

# APPLICATIONS OF NETWORK THEORY TO NORTH AMERICA STOCK MARKETS

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# ABSTRACT

This dissertation applies network theory to investigate the structure, direction, and dynamics of systemic risk in North American equity markets, focusing on the Toronto Stock Exchange (TSX 60) and Nasdaq 100. The research combines three complementary analyses to provide a deep understanding of market connectedness across pre-COVID, pandemic, and post-pandemic periods.

First, partial correlation and mutual information concepts are employed to identify edges in order to construct an undirected network for each stock market, namely the TSX 60 and the Nasdaq 100. We use several network analysis techniques, such as graphical LASSO (GLASSO), latent variable time-varying graphical lasso (LTGL), and minimum spanning tree (MST), to reveal key patterns of interconnectedness and shifts in the stock market across distinct sample periods: the “pre-COVID-19”, “COVID-19”, and “post-COVID-19” periods. Findings reveal shifts in sectoral centrality and the emergence of systemically important nodes during periods of market stress.

Second, the study transitions to directed network analysis, applying a range of methodologies, including net transfer entropy, the Diebold-Yilmaz (DY) spillover index and empirical mode decomposition, to identify edges and construct a directed volatility-spillover network. The findings reveal the distinctive structures of the two stock markets in both short-term and long-term frequency data. Both the TSX 60 and Nasdaq 100 networks demonstrate a robust interconnectedness and high volatility transmission during the pandemic, reflecting heightened market uncertainty. Furthermore, the study compares the net transfer entropy and the DY spillover index and finds that the DY spillover index is also capable of identifying the volatility spillover in a smoothed pattern from a long-term frequency data.

Finally, a novel integration of the directed network framework with a Susceptible - Infected - Recovered (SIR) epidemic model is used to simulate the dynamic propagation of financial contagion. By calibrating transmission ( $\beta$ ) and recovery ( $\gamma$ ) rates from empirical data, the model shows how network topology and sectoral positioning influence contagion speed and scale.

The dissertation advances the literature on financial networks, systemic risk, and contagion modeling by uniting static structure, directional influence, and dynamic simulation within a single framework. The results provide actionable insights for policymakers, regulators, and investors aiming to identify vulnerabilities, design targeted interventions, and strengthen market resilience against future shocks.

**Keywords:** Network Analysis, Systemic Risk, Financial Contagion, Volatility Spillovers, North American Stock Market, Mutual Information, Net Transfer Entropy, Diebold-Yilmaz Spillover Framework

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# DEDICATION

This thesis is dedicated, with love and gratitude, to my family.

To my parents, Guoyong Wang and Ping Chen, I extend my deepest thanks for your unconditional love and endless support. I would not be who I am today without you.

To my wife, Minsha He, thank you for your patience, understanding, and for always being by my side. Your love has been my enduring source of strength.

To my daughters, Christina Wang and Charlotte Wang, thank you for filling my life with love and joy. You are my inspiration, and I wish you both a bright and beautiful future.

To my son, expected in the summer of 2026, whose presence is already deeply felt, you represent new hope and beginnings. We eagerly await your arrival and look forward to the joy you will bring to us.

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# General Introduction

The connectedness of global financial markets has become increasingly complex and dynamic, leading to new challenges in understanding and managing systemic risks. The recent COVID-19 pandemic magnified these challenges, causing unprecedented market disruptions and highlighting the critical need for more sophisticated tools to analyze financial networks. This dissertation addresses this by applying advanced network theory and analysis to investigate the structure, dynamics, and risk propagation within North American stock markets, specifically focusing on the Toronto Stock Exchange (TSX 60) and the Nasdaq 100. The primary objective of this research is to investigate the structural characteristics of stock market networks and their implications for the identification and management of financial risks.

Market risk, as defined by the Basel Framework (2023), refers to the potential for losses arising from fluctuations in market prices. While market risk can be quantified and partially mitigated through strategies such as diversification and hedging, systematic risk, also known as non-diversifiable risk, remains inherent to the entire market, driven by broad macroeconomic factors. In contrast, systemic risk pertains to the potential collapse or severe disruption of the entire financial system, often triggered by the failure of one or more systemically important financial institutions. Such events can propagate contagion effects throughout highly interconnected markets, as exemplified by the 2008 global financial crisis.

This dissertation contributes to several strands of the existing literature. First, it advances the field of stock market network analysis by examining the structural properties of stock networks and applying various methodologies for network edge identification. The findings highlight key sectors that play central roles in market connectivity and potential risk propagation. Second, this research contributes to the risk management literature by

leveraging network topology to investigate both market risk and systemic risk, offering novel insights into the dynamics of risk transmission and potential mitigation strategies. Finally, this study makes a contribution to the literature on systemic risk simulation by integrating network structures with an epidemic modeling framework to simulate financial contagion. This innovative approach provides a deeper understanding of the mechanisms underlying systemic crises and offers practical implications for financial stability and policy intervention.

The chapter 1 of this dissertation employs undirected network models to explore the evolving structural relationships in stock markets over three distinct periods: before COVID-19, during the pandemic, and the post-pandemic period. Specifically, methodologies such as Graphical LASSO, latent variable time-varying graphical LASSO, and mutual information measures are applied to elucidate variations in network topology and the emergence of systemic risk factors. These techniques offer an alternative framework for capturing stock correlations, thereby providing valuable insights for market risk mitigation strategies.

Building on this foundation, the chapter 2 transitions to the analysis of directed networks, applying net transfer entropy and the Diebold-Yilmaz spillover framework to uncover the directionality and intensity of volatility spillovers among stocks. This approach provides insights into short-term and long-term risk transmission mechanisms, further enhancing the understanding of market dynamics under periods of stress and recovery.

In the final chapter 3 of this dissertation, the study introduces an integration of stock network analysis with the Susceptible-Infected-Recovered (SIR) epidemic model. This framework simulates the contagion dynamics of financial crises within the TSX 60 network, offering a novel perspective on how localized shocks can propagate and impact broader market stability. By modeling financial contagion through an epidemiological lens, the study underscores the critical role of network topology in both amplifying and mitigating systemic risk.

Overall, this dissertation offers a comprehensive examination of North American stock market networks, advancing both theoretical and practical understanding of financial interconnectedness, systemic risk, and contagion dynamics. The findings have significant

implications for policymakers, investors, and regulators seeking to enhance financial stability and resilience in an interconnected global economy.

# Chapter 1

## Undirected Network Analysis

### 1.1 Introduction

Market risk, as defined by the Basel Framework (2023), refers to the risk of losses resulting from fluctuations in market prices, including changes in interest rates, exchange rates, equity prices, and commodity prices. This type of risk is typically quantifiable and manageable through strategies such as hedging and diversification. Diversification reduces risk by holding positions in uncorrelated instruments, thereby minimizing exposure to idiosyncratic shocks. Hedging, on the other hand, involves offsetting long and short positions using instruments with strong negative correlations, thereby counterbalancing potential losses. Systemic risk relates to the potential collapse of an entire financial system or market, often triggered by the failure of a major financial institution, leading to contagion effects that propagate instability across interconnected entities, as exemplified by the 2008 global financial crisis. Meanwhile, systematic risk, also referred to as non-diversifiable risk, affects the entire market due to macroeconomic factors such as inflation, GDP growth, and interest rate fluctuations. Although systemic and systematic risks are often mentioned together, they represent distinct phenomena. Systematic risk is inherent to the entire market and cannot be eliminated through diversification, while systemic risk involves the breakdown of the financial system itself, usually through cascading failures in interconnected institutions. Some studies of systemic risk in the financial market have used network analysis,

among others, Diem et al. (2020), Zhang et al. (2020a), Zhang et al. (2020b), Ahelegbey and Giudici (2020), and Sharif et al. (2020).

This chapter uses network analysis to explore the network changes of the North America stock markets in the recent six years, including the global pandemic period. The COVID-19 virus was identified in China at the end of 2019 and spread rapidly throughout the world, causing a global pandemic. The pandemic has had a dramatic impact on social activities in North America. The governments implemented various measures to slow the spread of the virus, such as social distancing and lock downs. Due to government restrictions, many businesses changed their operating mode or operated at a low capacity in that period. Some industries, such as real estate and technology, experienced growth during the pandemic, resulting in a shift in investment strategies. In the period following the global pandemic, inflation, contractionary monetary policy and imbalanced global energy market have created many economic uncertainties, and further induced a volatile stock market. The COVID-19 pandemic is characterized by a high level of synchronicity among markets, with a rapid spread of risks and unprecedented uncertainty that has not been experienced before (e.g., Baker et al. 2020, Qureshi et al. 2021, Deng et al. 2022, and Sharif et al. 2020). Investigating the interdependent relationships of the stock market is important to understand the impact of COVID-19 on the Canadian and US stock markets.

The Canadian stock market, also known as the Toronto Stock Exchange (TSX), is the primary stock exchange in Canada. It is based in Toronto and is owned and operated by the TSX Group. The TSX is the ninth largest stock exchange in the world by market capitalization, with a total market capitalization of approximately 3.3 trillion CAD as of March 2023.<sup>1</sup> The Canadian stock market showed its resilience, in comparison to the US stock market, during the pandemics. For example, the TSX 60 index only dropped by around 6% in 2022.<sup>2</sup> On the contrary, the S&P 500 declined by more than 17%, and the

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<sup>1</sup>The data is retrieved as of 2023-03 at <https://www.tsx.com/listings/listing-with-us>.

<sup>2</sup>The S&P/TSX 60 is a key index for the Canadian equity market including the 60 largest companies listed on the Toronto Stock Exchange (TSX) based on market capitalization. It is considered a barometer of the Canadian economy. The companies in the TSX 60 represent a diverse range of industries, including financial services, energy, materials, and consumer goods. Some of the largest and most well-known Canadian companies in the index include Royal Bank of Canada, Shopify, Canadian National Railway, and Enbridge.

Nasdaq composite index tumbled by almost 30%. The Nasdaq stock market is a leading global electronic securities exchange including many prominent technology firms such as Apple, Microsoft, and Amazon. The Nasdaq 100 index is one of the most popular stock market indices in the world for high tech investors. Unlike many other major indices, the Nasdaq 100 excludes financial companies, which provides a unique focus on technology and growth-oriented sectors.

The chapter analyzes the evolution of the network structure of the TSX 60 and Nasdaq 100 using daily closing stock prices from January 2017 to September 2023. The main contributions of this chapter can be outlined in the following three aspects: one is to construct undirected networks for North America stock markets in different time periods (i.e. the “before COVID-19” period, the “COVID-19” period and the “Post COVID-19” period) and identify the change of network connections as a response to the market uncertainties. Secondly, it examines the connections and differences between the TSX 60 and the Nasdaq 100. Finally, the chapter demonstrates the usefulness of network analysis in the field of risk management.

The rest of the chapter is organized as follows. Section 1.2 reviews network analysis and risk management literature, specifically focusing on methodologies associated with undirected graphs and risk management. Section 1.3 briefly discusses the data used. Section 1.4 illustrates the edge identification strategy. Section 1.5 provides empirical results, and Section 1.6 concludes.

## **1.2 Literature Review**

Various methodologies and models have been developed over the years to study the systemic risk in the financial market. It has been shown that systemic risk is to a large extent related to the underlying network topology of financial exposures. Diem et al. (2020) demonstrate that systemic risk can be mitigated in a financial contract networks by rearranging a cross-sectional inter-bank loan network topology. Through the inter-bank loan network, they found that systemically important banks are highly interconnected, which

increases overall systemic risk. Zhang et al. (2020a) analyse the tail risk connectedness in Chinese stock market and find that stock market exposes to more systemic risk during market turmoil due to a high network density. Zhang et al. (2020b) establish a holistic framework to shed light on the topological dynamics and the evolving channels of connectedness network among 24 major stock markets. The recent COVID-19 pandemic is marked by a high level of synchronicity among markets (Zhang et al., 2020b). Sharif et al. (2020) find that the global pandemic increased US geopolitical risk and US economic uncertainty by analyzing the connectedness between stock market, oil price, the infected cases and policy indexes. Ahelegbey and Giudici (2020) construct a network-based turbulence score to analyze the relationship between financial markets in different jurisdictions and to identify systemically important markets with the highest contribution to financial turbulence. They also show that although the US market has played a critical role in most crises, it has not maintained its status as risk transmission hub in the COVID-19 crisis based on their network analysis. In addition, as shown by Zhang et al. (2021), COVID-19 has altered the relationship between the energy markets and stock markets. Before COVID-19, the energy markets were relatively independent of stock markets, indicating that normal price movements in stock markets had limited effect on energy markets. However, during COVID-19, the in-degree centrality of the global energy sector became extremely high, suggesting that energy markets faced significant systemic risks from stock markets. During the COVID-19, policymakers have implemented a range of measures to mitigate the health, political, and economic effects of the pandemics (Iyke 2020, Phan and Narayan 2021 and Deng et al. 2022). The lockdown order and low interest rate policy were two predominant measures. Governments implemented non-pharmaceutical interventions (NPIs) such as lockdown orders to control its spread. While these measures were effective, it indirectly impacted work environments and lifestyles (i.e. remote working). Additionally, the historically low policy rate has provided financial relief and helped alleviate the pandemic's impact on the economy. Deng et al. (2022) and Phan and Narayan (2021) shows that the lockdown order boosted investors' confidence and sentiment because they believed that policymakers had found a solution to the spread of the virus. These positive attitudes translated into a pos-

itive impact on the stock market. Deng et al. (2022) also demonstrate that a low interest rate policy is favorable for the stock market. However, we also observe that in the aftermath of COVID-19 both the Canadian and US stock markets have exceeded pre-COVID-19 levels, despite central banks rapidly raising interest rates to control inflation.

Network analysis (NA) has gained significant popularity in many different areas of research, such as financial market analysis (for example, see Niu and Hu 2021, Diebold and Yilmaz 2014 and Bekiros et al. 2017), music information visualization (Georges and Seckin, 2022) as well as public health (Harris and Clements, 2007) among many others. NA is an investigation process to explore the system structure through a network with two essential elements, nodes, and edges. Each node represents an analytic object. The edges capture the connection or relationship among the nodes. In addition, the network graph has two main categories, the undirected graph and the directed graph (Freeman, 2004). An undirected graph depicts a network system in which each edge measures the connectedness in the system without revealing the connection direction. The connections between nodes can be captured using various techniques. The Pearson correlation is a well-established and straightforward method for measuring connectedness. For example, see Guo et al. (2018), Sultornsanee et al. (2011), Eom and Park (2017) and Bekiros et al. (2017). However, applying this method in the financial market to detect connectedness may lead to misleading conclusions due to beta noise.<sup>3</sup> To address this issue, this study uses the precision matrix to measure the connection among stocks. The precision matrix is the inverse of the covariance matrix. With the assumption of normality of logarithmic stock returns, the precision matrix provides a conditional independence measurement. For instance, if the logarithmic returns of stock  $i$  and stock  $j$  are two elements from a multivariate normal distribution and conditionally independent, then the corresponding partial correlation will be zero. Friedman et al. (2008) have designed the graphical lasso algorithm (GLASSO) to eliminate non-zero elements in the precision matrix by applying  $L_1$  penalty. The GLASSO improves

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<sup>3</sup>Beta is a financial metric that measures the correlation between expected stock returns and market performance. Beta noise refers to the impact of market performance on stock correlation. For instance, consider two large listed companies from entirely different sectors that have a similar beta, indicating that their price movements are close to market performance. Therefore, the correlation between these two companies captures the similarity of price movements rather than an actual connection.

the interpretability of NA when the network contains many nodes. Millington and Niranjana (2017) construct a network by using the GLASSO to explore relationships between companies. Additionally, their findings indicate that an investment portfolio constructed using the GLASSO network exhibits lower return volatility and reduced investment risk, primarily due to enhanced diversification and hedging benefits.<sup>4</sup>

For many time series datasets, particularly financial data, understanding how the precision matrix/graph evolves over time is crucial in explaining shifts in underlying relationships. The study of time-varying graphical models has drawn great attention in recent years. Hallac et al. (2017) develop the time-varying graphical lasso (TVGL) based on the GLASSO to reveal the dynamics of the precision matrix over time. This method provides us with the ability to control both sparsity level and fluctuation of temporal precision matrices by introducing two regularization parameters for the optimization problem. Yang and Peng (2020) propose a local group graphical lasso model to reveal the change in interacting relationships among S&P 500 stocks during the global financial crisis of 2008. Tomasi et al. (2018b) propose latent variable time-varying graphical lasso (LTGL) to deal with latent factors that influence the system without being explicitly measured in a dynamics network. The model takes into account the impact of hidden or unmeasurable factors, estimating their contribution through both sparse and low-rank components for each time point.

Although the Pearson and partial correlations can capture connections between stocks' performance, the relationships between stocks' returns may not be linear. Information theory has been widely used to capture nonlinear correlations in the financial market (Ferreira and Dionísio 2017, Yang et al. 2014, He et al. 2022 and Barbi and Pratavia 2019). Shannon (1948) first proposed a metric to quantify a variable's information contained in a system. The entropy value measures the required information to identify a specific event. When the system has two discrete variables, the mutual information quantifies the information gained from one variable to another variable. Yang et al. (2014) employs mutual

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<sup>4</sup>According to the Basel framework, market risk is defined as the risk of losses resulting from fluctuations in market prices. Hedging and diversification are commonly used to mitigate market risk. Diversification reduces risk by holding positions in different instruments that are uncorrelated with each other. Hedging is a trading strategy that offsets the risks of exposures to long and short positions in highly negatively correlated instruments.

information to measure the correlation between stock returns and finds that the average and variance of mutual information exhibit a bell-shaped distribution during crises. This pattern suggests that stock return connections tend to strengthen during periods of financial turmoil. He et al. (2022) and Barbi and Prataiviera (2019) apply the minimum spanning tree (MST) algorithm to extract significant connection information in a stock market, and maintain a simple structure for the network of the stock market.

In this chapter, we split the data in three different periods to further investigate the network structure changes among the “before COVID-19”, “COVID-19” and “Post COVID-19” periods. Section 1.3 depicts data details. Section 1.4.1 discusses the precision matrix and GLASSO techniques; Section 1.4.2 briefly discusses the MST, and Section 1.4.3 deals with the mutual information identification strategy. A series of appendices are available to supplement the presentation of these concepts. Finally, empirical results are presented in Section 1.5.

## 1.3 Data

### 1.3.1 Stock Market Data

**Canadian Stock Market** While the TSX composite index is considered as a premier index for the Canadian market, it includes many small, less active companies whose stock prices remain relatively stable due to low trading volumes. If a stable price is driven by low trading volume rather than market conditions or the company’s performance, the stock return may contain little information in a network analysis. The TSX 60 is designed to separate the large capitalization segment of the Canadian equity market from the TSX composite constituents. These stocks have high trading volumes and are more attractive from an investment perspective. Figure 1.1 depicts the effective returns of TSX 60 and TSX composite index assuming 2017-01-03 as the starting point. The movements between these two indexes are consistent over time. Figure 1.2 illustrates that the TSX 60 is a subset of the TSX composite index with a similar sector coverage. Thus, the study uses

the performance of TSX 60 as a proxy of TSX composite performance.

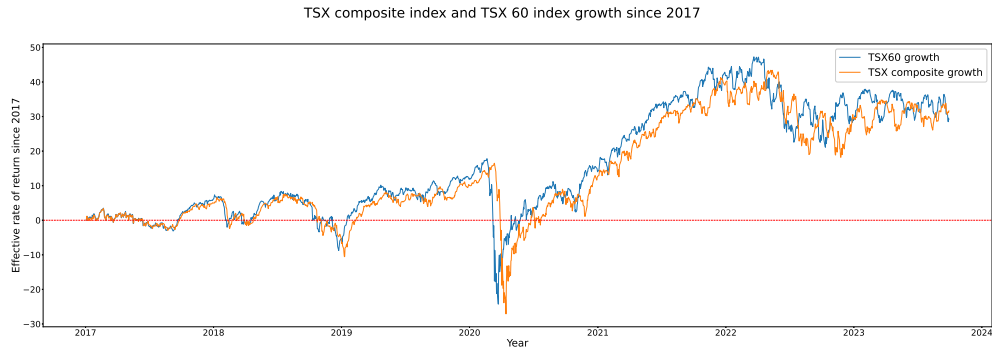


Figure 1.1: TSX Indexes Growth Since 2017. Data source: Yahoo Finance

### TSX Composite Index and TSX 60 Index Sector Distribution by Market Capitalization

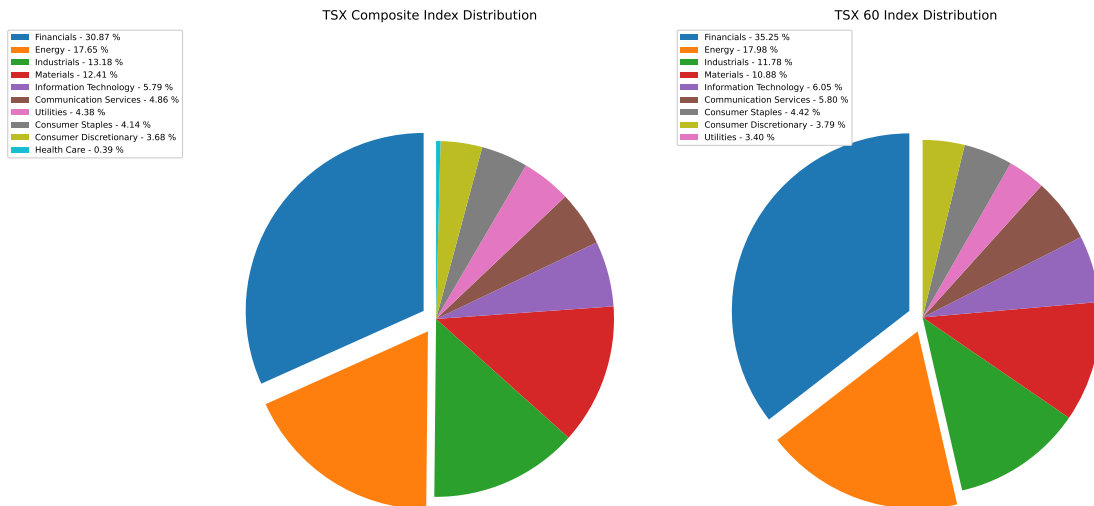


Figure 1.2: The Sectoral Distribution: TSX Composite Index and TSX 60 Index. Data source: retrieved from the TMX website as of 2023-03

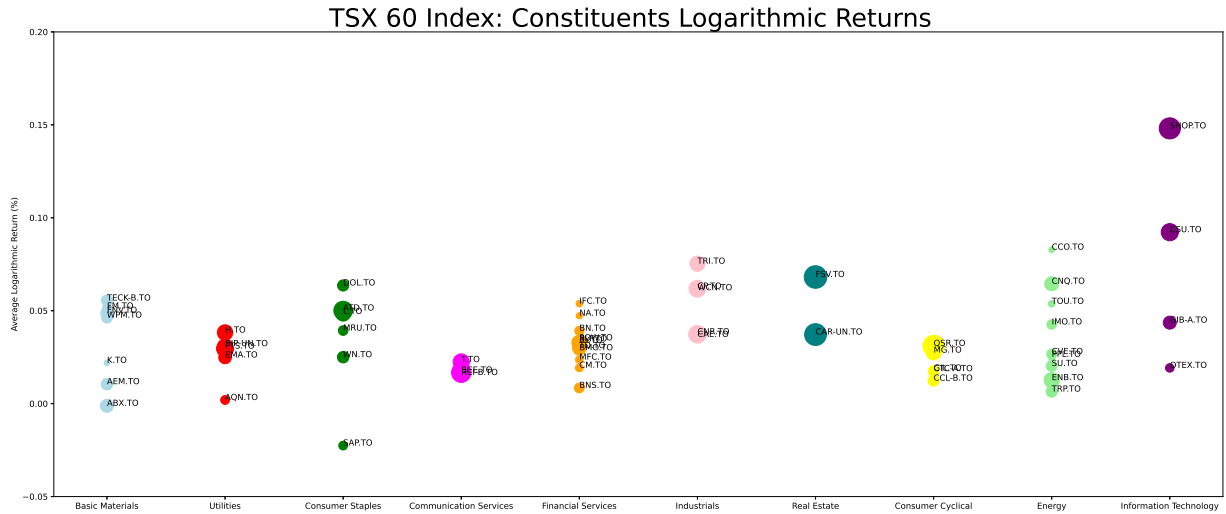


Figure 1.3: The TSX 60: Constituents Stock Average Daily Log Returns. Note: The size of points in the graph captures the rank of market capitalization of each stock.

The data used in this study contains daily closing prices with adjustments, from January 4, 2017 to September 29, 2023 for all TSX 60 constituents, except Nutrien Ltd (*NTR*).<sup>5</sup> Nutrien Ltd is a fertilizer company listed on the TSX since January 2018 but is excluded from the dataset because of missing data in 2017. Excluding *NTR* should not distort the analysis of the TSX 60 constituents network as the market capitalization of *NTR* is around 35 billion in 2023, which is relative small in comparison to the total market capitalization. Figure 1.3 provides the average of daily log returns over the entire sample period for each stock within its respective sector.

**US Stock Market** The performance of the US stock market is explored using data from the Nasdaq 100 constituents. This index excludes the financial sector, providing a unique focus on the high-tech and growing sector, unlike the TSX 60 composition. The data used for the U.S. market are daily closing prices with adjustments from January 4, 2017 to September 29, 2023 for the Nasdaq 100 constituents, excluding 11 companies listed on the Nasdaq over the sample period. The capitalization of these eleven companies was 470 billion in 2023, which is 2% of the total capitalization of the Nasdaq 100 constituents.<sup>6</sup>

<sup>5</sup>The daily closing stock price is adjusted for any split, dividend and merging.

<sup>6</sup>The eleven companies listed during the sample period present a missing value problem. Given that these companies account for only 2% of the Nasdaq 100 total capitalization, we have excluded them from our network analysis to ensure consistency over time.

Figure 1.4 depicts the average of daily log returns over the entire sample period for each stock regrouped by their corresponding sector.

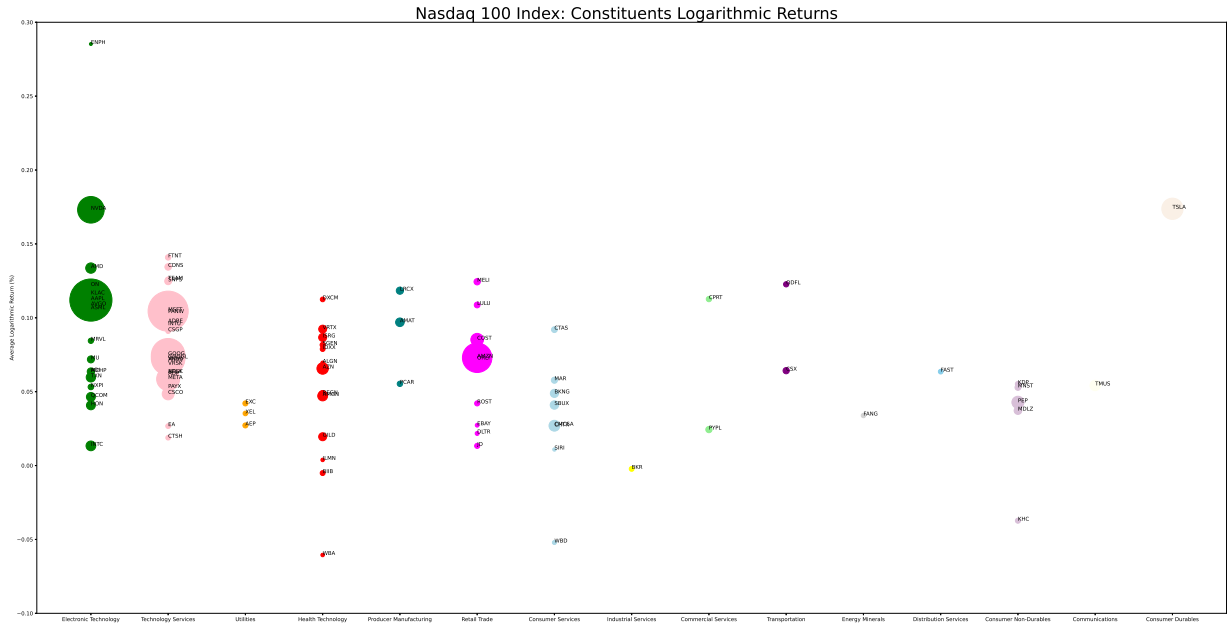


Figure 1.4: The Nasdaq 100 Constituents Stock Average Daily Log Returns. Note: The size of points in the graph captures the rank of market capitalization of each stock.

Figure 1.5 represents the sectoral distribution of the TSX 60 and the Nasdaq 100. The left panel shows that the technology service and electronic technology sectors comprise more than 70% of the total capitalization of the Nasdaq 100. Meanwhile, the right panel indicates that information technology accounts for only 6% of the TSX 60 capitalization. The capitalization of TSX 60 is primarily driven by the financials, energy, industrials, and basic materials sectors. Additionally, besides the TSX 60 and the Nasdaq 100, the dataset includes the West Texas Intermediate (WTI) oil price as an indicator of global energy prices to capture volatility in the global energy market.

## Nasdaq 100 Index and TSX 60 Index Sector Distribution by Market Capitalization

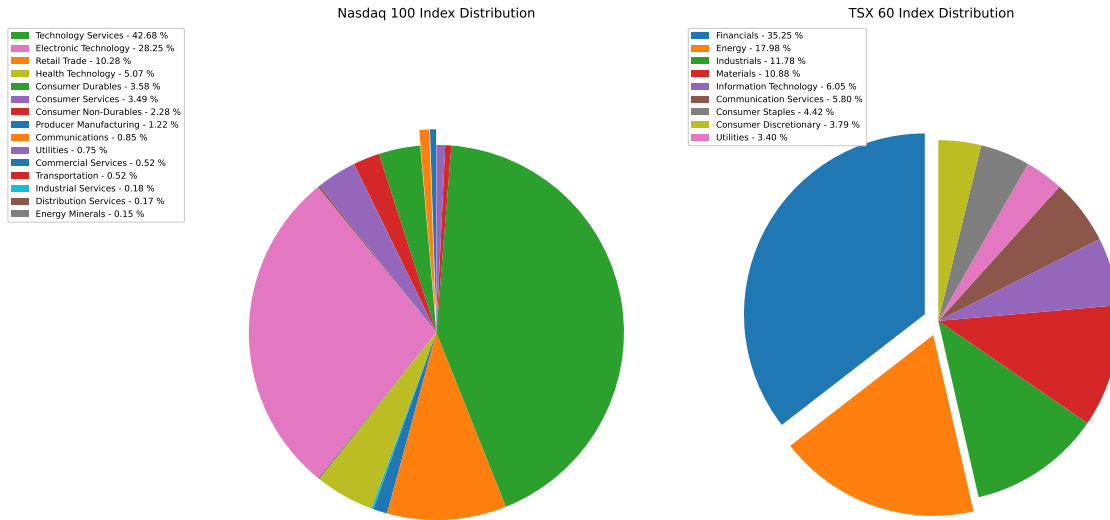


Figure 1.5: The Sectoral Distribution of The TSX 60 Constituents and The Nasdaq 100 Constituents

### 1.3.2 Data Processing

The logarithmic return is commonly used to assess stock performance because it is time-additive and symmetric. In this chapter, the stock return is defined as  $R_{t,i} = \log P_{t+1,i} - \log P_{t,i}$  where  $P_{t,i}$  is the daily adjusted closing price of the stock  $i$  at time  $t$ . Table 1.1 and Table 1.2 provide a summary of descriptive statistics for the logarithmic returns of the TSX 60 constituents and the Nasdaq 100 constituents. We explore the network structure evolution for both markets over a period including the COVID-19 crisis. The concept of volatility is used to detect the COVID-19 shock in the stock market. Volatility is a conventional measurement for quantifying a stock's underlying risk. The higher the volatility, the larger the price fluctuation. Figure 1.6 illustrates the sliding mean (left panel) and standard deviation (right panel) of the TSX 60 and Nasdaq 100 constituents over the sample period using a 30-day time window. The red lines represent the average of the sliding means and standard deviations for all individual stocks (blue lines). The graph shows a significant market fluctuation at the beginning of 2020 due to a dramatic change in the sliding standard deviations and sliding means. On 11 March 2020, the World

Health Organization (WHO) declared a global pandemic. This declaration coincides with the turbulence described in the data.

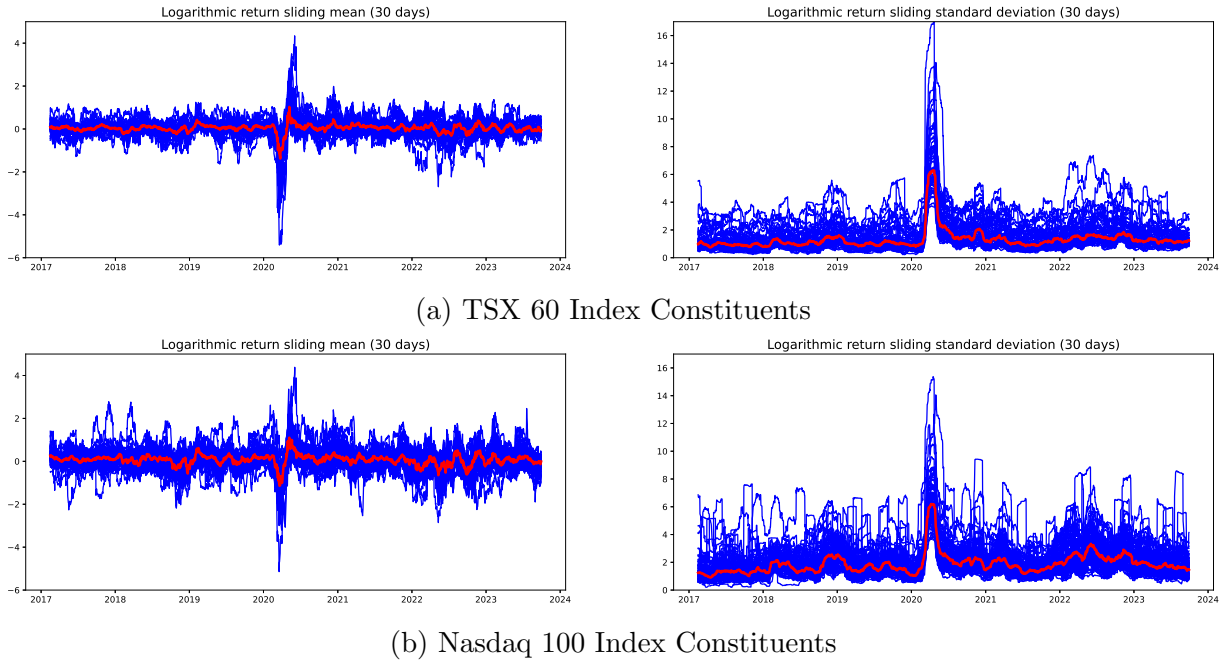


Figure 1.6: Detecting the Presence of Turbulence

Therefore, for our analysis, the data are divided into three sub-samples to examine the impact of COVID-19 on the North American stock market. Data from January 2017 to December 2019 (inclusive) are defined as the “before COVID-19” period; data from January 2020 to December 2021 (inclusive) are treated as the “COVID-19” period; the remaining data are used to measure the “post COVID-19” period or rate hiking period.<sup>7</sup> Figure 1.7 and 1.8 depict the average daily logarithmic return of each stocks over the selected periods for both markets. The comparison of different periods of stock returns suggests that the global pandemic has caused considerable impacts on the stock market, with different sectors exhibiting varying reactions. As illustrated in Figure 1.7, the IT, energy, real estate, financial services, and utilities sectors in the Canadian stock market experienced either positive or negative shocks during the three sample periods. While Figure 1.8 shows, for Nasdaq 100, that most high-tech giants have performed well during the COVID-19 period, and have quickly recovered from the market crash. The performance

<sup>7</sup>Both Bank of Canada and U.S. Federal Reserve initiated an aggressive rate hike to cool inflation in March 2022

of the electronic technology sector is showing a tendency to exceed that of the technology service sector in the post-COVID-19 period.

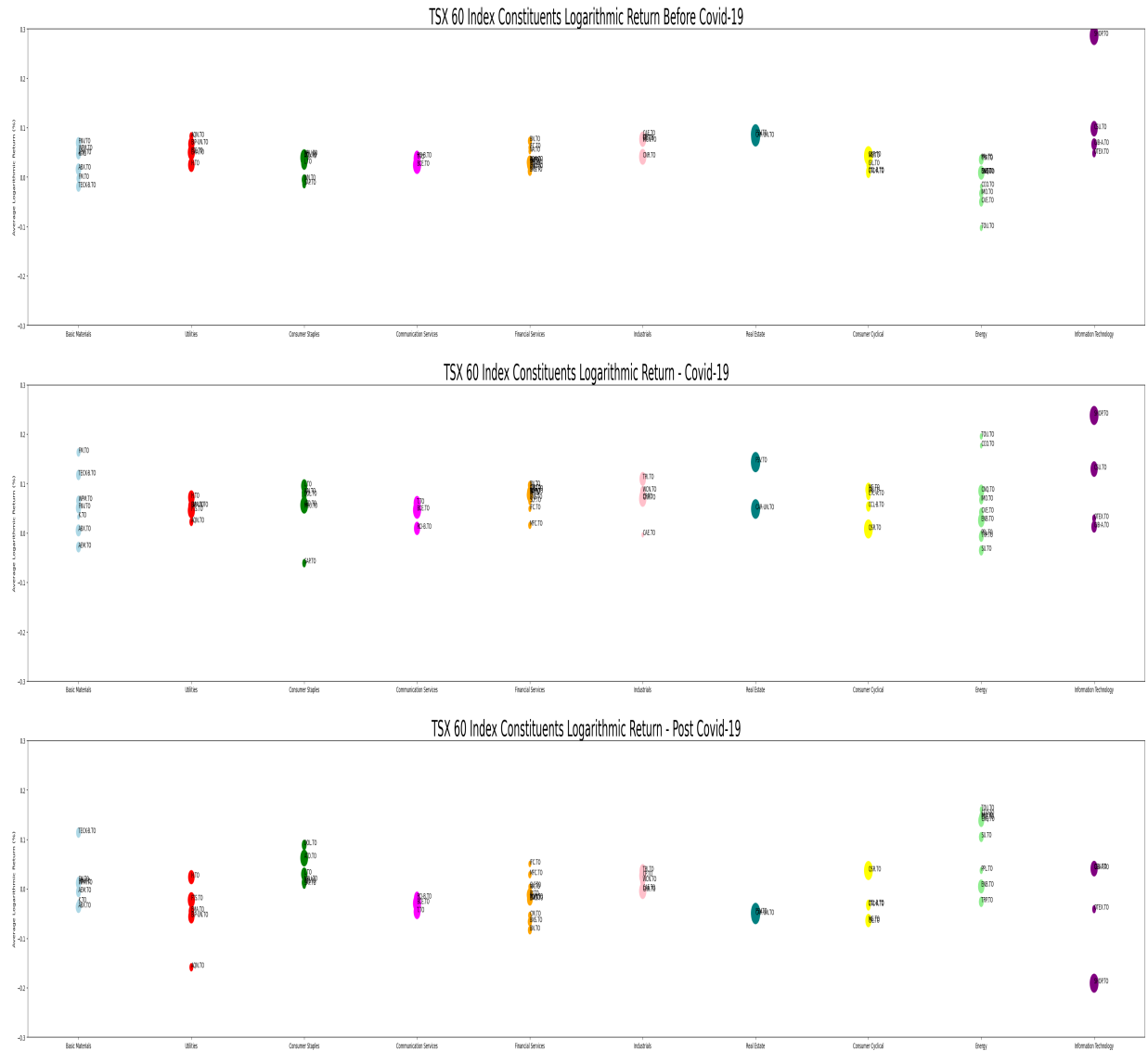


Figure 1.7: The TSX 60 Constituents Stock: Average Daily Log Returns across Different Periods.

Note: The size of points in the graph captures the rank of market capitalization of each stock.

