SIMULATION STUDY OF A SIGNALLING PROTOCOL
FOR MOBILE RADIO SYSTEMS

by

Michael J. McDonnell

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ABSTRACT

An essential component of any trunked, mobile radio system is the signalling protocol used for the purpose of call set-up. Presented herein are computer simulation results of one such protocol designed for use with dedicated signalling channels and incorporated within the Advanced Mobile Phone Service. The latter is a high capacity, cellular-structured mobile radio system currently being implemented by AT&T in Chicago, Illinois.

In particular this thesis examines (i) the signalling channel blocking probability, (ii) the effect of varying system parameters upon this blocking probability and (iii) the mean delays encountered in obtaining a signalling channel. Also of concern is the influence of the signalling protocol upon the subsequent voice channel blocking probability. To this end, a voice channel assignment algorithim is included in the two simulation models which are developed. The first model employs a fixed voice channel assignment scheme and an Erlang B service discipline. In the second model, the signalling protocol is modified slightly, a hybrid voice channel assignment scheme is used, and a Blocked Calls Held queuing discipline is introduced.

Principally, it is determined that if 2 or more automatic reattempts to gain access to the system are permitted, then only one signalling channel per cell is required in order to maintain the signalling channel blocking probability below 2% for all traffic loads likely to be encountered in practice.

ACKNOWLEDGEMENT

The author would like to express his appreciation to Dr. Nicolas D. Georganas -- firstly, for the assistance and advice he provided and secondly, for allowing the author to think and work in such an independent fashion. As a result, the time spent in the M.A.Sc. program has proved most enjoyable and profitable.

Perhaps the greatest debt of all is owed to my parents, Corinne and Thomas, who have worked so very hard all their lives. To them, this thesis is dedicated.

TABLE OF ABBREVIATIONS AND SYMBOLS

The following is a list of abbreviations which appear repeatedly throughout the thesis. A great many other abbreviations are peculiar to the simulation models and are not included here. They may be found in Sections 6.3 and 7.3.

A,B	(i)	Signifies two of the three time-multiplexed data streams transmitted over the paging channel
	(ii)	Refers to the two types of paging channels
ACS		Area Call Sign
AMPS		Advanced Mobile Phone Service
C		Number of duplex radio channels allotted to the entire system
C(acc)		First paging channel to scan when attempting to access the system
C(0),C(h)		First channel in a block of paging channels
CMAX		Total number of signalling channels in the mobile radio system

CPA . . . . Combined Paging/Access bit; used by the mobile unit for determining the first paging channel to scan

D . . . . Geographical separation between co-channel cell sites

DCC . . . . Digital Colour Code

FCAS . . . . Fixed (Voice) Channel Assignment Scheme

FIFO . . . . First-In First-Out

FM . . . . Frequency Modulation

HCAS . . . . Hybrid (Voice) Channel Assignment Scheme

i,j . . . . Shift parameters which uniquely determine the value of N

IF . . . . Intermediate Frequency

MIN . . . . Mobile Identification Number

MSA . . . . Mobile Service Area

MTSO . . . . Mobile Telephone Switching Office

n . . . . Path loss exponent

N . . . . Number of channel sets in entire mobile radio system

R . . . . Cell radius (from centre to any vertex)

RO . . . . Random Order

S . . . . Number of duplex radio channels per channel set (=C/N)

SAT . . . . Supervisory Audio Tone

S/I . . . . Ratio of mean RF signal power to interference power

λ . . . . Mean arrival rate of calls

μ . . . . Mean service rate of calls

CONTENTS

Chapter I

INTRODUCTION

It is the intent of this chapter to first explain the purpose and requirements of a signalling protocol and from there, proceed to a statement of the principal objectives for this investigation. The last section is devoted to an outline of the remaining chapters.

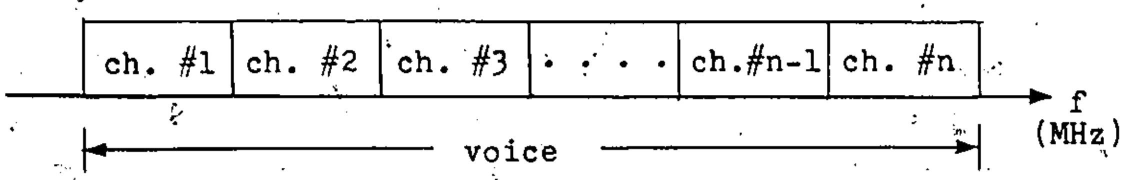
1.1 BACKGROUND

If a mobile customer is operating in some defined service area and he wishes to place a call, he must first of all make his presence known to the controlling system. That is, he must supply certain basic information such as his own identity and the dialed digits of the destination which he is attempting to reach. One function of a signalling protocol is simply to permit the exchange of this preliminary data which is necessary for the purpose of call set-up.

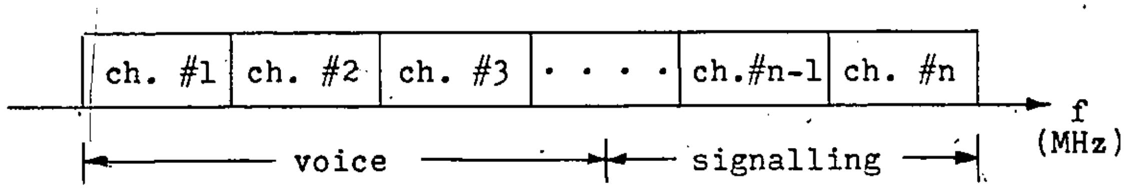
Now a problem arises because mobile subscribers will naturally attempt to place calls at random points in time. If two or more requests for service are transmitted sufficiently close in time over the same radio channel, they will overlap and be indistinguishable to the central controller of the system. Under such conditions a collision is said to have occurred. Thus an additional function of a signalling protocol is to minimize the frequency of occurrence of these collisions.

Basically, there are two types of signalling schemes: (i) the dedicated signalling channel method and (ii) the non-dedicated or idle tone method. In the latter scheme, an idle tone is transmitted over any vacant radio channel so as to indicate the availability of the channel -- first for signalling and subsequently for voice communication. For the purpose of explanation consider that all the channels available for communication form a solid block in the frequency spectrum as shown in Figure 1.1(a). Upon recognition of a mobile subscriber's request for service, the mobile's logic unit will begin scanning the channels sequentially, commencing with channel #1. The scan will continue and be repeated as many times as necessary until a free channel is found as indicated by the presence of an idle tone. At this point the mobile will submit it's call request. The central controller, upon reception of the call request, will remove the idle tone and conversation can begin. Once the conversation is finished the idle tone is reinstated.

Alternatively, dedicated signalling channels may be used as illustrated in Figure 1.1(b). As the name implies, channels are relegated to either the signalling function or voice communication. All the information necessary for control of the system is sent over a limited number of channels which are never used for speech transmission. When a service request is received over a signalling channel, the central controller will relay a voice channel assignment back to the mobile, again over a signalling channel. The mobile then tunes to the voice channel and conversation can begin. Termination of the call is noted by the controller, allowing the voice channel to be used for any new service requests.



(a) Non-dedicated Signalling Scheme



(b) Dedicated Signalling Scheme

Figure 1.1 Signalling Techniques

Both of these signalling methods have drawbacks. Firstly, for the idle tone method, much time may be wasted while scanning and this delay may become quite lengthy during periods of congestion.

Secondly, when an idle tone is used, the service received by the customer will be in random order for consider the case where all channels are occupied at a particular instant. There may be many mobile units scanning about seeking an idle tone but, when a channel does become free, only the first mobile to lock onto that channel and signal will obtain the channel although other mobiles may have been waiting for a much longer period of time. Thus, service will be in random order (RO). In the dedicated scheme, on the other hand, even if all the voice channels are busy, call requests can still be transmitted and queued centrally in a First-In First-Out (FIFO) fashion.

In comparing RO and FIFO service disciplines it is found that the *average* delay experienced by customers before obtaining a channel is the same [31]. This may be explained roughly by saying that the average delay depends only on the average rate of arrival of customers and the average rate at which they are served. However, both the variance of the delay and the probability of long delays are greater in the case of random service. Also, service in order of arrival has the added benefit that it is more equitable to the user insofar as his delay is entirely independent of all who arrive after him.

Thirdly, the idle tone method imposes restrictions on the volume of signalling data which may be transferred since it is imperative to minimize the amount of time the channels are engaged in signalling. In a dedicated scheme -- although it is important to keep the signalling messages as short as possible -- the problem is not as severe.

Finally, there is the question of reliability. A mobile must operate in an extremely hostile and complex interference environment. Signals undergo sudden and violent fluctuations in amplitude and phase as a result of multipath interference. Thus an idle tone could be temporarily lost or swamped by an interfering signal.

Despite its many disadvantages, the idle tone method is simple and inexpensive to implement and remains suitable for geographically small systems, using few channels, and served by a single, centrally located transmitter. A cellular mobile radio system, on the other hand, with hundreds of channels and a need for more sophisticated control, virtually demands the use of dedicated signalling channels. High redundancy and error detection and correction techniques can be used to combat the problem of multipath fading and to improve reliability. Many new and diverse functions can be implemented, including:

1. Automatic call handofr (changing channels as the mobile moves from one cell to an adjacent cell).

2. Automatic, mobile transmitter, power-level adjustment (upon command from the central controller).
3. Informing the mobile of the value of local system parameters; for example, the total number of signalling channels in the system. This feature enables a mobile operating outside its assigned Mobile Servie Area (MSA) to receive service.

This list is only a small sampling of the many commands and the sort of information which can be relayed to the mobile from the central controller.

Perhaps the sole disadvantage of dedicated control channels is that they subtract from the total reserve of channels available in the service area, leaving fewer channels for voice communication. Hence a question of paramount importance is the following: What is the minimum number of signalling channels required in order that no more than a small fraction (say 2%) of the calls offered to the system are blocked due to the unavailability of these channels? Any more than this minimum number would be wasteful in terms of spectrum utilization since generally the traffic load carried by the signalling channels is signifigantly less than is the case for the voice channels.

1.2 OBJECTIVES

The question formulated in the above paragraph constituted the major impetus in undertaking the research documented in this thesis. Of secondary concern was the impact of the signalling protocol upon the subsequent voice channel blocking probability. To this end, voice channel assignment logic was included in the simulation models. Parenthetically, this represents a rather novel approach since to date no known attempts have been made to simulate a complete system including both a signalling protocol and a voice channel assignment algorithm. Instead the investigations which have been carried out in this field [3,8-11,18,30] have concentrated primarily on the problem of voice channel assignment -- either assuming zero blocking over the signalling channels or choosing to ignore the signalling channel requirement altogether.

1.3 SCOPE OF THESIS

The Advanced Mobile Phone Service (AMPS) proposed by AT&T forms the *framework* for the development of the simulation models [42,44]. Any extensions or modifications to the AMPS protocol are clearly stated. The AMPS control plan calls for the use of dedicated signalling channels and consequently this is the only type of signalling technique considered here.

Chapter 2 provides a description of the control elements of a cellular-structured mobile radio system. Basic concepts and properties of such systems are introduced.

Use of the signalling channels is explained in some depth in Chapter 3. The last section of this chapter is concerned with the assignment of signalling channels in the frequency spectrum and the changes made to this assignment as a cellular mobile radio system matures and expands.

Chapter 4 describes how a mobile unit goes about finding a suitable signalling channel to use in the first place. The methods devised to minimize collisions and combat system disruption and overload are examined. Also included are scenarios for mobile-originated and mobile-completed call attempts.

The simulated system is presented in Chapter 5 along with the simplifying assumptions which were made during the course of model development.

The first of two simulation models is presented in Chapter 6. This model includes a Fixed Voice Channel Assignment Scheme and an Erlang B service discipline for the voice channels. (Blocked calls are cleared from the system and no further attempt is made to serve them).

The second model is detailed in Chapter 7. The signalling protocol is modified slightly, a Hybrid Voice Channel Assignment Scheme is used and a Blocked Calls Held service discipline for the voice channels is introduced. In this case callers who have signalled their desire to make a call are prepared to wait a short period of time for a voice channel designation. But if this time period expires, and the caller has not yet obtained a voice channel, then as before, his call attempt is blocked and no further attempts are made to serve him.

Simulation results and analysis of the separate models are included at the end of their respective chapters. In addition, Chapter 7 contains a selective comparison of the two sets of results. Chapter 8 is reserved for conclusions.

Finally, for those unfamiliar with queuing theory, a compendium of basic concepts can be found in Appendix A. Included is a brief, self-contained development of the various traffic formulas:

1. Erlang B (Blocked Calls Cleared)
2. Erlang C (Blocked Calls Delayed)
3. Blocked Calls Held,

and an introduction to telephone trunking.

Appendix B gives a listing of the simulation programs and an explanation of how they work.

Chapter II

COMPONENTS OF A CELLULAR MOBILE RADIO SYSTEM

The description given in this chapter and the terminology which is introduced are representative of any cellular-structured mobile system. Specific values of various parameters however, are taken from one particular system, namely the Advanced Mobile Phone Service [42,44]. The following discussion can perhaps be summarized by two key phrases: frequency reuse and cell splitting.

2.1 CONTROL ELEMENTS

The cellular concept entails dividing a large service area into smaller units called <u>cells</u> in order to handle localized radio traffic needs [16,20,42]. An orderly, methodical approach to the design and layout of such a system is greatly aided if all cells are visualized as being identical in shape. Invariably the shape which is chosen is a hexagon. This selection is motivated by the practical consideration that a hexagon closely approximates the circular coverage zone one would expect with omnidirectional antennas. Also, from the point of view of theory, a regular array of hexagonal cells does not produce any gaps or overlapping areas as would be the case with circular cells. A typical hexagonal pattern is illustrated in Figure 2.1.

Q_k : k'th cell using channel set Q

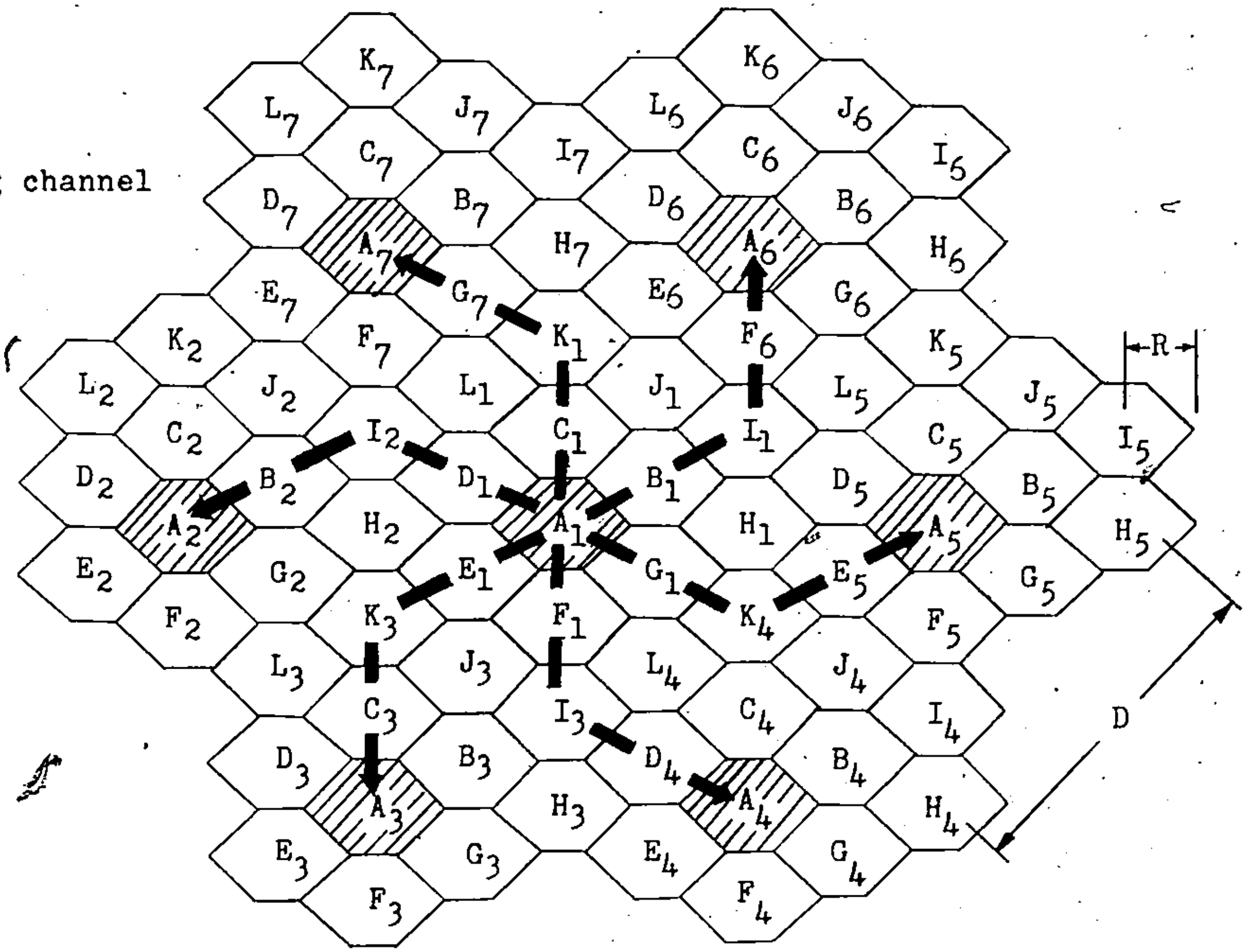


Fig. 2.1 Cellular Layout Illustrating Frequency Reuse Concept

Each cell is equipped with its own land-based transmitter- receiver station (termed a <u>cell site</u> in AMPS) and is assigned a distinct set of channels for communication solely with mobiles within the confines of its own borders. Also, there is a central controller called the Mobile Telephone Switching Office (MTSO) which is linked to each cell site by landline facilities. The responsibilities of the MTSO include:

1. Administering radio channel assignments.
2. Analyzing mobile location and signal strength data collected from the cell sites to determine, for example, if the mobile unit should boost or lower its transmitter power or if the mobile should switch to a new channel.
3. Assessment of charges.
4. Cell site fault recognition, reconfiguration of active and standby units to achieve a wholly functional system, and diagnostic tests designed to isolate probable failed units.
5. Serving as an interface between the mobile system and the conventional landline network. Since each mobile is given an ordinary ten-digit telephone number (composed of a 3-digit area code plus a 7-digit directory number), standard trunking and routing techniques can be employed.

As the above discussion indicates, there are three major control elements in a cellular mobile radio system, namely the MTSO, the cell site, and the mobile unit. The interconnection of these elements is shown in Figure 2.2.

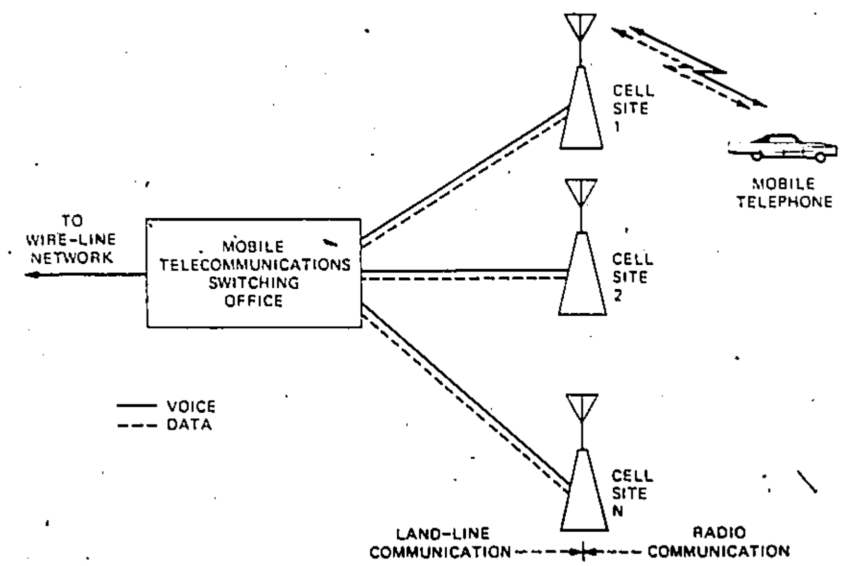


Figure 2.2 Control Elements in A Cellular Mobile Radio System [42]

2.2 FREQUENCY REUSE

High capacity is achieved through frequency reuse, i.e. cells sufficiently far apart so that co-channel interference does not pose a problem may use the same channel set. In this regard, a commonly employed figure of merit is the co-channel reuse ratio, D/R. Here "D" represents the physical distance between co-channel cell sites and "R", the cell radius (measured from the center to any vertex) as shown in

Figure 2.1. Once an acceptable signal-to-co-channel interference ratio has been determined (generally through a subjective testing program), the approximate value of D/R is automatically specified. This statement is explained with the aid of Figure 2.3.

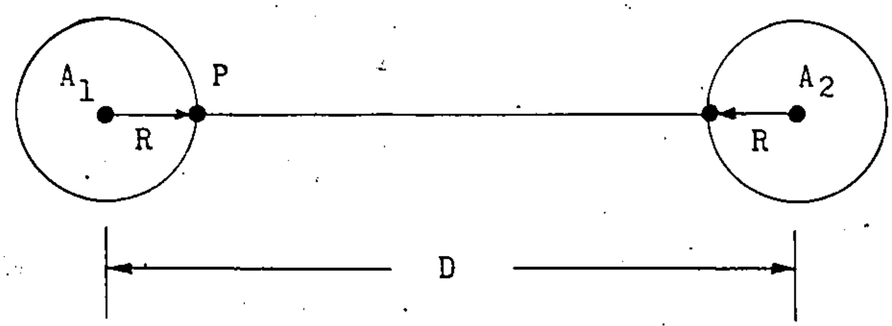


Figure 2.3 Frequency Reuse by Two Cell Sites Separated by a Distance "D".

In this diagram two isolated cell sites, labelled A_1 and A_2, are shown. They are separated by a distance D and are using the same channel set, A. Assume that the cell sites are equipped with omnidirectional transmitting and receiving antennas so that the coverage areas are roughly circular. A mobile is located at point P on the perimeter of the coverage area associated with cell site A_1. Furthermore, P has been purposely selected so as to lie between A_1 and A_2 and on, or near, the line connecting A_1 and A_2. This is obviously a worst-case situation since the mobile is not only located at the greatest distance possible from the serving cell site A_1 but is also located at the perigeal point with respect to the interfering (co-channel) cell site A_2. The mean signal power $\bar{S}$ received at point P from A_1 is proportional to R^{-n} where R is the distance from A_1 to P and n is the path loss exponent [7,20]. Likewise, the mean interference power $\bar{I}$ received at P from A_2 is proportional to $(D-R)^{-n}$. Thus:

$$\frac{\bar{S}}{\bar{I}} = \frac{R^{-n}}{(D-R)^{-n}} = \left[\frac{D}{R} - 1\right]^n \qquad (2.1(a))$$

This formula is only applicable in the case where A_1 has a single co-channel neighbour. In the general case where there are M co-channel cell sites located a distance D from A_1 and all received signals are independent at point P,[1] then:

$$\frac{\bar{S}}{\bar{I}} = \frac{1}{M}\left[\frac{D}{R} - 1\right]^n \qquad (2.1(b))$$

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[1] The assumption of independence is necessary so that the mean powers of the interfering signals received at A_1 are additive.

Figure 2.1 clearly shows that in an arrangement of hexagonal cells, A_1 will possess 6 co-channel neighbours A_2, $A_3 \ldots A_7$. Thus 2.1(b) becomes:

$$\frac{\bar{S}}{\bar{I}} = \frac{1}{6}\left[\frac{D}{R} - 1\right]^n \qquad (2.1(c))$$

Solving for D/R:

$$\frac{D}{R} = \left[6\,\frac{\bar{S}}{\bar{I}}\right]^{1/n} + 1 \qquad (2.1(d))$$

In AMPS, the RF signal-to-interference level which gives a sound quality comparable to ordinary nonradio telephone service is 18 dB = $10\log_{10}(\bar{S}/\bar{I})$. A typical value of n derived from actual measurements in an urban environment is 4 [7,20,42,45]. Using these figures, D/R is calculated to be 5.4.

Although the S/I requirement is the principal determinant of a suitable co-channel reuse ratio there are, of course, other factors. In addition to good transmission quality, cost and ultimate system capacity play important roles. In many instances these three factors form an opposing set of goals. The performance trade-offs, and the manner in which they influence the choice of D/R, are summarized below.

1. Minimizing D/R:

small D ==> a smaller number of channel sets required (refer to Equation (2.2) in Section 2.3). Since the total number of channels allocated to the system is fixed, the smaller number of sets implies more channels per set and per cell site. Each site can carry a higher traffic load.

large R ==> fewer cells are needed to span a given geographical area. The total number of cell sites is reduced and so is the cost associated with these installations.

Hence, ensuring that D/R is as small as possible helps to meet the objectives of low cost and large capacity.

2. Maximizing D/R:

large D ==> a larger separation between cochannel sites and consequently, lower co-channel interference levels.

small R ==> the mobile will be much closer to the serving cell site resulting in improved reception.

Hence, making D/R as large as possible benefits transmission quality.

The system designers of AMPS have arrived at a compromise amongst these objectives which calls for a co-channel reuse ratio of 6 during the early stages of system growth when omnidirectional antennas are used. In successive stages of development, each cell site will have 3 faces; each face serving a sector delineated by the main lobe of a 120° directional antenna as shown in Figure 2.4. This permits the co-channel reuse constraint to be relaxed to 4.6. The reason being that a 120° directional *transmitting* antenna can provide the same power level in the region that it serves as an omnidirectional antenna while at the same time introducing considerably less interference in co-channel cells which lie outside the 120° wedge of the main lobe. Similarly, a 120° directional *receiving* antenna will greatly attenuate interference from mobiles in co-channel cells which lie outside the 120° wedge.

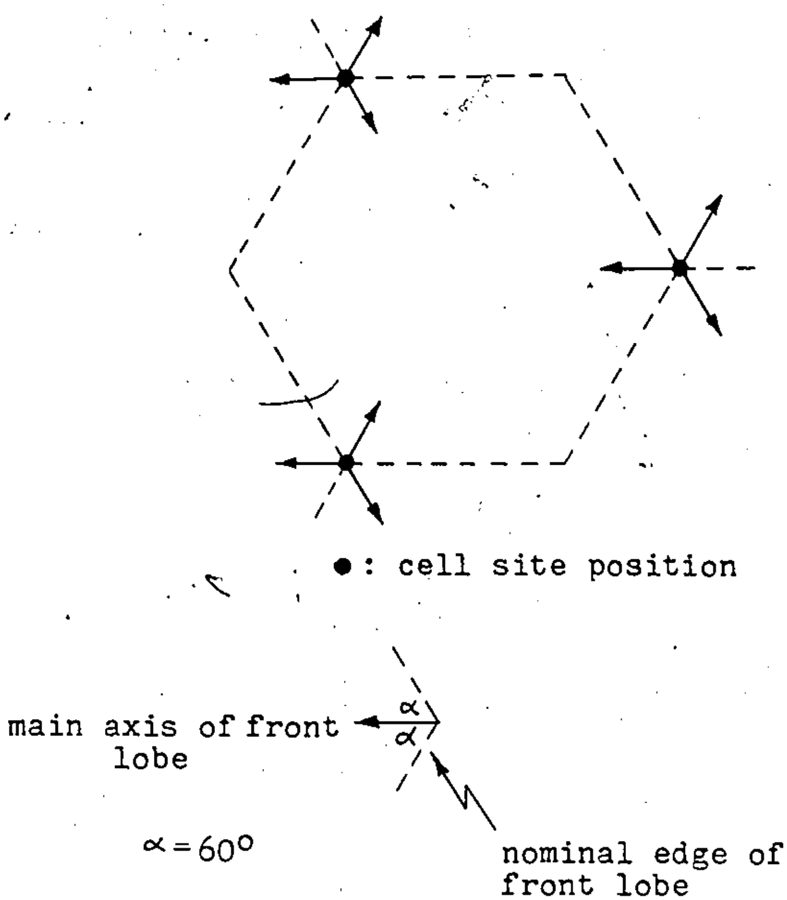


Figure 2.4 Orientation of Directional Antennas at Directional Cell Sites in AMPS [42]

2.3 CHANNEL SET DEPLOYMENT

Figure 2.1 illustrates a situation in which the number of disjoint channel sets, N, is equal to 12. These channel sets have been labelled A through L. The question now arises: Given "D/R" can "N" be computed? The answer is yes and the relationship is in fact quite simple. For an hexagonal cellular layout [42]:

$$N = \frac{1}{3}\left[\frac{D}{R}\right]^2 \tag{2.2}$$

Because of the nature of the geometry involved only certain values of N are realizable as given by:

$$N = i^2 + ij + j^2 \tag{2.3}$$

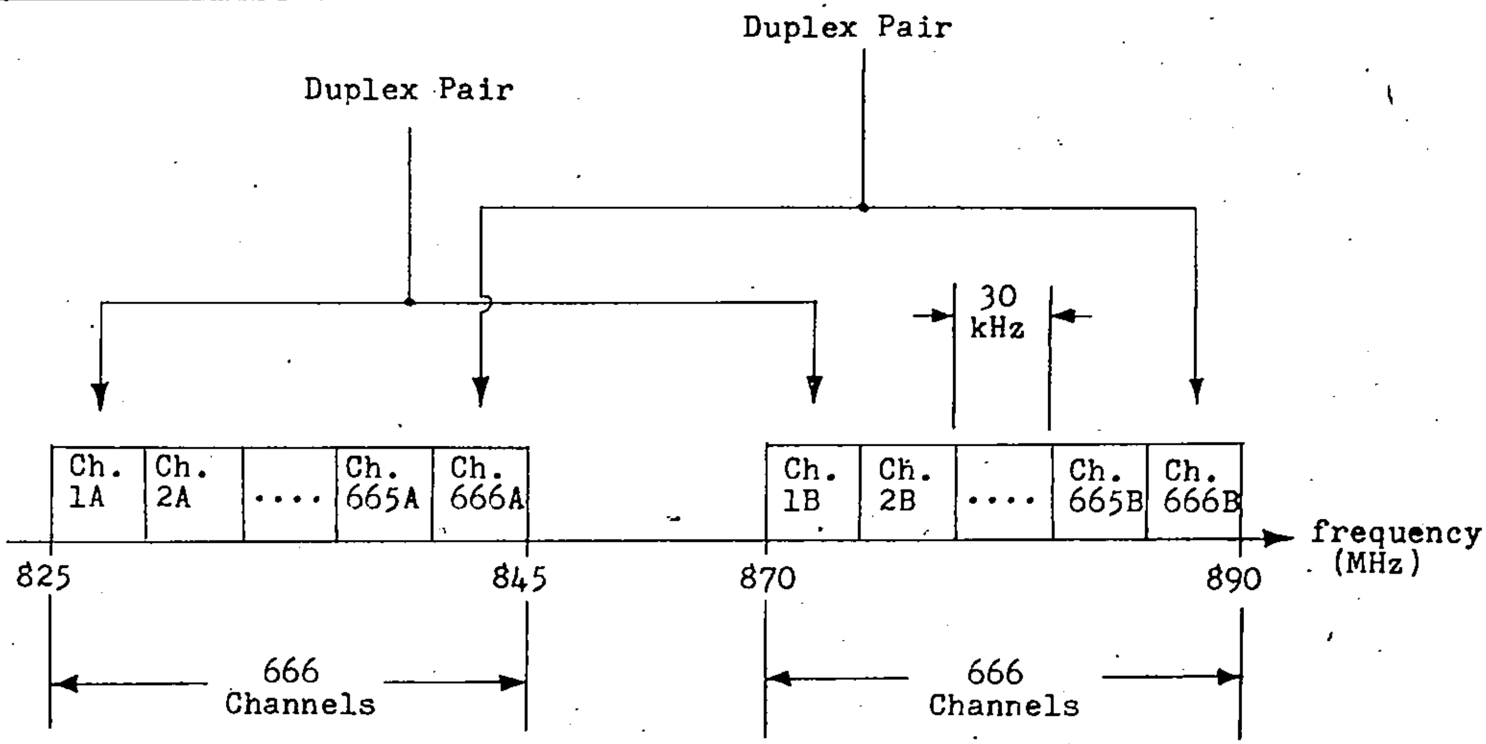
$$i = 1,2,3\ldots$$
$$j = 1,2,3\ldots$$

The cochannel reuse ratios of 6 and 4.6 cited in Section 2.2 correspond, by Equation (2.2), to 12 channel sets for omnidirectional sites and 7 channel sets for directional sites respectively. (Actually, in the latter case, each channel set is further subdivided into 3 subsets to serve the 3 separate faces of the directional site. It is therfore more accurate to speak of 21 channel subsets).

Figure 2.1 also demonstrates a method of determining the positions of the nearest co-channel neighbours of a cell. This method is based on the values chosen for i and j in (2.3). Starting with any cell as a reference -- here we have selected A_1 -- proceed outward i cells along any one of the six "chains" of hexagons emanating from A_1; turn counterclockwise 60°; now move j cells along the chain which lies on the new bearing. The j'th cell and the reference cell will be co-channel neighbours. Co-channel cells could also have been located by first moving j cells, turning, and then moving i cells or by turning clockwise 60° rather than counterclockwise. In this example i=j=2 and from (2.3), N=12.

If the total allocation of C duplex channels is now divided into N disjoint sets, each set will contain S = C/N duplex channels. The majority of these S channels are used for voice communication. A few channels in every set however, are set aside to perform the signalling function; they are used expressly for exchanging the information needed to establish and maintain a complete voice communication path.

In AMPS, duplex operation is achieved by dividing the RF band into two segments as shown in Figure 2.5. The 870 to 890 MHz range constitutes the "mobile receive"/"cell site transmit" range of frequencies while the 825 to 845 MHz band is used for the reciprocal operations of mobile transmission and cell site reception.



"A" denotes reverse (mobile-to-cell site) half of duplex channel

"B" denotes forward (cell site-to-mobile) half of duplex channel

Fig. 2.5 Duplex Communication in AMPS System

The individual channel bandwidth is 30 KHz.[2] Thus 666 duplex channels can be created out of the 40 MHz spectrum allocation. This, in turn, leads to approximately 55 channels per set for omnidirectional sites and 33 channels per subset for directional sites.

These calculations preclude any need for a spectral guard band which would normally be required to permit IF filters to reject adjacent channel interference adequately. Such a guard band would greatly reduce the number of available channels. Fortunately, in a cellular system, only a small fraction of the total allotment of channels is assigned to any given cell. Hence, with a suitable system design, it is possible to avoid the use of adjacent channels in the same cell. In fact a secondary source of interference can often be eliminated simply by prohibiting the assignment of adjacent channels to physically adjacent cells. Since there are no exigent demands imposed upon the receiver IF filter, no guard band is needed.

Every cell site is equipped with one transceiver for each voice or setup channel assigned to it. In addition, signal-level monitoring equipment attached to the transceivers is used to determine when a <u>handoff</u> is in order. A handoff is simply the transferral of a call-in-progress from one channel to another channel. This action is occasionally necessary during the course of a conversation since a mobile may

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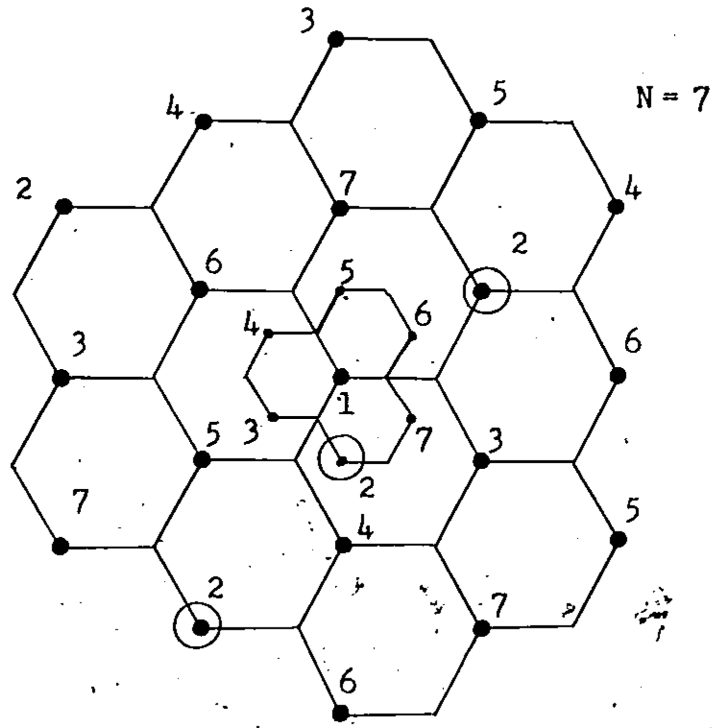
[2] FM with a peak frequency deviation of 12 KHz is employed.

move outside the sphere of influence of one cell and into another. Because disjoint channel sets are used in adjacent cells, the conversation can not continue on the same channel. A handoff to an appropiate channel in the new cell must be performed.

2.4 CELL SPLITTING

As telephone traffic demands increase, cell splitting is implemented; cells are subdivided into progressively smaller zones *without* augmenting the radio spectrum [42,46]. In the AMPS system, extra cell sites needed to mitigate excessive traffic demands on existing sites will be installed midway between adjacent old sites as shown in Figure 2.6. This simple expedient cuts the cell radius in half and reduces the cell area by a factor of 4. The system's traffic-carrying capacity is then increased by approximately the same factor of 4 since each cell site only needs to handle the traffic load in a greatly reduced geographical area.

In order to meet the requirement that the RF S/I ratio be 18 dB or higher over 90% of the service area [42], the AMPS designers have imposed an upper limit of 8 miles upon the cell radius (for a typical urban environment). A practical lower limit is considered to be 1 mile, making three stages of cell splitting feasible (from 8 to 4, from 4 to 2 and



Previously existing or "old" cell site

"New" cell site installed during cell splitting

Figure 2.6 Channel Set Deployment During Cell Splitting [42] lastly from 2 to 1 miles). There are no technical constraints preventing the use of smaller cells but, as the cell size is reduced, hand-offs become more and more frequent and the burden imposed upon the system's processing capacity may become prohibitively large.

Besides an increase in the system's traffic-handling capacity, there is a second benefit which accrues from cell splitting. This is the intrinsic capability to match cell size to traffic demand. Lower-demand regions (such as the suburbs of a city) can be served by larger cells while at the same time, higher-demand areas (such as the city core) can be served by smaller cells.

When cells of different sizes coexist in the same region, care must be taken to preserve the correct minimum separation D between cell sites which are assigned the same channel sets. As explained in Section 2.2, a certain co-channel reuse ratio D/R must be maintained throughout the system in order to ensure an acceptable signal-to-interference ratio, but when multiple cell sizes are used, the radius R will assume different values in different parts of the service area. In particular, near the transition region between cells of two different sizes, the applicable value for R may not be apparent.

This dilemma is best illustrated with a concrete example. In Figure 2.7, a system is shown in which D/R is 4.6 and the

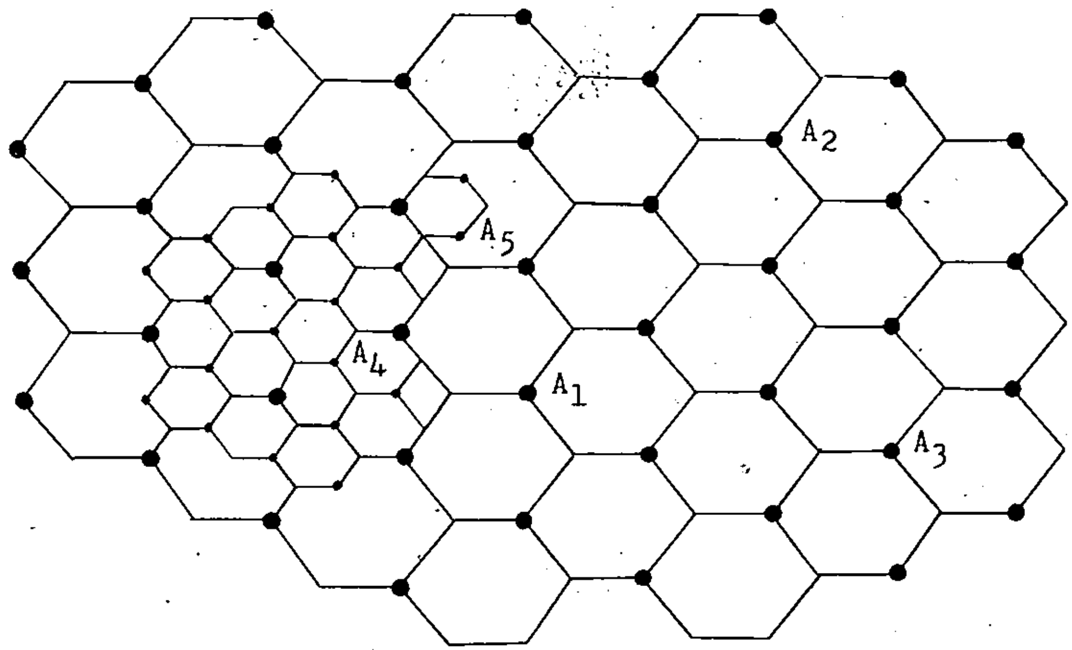


Figure 2.7 Illustration of Mixed Cell Sizes [42]

number of channel sets N is 7. $A_1, A_2, \ldots A_5$ are co-channel cell sites. Within the group of large cells, A_1, A_2 and A_3 are all separated by 4.6 *larger*-cell radii and within the group of small cells, A_4 and A_5 are separated by 4.6 *smaller*-cell radii. Thus, for these cases, the D/R constraint is satisfied. However, when considering sites on opposite sides of the transition region, such as A_4 and A_1 or A_5 and A_1, some ambiguity exists as to the appropiate choice for R. This uncertainty creates a very real problem. Say two mobiles, No. 1 and No. 4, are both using the *same* voice channel at the *same* time. Mobile 1 is being served by site A_1 and mobile 4 is being served by site A_4. Since mobile 4 lies within a region of smaller cells, it will naturally tend to be much closer to its serving cell site A_4 than mobile 1 will be to its serving cell site A_1. Correspondingly, the signal received by mobile 4 from A_4 will probably be much stronger than the signal received by mobile 1 from A_1. For this reason, mobile 4 should not experience any apparent degradation in sound quality due to its close proximity to the interfering cell site A_1. In fact, it is evident from Figure 2.7 that the cochannel reuse ratio is indeed satisfied for A_1 and A_4 *if* the smaller-cell radius is applied. Mobile 1, however, will probably experience a noticeable degradation is sound quality since interfering cell site A_4 is closer than the minimum permitted distance of 4.6 larger-cell radii.

The problems caused by a discontinuity in cell size are resolved by further partitioning the channel subsets assigned to any cell site face, lying in a larger-cell group and bordering upon the transition region, into two subsets. One subset of channels will continue to serve the large cell; the other subset will be restricted to smaller-cell use. (Whether the smaller-cell subset or the larger-cell subset is used to serve a mobile will depend upon the proximity of the mobile to the cell site as estimated by the MTSO on the basis of signal strength data). For example, in Figure 2.7, any channel installed in site A_4 or A_5 must be confined to small-cell use in A_1. As the telephone traffic load grows in the smaller cells, reassignment of more and more channels from the larger-cell subset to the smaller-cell subset will follow. Eventually the large cell will "disappear" entirely, to be replaced by smaller cells.

Chapter III

THE SIGNALLING CHANNELS

The signalling (or setup) channels can be subdivided into two groups. The terms paging and access have been coined to describe those signalling channels used in the forward (cell site-to-mobile) direction and reverse (mobile-to-cell site) direction respectively [13,30,34,44,46]. These two types of channels from a duplex pair, allowing information to be exchanged back and forth between the mobile and the cell site.

In this chapter, the purpose of the paging and access channels is explained and the data formats devised for AMPS are presented. The last section deals with the assignment of the signalling channels in the frequency spectrum and the manner in which this assignment is modified as the system grows and develops.

3.1 PAGING

When a call, directed toward a mobile, arrives at the MTSO from the land portion of the telephone network (or perhaps from another mobile), a paging message containing the mobile's identification is sent to all cell sites. Each cell site, in turn, broadcasts the message over the paging channel assigned to that cell. In this manner the page is made available to the entire service area. All mobiles not actively engaged in a call synchronize to and monitor the paging channel for the cell they are currently in.[3] Thus the mobile -- quite independent of its position -- should be able to respond via the nearest cell site.

In addition to its chief function of relaying the page per se to the mobile, the paging channel is also used for sending (i) voice channel assignments, (ii) descriptive information peculiar to the local system and (iii) a variety of commands which may be lumped together under the banner of "system control". Examples of the latter include commands to adjust the mobile power level, rescan the dedicated paging channels, or retransmit the dialed digits.

In connection with point (ii) an <u>overhead</u> <u>word</u> is transmitted periodically. The overhead word contains [44]:

1. An Area Call Sign (ACS) that uniquely identifies the mobile radio system.
2. The Digital Colour Code (DCC). There are three, distinct two-bit DCCs used for supervision purposes and which also serve to identify the cell site. When a mobile transmits a message over an access channel, a precursor containing the DCC is included; the appropiate DCC having been previously extracted from the

[3] The mobile unit is preprogrammed to find the proper paging channel by sampling the received signal strength on all channels reserved for the paging function. The channel with the strongest signal should be associated with the nearest cell site.

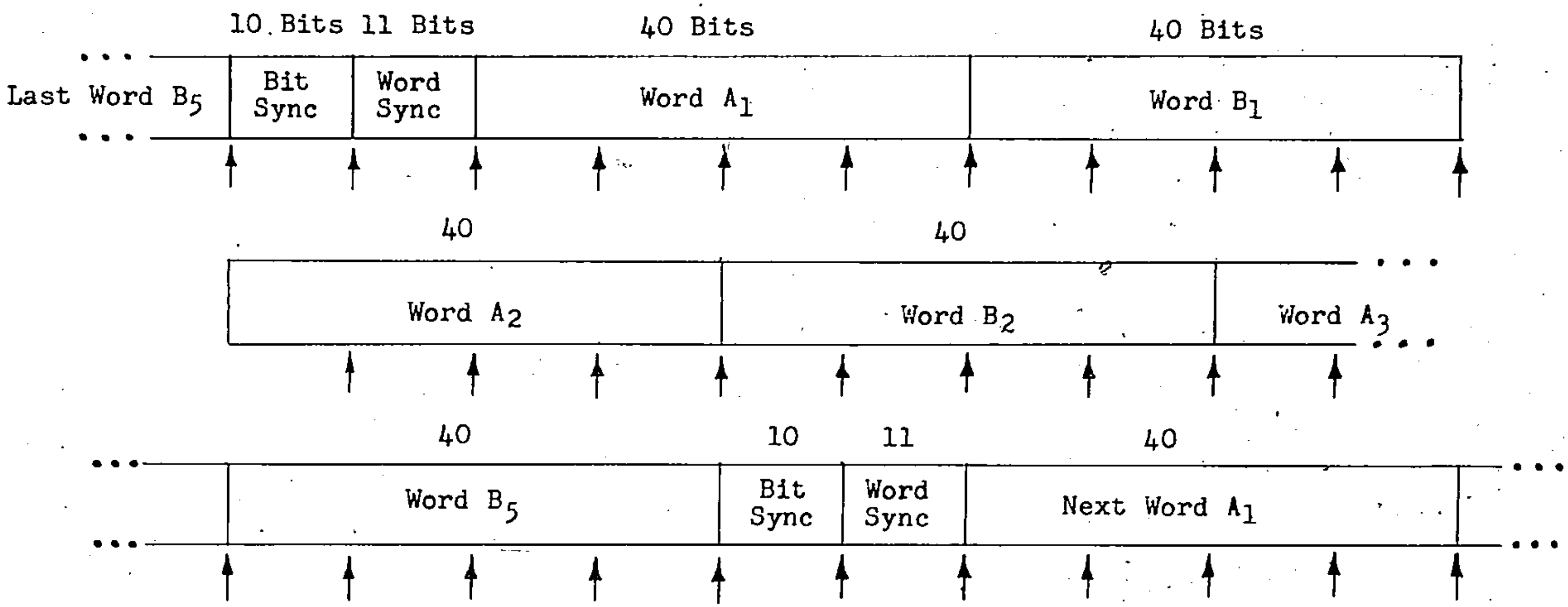
paging message. The reason for this action is illustrated by the following example. In a mature cellular system the cell radii are 1 mile and the co-channel reuse ratio is 4.7 so potential co-channel interferers are less than 5 miles away -- well within the mobile unit's transmission range of 5 to 10 miles. In the absence of a DCC, it is possible that a signalling message will be misdirected, i.e. received at a cell site other than the nearest. With a DCC, however, cell sites using the same signalling channel *and* the same DCC are much farther apart than is the case for cell sites using the same signalling channel alone. A cell site looks for the specific DCC it sent to be returned; if some other DCC is received, the incoming message is ignored.

3. The frequency reuse factor N which represents the number of channel sets.
4. A parameter called CMAX which specifies the total number of setup channels in the area that the mobile unit may use to access the land system.
5. A parameter called CPA which is used by the mobile unit in determining the first setup channel to scan when attempting to access the land system.

Finally, when there is no page to be transmitted, the cell site inserts filler text (1010...) in its place in order to preserve the synchronous nature of the cell site-to-mobile data stream.

The data format for the paging channels in AMPS is shown in Figure 3.1. In essence there are three entirely separate, time-multiplexed information streams termed the A, B and busy/idle streams. The mobile will monitor *either* stream A *or* stream B but not both, depending upon the value of the least signifigant bit of its Mobile Identification Number (MIN). Each word of the A or B stream is repeated five times. The mobile logic unit will record the five repeats of a word and perform a bit-by-bit,3-out-of-5 majority vote. The interleaving of the A and B streams which is evident in Figure 3.1 helps to ensure that a single burst of errors does not affect more than one of the five repeats of a word.

The busy/idle bits are inserted between the bit sync and word sync and occur every 11'th bit thereafter. As long as the MTSO perceives a message directed toward it over an *access* channel, it will ensure that the busy/idle bits are set to "busy" in the data stream of the corresponding *paging* channel. (Keep in mind the duplex nature of the signalling channels. For every paging channel there exists a counterpart used for access purposes). This is one mechanism for reducing the likelihood of collisions; a mobile may attempt to seize an access channel for its own use, but only after verifying that the channel is indeed idle.



Notes:

(i) ↑ = point of busy/idle bit insertion (after bit and word sync and after every 10 message bits)

(ii) A_i = i'th of five repeats of word from message stream A

(iii) B_i = i'th of five repeats of word from message stream B

(iv) a 40 bit word consists of 28 information bits and 12 parity bits

Information Content of Words:

(i) Pages

(ii) Channel Assignments

(iii) Overhead Words

(iv) Filler text (only required when there are no pages)

Fig. 3.1 AMPS Data Format For Paging Channel [44]

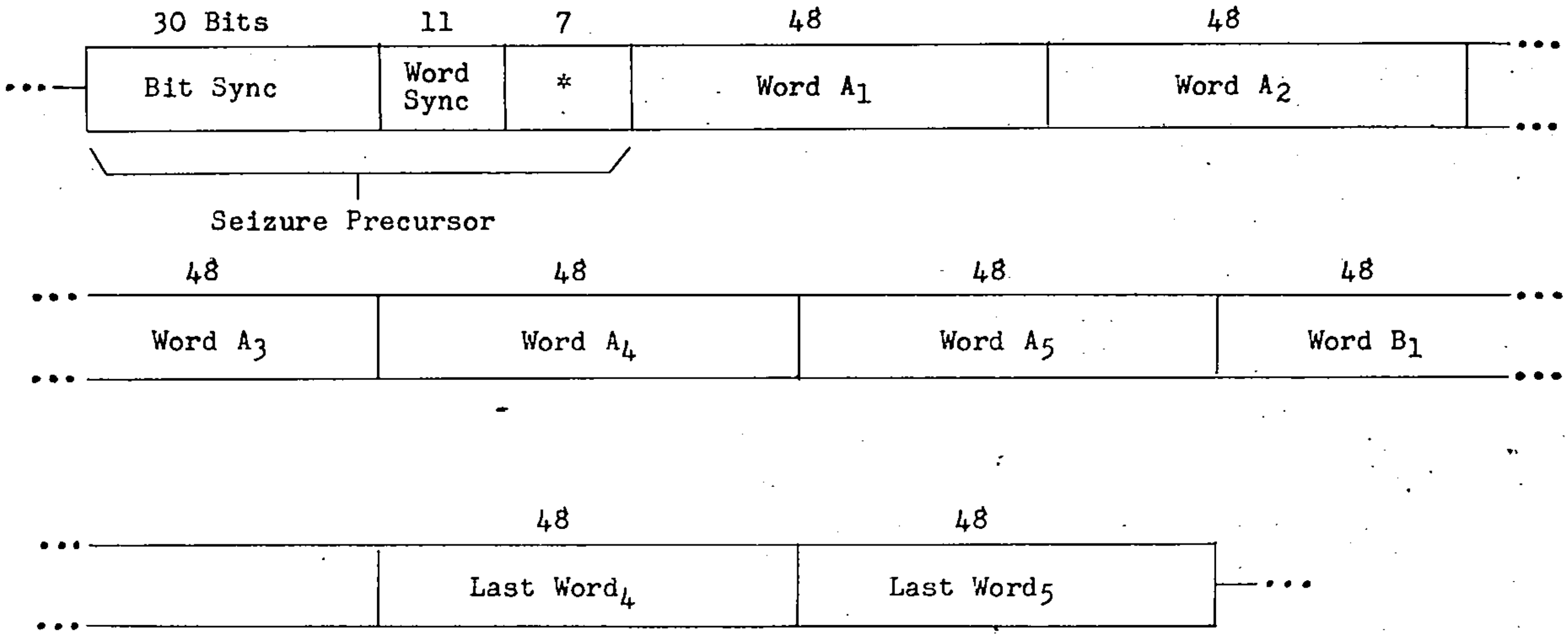
3.2 ACCESS

The telephone traffic directed *from* the mobile *to* the cell site consists of the following components:

1. Mobile-originated call attempts.
2. Page responses. (An acknowledgement by the mobile unit that it has recognized its page and is prepared to receive a call).
3. Order acknowledgements. (A confirmation by the mobile unit that a command from the MTSO has been received and correctly interpreted).

