SYSTEM IDENTIFICATION BY MEANS OF THE IMPLICIT SYNTHESIS METHOD

by

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ABSTRACT

This work is to make a study on the system identification problem. Special attention is paid to the parameter determination by the implicit synthesis method.

In Chapter 1, system identification problem is formulated. Preliminary considerations, such as mathematical model construction, test signal and stored energy, etc., are described.

In Chapter 2, the fundamental principles of system identification, such as correlation technique, model referenced technique, etc., are described. The concepts of generalized model and generalized error are explained. Based on these concepts, the identification schemes are classified as "using a physical model" and "using explicit mathematical relations".

In Chapter 3, several typical identification schemes of different methods, such as learning model, orthogonal filters, quasi-linearization method, regression analysis method, etc., are described in some detail. A brief description of some other methods is also given.

In Chapter 4, a detail investigation of the system identification by the implicit synthesis method is made. An extension of Clymer's method to the nonlinear system is presented. We have found that the parameters of a first-order nonlinear system whose nonlinearity ~~is~~ can be approximated by a polynomial ~~representable as a convergent power series~~ in the dependent variable can be determined by this method. Several examples are worked out on an EAI TR-48 analog computer and the values of the parameters thus determined are found to be satisfactorily accurate. A method of identifying an nth-order linear system with all simple and real characteristic roots is also proposed. This method involves transformation

of variables and can be implemented on an analog and digital hybrid computer system.

Despite the critical stability problem associated with the implicit synthesis method, it is still considered as very powerful and promising because of its simplicity.

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CHAPTER 1.

General Description of System Identification Problem

1. Introduction

System identification is one of the most important problems in achieving system optimization and/or adaptation. In this problem the system is considered as a black box (Fig. 1), neither the form nor the parameters of the system being known. The system configuration is to be determined from the known input and the measured output. If the form of the system is not accurately estimated, the system thus identified may be an equivalent system in a certain sense but not the exact system to be determined. Since the form or order of a system can be roughly estimated by dimensional analysis, transient analysis, etc., usually the form of the system is assumed to be known and the system is described by a proper mathematical model. Then the identification problem is reduced to a parameter determination problem and the only thing we have to do is to determine the parameters in the system. If the measured quantities used to determine the values of the parameters are corrupted by noise, then statistical analysis has to be used to achieve the best estimate for the parameter values. This type of problem is called the parameter estimation problem, and is covered in the system identification problem in the broader sense of the latter.

2. Preliminary Considerations

There are some important things which should be considered in the identification problem. They are the proper choice of the mathematical model which describes the system, the determination of whether a test signal can be used or simply the normal operating signal is to be used, and the consideration of the stored energy.

2.1 Mathematical model construction

The first problem in the system identification is the determination and construction of a proper mathematical model. Given a process, one should determine and know whether the system is linear or nonlinear, time-invariant or time-varying so that a proper mathematical model is constructed to describe the system. Once the system is described by a proper mathematical model in terms of differential equation or transfer function, then the characteristic parameters are determined from the experimental measurements of known quantities of the system input and output.

2.2 Test signals

Usually if a proper test signal can be applied at the input of the system, the solution of the identification problem can be achieved more easily. Very often white noise is used as a test signal [1-4]. But in many high performance systems, e.g., adaptive control systems, the injection of any test signal is intolerable or impossible and the identification should be done by using only the normal operating signals.

2.3 Stored energy

In the identification of a system, the stored energy should be considered. In the identification of non-inert system usually a long time is spent in making measurements, since the measurement can be made only after the stored energy is dissipated. In an adaptive control system the measurements should be carried out in a short time interval compared with the drift time of the system parameters. One cannot ignore the stored energy nor can he wait until it has been dissipated. There are two approaches to eliminate the stored energy effects: the first approach utilizes an identification procedure which is completely insensitive to the presence of the stored energy [2,5]. The second approach utilizes the measurement of the process output to compute the stored energy component of the output [5,6].

3. Mathematical Models

There are several mathematical models which can be used to describe a system. The proper choice of the mathematical model depends on the nature of the system and the method of measurement and analysis. Usually the following models are used:

1) Differential equation.
2) Transfer function.
3) Convolution integral.
4) Pulse transfer function.
5) State space equation.

3.1 Differential equation

The system input x(t) and output y(t) can be related by the differential equation

$$\sum_{i=o}^{n} a_i p^i y + N(y) = \sum_{j=o}^{m} b_j p^j x \tag{1}$$

where

a_i, b_j are system parameters to be determined.

N(y) is the nonlinear component of the system.

$p = \frac{d}{dt}$

For a linear system, N(y) = 0, (1) reduces to

$$\sum_{i=o}^{n} a_i p^i y = \sum_{j=o}^{m} b_j p^j x \tag{2}$$

The differential equation model applies to linear and nonlinear systems, time-varying and time-invariant systems.

3.2 Transfer function

A linear time-invariant system can be defined by the transfer function

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{j=o}^{m} b_j s^j}{\sum_{i=o}^{n} a_i s^i} \qquad n \geqslant m \tag{3}$$

where

Y(s) is the Laplace transform of the system output y(t).

X(s) is the Laplace transform of the system input x(t).

G(s) is the system transfer function.

If the system input signal is sinusoidal, $s = j\omega$, and (3) becomes

$$G(j\omega) = \frac{Y(j\omega)}{X(j\omega)} \tag{4}$$

In the identification problem the parameters a_i's and b_j's are to be determined in the frequency domain by measuring the amplitude and phase responses as functions of frequency ω.

3.3 Convolution integral

A linear system is completely characterized by the impulse response which relates the output and input by the convolution integral

$$y(t) = \int_{-\infty}^{t} g(t-\tau)\, x(\tau)\, d\tau = \int_{-\infty}^{t} g(\tau)\, x(t-\tau) d\tau$$

$$\text{(time-invariant system)} \tag{5}$$

$$g(t) = \int_{-\infty}^{t} g(t,\tau) x(\tau) d\tau \qquad \text{(time-varying system)} \tag{6}$$

where

$x(t)$ and $y(t)$ are the system input and the system output.

$g(t)$ is the impulse response of the system.

In the identification problem both the input $x(t)$ and the output $y(t)$ are known, and it is only the impulse response $g(t)$ that is to be determined.

3.4 Pulse transfer function

A sampled-data system or a discrete system can be defined by the pulse transfer function (in z-transform)

$$G(z) = \frac{Y(z)}{X(z)} = \sum_{n=o}^{\infty} g(nT)\, z^{-n} \tag{7}$$

where

$$z = e^{sT}$$

T is the sampling period.

G(z) is the pulse transfer function of the system.

In the identification problem the coefficients of the pulse transfer function are to be determined from the known quantities Y(z) and X(z).

3.5 State-space equation

By proper choice of state vector, a linear system can be defined by the state space equation

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{b}\,u(t) \tag{8}$$

where

$\underline{x}$ is the state vector.

$\underline{A}$ is an n x n matrix whose elements are parameters to be determined.

$\underline{b}$ is an n x 1 column matrix.

u(t) is the input signal.

The state space equation is in fact the same as the differential equation model, except that it represents the system by a set of n simultaneous first-order differential equations instead of the origional nth-order differential equations. This model has the advantage of simplifying the mathematical analysis of the system characteristics.

CHAPTER 2.

Fundamental Principles of System Identification

In the system identification problem, three sets of quantities are related by a proper mathematical equation with two sets of known quantities, the input and the output, and only one set of unknown quantities, the system function, is to be determined. Theoretically it is always possible to solve for this set of unknown quantities. The basic principles of most of the identification methods are correlation technique and model-referenced method.

1. Correlation Technique [1-4, 7-11]

A linear system is completely characterized by its impulse response. The output and input are related by the convolution integral

$$y(t) = \int_{-\infty}^{\infty} g(\tau)x(t-\tau)\, d\tau \qquad (9)$$

It is well known from the input-output cross-correlation theorem that the input-output cross-correlation of a linear system is the convolution of the unit impulse response and the input auto-correlation [1]

$$\phi_{xy}(\tau) = \int_{-\infty}^{\infty} g(\mu)\, \phi_{xx}(\tau-\mu)\, d\mu \qquad (10)$$

where

ϕ_{xy} is the input-output cross-correlation function

$$\phi_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t+\tau)\, dt$$

$\phi_{xx}(\tau)$ is the input auto-correlation function

$$\phi_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)\, dt$$

Comparison of (9) and (10) indicates that $\phi_{xy}(\tau)$ would be the response of the system if the input were $\phi_{xx}(\tau)$. If $\phi_{xx}(\tau)$ is an impulse function, $\phi_{xy}(\tau)$ is equal to the impulse response of the system, i.e.

$$\phi_{xy}(\tau) = 2\pi K g(\tau)$$

when

$$\phi_{xx}(\tau) = 2\pi K \delta(\tau) \quad \text{and} \quad \Phi_{xx}(\omega) = K = \text{constant}$$

where $\delta(\tau)$ is the unit impulse function.
$\Phi_{xx}(\omega)$ is the power density spectrum of $\phi_{xx}(\tau)$.

As is well known, the autocorrelation of white noise, a random Gaussian noise with constant power density spectrum over a wide band of frequency, is an impulse function. Consequently if white noise is applied to the input of a system, the cross-correlation function between the input and output is simply the system impulse response $g(\tau)$ at $t = \tau$. So the value of $g(t)$ can be obtained by varying the length of delay τ between the correlation functions. These conoepts provide a convenient method for measuring the impulse response $g(t)$. The block diagram and its mechanization for determining the system impulse response by the cross-correlation technique are shown in Fig. 2 and Fig. 3 respectively.

If the white noise test signal must be mixed with the normal operating input, the impulse response determined by the crosscorrelation technique is independent of the normal operating input $r(t)$ provided that the white noise test signal $n(t)$ and the normal operating input $r(t)$ are uncorrelated. A simple mechanication is shown in Fig. 4.

This method has the advantage that neither external disturbances nor noise generated within the system has any appreciable effect on the results, because of the action of the cross-correlator in rejecting all signals not correlated with the white noise. More generally if the system in operation has a stationary random input, not necessarily a white noise, the function we seek, g(t), is under the integral sign in (10). But in frequency domain, we have

$$G(j\omega) = \frac{\Phi_{xy}(j\omega)}{\Phi_{xx}(j\omega)}$$

The quantities that we measure are the input-output cross-correlation $\phi_{xy}(\tau)$ and the input autocorrelation $\phi_{xx}(\tau)$. These measurements are made with the actual input and actual output when the system is in normal operation. In such cases, the noise generated in the system itself can be utilized to find out the impulse response. Thus it becomes possible to identify the system under the actual operating conditions without any test signals. So the method has the great advantage that it can measure the system characteristics without interfering with the system's normal continuous operation. The main drawback of this method is to find out a suitable low-cost multiplier with averaging ability which will give faithful operation with small or large inputs. To overcome this drawback a random generator using Ecceles-Jordan circuit was proposed by Chatterzee and Bhattacharyya [4].

2. Model Referenced Techniques [2, 12-19]

In this method a model whose parameters can be adjusted is used. The model and the system are subjected to the same input. Their outputs are compared and the error is used to adjust the parameters in the model such that the error will be minimized by a certain criterion and thus the model will approximate the system as closely as possible. Then the parameters of the system can be determined from the parameters of the model. This is called the model referenced technique for identification. The block diagram of this method is shown in Fig. 5.

2.1 Generalized Model and Generalized Error

A parameter-estimating scheme using a generalized model and generalized error has been proposed by Eykhoff [12]. As shown in Fig. 6, the generalized model for a linear system is formed by a set of linear operators (transfer functions) $h_1, h_2, \dots h_m$ and $f_1, f_2, \dots f_n$; the potentiometers $\alpha_1 \alpha_2, \dots, \alpha_m$ and $\beta_1, \beta_2, \dots \beta_n$ and the adder, For nonlinear system the model is constructed by nonlinear filters and delay operators [12].

The generalized error is

$$\epsilon = \sum_{i=o}^{m} \alpha_i u_i + \sum_{j=o}^{n} \beta_j v_j \qquad (11)$$

where u_i and v_j are the outputs of linear operators.

One of the potentiometers can be chosen arbitrarily as reference. If $\alpha_o = -1$, then (11) becomes

$$\epsilon = -x + \sum_{i=1}^{m} \alpha_i \mu_i + \sum_{j=o}^{n} \beta_j v_j \qquad (12)$$

The potentiometers α_i's and β_j's are adjusted to minimize an error criterion which is functional of the generalized error

$$E = F(\epsilon) \qquad (13)$$

If $f_j = 0$ ($j = 1, 2, \ldots n$) and $\beta_0 = -1$ the generalized model becomes an ordinary model (Fig. 7) and the generalized error is simply the difference between the system output y and the model output z, i.e.

$$\epsilon = z - y \tag{14}$$

The generalized error is linearly related to each of the unknown coefficients:

$$u_i = \frac{\partial \epsilon}{\partial \alpha_i} \quad ; \quad v_j = \frac{\partial \epsilon}{\partial \beta_j}$$

This leads to the following two simple instrumentation schemes for $\frac{\partial E}{\partial \alpha_i}$ and $\frac{\partial E}{\partial \beta_j}$ [12].

(1). Using a physical model: A convergent adjustment scheme is constructed by properly choosing a relation between $\frac{\partial E}{\partial \alpha_i}$ and a function of α_i and between $\frac{\partial E}{\partial \beta_j}$ and a function of β_j. This is the model adjustment technique.

(2) Using an explicit mathematical relation: By putting $\frac{\partial E}{\partial \alpha_i} = \frac{\partial E}{\partial \beta_j} = 0$, a set of simultaneous linear equations in α_i and β_j, as many as the number of parameters to be determined, are obtained. The solution of such a set of simultaneous equations leads to the technique "using explicit mathematical relations".

2.2 Error Criteria

In the model adjustment technique, usually an even function of error is used as an error criterion. For examples

$$E_1 = \int_t^{t+T} \epsilon^2 \, dt \qquad \text{(integral squared error)}$$

$$E_2 = \int_t^{t+T} |\epsilon| \, dt \qquad \text{(integral absolute error)}$$

$$E_3 = \epsilon^2$$

$$E_4 = \frac{1}{2} \left[\sum_{i=o}^{n} q_i \frac{d^i\epsilon}{dt^i} \right]^2$$

etc. can be used. To minimize the error function the information of partial derivatives of system variables with respect to the system parameter to be adjusted, e.g. $\frac{\partial E}{\partial \alpha_i}$, $\frac{\partial E}{\partial \beta_j}$; $\frac{\partial \epsilon}{\partial \alpha_i}$, $\frac{\partial \epsilon}{\partial \beta_j}$ etc. should be known. They are not measurable explicitly, but they can be obtained by the following techniques:

1. Using parameter influence coefficients : The partial derivatives of problem variables with respect to system parameters, called parameter influence coefficients [20] , can be obtained from the solution of the sensitivity equations with respect to the parameters (see Appendix 1).

2. Using two models with different parameters: Two models with parameters α and $\alpha+\Delta\alpha$ respectively can be used to give approximation for the partial derivative. As shown in Fig. 8 the partial derivative is approximated by the difference of the outputs z_1 and z_2 of these models.

$$\frac{\Delta z}{\Delta \alpha} = - \frac{\Delta \epsilon}{\Delta \alpha} = - \frac{\partial \epsilon}{\partial \alpha}$$

3. Using one model with time-varying parameter : From the equation

$$\frac{dE}{dt} = \frac{\partial E}{\partial \alpha} \frac{d\alpha}{dt} + \frac{\partial E}{\partial t}$$

it is noted that the information $\frac{\partial E}{\partial \alpha}$ is available if the model parameter

$$E_1 = \int_t^{t+T} \epsilon^2 \, dt \qquad \text{(integral squared error)}$$

$$E_2 = \int_t^{t+T} |\epsilon| \, dt \qquad \text{(integral absolute error)}$$

$$E_3 = \epsilon^2$$

$$E_4 = \frac{1}{2} \left[\sum_{i=o}^{n} q_i \frac{d^i \epsilon}{dt^i} \right]^2$$

etc. can be used. To minimize the error function the information of partial derivatives of system variables with respect to the system parameter to be adjusted, e.g. $\frac{\partial E}{\partial \alpha_i}$, $\frac{\partial E}{\partial \beta_j}$; $\frac{\partial \epsilon}{\partial \alpha_i}$, $\frac{\partial \epsilon}{\partial \beta_j}$ etc. should be known. They are not measurable explicitly, but they can be obtained by the following techniques:

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$$\frac{\Delta z}{\Delta \alpha} = - \frac{\Delta \epsilon}{\Delta \alpha} = - \frac{\partial \epsilon}{\partial \alpha}$$

3. Using one model with time-varying parameter : From the equation

$$\frac{dE}{dt} = \frac{\partial E}{\partial \alpha} \frac{d\alpha}{dt} + \frac{\partial E}{\partial t}$$

it is noted that the information $\frac{\partial E}{\partial \alpha}$ is available if the model parameter is disturbed according to a known time function and the resulting disturbance $\frac{dE}{dt}$ is measured. $\frac{\partial E}{\partial t}$ acts as a noise-like disturbance in this problem (Fig. 9).

