

# Three Essays on Unconventional Monetary Policy at the Zero Lower Bound

A thesis submitted

*by*

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*to*

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## Abstract

- Chapter 1: Impact of Quantitative Easing at the Zero Lower Bound (with J. Dorich, R. Mendes)

We introduce imperfect asset substitution and segmented asset markets, along the lines of Andres et al. (2004), in an otherwise standard small open-economy model with nominal rigidities. We estimate the model using Canadian data. We use the model to provide a quantitative assessment of the macroeconomic impact of quantitative easing (QE) when the policy rate is at its effective lower bound. Our results suggest that a QE intervention of 4 per cent of GDP for 4 quarters has moderate impact on interest rates but small impact on output. More specifically, when the nominal short-term interest rate is constrained at the lower bound, following a decline of 10bps in the long-term bond yield through QE, we find a small impact on aggregate demand (real level of GDP increases by 0.06%, similar to the findings of Baumeister and Benati 2010) and a much larger impact on inflation (0.1%). The timing of the monetary stimulus, however, is much faster, with peak responses on output and inflation occurring after 4 and 3 quarters, respectively.

- Chapter 2: Impact of Forward Guidance at the Zero Lower Bound

This chapter addresses the use of conditional statement at the zero lower bound, along with other forms of policy guidance. I consider alternative monetary policy rules under commitment in a calibrated three-equation New Keynesian model and examine the extent to which forward guidance helps to mitigate the negative real impact of the zero lower bound (generated by a negative demand shock). The simulation results suggest that the conditional statement policy prolongs the zero lower bound duration for an additional 4 quarters and reverses half of the decline in inflation associated with the lower bound. It even generates a period of overshooting in inflation three quarters after the initial negative demand shock. Alternatively, the effect of price-level targeting as a forward guidance policy at the zero lower bound is slightly different. It shortens the zero lower bound duration by one quarter, but generates the highest inflation profile among the alternative policy rules considered.

- Chapter 3: Impact of Quantitative Easing on Household Deleveraging

I extend the DSGE model in the first chapter that features imperfect asset substitutability, segmented asset market with some financial frictions to match the great ratios and dynamics in the Canadian economy. I simulate the model to explore the effects of quantitative easing on asset prices, household balance sheet. There are two effects of QE on aggregate output originated from the model. First, QE leads to a decline in term premium, which increases current consumption relative to future consumption. Second, it creates a more favorable financing conditions for borrowers through an improvement in the household balance sheets arising from higher net worth. This leads to a lower loan to collateral value ratio and a decline in external finance premium. Favorable financing condition encourages further accumulation of household debt at cheaper rates, in turn, leads to an immediate higher household debt to income ratio. The effect of QE beyond the initial impact would depend on the elasticity of loan demand to the policy. If the increase in loan is greater than the increase in the overall collateral value, this would lead to an ultimate deeper leverage and therefore higher default rate. In the consideration of the future withdrawal of any stimulus provided from QE, this would pose greater challenges as it implies much intensive household deleveraging process. I provide some sensitivity analysis around key parameters of the model.

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# Chapter 1

## Introduction

In normal times, monetary policy steers the level of the key interest rates to ensure price and output stability over the medium term. This conventional approach has proven to provide sufficient monetary stimulus to the economy during downturns, preventing downward inflationary pressures and ensuring economic recovery. In the aftermath of the financial and economic crisis of 2007-2009, central banks in many advanced economies cut their policy rates very aggressively to counter rapidly deteriorating economic outlooks. During this process, many of these central banks found themselves hitting the effective lower bound on the policy rate. In this context, numerous unconventional monetary tools were considered in order to provide additional monetary stimulus.<sup>1</sup>

During abnormal times, unconventional monetary policy may be warranted under two possible scenarios (Smaghi 2009). First, the negative economic shock is severe enough such that the nominal interest rate needs to be lowered down to zero (or some effective lower bound). Second, unconventional monetary policy may be used when monetary policy transmission mechanism is significantly impaired, especially in an already "low-for-long" interest rate environment. Since it is impossible or ineffective to further reduce the policy rate at the lower bound, additional monetary stimulus can be provided in three ways: i) by guiding the long-term interest rate expectations; ii) by changing the composition of the central bank's balance sheet, and iii) by expanding the size of the central bank's balance sheet. I will discuss each approach in details below.

First, a central bank could reduce the spreads of longer-term interest rate over expected short-term policy rates through large-scale purchases of government securities. This approach is often referred to as quantitative easing (QE), which aims at easing fi-

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<sup>1</sup>For example, the Bank of England has engaged in large-scale asset purchases during 2009-2010 (Bean 2009). The Federal Reserve have conducted four rounds of quantitative easing with some recent forward guidance which links the monetary policy action conditional on the development of labour market (see FOMC release, September 2012).

nancing conditions of banks, households and firms. In practice, as analyzed by Krugman (1998) and Svensson (2004), since the cost of external finance is generally a premium over the short-term interbank lending rate, the central bank could reduce the spreads between various forms of external finance, thereby influencing the asset prices in the economy. When the nominal short-term interest rate remains constrained at the lower bound, the real interest rate falls as a result of higher expected inflation. Since long-term interest rates are average of expected short-term rates, the expectation channel would tend to flatten the yield curve, therefore stimulate spending.

The experience of the Bank of Japan during 2001 and 2006 provides an example of unconventional monetary policy of this type.<sup>2</sup> Recent studies of the Fed's first round of QE experience in 2009Q1 also provide more empirical evidence. Most of the studies on QE use event study or econometric analysis to quantify the impact of additional stimulus while only a few analysis are conducted within a DSGE framework.<sup>3</sup>

Alternatively, a central bank could reduce the economic costs of the zero lower bound by credibly signaling that short-term interest rates will remain low for a prolonged period of time, conditional on the development of a particular market (or indicator). This approach is often referred as conditional forward guidance (Evans 2011). Similarly, the central bank's forward guidance could also offset the impact of the zero lower bound by announcing that it will generate an overshooting of inflation or output growth once the lower bound is no longer a constraint (through targeting a nominal anchor like the price-level or output-level). To manage expectations efficiently and avoid time-inconsistency issue associated with policy guidance, monetary authority often announces clear conditions under which policy rates would begin to increase.

The practice of forward guidance has undergone significant development since the onset of the financial crisis in the formal statements by the Federal Open Market Committee (FOMC). Prior to the last guidance communicated in September 2012, commitment of

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<sup>2</sup>Ugai (2006) provides an evaluation of quantitative easing in Japan and similar to most papers on Japanese QE experience, it shows that although the unconventional policy has generated an accommodative environment for corporate financing, it did not lead to higher aggregate demand and prices. The insufficient functioning of the Japanese banking sector has significantly reduced the effectiveness of injecting liquidity into the economy.

<sup>3</sup>Among the most relevant literature, Caglar et al. (2011) examine QE in the UK using three separate DSGE models, including financial frictions. QE can correct the effects of a zero lower bound in DSGE models, by (i) offsetting the liquidity premium embedded in long-term bonds and/or (ii) adopting countercyclical subsidies to bank capital and/or (iii) the creation of central bank reserves that reduce the costs of loan supply. Using a portfolio balance model, they find limited role for QE (a fall in long-term interest rate of 100bp would require purchases of about 50-75% of debt outstanding). Chen et al (2011) analyze the impact of large-scale asset purchase programs in the U.S. using a DSGE model enriched with a preferred habitat framework. They estimate a moderate increase in GDP growth (by 0.4pp) following QE but the impact is persistent. The impact on inflation is also small due to the small degree of financial market segmentation in the model.

the Fed provides stimulus mainly through expectations of prolonged period of low nominal interest rates. The conditional forward guidance, on the other hand, not only works through expectation of long period of low nominal interest rate, but also through expectation of higher than expected future inflation. This further helps to reduce market expectation of future real interest rate and therefore effectively stabilizes the output gap and inflation.<sup>4</sup> Conditional guidance was recently used by the Bank of Canada in face of financial crisis in April 2009, given its experience of reaching the effective zero lower bound and the sluggish recovery of the world economy. Despite the wide use in practice, only a few studies have looked at the effectiveness of forward guidance policies at the zero lower bound and no studies have examined the effects of conditional forward guidance and guidance through a price-level targeting in a DSGE model framework.

Both accommodative conventional monetary policy stance or unconventional approaches (such as quantitative easing or forward guidance) aim at warranting a low interest rate environment in response to the global financial crisis and ensure a recovery of the global economic activity. While keeping interest rate low for longer would help to support aggregate output, the conduct of unconventional policies could also further strengthen the financial imbalances associated with household indebtedness (through an increasing household debt to income ratio) and intensify the misallocations of resources (e.g., overinvestment of residential structure) that were fueled by an already low-for-long interest rate environment. The concerns about the impact of house price volatility and the resulting changes in household net worth on amplification of the aggregate fluctuations have encouraged more research on household financial frictions.<sup>5</sup>

This thesis comprise three essays. In the first essay "Impact of quantitative easing at the zero lower bound" (chapter 2), we quantify the impact on aggregate demand from a central bank's open market purchases of long-term government bonds when the policy rate is at the zero lower bound. In the second essay "Measuring the effects of forward guidance at the zero lower bound", I consider two alternative monetary policy rules under commitment in a calibrated simple three-equation New Keynesian model to examine the extent to which forward guidance (either through a conditional statement or a price-level target) helps to mitigate the negative real impact of the zero lower bound. Finally, in the third essay, I extend the analysis of the first essay to explore the effects of quantitative easing on asset prices, household balance sheet as well as the implication of economy leverage level and monetary policy rules on ratios related to household financial imbalances. I discuss results and contribution of each essay in details in the section below.

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<sup>4</sup>see Woodford (1999), Eggertsson and Woodford (2003) and Walsh (2009) for more details.

<sup>5</sup>See Bernanke, Gertler and Gilchrist (1999), Christiano et al. (2010) and Iacoviello and Neri (2009) for prominent examples.

## 1.1 Contribution of the thesis

This thesis falls within the DSGE literature in evaluating the impact of unconventional monetary policies.

In the first essay, we provide a quantitative assessment of the macroeconomic impact of QE when the policy rate is at its effective lower bound. We make two key modifications to a standard NK model. In particular, we introduce imperfect asset substitution (IAS) and segmented asset markets (SAMs), along the lines of Andres et al. (2004), in an otherwise standard small open economy model with nominal rigidities. Both assumptions are crucial.<sup>6</sup> IAS allows the long-term rate to deviate from the level implied by the strict expectation theory of the term structure. In particular, it allows the model to generate a term premium that can be affected by monetary policy. SAMs allow the long-term interest rate to independently affect aggregate demand over and above short rates.

IAS is introduced by assuming that money and long term bonds have different degrees of liquidity. We capture this in the model by making utility depend on the ratio of money to long-term bond holdings. IAS allows the feasibility of implementing quantitative easing in the sense that the long-term rate could deviate from the level implied by the strict expectation theory of the term structure (through movements in the term premium). However, IAS alone only enables us to analyze the portfolio effects of quantitative easing. Only with additional feature of the asset market segmentation, QE program could affect the real economy as consumption is affected by both short-term and long-term interest rates.

Given our assumptions, we have two important predictions. First, the term premium is a decreasing function of the ratio of money to long-term bond holdings. As a result, QE has a direct impact on the term premium as long as it affects this ratio. Second, the macroeconomic impact of QE is determined by the sensitivity of aggregate demand to the long-term rate. In our framework, this sensitivity is influenced by the fraction of households that can only save in long-term assets.

We use this model to evaluate the effect of QE at the zero lower bound by simulating a large and persistent contractionary consumption demand shock. This shock is big enough that the desired policy rate becomes negative. As a result, the effective lower bound constraint becomes binding. We then assume that the central bank conducts open market purchases of long-term government bonds when the policy rate is at the lower

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<sup>6</sup>The standard New Keynesian (NK) model, commonly used in policy discussions, assumes that financial assets are perfect substitutes. Moreover, it predicts that aggregate demand is affected by the evolution of just one interest rate: the short term interest rate (i.e. the policy rate). Therefore, in the standard model there is no role for QE.

bound. The open market purchases support aggregate demand by reducing long-term interest rates.

The simulation result suggests a moderate impact of quantitative easing. When the nominal rate is constrained at the lower bound, following a decline of 10bps in the long-term bond yield through quantitative easing, we found a small impact on aggregate demand (real level of GDP increases by 0.06%, similar to the findings of Baumeister and Benati 2010) and a much larger impact on inflation (0.1%). The timing of the monetary stimulus, however, is much faster, with peak responses on output and inflation occurring after 4 and 3 quarters, respectively.

In the second essay, I explore forward guidance as an alternative unconventional monetary policy that the central bank uses to complement quantitative easing. I examine the implication of expectation-formation mechanism at the ZLB by considering alternative monetary policy rules under commitment. Even the nominal short-term interest rate remains constrained at zero, central banks can still provide further stimulus by shaping private agents' expectations about the future development of the economy. More specifically, I compare the performance of a state-contingent guidance (using conditional statement) and a price-level guidance in a calibrated simple three-equation New Keynesian model. I examine the extent to which forward guidance helps to mitigate the negative real impact of the zero lower bound.

The simulation results suggest that guidance using a conditional statement prolongs the zero lower bound duration for an additional 4 quarters and reverses half of the decline in inflation associated with the lower bound. In addition, it even generates a period of overshooting in inflation three quarters after the initial negative demand shock. As a result of higher more accommodative policy and higher inflation expectation, GDP increases by 1.8 per cent on peak with a short period of production expansion about a year and a half after the initial shock.

The effect of price-level targeting as a guidance policy at the zero lower bound is slightly different. The policy shortens the zero lower bound duration by one quarter, but generates the highest inflation profile among the alternative policy rules considered. Price level returns to pre-shock level and the overall loss is reduced to 6.4 per cent with the price-level targeting guidance, compared to 2.4 per cent with the conditional guidance under inflation targeting and 10.3 per cent with the case of zero lower bound.

In the third essay, I extend the small open-economy DSGE model with household financial frictions to analyze the effects of QE on household deleveraging. The model features heterogenous households and a financial intermediary. The patient savers can trade in both short- and long-term bonds markets and have self-imposed liquidity re-

quirements on their long-term investments (money and long-term bonds are imperfect substitutes). Impatient borrowers are middle-aged households who only trade in long-term bonds market and do not face liquidity requirements on their long-term investments. They are the borrowers in the economy, buying long-term government bonds and borrowing from the savers in the economy. The existence of borrowers allows for the conduct of QE policies as it guarantees long-term interest rate to matter over and above the short-term rates in the aggregate demand function (Andres et al. 2004). Borrowers purchase housing relying on mortgage loans provided by a financial intermediary. Borrowing is subject to an external financing premium due to loan default risk, which generates endogenous movement in borrowers' loan to value ratio.

I find two effects of QE on aggregate output originated from the model. First, QE leads to a decline in term premium, which increases current consumption relative to future consumption. On the other hand, it creates more favorable financing conditions for borrowers through an increase in collateral value and a decline in external finance premium. Lower finance premium encourages further accumulation of debt at cheaper rates and, in turn, leads to an immediate higher household debt to income ratio. Mortgage default declines on impact following QE.

The effect of QE beyond the initial impact depends on a few important factors. First, it depends on the elasticity of loan demand to the policy. If the increase in loan is greater than the increase in the overall collateral value, this would lead to an ultimate deeper leverage and therefore higher default rate. In the consideration of future withdrawal of any stimulus provided by QE, deeper leverage would pose greater challenges as it implies much more intensive household deleveraging process. In the baseline calibration, with the implementation of QE, there is a 1.7 per cent increase in the loan demand. This is reflected in a lower finance premium and an increase in collateral value. Overall, as household net worth returns to the steady state five years after the initial QE policy, default rate remains 5 percentage points higher than its steady state level.

The effect of QE also depends on the steady-state housing investment risk level (or the extent of borrowers' leverage). When the economy is in a high-leverage state, following QE, the increase in inflation of consumption goods is greater. QE thereby largely lowers borrowers' real debt, leading to greater increases in consumption, residential investment and demand for mortgage loans. Mortgage default five years after QE is slightly higher than the benchmark case level, suggesting higher housing investment risks post-QE.

The effect of QE on the economy is also subject to the monetary policy rule the central bank follows while conducting QE. For a strong inertia policy rule (higher smoothing parameter on the short-term nominal interest rate), short-term rate reacts less to the decline

of long-term interest rate (or remains constrained at the zero lower bound for longer) given there is more smoothing. As a result, under higher smoothing, borrowers increase their demand for consumption and housing more than the case when the policy rule is less inertia. Overall, keeping short-term interest rate more persistent helps to better stimulate the aggregate demand, but has less implications for household deleveraging as the decline in external finance premium is found to be smaller with more stable demand for housing under the lower long-term interest rate.

## Chapter 2

# Impact of Quantitative Easing at the Zero Lower Bound

### 2.1 Introduction

In normal times, monetary policy steers the level of the key interest rates to ensure the price or output stability over the medium term. This conventional measure has proven to provide sufficient monetary stimulus to the economy during downturns, preventing deflationary pressures and ensuring economic recovery. During abnormal times, unconventional monetary policy may be warranted under two possible scenarios (Smaghi, 2009). First, the negative economic shock is severe enough such that the nominal interest rate needs to be lowered down to zero.<sup>1</sup> Second, unconventional monetary policy may be used when monetary policy transmission mechanism is significantly impaired.

When economic slack emerges in the economy, the central bank can't move down the overnight interest rate below zero percent (or an effective zero lower bound). In this context, monetary policy could depart from targeting interest rates to targeting the amount of excess reserves held by banks. In another word, a central bank can target the quantity of currency in the financial system by purchasing financial assets (in form of long-term government treasury bonds in this model) in exchange for reserves. This approach is often referred to as quantitative easing (QE), which aims at easing financing conditions of banks, households and firms. In our model, QE is defined as the large purchase of government long-term treasury bonds that aims to reduce the spreads of

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<sup>1</sup>Since operationally it is impossible to further reduce the policy rate, additional monetary stimulus can be achieved in three complementary ways: i) by guiding the long-term interest rate expectations; ii) by changing the composition of central bank's balance sheet, and iii) by expanding the size of the central bank's balance sheet.

long-term interest rate over expected short-term overnight interest rate.

The standard New Keynesian model, commonly used in policy discussions, assumes that financial assets are perfect substitutes. Moreover, it predicts that aggregate demand is affected by the evolution of just one interest rate: the short-term interest rate (i.e. the policy rate). Therefore, in the standard model there is no role for QE. In this chapter, we provide a quantitative assessment of the macroeconomic impact of QE when the policy rate is at its effective lower bound. We make two key modifications to the standard model. In particular, we introduce imperfect asset substitution (IAS) and segmented asset markets (SAMs), along the lines of Andres et al. (2004), in an otherwise standard small open economy model with nominal rigidities. Both assumptions are crucial. IAS allows the long-term rate to deviate from the level implied by the strict expectation theory of the term structure. In particular, it allows the model to generate a term premium that can be affected by monetary policy. SAMs allow the long-term interest rate to independently affect aggregate demand over and above short-term rates.

IAS is introduced by assuming that money and long term bonds have different degrees of liquidity. We capture this in the model by making utility depend on the ratio of money to long-term bonds holdings. IAS allows the feasibility of implementing QE in the sense that the long-term rate could deviate from the level implied by the strict expectation theory of the term structure (through movements in the term premium). However, IAS alone only enables us to analyze the portfolio effects of QE. Without additional feature of market segmentation, QE program will not affect the real economy. This makes the assumption of SAMs also necessary.

We review the relevant literature and present the model structure and obtain some analytical solution. Given our assumptions, we have two important predictions. First, the term premium is a decreasing function of the ratio of money to long-term bonds holdings. As a result, QE has a direct impact on the term premium as long as it affects this ratio. Second, the macroeconomic impact of QE is determined by the sensitivity of aggregate demand to the long-term rate. In our framework, this sensitivity is influenced by the fraction of households that can only save in long-term assets.

In this chapter we use this model to evaluate the effect of QE at the lower bound by simulating a large and persistent contractionary consumption demand shock. This shock is big enough that the desired policy rate becomes negative. As a result, the effective lower bound constraint becomes binding. We then assume that the central bank conducts open market purchases of long-term government bonds when the policy rate is at the lower bound. The open market purchases support aggregate demand by reducing long-term interest rates.

Our work falls within the DSGE literature in evaluating the impact of QE through the portfolio balancing channel. This chapter contributes to the literature in two ways. First, it provides an empirical assessment of QE on key macroeconomic variables like GDP and inflation. Moreover, it provides some open-economy implications of the conduct of QE. Overall, our estimated small open economy model of Canada suggests a moderate impact of QE. When the nominal rate is constrained at the lower bound, following a decline of 10bps in the long-term bond yield through QE, our result suggests a small impact on aggregate demand (real level of GDP increases by 0.06%, similar to the findings of Baumeister and Benati 2010) and a much larger impact on inflation (0.1%). The timing of the monetary stimulus, however, is much faster, with peak responses on output and inflation occurring after 4 and 3 quarters, respectively.

This chapter is divided as follows. We start by discussing the relevant literature on QE. We then provide a description of the model's key structure, followed by presenting the steady state around which we linearize the model. We estimate some key parameters of the model using Bayesian methods and then assess the impact of QE when the nominal short-term interest rate is at its effective lower bound.

## 2.2 Relevant Literature on QE at the ZLB

The domestic transmission of QE operates through a number of channels. First, QE works through a direct interest rate channel by reducing long-term yields and real interest rates, thereby encouraging investment and consumption spending. Second, QE can also change the relative demand and prices of a targeted financial asset that is imperfect substitute with other securities, in turn, change the households' portfolio decisions. This is referred as the portfolio-balance channel as in Woodford (2012). The third channel through which QE influences the market is the signalling (or expectation) channel. By communicating the intention to a specific future interest rate path, the central bank shapes market expectations of the long-term yields and helps to stimulate aggregate demand. Fourth, the purchase of long-term asset may also support equity and house prices through the asset price channel. Finally, QE also works through the confidence channel as it may encourage investors to be more risk-taking.

Most of the studies on QE use approaches like event study or econometric analysis to quantify the impact of QE (Baumeister and Benati, 2010; Chung et al., 2011). Few work is done within a DSGE framework analyzing the impact of QE (e.g., Caglar et al., 2011; Chen et al., 2011). Our work falls within the DSGE literature in evaluating the impact of QE with a focus on the portfolio-balance channel. The approach used in this chapter

can be related to the segmented-market model of the term structure (e.g., Vayanos and Vila, 2009; Andres et al., 2004 for examples). This chapter contributes to the literature in two ways. First, it provides an empirical assessment of QE at the zero lower bound on key macroeconomic variables like GDP and inflation. More importantly, it provides some open-economy implications of the conduct of QE.

### 2.2.1 The Portfolio-Balance Effect of QE on term premium

Through open-market purchases of certain targeted securities (long-term government treasury bonds for example), a central bank creates the portfolio-balance effect. Woodford (2012) describes the effect that emerges as "the central bank holds less of certain assets and more of others, then the private sector responds by holding more of the former and less of the latter". A change in the relative prices of the financial assets is required to satisfy the new equilibrium portfolio for private agents.

The portfolio-balance effect requires a segmented long-term debt market and imperfect substitutability across the term structure. The implications of QE through the portfolio-balance channel has been reviewed in Thornton (2012). One way to introduce segmented market is by considering a market populated by two types of investors as in Vayanos and Vila (2009): the preferred-habitat investors demand only the bonds corresponding to their desired maturity, and the risk-averse arbitrageurs optimize the Markowitz risk-return trade-offs. Vayanos and Vila (2009) acknowledge that even though the assumption of the preferred-habitat investors is extreme, it is quantitatively important as the effect on the yield curve depends on the ratio of preferred-habitat investors to arbitrageurs.

Greenwood and Vayanos (2010) conduct further empirical analysis to examine the effectiveness of QE through the portfolio balance channel using Vayanos and Vila's (2009) model. Among three measures of the maturity structure of the public's holding of government debt (the average maturity of the debt, the fraction of the debt with maturity of or above 10 years, and the duration of the debt), they find the strongest evidence of a portfolio balance channel for longer-horizon excess returns.

D'Amico et al. (2010) also suggest that QE can affect long-term yield and term premium through a duration channel that has similar implications as the portfolio-balance channel.<sup>2</sup> By removing the aggregate duration from the outstanding stock of government

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<sup>2</sup>In addition to the duration channel, there are two other channels discussed in D'Amico et al. (2011). The first is scarcity channel through which the asset purchase of the Federal Reserve with a specific maturity leads to higher prices (and lower yields) of securities with similar maturities. The other channel is called the signaling channel since the purchases convey important information about the future path of the nominal short-term interest rate.

Treasury debt, QE reduces the term premium on securities across maturities. The duration channel is found to be statistically significant and accounts for about a third of the estimated total effect of QE.

Similarly, Gagnon et al. (2011) also suggest that QE reduces the yields on long-term asset because it removes a significant amount of financial assets with high duration from the market. As a result of lower duration risk of assets holding, the market demands a lower premium to hold that risk. Using two alternative measures of  $i$  (the 10-year Treasury bond yield and an estimate of the 10-year Treasury term premium), they estimate the equation

$$i = \alpha + X_t\beta_1 + Debt_t\beta_2 + \varepsilon_t \quad (2.1)$$

over January 1985 to June 2008 with a vector of key macroeconomic variables  $X_t$  and a measure of the public's holding of Treasury debt. Their analysis suggests that QE1 (\$1.75 trillion asset purchase) by the Fed would have reduced the term premium by about 52 basis points and the 10-year Treasury yield by about 82 basis points. Alternatively, Hamilton and Wu (2012) investigate the effects of QE1 by estimating a three factor affine term structure model and their estimates of the effect on the 10-year Treasury yield and term premium are smaller than that reported by Gagnon et al. (2011).

### 2.2.2 The Domestic Effects of QE

One potential issue as pointed out by Thornton (2012) of analyzing the effect of QE through portfolio-balance channel is that even if the effect of QE on long-term rate is large, the effect on domestic economic activity is comparatively small. Because the effectiveness of QE depends on the extent to which the long-term bond market being segmented from the rest of the market, the larger the effect of QE on long-term rate, the smaller the effect on interest rates that are more important for economic activity.

The experience of Bank of Japan during 2001 and 2006 provides an example of QE that has limited effectiveness in stimulating the aggregate demand. The unconventional policy features the purchase of Japanese government bonds to meet the target balance of current account deposits at the bank. This follows the practical approach of QE which traditionally focuses on buying long-term government bonds from banks. Ugai (2007) provides an evaluation of QE in Japan and similar to most papers on Japanese QE experience, he shows that although the unconventional policy has generated an accommodative environment for corporate financing, it did not lead to higher aggregate demand and prices. The insufficient functioning of the Japanese banking sector is be-

lieved to have significantly reduced the effect of injecting liquidity into the economy.

Recent studies of the purchases of long-term U.S. Treasury bonds by the Fed (QE1 or the first-round of large-scale asset purchase program, LSAP, in March 2009; and QE2 or Operation Twist for example) provide more positive evidence. Baumeister and Benati (2010) suggest a 4 percentage points positive impact on U.S. GDP in 2009Q1 and a 0.4 percentage point impact on U.S. inflation in 2009Q2. For a 50 bps decline in term premium, Chung et al. (2011) find that QE supports the level of U.S. real GDP by about 2% by early 2012 and inflation by 0.7 percentage points by 2011. Following studies on the macroeconomic effects of QE2 in 2010Q4, Chung et al. (2011) document that with additional 20 bps decline in term premium, without the unconventional measures, level of GDP would be lower by 1% and inflation would be lower by 0.3 percentage point. Similar quantitative impact is discussed by Macroeconomic Advisers (2010, 2011). With an assumption of 20 bps decline in 10-year bond yield, the level of real GDP increases by 0.4% after 8 quarters and inflation is 0.1 percentage point higher 2 years after the implementation of QE.

Among the most relevant DSGE literature on QE, Caglar et al. (2011) examine QE in the UK using three separate DSGE models, including one with financial frictions. More specifically, QE can correct the effects of a zero lower bound, by (i) offsetting the liquidity premium embedded in long-term bonds and/or (ii) adopting countercyclical subsidies to bank capital and/or (iii) the creation of central bank reserves that reduce the costs of loan supply. Using a portfolio balance model, they find limited role for QE (a fall in long-term interest rate of 100bp would require purchases of about 50-75% of debt outstanding). Chen et al. (2011) analyze the impact of large-scale asset purchase programs in the U.S. using a DSGE model enriched with a preferred habitat framework. They estimate a moderate but persistent increase in GDP growth (by 0.4 percentage point). The impact on inflation is also small due to the low degree of financial market segmentation in their model.

### **2.2.3 The Open-Economy Implications of QE**

For a small-open economy, unconventional monetary policy conducted domestically has some macroeconomic spillover effects (Chen et al. 2011; IMF 2013a). The transmission to the macroeconomy of foreign country stems primarily from conventional accommodative monetary conditions that support price and financial stability, stimulate aggregate demand and in some cases, restore full employment. When the nominal policy interest rate is constrained at the ZLB, a country conducts QE through large-purchase of government long-term treasury bonds with the objective to reduce the spreads of long-term

interest rate over expected short-term overnight interest rate. This is also the assumption we used throughout this chapter.

There are a few channels through which QE policy affects a foreign economy. First, through aggregate demand channel, lower borrowing costs resulted from a decline in the term premium encourages increases in domestic consumption, investment and as a result, domestic GDP. With some degree of trade openness, higher aggregate demand of QE country implies increase in demand for imports. The increase in imports, however, is partially offset by a relative depreciation of the domestic currency through the exchange rate channel originating from a negative interest rate differential versus the foreign country.

Third, QE also affects the foreign economy through portfolio-balancing channel. Large-purchase of illiquid assets (government long-term treasury bonds) increases cash holdings of households who then invest in riskier assets such as corporate bonds, equities and commodities, some of which are in form of capital outflows to foreign economies. This reduces the corporate risk premium in the foreign economy, lowering firms' borrowing cost and increases real investment in the foreign country. As foreign firms accumulate greater capital stock, they demand more labor. As a result, foreign household income increases following higher wage and hours of work, leading to higher consumption expenditure, higher inflation and output in the foreign economy. Monetary policy in the foreign country reacts by tightening to re-anchor inflation expectations and constraint aggregate demand. The increase in foreign policy rate further strengthens the appreciation, containing the increase in exports to QE countries. In this chapter, we have abstracted from discussing the implications of capital flows as we don't include investment in both domestic and foreign economies. Adding these feather would be an interesting extension of existing framework.

Finally, international spillover of QE could also operate through asset price channel. Bank lending channel can be considered as a further extension of the portfolio-balancing channel. Asset purchases in QE country increase cash holdings of investors, who deposit funds into banks, leading to more lending domestically or abroad. Lower financing costs boost asset prices in foreign economy, together with currency appreciation, leading to lower loan-to-value ratios of foreign households. This is viewed to pose some potential challenges especially during QE exit as foreign economy has higher household leverage, and greater financial vulnerability (IMF 2013b). We don't specify a banking sector in this chapter and therefore limit our discussion within a simple macroeconomic model where only monetary policy is considered as relevant policy instrument.

### 2.2.4 Our Approach

In this chapter, we introduce imperfect asset substitution (IAS) and segmented asset markets (SAMs), along the lines of Andres et al. (2004), in an otherwise standard small open economy model with nominal rigidities. Both assumptions are crucial for investigating the portfolio-balance channel of QE on the term premium. First, IAS is introduced by assuming that money and long term bonds have different degrees of liquidity. We capture this in the model by making utility of one type of households (who have self-imposed liquidity requirements on their long-term investments and regard money and long-term bonds as imperfect substitutes) depend on the ratio of money to long-term bonds holdings. IAS allows the long-term rate to deviate from the level implied by the strict expectation theory of the term structure. In particular, it allows the model to generate a term premium that can be affected by unconventional monetary policy. Given our assumptions, we show that the term premium is a decreasing function of the ratio of money to long-term bond holdings of this type of households. As a result, QE has a direct impact on the term premium as long as it affects this ratio.