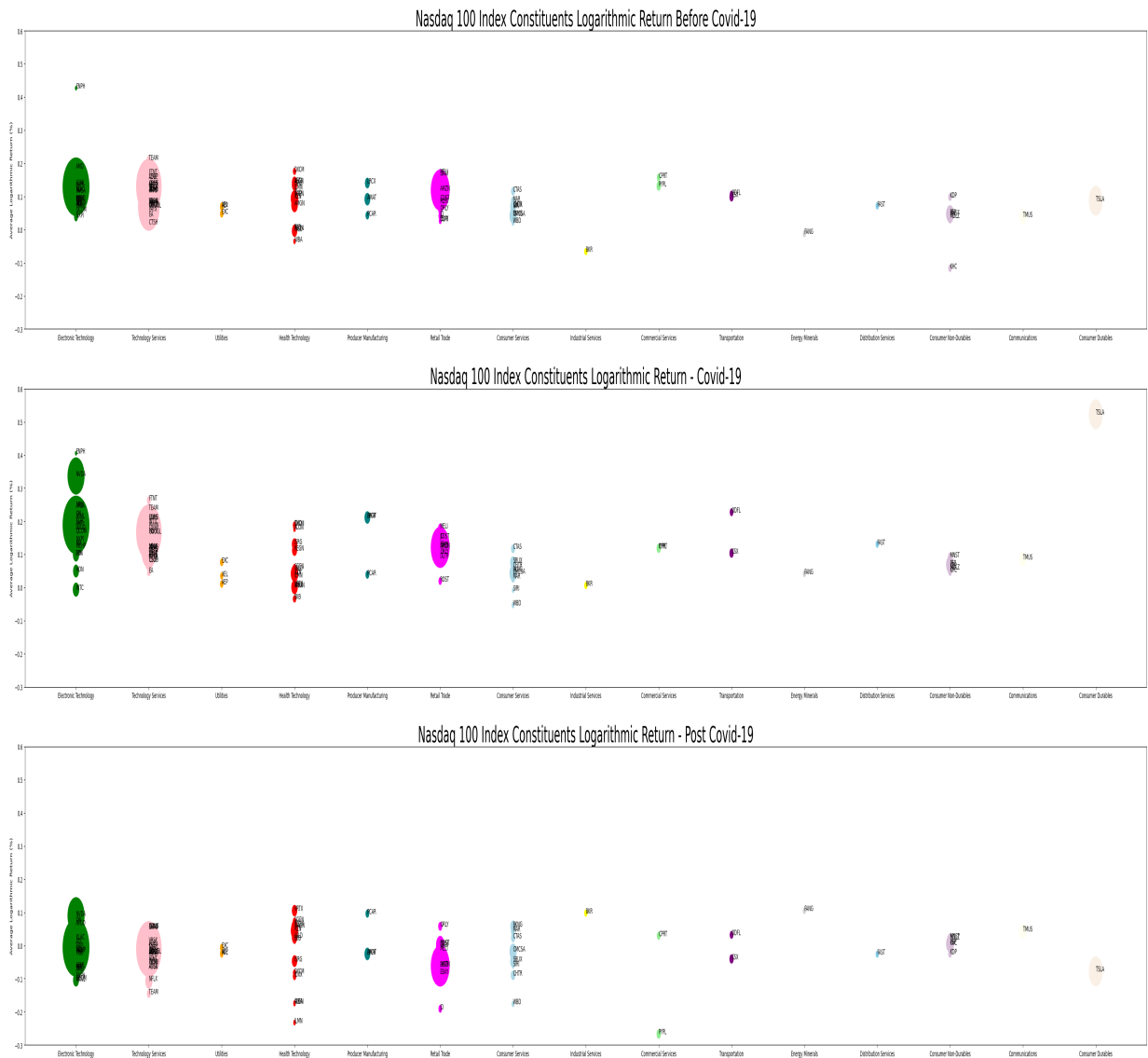


Figure 1.8: The Nasdaq 100 Constituents: Stock Average Daily Log Returns across Different Periods.

Note: The size of points in the graph captures the rank of market capitalization of each stock.

## 1.4 Edge Identification Strategy

### 1.4.1 Precision Matrix

#### Correlation versus Partial Correlation

Both Pearson and partial correlations are widely used measurements to assess linear relationships. The beta noise in financial market may alter the true correlation between equity returns if we only use Pearson correlation to capture the connection between stocks.

However, the partial correlation not only measures the relationship between two variables but also controls the influence of other variables. Assume a set of stock returns,  $V = \{R_1, \dots, R_n\}$ , where  $n$  refers to the  $n$ -th stock. We want to know the partial correlation  $\rho_{(R_i, R_j)}$  between variable  $R_i$  and variable  $R_j$  given all others returns in  $V$ , noted as  $V \setminus \{(R_i, R_j)\}$ . Suppose the covariance matrix  $\Sigma = (\sigma_{ij})$  is positive definite and invertible and given that the precision matrix is defined as  $\Omega = \Sigma^{-1}$ , then the partial correlation is derived as:

$$\rho_{(R_i, R_j | V \setminus \{(R_i, R_j)\})} = -\frac{[\Omega_{11}]_{12}}{\sqrt{[\Omega_{11}]_{22} [\Omega_{11}]_{11}}}. \quad (1.1)$$

Technically, the precision matrix is the inverse of the covariance matrix and is negatively proportional to the partial correlation.<sup>8</sup> The precision matrix is useful because it can be used to construct an undirected graphical network. The following numerical example illustrates the distinction between Pearson correlation and partial correlation. Assume  $X, Y, Z$  are three variables.  $X$  is normally distributed with  $\mu = 3$  and  $\sigma = 4$ ;  $Y$  is the sum of  $X$  and a normally distributed variable with  $\mu = 3$  and  $\sigma = 4$ ;  $Z$  is the sum of  $X$  and a normally distributed variable with  $\mu = 3$  and  $\sigma = 4$ . By construction,  $X$  is linearly correlated with  $Y$  and  $Z$  but  $Y$  and  $Z$  are less correlated. We randomly generated 100 numbers for  $X, Y$  and  $Z$  based on the above distribution settings. Then, we can easily derive the Pearson correlation matrix:

$$\begin{array}{ccc} X & Y & Z \\ \left( \begin{array}{ccc} 1 & 0.71 & 0.74 \\ 0.71 & 1 & 0.53 \\ 0.74 & 0.53 & 1 \end{array} \right) & \begin{array}{l} X \\ Y \\ Z \end{array} & , \end{array}$$

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<sup>8</sup>Appendix A.1 provides the details for deriving Equation 1.1 from a covariance matrix.

while the partial correlation matrix is, given Equation 1.1, computed as:

$$\begin{matrix} & X & Y & Z \\ \begin{pmatrix} 1 & 0.547 & 0.613 \\ 0.547 & 1 & 0.018 \\ 0.613 & 0.018 & 1 \end{pmatrix} & X \\ & Y \\ & Z \end{matrix} \cdot$$

The matrices above show that the Pearson correlation between  $Y$  and  $Z$  is 0.53, while the partial correlation is reduced to 0.018.<sup>9</sup> This indicates that the partial correlation can eliminate or reduce market impacts when examining the connections between stocks. The precision matrix enables the calculation of partial correlations, which can then be used to construct an undirected and weighted graph network. For our sample, we may construct the following network (see Figure 1.9) comprising the nodes  $X$ ,  $Y$ , and  $Z$ , with weighted edges representing the partial correlations between nodes.

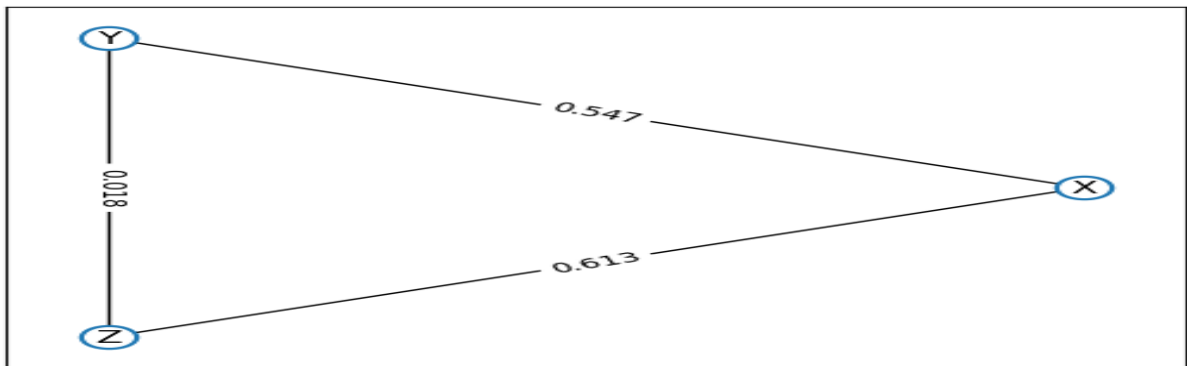


Figure 1.9: Sample Network

<sup>9</sup>Appendix A.2 outlines the details for deriving a partial correlation matrix from a covariance matrix.

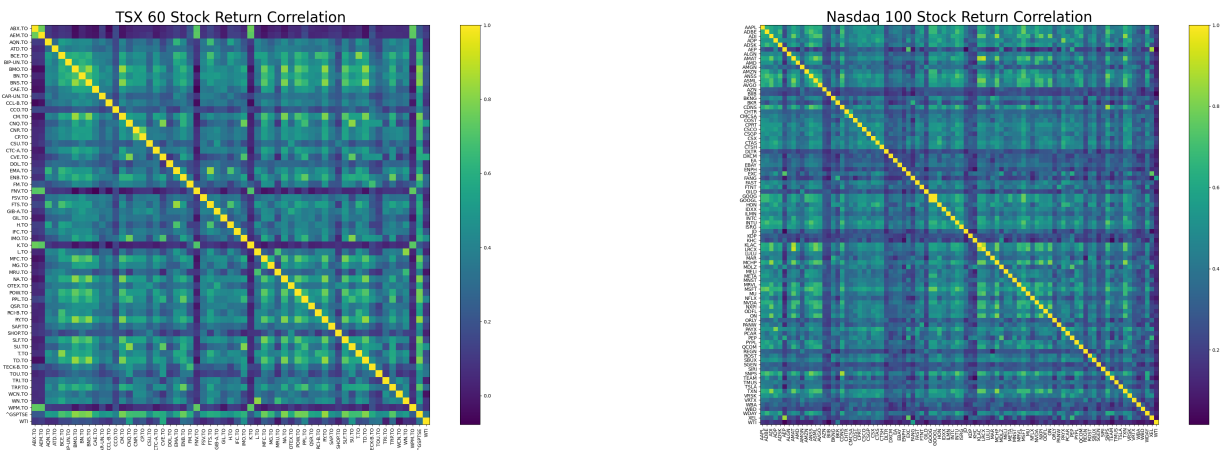


Figure 1.10: The Pearson Correlation Heatmap

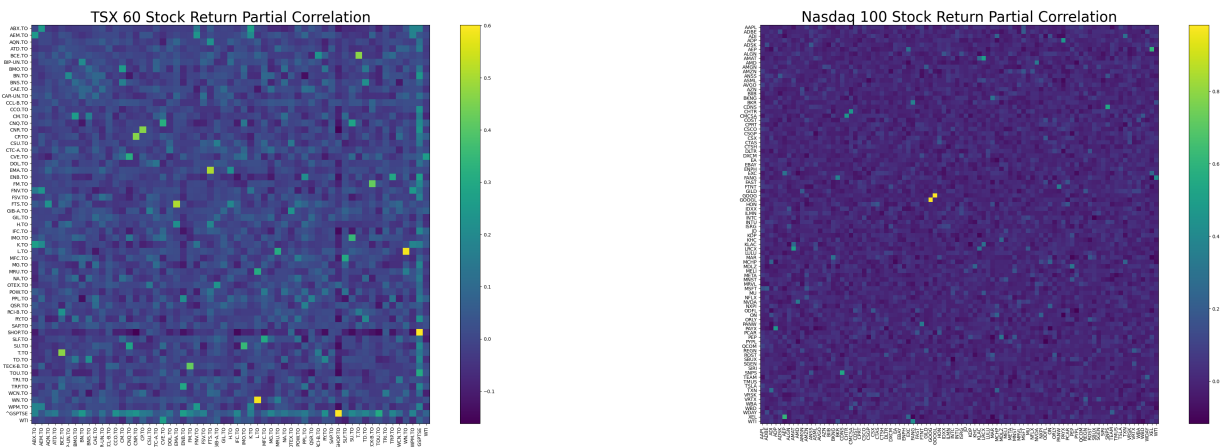


Figure 1.11: The Partial Correlation Heatmap

Figure 1.10 shows the Pearson correlation heatmap for the TSX 60 and Nasdaq 100. Many blocks are in brighter (e.g. light green) colours (i.e. Pearson correlation coefficient  $> 0.4$ ) which indicates high connectivity between stocks over the sample period. In contrast, Figure 1.11 shows that numerous partial correlation blocks are in darker colours (i.e. Partial correlation coefficient  $< 0.4$ ) for both markets.

### Precision Matrix Optimization

Although partial correlation measurements can filter out beta noise, they still imply complex networks due to numerous potential edges. For example, our Nasdaq 100 dataset has 90 stocks with a maximum of 4005 potential edges for an undirected network, making it

difficult to visualize and investigate.<sup>10</sup> Therefore, we use the GLASSO technique to remove insignificant connections. Friedman et al. (2008) propose the GLASSO method based on the normality assumption. This is a sparse network estimation algorithm using  $L_1$  regularization. Suppose the covariance matrix of logarithmic return data is  $\Sigma$ , then the precision matrix is  $\Omega = \Sigma^{-1}$ . Let  $S$  be the empirical covariance matrix, which is the covariance calculated from a sample of data rather than from a theoretical distribution. The GLASSO algorithm maximizes the  $L_1$  penalized log-likelihood function:<sup>11</sup>

$$\log \det(\Omega) - \text{tr}(S\Omega) - \alpha \|\Omega\|_1, \quad (1.2)$$

or equivalently, the algorithm searches for the precision matrix,  $\bar{\Omega}$ , which minimizes the following expression:

$$\bar{\Omega} = \arg \min_{\Omega} (\text{tr}(S\Omega) - \log \det(\Omega) + \alpha \|\Omega\|_1), \quad (1.3)$$

where  $\|\Omega\|_1$  is the sum of the absolute values of off-diagonal elements of  $\Omega$ , and  $\alpha$  is a deterministic vector of non-negative penalty parameters for the GLASSO that controls the sparsity of the network. Higher values of  $\alpha$  lead to a sparser network. This section uses a cross-validation methodology to find the optimal regularization parameter that balances sparsity and informative level in the constructed network.<sup>12</sup>

Although it is essential to investigate the network over the complete dataset (i.e. over the whole period considered), it might also be interesting to study subperiods. Hallac et al. (2017) propose the time-varying graphical lasso (TVGL) method to study the network

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<sup>10</sup>There are eleven companies that were listed during the sample period, resulting in a missing value problem (see section 1.3.1). Therefore, we removed these eleven companies while adding the WTI oil price, so that we have 90 nodes/stocks in the Nasdaq 100 network. The number of potential edges in an undirected network is given as  $\frac{n*(n-1)}{2}$ , where  $n = 90$  in our example, implying 4005 potential edges.

<sup>11</sup>Scikit-learn python package is used to implement the GLASSO algorithm (see Pedregosa et al. 2011).

<sup>12</sup>The  $\alpha$  is a predetermined parameter. If the value of the parameter  $\alpha$  is sufficiently large, the GLASSO will yield only a few connections in the network resulting in the elimination of some crucial connections within the network. In contrast, if  $\alpha$  is small which imposes a small penalty on the equation 1.3, then the network will contain many insignificant connections. The cross-validation uses the scikit-learn python package. The search for the optimal penalization parameter ( $\alpha$ ) is conducted on an iteratively refined grid. At the beginning of the process, the cross-validated scores on the grid are calculated. Subsequently, a new, refined grid is established, centered around the maximum. This process is repeated until convergence is achieved.

over time while controlling for the intertemporal network consistency. In other words, the algorithm optimizes each sub-sample network with the consideration of the overall network structure consistency. This approach allows for the creation of subperiod networks containing some degree of network structure similarity over time. Tomasi et al. (2018a) improve the convergence rate of TVGL by using a forward-backward splitting (FBS) procedure, which eventually permits a high dimensional network analysis. The TVGL network analysis is similar to the above GLASSO problem, but the analysis further decomposes the precision matrix over time period (i.e.  $1, 2, 3, \dots, T$ ) with the control of intertemporal smoothness. The identification strategy is defined as follows:

$$\min_{\Omega_1, \dots, \Omega_T} \sum_{t=1}^T [-n_t(\log \det(\Omega_t) - \text{tr}(S_t \Omega_t)) + \alpha \|\Omega_t\|_1] + \beta \sum_{t=1}^{T-1} \psi(\Omega_{t+1} - \Omega_t). \quad (1.4)$$

The TVGL minimizes equation 1.4 by choosing the temporal precision matrices  $\Omega = (\Omega_1, \dots, \Omega_T)$  and  $n_t$  represents the number of observations during each temporal period. Parameter  $\alpha$  controls the sparsity of the network, and  $\beta$  controls the degree of changes among the intertemporal precision matrices. The  $\psi(\ast)$  is a convex penalty function that controls the similarity between  $\Omega_{t+1}$  and  $\Omega_t$ . It has different selections reflecting various evolutionary scenarios. The  $L_1$  penalty is recommended to capture occasional changes in structure. When the  $ij$ -th edge of the network undergoes a break or a change at two neighboring times, this penalty still serves to encourage the remainder of the graph to remain unaltered (Hallac et al., 2017).

Additionally, the latent variable time-varying graphical lasso (LTGL) is proposed as a solution to address hidden or unmeasurable factors that influence the system without being explicitly measured in a dynamic network. LTGL essentially is an expansion of equation 1.4 of the TVGL representation by considering the dynamics of latent variables in an optimization process. The ground truth precision matrix  $\Sigma^{-1}$  is a combination of both latent and observed variable, such as  $\Sigma^{-1} = \Omega - L$ , where  $L$  provides the summary of marginalisation over latent variables on the observed ones (Tomasi et al., 2018b). The

optimization problem is defined as:

$$\begin{aligned} \min_{\substack{(\Omega, L) \\ L_t \succ 0}} & \sum_{t=1}^T [-n_t(\log \det((\Omega_t - L_t)) - \text{tr}(S_t(\Omega_t - L_t))) + \alpha \|\Omega_t\|_1 + \tau \|L_t\|_*] \\ & + \beta \sum_{t=1}^{T-1} \Psi(\Omega_{t+1} - \Omega_t) + \eta \sum_{t=1}^{T-1} \Phi(L_{t+1} - L_t), \end{aligned} \quad (1.5)$$

where  $\Psi$  and  $\Phi$  are the Laplacian penalty functions (i.e.  $\sum_{ij} (\cdot_{ij})^2$ ) to control the evolution of temporal precision matrices and latent variables over time.  $\|\Omega_t\|_1$  is the sum of the absolute values of off-diagonal elements of  $\Omega_t$ .  $\|L_t\|_*$  is called nuclear norm (a.k.a. trace norm), which is defined as the sum of the singular values of  $L_t$ . The parameters  $\beta$  and  $\eta$  control respectively the smoothness of temporal precision matrices, and the influence of the latent variables, while  $\alpha$  and  $\tau$  controls the sparsity of each temporal network including latent variables. The TVGL requires four penalty parameters (i.e.  $\beta, \alpha, \eta$  and  $\tau$ ) to analyze network dynamics. Therefore, we use Bayesian optimization to search for the model parameters.<sup>13</sup> Table 1.6 summarizes the optimized parameters for both the Nasdaq 100 and TSX 60 markets. The penalty parameters for sparsity ( $\alpha$  &  $\tau$ ) are much higher in the Nasdaq 100 market compared to the TSX 60, suggesting that a large number of insignificant edges are removed from the Nasdaq 100 network. We will provide network results from this approach in Section 1.5.1.

## 1.4.2 Minimum Spanning Tree

The GLASSO method explained in section 1.4.1 permits to filter out insignificant connections. Another procedure that permits to reduce the number of edges is a spanning tree. This is a subgraph of a connected undirected network comprising all the  $N$  nodes of the initial network with precisely  $N-1$  edges, and it is constrained by the requirement that the tree cannot form a cycle.<sup>14</sup> The minimum spanning tree (MST) is a spanning tree that

<sup>13</sup>Appendix B illustrates the details of optimization results.

<sup>14</sup>A cycle in a network is a closed loop. If we choose a node from the cycle as the start and end points, there is a path that connects the “start” and “end” points without revisiting any intermediate nodes.

must select the connected pairs of nodes so as to minimize the sum of the weights of the  $N - 1$  edges.<sup>15</sup> This feature has been widely used in analyzing financial networks (for example, see Niu and Hu 2021, Huang et al. 2016 and Kristoufek et al. 2012).

It is important to note that the weights in the analysed network are not the correlation/precision matrix of Section 1.4.1 (nor the mutual information matrix reviewed later in Section 1.4.3). Instead the weights are given by a distance metric. In other words, we have to convert the correlation matrix to a distance matrix so that the axioms of an Euclidean metric are satisfied (see Niu and Hu 2021, Huang et al. 2016 and Mantegna 1999). The transformation is defined as:

$$d_{i,j} = \sqrt{2(a - m_{i,j})}, \quad (1.6)$$

where  $m_{i,j}$  is either the partial correlation measurement or mutual information measurement (see Section 1.4.3) for stock  $i$  and stock  $j$ , and  $a$  is the maximum value over the measurements.<sup>16</sup> With the aforementioned transformation, the Kruskal's algorithm is then used to minimize the sum of the distance metrics  $d_{i,j}$  to generate a MST. The rationale behind this transformation is that it allows MST to provide a path that contains the largest correlation among stocks due to the negative sign in Equation 1.6. The pseudocode of Kruskal's algorithm and a graphical approach to the illustration of the algorithm are provided in Appendix C. The MST is applied to our stocks networks in both Section 1.5.1 and 1.5.2.

### 1.4.3 Mutual Information

Partial correlation measurement is limited to capture linear correlation among stocks. However, Yang et al. (2014) discuss the existence of nonlinear processes in the American stock market. Thus, we employ the mutual information measurement to identify any nonlinear correlations among stocks in both markets. The mutual information is a derivative of the concept of entropy in the field of information theory. According to Shannon (1948), discrete

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<sup>15</sup>It is important to note that a minimum spanning tree is not always unique. If all edge weights in the graph are equal, then any spanning tree of the graph is an MST.

<sup>16</sup>The partial correlation value is capped at 1, but the mutual information measurement depends on the complexity of the network.

entropy is defined as:

$$H = - \sum_{i=1}^n p_i \log p_i, \quad (1.7)$$

where  $p_i$  is the probability of occurrence for an event  $i$ . For a bivariate case, suppose the joint distribution of  $x, y$  is  $P(x, y), x \in X$  and  $y \in Y$ . The mutual information  $I(x, y)$  is used to quantify the information gained from one variable given another variable (Shannon (1948), Cover and Thomas (2006)), and is defined as:

$$I(x, y) = \sum_{x \in X} \sum_{y \in Y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)} = H(X) - H(X|Y). \quad (1.8)$$

This analysis is explained in detail in Appendix E from which Figure 1.12 is extracted. Figure 1.12 shows a Ven diagram to depict the relationship between mutual information and entropy. The mutual information,  $I(X, Y)$ , is the intersection of  $H(X)$  and  $H(Y)$ . Suppose  $X$  and  $Y$  are the stock returns. Then  $I(X, Y)$  captures how much information about the return of one stock can be inferred from the return of the other. In accordance with Equation 1.8, the quantity  $I(X, Y)$  can also be seen as the amount of reduced uncertainty regarding the stock performance  $X$  by knowing the stock performance of  $Y$ , and vice versa. A higher value of mutual information between two stocks indicates a stronger connection between these two stocks.

### Venn Diagram for Mutual Information

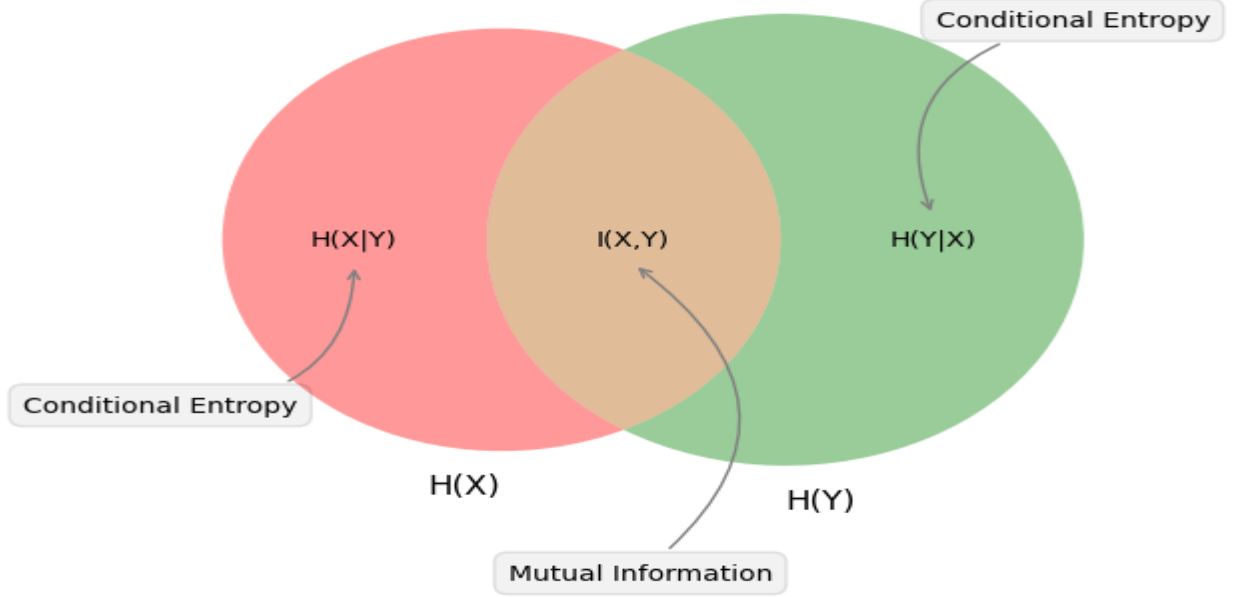


Figure 1.12: Entropy Ven Diagram

Note: Joint entropy,  $H(X, Y)$ , is the area of  $H(X) + H(Y) - I(X, Y)$ .

The stock logarithmic return is a continuous measurement, whereas the formula in Equation 1.8 measures discrete mutual information. Therefore, we have to discretize the data before calculating the mutual information. The selection of the number of bins is crucial in the context of discrete mutual information because it directly impacts the accuracy and bias of the mutual information estimate. If the number of bins is small, the discretization may fail to capture the intricate relationship between stock returns, resulting in a high degree of bias. On the other side, a large number of bins may be too granular to capture the true dependency, leading to a high degree of variance. Hacine-Gharbi et al. (2012) proposed a formula to determine the number of bins by minimizing the bias of entropy estimation. The formula for the number of bins,  $b$ , is defined as :

$$b = \text{round}\left(\frac{\vartheta}{6}\right) + \frac{2}{3\vartheta} + \frac{1}{3}, \quad (1.9)$$

where

$$\vartheta = \sqrt[3]{8 + 324N + 12\sqrt{36N + 729N^2}},$$

and  $N$  is the total number of data. We use this formula for the mutual information analysis

in Section 1.5.2.

## 1.5 Empirical Results

### 1.5.1 Partial Correlation

**TSX 60** According to the World Bank database, the value of stocks traded on the Canadian stock market was equivalent to 61.4% of GDP by the end of 2020, demonstrating its critical role in the Canadian economy.<sup>17</sup> The structure of the Canadian stock market is concentrated in the energy, basic materials, and financial services sectors, as these sectors represent more than 49% of market capitalization by the end of 2022 in both the TSX Composite Index and the TSX 60 Index. Following one month of the declaration of the COVID-19 pandemic by the World Health Organization (WHO) on March 11, 2020, the price of the WTI crude oil fell on April 20th by almost 300% and traded at -\$37 per barrel due to a collapse in oil demand.<sup>18</sup> Undoubtedly, the Canadian oil sector also entered a bear market. Later on, the global oil price peaked dramatically in the beginning of 2022 (i.e. during the “post COVID-19” period), which helped the energy sector get out of the bearish situation. Meanwhile, the change in the central bank’s monetary policy towards more restrictive stance, commencing in March 2022, enhanced the profitability of the commercial banking sector. This was due to the high mortgage rates, which increased the profit margins of the variable-rate mortgage business. Although the core constituents of the TSX 60 have experienced massive shocks over the study period (i.e. 2017-2023), the TSX 60 has delivered a stable return on investment since 2017.

Figure 1.13 depicts the network of TSX 60 constituents during the whole study period. Each node represents a stock, and the edges indicate connection between pairs of stocks. Figure 1.13a illustrates a partial correlation network. Although the GLASSO algorithm

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<sup>17</sup>The data is retrieved as of 2023-04-25 from <https://databank.worldbank.org/source/world-development-indicators/Series/CM.MKT.TRAD.GD.ZS>.

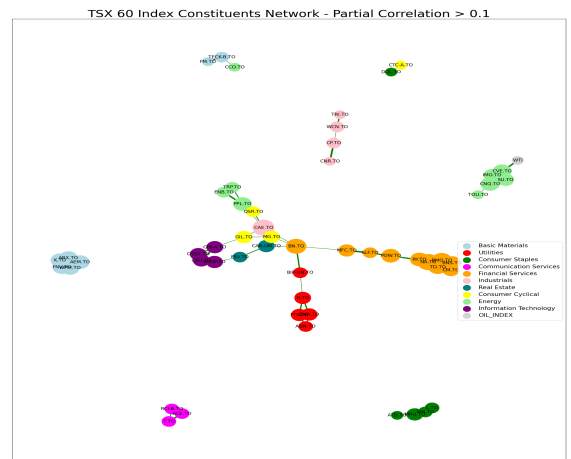
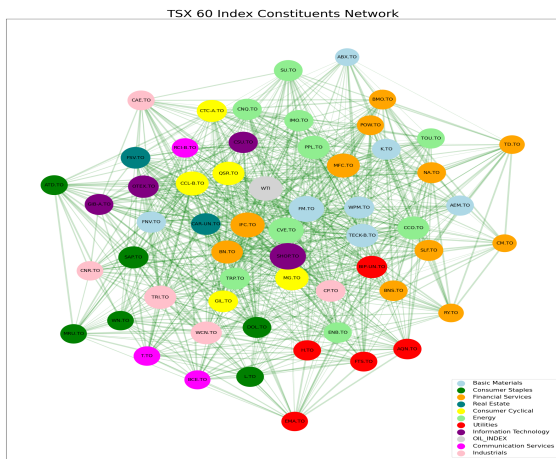
<sup>18</sup>During the COVID-19 pandemic, the demand for oil dropped sharply due to lockdown and travel restrictions, leading to an oversupply of oil. The storage facilities ran out of capacity while at the same time had to take physical possession of expiring WTI future contracts, pushing prices into negative values (for a very short time).

eliminates stocks with weak partial correlations among the TSX 60 constituents, there are still 829 edges among 58 nodes.<sup>19</sup> The left panel of Figure 1.13a provides the resulting undirected network with 829 edges. The right panel of Figure 1.13a depicts a sparse network when a threshold of 0.1 is applied to the partial correlation.<sup>20</sup> It shows several isolated sub-networks with stocks originating from the same sector. Although the largest sub-network in the center of the graph includes stocks from different sectors, the inter-sectoral connections rely on a few key nodes such as BN.TO and CAE.TO. Figure 1.13b shows a MST network with the path that captures the maximized partial correlation across stocks. The following sub-section will introduce the typical vocabulary employed in the context of networks, with a particular focus on the left panel in Figure 1.13a.

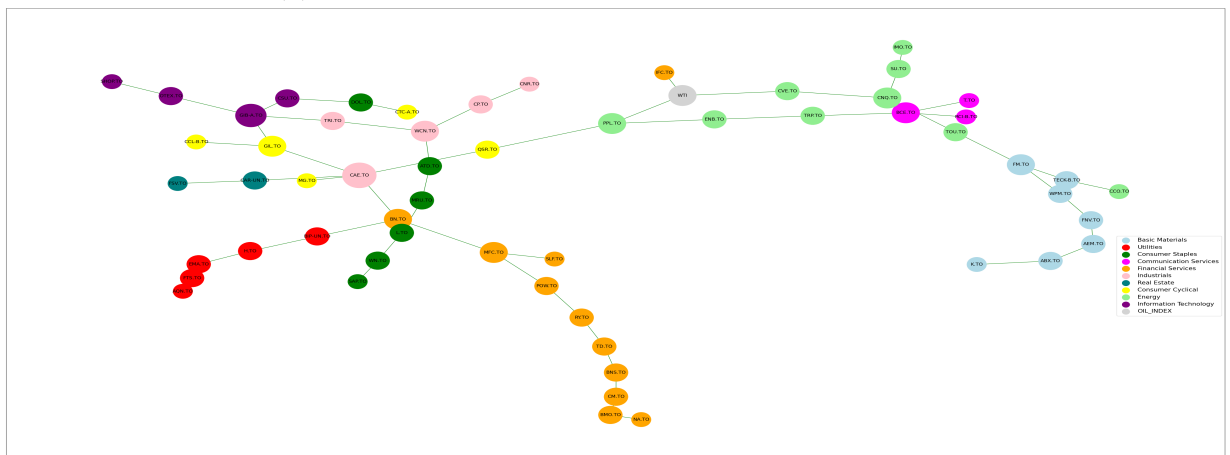
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<sup>19</sup>In an undirected network, there are  $\frac{n(n-1)}{2}$  potential edges. Since the TSX 60 has 59 nodes, the maximum number of potential edges is  $\frac{57*58}{2} = 1653$ . GLASSO yields 829 edges.

<sup>20</sup>The threshold of 0.1 was selected because it captures a meaningful partial correlation without creating pairwise networks where the majority of sub-networks would only contain two stocks. Table 1.7 shows a threshold-sensitive analysis.



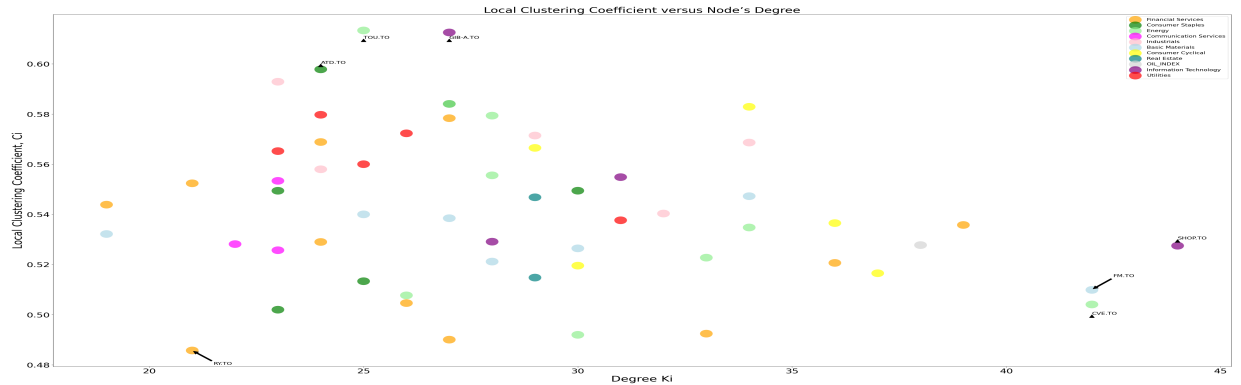
(a) TSX 60 Index Constituents Network by GLASSO



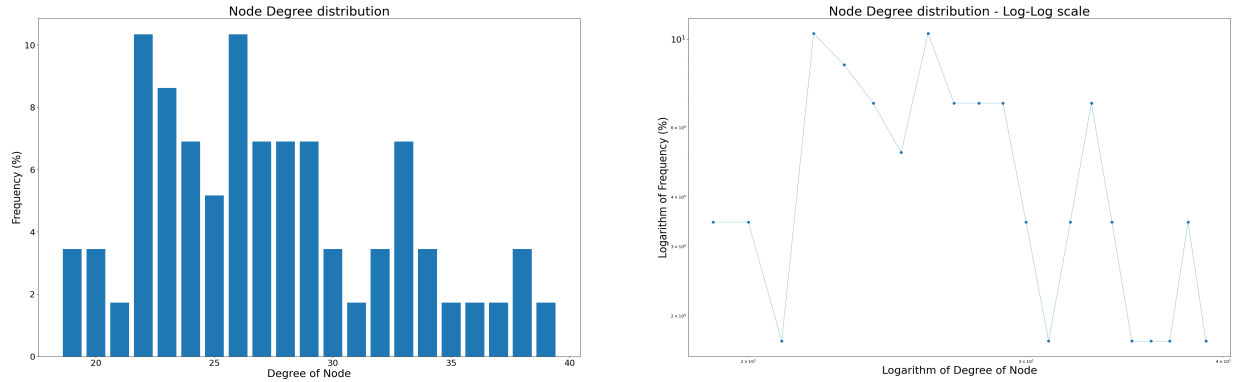
(b) TSX 60 Index Constituents Network by GLASSO and MST

Figure 1.13: TSX 60 Index Constituents Network

## Network Structure Analysis



(a) Local Clustering Coefficient versus Node Degree



(b) Distributions of Node Degree Note: mean: 4.76%, median: 3.45%

Figure 1.14: TSX 60 Index Constituents Network Node Analysis

**An analysis of the unweighted network** Quantitative metrics remain essential to shed further information that would be quite difficult to extract from the left panel of Figure 1.13a. The density of a network is the ratio of actual edges in the network, to all possible edges (an arrow between any two nodes) given by  $\frac{n*(n-1)}{2}$ , where  $n$  is the number of nodes. The network density (a number between 0 and 1) gives a sense of how closely knit the network is. In our case, we have 829 edges and 58 nodes so that the stock return network density is  $\frac{829}{58*57} = 0.50$ . Hence, the TSX 60 stock network is a relatively dense network because 50% of the edges are preserved during the optimization process (i.e., GLASSO).

The centrality measurement is widely used in the network analysis to reveal the role of a node within the network. Table 1.3 shows four centrality measurements that we used for the network analysis. First, the degree centrality is a measure of the influence of a node in a network, based on the number of direct connections, and normalized by dividing

by the maximum possible degree in a graph,  $N - 1$ , where  $N$  is the number of nodes in graph  $G$ , such as  $D_i = \frac{k_i}{n-1}$ , where  $D_i$  is the normalized degree while  $k_i$  is the non-normalized degree. Second, betweenness centrality measures the number of shortest paths that go through a node.<sup>21</sup> Intuitively, it attempts to identify nodes that are significant not because of their high degree centrality, but because of their role as a bridge between nodes, thereby providing the network with both connectivity and cohesion. Shopify serves as an illustrative example, exhibiting the highest degree centrality (0.772) yet displaying a markedly low betweenness centrality (0.021). This indicates that, despite its numerous connections with other market entities, Shopify exhibits a relatively low level of network connectivity. Third, eigenvector centrality measures the centrality of a node by considering not only the number of its connections (degree centrality) but also the importance of the nodes it is connected to.<sup>22</sup> This recursive definition implies that having a few connections to highly influential nodes can be more relevant than having many connections to less influential nodes. The last measurement is clustering coefficient. A clustering coefficient is a measure of the degree to which nodes in a graph tend to cluster together. The average clustering coefficient of the TSX 60 network is  $C = 0.543$  and is computed as the average of the local clustering coefficients of all nodes,  $C = \frac{\sum_{i=1}^N C_i}{N}$  where  $N$  is the total number of nodes. The local clustering coefficient of a node is itself defined as  $C_i = \frac{T_i}{\frac{k_i(k_i-1)}{2}}$ , where  $k_i$  is the degree of node (stock)  $i$  (i.e., the number of connections of  $i$  in the stock network) and  $\frac{k_i(k_i-1)}{2}$  is the maximum potential number of edges among the neighbours of node  $i$ .  $T_i$  is the actual number of edges among neighbours of  $i$ , and thus the number of triangles through stock  $i$ . Neighbours of  $i$  are stocks that are linked to stock  $i$ .

The local clustering coefficient can be viewed as a type of centrality measure, albeit one that takes small values for “important” nodes. For example, for  $i = \textit{Shopify}$ ,  $C_i = 0.527$  with  $T_i = 499$  edges among all neighbours of Shopify. When  $C_i$  is close to zero, node

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<sup>21</sup>The mathematical expression is  $BC(i) = \sum_{a,b \in V} \frac{\sigma_{ab}(i)}{\sigma_{ab}}$ , where  $V$  is the set of nodes,  $\sigma_{ab}$  is the total number of shortest paths from node  $a$  to node  $b$ , and  $\sigma_{ab}(i)$  is the number of shortest paths in  $\sigma_{ab}$  that pass through the node  $i$ . The shortest path is defined as the path between two nodes that minimizes the total distance (or sum of edge weights) among all possible paths connecting them.

<sup>22</sup>The mathematical expression to compute eigenvector centrality is  $Ax = \lambda x$ , where  $A$  is the adjacency matrix of the network,  $\lambda$  is the largest eigenvalue of the adjacency matrix, and  $x$  is a unique non-negative eigenvector, which provides centrality scores.

$i$  has a lot of neighbours but its neighbours are little connected between themselves and  $i$  is called a star. When  $C_i$  is close to 1, then the neighbourhood of  $i$  forms a clique of stocks ( $i$  is connected to its neighbours, and all the neighbours are connected to each others). A traditional method to assess real network results is to compare them to those obtained from a purely random network (e.g. an Erdos-Renyi graph) with same number of nodes and edges as the tested network. For instance, a well-established result is that  $C_i$  is independent of  $k_i$  in random networks while in real networks,  $C_i$  decreases with  $k_i$  (i.e., stocks with a large number of neighbours (edges) tend to have a lower value for  $C_i$  and vice-versa) (Ravasz and Barabási, 2003). Figure 1.14a depicts that Shopify (ticker: SHOP.TO) has the highest degree ( $k_i$ ) with one of the lowest local clustering coefficient ( $C_i$ ). In other words, the online shopping platform Shopify has a robust connection with other stocks in the market. However, its connected neighbours are less connected between themselves most likely because they originate from a variety of sectors. In addition, Figure 1.14a shows a somewhat loose negative relationship between individual stocks' local clustering coefficients and their degree. The intuition for this negative relationship is roughly as follows. On the one hand, stocks tend to group in a sector sharing mostly neighbours within the same sector. A stock with a low degree tends to be connected to nodes within its own sector, leading to a denser community and a relatively high local clustering coefficient. On the other hand, a stock  $i$  with a high degree tends to be connected to many stocks from several sectors, which increases the maximum (potential) number of edges among its neighbours, reducing this stock's local clustering coefficient  $C_i$ .

It is common in network analysis to provide a visual check of the node (stock) distribution of degree centrality. Sparsely connected networks have highly right-skewed degree centrality distribution – the bulk of the distribution occurring for low degree nodes with just a small number of nodes having a very high degree. They are often said to follow a Pareto or power law distribution (also known as scale-free distribution).<sup>23</sup> Scholz (2015) shows that the degree distribution of a densely connected network is very distinct from

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<sup>23</sup>A scale-free network is defined as one in which the node degree distribution follows a power law. The majority of nodes have very few connections, and a few important nodes have a huge number of connections, which enables them to exert considerable influence over the entire network.

power-law. Many densely connected networks show less difference in the distribution of low and high degree nodes. Figure 1.14b shows a histogram of node degree distribution (left panel) and log-log transformation (right panel) for the TSX 60 network. Although the left panel of Figure 1.14b indicates a loose right-skewed degree distribution, where the mean of frequency (4.76%) is greater than the median (3.45%), the distribution of low and high degree nodes is quite symmetric. This is consistent with a relatively dense network characteristic. Typically, if the distribution of node degree follows a power law, then the log-log transformation plot would show a downward slope. If this was the case, the majority of nodes in the TSX 60 network would have a relatively low number of connections, while some stocks would have a considerable large number of connections. The right panel of Figure 1.14b is a log-log plot of degree distribution. It is clear that the degree distribution of the TSX 60 network does not adhere to the characteristics of a power law distribution. Overall, these results demonstrate that the TSX 60 network exhibits a high level of connectivity and a balanced distribution of influence.

