The Balassa-Samuelson Model: A Test with A Panel of 5 OECD Countries

by

Hung Kuen Chan

(2635624)

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Supervisor: Professor Kathleen Day

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Abstract

Testing an extended Balassa-Samuelson model with a panel of data for 5 OECD countries for the period 1970-1998, this paper confirms that upward trend in the relative price of non-tradable to tradable goods is associated with faster productivity growth in tradable good sector within each country. But no evidence is found to support the theoretical relation between real exchange rates and productivity differentials across nations. Also, the evidence is not strong enough to let the government spending variable enter the model as a demand side factor.

1. Introduction

What fundamental factors determine differences in real exchange rates across nations, especially long-run movements in the real exchange rates? This problem is always a very intriguing topic in International Economics. Although there exist different propositions, Balassa(1964) and Samuelson(1964)'s productivity differential model is the most enduring one and still the dominant one.

The Balassa-Samuelson model rests on two components. First, the relative price of non-tradable to tradable goods within each country is positively related to the relative productivity of the tradable to the non-tradable good sector. Second, under the assumption of PPP (Purchasing Power Parity) for tradable goods, movements in the real exchange rate can be explained by the productivity differentials between countries, especially the productivity differentials in the tradable good sector between nations.

The original Balassa-Samuelson model only relies on supply side factors to explain the changes in the real exchange rate. But recent related theoretical and empirical studies tend to extend the model by including demand side factors to examine whether demand side factors also have impacts on the relative price of non-tradable goods within each country and on the real exchange rate between countries. Government spending is a commonly used demand side variable that can proxy the demand for non-tradable goods.

In this paper, an extended Balassa-Samuelson model is tested with a panel of data for 5 OECD countries for the period 1970-1998. Using techniques for dealing with pooled time-series cross-section data, the empirical work of this paper finds sufficient evidence to support the hypothesis that the increase in the relative price of non-tradable to tradable goods is associated with faster productivity growth in the tradable good sector. But no empirical evidence is found to relate movements in the real exchange rate with productivity differences across countries. Also, the empirical evidence is not strong enough to let the government spending variable enter into the original model.

The remaining parts of the paper are organized as follows. Section 2 illustrates the theoretical Balassa-Samuelson model. Section 3 is a review of the previous related empirical literature. The data used in this paper are discussed in section 4. Section 5 presents the empirical results. The conclusions are offered in section 6.

2. The Theoretical Model[1]

The Balassa-Samuelson model is constructed for a small open economy that has only two sectors: tradable goods and non-tradable goods. The production functions in both sectors are assumed to have constant returns to scale. The capital employed in production is mobile between the two sectors as well as internationally, while labor is mobile only across sectors. Because of the constant returns to scale production functions, we can express the sectoral output as

$$Y_T = A_T f(K_T)$$

and

$$Y_N = A_N g(K_N)$$

respectively, where the subscript T denotes the tradable sector, N denotes the non-tradable sector, Y is output per employed worker, K is the capital-labor ratio employed in production, and A represents productivity. Under perfect competition, profit-maximization implies that the producers in both sectors will choose the optimal levels of capital and labor so as to equate the marginal product of capital to the real interest rate and equate the

[1] The presentation of the theoretical model in this section is based on the lecture notes of the course *International Monetary theory and Policy* offered by Professor F. Demers of the Department of Economics of Carleton University.

marginal product of labor to the real wage rate.

Let's consider the tradable sector first. The first-order conditions for profit-maximization are

$$A_T f'(K_T)=r \qquad (1)$$

and

$$A_T f(K_T) - rK_T = W \qquad (2)$$

where the price of the tradable good is set to be equal to 1 (i.e. the price of tradable good is used as the numeraire.). In the small open economy, the interest rate r is determined by the outside world. So, given the exogenous interest rate r, the optimal capital-labor ratio K_T is determined by equation (1). Then, given the optimal K_T and the exogenous r, equation (2) determines sectoral wage rate W. Note that the key result here is that this tradable sector wage rate W will also be the economy-wide wage rate within the whole small open economy, since labor is mobile across sectors.

Similarly, in the non-tradable sector, profit-maximization implies

$$PA_N g'(K_N)=r \qquad (3)$$

and

$$PA_N g(K_N) - rK_N= W \qquad (4)$$

where P is the relative price of the non-tradable good in terms of the price of the tradable good. Now, given the exogenous interest rate r and the economy-wide wage rate W that has already been determined by the tradable sector, the relative price of the non-traded good, P and the capital-labor ratio K_N are determined by both the equations (3) and (4).

Since the movement or the change in the relative price P is of interest, we logarithmically differentiate both equations (2) and (4) and let $\hat{X}$ denote the rate of change for any variable X. Letting U_T and U_N be the ratio of labor's income (wage) to the sectoral output (the output generated in the tradable sector or non-tradable sector) respectively[2], then we get

$$\hat{A}_T = U_T\hat{W} \qquad (5)$$

and

$$\hat{P} + \hat{A}_N = U_N W \qquad (6)$$

Substituting the equation (5) into (6), we obtain the first component of the Balassa-Samuelson model:[3]

$$\hat{P} = (U_N / U_T)\hat{A}_T - \hat{A}_N \qquad (7)$$

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[2] $\hat{X} = d \log X = dX/X$, $U_T = W/A_T f(K_T)$, $U_N = W/A_N g(K_N)$.

[3] For the full process of derivation, see Appendix A.

which states that faster productivity growth in the tradable good sector than in the non-tradable good sector will lead to an increase in the relative price of the non-tradable to tradable goods provided the non-tradable sector is at least as labor-intensive as the tradable sector[4].

In order to derive the second component of the Balassa-Samuelson model, we need to extend the framework of the model to a world with two open economies: Home and Foreign (the variables for the foreign country are denoted with *). Suppose that the price of the traded good in each country is equal to 1 (this is a very important assumption in the Balassa-Samuelson model, because it implies that Purchasing Power Parity holds for traded goods across countries.), while the relative price of the non-traded good is P in the home country and P^* in the foreign country. If we construct price indices for both countries using the same geometric average of the tradable and non-tradable goods' prices, the price indices (denoted by C and C^*) for the home and foreign countries are

$$C = (1)^{\theta}(P)^{1-\theta} = P^{1-\theta}$$

and

$$C^* = (1)^{\theta}(P^*)^{1-\theta} = (P^*)^{1-\theta}$$

[4] Empirically, the non-tradable sector is more labor-intensive than tradable sector. So $U_N/U_T \geqslant 1$ usually holds in practice.

respectively[5]. The real exchange rate between the two countries is

$$C/C^* = (P/P^*)^{1-\theta} \tag{8}$$

Again, since the movement in the real exchange rate is our principal interest, we logarithmically differentiate equation (8) and substitute the first component of the theory, equation (7), into the log-differentiated equation (8). Then we finally obtain the second component of the Balassa-Samuelson model:

$$\hat{C} - \hat{C}^* = (1-\theta)\left[(U_N/U_T)(\hat{A}_T - \hat{A}_T^*) - (\hat{A}_N - \hat{A}_N^*)\right]$$

which shows how the movement in the real exchange rate is linked with the sectoral productivity differentials across countries.[6] It should be noted that, in practice, most productivity differentials across countries are observed in the tradable sector, or to put it in another way, there have been more positive shocks in the tradable sector than in the non-tradable sector. So the Balassa-Samuelson model implies a tendency for the country with higher productivity growth in the tradable sector to have a higher price level or to experience real exchange rate appreciation.