However, IAS alone only enables us to analyze the portfolio-balance effects of QE. Without additional feature of the market segmentation, QE program will not affect the real economy. To examine the effect of QE on the real economy, we introduce another type of households who has a specific preference for long-term bonds, in similar fashion as the preferred-habitat approach as in Vayanos and Vila (2009). SAMs allow the long-term interest rate to affect aggregate demand over and above short-term rates and the macroeconomic impact of QE is determined by the sensitivity of aggregate demand to the long-term rate. In our framework, this sensitivity is influenced by the fraction of households that can only save in long-term assets. In following sections we provide more detailed discussions of these two assumptions.

Besides separating the portfolio-balance effect of QE on the term premium from the effect of the long-term rate on the real economy, our work also provides new implications for conducting QE in a small open economy like Canada.

## 2.3 The Model

The following section develops a small open economy model with imperfect asset substitution and asset market segmentation. This model is built by merging two frameworks: the small open economy model proposed by Rabanal, Rubio-Ramírez and Tuesta (2011) with the imperfect asset substitution feature in a closed economy model proposed by Andres et al. (2004).

### 2.3.1 Households

We assume a continuum of infinitely-lived households, indexed by  $i \in [0, 1]$ . A fraction  $\omega$  of households can trade in both short- and long-term bonds markets. We use the term *unrestricted households* to refer to that subset of households. Moreover, these households have self-imposed liquidity requirements on their long-term investments (money and long-term bonds are imperfect substitutes). The remaining fraction  $1 - \omega$  of households can only trade in long-term bonds markets and do not face liquidity requirements on their long-term investments. We refer to these households as *restricted households*.

Before presenting the optimization problem of both types of households, we defend the realism of the heterogeneity in households we are assuming and discuss the practical implications of these assumptions. The unrestricted agents in our model can be thought of as representing the portion of the private sector that saves through commercial bank deposits (as commercial banks tend to have self-imposed liquidity requirements). The restricted households are assumed to represent those who save heavily through agencies such as pension funds and plan to cash them at maturity. They can be thought as investor clienteles with a specific preference for long-term bonds. This alternative interpretation is consistent with the preferred-habitat view, proposed by Culberston (1957) and Modigliani and Sutch (1966).<sup>3</sup>

The existence of restricted households allows long-term interest rate to matter over and above the short-term rates in the aggregate demand.<sup>4</sup> With only unrestricted households, even if they have access to long-term bonds market, they could always "bypass" this market altogether, and simply enforce their consumption plans by trading in sequences of short-term bonds. Imperfect substitutability between money and long-term bonds allows long-term rates to deviate from the level implied by the strict expectation theory of the term structure. In particular, given the way we model imperfect substitutability, this deviation in the long-term rate depends upon an exogenous term premium and an endogenous factor related to the ratio of money to long-term bonds. We will demonstrate this relationship in details using the first order conditions of households in following sections.

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<sup>3</sup>Greenwood and Vayanos (2009) discuss different market episodes supporting the preferred-habitat view.

<sup>4</sup>If restricted households consider money and long-term bonds as imperfect substitutes, the long-term interest rate will matter over and above the short term rate in the IS curve as long as the degree of imperfect substitutability faced by restricted households is lower than the one faced by unrestricted households. See Andres et al. (2004) for a detailed explanation of this point.

## Unrestricted households

A typical household of this type seeks to maximize the following expected discounted utility, taking similar form as in Andres et al. (2004)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ a_t \left\{ U \left( \frac{C_t^u}{\{C_{t-1}^u\}^h} \right) + V \left( \frac{M_t^u}{\xi_t P_t} \right) - \int_0^1 H(N_{jt}^u) dj \right\} - G(.) - F(.) - J(.) \right] \quad (2.2)$$

where  $0 < \beta < 1$  is a discount factor,  $a_t$  is a preference shock,  $C_t^u$  is the household's consumption,  $M_t^u/P_t$  is the household's end-of-period real money balances,  $P_t$  is the consumer price index (CPI),  $\xi_t$  is a shock to the household's demand for real money balances and  $N_{jt}^u$  is the quantity of type  $j$  labor supply (measured in hours of work). The period indirect utility is determined by the preference shock  $a_t$  and the following functions:  $U, V, H, G, F$  and  $J$ . In what follows, we impose separability among consumption, real money balances and hours by specifying these functions in the following forms

$$U(.) = \frac{1}{1-\sigma} \left( \frac{C_t^u}{\{C_{t-1}^u\}^h} \right)^{1-\sigma}, \quad V(.) = \frac{1}{1-\delta} \left( \frac{M_t^u}{\xi_t P_t} \right)^{1-\delta}, \quad H(.) = \frac{(N_{jt}^u)^{1+\varphi}}{1+\varphi},$$

$$G(.) = \frac{d}{2} \left\{ \exp \left[ c \left( \frac{M_t^u/P_t}{M_{t-1}^u/P_{t-1}} - 1 \right) + \exp \left[ -c \left( \frac{M_t^u/P_t}{M_{t-1}^u/P_{t-1}} - 1 \right) \right] - 2 \right] \right\},$$

$$F(.) = -\frac{v}{2} \left[ \frac{M_t^u}{B_t^{Lu}} \Phi - 1 \right]^2, \quad J(.) = -\frac{\tilde{v}}{2} \left[ \frac{M_t^u}{S_t B_t^{Lu*}} \tilde{\Phi} - 1 \right]^2.$$

The utility function displays internal habit formation and its quantitative importance is denoted by the size of  $h \in [0, 1]$ . The parameter  $\sigma > 0$  determines the degree of risk aversion and  $\varphi \geq 0$  represents the inverse of the Frisch labor supply elasticity. The unrestricted household face costs  $G(.)$  when adjusting their financial portfolios. The function is specified following Nelson (2002) and the parameters  $d$  and  $c$  in this function are strictly positive.

Before explaining the meaning of function  $F(.)$  and  $J(.)$ , we discuss two important assumptions about the features of long-term securities in the model. First, long-term bonds are modeled as zero-coupon bonds: there are no payments received by the households during the period they hold the bonds. The first assumption is in line with the treatment of long-term bonds in macroeconomic models and ensures the optimality condition for long-term bonds holding tractable.<sup>5</sup>

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<sup>5</sup>See Svensson (2000).

The second assumption is that there is no secondary market for long-term bonds. This assumption can be justified by the fact that a large fraction of the non-bank private sector holds long-term bonds with the intention of keeping them to maturity. The absence of a secondary market for long-term securities introduces a "loss" of liquidity for households when investing in long-term bonds market, relative to the same investment in short-term bonds. To mitigate this loss of liquidity, we assume that these households have self-imposed liquidity requirements on their holdings of long-term bonds. Formally, we specify this friction as a utility cost when purchasing long-term bonds. The liquidity cost functions of holding domestic and foreign long-term bonds are given by the functions  $F(\cdot)$  and  $J(\cdot)$ .  $B_t^{Lu}$  and  $B_t^{Lu*}$  denote holdings of the domestic and foreign currency denominated long-term (maturing  $L$ -period) bonds in period  $t$  respectively. These bonds are redeemed  $L$  periods after they are bought.  $S_t$  is the nominal exchange rate expressed in units of domestic currency required to buy one unit of foreign currency. Both of them are specified in terms of relative asset holdings<sup>6</sup>. The parameters  $v$  and  $\tilde{v}$  measure the degree of concern for liquidity of the households. The parameters  $\Phi$  and  $\tilde{\Phi}$  are chosen to be the inverses of the steady-state money to long-term bond ratios (domestic and foreign currency denominated long-term bonds respectively), ensuring zero liquidity cost at the steady state.

The maximization of the expected utility is subject to the sequence of budget constraints of the form

$$\begin{aligned}
& M_t^u + \frac{B_t^u}{(1 + R_t)} + \frac{S_t B_t^{u*}}{(1 + R_t^*)(1 + \kappa_t)} + \frac{(1 + \phi_t) B_t^{Lu}}{(1 + R_t^L)^L} + \frac{(1 + \phi_t^*) S_t B_t^{Lu*}}{(1 + R_t^{*L})^L E_t \left\{ \prod_{j=0}^{L-1} (1 + \kappa_{t+j}) \right\}} \\
& = M_{t-1}^{ul} + B_{t-1}^u + S_t B_{t-1}^{u*} + B_{t-L}^{Lu} + S_t B_{t-L}^{Lu*} + \int_0^1 W_{jt} N_{jt}^u dj - P_t C_t^u + P_t T_t^u + \Pi_t^u \quad (2.3)
\end{aligned}$$

where  $B_t^u$  and  $B_t^{u*}$  denote holdings of the domestic and foreign currency denominated short-term (maturing one-period) bonds in period  $t$  respectively. These bonds are redeemed one period after they are bought.  $R_t$  and  $R_t^*$  are the domestic and foreign currency short-term interest rates respectively. The term  $\kappa_t$  is interpretable as a debt

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<sup>6</sup>We consider an alternative version where short-term bonds are also considered as liquid assets in the numerator of the ratio. In this version, the deviation of the long-term interest rate from what the expectation theory of the term structure predicts is a complicated function of money and short term and long term bonds. In order to keep tractable the specification of the term structure, we decide to consider only money as liquid asset.

elastic interest rate premium and is given by

$$\kappa_t = -\varsigma \left[ \exp \left( \frac{S_t NFA_t}{P_{H,t} Y_t} - \overline{nfa} \right) - 1 \right] + \varepsilon_t^\kappa \quad (2.4)$$

where  $\varsigma > 0$  measures the sensitivity of the country-risk premium to changes in the aggregate net foreign asset position  $NFA_t$ . The parameter  $\overline{nfa}$  is the steady state value of net foreign asset expressed as a fraction of steady state nominal GDP ( $P_{H,t}$  is the composite good prices and  $Y_t$  is real GDP).  $\varepsilon_t^\kappa$  is a risk premium shock. The adopted functional form for  $\kappa_t$  ensures stationarity of the path of the net foreign asset to GDP ratio about its steady state value.

The terms  $\phi_t$  and  $\phi_t^*$  represent stochastic transaction costs in the domestic and foreign long-term bonds markets respectively. We assume that these costs have zero mean and represent either a pure loss or a benefit to the households in aggregate<sup>7</sup>.  $R_t^L$  and  $R_t^{*L}$  are the domestic and foreign currency long-term interest rates respectively.  $W_{jt}$  is the nominal wage paid for one hour of type  $j$  labor.  $T_t^u$  denotes lump-sum real transfer (or taxes if negative) households received from the government.  $\Pi_t^u$  are the dividends from ownership of firms. The household maximizes utility over consumption, money and bonds holdings with the following first order conditions summarizing its choices

$$\lambda_t^u = a_t U_{t,C_t^u} + \beta E_t \{ a_{t+1} U_{t+1,C_t^u} \} \quad (2.5)$$

$$\frac{(1 + \phi_t)}{(1 + R_t^L)^L} \left( \frac{\lambda_t^u}{P_t} \right) - v\Phi \frac{M_t^u}{(B_t^{Lu})^2} \left[ \frac{M_t^u}{B_t^{Lu}} \Phi - 1 \right] = \beta^L E_t \left( \frac{\lambda_{t+L}^u}{P_{t+L}} \right) \quad (2.6)$$

$$\frac{\lambda_t^u}{P_t} = \beta(1 + R_t) E_t \left( \frac{\lambda_{t+1}^u}{P_{t+1}} \right) \quad (2.7)$$

$$\begin{aligned} \frac{\lambda_t^u}{P_t} - \beta E_t \left( \frac{\lambda_{t+1}^u}{P_{t+1}} \right) = & a_t V_{t,M_t^u} - \{ G_{t,M_t^u} + \beta E_t (G_{t+1,M_t^u}) \} \\ & - \frac{v\Phi}{B_t^{Lu}} \left[ \frac{M_t^u}{B_t^{Lu}} \Phi - 1 \right] - \frac{\tilde{v}\tilde{\Phi}}{S_t B_t^{Lu*}} \left[ \frac{M_t^u}{S_t B_t^{Lu*}} \tilde{\Phi} - 1 \right] \end{aligned} \quad (2.8)$$

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<sup>7</sup>The main intention of adding these exogenous transaction costs is to account for exogenous movements of the domestic and foreign term premium.

$$\frac{(1 + \phi_t^*)S_t}{(1 + R_t^{*L})^L E_t \left\{ \prod_{j=0}^{L-1} (1 + \kappa_{t+j}) \right\}} \left( \frac{\lambda_t^u}{P_t} \right) - \tilde{v}\tilde{\Phi} \frac{M_t^u}{S_t (B_t^{Lu*})^2} \left[ \frac{M_t^u}{S_t B_t^{Lu*}} \tilde{\Phi} - 1 \right] \quad (2.9)$$

$$= \beta^L E_t \left( \frac{S_{t+L} \lambda_{t+L}^u}{P_{t+L}} \right)$$

$$\frac{S_t \lambda_t^u}{P_t} = \beta(1 + R_t^*)(1 + \kappa_t) E_t \left( \frac{S_{t+1} \lambda_{t+1}^u}{P_{t+1}} \right) \quad (2.10)$$

where  $U_{t,C_t} = \frac{\partial U_t}{\partial C_t}$ ,  $U_{t,C_{t+1}} = \frac{\partial U_{t+1}}{\partial C_{t+1}}$ ,  $V_{t,M_t} = \frac{\partial V_t}{\partial M_t}$ ,  $G_{t,M_t} = \frac{\partial G_t}{\partial M_t}$  and  $G_{t,M_{t+1}} = \frac{\partial G_{t+1}}{\partial M_{t+1}}$ .

Equation (2.5) links the Lagrange multiplier for the budget constraint,  $\lambda_t^u$ , to the marginal utility of wealth. With some degree of habit persistence,  $\lambda_t^u$  is affected by preference shocks at time  $t$  and  $t + 1$ .

Equation (2.6) and (2.7) correspond to the Euler equations for domestic long- and short-term bonds holding, respectively. From these two conditions, we can derive a term structure that links the long-term interest rate  $R_t^L$  to the short-term rates  $R_t$

$$\beta(1 + \phi_t) \frac{(1 + R_t)}{(1 + R_t^L)^L} E_t \left( \frac{\lambda_{t+1}^u}{P_{t+1}} \right) - v\Phi \frac{M_t^u}{(B_t^{Lu})^2} \left[ \frac{M_t^u}{B_t^{Lu}} \Phi - 1 \right] = \beta^L E_t \left( \frac{\lambda_{t+L}^u}{P_{t+L}} \right) \quad (2.11)$$

This relationship is subject to two frictions. First, the stochastic transaction costs of trading domestic long-term bonds  $\phi_t$  account for exogenous movements of the domestic term premium, following Tobin's "exogenous interest differentials". The more important friction is captured by the presence of the second term, which is a result of the assumption of absence of secondary market for long-term securities in the model. To mitigate the liquidity loss unrestricted households incur when investing in the long-term bonds market, they have self-imposed liquidity requirements. As a result, the term premium between long- and short-term interest rates is affected by the relative quantity of assets of different maturities. If some unconventional monetary policy targets to reduce the relative supply of more illiquid assets (long-term bonds  $B_t^{Lu}$  for example), the spread between illiquid assets and liquid assets is driven down. We can see if there exists a secondary long-term bonds market and the transaction costs is zero, this relationship collapse to the pure term-structure theory of interest rates.

Combining equation (2.8) with equation (2.7) we have the money demand function

$$\frac{\lambda_t^u}{P_t} \left( \frac{R_t}{1 + R_t} \right) = a_t V_{t,M_t^u} - \{G_{t,M_t^u} + \beta E_t (G_{t+1,M_t^u})\} - \frac{v\Phi}{B_t^{Lu}} \left[ \frac{M_t^u}{B_t^{Lu}} \Phi - 1 \right] - \frac{\tilde{v}\tilde{\Phi}}{S_t B_t^{Lu*}} \left[ \frac{M_t^u}{S_t B_t^{Lu*}} \tilde{\Phi} - 1 \right] \quad (2.12)$$

which links the marginal rate of substitution between money and wealth with the nominal interest rate  $R_t$ . Two frictions also arise from this money demand function. First, the presence of the portfolio adjustment costs  $G(\cdot)$  implies that both current and expectations of real income and nominal interest rate matter for unrestricted households' portfolio decision today. Second, money demand is also affected by the relative supply of long-term domestic bonds  $B_t^{Lu}$  (as well as foreign long-term bonds  $B_t^{Lu*}$ ). This friction is a result of the assumption of unrestricted households' self-imposed liquidity requirements. When there is an increase in the relative supply of more illiquid assets (long-term bonds), the demand for money also increases. The importance of this friction is governed by two factors: the degree of concern for liquidity of the households towards investing in domestic and foreign long-term bonds ( $v$  and  $\tilde{v}$  respectively) and the (inverse of) steady-state money to long-term bond ratios ( $\Phi$  and  $\tilde{\Phi}$  respectively).

Equation (2.9) and (2.10) correspond to the Euler equations for foreign long- and short-term bonds holding, respectively. Implicitly we can also derive a similar term structure for foreign long-term and short-term interest rates

$$\begin{aligned} \beta^L E_t \left( \frac{S_{t+L} \lambda_{t+L}^u}{P_{t+L}} \right) &= \beta(1 + \phi_t^*) \frac{(1 + R_t^*)(1 + \kappa_t)}{(1 + R_t^{*L})^L E_t \left\{ \prod_{j=0}^{L-1} (1 + \kappa_{t+j}) \right\}} E_t \left( \frac{S_{t+1} \lambda_{t+1}^u}{P_{t+1}} \right) \\ &\quad - \tilde{v} \tilde{\Phi} \frac{M_t^u}{S_t (B_t^{Lu*})^2} \left[ \frac{M_t^u}{S_t B_t^{Lu*}} \tilde{\Phi} - 1 \right] \end{aligned} \quad (2.14)$$

In addition to the stochastic transaction costs of trading foreign long-term bonds  $\phi_t^*$  and the liquidity loss associated with investing in foreign long-term bonds (second term), expectations of exchange rate  $S_{t+L}$  and country-risk premium  $\kappa_{t+j}$  also play roles in affecting the deviations of foreign long-term interest rate  $R_t^{*L}$  from the pure term-structure theory of interest rate. For simplicity, in this chapter, we abstract from discussing the policy implications when a foreign economy also conducts some unconventional monetary policy. Such experiment, however, is feasible extension of our specification.

We also abstract from discussing the labor market dynamics in this chapter by using the assumption of union to simplify the wage setting. The labor market structure is specified following Schmitt-Grohe and Uribe (2006). Following this setting, we assume that household does not maximize with respect to labor and aggregate wage is fixed by monopolistically competitive unions. As a result, hours worked are determined by labor demand. There is a separate subsection that describes the unions' problem and the labor demand that these unions face.

## Restricted households

We have shown from the optimization problem of unrestricted households that there exist some deviations of domestic long-term interest rate  $R_t^L$  from the pure term-structure theory of interest rate. Such deviation is a result of the presence of stochastic transaction costs of trading long-term assets and the self-imposed liquidity requirements by unrestricted households due to imperfect substitution between assets of different maturities. As a result, unconventional monetary policy like QE could affect the spread between illiquid and liquid assets by changing the relative quantity of assets of different maturities. In another word, IAS of unrestricted households ensures the effectiveness of QE on the term premium through the portfolio-balancing channel.

However, with only unrestricted households, even if they consider short- and long-term bonds as imperfect substitutes, they could always "bypass" the long-term bonds market altogether, and simply enforce their consumption plans by trading in sequences of short-term bonds. This suggests that even though IAS allows QE to affect the long-term interest rate through term premium, IAS alone does not guarantee any effect on aggregate demand from QE without additional link from long-term interest rate to aggregate demand. Therefore, to allow for an independent role of long-term interest rate over and above the short-term rates in the aggregate demand, we need to introduce some heterogeneity in households. We define the second type of households "restricted households" in that they have specific preference towards investing in long-term assets. The existence of restricted households can be justified following the preferred-habitat view as in Vayanos and Vila (2009).

A representative restricted household seeks to maximize the following expected discounted utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ a_t \left\{ U \left( \frac{C_t^r}{\{C_{t-1}^r\}^h} \right) + V \left( \frac{M_t^r}{\xi_t P_t} \right) - \int_0^1 H(N_{jt}^r) dj \right\} - G(\cdot) \right] \quad (2.15)$$

where  $C_t^r$  is the household's consumption,  $M_t^r/P_t$  is the household's end-of-period real money balances and  $N_{jt}^r$  is the quantity of type  $j$  labor supply (measured in hours). The period indirect utility is determined by the preference shock  $a_t$  and the functions  $U$ ,  $V$ ,  $H$ , and  $G$ , which have the same functional form of those adopted to solve the unrestricted households' optimization problem. Notice that these agents do not face any liquidity requirements on their long-term investments (money and long-term bonds are viewed as perfect substitutes). This explains why the functions  $F$  and  $J$  do not appear in the

period utility function, as it was the case of unrestricted households. If we relax this assumption and allow for some degree of imperfect substitutability for the restricted agents, then their relative holdings of money and long-term debt stocks will also matter over and above interest rates in the aggregate demand. Given that the conventional view is that asset quantities do not enter directly the aggregate demand, we decide to keep the assumption that money and long-term bonds are perfect substitutes for restricted agents.

The maximization of the expected utility is subject to the sequence of budget constraints of the form

$$M_t^r + \frac{(1 + \phi_t)B_t^{Lr}}{(1 + R_t^L)^L} = M_{t-1}^r + B_{t-L}^{Lr} + \int_0^1 W_{jt} N_{jt}^r dj - P_t C_t^r + P_t T_t^r \quad (2.16)$$

where  $B_t^{Lr}$  denote holdings of the domestic currency denominated long-term bonds in period  $t$ . For simplicity, we assume that restricted households do not trade in foreign long-term bonds.  $T_t^r$  denotes lump-sum real transfer (or taxes if negative) received from the government. Notice that we allow these transfers to differ from those received by unrestricted households. We consider the dividends from ownership of firms as a short-term asset. Therefore, by assumption, restricted households do not have access to dividends.

The first order conditions for the restricted households can be written as

$$\lambda_t^r = a_t U_{t,C_t^r} + \beta E_t \{ a_{t+1} U_{t+1,C_t^r} \} \quad (2.17)$$

$$\frac{(1 + \phi_t)}{(1 + R_t^L)^L} \left( \frac{\lambda_t^r}{P_t} \right) = \beta^L E_t \left( \frac{\lambda_{t+L}^r}{P_{t+L}} \right) \quad (2.18)$$

$$a_t V_{t,M_t^r} - \{ G_{t,M_t^r} + \beta E_t (G_{t+1,M_t^r}) \} = \frac{\lambda_t^r}{P_t} - \beta E_t \left( \frac{\lambda_{t+1}^r}{P_{t+1}} \right) \quad (2.19)$$

where the functions  $U_{t,C_t}$ ,  $U_{t,C_{t+1}}$ ,  $V_{t,M_t}$ ,  $G_{t,M_t}$  and  $G_{t,M_{t+1}}$  have the same forms as that of unrestricted households' optimization problem. Equation (2.17) determines the Lagrange multiplier for the budget constraint,  $\lambda_t^r$ , which is affected by preference shocks at time  $t$  and  $t + 1$ . Equation (2.18) corresponds to the Euler equations for domestic long-term bonds holding. Equation (2.19) corresponds to money demand function, subjecting to the portfolio adjustment costs  $G(\cdot)$ .

## Unions

The labor market structure is specified following Schmitt-Grohe and Uribe (2006). The economy has a continuum of unions, each representing a type of labor. Each union sets the wage rate for its members. In each union, there is a fraction  $\omega$  from unrestricted households and a fraction  $1 - \omega$  from restricted households. As a result, a continuum of measure one of labor services are sold to firms. All representative household of the same type supply the same amount of hours worked, i.e.,  $N_t^u = N_t^r$ . Firms allocate labor demand uniformly across different workers providing type  $j$  labor, independently of their household type. We abstract from discussing the labor market dynamics in this chapter by using the assumption of union to simplify the wage setting. Following this setting, we don't have to specify two different labor aggregators (for unrestricted and restricted labor types) and two different wages at the equilibrium. Given this assumption, it follows that  $N_{jt}^{ul} = N_{jt}^{rl} = N_{jt}$ .

Unions set wages in a staggered fashion. In each period, a union is allowed to reset the nominal wage rate  $W_{jt}^*$  with a probability  $1 - \theta_w$ . If the union is not allowed to re-optimize, it keeps the wage set in the previous period,  $W_{j,t-1}$ . Those unions that are allowed to reset its wage choose  $W_{jt}^*$  to maximize the following objective function

$$\mathbf{E}_t \sum_{s=t}^{\infty} (\beta \theta_w)^{s-t} \{ \omega \mathbb{F}_s^u + (1 - \omega) \mathbb{F}_s^r \} \quad (2.20)$$

subject to  $\omega$  constraints like(2.3),  $1 - \omega$  constraints like(2.16) and the following labor demand schedule

$$N_{jt} = \left( \frac{W_{jt}}{W_t} \right)^{-\epsilon_w} \widetilde{N}_t \quad (2.21)$$

$\mathbb{F}_s^u$  and  $\mathbb{F}_s^r$  represent the utility functions of the unrestricted and restricted households respectively in period  $s$ .  $\epsilon_w > 1$  and is the elasticity of substitution between differentiated labor service, which is assumed to be constant across types. The demand for different types of labor comes from a representative perfectly competitive labor packer that generates homogenous labor services according to the following technology

$$\widetilde{N}_t = \left[ \int_0^1 N_{jt}^{1 - \frac{1}{\epsilon_w}} dj \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \quad (2.22)$$

where  $N_{jt}$  denotes the quantity of type  $j$  labor employed by the labor packer in period  $t$ .

The labor packer then sells the homogenous labor  $H_t$  at the competitive price  $W_t$  to the differentiated good producers that are presented in the firms' section. The optimization process of the representative perfectly competitive labor packer implies that:

$$W_t = \left[ \int_0^1 W_{jt}^{1-\epsilon_w} dj \right]^{\frac{1}{1-\epsilon_w}} \quad (2.23)$$

Given our assumptions of fixed proportions of unrestricted/restricted labor types in each union, all unions resetting their wage in any given period will choose the same wage rate because they face an identical problem. In particular, it can be shown that the optimal wage  $W_t^*$  is given by the following equations:

$$\frac{W_t^*}{P_t} = \mathbf{E}_t \frac{\epsilon_w \Delta_t^{W1}}{(\epsilon_w - 1) \Delta_t^{W0}} \quad (2.24)$$

where

$$\Delta_t^{W1} = \left( \frac{W_t}{P_t} \right)^{\epsilon_w} a_t (\tilde{N}_t)^{1+\varphi} + \beta \theta_w (1 + \pi_{t+1})^{\epsilon_w} \Delta_{t+1}^{W1} \quad (2.25)$$

$$\Delta_t^{W0} = \left( \frac{W_t}{P_t} \right)^{\epsilon_w} \{ \omega \lambda_t^u + (1 - \omega) \lambda_t^r \} \tilde{N}_t + \beta \theta_w (1 + \pi_{t+1})^{\epsilon_w} \Delta_{t+1}^{W0} \quad (2.26)$$

Moreover, the fact that all unions resetting their wage choose the same wage implies that we can rewrite the aggregate wage index as:

$$W_t = \left[ \theta_w (W_{t-1})^{1-\epsilon_w} + (1 - \theta_w) (W_t^*)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}} \quad (2.27)$$

## Aggregation

Aggregate consumption, money and domestic long-term bonds are given by a weighted average of the corresponding variables for each consumer type. Formally,

$$C_t = \omega C_t^u + (1 - \omega) C_t^r \quad (2.28)$$

$$M_t = \omega M_t^u + (1 - \omega) M_t^r \quad (2.29)$$

$$B_t^L = \omega B_t^{Lu} + (1 - \omega) B_t^{Lr} \quad (2.30)$$

Similarly, aggregate short term bonds and foreign long term bonds are given by:

$$B_t = \omega B_t^u \quad (2.31)$$

$$B_t^* = \omega B_t^{u*} \quad (2.32)$$

$$B_t^{L*} = \omega B_t^{Lu*} \quad (2.33)$$

Regarding hours worked, the following relations hold:

$$N_{jt} = \omega N_{jt}^u + (1 - \omega) N_{jt}^r \quad (2.34)$$

$$N_t^u = \int_0^1 N_{jt}^u dj \quad (2.35)$$

$$N_t^r = \int_0^1 N_{jt}^r dj \quad (2.36)$$

$$N_t = \int_0^1 N_{jt} dj \quad (2.37)$$

$$N_t = \omega N_t^u + (1 - \omega) N_t^r \quad (2.38)$$

### 2.3.2 Firms

There are four types of domestic firms in the model: i) perfectly competitive firms that produce an homogenous final good that is used for private consumption  $C_t$ , ii) perfectly competitive firms that produce the composite goods  $Y_{H,t}$  and  $Y_{F,t}$ , used as inputs to produce the final good  $Y_t^C$ , iii) monopolistically competitive firms producing differentiated domestic goods, used as inputs to produce  $Y_{H,t}$  and iv) firms that import foreign differentiated goods that are used as inputs to produce  $Y_{F,t}$ .

#### Final good producers

The final good is produced by a representative, perfectly competitive firm with a constant returns technology of the form

$$Y_t^C = \left( (1 - \alpha)^{\frac{1}{\eta}} (Y_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (Y_{F,t})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (2.39)$$

where  $Y_{H,t}$  is the quantity of the composite of intermediate home goods,  $Y_{F,t}$  is the amount of the composite of intermediate foreign goods,  $\alpha$  is the share of foreign goods that are used as input and  $\eta > 0$  is the elasticity of substitution between the domestic

and imported goods. Profit maximization, taking as given the final good price  $P_t$  and the input prices  $P_{H,t}$  and  $P_{F,t}$ , yields the following demand functions

$$Y_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} Y_t^C \quad (2.40)$$

$$Y_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} Y_t^C \quad (2.41)$$

and the following zero profit condition:

$$P_t = \left\{ (1 - \alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right\}^{\frac{1}{1-\eta}} \quad (2.42)$$

### Composite goods producers

The composite goods are  $Y_{H,t}$  and  $Y_{F,t}$ . Both are produced by two different representative, perfectly competitive firms using the following technologies

$$Y_{H,t} = \left[ \int_0^1 Y_{H,t}(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.43)$$

and

$$Y_{F,t} = \left[ \int_0^1 Y_{F,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.44)$$

where  $Y_{H,t}(h)$  and  $Y_{F,t}(f)$  are home and foreign intermediate goods of type  $h$  and  $f$  respectively. The elasticity of substitution between any two types of intermediate goods is denoted by  $\varepsilon > 1$ . Profit maximization, taking as given the composite good prices  $P_{H,t}$  and  $P_{F,t}$  and the input prices for all intermediate goods  $P_{H,t}(h)$  and  $P_{F,t}(f)$  for all  $h \in [0, 1]$  and  $f \in [0, 1]$ , yields the following demand functions:

$$Y_{H,t}(h) = \left[ \frac{P_{H,t}(h)}{P_{H,t}} \right]^{-\varepsilon} Y_{H,t} \text{ for all } h \in [0, 1] \quad (2.45)$$

$$Y_{F,t}(f) = \left[ \frac{P_{F,t}(f)}{P_{F,t}} \right]^{-\varepsilon} Y_{F,t} \text{ for all } f \in [0, 1] \quad (2.46)$$

and the following zero profit conditions:

$$P_{H,t} = \left( \int_0^1 P_{H,t}(h)^{1-\varepsilon} dh \right)^{\frac{1}{1-\varepsilon}} \quad (2.47)$$

$$P_{F,t} = \left( \int_0^1 P_{F,t}(f)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}} \quad (2.48)$$

### Differentiated domestic good producers

We assume there are a continuum of monopolistically competitive firms producing differentiated goods. These differentiated goods are sold to the producers of the composite good  $Y_{H,t}$  or exported. Following Justiniano and Preston (2008) and Monacelli (2005), we adopt a linear production function in labor for a typical differentiated good producer. This function is given by:

$$Y_{H,t}(h) + Y_{H,t}^*(h) = Z_t \widetilde{N}_t(h) \quad (2.49)$$

where  $Y_{H,t}^*(h)$  denotes the quantity exported of good of type  $h$ ,  $Z_t$  denotes the labor productivity and  $N_t(h)$  is the hours demanded by a firm that produces the type  $h$ . We assume that labor productivity follows a simple stochastic auto-regressive process of the form:

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z$$

where  $0 \leq \rho_z \leq 1$  and  $\varepsilon_t^z$  is an i.i.d shock.