To carry out any of the above three tasks, the mobile unit must first verify that the selected access channel is indeed idle and then transmit its signalling (<u>seizure</u>) message. Verification of the busy/idle status of an access channel is accomplished by performing a 2-out-of-3 majority vote on the last three received busy/idle bits transmitted over the paging channel. Buried within the seizure message will be a code identifying its nature and purpose, that is, either a page response, an order response, or a call origination.

The format for the seizure message employed in AMPS is shown in Figure 3.2. Like the paging data stream, each word of a message is repeated five times. Each message may consist of up to five words; the exact length being dependent on the type of message and information received from the MTSO.



Notes:

(i) "*" signifies one of four Digital Colour Codes used to identify the cell site at which the message is directed

(ii) each word consists of 36 information bits and 12 parity bits

Fig. 3.2 AMPS Data Format for Access Channel [44]

In this thesis we will be concerned almost entirely with the access channels and only indirectly with the paging channels. It is the access channels which mobiles attempt to seize in a random and competitive fashion and it is these channels for which methods must be devised to minimize collisions. One such method has already been described, namely the use of busy/idle bits inserted in the paging data stream and which provide each use with "a priori" knowledge of the availability of an access channel.

3.3 DISTRIBUTION OF SIGNALLING CHANNELS IN SPECTRUM

Figure 3.3 depicts how the setup and voice channels will probably be distributed in the frequency spectrum during progressive stages of development in the AMPS system. Shown here is the band of frequencies used for transmission in the cell site-to-mobile direction. Since we are dealing with duplex channels, the same diagram may be applied to the 825 to 845 MHz range used from mobile to cell site merely by replacing the word "paging" by "access".

There are two types of paging channels which have been labelled A and B in Figure 3.3.[4] Overhead messages containing the local system parameters are sent over the type B channels while both the overhead message *and* the actual page (i.e. the MIN) are sent over the type A channels.

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[4] Not to be confused with the time-multiplexed A and B data streams described in Section 3.1.

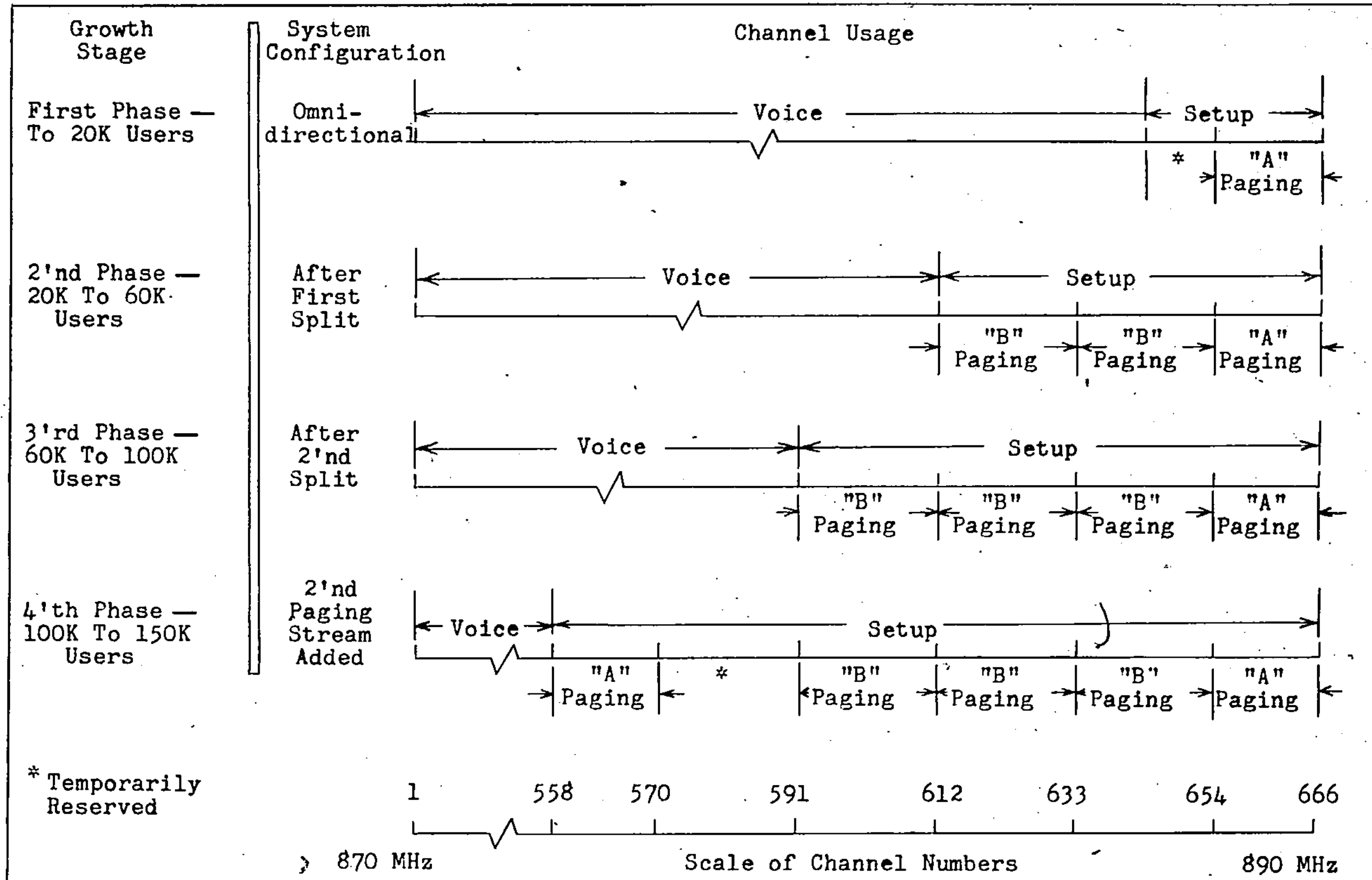

Fig. 3.3 Scenario for Setup Channel Usage as System Grows [42]

In the initial phase of development when onmidirectional antennas are used and the telephone traffic volume is small, 12 type A channels will suffice. In the second phase, with the switchover to directional sites, 21 type B paging channels will be added. At this stage then, there will be one type B paging channel for each of the 21 channel subsets; the 12 type A channels remain associated with the original omnidirectional sites.

As the system continues to grow, more and more type B channels will be required in order to cope with greater demands of an increasing number of mobile subscribers. Finally, in the fourth phase it is expected that an additional group of type A paging channels will be needed to handle the large volume of pages. It is unlikely though that this phase will be reached in any but the largest metropolitan centres and certainly not before the end of the century.

Chapter IV

LOGIC AND CONTROL FUNCTIONS OF THE MOBILE

PERTINENT TO THE SIGNALLING PROTOCOL

This chapter details the steps taken by the mobile logic unit in finding a suitable paging channel to monitor. The access channel which forms the reverse half of this paging channel will then be used when the mobile wishes to transmit a seizure message.

Also discussed are the techniques to reduce contention amongst users competing for the limited number of access channels. These techniques form the basis of the signalling protocol presented in Chapter 6. The description is confined entirely to AMPS although much of the call processing is typical of any cellular mobile system [16,17,28].

4.1 INITIALIZATION TASK

The initialization procedure consists of the following subtasks which are executed in sequential order after the mobile radio is switched on. The main function of the initialization task is to determine the proper paging channel to monitor. By "proper" is meant that channel belonging to the cell site (or cell site face in the case of directional

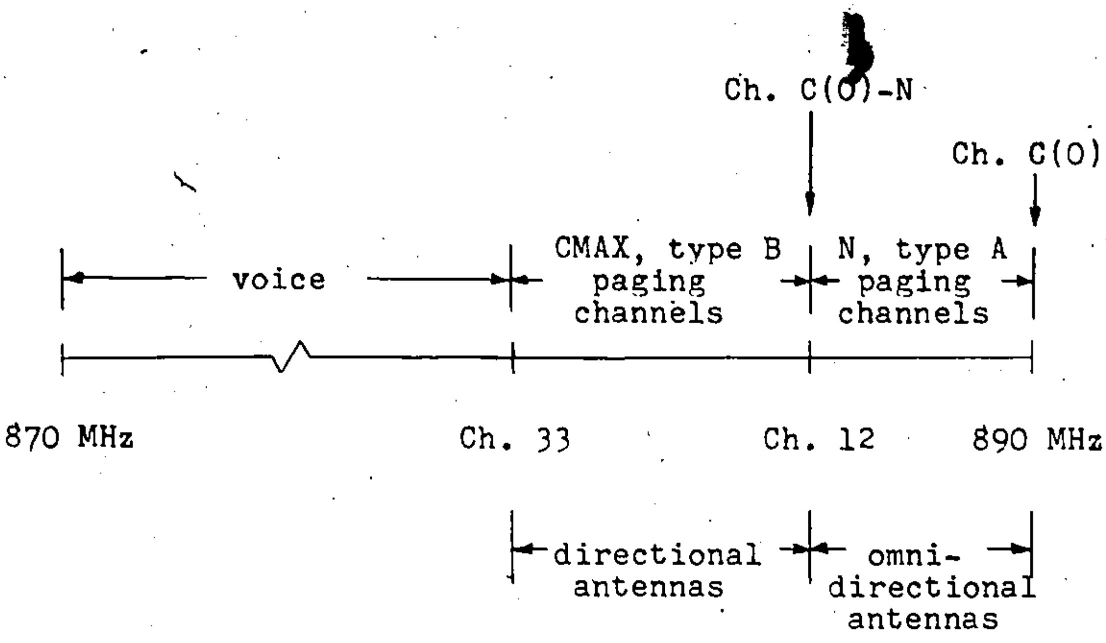

Figure 4.1 Significance of Terms Pertaining to Initialization Task

antennas) which is closest to the mobile. Figure 4.1 illustrates the signifigance of various terms and symbols.

Subtask 1. Scan Dedicated Paging Channels

The paging channels form a contiguous block in the spectrum. The mobile unit tunes to the first channel C(0) in this block, samples the received signal strength on each of two diversity antennas, and then tunes to the next channel, C(0)-1, where the measurement is repeated. When this has been done for the highest 21 channels, the mobile tunes to the channel deemed to have the strongest signal.

Subtask 2. Acquire Bit Sync and Word Sync

Subtask 3. Determine the System Parameters

The system parameters are: DCC, N, CMAX, CPA, ACS, and C(acc). With the exception of C(acc), these have all been defined previously. C(acc) represents the initial paging channel to sample when attempting to access the system. Its value is determined as follows:

i) If the CPA bit = 1, implying omnidirectional cell sites, then C(acc)=C(0).

ii) If the CPA bit = 0, implying directional antennnas, then C(acc)=C(0)-N.

Subtask 4. Make Preliminary Home/Roam Decision

While operating within the boundaries of its assigned MSA, a customer is termed a home mobile. Outside this area, the customer is termed a roamer. The ACS received over the paging channel is compared with the home ACS stored in the mobile unit's memory and a decision is made accordingly.

Subtask 5. Rescan N Paging Channels

Home mobiles commence scanning with channel C(h), the value of which is retained in the mobile unit's memory. (C(h) is the first channel within a group of type A paging channels to which the mobile is preassigned in the fourth phase of system development. In all earlier phases, C(h) and C(0) are identical). Roaming mobiles start scanning with channel C(0). All mobiles scan down in frequency until N channels have been sampled and then retune to the channel judged to have the strongest signal.

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Why is it necessary to scan the paging channels a second time? During the first scan (Subtask 1) the mobile may have landed on a type B paging channel. This will allow the mobile to receive the values of the system parameters but will not allow it to receive any pages. The second scan (Subtask 5), however, is confined to type A paging channels. Once this subtask is completed the mobile will be in a position to receive any pages directed toward it.

The initialization task is followed by the idle state wherein the mobile simply monitors the chosen paging channel and checks for the occurrence of each of the following events every 46.3 msec. This time span corresponds to the periodicity of messages sent in the paging data stream.

1. Page match

2. User request to place a call
3. Orders from the MTSO
4. Update of overhead information (eg. CMAX, N etc.)
5. Rescan timeout
6. Loss of word sync

If the rescan timer expires (Rescan Timeout referred to above), the idle state is exited and execution of the initialization task must begin anew. This ensures that as the mobile moves about, it is always monitoring the best possible paging channel in terms of signal strength.

4.2 SEQUENCE OF ACTIONS DURING A MOBILE-ORIGINATED CALL

Suppose now that the mobile subscriber wishes to place a call. He first enters the digits into a storage register and then depresses the SEND button (Figure 4.2(a)). The mobile logic unit recognizes this as a service request and initializes the value of an access timer to 6 seconds. This places a time limitation upon successful completion of the call origination sequence and as such, serves as a method for preventing continued collisions. The status of the timer is examined periodically before processing -- as outlined in the following steps -- is permitted to continue.

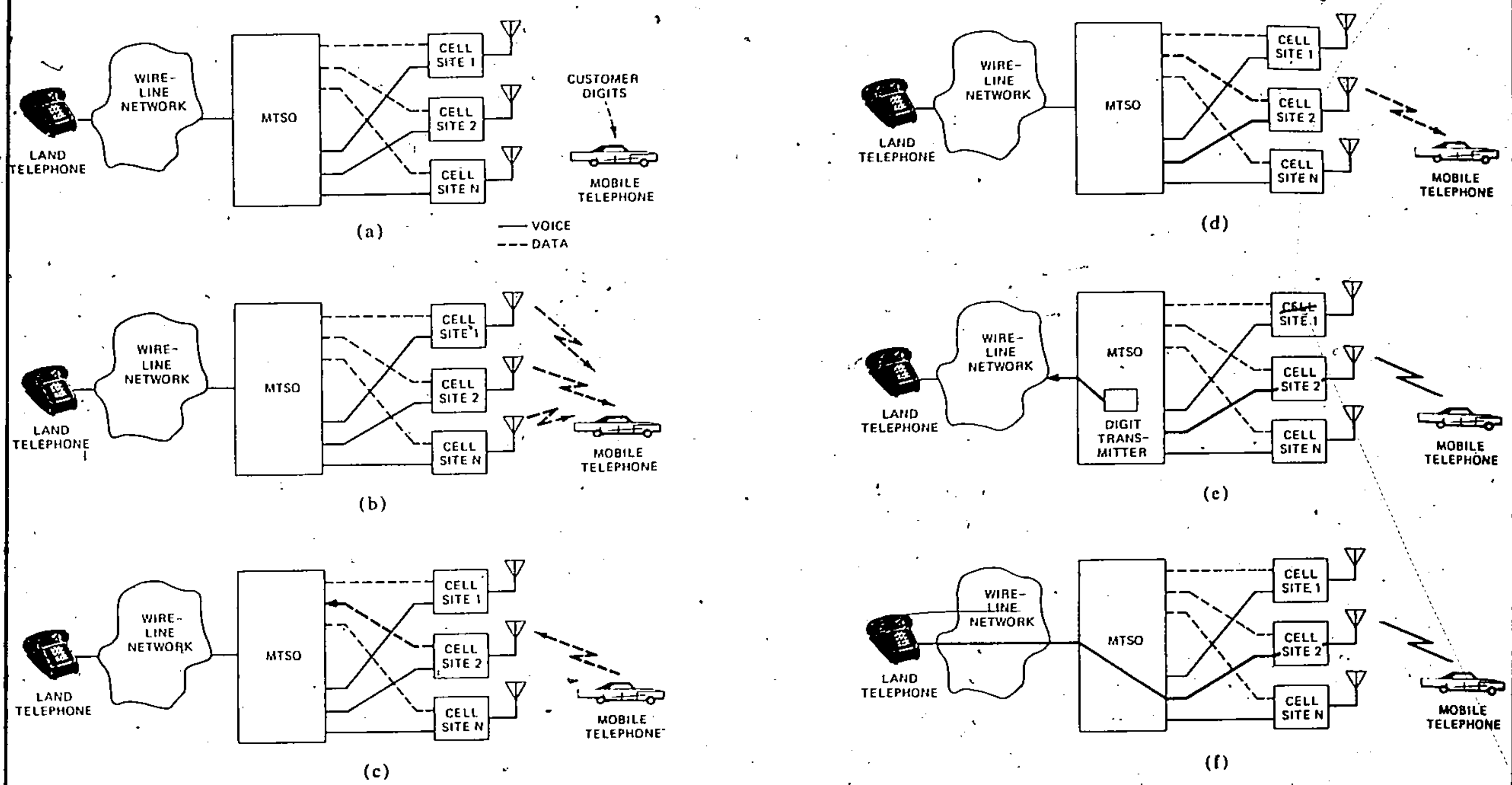

Figure 4.2 Mobile-originated Call Sequence. (a) Dialing (b) Cell Site Selection (c) Access Channel Seizure (d) Channel Designation (e) Digit Outpulsing (f) Conversation [42]

Step 1. Scan all paging channels in MSA (Figure 4.2(b)).

The mobile must scan CMAX paging channels beginning with C(acc) and lock onto the channel with the strongest signal.

Step 2. Acquire bit sync and word sync and decode the Digital Colour Code (Figure 4.2(c)).

Step 3. Seize Access Channel (Figure 4.2(c)).

Before attempting to seize the access channel, the mobile unit must determine the busy/idle status of the channel. If idle, transmission of the seizure data may begin; if busy, the mobile unit will wait a random amount of time before re-examining the busy/idle bits. Each time the mobile unit delays, a different random time -- uniformly distributed between 0 and 200 milliseconds -- is generated. However, to prevent system overload, a limit is placed on the number of automatic reattempts allowed.

If the busy/idle status changes to busy before 48 bits are sent or fails to change to busy by the time 96 bits are sent, the mobile assumes that the cell site did not receive (or recognize) the service request. In this event, the current seizure attempt is curtailed, a new random delay is calculated, and the busy/idle bits re-examined after the delay. Conversely, if at some point within the time window delimited by the 48'th and 96'th bits the busy/idle status does change to busy, then the mobile unit will continue to send the seizure message.

Step 4. Initial Voice Channel Designation (Figure 4.2(d)).

When transmission of the seizure message is completed, the mobile will wait for up to 5 seconds for a voice channel designation message to be returned over the paging channel *irrespective of the state of the access timer.* Suppose 5 seconds elapse and no such message is received. The mobile unit will now check the status of the access timer. If the timer has run out, the mobile subscriber will be made aware of this fact by means of a reorder tone. If the timer has not run out, the mobile will once again examine the busy/idle bits before reattempting to seize the access channel.

Assuming that there is at least one idle voice channel in the cell, the MTSO will choose a channel (and associated land-line trunk connecting the MTSO to the cell site) and inform the cell site of its selection. The serving cell site relays the channel assignment to the mobile over the paging channel and also begins transmitting a Supervisory Audio Tone (SAT) over the selected voice channel. There are three SATs with frequencies of 5970, 6000, and 6030 Hz just as there are three DCCs and there is a one-to-one correspondence between the two. The SATs are used for call supervision and confirmation of various actions over the voice channels and the DCCs perform the same function over the signalling channels.

The mobile tunes to the designated voice channel and transponds the SAT which it finds there. Upon recognition of the transponded SAT, the cell site places an off-hook condition on the landline trunk linking the cell site and MTSO. The MTSO interprets this condition as successful completion of the channel assignment task.[5]

Step 5. Digit Outpulsing (Figure 4.2(e)). The MTSO completes the call through the landline telephone network.

Step 6. Conversation (Figure 4.2(f)).

The MTSO establishes a complete communication link.

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[5] Along with its supervisory purpose, the SAT serves another function. Based on the round-trip delay of the SAT (cell site-to-mobile and back to cell site again) and with knowledge of the delay through the circuitry of the mobile transceiver, the MTSO can compute a rough estimate of the distance between the cell site and mobile. Combined with RF signal strength data, this gives the MTSO further information for deciding when a hand-off is needed.

4.3 SEQUENCE OF ACTIONS DURING A MOBILE-COMPLETED CALL

From the calling party's central office, the call is routed through the landline telephone network to the home MTSO of the mobile. The MTSO collects the digits, coverts them to the mobile's identification number, and instructs all cell sites equipped with type A paging channels to transmit the page. Upon recognizing its page, the mobile will follow a sequence of steps identical to Steps 1 through 4 described in Section 4.2.

When the mobile has tuned to the appropiate voice channel and transponded the SAT, the serving cell site will transmit a data message over this channel. The message will cause an audio tone to be generated in the mobile telephone alerting the mobile subscriber to the incoming call. Also, the mobile will begin transmitting a special 10 KHz tone over the reverse half of the voice channel. This 10 KHz tone confirms successful alerting to the MTSO which will then provide audible ringing to the calling party. When the mobile subscriber answers, transmission of the 10 KHz tone ceases. This is detected by the MTSO which removes the audible ringing which was supplied to the calling party. A complete connection is now established.

Chapter V

THE SIMULATED SYSTEM

In this chapter we describe the cellular mobile radio system that was simulated using the General Purpose System Simulator (GPSS) language [5,35,43]. We state the assumptions made and explain why they seem reasonable.

5.1 HEXAGONAL CELLULAR LAYOUT

For the simulation we considered a 40-cell mobile radio communication system. The choice of 40 cells was more or less arbitrary. A larger system could have been studied but at the expense of higher computing costs. Besides, it was observed early on in the study that a system of 40 cells produced results in which the cell-to-cell fluctuations in the collected statistics were sufficiently averaged out.

The number of channel sets N was taken to be 3 and from Equation (2.2), D/R is also 3. This cochannel reuse ratio applies to both the voice channels and the signalling channels. Once again, the choice of N is not critical. The reason for this is closely tied to the voice channel assignment schemes so we will postpone a discussion of this point until these schemes have been introduced.

A system such as just described might appear as in Figure 5.1.

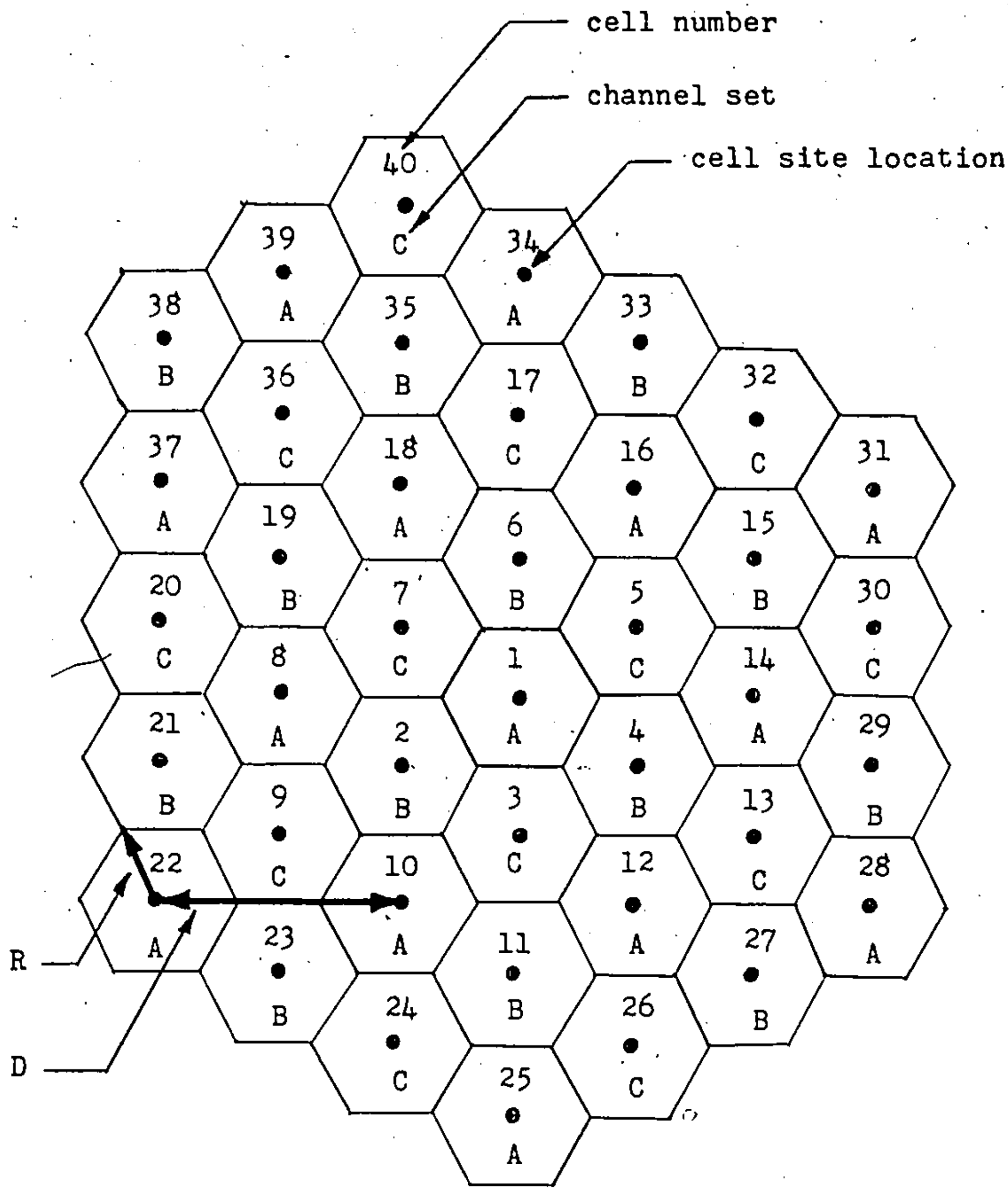
5.2 ASSUMPTIONS

The following general assumption was made:

1. The interarrival time of calls in each cell is exponentially distributed, with a mean value, $1/\lambda_1$ seconds. Equivalently, call generation forms a Poisson process with an average arrival rate of λ_1 calls/second/cell.

The following assumptions pertain to the access channels:

2. The access channel holding time is nominally 100 msec. This figure was arrived at by assuming a 10 kbit/sec transmission rate and an average seizure message length of 1000 bits [42].
3. The cell site always receives a seizure message when one is sent. In reality, the miss rate (probability of not detecting a message when one is sent) is expected to be about 10^{-4} for an RF S/I = 18 dB [42].
4. Because of the busy/idle bit feature, the probability of an actual collision between two mobiles simultaneously attempting to seize the same access channel is neglegible.



<u>Figure 5.1</u> Hexagonal Cellular Configuration Using Three Distinct Channel Sets

The following assumptions pertain to the voice channel assignment portion of the simulation models:

5. The voice channel holding time, $1/\mu_V$, is exponentially distributed with a mean value of 120 seconds.
6. Vehicles do not traverse cell boundaries while a call is in progress. This implies that callers use a given voice channel for the full holding time drawn from the exponential distribution and are not forced to terminate prematurely if no channels are available in the new cell.

The choice of an exponential distribution for both call interarrival times and voice channel holding times is so prevalent in the literature that it hardly seems necessary to justify Assumptions 1 and 5. Suffice it to say that the bulk of wireline telephone traffic theory is based on these very same assumptions and it is felt that they should apply equally well to radio channel trunking [12,14,18, 20-24,26,27].

The values cited in Assumptions 2 and 3 were garnered from an existing, representative system [42] and require no further justification.

Assumption 4 states that the probability of a collision between two callers is small enough to be ignored (compared to the blocking probability which results from the unavailability of an access channel). This statement will now be explained with the aid of Figure 5.2 which illustrates several typical cases where mobiles attempt to initiate calls very close in time. The following notes apply to Figure 5.2:

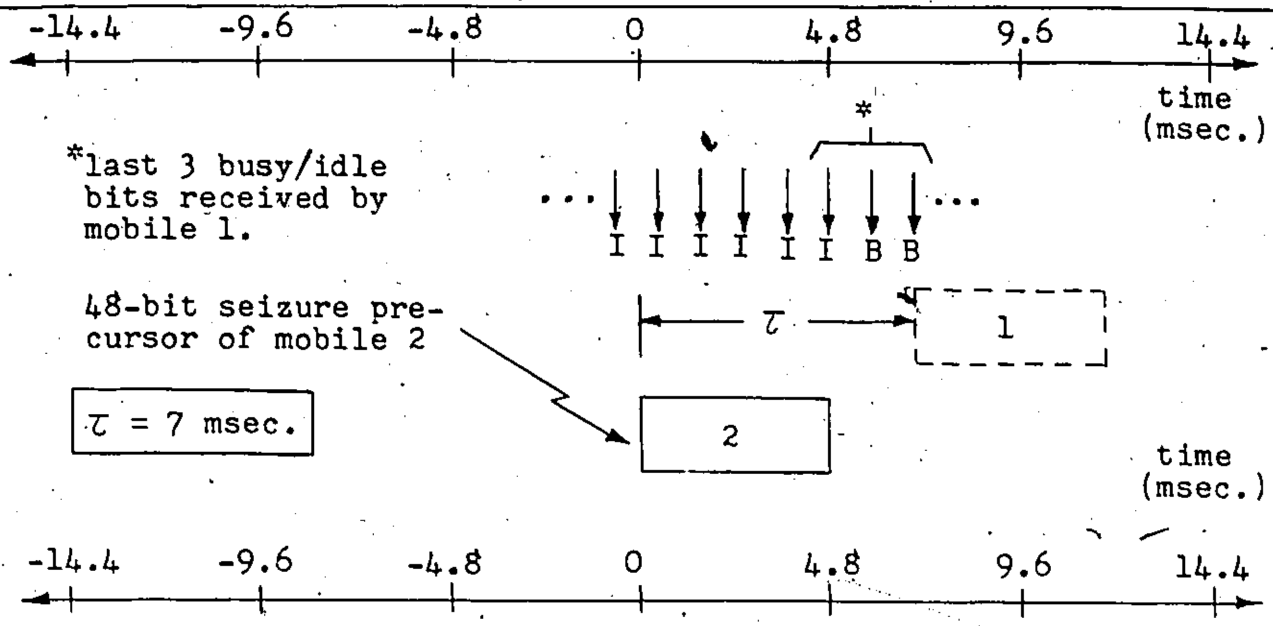
i) Only the first 48-bit portion of the seizure messages is shown.

ii) τ = interarrival time between seizure messages.

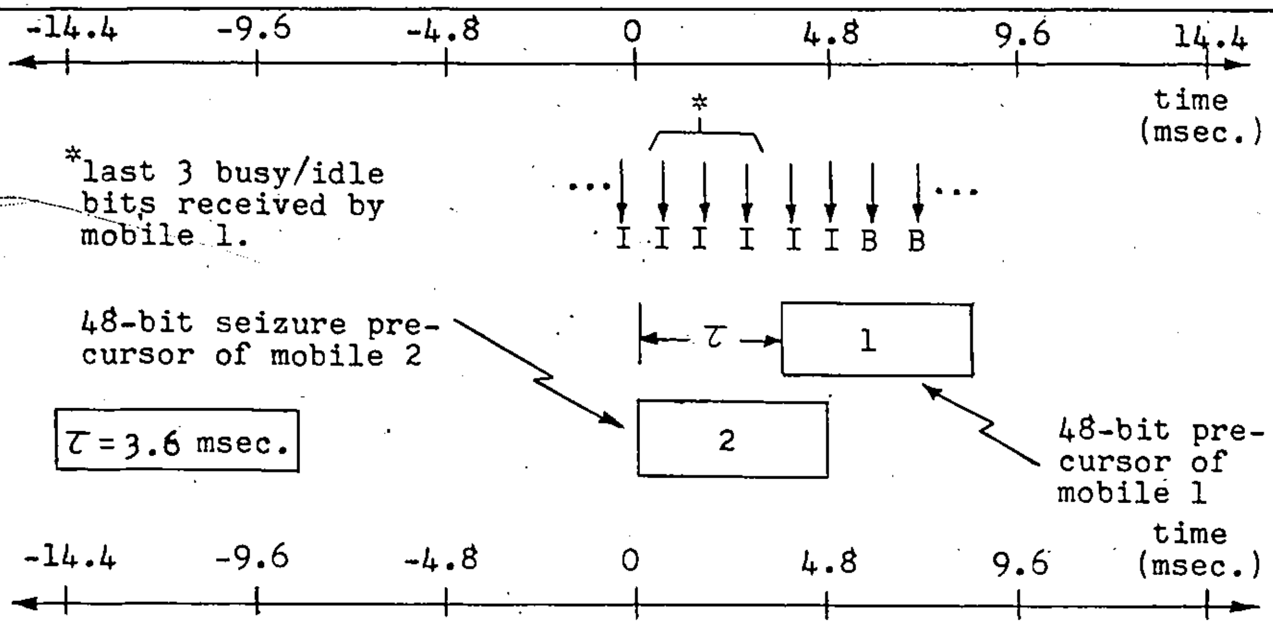
iii) $\downarrow$ = point of insertion of the busy/idle bit in the paging data stream. The paging channel is continuously monitored while transmitting or attempting to transmit over an access channel.

iv) "I" signifies an idle bit and "B" signifies a busy bit.

In AMPS every seizure message is preceded by a 48-bit precursor. On a time scale this corresponds to 4.8 msec for a 10 kbit/second data rate. Once this precursor is received intact, the cell site will begin to set the busy/idle bits to busy in the paging data stream thereby preventing any other mobiles from attempting to use the access channel until it is relinquished by the "seizing" mobile.

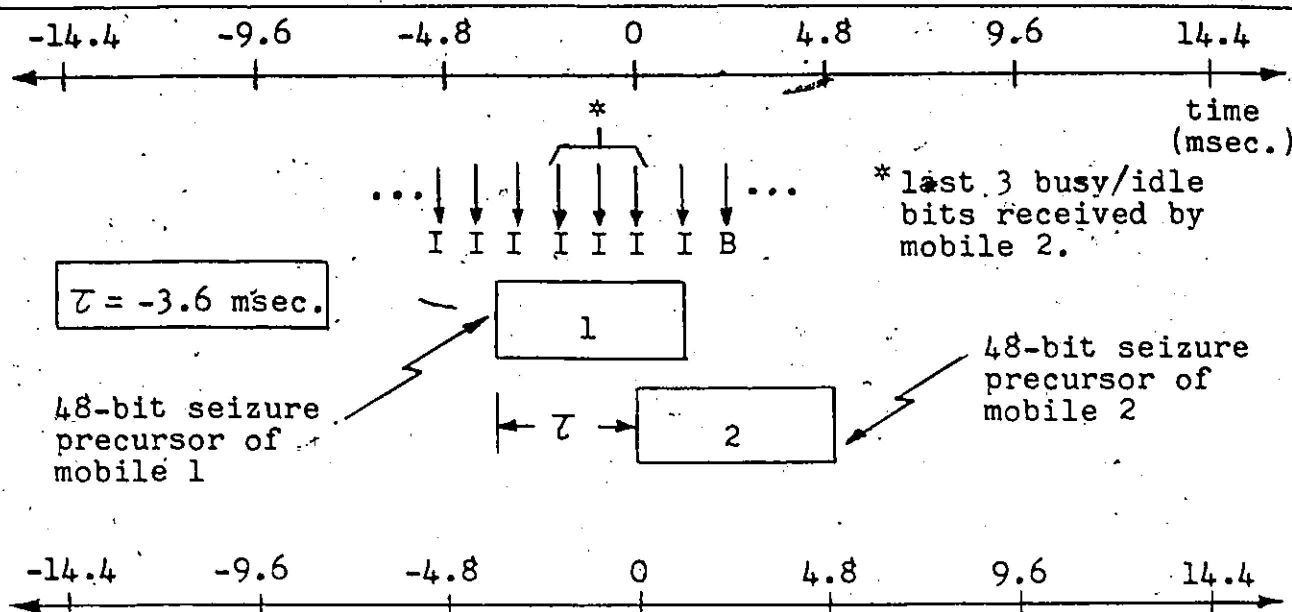


Case (a) Mobile 1 wishes to start transmitting at t = 7 msec. (over the same access channel that mobile 2 is currently using). However, he is prevented from doing so since a 2-out-of-3 majority vote of the last 3 received busy/idle bits indicates that the channel is busy. A collision does not occur.



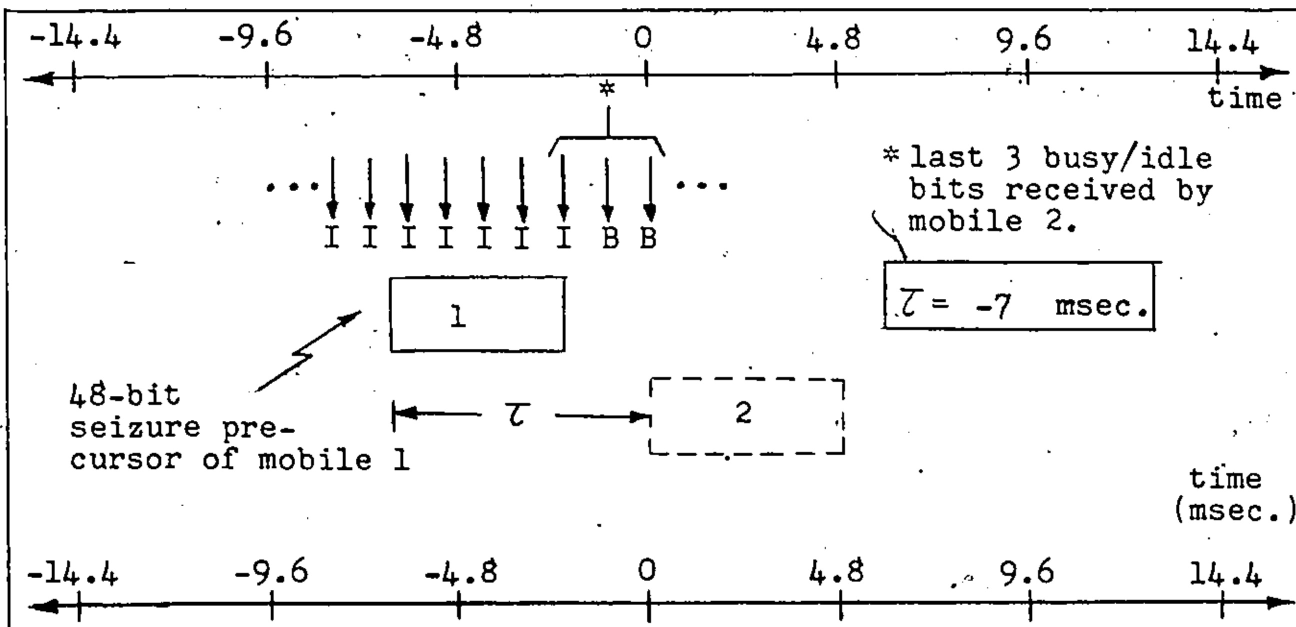
Case (b) Mobile 1 begins transmitting 3.6 msec after mobile 2. On the basis of the last 3 received busy/idle bits, mobile 1 perceives the access channel to be idle. A collision occurs.

Figure 5.2 Diagram for Discussion of Collision Problem



Case (c)

Mobile 2 begins transmitting 3.6 msec after mobile 1. On the basis of the last 3 received busy/idle bits, mobile 2 perceives the access channel to be idle. A collision occurs.



Case (d) Mobile 2 wishes to start transmitting at t = 0 msec. However, he is prevented from doing so since a 2-out-of-3 majority vote of the last 3 received busy/idle bits indicates that the channel is busy. A collision does not occur.

Figure 5.2 (cont'd) Diagram for Discussion of Collision Problem

With these facts in hand, the sequence of diagrams in Figure 5.2 now indicate that the vulnerable period during which one message may be corrupted by another lasts for 14 msec. If it is desired to limit the probability of a collision to some small value, it is likewise necessary to impose an upper limit upon the average rate at which calls are geneated, λ_1. In practice λ_1 is not expected to exceed 1 call per second in any cell [42]. If we now invoke Assumption 1, it is an easy matter to calculate the probability of a collision for this value of λ_1. Representing the random interarrival time by τ:

$$\begin{aligned}\text{Probability of a collision} &= \text{Prob}(\ \tau \leq t = 14 \text{ msec}) \\ &= 1 - e^{-\lambda_1 t} \\ &= 1 - e^{-(1)(.014)} \\ &= 0.0139\end{aligned}$$

Thus the probability of a collision will be less than 1.4% for all anticipated call arrival rates. As a side observation, this calculation assumed that a collision causes both seizure messages to be lost. In fact, because of the use of FM to transmit data in AMPS, if two messages collide, the message with the stronger signal will capture the cell site receiver and may be received without error. If this is the case, the vulnerable period for a collision will be only 7 msec and the probability of a collision will be halved.

Nevertheless, we will stick with the more pessimistic viewpoint.

Finally, regarding Assumption 6, vehicles crossing cell boundaries and requiring new channels produce the following effects:

i) The average call duration in each cell is shortened since a mobile exiting a cell vacates a channel prematurely.

ii) The net call attempt rate per cell is augmented by the boundary crossing rate since a call-in-progress entering a new cell requires a channel just as a mobile originating a call in the same cell does.

iii) Upon entering a new cell some small percentage of the calls will be forced to terminate prematurely due to a lack of free channels.

By definition, the offered telephone traffic load is the product of the mean channel holding time and the mean call attempt rate (see Appendix A). Considering points (i) and (ii) together, the traffic load per cell and the resultant grade of service are not likely to be greatly affected by boundary crossings. Partly for this reason, but primarily because our interest lies with the signalling channels, no effort was made to simulate vehicle motion.

Chapter VI

SIMULATION MODEL NO. 1

In Chapter 4 we described in some detail the features of the AMPS signalling channel protocol. In the present chapter we abstract the most pertinent of these features and present them in the concise form of a flowchart.

6.1 THE SIGNALLING PROTOCOL

The protocol for using the access channels is shown in Figure 6.1. The following statements apply:

1. The access channels belonging to a particular cell are assigned sequentially ordered numbers. The first time "x" is encountered in the flowchart it represents the largest of these numbers.
2. The actions described in the flowchart occur in every cell. Strictly speaking then, each box of the flowchart should have the additional qualifier: in cell i, i = 1,2,3...40.
3. It is best to consider a call as a dynamic entity which enters the flowchart at the top, moves about, and is acted upon by the various boxes.

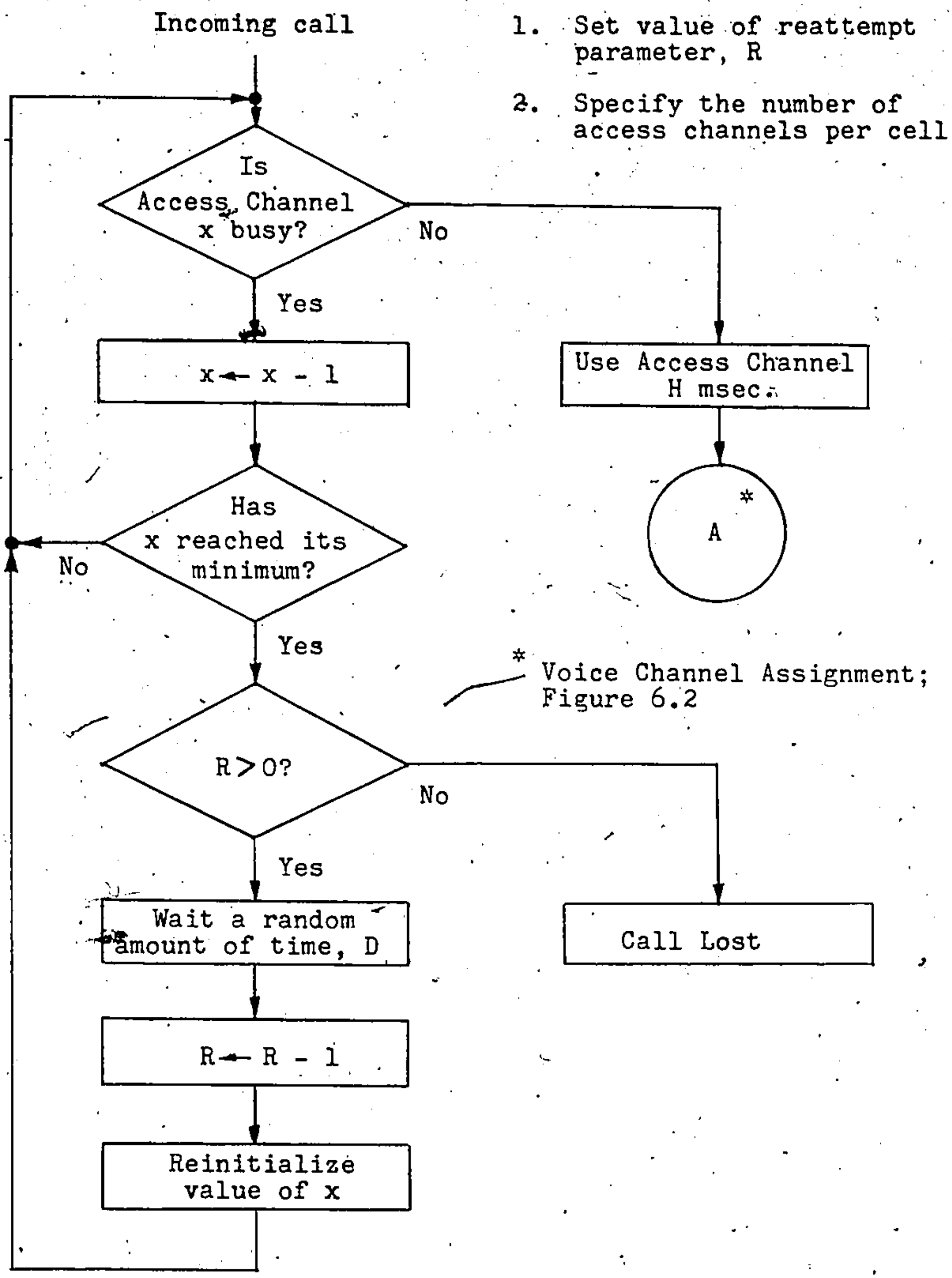


Figure 6.1 Signalling Protocol No. 1 for the Access Channels

Consider now the operation of the model. A call from a mobile in cell i first tests the busy/idle status of access channel x belonging to cell i. In actual practice this test would be performed by the mobile logic unit on the basis of the busy/idle bits in the paging data stream. If idle, the call uses the access channel for the allotted time before proceeding to the voice channel assignment segment of the model.

If, on the other hand, access channel x is busy, the busy/idle status of access channels x-1, x-2, etc. are tested in succession until either an idle channel is found or there are no further access channels to be tested in cell i. In the former case the call exits the loop, uses the selected access channel, and then continues on to the section of the program controlling voice channel assignment. In the latter case, the call also exits the loop but via the downward path. If the maximum number of reattempts permitted, R, is equal to zero, the call is lost; that is, it leaves the system and does not return. If R is greater than zero, the call will try again to obtain an access channel but only after a random delay. This random delay eliminates the synchronism which would otherwise persist between two or more calls entering the system at the same time. Also, and more simply, a call reattempting to gain access immediately would find the access channel still busy. After the delay, R for this call is decremented by one and the value of x is reinitialized. The call returns to the top of the flowchart where the sequence of events just outlined recommences.

The designers of AMPS predict that for most cities only one access channel per cell site will be required. To check the validity of this prediction we made the model sufficiently general to allow for more than one access channel per cell. The majority of the simulation trials, however, were confined to a single access channel per cell.

Absent from the flowchart is the access timer feature described in Section 4.2. The reason for not including the timer is best explained by means of a concrete example using representative values for various parameters:

i) Random delay D uniformly distributed between 0 and 200 msec.

ii) Maximum no. of automatic reattempts R = 10.

Examining the flowchart of Figure 6.1 it can be seen that, even in the extreme case where 10 reattempts are actually needed and a maximum delay of 200 msec is generated before each reattempt, only 2 seconds will be consumed out of the 6 seconds allowed by the access timer. In all instances then, R will be the limiting factor during an access channel seizure attempt. However, after obtaining an access channel and transmitting a seizure message the mobile unit is prepared to wait a maximum of 5 seconds for a voice channel assignment during which time the access timer continues to run. If the assignment is not received during this 5 second period, the mobile unit will examine the busy/idle bits before reattempting to gain access to the system. Thus it is possible for the timer to run out but only after the mobile has already obtained an access channel.

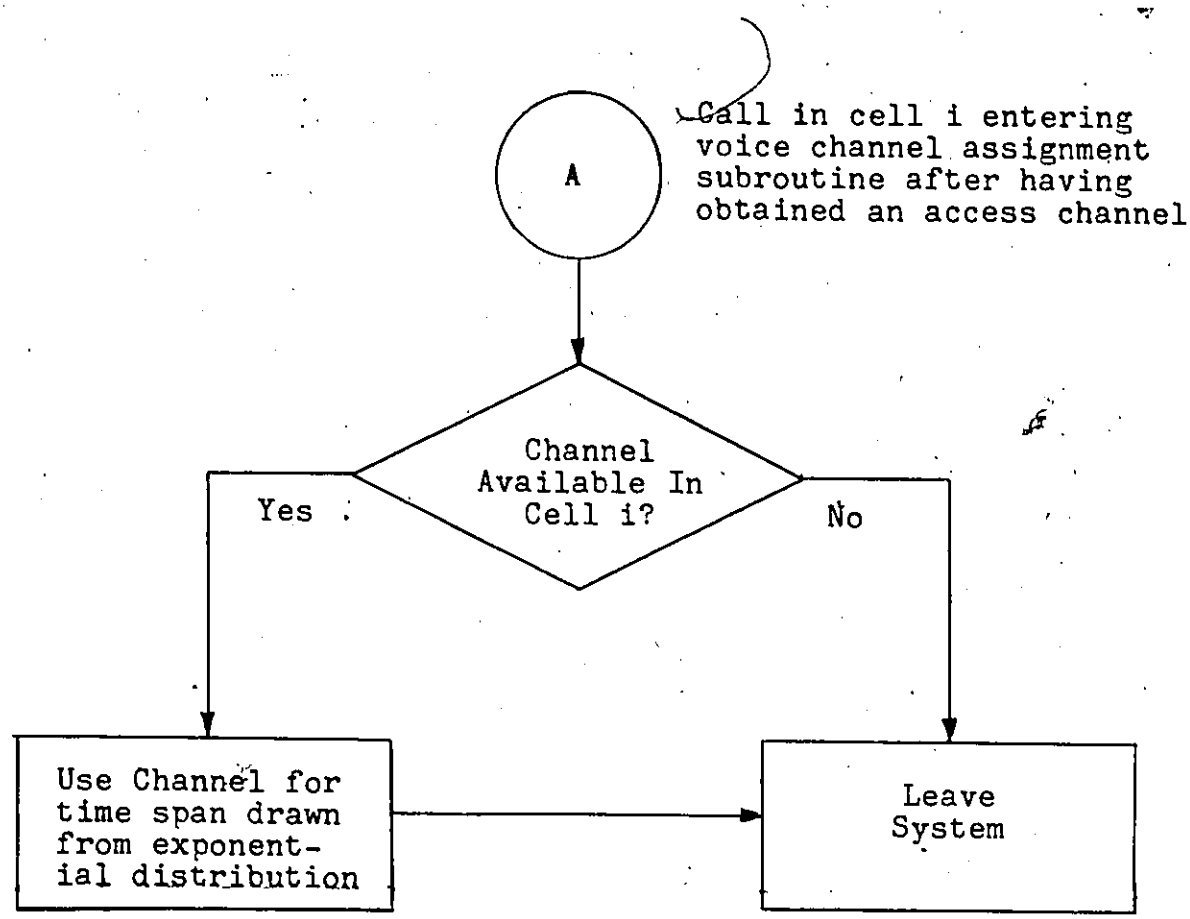
Since our main concern was the sequence of events which transpire during the initial attempts to seize an access channel, the access timer was not included in this particular simulation model. Also, the inclusion of two limiting factors (R and the access timer) would have greatly obscured the cause/effect relationship between the independent variables (eg. telephone traffic load) and the resulting grade of service (eg. blocking probability). However, an access timer was included in Simulation Model 2 and a far more complex model was the consequence as will become evident in Chapter 7.

6.2 THE VOICE CHANNEL ASSIGNMENT SCHEME

In this model we use a Fixed Voice Channel Assignment Scheme (FCAS). Such a scheme maintains a definite, static relationship between the channel sets and the cells where they are used. In fact, this is nothing new -- it is only scheme considered up to now.