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[5] All of the P, P^*, C, and C^* are expressed in a common currency (home country currency, foreign country currency or a

The original Balassa-Samuelson model does not allow any demand-side fundamentals to influence relative prices of nontradables to tradables within nations and real exchange rates across countries because of the strict assumption of perfect and instant mobility of production factors between the tradable and non-tradable sectors in a given country as well as perfect and instant mobility of capital across nations. Under this assumption, when increased demand in one sector drives the sectoral price upward, labor and capital will flow in instantly to satisfy the increased demand and dampen the sectoral price down to the previous level. However, this assumption is very hard to justify in reality, especially in the short-run. Therefore, some recent theoretical and empirical studies tend to relax the assumption and let demand-side fundamentals enter the model.

A typical demand-side fundamental that can represent the demand for nontradables would be government spending, since government spending is a demand that is more services intensive and thus more nontradables intensive (almost all services are concentrated in the non-tradable sector). Another similar nontradables demand fundamental can simply be per capita income, because demand for services is more income-elastic than demand in goods. Thus the demand shifts in nontradables can be modeled by the changes in

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third country's currency).

[6] For the full process of derivation, see Appendix A.

government spending or per capita income. In an extended Balassa-Samuelson model, an increase in government spending or per capita income in a country would lead to an increase in the relative price of nontradables and a real exchange rate appreciation.

In this paper, the extended Balassa-Samuelson model will be tested with a panel of data from OECD database.

3. A Review of the Previous Empirical Literature

Balassa was the first to empirically test the productivity-differential model. In Balassa (1964), using cross-section data for 12 industrial countries in the year 1960, he regresses the real exchange rate on the per capita income, which is used as the measure of productivity. As a result, the estimated coefficient for the per capita income is statistically different from zero and of the expected sign. Still using the same cross-section data but employing G.N.P. per head instead of per capita income as productivity variable, Balassa(1973) once again confirms that richer countries have higher nominal exchange rate adjusted price levels.

Hsieh (1982) is the first study that applies time-series data to the issue. This paper focuses on the real effective exchange rates of both Japan and German for the period 1954 –

1976.[7] Hsieh's regression equation is

$$e = \beta_1 + \beta_2(a_T - a_N) - \beta_3(a_T^* - a_N^*) + \beta_4(w - s - w^* + a_T^* - a_T) + \varepsilon$$

where the variables in lower case represent rate of change of the variables in upper case, e is the rate of change of the real effective exchange rate, s is the rate of change of the nominal exchange rate, and w is the nominal wage rate measured in local currency. Thus Hsieh regresses the rate of change of the real effective exchange rate on 3 independent variables: the difference in the growth rates of productivity between the tradable and non-tradable sectors in the home country, the difference in productivity between the two sectors abroad and the difference in the growth rates of unit labor costs of the tradable sectors in the home and foreign countries. Hsieh's division of sectors into tradable and non-tradable sectors is quite simple: the "manufacturing industry" is tradable, while all the other industries are categorized into the non-tradable sector. Productivity is measured by output per man-hour and the implicit GDP deflator is used to construct the real exchange rate.

In this study, Hsieh compares his empirical work with another cross-section data study, Officer (1976), whose cross-section regression does not support the productivity-

[7] The real effective exchange rate is used when multiple trading partners of the home country are considered. It is usually constructed as a geometric or arithmetic average of the home country's bilateral real exchange rates versus the home country's trading partners according to the trading weights. In Hsieh(1982), the real effective exchange rate of Germany is constructed as the geometric average of Germany's bilateral real exchange rates against Germany's trading partners. The real effective exchange rate of Japan is calculated in the same manner.

differential hypothesis. Although Hsieh's regression equation and sector division method are similar to those of Officer (1976), his regression results with time-series data provide a quite favorable confirmation of the productivity-differential model. Hsieh argues that cross-sectional regressions do not account for the individual characteristics of each country and when these differences between countries are large, the results of cross-section regression are not reliable.

In Edison and Klovland (1987), annual data for Norway and Britain spanning a century (1874-1971) are used to estimate an expanded PPP model that incorporates the Balassa-Samuelson hypothesis. The choice of Norway as home country and Britain representing the foreign country or the outside world is quite consistent with the theoretical setting in which the home country is a small open economy. According to the authors, " over the past century as a whole the United Kingdom has unambiguously been Norway's most important single trading partner." (Edison and Klovland 1987, 310)

In this study, two different proxy variables are used to measure productivity for each country. One is GDP per capita at constant market prices. The other is the ratio of average labor productivity in commodity sectors to service sectors (i.e. the ratio of real GDP divided by employment in commodity sectors to real GDP divided by employment in service sectors). Their empirical results show that in both cases, the productivity differential between the two countries is significant in explaining the movements in the real exchange rate.

Marston (1987) examines the yen/dollar real exchange rate for the period 1973-1983. Marston's approach to dividing industries into tradable and non-tradable industries is worth mentioning. He includes "manufacturing" and "agriculture, hunting, fishing and forestry" into the tradable sector. All the other industries except "mining, quarrying" and "electricity, gas, and water" are deemed non-tradable.[8] The excluded industries produce either energy or energy-intensive products. Marston believes that their prices are very sensitive to the OPEC price policies during the period of the data, whereas only the relative impact of technology change on relative price of non-tradable good is of interest to Marston.

Marston uses average labor productivity, which is calculated using sectoral employment data, as the measure for productivity variables. Finally, Marston's OLS regression with time-series data shows that productivity-differential variables are very significant in explaining the real appreciation of the yen against the U.S. dollar over the period 1973-1983.

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[8] According to the ISIC Rev.2 (International Standard Industrial Classification of All Economic Activities, Revision 2, adopted in the 1968 version of System of National Accounts) which Marston (1987) refers to, the whole economy is disaggregated into 10 industries: manufacturing; agriculture, hunting, fishing, and forestry; mining, quarrying; electricity, gas and water; construction; wholesale and retail trade; restaurants and hotels; transport, storage and communication; finance, insurance, real estate; business services, community, social and personal services; and government services.

Asea and Mendoza (1994) is also an important study. Using annual data for 14 OECD countries for the period 1975-1985, they first regress the relative prices of non-traded goods for each country on the productivity differentials between tradable and non-tradable sectors, then regress cross-country real exchange rates against the relative prices of non-traded goods. They find that although the productivity differentials between the tradable and non-tradable sectors are very significant in explaining changes in the relative prices of non-traded goods within each country, changes in the relative prices of non-traded goods are not significant in explaining changes in real exchange rates across countries; that is, their empirical research only supports the first component of the Balassa-Samuelson model.

In De Gregorio, Giovannini and Wolf (1994), an extended Balassa-Samuelson model is tested with panel data for 14 OECD countries over the period 1971-1985. The researchers add into the traditional Balassa-Samuelson model a demand side factor, government spending, which is believed to contain higher share of non-tradable goods than tradable goods. They want to examine whether demand factor has a short run impact on relative prices of non-tradable goods. Their pooled data regression results suggest that the coefficients of the productivity and government spending variables are both highly significant and have the correct signs. In order to see whether the demand factor matters in the long run, they also average the data over time for each country, then run a cross-section regression with the obtained average data. They find that in the long run, the productivity differential variable still remains quite significant, whereas the effects of the demand factor are diminished.