The partial derivatives thus obtained are used to minimize the error in the error-parameter space. Commonly used surface searching methods are:

1. Steepest descent.
2. Newton's method.
3. Southwell relaxation method.

Among them steepest descent method is the most popular one.

2.3 Classification of Model-Referenced Technique

Based on the concepts of generalized model and generalized error, the techniques of obtaining the partial derivatives and the main components used in the model, the model-referenced techniques can be classified as follows [21]:

A. Using a physical model (model adjustment technique):

(a). Classification according to the techniques of obtaining the partial derivatives:

1. Using parameter influence coefficients.
2. Using two models with different parameters.
3. Using one model with time-varying parameters.

(b). Classification according to the main components used in the model:

1. Using time delay elements.
2. Using orthogonal filters.
3. Using differentiators, etc.

B. Using explicit mathematical relations:

1. Using time delay elements.
2. Using orthogonal filters.
3. Using differentiators, etc.

CHAPTER 3.

Several Important Identification Schemes

Basically the identification schemes can be classified as "model adjustment schemes" and "identification schemes using explicit mathematical relations". So far a great number of papers have been published in the studies of system identification. We do not attempt to include all existing identification schemes in this thesis, so only a few typical ones are described in some detail. Many other identification schemes are briefly described. Most of the literature, however, will be given as references.

1. Learning Model Approach [13, 14]

In this method a parameter tracking servo consisting of (1) the physical process (2) the learning model (3) the adjusting mechanism is used to form a closed-loop self-adjusting system (Fig. 5). The model and the process are subjected to the same input. Their outputs are compared and the resultant error is fed to the adjusting mechanism where some error criteria are used to adjust the parameters of the learning model. The mechanism will continuously track the parameters of the physical process. The adjusting mechanism operates on an approximation to the method of steepest descent. A quadratic function of error is used as the error criterion and the parameter influence coefficient technique is used to give the partial derivatives for the minimization of the error function.

2. Auxiliary Models Approach [15].

An approach among the methods of using two models with different parameters is the method of auxiliary models [15].

For a system with n parameters, an n-dimensional parameter space defines a matrix of all possible combinations. By subdividing the parameter space and using the auxiliary models to scan each subspace, all values of the parameters may be tested rapidly. Logic operations select the proper model and initiate adjustments of the system parameters. The block diagram of system identification and adaptation using auxiliary models is shown in Fig. 10.

.3 Orthogonal Filters Method [18].

A model consisting of a set of orthogonal transfer function generators is used in this approach. The adjustable parameters correspond to the expansion coefficients of an orthogonal function expansion. Then the parameters become independent of each other and one adjustment for each parameter is sufficient in order to minimize the mean squared error ; therefore the same number of adjustments as the number of parameters are sufficient for the determination of all parameters.

Considering a system with impulse response g(t) and a model with impulse response h(t) (Fig. 5), the mean squared error is defined by

$$\overline{\{\epsilon(t)^2\}} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left[\int_{0}^{\infty}\{(g(\tau)-h(\tau)\}x(t-\tau)d\tau\right]^2 dt \quad (14)$$

where x(t) is the input signal to the system and the model.

Or

$$\overline{\epsilon(t)^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|G(j\omega)-H(j\omega)\right|^2 \Phi_{xx}(\omega)\,d\omega \quad (15)$$

where

G(jω) and H(jω) are the Fourier transforms of g(t) and h(t).

$\Phi_{xx}(\omega)$ is the Fourier transform of the input autocorrelation function

$$\phi_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\, x(t+\tau)dt$$

The orthogonal transfer function of the model with adjustable parameters is defined as

$$H(j\omega) = \sum_{i=1}^{N} a_i K_i(j\omega) \tag{16}$$

When the error criterion is minimized, the following condition must be satisfied

$$\sum_{i=1}^{N} a_i \int_{-\infty}^{\infty} \mathrm{Re}\,\{ K_i(j\omega)\, \overline{K_j(j\omega)} \}\, \Phi_{xx}(\omega)\, d\omega$$

$$= \int_{-\infty}^{\infty} \mathrm{Re}\,\{ G(j\omega)\, \overline{K_j(j\omega} \}\, \Phi_{xx}(\omega)\, d\omega \tag{17}$$

$$(j=1,\ 2, \ldots, N)$$

or

$$a_j = \frac{1}{p_{jj}} \left(p_{Gj} - \sum_{i\neq j} a_i p_{ij}\right) \tag{18}$$

where

$$p_{ij} = \int_{-\infty}^{\infty} \mathrm{Re}\,[K_i(j\omega)\, \overline{K_j(j\omega)}\,]\, \Phi_{xx}(\omega)\, d\omega$$

$$p_{Gj} = \int_{-\infty}^{\infty} \mathrm{Re}\,[\, G(j\omega)\, \overline{K_j(j\omega)}\,]\, \Phi_{xx}(\omega)\, d\omega$$

Because of the orthogonality of (16), $p_{ij} = 0$ when $i \neq j$, so we have

$$a_j = \frac{p_{Gj}}{p_{jj}} = \frac{\int_{-\infty}^{\infty} \mathrm{Re}\,[\,G(j\omega)\, \overline{K_j(j\omega)}\,]\, \Phi_{xx}(\omega)\, d\omega}{\int_{-\infty}^{\infty} \mathrm{Re}\,[K_j(j\omega)\, \overline{K_j(j\omega)}\,]\, \Phi_{xx}(\omega)\, d\omega} \tag{19}$$

It is noted that a_j can be determined independently of the other parameters to minimize the mean squared error. This is the greatest advantage of using orthogonal transfer functions as the model and Lagurre's functions, orthogonalized exponential functions, Legendre functions etc. can be used to give the orthogonal transfer functions. [1, 18].

If we put z(t) as a linear combination of $z_i(t)$ i.e.

$$z(t) = \sum_{i=1}^{N} a_i z_i(t) \tag{20}$$

where

$z_i(t)$ is the output of the element $K_i(s)$.

a_i's are constants.

When a_i's are adjusted to minimize the error criterion

$$\overline{e(t)^2} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} [y(t) - z(t)]^2 \, dt \tag{21}$$

we obtain the following condition

$$\sum_{i=1}^{N} a_i \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} z_i(t) z_i(t) dt = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t) z_j(t) dt \tag{22}$$

$$(i = 1, 2 \ldots, N)$$

where y(t) is the output of the transfer function to be measured. Using correlation notations, (22) can be written as

$$a_i \phi_{z_i z_i}(o) = \phi_{yz_i}(o) \qquad (i = 1, 2 \ldots, N) \tag{23}$$

So a_i's are obtained from (24)

$$a_i = \frac{\phi_{yz_i}(o)}{\phi_{z_i z_i}(o)} \qquad (i = 1, 2 \ldots, N) \tag{24}$$

And the transfer function of the system is

$$G(s) = \sum_{i=1}^{N} a_i K_i(s) = \sum_{i=1}^{N} \frac{\phi_{yz_i}(o)}{\phi_{z_i z_i}(o)} K_i(s) \qquad (25)$$

The mechanization of solving (24) is shown in Fig. 11.

4. Plant Observer

Staffin [22] proposed a scheme called plant observer for the identification of a linear system. Suppose the linear system is described by the differential equation

$$\sum_{i=o}^{n} a_i p^i y(t) = \sum_{j=o}^{m} b_j p^j x(t) \qquad (26)$$

where

$p = \frac{d}{dt}$

a_i, b_j are system parameters.

$x(t)$, $y(t)$ are the system input and output.

If the system input $x(t)$ and output $y(t)$ are sampled at $m+n+2$ instants of time, a set of linear simultaneous equations are obtained. Then the parameters a_i's and b_j's can be determined by solving these equations.

The mechanization of such a plant observer can be realized by differentiators, simultaneous equation solver, potentiometers and a positioning servomechanism. The differentiator can be constructed from analog components, using either operational amplifiers or delay line differentiators. A digital computer is used for the simultaneous equation solver. The outputs of the simultaneous equation solver are transformed into the potentiometer settings by the positioning servomechanism. The mechanization of the plant observer is shown in Fig. 12.

5. Quasi-linearization Method

The coefficients of the differential equation describing a linear or nonlinear system can be determined by a digital computer based on the quasi-linearization method (see Appendix 2) proposed by Kumar and Sridhar [23]. This method has the advantage that no external inputs are required and only the normal operating records are used. The identification procedure applies to nonlinear as well as linear systems.

To illustrate the method a first-order nonlinear system defined by (27) is considered.

$$\dot{x} = ax^3 + bu(t) \tag{27}$$

where x is the scalar state variable.

a , b are constants to be determined.

u is the system control variable (input).

By means of the quasi-linearization method, the (k+1) st approximation can be obtained from the kth, i.e.

$$\left\{ \begin{aligned} \dot{x}_{k+1} &= a_k x_k^3 + b_k u(t) + 3a_k x_k^2 (x_{k+1} - x_k) + (a_{k+1} - a_k) x_k^3 \\ &\qquad + (b_{k+1} - b_k) u(t) \\ \dot{a}_{k+1} &= 0 \\ \dot{b}_{k+1} &= 0 \end{aligned} \right. \tag{28}$$

The solution of (26) can be written as

$$\begin{pmatrix} x_{k+1} \\ a_{k+1} \\ b_{k+1} \end{pmatrix} = [\,\Phi\,] \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} + \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \tag{29}$$

where Φ is the fundamental matrix of (28). K_1, K_2, K_3 are constants properly chosen to satisfy the boundary conditions in (28) which can be obtained from the measurement of x(t) as

$$\begin{cases} x(t_1) = c_1 \\ x(t_2) = c_2 \\ x(t_3) = c_3 \end{cases} \tag{30}$$

The column vector $[p_1, p_2, p_3]$ is the particular solution of (28).

It has been shown that the convergence of the solution (29) is quadratic in nature. In practice, a large number of observations and a least-squares fit have to be used in order to determine the system parameters. If the order of the system is unknown, first a certain order of mathematical model is used as an initial guess. The quasi-linearization method will indicate the inaccuracy and give the direction of adjustment until an accurate mathematical model is obtained to represent the system.

6. Implicit Synthesis Method [17, 24-27]

The coefficients of an ordinary differential equation describing a linear or nonlinear system can be directly determined by Clymer's implicit synthesis method [17]. In this method the parameters to be determined are generated as the outputsof the high gain amplifiers which amplify the difference between the system actual output and the computed output of the implicit circuit. This method has the advantage of simplicity that it requires only one multiplier channel per unknown parameter, and can be implemented on a smaller scale analog computer.

The drawback of this method is the critical stability problem resulting from the use of differentiators in the implicit circuits.

7. Auxiliary Lag Networks Method

A technique using a series of auxiliary lag networks to generate as many independent signals as the unknown coefficients in the transfer function describing a system has been proposed by Puri and Waygant [28] for the identification of a linear time-varying system.

Consider a linear time-varying system described by the transfer function

$$F(s) = K \frac{N(s)}{D(s)} \tag{31}$$

where

$$N(s) = b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + 1 = 1 + \sum_{i=1}^{m} b_i s^i$$

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + \dots\dots + a_1 s + 1 = 1 + \sum_{j=1}^{n} a_j s^j$$

$$\frac{N(o)}{D(o)} = 1$$

$$n \geqslant m$$

(n+m+1) unknowns, the gain constant K and the coefficients a_j's, b_i's, completely characterize the system. They are assumed to be varying slowly with time in an unknown manner.

The impulse response of the system f(t) is defined as

$$f(t) = \mathcal{L}^{-1} [F(s)] \tag{32}$$

(n+m+1) signals required for the identification of the n+m+1 unknowns are generated by the equations:

$$e_i = \int_0^\infty f(t) g_i(t) dt \qquad i = 1, 2, \dots, n+m \tag{33}$$

From Parseval's Theorem, it is well known that

$$\int_{o}^{\infty} f(t)g_i(t)dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} F(s)\, G_i(-s)ds \qquad (34)$$

$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} F(-s)G_i(s)ds \qquad (35)$$

or

$$\int_{o}^{\infty} f(t)g_i(t)dt = \frac{1}{2\pi j}\int_{c} F(s)\, G_i(-s).ds \qquad (36)$$

$$= \frac{1}{2\pi j}\int_{c} F(-s)G_i(s)ds \qquad (37)$$

where in (36) and (37) the denominator of F(s)G(-s) is at least of degree 2 higher than its numerator. The contour C enclosed the entire left half plane.

The simplest series of $G_i(s)$ are lag networks of the form

$$F_i(s) = \frac{1}{s+\alpha_i} \qquad (i = 1,\ 2 \ldots.\ n+m) \qquad (38)$$

Then

$$e_i = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} K\,\frac{N(s)}{D(s)}\,\frac{1}{-s+\alpha_i}\,ds \quad (i = 1,\ 2, \ldots n+m) \qquad (39)$$

Suppose $n \geqslant m + 1$, then from the residue theorem

$$e_i = K\,\frac{N(\alpha_i)}{D(\alpha_i)} \qquad (i = 1,\ 2, \ldots.\ n+m) \qquad (40)$$

The gain constant K can be obtained by integrating the impulse response e_o (Fig. 13).

$$e_o = \lim_{s\to o}\left[s\,\frac{KN(s)}{sD(s)}\right] = K\,\frac{N(o)}{D(o)} = K \qquad (41)$$

From (40) and (41), we have

$$N(\alpha_i) - \frac{ei}{eo}\,D(\alpha_i) = 0 \quad (i = 1,\ 2,\ \ldots.n+m) \qquad (42)$$

where

$$N(a_i) = b_m a_i^m + b_{m-1} a_i^{m-1} + \dots + b_1 a_i + 1$$

$$D(a_i) = a_n a_i^n + a_{n-1} a_i^{n-1} + \dots, + a_1 a_i + 1$$

(42) can be put into the matrix form

$$\underline{A}\ \underline{U} = \underline{e} \tag{43}$$

where

$$\underline{U} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \\ a_1 \\ \vdots \\ a_n \end{bmatrix}_{(m+n)x1} \quad ; \quad \underline{e} = \begin{bmatrix} e_1/e_o - 1 \\ e_2/e_o - 1 \\ \vdots \\ e_{m+n}/e_o - 1 \end{bmatrix}_{(m+n)x1}$$

$$\underline{A} = \begin{bmatrix} a_1 & a_1^2 \dots\dots a_1^m & (-\frac{e_1}{e_o} a_1) & \dots\dots (-\frac{e_1}{e_o} a_1)^n \\ a_2 & a_2^2 \dots\dots a_2^m & (-\frac{e_2}{e_o} a_2) & \dots\dots (-\frac{e_2}{e_o} a_2)^n \\ \vdots & \vdots & \vdots & \vdots \\ a_m & a_m^2 \dots\dots a_m^m & (-\frac{e_m}{e_o} a_m) & \dots\dots (-\frac{e_m}{e_o} a_m)^n \\ a_{m+n} & a_{m+n}^2 \dots a_{m+n}^n & (-\frac{e_{m+n}}{e_o} a_{m+n}) & \dots (-\frac{e_{m+n}}{e_o} a_{m+n})^n \end{bmatrix}_{(m+n)x(m+n)}$$

and a_i should be so chosen that $\underline{A}$ is nonsingular.