Use of a FCAS, in conjunction with Assumption 6 which states that vehicles do not cross cell boundaries while a call is in progress, makes it possible to view each cell as an independent entity. Because there is no interaction between cells, the choice of N does not affect the operation of the simulator in any way. The use of cellular system merely provides a convenient means of smoothing out statistical fluctuations by taking an average over 40 cells.

The flowchart of the voice channel assignment subroutine is shown in Figure 6.2. An Erlang B service discipline has been implemented; calls are served immediately if at least one channel is unoccupied in cell i but are blocked and forced to leave the system if all voice channels are busy in cell i.



<u>Figure 6.2</u> Fixed Voice Channel Assignment Scheme and Erlang B Service Discipline

6.3 SYMBOLS AND DEFINITIONS

The following symbols and shorthand notations have been adopted. The phrase "calls which enter the system" is used frequently and and is meant to imply "calls generated by mobiles in all cells".

$a:v$ = channel division ratio
a = no. of access channels per cell
v = no. of duplex voice channels per cell

For example, if $a:v = 1:10$, the total number of channels assigned for use in a particular cell is 11. Of this number, 1 channel is reserved for signalling and the remaining 10 are used for voice communication. (Of course an access channel is but one-half of a duplex signalling channel; however, the paging channel which forms the other half is of no interest here).

$\bar{d}_1$ = mean delay encountered by all calls from the moment the call is generated to the moment the call obtains an access channel (or to the moment when the call is tabulated as blocked because an access channel was not available).

$\bar{d}_2$ = mean delay encountered by only those calls which do not obtain an access channel on the first attempt and consequently are forced to wait and reattempt.

D = random delay generated prior to making an access channel seizure reattempt. $D = (a,b)$ signifies that the delay in uniformly distributed between the limits "a" and "b", with mean value, $(a+b)/2$. The nominal values for a and b are 0 and 200 milliseconds respectively.

H = access channel holding time in milliseconds; nominally 100 msec.

P_{ba} = blocking probability for the access channels.

$$= \frac{\text{no. of calls which fail to obtain an access channel}}{\text{no. of calls which enter the system}}$$

P_{ba1} = probability of not obtaining an access channel on the first attempt or equivalently, the probability that at least one reattempt will be necessary.

$$= \frac{\left[\text{no. of calls which fail to find an access channel on the first attempt}\right]}{\text{no. of calls which enter the system}}$$

P_{bc} = combined blocking probability.

$$= \frac{\left[\text{total no. of calls blocked because they fail to find an access channel } \underline{\text{or}} \text{ a voice channel}\right]}{\text{no. of calls which enter the system}}$$

(The "or" is underlined to stress that this is a "one-or-the-other" situation).

P_{bv} = blocking probability of the voice channels.

$$= \frac{\text{no. of calls which fail to find a voice channel}}{\left[\text{no. of calls which have successfully obtained an access channel}\right]}$$

R = maximum number of reattempts permitted to obtain an access channel. (Also used in the flowchart to indicate the current number of reattempts a call has remaining out of this maximum value).

T_a = access channel traffic load expressed in the dimensionless unit of erlangs.

T_v = voice channel traffic load (erlang).

$1/\mu_v$ = mean voice channel holding time (120 seconds).

Notes:

1. Both $\bar{d}_1$ and $\bar{d}_2$ have signifigance only when R is nonzero.

2. The relationship between T_a and T_v is given by:

$$T_v = \frac{1}{\mu_v H} \cdot F \cdot (1-P_{ba}) \cdot T_a \quad \text{(erlang)} \qquad (6.1)$$

$$\wedge \quad 0 \leq F \leq 1$$

To explain (6.1), refer to Figure 6.3 which shows a schematic representation of call request flow rates through the system. In this diagram:

λ_1 = mean rate at which new calls arrive (calls/second).

λ_2 = λ_1 augmented by the fraction of calls which do not find an access channel immediately and are therefore forced to reattempt.

λ_3 = mean rate at which access channels are assigned to calls.

λ_4 = mean rate at which call requests for a voice channel arrive.

λ_5 = mean rate at which voice channels are assigned to calls.

F = fraction of all calls which fall into the categories of page responses or mobile originated call requests. The other component of the telephone traffic arriving at a cell site is the order acknowledgement. Unlike the first two, however, an order acknowledgement is not vying for a voice channel. An empirical value for F is not presently available; it has been set to unity for the purpose of simulation.

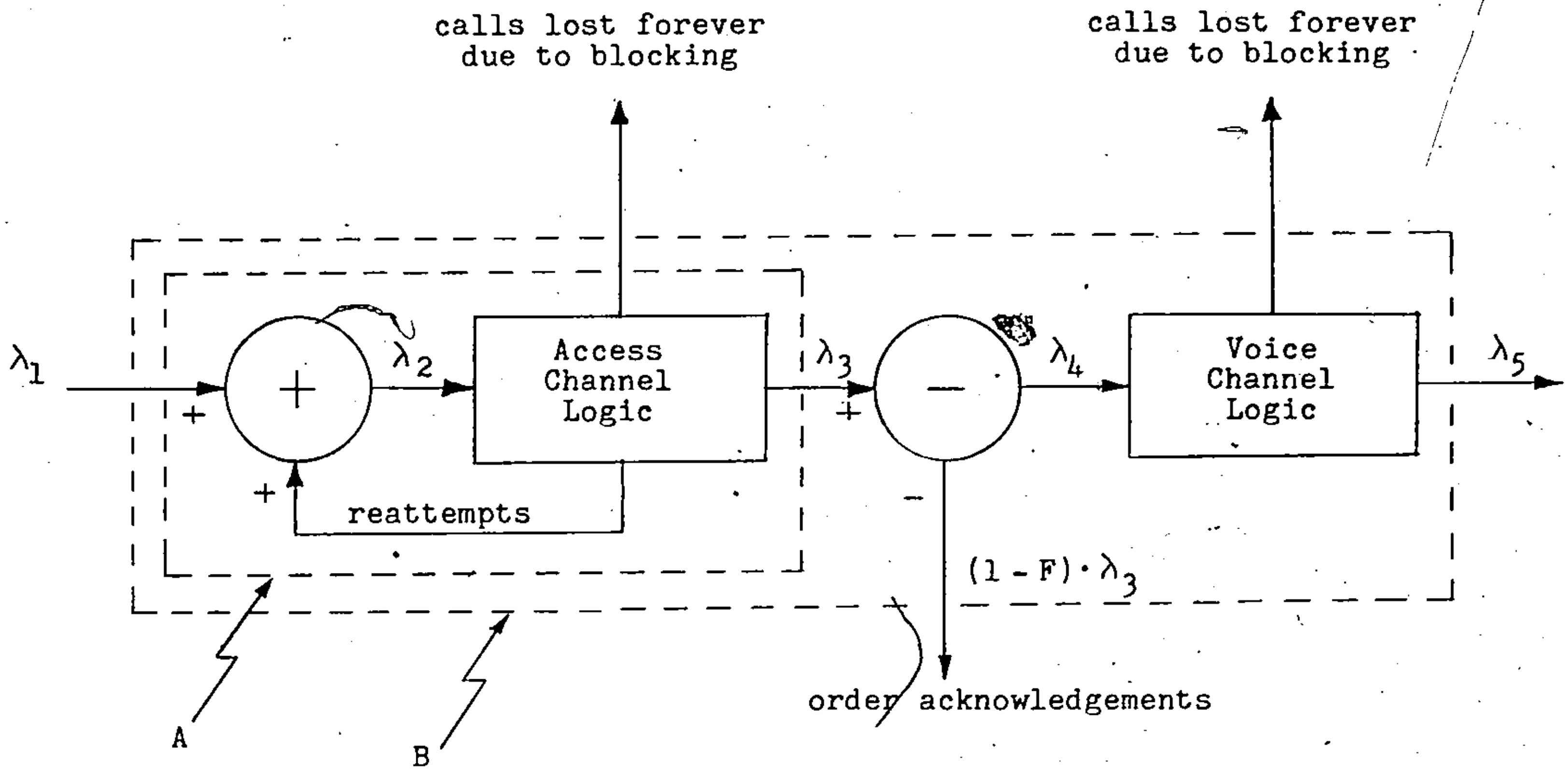


Figure 6.3 Call Flow Rates Through Simulation Model No. 1

In statistical equilibrium the average rate at which requests enter the system must equal the average rate at which channels are assigned [19,30]. Thus for surface A in Figure 6.3:

$$\lambda_3 = (1-P_{bal})(1-P_{ba})\cdot\lambda_2 \;, \quad R \neq 0 \tag{6.2}$$

$$\lambda_3 = (1-P_{ba})\cdot\lambda_2 \;, \quad R = 0 \tag{6.3}$$

$$\lambda_3 = (1-P_{ba})\cdot\lambda_1 \;, \quad R \geq 0 \tag{6.4}$$

Statement (6.2) says that the mean rate at which calls leave the access channel logic block (λ_3) is equal to the rate at which they arrive (λ_2) reduced by some fraction. This fraction, $(1-P_{bal})(1-P_{ba})$ may be thought of as a throughput factor representing that proportion of all calls which are *not* delayed because they are reattempting and are *not* lost forever because they are blocked.

Comparing (6.3) and (6.4) it is seen that $\lambda_1 = \lambda_2$ when R=0. This is consistent with the flow rate diagram since for the particular case of R=0 the feedback path from the access channel logic no longer exists.

We can also make the following statements:

$$\lambda_4 = F\lambda_3 \tag{6.5}$$

$$\lambda_5 = (1-P_{bv}) \cdot \lambda_4 \tag{6.6}$$

Finally, regarding surface B in Figure 6.3, we can state:

$$\lambda_5 = \lambda_1 , \tag{6.7}$$

for statistical equilibrium. Combining (6.4) and (6.5) we have:

$$\lambda_4 = F(1-P_{ba}) \cdot \lambda_1 , \; R \geq 0 \tag{6.8}$$

We now define the access channel traffic load as $T_a = \lambda_1 H$ and the voice channel traffic load as λ_4/μ_v. Equation (6.8) then leads directly to (6.1).

It would have been more proper to define the traffic offered to the access channels as $\lambda_2 H$ except for the fact that λ_2 is an endogenous variable. On the other hand, we can freely select any value we wish for λ_1 before simulation begins. A relationship obviously exists between λ_2 and λ_1 but we will not know what this relationship is until we have run the simulator and computed P_{ba} and P_{ba1}.

3. The various performance measures $\bar{d}_1$, $\bar{d}_2$, P_{ba}, etc., were first calculated for each of 40 cells. Average values were then computed, reflecting the system as a whole. Sufficient time was allowed for the system to reach a steady state; the simulation then continued for a further two hours (of simulated time) or until 10,000 calls had entered the system, whichever event occurred first. Statistics were gathered only during this steady state period.

6.4 THE SIMULATION RESULTS

The simulation results are shown in Figures 6.4 through 6.10. The values of those parameters which remain fixed for a particular set of simulation trials are indicated on the graphs. Notably, in those cases where the access channel traffic load and channel division ratio do not change, values of 0.5 erlangs and a:v=1:10 or 2:10 have been chosen. Generally, the greatest traffic level examined was 0.1 erlang corresponding to a maximum expected interarrival time of 1 call/second/cell on the average.

It may well be questioned why such a small number of voice channels per cell was considered when in AMPS the actual number is closer to 60. One reason is that the cost of simulation rises sharply as the number of channels per cell increases. A second reason is that early in the study it was observed that the voice channel blocking probability P_{bv} was only marginally affected by the access channel control logic. This being the case, it did not really matter whether 10 or even 100 channels were used since for any combination of v and T_v, P_{bv} could be readily calculated from the Erlang B formula (Appendix A).

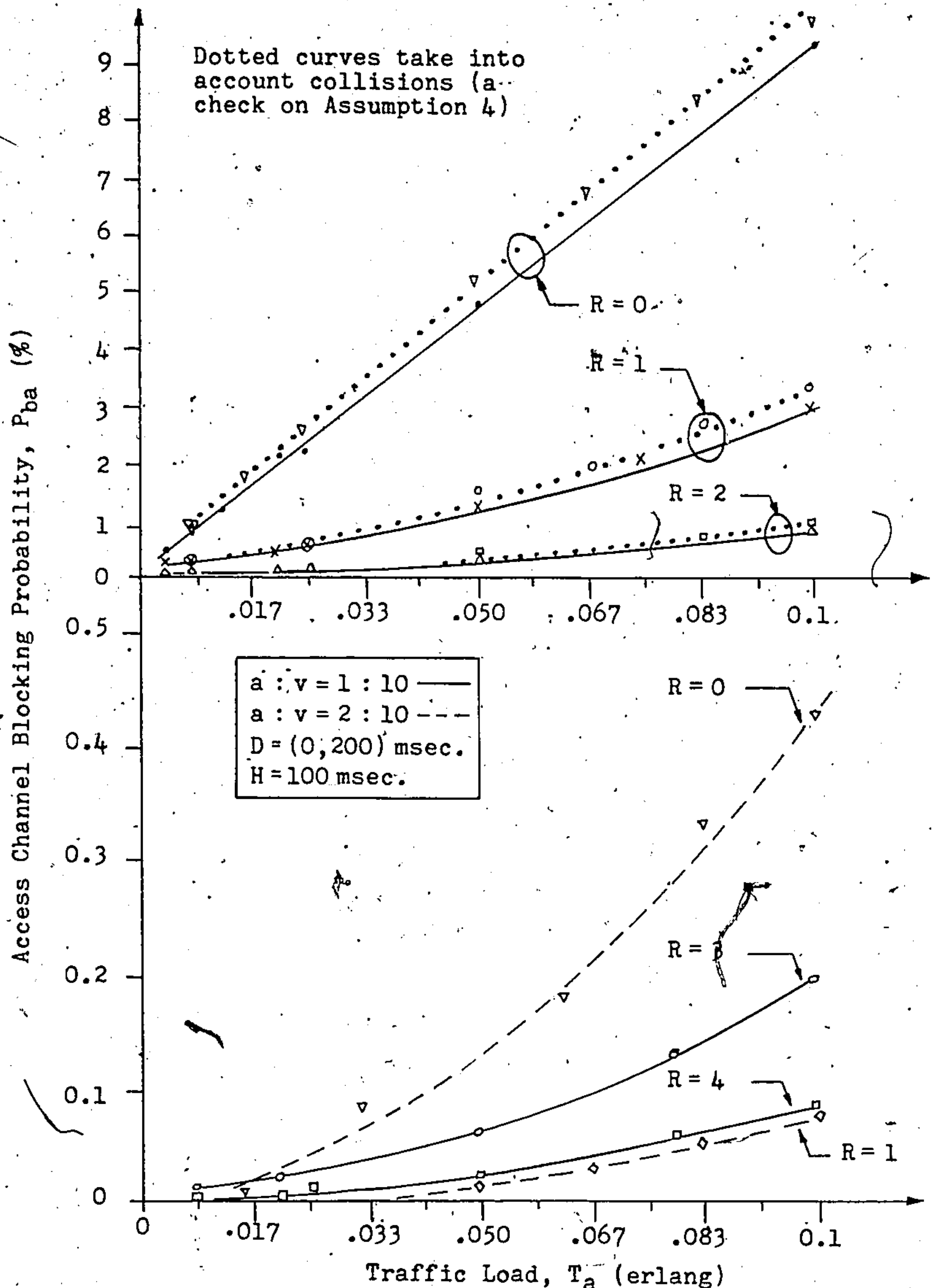


Figure 6.4 Response of Access Channel Blocking Probability to Increasing Traffic Loads

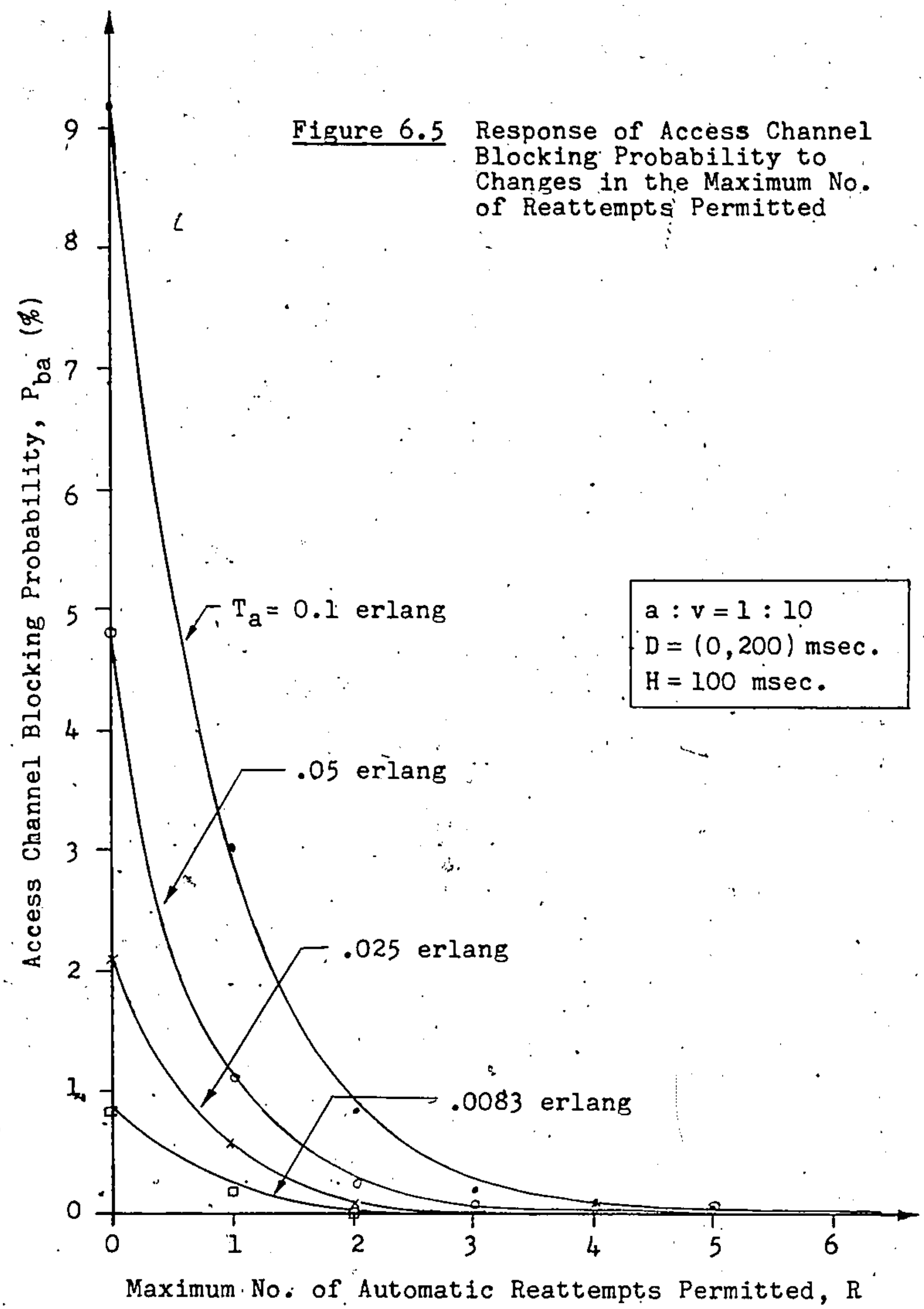


Figure 6.5 Response of Access Channel Blocking Probability to Changes in the Maximum No. of Reattempts Permitted
Access Channel Blocking Probability, P_{ba} (%)
Maximum No. of Automatic Reattempts Permitted, R
T_a = 0.1 erlang
.05 erlang
.025 erlang
.0083 erlang
a : v = 1 : 10
D = (0,200) msec.
H = 100 msec.
0
1
2
3
4
5
6
7
8
9

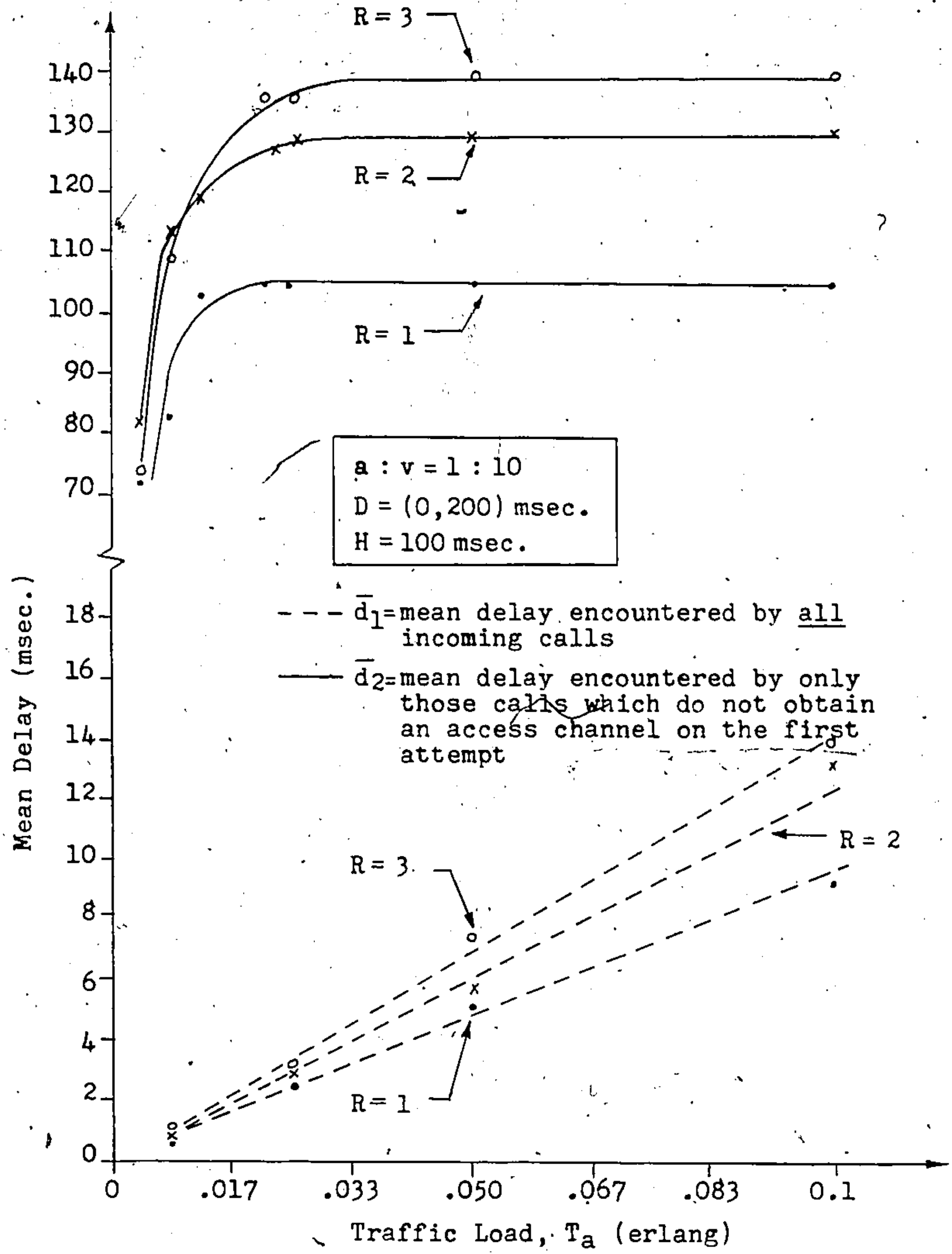
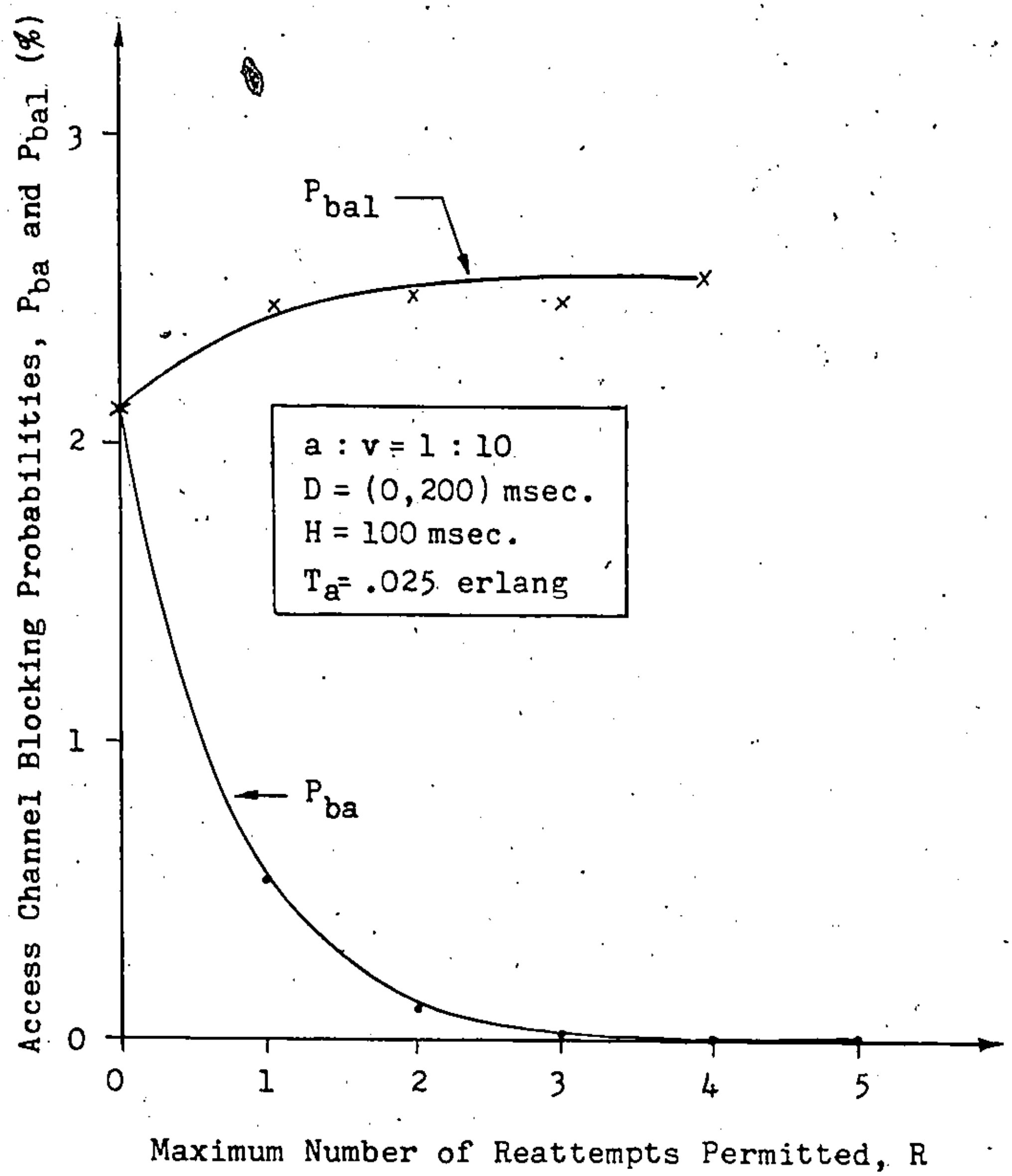


Figure 6.6 Variation in Mean Delay in Obtaining an Access Channel with Traffic Load



<u>Figure 6.7</u> Trade-off Between Access Channel Blocking Probabilities, P_{ba} and P_{bal}, As A Function of The Maximum Number of Reattempts Permitted

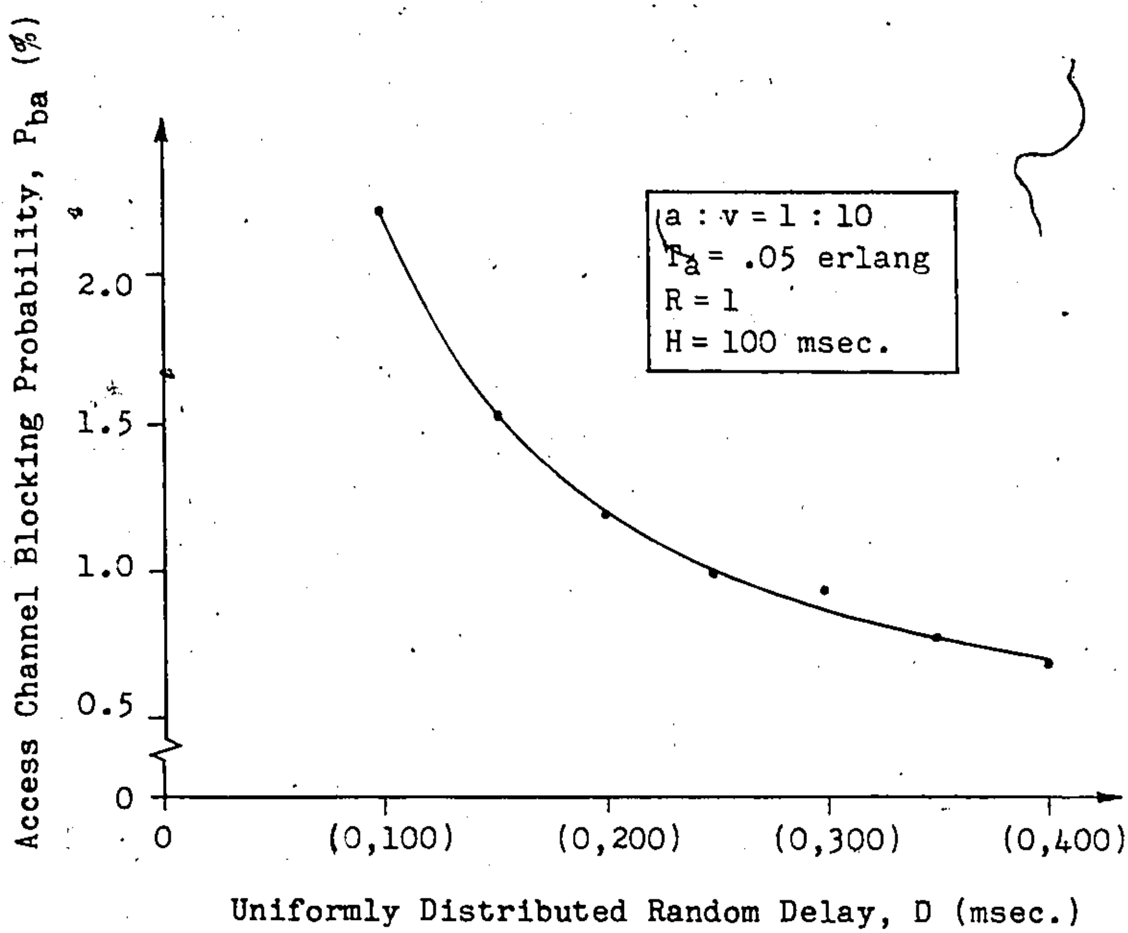


Figure 6.8 Change in The Access Channel Blocking Probability as the Random Delay Before Reattempting Access is Varied

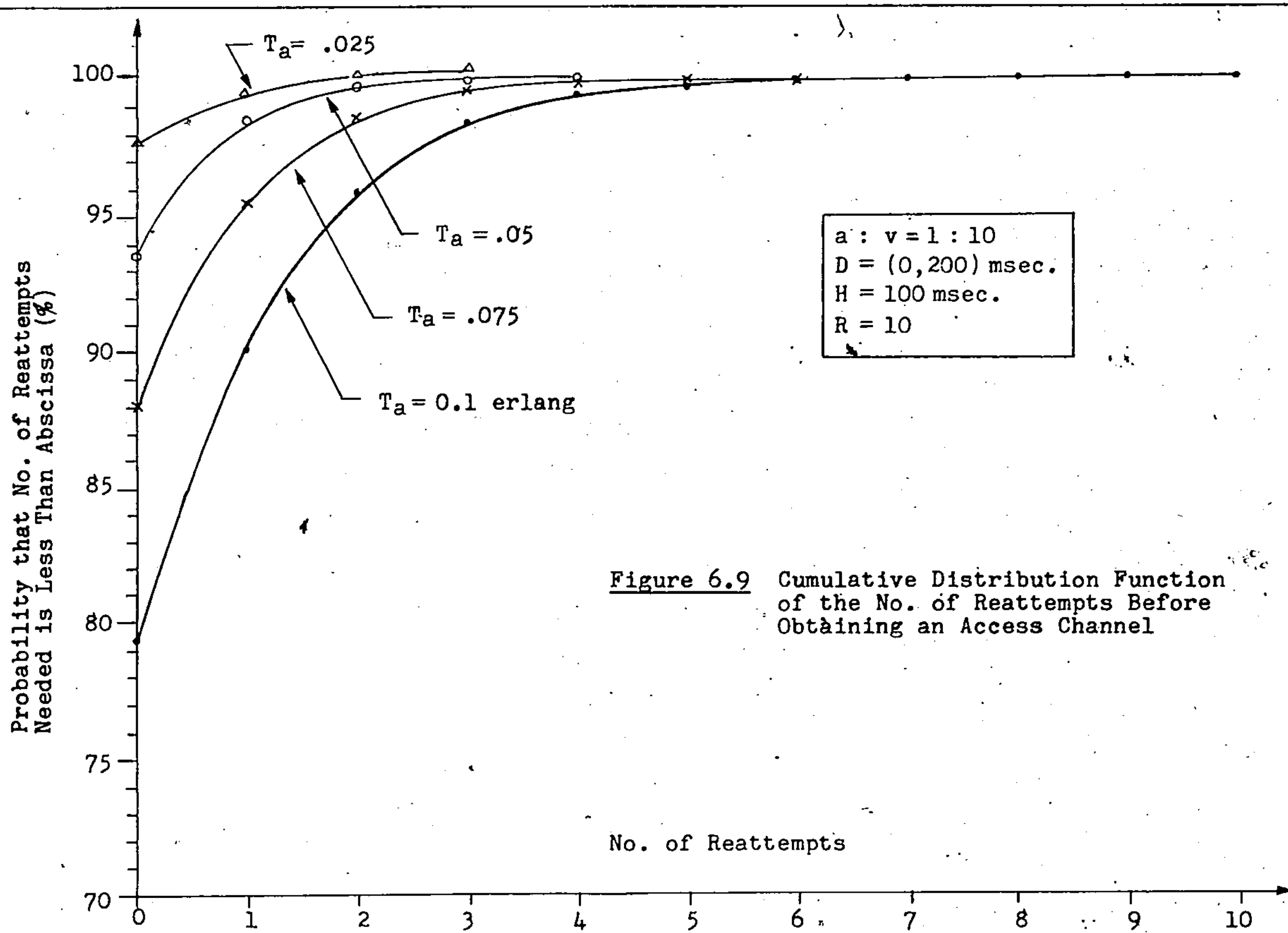


Figure 6.9 Cumulative Distribution Function of the No. of Reattempts Before Obtaining an Access Channel

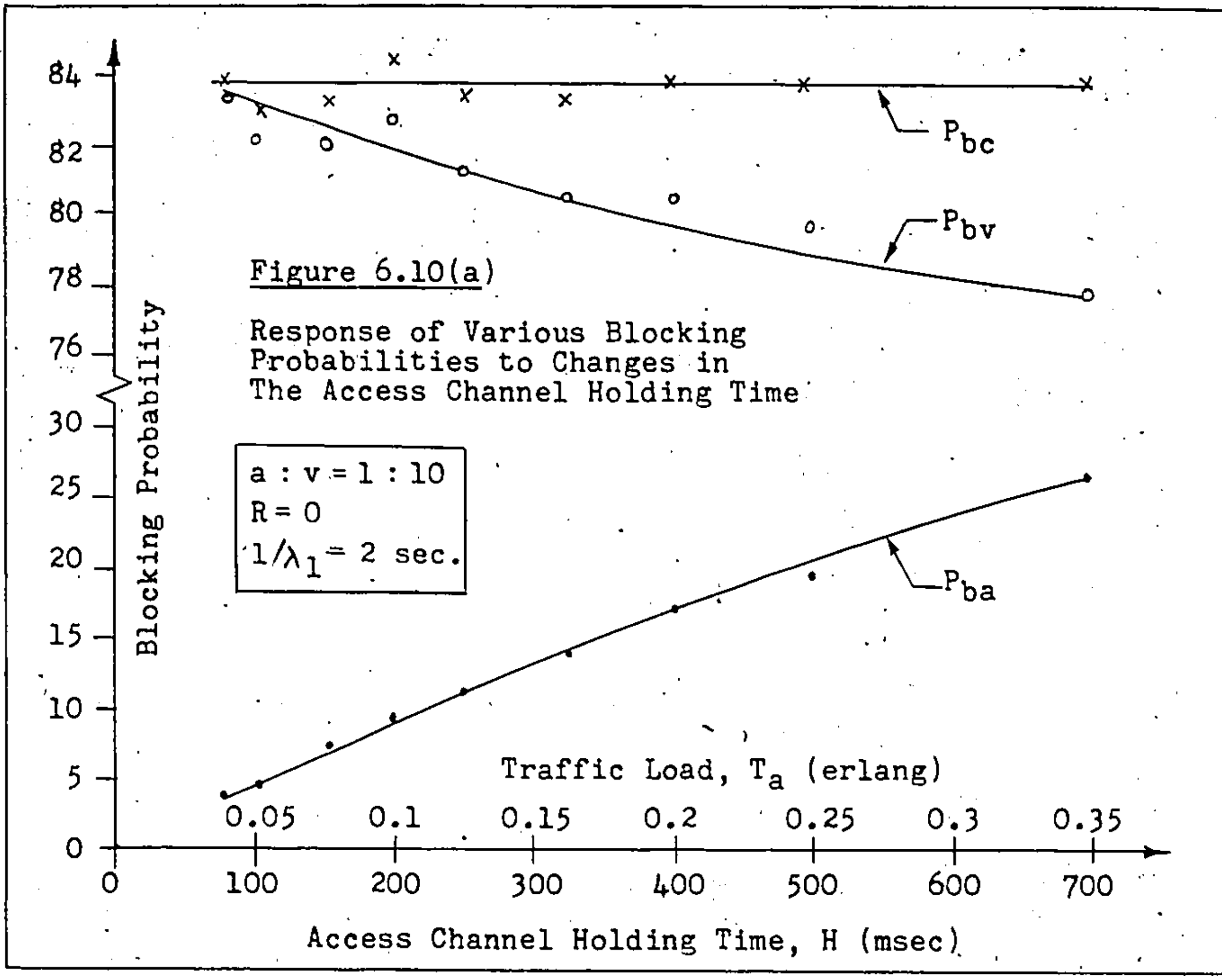


Figure 6.10(a)

Response of Various Blocking Probabilities to Changes in The Access Channel Holding Time

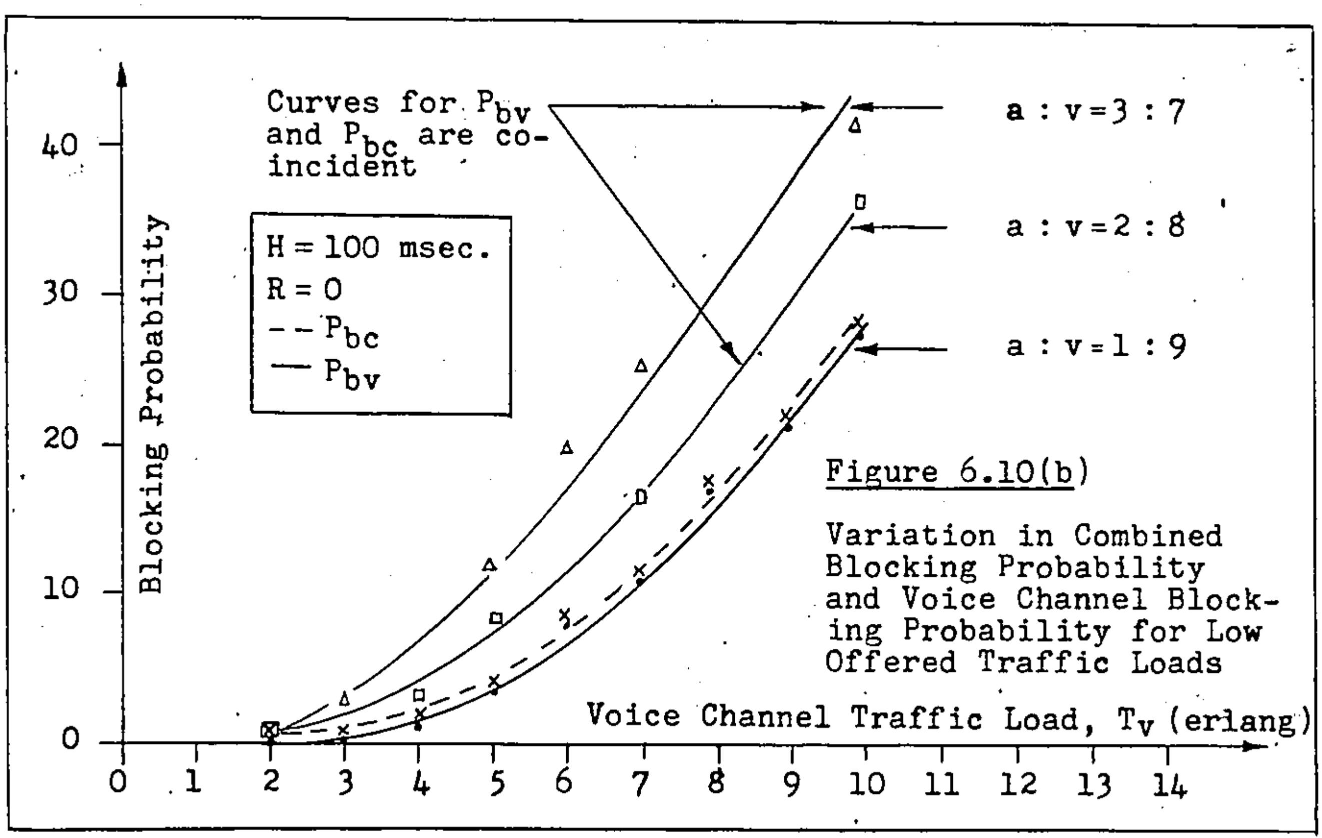


Figure 6.10(b)

Variation in Combined Blocking Probability and Voice Channel Blocking Probability for Low Offered Traffic Loads

Figure 6.4

Figure 6.4 shows the response of P_{ba} to increasing traffic loads for 1 or 2 access channels per cell. The case of 3 access channels per cell was also examined but found to produce inconsequentially small values (<0.01%) of P_{ba} for all traffic loads below 0.2 erlang.

At a later stage of the research the simulation model was modified to check for calls arriving within 7 msec of each other (corresponding to a vulnerable period of 14 msec as discussed in Section 5.2) Such calls are considered to have collided and must reattempt after a random delay D. The dotted curve in Figure 6.4 shows the results and it appears that Assumption 4 regarding collisions was fairly reasonable. A probability of collision (not blocking) was calculated from the simulation statistics. For R=0 and T_a=0.1 this probability was found to be 1.55% which compares favourably with the 1.4% determined theoretically in Section 5.2.

Theoretical results exist for two of the cases shown in Figure 6.4, namely:

(i) R=0; a=1

(ii) R=0; a=2

Case (i) is a special case of an $M/E_k/1/1$ queuing discipline

[19,41].[7] Here M signifies an exponential distribution for the interarrival pattern and E_k, an Erlang K distribution for the service time duration. The probability density function of the latter is given by [6,41]:

$$f(t) = \frac{(\mu k)^k t^{k-1} e^{-\mu k t}}{k!}, \quad t \geq 0 \tag{6.9}$$

The Erlang K distribution assumes many different forms when different values of k are used. Some of these forms are shown in Figure 6.11. We observe from Figure 6.11 or Equation (6.9) that the exponential (k=1) and constant (k=∞) service time models are particular cases within the Erlang K classification.

It is the k=∞ case which is of interest to us since the access channel holding time is constant -- fixed at 100 msec in most instances. It may be shown for the $M/E_k/1/1$ model that the blocking probability is simply equal to the traffic load [19]. Using the notation we have developed:

$$P_{ba} = T_a, \quad \left[\begin{matrix} R = 0 \\ a = 1 \end{matrix}\right] \tag{6.10}$$

[7] A/B/C/D is a shorthand notation for describing a queuing process. "A" indicates in some manner the interarrival time distribution, "B" the service pattern, "C" the number of servers (channels), and "D" the system capacity.

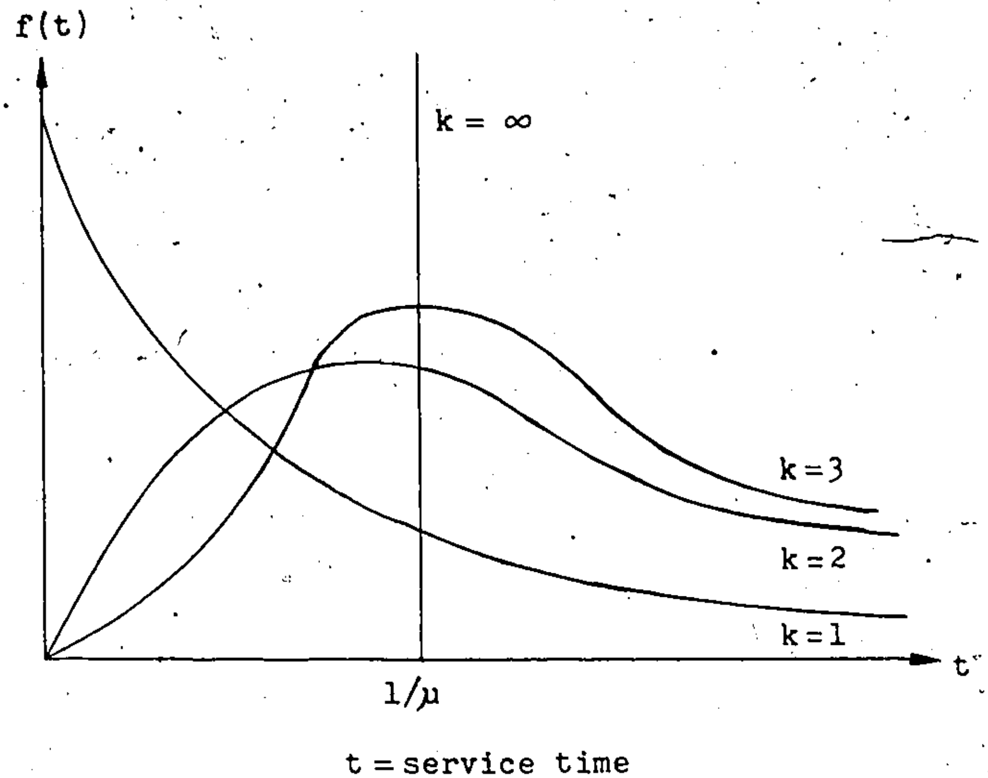


Figure 6.11 Erlang K Service Distribution with Mean Service Time $1/\mu$ [41]

In Figure 6.12(a) we have plotted (6.10) along with the corresponding simulation results. The curves coincide almost exactly. Some departure is evident at the higher levels of T_a but this *may* be attributable to the use of a truncated exponential function in the simulator. (In theory the exponential distribution extends to infinity so that there remains a finite probability of very long interarrival times. However, the maximum time between arrivals in the simulator is only about 10 seconds for a mean interarrival time, $1/\lambda_1$ = 1 second (T_a=0.1 erlang). So, as the traffic level nears 0.1 erlang and more values are drawn from this truncated distribution, inaccuracies become apparent.)

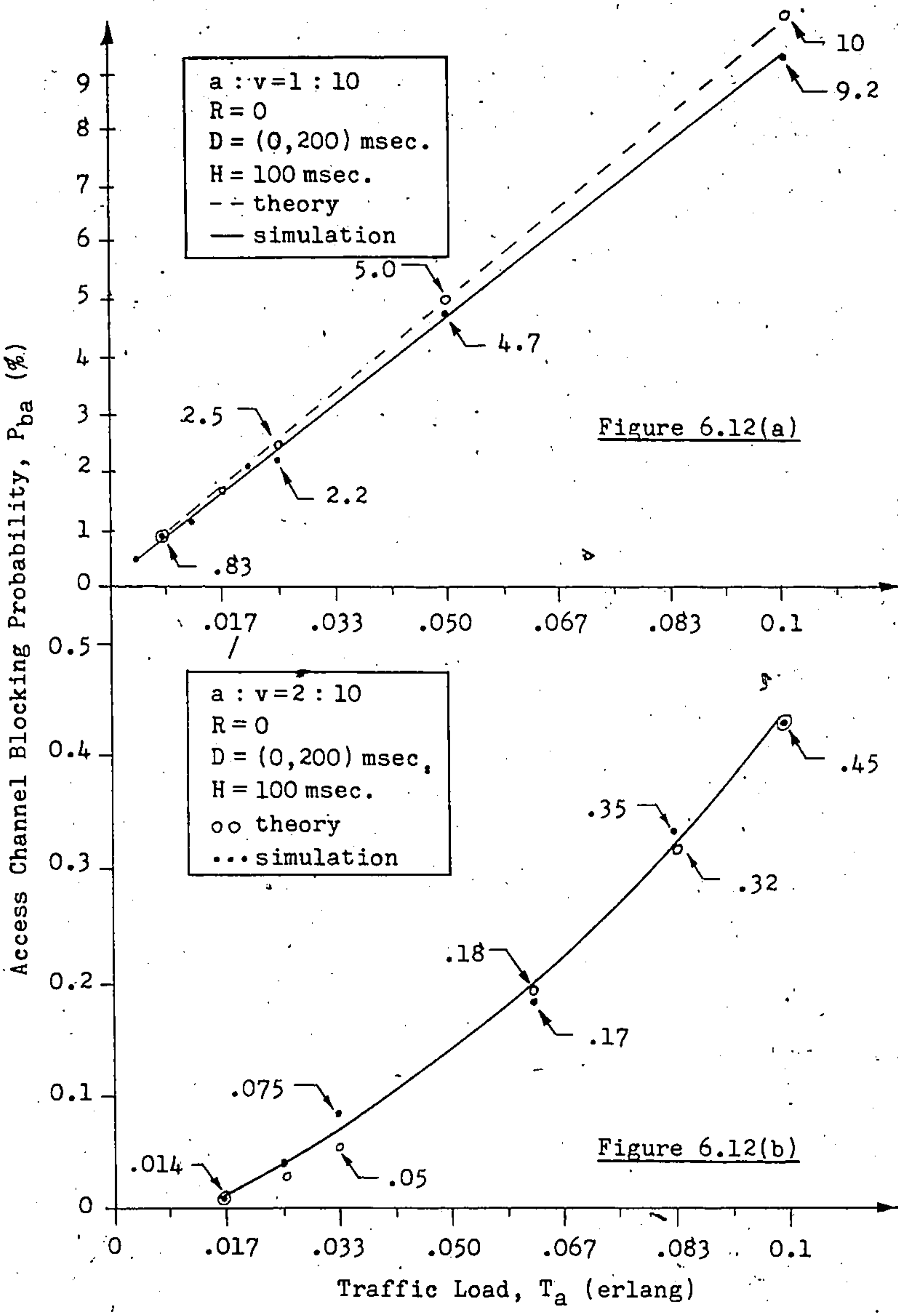


Figure 6.12 Comparison of Simulation and Theoretical Results for Access Channel Blocking Probability (For the Two Cases Where Theoretical Results Exist)

Case (ii) is a special case of an M/G/2/2 queuing discipline [19]. The "G" indicates a general distribution; that is, no assumption is made as to the precise form of the distribution. For the M/G/2/2 model it may be shown that the blocking probability is given by:

$$P_b = \frac{\lambda^2}{\mu^2+2\lambda\mu+2\lambda^2} \tag{6.11}$$

where λ is the arrival rate of customers and $1/\mu$ is the mean service time of these customers. For our purposes, $\lambda = \lambda_1 = \lambda_2$ and $1/\mu = H$. Since $T_a = \lambda_1 H$ $(\equiv \lambda/\mu)$ we may write (6.11) in the form:

$$P_{ba} = \frac{T_a^2}{2+2T_a+2T_a^2} \quad \begin{bmatrix} R=0 \\ a=2 \end{bmatrix} \tag{6.12}$$

In Figure 6.12(b) we have plotted (6.12) along with the simulation results. Agreement between the two is excellent.

Figure 6.5

Figure 6.5 contains all the information of Figure 6.4 but presented in a different manner. From this graph we can conclude that when the traffic load remains static, P_{ba} decreases very rapidly as the number of reattempts permitted increases. The following empirical formula has been found generally applicable:

$$P_{ba}\ (\%) = \frac{92T_a}{(R+1)^2}\ , \quad R=0,\cancel{1,2}\ldots m \qquad (6.13)$$

$$= 0\ , \quad R>m$$

(Where m is the largest integer less than or equal to $92T_a$. The nominal values of H and D, 100 msec and (0,200) msec respectively, are used and there is one access channel per cell).

Figure 6.6

The mean delays, $\bar{d}_1$ and $\bar{d}_2$, experienced by calls in their efforts to obtain an access channel follow the pattern shown in Figure 6.6. For a realistic measure of the delay these curves should be biased upward by about 500 msec. This 500 milliseconds is the time needed for the initial search for the correct signalling channel to use (Section 4.2). The search is performed once by the mobile logic unit; *not* before each and every reattempt to seize an access channel. For this reason we can tack on the half-second delay in an after-the-fact manner.

