THREE ESSAYS ON ENVIRONMENTAL ECONOMICS AND ON CREDIT MARKET IMPERFECTIONS

By
Muhammad Shahid Siddiqui

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FACULTY OF GRADUATE STUDIES
UNIVERSITY OF OTTAWA

To My Mother, Father, Wife, and Son

Table of Contents

3 Imperfect Credit Markets, Learning by Lending, and Output Dynamics 112

List of Tables

List of Figures

Abstract

This dissertation contains three essays on environmental economics and on credit market imperfections.

The literature on carbon tax incidence generally finds that carbon taxes have a regressive impact on the distribution of income. The main reason for that finding stems from the fact that poor households spend a larger share of their total expenditure on energy products than the rich households do. This literature, however, has ignored the impact of carbon taxes on income stemming from changes in relative factor prices. Yet, changes in household welfare depend not only on variations in commodity prices, but also on changes in income.

Chapter 1 provides a comprehensive analysis of the distributional impact of carbon taxes on inequality by considering both demand-side and supply-side channels. We use a multi-sector, multi-household general equilibrium model to analyze the distributional impact of carbon taxes on inequality. Using equivalent income as the household welfare metric, we apply the Shapley value and concentration index approaches to decomposing household inequality. Our simulation results suggest that carbon taxes exert a larger negative impact on the income of the rich than that of the poor, and are thereby progressive. On the other hand, when assessed from the use side alone (i.e., commodity prices alone), our results confirm previous findings, whereas carbon taxes are regressive.

However, due to the stronger incidence of carbon taxes on inequality from the income side, our results suggest that the carbon tax tends to reduce inequality. These findings further suggest that the traditional approach of assessing the impact of carbon taxes on inequality through changes in commodity prices alone may be misleading.

Chapter 2 investigates the economic impacts of creating an emissions bubble between Canada and the US in a context of subglobal participation in efforts to reduce pollution with market based-instruments. One of the advantages of an emissions bubble is that it can be beneficial to countries that differ in their production and consumption patterns. To address the competitiveness issue that arises from the free-rider problem in the area of climate-change mitigation, we consider the imposition of a border tax adjustment (BTA) - a commonly suggested solution in the literature.

We develop a detailed multisector and multi-regional general equilibrium model to analyze the welfare, aggregate, sectoral and trade impacts of the formation of an emissions bubble between Canada and the US with and without BTA. Our simulation results suggest that, in the absence of BTA, the creation of the bubble would make both countries better off through a positive terms-of-trade effect, and more importantly, through a significant reduction in Canada's marginal abatement cost. The benefits of these positive effects would spill over to the non-participating countries, leading them to increase their trade shares in non-emissions-intensive goods.

Moreover, the simulation results also indicate that a unilateral implementation of a BTA by any one of the two countries is welfare deteriorating in the imposing country and welfare improving in the other. In contrast, a joint implementation of a BTA by the two countries would make Canada better off and the US worse off.

Chapter 3 shows that learning by lending is a potential channel of understanding the business cycle fluctuation under an imperfect credit market. An endogenous link among the learning parameter, lending rates, and the size of investment makes it possible to generate an internal propagation even due to a temporary shock. The main finding of this chapter is the explanation of how *ex post* non-financial factors such as information losses by individual agents in a credit market may account for a persistence in real indicators such as capital stock and output.

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Introduction

This dissertation comprises three essays on climate change policies and on credit market imperfections. The first and second chapters provide comprehensive analysis on two very important issues related to pollution mitigation policies; The first chapter studies the distributional issue among households, and the second considers the formation of an emissions bubble as a means of improving regional cost-effectiveness. Chapter 2 also addresses in detail the issue of domestic competitiveness in the presence of a sub-global climate change policy. The third chapter introduces the learning by lending channel in a credit market which works under imperfect information. A brief introduction of each chapter is stated below.

Chapter 1 assesses the distributional impacts of carbon taxes on households by considering both the demand and the supply channels, through which an environmental policy might have an incidence on income inequality. Most papers in the literature find that carbon taxes are regressive. The main reason for that finding stems from the fact that poor households spend a larger share of their total expenditure on energy products than the rich households. However, these papers ignore the impact of carbon taxes on income brought about by changes in relative factor prices. Yet, changes in household welfare depend not only on the variations in commodity prices, but also on the changes in income.

Taking into account both the demand-side and supply-side channels, chapter 1 applies a mutlisector, multi-household general equilibrium model to analyze the distributional impact of carbon taxes on inequality within a static framework. The novelty of the analysis rests on the introduction of a formal decomposition approach to inequality, which breaks down the incidence of a pollution mitigation policy into individual elements of source and use sides of household income.

Our simulation results suggest that the negative incidence of carbon taxes stemming from the source-side of income (i.e., factor prices) concentrates among the rich households as carbon prices increase, implying that the pollution tax is progressive. Conversely, the incidence of pollution tax on the use-side of income (i.e., commodity prices alone) is regressive. Moreover, due to the stronger incidence of carbon taxes on inequality coming from the income side, our results also suggest that the carbon tax tends to reduce inequality. These findings demonstrate that the traditional approach of assessing the impact of carbon taxes on inequality through changes in commodity prices alone may be misleading.

Chapter 2 studies the economic impacts of creating an emissions bubble between Canada and the US in a context of sub-global participation in the efforts to reduce pollution with market based-instruments. One of the advantages of an emissions bubble is that it can be beneficial to countries that differ in their production and consumption patterns. To address the competitiveness issue that arises from the free-rider problem present in climate change mitigation efforts, we consider the imposition of a border tax adjustment (BTA) - a commonly suggested solution in the literature.

We develop a detailed multisector and multi-regional general equilibrium model to analyze the welfare, aggregate, sectoral and trade impacts of the formation of an emissions bubble between Canada and the US with and without BTA. Our simulation results suggest that, in the absence of BTA, the creation of the bubble would make both countries better off through a positive terms-of-trade effect, and more importantly, from a significant reduction in Canada's marginal abatement cost. The benefit of these positive effects will spill over to the non-participating countries, which would then increase their trade shares in non-emissions-intensive goods.

Our findings do not lend support to Copeland and Taylor's (2005) argument that under some conditions permits trading under free trade may lead to indirect negative terms-of-trade effects, which can offset the direct gains from permits trading. In the Canada-U.S. emissions bubble case, we find that the economic structures and trade patterns between the two countries lead to post-policy positive terms-of-trade effects. The increase in exports of emission-intensive goods to the U.S. benefits the latter country as it experiences an improvement in its terms of trade thereby reducing the negative impact of the carbon mitigation policy which would be observed in the absence of an emissions bubble.

In addition, the simulation results also indicate that a unilateral implementation of a BTA by any one of the two countries is welfare deteriorating in the imposing country and welfare improving in the other. In contrast, a joint implementation of a BTA by the two countries would make Canada better off and the US worst off.

Moreover, the impact of a BTA policy is also sensitive to the coverage of industries in the BTA implementation. For example, our results suggest that if a BTA is levied only on oil products, then its impact is sufficient to make the U.S. worse off relative to the non-BTA policy.

Chapter 3 introduces the learning by lending as a potential channel of understanding the business cycle fluctuation under an imperfect credit market. Understanding persistent output fluctuations with imperfect financial markets remains an important topic of macroeconomics. It is widely acknowledged that asymmetric information between borrowers and lenders creates deadweight losses. Most of the existing literature in this area addresses asymmetric information between lenders and borrowers by introducing collateral, while paying little attention to how lenders might learn about borrowers' private characteristics through the lending process. Intuitively, learning should happen in financial interactions where borrowers can be matched with the same lenders.

The main objective of this chapter is to determine whether the learning by lending channel can affect real macro variables. We develop a simple model in which lenders adopt a rational approach and learn about borrowers based on their choices of in vestment and production. However, this information acquisition is disrupted when a negative shock occurs, because a number of existing borrowers default. It takes several periods for accumulated information to rebuild. We show that this process can generate persistent investment and output effects. This chapter also offers a simple explanation for why the cost of external finance can fall over time, other than those explanations provided in net worth or collateral related models.

We also show that the impact of asymmetric information on aggregate output fluctuations is asymmetric. Changes in lender information have more notable impacts on the business cycle when bad shocks occur relative to good ones.

Chapter 1

Can Higher Carbon Taxes Be Progressive?

1.1 Introduction

It is generally perceived that the imposition of carbon taxes may not proportionally affect the metrics of individual household welfare. Asking whether a carbon tax is progressive can be seen as a provocative questioning. The reality is that the impact of a carbon tax on households (progressive or regressive) is still questionable in light of the differences in its incidence on inequality, particularly when assessed from either the demand or supply sides of household welfare metrics.

Indeed, most studies rely on the demand-side channel in their assessments of the impact of carbon taxes on relative prices of commodities. These studies generally find that carbon taxes exert a regressive impact on household welfare. The main reason behind these findings is that the increase in the prices of energy and energy-intensive goods, brought about by a carbon tax, hurts the poor more than the rich, as the former spend a larger proportion of their income on those goods than the latter do.

For example, Robinson (1985) finds that the incidence of industrial pollution abatement tax is heavily regressive.[1] Similarly, Hamilton and Cameron (1994) analyze the distributional effects of a carbon tax on Canadian households and find that the consequences of the tax are regressive. Wier et al. (2005) and Dinan and Rogers (2002) find similar results for Denmark and the Netherlands, respectively. Kerkhof et al. (2008) and Shammin and Bullard (2009) further confirm these results for the U.S. economy. All these studies have ignored the impact of carbon taxes on factor

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[1] See also Dubin and Henson (1988).

incomes, and ultimately on inequality. As long as a carbon tax policy carries differentiated implications for factor remunerations, the sources of income gain importance in the assessments of the the policy's impact on inequality. Taking into account the fact that the rich derive most of their income from capital, compared to the poor, assessing the impact of a carbon tax policy on inequality has the potential to provide completely new insights.

Within a general equilibrium setting, Fullerton and Heutel (2007) have shown that pollution control policies can harm the remuneration of capital more than that of labour. The main reason for this is that the polluting industries are more capital intensive than other industries (see Hettige et. al. (1992)). Implementing policies that affect negatively the former industries will be detrimental to the factor they use intensively. Under these circumstances, and looking from the factor income perspective alone, implementing a pollution control policy could have a progressive impact as the income of the affluent will be more negatively affected than that of the poor. It is important to note that we are not referring to the impact of the policy on social welfare, but rather on inequality; here, we are interested in its distributional impacts. Moreover, the progressiveness of the carbon tax policy, which we are referring to, does not stem from a revenue-recycling approach, as argued by Burtraw et al. (2009) and Bento et al. (2009)

Unfortunately, most of the studies found in the literature on the incidence of pollution control policies on inequality have exclusively considered the demand-side channel, i.e., their impact on the relative prices of commodities.[2] Yet, individual household welfare depends not only on commodity prices, but also on income. This suggests that most of the analyzes on the impact of carbon taxes on inequality are incomplete as they overlook an important channel, that is, their impact on inequality through factor incomes.

Our objective in this chapter is to offer a comprehensive analysis of the distributional impact of carbon taxes considering both the demand and the supply channels through which a policy might have an incidence on inequality. When viewed from this more holistic perspective, there is no definitive answer as to the impact of pollution control policies on inequality. This is attributed to the presence of two opposing

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[2] One notable exception to this is the recent paper by Araar et al. (2011) that analyzes the incidence of carbon mitigation policies on social welfare.

effects: a regressive impact through the relative prices of commodities, and a progressive impact through factor remuneration.

The final impact is an empirical matter that deserves to be investigated. We assess the possibility of a carbon tax being progressive, i.e., whether there exists some value of a carbon tax, where its positive impact on household welfare distribution through the supply side outweighs its negative impact induced through the demand side. We are not aware of any other paper that offers an analysis of the incidence of carbon taxes on inequality, while disentangling the differing effects arising from both the demand- and the supply-side channels. To do so, we combine general equilibrium analysis with income distribution analysis.[3]

Central to our analysis is the decomposition of the post-policy welfare metrics into different components, which include initial expenditure, the contribution of the changes induced by variations in commodity prices, and the contribution of the changes induced by variations in factor prices. For this purpose, we choose the equivalent income as the household welfare metrics to evaluate the incidence of changes in commodity and factor prices on household welfare. Later, we assess the contributions of the last two components in the change in inequality.

Among one of the earlier studies on the change in income inequality by factor components are Fei et al. (1978) who evaluate the U-shaped impact of growth on income distribution using an econometric approach to estimate the parameters. They argue that the weighted average estimator of the factor Gini coefficients, which they call “pseudo-ginis”, can represent the true Gini if their nonlinearity error term is small. Nevertheless, Shorrocks (1983) suggests that the “pseudo-gini” does not generate a unique decomposition formula; it rather represents only one decomposition method among other infinite possibilities. They also argue that the nonlinearity error term may not always be insubstantial. Lerman and Yithzaki (1985) introduce a new approach to estimate the income inequality effect by income sources. Using a natural approach, they show that each source’s contribution to Gini coefficient may be taken as the product of the source’s own Gini, its share of total income, and its correlation

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[3] A very recent US study by Metcalf et al. (2010) raises the issue of considering the income side in the analysis of the incidence of climate change policies. Nevertheless, their analytical framework is completely different from the one we suggest in this chapter. They analyze the effects of different emissions and revenue allocating approaches on household welfare measured in terms of equivalent variation. In contrast, we consider equivalent income as the household welfare metric and stress the importance of the distributional impact by analyzing inequality through a decomposition method of those elements considered important in income and spending decisions of households.

with the rank of total income.

Later on, Shorrocks (1999) and Chantreuil (1999) introduce the Shapley value as an exact substitute of the natural approach. The advantage of the Shapley decomposition approach is its simplicity and its avoidance of complex econometric techniques. However, some papers, such as Sastre and Trannoy (2002), have addressed one of the difficulties of using the Shapley decomposition method, i.e., its assumption of independency of inequality of a given income source from others.[4] Makdissi and Woddon (2004) show that the problem of interdependency can be controlled at some level.[5] An alternative approach showing income inequality by factor components is one introduced by Rao (1969). According to this approach, an inequality index of a variable can be decomposed as a weighted sum of the concentration indices of the component variables that add up to that variable (see also Araar (2008)). One advantage of using this approach is that it makes it possible to show the contribution of each component of a variable in an ex-post inequality index.

In this chapter, we apply both approaches, i.e. the Shapley value and the concentration indices. To assess change in inequality by demand and supply components, we first develop a static, multi-sector, and multi-household general equilibrium model of the Canadian economy and run several simulations with different values of the carbon tax. As equivalent income depends on both commodity and factor prices, we are able to assess the contribution of both demand- and the supply-side channels alluded to above in evaluating the impact of our chosen GHG control policy on inequality. Later, we use our CGE results to estimate inequality by each source of income variations.

1.2 Theoretical Background

This section presents a theoretical framework to our central argument that the pollution taxes may reduce the ratio of the rental rate of capital to the wage rate. For this purpose, we introduce a very simple model that will be useful for providing a good grasp of the impact of pollution taxes on factor prices. In particular, we assess

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[4] They argue that the contribution assigned to any income source is sensitive to the way in which other sources are grouped.

[5] They suggested the solutions of performance overlapping among many simultaneously implemented poverty reductions programs by introducing average measures of the marginal contribution of each program based on all the possible permutations of the various policies implemented by the government

the impact of a pollution tax on the relative price of capital and labor. The model will later help in building intuition on the distributional impact of carbon taxes on households. As argued before, a general equilibrium setting is the most appropriate framework for capturing the impact of a pollution tax on factor prices. The minimal representation of the environment required for this purpose consists of two firms, two production factors, and one household. The rationale for the sufficiency of one representative household at this stage rests on our assumption that households have Gorman preferences, whereby, without loss of generality, a representative consumer's preferences can be used to compute household demand.

In what follows in this section, we consider a closed economy that consists of two firms, one representative household and the government whose role is very basic. Each firm produces one good indexed by i= (1, the clean good, and, 2, the dirty good) by combining capital and labor with a constant-returns-to-scale technology. For the moment, we assume that no intermediate inputs are used in production activities.[6] The supplies of capital and labor are fixed and they are owned by the representative household. The latter has homothetic preferences over the two goods and derives income from the ownership of the primary factors and from tax revenue. Pollution is assumed to stem for the use of the dirty good alone according to a fixed proportion rule. As there are no intermediate inputs, pollution emanates from the consumption of the dirty good. The government's objective is to reduce pollution by imposing a tax on the consumption of the dirty good. For the sake of simplicity, and without loss of generality, we assume that the pollution tax is an ad valorem tax, t, that is imposed on the value of the dirty good. Alternatively, the pollution tax can be represented by the gross tax, $\tau = (1 + t)$. The proceeds of the taxes are accumulated in government savings so as to avoid a case of tax neutrality.[7] Finally, all agents operate in a competitive environment.

To achieve our objective, we will first characterize the consumer's and firms' behaviors and assess, in a general equilibrium setting, the impact of the pollution tax on the relative producer prices of the two goods as well as its impact on relative factor prices. As in most general equilibrium models, we are interested in the changes in relative prices; hence, our discussions below will mostly focus on the price and volume ratios instead of their levels.

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[6] We relax that assumption in a more broader and computable version of the model.

[7] For detailed explanations, see section 1.3.

1.2.1 The Consumer Problem

We assume that consumer preferences can be represented by a twice-continuously differentiable utility function. Let P_1 and P_2 be the producer prices of X_1 and X_2, which represent the demands for goods 1 and 2 by the consumer. As consumer preferences are homothetic, at the optimum, the ratio of the demand for the two goods $\frac{X_2}{X_1}$ is independent of the level of income; it depends on the relative price only.

$$\frac{X_2}{X_1} = f\left(\frac{P_1}{\tau P_2}\right) \qquad \textit{with } f' > 0 \tag{1.2.1}$$

By log-differentiation of Equation (3.2.1) we have:

$$d\ln\left(\frac{X_1}{X_2}\right) = -\sigma_d\left[d\ln\left[\frac{P_1}{P_2}\right] - d\ln\tau\right] \tag{1.2.2}$$

where σ_d is the elasticity of substitution between the two goods, which is a local measure of the curvature of their indifference curve. In Equation 1.2.2, an increase in the pollution tax, will reduce the ratio of the relative demand of the dirty good.

1.2.2 The Producer Problem

Each firm combines capital and labor to produce output, Q_i, using a constant-returns-to-scale technology that can be represented by a well-behaved twice-continuously differentiable cost function. The firm producing the dirty good is capital intensive, and we assume that there is no reversal in capital intensity, in the sense that the existing hierarchy in capital intensities holds for all vectors of positive input prices. Assuming that the two factors are fully used (equilibrium in both factor markets), the aggregate output in this economy is fixed. The combinations of the maximums of each good that can be produced is represented by a concave transformation curve. The concavity of the transformation curve stems from the assumptions of constant returns to scale technology and differing capital intensity ratios. As is well known, in a competitive environment, at a given ratio of output prices, the optimal supply of the two goods is such that the marginal rate of transformation is equal to the price ratio. In particular, the ratio of output depends on the ratio of output prices:

$$\frac{Q_1}{Q_2} = g\left(\frac{P_1}{P_2}\right) \qquad \textit{with } g' > 0 \tag{1.2.3}$$

By log differentiation of Equation (1.2.3) we have:

$$d\ln\left(\frac{Q_1}{Q_2}\right) = \sigma_s d\ln\left(\frac{P_1}{P_2}\right) \quad \textit{with } \sigma_s > 0 \tag{1.2.4}$$

The parameter σ_s is the elasticity of substitution in supply between the two goods. The parameter σ_s depends on the elasticities of substitution between capital and labor, the capital intensities in both firms, and on the shares of each input used in each firm. Its positive sign has bearing with the concavity of the transformation curve. Assuming equilibrium in the output markets, Equation (1.2.4) can thus be rewritten as follows:

$$\ln\left(\frac{X_1}{X_2}\right) = \sigma_s d\ln\left(\frac{P_1}{P_2}\right) \tag{1.2.5}$$

We thus have two equations, Equations 1.2.2 and 1.2.5, that can be used to assess the impact of the change in the pollution tax, τ on the relative price of the two goods. Hence our first proposition:

Proposition 1. *If the dirty and clean goods are substitutes in demand, under the assumptions that both firms use linear homogenous technologies and that the production of the dirty good is capital intensive, an increase in the pollution tax will increase the relative producer price of the clean good.*

Proof: The proof of Proposition 1 is straightforward. Combining Equations 1.2.2 and 1.2.5, it can easily be shown that:

$$\frac{d\ln\left(\frac{P_1}{P_2}\right)}{d\ln\tau} = \frac{\sigma_d}{\sigma_d + \sigma_s} \tag{1.2.6}$$

From Equation 1.2.2 the relative price of the clean good is positively related to the pollution tax when the elasticity of substitution in demand is positive.[8] The intuition behind this result is that an increase in the pollution tax increases the relative demand of the clean good that can only be achieved in equilibrium if the relative supply increases as well. Given the shape of the transformation curve, this can only occur with an increase in the relative price of the clean good. We are now left with the impact of the pollution tax on the relative factor prices.

With constant-returns-to-scale technology, the cost function, $C_i(w, r, Q_i)$, is linear in output and has the following general expression:

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[8] From our assumption on the technology of the two goods, σ_s is always positive

$$C_i(w, r, Q_i) = \min_{K_i, L_i} \{rK_i + wL_i : F_i(K_i, L_i) \geq Q_i\} \tag{1.2.7}$$

where w, r, and C_i respectively, the wage rate, the rental rate of capital, and the unit cost of production and $F_i(K_i, L_i)$ is a well-behaved production function.

$$C_i(w, r, Q_i) = Q_i c_i(w, r) \tag{1.2.8}$$

At the optimum, each firm sets its price equal to its marginal cost and determines the optimal level of input through cost minimization. The first-order conditions of their optimization problem are as follows.

$$P_i = c_i(w, r) \tag{1.2.9a}$$

$$L_i = \frac{dc_i(w, r)}{dw} Q_i = a_{L_i} Q_i \tag{1.2.9b}$$

$$K_i = \frac{dc_i(w, r)}{dr} Q_i = a_{K_i} Q_i \tag{1.2.9c}$$

Equation (1.2.9a) reflects the marginal cost pricing rule and Equations (1.2.9b-1.2.9c) are the conditional factor demands for labor and capital, respectively. a_{Li} and a_{Ki} are, respectively the quantities of labor and capital required to produce one unit of output i. Let us continue to apply the smokestack concept according to which the emission-intensive sector is also capital-intensive. Hence, the assumption that the ratio of capital to labor is higher in Firm 2 than the one in Firm 1, implies that $a_{K2}/a_{L2} > a_{K1}/a_{L1}$ or equivalently, $a_{K2}/a_{K1} > a_{L2}/a_{L1}$.

It is easy to show through total differentiation of Equation (1.2.9) that the following relations exist between the percentages changes in output and factor prices

$$d\ln(P_1) = \theta_{L1} d\ln(w) + (1 - \theta_{L1}) d\ln(r) \tag{1.2.10a}$$

$$dln(P_2) = \theta_{L2} d\ln(w) + (1 - \theta_{L2}) d\ln(r) \tag{1.2.10b}$$

where θ_{Li} is the share of labor in the cost of good i; it can also be shown that $\theta_{Li} = a_{Li} w / P_i$.

This system of two equations with two unknowns ($d\ln(w)$ and $d\ln(r)$) has a unique solution if and only if its determinant is different from zero. Yet, it can easily be shown that the determinant, which is equal to $(\theta_{L1} - \theta_{L2})$ is indeed different from

zero for all vectors of input prices, with the assumptions of different capital intensities in the two firms and no capital intensity reversal.
The solution to the system of equations yields:

$$d\ln(w) = (1-\theta_{L2})d\ln P_1 - (1-\theta_{L1})d\ln P_2 \tag{1.2.11a}$$

$$d\ln(r) = \theta_{L1}d\ln P_2 - \theta_{L2}d\ln P_1 \tag{1.2.11b}$$

We thus have the following proposition with regards to the impact of pollution tax on the relative price of factors:

Proposition 2. *If the dirty and clean goods are substitutes in demand, the dirty good is capital intensive, and both firms use linear homogenous technologies, an increase in the pollution tax decreases the relative price of capital.*

Proof The proof of this proposition is straightforward.

$$\frac{d\ln(w)-d\ln(r)}{d\ln(\tau)} = \frac{d\ln(w)-d\ln(r)}{d\ln(P_1)-d\ln(P2)}\frac{d\ln(P_1)-d\ln(P2)}{d\ln(\tau)} \tag{1.2.12}$$

Using equations (1.2.6,1.2.11a,1.2.11b), we have:

$$\frac{d\ln(w)-d\ln(r)}{d\ln(\tau)} = \frac{1}{\theta_{L1}-\theta_{L2}}\frac{\sigma_d}{\sigma_d+\sigma_s} \tag{1.2.13}$$

The right-hand side of expression (1.2.13) has a positive sign since the ratio of labor to capital is higher in the firm that produces the clean good than in the other. The intuition behind this result follows from the one discussed in the previous proposition and the Stolper-Samuelson theorem. The latter suggests that there is a positive relationship between the relative price of a good and the relative price of the factor used intensively in the production of that good.

1.2.3 Incidence on Equivalent Income

To explain the inequality and welfare consequences of pollution tax on income distribution, we apply the equivalent incomes approach as suggested by King (1983).[9] Assume that the reference or pre-policy budget constraint of a household is denoted as (P^0, M_h^0) where P is the vector of the household's consumption prices and M_h is its income. Throughout this section, the superscripts 0 and 1 represent the pre- and

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[9] An alternative measure of capturing welfare is living standard. For more detail, see Shorrocks (2004).

post-policy status of a variable, respectively. The equivalent incomes (total expenditure) can be defined as the value of total expenditure, M_h^E, that at the reference prices , leaves the household at the same level of utility as that obtained under the current or post-policy budget constraint (P^1, M_h^1):

$$v(P^0, M_h^E) = v(P^1, M_h^1) \tag{1.2.14}$$

We assume that the household has a Cobb-Douglas utility function, which it maximizes subject to a budget constraint.

$$\begin{aligned} \max_{X_1,X_2} U &= X_1^{\beta} X_2^{1-\beta} \\ \textit{subject to } M_h &\geq P_1X_1 + P_2X_2 \quad \textit{and} \quad \textit{with } 0 < \beta < 1 \end{aligned} \tag{1.2.15}$$

For a given vector of prices and income, the indirect utility function of household h has the following expression:

$$v(P, M_h) = \left(\frac{\beta}{P_1}\right)^{\beta} \left(\frac{1-\beta}{P_2}\right)^{1-\beta} \tag{1.2.16}$$

Considering Equations (1.2.14) and (1.2.16), we may write the following relationship between equivalent incomes and the household income after the reform:

$$M_h^E = M_h^1 \left(\frac{P_1^1}{P_1^0}\right)^{-\beta} \left(\frac{P_2^1}{P_2^0}\right)^{\beta-1} \tag{1.2.17}$$

The first-order approximation of Equation (1.2.17) can be rewritten in terms of the percentage changes in the variables (logarithmic derivatives) on its Right-hand-side after the reform.

$$M_h^E = M_h^0(1 + d\ln M_h)\left[1 - \beta d\ln P_1 - (1-\beta)d\ln P_2 + \xi\right] \tag{1.2.18}$$

where ξ is the combined effect of the commodity price changes.

Assuming that the consumer's total income M_h consists of labor and capital incomes, the percentage change in his total income can be derived as follows:

$$M_h = w\overline{L}_h + r\overline{K}_h \tag{1.2.19}$$

$$d\ln M_h = \theta_{Lh} d\ln w + \theta_{Kh} d\ln r \tag{1.2.20}$$

where $\overline{L}_h$ and $\overline{K}_h$ are, respectively, the consumer endowments of labour and capital, and θ_{Lh}, and θ_{Kh} represent, respectively the shares of labour and capital in total income.

Combining Equations (1.2.18 and 1.2.19), the equivalent incomes, M_h^E, can be decomposed as follows:

$$M_h^E = M_h^0(1+\theta_{Lh}d\ln w + \theta_{Kh}d\ln r)\,[1-\beta d\ln P_1 - (1-\beta)d\ln P_2 + \xi] \tag{1.2.21a}$$

$$M_h^E = M_h^0 + M_h^0(\theta_{Lh}d\ln w + \theta_{Kh}d\ln r) - \beta M_h^0 d\ln P_1 - (1-\beta)M_h^0 d\ln P_2 + \Psi_h \tag{1.2.21b}$$

$$M_h^E = M_h^0 + \delta_h^1 + \delta_h^2 + \delta_h^3 + \delta_h^4 \tag{1.2.21c}$$

where

$$\delta_h^1 = M_h^0(\theta_{Lh}d\ln w + \theta_{Kh}d\ln r) \tag{1.2.22}$$

$$\delta_h^2 = -\beta M_h^0 d\ln P_1 \tag{1.2.23}$$

$$\delta_h^3 = -M_h^0(1-\beta)d\ln P_2 \tag{1.2.24}$$

$$\delta_h^4 = M_h^E - (M_h^0 + \delta_h^1 + \delta_h^2 + \delta_h^3) \tag{1.2.25}$$

Equation (1.2.21c) suggests that the equivalent incomes after the reform can be decomposed into five components: (i) the equivalent incomes before the reform, M_h^0; (ii) the impact of the change in factor prices, δ_h^1; (iii) the impact of the change in the price of the clean good, δ_h^2; (iv) the impact of the change in the price of the dirty good δ_h^3; and (v) the residual that captures the combined impacts of changes in factor prices and in commodity prices, δ_h^4. Since the main focus of our analysis is to evaluate the contribution of each component to the change in the post-policy income inequality among households, we can reformulate Equation 1.2.21c as:

$$d_h \equiv \triangle M_h = \sum_{n=1}^{4} \delta_h^n \tag{1.2.26}$$

Where d_h refers to the post reform variation in the total equivalent incomes of the household of category h. However, it is worth mentioning that the impact of an individual component in the variation of equivalent incomes may be positive or negative, depending on the post-reform changes in factor and commodity prices. For example, Equation 1.2.22 implies that the post policy decline in factor prices accounts for a fall in the equivalent incomes of the household through this component.

A similar intuition can be extracted through Equation 1.2.24 if the post reform commodity prices of dirty goods increase as compared to their business as usual price levels. On the other hand, Equation 1.2.23 implies the post-policy decline in non-energy prices, as compared to the pre-policy levels, causes an increase in the equivalent incomes of the household by this component. The overall incidence of the pollution tax policy on the household's equivalent incomes is negative if the net impact of all component is negative. We will come back to a detailed analysis of the post reform impact of each component on total equivalent incomes of the household in section 1.5.

1.2.4 Income Inequality and Decomposition Rule

Consider a population equally distributed among H household groups. Households are ranked according to the size of their equivalent incomes (total expenditures) $M_1 \leq ..M_h \leq .. \leq M_H$ such that the mean of equivalent income across H households is $\mu(M)$. In addition, δ_h^n $(n = 1, 2, 3, 4)$ is the *post-reform* n^{th} source of variation in a household's equivalent income, which can be aggregated for H households in order to obtain its mean by component, $\mu(\delta^n) = \frac{1}{H}\sum_{h=1}^{H} \delta_h^n$.

The distributional impact of a pollution tax on household's equivalent income can be expressed as follows;

$$G(M^E) - G(M^0) = \sum_n G(\delta^n) \tag{1.2.27}$$

Where G^E and G^0 refer to Gini indices of equivalent income for *pre-* and *post-*policy reform, respectively. Since, for a pre-reform case, a Gini coefficient and concentration ratio, $C(M^0)$, will be the same, the left hand side of Equation 1.2.27 can be estimated by the difference between $G^E - C^0$. The variation in aggregate inequality can be decomposed by its sources. Notice that the right hand side of Equation 1.2.27 indicates a linear combination of Gini by different sources of variations in equivalent income. This additivity feature allows us to use the Shapley decomposition approach, that is often applied in cooperative game theory to evaluate the strategic contribution of each player to overall coalition. To capture player i's contribution in overall gains, there is a need to estimate the marginal payoff, i.e., the payoff from total collaboration relative to all possible sub-coalitions (that exclude player i). Since these sub-coalitions are random where each player (expect i) has equal chance to become part of sub-coalition. Hence, the contribution of player i in total payoff is simply the expected marginal contribution of player i relative to all possible sub-coalitions (excluding player i).

A similar, concept of the Shapley value has recently received attention in inequality analysis, where it has been used to derive the contribution of each component of income to the overall income inequality (see Shorrocks (1999), Chantreuil and Trannoy (1999), Kolenikov and Shorrocks (2005), Bibi and Duclos (2008)). In our case, we are interested to apply the Shapley approach to estimate the contribution of each component (i.e.,$G(\delta^n)$) in total post-policy income inequality i.e., $G(M^E)$. For this purpose, consider a set of all components N and a subset $S \subset N$. Assume that the right hand side of Equation 1.2.27 is $G(N) = \sum_n G(\delta^n)$. Also assume that a component δ^n is not included in S while the numbers of components in sets S and N are, respectively, s and k. In this case, one possible marginal contribution of component n in inequality is $\Delta G(n) = G(N) - G(n \notin S)$. To estimate the contribution of component n in total post-reform income inequality, there is need to estimate all possible marginal impacts of component n over the set of components in $n \notin S$. Given a random draw from set $n \notin S$, the expected marginal contribution (i.e., the Shapley value) of component n in total inequality subject to $n \notin S$ is:

$$G(\delta^n) = \sum_{S \subset N} \sum_{n \notin S} \frac{s!(k-s-1)!}{k(k-1)!} G(S \cup n) - G(n \notin S) \tag{1.2.28}$$

Equation 1.2.28 refers to the product of two probabilities. The first one refers to the probability that $n \notin S$ is $1/k$. The second probability is $\frac{s!(k-s-1)!}{(k-1)!}$, which means that (when n is excluded) there is permutation of s components (i.e., s!) from the possible permutation of (k-1)! components while for every permutation in the subset S, there are $(k-s-1)!$ permutations for the components that complement the subset S. Summing the contributions of all individual components equals to the right hand side of Equation 1.2.27, which is the variation in total Gini index due to a policy reform.

An alternative way to assess the *post policy* change in inequality of total equivalent income is the exact decomposition approach, explained by Araar (2008) and

Duclos and Araar (2006). Following them, the *policy-induced* change in total income inequality with respect to a variation in one of its component can be expressed as:

$$G(\delta^n) = \frac{\mu(\delta^n)}{\mu(M^0)}[G(M^0) - C(\delta^n)] \qquad (1.2.29)$$

Equation 1.2.29 indicates that the change in total income inequality due to a variation in the n^{th} source depends on the average size and on the concentration index belongs to the variation in income through that source relative to total *pre-reform* equivalent income. In other words, under a tax policy reform, if $G^0 < C^n$, it suggests that the incidence of the n^{th} component is more concentrated towards the higher income groups relative to that of the inequality through total income. In this case, the incidence of such *tax policy* on equivalent income through the n^{th} source would be progressive. Conversely, it would be regressive. Plugging Equation 1.2.29 in Equation 1.2.27 enables us to estimate the incidence of a pollution tax on income distribution.

1.3 The Numerical Model

We develop a small open, static, multisector, and multi-household general equilibrium model to assess the distributional impact of a carbon tax in light of the theoretical discussions presented in the section above. A noticeable departure of this numerical model, from the analysis we presented above is that, not only do we have more than two commodities, but we also consider an open economy that trades goods and services with the rest of the world. The model is in the tradition of computable general equilibrium models used to assess climate change policy options in several countries. In particular, it has bearing with some recent interesting contributions to the literature on the general equilibrium modeling of climate change in a multisector and static setting, such as that of Araar et al.(2011) and Böhringer and Rutherford (2010). A noteworthy difference in the model discussed here from that presented in the above-mentioned reference is that instead of using a single representative household, we consider several categories of households that are distinguished by their total income category. In addition to households, the economy consists of firms, the government and the rest of the world. The economy is disaggregated into 39 industries, indexed by j, and 43 commodities indexed by i. The number of commodities is larger than that of industries, because some industries, like the oil and gas industry produce more than one commodity (crude oil and natural gas in their case).[10]

We assume that the use of fossil fuels generates carbon dioxide (CO_2) whose level the government desires to regulate using a carbon tax as a policy instrument. All agents consider prices as given and must respect their budget constraints. It is worth mentioning that in addition to the impact of the tax on the consumption good as explained in our theoretical discussions, we also have a direct impact of the tax on fossil fuels that are used as intermediate inputs. Hence, the carbon tax will also have a direct impact on the producer prices of non-fossil-fuel goods. The extent of the producer price increase of the latter goods will depend on their carbon content. Finally, capital and labour are assumed to be mobile across industries.

In order to avoid the black-box syndrome related to CGE models, in what follows, we provide the key features of the behavioral aspects of the economic agents as well as the resource constraints they face. We believe that a discussion of these features will allow the reader to understand the intuition behind the numerical results of the model. Nevertheless, we provide in the appendix to this chapter a full listing of the model equations along with the definitions of variables and parameters.

1.3.1 Households

We disaggregate the household sector into 100 categories, indexed by h, according to a quantile ranking that is based on the total income of each household category. The preferences of the representative household of each category over the 43 commodities is represented by a Cobb-Douglas (C-D) utility function[11]. The household is assumed not to value leisure, hence, its labour supply, L_h^s, is inelastic. It follows that there is a fixed endowment of total labour supply available in the economy. In the same vein, each household category has a fixed endowment,[12] K_h, of the total supply of the capital stock in the economy. The household derives income from wages, capital income, transfers from the government, and net transfers from the rest of the world. It pays taxes on income and consumption goods and saves a fixed fraction of its disposable income. The representative household's problem is to choose its optimal combination of consumption goods by maximizing its utility function subject to the

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[10] See section 1.4 for details of industries and commodities.

[11] In the remainder of the text, we may use (for the sake of simplicity), the word household to designate the representative household of each category.

[12] However, the size of endowments is not uniformly distributed across households. In the next section, we will discuss about the allocation of endowments and the size of consumption baskets across households with more detail.

budget constraint:

$$\max_{C_i} \prod_{i=1}^{n} C_{hi}^{\beta_{hi}} \quad with \quad \sum_i \beta_{hi} = 1$$

subject to

$$\sum_i P_i^c C_{hi} \leq (1 - t_{fh})(wL_h^s + RK_h) + TR_h^G - eTR_h^{fr} - S_h \equiv M_h \tag{1.3.1}$$

where M_h, P_i^c, C_{hi}, are respectively, total expenditures, the price and the demand for the consumption good; t_{fh}, is the household income tax rate; w, is the wage rate; and, L_h^s is the household's labour supply. RK_h is the return to capital received by each household from the firms, while, TR_h^G, TR_h^{fr}, S_h, and e are respectively, transfers from the government, net transfers from the rest of the world (in world prices), savings, and the nominal exchange rate.[13] β_{hi} is a parameter of the utility function. As will be discussed below, the consumption good is a composite of the domestically produced and imported goods, and its price incorporates the effects of the carbon taxes that have been imposed on the uses of the intermediate inputs during its production.

