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Measuring the contribution of stratification and social class at birth to inequality of opportunity[*]

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Abstract

In this paper, we propose a new analytical approach to inequality of opportunity that allows for the decomposition of the impact of initial circumstances into the impact of social class at birth and the impact of social stratification based on an identity marker in a society in which there is a dominant and a marginalized group. We derive the dominance conditions associated with this approach. These dominance conditions allow for robust rankings of distributions. We also explore two potential views on the inequality of opportunity: the pro-poor and the meritocratic perspectives. To make the identification of all robust orderings implementable using survey data, we discuss an estimation approach and statistical inference for these dominance tests. Finally, to illustrate the empirical relevance of the proposed dominance conditions, we use data from the Egyptian Labor Market Panel Surveys for 1998, 2006, 2012, and 2018 to study the changes in inequality of opportunity and the change in the contributions of social class at birth and gender stratification over this period in Egypt.

Key words: *Inequality of opportunity, stochastic dominance, stratification.*

JEL Classification: I31, I32, O15, Z13.

1 Introduction

This paper is motivated by the recent development in stratification economics (see Chelwa, Hamilton, and Stewart, 2022; Darity, 2022), the increased public interest in intergroup identity-based inequalities, and the importance of stratification and identity in analyzing inequality of opportunity. Its objective is to understand how stratification and initial circumstances can contribute to the inequality of opportunity. In doing so, this paper links the stratification economics literature to the literature on inequality of opportunity (see Roemer, 1998; Bjerk, 2008; Ferreira and Gignoux, 2014; Lee and Seshadri, 2018). It presents two contributions to the literature on the inequality of opportunity. First, to link the equality of opportunity approach with the economics of stratification approach, it proposes a decomposition of the inequality of opportunity into an identity-based stratification component and a social class at birth component. Second, it develops new graphical tools and dominance conditions that allow comparing distributions using the proposed equality of opportunity approach. The dominance conditions we derive focus on three types of ethical principles. The first type of dominance conditions uses an adapted version of the standard ethical principles for inequality analysis and establishes the conditions that the inequality of opportunity indices should obey. The second and third types of dominance conditions are based on two ethical principles we introduce in the paper: the pro-poor and the meritocratic ethical principles. All these three principles apply to the identity-based stratification component and the social class at birth component.

The interest in issues related to stratification such as disparities and the extent of discrimination is not new in economics. Indeed there is a rich literature on intergroup inequalities that focuses on choices and human capital investments to explain the observed disparities. Nevertheless, the increased interest in stratification economics as a burgeoning area of research is recent (e.g., Chelwa, Hamilton, and Stewart, 2022; and Darity, 2022).

The core principles of the economics of stratification started with Darity's (2005) seminal paper where the author expands the focus on inequalities. Darity (2005) proposes an approach that goes beyond the conventional perspective to inequalities to focus on understanding the source of the persistence of intergroup inequalities. Specifically, stratification economics uses a perspective that includes assessing the global institutional sources that could be perpetuating disparities and focuses on measuring, understanding, and explaining intergroup inequalities from this perspective (see also Chelwa, Hamilton, and Stewart, 2022; and Darity, 2022). Indeed, Darity, Hamilton, and Stewart (2015) state that each identity group's political influence and social inclusion can be among the potential contributors to these disparities and should be scrutinized.

In explaining the persistence of these intergroup disparities, Darity (2005) implicitly distinguishes between the decisions for which an individual should be considered responsible, and decisions that are the consequence of the institutional context. The implicit assumption in this distinction is that these identity-based inequalities are unfair unless one thinks it is reasonable to assume that marginalized group members are systematically making choices that leave them worse than the dominant group. As a result, a natural way to approach these identity-based inequalities is from this fairness perspective. Using a fairness-based view, Seguino (2013) expands this idea of racial stratification to gender inequalities. She defines gender justice as a setting with equal probabilities of achieving all potential outcomes in all identified social domains regardless of gender.

The underlying basis of Seguino's (2013) definition of gender (identity) justice can be linked to the philosophical argument in Roemer's (1998) equality of opportunity model. More specifically, Seguino's (2013) concept of equal probabilities of achieving potential outcomes in all social domains and Roemer's (1998) concept of *accountable effort* have the same underlying fairness-based view. Indeed, Roemer's *accountable effort* concept is rooted in the idea that an individual's social background and institutional constraints may influence the capacity to exert the necessary raw *effort* to generate a social outcome. He also argues that the *effort level* for which a person is responsible should be purged from the effect of her initial conditions and social constraints. Roemer (1998) shows that if the residual luck is randomly assigned, the equality of opportunity condition requires that the quantile function of outcomes, conditional on initial circumstances, be the same for all potential combinations of initial circumstances. Conceptually, if we consider the individual's group identity as one of the initial circumstances, Roemer's (1998) condition of equality of opportunity and Seguino's (2013) concept of gender (racial) justice are then equivalent. Thus, by making the distinction between the outcome differences that are due to an individual's responsibility and those that are outside the individual's control (circumstances at birth), Roemer's (1998) model of equality of opportunities offers a powerful framework to analyze the intergroup identity-based inequalities. This paper, will adopt this framework.

Given that we decompose the inequality of opportunity into an identity-based stratification component and a social class at birth component, we need to adapt the inequality of opportunity framework. Thus, we propose a measurement framework that adjusts Temkin's (1986) idea of inequality as an aggregation of individuals' complaints when an individual is compared with *equally deserving* counterparts. Then, we adopt this notion of complaints and incorporate it into Roemer's (1998) framework by comparing outcomes at the same level of *accountable effort*.[1] Using Roemer's framework to analyze identity-based inequalities of opportunities is appropriate because its underlying philosophy overlaps with the stratification economics approach. Thus, we extend the inequality of opportunity approach and adapt it to the stratification-based approach. Indeed, in a stratified economy, circumstances at birth also include an identity marker that splits individuals into dominant and

[1]It is important to note that the framework we are proposing does not focus on individual outcomes but rather on the conditional distribution of outcomes linked with an individual's initial circumstances, including her identity.

marginalized groups. We, therefore, assess which part of inequality of opportunity is due to social class at birth and which part is due to identity-based stratification.

To estimate the inequalities of opportunities, we model the conditional distribution of outcomes by exploiting a distribution regression approach (see Chernozhukov, Fernandez-Vál, and Melly, 2013). Our estimation approach, which consists of a model of the conditional income distribution, is related to Pistolesi's (2009) approach; however, it is different in two respects. First, we estimate the conditional distribution of income using a distribution regression approach, which is more flexible than Pistolesi's (2009) Cox proportional hazard model. Second, we formulate the inequality of opportunity measurement framework and propose an approach compatible with the sets of inequality of opportunity indices that are functionals of the conditional distribution of income. Pistolesi (2009) uses the standard income inequality indices available in the literature.

Our estimation approach also relates to Brunori, Palmisano, and Peragine (2019) and Brunori, Ferreira, and Peragine (2021), who focus on identifying the circumstances that explain these inequalities of opportunities. Nevertheless, their estimation approach is different from the one we are adopting in this paper. In their paper, the authors compare conditional distributions of outcomes using random forest classification models, which is ideal for contexts where there are binary circumstances variables only. In this paper, by using a distribution regression modelling approach, we can exploit data that combines continuous and count circumstances variables such as the parent's income when the person was a child or the individual's age.

To substantiate the proposed approach's applicability, we provide an empirical application using data from the Egyptian Labor Market Panel Surveys for 1998, 2006, 2012, and 2018 (OAMDI, 2019). We study the changes in inequality of opportunity and the change in the contributions of gender stratification and social class at birth in Egypt from 1998 to

2018. Our results indicate a decrease in inequality of opportunity in Egypt between 2006 and 2012. During that same period, inequality of opportunity due to gender stratification decreased. Surprisingly, this decrease was not persistent as we observed an increase in inequality of opportunity due to gender stratification after 2012. However, the trends in the inequality of opportunities related to social class at birth between 2012-2018 and 2006-2012 were consistent. Combining the results from the gender stratification effects and the social class at birth effects suggests that equality of opportunity for the dominant group (men) has improved at the expense of equality of opportunity for the marginalized group (women) between 2012 and 2018.

The remainder of the paper runs as follows. Section 2 presents the measurement framework. Section 3 offers the dominance conditions for identifying robust orderings of distributions in regards to inequality of opportunity, inequality of opportunity due to identity-based stratification, and inequality of opportunity due to social class at birth. To apply these dominance conditions to survey data, Section 4 presents the estimation and statistical inference approach. Section 5 offers an application of the measurement approach using Egyptian data. Finally, Section 6 presents a brief conclusion and some directions for future research.

2 Measurement framework

The inequality of opportunity approach splits inequalities into two broad categories: inequalities due to individual's responsible decisions (i.e., accountable effort) and inequalities due to the birth lottery (i.e., initial circumstances at birth). Roemer (1998) shows that if one assumes that the distribution of residual luck[2] is independent of initial circumstances, then equality of opportunity translates into a condition on the quantile function. More specifically, Roemer's condition stipulates that the quantile function of an outcome conditional on a given vector of initial circumstances should be the same for all potential values for this

[2]The residual luck is the portion of luck not determined by the birth lottery.

vector of initial circumstances. Also, Roemer (1998) explains that under this assumption on residual luck, we can reduce these inequalities of opportunity by focusing on interventions that shift the lower contour of the set of conditional quantile functions.

In this paper, we adopt Roemer's view of inequality of opportunity and propose a new measurement framework based on Temkin's (1986) definition of inequality as the sum of complaints and the concept of *equally deserving individuals* (Cowell and Ebert, 2004). We think Temkin's concept of *equally deserving individuals* fits nicely in an equality of opportunity framework if one is willing to consider that individuals with the same level of accountable effort are *equally deserving*. From this perspective, we define the complaint as resulting from differences in the reward associated with each level of accountable effort.

As in Temkin (1986), when assessing the extent of a complaint, we specify a reference point. There are three possible reference points: (1) the average member, (2) the best-off member, and (3) all other people who are better off. This paper uses the *best-off equally deserving individual* as a reference point because it is consistent with Roemer's view of reducing inequalities of opportunity. Specifically, we define the reference point for a complaint as the point of the upper contour of the set of all conditional quantile functions at the same level of accountable effort. The upper contour is the function that gives the highest outcome possible for each level of accountable effort. Based on the proposed approach and choice of reference point, perfect equality of opportunity is reached when the lower contour is moved until it is identical to the upper contour.

An important particularity of the inequality of opportunity framework resides in that inequality is not assessed based on the observed individual outcome but on the conditional cumulative distribution of this outcome given the individual's initial conditions. In this context, Roemer (1998) shows that an appropriate index of accountable effort is the value of this conditional cumulative distribution. Its inverse function, the conditional quantile function, then associates a level of outcome with each level of accountable effort. In this sense, the conditional quantile function represents the opportunity set of individuals with a particular combination of initial circumstances.

2.1 Notation and definitions

To model this framework mathematically, we consider a society with a distribution of types $t \in \mathscr{T} \in \mathfrak{R}^{J+2}$ linked to individual initial circumstances, i.e., the aspects of the individual's environments that are beyond her control. These circumstances could include childhood and family environment. We assume that this society is also composed of two identity groups, a dominant group, D, and a marginalized group, M.[3] The definition of the group identity, $g \in \mathscr{G} := \{D, M\}$, is often determined by the society's specific institutional context and may be based on race, language, religion, or gender. In addition to their identity group, individuals differ in age and social class at birth. Let $a \in \mathscr{A}$ denote age. We group ages in different age cohorts, $c \in \mathscr{C}$, where $\mathscr{C}$ is a partition of $\mathscr{A}$ or $\mathscr{A}$ itself. Next, we consider social class as a marker for all the other circumstances at the birth $x \in \mathscr{X} \subseteq \mathfrak{R}^J$. These circumstances may include parents' characteristics during childhood (e.g., parent's education, employment statuses, earnings during childhood, and occupation).