In their paper, detailed input data including capital and labor are used to calculate total productivity. Also, a quantitative method that is quite different from previous (e.g., Marston's) qualitative approaches is used to classify tradable and non-tradable sectors. De Gregorio, Giovanni and Wolf calculate the ratio of total exports of a certain industry across the 14 sample countries to the total output of that industry and define an industry as tradable if the ratio is more than 10%. Despite the different approach, they reach very similar results to Marston's with respect to the classification of industries: "manufacturing", "agriculture", "mining", as well as "transportation" are classified as tradables. All the other industries are non-tradables.

Since the Balassa-Samuelson theory relates the long-run movement in the real exchange rate with productivity differentials, most related empirical studies focus on testing the long-run relations between the variables. With the development of applied time-series econometrics, some recent papers apply cointegration techniques to their empirical work:

either a three-step procedure[9] or, more recently, Johansen's (1991) maximum likelihood approach to cointegration and Horvath and Watson's (1995) tests are used.

For example, Strauss (1995), examining 14 bilateral Deutsche mark real exchange rates, confirms the existence of cointegration between real exchange rates and productivity differential variables in most cases. But the cointegrating vectors are not estimated.

Canzoneri, Cumby and Diba (1999) test the trend behavior of relative prices and relative productivity as well as nominal and PPP exchange rates with a panel of data for 13 OECD countries. Using the techniques for dealing with non-stationary data in heterogeneous panels and combining the data from the 13 countries, they find that relative prices and the relative productivity of non-tradables are cointegrated and that the slope of the cointegrating regression is close to 1. But the cointegrating regression for nominal and PPP exchange rates (against the U.S. dollar) does not support PPP for traded-goods. So the expected cointegrating relation between the real exchange rate and relative productivity can not be confirmed.

In Alexius and Nilsson (2000), quarterly real effective exchange rates and relative

[9] The first step is to test the real exchange rate and its fundamental variables for unit roots using the augmented Dickey-Fuller test. If the random walk for any of these variables can not be rejected, the second step is to estimate the cointegration regression using OLS. The third step is to test the restrictions imposed by the Balassa-Samuelson hypothesis.

productivity data for 15 OECD countries are used to investigate the two variables' systematic long-run relationships. In the paper, relative productivity is simply proxied by relative real GDP. This approach allows the observations of each country cover rather long time period (1960-1996).

As empirical results, the real effective exchange rate and relative productivity are found to be cointegrated in 11 of the 15 countries according to the Johansen (1991) approach, and 80% of the estimated parameters have the expected sign. In single-equation cointegration tests, cointegration is detected in all the countries and the estimated parameters have correct sign in two thirds of the cases. Finally, panel cointegration procedures also confirm cointegration in the panel data and the parameters of the estimated panel cointegration vector have the correct sign.

If comparing all the above mentioned empirical literature with each other, it seems that no consensus is reached regarding the relationship between real exchange rates and productivity differences across countries. Some studies find evidence to confirm the theoretically predicted relation, whereas some fail to find related empirical supports. With respect to the relationship between the relative price of nontradables to tradables and the relative productivity of tradable to non-tradable sector, there seems to exist a consensus in related studies, such as De Gregorio, Giovannini and Wolf (1994) and Asea and Mendoza (1994), that there exists a positive correlation between the relative price of non tradable to tradable goods and the relative productivity of the tradable to the non-tradable sector.

4. The Data

The data set used in this paper is balanced panel data[10] for 5 OECD countries for the period 1970-1998. The five countries are Canada, Finland, Italy, Norway (small countries) and the U.S.A.(large country). These countries are chosen because their sectoral employment data for constructing productivity variable cover a rather long time period so that a balanced panel with almost 30 observations for each country can be obtained. The data used to construct relative prices of non-tradable goods, sectoral productivity and government spending are from the OECD database. The data source for constructing real exchange rates is the IMF's *International Financial Statistics (IFS)*.[11]

4.1 The Division of Tradable and Non-tradable Sectors

The theoretical Balassa-Samuelson model relies on a clear division of all economic activities into tradable and non-tradable sectors. But in practice, such a clear dividing line does not exist. No matter what criterion for division is used, there always exist

[10] A panel in which the number of observations is exactly the same for each section is called a balanced panel.
[11] For more detailed data sources, see Appendix B.

some goods classified as non-tradable goods that enter into international trade and vice versa.

In most previous empirical studies, a certain sector or industry is classified as part of the tradable sector simply based on a qualitative judgement on the extent to which the sector participates into international trade. Some authors do specify a quantitative criterion (such as De Gregorio, Giovannini and Wolf (1994)). Although a quantitative criterion seems to be more desirable in the sense that research should be as rigorous as possible, the setting of the criterion could be quite arbitrary[12] and a particular quantitative criterion is not necessarily appropriate for different groups of countries. In addition, the quantitative approach of Degorio, Giovannini and Wolf (1994) has led to very similar results as that of quantitative judgement.

So this paper follows the qualitative approach by defining "agriculture, hunting and forestry", "fishing", "mining and quarrying", "manufacturing" and "electricity, gas and water" as tradable sectors. All the other sectors are deemed non-tradable sectors[13].

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[12] In De Gregorio, Giovannini, and Wolf(1994), a sector is defined as tradable sector if more than 10% of the 14 countries' total production in that sector is exported. But changing the 10% criterion would lead to different division results.

[13] According to the ISIC Rev.3(International Standard Industrial Classification of All Economic Activities, Revision3, adopted in 1993 Version of System of National Accounts)which is used in the National Accounts of OECD Countries Volume I and II, Version 2002, the whole economy is disaggregated into 17 sectors: agriculture, hunting and forestry; fishing; mining and quarrying; manufacturing; electricity, gas and water supply; construction; wholesale and retail trade, repair of motor vehicles, motorcycles and personal and household goods; hotels and restaurants; transport, storage and communication; financial intermediation; real estate, renting and business activities; public administration and defense and compulsory social security; education; health and social work; other community, social and personal service activities; private households with employed persons; extra-territorial organizations and bodies

Note that it is more reasonable to classify "electricity, gas and water supply" as non-tradable sector, but the data for this sector is aggregated into "manufacturing" in the table No.2 (the sectoral output data come from this table) of *National Accounts of OECD Countries*, Volume I and II, Version 2002, Electronic edition. Although the table No.7 in Volume II has disaggreated the "electricity, gas and water supply" from "manufacturing", the data in this table cover a much shorter time period. So "electricity, gas and water supply" has to be classified as tradable. For a similar reason, "transport, storage and communication", which seems more tradable, has to be categorized as non-tradable.

Therefore, the impossibility of a clear division between tradable and non-tradable sectors as required in the theory constitutes an important limitation for the related empirical research.

4.2 Proxy Variables

Almost all the variables needed for empirical work on Balassa-Samuelson model can not be directly obtained from available data. So one must construct the variables.

In the theory, the relative price of non-tradable goods in each country should be measured with appropriate domestic price indices of tradables and nontradables. Unfortunately, in reality, no country reports such price indices. This paper constructs these price indices using sectoral gross value-added in current and constant prices. First, the total output for the tradable (non-tradable) sector is obtained by summing up all the value-added produced in those industries classified as tradable (non-tradable) sector. Then, the price index for tradables (nontradables) is calculated by dividing the sectoral total output in current prices by the sectoral total value-added in constant prices. Finally, the relative price (denoted by P) of non-tradable goods is the ratio of the price index for nontradables to the price index for tradables.