From (43), we have

$$\underline{U} = \underline{A}^{-1}\ \underline{e} \tag{44}$$

The parameters $\underline{U}$ are determined from (44), which can be easily implemented on a digital computer. In case a_j, b_i are slowly varying with time, then they can be assumed as constant during the time signals e_i are obtained. Thus by exciting the system periodically with impulses, or white noise, a continuous tracking of the parameters is performed.

This method has the advantage that the order or form of the system need not be known. Its disadvantage is that a disturbance test signal of impulse or white noise has to be used in addition to the normal operating signal.

8. Regression Analysis Method

In regression analysis ~~is that, given~~, a set of random variables called observables, and a set of constants called regressors associated with each observable are given, ~~we want~~ it is required to find a functional dependence of the observables on the regressors. This method of analysis is widely used in ~~biometics~~ biometrics and econometrics. It has been found to be very useful in the system identification problem, especially when the observables are corrupted by noise. In the system identification problem we would identify the system by determining the functional dependence of the output on the input. There are two problems to be considered: (1) The functional form of the regression function is known and the problem is to find reasonable estimates for the unknown parameters to complete the specification of the function. That is, knowing the system to be linear or nonlinear, or the order of the system, we are to estimate its impulse response or unknown coefficients in the differential equation or transfer function to complete the identification (2). The functional form is unknown and the problem is to find the best function for a reasonable representation of the regression function. There is no a priori knowledge of the system properties available. It is required to determine a system model which best fits the observed data.

It can be shown that the best possible representation or fit for the functional dependence in the mean square sense is the regression function defined as the mean value of the observables for the associated set of regressors [29].

Linear unbiased estimate is used for the parameter estimation in linear regression model. Among them the least square estimate has the property that it is independent of the noise statistics and the Markov estimate has the property that it has the minimum covariance matrix. The maximum likelihood estimate (the likelihood function is defined as the logarithm of its probability density function) is identical to the Markov estimate if the noise is normally distributed (Gaussian distribution). It is also shown that [29] in the case of uncorrelated identically distributed noise components (white noise) Markov and least square estimates are identical, that is, least square estimates are optimum.

If the functional form of the regression is unknown, then we have to find a function which fits best in some sense for the set of observables. The most commonly used method of approximating unknown regression functions is the technique of orthogonal polynomial fitting [29].

This regression analysis method has the advantage that no information about the statistics of the noise distribution is required.

But it has computation difficulty of matrix inversion. It is noted that, for the purpose of regression analysis, the system is characterized by a single unknown parameter vector in the given sampling interval. It does not apply to time-varying systems. For time-varying system identification, multiple regression analysis has to be used [19].

9. Brief Description of Some Other Methods for Parameter Estimations.

In addition to the methods described in some detail in the previous sections, some other methods will be briefly described in this section.

1) J.A. Aseltine et. al. [31]: The impulse response is directly measured by using an impulse train as the input.

2) R.E. Kalman [9]: The pulse transfer function is determined from solving a set of pseudo-correlation functions by a digital computer.

3) L. Braun Jr. [5]: The impulse response is determined by solving the convolution integral. The system input, output and the impulse function are expanded in a MacLaurin series such that the coefficients of the impulse function expansion are determined from the corresponding coefficients in the input and output series expansions.

4) J.H. Westcott [8]: The impulse response is determined from the input autocorrelation function and the input-output crosscorrelation function.

5) M.J. Levin [32]: The parameters of the pulse transfer function are estimated by assuming additive Gaussian noise and utilizing the maximum likelihood method of estimates.

6) Ellington and McCallion [33]: The system characteristics are determined from transient response by nonlinear curve fitting techniques.

7) Corbin [34]; Lendaris [35]; Zaborszky and Berger [36]: The parameter estimates are obtained by solving sets of simultaneous linear equations in derivatives and integrals of the system input and output.

8) Kaya and Yamamura [37]; An analog computer is used for obtaining the approximate transfer function of the first- or second-order of a system continuously and automatically by solving the simultaneous equations whose coefficients are the filtered outputs of the system input and output.

9) K. Steighlitz [38]: The power spectrum identification is made by assuming that the spectrum has no zeros and is based on the theory of autoregressive schemes.

10) Hennig [39]: Crosscorrelation method is used to test for the transfer function of a noisy nonlinear process.

11) Shinbrot [40]: The output of a nonlinear system is characterized by a power series of its input and the coefficients of various terms in the power series are determined.

12) Kohr [41]: An approximate nonlinear differential equation is used as a model for the actual physical system and a set of linear filters are used to generate the derivatives of the output in the practical mechanization of the nonlinear system

identification. The parameters in the system are determined by the nulling procedure until a minimum-area closed curve is obtained for the output-input plot.

13) R.E. Kalman [42]: The optimal filtering theory also called the minimum variance technique, is used to identify the state of a system from noise measurement.

14) Ho and Whalen [43]: It has been shown that the necessary and sufficient condition for a system to be identifiable is that the successive state vectors span the n-dimensional state space. An recursion scheme is given for the estimation of system parameters.

15) Pottle [44]: Kalman's estimator theory based on the conditional expectation is used for the identification of the system state vector.

16) Coasert [45]: The problem of estimating the plant state, plant parameters and external disturbance is investigated through a noise channel.

17) Miller and Roy [46]: The learning technique and pattern recognition have been used for nonlinear process identification.

18) Cox [47]: Bellman's dynamic programming technique is used for the system states variables estimation.

19) Ho and Lee [48]: Bayesian decision theory is used in a stochastic estimation and control.

CHAPTER 4

System Identification by Means of Implicit Synthesis Method.

1. General Description:

The implicit synthesis technique was first proposed by Clymer [17] in 1959 for the determination of the parameters of a system. This method is based on the principle of the steepest descent, which assures the fastest minimization of the error in the error-parameter space until the desired answer is obtained. In Clymer's paper an example of determining a single parameter in a first-order linear system was given. Then in 1960, Grauppe [24] extended Clymer's implicit synthesis method and presented a computing circuit using an analog computer to solve a system of coupled equations of several unknown functions. The computing circuit is operated by using the least-squares method, or the least-magnitude method, or the determinant method [24]. All these methods are also based on the steepest descent principle. Thereafter in 1963, Madden [26] presented a modified least-magnitude method of steepest descent, which uses a periodic reversal of sign of the parameter-determining loops, coupled with independent step-wise adjustment of the computed value of each system parameter to improve the convergence. Two parameters in a second-order over-damped system were simultaneously determined by this method. An extension of this method is the steepest descent at constant velocity [26], which is less dependent on the switching-on time of the reverse-sign circuit.

2. Implicit Synthesis Circuits for First-Order Systems

In the implicit synthesis method the parameter of a system is determined by an implicit loop which performs automatic minimization of the error function and generates the desired value for the parameter by means of a high gain amplifier. For simplicity we shall use a first-order system to illustrate the fundamental theory and the circuits of this method. There is no loss of generality because the same principle applies to both linear and nonlinear high order systems.

A. First Circuit

Consider a first-order system described by the differential equation

$$\dot{y} + \lambda y = u(t) \tag{45}$$

where

u(t) is the system input.

y(t) is the system output.

λ is the parameter to be determined.

The block diagram of the implicit synthesis circuit is shown in Fig.14, where u(t) is the known input function of time, y(t) is the measurable output function of time, and λ_c is the computed answer of the parameter to be determined. If the computed parameter λ_c were not very close to the actual value, then $\lambda_c y$ would be wrong and thus so would $\dot{y}_c$, which would increase the error ϵ and hence in turn would change λ_c. The system then goes unstable. If the algebraic signs of the circuits are properly designed, the error will be minimized according to the steepest descent principle and will be driven by the implicit circuit to very nearly its correct value.

The implicit synthesis circuit of Fig.14 can be realized by the analog computer components as shown in Fig.15. In this circuit the parameter λ_c is directly obtained as the output of the high gain amplifier. Thus all operations are performed inside the implicit loop and the loop yields the answer directly. Instead of getting λ_c directly from the output of the high gain amplifier, it can be obtained by plotting $\lambda_c y$ against y, where $\lambda_c y$ is the output of the electronic multiplier. From the slope of the straight line connecting the starting point and the ending point, λ_c can be determined. λ_c camot be directly determined from the slope of the plot of $\lambda_c y$ vs y during the operating interval unless λ_c and y have nearly the same speed of transient response. Usually the response of λ_c is much faster and more complicated than that of y.

B. Second Circuit

There is another way of designing the implicit synthesis circuit. The block diagram is shown in Fig.16. The realization of Fig.16 by the analog computer components is shown in Fig.17. In this circuit $\lambda_c y$ is obtained directly as the output of the high gain amplifier and λ_c is then determined from the open-loop division of $\lambda_c y$ by y.

Since the output of the high gain amplifier can be any variable provided that it can make the whole closed loop of the implicit synthesis circuit satisfy the system relations, there are many ways of designing the implicit synthesis circuits. However, the basic principle of the various circuits is the same, although the transient responses will be different.

3. Transient Analysis of Implicit Synthesis Circuits

The implicit synthesis circuit involves the dynamic variables y, y_c, and λ_c, which are changing with time before coming to the steady state. So the circuit itself is a very complicated dynamic system, and this is why it is very difficult to keep it stable. The stability problem is in fact one of the pitfalls of this method.

In order to have better insight of the dynamic property of the implicit synthesis circuit, we shall study the transient responses of both circuits.

3-1. First Circuit

From Fig. 14, we have the following equations

$$\epsilon = y_c - y \tag{46}$$

$$\lambda_c = G\epsilon \tag{47}$$

$$\dot{y}_c + \lambda_c y = u(t) \tag{48}$$

$$\dot{y} + \lambda y = u(t) \tag{49}$$

where

y is the actual response of the system.

y_c is the computed response of the system at any instant.

ϵ is the error between y and y_c.

G is the gain of the high gain amplifier.

λ is the parameter to be determined.

λ_c is the computed value of the parameter at any instant.

u(t) is the input time function.

Eliminating ϵ between (46) and (47), we have

$$y_c = y + {}^1/_G\, \lambda_c \tag{50}$$

Next eliminating y_c from (48) and (50), we have

$$\dot{y} + {}^1/G\ \dot{\lambda}_c + \lambda_c y = u(t) \tag{51}$$

Finally, we eliminate u(t) between (49) and (51), obtaining

$$\dot{\lambda}_c + Gy\lambda_c = G\lambda y \tag{52}$$

(52) is the equation describing the transient response of the parameter λ_c.

Solving (52) for λ_c we have

$$\lambda_c = Ae^{-\int_0^t Gy(s)ds} + e^{-\int_0^t Gy(s)ds} \int_0^t G\lambda y(s) e^{\int_0^s Gy(\tau)d\tau} ds \tag{53}$$

where A is a constant depending on the initial value of λ_c.

From (53) we note that the transient response of λ_c is very complicated because y(t) is a changing variable in time. When y comes to a steady-state value, it becomes a constant. In this case (53) reduces to

$$\lambda_c = Ae^{-Gyt} + \lambda \tag{54}$$

Since λ_c is zero at t = 0, A is determined to be $-\lambda$. Thus we have the following equation

$$\lambda_c = \lambda(1 - e^{-Gyt}) \tag{55}$$

From (55) it is obvious that λ_c has a damped-exponential transient with a time constant equal to $(Gy)^{-1}$. It is seen that the time constant depends on the loop gain G and the system response y. The higher the gain G or/and the larger the response y, the faster the value λ_c comes to the correct answer. The response of λ_c is shown in Fig.18.

It is also noted that if y goes negative, λ_c diverges and the system becomes unstable. In order to maintain the system stable an absolute magnitude circuit (AMC) has to be inserted in the λ_c -line or the y-line to the multiplier [17]. We shall find out later from the experiment work that if a special test signal, such as a step function, is used so that y will never go negative, then the AMC can be omitted. The special choice of input or test signal is particularly important in the nonlinear system identification, because the insertion of the AMC introduces a time lag which makes the system more difficult to keep stable. This is particularly critical when the loop gain G is high.

3-2. Second Circuit

From Fig. 16 and Fig. 17, we have the following equations

$$\dot{y} + \lambda y = u(t) \tag{56}$$

$$\dot{y} + G(y_c - y) = u(t) \tag{57}$$

$$\lambda_c y = G(y_c - y) \tag{58}$$

$$\epsilon = y_c - y \tag{59}$$

Differentiating (58), we have

$$\dot{\lambda}_c y + \lambda_c \dot{y} = G(\dot{y}_c - \dot{y}) \tag{60}$$

Eliminating u(t) from (56) and (57), we have

$$\dot{y}_c + G(y_c - y) = \dot{y} + \lambda y \tag{61}$$

From (58), (60) and (61), with simplification and rearrangement we finally obtain

$$\dot{\lambda}_c + (G + \dot{y}/y)\,\lambda_c = G\lambda \tag{62}$$

(62) is the equation describing the transient response of the λ_c-system. Solving (62) we have

$$\lambda_c = Be^{-\int_0^t [G+\dot{y}(\tau)/y(\tau)]\,d\tau} + \left\{ e^{-\int_0^t [G+\dot{y}(\tau)/y(\tau)]\,d\tau} \cdot \int_0^t G\lambda e^{\int_0^\tau [G+\dot{y}(s)/y(s)]\,ds}\,d\tau \right\} \tag{63}$$

where B is a constant depending on the initial value of λ_c. When y comes to the steady state, it becomes a constant, and (63) reduces to

$$\lambda_c = Be^{-Gt} + \lambda \tag{64}$$

Since $\lambda_c = 0$ at $t = 0$, B is determined to be $-\lambda$. After the value of B is substituted into (64) we have

$$\lambda_c = \lambda(1 - e^{-Gt}) \tag{65}$$

(65) is the transient equation of λ_c when y is a constant. In (65) we observe that the time constant of the exponentially-decaying response is G^{-1} which is independent of y.

4. A Brief Comparison of the Two Implicit Synthesis Circuits.

In the previous sections we have described and analyzed two different forms of the implicit synthesis circuits for a first-order system (Fig.14 and Fig.16). Basically there is not much difference between these two circuits. From the previous analysis we can make a brief comparison as follows.

1) In the first circuit λ_c is directly determined as the output of the high gain amplifier. All operations are performed inside the implicit loop and the loop yields the answer directly. In the second circuit $\lambda_c y$ is the output of the high gain amplifier and λ_c is obtained from the open-loop division of $\lambda_c y$ by y. That is, the implicit loop is chiefly a differentiator and various operations are then performed open-loop on the result.

2) Both circuits have damped-exponential transient responses. But they have different forcing functions and time constants. In the first circuit the forcing function is $Gy\lambda$ and the time constant is $(Gy)^{-1}$, which is dependent on the value of y. In the

second circuit the forcing function is $G\lambda$ and the time constant is $(G+\dot{y}/y)^{-1}$, which depends on the values of y and $\dot{y}$. If y is a constant the time constant of the first circuit is still $(Gy)^{-1}$, but the time constant of the second circuit becomes G^{-1} which is independent of y.

3) In order to prevent the time constant from being negative or the system from going unstable, an absolute magnitude circuit (AMC) should be inserted in the y-line or λ_c -line of both circuits. But if a particular input or test signal u(t) is used, the AMC can be omitted. In the second circuit y is a divisor, and therefore cannot be zero at any time.

5. Identification of a First-Order Nonlinear System

As is obvious from the above explanation, the parameter λ_c can be obtained directly from the output of the high gain amplifier. If λ_c is plotted against y and a horizontal line is obtained, then λ_c is a constant; otherwise λ_c is a function of y. Clymer [17] mentioned that the nonlinearity of λ_c can be illustrated by plotting λ_c against y. But no method was proposed to determine the parameters in the nonlinearity. In this report a general solution for the identification of a first-order nonlinear system whose nonlinearity can be ~~represented~~ approximated ~~as a~~ by a ~~convergent power series~~ polynomial of the dependent variable is developed.