Note that the delay $\bar{d}_2$ rises sharply at first and then appears to saturate around 0.025 erlang. This is only temporary however, as the curves begin to rise again as T_a is increased beyond 0.1 erlang. Of course there will be an upper limit upon the delay. For example, if R=3 and D=(0,200) msec, the delay could never exceed 600 msec and would likely level off around 300 msec. (A mean delay of 100 msec is generated prior to each reattempt). In fact, it was observed that for an amount of traffic in the neighbourhood of 1 erlang and for the case of R=3, the delay $\bar{d}_2$ had reached 220 msec and was continuing to increase gradually.

Figure 6.7

Figure 6.7 illustrates an interesting result. The variation of P_{ba} and P_{ba1} with R for a fixed traffic load of 0.025 erlang has been plotted. Observe that P_{ba} declines rapidly as described by (6.13) while P_{ba1} rises slightly. The reason for this behaviour is not difficult to discover. As R is augmented, more and more calls are reattempting to gain access to the system. Therefore a newly-generated call is more likely to find the access channel(s) for its cell busy and will be forced to make at least one reattempt.

Thus a trade-off is apparent. Restrict the number of reattempts permitted ($R \leq 2$) and the probability of obtaining an access channel on the first attempt will be improved. Moreover the mean delays $\bar{d}_1$ and $\bar{d}_2$ will be smaller than otherwise. However, the long term blocking probabaility P_{ba} will be farly high -- approximately 1% or higher for one access channel per cell and a traffic greater than 0.02 erlang. Conversely, permitting a large number of reattempts will greatly reduce P_{ba} while degrading the system performance in terms of P_{ba1}, $\bar{d}_1$, and $\bar{d}_2$.

In connection with Figure 6.7, an attempt was made to check the validity of flow rate equations (6.2) to (6.8). At points within the simulation program corresponding to points in Figure 6.3 the mean arrival rates of calls λ_1, λ_2, λ_3 and λ_5, were recorded. The results are tabulated in Table 6.1 under the heading "Simulation". In the "Theory" columns we have applied (6.2) to (6.8) -- computing λ_1, λ_2 , and λ_5 from the observed values for λ_3 and the values found for P_{ba}, P_{ba1}, and P_{bv}. Agreement is excellent.

R	P_{bv} (%)	P_{ba} (%)	P_{bal} (%)	λ_3 (sec^{-1})	λ_1 (sec^{-1})		λ_2 (sec^{-1})		λ_5 (sec^{-1})	
				Simulation	Simulation	Theory*	Simulation	Theory**	Simulation	Theory***
0	67.1	2.24	2.24	.248	.254	.254	.254	.254	.082	.082
1	67.6	.606	2.45	.257	.258	.259	.266	.265	.080	.083
2	67.7	.087	2.54	.281	.281	.281	.294	.288	.087	.091
3	68.2	.02	2.5	.257	.257	.257	.267	.264	.083	.082

T_a = .025 erlang

* $\lambda_1 = (1 - P_{ba})^{-1} \cdot \lambda_3$

** $\lambda_2 = (1 - P_{ba})^{-1} \cdot (1 - P_{bal})^{-1} \cdot \lambda_3$, $R > 0$

$\lambda_2 = (1 - P_{ba})^{-1} \cdot \lambda_3$, $R = 0$

*** $\lambda_5 = (1 - P_{bv}) \cdot \lambda_3$

Table 6.1 A Check on Flow Rate Equations (6.2) to (6.8)

Figure 6.8

Figure 6.8 shows the variation in P_{ba} as a function of the random delay D which is generated just before each reattempt. Along the horizontal axis are plotted equal increments of the mean value of the uniform distribution for D. The reason for the decline in P_{ba} is simply that there are fewer calls reattempting to seize an access channel within any arbitrary time span as the random delay becomes lengthier.

If we make the assumption that as $D \rightarrow \infty$, the state of the system when a call makes its first reattempt is essentially independent of the state when a call made its first attempt to seize an access channel, we may compute the limiting value of P_{ba} in Figure 6.8. First from Figure 6.4, for T_a=.05 erlang, a:v=1:10 and R=0, we see that P_{ba} is roughly 5%. If we now allow one reattempt (R=1), as is the case in Figure 6.8, while keeping everything else the same then the blocking probability on the first attempt, P_{ba1}, will still be about 5%. (The rate at which access channel requests are made, λ_2, will be only slightly greater when R=1 compared to when R=0). The probability that a call is blocked on the 2'nd attempt (first reattempt) will also be about 5%. (This must be so since no distinction is made between a call making its first attempt and a call making its first reattempt or tenth reattempt for that matter. The total traffic incident on an access channel is merged without regard to the amount of time a call may have already waited).

The limiting value of P_{ba} in Figure 6.8 as $D \to \infty$ is then given by the product of the probabilities associated with the (assumed) independent events that a call is blocked on its first attempt and its first reattempt as well; that is:

$$\left.\begin{aligned} \lim_{D\to\infty} P_{ba} &= (P_{ba1})^2 \\ &= (.05)^2 \\ &= 0.25\% \end{aligned}\right] \quad R=1$$

Similar arguments may be applied for any value of R. In general:

$$P_{ba} = P_{ba1} \quad , \quad R=0$$

$$\lim_{D\to\infty} P_{ba} = (P_{ba_1})^{R+1} \quad , \quad \begin{array}{l} R=1,2,3... \\ T_a \text{ fixed} \end{array} \qquad (6.13)$$

Can we now find a formula for P_{ba1} (and hence P_{ba}) in terms of the access channel traffic load as $D \to \infty$?* To begin, refer to Figure 6.13 which shows a sequence of service

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* The basic idea for this development was suggested to the author by the description of the contention-based ALOHA system in References 1 and 2.

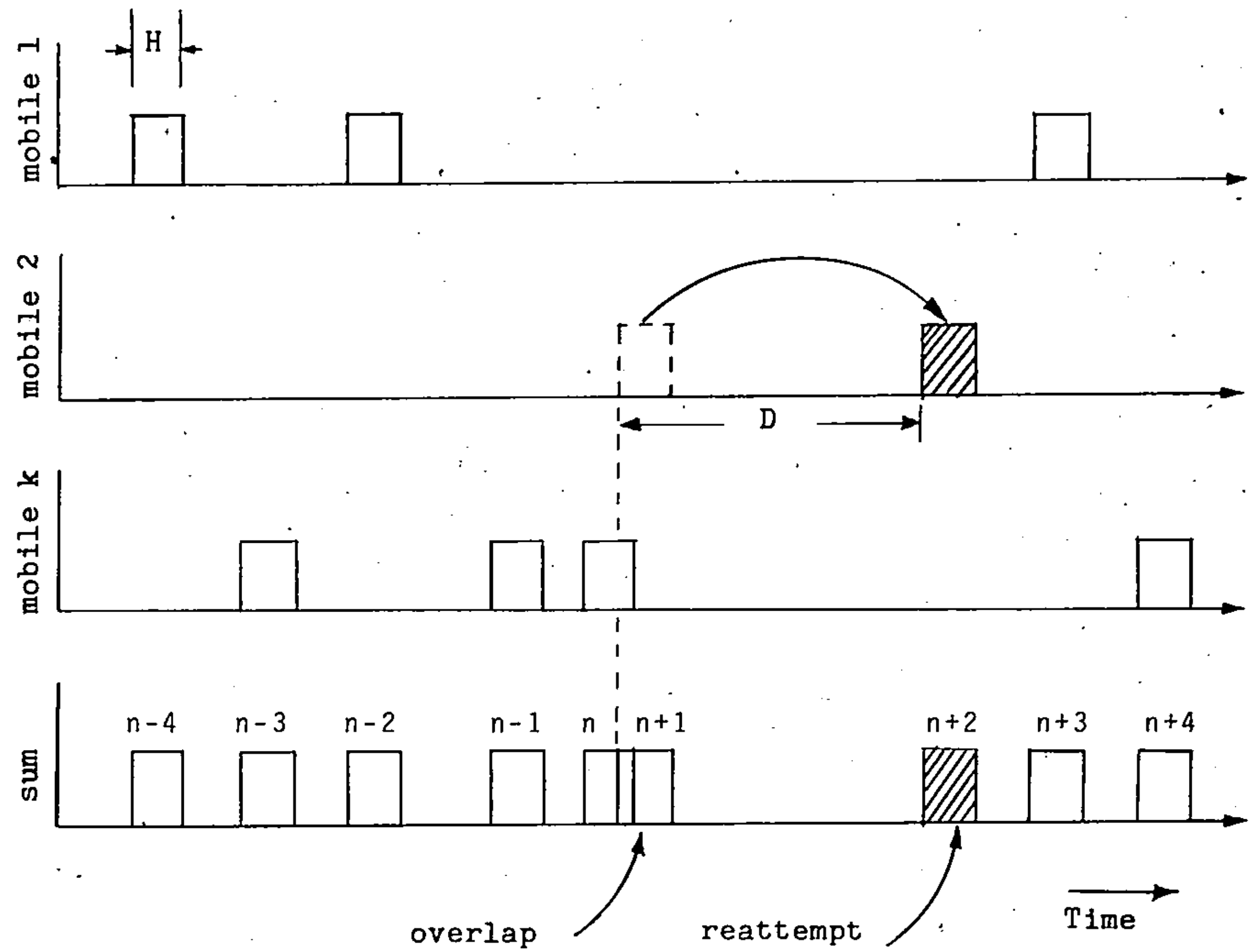


Figure 6.13 Seizure Messages Transmitted by Independent Mobile Callers

requests made by independent mobile callers at random points in time and within the same cell. Note the overlap of two seizure messages. Mobile k's request was made first; he may continue to use the channel. Mobile 2's request will have to repeated though, D msec later. (Do not confuse this with the collision problem where *both* requests had to reattempt. Mobile 2 here would have found the busy/idle bits for this signalling channel set to busy thereby preventing such a collision).

The net traffic incident on an access channel, consisting of initial call attempts and any reattempts is, by definition, $\lambda_2 H$. We will call this traffic T_a' to distinguish it from T_a $(=\lambda_1 H)$ which we have been using exclusively up to this point.

As $D \to \infty$, the point process defined by the start time of all attempts plus reattempts may be considered Poisson [1,19]. That is, our assumption of an exponential interarrival time distribution may still be applied. (Making such an assumption when D=(0,200) msec would be grossly incorrect since the time of a reattempt would still be highly correlated with the time of the previous (re)attempt).

With reference to Figure 6.13, we see that the probability of request n+1 from mobile 2 finding the access channel busy is simply the probability that this request is made within H msec of the last such request, n, from mobile k. Thus:

$$P_{ba1} = 1 - e^{-\lambda_2 H} \tag{6.14}$$
$$= 1 - e^{-T_a'}$$

Since P_{ba1} is the probability that a call will need to reattempt, the average number of reattempts per unit time is:

$$\lambda_2(1-e^{-T_a'}) \tag{6.15}$$

So that:

$$\lambda_2 = \lambda_1 + \lambda_2(1-e^{-T_a'}) \tag{6.16}$$

or:

$$T_a = T_a' \cdot e^{-T_a'} \tag{6.17}$$

It is clear that T_a/T_a' is the probability of a call successfully obtaining an access channel, therefore:

$$P_{ba1} = 1-\frac{T_a}{T_a'} = 1-e^{-T_a'} \tag{6.18}$$

Equation (6.18) can not be expressed explicity in terms of T_a; it is necessary to calculate T_a' for a given T_a using (6.17) and then insert this value of T_a' into (6.18).

We may now reformulate (6.13):

$$\left.\begin{array}{ll} P_{ba} = 1-e^{-T_a'} , & R=0 \\ \lim\limits_{D\to\infty} P_{ba} = (1-e^{-T_a'})^{R+1} , & R=1,2,3... \\ & T_a<0.1 \text{ erlang} \end{array}\right] \quad (6.19)$$

A number of simulation trials were carried out for D=(0,10000), for R=1, and for various traffic levels T_a, in order to test the validity of (6.19). The simulation and analytical results are compiled and compared in Table 6.2. We have also included a comparison when no reattempts are allowed. In such cases D has no relevance and T_a and T_a' are identical but it is interesting to note that the theoretical values for P_{ba} are still quite close to the values obtained by simulation. As T_a increases, however, the divergence in T_a and T_a' as computed from (6.17) will grow wider and (6.19) can no longer be used for R=0. (Equation (6.10) will continue to be valid though). This is why we have stipulated that T_a be less than 0.1 erlang in (6.19).

Finally, for the example which we have already worked out (R=1, T_a =.05 erlang) where it was predicted that $\hat{P}_{ba}$ would approach 0.25%, Table 6.2 shows that the simulation result was 0.28% while the theoretical result was 0.24%. So the three different approaches produce remarkably consistent results.

R	T_a (erlang)	T_a' (erlang)	P_{ba} (%) Theory	Simulation	
1	0.1	0.112	1.12	0.97	*
1	0.083	0.0908	0.76	0.73	*
1	0.067	0.072	0.48	0.45	*
1	0.05	0.0528	0.24	0.28	*
1	0.025	0.0257	0.063	0.068	*
1	0.0167	0.017	0.029	0.06	*
1	0.0083	0.00837	0.007	0	*
0	0.1	0.112	10.6	9.2	**
0	0.083	0.0908	8.7	7.9	**
0	0.067	0.072	6.95	6.0	**
0	0.05	0.0528	4.9	4.8	**
0	0.025	0.0257	2.5	2.1	**
0	0.0167	0.017	1.7	1.7	**
0	0.0083	0.00837	0.84	0.83	**

* in the limit as $D \to \infty$

** independent of D

Table 6.2 Comparison of Simulation and Theory for The Access Channel Blocking Probability

Figure 6.9

For this set of simulation trials we fixed R at 10, cognizant of the fact that few, if any, callers would actually use up their allotment of 10 reattempts and thus be denied service. The number of calls making i reattempts was recorded; i=0 implying that a call found the access channel idle during its initial seizure attempt.

The graphs indicate that even for the highest traffic load considered -- 0.1 erlang -- 80% of callers obtain an access channel on their very first try. And effectively 100% of callers require 4 attempts or less. This results is not too surprising since for T =0.1 erlang, the mean time between call arrivals is 1000 msec -- still ten times greater than the length of time a call occupies an access channel.

Figure 6.10

The last two figures in this chapter show the effects of the access channel logic upon the voice channel assignment scheme. Figure 6.10(a) serves to confirm a result which could have been predicted from Equation (6.1). Due to blocking over the access channels the mean interarrival time of calls to the voice channels is effectively lengthened. Since there are fewer calls competing for the voice channels, P_{bv} must fall. Just what is the extent of this effect? For the case shown in Figure 6.10(a), R=0 and from Equation (6.10), $P_{ba}=T_a$. Thus Equation (6.1) becomes:

$$\left.\begin{aligned} T_v &= \frac{F}{\mu_v H}\cdot(1-T_a)\cdot T_a \\ &= \frac{F}{\mu_v}\cdot(1-T_a)\cdot\lambda_1 \\ &= 60(1-T_a) \end{aligned} \qquad \left[\begin{matrix} 0\le T_a\le 1 \\ R=0 \\ a=1 \end{matrix}\right]\right] \qquad (6.20)$$

So as T_a continues to increase, T_v will continue to decrease. When T_a nears the <u>saturation level</u>' of 1 erlang for this single access channel, virtually no calls will be able to get past the access channel logic and P_{bv} will be almost zero.

In Table 6.3 we compare the simulation results for P_{bv} against the theoretical results computed from the Erlang B formula (Appendix A). Agreement is excellent. There is some discrepancy for the larger values of H but this is only because P_{ba} is actually less than T_a in this region while in (6.20) P_{ba} was set equal to T_a.

For larger values of R we can still expect P_{bv} to decline as T_a increases but the effect will become less noticeable simply because P_{ba} will be less.

' The saturation level of x channels would be x erlangs of traffic.

H (msec)	T_a (erlang)	T_v (erlang)	P_{bv} % Theory	P_{bv} % Simul.
75	.0375	57.75	83	83.4
100	.05	57	82.8	82.3
150	.075	55.5	82.4	82
200	.1	54	81.9	82.3
250	.125	52.5	81.4	81.4
325	.1625	50.25	80.6	80.7
400	.2	48	79.7	80.5
500	.25	45	78.4	79.8
700	.35	39	75.2	78.5

Table 6.3 Interaction Between T_a and T_v; Comparison of Simulation and Theoretical Results for The Voice Channel Blocking Probability ($1/\lambda_1 = 2$ seconds)

Lastly, we observe in Figure 6.10(a) that P_{bc} remains constant as T_a increases. This result seems quite reasonable in light of the previous discussion concerning the interaction between T_a and T_v and between P_{ba} and P_{bv}. If only a small percentage of the calls entering the system obtain an access channel, then a large percentage of those calls which filter through to the voice channel assignment logic should be able to obtain a voice channel. The converse is also true. Thus the relative magnitudes of the numerator and denominator in the definition of P_{bc} will remain fairly constant.

Figure 6.10(b) is included to emphasize the important point that as more channels within any cell are apportioned to the signalling function in an effort to reduce P_{ba}, there will be fewer channels available for the voice function, causing P_{bv} to rise.

Chapter VII

SIMULATION MODEL NO. 2

The signalling protocol of the present chapter differs slightly from the version already studied. Notably, an access timer is added.

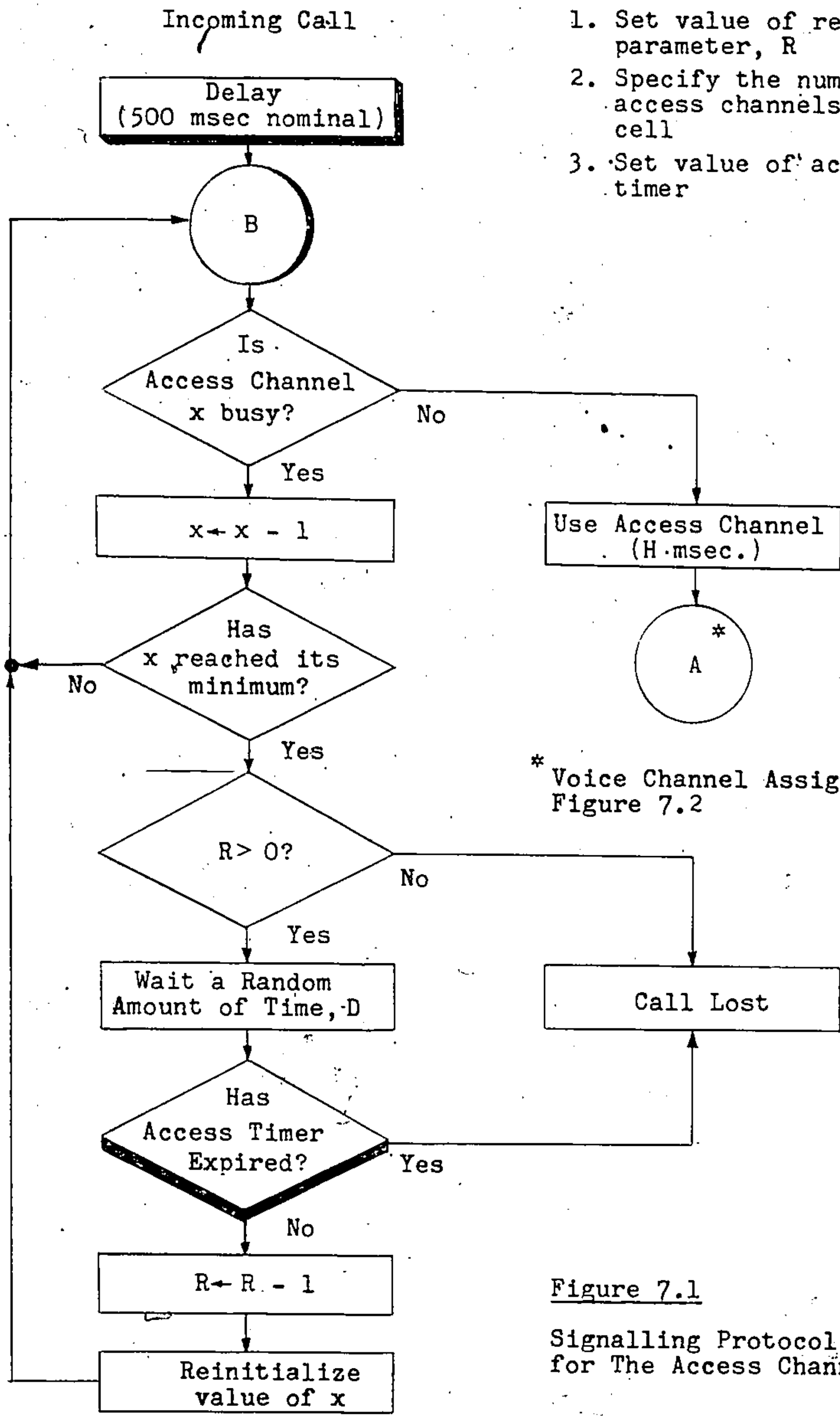
A hybrid voice channel assignment scheme is now used. The FCAS described earlier may be considered as a special case of this hybrid scheme. The service discipline for the voice channels can be termed "Blocked Calls Held". As the name implies, calls are prepared to wait a short time if necessary for a voice channel assignment.

7.1 THE SIGNALLING PROTOCOL

The protocol flowchart is shown in Figure 7.1. The same general comments made in the first paragraph of Section 6.1 also apply here. The differences between this flowchart and that of Figure 6.1 have been emphasized with a heavy line. These differences are:

1. An initial delay of 500 msec to model the time taken by the mobile logic unit to (i) scan the signalling channels, (ii) lock onto the one with the strongest signal and (iii) acquire word sync.

1. Set value of reattempt parameter, R
2. Specify the number of access channels per cell
3. Set value of access timer



* Voice Channel Assignment; Figure 7.2

Figure 7.1

Signalling Protocol No. 2 for The Access Channels

2. A 6-second access timer which keeps track of the amount of time each call has used up while attempting to seize an access channel (including the 500 msec delay) or while waiting for a voice channel designation.
3. A call which has obtained an access channel but was unable to find a voice channel may re-enter the system at point B.

Expanding upon items 2 and 3, a call which does not find a voice channel immediately will now queue for service, up to a maximum of 5 seconds. During this time a suitable voice channel may become free in which case the call is served. If after 5 seconds, however, the call is still waiting, it is purged from the queue and the status of the access timer is examined. If the timer has run out, the call is simply tabulated as blocked and leaves the system forever. If the timer has not run out, the call will attempt to re-access the system; that is, it will once again go through the necessary steps to obtain an access channel. The reattempt parameter, however, will not be reinitialized.

Given that a call entering the system for a second time is successful in finding an idle access channel it then searches anew for a free voice channel. It may find a channel right away or it may have to rejoin the queue as the last member. In the event that the call remains in the queue for the full 5 seconds it will be purged as before. At this stage the 6-second timer will have definitely run out and the call will simply be tabulated as blocked.

In summary, a call has at least one 5-second period, but no more than two such periods, in which to receive a voice channel assignment. Each time a call is purged from the queue due to the 5-second time limit, the status of the access timer is examined to see if processing should proceed any further. In addition, the status of the access timer and the reattempt parameter are checked prior to each reattempt to seize an access channel. If at any point either one of these constraints is exceeded then the call is tabulated as blocked and leaves the system forever.

The access channel logic and voice channel logic are now closely interlocked. The magnitude of P_{bv} will determine the number of calls which re-enter the system and these returning calls will place a greater load upon the access channels. The complexity of this interaction makes a detailed mathematical analysis intractable.

7.2 THE VOICE CHANNEL ASSIGNMENT SCHEME

For this simulation model, a Hybrid Channel Assignment Scheme (HCAS) is employed [18,24,36]. As before we have N=3 channel sets each of which is apportioned initially into "a" access channels and "v" voice channels. Every cell is assigned one of the three sets and the sets are reused at relative spacings, D/R=3. Thus far the situation is identical to that shown in Figure 5.1.

From the v voice channels, d channels are subtracted and donated to a central pool. Once this is accomplished there are f=v-d fixed voice channels remaining in each channel set and 3d dynamic channels belonging to the central pool. The dynamic channels may be used in any cell i as long as:

1. The D/R constraint for simultaneous usage is met. For example, in Figure 5.1, a dynamic channel in use in central cell 1 can not be re-used in any of the cells 2 through 7 which border directly on cell 1. In the same manner one can visualize an interference cell group for any cell x consisting of cell x itself and the ring of cells surrounding cell x.
2. A fixed voice channel is not available in cell i when a service request is made in cell i.

Since this channel assignment scheme is neither pure fixed nor pure dynamic (where any channel may be used in any cell provided the D/R rule is respected), the label "hybrid" is applied.

For the hybrid scheme the choice of N is important. No longer are the cells isolated from one another -- they now share a common resource, namely the dynamic voice channels and N directly affects the size of this resource. The choice of N=3 is in keeping with past studies [18,24,36].

The flowchart of the voice channel assignment algorithm is shown in Figure 7.2. It is reasonably self-explanatory so only the less obvious features will be discussed. One of these features is the channel reassignment technique: whenever a fixed channel becomes free in cell i, a call using a dynamic channel in cell i is switched over to the fixed channel. The motive for doing this is to try and pack channel assignments as close together in space as possible. For a system consisting of 40 cells and for N=3, no channel can be in simultaneous use in more than 13 or 14 cells. This limit is extremely unlikely to be reached for a dynamic channel; however; calls are initiated at random and invariably a dynamic channel will be assigned to serve calls in cells which are more than one reuse interval (D/R) away from each other. Fixed voice channels, on the other hand, have been assigned to cells beforehand so that the D/R constraint is *always* met but *never* exceeded. Channel reassignment frees a dynamic channel for possible further use elsewhere.

Once a call has completed its service time its "life" in the simulation program is not over. Call completion changes the state of the system so what better way to investigate the repercussions of these changes than to use the call which caused them? Thus we see in the flowchart of Figure 7.2 that call x, vacating a fixed or dynamic channel, "hangs around" for a while to perform the channel reassignment task

* A dynamic channel must satisfy the cochannel interference constraint for cell i. That is, it can not already be in use in cell i or any of the cells bordering directly on cell i. The first such channel found in the search is used.

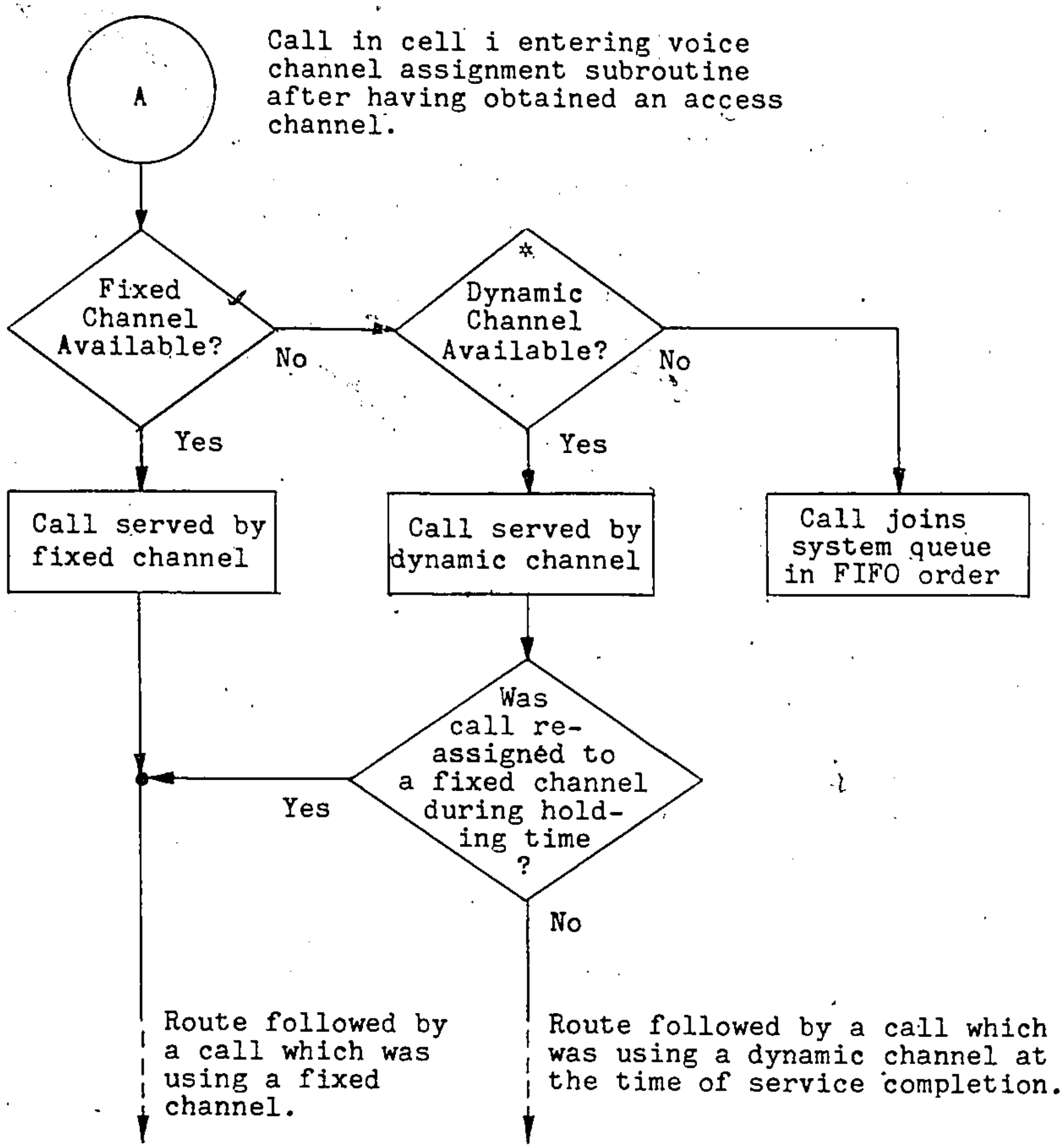


Figure 7.2 Hybrid Voice Channel Assignment Scheme and Blocked Calls Held Service Discipline

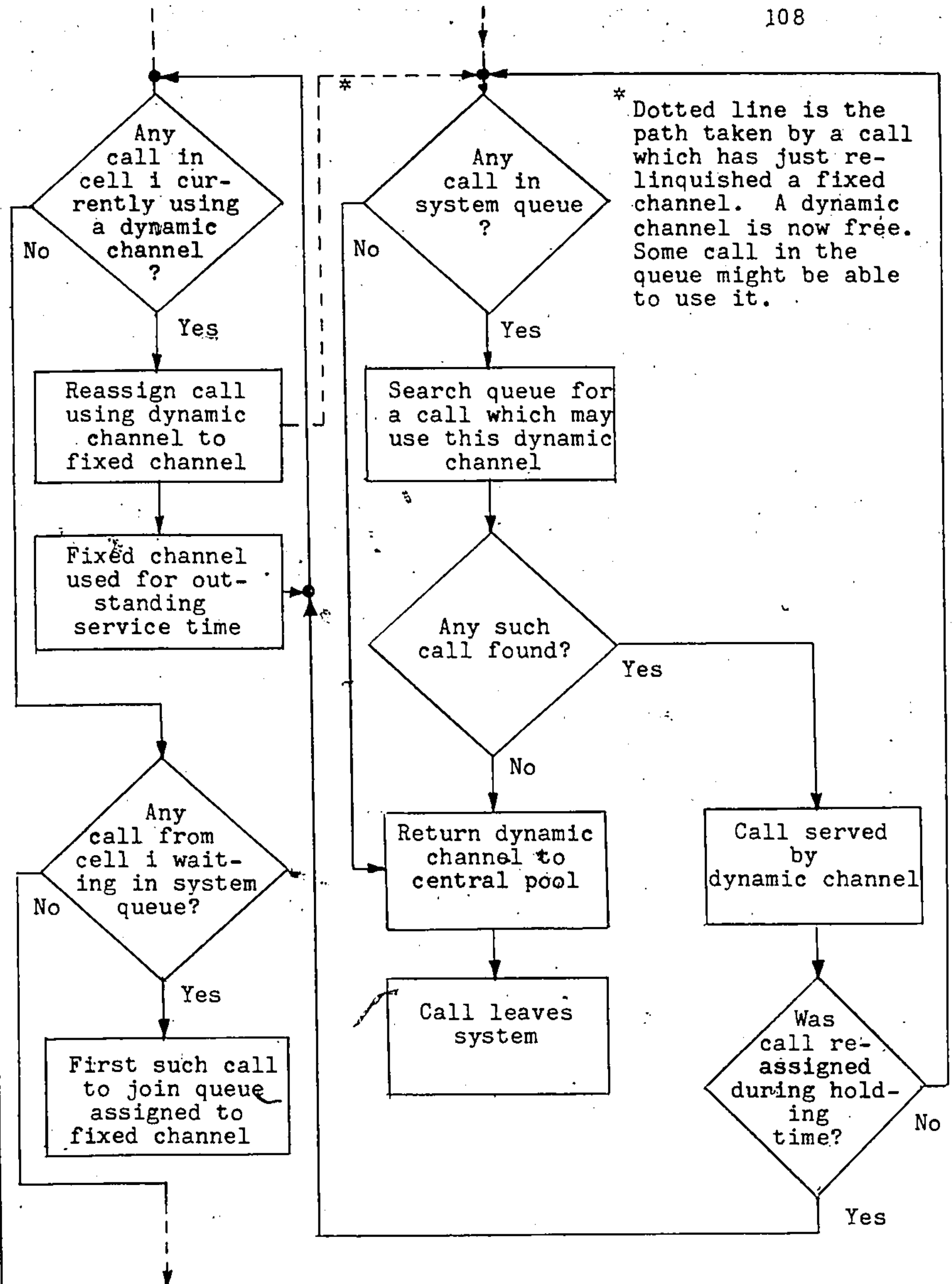


Figure 7.2 (cont'd)

Hybrid Voice Channel Assignment Scheme and Blocked Calls Held Service Discipline

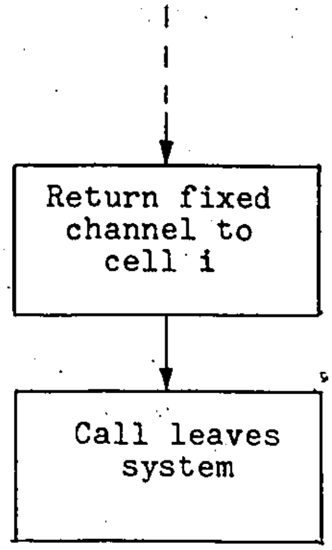


Figure 7.2 (cont'd)

Hybrid Voice Channel Assignment Scheme and Blocked Calls Held Service Discipline

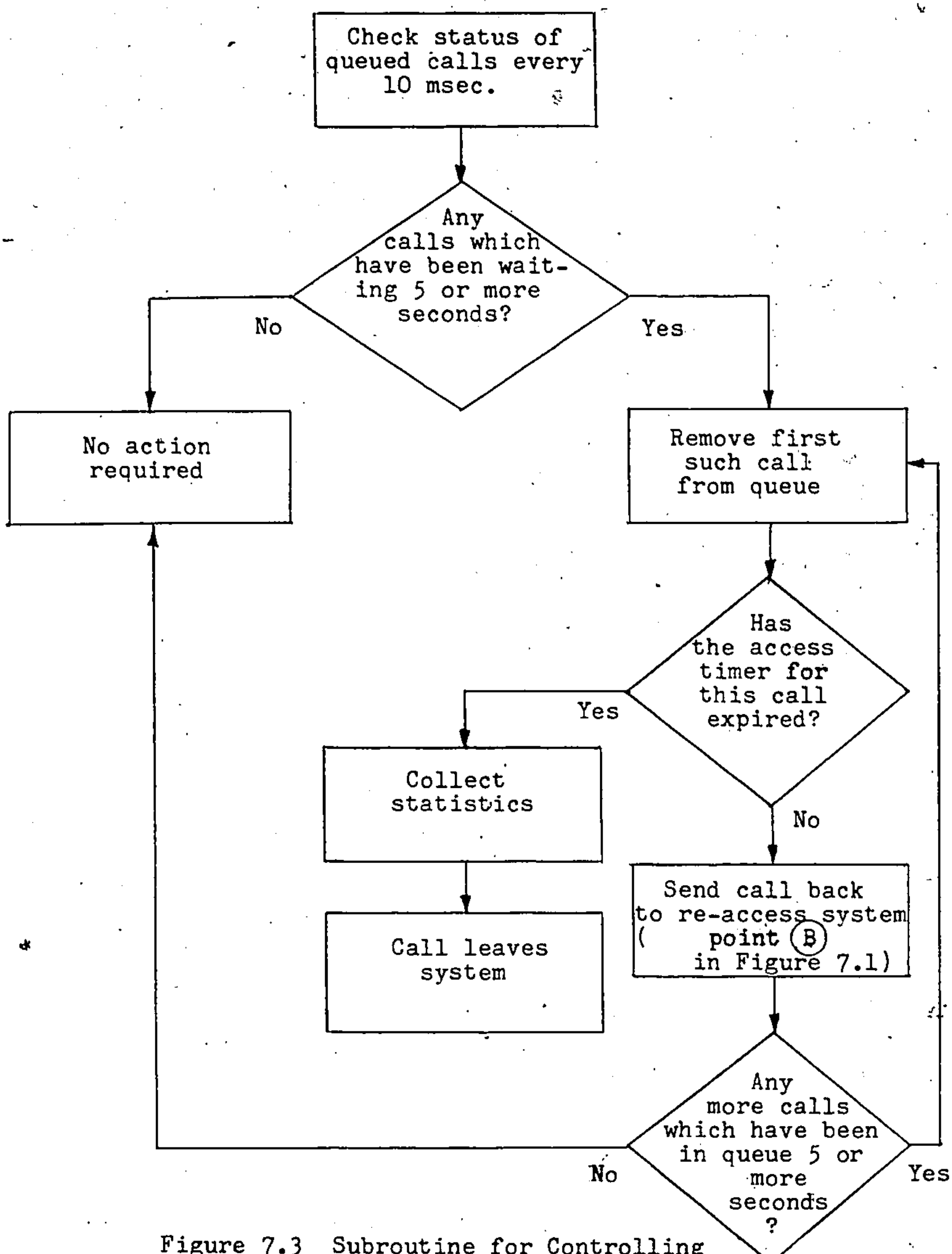


Figure 7.3 Subroutine for Controlling Time Spent by Calls in System Queue

or to check if some other call in the system queue, waiting for service, might be able to use the channel just relinquished by call x.

It is not evident from Figure 7.2 alone that a Blocked Calls Held service discipline has been implemented for the voice channels. The subroutine of Figure 7.3 operates independently of the main program and is responsible for removing calls from the queue which have been waiting for service for 5 seconds. Such calls are allowed to re-enter the system at point Ⓑ in Figure 7.1 provided the access timer has not run out; otherwise they are lost forever.

Lastly, why was the HCAS included here but not in the previous model? Simply because a great many HCASs have been studied already [8,10,11,15,18,24,36,38] in the absence of any signalling logic. Because the signalling logic of Model 1 really did not affect the voice channel assignment scheme, there wasn't much point in duplicating these past efforts. In the current model, though, its a different story as portended at the end of Section 7.1.

7.3 SYMBOLS AND DEFINITIONS

In addition to the definitions of the last chapter several new quantities are defined below. Subscripts have the following signifigance: b indicates blocking, q indicates queuing, s indicates service, "a" refers to the access channels, and v refers to the voice channels.

a:f:d = channel division ratio on a per cell basis; a=the number of access channels per cell, f=the number of fixed (voice) channels per cell and d=the *average* number of dynamic (voice) channels per cell (the number of channels in the system pool divided by N).

P'_{ba} = access channel blocking probability (because the reattempt allocation is consumed) upon second entry into the system.

$$= \frac{\left[\begin{array}{c}\text{no. of calls which fail to obtain an access channel}\\ \text{upon 2'nd entry due to the reattempt limit}\end{array}\right]}{\left[\begin{array}{c}\text{no. of calls purged from queue and which}\\ \text{subsequently attempt to re-enter the system}\end{array}\right]}$$

P''_{ba} = access channel blocking probability (because the access timer expires) upon second entry into the system.

$$= \frac{\left[\begin{array}{c}\text{no. of calls which do not obtain an access channel}\\ \text{upon 2'nd entry due to access timer limitation}\end{array}\right]}{\left[\begin{array}{c}\text{no. of calls purged from the queue and which}\\ \text{subsequently attempt to re-enter the system}\end{array}\right]}$$

P_{bv} = long term voice channel blocking probability.

$$= \frac{\text{no. of calls which fail to obtain a voice channel}}{\left[\begin{array}{c}\text{no. of calls which initially enter the voice}\\ \text{channel assignment subroutine}\end{array}\right]}$$

P_{qv1} = probability of queuing for a voice channel assignment upon first entry into the system.

$$= \frac{\left[\begin{array}{c}\text{no. of calls which join system queue upon 1'st} \\ \text{entry into voice channel assignment subroutine}\end{array}\right]}{\left[\begin{array}{c}\text{no. of calls which initially enter the voice channel} \\ \text{assignment subroutine}\end{array}\right]}$$

P_{qv2} = probability of queuing for a voice channel assignment upon second entry into the system.

$$= \frac{\left[\begin{array}{c}\text{no. of calls which join system queue upon 2'nd} \\ \text{entry into the voice channel assignment subroutine}\end{array}\right]}{\left[\begin{array}{c}\text{no. of calls which enter the voice channel} \\ \text{assignment subroutine a 2'nd time}\end{array}\right]}$$

P_{sv1} = probability of receiving a voice channel assignment upon first entry into the system.

$$= \frac{\left[\begin{array}{c}\text{no. of calls which obtain a voice channel} \\ \text{upon 1'st entry}\end{array}\right]}{\left[\begin{array}{c}\text{no. of calls which initially enter the voice channel} \\ \text{assignment subroutine}\end{array}\right]}$$

P_{sv2} = probability of receiving a voice channel assignment upon second entry into the system.

$$= \frac{\left[\begin{array}{c}\text{no. of calls which obtain a voice channel} \\ \text{upon 2'nd entry}\end{array}\right]}{\left[\begin{array}{c}\text{no. of calls which enter the voice channel} \\ \text{assignment subroutine a 2'nd time}\end{array}\right]}$$

Notes:

1. The definition of P_{ba} remains the same as in Chapter 6 but now applies only to those calls entering the system for the first time. So P_{ba} will still be due to the restriction upon R and a comparison can be made with the results of Chapter 6.

   What is the reason for distinguishing between calls coming into the system for the first or second time when defining P_{ba}, P'_{ba}, and P''_{ba}? Well, calls which are purged from the queue and re-enter the system do so with an unknown portion of their reattempt allocation remaining unlike newly-generated calls which all have the same initial value for R. In other words, calls re-entering the system are no longer on an equal standing and it would become very difficult to interpret the response of the access channel blocking probability to changing conditions if we did not filter out and examine separately the new calls and the returning calls.

2. Delays $\bar{d}_1$ and $\bar{d}_2$ reflect the delay experienced by newly-arriving calls alone.

3. No easy means exist for relating the traffic loads offered to the voice channels and the access channels as was the case in the last chapter. To simplify matters, assume that P''_{ba} is small enough to be neglected. The simulation results will later show that this is nearly always the case.

Refering to Figure 7.4, we know that λ_3 consists of two components: (a) calls entering the voice channel assignment subroutine for the first time and (b) calls which have been purged from the queue, manage to get past the access channel logic again and finally enter the voice channel assignment subroutine a second time. Label these two components λ_{3a} and λ_{3b} respectively, then:

$$\lambda_{3a} = \lambda_1(1-P_{ba}) \qquad (7.1(a))$$

$$\lambda_{3b} = \left[\lambda_1(1-P_{ba})(1-P_{vs1})\right](1-P'_{ba}) \qquad (7.1(b))$$

The quantity in square brackets is the rate at which calls re-enter the system. This number is then reduced

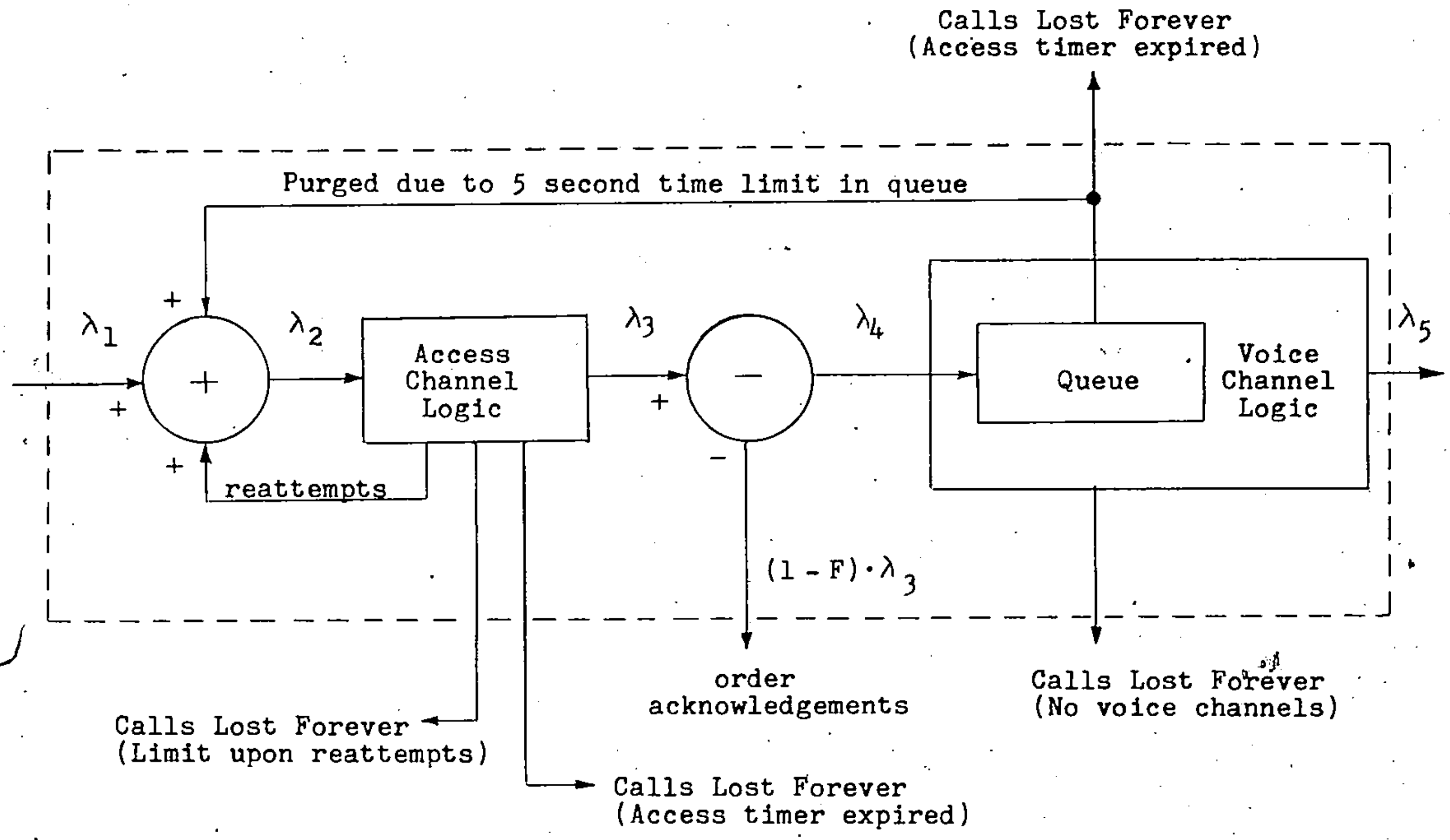


Figure 7.4 Call Flow Rates Through Simulation Model No. 2

by $1-P'_{ba}$ in order to eliminate the returning calls which are lost forever. The net rate of arrival of calls into the voice channel subroutine is:

$$\lambda_3 = \lambda_{3a} + \lambda_{3b}$$
$$= \lambda_1(1-P_{ba})(1+(1-P'_{ba})(1-P_{sv1})) \qquad (7.2)$$

Also:

$$\lambda_4 = F\lambda_3 \; , \qquad (7.3)$$

where F was defined in Chapter 6.

As before, $T_v = \lambda_4/\mu_v$ and $T_a = \lambda_1 H$ so that by combining (7.1) and (7.2) we now have:

$$T_v = \frac{F}{\mu_v H}\cdot(1-P_{ba})(1+(1-P'_{ba})(1-P_{sv1}))\cdot T_a \quad \text{(erlang)} \qquad (7.4)$$

Equation (7.4) is the relationship we seek between the access channel and the voice channel traffic loads. Finally, the following statements can also be made regarding Figure 7.4:

$$\lambda_5 = \lambda_{3a}\cdot P_{sv1} + \lambda_{3b}\cdot P_{sv2}$$
$$= \lambda_1(1-P_{ba})\left[P_{sv1} + P_{sv2}(1-P_{sv1})(1-P'_{ba})\right] \qquad (7.5)$$

$$\lambda_3 = (1-P_{ba1})(1-P_{ba})(1-P_{ba})\cdot\lambda_2 \ , \ R>0 \tag{7.6}$$

In deriving equations (7.1) to (7.6) we have assumed that the various arrival rates λ_i all describe exponential distributions. This is obviously true of λ_1 but we can not be certain that it is still true for say λ_5. This point will be considered further in the next section.

4. The various performance measures P'_{ba}, P''_{ba}, P_{bv}, etc., were calculated for only the central 20 cells when dynamic voice channels were involved. This was done because cells near the perimeter of the system have fewer neighbours and hence a greater likelihood of obtaining a given dynamic channel than cells near the center. The peripheral cells will therefore experience a lower blocking probability.

7.4 THE SIMULATION RESULTS

The simulation results are shown in Figures 7.5 to 7.18. Where possible, results from Chapter 6 have been included for comparison purposes. These results are drawn with a dashed line.