**An analysis of the weighted network** Section 1.4 outlined different methodologies for identifying the connections (i.e., edges) between stocks. The value of an edge (e.g., here the partial correlation), referred to as “weight”, contains useful information. The magnitude of the value quantifies the strength of connection, while the sign of the value indicates either a positive or negative correlation between a pair of stocks. Since the positive relationship and negative relationship are equally important in assessing the centrality of stock, the absolute value of partial correlation is used as the weight to calculate the weighted centrality. Everett and Borgatti (2014) show that eigenvector centrality has limited value for an undirected network with both positive and negative ties because the matrix may not have a dominant eigenvalue, resulting in an ambiguous rank. Therefore, the use of the absolute value of partial correlation as the weight also address the potential issue inherent in the eigenvector centrality measurement. In the MST analysis, as described in Section 1.4.2, Equation 1.6 also considers the sign of partial correlation when it is converted to a distance metric. Intuitively, if the partial correlation value is distant from the maximum of the partial

correlation value in the network, it is assigned a large distance value. As a result of the MST analysis, the node with a high distance value will be moved to the periphery of the MST (see Appendix C). It is noteworthy that mutual information, discussed in Section 1.4.3, is non-negative because it captures the strength of dependency without indicating a positive or negative direction.

This subsection compares the differences between the weighted network and the unweighted network based on the left panel of Figure 1.13a. Table 1.4 summarizes the non-normalized degree centrality, betweenness centrality, and eigenvector centrality in both weighted and unweighted networks. The unweighted centrality measurements are discussed in the preceding section. In the context of weighted centrality measurements, the absolute value of the partial correlation is employed instead of assuming that all edge weights are equal to “1”. To illustrate, the non-normalized weighted degree centrality calculates the total weight in all edges of a node. Similarly, the weighted betweenness centrality uses the absolute value of partial correlation as weights to calculate weighted shortest paths. The eigenvector centrality interprets the weight as the strength of the connection. Figure 1.15a provides a summary of the top 19 stocks with the highest value under each centrality measurement in Table 1.4. The highlighted cells indicate the presence of common stocks belonging to the same category of centrality assessment. While the weighted and unweighted centrality measurements demonstrate some discrepancy in determining the top-ranked stocks, the overall result exhibits a notable degree of overlap.

MST is another way to consider the impact of edge attributes in a weighted network. As described in Section 1.4.2, MST is an algorithm for identifying a path with the shortest distance. Table 1.5 presents a summary of centrality measurement on the TSX 60 MST. Similarly, the top 19 stocks with the highest value under each centrality measurement is depicted in Figure 1.15b. The figure indicates a notable degree of overlapping among top 19 stocks between weighted and unweighted centrality measurements, although the relative ranking of stocks exhibits minor discrepancies. The results are intuitive in two ways. Firstly, it demonstrates that Kruskal’s algorithm can incorporate weighted edge

information to construct a minimum spanning tree.<sup>24</sup> Secondly, the MST is a connected undirected network comprising N nodes and N-1 edges, wherein no cycle is formed. The weighted centrality measurement is constrained due to the significant reduction in the number of edges (i.e., connections) compared to the GLASSO-produced network. In other words, a reduction in the number of available edges results in a limitation of alternative connections for the centrality measurements.

| Degree Centrality | Betweenness Centrality | Eigenvector Centrality | Weighted Degree Centrality | Weighted Betweenness Centrality | Weighted Eigenvector Centrality |
|-------------------|------------------------|------------------------|----------------------------|---------------------------------|---------------------------------|
| SHOP              | CVE                    | SHOP                   | CVE                        | IFC                             | CVE                             |
| CVE               | FM                     | FM                     | SHOP                       | CTC-A                           | CNQ                             |
| FM                | SHOP                   | CVE                    | PPL                        | TRP                             | IMO                             |
| IFC               | IFC                    | IFC                    | FM                         | GIL                             | PPL                             |
| WTI               | WTI                    | WTI                    | FTS                        | ENB                             | SHOP                            |
| MG                | MG                     | MG                     | CNQ                        | CAR-UN                          | SU                              |
| CCL-B             | MFC                    | CCL-B                  | MFC                        | WPM                             | FM                              |
| MFC               | CCL-B                  | MFC                    | MG                         | H                               | TECK-B                          |
| PPL               | BN                     | QSR                    | IMO                        | EMA                             | WTI                             |
| QSR               | CCO                    | TECK-B                 | BN                         | CCO                             | MFC                             |
| TECK-B            | PPL                    | TRI                    | TECK-B                     | OTEX                            | MG                              |
| TRI               | TECK-B                 | PPL                    | WTI                        | FNV                             | K                               |
| CCO               | TRP                    | CCO                    | K                          | SHOP                            | FTS                             |
| BN                | TRI                    | WCN                    | L                          | WCN                             | BN                              |
| WCN               | WCN                    | BN                     | BMO                        | L                               | AEM                             |
| OTEX              | BIP-UN                 | OTEX                   | BCE                        | MFC                             | WPM                             |
| BIP-UN            | QSR                    | BIP-UN                 | AEM                        | CP                              | FNV                             |
| WPM               | CAR-UN                 | SAP                    | WPM                        | AEM                             | SLF                             |
| CTC-A             | CTC-A                  | GIL                    | SU                         | CVE                             | QSR                             |

(a) TSX 60 Network Top 19 Stocks in Centrality Analysis

| Degree Centrality | Betweenness Centrality | Eigenvector Centrality | Weighted Degree Centrality | Weighted Betweenness Centrality | Weighted Eigenvector Centrality |
|-------------------|------------------------|------------------------|----------------------------|---------------------------------|---------------------------------|
| CAE               | CAE                    | CAE                    | CAE                        | CAE                             | CAE                             |
| GIB-A             | PPL                    | GIL                    | GIB-A                      | PPL                             | GIL                             |
| MFC               | QSR                    | BN                     | GIL                        | QSR                             | BN                              |
| PPL               | GIL                    | GIB-A                  | WCN                        | GIL                             | GIB-A                           |
| BN                | GIB-A                  | QSR                    | BN                         | BN                              | QSR                             |
| GIL               | BN                     | CAR-UN                 | WTI                        | GIB-A                           | CAR-UN                          |
| WTI               | WTI                    | MG                     | PPL                        | WTI                             | MG                              |
| WCN               | CVE                    | MFC                    | MFC                        | CVE                             | MFC                             |
| BCE               | CNQ                    | PPL                    | FM                         | CNQ                             | CCL-B                           |
| FM                | MFC                    | BIP-UN                 | CNQ                        | MFC                             | TRI                             |
| CNQ               | TOU                    | TRI                    | BCE                        | TOU                             | PPL                             |
| ABX               | TRI                    | CCL-B                  | ATD                        | TRI                             | BIP-UN                          |
| AEM               | WCN                    | CSU                    | TOU                        | FM                              | CSU                             |
| ATD               | FM                     | OTEX                   | DOL                        | WCN                             | OTEX                            |
| BMO               | POW                    | WTI                    | CAR-UN                     | POW                             | FSV                             |
| CAR-UN            | RY                     | FSV                    | TRI                        | RY                              | WCN                             |
| FNV               | ATD                    | POW                    | CSU                        | BIP-UN                          | WTI                             |
| FTS               | BIP-UN                 | WCN                    | QSR                        | ENB                             | POW                             |
| H                 | WPM                    | ENB                    | POW                        | TD                              | DOL                             |

(b) TSX 60 MST Top 19 Stocks in Centrality Analysis

Figure 1.15: Weighted and Unweighted Centrality Analysis - TSX 60 Networks

**Group Centrality** Given the limited market influence of individual stocks, their connections provide little insight into the broader systemic relationships. To address this

<sup>24</sup>The pseudocode of Kruskal’s algorithm is provided in Appendix C

constraint, we aggregate stocks at the sectoral level to analyze their collective influence within the network. Throughout this study, group degree centrality and group betweenness centrality serve as key metrics for assessing these sectoral interactions. Unlike the stock level centrality measurement, the group centrality first defines a subset of nodes  $S$ , then calculate the centrality of  $S$ . For instance, the group degree centrality measures the importance of a selected group  $S$  in a network. It is defined as the number of non-group nodes that are connected to group members. The group betweenness centrality of  $S$  is the sum of the proportion of all-pairs shortest paths connecting pairs of non-group members that pass through a group  $S$ .<sup>25</sup> In the context of a weighted network, the weight in group betweenness centrality is interpreted as the length between two nodes. The length between two nodes represents the path connecting them within the network. Essentially, a high group betweenness centrality value implies that this group is an important “core bridge” within the network. The correlation between the group betweenness centrality and weighted group betweenness centrality for the TSX 60 network is 0.9, indicating that the betweenness centrality measurement at the aggregated level is less sensitive to the edge weight. Therefore, the group centrality measures presented in the following sections abstract away from weights computation.

### Partial Correlation Threshold and MST analysis

**TSX 60** Table 1.7 provides a summary of threshold analysis on the TSX 60 partial correlation network. As the partial correlation threshold increases from 0 to 0.1, the overall network average connectedness or network density (i.e.,  $\frac{edges}{\frac{n*(n-1)}{2}}$ ) decreases from 0.5 to 0.05. Figure 1.16 depicts the changes in the number of nodes and edges in the TSX 60 network under different thresholds. Figure 1.16a illustrates that the number of nodes (and, consequently, the number of potential edges) remains relatively constant across varying threshold settings. However, Figure 1.16b shows a notable decline in the number of actual edges as the threshold increases. Therefore, the density is largely driven by the deleted

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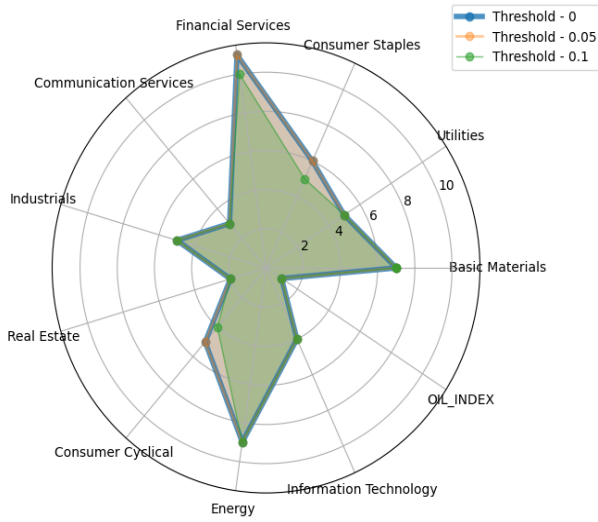
<sup>25</sup>The mathematical expression is  $GBC(S) = \sum_{a,b \in V \setminus S} \frac{\sigma_{a,b}(S)}{\sigma_{a,b}}$ , where  $V$  is the set of nodes (stocks),  $\sigma_{a,b}$  is the number of shortest  $(a, b)$  paths for a pair of  $(a, b)$  that is not number of the group  $S$  (i.e.,  $V \setminus S$ ).  $\sigma_{a,b}(S)$  is the number of those paths passing through some node in group  $S$ .

weak connections. In addition, it shows that numerous weak connections are associated with the financial services, industrials, basic materials, consumer cyclical and energy sectors, as they experience the greatest loss of edges as the threshold is increased. However, when the sectors are compared under the same thresholds, the data points on these sectors are markedly distant from the origin, and the shape of the radar chart remains largely unchanged as the threshold is increased (see also the zoom-in view in Figure 1.16c). This indicates that these sectors consistently exert a considerable influence on other stocks due to their large number of edges (connections) in the network. Figure 1.16 depicts the sectoral results derived from the calculation of stock level connections. However, it does not distinguish the connections in each sector that are either inter-sectoral or intra-sectoral. It is therefore worthwhile to investigate the cross-sector (or inter-sectoral) connections and centrality measures.

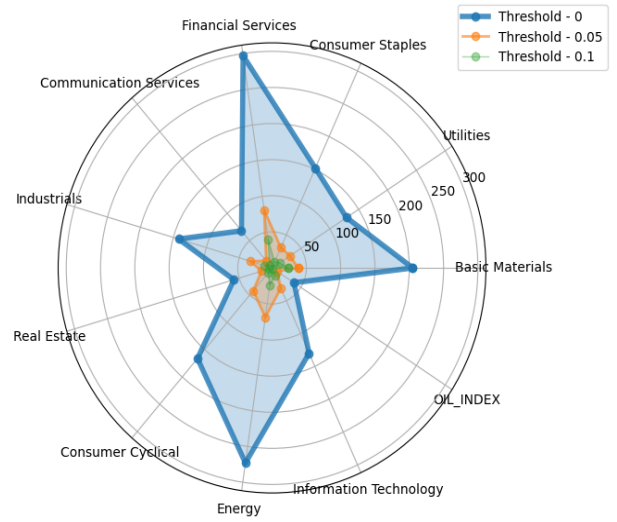
Table 1.7 summarizes the group degree centrality for each sector.<sup>26</sup> It shows that financial services, industrials, energy and consumer cyclical sectors have a relatively high degree of group degree centrality at each threshold setting. It corroborates the observation in Figure 1.16b that these sectors retain a considerable number of edges among other sectors at each threshold, despite also experiencing a notable loss of edges. It is notable that the majority of stocks within these sectors have a history of dividend payments. On the other side, a low group degree centrality suggests that the sector has a small number of connections with other sectors. For example, the communication services sector exhibits a group degree centrality of zero at a threshold of 0.1, whereas Figure 1.16c depicts a non-zero number of edges for this sector. It can thus be concluded that the connections in the communication services sector are intra-sectoral connections. In general, sectors with low group degree centrality lead to isolated sectoral sub-networks, such as the communication services, basic materials, and utilities sectors, which is also captured in Figure 1.13a, right panel, in Section 1.5.1. The performance of these sectors is driven more by their own sectoral outlook than by the influence of other sectors.

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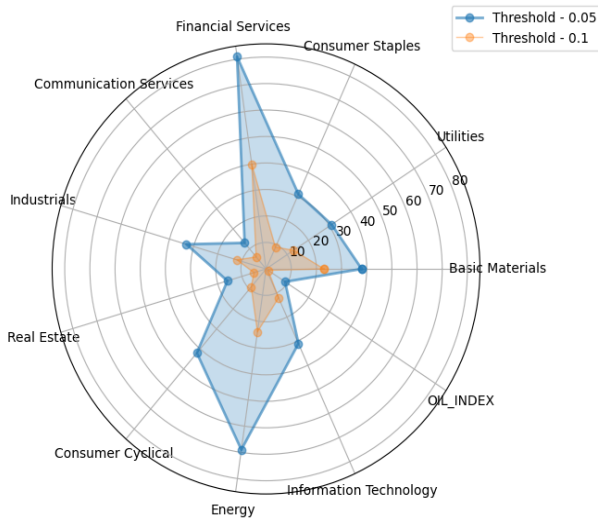
<sup>26</sup>As previously defined, the group degree centrality is a measure of the connections for a subset of nodes (i.e., nodes within a sector).



(a) Number of Nodes



(b) Number of Edges



(c) A zoomed-in view of Figure 1.16b

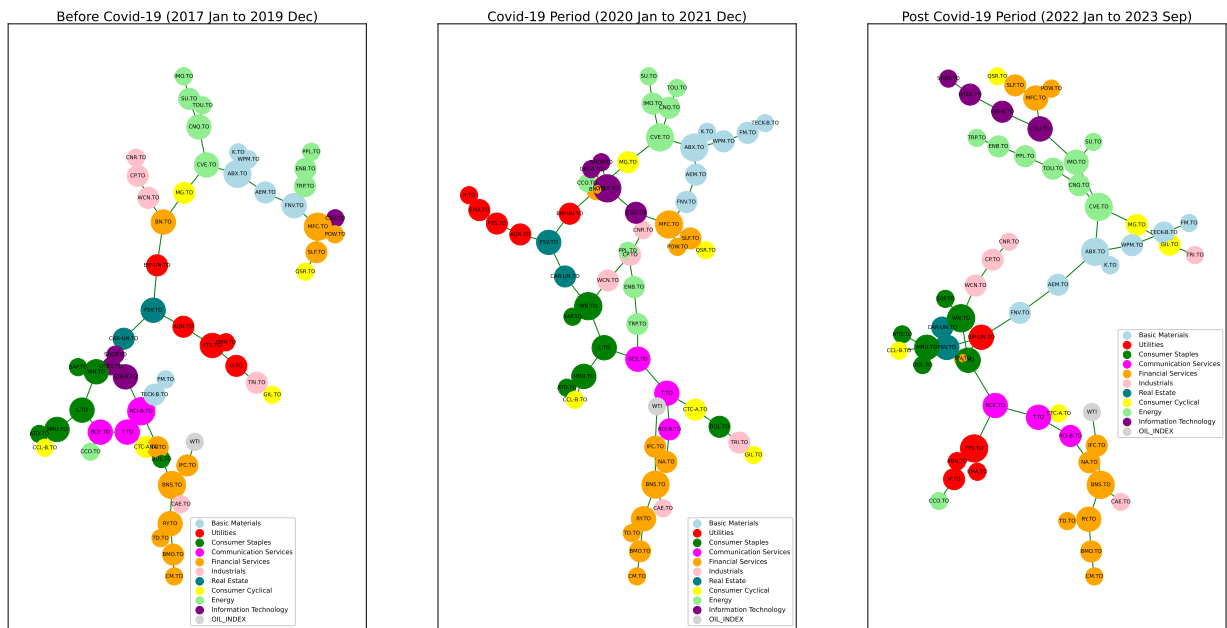
Figure 1.16: TSX 60 Network Component Analysis

**Time-Varying GLASSO Analysis** To comprehend the evolution of the TSX 60 network during the sample period, we use a latent variable time-varying GLASSO (LTGL) analysis. The predetermined timestamp break for estimating the temporal precision matrices ( $\Omega_1, \Omega_2$  and  $\Omega_3$ ) was set at 2019-12-31 and 2022-01-01. Figure 1.17 depicts the results of MST analysis based on the LTGL output. Figure 1.17a shows a visualization of MST in each sample period. Although each temporal network indicates a predominance of sector-wise connections (i.e., nodes with the same colour tend to be connected), the structure of MST networks underwent alterations over time, particularly with regard to pivotal con-

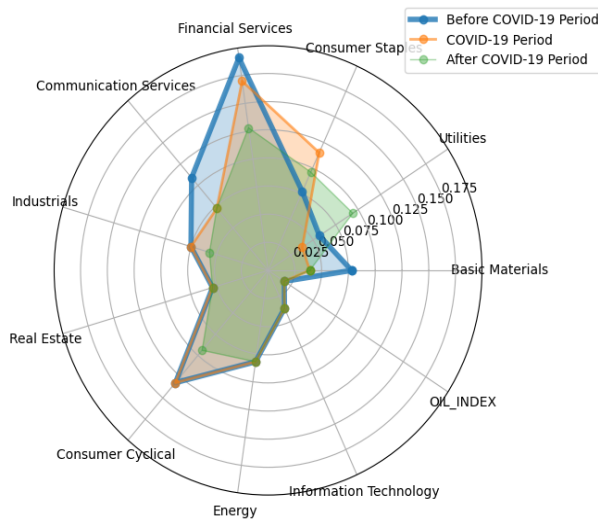
necting nodes. To illustrate, we present the group degree centrality and group betweenness centrality in Figure 1.17b and Figure 1.17c. For example, the financial services sector exhibits a considerable number of inter-sectoral/inter-group connections across all sample periods, as shown in Figure 1.17b. However, the role of this sector as a bridge with other sectors has undergone a change in the post COVID-19 era, as evidenced by a notable decline in group betweenness centrality compared to previous periods. Nonetheless, Figure 1.17c reveals that the high dividend sectors, including communication, energy, and utilities, play a key bridging role across all three periods. It means that these high-dividend sectors not only have a strong intra-sectoral connection but also exhibit a high inter-sectoral partial correlation. Another interesting finding is that the real estate sector plays a crucial role as a pivotal bridge throughout all periods, despite exhibiting a relatively low degree of centrality. This observation deviates from the patterns observed in the post COVID-19 period, during which the housing market exhibited a period of correction while the stock market demonstrated relative stability.<sup>27</sup> However, it is important to note that the TSX 60 real estate sector comprises only two companies that are primarily engaged in property management and rental property business. These companies are comparatively less vulnerable to the housing market correction due to the robust housing demand and resilient rental income.

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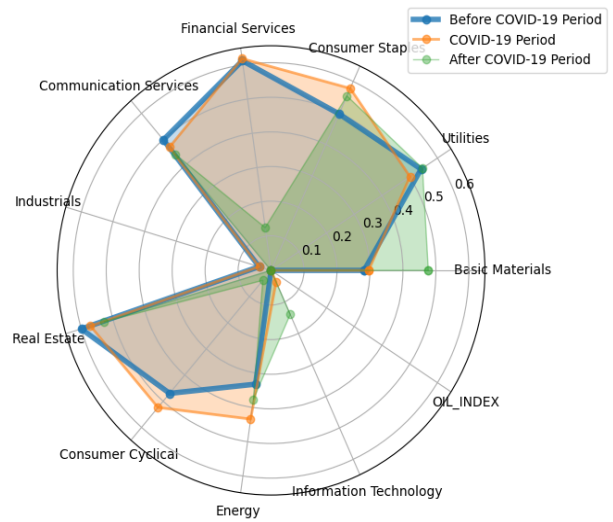
<sup>27</sup>From 2017 to 2021, the monthly growth rate of the Canadian housing price index was 0.4% and dropped to 0.1% from 2022 to 2023. If we remove the first quarter of 2022, prior to the Bank of Canada's rate increase, the monthly rate from 2022 to 2023 is -0.04%. The housing price index is retrieved from <https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810020501> on 2024-2-21.



(a) TSX 60 Index Constituents MST Network by LTGL



(b) Group Degree Centrality



(c) Group Betweenness Centrality

Figure 1.17: TSX 60 Index Constituents MST Analysis

Table 1.8 provides a summary of density and group degree centrality based on the LTGL results in different thresholds and sample periods. The network density remains stable over the three sample periods under each threshold, which reflects that the amount of nodes and edges in the each period are relatively constant. With regard to the three periods considered and the zero threshold assumption, the “COVID-19” period network density (0.796) is slightly higher than the “Before COVID-19” period (0.786). However, upon imposition of a threshold of 0.05, the density of the “COVID-19” period (0.078) is

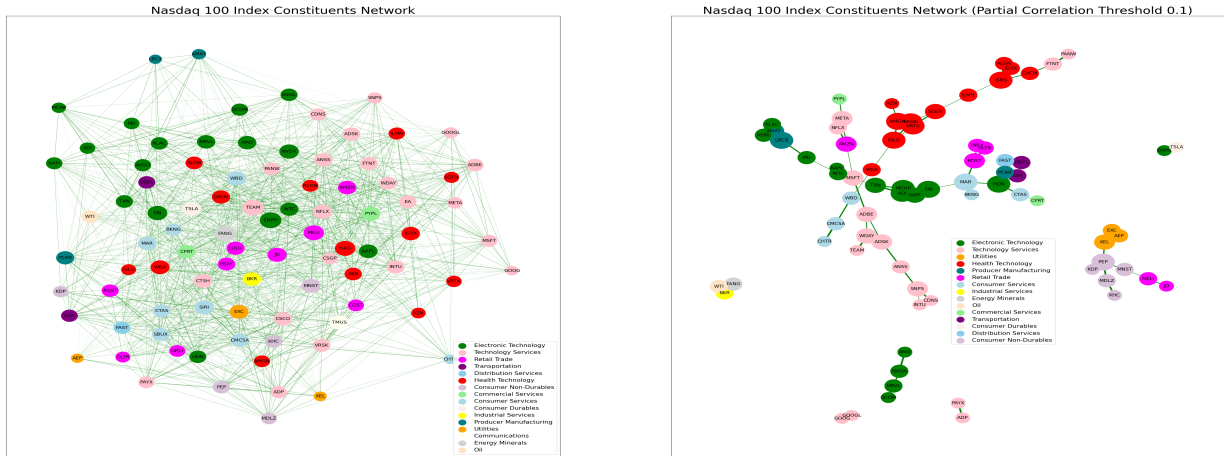
found to be lower than that of the “Before COVID-19” period (0.084). Furthermore, the “Post COVID-19” period density is restored to the pre-pandemic level. These findings suggest that the global pandemic and subsequent economic downturn have resulted in the formation of additional connections within the network. However, these connections are transient and tenuous, as they lack the strength and resilience to withstand the subsequent period (i.e. the “Post COVID-19” period). The group degree centrality shows that sectors are well connected because the majority of sectors exhibit a group degree centrality of 1 across the three periods. However, by filtering out the weak connections, the results indicate that the financial service sector has strong connections with other sectors. This is evidenced by the fact that its group degree centrality value is at an all-time high in different time periods and threshold settings. Furthermore, the results obtained with a threshold of 0.5 indicate that the significance of utilities, communication services, consumer cyclical, and information technology sectors in the network has increased over time, as evidenced by their elevated group degree centrality since the onset of the global pandemic. The common character of these sectors is that they are intimately connected to the daily lives of individuals. The global pandemic has led to a greater reliance on these sectors, as it has prompted changes in our lifestyle, including the adoption of virtual meetings and remote working.

To further reinforce the reliability of our findings, we employ two supplementary centrality metrics: group betweenness centrality and group eigenvector centrality. The betweenness centrality and eigenvector centrality are calculated for each stock in three sample periods. Table 1.9 shows that the centrality measurements are strongly correlated, which implies stocks with high degree centrality are also influential in connecting other parts of the network (high betweenness centrality) and are connected to other influential stocks (high eigenvector centrality). The high correlation among these measurements also facilitates the identification of key stocks, as stocks with the highest degree centrality also have high value in other aspects (e.g., betweenness and eigenvector). From an investment perspective, these stocks are also crucial for portfolio diversification or risk hedging.

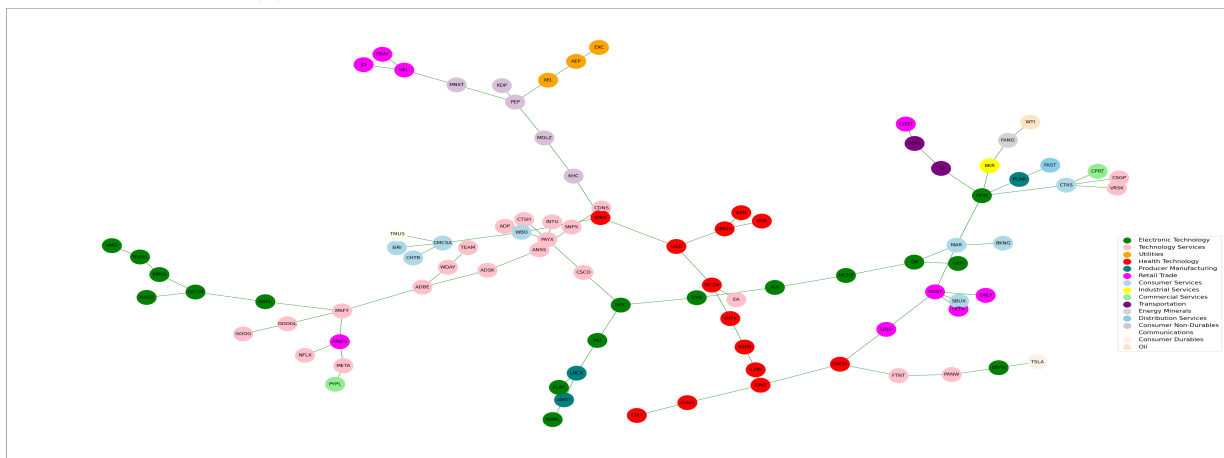
**Nasdaq 100** The Nasdaq 100 index is comprised mostly of high-tech companies and does not include financial companies. In contrast to the TSX 60, which is dominated by high dividend sectors, high-tech companies prioritize research and development over dividend payouts. As a result, investors tend to focus on long-term growth rather than immediate cash flows. This composition creates a unique research opportunity to explore the differences between the TSX 60 and Nasdaq 100. Figure 1.18 illustrates the Nasdaq 100 constituents network. The left panel of Figure 1.18a gives a partial correlation network among 90 stocks, where the GLASSO algorithm has been applied with  $L_1$  penalty to eliminate the weak connections. The right panel illustrates the connections with a partial correlation threshold of 0.1. A key observation from this network analysis is that chip companies, such as NVIDIA (NVDA) and Advanced Micro Devices (AMD), form an isolated subnetwork, which is disconnected from computer manufacturing stocks.<sup>28</sup> One possible explanation for this structural separation is that the expansion of semiconductor companies is no longer driven solely by traditional manufacturing demand. Instead, the rapid growth of artificial intelligence (AI) during the sample period has created new opportunities within the semiconductor industry. As a result, AMD and NVIDIA have emerged as dominant chip manufacturers, each with a market capitalization exceeding \$1.2 trillion at the time of this study. This shift indicates that the returns of chip companies are becoming increasingly interdependent, reinforcing the formation of a tightly connected subnetwork within the broader financial market. Figure 1.18b provides the MST network and highlights the electronic technology and technology services sectors as the root of the Nasdaq 100 spanning tree.

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<sup>28</sup>The chip companies are grouped into the electronic technology sector (green colour) and the computer producers are grouped into technology service (pink colour) sector according to Nasdaq category.



(a) Nasdaq 100 Index Constituents Network by GLASSO



(b) Nasdaq 100 Index Constituents Network by GLASSO and MST

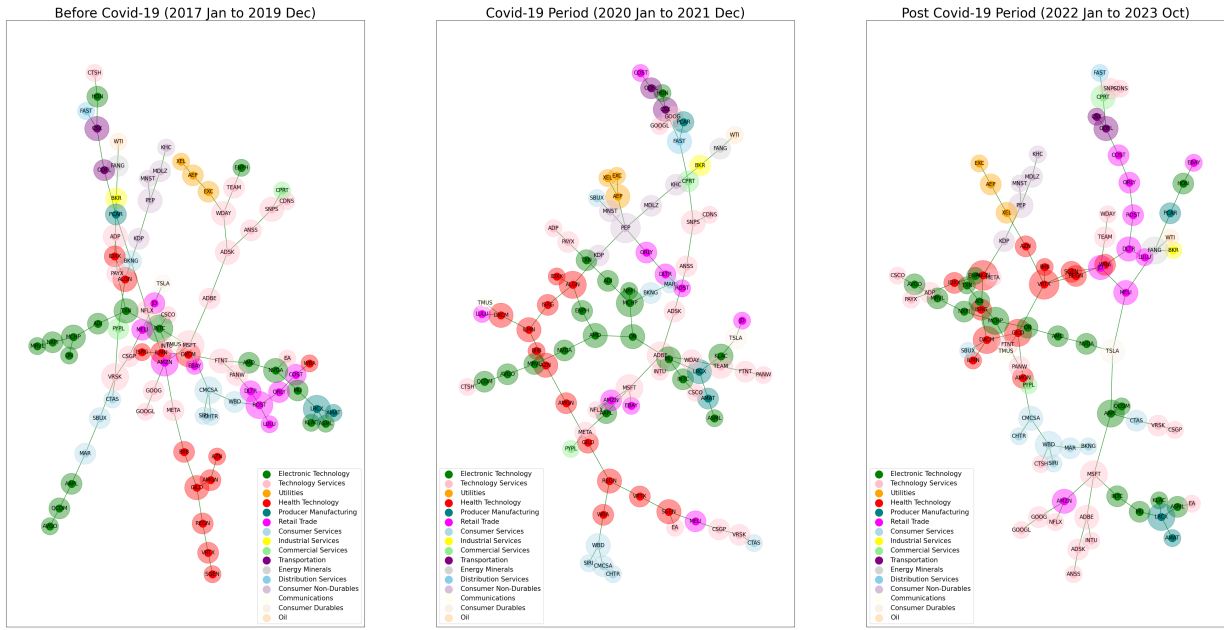
Figure 1.18: Nasdaq 100 Index Constituents Network

The group degree centrality analysis is summarized in Table 1.10. Overall, the network density of the Nasdaq 100 is lower than that of the TSX 60. Without a threshold, the network density is 0.326, in contrast to the 0.5 value observed for the TSX 60 network. It is notable that the Nasdaq 100 and the TSX 60 have distinct penalty parameters,  $\alpha$  (see Section 1.4.1). In the absence of a threshold, the observed density difference may be partially attributable to the optimization process of GLASSO. However, when a threshold is applied, the network is constrained to include only the absolute value of partial correlation exceeding or equal to the specified threshold, effectively removing the influence of the GLASSO implementation. When the effects of GLASSO optimization are excluded by imposing a threshold, the density of the Nasdaq 100 remains lower than that of the TSX 60. Despite the Nasdaq 100 network comprising a greater number of nodes (stocks) than the

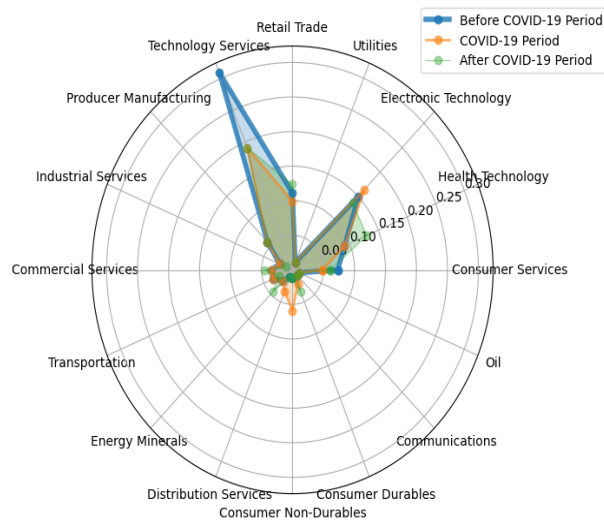
TSX 60 network, the connections among stocks are less dense than those observed in the TSX 60.<sup>29</sup> Furthermore, the group degree centrality result indicates an imbalanced group centrality distribution in the Nasdaq 100 network. In particular, the electronic technology, technology services, and health technology sectors have a much higher group degree centrality than other sectors. This is also reflected in the Nasdaq 100 capitalization distribution, where these sectors account for over 75% of its total capitalization. Moreover, Table 1.10 shows that the electronic technology sector's group degree centrality is less sensitive to the partial correlation thresholds than the technology services and health technology sectors.

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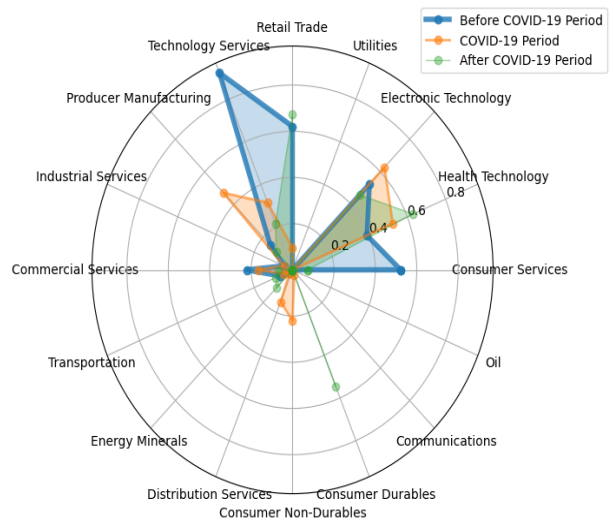
<sup>29</sup>The density differences between the Nasdaq 100 and TSX 60 networks can be explained by two factors. First, the Nasdaq 100 network has a greater number of potential edges due to its larger number of nodes ( $n$ ), as given by the formula  $\frac{n(n-1)}{2}$ . Second, the average partial correlation strength between stock pairs is weaker in the Nasdaq 100 network (0.0369) compared to the TSX 60 network (0.0427), leading to a sparser connectivity structure when weak links are filtered out.



(a) Nasdaq 100 Index Constituents MST Network by LTGL



(b) Group Degree Centrality



(c) Group Betweenness Centrality

Figure 1.19: Nasdaq 100 Index Constituents MST Analysis

Furthermore, we use the LTGL algorithm to investigate the development of the MST network over time. Figure 1.19 provides a visual representation of the MST analysis. Figure 1.19a gives a snapshot of the network across three sub-periods. Figure 1.19b and Figure 1.19c illustrate the group degree centrality and group betweenness centrality of the MST for all periods, as shown in Figure 1.19a. The technology service, health technology and electronic technology sectors consistently serve as the primary bridges of the network due to their high group betweenness centrality, as shown in Figure 1.19c.<sup>30</sup> From a risk man-

<sup>30</sup>These three sectors are ranked in the top five sectors with the highest group betweenness centrality

agement perspective, a high return volatility of these three sectors could expose the Nasdaq 100 network to significant uncertainty because the high partial correlation of their return with other stocks as reflected by their high group degree and betweenness centralities. Furthermore, the evolution of the network over time shows a trend in which many stocks in health technology sector (in red in Figure 1.19a) have shifted from the “periphery” to the “core” of MST. It is also evidenced by the increase of group degree centrality and group betweenness centrality of health technology section since the onset of COVID-19, as shown in Figure 1.19b and Figure 1.19c. This suggests that the COVID crisis has accelerated the development of health technology reflecting its increased influence on the network (see also Scott et al. (2020) and Kaplan (2020)). In the pre-COVID-19 period, AMD and NVDA (in green in Figure 1.19a) were relatively small companies with only a few connections to technology services on the “periphery” of the network. During the same period, TSLA (Tesla, consumer durable sector) was only connected to the retail trade sector rather than the technology sector. Over time, AMD and NVDA have become leaders in the electronics technology sector, while Tesla has emerged as a bridge in the electronic technology and retail industries (as illustrated in Figure 1.19a, third panel).

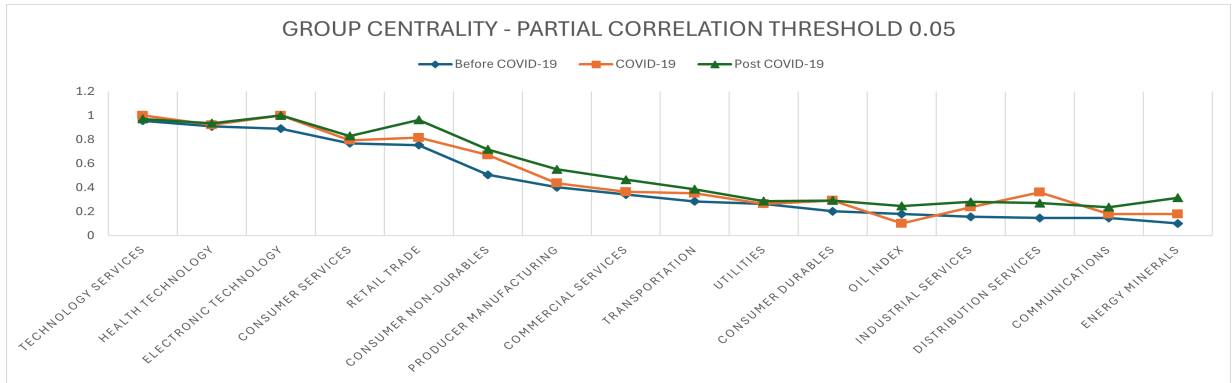
Table 1.11 summarizes the group centrality analysis by periods based on the LTGL output. It is worth noting that Tesla is the only stock in the consumer durables sector that invariably demonstrates its strong connections in the Nasdaq 100 network, regardless of the threshold used for partial correlation. Figure 1.20 compares the sectoral group degree centrality of the TSX 60 and the Nasdaq 100 using a partial correlation threshold of 0.05.<sup>31</sup> For each industry, stocks are grouped according to their underlying sector, and the fraction of non-group members connected to group members is calculated. Figure 1.20a illustrates that the health technology, electronic technology, and technology services sectors have a relatively high group degree centrality, indicating that these sectors have

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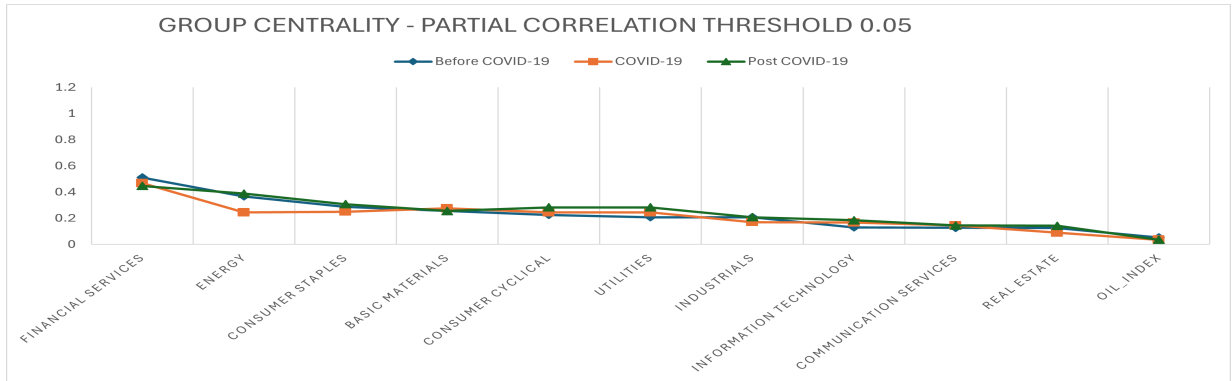
value across the three sub-periods.