Consider a first-order nonlinear system defined by the differential equation

$$\dot{y} + a_1 y + a_2 y^2 + \dots, + a_n y^n = u(t) \tag{66}$$

or

$$\dot{y} + (a_1 + a_2 y + \dots\dots, + a_n y^{n-1})y = u(t) \tag{67}$$

or

$$\dot{y} + \mu y = u(t) \tag{68}$$

where

$$\mu = a_1 + a_2 y + \ldots, + a_n y^{n-1} \tag{69}$$

We note that (68) is just the same as (45) with λ substituted by μ. So μ can be determined from the implicit synthesis circuit of Fig.14. If we use n different magnitudes of input or test signals we can obtain n different values of μ_c and n equations as follows

$$\begin{aligned} a_1 + a_2 y_1 + a_3 y_1^2 + \ldots\ldots, + a_n y_1^{n-1} &= \mu_1 \\ a_1 + a_2 y_2 + a_3 y_2^2 + \ldots\ldots, + a_n y_2^{n-1} &= \mu_2 \\ - \\ a_1 + a_2 y_n + a_3 y_n^2 + \ldots\ldots, + a_n y_n^{n-1} &= \mu_n \end{aligned} \tag{70}$$

or $$\sum_{j=1}^{n} a_j y_i^{j-1} = \mu_i$$

where $i = 1, 2, \ldots, n$. The n parameters a_j's can be obtained by solving the set of n simultaneous equations (70).

(70) can be put in the matrix form as

$$\underline{Y}\,\underline{A} = \underline{\mu} \tag{71}$$

where $\underline{Y}$ is an n x n matrix

$$\begin{bmatrix} 1 & y_1 & y_1^2 \ldots\ldots & y_1^{n-1} \\ 1 & y_2 & y_2^2 \ldots\ldots & y_2^{n-1} \\ - - - & - - - & - - - & - - - \\ 1 & y_n & y_n^2 \ldots\ldots & y_n^{n-1} \end{bmatrix}$$

and $\underline{A}$ is an n x 1 column vector such that

$$\underline{A}' = [a_1, a_2, \ldots\ldots a_n]$$

where prime denotes transpose of a matrix.

$\underline{\mu}$ is an n x 1 column vector such that

$$\underline{\mu}'=[\mu_1, \mu_2, \ldots\ldots \mu_n]$$

Then $\underline{A}$ can be determined as

$$\underline{A} = \underline{Y}^{-1} \underline{\mu} \tag{72}$$

(72) is the solution for the values of the parameters to be determined. In order to illustrate this method, some examples will be worked out (see Sec. 6-3).

Since for small y, functions such as sin y, e^y, sin h y, etc. can be expanded as a convergent power series in y, first-order nonlinear systems containing such functions can also be identified by this method.

Thus a general system

$$\dot{y} + f(y) = u(t) \tag{73}$$

can be identified, provided that f(y) is representable by a ~~convergnet power series in y~~ polynomial of y.

6. Experiment Work

In our experiment Clymer's first circuit is adopted because it yields the value of the parameter directly. Both the system model which generates the input and output signals required for the system identification and the implicit synthesis circuit are implemented on an EAI TR-48 general-purpose analog computer. Some work similar to Clymer's was repeated to confirm the results and to make a full investigation of the characteristics of such a circuit. An extension of this method to the nonlinear systems has been done and the results are used to illustrate the general method of identifying a first-order nonlinear system whose nonlinearity is representable as a convergent power series in y.

6-1 Computer Set-up.

1) Linear system with single parameter: A first-order linear system defined by

$$\dot{y} + \lambda y = u(t) \tag{74}$$

is to be identified. The system model and the implicit synthesis circuit simulated on the analog computer are shown in Fig.19.

In Fig.19. the amplifier A08 and A09 are used for the observation of the error variations. They can be omitted for saving of the operational amplifiers. Then -y and y_c can be directly applied to the input of the high gain amplifier A21.

2) Nonlinear system with single parameter: A first-order nonlinear system with single parameter, defined by

$$\dot{y} + a_2 y^2 = u(t) \tag{75}$$

is to be identified. The system model and the implicit synthesis circuit simulated on the computer are shown in Fig.20. The same implicit synthesis circuit as for the linear system is used.

From Fig. 20 it is seen that the only difference is that the system model used to generate the input and output signals is changed.

3) Nonlinear system with two parameters: A first-order nonlinear system with two parameters, defined by

$$\dot{y} + a_1 y + a_2 y^2 = u(t) \tag{76}$$

is to be identified. The system model used to generate the required input and output signals is shown in Fig.21. The same implicit circuit is used.

6-2 Curves and Data

1) For the system in (74), where a constant λ is to be determined, the following data have been obtained for various magnitudes of a step input and different values of loop gain G.

System : $\dot{y} + y = u(t)$; $\lambda = 1$

Table 1

G / u(t)	y		y_c		λ_c		$a_1=\lambda_c$ *	a_2 *
	1000	10	1000	10	1000	10		
3	3	3	3	3.15	0.998	0.998	1.010	0.001
5	5	5	5	5.14	1.005	1.005	0.988	0.003
8	8	8	8	8.14	1.002	1.002	1.015	0.001
least squares estimate					1.001	1.001	1.004	0.001

* a_1, a_2 are determined by the method of Sec. 5.

From Table 1, we observe that λ_c is independent of the loop gain. The value of G affects only the transient response of the implicit synthesis circuit and the steady-state error in y_c. The transient is a decaying exponential as is consistent with (55) derived in Sec. 3 . From Table 1 we also note that the higher the gain G, the smaller the error between y and y_c.

We have found that if $G \geq 200$ the steady-state error $\epsilon = y_c - y$ is practically zero. It means that λ_c will adjust itself to be equal to λ so that y_c is equal to y at all times. On the other hand if G is kept constant and u(t) is varied we note that λ_c is also independent of u(t) provided that u(t) is large enough to generate a system output y, which is in the operating range of the computer components. If u(t) is too small to actuate the variables in the normal operating range of the components, certain error will be introduced by noise. As to the transient speed we observe that the higher the gain G or/and the larger the input u(t), the faster the response (Fig. 22 and Fig. 23) will be.

Clymer [17] suggested that an absolute magnitude circuit should be inserted in the y-line or the λ_c -line to the multiplier so that a set of algebraic signs which keeps the system stable will be maintained at all times. Since we use positive step functions as the inputs, no change of sign in y will occur. Hence no need for such a circuit in our experiment. If such an AMC is used, as shown in Fig. 24, no effect will be produced on the values of λ_c and y_c which we have obtained. But in this case λ_c is no longer obtained directly from the output of the high gain amplifier (A21).
This is because imperfect diodes of the AMC introduce a voltage drop which makes the value obtained from the output of the high gain amplifier less than the actual value of the parameter to be determined. (The voltage drop is found to be 0.40v in the EAI TR-48 analog computer we used in our experiment). Since it is the value at the output of the AMC (also the input to the multiplier) that in turn determines the product $\lambda_c y$, which automatically adjusts to give a desired value of y_c, the value of λ_c obtained from the output of AMC is the same as that obtained directly from the high gain multiplier when no AMC is used.

When a sinusoidal input was used as the input signal we did not get a stable system even if AMC was used. Both sin t and sin 10t were tried, but no stable system was obtained. It is our conjecture that if sinωt has ω so ~~large~~ small that the system response can follow the high speed response of the high gain amplifier, then a stable system may be obtained. We have found that when we superimpose a step function to the original sinusoidal signal (which makes it a biased sinusoidal signal), a stable system has been obtained. Satisfactory result for the value of the parameter to be determined in a system is shown in Fig. 25.

2) The first-order nonlinear system defined in (75) with a_2 as a constant parameter is identified by the implicit synthesis circuit of Fig. 20. The data obtained and the value of a_2 thus determined is given in Table 2.

System : $\dot{y} + 2y^2 = u(t)$; $\mu = 2y$; $a_2 = 2$
Gain G : 1000 and 100
Step input : u(t)

Table 2

G / u(t)	y_c		y		μ_c		$a = \frac{\mu_c}{y}$	a_1^*	a_2^*
	1000	100	1000	100	1000	100			
4	1.41	1.46	1.41	1.41	2.78	2.78	1.99	0.16	1.85
8	2.03	2.08	2.03	3.93	3.93	3.93	1.94	-0.21	2.12
10	2.24	2.29	2.24	2.24	4.54	4.54	2.04	-	-
least squares estimates							1.99	0.02	1.99

* a_1, a_2 are determined by the method of Sec. 5.

In Table 2 we observe that μ_c is independent of gain G, same as in part 1 of this section. But μ_c is found to be dependent on the value of u(t), and μ cannot be a constant as λ in part 1. Since the dependent variable y is dependent on the value of the input u(t), we know that μ is a function of y and the system is nonlinear. The parameters in the nonlinear term will be calculated in the next section (6-3) using the general method of identifying a first-order nonlinear system described in Sec. 5.

3) The system defined in (76) with a_1 and a_2 as the constant parameters to be determined is identified by the use of the implicit synthesis circuit we used in part 1 and 2 of this section and the model in Fig. 21. The data obtained and the values of a_1 and a_2 thus determined are given in Table 3. The method of Sec. 5 is used for the calculation.

System : $\dot{y} + y + 2y^2 = u(t)$; $\mu = 1+2y$; $a_1=1$; $a_2=2$

Gain G : 1000

Step inputs : u(t)

Table 3

u(t)	y	y_c	μ_c	$a_1=1$	$a_2 = \frac{\mu_c - 1}{y}$	a_1^*	a_2^*
6	1.50	1.50	3.95	1	1.97	1.25	1.80
8	1.82	1.82	4.52	1	1.94	1.12	1.89
10	2.03	2.03	4.95	1	1.95	0.95	2.05
least squares estimate				1	1.95	1.11	1.91

* a_1, a_2 are calculated by the method of Sec. 5.

6-3 Computation

In order to illustrate the method developed in Sec. 5, the data obtained from the implicit synthesis circuits simulated on the computer will be used for the computation for each of the three cases presented in Sec. 6-2.

1) From Table 1 we take the following two sets of data for this calculation :

System : $\dot{y} + y = u(t)$; $a_1 = 1$; $a_2 = 0$

u(t)	y	μ_c
5	5.000	1.005
8	8.000	1.002

The matrices $\underline{A}$, $\underline{Y}$ and $\underline{\mu}$ are as follows

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} , \quad \underline{Y} = \begin{bmatrix} 1 & 5 \\ 1 & 8 \end{bmatrix} , \quad \underline{\mu} = \begin{bmatrix} 1.005 \\ 1.002 \end{bmatrix}$$

From (72) we have

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 8 & -5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1.005 \\ 1.002 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3.03 \\ 0.003 \end{bmatrix} = \begin{bmatrix} 1.01 \\ 0.001 \end{bmatrix}$$

$a_1 = 1.01$ and $a_2 = 0.001$ are very close to their actual values $a_1 = 1$ and $a_2 = 0$. The very small error caused by noise is negligible and the result is satisfactory within the experimental accuracy.

2) From Table 2 we take the following two sets of data in our calculation :

System : $\dot{y} + 2y^2 = u(t)$; $a_1 = 0$; $a_2 = 2$

u(t)	y	μ_c
4	1.41	2.78
10	2.24	4.54

The matrices are formed as follows

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad \underline{Y} = \begin{bmatrix} 1 & 1.41 \\ 1 & 2.24 \end{bmatrix} \qquad \underline{\mu} = \begin{bmatrix} 2.78 \\ 4.54 \end{bmatrix}$$

From (72) we have

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{0.83} \begin{bmatrix} 2.24 & -1.41 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2.78 \\ 4.54 \end{bmatrix} = \frac{1}{0.83} \begin{bmatrix} 0 \\ 1.76 \end{bmatrix} = \begin{bmatrix} 0 \\ 2.1 \end{bmatrix}$$

a_1 and a_2 are found to be accurately determined.

3) The detemination of $a_1=1$ and $a_2=2$ can be calculated from the data in Table 3 and (72) :

System: $\dot{y} + y + 2y^2 = u(t)$; $a_1 = 1$; $a_2 = 2$

$u(t)$	y	μ_c
8	1.82	4.52
10	2.03	4.95

Similar to part 1 and part 2, we have

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} ; \ \underline{Y} = \begin{bmatrix} 1 & 1.82 \\ 1 & 2.03 \end{bmatrix} ; \ \underline{\mu} = \begin{bmatrix} 4.52 \\ 4.95 \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{0.21} \begin{bmatrix} 2.03 & -1.82 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4.52 \\ 4.95 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 2.05 \end{bmatrix}$$

The computed values of the parameters, $a_1 = 0.95$ and $a_2 = 2.05$, are close to their actual values.

From the above examples we conclude that the generalized method of Sec. 5 can be used to identify any linear or nonlinear first-order systems provided that the nonlinearity is representable as a convergent power series in the dependent variable y.

6-4 Comment.

In the linear system an AMC ensures the system stability no matter what sign the dependent variable y may have and how large the gain G may be. But stability problems of the nonlinear systems are more severe. We have found that when an AMC is used the system becomes unstable if the gain G is larger than 500. This is due to the time lag introduced by the AMC. So the choice of a particular input signal so that no AMC is needed is more critical in nonlinear systems than in linear systems. Usually positive step functions can be used.

The output of the high gain amplifier in the implicit synthesis circuit can be made to be any variable provided that the closed loop relation satisfies the system equation. We have found that if the output of the high gain amplifier is made to be $10\lambda_c$ instead of λ_c the accuracy of the parameters thus determined is improved, especially when the parameters are small in value.

The parameters of a system can be determined by the method of Sec. 5 from the data obtained by the implicit synthesis circuit. These parameters can also be determined by the graphical method as shown in Fig. 26 and Fig. 27. In Fig. 26 the parameter a_2 is determined from the slope of the μ_c vs y curve and in Fig. 27 parameters a_2 and a_1 are determined from the slope and the intercept on the μ_c-axis, respectively. But the graphical method becomes difficult to apply when the number of parameters is greater than two.

Since noise introduces error in the data obtained, better accuracy can be achieved if least squares method is used in the analysis of the determined values.

7. Identification of High-Order Systems by the Implicit Synthesis Method.

Because of the pitfalls of the stability problems in the implicit synthesis circuit, so far no general method using implicit synthesis circuits to determine parameters in a system of order higher than two has been proposed. In Clymer's [17] paper only the method of determining a single parameter in first- and second-order systems have been given. He suggested several possibilities of attacking the high-order systems [17], but no definite method was presented. Madden [26] had used the least-magnitude method, which is an extension of implicit synthesis, to identify a second-order system. Two parameters in a second-order over-damped system were determined simultaneously, with the unit step function as the input. A symmetrical multivibrator was used to provide a periodic reversal of the sign of the parameter-determining loop to improve the convergence [26].

A possibility of using the implicit synthesis circuits to identify an n-th order system is suggested below.

Consider an n-th order system described by

$$\frac{d^n x}{dt^n} + a_{n-1}\frac{d^{n-1}x}{dt^{n-1}} + \ldots\ldots + a_1\frac{dx}{dt} + a_o x = u(t) \tag{77}$$

By letting

$$x_1 = x,\quad x_2 = \frac{dx}{dt},\ \ldots\ldots,\quad x_n = \frac{dx_{n-1}}{dt}$$

we obtain n simultaneous equations

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &------\\ \dot{x}_{n-1} &= x_n\\ \dot{x}_n &= -a_o x_1 - a_1 x_2 - \ldots\ldots - a_{n-1}x_n + u(t)\end{aligned} \tag{78}$$

If we use the state space notation and choose $\underline{X}$, an~~d~~ nx1 column vector such that

$$\underline{X}' = [x_1, x_2, \ldots\ldots, x_n]$$

as the state vector, then (78) can be represented by the state space equation

$$\dot{\underline{x}} = \underline{A}\cdot\underline{x} + \underline{b}\,u(t) \tag{79}$$

where

$\underline{A}$ is an nxn matrix

$$\begin{bmatrix} 0 & 1 & 0 & \ldots\ldots & 0 \\ 0 & 0 & 1 & \ldots\ldots & 0 \\ \hline -a_o & -a_1 & \ldots & \ldots\ldots & -a_{n-1} \end{bmatrix}$$

$\underline{b}$ is an nx1 column vector such that

$$\underline{b}' = [0, 0, \ldots\ldots, 1]$$

We observe that (79) is just similar to the typical first-order differential equation, so the implicit synthesis circuit can similarly be applied to (79). But here we have to determine n parameters by n first-order implicit synthesis circuits. Since interactions exist among those n equations, it is impossible to determine n parameters independently. Now let us consider a special case of a system having a stability matrix $\underline{A}$ whose n characteristic roots are all distinct and real. In this case there always exists a nonsingular matrix $\underline{T}$, such that the matrix $\underline{A}$ can be diagonalized.

