

HIGH-ENERGY YB-DOPED FEMTOSECOND FIBER LASERS

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Abstract

The main objective of the thesis is to understand the parameters that contribute in limiting the pulse energy and spectral bandwidth of the mode-locked femtosecond fiber lasers. I have focused on studying the impact of the parameters of the saturable absorber and the bandwidth of the lumped spectral filter on the temporal and spectral profiles of the pulse. Therefore, I developed two models that can help us to optimize the pulse characteristics such as the pulse energy, spectral bandwidth and de-chirped pulse width. I also introduce two techniques that result in increasing the pulse peak power and spectral bandwidth.

The nonlinear transmission coefficient of the saturable absorber is one of the main limitations to achieving high-energy pulses. Throughout my research, I have used two types of saturable absorbers. The first is a lumped semiconductor saturable absorber mirror (SESAM) and the second is based on the nonlinear polarization rotation (NPR) that is considered an artificial saturable absorber with distributed effect.

The first model introduced in this thesis is an analytical model, which provides closed form relations for the pulse characteristics of all-normal dispersion fiber laser. It shows how the spectral bandwidth of the lumped filter inserted inside the cavity affects the pulse characteristics. Also, it illustrates the influence of the saturable absorber parameters on the pulse characteristics. I show that increasing the small signal saturable absorber loss and decreasing the saturation power leads to the increase in pulse energy and spectral bandwidth. Numerical simulation and experimental results are in agreement with the results of the analytical models.

The second model, which is called the semi-vector model, is applicable to all-normal dispersion mode-locked fiber laser with high output coupling ratio. Nonlinear polarization rotation is employed for mode-locking. The model shows the relationship between the location of the overdriving point of the saturable absorber and the output pulse energy. The results of this model are in agreement with those of the full-vector model, but with a much reduced simulation time. In addition, the experimental results show the accuracy of the proposed model.

In this thesis, I mitigate the peak power limitation, caused by the accumulated nonlinear phase shift, by replacing the short high-doped Yb^{3+} fiber with a long low-doped one. This results in an increase of the peak power by a factor that depends on the ratio between the gain coefficient of the high- and low-doped Yb^{3+} fiber. The length of the nonlinear section is kept unchanged by reducing the length of the single mode fiber after the long low-doped Yb^{3+} fiber. Numerical simulation and experimental results validate the idea.

The location of narrow bandwidth lumped spectral filter, in an active Similariton laser, has proved to have a distinct effect on the pulse energy, spectral bandwidth and de-chirped pulse width and peak power. The proximity of the spectral filter to the input of the Yb^{3+} -doped fiber leads to increasing the pulse spectral bandwidth and peak power of the de-chirped pulse as well as shortening the de-chirped pulse, but at the expense of reducing the pulse energy.

In the name of God, the Most Gracious, the Most Merciful

In grateful dedication to my mother, my wife, Moustafa, and Yahia

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Glossary

AC	Autocorrelation
ASE	Amplified spontaneous emission
CARS	Coherent anti-Stokes Raman scattering
CGLE	Complex Ginzburg-Landau equation
CQCGLE	Complex Quintic Cubic Ginzburg-Landau equation
DDL	Dispersive delay line
DM	Dispersion managed
Er ³⁺	Erbium doped
FFT	Fast Fourier transform
FWHM	Full width at half maximum
GVD	Group velocity dispersion
HWP	Half wave plate
IFFT	Inverse fast Fourier transform
LED	Light emitting diode
LMA	Large mode area
MMF	Multimode fiber
MMI	Multimode interference
NLSE	Nonlinear Schrödinger equation
NOLM	Nonlinear optical loop mirror
NPR	Nonlinear polarization rotation
OSC	Oscilloscope
PBG	Photonics bandgap
PBS	Polarization beam splitter
PC	Polarization controller
PMF	Polarization maintaining fiber
Pol	Polarizer

QWP	Quarter wave plate
RMS	Root mean square
SA	Saturable absorber
SESAM	Semiconductor saturable absorber mirror
SF	Spectral filter
SGLE	Sinusoidal Ginzburg-Landau equation
Similariton	Self-similar
SMF	Single mode fiber
SPD	Spectral power density
SPM	Self-phase modulation
SRS	Stimulated Raman scattering
TOD	Third order dispersion
WDM	Wave division multiplexer
XPM	Cross phase modulation
Yb ³⁺	Ytterbium-doped

List of symbols

a	The total field
a_l	The small signal loss is
a_c	The core radius of the MMF
A_{eff}	The effective mode area of the fiber
b	The slant length between the gratings
b_l	The inverse of the saturation power of the SA.
B_p	The transmission coefficient
BW	The filter bandwidth
BW_{f_gauss}	The 3-dB bandwidth of Gaussian filter
BW_{min}	The value of the filter bandwidth at the threshold of stable pulse
BW_{th}	The spectral filter bandwidth at the border between lossy and stable regions in Hertz
c_o	The speed of light in free space
c_l	The overdriving coefficient of the SA
c_j	The field expansion coefficient.
$\tilde{c}_j$	The modified field expansion coefficient
C	The chirp parameter
d	The number of grooves per unit meter
Dis_{an}	The anomalous dispersion
Dis_g	The anomalous dispersion of the grating pairs
E	The pulse energy
E^*	The threshold pulse energy
$E_j^t(r, \theta)$	The j th transverse mode of MMF.
E_{sat}	The saturation energy of the Yb^{+3} fiber
E_{sat_av}	The average value of the saturation energy of the Yb^{+3} fiber
E_{sat_SA}	The saturation energy of the SESAM
E_{source}	The electric field at the facet of the MMF
F_s	Saturation fluence of the SESAM
g_o	The small signal gain coefficient
g_{o_l}	The small signal gain coefficient for the low-doped Yb^{+3} fiber
g_{o_av}	The average value of the small signal gain coefficient
g_{o_h}	The small signal gain coefficient for the high-doped Yb^{+3} fiber
$g(z)$	The saturated gain coefficient of the Yb^{+3} fiber
$g_{av}(E)$	The average saturated gain coefficient
IL_f	The insertion loss of the spectral filter

J	The area of the polarization ellipse
k_o	The free space propagation constant
l	The average loss coefficient including the power splitter and filter insertion loss
L	The total cavity length
L_B	The fiber beat length
L_D	The dispersion length
L'_D	The TOD dispersion length
L_{eff}	The effective length of the SMF
L_g	The net-gain length
L_N	The total length of the nonlinear region
L_{NL}	The nonlinear length
L_{ns}	The non-saturable loss
L_{SMF1}	The length of the long SMF
L_{SMF2}	The length of the SMF after the Yb ⁺³ -doped fiber
$L_{SMF2,h}$	The length of SMF ₂ for the cavity with high-doped Yb ⁺³ fiber
$L_{SMF2,l}$	The length of SMF ₂ for the cavity with low-doped Yb ⁺³ fiber
L_{Yb}	The length of the Yb ⁺³ -doped fiber
$L_{Yb,h}$	The length of the high-doped Yb ⁺³ fiber
$L_{Yb,l}$	The length of the low-doped Yb ⁺³ fiber
m	The diffraction order
M	The number of modes excited in the MMF
n_{core}	The core refractive index of the MMF
N	Soliton order
P	The pulse power
P_j	The j th mode power
P_p	The peak power of the pulse
$P_{p,h}$	The peak power the cavity with high-doped Yb ⁺³ fiber
$P_{p,l}$	The peak power the cavity with low-doped Yb ⁺³ fiber
$P_{p,sa}$	The power at the overdriving point of the SESAM
P_{sat}	The saturation power of SESAM
P_{source}	The source power
P_{TPA}	The two photons parameters
q_o	The small signal absorbance of the SESAM
q_{sa}	The saturable absorbance of the SESAM
Q	The second derivative of the spectral phase of the pulse
R_{ins}	The instantaneous reflectivity of the SESAM

R_{real}	The reflectivity of the SESAM
t	The retarded time
t_{NPR}	The field transfer function of the configuration
T_o	1/e intensity half width
T_A	The transfer function of NPR
T_f	The transfer function of the filter
T_{FWHM}	Full width at half maximum pulse width
T_{gauss}	The transfer function of the Gaussian filter
T_{MMI}	The power transfer function of the MMI filter
T_{sa}	The relaxation time of the SESAM
T_{SA}	The transmission transfer function of the average SA
TOD_g	The third order dispersion of the grating pairs
u	Horizontal polarized light
u_-	The left hand circularly polarized field
u_+	The right hand circularly polarized field
u_1	Input u-field to the optical fiber
u_2	Output u-field from the optical fiber
$U(z,t)$	The scalar field envelope
$\tilde{U}(0,\omega)$	the Fourier transform of the incident field envelope
v	Vertical polarized light
v_1	Input v-field to the optical fiber
v_2	Output v-field from the optical fiber
W_c	The matrix of the electric field coupling coefficient through the cavity of the coupler
W_f	The transfer matrix of the filter in frequency domain
W_{NL}	The nonlinear matrix of the fiber in linear polarization basis
W_{NL_c}	The nonlinear matrix in the circular polarization basis
W_p	The Jones matrix of the inline fiber polarizer
W_{PCn}	The Jones matrix for arbitrary oriented polarization controller
W_{rej}	The Jones matrix of the rejection port of polarization beam splitter
x	The normalized net saturation gain coefficient
y	The normalized spectral filtering term
Y	The modified field amplitude
z	The propagation distance through the fiber
z_{image}	The self-image length
α	The spectral filtering effect
α_o	The spectral filtering effect at the threshold of stable pulse

α_f	The linear power loss coefficient
α_n	The angle of rotation
β_2	The group velocity dispersion
β_{2av}	The average value of the group velocity dispersion
β_3	The third order dispersion
β_{3av}	The average value of the third order dispersion
β_c	The power coupling ratio
β_{MMF_j}	The j th mode propagation constant in MMF
γ	The nonlinear parameter
δ_n	The phase shift between the two polarizations
δ_s	Cubic term of SA
Δ	The half spectral bandwidth of the pulse
$\Delta\beta$	The linear birefringence of the fiber
ε	The ratio between the dispersion and dissipation effects
ζ	The normalized overdriving term of the SA
θ_d	The diffraction angle
λ	The central wavelength of the pulse
μ	The nonlinear transmission coefficient
μ_s	Quintic term of SA
σ	The net saturated gain coefficient
φ	The temporal phase of the pulse
φ_{max}	The maximum nonlinear phase shift due to SPM
φ_{NL_SPM}	The nonlinear phase shift due to SPM
ψ	The spectral phase of the pulse
ω	The angular optical frequency
ω_o	1/e intensity half width
Ω	The instantaneous frequency
Ω_f	The angular frequency at which the average filter transmission is null
Ω_{f_min}	The minimum value of Ω_f
Ω_g	The gain bandwidth of the Yb ⁺³ fiber

Chapter 1

1 Introduction

Ultrafast optics is a growing field with applications that span many disciplines from life sciences, medicine, and chemistry to industrial applications and optical communications. Compared to solid state lasers (e.g., titanium-sapphire lasers), Femtosecond fiber lasers are attractive sources for generating pulsed light as they can be designed to be compact, all-fiber devices that require no alignment or complicated maintenance.

Femtosecond fiber laser could play a critical role in biomedical applications such as nonlinear microscopy and endoscopy (e.g. multi-photons absorption, second harmonic generation, stimulated Raman scattering (SRS) and Coherent anti-Stokes Raman scattering (CARS)) [1- 6].

Our research group is focusing on developing a portable femtosecond fiber laser that can replace solid state lasers in performing various biomedical applications including nonlinear microscopy and endoscopy. Since the main constituent of biological tissues is water, the optical window around 1 μm is selected to be the center wavelength of the laser for these applications as the light experiences minimal absorption of water. Ytterbium-doped (Yb^{3+}) fiber provides large signal amplification at 1 μm with around 60% pump-to-signal efficiency. Furthermore, high-power pump lasers at 976 nm are commercially available thanks to their usage in telecommunication. Consequently, I have used Yb^{3+} -doped fiber as a gain medium in my research.

To enable nonlinear microscopy, the pulse energy generated by the fiber laser should be in the tens of nanojoules. My research has focused on femtosecond pulses as they have higher peak power for the same average power, which enhances the nonlinear effects of all microscopy modalities. Transform-limited picosecond pulses are sufficient to excite atoms in the vibrational states of a biological sample to generate SRS and CARS because the spectral bandwidth is lower than the linewidth of the vibrational states [1, 5]. However, femtosecond sources can be employed to generate a wide range of Stokes by passing the femtosecond pulses through a highly nonlinear element [7, 8]. Therefore, a wide range of vibrational states can be scanned. The main drawback of femtosecond sources is the reduction of spectral resolution; this can be overcome by using spectral focusing techniques [8- 10].

There are two approaches to developing high energy femtosecond lasers; the first is to implement a mode-locked fiber laser cavity generating a chirped pulse with energy in the range of picojoules followed by a chirped pulse amplifier and a pulse compressor [11].

The other approach is the implementation of a high energy mode-locked fiber laser generating pulse energy directly in the range of tens of nanojoules. In such a cavity, the output pulse width is in the range of picoseconds having temporal and spectral chirp, which can be compensated by a dispersion delay line to compress the pulse near its transform limit having a pulse width in the femtosecond region [12-14]. The advantage of the second approach is that it is cheaper and less complicated as the laser does not need any additional amplification. A detailed study on the factors that limit the pulse energy,

peak power, and de-chirped pulse width is important to reach an optimal design of high-energy femtosecond fiber lasers.

1.1 Novelty

The focus of my research is on developing models to study the impact of the parameters of the saturable absorber (SA) and the bandwidth of the lumped spectral filter on the temporal and spectral profiles of the pulse. I also demonstrate a technique to mitigate the effect of the excessive accumulated nonlinearity and increase the pulse peak power and spectral bandwidth. The four new outcomes of my research are summarized here:

I developed two novel models to study the parameters of the SA to order to maximize the pulse energy. One of them focuses on lumped SA, while the other focuses on the SA based on NPR.

Optimizing the SA design is key to achieving high-energy pulses. The main parameters of a SA are the small-signal absorption, saturation power, damage threshold and overdriving point. Previous simulation results showed that the small-signal absorption must be small to have high energy pulses [15]. Detailed investigation of the impact of the saturation power and overdriving point on the pulse energy and width has never been investigated. Simulation models are difficult to guide us in design optimization of the mode-locked laser cavity due to the excessive number of the parameters involved in shaping the pulse.

I developed a generalized analytical average model where the lumped saturable absorber can be optimized by selecting the best parameter values, resulting in an increase

in the pulse energy with wide spectral bandwidth. My model includes the overdriving effect of the SA that results from two-photon absorption, which occurs inside certain types of SA such the semiconductor saturable absorber mirror (SESAM) [16]. This model is capable of investigating the effect of the bandwidth of the lumped spectral filter as well as the parameters of the saturable absorber on the pulse energy and the temporal and spectral profile of the pulse.

I derived mathematical equations that relate the physical parameters of the SA to the average ones. All-normal dispersion mode-locked fiber lasers require a lumped spectral filter for the sake of pulse stabilization. I averaged the filtering effect of the lumped spectral filter over the cavity length that was not included in the previous analytical average models. Since improper selection of the spectral bandwidth of the lumped spectral filter leads to unstable mode-locking and multi-pulsing operation, my model enabled me to study of the relationship between the parameters of the mode-locked laser cavity and the spectral bandwidth of the filter, which is essential to optimize the performance of the cavity. I constructed the experimental set-ups and compared the results with my modeling results. Part of the simulation results was published in Conference of Photonics West 2014 [IV], while the whole work was recently submitted to the IEEE Journal of Selected Topics of Quantum Electronics [V].

The other type of SA is the nonlinear polarization rotation (NPR) that is considered an artificial SA with distributed effect. Appropriate selection of the orientation angles is required to maintain mode locking and generate low energy pulses. The study of the best combination of angles requires solving the vector complex Ginzburg-Landau equations (CGLEs). It is time-consuming compared to the scalar GLE.

Furthermore, detailed study of the effect of each parameter of the mode-locked fiber laser on the pulse characteristics (pulse energy, spectral bandwidth, and de-chirped pulse width) using this model is cumbersome.

Numerical average models provide a solution to the above drawbacks. However, the previous average models assume that the laser cavity consists of only a gain medium. The dispersion effect of the SMF fiber in the linear section after the output coupler was not included. Therefore, they do not simulate a realistic fiber laser cavity. To address this issue, I developed a new average model, which is called the semi-vector model. It has two major advantages: first, it contains all parameters required to simulate this type of laser cavity; second, the scalar GLE is solved instead of the vector CGLEs. The temporal and spectral profiles of the pulse created by this model are very close to the results of the full-vector model, but with the distinct advantage of a shorter simulation time. Using the semi-vector model, I studied the effect of the overdriving point of the SA on the pulse energy, spectral bandwidth, and de-chirped pulse width. The results show that pushing the overdriving point to a higher power value results in higher-energy mode-locked pulses. Two experimental set-ups were performed to validate my model. The results from my model and the experimental laser cavity are close. This work was published in IEEE Journal of Selected Topics of Quantum Electronics [III].

The accumulated nonlinear phase shift is another factor studied in this thesis. It leads to multi-pulsing operation or the wave-breaking of the pulse. As will be mentioned in subsection 2.2.2, the accumulated nonlinear phase shift broadens the spectral bandwidth of the pulse. If it approaches the spectral bandwidth of the Yb^{3+} -doped fiber, the pulse will suffer from spectral losses which make it unstable and the noise floor will

become energetic to generate another pulse. Wave-breaking of the pulse results from the propagation of a high peak power pulse in normal dispersion fiber. The pulse will have an interference pattern at its edges in which the intensity of the interference pattern increases during the pulse propagation until it causes the collapse of the pulse.

We developed a technique to enhance the pulse peak power by managing the accumulation of the nonlinear phase shift in the laser cavity. To the best of our knowledge, we are the first to introduce the effect of replacing high-doped short-length Yb^{3+} fiber with low-doped long length Yb^{3+} fiber on the accumulation of the nonlinear phase shift. I derived the theoretical limitation of this idea and constructed the experimental set-ups to verify the idea. It is successfully verified for large interval of net dispersion. This work was published in Conference of Photonics West 2012 [I] and IEEE Journal of Lightwave Technology [II].

The last proposed technique studies the effect of changing the position of the narrow bandwidth lumped spectral filter on the pulse energy, spectral bandwidth, de-chirped peak power and de-chirped pulse width in active Similariton fiber laser. It is mode-locked by NPR. To the best of our knowledge, this study has not been discussed before. A long piece of single mode fiber (SMF) was inserted between the pulse output port and the input of the Yb^{+3} -doped fiber. I addressed four locations of the lumped spectral filter in this research. Two of them are at the beginning and end of the SMF, while the other ones are at certain distances inside the SMF. The vector CGLEs were used to simulate the pulse propagation in the optical fiber. Both the simulation model and experimental validation show a distinct difference in the pulse energy, spectral bandwidth, de-chirped peak power, and de-chirped pulse width for each location of the

narrow bandwidth spectral filter. This work was recently submitted to Optics Letters [VI].

1.2 My publications

Parts of the results originated from my PhD thesis, under the supervision of Dr. Hanan Anis, are published in peer-reviewed journals and two international conferences:

- [I] Katherine J. Bock, **Hussein E. Kotb**, Mohamed A. Abdelalim, and Hanan Anis, “Increasing energy in an ytterbium femtosecond fiber laser with a longer gain medium and lower doping,” *Conference of Photonic West 2012, San Francisco, California, USA*, pp. 823731 (1-7).
- [II] **Hussein E. Kotb**, Mohamed A. Abdelalim, Katherine J. Bock, and Hanan Anis, “Peak power optimization of optical pulses using low-doped gain-medium in femtosecond fiber laser,” *IEEE Journal of Lightwave Technology*, vol. 31, no. 13, pp. 2230-2236, 2013.
- [III] **Hussein E. Kotb**, Mohamed A. Abdelalim, and Hanan Anis, “An efficient semi-vectorial model for all-fiber mode-locked femtosecond lasers based on nonlinear polarization rotation,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 20, no. 5, pp. 1100809 (1-9), 2014.
- [IV] **Hussein E. Kotb**, Mohamed A. Abdelalim, and Hanan Anis, “Effect of mode locking technique on the filtering bandwidth limitation in all normal dispersion femtosecond fiber laser,” *Conference of Photonic West 2014, San Francisco, California, USA*, pp. 89613A (1-11).

- [V] **Hussein E. Kotb**, Mohamed A. Abdelalim, and Hanan Anis, “Generalized analytical model for dissipative soliton in all normal dispersion mode locked fiber laser,” accepted in *IEEE Journal Selected Topics of Quantum Electronics*.
- [VI] **Hussein E. Kotb**, Mohamed A. Abdelalim, and Hanan Anis, “Effect of narrow spectral filter position on the characteristics of active Similariton mode-locked femtosecond fiber laser,” submitted to *Optics Express*.

1.3 Contribution of other members in our research group

The idea behind using low-doped long length Yb^{3+} fiber inside mode-locked fiber laser to increase the pulse peak power was proposed by Mohamed A. Abdelalim in his Ph.D. thesis. He also implemented the Matlab codes for the scalar GLE and vector CGLEs simulation models.

Katherine J. Bock, a Masters student, and I jointly measured all parameters of the mode-locked pulses published in [I].

1.4 Outline of the thesis

In chapter 2, the main theory of mode-locked fiber laser is explained. The propagation models for the pulse evolution inside the laser cavity are presented. The main parameters that affect the laser cavity are discussed. The main regimes of mode-locked fiber laser are discussed.

In chapter 3, a generalized analytical model is developed for all-normal dispersion mode-locked laser. This model takes into consideration the effect of the overdriving point of the SA. The lumped spectral filter as well as the lumped SA is

averaged along the whole length of the laser cavity. The results from this model are compared with a conventional simulation model based on scalar GLE and experimental results. The effect of the SA absorber parameters such as the small signal absorption, saturation power and location of the overdriving point on the pulse energy, spectral bandwidth and de-chirped pulse width are discussed.

In chapter 4, an efficient semi-vector model for a laser cavity, employing nonlinear polarization rotation as a SA and having a high output coupling ratio is introduced. A closed-form equation for the nonlinear transmission of the saturable absorber is derived showing the effects of the rotation angles and phase shift of the polarization controllers. Two different cases of mode-locked pulses with different temporal and spectral profiles are illustrated. Another case showing no mode-locking at certain orientations of the polarization controllers is shown. The results of the semi-vector model are compared with the conventional simulation model based on coupled vector GLE and the experimental results.

In chapter 5, the effect of using a long length of low-doped Yb^{3+} fiber on the accumulation of the nonlinear phase shift as well as the peak power of the pulse is discussed. A comparative study is done on two laser cavities. The total fiber length of both cavities is the same, in order to have the same dispersion effect. However, the length of the low-doped Yb^{3+} fiber in one cavity is 2.8 times the length of the high-doped Yb^{3+} fiber in the other one. Both simulation and experimental analysis were done at several values of net dispersion to ensure the validity of the idea.

In chapter 6, I study numerically and experimentally how the location of the narrow bandwidth spectral filter inside an active Similariton laser cavity affects the pulse characteristics.

In chapter 7, the conclusion and future directions are presented.

Chapter 2

2 Introduction to mode locking

This chapter provides a brief description of the basics of mode-locked fiber laser as well as the main components required in a laser cavity in order to generate stable pulses. Chapter 2 is divided into four sections; in section 2.1, a brief description of mode locked fiber laser is provided. The propagation models that are used to describe the evolution of the pulse inside the laser cavity are illustrated in the same section. The main parameters that affect the temporal and the spectral profile of the optical pulse in mode-locked fiber lasers are discussed in section 2.2. The main regimes of mode-locking fiber lasers are illustrated in section 2.3. Finally a conclusion is presented in section 2.4.

2.1 Mode-locked fiber laser

A fiber laser cavity whether it is ring or Fabry-Perot has multiple longitudinal modes separated by its repetition rate. If the phase relations between the neighboring modes are random, the cavity will generate continuous wave radiation. However, the cavity will generate optical pulses if the phase relations between the longitudinal modes are locked together. The optical pulses will be stable over time with periodicity given by the round trip of the laser cavity and the peak power will be much higher than the average power. There are two approaches to initiating mode locking; the first one is active mode locking that is performed by the insertion of an intensity or acousto-optical modulator. As shown in Fig. 2.1(a), it provides a periodic loss with periodicity equal to the cavity repetition rate. The disadvantage of this technique is that it does not generate an ultra-short pulse because of the speed limitation of the modulator. The other approach is the

passive mode locking that requires a passive saturable absorber, which introduces higher losses at lower power (see Fig. 2.1(b)). This forces the longitudinal modes of the cavity to lock together to generate high peak power pulses. In general, the pulse width generated by passive mode locking is narrower than the width that is generated by active mode locking [17, 18].

The mode-locked laser cavity mainly comprises of optical fiber. The models that govern the pulse evolution through it are introduced below.

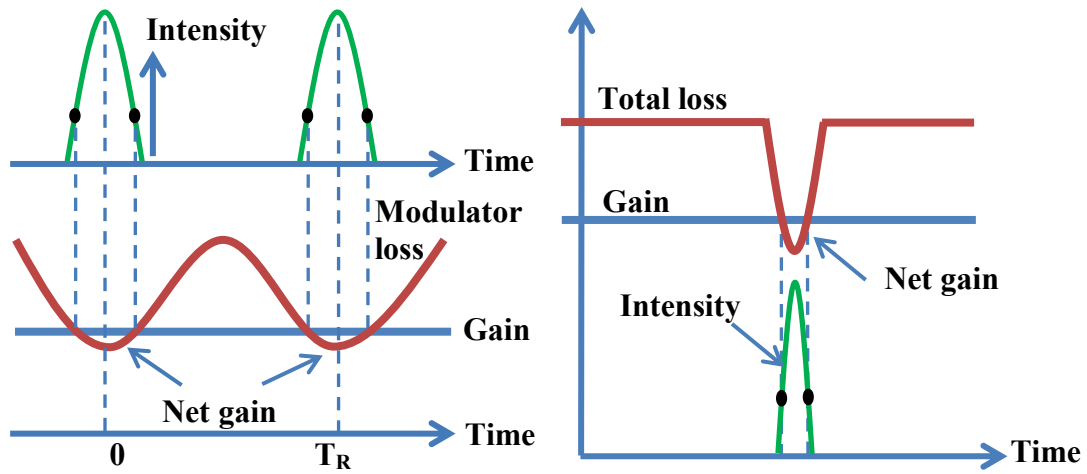


Fig. 2.1 Temporal profile of the mode-locked pulse and the dependent of the gain and loss of time for (a) active mode locking [18] and (b) passive mode locking (total loss comprises the SA and linear losses) [17].

2.1.1 Propagation models

Although the SA is important to initiate the laser pulses, it does not guarantee their stability. The laser cavity parameters such as the dispersion, nonlinear phase shift, gain, spectral selective elements (whether the gain bandwidth limitation or lumped spectral filter) and dispersion compensation element have a significant effect on pulse

stabilization and shaping [17, 18]. A schematic diagram of the laser cavity having all the necessary components for stable mode locking is shown in Fig. 2.2.

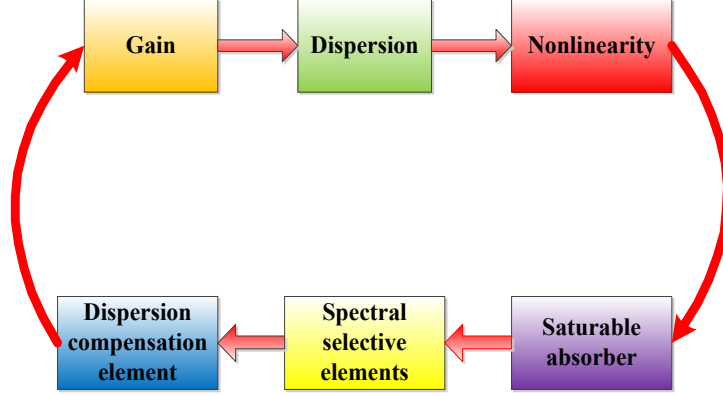


Fig. 2.2 Schematic diagram of mode-locked fiber laser showing the main components required to have stable pulse.

The GLE is used to describe the light propagation in the various fiber sections of a fiber laser cavity. The derivation of GLE from Maxwell's equation is well covered in [19]. Basically, two polarization axes are defined to give a full description of the pulse dynamics inside the fiber. Therefore, the evolution of optical pulses inside the gain fiber and the passive fiber is governed by vector CGLEs. These equations are applicable for optical fibers having single spatial mode. It is modeled as [20- 22]:

$$\frac{\partial u}{\partial z} - \frac{i\Delta\beta}{2}u + \frac{i\beta_2}{2}\frac{\partial^2 u}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 u}{\partial t^3} + \frac{\alpha_f}{2}u = \frac{g(z)}{2}\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right)u + i\gamma[|u|^2 + A|v|^2]u + i\gamma Bu^*v^2 \quad (2.1a)$$

$$\frac{\partial v}{\partial z} + \frac{i\Delta\beta}{2}v + \frac{i\beta_2}{2}\frac{\partial^2 v}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 v}{\partial t^3} + \frac{\alpha_f}{2}v = \frac{g(z)}{2}\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right)v + i\gamma[|v|^2 + A|u|^2]v + i\gamma Bv^*u^2 \quad (2.1b)$$

where u and v represent the two orthogonally polarized electric field envelopes ($\sqrt{W}$), the z axis denotes the propagation distance through the fiber, γ is the nonlinear parameter ($W^{-1}\text{m}^{-1}$) that is defined as: $\gamma = 2\pi n_2/(\lambda A_{eff})$, where n_2 is the nonlinear refractive index

(m^2/W), λ is the central wavelength of the pulse (e.g. 1030 nm), A_{eff} is the effective mode area of the fiber, $A = 2/3$ and $B = 1/3$ for silicate fibers, Ω_g is the gain bandwidth of the gain fiber (rad/s), β_2 (s^2/m) and β_3 (s^3/m) are the group velocity dispersion (GVD) and third order dispersion (TOD), α_f is the linear power loss coefficient (m^{-1}), $\Delta\beta$ is the linear birefringence of the fiber (rad/m), which is related to the fiber beat length (L_B) by: $\Delta\beta = 2\pi/L_B$. Since the frame of reference, which is called the retarded frame, is moving with the same group velocity of the pulse, t represents the retarded time. For the gain fiber, $g(z)$ is the saturated gain coefficient (m^{-1}) defined as:

$$g(z) = \frac{g_o}{1 + \left(\int_{-\infty}^{\infty} |a|^2 d\tau / E_{sat} \right)} \quad (2.2)$$

Here g_o is the small signal gain coefficient, $|a| = \sqrt{|u|^2 + |v|^2}$ is the magnitude of the total field ($\sqrt{W}$) and E_{sat} is the saturation energy of the gain fiber (J). The gain term is omitted for the pulse propagation inside the passive fiber. For simplicity, passive fiber is abbreviated as SMF (single mode fiber) throughout this thesis. If the mode-locked pulse evolving inside the cavity has a pulse width in the picosecond region, the higher order dispersion terms are omitted due to their minor role in the propagation of the pulses in optical fiber.

Two sources of nonlinearity are found in (2.1a) and (2.1b); the self-phase modulation (SPM) and the cross phase modulation (XPM). SPM is described by $(i\gamma(|u|^2)u)$ for the u -field and $(i\gamma(|v|^2)v)$ for the v -field while XPM is described by $(i\gamma A(|v|^2)u + i\gamma B u^* v^2)$ for the u -field and $(i\gamma A(|u|^2)v + i\gamma B v^* u^2)$ for the v -field.

Equations (2.1a) and (2.1b) cannot be solved analytically. Therefore, they are typically solved numerically using a fourth-order Runge-Kutta split-step Fourier transform algorithm [23]. Solving the vector CGLEs numerically is accurate but is time consuming and it is hard to apply it for guidance in the design of mode-locked fibers lasers. However, if the gain, dispersion, and linear loss terms are omitted, an analytical solution is found under certain conditions, which will be described in sub-section 2.2.6.2.

For laser cavities mode-locked by polarization insensitive SA, the polarization of light has minor role on the pulse dynamics. Therefore, the XPM terms in (2.1a) and (2.1b) can be neglected, and the pulse evolution can be modeled by scalar GLE expressed as [19, 24]:

$$\frac{\partial U}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 U}{\partial t^2} - \frac{\beta_3}{6} \frac{\partial^3 U}{\partial t^3} = -\frac{\alpha_f}{2} U + \frac{g_o/2}{1 + \int_{-\infty}^{\infty} (|U|^2) d\tau / E_{sat}} \left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t^2} \right) U + i\gamma |U|^2 U \quad (2.3)$$

where $U(z,t)$ is the scalar field envelope ($\sqrt{W}$). The same equation is used to model the pulse propagation inside SMF after omitting the gain term.

Equation (2.3) has to be solved numerically, similar to the CGLEs, but requires a fraction of the simulation time needed to solve the CGLEs [19, 24, 25]. However, it remains time-consuming and is not effective in providing guidance in mode-locked fiber laser design.

Another approach to studying the pulse propagation in mode-locked fiber laser is performed using average models. Such models are developed based on the vector CGLEs or the scalar GLE where all the parameters of the laser cavity are averaged along the cavity length and inserted in one equation called the average GLE. There are many

variations of the average GLE. The Complex Quintic Cubic Ginzburg-Landau equation (CQCGLE) is often used and it is given as [14, 23, 26]:

$$\frac{\partial U}{\partial z} = \left(\frac{g_0}{2} - l\right) U + \left(\frac{g_0/2}{\omega_g^2} - \frac{i\beta_2}{2}\right) \frac{\partial^2 U}{\partial t^2} + (\delta_s + i\gamma)|U|^2 U - \mu_s|U|^4 U \quad (2.4)$$

where l is the average linear loss (m^{-1}). The SA is defined by its cubic (δ_s) ($\text{W}^{-1}\text{m}^{-1}$) and (μ_s) ($\text{W}^{-2}\text{m}^{-1}$) quintic terms. Equation 2.4 can be solved, under certain conditions, analytically to have closed-form relations for the pulse energy and spectral bandwidth as a function of all parameters of the cavity and SA, which is considered to be an effective tool for detailed study of the laser cavity. Average models are covered in more detail in Chapters 3 and 4.

In the context of this thesis, models based on solving (2.1a), (2.1b) or (2.3) numerically are called simulation models, while the models based on averaging the cavity are called average models.

2.2 Laser cavity parameters

This section is organized as follows; in subsection 2.2.1, the impact of fiber dispersion on the pulse is discussed. The basics of SPM in SMF are covered in subsection 2.2.2. Wave breaking of optical pulses resulting from the propagation of high peak power pulses in normal dispersion SMF is illustrated in the same subsection. Background on Yb^{3+} -doped fiber, as an example of the gain medium, is depicted in subsection 2.2.3. Two kinds of dispersion compensation components are described in subsection 2.2.4. The main idea behind the insertion of a lumped filter in all normal dispersion fiber laser cavities is shown in subsection 2.2.5. Three different types of filters, which are used in

my research, are covered in the same subsection. The theory of a SA is explained in subsection 2.2.6. Two common types of SA are illustrated in the same subsection.

Before describing the main parameters of mode-locked fiber lasers, three characteristic lengths should be mentioned; they are used to show the weight of the dispersion relative to the self-phase modulation. Those are the GVD length (L_D), the TOD length (L'_D) and the nonlinear length (L_{NL}). They are defined as:

$$L_D = T_o^2 / |\beta_2| \quad (2.5a)$$

$$L'_D = T_o^3 / |\beta_3| \quad (2.5b)$$

where T_o is the pulse width. The nonlinear length (L_{NL}) is defined as:

$$L_{NL} = 1 / \gamma P_p \quad (2.6)$$

where P_p is the peak power of the pulse. The soliton order, $N^2 = L_D / L_{NL}$, describes the relation between dispersion and nonlinear lengths. In general, three cases describe the pulse propagation in optical fiber. They can be described as follows [19]:

- i. If the cavity length is much smaller than L_{NL} and at the same time comparable to the dispersion length (L_D or L'_D), the pulse evolution is governed by dispersion.
- ii. If the cavity length is much smaller than (L_D or L'_D) and at the same time comparable to L_{NL} , the pulse evolution is governed by SPM.
- iii. If the cavity length is comparable to the dispersion length and L_{NL} , dispersion and SPM have comparable effects on the pulse propagation.

In standard optical fiber lasing at a wavelength of 1 μm , the value of β_2 is greater than β_3 . Therefore, the value of L_D is smaller than L'_D for the same value of pulse width.

2.2.1 Fiber dispersion

As seen in (2.1a), (2.1b) and (2.3), the fiber dispersion includes both the GVD and TOD terms. Fiber dispersion does not change the spectral profile of the pulse. It changes only the temporal profile of the pulse and adds a instantaneous frequency (Ω) defined as $-\partial\varphi/\partial t$, where φ is the temporal phase of the pulse. The sign of the instantaneous frequency depends on whether the fiber has normal or anomalous dispersion. If the pulse has low peak power (case (i)), and the gain and loss terms are neglected, the field envelope at any distance z is given as [19]:

$$U(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}(0, \omega) \exp\left(\frac{i}{2}\beta_2\omega^2 z + \frac{i}{3}\beta_3\omega^3 z - i\omega t\right) d\omega \quad (2.7)$$

where $\tilde{U}(0, \omega)$ is the Fourier transform of the incident field envelope.

2.2.2 Self-phase modulation

If the pulse propagation is described by case (ii), the optical pulse propagating through the SMF is expressed as [19]:

$$U(z, t) = U(0, t) \exp(i\gamma |U(0, t)|^2 L_{eff}) \quad (2.8a)$$

$$L_{eff} = [1 - \exp(-\alpha_f z)] / \alpha_f \quad (2.8b)$$

where L_{eff} is defined as the effective length of the SMF. As shown in (2.8a), the temporal profile of the pulse does not change. However, an intensity-dependent phase shift is

created due to SPM; that is called the nonlinear phase shift (φ_{NL_SPM}). Its maximum value φ_{max} is defined as [19]:

$$\varphi_{max} = \gamma P_p L_{eff} \quad (2.9)$$

Fig. 2.3(a) shows the variation of $\Omega(t) = -\frac{\partial \varphi_{NL_SPM}}{\partial t} = -\gamma L_{eff} \frac{\partial |U(0,t)|^2}{\partial t}$. The SPM-induced frequency chirp of the Gaussian pulse is linear along the central part of the pulse, but it is non-linear near the trailing and leading edges of the pulse, which can cause wave breaking of the pulse.

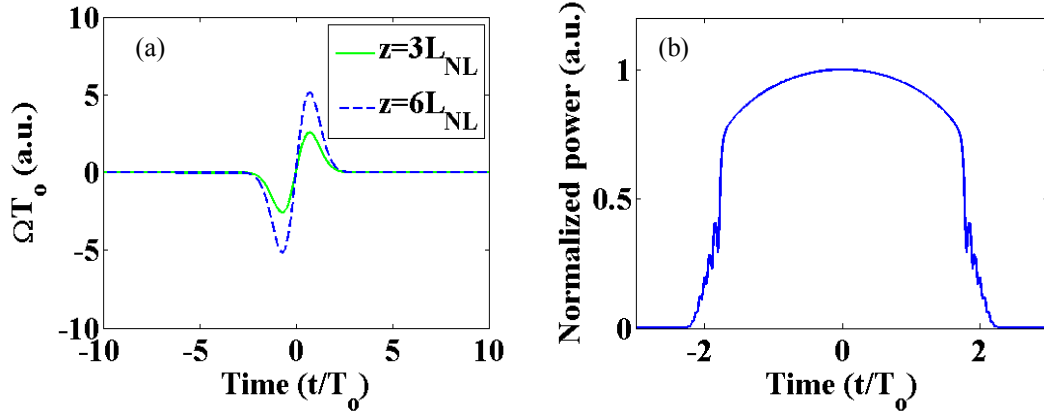


Fig. 2.3 (a) Plot of normalized frequency chirp ΩT_0 versus normalized time at z equal to $3L_{NL}$ and $6L_{NL}$ for an unchirped Gaussian pulse of pulse width 10 ps (b) Normalized temporal profile of an initially Gaussian pulse with $N=67$ at distance $z=46.7607 L_D$.

2.2.2.1 Wave breaking

Wave breaking is a phenomenon that occurs to pulses propagating in normal dispersion fibers due to SPM-induced nonlinear phase shift. It can be explained with the aid of Fig. 2.3(a) as follows; the red-shifted light near the leading edge travels faster and overtakes the non-shifted light in the forward tail of the pulse. The opposite occurs for the

blue-shifted light near the trailing edge. The leading and trailing regions of the pulse contain light at two different frequencies that produce oscillation at the pulse edges [17, 19] (see Fig. 2.3(b)).

2.2.3 Gain medium

Yb^{3+} -doped fiber is used as a gain medium because of its high gain coefficient around 1 μm . Optical pumping of Yb^{3+} -doped fiber creates population inversion of the Yb^{3+} ions, which leads to having gain by the means of stimulated emission [27]. The optical pulse, propagating through Yb^{3+} -doped fiber, experiences gain saturation modeled by $(\frac{g_o/2}{1+\int_{-\infty}^{\infty}(|U|^2)d\tau/E_{sat}})$ as well as spectral bandwidth limitation. The energy levels of Yb^{3+} in silica glass are shown in Fig. 2.4(a). The amorphous nature of silica is responsible for splitting energy levels into two bands. As the lifetime of the excited sub-levels is around 1 ms, the gain saturation is related to the average power of the pulse. The saturation power of the Yb^{3+} -doped fiber is related to the pump power. The saturation energy (E_{sat}) is given as the value of the saturation power divided by the round trip time. The gain bandwidth originates from the collective transitions between the upper and the lower bands in which the linewidth of each sub-level is dominated by homogeneous broadening. The Giles parameters, which are related to the emission and absorption cross sections of the Yb^{3+} -doped fibers [28], are plotted in Fig. 2.4(b).

The pump wavelength has the highest peak at 976 nm, which is used as the pump wavelength. The gain coefficient has its peak value at 1030 nm. The gain spectrum around 1030 nm can be modeled as a parabolic filter of gain bandwidth (Ω_g) having a value of 40 nm in either CGLE or scalar GLE [20]. The spectral effect has a negative

impact on the pulse as it limits the spectral bandwidth of the mode-locked pulse. However, as will be shown below, it has a positive impact as it filters out the nonlinear parts of phase shift induced by SPM, which gives high immunity to wave breaking.

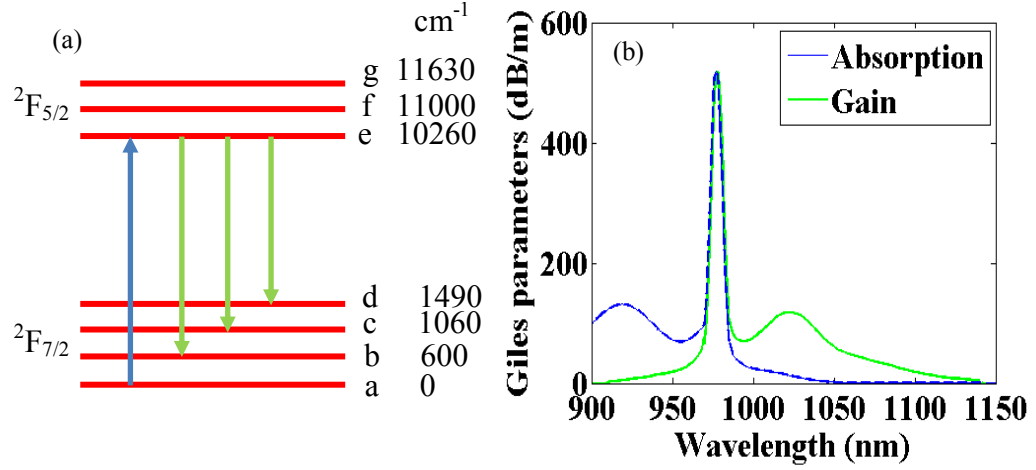


Fig. 2.4 (a) Yb³⁺ energy level structure inside silica [29]. The blue arrow represents the peak absorption at 977 nm. The emission around 1030 nm is depicted by the green arrows. (b) Giles parameters of Yb501 produced by CorActive [30].

2.2.4 Dispersion compensation elements

Throughout this thesis, two distinct dispersion compensation elements are used to produce anomalous dispersion to compensate the positive chirp of the pulse. The first one is a pair of diffraction gratings, while the second one is photonics bandgap (PBG) fiber.

2.2.4.1 Diffraction gratings

The diffraction gratings are inserted either inside the laser cavity for dispersion compensation or outside for pulse compression near its transform limit. Fig. 2.5(a) illustrates the schematic diagram of the diffraction gratings block. A retro-reflector mirror is used to reflect the diffracted beams back to the diffraction gratings at a higher level. The ray trace depicts the function of the diffraction grating pairs; the red-shifted

component on the optical pulse travels a longer distance than the blue-shifted component. Therefore, an anomalous dispersion is created that is a function of the separation between the gratings [17]. It is expressed as:

$$Dis_g = \frac{\partial^2 \psi}{\partial \omega^2} = -2 \frac{m^2 \lambda^3 b}{2\pi c_o^2 d^2 \cos^2 \theta_d} \quad (2.10)$$

where ψ is the spectral phase and m is the diffraction order. Most of the optical power is diffracted at m equals -1. c_o is the speed of light in free space. b is the slant length between the gratings. Its value is taken for the diffracted beam at the central wavelength of the pulse. d is the number of grooves per unit meter. θ_d is the diffraction angle calculated at the central wavelength of the pulse. A factor of two is added in (2.10) because the light diffracts from each grating twice.

Also, TOD is generated from the diffraction grating pairs. It has the opposite sign to that of the GVD. It is expressed in terms of Dis_g (2.10) as:

$$TOD_g = \frac{\partial^3 \psi}{\partial \omega^3} = -\frac{\partial^2 \psi}{\partial \omega^2} \frac{3\lambda}{2\pi c_o} \left[1 + \frac{m\lambda}{d} \frac{\sin \theta_d}{\cos^2 \theta_d} \right] \quad (2.11)$$

In mode-locked laser, the cubic phase resulting from the TOD causes an asymmetry of the optical pulse in the temporal and spectral domains [31, 32]. For simplicity, the TOD of the diffraction gratings is ignored throughout this thesis.

2.2.4.2 Photonics bandgap fiber

In general, the dispersion relation of PBG fiber can be tailored to be completely different from standard SMF. This can be done by engineering the diameter of the hollow cylinders, the periodicity of the photonic crystal lattice and the number of air-hole rings.

The light guidance inside the air core of the fiber is performed through a photonic bandgap effect, which prohibits certain frequency bands from propagating through the cladding [33].

PBG fiber (HC-1060) is utilized for dispersion compensation as it provides anomalous dispersion at our center wavelength. The image of its cross section area is shown in Fig. 2.5(b). The main parameters of this fiber are shown in Table 2.1.

