will overcome the difficulty and result in the accurate measurements of machine inductances assuming a new importance.

CHAPTER VIII

APPENDIX

Appendix I :-

Theory of Fluxmetric Bridge:-

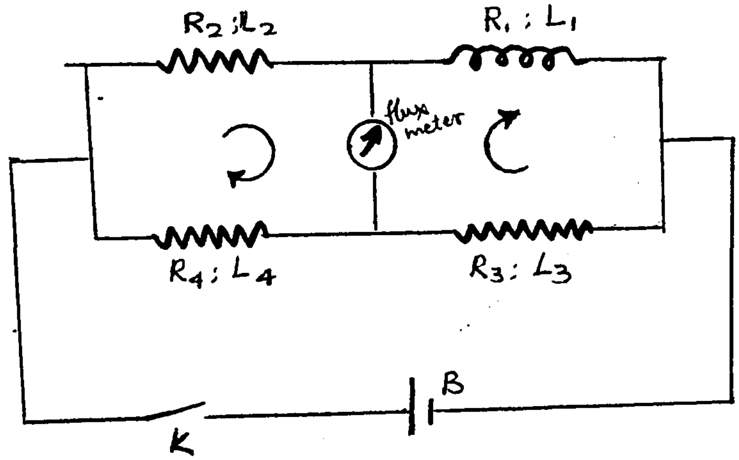


Fig. VIII-1

This bridge has been mentioned in chapter II and its theory is described below.

Initially with switch K closed the bridge is balanced. On opening the switch, the magnetic energy stored by the inductor L_1 is discharged and at the end of the transient it has deflected the fluxmeter by an angle θ ; let i^a and i^b be the instantaneous currents around the loops and i^f be the instantaneous current through the fluxmeter. Then,

$$i^f = i^b - i^a \tag{1}$$

Applying Kirchoff's second law to each loop,

$$K \frac{d\theta}{dt} = (R_2+R_4)\, i^a + (L_2+L_1) \frac{di^a}{dt} - R_f i^f - L_f \frac{di^f}{dt} \tag{2}$$

$$- K\frac{d\theta}{dt} = (R_1+R_3) i^b + (L_1+L_3) \frac{di^b}{dt} + R_f i^f + L_f \frac{di^f}{dt} \tag{3}$$

where K = fluxmeter constant,

L_2, L_3, L_4 = Residual inductances of the respective arms containing resistances R_2, R_3, R_4.

Eliminating i^a and i^b gives:-

$$- \ 1+ \frac{R_1 + R_3}{R_2 + R_4} \ K \frac{d\theta}{dt} = (R_1+R_3+R_f+ \frac{R_1+R_3}{R_2+R_4}) \, i^f$$

$$+ \ L_f \, (1+ \frac{R_1+R_3}{R_2+R_4}) \frac{di^f}{dt} - (L_2+L_4) \, \frac{R_1+R_3}{R_2+R_4} \, \frac{di^b}{dt}$$

$$+ \ (L_1+L_3) \, \frac{di^b}{dt} \qquad (4)$$

Integrating with respect to time over the transient period:

$$- \ (1+ \frac{R_1+R_3}{R_2+R_4}) \ K \, \Delta\theta \ = (R_1+R_3+R_f + \frac{R_1+R_3}{R_2+R_4}) \ \dot{q}^f$$

$$+ \ L_f(1 + \frac{R_1+R_3}{R_2+R_4}) \ (i_2^f \ - \ i_1^f)$$

$$- \ (L_2+L_4) \ \frac{R_1+R_3}{R_2+R_4} \, (i_2^a - i_2^b)$$

$$+ \ (L_1+L_3) \ (i_2^b \ - \ i_1^b) \qquad (5)$$

Subscripts (1) and (2) denote initial and final values. Separating the terms for each circuit element of the integrals of the type

$$- \, (1 + \frac{R_1+R_3}{R_2+R_4}) \, K\Delta\theta \ = \ (R_1+R_3+R_f + \frac{R_1+R_3}{R_2+R_4} \ R_f) \ q^f$$

$$+ \ L_f(1 + \frac{R_1+R_3}{R_2+R_4}) \ (i_2^f - i_1^f)$$

$$- \, \frac{R_1+R_2}{R_3+R_4} \, L_2(i_2^{L_4} \ - \ i_1^{L_2}) \ + L_4(i_2^{L_4} \ - \ i_1^{L_4} \ )$$

$$+ \ L_1(i_1^{L_1} \ - \ i_1^{L_1}) + L_3 \, (i_2^{L_3} \ - \ i_3^{L_3}) \qquad (6)$$

where q^f = charge through the fluxmeter.