It is straightforward to show that the demand for each commodity by each household has the following expression:

$$P_i^c C_{hi} = \beta_{hi} M_h \tag{1.3.2}$$

As saving is a fixed fraction of disposable income, it can be expressed as follows:

$$S_h = s_h \left[(1 - t_{fh})(wL_h^s + RK_h) + TR_h^G - eTR_h^{fr} \right] \tag{1.3.3}$$

Finally, the return to capital received by each household, RK_h, is a fixed proportion, β_h^K, of total dividend payments received from the firms. It is important to note that the sum of the dividends paid is lower than the total remuneration of capital in the economy, as firms keep a fixed proportion, β^{KE} for investment purposes. Hence RK_h has the following expression:

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[13] It is important to note that without income tax and saving, the total expenditures, M_h, will be identical to the concept of total income that we used in our analysis of the decomposition of household equivalent incomes in the previous section.

$$RK_h = \beta_h^K (1 - \beta^{KE}) \sum_j r K_j^d \tag{1.3.4}$$

where r, and K_j^d, are, respectively, the rental rate of capital, and the sectoral demand for capital by each industry, while t_b refers to the tax on entrepreneurial earnings.

1.3.2 Production

The representative firm in each industry has access to a linear homogeneous production function to produce a composite good that can be sold in the domestic and exports markets. The production technology is assumed to be weakly separable, uses capital, labour, fossil inputs and non-fossil inputs to produce the gross output. The weak separability property of the technology is captured by the nested structure of production as depicted in Figure 1. An interesting property of this separability is that it makes it possible to account for the substitution possibilities that are offered to the firms, as far as the use of energy products is concerned. This distinction is all the more important in the context of a pollution tax that is based on the carbon content of the fuel used.

As shown in Figure 1.1, at the top level of the production structure, the composite of output is a Constant Elasticity of Substitution (CES) of the aggregate of the composite of value-added and energy, KEL_j, and of the composite of intermediate inputs V_j. At the second level, the composite KEL_j is a Cobb-Douglas aggregate of labour, L_j^d, and a composite of capital and energy KE_j. At the third level of the nesting structure, the sectoral capital stock K_j is combined with the composite of energy inputs, E_j through a CES aggregation function to produce KE_j. The composite of energy input is a CES function of electricity and an aggregate of fossil fuels. The latter is another CES combination of coal, natural gas, and the composite of refined petroleum products. The composite of intermediate inputs is a CES function of a composite of a C-D aggregate of motive fuels and a Leontief aggregate of the other material inputs. An overview of the production structure is depicted in Figure 1.1.

The representative firm pays taxes net of subsidies on gross output and maximizes profits in order to determine the optimal levels of inputs. The traditional principle of equalizing the marginal product of the input to its price applies in this setting.

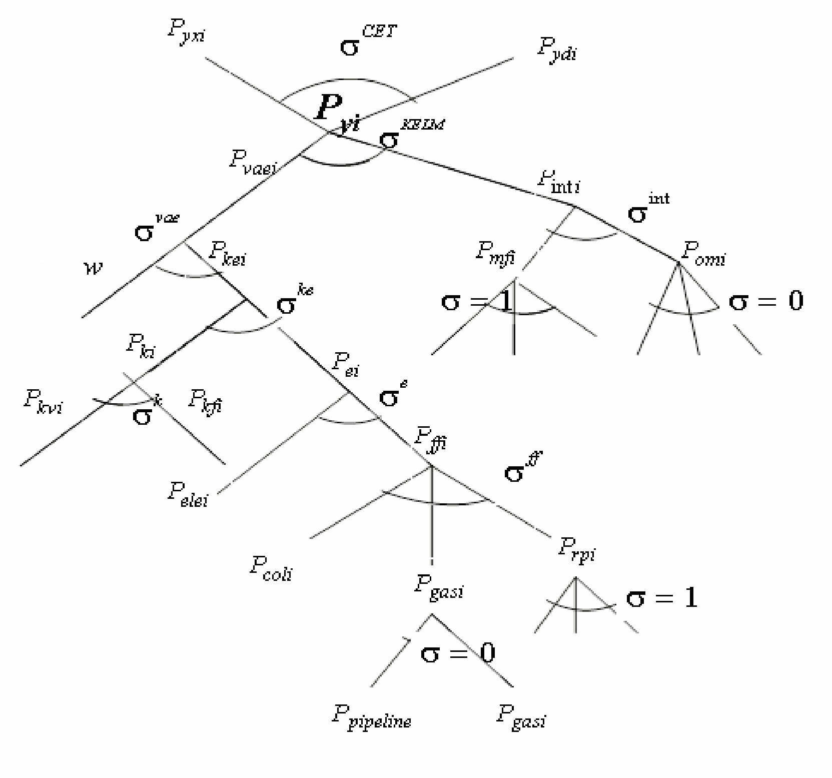

Figure 1.1: An overview of Production Structure of Sectors

Note that the price of a composite input is its dual price, which is obtained through a cost-minimization principle. An increase in the carbon tax will not only reduce the demand for the fossil fuel, but will also trigger a cascading increase in the dual prices of composite inputs and will eventually lead to an increase in the producer price of the composite good.

As mentioned above, the representative firm produces a composite good that is made of the domestic good and the export good. Both are imperfect substitutes and their production technology is captured by a concave transformation curve. Notably, a Constant Elasticity of Transformation (CET) function is used to transform the gross output into sales in domestic market and exports. The representative firm uses a revenue-maximizing principle subject to the technological constraint (represented by the transformation curve) to determine the optimal allocation of the gross output into sales in the two markets. For example, at the optimum, everything else equal, the increase in the relative price of exports will reduce the ratio of domestic sales to those in foreign markets.

Finally, the representative firm returns a fixed proportion, β^{KE}, of the return to physical capital to households as dividend payment, pays business income taxes and uses the remainder for investment purposes.

1.3.3 Government, Equilibrium Conditions and Closure Rules

The government collects direct and indirect taxes from households, firms, and international trade activities, including pollution activities. The government also provides subsidies and makes transfers to households excluding the revenues generated from pollution tax. It is compelled to balance its budget. However when a carbon tax in imposed, then to avoid a case of pollution tax neutrality, all revenues generated from carbon tax, go to the government saving. A similar approach of “No pollution tax neutrality” is adopted by Fullerton and Heutel (2007) so as to avoid the impact of pollution tax revenue recycling on the changes in relative factor or commodity prices. In addition, since our focus is the incidence of carbon tax on households, we keep government demand for goods exogenous.

The general equilibrium of this economy is represented by a vector of price and quantity variables such that all agents maximize their objective functions while respecting their respective budget constraints, and all markets clear. In equilibrium, the following conditions should be met: i) zero profit or no arbitrage conditions by all representative firms ii) income balance conditions, in which the household’s income must be equal to its expenditure and government revenue must be equal to its expenditure iii) markets clearing conditions, which implies equilibrium in the labour market, capital market, domestic goods market, and foreign exchange markets.

The model is real in the sense that changes in nominal variables do not have any impact on the real variables; only relative prices matter. For, this purpose, there is need to specify a numeraire, which is exogenous and all other prices need to be define relative to the numeraire price. It refers to a conversion factor that translate world prices of imported and exported goods into a domestic price. Literatures called this numeraire as “nominal exchange rate” and a model-based domestic prices relative to this fixed numeraire is called “real exchange rate” (see De Melo and Robinson (1989)), which is taken as a closure rule, meaning that foreign saving must be exogenous. In that case, current account balance is exogenous, implying that change in imports of one goods will change the real exchange rate belongs to that goods. Since foreign exchange is fixed, there is need an offset in the exports or imports of one or more goods.

Another important closure issue is to maintain saving-investment balance. Private savings (i.e., households and business savings), government savings, and foreign savings all are included into this balance. Therefore, under a model calibration with no carbon tax policy, the government transfer variable is used as closure rule to maintain saving-investment balance and all types of savings and government and investment demands are taken as exogenous.[14] It is worthy to recall again that we are not referring to the impact of the policy on social welfare, rather we focuss on changes in inequality at different carbon taxes.

Carbon emissions embodied in each commodity are represented by an emissions coefficient, which is the ratio of total emissions associated with a commodity divided by the total consumption of this commodity by all households and firms. The multiplication of the emissions coefficients of each commodity by the commodity demands by all households and firms gives us the share of that commodity in total emissions. Assume that $Emit_i$ is an emissions factor (i.e., the quantity of CO_2, in tons, which emits in the burning of fossil fuels), P^q is the unit carbon tax, denoted in \$ per ton of CO_2, and $\overline{P}_i^c$ is the composite price of domestic and imported good. The post-policy price of a polluted good will, therefore, be $P_i^c = \overline{P}_i^c + P^q Emit_i$. At a given price of carbon, the equilibrium level of cap of emissions from commodity i is determined endogenously.

1.4 Data Structure and CGE Results

To simulate the base case, we construct a social accounting matrix (SAM), which is based on the Canadian national economic accounts of 2004. The SAM is a reconciliation of input-output (IO) accounts with macroeconomic accounts and captures a balance income and expenditure flows among industries, households, government, and the rest of the world for each accounts. For example for households accounts, the SAM contains a balanced between households income from all factor remunerations and transfers and their expenditures on goods and services, payment of direct taxes, and households savings. Once we construct a balanced social account matrix (SAM), it is a general practice in CGE modelling to use this balanced information to calculate the tax rates by using the database information. For example to calculate

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[14] All other variables listed in the appendix and are otherwise not discussed in this section are taken as endogenous.

direct tax rate, we use total revenue collected through direct taxes divided by the total relevant income.

The IO accounts contain 39 industries and 43 commodities. For analytical reasons, we categorize all industries either into energy-related or non energy-related sectors. *Energy-related industries* (EJ) include fossil fuels (contain coal, oil, and gas), gas pipeline, refineries (comprise gasoline, diesel, liquefied petroleum, other refineries), and electricity. *Non-Energy related industries* (NEJ) encompass Energy-Intensive (EIS) industries (contain pulp-paper, printing, chemical, mining, plastic & rubber, non metallic metals, steel and other metals, machinery, transport services) and Non-Energy-Intensive (NEIS) industries (contain agriculture, food, construction, textile, wood, transport equipment, other manufacturing, wholesale, retail trade, communication, education, health, accommodation, electric products, storage, fire, business service, and other services). The inventory values of carbon dioxide (CO2) emissions are taken from Statistics Canada. As indicated in the previous section, we align emissions inventory with economic data by calculating the emissions factor of each commodity.

We obtain households income and expenditures information from the Canadian “Social Policy Simulation Database” (SPSD), which is a database that comprises a transformation of four data sources into a single non-confidential and publicly used micro data file. The SPSD process uses the Canadian Survey of Labour and Income Dynamics (SLID) as a host dataset and then maps it with the donor datasets (i.e., Personal Income Tax Return data, Employment Insurance (EI), and the Survey of Household Spending) based on some similar records or categorical matchings among different datasets.[15] The database produces 82,753 observations of households incomes and expenditures for the base year 2004. These households are then ranked according the total spending of each household group (see also Rausch et al. 2009).

Finally, the sorted observation is distributed in a quintile group. To avoid the impact of any influential or irregular value in households database, we apply non parametric estimation regression smoother (see for example Fan (1993)) for smoothing all households income and expenditure data. Once we managed the above households database, we align these database (e.g., labour and capital incomes, foreign and governments transfers, direct taxes, households savings, and their consumptions on 39

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[15] A detail of categorical matching is explained in the ”Database Creation Guide” of SPSD/M package of Statistics Canada.

commodities) with the national SAM data by using the approach that each household group is allocated the value of endowments, transfers, savings, tax burden, and expenditures of each goods according to it's share in aggregate of 100 households.[16]

The model is numerically run using the GAMS software with the CONOPT solver. The business-as usual (BAU) or base case refers to a calibration with no carbon tax policy. The analytical structure of the model enables it to calibrate the equilibrium prices and quantities of goods, which are very consistent with the actual data. Table 1.1 depicts the database values of Gross Domestic Product (GDP) and its components, CO2 emissions, total earnings of all households from capital and labour services. In fact, the base case[17] model calibrates precisely the same values of selected indicators depicted in Table 1.1.

Table 1.1: Base Case Calibration of Real Aggregate Economic Indicators(In Trillion $CN)

Indicators	Database Value
GDP at Market Price	1.288
Consumption	0.720
Investment	0.267
Exports	0.495
Imports	0.441
Government Expenditure	0.249
CO2 Emissions(MT)	588
Households Total Capital Income	0.138
Households Total Labour Income	0.705

1.4.1 Counterfactual Analysis

The model can examine the counterfactual scenarios by imposing different values for the carbon tax. However in this section, we illustrate the impacts of a $15, $30, $50, and $100 carbon tax on the economy as a whole, and on its individual sectors.

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[16] Therefore, each household sector faces different income tax rate, depending its contribution of direct taxes out of its total gross earned income.

[17] Throughout this chapter, we interchangeably use the words ”BAU”, ”base case” for no carbon tax policy case

Aggregate Impact

The aggregate impact of the imposed carbon taxes in the economy is represented by the variation in real GDP relative to the level of BAU. As expected, the post-policy[18] real GDP declines ranging from 0.11% to an almost 1% (see Table 1.2). To investigate the aggregate impact of a carbon tax more precisely, we decompose the incidence on real GDP by its components and by categories of commodities, which we discussed above.

The real consumption is the most effected component of real GDP showing a decline in total final consumptions ranging from 0.28% to 1.32% when the carbon tax increases from $ 15 to $100. As anticipated, the major decline in real consumption is due to the fall in demands for energy goods. This is because of the substitution effect, which represents a change in the composition of final demand due to a change in relative prices. Table 1.2 shows that the policy induced distortion in relative price of energy goods is higher, the higher is the carbon tax. The falls in demands for energy goods cause no investment growth in energy sectors. However, the real investment, at aggregate level, increases by 0.17% at $15 to 0.41% at $100 tax, mainly due to the high investment activities in non-Energy-Intensive sectors. Notice that the investment in energy-intensive products also increases, possibly because of the substitution of technology from emission-intensive to a relatively low-emission intensive energy goods.

The post reform changes in trade activities also show some interesting results. For example, Table 1.2 shows that the energy-intensive industries face the competitiveness issues from their foreign competitors. The exports of energy-intensive products decline by 0.4% at $15 to 1.5% at $100 tax, while the imports of the similar products increase by 0.1$ at $15 to 0.3% at $100 tax. On the other hand the exports of non-energy intensive products increase ranging from 0.3% to 1.4%, with no changes in the imports of similar products relative to the BAU level. However, the net exports of Canada is balanced due to the exogenous capital inflow explained in the last section.

It is also interesting to note that the relative factor prices (i.e., the ratio of the rental rate of capital to the wage rate) decline monotonically ranging from -0.93% to -4.34% as the value of the carbon tax increases from $15 to $100. Intuitively, this aligns with our argument in the earlier section that the energy intensive goods

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[18] Throughout this chapter we interchangeably use the words *post-policy* and *post-reform.*

Table 1.2: Aggregate Impacts of Different Values of Carbon Tax ($ Per Ton of CO_2)

	$15	$30	$50	$100
% Change Relative to Base Case				
○*Real GDP at Market Price*	-0.11	-0.24	-0.43	-0.91
A. Real Consumption	-0.28	-0.51	-0.78	-1.32
i. Energy Goods(EJ)	-0.21	-0.37	-0.55	-0.88
ii. Non-Energy Goods(NEJ)	-0.07	-0.14	-0.22	-0.44
a. Energy-Intensive Goods(EIS)	-0.04	-0.07	-0.12	-0.20
b. Non-Energy-Intensive Goods(NEIS)	-0.03	-0.06	-0.11	-0.24
B. Real Investment	0.17	0.27	0.35	0.41
i. Energy Goods(EJ)	0.00	0.00	0.00	0.01
ii. Non-Energy Goods(NEJ)	0.17	0.27	0.35	0.40
a. Energy-Intensive Goods(EIS)	0.03	0.05	0.07	0.10
b. Non-Energy-Intensive Goods(NEIS)	0.14	0.22	0.27	0.30
C. Real Exports	-0.16	-0.26	-0.35	-0.49
i. Energy Goods(EJ)	-0.11	-0.19	-0.28	-0.42
ii. Non-Energy Goods(NEJ)	-0.05	-0.07	-0.07	-0.07
a. Energy-Intensive Goods(EIS)	-0.37	-0.64	-0.94	-1.49
b. Non-Energy-Intensive Goods(NEIS)	0.31	0.58	0.87	1.42
D. Real Imports	-0.16	-0.26	-0.35	-0.49
i. Energy Goods(EJ)	-0.24	-0.41	-0.57	-0.85
ii. Non-Energy Goods(NEJ)	0.08	0.15	0.23	0.36
a. Energy-Intensive Goods(EIS)	0.07	0.13	0.20	0.34
b. Non-Energy-Intensive Goods(NEIS)	0.01	0.02	0.02	0.02
○*Rental rate of capital*	-1.28	-2.37	-3.61	-6.10
○*Nominal wage rate*	-0.36	-0.66	-1.03	-1.84
○*Ratio of rental rate of capital to wage rate*	-0.93	-1.71	-2.61	-4.34
○*Consumer Price Index*	0.30	0.63	1.10	2.31
○*Consumer Price Index (Non − Energy Goods)*	-0.05	-0.07	-0.09	-0.07
○*Consumer Price Index (Energy Goods)*	0.17	0.34	0.57	1.50
○*Reduction in Total Emissions*	-14.51	-23.31	-31.59	-44.75

are relatively capital intensive. Hence on the income-side, the incidence of a carbon tax is likely to be higher across those households that derive a higher share of their income through capital earning. We will discuss, in detail, the distributional aspect arising from the income-side channel in the next section.

Table 1.2 also shows that the composite price of all commodities, on average, increases monotonically with the increase in the carbon tax (from 0.3% at $15 to 2.3% at $100) indicating that the relative commodity prices would be stronger at relatively higher carbon tax levels. Therefore, on the consumption-side, the incidence of a carbon tax will reduce the purchasing power of households—those households having a higher share of their earned income on the consumption of the goods will be more negatively affected due to the higher relative levels of commodity prices. We will return to discussing the distributional aspect emerging from the demand-side channel in more detail in section 1.5.

Sectoral Impact

The carbon tax can affect the economy's sectors through at least two channels—direct and indirect. The direct channel refers to an increase in the cost of fossil-fuel energy in the production process. The extent of this increase depends on the carbon content of the fossil fuel used and its share in the production cost. This effect further transmits to the downstream sectors, which use emissions-intensive goods as intermediate inputs. The cost of production could vary depending on the content of emissions embodied in energy related intermediate goods. A potential indirect channel through which the carbon tax affects the economy is represented by the substitution possibilities among energy sources, as well as between energy and non-energy inputs. The ultimate effects of these two channels have a bearing on the supply and the demand for each commodity, which determine the qualitative impact of a carbon tax policy on sectoral prices. Let us explain the above direct and indirect sectoral effects more explicitly through our empirical results.

By grouping industries into three sets of sectors, namely the fossil fuel sector, the non-fossil fuel energy-intensive sector, and the non energy-intensive sector, one can easily explain the policy impacts through the different channels stated above. For example, the fossil-fuel producing industries experience a significant increase in their post-policy cost of production. This direct effect causes a rise in the user prices (following the imposition of a carbon tax) of these sectors. In response, the demands for their products considerably decline (see Table 1.3 and Table 1.4). The fall in the demand for fossil fuel forces these sectors to reduce their producer prices. Hence, the producer price of oil and gas, for instance, falls below its BAU level. Nevertheless, because of its high emissions intensity, the coal industry experiences an increase in its production cost and its producer price increases despite the fall in the demand for its product.

The indirect effects are mainly represented by the substitution of high energy-intensive goods to low energy-intensive ones. Households and industries substitute away from fossil fuel energy to electricity (See Table 1.4), as the user price of electricity increases relatively less drastically than the fossil fuels energy products (see

Table 1.3). Even the substitution possibility among energy sources may not help the downstream industries (such as chemical, steel and other metals, non-metallic metals, pulp-papers-printing, and transport services) to reasonably reduce their post-policy cost of production. They also experience a percentage increases in their post-policy producer prices relative to that of base case level (See Table 1.3).

The indirect channel also reveals that the substitution effect between energy-intensive and non energy-intensive sectors causes a fall in the demands for energy-intensive goods, which in turn, leads to a reduction in the demand for the factors used in the production of these commodities (see Table 1.5). Hence, we see in Table 1.5 that the post-policy demands for capital and labour, in almost all of the energy-intensive sectors, falls relative to their BAU levels. In most of the energy and energy-intensive sectors, the incidence of the carbon tax on factor incomes is negative. In contrast, the demands for some non-energy intensive products such as construction, textile and wood, plastic and rubber, machinery, transport equipment, and communication increase from their base case (see Table 1.4). Consequently, the demand for capital and labour, in these industries increases relative to the base case (see Table 1.5). However, due to the low factor price, household earning from these sectors may not overcome the loss in household income brought about by a contraction in energy and energy-intensive sectors.

1.5 Distributional Impact

To assess the distributional impact of a carbon tax policy, we calibrate the base case model by decomposing household sector into 100-quantile groups. It is interesting to first review the pre-policy contribution of each factor of equivalent incomes (expenditures) in the household's income and consumption patterns. For example, Figure 1.2, suggests that in the base case, the share of capital earnings to equivalent incomes is higher for rich households in comparison to poor households. In contrast, the share of energy consumption in total expenditures is higher for the latter category of households than for the former one (see Figure 1.3). As expected, the distribution of each component, among household groups, is skewed even in the BAU case.

Figures 1.2 and 1.3 support our argument that the incidence of a carbon tax policy on households will have opposite impacts through two sides, i.e., the source vs. the use side. On the source-side, the negative impact of the underlying policy

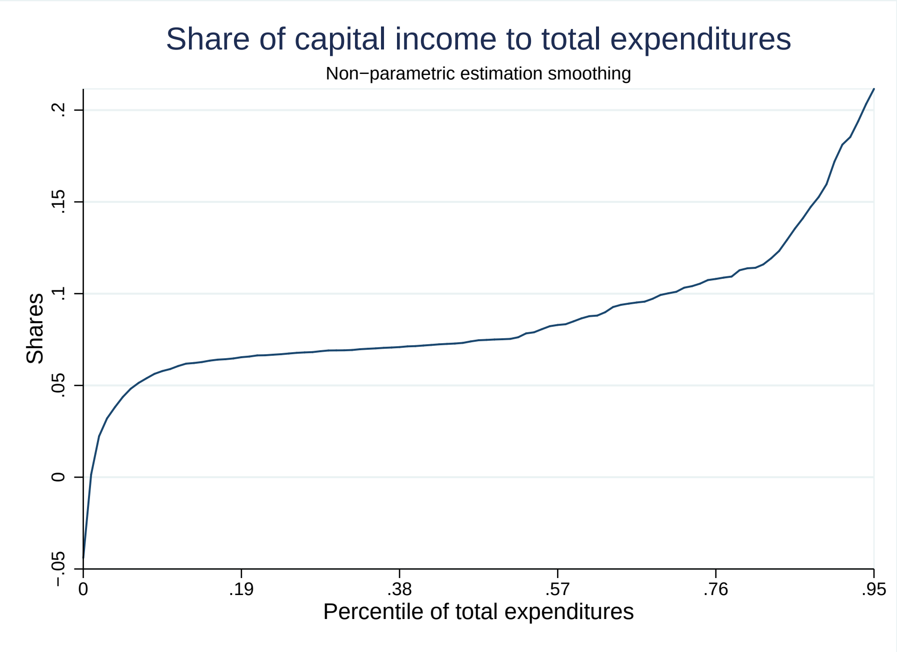


Figure 1.2: Share of Capital Income to Total Expenditures among Different Household Groups

would be relatively larger for the higher income groups because the capital earning is more heavily affected than the labour income. On the use-side, the policy would negatively affect low income quantiles because energy goods become more expensive than non-energy goods. Let us examine these two sides of policy incidence through some counterfactual analysis.

Figures 1.4 and 1.5 show the impacts of a carbon tax valued at $30 and $100, respectively on the components of households incomes and expenditures channels. On the income-side (i.e., through the variation in relative factor prices), one can assess the incidence of the carbon tax on equivalent income by calculating the ratio of decline in capital to wage earnings. Figure 1.4 shows that the losses in households' income through capital earnings are considerably high among the higher income groups, causing these groups be more affected by the pollution tax policy relative to the lower income groups. This is consistent with the results we inferred above in the BAU case (see Figure 1.2). Similarly on the expenditure-side (i.e., through the variation in relative commodity prices), we can assess the incidence of the carbon tax by calculating the ratio of the changes in energy and non-energy consumptions to the total variations in factors earnings among income quantile groups. For instance, Figure 1.5 shows that the incidence of the carbon tax through increase in energy prices is larger across the low income groups relative to the high income groups. This is also

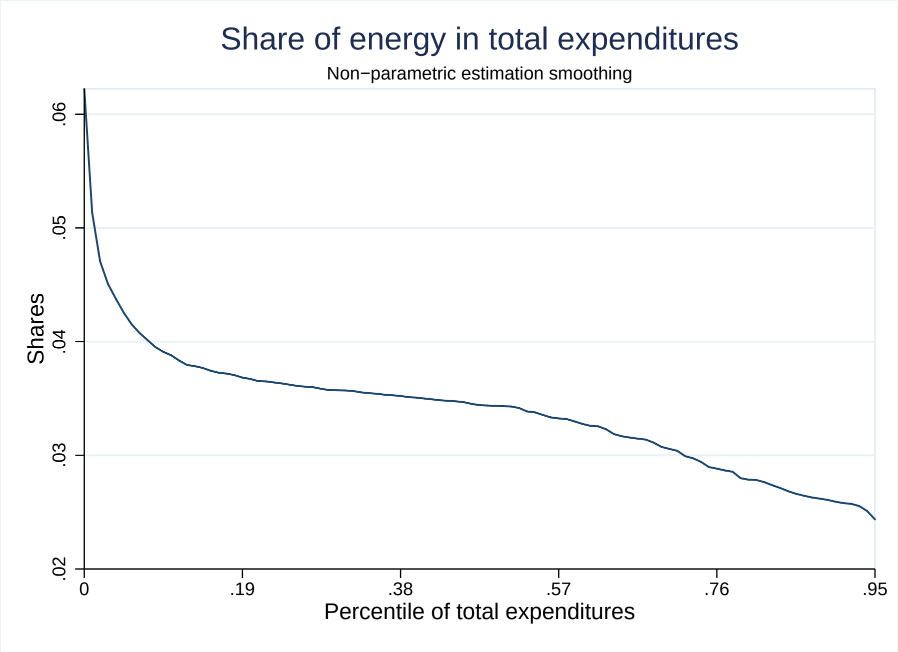
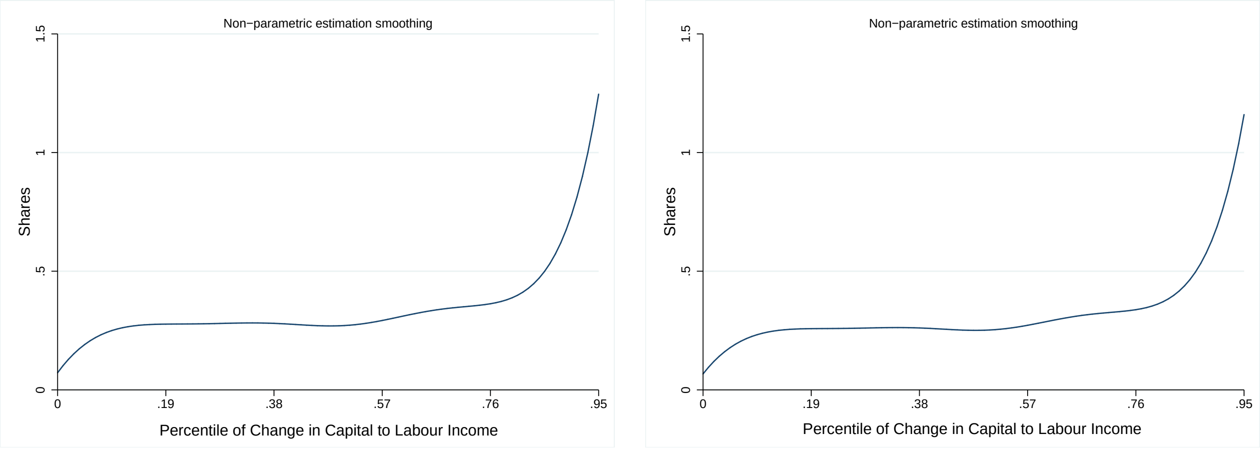


Figure 1.3: Share of Energy Consumption in total Expenditures among Different Household Groups

consistent with the results we inferred in the base case (see Figure 1.3). Figure 1.5 also shows that the variation in equivalent incomes through the changes in energy price is around two times larger than the variation in equivalent incomes through factor earnings among the low income groups, indicating that the expenditure-side effect is more influential within this income group.

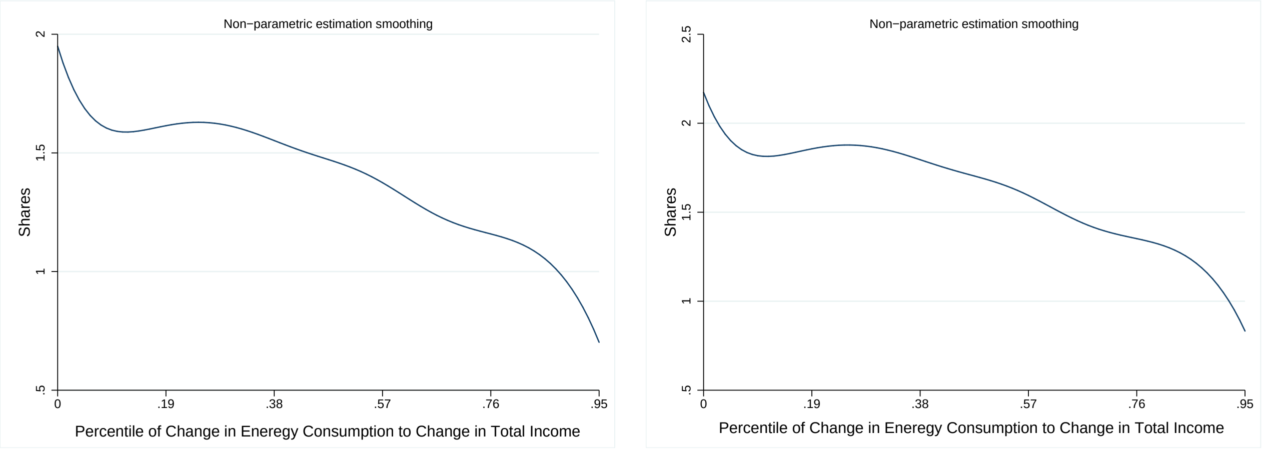


(a) Carbon Tax at $30

(b) Carbon Tax at $100

Figure 1.4: Ratio of losses in Capital to Labour Incomes in total Losses in Equivalent Incomes.

More precisely, the above discussion provides reasonable supports to our argument that the incidence of carbon tax through income-side channel is non-trivial and that



(a) Carbon Tax at $30

(b) Carbon Tax at $100

Figure 1.5: Ratio of Decline in Energy Consumptions to Declines in Total Expenditures.

the incidence through two channels has opposing effects on households welfare.

1.5.1 Income Inequality

To investigate the final impact of an environmental tax on households welfare more precisely, let us first examine the incidence of carbon tax on the change in total Gini relative to the level of base case. In Table 1.6, we show the estimates of Gini coefficients of equivalent incomes in the base case and at different carbon taxes at 95% confidence interval. While increasing the carbon tax from zero to $100, the Gini coefficient declines, indicating that the overall impact of carbon tax on household equivalent incomes is progressive. Notice that the standard error associated with the confidence interval at a higher carbon tax also declines, which is the notion of a stability in the estimated Gini coefficients.

Central of our analysis, in this chapter, is to assess the contributions of the demand and supply channels in the change in total inequality. For this purpose, we decompose the total change in Gini, indicated in Table 1.6, into its different components by using two different approaches explained in Equations 1.2.28-1.2.29.

Table 1.7 shows the Shapley decomposition of change in total inequality into the contribution of relative factors and commodity prices channels. Recall that the Shapley decomposition refers to the contribution of each channel to the change in overall inequality. It is interesting to note that the magnitudes of the Shapley contributions of relative factors and commodities prices in overall inequality are monotonic, but as expected, are in opposite directions. However, the contribution of factor prices are larger, making the overall effect progressive at different carbon taxes.[19] The intuition of this finding is directly linked to our argument and the results we showed in earlier sections that: i) energy intensive sectors are capital intensive, ii) high income households own a higher share of capital earnings in their total earned incomes relative to the low income groups and, iii) low income households spend more of their earned incomes on energy related goods.

Notice that our results shows that the Shapley contribution of commodity prices to overall inequality is regressive. This result is mostly discussed in the distributional related literature although through the use of different household welfare measures (see for example, Metcalf (2009) and Wier et al. (2005)). In our case, this result is consistent with the result in Figure 1.3, which shows that the low income groups spend a higher proportion of their earned income on energy related goods.[20] A post-policy increase in energy prices would cause an increase in the demand for a substituted non-energy good, as long as the post-policy consumer prices of substitute goods are relatively low. Hence, the purchasing power of households to buy those goods increases. An interesting point to note is that the Shapley contribution of non-energy intensive goods to total inequality also increases when we move from a low to a high carbon tax, implying that the overall impact of a change in commodity prices is more concentrated towards the lower income group as carbon prices increase. However, due to the stronger the Shapley contribution of factor prices on overall inequality, the combined effects of factor and commodity prices tend to reduce inequality following an increase in the carbon tax.

To assess the robustness of above findings, we estimate the decomposition by using an alternative approach, i.e. concentration indices approach explained in Equation 1.2.29. Recall that a negative estimated value of a component indicates that $G^0 < C^n$, which means that the contribution of the n^{th} component is more concentrated towards rich people relative to the overall inequality in the base case level. Hence, the negative

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[19] In a related paper, Oladosu and Rose (2007), also find a similar result that the incidence of carbon tax is mildly progressive while they use cost-side income distribution for in the Susquehanna River Basin regions of the USA. They estimated the incidence by using income bracket changes, per capita equivalent variation, and Gini coefficient changes based on consumption pattern. However their methodology is different from us as they assume a fixed rental rate of capital to wage rate ratio in their model. In other related papers, Metcalf et al. (2010) and Rausch et al. (2009) also find that the distributional impact of \$15 CO2eq tax on households is progressive while considering a combination of use and source sides. Our methodology differs from them as we explicitly show role of each component of equivalent incomes affecting households' income and consumption patterns.

[20] As we explained earlier, we consider household's earned income in Figure 1.3 rather than their total consumption so as to avoid the impact of any transfer.

coefficients of factor prices in Table 1.7 shows that the loss in equivalent income through factor earnings concentrates towards the rich households as the carbon tax increases, implying that the incidence of the pollution tax on the source-side of income is progressive. Conversely, the incidence of pollution tax on the use-side (i.e., through commodity price changes) of income is regressive. Again, due to the larger effect from the source-side, the combined effects of source and use-sides reduce inequality following an increase in the carbon tax.

1.6 Conclusions

In this chapter, we have investigated the incidence of pollution control policies on households' income while incorporating equivalent incomes as the household welfare metric and the Shapley value and concentration indices as the decomposition methods of illustrating inequality by components. The strength of equivalent incomes, for this analysis, rests on their incorporation of both commodity and factor prices, while the Shapley value and the concentration ratio approaches allow for an assessment of the contribution of each component of equivalent incomes to total inequality. The main focus of this chapter has been to develop a methodology to disentangle the components of the source and use sides of households' equivalent incomes and to examine the continuation of individual factors in the total incidence of a pollution tax policy.

We have developed a multi-sector, multi-household general equilibrium model of the Canadian economy to assess the distributional impact of a GHG mitigation policy in Canada by using a carbon tax as the policy instrument. The simulation results are then used to calculate the Shapley contributions and concentration indices of the components of equivalent incomes. In calibrating the model to different carbon tax levels, our CGE results suggest strong impacts of pollution control policies on relative factors and commodity prices. On the income-side, a carbon tax reduces both the wage rate and the rental rate on capital. However, the capital earnings are more heavily affected than the wages as the ratio of the rental rate of capital to the wage rate declines monotonically when we switch from a low (e.g., \$15) to a high (e.g., \$100) carbon tax. On the use-side, a carbon tax reduces the consumer prices of non energy-intensive goods, while it increases the consumer prices of energy-intensive goods relative to their BAU levels. However, due to a strong impact on energy prices, the composite price of all commodities, on average, is monotonically increased with an increase in the carbon tax.

The CGE results have also addressed the impacts of post-policy variations in factor and commodity prices across 100 household groups. The results show that the post-policy decline in relative factor prices have higher negative impacts on upper quintile groups because they derive a larger share of their income from capital earnings source in comparison to lower quintile groups. On the other hand, the impact of a change in energy prices seems to have a relatively stronger effect on low income groups than the higher income groups as the former spend more of their total expenditure on energy related goods than the latter. As expected, these two channels have influence but opposing effects on households welfare.

Finally to investigate the net impact on households income, we have estimated Gini coefficients as inequality measures. Our estimation shows that the Gini values of post-policy equivalent incomes decline as carbon tax increase, implying that the overall impact of a pollution tax on households' equivalent incomes is progressive.

Later, we decomposed total inequality by components of factor and commodity prices. Our estimates show that the consideration of increases in the prices of energy goods alone contribute to increasing inequality. The same result holds when the whole vector of commodity prices is considered. This result is in conformity with the conventional wisdom that suggests that carbon taxes are regressive. However, when the contributions of changes in factor prices are taken into consideration, the total effect of the carbon tax tends to be progressive. This suggests that the changes in factor incomes tends to reduce inequality. In other words, the effects of carbon tax on factor incomes tend to reduce the gap between poor and rich, and hence contribute to reduce inequality. These findings suggest that the traditional approach of assessing the impact of carbon taxes on inequality through changes in commodity prices alone may be misleading.

Certainly, there are some potential questions can arise in both use- and source-sides of assumptions we made in the model. On the use-side, for example, one option is the replacement of our assumption of homothetic preferences with non-homothetic one. However, given the fact that rich people drive more of their income from capital than the poor households do, and that the post-policy rental rate of capital falls more than the wage rate, the incidence through changes in factor prices channel will still be progressive, which is the main finding in this chapter. On the source-side, there is a possibility to decompose the labour supply into skilled/unskilled categories. Again, since the energy-intensive industries are capital intensive and the fact that rich people have higher shares in capital earnings than the that of poor, a decomposition of average wage rate into high and low wage rates will not change our result of progressivity as long as the percentage decline in rental rate of capital is larger than the two types of wage rates. Also the evidence of capital-skill labour Complementarity (see FitzRoy and Funke (1995)) supports our results that in case of the decomposition of labour supply, the wage rate of skilled labour may be affected more than that of the unskilled labour.