Real marginal cost is common to all firms and is given by:

$$MC_t = \frac{W_t}{P_{H,t} Z_t} \quad (2.50)$$

Differentiated domestic good producers are assumed to set nominal prices in a staggered fashion, according to the stochastic time-dependent rule proposed by Calvo (1983). Each firms resets its price with probability  $1 - \theta_H$  every period. For simplicity, we assume that the reset price for good  $h$  is the same for both the domestic market and the foreign market. Moreover, in order to have a symmetric export demand for good  $h$ , we assume that the law of one price holds for type  $H$  goods<sup>8</sup>. All firms having the opportunity to

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<sup>8</sup>Given that differentiated domestic goods prices are sticky in the domestic currency, the law of one

reset their price in period  $t$  face the same decision problem. Therefore, all of them set a common price  $\overline{P_{H,t}}$  that solves the following optimization problem:

$$\max_{\overline{P_{H,t}}} \sum_{k=0}^{\infty} \theta_H^k E_t \left\{ Q_{t,t+k} (Y_{H,t+k}(h) + Y_{H,t+k}^*(h)) \left[ \frac{\overline{P_{H,t}}}{P_{H,t+k}} - MC_{t+k} \right] \right\} \quad (2.51)$$

subject to the sequence of demand constraints

$$Y_{H,t+k}(h) = \left[ \frac{\overline{P_{H,t}}}{P_{H,t+k}} \right]^{-\varepsilon} Y_{H,t+k} \quad (2.52)$$

and

$$Y_{H,t+k}^*(h) = \left[ \frac{\overline{P_{H,t}}}{P_{H,t+k}} \right]^{-\varepsilon} Y_{H,t+k}^* \quad (2.53)$$

where  $Q_{t,t+k}$  is the stochastic discount factor used by unrestricted households<sup>9</sup>. The first order condition of this optimization problem is given by:

$$\sum_{k=0}^{\infty} \theta_H^k E_t \left\{ Q_{t,t+k} (Y_{H,t+k}(h) + Y_{H,t+k}^*(h)) \left[ \frac{\overline{P_{H,t}}}{P_{H,t+k}} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k} \right] \right\} = 0 \quad (2.54)$$

The equation describing the dynamics of the aggregate price level of differentiated domestic goods is given by

$$P_{H,t} = \left[ \theta_H P_{H,t-1}^{1-\varepsilon} + (1 - \theta_H) \overline{P_{H,t}}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (2.55)$$

### Importers of foreign differentiated goods

We assume there are a continuum of monopolistically competitive firms importing foreign differentiated goods for which the law of one price holds at the docks. These differentiated goods are sold to the producers of the composite good  $Y_{F,t}$ . In setting the domestic currency price of the differentiated goods, these firms solve an optimal dynamic mark-up problem. This leads to a violation of the law of one price in the short run.

price implies that the price expressed in foreign currency is flexible.

<sup>9</sup>Notice that when presenting the export demand for good  $h$ , we have used the fact that the law of one price. Formally,  $Y_{H,t}^*(h) = \left[ \frac{P_{H,t}^*}{P_{H,t}} \right]^{-\varepsilon} Y_{H,t}^*$  where  $P_{H,t}^*$  is the price of the domestic good expressed in units of the foreign currency and the second equality has made use of the law of one price assumption.

Real marginal cost of firm importing the type  $f$  good is given by:

$$MC_t(f) = S_t \frac{P_{F,t}^*(f)}{P_{F,t}} \quad (2.56)$$

where  $P_{F,t}^*(f)$  is the rest-of-the-world price of type  $f$  good. Notice that this cost is firm-specific. In order to make this cost common to all firms, we assume that there is no price dispersion in the rest of the world (i.e.,  $P_{F,t}^*(f) = P_{F,t}^*$ ).<sup>10</sup>

These importers are assumed to set nominal prices in a staggered fashion, like the producers of the differentiated domestic goods. Each firm resets its price with probability  $1 - \theta_F$  every period. Given that there is no price dispersion in the rest of the world, all importers having the opportunity to reset their price in period  $t$  face the same decision problem. Therefore, all of them set a common price  $\overline{P_{F,t}}$  that solves the following optimization problem:

$$\max_{\overline{P_{F,t}}} \sum_{k=0}^{\infty} \theta_F^k E_t \left\{ Q_{t,t+k} Y_{F,t+k}(f) \left[ \frac{\overline{P_{F,t}}}{P_{F,t+k}} - S_{t+k} \frac{P_{F,t+k}^*}{P_{F,t+k}} \right] \right\} \quad (2.57)$$

subject to the sequence of demand constraints

$$Y_{F,t+k}(f) = \left[ \frac{\overline{P_{F,t}}}{P_{F,t+k}} \right]^{-\varepsilon} Y_{F,t+k} \quad (2.58)$$

The first order condition of this optimization problem is given by:

$$\sum_{k=0}^{\infty} \theta_F^k E_t \left\{ Q_{t,t+k} Y_{F,t+k}(f) \left[ \frac{\overline{P_{F,t}}}{P_{F,t+k}} - \frac{\varepsilon}{\varepsilon - 1} S_{t+k} \frac{P_{F,t+k}^*}{P_{F,t+k}} \right] \right\} = 0 \quad (2.59)$$

The equation describing the dynamics of the aggregate price level of differentiated imported goods is given by:

$$P_{F,t} = \left[ \theta_F P_{F,t-1}^{1-\varepsilon} + (1 - \theta_F) \overline{P_{F,t}}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (2.60)$$

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<sup>10</sup>There are two alternative stories to defend this assumption. The first one is that prices are flexible and firms face the same technology and demand functions in the rest of the world. The second story is that prices are sticky but monetary policy eliminates this distortion by reaching price stability and implement the flexible price allocation.

### 2.3.3 Government

The government budget constraint is specified as:

$$M_t + \frac{B_t}{(1 + R_t)} + \frac{B_t^L}{(1 + R_t^L)^L} - (B_{t-1} + B_{t-1}^L + M_{t-1}) = P_t T_t \quad (2.61)$$

where  $T_t = \omega T_t^u + (1 - \omega) T_t^r$ . Given that the focus of the chapter is not on fiscal policy, we abstract from modeling government expenditures and taxes.<sup>11</sup> For simplicity, we assume that long-term bonds follow a simple AR(1) process of the form:

$$\frac{B_t^L}{P_t} = \left( \frac{B_{t-1}^L}{P_{t-1}} \right)^{\rho_B} \exp(\epsilon_t^{B^L}) \quad (2.62)$$

where  $\rho_B \in [0, 1]$  and  $\epsilon_{B^L}$  is an i.i.d exogenous disturbance. Therefore, short-term debt is used as a residual means of public financing. The government in this model finances its transfers by issuing short-term bonds in order to maintain a desired debt-to-GDP ratio,  $(\frac{B}{P})$ , over the medium term. To guarantee dynamic stability and a unique equilibrium in the model, we also assume that aggregate transfers and transfers to the restricted households are set according to the following rules:

$$T_t - \bar{T} = -\varkappa_B \left[ \frac{B_{t-1}}{P_{t-1}} - \frac{B}{P} \right] + \epsilon_t^T \quad (2.63)$$

$$T_t^r - \bar{T}^r = -\varkappa_B \left[ \frac{B_{t-1}}{P_{t-1}} - \frac{B}{P} \right] + \epsilon_t^{T_r} \quad (2.64)$$

where  $\varkappa_B \in [0, 1]$ .  $\bar{T}$ ,  $\bar{T}^r$  and  $\frac{B}{P}$  are the steady-state values of aggregate transfers, transfers to restricted households and short-term government bonds respectively.  $\epsilon_t^T$  and  $\epsilon_t^{T_r}$  are temporary aggregate transfer shock and temporary transfer shock to the restricted households, respectively.

### 2.3.4 Monetary Policy

The instrument of monetary policy is the short-term nominal interest rate (or overnight interbank lending interest rate). Under the normal circumstances, the central bank cuts the target for the overnight interest rate in order to stimulate lending activity, thereby supporting consumption and investment. This is considered as "conventional monetary

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<sup>11</sup>Conventionally, government is also assumed to carry function of purchasing goods and services for the government and collecting taxes on labour income and consumption. We don't, however, discuss these features in this model.

policy" in this model.

We follow Ireland (2004) and assume that the monetary authority sets the nominal interest rate following an augmented Taylor rule. In addition to responding to the lagged interest rate and deviations of output and inflation relative to their steady-state, the nominal interest rate also responds to nominal money growth. The presence of changes in real money balance in the monetary policy reaction function could indicate the existence of money growth variability in the central bank's loss function, as suggested in Rudebusch and Svensson (2002).

The policy rule is specified as:

$$\frac{(1 + R_t)}{1 + \bar{R}} = \left[ \frac{(1 + R_{t-1})}{1 + \bar{R}} \right]^{\rho_r} \left\{ \left( \frac{P_t}{P_{t-1}} \right)^{\rho_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\rho_y} \left( \frac{M_t}{M_{t-1}} \right)^{\rho_m} \right\}^{(1-\rho_r)} \exp(\varepsilon_t^R) \quad (2.65)$$

where  $\bar{R}$  is the steady-state nominal interest rate.  $\rho_r$  is the interest rate smoothing parameter,  $\rho_\pi$  is the sensitivity of the short-term risk-free interest rate to inflation deviation from the target,  $\rho_y$  is the sensitivity of the short-term risk-free interest rate to output gap, and  $\rho_m$  is the sensitivity of the short-term risk-free interest rate to money growth.  $Y_t$  is the aggregate output level and  $\varepsilon_t^R$  is normally distributed with standard deviation of  $\sigma^R$ .

### 2.3.5 Market Clearing

The clearing of labor and goods markets requires that the following conditions are satisfied for every  $t$ <sup>12</sup>:

$$\tilde{N}_t = \int_0^1 \tilde{N}_t(h) dh \quad (2.66)$$

$$Y_t^C = C_t \quad (2.67)$$

$$Y_t = Y_{H,t} + Y_{H,t}^* \quad (2.68)$$

where  $Y_{H,t}^*$  is given by

$$Y_{H,t}^* = \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} Y_t^* \quad (2.69)$$

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<sup>12</sup>The market clearing conditions for the domestic and foreign differentiated goods have been already taken into account when solving the dynamic optimization problem of the price setters and when characterizing the evolution of the aggregate price levels  $P_{H,t}$  and  $P_{F,t}$ .

Finally, combining the flow budget constraints for both types of households with that of the government, and using the definitions of dividends, we get the following equation describing the evolution of the balance of payments denominated in domestic currency:

$$\begin{aligned} & \frac{S_t B_t^*}{(1 + R_t^*)(1 + \kappa_t)} + \frac{S_t B_t^{L*}}{(1 + R_t^L)^L E_t \left\{ \prod_{j=0}^{L-1} (1 + \kappa_{t+j}) \right\}} \\ &= S_t B_{t-1}^* + S_t B_{t-L}^{L*} + P_{H,t} Y_{H,t}^* - S_t P_{F,t}^* Y_{F,t} \end{aligned} \quad (2.70)$$

## 2.4 The Transmission Mechanism of QE in The Model

When economic slack emerges in the economy, the central bank cannot move down the overnight interest rate below zero percent (or an effective zero lower bound). As an alternative, the central bank can target the quantity of currency in the financial system by purchasing financial assets (in form of long-term government treasury bonds in this model) in exchange for reserves. In our model, QE is defined as the large-purchase of government long-term treasury bonds to reduce the spreads of long-term interest rate over expected short-term overnight interest rate. QE aims at easing financing conditions of banks, households and firms.

Both of the domestic term structure and the aggregate consumption Euler equation play key roles in the transmission mechanism of QE. The log-linearized versions of these two equations make it more transparent how QE operates in the model.<sup>13</sup>

### Term Structure

The relation between QE and the long-term real interest rate can be seen in the following domestic term structure derived from the model:

$$\widehat{r}_{L,t} = \frac{1}{L} \sum_{j=0}^{L-1} \widehat{r}_{t+j} + \frac{1}{L} tp_t \quad (2.71)$$

where  $\widehat{r}_{L,t}$  is the long-term real interest rate,  $\widehat{r}_{t+j}$  is the short-term real interest rate.

The term premium  $tp_t$  is expressed as

$$tp_t \equiv \widehat{\phi}_t - \tau \left( \widehat{m}_t^u - \widehat{b}_t^{Lu} \right) \quad (2.72)$$

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<sup>13</sup>Andres et al. (2004) provides details of the log-linear approximation of the first order conditions.

where the terms  $\phi_t$  represent the disutility in a form of transaction cost the unrestricted households have to pay for purchasing one unit of long-term bond.  $\widehat{m}_t^u$  and  $\widehat{b}_t^{Lu}$  are unrestricted households' real balances and long-term bonds holding (in log deviations). The elasticity of term premium to the unrestricted households' money to long-term bonds holding ratio is governed by parameter  $\tau$ , expressed as

$$\tau \equiv \frac{v (1 + r^L)^L r}{(1 + r) b^{Lu} (m^u)^{-\delta}} \quad (2.73)$$

where  $r^L$  and  $r$  are the steady-state real long- and short-term interest rate,  $b^{Lu}$  and  $m^u$  are the long-term bonds holdings and real balances of unrestricted households at the steady state. Parameter  $v$  affects the degree of concern for liquidity of the households towards investing in domestic long-term bonds.  $\delta$  represents the inverse elasticity of substitution between consumption and real balances.

This relationship based on imperfect asset substitution breaks the perfect arbitrage opportunity between the short- and long-term assets and allows the long-term rate to deviate from the level implied by the pure expectations theory of the term structure.<sup>14</sup> The deviation is modelled as the term premium and its presence implies that long-term rates can vary independently of the expected path of short-term rates. Equation (2.71) shows that an increase in the unrestricted households' money holdings,  $\widehat{m}_t^u$ , or a decrease in their long-term bond holdings,  $\widehat{b}_t^{Lu}$ , reduces the term premium, and consequently long-term real interest rates. With zero long-term bonds transaction costs ( $\phi_t = 0$ ) and with no concern of liquidity loss when investing in illiquid assets ( $v = 0$ ), equation (2.71) collapses to the standard term-structure:

$$\widehat{r}_{L,t} = \frac{1}{L} \sum_{j=0}^{L-1} \widehat{r}_{t+j} \quad (2.74)$$

QE operations in the model affect the long-term real interest rate through the term structure relationship. Since the medium- to long-term expected real interest rate matters the most for investment and consumption decisions, QE consequently affects consumption and output in the model.

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<sup>14</sup>See Andres, Lopez-Salido and Nelson (2004).

## Euler Equation

The log-linearized aggregate consumption Euler equation is given by:

$$\widehat{c}_t - E_t \widehat{c}_{t+L} = -\frac{\omega}{\varkappa_2} \left[ \sum_{j=0}^{L-1} \widehat{r} \widehat{r}_{t+j} \right] - \frac{(1-\omega)L}{\varkappa_2} \widehat{r} \widehat{r}_{L,t} + \frac{\varkappa_1}{\varkappa_2} \widehat{c}_{t-1} \quad (2.75)$$

$$+ \beta \frac{\varkappa_1}{\varkappa_2} E_t \widehat{c}_{t+1} - \frac{\varkappa_1}{\varkappa_2} \widehat{c}_{t+L-1} + \dots \quad (2.76)$$

$$- \beta \frac{\varkappa_1}{\varkappa_2} E_t \widehat{c}_{t+L+1} + \frac{(1-\beta h \rho_a)(1-\rho_a^L)}{\varkappa_2(1-\beta h)} \widehat{a}_t \quad (2.77)$$

where  $\widehat{r}_t$  and  $\widehat{r}_{L,t}$  denote the short- and long-term real interest rates respectively (both expressed in quarterly terms).  $\varkappa_1$  and  $\varkappa_2$  are non-linear functions of the structural parameters, which are given by:

$$\varkappa_1 = \frac{(\sigma-1)h}{1-\beta h}, \quad \varkappa_2 = \frac{\sigma + (\sigma-1)\beta h^2 - \beta h}{1-\beta h}$$

Equation (2.75) shows that the long-term rate,  $\widehat{r}_{L,t}$ , matters over and above short-term rates,  $\widehat{r}_{t+j}$ . In the model, this is a consequence of the asset market segmentation assumption that the intertemporal substitution decisions of restricted households are driven by long-term rates. The importance of the asset market segmentation is influenced by the share of restricted households ( $1-\omega$ ), the discount factor  $\beta$ , the degree of risk aversion  $\sigma$  and the habit persistence parameter  $h$ .<sup>15</sup>

## 2.5 Model Estimation

Since the impact of QE largely depends on the few key parameters of the model (the degree of concern when investing in domestic long-term bonds,  $v$ ; the share of restricted households ( $1-\omega$ ), the degree of risk aversion  $\sigma$  and the habit persistence parameter  $h$  etc), we use a combination of calibration and estimation to select parameters such that the model matches some historical empirical observations. Because estimation involves a few parameters which enter the likelihood non-linearly, we use a genetic evolutionary heuristic to select parameters which maximize the Bayesian likelihood (An and Schorfheide 2007). We construct the likelihood using the Kalman filter with a non-linear step based on the state space representation of the rational expectation solution of the model.<sup>16</sup> In the remainder of this section, we first describe the data used in estimation,

<sup>15</sup>Estimation of the key parameters of the Euler equation is discussed in later sections.

<sup>16</sup>The Kalman filter is ran twice in the estimation. In the first run, the Kalman filter uses the linearized model to generate a one-step-ahead predictions in each of its steps. In the second run, the

followed by the parameter prior and posterior distributions.

### 2.5.1 Data

We use quarterly data for Canada from 1980q1 to 2012q3 for calibration and estimation.<sup>17</sup> We obtain National Accounts variables such as GDP and consumption. The quarterly log-difference in the core consumer price index is our measure of inflation for the policy rule. We used 90-day risk-free bank rate as our measure of nominal short-term rate. The long-term risk-free rate is defined as the average 3-5 year yield on Government of Canada marketable bonds. We also include money supply in the estimation.

Data are de-trended using the LRX filter (see Berg, Karam, Laxton 2006 for a detailed description) and the estimation is performed on the stationary series.

### 2.5.2 Calibration and prior choice

The value of the discount factor  $\beta$  is chosen to respect the model's steady-state stability condition

$$\beta = (1 + r)^{-1}.$$

Conditional on the choices for the real quarterly interest rate,  $r = 0.008$ , we choose a discount factor  $\beta = 0.99$ .

We calibrate the parameters in the long-term domestic and foreign bonds transaction costs functions,  $c$  and  $d$  to be 1 and 0.105197, following Nelson (2002). The parameters measuring the degree of concern for liquidity of the households,  $v$  and  $\tilde{v}$ , are chosen to be 0.003 and 0.005. The sensitivity of the country-risk premium to changes in the aggregate net foreign asset position  $\zeta$  is calibrated to be 0.008.

Parameters related to the wage setting problems are set according to An and Schorfheide (2007). In each period, the probability of a union not to reoptimize the nominal wage rate,  $\theta_w$  is set to be 0.5, reflecting moderate wage stickiness in the Canadian data. The elasticity of substitution between differentiated labor service,  $\epsilon_w$  is set to be 6.

On firm's side, the share of foreign goods that are used as input,  $\alpha$  is calibrated to be 0.3, according to the import importance of Canadian domestic goods production, suggesting modest degree of trade openness in Canada. The elasticity of substitution

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Kalman filter calls a simulation in an exact non-linear mode to produce one-step-ahead predictions.

<sup>17</sup>Since the sample for estimation includes a short period of the zero lower bound, we conduct sensitivity analysis by estimating the model over only normal time to examine the robustness of the parameter estimates. We find little change of the result when the lower bound period is excluded.

between the domestic and imported inputs for final goods production,  $\eta$  is set to be 0.8. This value is also used to calibrate the elasticity of substitution between any two types of intermediate goods, denoted by  $\varepsilon$ . Finally, the proportion of firm not resetting its price  $\theta_H$  is chosen to be 0.7, implying that domestic prices are re-optimized, on average, once every three quarters.

To pin down the steady-state, we assume that domestic and foreign inflation are zero, which implies that there is no price dispersion in steady state. We assume that steady state consumption is the same across households types, i.e.,  $C^u = C^r = C$ . This outcome can always be guaranteed by an appropriate choice of government's real transfers  $T^u$  and  $T^r$ . Because the focus of the chapter is on the different responses of the economy to different shocks, as opposed to steady state differences across households, we think this assumption is reasonable. Given that both types of households face the same aggregate shocks, our assumption of competitive labor markets combined with symmetry in consumption in steady state, imply that hours worked are also the same across households (i.e.  $N^u = N^r = N$ ). Our assumption on the functions  $F(\cdot)$  and  $J(\cdot)$ , combined with our assumption on the random transaction costs  $\phi_t$  and  $\phi_t^*$ , imply that term premium is equal to zero in steady state. This implies  $R = R^L$  and  $R^* = R^{*L}$ . The steady-state values for annualized short- and long-term nominal interest rates is 3 per cent. This implies a 3 per cent of the real short- and long-term interest rate around the zero inflation rate. Given that there is no distinction between short- and long-term rates in steady state and that there are no differences in consumption across households, money demands are also the same across households (i.e.  $m = m^u = m^r$ ). We assume that both types of households hold the same amount of domestic and foreign long-term bonds.

To pin down the aggregate real quantities of domestic short-term and long-term bonds, we solve the following system of two equations with two unknowns:

$$gdebt = b + Lb^L \tag{2.78}$$

$$b^L = \frac{T - \frac{R}{1+R}gdebt}{\left\{ \frac{1-(1+R)^L}{(1+R)^L} - \frac{R}{1+R}L \right\}} \tag{2.79}$$

where eq(2.78) is the definition of real government debt around a zero inflation steady state and eq (2.79) is the government budget constraint around a zero inflation steady state. Notice that when writing (2.79), we have used the definition of government debt. We set  $L = 20$ , corresponding to a maturity of long-term bonds of 5 years in the model. We set  $T = -0.009Y$ , which means that the government runs a primary surplus of 0.9

percent of GDP in steady state. We assume that government debt is equal to 100 percent of quarterly output (i.e  $gdebt = Y$ ), which is consistent with the actual level of Canadian government debt. Given all these assumptions, we find the steady state values for  $b$  and  $b^L$ .

We assume that the terms of trade are equal to 1 in steady state. This implies  $P^H = P^F = P$ . Moreover, we assume a balanced trade. The specification of the country risk premium implies that this is equal to zero in steady state. To pin down the aggregate quantity of foreign short- and long-term bonds, we solve the following system of two equations with two unknowns:

$$b^* = nfa - Lb^{L*} \tag{2.80}$$

$$b^{L*} = \frac{-Y \frac{(\pi^* - R^* - \kappa - R^* \kappa)}{(1+R^*)(1+\pi^*)(1+\kappa)} nfa}{\left\{ \frac{\{(1+\pi^*)^L - (1+R^*)^L(1+\kappa)^L\}}{(1+R^*)^L(1+\pi^*)^L(1+\kappa)^L} - \frac{(\pi^* - R^* - \kappa - R^* \kappa)}{(1+R^*)(1+\pi^*)(1+\kappa)} L \right\}} \tag{2.81}$$

where equation(2.80) is the definition of net foreign asset position around a zero foreign inflation steady and equation (2.81) is the definition of the balance of payments around a zero foreign inflation steady state. Notice that when writing (2.81), we have used the definition of net foreign asset position. We set  $nfa = 0.1$ , which means that the country has a net foreign asset position of 10 percent of quarterly GDP in steady state. Given all these assumptions, we find the steady state values for  $b^*$  and  $b^{L*}$ . Given that the real exchange rate is constant in steady state, and domestic and foreign inflation are equal, this means that nominal depreciation in the model is zero.

Table 1 reports the prior and posterior distribution for the parameters being estimated. Prior distributions are consistent with the literature and the means of the distribution are taken largely from Justiniano and Preston (2010). The autoregressive coefficients in the shock processes have a beta distribution with a mean of 0.5 and standard deviation of 0.25. The priors on standard deviations of the shocks have inverse gamma distribution with mean 0.5 and an infinite variance. Monetary policy rule parameters have priors with gamma distributions centered around the values used in the literature (see Hofmann and Bogdanova 2012 for a review).

The estimated habit persistence in consumption,  $h$ , is moderate at a value of 0.51, suggesting somewhat delayed peak response of consumption to movements in real interest rates. The intertemporal elasticity of substitution  $\sigma$  and the inverse of the labor supply elasticity  $\varphi$  are estimated to be 1.45 and 0.58, respectively. The monetary policy parameters are estimated to indicate moderate degree of interest rate smoothing, with a value of 0.66 for  $\rho_r$ . The response to core inflation is significant ( $\rho_\pi$  is estimated to be

1.4). We also estimated a significant response to output gap ( $\rho_y$  is estimated to be 0.25).

The share of unrestricted households that have access to both short- and long-term asset markets,  $\omega$ , is an important parameter for the property of the dynamics in the model. It helps to identify the degree of bond market segmentation in the model. We chose a beta distribution with mean 0.5 and standard deviation 0.25. We estimate the share of unrestricted household,  $\omega$ , to be 0.36, suggesting there is significant degree of segmentation in the bond market in Canada. This result also lies within the range of 0.7 estimate from Chen, Curdia and Ferrero (2011) and 0.29 estimated by Andres et al. (2004).

**Table 1: Prior and Posterior Distribution of Estimated Parameters**

Estimates	Description	Posterior	Std. Dev	Prior Dist. (mean, Std. Dev)
$\sigma$	(inverse) consumption elasticity	1.45	0.2985	<i>Gamma</i> (1.2, 0.2)
$\varphi$	(Inverse) elasticity of labor supply	0.58	0.0486	<i>Beta</i> (0.5, 0.25)
$h$	Consumption habit	0.51	0.0026	<i>Beta</i> (0.8, 0.2)
$\omega$	Share of unrestricted households	0.36	0.0009	<i>Beta</i> (0.2, 0.1)
$\rho_R$	Policy rule - smoothing	0.66	0.0004	<i>Normal</i> (0.5, 0.25)
$\rho_\pi$	Policy rule - inflation	1.38	0.0209	<i>Normal</i> (1.5, 0.25)
$\rho_y$	Policy rule - output gap	0.25	0.0049	<i>Normal</i> (0.25, 0.125)

## 2.6 Assessing the Impact of QE

In order to assess the quantitative implications of QE in the model, we begin by assuming that a large and persistent negative demand shock hits the steady state economy so that the short-term nominal rate is constrained at the ZLB for a year. In particular, we use the intertemporal preference shock,  $a_t$ , to simulate the effects of the negative demand shock. We choose to use preference shock given its important role in explaining aggregate fluctuations as an unobserved demand shock in the literature (for example, Rotemberg and Woodford 1997, Galí and Rabanal 2004). We choose a value of 0.2 for the preference shock, a size lies within the empirical estimation of the standard deviation of the preference shock in the literature. In the experiment, QE is defined as the large-purchase of government long-term treasury bonds to reduce the spreads of long-term interest rate over expected short-term overnight interest rate. QE aims at easing financing conditions of banks, households and firms.

The dashed lines in Figure 2.1 show the impact of the preference shock when no unconventional monetary policy is implemented and policy rate is subject to the ZLB. Note that all variables are expressed in percentage deviations from steady-state, with

the exception of the inflation and interest rate variables, which are expressed in (annual) percentage point deviations. The shock causes output and inflation to fall substantially. Monetary policy reacts by cutting the short-term nominal interest rate, reaching its lower bound and remaining constrained for five quarters.

The solid lines in Figure 2.1 show the impact of the same shock when the conventional monetary policy is augmented with QE. In this case, the size of the purchase of government long-term treasury bond is equal to four per cent of GDP for four quarters. Figure 2.2 shows the marginal impact of QE (the difference between the solid and dashed lines in Figure 2.1). The change in the relative supply of assets generates a significant decline in the term premium by roughly 60 bps. QE also leads to the expectation that short-term rates will rise more quickly as it generates some growth and inflation. These two effects combined lead to an immediate decline in the long-term rate by roughly 40 bps.

The movement in the term premium is a result of our assumption of IAS, which breaks the perfect arbitrage opportunity between the short- and long-term assets and allows the long-term rate to deviate from the level implied by the pure expectations theory of the term structure. Such deviation is a result of the presence of stochastic transaction costs of trading long-term assets and the self-imposed liquidity requirements by unrestricted households due to imperfect substitution between assets of different maturities. Since there is no secondary market for long-term securities in the model, to mitigate the liquidity loss unrestricted households incur when investing in the long-term bonds market, they have self-imposed liquidity requirements. As a result, the term premium between long- and short-term interest rates is affected by the relative quantity of assets of different maturities. Through open-market purchases of certain targeted securities (long-term government treasury bonds for example), the central bank creates the portfolio balance effect as the private sector responds to such purchase by holding less of the long-term bonds. More specifically, since the term premium  $tp_t$  depends on unrestricted households' real balances and long-term bonds holding, any unconventional monetary policy that affects the money/long-term bonds stock ratio matters for the term premium. Equation (2.71) shows that a decrease in their long-term bond holdings,  $\widehat{b}_t^{Lu}$ , or an increase in the unrestricted households' money holdings,  $\widehat{m}_t^u$ , reduces the term premium, and consequently long-term real interest rates.