The initial currents are: $i_1^f = 0$; $i_2^{L_1} = i_1^{L_2} = I_1$

$$i_1^{L_3} = i_1^{L_4} = I_o$$

The final currents i^2 are zero.

The charge q^f is proportional to the viscous damping of the meter and if this is small it, together with the expression $(R_1+R_3+R_1+\frac{R_1+R_3}{R_2+R_4})$.

may be neglected from equation (6).

The final expression then, is

$$L_1I_1 - L_3I_o + \frac{R_1+R_3}{R_2+R_4}(L_4I_o - L_2I_1) = (1+\frac{R_1+R_3}{R_2+R_4})K \qquad (7)$$

which accounts for the residual inductances of the resistors.

Appendix II:-

Transient equations for a separately-excited d. c. shunt generator. Sign Conventions:-

(1) Clockwise rotation is positive

(2) Positive m. m. f. s are in the directions shown in Fig VIII-2 along the direct and quadrature axes.

(3) Applied voltage = Σ voltage drops

(4) For a motor, e = Z.i. For a generator, the voltages are reversed or the currents are reversed. Thus, the equations for a generator become (-e) = Z.i or e = (-Z).i. Therefore, the signs of all terms in the motor impedance matrix are changed to give the generator impedance matrix.

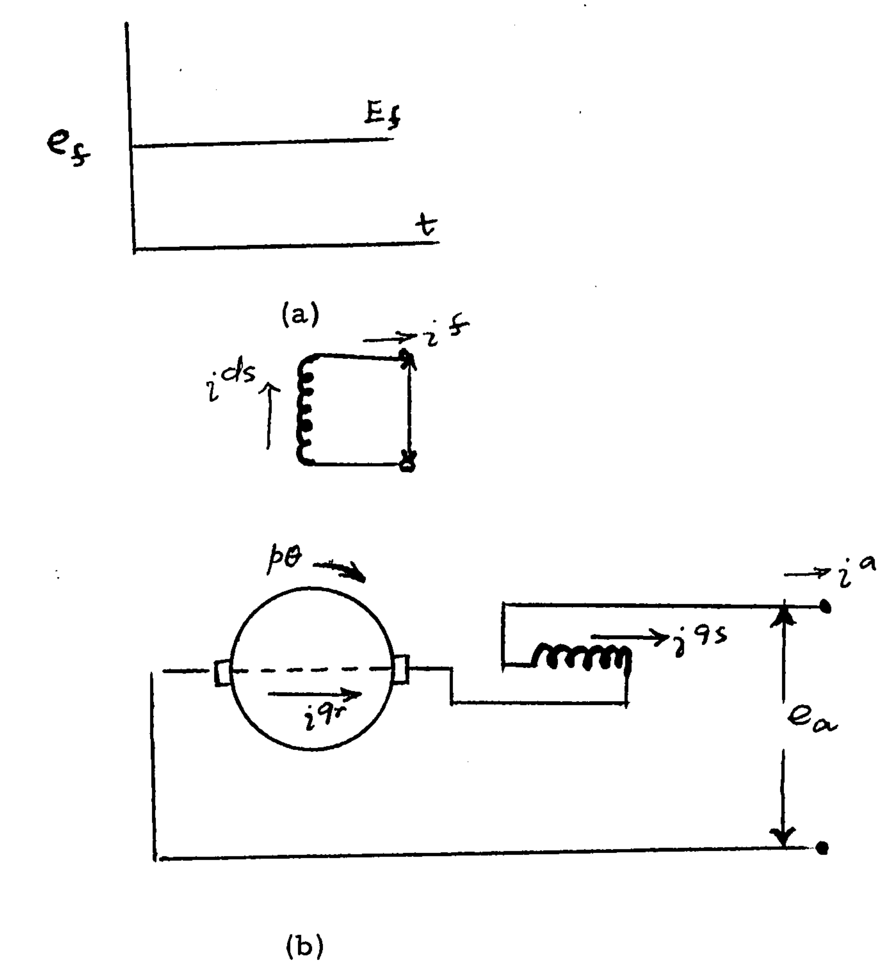


Fig. VIII-2

From figure VIII-2, impedance matrix can be written down as:-

A_{mn} =

	ds	qr	qs
ds	$-r_f - L_f p$		
qr	$M'_{fa} p\theta$	$-r_a - L_a p$	$-M_{ai} p$
qr		$-M_{ai} p$	$-r_i - L_i p$

$C^n_{n'}$ =

	f	a
ds	1	
qr		1
qs		-1

$C^m_{m'}$ =

ds	qr	qs
1		
	1	-1

On transformation,

$Z_{m'n'}$ =

	f	a
f	$-r_f - L_f p$	
a	$M'_{fa} p\theta$	$-r_{(a+i)} - L_{(a+i)} p$

(1)

The voltage equation can now be written as:

$$\begin{bmatrix} e_f \\ e_a \end{bmatrix} = \begin{array}{c|cc} & f & a \\ \hline f & -r_f - L_f p & \\ a & M'_{fa} p\theta & -r_{(a+i)} - L_{(a+i)} p \end{array} \begin{bmatrix} i^f \\ i^a \end{bmatrix} \qquad (2)$$

where, $r_{(a+i)} = r_a + r_i$

$$L_{(a+i)} = L_a + L_i - 2M_{ai}$$

From (2) :-

$$e_f = (-r_f - L_f p)\, i^f \qquad (3)$$

$$e_a = [-r_{(a+i)} - L_{(a+i)} p\,]\, i^a + M'_{fa} p\theta\, i^f \qquad (4)$$

Solution of equation (3) is:

$$i^f = \frac{E_f}{r_f}\left(1 - e^{-t/T_f}\right) + I_f(0)\; e^{-t/T_f} \qquad (5)$$

If the initial field current is zero:

$$i^f = - \frac{E_f}{r_f}\left(1 - e^{-t/T_f}\right) \qquad (6)$$

Consider, next, equation (4):

On load, $e_a = i^a r_1$ where

r_1 = load resistance

For constant speed, $M'_{fa} p\theta = K_f$, where K_f is a proportionality factor for the speed voltage.

Equation (4) can now be rewritten as:-

$$i^a r_1 = - r_{(a+i)} i^a - L_{(a+i)} p i^a + K_f i^f$$

Substituting equation (5) for the value of i^f and rearranging,

$$r_1 + r_{(a+i)}\; i^a + L_{(a+i)} p i^a = K_1\left(1 - e^{-t/T_f}\right) - K_2 e^{-t/T_f}$$

where $K_1 = - \frac{K_f E_f}{r_f}$

$K_2 = - K_f I_f(0)$

Solution of the above equation gives:

$$i^a = -\frac{K_1}{L_{(a+i)}} \frac{T_a T_f}{T_f - T_a} (e^{-t/T_f} - e^{-t/T_a})$$

$$-\frac{K_1}{r_{(a+i)}+r_1}(1-e^{-t/T_a}) - \frac{K_2}{L_{(a+i)}} \frac{T_a T_f}{T_f - T_a} (e^{-t/T_f} - e^{-t/T_a})$$

$$+ I_a(o)\, e^{-t/T_a} \tag{7}$$

where T_a = armature time constant = $\frac{L_{(a+i)}}{r_{(a+i)}}$

The time constant of the armature circuit is small compared with the time constant of the field circuit, Neglecting T_a would yield:-

$$i^a = \frac{K_1 + K_2}{r_{(a+i)}+r_1} (1-e^{-t/T_f}) - \frac{K_1}{r_{(a+i)}+r_1} + I_a(o)e^{-t/T_a} \tag{8}$$