Another potential question can be to incorporate minimum wage rigidity in the labour market. Intuitively, the feature of constraining the lower bound of wage rate will be in favour of our results of supply-side progressivity. Certainly, there are several other features that might be interesting to asses within the framework we introduced in this chapter. The decline in inequality reported in this study is not systematic. It can still be the case that the regressive impact of the change in commodity prices overrides the progressive impact coming the changes in factor income. This study has just showed that carbon taxes can be progressive.

Nevertheless, there are some caveats that need to be mentioned. For example, the analysis does not take into consideration the environmental impact of reduced pollution on inequality and welfare. In addition, a distinction must be made between a decrease in inequality and a fall in welfare. Despite the decline in inequality, and abstracting for the environmental effects, welfare does fall for all households.

Table 1.3: Impacts of Carbon Taxes on Different Prices

Sectors	$30			$50			$100		
	Value-Added Price	Producer Price	User Price*	Value-Added Price	Producer Price	User Price*	Value-Added Price	Producer Price	User Price*
	Percentage Change from Base case								
Energy									
Electricity	-0.38	4.32	4.41	-0.49	6.25	6.38	-0.63	10.01	10.18
Oil and gas	-0.13	-1.20	5.24	-0.19	-1.52	8.95	-0.32	-3.06	18.59
Coal	-0.11	0.07	166.14	-0.16	0.17	276.94	-0.27	0.48	554.00
Refineries	-0.08	0.57	22.31	-0.12	0.89	37.30	-0.20	1.53	75.29
Non-energy									
Agriculture	-0.10	0.02	0.02	-0.15	0.10	0.10	-0.25	0.40	0.41
Food	-0.10	-0.2	-0.21	-0.15	-0.25	-0.27	-0.25	-0.25	-0.27
Construction	-0.16	0.79	0.79	-0.24	1.3o	1.30	-0.38	2.51	2.51
Textile and Wood	-0.09	-0.3	-0.42	-0.13	-0.42	-0.59	-0.21	-0.62	-0.85
Pulp, Paper, Printing	-0.09	0.25	0.45	-0.14	0.43	0.74	-0.22	0.89	1.45
Chemical	-0.11	2.59	1.97	-0.16	4.12	3.06	-0.28	7.40	5.22
Mining	-0.11	0.09	0.14	-0.38	-0.17	0.33	-0.37	0.66	0.95
Plastic and Rubber	-0.08	0.18	0.20	-0.12	0.29	0.33	-0.28	0.52	0.58
Non Metallic Metals	-0.11	0.41	0.36	-0.16	0.68	0.60	-0.26	1.36	1.20
Steel and other metals	-0.15	1.08	1.17	-0.21	1.56	1.68	-0.32	2.53	2.63
Machinery	-0.12	-0.11	-0.07	-0.18	-0.18	-0.12	-0.30	-0.30	-0.20
Transport Equipment	-0.12	-0.22	-0.25	-0.18	-0.33	-0.38	-0.29	-0.55	-0.64
Other Manufacturing	-0.15	-0.10	-0.06	-0.22	-0.13	-0.08	-0.35	-0.14	-0.09
Wholesale	-0.12	-0.41	-0.46	-0.18	-0.59	-0.66	-0.29	-0.89	-1.00
Retail Trade	-0.07	-0.54	-0.54	-0.11	-0.81	-0.82	-0.18	-1.36	-1.36
Transport Services	-0.11	1.19	1.44	-0.17	1.94	2.33	-0.28	3.62	4.30
Communication	-0.12	-0.92	-0.91	-0.18	-1.39	-1.38	-0.29	-2.33	-2.31
Education	-0.08	-0.46	-0.48	-0.13	-0.68	-0.72	-0.22	-1.13	-1.20
Health	-0.08	-0.7	-0.69	-0.12	-1.07	-1.06	-0.20	-1.83	-1.81
Accommodation	-0.07	-0.54	-0.53	-0.10	-0.81	-0.79	-0.17	-1.33	-1.30
All Others	-0.10	-0.44	-0.47	-0.15	-0.64	-0.69	-0.25	-1.02	-1.09

* Including carbon tax

Table 1.4: Impacts of Carbon Tax on Commodities Demand, Supply and Prices

Sectors	$30			$50			$100		
	Total Supply	Total Demand	Domestic Price	Total Supply	Total Demand	Domestic Price	Total Supply	Total Demand	Domestic Price
				Percentage Change from Base case					
Energy									
Electricity	-4.54	-3.74	4.58	-6.32	-5.20	6.63	-9.35	-7.69	10.59
Natural gas	-4.84	-11.60	-2.94	-7.60	-17.92	-4.62	-12.56	-30.14	-8.28
Coal	-54.55	-55.32	0.17	-65.61	-66.25	0.42	-77.96	-78.21	1.20
Gasoline	-14.34	-13.30	1.76	-21.23	-19.96	2.53	-33.37	-32.13	3.63
Other fuels for vehicles	-16.43	-22.30	-3.00	-24.00	-32.01	-4.54	-36.77	-47.46	-7.32
Non-energy commodities									
Agriculture	-0.07	-0.06	0.02	-0.37	-0.30	0.12	-1.45	-1.18	0.48
Food	-0.10	-0.35	-0.27	-0.33	-0.63	-0.34	-1.10	-1.40	-0.33
Construction	0.65	0.65	0.79	0.68	0.69	1.31	0.27	0.28	2.51
Textile & Wood	1.49	0.24	-0.67	2.02	0.22	-0.95	2.67	-0.07	-1.41
Pulp, Paper, Printing	-2.30	-1.05	0.67	-3.67	-1.68	1.12	-6.67	-3.06	2.20
Chemical	-12.50	-4.00	4.69	-18.54	-5.98	7.38	-29.08	-9.62	12.98
Mining	-6.47	-6.06	0.19	-9.55	-8.42	0.46	-15.94	-12.67	1.33
Plastic & Rubber	-0.21	0.65	0.40	-0.46	0.97	0.66	-1.13	1.49	1.20
Non Metallic Metals	-0.33	0.20	0.52	-0.69	0.18	0.86	-1.87	-0.13	1.72
Steel and other metals	-4.26	-0.24	2.21	-5.80	-0.18	3.19	-8.60	-0.08	5.07
Machinery	1.87	0.66	-0.39	2.86	0.90	-0.63	4.58	1.19	-1.07
Transport Equipment	4.80	2.05	-0.90	7.45	3.14	-1.38	12.67	5.16	-2.31
Other Manufacturing	0.39	-0.26	-0.24	0.42	-0.44	-0.31	0.08	-0.91	-0.36
Wholesale	0.10	-0.15	-0.50	0.08	-0.28	-0.71	-0.12	-0.66	-1.08
Retail Trade	-0.12	-0.14	-0.54	-0.22	-0.24	-0.82	-0.49	-0.53	-1.37
Transport Services	-2.53	-1.18	1.67	-4.00	-1.87	2.70	-7.08	-3.32	5.00
Communication	0.48	0.03	-1.02	0.70	0.01	-1.55	1.07	-0.09	-2.61
Education	0.00	-0.16	-0.51	-0.03	-0.28	-0.76	-0.17	-0.59	-1.27
Health	-0.08	-0.10	-0.70	-0.13	-0.15	-1.07	-0.26	-0.31	-1.83
Accommodation	0.16	-0.28	-0.64	0.21	-0.45	-0.96	0.22	-0.87	-1.59
All Others	-0.32	-1.49	-0.74	-0.59	-2.30	-1.09	-1.22	-3.95	-1.75

Table 1.5: Impacts of Carbon Tax on Different Inputs of Productions

	$30			$50			$100		
	Capital	Labour	Energy	Capital	Labour	Energy	Capital	Labour	Energy
				Percentage Change from Base case					
Energy									
Electricity	-0.76	0.56	-30.9	-0.81	1.00	-40.44	-0.53	2.17	-53.98
Oil and gas	-3.46	-5.40	-21.48	-5.47	-8.36	-31.71	-9.04	-13.68	-49.09
Coal	-53.82	-54.32	-57.50	-64.75	-65.31	-69.03	-76.98	-77.57	-81.67
Refineries	-8.71	-13.30	-27.47	-13.01	-19.62	-38.98	-20.82	-30.69	-56.73
Non-energy									
Agriculture	1.29	0.30	-2.86	1.74	0.27	-4.64	2.21	-0.09	-8.71
Food	0.80	0.01	-3.49	1.06	-0.09	-5.61	1.20	-0.47	-10.34
Construction	0.63	2.49	-11.18	0.56	3.67	-17.40	-0.23	5.87	-29.60
Textile & Wood	2.45	1.53	-1.48	3.50	2.11	-2.60	5.11	2.98	-5.37
Pulp, Paper, Printing	-2.14	-1.55	-4.77	-3.45	-2.51	-7.44	-6.38	-4.56	-13.05
Chemical	-4.85	-8.61	-24.30	-7.12	-12.75	-35.28	-10.77	-19.90	-53.16
Mining	-5.00	-5.88	-13.15	-7.30	-8.57	-19.49	-12.24	-14.08	-31.70
Plastic & Rubber	1.38	0.08	-6.29	1.96	0.01	-9.60	2.91	-0.24	-16.26
Non Metallic Metals	0.08	0.70	-5.08	-0.07	0.93	-7.83	-0.91	1.16	-13.78
Steel and other metals	-3.87	-2.39	-10.61	-5.18	-3.14	-14.73	-7.50	-4.37	-22.00
Machinery	2.97	1.97	-2.47	4.54	3.05	-3.92	7.37	5.01	-7.35
Transport Equipment	5.88	4.84	0.56	9.13	7.54	0.72	15.57	12.96	0.46
Other Manufacturing	1.34	0.58	-5.11	1.83	0.74	-8.07	2.39	0.80	-14.30
Wholesale	1.35	0.09	-6.75	2.01	0.11	-10.51	3.16	0.08	-18.25
Retail Trade	0.97	-0.14	-3.52	1.45	-0.23	-5.42	2.29	-0.44	-9.49
Transport Services	-0.86	-1.84	-7.36	-1.44	-2.89	-11.31	-2.85	-5.08	-19.21
Communication	1.55	-0.06	-6.45	2.36	-0.11	-9.91	3.93	-0.22	-16.86
Education	0.83	0.16	-5.44	1.20	0.22	-8.49	1.77	0.36	-14.93
Health	1.22	-0.23	-4.56	1.86	-0.35	-6.95	3.10	-0.59	-11.84
Accommodation	1.37	0.15	-3.74	2.07	0.21	-5.69	3.39	0.30	-9.74

Table 1.6: Gini Indices of Household Equivalent Income ($ per Ton of CO2 eq.)

	Estimate	Standard Error	Lower Bound	Upper Bound
Base Case	0.312021	0.026195	0.260044	0.363997
$ 15	0.311420	0.025981	0.259868	0.362971
$ 30	0.310910	0.025799	0.259718	0.362101
$ 50	0.310317	0.025589	0.259542	0.361091
$ 100	0.309109	0.025162	0.259182	0.359036

Table 1.7: Decomposition of Changes in Total Gini Indices of Household Equivalent Income at Different levels of Carbon Tax ($ per Ton of CO2 eq.)

	$15		$30		$50		$100	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
Shapley Decomposition Approach								
Factor prices	-0.000764	126.7%	-0.001412	126.8%	-0.002159	126.4%	-0.003653	125.1%
Prices EIS	0.000136	-22.6%	0.000239	-21.5%	0.000338	-19.8%	0.000471	-16.1%
Prices NEIS	0.000025	-4.1%	0.000059	-5.3%	0.000113	-6.6%	0.000263	-9.0%
Total	-0.000603	100.0%	-0.001114	100.0%	-0.001708	100.0%	-0.002919	100.0%
Exact Decomposition Approach								
Factor prices	-0.000759	126.9%	-0.001398	127.0%	-0.002129	126.8%	-0.003570	125.9%
Prices EIS	0.000137	-22.9%	0.000241	-21.9%	0.000344	-20.5%	0.000491	-17.3%
Prices NEIS	0.000024	-4.0%	0.000056	-5.1%	0.000106	-6.3%	0.000244	-8.6%
Total	-0.000598	100.0%	-0.001101	100.0%	-0.001679	100.0%	-0.002835	100.0%

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1.7 Appendix

Consumption

$$C_h = (1 - s_h) M_h^d \tag{1.7.1}$$

Disposable Income

$$M_h^d = (1 - t_{fh})(wL_h^s + RK_h) + TR_h^G - eTR_h^{fr} \tag{1.7.2}$$

Capital Income

$$RK_h = \beta_h^K (1 - t_b) \sum_j (P_{kvj} KV_j + RKF_j) \tag{1.7.3}$$

Household saving

$$S_h = s_h M_h^d \tag{1.7.4}$$

Business saving

$$S_b = (1 - \beta^K)(1 - t_b) \sum_j (P_{kvj} KV_j + RKF_j) \tag{1.7.5}$$

Final demand

$$P_i^c C_{hi} = \beta_{hi}^c C_h \tag{1.7.6}$$

Production Technology

Top level nest *CES nest between composite of value-add-energy and a composite of intermediate goods*

For $\sigma_{1j} \leq 1$,

$$P_{yj}(1 - t_{yj}) = \frac{1}{A_{1j}} \{\alpha_{1j}^{\sigma_{1j}} P_{intj}^{1-\sigma_{1j}} + (1 - \alpha_{1j})^{\sigma_{1j}} P_{vaej}^{1-\sigma_{1j}}\}^{\frac{1}{1-\sigma_{1j}}} \tag{1.7.7}$$

$$VAEj = A_{1j}^{\sigma_{1j}-1} Y_j \{\frac{(1 - \alpha_{1j}) P_{yj}(1 - t_{yj})}{P_{vaej}}\}^{\sigma_{1j}} \tag{1.7.8}$$

$$INTj = A_{1j}^{\sigma_{1j}-1} Y_j \{\frac{\alpha_{1j} P_{yj}(1 - t_{yj})}{P_{intj}}\}^{\sigma_{1j}} \tag{1.7.9}$$

2nd level nest *Cobb-Douglas nest between labour and composite of capital-energy goods*

$$P_{vaej} = \frac{1}{A_{2j}} \{(\frac{\alpha_{2j}}{P_{kej}})^{-\alpha_{2j}} (\frac{1 - \alpha_{2j}}{w})^{-(1-\alpha_{2j})}\} \tag{1.7.10}$$

$$P_{kej}KEj = \alpha_{2j}P_{vaej}VAEj \tag{1.7.11}$$

$$wL_j^d = (1-\alpha_{2j})P_{vae}VAEj \tag{1.7.12}$$

3rd level nest *CES nest between a composite of capital and a composite of energy goods*

$$P_{kej} = \frac{1}{A_{3j}}\{\alpha_{3j}^{\sigma_{3j}}P_{kj}^{(1-\sigma_{3j})} + (1-\alpha_{3j})^{\sigma_{3j}}P_{ej}^{(1-\sigma_{3j})}\}^{\frac{1}{1-\sigma_{3j}}} \tag{1.7.13}$$

$$K_j = A_{3j}^{\sigma_{3j}-1}KE_j\{\frac{\alpha_{3j}P_{kej}}{P_{kj}}\}^{\sigma_{3j}} \tag{1.7.14}$$

$$E_j = A_{3j}^{\sigma_{3j}-1}KE_j\{\frac{(1-\alpha_{3j})P_{kej}}{P_{ej}}\}^{\sigma_{3j}} \tag{1.7.15}$$

4th level nest *CES nest between variable and fixed physical capital*

$$K_j = A_{4j}\{\alpha_{4j}KV_j^{-\rho_{4j}} + (1-\alpha_{4j})KF_j^{-\rho_{4j}}\}^{-\frac{1}{\rho_{4j}}} \tag{1.7.16}$$

$$P_{kv} = P_{kj}\alpha_{4j}A_{4j}^{-\rho_{4j}}(\frac{K_j}{KV_j})^{\frac{1}{\sigma_{4j}}} \tag{1.7.17}$$

$$RKF_j = P_{kj}K_j - P_{kv}KV_j \tag{1.7.18}$$

5th level nest *CES nest between a composite of fossil fuel products and electricity input*

$$P_{ej} = \frac{1}{A_{5j}}\{\alpha_{5j}^{\sigma_{5j}}P_{ffj}^{(1-\sigma_{5j})} + (1-\alpha_{5j})^{\sigma_{5j}}PV_{elej}^{(1-\sigma_{5j})}\}^{\frac{1}{1-\sigma_{5j}}} \tag{1.7.19}$$

$$FF_j = A_{5j}^{\sigma_{5j}-1}E_j\{\frac{\alpha_{5j}P_{ej}}{P_{ffj}}\}^{\sigma_{5j}} \tag{1.7.20}$$

$$VELE_j = A_{5j}^{\sigma_{5j}-1}E_j\{\frac{(1-\alpha_{5j})P_{ej}}{PV_{elej}}\}^{\sigma_{5j}} \tag{1.7.21}$$

6th level nest *CES nest among refined petroleum products, natural gas, and coal*

$$P_{ffj} = \frac{1}{A_{6j}}\{\alpha_{61j}^{\sigma_{6j}} P_{rpj}^{(1-\sigma_{6j})} + \alpha_{62j}^{\sigma_{6j}} P_{gasj}^{(1-\sigma_{6j})} + \alpha_{63j}^{\sigma_{6j}} PV_{colj}^{(1-\sigma_{6j})}\}^{\frac{1}{1-\sigma_{6j}}} \tag{1.7.22}$$

$$RP_j = A_{6j}^{\sigma_{6j}-1} FF_j\{\frac{\alpha_{61j} P_{ffj}}{P_{rpj}}\}^{\frac{1}{1-\sigma_{6j}}} \tag{1.7.23}$$

$$GAS_j = A_{6j}^{\sigma_{6j}-1} FF_j\{\frac{\alpha_{62j} P_{ffj}}{P_{gasj}}\}^{\frac{1}{1-\sigma_{6j}}} \tag{1.7.24}$$

$$VCOL_j = A_{6j}^{\sigma_{6j}-1} FF_j\{\frac{\alpha_{63j} P_{ffj}}{PV_{colj}}\}^{\frac{1}{1-\sigma_{6j}}} \tag{1.7.25}$$

$$P_{gasj} = \sum AIJ_{gasj} PV_{gasj} \tag{1.7.26}$$

$$VGAS_j = AIJ_{gasj} GAS_j \tag{1.7.27}$$

7th level nest *Cobb-Douglas nest among different refined petroleum products*

$$P_{rpj} = \frac{1}{A_{7j}} \prod_{rp} (\frac{PV_{rpj}}{\alpha_{7j,rp}})^{\alpha_{7j,rp}} \tag{1.7.28}$$

$$PV_{rpj} = \alpha_{7j,rp} P_{rpj} RP_j \tag{1.7.29}$$

8th level nest *CES nest among a composite of motive fuels (i.e, gasoline and diesel) and other intermediate materials*

$$P_{intj} = \frac{1}{A_{8j}}\{\alpha_{8j}^{\sigma_{8j}} P_{mfj}^{(1-\sigma_{8j})} + (1-\alpha_{8j})^{\sigma_{8j}} P_{omj}^{(1-\sigma_{8j})}\}^{\frac{1}{1-\sigma_{8j}}} \tag{1.7.30}$$

$$MF_j = A_{8j}^{\sigma_{8j}-1} INT_j\{\frac{\alpha_{8j} P_{intj}}{P_{mfj}}\}^{\sigma_{8j}} \tag{1.7.31}$$

$$OM_j = A_{8j}^{\sigma_{8j}-1} INT_j\{\frac{\alpha_{8j} P_{intj}}{P_{omj}}\}^{\sigma_{8j}} \tag{1.7.32}$$

$$P_{mfj} = A_{9j}^{-1} \prod_{mf} (\frac{PV_{mfj}}{\alpha_{9j,mf}})^{\alpha_{9j,mf}} (1.7.33)$$

$$PV_{omj} VMF_j = \alpha_{9j,mf} P_{mfj} MF_j \tag{1.7.34}$$

$$P_{omj} = \sum_{om} AIJ_{j,om} PV_{omj} \tag{1.7.35}$$

$$VOM_j = AIJ_{j,om} OM_j \tag{1.7.36}$$

Constant Elasticity of Transformation (CET) *CET nest between domestically produced goods and foreign goods*

$$P_{yj} = A_{Tj}^{-1} [\sum_i \delta_{ij}^{-\sigma_{Tj}} P_{yti}^{1+\sigma_{Tj}}]^{-(1+\sigma_{Tj})} \tag{1.7.37}$$

$$Y_{ij}^c = A_{Tj}^{-1(1+\sigma_{Tj})} Y_j [\frac{P_{yti}}{\delta_{ij} P_{yj}}]^{\sigma_{Tj}} \tag{1.7.38}$$

$$Y_i^{ct} = \sum_j Y_{ij}^c \tag{1.7.39}$$

$$P_{yti} = A_{Xi}^{-1} [\delta_{Xi}^{-\sigma_{Xi}} P_{yxi}^{1+\sigma_{Xi}} + (1 - \delta_{Xi})^{-\sigma_{Xi}} P_{ysdi}^{1+\sigma_{Xi}}]^{-1(1+\sigma_{Xi})} \tag{1.7.40}$$

$$YX_i = A_{Xi}^{-1(1+\sigma_{Xi})} Y_i^{ct} [\frac{P_{yxi}}{\delta_{xi} P_{yti}}]^{\sigma_{Xi}} \tag{1.7.41}$$

$$YSD_i = A_{Xi}^{-1(1+\sigma_{Xi})} Y_i^{ct} [\frac{P_{ysdi}}{(1 - \delta_{xi}) P_{yti}}]^{\sigma_{Xi}} \tag{1.7.42}$$

$$\overline{P}_{ci} = A_{Mi}^{-1} [\delta_{Mi}^{\sigma_{Mi}} P_{ymi}^{1-\sigma_{Mi}} + (1 - \delta_{Mi})^{\sigma_{Mi}} P_{yddi}^{1-\sigma_{Mi}}]^{-1(1+\sigma_{Mi})} \tag{1.7.43}$$

$$YDD_i = A_{Mi}^{(\sigma_{Mi}-1)} Y_{ci} \{\frac{(1 - \delta_{Mi}) \bar{P}_{ci}}{P_{yddi}}\}^{\sigma_{Mi}} \tag{1.7.44}$$

$$YM_i = A_{Mi}^{(\sigma_{Mi}-1)} Y_{ci} \{\frac{\delta_{Mi} \bar{P}_{ci}}{P_{ymi}}\}^{\sigma_{Mi}} \tag{1.7.45}$$

Government income

$$\begin{aligned} YG = &\sum_j t_{yj}P_{yi}Y_j + \sum_h t_{ci}P_i^c C_{hi} + \sum_i t_{inv,i}\bar{P}_{ci}INV_i \\ &+ \sum_i t_{xi}P_{yxi}YX_i + \sum_i t_{mi}eP_{ymi}YM_i + \sum_h t_{fh}(wL_h^d + RK_h) \\ &+ t_b\sum_j P_{kv}KV_j + P^q EMIT \end{aligned} \tag{1.7.46}$$

Foreign saving

$$S^{fr} = \sum_i (P_{ymi}YM_i - P_{yxi}YX_i) \tag{1.7.47}$$

Government transfers

$$TR_h^G = TRG\beta_h^{TRG} \tag{1.7.48}$$

Government saving

$$GS = YG - \sum_i \overline{P}_{ci}G_i - TRG \tag{1.7.49}$$

Total saving

$$Total\ saving = \sum_h S_h + GS + S_b + e * S^{fr} \tag{1.7.50}$$

Domestic absorption

$$Y_{ci} = \sum_h C_{hi} + G_i + INV_i + \Delta N_i + \sum_j V_{ij} \tag{1.7.51}$$

Demand for investment

$$INV_i\overline{P}_{ci}(1 + t_{inv,i}) = \beta_i(Total\ saving - \sum_i(\overline{P}_{ci}\Delta N_i)) \tag{1.7.52}$$

Imports good price including tariffs)

$$P_{ymi} = PMO_i * e * (1 + t_{mi}) \tag{1.7.53}$$

Exports good price including tariffs)

$$PXO_I * e = P_{yxi} * (1 + t_{xi}) \tag{1.7.54}$$

Price of intermediate goods

$$P_{ci} = \overline{P}_{ci} + P^q Emit_i \tag{1.7.55}$$

Price of intermediate goods including tariffs)

$$PV_{ij} = P_{ci} * (1 + t_{vij}) \tag{1.7.56}$$

Consumer price including tariffs

$$P_i^c = P_{ci} * (1 + t_{ci}) \tag{1.7.57}$$

Equilibrium condition on domestic market

$$YSD_i = YDD_i \tag{1.7.58}$$

Equilibrium condition on labour market

$$\sum_j L_j^d = \sum_h L_h^s \tag{1.7.59}$$

Equilibrium condition on capital market

$$KVT = \sum_j KV_j \tag{1.7.60}$$

Total CO2 emissions

$$EMIT = \sum_{ih} Emit_i C_{hi} + \sum_{ij} Emit_i V_{ij} \tag{1.7.61}$$

1.7.1 Tables of Sets, Variables, Elasticities, and Share Coefficients

Table 1.8: Sets

Sets	Description
i	goods
j	Sectors
h	Households

Table 1.9: Activity Variables

Activity variables	**Description**
M_h^d	Disposable income by household h
C_h	Consumption by household h
S_h	Household saving
L_h^s	Labour supply by households
L_j^d	Labour demand by sector j
RK_h	Returns on capital by households
TR_h^G	Government transfer received by a household
TR_h^{fr}	Foreign transfer contributed by a household
KV_j	Demand for a variable physical capital by sector j
RKF_j	Return to fixed capital in sector j
Y_j	Composite output j
VAE_j	Composite of value added and energy for output j
INT_j	Composite of intermediate goods for output j
KE_j	Composite of capital and energy for output j
K_j	Physical Capital used in the production of j
E_j	Energy consumed in the production of j
KV_j	Variable physical capital used in the production of j
KF_j	Fixed capital used in the production of j
FF_j	Fossil fuel used in the production of j
VELE_j	Electricity used in the production of j

Table 1.10: Activity Variables (CONT')

Activity variables	**Description**
RP_j	Refined petroleum product used in j
GAS_j	Natural gas used as an energy input of j
$VCOL_j$	Coal is used as intermediate goods in the production of j
$VGAS_j$	Gas is used as intermediate goods in the production of j
AIJ_{ij}	Autonomous efficiency improvement in input i for production of j
MF_j	Consumption of motive fuels as intermediate input of j
OM_j	Composite of materials used by sector j
VMF_j	Demand for motive fuels as intermediate goods by sector j
VOM_j	Demand for materials as intermediate goods by sector j
Y^c_{ij}	Supply of commodity i by industry j
Y^{ct}_i	Total supply of commodity i
YSD_i	Supply of domestic good i
YDD_i	Demand for domestic good i
YX_i	Supply of good i for Exports
YM_i	Imports of good i
TRG	Total government transfer
YG	Government total income
INV_i	Demand for good i for investment purpose
N_i	Amount of good i as an inventory
G_i	Government demand for good i
GS	Total government saving
V_{ij}	Demand for good i as intermediate good by sector j
Y_{ci}	Total demand or domestic absorption of good i
$Emit_i$	Emission of CO2 embodied with good i
EMIT	Total CO2 emissions
KVT	Stock of total variable physical capital

Table 1.11: Price Variables

Price variables	**Description**
w	Wage rate
e	Exchange rate
P_{yj}	Price of output j
P_{intj}	Price of composite of intermediate inputs for sector j
P_{vaej}	Price of composite input of value-added-energy for sector j
P_{kej}	Price of composite capital and energy demand by sector j
P_{kj}	Price of composite of total physical capital for sector j
P_{ej}	Price of composite input of energy for sector j
P_{kvj}	Price of variable physical capital for sector j
P_{ffj}	Price of composite input for fossil energy for sector j
PV_{elej}	Price of electricity used by sector j
P_{rpj}	Price of composite input of refined petroleum for sector j
P_{gasj}	Price of natural gas used as energy by sector j
PV_{colj}	Price of coal used as intermediate input by sector j
PV_{gasj}	Price of gas used as intermediate input by sector j
PV_{rpj}	Price of refined petroleum used as intermediate
P_{mfj}	Price of composites of motive fuels used by sector j
P_{omj}	Price of composites of materials used by sector j
PV_{mfj}	Price of motive fuels as intermediate input for sector j
PV_{omj}	Price of materials as intermediate input for sector j
P_{yti}	Price of composite of total supply i
P_{yxi}	Price of composite of exports supply i
P_{ydi}	Price of domestically supply good i
P_{ymi}	Price of imported good i

Table 1.12: Price Variables (CONT')

Price variables	**Description**
$\overline{P}_{ci}$	Composite consumption price of domestic and import commodities with permit price
PMO_i	Mondial exports Price
PXO_i	Mondial exports Price
P_{ci}	Price of intermediate goods before tax
P_i^c	Price of consumer goods after tariffs
P^q	permit price of CO2 emission

Table 1.13: Substitution and Shares Parameters

Substitution and shares parameters	**Description**
α_{1j}	Substitution parameter in the 1st level of nested CES production functions
α_{2j}	Substitution parameter in the 2nd level of nested CES production functions
α_{3j}	Substitution parameter in the 3rd level of nested CES production functions
α_{4j}	Substitution parameter in the 4th level of nested CES production functions
α_{5j}	Substitution parameter in the 5th level of nested CES production functions
α_{61j}	Substitution parameter in the 6th level of nested CES production functions
α_{62j}	Substitution parameter in the 6th level of nested CES production functions
α_{63j}	Substitution parameter in the 6th level of nested CES production functions
$\alpha_{7j,i}$	Substitution parameter in the 7th level of nested CES production functions
α_{8j}	Substitution parameter in the 8th level of nested CES production functions
$\alpha_{9j,i}$	Substitution parameter in the 9th level of nested CES production functions
s_h	Share of disposable income that a household saves
β_h^K	Share of total capital stock owned by a household
β^K	Share of total capital stock owned by all households
δ_{ij}	Share parameter in the make CET functions
δ_{Xi}	Share parameter in the 1st level of nested CET functi
δ_{Mi}	Share parameter in the 1st level of nested Armington functions
ecof_i	Emission coefficient for good i

Table 1.14: Shift Parameters

Shift parameters	**Description**
A_{1j}	Shift parameter in the 1st level of nested CES production functions
A_{2j}	Shift parameter in the 2nd level of nested CES production functions
A_{3j}	Shift parameter in the 3rd level of nested CES production functions
A_{4j}	Shift parameter in the 4th level of nested CES production functions
A_{5j}	Shift parameter in the 5th level of nested CES production functions
A_{6j}	Shift parameter in the 6th level of nested CES production functions
A_{7j}	Shift parameter in the 7th level of nested CES production functions
A_{8j}	Shift parameter in the 8th level of nested CES production functions
A_{9j}	Shift parameter in the 9th level of nested CES production functions
A_{Tj}	Shift parameter in the make CET function
A_{Xi}	Shift parameter in the first level of CET function
A_{Mi}	Shift parameter in the first level of Armington function

Table 1.15: Tax Rates

Tax rates	**Description**
t_{fh}	Tax on factor income imposed to households
t_b	Tax on entrepreneurial earning
t_{yj}	Tax on production output of j
t_{ci}	Tax on consumption of good i
$t_{inv,i}$	Tax on investment good i
t_{xi}	Tax or subsidies on exports goods
t_{mi}	Tax or subsidies on imported goods
t_{vij}	Tax or subsidies on intermediate goods i

Table 1.16: Elasticities

Elasticities	**Description**
ρ_{4j}	Substitution parameter in the 4th level of nested CES production functions
σ_{1j}	Elasticity of substitution in the 1st level of nested CES production functions
σ_{2j}	Elasticity of substitution in the 2nd level of nested CES production functions
σ_{3j}	Elasticity of substitution in the 3rd level of nested CES production functions
σ_{4j}	Elasticity of substitution in the 4th level of nested CES production functions
σ_{5j}	Elasticity of substitution in the 5th level of nested CES production functions
σ_{6j}	Elasticity of substitution in the 6th level of nested CES production functions
σ_{7j}	Elasticity of substitution in the 7th level of nested CES production functions
σ_{8j}	Elasticity of substitution in the 8th level of nested CES production functions
σ_{Tj}	Elasticity of substitution in the make CET functions
σ_{Xi}	Elasticity of substitution in the 1st level of nested CET functions
σ_{Mi}	Elasticity of substitution in the 1st level of nested Armington functions

Chapter 2

Regional Trade Agreement, Emissions Bubble, and Carbon Tariff Harmonization

2.1 Introduction

This chapter analyzes the economic impact of the creation of an emissions bubble between Canada and the U.S. in the context of sub-global participation in the efforts to reduce pollution with market based-instruments. It is well known in the literature that there are several methods for improving the cost-effectiveness of pollution control policies. One of these methods is the creation of an emissions bubble among partners as allowed by UNFCCC (U.N. Framework Convention on Climate Change). According to that convention, countries are allowed to combine their individual targets to form a joint target. This policy has been experimented in the European Union (EU) that established a bubble in 1998. The EU committed to a joint abatement target of 8 percent below the level of 1990, given their individual targets.

The benefits of an emissions bubble is all the more important when a free trade agreement exists among the partners, and when the latter have different economic structures, as far as their consumption and production of emissions-intensive goods are concerned. Frankel (1999) argues[1] that within a bubble, a member country that has a comparatively low abatement cost could set an effectively higher reduction target and export additional allowances to the other members. In response, it could import emissions-intensive goods from its members.

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[1] Although he focusses on gains from emissions trading between developing and industrialized countries, but the tools he refers can also be applied any two regions follow comparative advantage.

Recent studies on carbon footprints and life cycle assessments by Peters and Hertwich (2008) and Ahmad and Wyckoff (2003) show that countries do differ in terms of their consumption and production of carbon embodied in goods. In particular, they show that Canada is a net producer of high emissions-intensive goods, while the U.S. is a net consumer of the same goods. Under these circumstances, the U.S. will have a lower abatement cost in comparison to Canada, making it possible for the former to sell permits to the latter. In return, Canada can be made better off by increasing its production of emissions-intensive goods and selling the surplus to the U.S.

Several studies have been recently carried out to assess the economic impact of the creation of an emissions bubble in the EU. For example, Kemfert et al. (2006) use a multi-sector, multi-regional general equilibrium model to analyze the economic impact of the bubble. They find that all member countries gain when the emissions trading is restricted to emissions-intensive industries alone. In contrast, they find that an economy-wide implementation of emissions trading leads to asymmetric gains among the participating countries.

In a similar vein, Babiker et al. (2004), using a recursive dynamic multi-regional general equilibrium model to assess the European emissions trading system, find that distributional problems may arise when pre-existing distortionary taxes exists in some participating countries. A permit trading can be welfare deteriorating due to the tax-interaction effect[2] and the term of trade effect. The latter refers to the "immiserizing" growth effect raised by Bhagwati (1958, 1968), which suggests that the direct gain of a member country from selling permits can be outweighed by the negative terms of trade effects. The same insight has been provided by Klepper and Peterson (2006) who find that asymmetric gains among participating members could arise if an emissions tax is imposed on an industry that is not included in the emissions trading system.[3]

Still, in a theoretical paper, Copeland and Taylor (2005) argue that direct gains from exports of emissions allowances could be offset by the negative, indirect terms of trade effects if total output of the emissions-intensive goods among trading partners

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[2] It refers to a situation where primary income gains from selling emissions permits can be offset by possible losses of households' real income in selling country due to its post-trading high cost of production and therefore, increases in the relative prices of consumption goods (see for example, Goulder et al. (1996)).

[3] See also Eichner and Pethig (2009)

declines. In this case, a decline in output of such goods would push their prices up and make at least one of the member countries worse off through the deterioration of its terms of trade.

From another perspective, climate change is a global issue, and as such, it entails the well-known free-rider problem, whereby some countries have incentives not to implement stringent pollution control policies. This sub-global participation in the efforts to reduce GHG (greenhouse gases) pollutants creates certain competitiveness concerns in countries that implement emissions abatement policies vis - a -vis those that do not.[4]

A common solution to addressing the competitiveness issue is to implement the so-called carbon border tax adjustment (BTA), which is a levy on the imports of emissions-intensive goods based on their carbon contents. Johnson and Krauss (1970) define the border tax adjustment as a tool which equalizes the conditions of competition between domestic and foreign producers, thereby allowing comparative cost to govern trade patterns. Hoel (1994) shows that, in the presence of economic externalities, if the marginal environmental cost of total emissions is sufficiently high in compliant regions then both the production and consumption of carbon should be taxed at positive rates. Some other studies (e.g., see Stiglitz (2006), Ismar and Neuhoff (2007), and Frankel (2009)) also suggest applying tariffs on the international trade framework in order to internalize the economic externalities embedded in the sub-global climate change policy.

However, drawing on previous papers, Lockwood and Whalley (2010) show, in a theoretical discussion, that if prices are flexible and labor is immobile between countries, any real effect of a border tax adjustment would be fully offset by changes in exchange rates, with no effects on trade.[5]

Thus far, there exist a number of strong opposite views in the literature as to the legality of the border tax adjustment with regards to the WTO (World Trade Organization) rules.[6] Several studies have been carried to assess the sectoral and welfare implications of this policy. Recent studies in this line, such as that of Dissou and Eyland (2010) and Babiker and Rutherford (2005), use a single or multi-country

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[4] Frankel (2008 and 2009), are some recent studies on the subject.

[5] Johnson and Krauss (1970) refer that the trade-neutrality is based on a uniform or general condition where rebates on exporters and tariffs imposed on importers, under the destination principle, are exactly adjusted to the taxes paid on consumptions of domestically produced goods.

[6] See Goh (2004), Cendra (2006), Ismer and Neuhoff (2007) and Fischer and Fox (2008) for discussions on that topic.

general equilibrium framework to assess the economic impacts of implementing a BTA. They find that, while the BTA can partially or fully address the competitiveness of a sub-global participation in GHG mitigation efforts, its welfare cost could prove to be small or significant depending on the situation.

There is a need for empirical research on the feasibility of the Canada-USA emissions bubble and its potential economic effects. We are not aware of any study on the economic impact of the formation of an emissions bubble between Canada and the U.S., using a detailed multi-sector, multi-regional general equilibrium model. In the same vein, there is no study, to our knowledge, on the economic impact of implementing a BTA in the framework of a Canada-US emissions bubble. The latter issue raises the natural question as to the manner in which best to implement a BTA. Should each country implement a BTA policy individually or should they jointly implement a harmonized BTA policy?

The objective of this chapter is to analyze the welfare, aggregate, sectoral and trade impacts of the formation of an emissions bubble between Canada and the U.S. with and without a BTA. For this purpose, we develop a multi-sector, and multi-region, static general equilibrium model to assess several scenarios related to the above-mentioned objectives. A general equilibrium setting has not only been recognized to be the most appropriate framework for assessing pollution control policies with market-based instruments, but also for studying bilateral trade impacts of major policy reforms.

The remainder of the chapter is organized as follows. The next section presents an overview of the model, while the third discusses the calibration procedures. The simulation results and sensitivity analysis are discussed in sections four and five. The last section shows conclusions.