Through portfolio balance channel, when QE targets to reduce the relative supply of more illiquid assets, the spread between illiquid assets and liquid assets is driven down. This finding is in line with the "duration channel of QE" as in Gagnon et al. (2011). When long-term bonds are purchased by the central bank, investors' portfolios become

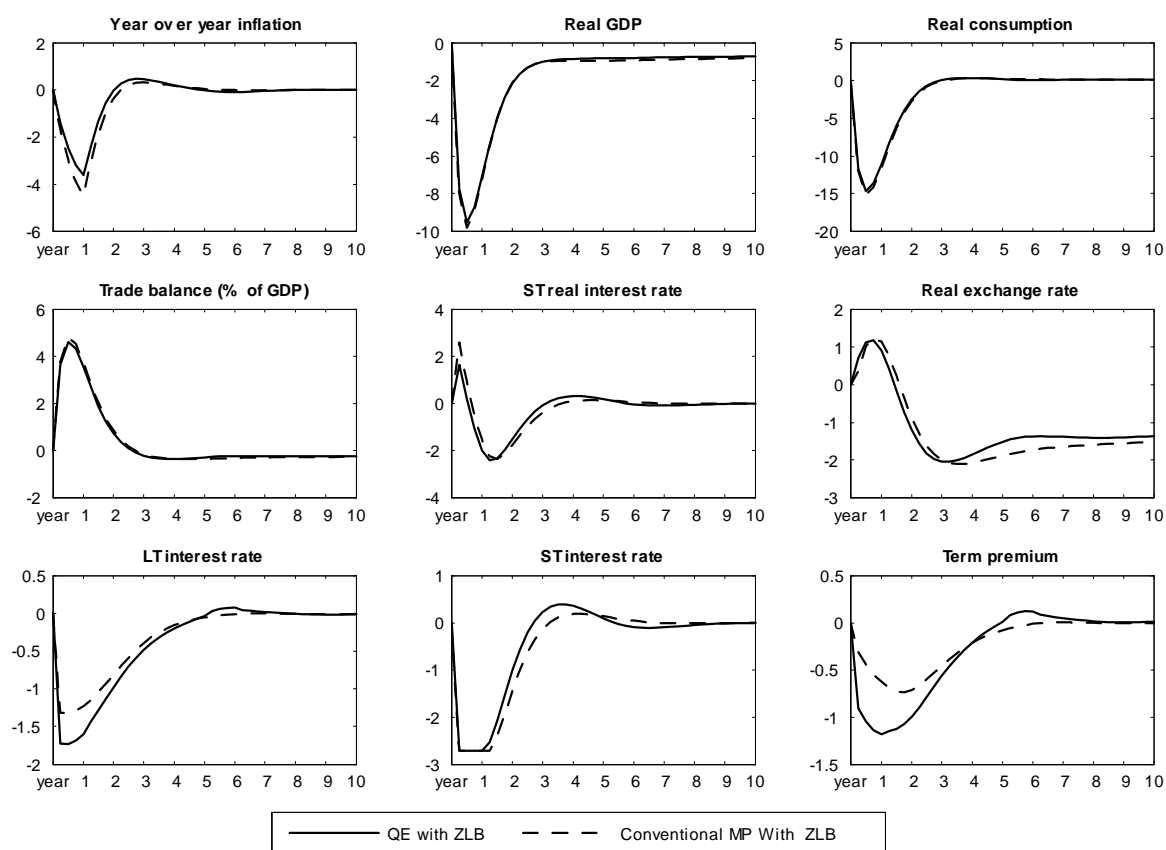
safer as it removes a significant amount of financial assets with high duration from the market. As a result of lower duration risk (or less exposed interest rate risk) of assets holding, the price of risk decreases. The market in turn is willing to demand a lower premium to hold the remaining bonds.

We have shown that unconventional monetary policy like QE could affect the spread between illiquid and liquid assets by changing the relative quantity of assets of different maturities. In another word, IAS of unrestricted households ensures the effectiveness of QE on the term premium through the portfolio-balancing channel. However, with only unrestricted households, even if they consider short- and long-term bonds as imperfect substitutes, they could always "bypass" the long-term bonds market altogether, and simply enforce their consumption plans by trading in sequences of short-term bonds. This suggests that even though IAS allows QE to affect the long-term interest rate through term premium, IAS alone does not guarantee any effect on aggregate demand from QE without additional link from long-term interest rate to aggregate demand.

Therefore, to allow for an independent role of long-term interest rate over and above the short-term rates in the aggregate demand, it's necessary to have the second type "restricted households" who have specific preference towards investing in long-term assets. Overall, QE generates a significant decline in the term premium by roughly 60 bps. This is translated to an increase in consumption by nearly 0.5 per cent and year over year inflation rises to a peak of 0.8 per cent above its no QE level (this prevents the short-term real rate from rising). Output also rises by about 0.3 per cent above its no QE level.

In addition to the domestic effect, QE policy also affects a foreign economy through aggregate demand channel. Lower borrowing costs originated from a decline in the term premium encourages increases in domestic consumption and as a result, domestic GDP. With some degree of trade openness, higher aggregate domestic demand implies increase in demand for imports from the foreign country. The increase in imports, however, is partially offset by a relative depreciation of the domestic currency through the exchange rate channel originating from a negative interest rate differential versus the foreign country. The exchange rate depreciates for over a year, supporting Canadian exports. On net, the aggregate demand channel dominates the exchange rate channel in the short-run, leading to a deterioration of the trade balance. In the near-term, as QE leads to a faster increase in the policy rate relative to the ZLB case, exchange rate appreciates in year 2 and 3, containing the increase in exports of the QE country. In the long-run, asset purchases in QE country increase cash holdings of investors, who deposit funds into banks, leading to more lending domestically or abroad. Current account of QE country

Figure 2.1: Impulse Response Functions

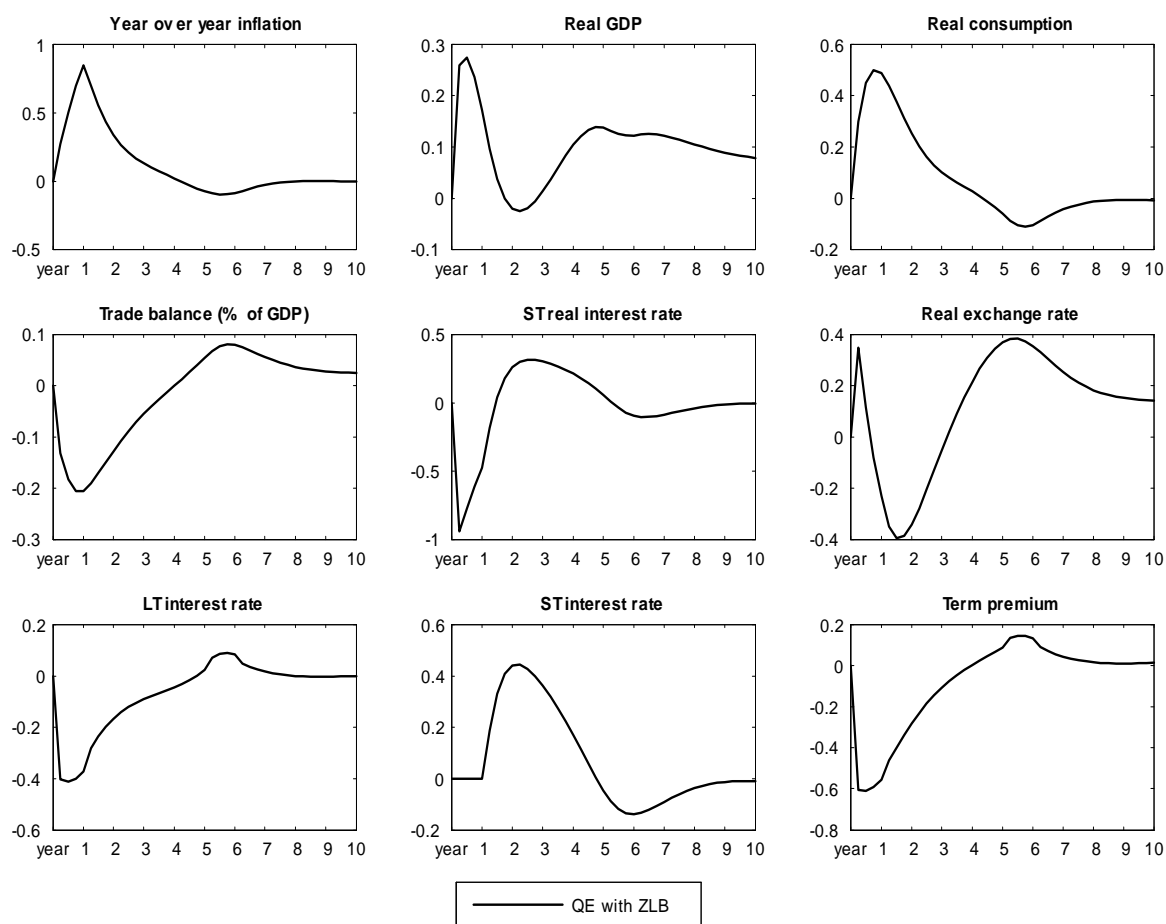


deteriorates as a result of increase in consumption and imports, leading to a long-run depreciation of the exchange rate. This finding is similar to the conventional view of the exchange rate impact from an expansionary monetary policy. The dynamics of exchange rate, however, are more volatile given the richer specifications of the interest rates that matter for the exchange rate determination.

## 2.7 Concluding Remarks

The standard New Keynesian model predicts that financial assets are perfect substitutes and aggregate demand is just affected by the evolution of one-single interest rate: the short-term interest rate. However, there exists some evidence as presented by Andres et al. (2004) showing that these two predictions do not hold in the data. In practice, the explosive growth of base money in the United States since September 2008 has also suggested that the Fed's policy has started shifting from an interest rate policy to

Figure 2.2: Marginal Impact of QE



one often described as "quantitative easing". With the federal funds rate reaching the effective zero lower bound since December 2008, significant developments have also been observed in the changing composition of the asset side through the Fed's unconventional balance sheet operations. The conduct of unconventional policy largely depends on the imperfect substitution among various financial assets.

In this chapter, we extend the analysis of Andres et al. (2004). In particular, we develop a small open economy model with imperfect asset substitution and segmented asset markets. Imperfect substitutability allows long-term rates in the model to deviate from the level implied by the strict expectation theory of the term structure. Segmented asset markets allow long-term interest rate to matter over and above the short-term rates in the aggregate demand. The modified model nests the standard version of the New Keynesian model, which does not allow for any independent movements in long-term rates. As in Andres et al. (2004), our results from a small open economy model also support the presence of imperfect substitutability and segmented asset markets.

We estimate the model with IAS and SAMs using Canadian data. We then use this model to evaluate the effect of QE when the policy rate is at its effective lower bound. Our results suggest that a QE intervention of 4 per cent of GDP for 4 quarters has moderate impact on rates but small impact on output. The key contribution of this approach is that the structural nature of the model allows us to isolate the impact of independent variation in long-term rates (unrelated to changes in expectations of future short-term rates) when the effectiveness of normal monetary policy has been limited by the zero lower bound.

There are several directions in which the model presented in this chapter can be improved. First, adding nominal wage stickiness would help the model to generate lower movements in inflation and nominal wages. Second, introducing a more active fiscal policy could be interesting to study the interactions between fiscal policy and unconventional monetary policy. Third, introducing investment into the model could be useful to study the interactions between credit easing and quantitative easing. Finally, a well-specified rest of the world model could also be extended in order to study how unconventional policies abroad affect the small open economy model and the implication of policy coordinations among countries.

# Chapter 3

## Effects of Forward Guidance at the Zero Lower Bound

### 3.1 Introduction

In the aftermath of the financial and economic crisis of 2007-2009, central banks in many advanced economies cut their policy rates very aggressively to counter rapidly deteriorating economic outlooks. During this process, many of these central banks found themselves hitting the effective lower bounds on the policy rate. In this context, numerous unconventional monetary tools were considered in order to provide additional monetary stimulus.<sup>1</sup> At the zero lower bound (ZLB), a central bank could reduce the spreads of longer-term interest rate over expected policy rates through large-scale purchases of government securities (often referred to as quantitative easing, QE). Alternatively, a central bank could reduce the economic costs of the lower bound by credibly signaling that short-term interest rates will remain low for a prolonged period of time, either till a fixed date or conditional on the development of a particular indicator (unconditional or conditional forward guidance). Finally, it could also offset the impact of ZLB by announcing that it will generate an overshooting of inflation or output growth once the lower bound is no longer a constraint (forward guidance through targeting a nominal anchor like price-level or output-level). To manage expectations efficiently and avoid time-inconsistency issue associated with policy guidance, monetary authority often announces clear conditions under which policy rates would begin to increase. In this chapter, I provide a quanti-

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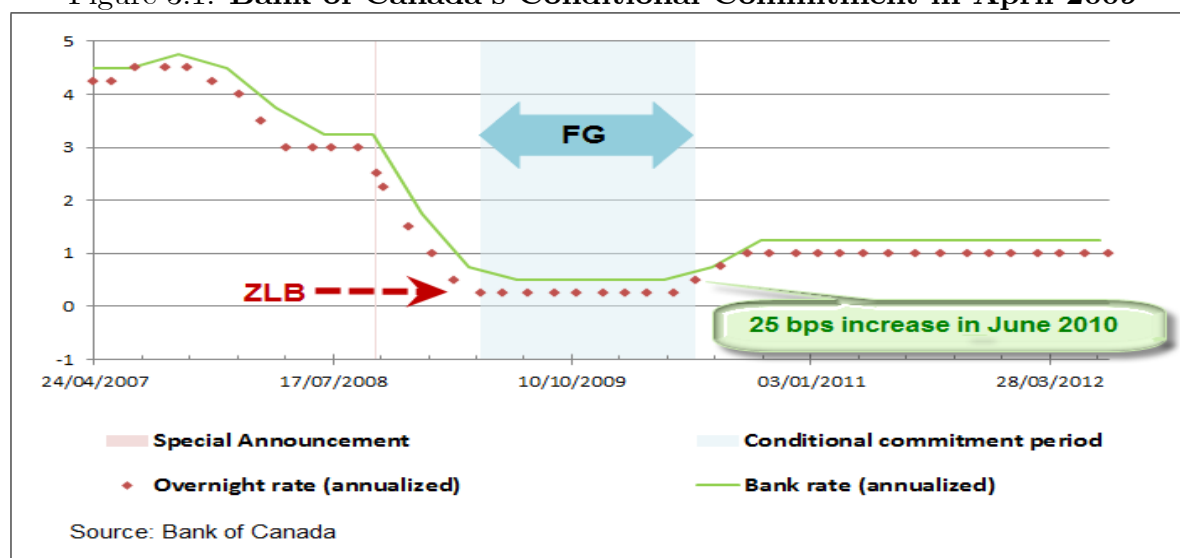
<sup>1</sup>For example, the Bank of England has engaged in large-scale asset purchases during 2009-2010 (see Bean 2009). The Federal Reserve have conducted four rounds of quantitative easing with some recent forward guidance which links the monetary policy action conditional on the development of labour market. (see FOMC release, September 2012).

tative assessment of the macroeconomic impact of different types of forward guidance when the policy rate is constrained at its effective lower bound.

The practice of forward guidance has undergone significant development since the onset of the financial crisis in the formal statements by the Federal Open Market Committee (FOMC). In December 2008 it stated "the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time." In March 2009, when the first round of large-scale assets purchase program was announced, it reemphasized that "economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period." Given the sluggish global recovery from a deep, synchronous recession, the FOMC stated in November 2011 with an explicit timing of its policy such that "economic conditions—including low rates of resource utilization and a subdued outlook for inflation over the medium run—are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013." In face of the recent strains in global financial markets and its associated downside risks, the FOMC states in August 2012 that it "anticipates that economic conditions—including low rates of resource utilization and a subdued outlook for inflation over the medium run—are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014." Finally, along with the fourth round of QE, the FOMC stated in September 2012 that the Fed's fund rate could be expected to remain at exceptionally low levels until mid-2015 and monetary policy reaction is *conditional* on the future development of the U.S. labor market.

Prior to the last guidance, commitment of the Fed provides stimulus mainly through expectations of prolonged period of low nominal interest rates. The conditional forward guidance, on the other hand, not only works through expectation of long period of low nominal interest rate, but also through expectation of higher than expected future inflation. This further helps to reduce market expectations of future real interest rate and therefore effectively stabilizing the output gap and inflation (see Woodford 1999, Eggertsson and Woodford 2003 and Walsh 2009). Conditional guidance was recently used by the Bank of Canada in face of financial crisis, given its experience of reaching the effective zero lower bound and the sluggish recovery of the world economy. The Bank has employed new tools to communicate the likely nature of future policy stance. The most explicit communication was in April 2009, when the Bank lowered its target for the overnight rate by one-quarter of a percentage point to 1/4 per cent, which is the effective lower bound for the policy rate. Accompanying the policy action, the Bank states "*conditional* on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve

Figure 3.1: Bank of Canada's Conditional Commitment in April 2009



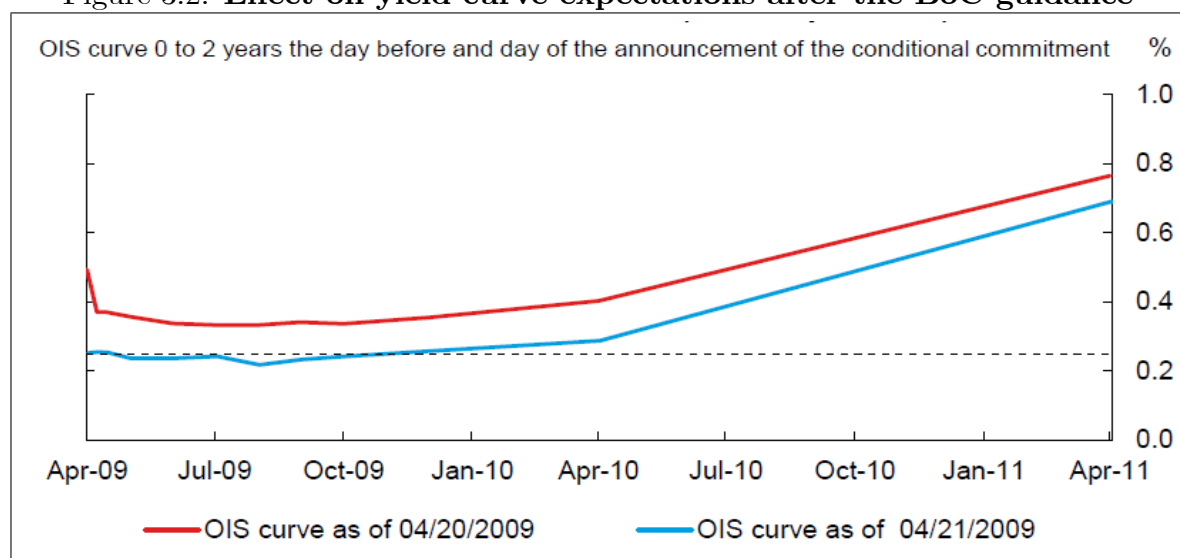
the inflation target." The relevant targeting horizon of such guidance is shown below in Figure 3.1.

In the April 2009 Monetary Policy Report, the Bank of Canada also provided guidance through two additional channels. First, it published the confidence intervals for the first time to illustrate the uncertainty associated with the economic projections of year-over-year core and total CPI inflation over the horizon from 2009Q2 to 2011Q4. This reinforces the conditionality of the statement and provided a clear band of estimates of the uncertainty associated with the development of the economy, as well as the magnitude and persistence of ongoing economic shocks. Second, the Bank also presented a framework of conducting monetary policy at low interest rates by listing three main instruments that it could consider using to achieve its monetary policy objective at the effective lower bound: i) a conditional statement about the future path of policy rates; ii) quantitative easing and iii) credit easing.<sup>2</sup>The adopted conditional commitment by the Bank of Canada is considered successful as it has provided the desired stimulus effects through the shift of the yield curve (Carney 2012)<sup>3</sup>.

Several studies have examined the effects of central bank communication more generally (see Gurkaynak et al. 2005, Kohn and Sack 2004, and Bernanke et al. 2004). They found that the Federal Reserve's policy statements have significant effects on financial market expectations of future policy actions as well as Treasury yields. Only a few studies have looked at the effectiveness of forward guidance policies at the zero lower bound.

<sup>2</sup>See Bank of Canada, "Framework for Conducting Monetary Policy at Low Interest Rates," MPR, April 2009.

<sup>3</sup>Figure source: Bloomberg and Bank of Canada speech "Guidance".

Figure 3.2: **Effect on yield curve expectations after the BoC guidance**

Reifschneider and Williams (1999) use the FRB/US model to quantify the effects of the ZLB on macroeconomic stabilization and find that simple modified Taylor rule yields a dramatic reduction in the detrimental effects of the ZLB. However, since the model is highly stylized, there is no discussion incorporating any conditionality into the policy rule.

Eggertsson and Woodford (2003) examine the optimal policy under commitment at the ZLB and find that an optimal commitment involves a sort of history-dependent policy that leads to higher inflation expectations in response to a binding zero bound on nominal interest rate. However, it is not optimal to commit to so much future inflation that the zero bound ceases to bind. To achieve the desirable outcome of the unconventional policy, it is essential for a central bank to conduct effective communication with the public about its policy commitments.

Campbell et al. (2012) use the DSGE model developed at the Federal Reserve Bank of Chicago to examine the macroeconomic effects of FOMC forward guidance. The authors distinguish between two kinds of FOMC forward guidance: Odyssean forward guidance (that changes private expectations by publicly committing the FOMC to future deviations from its underlying policy rule) and Delphic forward guidance (that merely forecasts the future outcome of interested macroeconomic variables). The authors augment the event-study approach by identifying the Odyssean forward guidance with a parsimonious rule for setting the policy rate as a function of current or expected economic condition. The identified anticipated deviation of public expectations from the monetary policy rule is referred to as the Odyssean component. They find that Odyssean forward guidance

helps to provide more accommodation at the zero lower bound.

In addressing new and nontraditional policy responses when the economic slackness is enormous, Evans (2011) has proposed policy accommodation through making simple conditional guidance on future development of unemployment and inflation expectations. The objective of such guidance is to allow the market to fully establish the conditionality of the future policy action by the FOMC such that the federal funds rate will remain at extraordinary low level until the unemployment rate falls substantially, or inflation rises to a certain target. The conditional nature of the guidance allows more flexibility of the policy conduct. On one hand, even when the economy starts to recover, such policy helps to maintain a gradual increase of the policy rate until the prescribed condition is met. On the other hand, it doesn't have the hurdle for early policy tightening as associated with unconditional guidance, which largely maintains a central bank's credibility in an environment with large amount of uncertainty.

There are more aggressive approaches that could establish clearer communication of policy conditionality with observable data. Evans (2012) discusses how state-contingent price-level targeting could help to provide further monetary accommodation even at the zero lower bound. This is in line with other level-targeting proposal as suggested by Romer (2011) in favor of targeting the nominal GDP level. However, Woodford (2012) argues that a simple nominal GDP targeting regime doesn't achieve quite the full welfare gains associated with a credible price-level targeting regime, based on the theoretical analysis of Eggertsson and Woodford (2003). Nevertheless, the nominal GDP target path can well serve as a compromise between the objective to choose a target that leads to optimal equilibrium and a target that's widely observable and easy to communicate.

Chehal and Trehan (2009) analyze the practice of the Bank of Canada's conditional guidance by using measures derived from overnight index swaps.<sup>4</sup> They find that the guidance has an immediate effect on financial market expectations regarding short-term interest rates in Canada. The authors also emphasize the importance of the conditionality of the guidance. In particular, they document that when the Canadian economy appeared to be recovering from the recession more quickly than anticipated, market participants began to expect interest rates to rise ahead of the previously announced date.

While the empirical evidence is in support of the implementation of forward guidance at the zero lower bound, its effectiveness still has some limitations. For example, Levin et al. (2009) consider optimal policy in a New Keynesian model when the nominal interest rate is constrained at its lower bound. Using U.S. aggregate time series data, they find

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<sup>4</sup>In particular, they use the overnight index swaps that are regarded as proxy for market expectations of the policy rate for the 30-day period ending 12 months in the future.

that although forward guidance is effective in offsetting natural rate shocks of moderate size and persistence, it is not sufficient to restore output and inflation when the shock is large and highly persistent. Therefore, other unconventional monetary policies are called for as complements to forward guidance. Similarly, focusing on the Canadian experience, Fay and Gravelle (2010) investigate the reaction of Canadian financial markets to the Bank of Canada's communications and find evidence that the inclusion of forward-looking policy rate guidance does not significantly impact market rates over the more recent periods. The authors find evidence that even the forward-looking statements are designed to be conditional, they have only made the Bank's decision on the policy rate more predictable, but not necessarily more transparent. This limits the effectiveness of guidance on minimizing the negative effects of the ZLB.

The discussion in this chapter addresses the use of conditional statement at the ZLB, along with another form of policy guidance. In particular, I compare the performance of a state-contingent guidance (using conditional statement) and a price-level guidance.<sup>5</sup> Throughout this analysis, I assume that aggregate demand is affected by the evolution of just one interest rate: the short-term interest rate (i.e. the policy rate). Therefore, there is no role for other unconventional monetary policies (for example, quantitative easing or credit easing). I consider alternative monetary policy rules under commitment in a calibrated simple three-equation New Keynesian model and examine the extent to which forward guidance helps to mitigate the negative real impact of the ZLB.

The remainder of this chapter is organized as follows. Section 2 highlights some key features of a small closed economy model. Section 3 discusses the basic mechanics of forward guidance at the ZLB in this model. In Section 4, I conduct some experiment to understand the transmission mechanism of different types of forward guidance in the model. Section 5 quantifies some limitation of forward guidance through some sensitivity analysis of the key parameters. Section 6 concludes.

## 3.2 The New Keynesian Model

In this model, I introduce a prototypical closed economy New Keynesian DSGE (dynamic stochastic general equilibrium) model with nominal price rigidities and no capital. This model falls into the class of "miniature" DSGE models (e.g., Clarida, Gali and Gertler

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<sup>5</sup>Adopting a nominal GDP-level targeting framework could achieve similar effect as the price-level targeting. Both framework rely on the "history dependence" of targeting a nominal anchor in the sense that the monetary authority is required to make up for past deviation on the path of nominal anchors. The expectations for future overshooting through prolonged accomodative policy action helps to stimulate the economy.

1999 and Woodford 2003, among others) that are based on optimizing households and firms, rational expectations and nominal price rigidities, that are introduced in the form of quadratic costs in price adjustment. The model consists of three linearized equations: a forward-looking "IS curve" that relates output to the real interest rates, a New Keynesian "Phillips curve" that relates inflation rate to past inflation and the real marginal cost, and a central bank that sets monetary policy according to an augmented Taylor-type rule.<sup>6</sup> I abstract from the specification of money demand in this chapter and assume money supply is adjusted to money demand at a given interest rate.

### 3.2.1 Households

The economy is populated by a unit measure of identical and infinitely-lived households. The representative household maximizes lifetime utility function over consumption,  $C_t$ , and quantity of labor,  $H_t$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \Theta_t \left\{ \frac{(C_t - \zeta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\varphi}}{1+\varphi} \right\} \right] \quad (3.1)$$

subject to the budget constraint:

$$C_t + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} H_t + I_{t-1} \frac{B_{t-1}}{P_t} + \frac{\Pi_t}{P_t} \quad (3.2)$$

where  $\beta$  is the discount factor,  $\sigma$  is the coefficient of relative risk aversion, and  $1/\varphi$  is the Frisch elasticity of labor supply.  $\zeta$  is a habit parameter, and the habit level of consumption is assumed to be external to the agent.  $P$  is the aggregate price level,  $W$  is the nominal wage rate, and  $\Pi$  is the dividends received from the ownership of intermediate goods producers. Households hold 1-period domestic bonds,  $B$ , which pay a gross nominal interest of  $I$ .

$\Theta_t$  is an exogenous intertemporal preference shock that affects the marginal utility of consumption. This shock evolves according to an autoregressive process in the form of

$$\log \Theta_t = (1 - \rho_{\Theta}) \log \bar{\Theta} + \rho_{\Theta} \log \Theta_{t-1} + \varepsilon_{\Theta,t}, \quad \varepsilon_{\Theta,t} \sim NID(0, \sigma_{\Theta}^2) \quad (3.3)$$

where  $\rho_{\Theta} \in [0, 1]$  is the autoregressive coefficient and the innovation  $\varepsilon_{\Theta}$  is an i.i.d shock.

In equilibrium, the marginal utility of increasing current consumption is equal to the marginal disutility from reducing current income. The first order condition for consump-

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<sup>6</sup>Properties of this type of three-equation New Keynesian model are discussed in Ireland (2004).

tion is given as

$$\Theta_t (C_t - \zeta C_{t-1})^{-\sigma} = \Lambda_t \quad (3.4)$$

where  $\Lambda$  is the Lagrangian multiplier on the household's budget constraint. Similarly, the households determine their labor supply by equating the marginal disutility from work to the marginal utility gain from increasing wage income,

$$\Theta_t H_t^\varphi = \Lambda_t \frac{W_t}{P_t}. \quad (3.5)$$

where  $W_t$  denotes the aggregate nominal wage rate at time  $t$ .

The first-order-condition with respect to  $B$  yields the asset pricing equation for domestic bonds:

$$\frac{\Lambda_t}{P_t} = \beta E_t \left[ \frac{\Lambda_{t+1}}{P_{t+1}} I_t \right]. \quad (3.6)$$

or

$$\Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{I_t}{\pi_{t+1}} \right]. \quad (3.7)$$

where the consumer-price index inflation rate is defined as  $\pi_t = P_t/P_{t-1}$ .

### 3.2.2 Firms

There are two types of firms in the model: i) perfectly competitive final goods producers that produce an homogenous final good that is used for private consumption, and ii) monopolistically competitive intermediate goods producers that rent labor from the households to produce differentiated products.

#### Final Goods Firms

At time  $t$ , a final consumption good,  $Y_t$  is produced by a perfectly competitive representative firm. The firm produces the final good by combining a continuum of intermediate differentiated goods, indexed by  $i \in [0, 1]$ , using technology

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t-1}} \quad (3.8)$$

where  $\theta_t$  is the time-varying elasticity of substitution across the differentiated intermediate goods. The elasticity evolves over time subject to an exogenous mark-up shock process. Let  $\mu_t = \theta_t / (\theta_t - 1)$  denote the price mark-up over marginal cost, the shock evolves according to a process as given by

$$\log \mu_t = (1 - \rho_\mu) \log \mu + \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}, \quad \varepsilon_{\mu,t} \sim NID(0, \sigma_\mu^2) \quad (3.9)$$

where  $\rho_\mu$  is the persistence parameter and the innovations  $\varepsilon_\mu$  is an i.i.d shock.

These final goods are only used by households for private consumption; hence  $Y_t = C_t$ . The firm takes its output price,  $P_t$ , and the price of the intermediate good,  $P_t(i)$  as given to maximize its profit.

$$\max P_t Y_t - \int_0^1 P_t(i) Y_t(i) dj \quad (3.10)$$

where  $P(i)$  is the price of the intermediate good  $i$ .

Profit maximization of the final goods firms, taking the final good price and the input prices, yields the demand function for the intermediate good  $i$  :

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t \quad (3.11)$$

### Intermediate Goods Firms

Intermediate good  $i \in [0, 1]$  is produced by a monopolistically competitive intermediate goods producer following a production technology

$$Y_t(i) = Z_t H_t(i) \quad (3.12)$$

where  $H(i)$  is the amount of labor input used in the production of intermediate good  $j$ . The variable  $Z$  represents a time  $t$  shock to the technology for producing intermediate output. The aggregate productivity shock also follows a simple stochastic autoregressive process of the form:

$$\log Z_t = (1 - \rho_z) \log \bar{Z} + \rho_z \log Z_{t-1} + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim NID(0, \sigma_z^2) \quad (3.13)$$

where  $\rho_z$  is the persistence parameter and the innovations  $\varepsilon_z$  is an i.i.d shock.

The intermediate goods firms take the demand function of the final goods producers,

$Y_t(i)$ , as given and set prices  $P_t(i)$ , to maximize the present discounted value of profits:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left[ \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} H_t(i) - \frac{\kappa}{2} \left( \frac{P_t(i)}{\pi_{t-1}^{\theta_\pi} P_{t-1}(i)} - 1 \right)^2 Y_t \right] \quad (3.14)$$

where  $\Lambda_t$  is the owners' marginal utility of an additional dollar of profits in period  $t$ . The sticky price specification follows Ireland (2003) and others as based on Rotemberg (1982) quadratic cost of nominal price adjustment.<sup>7</sup> More specifically, each intermediate good firm pays an increasing and convex cost  $\frac{\kappa}{2} \left( \frac{P_t(i)}{\pi_{t-1}^{\theta_\pi} P_{t-1}(i)} - 1 \right)^2 Y_t$  scaled by the aggregate output level. The parameter  $\kappa$  regulates the magnitude of the price adjustment costs. When  $\kappa = 0$ , this collapses to a flexible-price system. The parameter  $\theta_\pi$  determines the extent to which current price changes are indexed to past inflation, when its price increase deviates from the steady state inflation rate. The firms discount future earnings at the same rate as households and value their distributions according to households' marginal rate of substitution.