If initial field and armature currents are zero, equation (8) simplifies to:

$$i^a = -\frac{K_1}{r_{(a+i)}+r_1} (1-e^{-t/T_f}) \tag{9}$$

Equation (9) shows that the value of i^a depends largely upon the parameters of the field circuit. In fact, $I_a(o)$ is never zero due to the e.m.f. generated from residual flux. The exact solution must take into account the value of $I_a(o)$ and then the following equation must be used:-

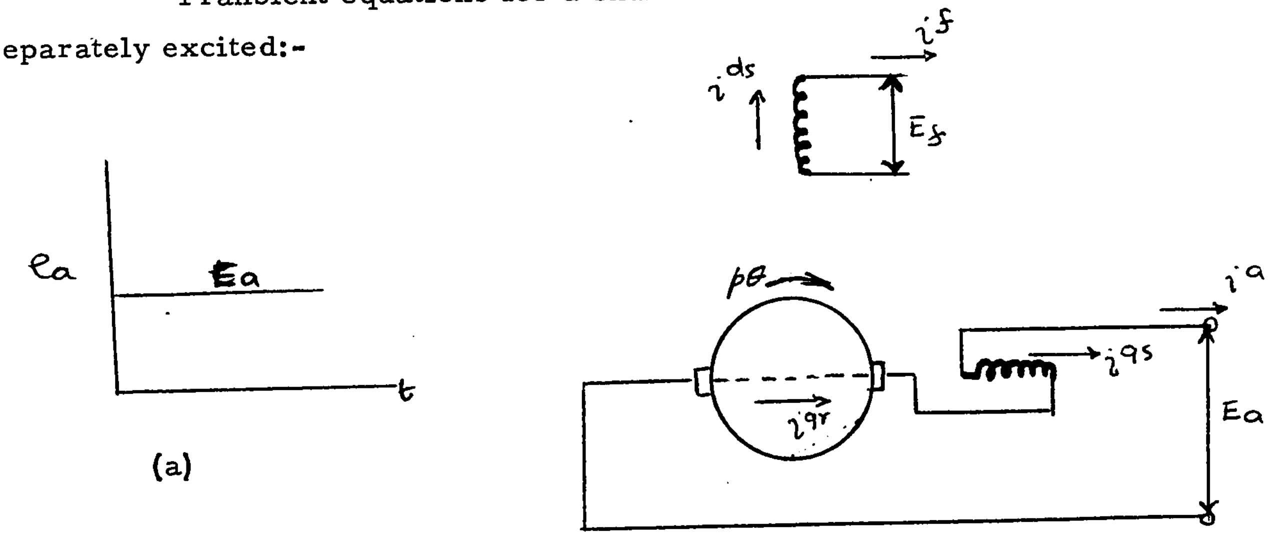
$$i^a = \frac{K_1}{r_1 + r_{(a+i)}} e^{-t/T_f} + I_a(o)e^{-t/T_a} \tag{10}$$

For the special case, at time $t = T_f$, equation (9) becomes:

$$i^a = \frac{0.632K_1}{r_1+r_{(a+i)}} \tag{11}$$

Appendix III :-

Transient equations for a shunt motor: field circuit separately excited:-



(b)

(Fig. VIII-3)

Reference to the above figure enables the impedance matrix to be written

Z_{mn} =

	ds	qr	qs
ds	$+r_f+L_f p$		
qr	$-M'_{fa}p\theta$	$r_a + L_a p$	$M_{ai}p$
qs		$M_{ai}p$	$r_i + L_i p$

