Experimental super-resolution with prolate spheroidal functions

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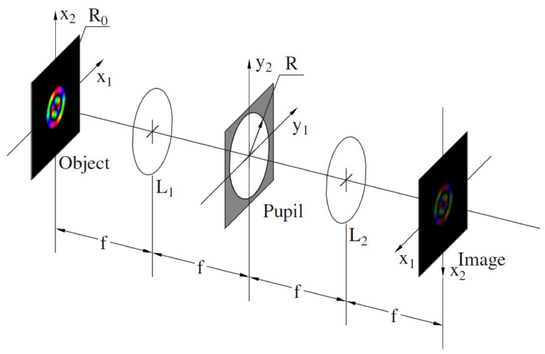
Introduction



Simple figure of a 4-f system.

- Diffraction: Distortion of images passing through an optical system.
- Super-resolution: Class of techniques that resolve images passed the diffraction or Rayleigh limit [1], i.e. reduce the effects of diffraction.
- 4-f system: Basic diffraction limited optical system.

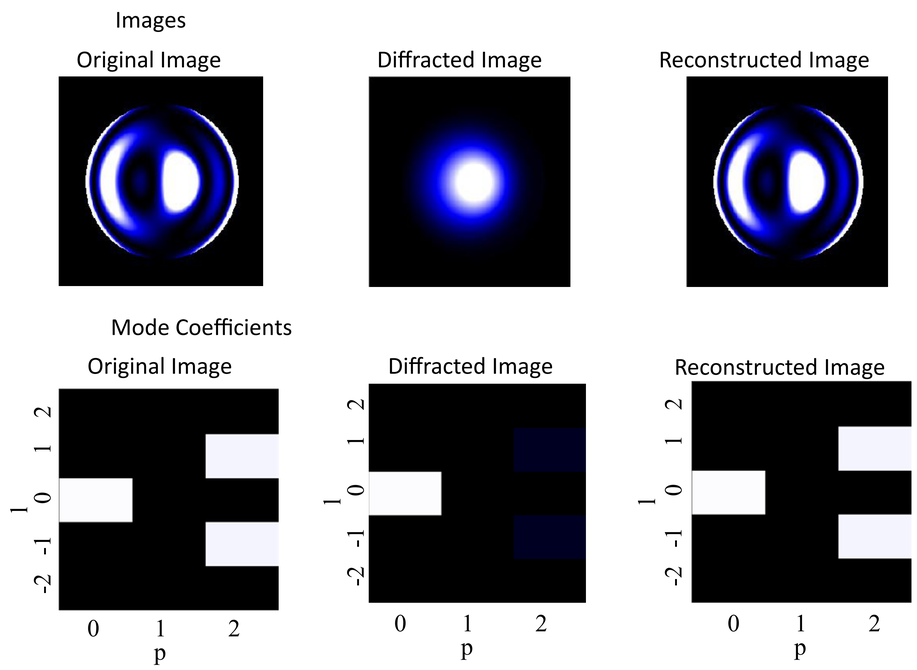
Theory

- Eigenmode: Image that passes through an optical system with a change in intensity, but is otherwise unchanged.
- All images are superpositions of eigenmodes, $\boldsymbol{P_{\ell,p}(r,\theta)}$.
- Super-resolution: Divide the measured coefficients, $\boldsymbol{D_{\ell,p}}$ by the transmittance.

$$\boldsymbol{I(r,\theta)} = \sum_{\ell=-\infty}^{\infty}\sum_{p=0}^{\infty} \boldsymbol{I_{\ell,p}P_{\ell,p}(r,\theta)}$$
$$= \sum_{\ell=-\infty}^{\infty}\sum_{p=0}^{\infty} \frac{\boldsymbol{D_{\ell,p}}}{\boldsymbol{\lambda_{\ell,p}}}\boldsymbol{P_{\ell,p}(r,\theta)}$$