Converting the firm's profit maximization problem into an unconstrained maximization problem by substituting the profit equation and the demand function for  $Y_t(i)$ , we have:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left[ \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta_t} Y_t - \frac{W_t}{P_t} \frac{1}{Z_t} \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t - \frac{\kappa}{2} \left( \frac{P_t(i)}{\pi_{t-1}^{\theta_\pi} P_{t-1}(i)} - 1 \right)^2 Y_t \right]. \quad (3.15)$$

Solving the problem with respect to  $P_t(i)$  yields the following first order condition for an intermediate goods firm:

$$\begin{aligned} 0 = & (1 - \theta_t) \frac{\Lambda_t}{\Lambda_0} \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} \left( \frac{Y_t}{P_t} \right) + \theta_t \frac{\Lambda_t}{\Lambda_0} \frac{W_t}{P_t} \frac{1}{Z_t} \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t-1} \left( \frac{Y_t}{P_t} \right) \\ & - \kappa \frac{\Lambda_t}{\Lambda_0} \left( \frac{P_t(i)}{\pi_{t-1}^{\theta_\pi} P_{t-1}(i)} - 1 \right) \left( \frac{Y_t}{\pi_{t-1}^{\theta_\pi} P_{t-1}(i)} \right) \\ & + \beta E_t \left[ \kappa \frac{\Lambda_{t+1}}{\Lambda_0} \left( \frac{P_{t+1}(i)}{\pi_t^{\theta_\pi} P_t(i)} - 1 \right) \left( \frac{P_{t+1}(i) Y_{t+1}}{\pi_t^{\theta_\pi} P_t(i)^2} \right) \right] \end{aligned} \quad (3.16)$$

Given  $W_t$  and  $P_t$  are the same for all intermediate firms, every firm sets the same price

<sup>7</sup>The quadratic price-adjustment cost function is commonly used in New Keynesian models. It suggests that if a firm chooses to set a price  $P_t(i)$  that is different from the price of previous period  $P_{t-1}(i)$ , it incurs a real cost. The larger the deviation of the current price from the reference level price, the greater is the cost. Due to the quadratic nature of the function, the cost associated with adjusting prices are symmetric with respect to both price increases and declines.

$P_t(i) = P_t$ . We then simplify the first order condition as:

$$0 = (1 - \theta_t) + \theta_t \frac{W_t}{P_t} \frac{1}{Z_t} - \kappa \left( \frac{\pi_t}{\pi_{t-1}^{\theta_\pi}} - 1 \right) \left( \frac{\pi_t}{\pi_{t-1}^{\theta_\pi}} \right) + \beta E_t \left[ \kappa \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{\pi_{t+1}}{\pi_t^{\theta_\pi}} - 1 \right) \left( \frac{\pi_{t+1}}{\pi_t^{\theta_\pi}} \right) \left( \frac{Y_{t+1}}{Y_t} \right) \right] \quad (3.17)$$

where  $\pi_t = P_t/P_{t-1}$ .

Rearranging terms yields the following New Keynesian Phillips curve:

$$\kappa \left( \frac{\pi_t}{\pi_{t-1}^{\theta_\pi}} - 1 \right) \left( \frac{\pi_t}{\pi_{t-1}^{\theta_\pi}} \right) = \kappa E_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{\pi_{t+1}}{\pi_t^{\theta_\pi}} - 1 \right) \left( \frac{\pi_{t+1}}{\pi_t^{\theta_\pi}} \right) \left( \frac{Y_{t+1}}{Y_t} \right) \right] + \left[ (1 - \theta_t) + \theta_t \frac{W_t/P_t}{Z_t} \right]. \quad (3.18)$$

Note that in the absence of price adjustment costs,  $\kappa = 0$ , the above expression simplifies to the familiar price mark-up equation

$$P_t = \left( \frac{\theta_t}{\theta_t - 1} \right) \frac{W_t}{Z_t} \quad (3.19)$$

where  $W_t/Z_t$  is the marginal cost of production and  $\theta_t/(\theta_t - 1)$  is the gross mark-up on price over marginal cost.

### 3.2.3 Monetary Policy

The monetary authority follows a Taylor-type policy rule by setting the short-term nominal interest rate:

$$\log I_t = \rho_I \log I_{t-1} + (1 - \rho_I) \left[ \rho_\pi \log(\pi_t) + \rho_y \left( \log \frac{Y_t}{Y_t^n} \right) + \log \bar{R} \right] + \varepsilon_{I,t}, \quad (3.20)$$

where  $\bar{R}$  is the steady-state value of the nominal interest rate,  $\rho_I$  determines the extent of interest rate smoothing, and the parameters  $a_\pi$  and  $a_y$  determine the importance of inflation and the output gap in the Taylor rule. Output gap is defined as the log deviation of the actual output  $Y_t$  from the natural rate of output  $Y_t^n$  following the conventional definition.<sup>8</sup>  $\varepsilon_I$  is monetary policy shock and is i.i.d  $N(0, \sigma_I^2)$ .

<sup>8</sup> $Y_t^n$  is the natural rate of output, which is the level of output that would prevail under imperfectly competitive markets but with flexible wages and prices. It reflects the static distortions originated from imperfect competition and the dynamic distortions due to exogenous variations in the degree of market competitiveness, as captured by markup shocks in goods and labor markets in DSGE models (Justiniano and Primiceri 2008).

### 3.2.4 Market Clearing Conditions

The goods market clearing condition is given by

$$C_t = Y_t \quad (3.21)$$

Similarly, the labor market clearing condition is given by

$$H_t = \int_0^1 H_t(j) dj. \quad (3.22)$$

In equilibrium, I assume that domestic bonds are in zero-supply (they are inside bonds between private agents):

$$B_t = 0 \quad (3.23)$$

for all  $t$ . The profits received by households are equal to the dividend payments of the intermediate firms:

$$\Pi_t = \int_0^1 \Pi_t(j) dj. \quad (3.24)$$

Note that the national income in the model is defined as:

$$\frac{W_t}{P_t} H_t + \frac{\Pi_t}{P_t} = C_t = Y_t. \quad (3.25)$$

### 3.2.5 Mechanism of assessing forward guidance at the zero lower bound

Log-linearized the dynamic equations around the steady-state of the model, we obtain the three-equation NK model with the IS curve is given by

$$\hat{y}_t = \frac{1}{1+\zeta} E_t [\hat{y}_{t+1}] + \frac{\zeta}{1+\zeta} \hat{y}_{t-1} - \frac{(1-\zeta)}{\sigma(1+\zeta)} \left( \hat{i}_t - E_t [\hat{\pi}_{t+1}] \right) + \hat{d}_t. \quad (3.26)$$

The New-Keynesian Phillips curve is given by

$$\hat{\pi}_t = \frac{\theta-1}{\kappa(1+\beta\theta_\pi)} \hat{m}c_t + \frac{\beta}{1+\beta\theta_\pi} E_t [\hat{\pi}_{t+1}] + \frac{\theta_\pi}{1+\beta\theta_\pi} \hat{\pi}_{t-1} + \hat{m}_t. \quad (3.27)$$

Finally, the policy rule is

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1 - \rho_i) [\rho_\pi \widehat{\pi}_t + \rho_y (\widehat{y}_t - \widehat{y}_t^n)] + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim NID(0, \sigma_i^2) \quad (3.28)$$

As in standard NK models, there are two key assumptions embedded in this simple version of closed economy model. First, aggregate demand is affected by the evolution of just one path of the short-term, risk-free real interest rate.

$$r_t = i_t - \pi_t \quad (3.29)$$

This interest rate determines the degree of intertemporal substitution by firms and households in their decisions to produce, spend, save and work. Second, short- and long-term, risk-free assets are assumed to be perfect substitutes. This implies that the expected rates of return on these two types of asset will be equalized by arbitrage. Assuming the long-term asset under consideration has a 5-year maturity ( $T = 20$  quarters) and the short-term asset has a maturity of 1 quarter, under the pure expectation theory of the term structure, the long-term rate is exactly equal to the average path of expected future short-term rates (over the subsequent 20 quarters):

$$i_t^T = \frac{1}{T} \sum_{j=0}^{T-1} E_t i_{t+j}. \quad (3.30)$$

In response to large negative shocks, central bank follows its conventional policy rule and cut nominal interest rate to stimulate the aggregate demand. If the shocks are very large and persistent, there is a possibility of nominal interest rate reaching the ZLB. Once the nominal short-term interest rate reaches the lower bound, through effective guidance, central banks may still be able to influence expectations of the policy rate beyond the point at which the bound ceases to bind. Under the expectations hypothesis of the term structure, a credible promise to keep future short-term interest rates lower than they otherwise would be will lower long-term interest rates today, thereby stimulating the economy. In addition, by influencing inflation expectations, the alternative policies can boost current inflation, even when nominal short-term rates are constrained at the lower bound. In practice, several approaches can be used to signal that nominal short-term interest rates will remain lower for a longer period of time after the lower bound ceases to bind (for example, FOMC's communication of a prolonged period of low interest rate environment and the Bank of Canada's conditional statement) or that central bank will engineer an overshooting of inflation once the lower bound is not a constraint (price-level

targeting).

In the following analysis, I examine two inflation-targeting rules and one price-level-targeting rule. For easy comparison with other work in an NK model with forward guidance, I consider the following augmented interest rate rule as in Campbell et al. (2012) and Laseen and Svensson (2011):

$$i_t = \rho_i i_{t-1} + \{(1 - \rho_i)\rho_\pi \tilde{\pi}_t + \rho_y \tilde{y}_t\} + \sum_{j=0}^T \exp(\varepsilon_{t-j,j}^i) \quad (3.31)$$

where  $\rho_i$  determines the degree of interest rate smoothing, and the parameters  $\rho_\pi$  and  $\rho_y$  determine how policy rate responds to typical movements in macroeconomic conditions (measured by inflation rate and the output gap).

I rewrite the policy shocks process into two components, an unanticipated component  $\varepsilon_{t,0}^i$ , and an anticipated component  $\varepsilon_t^{i,fg}$ , following the form

$$\sum_{j=0}^T \exp(\varepsilon_{t-j,j}^i) = \varepsilon_t^{i,0} + \varepsilon_t^{i,fg} \quad (3.32)$$

where  $\varepsilon_{t,0}^i$  is the usual monetary policy disturbance used in conventional policy rules. The anticipated component are the forward guidance shocks defined as  $\varepsilon_t^{i,fg} = \sum_{j=1}^T \exp(\varepsilon_{t-j,j}^i)$ , where  $\varepsilon_{t-j,j}^i$  is the T-period ahead innovation perceived by the agents in the model about the deviations of central bank's policy from the normal policy prescription. The inclusion of both components in the policy shock is motivated by recent experience that suggests that financial markets not only understand and react to a shift in monetary policy quickly, but also react to anticipated policy action (for example, market partially priced in the impact of QE2 during 2010-11). Here I make the assumptions about expectations that all agents fully understand how forward guidance is implemented under the ZLB and its transmission mechanism in the economy. From this perspective, this would be a credible assumption either when a policy has been in place for some time or the central bank conducts fully transparent communication about the nature and horizon of its future policy (for example, the practice of publishing policy rate path by central banks). The public observe  $\varepsilon_{t,j}^i$  in quarter t, and the central bank applies it to the policy rule. The expected path of interest rates is determined by the realization of all the anticipated shocks revealed in quarter t. I assume the innovations are i.i.d and are uncorrelated across time and horizons.



where

$$J = \frac{\partial f_t}{\partial y'_t}, F_t^i = \frac{\partial f_t}{\partial y'_{t+i}}, B_t^i = \frac{\partial f_t}{\partial y'_{t-i}}, \quad (3.34)$$

are  $n * n$  matrices.

When applying Newton's method to the model equations stacked over time, the algorithm takes advantage of the special sparse structure of the Jacobian matrix to solve the linear Newton step equations in a more efficient fashion. The algorithm involves iterating on the set of  $nT$  equations

$$J(y^s - y^{s-1}) = -f(y^{s-1}) \quad (3.35)$$

where  $y^s$  is the  $nT * 1$  vector of stacked values of the endogenous variables in iteration  $s$  and  $f$  is the  $nT * 1$  vector valued function formed by stacking  $f \dots f_T$ . Iteration starts from an initial guess at the solution,  $y^0$ , and terminate when a convergence criteria is satisfied for some small value of  $\varepsilon$

$$\max_j \left| \frac{y^s - y^{s-1}}{y^{s-1}} \right| = \varepsilon. \quad (3.36)$$

The unique structure of the Jacobian matrix allows to solve the equations efficiently, using Gaussian pivoting on the blocks. The Jacobian matrix is transformed to an upper block-triangular structure, then this structure is simply solved recursively, block by block, from period  $T$  down to period 1. The horizon is chosen to be 400 quarters to minimize the impact of horizon on the solution. The model results are produced in Troll software.<sup>9</sup>

### 3.2.7 Parameterization

To simplify the analysis of different forward guidance policies at the ZLB, I calibrate the model to reflect the most important features of the Canadian economy. I assume that steady-state inflation is zero, which implies that there is no price dispersion in steady state. Similarly, I also assume that there is no wage dispersion in steady state ( $N = \tilde{N}$ ). I assume that steady state consumption is the same across all households types. The assumption that short- and long-term, risk-free assets are perfect substitutes implies that term premium is equal to zero in steady state. This implies  $i = i^L$ . The discount factor  $\beta$  is set at a value of 0.99, whose value corresponds to  $\beta = (1 + r)^{-1}$ , given the choice for

<sup>9</sup>See Hollinger (1996), Juillard (1996) for detailed discussion of the stacked-time algorithm for solving forward-looking models.

the real quarterly steady-state interest rate of 1%.

I calibrate the parameters over the sample period (1980:I - 2012:3). I set the elasticity of substitution among goods  $\sigma$  equal to 5. The parameter relevant for the Frisch elasticity of labor supply,  $\varphi$ , is set at 1.5. The elasticity of substitution between the differentiated intermediate goods  $\theta$  is set at a value of 6. I assume the price adjustment costs parameter  $\kappa$  takes a value of 100. The parameter  $\theta_\pi$  regulates the extent to which current price changes are indexed to past inflation. In the benchmark case, I assume that there is no inflation indexation, therefore  $\theta_\pi$  is set to zero. This assumption implies no role of lagged inflation in determining current inflation. This allows monetary policy to exert important influence on current inflation through affecting the expectations of future marginal cost even if it doesn't affect the current value of real marginal cost. With some degree of indexation, the effect of expected real marginal costs on inflation will decline, and the effectiveness of guidance through the expectation channel will also weaken. Similarly, the habit parameter  $\zeta$  is set to be zero in the benchmark case, this is a simplified assumption. With a higher degree of habit formation in consumption, we will observe a delayed peak response of consumption to movements in real interest rates and policy guidance. I will examine its relevance later on.

The parameters for the policy rule are set as follows. The high degree of interest-rate smoothing ( $\rho_i = 0.8$ ) could reflect the central bank's concern with hitting the lower bound (Woodford 1999). Indeed, by making interest-rate changes highly persistent, the monetary authority is better able to influence longer-term interest rates, a feature that is welcomed especially when the lower bound has become a constraint. With this built-in history dependence feature, this rule can be seen as partly dealing with the lower bound. The policy response to inflation  $\rho_\pi$  is calibrated at 1.5, this is in the mid-range of previous estimates (see review by Hofmann and Bogdanova 2012). The response to output gap  $\rho_y$  is much weaker, set at a value of 0.25.

### 3.3 Assessing the Impact of Forward Guidance

In this section, I use the simple NK model to explore the implications of forward guidance at the zero lower bound under alternative policy mechanisms. The lower bound on nominal interest rate is of concern to the central banks because the traditional channel of monetary policy to stimulate weak aggregate demand through lowering short-term interest rates is no longer available. Most of the guidance is implemented through influencing agents' expectation of current real long-term interest rates (therefore current spending and inflation) by promising to keep future nominal interest rates lower than

they otherwise would be. Alternatively, since future real short-term interest rates are the difference between expected nominal rates and expected inflation rate, central bank can also try to raise inflation expectations by promising a more accommodative policy in the future. I show the difference in guidance through these two alternative channels in the policy simulations below.

### 3.3.1 Macroeconomic impact of the zero lower bound

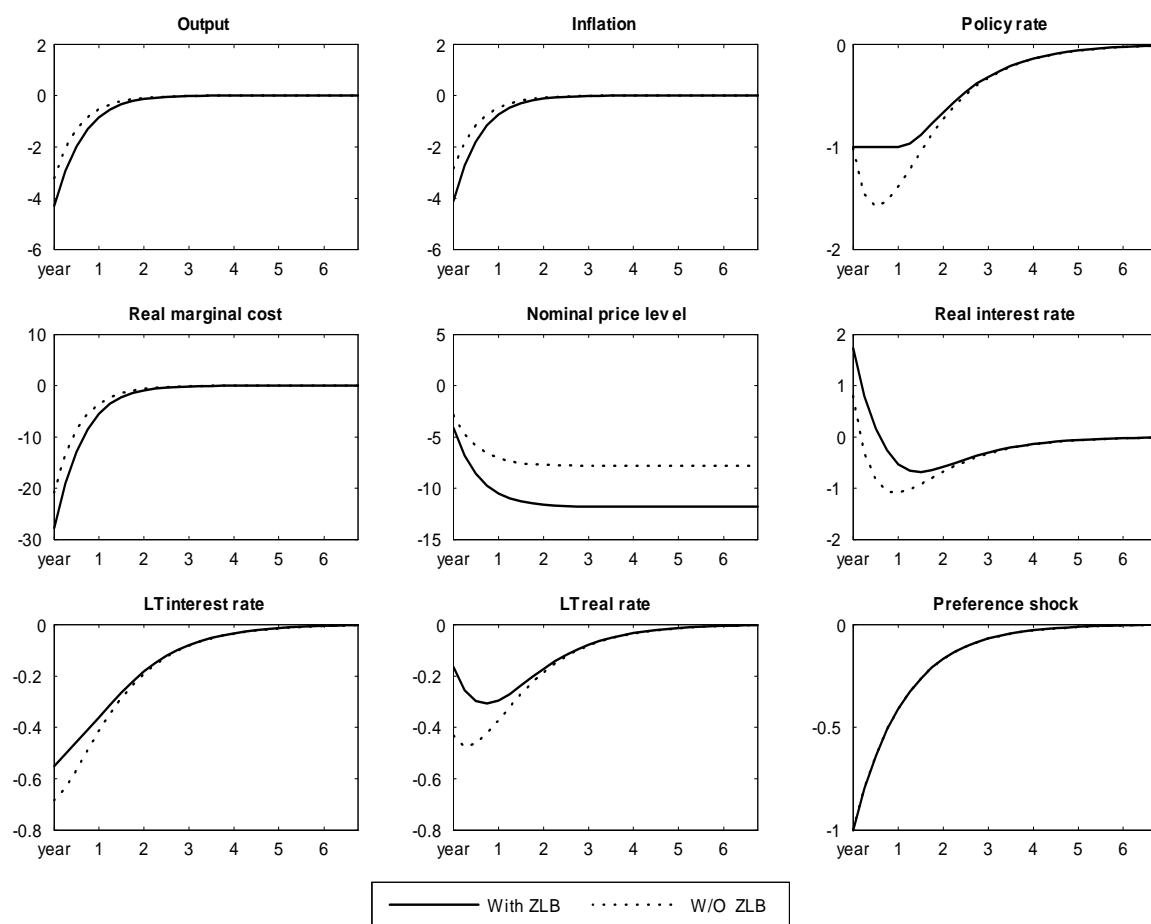
The lower bound on nominal interest rates implies that the effect of any macroeconomic shock will be amplified by the absence of a monetary policy response once the interest rate floor is reached, following a policy rule

$$i_t = \max[\rho_i i_{t-1} + (1 - \rho_i) \{ \rho_\pi \tilde{\pi}_t + \rho_y \tilde{y}_t \} + \sum_{j=0}^T \exp(\varepsilon_{t-j,j}^i), 0]. \quad (3.37)$$

To illustrate this, I begin by examining the impact of a particularly large and persistent shock to household spending with an initial negative impact of 1 per cent decline in aggregate consumption. Figure 3.3 shows the impulse responses of the shock with and without the restriction of the zero lower bound on nominal interest rates, labelled as ZLB and NOLB cases, respectively. For discussion purpose only, I arbitrarily chose -1 per cent as the effective lower bound. When the lower bound is not imposed, output gap widens to -3 percent at its trough and the impact to household demand gradually fades away over the next four years. Inflation declines by nearly 3 per cent following the sharp decline in the real marginal cost. However, because household and firms in the economy react to the innovation gradually (due to consumption habit or price adjustment costs), the decline in output level and inflation persist for a year or two even after the ZLB ceases to bind. Central bank needs to create an accommodative policy environment by cutting the nominal short-term interest rate until the real activity gets restored. The nominal policy rate falls to around -1.6 per cent following the demand shock, reaching a trough after 3 quarters.

The lower bound exacerbates the recession considerably: with monetary policy constrained for five quarters, real interest rates increase further than the NOLB case and the output gap widens even further, to -4.3 per cent at the trough and remains persistently lower. There is also considerably further downward pressure on inflation under the ZLB case: inflation rate reaches -4 per cent at the trough, 1.3 per cent lower than in the NOLB case. Although there is no different long run implication on inflation between the two cases, relative to the NOLB scenario, the peak impact on nominal price of the negative

Figure 3.3: Impact of zero lower bound



demand shock is nearly doubled due to the presence of the lower bound. Consumer prices fall by 12 per cent in the ZLB case, 4 percent lower than in the NOLB case.

Since the agents are forward-looking and fully anticipate the future development of the short-term interest rates, long-term interest rate falls immediately following the negative demand shock. Consistent with agent's expectation of the future path of short-term nominal interest rate, long-term rate falls by 47 percentage points in the NOLB case, compared to the decline of 30 percentage points in the ZLB case.

### 3.3.2 Policy simulations of alternative guidance

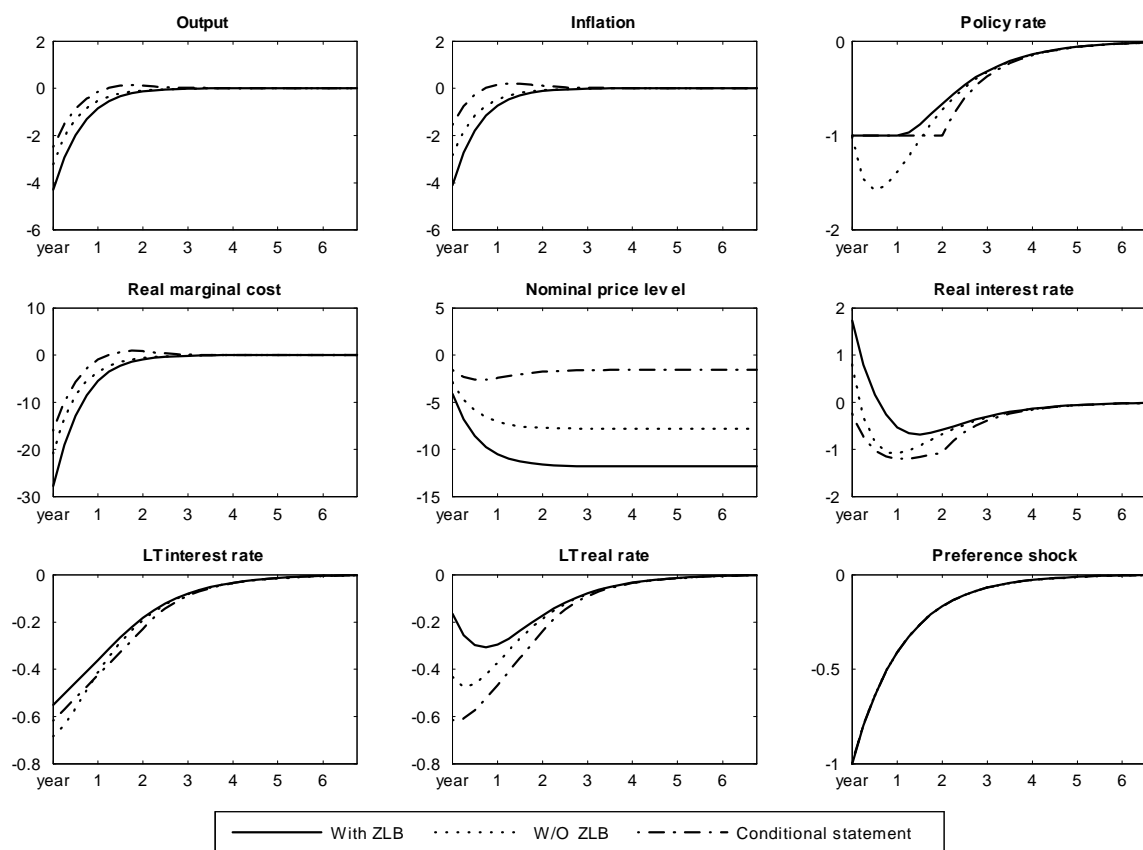
I consider alternative monetary policy rules under commitment in the model and examine the extent to which forward guidance helps to mitigate the negative real impact of the ZLB. In particular, I compare the performance of a state-contingent guidance (using conditional statement) and a price-level guidance. The conditional forward guidance not only works through expectation of long period of low nominal interest rate, but also through expectation of higher than expected future inflation. The latter channel further helps to reduce market expectations of future real interest rate and therefore effectively stabilizing the output gap and inflation in response to negative shocks (Woodford 1999, Eggertsson and Woodford 2003 and Walsh 2009). Price-level targeting, on the other hand, could also offset the impact of ZLB when a central bank announces that it will generate an overshooting of inflation once the lower bound is no longer a constraint. It mainly works through the inflation expectation channel. I present the simulation results of both cases below.

#### Conditional statement in Inflation Targeting

A central bank could provide forward guidance by pledging to keep policy rates near zero until a condition is met. Such condition would commit the central bank to start raising rates if some pre-announced outcome on a measurable threshold is achieved (for example inflation rises above zero, or unemployment rate rises above a target level). The rule can be specified as

$$i_t = \left\{ \begin{array}{l} 0, \text{ if } [\rho_i i_{t-1} + (1 - \rho_i) \{ \rho_\pi \tilde{\pi}_t + \rho_y \tilde{y}_t \} + \sum_{j=0}^T \exp(\varepsilon_{t-j,j}^i) \leq 0] \text{ or } \tilde{\pi}_t < \tilde{\pi}^*, \\ \rho_i i_{t-1} + (1 - \rho_i) \{ \rho_\pi \tilde{\pi}_t + \rho_y \tilde{y}_t \} + \sum_{j=0}^T \exp(\varepsilon_{t-j,j}^i), \text{ otherwise.} \end{array} \right\} \quad (3.38)$$

Figure 3.4: Forward guidance with conditional statement



More precisely, the central bank follows a policy rule in normal times but, once the lower bound on nominal interest rate starts to bind, keeps interest rates at the lower bound as long as the observed value of inflation  $\tilde{\pi}_t$  has not returned to its target  $\tilde{\pi}^*$ .

This strategy was recently used by the Bank of Canada in its statement in April 2009 about future policy when its monetary policy was operating at the effective lower bound for the policy rate. The Bank of Canada states: "conditional on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve the inflation target." I implement this conditional statement in the model from a zero lower bound environment. The impact of the forward guidance is plotted by dashed line in Figure 3.4.

The conditional guidance policy prolongs the period that the nominal short term rate remains at its effective zero lower bound (-1 percent) for an additional 4 quarters. This additional easing leads to considerably higher inflation relative to the ZLB case (inflation falls to -1 percent at its trough) and also generates a period of overshooting in inflation

three quarters after the initial negative demand shock. As a result of the anticipated higher average inflation, real long-term interest rate falls more sharply than the ZLB case, stimulating real activity. In fact, under the conditional guidance, the peak impact on output gap is reduced by nearly a half relative to the ZLB case (from -4.3 percent to -2.5 percent) and the output level in the near-term is even higher than that of the NOLB case. Moreover, the conditional guidance policy also generates a short period of production expansion about a year and a half after the initial shock. Clearly, the implementation of the forward guidance through the communication of the conditionality of the inflation outlook largely offsets the negative real impact from the presence of the zero lower bound. Practical evidence suggests that, with explicit commitment to the conditionality nature of its guidance, the action by the Bank of Canada has led to a significant decline in implied yields on government bonds out to one year by a range of 10- to 20 basis points (Murray 2009).

### **Guidance through State-contingent Price-level targeting**

I now adopt a price-level targeting regime in this alternative experiment to assess the extent such policy could help to mitigate the negative impact associated with the presence of the ZLB. The merit underlying price-level targeting rule as an unconventional policy lies in the long-run predictability of the price level under such regime (Coulombe 1998a, 1998b). The superior performance of price-level targeting over inflation targeting on dealing with the zero lower bound has been analyzed in various literature (see Eggertsson and Woodford 2003, Wolman 2005, Mankiw 2008 and Ambler 2009 for examples). In this rule, inflation is replaced by the deviation of the price level from a target level:

$$i_t = \max[\rho_i i_{t-1} + (1 - \rho_i) \{ \rho_p (p_t - \bar{p}_t) + \rho_y \tilde{y}_t \} + \sum_{j=0}^T \exp(\varepsilon_{t-j,j}^i), 0] \quad (3.39)$$

where  $P$  is the log price level and  $\bar{p}$  is the log target price level. Similar to the analysis of Eggertsson and Woodford (2003), I assume that the long-run inflation target is zero and the target for gap-adjusted price level is constant.

Under the price-level targeting rule, the central bank would pledge to maintain the policy rate at its ZLB as long as price level remains below the desired predetermined path (that is consistent without the presence of the lower bound). Any shortfall in inflation associated with price decline during a period of weak demand will be made up later by price increase, thereby increasing inflation expectation. Once price level reaches back to the deterministic path, it becomes appropriate for the central bank to start raising the

policy rate and follow its conventional policy rule again. This mechanism differs from aiming a target rate of inflation as a forward-looking inflation target accommodates a permanent decline in the price level after a period of one-sided target misses due to a binding ZLB (Cúrdia and Woodford 2009). The effectiveness of the price-level targeting regime relies on two important conditions: i) that the policy is fully credible, and ii) that private sector fully anticipates that any price changes under a price-level targeting regime will be undone. If both conditions are met, even in presence of the ZLB, a central bank can still effectively manage expectations of future inflation since there is less inflation variability under the price-level targeting regime when the central bank targets directly the price level path.