For each individual, we observe a social outcome. In this paper, we will be focus on income, y, nevertheless, the analyst can consider any other social outcome such as education (Ferreira and Gignoux, 2014) or health (Davillas and Jones, 2020). Following Roemer (1998), we assume that income, $y \in \mathscr{Y} \in \mathfrak{R}_+$, has a production function $\phi(\cdot)$. The production function, $\phi(e_R, a, x, g, \ell)$, depends on a measure of raw effort, e_R, the age cohort of the individual, the social class at birth, the identity group, and the residual luck, ℓ (i.e., luck not determined by the birth lottery). We also assume that $\phi(\cdot)$ is strictly monotone

[3] In this paper, we keep the stratification of the economy into two groups for ease of exposition. All methods and results presented in the paper can be generalized to multiple groups and even account for the intersectionality of stratification.

increasing in the raw level of effort, e_R. This raw effort variable may include, among other things, years of schooling, type of training, and labor supply. Roemer (1998) explains that an individual's capacity to produce a given level of raw effort may also depend on birth circumstances. For instance, a parent with a higher education level can offer more support to her children in their learning activities. A wealthier parent can also pay for private tutoring, which poor parents cannot afford. For this reason, Roemer suggests that we should purge the impact of circumstances at birth from the level of raw effort. To do so, he defines the concept of accountable effort, e, as the individual's rank in the distribution of effort of her type. Suppose an individual is of type $t = (a, x, g)$. In that case, her accountable effort is $e = G_{E_R|A,X,G}(e_R|a, x, g)$, where $G_{E_R|A,X,G}(e_R|a, x, g)$ represents the cumulative distribution of raw effort conditional on $A = a$, $X = x$, and $G = g$. Given that we cannot fully observe accountable effort, it is difficult to account for it without additional assumptions. Nevertheless, suppose we assume that $\ell \perp\!\!\!\perp A, X, G$, i.e., residual luck is statistically independent of types. In that case, the strict increasing monotonicity of $\phi(\cdot)$ in e_R implies that an individual will have the same rank in $G_{E_R|A,X,G}(e_R|a, x, g)$ as in $F_{Y|A,X,G}(y|a, x, g)$. The $F_{Y|A,X,G}(y|a, x, g)$ is the distribution of the social outcome conditional to type $t = (a, x, g)$. This strict monotonicity assumption imposed on $\phi(e_R, c, x, g, \ell)$ is a key identification assumption of our model. This assumption allows us to overcome the need for a specific model of $G_{E_R|A,X,G}(e_R|a, x, g)$ because the ranks in the distribution of raw effort will be mapped in the distribution of the social outcome. Thus, the only object that needs to be estimated is $F_{Y|A,X,G}(y|a, x, g)$ and/or its inverse function $Q_{e|A,X,G}(e|a, x, g)$. It is essential to mention that while we are naturally inclined to consider an individual's level of accountable effort as the main focus in this framework, empirically, it is impossible to do so. Specifically, we cannot identify an individual's level of accountable effort separately because the individual's i outcome, y_i, is a function of two unobserved factors: the level of raw effort, e_{Ri}, and the unobserved realization of residual luck, ℓ_i. These two unobserved factors cannot be disentangled from each other because they are both unobserved. Thus, while we cannot identify the individual's level of accountable effort separately, we can estimate the mathematical objects $F_{Y|A,X,G}(y|a,x,g)$ and $Q_{e|A,X,G}(e|a,x,g)$ and conduct an inequality of opportunity analysis.

Roemer (1998) argues that perfect equality of opportunity requires the conditional quantile functions to be the same for all types at birth. Nevertheless, for some social outcomes such as wealth, income, wage, and health, differences between people of different age cohorts may be acceptable because these outcomes vary with a life cycle for the same level of accountable effort. Given that this paper does not focus on inequities across cohorts, we will condition on these differences. In this case, measuring inequality of opportunity requires that the conditional quantile functions $Q_{Y|A,X,G}(e|a,x,g) = \inf\left\{ y \in \mathscr{Y} \,|\, F_{Y|A,X,G}(y|a,x,g) \geq e\right\}$ be the same for all types at birth for all $(a,x,g) \in \{c\} \times \mathscr{X} \times \{D,M\}$, i.e. all social classes and all identity groups within a same age cohort c. It is important to note that we may relax the assumption if the objective is to measure the inequities across cohorts. In this case, one may also conceive outcomes (such as access to a clean environment) for which differences between age cohorts are not acceptable. In such cases, one should redefine $\mathscr{C} := \mathscr{A}$.[4]

2.2 Inequality of opportunity as a complaint

Let us assume that the outcome of interest is income. Then in a canonical income inequality measurement framework, the individual's outcome (y_i) is a function of the effort level chosen by this individual (e_{Ri}), her age (a_i), her social class at birth (x_i), her group identity (g_i) and residual luck (ℓ_i) befalling her. From an inequality of opportunity framework perspective, the analyst's main object of interest is the opportunities offered to individuals. These opportunities are reflected in the distribution of income conditional on initial conditions

[4]This is the reason that we assumed that $\mathscr{C}$ could be $\mathscr{A}$ or a partition of $\mathscr{A}$. Since a set cannot be a partition of itself, we had to include this possibility to allow for the analysis in such situations.

(a, x, g), for all possible realization of residual luck, ℓ. In order words, from an inequality of opportunity perspective, the mathematical objects of interest are

$$F_{Y|A,X,G}(y|a,x,g) = \int_{-\infty}^{\infty} F_{Y|A,X,G,L}(y|a,x,g,\ell)\mathrm{d}F_L(\ell), \tag{1}$$

and

$$Q_{e|A,X,G}(e|a,x,g) = \inf\{y \in \mathfrak{R}_+ | e \leq F_{Y|A,X,G}(y|a,x,g)\}. \tag{2}$$

For each possible level of accountable effort, the complaint should be defined for each combination of initial conditions (a, x, g). This complaint is defined as the relative difference between the expected outcome at accountable effort level e conditional on the initial conditions (a, x, g) and the reference expected outcome associated with an accountable effort e for age cohort $c = [a]$, where $[a]$ is the cell c of the partition $\mathscr{C}$ such that $a \in c$. In the context of this paper, the reference expected outcome represents the expected outcome had the opportunities been at the same level as the ones generating the best outcome at this level of accountable effort in this age cohort (i.e., the reference point). We define a function that represents this reference point for all accountable effort levels $e \in [0, 1]$ in age cohort c as follows:

$$\rho(e, c) = \max_{a,x,g \in \{c\} \times \mathscr{X} \times \{D,M\}} Q_{Y|A,X,G}(e|a,x,g), \qquad \forall e \in [0,1]. \tag{3}$$

In other terms, $\rho(e, c)$ represents the upper contour of all possible quantile functions within age cohort c.

Figure 1 represents the idea of this upper contour function. In this figure, we consider a society for which only four potential combinations of initial conditions for individuals in age cohort c exist. Each conditional quantile function, $Q_{e|A,X,G}(e|a_k, x_k, g_k)$, $k \in \{1, 2, 3, 4\}$ is depicted using a different color. The upper contour, $\rho(e, c)$, is depicted with a dotted blue line that takes the maximum value of the outcome of interest for each level of accountable effort. In this particular cases, $\rho(e,c) = Q_{e|A,X,G}(e|a_1,x_1,g_1)$ for $e \in [0,e_1]$ and $e \in [e_2,1]$ and $\rho(e,c) = Q_{e|A,X,G}(e|a_2,x_2,g_2)$ for $e \in [e_1,e_2]$.

Having defined the upper contour function, we use it in Figure 2 to illustrate the concept of a complaint at a level of accountable effort e_1. The higher curve represents the upper contour of all possible quantile functions within the age cohort $[a]$. The lower curve represents the quantile function conditional to (a,x,g). The absolute loss at a level of accountable effort e_1 is $\rho(e_1,[a]) - Q_{Y|A,X,G}(e_1|a,x,g)$. This concept of absolute loss can be used to define $\kappa(e,a,x,g)$, the complaint of a person of type (a,x,g) at a level of accountable effort e, as the proportion of income lost due to inequality of opportunities:

$$\kappa(e,a,x,g) = \frac{\rho(e,[a]) - Q_{Y|A,X,G}(e|a,x,g)}{\rho(e,[a])}. \tag{4}$$

The overall complaint associated with the type (a,x,g) is a socially weighted sum of complaints at all effort levels:

$$\widetilde{\kappa}(a,x,g) = \int_0^1 \omega(e)\kappa(e,a,x,g)\mathrm{d}e, \tag{5}$$

where $\omega(e)$ is a social weight function for complaints at a given accountable effort level e, we assume that $\omega(e) \geq 0$ for all $e \in [0,1]$ and that $\int_0^1 \omega(e)\mathrm{d}e = 1$ (for $\widetilde{\kappa}(a,x,g)$ to be socially weighted average). An example of such social weight function is the case in which $\omega(e) = 1$ for all $e \in [0,1]$. In this case, $\widetilde{\kappa}(a,x,g)$ is the average complaint of type (a,x,g) or the average proportion of income lost due to unequal opportunities. In the empirical application in Section 5, the only mathematical objects that are estimated at the individual level are the expected complaint function $\kappa(e,a_i,x_i,g_i)$ (for all $e \in [0,1]$) and the expected overall complaint $\mathrm{E}[\widetilde{\kappa}(a_i,x_i,g_i)]$.

Let us define the effort-dependent inequality of opportunity index as an average of these complaints over the distribution of types and effort

$$I(F_{Y,A,X,G}) = \mathrm{E}[\widetilde{\kappa}(a,x,g)] = \sum_{g \in \{D,M\}} \Pr[G=g] \int_{\mathscr{X}} \widetilde{\kappa}(a,x,g)\mathrm{d}F_{X|A,G}(x|a,g). \tag{6}$$

One interesting property of the effort-dependent inequality of opportunity indices is that they are subgroup-perfectly decomposable. This perfect decomposability means that if one defines

$$I_g(F_{Y,A,X,G}) \;=\; \mathrm{E}[\widetilde{\kappa}(a,x,g)|G=g] = \int_{\mathscr{X}} \widetilde{\kappa}(a,x,g)\mathrm{d}F_{X|A,G}(x|a,g), \tag{7}$$

as the effort-dependent inequality of opportunity index of group g, then the overall index is the sum of subgroup indices weighted by their population shares:

$$I(F_{Y,A,X,G}) = \sum_{g\in\{D,M\}} \Pr[G=g] I_g(F_{Y,A,X,G}). \tag{8}$$

Given that our main interest is to capture which part of the inequality of opportunity is attributable to stratification and which part of inequality is associated with social class at birth, we decompose the complaints of the marginalized group as follows:

$$\kappa(e,a,x,M) = \frac{\left[\rho(e,[a]) - Q_{Y|A,X,G}(e|a,x,D)\right] + \left[Q_{Y|A,X,G}(e|a,x,D) - Q_{Y|A,X,G}(e|a,x,M)\right]}{\rho(e,[a])}, \tag{9}$$

where $Q_{Y|A,X,G}(e|a,x,D)$ is the income of a person of the same age from the dominant group at the same level of accountable effort and the same initial conditions, except group identity. The overall complaint is decomposed into a stratification component and another due to the social class at birth. Figure 3 illustrates this decomposition. The blue double-arrow segment indicates the absolute loss due to stratification at accountable effort level e_1. We define the stratification complaint function for all effort levels $e \in [0,1]$ as this loss in relative term, i.e.

$$\kappa^{Strat}(e,a,x,M) = \frac{Q_{Y|A,X,G}(e|a,x,D) - Q_{Y|A,X,G}(e|a,x,M)}{\rho(e,[a])}. \tag{10}$$

The green double-arrow segment in Figure 3 indicates the absolute loss due to social class at birth. The social class complaint function is defined for all effort levels as this loss in relative terms, i.e.

$$\kappa^{Class}(e,a,x,M) = \frac{\rho(e,[a]) - Q_{Y|A,X,G}(e|a,x,D)}{\rho(e,[a])}. \tag{11}$$

In this context, we can define inequality of opportunity that is due to stratification as

$$I^{Strat}(F_{Y,A,X,G}) = \Pr[G = M] \int_{\mathscr{X}} \int_0^1 \omega(e)\kappa^{Strat}(e,a,x,M)\mathrm{d}e\mathrm{d}F_{A,X|G}(a,x|M). \quad (12)$$

Similarly, we can define the inequality of opportunity that is due to social class at birth as:

$$\begin{aligned} I^{Class}(F_{Y,A,X,G}) &= \Pr[G = D] \int_{\mathscr{X}} \int_0^1 \omega(e)\kappa(e,a,x,D)\mathrm{d}e\mathrm{d}F_{X|A,G}(x|a,D) \\ &+ \Pr[G = M] \int_{\mathscr{X}} \int_0^1 \omega(e)\kappa^{Class}(e,a,x,M)\mathrm{d}e\mathrm{d}F_{X|A,G}(x|a,M). \quad (13) \end{aligned}$$

The subgroup decomposability of $I(F_{Y,A,X,G})$ and the linearity of $\kappa(e,a,x,g)$ implies that

$$I(F_{Y,A,X,G}) = I^{Strat}(F_{Y,A,X,G}) + I^{Class}(F_{Y,A,X,G}). \quad (14)$$

In other words, total inequality of opportunity in a population is the sum of inequality of opportunity due to social class at birth and inequality of opportunity due to stratification based on identity markers.