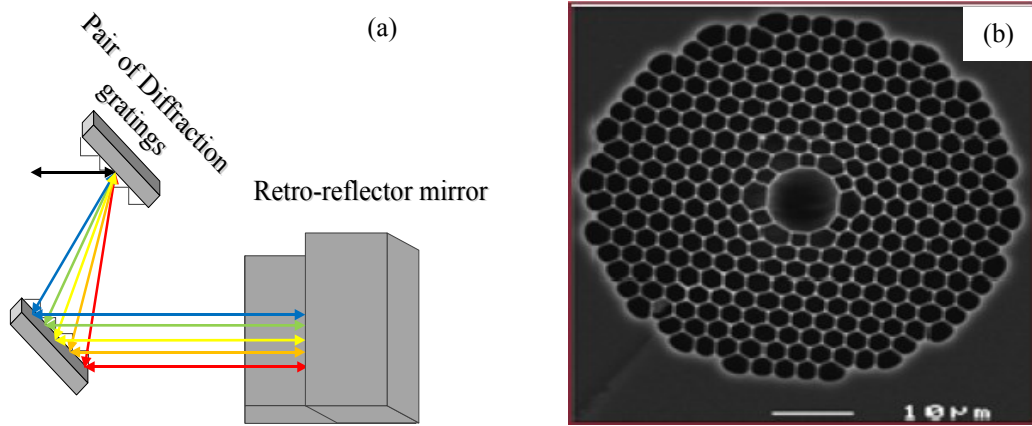


Fig. 2.5 (a) Schematic diagram of the diffraction gratings pair combined with a retro-reflector mirror. (b) SEM image of PBG fiber (HC-1060) [34].

Table 2.1 Main parameters of HC-1060 [34]

Parameter	Value
Core diameter	$10 \mu\text{m} \pm 1 \mu\text{m}$
Mode field diameter	$7.5 \mu\text{m} \pm 1 \mu\text{m}$
Cladding diameter	$123 \mu\text{m} \pm 5 \mu\text{m}$
Coating diameter	$220 \mu\text{m} \pm 50 \mu\text{m}$
Dispersion at 1030 nm	80 ps/nm/Km
Dispersion at 1060 nm	120 ps/nm/Km
Effective mode index	0.99

2.2.5 Lumped spectral filter

Spectral filtering plays a critical role in stabilizing mode-locked all normal dispersion fiber lasers. Fig. 2.6 illustrates the impact of the spectral filter on the temporal profile of a chirped pulse. As an optical pulse propagates in normal dispersion fiber, it acquires a positive frequency chirp due to dispersion and self-phase modulation. Its pulse width also broadens with the propagation distance. A SA by itself cannot compress the pulse back to its initial value before entering the SMF. Another element should be inserted inside the cavity to assist the saturable absorber; that is the lumped spectral filter. Spectral filtering of the pulse in the frequency domain is mapped into temporal cutting of the pulse edges, which mainly acts as a pulse compressor [35].

Three types of spectral filter s are used in this thesis. Those are the ideal Gaussian filter, a multimode interference (MMI) filter, and an interference filter. A brief description of each of them is illustrated below.

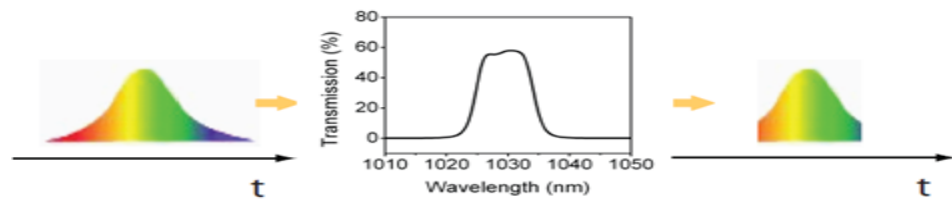


Fig. 2.6 Temporal compression of a chirped pulse after passing through a spectral filter [35, 36].

2.2.5.1 Ideal Gaussian filter

An ideal Gaussian filter is implemented in the models depicted in this thesis. The power transfer function of the filter, T_{gauss} , is given as:

$$T_{gauss} = \exp\left(-\frac{\omega^2}{\omega_0^2}\right) \quad (2.12)$$

where ω_o is $1/e$ intensity half width. The 3-dB bandwidth of the filter is related to ω_o by:

$$BW_{f_gauss} = 2(\ln 2)^{1/2} \omega_o \approx 1.665 \omega_o \quad (2.13)$$

2.2.5.2 Multimode interference filter

This is physically constructed by inserting a section of step index multimode fiber (MMF) between two SMFs. The schematic diagram of this filter is shown in Fig. 2.7(a). The theory of operation of the MMI filter is explained as follows; the light at the end of the input SMF, which has an approximately Gaussian-shaped field profile, excites some modes inside the MMF. If the core diameter of the MMF is large, the radiation modes of the MMF are not excited. In addition if the axes of symmetry of the input SMF, MMF and the output SMF are collinear, the mode field distributions with azimuth components are not excited. That is because the coupling coefficient, which depends on the overlap integral between the Gaussian field profile of the input SMF and the spatial profile of the modes of the MMF, is equal to zero for the azimuth mode [37]. Therefore, the number of modes inside the MMF is reduced to the radial modes. As shown in Fig. 2.7(b), these modes interfere with each other through the length of the MMF. Past a certain distance inside the MMF, a constructive interference between these modes occurs to create an image of the mode field distribution of the input SMF [38- 40]. It is called the self-image length, z_{image} , which is defined as:

$$z_{image} = \frac{8n_{core}k_o a_c^2}{\pi} \quad (2.14)$$

where n_{core} is core refractive index of the MMF. k_o is the free space propagation constant. a_c is the core radius of the MMF. It is clear from (2.14) that the location of the image is a function of the wavelength of the light. Hence, the SMF-MMF-SMF configuration can

act as a spectral filter. The output SMF should be spliced at the position of the self-image constructed by our desired center wavelength.

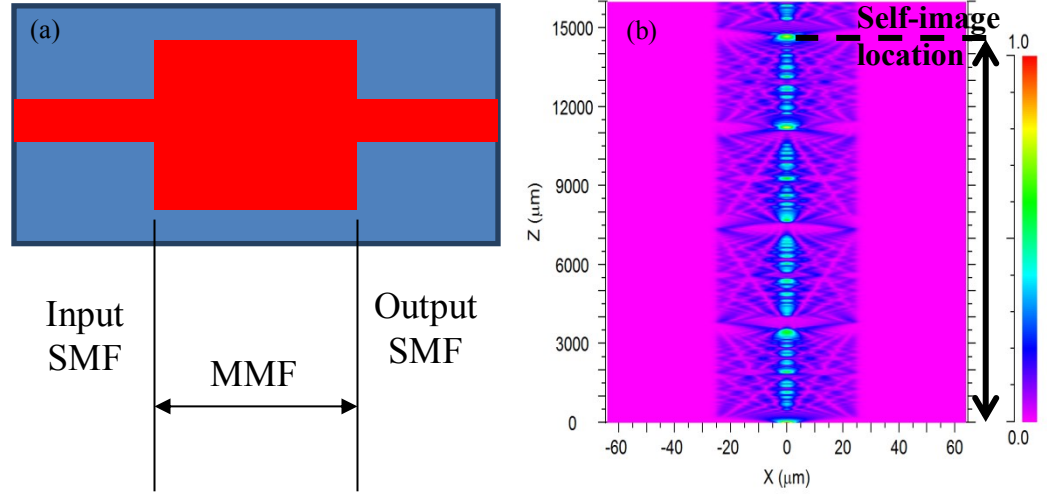


Fig. 2.7 (a) Schematic diagram of the MMI fiber (b) Calculated MMI intensity pattern inside MMF of core radius 25 μm and numerical aperture of 0.22. The location of the self-image is mentioned in the figure.

The mathematical model governing the design of the MMI filter is illustrated as follows; the electric field, E_{source} , at the facet of the MMF can be written as summation of the radial modes of the MMF:

$$E_{source}(r, \theta) = \sum_{j=0}^{M-1} c_j E_j^t(r, \theta) \quad (2.15)$$

where c_j is the field expansion coefficient. M is the number of modes excited in the MMF. $E_j^t(r, \theta)$ is the j th transverse mode of the MMF.

The power transfer function of the MMI filter is given as [39, 40]:

$$T_{MMI} = \left| \sum_{j=0}^{M-1} \tilde{c}_j^2 \exp(i\beta_{MMF_j} z) \right|^2 \quad (2.16)$$

where β_{MMF_j} is the j th mode propagation constant in MMF, and $\tilde{c}_j$ is the modified field expansion coefficient, which is related to c_j by:

$$\tilde{c}_j = c_j \left(\frac{P_j}{P_{source}} \right)^{1/2} \quad (2.17)$$

Here P_j and P_{source} are the j th mode and source power, respectively. In our design, the center wavelength of the MMF is set to 1030 nm. Standard SMF (HI-1060) is used as the input and output SMFs. The part number of the MMF bought from Thorlabs is FG050LGA. It has a numerical aperture of 0.22 and a core radius of 25 μm . The value of z_{img} was calculated to be 13.88 mm. Multiples of this length can be selected, but at the expense of reducing the spectral bandwidth [39]. The length of the MMF was adjusted to be three times z_{img} (41.64 mm) to have a spectral filter with 3-dB bandwidth of 5 nm.

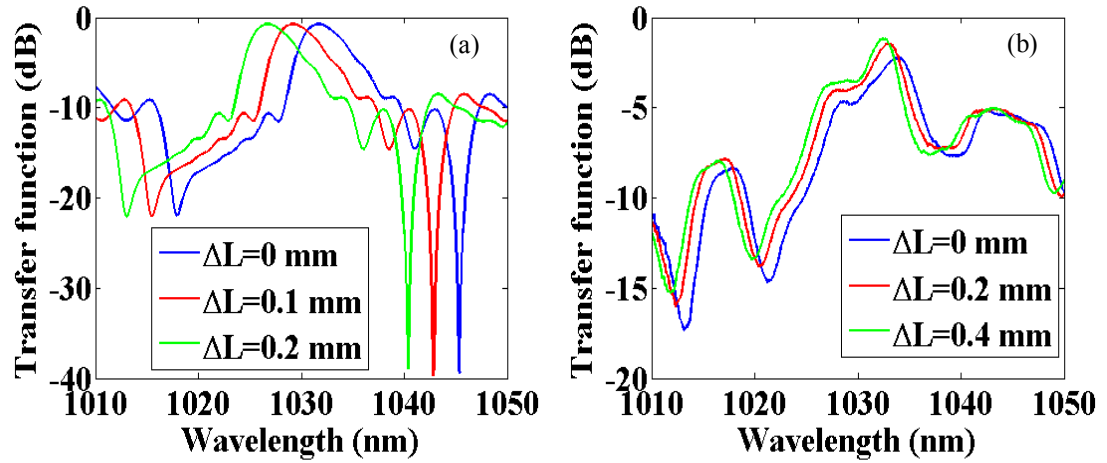


Fig. 2.8 (a) Theoretical and (b) experimental transfer functions of the MMI-filter.

An interesting feature in this filter is its tuning by stretching the MMF section [41, 42]. The center wavelength of the filter is blue-shifted by 2.46 nm per 100 micrometer stretch of the MMF. Fig. 2.8(a) shows the theoretical transfer function of the filter at

different stretches of the MMF. For all lengths of the MMF, the theoretical value of filter insertion loss is 0.73 dB, and the 3-dB bandwidth of the filter is kept the same.

The experimental transmission spectrum of the MMI filter is plotted in Fig. 2.8(b). Yb^{3+} -doped fiber amplified spontaneous emission (ASE) source was passed through the MMI filter to test its performance. The experimental values of insertion loss and 3-dB bandwidth are 2 dB and 6.24 nm. The difference between the theoretical and experimental values is due to imperfect splicing of the SMF to the MMF. The asymmetry observed in the filter experimental transfer function is mainly due to the transverse misalignment between the SMF and MMF, which excites the high order azimuth modes in the MMF. The tuning procedure was done by clamping the input and output SMFs with rubber taps. An axial strain was applied to the MMF using a single-axis translation stage.

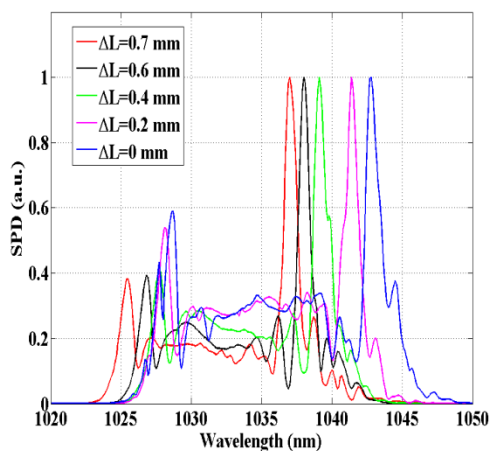


Fig. 2.9 Tuning results for all-fiber all-normal dispersion mode-locked pulse.

Tuning of the center wavelength of all-fiber all-normal-dispersion fiber laser can be performed with the aid of the MMI filter. The experimental results of the tuning of the

center wavelength of the filter are illustrated in Fig. 2.9. The tuning range does not match the theoretical value because the displacement applied by the translation stage is not exactly interpreted as the same value of MMF elongation.

2.2.5.3 Interference filter

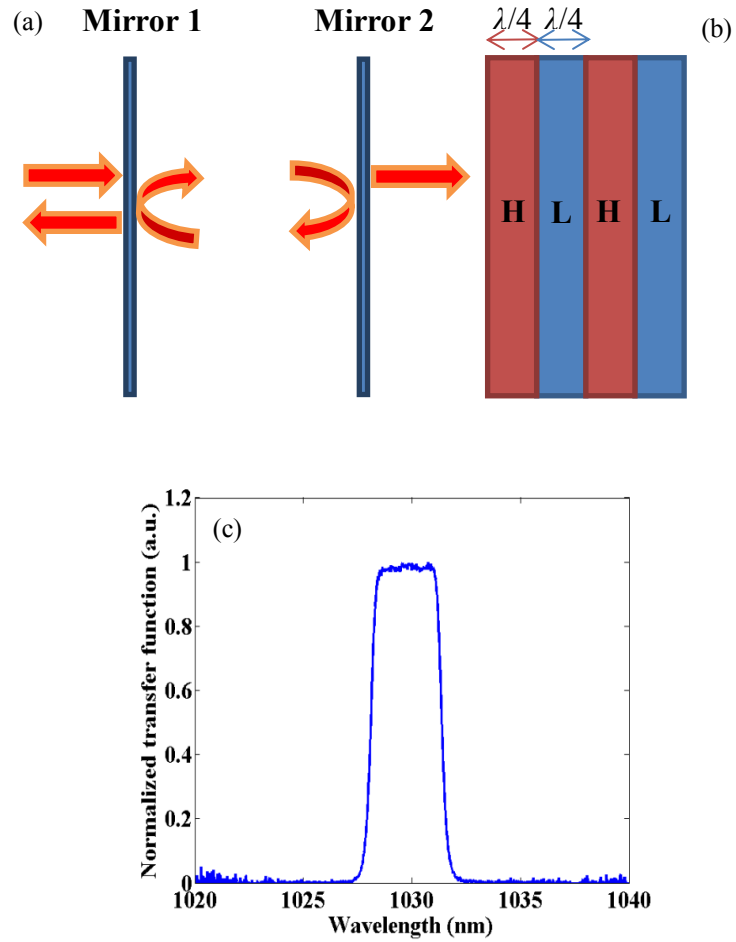


Fig. 2.10 (a) Schematic diagram of Fabry Perot filter. (b) Construction of Bragg-mirror; H: high refractive index material and L: low refractive index material (c) Measured transfer function of the spectral filter purchased from Fiber Logix and having a part number is FL-BDF-1030-02-10-N-B-1.

A Fabry Perot filter is a kind of interference filter, which is used as a spectral filter in this thesis. As shown in Fig. 2.10(a), it consists of two partially reflected mirrors between a medium that is ideally transparent to incident optical wavelengths. Its theory of

operation is briefly explained as follows; the incident light is resonantly bounced between two partially reflective mirrors. The center wavelength of the transmission transfer function of the filter is adjusted by the optical thickness (the physical thickness multiplied by its refractive index) of the medium between the two mirrors. It must be a multiple of $\lambda/2$, where λ is the center wavelength. The bandwidth of the filter is a function of the reflectivity of the mirrors [43]. The mirrors of the filter used in this thesis are distributed Bragg mirrors. They are constructed by periodically stacking $\lambda/4$ pairs of high and low refractive indices (see Fig. 2.10(b)). The bandwidth of these mirrors decreases by increasing the number of pairs. Therefore, the overall response of the filter is a function of the number of pairs of the Bragg mirrors [44].

The experimental transfer function of a Fabry-Perot filter with Bragg-mirrors is shown in Fig. 2.10(c). It has a 3-dB bandwidth of 3.2 nm. A light emitting diode (LED) having a center wavelength 1030 nm was used to characterize the filter.

2.2.6 Saturable absorber

The SA is considered to be the heart of a passive mode-locked laser. It causes the amplitude modulation of the pulse. It forces the fiber laser to operate in pulsed mode operation rather than continuous wave. It provides a higher loss to the light having lower power than the light having higher power [18]. As shown in Fig. 2.11, if a wide pulse passes through the SA, its output temporal profile will be affected by the nonlinear response of the SA. If the peak power of the pulse (denoted by point a) is lower than the overdriving point of the SA, the high-power portions of the pulse will experience lower loss than the low-power ones. Therefore, the pulse will be compressed (blue pulse). However, if the peak power of the pulse (denoted by point b) is higher than the

overdriving of the SA, the central part of the pulse will suffer from higher losses than parts of the leading and trailing edges. Therefore, the pulse will be distorted (green pulse).

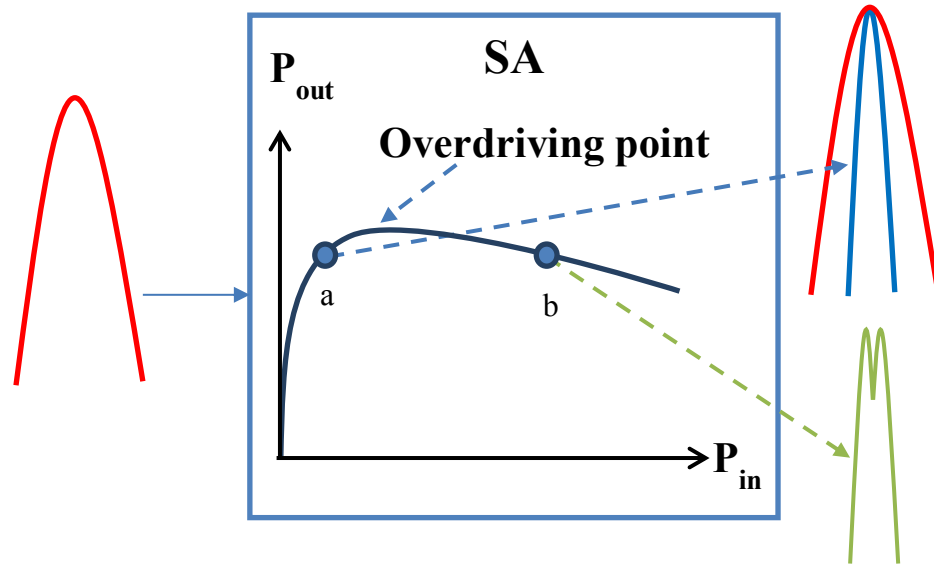


Fig. 2.11 Cartoon of the effect of the SA on a wide pulse (red pulse) to convert it narrow one (blue pulse) if the input power is before the overdriving point or narrow pulse with notch in the middle (green pulse) if power is after the overdriving point.

Two types of SA can be used in mode-locked fiber lasers. The first one is the lumped SA. The other is an artificial SA that employs NPR. Both SPM and XPM cause nonlinear rotation of the ellipse of an optical pulse while propagating through the fiber.

2.2.6.1 Lumped saturable absorber

In mode-locked fiber laser, a lumped SA can be implemented by inserting carbon nanotubes between two fiber ferrules in a standard SMF connector [45] or around tapered SMF [46- 49]. Graphene oxide is a kind of material used as a SA [50, 51]. A SESAM is a common type of lumped SA. Since a SESAM-based mode-locking technique is used in

two experimental set-ups in Chapters 3 and 5, a brief description of its design is addressed in this section.

The SA layer of the SESAM is typically made from layers of InGaAs quantum wells. Electron hole pairs are created in these layers due to the absorption of the injected photons. However, the SA layer has a limited number of electron-hole pairs created. Therefore, only a small percentage of the injected photons is absorbed, and the rest of them are reflected back from the Bragg mirror.

Figures 2.11(a) and 2.11(b) show two possible layouts of a SESAM [52]; resonant and anti-resonant. For resonant SESAM (see Fig. 2.12(a)), the SA layer is inserted between two reflectors. The one to the right of the absorber layer has 100% reflectivity while the one to left, where the light is incident on, has partial reflectivity. In this configuration, the wavelength of the incident light coincides with the resonance wavelength. As the light is trapped inside the resonant SESAM, it has low saturation power and high modulation depth (difference between maximum and minimum transmission of the SA). However, it has a low damage threshold and a reduced bandwidth because of the limited resonance bandwidth.

Fig. 2.12(b) depicts an anti-resonant SESAM where the wavelength of the incident light coincides with the center of the anti-resonance region. In this configuration, one of the mirrors has 100% reflection (right) while the other side is a partial reflector through the air-semiconductor interface. Therefore, it has larger bandwidth and higher saturation power and damage threshold, but has smaller value of modulation depth than resonant SESAM. Since the optical power of the incident light should be much higher

than the saturation power in order to have a saturable absorption effect [53, 54], the higher damage threshold of Anti-resonant SESAM is not practically an advantage because the optical power of the light will be close to the damage threshold. It is worth mentioning that we only use anti-resonant SESAM in this thesis.

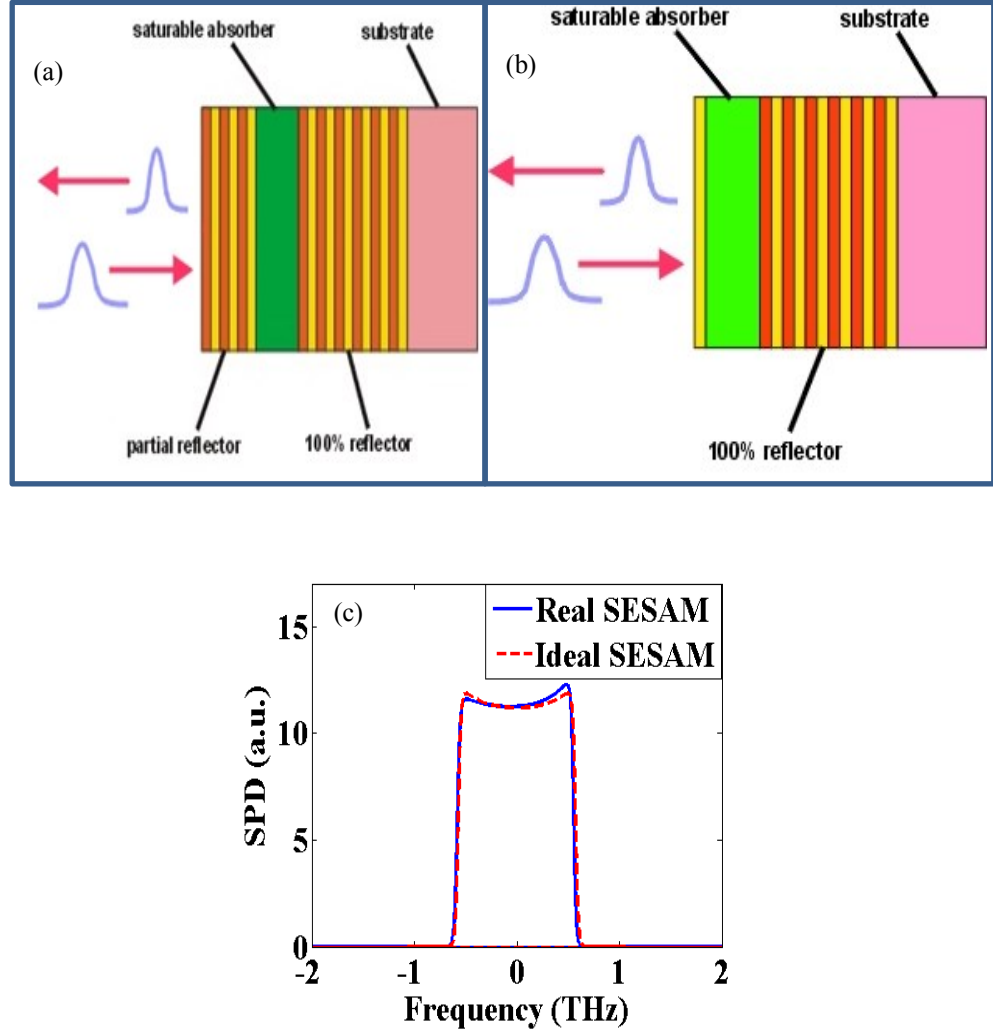


Fig. 2.12 Layouts of SESAM having mainly a reflector made of Bragg-mirror and SA layer [52]: (a) resonant (b) anti-resonant. (c) SPD of mode-locked chirped pulse propagating inside a cavity having a real (solid blue line) and ideal (dashed red line) SESAM; its relaxation time is 500 fs.

The reflectors of the SESAM are composed of a Bragg mirror grown on a semiconductor substrate. The Bragg mirror is fabricated by periodically stacking layers of GaAs and AlAs.

The recovery time of the SESAM should be much shorter than the pulse width in order not to cause distortion in the pulse. For the case of chirped pulse oscillator, if the recovery time is of the same order as the pulse width, the spectral profile of the pulse will be asymmetric (see Fig. 2.12(c)). The recovery time of the SESAM can be reduced by introducing lattice defects in the absorber layer, which enhances the non-radiative relaxation time of the carriers. SESAMs are well known to have bi-temporal recovery time. The shorter recovery time is in the range of sub-picosecond, which is essential for short pulses in mode-locked fiber laser. The longer one is important to initiate mode locking [54].

The mathematical model that governs the dynamics of the SESAM is described as follows; its saturable absorption (q_{sa}) is modeled as [55, 56]:

$$\frac{\partial q_{sa}}{\partial t} = -\frac{q_{sa}-q_o}{T_{sa}} - \frac{P(t)}{E_{sat_SA}} q_{sa} \quad (2.18)$$

where q_o is the small signal absorption of the SESAM. E_{sat_SA} is the saturation energy. T_{sa} is the relaxation time. $P(t)$ is the pulse power. The power reflectivity (R_{real}) of the SESAM is defined as:

$$R_{real} = 1 - q_{sa} - L_{ns} - \frac{P(t)}{P_{TPA}} \quad (2.19)$$

where L_{ns} is non-saturable loss. P_{TPA} is the two-photon parameter, which is responsible for the overdriving of the SA [16, 57]. Equation (2.18) can be simplified by assuming an

instantaneous transmission transfer function of the SA. This approximation is valid as long as the pulse width is longer than the relaxation time, which matches the Similariton and dissipative soliton regimes. Hence, the instantaneous power reflectivity (R_{ins}) of the SESAM is expressed as [18]:

$$R_{ins} = 1 - \left(\frac{q_o}{1 + \frac{P}{P_{sat}}} + \frac{P}{P_{TPA}} + L_{ns} \right) \quad (2.20)$$

where P_{sat} is the saturation power of SESAM defined as E_{sat_SA}/T_{sa} .

2.2.6.2 Nonlinear Polarization Rotation SA

NPR-based SA has important advantages over SESAM. The first one is its high value of damage threshold, which is related to self-focusing of the optical fiber (4.677 MW) [58]. The second advantage is the fast Kerr nonlinearity response time (<10 fs) [19]. The third advantage is the ability to change the transfer function of the SA by adjusting the parameters of the polarization controller. However, the random birefringence of the fiber destabilizes the nonlinear transmission response of SA, which converts the pulses into noise-like pulses [59, 60]. The angles of the polarization controllers and the polarization ellipse rotation through the optical fiber must provide a lower loss to the light having higher power than the light having lower power. Two schematic diagrams of the main blocks of NPR-based SA are shown in Fig. 2.13. Two polarization controllers are inserted on either sides of the Kerr medium as shown in Fig. 2.13(a). It starts with a polarizer that only permits one polarization of light (Horizontal polarization is selected here). The polarization changes to elliptical through the second polarization controller (PC_2). The light passes through a Kerr medium represented by an optical fiber. The rotation of the ellipse is a function of the power of propagating light.

The light emerging from the optical fiber passes through the first polarization controller (PC₁) that changes the polarization of the light. Finally, it passes through the polarizer, which only selects one polarization.

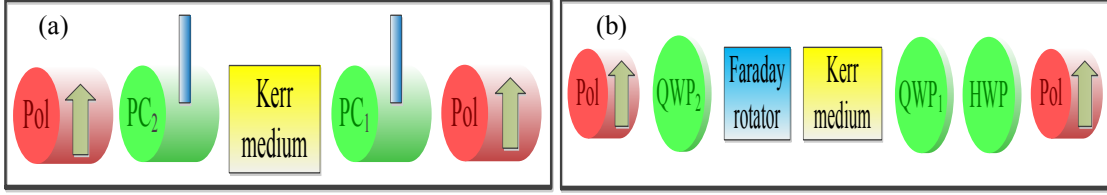


Fig. 2.13 Schematic diagrams of the main blocks of NPR-based SA. The Kerr medium is inserted between (a) two polarization controllers and (b) wave plates and Faraday rotator. Pol: polarizer, PC₁: first polarization controller, PC₂: second polarization controller, QWP₁: first quarter wave plate, HWP: half wave plate, QWP₂: second quarter wave plate.

To get an analytical expression to the nonlinear transmission of the NPR, all above components are defined by their transmission matrices. The Jones matrix for an arbitrary oriented PC_n is:

$$W_{PCn} = \begin{pmatrix} \cos(\alpha_n) & -\sin(\alpha_n) \\ \sin(\alpha_n) & \cos(\alpha_n) \end{pmatrix} \begin{pmatrix} e^{i\delta_n} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\alpha_n) & \sin(\alpha_n) \\ -\sin(\alpha_n) & \cos(\alpha_n) \end{pmatrix} \quad (2.21)$$

where δ_n is the phase shift between the two polarizations, and α_n is the angle of rotation (i.e. n refers to the number of the polarization controller).

The matrix that relates the output fields (u_2 and v_2) from the optical fiber to its input (u_1 and v_1) field is analytically driven from equations (2.1a) and (2.1b), but in the condition that we have only linear birefringence, SPM and XPM. The solution is in terms of elliptical functions [19, 61]. It can be simplified further by assuming non-birefringence fiber. The equations are rewritten as:

$$\frac{\partial u}{\partial z} = i\gamma[|u|^2 + A|v|^2]u + i\gamma B u^* v^2 \quad (2.22a)$$

$$\frac{\partial v}{\partial z} = i\gamma[|v|^2 + A|u|^2]v + i\gamma B v^* u^2 \quad (2.22b)$$

They have closed form relations under two conditions [62]:

(i) The optical power ($P = |u|^2 + |v|^2$) is constant along the whole length of the fiber.

(ii) $\text{Im}(u v^*)$ should be constant. This term represents the area of the polarization ellipse and it will be denoted as J .

The final solution of equations (2.22a) and 2.22b) results in a nonlinear transmission matrix (W_{NL}) defined as [23, 62, 63]:

$$\begin{pmatrix} u_2 \\ v_2 \end{pmatrix} = W_{NL} \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} \quad (2.23a)$$

$$W_{NL} = e^{i\gamma P z} \begin{pmatrix} \cos\left(\frac{2}{3}\gamma J z\right) & \sin\left(\frac{2}{3}\gamma J z\right) \\ -\sin\left(\frac{2}{3}\gamma J z\right) & \cos\left(\frac{2}{3}\gamma J z\right) \end{pmatrix} \quad (2.23b)$$

It shows the intensity-dependent polarization rotation of the polarization ellipse as it evolves along the fiber. It is more illustrative to express the nonlinear transmission matrix in the circular polarization basis:

$$W_{NL_c} = e^{i\gamma P z} \begin{pmatrix} \exp(i\varphi_{NL_NPR}) & 0 \\ 0 & \exp(i\varphi_{NL_NPR}) \end{pmatrix} \quad (2.24)$$

where $\varphi_{NL_NPR} = \gamma z(|u_+|^2 - |u_-|^2)/3$ is the nonlinear phase shift due to NPR; u_+ is a right-hand circularly polarized field defined as $u_+ = (u + iv)/\sqrt{2}$, and u_- is a left-hand circularly polarized field defined as $u_- = (u - iv)/\sqrt{2}$ [19, 59]. The rotation of the

elliptical polarization is physically explained in terms of the right and left circular polarized fields. A nonlinear birefringence is induced in the optical fiber that depends on φ_{NL_NPR} . Hence, the right and left circularly polarized fields propagate with different phase velocities that will lead to the rotation of the polarization ellipse through the optical fiber [58]. Whenever the polarization state is linear, $|u_+|^2 = |u_-|^2$, φ_{NL_NPR} equals to zero. Thus linear polarized light does not experience intensity-dependent polarization rotation [58].

The polarization discrimination is performed in the polarizer. Without loss of generality, the Jones matrix of a polarizer that passes only the horizontal polarization is:

$$W_p = \begin{pmatrix} B_p & 0 \\ 0 & 0 \end{pmatrix} \quad (2.25)$$

where B_p is the transmission coefficient. As it will be seen in Chapter 6, a polarization beam splitter (PBS) can be used instead of the polarizer. Therefore, another matrix for the rejection port of the PBS will be given as:

$$W_{rej} = \begin{pmatrix} 0 & 0 \\ 0 & B_p \end{pmatrix} \quad (2.26)$$

The field nonlinear transmission is determined by multiplying the matrices of components shown in Fig. 2.12(a):

$$\begin{pmatrix} u_{out} \\ 0 \end{pmatrix} = W_p W_{PC1} W_{NL} W_{PC2} \begin{pmatrix} u_{in} \\ 0 \end{pmatrix} \quad (2.27)$$

The closed form of the field nonlinear transmission $t_{NPR_PC} = u_{out}/u_{in}$ is determined after solving (2.27) to be:

$$t_{NPR_PC} = B_p e^{i\gamma Pz} e^{i((\delta_1 + \delta_2)/2)} \left[\cos\left(\frac{2\gamma Jz}{3}\right) \cos\left(\frac{\delta_1}{2}\right) \cos\left(\frac{\delta_2}{2}\right) + i \cos\left(\frac{2\gamma Jz}{3} - 2\alpha_2\right) \cos\left(\frac{\delta_1}{2}\right) \sin\left(\frac{\delta_2}{2}\right) + i \cos\left(\frac{2\gamma Jz}{3} + 2\alpha_1\right) \sin\left(\frac{\delta_1}{2}\right) \cos\left(\frac{\delta_2}{2}\right) - \cos\left(\frac{2\gamma Jz}{3} + 2\alpha_1 - 2\alpha_2\right) \sin\left(\frac{\delta_1}{2}\right) \sin\left(\frac{\delta_2}{2}\right) \right] \quad (2.28a)$$

$$J = -\frac{1}{2} \sin(\delta_2) \sin(2\alpha_2) P \quad (2.28b)$$

where (δ_1, δ_2) are the phase shifts induced by the two polarization controllers and the orientation of these controllers with respect to the principle axes of the fiber are (α_1, α_2) . The effect of the characteristic angles of the polarization controller on the temporal and spectral profile of the mode-locked pulse is explained in more detail in Chapter 4.

Another way to implement the NPR-based SA is shown in Fig. 2.13(b). Two quarter wave plates (QWPs) and one half wave plate (HWP) can replace the polarization controllers to give similar performance. The Faraday rotator shown in Fig. 2.12(b) refers to the polarization dependent Faraday isolator, which is inserted to have unidirectional light propagation [18, 64].

The wave-plates and Faraday isolator are modeled by their Jones matrices. For arbitrary orientation with respect to the fast axis of the fiber, the matrices of the wave-plates are given as [23, 63]:

$$W_{QWPn} = \begin{pmatrix} \cos(\theta_n) & -\sin(\theta_n) \\ \sin(\theta_n) & \cos(\theta_n) \end{pmatrix} \begin{pmatrix} e^{-i\frac{\pi}{4}} & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \begin{pmatrix} \cos(\theta_n) & \sin(\theta_n) \\ -\sin(\theta_n) & \cos(\theta_n) \end{pmatrix} \quad (2.29a)$$

$$W_{HWP} = \begin{pmatrix} \cos(\theta_n) & -\sin(\theta_n) \\ \sin(\theta_n) & \cos(\theta_n) \end{pmatrix} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \cos(\theta_n) & \sin(\theta_n) \\ -\sin(\theta_n) & \cos(\theta_n) \end{pmatrix} \quad (2.29b)$$

Here θ is the angle of rotation of each wave-plate with respect to the fiber fast axis. The Faraday isolator is modeled as a rotation matrix of 45° with respect to the fast axis of the fiber.

$$W_F = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad (2.30)$$

Following the same procedure performed above, the field nonlinear transmission is determined by multiplying the matrices of the components shown in Fig. 2.13(b):

$$\begin{pmatrix} u_{out} \\ 0 \end{pmatrix} = W_{QWP2} W_F W_{NL} W_{QWP1} W_{HWP} \begin{pmatrix} u_{in} \\ 0 \end{pmatrix} \quad (2.31)$$

The closed form of the field nonlinear transmission $t_{NPR_WP} = u_{out}/u_{in}$ is determined after solving (2.31) to be:

$$t_{NPR_WP} = B_p e^{i\gamma Pz} \left[i \sin\left(\frac{2\gamma Jz}{3} - \theta_2 + \theta_1\right) \sin(-\theta_2 + \theta_1 + 45^\circ - 2\theta_h) + \cos\left(\frac{2\gamma Jz}{3} - \theta_2 + \theta_1\right) \cos(-\theta_2 - \theta_1 + 45^\circ + 2\theta_h) \right] \quad (2.32a)$$

$$J = \frac{1}{2} \sin(2\alpha_2 - 90^\circ) P \quad (2.32b)$$

where $(\theta_1, \theta_h$ and $\theta_2)$ are the rotation angles of QWP₁, HWP and QWP₂, respectively. Equations (2.32a) and (2.32b) are used in Chapter 6 to find the values of the rotation angles of the wave-plates resulting in a stable mode-locked pulse.

2.3 Mode-locked fiber laser regimes

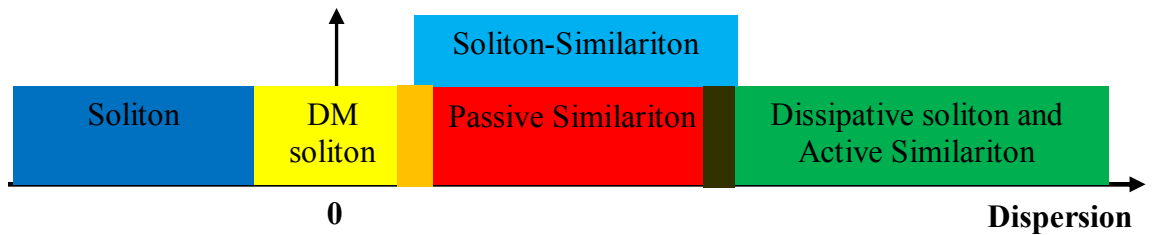


Fig. 2.14 Various regimes of mode-locked fiber laser classified with respect to dispersion. The orange and brown regions are the overlap between regimes.

This section gives a brief description of the main regimes of mode-locked fiber laser operating around 1 μm . The main regimes of mode-locked fiber laser are shown in Fig. 2.14. The soliton regime requires anomalous dispersion for propagation [65-69]. Since standard fiber has a normal dispersion around 1 μm , a dispersion compensation element can be inserted inside the cavity such that the normal dispersion of the fiber is neglected with respect to the anomalous dispersion, and in such a case, solitonic propagation can be achieved [70, 71]. Equation (2.3) is converted to the standard Nonlinear Schrödinger equation (NLSE) if the gain and loss terms are set to zero. An analytical solution exists for pulse propagation in anomalous dispersion medium and it has the form [19]:

$$U(t, z) = U_o \text{sech}\left(\frac{t}{T_o}\right) \exp\left(\frac{iz}{2}\right) \quad (2.33)$$

The pulse energy is related to the pulse width T_o by a well-known relation called the area theorem. It is expressed as [19]:

$$ET_o = \frac{|\beta_2|}{\gamma} \quad (2.34)$$

Because of this area theorem, the pulse energy is limited to picojoules in a cavity using standard SMF. With large mode area (LMA) fiber, higher values of pulse energy have been reported. [71]. However, an all-fiber based soliton regime is not possible in Yb^{3+} -based laser cavities.

For dispersion managed (DM) soliton or stretched pulse laser, a dispersion compensation element is inserted inside the cavity and the net dispersion of the laser cavity is close to zero. As shown in Fig. 2.15, this regime is characterized by two

locations of local minima of the pulse width around the midpoint of the normal and anomalous dispersion media. The frequency chirp flips from positive to negative during the pulse propagation in the normal and anomalous dispersion media, respectively. The output energy is one order of magnitude higher than soliton laser. This regime of mode-locking was first demonstrated by K. Tamura et al. at a central wavelength of 1.55 μm [72, 73]. The laser cavity was mode-locked by NPR. A DM soliton can also be created in a laser cavity mode locked by a SESAM or nonlinear optical loop mirror (NOLM) [74, 75]. However, this wavelength is not suitable for biomedical applications. Wise's research group pioneered the development of DM soliton mode-locking at 1.03 μm [35, 70, 76, 77]. B. Ortaç et al. then showed the possibility of having a DM soliton at 1.06 μm using double cladding LMA fiber [78]. At 1 μm , a pair of diffraction gratings can be used for dispersion compensation [35, 70, 76- 78].

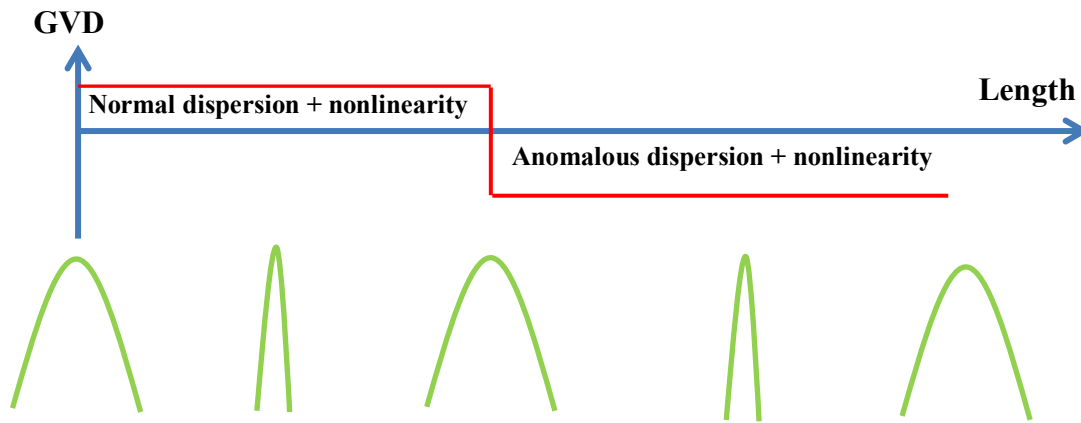


Fig. 2.15 Cartoon of the pulse evolution in a DM soliton showing the type of dispersion in each fiber section [35]. GVD stands for group velocity dispersion.

Passive Similariton pulses have been generated in mode-locked fiber laser cavities at positive net dispersion. The initial idea to generate Similariton pulses, which have

parabolic shape, was reported by D. Anderson et al. They theoretically showed the possibility of having a parabolic pulse with linear frequency chirp in a normal dispersion optical fiber. As a result, pulses can be very effectively compressed at the fiber output close to its transform-limited values [79]. However, it was very difficult to generate such pulses in mode-locked fiber lasers due to the monotonic increase of its temporal and spectral width, which is challenging with the periodic boundary condition of ring lasers. Ilday et al. succeeded in generating a passive Similariton in a mode-locked fiber laser having diffraction gratings inside the cavity for pulse compression. A short length of Yb^{3+} -doped fiber was used to produce sufficient gain and spectral filtering decoupled from any dispersion or nonlinear effect [80]. A passive Similariton operates at a higher value of net dispersion. All cavity parameters contribute in the formation of a passive Similariton pulse. Distinct features differentiate a passive Similariton from a DM soliton. The pulse exhibits one location of absolute minimum pulse width near the end of the anomalous dispersion medium (see Fig. 2.16). The pulse has positive frequency chirp over all the sections of the laser cavity. The absolute value of the anomalous dispersion required to compress the pulse near its transform limit is higher than the absolute value of intra-cavity anomalous dispersion. Another important feature observed in a laser cavity mode-locked by nonlinear polarization evolution (NPR) is that the spectral profiles of the intra-cavity pulse and the pulse at the rejection port of the NPR have almost the same parabolic shape. This means that the SA plays a minor role in the pulse shaping [12, 35, 80- 84]. An environmentally-stable passive Similariton was generated in a Fabry-Perot laser cavity constructed by polarization maintaining fibers (PMFs) and mode-locked by a SESAM [85]. A passive Similariton and a DM soliton can be observed simultaneously at

a certain range of positive net dispersion. Switching between these two regimes is performed by changing the nonlinear transmission coefficient of the SA [86]. However, the former one shows some kind of instability around zero dispersion [87]. Due to its robustness against optical wave breaking, the energy per pulse for a Similariton is higher than that of many other types such as Soliton and Stretched pulse regimes.

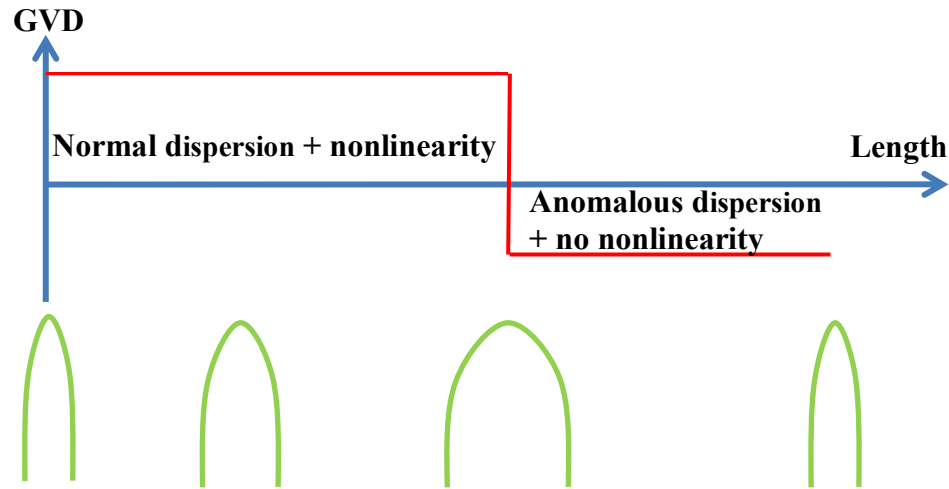


Fig. 2.16 Cartoon of the pulse evolution in a passive Similariton showing the type of dispersion in each section [35].