Figure 3.5 reports the macroeconomic performance of the price-level targeting rule following the negative demand shock. The nominal overnight rate is pinned at the lower bound for four quarters, one quarter shorter than in the ZLB scenario. Despite a shorter lower bound period, the price-level targeting policy is very stimulative. The overall inflation profile is the highest among the alternative policy rules considered. Real rates experience a smaller increase and remain lower than in the ZLB case throughout the projection period, helping to dampen the decline in the output gap. The macroeconomic loss is reduced by 6.4 per cent with the price-level targeting rule, compared to 2.4 per cent with the conditional inflation targeting rule and 10.3 per cent with the ZLB case.<sup>10</sup>

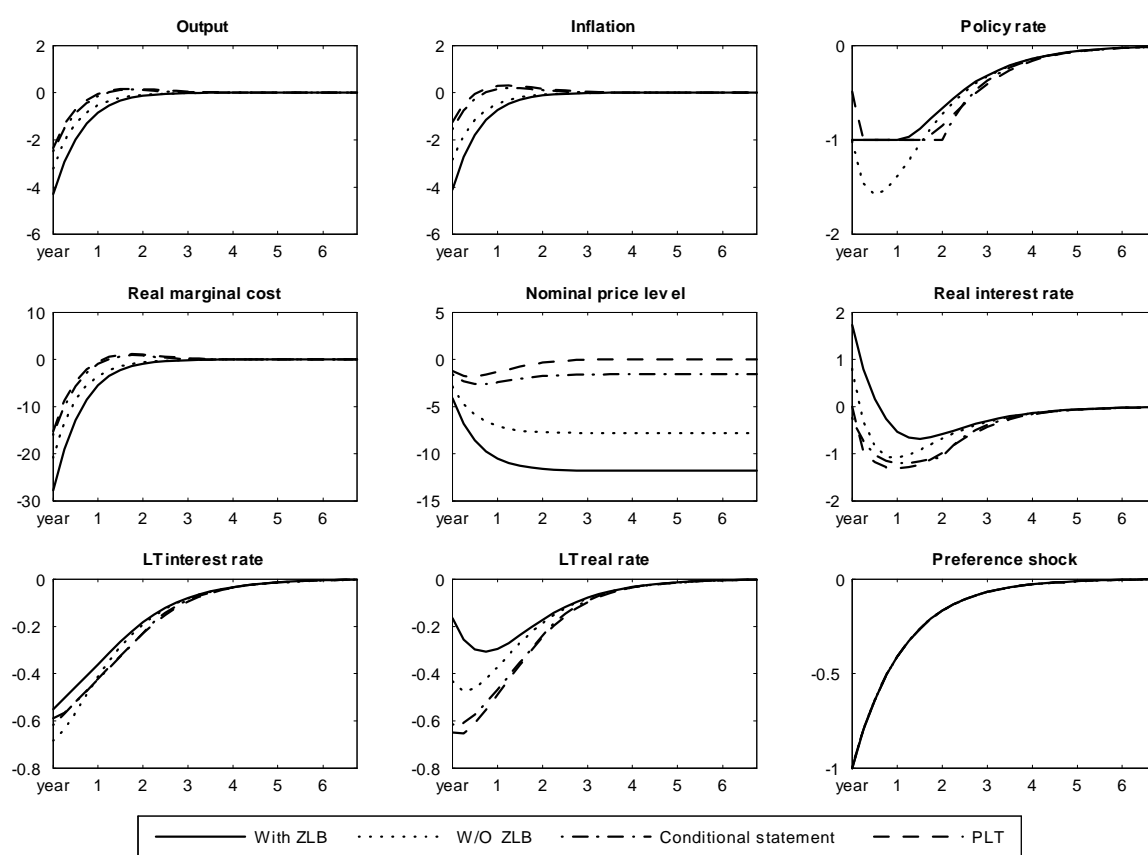
The finding in this chapter is consistent with other work on the superior performance of price-level targeting at dealing with the ZLB. For example, Ambler (2009) finds that the economy is less likely to hit the ZLB under a price-level targeting regime and monetary policy has more leverage at or near the zero bound under the price-level targeting regime. Other studies also show how price-level targeting can mitigate the ZLB constraint on the policy interest rate and help to offset some negative real impact on the economy (including Eggertsson and Woodford 2003, Gaspar, Smets and Vestin 2007, McCallum 2000, Nakov 2008, Svensson 2003 and Wolman 2005). The advantage of price-level targeting as a forward guidance tool comes from its ability to increase market expectation of inflation. Under inflation targeting regime, if inflation rate gap is expected to remain below zero over the zero lower bound horizon, and gradually returns to its target, the

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<sup>10</sup>I use a loss function based on the discounted cumulative sum of squared deviations of the output gap from zero and core inflation from its target level over the first twenty-five years of simulation. I assign equal weights to output and inflation fluctuations following the general form in Woodford (2003):

$$L = \sum_{i=0}^T \beta^i [\lambda_y ygap^2 + \lambda_\pi (\pi_t - \pi^*)^2] : \lambda_y = \lambda_\pi = 1.$$

Figure 3.5: Forward guidance with price-level targeting



average expected inflation remains negative. Under a credible price-level targeting commitment, the market is convinced that if deflation occurs in the short run, the central bank will vigorously reverse it by subsequently generating higher inflation. This leads to higher average expected inflation than that under the inflation-targeting regime.

For price-level targeting to be effective in mitigating the ZLB effect, it requires that the central bank fully commits to the policy and that private agents form inflation expectations in a manner that is fully consistent with the policy. This implies that if private agents expect the central bank to use the alternative price-level targeting rule only as a short-term strategy to deal with the lower bound problem, they may incorporate the expectation that the central bank will also renege on the high inflation target once the economy begins to recover (and the ZLB is no longer binding). Accounting for such future possible reversal in policy, the performance of price-level targeting is discounted as private agents are less likely to expect that future inflation will remain high. This credibility issue is even more relevant when the regime shift from inflation-targeting to price-level targeting occurs during a period when the ZLB is present. In addition, private sector expectations regarding the new policy regime are also likely to be slow to adjust. Failure to reach the price-level target within the communicated (expected) horizon would likely pose further challenges towards the credibility of the new policy and slow down the adjustment of inflation expectations by the private agents. As a result of some of the aforementioned challenges, the application of the price-level targeting as a forward guidance tool is largely limited in practice.

### 3.4 Concluding Remarks

I calibrate a standard version of the closed economy New Keynesian DSGE model using Canadian data. I then use this model to provide a quantitative assessment of the macroeconomic impact of forward guidance (in forms of conditional guidance and price-level targeting) when the policy rate is constrained at its effective lower bound. The simulation results suggest that the conditional statement policy prolongs the zero lower bound duration for an additional 4 quarters and reverses half of the decline in inflation associated with the lower bound and even generates a period of overshooting in inflation three quarters after the initial negative demand shock. As a result of more accommodative policy and higher inflation expectation, GDP increases by 1.8 per cent on peak with a short period of production expansion about a year and a half after the initial shock.

The effect of price-level targeting as an unconventional policy at the zero lower bound is slightly different. The policy shortens the zero lower bound duration by one quarter,

but generates the highest inflation profile among the alternative policy rules considered. Price level returns to pre-shock level and the overall loss is reduced to 6.4 per cent with the price-level targeting rule, compared to 2.4 per cent with conditional guidance and 10.3 per cent with the case of zero lower bound.

A central bank could reduce the economic costs of the lower bound by credibly signaling longer accommodative policy conditional on a numerical target or by announcing that it will generate an overshooting of inflation once the lower bound is no longer a constraint. The impact of forward guidance is largely conditional on the credibility of the monetary policy and expectation of the private agents. For example, when agents become more backward-looking or when the price adjustment becomes more costly (associated with higher uncertainty at the lower bound), the effect of forward guidance would be limited to some extent. Some sensitivity analysis can be conducted to provide further analysis around these crucial assumptions.

Table 3.1: **Model Calibration**

Parameter	Definition	Value
$\beta$	discount factor	0.99
$\sigma$	elasticity of substitution among consumption goods	5
$\varphi$	Frisch elasticity of labor supply parameter	1.5
$\theta$	elasticity of substitution between intermediate goods	6
$\kappa$	price adjustment costs parameter	100
$\theta_\pi$	price indexation	0
$\zeta$	Consumption habit	0
$\rho_i$	Policy rule - smoothing	0.8
$\rho_\pi$	Policy rule - inflation	1.5
$\rho_y$	Policy rule - output gap	0.25

# Chapter 4

## Impact of Quantitative Easing on Household Deleveraging

### 4.1 Introduction

The recent global financial crisis and the resulted fragile global economic recovery has led policy institutions to maintain either very accommodative conventional monetary policy stance (Bank of England, Bank of Canada) or undertake unconventional approaches (such as quantitative easing or forward guidance) to warrant a low interest rate environment (see the recent announcement of QE4 by the Fed). The “low for long” interest rate environment has potentially contributed to the risks associated with financial imbalances as reflected through house price overvaluation and excessive household indebtedness. The concerns about the impact of house price volatility and the resulting changes in household net worth on amplification of the aggregate fluctuations have encouraged more research on household financial frictions (see Bernanke, Gertler and Gilchrist 1999, Christiano et al. 2010, Iacoviello and Neri 2009 for prominent examples).

If the downside risks associated with global headwinds are realized, or domestic demand experiences unexpected large negative shock, further monetary policy accommodation is required to continue generating sustained economic growth. While keeping interest rate low for longer would help to support aggregate output, the conduct of unconventional policies could also further strengthen the financial imbalances related to household indebtedness (through an increasing household debt to income ratio) and intensify the misallocations of resources (e.g., overinvestment of residential structure) that were fueled by a low-for-long interest rate environment. To this end, there is some room of using macroprudential policies (such as household loan to value regulations, property taxes) along with the monetary policies to jointly tackle issues related to stimulating

aggregate economy and contain household indebtedness.

This chapter builds upon a small open economy dynamic stochastic general equilibrium (DSGE) model with real and household financial frictions to analyze the effects of quantitative easing on household deleveraging. The model features heterogeneous households and a financial intermediary. The patient savers can trade in both short- and long-term bond markets and have self-imposed liquidity requirements on their long-term investments (money and long-term bonds are imperfect substitutes).

Impatient borrowers are middle-aged households who only trade in long-term bond markets and do not face liquidity requirements on their long-term investments. They are the borrowers in the economy, buying long-term government bonds and borrowing from the savers in the economy. The existence of borrowers allows for the conduct of quantitative easing policies as it allows long-term interest rate to matter over and above the short-term rates in the aggregate demand (Andres et al. 2004, Dorich et al. 2012). Borrowers purchase housing relying on mortgage loans provided by the intermediary. Borrowing is subject to an external financing premium due to loan default risk, which generates endogenous movement in borrowers' loan to value ratio.

I calibrate the model to match the great ratios and dynamics in the Canadian economy and simulate the model to explore the effects of quantitative easing on asset prices, household balance sheet as well as the implication of ratios related to household financial imbalances. For example, with QE, the risk-free long-term rate declines as a result of lower term premium, providing support to both output and inflation as well as house prices. On impact, the increase in house prices leads to an improvement in the household balance sheets through an increasing of net worth. Higher household net worth, in turn, leads to a declining loan to collateral value ratio. Since the borrower's default rate is increasing in leverage, lower loan to value ratio implies a lower external financing premium faced by borrowers.

Overall, there are two effects of QE on aggregate output originated from the model. First, QE leads to a decline in term premium, which increases current consumption relative to future consumption. On the other hand, it creates more favorable financing conditions for borrowers through an increase in the value of collateral, therefore encourages further accumulation of debt at cheaper rates and leads to an immediate higher household debt to income ratio. The effect of QE beyond the initial impact would depend on the elasticity of loan demand to the policy. If the increase in loan is greater than the increase in the overall collateral value, QE would lead to an ultimately deeper leverage and therefore higher default rate in the economy. In the consideration of future withdrawal of any stimulus provided from QE, this would pose greater challenges as it

implies much more intensive household deleveraging process.

From a financial stability perspective, there is some room for using regulation policies in reducing household leveraging ratio, or imposing taxes on housing consumption along with the QE policy. This would help to preserve the stimulating effect from the prolonged low interest rate environment while preventing situations with deep household leverage and higher default rate down the road.

The chapter is organized as follows. I first present the key features of the model, with special focus on financial frictions on household side. I then discuss the calibration, followed by some simulated experiments of the conduct of quantitative easing in the model. I provide some sensitivity analysis around the key parameter calibrations as a robustness check of the results. Section 5 concludes.

## 4.2 The Model

The following section develops a small open economy DSGE model with real and household financial frictions to analyze the effects of quantitative easing on household deleveraging. This model is built upon the small open economy model of Dorich, Mendes and Zhang (2012) with additional features of heterogenous households and a financial intermediary.

### 4.2.1 Households

There are two types of households in the economy and their discount factor differ. The discount factor for borrowers is lower than that of savers  $0 < \beta^b < \beta^s < 1$ , making them more impatient. Borrowers are the home buyers in the economy who rely on mortgage loans to finance their purchases of housing. One can think of the borrowers as young homeowners, young families with children, the better-educated and those with high household incomes. This assumption can be justified as In Canada, over 60% of household debt was held by those under 45 years of age, and nearly one-half was held by couples with children.<sup>11</sup> In terms of assets composition, I assume that borrowers only trade in long-term bond markets and do not face liquidity requirements on their long-term investments. This assumption follows the preferred-habitat view, proposed by Culberston (1957) and Modigliani and Sutch (1966) as well as Greenwood and Vayanos (2009). The existence of borrowers whose preference to only long-term assets allows for

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<sup>11</sup>For more details on household debt distribution in Canada, see Raj K. Chawla and Sharanjit Uppal "Household debt in Canada", Component of Statistics Canada Catalogue no. 75-001-X, Perspectives on Labour and Income, March 23, 2012.

the conduct of quantitative easing policies as it allows long-term interest rate to matter over and above the short-term rates in the aggregate demand.

On the other hand, savers in the economy can be thought of as middle-aged household with low expected income growth and higher savings. We assume that savers don't enter the housing market. Moreover, savers can trade in both short- and long-term bond markets and they have self-imposed liquidity requirements on their long-term investments (liquid and illiquid assets are imperfect substitutes). I start with the optimization problem of savers, followed by the borrowers' problem and the specification of financial frictions.

### Savers

Since savers don't enter the housing market, they maximize utility over consumption, money and bonds holdings following similar optimization problem as in Dorich, Mendes and Zhang (2012). Moreover, savers can trade in both short- and long-term bond markets and they have self-imposed liquidity requirements on their long-term investments (liquid and illiquid assets are imperfect substitutes). A typical saver seeks to maximize the following expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} (\beta^s)^t \left[ a_t \left\{ U \left( \frac{C_t^s}{\{C_{t-1}^s\}^{h_c}} \right) + V \left( \frac{M_t^s}{\xi_t P_t} \right) - \int_0^1 L(N_{jt}^s) dj \right\} - G(\cdot) - F(\cdot) - J(\cdot) \right] \quad (4.1)$$

where  $\beta^b \in (0, 1)$  is the subjective discount factor,  $a_t$  is a preference shock,  $C_t^s$  is the household's consumption,  $M_t^s/P_t$  is the household's end-of-period real money balances,  $P_t$  is the consumer price index (CPI), and  $N_{jt}^s$  is the quantity of type  $j$  labor supply (measured in hours). The period indirect utility is determined by the preference shock  $a_t$  and the following functions:  $U, V, L$  and  $G$ . In what follows, I specify these functions as:

$$U(\cdot) = \frac{1}{1 - \sigma_c} \left( \frac{C_t^s}{\{C_{t-1}^s\}^{h_c}} \right)^{1 - \sigma_c}, \quad V(\cdot) = \frac{1}{1 - \delta} \left( \frac{M_t^s}{\xi_t P_t} \right)^{1 - \delta}, \quad H(\cdot) = \frac{(N_{tj}^s)^{1 + \varphi}}{1 + \varphi},$$

$$G(\cdot) = \frac{d}{2} \left\{ \exp \left[ c \left( \frac{M_t^s/P_t}{M_{t-1}^s/P_{t-1}} - 1 \right) \right] + \exp \left[ -c \left( \frac{M_t^s/P_t}{M_{t-1}^s/P_{t-1}} - 1 \right) \right] - 2 \right\}.$$

We impose separability among consumption, real money balances and hours. The utility function displays internal habit formation and its quantitative importance is de-

noted by the size of  $h_c \in [0, 1]$ . The parameter  $\sigma_c > 0$  determines the degree of risk aversion and  $\varphi \geq 0$  represents the inverse of the Frisch labor supply elasticity. The household face portfolio adjustment cost, which are given by the function  $G(\cdot)$ . Such cost can represent the direct cost of financial portfolio adjustment through payments to a financial intermediary, or capture the total cost of making an optimal choice about the asset holdings (Bonaparte et al. 2012). The parameters  $d$  and  $c$  in this function are strictly positive.

To introduce the possibility of conducting QE in the model, I make two important assumptions about the features of long-term securities. First, long-term bonds are modeled as zero-coupon bonds: there are no payments received by the households during the period they hold the bond. Second, there is no secondary market for long-term bonds. The first assumption is in line with the treatment of long-term bonds in macroeconomic models.<sup>12</sup> The second assumption can be justified by the fact that a large fraction of the non-bank private sector holds long-term bonds with the intention of keeping them to maturity.

The absence of a secondary market for long-term securities introduces a "loss" of liquidity for households when investing in long-term bonds market, relative to the same investment in short-term bonds. To mitigate this loss of liquidity, I assume that these households have self-imposed liquidity requirements on their relative asset holdings (liquid asset versus illiquid asset). The utility costs functions of holding domestic and foreign long-term bonds are given by the functions  $F(\cdot)$  and  $J(\cdot)$ . Both of them are specified in terms of

$$F(\cdot) = -\frac{v}{2} \left[ \frac{M_t^s}{B_t^{Ls}} \Phi - 1 \right]^2, \quad J(\cdot) = -\frac{\tilde{v}}{2} \left[ \frac{M_t^s}{S_t B_t^{Ls*}} \tilde{\Phi} - 1 \right]^2$$

where  $B_t^{Lb}$  and  $B_t^{Lb*}$  denote holdings of the domestic and foreign currency denominated long-term bonds in period  $t$  respectively. These bonds are redeemed  $L$  periods after they are bought.  $S_t$  is the nominal exchange rate expressed in units of domestic currency needed to buy one unit of foreign currency. The parameters  $v$  and  $\tilde{v}$  measure the degree of concern for liquidity of the households. The parameters  $\Phi$  and  $\tilde{\Phi}$  are the inverses of the steady-state money to long-term bond ratios (domestic and foreign currency denominated long-term bonds respectively) and can be considered as the targets of savers' relative assets holdings. The functions are quadratic to reflect the assumption of symmetric portfolio adjustment costs.

The maximization of the expected utility is subject to the sequence of budget con-

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<sup>12</sup>See Svensson (2000).

straints of the form:

$$\begin{aligned}
& M_t^s + \frac{B_t^s}{(1+R_t)} + \frac{S_t B_t^{s*}}{(1+R_t^*)(1+\kappa_t)} + \frac{(1+\phi_t)B_t^{Ls}}{(1+R_t^L)^L} + \frac{(1+\phi_t^*)S_t B_t^{Ls*}}{(1+R_t^{*L})^L E_t \left\{ \prod_{j=0}^{L-1} (1+\kappa_{t+j}) \right\}} \\
& = M_{t-1}^{sl} + B_{t-1}^s + S_t B_{t-1}^{s*} + B_{t-L}^{Ls} + S_t B_{t-L}^{Ls*} + \int_0^1 W_{jt} N_{jt}^s dj - P_t C_t^s + P_t T_t^s + \Pi_t^s
\end{aligned}$$

where  $B_t^u$  and  $B_t^{u*}$  denote holdings of the domestic and foreign currency denominated short-term bonds in period  $t$  respectively. These bonds are redeemed 1 period after they are bought.  $R_t$  and  $R_t^*$  are the domestic and foreign currency short-term interest rates respectively. The term  $\kappa_t$  is interpretable as a debt elastic interest rate premium and is given by:

$$\kappa_t = -\varsigma \left[ \exp \left( \frac{S_t NFA_t}{P_{H,t} Y_t} - \overline{nfa} \right) - 1 \right] + \varepsilon_t^\kappa \quad (4.2)$$

where  $\varsigma > 0$  measures the sensitivity of the country-risk premium to changes in the aggregate net foreign asset position  $NFA_t$ . The parameter  $\overline{nfa}$  is the steady state value of  $NFA_t$  expressed as a fraction of steady state nominal GDP.  $\varepsilon_t^\kappa$  is a risk premium shock. The adopted functional form for  $\kappa_t$  ensures stationarity of the path of the net foreign asset to GDP ratio about its steady state value.

The terms  $\phi_t$  and  $\phi_t^*$  represent stochastic transaction costs in the domestic and foreign long-term bond markets respectively. These costs are assumed to have zero mean and represent either a pure loss or a benefit to the households in aggregate.  $R_t^L$  and  $R_t^{*L}$  are the domestic and foreign currency long-term interest rates respectively.  $W_{jt}$  is the nominal wage paid for one hour of type  $j$  labour.  $T_t^s$  denotes lump-sum real transfer (or taxes if negative) received from the government.  $\Pi_t^u$  are the dividends paid to households from ownership of firms. The following first order conditions summarize savers' choices:

Consumption  $C_t^s$ :

$$\lambda_t^s = a_t \left( \frac{C_t^s}{\{C_{t-1}^s\}^{h_c}} \right)^{1-\sigma_c} \frac{1}{C_t^s} - \beta a_{t+1} h_c \left( \frac{C_{t+1}^s}{\{C_t^s\}^{h_c}} \right)^{1-\sigma_c} \frac{1}{C_t^s} \quad (4.3)$$

LT bonds  $B_t^{Ls}$ :

$$\frac{(1+\phi_t)}{(1+R_t^L)^L} \left( \frac{\lambda_t^s}{P_t} \right) - v\Phi \frac{M_t^s}{(B_t^{Ls})^2} \left[ \frac{M_t^s}{B_t^{Ls}} \Phi - 1 \right] = \beta^L E_t \left( \frac{\lambda_{t+L}^s}{P_{t+L}} \right) \quad (4.4)$$

ST bond  $B_t^s$ :

$$\frac{\lambda_t^s}{P_t} = \beta(1 + R_t)E_t \left( \frac{\lambda_{t+1}^s}{P_{t+1}} \right) \quad (4.5)$$

Money demand  $M_t^s$ :

$$\begin{aligned} & a_t V_{t, M_t^s} - \{G_{t, M_t^s} + \beta E_t (G_{t+1, M_t^s})\} - \frac{v\Phi}{B_t^{L^s}} \left[ \frac{M_t^s}{B_t^{L^s}} \Phi - 1 \right] - \frac{\tilde{v}\tilde{\Phi}}{S_t B_t^{L^{s*}}} \left[ \frac{M_t^s}{S_t B_t^{L^{s*}}} \tilde{\Phi} - 1 \right] \\ &= \frac{\lambda_t^s}{P_t} - \beta E_t \left( \frac{\lambda_{t+1}^s}{P_{t+1}} \right) \end{aligned} \quad (4.6)$$

Foreign LT bond  $B_t^{L^{s*}}$ :

$$\frac{(1 + \phi_t^*)S_t}{(1 + R_t^{*L})^L E_t \left\{ \prod_{j=0}^{L-1} (1 + \kappa_{t+j}) \right\}} \left( \frac{\lambda_t^s}{P_t} \right) - \tilde{v}\tilde{\Phi} \frac{M_t^s}{S_t (B_t^{L^{s*}})^2} \left[ \frac{M_t^s}{S_t B_t^{L^{s*}}} \tilde{\Phi} - 1 \right] \quad (4.7)$$

$$= \beta^L E_t \left( \frac{S_{t+L} \lambda_{t+L}^s}{P_{t+L}} \right) \quad (4.8)$$

Foreign ST bond  $B_t^{s*}$ :

$$\frac{S_t \lambda_t^s}{P_t} = \beta(1 + R_t^*)(1 + \kappa_t) E_t \left( \frac{S_{t+1} \lambda_{t+1}^s}{P_{t+1}} \right) \quad (4.9)$$

where  $U_{t, C_t} = \frac{\partial U_t}{\partial C_t}$ ,  $U_{t, C_{t+1}} = \frac{\partial U_{t+1}}{\partial C_{t+1}}$ ,  $V_{t, M_t} = \frac{\partial V_t}{\partial M_t}$ ,  $G_{t, M_t} = \frac{\partial G_t}{\partial M_t}$  and  $G_{t, M_{t+1}} = \frac{\partial G_{t+1}}{\partial M_{t+1}}$ .

## Borrowers

Borrowers are the home buyers in the economy who rely on mortgage loans to finance their purchases of housing. They only trade in long-term bonds market and do not face liquidity requirements on their long-term investments. The existence of borrowers whose preference to only long-term assets allows for the conduct of quantitative easing policy as it allows long-term interest rate to matter over and above the short-term rates in the aggregate demand.

The preferences of infinitely-lived borrowers are given by:

$$E_0 \sum_{t=0}^{\infty} (\beta^b)^t \left[ a_t \left\{ U \left( \frac{C_t^b}{\{C_{t-1}^b\}^{hc}} \right) + H \left( \frac{RS_{t+1}^b}{\{RS_t^b\}^{hrs}} \right) + V \left( \frac{M_t^b}{\xi_t P_t} \right) - \int_0^1 L(N_{jt}^b) dj \right\} - G(\cdot) \right] \quad (4.10)$$

where  $E_0[\cdot]$  denotes the expectation operator conditional on the time 0 information.  $C_t^b$  is the borrower's consumption of non-durable goods,  $RS_t^b$  is the level of residential structures at the beginning of period  $t$  and it's assumed to be equal to the housing service demand by borrowers.<sup>13</sup>  $M_t^b/P_t$  is the borrower's end-of-period real money balances and  $\xi_t$  is a shock to the borrower's demand for real money balances.  $N_{jt}^b$  is the fraction of the time that type  $j$  labor work at  $t$ . In each period the borrower derives income from supplying the labor services at a wage rate  $W_{jt}$ .

I impose separability among consumption, housing, real money balances and hours of work by specifying these functions in the following forms

$$U(\cdot) = \frac{1}{1 - \sigma_c} \left( \frac{C_t^b}{\{C_{t-1}^b\}^{h_c}} \right)^{1 - \sigma_c}, H(\cdot) = \frac{1}{1 - \sigma_{rs}} \left( \frac{RS_{t+1}^b}{\{RS_t^u\}^{h_{rs}}} \right)^{1 - \sigma_{rs}},$$

$$V(\cdot) = \frac{1}{1 - \delta} \left( \frac{M_t^b}{\xi_t P_t} \right)^{1 - \delta}, L(\cdot) = \frac{(N_{tj}^b)^{1 + \varphi}}{1 + \varphi},$$

$$G(\cdot) = \frac{d}{2} \left\{ \exp \left[ c \left( \frac{M_t^b/P_t}{M_{t-1}^b/P_{t-1}} - 1 \right) \right] + \exp \left[ -c \left( \frac{M_t^b/P_t}{M_{t-1}^b/P_{t-1}} - 1 \right) \right] - 2 \right\},$$

The internal habit persistence of consumption and housing service are denoted by parameters  $h_c$  and  $h_{rs}$ , respectively ( $h_c, h_{rs} \in [0, 1]$ ).  $\sigma_c$  and  $\sigma_{rs}$  are intertemporal elasticities of substitution and  $\varphi$  is the inverse wage elasticity of labour supply. The borrowers face costs  $G(\cdot)$  when adjusting their financial portfolios. The function is specified following Nelson (2002) and the parameters  $d$  and  $c$  in this function are strictly positive. The period indirect utility is determined by the preference shock  $a_t$ .

Housing consumption is financed by taking 1-period mortgage loans  $L_{t+1}$  from financial intermediaries if needed (loan will be paid back in period  $t+1$ ). This is a simplification of the mortgage contract as commonly used in DSGE models.<sup>14</sup> Each period, the net worth of borrower (excluding income) before debt repayment is:

$$NW_t^b = P_t^h (1 - \delta_h) RS_t^b \tag{4.11}$$

<sup>13</sup>This assumption simplifies the housing producer's problem and similar utility function can be found in Forlati and Lambertini (2011).

<sup>14</sup>Recent work with modeling multi-period contracts include Calza, Monacelli and Stracca (2011) and Forlati and Lambertini (2012). Calza, Monacelli and Stracca (2011) study how monetary policy transmission mechanism is affected by the structure of housing finance. They build a DSGE model with one- and two-period contracts, with the latter features a fixed interest rate and equal repayments in the first and second period. They show that consumption falls more with 1-period contracts following a monetary policy tightening. Similar, Forlati and Lambertini (2012) also defines a two-period contact with a focus on the role of different amortization schedules. Using a two-sector DSGE model, they allow for the fraction of principal to be repaid in the first period to vary. They show that as this fraction falls, leverage increases and the impact of a housing risk shock on consumption and output is amplified.

where  $NW_t^b$  is the net worth of borrowers,  $P_t^h$  is house price and  $\delta_h$  is the depreciation rate of housing stock. Depending on the net worth position, borrowers can repay the loans or default the debt. The asymmetric information between the borrowers and savers makes the external finance costly.

**Financial friction in housing consumption of borrowers** To introduce financial friction through household borrowing and housing consumption, following the costly state verification literature, I assume that an idiosyncratic shock  $\xi_{t+1}^{df}$  to the value of borrower's housing stock  $P_t^h(1 - \delta_h)RS_t^b$ . At the beginning of the period both borrowers and intermediary know the distribution of  $\xi_t^{df}$  and the default risk but the borrowers learn their types in the end of the period without incurring any cost. As a result, lending must be intermediated by banks that seizes a collateral,  $CLT_t^b$ , defined as an enforcement technology to the loans following:

$$CLT_t^b = \xi_t^{df} NW_t^b = \xi_t^{df} P_t^h (1 - \delta_h) RS_t^b \quad (4.12)$$

where  $\xi_{t+1}^{df}$  is an idiosyncratic shock that follows a log-normal cumulative distribution function as in Bernanke, Gertler and Gilchrist (1999) and Medina (2004). More specifically,  $\xi_{t+1}^{df}$  is a unit mean lognormal random variable distributed independently over time and across households. The standard deviation of  $\ln(\xi_{t+1}^{df})$ ,  $\sigma^{df}$  is itself a stochastic process. The idiosyncratic shock  $\xi_{t+1}^{df}$  is privately observed by the borrower and has a unit mean  $E(\xi_t^{df}) = \int_0^\infty \xi_t^{df} dF(\xi_t^{df}) = 1$  and variance  $var(\xi_t^{df}) = (\sigma^{df})^2$ . This implies the following:

$$\ln(\xi_t^{df}) \sim N\left(-\frac{1}{2}(\sigma^{df})^2, (\sigma^{df})^2\right).$$

The density function of  $\xi_t^{df}$  can be given by:

$$f(\xi_t^{df}) = \frac{1}{\sqrt{2\pi}\xi_t^{df}\sigma^{df}} \exp\left[-\frac{1}{2}\left(\frac{\ln(\xi_t^{df}) + \frac{1}{2}(\sigma_{df,t}^{df})^2}{\sigma_{df,t}^{df}}\right)^2\right]. \quad (4.13)$$

Financial intermediaries in the model allocate household savings by financing borrowers' housing consumption. By funding a large pool of borrowers, the intermediaries diversify project specific risk and guarantee a safe return to the savers. Here I assume there is no aggregate risk during the life of the mortgage contract. After idiosyncratic shocks are realized, borrowers decide whether to repay the mortgage loan or default. The financial contact in the model determines a default threshold  $\bar{\xi}_{t+1}^{df}$  such that:

· if  $\xi_{t+1}^{df} \leq \bar{\xi}_{t+1}^{df}$ , or  $\xi_{t+1}^{df} \in [0, \bar{\xi}_{t+1}^{df})$ , the realization of  $\xi_{t+1}^{df}$  is too low (i.e., the borrower experiences sufficiently bad shock), therefore defaults mortgage and loses the full collateral value of their housing stock. For the lender, it pays a cost of  $\tau^{df}$  units of capital goods to financial intermediary to monitor the mortgage loans and in case the borrower defaults, the bank can seize the collateral. The expected value of the idiosyncratic shock conditional on the borrower defaults is  $G(\bar{\xi}_{t+1}^{df})$ , where:

$$G(\bar{\xi}_{t+1}^{df}) = \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} dF = \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df}. \quad (4.14)$$

The presence of monitoring costs  $\tau^{df} G(\bar{\xi}_{t+1}^{df})$  induce borrowers to reveal the value of the idiosyncratic shock.