If one imposes a specific form on the social weight function, using these indices allows for a complete ordering of all joint distributions $F_{Y,A,X,G}$ in terms of inequality of opportunity, inequality of opportunity due to stratification, and inequality of opportunity due to social classes.

3 Identifying robust orderings in terms of inequality of opportunity

When using a specific index belonging to the class of effort-dependent inequality of opportunity indices, it is possible to have a complete ordering of distributions. However, this ordering will be contingent on the specific mathematical form of the index, i.e., the structure imposed on the social weight function $\omega(e)$. Nevertheless, it is always possible to test whether some rankings are robust to all potential functional forms the analyst may impose on the social weight function. This section aims to lay down the conditions for identifying such robust orderings.

To identify robust orderings of inequality of opportunity the analyst can use a dominance approach analogous to the one used in income inequality. Indeed, in the income inequality literature, the non intersection of the Lorenz curves can be used to identify robust orderings of income inequality. In such a case, the distribution with a Lorenz curve closer to the 45-degree line has the lowest inequality. This result holds for any inequality index.

In the inequality of opportunity framework, the object of interest is not the individual's income but the complaint function, $\kappa(e,a,x,g)$ associated with given initial circumstances. For this reason, in what follows we define new curves based on these complaint functions.

3.1 Inequality of opportunity orderings

First, we define is the complaint incidence curve. For each level of accountable effort, this curve displays the expected complaint in the population. The formal mathematical definition of this curve is

$$CI(e,F_{Y,A,X,G}) = \sum_{g\in\{D,M\}} \Pr[G=g] \int_{\mathscr{X}} \kappa(e,a,x,g)\mathrm{d}F_{X|A,G}(x|a,g). \tag{15}$$

Analogously, one can define the stratification and the social class complaint incidence curves as:

$$CI^{Strat}(e,F_{Y,A,X,G}) = \Pr[G=M] \int_{\mathscr{X}} \kappa^{Strat}(e,a,x,M)\mathrm{d}F_{X|A,G}(x|a,M), \tag{16}$$

and

$$\begin{aligned} CI^{Class}(e,F_{Y,A,X,G}) &= \Pr[G=D] \int_{\mathscr{X}} \kappa(e,a,x,D)\mathrm{d}F_{X|A,G}(x|a,D) \\ &\quad +\Pr[G=M] \int_{\mathscr{X}} \kappa^{Class}(e,a,x,M)\mathrm{d}F_{X|A,G}(x|a,M). \end{aligned} \tag{17}$$

The curves described in 15, 16, and 17 provide a graphical representation of the distribution of expected complaint for all accountable effort levels. In addition, these curves can be used to identify robust rankings of inequality of opportunity.

Let us denote by Ω the set of all effort-dependent inequality of opportunity indices defined by equation (6). This class of indices is formally defined as

$$\Omega := \left\{ I(\cdot) \;\middle|\; \omega(e) \geq 0 \;\forall e \in [0,1], \text{ and } \int_0^1 \omega(e)\mathrm{d}e = 1 \right\}. \tag{18}$$

In our framework, robust rankings of inequality of opportunity are analogous to robust rankings of income inequality. For income inequality, it is well known that a robust ranking is identified when two Lorenz curves do not intersect. In our framework, we identify robust rankings of inequality of opportunity for all indices $I(\cdot) \in \Omega$ when complaint incidence curves do not intersect.

Let $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) = I(F^1_{Y,A,X,G}) - I(F^0_{Y,A,X,G})$ denotes the difference in inequality of opportunity between distribution $F^0_{Y,A,X,G}$ and $F^1_{Y,A,X,G}$.

Theorem 1. $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ *for all indices* $I(\cdot) \in \Omega$ *if and only if*

$$CI(e, F^1_{Y,A,X,G}) - CI(e, F^0_{Y,A,X,G}) \leq 0 \;\forall e \in [0,1].$$

If we define analogously $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G})$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G})$ and use CI^{Strat} and CI^{Class} similar results hold. If the conditions in Theorem 1 do not allow for a ranking between two distributions, it is possible to consider subsets of indices. Thus an analyst can impose an additional structure on the ethical principles and check if a robust ordering exists for these subsets of the indices $I(\cdot) \in \Omega$. In this paper, we propose to consider two alternative normative views: a pro-poor normative view and a meritocratic normative view.

3.2 Pro-poor inequality of opportunity orderings

When an analyst has a pro-poor view of inequality of opportunity, she puts a higher weight on complaints associated with low levels of accountable effort. Thus, pro-poorness implies a non-increasing $\omega(e)$ function. A well-known example of such a function is $\omega(e) = \nu(1 -$

$e)^{\nu-1}$, which represents the social weights of the rank-dependent social welfare function associated with the extended class of Gini indices. The parameter ν is an inequality aversion parameter in the Gini social welfare function. In our framework, since the ranking variable is accountable effort, such a parameter would be a parameter of pro-poor inclination. An infinity of other potential functional forms that would satisfy a pro-poor view of the social weight function. Let us denote by Ω_P the set of all pro-poor effort-dependent inequality of opportunity indices. This class of indices is formally defined as follows:

$$\Omega_P := \left\{ I(\cdot) \in \Omega \;\middle|\; \frac{\mathrm{d}\omega(e)}{\mathrm{d}e} \leq 0 \;\forall e \in [0,1], \text{ and } \omega(1) = 0 \right\}. \tag{19}$$

Drawing from the tools developed in the literature on progressive income taxation and the literature on socioeconomic health inequality, we use the generalized concentration curve (see Schechtman, Shelef, Yitzhaki, and Zitikis, 2008; Khaled, Makdissi, and Yazbeck, 2018).[5] A generalized concentration curve is a graphical tool that can be used to identify robust orderings of inequality of opportunities for all pro-poor effort-dependent inequality of opportunity indices. In the context of this paper, we refer to these curves as the pro-poor complaint concentration curves. The formal mathematical definition of the pro-poor complaint concentration curves is as follows:

$$CC_p(e, F_{Y,A,X,G}) = \int_0^e CI(s, F_{Y,A,X,G})\mathrm{d}s, \tag{20}$$

$$CC_p^{Strat}(e, F_{Y,A,X,G}) = \int_0^e CI^{Strat}(s, F_{Y,A,X,G})\mathrm{d}s, \tag{21}$$

and

$$CC_p^{Class}(e, F_{Y,A,X,G}) = \int_0^e CI^{Class}(s, F_{Y,A,X,G})\mathrm{d}s. \tag{22}$$

For a given level of accountable effort, e, the pro-poor complaint concentration curve depicts the average complaint in the distribution for those whose effort levels are lower than e. When

[5] Schechtman, Shelef, Yitzhaki, and Zitikis (2008) use the term absolute concentration curve instead of generalized concentration curve.

$e = 1$, the pro-poor complaint concentration curve is equal to the average complaint level in the distribution.

The analyst can use these curves to identify robust rankings of distributions for all pro-poor inequality of opportunity indices belonging to Ω_P.

Theorem 2. $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ *for all indices* $I(\cdot) \in \Omega_P$ *if and only if*

$$CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G}) \leq 0 \;\; \forall e \in [0,1].$$

We can derive similar results for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G})$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G})$ using CC_p^{Strat} and CC_p^{Class}.

3.3 Meritocratic inequality of opportunity orderings

When an analyst has a meritocratic view of inequality of opportunity, she puts a higher weight on complaints associated with high levels of accountable effort. The meritocratic view implies a non-decreasing $\omega(e)$ function. One example of such a function would be $\omega(e) = \alpha e^{\alpha-1}$, where the parameter α is a parameter of meritocratic inclination. Nevertheless, one may select another functional form as there is an infinity of functional forms for $\omega(e)$ that would satisfy a meritocratic view. Let us denote by Ω_M the set of all meritocratic effort-dependent inequality of opportunity indices defined by equation (6). The mathematical definition of this class of indices is

$$\Omega_M := \left\{ I(\cdot) \in \Omega \;\middle|\; \frac{\mathrm{d}\omega(e)}{\mathrm{d}e} \geq 0 \;\; \forall e \in [0,1], \text{ and } \omega(0) = 0 \right\}. \tag{23}$$

How can we reconcile a meritocratic normative view with the well-known aversion to inequality concept? To do so, one needs to remember that the complaint function, $\kappa(e, a, x, g)$, assigns a complaint with each possible level of accountable effort, $e \in [0,1]$, a person could have exerted given the initial circumstances (a, x, g). This complaint function captures inequality aversion. Thus, if one accepts the assumption that, from the individual's perspective, making an effort is costly and that the cost of effort increases with the effort level, it would be natural to put more weight on complaints associated with a higher level of effort. This ethical principle, combined with the identity stratification component, is also linked with an aversion to glass ceilings (see Bjerk, 2008).

Similarly to pro-poor complaint concentration curves, one can define the meritocratic complaint concentration curves as follows:

$$CC_m(e, F_{Y,A,X,G}) = \int_e^1 CI(s, F_{Y,A,X,G})\mathrm{d}s, \tag{24}$$

$$CC_m^{Strat}(e, F_{Y,A,X,G}) = \int_e^1 CI^{Strat}(s, F_{Y,A,X,G})\mathrm{d}s, \tag{25}$$

and

$$CC_m^{Class}(e, F_{Y,A,X,G}) = \int_e^1 CI^{Class}(s, F_{Y,A,X,G})\mathrm{d}s. \tag{26}$$

For a given level of accountable effort e, the meritocratic complaint concentration curve depicts the average complaint in the distribution if the only complaints considered were those of effort levels higher than e. When $e = 0$, the meritocratic complaint concentration curve equals the average complaint level in the distribution. In other terms, $CC_m(0, F_{Y,A,X,G}) = CC_p(1, F_{Y,A,X,G})$.

As in the case of the pro-poor complaint concentration curves, these curves can be used to identify rankings that are robust to all meritocratic inequality of opportunity indices belonging to Ω_M.

Theorem 3. $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ *for all indices* $I(\cdot) \in \Omega_M$ *if and only if*

$$CC_m(e, F^1_{Y,A,X,G}) - CC_m(e, F^0_{Y,A,X,G}) \leq 0 \ \forall e \in [0,1].$$

We can derive similar results for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G})$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G})$ using CC_m^{Strat} and CC_m^{Class}.

4 Estimation and inference

In previous sections, we introduced new inequality of opportunity measurement tools and dominance conditions for the robust ordering of inequality of opportunity and the impact of stratification and social class at birth on this inequality of opportunity. However, we assumed that the joint distributions, $F_{Y,A,X,G}$, were known to researchers, and we did not discuss uncertainty. This section will provide details regarding this paper's estimation and inference approach. It is important to note that while the measurement framework, the graphical tools, and the dominance conditions proposed in this paper are new, the estimation and the inference methodology we use are already available in the econometric literature.