The bulky nature of the pair of diffraction gratings acting as a dispersion compensation element is a major hurdle to have all-fiber femtosecond laser. Furthermore, the pair of diffraction gratings needs extreme care in light alignment. This problem could be partially solved by replacing this bulky anomalous dispersion element by a PBG fiber. However, this type of laser cavity suffers from a self-starting problem that leads to the insertion of an acousto-optical modulator to initiate mode locking [88]. This problem arises from the Fresnel back reflection at both ends of the PBG fiber and the embedded birefringence of the fiber, which results in an increased effect of polarization mode

dispersion. The Fresnel back reflection problem was solved in [89] at the expense of greater complication of the laser cavity by having a SESAM and NPR to initiate and stabilize the mode-locking.

A crucial step towards developing an all-fiber femtosecond laser is achieved by replacing the dispersion compensation element by a spectral filter. As mentioned in section 2.2.5, the spectral filter plays a critical role in the pulse stabilization as it cuts its temporal edges. This sustains the periodic boundary condition in the laser cavity. J. Buckley et al. investigated the stabilization effect of the spectral filter and how it can assist the mode locking inside the cavity [90]. A. Chong et al. demonstrated a stable all normal dispersion mode-locked fiber laser [91]. The length of the Yb^{3+} -doped fiber and the bandwidth of the spectral filter define the shape of the pulse.

Active Similariton pulses are generated in a mode-locked laser cavity having a long Yb^{3+} -doped fiber and narrow bandwidth of spectral filter (2-4 nm). It is based on the work done by M. E. Fermann et al. They reported the conversion of arbitrary pulses into parabolic pulses with linear frequency chirp after its propagation through a long section of gain medium having normal dispersion [92]. More work was done to achieve parabolic pulses with linear chirp starting with any arbitrary pulse [93- 95]. This regime is different from a passive Similariton in one important point. A passive Similariton has a short length of Yb^{3+} -doped fiber. Therefore, the pulse is not converted to a parabolic pulse inside it. In contrast, the length of Yb^{3+} -doped fiber in an active Similariton regime is much longer, fully converting the pulse into a parabolic shape. A number of mode-locked lasers were constructed based on this idea. The first one is a hybrid laser cavity having a long section of Er^{3+} -doped fiber with normal dispersion and standard SMF with

anomalous dispersion at 1.5 μm . The pulse evolves as an active Similariton in the normal dispersion section, while it evolves as a soliton in the anomalous dispersion section. As the temporal and spectral profiles broaden monotonically in the normal dispersion section, a lumped spectral filter is inserted between the two sections to cut the pulse edges before entering the anomalous dispersion section [96, 97]. At 1 μm , an all-normal dispersion active Similariton was built based on this idea [98]. As shown in Fig. 2.17, the temporal profile of the pulse broadens during its propagation inside the cavity. Also, the spectral profile of the pulse broadens in a similar manner. Therefore, a narrow bandwidth (2-4 nm) lumped spectral filter is required to return the pulse back to its initial profile at the beginning of the Yb^{3+} -doped fiber. The pulse energy is not much higher than what is achieved by a passive Similariton if the cavity is constructed using standard Yb^{3+} -doped fiber. However, it is increased by around one order of magnitude if it is replaced by LMA Yb^{3+} -doped fiber [99, 100].

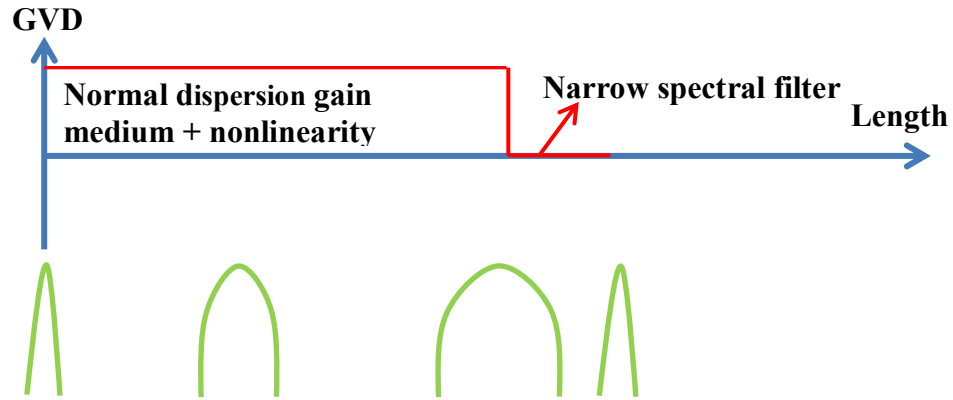


Fig. 2.17 Cartoon of the pulse evolution in an active Similariton showing the type of dispersion in the gain medium [35].

Dissipative soliton pulses are generated in a mode-locked laser cavity having a relatively smaller length of Yb^{3+} -doped fiber and wider bandwidth of the spectral filter.

In contrast to active Similariton, the pulse shape in dissipative soliton pulse depends on the parameters of the laser cavity (see Fig. 2.18). The highest value of pulse energy is reported in the dissipative soliton regime either with standard fiber [13, 101] or LMA fiber [102-109]. In my research, dissipative soliton is also observed in a laser cavity with a dispersion map, but at a high value of net dispersion. This shows some kind of overlap between a passive Similariton and a dissipative soliton. Switching between the two regimes is done by changing the parameters of the saturable absorber. V. G. Bucklew et al. reported that, for small SA nonlinear loss, passive Similariton is generated when the SA has much lower saturation power [110].

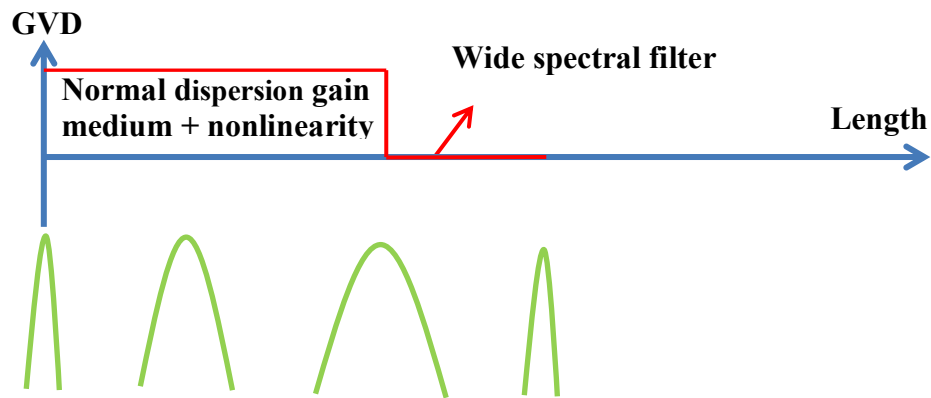


Fig. 2.18 Cartoon of the pulse evolution in a dissipative soliton showing the type of dispersion in the gain medium [35].

Average propagation models can be used in any mode-locked laser where the pulse propagation relies on a complete balance between the dispersion, nonlinearity, spectral filtering and saturable absorption through the whole cavity [111]. They can describe all the regimes of the mode-locked lasers except for modeling active Similariton lasers. This is because for active Similariton lasers, the pulse evolution relies on local

nonlinear attractor in the Yb^{3+} -doped fiber, which forces the pulses to evolve towards a parabolic profile [98, 111].

Average models can be analytical or numerical. In the case of the analytical average model, a close form relation between the temporal and spectral profiles is developed. It cannot show the pulse dynamics inside the cavity. In the case of the numerical average model, the evolution of the pulse amplitude, pulse width and spectral bandwidth through the cavity are studied at the expense of not having closed form relations. More details on these types of average models will be presented in Chapters 3 and 4.

2.4 Conclusion

This chapter summarizes the background of mode-locked fiber laser. The propagation models such the vector GLEs and scalar GLE involved in the numerical simulation of the mode-locked laser cavity are depicted. The average model is also discussed. The main parameters that participate in the temporal and spectral shaping of the pulse, such as fiber dispersion, SPM, gain filtering, dispersion compensation, lumped spectral filter and SA, are discussed in this chapter. The main mode-locking regimes are illustrated. A brief description of the types of the average propagation models applicable to all mode-locking regimes except for active Similariton is provided.

Chapter 3

3 Generalized analytical model for dissipative soliton in all-normal dispersion mode-locked fiber laser¹

In this chapter, an analytical average model is developed and used to study the dependence of the cavity parameters and the small signal absorption, the saturation power and the location of the overdriving point of a lumped SA on the spectral bandwidth of the lumped filter.

The chapter is organized as follows; in section 3.1, a literature review on analytical average models is discussed, our analytical model is developed in section 3.2 and validation of the analytical model is presented in section 3.3. A detailed study on the relationship between the parameter of the SA and the pulse energy, spectral bandwidth and de-chirped pulse width is depicted in section 3.4. Finally, the conclusion is presented in section 3.5.

3.1 Literature review on analytical average models

In this section, I provide a literature survey for analytical average models. A simple analytical average model was initially used, by H. A. Haus et al. to obtain the temporal profile of the pulse in the soliton regime. However, the SA effect was only limited to the cubic term [18, 64, 112]. This model was modified to study mode-locked pulse in DM fiber laser. Time varying nonlinearity and cubic self-amplitude modulation parameters replaced the constant ones to account for the temporal breathing of the pulse [113, 114].

¹ Most of the results in this chapter have been published in [III] and [IV].

N. Akhmediev et al. developed a model for laser cavities with anomalous dispersion (soliton regime). Specific antaz (establishment of a starting equation that solves a mathematical problem taking into consideration some assumptions) was used to get the closed form relation of the pulse temporal profile [26]. The SA transmission coefficient, mentioned in (2.4), was truncated to only its cubic and quantic terms [26, 115]. This model was modified to fit with normal dispersion (dissipative soliton) laser cavities [116, 117]. However, the analytical solution was not stable, except for the case of flat-top pulses or what is called dissipative soliton resonance [116, 117]. Nevertheless, this model was used to describe several pulse regions depending on the transmission coefficient of the SA [118, 119]. The main reason for instability in the above model was that the gain medium was modeled as linear term without gain saturation [119]. All of the above models give a general description of the mode-locked fiber lasers whether the mode-locking is initiated by NPR based-SA or lumped SA. However, they have a common problem; there is no relation between the mathematical expression used and physical parameters of the SA.

H. Leblond et al. addressed this problem for fiber lasers mode-locked by NPR [120]. The vector CGLEs were solved analytically by neglecting the dispersion, the gain filtering and the nonlinearity of the fiber. Then, they were added as perturbation parameters. This analysis resulted on having a scalar GLE model with only cubic term representing the SA. They developed mathematical relations between the physical parameters of the SA (the angles of the polarization controllers) and the cubic term of the SA [120]. They also explored the dependence of the pulse characteristics on the angles of the polarization controllers. This model investigated the effect of the wave plates on the

stability of the mode-locked fiber lasers in the dissipative soliton, soliton, and dispersion-managed soliton (stretched pulse) regimes [120-122]. It included the effects of GVD, linear gain, linear birefringence, and optical Kerr nonlinearity. For higher energetic pulses, the quintic saturation term has been added to express the saturation of the SA forming the CQCGLE model [123]. The main problems in the models discussed above [120- 123] in describing fiber laser mode-locked by NPR are the following; 1- The gain saturation, which is important for pulse stabilization, is not included. 2-The nonlinear transmission of the SA based on NPR is truncated to either the cubic or the quintic terms, which is not an acceptable approximation for high energy pulses [63]. 3-The effect of the bandwidth of the lumped spectral filter is not included. 4-The cavity is assumed to have only a gain fiber, which is not the case for realistic all fiber laser cavities. They always have a long piece of SMF between the output coupler and the input of the gain fiber.

V. L. Kalashnikov et al. developed another model for fiber lasers mode-locked by lumped SA. This model did not assume any prior antaz for the pulse temporal profile. The SA parameters are not truncated to the cubic and quintic terms. This model can describe the mode-locked pulse propagating in normal dispersion medium, which is the case for dissipative soliton laser. The gain saturation of the gain medium was inserted into the model [124]. Closed-form relations for the temporal and spectral profile of the pulse were shown with the border between stable and unstable pulse regions. However, the transmission coefficient of the SA was modeled as an ideal response without any overdriving point, which results mainly from the two-photon absorption inside the medium of the SA [16].

Based on the above illustration, to the best of our knowledge, average analytical models for fiber lasers mode-locked by lumped SA do not include relations between the average and physical parameters of SA. Also, the models describing pulse propagation in normal dispersion medium do not include the effect of the lumped spectral filter bandwidth.

Our analytical model solves all of the above problems. It includes the overdriving of the transmission coefficient of the lumped SA in the average GLE. It provides a mathematical relation between the physical parameters of the SA and the average ones. It also averages the bandwidth of the lumped spectral filter through the cavity length. As it will be shown below, all these parameters have significant impact on the temporal and spectral profiles of the mode-locked pulse.

3.2 Our generalized average analytical model

An average model describing the pulse evolution inside the cavity is developed to provide a closed-form relation between the pulse energy, de-chirped pulse width, spectral power density (SPD), and spectral chirp. This model is applicable under two assumptions; the first is that the effects of all components inside the cavity are averaged over its length. This means the pulse experiences a minor change by each component through each round trip [26, 115-117]. The other assumption is that no local nonlinear attractor exists in the laser cavity, which influences the pulse solution [98]. These assumptions are valid for dissipative soliton pulse in a normal dispersion laser cavity.

In this chapter, it is assumed that the laser cavity is mode-locked by a lumped SA. The instantaneous transmission transfer function of an SA is used in this model, where

the SA relaxation time is lower than the pulse width. The complex field amplitude $U(z,T)$ is determined by solving the average GLE written as:

$$\frac{\partial U}{\partial z} = \sigma U + \left(\alpha - \frac{i\beta_2}{2} \right) \frac{\partial^2 U}{\partial t^2} + \left(\frac{a_1 b_1}{1+|U|^2 b_1} - c_1 + i\gamma \right) |U|^2 U \quad (3.1)$$

where σ is the net saturated gain coefficient; it has the form of $\sigma = g_{av}(E)/2 - a_1 - l$; here $g_{av}(E)$ is the average saturated gain coefficient along the cavity length; it can be written as $\frac{g_{o_av}}{2} / \left(1 + E/E_{sat_av} \right)$; the value of g_{o_av} is equal to g_o (small signal power gain coefficient of Yb^{3+} -doped fiber) multiplied by ratio between Yb^{3+} length (L_{Yb}) and the total cavity length (L). E_{sat_av} is the average value of the saturation energy; its mathematical formula is shown in this section (see (3.27)). a_l is the small signal saturable loss coefficient, which is related to the SA modulation depth. l includes the power splitter and filter insertion loss. b_l represents the inverse of the saturation power of the SA. The effect of overdriving the SA is represented by inserting the coefficient c_l . To consider all sources of spectral filtering effects, the term α is defined as:

$$\alpha = (g_{av}(E)/2)/\Omega_g^2 + 1/L\Omega_f^2 \quad (3.2)$$

Here the first part of α ($(g_{av}(E)/2)/\Omega_g^2$) is related to the gain bandwidth of the Yb^{3+} -doped fiber defined in (2.3). I add the other part of α to average the effect of the lumped spectral filter along the whole length of the cavity. Since the gain filtering of the Yb^{3+} -doped fiber is modeled to have a parabolic spectral profile [20], the average filtering effect of the lumped spectral filter is assumed to have same profile. Ω_f is the angular frequency at which the average filter transmission is null. It is related to lumped spectral filter bandwidth (BW) by:

$$BW = \sqrt{2}\Omega_f/2\pi \quad (3.3)$$

The other parameters are described in sub-section (2.1.1).

Without any restrictions on the pulse phase $\phi(t)$, the pulse profile as a possible solution of (3.1) is [124]:

$$U(z, t) = \sqrt{P(t)} \exp(i\phi(t) + iqz) \quad (3.4)$$

where q accounts for the slip of the carrier phase with respect to the pulse envelope. Since the temporal width (T) of the dissipative soliton pulse, propagating inside the cavity, is in the order of picoseconds [13, 106, 109, 125], the dispersion length defined in (2.5a) is much longer than the cavity length; (i.e. $\beta_2/T^2 \ll 1$). Consequently this leads to $\lim_{\beta_2/T^2 \rightarrow 0} d^2\sqrt{P}/dt^2 = 0$ [110]. Employing this approximation, and substituting (3.4) into (3.1), one can decompose the resulting equation into imaginary and real parts:

$$\frac{\Omega^2 \beta_2}{2} (1 - \varepsilon) + \gamma P = q \quad (3.5a)$$

$$\sigma\sqrt{P} - \sqrt{P}\alpha\Omega^2 - \frac{\beta_2}{2}\sqrt{P}\frac{d\Omega}{dt} - \frac{\beta_2}{2\sqrt{P}}\frac{dP}{dt}\Omega + \left(\frac{a_1 b_1}{1 + P b_1} - c_1\right)P\sqrt{P} = 0 \quad (3.5b)$$

Here $\Omega = -d\phi/dt$ represents the instantaneous frequency of the circulating pulse inside the cavity and. In all-normal dispersion fiber laser regime, the GVD effect (β_2) dominates the spectral filtering effect (α) [124]. The relationship between β_2 and α is included in ε , which is given as:

$$\varepsilon = \frac{2\alpha}{\Omega^2 \beta_2} \left[\frac{d\Omega}{dt} + \frac{\Omega}{P} \frac{dP}{dt} \right] \quad (3.6)$$

The inequality $\varepsilon \ll 1$ is used as a second approximation to simplify (3.5a). Based on the above approximations, the pulse power is given as:

$$P(t) = \frac{\beta_2}{2\gamma} (\Delta^2 - \Omega(t)^2) \quad (3.7)$$

where $\Delta^2 = 2q/\beta_2$. This implies that the SPD of the optical pulse is truncated at $\pm\Delta$, and that the optical bandwidth is 2Δ . Using (3.7) in (3.5b) results in the following ordinary differential equation for Ω :

$$\frac{\beta_2}{2} \frac{d\Omega}{dt} = \frac{(\Delta^2 - \Omega^2) \left[\left(\sigma + a_1 - \frac{c_1}{b_1} \right) (\Delta^2 - \Omega^2) + \frac{2\sigma\gamma}{b_1\beta_2} - \alpha\Omega^2 \left(\Delta^2 - \Omega^2 + \frac{2\gamma}{\beta_2 b_1} \right) - \frac{c_1\beta_2}{\gamma} (\Delta^2 - \Omega^2) \right]}{(\Delta^2 - 3\Omega^2) \left(\Delta^2 - \Omega^2 + \frac{2\gamma}{\beta_2 b_1} \right)} \quad (3.8)$$

The temporal and spectral profiles must not have any singularity. Therefore, the poles in (3.8) have to be removed. This imposes what is called the regularity condition; $d\Omega/dt < \infty$ [124]. Equation (3.8) has two poles; one of them is at $\Delta^2 + \frac{2\gamma}{\beta_2 b_1}$, and the other one is at $\Delta^2/3$. The first pole is located outside the SPD of the pulse, therefore it is discarded. However, it is not the case for the latter one. This pole can be removed by dividing the square bracket of (3.8) by $(\Delta^2 - 3\Omega^2)$. The remainder of the division should equal zero to get rid of this pole. It is given as:

$$-\frac{2}{9} \left[\alpha + \frac{c_1\beta_2}{\gamma} \right] \Delta^4 + \frac{1}{3} \left[2 \left(\sigma + a_1 - \frac{c_1}{b_1} \right) - \frac{2\gamma\alpha}{\beta_2 b_1} \right] \Delta^2 + \frac{2\gamma\alpha}{\beta_2 b_1} = 0 \quad (3.9)$$

By solving (3.9), an expression of Δ^2 is given as:

$$\Delta^2 = a_1 \left[\frac{[2(x+1-\zeta)-y] \pm \sqrt{(y-2)^2 + 4x(x+2+y) + 4\zeta(y+\zeta+2(x-1))}}{\frac{4}{3}\alpha \left(1 + \frac{2\zeta}{y} \right)} \right] \quad (3.10)$$

where $x = \sigma/a_1$ is the normalized net saturation gain coefficient. The normalized spectral filtering term is defined as:

$$y = 2\alpha\gamma/\beta_2 a_1 b_1 \quad (3.11)$$

$\zeta = c_1/a_1 b_1$ represents the normalized overdriving term of the SA. To have a real spectral bandwidth 2Δ , the normalized spectral filtering term y has to satisfy the following relation:

$$y \leq 2(1 - x - \zeta) - 4\sqrt{-x} \quad (3.12)$$

For y to be real, x and ζ are restricted to certain ranges. x is restricted to values less than zero. This means that the net saturated gain should not be greater than zero, which means that the noise background is not energetic to generate other pulses [119, 124]. ζ should be less than one to have a SA with a positive slope for its transmission coefficient around zero input power.

It is worth mentioning that (3.10) has two solutions, called the positive and the negative branches. This corresponds to the positive and negative signs, respectively. However, only the positive branch exists at the stability threshold ($x=0$) [124].

Equation (3.8) is modified after employing the regularity condition ($d\Omega/dt < \infty$).

It is expressed as:

$$\frac{d\Omega}{dt} = \frac{2\alpha\left(1-\frac{\zeta}{y}\right)(\Delta^2-\Omega^2)[S^2-\Omega^2]}{3\beta_2\left(\Delta^2-\Omega^2+\frac{2\gamma}{\beta_2 b_1}\right)} \quad (3.13)$$

where S is given by:

$$S^2 = \frac{1}{\alpha\left(1-\frac{\zeta}{y}\right)} \left[a_1(x + 1 - \zeta) + \frac{2\alpha}{3} \left[1 - \frac{5\zeta}{2y} \right] \Delta^2 + y a_1 \right] \quad (3.14)$$

Equation (3.13) can be solved analytically to give the implicit relation of the instantaneous frequency (Ω) and pulse profile (using (3.7)). It is written as:

$$t = \frac{3}{\alpha(1-\frac{\xi}{y})b_1S\Delta(S^2-\Delta^2)} \left[S \gamma \tanh^{-1} \left(\frac{\Omega}{\Delta} \right) + \left[(S^2 - \Delta^2)b_1 \frac{\beta_2}{2} - \gamma \right] \Delta \tanh^{-1} \left(\frac{\Omega}{S} \right) \right] \quad (3.15)$$

The full width at half maximum (FWHM) of this chirped pulse (called the chirped pulse width in short) is expressed as:

$$T_{FWHM} = \frac{6}{\alpha(1-\frac{\xi}{y})b_1S\Delta(S^2-\Delta^2)} \left[S \gamma \tanh^{-1} \left(\frac{1}{\sqrt{2}} \right) + \left[(S^2 - \Delta^2)b_1 \frac{\beta_2}{2} - \gamma \right] \Delta \tanh^{-1} \left(\frac{\Delta}{\sqrt{2}S} \right) \right] \quad (3.16)$$

The stationary phase method explained in Appendix A is applied to (3.7) to have an expression of SPD of the pulse [79, 124], as follows:

$$P(\omega) = \frac{3\pi\beta_2^2}{2\alpha\gamma} \frac{1}{(1-\frac{\xi}{y})} \left[\frac{\Delta^2 + \frac{2\gamma}{b_1\beta_2} - \omega^2}{S^2 - \omega^2} \right] H(\Delta^2 - \omega^2) \quad (3.17)$$

Here $H(\Delta^2 - \omega^2)$ is the Heaviside function. An expression of the pulse energy is obtained by applying Parseval's theorem ($E = \frac{1}{2\pi} \int_{-\Delta}^{\Delta} P(\omega) d\omega$) [126] on (3.17). It is given as:

$$E = \frac{3\pi\beta_2^2}{2\alpha\gamma} \frac{\Delta}{(1-\frac{\xi}{y})} \left[1 - \frac{1}{\Delta S} \left(S^2 - \Delta^2 - \frac{2\gamma}{b_1\beta_2} \right) \tanh^{-1} \left(\frac{\Delta}{S} \right) \right] \quad (3.18)$$

The spectral phase of the pulse ψ is defined as [127]:

$$\psi = -\phi(t^*(\omega)) + \omega t^*(\omega) \quad (3.19)$$

Since the stationary phase method, explained in Appendix A, implies that $\Omega(t_o) = -\omega$ (A.3), $t^*(\omega)$ is the same as (3.15) with the replacement of Ω by $-\omega$. Therefore, a closed form relation for the spectral phase (ψ) and its second derivative (Q) is given as:

$$\psi = \frac{3}{\alpha(1-\frac{\zeta}{y})b_1S\Delta(S^2-\Delta^2)} \left[S\gamma \left(\omega \tanh^{-1} \left(\frac{\omega}{\Delta} \right) + \frac{\Delta}{2} \ln(\Delta^2 - \omega^2) \right) + \left[(S^2 - \Delta^2)b_1 \frac{\beta_2}{2} - \right. \right. \\ \left. \left. \gamma \right] \Delta \left(\omega \tanh^{-1} \left(\frac{\omega}{S} \right) + \frac{S}{2} \ln(S^2 - \omega^2) \right) \right] H(\Delta^2 - \omega^2) \quad (3.20a)$$

$$Q = \frac{d^2\psi}{d\omega^2} = \frac{3\beta_2(\Delta^2 - \omega^2 + \frac{2\gamma}{\beta_2 b_1})}{2\alpha(1-\frac{\zeta}{y})(\Delta^2 - \omega^2)[S^2 - \omega^2]} \quad (3.20b)$$

The value of anomalous dispersion (Dis_{an}) required to compress the pulse near its transform limit should be exactly equal to the magnitude of Q in (3.20b) around ω equal to zero, but with the opposite sign ($\psi_{an} = -\frac{1}{2}Dis_{an}\omega^2$). It is expressed as:

$$Dis_{an} = \frac{3}{2} \frac{\beta_2(\Delta^2 + \frac{2\gamma}{b_1\beta_2})}{\alpha(1-\frac{\zeta}{y})\Delta^2 S^2} \quad (3.21)$$

It is clear that the pulse characteristics mainly depend on three parameters x , y and ζ . This is a general form of that presented in [124] by introducing the two-photon absorption term (c_l , representing the overdriving point) in the SA transmission coefficient. The dependence of the pulse characteristics on c_l is illustrated in section 3.4.

For a positive and bounded energy, y should be less than ζ (see (3.17) or (3.18)). Substituting the lower bound of y into (3.12), one gets $x > -\frac{1}{2}(2 + 3\zeta) + \sqrt{6\zeta}$, which is identical to the expression obtained in [124] ($x > -1$) at ζ equals zero.

At $x=0$, the saturated gain $g_{av}(E)/2$ is equal to the total losses $l + a_1$ and the value of α in (3.2) becomes:

$$\alpha_o = (l + a_1)/\Omega_g^2 + 1/L\Omega_{f_min}^2 \quad (3.22)$$

where Ω_{f_min} is the minimum value of Ω_f (3.3). The value of the lumped spectral filter bandwidth (3.3) at the border of a stable pulse can be rewritten as:

$$BW_{min} = \sqrt{2} \Omega_{f_min} / 2\pi \quad (3.23)$$

BW_{min} is obtained after substituting α_o (3.22) in (3.11) and (3.23). It is expressed as:

$$BW_{min} = \frac{\sqrt{2}}{2\pi} \frac{1}{\sqrt{L \left[\frac{y\beta_2 a_1 b_1}{2\gamma} - \frac{(l+a_1)}{\Omega_g^2} \right]}} \quad (3.24)$$

The value of y is bounded between ζ and $2(1-\zeta)$. This maximum value of y is given by substituting $x=0$ in (3.12). It corresponds to the maximum value of α_o (3.11), which is located at the boundary between the lossy and stable regions. Hence by substituting the upper bound of y into (3.24), the minimum value of spectral filter bandwidth at this location is expressed as:

$$BW_{th} = \frac{\sqrt{2}}{2\pi} \frac{1}{\sqrt{L \left[\frac{(1-\zeta)\beta_2 a_1 b_1}{\gamma} - \frac{(l+a_1)}{\Omega_g^2} \right]}} \quad (3.25)$$

The value of σ around zero is expressed as [124]:

$$\sigma(E) = \delta (E - E^*) \quad (3.26)$$

Here E^* is the threshold pulse energy at x (or σ) =0, and δ is $d\sigma/dE|_{E=E^*}$. The value of E_{sat_av} can be given in terms of E^* as:

$$E_{sat_av} = \frac{E^*(l+a_1)}{(g_{o_av}/2)-l-a_1} \quad (3.27)$$

Next, the connection between the average and lumped parameters of the SA is derived to obtain a full description of the laser cavity in the analytical model.

3.2.1 Relation between the average and physical parameters of a lumped SA

The lumped SESAM can be modeled as an instantaneous intensity dependence medium; its reflectivity (R_{ins}) is given in (2.20). The transmission transfer function of the average SA is expressed as [110]:

$$T_{SA} = \exp(-2a_1L)\exp\left(2\left[\frac{a_1b_1P}{1+b_1P} - c_1P\right]L\right) \quad (3.28)$$

It should have the same transmission profile as the equation of SA defined in (2.20). The values of the small signal saturable loss coefficient (a_l), the inverse of the saturation power of the SA (b_l) and its overdriving point (c_l) are calculated so that the two curves coincide at three key points; at $P = 0$, $P = P_{sat}$, and the location of the overdriving point. This results in the following:

$$a_1 = -\frac{1}{2L}\ln(1 - q_o - L_{ns}) \quad (3.29a)$$

$$kP_p^2P_{sat}b_1^3 + [k(P_{p_sa}^2 + 2P_{p_sa}P_{sat}) - (P_{p_sa}^2 + P_{sat}^2)]b_1^2 + [(2P_{p_sa} + P_{sat})(k - 1)]b_1 + (k - 1) = 0 \quad (3.29b)$$

$$c_1 = \frac{a_1b_1}{(1+b_1P_{p_sa})^2} \quad (3.29c)$$

Here $k = \ln(1 - q_o/2 - P_{sat}/P_{TPA} - L_{ns})/\ln(1 - q_o - L_{ns})$, and $P_{p_sa} = P_{sat}[\sqrt{q_oP_{TPA}/P_{sat}} - 1]$. The numerical values of the parameters, except for P_{TPA} , of the SESAM are taken from the datasheet of SESAM-1040-30-500fs-FC-HI1060 purchased from Batop. The values of q_o , F_s , T_{sa} and L_{ns} are given as: 0.3, 120 $\mu\text{J}/\text{cm}^2$, 500 fs and 0.1, respectively. The SESAM is mounted on a ferrule a SMF connector. The mode field diameter of the light inside the SMF (HI1060) is 6 μm . Therefore, the value of A_{eff} is calculated to be 28.27 μm^2 . The value of P_{TPA} is not given in the datasheet of the SESAM. Therefore, it is assumed to be 5 KW, which is close to the value used in [16]. These values are substituted in (3.29a)-(3.29c) to get the values of a_l , b_l and c_l and are

found to be to be 0.043 m^{-1} , 0.012 W^{-1} , and $3.18 \times 10^{-5} \text{ m}^{-1} \text{ W}^{-1}$, respectively. These values will be used below to validate the accuracy of the analytical model.

Fig. 3.1(a) shows a plot of the reflectivity of the SESAM compared to the transmission transfer function of the SA calculated by the analytical model. There is a good agreement between the two curves that shows the ability of the above method to describe the SA transmission profile.

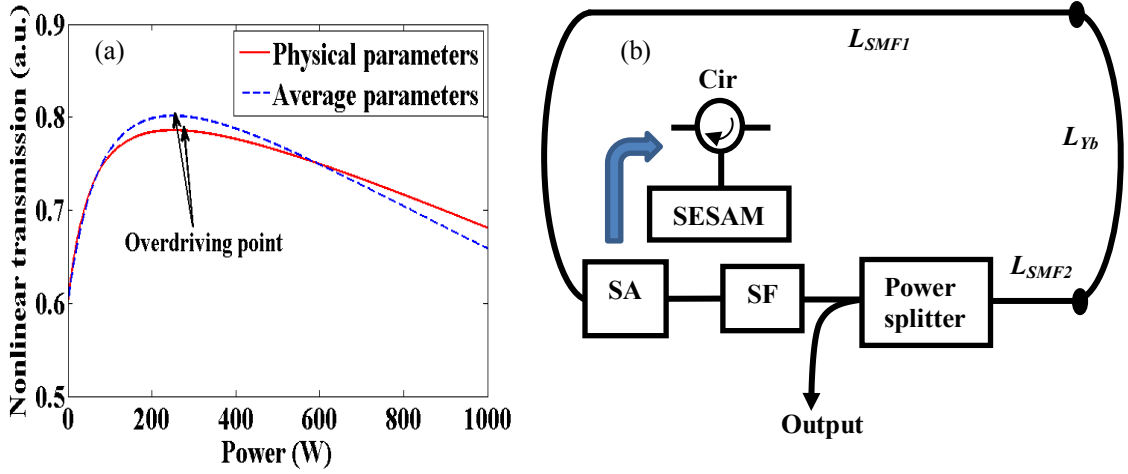


Fig. 3.1 (a) Plot of the reflectivity of the SESAM (solid line) and transmission coefficient of the SA coming from the analytical model (dotted line) versus the input power. (b) Schematic diagram of the laser cavity used in the simulation and experimental work: SF-spectral filter, Cir-three ports circulator and SESAM-semiconductor saturable absorber mirror.

3.3 Validation of the analytical model

As mentioned in section 2.3, a dissipative soliton pulse evolves in an all normal dispersion laser cavity. Therefore, all parameters of the laser cavity shown in Fig. 2.2 exist in Fig. 3.1(b) except for the dispersion compensation. To check the validity of the analytical model, the fiber sections in the laser cavity shown in Fig. 3.1(b) are

numerically simulated using the techniques illustrated in subsection 2.1.1. Equation (2.20) is used to model the nonlinear transfer function of the SESAM. In addition, a laser cavity similar to the one shown in Fig. 3.1(b) is experimentally implemented to validate our analysis. The length of each fiber section is selected to be close to the nominal values found in the literature [13, 101]. The length of each section is selected as follows; the Yb^{3+} -doped fiber has a length (L_{Yb}) of 0.45 m. The SMF (L_{SMF1}) preceding L_{Yb} has a length of 5 m. The SMF (L_{SMF2}) has a length of 0.5 m. A power splitter is inserted after L_{SMF2} , which couples 70% of the input light outside the cavity. The spectral filter is modeled as a Gaussian filter with an insertion loss of 2 dB. A three-port circulator (Cir) is also inserted inside the cavity to force unidirectional propagation and to couple the light reflected from the SESAM into the ring cavity.

3.3.1 Comparison between the simulation and analytical results

For the simulation model, the filter bandwidth at the threshold between the loss and stable single pulse region is 6 nm (see Fig. 3.2(a)). This corresponds to the minimum value of a small signal power gain coefficient ($g_0=21$ dB/m). The same parameter was calculated to be equal to 6.7 nm using the analytical model (3.25). This shows the capability of the analytical model to give a good estimation of the spectral filter bandwidth.

Fig. 3.2(b) shows the plot of the chirped pulse widths for both the simulation and analytical models. The two curves qualitatively agree in behavior. In both cases, two regimes are observed. In the first regime, the chirped pulse width decreases with BW_{min} till it reaches 7.4 ps ($g_0=25$ dB/m) in the simulation model, and 6.5 ps in the analytical model. The corresponding values of BW_{min} are 7.3 nm and 8.6 nm in the simulation and

the analytical models, respectively. As observed in Fig. 3.2(d), the pulse energy increases monotonically with BW_{min} . Therefore, the relationship between the pulse energy and the chirped pulse width follows the solitonic pulse dynamics [18, 114]. Also, this behavior is noticed at certain parameters of the laser cavity [118]. In the second regime, the chirped pulse width increases with BW_{min} ; this is one of the main characteristics of a dissipative soliton pulse in a normal dispersion region [118]. The spectral filter acts as a temporal window for this chirped pulse. The spectral cutting of the edges of the pulse through the spectral filter is mapped as temporal cutting of pulse edges. It is well known that any system tries to exist with the minimum permissible losses. Accordingly, if the spectral filter bandwidth is small, the chirped pulse will adapt itself to propagate inside the cavity with a narrow pulse width to minimize the losses resulting from the filter spectral and temporal cutting. To have a high energetic single pulse with a wide SPD, a balance between the spectral filter bandwidth and the chirped pulse width has to be achieved [128].

While the simulation model does not show stable pulses for $g_o > 120$ dB/m (Fig. 3.2(a)) due to the excessive nonlinear phase shift, the analytical model shows stable pulses at higher values. This is due to the nature of the two models. In the simulation model, the periodic boundary condition must be satisfied to have stable pulses. However, in the analytical model, (3.1) is solved without applying any boundary conditions.

The variation of anomalous dispersion, required to compress the pulse near its transform limit, as a function of BW_{min} is plotted in Fig. 3.2(b) for both models. A linear dispersion delay line is used to perform the pulse compression in the simulation model while (3.21) was used for the analytical model. There is a good agreement in the

anomalous dispersion between the two models. In both models, the absolute values of the anomalous dispersion sharply decrease in the soliton region and are constant ($\sim 0.65 \text{ ps}^2$) in the dissipative soliton region. In the dissipative soliton region, the increase of the pulse width is accompanied by adding new frequency components at the pulse edges. This does not significantly affect the value of the second derivative of the spectral phase with respect to the angular frequency calculated at the center of the pulse ($\frac{d^2\psi(\omega)}{d\omega^2} \big|_{\omega=0}$).

The bandwidth of SPD of the chirped pulses versus BW_{min} is plotted for the simulation and analytical models in Fig. 3.2(c). For the former one, it is calculated at the power splitter output, while for the latter one, it is equal to 2Δ in (3.10). The two curves have the same qualitative variation with BW_{min} . Both curves have a monotonic increase of the spectral bandwidth of the chirped pulse, and they tend to saturate at larger values of BW_{min} . However, there is a difference between the pulse spectral bandwidth in the simulation and analytical models. This difference results from the offset between the values of BW_{min} extracted from the simulation model compared to the ones calculated from the analytical model.

The de-chirped pulse width versus BW_{min} , for both models is plotted in Fig. 3.2(c), using the dispersion delay line for the simulation model and (3.17), (3.20(a)) and (3.21) for the analytical model. Both models show good qualitative agreement. The de-chirped pulse width tends to saturate at high values of BW_{min} in the dissipative soliton regime. This saturation is due to two factors; the stationary spectral chirp (Fig. 3.2(b)) and the saturation tendency of the spectral bandwidth of the chirped pulse at high values of BW_{min} (g_0) (Fig. 3.2(c)).

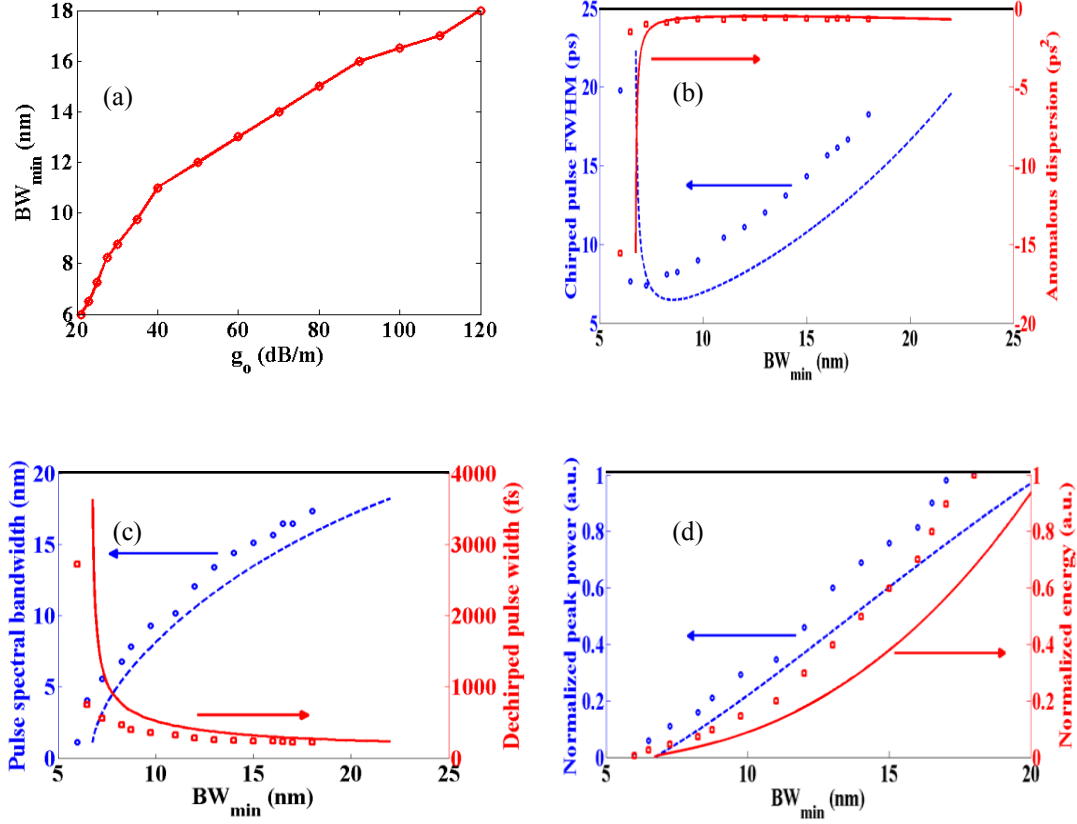


Fig. 3.2 (a) The simulation results for the change of BW_{min} with g_0 . The plot of (b) the chirped pulse width, and the anomalous dispersion of the dispersion delay line, (c) the spectral bandwidth of the chirped pulse, and the de-chirped pulse, (d) Normalized pulse energy, and the normalized peak power versus BW_{min} . For the left blue y-axis, simulation: circles and analytical: dashed line, while for the right blue y-axis, simulation: squares and analytical: solid line.

The energy and peak power of the chirped pulse are normalized to their respective maximum values for the simulation model (at $BW_{min} = 18$ nm). For the analytical model, energy and peak power are normalized to the values at $BW_{min} = 20.4$ nm due to the difference in BW_{min} in both two models. Both models are plotted in Fig 3.2(d). Both energy and peak power increase as the bandwidth increases. Nevertheless, the rate of increase of energy is higher than that of the peak power due to the increase of the chirped pulse width with BW_{min} (Fig. 3.2(b)).

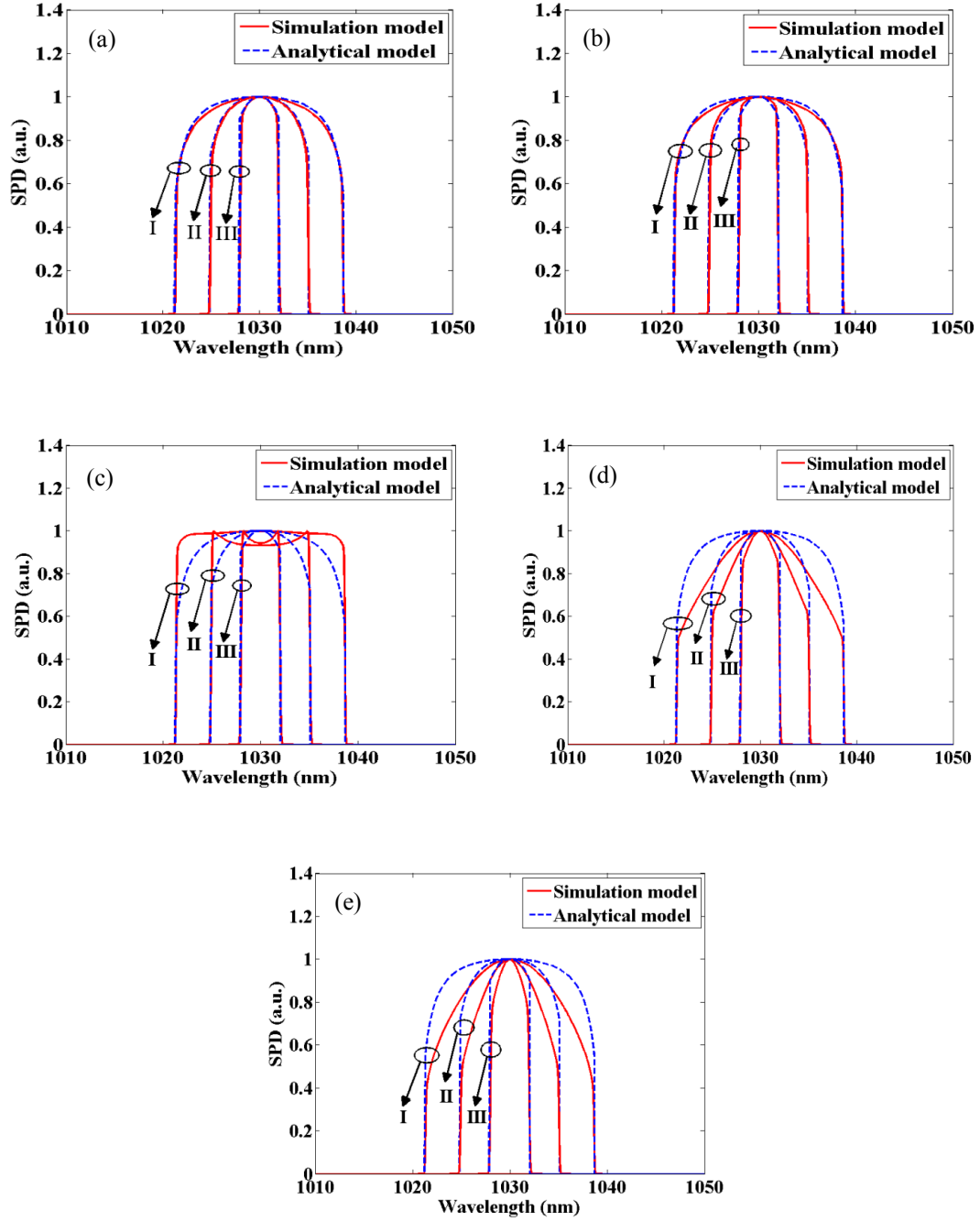


Fig. 3.3 Normalized SPD of the chirped pulse at the end of (a) SMF₁, (b) Yb⁺³ fiber, (c) SMF₂, (d) spectral filter, and (e) SAM. For the simulation results, I, II, and III represent $BW_{min}=18, 11$, and 6.5 nm, which corresponds to $g_o=120, 40$, and 23 dB/m, respectively. The normalized SPD using (3.15a) in the analytical model is plotted for the sake of comparison, the bandwidths of the spectral filter are $20.43, 11.63$, and 7.59 nm for I, II, and III, respectively.

The SPD profiles of the chirped pulse at the end of each element of the cavity at different values of BW_{min} (I, II and III) are plotted in Fig. 3.3 for both the simulation and analytical model. To provide a fair comparison, the shift in values of BW_{min} is taken into consideration. In the simulation model, BW_{min} equals 6.5 and 11 nm (I and II), the SPD has a parabolic top everywhere except at the end of L_{SMF2} , which has an M-like shaped profile. However at $BW_{min} = 18$ nm (III), the SPD has a flat top at the end of L_{SMF2} and a parabolic top elsewhere. For the analytical model, the SPD of the chirped pulse always has a parabolic top at any value of the spectral filter bandwidth [124]. This is because the analytical model is an average model that gives the average profile of the SPD in the simulation model.