· if  $\xi_{t+1}^{df} > \bar{\xi}_{t+1}^{df}$ , or  $\xi_{t+1}^{df} \in [\bar{\xi}_{t+1}^{df}, \infty]$ , the borrower repays the loan rather than loses the collateral. The expected value of the idiosyncratic shock conditional on the borrower repays the loan is  $(1 - F(\bar{\xi}_{t+1}^{df}))\bar{\xi}_{t+1}^{df}$ . Let  $R_t^l$  denotes the lending rate of risky debt charged by banks at time  $t$  and  $L_{t+1}$  denotes the mortgage borrowers take at time  $t$  and will be repaid in period  $t + 1$ . The threshold value of the idiosyncratic shock for which the borrower is willing to repay the loan at the contract mortgage rate satisfies the following relationship:

$$\bar{\xi}_{t+1}^{df} NW_{t+1}^b = (1 + R_t^l) L_{t+1} \quad (4.15)$$

Over the full distribution, the expected value of the housing value that goes to lenders, gross of monitoring cost, can be defined as the repayment share  $RP(\bar{\xi}_{t+1}^{df})$ :

$$\begin{aligned} RP(\bar{\xi}_{t+1}^{df}) &= \left[ \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} dF + (1 - F(\bar{\xi}_{t+1}^{df}))\bar{\xi}_{t+1}^{df} \right] \\ &= \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} + \bar{\xi}_{t+1}^{df} \int_{\bar{\xi}_{t+1}^{df}}^{\infty} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df}. \end{aligned} \quad (4.16)$$

With some assumption of insurance coverage of the loans, the borrower repays the lender with the total amount of  $RP(\bar{\xi}_{t+1}^{df})NW_t^b$ . For lenders to participate in the loan, it must require a positive profit such that the repayment on total loans net of monitoring costs  $\tau^{df} G(\bar{\xi}_{t+1}^{df})$  and depreciation of housing stocks is greater or equal to the opportunity cost of the lending. At time  $t$  the lenders make total loan  $L_{t+1}$  to borrowers at a non-state contingent rate of return  $R_t$ . The participation constraint of the lenders can be given

by:

$$\left[ RP(\bar{\xi}_{t+1}^{df}) - \tau^{df} G(\bar{\xi}_{t+1}^{df}) \right] NW_{t+1}^b = (1 + R_t) L_{t+1} \quad (4.17)$$

or

$$\begin{aligned} & \left[ \left( (1 - F(\bar{\xi}_{t+1}^{df})) \bar{\xi}_{t+1}^{df} + \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \right) - \tau^{df} \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \right] NW_{t+1}^b \\ &= (1 + R_t) L_{t+1} \\ & \left[ \left( (1 - F(\bar{\xi}_{t+1}^{df})) \bar{\xi}_{t+1}^{df} \right) + (1 - \tau^{df}) \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \right] NW_{t+1}^b = (1 + R_t) L_{t+1} \\ & \left[ \bar{\xi}_{t+1}^{df} \int_{\bar{\xi}_{t+1}^{df}}^{\infty} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} + (1 - \tau^{df}) \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \right] NW_{t+1}^b = (1 + R_t) L_{t+1} \quad (4.18) \end{aligned}$$

Given the relationship between the threshold idiosyncratic shock and the mortgage loan rate:

$$\bar{\xi}_{t+1}^{df} NW_{t+1}^b = (1 + R_t^l) L_{t+1}$$

Rewrite the participation constraint as:

$$\begin{aligned} & \int_{\bar{\xi}_{t+1}^{df}}^{\infty} (1 + R_{t+1}^l) L_{t+1} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} + (1 - \tau^{df})(1 - \delta_h) \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} P_{t+1}^h R S_{t+1}^b f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \\ &= (1 + R_t) L_{t+1} \end{aligned}$$

In equilibrium, I obtain the lenders' participation constraint as:

$$LTV(\bar{\xi}_{t+1}^{df}) = \frac{(1 + R_t) L_{t+1}}{NW_{t+1}^b} \quad (4.19)$$

where  $LTV(\bar{\xi}_{t+1}^{df})$  is the loan to collateral value which measures the degree of household leverage, follows:

$$\begin{aligned} LTV(\bar{\xi}_{t+1}^{df}) &= \left[ \left( \bar{\xi}_{t+1}^{df} \int_{\bar{\xi}_{t+1}^{df}}^{\infty} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} + \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \right) \right. \\ & \quad \left. - \tau^{df} \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \right] \\ &= RP(\bar{\xi}_{t+1}^{df}) - \tau^{df} G(\bar{\xi}_{t+1}^{df}) \quad (4.20) \end{aligned}$$

**Optimization problems of borrowers** The maximization of borrower's expected utility is subject to two constraints: i) the lender's participation constraint

$$\left[ RP(\bar{\xi}_{t+1}^{df}) - \tau^{df} G(\bar{\xi}_{t+1}^{df}) \right] NW_{t+1}^b = (1 + R_t^r) L_{t+1} \quad (4.21)$$

and ii) the sequence of budget constraints:

$$\begin{aligned} & P_t C_t^b + P_t^h RS_{t+1}^b + RP(\bar{\xi}_t^{df}) NW_t^b + M_t^b + \frac{(1 + \phi_t) B_t^{Lb}}{(1 + R_t^L)^L} + \\ & = NW_t^b + L_{t+1}^b + M_{t-1}^b + B_{t-L}^{Lb} + \int_0^1 W_{jt} N_{jt}^b dj + P_t T_t^b \end{aligned} \quad (4.22)$$

where  $RP(\bar{\xi}_{t+1}^{df}) NW_t^b$  is the total repayment of loans,  $NW_t^b$  is the net worth,  $L_{t+1}^b$  is the mortgage loans.  $B_t^{Lb}$  denotes holdings of the domestic currency denominated long-term bonds in period  $t$ .  $R_t^L$  is the domestic long-term interest rates respectively. For simplicity, I assume that borrowers do not trade in foreign long-term bonds.  $W_{jt}$  is the nominal wage paid for one hour of type  $j$  labor.  $T_t^b$  denotes lump-sum real transfer (or taxes if negative) received from the government.<sup>15</sup> I exclude the dividends from ownership of firms to borrowers by assuming that such return is categorized as a short-term asset.

Rewriting the budget constrain as:

$$\begin{aligned} & P_t C_t^b + P_t^h RS_{t+1}^b + (1 + R_t) L_t^b + M_t^b + \frac{(1 + \phi_t) B_t^{Lb}}{(1 + R_t^L)^L} \\ & = (1 - \delta_h) \left[ 1 - \tau^{df} G(\bar{\xi}_t^{df}) \right] P_t^h RS_t^b + L_{t+1}^b + M_{t-1}^{bl} + \\ & + B_{t-L}^{Lb} + \int_0^1 W_{jt} N_{jt}^b dj + P_t T_t^b \end{aligned} \quad (4.23)$$

The borrowers maximize their utility subject to the budget constraint and the participation constrain:

$$(1 + R_{t+1}) L_{t+1}^b = \left[ RP(\bar{\xi}_{t+1}^{df}) - \tau^{df} G(\bar{\xi}_{t+1}^{df}) \right] (1 - \delta_h) P_{t+1}^h RS_{t+1}^b \quad (4.24)$$

The optimal choices of the borrowers can be summarized by the following first order conditions:<sup>16</sup>

<sup>15</sup>I allow these transfers to adjust endogenously and it may differ from those received by savers.

<sup>16</sup>See Appendix 2 for a detailed derivation of borrowers' first order conditions.

Consumption  $C_t^b$ :

$$\lambda_t^b = a_t \left( \frac{C_t^b}{\{C_{t-1}^b\}^{h_c}} \right)^{1-\sigma_c} \frac{1}{C_t^b} - \beta a_{t+1} h_c \left( \frac{C_{t+1}^b}{\{C_t^b\}^{h_c}} \right)^{1-\sigma_c} \frac{1}{C_t^b} \quad (4.25)$$

Default threshold  $\bar{\xi}_{t+1}^{df}$ :

$$\lambda_{t+1}^{pc} = \lambda_{t+1}^b \beta^{\tau^{df}} \frac{G'(\bar{\xi}_{t+1}^{df})}{\left[ RP'(\bar{\xi}_{t+1}^{df}) - \tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right]} = \lambda_{t+1}^b \beta^{\tau^{df}} prem_{t+1} \quad (4.26)$$

Loan  $L_{t+1}^b$ :

$$\frac{\lambda_t^b}{P_t} = (1 + R_t) E_t \left[ \beta^b \frac{\lambda_{t+1}^b}{P_{t+1}} + \frac{\lambda_{t+1}^{pc}}{P_{t+1}} \right] \quad (4.27)$$

Housing stock  $RS_{t+1}^b$ :

$$\begin{aligned} \frac{\lambda_t^b}{P_t} P_t^h &= \left( \frac{RS_{t+1}^b}{\{RS_t^b\}^{h_{rs}}} \right)^{1-\sigma_h} \frac{1}{RS_{t+1}^b} - \beta^b h_{rs} \left( \frac{RS_{t+2}^b}{\{RS_{t+1}^b\}^{h_{rs}}} \right)^{1-\sigma_h} \frac{1}{RS_{t+1}^b} \\ &+ \beta^b E_t (1 - \delta_h) P_{t+1}^h \left[ \frac{\lambda_{t+1}^b}{P_{t+1}} \left[ 1 - \tau^{df} G'(\bar{\xi}_t^{df}) \right] + \frac{\lambda_{t+1}^{pc}}{P_{t+1}} \left[ RP(\bar{\xi}_t^{df}) - \tau^{df} G(\bar{\xi}_t^{df}) \right] \right] \end{aligned} \quad (4.28)$$

LT bonds  $B_t^{Lb}$ :

$$\frac{(1 + \phi_t)}{(1 + R_t^L)^L} \left( \frac{\lambda_t^b}{P_t} \right) - v\Phi \frac{M_t^b}{(B_t^{Lb})^2} \left[ \frac{M_t^b}{B_t^{Lb}} \Phi - 1 \right] = \beta^L E_t \left( \frac{\lambda_{t+L}^b}{P_{t+L}} \right) \quad (4.29)$$

Money demand  $M_t^b$ :

$$\begin{aligned} &a_t V_{t, M_t^b} - \left\{ G_{t, M_t^b} + \beta E_t \left( G_{t+1, M_t^b} \right) \right\} - \frac{v\Phi}{B_t^{Lb}} \left[ \frac{M_t^b}{B_t^{Lb}} \Phi - 1 \right] - \frac{\tilde{v}\tilde{\Phi}}{S_t B_t^{Lb*}} \left[ \frac{M_t^b}{S_t B_t^{Lb*}} \tilde{\Phi} - 1 \right] \\ &= \frac{\lambda_t^b}{P_t} - \beta E_t \left( \frac{\lambda_{t+1}^b}{P_{t+1}} \right) \end{aligned} \quad (4.30)$$

where  $U_{t, C_t} = \frac{\partial U_t}{\partial C_t}$ ,  $U_{t, C_{t+1}} = \frac{\partial U_{t+1}}{\partial C_t}$ ,  $V_{t, M_t} = \frac{\partial V_t}{\partial M_t}$ ,  $G_{t, M_t} = \frac{\partial G_t}{\partial M_t}$  and  $G_{t, M_{t+1}} = \frac{\partial G_{t+1}}{\partial M_t}$ .

**Key first order conditions related to deleveraging** There are two important relationship derived from the borrowers' first order conditions.

1. The links between time-varying default rate  $\bar{\xi}_t^{df}$  and total debt repayment  $RP'(\bar{\xi}_{t+1}^{df})$  as well as the loan to collateral ratio  $LTV'(\bar{\xi}_{t+1}^{df})$ .

Note that:

$$RP(\bar{\xi}_{t+1}^{df}) = \left[ (1 - F(\bar{\xi}_{t+1}^{df}))\bar{\xi}_{t+1}^{df} + \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} dF \right] \text{ and}$$

$$LTV(\bar{\xi}_{t+1}^{df}) = \left[ RP(\bar{\xi}_{t+1}^{df}) - \tau^{df} \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} dF \right]$$

and

$$\int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} dF = \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} F'(\xi_{t+1}^{df}) d\xi_{t+1}^{df} = \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df}$$

We have:

$$\begin{aligned} RP'(\bar{\xi}_{t+1}^{df}) &= 1 - F(\bar{\xi}_{t+1}^{df}) - \bar{\xi}_{t+1}^{df} F'(\bar{\xi}_{t+1}^{df}) + \frac{d}{d\bar{\xi}_{t+1}^{df}} \int_0^{\bar{\xi}_{t+1}^{df}} \xi_{t+1}^{df} f(\xi_{t+1}^{df}) d\xi_{t+1}^{df} \\ &= 1 - F(\bar{\xi}_{t+1}^{df}) - \bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df}) + \bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df}) \\ &= 1 - F(\bar{\xi}_{t+1}^{df}) \end{aligned} \quad (4.31)$$

$$\begin{aligned} LTV'(\bar{\xi}_{t+1}^{df}) &= 1 - F(\bar{\xi}_{t+1}^{df}) - \tau^{df} \bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df}) \\ &= \left( 1 - \tau^{df} \frac{\bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df})}{1 - F(\bar{\xi}_{t+1}^{df})} \right) (1 - F(\bar{\xi}_{t+1}^{df})) \end{aligned} \quad (4.32)$$

Since  $\xi_t^{df}$  follows a log-normal distribution, I obtain:

$$\frac{d}{d\bar{\xi}_{t+1}^{df}} = \left( \frac{\bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df})}{1 - F(\bar{\xi}_{t+1}^{df})} \right) > 0 \quad (4.33)$$

which implies that

$$\frac{\partial RP(\bar{\xi}_{t+1}^{df})}{\partial \bar{\xi}_{t+1}^{df}} < 0, \text{ and } \frac{\partial LTV(\bar{\xi}_{t+1}^{df})}{\partial \bar{\xi}_{t+1}^{df}} > 0, \quad (4.34)$$

This suggests that as default rate increases, debt repayment declines and the loan to collateral ratio increases. In another word, default rate is positively correlated with the loan to collateral value ratio.

2. The relationship between default rate and the financing premium  $prem_t$ .

From the lender's participation constraint

$$LTV(\bar{\xi}_{t+1}^{df})NW_{t+1}^b = (1 + R_{t+1})L_{t+1}$$

one can show that

$$\frac{\partial R_{t+1}}{\partial \bar{\xi}_{t+1}^{df}} > 0. \quad (4.35)$$

Holding constant the housing collateral value and the outstanding loan, this suggests that higher default rate also leads to higher predetermined rate of return on lending, which worsens the financing condition and negatively affect the non-defaulting borrowers' decisions.

We can verify this statement from the first order condition of the default threshold  $\bar{\xi}_{t+1}^{df}$ :

$$\begin{aligned} \lambda_{t+1}^{pc} &= \lambda_{t+1}^b \beta^b \frac{\tau^{df} G'(\bar{\xi}_{t+1}^{df})}{\left[ RP'(\bar{\xi}_{t+1}^{df}) - \tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right]} = \lambda_{t+1}^b \beta^b \frac{1}{\left[ \frac{RP'(\bar{\xi}_{t+1}^{df})}{\tau^{df} G'(\bar{\xi}_{t+1}^{df})} - 1 \right]} \\ &= \lambda_{t+1}^b \beta^b prem_t \end{aligned} \quad (4.36)$$

Evaluating

$$\frac{RP'(\bar{\xi}_{t+1}^{df})}{\tau^{df} G'(\bar{\xi}_{t+1}^{df})} = \frac{1 - F(\bar{\xi}_{t+1}^{df})}{\tau^{df} \bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df})} \quad (4.37)$$

Given  $\frac{d}{d\bar{\xi}_{t+1}^{df}} \left( \frac{\bar{\xi}_{t+1}^{df} f(\bar{\xi}_{t+1}^{df})}{1 - F(\bar{\xi}_{t+1}^{df})} \right) > 0$ ,  $\frac{RP'(\bar{\xi}_{t+1}^{df})}{\tau^{df} G'(\bar{\xi}_{t+1}^{df})}$  is decreasing in  $\bar{\xi}_{t+1}^{df}$ , which implies that  $prem_t = \frac{1}{\left[ \frac{RP'(\bar{\xi}_{t+1}^{df})}{\tau^{df} G'(\bar{\xi}_{t+1}^{df})} - 1 \right]}$  is increasing in  $\bar{\xi}_{t+1}^{df}$ . Therefore, as default threshold value increases, the financing premium increases.

## Aggregation

Aggregate consumption, housing, money and domestic long-term bonds are given by a weighted average of the corresponding variables for each consumer type.

Consumption:

$$C_t = \omega C_t^b + (1 - \omega) C_t^s \quad (4.38)$$

Housing demand:

$$RS_t = [RS_{t+1}^b - (1 - \delta_h)(1 - \tau^{df} \int_0^{\bar{\xi}_t^{df}} \xi_t^{df} dF) RS_t^b] \quad (4.39)$$

Money:

$$M_t = \omega M_t^b + (1 - \omega) M_t^s \quad (4.40)$$

Long-term bond:

$$B_t^L = \omega B_t^{Lb} + (1 - \omega) B_t^{Ls} \quad (4.41)$$

Similarly, aggregate short-term bonds and foreign long-term bonds are related only to the borrowers demand, given by:

$$B_t = \omega B_t^b \quad (4.42)$$

$$B_t^* = \omega B_t^{b*} \quad (4.43)$$

$$B_t^{L*} = \omega B_t^{Lb*} \quad (4.44)$$

Finally, total hours worked is given by:

$$N_t = \omega N_t^b + (1 - \omega) N_t^s \quad (4.45)$$

### 4.2.2 Firms

The firm's side of the model is specified similarly as in Dorich, Mendes and Zhang (2012). Monopolistically competitive firms produce differentiated domestic goods, which are then used as inputs to produce domestic consumption goods or directly exported. A perfectly competitive firm then produces the composite goods  $Y_{H,t}$  and  $Y_{F,t}$ , used as inputs to produce the final good for private consumption.

### 4.2.3 Government

The government budget constraint is specified as:

$$M_t + \frac{B_t}{(1 + R_t)} + \frac{B_t^L}{(1 + R_t^L)^L} - (B_{t-1} + B_{t-1}^L + M_{t-1}) = P_t T_t \quad (4.46)$$

where  $T_t = \omega T_t^b + (1 - \omega)T_t^s$ . Short-term debt is used as a residual means of public financing and the long-term bonds are assumed to follow a simple AR(1) process:

$$\frac{B_t^L}{P_t} = \left( \frac{B_{t-1}^L}{P_{t-1}} \right)^{\rho_B} \exp(\epsilon_t^{B^L}) \quad (4.47)$$

where  $\rho_B \in [0, 1]$  and  $\epsilon_{B^L}$  is an i.i.d exogenous perturbation. Aggregate transfers and transfers to the savers households are set according to the following rules:

$$T_t - \bar{T} = -\varkappa_B \left[ \frac{B_{t-1}}{P_{t-1}} - \frac{B}{P} \right] + \epsilon_t^T \quad (4.48)$$

$$T_t^s - \bar{T}^s = -\varkappa_B \left[ \frac{B_{t-1}}{P_{t-1}} - \frac{B}{P} \right] + \epsilon_t^{T^s} \quad (4.49)$$

where  $\varkappa_B \in [0, 1]$ .  $\bar{T}$ ,  $\bar{T}^s$  and  $\frac{B}{P}$  are the steady state values of aggregate transfers, transfers to savers and short-term government bonds respectively.

### 4.2.4 Monetary Policy

I assume that monetary policy follows an extended Taylor-type rule when setting the nominal interest rate according to the log deviation of inflation, output and money growth from their steady-state values:

$$\frac{(1 + R_t)}{1 + \bar{R}} = \left[ \frac{(1 + R_{t-1})}{1 + \bar{R}} \right]^{\rho_R} \left\{ \left( \frac{P_t}{P_{t-1}} \right)^{\rho_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\rho_y} \left( \frac{M_t}{M_{t-1}} \right)^{\rho_m} \right\}^{(1-\rho_R)} \exp(\epsilon_t^R) \quad (4.50)$$

where  $\bar{R}$  is the steady-state nominal interest rate,  $\rho_R$  is the interest rate smoothing parameter,  $\rho_\pi$  is the coefficient on the inflation rate,  $\rho_y$  and  $\rho_m$  are coefficients on the output gap and money growth, respectively.  $\epsilon_t^R$  is a monetary policy shock that is normally distributed with standard deviation of  $\sigma^R$ .

### 4.2.5 Market Clearing

The equilibrium of the good market requires that production of the final goods equals aggregate demand:

$$Y_t^C = C_t + P_t^h R S_t \quad (4.51)$$

The clearing of the labor market requires

$$\widetilde{N}_t = \int_0^1 \widetilde{N}_t(h) dh \quad (4.52)$$

Domestic output is equal to the domestic demand and foreign demand

$$Y_t = Y_{H,t} + Y_{H,t}^* \quad (4.53)$$

where  $Y_{H,t}^*$  is given by

$$Y_{H,t}^* = \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} Y_t^* \quad (4.54)$$

The evolution of the balance of payments denominated in domestic currency:

$$\begin{aligned} & \frac{S_t B_t^*}{(1 + R_t^*)(1 + \kappa_t)} + \frac{S_t B_t^{L*}}{(1 + R_t^L)^L E_t \left\{ \prod_{j=0}^{L-1} (1 + \kappa_{t+j}) \right\}} \\ &= S_t B_{t-1}^* + S_t B_{t-L}^{L*} + P_{H,t} Y_{H,t}^* - S_t P_{F,t}^* Y_{F,t} \end{aligned} \quad (4.55)$$

### 4.2.6 Exogenous Shocks

There are eight exogenous shocks in the model. The idiosyncratic shock to the value of borrower's housing stock  $\xi_t^{df}$  is observed only by the borrower at the end of each period and is specified to follow a lognormal distribution as in Bernanke, Gertler, and Gilchrist (1999), Medina (2004) and Forlati and Lambertini (2011). The set of parameters that determine the distribution can be reduced to the mean ( $\mu_{df,t}$ ) and variance ( $\sigma_{df,t}^2$ ). I assume the standard deviation of  $\ln \xi_t^{df}$  is an exogenous shock subject to a first-order autoregressive process of the form:

$$\ln \sigma_{df,t} - \ln \bar{\sigma}_{df,t} = \rho_z (\ln \sigma_{df,t-1} - \ln \bar{\sigma}_{df,t-1}) + \varepsilon_{\sigma_{df,t}}$$

Household real money balances, consumption and housing preference as well as labor productivity all follow a simple stochastic autoregressive process:

$$\begin{aligned}\ln \xi_t &= \rho_\xi \ln \xi_{t-1} + \varepsilon_t^\xi \\ \ln A_t^c &= \rho_c \ln A_{t-1}^c + \varepsilon_t^c \\ \ln A_t^h &= \rho_h \ln A_{t-1}^h + \varepsilon_t^h \\ \ln Z_t &= \rho_z \ln Z_{t-1} + \varepsilon_t^z\end{aligned}$$

The debt elastic interest rate premium is also subject to an additive shock, following process:

$$\kappa_t = -\varsigma \left[ \exp \left( \frac{S_t N F A_t}{P_{H,t} Y_t} - \overline{nfa} \right) - 1 \right] + \varepsilon_t^\kappa \quad (4.56)$$

The long-term bonds follow a simple AR(1) process of the form:

$$\frac{B_t^L}{P_t} = \left( \frac{B_{t-1}^L}{P_{t-1}} \right)^{\rho_B} \exp(\varepsilon_t^{B^L}) \quad (4.57)$$

where  $\rho_B \in [0, 1]$  and  $\varepsilon_t^{B^L}$  is an i.i.d exogenous disturbance.

Finally, nominal interest rate is subject to a monetary policy shock  $\varepsilon_t^R$ , which is normally distributed with standard deviation of  $\sigma^R$ .

$$\ln \varepsilon_t^R = \rho_R \ln \varepsilon_{t-1}^R + \sigma_t^R$$

### 4.3 Calibration

I calibrate the parameter values in the benchmark model following Monacelli (2009) and Andres, Lopez-Salido and Nelson (2004). The savers' discount factor is set equal to 0.99 and the borrowers' discount factor is set to be 0.98. The quarterly depreciation rate of housing is set at 0.25 percentage points. Housing prices are assumed to be fully flexible, and the habit persistence parameter of residential investment is set to be 0.5. I set the standard deviation of the distribution of the idiosyncratic housing investment shock to be 0.2 to correspond to an annual default rate of 2.34 percentage points. Following Christiano, Motto and Rostagno (2009), I set the persistence of the idiosyncratic shock to be 0.9. The monitoring cost for loans is set at 0.12, following Bernanke, Gertler and

Table 4.1: **Benchmark Calibration**

Parameter Description	Value
Discount factor of savers	0.99
Discount factor of borrowers	0.98
Share of borrowers	0.6
Habit persistent in housing investment	0.5
Standard deviation of idiosyncratic shock	0.2
Loan monitoring cost	0.12
Depreciation rate of housing investment	0.25
Persistence of idiosyncratic shock	0.9
Steady-state loan-to-value ratio	0.8
Steady-state external finance premium	0.4
Taylor-rule coefficient on interest rate smoothing	0.74
Taylor-rule coefficient on inflation	2
Taylor-rule coefficient on output gap	0.67
Steady-state default rate	2.34

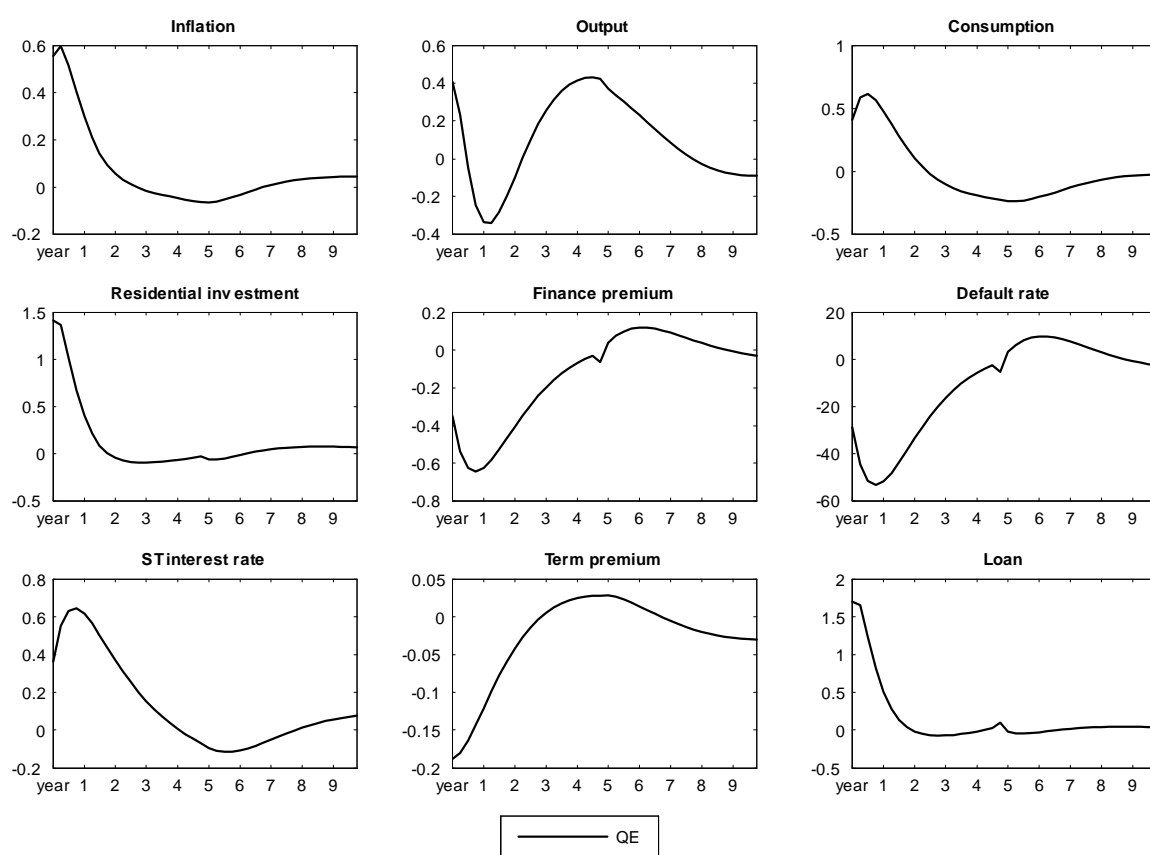
Gilchrist (1999). The steady-state loan-to-value ratio is calibrated to be 80 percentage points. The steady-state external financing premium is calibrated to 0.4 percentage points on an annual basis. Table 3.1 summarizes the calibration.

## 4.4 Simulation results

In order to assess the quantitative implications of QE on household deleveraging in the model, I simulate the model with a size of the QE intervention that is equal to 10 per cent of nominal GDP for one quarter. As shown in Figure 4.1, the marginal impact of QE on the term premium is about 20 bps. There are two effects of QE on aggregate output originated from the model. On one hand, QE leads to a decline in the risk-free long-term rate as a result of lower term premium, which increases current consumption relative to future consumption. This provides stimulus to both output and inflation. Year over year inflation rises to a peak of 60 percentage points above the steady-state.

On the other hand, QE also leads to an increase in house prices thereby an improvement in the household balance sheets through higher household net worth. As a result of increase in the value of collateral and a declining loan to collateral value ratio, QE generates a more favorable financing condition for borrowers through lower external financing premium. The low financing premium, coupled with low risk-free long-term rate and positive wealth effect from higher house prices and housing investment, encourage

Figure 4.1: Benchmark quantitative easing



household to continue investing in housing, thereby further accumulating household debt at cheaper rates.

The lower loan-to-value ratios following QE also allows lenders in the economy to consider higher-risk borrowers, such as those with low credit scores, high debt-to-disposable income ratio groups, as well as those who had previous late payment in their mortgages. Residential investment therefore has positive co-movement with consumption in response to QE. This result is consistent to a general finding in Erceg and Levin (2006) that sectoral output moves in the same directions in response to a monetary policy type shock. Household loan increases following QE by about 1 per cent, before returning to the steady-state after nearly three years.

Aggregate domestic demand increases as a result of increases in consumption and residential investment. Total output increases by 0.5 per cent one year after the implementation of QE.

The threshold value of the idiosyncratic shock below which borrowers default their mortgages declines in response to QE. Since the borrowers default rate is an increasing function in leverage, QE lowers the mortgage default rate on impact. As the effect of QE gradually dissipates, default rates rises over the steady state starting from the 6th year. This is accompanied by higher financing premium and an increase of loan-to-value ratio. As the financing condition worsens, there is increasing housing investment risks in the post-QE economy. Should the housing market experiences an unexpected negative shock in the future, the overall impact of the shock would be greater than in an economy without QE intervention.

## 4.5 Sensitivity analysis

The effect of QE beyond the initial impact depends on a few important factors. First, it depends on the elasticity of loan demand to the policy. If the increase in loan is greater than the increase in the overall collateral value, this would lead to an ultimate deeper leverage and therefore higher default rate. In the consideration of the future withdrawal of any stimulus provided by QE, this would pose greater challenges as it implies much more intensive household deleveraging process. In the baseline calibration, with the implementation of QE, there is a 1.7 per cent increase in the loan demand. This is reflected in a lower financing premium and the increase in collateral value. Overall, as the net worth of household returns to the steady state five years after the initial QE policy, default rate remains 5 percentage points higher than the steady state.

The effect of QE is also subject to the monetary policy rule the central bank follows

while conducting QE. For a strong inertia policy rule (higher smoothing parameter on the short-term nominal interest rate), short-term rate would react less to the decline of long-term interest rate (or remains constrained at the zero lower bound for longer duration) since there is more smoothing. This implies greater overall effect on the long-term rate from QE. As a result, under higher smoothing, borrowers increase their demand for consumption and housing more than the case when there is less smoothing on the nominal interest rate.

In the following simulations, I show quantitatively how the effect of QE on household deleveraging changes in response to the varying economy leverage levels and the extent of interest rate smoothing in the policy rule.

#### **4.5.1 QE with higher housing investment risk**

I simulate the same QE intervention in an alternatively calibrated economy with higher idiosyncratic housing investment risk (captured through higher steady-state standard deviation of the distribution of housing investment risk at 0.3). The rest of calibration is identical to the benchmark economy. For the same mean value of the idiosyncratic shock, higher standard deviation implies a higher cumulative distribution function and a higher steady-state default rate on mortgages.