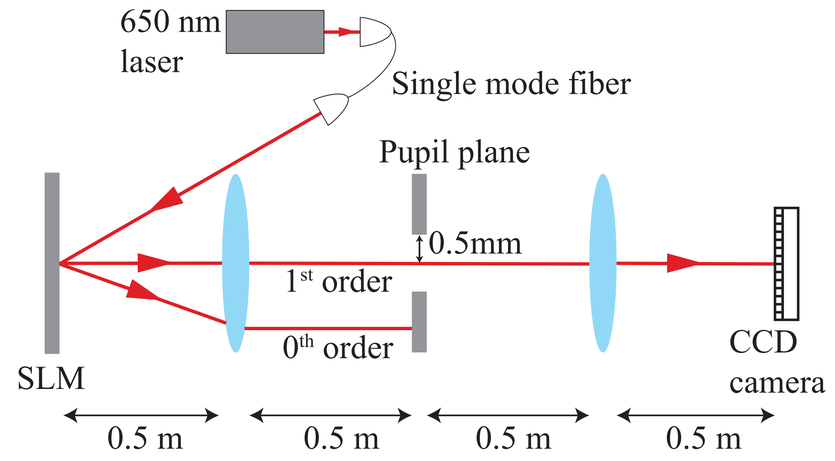
$\boldsymbol{\ell}$ and $\boldsymbol{p}$ are the angular and radial indexes, respectively. The eigenvalue $\boldsymbol{\lambda_{\ell,p}}$ is the transmitance of the respective eigenmode. i.e. the fraction by which the intensity changes.

Simulation



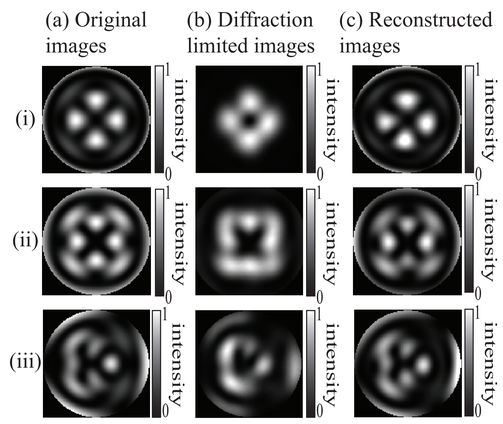
Computer simulations of eigenmode super-resolution.

Experiment

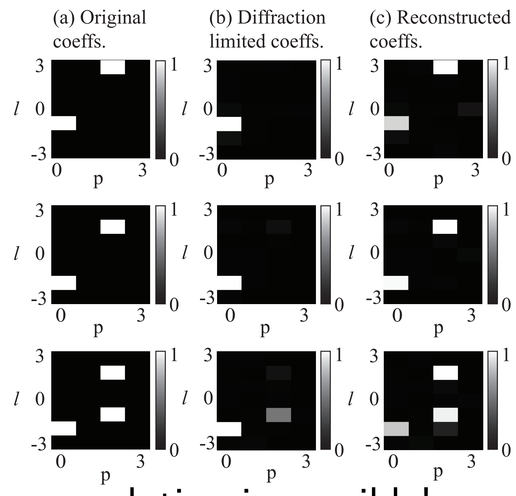


Results

Sample Images



Coefficients



Experiments show that super-resolution is possible!

Conclusions

We have demonstrated that super-resolution is experimentally possible. The next step to this research is to generalise this to more complicated optical systems and ultimately to reach the quantum limit to resolution due to quantum fluctuations [2].

References

[1] J. W. S. Rayleigh, *Collected Optics Papers of Lord Rayleigh*. Optical Society of America, 1994.

[2] V. N. Beskrovny and M. I. Kolobov, "Quantum-statistical analysis of superresolution for optical systems with circular symmetry," *Phys. Rev. A*, vol. 78, p. 043824, Oct 2008.