Assume that we have two data sets of n_0 and n_1 observations with n_{0D}, n_{1D}, n_{0M}, and n_{1M} observations from the dominant and marginalized groups. All graphical tools and dominance conditions introduced in the previous two sections do not focus on the marginal cumulative distribution of a specific outcome y but instead on the conditional distribution that is associated with initial circumstances, $F_{Y|A,X,G}(y|a,x,g)$. To estimate the graphical tools and conduct the dominance tests for conditions provided in Theorems 1, 2, and 3, the analyst needs an econometric framework to estimate this conditional distribution. Chernozhukov, Fernandez–Vál, and Melly (2013) offer an appropriate framework for this estimation, and Schechtman, Shelef, Yitzhaki, and Zitikis's (2008) framework matches the testing object of interest in this paper.

4.1 Estimation

In the first estimation step, which consists of estimating the conditional distribution, we follow Chernozhukov, Fernandez-Vál, and Melly (2013) and estimate for each group $g \in \{D, M\}$, a model of the conditional distribution of outcome on a grid of pre-specified thresholds, $\{z_1, z_2, \dots, z_K\}$.

$$\widehat{F}_{Y|A,X,G=g}(z_k|a,x,g) = \Lambda\left(P(a,x,g)'\widehat{\beta}(z,g)\right) \quad \forall z \in \{z_1, z_2, \dots, z_n\}, \tag{27}$$

where

$$\widehat{\beta}(z_k, g) = \underset{b\in\Re^{d_p}}{\operatorname{argmax}} \sum_{i=1}^{n_g} \{\mathbb{1}[y_i \le z_k] \ln[\Lambda(P(a_i,x_i,g_i)'b)] + \mathbb{1}[y_i > z_k] \ln[1 - \Lambda(P(a_i,x_i,g_i)'b)]\} \tag{28}$$

and Λ is some link function. Some examples of link functions that could be considered are the probit, logit, linear, log-log, and Gosset functions. $P(\cdot)$ is some vector transformation of (a,x,g), and $d_p = dim\ P(a,x,g)$.[6] The advantage of Chernozhukov, Fernandez-Vál, and Melly's (2013) approach is that it is flexible enough to allow for the estimation of mixed discrete/continuous distributions. This is particularly useful in situations where the distribution has a mass at 0, such as outcomes for non-participants in the labor market.

In the second estimation step, we use the estimated models and predict the value of the conditional quantile function for each observation i based on a grid of pre-specified accountable effort levels $\{e_1, e_2, \dots, e_L\}$:

$$\widehat{Q}_{Y|A,X,G}(e_\ell, a_i, x_i, g_i) = \widehat{F}^{-1r}_{Y|A,X,G}(e_\ell, a_i, x_i, g_i),\ \forall e_\ell \in \{e_1, e_2, \dots, e_L\}, \tag{29}$$

where $\widehat{F}^{-1r}_{Y|A,X,G}$ is the rearrangement of $\widehat{F}^{-1}_{Y|A,X,G}$ if $\widehat{F}^{-1}_{Y|A,X,G}$ is non-monotone (for details, see Chernozhukov, Fernandez-Vál, and Galichon, 2010).[7] For observation belonging to the marginalized group (i.e. if $g_i = M$), we also need a predicted value of the counterfactual conditional quantile function for each observation:

$$\widehat{Q}_{Y|A,X,G}(e_\ell, a_i, x_i, D) = \widehat{F}^{-1r}_{Y|A,X,G}(e_\ell, a_i, x_i, D),\ \forall e_\ell \in \{e_1, e_2, \dots, e_L\}. \tag{30}$$

In the third step, we estimate for each age cohort $c \in \mathscr{C}$ the maximum value of the conditional quantile functions on the grid of pre-specified accountable effort levels $\{e_1, e_2, \dots, e_L\}$.

[6] P(a,x,g) can be (a,x,g) itself if the expression is linear.

[7] Chernozhukov, Fernandez-Vál, and Galichon (2010) explains that predicted values of $F_{Y|A,X,G}$ may be non-monotonic for a given combinations (a,x,g). In these cases, a rearrangement of these values is required.

Formally,

$$\widehat{\rho}_i(e_\ell, c) = max_i \left[\widehat{Q}_{Y|A,X,G}(e_\ell, a_i, x_i, g_i) \cdot \mathbb{1}(a_i \in c)\right], \ \forall e_\ell \in \{e_1, e_2, \dots, e_L\}, \text{ and } \forall c \in \mathscr{C}. \tag{31}$$

Finally, the fourth step consists of using (29) and (31) to predict on the grid of pre-specified accountable effort levels $\{e_1, e_2, \dots, e_L\}$ a value of the complaint function for each observation i:

$$\widehat{\kappa}(e_\ell, a_i, x_i, g_i) = \frac{\widehat{\rho}(e_\ell, [a_i]) - \widehat{Q}_{Y|A,X,G}(e_\ell|a_i, x_i, g_i)}{\widehat{\rho}(e_\ell, [a_i])}, \ \forall e_\ell \in \{e_1, e_2, \dots, e_L\}. \tag{32}$$

For observations belonging to the marginalized group (i.e., if $g_i = M$), (29), (30), and (31) can be used to estimate the predicted values of the complaint due to stratification and the complaint due to social class at birth:

$$\widehat{\kappa}^{Strat}(e_\ell, a_i, x_i, g_i) = \frac{\widehat{Q}_{Y|A,X,G}(e_\ell|a_i, x_i, D) - \widehat{Q}_{Y|A,X,G}(e_\ell|a_i, x_i, g_i)}{\widehat{\rho}(e_\ell, [a_i])}, \ \forall e_\ell \in \{e_1, e_2, \dots, e_L\}, \tag{33}$$

and

$$\widehat{\kappa}^{Class}(e_\ell, a_i, x_i, g_i) = \frac{\widehat{\rho}(e_\ell, [a_i]) - \widehat{Q}_{Y|A,X,G}(e_\ell|a_i, x_i, D)}{\widehat{\rho}(e_\ell, [a_i])}, \ \forall e_\ell \in \{e_1, e_2, \dots, e_L\}. \tag{34}$$

The set of complaint incidence curves can then be estimated on each point $e_\ell \in \{e_1, e_2, \dots, e_L\}$ using the values obtained in (32), (33), and (34):

$$\widehat{CI}(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{N}\sum_{i=1}^{N} \widehat{\kappa}(e, a_i, x_i, g_i), \tag{35}$$

$$\widehat{CI}^{Strat}(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{N}\sum_{i=1}^{N} \widehat{\kappa}^{Strat}(e, a_i, x_i, g_i)\mathbb{1}[g_i = M], \tag{36}$$

$$\begin{aligned} \widehat{CI}^{Class}(e, \widehat{F}_{Y,A,X,G}) &= \frac{1}{N}\sum_{i=1}^{N} \widehat{\kappa}(e, a_i, x_i, g_i)\mathbb{1}[g_i = D] \\ &+ \frac{1}{N}\sum_{i=1}^{N} \widehat{\kappa}^{Class}(e, a_i, x_i, g_i)\mathbb{1}[g_i = M], \end{aligned} \tag{37}$$

We estimate the pro-poor and meritocratic complaint concentration curves by integrating the expressions in equations (35), (36), and (37) using a Riemann sum approximation (see Appendix B for details).

4.2 Testing for dominance conditions

Chernozhukov, Fernandez-Vál, and Melly (2013) prove the validity of the exchangeable bootstrap for the cumulative distribution model and its counterfactuals. In addition, they show the validity of the exchangeable bootstrap for smooth functionals of the cumulative distribution model and its counterfactuals. These smooth functionals include Kolmogorov-Smirnov type of statistics. This section adopts a testing procedure that builds on Schechtman, Shelef, Yitzhaki, and Zitikis (2008) and Khaled, Makdissi, and Yazbeck (2018). This testing procedure uses a directional version of a statistic akin to the Kolmogorov-Smirnov statistic.

Let us denote by $C(e)$ one of the curves defined in the preceding sections. $C(e)$ can be $CI(e)$, $CC_p(e)$, $CC_m(e)$, or one of their stratification or social class versions. Let $C_0(e)$ and $C_1(e)$ be the curve of two populations, 0 and 1. Assume that we have two i.i.d samples, S_0 and S_1, of size n_0 and n_1 from these two populations. We are interested in testing one of the dominance conditions in Theorems 1, 2, and 3. Formally, the null and alternative we are interested in are:

$$\begin{aligned} H_0 &: C_1(e) - C_0(e) \leq 0, \forall e \in [0,1] \\ H_1 &: C_1(e) - C_0(e) > 0, \text{ for some } p \end{aligned}$$

It is important to mention that in the above test, we are not trying to establish dominance by imposing a null of non-dominance. Instead, we impose a null of dominance and test if this null can be rejected. There are two reasons why we adopt this testing approach which may seem counterintuitive. First, in a similar context, Davidson and Duclos (2013) have shown that testing a null hypothesis of non-dominance requires strong evidence against the null, which may be challenging to obtain. Second, since the conditions in Theorems 1, 2, and 3 only require weak dominance, we follow the usual practice from the empirical literature on stochastic dominance and test for $H_0^1 : C_1(e) - C_0(e) \leq 0, \forall e \in [0,1]$ and for $H_0^2 : C_0(e) - C_1(e) \leq 0, \forall e \in [0,1]$. Table 1 displays the decision rules for the dominance tests. For a level of significance α, we will consider that we have strong evidence in favor of dominance if the p-values of one of the aforementioned nulls are larger or equal to α while the p-values of the other are strictly lower.

Let $\tau = \sup_e [C_1(e) - C_0(e)]$, it is straightforward to construct a KS type directional test statistic $\widehat{\tau}$ that is a non-parametric estimator of τ:

$$\widehat{\tau} = \sqrt{\frac{n_0 n_1}{n_0 + n_1}} \sup_e \left[\widehat{C}_1(e) - \widehat{C}_0(e)\right] \tag{38}$$

The asymptotic distribution of $\widehat{\tau}$ will be that of a functional of a two-dimensional Gaussian process that is very complicated to compute.[8] To overcome this computational issue, we rely on a recentered bootstrap procedure, as in Schechtman, Shelef, Yitzhaki, and Zitikis (2008). For a detailed description of the bootstrap procedure, please refer to Appendix C.

5 Empirical illustration: Gender stratification, social class, and inequality of opportunity in Egypt

5.1 Political Context

In this section, we analyze the evolution of inequality of opportunity in Egypt from 1998 to 2018. In 1998, Egypt has been under an authoritarian president, Hosni Mubarak, since 1981. The Mubarak years accelerated the transformation of the Egyptian economy that started under Anwar el-Sadat. Privatizations and financial liberalizations accelerated during that period. In the 1990s, these economic reforms favored a class of politically connected capitalists that grew very rich (see Diwan, Keefer, and Schiffbauer, 2019). During the *Arab Spring* in 2011, there were massive protests demanding more justice. These protests led to Mubarak's resignation in the same year. The demand for more justice is substantiated in the data from the World Value Survey. Indeed, the support for redistribution in Egypt grew

[8] Chernozhukov, Fernandez-Vál, and Melly (2013) refers to this issue as the Durbin problem.

from 22% in 2008 to 59% in 2012 (see El Rafhi and Volle, 2020). Not long after Moubarak's resignation (i.e., in June 2012), Mohamad Morsi, the leader of the Freedom Justice Party,[9] was elected president. In June 2013, there were protests again in Egypt. Protesters accused Morsi of pushing an Islamist agenda disregarding the secular opposition and the rule of law. In July 2013, there was a military coup removing Morsi from office. The following March, Abdel Fattah el-Sisi became president of Egypt, bringing back the country under an authoritarian regime.