The pulse energy is plotted as a function of the spectral filter bandwidth (BW) at various values of the normalized net saturation gain coefficient (x) in Fig. 3.4(a). First, I explored the impact of fixing the BW on the characteristics of the generated optical pulse. It is set at 11 nm and 11.6 nm in the simulation and the analytical models, respectively. As shown in Fig. 3.4(a), two values of the pulse energy exist for each value of BW . The first is the negative branch (lower pulse energy) and the other is a positive branch (higher pulse energy). The minimum value of x is found to be -0.146 at which the energy-vs- BW curve is tangent to the line of fixed BW . There is a good agreement in the main pulse characteristics between the two models. The chirped pulse width monotonically decreases with the normalized energy in Fig. 3.4(b) for both models. The change of the spectral bandwidth of the pulse with the normalized energy is shown in Fig. 3.4(c). The monotonic increase in the spectral bandwidth is due to SPM. The chirped pulse is either compressed by the dispersion delay line in the simulation model or through (3.17),

(3.20(a)) and (3.21) in the analytical model. A comparison between the de-chirped pulse widths in the two models is shown in Fig. 3.4(c). The monotonic decrease in the de-chirped pulse width is mainly due to the increase in the spectral bandwidth of the pulse.

Next, I explored the effect of fixing the small signal gain coefficient (g_o) on the pulse characteristics. It is set to 30 dB/m for the simulation model, which corresponds to g_{o_av} ($g_o L_{Yb}/L=2.27$ dB/m) for the analytical model. It is observed that in the simulation model, the pulse energy at the power splitter output is slightly increased with the increase of BW . Given the values of g_{o_av} , a_l , l , and E^* , equation (3.27) is used to get the value of E_{sat_av} to be 4.93 nJ. The value of x is calculated for each energy value using the formula: $x = \left(\frac{g_{o_av}}{2} / \left(1 + E/E_{sat_av} \right) - l - a_1 \right) / a_1$. For simplicity the spectral filtering loss is neglected in the total loss calculation. The curve represents a constant value of g_o in the pulse energy-vs- BW diagram is shown in Fig. 3.4(a). It intersects the positive and negative branches at low and high values of BW , respectively. The value of x at the border between the positive and negative branches is -0.186. Although there is a shift between the simulation and the analytical results (Figures 3.4(d)-3.4(f)), a qualitative agreement is observed between both curves. The offset between the values of BW_{min} extracted from the simulation and the analytical models is the source for this numerical discrepancy.

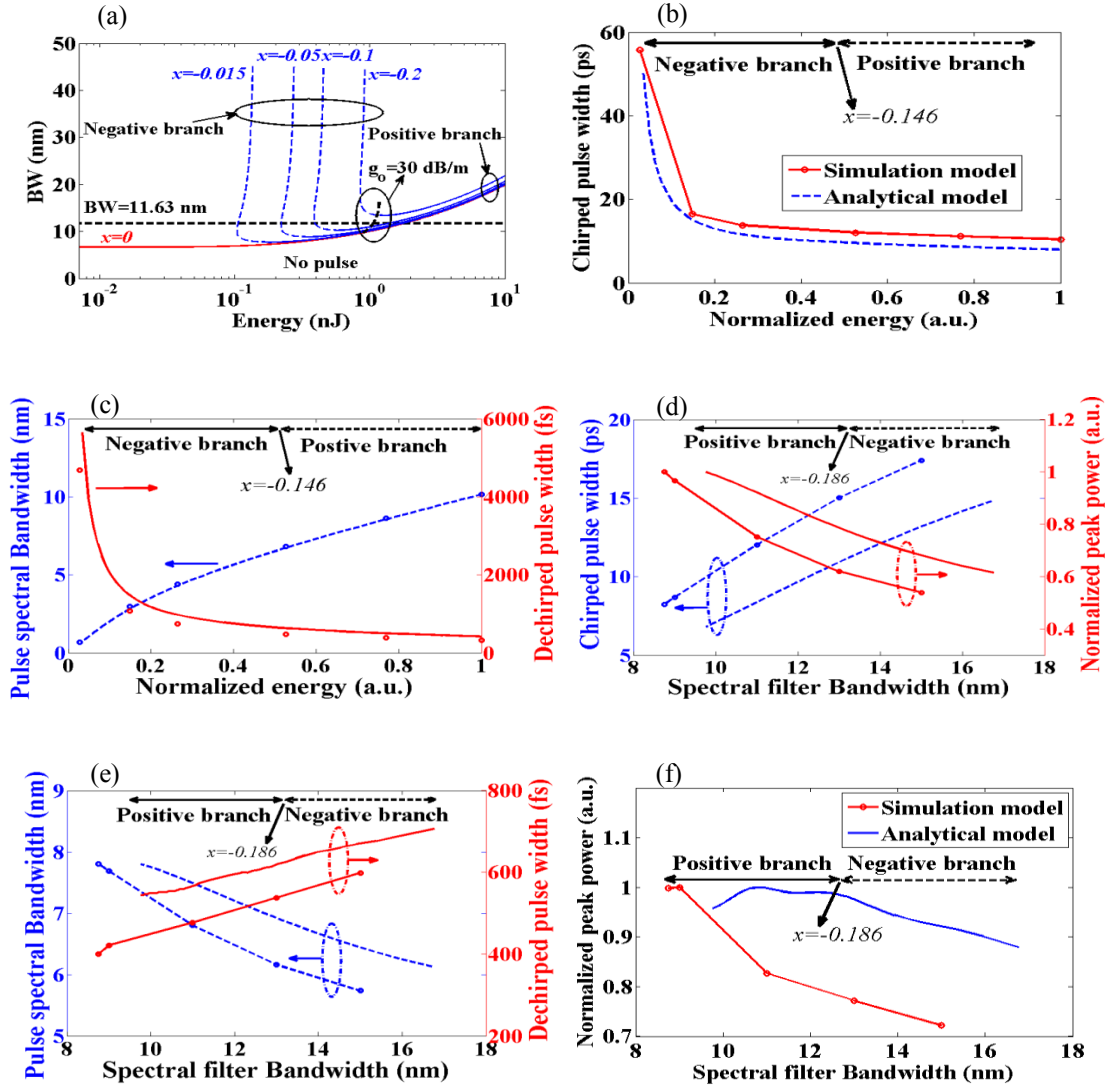


Fig. 3.4 Plot of (a) bandwidth of the spectral filter (BW) versus pulse energy at various values of x showing the constant spectral filter bandwidth, and the constant gain lines. For constant spectral filter bandwidth, the plot of (b) the chirped pulse width, (c) the spectral bandwidth of the chirped pulse, and the de-chirped pulse width versus the normalized value of energy. For the left blue y-axis, simulation: circles and analytical: dashed line, while for the right red y-axis, simulation: squares and analytical: solid line. For $g_0=30$ dB/m, The plot of (d) the FWHM of the chirped pulse, the normalized peak power of the chirped pulse, (e) the spectral bandwidth of the chirped pulse, the FWHM of the de-chirped pulse, and (f) the normalized peak power of the de-chirped pulse versus the spectral filter bandwidth. For the left blue y-axis, simulation: dashed line with circles and analytical: dashed line, while for the right red y-axis, simulation: solid line with squares and analytical: solid line.

The chirped and de-chirped pulse widths are plotted in Figures 3.4(d) and 3.4(e), showing a monotonic increase with larger values of BW . In contrast, the spectral bandwidth and the peak power of the chirped pulse decrease with larger values of BW (Figures 3.4(d) and 3.4(e)). Fig. 3.4(f) depicts the decrease of the peak power of the de-chirped pulse with the increase of the SF bandwidth.

The analytical model demonstrates two well-known effects on the pulse characteristics shown in previous simulation models [24, 101, 129]: 1-increasing g_o at constant value of BW of the SF and 2- increasing BW of the SF at constant value of g_o . The major advantage of our model is having a closed form relations between all parameters of the mode-locked laser cavity.

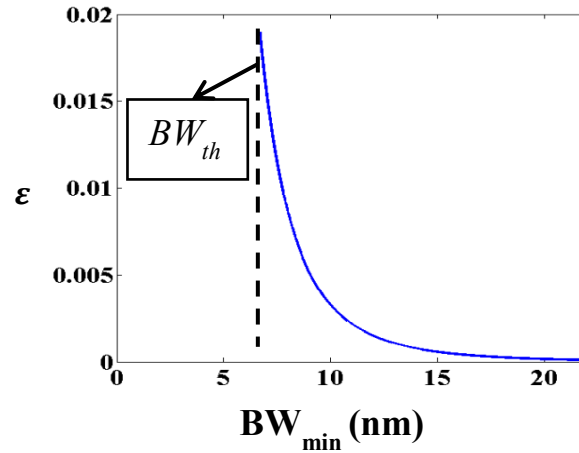


Fig. 3.5 Plot of the ratio ε calculated at the FWHM of the chirped pulse versus BW_{min} , the value of x chosen to be zero.

The validity of the two main assumptions, mentioned in section 3.2, is explored. For the first assumption, the value of the inverse of the dispersion length in (2.5a) (β_2/T_{FWHM}^2) should be much less than one. Based on Fig. 3.2(b), the minimum value of

chirped pulse width in the simulation model and the analytical model is selected to calculate β_2/T_{FWHM}^2 . It has a value of $4.5 \times 10^{-4} \text{ m}^{-1}$ and $5.9 \times 10^{-4} \text{ m}^{-1}$ in the simulation and analytical model, respectively.

For the second assumption, ε defined in (3.6) must be much smaller than one. It is clear from Fig. 3.5 that the value of ε decreased monotonically with the increase of BW_{min} , where the maximum value of ε ($0.019 \ll 1$) occurs at BW_{th} which is the border between the stable and lossy regions.

3.3.2 Experimental results

The Yb^{3+} fiber used in the experiment is Yb214 from CorActive. Standard SMF (HI1060) is used in this experiment. A 3-port fiber circulator from General Photonics (CIR-1030-NC-90-SS-P) is inserted inside the cavity. An inline fiber polarization controller was inserted before the power splitter to stabilize the mode-locking and have equal amplitude of the pulse train [56]. Its damage fluence is 1 mJ/cm^2 (the damage energy is 0.3 nJ).

Two different SFs were used in the experiment. The first filter has a 3-dB bandwidth of 3.2 nm (see Fig. 2.10(c)). The second one is designed based on the MMI and made in our lab. MMI-based filter design theory is described in detail in sub-section 2.2.5.2. The filter dimensions are mentioned in the same sub-section. The filter has a 3 dB spectral width of 6.24 nm and its insertion loss is 2 dB. Its spectral profile is shown in Fig. 2.8(b).

No mode-locking was observed using a 3.2 nm spectral filter in the cavity. In contrast, 30 mW-average power stable pulses were observed when a 6.2 nm spectral filter was used. These results are in agreement with both the simulation and analytical results.

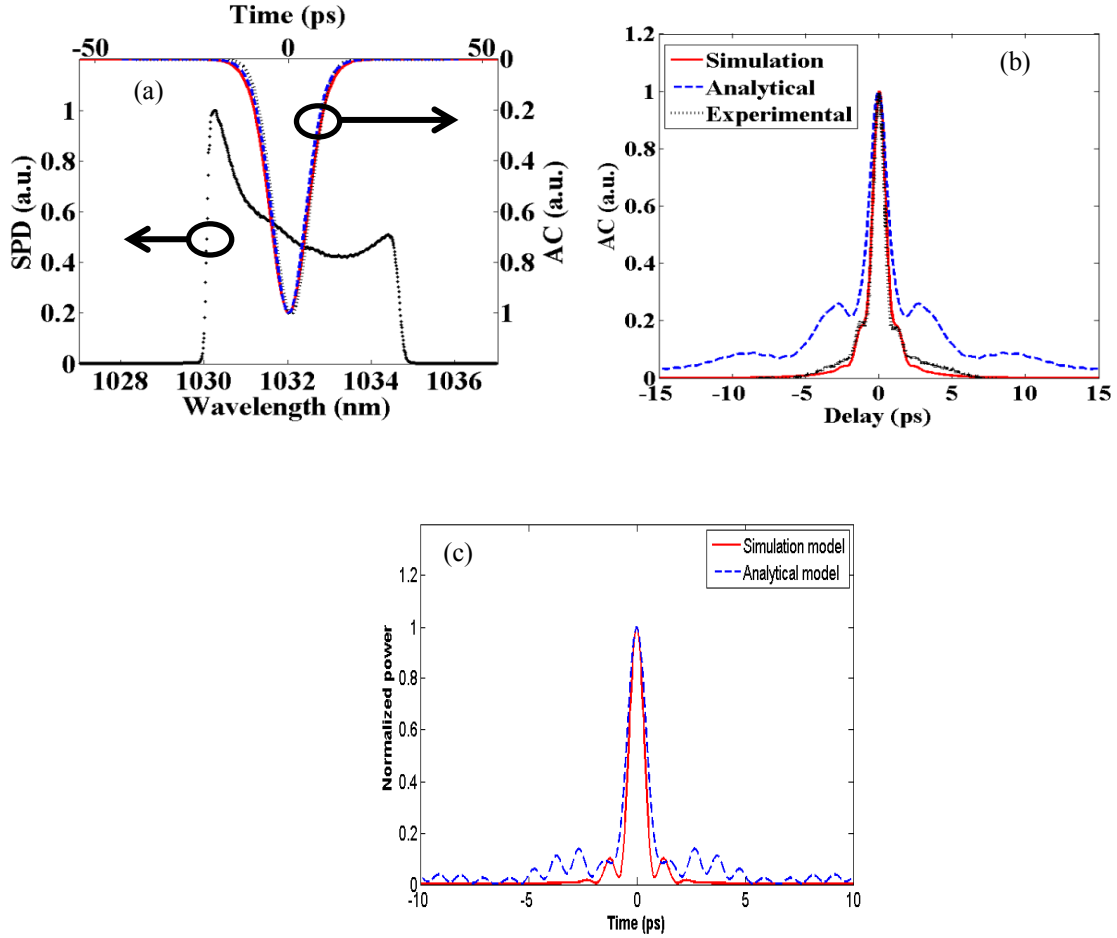


Fig. 3.6 Plot of (a) the experimental SPD of the chirped pulse at the power splitter output, the autocorrelation (AC) of the chirped pulses, (b) the AC of the de-chirped pulses and (c) the temporal profiles of the de-chirped pulses.

The length of the cavity was extended to achieve higher energy, the repetition rate was measured to be 18 MHz, and the pulse energy was 1.7 nJ. Even with this cavity length, the value of BW_{min} was calculated in the analytical model to be 4.7 nm, which is still larger than the spectral bandwidth of first filter. As explained in Appendix B, I

implemented an experimental set-up to have full characterization of the pulse. The SPD at the power splitter output is shown in Fig. 3.6(a), showing a peak-to-peak spectral bandwidth of 4.2 nm. The spectral bandwidth is close to the simulation and analytical results shown in Fig. 3.3 (case III). The autocorrelation of the chirped pulse is plotted in Fig. 3.6(a); the FWHM of the autocorrelation of the chirped pulse is 10.7 ps. Using the simulation model, the autocorrelation of the chirped pulse calculated at the output of the power splitter is plotted in Fig. 3.6(a). The autocorrelation of the chirped pulse calculated from the analytical model is plotted in the same figure. There is a good agreement between the results. The convolution factor between the chirped pulse and its autocorrelation is calculated to be 0.67 and 0.66 in the simulation and analytical models, respectively.

For the experimental results, the chirped pulse width is calculated, using the above convolution factors, to be 7.2 ps. A pair of diffraction gratings having 1200 lines per groove is used to compress the pulse to near its Fourier-transform limit. The value of the anomalous dispersion was found to be -2 ps^2 . This value is not close to the values calculated in the simulation (-1.5 ps^2) and the analytical (-1.13 ps^2) models because of the high accompanied value of the TOD of the diffraction gratings. The autocorrelation of the de-chirped pulses is shown in Fig. 3.6(b). Its pulse width was measured to be 956.7 fs. A pedestal appeared in each of the simulation, analytical, and experimental models. This is due to the side pulses observed in the de-chirped temporal profile (see Fig. 3.6(c)). However, in the analytical model, the de-chirped temporal profile shows a larger number of side pulses that affect the profile of its autocorrelation. The main reason is the deviation of the spectral phase (3.13b) from being parabolic at the spectral edges. The

convolution factor is calculated to be 0.73 in the simulation model, while it is 0.61 in the analytical model. Therefore, the de-chirped pulse width cannot exceed 695 fs. The value of energy at the SESAM was found to be slightly lower than its damage energy. Therefore, this is the maximum energy that can be extracted from the laser cavity. However, when the pump power was increased, I did not notice any damage or degradation in the SESAM response. That is because I had a multi-pulsing operation that cut any increase in the pulse energy.

3.4 Impact of SA parameters on the pulse characteristics

The major advantage of our analytical model is that it can guide the design and provides an understanding of the relationship between all parameters of the mode-locked laser cavity. Therefore, based on our analytical model, the values of the small signal saturable loss coefficient (a_I), the inverse of the saturation power of the SA (b_I), and the overdriving of the SA (c_I) are varied to maximize the pulse energy and reduce the de-chirped pulse width while keeping all of the laser cavity parameters mentioned in the previous section the same. To have a fair comparison with the results mentioned in the previous section, the bandwidth of the spectral filter was selected to be 7.6 nm in the analytical model, which corresponds to 6.5 nm in the simulation one. First, I fixed the value of c_I at the value previously mentioned ($3.18 \times 10^{-5} \text{ m}^{-1} \text{ W}^{-1}$) to study the effects of a_I , and b_I .

Fig. 3.7 is a 3D plot showing the effect of a_I , and b_I on the pulse normalized energy, SPD, and de-chirped pulse width for the analytical models. As shown in Figures 3.7(a) and 3.7(b), the increase of a_I , and b_I leads to an increase in the pulse energy and the spectral bandwidth of the chirped pulse. Keeping the non-saturable loss (L_{ns}) fixed at

0.1, as seen in (3.29a), the value of small signal absorption (q_o) increases with a_I . This leads to the increase of the small signal saturable loss of the SA. The saturation power of the SA is inversely proportional to b_I . This means that the pulse energy and spectral bandwidth of the chirped pulse is enhanced by reducing the value of saturation power. However, it is found that a_I has higher effect on the pulse energy and spectral bandwidth than b_I .

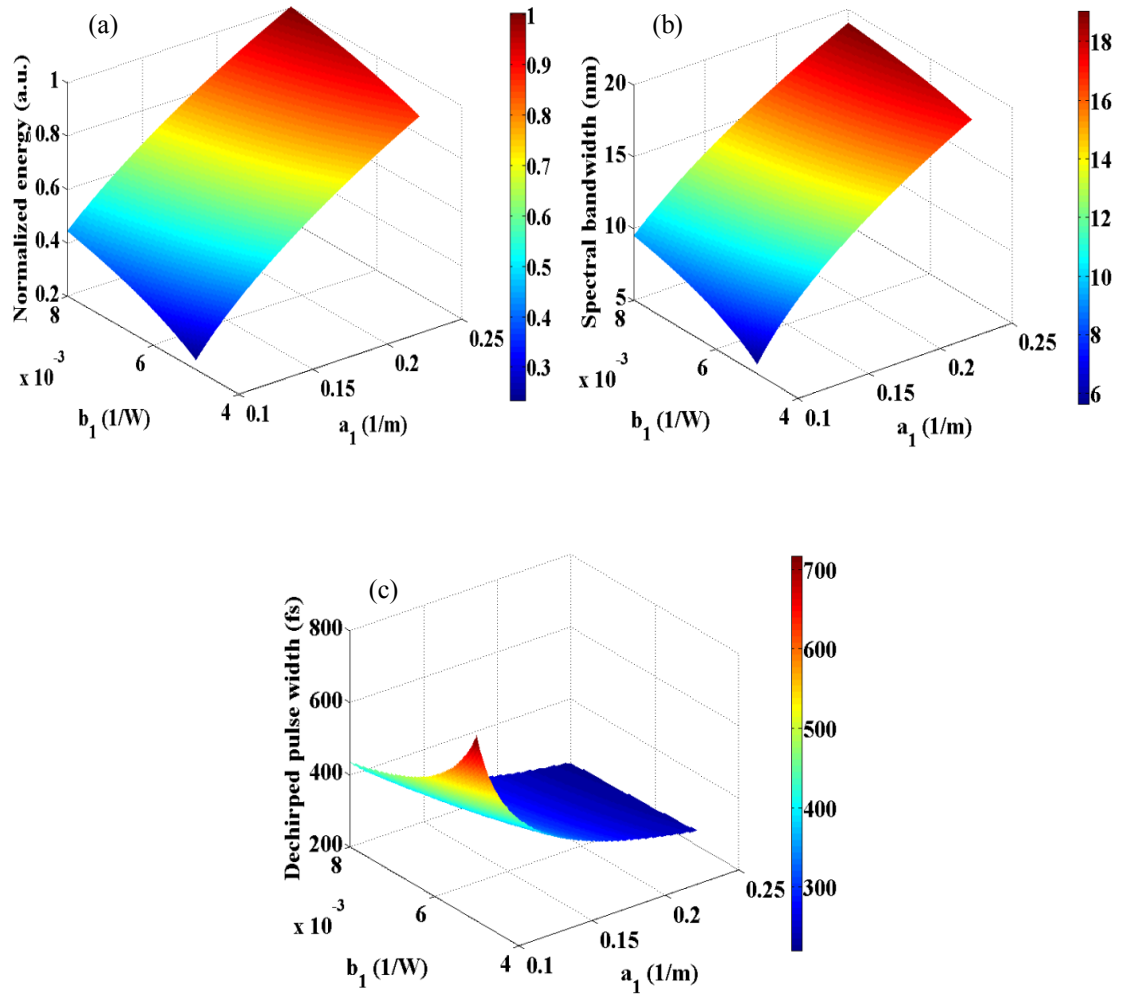


Fig. 3.7 Plot of (a) the pulse energy normalized to its maximum value, (b) spectral bandwidth of the chirped pulse, and (c) the de-chirped pulse width versus a_I , and b_I .

Fig. 3.7(c) shows the plot of the de-chirped pulse width. Compared to results shown in section 3.3, at $a_I = 0.25 \text{ m}^{-1}$, and $b_I = 0.008 \text{ W}^{-1}$, the pulse energy is increased by 5.82 times. The spectral bandwidth is increased to 19 nm. The de-chirped pulse width is decreased to 220 fs. The absolute value of the anomalous dispersion required for pulse compression is decreased to 0.07 ps^2 . Practically, we cannot go to higher value of a_I because the transmission coefficient of the SA will approach null at zero input power. In this case, the value of small signal gain required to have CW oscillation, which is required to initialize the mode locking, will be extremely high [130].

To show that our analytical model is valid, the two assumptions shown in section 3.2 are checked. The values of the inverse of the dispersion length (β_2/T_{FWHM}^2) and ε given in (3.6) are calculated to be 0.007, and 0.009, respectively.

As a confirmation of the results of the analytical model, a simulation of the laser cavity was performed for the new values a_I , and b_I . The values of the physical parameters of the SA were calculated using a method similar to that shown in sub-section 3.2.1; the only difference was the selection of P to be equal to b_I instead to P_{sat} . This resulted in:

$$q_o = 1 - L_{ns} - \exp(-2a_1L) \quad (3.30a)$$

$$P_{sat}^3 \left(\frac{b_1^2 k_1}{q_o} - b_1^2 \right) + P_{sat}^2 \left[\frac{b_1 k_1}{q_o} + \frac{2P_p b_1^2 k_1}{q_o} - 2P_p b_1^2 - b_1 \right] + P_{sat} \left[\frac{2P_p b_1 k_1}{q_o} + \frac{b_1^2 P_p^2 k_1}{q_o} - b_1^2 P_p^2 - 1 \right] + \frac{P_p^2 b_1^2 k_1}{q_o} = 0 \quad (3.30b)$$

$$P_{TPA} = \frac{P_{sat}}{q_o} \left(\frac{P_p}{P_{sat}} + 1 \right)^2 \quad (3.30c)$$

Here $k = \ln(1 - q_o/2 - P_{sat}/P_{TPA} - L_{ns})/\ln(1 - q_o - L_{ns})$.

The values of q_o , P_{sat} , and P_{TPA} were calculated to be 0.85, 429 W, and 4.6 KW, respectively. The reflectivity of SESAM based on these values is plotted in Fig. 3.8(a). It is compared to the transmission coefficient of the SA calculated using (3.28). There is a slight difference in the value of the modulation depth (the difference between the maximum and the minimum values of the curve). It was calculated to be 39.26%, and 44.41% in the simulation, and analytical models, respectively.

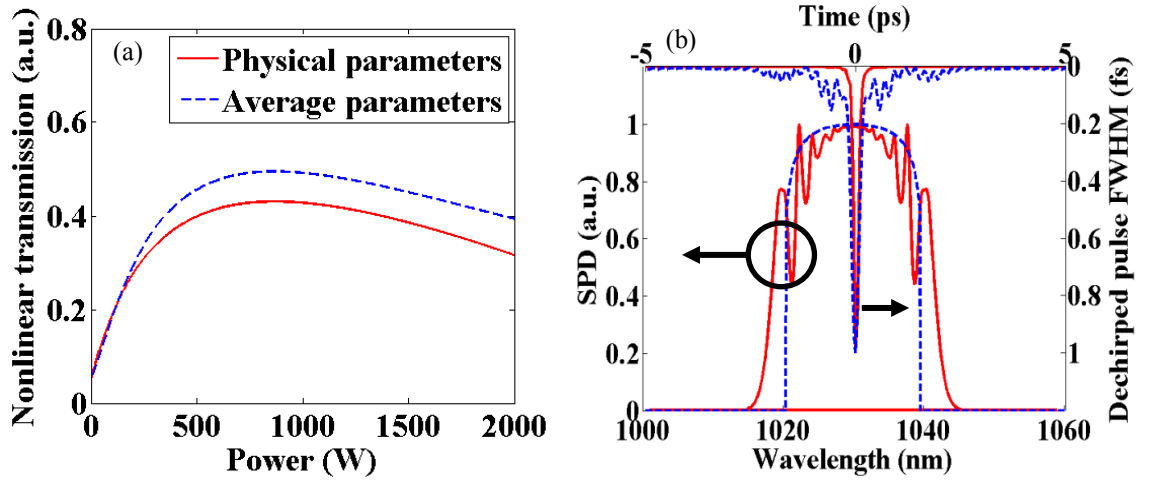


Fig. 3.8 (a) Plot of the reflectivity of the SESAM (solid line) and transmission coefficient of the SA come from the analytical model (dotted line) versus the input power. (b) Plot of normalized SPD, and temporal profile of the de-chirped pulse at the output of the power splitter. For the analytical model, the normalized SPD, and temporal profile of the chirped pulse is shown for comparison. The bandwidth of the SF is 7.59 nm. Simulation: solid red line, and analytical: blue dashed line.

The value of the exposure area ($A_{eff} = P_{sat}T_{sa}/F_s$) was calculated at the same value of F_s and damage fluence to be $178.7 \mu\text{m}^2$. Therefore, the damage energy is 1.79 nJ that is 6.32 times more than that shown in the previous section. The simulation model reveals that the maximum value of g_o for single pulse operation is 77.5 dB/m and the

pulse energy at the output of power splitter is 12.5 nJ. The SPD of the chirped pulse at the output of the power splitter is plotted in Fig. 3.8(b). It has some structures at its edges; its 3-dB spectral bandwidth was calculated to be 22.8 nm. The de-chirped pulse was compressed via the dispersion delay line to have a value of 108.5 fs (Fig. 3.8(b)). The absolute value of the anomalous dispersion of the dispersion delay line was calculated to be 0.11 ps². The value of pulse energy incident on the SESAM was calculated to be 1.2 nJ, which is lower than the damage energy. Therefore, this is considered the best design of SESAM having a high immunity for damage and generating the highest pulse energy, widest spectral bandwidth, and shortest de-chirped pulse width.

For the sake of comparison, the SPD of the chirped pulse and temporal profile of the de-chirped pulse calculated from the analytical model are shown in Fig. 3.8(b). The SPD obtained from the simulation model shows some structures. However, the SPD from the analytical model does not show any of these structures. Nevertheless, the temporal pulse profile and pulse width are close for both the simulation and the analytical models.

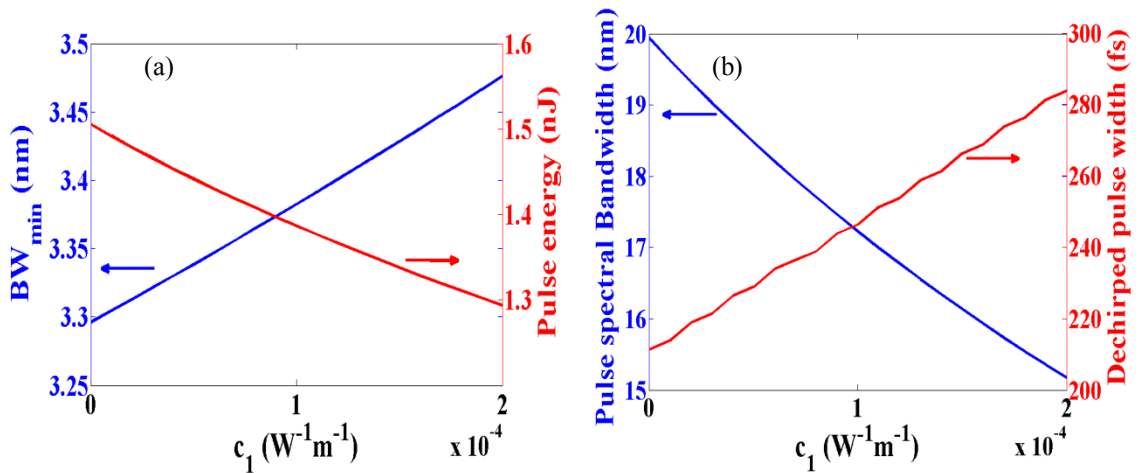


Fig. 3.9 The change of (a) pulse energy, BW_{min} (b) spectral bandwidth, and FWHM of the de-chirped pulse with c_1 , which is the two-photon absorption parameter (TPA) in the analytical model.

The effect of overdriving the SA (c_I) on BW_{min} , pulse energy, spectral bandwidth, and de-chirped pulse width is examined at the new values of a_I , and b_I , and is plotted in Fig. 3.9. It is clear from Figures 3.9(a)-3.9(b) that the maximum pulse energy, spectral bandwidth and minimum de-chirped pulse width occurs at c_I equal zero, which theoretically means no overdriving point. While reducing it to zero is not practical [16, 52, 131], c_I could be reduced to have mode-locked pulse with high energy, wide SPD and narrow de-chirped pulse width.

In summary, this proposed analytical model is more generalized than the model illustrated in [124] because of the addition of the overdriving of the SA and the effect of the lumped SF. Furthermore it shows the border between the stable and unstable mode-locking regions, which is not introduced in [116, 117]. This analytical model can provide a guidance to the simulation model, as it can predict the main characteristics of a mode-locked pulsed fiber laser. The values of the BW_{min} of the lumped SF, the spectral bandwidth of the pulse, and the chirped and de-chirped pulse widths extracted from the analytical model are close to that of the simulation model. However, the values of the pulse energy and peak power calculated from the analytical model are far from the corresponding values in the simulation model. This could be due to the fact that this model is an average model and gives the value of the average pulse energy, and peak power. Nevertheless, it gives a qualitative estimate of the flow of the pulse energy with the change of the cavity or SA parameters.

3.5 Conclusion

In the first part of the chapter, an average analytical model has been developed. The novelty of the model is that it takes into account the overdriving point of the SA and

the effect of the lumped SF bandwidth. The model shows a very good agreement with the numerical and experimental results in different cases. In the second part, this model is used to study the effect of the design parameters of the SA on the pulse characteristics in normal dispersion femtosecond fiber laser to maximize the energy and minimize the temporal pulse width. It is found that increasing the small signal absorption, and reducing the saturation power of the SA enhances the output pulse parameters. More specifically, it increases the pulse energy and spectral bandwidth and decreases the temporal pulse width.

Chapter 4

4 An efficient semi-vector model for all-fiber mode-locked femtosecond lasers based on nonlinear polarization rotation²

In this chapter, a novel numerical average model is represented on all-fiber femtosecond laser mode locked by NPR. As mentioned in sub-section 2.1.1, the simulation model relying on solving vector CGLEs is not the optimum solution to studying the impact of the parameters of the NPR-based SA on the pulse energy, spectrum bandwidth, and de-chirped pulse width. The average model is an effective tool for performing this study. In contrast to Chapter 3, closed form relations for the pulse energy and SPD are impossible to have due to the complicated nonlinear transmission function of the NPR-based SA (see e.g. (2.28a) and (2.28b)). However, the closed form relation of NPR enables us to study the effect of the characteristic angles (phase and orientation angles) of the polarization controllers on the parameters of the SA such as small signal transmission, slope of the NPR transmission curve and the location of the overdriving point. Consequently, the average GLE is numerically solved based on the selection of these angles.

This chapter is organized as follows: survey on the previous numerical average models dedicated to mode-locked fiber lasers is represented in section 4.1. The semi-vector model is explained in section 4.2. A comparison of its results with those of a vector simulation model is discussed in section 4.3. In section 4.4, the experimental results are introduced to check the validity of our model and show how close it is to

² Most of the results in this chapter have been published in [II].

experimental results. Finally, the conclusion is presented.

4.1 Literature survey on numerical average models

As mentioned in Section 3.1, the analytical average model for fiber lasers mode-locked by NPR does not include the effect of gain saturation. E. Ding et al. included this effect in the average propagation model defined by CQCGLE. It was solved numerically to study the effect of the wave plates on the stability of pulse inside the cavity. It also studied the effect of the cavity parameters on the existence of multi-pulse operation [132].

An attempt to insert the effect of lumped SF bandwidth was performed by A. Komarov et al., where the SF has been added as a phenomenological term in the scalar GLE. Therefore, it does not have any physical relation to the lumped SF. In this model, the vector CGLEs were solved analytically in the absence of gain and dispersion. Then, the laser cavity was divided into a lumped block for the SA based on NPR and the gain medium, which is numerically solved using the scalar GLE [62].

Another attempt to physically insert the effect of the lumped SF was performed by B. G. Bale et al.; they divided the laser cavity into two sections; the gain medium and lumped SF. The SA parameters were truncated to the quintic term. The resultant CQCGLE was solved numerically for the pulse propagation in the gain section. A qualitative description of the effect of the filter bandwidth on the pulse characteristics is illustrated without relating the cubic and quintic terms to the actual parameters of the SA [129].

Another model including the sinusoidal nature of the NPR curve in the master equation model was introduced, named the Sinusoidal Ginzburg-Landau equation (SGLE) model. This model has included the whole effect of the NPR curve without any of the truncation of high order terms that was done in the previous models. This equation was solved numerically to study the relationship between cavity parameters and the existence of single or multi-pulse operation [63].

Based on the above discussion, no previous numerical average models have a complete description of all fiber mode-locked laser operating in the normal dispersion region that has gain medium, lumped SF, nonlinear transmission profile of the SA without ignoring any high order terms and the long piece of SMF between the output coupler and the input of the gain fiber.

In this chapter, I developed a model that takes into account all these effects. This results in an average model that can have a realistic representation of all fiber femtosecond laser. I call it a semi-vector model because it takes into consideration the characteristic angles of the polarization controllers and the NPR inside the Yb^{3+} -doped fiber and at the same time numerically solving the scalar GLE. The semi-vector model takes less than 5% of the time needed by the vector simulation model for the same computational platform.

This model is then used to study two effects. The first is the impact of the overdriving of NPR based SA on the pulse characteristics (pulse energy, spectral bandwidth and de-chirped pulse width) [81, 133]. The second effect is the change of the order of the components inside the laser cavity.

4.2 Laser cavity structure

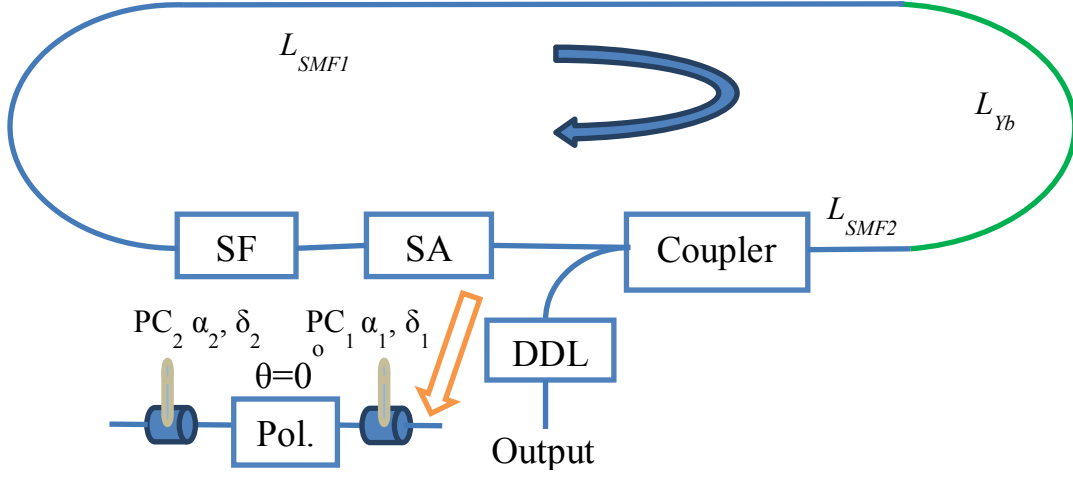


Fig. 4.1 Schematic description of all fiber femtosecond laser cavity used in theoretical and experimental work: Pol-fiber polarizer, PC-fiber polarization controller, DDL-dispersive delay line and SF-spectral filter.

A general mode-locked laser cavity structure having all the parameters shown in Fig. 2.2, except for the dispersion compensation block, is used in this work. The cavity structure is depicted in Fig. 4.1. PC₁ and PC₂ having a fiber polarizer between them are inserted inside the cavity to initiate mode-locking through NPR.

4.2.1 Model description

Any laser cavity can be divided into two regions based on the pulse energy in that region; linear (low energy pulses) and nonlinear regions (high energy pulses). Because most of the pulse energy is usually directed outside the cavity by the coupler, the Yb³⁺-doped fiber (L_{Yb}) and the SMF (L_{SMF2}) constitute the nonlinear region, while the rest of the cavity forms the linear region for the laser cavity shown in Fig. 4.1.

The light propagation through the polarization controllers, polarizer, SF and coupler is modeled by their Jones transfer matrices [62, 63]. The gain in the Yb³⁺-doped

fiber is expressed in (2.2). The linear birefringence $\Delta\beta$ is set to zero to only take into account the nonlinear birefringence caused by NPR. In the vector model, many round trips (up to thousands) are required to check whether there is stable mode-locking for every combination of polarization angles, which is a very time consuming process.

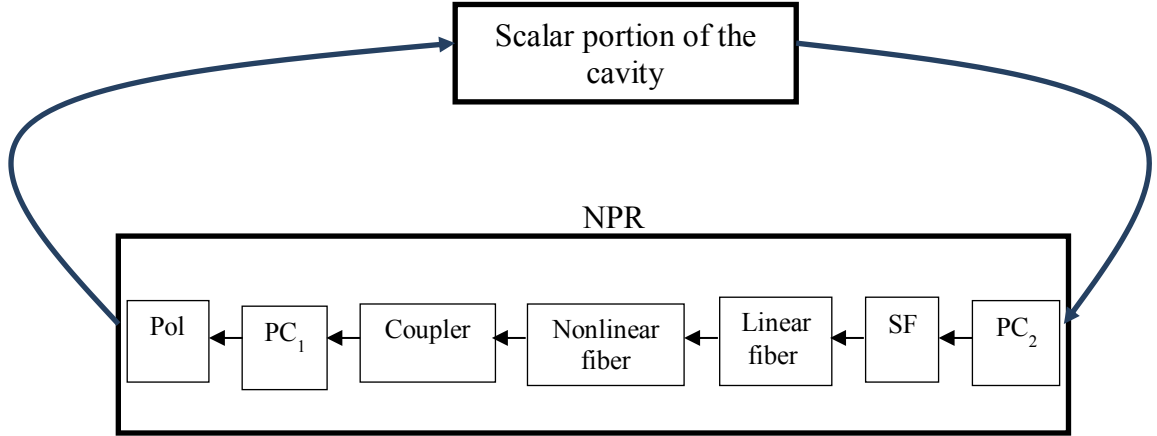


Fig. 4.2 Equivalent schematic diagram of the proposed semi-vector model including two blocks; NPR and Scalar portion of the cavity.

The aim of this work is to develop an efficient model that can be used to study femtosecond fiber laser cavities that are locked by NPR. The model will convert the simulation model using vector CGLEs (2.1a) and 2.1b) into a fast average model that includes a NPR block having the transfer matrices of the nonlinear region, polarization controllers, lumped SF, coupler, and the dispersion effect of the linear region. The gain, self-phase modulation, and dispersion of the nonlinear region are included in the scalar GLE, which is relatively faster than the vector CGLEs (see Fig. 4.2).

As a first step in the semi-vector method presented here, the dispersion and the gain coefficient of the nonlinear region are averaged over its length. The averaged values of these parameters; β_{2av} , β_{3av} , and g_{o_av} are defined as:

$$\beta_{2av} = \frac{1}{L_N} [\beta_{2Yb} L_{Yb} + \beta_{2SMF2} L_{SMF2}] \quad (4.1a)$$

$$\beta_{3av} = \frac{1}{L_N} [\beta_{3Yb} L_{Yb} + \beta_{3SMF2} L_{SMF2}] \quad (4.1b)$$

$$g_{o_av} = \frac{1}{L_N} g_o L_{Yb} \quad (4.1c)$$

where $L_N = L_{Yb} + L_{SMF2}$ is the total length of the nonlinear region. Since the peak power of the pulse is large in the nonlinear medium, the Kerr nonlinearity has the dominant effect in CGLE. The effects of dispersion, fiber amplification, linear loss, and frequency gain filtering, are neglected as a first order approximation and their effects will be taken into account later on as perturbation factors. This assumption is valid as long as the following two conditions are satisfied:

- (i) The dispersion lengths (L_D) defined in (2.5a) and the net-gain length ($L_g = (\exp((g_o - \alpha_f)L) - 1)/(g_o - \alpha_f)$) are longer than L_N .
- (ii) The spectral bandwidth of the pulse is narrower than the gain filtering bandwidth. Therefore, it has little effect on the pulse dynamics, and it can be assumed as a perturbation parameter.

Following these assumptions, the CGLE are converted to (2.22a) and (2.22b). A closed-form solution of these two equations for an optical fiber is illustrated in (2.23b) as W_{NL} .

The Jones matrix for an arbitrary oriented polarization controller (W_{PCn} , where n is 1 for PC_1 and 2 for PC_2) and the inline fiber polarizer (W_P) are shown in (2.24) and (2.25), respectively. The lumped filter is a linear element, which plays a role in stabilizing the optical chirped pulse inside the cavity by cutting its edges [13, 91, 101]. It also has an insertion loss (IL_f). The transfer matrix of the filter in the frequency domain is:

$$W_f = \begin{pmatrix} T_f & 0 \\ 0 & T_f \end{pmatrix} \quad (4.2)$$

where T_f is the transfer function of the filter. The electric field coupling coefficient through the cavity of the coupler is defined by W_c , as follows:

$$W_c = \sqrt{\beta_c} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.3)$$

where β_c is the power coupling ratio. SMF located in the linear region (L_{SMF1}) acts as a dispersive element. In this case the CGLE equations through L_{SMF1} turn to two uncoupled differential equations having the following transfer matrix [19].

$$W_L = \begin{pmatrix} \exp\left(i\left(\omega^2 \frac{\beta_{2SMF1}}{2} - \omega^3 \frac{\beta_{3SMF1}}{6}\right)L_{SMF1}\right) & 0 \\ 0 & \exp\left(i\left(\omega^2 \frac{\beta_{2SMF1}}{2} - \omega^3 \frac{\beta_{3SMF1}}{6}\right)L_{SMF1}\right) \end{pmatrix} \quad (4.4)$$

where ω is the angular optical frequency. After the polarizer the amplitude of the electric field at the l^{th} round trip can be defined as $u_l(t)$. Therefore, $u_{l+1}(t)$ is calculated as a function of the amplitude at the l^{th} round trip by multiplying the matrices of all components in the following order:

$$\begin{pmatrix} u_{l+1}(t) \\ 0 \end{pmatrix} = W_P W_{PC1} W_c W_{Yb} iFFT \left[W_L W_f FFT \left[W_{PC2} \begin{pmatrix} u_l(t) \\ 0 \end{pmatrix} \right] \right] \quad (4.5)$$

where FFT and iFFT stand for fast Fourier transform and inverse fast Fourier transform, respectively. The polarizer's output is selected to be the starting point as its output polarization is well known and hence a scalar equation can be used to describe the pulse evolution. Equation (4.5) acts as a fast SA resulting from the combination of NPR in the nonlinear medium, polarization controllers, polarizer, coupler, filter and SMF acting as a dispersive element in the linear region. It is worth mentioning that (4.5) includes also the

effect of self-phase modulation. That will be eliminated below to have a saturable absorber with only self-amplitude modulation.

In the above derivation, the dispersion effect, Yb^{3+} -doped fiber gain, spectral gain filtering, and linear loss coefficient are neglected. These effects are considered as a perturbation on one round trip but for a large number of round trips, they can be written as a differential equation for the amplitude of the electric field just after the polarizer. This differential equation can be transformed into a scalar GLE using the following transformation [62]:

$$Y = u e^{i\gamma \int_0^z P(z') dz'} \quad (4.6)$$

In this case the scalar GLE is:

$$\frac{\partial Y}{\partial z} + \frac{i\beta_{2av}}{2} \frac{\partial^2 Y}{\partial t^2} - \frac{\beta_{3av}}{6} \frac{\partial^3 Y}{\partial t^3} + \frac{\alpha_f}{2} Y = \frac{g_{o,av}(z)}{2} \left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t^2} \right) Y + i\gamma |Y|^2 Y \quad (4.7)$$

Equation (4.5) can be rewritten as:

$$u_{l+1} = e^{i\gamma PL_N} T_A u_l \quad (4.8)$$

Here, the effect of the SPM is represented by the term: $e^{i\gamma PL_N}$. T_A refers to the nonlinear transmission ratio of NPR defined by the matrices in (4.5).

Since most of the power is confined in the nonlinear region with constant value, then the $\int_{z_n}^{z_{n+1}} P(z') dz' \approx PL_N$ [62]. In this case, the nonlinear transmission coefficient in (4.8) representing NPR can be written as:

$$Y_{l+1} = T_A(\delta_1, \delta_2, \alpha_1, \alpha_2, B_p, \beta_c, T_f, P) Y_l \quad (4.9)$$

Equations (4.7 and 4.9) model the pulse propagation shown in Fig. 4.2 through the nonlinear region and the lumped fast saturable absorber.