The gross value-added data used here are evaluated at "Basic Price".[14]The Basic Price excludes any influence of taxes, subsidies and transport charges on producer's prices. So compared with the data evaluated at other price bases, such as market price, the Basic-Price data are more appropriate for evaluating the impacts of technology or productivity changes on producers' prices. The main shortcoming of this method of constructing the relative price of nontradables results from, once again, the operational impossibility of a clear division into tradable and non-tradable sectors.

In previous empirical work, two main approaches are employed to proxy

[14] According to the definition in National Accounts of OECD Countries, Volume I and II, Version 2002, the "Basic Price" is the amount receivable by the producer from the purchaser for a unit of good or service produced as output minus any tax payable, and plus any subsidy receivable, on that unit as a consequence of its production or sale; it excludes any transport charges invoiced separately by the producer.

productivity variables. One is using total inputs data including labor and capital to calculate total productivity. The other is constructing labor productivity only with employment data. This paper chooses to follow the labor productivity approach, since sectoral capital data is usually believed to be much less reliable than employment data. The productivity variable (denoted by A_T and A_N) is measured as average labor productivity in each sector, defined as the ratio of sectoral (tradable or non-tradable) total value-added in constant prices to the sectoral (tradable or non-tradable) total employment.

The government spending variable (denoted by G) in this paper is the share of final government expenditure in total output (GDP). Both the government expenditure and total outputs are in constant prices.

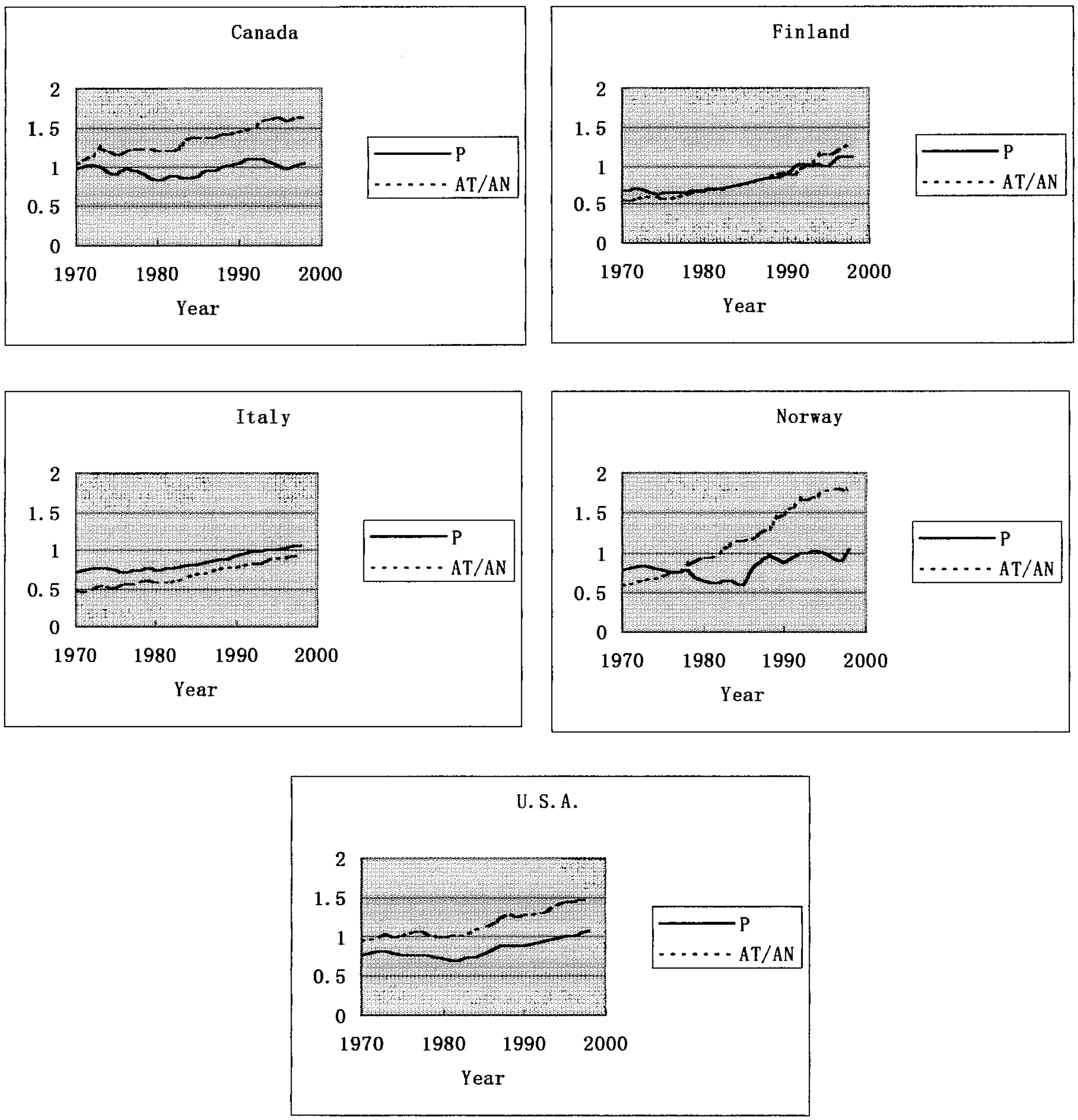
The data on nominal exchange rates and CPIs (Consumer Price Index) are drawn from the IMF's *International Financial Statistics(IFS)*. The nominal exchange rates are bilateral rates against the U.S. Dollar and are period average rates (i.e. the average of one year's monthly rates). The CPIs are chosen to construct real exchange rates because the CPI is usually broadly representative of both tradable and non-tradable goods, which in this sense is quite consistent with the requirement of the theoretical model. But compared with the theoretical variable, the real exchange rate constructed with the CPI still has limitations. First, the CPI does not exclude the components of prices controls, subsidies and indirect taxes, therefore the changes in these factors can influence its level, whereas only the impact of technology or productivity changes on price levels is the object of the research. Second, CPIs for different countries are neither based on the same baskets of goods and services nor constructed with same weights for tradable (non-tradable) goods. But in the theory, national price indices are constructed with exactly the same composites of tradables and nontradables and with the same weights.

Finally, in this paper, while testing the second component of the Balassa-Samuelson model, the United States is taken to be foreign country (denoted by *). The real exchange rate (denoted by RER) is the ratio of home country's CPI to the foreign country's CPI adjusted by the nominal exchange rate. Thus an increase in the real exchange rate means a real exchange rate appreciation for the home country.

4.3 Some Time Series Properties of the Data

This section examines some time-series properties of the data before presenting the econometric analysis. Figure 1 plots the relative price of nontradables to tradables along with the relative productivity of tradable sector to non-tradable sector for each of the five countries. The relative productivity of the traded good sector has been steadily increasing in all the countries since 1970. This trend confirms that in practice, there has been indeed

Figure 1. Relative Price of Non-tradable to Tradable Goods and Relative Productivity of Tradable to Non-tradable Sector



higher productivity growth in the tradable sector than in the non-tradable sector for decades. The increasing trend is especially strong in Norway and Finland. Figure 1 also shows that the relative price of nontradables in each country exhibits the same increasing trend over time. So it seems that the relative price of nontradables and the relative productivity of tradable-sector have the correlation predicted by the Balassa-Samuelson model. But the examination of the causation relation between the two variables requires further econometric analysis of the data.