2.2 Model Overview

We use a top-down competitive (Walrasian) general equilibrium model that comprises a standard format of market participants, including representative consumers, firms, governments, and international trade partners to analyze the effect of implementing a BTA within a Canada-US emissions-bubble. A detailed diagrammatic view of the model is shown in Figure 2.1. The representative consumer in region r, henceforth

RA_r, has initial endowments of capital $(\overline{K}_r)$, labour $(\overline{L}_r)$, and energy resources[7] $(\overline{Q}_{ff,r})$, and receives rents on these factors. The RA maximizes its utility (U_r) from the consumption of composite goods C_r which combines demands for energy and non-energy commodities within a Constant-Elasticity-of-Substitution (CES) function.

Firms maximize their profits and have access to a constant-returns-to- scale production technology. Production activities for goods i in region r are decomposed into the production of non-fossil fuel, $Y_{i \notin ff,r}$ and fossil fuel, $Y_{i \in ff,r}$. Primary factors are immobile across regions; however, intermediate goods, M^s_{ir}, can be substituted between domestic $(s = d)$ and imported $(s = m)$ goods through a CES function.

The government, in each region, is assumed to run a balanced budget. It collects revenues from households, firms, pollution activities, and emissions and goods trade activities. It recycles these revenues net of output subsidies to the representative consumer on a lump-sum basis. Therefore, the distortion arising through the tax-interaction effect may not be significant in our model framework. International trade in the model involves intermediate (as indicated above) and final goods levels. Final demands for domestic goods, i.e. C^d_{ir}, are also substituted with the imports of similar goods, C^m_{ir}, through a CES function.

The main features of market participants and sectors' activities are briefly reviewed below.

2.2.1 Representative Consumer's Behavior

As shown in Figure 2.2, the final demand for good i, in region r, is represented by a nested CES function between energy, $i \in e$, and non-energy, $i \notin e$, composites while each nest is further represented by a Cobb-Douglas utility function.

The representative consumer's problem, in region r, is to choose a consumption level, C_{ir}, in order to maximize its utility function subject to the associated budget constraint i.e.,

$$\max_{C_{ir}} U(C_{ir})$$

subject to

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[7] The subscript ff represents a set of coal, oil, and gas.

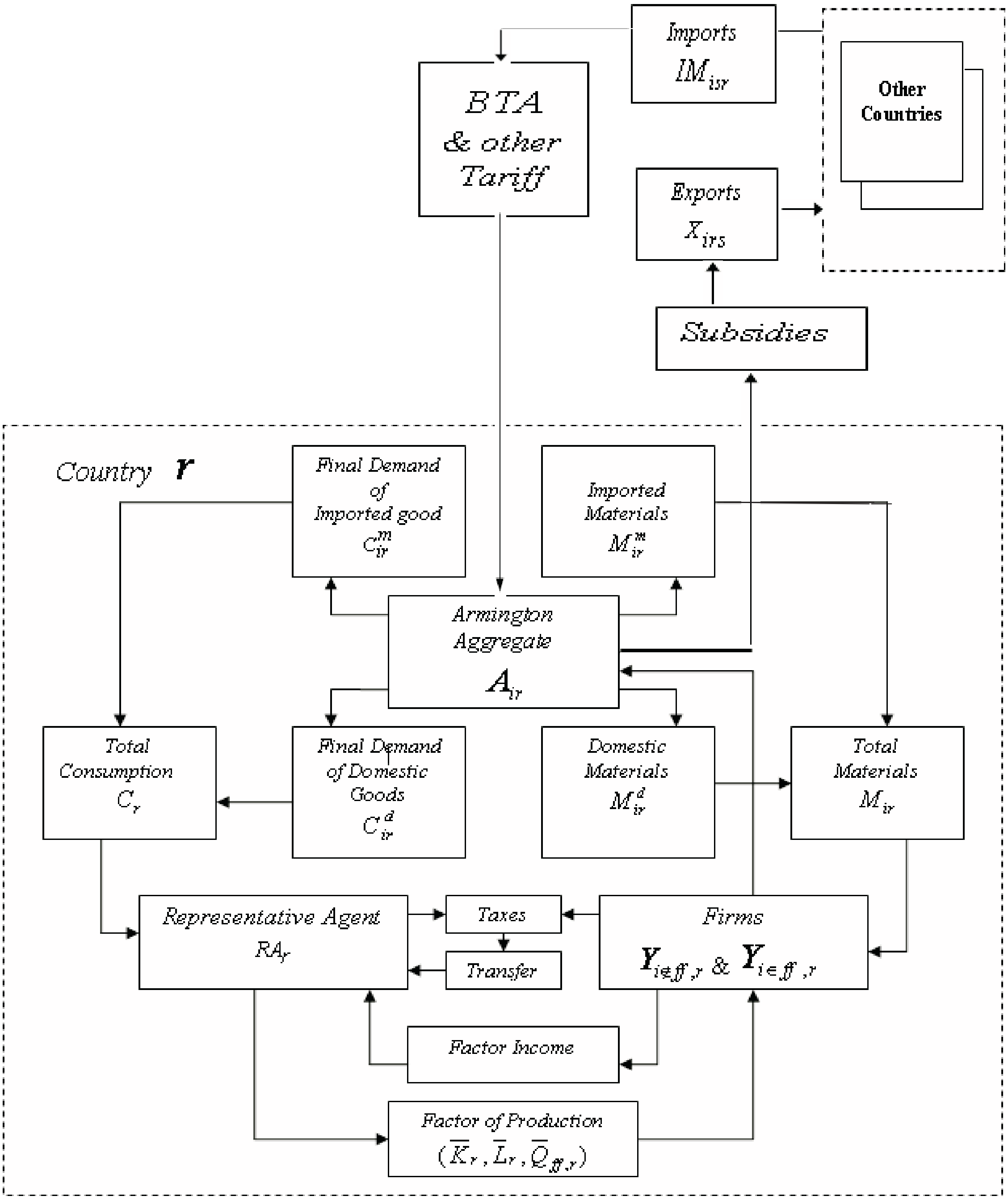

Figure 2.1: Diagrammatic View of the Model Structure

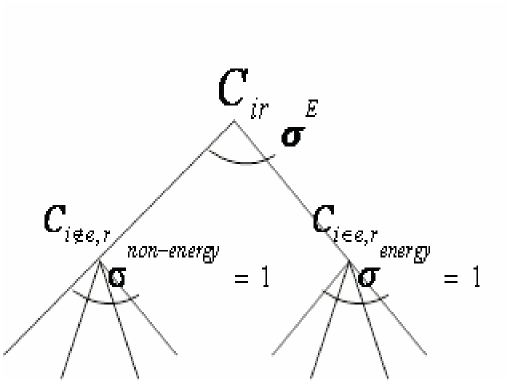

Figure 2.2: Final Consumptions of Goods

$$\sum_i P^C_{ir} C_{ir} \le P^l_r \bar{L}_r + P^K_r \bar{K}_r + \sum_{ff} P^Q_{ff,r} \bar{Q}_{ffr} + P^{CO_2}_r \overline{CO}_{2,r} - \sum_i P^G_{ir} G_{ir} - P^I_r I_r + \bar{B}_r + Tr_r \tag{2.2.1}$$

where P^C_{ir}, P^l_r, P^K_r, P^Q_{ffr}, $P^{CO_2}_r$, P^G_{ir}, and P^I_r, in Equation (2.2.1), represent, respectively, the prices of consumption goods, C_{i_r}, labor services, $\overline{L}_r$, capital services, $\overline{K}_R$, rent on natural resources, $\overline{Q}_{ff,r}$, $\overline{CO}_2$ emissions, government expenditure, and new vintage capital. In addition, G, I, $\bar{B}$, and Tr represent, respectively, government expenditure, investment, and capital flows, and lump-sum transfers to representative consumer.

2.2.2 Producer Behavior

Producer activities are decomposed into the production of non-fossil fuel and fossil fuel goods. As depicted in Figure 2.3, the production index of the non-fossil fuel good i in region r i.e., $Y_{i\notin e,r}$, is characterized by a nested, separable CES function between a composite of capital-labor-energy inputs and non-energy materials (M). The energy aggregation is decomposed into electricity and non-electricity sectors through a CES elasticity of 0.5. Non-electricity sectors are further decomposed into fossil fuel sectors. At this point, the model incorporates carbon emissions, which are linked to the use of energy inputs in a fixed proportion.

The production index of fossil fuel goods $i \in ff$ in region r i.e. $Y_{i\in ff,r}$ is produced by a top level nested CES function between sector-specific natural resources and a composite of value-added, energy, and non-energy materials (see Figure 2.4). All

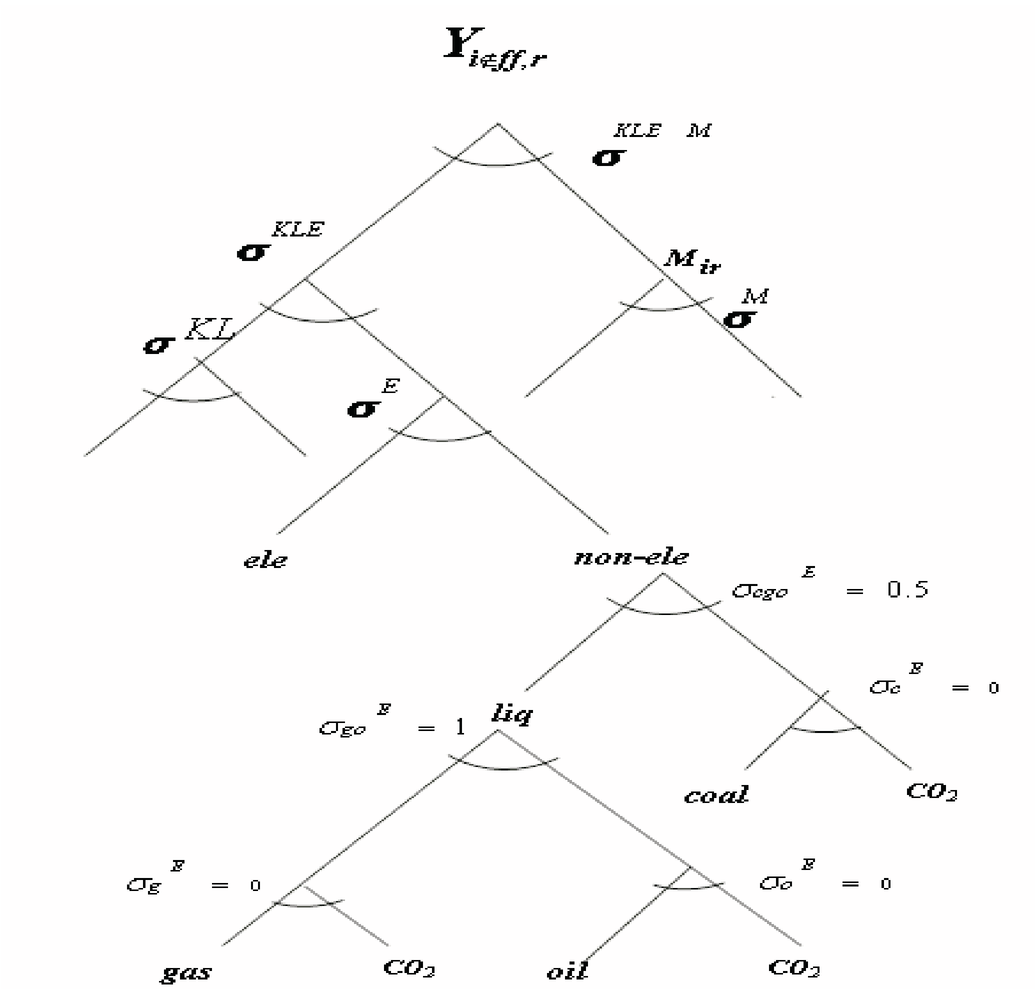

Figure 2.3: Production Structure of Non-fossil Fuel Sectors

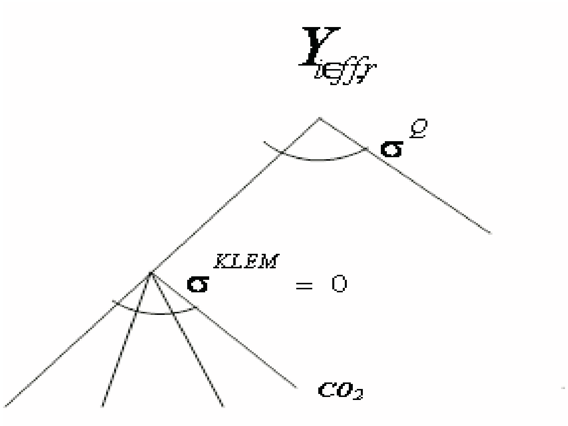

Figure 2.4: Production Structure of Fossil Fuel Sectors

other inputs are entered in a fixed proportion into the lower nested level.

The sector-specific resources , ff, are used as inputs in the production of fossil fuels. Here, carbon emissions are linked in fixed proportions to the use of fossil fuels in their productions. However, we allow the coefficients of emissions to differ by fossil fuels categories.

2.2.3 International Trade

International trade components are included into the model such that when one region is involved in international trade, the same class of intermediate (for production) and final goods (for consumption) from different regions are treated as imperfect substitutes (see Armington (1969)). This means that domestic and foreign goods of the same variety are distinguished only by their country of origin. As shown in Figure 2.5, the top level nest is an Armington aggregate of domestic and foreign composite goods, while the second level nest is an Armington composite of regions. In addition, it is assumed that each imported good incurs a transportation service cost with a fixed proportion. However, the choices among different international transportation services, YT_i, are made through a Cobb-Douglas composite. In the counterfactual analysis, international competitiveness issues brought about by emissions mitigation policies, can be offset by imposing a BTA as a harmonization policy.

2.2.4 Carbon Border Tariff Adjustment

Following Babiker and Rutherford (2005), the following constraint is endogenously imposed on the model:

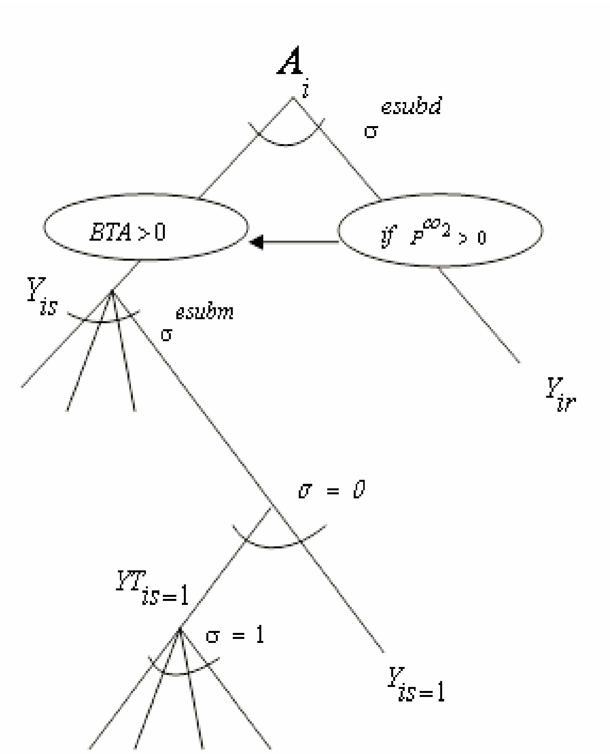

Figure 2.5: Armington Aggregation

$$CTA_{si} = \frac{P_r^{CO_2} * Emint_{s,i}^{CO_2}}{P_{s,i}} \tag{2.2.2}$$

where CTA indicates a measure of carbon tariff adjustment and $Emint^{CO_2}$ refers to the production-based emissions intensity by sector and region. The calculation of $Emint^{CO_2}$ requires information on carbon contents in the use of different fossil fuels mixed, λ_{ff}. It is calculated based on the knowledge of stoichiometry and information provided by the U.S. Department of Energy, the Energy Information Administration. We then calculate the emissions intensity for electricity as follows:

$$\lambda_{ele} = \frac{\sum_{ff} \lambda_{ff} * D_{ele,r}^{ff}}{\sum_{g} D_{g,r}^{ele}} \tag{2.2.3}$$

where notation D stands for demand, for instance, for energy or fossil fuels, and so on. Equation 2.2.3 shows the contents of CO_2 embodied in the consumption of electricity by different sectors which use a mix of fossil fuels.[8] Using the coefficients of emissions for energy input shown in Tables 2.1 and 2.2, we calculate emissions intensities by sector and region:

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[8] Of course, the value of λ_{ele} varies across regions depending on their choice of fossil fuel mix in the electricity production (see Table 2.2).

Table 2.1: CO2 Emission Content in Fossil Fuel use (in Gt carbon per ej)

Fossil Fuel	OIL	GAS	COL
λ_{ff}	0.066	0.050	0.088

Table 2.2: CO2 Emissions Intensity in Electricity Consumption by Regions for a Composite of Fossil Fuels Mix (in ton per US Dollar 2004)

Region	λ_{ele}
Australia-Newzealand	0.189
Brazil	0.016
Canada	0.056
China	0.226
European Union†	0.091
India	0.252
Japan	0.091
Mexico	0.131
Russia	0.079
Rest of the World	0.123
South Korea and Indonesia	0.125
United States	0.152

† Including EFTA

$$Emint_{r,i}^{CO_2} = \frac{\sum_e \lambda_e * D_{i,r}^{e}}{Y_{i,r}} \tag{2.2.4}$$

Plugging Equation 2.2.4 into 2.2.2 enables us to calculate the *CTA* which is equal to the costs of emissions by sector and region in total supply values of good i.

The subsidy rate to the exporters, who incur cost of emissions abatement, can be estimated as:

$$Rebate_{ir} = \frac{P_{ir}^{CO_2}\overline{CO}_{2,ir}}{P_{ir}Y_{ir}} \tag{2.2.5}$$

where *Rebate*, in Equation 2.2.5, is equivalent to the cost of emissions abatement that a sector i incurs in its total cost of production. Combining Equations 2.2.2 and 2.2.5 entails the concept of border tax adjustment generally discussed in literatures (see Johnson and Krauss (1970)). In the context of harmonizing the carbon abatement cost endured by a domestic producer, here we call it *BTA*.

Using a mixed complimentarity problem (MCP) approach that accounts for household and firm behaviors as well as international trade, the model solves for all equilibrium prices and goods demands in each market, such that the equilibrium conditions (i.e., zero profit, market clearing, and income balance conditions) are met. The U.S. consumer price index is taken as ”nominal exchange rate” while the real exchange rate is endogenously determined by the model, meaning that the current account balance is exogenously fixed.[9] The regional resource supplies are determined by the price of individual resources and the value of real exchange rate. To balance macroeconomic constraint saving-investment, government transfer is taken as endogenous, meaning that foreign and domestic savings, private and public investment demands are also exogenously fixed. The full listing of the model equations is provided in the Appendix. Numerically, the model is run using GAMS/MPSGE and solved using PATH.

2.2.5 Data Structure

This study uses a GTAP7 database which contains a hybrid of energy data in physical units along with detailed accounts of regional production and bilateral trade flows for the year 2004 (see Narayanan and Walmsley (2008)). All benchmark data for commodities and prices, and Armington elasticities are taken from GTAP7, while the elasticities of substitutions for value-added, energy and materials (i.e., KL, KLE, and KLEM) are taken from Okagawa and Ban (2008). Since the focus of the current analysis is on the target year of 2020, Okagawa and Ban (2008) provide more updated elasticities information for such a medium term period (horizon). Within the modeling structure, the world is broadly divided into two groups of regions. Annex 1 countries include Australia-New Zealand(ANZ), Canada(CAN), the European Union(EU), Japan(JPN), the United States (U.S.), and Russia(RUS). non-Annex 1 countries include Brazil(BRA), China(CHN), India(IND), Mexico(MEX), Indonesia and South Korea(SIN), and the Rest of the World(ROW).

The model incorporates various sectors which can be broken down into energy-intensive and non energy-intensive sectors. *Energy-intensive sectors* (EIS) include coal, crude oil, natural gas, refined oil products, electricity, mining, chemical products, air transport, other transport, non-metallic minerals, iron and steel products,

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[9] For details on external closure see De Melo and Robinson (1989).

non-ferrous metals, and paper-pulp-print. *Non-Energy Intensive Sectors* (NEIS) contain transport equipment, other machinery, food products, wood and wood-products, construction, textiles-wearing apparel-leather, other manufacturing, agricultural products, commercial and public services, and dwellings.

2.3 Baseline Calibration

Using the projected values of GDP growth, future energy demand and energy prices from the International Energy Outlook (IEO) 2008 database, the baseline year 2020 is chosen as a potential target year.

Using the IEO forecasted values of the growth of GDP and energy demand for 2020, we calculate the adjustment factors which are consistent with the IEO growth projections. We then use these multipliers to readjust the input-output dataset in such a way as to obtain a consistent social account matrix for the baseline year. Here, we calculate the autonomous energy efficiency index (AEEI) for the baseline year 2020 based on the same technique used by Babiker (2005). However, the dataset and IEO projections are different in the two models. The value of AEEI in Table 2.3 indicates the capacity of energy efficiency improvement among different regions by 2020 relative to the base year 2004.

The forecasted values of IEO energy prices for 2020 (see Table 2.4) are used to readjust the calibrated prices and the supply of specific resources in productions of fossil fuel sectors. The baseline calibration values satisfy all three equilibrium conditions indicated in the previous section.[10]

Table 2.5 shows the distribution of sectoral GDP, income level, and aggregate emissions intensity among the regions in the Business-as-Usual (BAU) case. Sectoral GDP is defined in terms of factor earnings, for example, earnings from capital, labor, and natural resources. As expected, the leading oil exporting Annex 1 regions, that is, Russia and Canada, have the highest shares of energy products in their total GDP among all regions selected for this study.[11] In contrast, the large Annex 1 regions such as Japan, the EU, and the U.S., have the lowest shares of energy production in their total sectoral GDP. Since income per capita in Annex 1 regions (excluding Russia) is not polarized, it seems that those regions which produce less of energy-intensive

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[10] For detail of algebra, see Appendix.

[11] Note that all OPEC regions are included into the ROW. Hence these regions are represented as an average of the Rest of the world.

Table 2.3: Baseline Projection Based on Energy Efficiency and GDP Growth Indices (2004-2020)

Region	Autonomous energy efficiency indices			GDP growth
	OIL	GAS	COL	Index
Australia-Newzealand	0.769	0.874	0.915	1.611
Brazil	0.701	1.240	1.534	2.146
Canada	0.767	0.972	0.942	1.492
China	0.485	0.845	0.702	4.460
European Union†	0.783	0.933	0.819	1.399
India	0.472	1.061	0.650	5.593
Japan	0.789	1.008	0.850	1.253
Mexico	0.881	1.061	0.953	1.695
Russia	0.580	0.663	0.733	2.839
South Korea and Indonesia	0.740	1.083	0.792	2.317
Rest of the World	0.702	0.937	0.752	2.394
United States	0.746	0.803	0.981	1.497

Source The computed values are based on GTAP7 and U.S. Department of Energy, Energy Information Administration, EIA (2008)
† Including EFTA

Table 2.4: World Crude Oil Prices (In US Dollars per barrel)

Year	2004	2020
Price	34.620	61.220

Source U.S. Department of Energy, Energy Information Administration, EIA (2008)

goods could meet their net energy demand by importing similar products. Table 2.6 shows that the U.S., Japan, and the EU are net importers of energy related goods while Canada, Russia, and Australia-New Zealand are net exporters. According to Figure 2.6, it is further observed that the current economic structures existing within Annex 1 regions may be distinguished between those which are net producers and those which are net consumers of GHG emissions-intensive goods. This implies that the differences in economic structure, resource returns, and trade patterns within Annex 1 countries need to be taken into account in the analysis of economic impacts of climate change policies. This is particular true in the context of Canada and the U.S. where the differences in production activities of energy-related goods may influence policy choices and their associated effects.

Although our main focus is on the competitiveness issue, we note an interesting

Table 2.5: Distribution of Sectoral GDP in Energy and Non-Energy Intensive Sectors Activities

Region	% of Energy* Sectors in GDP	%of Energy-Intensive Sectors in GDP	%of Non Energy Energy-Intensive	GDP per Capita in US Dollar 2004	Emission Intensity kg/US Dollar 2004
Australia-Newzealand	5.70	20.16	79.84	43091	0.58
Brazil	8.61	15.25	84.75	6172	1.07
Canada	11.99	21.20	78.80	43109	0.62
China	6.19	26.50	73.50	4685	1.78
European Union+EFTA	2.75	15.19	84.81	37804	0.31
India	5.03	18.67	81.33	1904	1.14
Japan	1.64	13.01	86.99	47899	0.24
Mexico	9.16	21.61	78.39	9234	0.75
Russia	30.27	40.83	59.17	10090	1.41
South Korea and Indonesia	9.75	21.52	78.48	6782	0.89
Rest of the World	21.87	32.79	67.21	3402	1.18
United States	3.85	12.86	87.14	52683	0.43

* This column includes energy goods which are oil, gas, coal, and electricity.

Table 2.6: Distribution of Regional Exports and Imports

Distribution of energy-intensive trade by regions (In Billion US2004)

Region	CAN		U.S.		EU		JPN		ANZ		RUS		ROW	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
CAN	-	-	78.5	131.7	17.1	16.5	1.3	3.0	1.3	0.7	1.1	0.3	32.9	34.0
U.S.	131.7	78.5	-	-	166.1	125.1	23.0	26.7	7.3	9.7	16.8	3.0	462.1	292.2
EU	16.5	17.1	125.10	166.14	1172.90	1172.90	21.43	30.04	11.69	17.74	115.97	43.98	645.35	625.08
JPN	3.0	1.3	26.7	23.6	30.0	21.4	-	-	15.5	2.9	4.3	1.1	215.4	187.3
ANZ	0.7	1.3	9.7	7.3	17.7	11.7	2.9	15.5	-	-	0.1	0.3	48.5	84.5
RUS	0.3	1.1	3.0	16.8	44.0	116.0	1.1	4.3	0.3	0.1	-	-	37.2	118.5
ROW	34.0	32.9	292.2	462.1	625.1	645.4	187.3	215.4	84.5	48.5	118.5	37.2	1820.6	1816.3
Total	186.2	132.2	535.3	807.7	2073.0	2108.9	237.1	295.0	120.6	79.6	256.8	86.0	3262.0	3158.0

Percentage share of energy-intensive goods in total trade by regions

Region	CAN		U.S.		EU		JPN		ANZ		RUS		ROW	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
CAN	-	-	26.3	38.3	37.2	37.4	12.7	30.0	43.7	22.6	80.0	17.2	29.4	40.0
U.S.	38.3	26.3	-	-	36.2	28.0	17.2	23.2	20.1	46.1	85.3	14.5	31.6	28.6
EU	37.4	37.2	28.0	36.2	34.4	34.4	17.7	32.0	22.7	60.1	86.6	30.2	32.6	34.7
JPN	30.0	12.7	23.2	17.6	32.0	17.7	-	-	84.2	11.0	61.3	11.6	39.2	32.0
ANZ	22.0	44.9	26.6	34.7	34.4	39.6	15.8	58.9	-	-	56.1	33.3	29.8	66.9
RUS	17.2	80.0	14.5	85.3	30.2	86.6	11.6	61.3	33.3	56.1	-	-	24.6	71.8
ROW	40.0	29.4	28.6	31.6	34.7	32.6	32.0	39.2	66.9	29.8	71.8	24.6	41.0	40.4
EIS as % of total Trade	38.1	28.1	27.6	33.1	34.5	34.2	27.0	36.8	51.0	32.8	78.4	26.1	36.8	38.1

fact that can help improve our understanding of the causes of carbon leakages. As depicted in Figure 2.6, China and India are net exporters of CO_2 embodied goods in international trade, but according to Table 2.7 these countries are net importers of energy-intensive goods in the case of trade with individual Annex 1 regions, with the exception being trade between China and the U.S. In this case, China emerges as a net exporter of energy-intensive goods to the U.S. In fact, there seems to be no conflict between what we show in Figure 2.6 and Table 2.7 because non-Annex 1 regions have higher emissions intensities. These results are summarized in Table 2.5. Our results are consistent with the Dong and Whalley's (2008) argument that there are greater differences in emissions intensities across regions rather than across products.

Hence in the baseline case, the trade patterns would be such that non-Annex 1 regions (except ROW[12]) would specialize in non-energy intensive goods, while Annex 1 regions would specialize in energy-intensive goods (see Table 2.7). However, when Annex 1 regions adopt their domestic mitigation policies, the non-Annex 1 regions would increase production of energy related goods not only to meet their own demands for energy-intensive goods, but also to meet the partial demand from Annex 1 regions. The non-Annex 1 countries would then reduce their share in NEIS goods and cause higher emissions per unit of production due to the existing technology differences with Annex 1 regions.

To see the feasibility of a creation of Canada-USA emissions bubble as a potential policy (see section 2.4.2), we report the facts about the aggregate economic behavior, industrial structure, and the trade patterns of two regions (see Table 2.8).

The economic behavior suggests that Canada is a net producer of energy-intensive goods, while the U.S. is a net consumer. The industrial structures between the two regions indicate that Canada is relatively more emissions-intensive than the U.S. The trade patterns between the two regions show that Canada is the major net exporter of energy-intensive goods to the U.S., which in turn is the major net exporter of non-energy-intensive goods to Canada. In addition, Canada is a net exporter of emissions-intensive goods, while the U.S. is a net importer of these goods. The differences in economic behavior, industrial structure, and trade patterns between the two regions are the main motivations examining the scope of a potential emissions bubble between Canada and the U.S.

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[12] See footnote 11.

Table 2.7: Trade Pattern of Selected non-Annex 1 Countries at BAU level (In Billion US $ 2004)

Trade in energy-intensive goods

	Annex 1		ANZ		CAN		EU		JPN		RUS		U.S.	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
CHN	274.95	197.17	26.582	9.55	12.91	6.97	81.81	72.21	73.77	37.91	22.12	3.96	57.76	66.58
IND	99.46	30.14	15.69	0.72	1.75	0.68	61.84	15.71	4.01	2.22	4.46	1.79	11.71	9.01
MEX	74.81	68.11	0.21	0.15	1.10	1.32	9.64	7.83	1.63	0.62	0.28	0.05	61.95	58.15
BRA	35.01	31.37	1.11	0.27	1.41	0.97	18.18	13.40	1.38	2.38	1.11	0.26	11.82	14.09
SIN	139.06	76.06	13.44	7.75	4.11	1.45	36.75	16.93	44.44	31.93	6.84	2.14	33.49	15.88

Trade in non energy-intensive goods

	Annex 1		ANZ		CAN		EU		JPN		RUS		U.S.	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
CHN	548.69	1118.46	17.47	33.32	12.27	33.44	217.87	448.80	161.33	182.96	8.07	27.25	131.68	392.69
IND	80.69	191.57	1.55	4.25	2.03	5.62	47.49	109.89	6.25	9.60	2.69	4.22	20.67	57.99
MEX	153.02	191.21	0.62	0.63	3.91	7.79	21.37	10.52	5.13	1.97	0.04	0.66	121.94	169.64
BRA	52.52	61.34	0.38	0.70	0.94	1.45	27.56	32.62	3.23	2.50	0.21	4.59	20.21	19.48
SIN	252.33	282.35	11.76	10.74	4.26	7.09	77.56	111.08	62.97	39.56	2.67	7.49	93.09	106.39

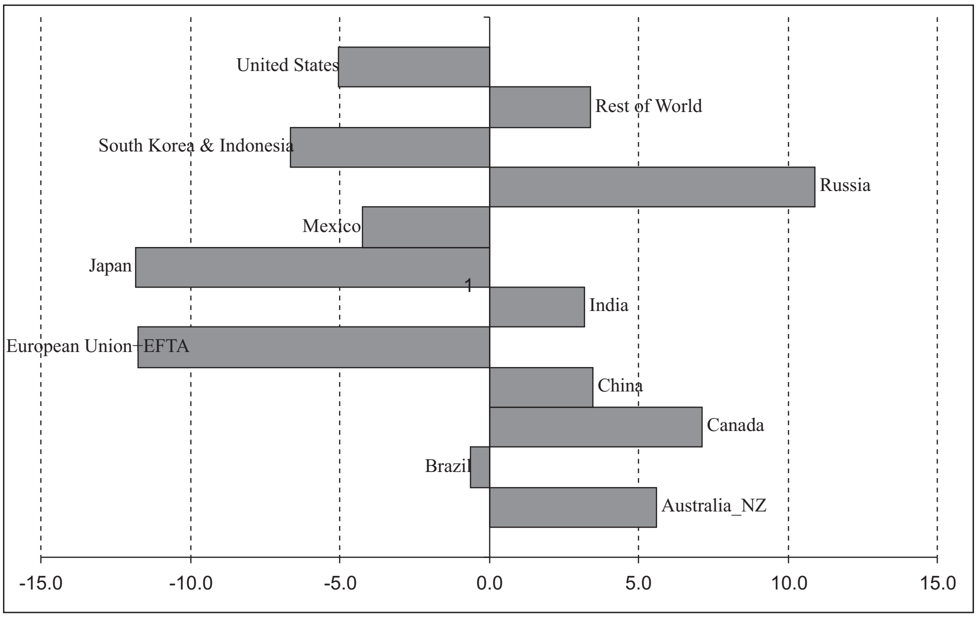

Figure 2.6: Trade Balance in CO2 Emission (As % of Domestic Production)

Table 2.8: Indicators of Consumption Behavior, Industrial Structure, and Trade Patterns of Canada and the U.S.

Indicators	Canada	U.S.
Trade balance of CO_2 emission*	7.00	-5.00
Emission intensity	0.62	0.43
% of emission-intensive goods in GDP	21.22	12.86
Net exports of energy-intensive goods**	+53.23	-53.23
Net exports of non-energy-intensive goods**	-8.25	+8.25

* As % of domestic production **In billion US$ 2004

2.4 Policy Scenarios and Results

In this analysis, we assume that each Annex 1 region sets a uniform emissions reduction target of 20 % below their 2005 level by 2020. We then evaluate the economic and environmental consequences of the formation of a potential Canada-USA emissions bubble and its effectiveness with and without the carbon border tax adjustment policy, whether it is imposed by an individual member of the bubble or by all Annex 1 regions. We also examine the case of the imposition of a common external BTA by Canada and the U.S.

The descriptions of all scenarios are shown in Table 2.9. In the reference (REF) scenario, all Annex 1 regions adopt domestic abatement actions without any emissions bubble and BTA policies. In scenario I, we compare the REF case with the

Table 2.9: Description of Policy Scenarios

	Policies without BTA
REF	All Annex 1 regions take domestic abatement actions without any emissions bubble and BTA policies
I	Canada & the U.S. adopt emissions bubble and other Annex 1 regions implement domestic action
	Policies with BTA
II	Canada & the U.S. make emissions bubble, other Annex 1 regions adopt REF scenario, and only Canada levies BTA
III	Canada & the U.S. make emissions bubble, other Annex 1 regions adopt REF scenario, and only the U.S. levies BTA
IV	Canada & the U.S. adopt emissions bubble along with the common BTA while other Annex 1 regions adopt REF scenario

Canada-USA emissions bubble. Scenarios II and III observe the Canada-USA emissions bubble policy when the BTA is imposed either by Canada or the U.S., and scenario IV covers the Canada-USA bubble along with a common external carbon tariff harmonization policy.

To understand more exhaustively the effect of trade protection brought about by a domestic pollution control policy, we examine sections II-IV by assessing the impact of carbon tariff with and without subsidies granted to the exporters who incurs emissions abatement cost in their production process. However, the results of scenarios II to IV, we showed in all Tables and Figures, are reflecting the case of full BTA policy.

To avoid any ad hoc decisions over the choices of sectors in our carbon border tax adjustment analysis, rather we consider a simple approach which involves the imposition of a BTA on the content of carbon embodied in imports and exports activities of each sector in this study.

2.4.1 Reference (REF) Scenario: all Annex 1 regions adopt domestic abatement actions without any emissions bubble and BTA policies

The implication of the REF case remains a challenge for regions experiencing a high energy intensity and a low opportunity of fuel switching. Hence, we incorporate caps and trade as a domestic abatement policy option. According to the BAU analysis, Canada is a highly energy-intensive economy and has a low opportunity of fuel switching (see Table 2.10).

Table 2.10: Decomposition of Fossil Fuels Mix (In %)

Regions	Coal	Gas	Oil
ANZ	51.00	9.09	39.91
CAN	18.78	31.05	50.17
EUR	27.22	18.89	53.89
JPN	27.34	13.62	59.04
RUS	29.23	38.21	32.56
USA	44.24	6.84	48.92

Therefore, it incurs the highest marginal cost of abatement due to its high emissions intensity among all specified Annex 1 regions, except for Russia[13] (see Table 2.13). In contrast, regions such as the U.S. have a high opportunity to switch to fuel and can abate with a lower welfare cost. A comparison of BAU fossil fuels mix used by all sectors in each Annex 1 region indicates that the U.S. has the least share of natural gas in the composition of fossil fuels mix and thus has the highest occasion of efficiency improvement among all Annex 1 regions (as shown in Table 2.10).

The REF policy exerts a significant impact on terms of trade (see Table 2.12), such that Annex 1 regions increase imports of EIS goods from non-Annex 1 regions. Some major trading partners, including the EU & China, even invert their trade patterns in such a way that the EU becomes a net importer of EIS goods from China, while it was a net exporter in the BAU case (see Tables 2.7 & 2.21). Similarly, the U.S. increases imports of EIS goods from China by around 8% relative to its BAU level (see Tables 2.7 & 2.21). Despite a considerable increase in the imports of EIS goods from China, the overall effects of such trade adjustments still have a negative impact on Chinese welfare (see Table 2.16). One possible reason may be the international trade spillover. Annex 1 regions not only increase their imports of energy-intensive products from China, but they also reduce imports of non energy-intensive goods in which China had *ex ante* trade specialization. The net welfare effect of a change in the Chinese trade pattern is negative. In our example, an increase in the Chinese energy-intensive production (see Table 2.14) explains part of carbon leakage[14] (see

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[13] In the recent decade, the dollar value of rubles has declined considerably. Hence, replacing GDP, at the market exchange rate, (which we used in this analysis) with purchasing power parity can reduce the emissions intensity of Russia. In addition, the opportunity of replacing old technologies with new ones combined with an increase in the share of nuclear energy consumption may provide a high efficiency improvement avenue for Russia.

[14] Leakage is defined as the ratio of increase in GHG emissions in non-Annex 1 regions to a decline in GHG emissions in Annex 1 countries. Notice that the capital is immobile across regions. Relaxing this assumption may lead to a larger impact of leakage.

Table 2.15).

A similar effect of the policy shock is observed in the Canada-USA bilateral trade pattern. In the baseline case, Canada was a net exporter of energy-intensive goods and a net importer of non energy-intensive products from the U.S. (see Table 2.11). However, in the REF scenario, the direction of their trade patterns changes. According to Table 2.13, a high cost of abatement causes a competitive disadvantage to Canada over its competitors in the U.S. market. Therefore, the *ex post* differences in EIS prices (see Table 2.12) between Canada and its competitors induce the U.S. to increase its import shares from other regions especially from China. Hence, Canada experiences a significant decline in its production of EIS goods (see Table 2.14) and becomes a net importer of energy-intensive goods (excluding fossil fuels) from the U.S. (see Table 2.11). On the other hand, an increase in EIS production in non-Annex 1 regions accounts for carbon leakages because of the differences in emissions intensity across regions.