<sup>31</sup>In order to facilitate cross-market comparisons, a less stringent threshold (0.05) is employed to balance the significance of edges and the network structure. This approach is intended to prevent the emergence of misleading scenarios, such as that observed in the TSX 60 communication services sector, which exhibits zero group centrality under the threshold of 0.1. The visualization depicted in Figure 1.13a (right panel) suggests that this zero centrality is primarily influenced by intra-sector connections, rather than the elimination of sectors.

robust inter-sectoral connections within the Nasdaq 100 network. Similarly, Figure 1.20b demonstrates that the financial services and energy sectors exhibit a distinct inter-sectoral connectedness within the TSX 60 network. In addition, the sub-period analysis reveals that the Nasdaq 100 exhibits a higher average group degree centrality than the TSX 60, indicating that sectors within the Nasdaq 100 are more densely connected to other sectors in the network. The average sectoral degree centrality is calculated by averaging the sector-wise centrality. This finding contrasts with the whole-period analysis, where the TSX 60 demonstrated stronger density. The observed discrepancy between the whole-period and sub-period analyses can be attributed to differences in partial correlation strength, which is fundamentally determined by the inverse of the covariance matrix. The sub-period analysis captures stock return variations at a more granular level, thereby mitigating the influence of stable periods and allowing for a more refined assessment of dynamic market relationships. As a result, the partial correlations in the Nasdaq 100 are stronger in the sub-period analysis than in the TSX 60, leading to a denser sectoral connectivity structure. This highlights the importance of temporal segmentation in network-based financial contagion analysis because the dependence structure among asset returns evolves over time, analyses based on long, aggregated samples may obscure short-lived but economically meaningful inter-dependencies. In contrast, temporally segmented networks constructed using LTGL are able to uncover transient changes in partial correlations, capturing periods of intensified connectedness that often coincide with market stress. These short-term fluctuations reveal latent dependency structures that remain hidden in full-sample analyses, underscoring the necessity of time-resolved network modelling for accurately assessing dynamic systemic risk. Similarly to the TSX 60 centrality robustness check, we calculate the correlation among three centrality measurements, degree centrality, betweenness centrality and eigenvector centrality in Table 1.12. It shows a very high correlation among these three centrality measurements, which enhances our centrality analysis.



(a) Nasdaq 100 Index Group Degree Centrality - Partial Correlation 0.05



(b) TSX 60 Index Group Degree Centrality - Partial Correlation 0.05

Figure 1.20: Group Degree Centrality with Partial Correlation of 0.05

### International Connection

Connection between the Canadian stock market and the US stock market is expected to be important as the two countries have an extensive trade relationships and considerable policy synergies.<sup>32</sup> Many Canadian companies are also traded on the US stock market, and vice versa. In this section, we mix sectoral returns in the two markets to explore the international connection. The sectoral logarithmic return is calculated by regrouping each individual stock return of a same sector in either the US or Canada. Given that the performance of stocks with a relatively small capitalization is more volatile than that of the overall market, the sectoral logarithmic return is also weighted by the capitalization of each stock in order to mitigate the bias that arises from the volatility of small capitalization stocks. Figure 1.21 depicts the Pearson correlation of the dataset. The graph shows a discernible correlation between the sectors of the TSX 60 and Nasdaq 100, although the

<sup>32</sup>Appendix D investigates the impact of economic uncertainty indexes in the stock return networks.

inter-sectoral correlation within a specific country is stronger (i.e., the upper left block and the bottom right block have darker colours than the other two areas).

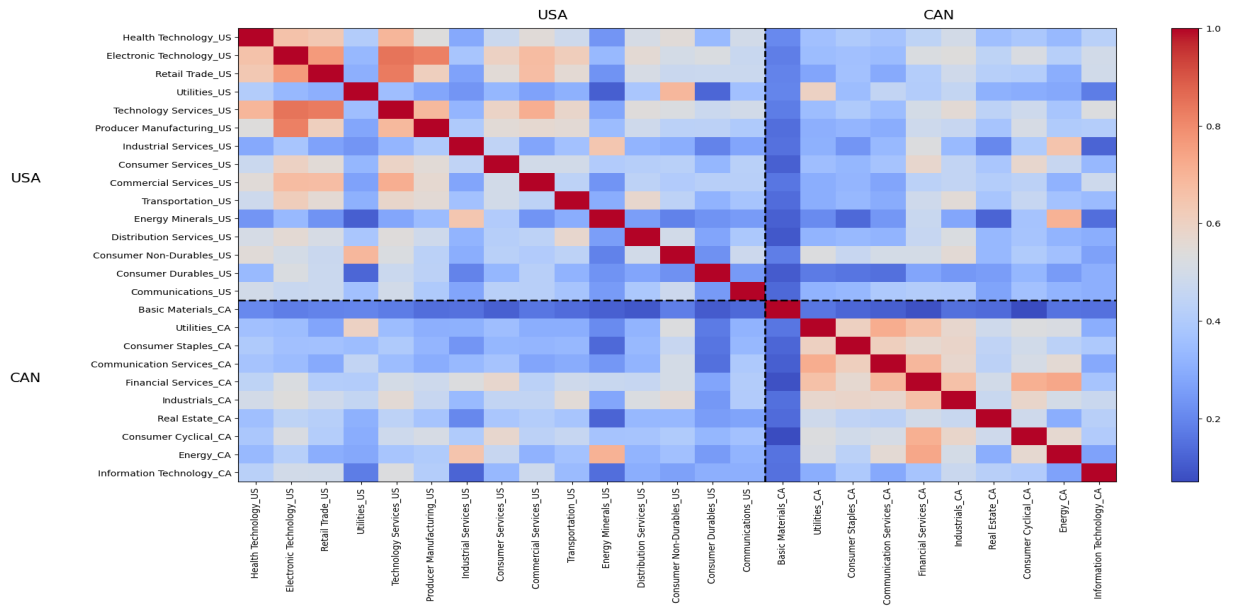
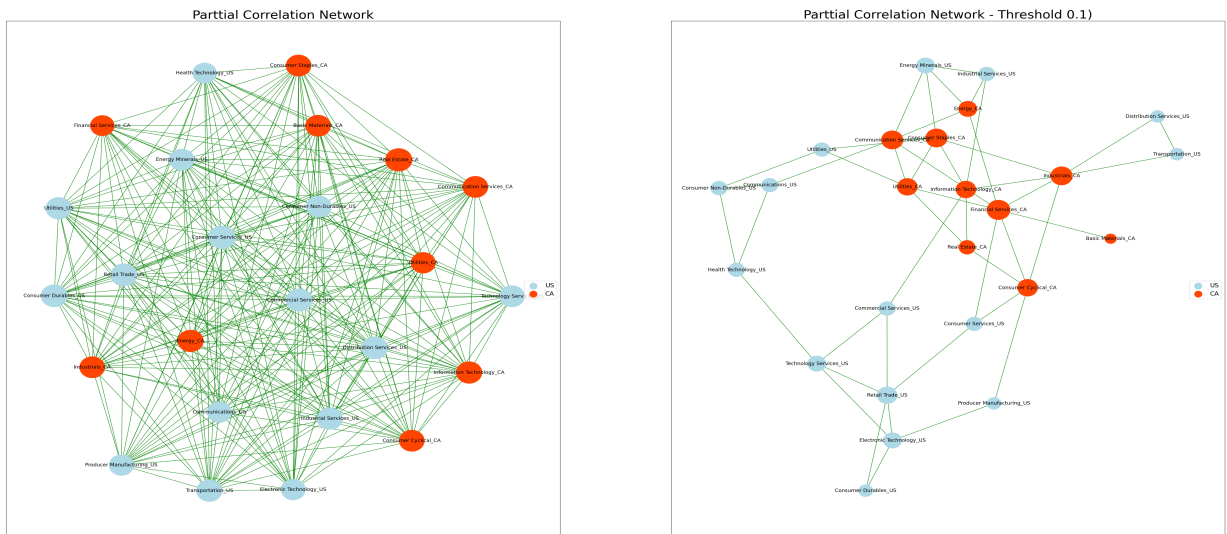


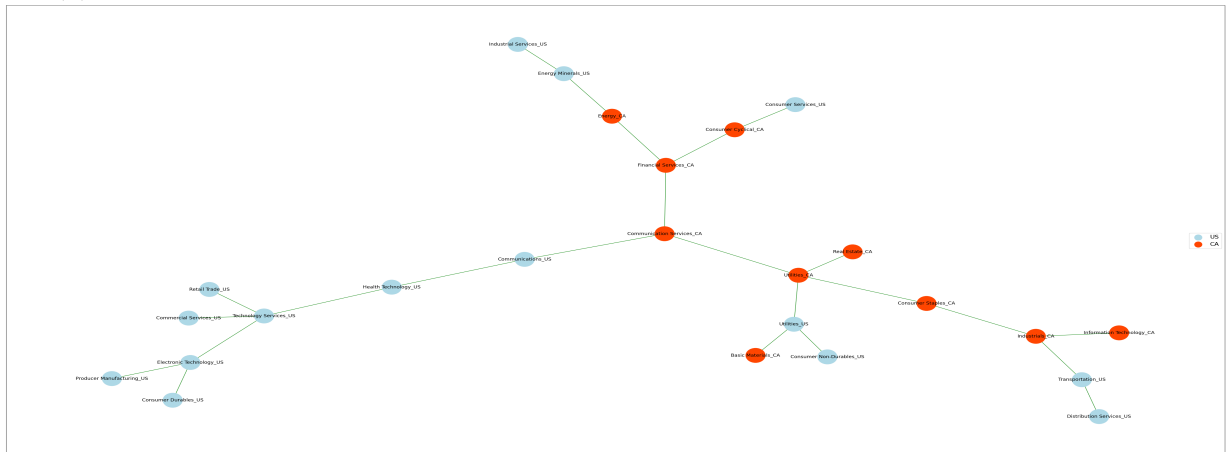
Figure 1.21: TSX 60 and Nasdaq 100 Sectoral Returns Correlation

Figure 1.22 provides a visualization of the connections between the TSX 60 stocks and the Nasdaq 100 stocks. Figure 1.22a provides a visualization of GLASSO optimized network. The right panel of Figure 1.22a illustrates that intra-country partial correlation is prominent, although weak inter-country connections are observed. In Figure 1.22b, the MST provides further corroboration of our findings, demonstrating that the majority of sectors are connected within a given country, rather than the same sectors (e.g., technology/ information technology) being connected across countries. Nonetheless, it shows that the US energy, communication services, and utility sectors have close connections with their Canadian counterparts. Similarly, the US transportation and distribution sectors are linked to the Canadian industrial sector. It is reasonable to conclude that the interconnections between the return of the energy sectors and utility sectors of both countries are influenced by global commodity and energy prices. The observed relationships between the US transportation and distribution sectors and the Canadian industrial sector suggest a high level of trade between the two countries. Table 1.13 summarizes the group degree centrality of the North American network, as derived from the left panel of Figure 1.22a. The results indicate that the Nasdaq 100 exhibits a more robust group degree centrality

than the TSX 60, consistently maintaining higher values across different threshold levels. Table 1.14 presents the correlation among the degree centrality, betweenness centrality, eigenvector centrality and clustering for the North America network. The results highlight that groups with a high degree centrality, betweenness centrality, and eigenvector centrality tend to have a lower “rank” for the clustering coefficient (although the magnitude of the clustering coefficient is not low, with an average of 0.89). A node with a high degree centrality and a relatively low clustering coefficient can rapidly disseminate information among its numerous neighbours, but the information is not spread directly between neighbors themselves due to the lack of connections among them. This type of structure is consistent with the contents of Figures 1.21 and 1.22, which illustrate the strong country-oriented connections. Table 1.15 summarizes the sectoral centrality measurements including degree centrality, betweenness centrality, eigenvector centrality, and clustering.



(a) TSX 60 and Nasdaq 100 Sectoral Returns Partial Correlation Network. Note:  $\alpha = 0.01$ .



(b) TSX 60 and Nasdaq 100 Sectoral Returns MST Network

Figure 1.22: TSX 60 and Nasdaq 100 Sectoral Returns Partial Correlation Network

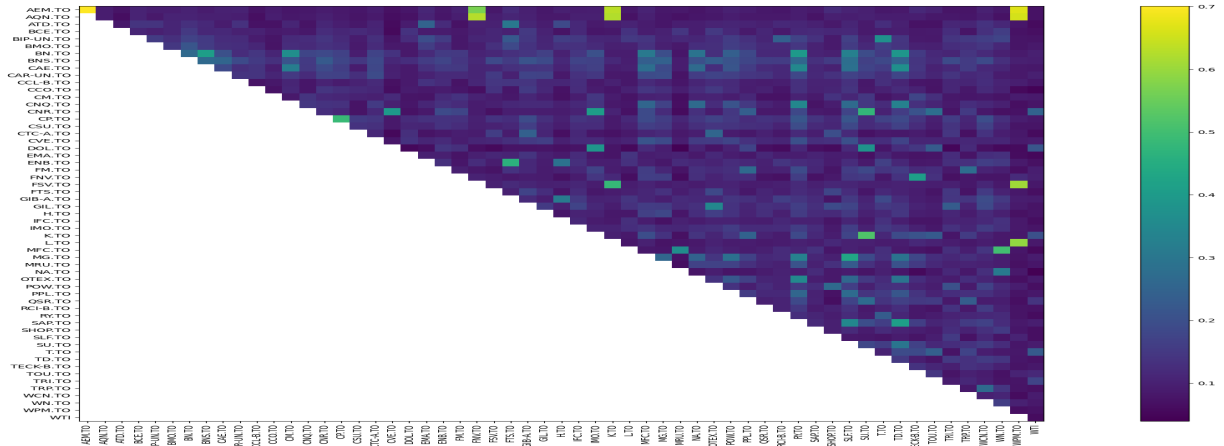
## 1.5.2 Mutual Information

Although the Pearson correlation and partial correlation can be employed to identify connections between stocks' performance, the relationships between stocks' returns may not be linear. The mutual information metric quantifies the total dependency between two variables based on the joint probability distribution of the variables, which captures the nonlinear relationship. In this section, we use the mutual information metric to explore the stock network structure.

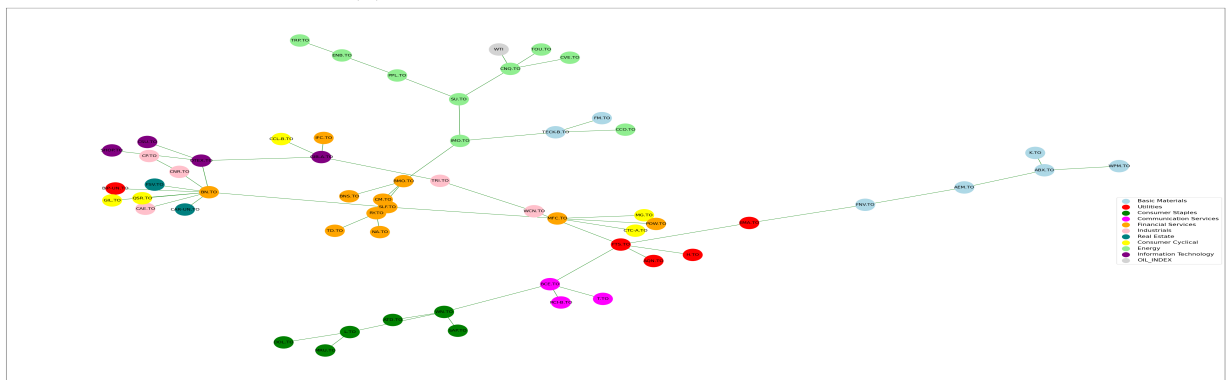
Figure 1.23 provides a visualization of mutual information analysis. Figure 1.23a is a heatmap that depicts the magnitude of mutual information among stocks. As an undirected

network implies a symmetric matrix, only the upper triangle matrix is shown. Lighter yellow and green colours represent a high mutual information between a pair of stocks, while darker colours indicate that a pair of stocks contains little information between each other. Figure 1.23b uses the mutual information matrix to build the MST networks. It shows that the large majority of connections are sector-oriented. Most high dividend payout sectors, such as energy, financial services, basic materials, and communication services sectors, have their own sector-wide clusters. This structure strengthens the network against systemic risks, as the performance of these sectors highly depends on their own sector performance. In addition, Table 1.16 summarizes the group centrality of Figure 1.23b. The results indicate that the financial services sector has the highest group degree centrality and group betweenness centrality. This observation is intuitive, as the financial services sector plays a pivotal role within the network due to its extensive connections with other sectors and its capacity to facilitate the transfer of information (i.e., high group betweenness centrality). Furthermore, the utility, industrials, and information technology sectors demonstrate relatively high values in both group betweenness centrality and group degree centrality, as shown in Table 1.16. The presence of a few sectors with high group betweenness centrality and group degree centrality in a network can have implications for the network's resilience to systemic risk. In normal circumstances, the resilience of sectors with high centrality contributes to the market's resilience. In the event of a crisis, the existence of multiple sectors with high group centrality can result in the partitioning of market risk. To illustrate, the financial services sector may experience a decline in performance during a financial contagion. If the financial services sector is the only bridge in the network, other sectors would also experience a downward pressure immediately due to high betweenness centrality of the financial services sector. However, the presence of multiple sectors with high group centrality may mitigate the impact of the financial services sector, particularly for sectors that are not directly linked to the financial services sector (e.g., the consumer staples and basic materials sectors in Figure 1.23b). From a risk management perspective, understanding the network structure of stock returns offers valuable insights for investment diversification. By identifying sectors that serve as key nodes in financial networks,

investors can strategically allocate assets to reduce exposure to systemic risk. The centrality measures across sectors thus reveals core inter-dependencies among stocks, providing a framework for constructing more resilient investment portfolios.

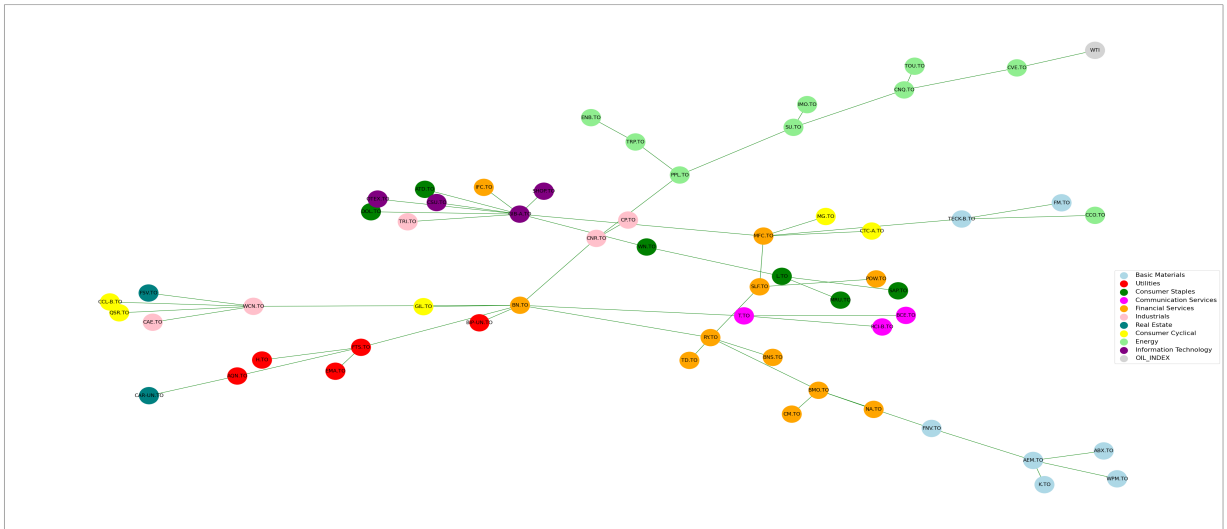


(a) TSX 60 Mutual Information Heatmap

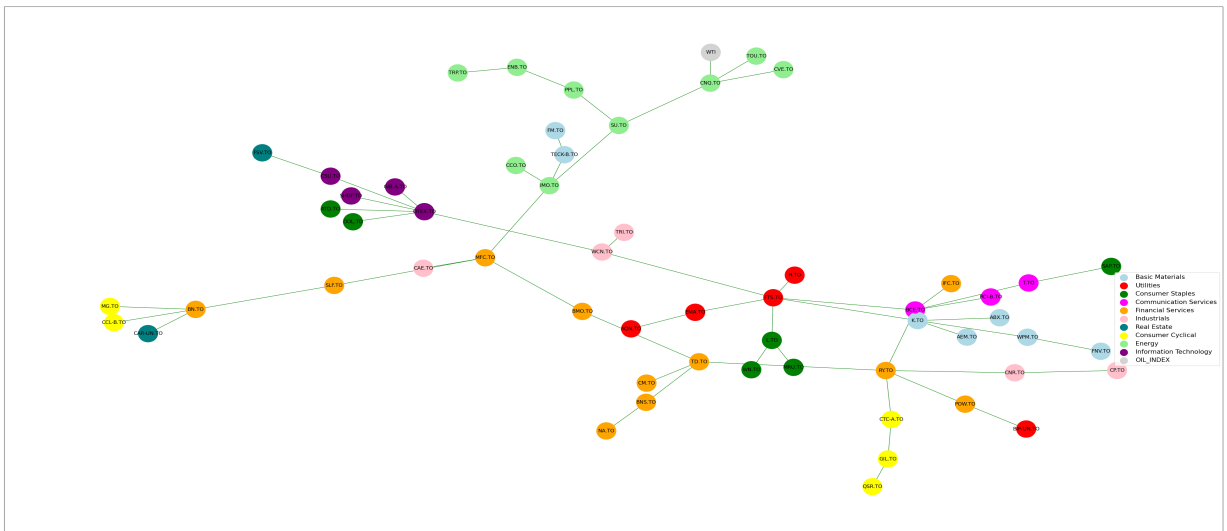


(b) TSX 60 Mutual Information MST Network.

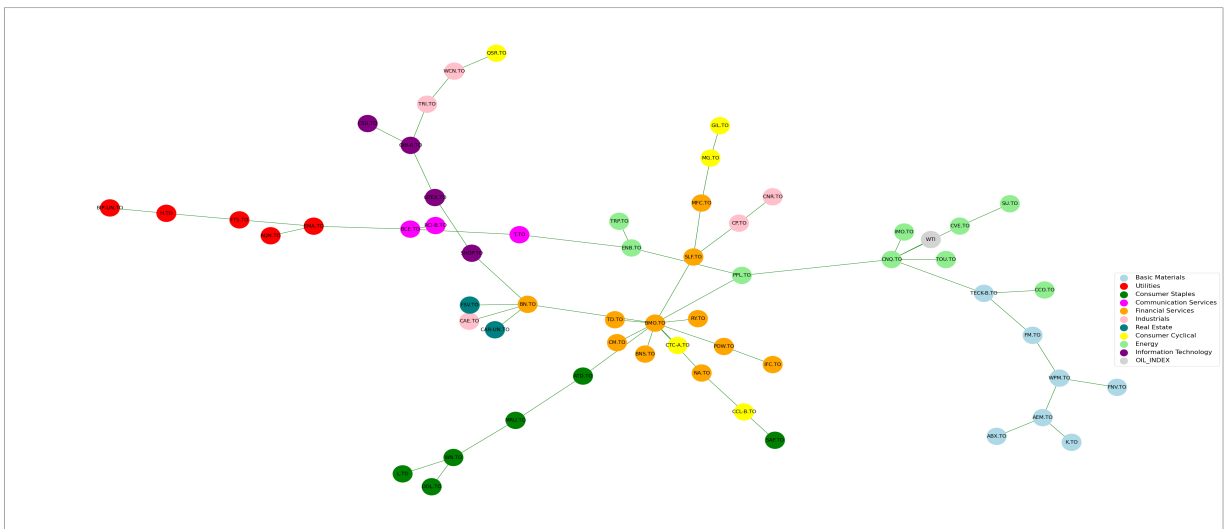
Figure 1.23: TSX 60 Mutual Information Network



(a) TSX 60 Mutual Information MST - Before COVID-19



(b) TSX 60 Mutual Information MST - COVID-19 Period



(c) TSX 60 Mutual Information MST Network - Post COVID-19

Figure 1.24: TSX 60 Mutual Information Network By Periods

Figure 1.24 illustrates the TSX 60 network structure at selected time periods. As shown in Figure 1.24a, the high dividend sectors, such as financial services (orange), energy (green), basic materials (light blue), utilities (red), and communication services (pink), form well connected sector-oriented clusters. Financial services stocks (i.e. BN.TO, RY.TO and MFC.TO) also serve as bridges connecting with stocks from other sectors in the spanning tree. During the COVID-19 pandemic, financial services sector continued showing a strong connection with stocks from other sectors, as shown in Figure 1.24b.<sup>33</sup> At the end of the global pandemic, the intra-sector clusters were restored. It is worth noting that the basic material and energy sectors have demonstrated strong sector coherence because their clusters have persisted over the three distinct periods. This observation can be attributed to these sectors' business model, which is more reliant on global commodity prices than on the other sectors' performance. Overall, Figure 1.24 indicates that the fundamental TSX 60 network structure has remained unchanged over the three subperiods, even if the volatile market led to some restructuring of connections during the COVID-19 period.

Similarly, we apply the mutual information analysis to the Nasdaq 100 constituents. Figure 1.25a illustrates the magnitude of mutual information. The overall stock interdependency is relatively consistent because the heatmap has only a few light colours (i.e., high mutual information value) when compared to the TSX 60 heatmap (see Figure 1.23a). This suggests that the strength of connections among stocks in the Nasdaq 100 is distributed more evenly, whereas the TSX 60 mutual information heatmap indicates quite a few strong connections between stocks that exceed the average strength. Table 1.17 presents a summary of descriptive statistics mutual information for both the TSX 60 and the Nasdaq 100 over the three sample periods. Overall, the average mutual information in the Nasdaq 100 are stronger than those in the TSX 60.<sup>34</sup> The standard deviation of mutual information in the TSX 60 is relatively stable compared to that in the Nasdaq 100,

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<sup>33</sup>The betweenness centrality of the financial services sector was 0.86 in the pre-COVID-19 period, 0.66 in the COVID-19 period, and 0.69 in the post-COVID-19 period.

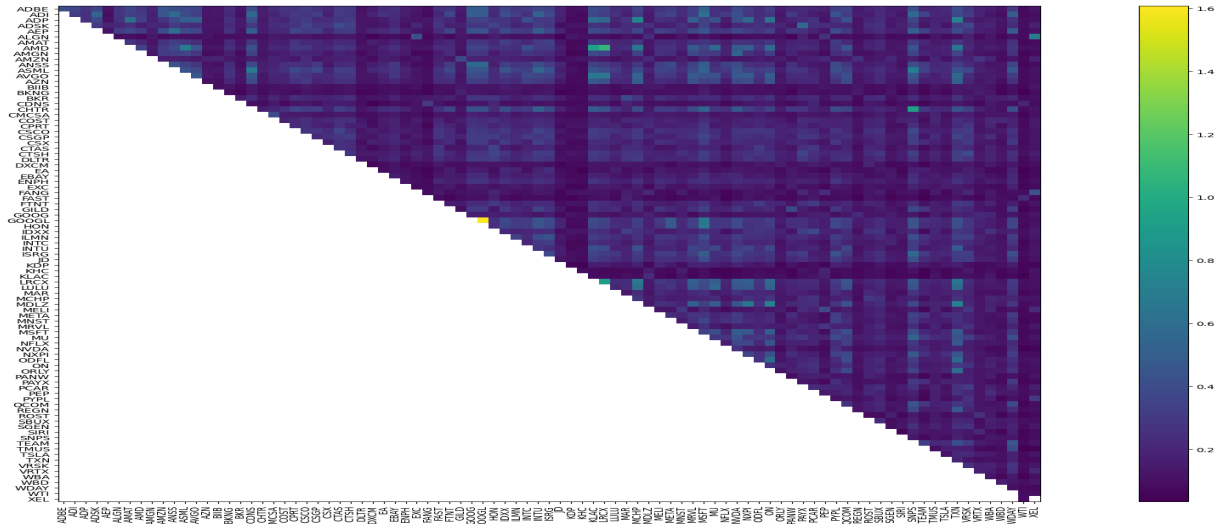
<sup>34</sup>As MI quantifies the shared information between variables, the objective of this comparison is to illustrate the existence of a discrepancy in the average quantity of shared information between two distinct markets. The results of this comparison may be inconclusive due to the sensitivity of MI to the scale of the datasets and the complexity of the relationships between variables. However, as the sample period and data treatments used in the MI analysis are consistent across both markets, the results may still offer insights into the information shared between stocks in these two markets.

even during the COVID-19 period. The mutual information between the returns of two stocks captures how much information about the return of one stock can be inferred from the return of the other. A relatively small variation of mutual information during the crisis periods as is the case for TSX 60, implies that the market turmoil does not amplify the uncertainty of stocks' performance. In other words, a stable connection framework will strengthen the network and make it more resilient to market risk.

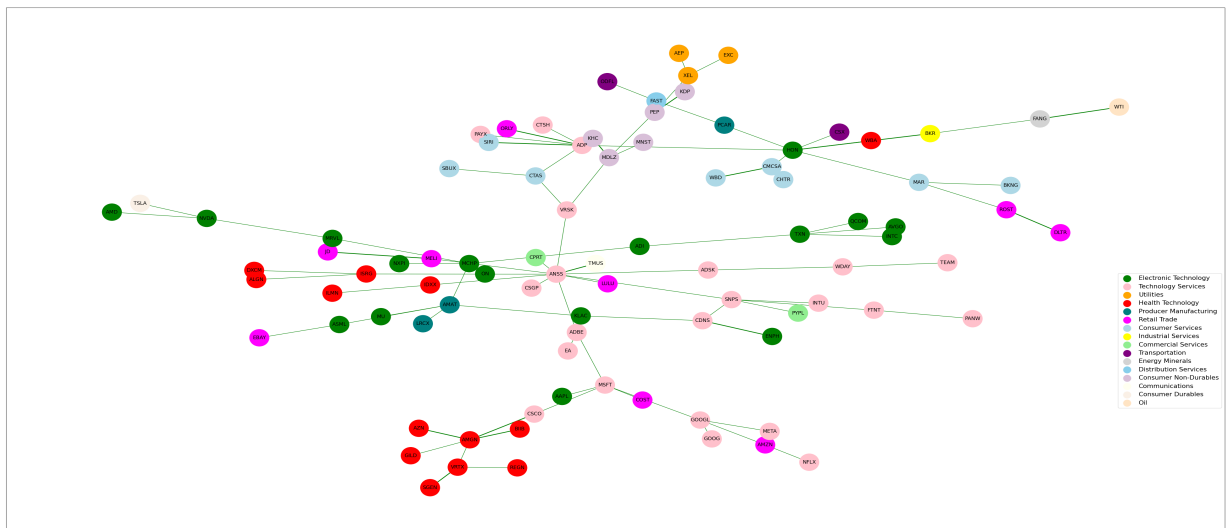
Figure 1.25b shows a MST, maximizing the mutual information among the Nasdaq 100 constituents. The technology services and electronic technology sectors establish numerous inter-sectoral connections, while the health technology sector forms two small intra-sectoral clusters. This network structure has significant implications for systemic risk, as the dense interconnections within the technology-related sectors increase the potential for shock propagation across the broader market.

Table 1.18 summarizes the group centrality analysis for the Nasdaq 100 MST network. The results indicate that technology services and electronic technology sectors are the only two sectors with relatively high values in both group degree centrality and group betweenness centrality. In other words, these two sectors not only demonstrate extensive connections with other sectors but also play a pivotal role in the transmission of information. A comparison with Table 1.16 reveals that the TSX 60 network has a higher degree of group betweenness centrality in four out of ten sectors, with values exceeding 0.4. In contrast, the Nasdaq 100 network shows only two sectors, technology services and electronic technology, with comparable levels of betweenness centrality. Moreover, the business models of these sectors are significantly correlated. The limited number of sectors with high group betweenness centrality and the lack of diversity among those sectors renders the Nasdaq 100 more susceptible to a market shock. For example, a market shock affecting technology services and electronic technology will rapidly spread return volatility to other sectors due to the extensive inter-sectoral connections between these two sectors and the rest of the network. In contrast to the TSX 60 network, the Nasdaq 100 network lacks a diversified sector of high betweenness centrality, which could help to mitigate the impact of a crisis in the technology services and electronic technology sectors. This discrepancy in

network structure may be a key factor in explaining the heightened volatility observed in the Nasdaq 100 during the global pandemic crisis.



(a) Nasdaq 100 Mutual Information Heatmap



(b) Nasdaq 100 Mutual Information MST Network.

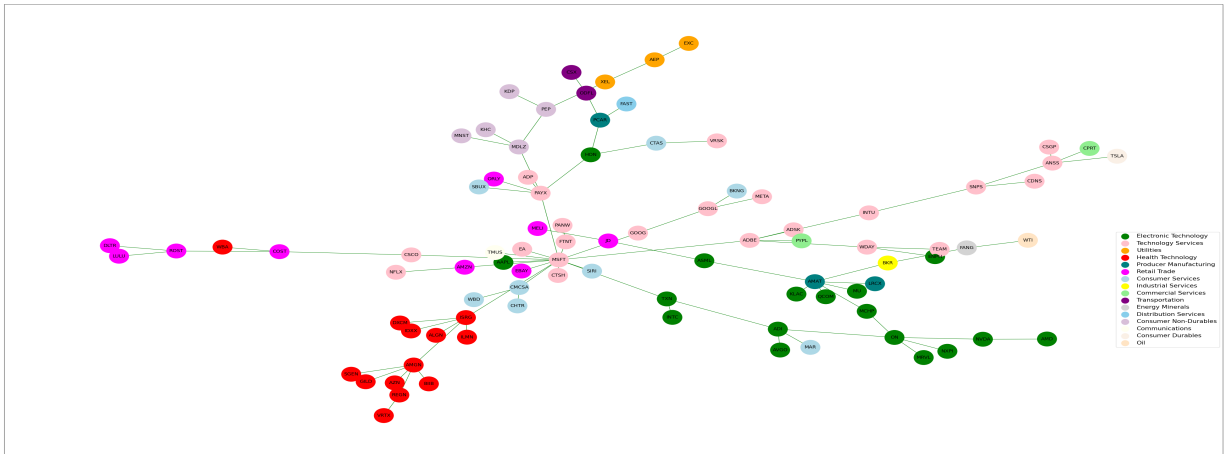
Figure 1.25: Nasdaq 100 Mutual Information Network

Finally, Figure 1.26 illustrates the evolution of Nasdaq 100 network during three sub-sample periods. The structure of the Nasdaq 100 MST network underwent some notable changes over the study period. Figure 1.26a, which depicts the situation prior to the onset of the COVID-19 pandemic, illustrates that the technology sector (shown in pink) played a pivotal role in facilitating connectivity between various sectors. The health technology and electronic technology sectors, on the other hand, are represented by sectoral clusters at the periphery of the network. Figure 1.26b shows that, during the period of the global

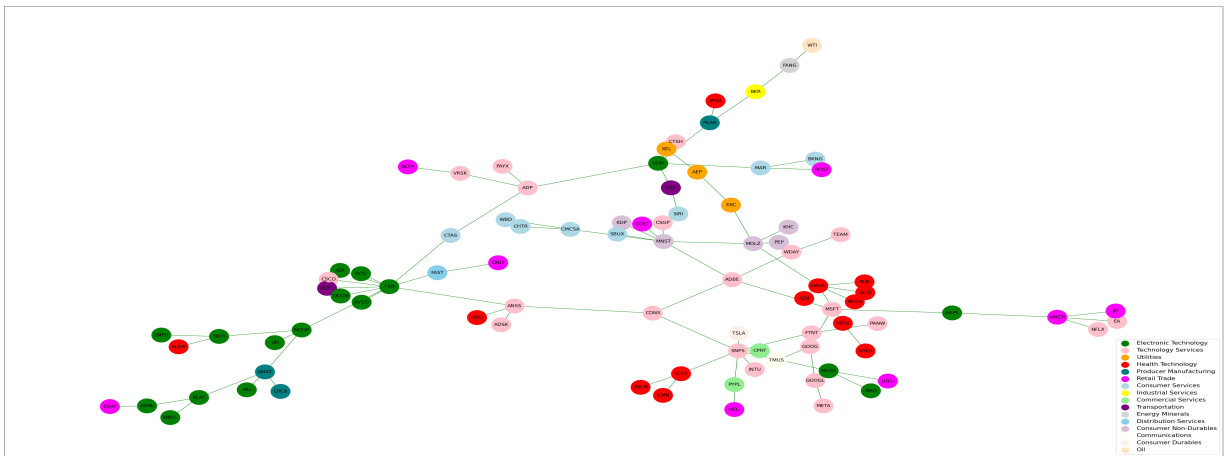
pandemic, the electronic technology sector (e.g. TXN, and NVDA), consumer non-durable sector (e.g. MNST) and consumer service sector (e.g. MAR) demonstrated an important role in transmitting the information, although the technology sector remained the primary bridge within the network. In the period following the onset of the global pandemic, the health technology sector shows a relatively diversified set of connections with other sectors, in contrast to what is observed in previous periods. It is also worth noting that the semiconductor industry demonstrates a relatively similar position in the three periods networks, indicating that the industry's performance is largely driven by its own outlook rather than the performance of other sectors.<sup>35</sup> A possible explanation is that the semiconductor industry's growth is driven by an exponential demand for chips due to the advancement of artificial intelligence. Figure 1.27 presents the sectoral betweenness centrality information based on the information presented in Figure 1.26. The consumer durable, communications, and oil sectors indicate a zero sectoral betweenness centrality, which is attributable to the fact that each sector is comprised of a single company on the periphery of the network. The changes in betweenness centrality over the sample periods reflect the findings presented in Figure 1.26. For example, the electronic technology, consumer non-durable and consumer service sectors show a notable increase in the sectoral betweenness centrality during the COVID-19 period (shown in orange). The health technology sector only shows a moderate degree of betweenness centrality in the post COVID-19 period, which corroborates the observation in Figure 1.26c that it has a relatively diversified set of connections with other sectors in the aftermath of pandemics.

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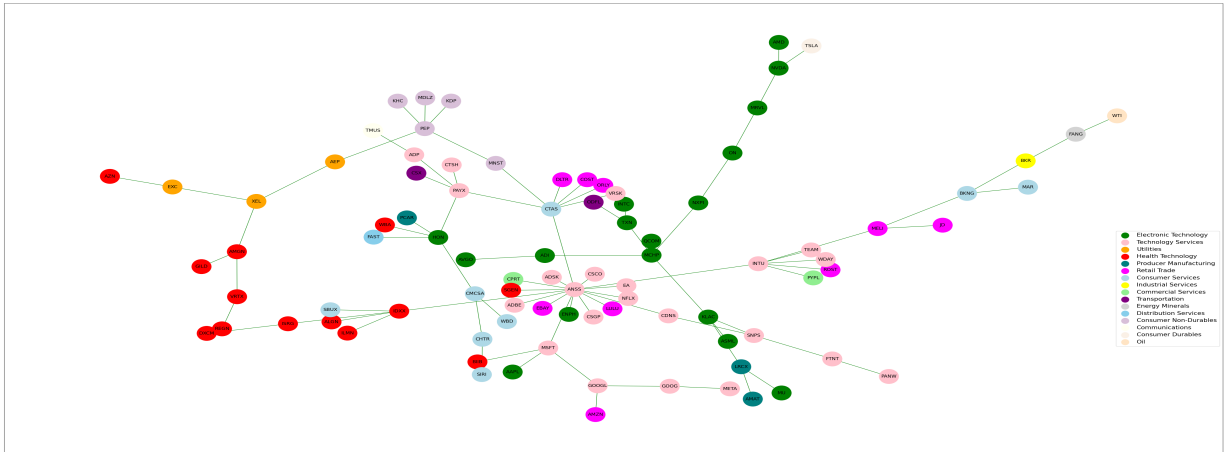
<sup>35</sup>The semiconductor industry covers two sectors, electronic technology sector (i.e. NVDA, MU, MRVL, ASML, AMD, QCOM, INTC, ON, and MCHP) and producer manufacturing sector (i.e. AMAT, and LRCX)



(a) Nasdaq 100 Mutual Information MST - Before COVID-19



(b) Nasdaq 100 Mutual Information MST - COVID-19 Period



(c) Nasdaq 100 Mutual Information MST Network - Post COVID-19

Figure 1.26: Nasdaq 100 Mutual Information Network By Periods

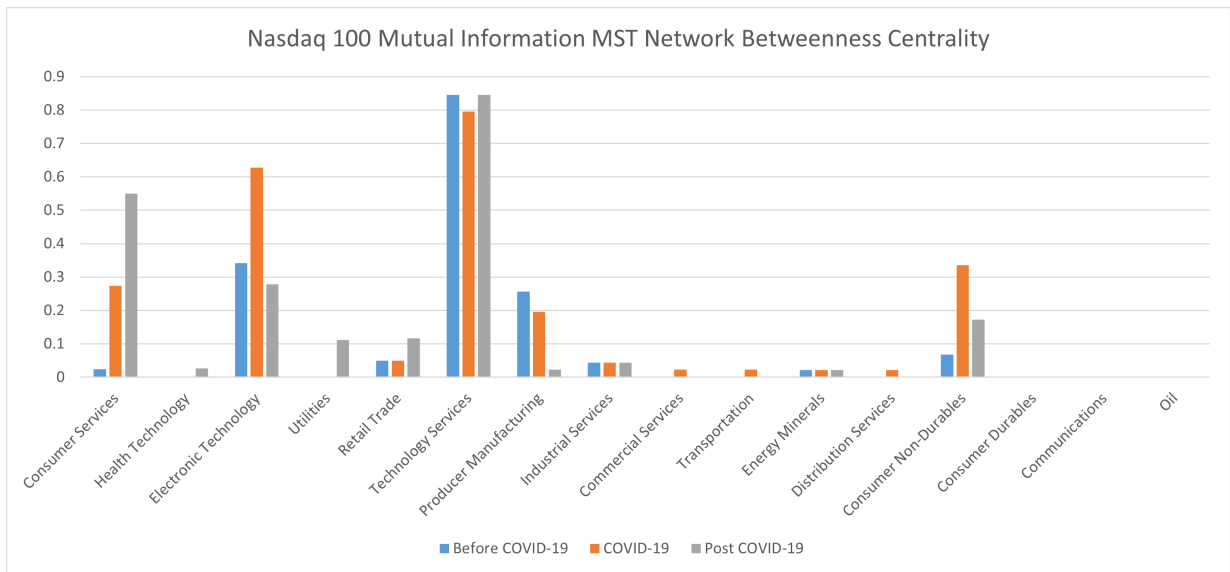


Figure 1.27: Nasdaq 100 Mutual Information MST Network Betweenness Centrality

In section 1.5.1, we have tested the connections between the Canadian and US markets in a partial correlation network. Similarly, we establish a MST mutual information network by using the mutual information metric. Figure 1.28 illustrates the mutual information MST network between the TSX 60 and the Nasdaq 100 sectors. It corroborates the international connections of the energy and utility sectors, which is also identified in Figure 1.22b.