Figure 2. Real Exchange Rate and Productivity Relative to U.S.A.[15]

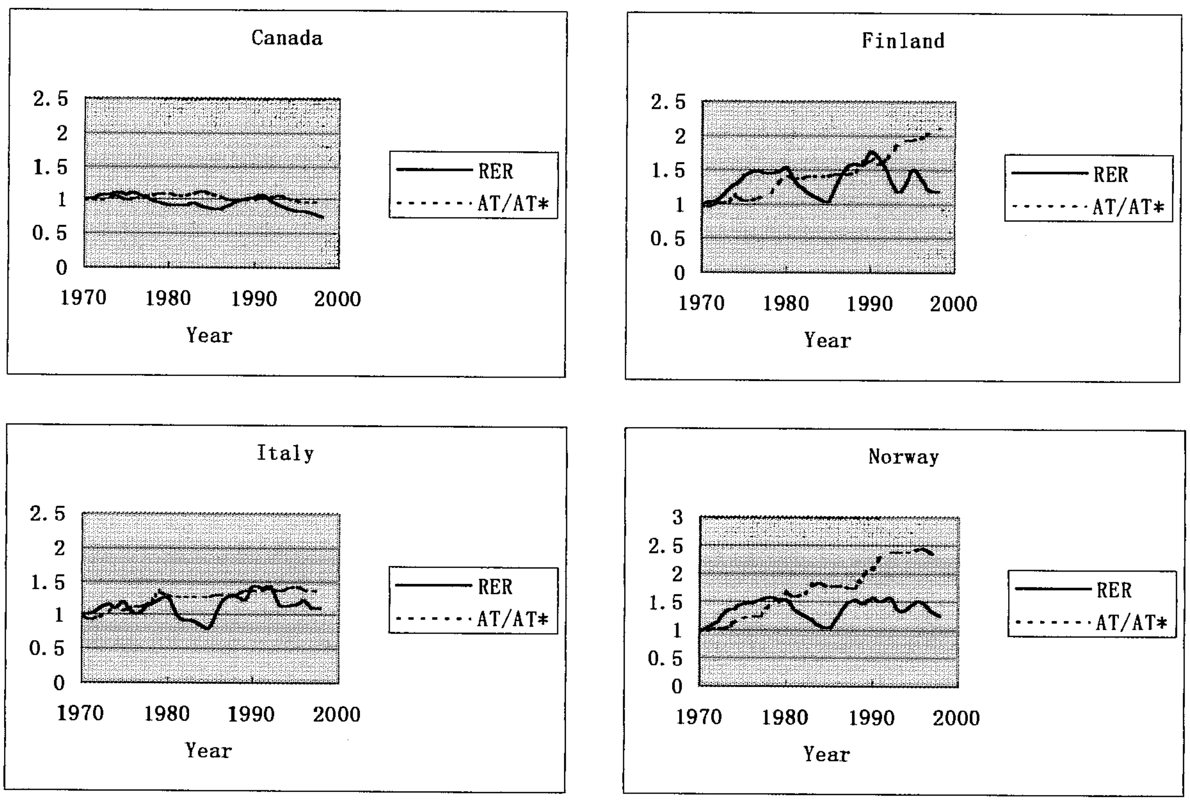


Figure 2 plots the bilateral real exchange rate of each country along with the productivity of each country relative to the U.S.A. It can be seen from the figure that there

[15] The data for the year 1970 have been normalized to 1

are no comovements between the relative productivity and real exchange rates as the second component of the theory model predicts, and the lack of correlation is very obvious, at least in the short-run. The real exchange rate is more volatile than the relative productivity in all of the countries except Canada. But, to investigate whether there exists a long-run relationship between the variables, an econometric analysis of the data is necessary.

5. The Empirical Results

In this section, the techniques dealing with time-series cross-section data in both single equation and systems of equations are applied to do estimations and hypotheses testing. The empirical work begins with pooling the data of all the countries and assuming a common intercept and a common set of slope coefficients for all countries at all time periods. Under this assumption, this paper specifies the theoretical model's econometric counterparts as:

$$lnP_{i,t} = \beta_1 + \beta_2 ln(A_T/A_N)_{i,t} + \beta_3 G_{i,t} + \varepsilon_{i,t}$$

and

$$lnRER_{i,t} - lnRER_{i,t-1} = \alpha_1 + \alpha_2 [ln(A_T/A^*_T)_{i,t} - ln(A_T/A^*_T)_{i,t-1}] + \alpha_3 [ln(G/G^*)_{i,t} - ln(G/G^*)_{i,t-1}] + u_{i,t}$$

where i denotes the country, t the year, P is the relative price of non-tradables to tradables, A represents average labor productivity, and G is the government spending variable.

First applying OLS to pooled data leads to the OLS estimates. The appropriate diagnostic tests (Lagrange Multiplier Test, Breuch-Pagan (1980) Lagrange Multiplier Test) are then employed to detect cross-section heteroskedasticity and cross-section correlation in the panel data. If the assumption of independently and/or identically distributed disturbances is rejected, an appropriate correction matrix is used to test the significance of the OLS estimates. Since the data within each section is time series data, within-section autocorrelation should also be tested. Then, given all the diagnostic test results, appropriate feasible GLS estimators are applied to get more efficient estimates. The significance of the GLS estimates and whether they are of the theoretically predicted sign are also examined.

For a panel of data with 5 countries with almost 30 observations in each country, the assumption of common coefficients across countries is a quite strict one. The empirical work then turns to relaxing the assumption and allowing all the coefficients differ across countries. The econometric models now change to:

$$LnP_{i,t} = \beta_{1i} + \beta_{2i} ln(A_T/A_N)_{i,t} + \beta_{3i} G_{i,t} + \varepsilon_{i,t}$$

and

$$lnRER_{i,t} - lnRER_{i,t-1} = \alpha_{1i} + \alpha_{2i} [ln(A_T/A^*_T)_{i,t} - ln(A_T/A^*_T)_{i,t-1}] + \alpha_{3i} [ln(G/G^*)_{i,t} - ln(G/G^*)_{i,t-1}] + u_{i,t}$$

The Seemingly Unrelated Regression (SUR) is then employed to estimate different set of coefficients for each country.

5.1 Testing the First Component of the Balassa-Samuelson Model

The first component of the Balassa-Samuelson Model explains the increase in the relative price of non-tradable goods by faster productivity growth of the tradable good sector. To test this hypothesis, the empirical work begins with pooling the data for all the five countries and doing OLS estimation.

Table 1 reports the pooled OLS estimation results. The estimated coefficient for the relative productivity variable has the correct sign, while the government spending variable does not enter the equation with expected sign. Since the paper is dealing with panel data, before testing the significance of these OLS estimates, whether there exists cross-section heteroskedasticity or cross-section correlation should be investigated.