Let $\underline{x} = \underline{T}\ \underline{z}$. From (79) we have

$$\dot{\underline{z}} = \underline{T}^{-1}\,\underline{A}\ \underline{T}\ \underline{z} + \underline{T}^{-1}\,\underline{b}\ u(t) \tag{80}$$

Since n characteristic roots of $\underline{A}$ are distinct, $\underline{A}$ can be diagonalized and the resulting diagonal matrix $\underline{\lambda}$ is defined as

$$\underline{\lambda} = \underline{T}^{-1} \underline{A}\, \underline{T} \tag{81}$$

then (80) can be reduced to

$$\dot{\underline{z}} = \underline{\lambda}\, \underline{z} + \underline{b}^{1} u(t) \tag{82}$$

where

$$\underline{\lambda} = \begin{bmatrix} \lambda_1 & 0 \ldots\ldots\ldots & 0 \\ 0 & \lambda_2 \ldots\ldots\ldots & 0 \\ \ldots & \ldots\ldots\ldots\ldots & \ldots \\ 0 & 0 \ldots\ldots\ldots & \lambda_n \end{bmatrix}; \quad \underline{b}^{1} = \underline{T}^{-1} \underline{b}$$

and $\lambda_1, \lambda_2, \ldots, \lambda_n$ are n distinct roots of $\underline{A}$. Since (82) consists of n simultaneous first-order differential equations which do not have interactions on one another, each λ_c can be determined independently from each equation. Here the signals used for the identification are not the original input $\underline{b}\, u(t)$ and $\underline{x}$, but the transformed variables $\underline{z}$ ($\underline{z} = \underline{T}^{-1} \underline{x}$) and $\underline{b}^{1} u(t)$. Also the answers obtained are $\underline{\lambda}$ instead of the parameters $\underline{A}$ to be determined, but they are related by (81), or

$$\underline{A} = \underline{T}\, \underline{\lambda}\, \underline{T}^{-1}$$

Therefore transformation of variables should be made such that $\underline{x}$ will be changed to $\underline{z}$ and $\underline{\lambda}$ changed back to $\underline{A}$. We believe this can be implemented by a hybrid system of analog and digital computers. Owing to the increased complexity ~~complicatedness~~ of the circuit, it is expected that the stability problem will be more severe, and the practicability of this method is difficult to predict. However, theoretically at least, this seems to be a possible approach.

The suggested schematic diagram is shown in Fig. 28. In the diagram double-line arrows denote multivariable signal transmission. $\underline{T}$ is an nxn matrix whose columns are the eigenvectors of the matrix $\underline{A}$.

$$\underline{T} = \begin{pmatrix} 1 & 1 \ldots\ldots & 1 \\ \lambda_1 & \lambda_2 \ldots\ldots & \lambda_n \\ \lambda_1^2 & \lambda_1^2 \ldots\ldots & \lambda_n^2 \\ \ldots & \ldots\ldots\ldots & \ldots \\ \lambda_1^{n-1} & \lambda_2^{n-1} \ldots & \lambda_n^{n-1} \end{pmatrix}$$

$\underline{T}^{-1}$ is the inverse of the matrix $\underline{T}$.

M is the multiplier with number 1, 2 denoting order of matrix multiplication.

Example:

Consider a third-order system defined by

$$\dddot{x} + 6\ddot{x} + 11\dot{x} + 6x = u(t) \quad ; \quad u(t) = k \text{ (constant)}$$

where $a_0 = 6$, $a_1 = 11$ and $a_2 = 6$ are parameters to be determined.

The system can be defined by the state space equation

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{b}\ u(t)$$

where

$$x_1 = x\ , \quad x_2 = \dot{x} \quad x_3 = \ddot{x}$$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix} \ ; \quad \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \ ; \quad \underline{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The system has three distinct roots -1, -2, and -3, which are the roots of the characteristic equation

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{vmatrix} = (\lambda+1)(\lambda+2)(\lambda+3) = 0$$

So the transforming matrix $\underline{T}$ is

$$T = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

and its inverse $\underline{T}^{-1}$ is

$$\underline{T}^{-1} = -\frac{1}{2} \begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$$

The state vector $\underline{X}$ is

$$\underline{X} = \begin{bmatrix} -k/2\, e^{-t} + k/2\, e^{-2t} - k/6\, e^{-3t} + k/6 \\ k/2\, e^{-t} - k\, e^{-t} + k/2\, e^{-3t} \\ -k/2\, e^{-t} + 2ke^{-2t} - 3k/2\, e^{-3t} \end{bmatrix}$$

After the transformation the new variables $\underline{z}$ will be

$$\underline{z} = \begin{bmatrix} k/2\,(1-e^{-t}) \\ -k/2\,(1-e^{-2t}) \\ k/6\,(1-e^{-3t}) \end{bmatrix}$$

and now matrices $\underline{\lambda}$ and $\underline{b}^1$ are

$$\underline{\lambda} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad ; \quad \underline{b}^1 = \begin{bmatrix} 1/2 \\ -1 \\ 1/2 \end{bmatrix}$$

The new system after transformation is

$$\dot{\underline{z}} = \underline{\lambda}\,\underline{z} + \underline{b}^1 u(t)$$

or

$$\dot{z}_1 + z_1 = k/2$$

$$\dot{z}_2 + 2z_2 = -k$$

$$\dot{z}_3 + 3z_3 = k/2$$

Finally we get the new system which consists of three independent first-order systems. The new parameters $\lambda_1 = -1$, $\lambda_2 = -2$ and $\lambda_3 = -3$ can be determined independently by the implicit synthesis circuits. The actual parameters $a_0 = 6$, $a_1 = 11$ and $a_2 = 6$ will be obtained by transforming $\underline{\lambda}$ back to $\underline{A}$ as follows.

$$\underline{A} = \underline{T}\underline{\lambda}\,\underline{T}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

From the computed matrix $\underline{A}$ we obtain the parameters $a_0=6$, $a_1=11$ and $a_2=6$.

Conclusion:

Intelligent system design and system optimization and adaptation require the knowledge of the process dynamics. System identification provides the required information about the process dynamics. The fundamental approaches of the identification schemes are correlation technique, model referenced method and regression analysis method. Each approach has its own advantages and disadvantages. How to choose a proper mathematical model and which approach should be used to identify the given system depend on the nature of the system and the a priori information about the system.

In the first three chapters the identification problem is formulated. Some preliminary considerations and the mathematical models of characterizing a system are described. From the descriptions in the previous chapters we know that all three fundamental principles: correlation method, model referenced method and regression analysis method are powerful and promising. In the correlation method if the noise existing in the system itself can be utilized effectively to determine the impulse response, it will be a very practical method. In the model-referenced method, the orthogonalized model seems to be the most promising one, because this approach has the great advantage that each parameter can be adjusted independently to minimize the error criterion. But how to construct the orthogonalized model with the minimum number of orthogonal filters is still a problem. The regression analysis method has the advantage that no information about the noise statistics is required. It is especially useful in the identification of the system when the measured signals are corrupted by noise. But this method requires the inversion of a matrix, which causes a great difficulty in

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computation. It is also interesting to note that some new techniques, such as pattern recognition [46], dynamic programming [47], decision theory [48] have been introduced into this problem. In recent years the most significant advancement in the identification problem are the approaches based on Kalman's optimal filtering theory (or the minimum variance technique). Its application is not far behind the theory and a digital computer programs based on this theory ~~is~~ are being considered as the basis for the implementation of the on-board computer to direct the Appolo vehicle to and around the moon [49].

In this thesis special attention has been given to ~~the~~ system identification by the implicit synthesis method. This method has the great advantage of simplicity that it requires only one multiplier channel per unknown parameter and can be programmed on the smaller analog computer. In Chapter 4 a detailed investigation of the implicit synthesis circuit has been made. A general method of identifying a first-order nonlinear system whose nonlinearity ~~is~~ can be approximated by a polynomial ~~representable as a convergent power series~~ in the dependent variable has been presented. Several examples are worked out to illustrate the method. The values of parameters thus determined from the data obtained on an EAI TR-48 analog computer and the presented method are found to be satisfactorily accurate. A method of identifying an nth-order linear system with simple and real characteristic roots is also proposed. An example is given to illustrate this method. The use of this method for the identification of more general high-order systems may be possible if more advanced techniques of hybrid computer systems and interactions among multivariable system are better understood. Although the implicit synthesis method has the pitfalls of critical stability and accuracy problems, it is still very promising and practical because of its simplicity.

Appendix 1. Parameter Influence Coefficients.

Let the system be described by a set of differential equations

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n, t; \alpha_1, \alpha_2, \ldots \alpha_m) \qquad \text{(A-1)}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_m$ are system parameters. The solution of (A-1) is expressed as

$$x_{io} = x_{io}(t; \alpha_1, \alpha_2, \ldots, \alpha_m) \qquad \text{(A-2)}$$

The partial derivatives of the problem variables with respect to pertinent system parameters $\frac{\partial x_{io}}{\partial \alpha_j}$ ($i=1, 2, \ldots, n$; $j = 1, 2, \ldots m$) are called the parameter influence coefficients. These parameter influence coefficients can be obtained, simultaneously with the solution of the original system differential equations, by solving the auxiliary differential equations, known as sensitivity equations, which are derived from the system equations by a simple differentiation process.

Consider an nth-order system with m parameters α_1, $\alpha_2, \ldots, \alpha_m$, as defined by (A-1). The sensitivity equation with respect to α_k yielding $u_{ik} = \frac{\partial x_i}{\partial \alpha_k}$ may be formulated as follows

$$\frac{\partial}{\partial \alpha_k}\left(\frac{\partial x_i}{\partial t}\right) = \frac{\partial u_{ik}}{\partial t} = \frac{\partial f_i}{\partial x_1} u_{1k} + \frac{\partial f_i}{\partial x_2} u_{2k} + \dots + \frac{\partial f_i}{\partial x_n} u_{nk} \qquad (A-3)$$

All initial conditions are zero (i.e., $u_{ik}(o) = o$) provided that none of the initial conditions of the original system are considered as parameters. This formulation applies to linear as well as nonlinear systems.

Example:

A second-order system is defined by

$$\frac{d^2x}{dt^2} + \mu \frac{dx}{dt} + \lambda x = f(t) \qquad (A-4)$$

with initial conditions $x(o) = a, \quad \frac{dx}{dt}(o) = b$.

Differentiation of (A-4) with respect to λ yields

$$\frac{\partial^3 x}{\partial\lambda\partial^2 t} + \mu \frac{\partial^2 x}{\partial\lambda\partial t} + \lambda \frac{\partial x}{\partial\lambda} + x = 0 \qquad (A-5)$$

or

$$\frac{\partial^2 u}{\partial t^2} + \mu \frac{\partial u}{\partial t} + \lambda u = -x \qquad (A-6)$$

where $u = \frac{\partial x}{\partial \lambda}$, also $u(o) = 0$, $\frac{\partial u(o)}{\partial t} = 0$, since $x(0)$ and $\frac{\partial x(o)}{\partial t}$ do not depend on λ.

(A-6) is called the sensitivity equation of the system with respect to the parameter λ. Similarly, we can obtain the sensitivity equation with respect to μ as

$$\frac{\partial^2 v}{\partial t^2} + \mu \frac{\partial v}{\partial t} + \lambda v = -\frac{\partial x}{\partial t} \qquad (A-7)$$

where $V = \frac{\partial x}{\partial \mu}$ and $V(o) = 0$, $\frac{\partial v}{\partial t}(0) = 0$

The computer circuits which consist of a master system and a slave system to yield x, u, v is shown in Fig. A-1.

Appendix 2. Method of Quasi-linearization.

Suppose a vector differential equation

$$\dot{\underline{x}} = f(\underline{x}, t) \qquad t_o \leq t \leq t_T \tag{A-8}$$

be given with the initial conditions

$$\langle \underline{c}(t_i), \underline{x}(t_c) \rangle = b_i \; ; \; i = 1, 2, \ldots, n \tag{A-9}$$

$$t_o \leq t_1 \leq t_2 \; \cdots\cdots \leq t_n \leq t_T$$

where $\underline{c}$ and $\underline{x}$ are n vectors.

It is assumed that (A-8) and (A-9) have a unique solution on $[t_o, t_T]$. If x_o be the initial guess to the solution of (A-8) on $[t_o, t_T]$, then the (k+l) st approximation can be obtained from the kth via

$$\dot{\underline{x}}_{k+1} = f(\underline{x}_k, t) + \underline{J}\, f(\underline{x}_k, t)\,(\underline{x}_{k+1} - \underline{x}_k) \tag{A-10}$$

and x_{k+1} satisfies (A-9), where J is the Jacobian matrix whose ijth element, $\frac{\partial f}{\partial x_k}$, is the partial derivative of the ith component of f with respect to the jth component of x.

The initial approximation vector $x_o(t)$, whose components may be constants or function of time, is used to calculate the first approximation $x_1(t)$ which is the solution of

$$\begin{aligned} \dot{\underline{x}}_1 &= f(\underline{x}_o, t) + \underline{J}\, f(\underline{x}_o, t)\,(\underline{x}_1 - \underline{x}_o) \\ &= \underline{J}\, f(\underline{x}_o, t)\,\underline{x}_1 + f(\underline{x}_o, t) - J\, f(\underline{x}_o, t)\,\underline{x}_o \end{aligned} \tag{A-11}$$

and satisfying (A-9).

Let $\Phi_1(t)$ be the fundamental solution matrix of

$$\dot{\underline{\Phi}}_1(t) = \underline{J}\ f(\underline{x}_0,\ t)\ \underline{\Phi}_1(t)\ ;\ \underline{\Phi}_1(0) = \underline{I} = \text{Identity matrix} \quad \text{(A-12)}$$

and also $p_1(t)$ be the particular solution vector of

$$\dot{\underline{p}}_1(t) = \underline{J}\ f(\underline{x}_0, t)\ \underline{p}_1(t) + f(\underline{x}_0, t) - \underline{J}\ f(\underline{x}_0, t)\underline{x}_0\ ;\ \underline{p}_1(0) = 0 \quad \text{(A-13)}$$

Then the general solution of (A-11) is

$$\underline{x}_1(t) = \underline{\Phi}_1(t)\underline{k}_1 + \underline{p}_1(t) \quad \text{(A-14)}$$

where k_1 is a constant vector properly chosen to satisfy the initial condition

$$\langle\ \underline{c}(t_i),\ (\underline{\Phi}_1(t_i)\,\underline{k}_1 + \underline{p}_1(t))\ \rangle = b_i$$

$$(i = 1,\ 2,\ \ldots,\ n) \quad \text{(A-15)}$$

All calculations can be easily carried out on the digital computer and the convergence of this scheme is quadratic in nature.

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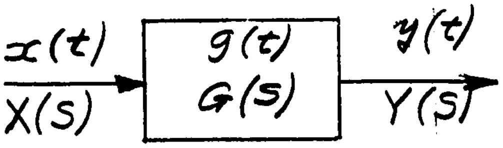

Fig. 1. Black box.