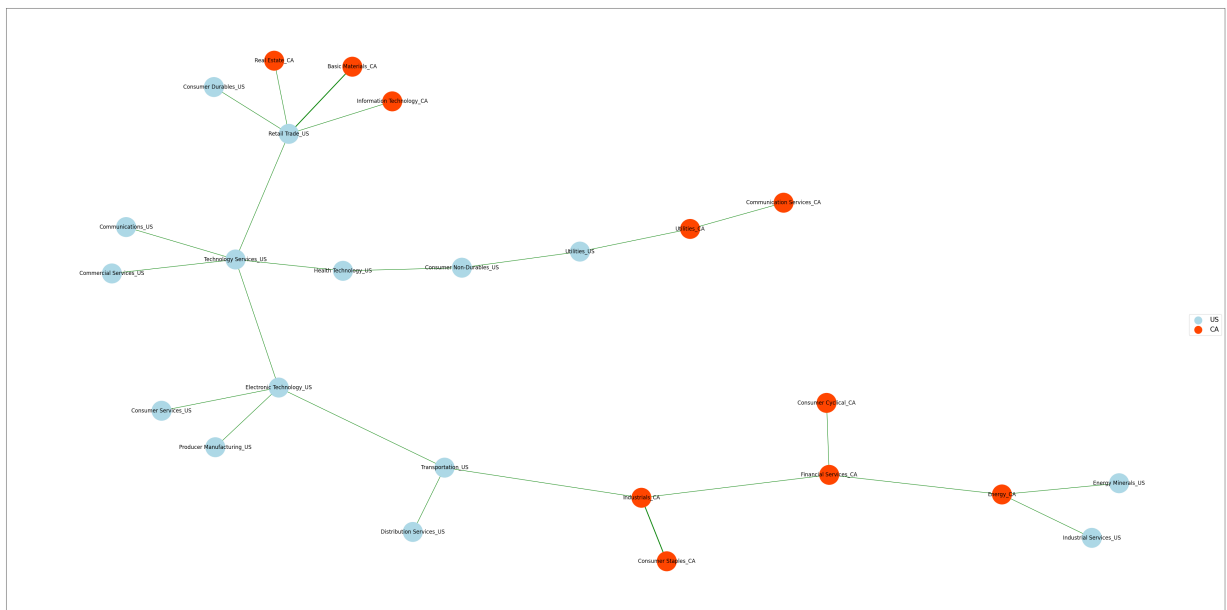


Figure 1.28: TSX 60 and Nasdaq 100 Sectoral Returns Mutual Information MST Network

## 1.6 Conclusion

This chapter uses network analysis to examine the undirected networks in both the US and Canadian stock markets over the past six years, including the COVID-19 pandemic period. For the Canadian stock market, we investigated the network structure of the TSX 60. Dividend-driven sectors have shown significant interconnections throughout all sample periods. These sectors typically have a strong history of capital accumulation, regular dividend payouts, and robust cash flow. Furthermore, the presence of multiple high group betweenness centrality sectors and well-defined sectoral clusters enhances the network's resilience to systemic risk by ensuring both strong intra-sectoral cohesion and diversified inter-sectoral linkages. For the US market, we examined the evolution of the Nasdaq 100 network. The Nasdaq 100 is a growth-driven index, which heavily focuses on the technology sectors. Although the technology service and electronic technology sectors act as pivotal bridges within the network, it is evident that the network itself has undergone notable changes over time. One such change is the growing importance of the health technology sector. The evolving network topology suggests that the Nasdaq 100 is more flexible than the TSX 60, allowing for greater adaptability to technological advancements and market innovations. The explosion of artificial intelligence and technological development during the sample period has further reinforced the Nasdaq 100's growth trajectory.

Diversification remains a fundamental strategy for mitigating systemic risk. By holding positions in assets that exhibit low correlation with market portfolio, investors can effectively manage exposure to market downturns. This study contributes to risk management practices by providing a network-based framework for identifying correlations among stock returns. Through various correlation identification strategies and in-depth analysis of stock return network (i.e. centrality analysis, minimum spanning tree analysis), investors and portfolio managers can extract valuable insights into market structure and refine their investment strategies. For example, from the TSX 60 analysis, we know that the dividend-driven sectors, such as energy, utility, and communication services sectors, form the foundation of TSX 60 network, although the financial sector accounts for 30% of TSX

60 capitalization. Additionally, stocks positioned at the periphery of the MST network offer potential opportunities for diversification, as they are comparatively less influenced by dominant sectors.

Overall, this study demonstrates the applicability of network analysis in financial research, particularly in the context of market risk management. By leveraging financial networks, investors and policymakers can gain a deeper understanding of sectoral interdependencies, and optimal diversification strategies. These insights provide a foundation for future research exploring the dynamic nature of financial networks and their implications for market stability.

## 1.7 Tables

Table 1.1: TSX 60 Constituents Descriptive Statistics

| Stock  | Sector                 | Average Return | Standard Deviation | Minimum Return | Maximum Return |
|--------|------------------------|----------------|--------------------|----------------|----------------|
| ABX    | Basic Materials        | -0.001%        | 2.12               | -11.62%        | 11.77%         |
| AEM    | Basic Materials        | 0.01%          | 2.21               | -16.99%        | 14.57%         |
| AQN    | Utilities              | 0.002%         | 1.59               | -21.44%        | 11.81%         |
| ATD    | Consumer Staples       | 0.05%          | 1.57               | -15.07%        | 9.97%          |
| BCE    | Communication Services | 0.017%         | 1.06               | -13.35%        | 11.14%         |
| BIP-UN | Utilities              | 0.031%         | 1.65               | -18.92%        | 18.71%         |
| BMO    | Financial Services     | 0.029%         | 1.4                | -17.93%        | 15.68%         |
| BN     | Financial Services     | 0.039%         | 1.77               | -15.05%        | 15.52%         |
| BNS    | Financial Services     | 0.008%         | 1.27               | -14.35%        | 15.56%         |
| CAE    | Industrials            | 0.036%         | 2.38               | -24.03%        | 16.73%         |
| CAR-UN | Real Estate            | 0.037%         | 1.32               | -15.21%        | 6.1%           |
| CCL-B  | Consumer Cyclical      | 0.012%         | 1.61               | -18.71%        | 14.25%         |
| CCO    | Energy                 | 0.083%         | 2.77               | -18.53%        | 16.85%         |
| CM     | Financial Services     | 0.019%         | 1.32               | -18.79%        | 17.36%         |
| CNQ    | Energy                 | 0.064%         | 2.64               | -34.5%         | 20.39%         |
| CNR    | Industrials            | 0.037%         | 1.34               | -10.88%        | 9.88%          |
| CP     | Industrials            | 0.062%         | 1.53               | -17.1%         | 12.52%         |
| CSU    | Information Technology | 0.092%         | 1.61               | -9.41%         | 13.32%         |
| CTC-A  | Consumer Cyclical      | 0.017%         | 1.75               | -22.06%        | 13.61%         |
| CVE    | Energy                 | 0.027%         | 3.54               | -72.66%        | 20.83%         |
| DOL    | Consumer Staples       | 0.064%         | 1.6                | -18.86%        | 10.73%         |
| EMA    | Utilities              | 0.025%         | 1.15               | -12.21%        | 9.44%          |
| ENB    | Energy                 | 0.013%         | 1.56               | -18.04%        | 18.8%          |
| FM     | Basic Materials        | 0.051%         | 3.78               | -25.12%        | 34.97%         |
| FNV    | Basic Materials        | 0.049%         | 1.71               | -9.93%         | 10.97%         |
| FSV    | Real Estate            | 0.068%         | 1.71               | -14.27%        | 16.37%         |
| FTS    | Utilities              | 0.03%          | 1.12               | -11.55%        | 10.07%         |
| GIB-A  | Information Technology | 0.044%         | 1.27               | -11.4%         | 6.71%          |
| GIL    | Consumer Cyclical      | 0.018%         | 2.18               | -29.74%        | 19.55%         |
| H      | Utilities              | 0.038%         | 1.05               | -11.46%        | 9.57%          |
| IFC    | Financial Services     | 0.054%         | 1.18               | -9.44%         | 11.1%          |
| IMO    | Energy                 | 0.043%         | 2.27               | -19.51%        | 16.08%         |
| K      | Basic Materials        | 0.022%         | 2.81               | -16.63%        | 18.11%         |
| L      | Consumer Staples       | 0.048%         | 1.19               | -11.16%        | 11.16%         |
| MFC    | Financial Services     | 0.024%         | 1.71               | -19.76%        | 18.33%         |
| MG     | Consumer Cyclical      | 0.027%         | 2.15               | -21.54%        | 18.13%         |
| MRU    | Consumer Staples       | 0.039%         | 1.08               | -8.85%         | 10.58%         |
| NA     | Financial Services     | 0.047%         | 1.42               | -18.34%        | 17.87%         |
| OTEX   | Information Technology | 0.019%         | 1.6                | -14.75%        | 11.98%         |
| POW    | Financial Services     | 0.034%         | 1.57               | -19.66%        | 18.69%         |
| PPL    | Energy                 | 0.025%         | 2.24               | -31.1%         | 30.75%         |
| QSR    | Consumer Cyclical      | 0.031%         | 1.75               | -23.22%        | 16.55%         |
| RCI-B  | Communication Services | 0.015%         | 1.37               | -14.47%        | 12.86%         |
| RY     | Financial Services     | 0.033%         | 1.14               | -11.14%        | 13.89%         |
| SAP    | Consumer Staples       | -0.023%        | 1.43               | -11.85%        | 11.49%         |
| SHOP   | Information Technology | 0.148%         | 3.68               | -18.76%        | 20.83%         |
| SLF    | Financial Services     | 0.034%         | 1.32               | -16.1%         | 12.09%         |
| SU     | Energy                 | 0.02%          | 2.39               | -23.14%        | 22.05%         |
| T      | Communication Services | 0.022%         | 1.1                | -14.23%        | 11.66%         |
| TD     | Financial Services     | 0.03%          | 1.28               | -13.16%        | 16.45%         |
| TECK-B | Basic Materials        | 0.055%         | 3.06               | -24.7%         | 19.07%         |
| TOU    | Energy                 | 0.054%         | 2.42               | -12.8%         | 13.99%         |
| TRI    | Industrials            | 0.075%         | 1.26               | -10.59%        | 10.21%         |
| TRP    | Energy                 | 0.006%         | 1.6                | -24.66%        | 18.19%         |
| WCN    | Industrials            | 0.061%         | 1.23               | -9.94%         | 7.74%          |
| WN     | Consumer Staples       | 0.025%         | 1.21               | -12.48%        | 11.12%         |
| WPM    | Basic Materials        | 0.046%         | 2.06               | -10.7%         | 13.41%         |
| WTI    | <i>OILINDEX</i>        | 0.067%         | 3.03               | -28.22%        | 31.96%         |

The total number of observations for each selected stock/index is 1725 (i.e., 6 years and 9 months). The data is retrieved from Yahoo Finance as of 2023-10.

Table 1.2: Nasdaq 100 Constituents Descriptive Statistics

| <b>Stock</b> | <b>Sector</b>          | <b>Average Return</b> | <b>Standard Deviation</b> | <b>Minimum Return</b> | <b>Maximum Return</b> |
|--------------|------------------------|-----------------------|---------------------------|-----------------------|-----------------------|
| Stock        | Sector                 | Average Return        | Standard Deviation        | Minimum Return        | Maximum Return        |
| AAPL         | Electronic Technology  | 0.11%                 | 1.92                      | -13.77%               | 11.32%                |
| ADBE         | Technology Services    | 0.1%                  | 2.22                      | -18.38%               | 16.31%                |
| ADI          | Electronic Technology  | 0.06%                 | 2.05                      | -18.17%               | 13.71%                |
| ADP          | Technology Services    | 0.06%                 | 1.66                      | -16.55%               | 11.16%                |
| ADSK         | Technology Services    | 0.06%                 | 2.45                      | -17.27%               | 15.01%                |
| AEP          | Utilities              | 0.03%                 | 1.37                      | -12.31%               | 11.05%                |
| ALGN         | Health Technology      | 0.07%                 | 3.22                      | -31.45%               | 29.99%                |
| AMAT         | Producer Manufacturing | 0.1%                  | 2.72                      | -22.76%               | 12.06%                |
| AMD          | Electronic Technology  | 0.13%                 | 3.54                      | -27.75%               | 18.19%                |
| AMGN         | Health Technology      | 0.05%                 | 1.52                      | -8.62%                | 10.34%                |
| AMZN         | Retail Trade           | 0.07%                 | 2.13                      | -15.14%               | 12.69%                |
| ANSS         | Technology Services    | 0.07%                 | 2.04                      | -13.18%               | 14.1%                 |
| ASML         | Electronic Technology  | 0.11%                 | 2.39                      | -19.05%               | 13.6%                 |
| AVGO         | Electronic Technology  | 0.11%                 | 2.22                      | -22.21%               | 14.7%                 |
| AZN          | Health Technology      | 0.07%                 | 1.61                      | -16.14%               | 8.89%                 |
| BIIB         | Health Technology      | -0.01%                | 2.76                      | -34.57%               | 36.45%                |
| BKNG         | Consumer Services      | 0.05%                 | 2.14                      | -14.53%               | 17.19%                |
| BKR          | Industrial Services    | 0%                    | 2.66                      | -25.18%               | 18.71%                |
| CDNS         | Technology Services    | 0.13%                 | 2.1                       | -13.47%               | 14.29%                |
| CHTR         | Consumer Services      | 0.03%                 | 1.93                      | -17.89%               | 13.27%                |
| CMCSA        | Consumer Services      | 0.03%                 | 1.69                      | -9.57%                | 11.84%                |
| COST         | Retail Trade           | 0.09%                 | 1.45                      | -13.3%                | 9.49%                 |
| CPRT         | Commercial Services    | 0.11%                 | 1.84                      | -15.15%               | 16%                   |
| CSCO         | Technology Services    | 0.05%                 | 1.69                      | -14.77%               | 12.55%                |
| CSGP         | Technology Services    | 0.09%                 | 2.02                      | -16.25%               | 14.57%                |
| CSX          | Transportation         | 0.06%                 | 1.88                      | -16.9%                | 21.03%                |
| CTAS         | Consumer Services      | 0.09%                 | 1.82                      | -18.47%               | 13.68%                |
| CTSH         | Technology Services    | 0.02%                 | 1.86                      | -18.85%               | 13.94%                |
| DLTR         | Retail Trade           | 0.02%                 | 2.26                      | -17.34%               | 19.77%                |
| DXCM         | Health Technology      | 0.11%                 | 3.1                       | -39.53%               | 26.73%                |
| EA           | Technology Services    | 0.03%                 | 1.87                      | -14.28%               | 14.89%                |
| EBAY         | Retail Trade           | 0.03%                 | 1.96                      | -12.47%               | 12.95%                |
| ENPH         | Electronic Technology  | 0.29%                 | 4.93                      | -30.12%               | 35.38%                |

Continued on next page

Table 1.2 – continued from previous page

| Stock | Sector                 | Average Return | Standard Deviation | Minimum Return | Maximum Return |
|-------|------------------------|----------------|--------------------|----------------|----------------|
| EXC   | Utilities              | 0.04%          | 1.6                | -17.54%        | 16.55%         |
| FANG  | Energy Minerals        | 0.03%          | 3.57               | -59.14%        | 26.97%         |
| FAST  | Distribution Services  | 0.06%          | 1.75               | -11.68%        | 15.83%         |
| FTNT  | Technology Services    | 0.14%          | 2.62               | -28.86%        | 19.25%         |
| GILD  | Health Technology      | 0.02%          | 1.59               | -9.01%         | 12.15%         |
| GOOG  | Technology Services    | 0.07%          | 1.86               | -11.77%        | 9.94%          |
| GOOGL | Technology Services    | 0.07%          | 1.86               | -12.37%        | 9.19%          |
| HON   | Electronic Technology  | 0.04%          | 1.55               | -12.88%        | 14.04%         |
| IDXX  | Health Technology      | 0.08%          | 2.05               | -12.71%        | 12.44%         |
| ILMN  | Health Technology      | 0%             | 2.57               | -17.57%        | 15.68%         |
| INTC  | Electronic Technology  | 0.01%          | 2.26               | -19.9%         | 17.83%         |
| INTU  | Technology Services    | 0.09%          | 2.14               | -15.64%        | 18.3%          |
| ISRG  | Health Technology      | 0.09%          | 2.12               | -15.47%        | 16.08%         |
| JD    | Retail Trade           | 0.01%          | 3.09               | -17.24%        | 33.19%         |
| KDP   | Consumer Non-Durables  | 0.06%          | 1.53               | -18.02%        | 20.21%         |
| KHC   | Consumer Non-Durables  | -0.04%         | 1.86               | -32.1%         | 17.81%         |
| KLAC  | Electronic Technology  | 0.12%          | 2.6                | -14.5%         | 15.86%         |
| LRCX  | Producer Manufacturing | 0.12%          | 2.81               | -20.37%        | 18.49%         |
| LULU  | Retail Trade           | 0.11%          | 2.51               | -26.71%        | 15.11%         |
| MAR   | Consumer Services      | 0.06%          | 2.26               | -16.48%        | 17.8%          |
| MCHP  | Electronic Technology  | 0.06%          | 2.55               | -22.68%        | 14.48%         |
| MDLZ  | Consumer Non-Durables  | 0.04%          | 1.3                | -12.14%        | 10.69%         |
| MELI  | Retail Trade           | 0.12%          | 3.35               | -18.49%        | 18.97%         |
| META  | Technology Services    | 0.06%          | 2.59               | -30.64%        | 20.93%         |
| MNST  | Consumer Non-Durables  | 0.05%          | 1.71               | -15.59%        | 12.01%         |
| MRVL  | Electronic Technology  | 0.08%          | 2.91               | -15.82%        | 28.08%         |
| MSFT  | Technology Services    | 0.1%           | 1.81               | -15.95%        | 13.29%         |
| MU    | Electronic Technology  | 0.07%          | 2.86               | -22.09%        | 12.52%         |
| NFLX  | Technology Services    | 0.06%          | 2.85               | -43.26%        | 15.58%         |
| NVDA  | Electronic Technology  | 0.17%          | 3.17               | -20.77%        | 21.81%         |
| NXPI  | Electronic Technology  | 0.05%          | 2.52               | -21.54%        | 16.47%         |
| ODFL  | Transportation         | 0.12%          | 2.01               | -13.9%         | 12.04%         |
| ON    | Electronic Technology  | 0.12%          | 3.2                | -31.25%        | 21.82%         |
| ORLY  | Retail Trade           | 0.07%          | 1.83               | -20.94%        | 12.27%         |
| PANW  | Technology Services    | 0.1%           | 2.41               | -27.65%        | 17.06%         |

Continued on next page

**Table 1.2 – continued from previous page**

| <b>Stock</b> | <b>Sector</b>          | <b>Average Return</b> | <b>Standard Deviation</b> | <b>Minimum Return</b> | <b>Maximum Return</b> |
|--------------|------------------------|-----------------------|---------------------------|-----------------------|-----------------------|
| PAYX         | Technology Services    | 0.05%                 | 1.68                      | -21.67%               | 16.72%                |
| PCAR         | Producer Manufacturing | 0.06%                 | 1.64                      | -7.92%                | 13.28%                |
| PEP          | Consumer Non-Durables  | 0.04%                 | 1.26                      | -12.14%               | 12.17%                |
| PYPL         | Commercial Services    | 0.02%                 | 2.55                      | -28.22%               | 13.2%                 |
| QCOM         | Electronic Technology  | 0.05%                 | 2.44                      | -16.19%               | 20.87%                |
| REGN         | Health Technology      | 0.05%                 | 2                         | -11.08%               | 17.27%                |
| ROST         | Retail Trade           | 0.04%                 | 2.22                      | -25.45%               | 15.38%                |
| SBUX         | Consumer Services      | 0.04%                 | 1.79                      | -17.68%               | 13.74%                |
| SGEN         | Health Technology      | 0.08%                 | 2.55                      | -16.7%                | 17.38%                |
| SIRI         | Consumer Services      | 0.01%                 | 2.18                      | -16.55%               | 35.25%                |
| SNPS         | Technology Services    | 0.12%                 | 1.97                      | -14.14%               | 10.62%                |
| TEAM         | Technology Services    | 0.13%                 | 3.25                      | -34.19%               | 22.03%                |
| TMUS         | Communications         | 0.05%                 | 1.71                      | -11.89%               | 11.14%                |
| TSLA         | Consumer Durables      | 0.17%                 | 3.84                      | -23.65%               | 18.14%                |
| TXN          | Electronic Technology  | 0.06%                 | 1.87                      | -12.59%               | 12.69%                |
| VRSK         | Technology Services    | 0.07%                 | 1.53                      | -11.29%               | 12.47%                |
| VRTX         | Health Technology      | 0.09%                 | 2.09                      | -23.19%               | 18.92%                |
| WBA          | Health Technology      | -0.06%                | 1.97                      | -13.7%                | 11.87%                |
| WBD          | Consumer Services      | -0.05%                | 2.96                      | -32.08%               | 15.59%                |
| WDAY         | Technology Services    | 0.07%                 | 2.48                      | -13.64%               | 15.84%                |
| XEL          | Utilities              | 0.04%                 | 1.4                       | -13.57%               | 10.67%                |
| WTI          | Oil                    | 0.07%                 | 3.06                      | -28.22%               | 31.96%                |

The total number of observations for each selected stock/index is 1694 (i.e., 6 years and 9 months). The data is retrieved from Yahoo

Finance as of 2023-10.

Table 1.3: TSX 60 Constituents Centrality

| Stock  | Degree Centrality | Betweenness Centrality | Eigenvector Centrality | Clustering |
|--------|-------------------|------------------------|------------------------|------------|
| SHOP   | 0.772             | 0.021                  | 0.198                  | 0.527      |
| CVE    | 0.737             | 0.021                  | 0.184                  | 0.504      |
| FM     | 0.737             | 0.021                  | 0.186                  | 0.51       |
| IFC    | 0.684             | 0.016                  | 0.176                  | 0.536      |
| WTI    | 0.667             | 0.016                  | 0.171                  | 0.528      |
| MG     | 0.649             | 0.015                  | 0.167                  | 0.517      |
| MFC    | 0.632             | 0.014                  | 0.159                  | 0.521      |
| CCL-B  | 0.632             | 0.014                  | 0.162                  | 0.537      |
| QSR    | 0.596             | 0.01                   | 0.158                  | 0.583      |
| TRI    | 0.596             | 0.011                  | 0.155                  | 0.569      |
| PPL    | 0.596             | 0.012                  | 0.154                  | 0.535      |
| TECK-B | 0.596             | 0.011                  | 0.156                  | 0.547      |
| BN     | 0.579             | 0.013                  | 0.145                  | 0.492      |
| CCO    | 0.579             | 0.012                  | 0.147                  | 0.523      |
| WCN    | 0.561             | 0.011                  | 0.146                  | 0.54       |
| OTEX   | 0.544             | 0.009                  | 0.143                  | 0.555      |
| BIP-UN | 0.544             | 0.01                   | 0.14                   | 0.538      |
| SAP    | 0.526             | 0.009                  | 0.136                  | 0.549      |
| TRP    | 0.526             | 0.011                  | 0.13                   | 0.492      |
| WPM    | 0.526             | 0.009                  | 0.134                  | 0.526      |
| CTC-A  | 0.526             | 0.01                   | 0.135                  | 0.52       |
| CP     | 0.509             | 0.008                  | 0.135                  | 0.571      |
| FSV    | 0.509             | 0.008                  | 0.132                  | 0.547      |
| GIL    | 0.509             | 0.008                  | 0.136                  | 0.567      |
| CAR-UN | 0.509             | 0.01                   | 0.127                  | 0.515      |
| IMO    | 0.491             | 0.007                  | 0.131                  | 0.579      |
| CNQ    | 0.491             | 0.007                  | 0.13                   | 0.556      |
| CSU    | 0.491             | 0.008                  | 0.127                  | 0.529      |
| FNV    | 0.491             | 0.009                  | 0.126                  | 0.521      |
| DOL    | 0.474             | 0.007                  | 0.125                  | 0.584      |
| SLF    | 0.474             | 0.007                  | 0.124                  | 0.578      |
| GIB-A  | 0.474             | 0.006                  | 0.126                  | 0.613      |
| SU     | 0.474             | 0.006                  | 0.126                  | 0.584      |
| NA     | 0.474             | 0.009                  | 0.117                  | 0.49       |
| K      | 0.474             | 0.007                  | 0.122                  | 0.538      |
| BNS    | 0.456             | 0.009                  | 0.11                   | 0.505      |
| ENB    | 0.456             | 0.008                  | 0.115                  | 0.508      |
| AQN    | 0.456             | 0.006                  | 0.119                  | 0.572      |
| AEM    | 0.439             | 0.007                  | 0.113                  | 0.54       |
| L      | 0.439             | 0.007                  | 0.11                   | 0.513      |
| TOU    | 0.439             | 0.005                  | 0.121                  | 0.613      |
| H      | 0.439             | 0.006                  | 0.112                  | 0.56       |
| ATD    | 0.421             | 0.005                  | 0.112                  | 0.598      |
| BMO    | 0.421             | 0.007                  | 0.103                  | 0.529      |
| FTS    | 0.421             | 0.006                  | 0.108                  | 0.58       |
| CAE    | 0.421             | 0.005                  | 0.111                  | 0.558      |
| POW    | 0.421             | 0.005                  | 0.111                  | 0.569      |
| WN     | 0.404             | 0.007                  | 0.097                  | 0.502      |
| RCI-B  | 0.404             | 0.005                  | 0.103                  | 0.553      |
| CNR    | 0.404             | 0.004                  | 0.107                  | 0.593      |
| MRU    | 0.404             | 0.005                  | 0.104                  | 0.549      |
| T      | 0.404             | 0.006                  | 0.103                  | 0.526      |
| EMA    | 0.404             | 0.005                  | 0.099                  | 0.565      |
| BCE    | 0.386             | 0.006                  | 0.093                  | 0.528      |
| RY     | 0.368             | 0.006                  | 0.088                  | 0.486      |
| TD     | 0.368             | 0.004                  | 0.093                  | 0.552      |
| CM     | 0.333             | 0.004                  | 0.084                  | 0.544      |
| ABX    | 0.333             | 0.004                  | 0.083                  | 0.532      |

Table 1.4: TSX 60 Constituents Centrality and Weighted Centrality Measurement

| Stock  | Degree Centrality | Betweenness Centrality | Eigenvector Centrality | Weighted Degree Centrality | Weighted Betweenness Centrality | Weighted Eigenvector Centrality |
|--------|-------------------|------------------------|------------------------|----------------------------|---------------------------------|---------------------------------|
| SHOP   | 44                | 0.021                  | 0.198                  | 1.828                      | 0.076                           | 0.184                           |
| CVE    | 42                | 0.021                  | 0.184                  | 1.929                      | 0.057                           | 0.225                           |
| FM     | 42                | 0.021                  | 0.186                  | 1.586                      | 0.029                           | 0.179                           |
| IFC    | 39                | 0.016                  | 0.176                  | 0.983                      | 0.165                           | 0.103                           |
| WTI    | 38                | 0.016                  | 0.171                  | 1.443                      | 0.03                            | 0.172                           |
| MG     | 37                | 0.015                  | 0.167                  | 1.507                      | 0.004                           | 0.155                           |
| MFC    | 36                | 0.014                  | 0.159                  | 1.517                      | 0.059                           | 0.168                           |
| CCL-B  | 36                | 0.014                  | 0.162                  | 0.962                      | 0.002                           | 0.098                           |
| PPL    | 34                | 0.012                  | 0.154                  | 1.693                      | 0.048                           | 0.187                           |
| TECK-B | 34                | 0.011                  | 0.156                  | 1.453                      | 0.001                           | 0.172                           |
| TRI    | 34                | 0.011                  | 0.155                  | 1.16                       | 0.015                           | 0.117                           |
| QSR    | 34                | 0.01                   | 0.158                  | 1.288                      | 0.021                           | 0.134                           |
| BN     | 33                | 0.013                  | 0.145                  | 1.492                      | 0.001                           | 0.149                           |
| CCO    | 33                | 0.012                  | 0.147                  | 0.884                      | 0.105                           | 0.103                           |
| WCN    | 32                | 0.011                  | 0.146                  | 1.296                      | 0.064                           | 0.126                           |
| BIP-UN | 31                | 0.01                   | 0.14                   | 1.064                      | 0.028                           | 0.104                           |
| OTEX   | 31                | 0.009                  | 0.143                  | 1.259                      | 0.093                           | 0.13                            |
| TRP    | 30                | 0.011                  | 0.13                   | 1.099                      | 0.136                           | 0.117                           |
| CTC-A  | 30                | 0.01                   | 0.135                  | 1.05                       | 0.151                           | 0.103                           |
| WPM    | 30                | 0.009                  | 0.134                  | 1.302                      | 0.118                           | 0.144                           |
| SAP    | 30                | 0.009                  | 0.136                  | 0.743                      | 0.027                           | 0.065                           |
| CAR-UN | 29                | 0.01                   | 0.127                  | 0.873                      | 0.12                            | 0.089                           |
| FSV    | 29                | 0.008                  | 0.132                  | 1.158                      | 0.027                           | 0.117                           |
| CP     | 29                | 0.008                  | 0.135                  | 1.142                      | 0.059                           | 0.108                           |
| GIL    | 29                | 0.008                  | 0.136                  | 1.035                      | 0.132                           | 0.104                           |
| FNV    | 28                | 0.009                  | 0.126                  | 1.281                      | 0.08                            | 0.14                            |
| CSU    | 28                | 0.008                  | 0.127                  | 1.099                      | 0.003                           | 0.11                            |
| CNQ    | 28                | 0.007                  | 0.13                   | 1.525                      | 0.013                           | 0.202                           |
| IMO    | 28                | 0.007                  | 0.131                  | 1.501                      | 0                               | 0.192                           |
| NA     | 27                | 0.009                  | 0.117                  | 1.187                      | 0.046                           | 0.117                           |
| K      | 27                | 0.007                  | 0.122                  | 1.393                      | 0.05                            | 0.151                           |
| SLF    | 27                | 0.007                  | 0.124                  | 1.226                      | 0.004                           | 0.135                           |
| DOL    | 27                | 0.007                  | 0.125                  | 0.877                      | 0                               | 0.082                           |
| SU     | 27                | 0.006                  | 0.126                  | 1.297                      | 0.025                           | 0.179                           |
| GIB-A  | 27                | 0.006                  | 0.126                  | 1.268                      | 0.006                           | 0.123                           |
| BNS    | 26                | 0.009                  | 0.11                   | 1.19                       | 0.004                           | 0.115                           |
| ENB    | 26                | 0.008                  | 0.115                  | 1.19                       | 0.121                           | 0.129                           |
| AQN    | 26                | 0.006                  | 0.119                  | 0.946                      | 0.004                           | 0.095                           |
| L      | 25                | 0.007                  | 0.11                   | 1.37                       | 0.062                           | 0.114                           |
| AEM    | 25                | 0.007                  | 0.113                  | 1.305                      | 0.058                           | 0.148                           |
| H      | 25                | 0.006                  | 0.112                  | 0.956                      | 0.115                           | 0.098                           |
| TOU    | 25                | 0.005                  | 0.121                  | 0.868                      | 0.009                           | 0.11                            |
| BMO    | 24                | 0.007                  | 0.103                  | 1.342                      | 0                               | 0.134                           |
| FTS    | 24                | 0.006                  | 0.108                  | 1.581                      | 0.019                           | 0.149                           |
| POW    | 24                | 0.005                  | 0.111                  | 1.172                      | 0.001                           | 0.126                           |
| ATD    | 24                | 0.005                  | 0.112                  | 0.865                      | 0.035                           | 0.075                           |
| CAE    | 24                | 0.005                  | 0.111                  | 1.164                      | 0.013                           | 0.12                            |
| WN     | 23                | 0.007                  | 0.097                  | 1.22                       | 0.026                           | 0.101                           |
| T      | 23                | 0.006                  | 0.103                  | 1.06                       | 0.013                           | 0.088                           |
| RCI-B  | 23                | 0.005                  | 0.103                  | 0.815                      | 0.008                           | 0.076                           |
| MRU    | 23                | 0.005                  | 0.104                  | 0.992                      | 0.026                           | 0.086                           |
| EMA    | 23                | 0.005                  | 0.099                  | 1.235                      | 0.111                           | 0.121                           |
| CNR    | 23                | 0.004                  | 0.107                  | 1.069                      | 0                               | 0.097                           |
| BCE    | 22                | 0.006                  | 0.093                  | 1.333                      | 0.044                           | 0.108                           |
| RY     | 21                | 0.006                  | 0.088                  | 1.223                      | 0                               | 0.121                           |
| TD     | 21                | 0.004                  | 0.093                  | 1.133                      | 0.004                           | 0.113                           |
| ABX    | 19                | 0.004                  | 0.083                  | 1.185                      | 0.048                           | 0.132                           |
| CM     | 19                | 0.004                  | 0.084                  | 1.178                      | 0                               | 0.12                            |

Table 1.5: TSX 60 Constituents Centrality and Weighted Centrality Measurement - MST

| Stock  | Degree Centrality | Betweenness Centrality | Eigenvector Centrality | Weighted Degree Centrality | Weighted Betweenness Centrality | Weighted Eigenvector Centrality |
|--------|-------------------|------------------------|------------------------|----------------------------|---------------------------------|---------------------------------|
| CAE    | 5                 | 0.704                  | 0.53                   | 4.731                      | 0.697                           | 0.55                            |
| GIL    | 3                 | 0.42                   | 0.366                  | 2.899                      | 0.414                           | 0.389                           |
| BN     | 3                 | 0.405                  | 0.329                  | 2.748                      | 0.403                           | 0.322                           |
| GIB-A  | 4                 | 0.409                  | 0.294                  | 3.644                      | 0.402                           | 0.292                           |
| QSR    | 2                 | 0.474                  | 0.264                  | 1.839                      | 0.47                            | 0.262                           |
| CAR-UN | 2                 | 0.035                  | 0.235                  | 1.871                      | 0.035                           | 0.245                           |
| MG     | 1                 | 0                      | 0.201                  | 0.947                      | 0                               | 0.212                           |
| MFC    | 3                 | 0.25                   | 0.183                  | 2.582                      | 0.249                           | 0.166                           |
| CCL-B  | 1                 | 0                      | 0.139                  | 0.992                      | 0                               | 0.157                           |
| TRI    | 2                 | 0.246                  | 0.143                  | 1.87                       | 0.238                           | 0.145                           |
| PPL    | 3                 | 0.511                  | 0.165                  | 2.661                      | 0.508                           | 0.141                           |
| BIP-UN | 2                 | 0.133                  | 0.151                  | 1.793                      | 0.132                           | 0.132                           |
| CSU    | 2                 | 0.069                  | 0.134                  | 1.852                      | 0.069                           | 0.128                           |
| OTEX   | 2                 | 0.035                  | 0.131                  | 1.672                      | 0.035                           | 0.111                           |
| FSV    | 1                 | 0                      | 0.089                  | 0.929                      | 0                               | 0.093                           |
| WCN    | 3                 | 0.226                  | 0.083                  | 2.82                       | 0.217                           | 0.079                           |
| WTI    | 3                 | 0.385                  | 0.094                  | 2.713                      | 0.383                           | 0.075                           |
| POW    | 2                 | 0.192                  | 0.084                  | 1.808                      | 0.191                           | 0.071                           |
| DOL    | 2                 | 0.035                  | 0.06                   | 1.909                      | 0.035                           | 0.059                           |
| H      | 2                 | 0.102                  | 0.07                   | 1.778                      | 0.101                           | 0.058                           |
| ENB    | 2                 | 0.133                  | 0.077                  | 1.442                      | 0.132                           | 0.05                            |
| SLF    | 1                 | 0                      | 0.07                   | 0.734                      | 0                               | 0.049                           |
| SHOP   | 1                 | 0                      | 0.05                   | 0.845                      | 0                               | 0.038                           |
| ATD    | 2                 | 0.133                  | 0.038                  | 1.926                      | 0.125                           | 0.038                           |
| CP     | 2                 | 0.035                  | 0.037                  | 1.301                      | 0.035                           | 0.031                           |
| RY     | 2                 | 0.163                  | 0.039                  | 1.749                      | 0.162                           | 0.031                           |
| IFC    | 1                 | 0                      | 0.036                  | 0.985                      | 0                               | 0.03                            |
| CVE    | 2                 | 0.338                  | 0.046                  | 1.434                      | 0.336                           | 0.026                           |
| CTC-A  | 1                 | 0                      | 0.023                  | 0.942                      | 0                               | 0.023                           |
| EMA    | 2                 | 0.069                  | 0.032                  | 1.272                      | 0.069                           | 0.02                            |
| TRP    | 2                 | 0.102                  | 0.036                  | 1.601                      | 0.101                           | 0.016                           |
| MRU    | 2                 | 0.102                  | 0.018                  | 1.669                      | 0.093                           | 0.016                           |
| TD     | 2                 | 0.133                  | 0.018                  | 1.653                      | 0.132                           | 0.012                           |
| CNQ    | 3                 | 0.328                  | 0.027                  | 2.304                      | 0.327                           | 0.01                            |
| BCE    | 3                 | 0.07                   | 0.019                  | 2.297                      | 0.069                           | 0.007                           |
| TOU    | 2                 | 0.246                  | 0.013                  | 1.914                      | 0.244                           | 0.005                           |
| BNS    | 2                 | 0.102                  | 0.008                  | 1.706                      | 0.101                           | 0.005                           |
| L      | 2                 | 0.069                  | 0.008                  | 0.719                      | 0.103                           | 0.005                           |
| CNR    | 1                 | 0                      | 0.014                  | 0.349                      | 0                               | 0.004                           |
| FTS    | 2                 | 0.035                  | 0.014                  | 1.29                       | 0.035                           | 0.004                           |
| SU     | 2                 | 0.035                  | 0.012                  | 1.316                      | 0.035                           | 0.003                           |
| RCI-B  | 1                 | 0                      | 0.007                  | 0.833                      | 0                               | 0.002                           |
| FM     | 3                 | 0.226                  | 0.008                  | 2.401                      | 0.224                           | 0.002                           |
| CM     | 2                 | 0.069                  | 0.004                  | 1.645                      | 0.069                           | 0.002                           |
| T      | 1                 | 0                      | 0.007                  | 0.504                      | 0                               | 0.002                           |
| AQN    | 1                 | 0                      | 0.005                  | 0.844                      | 0                               | 0.001                           |
| WPM    | 2                 | 0.133                  | 0.004                  | 1.735                      | 0.132                           | 0.001                           |
| IMO    | 1                 | 0                      | 0.005                  | 0.634                      | 0                               | 0.001                           |
| BMO    | 2                 | 0.035                  | 0.002                  | 1.674                      | 0.035                           | 0.001                           |
| TECK-B | 2                 | 0.035                  | 0.003                  | 1.421                      | 0.035                           | 0.001                           |
| FNV    | 2                 | 0.102                  | 0.002                  | 1.568                      | 0.101                           | 0                               |
| NA     | 1                 | 0                      | 0.001                  | 0.912                      | 0                               | 0                               |
| CCO    | 1                 | 0                      | 0.001                  | 0.956                      | 0                               | 0                               |
| AEM    | 2                 | 0.069                  | 0.001                  | 1.561                      | 0.069                           | 0                               |
| ABX    | 2                 | 0.035                  | 0                      | 1.478                      | 0.035                           | 0                               |
| K      | 1                 | 0                      | 0                      | 0.725                      | 0                               | 0                               |
| WN     | 2                 | 0.035                  | 0.004                  | 0.98                       | 0.053                           | 0                               |
| SAP    | 1                 | 0                      | 0.001                  | 0.98                       | 0                               | 0                               |

Table 1.6: Latent Variable Time-varying Graphical Lasso (LTGL)

|            | Parameter | Value |
|------------|-----------|-------|
| TSX 60     | $\alpha$  | 0.03  |
|            | $\beta$   | 7     |
|            | $\eta$    | 2     |
|            | $\tau$    | 0.12  |
| <hr/>      |           |       |
|            | Parameter | Value |
| Nasdaq 100 | $\alpha$  | 0.14  |
|            | $\beta$   | 10    |
|            | $\eta$    | 4     |
|            | $\tau$    | 3.46  |

The parameters is optimized via 5-fold cross validation.  $\alpha$  search space is  $[0.01, 1]$ ,  $\tau$  search space is  $[0.1, 10]$ ,  $\beta$  search space is  $[0, 15]$ , and  $\eta$  search space is  $[0, 5]$ .