Table 1. Pooled OLS Estimates with Panel-Corrected Standard Errors

Equation: $lnP_{i,t} = \beta_1 + \beta_2 ln(A_T/A_N)_{i,t} + \beta_3 G_{i,t} + \varepsilon_{i,t}$

	β_1	β_2	β_3
Pooled OLS estimates (Panel-corrected standard errors)	-0.1449** (0.0548)	0.2907** (0.0354)	-0.0819 (0.2576)
No. of observations = 145 R-Squared = 0.4195 F (2, 142) = 50.55 (P-Value = 0.000)			

#: Significant at 10% level

*: Significant at 5% level

**: Significant at 1% level

The results of related diagnostic tests are reported in table 2. Based on the pooled OLS results, under the null hypothesis of no cross-section heteroskedasticity, the LM test statistic follows a chi-squared distribution with 4 degrees of freedom. Since the p-value for the obtained LM test statistic is 0.0046, the null hypothesis of no-cross section heteroskedaticity can be rejected at the 1% significance level. Similarly, based on the pooled OLS results, the LM test statistic for testing no cross-section correlation follows a chi-squared distribution with 10 degrees of freedom. Due to very low p-value (0.0000) for the obtained test statistic, the null hypothesis of no cross-section correlation in the disturbances of the panel data should be rejected with a high degree of confidence. So table 2 provides strong evidence of cross-section heteroskedasticity and cross-section correlation in the disturbance terms of the pooled data.

Table 2. Diagnostic Tests Based on Pooled OLS Estimation

Equation: $lnP_{i,t} = \beta_1 + \beta_2 ln(A_T/A_N)_{i,t} + \beta_3 G_{i,t} + \varepsilon_{i,t}$

	H_0	Test statistic	P-Value
LM Test for cross-section heteroskedasticity	No cross-section heteroskedasticity	15.0480	0.0046
Breusch-Pagan LM test for Cross-section correlation	No cross-section correlation	58.5860	0.0000

Given the findings of the diagnostic tests, it should be expected that the t-tests for the pooled OLS estimates based on OLS standard errors are no longer valid. So, instead of OLS standard errors, table 1 presents the panel-corrected (Beck and Katz (1995)) standard errors that accounts for cross-section heteroskedasticity and cross-section correlation. Based on the panel-corrected standard errors, the coefficients of the relative productivity variable is highly significant (at the 1% level) in the pooled OLS estimation. The coefficient of the government spending variable does not have the theoretically expected sign, but it is not significant either. Therefore, according to the pooled OLS estimation, a 1% increase in the relative productivity of the tradable to the non-tradable sector leads to a 0.29% increase in the relative price of the non-tradable to the tradable good, while the change in government spending share has no statistically significant impact on the relative price of nontradables.

However, the analysis based on pooled OLS estimation with panel-corrected errors ignores the possibility of within-section autocorrelation, since the data set within each country is time-series data. Table 3 contains the results of diagnostic tests for within-section autocorrelation and the estimated autocorrelation coefficients. The Durbin-Watson statistic for each country suggests that there is indeed a strong autocorrelation process in the disturbance terms in each of the five data sets. The table also reports the estimated autocorrelation coefficients based on separate OLS estimation for each country and based on pooled OLS estimation. These coefficients also provide strong evidence of an AR(1) process in each section. Furthermore it can be seen from the table that the estimated AR(1) coefficients based on pooled OLS are very close to 1.

Since cross-section heteroskedasticity and cross-section correlation as well as within-section autocorrelation have all been detected in the panel data, the empirical work then proceeds to re-estimate the model with FGLS (Feasible Generalized Least Squares). Table 4 reports the FGLS estimates for the model with cross-section heteroskedasticity, cross-section correlation and country-specific first-order autocorrelation. In table 4, the coefficients of both the relative productivity and government spending variables are significant at the 1% level. According to the FGLS estimates, a 1% increase in the relative productivity of tradable to non-tradable good sector would cause a 0.39% increase in the

Table 3. Diagnostic Tests For Autocorrelation within Each Country And Estimated Autocorrelation Coefficients

		Canada	Finland	Italy	Norway	U.S.A.
Based on separate OLS estimation for each country	Durbin-Watson Statistic (P-Value for positive autocorrelation test)	0.3122 (0.0000)	0.6987 (0.0000)	0.5315 (0.0000)	0.3345 (0.0000)	0.3852 (0.0000)
	Autocorrelation coefficient	0.8007	0.6277	0.7219	0.8170	0.7041
Based on pooled OLS estimation	Autocorrelation coefficient	0.8805	0.9509	0.9844	0.8881	0.9736

relative price of non-tradable to tradable good. A 1% increase in the government share in total GDP would lead to almost the same percentage increase in the relative price of nontradables to tradables.

Table 4. Pooled FGLS Estimates for the Model with Cross-section Heteroskedasticity, Cross-section Correlation And AR(1)

Equation: $lnP_{i,t} = \beta_1 + \beta_2 ln(A_T/A_N)_{i,t} + \beta_3 G_{i,t} + \varepsilon_{i,t}$

	β_1	β_2	β_3
FGLS estimates (standard errors)	-0.3535 (0.1001)	0.3394** (0.0521)	1.0004** (0.3879)
No. of observations = 145 Buse Raw-Moment R-Squared = 0.2447 F (2, 142) = 22.714 (P-Value = 0.000)			

#: Significant at 10% level

*: Significant at 5% level

**: Significant at 1% level

Till now, all the estimated models have assumed the same set of coefficients for all of the 5 countries. This is a strict constraint given that there are 29 observations for each country. The following analysis then relaxes the constraint, allowing coefficients differ across countries and examining whether the estimation result in each country supports the theoretical hypotheses.

The FGLS estimates of SUR (Seemingly Unrelated Regressions) are presented in table 5. Because of the AR(1) process within each country, the data used for SUR is the transformed data correcting for AR(1). The data are transformed with estimated autocorrelation coefficients based on separate OLS estimation for each country. The first observations of the transformed data sets have been dropped. A quick examination of the table finds that the estimated coefficients of the relative productivity variable have the expected sign for all of the 5 countries. Whereas only 3 of the 5 estimated coefficients for the government spending variable have the correct sign. Also, the coefficient of the relative productivity variable is significant in Canada, Finland, Italy and the U.S.A. at the 10% significance level, and in Finland, Italy, and the U.S.A. the coefficient of the relative

Table 5. SUR Estimates

(Standard Errors in Parentheses)

Equation: $LnP_{i,t} = \beta_{1i} + \beta_{2i} ln(A_T/A_N)_{i,t} + \beta_{3i} G_{i,t} + \varepsilon_{i,t}$

	β_1	β_2	β_3
Canada	-0.0679 (0.0466)	0.2837# (0.1716)	0.9552 (0.8692)
Finland	-0.2366** (0.0576)	0.6233** (0.0469)	2.4721** (0.6845)
Italy	-0.0718 (0.0492)	0.5803** (0.0678)	1.6702# (0.9876)
Norway	-0.0098 (0.1001)	0.2224 (0.1717)	-0.8284 (2.4791)
U.S.A.	-0.0502 (0.0405)	0.7598** (0.0959)	-0.7603 (0.7389)
No. of observations = 28 for each country System R-squared = 0.9115			

#: Significant at 10% level

*: Significant at 5% level

**: Significant at 1% level

productivity is also significant at the 1% level. While only in Finland and Italy, the coefficient of the government spending variable is significant at the 10% level, and only in Finland it is also significant at the 1% level.

Based on all the above empirical results, it can be concluded that the relative productivity of the tradable to non-tradable sector has positive impact on the relative price of nontradables to tradables (except for Norway in the SUR estimation), as the Balassa-Samuelson model predicts. This finding is consistent with those in almost all the previous similar studies using panel data, such as Asea and Mendoza (1994), De Gregorio, Giovannini and Wolf (1994) and Canzoneri, Cumby and Diba (1999). But the empirical evidence here is not enough to support the inclusion of the government spending variable in the model. This result is only partly consistent with that of De Gregorio, Giovannini and Wolf (1994a), which concludes that government spending has a short-run impact on relative price of nontradables but has no long-run effect.