4.2.2 Effect of the angles of the polarization controllers on the SA nonlinear transmission

One of the challenges in the vector models for NPR-based mode-locked lasers is to find a state of the polarizers' angles that produces pulsed operation. Trial-and-error is usually used; however, it is very time-consuming. To have stable mode-locking, the transfer function of the SA should have a positive slope around zero input peak power [62]. To get a closed form of the saturable absorber's transfer function for our proposed model, the filtering bandwidth limit and the dispersion effect of the SMF located in the linear region are neglected in (4.9). Neither of these neglected parameters affects the slope of the NPR curve near zero input power. Hence, the transfer function of the NPR is given as (see section 2.2.6.2 for more details):

$$u_{l+1} = \sqrt{IL_f} B_p \sqrt{\beta_c} e^{i\gamma PL_{NL}} e^{i((\delta_1 + \delta_2)/2)} \left[\cos\left(\frac{2\gamma JL_N}{3}\right) \cos\left(\frac{\delta_1}{2}\right) \cos\left(\frac{\delta_2}{2}\right) + i \cos\left(\frac{2\gamma JL_N}{3} - 2\alpha_2\right) \cos\left(\frac{\delta_1}{2}\right) \sin\left(\frac{\delta_2}{2}\right) + i \cos\left(\frac{2\gamma JL_N}{3} + 2\alpha_1\right) \sin\left(\frac{\delta_1}{2}\right) \cos\left(\frac{\delta_2}{2}\right) - \cos\left(\frac{2\gamma JL_N}{3} + 2\alpha_1 - 2\alpha_2\right) \sin\left(\frac{\delta_1}{2}\right) \sin\left(\frac{\delta_2}{2}\right) \right] u_l \quad (4.10a)$$

$$u_{l+1} = e^{i\gamma PL_{NL}} \mu(\delta_1, \delta_2, \alpha_1, \alpha_2, P) u_l \quad (4.10b)$$

$$J = -\frac{1}{2} (IL_f) \sin(\delta_2) \sin(2\alpha_2) |u_l|^2 \quad (4.10c)$$

where (δ_1, δ_2) are the phase shifts induced by the two polarization controllers, the orientation of these controllers with respect to the principal axes of the fiber are (α_1, α_2) , IL_f is the insertion loss of the SF and μ is the nonlinear transmission coefficient. The slope of $|\mu|^2$ with respect to total power (P) at a certain value P_0 determines the possibility of having mode-locking.

The nonlinear transmission ratios ($|\mu|^2/(\beta_c(IL_f)B_p^2)$) are plotted versus the value of total power of the pulse (P) for three sets of angles. Set 1: ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 50^\circ$), Set 2 ($\delta_1 = 70^\circ$, $\delta_2 = 135^\circ$, $\alpha_1 = 160^\circ$ and $\alpha_2 = 65^\circ$) and Set 3: ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 20^\circ$) in Fig. 4.3(a). $|\mu|^2/(\beta_c(IL_f)B_p^2)$ has three key parameters: The location of the overdriving point, the slope at zero input power, and the small signal transmission coefficient.

$|\mu|^2$ is a sinusoidal function with an argument that depends on J , δ_1 and α_1 . The overdriving point, which is the peak of the sinusoidal wave, occurs when its argument is equal to 90° . By fixing δ_1 and α_1 , the location of the overdriving point (the black dots shown in Fig. 4.3(a)) will depend on J and hence will depend on δ_2 , α_2 and P .

Based on the above angles, the value of $\sin(\delta_2)\sin(2\alpha_2)$ for Set 2 is 1.64 times that of Set 1. Since J must be the same for both Set 1 and Set 2 at the overdriving point, the corresponding value of P , at Set 1, is 1.64 times that of Set 2.

The values of δ_1 and α_1 were selected for Set 1 and Set 2 to have positive slope around zero input power to ensure mode locking [62]. In Figures 4.3(b) and 4.3(c), δ_2 and α_2 , are fixed and the slope of the nonlinear transmission ratio ($|\mu|^2$) around zero power is plotted as a function of δ_1 and α_1 .

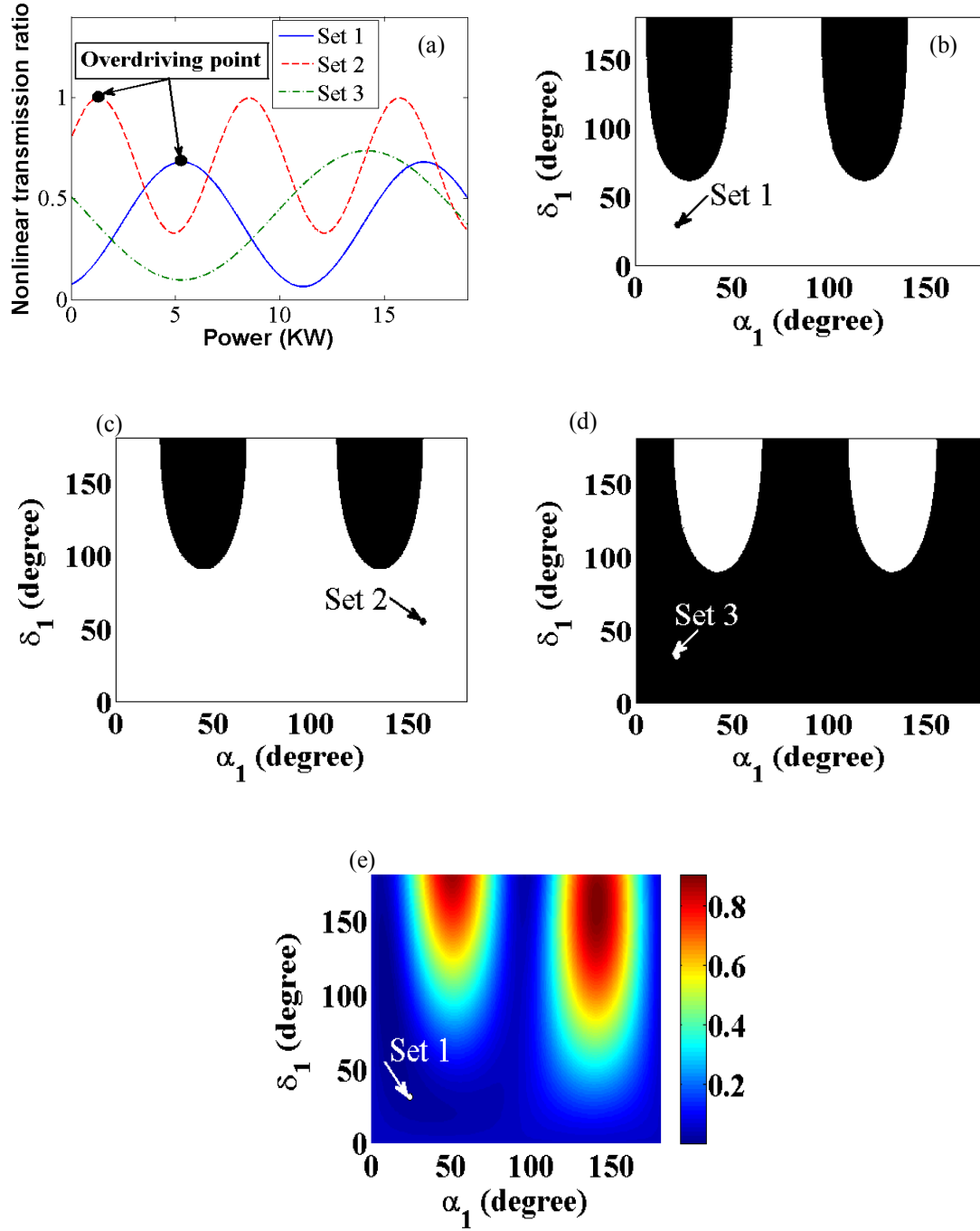


Fig. 4.3 (a) Variation of the nonlinear transmission coefficient with total power at three sets of polarization controller angles (δ_1 , δ_2 , α_1 and α_2): solid line (50° , 160° , 40° and 50°), dashed line (70° , 135° , 160° and 65°) and dashed-dot line (50° , 160° , 40° and 20°). Plot of the sign of the slope of $|\mu|^2$ at zero input power versus δ_1 and α_1 for (b) Set 1, (c) Set 2 and (d) Set 3. The white region represents the positive slope, while the black region represents the negative slope. (e) Plot of $|\mu(0)|^2$ as a function of δ_1 and α_1 for Set 1.

The white and black regions represent positive and negative slopes, respectively. The arrows points to the selected values of δ_1 and α_1 , which has positive slope of $|\mu|^2$ at zero input power. As shown in Fig. 4.3(a), negative slope regions may lead to either multi-pulsing or unstable mode-locking states. Violation of the proper SA is pronounced at its negative slope regions. For Set 3, the values of δ_1 and α_1 were selected so that the curve has negative slope at zero input power (see Figures 4.3(a) and 4.3(d)); this means that no mode-locking exists.

Pushing the location of the overdriving point to high input power leads to having a high value of saturation power of the SA, which has a negative impact on the pulse energy and spectral bandwidth. Based on the results in section 3.4, reducing the value of the small signal transmission coefficient $|\mu(0)|^2$ can counteract this negative effect. As shown in Fig. 4.3(e), the values of δ_1 and α_1 were also chosen to give a small value of $|\mu(0)|^2$ for Set 1. In the next section, I will show the effect of the shape of $(|\mu|^2/(\beta_c(IL_f)B_p^2))$ on the temporal and spectral profiles of the pulse.

4.3 Simulation results

The length of the Yb^{3+} fiber (L_{yb}) is 37.5 cm, followed by SMF (L_{SMF2}) of length 39.5 cm. β_2 and β_3 are 24690.1 fs²/m and 75.3 fs³/m for the SMF, and 32550.1 fs²/m and 79.6 fs³/m for Yb^{3+} fiber, respectively. The values of β_{2av} and β_{3av} are calculated using (4.1a) and (4.1b) to be 28543.3 fs²/m and 77.41 fs³/m. SMF (L_{SMF1}) is 8.02 m. The power coupling ratio of the coupler through the cavity is 1:10. The transmission efficiency (B_p) of the polarizer is 0.95. The SF is modeled as a Gaussian filter of 3-dB bandwidth 6.24 nm and insertion loss is 2 dB. The parameters of the SF are specified from its

experimental characterization. The small signal gain (g_o) is 91.8 dB/m, and g_{o_av} is 45 dB/m (4.1c).

White noise has been used as an initial condition. Therefore, the temporal location of the pulses in both models is arbitrary. Thousands of roundtrips are used to ensure that a stable mode-locked pulse operation has been reached. Comparisons between the output pulses from the semi-vector and vector models at three different sets are reported.

For Set 1, E_{sat} is taken as 5.75 nJ and 10 nJ for the vector and semi-vector models, respectively, to obtain single-pulse operation. The discrepancy between E_{sat} in the vector and semi-vector models is due to the averaging nature of the semi-vector model [63]. Fig. 4.4 shows the evolution of the temporal profiles of the self-started output pulse at different values of round trips for both two models. Stable single pulse operation is verified by plotting the change of the pulse peak power and root mean square (RMS) pulse width with the number of round trips. As shown in Fig. 4.5, they have constant values after hundreds of round trips. For the same computing platform (Intel I5 CPU of speed 3.2 GHz and 6 GB RAM), the simulation time of 15000 round trips in the vector and the semi-vector model is 45 and 2.5 hours, respectively. This depicts the efficiency of the semi-vector model.

The SPD of the output pulse at steady state for the two models is plotted in Fig. 4.6. The steep edges of the SPD are evidence of dissipative Soliton pulse [13, 91, 101, 118, 134, 135]. The M-like shape of SPD of the semi-vector model shown in Fig. 4.6(a) is similar to that of the v field shown in Fig. 4.6(b). The SPD of the u field is different from that of the semi-vector model; it has a rounded top with a small notch in the middle.

However, the peak-to-peak spectral bandwidths for both semi-vector and vector models are in good agreement, 11.7 nm and 12.2 nm, respectively. This shows that the semi-vector model can give a qualitative description of the SPD, but it cannot depict all details.

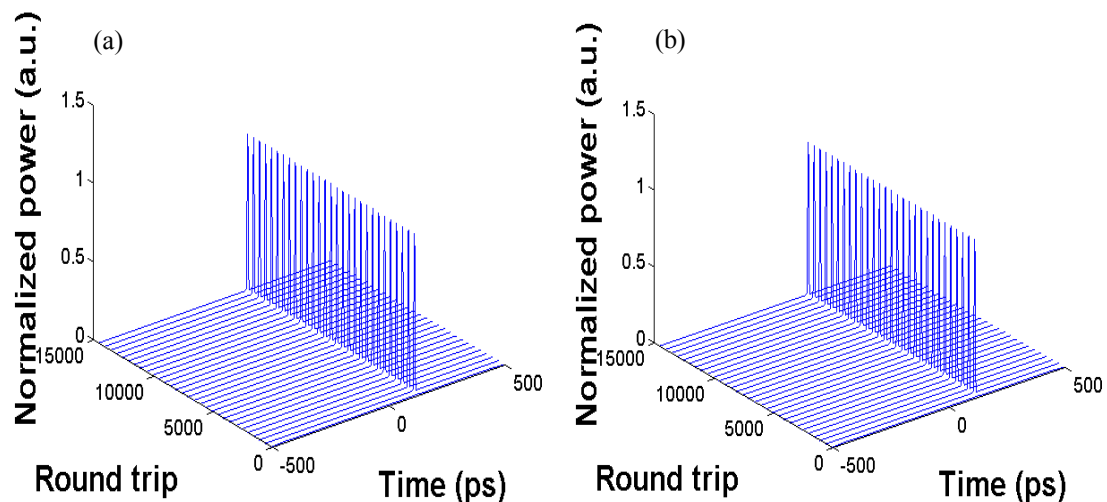


Fig. 4.4 Numerical simulation of a self-starting mode-locked cavity at different round trips using (a) semi-vector and (b) vector model. The location of the pulses is random as it is started from noise. The peak power is normalized to unity in both models. The polarization controllers characteristic angles are ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 50^\circ$).

The evolution of the pulse profiles at Set 2 is plotted in Fig. 4.7. Again, for single pulse operation, E_{sat} was adjusted to be 1.25 nJ and 3.25 nJ for the vector and semi-vector model, respectively. Stable single pulse operation is also demonstrated by plotting the change of the pulse peak power and RMS pulse width with the number of round trips see (Fig. 4.8).

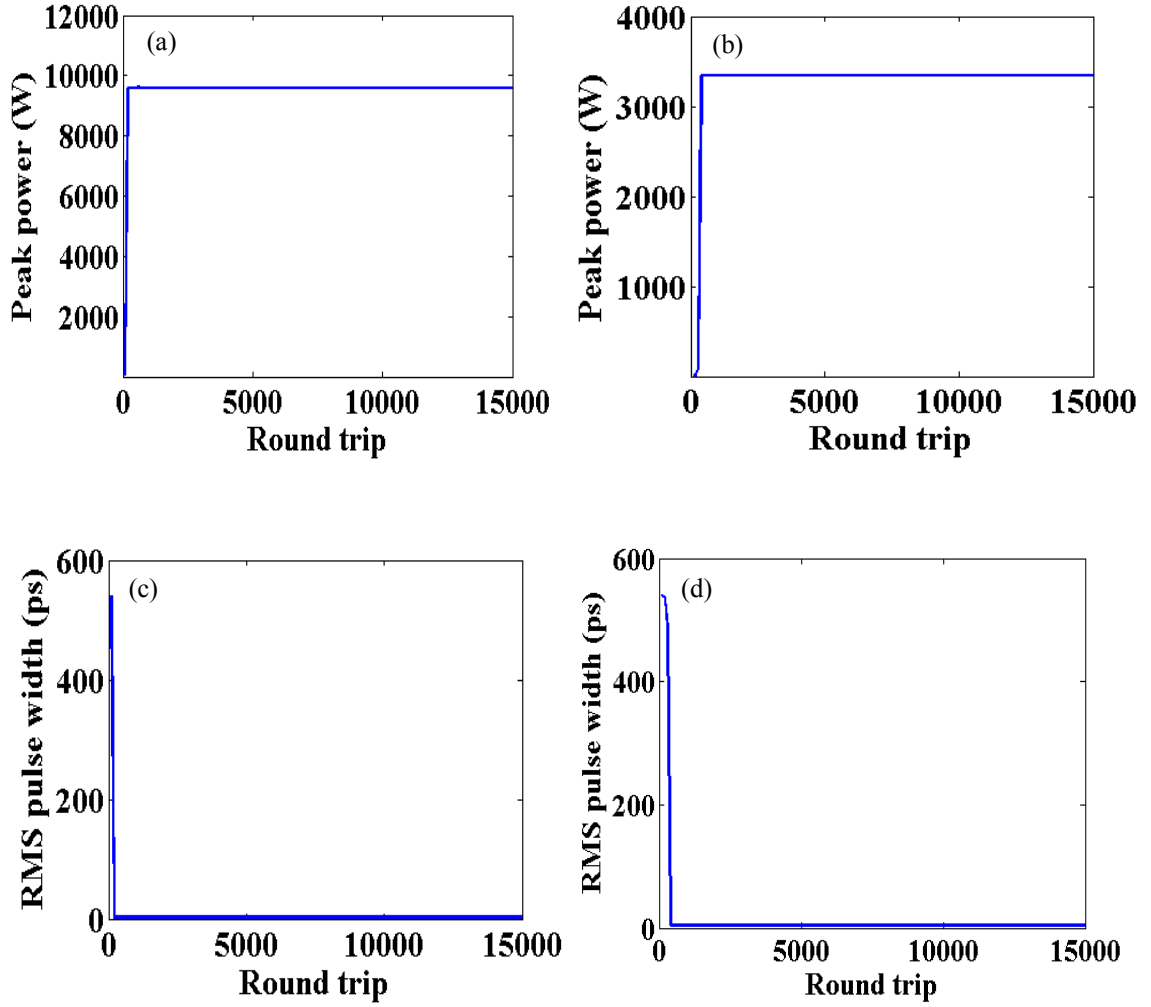


Fig. 4.5 Plot of the pulse peak power (a) semi-vector and (b) vector versus the number of round trips. Plot of the RMS pulse width (c) semi-vector and (d) vector versus the number of round trips.

For both models, the SPDs are plotted in Fig. 4.9. The spectral bandwidths are 5.1 nm and 5.2 nm for the semi- and vector model, respectively. The SPD of the mode-locked pulses extracted from the semi-vector model is quite similar to the SPD of the total field using the vector model.

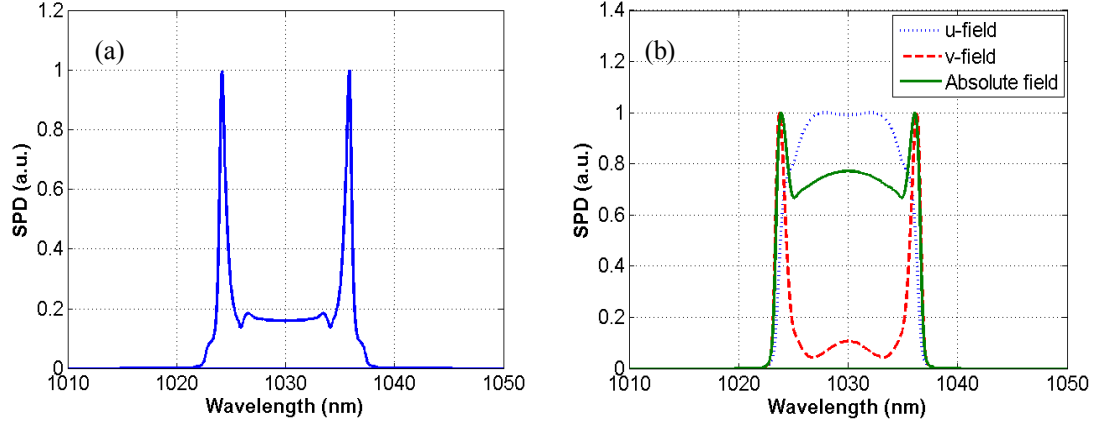


Fig. 4.6 Normalized SPD at coupler output using (a) semi-vector model (b) vector model; (dotted line) SPD of the u component, (dashed line) SPD of the v component and (solid line) SPD of the total field. The polarization controllers characteristic angles are ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 50^\circ$).

For the above two cases, the output chirped pulses from both models were fitted to Gaussian and sech^2 curves. Both functions were close to describing the output pulse. However, the Gaussian profile is much closer especially at the full width at half the maximum (FWHM) of the pulse.

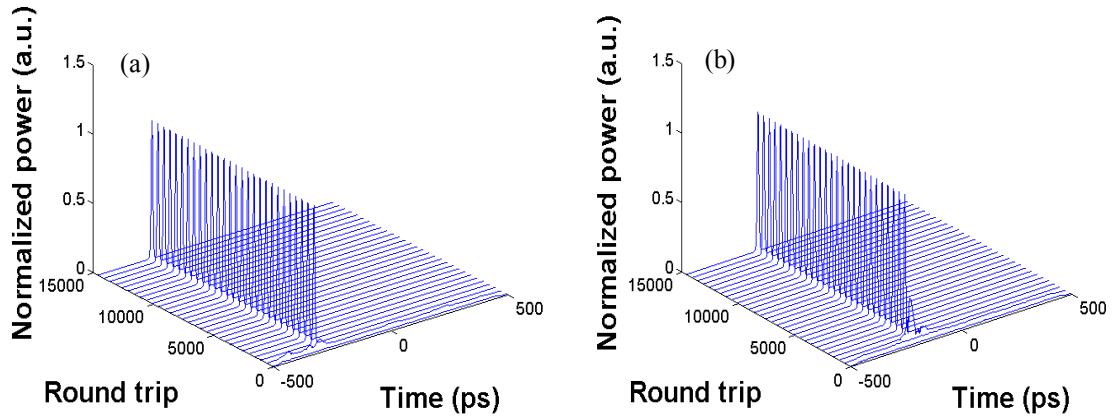


Fig. 4.7 Numerical simulation of self-starting mode-locked cavity at different round trips using the (a) semi-vector and (b) vector models. The location of the pulses is random as it is started from noise. In both models, the peak power is normalized with respect to the peak power in the previous case. The polarization controllers characteristic angles are ($\delta_1 = 70^\circ$, $\delta_2 = 135^\circ$, $\alpha_1 = 160^\circ$ and $\alpha_2 = 65^\circ$).

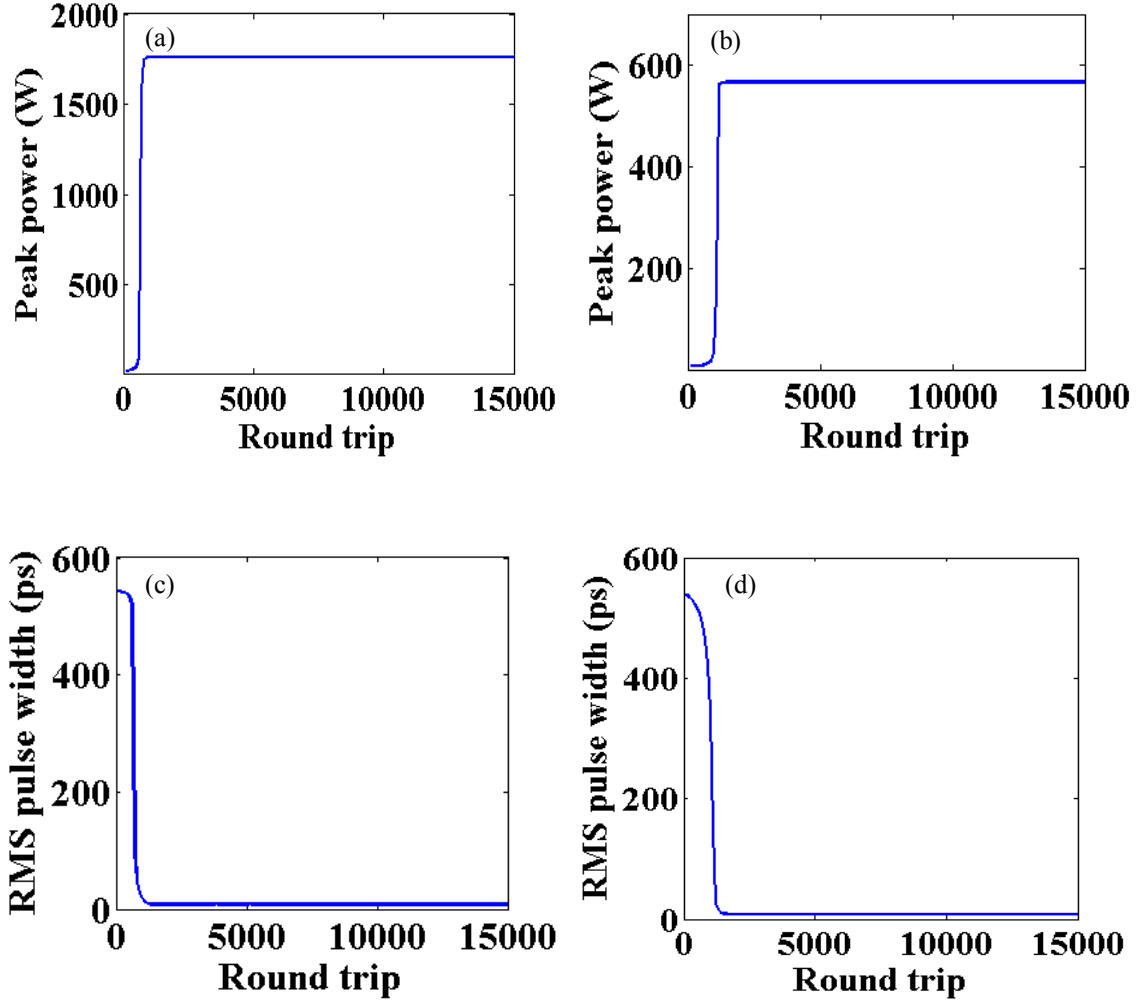


Fig. 4.8 Plot of the pulse peak power (a) semi-vector and (b) vector versus the number of round trips. Plot of the RMS pulse width (c) semi-vector and (d) vector versus the number of round trips.

For the same characteristic angles, double pulsed operation was observed at $E_{sat} = 6.25$ nJ for the semi-vector model, while for the vector model $E_{sat} = 2.25$ nJ. The evolution of the temporal profile with round trips for each of the two models at double pulse regime is plotted in Fig 4.10.

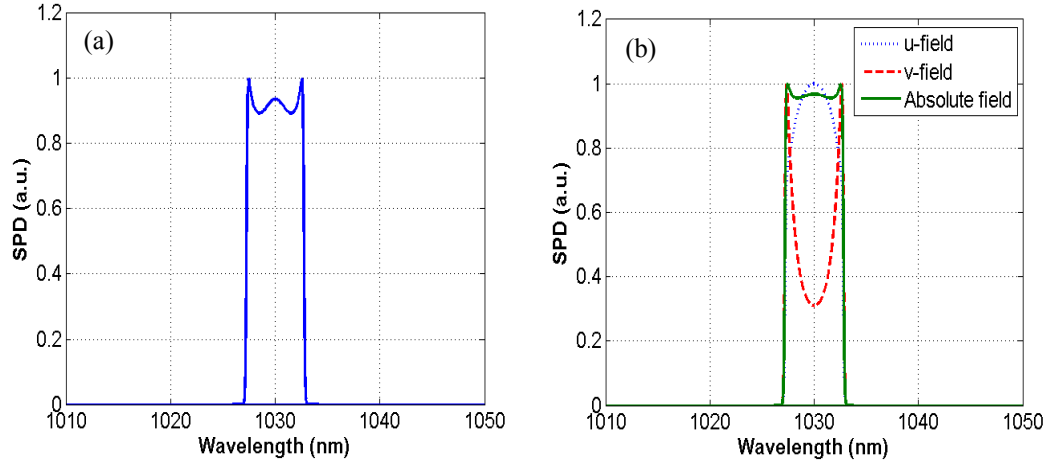


Fig. 4.9 Normalized SPD at coupler output using the (a) semi-vector model (b) vector model; (dotted line) SPD of the u component, (dashed line) SPD of the v component and (solid line) SPD of total field. The polarization controllers characteristic angles are $(\delta_1 = 70^\circ, \delta_2 = 135^\circ, \alpha_1 = 160^\circ$ and $\alpha_2 = 65^\circ)$.

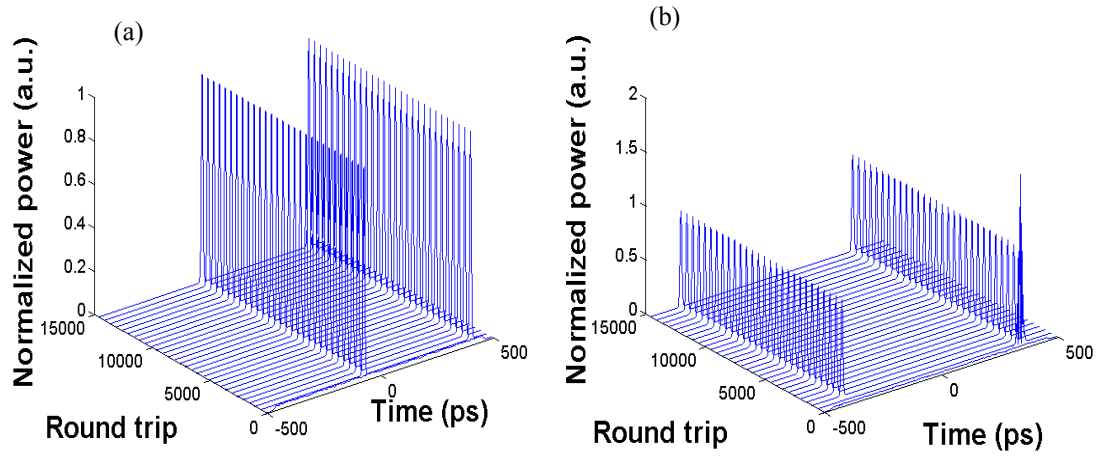


Fig. 4.10 Numerical simulation of self-starting mode-locked cavity at the double pulse regime (a) semi-vector and (b) vector model. In both models, the peak power is normalized with respect to the single pulse operation. The location of the pulses is random as it is started from noise. The polarization controllers characteristic angles are $(\delta_1 = 70^\circ, \delta_2 = 135^\circ, \alpha_1 = 160^\circ$ and $\alpha_2 = 65^\circ)$.

The dispersion delay line is a linear element which provides an anomalous dispersion required to compress the output chirped pulse to near its transform limit. The

normalized de-chirped pulses are plotted in Fig. 4.11(a) for Set 1. The FWHM was calculated to be 192.3 fs for the semi-vector model. For the vector model, the FWHM of the u- and v-components were calculated to be 247.2 fs and 189.2 fs, respectively. Fig. 4.11(b) shows the normalized de-chirped pulses of Set 2 at E_{sat} corresponding to the single pulse operation regime. The semi-vector model gave 564.6 fs, while the vector model gave 579.8 fs and 494.4 fs for the u- and v- components, respectively.

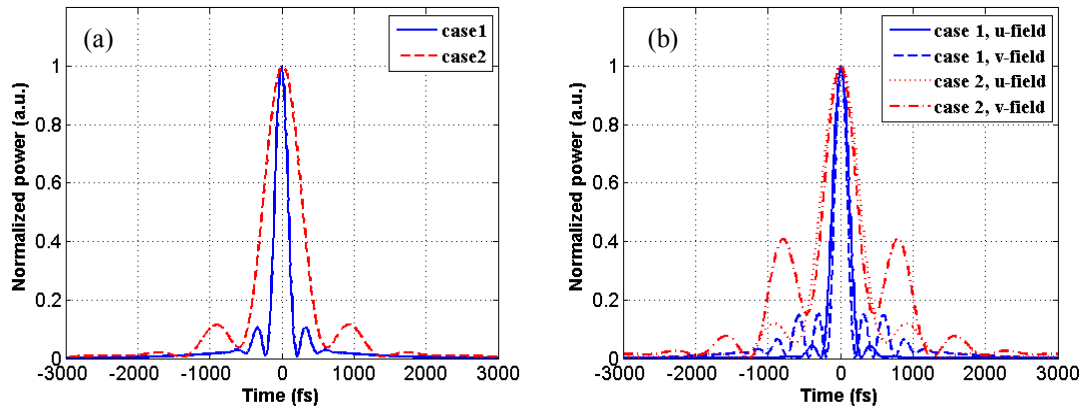


Fig. 4.11 Numerical temporal profile of the de-chirped pulses (a) semi-vector; (solid line) Set 1, (dashed line) Set 2 and (b) vector model. For Set 1; (solid line) u-component, (dashed line) v-component. For Set 2, (dotted line) u-component and (dashed and dot line) v-component.

For Set 3, E_{sat} is 5.75 nJ and 10 nJ for the vector and semi-vector model, respectively. No stable pulse was extracted from initial noise from either model. To confirm that stable mode locking at this set is impossible, a unity pulse was used to propagate through the different cavity components for 500 round trips to check if the cavity will lock to stable pulse or not. As shown in Fig. 4.12, the amplitude of the pulse decays with the increase of its background and finally, the pulse totally disappears.

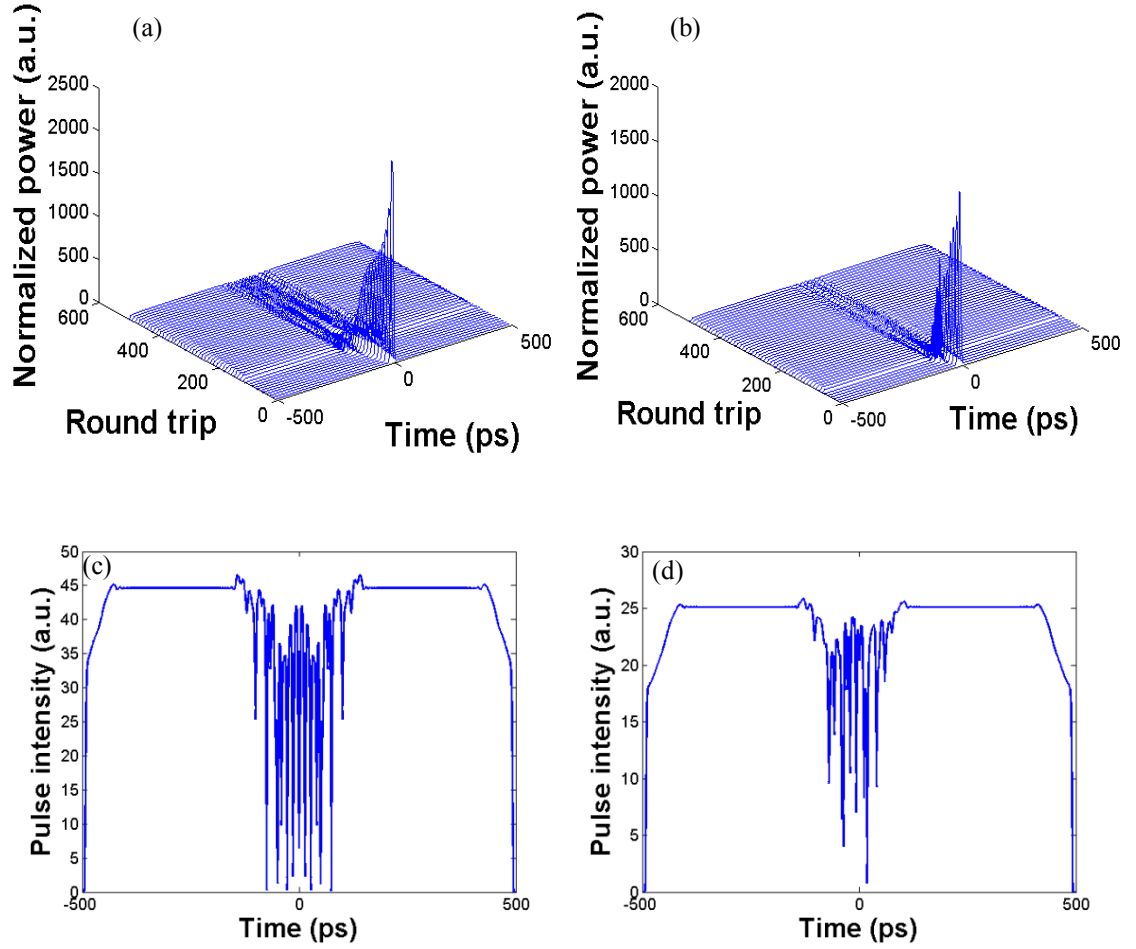


Fig. 4.12 Numerical simulation of the cavity with initial unity pulse and no mode locking (a) semi-vector and (b) vector model. The polarization controllers characteristic angles are ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 20^\circ$). The temporal profile at the last round trip is plotted for (c) semi-vector and (d) vector model.

4.3.1 Analysis of results and discussion

The effect of the NPR curve on the temporal and spectral characteristics of the mode-locked pulses is discussed in this section. The overdriving point of the NPR curve for Set 2 occurs at a lower value of the peak power than that of Set 1 (see Fig. 4.3(a)). As the peak power of the mode-locked pulses has a direct relation with the spectral bandwidth via SPM [19], the reduced permissible peak power of Set 2 shrinks the spectral bandwidth (well below the gain bandwidth of the Yb^{3+} -doped fiber) and widens

the temporal width of the generated pulse. If the pulse peak power is increased beyond the overdriving point, the mode-locked pulses enter an unstable regime, and the background noise is more energetic to form the second pulse. As a result, I have double pulse mode-locking and the peak power of each pulse is below the overdriving point. The same scenario happens to the n -pulse operation ($n > 2$). These results are in contrast to the work by Feng Li et al. who mentioned that the bandwidth limitation of the gain spectrum of the Yb^{3+} -doped fiber is the source of multi-pulsing [128].

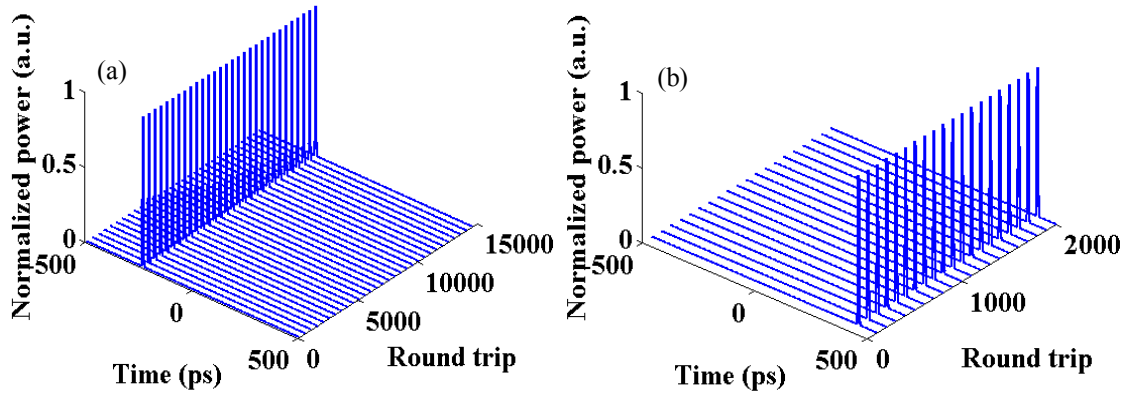


Fig. 4.13 Numerical simulation of a self-starting mode-locked cavity at different round trips using (a) semi-vector and (b) vector models. The spectral filter and The L_{SMFI} are swapped. The location of the pulses is random as it is started from noise. In both models, the peak power is normalized with respect to the peak power in the previous case. The polarization controllers characteristic angles are ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 50^\circ$).

The ratio of the peak power of the pulses in Set 1 to that of Set 2 is calculated to be 5.5 and 5.9 using the semi-vector and vector models, respectively. As explained in chapter 3, the spectral bandwidth of the filter affects the pulse dynamics inside the cavity. Above a certain spectral filtering bandwidth threshold, the narrower the spectral filtering bandwidth, the wider the spectrum bandwidth of the mode-locked pulse. However, below

this threshold multi-pulsing operation or unstable mode locking occurs, depending on the cavity parameters such as the length of each fiber section, the value of g_o and E_{sat} for the Yb^{3+} fiber and the characteristic angles of the polarization controllers.

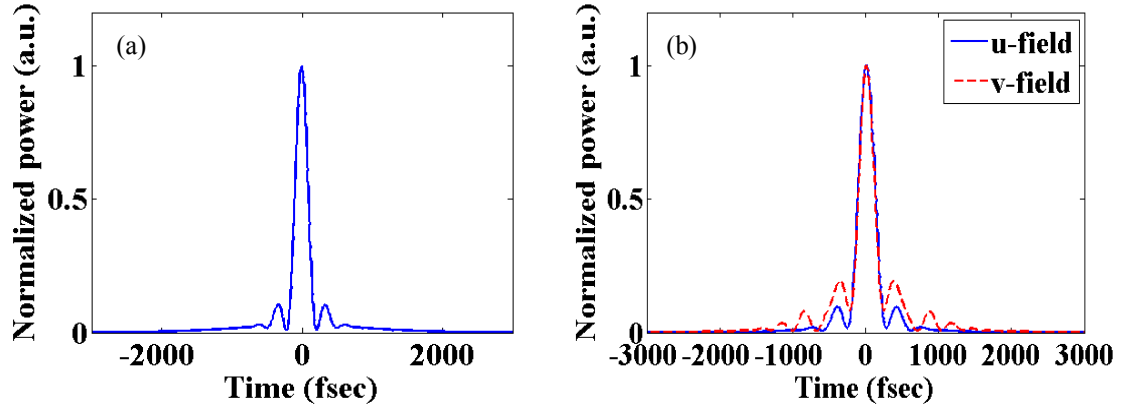


Fig. 4.14 Numerical temporal profile of the de-chirped pulses; (a) semi-vector and (b) vector models. The spectral filter and The L_{SMFI} are swapped. The polarization controllers characteristic angles are ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 50^\circ$). For this set of angles; the solid line is the u-component, and the dashed line is the v-component.

To verify the sensitivity of the semi-vector model to the cavity's component order, the positions of the SMF (L_{SMFI}) and the SF were exchanged. At Set 1, the maximum value of E_{sat} for stable mode locking becomes 9.5 nJ. The temporal as well as the spectral profile of the pulse is shown in Figures 4.13(a), 4.14(a) and 4.15(a). The temporal and the spectral profiles are similar to Figures 4.4(a), Fig. 4.6(a) and Fig. 4.11(a). However, the peak-to-peak spectral width is 11.4 nm and the de-chirped FWHM pulse width increases to 201.4 fs. For comparison, this modified cavity structure is also simulated using the vector model (see Figures 4.13(b), Fig. 4.14(b), and 4.15(b)). The maximum value of E_{sat} for stable mode locking is reduced to be 5.3 nJ. Also, the peak-to-peak spectral width is reduced to 10.5 nm. The FWHM of the u-component is kept the

same as was found in the original cavity. However, the v -component is increased to 228.9 fs. These results show that the semi-vector model is sensitive to the order of the cavity's components, which is in good agreement with the vector model.

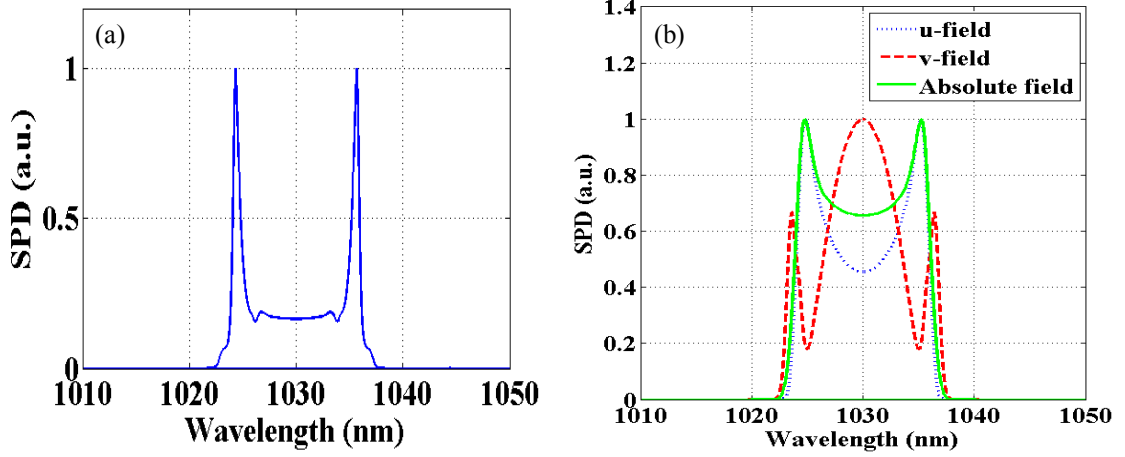


Fig. 4.15 Normalized SPD at coupler output using (a) semi-vector model (b) vector models; (dotted line) SPD of the u component, (dashed line) SPD of the v component and (solid line) SPD of total field. The spectral filter and The L_{SMFI} are swapped. The polarization controllers characteristic angles are ($\delta_1 = 50^\circ$, $\delta_2 = 160^\circ$, $\alpha_1 = 40^\circ$ and $\alpha_2 = 50^\circ$).

The assumptions shown in sub-section 4.2.1 are tested to check the validity of the semi-vector model. The same assumptions are also checked in the vector model as it is considered our reference. For the semi-vector model, the GVD length is 77 m and 935 m for Set 1 and Set 2, respectively. For the full vector model, this length is 437 m and 1266 m for Set 1 and Set 2, respectively. The TOD dispersion length is longer than the GVD lengths. The net gain length L_g is 260.3 m and 129.3 m using the semi- and vector models, respectively. All these values are much longer than L_N which validates the assumptions of our semi-vector model.

To be able to ignore the nonlinearity in the linear region, the ratio of the nonlinear length in the linear region to the nonlinear length in the nonlinear region should be greater than 10. It should be calculated in the vector model because the SPM and XPM are not neglected. Two types of nonlinear length are found in this type of laser cavity. The first one is due to SPM and the other one is due to NPR resulting from XPM. They are given as:

$$L_{NL_SPM} = L_h / \varphi_{NL_SPM} \quad (4.15a)$$

$$L_{NL_NPR} = L_h / \varphi_{NL_NPR} \quad (4.15b)$$

where h denotes Yb, SMF₂ and SMF₁. φ_{NL_SPM} , and φ_{NL_NPR} are maximum nonlinear phase shift due to SPM, and NPR, respectively. The former one is defined as:

$$\varphi_{max} = \gamma \int_{Fiber\ length} P_p(z) dz \quad (4.16)$$

while the later one is defined as:

$$\varphi_{NL_NPR} = \frac{\gamma}{3} \int_{Fiber\ length} (|u_+|^2 - |u_-|^2) dz \quad (4.17)$$

Here $|u_{\pm}|^2$ is the electric field in the circular polarization basis; (+) is the right hand circular and (-) is the left hand circular polarization. As shown in Table 4.1 and 4.2, this ratio is calculated for several values of the coupling ratio β_c using the vector model. It is found that β_c should be kept < 0.1 to satisfy the above condition.