Overall, the magnitude of the effect of an *ex post* policy distortion on regional welfare varies across countries depending on their costs of industrial restructuring, economic behavior, and terms of trade effects.

2.4.2 Scenario I: Canada & the U.S. adopt emissions bubble while other Annex 1 regions implement REF scenario.

There is no pre-existing emissions tax distortion (see Babiker et al. (2004)) in our emissions trading mechanism.[15] As described earlier, an emissions bubble refers to a collection of emissions by a group of regions which commit to a single emissions limit. In the Canada-USA case, due to the abatement cost advantage, the U.S. abates more and sells its emissions allowances to Canada which, in turn, covers the energy related consumption demand for their economy. This is shown in Table 2.14 where Canada increases and the U.S. reduces their production of energy-intensive goods relative to the REF case. The common target is allocated between the two regions such that Canada and the U.S. set domestic targets by 16.32% and 23.19%, respectively. The market clearing permit price, determined by the demand and the supply of permits, is set at US\$36 per ton of CO_2 equivalent (see Table 2.13).

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[15] Here we refer that the existing lump-sum transfers of all tax revenues net of output subsidies to representative consumers make the tax-interaction effect less significant, thereby we are more focussed on term of trade effect.

Table 2.11: Net Exports of Canada to the U.S.(In Billion US$ 2004)

Sectors	BAU			REF			Scenario I		
	Export	Import	Net	Export	Import	Net	Export	Import	Net
Aggregate	343.90	299.02	44.88	339.73	293.97	45.77	343.82	294.23	49.60
EIS	131.68	78.55	53.23	114.01	78.20	35.81	126.46	76.69	49.77
EIS (excluding *ff*†)	79.46	72.87	6.59	63.04	74.21	-11.17	77.05	71.94	5.11
Fossil Fuels	52.11	4.99	50.96	50.93	3.77	47.16	49.46	4.37	45.09
NEIS	212.22	220.47	-8.25	225.72	215.76	9.96	217.36	217.54	-0.17

†*ff* is a set of fossil fuels i.e. oil, gas, and coal.

Table 2.12: Equilibrium Price Indices of Some Energy-Intensive Goods

	CANADA		USA		EUR		JPN		ANZ		RUS	
	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In % change)
Refined oil products	1.582	15.474	1.642	1.082	1.619	10.493	1.585	13.552	1.629	4.822	1.617	-0.872
Electricity	1.053	21.593	1.085	14.021	1.085	16.416	1.129	10.874	1.059	33.009	1.392	5.706
Chemical Products	1.085	8.752	1.053	1.250	1.074	1.845	1.118	2.851	1.041	-0.423	1.175	2.697
Iron and steel	1.055	4.603	1.027	0.664	1.069	2.632	1.105	2.905	1.053	1.632	1.045	0.493
Mining	1.149	8.477	1.059	0.137	1.157	0.301	1.121	1.742	1.162	-2.484	0.867	-0.882
Non-ferrous metals	1.045	4.787	1.019	0.831	1.049	1.521	1.066	0.645	1.077	5.089	0.995	0.219
Non-metallic minerals	1.023	1.771	1.027	0.917	1.067	1.914	1.092	1.601	1.049	0.304	1.057	0.132
Paper-pulp-print	1.025	1.662	1.001	0.059	1.035	-0.137	1.075	0.121	1.015	-1.394	0.929	-1.316

	CHN		IND		MEX		BRA		SIN		ROW	
	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In %change)	BAU	REF (In % change)
Refined oil products	1.492	-2.324	1.599	-2.775	1.629	-2.542	1.516	-2.062	1.629	-2.882	1.617	-3.024
Electricity	0.940	-0.817	0.870	-0.931	1.446	-0.205	1.005	-0.301	1.130	-1.652	1.152	-1.653
Chemical Products	0.946	-0.164	0.986	-0.377	1.027	-0.416	1.069	-0.170	1.113	-0.511	1.058	-0.380
Iron and steel	0.941	-0.305	0.865	-0.274	1.111	-0.356	1.049	-0.496	1.035	-0.235	1.030	-0.391
Mining	0.948	0.106	0.951	-0.058	1.426	-0.444	1.087	-0.204	1.017	0.049	1.001	0.088
Non-ferrous metals	0.927	-0.125	0.884	0.010	0.987	-0.350	1.053	-0.305	1.003	0.051	0.984	-0.139
Non-metallic minerals	0.910	-0.311	0.883	-0.322	1.028	-0.255	1.057	-0.358	1.027	-0.555	0.993	-0.530
Paper-pulp-print	0.879	-0.228	0.815	-0.303	0.976	-0.323	1.004	-0.187	1.007	-0.295	0.971	-0.289

Table 2.13: Marginal Abatement Costs (US$ 2004 per ton of CO_2 eq.)

	REF	I	II	III	IV
Australia-Newzealand	59.68	59.58	59.57	59.51	59.49
Canada	125.42	36.27	36.80	36.66	37.26
European Union (Including EFTA)	76.62	76.43	76.36	78.13	77.96
Japan	75.32	75.26	75.20	75.42	75.35
Russia	16.56	16.51	16.50	16.92	16.88
United States	29.78	36.27	36.80	36.66	37.26

Table 2.14: Energy-Intensive Production (% Change from BAU level)

	REF	I	II	III	IV
Australia-Newzealand	-3.50	-3.54	-3.54	-3.51	-3.52
Brazil	1.52	1.46	1.42	0.92	0.89
Canada	-17.29	-4.15	-1.97	-3.39	-1.11
China	1.14	1.09	1.04	0.77	0.71
European Union (Including EFTA)	-3.50	-3.55	-3.57	-3.49	-3.51
India	1.06	1.04	1.02	0.84	0.82
Japan	-3.51	-3.56	-3.57	-3.52	-3.53
Mexico	1.30	1.15	1.07	0.59	0.51
Russia	-1.55	-1.57	-1.57	-1.37	-1.38
South Koera and Indonesia	2.97	2.91	2.85	2.58	2.52
Rest of the World	2.51	2.45	2.39	1.91	1.85
United States	-1.60	-2.54	-2.56	-2.19	-2.20

Forming the Canada-USA emissions bubble, in such a way, allows both trading partners to improve their trade balance relative to the REF case (see Table 2.11). The combination of free trade in goods with free trade in emissions causes Canada to remain a net importer of emissions permits, and the U.S. to remain a net importer of energy-intensive goods (see Tables 2.11 & 2.17). Hence, the *ex-post* trade patterns are consistent with their *pre-policy* directions of trade. Consequently, the combination of an emissions bubble with a policy of free trade in goods makes both regions better off and leads to a welfare improvement relative to the REF case.

In the context of the Canada-USA bubble, our finding does not support the existence of nontrivial negative terms of trade effects in a fashion of East-West regions' permit trading theory explained by Copeland and Taylor (2005). They assume that the East and the West are two emissions binding regions. Both regions produce

Table 2.15: Leakage (In %)

	REF	I	II	III	IV
Brazil	0.45	0.45	0.44	0.24	0.23
China	3.83	3.73	3.54	2.84	2.67
India	1.18	1.19	1.18	1.04	1.04
Mexico	0.20	0.19	0.18	0.03	0.02
South Korea and Indonesia	1.58	1.57	1.56	1.43	1.41
Rest of the World	8.76	8.55	8.39	6.99	6.84
Global	15.91	15.68	15.29	12.56	12.20

and trade emissions-intensive as well as non-emissions intensive goods. The West is specialized in clean goods, while the East is diversified. Under domestic abatement action, the West has a higher permit price than that of the East. Hence, if the East-West begin permits trading, the West would be a net importer of the emissions permit. In addition, if the West increases *ex post* production of clean goods in which it had an *ex ante* specialization, then the total production of emissions-intensive goods in the two regions would decline, which in turn would account for a rise in international prices of emissions-intensive goods. This would have a negative terms of trade effect on at least one of the trading regimes.

Contrary to Copeland and Taylor's (2005) assumption, our results suggest that the U.S. is not only a net exporter of clean goods to Canada, but it also has a lower cost of abatement than Canada. Moreover, the economic behaviors of the two countries are such that Canada is a net producer of energy-intensive goods, while the U.S. is a net consumer. Under these circumstances, the overall efficiency in terms of real GDP might improve in both energy-intensive and non-energy-intensive sectors (see Table 2.18). This reflects that both regions would be better off in scenario 1 relative to the REF case.

The increase in trade activities (see Table 2.11) between Canada and the U.S., brought about by an emissions bubble, may ultimately project a positive effect on international trade patterns such that non-Annex 1 regions increase their trade share in non energy-intensive goods in which they had *ex ante* trade specialization. Hence, the development towards the Canada-USA emissions trading would not only make these regions better off, but it would also improve global welfare (see Table 2.16). The environmental effect would also be positive due to the differences in the emissions

Table 2.16: Welfare Cost of GHG Abatement (% Change from BAU Level)

	REF	I	II	III	IV
Australia-Newzealand	-1.06	-1.07	-1.07	-1.06	-1.06
Brazil	0.00	-0.01	-0.01	-0.11	-0.11
Canada	-2.24	-1.65	-1.83	-0.99	-1.19
China	-0.15	-0.13	-0.15	-0.38	-0.40
European Union (Including EFTA)	-0.63	-0.63	-0.62	-0.60	-0.60
India	0.17	0.19	0.18	0.01	0.01
Japan	-0.48	-0.48	-0.48	-0.48	-0.48
Mexico	-0.48	-0.52	-0.52	-0.51	-0.51
Russia	-1.84	-1.87	-1.88	-1.83	-1.84
South Koera and Indonesia	-0.03	-0.03	-0.04	-0.17	-0.18
Rest of the World	-0.60	-0.62	-0.63	-0.69	-0.70
United States	-0.10	-0.08	-0.06	-0.25	-0.24
the Canada-USA	-0.25	-0.18	-0.18	-0.30	-0.30
All Annex 1 regions	-0.49	-0.46	-0.46	-0.50	-0.50
Global	-0.43	-0.411	-0.413	-0.48	-0.49

Table 2.17: Net Imports of Emissions between the Canada-USA(In Million of tons eq. of CO2)

	REF	I	II	III	IV
Canada	-	170	181	177	188
United States	-	-170	-181	-177	-188

intensity across countries. This implies that the pre-existence of regional free trade in goods gives rise to a potential formulation of a Canada-USA emissions bubble.

2.4.3 Scenario II: Canada & the U.S. make emissions bubble, other Annex 1 regions adopt REF scenario, and only Canada levies BTA.

Carbon Tariff along with Exports Subsidies

Contrary to policy anticipation, Canada experiences a welfare loss under a self-implemented BTA policy regardless of whether it is imposed under an emissions bubble or in the REF case. Let us first consider the case where Canada imposes a BTA under an emissions bubble. Recall that under this policy, Canada harmonizes the *post-policy* differences between the prices of domestically produced goods and the foreign goods belonging to the non-Annex1 regions. In other words, it imposes carbon harmonized tariffs (subsidies) on imported goods (to exported goods) from non-Annex1 regions (to its exports markets). Due to this policy, Canada's output and

Table 2.18: Total real GDP* of the Canada-USA from two Groups of Sectors (In Billion US$ 2004)

	REF	Scenario I	% change from REF
EIS	2,033	2,046	0.64
NEIS	14,961	14,973	0.08

* GDP is defined in terms of factor earnings, for example, earnings from capital, labor, and natural resources.

net exports of EIS goods are increased respectively by 1.8 percent and 15 percent[16] relative to the policy of emissions bubble without BTA. Canada and the U.S. both improve their exports of EIS goods to each others relative to scenario I. However, Canada gains net exports to the U.S. by 0.5 percent. The U.S. trade activities with other regions (especially with non-Annex1 regions) remain almost stable at scenario I level (see Table 2.24).

Since Canada has emissions constraints, the BTA-induced additional productions can be possible only by purchasing more emissions from the Canada-USA emissions market. Therefore, we see in Table 2.17 that Canada purchases more emissions from the U.S. at relatively high prices (see Table 2.13). This additional transfers to the U.S. are allocated to its representative consumer. On the other hand, the carbon tax-adjusted subsidies to the Canadian exporters reduces a portion of transfers amounts which were returned to the domestic consumer in the emissions bubble policy. Under these circumstance, the Canadian BTA policy with Canada-USA emission bubble makes Canada's welfare worse off and the U.S's better off (see Table 2.16).

If Canada adopts a BTA in line with the REF case, it would even more worse off (see Table 2.19). In the absence of emissions bubble, an increase in the domestic production of EIS goods would further increase the domestic auction price of emissions, and so would the tariffs levied on the EIS goods imported from non-Annex1 regions. Therefore, the domestic consumer will suffer more relative to the emissions bubble scenario (see Table 2.19). On the other hand, due to high permit prices, the domestic producers will get high subsidies on EIS goods export to other regions, which in turn makes foreign consumers better off and the domestic consumers worse off as they will loose part of the government transfers due to the subsidies given to exporters.

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[16] Canada's net exports increase mainly due to a decline of -16 percent imports of EIS goods from non-Annex1 regions.

Table 2.19: Effects of Imposing BTA by Canada Without Emissions Bubble

Region	Indicators	REF	BTA-Canada
Canada	Permit-Price	125.42	151.49
	Welfare*	-2.24	-2.80
	Energy-Intensive output*	-17.29	-13.94
	Net exports of EIS to Annex 1**	33.07	30.21
	Export	130.22	145.02
	Import	97.99	114.48
	Net exports of EIS to USA**	35.81	35.04
	Export	114.02	126.05
	Import	78.21	91.01
	Net exports of EIS to non-Annex 1**	-5.54	12.75
	Export	26.85	31.35
	Import	32.39	18.60
USA	Permit-Price	29.78	29.81
	Welfare*	-0.10	-0.07
	Energy-Intensive output*	-1.60	-1.48
	Net exports of EIS to Annex 1**	-77.70	-75.74
	Export	242.50	254.34
	Import	320.20	330.08
	Net exports of EIS to non-Annex 1**	-181.09	-183.34
	Export	284.37	282.28
	Import	465.46	465.62

*% change from BAU level **In billion US$ 2004

Carbon Tariff only

In this case, Canada's imposition of a carbon tariff does not improve the competitiveness of its domestic energy-intensive sectors, no matter if it is imposed under an emissions bubble or in the REF case. Nonetheless, Canada experiences a welfare loss under a self-implemented carbon tariff policy.

If Canada imposes carbon tariff under an emissions bubble, the production of energy-intensive goods, relative to the BAU level, will decline by -4.34 percent, which is larger than the case of full BTA as well as scenario I (see Table 2.14). A decline in the Canadian EIS production is a result of a drop in its exports shares of energy-intensive goods to both Annex 1 and non-Annex 1 countries. A possible explanation of such post-policy trade effects can be deduced from the *ex post* changes in the prices of EIS goods among Annex 1 and non-Annex 1 regions, i.e., the terms of trade effects. Table 2.20 shows that the qualitative effects of a tariff imposition by Canada on price changes are different for Canada and China. Although we do not analyze the decomposition of cost structure of EIS, it may be possible for the carbon tariff to increase the cost of intermediates used in the production of EIS-related goods, which could ultimately increase the prices of energy related final goods (see Table 2.20). On the other hand, the *ex post* prices of intermediate and related EIS goods could also decline in the non-Annex 1 regions due to the imposition of tariff by Annex 1 regions, such as Canada. Since Canada has to face competition in its exports market from both compliant and non-compliant countries, this could lead to lower prices of intermediate and EIS-related goods, forcing Canada to reduce its exports shares.

Table 2.20: Comparison of Price Changes in some EIS Goods in Canada & China(*In %change relative to a case when Canada-USA adopt emissions bubble but not a carbon tariff policy*)

	Canada	China
Refined oil Products	3.93	0.10
Electricity	1.76	-1.59
Chemical Products	1.64	-1.09
Iron and Steel	1.54	-1.27
Mining	2.06	-1.46
Non-ferrous metals	1.50	-1.23
Non-metallic minerals	1.10	-1.33
Paper-pulp-print	0.97	-1.28

On the other hand, if Canada imposes only Carbon tariff, the U.S. increases its exports of EIS to Canada by 4.3%, while it decreases the imports of the same goods from Canada by -0.5% relative to scenario I. Since the U.S. slightly increases its imports of EIS goods from China (see Table 2.21), under these circumstances, an increase in the Canadian imports of EIS goods from the U.S. implies that part of the Canadian imports from non-Annex 1 regions could pass through the U.S. border at a price margin that is different from that between the U.S. and non-Annex 1 regions. It is interesting to note that under carbon tariff case, the welfare loss of Canada relative to the BAU is -1.68%, which is lower (higher) than the case when Canada also grants subsidies to its EIS goods exporters (see scenario II in Table 2.16). The intuition, as we explained earlier, that the transfers to the Canadian households will be less affected in tariff case as compare to that of full BTA. Conversely, we see that the welfare loss of the U.S. is slightly high (i.e. -0.07%) in tariff case relative to the full BTA (i.e. scenario II), in which case it was -0.06%. This is because in tariff case, Canada purchases less of emissions permit from the U.S.[17] A similar result is observed if Canada adopts a carbon tariff under the REF case, implying that the carbon tariff will be less welfare deteriorating than the case when it is imposed with exports subsidies.

2.4.4 Scenario III: Canada & the U.S. make emissions bubble, other Annex 1 regions adopt REF scenario, and only the U.S. levies BTA.

Carbon Tariff along with Exports Subsidies

The implementation of a full BTA by the U.S., either in an emissions bubble or in the REF case, nonetheless would improve its output and trade balance of energy-intensive goods, but at the cost of its welfare loss due to a similar reason we discussed in the previous subsection.

If the U.S. implements carbon tariff on imports of EIS goods from non-Annex1 regions in line with an emissions bubble, it reduces its imports shares of EIS goods from these regions by 11 percents relative to scenario I (see Table 2.24). Since the U.S. also provides subsidies to its Energy-Intensive Trade Exposed (EITE) industries, causing its output EIS goods to increase relative to scenario I (see Table 2.14). However, it could not meet the domestic excess demand for EIS that was *ex-ante* covered through imports from non-Annex1 regions. In response, Canada would increase the production of EIS goods to partially bridge the net demands of the U.S. for EIS goods. Since the U.S. provides subsidies to it EIS goods producers, Canada could not purchase as much emissions in the Canada-USA market as it would receive in scenario II (see Table 2.17). Therefore, we see in Tables 2.23 and 2.24 that Canada not only increases its exports[18] of EIS goods to the U.S. but it also increases the imports of energy-intensive goods from non-Annex 1 regions. Other Annex 1 regions, such as the EU also takes advantage of such a trade distortion and increases it net exports of EIS goods to the U.S. by around 64 percent relative to scenario I. Almost half of its next exports of EIS goods to the U.S. is met through increases in its net imports of EIS goods from non-Annex 1 regions. Since the U.S. has a large market of EIS goods, the E.U enables to improve its competitiveness as well as its welfare relative to scenario I (see Tables 2.14 and 2.16). On the other hand, the consumer

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[17] In carbon tariff case, Canada purchases net quantity of 170MT of CO2 from the U.S, which is smaller than that of scenario II mentioned in Table 2.17.

[18] In comparison with scenario I, Canada increases the net exports of EIS goods to the U.S. by 21% while it also increases the imports of the same composite goods from non-Annex 1 regions by more than 7%.

in the U.S. has to pay higher prices on energy-intensive goods to Annex 1 countries, that is the same goods which were exported *ex ante* by non-Annex 1 regions (e.g., China) at relatively low prices. In addition, they receive less transfers as some part of government tax revenue is now used as subsidies to EIS good producers.

Hence in this case, the imposition of a BTA simply redistributes welfare among trading partners at the cost of a welfare loss of the BTA imposing region. However, since the U.S. is a large net consumer of emissions related goods, an *ex post* increase in the prices of EIS goods would put downward pressure on their domestic demand for EIS goods. This implies that leakages would be relatively weak in this case. Here, we see that the global leakages decline from 15.91 % in the REF case to 12.96 % in Scenario III.

In the above discussion, we analyzed the implementation of a BTA policy within a Canada-USA emissions bubble. If the U.S. imposes a BTA within the REF case, the impact of price arbitrage becomes even more severe. Since each Annex 1 region is bounded to its target, it has more incentives to gain from international trade. Therefore, we see in Table 2.22 that Canada increases its imports of EIS from non-Annex 1 regions, while it also increases its exports of the same products to Annex 1 regions without increasing its own production. Moreover, the U.S. consumers will not get transfers, which they were receiving by selling emissions permit in emissions bubble case. As a result, the U.S. observes a large welfare loss relative to the BTA policy under emissions bubble.

Carbon Tariff only

If the U.S. imposes only a carbon tariff along with emissions bubble policy, then Canada will be able to purchase more emissions permit in the bubble market[19] relative to the case where the U.S. EITE sectors also receive subsidies. Hence, Canada's production of EIS goods, relative to its BAU level, will decline by -3.31 percent, which is smaller than scenarios I and III (see Table 2.14). Still it could partially bridge the net demands of the U.S. for EIS goods which were *ex ante* met through imports from non-Annex 1 regions, especially from China. Therefore, with the similar reason explained above, Canada not only increases its exports of EIS goods to the U.S. but it also increases the imports of energy-intensive goods from non-Annex 1

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[19] In this case, Canada purchase emissions of 181 MT of CO2 eq., which is larger than scenario III (i.e. a full BTA case) in Table 2.17.

Table 2.21: Trade Pattern of Annex 1 Regions with China (In Billion US$) 2004

Trade in energy-intensive goods

	Annex 1		ANZ		CAN		EUR		JPN		RUS		USA	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
REF	255.75	211.54	25.74	9.84	10.24	7.70	75.77	79.07	66.27	38.94	22.26	4.15	55.46	71.84
I	256.47	210.02	25.69	9.84	12.22	7.47	75.62	78.98	66.13	38.92	22.25	4.15	54.57	70.65
II	256.61	207.49	25.65	9.85	12.92	5.13	75.48	79.02	66.02	38.94	22.22	4.15	54.32	70.39
III	253.67	193.86	25.52	9.90	11.68	7.60	74.57	79.79	65.44	39.15	21.64	4.20	54.84	53.20
IV	253.79	191.15	25.48	9.90	12.22	5.21	74.43	79.83	65.31	39.16	21.62	4.20	54.62	52.86

Trade in non energy-intensive goods

	Annex 1		ANZ		CAN		EUR		JPN		RUS		USA	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
REF	550.95	1093.28	17.94	32.06	12.91	32.08	219.89	437.63	161.37	180.11	8.26	26.30	130.58	385.10
I	550.61	1095.04	17.95	32.07	12.49	32.53	219.98	437.68	161.33	180.16	8.27	26.28	130.58	386.32
II	549.71	1095.37	17.93	32.10	12.31	31.00	219.81	438.16	161.21	180.31	8.27	26.29	130.18	387.50
III	543.60	1093.30	17.80	32.31	12.12	33.38	218.14	441.75	160.02	181.36	8.20	26.47	127.34	377.92
IV	542.60	1093.49	17.78	32.35	11.93	31.80	217.94	442.30	169.87	181.54	8.19	26.48	126.87	379.03

Table 2.22: Effects of Imposing BTA by the U.S. Without Emissions Bubble

Indicators	Scenario	Canada	USA
Welfare*	REF	-2.24	-0.10
	BTA-USA	-1.61	-0.29
Energy Intensive Output*	REF	-17.29	-1.60
	BTA-USA	-17.31	-1.36
Net Exports** of EIS to Annex 1	REF	33	-77
	BTA-USA	39	-114
Net Exports** of EIS to Non-Annex 1	REF	-5	-181
	BTA-USA	-9	-133

*% change from BAU level **In billion US$ 2004

Table 2.23: Trade of Energy-Intensive Goods with Annex 1 Regions (In US (2004) billion dollars)

Exports

	BAU	REF	I	II	III	IV
Australia-Newzealand	43	39	39	39	39	39
Canada	152	130	146	152	159	165
European Union (Including EFTA)	1448	1399	1395	1395	1417	1417
Japan	50	48	48	48	51	50
Russia	138	131	131	131	136	136
United States	243	242	238	242	246	251

Imports

	BAU	REF	I	II	III	IV
Australia-Newzealand	38	36	36	36	37	36
Canada	99	98	97	103	99	106
European Union (Including EFTA)	1463	1416	1416	1416	1417	1417
Japan	80	74	74	74	74	74
Russia	49	46	46	46	46	46
United States	346	320	329	333	376	380

Table 2.24: Trade of Energy-Intensive Goods by Non-Annex 1 Regions (In US (2004) billion dollars)

Exports	BAU	REF	I	II	III	IV
Australia-Newzealand	78	72	72	72	72	71
Canada	34	27	32.4	34	31	33
European Union (Including EFTA)	625	582	582	581	575	575
Japan	187	170	170	170	168	168
Russia	119	121	121	121	117	117
United States	292	284	280	279	284	283

Imports	BAU	REF	I	II	III	IV
Australia-Newzealand	42	41	41	41	41	41
Canada	33	32	33	28	36	30
European Union (Including EFTA)	645	648	648	648	658	658
Japan	215	206	206	206	206	207
Russia	41	42	42	42	43	43
United States	462	465	459	459	407	409

regions.[20]

It is interesting to note that the welfare loss of the U.S under carbon tariff case is -0.22%, which is less than the loss in full BTA case (see scenario III in Table 2.16). This is because the revenue transfers to the U.S. consumer through emissions selling is higher than the case when the subsidies are given to the EITE industries. In contrast, the welfare loss for Canada is -1.05%, which is greater than the welfare loss under scenario III. This is because some part of the transfers, which was received by the Canadian consumer in full BTA case is now received by the U.S. consumer due to increase in the size of emissions permit bought by Canada.[21] Again an analogous result is obtained if the U.S. adopts a carbon tariff under the REF case, suggesting that the carbon tariff will be less welfare deteriorating than the case when it is imposed with exports subsidies.

Precisely, we show that the imposition of a BTA by Canada or the U.S. alone,

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[20] In comparison with scenario I, Canada increases the net exports of EIS goods to the U.S. by 24% while it also increases the imports of the same composite goods from non-Annex 1 regions by more than 8%.

[21] Under carbon tariff by the U.S, Canada purchases net quantity of 181MT of CO2 from the U.S, which is larger than that of scenario III mentioned in Table 2.17.

Table 2.25: Effects of Imposing Common BTA by Canada and the U.S. with and Without Emissions Bubble

Region	Indicators	REF	REF-Common BTA	Bubble-Common BTA
	Permit-Price†	125.42	160.57	37.70
Canada	Welfare*	-2.24	-2.29	-1.19
	Energy-Intensive output*	-17.29	-13.84	-1.11
	Pemit-Price	29.78	29.87	37.26
USA	Welfare*	-0.10	-0.25	-0.24
	Energy-Intensive output*	-1.60	-1.23	-2.20

†In US$ 2004 *% change from BAU level

under both scenarios—with and without emissions bubble, may increase competitiveness issue (i.e. it increases output and exports of energy-intensive goods), but at the cost of their welfare loss. On the other hand, if either of the regions imposes carbon tariff only, then the tariff imposing region produces less and its trading partner region produces more EIS goods relative to the full BTA case. However, the welfare impact goes in favor of the tariff imposing regions relative to the case when it imposes full BTA.

2.4.5 Scenario IV: Canada & the U.S. adopt emissions bubble along with the common BTA while other Annex 1 regions employ REF scenario.

Carbon Tariff along with Exports Subsidies

In this scenario, we apply a minimum of two countries' BTA for each product as a common external carbon tariff rate. Using this approach, we find that Canada would be better off, while the U.S. would be worse off relative to the REF and scenario I (see Table2.16). Due to a common BTA policy, both countries would be able to improve the net exports of EIS goods.[22] The two regions also increase their bilateral trade in EIS goods.[23] However, as discussed earlier that the U.S. is a large net consumer of energy-intensive goods, the increase in the production of EIS goods in Canada and the U.S., would partially cover the net demand for energy related goods in the U.S., which was *ex ante* met through imports from non-Annex1 regions. Therefore, other Annex 1 regions such as EU and Russia will capture some share of the north

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[22] In comparison with scenario I, Canada increases it net exports of EIS goods by around 28% while the U.S. reduces it trade deficit of EIS goods by 4%.

[23] Canada and the U.S. have increased their bilateral size of exports by 15% and 10%, respectively while comparing with scenario I.

American market with the similar intuition we mentioned in the scenario III.

Notice that the common BTA, under emissions bubble allows, Canada to purchase more emissions from the bubble market (see Table 2.17) and so it produces more EIS goods relative to the other scenarios. The U.S. consumer receives transfers generated through permit trading that makes them slightly better off than that of scenario III (see Table 2.16). However, the loss of some transfers due to subsidies given to the EIS goods' exporters and tariffs imposed on imported goods keeps the welfare of the U.S. consumer still lower than the scenario I. On the other hand, the U.S. consumer and industries use their transfer from permit selling and subsidies on exports to purchase the EIS goods from Canada, which enables to increase its net exports of EIS goods to the U.S. by 23% relative to scenario I. This is one potential intuition making Canada better off in common BTA relative to the case of emissions bubble alone.

If Canada and the U.S. implement BTA under REF case, both will be worse off. Since Canada has high abatement cost, subsides to EITE sectors make the permit prices even higher than the REF case (see Table 2.25). The higher permit prices enables EITE industries to get higher subsidies, which are the foregone transfers that the Canadian consumer was receiving in REF case. This is one possible reason that makes Canada's welfare worse off relative to the REF case only. A similar intuition can be applied for the U.S. welfare loss in BTA under REF relative to the REF only.

Carbon Tariff only

If both regions impose only carbon tariff along with emissions bubble, Canada and the U.S. respectively observe -1.08% and -0.22% welfare loss relative to their BAU level. Notice that both regions are better off relative to the case when they apply full BTA (see scenario IV in Table 2.16). Under this case, the U.S. will abate more and sell its emissions permits to Canada, so the consumer in the U.S. will receive more transfers through permit selling and along with no subsidies to EITE exporters. However, the U.S. will still be worse off relative to scenario I, in which there was no tariff on imports of EIS from non-Annex1 regions.

As discussed in the above subsection that Canada will be better off under common BTA relative to scenario I, it can further improve it welfare if both regions impose carbon tariff in place of full BTA. The reasons is that in both types of trade protection policies, Canada receives approximately the same level of exports to output ratio for its EIS goods, however in the carbon tariff case, the consumer, in Canada, will receive a larger transfers relative to the case when these countries grant subsidies to EITE goods exporters.

If both regions impose common tariff under REF case, Canada and the U.S. respectively observe -1.81% and -0.25% welfare loss relative to their BAU level, which indicates that Canada will be better off while the U.S. will be worse off relative to the REF without tariff (see Table 2.16) even though Canada's output, relative to BAU, declines by 17.59%, which is even higher than the REF case (see Table 2.14). The result is not surprising as Canada, due to emissions constrained is limited to produce EIS goods under REF case. However, since the U.S also imposes carbon tariff, Canada becomes able to increase its exports of EIS goods to the U.S market by 6% relative to the REF case even if it experiences domestic permit price at $132 per ton of CO2 equivalent. On the other hand, Canada increases imports of NEIS goods from the U.S. by around $5 billion relative to the REF case. Remember that, as we discussed earlier, non-Annex1 regions are $ex-ante$ specialized in NEIS goods, a carbon tariff imposed on these regions will increase their productions of NEIS goods,[24] which accounts for a downward pressure in the prices of NEIS goods. Therefore, the absorption of NEIS goods and exports of EIS goods are the potential reasons of making Canada better off in the REF with common tariff relative to the only REF case.

More concisely, the imposition of a common carbon tariff or BTA policy, whether it is imposed with or without emissions bubble, makes Canada better off at the cost of the U.S. is worse off. The intuition behind this surprising result is that Canada's firms benefit more from the higher protection afforded by the BTA. They are able to increase their exports of EIS goods to the U.S. where consumers loose cheap opportunities of imports of emissions-intensive goods from non-Annex 1 countries.

2.5 Sensitivity Analysis

We conduct some sensitivity analyzes of our simulation results. First, we calibrate scenario V in which Canada and the U.S. form an emissions bubble, while other Annex 1 regions employ REF case and all Annex 1 regions levy BTA. It is interesting

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[24] We will discuss more on it in sensitivity analysis.

to note that even if all Annex 1 regions impose a BTA,[25] it would still not be sufficient to motivate Annex 1 regions to increase their production of energy-intensive goods. We only see an *ex post* redistribution of welfare among producer- and consumer-based economies within the Annex 1 group (see Table 2.26 along with Tables 2.16 & 2.14). The producer-based economies such as Russia, Australia-New Zealand, and Canada would gain from trade while consumer-based economies such as the European Union, Japan, and the U.S. would suffer from trade. Table 2.26 supports the evidence of pecuniary effects regardless of whether it is full BTA (scenario V) or only a carbon tariff policy (scenario VI). For example, in comparison with the scenario V, Australia and Russia are still depicting positive welfare changes relative to the BAU level.

Table 2.26: Economic Impact of External Carbon Tariff w/without Exports Rebate

	Scenario V			Scenario VI		
	Permit Price†	Welfare Loss∗	Output Loss∗	Permit Price†	Welfare Loss∗	Output Loss∗
Australia-Newzealand	56.66	0.19	-5.45	54.09	0.20	-5.78
Canada	37.70	-0.92	-1.54	36.53	-0.79	-3.45
European Union	81.94	-1.07	-2.07	73.41	-1.02	-2.81
Japan	63.57	-2.08	-5.17	62.22	-2.07	-5.66
Russia	15.50	3.06	-2.72	15.51	3.09	-2.73
United States	37.70	-0.168	-1.98	36.53	-0.168	-2.36

† In $US 2004 ∗ % change in EIS goods relative to their BAU level

Table 2.27: Comparison of Scenario V Relative to REF

Region	Sectors	%Qty. change from REF	% Price change from REF
Annex 1	EIS	+0.40	+0.60
	NEIS	-0.09	+0.06
Non-Annex 1	EIS	-0.50	-0.35
	NEIS	+0.10	-0.32

Notice that we showed in Tables 2.23 & 2.24 that Russia and Australia export, respectively 53.7% and 35.5% of their total EIS output to compliant regions in the BAU case. We also see in Table 2.27 that the BTA policy, by all Annex 1 regions, accounts for the average price and total output of EIS goods, in these regions, to

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[25] Although we do not show the case in which all Annex 1 regions implement BTA under the REF case, however we obtain similar qualitative results as those discussed in scenario V.

Table 2.28: Effects of Imposing BTA on Alternative Sets of Industries by the U.S.

Indicators	Scenario	Canada	USA
Welfare*	Scenario VII	-1.529	-0.063
	Scenario VIII	-1.436	-0.258
Energy Intensive Output*	Scenario VII	-3.340	-1.770
	Scenario VIII	-3.518	-2.791
Net Exports** of EIS to Annex 1	Scenario VII	53	-103
	Scenario VIII	54	-113
Net Exports** of EIS to Non-Annex 1	Scenario VII	-2	-146
	Scenario VIII	-3	-167

*% change from BAU level **In billion US$ 2004

increase by 0.4% and 0.6%, respectively. The *post policy* price margin induces these producer-based countries to increase their exports share within compliant regions. Therefore, Russia and Australia increase their shares of total exports of EIS goods to Annex 1 regions by up to 63.7% and 47.5%, respectively.

On the other hand, since non-Annex 1 regions are highly emissions intensive, the imposition of a BTA (i.e. scenario V) on emissions embodied in imports of EIS goods from these regions leads to decline in their EIS exports, causing these regions to increase their production of NEIS goods. This is shown Table 2.27 where the total output of NEIS goods in non-Annex 1 regions increases by 0.10% while their price, on average, declines by -0.32%. Under these situations, the producer-based Annex 1 regions receive double dividends in the sense that they enjoy a high price margin advantage on the exports of EIS goods to consumer-based Annex 1 regions as well as a low import price advantage of NEIS goods from non-Annex 1 regions. Overall, we do not find evidence that a BTA, even if it is imposed by all Annex 1 regions (i.e. scenario V), resolves the competitiveness issue. What is more, a BTA generates a welfare loss not only at the overall Annex 1 level, but also at the global level.

Recently, Böhringer et al. (2010) conduct an analysis in which they consider a narrow set of industries in their setting of EITE sectors. We apply this sensitivity analysis in scenario III addressed in the previous section. Following Böhringer et al. (2010), we set scenario VII, in which we assume that only chemicals, non-metallic minerals, pulp-paper-print, iron and steel, and nonferrous metals commodities are levied a BTA. In addition, we set another scenario VIII, in which only petroleum products are levied a BTA.

Table 2.28 shows that the impact of a BTA policy is sensitive to the coverage of industries under the BTA policy. In the context of the U.S., scenario VII covers five downstream industries, in which three products (i.e., chemicals, pulp-paper-print, and nonferrous metals) that are mostly imported from Annex 1 regions,[26] with the U.S. being a net exporter of the chemical product. Hence, the impact of a BTA policy on these industries makes the U.S. better off relative to the other case we discussed earlier. In contrast, Table 2.28 depicts that the imposition of a BTA only on the oil product industries (scenario VIII) makes the U.S. worse off relative to the case when other products are also covered under the BTA policy. The intuition behind this result is that since the U.S. is a net consumer of oil products, a BTA policy reduces the consumption of those intermediate and final goods which are associated with oil, as they are made more expensive. This reduces the demand and, thus the production of those goods. Therefore, the economy is worse off than in the other cases.

Notice that our findings related to the emissions bubble are sensitive to the differences in the marginal abatement costs between Canada and the U.S. One influential parameter that can affect the policy results, in this context, is elasticity of substitution between energy and non-energy composites as well as within the energy goods and are elasticity of fossil fuel supply. As is convention in the CGE analysis, we choose the value 0.5 for the above elasticity parameter, as this value is also used by Babiker et. al (2004) and Babiker and Rutherford (2005). To test the robustness, we assume a hypothetical situation, in which Canada has more flexible avenues to switch among different fossil fuels in its electricity production. For this purpose, we change the relevant elasticity value only for Canada from 0.5 to 2. We notice in Table 2.29 that even at such high level of elasticity of substitution, our results hold, nonetheless we show in Table 2.10 that the substitution possibility is as difficult for Canada as it is for the U.S.[27]

Our central focus in this chapter remains on the feasibility of Canada-USA emissions bubble and the impact of carbon border tax adjustment under the implication of Canada-USA emissions bubble. However, wherever flexibility, we attempted to assess the alternative policy options and comparison with our core scenario. It would

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[26] Based on our BAU calibration, the U.S. imports 68% of its total imports of pulp-paper-print, 61% of chemical, and 52% of nonferrous metals from Annex 1 regions.

[27] Also see Babiker and Rutherford (2005) and Böhringer et al. (2008) for empirical comparisons of the abatement cost of Canada in different scenarios.