$C^n_{n'}$ =

	f	a
ds	1	
qr		1
qs		-1

$C^m_{m'}$ =

	ds	qr	qs
f	1		
a		1	-1

Transformation yields:

$$Z_{m'n'} = \begin{array}{c|c|c|} & f & a \\ \hline f & r_f + L_f p & \\ \hline a & -M'_{fa} p\theta & r_{(a+i)} + L_{(a+i)} p \\ \hline \end{array} \qquad (1)$$

where $r_{(a+i)} = r_a + r_1$

$$L_{(a+i)} = L_a + L_i = 2M_{ai}$$

The voltage equation can now be written as:-

$$\begin{array}{|c|} \hline e_f \\ \hline e_a \\ \hline \end{array} = \begin{array}{c|c|c|} & f & a \\ \hline f & r_f + L_f p & \\ \hline a & -M'_{fa} p\theta & r_{(a+i)} + L_{(a+i)} p \\ \hline \end{array} \begin{array}{|c|} \hline i^f \\ \hline i^a \\ \hline \end{array} \qquad (2)$$

The acceleration matrix follows from the torque equation for a motor i.e.

$$T = Jp^2\theta + Rp\theta - i^f G_{m'n'} i^a$$

where, J = Polar moment of inertia (kg-m^2)

$Rp\theta$ = Accelerating torque (N-m)

$i^f G_{m'n'} i^a$ = Electromagnetic torque (N-m)

From equation (1),

$$G_{m'n'} = \begin{array}{c|c|c|} & f & a \\ \hline f & & \\ \hline a & -M'_{fa} & \\ \hline \end{array}$$

Then,

$$i^f G_{m'n'}\ i^a = -i^f M'_{fa}\ i^a$$

and $$T = Jp^2\theta + Rp\theta + i^f M'_{fa}\ i^a \qquad (3)$$

Adding one more row and column 's' to voltage equation (2), thus separting the speed e.m.f. terms i.e. moving 'pθ' terms to the new 's' column, we have,

$$\begin{bmatrix} e_f \\ e_a \\ T \end{bmatrix} = \begin{array}{c} f \\ a \\ s \end{array} \begin{array}{c} \begin{array}{ccc} f & a & a \end{array} \\ \begin{bmatrix} r_f + L_f p & & \\ & r_{(a+i)} + L_{(a+i)}p & -M'_{fa}\cdot i^f \\ & i^f M'_{fa} & Jp+R \end{bmatrix} \end{array} \begin{bmatrix} i^f \\ i^a \\ p\theta \end{bmatrix} \qquad (4)$$

The impedance in equation (4) is the acceleration impedance $L_{m'n'}$ where:

$$L_{m'n'} = \begin{bmatrix} r_f + L_f p & & \\ & r_{(a+i)} + L_{(a+i)}p & -M'_{fa}\cdot i^f \\ & i^f\ M'_{fa} & Jp+R \end{bmatrix} \qquad (5)$$

From equation (4) the following equations are obtained:-

$$e_f = (r_f + L_f p)\ i^f \qquad (6)$$

$$e_a = [\ r_{(a+i)} + L_{(a+i)}p]\ \ i^a = M'_{fa} p\theta\ i^f \qquad (7)$$

$$T = Jp^2\theta + Rp\theta + i^f M'_{fa}\ i^a \qquad (8)$$

Solution of equation (6) gives:

$$i^f = \frac{E_f}{r_f}(1 - e^{-t/T_f}) = I_f(0)\, e^{-t/T_f} \tag{6a}$$

Equations (7) and (8) are to be solved simultaneously. Here, there are two alternatives. Either a step in armature voltage can cause a transient change in the speed of the machine, or, a step in the field voltage can change the speed in a transient fashion. The former case is considered first i.e. the field current is kept constant.

$$e_a = r_{(a+i)} i^a + L_{(a+i)} p i^a + k_1 p\theta \tag{9}$$

$$T = jp^2\theta + Rp\theta - k_2 i^a \tag{10}$$

where $K_1 = -M'_{fa} \cdot i^f$

$K_2 = - k_t \cdot i^f$ where K_t is a constant depending upon the units used. In our case, $K_1 = K_2$ but the symbol K_2 will be used for generality. Also, since the quantity 'R' is very small, it can be omitted from equation (10). [For the present machine $R = 3.66 \times 10^{-3}$ for $\omega = 60\pi$ rad/sec (1800 r.p.m.)]. Equation (10) can be rewritten as:

$$i^a = \frac{Jp^2\theta - T}{K_2} \tag{10a}$$

Substituting equation (10a) in equation (9) and rearranging:

$$p\theta = \frac{\frac{e_a}{K_1} + \frac{T_m}{J}(1 + T_a p) \cdot T}{1 + T_m p(1 + T_a p)} \tag{11}$$

where T_m = mechanical time constant

$$= \frac{J \cdot r_{(a+i)}}{K_1 K_2}$$

The transient analysis may be made for the condition when the applied torque is zero and substitution of T = 0 in equation (II) simplifies it to equation (12). However, if there is an initial constant applied torque, the variables in equation (II) are taken to be superimposed changes of voltage, torque and speed. Thus, in either case T = 0 may be substituted for the acceleration component of motion. Equation (II) then, simplifies to:

$$(p\dot{\theta}) = \frac{e_a}{K_1 \,[1+T_m p(1+T_a p)]} \tag{12}$$

Solution by the rigorous method gives:

$$(p\theta) = \frac{E_a \,[\,K_4(1-e^{-K_3 t}) - K_3(1-e^{-K_4 t})]}{K_1 \, T_m T_a K_3 K_4 (K_4 - K_3)}$$

$$+ \quad \frac{p\theta(0)}{K_3 K_4} \quad \frac{1-K_4 T_a}{T_a} \cdot e^{-K_4 t} - \frac{1-K_3 T_a}{T_a} e^{-K_3 t}$$

$$+ \quad \frac{p\theta(0) \;[\,e^{-K_3 t} - e^{-K_4 t}\,]}{K_4 - K_3} \tag{12}$$

where

$$K_1 = M'_a \, i^f$$

$$K_2 = K_t i^f$$

$$K_3 = \frac{1}{2T_a}\left(1 + \frac{T_m - 4T_a}{T_m}\right) \tag{13}$$

$$K_4 = \frac{1}{2T_a}\left(1 - \frac{T_m - 4T_a}{T_m}\right)$$

From equation (10a),

$$ia = \frac{J}{K_2} \quad \frac{d}{dt}(p\theta)$$

Then:

$$i_a = \frac{E_a J(e^{-K_4 t} - e^{-K_3 t})}{T_m T_a K_1 K_3 K_4 (K_4 - K_3)}$$

$$+ \frac{Jp\theta(0)}{K_2 K_4 T_a} (1 - K_4 T_a) e^{-K_3 t}$$

$$- \frac{Jp\theta(0)}{K_2 K_3 T_a} (1 - K_4 T_a) e^{-K_4 t}$$

$$+ \frac{Jp^2\theta(0)}{K_2} \quad \frac{K_4 e^{-K_4 t} - K_3 e^{-K_3 t}}{K_3 - K_4} \tag{14}$$

A simpler way is to solve equation (12) as follows: Since the time constant of the armature circuit is small compared to the mechanical time constant, equation (12) can be written as:

$$(p\theta) \quad \frac{e_a}{K_1 \quad 1 + T_m p(1 + T_a p) + T_m p}$$

$$= \frac{e_a}{K_1(1 + T_m p)(1 + T_a p)} \tag{15}$$

But T_a is small and equation (15) may be simplified to

$$(p\theta) = \frac{e_a}{K_4 K_1 (1 + T_m p)} \tag{16}$$

The solution of equation (16) is straight forward.

$$(p\theta) = \frac{E_a}{K_1} (1 - e^{-t/T_m}) - p\theta(0) e^{-t/T_m} \tag{17}$$

A simple and quite accurate approach would be to solve equation (12) by using an analogue computer, substituting the various values of the time constants.

The second case concerns the transients in speed when a step voltage is applied to both field and armature circuits of a d.c. shunt motor. The method of analysis is similar to that given above but requires extending along the following lines: The field current is not constant, as it was in the above case, but is described by equation (6a) and this equation should be substituted for the value of i^f in equations (7) and (8). The curves describing the transients in field current are obtained in exactly the same way as for the shunt generator. The value of the field current at any instant can then be obtained from curves such as those on pages VI-9 and VI-10. These values, when substituted in equations (9) and (10) would give the transient characeteristics of the armature current and speed.

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