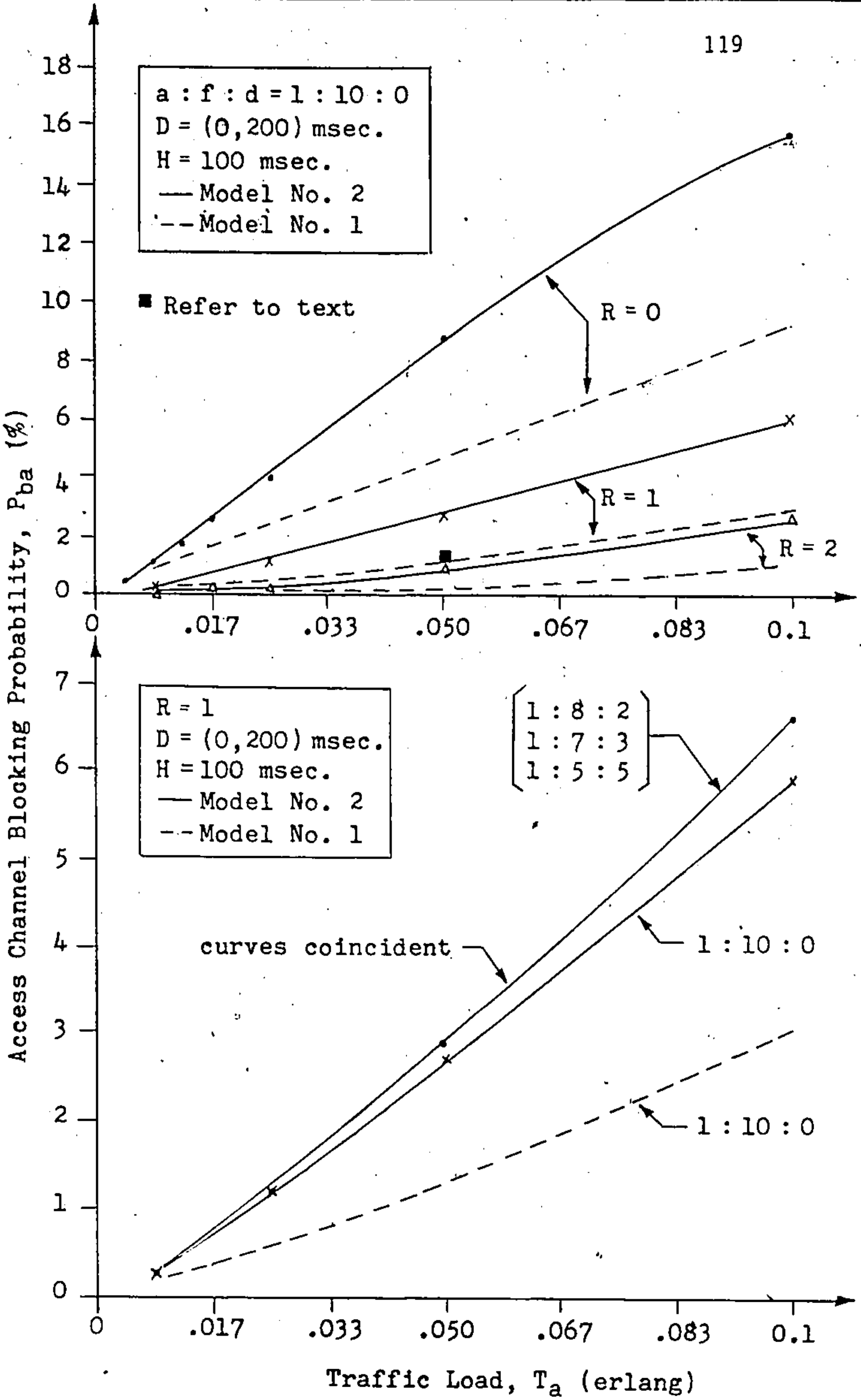


Figure 7.5 Response of Access Channel Blocking Probability to Increasing Traffic Loads; 10 Voice Channels & 1 Access Channel Per Cell On The Average

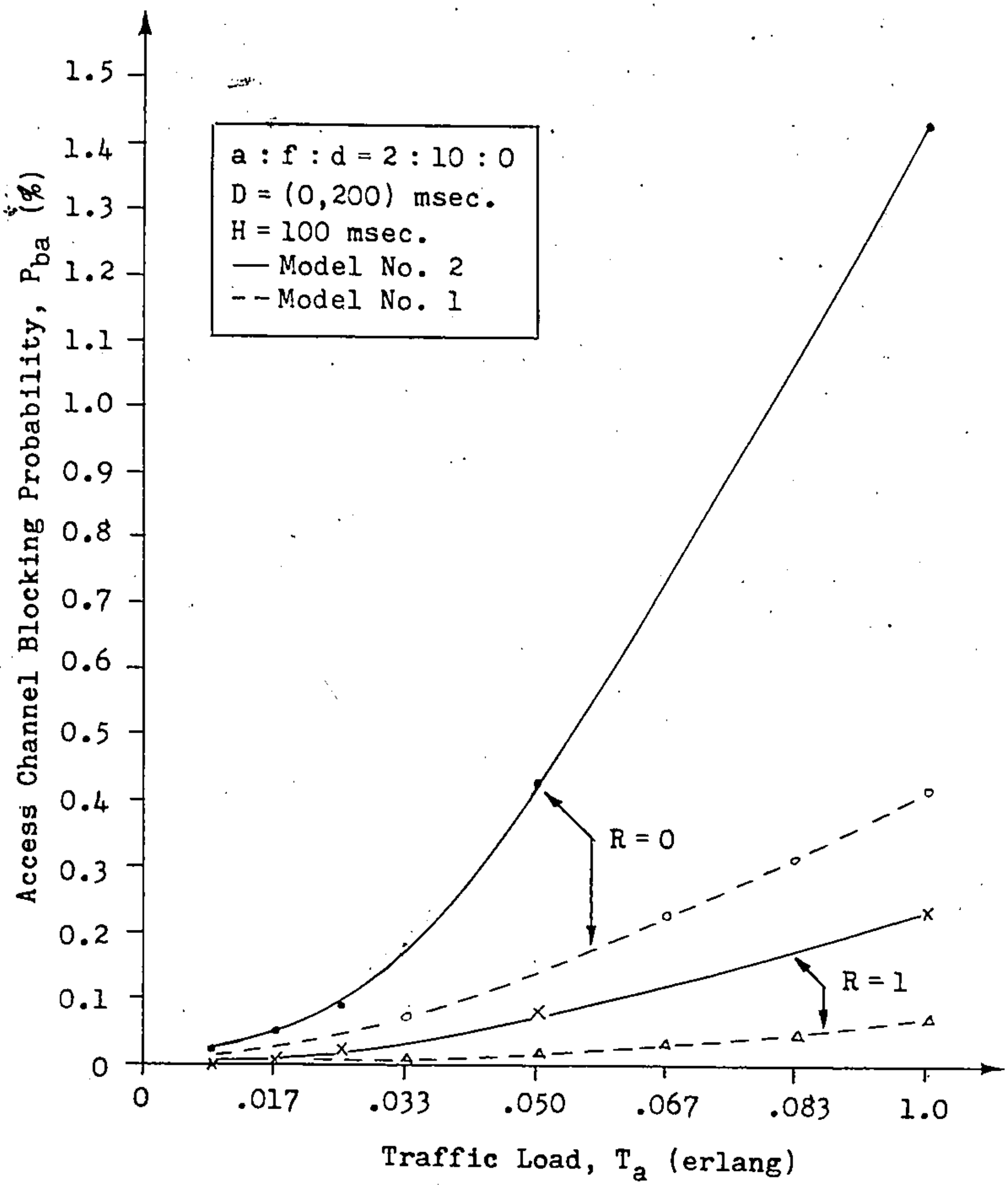


Figure 7.6 Response of Access Channel Blocking Probability to Increasing Traffic Loads; 10 Voice Channels & 2 Access Channels Per Cell

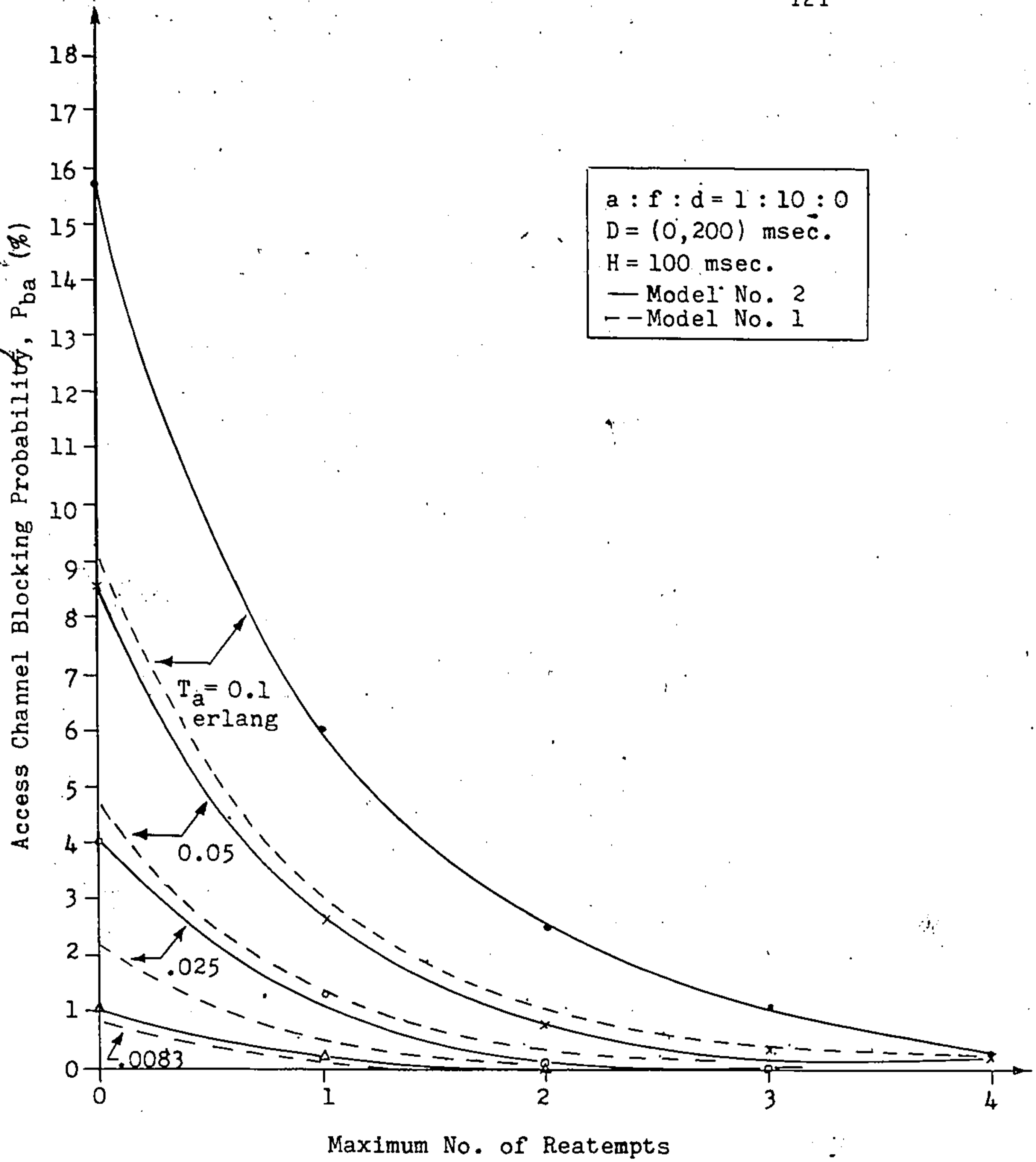


Figure 7.7 Response of the Access Channel Blocking Probability to Changes in The Maximum No. of Reattempts Permitted

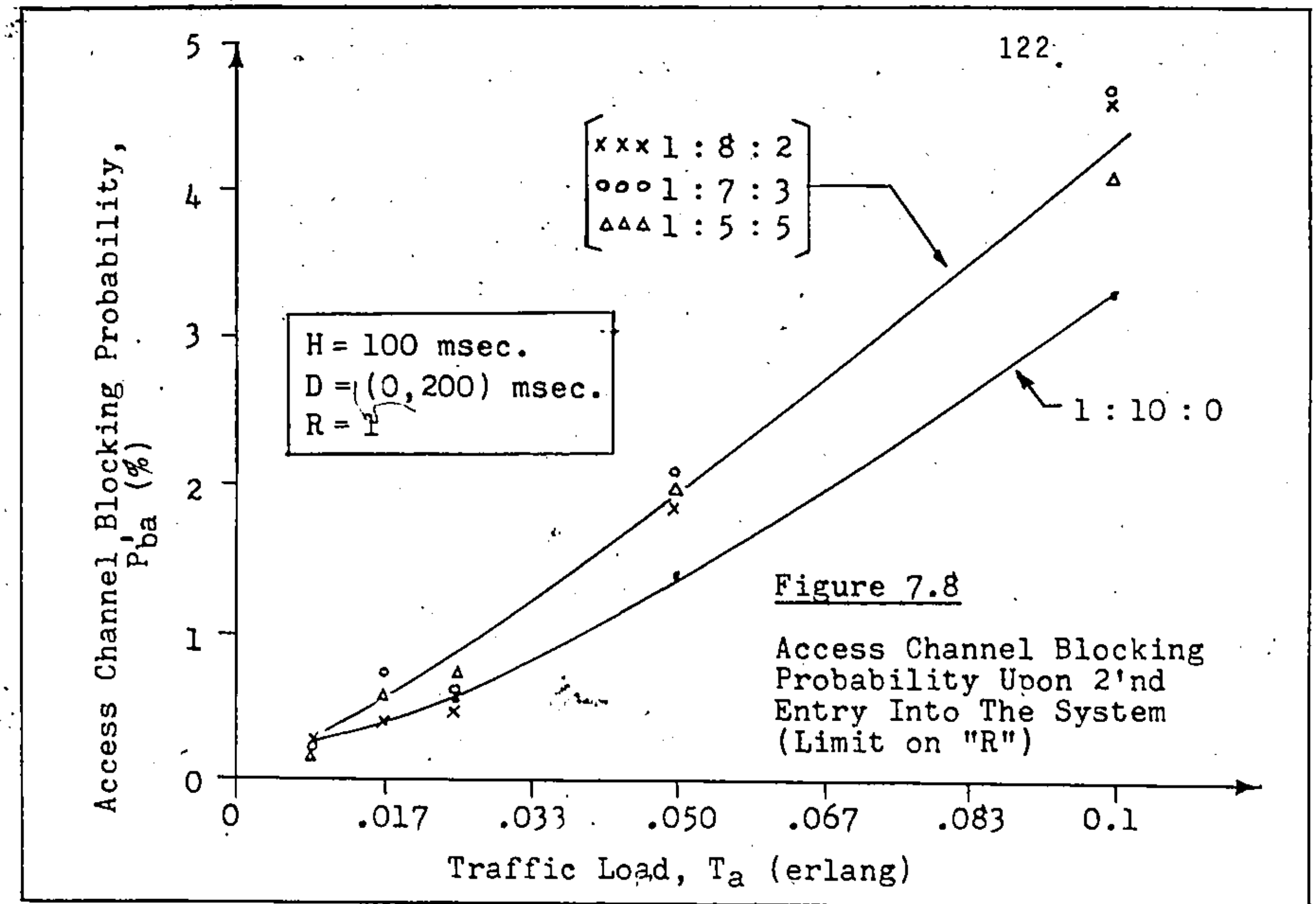


Figure 7.8

Access Channel Blocking Probability Upon 2'nd Entry Into The System (Limit on "R")

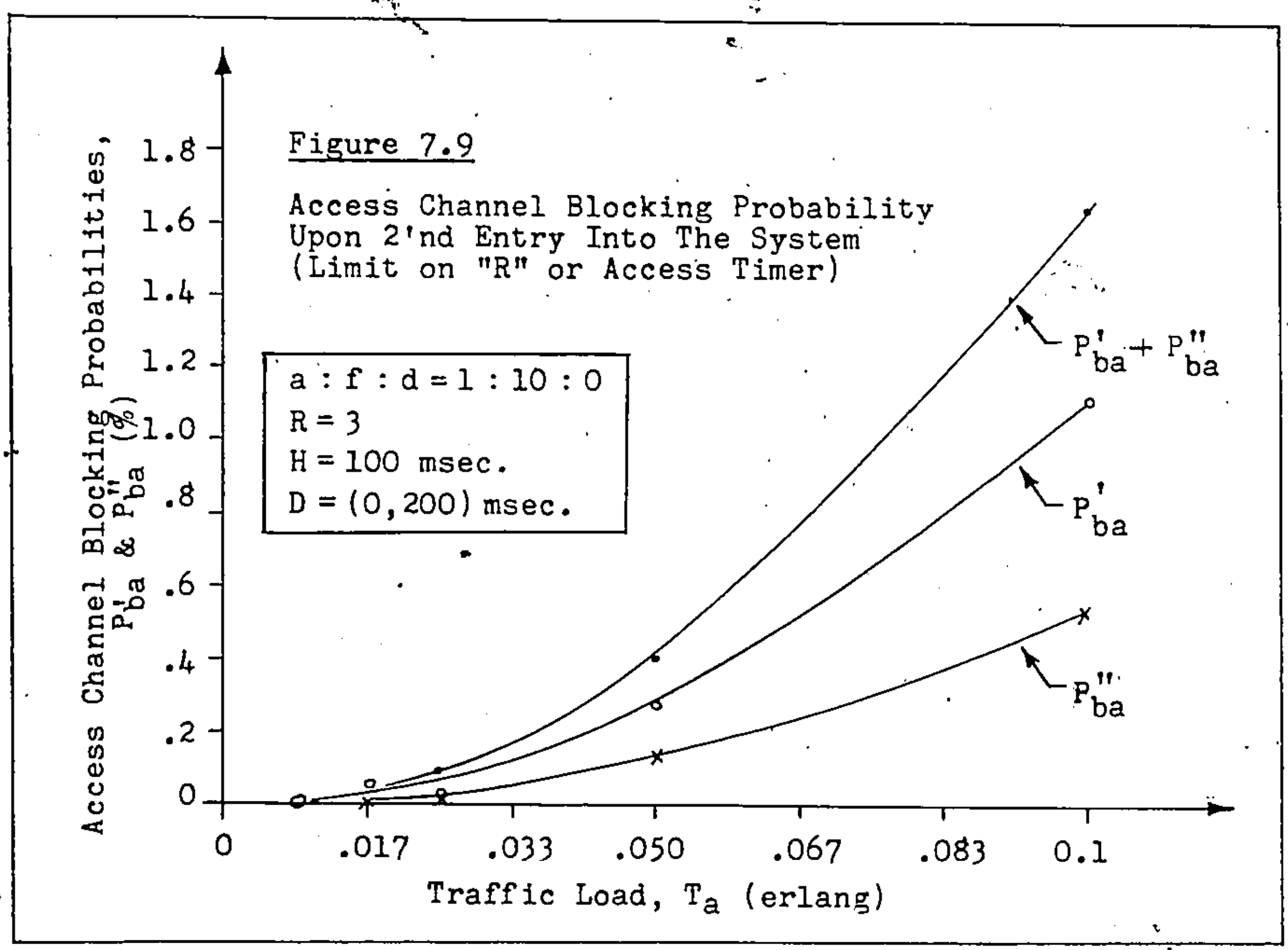


Figure 7.9

Access Channel Blocking Probability Upon 2'nd Entry Into The System (Limit on "R" or Access Timer)

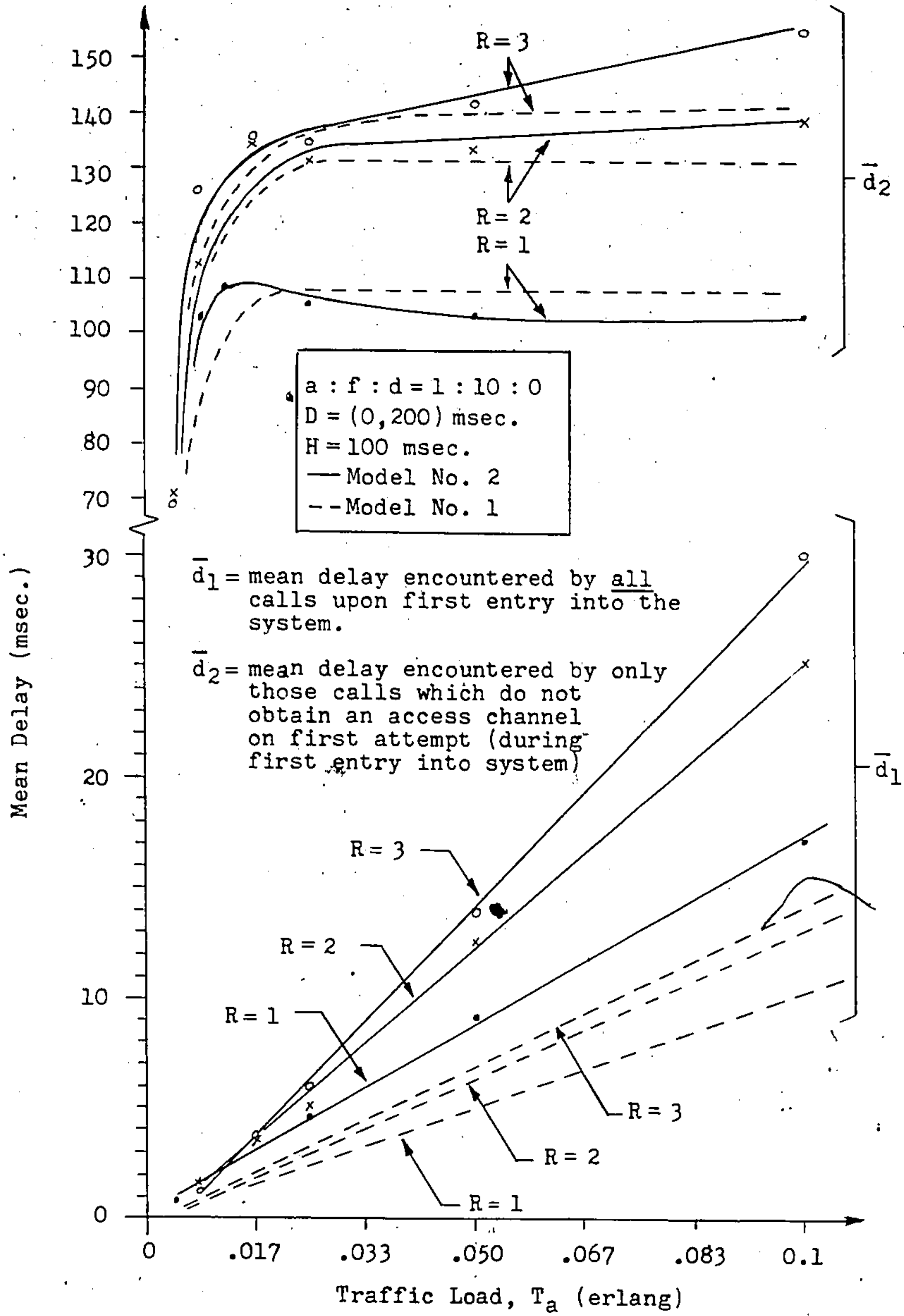


Figure 7.10 Variation in the Mean Delay in Obtaining an Access Channel with Traffic Load

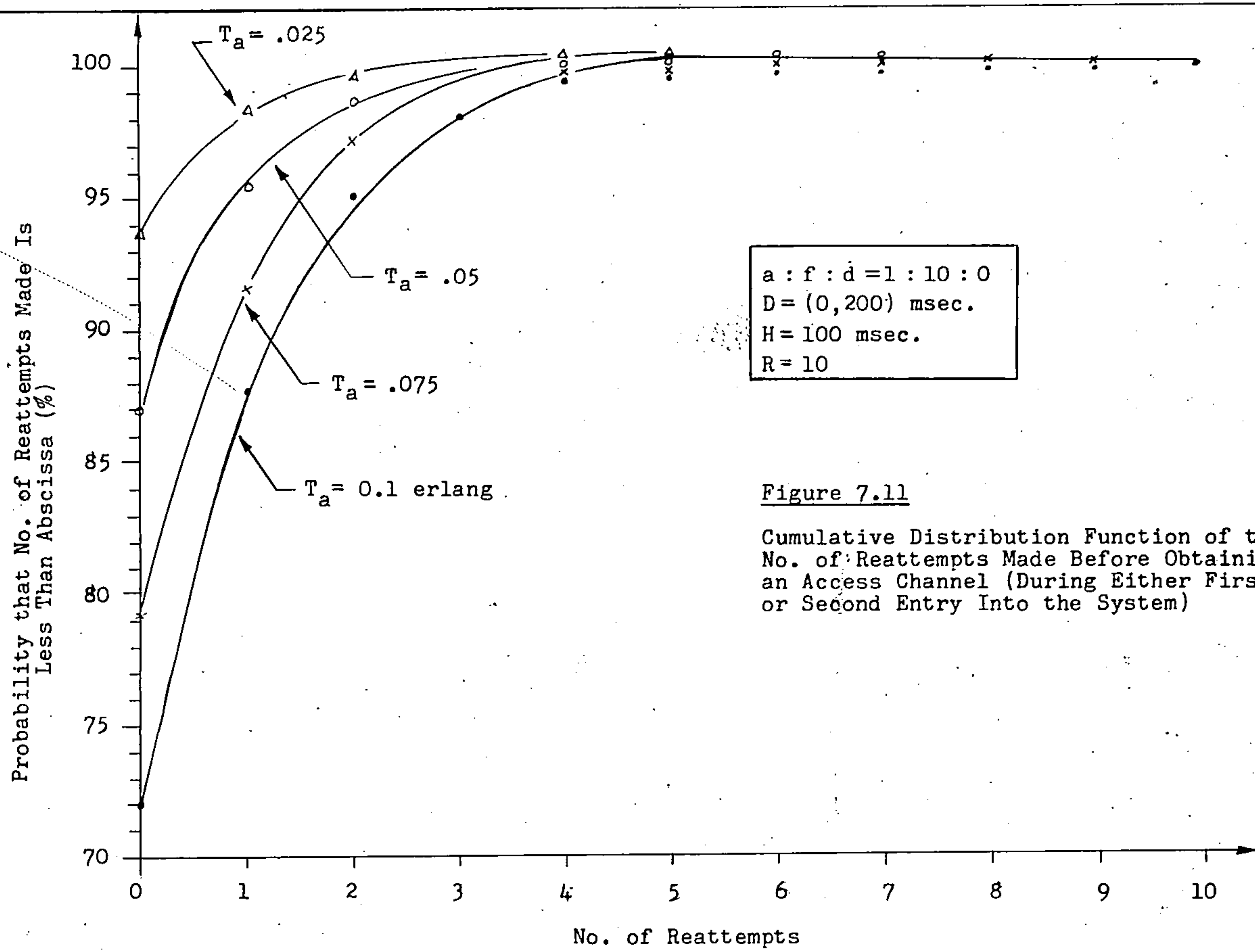


Figure 7.11

Cumulative Distribution Function of the No. of Reattempts Made Before Obtaining an Access Channel (During Either First or Second Entry Into the System)

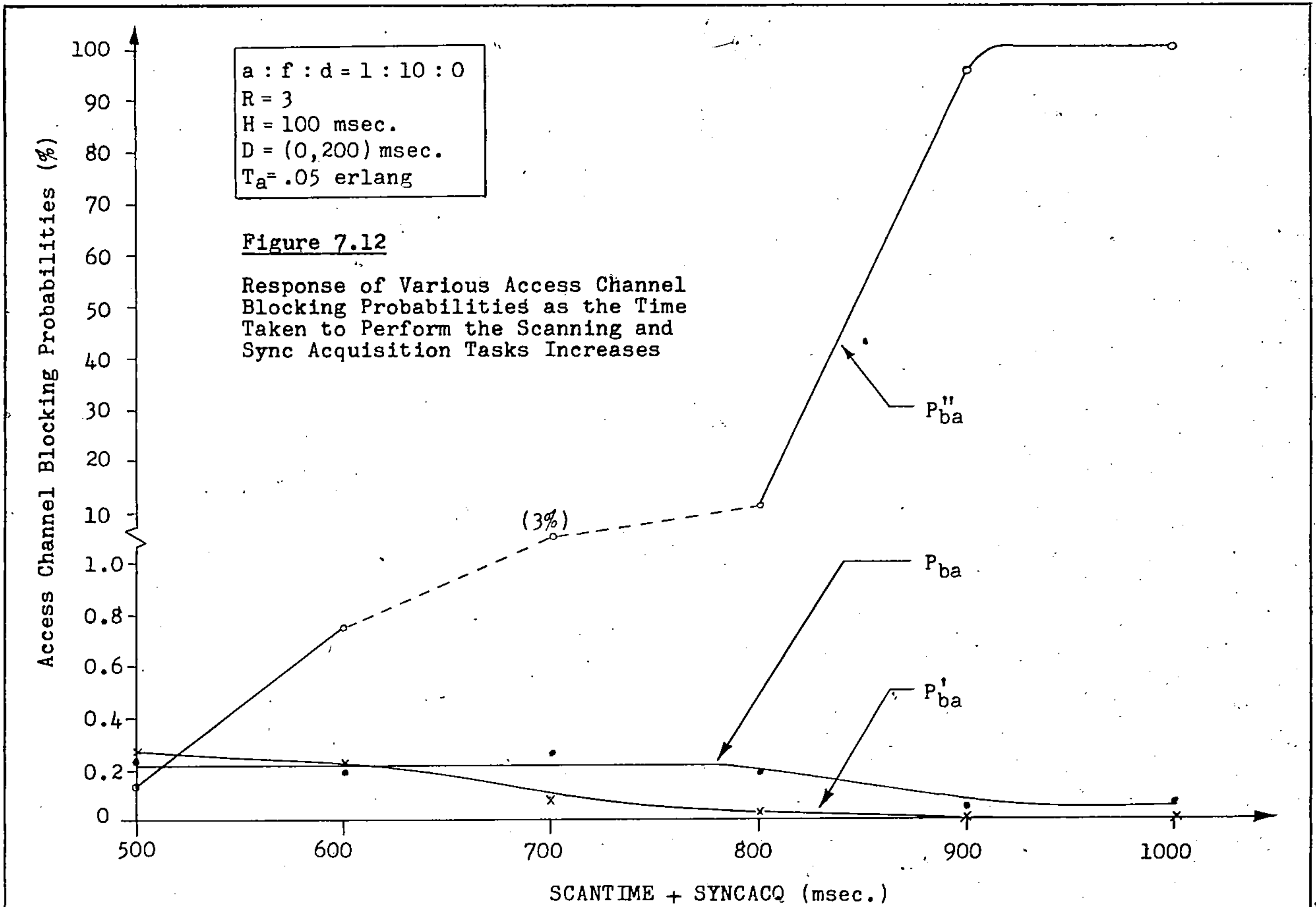


Figure 7.12

Response of Various Access Channel Blocking Probabilities as the Time Taken to Perform the Scanning and Sync Acquisition Tasks Increases

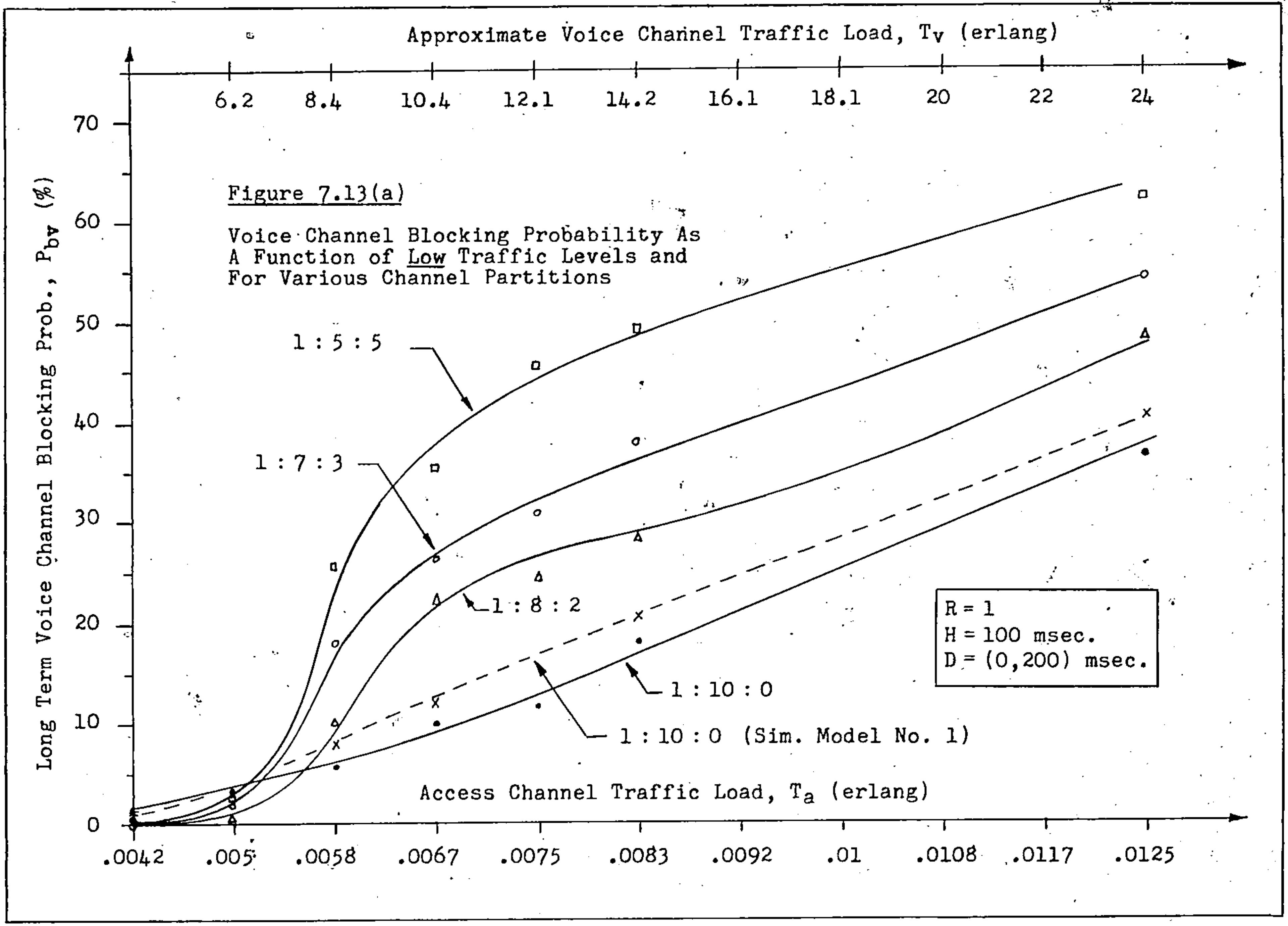


Figure 7.13(a)

Voice Channel Blocking Probability As A Function of Low Traffic Levels and For Various Channel Partitions

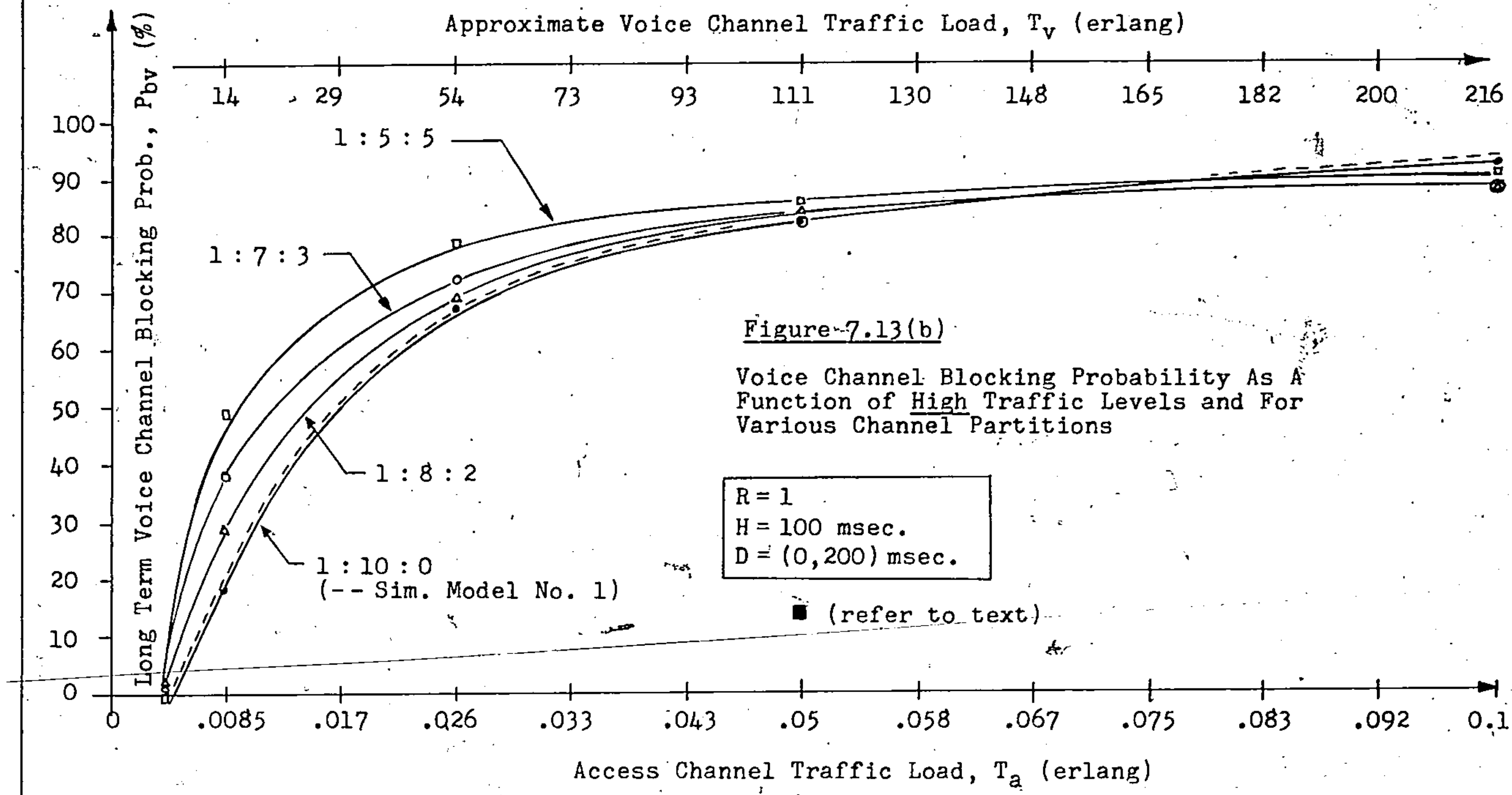


Figure 7.13(b)

Voice Channel Blocking Probability As A Function of High Traffic Levels and For Various Channel Partitions

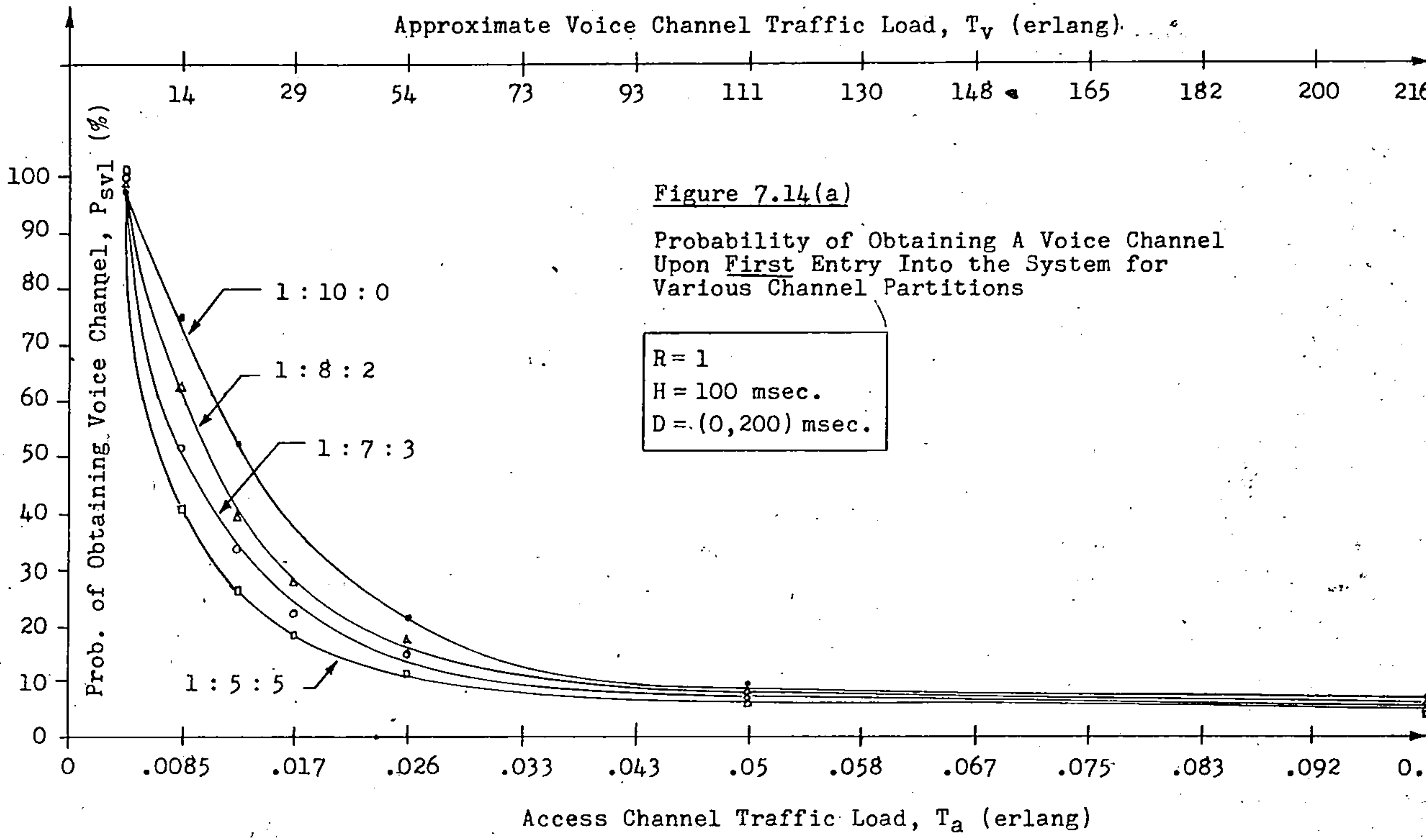


Figure 7.14(a)

Probability of Obtaining A Voice Channel Upon First Entry Into the System for Various Channel Partitions

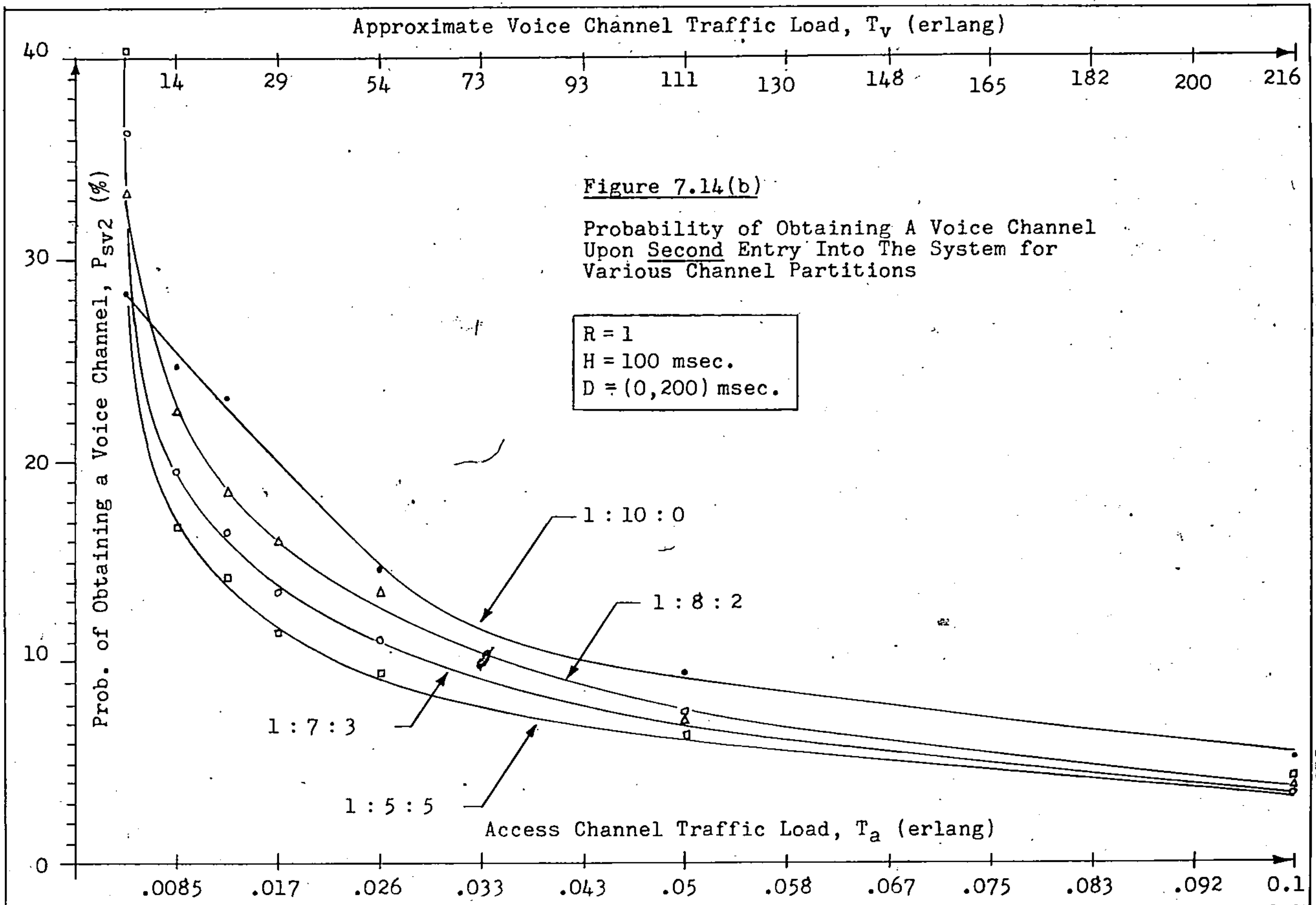


Figure 7.14(b)

Probability of Obtaining A Voice Channel Upon Second Entry Into The System for Various Channel Partitions

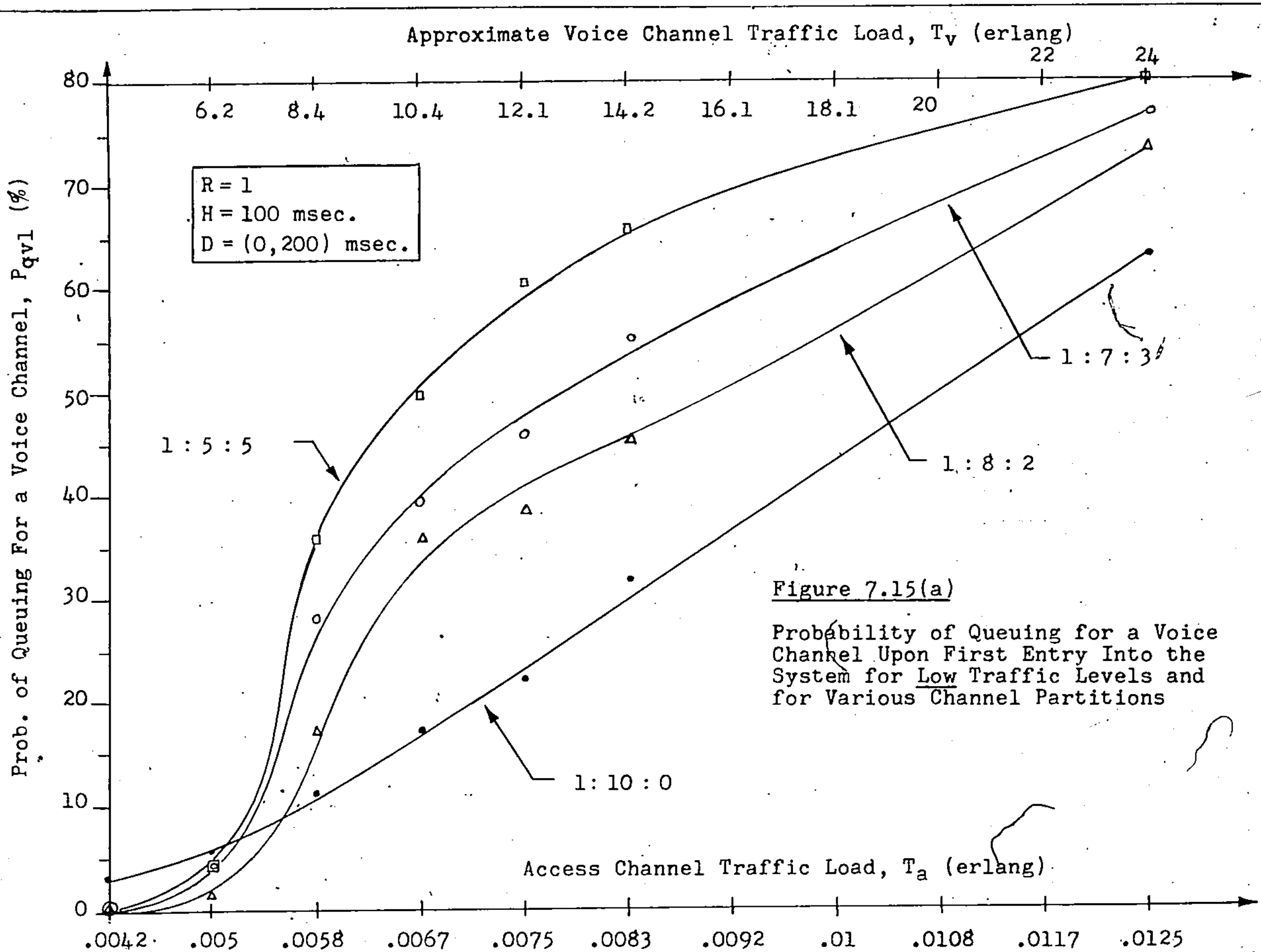


Figure 7.15(a)

Probability of Queuing for a Voice Channel Upon First Entry Into the System for Low Traffic Levels and for Various Channel Partitions

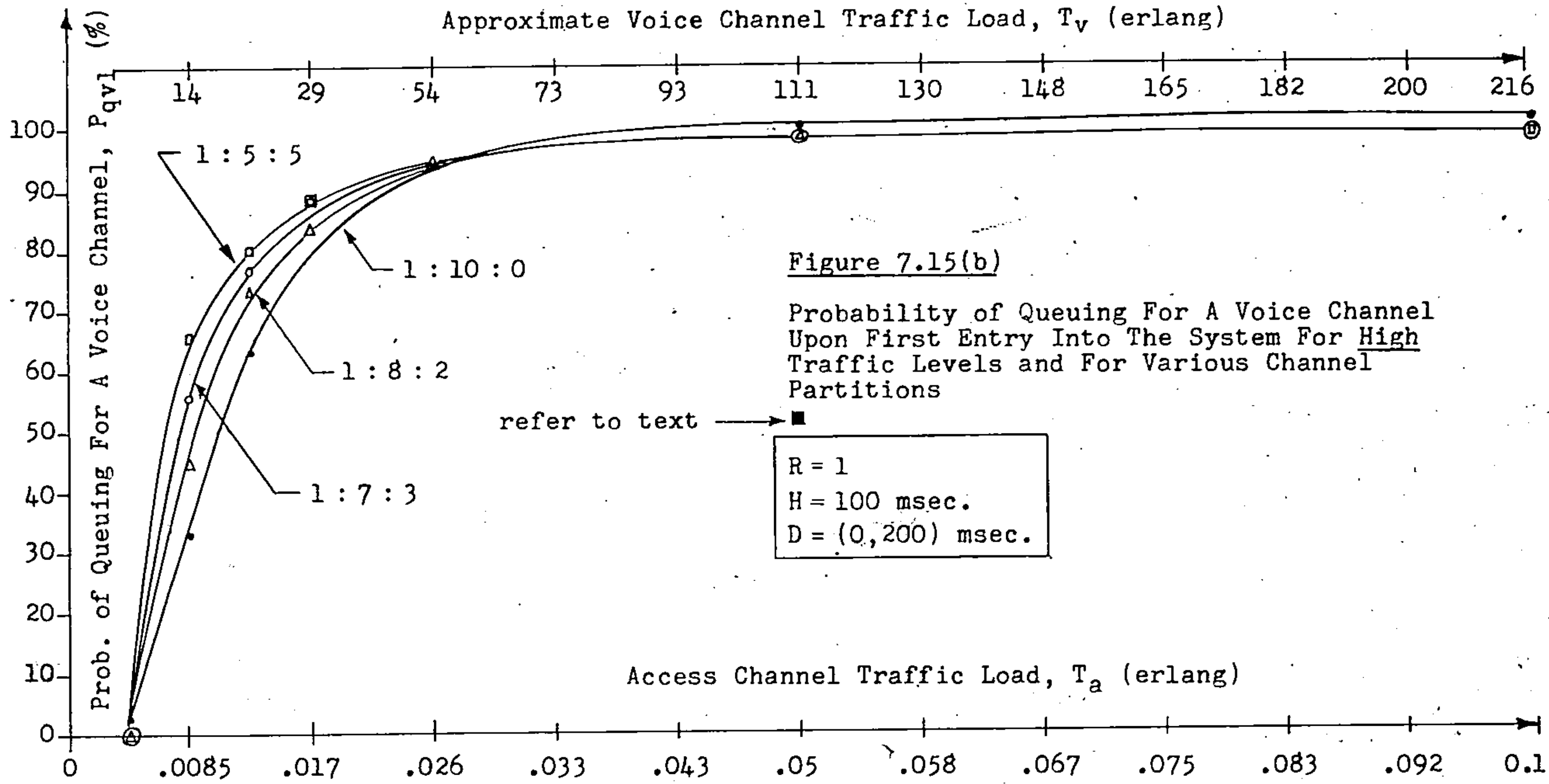


Figure 7.15(b)

Probability of Queuing For A Voice Channel Upon First Entry Into The System For High Traffic Levels and For Various Channel Partitions

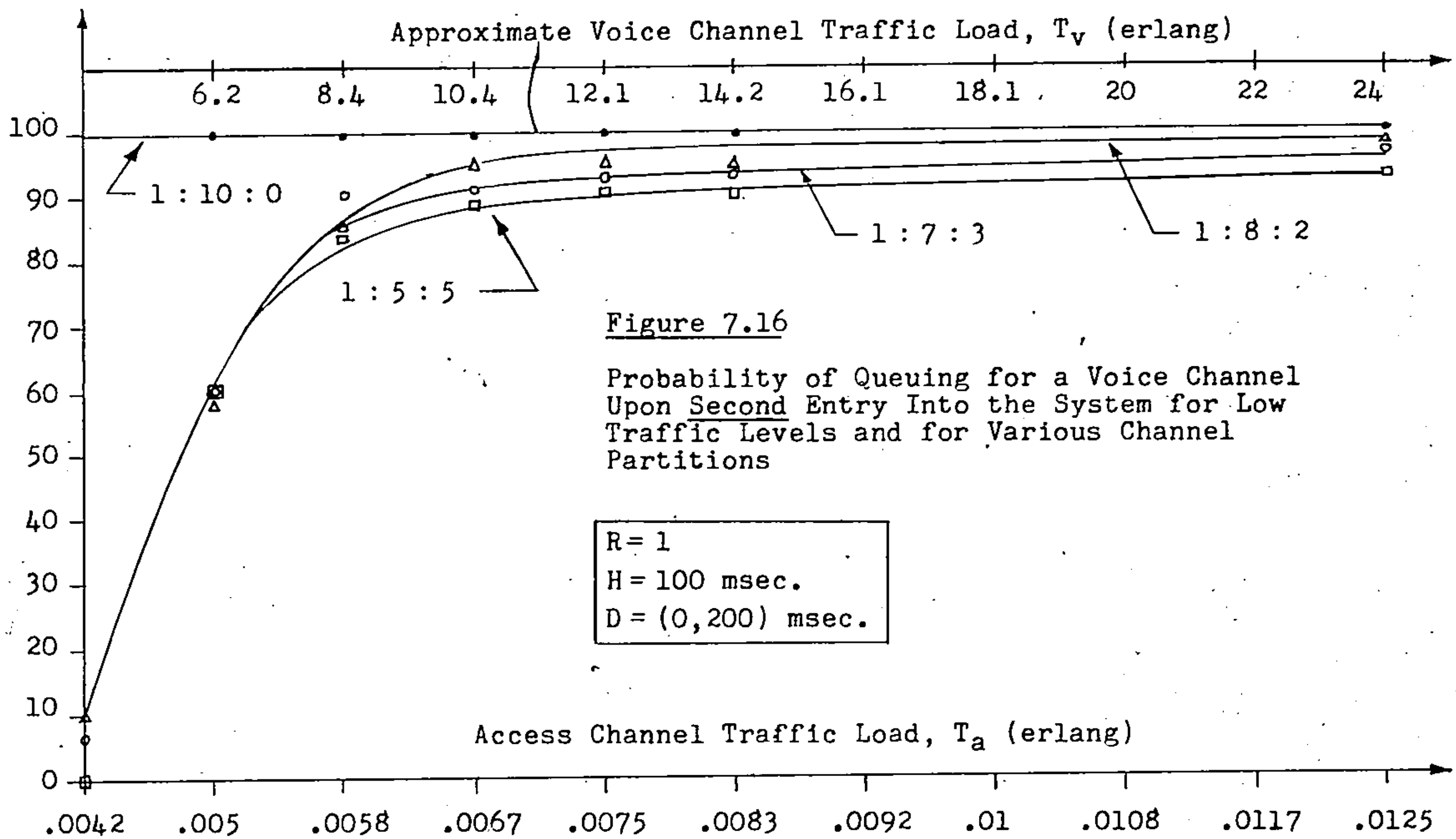


Figure 7.16

Probability of Queuing for a Voice Channel Upon Second Entry Into the System for Low Traffic Levels and for Various Channel Partitions

R = 1
H = 100 msec.
D = (0,200) msec.