Table 2.29: Sensitivity at Intra-fuel Elasticity for Electricity Generation for Canada

Indicators	Canada	USA
Permit Price (REF)	76.07	29.8
Permit Price(scenario I)	34.00	34.00
Welafre Loss(REF)	-1.42	-0.10
Welafre Loss(scenario I)	-1.34	-0.08

be interesting if we shed light on the possibility of a policy scenario in which all compliant regions access to a common emissions trading market along with a common BTA policy for non-compliant regions.

Table 2.30: Comparison Between the Polices of Annex 1 Emissions Trading along with BTA and REF case)

	ANZ	CAN	EU	JPN	RUS	USA
	%Change in EIS output relative to BAU level					
REF	-3.50	-17.29	-3.50	-3.51	-1.55	-1.60
Annex1 trading-BTA	-4.99	-2.39	-0.89	-4.48	-6.96	-3.00
	%Change in welfare relative to BAU level					
REF	-1.06	-2.24	-0.63	-0.48	-1.84	-0.10
Annex1 trading-BTA	0.21	-1.17	-0.91	-1.98	3.89	-0.118

If emissions trading market is set up at Annex 1 level, then Australia, Canada, Europe, and Japan will the net purchaser of emissions permits from the U.S and Russia. The market clearing permit price is achieved at $45 with the tradeable emissions around 625MT of CO2 equivalent. Since all Annex 1 regions have imposed BTA on the imports of EIS goods from non-compliant regions, the advantage of such common policy would go the regions who are the net producers of energy related goods. Under the Annex 1 emissions trading, the U.S. would export emissions to other Annex 1 regions, which in turn export EIS goods to the U.S. So its consumers would pay more for the goods they used to purchase at less prices from the non-Annex 1 regions. On the other hand Europe, which is also a net consumer of EIS goods, the EITE goods exporters, in this region, produce more of emissions intensive goods by purchasing emissions right from the U.S. and Russia and sell their products to the U.S market. Therefore, we see that the EU experiences a larger welfare loss in BTA with Annex 1 emissions trading than the U.S. while as compare to the scenario with the REF case (see Table 2.30).

So far, we have focused on the economic aspects of the emissions bubble and the carbon border adjustment policies. It will be interesting to asses the scenarios we discussed earlier while keeping the global emissions reduction fixed at certain level in all alternative policy options. To do so, we design policy simulations that produce an equivalent reduction in global emissions and compare the cost of achieving that reduction using the policy instruments we have so far discussed.

Table 2.31: Comparison among Alternative Scenarios under a Fixed Global Reduction Target)

	REF	I	II	III	IV	V	Annex1 trading-BTA
	%Change in EIS output relative to BAU level						
ANZ	-3.82	-3.85	-3.83	-3.59	-3.58	-5.61	-5.13
CAN	-19.94	-5.08	-2.34	-4.03	-1.21	-1.63	-2.55
EU	-4.51	-4.57	-4.56	-4.26	-4.26	-2.18	-0.90
JPN	-4.33	-4.37	-4.36	-4.14	-4.14	-5.29	-4.58
RUS	-2.74	-2.75	-2.71	-2.26	-2.24	-2.62	-7.33
USA	-2.28	-3.05	-3.07	-3.31	-2.49	-2.12	-3.17
	%Change in welfare relative to BAU level						
ANZ	-1.23	-1.23	-1.22	-1.14	-1.13	0.13	0.19
CAN	-2.87	-2.08	-2.28	-1.35	-1.58	-1.07	-1.28
EU	-0.89	-0.88	-0.87	-0.78	-0.77	-1.12	-0.94
JPN	-0.65	-0.64	-0.64	-0.61	-0.60	-2.11	-2.02
RUS	-2.44	-2.45	-2.44	-2.29	-2.28	2.93	3.80
USA	-0.17	-0.116	-0.10	-0.29	-0.26	-0.17	-0.119

Table 2.31 shows the output and welfare effects among different scenarios while keeping the global emissions target exogenously fixed among all policies. Under this circumstance, a BTA policy (let us consider scenario V) would be deteriorating to the net emissions consumers regions such as the U.S, the E.U, and Japan. The intuition is that in case of a global emissions target, the total world production of emissions intensive goods will not exceed the ex-ante level. Even it might be less due to the difference in emissions intensities between developed and non-developed regions. So in BTA policy, for example scenario V, the U.S produces more to export to other regions and observes a higher welfare loss relative to the scenario I (i.e. emissions bubble).[28] Australia and Russia, which have a large exports market in non-Annex 1 regions in BAU, they get advantage of such BTA-induced increasing demand of EIS goods by some Annex 1 regions such as Japan and the EU. They increase their

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[28] Remember that scenario V assume that Canada-USA, therefore we compare it with scenario I. However, this result is valid even if we create a scenario in which all Annex 1 regions implement BTA under REF case. In this case, the welfare loss of the U.S. relative to its BAU level is -0.18%, which is higher than the welfare loss of the U.S. under REF case (see Table 2.31), meaning that the U.S. will be worse off even if we compare REF with and without the BTA in a fixed global emissions target.

exports to these regions and reduce their exports share to non-annex regions. As a result, they observe a gain in their welfare. It is interesting to note that Australia has even reduced its production of EIS goods in scenario V relative to scenario I, indicating that it improves its welfare mainly through trade activities. Conversely, Japan had to switch parts of its EIS imports from non-annex 1 to Annex 1 regions while these regions are constrained by emissions target. Therefore, Japan observes a large welfare loss relative to scenario 1. More precisely, the net consumers of EIS goods such as the U.S, Japan, and Europe reduce their welfare relative to scenario I if they also introduce BTA policy on it.

Interestingly, incorporation of Annex 1 emissions trading (i.e. the last column in Table 2.31) makes the U.S. better off relative to the REF case while Japan and the EU are still worse off. Indeed the Annex 1 emission trading gives the U.S. an opportunity to abate and sell the emissions permits more to the Annex 1 regimes, and enjoy more consumptions by importing the EIS goods especially from Canada and Europe. However, for other consumer-based regions such as the E.U. experience a similar situation we explain in scenario I where the production and exports of a BTA imposing regime increases, but at the cost of a decline in domestic consumption.

2.6 Conclusions

In this chapter, we have analyzed the potential economic implications of the creation of an emissions bubble between Canada and the U.S. in the context of sub-global participation in the efforts to curb GHG emissions. We have assessed the feasibility of a potential emissions bubble between Canada and the U.S. based on their existing economic behaviors, industrial structures, and trade patterns.

We have developed a multisector, multi-regional general equilibrium model to analyze the welfare, aggregate, sectoral and trade impacts of the formation of an emissions bubble between Canada and the U.S. The model features a detailed representation not only of Canada's and the U.S.'s economy, but also of other major Annex 1 and non-Annex 1 countries. We have run several scenarios related to the creation of the emissions bubble in the presence and non-presence of a BTA. As well, we ran a reference scenario in the absence of an emissions bubble, which we used for the comparison purpose. In the reference case, Annex-1 countries undertake domestic actions individually to reduce GHG emissions in the absence of an emissions bubble and a BTA. Because of the lower opportunity of fuel switching due to a lower share of coal in energy use in Canada than in the U.S., the marginal cost of abatement in the former is more than four-fold than that of the latter. While almost all countries experience welfare deterioration, we observe, as in previous studies, an emissions leakage toward non-Annex 1 countries. These countries increase their emissions by more than 15% in the reference case in comparison to the BAU case.

The creation of an emissions bubble between Canada and the U.S. without the imposition of a BTA improves welfare in both countries in comparison to the reference case, but with a significant advantage for the former country in comparison to the latter. The reason for this is that Canada is able to benefit significantly from the reduction of its marginal abatement cost induced by the purchase of permits from the U.S., and it is also able to increase its net exports of emissions-intensive goods. Our results do not support Copeland and Taylor's (2005) argument that under some conditions permits trading under free trade may lead to an indirect negative terms-of-trade effects which can offset the direct gains from permits trading. In the Canada-USA emissions bubble case, we find that the economic structures and trade patterns between the two regions lead to a post-policy positive term-of-trade effects. The increase in exports of emissions-intensive goods to the U.S. benefits the latter country as it experiences an improvement in its terms of trade and thus reduces the negative impact of the carbon mitigation policy observed in the reference case. Besides, the creation of the bubble makes it possible for non-Annex 1 countries to increase their trade shares in non-energy-intensive goods in comparison to the reference case.

Our results also indicate that, within the emission bubble, whenever any one of the two countries implements a unilateral BTA on the imports of energy intensive goods from non-Annex 1 countries, it suffers from a welfare loss at the benefit of its partner. Also, the BTA will not help the BTA imposing country to deal with its competitiveness issues because the *post-policy* decline in output and net exports of EIS goods are larger than in the case of the emissions bubble without a BTA. The rationale for this is that the existence of a free trade agreement between the two countries, allows the non-imposing BTA country to increase its exports to the other thanks to the increase in cheap imports from the non-Annex 1 countries. The above results are consistent with the case when the two regions implement a BTA individually under their reference scenario.

When both countries jointly implement a BTA on the non-Annex 1 imports of emissions-intensive goods, welfare improves in Canada in comparison to the reference case, while it deteriorates in the U.S. The intuition behind this surprising result is that Canada's firms benefit more from the higher protection afforded by the BTA. They are able to increase their exports to the U.S. where consumers loose cheap opportunities of imports of emissions-intensive goods from non-Annex 1 countries.

In the sensitivity analysis, we show that the policy impact of a BTA is sensitive to the coverage of industries under the BTA policy. For example, we show that imposing BTA only on oil products is sufficient to make the U.S. worse off relative to the non BTA policy scenario. On the other hand, a narrow coverage may cause the U.S.to become relatively better off than in broad coverage we discussed in scenario II-VI. However, to avoid the complexity in the choice of which industries are to be covered under the BTA policy, we consider a simple approach which involves the imposition of a BTA on the content of carbon embodied in imports. Based on that approach, our findings suggest that a BTA is not a helpful tool for resolving the competitiveness issues.

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2.8 Appendix

2.8.1 Optimal Solution in Different Activities

Consumption Activities

$$exp_r = [\mu_r\{\prod_{i\in e}(\frac{P_i}{\phi_i})^{\phi_i}\}^{1-\alpha_E} + \nu_r\{\prod_{i\notin e}(\frac{P_i}{\varphi_i})^{\varphi_i}\}^{1-\alpha_E}]^{\frac{1}{1-\alpha_E}} \tag{2.8.1}$$

Where $\sum_i \phi_i = \sum_i \varphi_i = 1$.

Production Activities

- Production of non-fossil fuel goods

$$P^{Y}_{i\notin ff,r} = [\gamma^{M}_{i}\{(\sum_{j\notin e}\gamma^{Z}_{j}P^{Z^{1-\alpha_{Z}}}_{j})^{\frac{1}{1-\alpha_{Z}}}\}^{1-\alpha_{KLEM}} + (1-\gamma^{M}_{i})[[\gamma^{E}_{i}P^{E^{1-\alpha_{KLE}}}_{j\epsilon e,r}$$
$$+ (1-\gamma^{E}_{i})[[\gamma^{K}_{i}P^{K^{1-\alpha_{KL}}}_{ir} + (1-\gamma^{K}_{i})P^{l^{1-\alpha_{KL}}}_{ir}]^{\frac{1}{1-\alpha_{KL}}}]^{1-\alpha_{KLE}}]^{\frac{1}{1-\alpha_{KLE}}}]^{1-\alpha_{KLEM}}]^{\frac{1}{1-\alpha_{KLEM}}} \tag{2.8.2}$$

In the above equation, P^{E}_{i} is represented as;

$$P^{E}_{i} = \{\gamma^{ele}_{i}P^{ele^{1-\alpha_{E}}}_{ir} + (1-\gamma^{ele}_{i})(\sum_{i\in nele}\gamma^{nele}_{i}P^{nele^{1-\alpha_{E}}}_{ir})\}^{\frac{1}{1-\alpha_{E}}} \tag{2.8.3}$$

Where P^{nele}_{ir} is the nest of;

$$P^{nele}_{ir} = [\sum_{j\epsilon liq}\gamma^{liq}_{j}\{(\prod_{j\in liq}\frac{P^{liq}_{j}}{\psi_{j}})^{\psi_{j}}]^{1-\alpha^{cgo}_{E}} + (1-\sum_{j\in liq}\gamma^{liq}_{j})P^{col^{1-\alpha^{cgo}_{E}}}_{i}]^{\frac{1}{1-\alpha^{cgo}_{E}}} \tag{2.8.4}$$

Where $\sum_{j}\psi_{j} = 1$.

In case of CO_2 tax, the *ex post* Eq 2.8.3 will be

$$P^{E^{ex\ post}}_{i} = \{\sum_{j\in e}\gamma^{E}_{j}(P^{CO_2}_{r}E^{emit}_{j} + P^{E}_{j})^{1-\alpha_{E}}\}^{\frac{1}{1-\alpha_{E}}} \tag{2.8.5}$$

- Production of fossil fuel goods

$$P^{Y}_{i\epsilon ff} = [\gamma^{Q}_{ir}P^{Q^{1-\alpha_{q}}}_{ir} + (1-\gamma^{Q}_{i})(\gamma^{K}_{i}P^{K}_{i} + \gamma^{L}_{i}P^{L}_{i} + \gamma^{E}_{i}P^{E}_{i} + \sum_{j\in e}\gamma^{M}_{j}P^{M}_{j})^{1-\alpha_{q}}]^{\frac{1}{1-\alpha_{q}}} \tag{2.8.6}$$

International Trade Activities

- Armington Aggregate

$$P^{A}_{ir} = [\gamma^{D_y}_{i}P^{D_y^{1-\alpha_{esubd}}}_{ir} + (1-\gamma^{D_y}_{i})P^{IM_y^{1-\alpha_{esubd}}}_{ir}]^{\frac{1}{1-\alpha_{esubd}}} \tag{2.8.7}$$

- Aggregate Imports across imports regions

$$P_{ir}^{IM_y} = [\sum_{s} \gamma_{is}^{IM_y} \{P_{is}^{IM_y} + (\prod_{js} \frac{P_{js}^{YT}}{\nu_{js}})^{\nu_{js}}\}^{1-\alpha_{esubm}}]^{\frac{1}{1-\alpha_{esubm}}} \tag{2.8.8}$$

Where $\sum_j \nu_j = 1$.

2.8.2 Zero Profit Condition

After solving the representative agent and firms' problem by dual approach[29], we get per unit expenditure i.e. $exp()$ and cost functions i.e. $c()$. Accordingly, the following complimentarity slackness conditions are hold:

$$[P_r^C - exp_r(P_{i \notin e}, P_{i \in e})]C_r = 0 \;\Rightarrow\; C_r > 0 \;\&\; P_r^C = exp_r$$

$$[(P_{i \notin ff,r}^Y - c_r(P_i^E, P_i^l, P_i^K, P_i^Z))]Y_{i \notin ff,r} = 0 \;\Rightarrow\; Y_{i \notin ff,r} > 0 \;\&\; P_{i \notin e,r}^Y = c_r(..)$$

$$[(P_{i \in ff,r}^Y - c_r(P_i^Q, P_i^E, P_i^l, P_i^K, P_i^M))]Y_{i \in ff,r} = 0 \;\Rightarrow\; Y_{i \in ff,r} > 0 \;\&\; P_{ir}^Y = c_r(..)$$

$$[P_{i \in e,r}^E - c_r(P_i^{ele}, P_i^{nele}, P^{CO_2})]E_{i \in e,r} = 0 \;\Rightarrow\; E_{i \in e,r} > 0 \;\&\; P_{i \in e,r}^E = c_r(..)$$

$$[(P_{i,r}^A - c_r(P_i^D, P_i^{IM}))]A_{i,r} = 0 \;\Rightarrow\; A_{i,r} > 0 \;\&\; P_{i,r}^A = c_r(..)$$

$$[P_{i,sr}^{IM} - c_{sr}(P_i^Z, P_i^{YT})]IM_{i,sr} = 0 \;\Rightarrow\; IM_{i,sr} > 0 \;\&\; P_{i,sr}^{IM} = c_r(..)$$

The zero profit conditions refers to the positive activities as along as price are not equal to the marginal costs.

2.8.3 Market Clearing Condition

The existence of non-zero prices for each market product requires for market clearing condition.

1) For each commodity, market supply (S) equals market demand (D) .

$$[\sum_{j} M_{ijr}^D(P_{ir}^M, Y_{ir}) + C_r^D(P_r^C) + I_r^D(P_r^I) + IM_{ir}^D(P_{ir}^{IM}) - X_{ir}^D(P_{ir}^X) - Y_{ir}^S]P_{ir}^Y = 0 \tag{2.8.9}$$

Hence, the equilibrium solution with $P_{ir}^Y > 0$ requirers that $Y_{ir}^D = Y_{ir}^S$

2) For each factor market (i.e., labor, capital, and energy resources), supply equals demand.

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[29] For detail see Dixit and Norman (1980).

$$[\sum_j F^D_{jr}(P^F, Y_{jr}) - F^S_{ir}]P^F_{ir} = 0 \tag{2.8.10}$$

where $F = K, L, Q$ and equilibrium solution with $P^F_{ir} > 0$ requires that$F^D_{ir} - F^S_{ir} = 0$

3) Regional exports (X) supplies of i from region r must be equal to total imports of i from r by all other regions s. This implies that $P^A_i > 0$

$$X_{ir} = \sum_s IM_{isr} \tag{2.8.11}$$

4) International trade in each commodity must be

$$\sum_r X_{ir} = \sum_r IM_{ir} \tag{2.8.12}$$

2.8.4 Income-Expenditure Balance Condition

Following income balance conditions must hold: 1) representative agent's income must be equal to the expenditure 2) Government revenue must be equal to its expenditure.

2.8.5 Tables of Sets, Variables, Elasticities, and Share Coefficients

Table 2.32: Sets

Sets	**Description**
i & j	Sectors and goods
r & s	Regions
f	Value added i.e capital and labor
ff	Fossil fuels i.e. coal, refined oil, and natural gas
e	Energy inputs i.e., ff plus electricity

Table 2.33: Activity Variables

Activity Variables	**Description**
C_{ir}	Household consumption of good i in region r
Y_{ir}	Production in sector i and region r
E_{ir}	Aggregate energy input in sector i and region r
IM_{ir}	Aggregate imports of good i in region r
A_{ir}	Armington aggregate of good i in region r
C_{r}	Aggregate household consumption in region r
X_{ir}	Exports of good i from region r
E_{j}^{mit}	CO_2 emission in the production of good i

Table 2.34: Price Variables

Price Variables	**Description**
P_{ir}	Output price of good i consumed in region r
P_{i}^{Y}	Output price of good i produced in region r
P_{ir}^{E}	Price of aggregate energy goods sector i and region r
P_{r}^{K}	Price of capital services in region r
P_{r}^{L}	Wage rate in region r
P_{ir}^{ele}	Price of electricity used in good i and in region r
P_{ir}^{nele}	Price of non-electricity energy goods used in goods i and in region r
P^{liq}	Price of liquid non-electricity energy goods used in good i and in region r
P_{i}^{col}	Price of coal used in good i and in region r
P_{ir}^{Q}	Rent to natural resources in region r (i $\in ff$)
P_{r}^{CO2}	CO_2 tax in region r
P_{j}^{Z}	Price of good j within the composite of non-energy intermediate goods
P_{i}^{M}	Price of good energy related intermediate goods
P_{ir}^{A}	Price of Armington good i in region r
P_{ir}^{Dy}	Price of domestic good i in region r
P_{ir}^{IMy}	Price of imported goods i in region r

Table 2.35: Cost Shares

Cost Shares	**Description**
μ_r	Share of energy composites in total consumption in region r
ν_r	Share of non-energy composites in total consumption in region r
ψ_j	Share of product j in a composite of liquid fossil fuel
ν_{js}	Share of product j in a composite of transportation service from region s
ϕ_i	Share consumption of good i in a composite of energy goods
φ_i	Share consumption of good i in a composite of non-energy goods
γ_{isr}^{IM}	Share imports of good i from region s
γ_j^{Z}	Share of good j in a composite of non-energy intermediate good
γ_i^{M}	Share of material goods in the production of good i
γ_i^{E}	Share of energy composite in energy demand by sector i
γ_i^{K}	Share coefficient of Capital in value-added of good i
γ_i^{ele}	Share of electricity in a composite of energy demand by sector i
γ_i^{nele}	Share of non-electricity in a composite of energy demand by sector i
γ_j^{liq}	Share of good j in a composite of liquid fossil fuel demand
γ_{ir}^{Q}	Share of sector-specific resources in production of fossil fuel i $\in ff$
γ_i^{L}	Share of labor input in value-added of good i
γ_i^{Dy}	Share of domestic production in aggregate supply of good i
γ_i^{IMy}	Share of import in aggregate supply of good i

Table 2.36: Elasticities

Elasticities	**Description**	Values*
α_{E}	Substitution between energy and	0.5 for final demand
	non-energy composites	O&B for intermediate demand
α_{Z}	Substitution within the composite of	O&B
	non-energy intermediate goods	
α_{KLEM}	Substitution between materials	O&B
	and non-materials production inputs	
α_{KLE}	Substitution between value-added	O&B
	and energy inputs	
α_{KL}	Substitution between	O&B
	Labor and Capital	
α_{E}^{cgo}	Substitution between	1 for final demand
	liquid and coal fossil fuel inputs	0.5 for intermediate demand
α_{q}	Substitution between natural resources	estimate is based on supply elasticities
	and other inputs in fossil fuel production	i.e., coal=1, gas=0.5, crudeoil=0.25
α_{esubd}	Substitution between the	2 for energy goods
	import aggregate and the domestic input	GTAP values for other goods
α_{esubm}	Substitution between	4 for energy goods
	imports from different regions	GTAP values for other goods

*O&B refers to Okagawa, A. and K. Ban, (2008)

Chapter 3

Imperfect Credit Markets, Learning by Lending, and Output Dynamics

3.1 Introduction

Understanding persistent output fluctuations with imperfect financial markets remains an important topic of macroeconomics. It is widely acknowledged that asymmetric information between borrowers and lenders creates deadweight losses.[1] Most of the existing literature in this field addresses asymmetric information between lenders and borrowers by introducing collateral, while paying little attention to how lenders might learn about borrowers' private characteristics through the lending process. For example, in collateral-based models, a bad shock reduces the prices of assets, the impact of which is a transmission to borrowers' net worth and their investment and output decisions. However, these approaches have ignored the lenders' capacity to alleviate the informational friction. Intuitively, learning should happen in financial interactions where borrowers can be matched with the same lenders. For example, if borrowers keep repaying debts, they must be considered good customers and must get good lending offers on this basis. We term this form of information

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[1] Jaffee and Russell (1976), Stiglitz and Weiss (1981) are among the earliest papers showing that an *ex ante* adverse selection problem in credit market entails the existence of credit rationing. They use interest rates as a sorting mechanism. Bester (1985) modifies the above sorting mechanism by considering interest rates and collateral simultaneously, but inversely. This allows his model to obtain separating equilibria. Townsend (1979), Gale and Hellwig (1985), Williamson (1987), and Bernanke and Gertler (1989) are among those who show that *ex post* costly state of verification (or moral hazard problem) in financial markets causes borrowers to pay a premium on their external finances. Some papers (such as Stiglitz and Weiss (1992) and Boyd and Smith (1993)) attempts to show the intensified effects of asymmetric information when the combined effect of *ex ante* and *ex post* information asymmetries are included in credit markets. Carlier and Renou (2006) also combine these two asymmetric effects and characterize the existence of separating and pooling equilibria on the basis of the intensity of costly state of verification.

acquisition 'learning by lending,' which at a micro level has not received much attention, but its existence in credit markets might have important implications for the macroeconomy.

The main objective of this chapter is to determine whether the learning by lending channel can affect real macro variables. We develop a simple model in which lenders adopt a rational approach and learn about borrowers based on their choices of investment and production. However, this information acquisition is disrupted when a negative shock occurs, because a number of existing borrowers default. It takes several periods for accumulated information to rebuild. We show that this process can generate persistent investment and output effects. Up till now only few papers (for example, Carlstrom and Fuerst (1997)) have discussed this channel in output fluctuation, but have not built this approach into their empirical investigation. We are not aware of any study which investigates persistence in aggregate fluctuations through a learning by lending channel.

The theoretical framework of learning by lending is based on a competitive credit market where a typical lender is endowed with two units of capital. She is matched with a borrower who has an investment opportunity with a decreasing return to scale technology, but no capital endowment. Borrowers can be either low or high quality or ability types, and this type is their private information. Lenders form consistent beliefs on borrower's quality and offer lending contracts (for either one or two units of capital) based on these beliefs. In response, the borrower decides whether to invest or not subject to the offered contacts, his outside option, and his observation of an idiosyncratic productivity shock. The idiosyncratic shock is distributed such that, *ex post*, a large fraction of the possible value of output could have been produced by either type. Hence, there is a possibility that upon a draw of an idiosyncratic shock either type of borrowers can accept or decline a particular contract. This avoids any revealing or signalling opportunity by the agents based on their choice of contract and production levels. After the borrower makes investment and production decisions, the lender rationally updates her previous beliefs on borrower's quality using Bayes' Rule. This generates a dynamic process such that the learning in the current period will affect the equilibrium lending rates and investment levels in the next period.[2]

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[2] Among the pioneer studies on dynamics aspects of financial contracts & credit market equilibrium are Stiglitz and Weiss (1983) and Gertler (1992). The former is based on *ex ante* information asymmetries while the latter is on *ex post* asymmetries. The limitation of these studies is that they

This learning process is, however, disrupted when an aggregate shock is observed. The mechanism is that a negative shock causes borrowers' bankruptcy and exit. With exit comes a loss of information. Until the negative-shock period, the lender has interacted a number of times with the borrower and has learnt about the borrowers' underlying productivity. When default occurs, this information is lost. New (high quality) borrowers now face a higher average interest rate than those who just exited. Such borrowers now rationally choose to borrow less in equilibrium, leading to a lower choice of capital stock in subsequent periods and therefore lower output. This result follows from information asymmetries about borrower quality i.e. the lender can not observe whether a bad outcome happened to the borrower's ability or a bad idiosyncratic shock. Therefore, analogous to Bernanke and Gertler (1989) under a networth channel, the learning by lending channel shows how aggregate productivity shocks can have asymmetric effects. Specifically, there is greater propagation when a bad shock occurs relative to a good one. The intuition is that negative shocks lead to bankruptcies and so stocks of information about borrowers are wiped out. With a positive shock there is more repayment (less bankruptcies) and so a relatively small increase in the stock of information. Contrary to the real variable such as information loss, a monetary variable such as interest rate — which is perfectly revealed to both sides of the market — creates no persistence in the real macroeconomic variables. For example, following a temporary increase in interest rate, the lender raises the gross lending rate while the borrower chooses less investment and output. Once both agents readjust their lending and borrowing decisions, there will be no further impact of this exogenous policy change in our model.

Previous literature in this area focuses mainly on entrepreneurs' equity or net worth to generate output persistence. In an earlier paper, Bernanke and Gertler (1989) represent the condition of borrowers' balance sheets (such as net worth) as the source of output dynamics. Since the borrower's net worth is likely to be procyclical (i.e. the borrower is more solvent during good times), there will be decline in agency cost during booms and an increase in recessions. This phenomenon implies the emergence of a financial accelerator. Carlstrom and Fuerst (1997) develop a computable general equilibrium model to investigate the role of agency cost as a source of internal propagation. They find that the source of output dynamics in a standard

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consider multi-period but not multi-typed (heterogenous) agents in their models.

real business cycle model comes from households' delay in their investment decisions when the agency cost of external finance is high. The finding in this chapter is similar to that of Carlstrom and Fuerst (1997), but with a different channel; namely, a delay in investment decisions by high types — unless the impact of information losses alleviates — is the main source of endogenous propagation. Bernanke et al. (1999) derive a similar result in a monopolistic competition framework. Greenwald and Stiglitz (1993) show that in the presence of imperfect information and equity market rationing, the risk associated with future market conditions and production activities may be a source of real macroeconomic fluctuations. Kiyotaki and Moore (1997) introduce propagation through the fluctuations in demand for durable goods and production factors such as land and machinery, which can also be used as collateral. Kurlat (2010) introduces heterogeneity in the quality of collateral in Kiyotaki and Moore's (1997) type of model to generate an additional *ex post* internal propagation.

There are some papers which specifically examine information asymmetries in a macro context. For example, Lang and Nakamura (1989) introduce learning as a source of output dynamics. They show that the aggregate returns of current borrowers contain information about the mean returns to future borrowers and thus can act as a propagation mechanism. The essential difference between Lang and Nakamura (1989, 1990) and our model is that they introduce a learning process in the credit market through public information,[3] with no matching between existing individual agents after one period. By contrast, we stress the impact of private information when there is a likelihood of repeated lending. DellAriccia and Marquez (2004) incorporate information asymmetries among lenders and show that the informational advantage of informed lenders provides them with some degree of market power. This leads to borrower capture, since adverse selection makes it difficult for borrowers to obtain credit from outside lenders. Our model differs from theirs in the sense that they focus mainly on the credit allocation between captive and non-captive borrowers while we focus on how changes in the size of information that lenders have about borrowers can serve as a propagation mechanism for aggregate shocks.

Nieuwerburgh and Veldkamp (2006) show that the learning about the aggregate technology can explain business fluctuations. In a closer paper, Schivardi (2003)

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[3] In other words, the information set available to both agents is the same. Also their papers do not explain the dynamics of the credit market at the level of individual borrowers.

shows that a negative aggregate shock can be a source of information accumulation and endogenous revelation in an economy where learning about a technological efficiency is a gradual Bayesian process and shutting down is costly.[4] A negative aggregate shock makes delay more costly, which breaks the inertia to exit and has multiplier effects. Schivardi's environment can generate an information accumulation process which amplifies relatively modest aggregate shocks. While also using learning as a propagation mechanism, this chapter differs from Schivardi (2003) by showing how negative aggregate shocks can produce a loss of information (accumulated by private agents through learning by lending) via default. Thus for new borrowers, lenders charge an extra premium that causes some high quality borrowers to either invest less or stay out of the lending market altogether. Several periods are needed to restore the information loss, which produces internal propagation endogenously as an interaction between learning and capital accumulation.

This chapter also offers a simple explanation for why the cost of external finance can fall over time, other than those explanations provided in networth or collateral related models. In networth-based models (for example see Bernanke and Gertler (1989) and Bernanke et al. (1996)), a low networth is associated with a high agency cost and ultimately with a high external finance premium. Hence an entrepreneur has incentives to accumulate networth and to discount its current consumption more heavily. For instance, in Carlstrom and Fuerest's (1997) model, the entrepreneur has a tradeoff between the benefit of current consumption and the future returns on internal funds. Intuitively, in our model, an entrepreneur gets the benefit of consuming today as well as reducing the cost of external financing through learning by lending. In other words, the long-term equilibrium financing in their model is equity financing[5] while in our model it is debt financing.

3.2 Environment

Consider a credit market with two types of long-lived and risk neutral agents: borrowers and lenders. There are a total N borrowers. Half of them have high ability and half have low ability. This ability or quality is denoted by θ_h or θ_l, which does not vary over time. Each period a borrower has to decide whether to borrow and

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[4] The agents have an incentive to wait for somebody else to exit in order to make a better informed decision.

[5] For a similar results see Bernanke et al. (1999).

invest in a risky project or to take a pre-determined outside option. If they invest in a risky project, they receive an idiosyncratic shock, denoted e_{it}. The distribution from which this shock is drawn is the same for both types. The output of each borrower is also affected by an aggregate shock ϵ_t. In period t agent i produces output according to:

$$Y_{it} = A_{it} K_{it}^{\alpha} \tag{3.2.1}$$

Where $A_{it} = \theta_i e_{it} \epsilon_t$ is a productivity factor, K_{it} is the amount of capital employed by agent i in period t, and $\alpha \in (0, 1)$ refers to a decreasing return to scale (DRS) technology function. Notice that A_{it} contains two individual specific factors (i.e. borrower's quality and his observation of an idiosyncratic shock) as well as the aggregate shock. As the individual factors are the borrower's private information, there is a possibility that a low quality borrower can produce more due to a favorable idiosyncratic shock. It is possible because the distribution of e_i is such that a large fraction of the possible value of output could have been produced by either type. This restricts lenders to reveal borrowers' types as long as repayments occur. Assume that borrowers have no endowment. Therefore, to avail investments opportunities they approach to lenders at the beginning of every period.

3.2.1 Information

Both agents commonly know the fraction of high quality borrowers in the population (i.e., half-half), the production technology, levels of θ_h and θ_l, the distributions of idiosyncratic shocks, and the distribution of the current period's aggregate shock. Both agents also observe an aggregate shock when it happens. However, an idiosyncratic shock is only observed by the borrowers. In addition, if the borrower repays, the lender observes the repayment of a loan it makes but not the borrowers' output, which is observed by a lender only when a borrower defaults.

3.2.2 Lender's Problem

Lenders are identical and each lender enters into the credit market with two units of capital. She can lend one, two or none of these units to the borrowers. The capital which is not lent to borrowers earns the risk-free rate r_f. With the borrower's production technology exhibiting diminishing returns to capital, the lender recognizes that the borrower is more likely to default on a loan of two units than on a loan of one unit. Therefore, she offers the borrower two contracts, a loan of $K = 1$ *or* 2 at gross interest rates R_{itK}. If the borrower is unable to repay the loan, the lender receives the borrower's production of output, Y_{it}. Since the lender is risk neutral, she sets R_{itK} for $K = 1,\ 2$ such that the expected return to making the loan is equal to her outside option:

$$\begin{aligned} K(1+r_f) = \phi_{it}\{R_{htK}P(Y_{ht} \geq K_{ht}R_{htK}) + E(Y_{ht}|Y_{ht} < K_{ht}R_{htK})P(Y_{ht} < K_{ht}R_{htK})\} \\ + (1-\phi_{it})\{R_{ltK}P(Y_{lt} \geq K_{lt}R_{ltK}) + E(Y_{lt}|Y_{lt} < K_{lt}R_{ltK})P(Y_{lt} < K_{lt}R_{ltK})\} \end{aligned} \tag{3.2.2}$$

Where the solution of Equation 3.2.2 gives an equilibrium gross lending rate, R_{itK}, which depends on r_f and ϕ_{it}. An exogenous change in r_f accounts for a shift in gross lending rate along the same direction of r_f. The notation ϕ_{it} denotes the probability that the lender, who is matched with borrower i, assigns to the event that borrower i is of high quality given what she has observed prior to the start of period t. The equilibrium lending rate is a continuous but inverse function of ϕ_{it}. Using some calculations (see Appendix A), it can be shown that the first and second derivatives of R_K with respect to ϕ are $R'_K(\phi) < 0$ & $R''_K(\phi) > 0$, meaning that the premium rate is monotonically decreasing with the lender's belief about borrower's quality.

Hence, if $K_{it} = 0$, then the lenders possible payoff will be $\pi^L_t = 2(1 + r_f)$. For $K_{it} = 2$, her two units of endowments will earn:

$$\pi^L_t = \begin{cases} Y_{it} & \text{if} \quad Y_{it} < 2R_{it2} \\ 2R_{it2} & \text{if} \quad Y_{it} \geq 2R_{it2} \end{cases} \tag{3.2.3}$$

Equation 3.2.3 indicates the possible payoffs of the lender if she lends both of the units of capital.

3.2.3 Borrower's Problem

Faced with the loan offers from the lender, the borrower can choose $K_{it} = 0, 1$ *or* 2. He makes this choice after observing e_{it}. This is an important assumption to restrict any revealing by the lender or any signalling by the borrower. Therefore, it is possible that at a given interest rate a high quality borrower may decline a loan that would be accepted by a low quality borrower receiving a more favorable draw for e_{it}.

If the borrower chooses $K_{it} > 0$ then he produces according to Equation (3.2.1). If he chooses $K_{it} = 0$ then he receives a certain income of $w\theta_i$, where w is the productivity for the borrower's outside option. A natural interpretation of w is the labour income the borrower could earn if he chooses not to pursue his own production opportunity.[6] The borrower chooses K_{it} to maximize his expected income minus any debt repayment. Therefore, the possible *ex post* payoff of borrower i is denoted by:

$$\pi_{it}^{B} = \begin{cases} 0 & \text{if} \quad Y_{it} \leq K_{it}R_{itK} \quad \text{and} \quad K_{it} > 0 \\ Y_{it} - K_{it}R_{itK} & \text{if} \quad Y_{it} > K_{it}R_{itK} \quad \text{and} \quad K_{it} > 0 \\ w\theta_i & \text{if} \quad K_{it} = 0 \end{cases} \tag{3.2.4}$$

3.2.4 Timing within Period t

The timing of events is as follows;

1. Beginning of period t.
    (a) Borrower i is matched with a lender. The lender forms her beliefs ϕ_{it} and offers loan contracts $\{K_{it} = 1, R_{it1}\}$ and $\{K_{it} = 2, R_{it2}\}$ to borrower i.
    (b) The borrower observes e_{it} and chooses $K_{it} = 0, 1,$ *or* 2.
    (c) The lender updates ϕ_{it} based on borrower's choice of K_{it}. If the borrower chooses $K_{it} = 0$, both parties receive their outside options. In that case, the lender updates her beliefs and makes contract offers to the borrower next period.[7]
2. If borrower i chooses $K_{it} = 1$ *or* 2.
    (a) Production takes place.
    (b) The aggregate shock is observed by both parties.[8]
3. End of period t.

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[6] The labour income also depends on the borrower's quality/ability.

[7] In section 3.3, we examine the possibility that if the borrower chooses $K_{it} = 0$, the relationship ends and the borrower is matched with a new lender next period.

[8] In section 3.3, we also consider the case where only the borrower can observe the aggregate shock.

(a) The borrower either repays the loan or defaults.

(b) If the borrower repays, the lender is matched with the same borrower next period. In this case, the lender simply updates ϕ_{it}.

(c) If the borrower does not repay, he is replaced by a new borrower of the same type. Based on this assumption, the model remains in stationary state. New borrowers are randomly matched with lenders. Finally, the lender forms beliefs about the quality of new borrowers by using the expected fraction of occurring bankruptcies that would have been considered high type.[9]

3.2.5 Learning Process

Beliefs are formed rationally and updated using Bayes' Rule. To illustrate how the lender updates her beliefs about borrower's type, we consider a simple scenario. Certainly, there are other scenarios under which the lenders update their beliefs. In a later section, we discuss some of them.

Base scenario In the base scenario we assume that a lender only updates beliefs after a borrower's choice of K_i and production. If $K_i = 0$, the lender and borrower do not separate, but the lender updates her beliefs. Both parties observe the aggregate shock when production takes place. In order to form beliefs for new borrowers a lender uses the expected fractions of defaults to form beliefs about the quality of new borrowers.