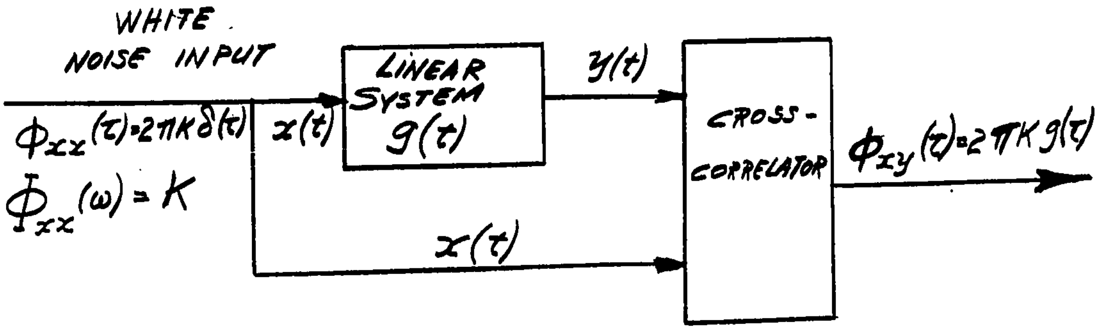

Fig. 2. Determination of the impulse response of a linear system by cross-correlation technique.

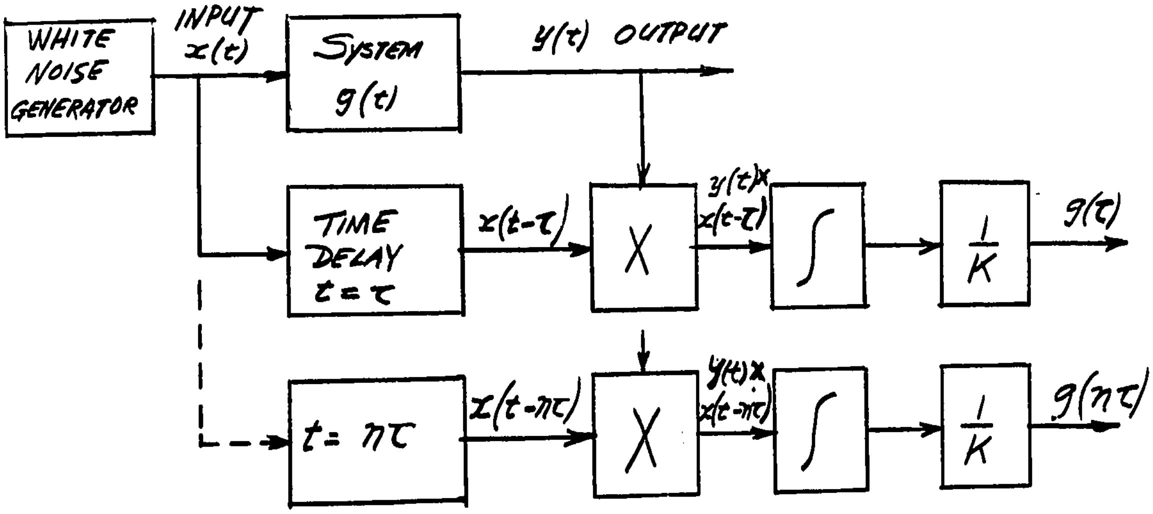

Fig. 3. Mechanization of identification with cross-correlation.

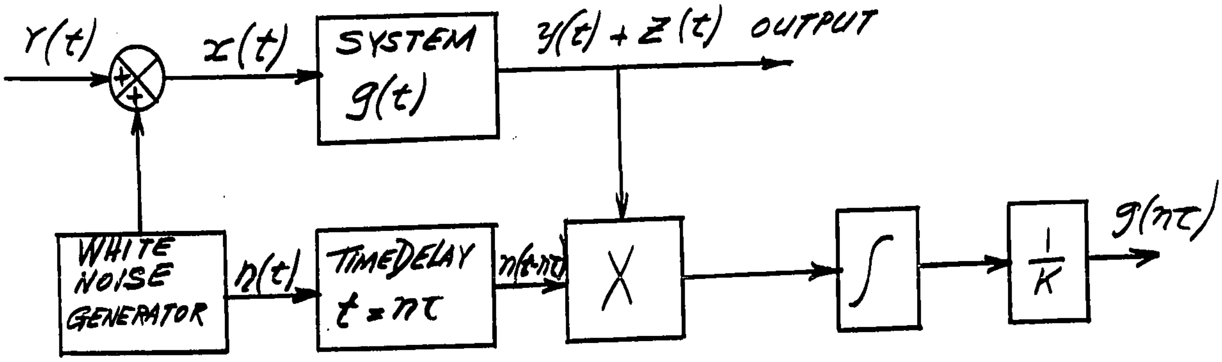

Fig. 4. Identification when normal input Y(t) is present.

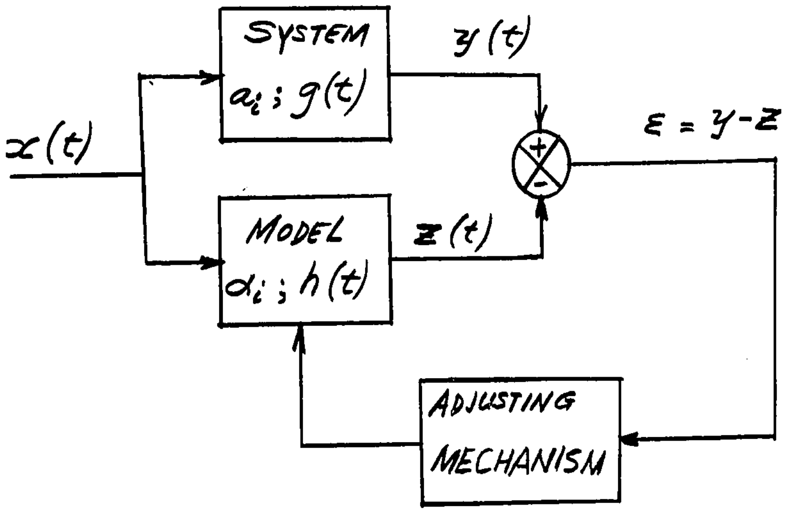

Fig. 5. Model reference technique

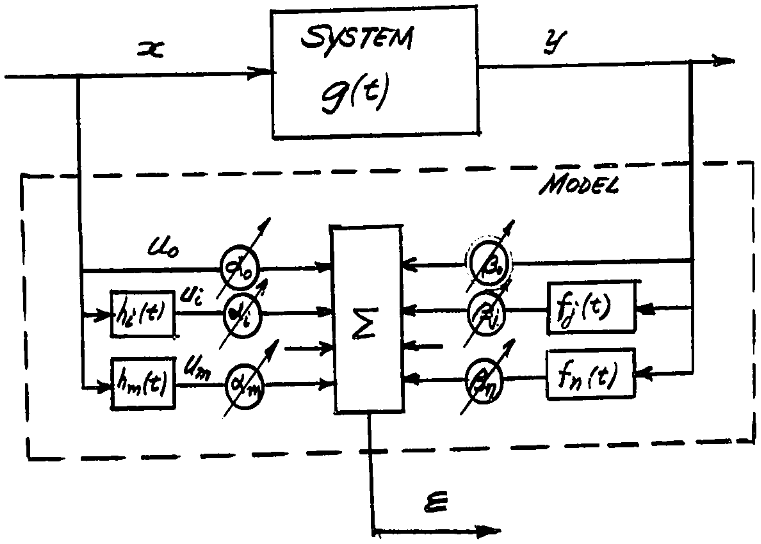

Fig. 6. Generalized model

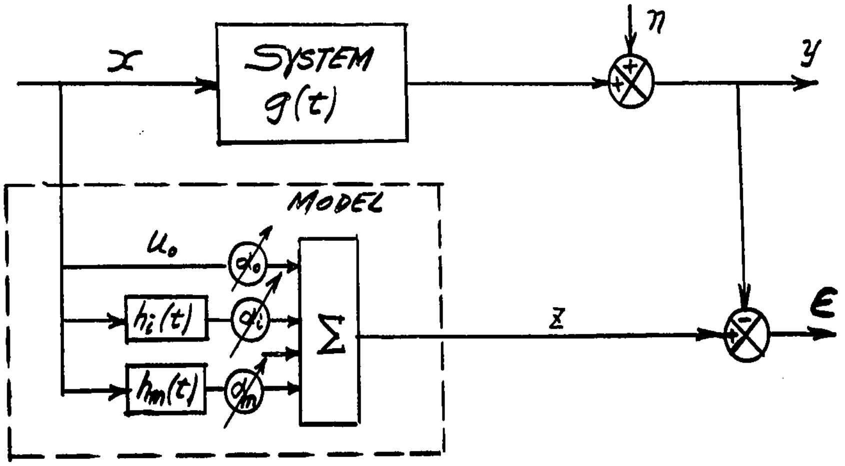

Fig. 7. Ordinary model.

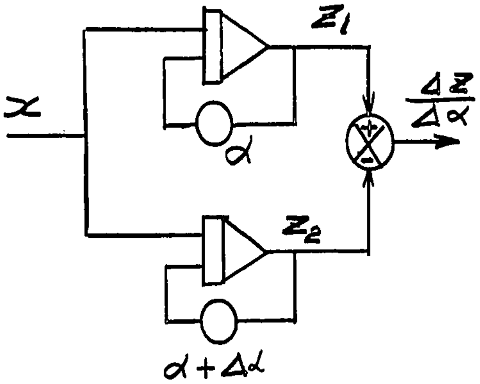

Fig. 8. Finding partial derivatives by using two models with different parameters.

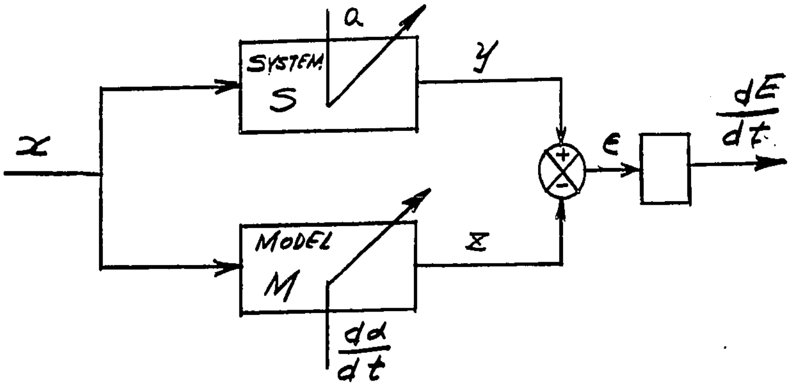

Fig. 9. Using a model with time varying parameter.

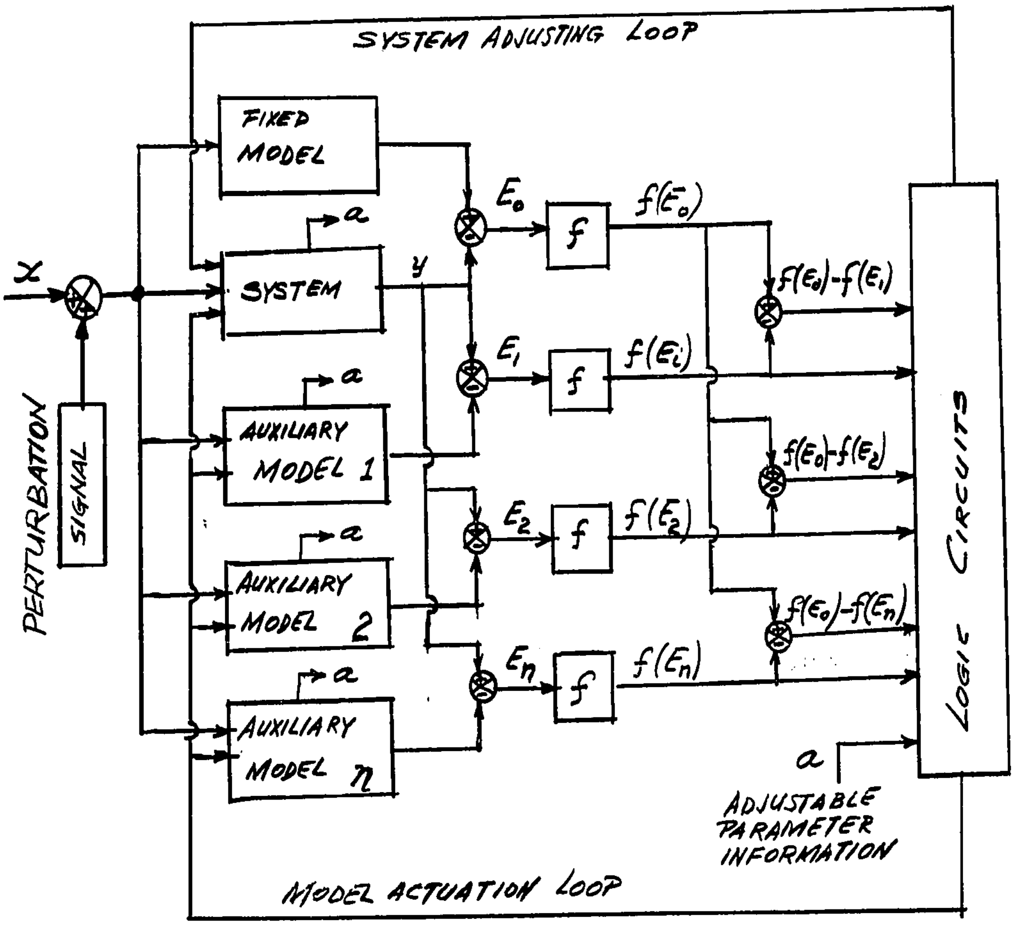

Fig. 10. Auxiliary models for identification

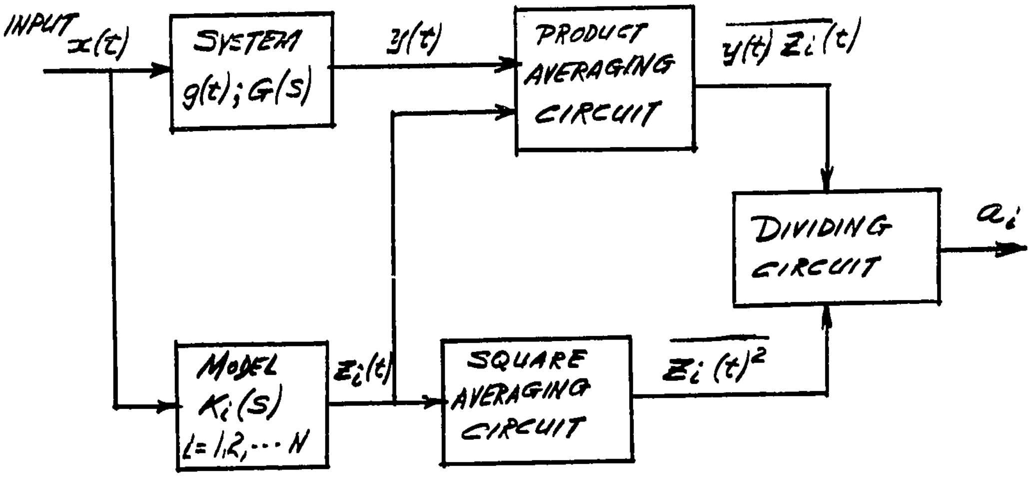

Fig. 11. A measuring system suitable for self-optimizing systems.

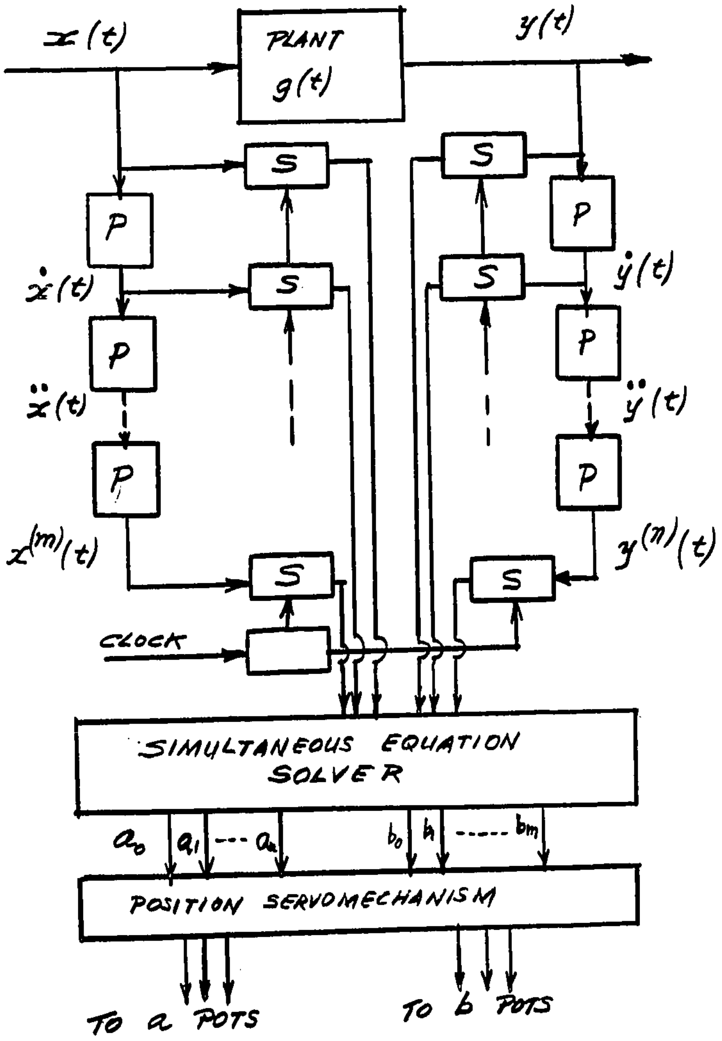

Fig. 12. Plant observer.