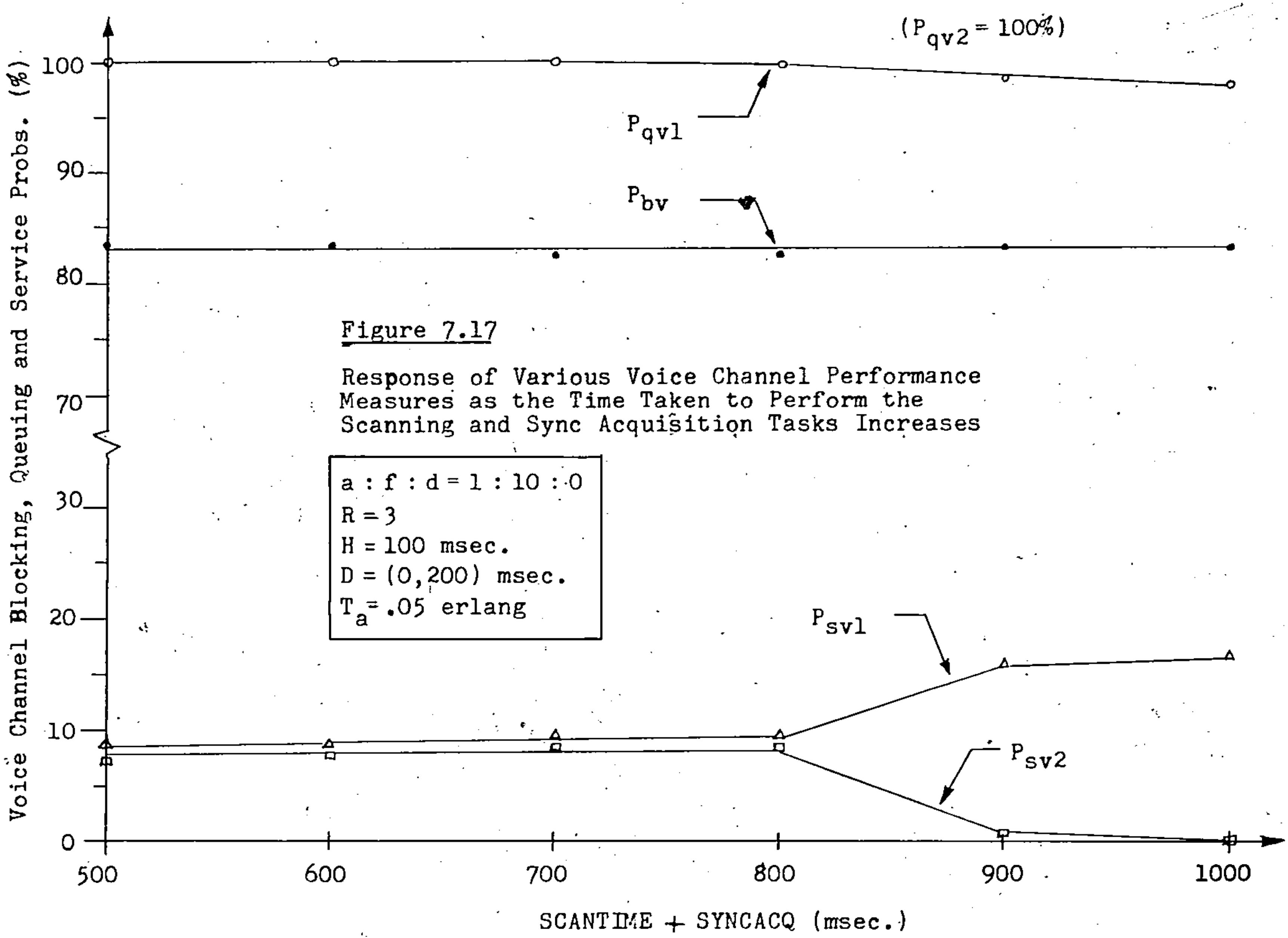


Figure 7.17

Response of Various Voice Channel Performance Measures as the Time Taken to Perform the Scanning and Sync Acquisition Tasks Increases

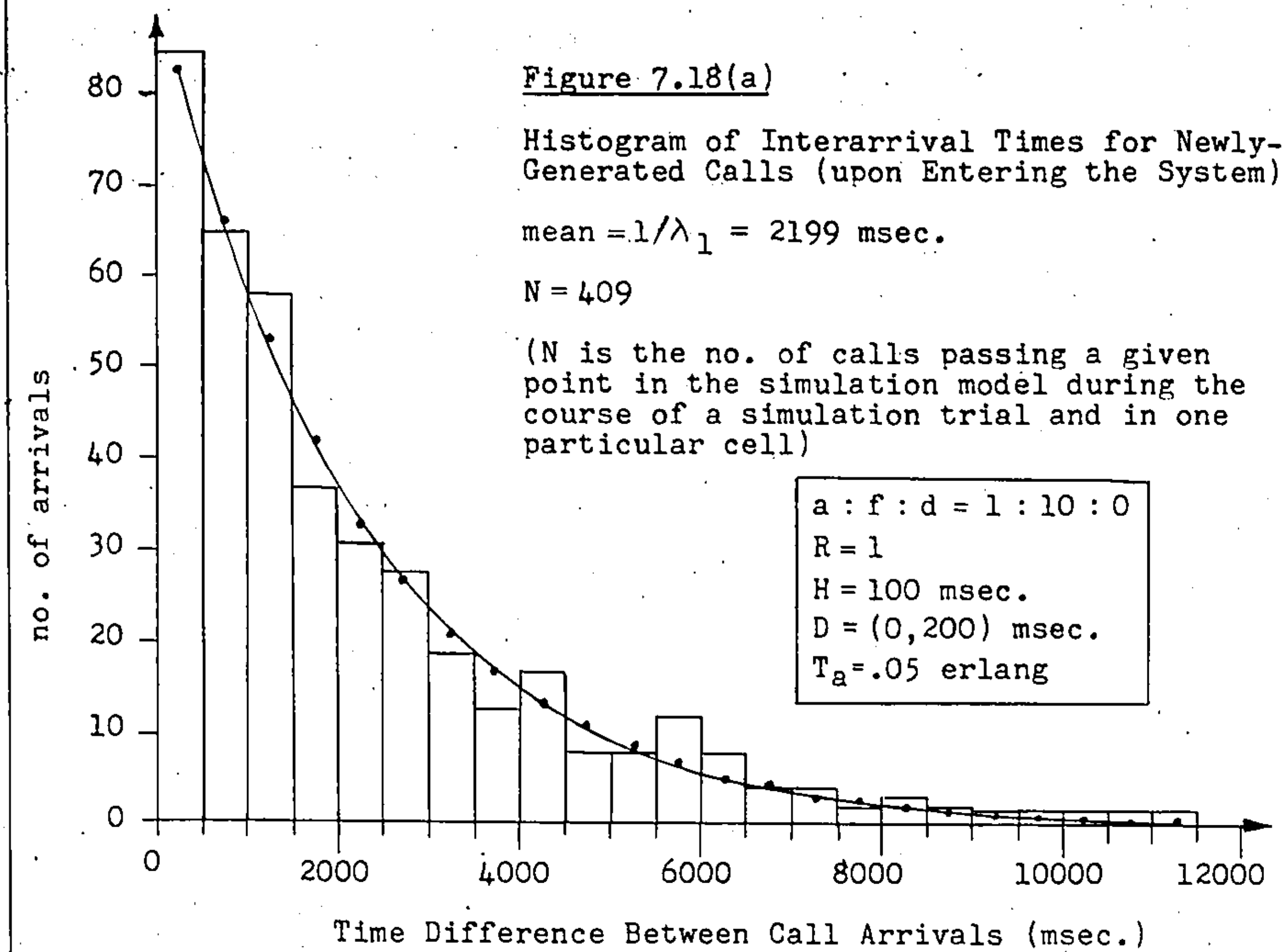


Figure 7.18(a)

Histogram of Interarrival Times for Newly-Generated Calls (upon Entering the System)

mean $= 1/\lambda_1 = 2199$ msec.

N = 409

(N is the no. of calls passing a given point in the simulation model during the course of a simulation trial and in one particular cell)

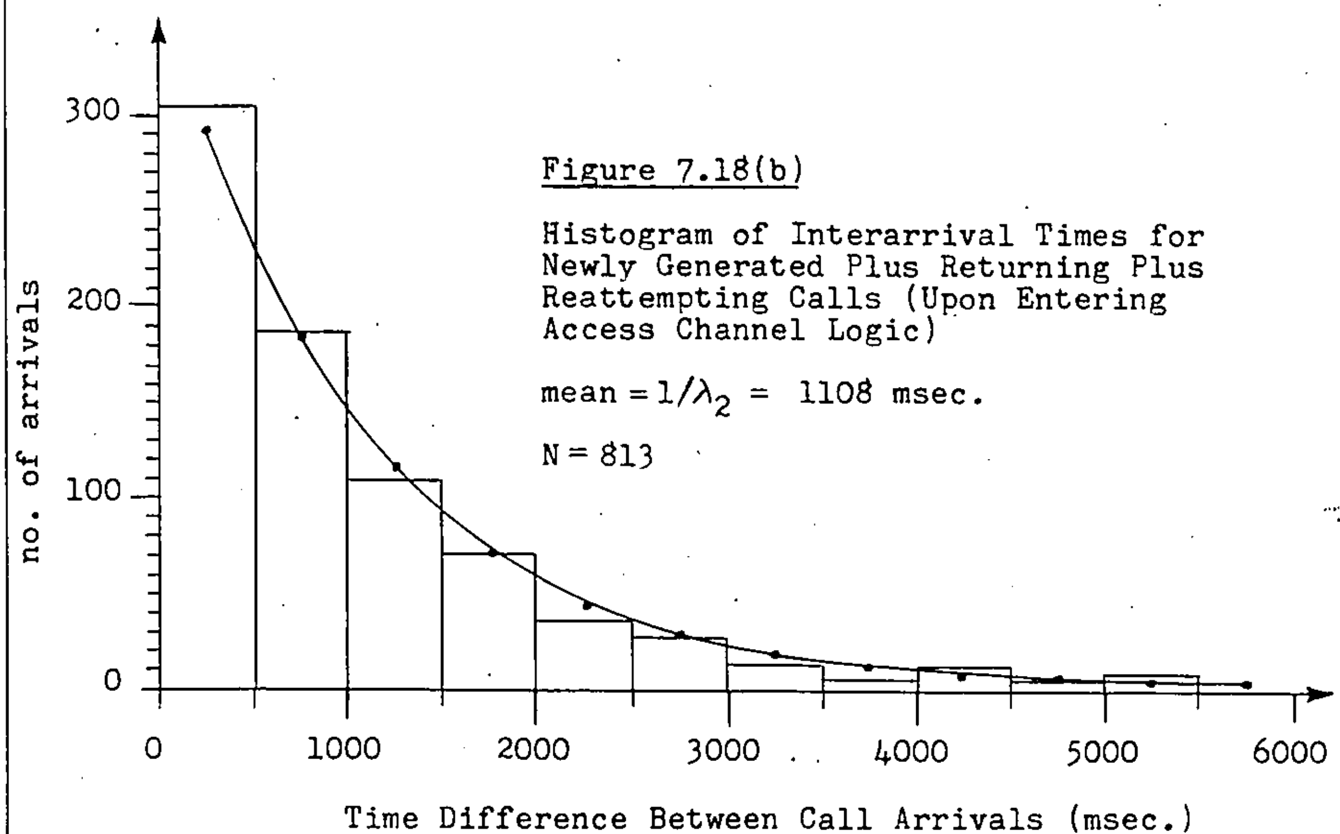


Figure 7.18(b)

Histogram of Interarrival Times for Newly Generated Plus Returning Plus Reattempting Calls (Upon Entering Access Channel Logic)

mean $= 1/\lambda_2 = 1108$ msec.

N = 813

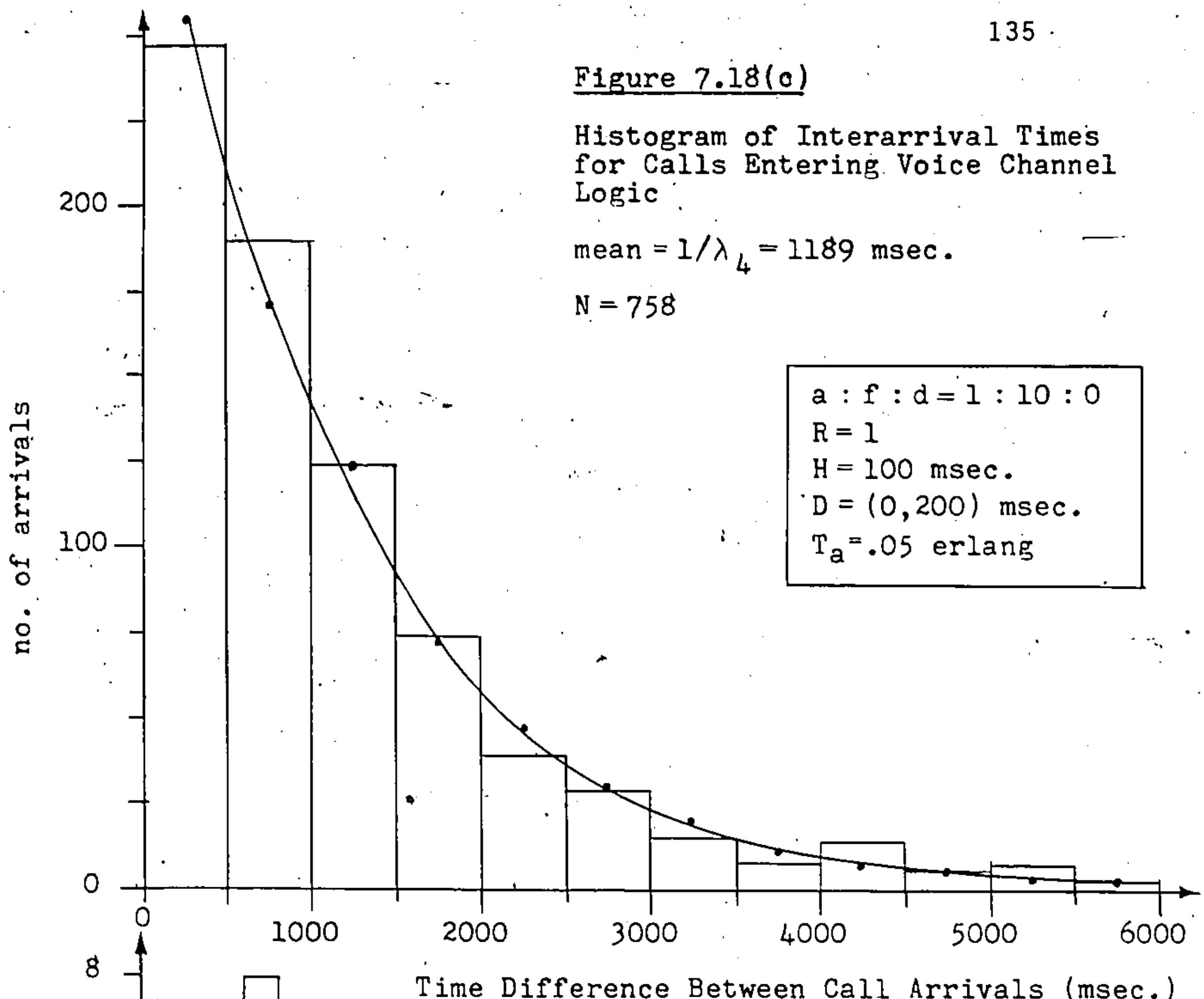


Figure 7.18(c)

Histogram of Interarrival Times for Calls Entering Voice Channel Logic

mean = $1/\lambda_4$ = 1189 msec.

N = 758

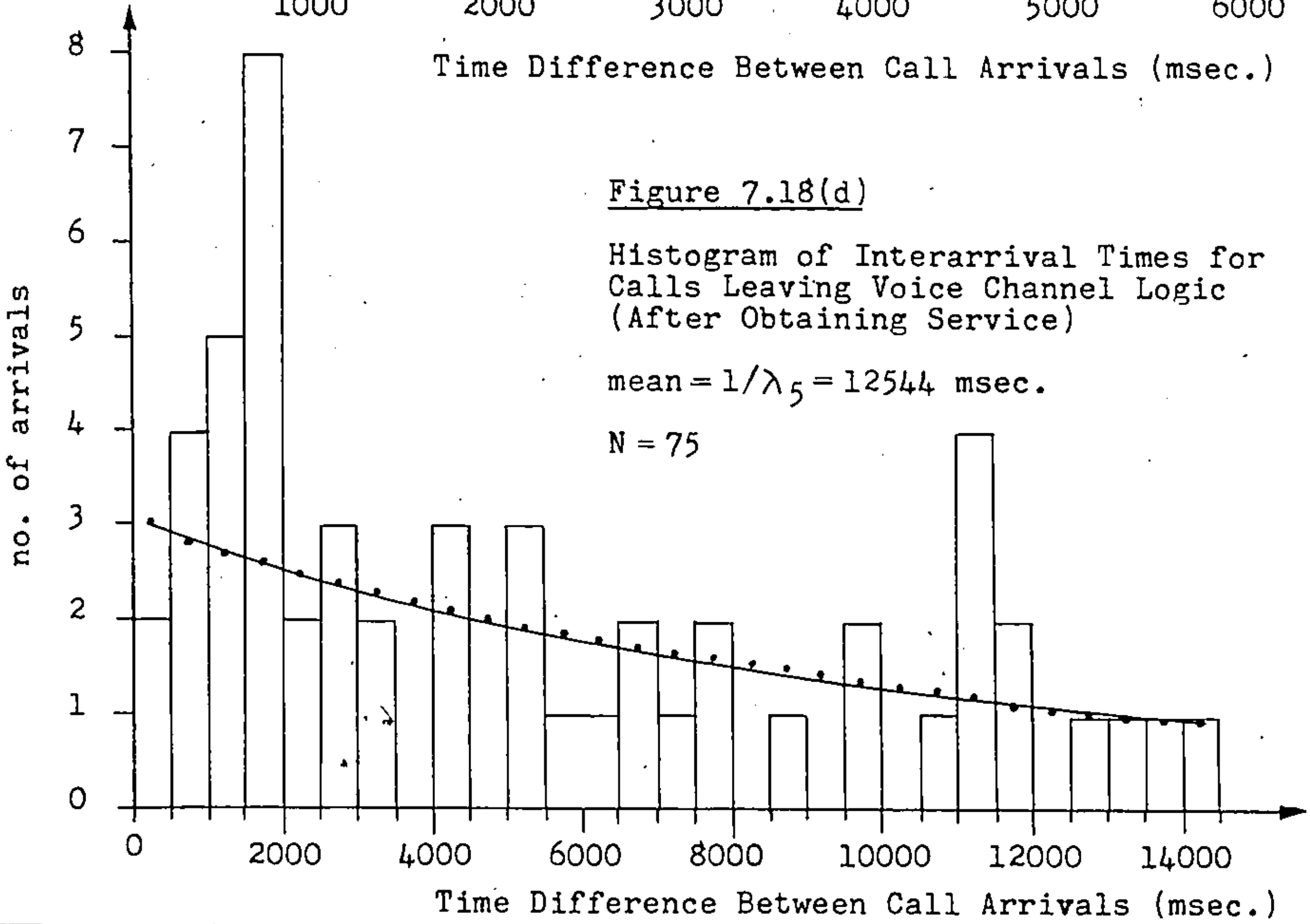


Figure 7.18(d)

Histogram of Interarrival Times for Calls Leaving Voice Channel Logic (After Obtaining Service)

mean = $1/\lambda_5$ = 12544 msec.

N = 75

Figure 7.5

In Figure 7.5, P_{ba} is plotted as a function of the access channel traffic load. In the top set of curves we concentrate upon the FCAS and use R as a parameter. As anticipated, P_{ba} has gone up substantially compared to Model 1 because of the greater burden placed on the access channels by returning calls. From our earlier discussion of Equation 7.2(b), this additional traffic load is:

$$\left.\begin{aligned} T_a^+ &= \lambda_1(1-P_{ba})(1-P_{svl})H \\ &= T_a\cdot(1-P_{ba})(1-P_{svl}) \end{aligned}\right] \qquad (7.7)$$

As an example, for T_a=.05 erlang and R=1, a value of P_{ba}=2.7% can be read off from Figure 7.5. Jumping ahead to Figure 7.14(a), the corresponding value for P_{svl} is 9.6% so that:

$$T_a^+ = (1-.027)(1-.096)(.05) = .044 \text{ erlang}$$

The net traffic load is:

$$(T_a)_{net} = T_a + T_a^+ = .094 \text{ erlang}$$

Now consider the dashed curve for R=1 in Figure 7.5. At T_a=.094 erlang we see that P_{ba} is just about 2.7%. So it appears that once we take into account the greater traffic level due to returning calls, the new simulation results are quite consistent with those of Model 1.

A single simulation trial was performed for the case of T_a=.05 erlang and R=1 and for 55 (fixed) voice channels per cell. The isolated data point (■) is the measured value for P_{ba}. For a realistic number of voice channels then, relatively few calls re-enter the system and P_{ba} very nearly equals the value found in Chapter 6.

The bottom set of curves in Figure 7.5 compares the performance of the hybrid scheme with the fixed scheme (1:10:0) for R=1. (The two curves labelled 1:10:0 were simply extracted from the top set of curves). The hybrid scheme is seen to perform less well than the fixed scheme. The reason is that, for the traffic range plotted, P_{bv} for the hybrid scheme is greater than for the fixed scheme. So more calls re-enter the system causing $(T_a)_{net}$, and hence P_{ba}, to increase.

Figure 7.6

Figure 7.6 is again a plot of P_{ba} versus T_a but with 2 access channels per cell this time. P_{ba} has dropped quite dramatically in much the same manner observed in Model 1 although the magnitude is, of course, greater than in the earlier model. The case of 3 access channels per cell was also examined but still produces zero or very small blocking probabilities.

Figure 7.7

Figure 7.7 is the counterpart of Figure 6.5. The following relationship gives a fairly good fit to these curves for small values of R but produces somewhat pessimistic results for R > 3 or thereabouts:

$$P_{ba}\,(\%) = \frac{160T_a}{(R+1)^{3/2}} \quad , \quad R=0,1,2\ldots m \qquad (7.8)$$

$$= 0 \quad , \quad R>m$$

The same restrictions as Equation (6.13) apply with the additional proviso that there be 10 voice channels per cell.

Figure 7.8

Figure 7.8 shows the blocking probability P'_{ba} (due to the limit upon R) for those calls entering the system a second time. The general form of the curves is the same as Figure 7.5 but for a given T_a, P'_{ba} is less than P_{ba}. This seems perplexing at first. After all, a returning call may very well have consumed all, or some portion, of its reattempt allocation during its first passage through the access channel logic and thus would be more prone to failure upon second entry into the system. Moreover, a returning call must compete with newly-arriving calls for the same access channel. So, if anything, P'_{ba} should be greater than P_{ba}. The reason why this is not the case probably lies in an observation made in the last chapter. There it was noted that, as the mean value of D increased, P_{ba} fell markedly. If we think of the 5 second period spent in the voice channel queue as being analagous to D, then we have a similar situation. Calls are reattempting to obtain an access channel at a time far removed from their initial attempt. Correspondingly, the blocking probability for these calls has fallen.

Figure 7.9

In Figure 7.9 we focus our attention upon returning calls which have been ejected from the voice channel queue. The choice of R=3 is intentional since for smaller values of R and for $T_a \leq 0.1$ erlang, P''_{ba} is always equal to zero. Apparently when R<3 calls do not remain in the system long enough for the access timer limit to come into play.

For R>3 and for a given T_a, P''_{ba} continues to increase but only very, very slightly, (For large R, most calls do not require their entire reattempt allocation anyways).

Figure 7.10

The mean delays, $\bar{d}_1$ and $\bar{d}_2$, experienced by newly-arriving calls are plotted in Figure 7.10. These curves should all be biased upward by about 500 msec to account for the time needed to scan the paging channels.

Compared to the results of Model 1, both $\bar{d}_1$ and $\bar{d}_2$ are larger but this is only natural since for a given T_a, the actual traffic impinging on the access channels $(T_a)_{net}$ is greater in the current model. It also appears that $\bar{d}_1$ is affected to a greater extent than $\bar{d}_2$ by the modifications made to Model 1. This is an indication that many more calls are now forced to make at least one reattempt before obtaining an access channel. (Note that if *all* calls made at least one reattempt then $\bar{d}_1$ would equal $\bar{d}_2$ by definition).

Figure 7.11

The graphs of Figure 7.11 indicate that virtually 100% of all callers make 5 or less reattempts before finding the access channel idle. Thus R=5 may be considered a cut-off point. Values of R greater than 5 will not signifigantly reduce P_{ba} or P'_{ba} any further. (In Figure 7.11, no distinction is made between newly arriving and returning calls. For example, a call making 5 reattempts may have done so upon first entry into the system or perhaps only 2 reattempts were needed upon first entry while the remaining 3 were used up after being ejected from the voice channel queue).

Figure 7.12

As stated at the start of Section 7.1, the composite time taken by the mobile logic unit to scan the paging channels and acquire word sync is assumed to be 500 msec. All the results presented thus far are based on a delay of this length. However, this is only an estimate. While the AT&T spec [44] states minimum values which should not be exceeded there is no mention of absolute values.[10] For this reason several simulation trials were performed where this delay (call it SCANTIME+SYNCACQ) was varied. Figure 7.12 shows the effect upon the access channel blocking probabilities.

P''_{ba} climbs sharply as SCANTIME+SYNCACQ increases because a call has progressively less time in which to find an access channel or a voice channel before expiration of the access timer. When the delay reaches 900 msec, P''_{ba} is seen to be very nearly 100%. To explain why this occurs, consider a call which is purged from the voice channel queue after 5 seconds. Such a call has been in the system for 0.9+5+0.1=6 seconds, where the added 100 msec represents the access

[10] These time limits are 700 msec for scanning and 400 msec for sync acquisition.

channel holding time. In fact, if one or more reattempts were originally needed to obtain an access channel, the figure will be even greater than 6 seconds. In any event, the important point is that the test made on the access timer in the flowchart of Figure 7.3 will indicate that the timer has run out and so the call will be terminated. Because calls are prevented from re-entering the system, then by its very definition, P''_{ba} must be 100%.

P_{ba} and P'_{ba}, on the other hand, fall as SCANTIME+SYNCACQ increases. This occurs because there are fewer calls competing for the single access channel in any given cell. For SCANTIME+SYNCACQ=900 msec, P'_{ba} will be zero for reasons cited above and as is evident from its definition. P_{ba} should attain the same value observed in Model 1 for a traffic load of .05 erlang and R=3. There P_{ba} was measured to be .05% and this is almost precisely the value observed here.

Figure 7.13

In this figure and in most of the figures to follow, two horizontal scales are shown -- one is for the access channel traffic load while the other gives the corresponding voice channel traffic load as calculated from (7.4).

For low offered traffic loads (T_V<7 erlang) the various channel partitions of the hybrid scheme outperform the fixed scheme -- producing lower blocking probabilities. Further, for traffic levels at or below 5 erlang, the channel partitions which employ the most dynamic channel do better than those which employ the least; that is, 1:5:5 is better than 1:7:3 which is in turn better than 1:8:2. (Because of the scale used, this is difficult to see in Figure 7.13(a)). This behaviour reflects the ability of dynamic channels to move about from cell to cell in order to serve the random fluctuations in traffic demands. To illustrate, say 10 channels are in use in cell x and a new service request is received. In a fixed scheme (1:10:0) nothing can be done since there are no more voice channels available in the cell. The request has no choice but to queue up and wait for a channel to come free. But in a hybrid scheme, cell site x can call upon any one of the dynamic channels in the central pool in addition to the fixed voice channels reserved for its own use. If we take the 1:5:5 partition as an example, cell site x has, ideally, 20 channels at its disposal -- 5 fixed and 15 dynamic voice channels. We say "ideally" because a dynamic channel must meet the cochannel interference constraint for cell x. So the new service request which is rejected in the fixed scheme can conceivably be handled by the hybrid scheme.

This scenario remains valid as long as the traffic demands in each cell remain low. As traffic levels increase the hybrid scheme becomes less efficient. The dynamic channels will invariably be assigned to cells at relative distances greater than D/R. In fact, for T_v=7 erlang and a:f:d=1:5:5, it was observed that the dynamic channels were in use in 11 cells on the average while for the fixed scheme, 1:10:0, the fixed channels in channel set A (see Figure 5.1) were in use in 13 cells on the average. As traffic levels increased further, the average dynamic usage remained the same and the average usage of fixed channels in set A approached its maximum achievable value of 14.

In Figure 7.13(a) we have also included the results from Model 1. Evidently, allowing calls to wait for a voice channel assignment -- even for only 5 seconds -- brings about a noticeable improvement in the voice channel blocking probability. The penalty, though, is a poorer grade of service over the access channels.

In Figure 7.13(b) we've plotted P_{bv} for high traffic levels. Actually, these traffic levels are totally unrealistic for 10 voice channels per cell on the average but the curves still highlight the relative performance of the different channel partitions. The isolated data point (■) is the resultant blocking probability for 55 fixed voice channels per cell.

Figure 7.14

The general response of P_{sv1} and P_{sv2} to increasing traffic levels are quite similar as illustrated in Figure 7.14(a) and (b) respectively. In both cases, the fixed scheme is superior to the hybrid scheme except at the lowest end of the traffic scale for reasons which parallel those given for Figure 7.13. However, P_{sv2} is lower than P_{sv1}. To explain, say a call is ejected from the voice channel queue due to the 5 second time limit. The call may have been quite close to the top of the queue at the time but if it does manage to get past the access channel logic again and re-enter the voice channel assignment subroutine it will have lost its position in the queue and must start all over again. The problem is compounded by the high probability of queuing upon second entry into the voice channel assignment subroutine. Far fewer returning calls find a voice channel right off compared to new calls and consequently P_{sv2} will be lower than P_{sv1}.

Figures 7.13 and 7.14 reveal the following relationship for a given channel partition and traffic load:

$$P_{bv} = (1-P_{sv1})(1-P_{sv2}) \qquad (7.9)$$

Equation (7.9) implies that the state of the system when a call makes its first request for a voice channel is more or less independent of the state when the same call makes its second request for a voice channel (after re-entering the system).

Figure 7.15

The response of P_{qv1} to increasing traffic levels is shown in Figures 7.15(a) and (b). The curves are very similar to those for P_{bv} and the explanation for the relative performance of the different channel partitions remains the same. The isolated data point (■) in Figure 7.15(b) is for the case of 55 fixed voice channels per cell.

Figure 7.16

The response of P_{qv2} to increasing traffic levels is shown in Figure 7.16. Why is P_{qv2} so high? When a call is ejected from the voice channel queue (the first time) it must successfully pass through the access channel logic again in order to bid (a second time) for a voice channel. But any delays which it may have encountered in the access channel logic (measurable in hundreds of milliseconds) would be far less in comparison to the mean voice channel holding time of 120 seconds. Therefore a voice channel probably will not have become available in the short period of time which has gone by since the call was last ejected from the queue.

This is the one instance where the hybrid scheme is always superior to the fixed scheme and moreover the channel partitions whith the most dynamic channels are better than the partitions with the fewest. The reason for this is again linked to the greater ability of the dynamic channels to handle random variations in traffic throughout the system. In the fixed scheme, whether or not a call in cell x can find a voice channel is dependent solely on the prevailing state of cell x. In the hybrid scheme, however, a call's ability to find a channel depends as well on the state of all dynamic channels in all cells. In other words, it is more likely that the state of the system will change in a short period of time than it is likely that the state of cell x alone will change.

Figure 7.17

Figure 7.17 illustrates that, as the composite time taken to perform the scanning and sync acquisition functions increases, P_{bv} is hardly affected. An explanation is best given in terms of Equation (7.9). First of all, P_{sv2} falls sharply as SCANTIME+SYNCACQ approaches 900 msec because fewer calls are re-entering the system -- most find that the access timer has run out when they are ejected from the voice channel queue. Concurrently, P_{svl} begins to rise. New calls just entering the voice channel assignment subroutine for the first time are no longer competing with the returning calls. These two facts considered together and Equation (7.9) explain the constancy of P_{bv}.

Figure 7.18

In an effort to validate the equations of Section 7.3, the mean interarrival times λ_1, λ_2, λ_4 and λ_5 and the call interarrival time distributions were recorded at points in the simulation program corresponding to points in Figure 7.4. Figure 7.18 shows the observed distributions for one particular case, namely T_a=.05 erlang (roughly). Overlaid on each histogram is the theoretical exponential curve for the observed mean interarrival time. Points on this curve were computed using:

$$\text{no. of calls arriving in } (t_1,t_2) = N\cdot\int_{t_1}^{t_2} \lambda_i e^{-\lambda_i t}dt$$

$$= N\cdot(e^{-\lambda_i t_1} - e^{-\lambda_i t_2}) \qquad (7.10)$$

where N = total number of calls passing a given point in the simulation model during the course of a simulation trial and in one particular cell.

For example, in Figure 7.18(a), the number of calls generated in some cell x was 409 and of this number, 83 arrived within 500 msec of each other, that is:

$$\text{no of calls arriving in } (0,500) = 409(e^{-0} - e^{-\frac{500}{2199}})$$

$$= 83$$

All the distributions are obviously exponential in nature except for the last one which describes the rate at which calls vacate voice channels (λ_5). Despite this one exception, the equations of Section 7.3 still seem to be quite valid as evidenced by Table 7.1. The data in this table was collected from 4 separate simulation runs in which the independent variable λ_1 (and hence $T_a = \lambda_1 H$) was varied. Included is the case shown in Figure 7.18. Mean interarrival times λ_5 and λ_3 were computed from the observed values of λ_1 using relationships (7.2) and (7.5) and compared with their observed values. Similarly, λ_2 was computed from the observed value of λ_3 using (7.6). Agreement is quite good.

T_a (erl)	P_{ba} (%)	P'_{ba} (%)	P_{bal} (%)	P_{svl} (%)	P_{sv2} (%)	λ_1 (sec^{-1})	λ_5 (sec^{-1})		λ_3 (sec^{-1})		λ_2 (sec^{-1})	
						Simulation	Simulation	Theory*	Simulation	Theory**	Simulation	Theory***
.025	1.1	0.83	3.4	21.	14.8	.257	.094	.083	.4405	.454	.466	.465
.05	2.6	1.9	6.8	9.5	8.7	.455	.0797	.0764	.841	.836	.903	.945
.075	4.6	3.04	10.2	5.9	6.1	.77	.0998	.084	1.404	1.34	1.62	1.69
0.1	6.5	4.4	13.4	4.7	4.8	1.027	.0908	.087	1.817	1.834	2.210	2.347

$$*\lambda_5 = (1-P_{ba})\left[P_{svl} - (1-P_{svl})(1-P'_{ba})(P_{sv2})\right]\cdot\lambda_1$$

$$**\lambda_3 = (1-P_{ba})\left[1 - (1-P_{ba})(1-P_{svl})\right]\cdot\lambda_1$$

$$***\lambda_2 = \left[(1-P_{bai})(1-P_{ba})(1-P'_{ba})\right]^{-1}\cdot\lambda_3$$

R = 1
H = 100 msec.
D = (0, 200) msec.
a : f : d = 1 : 10 : 0

Table 7.1 A Check on Flow Rate Equations (7.1) to (7.6)

Chapter VIII

CONCLUSIONS

The results of Simulation Model 1 indicate that, if 2 or more automatic reattempts (R) are permitted, only one signalling channel per cell is required in order to maintain the access channel blocking probability P_{ba} below 2% for all traffic loads T_a up to 0.1 erlang. This traffic corresponds to a mean interarrival time of 1 second and it is doubtful that the traffic demands in even the most heavily populated cell will exceed this amount. Furthermore, P_{ba} drops by a factor of at least 30 if 2 access channels per cell are used and all other quantities (H,D,T ,etc.) remain static. Three or more access channels per cell produce inconsequentially small call rejection rates ($<10^{-4}$) for all traffic loads up to 0.1 erlang.

We have also concluded from the results of Model 1 that the access channel logic does not appreciably influence the blocking probabilities of the voice channels (or vice versa).

The above conclusions are also quite valid in the context of Model 2 as long as there is a realistic number of voice channels per cell. (By realistic is meant 50 or more). Otherwise many calls are expelled from the voice channel queue and try to seize an access channel again. These returning calls greatly increase the load on the access channels, causing the blocking probabilities to go up as well. Unfortunately, due to cost constraints, the cases considered for study in Model 2 were largely limited to 10 voice channels per cell on the average. The resulting dilemma was that a reasonable access channel traffic level (<0.1 erlang) constituted, in most instances, a totally unreasonable voice channel traffic level (>10 erlang) for 10 voice channels.

Two empirical formulas, (6.13) and (7.8), have been deduced from the simulation results of Models 1 and 2 respectively. These formulas relate P_{ba}, T_a and R and indicate that for a given T_a, P_{ba} is inversely proportional to the n'th power of (R+1) where n lies in the range 1.5 to 2. Formula (7.8) is really only valid for the case of 10 voice channels per cell while (6.13) is entirely independent of the number of voice channels and may be considered the more useful of the two. (For a realistic number of voice channels, (6.13) would accurately reflect either Model 1 or 2).

For two special cases, namely:

(i) R=0; 1 access channel per cell

and (ii) R=0; 2 access channels per cell,

we were able to corroborate the simulation results for P_{ba} with theoretical results based on general queuing models (Equations (6.10) and (6.12)). Further, a theoretical formula, (6.19), was derived which expresses P_{ba} as a function of R and T_a in the limit as the random delay D inserted before each attempt to seize an access channel, approaches infinity. All 3 formulas are of general utility provided the actual traffic impinging on the access channels is used; that is, new, returning, and reattempting calls must all be included.

The nature of the delays encountered in obtaining an idle access channel has been examined as a function of traffic. The delay is less than 600 msec for most new calls (including the time needed to scan the paging channels before undertaking an access channel seizure attempt).

We have illustrated the type of trade-offs which can occur while juggling the values of certain system parameters. For example, we demonstrated that values of $R \leq 2$ improve the odds of obtaining an access channel on the first attempt while also minimizing the delays. Values of $R>2$, on the other hand, drastically reduce the access channel blocking probability.

The 6-second access timer does not affect the access channel blocking probabilities unless:

(i) $R \geq 3$.

or (ii) the time taken to scan the paging channels and synchronize to the one with the strongest signal (SCANTIME+SYNCACQ) approaches 1 second.

Finally, for low offered voice channel traffic loads (say less than 60% of the saturation level of the voice channels) the hybrid scheme is superior to the fixed scheme. Moreover, for very low traffic loads (less than 50% of the saturation level) the channel partitions of the hybrid scheme with the most dynamic channels perform better than the partitions with the least number of dynamic channels.

Appendix A

APPLICATIONS OF QUEUING THEORY

TO TELEPHONE TRAFFIC ANALYSIS

A.1 BASIC ELEMENTS OF QUEUING THEORY

The basic model used as a starting point for virtually any discussion of queuing theory is shown in Figure A.1.

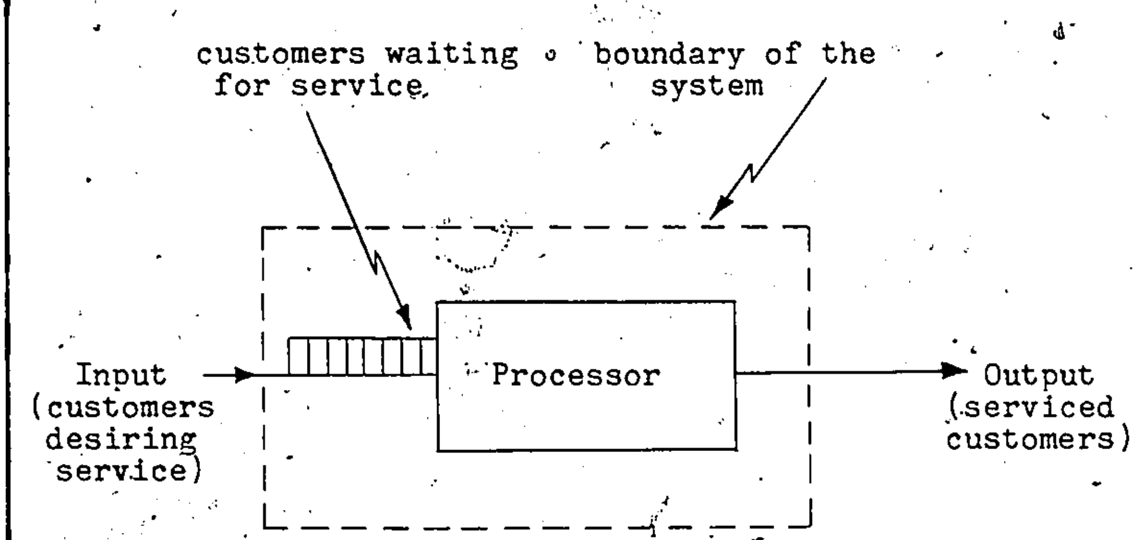


Figure A.1 Basic Queuing Configuration

An everyday example of a queuing system might be cars arriving at an intersection controlled by traffic lights. In this case, the traffic lights would represent the processor, the customers would be the arriving cars, the queue would be the waiting cars when the light is red and the output would be those cars which have crossed the intersection once the light has turned green..

The model of Figure A.1 has certain basic characteristics [19].

1. Arrival Pattern. Is the rate of arrival of customers deterministic or random?
2. Processor. How many service channels are there? In a given service channel, how many stages of service are necessary? Is the service time deterministic or random?
3. Queuing Discipline. How does the processor select customers from the queue? Examples include:

   a) First-In First-Out (FIFO)
   b) Last-In First-Out (LIFO)
   c) Priority Service
   d) Random Service
4. Capacity. Does the system have a finite capacity imposed by physical limitations or is the capacity, for all intents and purposes, unlimited?

Generally, these four characteristics will provide an adequate description of a queuing system. In terms of our previous example of the traffic lights, the arrival pattern is random, there is only a single server, the service time is probably deterministic (a fixed time is needed to cross the intersection), the queuing discipline is FIFO and there is no apparent limitation on capacity.

Analysis of deterministic queuing models is relatively straightforward and exact solutions can always be found. The same, however, is not true for stochastic models which are generally of the greatest practical interest. For the random case the analysis usually proceeds along the following lines.

The arrival pattern is first characterized by considering the interarrival time between consecutive customers as illustrated in Figure A.2. Here, the T_i are the random arrival times and the $tt_i = T_i - T_{i-1}$ are the random interarrival times [20].

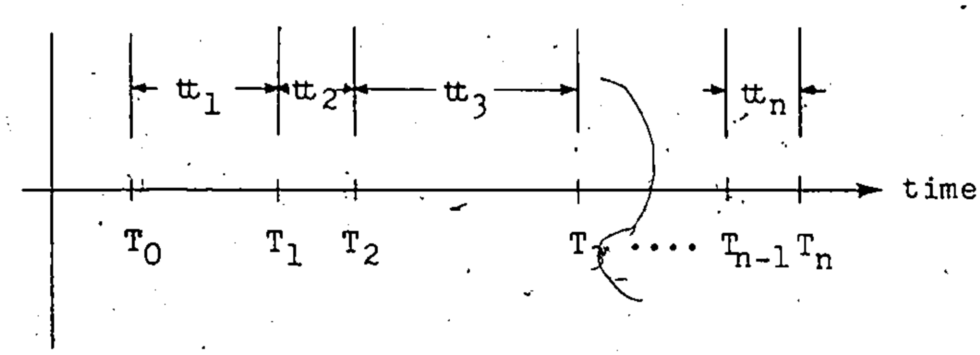


Figure A.2 Random Arrival Pattern

If we let $A_i(t)$ represent the cumulative distribution function of the i'th interarrival time, then:

$A_i(t) = \text{Prob}(tt_i \leq t)$ = probability that the time period between the arrivals of two consecutive customers is less than or equal to t.

Further, if $a_i(t)$ represents the probability density function of the i'th interarrival time, then:

$$a_i(t) = \frac{dA_i(t)}{dt} = \text{Prob}(t \leq tt_i \leq t+dt)$$

= probability that the interarrival time lies in the infinitesimal range [t,t+dt].

Generally it is assumed that the interarrival times are statistically independent and identically distributed; if so $A_i(t) = A(t)$ and $a_i(t) = a(t)$.

Let us now define an arrival rate function f(t) as follows:

f(t)dt = conditional probability of an arrival in [t,t+dt] given no arrival during the preceding "t" time units.

$$= \frac{a(t)dt}{1-A(t)}$$

Therefore:

$$f(t) = \frac{a(t)}{1-A(t)} \tag{A.1}$$

Substituting $dA(t)/dt$ for $a(t)$ and recognizing that $A(0)=0$ since t can never be negative, it is found that:

$$A(t) = 1 - \exp\left[- \int_0^t f(t)\,dt\right], \quad t \geq 0 \qquad (A.2)$$

If we now take a case of common interest, namely $f(t)=\lambda$, then:

$$A(t) = 1 - e^{-\lambda t}, \quad t \geq 0$$

And

$$a(t) = \frac{dA(t)}{dt} = \lambda e^{-\lambda t}, \quad t \geq 0$$

Therefore, if the conditional arrival rate is a constant, the implication is that the interarrival times are exponentially distributed. The converse is also true. The mean interarrival time, $\overline{\mathrm{tt}}$, is:

$$\overline{\mathrm{tt}} = \lambda \int_0^\infty t e^{-\lambda t}\,dt = 1/\lambda$$

Similar arguments can be applied to the service pattern, resulting in the following analagous definitioms (tt_s signifies the service time random variable):

$B(t) = \text{Prob}(t_s \leq t)$ = Probability that the time period required to serve a customer will be less than or equal to t.

$b(t) = \frac{dB(t)}{dt}$ = $\text{Prob}(t \leq t_s \leq t+dt)$

= Probability density function of the service time

$g(t)dt$ = the conditional probability that the service time will lie between t and t+dt given that service has already lasted for a time t.

$g(t)$ = service rate function.

$$= \frac{b(t)}{1-B(t)} \tag{A.3}$$

$$B(t) = 1 - \exp\left[-\int_0^t g(t)\,dt\right], \quad t \geq 0 \tag{A.4}$$

As before, if $g(t)=\mu$, then the service time will be exponentially distributed with mean value, $t_s = 1/\mu$.

System behaviour is often described by means of the following performance measures [19].

$p_n(t)$ = probability that there are n customers in the system at an arbitrary time t.

$L(t)$ = mean no. of customers in the system at time t.

$L_q(t)$ = mean no. of customers in the queue at time t.

W = mean waiting time in the system.

W_q = mean waiting time in the queue.

C(t) = mean no. of busy servers at time t.

$\bar{C}(t)$ = mean no. of idle servers at time t.

To illustrate the manner in which these quantities may be calculated, consider the system of Figure A.3 which is composed of m parallel service channels. Suppose we know $p_n(t)$,[11] then from the basic definition for the expectation of a random variable:

$$L(t) = \sum_{n=0}^{\infty} np_n(t) \tag{A.5}$$

$$L_q(t) = \sum_{k=1}^{\infty} kp_{m+k}(t) \tag{A.6}$$

Equation (A.6) follows since if there are m or less customers in the system, they will all be in the processor and

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[11] A means of finding $p_n(t)$ is discussed in Section A.2.

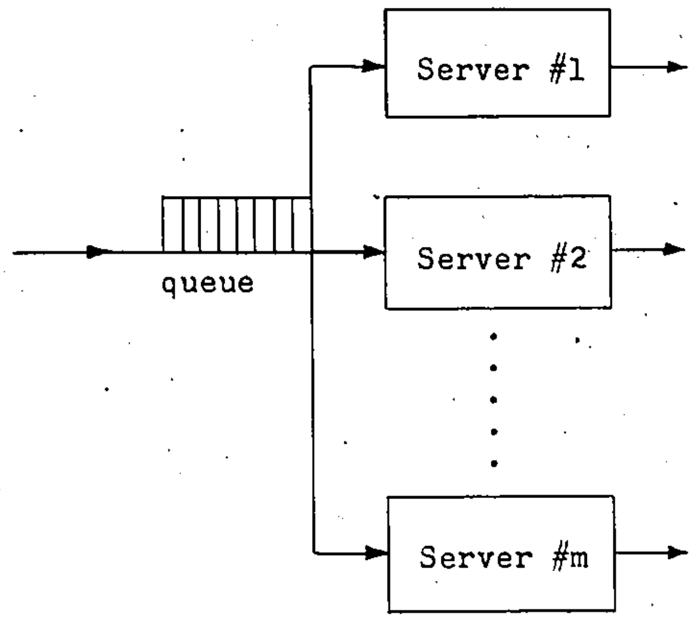


Figure A.3 Example System for Calculating Queuing Performance Measures

the queue will be empty. The probability that there are m+1 customers in the system, for example, is the same as the probability that there is a single customer in the queue. Continuing:

$$C(t) = \sum_{n=0}^{m} np_n(t) + m \cdot \sum_{n=m+1}^{\infty} p_n(t) \tag{A.7}$$

$$\overline{C}(t) = m - C(t) \tag{A.8}$$

In many instances, W and W_q may be calculated from L and L_q respectively using a relationship generally referred to as Little's formula [19,26]:

$$W = \frac{L}{\lambda'} \tag{A.9}$$

$$W_q = \frac{L_q}{\lambda'} \tag{A.10}$$

The symbol λ' represents the mean arrival rate of customers who actually join the system and are not rejected because the system is full. ~~If P_b~~ is the probability that the system is full, then $1-P_b$ is the probability that there is at least one idle server or one available position in the queue. Equivalently, $1-P_b$ is the fraction of arriving customers which enter the system. In terms of the total mean arrival rate:

$$\lambda' = \lambda(1-P_b) \tag{A.11}$$

Intuitively, Little's formula is easy to understand. Consider a (human) customer who has just arrived; on the average he enters into service after a time W_q. If, as his service begins, this individual were to glance back and count the number of people waiting behind him, then on the average this number would be L_q. It also took, on the average, $1/\lambda$ for each of these L_q individuals to arrive, and the total time it took for the L_q arrivals to queue up behind him must equal his own waiting time, i.e. $L_q(1/\lambda) = W_q$. Similar arguments can be applied to (A.9) by observing that when a customer is just completing his service, on the average, L people will be in the system and his average total waiting time W is equivalent to the average time it took the L to arrive.

Sufficient condtions for the validity of (A.9) and (A.10) are rather general and the reader is referred to Gross and Harris [19] for a more in-depth exploration of this subject. Suffice it to say that Little's formula is valid for the common case of exponentially distributed interarrival times, general service,[12] and a FIFO queuing discipline.

Finally, one further relationship exists between W and W_q:

$$W = W_q + \overline{tt}_s \qquad \text{(A.12)}$$

That is, the mean waiting time in the system is simply the sum of the mean waiting times in the queue and the mean service time. The proof of (A.12) is simple. Let tt and tt_q be random variables representing the waiting time in the

[12] No assumption is made regarding the exact form or nature of the service time distribution.

system and the queue respectively. We have $tt = tt_q + tt_s$ and hence $E[tt]=E[tt_q]+E[tt_s]$ which yields (A.12) directly.

A.2 TELEPHONE TRAFFIC FORMULAS

The Erlang B, Erlang C and Blocked Calls Held formulas are firmly established and widely used within the realm of telephone traffic engineering, having been found to give good overall experimental agreement. These formulas furnish a relationship between the offered traffic load (defined later), the number of communication channels (trunks) and the resulting blocking or queuing probability. The principal difference between the three lies in the assumed behaviour or reaction of customers who arrive at a time when all the service channels are busy.