Let us now explain the learning by lending mechanism under the base scenario. Consider borrower i who is matched with a lender and receives an offer of two contracts. He makes an investment decision after observing his idiosyncratic shock. Assume that his observation is $\hat{e}_{it}$, which is drawn from a uniform distribution $[\underline{e}, \overline{e}]$. Hence, he would choose $K_{it} = 2$ if his observed $\hat{e}_{it}$ makes his net expected payoff from $K_{it} = 2$ at least equal to what he would expect to get from $K_{it} = 1$. If not, he will choose $K_{it} = 1$ if his net expected payoff is at least equal to his income from outside option. Otherwise, he chooses $K_{it} = 0$ and avails his outside option. His choice of K_{it}, nonetheless, is subject to the lender's offer, technology, his outside option, and

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[9] In section 3.3, we discuss other possibilities for forming beliefs about new lender quality post-default.

his observation of the idiosyncratic shock. As discussed earlier, it is possible that, *ceteris paribus*, a low quality borrower may observe a favorable draw of e_{it} and accept $K_{it} = 2$, but this same offer could be declined by a high quality borrower who receives a low e_{it}. This suggests that the choice of K_t by borrowers does not necessarily mean that the lenders reveal borrowers types.

If borrower i makes an investment decision, the lender uses the information of borrowers' choice of K_{it} to deduce the cutoff values $\hat{e}_{1it}$ and $\hat{e}_{2it}$ that borrower i would require to choose $K_i = 1$ *and* 2 contingent on his private quality-type (for a detailed discussion, see Appendix B). Otherwise, the borrower chooses $K_i = 0$ and avails his outside option. Given the information of $\underline{e}$, $\overline{e}$, $\hat{e}_{1it}$, and $\hat{e}_{2it}$, the lender updates her beliefs about the borrower's type as $\hat{\phi}_{it} = \frac{\phi_{it}\rho_{Kht}}{\rho_{Kit}}$ such that $\hat{\phi} \in [0, 1]$, where ρ_{Kht} is the probability of borrower $i \in l, h$ being high quality given his choice of $K_{it} = 0, 1, or\ 2$ and $\rho_{Kit} = \phi_{it}\rho_{Kht} + (1 - \phi_{it})\rho_{Klt}$ is the probability that borrower i chooses $K_{it} = 0, 1, or\ 2$ (for a detailed explanation, see Appendix B).

If borrower i chooses $K_i > 0$ and undertakes production activities, he will repay if:

$$\theta_i \hat{e}_{it} \epsilon_t K_{it}^{\alpha} \geq K_{it} R_K \tag{3.2.5}$$

or else he will default.

Hence, the lender updates beliefs that borrower i, who has repaid, is of high type. She uses Bayes' Rule for $K_i = 1, 2$ as follows:

$$\phi_{it+1} = \frac{\hat{\phi}_{it} P(Y_{it} \geq K_{it} R_{itK} | h)}{\hat{\phi}_{it} P(Y_{it} \geq K_{it} R_{itK} | h) + (1 - \hat{\phi}_{it}) P(Y_{it} \geq K_{it} R_{itK} | l)}. \tag{3.2.6}$$

Where Equation 3.2.6 is a Bayes' theorem, in which the lender updates her prior probability, $\hat{\phi}_{it}$, by using new information conditional on the borrower's repayment of loan schedule, $P(.)$, in current period. Here ϕ_{it+1} is her posterior probability that borrower i, who repayed a particular loan ($K = 1$ *or* 2) in period t, is a high quality type. Since a lender now knows all of the parameters in Equation 3.2.5 except $\hat{e}_{it}$, she estimates the possible minimum values of borrower $i's\ \in h, l$ idiosyncratic shock for $K_i = 1, 2$ as $\frac{K_{it} R_{itK}}{\theta_i \epsilon_t K_{it}^{\alpha}}$. Using this cutoff value, she estimates the probability that borrower i will repay the loan in $t + 1$ based on his repayment in period t as follows:

$$\begin{aligned}
If\ \frac{K_{it}R_{itK}}{\theta_i\epsilon_t K_{it}^{\alpha}} &< \underline{e},\ then\ P(Y_{it}\geq K_{it}R_{itK}) = 1\\
If\ \frac{K_{it}R_{itK}}{\theta_i\epsilon_t K_{it}^{\alpha}} &> \overline{e},\ then\ P(Y_{it}\geq K_{it}R_{itK}) = 0\\
If\ \overline{e} > \frac{K_{it}R_{itK}}{\theta_i\epsilon_t K_{it}^{\alpha}} &> \underline{e},\ then\ P(Y_{it}\geq K_{it}R_{itK}) = \frac{\overline{e}-\frac{K_{it}R_{itK}}{\theta_i\epsilon_t K_{it}^{\alpha}}}{\overline{e}-\underline{e}}.
\end{aligned} \tag{3.2.7}$$

The first two cases in Equation 3.2.7 represent extreme situations. If the estimated cutoff value for borrower $i's$ idiosyncratic shock is less than $\underline{e}$, there is probability one that the borrower will repay regardless of his type. Conversely, the second case depicts a situation that the borrower will default. The third case indicates that the probability of repayment differs between borrowers according to their types. If borrower i defaults, the lender still updates her beliefs that a borrower, being defaulted, is of high type:

$$\phi_{it+1} = \frac{\hat{\phi}_{it}P(Y_{it} < K_{it}R_{itK}|h)}{\hat{\phi}_{it}P(Y_{it} < K_{it}R_{itK}|h) + (1-\hat{\phi}_{it})P(Y_{it} < K_{it}R_{itK}|l)} \tag{3.2.8}$$

Finally, when default occurs, the lender receives output Y_{it} and is matched with a new borrower next period. The lender updates her beliefs for a new borrower by using Equation 3.2.8 to estimate the expected proportion of defaulted borrowers who were high type as:

$$\phi_{it+1} = \sum_{i=1}^{N} \frac{\phi_{it+1}|P(Y_{it} < K_{it}R_{itK}|h)}{\phi_{it+1}|P(Y_{it} < K_{it}R_{itK}|l,h)} \tag{3.2.9}$$

The lender uses this information to set her beliefs for new borrowers in the next period. Equations 3.2.6 and 3.2.9 refer to the learning process by the lender for existing and new borrowers.

3.3 Simulation

Given the learning environment described in the previous section, our primary objective is to identify the existence of persistent effects on real macro-level variables due to aggregate shocks. To do so, we perform a series of numerical simulations using the base scenario outlined above, and proceed to alter these assumptions to test the robustness of our results.[10] To conduct numerical analysis, we assume some parameter values, while in section 3.3. we examine sensitivity analysis to see the

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[10] We simulate all scenarios by using MATLAB software.

qualitative effects associated with them.

To begin, assume that the qualities of borrowers are given by $\theta_l = 1$ and $\theta_h = 1.15$. The productivity of borrower's outside option is $w = 0.04$ and the parameter value of the production function is $\alpha = 0.88$. The outside option for a lender is $r_f = 0.04$. The idiosyncratic shock is drawn from a uniform distribution over $[0.9, 1.3]$, and a random aggregate shock is drawn from the normal distribution $N(1, 0.099)$ in each period so that the volatility of output in the model matches that of U.S. output. Real G.N.P. of the U.S. is used to approximate the mean standard deviation (hereafter, SD) of calibrated output. It is worth mentioning that the choices for θ and the supports of the uniform distribution ensure that there is a range of output that could be produced by either the high or low quality borrower. Therefore, meaning that the lender will not always be able to perfectly infer borrower type.

The dynamic impacts of the learning process at the aggregate level are tracked for an average of 1000 economies (each containing 1000 borrowers) for 21 periods. The simulated mean SD of output across 1000 economies is 0.0309, which is very close to the mean SD of real GNP of the USA (i.e. 0.0302) using annual data for 1946-2009. By tracking the simulation results we obtain a relationship between lenders' beliefs about borrowers' quality and the offered lending rates over time. The impact of the learning process on financial contracts can be seen from the inverse relationship between ϕ and R_K in Figure 3.3. This figure supports our argument that learning by lending can make a borrower better off without making the lender worse off.

3.3.1 Output Dynamics

To analyze output persistence, a negative or positive SD aggregate shock is introduced in period 11. It is worth mentioning that the aggregate shock itself is a white noise process that contributes no trend effects in the number of defaults in the credit market (see Figure 3.1). On the other hand, if learning by lending is fast in the initial period, this reduces the gross lending rate that motivates high quality borrowers to invest more. This in turn increases default by high types when a negative shock occurs, but it also entails an increase in average aggregate output in those periods when gains from learning are the greatest (see Figures 3.2-3.8). This indicates that learning by lending acts as a potential source of internal propagation in the model.

Now consider the introduction of a negative or positive SD aggregate shock of different strengths in period 11. Figure 3.9 depicts the impact of ±1 and ±2 SD impulse of aggregate shocks on average aggregate output. It shows that the dynamics of output are sensitive to the magnitude of the shock but, interestingly, this impact is asymmetric. In other words, the impact of two sides of an aggregate shock does not lead to an equal strength of internal propagation. Let us discuss more the sources of propagation and causes of asymmetries brought about by negative and positive aggregate shocks.

First, assume that a negative aggregate shock of $-2SD$ is observed by both agents in period 11. An instantaneous effect is a fall in borrowers' output that accounts for approximately 60% of existing borrowers to default. This entails a loss of information in the credit market.

Table 3.1: Impact of an Amplified Aggregate Shock in Period 11 on the Size of Defaults

Indicators	Full information	Base Scenario				
	$-2SD$	Normal shock*	$-1SD$	$-2SD$	$+1SD$	$+2SD$
High	351	39	79	365	0	0
Low	224	35	96	242	0	0
Total	575	74	173	607	0	0

* Here normal shock refers to a random aggregate shock, which is observed each period and characterizes as white noise process.

Therefore, the average belief held by lenders that a borrower is of high type will decline(see Figure 3.10). Given the inverse relationship between ϕ and R_{tK}, the average gross lending rates will rise, which causes the average aggregate capital stock to decline in the following period. Since the lender takes the borrower's choice of K into account when she updates beliefs on borrowers quality, the average ϕ and K will start rising in the succeeding periods and will return to *normal aggregate shock* levels over time (see Figure 3.10). The above endogenous process, brought about by a $-2SD$ aggregate shock, accounts for a persistent average aggregate output fluctuation relative to the *normal aggregate shock* case (see Figure 3.9). This is contrary to what would occur in a perfect information market where the lender knows borrowers types. Here, an amplified aggregate shock does not influence the gross lending rate, nor the choice of capital made by a borrower. Therefore, the impact of such a shock in the perfect information case is exclusively transitory. This can be seen in Figure 3.11 where nonetheless a large aggregate shock causes the same number of defaults

Table 3.2: Impact of +2SD Shock on Average Beliefs and Capital Stocks in Period 11

	Base scenario	+2SD shock
Average ϕ among high types	0.90	0.92
Average ϕ among low types	0.27	0.23
Capital stock among high types	668	670
Capital stock among high types	242	239

as in the base scenario (see Table 3.1), but the average aggregate output returns to *normal* level in the next period.

As expected, a large negative aggregate shock will cause a large number of defaults (see Table 3.1), a large information loss, and a large increase in gross lending rates. This would cause a large decline in average aggregate capital stock relative to an small shock (see Figure 3.12). Hence, the larger is a negative shock, the larger is output persistence likely to be (see Figure 3.9).

In the positive shock case, the likelihood is that both high and low types will be increasingly able to repay (see Table 3.1). That is, lenders infer that the minimum value of idiosyncratic shocks required for both types of borrowers to repay will fall, and as a result their updated beliefs about quality after repayment are weakened. No further information is available to lenders in that particular period. However, what the lenders have so far learned about a borrowers' type through the lending process would affect on their updating process in that period. Table 3.2 shows the *post shock* average value of ϕ among high quality borrowers is slightly increased while it is decreased among the low quality borrowers. Consequently, the increase in investment activities among high quality borrowers is traded off with its decline among low quality borrowers. The overall effect vanishes after one period (see Figure 3.9).

3.3.2 Robustness

The output persistence of a temporarily amplified aggregate shock, so far discussed, is simulated under the assumptions of the base scenario. We analyze the robustness of the persistency results by changing various assumptions of the base scenario in turn.

Scenario 1 *Change the base scenario with the assumption that the lender updates beliefs only after production activities take place.*

In scenario 1, the lender updates beliefs only for those borrowers who choose $K_i > 0$. Therefore, the average values of ϕ assigned to successful borrowers are higher in scenario 1 relative to the base scenario, where a lender also considers the borrower's choice of $K_i = 0$ when updating (see Figure 3.2). This is intuitively because the beliefs of the lenders, in this case, are linked with those borrowers (who can be either low or high quality type), who observe good idiosyncratic shocks. The higher average value of ϕ causes the gross lending rate to decline. This motivates high types to invest more on average. As a result, the number of defaults by high quality borrowers relative to the base scenario increases initially and then declines slightly (see Figure 3.6). Therefore, when a large negative aggregate shock of $-2SD\%$ is observed by both agents in period 11, it causes a larger number of defaults compared to the base scenario. However, due to the overall high average value of ϕ, the average aggregate capital stock and output is not grossly affected in this scenario (see Figure 3.13).

Table 3.3: Size of Defaults Due to $\Delta\epsilon_{11} = -2SD$ Under Alternative Scenarios

Indicators	Base Scenario	Scenarios 1-4			
		1	2	3	4
High	365	373	354	360	364
Low	242	268	254	241	241
Total	607	641	608	601	605

Scenario 2 *Change the base scenario with the assumption that if $K_i = 0$, the lender and borrower separate. In addition, the lender use her information of $K_i = 0$ to form beliefs for new borrowers accompanied with the assumption in base scenario that she uses the expected fractions of defaults to form beliefs about the quality of new borrowers.*

Scenario 2 refers to an environment where the lenders, though still updating their beliefs based on borrowers' choices of $K_{it} > 0$ and production, do not keep the history of those borrowers who choose $K_i = 0$. In other words, the lender uses information she gains from a borrower's choice of $K_{it} = 0$ to update her beliefs for a new borrower. This is in addition to that we discussed in base scenario that the lender uses the expected fractions of defaults to form beliefs about the quality of new borrowers. Intuitively, the impact of the use of $K_{it} = 0$ information for new borrowers goes in favour of low quality borrowers, who will receive low lending rate. Therefore on average, the learning about low quality borrowers will be lower in this scenario as compared to the base scenario (see Figure 3.4). On the other hand, a high quality borrower who decides to choose $K_{it} = 0$ due to a bad idiosyncratic shock in period t, will receive a higher lending rate in period $t + 1$ relative to that which he would have received in the base scenario. In other words, the average learning about high quality borrowers will also be lower in this scenario as compared to the base scenario (see Figure 3.5). The overall impact of the misrepresentation of $K_i = 0$ by the lender[11] is that the average capital investment and output by high quality borrowers will be smaller than the base scenario (see Figure 3.8). Therefore, when an amplified negative aggregate shock is observed in period 11, the number of defaults by high types would also be smaller relative to the base scenario (see Table 3.3). Moreover, due to higher average values of ϕ, compared to the base scenario, new high quality borrowers will be less affected in scenario 2. Hence, the *post shock* percentage decline in average aggregate capital stock and output will be less affected in scenario 2 (see Figure 3.14).

Scenario 3 *Change the base scenario with the assumption that only the borrower observes the aggregate shock when production takes place.*

Scenario 3 represents a case in which only borrowers can observe the aggregate shock. This makes the lenders's learning process weaker than the base scenario, in which they were able to observe the aggregate shock. Therefore, when lenders update their beliefs after the production has taken place, they underestimate their updated beliefs about borrowers being high quality for both non-defaulted and defaulted borrowers (see Figure 3.2). Since the gross lending rate is an inverse function of the lender's beliefs, it will be higher for exiting as well as new borrowers.[12] In response, both types borrowers will choose less investment and output. Therefore, we see that the average gross output of the economy is less in scenario 3 relative to the base case (see Figure 3.8). Since the lenders also learn by observing borrowers' choices

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[11] Here the word "misrepresentation" refers to case where the lenders use the information belongs to $K_i = 0$ for other borrowers.

[12] Remember that here lenders use the expected fractions of defaulted borrowers, who were believed as high types, to form beliefs about the quality of new borrowers. In that case, the underestimation of beliefs for defaulted borrowers, accounts for higher gross lending rates offered by lenders relative to that of the base case.

of capital investment, the impact of missing information about the aggregate shock in learning about borrowers' quality is somewhat alleviated. Figure 3.5 shows that the lenders' learning about borrower quality among two types tends to gradually improve.

When a negative aggregate shock occurs in period 11 causing the comparable number of defaults relative to the base scenario, the average value of ϕ relative to the base scenario declines sharply (see Figure 3.15). As we addressed above, the lack of information regarding aggregate shock, makes the learning process of lenders weaker than the base scanrio. Lenders link such defaults to the unobserved quality of borrowers. The ultimate effect is a higher offered lending rate and a lower average aggregate capital stock and output in an economy. Hence, we see that persistence is larger in this case relative to the base scenario (see Figure 3.16).

Scenario 4 *Change the base scenario with the assumption that the lender uses 50-50% (or naive) beliefs for the quality of new borrowers.*

Scenario 4 introduces naive expectations on the lender's part for new borrowers. In this scenario, the lender assigns a probability for a new borrower being of high type consistent with the unconditional probabilities (i.e. 50-50). Since a random aggregate shock is observed each period, the impact of naive expectations over time will be a slower learning process relative to the base scenario (see Figures 3.2-3.7). This impact is also reflected in the average aggregate output (see Figure 3.8). Hence, following an aggregate shock causing the same number of defaults relative to the base scenario (see Table 3.3), there will be a larger negative effect on the average belief in the economy about high quality types (see Table 3.15) relative to the base scenario. The impact is to generate a larger persistence in average aggregate output relative to the base scenario (see Figure 3.17).

3.3.3 Sensitivity Analysis in Parameters

The main result concerning output persistence can be strengthened by conducting a sensitivity analysis on parameters values. These simulations are run under the base scenario.

Size of quality parameters. The impact of the aggregate shock in the imperfect information case also depends on the productivities of high and low quality types in the credit market. This impact can be seen in Figures 3.18 and 3.19, which show a difference in the output persistence of the same size of aggregate shock using different values of θ_h and θ_l. This differential effect is not seen in perfect information case (see Figure 3.20).

A larger gap in the values assigned to high and low qualities can lead to faster learning and thus accounts for a lower premium on lending relative to the case where qualities are closer to each other. Thus, investment decisions will be less affected by an aggregate shock and will cause relatively smaller propagation into output fluctuation. Figure 3.21 shows that the values of ϕ and K for high quality borrowers are larger when the gap between qualities are larger. A similar result is shown in Figures 3.22 and 3.23 where, at a same level of aggregate shock, the average value of ϕ is less affected given a larger differential in qualities.

Conversely, if borrowers' qualities are closer to each other along with their low levels (e.g., when θ_h=1.10 and θ_l=1), then a higher premium on lending due to slower learning, will cause high quality borrowers to invest less relative to the base scenario (see Figure 3.21). This may be a potential reason for low output persistence in the case of θ_h=1.10 and θ_l=1, relative to the base scenario.

The sensitivity of results to changes in quality parameters follows from the fact that output persistence depends on the endogenous response of lenders. What matters is not only the gap between high and low type productiveness, but also their levels.

Changes in outside options. In our model framework, a change in the magnitude of lenders' and borrowers' outside options do not by themselves generate any output persistence. For illustration, suppose that the risk free market interest rate increases by 50% relative to the base scenario only in period 11. The first response comes from the lender: namely, to raise the gross lending rate. As a second response, borrowers would choose less investment. The overall impact is a lower average aggregate output relative to the 'normal' market interest rate. Since the monetary shock was very transitory, i.e. only for period 11, and perfectly observed by both parties. Therefore, when the risk free market interest rate returns to its previous level in the next period, the lending rate, investment, and output activities will also be re-adjust accordingly. So, there will be no persistent effect of this exogenous policy change (see Figure 3.24).

Table 3.4: Effect of Monetary Tightening on Credit Market and Output in Period 11

	Default		**Output**	
	Rf=1.04	Rf=1.06	Rf=1.04	Rf=1.06
Base Scenario	73	72	1139	1015
Scenario 2	72	71	1108	983
Scenario 4	73	72	1131	1003

Table 3.5: Effect of Wage Increase on Credit Market and Output in Period 11

	Default		**Output**	
	w=0.04	w=0.06	w=0.04	w=0.06
Base Scenario	73	58	1139	1094
Scenario 2	72	57	1108	1062
Scenario 4	73	58	1131	1085

Table 3.4 shows that, in all three scenarios, the number of defaults are the same as we see in base scenario. However, average aggregate output declines somewhat. Note that it is possible for the above monetary impact to intensify the effect of an aggregate productivity shock discussed earlier, but this constitutes an external source of propagation which is not of interest for our analysis.

A similar result is obtained if the outside option of borrowers (i.e., the wage rate w) increases (see Table 3.5). Here due to a high outside option, the investment and output activities and, therefore, the number of defaults are low in period 11. However, this impact disappears in the next period.

3.4 Conclusions

This chapter shows that 'learning by lending' is a potential channel to understand business cycle fluctuations under imperfect credit markets. By introducing a simple learning mechanism we can generate persistence in real indicators such as the aggregate capital stock and output of an economy. The *post shock* speed of learning depends on how lenders forms their beliefs about the underlying quality of existing and new borrowers. We show that persistence will be magnified when (i) lenders take into account borrowers' previous choices of investment when offering lending contracts, (ii) lenders maintain the history of those borrowers who do not invest in the current period (but may do so in the future), (iii) lenders can not observe an aggregate productivity shock, but borrowers can and (iv) lenders apply naive expectations for new borrowers. Obviously, there could be a number of other possible scenarios to investigate persistence. Here, our main intension was to verify that learning by lending can be a potential channel to propagate real shocks. We derive an endogenous link between the learning effectiveness, lending rates, and investment levels that allows us to obtain internal propagation even due to a temporary shock.

The strength of propagation is sensitive to variations in borrowers' qualities in an economy. We find that output persistence is affected not only by the magnitude of quality gaps but also by the levels of borrower productivities. A large gap between low and high qualities of borrowers leads to faster learning and weaker internal propagation. On the other hand, if the levels of borrower qualities are low, then investment activities will be inherently sluggish and thus less responsive to the *post shock* learning disruption. We also show that the impact of asymmetric information on aggregate output fluctuations is asymmetric, which is also found in several empirical literatures. One particular example is Beaudy and Koop (1993) who use a time-series econometric approach to show that the impulse response of GNP is asymmetric, negative innovations to GNP are observed to be much less persistent than positive ones. Nonetheless, we also show an asymmetric output response, our approach is totally different, in particular we showed in this chapter that the changes in lender information have more notable impacts on the business cycle when bad shocks occur relative to good ones. This result is aligned with the previous literatures on collateral-based model. For example, Bernanke and Gertler (1989) conclude that the dynamic effects of productivity shock may be asymmetric i.e. sharp investment downturns are more likely than the sharp upturn.

Our analysis opens a new avenue for understanding business cycle fluctuation. There is potential to extend our analysis both theoretically as well as empirically. For instance, one can alter the severity of asymmetric information by introducing opportunities for lenders to screen borrowers by type, or for borrowers to signal type by their choices. In addition, one can embed our partial equilibrium analysis into a general equilibrium framework. We leave these extensions for future research.

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3.6 Appendix A

Proposition 3. *The premium on the gross lending rate is monotonically decreasing with the lender's belief i.e.,*$R'_K(\phi) < 0$ *&* $R''_K(\phi) > 0$.

Proof Consider a simplified form of Equation 3.2.2:

$$K(1 + r_f) = \phi_{it}\Theta_{ht} + (1 - \phi_{it})\Theta_{lt} \tag{3.6.1}$$

Using Equation 3.6.1, the first and second derivatives of R_K with respect to P are:

$$R'_{itK}(\phi) = \frac{\Theta_{lt} - \Theta_{ht}}{\phi_{it} P(Y_{ht} > K_{ht}R_{itK}) + (1-\phi_{it})P(Y_{lt} > K_{lt}R_{itK})} \tag{3.6.2}$$

$$\begin{aligned} R''_{itK}(\phi) = {} & [P(Y_{lt} > K_{lt}R_{itK})R'_{itK}(\phi) - P(Y_{ht} > K_{ht}R_{itK})R'_{itK}(\phi)][\phi_{it}P(Y_{ht} > K_{ht}R_{itK}) \\ & + (1-\phi_{it})P(Y_{lt} > K_{lt}R_{itK})]^{-1} - (\Theta_{lt} - \Theta_{ht})[\phi_{it}P(Y_{ht} > K_{ht}R_{itK}) \\ & + (1-\phi_{it})P(Y_{lt} > K_{lt}R_{itK})]^{-2}[P(Y_{ht} > K_{ht}R_{itK}) - P(Y_{lt} > K_{lt}R_{itK})] \end{aligned} \tag{3.6.3}$$

Assume that $q_h > q_l$. This implies that, ceteris paribus, $P(Y_{ht} > K_{ht}R_{itK}) > P(Y_{lt} > K_{lt}R_{itK})$ and therefore, $E(Y_{ht}|Y_{ht} < K_{ht}R_{itK}) > E(Y_{lt}|Y_{lt} < K_{lt}R_{itK})$. Hence, $q_h > q_l$ is the necessary and sufficient condition to show that Equations 3.6.2 and 3.6.3 represent $R'_K(\phi) < 0$ & $R''_K(\phi) > 0$.

3.7 Appendix B

Assume that $\hat{e}_{it}$ is a realization of the idiosyncratic shock, which is drawn from a uniform distribution $[\underline{e}, \overline{e}]$. After simplifying the expressions: $\theta_i \hat{e}_{it}\epsilon_t 2^\alpha - 2R_{it2} \geq \theta_i \hat{e}_{it}\epsilon_t - R_{it1}$ and $\theta_i \hat{e}_{it}\epsilon_t - R_{it1} \geq w\theta_i$, the lender sets the cutoff values that borrower i may choose $K_i = 0, 1 or\ 2$. Borrower i chooses $K_i = 2$ if $\hat{e}_{it} \geq \frac{2R_2 - R_1}{\theta_i\epsilon_t(2^\alpha - 1)} (\equiv \hat{e}_{2it})$, or $K_i = 1$ if $\hat{e}_{it} \geq \frac{w\theta_i + R_1}{\theta_i \epsilon_t} (\equiv \hat{e}_{1it})$. Otherwise, he chooses $K_i = 0$ and avails his outside option.

Having the knowledge of $\underline{e}$, $\overline{e}$, $\hat{e}_{1it}$, and $\hat{e}_{2it}$, the lender estimates the possibility that borrower i is of high type given his choice of K_{it} as follows:

$$\begin{gathered} If\ \hat{e}_{2it} < \underline{e},\ then\ \rho_{2it} = 1,\ \rho_{1it} = 0,\ \rho_{0it} = 0 \\ If\ \hat{e}_{2it} < \overline{e}\ and\ \hat{e}_{1it} < \underline{e},\ then\ \rho_{2it} = \frac{\overline{e} - \hat{e}_{2it}}{\overline{e} - \underline{e}},\ \rho_{1it} = 1 - \rho_{2it},\ \rho_{0it} = 0 \\ If\ \hat{e}_{2it},\ \hat{e}_{1it} < \overline{e}\ then\ \rho_{2it} = \frac{\overline{e} - \hat{e}_{2it}}{\overline{e} - \underline{e}},\ \rho_{1it} = \frac{\hat{e}_{2it} - \hat{e}_{1it}}{\overline{e} - \underline{e}},\ \rho_{0it} = \frac{\hat{e}_{1it} - \underline{e}}{\overline{e} - \underline{e}} \\ If\ \hat{e}_{2it} > \overline{e}\ and\ \hat{e}_{1it} < \underline{e},\ then\ \rho_{2it} = 0,\ \rho_{1it} = 1,\ \rho_{0it} = 0 \\ If\ \hat{e}_{2it} > \overline{e}\ and\ \hat{e}_{1it} < \overline{e},\ then\ \rho_{2it} = 0,\ \rho_{1it} = \frac{\overline{e} - \hat{e}_{1it}}{\overline{e} - \underline{e}},\ \rho_{0it} = \frac{\hat{e}_{1it} - \underline{e}}{\overline{e} - \underline{e}} \\ If\ \hat{e}_{2it} > \overline{e}\ and\ \hat{e}_{1it} > \overline{e},\ then\ \rho_{2it} = 0,\ \rho_{1it} = 0,\ \rho_{0it} = 1 \end{gathered} \tag{3.7.1}$$

For an illustration, assume that the lender learns that $\underline{e} > \hat{e}_{2ht}$, $\hat{e}_{2lt} > \overline{e}$, and $\underline{e} > \hat{e}_{1lt}$.

Given this information, if the borrower chooses $K_{it} = 2$, then the lender knows from above distribution that $\rho_{2ht} = 1$ and $\rho_{2lt} = 0$. Plugging these values into $\rho_{Kit} = \phi_{it}\rho_{Kht} + (1 - \phi_{it})\rho_{Klt}$ and $\hat{\phi}_{it} = \frac{\phi_{it}\rho_{Kht}}{\rho_{Kit}}$, she can reveal that $\hat{\phi}_{it} = 1$, which means borrower i is of high type based on his choice of K_{it}. If borrower chooses $K_{it} = 1$, then she gains information from the above distribution that $\rho_{1ht} = 0$ and $\rho_{1lt} = 1$. Using these values, she can know that the borrower i is of low type because of $\hat{\phi}_{it} = 0$.

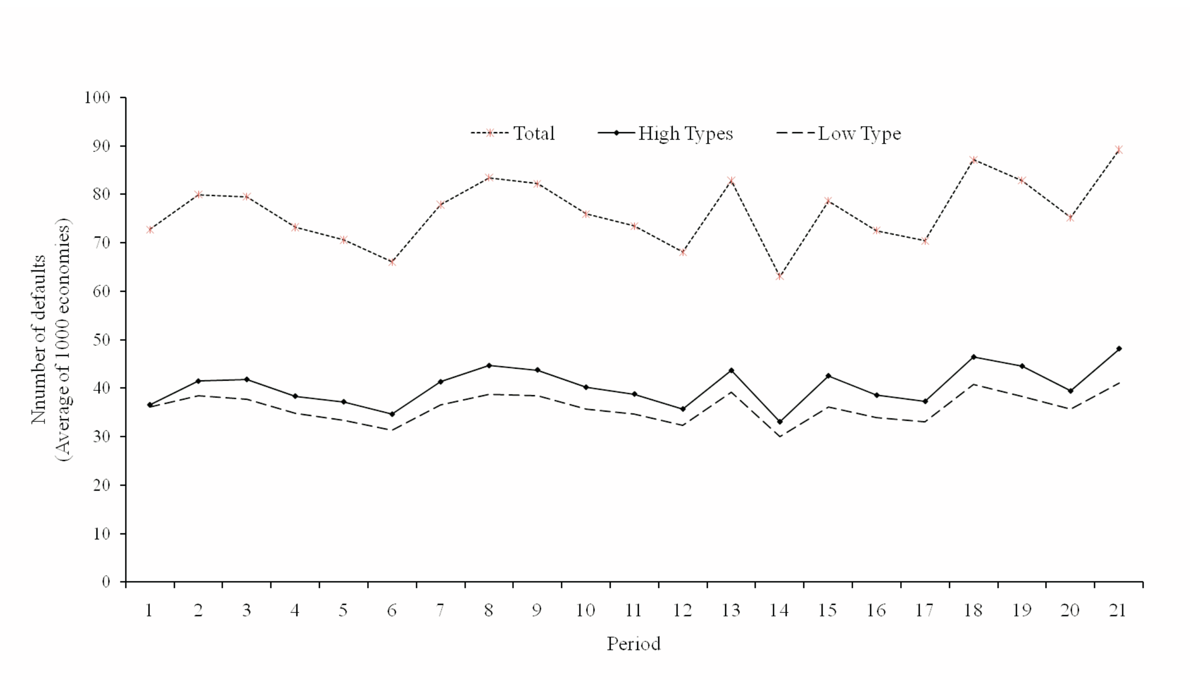
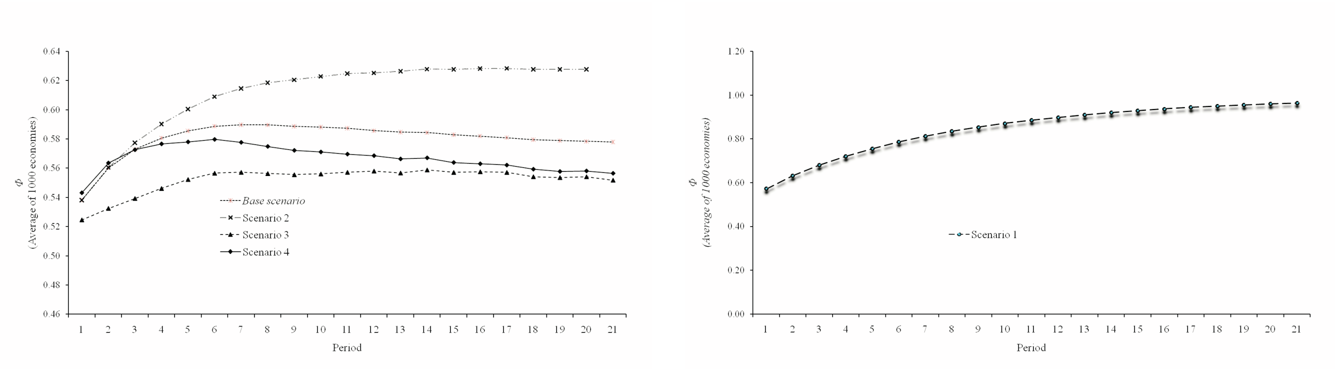
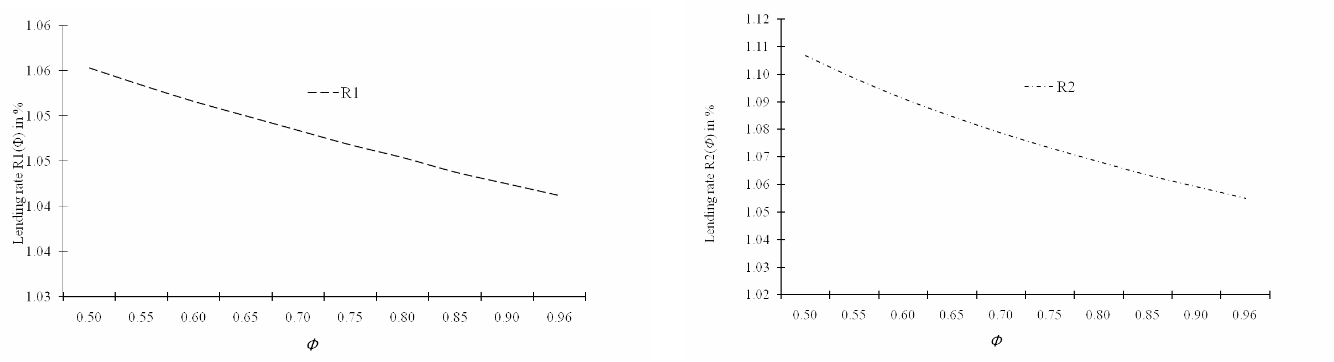


Figure 3.1: Impact of a Normal Random Aggregate Shock on the Size of Defaults in Base Scenario



(a) (b)

Figure 3.2: Value of ϕ (Average Among all Borrowers and Across 1000 Economies) in Alternative Scenarios



(a) (b)

Figure 3.3: Links Between Lender's Beliefs and Offered Lending Rates

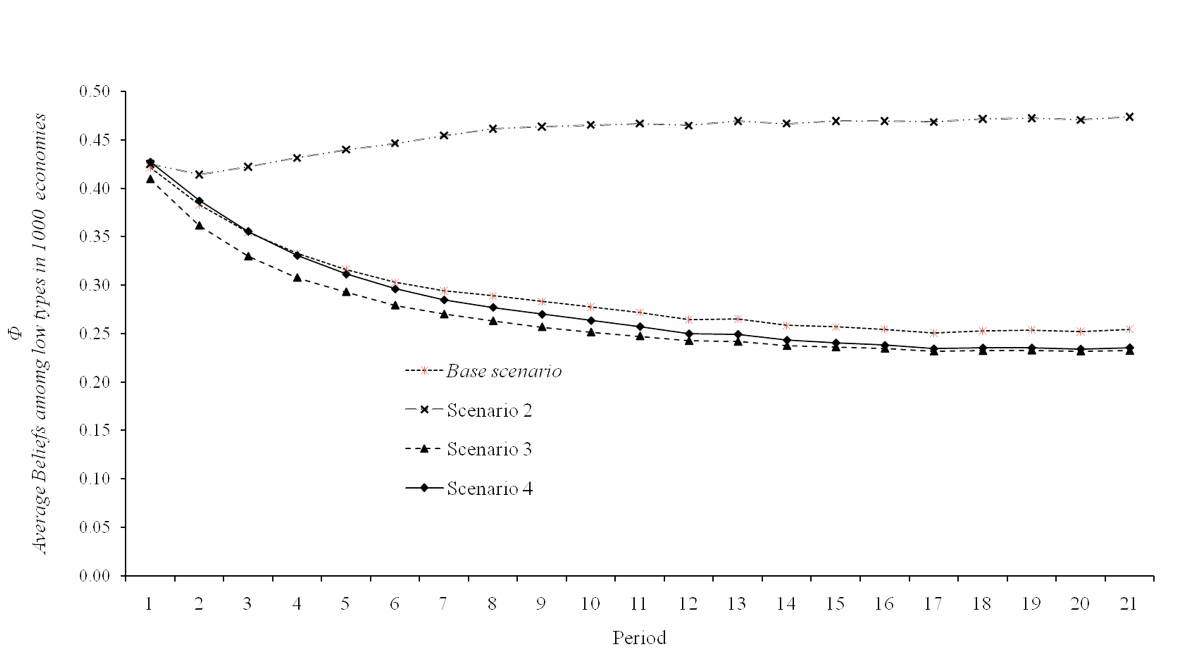


Figure 3.4: Value of ϕ (Average Among Low Quality Borrowers and Across 1000 Economies) in Alternative Scenarios

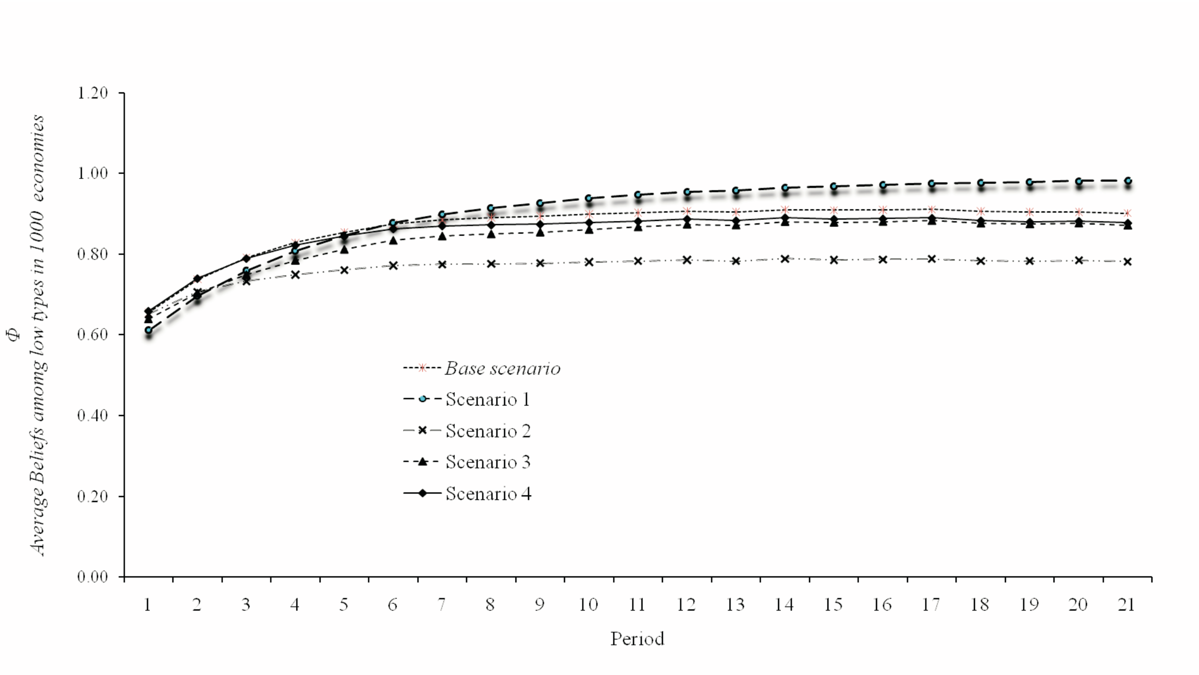


Figure 3.5: Value of ϕ (Average Among High Quality Borrowers and Across 1000 Economies) in Alternative Scenarios