Table 4.1 The ratio $L_{NL_SPM_SMF1}/(L_{NL_SPM_Yb} + L_{NL_SPM_SMF2})$ for various values of coupling ratio β_c

β_c	$\varphi_{NL_SPM_SMF1}$	$L_{NL_SPM_SMF1}$	$\varphi_{NL_SPM_Yb}$	$L_{NL_SPM_Yb}$	$\varphi_{NL_SPM_SMF2}$	$L_{NL_SPM_SMF2}$	Ratio
0.05	1.8286	4.3858	2.2540	0.1664	6.99	0.0558	19.742
0.1	3.8238	2.0974	2.6875	0.1395	7.3175	0.0533	10.877
0.15	5.2496	1.5277	2.8303	0.1325	7.2776	0.0536	8.2099
0.2	6.3181	1.2694	2.8964	0.1295	7.1953	0.0542	6.9110

Table 4.2 The ratio $L_{NL_NPR_SMF1}/(L_{NL_NPR_Yb} + L_{NL_NPR_SMF2})$ for various values of coupling ratio β_c

β_c	$\varphi_{NL_NPR_SMF1}$	$L_{NL_NPR_SMF1}$	$\varphi_{NL_NPR_Yb}$	$L_{NL_NPR_Yb}$	$\varphi_{NL_NPR_SMF2}$	$L_{NL_NPR_SMF2}$	Ratio
0.05	0.2075	38.65	0.26	1.442	0.8087	0.482	20.1
0.1	0.4356	18.41	0.3137	1.1195	0.8567	0.455	11.158
0.15	0.5988	13.39	0.3317	1.131	0.8551	0.456	8.4373
0.2	0.7209	11.125	0.3397	1.104	0.8463	0.461	7.109

4.4 Experimental results

An experimental set-up, similar to that shown in Fig. 4.1, was constructed to investigate the above simulation results. The Yb^{3+} fiber used in the experiment is Yb406 from CorActive. Standard SMF (HI1060) was used in this experiment. A fiber isolator (IO-K-1030 from Thorlabs) was inserted inside the cavity to ensure unidirectional propagation. A SF based on MMI was designed and constructed in our lab. The design background is shown in detail in section 2.2.5.2. A MMF section has length of 41.6 mm, the filter has 3 dB spectral width of 6.2 nm and its insertion loss is 2 dB (Fig. 2.8). Although the spectral profile of the filter does not have steep edges, it is sufficient to sustain the stability of the mode locked chirped pulse.

At the maximum rating of pump power (420 mW), the polarization controllers were adjusted to have self-starting stable single pulse mode-locking operation, which has the spectral characteristics of dissipative Soliton pulses in all normal dispersion regimes. Its SPD (shown in Fig. 4.16(a)) is very close to M-like shape spectrum with very steep edges. The peak-to-peak spectral bandwidth was measured to be 10 nm. This spectral profile is similar to simulation results shown in Fig. 4.6. The asymmetry in Fig. 4.16(a) is mainly due to the asymmetry in the SF as shown in Fig. 2.8(b). This means that the characteristic angles provided by the inline fiber polarization controllers have the same impact as the angles used to get the temporal and spectral profile in Fig 4.4 and Fig. 4.6, respectively. The experimental set-up implemented to characterize the temporal and spectral profiles of the pulse is described in Appendix B. The output chirped pulse is directed at the 90% output port of the coupler. The average power is 180.9 mW, the repetition rate is 23.7 MHz, and the energy is 7.6 nJ. The autocorrelation of the chirped pulse is presented in Fig. 4.17(a). The FWHM pulse width of the autocorrelation signal is 9.438 ps. With prior knowledge of having Gaussian pulse, the pulse peak power was calculated to be 1.02 KW [48]. No multi-pulsing operation was observed under these conditions.

At another setting of the polarization controllers, another mode-locking spectrum was found; its SPD is shown in Fig. 4.16(b). It has spectral profile near to the results shown in Fig. 4.9. The peak-to-peak bandwidth was measured to be 4.1 nm. As stated above, the setting of the polarization controllers gave the same effect as the results in Fig. 4.7 and Fig. 4.9. The maximum pump power for single pulse operation was 210 mW. When the pump power was increased above this value, the second harmonic of the

optical pulses (with repetition rate 47.42 MHz) would appear. This means that the cavity parameters cannot support one pulse per period. This agrees with the simulation models which show that the maximum value of E_{sat} for stable mode-locking is lower than the mode-locking regime with wider spectrum. The average power at the coupler output is 56.6 mW, hence the energy is 2.4 nJ. The autocorrelation of the chirped pulse is plotted in Fig. 4.17(b).

The pulse width of the autocorrelation signal was measured to be 12.9 ps. With prior knowledge of having Gaussian pulse, the pulse peak power was calculated to be 244.5 W. The peak power ratio was around 4.4:1 for the two settings of the polarization controllers. Although this ratio was lower than the calculated in the simulation results, it showed the same trend as in the simulation models.

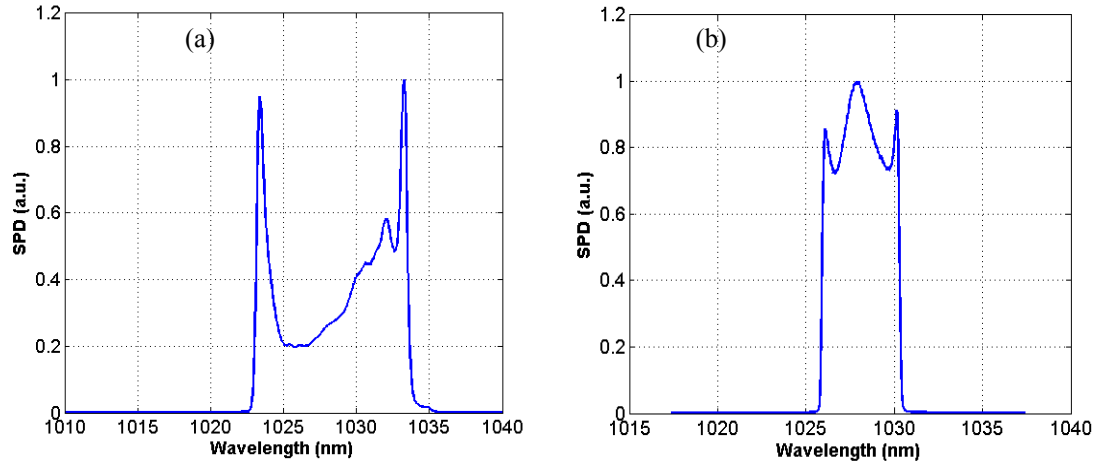


Fig. 4.16 (a) M-shaped spectrum of bandwidth 9.96 nm, (b) narrow spectrum with two peaks at its edges of bandwidth 4.1 nm. The characteristic angles of the polarization controllers control the spectral profile of the pulse.

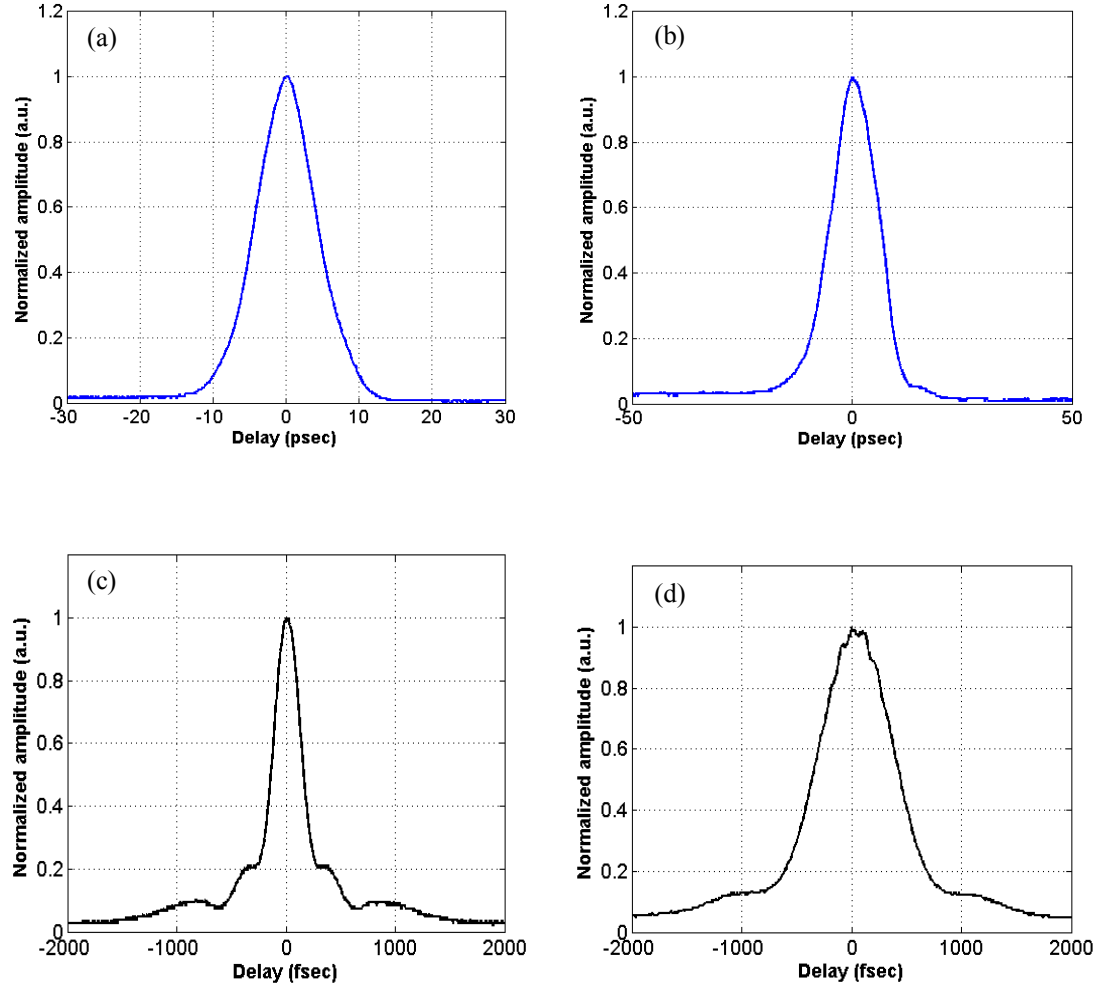


Fig. 4.17 Experimental autocorrelation signal of the chirped and de-chirped pulses at different characteristic angles; the autocorrelation signals of the chirped pulse (a) and de-chirped pulse (c) belong to the results in Fig. 4.16(a), while the autocorrelation signals of the chirped pulse (b) and de-chirped pulse (d) belong to the results in Fig. 4.16(b).

The coupler output chirped pulses were compressed via a pair of diffraction gratings outside the cavity providing anomalous dispersion. The autocorrelation profiles of the de-chirped pulses are shown in Figures 4.17(c) and 4.17(d). Assuming Gaussian profile, the FWHM pulse width of the de-chirped pulse was measured to be 217 fs and 574 fs for Set 1 and Set 2, respectively.

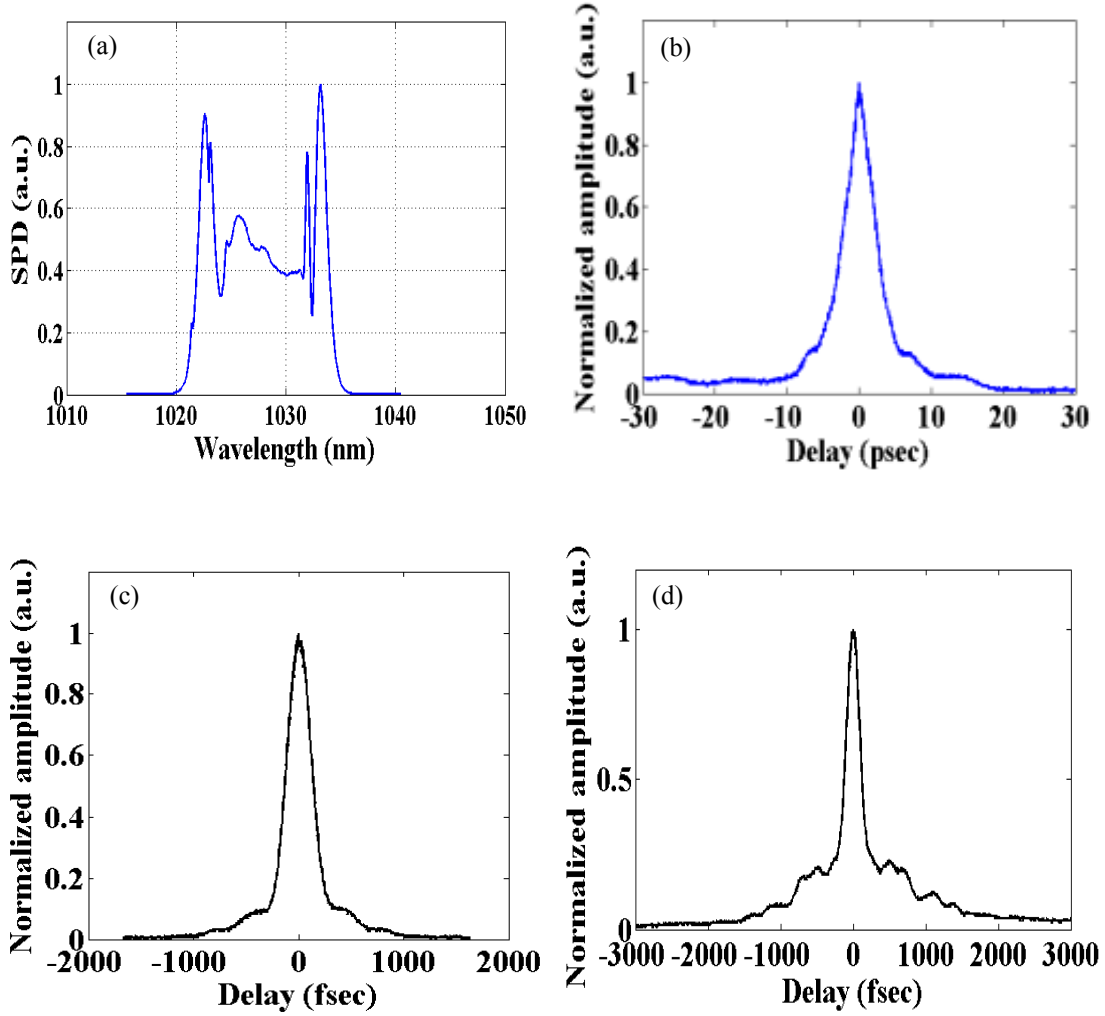


Fig. 4.18 (a) M-shaped spectrum of bandwidth 10.96 nm and (b) the autocorrelation signal of the chirped pulse. The autocorrelation signal of the de-chirped pulse using (c) a pair of diffraction gratings and (d) PBG fiber.

Finally, the spectral filter and L_{SMFI} were swapped to check the impact on the spectral and temporal characteristics of the pulse. At the maximum rating of the pump power, the polarization controllers were adjusted to get self-starting single pulse mode locking, and the peak-to-peak spectral bandwidth was measured to be 11 nm (Fig. 4.18(a)). Fig. 4.18(b) shows the autocorrelation of the chirped pulse. The autocorrelation of the chirped pulse was measured to 6.5 ps. A pair of diffraction gratings having 900

grooves per mm is used to compress the chirped pulse shown in Fig. 4.18(c). The FWHM pulse width of the de-chirped pulse was calculated to be 192.3 fs assuming a Gaussian profile. To have an all-fiber laser cavity, a PBG fiber, HC-1060, of length 3.9 m replaced the pair of diffraction gratings. Its length was selected to compensate for the positive chirp of the pulse. The splicing method illustrated in [136] was used to minimize the splicing loss (~ 0.1 dB) with HI-1060. Fig. 4.18(d) shows the autocorrelation of the pulse. The FWHM of the de-chirped pulse was calculated by assuming the Gaussian profile to be 170.8 fs.

It is worth mentioning that it is quite difficult to adjust the inline fiber polarization controllers to have the same angles mentioned in the simulation results. Consequently, I scanned all possible orientations of the polarization controllers till I found the settings of the spectral profiles that are similar to what is shown in Figures 4.6, 4.9 and 4.15. Therefore, with the help of the semi-vector model, I was able to understand the reasons behind having different temporal and spectral profiles of mode locked pulses. If the manual polarization controllers are replaced by electrically driven ones, they can, potentially, be calibrated to have a relation between the applied voltages and the characteristic angles.

4.5 Conclusion

An efficient and fast semi-vector model for mode-locking based on NPR in all-fiber femtosecond laser is presented in this chapter. The model is based on simplifying the laser cavity to consist of only two blocks; the NPR and the scalar portion of the cavity. The NPR block includes the following: 1-the nonlinear transfer function of the nonlinear fiber section of the mode-locked laser cavity, 2-the Jones matrices of the polarization

controllers and polarizer, 3-the dispersion effect of the linear section, and 4-the filter transfer function. The nonlinear transfer function of the nonlinear fiber section inside the NPR block does not contain the SPM, dispersion, and gain parameters. They are added to the scalar portion of the cavity, which solves numerically the scalar GLE for the nonlinear section. The effects of the gain, dispersion of the fiber in the nonlinear and linear regions, and the lumped spectral filter are omitted to derive a closed form relation of NPR between the field circulating the mode-locked laser cavity at instant (n) and instant (n+1). This enables us to investigate different NPR curves. Three cases at different sets of polarization controllers' angles using both the semi- and the full-vector models for comparison are studied. The semi-vector model enables us to show the effect of the location of the overdriving point on the pulse energy, spectral bandwidth, and de-chirped pulse width. Changing the order between the linear fiber and the lumped SF is also studied using the semi-vector model. Good agreement is observed between the results of the proposed model and both the vector model and the experimental set-up, which demonstrates its efficiency. However, as it is under the category of average models, the full details of the temporal and spectral profile of the mode-locked pulse cannot be depicted in the semi-vector model. The new method takes 1/18 of the time needed by the vector method for the same computational platform.

Chapter 5

5 Peak power optimization of optical pulses using low-doped gain-medium in femtosecond fiber laser³

Chapter 2 and 3 mainly focused on the effect of the SA parameters on the pulse energy, spectral bandwidth and de-chirped pulse width. However, understanding the SA and optimizing its design does not prevent the occurrence of optical wave breaking due to accumulated nonlinearity. Therefore, the scope of this chapter is designing a femtosecond fiber laser that pushes the onset of wave breaking, which results in an increase in power and energy. In contrast to the conventional approach discussed in section 2.3 where the gain medium consists of a short length of high-doped gain fiber [12, 80], this chapter shows both numerically and experimentally that we can use low-doped long fiber length to increase the peak power of the generated laser pulses. Conceptually, as the nonlinearity is gradually accumulated inside the low-doped Yb^{3+} fiber, the peak power of the generated pulse is increased before hitting the threshold of multi-pulsing or wave breaking. The accumulated nonlinear phase shift as well as its threshold value for single-pulse operation was studied and used to explain the robustness of such a cavity against wave breaking.

Two cavities were implemented; one used a high-doped short length of Yb^{3+} fiber while the other cavity used a low-doped long length of Yb^{3+} fiber. Passive Similariton femtosecond fiber laser is selected for two reasons: (1) we can implement the mode-locked laser cavity at different values of net GVD at fixed repetition rate. (2) Similariton

³ Most of the results in this chapter have been published in [I].

pulses have linear chirp, which results in narrow compressed pulses close to the transform limit [80].

Neither the average model implemented by V. G. Bucklew *et al.* [110] nor the one described in Chapter 3 can be used to demonstrate the effect of low-doped long-length Yb^{3+} fiber on the pulse peak power because these models average all cavity parameters along the whole length of the cavity. Therefore, the scalar simulation model, discussed in sub-section 2.1.1, is used instead.

The chapter is organized as follows; section 5.1 explains the concept of increasing the peak power. Simulation results of both cavities are presented in section 5.2. Section 5.3 focuses on the experimental set-ups and the measurement results. In this section the two experimental set-ups of the ring fiber cavity are demonstrated, both of them use a SESAM as real mode-locker. A comparison between the low-doped long-length Yb^{3+} fibers and high-doped short-length Yb^{3+} fibers is also reported in this section.

5.1 Theoretical background

The accumulated nonlinear phase shift (φ) is the main obstacle that hinders the generation of high-energetic pulses and causes optical wave breaking. Its maximum value (φ_{max}) for single mode operation is given in (4.16).

The peak power should increase when the low-doped long-length gain fiber is used because the rate of accumulation of the nonlinear phase shift gradually ramps inside the gain fiber. Fig. 5.1 is a cartoon depicting a comparison between the evolutions of peak power for the case of high- and low-doped Yb^{3+} gain fiber operating on the edge of the single pulse operation. For simplicity, the input peak power P_{in} of Yb^{3+} -doped fibers

after SMF₁ in the two cases is assumed to be equal. The small signal gain coefficient is used to derive a closed form relation for $\varphi_{\max}$ in the high- and low-doped cases, to be respectively:

$$\varphi_{\max_h} \approx \gamma P_{\text{in}} \exp(g_h L_{Yb_h}) \left[\frac{1}{g_{o_h}} + L_{SMF_{2h}} \right] \quad (5.1)$$

$$\varphi_{\max_l} \approx \gamma P_{\text{in}} \exp(g_l L_{Yb_l}) \left[\frac{1}{g_{o_l}} + L_{SMF_{2l}} \right] \quad (5.2)$$

where, $g_{o_h(l)}$ is the small signal gain coefficient for the high (low) doped Yb³⁺-doped fiber, $L_{Yb_h(l)}$ is the length of the high (low) doped Yb³⁺-doped fiber, and $L_{SMF_{2_h(l)}}$ is the length of SMF₂ for the cavity with high (low) doped Yb³⁺-doped fiber.

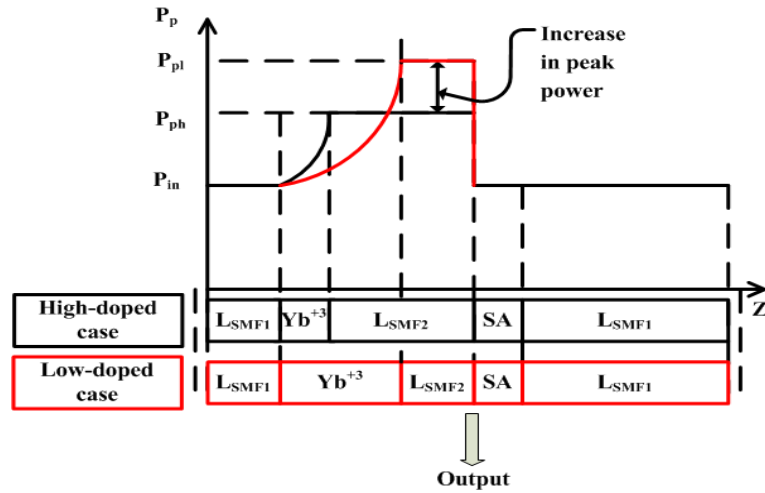


Fig. 5.1 Cartoon of peak power evolution in the cavity through the Yb³⁺, SMF₁, saturable absorber (SA) and SMF₂ for the two cases. The short Yb³⁺ fiber with length L_{Yb_h} has higher gain per unit length than long Yb³⁺ fiber with length L_{Yb_l} . The low-doped case can reach higher peak powers due to the slow accumulation of nonlinear phase shift. The SA effect is neglected to emphasize the proposed concept.

To have a fair comparison, it is assumed that, $\varphi_{\max_l} = \varphi_{\max_h}$ for single pulse operation hence the area under both curves should be the same, in this case the peak power before the cavity output is found to be higher for the low-doped Yb³⁺ fiber than

that of the high-doped one. However, as will be shown in simulation and experimental results, the cavity with low-doped Yb^{3+} fiber can tolerate higher values of nonlinear phase shift than the one with high-doped Yb^{3+} fiber.

Having equal $\varphi_{\max}$, the ratio between the peak powers of the two cases after the gain medium is expressed as

$$\frac{P_{p,l}}{P_{p,h}} = \frac{\left[\frac{1}{g_{o,h}} + L_{\text{SMF}_{2h}} \right]}{\left[\frac{1}{g_{o,l}} + L_{\text{SMF}_{2l}} \right]} \quad (5.3)$$

Hence the following condition has to be satisfied to get higher peak power using lower doping gain fiber at the same cavity conditions;

$$g_{o,h} > g_{o,l} > \frac{1}{L_{\text{SMF}_{2h}} - L_{\text{SMF}_{2l}}} \quad (5.4)$$

5.2 Simulation results

5.2.1 Numerical model description

The laser cavity structure used in the simulation model is similar to Fig. 3.1(b), except that a dispersion compensation element replaces the SF. It consists of an Yb^{3+} -doped gain medium, followed by nonlinear SMF and following linear SMF. Equation (2.12) for the pair of diffraction gratings is used to model the dispersion compensation element. The net cavity GVD is specified by the anomalous dispersion of the pair of diffraction gratings as well the total length of the fibers. The net cavity GVD is specified by the anomalous dispersion of the pair of diffraction gratings as well the total dispersion of the fibers. Mode locking is obtained using a lumped SA. The light propagating in the Yb^{3+} -doped fiber or SMF is modeled by (2.3) [19, 24, 80].

The laser cavity parameters are selected as: α_f is the linear loss taken as 0.04 m^{-1} , β_2 is the GVD parameter taken as $24690.6 \text{ fs}^2/\text{m}$ for SMF, and $24537.2 \text{ fs}^2/\text{m}$, and $23651.5 \text{ fs}^2/\text{m}$ for high- and low-doped Yb^{3+} fibers, respectively. For simplicity, the TOD is assumed to be zero. The nonlinear parameter γ is taken as $0.005 \text{ W}^{-1}\text{m}^{-1}$. L_{SMF1} is taken to be 3.53 m . L_{SMF2} was varied to ensure equal cavity length and hence the same repetition rate. It has value of 1.43 m for the laser cavity having high-doped Yb^{3+} fiber of length 0.25 m , while its value is 0.97 m for the laser cavity having low-doped Yb^{3+} fiber of length 0.73 m . E_{sat} is the gain medium saturation energy taken as 4 nJ . g_0 is the small signal power gain coefficient with parabolic frequency dependence and a bandwidth Ω_g of 50 nm .

The SA is modeled as an instantaneous intensity dependence transmission medium, as described in (2.20). Two different parameters of the SA were used; the first one does not have an overdriving point ($P_{TPA} \rightarrow \infty$), as was used in previous models of passive Similariton lasers [80, 83]. The values of q_o , L_{ns} and P_{sat} are taken from [80] to be 0.3 , zero and 2 KW . The parameters of the other SA are taken from the datasheet of SAM-1030-30-500fs-4.0-25.4s-c purchased from Batop, which is used in the experimental set-up. The values of q_o , F_s , T_{sa} and L_{ns} are given as: 0.3 , $120 \mu\text{J}/\text{cm}^2$, 500 fs and 0.1 , respectively. The value of P_{TPA} is taken to be 5 KW . A_{eff} is calculated to be $93.8 \mu\text{m}^2$, which leads to a value of P_{sat} equal to 225.13 W . For the rest of this chapter, the mode-locked laser cavity having the first SA is labeled as SA_1 , while the one having the other SA is labeled as SA_2 .

The relaxation time of the SA has a negligible effect on such a wide chirped pulse propagating inside the cavity. The total insertion loss of all cavity components including

that of the gratings and the light output port are lumped at the diffraction gratings; it ranges from 8 to 10 dB. A very small noise pulse has been used to propagate through the different cavity components for a few thousand roundtrips to reach a stable mode-locked pulse operation at certain cavity conditions.

5.2.2 Simulation results and discussion

To ensure that the nonlinear phase shift is the same for both the low- and high-doped cases, g_{o_l} value is calculated from (5.1) through (5.4) based on the corresponding g_{o_h} value at its maximum nonlinear phase shift $\varphi_{\max_h}$. The threshold of wave breaking is higher for high values of net GVD as the pulse is widely chirped, so the cavity can operate at single-pulse operation with higher values of g_o [80].

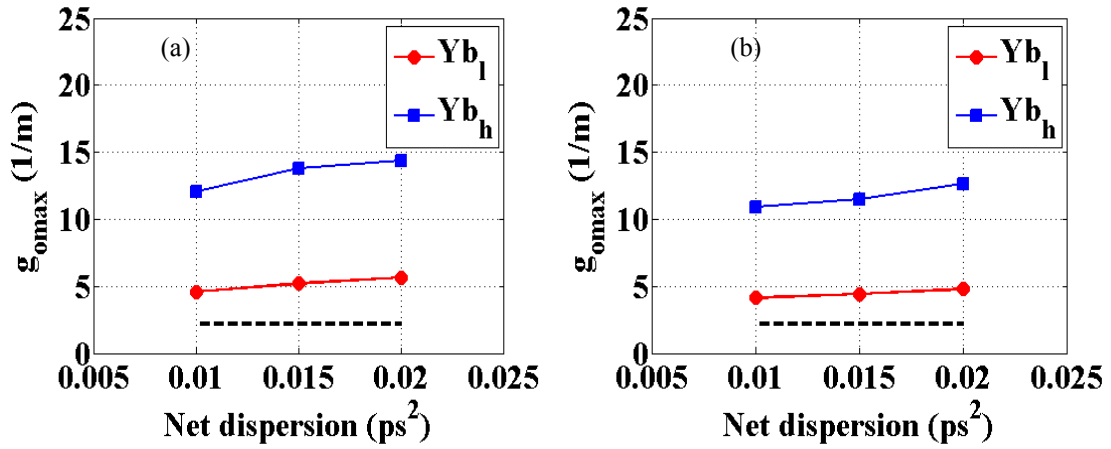


Fig. 5.2 Change of the maximum small signal gain ($g_{\max}$) with net dispersion for (a) SA₁ and (b) SA₂ having low- (Yb_l) and high- (Yb_h) doped Yb³⁺ fibers. The dashed line represents the lower limit of (5.4) ($1/(L_{\text{SMF2h}} - L_{\text{SMF2l}})$).

The change of the maximum small signal power gain coefficient ($g_{\max}$) for single pulse operation just before optical wave breaking with the net dispersion is plotted in Figures 5.2(a) and 5.2(b) for SA₁ and SA₂, respectively. For comparison purposes, the

variation of the maximum energy of the single-pulse operation ($E_{\max}$) with different values of net dispersion for the two types of Yb^{3+} -doped fiber is plotted in Figures 5.3(a) and 5.3(b) for SA_1 and SA_2 , respectively. Fig. 5.4(a) shows the variation of the maximum peak power ($P_{\max}$) at the end of L_{SMF2} with net dispersion for SA_1 , while $P_{\max}$ is plotted as a function of net dispersion for SA_2 in Fig. 5.4(b).

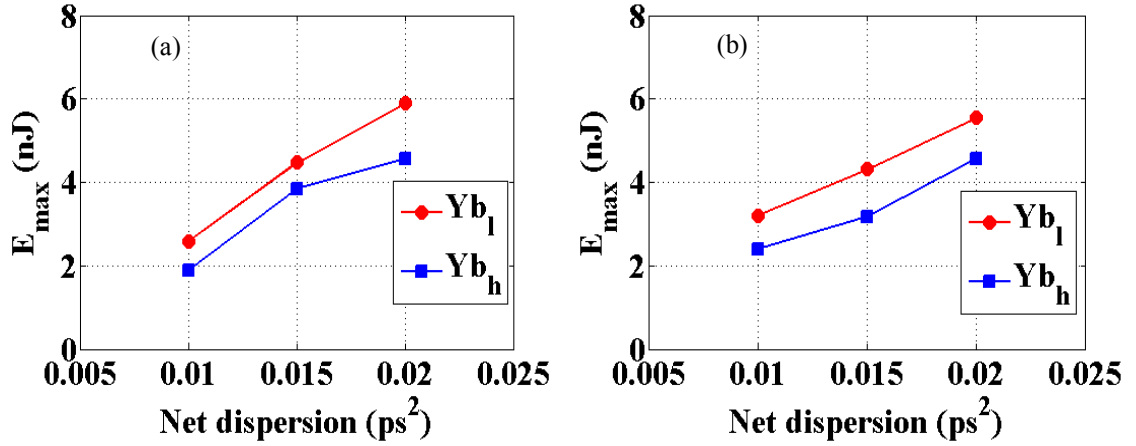


Fig. 5.3 Change of the maximum energy ($E_{\max}$) with net dispersion for (a) SA_1 and (b) SA_2 having low- (Yb_l) and high- (Yb_h) doped Yb^{3+} fibers.

The values of $g_{\max}$ for the cavity with the high-doped Yb^{3+} fiber are higher than the corresponding values for the low-doped one. Also, the values of $g_{\max}$ are higher than the lower bound of (5.4) (see Figures 5.2(a) and 5.2(b)). This agrees with the inequality given in (5.4). The cavity with low-doped Yb^{3+} fiber provides higher energy and peak power for single pulse operation compared to that of the high-doped one. Although the maximum energy in Figures 5.3(a) and 5.3(b) increases monotonically with the net dispersion, the peak power in Figures 5.4(a) and 5.4(b) does not. This is because both the energy and chirped pulse width are increasing with net dispersion at different rates (see Figures 5.5(a) and 5.5(b)). For the energy variation, a trend similar to these results is

found in the work of Ilday, *et al.* [80].

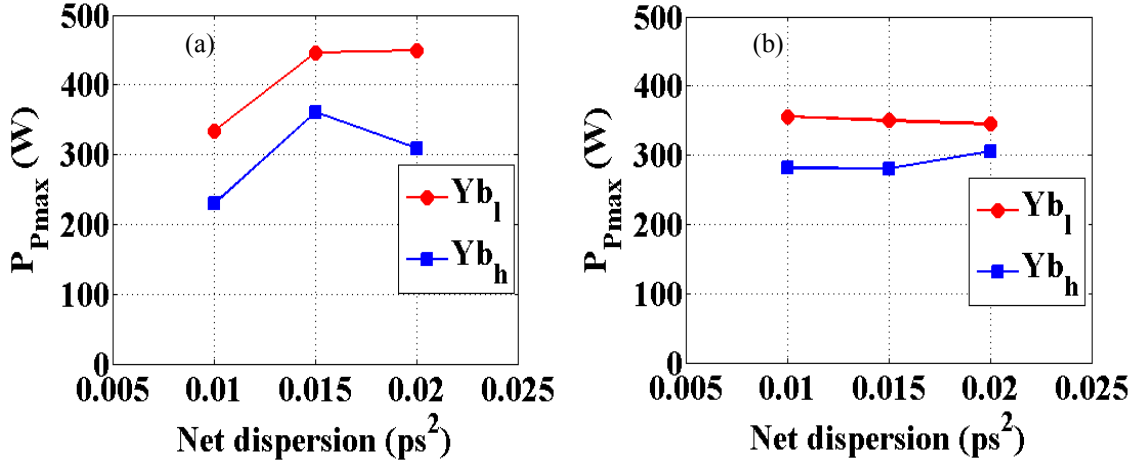


Fig. 5.4 Change of the maximum peak power (P_{pmax}) with net dispersion for (a) SA_1 and (b) SA_2 having low- (Yb_l) and high- (Yb_h) doped Yb^{3+} fibers.

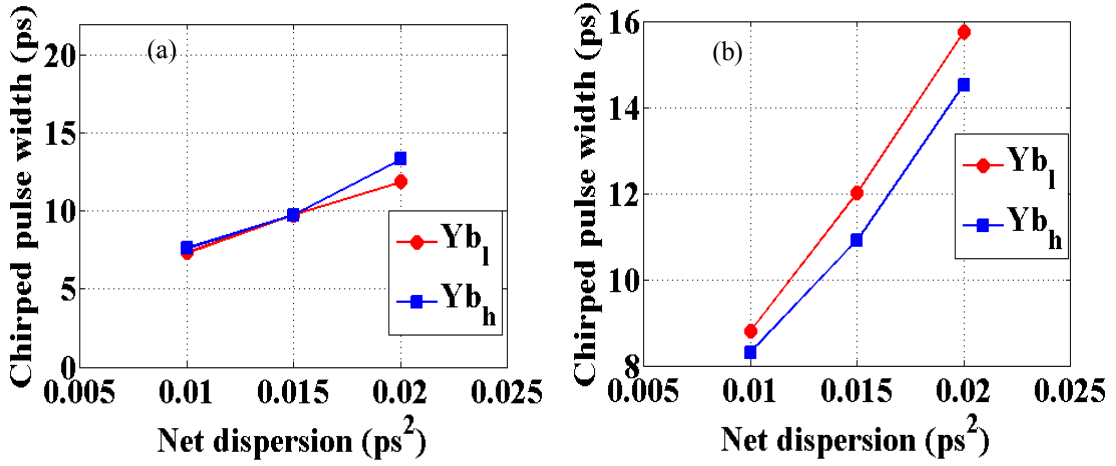


Fig. 5.5 Change of the full width at half maximum of the chirped pulse with net dispersion for (a) SA_1 and (b) SA_2 having low- (Yb_l) and high- (Yb_h) doped Yb^{3+} fibers.

The ratio between the values of P_{pmax} , extracted from the simulation model, in the laser cavity with low-doped Yb^{3+} fiber to the corresponding values in the high-doped one is plotted in Figures 5.6(a) and 5.6(b) for SA_1 and SA_2 , respectively. Equation (5.3) is plotted in the two figures for the sake of comparison. It is clear that (5.3) provides a good

estimation of the ratio between P_{pmax} in both cavities.

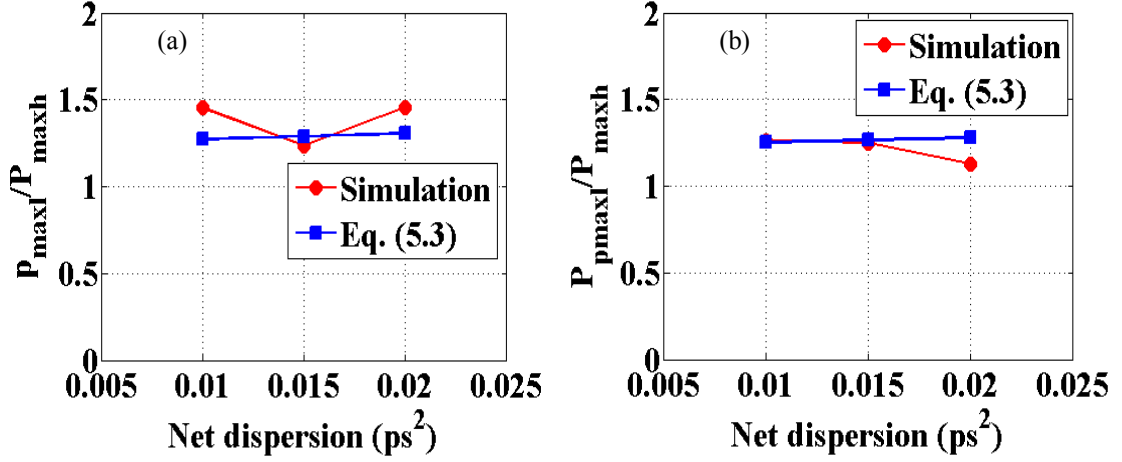


Fig. 5.6 Change of the ratio between P_{pmax} of the low- to the high- doped Yb^{3+} fibers with net dispersion for (a) SA_1 and (b) SA_2 having low- (Yb_l) and high- (Yb_h) doped Yb^{3+} fibers.

The spectral profiles of the laser pulses at net dispersion equal to 0.02 ps^2 after the L_{SMF2} for the two types of the Yb^{3+} fibers are plotted in Fig. 5.7(a) for SA_1 and Fig. 5.7(b) for SA_2 . The steep edges of the spectra with parabolic tops as well as the positive chirp of the cavity overall are signatures of Similariton operation [80, 85]. It is clear that the SPD of SA_2 is closer to a parabolic spectrum compared to that of SA_1 . As mentioned in [110], the saturation power of the SA plays an important role in the formation of the spectral profile of the pulse. As the SA saturation power decreases, the pulse tends to evolve more like a parabolic pulse. The excess kurtosis factor is used to indicate the profile of the pulse in the cavity. The excess kurtosis is -1.2, -0.86, 0, and 1.2 for rectangular, ideal parabola, Gaussian and Sech^2 profiles, respectively [83]. The excess kurtosis of the SPD is plotted for SA_1 and SA_2 in Figures 5.7(c) and 5.7(d), respectively. Apparently from Fig. 5.7(d), the spectral evolution is closer to -0.86 than the corresponding spectral

evolution in Fig. 5.7(c). Therefore, the SPD in SA₂ approaches the parabolic profile through all parts of the cavity.

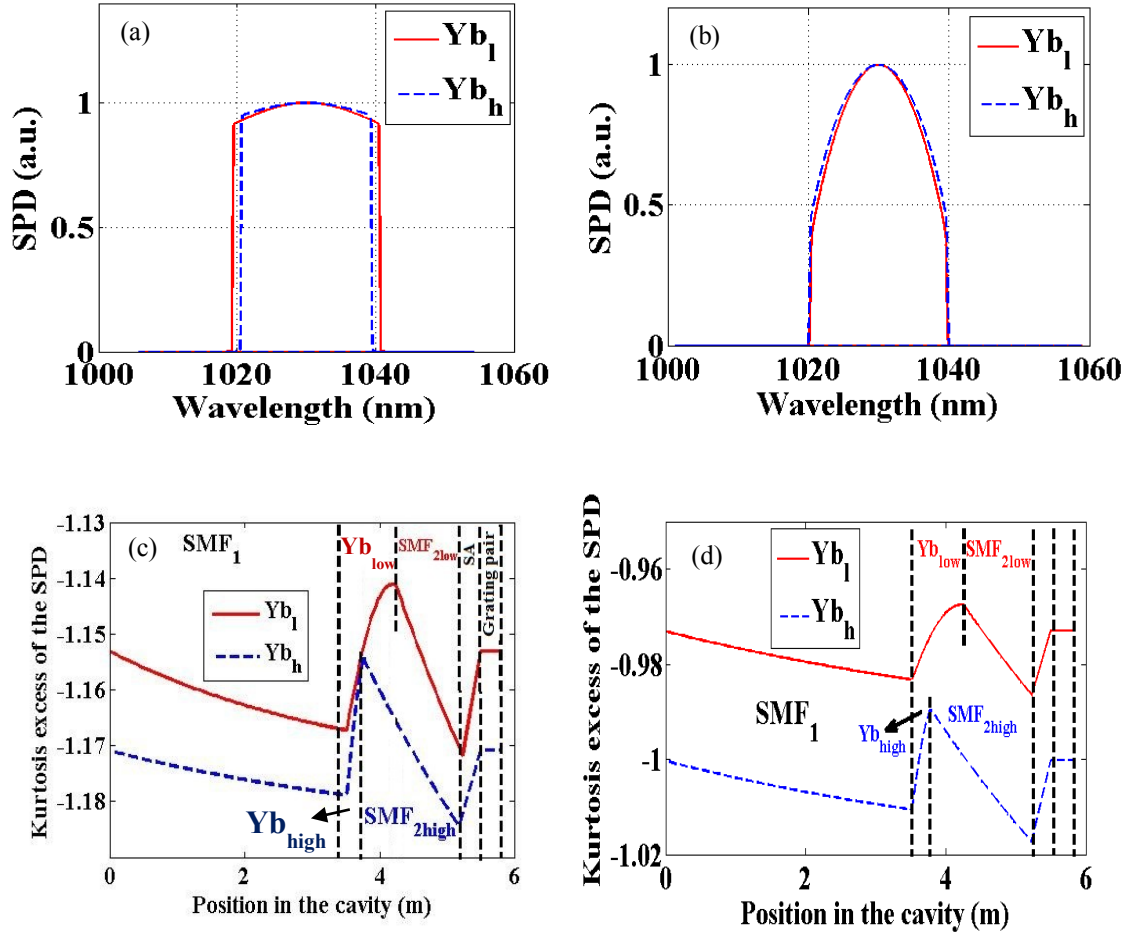


Fig. 5.7 Normalized SPD of the two cavities at net dispersion 0.02 ps^2 : (a) SA₁ and (b) SA₂. Evolution of the excess kurtosis factor of the spectrum power density (SPD) for both cavities: (c) SA₁ and (d) SA₂.

At different values of net dispersion and at the edge of single pulse operation, φ_{max_h} and φ_{max_l} are also calculated based on (4.16) and plotted in Figures 5.8(a) and 5.8(b) for SA₁ and SA₂, respectively. Except for SA₂ at net dispersion equal to 0.02 ps^2 , the cavity with low-doped Yb³⁺ fiber can accumulate higher values of φ_{max} before wave breaking (i.e. $\varphi_{\text{max}_l} > \varphi_{\text{max}_h}$). The reduction of φ_{max} at net dispersion equal to 0.02 ps^2

for SA₂ is due the overdriving point of the SA (see Fig. 5.8(c) point a). It is located at an input power of 328 W. As shown in (4.16), $\varphi_{\max}$ is directly proportional to the pulse peak power. Therefore, the peak power of the pulse cannot exceed the overdriving point. Otherwise, as shown in Fig. 2.11, the pulse will be distorted. The diffraction gratings and SA are swapped to check the validity of the above claim. As mentioned above, the loss of the cavity is lumped at the diffraction grating. Therefore, the peak power of the pulse at the input of the SA is lower than the location of the overdriving point. As shown in Fig. 5.8(d), $\varphi_{\max}$ has a higher value for the cavity with the low-doped Yb³⁺ fiber than that of the other cavity. This shows that, indeed, the SA can limit the possible maximum achievable power. For the sake of comparison, the nonlinear transmission of the SA for SA₁ is shown in Fig. 5.8(c); it does not have any overdriving point that limits the pulse peak power.

Another limitation for increasing the pulse peak power using a long piece of a low-doped Yb³⁺ fiber is occurred when the spectral bandwidth of the pulse becomes very close to the gain bandwidth of the Yb³⁺ fiber. In this case the cavity will become unstable to have a single pulse operation and operate in the multi-pulse regime [128].

The most interesting part of the simulation is the ability to monitor the peak power evolution inside the fiber. Figures 5.9(a) and 5.9(b) show the peak power evolution inside the nonlinear part of the cavity including Yb³⁺ and L_{SMF2} at net dispersion of 0.02 ps² for SA₁ and SA₂, respectively. The peak power reduction in L_{SMF2} results mainly from the fiber dispersion. However this reduction is not large at high net dispersion due to a small breathing ratio [80]. The peak power at the end of L_{Yb} is higher for the low-doped fiber compared to the high-doped one. This also verifies that the increase in the peak

power at the end of L_{SMF2} is mainly due to the low-doped Yb^{3+} fiber, not due to the dispersion in the long L_{SMF2} after the high-doped Yb^{3+} fiber.