Since the demand for more justice and equality was at the core of the demands during the Arab Spring, it is interesting to analyze the evolution of inequality. This broad popular perception of high-level inequality does not seem to be confirmed by empirical analysis based on household surveys. In Egypt, as in the rest of the Arab countries, studies based on household surveys indicated that the overall level of inequality in the country was mild and even declining (see Belhaj Hassine, 2011, 2015; Bibi and Nabli, 2009). Similarly, inequality of opportunity was declining in the period preceding the Arab spring (Assaad, Krafft, Roemer, and Salehi-Isfahani, 2018). Nevertheless, in their discussion, Assaad, Krafft, Roemer, and Salehi-Isfahani (2018) suggest that this result could be masking some undesirable changes within the distribution of income. Specifically, individuals from the middle class are moving closer to people from poor backgrounds widening the distance between cronies that are privileged by the regime who remain at high levels and the new poor class. Alverado, Assouad, and Piketty (2019) confirm this intuition. Their results indicate that the top 1%'s share of total income (19%) was equivalent to the share of the bottom 50% (18%) in 2015. These proportions remained stable throughout their study (1999 to 2015).[10]

Even if studies done using household surveys cannot capture this essential part of the

[9] The FJP is an Islamist party with strong links with the Muslim Brotherhood of Egypt.

[10] Assaad and Saleh (2018) have suggested that the Arab Spring may be explained by the rapid increase in intergenerational mobility in education in the Arab region that has failed to yield similar mobility in either income or social status in the region.

inequality story, it is still interesting to decompose the underlying trends in inequality of opportunity into components that are due to social class at birth and stratification. The Egyptian labor market has a low female labor market participation rate. This low female labour market participation is reflected in Table 2 where we report the ILO's estimates of (paid) labor force participation of individuals over 15. One potential reason behind this low participation is the presence of a clear stratification in gender roles in Egypt. Indeed, Assaad, Krafft, and Selwaness (2022) find that marriage has a negative impact on women's employment: it induces employed women to leave their position except for those with government jobs. Since most women eventually marry in Egypt (Assaad and Krafft, 2015), this gender norm strongly impacts women's labor market participation levels. However, it is worth mentioning that even if women do not participate in the paid labor market per se, many women still have a primary labor activity that is akin to employment, but this activity takes the form of being an *unpaid family worker*.

5.2 Data and Modeling Choice

In the empirical application, we analyze the dynamics of inequality of opportunity in Egypt between 1998 and 2018. We use the 1998, 2006, 2012, and 2018 rounds of the Egypt Labor Market Panel Survey (ELMPS).[11] The dynamics of inequality of opportunity in Egypt between 1998 and 2012 has been previously analyzed by Assaad, Krafft, Roemer, and Salehi-Isfahani's (2018) who use the Ferreira and Gignoux's (2011) inequality of opportunity measurement approach.

While our empirical application may seem similar to the one by Assaad, Krafft, Roemer, and Salehi-Isfahani's (2018), it remains different in two important aspects. First, given that we have an additional survey round in 2018, we can provide the post *Arab Spring* portrait. The massive *Arab Spring* protests clearly demanded more equality. Therefore it

[11] The Economic Research Forum for the Arab Countries, Iran, and Turkey (ERF) collaborated with the Egyptian Central Agency for Public Mobilization and Statistics (CAPMAS) to carry out the ELMPS.

is important to assess how the Egyptian economy has adapted to this increased demand for equality. Second, given that our model allows for the estimation of mixed distributions with some probability mass, it also allows modeling a cluster of observations at 0 which is very convenient for modeling women who are unpaid family workers. Thus, this modeling choice allows for the inclusion of women in our analysis which allows us to assess gender stratification's contribution to inequality of opportunity. The proportion of women having unpaid labor market activities exceeds that of men by far. Indeed, when women participate in other labor activities in Egypt, most often, they do as *unpaid family worker*. The ELMPS has an explicit question on the main labor activity of the individual that includes this category of *unpaid family worker*. Table 3 displays the proportion of individuals declaring *unpaid family worker* as their primary labor activity by gender. For all the years, we note that more than 50% of women who participate in the labour market have an income of zero. For this reason, labor economists studying the labor market in Egypt introduced the idea of an extended definition of employment that includes these unpaid employment activities (see Assaad and Kraft, 2015; Nazier and Ramadan, 2018). Our analysis will focus on inequality of opportunity conditional on being employed using this extended definition of employment.

This paper's measurement and estimation framework can be used to model labor market participation. While it is commonly thought that, modeling labor market participation is desirable, it is more appropriate for developed economies. In fact, unemployment in Egypt is primarily a privileged, educated, and new entrant phenomenon (Assaad, Krafft, Roemer, and Salehi-Isfahani, 2018). Therefore, the inclusion of non-participants with a zero labor income may distort the picture of inequality of opportunity for some developing countries who share similar unemployment phenomena. We make this analytical choice to avoid taking arbitrary decisions regarding exclusions and inclusions of non-labor market participants.

The subsample we use for our analysis is all participants in the age groups between

25 and 54 who are considered employed in the extended definition of the labor market.[12] This modeling choice includes those declaring being an *unpaid family worker* as their main labor activity. The outcome of interest is monthly wage earnings in 2018 PPP international dollars. Monthly wages are set to 0 for all individuals declaring *unpaid family worker* as their main labor activity (unless they report themselves some income). As a result, sample sizes are 4,754 for 1998, 7,420 for 2006, 9,374 for 2012, and 9,393 for 2018. Our analysis defines males as the dominant identity group (D) and females as the marginalized identity group (M). We define circumstances at birth with both parents' education level, the father's occupation when the respondent was 15, region of birth, and the age of the individual. Each parent's educational attainment is coded as (1) illiterate (2) reads and writes (3) basic (4) intermediate and above intermediate (upper secondary and two-year higher education programs), or (5) university (four-year higher education programs) and above. We follow Assaad, Kraft, Roemer, and Salehi-Isfahani (2018) and define a new variable into four basic types: parental education of (1) sum of 1–2, (2) sum of 3–5, (3) sum of 6–7, or (4) sum of 8–10. Father's occupation is defined as white-collar, blue-collar, or agricultural worker. If the father was not working or absent when the respondent was 15, the information is coded as blue-collar. Region of birth is defined as metropolitan, provincial urban, or provincial rural. Table 4 present the average wage earnings by partitioning the population by each of these covariates. We use the exact age in the estimation of distribution regression. However, given the size of the relatively small size of datasets, we use the following partition of the age set for the estimations of complaints: $\mathscr{C} := \{[25, 29], [30, 34], [35, 39], [40, 44], [45, 49], [50, 54]\}$.

[12]The ILO definition of prime-age workers is workers between the age of 25 and 54.

5.3 Results

5.3.1 Inequality of opportunity

In the first round of estimation, we estimate tests for Theorem 1, 2, and 3 dominance relations of inequality of opportunity between years of the survey. For these tests and all other estimations in this section, we use 999 bootstrap replications. Figure 4 displays the most important comparison over this period. It displays the complaint incidence curves of 2012 and 2006. From visual inspection of Figure 4, there seems to be a robust reduction of inequality of opportunity between these two years and the p-values in Table 5 confirm this interpretation. Table 5 also displays the p-values of all the other comparisons.[13] The results and p-values in Table 5 indicate that the most important change in inequality of opportunity occured between 2006 and 2012. Indeed, inequality of opportunity decreased between these two years. Table 6 presents the robust orderings identified based on the p-values in Table 5. There is less inequality of opportunity for all indices in Ω in 2012 than in 2006 (p-value < 0.01) and less inequality in 2012 than in 1998 (p-value < 0.05). If we restrict our focus on pro-poor indices in Ω_P or meritocratic indices in Ω_M, we have the same rankings for 2012 compared to 1998 with results being significant at the 1% level and an additional ranking for the inequality of opportunity. Specifically, inequality of opportunity is lower in 2018 than in 2006 and 1998 (p-value 0.01 level). Although there exist some specific indices that may well capture variation in inequality of opportunity between 1998 and 2006 or between 2012 and 2018, the fact that there are no robust rankings between these years means that there will always be a specific form of the social weight function $\omega(e)$ that will produce the opposite result. This decrease in inequality of opportunity is in line with the results obtained by Assaad, Krafft, Roemer, and Salehi-Isfahani's (2018).

[13] In this discussion, we will focus on dominance results at the 0.05 level or better (i.e., those marked by $**$, and $***$).

5.3.2 Inequality of opportunity decomposed

We decompose our main analysis and estimate tests for Theorems 1, 2 and 3 dominance conditions of inequality of opportunity that are due to stratification and social class at birth. Figure 5 displays the stratification pro-poor (upper left panel), the meritocratic (upper right panel) complaint concentration curves for 2012 and 2018. It also displays the stratification pro-poor (lower left panel) and meritocratic (lower right panel) complaint concentration curves for 2006 and 2012. Figure 6 displays the social class pro-poor (left panel) and the meritocratic (right panel) complaint concentration curves for 2012 and 2018. A visual inspection of Figure 5 indicates that, although inequality of opportunity that is associated with gender stratification is decreasing between 2006 and 2012, it is increasing between 2012 and 2018. This result is valid for all pro-poor and meritocratic indices. A visual inspection of Figure 6 shows a different pattern, it indicates that during the same 2012 to 2018 period, inequality of opportunity due to social class at birth was going down for all pro-poor and meritocratic indices. Table 7 displays the ranking associated with the dominance tests for all survey years considered in this study. It indicates no robust ranking if one considers all indices in Ω. However, if we focus on pro-poor indices in Ω_P or meritocratic indices in Ω_M, it confirms the interpretation of the visual inspection. Indeed, inequality of opportunity associated with gender stratification increased between 2012 and 2018 (p-value < 0.01). However, inequality of opportunity associated with gender stratification decreased between 2006 and 2012 (p-value < 0.05). Also, Table 7 confirms the conclusions derived from the visual inspection of Figure 6 above. Inequality of opportunity due to social class at birth decreased between 2012 and 2018 (p-value < 0.01) and between 2006 and 2012 (p-value < 0.05).

The results above indicate that the evolution of the overall inequality of opportunity associated with gender stratification and inequality of opportunity due to social class at birth are trending in the opposite direction between 2012 and 2018. Since the marginalized group's inequality of opportunity is affected by these two components, the net impact of this evolution is not clear. To get insights into whether this evolution impacted inequality of opportunity for women positively or negatively, we test Theorems 1, 2, and 3 to assess the inequality of opportunity for the dominant and marginalized groups separately (Table 8). Results for the dominant group (i.e., men) (seeTable 8) indicate that there is no robust ranking of inequality of opportunity for all indices in Ω. If we restrict the analysis to pro-poor and meritocratic inequality of opportunity indices in Ω_P and Ω_M, Table 8 indicates that there is less inequality of opportunity for men in 2018 than in 2012, 2006, and 1998 (p-value < 0.01 level). There is also less inequality of opportunity for men in 2012 compared to 2006 (p-value < 0.05) and 1998 (p-value < 0.01).

The conclusions are not the same for women. In fact, results in Table 8 are trending in the opposite direction. Inequality of opportunity is higher in 2018 than in 2012 (p-value $<$ 0.01) for any index in Ω. If we compare 2018 to 2006, the same result holds for all indices in Ω_P (p-value <0.05). Nevertheless, between 2006 and 2012, the evolution of inequality of opportunity for women was trending in the right direction as there is less inequality of opportunity in 2012 than in 2006 for any index in Ω (p-value < 0.01). Similarly, inequality of opportunity was decreasing between 1998 and 2012 (p-value <0.05) for any index in Ω and (p-value <0.01) for any index in Ω_P and Ω_M. The conclusions derived from comparisons of inequality of opportunity between identity groups are interesting. The results reveal that when a society is facing an increasing demand for equality in context of scarcity, the adopted adjustments may come at a high price. Indeed, these adjustments may be decreasing the burden of inequality of opportunity for members of the dominant group while increasing the burden for the marginalized group. While it is interesting to understand what are the potential mechanisms that led to such substitutions, the explanation of the underlying mechanism (intended or not) is beyond the scope of this empirical illustration but is undoubtedly an important subject for further research.[14]

6 Conclusion

In this paper, we developed a framework for measuring inequality of opportunity using the definition of inequality of opportunity proposed by Roemer (1998) and the complaint definition by Temkins (1986). We then show how to use this framework to decompose these inequalities into two components: the first due to social class at birth and the second due to stratification based on an identity marker. This decomposition provides a valuable measurement framework for the theories of stratification economics. We identify dominance conditions for all indices, pro-poor indices, and meritocratic indices. We then show how to use the econometric models available in the literature to estimate the model and test the dominance conditions.