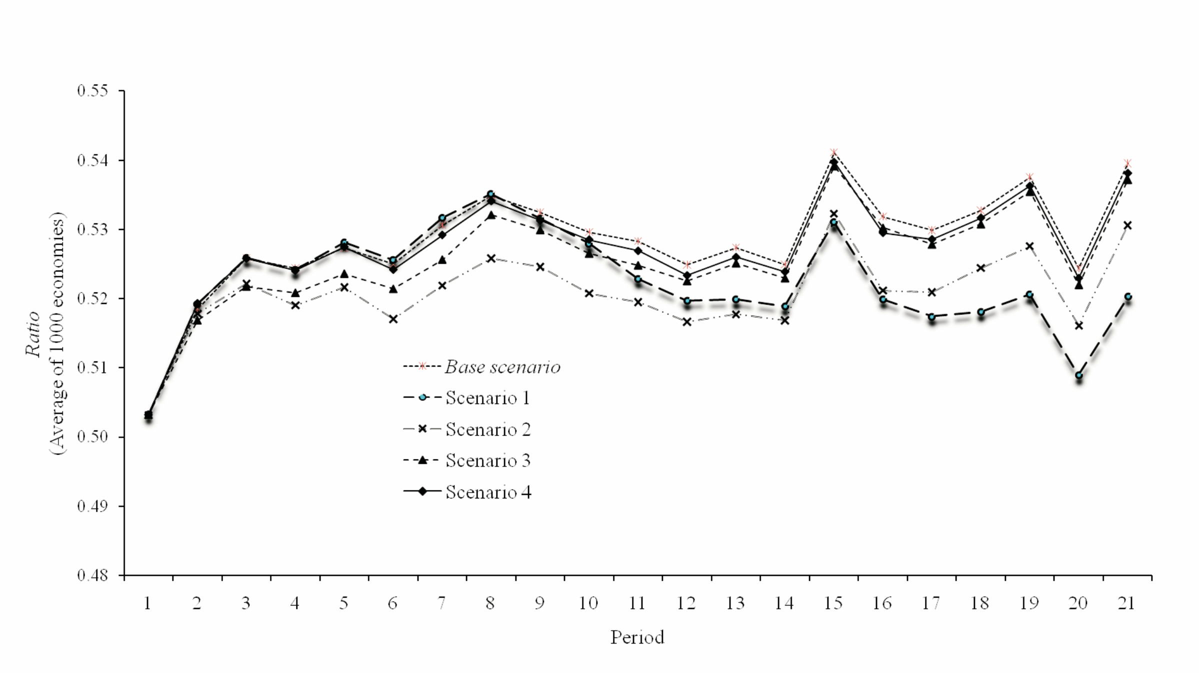


Figure 3.6: Value of ϕ (Share of High Quality Types in Total Defaults (Average Across 1000 Economies) Under Alternative Scenarios

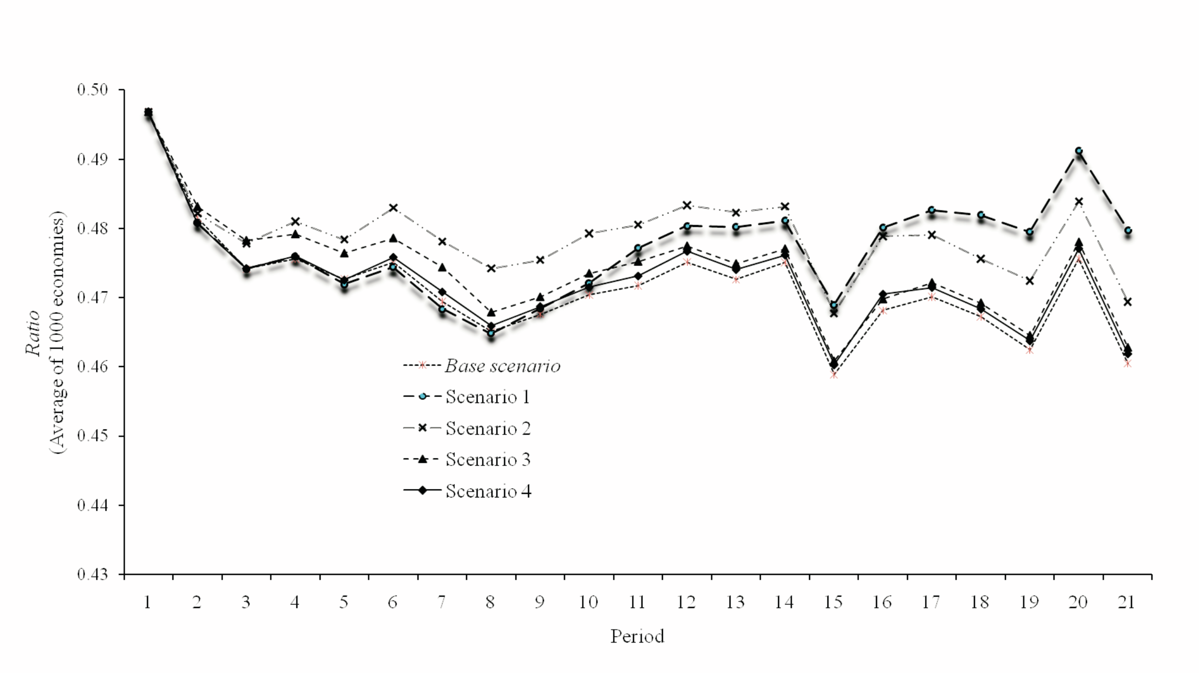


Figure 3.7: Value of ϕ (Share of Low Quality Types in Total Defaults (Average Across 1000 Economies) Under Alternative Scenarios

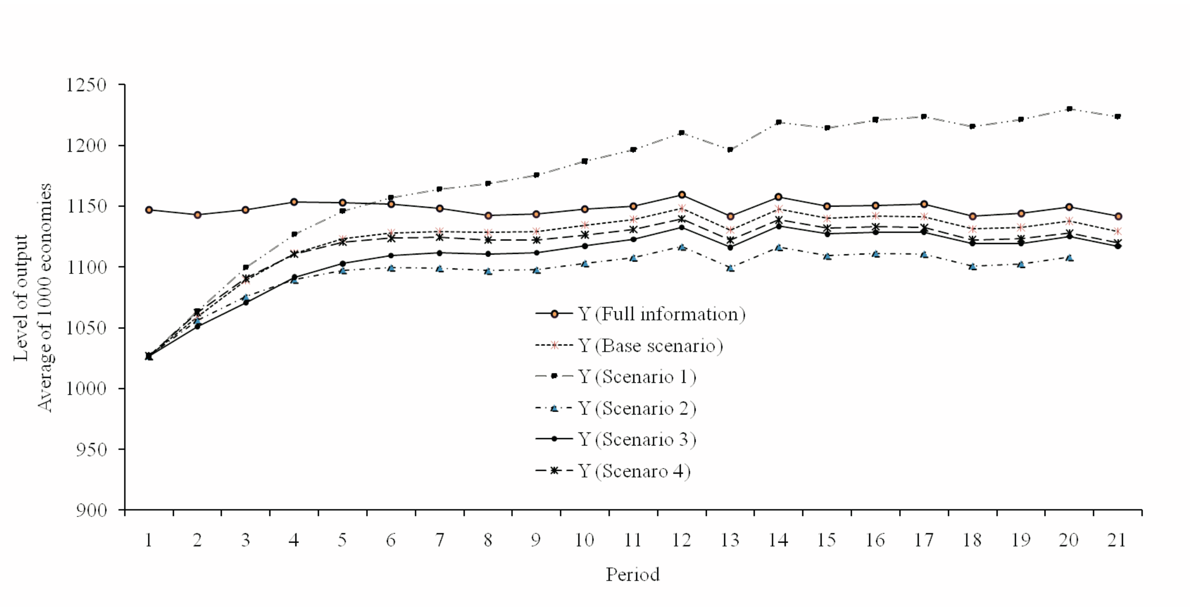


Figure 3.8: Impact of Learning Process on Average Aggregate Output in Alternative Scenarios

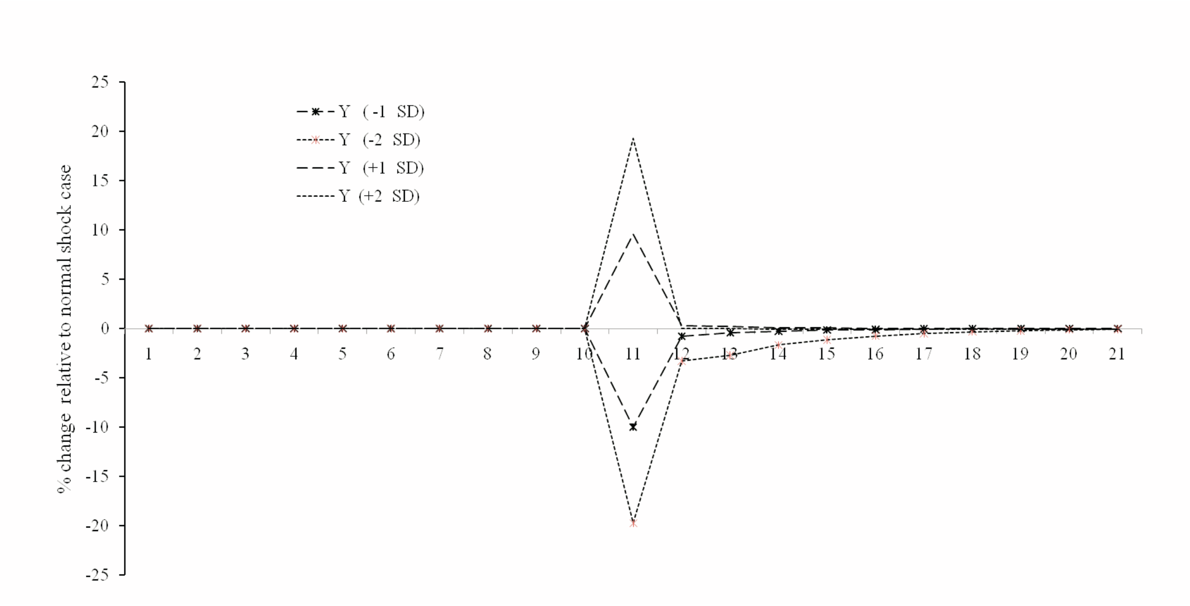


Figure 3.9: Impact of $\Delta\epsilon_{11} = \pm 1SD$ & $\pm 2SD$ on Average Aggregate Output in Base Scenario

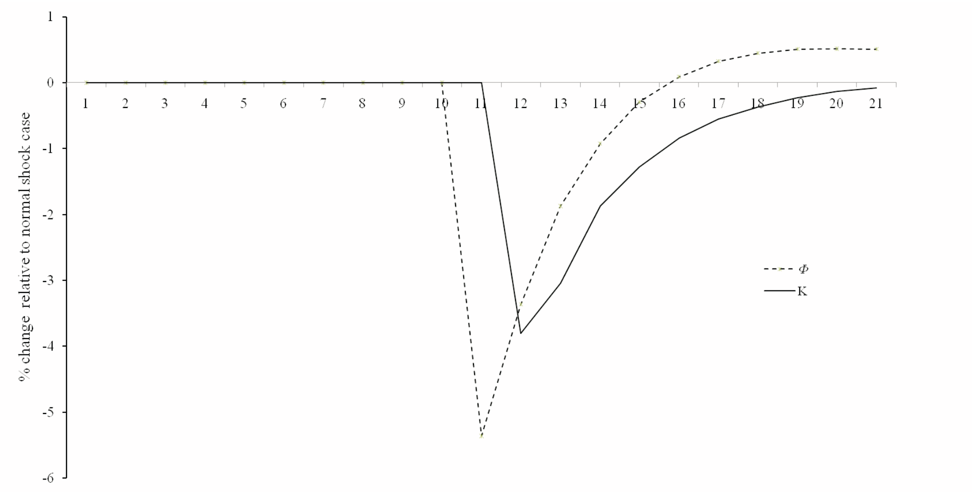


Figure 3.10: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Capital Stock and General Belief of Being a High Type Borrower–Base Scenario

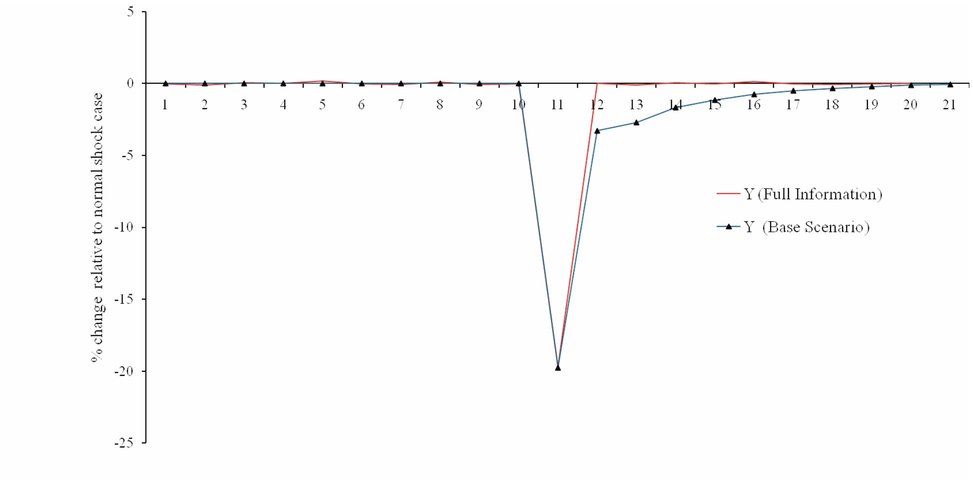


Figure 3.11: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output in Alternate Credit Markets

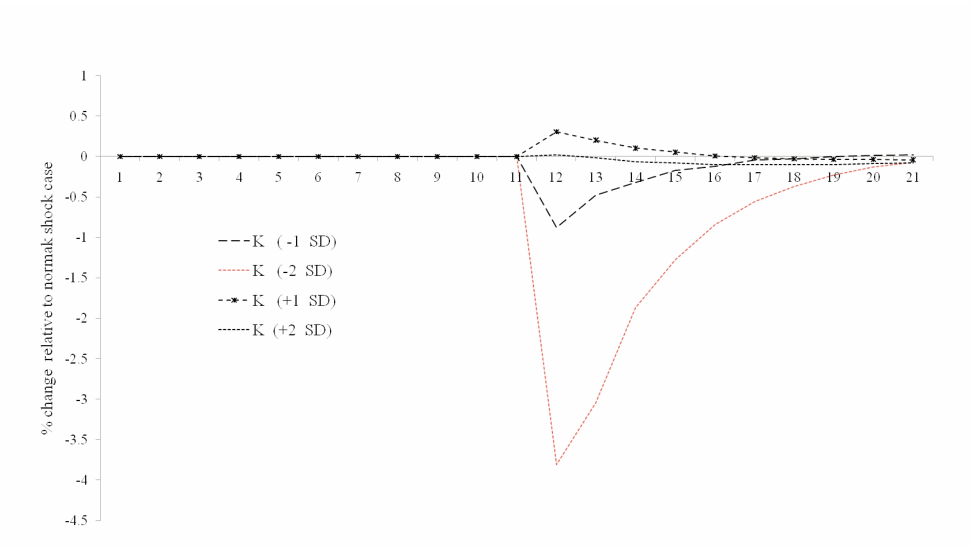


Figure 3.12: Impact of $\Delta\epsilon_{11} = \pm 1SD$ & $\pm 2SD$ on Average Aggregate Capital Stock in Base Scenario

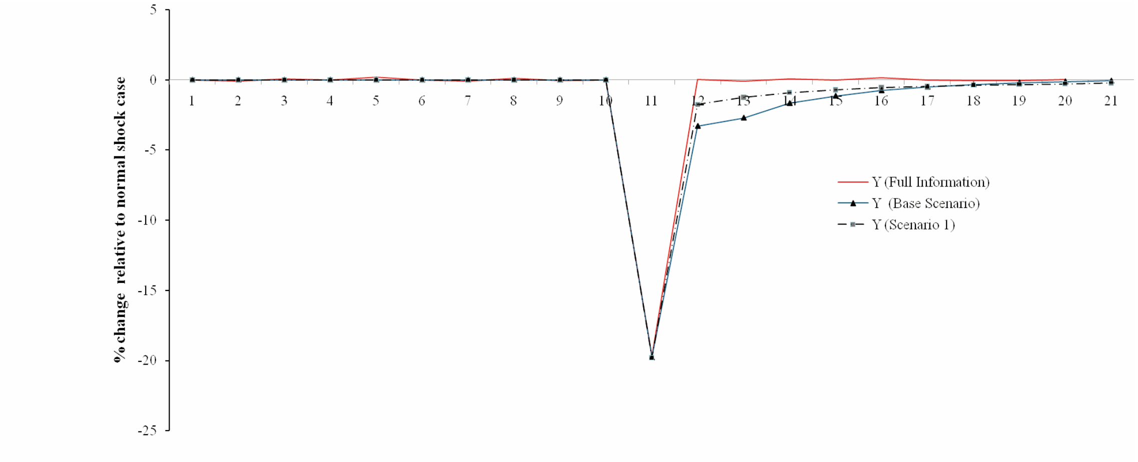


Figure 3.13: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output in Scenario1

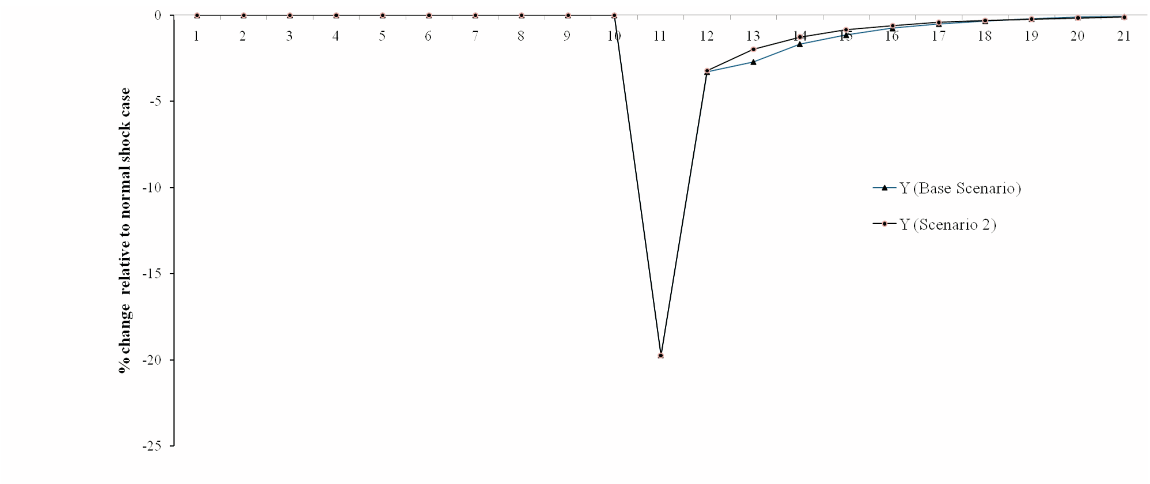


Figure 3.14: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output in Scenario 2

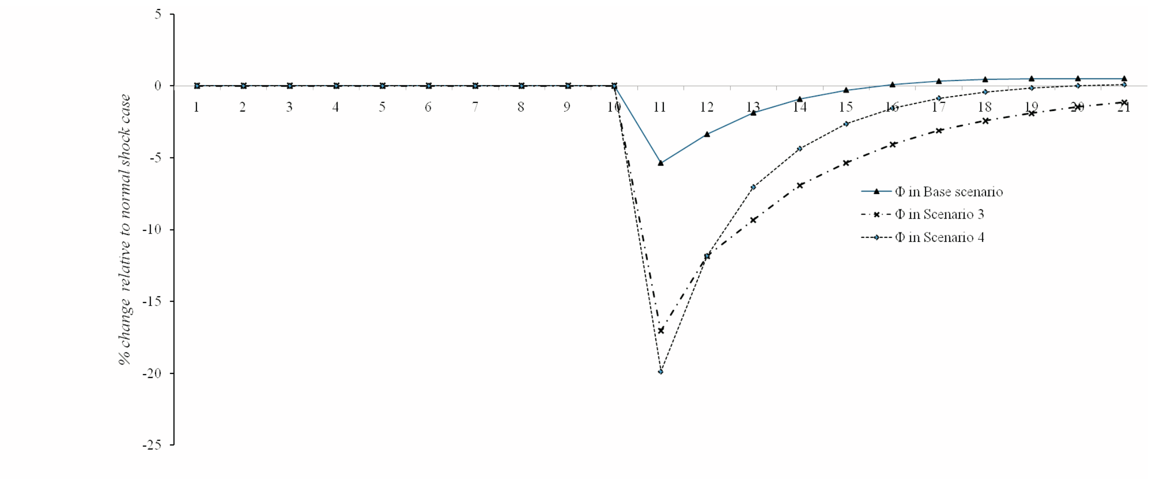


Figure 3.15: Impact of $\Delta\epsilon_{11} = -2SD$ on General Belief of a Borrower Being a High Type in Alternative Scenarios

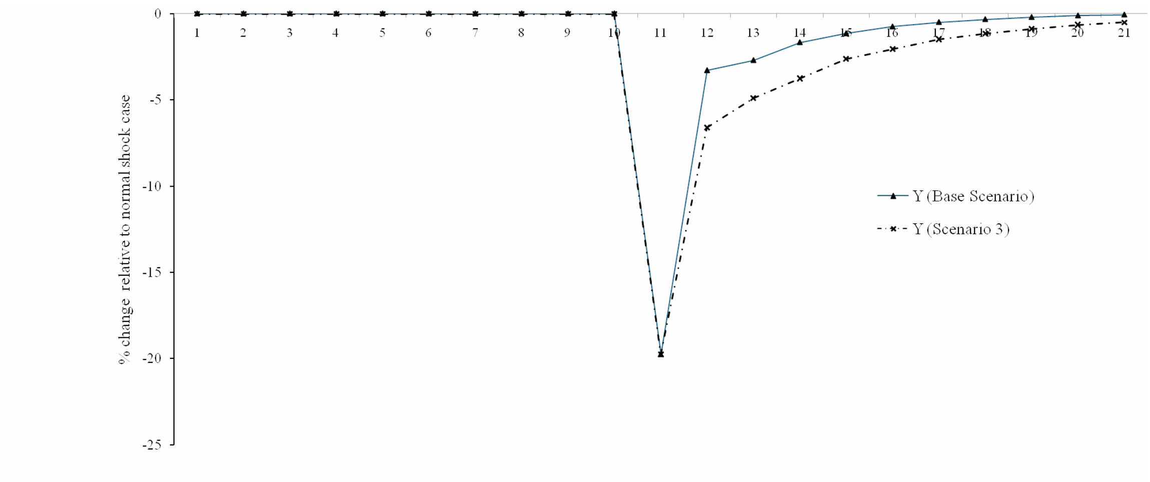


Figure 3.16: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output in Scenario 3

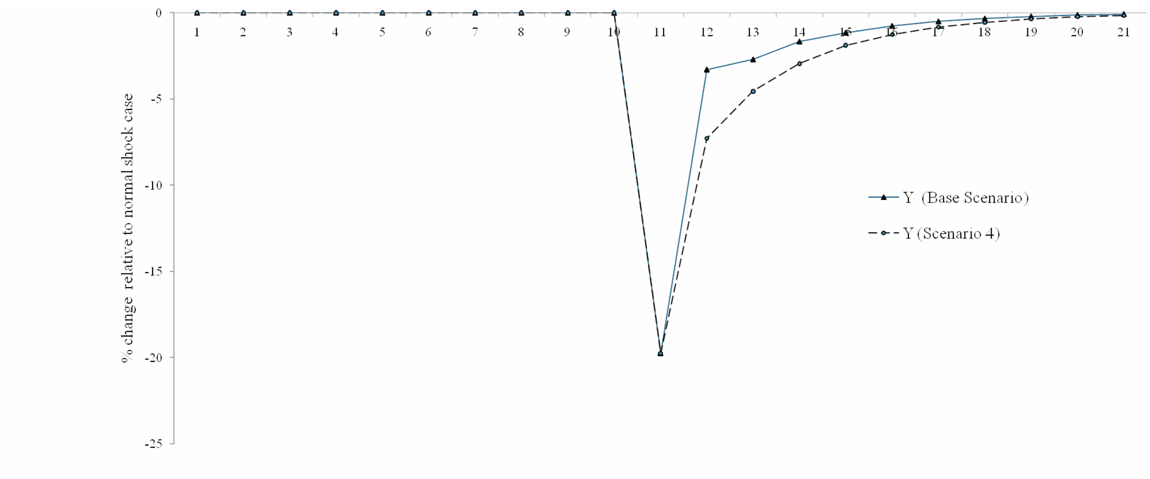


Figure 3.17: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output in Scenario 4

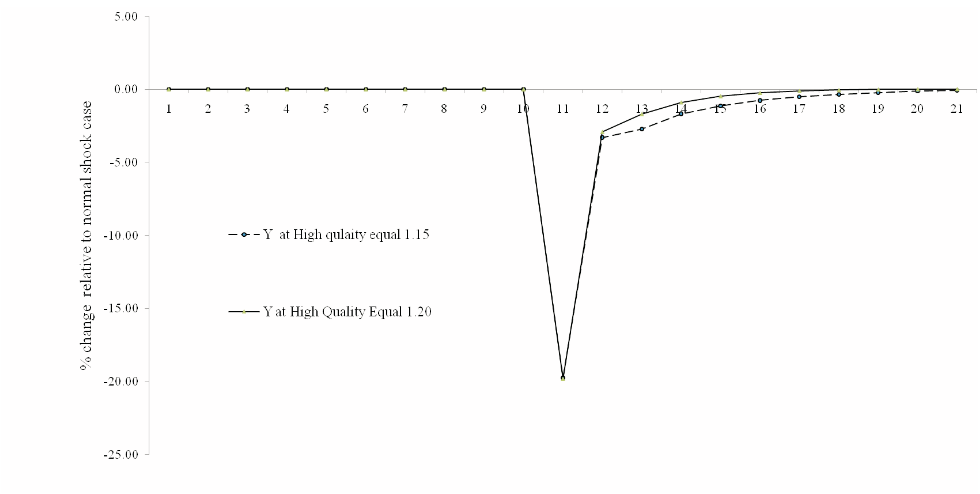


Figure 3.18: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output at Different Levels of High Qualities—Base Scenario

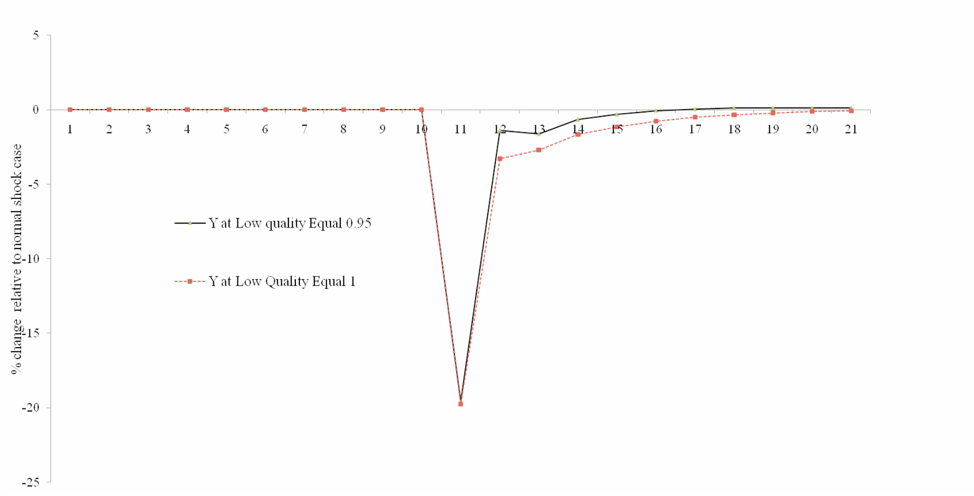


Figure 3.19: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output at Different Levels of Low Qualities—Base Scenario

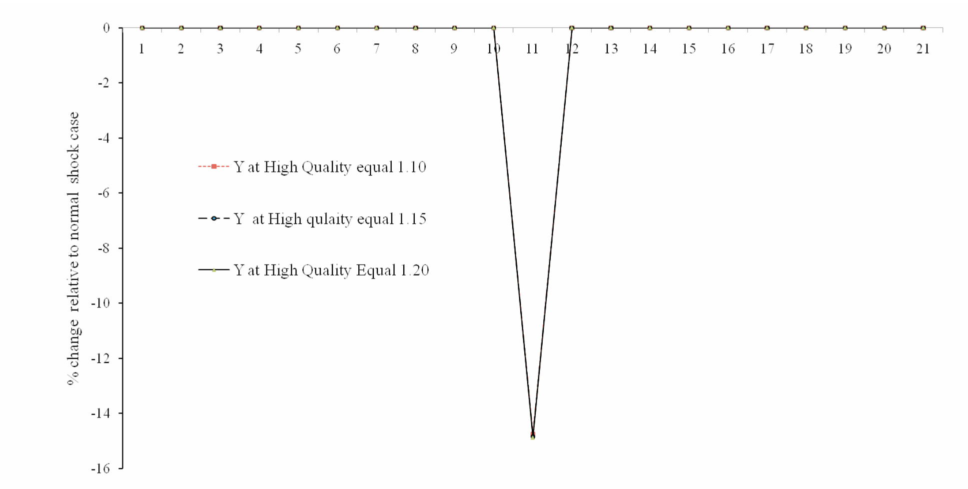
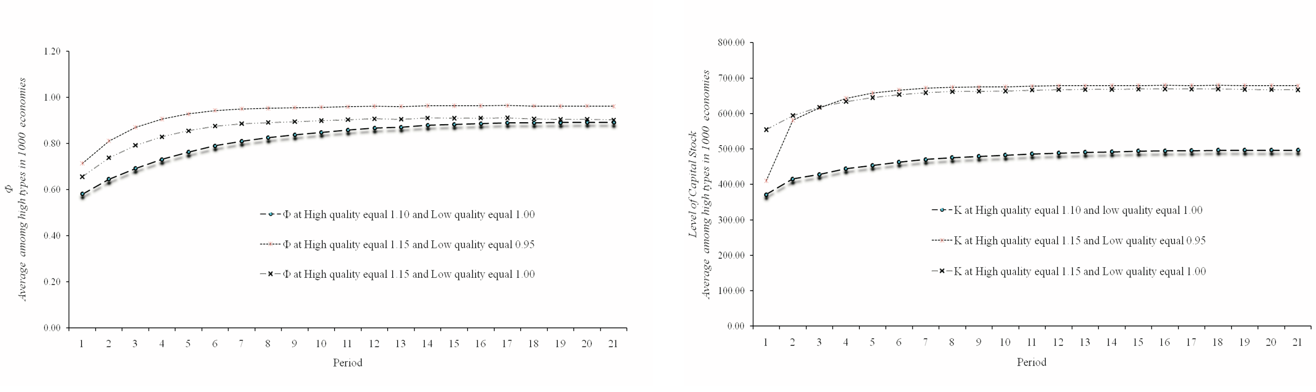


Figure 3.20: Impact of $\Delta\epsilon_{11} = -2SD$ on Average Aggregate Output at Different Levels of High Qualities—Perfect Information Scenario



(a) ϕ

(b) K

Figure 3.21: Value of ϕ and K (Average Among High Quality Groups of Borrowers Across 1000 Economies) at Different Levels of Quality Parameters in Base Scenario

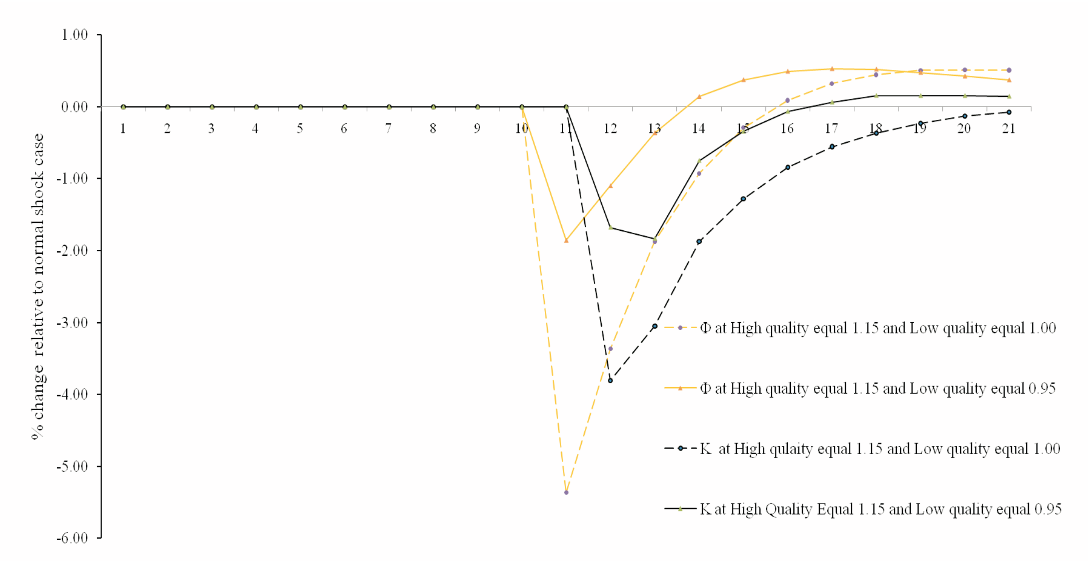


Figure 3.22: Impact of $\Delta\epsilon_{11} = -2SD$ on General Belief of Being a High Type Borrower and on Average Aggregate Capital Stock Under Base Scenario

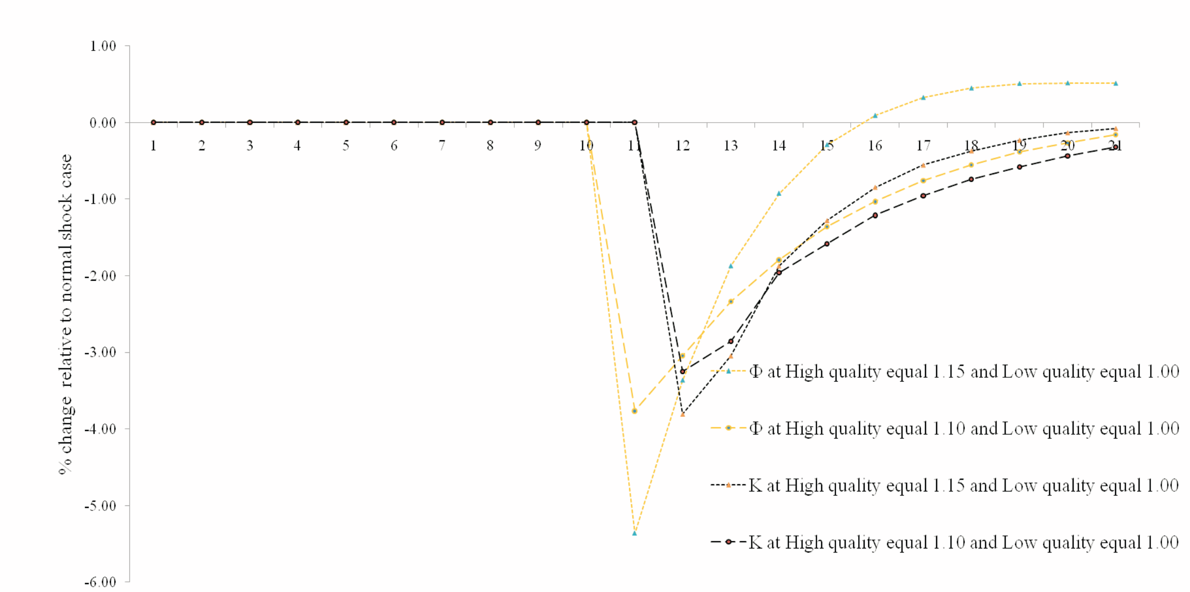


Figure 3.23: Impact of $\Delta\epsilon_{11} = -2SD$ on General Belief of Being a Low Type Borrower and on Average Aggregate Capital Stock Under Base Scenario

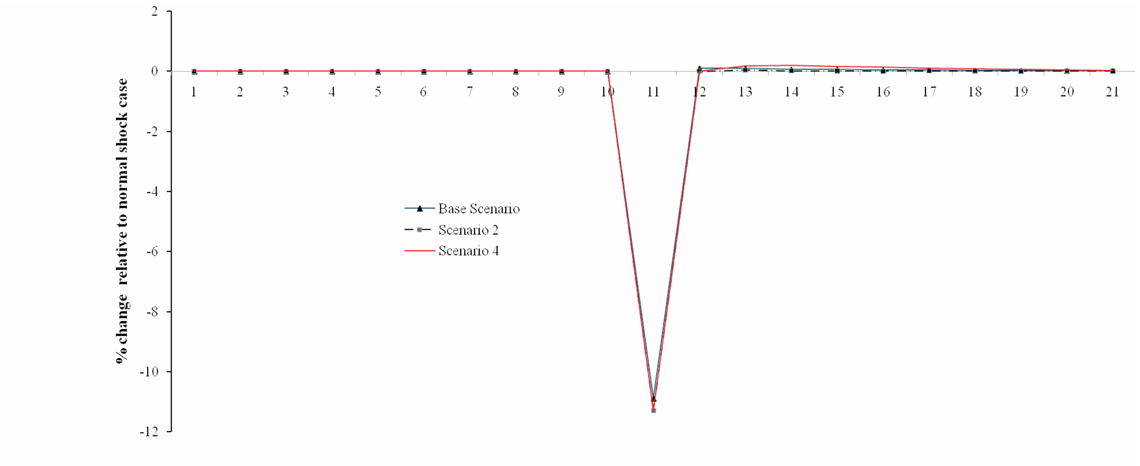


Figure 3.24: Impact of 50% increase in r_f on on Average Aggregate Output at Alternative Scenarios