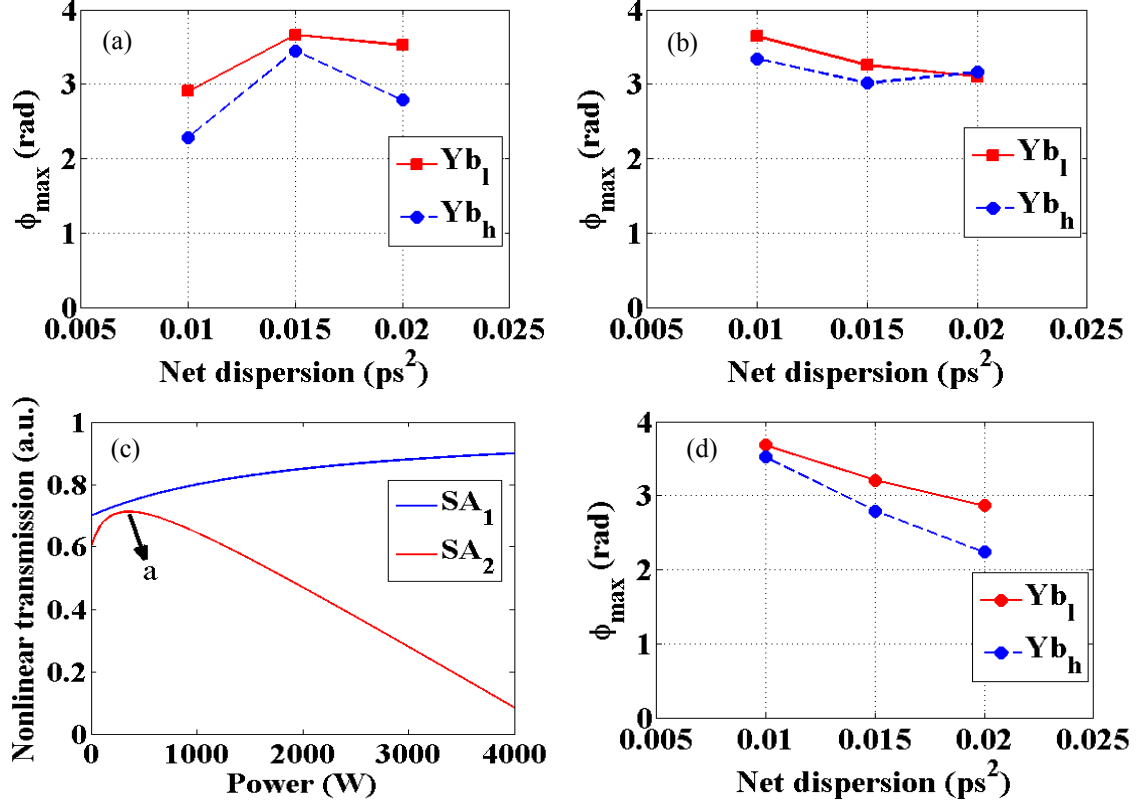


Fig. 5.8 Plot of $\phi_{\max}$ with net dispersion for the two laser cavities based on the simulation results: (a) SA₁ and (b) SA₂. (c) Nonlinear transmission of the two SAs used in the laser cavities. (d) Plot of $\phi_{\max}$ as function of dispersion for SA₂ after exchanging the positions of the SA and diffraction gratings.

For SA₁, Fig. 5.10(a) (Fig. 5.10b)) shows the evolution of the temporal and the spectral RMS pulse widths for high- (low-) doped Yb^{3+} fiber cavities at 0.02 ps^2 net dispersion. The temporal breathing ratio (the ratio between maximum and minimum pulse width) (1.29 and 1.22 for low- and high-doped Yb^{3+} fiber cavities, respectively) is higher than the spectral breathing ratio (1.016 for both low- and high-doped Yb^{3+} fiber cavities), which is in agreement with previous reports for passive Similariton [80, 83].

For SA₂, the evolution of the temporal and the spectral RMS pulse widths for high- (low-) doped Yb³⁺ fiber cavities at the same value of net dispersion is plotted in Fig. 5.10(c) (Fig. 5.10(d)). The temporal and spectral breathing ratios are very close to what is shown in SA₁.

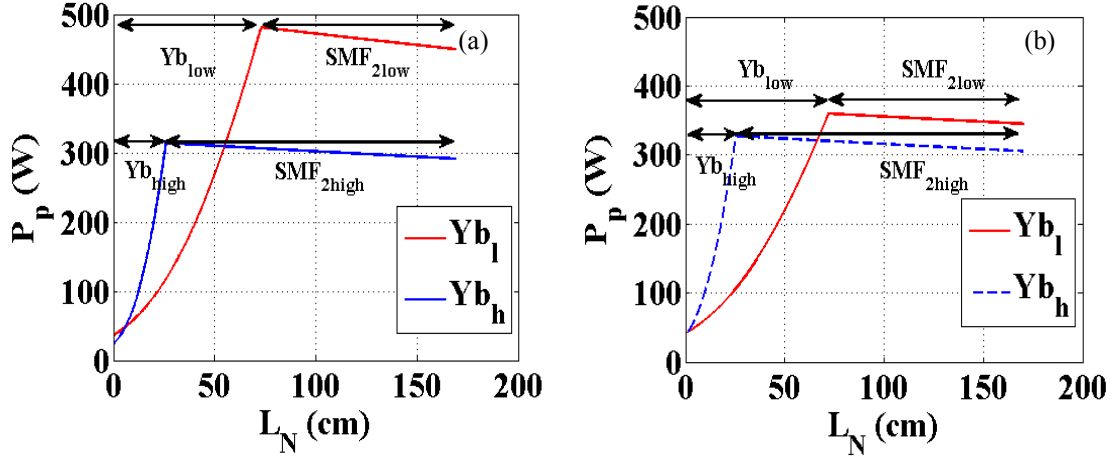


Fig. 5.9 Peak power evolution in the two cavities at net dispersion 0.02 ps²: (a) SA₁ and (b) SA₂. L_N denotes to the nonlinear section of the mode-locked laser cavity.

As shown in Fig. 5.10, the mode-locked laser cavities with high-doped Yb³⁺ fiber have slight compression in Yb³⁺ doped fiber, which is also observed in [83]. However, there is only one location of minimum pulse width at the end of the diffraction gratings pair for the mode-locked laser cavities with low-doped Yb³⁺ fiber. This can be explained as follows; the gain spectral filtering and the dispersion affect the pulse dynamics in Yb³⁺-doped fiber. For the mode-locked laser cavities with high-doped short-length Yb³⁺ fiber, the gain spectral filtering of the chirped pulse is dominant, which appeared as a pulse compression. However, for the mode-locked laser cavities with low-doped long-length Yb³⁺ fiber, the dispersion is dominant which leads to temporal pulse broadening.

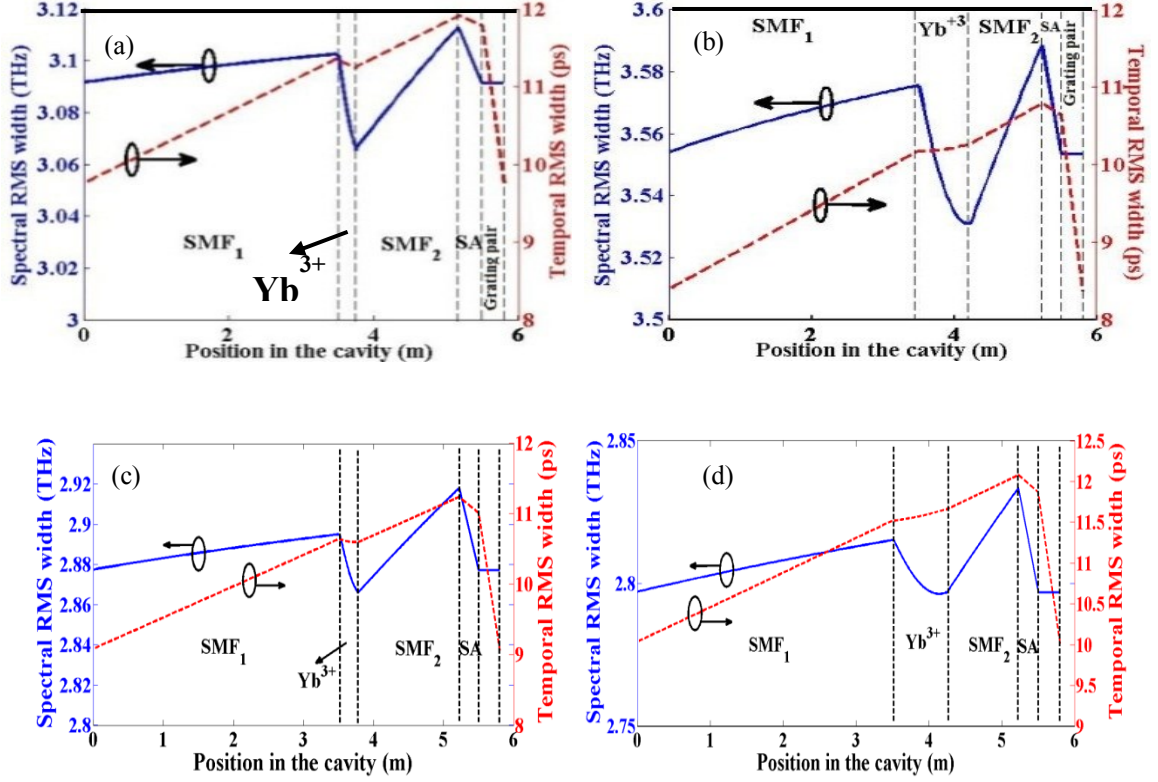


Fig. 5.10 Evolution of the spectral and temporal RMS widths in the laser cavities at 0.02 ps^2 net dispersion for (a) SA₁ high-, (b) SA₁ low-, (c) SA₂ high- and (d) SA₂ low-doped Yb^{3+} fiber.

For SA₁, the RMS spectral widths at the end of L_{SMF2} are 3.6 THz and 3.1 THz for the low- and high-doped Yb^{3+} fiber cavities, respectively. For SA₂, we have lower values; 2.8 THz and 2.9 THz for the low- and high-doped Yb^{3+} fiber cavities, respectively. For all cavities, the evolution of the spectral RMS pulse width is very similar to that reported in [80, 83].

5.3 Experimental work

5.3.1 Experimental set-up description

Fig. 5.11 depicts the experimental set-up employing a SESAM as a mode-locker. Its parameters are shown in sub-section 5.2.1. In this set-up, two 980 nm pump-sources

are combined using a polarization combiner to increase the available pump power. The gain medium (Yb^{3+} -doped fiber) is pumped through the output of the wave division multiplexer (WDM) and is followed by SMF_2 . The optical signal is coupled to/from the free space through angled-cleaved collimators. A free-space Faraday isolator (OFR, IO5-1030 HP) is inserted inside the cavity for unidirectional propagation. The net cavity GVD is adjusted by a pair of diffraction gratings and the output light is captured from the PBS_1 . The energy fluence incident on the SESAM is controlled by QWP_1 and HWP_1 [105, 137]. PBS_2 acts as a polarization director, which passes the horizontally-polarized light to the SESAM and directs the reflected light from the SESAM whose polarization is vertical to the HWP_2 [105, 137]. The polarization of the reflected light from the SESAM is vertical due to the double effect of the QWP_2 on the incident horizontally-polarized light to the SESAM (which is polarization-insensitive). As the efficiency of the diffraction gratings is maximal for horizontally-polarized beams, another HWP (HWP_2) is inserted to convert the polarization to be horizontal again before hitting the gratings. The SESAM is inserted at the focal point of a converging lens (focal length equal to 2 cm) to increase the fluence of the optical pulses incident on the SESAM. The mode-locking is self-starting and the wave plates (QWP_1 and HWP_1) are used to adjust the value of the incident SESAM's fluence required to initiate the mode-locking. The mode-locking operation is lost when SESAM is replaced by a mirror. This verifies that the wave plates' angles cannot initiate any mode-locking from NPR. The pulse is de-chirped outside the cavity.

The details of the experimental set-up that was used to characterize the spectral and temporal profiles of the mode-locked pulses are demonstrated in Appendix B. The zero order reflection of the intra-cavity grating is employed to capture the properties of

the pulse circulating inside the cavity. The pump power is adjusted at each value of net dispersion to have a single pulse operation with the maximum output energy.

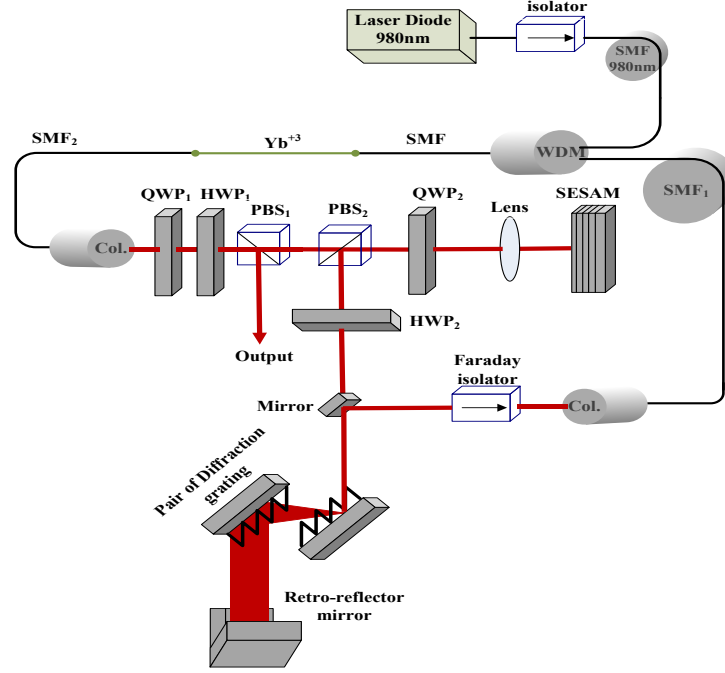


Fig. 5.11 Schematic diagram of the experimental set-up femtosecond fiber laser.

5.3.2 Experimental results and discussion

To experimentally verify that low-doped long gain fiber length increases the peak power of the generated laser pulses, two available types of Yb³⁺-doped fiber (Coractive Yb214, and CorActive Yb501) having doping concentration contrast are selected in the experimental set-up shown in Fig. 5.11. Two laser cavities are built having the same dimensions in sub-section 5.2.1. The average repetition rate for the two cases is about 30.21 MHz. Detailed specifications of the Yb³⁺-doped fiber used are reported in Table 5.1, as provided from CorActive Inc.

Fig. 5.12(a) shows the maximum pump power that can be used for single pulse operation at each net dispersion value, for the two Yb^{3+} -doped fiber types. The cavity with low-doped Yb^{3+} fiber needs lower pump power than the cavity with high-doped Yb^{3+} fiber. Since the pump power of the Yb^{3+} -doped fiber is directly proportional to its gain, we believe that the experimental values of the small signal gain have the same behavior as in the simulation results (see Fig. 5.2). The energy and the peak power of the output pulse at different values of net dispersion are shown in Fig. 5.12(b) and Fig. 5.12(c), respectively. Obviously, the maximum energy monotonically increases with net dispersion. The cavity with a low-doped Yb^{3+} fiber results in a higher energy and peak power than that with a high-doped Yb^{3+} fiber. The enhancement factor in this specific case is around two. The chirped pulse width increases with net dispersion as shown in Fig. 5.12(d). These experimental results are qualitatively in agreement with the simulation results presented in Figures 5.3 to 5.5.

Table 5.1 Physical parameters of the Yb^{3+} fibers

Parameter	Yb501	Yb214
Core radius	2.65 μm	2.45 μm
Doping radius	1.65 μm	2.497 μm
Doping concentration	1.26×10^{26} ion/ m^3	1.85×10^{26} ion/ m^3
Peak absorption @915 nm (dB/m)	139	308
Overlap factor	0.33 @ 1060 nm	0.75
Carrier life time	0.0011 s	0.85 ms
Measured Saturation factor	2.91×10^{18} (1/m.sec)	NA
Mode field diameter	5.69 μm	6 μm
Numerical aperture	0.13	0.14

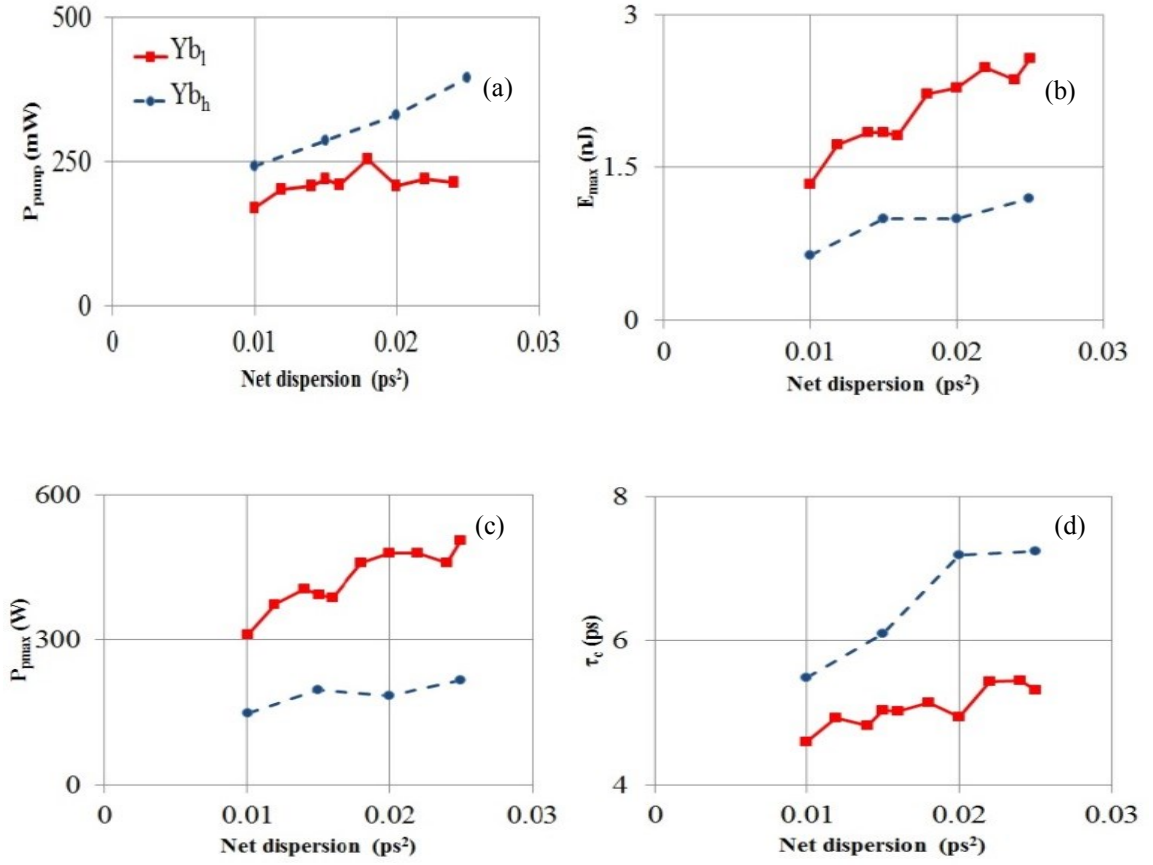


Fig. 5.12 (a) Pump power (P_{pump}) (b) Maximum pulse energy (E_{max}) (c) Maximum peak power (P_{pmax}) (d) Chirped pulse width τ_c versus net dispersion.

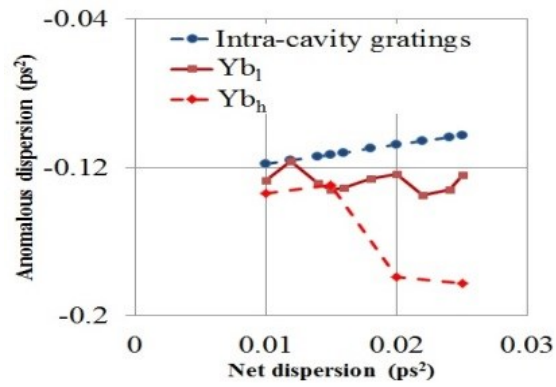


Fig. 5.13 Comparison of dispersion contributed from intra-cavity grating and de-chirping grating for cavity with high and low doped Yb³⁺ fiber.

The pulses are confirmed to be Similariton by checking the relation between the anomalous dispersion required to de-chirp the pulses compared to the intra-cavity grating dispersion. Fig. 5.13 shows that the pulses are Similariton because the magnitude of the anomalous dispersion required to de-chirp the output pulse near the transform limit is larger than the intra-cavity grating dispersion.

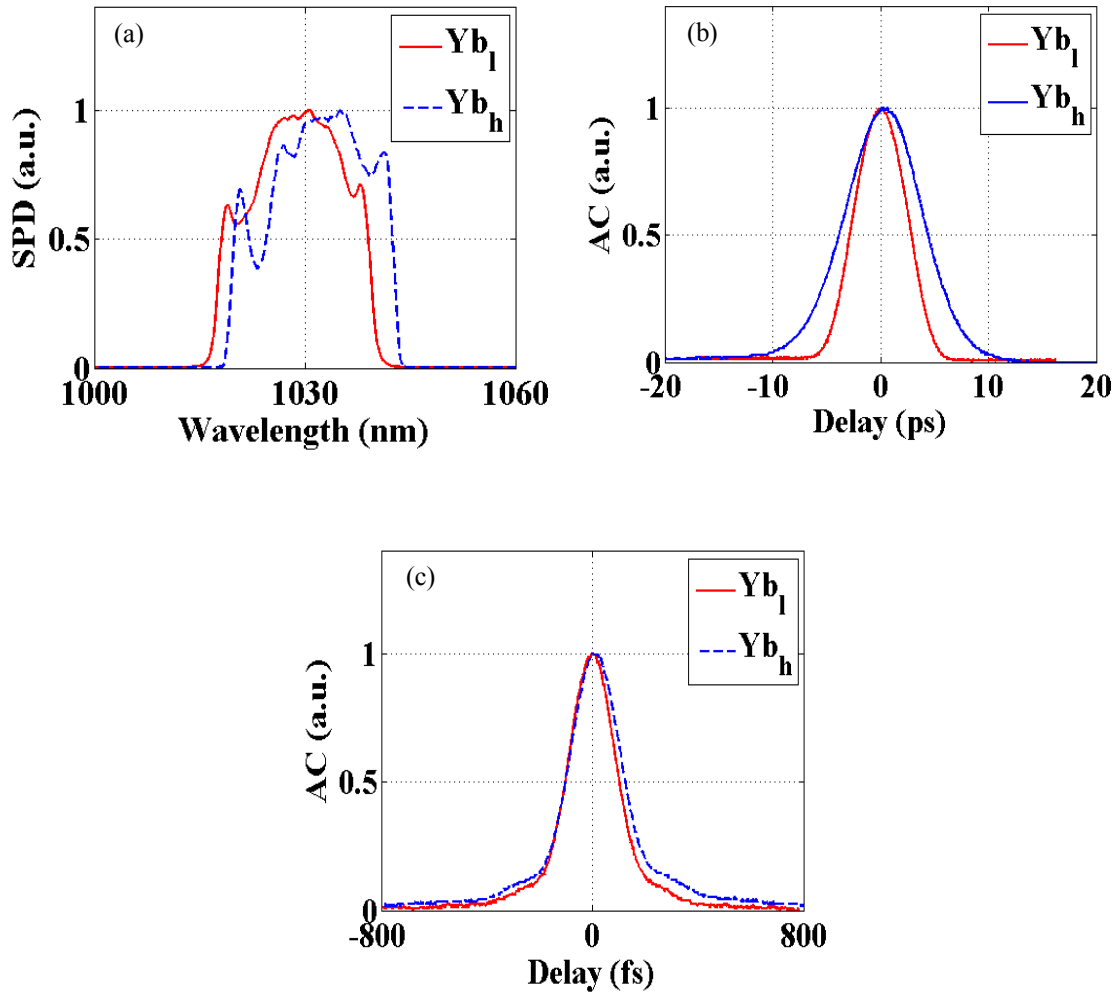


Fig. 5.14 Output pulse characteristics of the cavity at a net dispersion of 0.02 ps^2 ; (a) parabolic spectrum for the two cavities with high- and low-doped Yb^{3+} fiber (b) autocorrelation (AC) of the chirped pulse of width 8.3 ps (5.7 ps) for cavity with high (low) doped Yb^{3+} fiber. (c) AC of the de-chirped pulse of width 221.76 fs (205.7 fs) for the cavity with high (low) doped Yb^{3+} fiber.

The spectral profile of the chirped pulse and the temporal profile of the chirped and de-chirped pulses at 0.02 ps^2 net dispersion for the two cavities are presented in Fig. 5.14. The spectral bandwidth of the pulse generated from high-doped Yb^{3+} fiber cavity is approximately the same as the one generated from the low-doped one. The two spectra are fitted to sech^2 and parabolic curves in Fig. 5.15. It is clear that the parabolic fitting is close to the measured spectra, and is in agreement with the simulated spectral profile in Fig. 5.7(b). The Fourier transform limited pulse profiles are calculated from the measured spectra for the two cavities at 0.02 ps^2 net dispersion. They are also fitted to sech^2 and parabolic curves as shown in Fig. 5.16. The pulses are more parabolic especially at their edges.

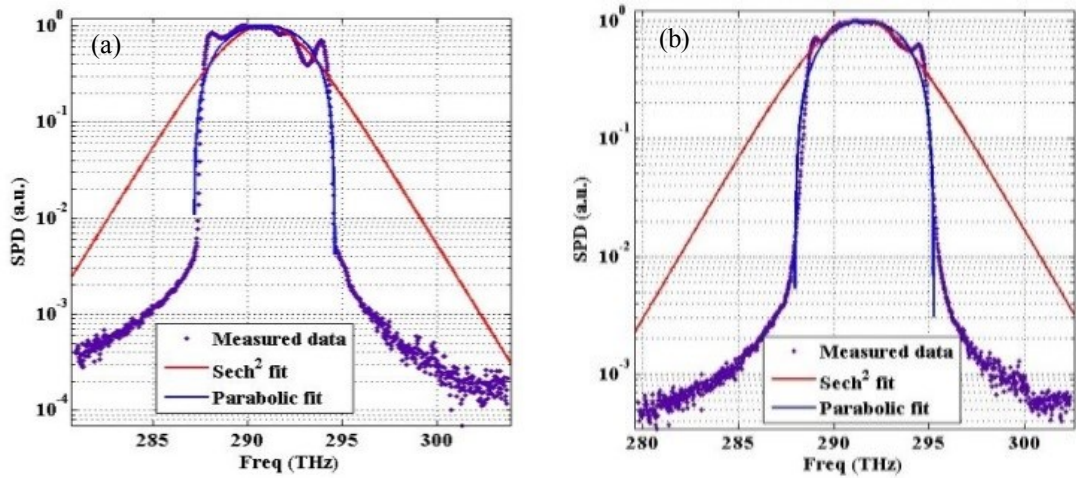


Fig. 5.15 Output spectrum fitted to sech^2 and parabolic curves at net dispersion 0.02 ps^2 for the cavity with (a) high- (b) low-doped Yb^{3+} fiber.

The de-chirped parabolic pulse widths calculated from the autocorrelation are 193.3 fs and 179.3 fs for the cavity with high- and low-doped Yb^{3+} fiber, respectively. The time bandwidth product is found to be 0.84 and 0.92 for high- and low-doped Yb^{3+} fiber, respectively. The transform limit to the parabolic pulse is exceeded by $\sim 25\%$ and

~31%, respectively. These values are close to the values reported in [12, 82] and indicate linearly chirped pulse to a great extent.

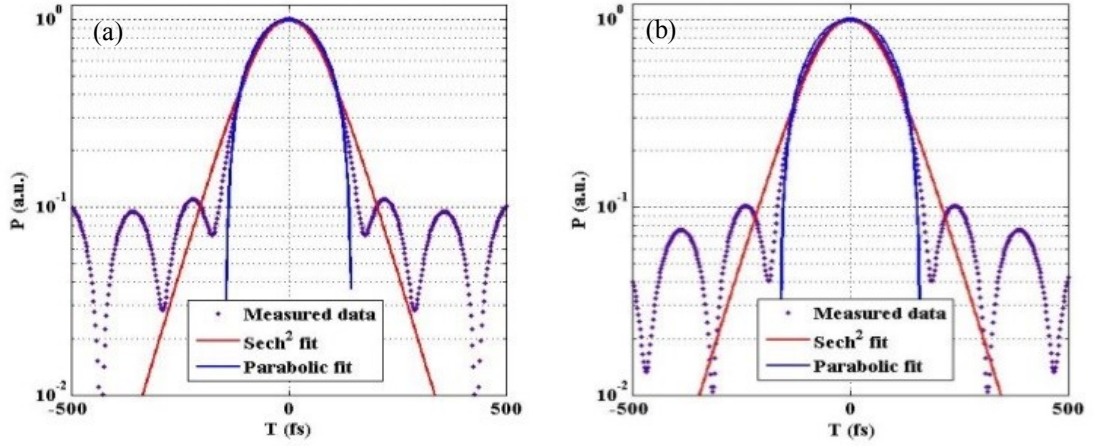


Fig. 5.16 Calculated intensity fitted to sech^2 and parabolic curves at net dispersion 0.02 ps^2 for the cavity with (a) high- (b) low-doped Yb^{3+} fiber.

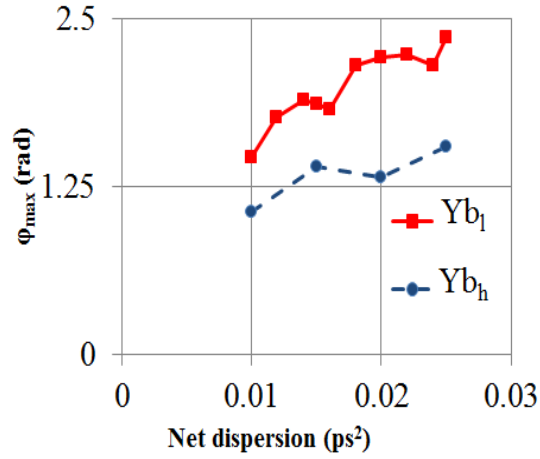


Fig. 5.17 The variation of ϕ_{max} with net dispersion for the cavities having Yb501 and Yb214.

For the two cavities, ϕ_{max} is calculated for each value of net dispersion using (5.1) through (5.3), and plotted in Fig. 5.17. ϕ_{max} is higher for the cavity with low-doped Yb^{3+} fiber (Yb501) compared to that of the other cavity with Yb^{3+} fiber (Yb214), (i.e

$\varphi_{\max_l} > \varphi_{\max_h}$). Moreover this cavity can support higher values of $\varphi_{\max}$ for single pulse operation before the onset of multi-pulsing or wave breaking.

5.4 Conclusion

In conclusion, we have demonstrated, for the first time, that decreasing the doping level and increasing the length of the gain medium results in an increase in the peak power. A comparison between high-doped short-length and low-doped long-length gain medium in fiber laser cavity is carried out both theoretically and experimentally. Moreover, we demonstrate that the ability of the low-doped gain medium to generate high peak power in the fiber laser cavity is attributed to two reasons: one is direct and the other is indirect. The direct reason is the gradual increase of the nonlinear phase shift accumulated during the pulse propagation in the gain medium. The indirect reason is the ability of the low-doped fiber to tolerate a higher maximum nonlinear phase shift for single-pulse operation. This arises from the larger effect of the nonlinearity per unit length in the high-doped fiber compared to that of the low-doped fiber as a function of dispersion. In the high-doped gain medium, the nonlinearity effect per unit length dominates the dispersion effect on the pulse propagation. However, in the low doped gain medium, their effect per unit length is similar. As a result the threshold nonlinear phase shift for single-mode operation has a higher value in low-doped fiber than that in a high-doped fiber cavity. Consequently the available output peak power is higher in the case of the low-doped fiber. So, in conclusion, the selection of the lowest-doped fiber results in the highest peak power of the generated pulse provided that the cross-sections of all fibers are the same.

Chapter 6

6 Effect of narrow spectral filter position on the characteristics of active Similariton mode-locked femtosecond fiber laser⁴

The generation of parabolic pulses, with linear frequency chirp in normal dispersion gain medium, results in high immunity to wave breaking [92, 93]. The insertion of a narrow-bandwidth SF (2 nm – 4 nm) assists in the generation of the parabolic pulse given that the length of Yb³⁺-doped fiber is sufficient to convert any arbitrary pulse into a parabolic pulse with linear frequency chirp [98-100].

W. H. Renninger et al. reported that injecting a part of the first order diffraction beam from a diffraction grating into a fiber collimator can act as a narrow-bandwidth SF. However, such a configuration introduces a lot of coupling loss that requires a high gain value to compensate for it [98-100]. This filter can be replaced by an inline fiber interference SF with a much lower insertion loss. A long piece of SMF is often placed between the output port and Yb³⁺-doped fiber to increase the cavity length in order to increase the pulse energy [125].

In this chapter, our objective is to study the effect of the location of the lumped SF, with respect to the long SMF, on the pulse energy, de-chirped pulse width, and peak power.

Based on the simulation and experimental results shown in sections 4.3 and 4.4, the position of a wide bandwidth SF (>6 nm) either before or after the SMF in dissipative

⁴ Most of the results in this chapter have been published in [V].

soliton fiber laser does not change the pulse characteristics (spectral bandwidth and de-chirped pulse width) significantly.

However, as will be seen below, I observe a significant change in the pulse characteristics both numerically and experimentally as a function of the location of the lumped SF. The closer the SF is to the input of the Yb^{3+} -doped fiber, the higher the peak power and shorter the de-chirped pulse width.

Average models cannot be used to model active Similariton lasers because the pulse evolution relies on a local nonlinear attractor in the Yb^{3+} -doped fiber, which forces the pulses to evolve towards a parabolic profile [98, 111]. Therefore, the vector CGLEs are used to simulate the pulse propagation in the Yb^{3+} -doped fiber and SMFs.

This chapter is organized as follows: the mode-locked laser cavity is described in section 6.1. In section 6.2, I will illustrate the simulation model. Discussion of the simulation results are depicted in section 6.3. The experimental results are demonstrated in section 6.4. Finally, the conclusion is presented.

6.1 Laser cavity description

Fig. 6.1 shows the mode-locked laser cavity designed to investigate the idea; it has almost all the blocks shown in Fig. 2.2, except for the dispersion compensation block. It is mode-locked by NPR. Two QWPs, one HWP and PBS are inserted inside the cavity to initiate the mode-locking.

Four cases are illustrated to study the effect of the position of the lumped SF. For all cases, the length of the Yb^{3+} -doped fiber (L_{yb}) is taken to be 1.98 m. The length of SMF (L_{SMF2}) proceeding L_{yb} is 0.4 m. Also, the whole length of SMF (6.355 m) between

the input of the Yb^{3+} -doped fiber and QWP_2 is kept constant. For Case 1, the SF is located right after QWP_2 , while, for Case 2, a piece of SMF of length 1.6 m is inserted between QWP_2 and the SF. For Case 3, this piece of SMF is extended to be 4.53 m. Finally, for Case 4, the SF is inserted at the input of the Yb^{3+} doped fiber.

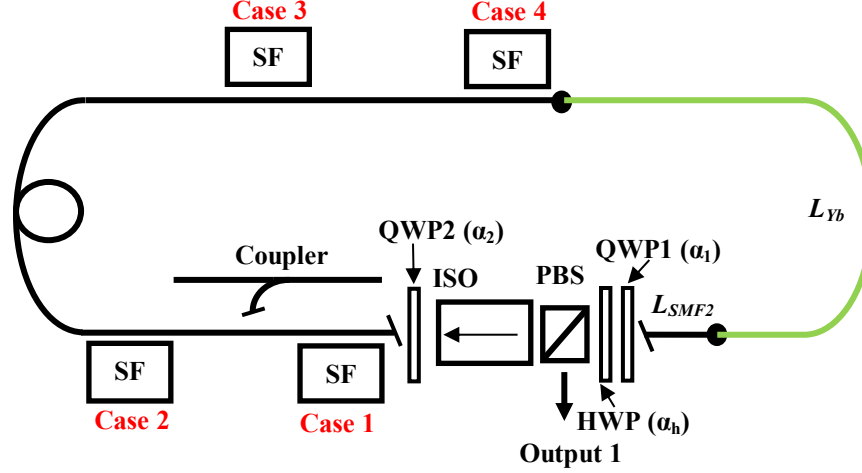


Fig. 6.1 Schematic diagram of the active Similariton laser cavity having the spectral filter (SF) at four positions to test its effect on the temporal and spectral properties of the pulse; QWP is the quarter wave plate, HWP is the half wave plate and ISO-is the Faraday isolator.

It is worth mentioning that the experimental implementation of Case 1 and Case 4 is difficult because of the leads of the WDM and the spectral filter. Therefore, only Case 2 and Case 3 are experimentally constructed. To match with the experimental set-up, a coupler of output coupling ratio 0.05 is inserted at 0.81 m from QWP_2 . The main output of the mode-locked laser cavity is at the rejection port of the PBS, which is named as output 1 throughout this chapter.

6.2 Simulation model parameters

The evolution of the optical pulse in Yb^{3+} -doped fiber and SMFs is modeled by CGLEs shown in (2.1a) and (2.1b). The gain in the Yb^{3+} -doped fiber is expressed in (2.2). The linear birefringence $\Delta\beta$ is set to zero to only take into account the nonlinear birefringence caused by NPR. The lumped spectral filter is modeled as a Gaussian filter with insertion loss 0 dB and 3-dB bandwidth of 3.2 nm. β_2 and β_3 are 24690.1 fs²/m and 75.3 fs³/m for the SMFs, and 22752.5 fs²/m and 74.2 fs³/m for Yb^{3+} -doped fiber, respectively. The nonlinear parameter γ is taken as 0.005 W⁻¹m⁻¹ and 0.0041 W⁻¹m⁻¹ for the SMFs and Yb^{3+} -doped fiber, respectively. g_0 is the small signal power gain coefficient with parabolic frequency dependence and a bandwidth $\Omega_g=40$ nm; it is selected as 17.5 dB/m. The value of E_{sat} is adjusted for each case to have stable single pulse operation. For Case 1 and Case 4, the value of E_{sat} has a value of 1.25 nJ. To account for the coupling loss in Case 2 and Case 3, E_{sat} has a slightly higher value of 1.3 nJ. To have a fair comparison, the orientation angles of the wave plates are kept the same for the all cases. Their values are selected as follows: the orientation angles of QWP₁, HWP and QWP₂ are 121.5°, 151° and 43.65°. These values are substituted in (2.32a) and (2.32b) to ensure proper nonlinear transmission of SA. The through and rejection ports of the PBS, the wave-plates and the Faraday isolator are modeled by their Jones matrices. Equations (2.25) and (2.26) show the matrices of the through and rejection ports of the PBS, respectively. Equations (2.29a), (2.29b) and (2.30) define the transfer functions of the QWP, HWP and the Faraday isolator. The simulation model is initiated by a small pulse of unity amplitude. 1500 roundtrips were sufficient to have a stable single pulse mode-locked operation.

6.3 Simulation results and discussion

For all cases, the normalized temporal profiles of the chirped pulses at the end of L_{SMF2} , the through port of the PBS and output 1 are shown in Fig. 6.2. The excess kurtosis factor, mentioned in sub-section 5.2.2, is used to verify if the pulse approaches the parabolic profile. It has a value of -1.2, -0.86, 0 and 1.2 for rectangular pulse, ideal parabola, Gaussian, and Sech^2 profiles, respectively [83]. They are reported in Table 6.1 at various locations inside the mode-locked laser cavity. These values show that the chirped pulses for the four cases are close to having a parabolic temporal profile only at the end of L_{SMF2} . This can be explained as follows: the long length of the Yb^{3+} -doped fiber has the capability to convert the input pulse into a parabolic one [92]. However, the through port of the PBS is affected by the nonlinear transmission of the NPR that filters out the low power edges of the pulse and converts it to the shape shown in Fig. 6.2. In contrast, the low power portions of the pulse at the end of L_{SMF2} that contain the sharp edges are passed through output 1. Then, the temporal profile of the pulse tends to have a rectangular shape.

Table 6.1 Excess Kurtosis factor of the chirped pulses

Laser cavity	The end of L_{SMF2}	the through port of the PBS	Output 1
Case 1	-0.91	-0.64	-0.97
Case 2	-0.89	-0.65	-0.97
Case 3	-0.89	-0.66	-1.03
Case 4	-0.9	-0.72	-1.05

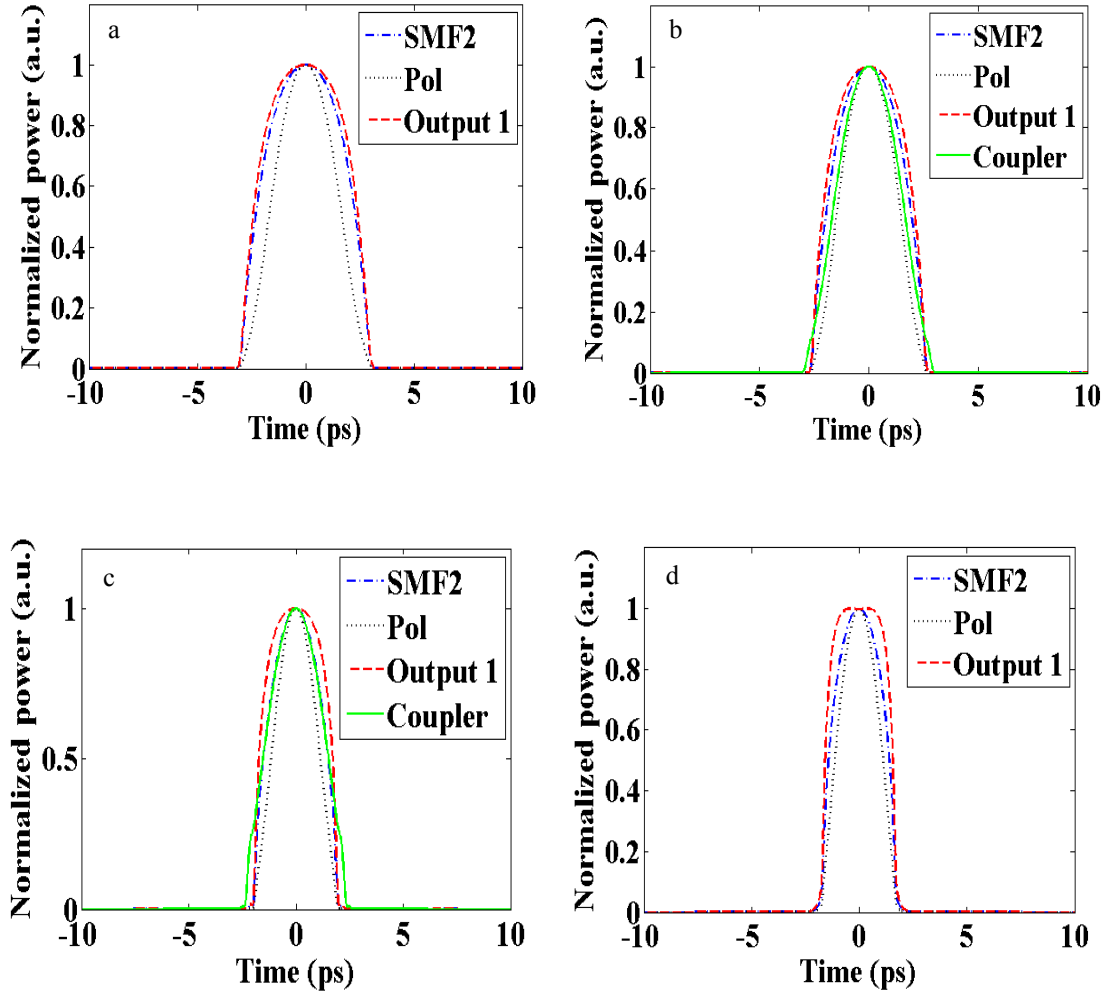


Fig. 6.2 The numerical normalized temporal profile of the chirped pulse at the end of L_{SMF2} (SMF2), the through port of the PBS (Pol), the rejection port of the PBS (output 1) and the coupler output: (a) Case 1, (b) Case 2, (c) Case3 and (d) Case 4.

As shown in Fig. 6.2(c), the chirped pulse at the coupler output for Case 3 has some distortion at its edges that is not seen in Case 2. It can be explained as follows: the peak power of the pulse at the through port of the PBS for Case 3 is 2.66 times higher than the corresponding peak power in Case 2. This factor enhances the interplay between the dispersion and the self-phase modulation, which results in temporal distortion and interference structure in the spectral profile.

The Full width at half maximum (FWHM) of the chirped pulses at the end of L_{SMF2} , through the port of the PBS and output 1 is plotted in Fig. 6.5(a). It is clear that the FWHM decreases with the proximity of the lumped spectral filter to the input of L_{yb} . For Case 2 and Case 3, the FWHM was calculated to be 3.4 ps and 3 ps, respectively.

Fig. 6.3 shows the spectral power densities (SPDs) of the pulses at the end of L_{SMF2} , the through port of the PBS and output 1 for all cases. The SPDs of the pulses at the coupler output for Case 2 and Case 3 are shown as well. The SPDs of the pulses do not have any sharp edges, which is a signature of having active Similariton mode-locking [98]. The SPDs of Case 3 and Case 4 have more oscillatory structures compared to the corresponding SPDs of Case 1 and 2. As seen in Fig. 6.8, the higher values of peak power at the end of L_{SMF2} enhance the effect of SPM that causes spectral broadening along with these oscillatory structures. As shown in Fig. 6.5(b), the SPD of the chirped pulses is broadened with the proximity of the lumped spectral filter to the input of L_{yb} . For Case 2 and Case 3, the spectral bandwidths were calculated to be 11.4 nm and 20.1 nm, respectively.

For Case 2 and Case 3, the autocorrelation profiles of the chirped pulses at output 1 are plotted in Fig. 6.4(a) to be compared with the measured counterpart. The autocorrelation deconvolution factors were calculated to be 0.93 and 0.96, respectively.

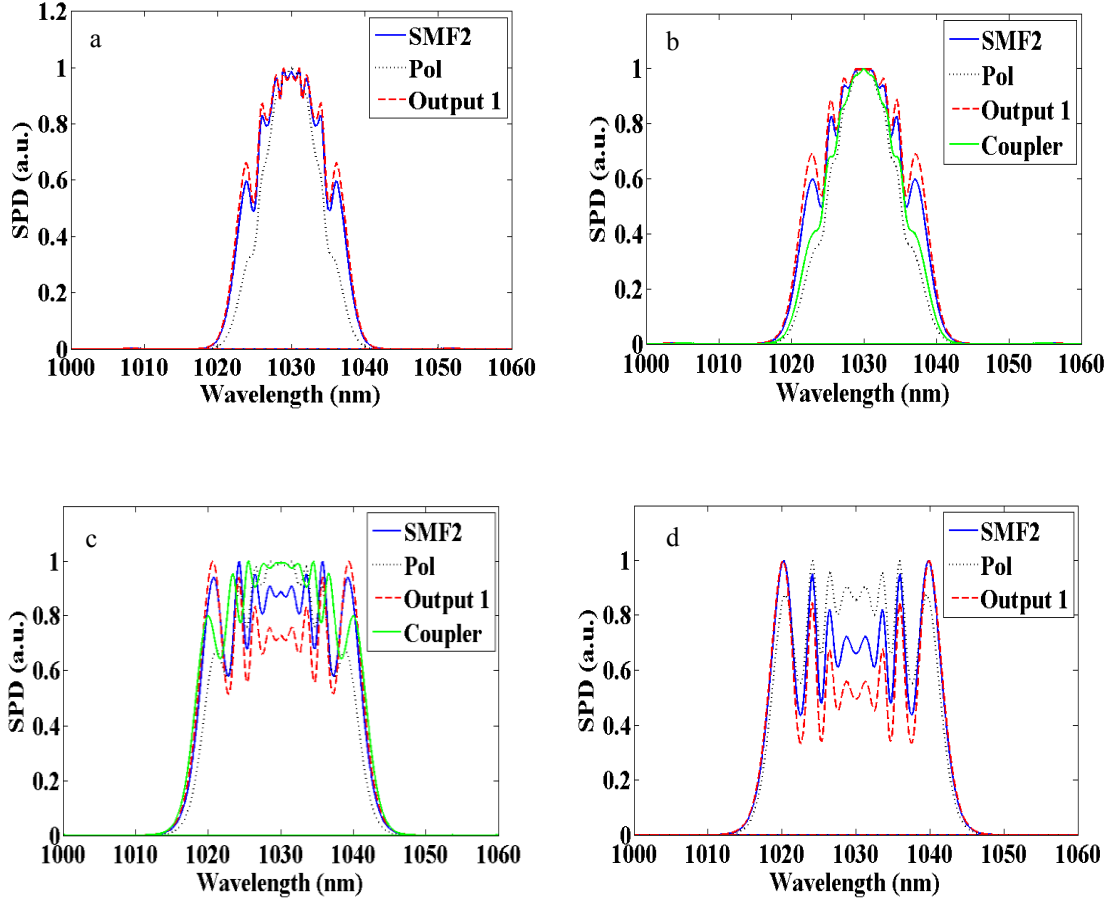


Fig. 6.3 Numerical normalized SPD of the chirped pulse at the end of L_{SMF2} (SMF2), the through port of the PBS (Pol), output 1 and the coupler output: (a) Case 1, (b) Case 2, (c) Case3 and (d) Case 4.