We also offer an empirical illustration analyzing the changes in the contribution of gender stratification and social class at birth to inequality of opportunity in Egypt between 1998 and 2018. This twenty-year period in Egypt is an interesting case because it witnessed the revolutions linked with the Arab Spring and increased demand for more equity in the population. In the face of this greater demand for equality, the social system has adjusted by decreasing the burden of inequality of opportunity among the dominant group, men, while increasing it among the marginalized group, women.

Finally, it is worth mentioning that it is possible to adapt the method of this paper to other contexts in which stratification is along with another identity marker. It can also be adapted to incorporate multiple identities and eventually assess the impact of the intersectionality of discrimination.

[14] In Yemen, Tandon (2019) has explored the perception of safety outside the household as a potential source of explanation in variation in the gender gap in employment opportunities.

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A Proof of theorems

Proof of Theorem 1. First note that equation (6) can be rewritten as

$$I(F_{Y,A,X,G}) = \int_0^1 \omega(e) \sum_{g\in\{D,M\}} \Pr[G=g] \int_{\mathscr{X}} \kappa(e,a,x,g) \mathrm{d}F_{X|A,G}(x|a,g)\mathrm{d}e. \tag{39}$$

Combining equations (15) and (39) we get

$$I(F_{Y,A,X,G}) = \int_0^1 \omega(e) CI(e, F_{Y,A,X,G})\mathrm{d}e. \tag{40}$$

Similarly, equations (12) and (13) can be rewritten as

$$I^{Strat}(F_{Y,A,X,G}) = \int_0^1 \omega(e) CI^{Strat}(e, F_{Y,A,X,G})\mathrm{d}e. \tag{41}$$

$$I^{Class}(F_{Y,A,X,G}) = \int_0^1 \omega(e) CI^{Class}(e, F_{Y,A,X,G})\mathrm{d}e. \tag{42}$$

Using equation (40), we can write

$$\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) = \int_0^1 \omega(e) \left[CI(e, F^1_{Y,A,X,G}) - CI(e, F^0_{Y,A,X,G})\right]\mathrm{d}e. \tag{43}$$

Since $\omega(e) \geq 0$ for all $e \in [0,1]$, $CI(e, F^1_{Y,A,X,G}) - CI(e, F^0_{Y,A,X,G}) \leq 0 \ \forall e \in [0,1]$ implies that $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. Similar results hold for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. This proves the sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. In order to prove necessity, consider the following social weight function:

$$\omega(e) = \begin{cases} 0 & 0 \leq e_c \\ 1/\varepsilon & e_c \leq e \leq e_c + \varepsilon \\ 0 & e \geq e_c + \varepsilon \end{cases} \tag{44}$$

The inequality of opportunity index $I_\varepsilon(\cdot)$ having the social weight function in (44) belongs to Ω. Now assume that for a arbitrary small $\varepsilon > 0$, we have $CI(e, F^1_{Y,A,X,G}) - CI(e, F^0_{Y,A,X,G}) \leq 0 \ \forall e \in [0, e_c] \cup [e_c + \varepsilon, 1]$ and $CI(e, F^1_{Y,A,X,G}) - CI(e, F^0_{Y,A,X,G}) > 0 \ \forall e \in [e_c, e_c + \varepsilon]$. In this case, $\Delta I_\varepsilon(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) > 0$. Similar results also hold for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. This proves the necessity of the condition. □

Proof of Theorems 2 and 3. Integrating by parts equation (43) yields

$$\begin{aligned}\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) &= \omega(e)\left[CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G})\right]\Big|_0^1 \\ &\quad - \int_0^1 \frac{\mathrm{d}\omega(e)}{\mathrm{d}e}\left[CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G})\right]\mathrm{d}e. \qquad (45)\end{aligned}$$

or

$$\begin{aligned}\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) &= -\omega(e)\left[CC_m(e, F^1_{Y,A,X,G}) - CC_m(e, F^0_{Y,A,X,G})\right]\Big|_0^1 \\ &\quad + \int_0^1 \frac{\mathrm{d}\omega(e)}{\mathrm{d}e}\left[CC_m(e, F^1_{Y,A,X,G}) - CC_m(e, F^0_{Y,A,X,G})\right]\mathrm{d}e. \qquad (46)\end{aligned}$$

Since $CC_p(0, F_{Y,A,X,G}) = 0$ and $\omega(1) = 0$ for all $I(\cdot) \in \Omega_P$, equation (45) can be rewritten as

$$\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) = -\int_0^1 \frac{\mathrm{d}\omega(e)}{\mathrm{d}e}\left[CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G})\right]\mathrm{d}e. \qquad (47)$$

Similarly, since $CC_m(1, F^1_{Y,A,X,G}) = 0$ and $\omega(0) = 0$ for all $I(\cdot) \in \Omega_M$, equation (46) can be rewritten as

$$\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) = \int_0^1 \frac{\mathrm{d}\omega(e)}{\mathrm{d}e}\left[CC_m(e, F^1_{Y,A,X,G}) - CC_m(e, F^0_{Y,A,X,G})\right]\mathrm{d}e. \qquad (48)$$

Let us start with equation (47). For indices $I(\cdot) \in \Omega_P$, $\frac{\mathrm{d}\omega(e)}{\mathrm{d}e} \leq 0$ for all $e \in [0,1]$. This implies that if $CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G}) \leq 0$ $\forall e \in [0,1]$ then $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. Let us now turn to equation (48). For indices $I(\cdot) \in \Omega_M$, $\frac{\mathrm{d}\omega(e)}{\mathrm{d}e} \geq 0$ for all $e \in [0,1]$. This implies that if $CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G}) \leq 0$ $\forall e \in [0,1]$ then $\Delta I(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. Similar results hold for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. This proves the sufficiency of the conditions in Theorems 2 and 3.

Having provided a sufficiency condition let us now prove for the necessity of the condition. In order to prove necessity of the conditions in Theorem 2, consider the following social weight function:

$$\omega(e) = \begin{cases} \frac{2}{2e_c+\varepsilon} & 0 \leq e_c \\ \frac{2(e_c+\varepsilon-e)}{\varepsilon(2e_c+\varepsilon)} & e_c \leq e \leq e_c+\varepsilon \\ 0 & e \geq e_c+\varepsilon \end{cases} \qquad (49)$$

The inequality of opportunity index $I_\varepsilon(\cdot)$ having the social weight function in (49) belongs to Ω_P. Now assume that for a arbitrary small $\varepsilon > 0$, we have $CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G}) \leq 0 \ \forall e \in [0, e_c] \cup [e_c + \varepsilon, 1]$ and $CC_p(e, F^1_{Y,A,X,G}) - CC_p(e, F^0_{Y,A,X,G}) > 0 \ \forall e \in [e_c, e_c + \varepsilon]$. In this case, $\Delta I_\varepsilon(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) > 0$. Similar results also hold for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. This proves the necessity of the condition for Theorem 2.

In order to prove necessity of the conditions in Theorem 3, consider the following social weight function:

$$\omega(e) = \begin{cases} 0 & 0 \leq e_c \\ \frac{2(e-e_c)}{\varepsilon(2e_c+\varepsilon)} & e_c \leq e \leq e_c + \varepsilon \\ \frac{2}{2-2e_c-\varepsilon} & e \geq e_c + \varepsilon \end{cases} \tag{50}$$

The inequality of opportunity index $I_\varepsilon(\cdot)$ having the social weight function in (50) belongs to Ω_P. Now assume that for a arbitrary small $\varepsilon > 0$, we have $CC_m(e, F^1_{Y,A,X,G}) - CC_m(e, F^0_{Y,A,X,G}) \leq 0 \ \forall e \in [0, e_c] \cup [e_c + \varepsilon, 1]$ and $CC_m(e, F^1_{Y,A,X,G}) - CC_m(e, F^0_{Y,A,X,G}) > 0 \ \forall e \in [e_c, e_c + \varepsilon]$. In this case, $\Delta I_\varepsilon(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) > 0$. Similar results also hold for $\Delta I^{Strat}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$ and $\Delta I^{Class}(F^0_{Y,A,X,G}, F^1_{Y,A,X,G}) \leq 0$. This proves the necessity of the condition for Theorem 3. □

B Details on the Riemann sum approximations for the complaint concentration curves

The Riemann sum approximations for the complaint concentration curves are:

$$\widehat{CC}_p(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{L}\sum_{\ell=1}^{L} \widehat{CI}(e_\ell, \widehat{F}_{Y,A,X,G}) \mathbb{1}[e_\ell \leq e], \tag{51}$$

$$\widehat{CC}_p^{Strat}(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{L}\sum_{\ell=1}^{L} \widehat{CI}^{Strat}(e_\ell, \widehat{F}_{Y,A,X,G}) \mathbb{1}[e_\ell \leq e], \tag{52}$$

$$\widehat{CC}_p^{Class}(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{L}\sum_{\ell=1}^{L} \widehat{CI}^{Class}(e_\ell, \widehat{F}_{Y,A,X,G}) \mathbb{1}[e_\ell \leq e], \tag{53}$$

$$\widehat{CC}_m(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{L}\sum_{\ell=1}^{L} \widehat{CI}(e_\ell, \widehat{F}_{Y,A,X,G}) \mathbb{1}\left[e_\ell \geq e\right], \tag{54}$$

$$\widehat{CC}_m^{Strat}(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{L}\sum_{\ell=1}^{L} \widehat{CI}^{Strat}(e_\ell, \widehat{F}_{Y,A,X,G}) \mathbb{1}\left[e_\ell \geq e\right], \tag{55}$$

and

$$\widehat{CC}_m^{Class}(e, \widehat{F}_{Y,A,X,G}) = \frac{1}{L}\sum_{\ell=1}^{L} \widehat{CI}^{Class}(e_\ell, \widehat{F}_{Y,A,X,G}) \mathbb{1}\left[e_\ell \geq e\right]. \tag{56}$$

C Bootstrap algorithm

The bootstrap algorithm is constructed as follows. Assume that we have an i.i.d. sample of size n_0 from the random variable corresponding to first theoretical curve L_0 and and and i.i.d. sample of size n_1 from the random variable corresponding to the second theoretical curve C_1. Denote those samples by $\mathcal{S}_0$ and $\mathcal{S}_1$ respectively. Let $\widehat{C}_0$ and $\widehat{C}_1$ be the nonparametric estimators of C_0 and C_1 respectively, constructed from those two samples. Let

$$\widehat{\tau} = \sqrt{\frac{n_0 n_1}{n_0 + n_1}} \sup_e \left[\widehat{C}_1(e) - \widehat{C}_0(e)\right]$$

1. Repeat for $b = 1, \ldots, B$

   (a) Draw a sample of size n_0 from $\mathcal{S}$. Compute the nonparametric estimator $\widehat{C}_{0b}$.

   (b) Draw a sample of size n_1 from $\mathcal{S}$. Compute the nonparametric estimator $\widehat{C}_{1b}$.

   (c) Compute $\widehat{\tau}_b = \sqrt{\frac{n_0 n_1}{n_0+n_1}} \sup_e \left[\widehat{C}_{1b}(e) - \widehat{C}_{0b}(e) - \widehat{C}_1(e) + \widehat{C}_0(e)\right]$.