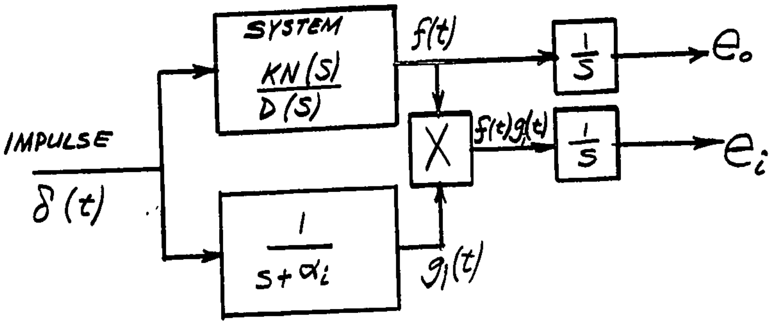

Fig. 13. Auxiliary networks.

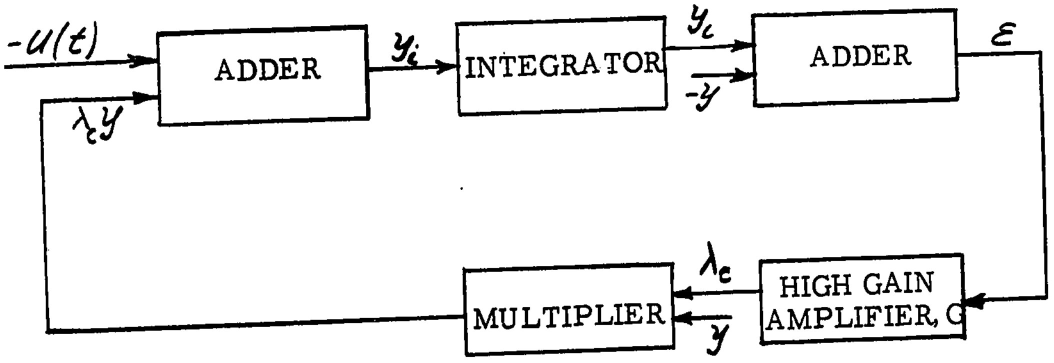

Fig. 14. Block diagram of first implicit synthesis circuit.

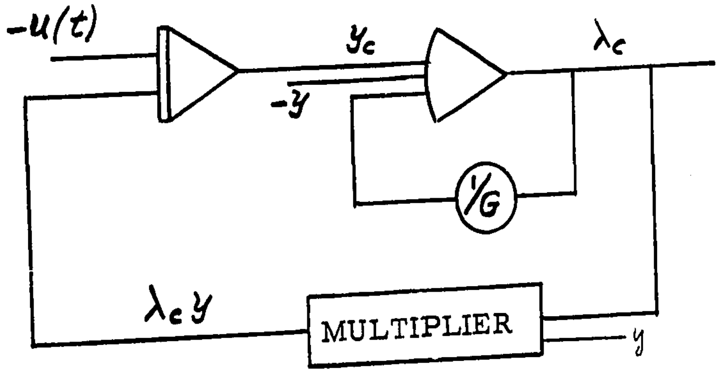

Fig. 15. Analog computer realization of first implicit synthesis circuit.

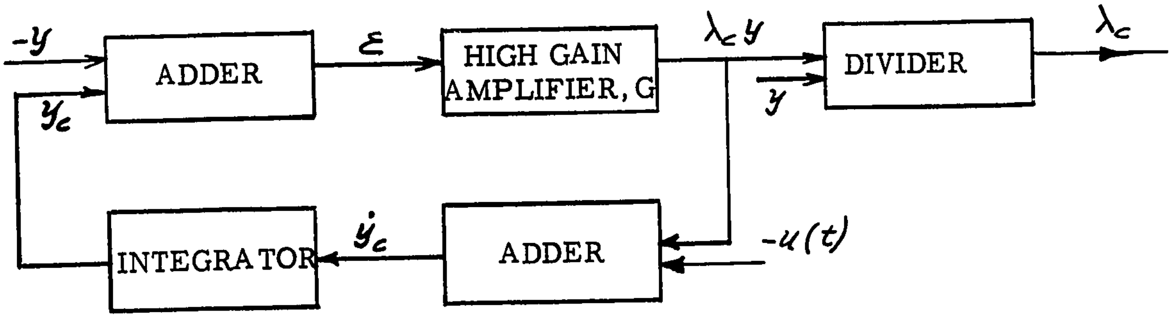

Fig. 16. Block diagram of second implicit synthesis circuit.

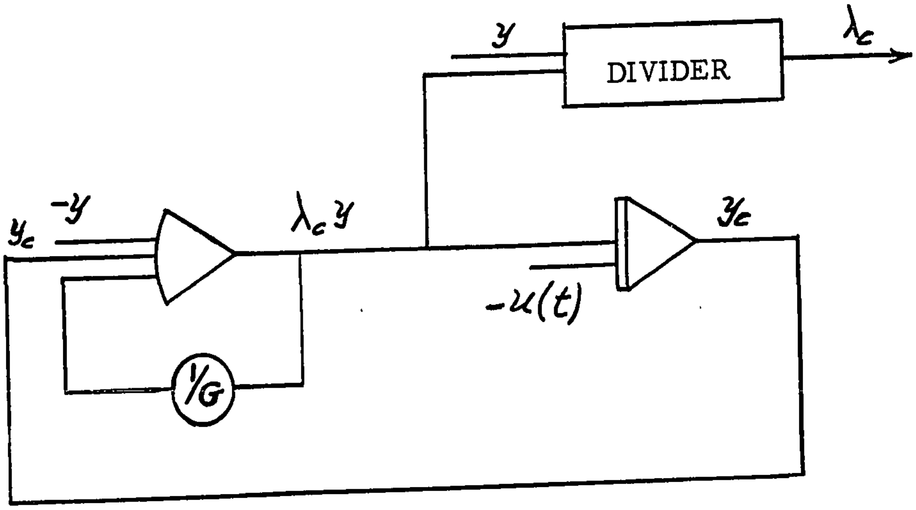

Fig. 17 Analog computer realization of second implicit synthesis circuit.

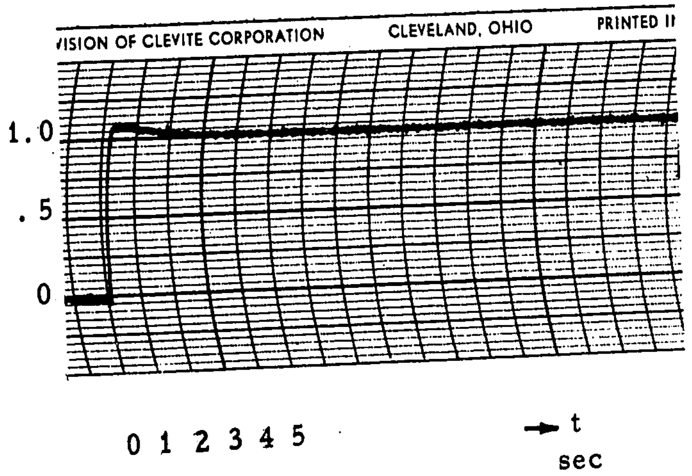

Fig.18. λ-response for G = 100 ; u(t) = 1

$\dot{y} + y = u(t)$

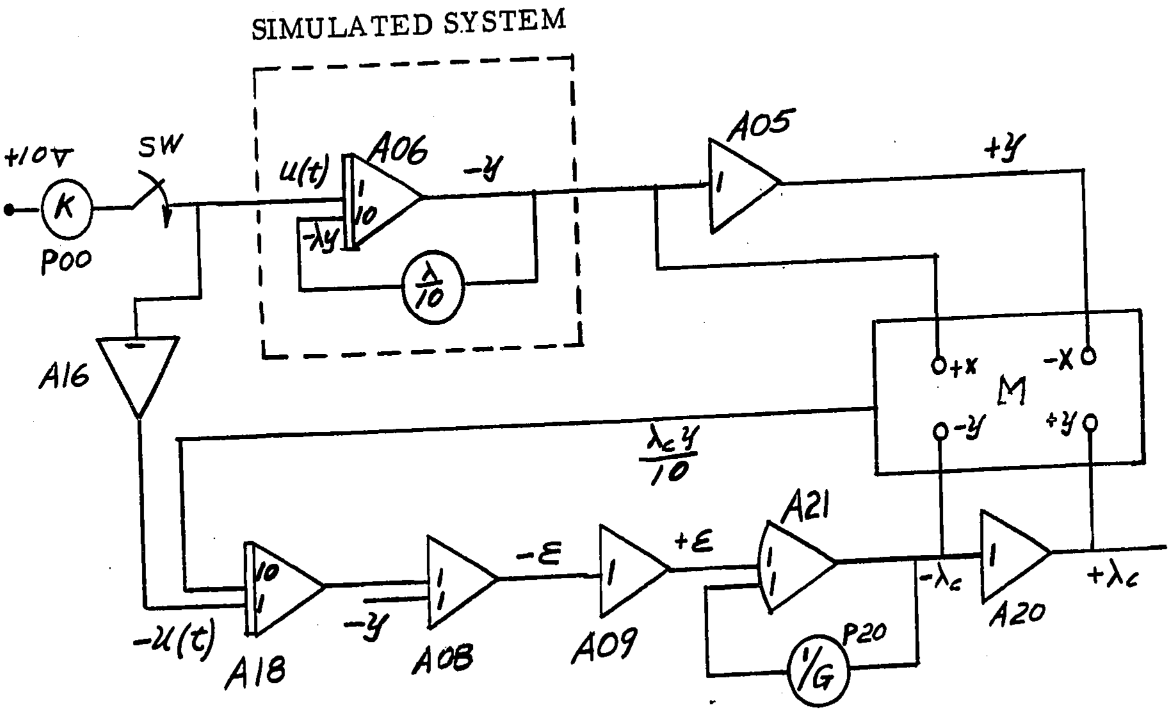

Fig. 19. Computer set-up for the identification of the system $\dot{y} + \lambda y = u(t)$.

SIMULATED SYSTEM

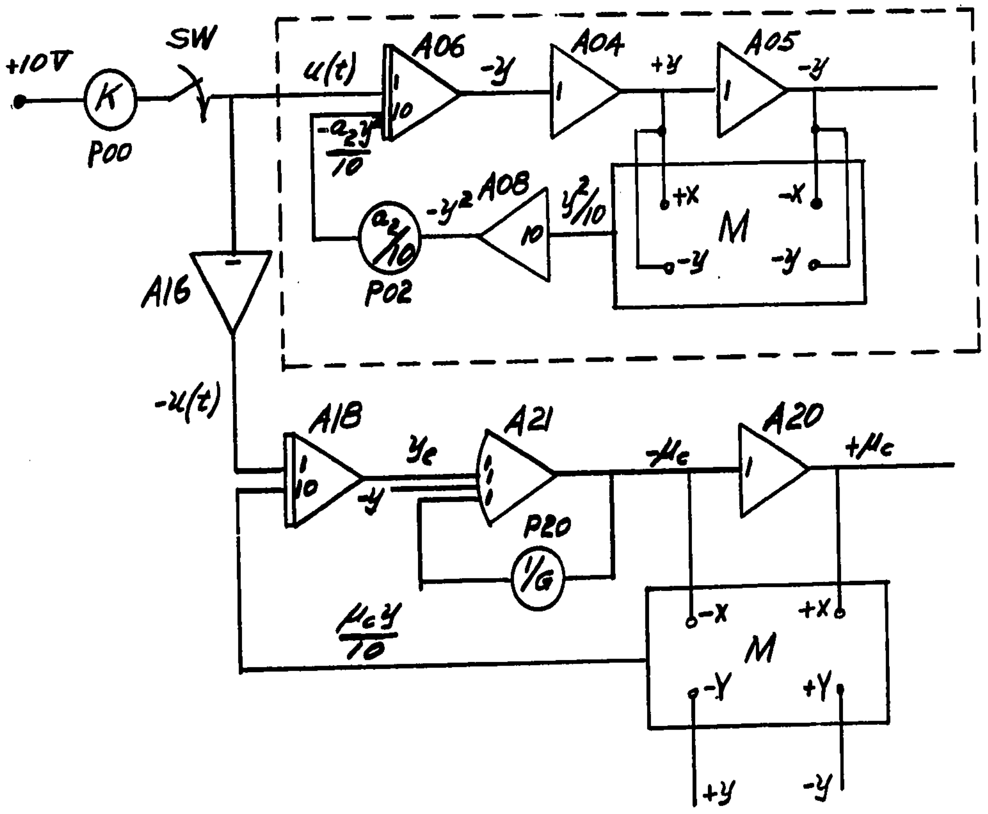

Fig. 20. Computer set-up for the identification of the system $\dot{y} + a_2 y^2 = u(t)$

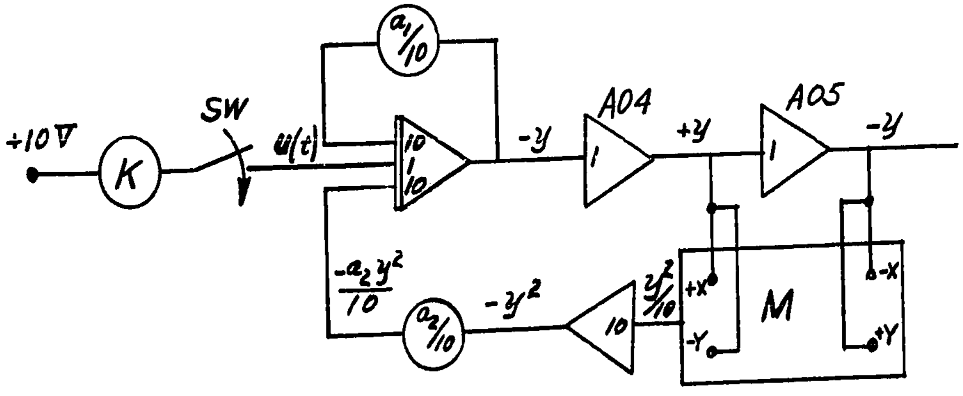

Fig. 21. Simulation of system $\dot{y} + a_1 y + a_2 y^2 = u(t)$.