Table 1.7: The Degree Centrality of TSX 60 Network and Group's Centrality Sensitivity Analysis

| <i>Sector</i>                       | Partial Correlation Thresholds |       |       |
|-------------------------------------|--------------------------------|-------|-------|
|                                     | 0                              | 0.05  | 0.1   |
| TSX 60 Network (average centrality) | 0.502                          | 0.119 | 0.052 |
| Basic Materials                     | 0.98                           | 0.176 | 0.021 |
| Utilities                           | 0.774                          | 0.226 | 0.02  |
| Consumer Staples                    | 0.942                          | 0.269 | 0.020 |
| Financial Services                  | 1.000                          | 0.404 | 0.089 |
| Communication Services              | 0.709                          | 0.127 | 0.000 |
| Industrials                         | 0.849                          | 0.377 | 0.120 |
| Real Estate                         | 0.75                           | 0.232 | 0.057 |
| Consumer Cyclical                   | 1.000                          | 0.340 | 0.098 |
| Energy                              | 1.000                          | 0.408 | 0.087 |
| Information Technology              | 0.963                          | 0.296 | 0.039 |
| OIL INDEX                           | 0.667                          | 0.158 | 0.019 |

The centrality of a node is calculated as the number of edges connecting to the node. The degree centrality values are normalized by dividing them by the maximum possible degree in a simple graph, which is  $n-1$ , where  $n$  represents the number of nodes in a network. The network centrality of TSX 60 is the average of individual stock degree centrality. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes (i.e. The total number of nodes - the number of nodes in the group). The maximum degree centrality is 1. Note: GLASSO uses the optimized  $\alpha$  of 0.05.

Table 1.8: The Degree Centrality of TSX 60 Network and Group's Centrality by Periods

| <i>Partial Correlation Thresholds</i> | <i>Sector</i>          | <i>Sample Periods</i> |          |               |
|---------------------------------------|------------------------|-----------------------|----------|---------------|
|                                       |                        | Before COVID-19       | COVID-19 | Post COVID-19 |
| 0                                     | TSX 60 Network         | 0.786                 | 0.796    | 0.785         |
|                                       | Basic Materials        | 1.000                 | 1.000    | 1.000         |
|                                       | Utilities              | 1.000                 | 1.000    | 1.000         |
|                                       | Consumer Staples       | 1.000                 | 1.000    | 1.000         |
|                                       | Financial Services     | 1.000                 | 1.000    | 1.000         |
|                                       | Communication Services | 1.000                 | 1.000    | 1.000         |
|                                       | Industrials            | 1.000                 | 1.000    | 1.000         |
|                                       | Real Estate            | 0.946                 | 0.929    | 0.964         |
|                                       | Consumer Cyclical      | 1.000                 | 1.000    | 1.000         |
|                                       | Energy                 | 1.000                 | 1.000    | 1.000         |
|                                       | Information Technology | 0.944                 | 1.000    | 1.000         |
|                                       | OIL INDEX              | 0.825                 | 0.684    | 0.754         |
| 0.05                                  | TSX 60 Network         | 0.084                 | 0.078    | 0.086         |
|                                       | Basic Materials        | 0.255                 | 0.275    | 0.255         |
|                                       | Utilities              | 0.208                 | 0.245    | 0.283         |
|                                       | Consumer Staples       | 0.288                 | 0.25     | 0.308         |
|                                       | Financial Services     | 0.511                 | 0.468    | 0.447         |
|                                       | Communication Services | 0.127                 | 0.145    | 0.145         |
|                                       | Industrials            | 0.208                 | 0.170    | 0.208         |
|                                       | Real Estate            | 0.125                 | 0.089    | 0.143         |
|                                       | Consumer Cyclical      | 0.226                 | 0.245    | 0.283         |
|                                       | Energy                 | 0.367                 | 0.245    | 0.388         |
|                                       | Information Technology | 0.130                 | 0.167    | 0.185         |
|                                       | OIL INDEX              | 0.053                 | 0.035    | 0.035         |
| 0.1                                   | TSX 60 Network         | 0.043                 | 0.043    | 0.043         |
|                                       | Basic Materials        | 0.077                 | 0.051    | 0.049         |
|                                       | Utilities              | 0.000                 | 0.000    | 0.023         |
|                                       | Consumer Staples       | 0.000                 | 0.024    | 0.047         |
|                                       | Financial Services     | 0.083                 | 0.111    | 0.132         |
|                                       | Communication Services | 0.000                 | 0.000    | 0.000         |
|                                       | Industrials            | 0.023                 | 0.023    | 0.022         |
|                                       | Real Estate            | 0.000                 | 0.000    | 0.000         |
|                                       | Consumer Cyclical      | 0.070                 | 0.070    | 0.091         |
|                                       | Energy                 | 0.081                 | 0.053    | 0.050         |
|                                       | Information Technology | 0.023                 | 0.023    | 0.022         |
|                                       | OIL INDEX              | 0.000                 | 0.000    | 0.000         |

The centrality of a node is calculated as the number of edges connecting to the node. The degree centrality values are normalized by dividing them by the maximum possible degree in a simple graph, which is  $n-1$ , where  $n$  represents the number of nodes in a network. The network centrality of TSX 60 is the average of individual stock degree centrality. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes (i.e. The total number of nodes - the number of nodes in the group). The maximum degree centrality is 1. Note: The optimized parameters are summarized in Table 1.6.

Table 1.9: The Correlation of Centrality Measurements - TSX 60

| <i>Correlation of Centrality Measurements</i> |                        |                   |                        |                        |       |
|---|------------------------|-------------------|------------------------|------------------------|-------|
| <i>Sample Period</i>                          |                        | Degree Centrality | Betweenness Centrality | Eigenvector Centrality |       |
| Before COVID-19                               | Degree Centrality      | 1.000             | 0.953                  | 0.997                  | 0.997 |
|   | Betweenness Centrality | 0.953             | 1.000                  | 0.929                  | 0.929 |
|   | Eigenvector Centrality | 0.997             | 0.929                  | 1.000                  | 1.000 |
| COVID-19                                      | Degree Centrality      | 1.000             | 0.956                  | 0.998                  | 0.998 |
|   | Betweenness Centrality | 0.956             | 1.000                  | 0.938                  | 0.938 |
|   | Eigenvector Centrality | 0.998             | 0.938                  | 1.000                  | 1.000 |
| Post COVID-19                                 | Degree Centrality      | 1.000             | 0.960                  | 0.998                  | 0.998 |
|   | Betweenness Centrality | 0.960             | 1.000                  | 0.941                  | 0.941 |
|   | Eigenvector Centrality | 0.998             | 0.941                  | 1.000                  | 1.000 |

Degree centrality, betweenness centrality, and eigenvector centrality are calculated at the stock level for each period. The centrality measurement correlation is calculated for each sample period.

Table 1.10: The Degree Centrality of Nasdaq 100 Network and Group's Centrality

| <i>Sector</i>          | Partial Correlation Thresholds |       |       |
|------------------------|--------------------------------|-------|-------|
|                        | 0                              | 0.05  | 0.1   |
| Nasdaq 100 Network     | 0.326                          | 0.072 | 0.029 |
| Consumer Services      | 0.902                          | 0.305 | 0.070 |
| Health Technology      | 0.987                          | 0.299 | 0.031 |
| Electronic Technology  | 1.000                          | 0.370 | 0.131 |
| Utilities              | 0.552                          | 0.080 | 0.014 |
| Retail Trade           | 0.926                          | 0.333 | 0.070 |
| Technology Services    | 1.000                          | 0.456 | 0.067 |
| Producer Manufacturing | 0.529                          | 0.115 | 0.068 |
| Industrial Services    | 0.371                          | 0.045 | 0.026 |
| Commercial Services    | 0.58                           | 0.136 | 0.027 |
| Transportation         | 0.523                          | 0.080 | 0.027 |
| Energy Minerals        | 0.494                          | 0.112 | 0.026 |
| Distribution Services  | 0.303                          | 0.090 | 0.026 |
| Consumer Non-Durables  | 0.741                          | 0.129 | 0.028 |
| Consumer Durables      | 0.404                          | 0.079 | 0.013 |
| Communications         | 0.404                          | 0.022 | 0.000 |
| OIL INDEX              | 0.315                          | 0.034 | 0.026 |

The centrality of a node is calculated as the number of edges connecting to the node. The degree centrality values are normalized by dividing them by the maximum possible degree in a simple graph, which is  $n-1$ , where  $n$  represents the number of nodes in a network. The network centrality of TSX 60 is the average of individual stock degree centrality. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes (i.e. The total number of nodes - the number of nodes in the group). The maximum degree centrality is 1. Note: GLASSO uses the optimized  $\alpha$  of 0.167.

Table 1.11: The Degree Centrality of Nasdaq 100 Network and Group's Centrality by Periods

| <i>Partial Correlation Thresholds</i> | <i>Sector</i>          | <i>Sample Periods</i> |          |               |
|---------------------------------------|------------------------|-----------------------|----------|---------------|
|                                       |                        | Before COVID-19       | COVID-19 | Post COVID-19 |
| 0                                     | Nasdaq 100 Network     | 0.931                 | 0.939    | 0.929         |
|                                       | Consumer Services      | 1.000                 | 1.000    | 1.000         |
|                                       | Health Technology      | 1.000                 | 1.000    | 1.000         |
|                                       | Electronic Technology  | 1.000                 | 1.000    | 1.000         |
|                                       | Utilities              | 1.000                 | 1.000    | 1.000         |
|                                       | Retail Trade           | 1.000                 | 1.000    | 1.000         |
|                                       | Technology Services    | 1.000                 | 1.000    | 1.000         |
|                                       | Producer Manufacturing | 1.000                 | 1.000    | 1.000         |
|                                       | Industrial Services    | 0.933                 | 0.899    | 0.921         |
|                                       | Commercial Services    | 0.989                 | 1.000    | 0.989         |
|                                       | Transportation         | 1.000                 | 1.000    | 1.000         |
|                                       | Energy Minerals        | 0.876                 | 0.899    | 0.944         |
|                                       | Distribution Services  | 0.921                 | 0.966    | 0.978         |
|                                       | Consumer Non-Durables  | 1.000                 | 1.000    | 1.000         |
|                                       | Consumer Durables      | 0.876                 | 0.921    | 0.910         |
|                                       | Communications         | 0.955                 | 0.933    | 0.966         |
| OIL INDEX                             | 0.933                  | 0.888                 | 0.865    |               |
| 0.05                                  | Nasdaq 100 Network     | 0.175                 | 0.205    | 0.258         |
|                                       | Consumer Services      | 0.768                 | 0.793    | 0.829         |
|                                       | Health Technology      | 0.909                 | 0.922    | 0.935         |
|                                       | Electronic Technology  | 0.890                 | 1.000    | 1.000         |
|                                       | Utilities              | 0.264                 | 0.264    | 0.287         |
|                                       | Retail Trade           | 0.753                 | 0.815    | 0.963         |
|                                       | Technology Services    | 0.956                 | 1.000    | 0.971         |
|                                       | Producer Manufacturing | 0.402                 | 0.437    | 0.552         |
|                                       | Industrial Services    | 0.157                 | 0.236    | 0.281         |
|                                       | Commercial Services    | 0.341                 | 0.364    | 0.466         |
|                                       | Transportation         | 0.284                 | 0.352    | 0.386         |
|                                       | Energy Minerals        | 0.101                 | 0.180    | 0.315         |
|                                       | Distribution Services  | 0.146                 | 0.360    | 0.270         |
|                                       | Consumer Non-Durables  | 0.506                 | 0.671    | 0.718         |
|                                       | Consumer Durables      | 0.202                 | 0.292    | 0.292         |
|                                       | Communications         | 0.146                 | 0.18     | 0.236         |
| OIL INDEX                             | 0.18                   | 0.101                 | 0.247    |               |
| 0.1                                   | Nasdaq 100 Network     | 0.036                 | 0.035    | 0.052         |
|                                       | Consumer Services      | 0.165                 | 0.063    | 0.256         |
|                                       | Health Technology      | 0.189                 | 0.095    | 0.442         |
|                                       | Electronic Technology  | 0.197                 | 0.314    | 0.452         |
|                                       | Utilities              | 0.012                 | 0        | 0.023         |
|                                       | Retail Trade           | 0.167                 | 0.076    | 0.296         |
|                                       | Technology Services    | 0.313                 | 0.323    | 0.397         |
|                                       | Producer Manufacturing | 0.107                 | 0.083    | 0.115         |
|                                       | Industrial Services    | 0.035                 | 0.047    | 0.079         |
|                                       | Commercial Services    | 0.071                 | 0.035    | 0.08          |
|                                       | Transportation         | 0.035                 | 0.071    | 0.091         |
|                                       | Energy Minerals        | 0.023                 | 0.023    | 0.067         |
|                                       | Distribution Services  | 0.047                 | 0.058    | 0.056         |
|                                       | Consumer Non-Durables  | 0.012                 | 0.073    | 0.188         |
|                                       | Consumer Durables      | 0.012                 | 0.023    | 0.09          |
|                                       | Communications         | 0.012                 | 0        | 0.034         |
| OIL INDEX                             | 0.035                  | 0.023                 | 0.034    |               |

The centrality of a node is calculated as the number of edges connecting to the node. The degree centrality values are normalized by dividing them by the maximum possible degree in a simple graph, which is  $n-1$ , where  $n$  represents the number of nodes in a network. The network centrality of TSX 60 is the average of individual stock degree centrality. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes (i.e. The total number of nodes - the number of nodes in the group). The maximum degree centrality is 1. Note: The optimized parameters are summarized in Table 1.6.

Table 1.12: The Correlation of Centrality Measurements - Nasdaq 100

| <i>Correlation of Centrality Measurements</i> |                        |                   |                        |                        |       |
|---|------------------------|-------------------|------------------------|------------------------|-------|
| <i>Sample Period</i>                          |                        | Degree Centrality | Betweenness Centrality | Eigenvector Centrality |       |
| Before COVID-19                               | Degree Centrality      | 1.000             | 0.932                  | 1.000                  | 1.000 |
|   | Betweenness Centrality | 0.932             | 1.000                  | 0.922                  | 0.922 |
|   | Eigenvector Centrality | 1.000             | 0.922                  | 1.000                  | 1.000 |
| COVID-19                                      | Degree Centrality      | 1.000             | 0.943                  | 1.000                  | 1.000 |
|   | Betweenness Centrality | 0.943             | 1.000                  | 0.934                  | 0.934 |
|   | Eigenvector Centrality | 1.000             | 0.934                  | 1.000                  | 1.000 |
| Post COVID-19                                 | Degree Centrality      | 1.000             | 0.947                  | 1.000                  | 1.000 |
|   | Betweenness Centrality | 0.947             | 1.000                  | 0.938                  | 0.938 |
|   | Eigenvector Centrality | 1.000             | 0.938                  | 1.000                  | 1.000 |

Degree centrality, betweenness centrality, and eigenvector centrality are calculated at the stock level for each period. The centrality measurement correlation is calculated for each sample period.

Table 1.13: Degree Centrality of International Network by Country

| <i>Sector</i> | Partial Correlation Thresholds |       |       |
|---------------|--------------------------------|-------|-------|
|               | 0                              | 0.05  | 0.1   |
| Overall       | 0.893                          | 0.383 | 0.157 |
| Nasdaq 100    | 1                              | 1     | 0.8   |
| TSX 60        | 1                              | 0.93  | 0.6   |

The centrality of a node is calculated as the number of edges connecting to the node. The degree centrality values are normalized by dividing them by the maximum possible degree in a simple graph, which is  $n-1$ , where  $n$  represents the number of nodes in a network. The network centrality of TSX 60 is the average of individual stock degree centrality. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes (i.e. The total number of nodes - the number of nodes in the group). The maximum degree centrality is 1. Note: GLASSO uses the optimized  $\alpha$  of 0.01.

Table 1.14: The Correlation of Centrality Measurements - International Network

| <i>Correlation of Centrality Measurements</i> |                   |                        |                        |            |
|---|-------------------|------------------------|------------------------|------------|
|   | Degree Centrality | Betweenness Centrality | Eigenvector Centrality | Clustering |
| Degree Centrality                             | 1.000             | 0.932                  | 1.000                  | -0.650     |
| Betweenness Centrality                        | 0.932             | 1.000                  | 0.914                  | -0.875     |
| Eigenvector Centrality                        | 1.000             | 0.914                  | 1.000                  | -0.614     |
| Clustering                                    | -0.650            | -0.875                 | -0.614                 | 1          |

Degree centrality, betweenness centrality, and eigenvector centrality are calculated at the sector level.

Table 1.15: Centrality Measurement of International Network

| <i>Sector</i>             | <i>Degree Centrality</i> | <i>Betweenness Centrality</i> | <i>Eigenvector Centrality</i> | <i>Clustering</i> |
|---------------------------|--------------------------|-------------------------------|-------------------------------|-------------------|
| Consumer Services US      | 1                        | 0.006                         | 0.222                         | 0.884             |
| Real Estate CA            | 1                        | 0.006                         | 0.222                         | 0.884             |
| Retail Trade US           | 0.958                    | 0.006                         | 0.213                         | 0.885             |
| Commercial Services US    | 0.958                    | 0.006                         | 0.213                         | 0.885             |
| Distribution Services US  | 0.958                    | 0.006                         | 0.213                         | 0.885             |
| Consumer Durables US      | 0.958                    | 0.005                         | 0.213                         | 0.889             |
| Information Technology CA | 0.917                    | 0.004                         | 0.205                         | 0.9               |
| Utilities US              | 0.917                    | 0.005                         | 0.204                         | 0.883             |
| Transportation US         | 0.917                    | 0.005                         | 0.204                         | 0.883             |
| Industrial Services US    | 0.917                    | 0.004                         | 0.205                         | 0.9               |
| Energy CA                 | 0.917                    | 0.005                         | 0.205                         | 0.896             |
| Communication Services CA | 0.875                    | 0.005                         | 0.195                         | 0.886             |
| Consumer Cyclical CA      | 0.875                    | 0.004                         | 0.196                         | 0.895             |
| Industrials CA            | 0.875                    | 0.004                         | 0.196                         | 0.9               |
| Consumer Non-Durables US  | 0.875                    | 0.005                         | 0.195                         | 0.886             |
| Consumer Staples CA       | 0.875                    | 0.005                         | 0.194                         | 0.876             |
| Basic Materials CA        | 0.875                    | 0.004                         | 0.196                         | 0.9               |
| Energy Minerals US        | 0.875                    | 0.004                         | 0.196                         | 0.9               |
| Technology Services US    | 0.875                    | 0.005                         | 0.195                         | 0.881             |
| Communications US         | 0.833                    | 0.004                         | 0.187                         | 0.9               |
| Utilities CA              | 0.833                    | 0.003                         | 0.188                         | 0.905             |
| Producer Manufacturing US | 0.833                    | 0.004                         | 0.188                         | 0.9               |
| Electronic Technology US  | 0.833                    | 0.004                         | 0.187                         | 0.9               |
| Financial Services CA     | 0.792                    | 0.003                         | 0.179                         | 0.912             |
| Health Technology US      | 0.792                    | 0.003                         | 0.179                         | 0.906             |

Table summarizes four centrality measurement, degree centrality, betweenness centrality, eigenvector centrality, and clustering.

Table 1.16: TSX 60 Mutual Information MST Network Group Centrality

| <i>Sector</i>          | <i>Group Degree Centrality</i> | <i>Group Betweenness Centrality</i> |
|------------------------|--------------------------------|-------------------------------------|
| Basic Materials        | 0.058                          | 0.039                               |
| Utilities              | 0.075                          | 0.428                               |
| Consumer Staples       | 0.019                          | 0.000                               |
| Financial Services     | 0.255                          | 0.660                               |
| Communication Services | 0.036                          | 0.197                               |
| Industrials            | 0.056                          | 0.457                               |
| Real Estate            | 0.017                          | 0.000                               |
| Consumer Cyclical      | 0.056                          | 0.000                               |
| Energy                 | 0.061                          | 0.119                               |
| Information Technology | 0.074                          | 0.520                               |
| OIL INDEX              | 0.017                          | 0.000                               |

Table summarizes group degree centrality and group betweenness centrality.

Table 1.17: TSX 60 and Nasdaq 100 Mutual Information Descriptive Statistics By Periods

|                    | Before COVID-19 |            | COVID-19 |            | Post COVID-19 |            |
|--------------------|-----------------|------------|----------|------------|---------------|------------|
|                    | TSX 60          | Nasdaq 100 | TSX 60   | Nasdaq 100 | TSX 60        | Nasdaq 100 |
| Mean               | 0.126           | 0.169      | 0.219    | 0.254      | 0.194         | 0.267      |
| Standard Deviation | 0.074           | 0.098      | 0.077    | 0.102      | 0.106         | 0.150      |
| Minimum            | 0.021           | 0.019      | 0.082    | 0.055      | 0.051         | 0.031      |
| Maximum            | 0.724           | 1.400      | 0.839    | 1.380      | 0.929         | 1.961      |

Table 1.18: Nasdaq 100 Mutual Information MST Network Group Centrality

| <i>Sector</i>          | <i>Group Degree Centrality</i> | <i>Group Betweenness Centrality</i> |
|------------------------|--------------------------------|-------------------------------------|
| Consumer Services      | 0.048                          | 0.310                               |
| Health Technology      | 0.038                          | 0.000                               |
| Electronic Technology  | 0.164                          | 0.449                               |
| Utilities              | 0.011                          | 0.000                               |
| Retail Trade           | 0.086                          | 0.024                               |
| Technology Services    | 0.264                          | 0.856                               |
| Producer Manufacturing | 0.068                          | 0.336                               |
| Industrial Services    | 0.022                          | 0.044                               |
| Commercial Services    | 0.022                          | 0.000                               |
| Transportation         | 0.022                          | 0.000                               |
| Energy Minerals        | 0.022                          | 0.022                               |
| Distribution Services  | 0.022                          | 0.022                               |
| Consumer Non-Durables  | 0.023                          | 0.068                               |
| Consumer Durables      | 0.011                          | 0.000                               |
| Communications         | 0.011                          | 0.000                               |
| OIL INDEX              | 0.011                          | 0.000                               |

Table summarizes group degree centrality and group betweenness centrality.

# Chapter 2

## Directed Network Analysis

Chapter 1 examined the structural connectedness of the TSX 60 and Nasdaq 100 through undirected network, using methodologies such as Graphical LASSO, latent variable time-varying graphical LASSO, and mutual information measures. This approach provided valuable insights into the evolution of market structures before, during, and after the COVID-19 pandemic. However, undirected networks treat relationships as symmetric and therefore cannot capture the directionality or asymmetry in risk transmission.

The next step is to explore how shocks travel in specific directions between market participants. In particular, identifying whether certain stocks or sectors tend to act as net transmitters or receivers of volatility, is crucial for understanding systemic vulnerabilities. Chapter 2 addresses this gap by employing directed network methodologies, specifically net transfer entropy and the Diebold - Yilmaz spillover framework, to quantify and compare directional volatility spillovers. This enables a deeper examination of how information and risk propagate in North American equity markets over different time horizons, enhancing the ability to design targeted risk mitigation strategies.

### 2.1 Introduction

Systemic risk involves the potential failure or instability of the entire financial system or significant portions of it, often triggered by interdependencies among institutions or mar-

kets. Events such as financial crises, widespread bank failures, or severe market disruptions exemplify systemic risk, as they can cause cascading failures with profound economic consequences. Conversely, systematic risk, reflects the volatility inherent in the stock market due to macroeconomic factors such as changes in interest rates, inflation, or geopolitical tensions. Unlike systemic risk, which highlights the fragility of interconnected financial networks, systematic risk cannot be diversified away because it influences the entire market. For investors, understanding both risks is essential: systemic risk underscores the vulnerabilities within financial systems, while systematic risk emphasizes market-wide exposure to economic fluctuations.

Network analysis (NA) has gained popularity in academic research focusing on systemic risk (Diebold and Yilmaz 2014, Zhang et al. 2020a, Niu and Hu 2021, Ahelegbey and Giudici 2020, and Sharif et al. 2020). This chapter uses NA to explore the structural changes in the North America stock markets in the most recent six years, including the global pandemic period. The COVID-19 virus was identified in China at the end of 2019 and spread rapidly throughout the world, causing a global pandemic. The pandemic has had a dramatic impact on social activities in North America. In response, governments implemented various containment measures, including social distancing mandates and lockdown, which significantly disrupted business operations. Many industries adapted by modifying their business models or operating at reduced capacity. Some industries, such as real estate and technology, experienced growth during the pandemic, resulting in a shift in investment strategies. Following the global pandemic, inflation, contractionary monetary policy and imbalanced global energy market have created many economic uncertainties, and further induced a volatile stock market. The COVID-19 pandemic has been characterized by a high level of synchronicity among markets, with a rapid spread of risks and unprecedented uncertainty (Baker et al. 2020, Qureshi et al. 2021, Deng et al. 2022, and Sharif et al. 2020). An investigation into the transmission of volatility among stocks will facilitate an understanding of the impact of COVID-19 on the stock market performance.

During the COVID-19 pandemic, the TSX demonstrated greater resilience compared to its U.S. counterparts, as the TSX 60 index declined by only 6% in 2022, whereas the

S&P 500 fell by more than 17% and the Nasdaq composite index dropped nearly 30%. This relative stability can be attributed to the TSX's sectoral composition, which includes a strong presence of commodity-based industries, financial services, and dividend-paying stocks, all of which provided defensive characteristics during market downturns. In contrast, the Nasdaq stock market, home to some of the world's largest technology firms such as Apple, Microsoft, and Amazon, exhibited significantly higher volatility due to its growth-oriented structure. The Nasdaq 100 index, which excludes financial companies, offers a unique focus on technology and innovation-driven firms, making it highly sensitive to macroeconomic fluctuations, including interest rate changes, inflationary pressures, and shifts in investor sentiment. The performance divergence between the TSX and Nasdaq 100 highlights the role of market composition in determining systemic risk exposure, reinforcing the importance of network-based approaches in understanding financial contagion and risk transmission across stock markets.

This chapter analyzes the evolution of the network structure of TSX 60 and Nasdaq 100 using annualized daily stock volatility from January 2017 to September 2023. This chapter's main contributions are threefold. First, this chapter establishes the directed networks for North America stock markets in different time periods (i.e. the "before COVID-19", the "COVID-19", and the "Post COVID-19 " periods) and identify the change in network connections as a response to the market uncertainties in both short term and long term. Secondly, this study compares two popular methodologies for analyzing volatility: Net transfer entropy and the Diebold-Yilmaz spillover index. The results show that both methods complement each other's analysis. Finally, this study demonstrates the usefulness of network analysis in the field of systemic risk management and highlights the importance of combining volatility analysis and network analysis.

The rest of this chapter is organized as follows. Section 2.2 reviews network analysis and risk management literature, specifically focusing on non-parametric and parametric methodologies associated with directed network analysis. Section 2.3 briefly discusses the data used. Section 2.4 illustrates the network edge identification strategies. Section 2.5 provides empirical results, and Section 2.6 concludes.

## 2.2 Literature Review

A growing body of literature underscores the critical role of network topology in shaping systemic risk, highlighting that the interconnectedness of financial institutions can significantly exacerbate risk transmission. In highly centralized networks, where financial exposures are concentrated, financial contagion can be amplified, increasing systemic fragility. NA provides a quantitative framework for assessing systemic vulnerabilities, allowing for a more comprehensive understanding of the structural dependencies among financial entities. Diem et al. (2020) demonstrate that systemic risk in interbank loan networks can be mitigated through topological restructuring, which reduces the concentration of financial dependencies. Their findings indicate that systemically important banks exhibit high levels of interconnectedness, which intensifies contagion and heightens financial instability. As a methodological approach, network analysis enables the exploration of a system's structure through a graph-based representation, consisting of two fundamental components: nodes and edges. Each node represents an analytic object, for example, an individual in a social network, or a stock in a stock market network. The edges capture the existence of a connection or relationship among the nodes. Networks can be classified as undirected or directed (Freeman, 2004). Chapter 1 was dedicated to the examination of undirected networks, while chapter 2 concentrates on directed networks. A directed network not only provides a connectedness of the system but also illustrates the direction of the connection between nodes. The directional dependence measurement is a crucial element in constructing the directed network.

The existing literature demonstrates two core approaches. The parametric approach utilizes an econometric model, vector autoregression (VAR), to construct a connectedness table. Diebold and Yilmaz (2009) proposed a forecast error variance decompositions (FEVDs) method to allow the application of VAR identifications on the directed network analysis. The Diebold and Yilmaz (DY) “2009” framework has two limitations. First, the Cholesky factor identification of VARs results in variance decompositions that depend on the ordering of variables. It is preferable to have a spillover measure that is invariant

to ordering. Second, the framework only captures aggregated spillovers and not pairwise spillovers. Diebold and Yilmaz (2012) and Diebold and Yilmaz (2014) use the generalized VAR framework of Sims (1980), which is invariant to ordering, to address the limitations of DY framework. Since then, the DY framework has been widely used in the financial spillover analysis (see Brunner and Hipp 2021, Demirer et al. 2018, Balcilar and Bekun 2020, and Meglioli and Gauci 2021). Demirer et al. (2018) develop a global bank network using FEVDs techniques and introduce the LASSO method in a high-dimensional network setting. Another application of VAR is for identifying the granger causality in a network. Tian et al. (2019) construct causal networks and use the centrality of nodes in the Chinese and American stock markets on the multiple timescales to quantify the influence of market participants. Their study shows that the impact of dominant sectors on various time scales is relatively consistent in both the Chinese and American markets.

The second approach is a non-parametric method, using entropy concept and empirical mode decomposition. Shannon (1948) proposes a metric to quantify a variable's information contained in a system. The entropy value measures the required information to identify a specific event. When the system has two discrete variables, the mutual information quantifies the information gained from one variable to another variable. Schreiber (2000) advances the study of mutual information and proposes transfer entropy to capture the directional information flow in a system. The transfer entropy has been extensively applied in the directed financial network analysis. Niu and Hu (2021) shows that the transfer information transmission between the commodity futures market and the stock market decreases gradually as the time scale increases. He et al. (2022) establish an entropy network of the stock market to identify the systemic risk contributions. Bekiros et al. (2017) examine the causal relationships between US equity and commodity futures markets via an entropy network analysis.

Investigating financial data is a complex process as different risk appetites lead to different investment strategies; different events trigger different market reactions. The study of a financial dataset might lead to different conclusions depending on data frequency time horizons, such as short-term, middle-term and long-term. Huang et al. (1998) design

the empirical mode decomposition method (EMD), a non-parametric method, to analyse non-linear and non-stationary data. The intrinsic mode functions (IMFs) define the instantaneous frequencies from the original data, which are integral parts of EMD. Torres et al. (2011) present the complete empirical mode decomposition based on adaptive noise (CEEMDAN) to solve the EMD mode mixing problem. In essence, EMD permits to implement financial data mining on various time scales (see Niu and Hu 2021 and Tian et al. 2019).

As the aforementioned edge identification strategy is not a binary (0/1) measurement, the value of edges reflects the strength of the connection. In order to identify the most critical connections among stocks, a minimum spanning tree (MST) is employed to filter out the most robust connections. MST is a connected network comprising a link between all  $n$  nodes of the network with exactly  $n - 1$  edges, with the objective of minimizing the total weight of the edges. It is a popular method in graph theory for extracting from a complex weighted network, a spanning path with optimized edge weights. This feature has been widely used in analyzing financial networks. He et al. (2022), and Barbi and Pratavia (2019) apply the minimum spanning tree (MST) algorithm to extract significant connection information in a stock market, and maintain the simplest structure of the stock market network. Niu and Hu (2021) uses a MST to identify the connections between the Chinese stock market and the commodity futures market. We apply this algorithm using both the Diebold-Yilmaz spillover index and the transfer entropy measure, as a way to characterizing the weight of the network, and eventually to build the MST that permits us to investigate the volatility network in both the TSX 60 and the Nasdaq 100 networks.

## **2.3 Data**

### **2.3.1 Data processing**

The return and volatility of the stock market are potential metrics to capture relevant stock connections because they reflects forward-looking assessments of all available information

(Diebold and Yilmaz, 2014). In this chapter, we use the daily highest/lowest stock prices with adjustment from January, 2017 to September, 2023 for TSX 60 and Nasdaq 100. Table 2.1 and Table 2.2 provide a summary of descriptive statistics for the annualized daily volatility of the TSX 60 constituents and the Nasdaq 100 constituents. In addition, the study employs the stock daily realized variance to measure volatility spillovers among stocks listed in the TSX 60 and the Nasdaq 100. Parkinson (1980) proposes a direct daily volatility measurement by using daily highest and lowest prices. In this chapter, the log of annualized daily volatility, variance of stock  $i$  on day  $t$  ( $\sigma_{i,t}$ ), is defined as:

$$\sigma_{i,t} = 100 * \sqrt{365 * \underbrace{(0.361 * [\ln(P_{i,t}^{Max}) - \ln(P_{i,t}^{Min})])^2}_{\text{Part 1}})}, \quad (2.1)$$

where  $\ln(P_{i,t}^{Max})$  is the log of the highest price of stock  $i$  on day  $t$ ;  $\ln(P_{i,t}^{Min})$  is the log of lowest price of stock  $i$  on day  $t$ ; Part 1 is an estimator of the daily volatility that is originally proposed by Parkinson. 0.361 captures the relationship between the diffusion constant and the square of stock daily variation.<sup>1</sup> The log of annualized daily volatility ensures the normality assumption. The daily highest/lowest stock prices are adjusted for any split, dividend and merging.

Figure 2.1 represents the sectoral distribution of the TSX 60 and the Nasdaq 100. The left panel shows that the technology service and electronic technology sectors comprise more than 70% of the total capitalization of the Nasdaq 100. Meanwhile, the right panel indicates that information technology accounts for only 6% of the TSX 60 capitalization. The capitalization of the TSX 60 is primarily driven by the Financials, energy, industrials, and basic materials sectors.

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<sup>1</sup>It is generally accepted that a random walk process provides a good approximation of the movement of the log of stock price. Parkinson (1980) investigates the relationship between the movement of the log stock price and the diffusion constant in a continuous random walk. The diffusion constant is the variance of the displacement of stock price after a unit time interval. The study shows that the extreme value estimate for the diffusion constant ( $D$ ) can be written as  $D = \frac{0.361}{n} \sum_{i=1}^n (\ln \frac{\ln(P_i^{Max})}{\ln(P_i^{Min})})$ , where  $n$  is the unit time interval (i.e.  $n = 1$  in our case).

## Nasdaq 100 Index and TSX 60 Index Sector Distribution by Market Capitalization

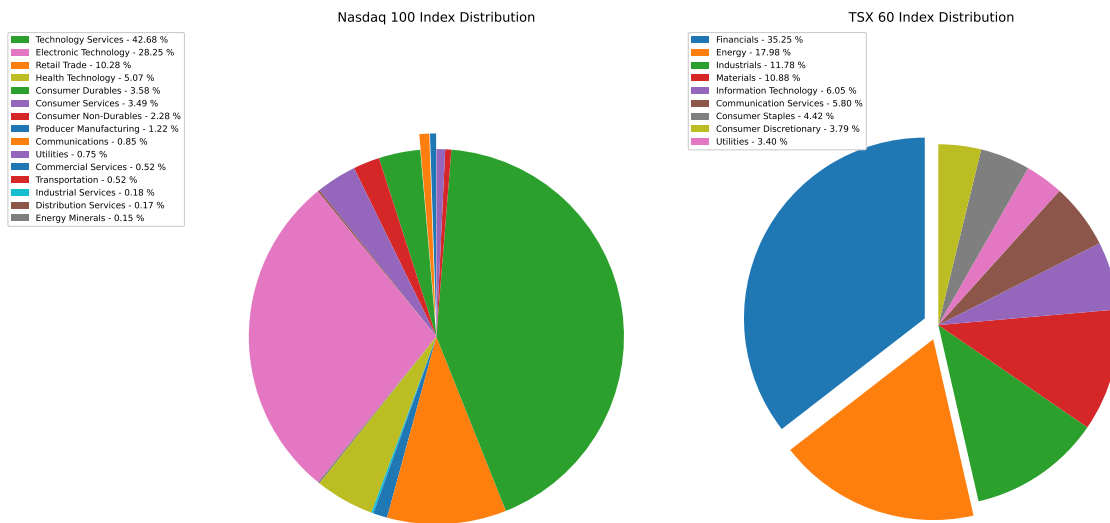
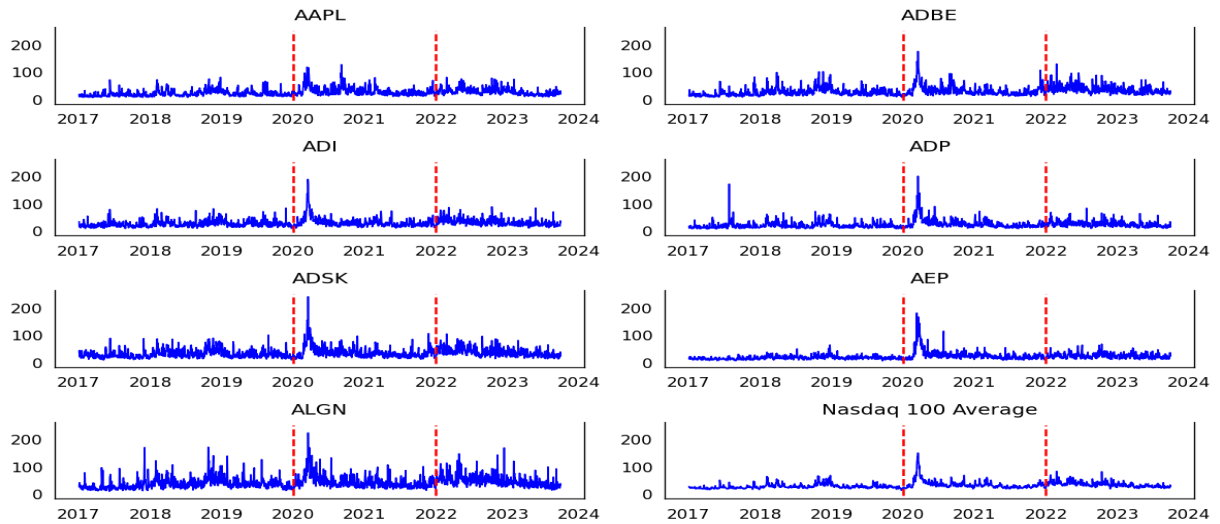


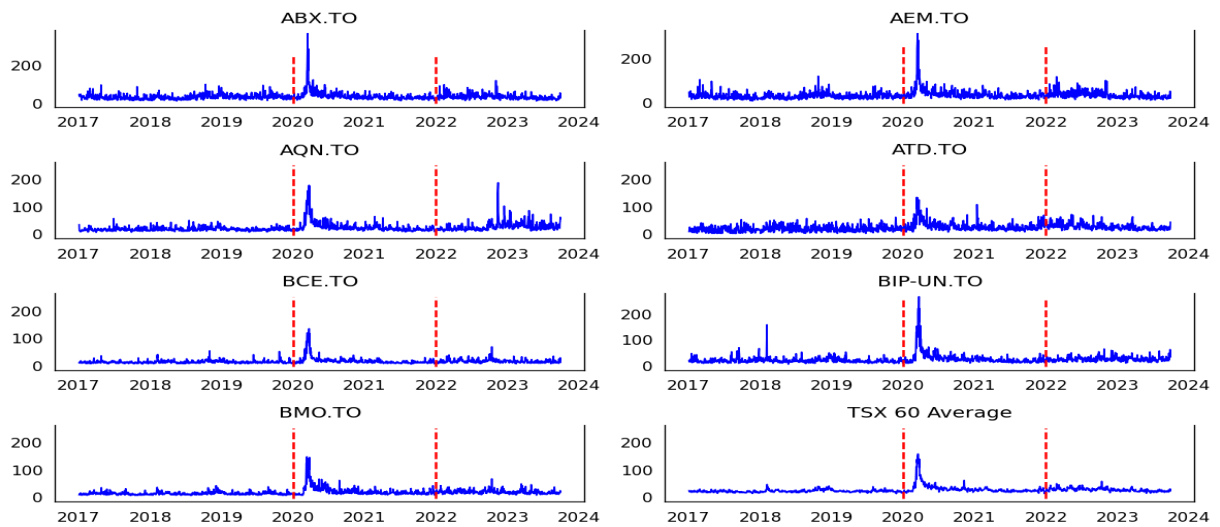
Figure 2.1: The Sector Distribution of the TSX 60 and the Nasdaq 100 Constituents

The data is divided into three sub-samples to analyze the impact of COVID-19 on the North American stock market. The period from January 2017 to December 2019 (inclusive) is defined as the “before COVID-19” period; The period from January 2020 to December 2021 (inclusive) refers the “COVID-19” period; The remaining data was used to measure the “post-COVID-19” period. Figure 2.2 randomly selects a few stocks from both markets and depicts their annualized daily volatility and the average of annualized daily volatility for each market constituent over the entire sample period. Figure 2.2a shows the annualized daily volatility of eight selected stocks listed on the Nasdaq 100. Figure 2.2b shows the eight chosen stocks in the TSX 60. Both markets indicate a high annualized price variance in the beginning of global pandemic. Additionally, the pre COVID-19 period illustrates a relative more stable price movement than the post-COVID-19 period. The rationale behind these date selections is driven by two key aspects. First, the market initial market crash occurred at the end of February 2020, reaching a bottom in early March 2020. Figure 2.2 also shows the considerable volatility that was observed at the outset of 2020. In early 2022, both Canada and the United States observed a high rate of inflation. Both countries initiated a contractionary monetary policy in March 2022. The “post-COVID” period, therefore,

represents a distinct economic environment from both the “COVID-19” and the “before COVID-19” periods. The second rationale for these dates is that the study uses the EMD to investigate both short-term and long-term patterns. To prevent the data decomposition from starting at the event’s inception (i.e. end of February, 2020), each observation period was extended by a few weeks.



(a) Nasdaq 100 - selected stocks and Nasdaq 100 average



(b) TSX 60 - selected stocks and TSX 60 average

Figure 2.2: Annualized Daily Volatility The red dashed lines indicate the dates on which the data were separated, leading to three distinct periods.