2. Using the sample $\widehat{\tau}_1, \ldots, \widehat{\tau}_B$, compute the bootstrap p-value

$$\frac{1}{B}\sum_{b=1}^{B} \mathbb{1}(\widehat{\tau}_b > \widehat{\tau}).$$

Figures and Tables

Figure 1: Upper countour $\rho(e, c)$

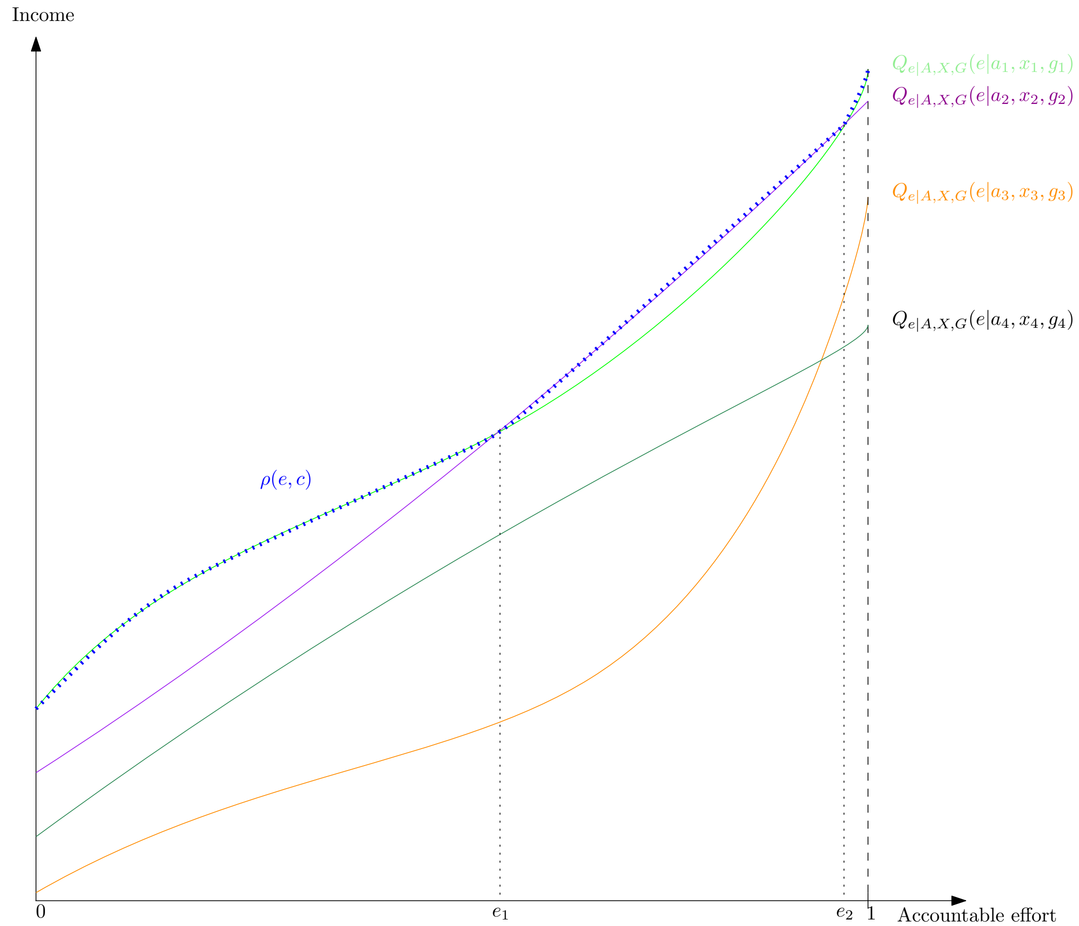

Figure 2: Complaint associated with an effort e_1 for type (a, x, g)

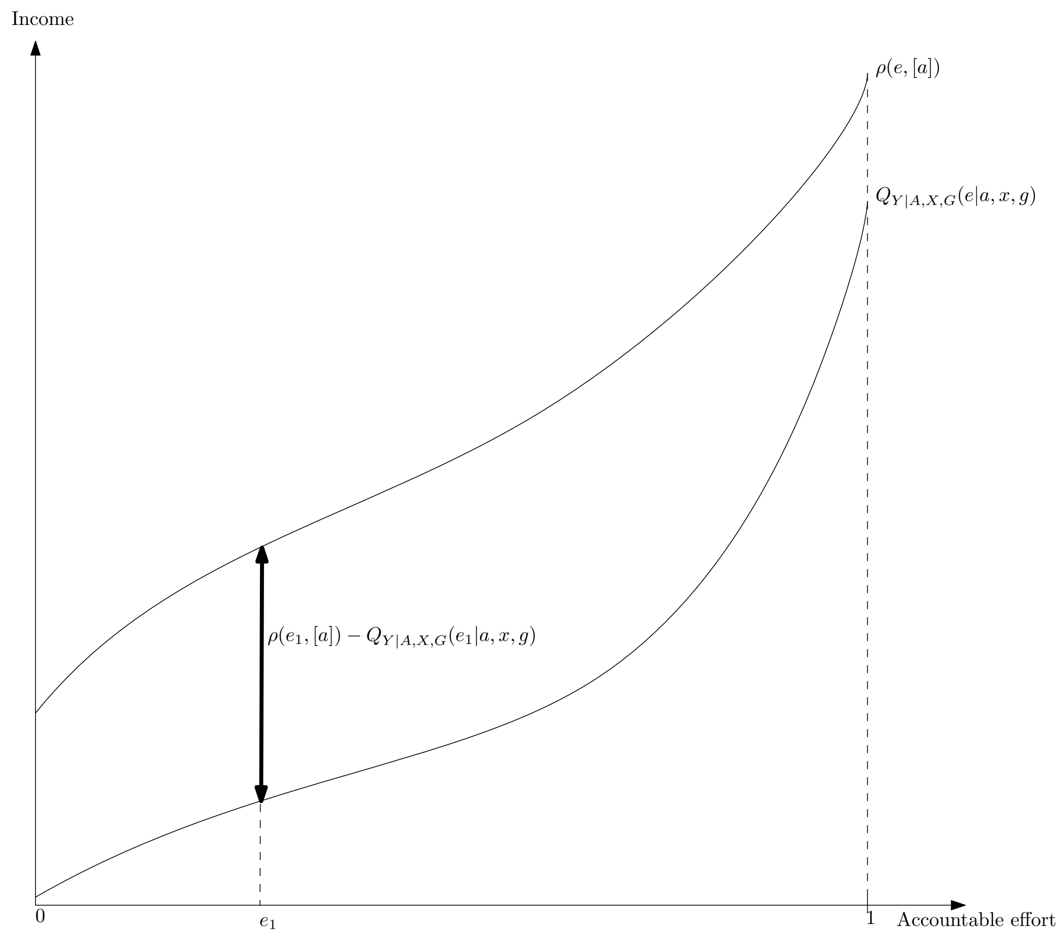

Figure 3: Decomposition of the complaint associated with an effort e_1 of type (a, x, M)