5.2 Testing the Second Component of the Balassa-Samuelson Model

Now the empirical work turns to testing whether the movement in the real exchange rate is related to the productivity differentials in the tradable sector across countries. The following empirical work will also test whether the government spending differentials across countries are linked with the movement in the real exchange rate. The empirical methods used here are basically the same as that used in the section 5.1.

Table 6 reports OLS estimation results using the pooled data of 4 countries (since the U.S.A. is taken to be the foreign country, the variables of the U.S.A. are the reference variables). First, it can be noticed that the coefficient of the productivity differential variable does not have the correct sign (the theoretical model predicts a positive sign for both productivity differential and government spending differential variables). But before testing its significance, the results of diagnostic tests should be examined.

The diagnostic test results are reported in table 7. Under the null hypothesis of no cross-section heteroskedasticity, the LM test statistic follows a chi-squared distribution with 3 degrees of freedom. Based on the p-value for the observed test statistic, the null hypothesis

Table 6. Pooled OLS Estimates with Panel-Corrected Standard Errors

Equation: $lnRER_{i,t} - lnRER_{i,t-1} = \alpha_1 + \alpha_2[ln(A_T/A^*_T)_{i,t} - ln(A_T/A^*_T)_{i,t-1}] + \alpha_3[ln(G/G^*)_{i,t} - ln(G/G^*)_{i,t-1}] + u_{i,t}$

	α_1	α_2	α_3
Pooled OLS estimates (Panel-corrected standard errors)	0.0073 (0.0149)	-0.1055 (0.2788)	0.0111 (0.4241)
No. of observations = 112			

#: Significant at 10% level

*: Significant at 5% level

**: Significant at 1% level

can be rejected at the 1% significance level. Similarly, under the null hypothesis of no cross-section correlation, the LM test statistic follows a chi-squared distribution with 6 degrees of freedom. According to the p-value for the obtained test statistic, the null hypothesis of no cross-section correlation can be rejected. Therefore, table 7 suggests that both cross-section heteroskedasticity and cross-section correlation have been detected in the pooled data. Given the results of table 7, panel-corrected standard errors are used to test the significance of the pooled OLS estimates. It can be seen from table 6 that the coefficients of neither the productivity differential nor the government spending differential variable are significant at any desired significance levels.

Table 7. Diagnostic Tests Based on Pooled OLS Estimation

Equation: $lnRER_{i,t} - lnRER_{i,t-1} = \alpha_1 + \alpha_2[ln(A_T/A^*_T)_{i,t} - ln(A_T/A^*_T)_{i,t-1}] + \alpha_3[ln(G/G^*)_{i,t} - ln(G/G^*)_{i,t-1}] + u_{i,t}$

	H_0	Test statistic	P-Value
LM Test for cross-section heteroskedasticity	No cross-section heteroskedasticity	11.8320	0.0080
Breusch-Pagan LM test for Cross-section correlation	No cross-section correlation	60.4450	0.0000

Table 8 contains the results of diagnostic tests for within-section autocorrelation and estimated autocorrelation coefficients for each country. The presence of a within-section autocorrelation process is obvious except in Italy.

Given the findings of all the above diagnostic tests, table 9 presents the pooled FGLS estimates that account for cross-section heteroskedasticity, cross-section correlation and AR(1) errors within each section. It can be shown from table 9 that the coefficient of the

Table 8. Diagnostic Tests For Autocorrelation within Each Country And Estimated Autocorrelation Coefficients

		Canada	Finland	Italy	Norway
Based on separate OLS estimation for each country	Durbin-Watson Statistic (P-Value for positive autocorrelation test)	1.1592 (0.0072)	1.2938 (0.0188)	1.7775 (0.2559)	1.3491 (0.0279)
	Autocorrelation coefficient	0.3703	0.3497	0.1098	0.3086
Based on pooled OLS estimation	Autocorrelation coefficient	0.5181	0.3449	0.1273	0.3243

productivity differential variable still does not have the theoretically predicted sign and that none of the coefficients is significant at desired significance levels.

Table 9. Pooled FGLS Estimates for the Model with Cross-section Heteroskedasticity, Cross-section Correlation And AR(1)

Equation: $lnRER_{i,t} - lnRER_{i,t-1} = \alpha_1 + \alpha_2[ln(A_T/A^*_T)_{i,t} - ln(A_T/A^*_T)_{i,t-1}] + \alpha_3[ln(G/G^*)_{i,t} - ln(G/G^*)_{i,t-1}] + u_{i,t}$

	α_1	α_2	α_3
FGLS estimates (standard errors)	-0.0033 (0.0132)	-0.1569 (0.1196)	0.0552 (0.2102)
No. of observations = 112			

#: Significant at 10% level

*: Significant at 5% level

**: Significant at 1% level

The FGLS estimates of a SUR model of the real exchange rate are reported in table 10. Again, the data used for SUR are the transformed data correcting for AR(1). The data are transformed with estimated autocorrelation coefficients based on separate OLS estimation for each country. The first observations of the transformed data sets have been dropped. In table 10, 3 of the 4 estimated coefficients for the productivity differential variable have wrong signs, but only 1 of them is significant. One estimated coefficient for government spending differential variable does not have expected sign and it is not significant either. Only one estimated coefficient for government spending variable is significant and also of the correct sign.

Table 10. SUR Estimates

(Standard Errors in Parentheses)

Equation: $lnRER_{i,t} - lnRER_{i,t-1} = \alpha_{1i} + \alpha_{2i}[ln(A_T/A^*_T)_{i,t} - ln(A_T/A^*_T)_{i,t-1}] + \alpha_{3i}[ln(G/G^*)_{i,t} - ln(G/G^*)_{i,t-1}] + u_{i,t}$

	α_1	α_2	α_3
Canada	-0.0105 (0.0065)	0.2707 (0.2346)	1.0508** (0.3512)
Finland	0.0176 (0.0200)	-0.4017 (0.2873)	-0.3529 (0.3650)
Italy	0.0088 (0.0220)	-0.2286 (0.3039)	0.1667 (0.4684)
Norway	0.0122 (0.0167)	-0.3398* (0.1444)	0.2366 (0.2744)
No. of observations = 27 for each country			

#: Significant at 10% level

*: Significant at 5% level

**: Significant at 1% level

Based on the above empirical analysis, no evidence has been found to support the theoretically predicted relation between the real exchange rate and productivity differentials across countries. The empirical evidence is also not strong enough to support the expected link between the real exchange rates and the government spending differentials across countries. These findings are consistent with those of Asea and Mendoza (1994), who regressed cross-country real exchange rates on the relative price of nontradables for a panel of 14 OECD countries, and could not confirm the second component of the Balassa-Samuelson model either. In comparison with those panel-data studies using cointegration techniques, the results of this paper are consistent with those of Canzoneri, Cumby and Diba (1999) but contrast with those of Strauss (1995) and Alexius and Nilsson (2000).