For all cases, the chirped pulses at output 1 were passed through a dispersion delay line that provides the required anomalous dispersion to compress the pulse near its transform limit. The temporal profiles of de-chirped pulses are shown in Figures 6.4(b) and 6.4(c). The autocorrelation profiles of these pulses are shown in Figures 6.4(d) and 6.4(e). The autocorrelation deconvolution factor was calculated to be on average 0.75. This indicates that the temporal profile of the pulse is close to having a Gaussian shape (the autocorrelation deconvolution factor of a Gaussian pulse is 0.709). The FWHM of the de-chirped pulse is plotted in Fig. 6.5(c). The pulse is shortened with the proximity of

the lumped spectral filter to the input of L_{yb} , which is logical as the spectral bandwidth increases in the same direction.

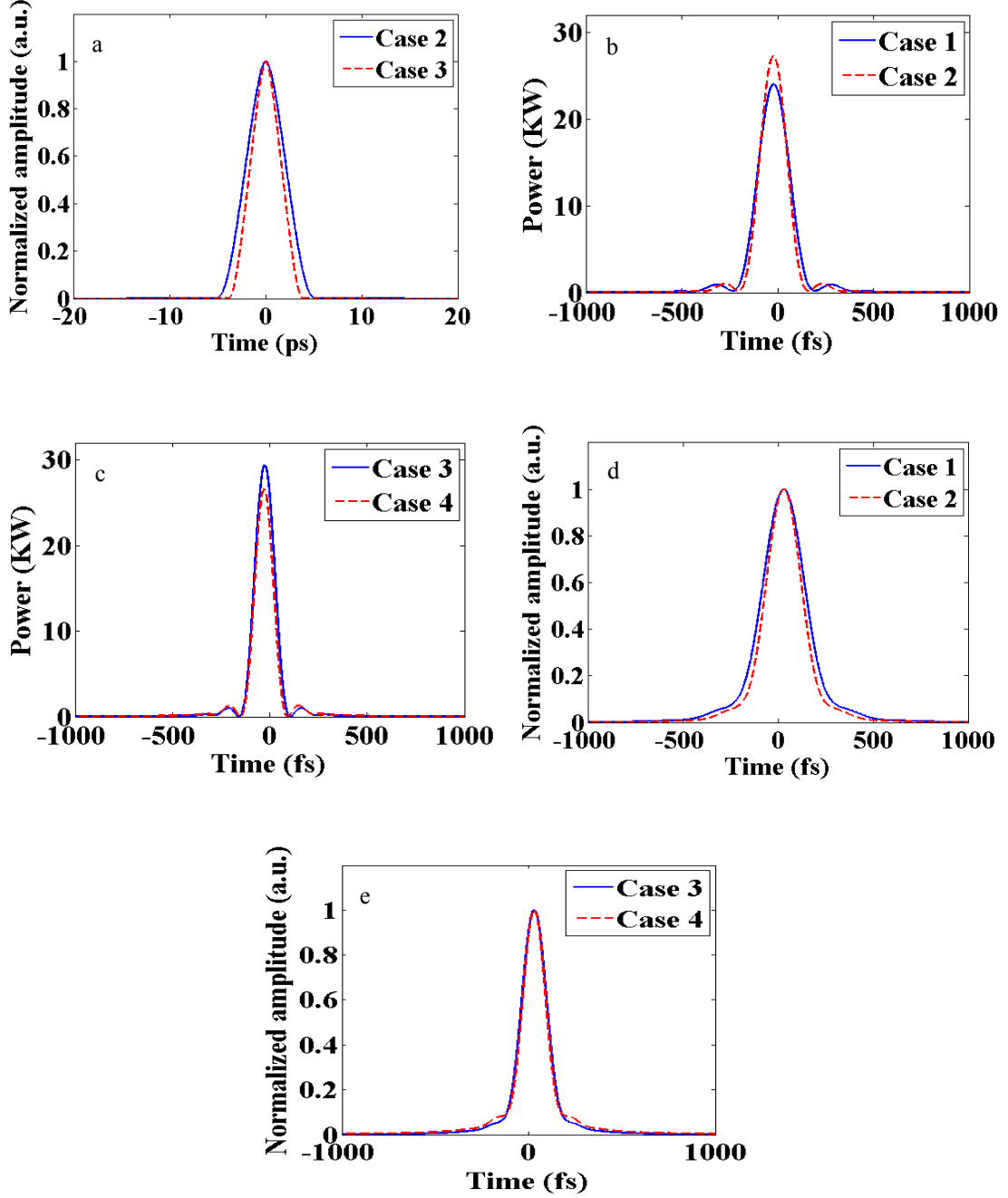


Fig. 6.4 (a) Normalized autocorrelation of the chirped pulse at output 1 for Case 3 and Case 4. Temporal profile of the de-chirped pulses: (b) Case 1 and Case 2 and (c) Case 3 and Case 4. Normalized autocorrelation of the de-chirped pulses: (d) Case 1 and Case 2 and (e) Case 3 and Case 4.

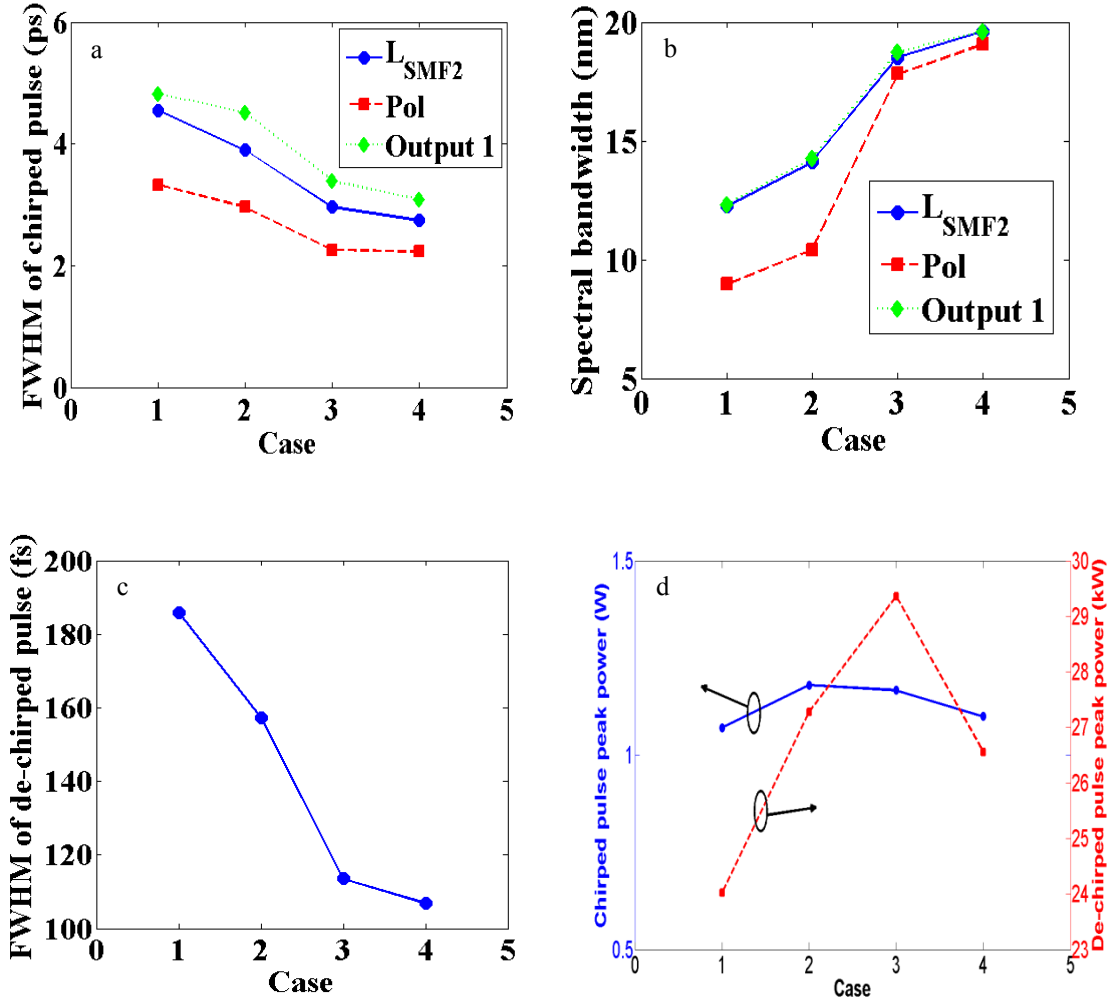


Fig. 6.5 Plot of (a) FWHM of the chirped pulse and (b) spectral bandwidth at various locations inside the cavity. (c) Plot of FWHM of the de-chirped pulse. (d) Plot of peak power of the chirped and de-chirped pulses.

The peak power of the chirped and de-chirped pulses is plotted in Fig. 6.5(d). The peak power of the chirped pulse changes slightly from one case to another. However, the peak power of the de-chirped pulse has its highest value in Case 3. Then, it decreases again at Case 4. If the pulse energy is the same for each case, the peak power of the de-chirped pulse should increase because of the reduction of the pulse width. However, the simulation results show a monotonic reduction of the pulse energy with the lumped

spectral filter approaching the input of L_{yb} (see Fig. 6.6(a)). Furthermore, as shown in Fig. 6.3, the SPD of Case 4 depicts an oscillating structure, which leads to having the highest percentage of energy in the side pulses (see Fig. 6.6(b)).

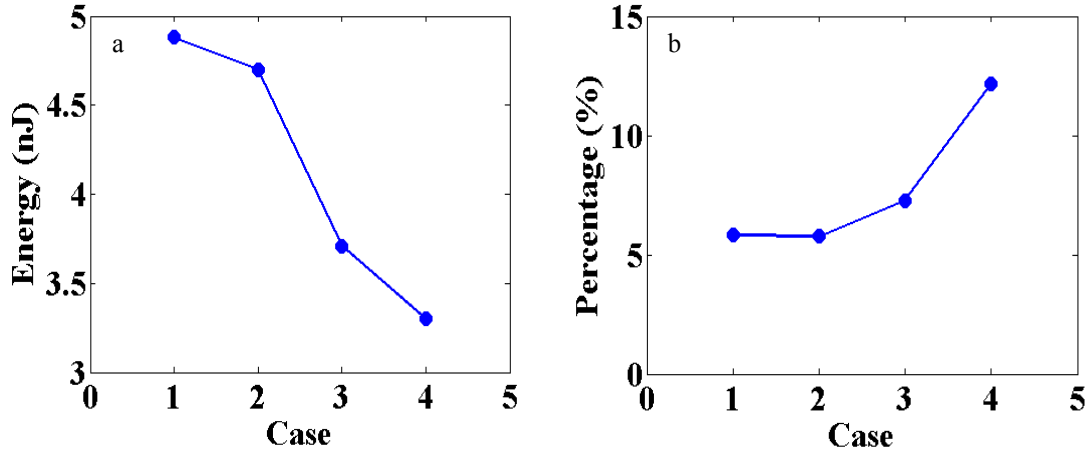


Fig. 6.6 Plot of (a) pulse energy at output 1 and (b) percentage of the pulse energy of the side pulses to the total energy.

To have a clear understanding of the simulation results, the evolution of the RMS temporal and spectral widths for the u-field (horizontally polarized light) and the v-field (vertically polarized light) through the four mode-locked laser cavities are plotted in Fig. 6.7. Also the evolution of the total peak power through each cavity is plotted in Fig. 6.8. For all cases, the lumped spectral filter reshapes the chirped pulse to have approximately the same value of RMS for temporal and spectral widths and peak power.

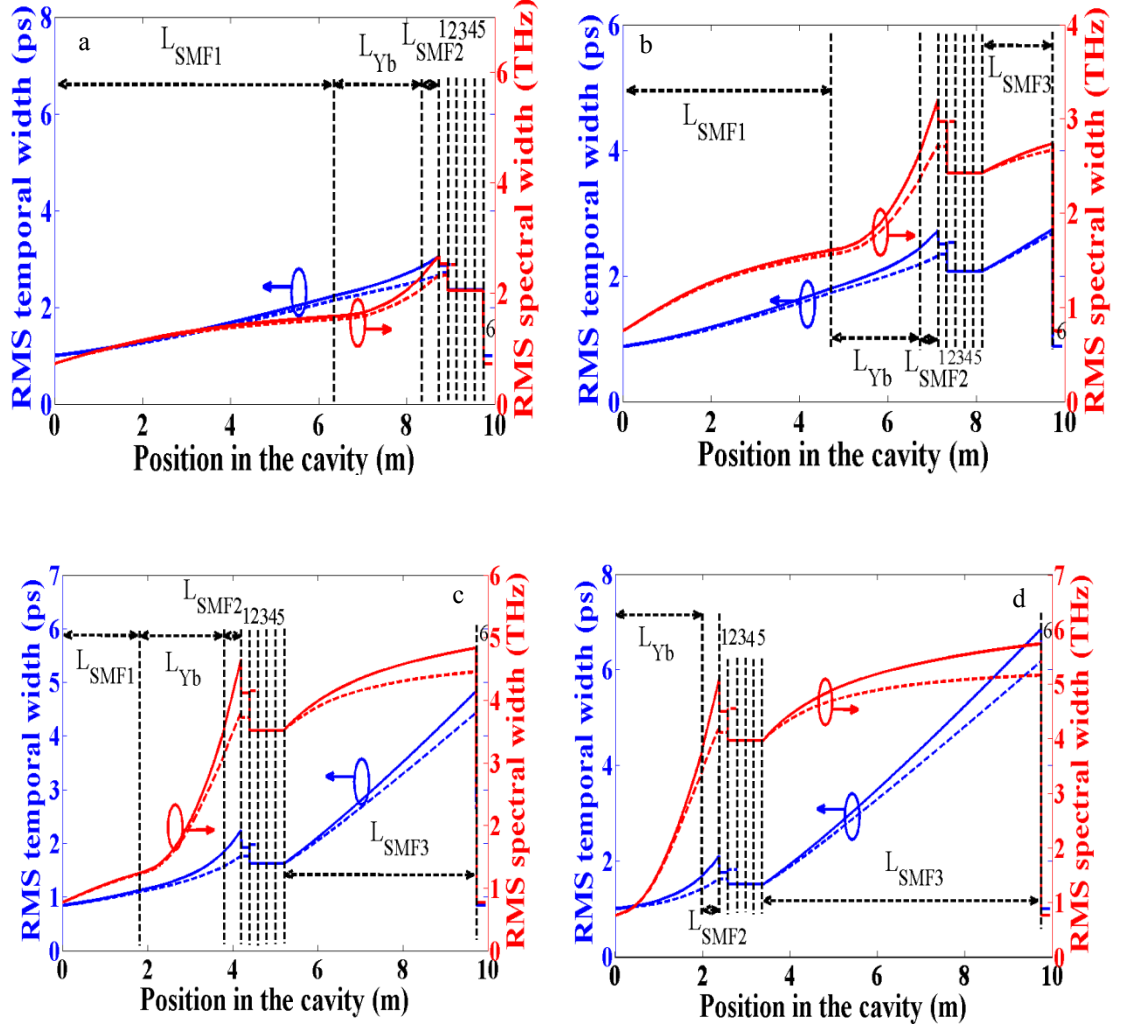


Fig. 6.7 Evolution of the spectral and temporal RMS widths through the four mode-locked laser cavities: (a) Case 1, (b) Case 2, (c) Case 3 and (d) Case 4. The solid lines represent the u-field, while the dashed lines represent the v-field. 1: QWP₁; 2 HWP; 3: the through port of the PBS; 4: Faraday isolator; 5: QWP₂; 6: Lumped spectral filter. Artificial lengths are set to these components for the sake of illustration.

For Case 1 and Case 2, the pulse propagates through a long length of SMF (L_{SMF1}) (6.355 m or 4.755 m, respectively) that leads to an increase in RMS temporal and spectral widths due to dispersion and nonlinear effects. Therefore, the RMS temporal width has the highest value at end of L_{SMF2} . However, the RMS spectral width tends to saturate at

the middle of L_{SMF1} because of the reduction of the pulse peak power (Figures 6.7(a) and 6.7(b)).

As shown in Figures 6.7(b), 6.7(c) and 6.7(d), the RMS temporal and spectral widths increase through the SMF after QWP_2 (L_{SMF3}). However, the RMS spectral width for Case 3 and Case 4 follows a saturation behavior because of the reduction of the pulse peak power (see Fig. 6.8) that results from the dispersion effect.

For Case 3 and Case 4, the pulse propagates through a short length of SMF ($L_{SMF1}=1.825$ m) or directly through the Yb^{3+} -doped fiber, respectively. The pulse peak power at the beginning of the Yb^{3+} -doped fiber is higher than the corresponding values in Case 1 and Case 2 (see Fig. 6.8). Because of the self-phase modulation, the SPD of Case 4 has the broadest spectral bandwidth with deep oscillations. No sign of wave breaking was seen in either Case 3 or Case 4 through the Yb^{3+} -doped fiber and L_{SMF2} because of the linearization of the frequency chirp produced by the conversion of the pulse to Similariton.

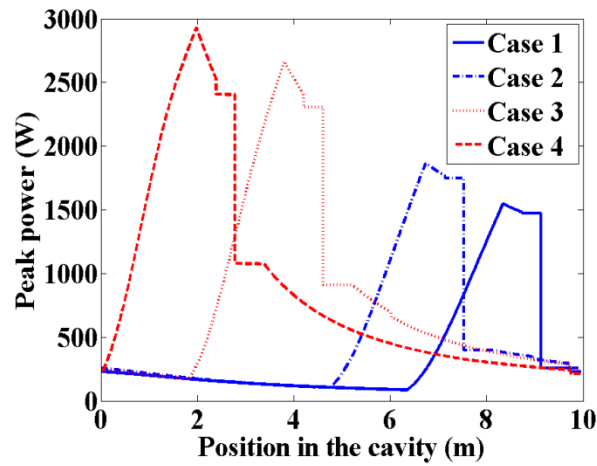


Fig. 6.8 Evolution of the total peak power through the four mode-locked laser cavities.

To explain the reason behind the decrease of the pulse energy from Case 1 to Case 4 (see Fig. 6.6(a)), we can define two sources of energy loss inside each laser cavity; the spectral filter loss and the loss induced by the rejection port of the PBS. The temporal and spectral dynamics of the chirped pulse are affected by the spectral filter location. The spectral filter produces an energy loss to the chirped pulse because of the spectral and temporal filtering. The value of energy loss for each case is reported in Table 6.2. It is clear that the spectral filter produces the lowest value of energy loss for Case 1, while the highest value is for Case 4. As shown in Fig. 6.7(a), the RMS temporal and spectral widths of the chirped pulse have the lowest values at the input of the spectral filter for Case 1. In contrast, they have the highest value for Case 4. As shown in Table 6.2, the pulse energy for each case has approximately the same value at the end of L_{SMF2} and after the spectral filter. This means that the value of the saturated gain is approximately the same. As the net gain of any oscillator should equal zero at steady state, the amount of energy loss induced by the rejection port of the PBS changes in the cavity to satisfy this condition. As a result, Case 1 has the highest value of loss at output 1, which is interpreted as the highest value of output energy. As the value of spectral filter loss increases, the value of energy loss at output 1 decreases, which corresponds to the reduction of the output pulse energy.

Table 6.2 Chirped pulse energy at the end of L_{SMF2} and spectral filter loss for each laser cavity

Laser Cavity	Pulse energy at the end of L_{SMF2} (nJ)	Pulse energy before the spectral filter (nJ)	Pulse energy after the spectral filter (nJ)	Spectral filter loss (dB)
Case 1	6.36	0.85	0.29	4.7
Case 2	6.51	1.17	0.28	6.21
Case 3	6.4	2.07	0.26	9.08
Case 4	6.24	2.33	0.27	9.44

6.4 Experimental results

The transfer function of the spectral filter used in this experiment is shown in Fig. 2.10(c). It has the same 3-dB bandwidth of the spectral filter in simulation model. A lightly-doped Yb^{3+} fiber (its part number is SCF-YB25-6/125-13) purchased from CorActive is used as the gain medium; its core radius is 3 μm ; its doping concentration is $1.28 \times 10^{25} \text{ ion/m}^3$; its core absorption at 915 nm is 30 dB/m. The optical signal is coupled to/from the free space through angled-cleaved collimators. A free-space Faraday isolator (OFR, IO-5-1030 HP) is inserted inside the cavity for unidirectional propagation. The pulse at output 1 is de-chirped outside the cavity using a pair of diffraction gratings serving as a dispersion delay line. The experimental set-up used to have full characterization of the pulse is described in Appendix B.

For Case 2, the pump power for single pulse operation was found to be 400 mW. The repetition rate of the pulses was measured to be 25 MHz. The average power at output 1 was measured to be 136 mW. Therefore the pulse energy was calculated to be 5.4 nJ. The SPDs of the mode locked pulse at output 1 and the coupler output are plotted in Fig. 6.9(a). The SPDs do not have sharp edges as seen in Fig. 6.3(b). It is clear that the SPD at output 1 has more structure than the corresponding SPD at the coupler output, which is also observed in the simulation results. The spectral bandwidth of the SPD at output 1 has a value of 12.2 nm, while it has a value of 11.9 nm at the coupler output. The values are in good agreement with the simulation results.

The autocorrelation profiles of the chirped and de-chirped pulses at output 1 are plotted in Figures 6.10(a) and 6.10(b), respectively. The values of the FWHM of the chirped and de-chirped pulses were measured to be 2.9 ps and 194 fs, respectively. The

convolution factor calculated in the simulation model is used to get the values of the FWHM of the chirped and de-chirped pulses to be 2.7 ps and 144.1 fs.

For Case 3, the pump power for single pulse operation and the repetition rate were approximately the same. The average power at output 1 was measured to be 116.6 mW. Therefore the pulse energy was calculated to be 4.7 nJ. The pulse energy is lower than its value in Case 2, which matches the simulation results. The SPD of the mode-locked pulse at output 1 and the coupler output are plotted in Fig. 6.9(b). Similar to the SPDs plotted in Fig. 6.3(c), the spectral edges are not so sharp. The spectral bandwidth was measured to be 21.3 nm and 22.5 nm for both outputs, respectively. The values of the spectral bandwidth are close to the numerical results. The spectral profiles at both outputs have structures similar to the simulation results.

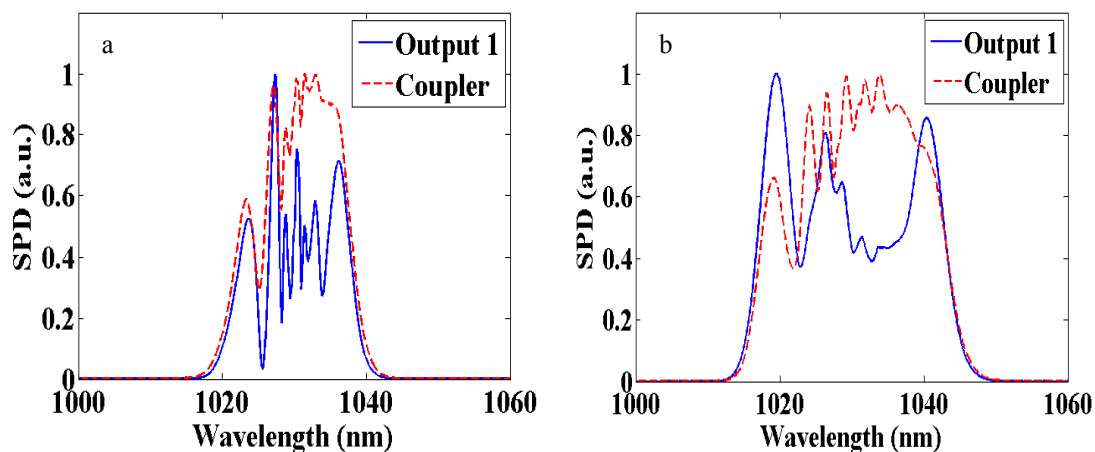


Fig. 6.9 Experimental mode locking SPD at output 1 (blue solid line) and the coupler output (red dashed line) for (a) case 2 and (b) case 3.

The autocorrelation profiles of the chirped and de-chirped pulses at output 1 are plotted in Figures 6.10(a) and 6.10(b), respectively. The values of the FWHM of the

chirped and de-chirped pulses were measured to be 2. ps and 148 fs, respectively. Using the convolution factor calculated in the simulation model, the values of the FWHM of the chirped and de-chirped pulses are 2.5 ps and 109.4 fs. Based on the relation shown in [48], the peak power of the de-chirped pulse was calculated to be 35.5 KW and 40 KW for Case 2 and Case 3, respectively.

The experimental values of the FWHM of the chirped and de-chirped show the same qualitative behavior with the variation of the position of the spectral filter within the cavity.

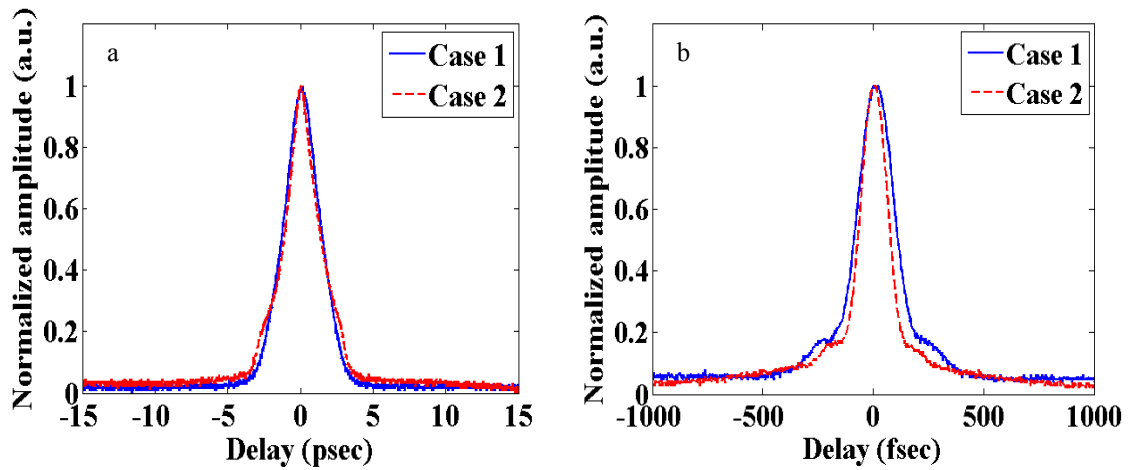


Fig. 6.10 Experimental autocorrelation profile of (a) the chirped pulse and (b) de-chirped pulse at output 1. Case 2: blue solid line and Case 3: red dashed line.

Both the simulation and experimental results demonstrate that the position of the spectral filter plays a significant role in the energy and the peak power of the de-chirped pulses. In applications where high pulse energy is required, it is advantageous to insert lumped spectral filter close to QWP₂ (Case 1), the pulse will have higher output energy (see Fig. 6.6) with reduced value of the spectral bandwidth (see Fig. 6.5(b)) and wider de-

chirped pulse width (see Fig. 6.5(c)). In contrast, if the high peak power of the de-chirped pulses is required, and the lumped spectral filter is located as in Case 3, the de-chirped pulse will have higher peak power (see Fig. 6.5(d)) and spectral bandwidth (see Fig. 6.5(b)) and narrower pulse width (see Fig. 6.5(c)) but with lower value of energy (see Fig. 6.6).

The insertion of a power splitter at the end of Yb^{3+} doped fiber with a high output coupling ratio can fix the value of output pulse energy with the advantage of having a high value of peak power as shown in Case 3.

Finally, the bulky wave plates and polarizer can be replaced with an inline fiber polarization controller and inline fiber PBS to open the way to have all fiber all normal Similariton mode-locked fiber laser.

6.5 Conclusion

In this chapter, it is shown that the location of the lumped spectral filter affects the pulse dynamics inside the mode-locked laser cavities. Four laser cavities were numerically simulated and two of them were experimentally validated. The spectral bandwidth of the pulse is increased when the narrow bandwidth spectral filter is located close to the Yb^{3+} -doped fiber, but at the expense of the pulse energy. The peak power of the de-chirped pulse depends on the pulse energy, de-chirped pulse width and the degree of the oscillatory structures in the SPD. The combination of these factors results in Case 3 having the highest value of peak power. The experimental results support the analysis made in the simulation model.

Chapter 7

7 Conclusion and future directions

7.1 Conclusion

This thesis addresses the main problems that limit the value of pulse energy and peak power. These problems are the parameters of the SA, the accumulated nonlinear phase shift and the lumped spectral filter.

Understanding the influence of the SA on the pulse characteristics, such as the pulse energy, peak power, spectral bandwidth, and de-chirped pulse, guides the design of mode-locked fiber lasers. Average models can provide clear understanding of the relationship between the SA and cavity parameters.

In Chapter 3, an analytical average model is developed for a lumped SA. This model provides closed form relations for the temporal and spectral characteristics of the mode-locked pulse in a normal dispersion regime. The model includes the complete description of the SA and shows the effect of the lumped spectral filter on the pulse characteristics. The results of this model are in good agreement with the simulation and experimental results. Based on this model, the values of the small signal absorption coefficient, the saturation power, and the location of the overdriving point of the SA were scanned to explore their effects on the pulse characteristics. The analytical results show that increasing the small signal absorption coefficient, decreasing the saturation power and pushing the overdriving point to a higher power value leads to an increase in the pulse energy and spectral bandwidth as well as a reduction in the de-chirped pulse width.

In Chapter 4, a semi-vector numerical average model for femtosecond laser mode-locked by NPR is introduced. The model includes the lumped spectral filter and the dispersion effect of the linear section of the mode-locked laser cavity after the coupler. I divide the mode-locked laser cavity into two blocks: the NPR and the scalar portion of the cavity. The NPR block consists of the nonlinear transfer function of the nonlinear fiber section of the mode-locked laser cavity, the Jones matrices of the polarization controllers and polarizer, the dispersion effect of the linear section, and the filter transfer function. The nonlinear transfer function of the nonlinear fiber section inside the NPR block does not contain SPM, dispersion, and gain parameters. They are included in the scalar portion of the cavity, which solves numerically the scalar GLE for the nonlinear section. The semi-vector model takes 1/18 of the time needed by the vector method for the same computational platform. Three different sets of polarization controllers' angles were studied with the semi-vector model showing that the location of the overdriving point has a distinct effect on the pulse characteristics. The effect of exchanging the order of the linear fiber and the lumped spectral filter is also studied using the semi-vector model. Good agreement is shown between the results of the semi-vector model, the full vector model and the experimental set-ups. However, I observed that the semi-vector model cannot show the full details of the temporal and spectral profiles of the pulse.

The accumulated nonlinear phase shift is a major hurdle that limits the pulse peak power. In Chapter 5, we have demonstrated both theoretically and experimentally that the insertion of low-doped long-length Yb^{3+} fiber inside mode-locked fiber laser results in increasing the allowed peak power of the pulse before reaching the onset of wave breaking. The gradual increase of the accumulated nonlinear phase shift during the pulse

propagation is one reason. The effect of the dispersion and the nonlinearity per unit length is another reason. The nonlinear effect in high-doped short-length Yb^{3+} fiber and the following SMF dominate the dispersion effect while both dispersion and nonlinearity have comparable weight in the case of using low-doped long-length Yb^{3+} fiber. This leads to higher immunity for wave breaking that results in higher maximum nonlinear phase shift for single-pulse operation. Therefore, the peak power of the mode-locked pulse is higher in a cavity having most of its length comprised of low-doped long-length Yb^{3+} fiber.

Another way to increase the peak power of the de-chirped pulse was demonstrated in Chapter 6. I have shown both numerically and experimentally that the position of the narrow bandwidth lumped spectral filter has a distinct effect on the pulse energy, spectral bandwidth, and de-chirped pulse peak power in active Similariton fiber laser. The spectral bandwidth broadens, and the de-chirped pulse width becomes narrower with the proximity of the lumped spectral filter to the input of the Yb^{3+} -doped fiber, but at the expense of the pulse energy. Three factors control the peak power of the de-chirped pulse, which are the pulse energy, de-chirped pulse width, and the degree of the oscillatory structures in the SPD. I find the maximum value of the de-chirped pulse power at a location of the lumped spectral filter which is close to the Yb^{3+} -doped fiber (1.825 m away).

7.2 Future directions

7.2.1 Short term

In the short term, further effort is needed to improve the SA, increase the pulse energy and make the laser more environmentally stable.

As shown in Chapter 3, we need a SA with a high value of the small signal absorption coefficient and a low value of saturation power to have high-energy mode-locked pulses. Commercial SESAM and carbon nanotubes SA cannot provide these specifications. MMI configuration with step index MMF between two SMFs can potentially be used as a SA if the input power is sufficiently high and the step index MMF has high nonlinearity parameter. The idea of operation of this SA is based on the change of the self-imaging length inside the MMF as a function of the input power. Therefore, the length of the multi-mode fiber will be selected to give high value of loss for the light having low power than the light having high power. It will act like Kerr-lens SA. Recently, E. Nazemosadat et al. showed theoretically that the capability of implementing this SA by inserting a graded index multimode fiber between two SMFs. Promising results were reported that can be built on in the future [138]. Furthermore, as mentioned in subsection 2.2.5.2, the MMI-based SA can also work as a spectral filter, which is needed in all-normal dispersion mode-locked fiber laser. This SA has three major advantages: 1—we will have some flexibility to design the best SA that results in having a high energy pulse with wide spectral bandwidth and narrow de-chirped pulse width, 2—The damage threshold is expected to be high since the SA is made of fiber and 3—The relaxation time of the SA will be in the femtosecond region as it is for the case of the SA based on NPR.

The maximum value of pulse energy in all-fiber mode-locked laser reported in the literature is 20 nJ [125]. The pulse energy can approach 100 nJ if we replace the standard Yb^{3+} -doped fiber with double-cladding fiber. However, previous reports show a difficulty in integrating Yb^{3+} -double-cladding fiber in all-fiber mode locked laser [105, 106]. Therefore, based on the results shown in Chapter 4, we can investigate a new mode-locked laser cavity configuration where the cavity can be divided into nonlinear and linear sections based on the power of the pulse propagating in that section. In the nonlinear section, whereas a high power pulse is propagating; the optical wave-breaking can be avoided by utilizing low nonlinearity fibers that have a large mode area (LMA). The linear section can be constructed from a standard single-mode fiber of mode field diameter 6 μm . Mode-locking can be achieved by NPR or the MMI-based SA.

The main problem in fiber laser mode-locked by its NPR is stability against any environmental changes. C. K. Nielsen et al. reported the implementation of environmentally stable mode-locked Fabry-Perot all-fiber laser using PMF [130]. The mode-locked pulses fall under the category of dissipative soliton pulses in an all normal dispersion region. The linear birefringence of the PMF was cancelled by having a Faraday mirror. However, this configuration suffers from narrow spectral bandwidth (2 nm) and low average power (8 mW) [130]. We propose a new design of the mode-locked laser cavity that replaces the Fabry-Perot cavity with a Sigma cavity. This new design will result in wider spectral bandwidth and higher average power. It will consist of a loop having the gain medium, the output coupler and the spectral filter. It will couple to the other part of the cavity through a PBS. This part will be comprised of the SA. The

location of the overdriving point of the SA and spectral bandwidth of the spectral filter will be optimized to maximize the average power and spectral bandwidth of the pulse.

7.2.2 Longer term

Divided pulse amplification is an efficient technique to produce pulse energy in the range of microjoules. The mode-locked pulse is divided into number of small pulses. The nonlinearity has a minor effect on the small pulses. Each of them is amplified individually inside the gain medium, and then they will be combined coherently outside the gain medium to have a high energy pulse [139]. Divided pulse amplification was applied to soliton fiber laser, but the pulse energy was in the nanojoules range [140]. However, the pulse energy can approach microjoules in mode-locked fiber laser without any external amplification if the pulse follows the dissipative soliton regime. Previous configurations of a divided pulse amplifier have a lot of bulky optics components, which prevent it from being portable. The main challenge is to have an all-fiber configuration for divided pulse amplification as a part of an all-fiber mode-locked laser. The design should rely on PMF components in order to have an environmentally stable device.

Appendix A

A. Stationary phase method

The stationary phase method is used to get a closed form of the spectral power density and spectral phase of the mode-locked pulse defined by equations (3.5) and (3.11). The Fourier transform of the temporal profile of the field is given as:

$$I(\omega) = \int_{-\infty}^{\infty} \sqrt{P(t)} \exp(i\phi(t) - i\omega t) dt \quad (\text{A.1})$$

The phase term $(\phi(t) - \omega t)$ is a rapidly varying function of t over most of the integration range, $\sqrt{P(t)}$ is a slow varying function. Rapid change of the exponential term leads to having a zero value of $I(\omega)$ over most of the integration range, except to the locations where $d(\phi(t) - \omega t)/dt = 0$; point of stationary phase [141]. These points (t_o) are defined as:

$$\left. \frac{d\phi(t)}{dt} \right|_{t_o} - \omega = 0 \quad (\text{A.2})$$

Since I defined in section 3.2 $\Omega = -d\phi/dt$, the value of phase of the pulse at the stationary point is given as:

$$\Omega(t_o) = -\omega \quad (\text{A.3})$$

As mentioned above, $\sqrt{P(t)}$ is a slow varying function. Therefore, $P(t) \approx P(t_o)$ in the neighbourhood of these stationary points. By substituting (A.3) into (3.5), the pulse power will be expressed as:

$$P(t_o) = \frac{\beta_2}{2\gamma} (\Delta^2 - \omega^2) \quad (\text{A.4})$$

Equation (A.4) is considered a constant function with respect to time. Therefore, it will be taken outside the integration in (A.1). By expanding the phase term $\phi(t) - \omega t$ by Taylor series to only the second term around t_o , it will be given as:

$$\phi(t) - \omega t = \phi(t_o) - \omega t_o + \frac{1}{2} \frac{d^2 \phi(t)}{dt^2} \Big|_{t_o} (t - t_o)^2 \quad (\text{A.5})$$

Substituting (A.4) and (A.5) into (A.1), we will have:

$$I(\omega) = \sqrt{\frac{\beta_2}{2\gamma} (\Delta^2 - \omega^2)} \exp(\phi(t_o) - \omega t_o) \int_{-\infty}^{\infty} \exp\left(i \frac{1}{2} \frac{d^2 \phi(t)}{dt^2} \Big|_{t_o} (t - t_o)^2\right) dt \quad (\text{A.6})$$

Equation (A.6) has a closed form of:

$$I(\omega) = \sqrt{\frac{\pi \beta_2}{i \gamma \frac{d^2 \phi(t)}{dt^2} \Big|_{t_o}} (\Delta^2 - \omega^2) \exp(\phi(t_o) - \omega t_o)} \quad (\text{A.7})$$

As $\frac{d\Omega(t)}{dt} \Big|_{t_o} = -\frac{d^2 \phi(t)}{dt^2} \Big|_{t_o}$, equation (3.11) can be modified after substituting (A.3):

$$\frac{d^2 \phi(t)}{dt^2} \Big|_{t_o} = -\frac{2\alpha \left(1 - \frac{\zeta}{y}\right) (\Delta^2 - \omega^2) [S^2 - \omega^2]}{3\beta_2 \left(\Delta^2 - \omega^2 + \frac{2\gamma}{\beta_2 b_1}\right)} \quad (\text{A.8})$$

Substituting (A.8) into (A.7), the spectral power density of the pulse $P(\omega) = |I(\omega)|^2$ is expressed as:

$$P(\omega) = \frac{3\pi \beta_2^2}{2\alpha \gamma} \frac{1}{\left(1 - \frac{\zeta}{y}\right)} \left[\frac{\Delta^2 + \frac{2\gamma}{b_1 \beta_2} - \omega^2}{S^2 - \omega^2} \right] H(\Delta^2 - \omega^2) \quad (\text{A.9})$$

The Heaviside function is added to (A.9) in order not to have any singularity in the equation. Finally, it is obvious that (A.9) is exactly the same as (3.15a).

Appendix B

B. Temporal and spectral characterization of mode-locked fiber laser

Fig. B.1 shows the schematic diagram of the experimental set-up used to characterize output of the mode-locked chirped pulse. The average power of the chirped pulse at the main output is measured by a power meter (PM100 or PM100D) from Thorlabs. The chirped pulse passes through a beam splitter (BS) with a 50% splitting ratio. The chirped pulse at output 1 of the BS is incident on an Autocorrelator with scanning range 120 ps; its part number is FR-103XL from Femtochrome Research, Inc. It measures the autocorrelation of the pulse. A flip mirror is inserted in the path of the chirped pulse to direct it to the dispersion delay line (DDL); it is implemented by using a pair of diffraction gratings and a retro-reflector as explained in sub-section 2.2.4. In Chapters 3 and 4, we used the same Autocorrelator to measure the autocorrelation of the pulse passed through the dispersion delay line, while we used another Autocorrelator (FR-103MN from the same company) having a lower scanning range of 20 ps to measure the autocorrelation of the pulse passed through the dispersion delay line in Chapters 5 and 6. Since vertical polarization is favoured by both these Autocorrelators, a half-wave plate (HWP) is inserted in front of them to maximize the input power to the second harmonic generation crystal inside them. Both Autocorrelators have a wavelength spanning from 410 to 1800 nm and a resolution of 5 fs.

An angle fiber collimator is used to inject the light from output 2 to an SMF (HI1060). The other terminal of the fiber is connected to an Optical spectrum analyzer (OSA) via an FC/PC connector. The part number of the OSA is AQ-6315E from ANDO;

it has scanning wavelength span from 350 nm to 1750 nm and a minimum resolution bandwidth of 0.05 nm. The OSA is used to plot the spectral power density of the pulse. The interference pattern in the SPD is a signature of multi-pulsing operation.

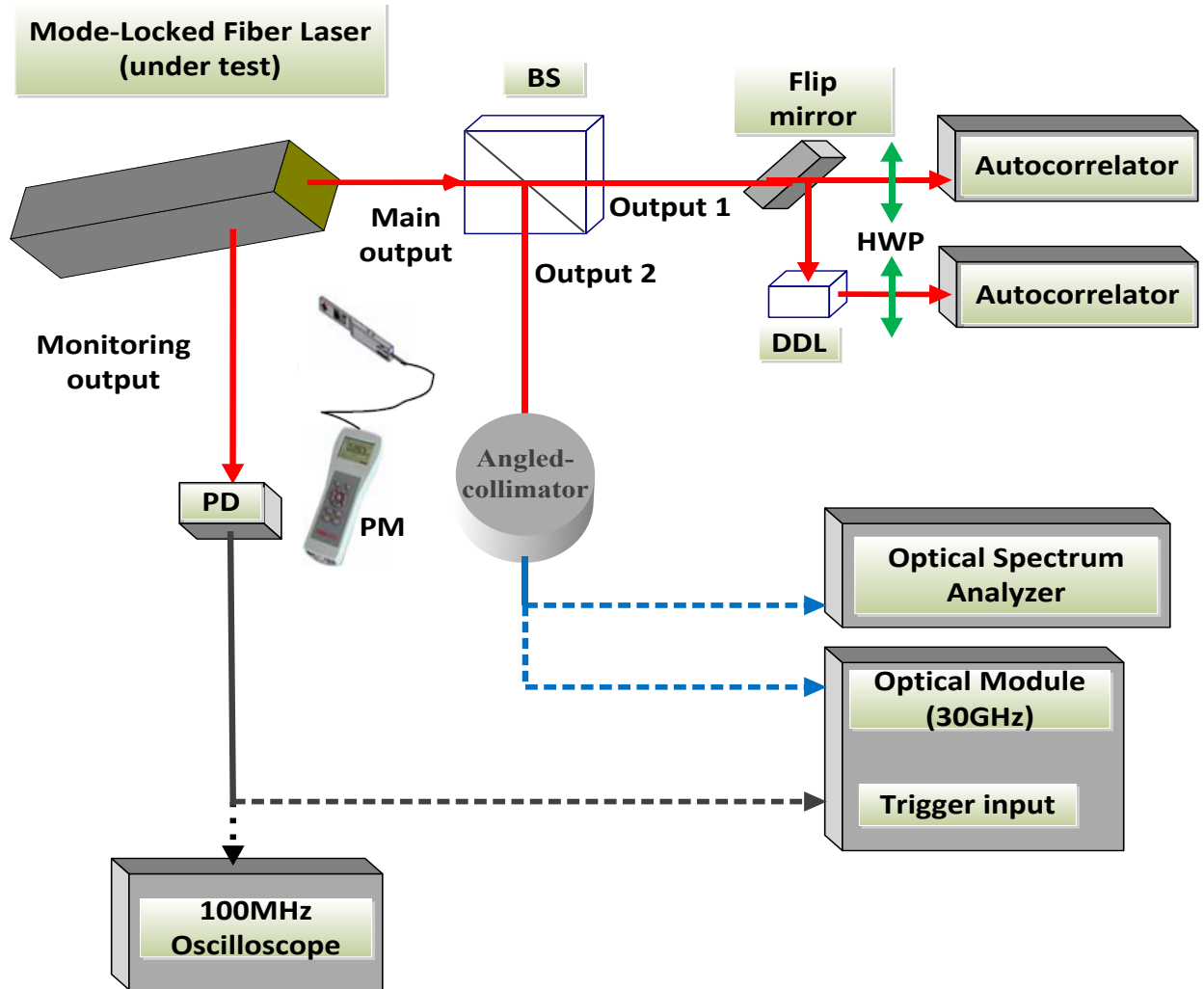


Fig. B.1 Schematic diagram of the experimental set-up used to characterize the mode-locked pulse. The red lines represent free space collimated optical light. The blue lines denote SMF. The black lines denote electric cables. Dashed lines mean that the two instruments are not connected simultaneously. BS: beam splitter, PD: photodetector; DDL: dispersion delay line, PM: optical power meter and HWP: half-wave plate.

The monitoring output of the mode-locked fiber laser is directed to an amplified Si PIN photodetector; its part number is 818-BB-21A from Newport. Its wavelength span is 300-1100 nm and the electrical bandwidth is 1.2 GHz. An electrical cable connects the electrical output of the PIN photodetector to an oscilloscope (OSC) with a bandwidth of 200 MHz and a sampling rate of 2 GHz. It has part number of TDS2022C from Tektronix. This OSC serves as real time monitoring of the mode-locked pulse.

Since the maximum separation between pulses that can be measured with the Autocorrelator (FR-103XL) is only 120 ps, another OSC is used in order to have larger scanning range. It is a sampling-based oscilloscope with part number HP 83480A. It has a fast optical photodetector (30 GHz) module; its part number is 83482A. The SMF attached to the angled collimator is connected to the optical input of the fast photodetector, and the electrical cable from the PIN photodetector is connected to the trigger input of the OSC.

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