The following assumptions apply to all three formulas:

1. There is a finite number of channels = m.
2. The interarrival time of calls is exponentially distributed with a mean value = $1/\lambda$.
3. The length of time a call engages a channel (the <u>holding time</u>) is exponentially distributed with a mean value = $1/\mu$.

In terms of the basic queuing model developed in Appendix A.1, the call requests are the customers and the communication channels are the servers. Table A.1 defines the system states used in the derivation of the formulas.

State	Interpretation
0	no calls in system; all channels idle
1	1 call in system; 1 channel busy
2	2 calls in system; 2 channels busy
.	.
.	.
.	.
m	m calls in the system; m channels busy; processing capacity reached. The disposition of any new arrivals in now dependent upon the service discipline; calls will be blocked, queued indefinitely or held for a specified period of time.
m+1	m+1 calls in the system; m channels busy. This state and higher states do not apply to the Erlang B service discipline.
.	.
.	.
.	.

Table A.1 Definition of System States for Use in Development of Traffic Formulas

A.2.1 Erlang B (Blocked Calls Cleared) Discipline

Under this service policy, calls which arrive when a channel is idle are served immediately. An arrival that occurs when all channels are busy however, is blocked, leaves the system and does not return. The situation is shown conceptually in Figure A.4.

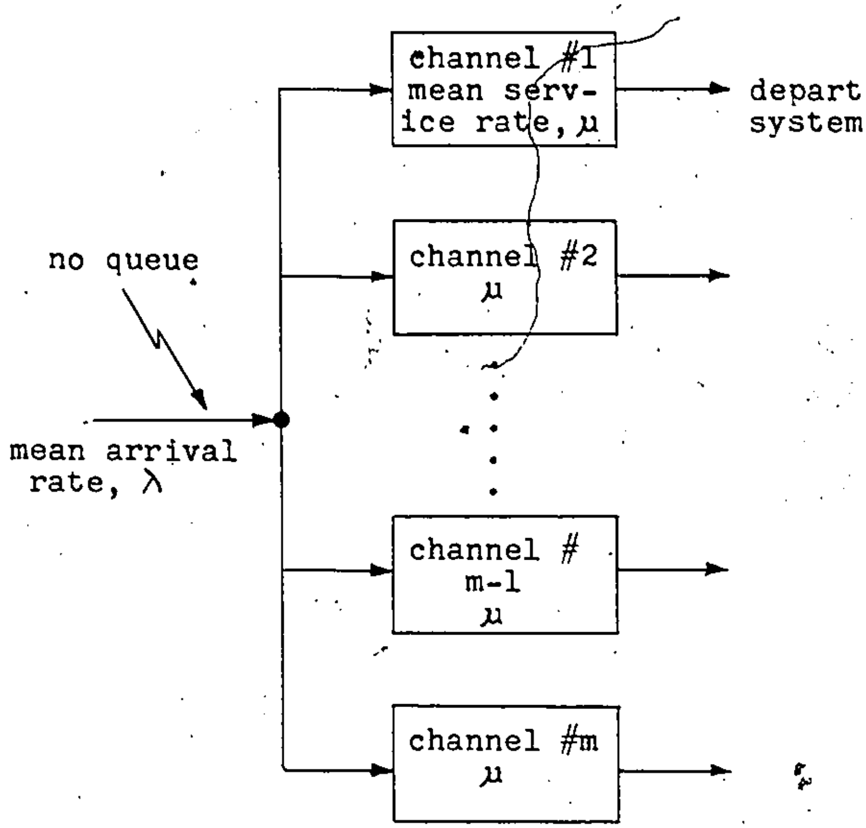


Figure A.4 Blocked Calls Cleared Service Discipline (System capacity = no. of servers = m)

Let $p_n(t)$ be the probability that the system is in state n at time t where state n ($<m$) is defined in Table A.1. An analytical approach to solving for $p_n(t)$ generally involves writing a set of differential-difference equations [19,20,40,41]. The methodology is quite involved but fortunately we are primarily interested in the system at steady state and, as we shall see, this leads to a simpler set of linear equations.

The signifigance of λ and μ remains the same as in Appendix A.1. Also, it is assumed that the probability of more than one arrival in time interval Δt or more than one service completion in Δt is given by $0(t)$, where:

$$\lim_{\Delta t \to 0} \frac{0(\Delta t)}{\Delta t} = 0 \qquad \text{(A.13)}$$

In words, (A.13) states that, as Δt becomes smaller, the probability of observing more than one event in Δt will likewise become smaller, approaching zero in the limit as $\Delta t \to 0$. With these considerations in mind we can make the following statement concerning state "0":

$$p_0(t+\Delta t) = (1-\lambda\Delta t)p_0(t) + \mu\Delta t p_1(t) \qquad \text{(A.14)}$$

where: $p_0(t+\Delta t) \triangleq$ probability that the system is in state 0 at time $t+\Delta t$.

$(1-\lambda\Delta t)$ = conditional probability that the system remains in state 0 during the interval $[t,t+\Delta t]$ given that the system was in state 0 at time t.

$\mu\Delta t$ = conditional probability that the system enters state 0 from state 1 during the interval $[t,t+\Delta t]$ as a result of a service completion and given that the system was in state 1 at time t.

Note in (A.14) that all higher order transitions (for example, from state 2 to state 0) which would require more than one service completion during Δt are excluded. Rearranging the terms of (A.14) and taking the limit as Δt approaches zero:

$$\lim_{\Delta t \to 0} \frac{p_0(t+\Delta t)-p_0(t)}{\Delta t} = \frac{dp_0(t)}{dt} \tag{A.15}$$

$$= -\lambda p_0(t)+\mu p_1(t)$$

Similarly if we consider state 1 and ask ourselves how it is possible to move into or out of state 1 we arrive at the following expression:

$$p_1(t+\Delta t) = (1-\lambda\Delta t)(1-\mu\Delta t)p_1(t) + \lambda\Delta t p_0(t) + 2\mu\Delta t p_2(t) \qquad (A.16)$$

For example, the first term on the right-hand side of (A.16) is the probability that we remain in state 1 during Δt (no new arrivals and no service completions). Also notice in this equation that the service rate in proceeding from state 2 to state ~~1 is~~ 2μ rather than simply μ. Thus with two servers engaged, the service rate is twice as great as when only one server is working. In general the service rate in state n will be $n\mu$. Again, taking the limit as $\Delta t \to 0$ (noting that the term involving $(\Delta t)^2$ is $0(\Delta t)$), (A.16) becomes:

$$\lim_{\Delta t \to 0} \frac{p_1(t+\Delta t)-p_1(t)}{\Delta t} = \frac{dp_1(t)}{dt}$$

$$= -(\mu+\lambda)p_1(t) + \lambda p_0(t) + 2\mu p_2(t) \qquad (A.17)$$

For the general state $n<m$:

$$\frac{dp_n(t)}{dt} = -(n\mu+\lambda)p_n(t) + \lambda p_{n-1}(t) + (n+1)\mu p_{n+1}(t)$$

$$n = 1,2,3\ldots m-1 \qquad (A.18)$$

Finally, for the highest state m, we have (recognizing that no new arrivals *into* the system are possible due to the blocked-calls-cleared service policy):

$$\frac{dp_m(t)}{dt} = -m\mu p_m(t) + \lambda p_{m-1}(t) \qquad \text{(A.19)}$$

Equations (A.15), (A.18) and (A.19) are the differential-difference equations we seek -- differential in time; difference equations in n. This set of m+1 equations may be written in matrix form:

$$\frac{d\,[p(t)]}{dt} = [p(t)]\; \begin{array}{c|l} \begin{bmatrix} -\lambda & \lambda & & & & & & & \\ \mu & -\lambda-\mu & \lambda & & & & 0 & & \\ & 2\mu & -\lambda-2\mu & \lambda & & & & & \\ & & & \ddots & & & & & \\ & & & & n\mu & -\lambda-n\mu & \ddots & & \\ & & 0 & & & \ddots & & & \\ & & & & & & & & \lambda \\ & & & & & & & m\mu & -m\mu \end{bmatrix} & \begin{array}{l} \text{state:} \\ 0 \\ 1 \\ 2 \\ \\ n \\ \\ m-1 \\ m \end{array} \\ \text{state: } \; 0 \quad 1 \quad 2 \qquad\qquad n \qquad\qquad m-1 \quad m & \end{array}$$

(A.20)

where $[p(t)] = [p_0(t),\ p_1(t), \ldots p_n(t)\ \ldots\ p_{m-1}(t),\ p_m(t)]$

Employing a convenient shorthand notation, (A.20) becomes:

$$\frac{d\left[p(t)\right]}{dt} = \left[p(t)\right]\left[R\right] \tag{A.21}$$

Matrix [R] is termed the transition rate matrix. Each off-diagonal element of [R], r_{ij} ($i \neq j$), may be thought of as the rate of transition from state i to state j. Figure A.5 is a visual interpretation of (A.20).

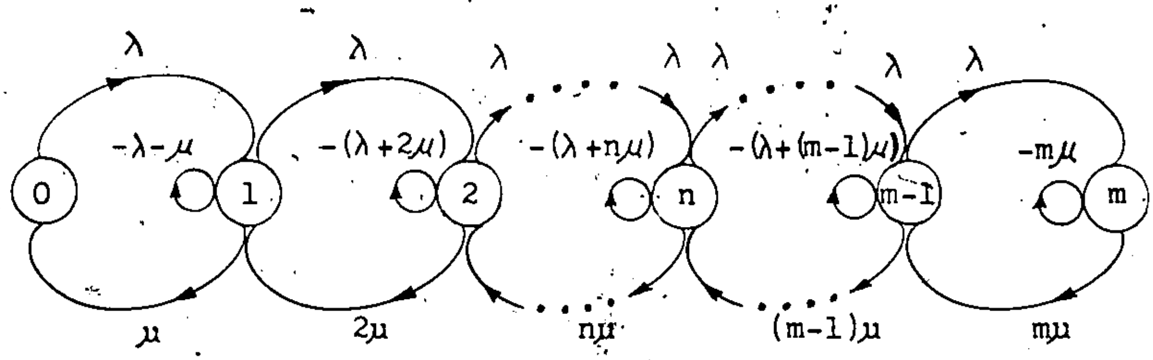


Figure A.5 Probability Transition Diagram for Erlang B Service Discipline

The nodes in this diagram represent system states and the transition rates are indicated on the directed branches. For example, one moves from state 0 where there are no calls in the system to the next higher state where there is one call in the system at a rate, λ . Conversely, one returns to the original state 0 once a call is completed and the channel is freed. Such a transition occurs at a rate μ . The rate associated with a self-loop is equal to the negative of the sum of the outgoing rates for the node under consideration. These correspond to the elements along the principal diagonal of (A.20).

A knowledge of the state of the system at t=0 is equivalent to knowing $p_i(0)$, i=0,1,2...m, and this is sufficient to allow the set of m+1 interrelated differential equations represented by (A.21) to be solved. However, if our sole interest lies in the steady state probabilities we can proceed in another way and avoid this chore.

Firstly, what is meant by the term "steady state"? Mathematically speaking, we wish to find $p_n = \lim_{t\to\infty} p_n(t)$. Practically speaking, we envision that the system will eventually reach a point where its behaviour is independent of its initial state and that no matter what time we observe the system, the probability of finding n customers will be a constant. By this time, any transient components will, hopefully, have been long since washed out.

Although not immediately apparent, the idea of a steady state is closely linked to the concept of ergodicity [6]. When applied to a random process, the descriptor "ergodic" infers that statistical measures such as the mean or variance may be determined equally well from a time average of any one sample function or from an ensemble average at a particular instant of time. In a queuing model the random process is $\{N(t)\}$ where $N(t)$ represents the (random) number of customers in the system at time t and the brackets indicate a collection or ensemble of such functions. Realizations from $\{N(t)\}$ might appear as in Figure A.6.

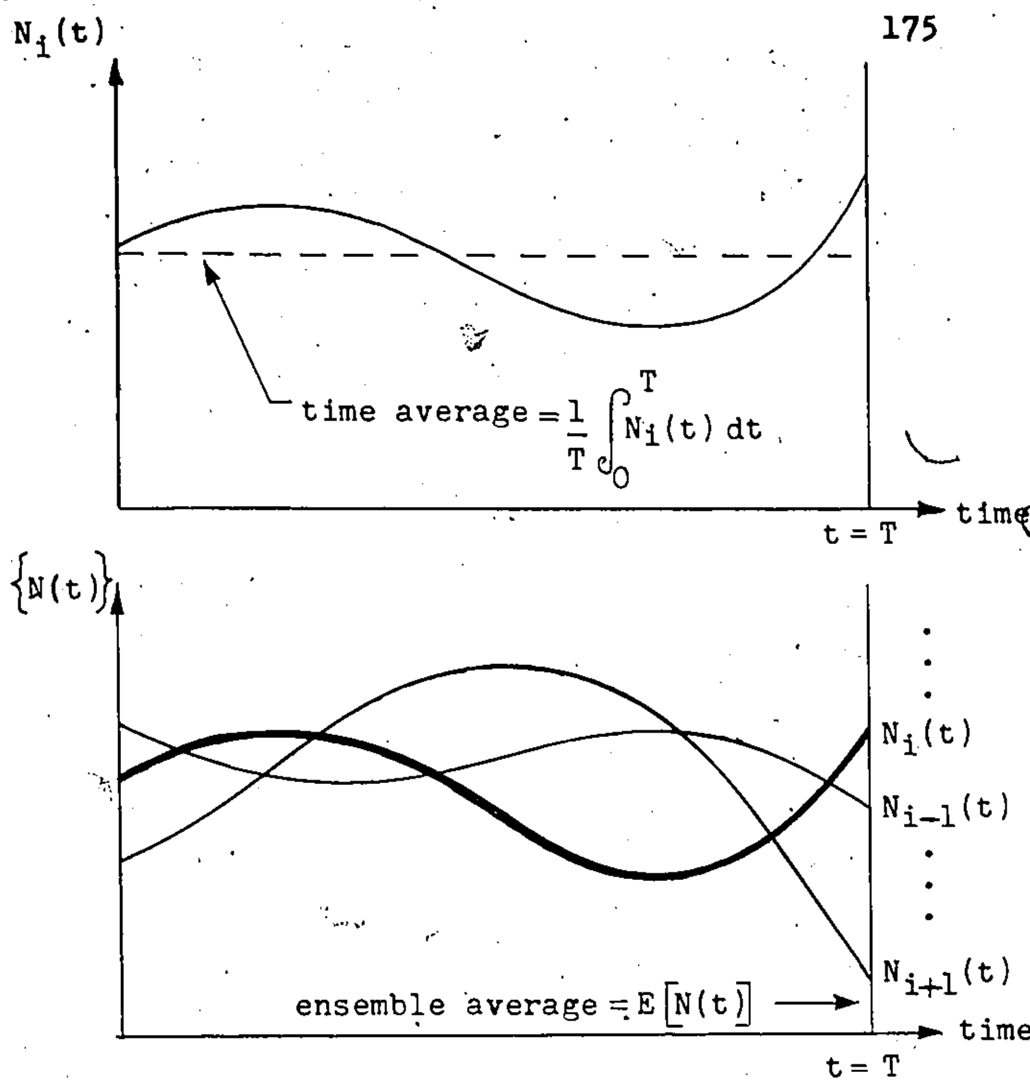
The time average of one particular sample function, $N_i(t)$, is:

$$N_i = \frac{1}{T}\int_0^T N_i(t)\,dt = \text{constant} \tag{A.22}$$

Obviously, the accuracy of this average will improve as T grows larger. The ensemble average or expectation is:

$$m(t) = \lim_{n\to\infty} \sum_{i=0}^{n} N_i(t)/n$$

$$\equiv \sum_{n=0}^{\infty} np_n(t) = \text{function of time} \tag{A.23}$$



<u>Figure A.6</u> The Concept of Ergodicity [19]

Now, if $\{N(t)\}$ is indeed ergodic then the time average must equal the ensemble average:

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T N_i(t)\,dt = \lim_{t\to\infty} m(t) \qquad (A.24)$$

Since the LHS of (A.24) is a constant, $m(t)$ must become independent of time as $t \to \infty$ and from (A.23), the same must apply to $p_n(t)$. That is, $\lim_{t\to\infty} p_n(t) = p_n$.

Returning now to our objective of finding a solution to (A.21), let us proceed under the assumption that the system of Figure A.5 is ergodic and later determine the conditions under which this assumption is true. In the steady state, $d[P(t)]/dt = d[p]/dt = 0$ and (A.21) becomes:

$$\left[P\right]\left[R\right] = 0 \tag{A.25}$$

Or, in expanded form:

$$\left[p_o, p_1 \cdots p_{m-1}, p_m\right] \begin{bmatrix} -\lambda & \lambda & & & & & & & \\ u & -\lambda-u & \lambda & & & & 0 & & \\ & 2u & -\lambda-2u & \lambda & & & & & \\ & & \ddots & \ddots & \ddots & & & & \\ & & & & nu & -\lambda-nu & \lambda & & \\ & & & & & \ddots & \ddots & \ddots & \\ & & 0 & & & & & & \lambda \\ & & & & & & & mu & -mu \end{bmatrix} = 0$$

(A.26)

Equation (A.26) may be considered as a set of equations governing the exchanges of the steady state probabilities amongst the various states. These exchanges may be represented by a steady-state probability exchange diagram as shown in Figure A.7.

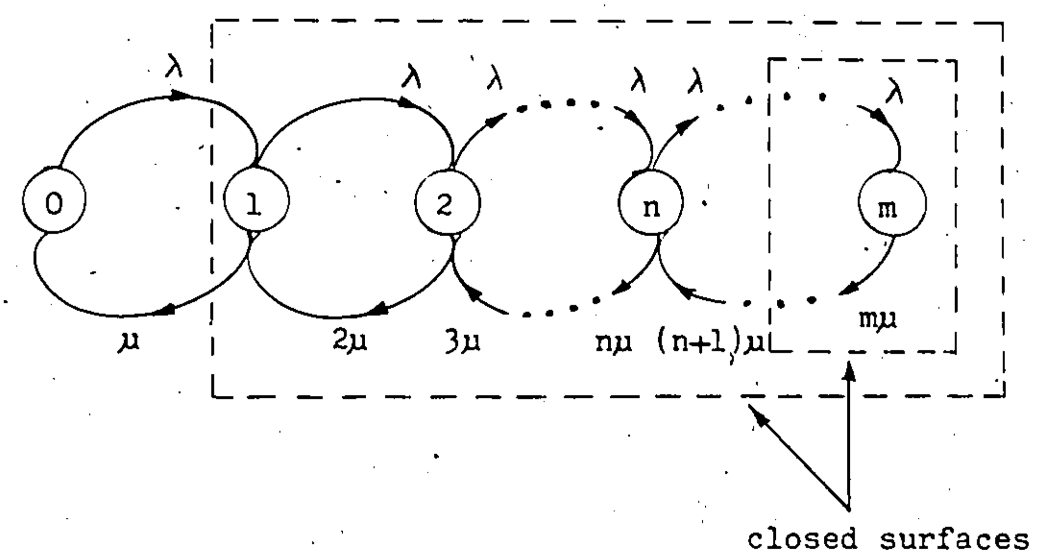


Figure A.7 Steady-state Probability Exchange Diagram for Erlang B Service Discipline

The only difference between this diagram and the diagram of Figure A.5 is that the self loops have been removed. A balance equation may be written by considering a surface which encloses all states above state n:

$$\left[\begin{array}{c}\text{the steady state probability}\\ \text{flowing out of the surface}\end{array}\right] = \left[\begin{array}{c}\text{steady-state probability}\\ \text{flowing into the surface}\end{array}\right] \quad (A.27)$$

If we apply (A.27) to state 0 as indicated in Figure A.7, the result is:

$$\lambda p_0 = \mu p_1$$

This is simply the first equation of (A.26). If we apply (A.27) to state n as is also indicated in Figure A.7, the result is:

$$(n+1)\mu p_{n+1} = \lambda p_n \qquad \text{(A.28)}$$

Equation (A.28) is a recursive relationship and by writing out a few sample terms:

$$p_1 = \frac{\lambda}{\mu} p_0 = \frac{\lambda/\mu}{1!} p_0$$

$$p_2 = \frac{\lambda}{\mu} \cdot \frac{p_1}{2} = \frac{(\lambda/\mu)^2 p_0}{2!}$$

$$p_3 = \frac{\lambda}{\mu} \frac{p_2}{3} = \frac{(\lambda/\mu)^2 p_0}{3!} ,$$

it is apparent that the general formula, expressed in terms of p_0, is:

$$p_n = \frac{(\lambda/\mu)^n p_0}{n!} \qquad \text{(A.29)}$$

The m+1 states are mutually exclusive and exhaustive, thus

$$\sum_{n=0}^{m} p_n = 1 \qquad \text{(A.30)}$$

That is:

$$p_0 + \frac{\lambda}{\mu}p_0 + \left(\frac{\lambda}{\mu}\right)^2\frac{p_0}{2!} + \ldots + \left(\frac{\lambda}{\mu}\right)^m\frac{p_0}{m!} = 1$$

$$p_0 = \left(\sum_{n=0}^{m} (\lambda/\mu)^n/n!\right)^{-1} \qquad \text{(A.31)}$$

And from (A.30):

$$p_n = \frac{(\lambda/\mu)^n/n!}{\sum_{n=0}^{m}(\lambda/\mu)^n/n!}, \quad n=0,1,2\ldots m \qquad \text{(A.32)}$$

The <u>blocking</u> <u>probability</u>, P_b, is simply the probability that the system is in state m since while in this state any new arrivals will be turned away, therefore:

$$P_b = \frac{(\lambda/\mu)^m/m!}{\sum_{n=0}^{m}(\lambda/\mu)^n/n!} \qquad \text{(A.33)}$$

Equation (A.33) is the well known Erlang B formula. In the context of telephone traffic analysis, (A.33) specifies the relationship between the offered traffic load, λ/μ, the number of channels (trunks), m, and the anticipated blocking probability, P_b.

Before going any further, the concept of an offered traffic load could probably use some clarification. Consider the case of only one channel (m=1). If, for example, $\lambda = \mu$ (the mean arrival rate = the mean service rate) then calls are arriving at a rate which just equals the channel's ability to handle these calls. The condition $\lambda/\mu = 1$ therefore seems to represent some sort of saturation point. In theory, if perfect scheduling of calls could be achieved (the start of one calls coinciding exactly with the end of the previous call) so that the channel is engaged constantly, then $\lambda/\mu=1$. In practice, of course, the random nature of customer requests for service will prevent this from occurring.

Note also from (A.33) that theory does allow values of λ/μ in excess of unity. The limitation on system capacity results in the rejection of a certain fraction P of customers so that overload is prevented. Thus a steady state solution always exists and the assumption of ergodicity applies. However, as $m \to \infty$, (A.31) becomes an infinite series, requiring $\lambda/\mu<1$ for convergence. In this case, $\lambda/\mu<1$ is the necessary condition for existence of a steady state solution.

The quantity λ/μ has a unit associated with it. This unit is dimensionless and is generally called an "erlang". Alternatively, traffic load may be expressed in terms of "call seconds per second". This latter unit is more practically-oriented as illustrated by the following scenario. An individual is monitoring ("tapping") a telephone line. At some arbitrary time t=0 he connects his recording equipment and at some later time t=T he disconnects the equipment. The period T will be called the interval of study as shown in Figure A.8. During this interval he records three con-

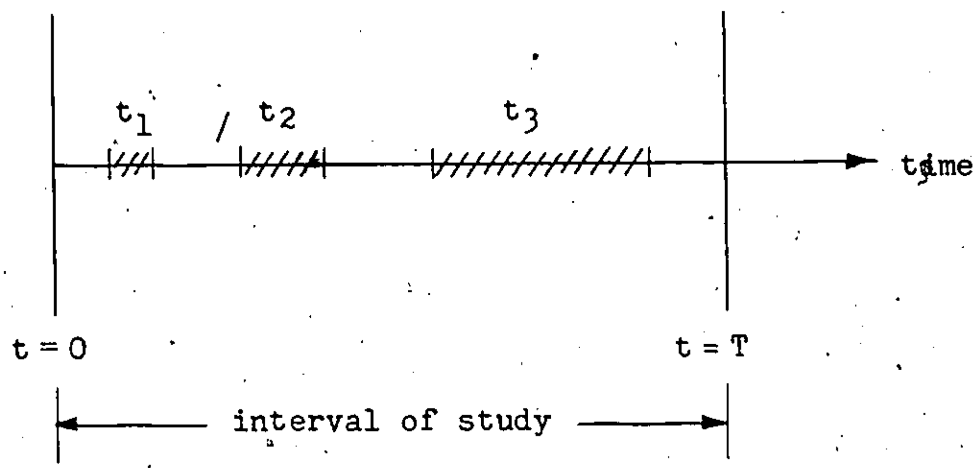


Figure A.8 A Practical Interpretation of "Traffic Load"

versations of varying durations t_1, t_2 and t_3. The traffic load or channel utilization is defined as

$$\frac{t_1 + t_2 + t_3}{T}, \qquad \text{(call sec/sec)}$$

and is obviously less than or equal to unity. (If the users pause to consider what they are discussing the call is still in progress and the channel still in use). It is apparent that 1 erlang is equivalent to 1 call second per second.

A.2.2 Blocked Calls Held Service Discipline

A second traffic formula is obtainable from the Erlang B model by allowing $m \to \infty$. This is interpreted as meaning that no customer is rejected due to a lack of channels. Rather, they are prepared to wait for a time $1/\mu$ on the average if upon entry into the system, all channels are found to be busy [20]. Should a channel become available while waiting it is seized and used for the residual portion of the call holding time. In any case, no customer remains in the system for longer than one complete call holding time.

As before, the steady state probabilities are:

$$p_n = \frac{(\lambda/\mu)^n}{n!} \cdot p_0$$

From (A.31), with $m = \infty$:

$$p_0 = \left(\sum_{n=0}^{\infty} (\lambda/\mu)^n / n!\right)^{-1} = e^{-\lambda/\mu} \tag{A.34}$$

Consequently:

$$p_n = \frac{(\lambda/\mu)^n}{n!} \cdot e^{-\lambda/\mu} \quad , \quad \lambda/\mu < 1 \tag{A.35}$$

The blocking probability is now defined as the probability that the system is in state m or higher, that is:

P_b = Prob(the system is in state m or in state m+1 or in state m+2 or ...)

Since this represents a union of disjoint events:

$$P_b = \sum_{n=m}^{\infty}((\lambda/\mu)^n/n!) \cdot e^{-\lambda/\mu} \qquad (A.36)$$

A.2.3 Erlang C (Blocked Calls Delayed) Discipline

As shown in Figure A.9, the model for the Erlang C service discipline consists of m channels and an unlimited queue. Calls arriving to find an idle channel are served immediately while calls which arrive when all channels are occupied join the queue in FIFO order and wait indefinitely for service.

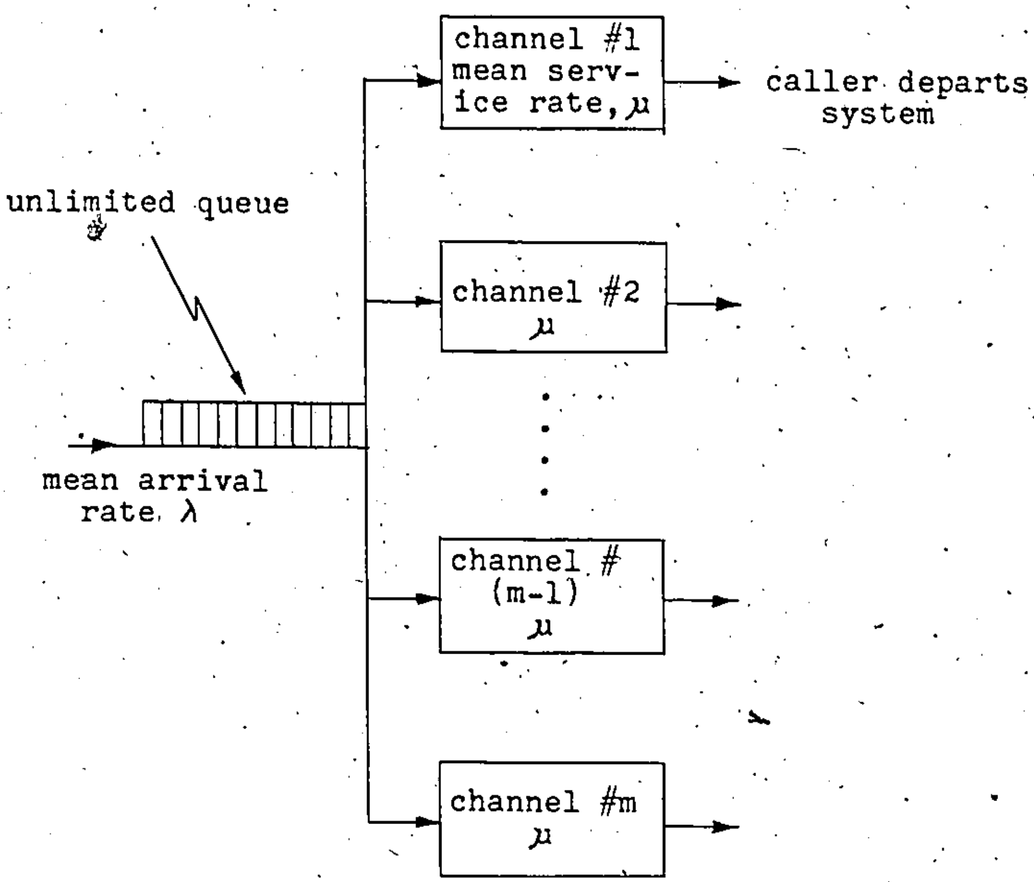


Figure A.9 Erlang C Service Discipline (system capacity = ∞)

Consider the surface A enclosing all states above state n ($n \le m-1$) in Figure A.10(a). The balance equation is:

$$(n+1)\mu p_{n+1} = \lambda p_n \ , \quad n=0,1,2...m-1 \qquad (A.37)$$

The general relationship in terms of p_0 is:

$$p_n = \frac{(\lambda/\mu)^n}{n!} p_0 \ , \quad n=0,1,2...m \qquad (A.38)$$

Now consider the surface B enclosing all states above state n ($n \ge m$) in Figure A.10(b). The balance equation in this case is:

$$m\mu p_{n+1} = \lambda p_n \ , \quad n=m,m+1,m+2... \qquad (A.39)$$

Writing out a few terms of (A.39) and employing (A.38) when n=m:

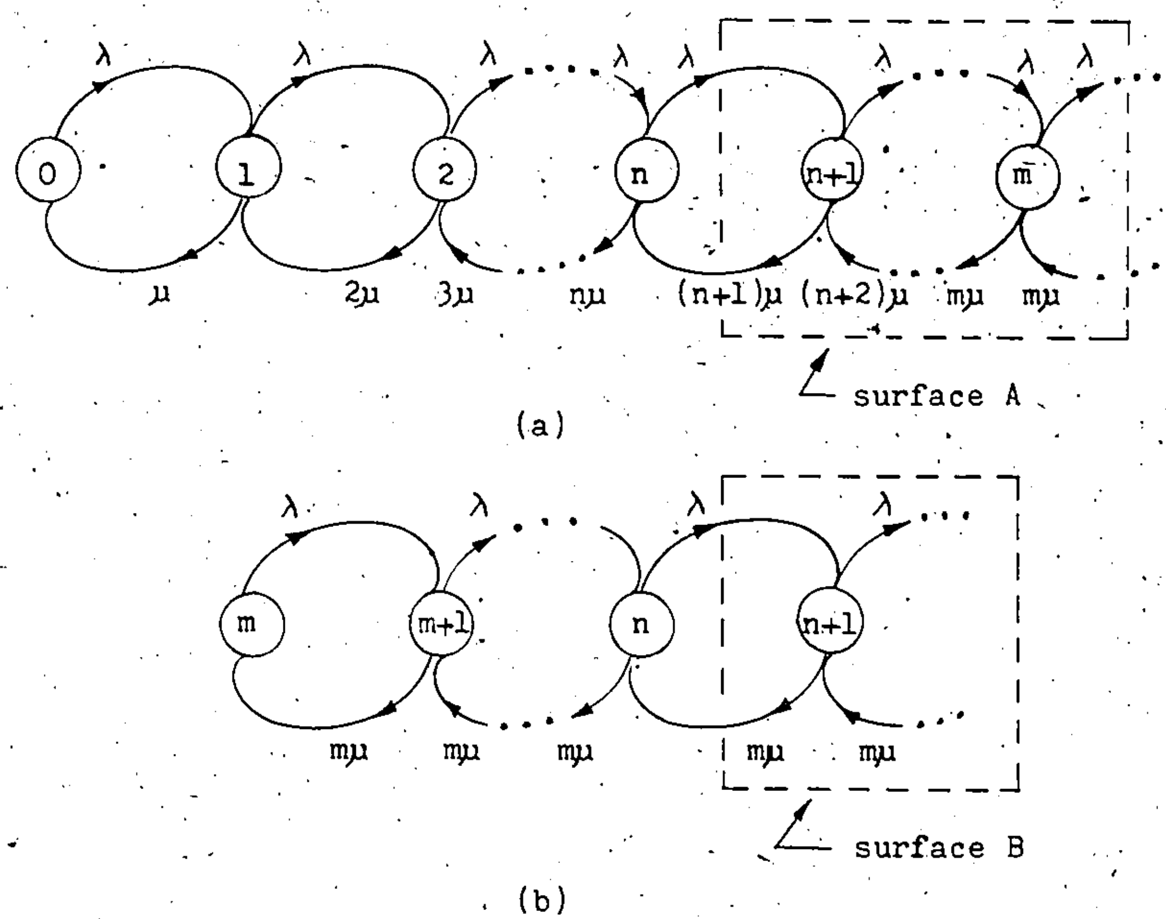
$$p_{m+1} = \frac{\lambda}{\mu}.\frac{p_m}{m} = \frac{(\lambda/\mu)^{m+1}}{m \cdot m!}.p_0$$

$$p_{m+2} = \frac{\lambda}{\mu}.\frac{p_{m+1}}{m} = \frac{(\lambda/\mu)^{m+2}}{m^2 \cdot m!}.p_0$$

In general:

$$p_n = \frac{(\lambda/\mu)^n \cdot m^{m-n}}{m!} \cdot p_0 \qquad (A.40)$$

$$n = m+1, m+2, m+3....$$



<u>Figure A.10</u> Steady-state Probability Exchange Diagram for Erlang C Service Discipline

Since

$$\sum_{n=0}^{\infty} p_n = 1 ,$$

it is found after much simplification that:

$$p_0 = \left(\sum_{n=0}^{m-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^m}{m!} \cdot \frac{\mu m}{\mu m-\lambda}\right)^{-1} \qquad \text{(A.41)}$$

Inasmuch as no calls are denied service, it is no longer possible to speak of a blocking probability. However, we can define a probability of delay P_q associated with finding m or more calls in the system. Thus:

$$P_q = p_0 \sum_{n=m}^{\infty} \frac{(\lambda/\mu)^n}{m!} \cdot m^{m-n} \qquad \text{(A.42)}$$

Let us temporarily introduce the symbol $\rho=\lambda/m\mu$ which, incidently, represents the relative traffic load per channel. Then:

$$P_q = p_0 \sum_{n=m}^{\infty} \frac{m^m}{m!} \cdot \rho^n$$

$$= p_0 \frac{m^m}{m!}\left(\sum_{n=0}^{\infty} \rho^n - \sum_{n=0}^{m-1} \rho^n\right)$$

$$= p_0 \frac{m^m}{m!}\left(\frac{1}{1-\rho} - \frac{1-\rho^m}{1-\rho}\right)$$

$$= p_0 \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \cdot \frac{\mu m}{\mu m-\lambda}$$

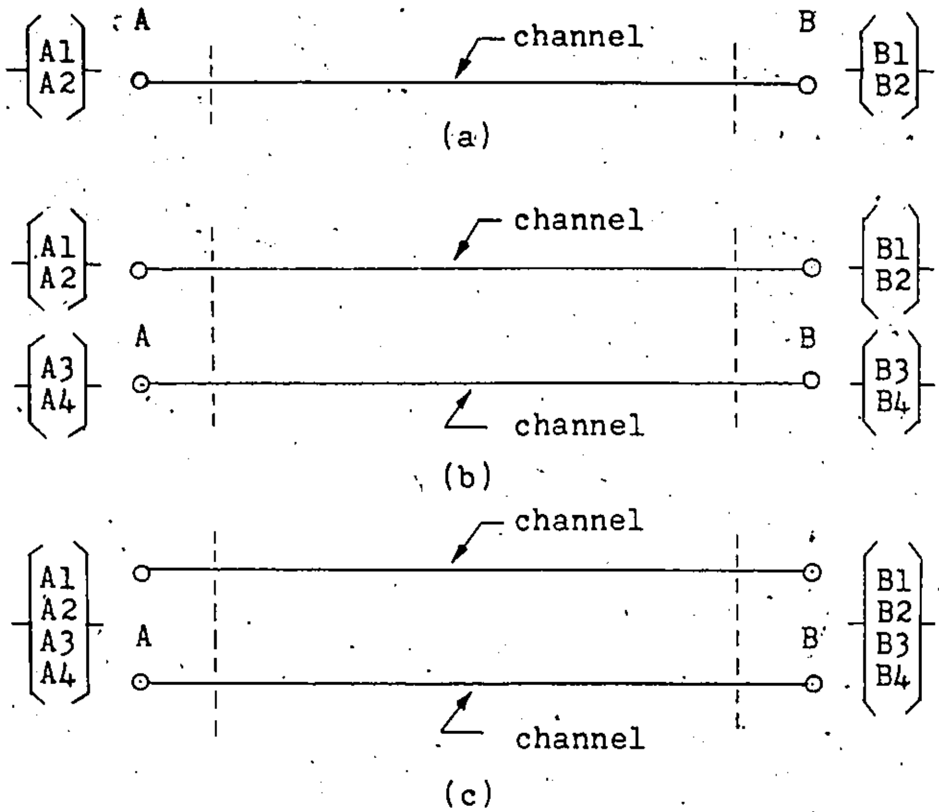
Finally, substituting for p_0 from (A.41), we arrive at the Erlang C formula for the probability of delay:

$$P_q = \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \left(\frac{\mu m-\lambda}{\mu m} \cdot \sum_{n=0}^{m-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^m}{m!}\right)^{-1} \qquad \text{(A.43)}$$

A.3 TRUNKING

Trunking is a system design concept usually discussed in the context of conventional landline telephone service but which is also quite applicable to radio communications. Under some circumstances it offers considerable potential for increasing channel utilization and reducing delays for service [45].

The premise upon which trunking is based may be understood with the aid of Figure A.11. At the top of this diagram we show two users, A1 and A2, at point A connected by a single communication channel to two other users, B1 and B2, at point B. Obviously, if someone at A is talking to someone at B then the other users at these two sites, who might also wish to communicate, are forced to wait.



<u>Figure A.11</u> The Trunking Concept for Two Groups of Users at Two Isolated Points A and B

Suppose now that two more users appear at each point and that an additional channel is dedicated to their use as shown in Figure A.11(b). From the users' standpoint, the quality of service has in no way changed. If on one occasion, A1 is talking to B1 and A2 wants to talk to B2, then A2 has no choice but to wait until A1 and B1 finish. All the while, the second channel may be idle; no member of the second group of users needs it at that moment. However, if the second channel could be made accessible to the first group of users, A2 could place his call without delay. Suppose this is done. The final configuration is shown in Figure A.11(c).

The service available to the users is much improved; the probability of having to wait for a channel is clearly reduced. Moreover the utilization of the channels has increased; an idle one can now be put to work. This is the essence of the trunking concept.

The formulations of Appendix A permit us to replace this simple scenario with a more precise description. Figures A.12 and A.13 illustrate some typical results for the Erlang C service discipline. The graphs of Figure A.12 relate waiting time (W_q) to channel usage ($\rho = \lambda/m\mu$), with the number of trunked channels (m) serving as a parameter. Channel usage is indicated in call seconds/second (see discussion of

Fig. A.12 Average Waiting Time Vs. Channel Usage for "m" Trunked Channels

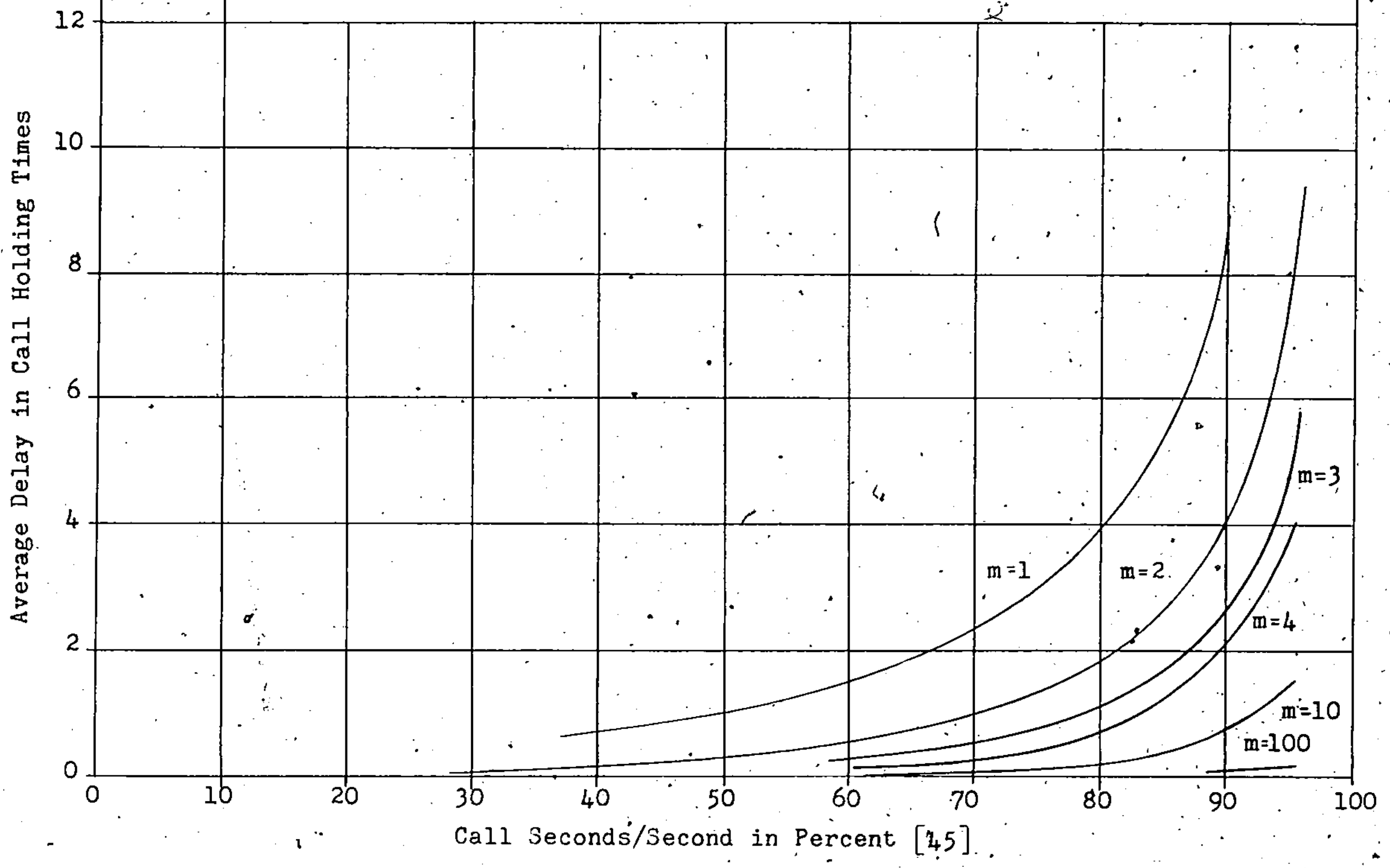


Fig. A.13′ Probability of Finding a Clear Channel Vs. Channel Usage for "m" Trunked Channels

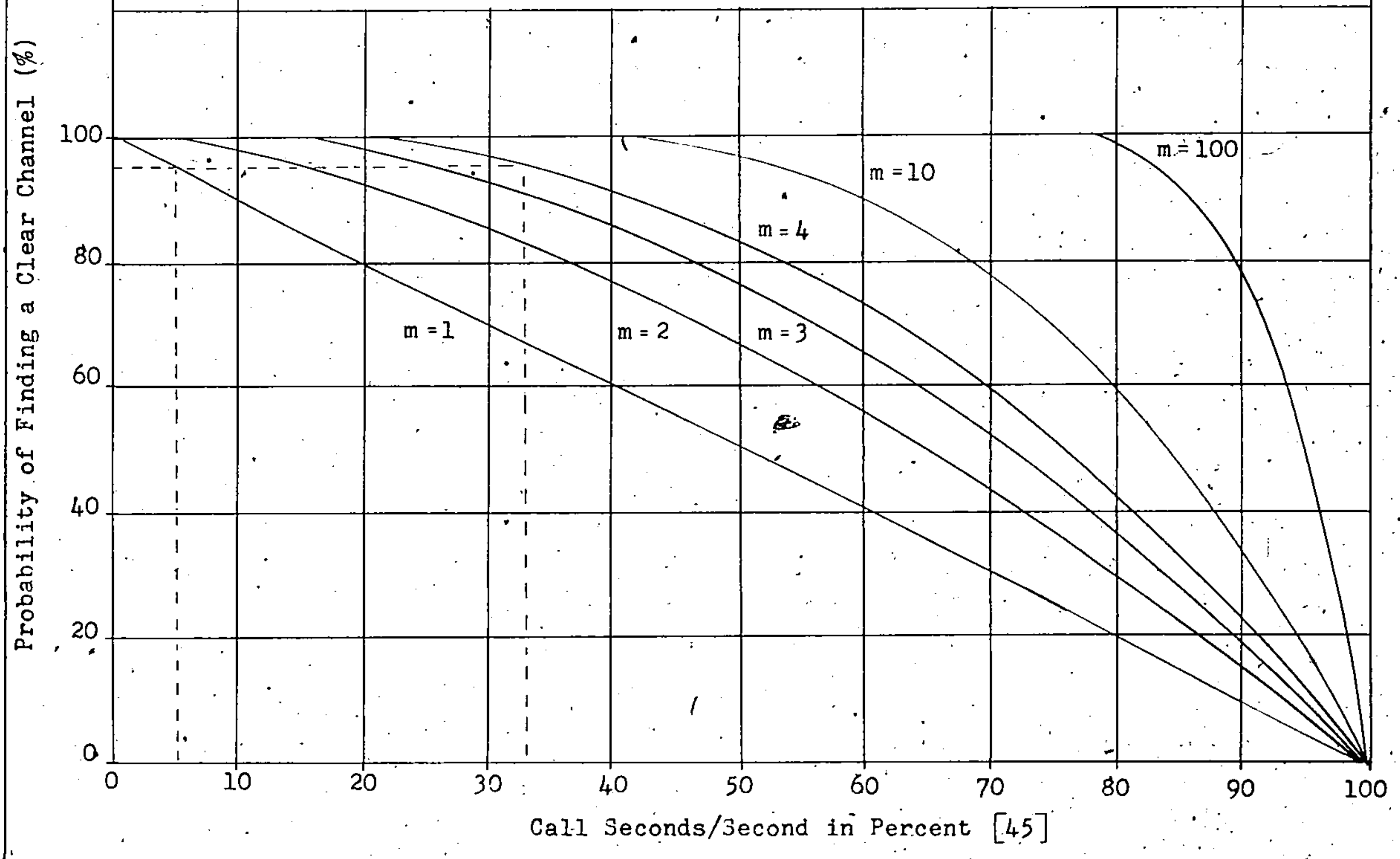


Figure A.8). Waiting time is expressed in multiples of the mean channel holding time ($1/\mu$) -- solely as a mathematical convenience. Figure A.13 shows the probability of finding a clear channel ($1-P_q$) as a function of channel usage with the number of trunked channels as a parameter.

The relative benefits of trunking depend heavily upon the quality of service demanded by the users. For example, take a user who must have a probability of 0.95 of finding a single channel idle whenever there is a message to be transmitted. From Figure A.13 such a channel can be in use, on the average, only 5% of the time. Next consider three more users with identical needs, each with a channel of his own. If all agree to trunk their channels, the system of four trunked channels can carry six times as much traffic while maintaining the same probability (95%), for all users, of finding a channel available when a message is to be sent. This is indicated in Figure A.13.

Now consider a different situation. If a group of users can tolerate short delays then a single channel, loaded nearly to the 70% level will suffice.[13] (See Figure A.12). Under these conditions trunking affords little improvement. Four such channels, trunked, would have a much reduced delay time but what value would this be to users prepared to accept the delay associated with the non-trunked channels? Also, some small improvement in channel utilization could be

[13] By a short delay is meant an average delay of one or two average call holding times.

achieved but the system, already at 70% capacity, in practice could not be pushed to much higher levels.

In general, the advantages of trunking are greatest when the channels are lightly loaded. When channels are already heavily loaded, the advantages of applying the trunking principle may be small or nonexistent.

Appendix B

GPSS SIMULATION PROGRAMS

B.1 GPSS PROGRAM: MODEL 1

The General Purpose System Simulator (GPSS) program listing is contained in Box B.1. The heart of the program is control matrix MH1 which keeps track of the availability of the fixed (voice) channels and the access channels in all 40 cells. This matrix has dimensions 40 X XH1 as illustrated in Figure B.1. The number of rows corresponds to the number of cells while XH1 is a savevalue location containing the number of fixed plus access channels per cell. XH1 is initialized before simulation begins.

Any entry (x,y) in MH1 (where x specifies the row and y, the column) is either a 0 or a 1 indicating the availability of nonavailability respectively of voice/access channel y in cell x. One can visualize the elements of MH1 changing in a random fashion as the simulation progresses and new calls join the system while completed calls leave.