## 2.3.2 Stock Market Data

**Canadian Stock Market** Although the TSX composite index is considered as a premier index for the Canadian market, it includes many small, less active companies whose stock prices remain relatively stable due to low trading volumes. If a stable price is driven by low trading volume rather than market conditions or the company's performance, the daily volatility may contain little information in a network analysis. The TSX 60, however, is designed to separate the large capitalization segment of the Canadian equity market from the TSX composite constituents. These stocks have high trading volumes and are more attractive from an investment perspective. Figure 2.3 depicts the effective returns of TSX 60 and TSX composite index assuming 2017-01-03 as the starting point. The movements between these two indexes are consistent over time. Figure 2.4 illustrates that the TSX 60 is a subset of the TSX composite index with a similar sector coverage. Thus, this chapter uses the performance of TSX 60 as a proxy of TSX composite performance.

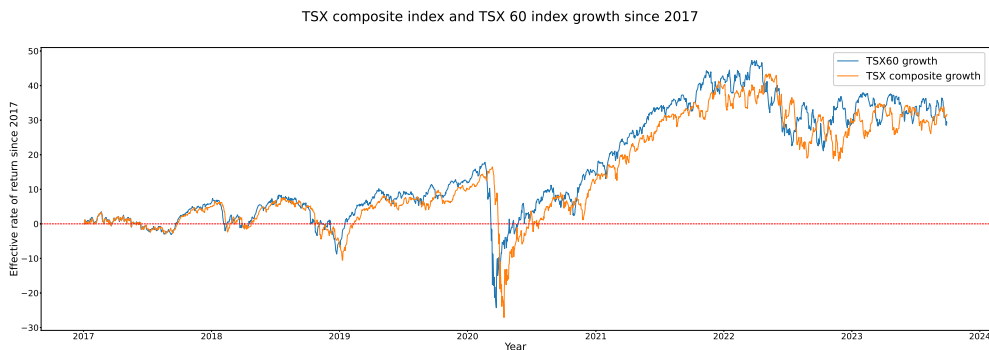


Figure 2.3: TSX Indexes Growth Since 2017. Data source: Yahoo Finance

## TSX Composite Index and TSX 60 Index Sector Distribution by Market Capitalization

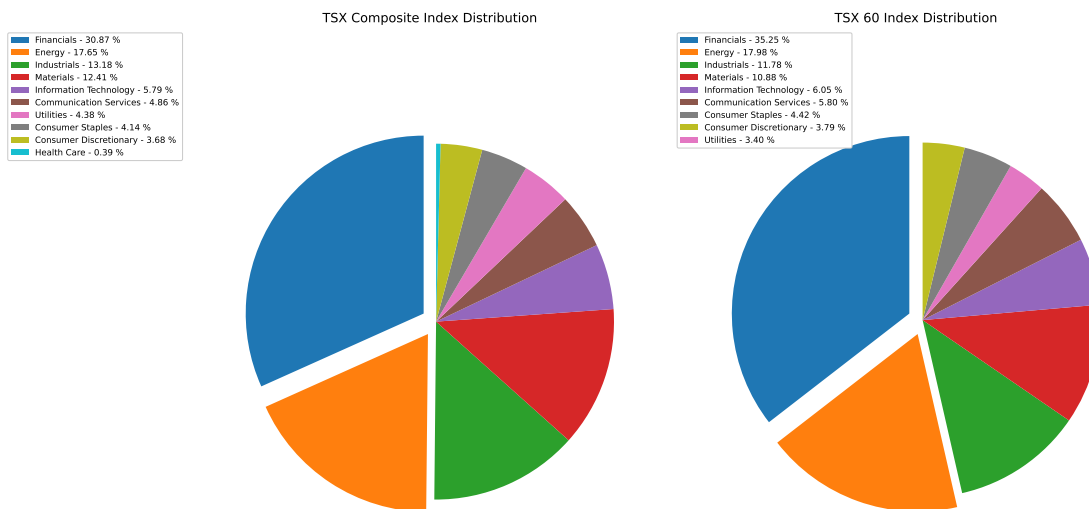


Figure 2.4: The Sector Distribution: TSX Composite Index and TSX 60 Index. Data Source: The data is retrieved from the TMX website as of 2023-03

We exclude Nutrien Ltd (*NTR*), Brookfield Asset Management LTD (*BAM – A*) and Shaw Communications Inc (*SJR – B*) data because these companies do not have the full observations over the entire sample period. Figure 2.5 provides the annualized average daily variance of each stock within their own sector during the selected periods. The comparison of different periods of stock volatility suggests that the global pandemic has caused considerable impacts on the stock market. The IT, energy, and basic materials sectors are the most volatile sectors during the three sample periods.



Figure 2.5: The TSX 60 Constituents Average Annualized Daily Volatility.  
 Note: The size of points in the graph captures the rank of market capitalization of each stock. The red line is the average of TSX 60 volatility.

**US Stock Market** The Nasdaq 100 index excludes the financial sector, providing a distinctive focus on high-tech and growing industries. This is in contrast to the composition of the TSX 60 index. The data used in this chapter for the Nasdaq 100 network analysis excludes twelve companies due to incomplete data over the sample period.<sup>2</sup> Figure 2.6 depicts the average annualized daily volatility for each stock, grouped by sector. The electronic technology, technology services and health technology sectors are relatively stable with respect to the rest of the sectors.

<sup>2</sup>The capitalization of these twelve companies was 513 billion in 2023, which is 2.6% of the total capitalization of the Nasdaq 100 constituents. One stock has been removed from the Nasdaq 100 index since writing of the first chapter. The removed stock is Seagen (SGEN), which merged with Aris Merger Sub, Inc. on December 14, 2023.

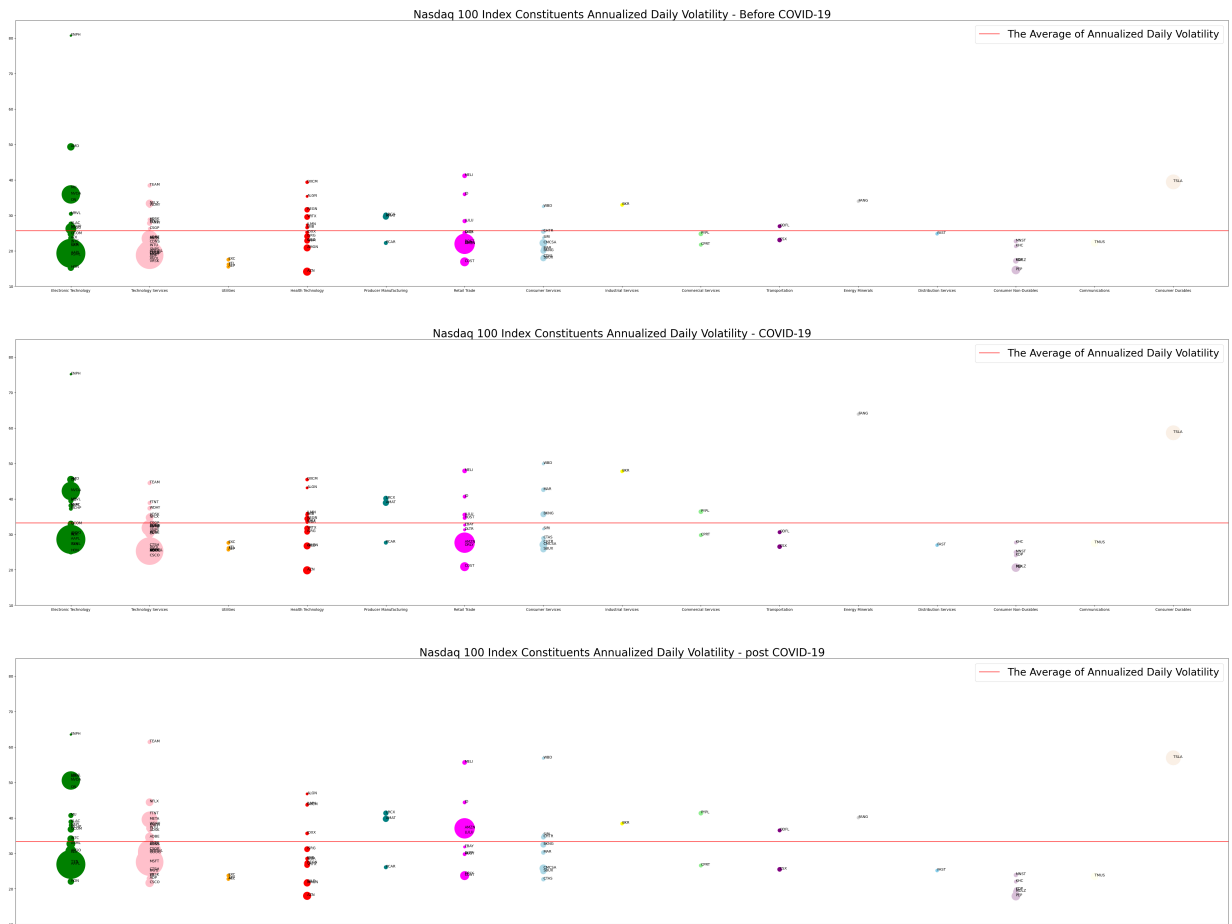


Figure 2.6: The Nasdaq 100 Constituents Average Annualized Daily Volatility.  
 Note: The size of points in the graph captures the rank of market capitalization of each stock.

Table 2.1 and Table 2.2 present the descriptive statistics for each stock market dataset. Furthermore, the Augmented Dickey-Fuller test (ADF) is used to ascertain the stationarity of the data. The last two columns of each table show the results of the ADF test for each stock. The null hypothesis of the ADF test is that a unit root is present in a time series sample. Given that the p-value is less than 5% significance level for each stock, we can reject the null hypothesis. The stationary time series permits the use of vector autoregression (VAR) in the section 2.5.2 for the Diebold-Yilmaz (DY) spillover analysis.

## 2.4 Edges Identification Strategy

This section presents the methodologies employed for the identification of connections in the directed network analysis. Subsection 2.4.1 presents the application of transfer entropy, while Subsection 2.4.2 describes the methodology underlying the DY spillover index. Subsection 2.4.3 illustrates the EMD algorithm, which is used to decompose data into short-term and long-term frequencies. Finally, Subsection 2.4.4 outlines the MST algorithm used for network analysis.

### 2.4.1 Transfer Entropy

For the measurement of information transmission between the market participants, we use the entropy concept, initially proposed by Shannon (1948). The mathematical expression for discrete entropy is defined as:

$$H = - \sum_{i=1}^n p_i \log p_i, \quad (2.2)$$

where  $p_i$  is the probability of occurrence for an event  $i$ . For a bivariate case, suppose the joint distribution of  $x, y$  is  $P(x, y)$ ,  $x \in X$  and  $y \in Y$ . The mutual information  $I(x, y)$  is used to quantify the information gained from one variable given another variable (Shannon 1948, and Cover and Thomas 2006):

$$I(x, y) = \sum_{x \in X} \sum_{y \in Y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)} = H(X) - H(X|Y). \quad (2.3)$$

where  $0 \leq I(x, y) \leq \min(H(x), H(y))$  and  $I(x, y) = 0$  if  $x$  is independent of  $y$ . Figure 2.7 shows a Ven diagram depicting the relationship between mutual information and entropy. The mutual information is the intersection of  $H(X)$  and  $H(Y)$ . Suppose  $X$  and  $Y$  are the stock returns. Then  $I(X, Y)$  captures how much information about the return of one stock can be inferred from the return of the other.

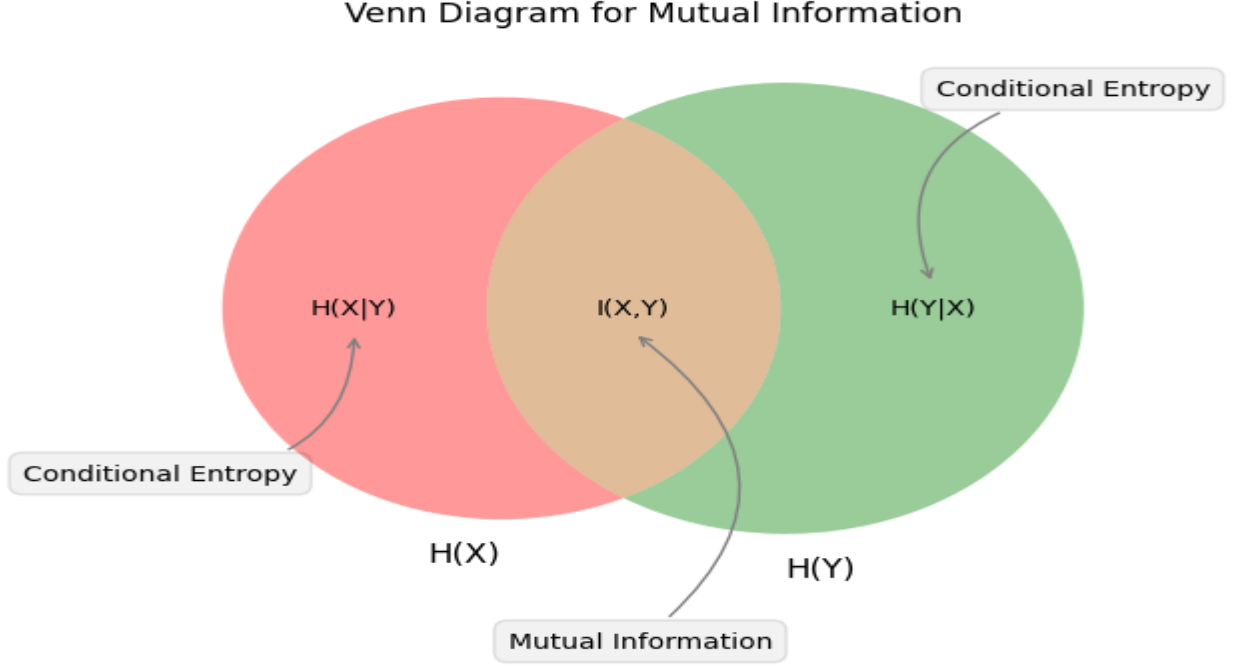


Figure 2.7: Entropy Ven Diagram The joint set of  $H(X | Y)$  and  $H(Y | X)$  is the joint entropy  $H(X, Y)$

Schreiber (2000) proposes the concept of transfer entropy  $T_{y \rightarrow x}$  by using the Kullback-Leibler divergence, also known as relative entropy (Kullback and Leibler, 1951).<sup>3</sup> The transfer entropy  $T_{y \rightarrow x}$  incorporates the Markov process of order  $k$  measuring the transition probabilities:

$$T_{y \rightarrow x} = \sum p(x_{t+1}, x_t^{(k)}, y_t^{(l)}) \log \frac{p(x_{t+1} | x_t^{(k)}, y_t^{(l)})}{p(x_{t+1} | x_t^{(k)})}, \quad (2.4)$$

where  $x_t \in X$  is a stationary Markov process of order  $k$  and  $y_t \in Y$  is a stationary Markov process of order  $l$ .  $T_{y \rightarrow x}$  measures the information spillover from variable  $y$  to variable  $x$ .<sup>4</sup> Assuming a well-developed market and a random walk behaviour for stock prices, we assume  $k = l = 1$ , that is, the current stock price is only significantly affected by the previous day's price (Bekiros et al., 2017). Similarly, the information transmitted from variable  $x$  to variable  $y$  is defined as:

$$T_{x \rightarrow y} = \sum p(y_{t+1}, y_t^{(k)}, x_t^{(l)}) \log \frac{p(y_{t+1} | y_t^{(k)}, x_t^{(l)})}{p(y_{t+1} | y_t^{(k)})}. \quad (2.5)$$

<sup>3</sup>The relative entropy between two probability distributions,  $p(x)$  and  $q(x)$ , is  $D_{\text{KL}}(p(x) \parallel q(x)) = \sum_x p(x) \log \frac{p(x)}{q(x)}$ .

<sup>4</sup>For further insight into the derivation of mutual information, please refer to Appendix E.

Then the net transfer entropy is calculated as:

$$\begin{cases} NT_{x \rightarrow y} = T_{x \rightarrow y} - T_{y \rightarrow x} & \text{if } T_{x \rightarrow y} > T_{y \rightarrow x} \\ NT_{x \rightarrow y} = 0 & \text{if } T_{x \rightarrow y} = T_{y \rightarrow x} \\ NT_{y \rightarrow x} = T_{y \rightarrow x} - T_{x \rightarrow y} & \text{if } T_{x \rightarrow y} < T_{y \rightarrow x} \end{cases}, \quad (2.6)$$

where  $NT_{x \rightarrow y}$  measures the net information flow from variable  $x$  to variable  $y$ .

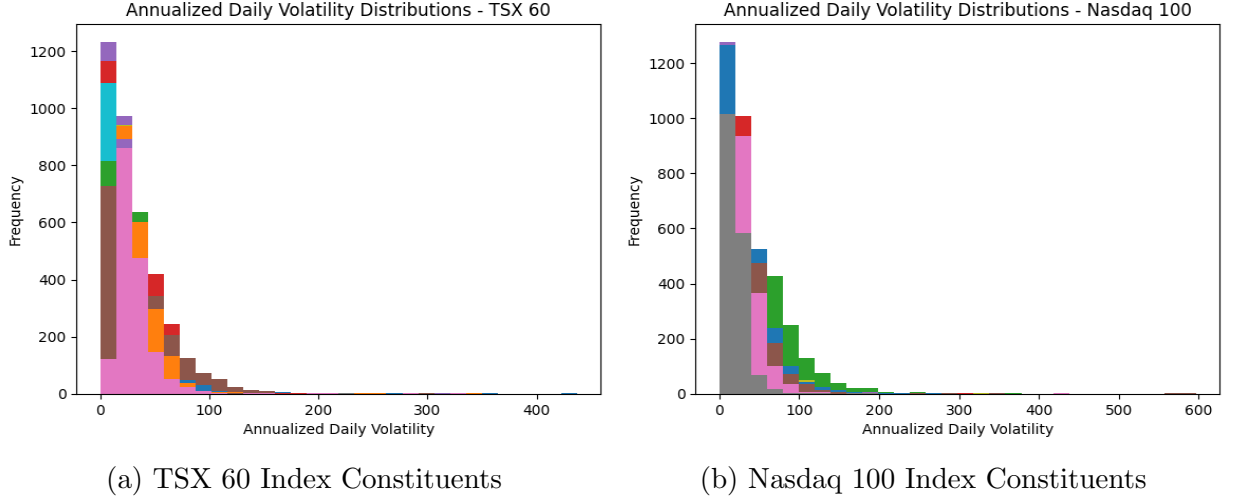


Figure 2.8: Annualized Daily Volatility Distributions

The selection of the number of bins for the annualized daily volatility is of great importance in the context of discrete transfer information, as it directly affects the accuracy and bias of the transfer information estimate. If the number of bins is small, the discretization may fail to capture the intricate relationship between stocks' volatility, resulting in a high degree of bias. On the other side, a large number of bins may be too granular to capture the true dependency, leading to a high degree of variance. Hacine-Gharbi et al. (2012) proposed a formula for determining the number of bins by minimizing the bias of entropy estimation. The formula is defined as :

$$b = \text{round}\left(\frac{\vartheta}{6}\right) + \frac{2}{3\vartheta} + \frac{1}{3} \quad (2.7)$$

where

$$\vartheta = \sqrt[3]{8 + 324N + 12\sqrt{36N + 729N^2}},$$

and  $N$  is the total number of data. Figure 2.8 shows that the datasets in both markets are highly skewed. Therefore, we discretize each stock's annualized daily volatility into equal-sized buckets based on sample quantiles to ensure a more comprehensive understanding of the data distribution. Table 2.3 depicts the transfer entropy measurement for the first 19 stocks in TSX 60, while Table 2.5 shows the results for the first 19 stocks in Nasdaq 100. Table 2.4 and Table 2.6 provide an example of net transfer entropy calculation for each market by using Equation 2.6.<sup>5</sup>

## 2.4.2 Diebold-Yilmaz Spillover Index Framework

Vector autoregression (VAR) is one conventional econometric model in which each variable has an identification equation including the variable's past values, the past values of other variables, and an error term. The  $p^{\text{th}}$ -order VAR (VAR(P)) model is written as:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t \quad (2.8)$$

Here  $c$  denotes an  $(n \times 1)$  vector of constants, and  $A_j$  a time-invariant  $(n \times n)$  matrix for  $j = 1, 2, \dots, p$ . The  $(n \times 1)$  vector  $e_t$  is a vector generalization of white noise:

$$E(e_t) = 0 \quad (2.9)$$

$$E(e_t e'_\tau) = \begin{cases} \Sigma & \text{for } t = \tau \\ 0 & \text{otherwise,} \end{cases} \quad (2.10)$$

For example, the VAR(1) two-variables model is defined as:

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + e_t, \quad (2.11)$$

---

<sup>5</sup>The results for TSX 60 are summarized in Table 2.4, while those for Nasdaq 100 are summarized in Table 2.6.

where  $\mathbf{y}_t = (y_{1,t}, y_{2,t})$  and  $\mathbf{A}$  is a  $2 \times 2$  coefficients matrix. Assume the data covariance is stationary, the moving average representation of VAR exists and is defined as:

$$\mathbf{y}_t = \Theta(L)e_t,$$

where  $\Theta(L) = (\mathbf{I} - \Phi L)^{-1}$ . Let  $\mathbf{Z}(L) = \Theta(L)\mathbf{Q}_t^{-1}$ ,  $\epsilon_t = \mathbf{Q}_t e_t$ ,  $E(\epsilon_t \epsilon_t') = \mathbf{I}$ , and  $\mathbf{Q}_t^{-1}$  be the lower-triangular Cholesky factor of the covariance matrix of  $e_t$ . Then we have  $\mathbf{y}_t = \mathbf{Z}(L)\epsilon_t$ . Now considering the mean squared error of the 1-step ahead forecast, Equation 2.11 can be written as:

$$\mathbf{y}_{t+1} - \hat{\mathbf{y}}_{t+1|t} = \mathbf{Z}_0 \epsilon_{t+1} = \begin{bmatrix} z_{0,11} & z_{0,12} \\ z_{0,21} & z_{0,22} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \end{bmatrix},$$

where the covariance matrix is:

$$E(\mathbf{e}_{t+1,t} \mathbf{e}_{t+1,t}') = \mathbf{Z}_0 \mathbf{Z}_0'.$$

Diebold and Yilmaz (2009) define that  $z_{0,21}^2$  quantifies the contribution from the  $\mathbf{y}_{1,t}$  shocks to the forecast error variance of  $\mathbf{y}_{2,t}$ .  $z_{0,12}^2$  measures the contribution from the  $\mathbf{y}_{2,t}$  shocks to the forecast error variance of  $\mathbf{y}_{1,t}$ .

Although Cholesky factorization achieves orthogonality, the variance decomposition is not invariant to the ordering of the variables in the VAR. Diebold and Yilmaz (2012) and Diebold and Yilmaz (2014) use the generalized VAR framework, proposed by Pesaran and Shin (1998) and Koop et al. (1996), to compute an order-invariant variance decomposition. This method uses the error distribution that has been historically realized to account for the correlated shocks. The H-step ahead forecast error variance decomposition ( $\mathbf{d}_{ij}^g(H)$ ) under the generalized VAR framework for  $N$  stocks is written as:

$$\mathbf{d}_{ij}^g(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (i_i' \theta_h \Sigma i_j)^2}{\sum_{h=0}^{H-1} (i_i' \theta_h \Sigma \theta_h' i_i)}, \quad (2.12)$$

where  $\Sigma$  is the variance matrix for the error vector ( $e_t$ ),  $\sigma_{jj}$  is the standard deviation of the error term for the  $j$ th variable equation, and  $i_j$  is a selection vector to select the  $j$ th

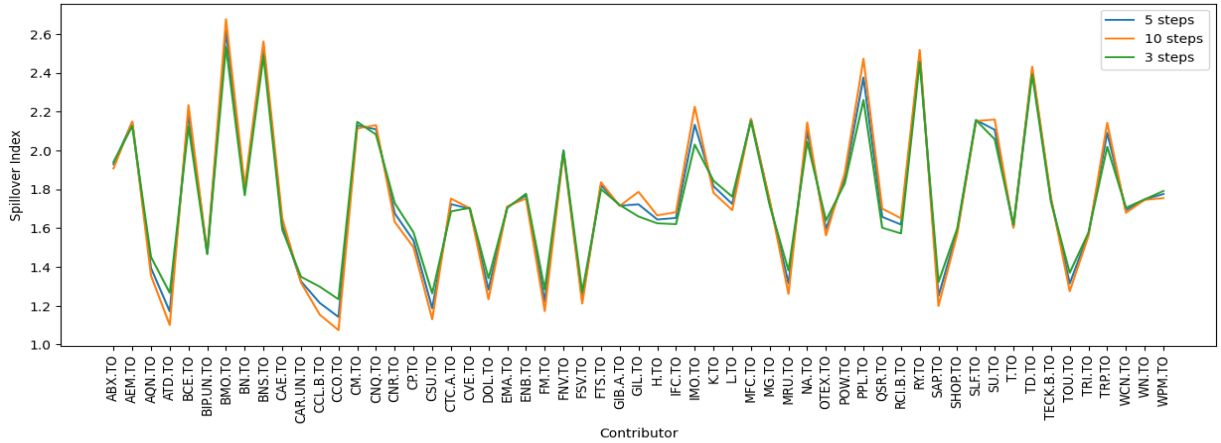
element. Since  $\sum_{j=1}^N \mathbf{d}_{ij}^g(H) \neq 1$ , Diebold-Yilmaz (DY) framework normalizes the variance decomposition matrix as:

$$\tilde{\mathbf{d}}_{ij}^g(H) = \frac{\mathbf{d}_{ij}^g(H)}{\sum_{j=1}^N \mathbf{d}_{ij}^g(H)},$$

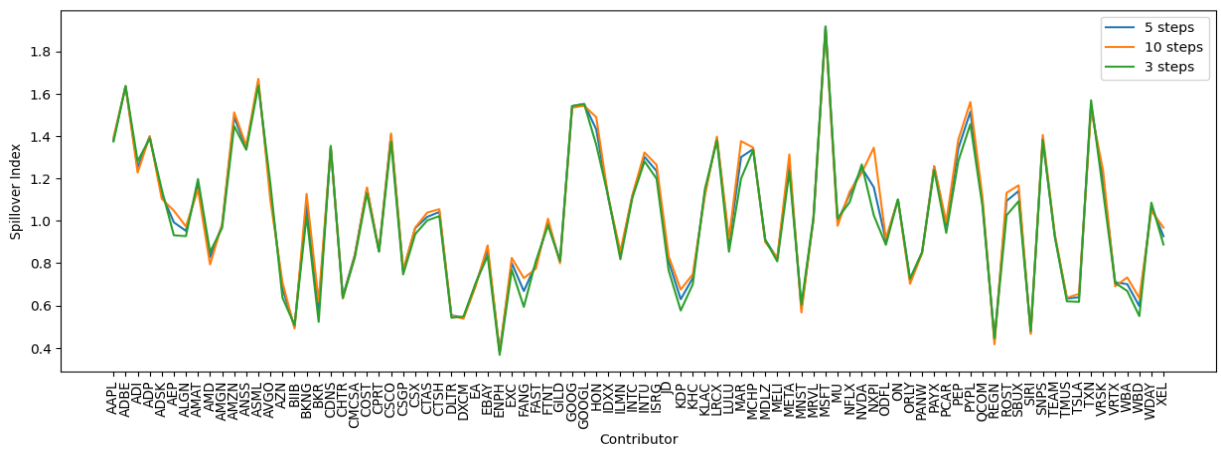
where  $\sum_{j=1}^N \tilde{\mathbf{d}}_{ij}^g(H) = 1$  and  $\sum_{i,j=1}^N \tilde{\mathbf{d}}_{ij}^g(H) = N$ . Since this chapter focuses on the directed network analysis, we defined the net spillover as:

$$\mathbf{Nd}_{ij}^g(H) = \begin{cases} \tilde{\mathbf{d}}_{ji}^g(H) - \tilde{\mathbf{d}}_{ij}^g(H) & \text{if } \tilde{\mathbf{d}}_{ji}^g(H) > \tilde{\mathbf{d}}_{ij}^g(H) \\ 0 & \text{if } \tilde{\mathbf{d}}_{ij}^g(H) = \tilde{\mathbf{d}}_{ji}^g(H) \\ \tilde{\mathbf{d}}_{ij}^g(H) - \tilde{\mathbf{d}}_{ji}^g(H) & \text{if } \tilde{\mathbf{d}}_{ij}^g(H) > \tilde{\mathbf{d}}_{ji}^g(H) \end{cases}. \quad (2.13)$$

Furthermore, the log of annualized daily volatility is utilized in the construction of the DY spillover index to ensure the normality assumption. Table 2.7 summarizes the spillover index of the first 19 constituents of TSX 60, while Table 2.9 summarizes the results of the first 19 constituents of Nasdaq 100. Tables 2.8 and 2.10 present the first 19 stock net spillover index using Equation 2.13. The VAR model uses a lag of 1, as we assume that the daily stock volatility is only affected by previous day's situation. In addition, we choose the 5-steps ahead variance decomposition to construct the DY spillover index. We test the model by using 3, 5, 10 steps ahead variance decomposition. Figure 2.9 shows that the DY spillover index exhibits minimal sensitivity to the number of steps selected.



(a) TSX 60 Index Constituents H-steps Ahead Variance Decomposition Test



(b) Nasdaq 100 Index Constituents H-steps Ahead Variance Decomposition Test

Figure 2.9: The Aggregated Spillover Index by Contributor

### 2.4.3 Empirical Mode Decomposition

The empirical mode decomposition (EMD), proposed by Huang et al. (1998), is a non-parametric method for decomposing non-linear and non-stationary time series into different oscillating components. It permits to extract short-term and long-term effects from the inherent features of a given data. The EMD generates a set of intrinsic mode functions (IMFs) that contain different time scale oscillating components. In a univariate case, the IMFs are indirectly determined by the mean of an upper and lower envelope in an iterative algorithm. The sum of IMFs and the residual term restores the original time series. However, the classic EMD leads to the mode mixing problem: an IMF contains different frequency components. Torres et al. (2011) introduce the complete empirical mode decom-

position based on the adaptive noise (CEEMDAN) method, addressing the mode mixing problem and reducing the computational cost. The CEEMDAN algorithm can be described as follows:

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**Algorithm 1:** The algorithm for the complete empirical mode decomposition based on adaptive noise (CEEMDAN)

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- 1 Let  $X(i), i = 1, 2, \dots, N$  be the  $i$ -th company stock annualized daily volatility.
- 2 Generate  $X(i)_l = X(i) + \beta_0 \omega_l, l \in (1, 2, \dots, L), i \in (1, 2, \dots, N)$ , where  $\omega_l$  is a white noise and  $\beta_0 > 0$ .  $L$  is the total number of decompositions for each IMF.
- 3 For every  $l = 1, 2, \dots, L$ , each  $X(i)_l$  is decomposed by EMD. Then the first IMF is the average of  $L$  times  $IMF_{1,L}$ :

$$IMF_1(i) = \frac{1}{L} \sum_{l=1}^L IMF_{1,l}(i) = \frac{1}{L} \sum_{l=1}^L E_1(X(i)_l), i \in (1, 2, \dots, N),$$

where  $E_k(*)$  is the EMD operator that produces the  $k$ -th IMF.

- 4 When  $k = 1$ , the first residue is calculated as  $r(i)_1 = X(i) - IMF_1(i)$ .
- 5 Implement  $L$  times EMD on  $r(i)_1 + \beta_1 E_2(w_l)$  and obtain the second IMF:

$$IMF_2(i) = \frac{1}{L} \sum_{l=1}^L E_1(r(i)_1 + \beta_1 E_1(w_l)), i \in (1, 2, \dots, N).$$

- 6 The  $k$ th residue is:  $r(i)_k = r(i)_{k-1} - IMF_k(i)$ . Similar to step (3), the  $k$ th IMF is:

$$IMF_k(i) = \frac{1}{L} \sum_{l=1}^L E_{k-1}(r(i)_{k-1} + \beta_{k-1} E_{k-1}(w_l)), i \in (1, 2, \dots, N).$$

- 7 Repeat step (4) for the next IMF until finding all IMFs, such that the number of extrema is less than 2.
  - 8 The residual term is  $R(i) = X(i) - \sum_{k=1}^K IMF_k(i), i \in (1, 2, \dots, N)$ , where  $K$  is the total number of IMFs.
-

The data is decomposed into different time scales by applying the above algorithm. The existing literature combines the IMFs with predetermined frequencies to implement the short-term, mid-term and long-term analysis (e.g., Zhang et al. 2009, Dimpfl and Peter 2018, and Niu and Hu 2021). Niu and Hu (2021) and Tian et al. (2019) define three financial data frequencies, such as 1-90 trading days, 91-250 trading days, and more than 250 trading days. The 1-90 trading days frequency is a short-term measurement capturing the fluctuations caused by the market/information inefficiency. The mid-term frequency category evaluates event-driven impact on the market. The macroeconomic influence or the fundamental change is reflected in the long-term frequency category from 250 trading days and beyond. The impact of the global pandemic is more profound than that of a classical event or policy change, and therefore it does not fit well into the mid-term frequency category. To better fit the underlying dataset in different periods, we have adjusted the combination criteria. When conducting the subperiod analysis, a 30-day cut-off point is used to distinguish between short-term and long-term frequency.

Let  $n_{i,k}, k \in (1, 2, \dots, K), i \in (1, 2, \dots, N)$  be the total number of extrema in each IMF and  $l_{i,k}, k \in (1, 2, \dots, K), i \in (1, 2, \dots, N)$  be the length of each IMF. Then the strategy to reassemble the annualized daily volatility in a specific frequency category is defined as follows:

$$\begin{cases} G_{1,i} = \sum IMF_{i,k} & \text{if } \frac{l_{i,k}}{n_{i,k}} \leq \mathbf{a}, k \in (1, 2, \dots, K), \\ G_{2,i} = \sum IMF_{i,k} & \text{if } \frac{l_{i,k}}{n_{i,k}} > \mathbf{a}, k \in (1, 2, \dots, K), \end{cases} \quad (2.14)$$

where  $\mathbf{a}$  is the threshold based on the underlying dataset (i.e. 30 days for each sub-sample). The analysis of  $G_{1,i}$  is assumed to reveal the market's short-term reactions, while  $G_{2,i}$  depicts long-term market reactions. Figure 2.10 provides an example of CEEMDAN decomposition for Apple, whose annualized daily volatility is decomposed into 7 IMFs and one residual. For instance, based on the reassembling strategy of the Apple daily volatility decomposition, the Apple IMFs 1-5 are treated as short-term data, while the rest of the IMFs and the residual are classified as long-term information. By applying similar procedures to the rest of the stocks, the original data is reassembled so that the IMFs with high frequency are grouped into the short-term data, and the IMFs with low frequency are

classified as long-term data.

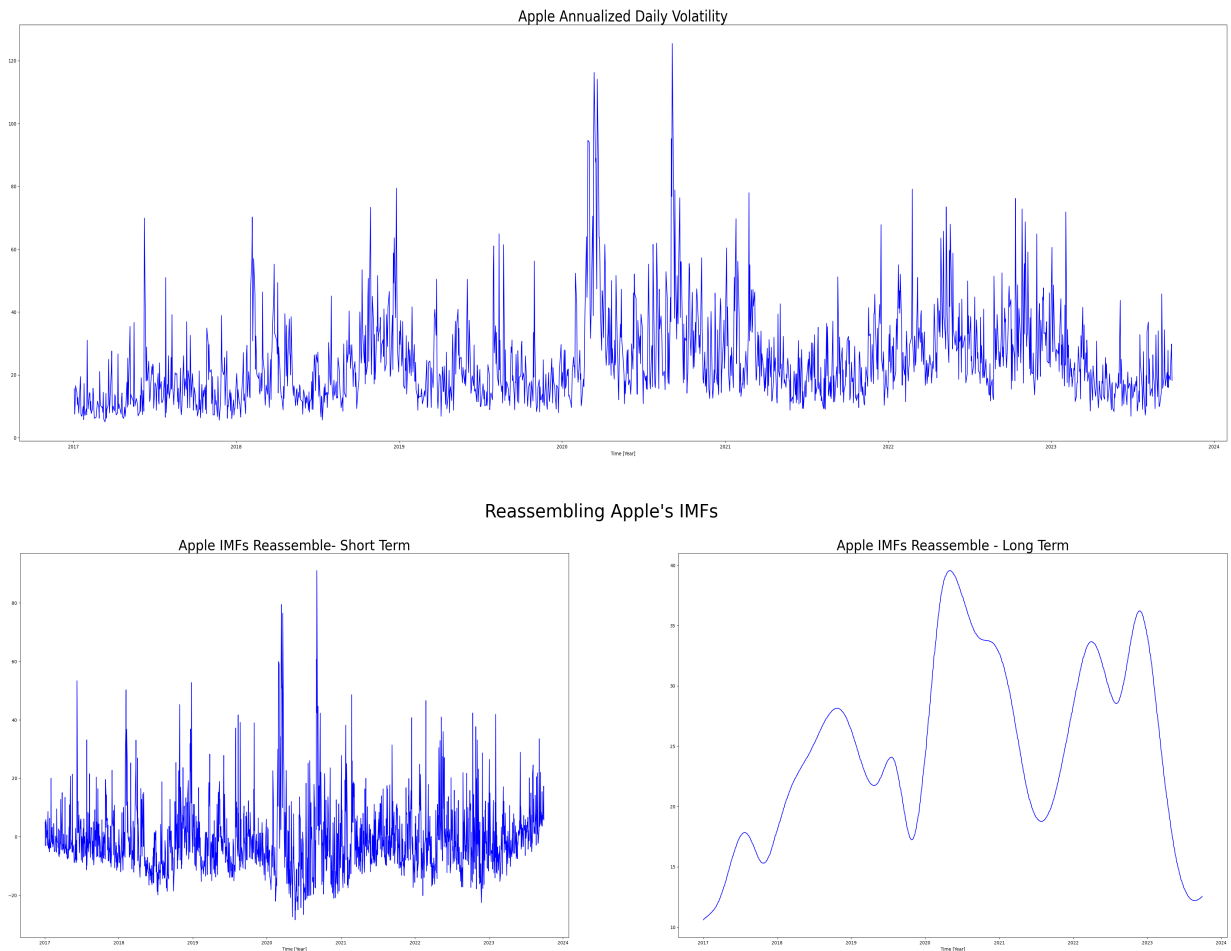


Figure 2.10: EMD Example - Apple Annualized Daily Volatility

It is worth mentioning that the CEEMDAN methodology is more efficient than conventional moving average measurements in identifying trends. Figure 2.11 provides a case study to compare both the CEEMDAN and the moving average methodologies. The orange line represents the 30-day moving average of annualized daily volatility data for Royal Bank of Canada (RBC) bank, while the blue line represents the RBC long-term annualized daily volatility data obtained using the CEEMDAN and reassembly strategy. The moving average clearly shows a lag in measuring the long-term volatility pattern, which is a result of the calculation design, while the CEEMDAN method does not require lags.

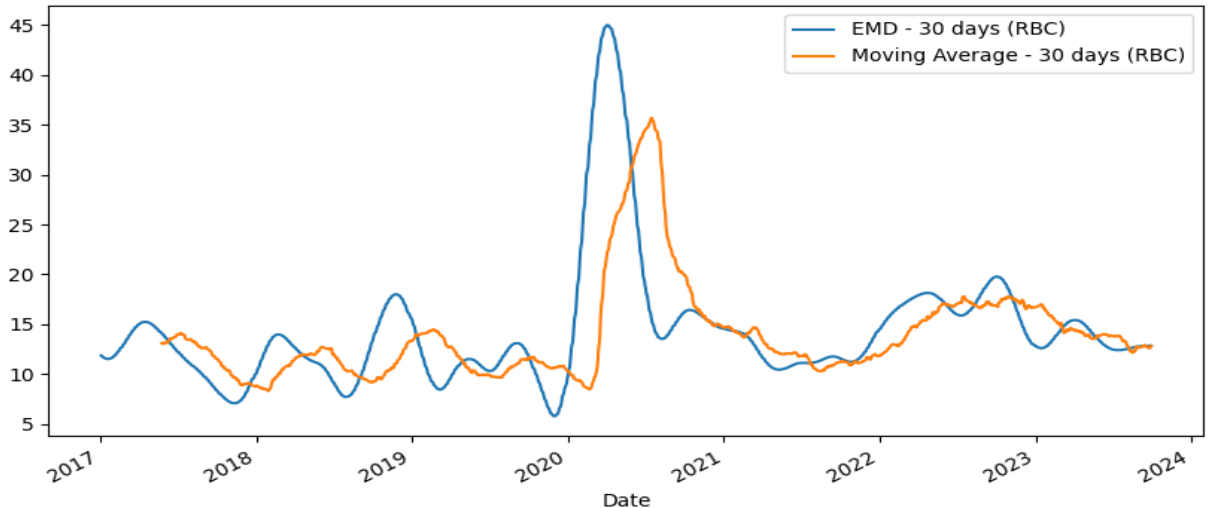


Figure 2.11: EMD v.s Moving Average

#### 2.4.4 Minimum Spanning Tree

The edge identification strategies explained in section 2.4.1 and 2.4.2 do not have a well-defined method for removing insignificant connections. A spanning tree is used to remove the number of edges in a network. It is a network constructed from a connected graph comprising all  $N$  nodes with precisely  $N-1$  edges. The tree is constrained by the requirement that it cannot form a cycle.<sup>6</sup> The minimum spanning tree (MST) is a spanning tree that must select the connected pairs of nodes so as to minimize the sum of the weights of the  $N - 1$  edges.<sup>7</sup> This feature has been widely used in analyzing financial networks (for example, see Niu and Hu 2021, Huang et al. 2016, and Kristoufek et al. 2012).

When deriving the MST network, the DY spillover index and transfer entropy cannot be used as a distance metric because they do not satisfy axioms defining an Euclidean metric. We have to convert the measurement metrics to a distance metric so that the axioms are satisfied (e.g., Niu and Hu 2021, Huang et al. 2016, and Mantegna 1999). The

<sup>6</sup>A cycle in a network is a closed loop. If we choose a node from the cycle as the start and end points, there is a path that connects the “start” and “end” points without revisiting any intermediate nodes.