As in the benchmark case, the implementation of QE leads to a decline in the risk-free long-term interest rate and lower financing cost for borrowers. On one hand, as real house prices increase, one can observe greater decline in mortgage defaults under the high-leverage state as households are more sensitive to the long-term interest rate decline. As more borrowers now repay their loans, the state-contingent mortgage interest rate falls. On the other hand, short-term risk-free nominal interest rate rises in response to QE. Overall, the decline in mortgage interest rate largely offsets the increase in the nominal risk-free interest rate, leading to a greater fall in the external finance premium. Consequently, when the economy is in a high-leverage state, increase in inflation of consumption goods is greater and largely lowers borrowers' real debt, leading to greater increases in consumption, residential investment and demand for mortgage loans. Mortgage default five years after QE is slightly higher than the benchmark case level, suggesting higher housing investment risks post-QE.

#### **4.5.2 QE with higher interest rate smoothing**

I now analyze the implications of adopting an interest rate policy that has higher degree of smoothing.

Figure 4.2: Impact of deeper leverage on QE

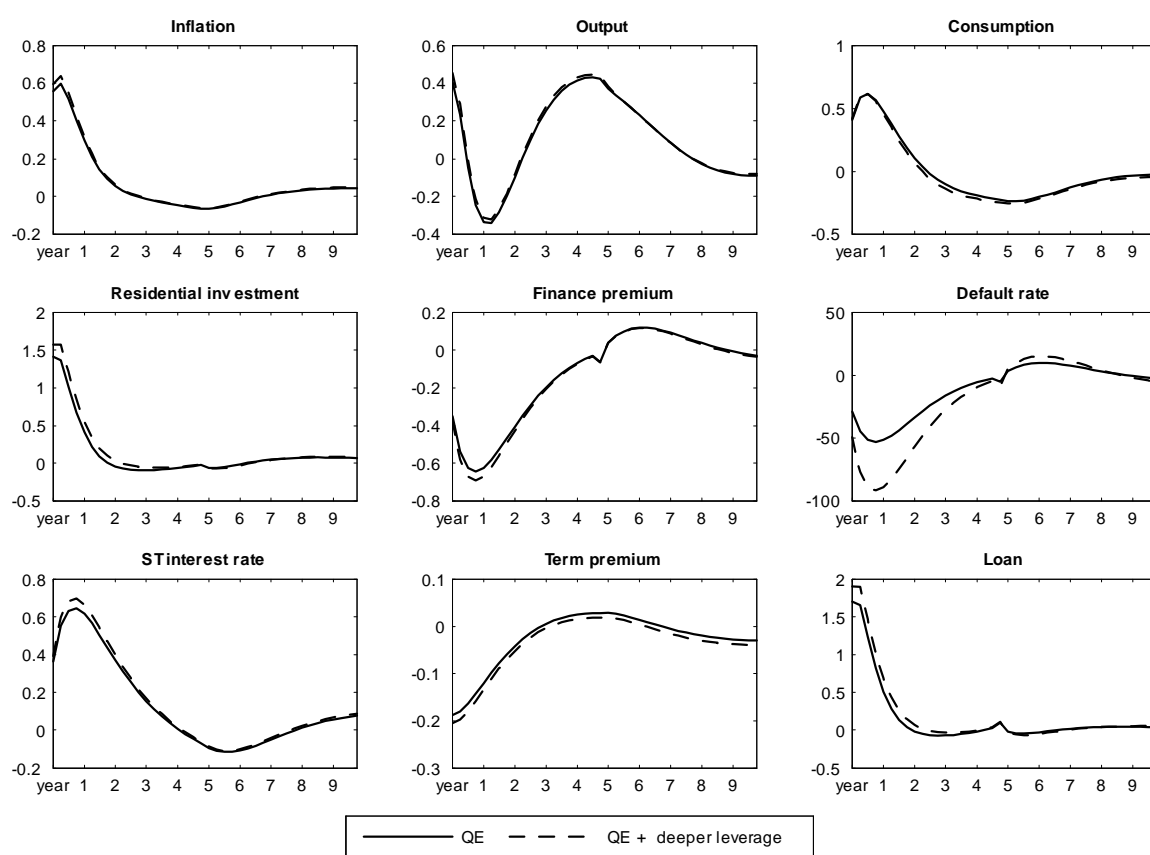
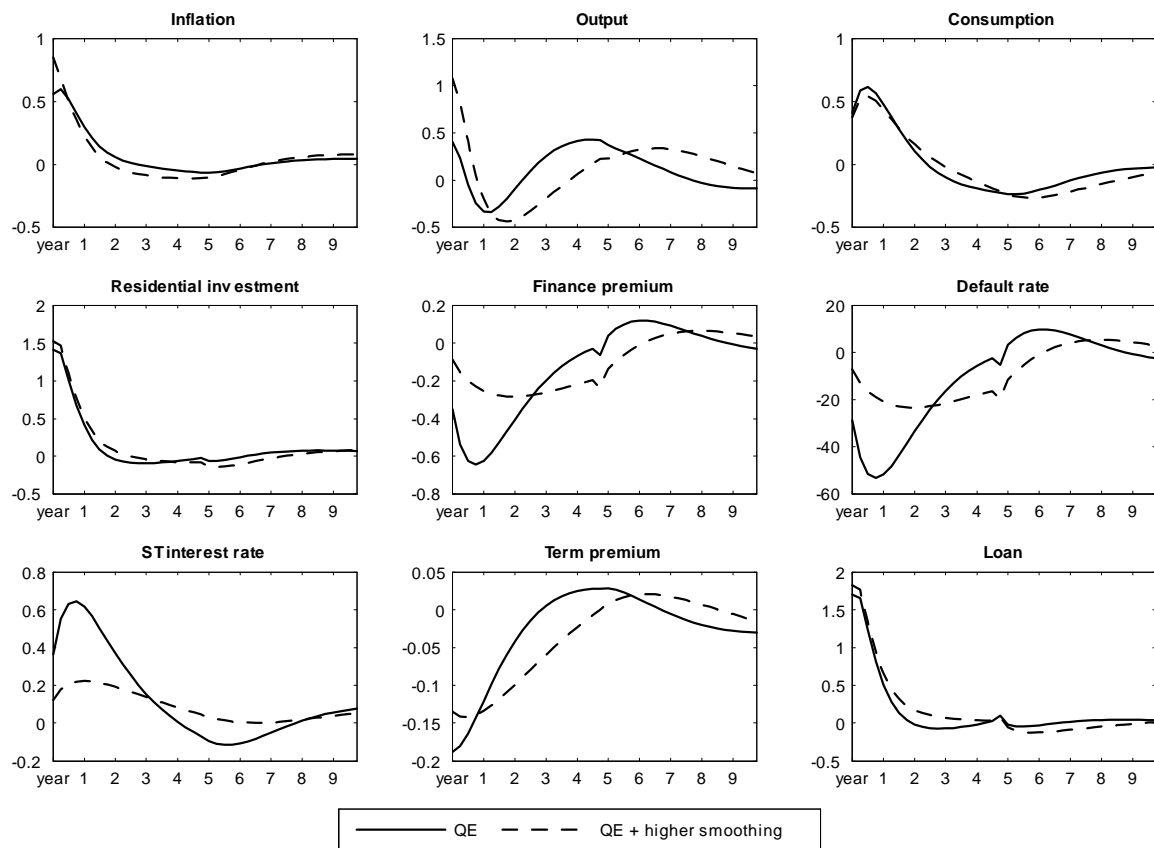


Figure 4.3: Impact of higher interest rate smoothing on QE



Conditional on the same size of QE intervention, greater smoothing induces much more persistence in short-term interest rates as well as greater decline in the long-term risk-free interest rate. Similar to the benchmark case, long-term interest rates clearly influence aggregate demand beyond the impact of the expected path of short-term interest rates. As a result, under higher smoothing, output departs from its long-run value longer than in the benchmark case with nearly doubled effect of QE on impact, generating higher volatility in the economy. Housing demand is well-supported under the inertia monetary policy, along with the conduct of QE. As the loan demand rises slightly higher on impact relative to the benchmark case, the fall in the state-contingent mortgage interest rate is smaller under the high smoothing case. On net, the external finance premium falls by less under the smoothing case, accompanied by less decline in mortgage defaults compared to the benchmark case. Overall, keeping short-term interest rate more persistent helps to better stimulate the aggregate demand, but has less impact on household deleveraging as the decline in external finance premium is also smaller with stable demand for housing under the lower long-term interest rate.

## 4.6 Concluding remarks

In this chapter, I use a small open economy DSGE model with real and household financial frictions to analyze the effects of QE on household deleveraging. The model features heterogeneous households and a financial intermediary. The existence of borrowers in the economy allows for the conduct of QE as it allows long-term interest rate to matter over and above the short-term rates in the aggregate demand. Borrowers purchase housing relying on mortgage loans provided by the intermediary. Borrowing is subject to an external financing premium due to loan default risk, which generates endogenous movement in borrowers' loan to value ratio.

I find two effects of QE on aggregate output originated from the model. First, QE leads to a decline in term premium, which increases current consumption relative to future consumption. On the other hand, it creates a more favorable financing conditions for borrowers through an increase in the value of collateral and a decline in external finance premium, therefore encourages further accumulation of debt at cheaper rates. This, in turn, leads to an immediate higher household debt to income ratio. Mortgage default declines on impact following QE.

Sensitivity analysis results suggest that the effect of QE beyond the initial impact depends on a few important factors. First, it depends on the elasticity of loan demand to the policy. If the increase in loan is greater than the increase in the overall collateral

value, this would lead to an ultimate deeper leverage and therefore higher default rate. In the consideration of future withdrawal of any stimulus provided by QE, this would pose greater challenges as it implies much more intensive household deleveraging process. In the baseline calibration, with the implementation of QE, there is a 1.7 per cent increase in the loan demand. This is reflected in a lower finance premium and an increase in collateral value. Overall, as the net worth of household returns to the steady state five years after the initial QE policy, default rate remains 5 percentage points higher than the steady state.

The effect of QE also depends on the steady-state housing investment risk level (or the extent of borrowers' leverage). When the economy is in a high-leverage state, following QE, the increase in inflation of consumption goods is greater, thereby largely lowering borrowers' real debt, leading to greater increases in consumption, residential investment and demand for mortgage loans. Mortgage default five years after QE is slightly higher than the benchmark case level, suggesting higher housing investment risks post-QE.

Finally, the effect of QE is also subject to the monetary policy rule the central bank follows while conducting QE in the economy. For a policy rule with great inertia (higher smoothing parameter on the short-term nominal interest rate), short-term rate would react less to the decline of long-term interest rate (or remains constrained at the zero lower bound for longer) if there is more smoothing. As a result, under higher smoothing, borrowers increase their demand for consumption and housing more than the case when the policy rule is less inertia. Overall, keeping short-term interest rate more persistent helps to better stimulate the aggregate demand, but has less implications for household deleveraging as the decline in external finance premium is smaller with more stable demand for housing with lower long-term interest rate.

# Chapter 5

## Conclusion

In normal times, central banks conduct monetary policy by steering the level of the key policy instrument (usually short-term nominal interest rates) to achieve their objectives on price and output stability over the medium term. This conventional approach has proven to provide sufficient monetary stimulus to the economy during downturns, preventing deflationary pressures and ensuring economic recovery. In the wake of the financial and economic crisis of 2007-2009, central banks in many advanced economies aggressively responded by cutting their policy rates aiming to counter rapidly deteriorating economic outlooks. During this process, many of these central banks found themselves hitting the effective lower bounds on the policy rates. In this context, numerous unconventional monetary tools were considered in order to provide additional monetary stimulus.

Unconventional policy may be warranted when the nominal interest rate is at the effective zero lower bound. Alternatively, it may also be used when monetary policy transmission mechanism is significantly impaired especially in an already "low-for-long" interest rate environment. Despite of growing branch of literature on the use of unconventional monetary policies and the evaluation their performance, analysis of quantifying the effects of these policies in a structural model is still quite limited. This thesis is an attempt towards this direction, presenting a quantitative exploration of different alternative unconventional monetary policies in a dynamic stochastic general equilibrium model.

In the first chapter, we provide a quantitative assessment of the macroeconomic impact of quantitative easing (QE) when the policy rate is at its effective lower bound. Under QE, a central bank reduces the spreads of long-term interest rate over expected short-term policy rates through large-scale purchases of government securities. Since the cost of external finance is generally a premium over the short-term interbank lending rate,

central bank could reduce the spreads between various forms of external finance, thereby influencing the asset prices in the economy. When the nominal short-term interest rate remains constrained at the lower bound, the real interest rate falls as a result of higher expected inflation. Since long-term interest rates are average of expected short-term rates, the expectation channel would tend to flatten the yield curve, therefore stimulate spending.

In particular, we extend a simple Keynesian model of a small open economy to allow for two additional frictions. First, we assume that long-term and short-term assets are imperfect asset substitutes. This allows the long-term rate to deviate from the level implied by the strict expectations theory of the term structure and generates a term premium in the model that can be affected by unconventional monetary policy. We also assume there is some degree of asset market segmentation in the economy. With some households' preferences towards only long-term assets, the long-term interest rate is allowed to independently affect aggregate demand over and above short-term rates. Given the open economy setting, the portfolio rebalancing effect from the implementation of QE also implies deviations from the uncovered interest rate parity, providing some implications for the determinations of the exchange rate.

We estimate the model using Canadian data and use it to evaluate the effect of QE at the lower bound by simulating a large and persistent contractionary consumption demand shock. This shock is big enough that the desired policy rate becomes negative. As a result, the effective lower bound constraint becomes binding. We then assume that the central bank conducts open market purchases of long-term government bonds when the policy rate is at the lower bound. The open market purchases support aggregate demand by reducing long-term interest rates. The simulation result suggests a moderate impact of quantitative easing. In particular, when the nominal rate is constrained at the lower bound, following a decline of 10bps in the long-term bond yield through QE, there is a small impact on aggregate demand (real level of GDP increases by 0.06%, similar to the findings of Baumeister and Benati 2010) and a much larger impact on inflation (0.1%). The timing of the monetary stimulus, however, is much faster, with peak responses on output and inflation occurring after 4 and 3 quarters, respectively.

In the second chapter, I explore forward guidance as an alternative option a central bank uses to complement other unconventional monetary policies. More specifically, a central bank could reduce the economic costs of the lower bound by credibly signaling that short-term interest rates will remain low for a prolonged period of time, unconditionally or conditional on the development of a particular indicator (i.e., conditional forward guidance). Similarly, a central bank could also offset the impact of ZLB by announcing

that it will generate an overshooting of inflation once the lower bound is no longer a constraint (i.e., guidance through price-level targeting). The practice of forward guidance has undergone significant development since the onset of the financial crisis by the Federal Open Market Committee (FOMC) and was recently used by the Bank of Canada in April 2009. Despite the wide use in practice, only a few studies have looked at the effectiveness of forward guidance policies at the zero lower bound.

I examine the implication of expectation-formation mechanism at the ZLB by comparing the performance of a state-contingent guidance (using conditional statement) and a price-level guidance in a calibrated simple three-equation New Keynesian model. I examine the extent to which forward guidance helps to mitigate the negative real impact of the zero lower bound. The simulation results suggest that guidance using a conditional statement prolongs the zero lower bound duration for an additional 4 quarters and reversed half of the decline in inflation associated with the lower bound. In addition, it even generates a period of overshooting in inflation three quarters after the initial negative demand shock. As a result of more accommodative policy and higher inflation expectation, GDP increases by 1.8 per cent on peak with a short period of production expansion about a year and a half after the initial shock.

The effect of price-level targeting as a guidance policy at the zero lower bound is slightly different. The policy shortens the zero lower bound duration by one quarter, but generates the highest inflation profile among the alternative policy rules considered. Price level returns to pre-shock level and the overall loss is reduced to 6.4 per cent with the price-level targeting rule, compared to 2.4 per cent with the conditional inflation targeting rule and 10.3 per cent with the case of zero lower bound. It's useful to relate this result with those in Eggertsson and Woodford (2003) where they identify the relevant channel to escape from a ZLB is through affecting the expectations about future inflation. In addition, they show that the optimal policy in a ZLB environment is targeting the price level. Similarly, Evans (2012) also discusses how state-contingent price-level targeting could help to provide further monetary accommodation even at the zero lower bound. In the second chapter, although I don't specify any objective functions to solve for the optimal policy at the ZLB, the relative advantage of using price-level targeting as a tool for forward guidance highlights the effectiveness of unconventional policy through the expectation channel.

In the third chapter, I analyze the effects of quantitative easing on household deleveraging. This is motivated by the fact that while quantitative easing helps to lower long-term interest rate, support aggregate output, it may also further strengthen the potential financial imbalances related to household indebtedness (through an increasing household

debt to income ratio) and intensify the misallocations of resources (e.g., overinvestment of residential structure) that were fueled by a low interest rate environment.

I extend the model in the first essay with features of heterogeneous households and additional real and household financial frictions. The patient savers view short- and long-term assets as imperfect substitutes, therefore allowing the long-term rate to deviate from the level implied by the strict expectations theory of the term structure. With the existence of imperfect asset substitutability, the model contains a term premium in the model that can be affected by quantitative easing policy. Impatient households are middle-aged households who only trade in long-term bond markets and borrow from the savers in the economy. With borrowers' preferences towards only long-term assets, long-term interest rate is allowed to independently affect aggregate demand over and above short-term rates. Borrowers purchase housing relying on mortgage loans provided by an intermediary and borrowing is subject to an external financing premium due to loan default risk, which generates endogenous movement in borrower's loan to value ratio.

I find two effects of QE on aggregate output originated from the model. First, QE leads to a decline in term premium, which increases current consumption relative to future consumption. On the other hand, it creates a more favorable financing conditions for borrowers through an increase in the value of collateral and a decline in external finance premium, therefore encourages further accumulation of household debt at cheaper rates. This, in turn, leads to an immediate higher household debt to income ratio. Mortgage default declines on impact following QE.

Sensitivity analysis results suggest that the effect of QE beyond the initial impact depends on a few important factors. First, it depends on the elasticity of loan demand to the policy. If the increase in loan is greater than the increase in the overall collateral value, this would lead to an ultimately deeper leverage and therefore higher default rate. In the consideration of future withdrawal of any stimulus provided by QE, this would pose greater challenges as it implies much more intensive household deleveraging process. The effect of QE also depends on the steady-state housing investment risk level (or extent of borrower's leverage). When the economy is in a high-leverage state, following QE, mortgage default five years after QE is slightly higher than the benchmark case level, suggesting higher housing investment risks post-QE. Finally, the effect of QE is also subject to the monetary policy rule the central bank follows while conducting QE in the economy. A policy rule with great inertia helps to better stimulate the aggregate demand, but has less implications for household deleveraging as the decline in external finance premium is smaller with more stable demand for housing under the lower long-term interest rate.

# Appendix A

## Appendix

### A.1 Linearized Equilibrium Dynamics in Chapter 3

Below, I log-linearized the dynamic equations around the steady-state of the model to yield the following equilibrium conditions.

The IS curve is given by

$$\widehat{y}_t = \frac{1}{1+\zeta} E_t [\widehat{y}_{t+1}] + \frac{\zeta}{1+\zeta} \widehat{y}_{t-1} - \frac{(1-\zeta)}{\sigma(1+\zeta)} \left( \widehat{i}_t - E_t [\widehat{\pi}_{t+1}] \right) + \widehat{d}_t \quad (\text{A.1})$$

where the demand shock  $d_t$  is

$$\widehat{d}_t = \frac{(1-\zeta)(1-\rho_\Theta)}{\sigma(1+\zeta)} \widehat{\Theta}_t \quad (\text{A.2})$$

and follows an AR(1) process

$$\widehat{d}_t = \rho_\Theta \widehat{d}_{t-1} + \varepsilon_{d,t}, \quad \varepsilon_{d,t} \sim NID(0, \sigma_d^2) \quad (\text{A.3})$$

The natural output can be derived from a flexible prices equilibrium

$$\widehat{y}_t^n = \left[ \frac{\sigma\zeta}{\varphi(1-\zeta) + \sigma} \right] \widehat{y}_{t-1}^n + \frac{(1-\zeta)(1+\varphi)}{\varphi(1-\zeta) + \sigma} \widehat{z}_t. \quad (\text{A.4})$$

The New-Keynesian Phillips curve is given by

$$\widehat{\pi}_t = \frac{\theta-1}{\kappa(1+\beta\theta_\pi)} \widehat{m}c_t + \frac{\beta}{1+\beta\theta_\pi} E_t [\widehat{\pi}_{t+1}] + \frac{\theta_\pi}{1+\beta\theta_\pi} \widehat{\pi}_{t-1} + \widehat{m}_t \quad (\text{A.5})$$

where  $\widehat{m}c_t$  is the marginal cost

$$\widehat{m}c_t = \left( \varphi + \frac{\sigma}{1-\zeta} \right) (\widehat{y}_t - \widehat{y}_t^n) - \frac{\sigma\zeta}{1-\zeta} (\widehat{y}_{t-1} - \widehat{y}_{t-1}^n) \quad (\text{A.6})$$

The output gap is defined as

$$\widehat{y}_t^{gap} = \widehat{y}_t - \widehat{y}_t^n \quad (\text{A.7})$$

and the mark-up shock is specified as

$$\widehat{m}_t = \frac{\theta - 1}{\kappa (1 + \beta\theta_\pi)} \widehat{\mu}_t \quad (\text{A.8})$$

which follows an AR(1) process:

$$\widehat{m}_t = \rho_\mu \widehat{m}_{t-1} + \varepsilon_{m,t}, \quad \varepsilon_{m,t} \sim NID(0, \sigma_m^2). \quad (\text{A.9})$$

The policy rule is

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1 - \rho_i) [a_\pi \widehat{\pi}_t + a_y (\widehat{y}_t - \widehat{y}_t^n)] + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim NID(0, \sigma_i^2) \quad (\text{A.10})$$

## A.2 Households' optimality conditions in Chapter 4

Borrower households (denoted by  $b$ ) with future discount rate  $\beta^b$  choose  $C_t^b, RS_{t+1}^b, N_{jt}^b, M_t^b$  and  $B_t^b, B_t^{b*}, B_t^{Lb}, B_t^{Lb*}, L_{t+1}, \bar{\xi}_{t+1}^{df}$  to maximize

$$\max_{\{\}} E_0 \sum_{t=0}^{\infty} (\beta^b)^t \left[ a_t \left\{ U(C_t^b) + H(RS_{t+1}^b) + V(M_t^b) - \int_0^1 L(N_{jt}^b) dj \right\} - G(\cdot) \right] \quad (\text{A.11})$$

subject to the budget constrain:

$$\begin{aligned} & P_t C_t^b + P_t^h RS_{t+1}^b + RP(\bar{\xi}_t^{df}) NW_t^b + M_t^b + \frac{(1 + \phi_t) B_t^{Lb}}{(1 + R_t^L)^L} + \\ & = NW_t^b + L_{t+1}^b + M_{t-1}^b + B_{t-L}^{Lb} + \int_0^1 W_{jt} N_{jt}^b dj + P_t T_t^b \end{aligned} \quad (\text{A.12})$$

or:

$$\begin{aligned}
& P_t C_t^b + P_t^h R S_{t+1}^b + (1 + R_{t-1}) L_t^b + M_t^b + \frac{(1 + \phi_t) B_t^{Lb}}{(1 + R_t^L)^L} \\
= & (1 - \delta_h) \left[ 1 - \tau^{df} G(\bar{\xi}_t^{df}) \right] P_t^h R S_t^b + L_{t+1}^b + M_{t-1}^{bl} + \\
& + B_{t-L}^{Lb} + \int_0^1 W_{jt} N_{jt}^b dj + P_t T_t^b
\end{aligned} \tag{A.13}$$

and the participation constrain:

$$(1 + R_{t-1}) L_t^b = \left[ R P(\bar{\xi}_t^{df}) - \tau^{df} G(\bar{\xi}_t^{df}) \right] (1 - \delta_h) P_t^h R S_t^b. \tag{A.14}$$

We can set up the Lagrangian as

$$\begin{aligned}
L = & : E_0 \sum_{t=0}^{\infty} (\beta^b)^t \left[ a_t \left\{ U \left( \frac{C_t^b}{\{C_{t-1}^b\}^{h_c}} \right) + H \left( \frac{R S_{t+1}^b}{\{R S_t^b\}^{h_{rs}}} \right) + V \left( \frac{M_t^b}{\xi_t P_t} \right) - \int_0^1 L(N_{jt}^b) dj \right\} \right. \\
& \left. - G(\cdot) \right] \\
& - \lambda_t^u \left[ \begin{aligned} & P_t C_t^b + P_t^h R S_{t+1}^b + (1 + R_{t-1}) L_t^b + M_t^b + \frac{(1 + \phi_t) B_t^{Lb}}{(1 + R_t^L)^L} \\ & - [(1 - \delta_h) \{ 1 - \tau^{df} G(\bar{\xi}_t^{df}) \}] P_t^h R S_t^b + L_{t+1}^b + M_{t-1}^{bl} \\ & + B_{t-L}^{Lb} + \int_0^1 W_{jt} N_{jt}^b dj + P_t T_t^b \end{aligned} \right] \\
& - \lambda_t^{pc} \left[ (1 + R_{t-1}) L_t^b - \left\{ R P(\bar{\xi}_t^{df}) - \tau^{df} G(\bar{\xi}_t^{df}) \right\} (1 - \delta_h) P_t^h R S_t^b \right]
\end{aligned} \tag{A.15}$$

Consumption  $C_t^u$ :

$$\lambda_t^b = a_t \left( \frac{C_t^b}{\{C_{t-1}^b\}^{h_c}} \right)^{1-\sigma_c} \frac{1}{C_t^b} - \beta a_{t+1} h_c \left( \frac{C_{t+1}^b}{\{C_t^b\}^{h_c}} \right)^{1-\sigma_c} \frac{1}{C_t^b} \tag{A.16}$$

Default threshold  $\bar{\xi}_{t+1}^{df}$ :

$$\begin{aligned}
\beta^b E_t \lambda_{t+1}^b (1 - \delta_h) P_{t+1}^h R S_{t+1}^b \left[ -\tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right] &= -E_t \lambda_{t+1}^{pc} (1 - \delta_h) P_{t+1}^h R S_{t+1}^b \left[ R P'(\bar{\xi}_t^{df}) - \tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right] \\
-\beta^b \lambda_{t+1}^b \tau^{df} G'(\bar{\xi}_{t+1}^{df}) + \lambda_{t+1}^{pc} \left[ R P'(\bar{\xi}_{t+1}^{df}) - \tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right] &= 0 \\
\lambda_{t+1}^{pc} = \lambda_{t+1}^b \beta^b \tau^{df} \frac{G'(\bar{\xi}_{t+1}^{df})}{\left[ R P'(\bar{\xi}_{t+1}^{df}) - \tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right]} &= \lambda_{t+1}^b \beta^b prem_{t+1}
\end{aligned} \tag{A.17}$$

$$prem_{t+1} = \frac{\tau^{df} G'(\bar{\xi}_{t+1}^{df})}{\left[ RP'(\bar{\xi}_{t+1}^{df}) - \tau^{df} G'(\bar{\xi}_{t+1}^{df}) \right]} \quad (\text{A.18})$$

Loan  $L_{t+1}^b$ :

$$\begin{aligned} \frac{\lambda_t^b}{P_t} - (1 + R_t) E_t \beta^b \frac{\lambda_{t+1}^b}{P_{t+1}} - (1 + R_t) E_t \frac{\lambda_{t+1}^{pc}}{P_{t+1}} &= 0 \\ \frac{\lambda_t^b}{P_t} &= (1 + R_t) E_t \left[ \beta^b \frac{\lambda_{t+1}^b}{P_{t+1}} + \frac{\lambda_{t+1}^{pc}}{P_{t+1}} \right] \end{aligned} \quad (\text{A.19})$$

Housing stock  $RS_{t+1}^b$ :

$$\begin{aligned} &U_{t,RS_{t+1}^b} + \beta^b E_t \left\{ U_{t+1,RS_{t+1}^b} \right\} - \frac{\lambda_t^b}{P_t} P_t^h \\ &+ \beta^b E_t \frac{\lambda_{t+1}^b}{P_{t+1}} (1 - \delta_h) \left[ 1 - \tau^{df} G'(\bar{\xi}_t^{df}) \right] P_{t+1}^h \\ &+ \beta^b E_t \frac{\lambda_{t+1}^{pc}}{P_{t+1}} (1 - \delta_h) \left[ RP(\bar{\xi}_t^{df}) - \tau^{df} G(\bar{\xi}_t^{df}) \right] P_{t+1}^h = 0 \\ &\left( \frac{RS_{t+1}^b}{\{RS_t^b\}^{h_{rs}}} \right)^{1-\sigma_h} \frac{1}{RS_{t+1}^b} - \beta^b h_{rs} \left( \frac{RS_{t+2}^b}{\{RS_{t+1}^b\}^{h_{rs}}} \right)^{1-\sigma_h} \frac{1}{RS_{t+1}^b} - \frac{\lambda_t^b}{P_t} P_t^h \\ &+ \beta^b E_t (1 - \delta_h) P_{t+1}^h \left[ \frac{\lambda_{t+1}^b}{P_{t+1}} \left[ 1 - \tau^{df} G'(\bar{\xi}_t^{df}) \right] + \frac{\lambda_{t+1}^{pc}}{P_{t+1}} \left[ RP(\bar{\xi}_t^{df}) - \tau^{df} G(\bar{\xi}_t^{df}) \right] \right] \\ \frac{\lambda_t^b}{P_t} P_t^h &= \left( \frac{RS_{t+1}^b}{\{RS_t^b\}^{h_{rs}}} \right)^{1-\sigma_h} \frac{1}{RS_{t+1}^b} - \beta^b h_{rs} \left( \frac{RS_{t+2}^b}{\{RS_{t+1}^b\}^{h_{rs}}} \right)^{1-\sigma_h} \frac{1}{RS_{t+1}^b} \\ &+ \beta^b E_t (1 - \delta_h) P_{t+1}^h \left[ \frac{\lambda_{t+1}^b}{P_{t+1}} \left[ 1 - \tau^{df} G'(\bar{\xi}_t^{df}) \right] + \frac{\lambda_{t+1}^{pc}}{P_{t+1}} \left[ RP(\bar{\xi}_t^{df}) - \tau^{df} G(\bar{\xi}_t^{df}) \right] \right] \end{aligned} \quad (\text{A.20})$$

LT bonds  $B_t^{Lu}$ :

$$\frac{(1 + \phi_t)}{(1 + R_t^L)^L} \left( \frac{\lambda_t^b}{P_t} \right) - v \Phi \frac{M_t^b}{(B_t^{Lb})^2} \left[ \frac{M_t^b}{B_t^{Lb}} \Phi - 1 \right] = \beta^L E_t \left( \frac{\lambda_{t+L}^b}{P_{t+L}} \right) \quad (\text{A.21})$$

Money demand  $M_t^u$ :

$$\begin{aligned}
 & a_t V_{t, M_t^b} - \left\{ G_{t, M_t^b} + \beta E_t \left( G_{t+1, M_t^b} \right) \right\} - \frac{v\Phi}{B_t^{Lb}} \left[ \frac{M_t^b}{B_t^{Lb}} \Phi - 1 \right] - \frac{\tilde{v}\tilde{\Phi}}{S_t B_t^{Lb*}} \left[ \frac{M_t^b}{S_t B_t^{Lb*}} \tilde{\Phi} - 1 \right] \\
 = & \frac{\lambda_t^b}{P_t} - \beta E_t \left( \frac{\lambda_{t+1}^b}{P_{t+1}} \right) \tag{A.22}
 \end{aligned}$$

This is the Appendix.

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