A straightforward linear search is conducted when assigning channels to calls. Thus in Figure B.1, when looking for a free access channel to serve a call in cell i, we

```
        CREATE
        REALLOCATE BLO,250,BVR,2,CHA,1,COM,150000,FMS,1
        REALLOCATE FSV,1,FUN,2,GRP,41,HMS,7,HSV,10,LOG,41
        REALLOCATE QUE,42,STO,42,TAB,9,VAR,87,XAC,1500

*LOC    OPERATION  A,B,C,D,E,F,G              COMMENTS
        SIMULATE   10
*
*INITIALIZATION
*
        INITIAL    XH1,11      # OF FIXED CHANNELS/CELL (VOICE+ACCESS)
        INITIAL    XH2,10      # OF FIXED CHANNELS/CELL (VOICE)
        INITIAL    XH3,0       # OF DYNAMIC CHANNELS IN SYSTEM
        INITIAL    XH4,3       # OF REATTEMPTS
 1      MATRIX     H,40,11     # OF COLUMNS = XH1
 2      MATRIX     H,40,6      # OF COLUMNS = XH3
 3      MATRIX     H,1,10      # OF COLUMNS > XH4
 4      MATRIX     H,40,10     # OF COLUMNS > XH4
*
*STORAGE AND QUEUE DEFINITION
*
        STORAGE    S1-S40,50   STORAGES 1 THRU 40 ARE ASSIGNED A
 SYSTM  EQU        41,S        CAPACITY OF 50 CALLS.  STORAGE
 SYSTM  STORAGE                "SYSTM" HAS UNLIMITED CAPACITY.
 ACCES  EQU        41,Q
*
*TABLE DEFINITION
*
 SRVDA  TABLE      P1,1,1,41   SRVDA AND FAILA TABULATE THE NUMBER
 SRVDB  TABLE      P1,1,1,41   OF CALLS WHICH OBTAIN OR DON'T OBTAIN
 FAILA  TABLE      P1,1,1,41   AN ACCESS CHANNEL. SRVDB AND FAILB
 FAILB  TABLE      P1,1,1,41   SERVE THE SAME PURPOSE FOR THE VOICE
*                              CHANNELS.
 TIME1  TABLE      IA,0,500,30
 TIME2  TABLE      IA,0,500,30
 TIME3  TABLE      IA,0,500,30
 TIME4  TABLE      IA,0,500,30
*
*FUNCTION DEFINITION
*
 EXPON  FUNCTION   RN4,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
```

Box B.1 Program Listing for Simulation Model No. 1

```
*
*VARIABLE DEFINITION
*
    81     VARIABLE    4500000     MAXIMUM SIMULATION TIME
    82     VARIABLE    120000     MEAN CALL HOLDING TIME
    83     VARIABLE    4000      MEAN INTERARRIVAL TIME OF CALLS
    84     VARIABLE    XH4-P4+1
    1      BVARIABLE   C1'LE'V81*N$CALLS'LE'9000
*
*CALL GENERATION
*
1   CEL1   GENERATE    V83,FN$EXPON,,,,10,F
2          ASSIGN      1,1
3          TRANSFER    ,CALLS
4   CEL2   GENERATE    V83,FN$EXPON,,,,10,F
5          ASSIGN      1,2
6          TRANSFER    ,CALLS
7   CEL3   GENERATE    V83,FN$EXPON,,,,10,F
8          ASSIGN      1,3
9          TRANSFER    ,CALLS
10  CEL4   GENERATE    V83,FN$EXPON,,,,10,F
11         ASSIGN      1,4
12         TRANSFER    ,CALLS
13  CEL5   GENERATE    V83,FN$EXPON,,,,10,F
14         ASSIGN      1,5
15         TRANSFER    ,CALLS
16  CEL6   GENERATE    V83,FN$EXPON,,,,10,F
17         ASSIGN      1,6
18         TRANSFER    ,CALLS
19  CEL7   GENERATE    V83,FN$EXPON,,,,10,F
20         ASSIGN      1,7
21         TRANSFER    ,CALLS
22  CEL8   GENERATE    V83,FN$EXPON,,,,10,F
23         ASSIGN      1,8
24         TRANSFER    ,CALLS
25  CEL9   GENERATE    V83,FN$EXPON,,,,10,F
26         ASSIGN      1,9
27         TRANSFER    ,CALLS
28  CEL10  GENERATE    V83,FN$EXPON,,,,10,F
29         ASSIGN      1,10
30         TRANSFER    ,CALLS
31  CEL11  GENERATE    V83,FN$EXPON,,,,10,F
32         ASSIGN      1,11
33         TRANSFER    ,CALLS
34  CEL12  GENERATE    V83,FN$EXPON,,,,10,F
35         ASSIGN      1,12
36         TRANSFER    ,CALLS
37  CEL13  GENERATE    V83,FN$EXPON,,,,10,F
38         ASSIGN      1,13
39         TRANSFER    ,CALLS
40  CEL14  GENERATE    V83,FN$EXPON,,,,10,F
41         ASSIGN      1,14
42         TRANSFER    ,CALLS
43  CEL15  GENERATE    V83,FN$EXPON,,,,10,F
44         ASSIGN      1,15
```

<u>Box B.1</u> (cont'd)

Program Listing for Simulation Model No. 1

```
             TRANSFER    ,CALLS
     CEL16   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,16
             TRANSFER    ,CALLS
     CEL17   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,17
             TRANSFER    ,CALLS
     CEL18   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,18
             TRANSFER    ,CALLS
     CEL19   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,19
             TRANSFER    ,CALLS
     CEL20   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,20
             TRANSFER    ,CALLS
     CEL21   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,21
             TRANSFER    ,CALLS
     CEL22   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,22
             TRANSFER    ,CALLS
     CEL23   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,23
             TRANSFER    ,CALLS
     CEL24   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,24
             TRANSFER    ,CALLS
     CEL25   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,25
             TRANSFER    ,CALLS
     CEL26   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,26
             TRANSFER    ,CALLS
     CEL27   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,27
             TRANSFER    ,CALLS
     CEL28   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,28
             TRANSFER    ,CALLS
     CEL29   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,29
             TRANSFER    ,CALLS
     CEL30   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,30
             TRANSFER    ,CALLS
     CEL31   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,31
             TRANSFER    ,CALLS
     CEL32   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,32
             TRANSFER    ,CALLS
     CEL33   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,33
             TRANSFER    ,CALLS
     CEL34   GENERATE    V83,FN$EXPON,,,,10,F
             ASSIGN      1,34
```

<u>Box B.1</u> (cont'd)

Program Listing for Simulation Model No. 1

```
102            TRANSFER     ,CALLS
103      CEL35 GENERATE     V83,FN$EXPON,,,,10,F
104            ASSIGN       1,35
105            TRANSFER     ,CALLS
106      CEL36 GENERATE     V83,FN$EXPON,,,,10,F
107            ASSIGN       1,36
108            TRANSFER     ,CALLS
109      CEL37 GENERATE     V83,FN$EXPON,,,,10,F
110            ASSIGN       1,37
111            TRANSFER     ,CALLS
112      CEL38 GENERATE     V83,FN$EXPON,,,,10,F
113            ASSIGN       1,38
114            TRANSFER     ,CALLS
115      CEL39 GENERATE     V83,FN$EXPON,,,,10,F
116            ASSIGN       1,39
117            TRANSFER     ,CALLS
118      CEL40 GENERATE     V83,FN$EXPON,,,,10,F
119            ASSIGN       1,40
120            TRANSFER     ,CALLS
121      CALLS ENTER        SYSTM
122            ASSIGN       2,XH1
123            ASSIGN       3,XH2
124            ASSIGN       4,XH4
125            TEST NE      P2,P3,FIXD1  (DYN01)
      *
      *ACCESS CHANNEL SEIZURE ATTEMPT
      *
126            QUEUE        ACCES
127            QUEUE        P1
128      ACES4 ASSIGN       2,XH1
129      ACES5 TEST E       MH1(P1,P2),0,NEXT
130            DEPART       P1
131            DEPART       ACCES
132            TABULATE     SRVDA
133            MSAVEVALUE   1,P1,P2,1,H
134            PRIORITY     1
135            ADVANCE      100
136            MSAVEVALUE   1,P1,P2,0,H
137            LEAVE        SYSTM
138            TERMINATE
      *
      *FIXED CHANNEL ASSIGNMENT
      *
139      FIXD1 GATE LR      P1,FAILB    (DYN01)
140      FIXD2 TEST E       MH1(P1,P3),0,SERCH
141            MSAVEVALUE   1,P1,P3,1,H
142            TABULATE     SRVDB
143            ENTER        P1
144            PRIORITY     2
145            ADVANCE      V82,FN$EXPON
146            TEST E       G*1,0,FREE
147            MSAVEVALUE   1,P1,P3,0,H
148      RESET LOGIC R      P1
149            TRANSFER     ,EXIT
      *
      *SEARCH FOR A FIXED CHANNEL
      *
```

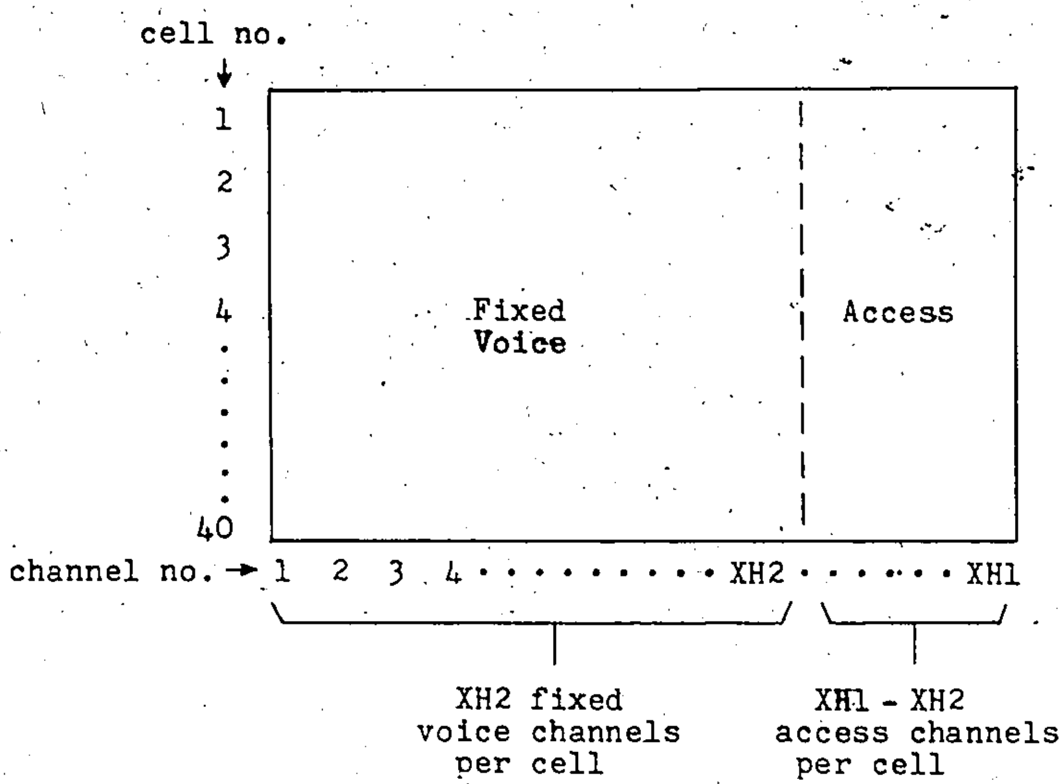
<u>**Box B.1**</u> **(cont'd)**

Program Listing for Simulation Model No. 1

```
150        SERCH LOOP          3,FIXD2
151              LOGIC S       P1
152              TRANSFER      ,FAILB    (DYNO1)
           *
           *SEARCH FOR AN ACCESS CHANNEL, WAIT & REATTEMPT IF NECESSARY
           *
153        NEXT  ASSIGN        2-,1
154              TEST E        P2,P3,ACES5
155              TEST G        P4,0,FAILA
156              ADVANCE       5000,5000
157              MSAVEVALUE    3+,1,V34,1,H
158              MSAVEVALUE    4+,P1,V84,1,H
159              ASSIGN        4-,1
160              TRANSFER      ,ACES4
           *
           *CALL UNABLE TO FIND ACCESS CHANNEL
           *
161        FAILA TABULATE      FAILA
162              DEPART        P1
163              DEPART        ACCES
164              LEAVE         SYSTM
165              TERMINATE
           *
           *
           *CALLS LEAVE SYSTEM
           *
193        EXIT  LEAVE         P1
194              LEAVE         SYSTM
195              TERMINATE
           *
           *AUXILIARY CLOCK (CONTROL OF SIMULATION DURATION)
           *
196              GENERATE      300000
197              TEST E        BV1,1,STOP
198              TERMINATE     1
199        STOP  TERMINATE     24
                 START         1,NP
                 RESET         S41
                 INITIAL       MH3(1,1-10),0/MH4(1-40,1-10),0
                 START         25
                 END
```

<u>Box B.1</u> (cont'd)

Program Listing for Simulation Model No. 1



<u>Note</u>: Elements of MH1 can be 1 or 0; a 1 indicates the channel is in use while a zero indicates the channel is idle

<u>Figure B.1</u> Fixed Voice and Access Channel Control Matrix, MH1

start at the extreme right of row i and move leftward until a 0 in encountered. The 0 is changed to a 1 and the call then uses the channel whose identity is given by the column where the 0 was found. It is not permitted to go past the column represented by the dashed line during an access channel search.

The same procedure is followed in assigning a voice channel except that the search now begins at the partition separating the voice channel control from the access channel control. If we reach column 1 without encountering a 0, the call is blocked and is tabulated as such.

Calls are generated independently in each cell in the section of the program titled CALL GENERATION. During the program assembly phase each of the forty GENERATE blocks is primed; that is, arrival times t_i, $1<i<40$, are drawn from the exponential distribution. (Really, the initial t_i are the interarrival times relative to $t=0$). Calls are then scheduled to enter the system at each time t_i.

When program execution actually begins, the simulation clock jumps to the minimum t_i. A call leaves GENERATE block i, picks up its cell number (i) in the subsequent ASSIGN block and TRANSFERs to the main program at address CALLS, line 121. While this is going on, a new interarrival time t_i' is computed and another call is scheduled to leave GENERATE BLOCK i at time t_i+t_i'. This process is repeated over and over again for each GENERATE block. In this way the passage of time is modelled -- the simulation clock being continually updated to the time of occurrence of the next event.

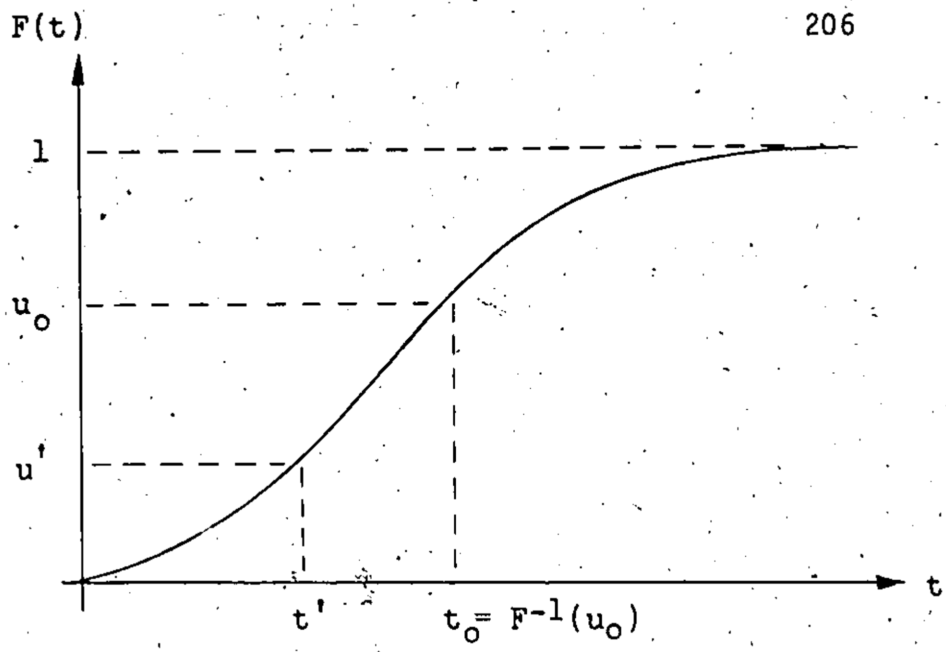
Processing of a call continues until (i) the call encounters some non-zero delay such as the time spent "holding" an access channel or (ii) the call finally LEAVEs the system. In the latter case, all references to the call are destroyed and the simulation clock updated. In the former case, processing of the call is suspended and a note is made that this call should not be re-examined until some future time. (if we take the access channel holding time as an example, this future time would be H=100 msec from now). The simulation clock is then updated and processing of the next call in line begins (or resumes at the point in the model where it was suspended).

The concept of some dynamic entity -- in this case, a call -- moving through the program blocks is simply a convenient artifice. As far as the GPSS processor is concerned, a call is nothing more than a number in a memory location. Associated with this number or call identity will be a great deal of information; for example, the time at which the call is next scheduled to move or its current location in the model. As well, each call has its own individual parameters. In our model we have used these parameters to store the cell number (GENERATE block) where the call was generated, identity of the channel which the call is using, or the portion of the reattempt allotment R which still remains, amongst other things.

Most of the laborious tasks such as the scheduling of events in sequential order, updating the simulation clock or producing and printing output statistics are handled automatically by the GPSS Processor. The programmer's principal responsibilities (besides writing a logically consistent program of course) are to specify (i) the desired interarrival time distribution for events, (ii) the conditions under which simulation is to cease and (iii) the form which the output statistics should take. Let's look at these one-by-one.

(i) How are random interarrival times computed?

Sample values from a user-specified cumulative distribution function, $F(t)$, are generated according to the inverse distribution method [5]. This method selects a random number u, uniformly distributed between 0 and 1, sets $F(t)=u$ and solves for t. For a particular value u_0 of u, Figure B.2 shows that a value t_0 is returned which is a particular sample of the random variable T, that is $t_0=F^{-1}(u_0)$. We need to prove that the values of T produced in this manner actually have the distribution $F(t)$.



<u>Figure B.2</u> Example Cumulative Distribution Function (for Discussion of The Method of Generating Sample Values of a Random Variable)

Say t_i' are the observed values of T' and F'(t') the corresponding cdf. If we can show that F'(t') = F(t') then the observed distribution will be identical to the specified distribution and T' = T.

For $T' \leq t'$, we must have $u \leq u'$ as in Figure B.2, so:

$$F'(t') = \text{Prob}(T' \leq t') = \text{Prob}(u \leq u')$$

But $u' = F(t')$, so

$$F'(t') = \text{Prob}(u \leq F(t')) = \text{Prob}\ (u \leq k)$$

where $F(t') = k$.

Since the values of u are uniformly distributed between 0 and 1, Prob($u \leq k$)=k and

$$F'(t')=F(t')$$

Therefore the observed values of t' are genuine samples of the specified distribution F(t).

When T is exponentially distributed, the cdf is:

$$F(t) = 1 - e^{-\lambda t} \quad , \quad t \geq 0$$

where λ=mean arrival rate.

If we set F(t)=u and solve for t as described above:

$$t = -\frac{1}{\lambda} \cdot \ln(1-u)$$

The function $t = -\ln(1-u)$ is tabulated before simulation starts and is included in the program listing (under the heading FUNCTION DEFINITION). The format is:

$$u_1, t_1 / u_2, t_2 / u_3, t_3 / \ldots$$

Now whenever a random interarrival time is needed the steps followed are:

1. Generate a value u_0, $0<u_0<.9999$

2. From the tabulated ordered pairs of function values compute $t_0 = -\ln(1-u_0)$ by linear interpolation.
3. Multiply t_0 by $1/\lambda$; the result is the desired inter-arrival time. (Usually, $1/\lambda$ is an independent variable, assigned a value just before a simulation trial).

Exactly the same steps are followed for the generation of random service times except that t_0 is multipled by $1/\mu$ -- the mean service time.

(ii) When is a simulation trial finished?

Control of the simulation duration is by means of short subprogram labelled AUXILIARY CLOCK in the program listing. This subroutine has two pruposes. First, after 5 minutes of simulated time, all the statistics accumulated up to this point are cleared and the model RESET. However, matrix MH1 remains unchanged and all the calls in the model at the time the RESET was initiated are left untouched. From this steady state point, the simulation is STARTed once again whereupon the second function of the subprogram comes into play. After every 5 minutes of simulated time a test is performed to see if either 1½ hours of simulated time have gone by or if 10,000 calls have been generated. Fulfillment of either condition is a signal to end the simulation.

(iii) How are the output statistics produced?

The chief means for accumulating statistics (in this particular model) are the TABLEs which have been assigned the names: SRVDA, FAILA, SRVDB, FAILB. A sample output for table SRVDA is shown in BOX B.2. When a call in cell i obtains an access channel, the entry in row i and column 2 of SRVDA is increased by 1. Similarly when a call fails to obtain an access channel a corresponding entry in FAILA is increased by 1. SRVDB and FAILB serve the same purpose for the voice channels. These 4 tables allow calculation of the access channel and voice channel blocking probabilities P_{ba} and P_{bv}. The remaining data in Box B.2 is produced automatically by GPSS and is not relevant to the discussion here.

TABLE SRVDA

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION	SUM OF ARGUMENTS
11775	20.439	11.527	240674.000

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE	CUMULATIVE REMAINDER	MULTIPLE OF MEAN
1	287	2.43	2.4	97.5	.048
2	330	2.80	5.2	94.7	.097
3	313	2.65	7.8	92.1	.146
4	300	2.54	10.4	89.5	.195
5	269	2.28	12.7	87.2	.244
6	277	2.35	15.0	84.9	.293
7	283	2.40	17.4	82.5	.342
8	296	2.51	19.9	80.0	.391
9	304	2.58	22.5	77.4	.440
10	291	2.47	25.0	74.9	.489
11	298	2.53	27.5	72.4	.538
12	314	2.66	30.2	69.7	.587
13	301	2.55	32.8	67.1	.636
14	280	2.37	35.1	64.8	.684
15	300	2.54	37.7	62.2	.733
16	271	2.30	40.0	59.9	.782
17	294	2.49	42.5	57.4	.831
18	285	2.42	44.9	55.0	.880
19	311	2.64	47.5	52.4	.929
20	306	2.59	50.1	49.8	.978
21	252	2.14	52.3	47.6	1.027
22	309	2.62	54.9	45.0	1.076
23	289	2.45	57.4	42.5	1.125
24	301	2.55	59.9	40.0	1.174
25	309	2.62	62.5	37.4	1.223
26	287	2.43	65.0	34.9	1.272
27	310	2.63	67.6	32.3	1.320
28	284	2.41	70.0	29.9	1.369
29	315	2.67	72.7	27.2	1.418
30	280	2.37	75.1	24.8	1.467
31	320	2.71	77.8	22.1	1.516
32	326	2.76	80.6	19.3	1.565
33	287	2.43	83.0	16.9	1.614
34	270	2.29	85.3	14.6	1.663
35	274	2.32	87.6	12.3	1.712
36	303	2.57	90.2	9.7	1.761
37	320	2.71	92.9	7.0	1.810
38	252	2.14	95.0	4.9	1.859
39	288	2.44	97.5	2.4	1.908
40	289	2.45	100.0	.0	1.957

REMAINING FREQUENCIES ARE ALL ZERO

cell no. — no. of calls obtaining an access channel

Box B.2 Sample Output for Table SRVDA Which Records The No. of Calls Obtaining An Access Channel In Each Cell

A record of the reattempts made by calls while seeking an idle access channel is provided by matrices MH3 and MH4. Any element in MH4 -- symbolized by MH4(x,y) -- represents the number of calls in cell x which had to make y reattempts ($y<R$). Any element in MH3 -- symbolized by MH3(1,y) - represents the number of calls entering the system which needed to make y reattempts. These matrices allow calculation of the reattempt cdf and P_{bal} -- the probability of making at least one reattempt.

Finally, the mean delays $\bar{d}_1$ and $\bar{d}_2$ are measured by means of the GPSS QUEUE entity. The time which elapses between entry of a call into a QUEUE block and exit from a complementary DEPART block located elsewhere in the model is recorded and average values are computed once simulation ends. (The label "QUEUE" is really a misnomer -- no calls are actually delayed by the QUEUE block).

A sample of the queue statistics is illustrated in Box B.3. There is one queue for each cell as well as a system queue called ACCESS. The delays listed in the last two columns are in milliseconds. Zero entries refers to calls which experienced a zero delay between complementary QUEUE/DEPART blocks.

QUEUE	MAXIMUM CONTENTS	AVERAGE CONTENTS	TOTAL ENTRIES	ZERO ENTRIES	PERCENT ZEROS	AVERAGE TIME/TRANS	$AVERAGE TIME/TRANS
1	2	.041	286	275	96.1	172.479	4484.453
2	2	.054	308	292	94.8	211.422	4069.875
3	2	.018	279	274	98.2	79.455	4433.597
4	2	.031	293	287	97.9	127.682	6235.164
5	2	.030	282	276	97.8	130.297	6124.000
6	2	.020	270	265	98.1	92.911	5017.199
7	2	.016	285	281	98.5	70.614	5031.250
8	2	.019	303	298	98.3	77.564	4700.398
9	2	.032	281	273	97.1	137.740	4838.125
10	2	.047	317	308	97.1	179.735	6330.664
11	2	.031	275	268	97.4	138.614	5445.570
12	3	.049	329	320	97.2	179.179	6550.000
13	2	.027	305	300	98.3	109.560	6683.199
14	2	.049	320	310	96.8	186.809	5977.898
15	3	.015	298	292	97.9	60.875	3023.500
16	2	.036	314	306	97.4	141.031	5535.500
17	2	.038	305	296	97.0	152.672	5173.886
18	2	.030	288	280	97.2	126.781	4564.125
19	2	.018	303	298	98.3	73.046	4426.597
20	2	.026	312	307	98.3	102.060	6368.597
21	2	.035	301	293	97.3	143.139	5385.625
22	2	.024	282	276	97.8	104.283	4901.332
23	2	.036	329	322	97.8	132.158	6211.425
24	2	.030	283	277	97.8	130.462	6153.500
25	2	.046	305	295	96.7	183.665	5601.796
26	3	.037	313	305	97.4	145.281	5684.125
27	1	.005	269	268	99.6	24.007	6458.000
28	2	.035	308	302	98.0	138.220	7095.332
29	2	.038	301	290	96.3	154.830	4236.726
30	2	.031	291	285	97.9	130.828	6345.164
31	2	.016	307	302	98.3	63.716	3912.199
32	2	.016	292	288	98.6	66.215	4833.750
33	3	.045	289	279	96.5	190.854	5515.699
34	2	.010	270	267	98.8	47.185	4246.664
35	2	.023	305	298	97.7	94.311	4109.285
36	2	.014	256	251	98.0	66.839	3422.199
37	3	.015	277	274	98.9	69.007	6371.664
38	2	.025	306	299	97.7	101.114	4420.140
39	2	.040	298	290	97.3	162.500	6053.125
40	2	.042	301	292	97.0	171.378	5731.664
ACCES	7	1.213	11836	$\boxed{11559}$	97.6	122.991	5255.347

$AVERAGE TIME/TRANS = AVERAGE TIME/TRANS EXCLUDING ZERO ENTRIES

total entries into system

Box B.3 Queue Statistics (Measurement of the Delay Experienced by Calls Before Obtaining an Access Channel)

B.2 GPSS PROGRAM: MODEL 1 (MODIFIED)

Box B.4 contains a listing of that segment of Model 1 which was modified to take into account collisions resulting from calls arriving within 7 msec of each other. The new statements have been highlighted. This modification, although it consists of only a few additional lines, turned out to be a very formidable task. GPSS can only handle one call at a time so some artificial means had to be devised for circumventing this limitation and to allow for consideration of two or more calls at the same time.

Now, instead of setting an entry in matrix MH1 to 1 immediately when a call seizes an access channel, this task is held in abeyance for 7 msec. The call, though, begins to serve out its holding time. After 7 msec the call checks to see it some other call in the same cell has arrived during the last 7 msec. If so, the calls are considered to have collided and are forced to reattempt seizure of the access channel at a later time. The channel remains idle. If no collision occurred, the access channel status is changed to busy and the call serves out its remaining service time, (H-7) msec.

```
*
*ACCESS CHANNEL SEIZURE ATTEMPT
*
126              MARK         5   ←
127              MARK         6   ←
128       ACES4  ASSIGN       2,XH1
129       ACESS  TEST E       MH1(P1,P2),0,NEXT
130              JOIN         41  ←
131              ADVANCE      7   ←
132              REMOVE       41  ←
133              ALTER        41,1,10,1,1,P1,CRASH  ←
134              TRANSFER     ,TEST2  ←
135       CRASH  TEST NE      P10,1,TEST2  ←
136              TABULATE     SRVDA
137              TEST G       MP6,8,BYPAS  ←
138              TABULATE     TIME1        EXCLUDES ZERO ENTRIES  ←
139       BYPAS  TABULATE     TIME2        INCLUDES ZERO ENTRIES  ←
140              MSAVEVALUE   1,P1,P2,1,H
141              PRIORITY     1
142              ADVANCE      92
143              MSAVEVALUE   1,P1,P2,0,H
144              LEAVE        SYSTM
145              TERMINATE
*
*FIXED CHANNEL ASSIGNMENT
*
146       FIXD1  GATE LR      P1,FAILB   (DYN01)
147       FIXD2  TEST E       MH1(P1,P3),0,SERCH
148              MSAVEVALUE   1,P1,P3,1,H
149              TABULATE     SRVDB
150              ENTER        P1
151              PRIORITY     2
152              ADVANCE      V82,FN$EXPON
```

Box B.4 Modified Segment of Model No. 1 (To Check for Collisions)

← new statements

B.3 GPSS PROGRAM: MODEL 2

The program listing for Simulation Model 2 is contained in Box B.5. All the features of Model 1 are included in Model 2 with the exception that TABLEs are no longer used for collecting statistics. Instead, several matrices are employed as will be described later.

In additon to matrix MH1 which controls fixed voice channel and access channel assignments, a second matrix, MH2, is now responsible for keeping an up-to-the-minute record of dynamic channel usage throughout the system. Any position (x,y) in MH2 can only assume the values 0 and 1 indicating the non-use or use of dynamic channel y in cell x.

VARIABLEs 1 to 40 (under the heading VARIABLE DEFINTION) are defined in terms of MH2 and are used to determine if a given dynamic channel may be used in a given cell. (VARIABLEs 41 to 80 are merely continuations of the expressions for VARIABLEs 1 to 40). To explain how this works, consider the situation where all fixed voice channels are busy in cell 1 in Figure 5.1 and a new call is generated in this cell. This call's only hope for obtaining service is to find a dynamic channel. The channel can not be presently in use in cell 1 or any of the six surrounding cells, 2 through 7, if the cochannel reuse ratio D/R is to be maintained.

```
      CREATE
      REALLOCATE BLO,250,BVR,4,CHA,2,COM,250000,FMS,1
      REALLOCATE FSV,1,FUN,2,GRP,42,HMS,12,HSV,10,LOG,81
      REALLOCATE QUE,42,STO,42,TAB,1,VAR,88,XAC,2500


*LOC  OPERATION  A,B,C,D,E,F,G              COMMENTS
      SIMULATE   18
*
*INITIALIZATION
*
      INITIAL    XH1,11      # OF FIXED CHANNELS/CELL (VOICE+ACCESS)
      INITIAL    XH2,10      # OF FIXED CHANNELS/CELL (VOICE)
      INITIAL    XH3,0       # OF DYNAMIC CHANNELS IN SYSTEM
      INITIAL    XH4,10      # OF REATTEMPTS
 1    MATRIX     H,40,11     # OF COLUMNS = XH1
 2    MATRIX     H,40,1      # OF COLUMNS = XH3
 3    MATRIX     H,1,10      # OF COLUMNS > XH4
 4    MATRIX     H,40,10     # OF COLUMNS > XH4
 5    MATRIX     H,40,3      CALLS WHICH ENTER VOICE CH. SEGMENT
 6    MATRIX     H,40,3      CALLS WHICH ENTER SYSTEM QUEUE
 7    MATRIX     H,40,3      CALLS WHICH OBTAIN ACCESS CHANNEL
 8    MATRIX     H,40,3      CALLS WHICH OBTAIN VOICE CHANNEL
 9    MATRIX     H,40,3      CALLS WHICH DON'T OBTAIN ACCESS CH.
*                            DUE TO REATTEMPT LIMITATION
 10   MATRIX     H,40,3      CALLS WHICH DON'T OBTAIN ACCESS CH.
*                            DUE TO ACCESS TIMER LIMITATION
 11   MATRIX     H,40,3      CALLS WHICH ARE PURGED FROM QUEUE
*
*STORAGE AND QUEUE DEFINITION
*
      STORAGE    S1-S40,50   STORAGES 1 THRU 40 ARE ASSIGNED A
 SYSTM EQU       41,S        CAPACITY OF 50 CALLS.  STORAGE
 SYSTM STORAGE               "SYSTM" HAS UNLIMITED CAPACITY.
 ACCES EQU       41,Q
*
*FUNCTION DEFINITION
*
 EXPON FUNCTION  RN4,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*VARIABLE DEFINITION
*
 1    VARIABLE   MH2(1,P3)+MH2(2,P3)+MH2(3,P3)+MH2(4,P3)+V41
 2    VARIABLE   MH2(2,P3)+MH2(9,P3)+MH2(10,P3)+MH2(3,P3)+V42
 3    VARIABLE   MH2(3,P3)+MH2(10,P3)+MH2(11,P3)+MH2(12,P3)+V43
 4    VARIABLE   MH2(4,P3)+MH2(3,P3)+MH2(12,P3)+MH2(13,P3)+V44
 5    VARIABLE   MH2(5,P3)+MH2(1,P3)+MH2(4,P3)+MH2(14,P3)+V45
 6    VARIABLE   MH2(6,P3)+MH2(7,P3)+MH2(1,P3)+MH2(5,P3)+V46
 7    VARIABLE   MH2(7,P3)+MH2(8,P3)+MH2(2,P3)+MH2(1,P3)+V47
 8    VARIABLE   MH2(8,P3)+MH2(21,P3)+MH2(9,P3)+MH2(2,P3)+V48
 9    VARIABLE   MH2(9,P3)+MH2(22,P3)+MH2(23,P3)+MH2(10,P3)+V49
 10   VARIABLE   MH2(10,P3)+MH2(23,P3)+MH2(24,P3)+MH2(11,P3)+V50
 11   VARIABLE   MH2(11,P3)+MH2(24,P3)+MH2(25,P3)+MH2(26,P3)+V51
 12   VARIABLE   MH2(12,P3)+MH2(11,P3)+MH2(26,P3)+MH2(27,P3)+V52
 13   VARIABLE   MH2(13,P3)+MH2(12,P3)+MH2(27,P3)+MH2(28,P3)+V53
 14   VARIABLE   MH2(14,P3)+MH2(4,P3)+MH2(13,P3)+MH2(29,P3)+V54
 15   VARIABLE   MH2(15,P3)+MH2(5,P3)+MH2(14,P3)+MH2(30,P3)+V55
 16   VARIABLE   MH2(16,P3)+MH2(6,P3)+MH2(5,P3)+MH2(15,P3)+V56
 17   VARIABLE   MH2(17,P3)+MH2(18,P3)+MH2(6,P3)+MH2(16,P3)+V57
```

Box B.5 Program Listing for Simulation Model No. 2

```
18      VARIABLE    MH2(18,P3)+MH2(19,P3)+MH2(7,P3)+MH2(6,P3)+V58
19      VARIABLE    MH2(19,P3)+MH2(20,P3)+MH2(8,P3)+MH2(7,P3)+V59
20      VARIABLE    MH2(20,P3)+MH2(39,P3)+MH2(21,P3)+MH2(8,P3)+V60
21      VARIABLE    MH2(21,P3)+MH2(40,P3)+MH2(22,P3)+MH2(9,P3)+V61
22      VARIABLE    MH2(22,P3)+MH2(23,P3)+MH2(9,P3)+V62
23      VARIABLE    MH2(23,P3)+MH2(24,P3)+MH2(10,P3)+V63
24      VARIABLE    MH2(24,P3)+MH2(25,P3)+MH2(11,P3)+V64
25      VARIABLE    MH2(25,P3)+MH2(26,P3)+V65
26      VARIABLE    MH2(26,P3)+MH2(27,P3)+MH2(12,P3)+V66
27      VARIABLE    MH2(27,P3)+MH2(28,P3)+MH2(13,P3)+V67
28      VARIABLE    MH2(28,P3)+MH2(29,P3)+V68
29      VARIABLE    MH2(29,P3)+MH2(30,P3)+MH2(14,P3)+V69
30      VARIABLE    MH2(30,P3)+MH2(31,P3)+MH2(15,P3)+V70
31      VARIABLE    MH2(31,P3)+MH2(32,P3)+V71
32      VARIABLE    MH2(32,P3)+MH2(33,P3)+MH2(16,P3)+V72
33      VARIABLE    MH2(33,P3)+MH2(34,P3)+MH2(17,P3)+V73
34      VARIABLE    MH2(34,P3)+MH2(35,P3)+V74
35      VARIABLE    MH2(35,P3)+MH2(36,P3)+MH2(18,P3)+V75
36      VARIABLE    MH2(36,P3)+MH2(37,P3)+MH2(19,P3)+V76
37      VARIABLE    MH2(37,P3)+MH2(38,P3)+MH2(20,P3)+V77
38      VARIABLE    MH2(38,P3)+MH2(39,P3)+V78
39      VARIABLE    MH2(39,P3)+MH2(40,P3)+V79
40      VARIABLE    MH2(40,P3)+MH2(22,P3)+V80
41      VARIABLE    MH2(5,P3)+MH2(6,P3)+MH2(7,P3)
42      VARIABLE    MH2(1,P3)+MH2(7,P3)+MH2(8,P3)
43      VARIABLE    MH2(4,P3)+MH2(1,P3)+MH2(2,P3)
44      VARIABLE    MH2(14,P3)+MH2(5,P3)+MH2(1,P3)
45      VARIABLE    MH2(15,P3)+MH2(16,P3)+MH2(6,P3)
46      VARIABLE    MH2(16,P3)+MH2(17,P3)+MH2(18,P3)
47      VARIABLE    MH2(6,P3)+MH2(18,P3)+MH2(19,P3)
48      VARIABLE    MH2(7,P3)+MH2(19,P3)+MH2(20,P3)
49      VARIABLE    MH2(2,P3)+MH2(8,P3)+MH2(21,P3)
50      VARIABLE    MH2(3,P3)+MH2(2,P3)+MH2(9,P3)
51      VARIABLE    MH2(12,P3)+MH2(3,P3)+MH2(10,P3)
52      VARIABLE    MH2(13,P3)+MH2(4,P3)+MH2(3,P3)
53      VARIABLE    MH2(29,P3)+MH2(14,P3)+MH2(4,P3)
54      VARIABLE    MH2(30,P3)+MH2(15,P3)+MH2(5,P3)
55      VARIABLE    MH2(31,P3)+MH2(32,P3)+MH2(16,P3)
56      VARIABLE    MH2(32,P3)+MH2(33,P3)+MH2(17,P3)
57      VARIABLE    MH2(33,P3)+MH2(34,P3)+MH2(35,P3)
58      VARIABLE    MH2(17,P3)+MH2(35,P3)+MH2(36,P3)
59      VARIABLE    MH2(18,P3)+MH2(36,P3)+MH2(37,P3)
60      VARIABLE    MH2(19,P3)+MH2(37,P3)+MH2(38,P3)
61      VARIABLE    MH2(8,P3)+MH2(20,P3)+MH2(39,P3)
62      VARIABLE    MH2(21,P3)+MH2(40,P3)
63      VARIABLE    MH2(9,P3)+MH2(22,P3)
64      VARIABLE    MH2(10,P3)+MH2(23,P3)
65      VARIABLE    MH2(11,P3)+MH2(24,P3)
66      VARIABLE    MH2(11,P3)+MH2(25,P3)
67      VARIABLE    MH2(12,P3)+MH2(26,P3)
68      VARIABLE    MH2(13,P3)+MH2(27,P3)
69      VARIABLE    MH2(13,P3)+MH2(28,P3)
70      VARIABLE    MH2(14,P3)+MH2(29,P3)
71      VARIABLE    MH2(15,P3)+MH2(30,P3)
72      VARIABLE    MH2(15,P3)+MH2(31,P3)
73      VARIABLE    MH2(16,P3)+MH2(32,P3)
74      VARIABLE    MH2(17,P3)+MH2(33,P3)
```

Box B.5 (cont'd)

Program Listing for Simulation Model No. 2

```
75      VARIABLE     MH2(17,P3)+MH2(34,P3)
76      VARIABLE     MH2(18,P3)+MH2(35,P3)
77      VARIABLE     MH2(19,P3)+MH2(36,P3)
78      VARIABLE     MH2(20,P3)+MH2(37,P3)
79      VARIABLE     MH2(21,P3)+MH2(20,P3)
80      VARIABLE     MH2(21,P3)+MH2(39,P3)
81      VARIABLE     4500000      MAXIMUM SIMULATION TIME
82      VARIABLE     120000       MEAN CALL HOLDING TIME
83      VARIABLE     2000         MEAN INTERARRIVAL TIME OF CALLS
84      VARIABLE     XH4-P4+1
85      VARIABLE     P1+40
1       BVARIABLE    C1'LE'V81*N$CALLS'LE'9000
2       BVARIABLE    V*1'E'0
3       BVARIABLE    MP9'GE'5000
*
*CALL GENERATION
*
 CEL1   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,1
        TRANSFER     ,CALLS
 CEL2   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,2
        TRANSFER     ,CALLS
 CEL3   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,3
        TRANSFER     ,CALLS
 CEL4   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,4
        TRANSFER     ,CALLS
 CEL5   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,5
        TRANSFER     ,CALLS
 CEL6   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,6
        TRANSFER     ,CALLS
 CEL7   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,7
        TRANSFER     ,CALLS
 CEL8   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,8
        TRANSFER     ,CALLS
 CEL9   GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,9
        TRANSFER     ,CALLS
 CEL10  GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,10
        TRANSFER     ,CALLS
 CEL11  GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,11
        TRANSFER     ,CALLS
 CEL12  GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,12
        TRANSFER     ,CALLS
 CEL13  GENERATE     V83,FN$EXPON,,,,10,F
        ASSIGN       1,13
        TRANSFER     ,CALLS
 CEL14  GENERATE     V83,FN$EXPON,,,,10,F
```

Box B.6 (cont'd)

Program Listing for Simulation Model No. 2

```
        ASSIGN      1,14
        TRANSFER    ,CALLS
CEL15   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,15
        TRANSFER    ,CALLS
CEL16   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,16
        TRANSFER    ,CALLS
CEL17   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,17
        TRANSFER    ,CALLS
CEL18   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,18
        TRANSFER    ,CALLS
CEL19   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,19
        TRANSFER    ,CALLS
CEL20   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,20
        TRANSFER    ,CALLS
CEL21   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,21
        TRANSFER    ,CALLS
CEL22   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,22
        TRANSFER    ,CALLS
CEL23   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,23
        TRANSFER    ,CALLS
CEL24   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,24
        TRANSFER    ,CALLS
CEL25   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,25
        TRANSFER    ,CALLS
CEL26   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,26
        TRANSFER    ,CALLS
CEL27   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,27
        TRANSFER    ,CALLS
CEL28   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,28
        TRANSFER    ,CALLS
CEL29   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,29
        TRANSFER    ,CALLS
CEL30   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,30
        TRANSFER    ,CALLS
CEL31   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,31
        TRANSFER    ,CALLS
CEL32   GENERATE    V83,FN$EXPON,,,,10,F
        ASSIGN      1,32
        TRANSFER    ,CALLS
CEL33   GENERATE    V83,FN$EXPON,,,,10,F
```

<u>Box B.5</u> (cont'd)

Program Listing for Simulation Model No. 2

```
98              ASSIGN      1,33
99              TRANSFER    ,CALLS
100      CEL34  GENERATE    V83,FN$EXPON,,,,10,F
101             ASSIGN      1,34
102             TRANSFER    ,CALLS
103      CEL35  GENERATE    V83,FN$EXPON,,,,10,F
104             ASSIGN      1,35
105             TRANSFER    ,CALLS
106      CEL36  GENERATE    V83,FN$EXPON,,,,10,F
107             ASSIGN      1,36
108             TRANSFER    ,CALLS
109      CEL37  GENERATE    V83,FN$EXPON,,,,10,F
110             ASSIGN      1,37
111             TRANSFER    ,CALLS
112      CEL38  GENERATE    V83,FN$EXPON,,,,10,F
113             ASSIGN      1,38
114             TRANSFER    ,CALLS
115      CEL39  GENERATE    V83,FN$EXPON,,,,10,F
116             ASSIGN      1,39
117             TRANSFER    ,CALLS
118      CEL40  GENERATE    V83,FN$EXPON,,,,10,F
119             ASSIGN      1,40
120             TRANSFER    ,CALLS
121      CALLS  ENTER       SYSTM
122             ASSIGN      4,XH4
123             ASSIGN      10,1
       *
       *ACCESS CHANNEL SEIZURE ATTEMPT
       *
124             MARK        5
125             ADVANCE     500
126             QUEUE       ACCES
127             QUEUE       P1
128      ACES4  ASSIGN      3,XH2
129      ACES5  ASSIGN      2,XH1
130      ACES6  TEST E      MH1(P1,P2),0,WAIT1
131             TEST E      P10,1,ACES9
132             DEPART      P1
133             DEPART      ACCES
134      ACES9  MSAVEVALUE  7+,P1,P10,1,H
135             MSAVEVALUE  1,P1,P2,1,H
136             PRIORITY    1
137             ADVANCE     100
138             MSAVEVALUE  1,P1,P2,0,H
       *
       *FIXED CHANNEL ASSIGNMENT
       *
139      FIX0   MSAVEVALUE  5+,P1,P10,1,H
140      FIX1   GATE LR     P1,DYN24 ...........IF NO FIXED CHANNEL AVAILABLE
141      FIX2   TEST E      MH1(P1,P3),0,SERCH   GO TO ADDRESS "DYN1". IF
142             MSAVEVALUE  1,P1,P3,1,H          NEITHER FIXED NOR DYNAMIC CH.
143      FIX4   MSAVEVALUE  6+,P1,P10,1,H        AVAILABLE JOIN SYSTEM QUEUE.
144             ENTER       P1                   AT ADDRESS "DYN24".
145             PRIORITY    2               .....IF LOGIC SWITCH SPECIFIED BY
146             ADVANCE     V82,FN$EXPON    .    VARIABLE 85 IS RESET THEN
147      FIX8   TEST E      G*1,0,FREE      .    THERE ARE NO CALLS FROM CELL
148      FIX9   GATE LR     V85,FIX13 .......    "I" WAITING IN SYSTEM QUEUE.
```

Box B.5 (cont'd)

Program Listing for Simulation Model No. 2

```
149           MSAVEVALUE 1,P1,P3,0,H
150           LOGIC R    P1
151           TRANSFER   ,EXIT
152     FIX13 ALTER      41,ALL,3,P3
153           UNLINK     WAIT,FIX19,1,1,,FIX17 ....P1 OF THE PASSING XAC IS
154           PRIORITY   0,BUFFER               MATCHED WITH THE FIRST MEMBER
155           SCAN       41,1,P1,,,FIX17        OF SYSTEM QUEUE WHICH HAS THE
156           TRANSFER   ,EXIT                  SAME VALUE OF P1. THE UNLINK-
157     FIX17 LOGIC R    V85                    ED CALL THEN PROCEEDS TO  USE
158           TRANSFER   ,EXIT                  THE FIXED CH. WHICH HAS  JUST
159     FIX19 REMOVE     41                     BEEN RELEASED.
160           TRANSFER   ,FIX4
161     FIX21 LOGIC R    P1
162           TRANSFER   ,FIX9
       *
       *SEARCH FOR A FIXED CHANNEL
       *
163     SERCH LOOP       3,FIX2
164           LOGIC S    P1
165           TRANSFER   ,DYN24
       *
       *SEARCH FOR AN ACCESS CHANNEL, WAIT & REATTEMPT IF NECESSARY
       *
166     WAIT1 ASSIGN     2-,1
167           TEST E     P2,P3,ACES6
168           TEST G     P4,0,MTX9
169           ADVANCE    100,100
170           TEST LE    MP5,6000,MTX10
171           MSAVEVALUE 3+,1,V84,1,H
172           MSAVEVALUE 4+,P1,V84,1,H
173           ASSIGN     4-,1
174           TRANSFER   ,ACESS
       *
       *CALL UNABLE TO FIND ACCESS CHANNEL
       *
175     MTX9  MSAVEVALUE 9+,P1,P10,1,H
176           TEST E     P10,1,LEAV
177           DEPART     P1
178           DEPART     ACCES
179     LEAV  LEAVE      SYSTM
180           TERMINATE
181     MTX10 MSAVEVALUE 10+,P1,P10,1,H
182           LEAVE      SYSTM
183           TERMINATE
       *
       *DYNAMIC CHANNEL ASSIGNMENT
       *
184     DYN1  ASSIGN     3,XH3
185     DYN2  TEST E     V*1,0,DYN23 .........DOES THE DYNAMIC CHANNEL
186           MSAVEVALUE 2,P1,P3,1,H            SPECIFIED BY THE CURRENT
187     DYN4  MSAVEVALUE 8+,P1,P10,1,H          VALUE OF P3 SATISFY  THE
188           ASSIGN     6,1                    COCHANNEL    INTERFERENCE
189           JOIN       P1                     CONSTRAINT?
190           ENTER      P1
191           PRIORITY   2
192           ADVANCE    V32,FN$EXPON
193           TEST E     P7,0,FIX8  ..........WAS CALL REASSIGNED DURING
```

Box B.5 (cont'd)

Program Listing for Simulation Model No. 2

```
194            REMOVE          P1                            CALL HOLDING TIME?
195   DYN12    TEST NL         CH$WAIT,0,DYN19 .....ARE THERE ANY CALLS WAITING
196            ALTER           41,ALL,3,P3                   IN SYSTEM QUEUE?
197            UNLINK          WAIT,DYN21,1,BV2,,DYN19 ....UNLINK THE FIRST CALL
198            PRIORITY        0,BUFFER                      IN QUEUE WHICH SATISFIES THE
199            SCAN            41,1,P1,,,DYN17               INTERFERENCE CONSTRAINT  AND
200            TRANSFER        ,EXIT                         ASSIGN THE JUST RELEASED DYN-
201   DYN17    LOGIC R         V85                           AMIC CHANNEL TO IT.
202            TRANSFER        ,EXIT
203   DYN19    MSAVEVALUE      2,P1,P3,0,H
204            TRANSFER        ,EXIT
205   DYN21    REMOVE          41
206            TRANSFER        ,DYN4
*
207   DYN23    LOOP            3,DYN2
208   DYN24    MARK            9
209            LOGIC S         V85 ................SET SWITCH 85 TO INDICATE
210            JOIN            41                            THAT THERE IS AT LEAST ONE
211            MSAVEVALUE      6+,P1,P10,1,H                 CALL IN SYSTEM QUEUE FROM
212            LINK            WAIT,FIFO                     THE CELL SPECIFIED.
*
*CHANNEL REASSIGNMENT
*
213   FREE     SCAN            P1,6,1,3,8,FIX21
214            ALTER           P1,1,7,1,3,P8
215            ALTER           P1,1,3,P3,3,P8
216            REMOVE          P1,1,,3,P3
217            ASSIGN          3,P8
218            TRANSFER        ,DYN12
*
*CALLS LEAVE SYSTEM
*
219   EXIT     LEAVE           P1
220            LEAVE           SYSTM
221            TERMINATE
*
*CALLS PURGED FROM QUEUE DUE TO TIME LIMITATION
*
222   PURG1    REMOVE          41
223            SCAN            41,1,P1,,,PURG7
224   PURG3    MSAVEVALUE      11+,P1,P10,1,H
225            ASSIGN          10+,1
226            TEST LE         MP5,6000,MTX10
227            TRANSFER        ,ACES4
228   PURG7    LOGIC R         V85
229            TRANSFER        ,PURG3
*
*AUXILIARY CLOCK #1 (CONTROL OF CALLS IN QUEUE "WAIT")
*
230            GENERATE        20,,,3,1,H
231            UNLINK          WAIT,PURG1,ALL,BV3
232            TERMINATE
*
*AUXILIARY CLOCK #2 (CONTROL OF SIMULATION DURATION)
*
233            GENERATE        300000
234            TEST E          BV1,1,STOP
```

Box B.5 **(cont'd)**

Program Listing for Simulation Model No. 2

```
235                TERMINATE   1
236         STOP   TERMINATE   24
                   START       1,NP
                   RESET       S41
                   INITIAL     MH3(1,1-10),0/MH4(1-40,1-10),0
                   INITIAL     MH5(1-40,1-3),0/MH6(1-40,1-3),0
                   INITIAL     MH7(1-40,1-3),0/MH8(1-40,1-3),0
                   INITIAL     MH9(1-40,1-3),0/MH10(1-40,1-3),0/MH11(1-40,1-3),0
                   START       25
                   END
```

Box B.5 (cont'd)

Program Listing for Simulation Model No. 2

The test for a suitable dynamic channel is made in line 185 of the program where the variable specified by parameter 1 (V*1) is referenced. As mentioned before, each call has a set of parameters associated with it; parameter 1 (P1) contains the number of the cell where the call was generated. In this example P1=1 and the corresponding variable is V*P1=V1:

$$V1 = MH2(1,P3)+MH2(2,P3)+MH2(3,P3)+MH2(4,P3)+V41$$

$$= MH2(1,P3)+MH2(2,P3)+MH2(3,P3)+MH2(4,P3)+ MH2(5,P3)+MH2(6,P3)+MH2(7,P3)$$

Note that the column of MH2 is specified by parameter 3. P3 is initially assigned a value equal to the total number of dynamic channels in the system pool. Now if V1=0, then dynamic channel P3 is not currently being used in cell 1 or any of the six neighbouring cells and can be assigned to the call. MH2(1,P3) is set to 1 indicate this fact. If V1>1, the search must continue. In this case P3 is decremented by 1 and V1 is tested for the new channel, P3-1. The procedure is repeated until a dynamic channel meeting the cochannel interference constraint for cell 1 (V1=0) is found. If after testing all the dynamic channels in the system pool, no suitable channel is found, the call joins the system queue. (The fixed voice channel search precedes the dynamic channel search so at this stage the call has no choice but to join the queue).

The system queue is modelled by means of the GPSS USER CHAIN entity. A USER CHAIN is a queue in the true sense of the word. Calls are LINKed to the chain in a FIFO manner and once linked, they are for all intents and purposes inactive. No attempts will be made by the GPSS processor to move these calls any further unless logic is explicity included to UNLINK the calls when some aspect of the system state changes. In our model this unlinking occurs when an active call has finished its service time thereby freeing a fixed or dynamic voice channel for use by a waiting call or when the 5 second time limitation for the queued call has expired.

Eleven matrices are used to record various statistics. They have the following signifigance:

MH3(1,y): no. of calls in the entire system which make y reattempts (y<10) to obtain an access channel.

MH4(x,y): no. of calls in cell x (x=1,2,3...40) which make y reattempts (y<10) to obtain an access channel.

MH5(x,y): no. of calls in cell x which initially enter the voice channel assignment subroutine (y=1) or which enter a second time (y=2) after having been expelled from the system queue the first time around.

MH6(x,y): no. of calls in cell x which queue for service upon first (y=1) or second (y=2) entry into the voice channel assignment subroutine.

MH7(x,y): no. of calls in cell x obtaining an access channel upon first (y=1) or second (y=2) entry into the system.

MH8(x,y): no. of calls in cell x obtaining a voice channel upon first (y=1) or second (y=2) entry into the voice channel assignment subroutine.

MH9(x,y): no. of calls in cell x failing to obtain an access channel upon first (y=1) or second (y=2) entry into the system due to the reattempt limitation.

MH10(x,y): same as MH9 except failures are due to the access timer limitation.

MH11(x,y): no. of calls in cell x purged from queue upon first (y=1) or second (y=2) entry into the voice channel assignment subroutine due to the 5 second time limit. Calls purged from the queue a second time are considered to be blocked.

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