6. Conclusions

This paper uses techniques for dealing with pooled time-series cross-section data to test an extended Balassa-Samuelson model. The empirical evidence from a panel data of 5 OECD countries support the hypothesis that the relative price of non-tradable to tradable goods is positively related to the relative productivity of tradable to non-tradable good sector. That is, faster growth in the tradable good sector's productivity can indeed explain the upward trend of the relative price of nontradables to tradables over time. The Balassa-Samuelson model also predicts that under the assumption of PPP for tradable goods, the movement in the real exchange rate is associated with the productivity differentials between countries, especially the productivity differences in the tradable sector across countries. However, the empirical work of this paper finds no evidence to support this theoretical hypothesis.

Government spending is a demand side variable that can proxy the demand for non-tradable goods in the extended Balassa-Samuelson model. It is theoretically predicted to be positively related to the relative price of the non-tradable to tradable good. But the empirical evidence in this paper are not strong enough to support including either the government spending variable or the government spending differential (across countries) variable into the original model.

There are probably two reasons for the failure to find empirical evidence of the theoretically predicted relation between the real exchange rate and the productivity differential variable. First, PPP for tradable goods does not hold in practice as the theory requires. Whether PPP holds for tradable goods across countries has not been tested in this study. This kind of test could be another effective empirical approach to test the second component of the Balassa-Samuelson model. Second, the empirical results could be very sensitive to the choice of reference country and reference currency (i.e. foreign country and foreign currency), because the theoretical model also relies on close international trade links between home and foreign country. In this paper, the U.S.A is taken to be foreign country and thus US$ is chosen to be reference currency for all the home countries. But America is not necessarily the most important trading partner for all the home countries in the case of this paper. So choosing more appropriate reference country and reference currency for each home country or using effective real exchange rate data may lead to more accurate empirical results.

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Appendix A

1. Logarithmically differentiate the equation (2):

The equation (2) is $A_T f(K_T) - rK_T = W$

⇨ $A_T f(K_T) = rK_T + W$

⇨ $\mathrm{Log} A_T f(K_T) = \log(rK_T + W)$

⇨ $\log A_T + \log f(K_T) = \log(rK_T + W)$

⇨ $d \log A_T + d \log f(K_T) = d \log(rK_T + W)$

⇨ $dA_T/A_T + d\, f(K_T)/f(K_T) = d(rK_T + W)/(rK_T + W)$

⇨ $dA_T/A_T + f'(K_T)dK_T/f(K_T) = (rdK_T + dW)/(rK_T + W)$

Since r is exogenous, it can be treated as a constant here.

⇨ $dA_T/A_T + f'(K_T)dK_T/f(K_T) = (rdK_T + dW)/A_T f(K_T)$

Since $A_T f(K_T) - rK_T = W$ [equation (2)]

⇨ $dA_T/A_T + f'(K_T)dK_T/f(K_T) = rdK_T/A_T f(K_T) + dW/A_T f(K_T)$

⇨ $dA_T/A_T + f'(K_T)dK_T/f(K_T) = A_T f'(K_T)dK_T/A_T f(K_T) + dW/A_T f(K_T)$

Since $A_T f'(K_T) = r$ [equation (1)]

⇨ $dA_T/A_T + f'(K_T)dK_T/f(K_T) = f'(K_T)dK_T/f(K_T) + dW/A_T f(K_T)$

⇨ $dA_T/A_T = dW/A_T f(K_T)$

⇨ $dA_T/A_T = [W/A_T f(K_T)]\, dW/W$

⇨ $\hat{A}_T = U_T \hat{W}$ Since $\hat{X} \equiv dX/X$ for any variable X and $U_T \equiv W/A_T f(K_T)$

2. Logarithmically differentiate the equation (4):

The equation (4) is $PA_N g(K_N) - rK_N = W$

⇨ $PA_N g(K_N) = rK_N + W$

⇨ $\log[PA_N g(K_N)] = \log(rK_N + W)$

⇨ $\log P + \log A_N + \log g(K_N) = \log(rK_N + W)$

⇨ $d \log P + d \log A_N + d \log g(K_N) = d \log(rK_N + W)$

⇨ $dP/P + dA_N/A_N + d\, g(K_N)/g(K_N) = d(rK_N + W)/(rK_N + W)$

⇨ $dP/P + dA_N/A_N + g'(K_N)dK_N/g(K_N) = (rdK_N + dW)/(rK_N + W)$

Since r is exogenous, it can be treated as a constant here.

⇨ $dP/P + dA_N/A_N + g'(K_N)dK_N/g(K_N) = (rdK_N + dW)/(PA_N g(K_N))$

Since $PA_N g(K_N) - rK_N = W$ [equation (4)]

⇨ $dP/P + dA_N/A_N + g'(K_N)dK_N/g(K_N) = rdK_N/PA_N g(K_N) + dW/PA_N g(K_N)$

⇨ $dP/P + dA_N/A_N + g'(K_N)dK_N/g(K_N) = PA_Ng'(K_N)dK_N/PA_Ng(K_N) + dW/PA_Ng(K_N)$

Since $PA_Ng'(K_N) = r$ [equation (3)]

⇨ $dP/P + dA_N/A_N + g'(K_N)dK_N/g(K_N) = g'(K_N)dK_N/g(K_N) + dW/PA_Ng(K_N)$

⇨ $dP/P + dA_N/A_N = dW/PA_Ng(K_N)$

⇨ $\hat{P} + \hat{A}_N = U_N\hat{W}$ Since $\hat{X} \equiv dX/X$ for any variable X and $U_N \equiv X/PA_Ng(K_N)$

3. Logarithmically differentiate the equation (8):

The equation (8) is $C/C^* = (P/P^*)^{1-\theta}$

⇨ $\log(C/C^*) = \log(P/P^*)^{1-\theta}$

⇨ $\log C - \log C^* = (1 - \theta)(\log P - \log P^*)$

⇨ $d\log C - d\log C^* = (1 - \theta)\, d\log P - d\log P^*$

⇨ $dC/C - dC^*/C^* = (1 - \theta)(dP/P - dP^*/P^*)$

⇨ $\hat{C} - \hat{C}^* = (1 - \theta)(\hat{P} - \hat{P}^*)$ Since $\hat{X} \equiv dX/X$ for any variable X.

⇨ $\hat{C} - \hat{C}^* = (1 - \theta)\{(U_N/U_T)\hat{A}_T - \hat{A}_N - [(U_N/U_T)\hat{A}^*_T - \hat{A}^*_N]\}$

Since $\hat{P} = (U_N/U_T)\hat{A}_T - \hat{A}_N$, $\hat{P}^* = (U_N/U_T)\hat{A}^*_T - \hat{A}^*_N$ [equation (7)]

⇨ $\hat{C} - \hat{C}^* = (1 - \theta)[(U_N/U_T)(\hat{A}_T - \hat{A}^*_T) - (\hat{A}_N - \hat{A}^*_N)]$

Appendix B: The Data Sources

The data used to construct the relative price of non-tradable goods, sectoral productivity and government spending are drawn from the *Nation Accounts of OECD Countries*, Volume I and II, Version 2002, Electronic edition: The sectoral gross value-added data are from Table 2-Gross domestic product: output approach. The sectoral employment data are from Table 6-Employment in persons and in full-time equivalents by industry: domestic concept. The final government expenditure and total output data are from Table 1-Gross domestic product: expenditure approach.

The data on nominal exchange rates and CPIs (Consume Price Index) are drawn from IMF's *International Financial Statistics*, Electronic resource, CD-ROM. The nominal exchange rates are bilateral nominal exchange rates against the U.S.$ and are period average rates.