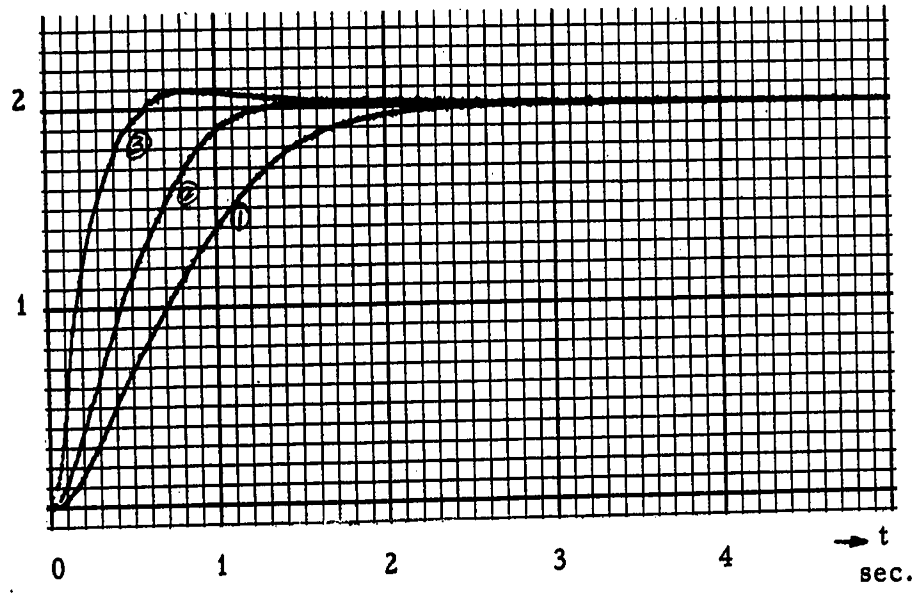

Fig. 22. Response speed as a function of gain G

$\dot{y} + 2y = u(t)$; $u(t) = 4.5$

1. G = 1
2. G = 2
3. G = 10

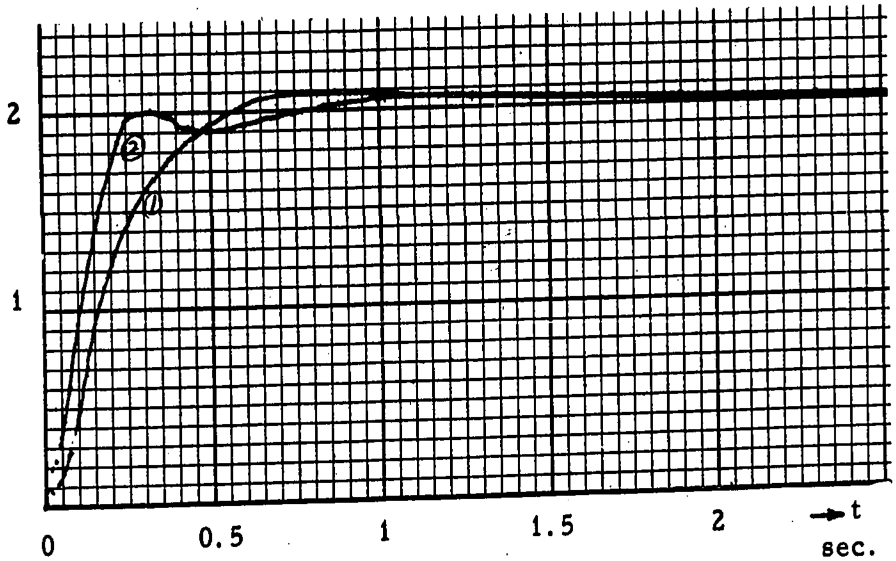

Fig. 23. Response speed as a function of input u(t)

$\dot{y} + 2y = u(t)$; G = 10

1. u(t) = 4.5
2. u(t) = 10

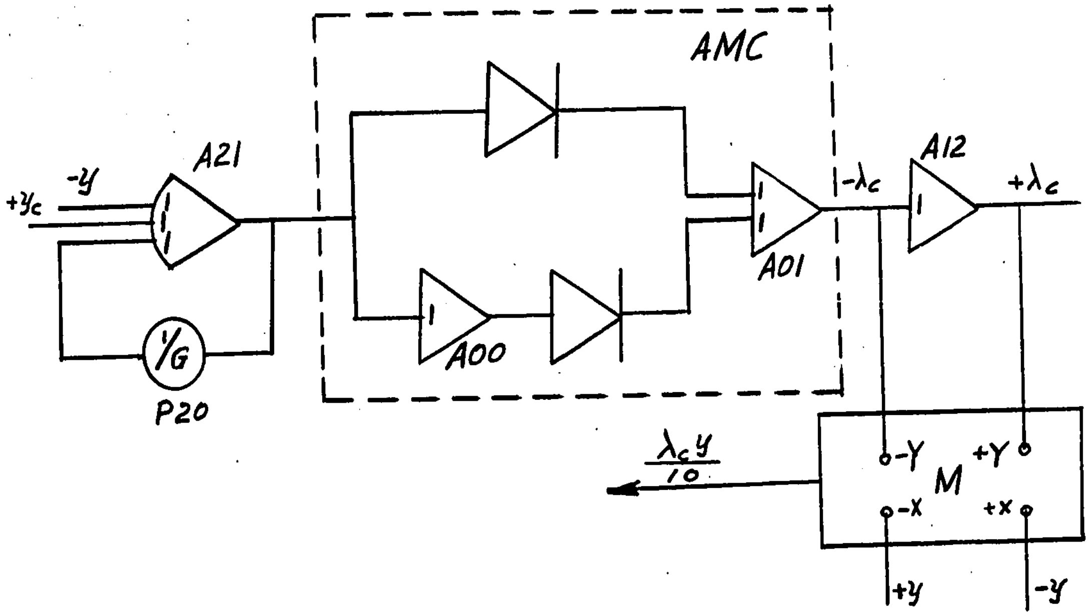

Fig. 24. Absolute magnitude circuit.

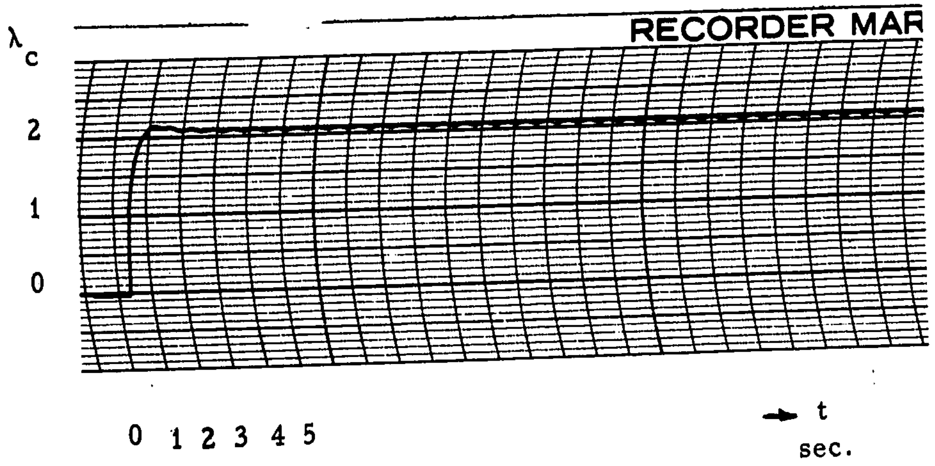

Fig. 25. λ-response with biased sinusoidal input

$\dot{y} + 2y = u(t)$; $u(t) = 4.5 + \sin 10t$; $G=100$.

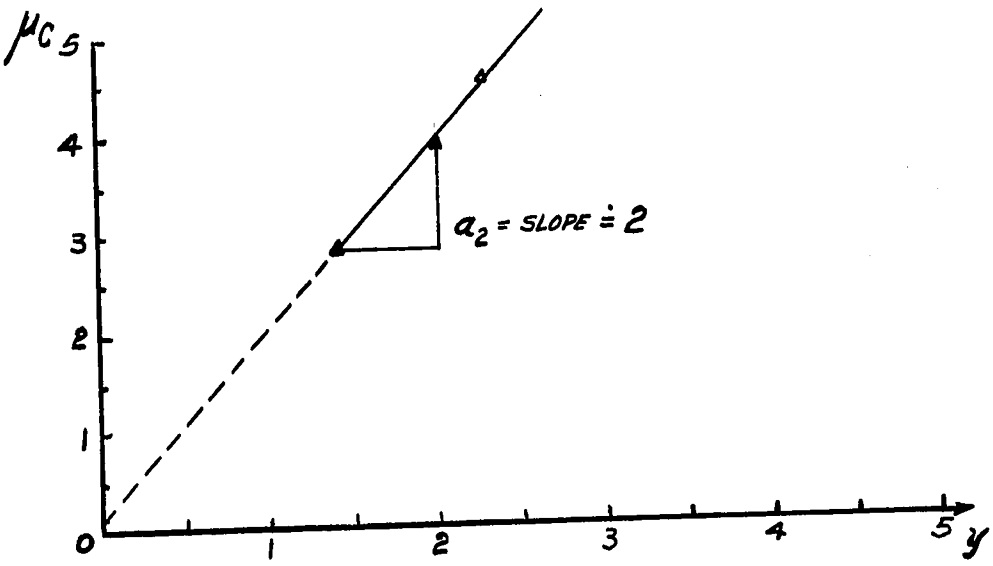

Fig. **26**. Graphical determination of a_2 in system $\dot{y} + a_2 y^2 = u(t)$.

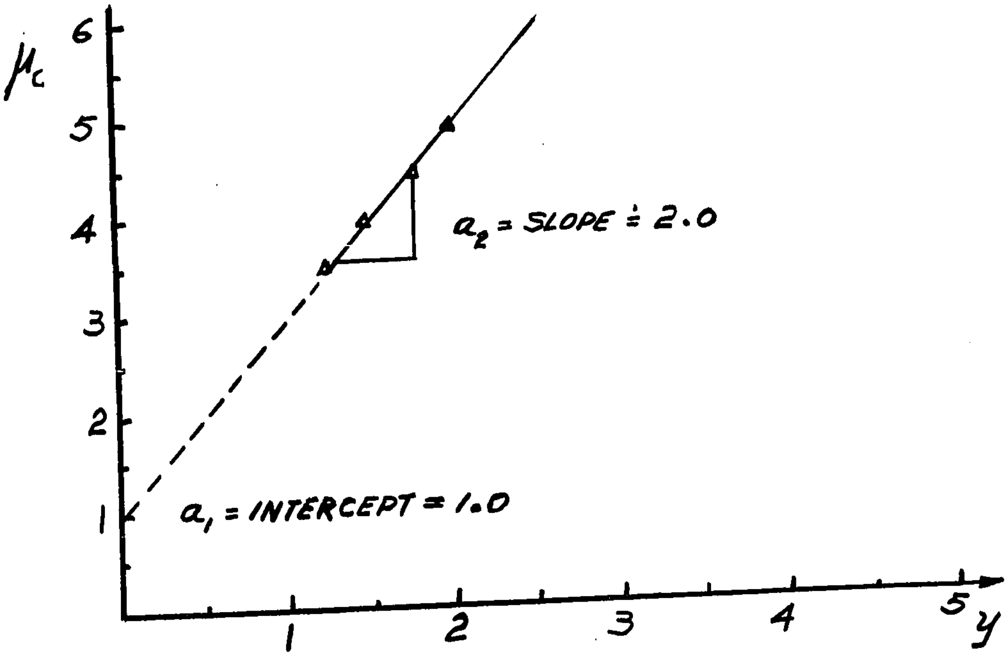

Fig. **27**. Graphical determination of a_1 and a_2 in system $\dot{y} + a_1 y + a_2 y^2 = u(t)$.

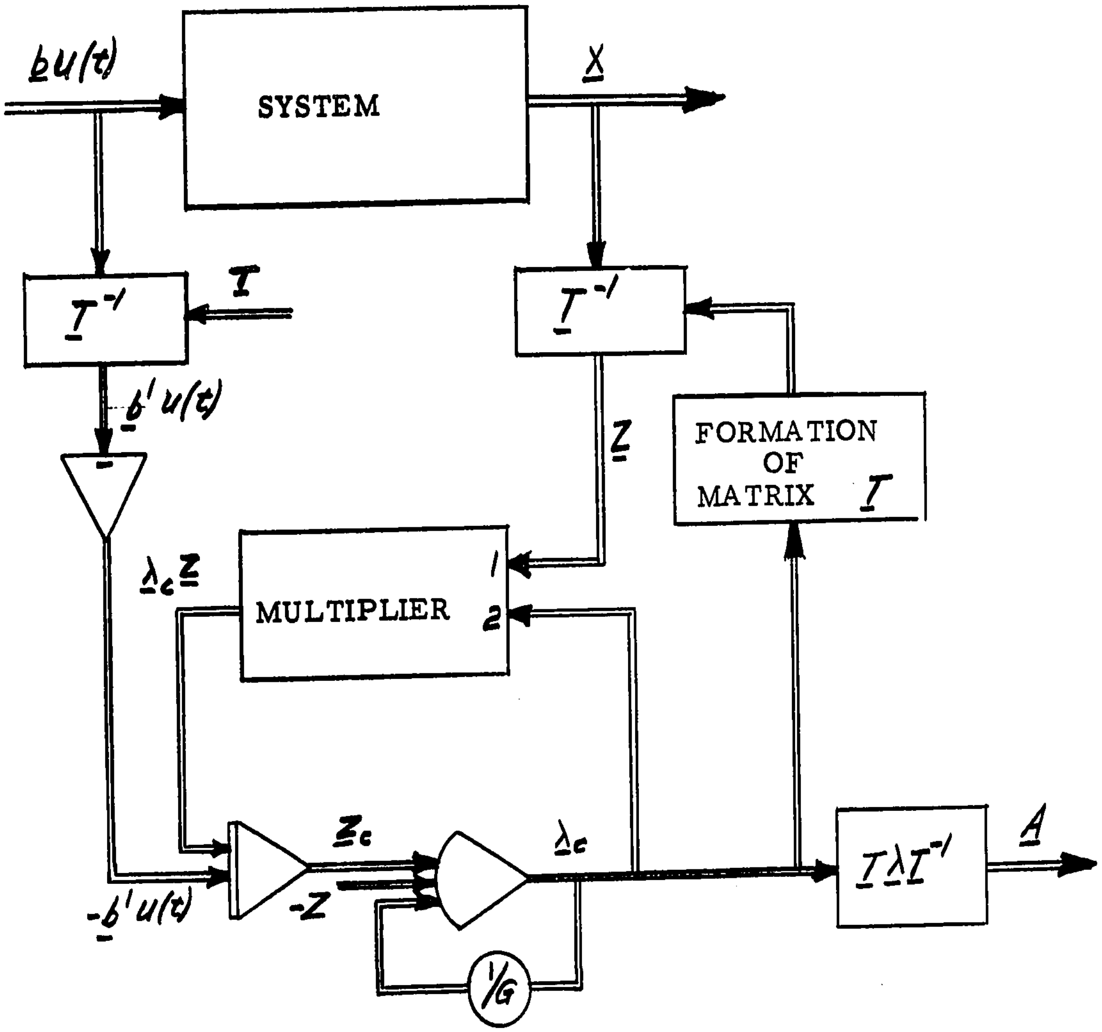

Fig. 28. Implicit synthesis method for high-order system identification.

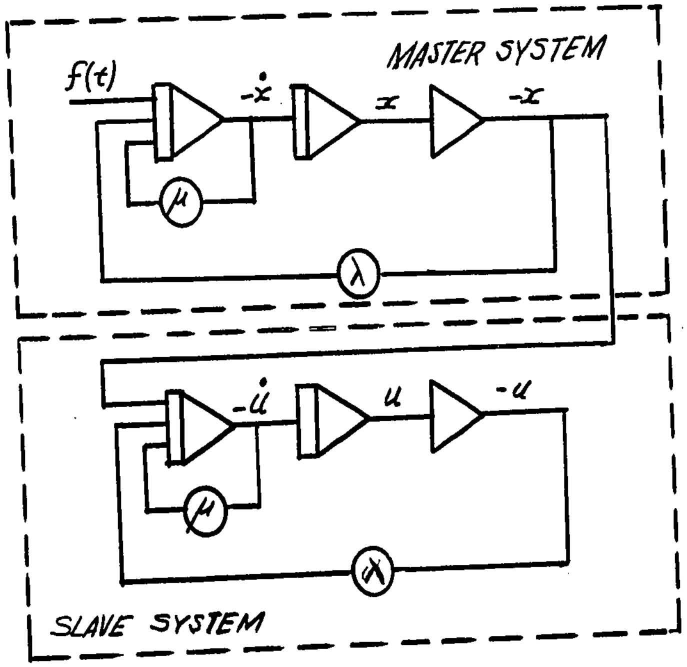

Fig. A-1. Parameter influence coefficient.

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