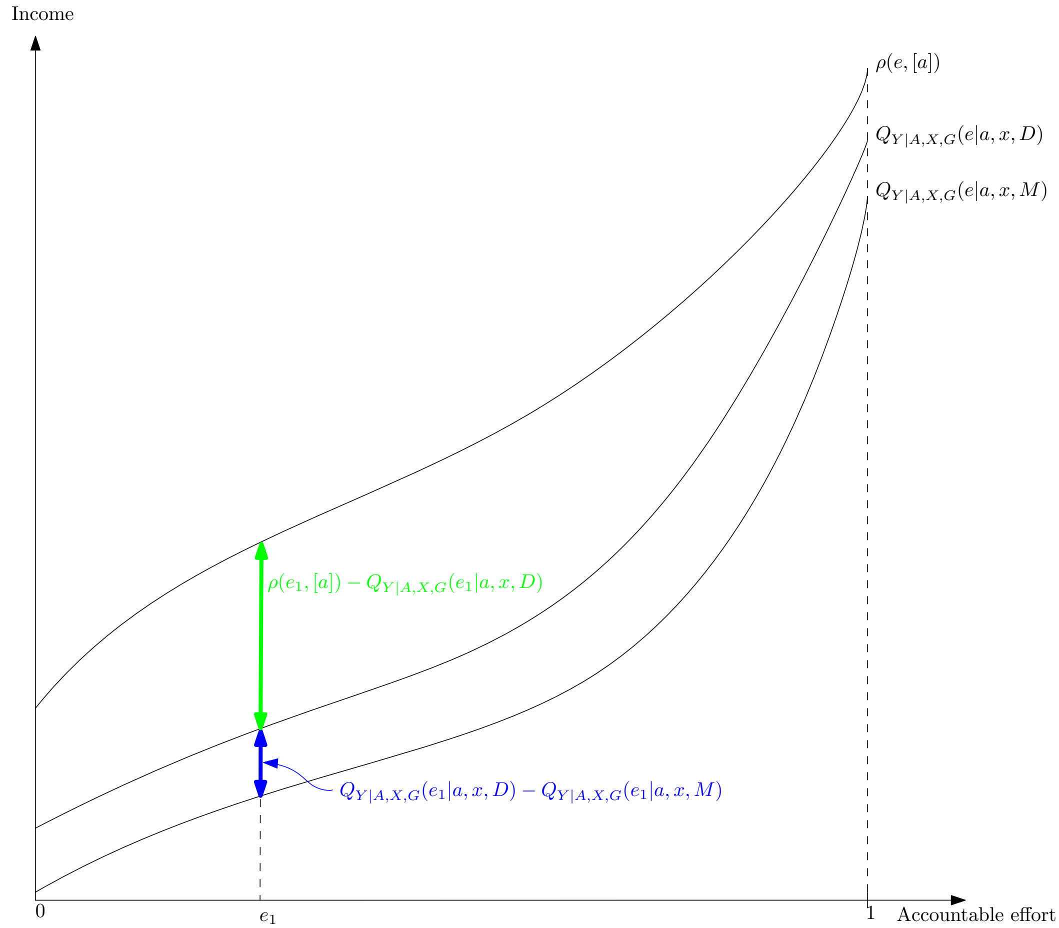

Figure 4: Complaint curves, Egypt 2006 and 2012

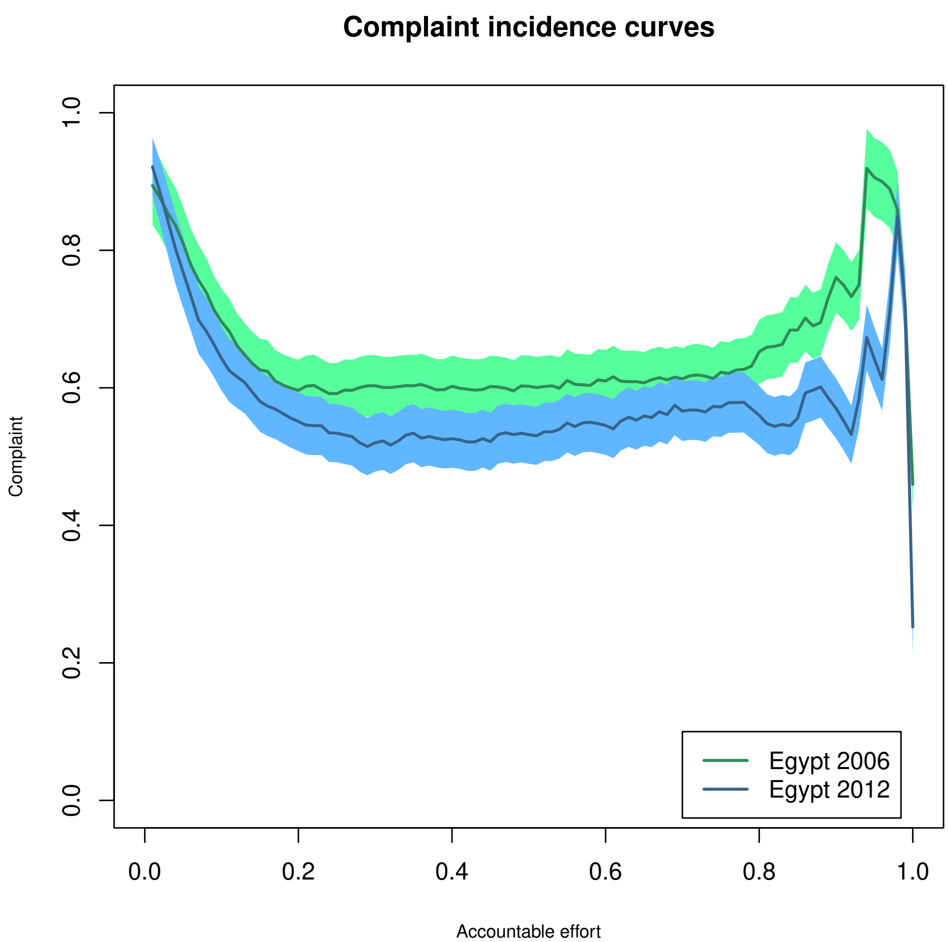

Figure 5: Stratification complaint curves

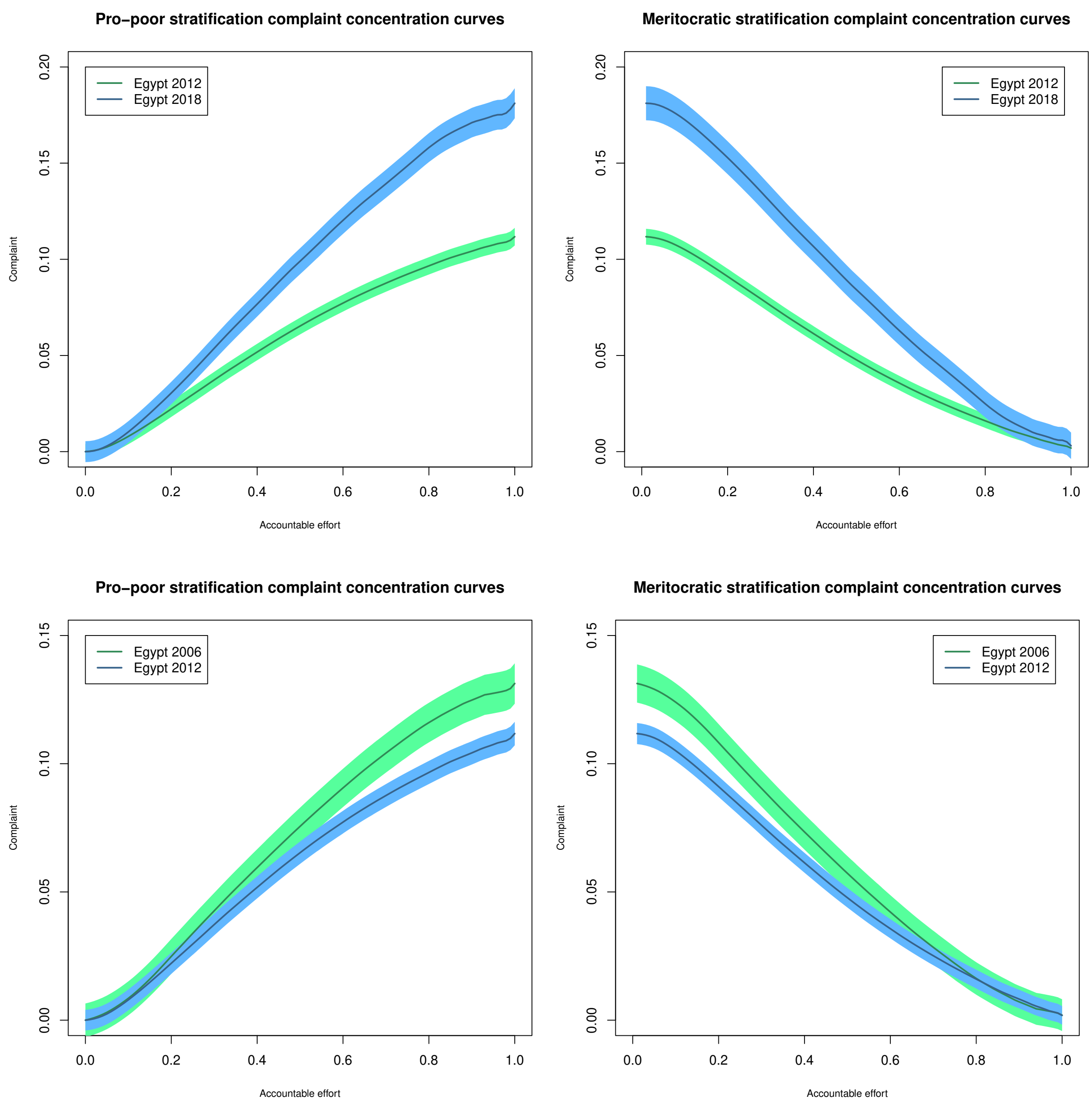

Figure 6: Social class complaint curves

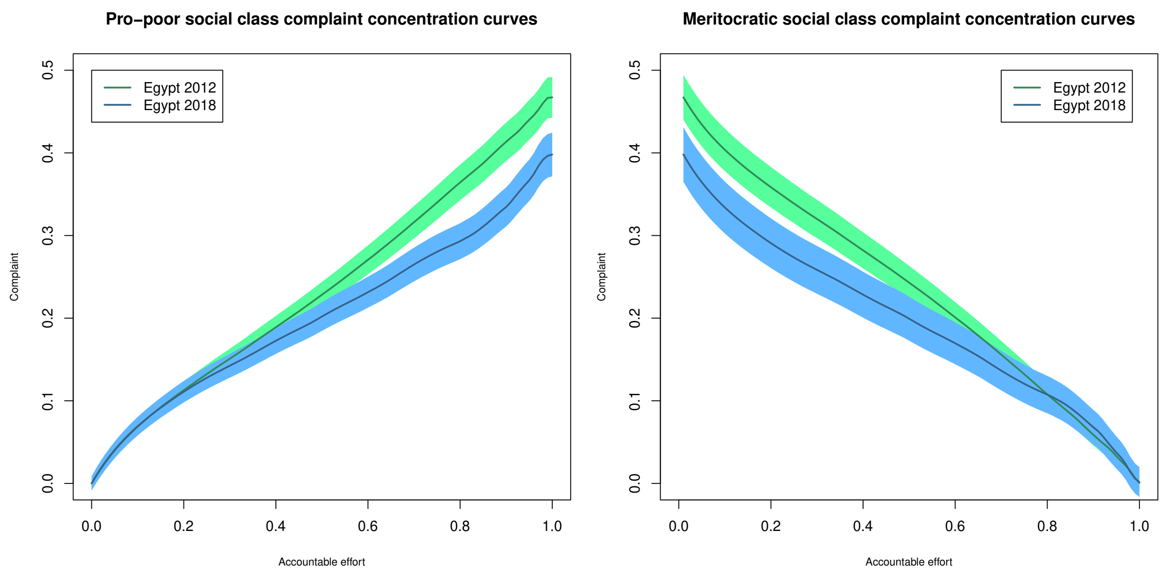

Table 1: Interpretation of dominance tests for a level of significance α

p-values	Interpretation
$p_1 \geq \alpha$ and $p_2 \geq \alpha$	$C_1(e) = C_0(e)$
$p_1 < \alpha$ and $p_2 \geq \alpha$	$C_0(e) \leq C_1(e), \forall e \in [0,1]$
$p_1 \geq \alpha$ and $p_2 < \alpha$	$C_1(e) \leq C_0(e), \forall e \in [0,1]$
$p_1 < \alpha$ and $p_2 < \alpha$	$C_0(e)$ and $C_1(e)$ intersect

Table 2: Labor force participation by gender

Year	Female	Male
1998	19.84%	72.46%
2006	21.34%	72.61%
2012	22.55%	74.37%
2018	18.48%	68.18%

Source: ILOSTAT database.

Table 3: Share of individuals declaring *unpaid family worker* as main labor activity

	1998	2006	2012	2018
Men	4.51%	5.30%	2.93%	2.23%
Women	66.67%	63.53%	51.54%	66.38%
Number of observations	4,574	7,420	9,374	9,393

Source: Authors' own estimation, ELMPS.

Table 4: Average wage earnings in 2018 PPP international dollars

	1998	2006	2012	2018
Parents education				
Lower class	287.67	396.88	516.47	505.31
Middle class	437.28	621.96	671.01	559.52
Upper middle class	647.43	779.53	798.33	625.84
Upper class	891.49	1,258.97	1,139.76	1,436.55
Father's employment				
Agriculture	258.18	340.35	467.11	493.85
Blue collar	428.99	673.62	672.62	541.22
White collar	517.87	726.29	811.99	727.84
Gender				
Male	560.44	763.78	792.89	829.51
Female	164.03	244.44	315.49	211.60
Number of observations	4,574	7,420	9,374	9,393

Source: Authors' own estimation, ELMPS.

Table 5: Complaints dominance tests p-values

Year A | Year B	2018 | 2012	2018 | 2006	2018 | 1998	2012 | 2006	2012 | 1998	2006 | 1998
				$***$	$**$	
$C_A(e) \le C_B(e)$	0.1902	0.7387	0.1622	0.9960	0.8699	0.5405
$C_B(e) \le C_A(e)$	0.9479	0.4044	0.4665	0.0000	0.0180	0.6136
		$***$	$***$	$***$	$***$	$*$
$PC_A(e) \le PC_B(e)$	0.3644	0.4765	0.6066	0.5836	0.7127	0.8298
$PC_B(e) \le PC_A(e)$	0.4014	0.0000	0.0000	0.0000	0.0000	0.0521
		$***$	$***$	$***$	$***$	
$MC_A(e) \le MC_B(e)$	0.2052	0.8118	0.5536	1.0000	0.9990	0.5115
$MC_B(e) \le MC_A(e)$	0.8959	0.0000	0.0000	0.0000	0.0000	0.0971

$*$: Dominance at the 0.1 level
$**$: Dominance at the 0.05 level
$***$: Dominance at the 0.01 level

Table 6: Inequality of opportunity orderings

	2018	2012	2006	1998
2018	-	ND	Ω_P^{***} and Ω_M^{***}	Ω_P^{***} and Ω_M^{***}
2012	ND	-	Ω^{***}	Ω^{**}, Ω_P^{***}, and Ω_M^{***}
2006			-	ND
1998			ND	-

$**$: Dominance at the 0.05 level for the indicated set of indices
$***$: Dominance at the 0.01 level for the indicated set of indices
ND: No dominance

Table 7: Stratification and Social Class induced inequality of opportunity orderings

Stratification Dominance				
	2018	2012	2006	1998
2018	-			
2012	Ω_P^{***} and Ω_M^{***}	-	Ω_P^{**} and Ω_M^{**}	Ω_P^{**} and Ω_M^{**}
2006	Ω_P^{***} and Ω_M^{***}		-	ND
1998	Ω_P^{***} and Ω_M^{***}		ND	-
Social Class Dominance				
	2018	2012	2006	1998
2018	-	Ω_P^{***} and Ω_M^{***}	Ω_P^{***} and Ω_M^{***}	Ω_P^{***} and Ω_M^{***}
2012		-	Ω_P^{**} and Ω_M^{**}	Ω_P^{**} and Ω_M^{**}
2006			-	ND
1998			ND	-

$**$: Dominance at the 0.05 level for the indicated set of indices

$***$: Dominance at the 0.01 level for the indicated set of indices

ND: No dominance

Table 8: Inequality of opportunity orderings by gender

Men				
	2018	2012	2006	1998
2018	-	Ω_P^{***} and Ω_M^{***}	Ω_P^{***} and Ω_M^{***}	Ω_P^{***} and Ω_M^{***}
2012		-	Ω_P^{**} and Ω_M^{**}	Ω_P^{***} and Ω_M^{***}
2006			-	ND
1998			ND	-
Women				
	2018	2012	2006	1998
2018	-			ND
2012	Ω^{***}	-	Ω^{***}	Ω^{**}, Ω_P^{***}, and Ω_M^{***}
2006	Ω_P^{**}		-	ND
1998	ND		ND	-

$**$: Dominance at the 0.05 level for the indicated set of indices

$***$: Dominance at the 0.01 level for the indicated set of indices

ND: No dominance