<sup>7</sup>It is important to note that a minimum spanning tree is not always unique, and all edge weights in the MST will be distinct if the edges’ weights are unique. If all edge weights in the graph are equal, then any spanning tree of the graph is an MST.

transformation is defined as:

$$\begin{aligned} m_{ij} &= T_{i \rightarrow j} + T_{j \rightarrow i} \\ d_{i,j} &= \sqrt{2(a - m_{i,j})} \end{aligned} \tag{2.15}$$

where  $T_{i \rightarrow j}$  is either the DY spillover index or transfer entropy measurement for the influence from stock  $i$  to stock  $j$ , and  $a$  is the maximum value over the respective measurements.

## 2.5 Empirical Results

### 2.5.1 Transfer Entropy Analysis

The transfer entropy  $T_{x \rightarrow y}$  essentially quantifies the amount of uncertainty reduced in predicting the future value of  $y$  given variable  $x$ . A higher transfer entropy from  $x$  to  $y$  indicates that  $x$  contains essential information regarding the future value of  $y$ . In the context of daily volatility, transfer entropy quantifies the information in predicting the volatility of stocks. When stock  $x$  is a net transfer entropy sender (for example,  $NT_{x \rightarrow y} > 0$ ), this indicates that the volatility of stock  $x$  exerts a stronger influence on the volatility of stock  $y$ . This is because the volatility of stock  $x$  contains more information regarding the future volatility of stock  $y$  than stock  $y$  contains regarding the future volatility of stock  $x$ . In this case, stock  $y$  is a net transfer entropy receiver, indicating that the volatility of stock  $y$  is influenced by the volatility of stock  $x$ .

**TSX 60** This section uses the TSX 60 constituents' daily volatility to construct the transfer entropy directed network. We use heatmaps to illustrate some of the transfer entropy tables described earlier. For example, the left panel of Figure 2.12 is a heatmap representation of Table 2.3, while the right panel illustrates Table 2.4. Lighter colored blocks in the left panel indicate a larger amount of transfer entropy across pair of stocks, while darker colored blocks indicate a smaller amount of transfer entropy. White blocks in the the right panel of Figure 2.12 indicate zero net transfer entropy between a pair of stocks, meaning that the amount of information sent from stock  $i$  to stock  $j$  is offset by the amount

of information received from stock  $j$  to stock  $i$ . In addition, we calculate the sector-wise net transfer entropy and summarize it in Table 2.11.<sup>8</sup> The financial services, energy, utilities and industrials sectors are the top four contributors to market volatility, while the basic materials, information technology, consumer staples and industrials sectors are the top four sectors most affected by market volatility. It is worth noting that the industrials sector demonstrates a greater transfer entropy outflow (0.056) than inflow (0.05), despite its designation as a top 4 influential sector with respect to both inflow and outflow. This suggests that the influence of daily volatility in the industrials sector is ambiguous, as it may either influence other stocks' volatility or be influenced by others. In addition, the sectors that contribute the most to market volatility are also those with high dividend yields, making them a hub of market stability or instability depending on whether their dividend are stable or not. In other words, a stable (unstable) dividend yield in these sectors can help stabilize (destabilize) the markets.

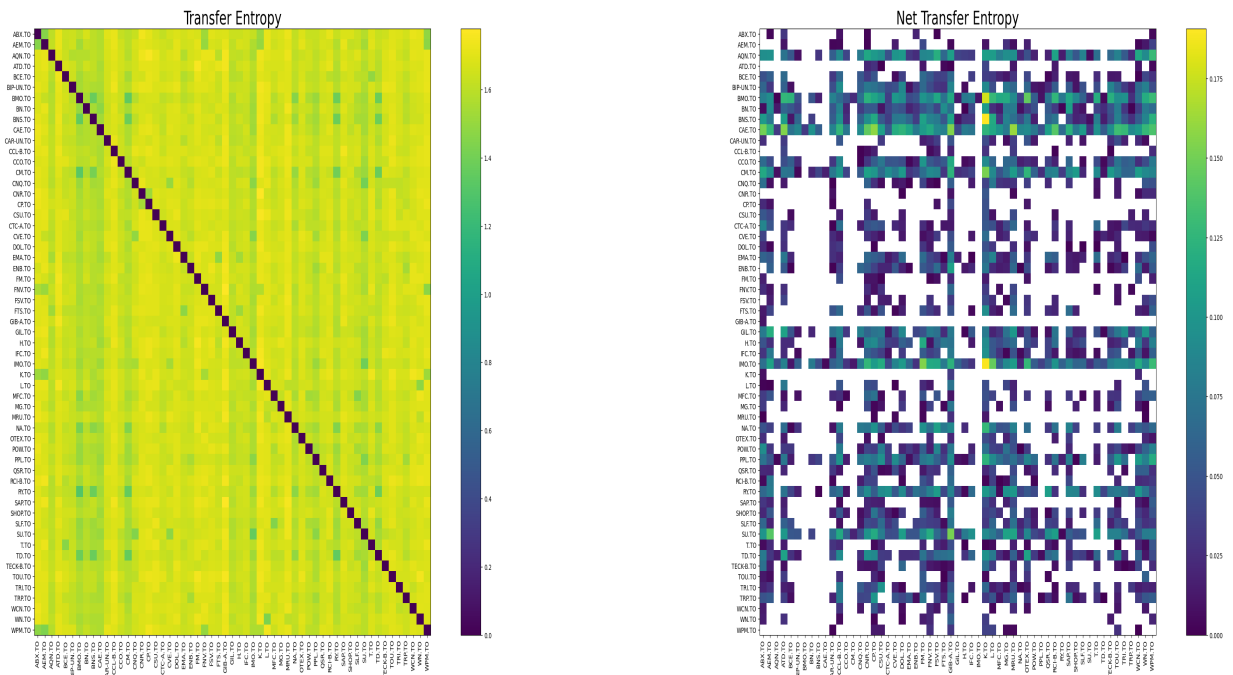


Figure 2.12: Transfer Entropy Analysis - TSX 60

Figure 2.13 provides a visualization of the net transfer entropy network at the level of individual stocks. Figure 2.13a depicts a directed net transfer entropy network with a

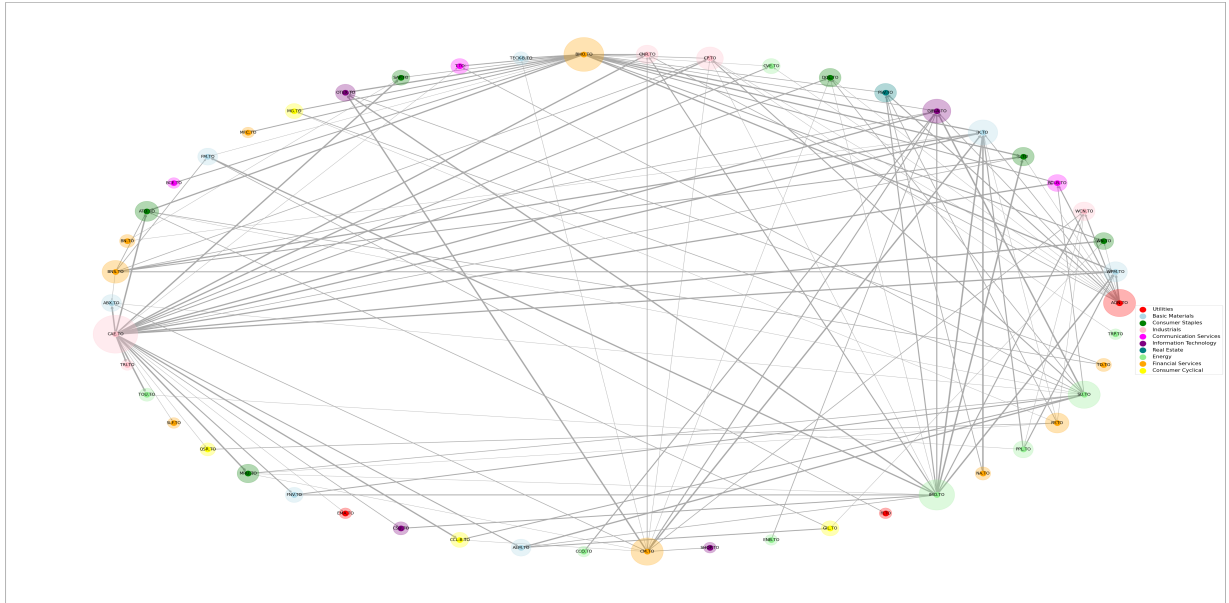
<sup>8</sup>The average transfer entropy sent and received by each sector is calculated by taking the average net transfer entropy for stocks in each sector.

threshold of 0.1.<sup>9</sup> The size of the nodes represents their level of degree centrality, with larger nodes indicating higher centrality.<sup>10</sup> The graph indicates that the financial service, energy, and industrials sectors (all with large size of nodes) have high out-degree centrality. Figure 2.13b shows a minimum spanning tree (MST) by using the transformation outlined in Equation 2.15 to capture the total transfer entropy (i.e.,  $T_{i \rightarrow j} + T_{j \rightarrow i}$ ). The basic materials, information technology, and consumer staples sectors serve as the primary bridges connecting other stocks within the MST. In particular, Kinross Gold Corp (K) and CGI Inc (GIB.A) serve as key connecting nodes within the TSX 60 volatility network, as illustrated in Figure 2.13b. These two stocks exhibit strong linkages with a large number of other stocks, indicating their central role in volatility transmission across the network.

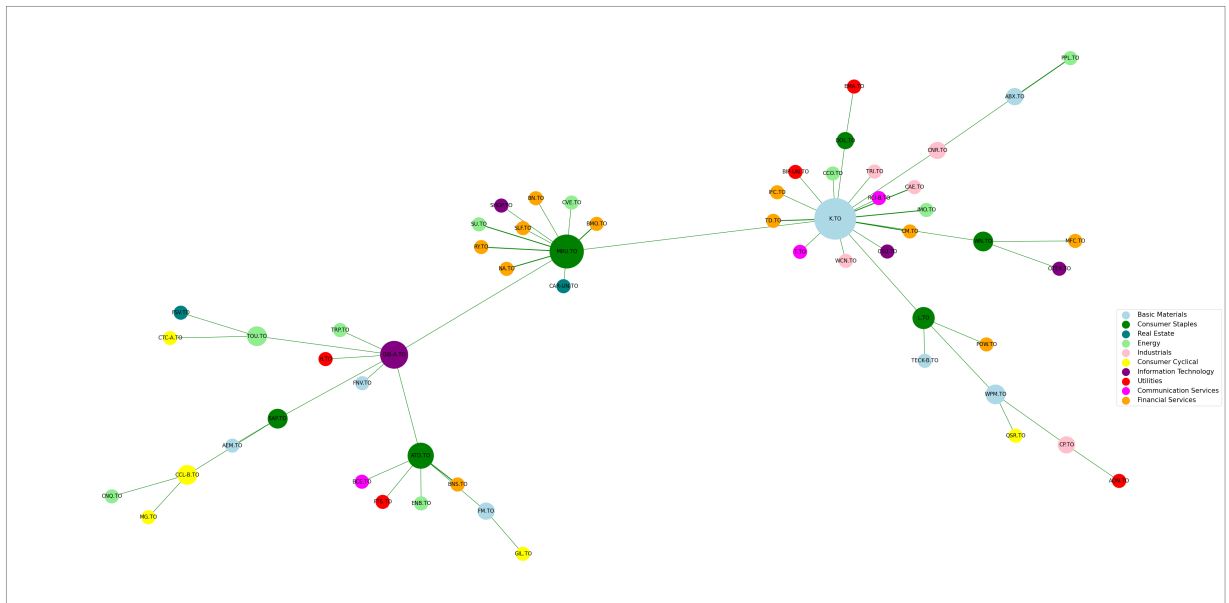
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<sup>9</sup>A threshold of 0.1 is applied to eliminate insignificant volatility transmission connections. If an edge attribute, specifically the Net Transfer Entropy (NTE), falls below 0.1, the corresponding edge is excluded from the network, ensuring that only meaningful relationships are retained in the analysis.

<sup>10</sup>In a directed network, in-degree centrality counts incoming edges, indicating how much a node is influenced by others, while out-degree centrality counts outgoing edges, reflecting its potential to transmit influence or shocks.



(a) TSX 60 Index Constituents Net Transfer Entropy Network - Threshold 0.1



(b) TSX 60 Index Constituents Total Transfer Entropy ( $T_{i \rightarrow j} + T_{j \rightarrow i}$ ) - MST

Figure 2.13: TSX 60 Index Constituents Transfer Entropy

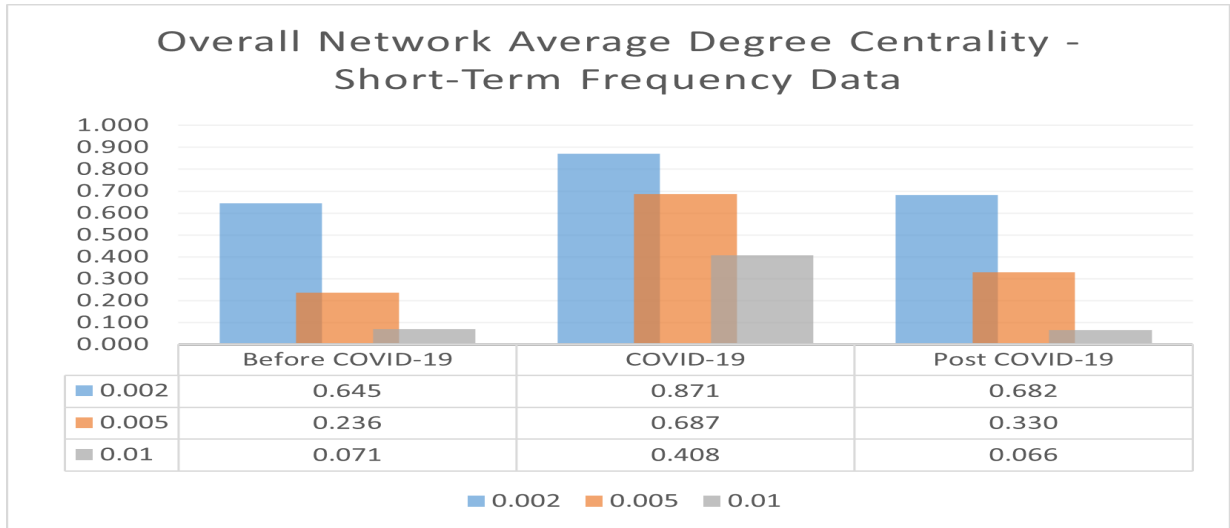
In addition, we conduct a sub-period analysis with a focus on the short term and long term effects. As outlined in Section 2.4.3, a threshold of 30 trading days is employed to reassemble IMFs and construct the short-term and long-term data frequencies sets. Figure 2.14 shows the overall network average degree centrality or network density for different net transfer entropy thresholds.<sup>11</sup> The higher average degree centrality during the COVID-19

<sup>11</sup>The average degree centrality is defined as  $\frac{\sum D_i}{\text{Number of Nodes}}$  for  $i \in (1, 2, \dots, N)$ , where  $D_i$  is the normalized degree centrality (i.e., in-degree and out-degree) for node  $i$ .

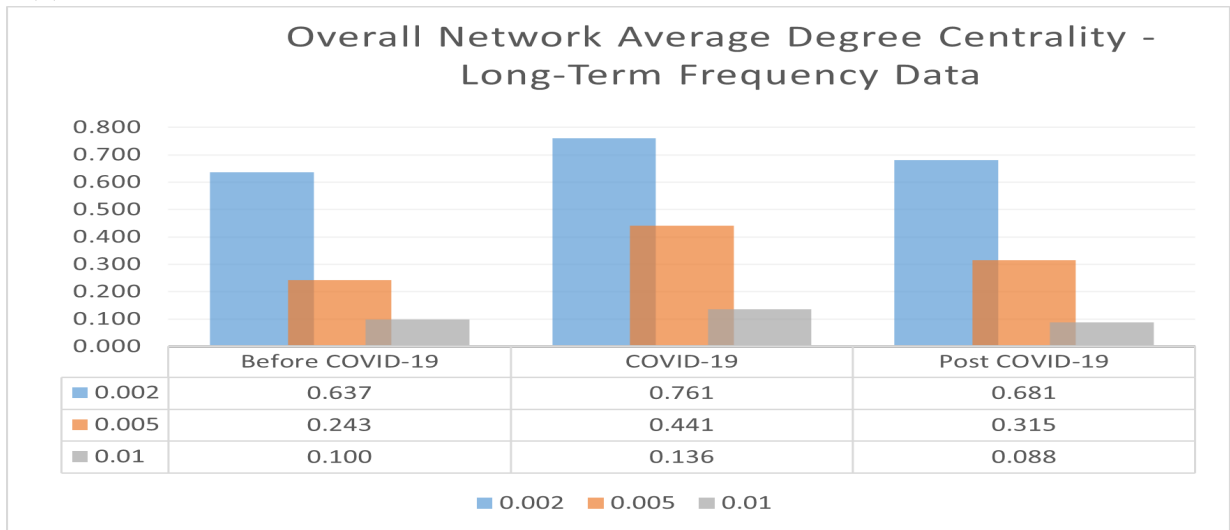
crisis suggests a stronger density and thus a stronger volatility (uncertainty) transmission during the pandemic in comparison to other periods, reflecting heightened market instability. As expected, average density decreases as the threshold value increases, since stricter filtering removes weaker connections. However, the rate of change in centrality varies across different sample periods, indicating structural differences in volatility transmission across time. Figure 2.14a further reveals that volatility transfer intensified during the crisis period in the short term, as evidenced by higher overall network density (average degree centrality) relative to other periods. The overall connectedness still remains around 0.4 when the threshold value is increased to 0.01. Figure 2.14b shows a more stable network connection for long-term frequency data across different periods when compared to the short-term frequency data results. However, there is also a significant volatility spillover risk during the COVID period due to the higher density of the network.<sup>12</sup> The results suggest that the Canadian stock market maintains an efficient information transmission, as evidenced by degree centrality, which reflects a barrier-free information spillover network during the crisis.

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<sup>12</sup>Appendix F.1 provide a visualization of TXS 60 net transfer entropy directed networks with a threshold of 0.1



(a) TSX 60 Index Constituents Net Transfer Entropy Network - Short-Term Frequency Data



(b) TSX 60 Index Constituents Net Transfer Entropy Network - Long-Term Frequency Data

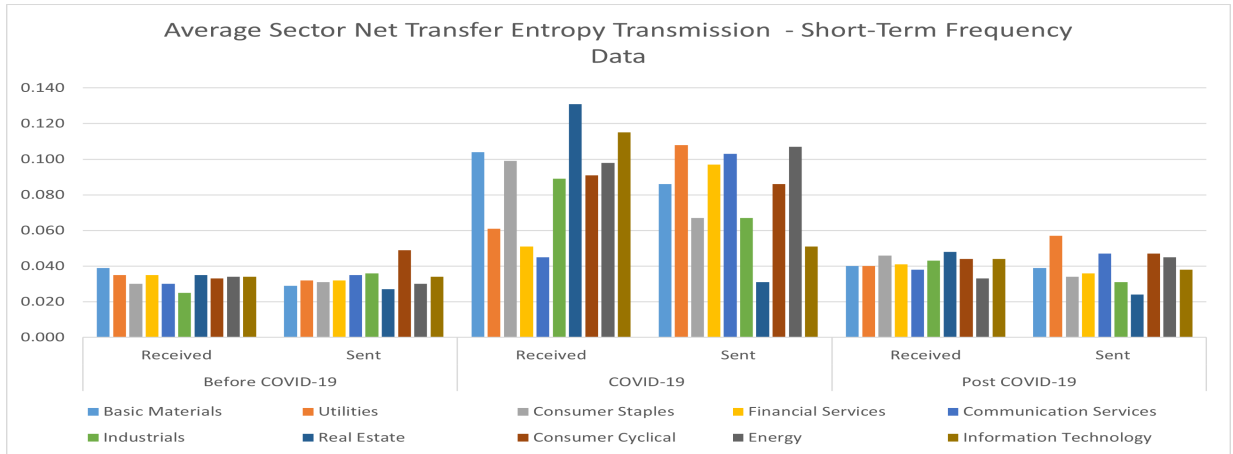
Figure 2.14: TSX 60 Index Constituents Overall Sector Degree Centrality - Sub-periods.  
 Note: Node degree centrality measures the number of nodes that a node is connected to. The measurement is typically normalized by dividing the maximum possible degree in a simple graph, which is  $2(n - 1)$ , where  $n$  represents the number of nodes in a network. The average centrality of a network is the sum of the individual node centrality divided by  $n$ . The maximum average network degree centrality is 1. [0.002, 0.005, 0.01] is a set of threshold values.

In the following, we pursue our analysis from the level of individual stocks to their corresponding sectors. To this end, the average net transfer entropy for each sector is calculated over different time periods by grouping nodes based on their underlying sectors (see Figure 2.15). As shown in Figure 2.15a, for the short-term frequency data analysis, sectoral average net transfer entropy demonstrates a substantial increase in the COVID-19 period and a modest increase in the post COVID-19 period in comparison to the before COVID-19 period. The net information transmission is intensified during the crisis both

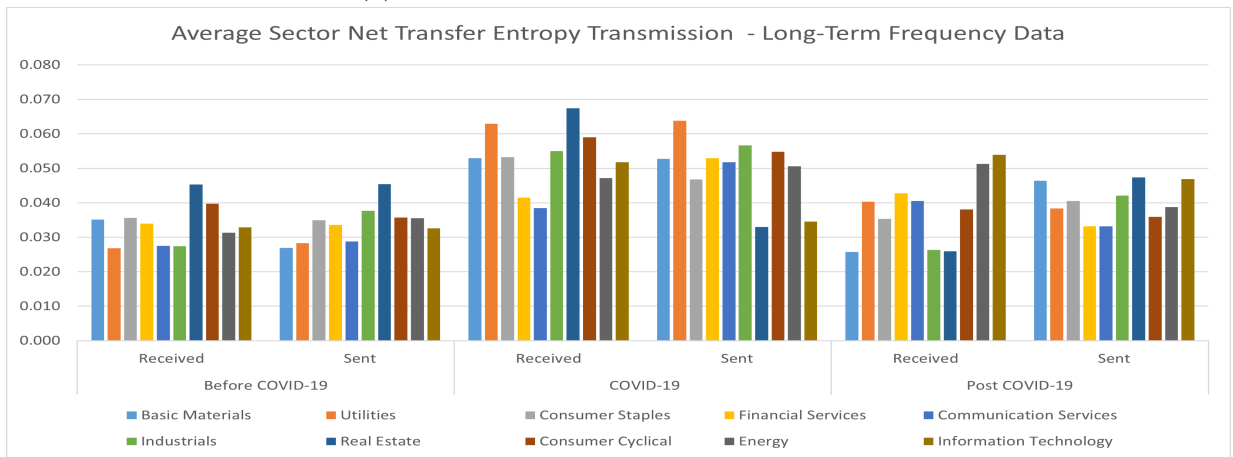
from the large amount of information sent and received. In addition, the real estate sector experienced considerably greater volatility impacts from other sectors than it exerted, therefore being a net information receiver, during the “ COVID-19” period, as evidenced by the significantly higher “Received” bar in comparison to the “Sent” bar.<sup>13</sup> On the other hand, high dividend yield sectors such as financial services, utilities, and communication services have played a crucial role in being net information senders during that same period. This is evidenced by the magnitude of net transfer entropy transmitted exceeding the one received within these sectors. Intuitively, market volatility appears to be largely driven by these sectors, given their substantial volatility outflows during the crisis period. Figure 2.15b illustrates the net transfer entropy spillover among sectors in long-term frequency data. The financial services and communication services sectors demonstrate a notable net outflow of volatility during the crisis in the long run, which aligns with the observation in the short-term frequency data analysis. In contrast, the utility sector has had a relatively stable volatility spillover with respect to the amount of information sent and received in the long run. Overall, sectors with high dividend yields (i.e., basic materials, utilities, financial services, energy, and communication services) show a dominant influence on information transfer in the long term across different periods.

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<sup>13</sup>The companies included in the TSX 60 real estate sector are primarily property management firms. In the context of the crisis, investors may have sought out companies that could provide essential, recurring revenue-based services to mitigate the market uncertainties. As a result, property management became a more appealing investment opportunity. The short term volatility of inflows to the real estate sector may be driven by a shift in investor preferences.



(a) Short Term Sector-wise Transmission



(b) Long Term Sector-wise Transmission

Figure 2.15: TSX 60 Index Constituents Average Net Transfer Entropy - Sub-periods. Note: Scale is different across both panels.

Tables 2.12 and 2.13 use the sectoral in/out degree centrality measurements with different net transfer entropy thresholds to conduct a robustness check. Recall that group in/out degree centrality measures the connectedness of a particular sector. A higher out-degree centrality signifies stronger volatility spillovers to other sectors, whereas a lower out-degree centrality implies more limited outward transmission of volatility. Under the traditionally measured sectoral in/out degree centrality which solely considers the change in the number of edges over time, we now calculate the in/out degree centrality under varying thresholds taking into account the weight of edges in the network. From the perspective of risk management, a sector with a high out-degree has two implications. First, sectors with a high out-degree centrality exert a strong influence on market volatility. Therefore, the stability of these sectors plays a significant role in stabilizing the market. Second, during a crisis,

market volatility is high, and investors can leverage the performance of these sectors to gauge the severity of the impact of the crisis on the market. In contrast, a sector with a high in-degree (i.e. high volatility inflow) is vulnerable as its volatility will be influenced by many other sectors. Table 2.12 shows that in-degree/out-degree centrality measurements vary with time and thresholds, but the sector's rank remains relatively stable in short-term frequency data. Prior to the COVID-19 pandemic, a majority of sectors had similar in-degree and out-degree centralities, indicating a diversified market where each sector contributes to market volatility. Since the onset of COVID-19, including the post-COVID-19 period, the high-yield sectors, namely utilities, financial services, and energy, exhibit a high out-degree centrality. This pattern indicates that shocks originating in these sectors are likely to exert a disproportionate influence on short-term market volatility. In the long-term frequency data analysis, Table 2.13 shows that the role of sectors within the network varies between the pre-COVID-19 and post-COVID-19 periods. However, these sectors share a common characteristic, which is a stable dividend yield. The long-term transmission of information heavily relies on dividend-driven sectors because of their high in-degree or out-degree centralities.

**Nasdaq 100** The Nasdaq 100 index is comprised mostly of high-tech companies and does not include any financial companies. In contrast to the TSX 60, which is dominated by high dividend sectors, high-tech companies prioritize research and development over dividend payouts. As a result, Nasdaq 100 investors tend to focus on long-term growth rather than immediate cash flows. This creates a unique research opportunity to explore the differences between the TSX 60 and Nasdaq 100. Figure 2.16 depicts the transfer entropy and net transfer entropy of Nasdaq 100 companies. The left panel of Figure 2.16 is a heatmap representation of Table 2.5, while the right panel illustrates Table 2.6. White blocks on the right panel correspond to zero net transfer entropy between a pair of stocks. NXP Semiconductors NV, a company in the electronic technology sector, appears to transmit a significant amount of net information to other sectors, as shown by the light green-coloured row. Using the net transfer entropy calculation outlined in Equation 2.6, NXP

Semiconductors NV was identified as a pure information sender. The Table 2.14 shows the sector-wise average net transfer entropy. The electronic technology, energy minerals, and commercial services sectors are the top three contributors to market volatility, while the distribution services, health technology, and communications sectors are the top three sectors most affected by other sectors' volatility.

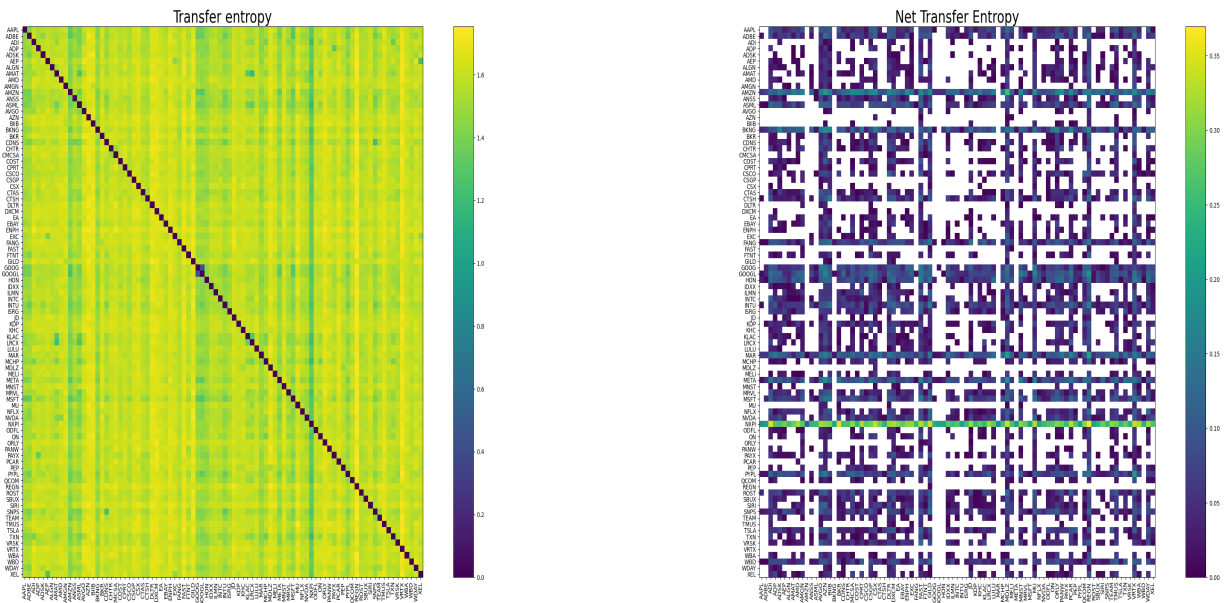
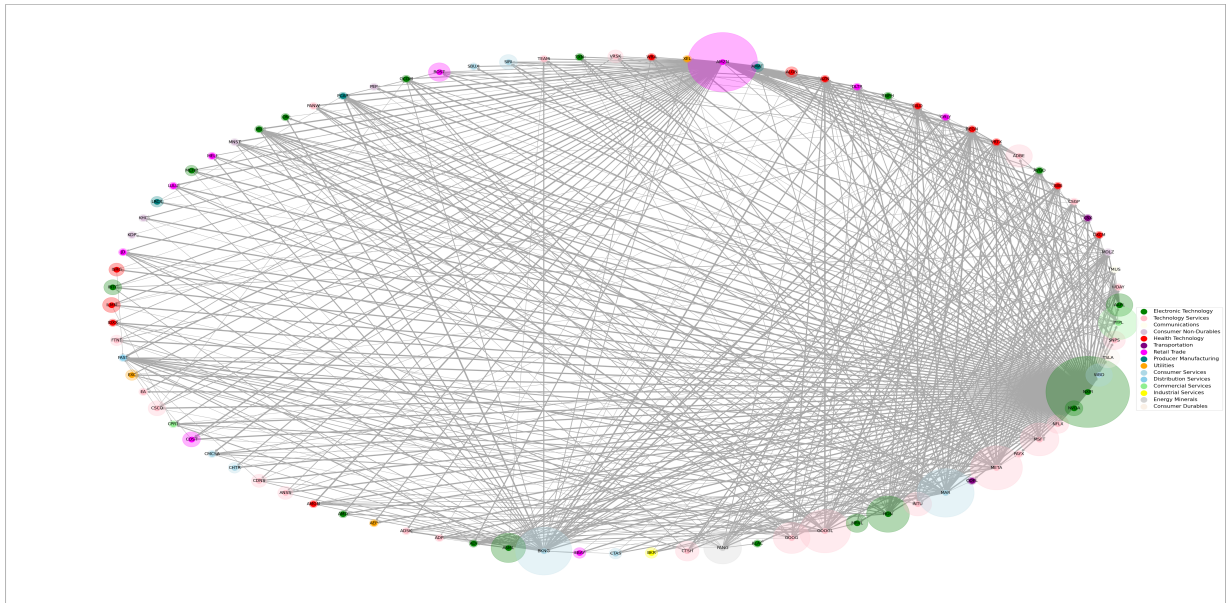


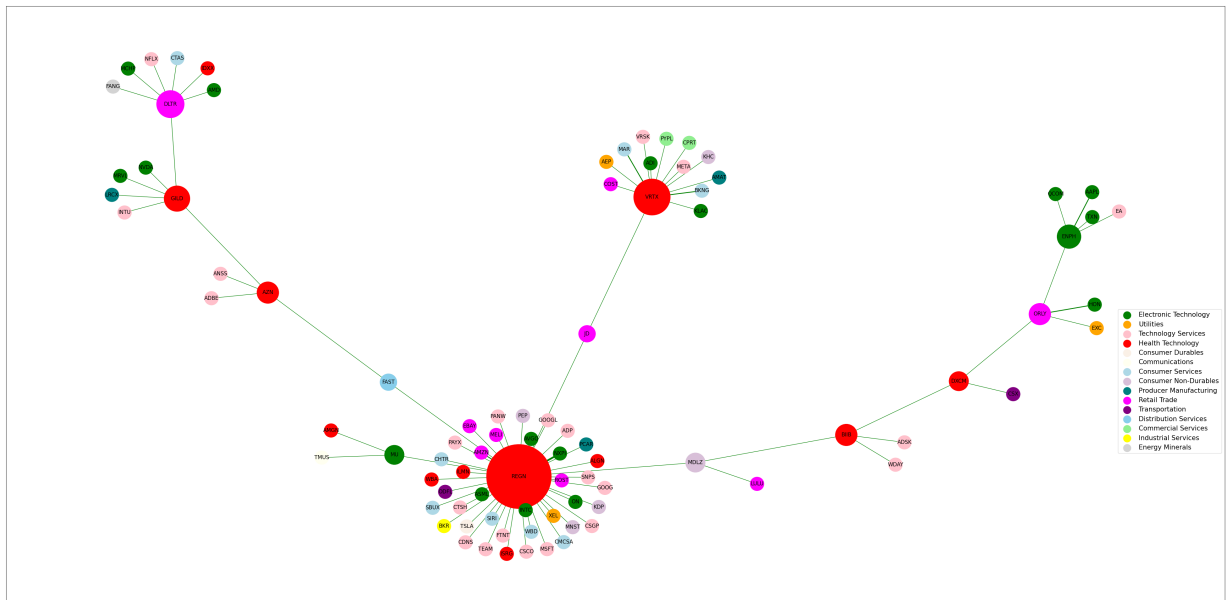
Figure 2.16: Transfer Entropy Analysis - Nasdaq 100

Figure 2.17 presents a visualization of the net transfer entropy network for the Nasdaq 100. Figure 2.17a depicts a directed net transfer entropy network with a threshold of 0.1. The Nasdaq 100 network has a higher average node centrality (i.e. density) than the TSX 60 network.<sup>14</sup> The electronic technology, and technology services sectors demonstrate a high out-degree of connectedness, and are depicted with larger nodes to reflect the large number of out-flow connections. Amazon is the only company from the retail trade sector represented with a large node size. Figure 2.17b shows a MST network based on the transfer entropy transformation. The health technology sector forms a number of clusters in the network, while the distribution service, retail trade, and consumer non-durable sectors play a critical role in connecting the entire network.

<sup>14</sup>The network in Figure 2.17a has a density of 0.07, while the network in Figure 2.13a has a density of 0.05



(a) Nasdaq 100 Index Constituents Net Transfer Entropy Network - Threshold 0.1

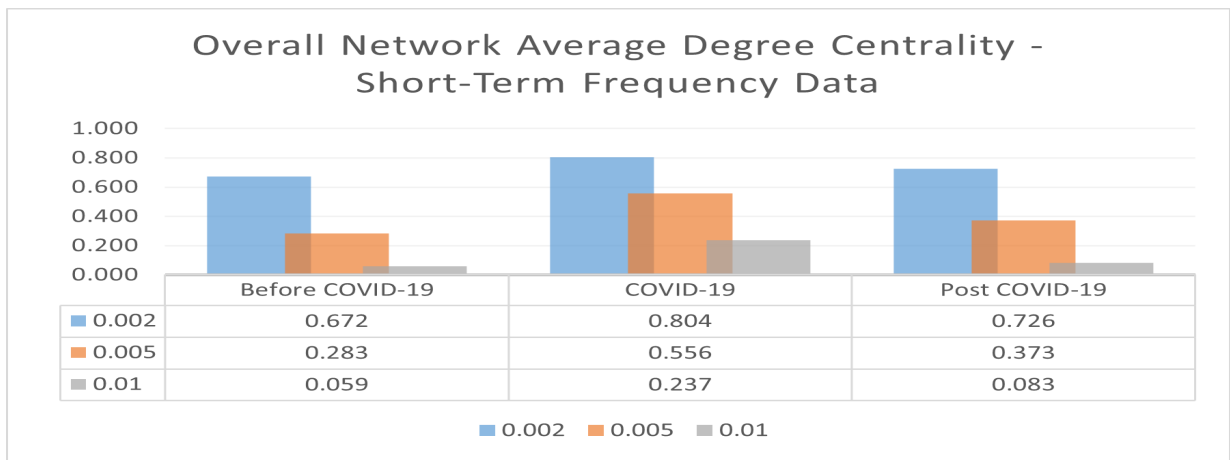


(b) Nasdaq 100 Index Constituents Net Transfer Entropy MST

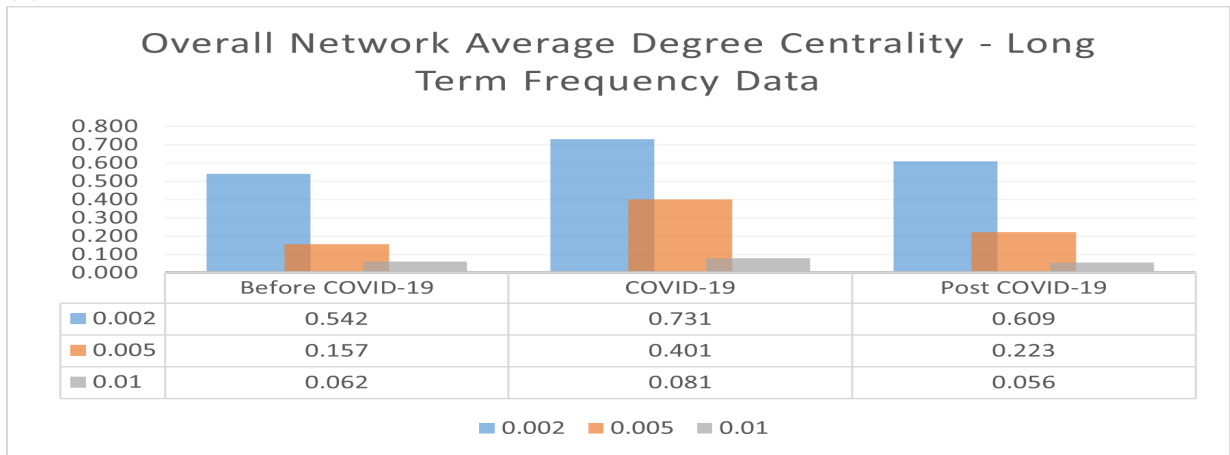
Figure 2.17: Nasdaq 100 Index Constituents Net Transfer Entropy

Moving next to subperiod analysis, the Nasdaq 100 net transfer entropy network shows an intensified transmission during the COVID-19 period as the overall degree centrality is relative higher in this period than in the other two sample periods (see Figure 2.18). The short-term frequency information spillover is also slightly stronger than the long-term frequency information transmission (Figure 2.18a v.s. Figure 2.18b). Figure 2.18a indicates that the short-term information transfer is more robust over all thresholds during a crisis

period than in other periods. This is evidenced by the fact that the degree average centrality is less sensitive to the threshold values, which suggests that the short-term volatility spillover during the crisis is significant, with stocks demonstrating a clear reaction to the impacts of the global pandemic. In contrast, Figure 2.18b shows that the overall centrality degree is sensitive to threshold values, even during the crisis period.<sup>15</sup> This suggests that the long-term volatility spillover network contains many weak connections. At each threshold, the long-term spillover network exhibits a lower level of connectivity than the short-term network.



(a) Nasdaq 100 Index Constituents Net Transfer Entropy Network - Short-Term Frequency Data



(b) Nasdaq 100 Index Constituents Net Transfer Entropy Network - Long-Term Frequency Data

Figure 2.18: Nasdaq 100 Constituents Overall Sector Degree Centrality - Sub-periods.

Note: Node degree centrality measures the number of nodes that a node is connected to. The measurement is typically normalized by dividing the maximum possible degree in a simple graph, which is  $2(n - 1)$ , where  $n$  represents the number of nodes in a network. The average centrality of a network is the sum of the individual node centrality divided by  $n$ . The maximum average network degree centrality is 1. [0.002, 0.005, 0.01] is a set of threshold values.

<sup>15</sup>Appendix F.2 provide a visualization of Nasdaq 100 net transfer entropy directed networks with a threshold of 0.1

Table 2.15 summarizes the sector-wise average information transmission across different periods and data frequencies. Figure 2.19 provides a visualization of Table 2.15. During the crisis, the energy minerals and transportation sectors, as net transfer entropy senders, contributed to a considerable volatility to the rest of the sectors in both the short and long term. This observation also reflects the impacts of COVID-19 measures on these sectors, as they were heavily affected by quarantine and social distance policies. On the other hand, the retail trade, consumer non-durables, and consumer durables sectors were net receivers of information during this period as downstream sectors relying on transportation. In the post COVID-19 period, the consumer services and consumer non-durables sectors have the highest information outflow (i.e., highest “Sent” bar) in the short term. Meanwhile, the producer manufacturing, industrial services, and commercial services are the top sectors with the highest information outflow in the long term. It is also worth noting that the health technology sector is one of the top information receivers in both the short and long term frequency data. To a certain extent, this reflects that the healthcare sector became a key attention-receiving sector during the global pandemic.

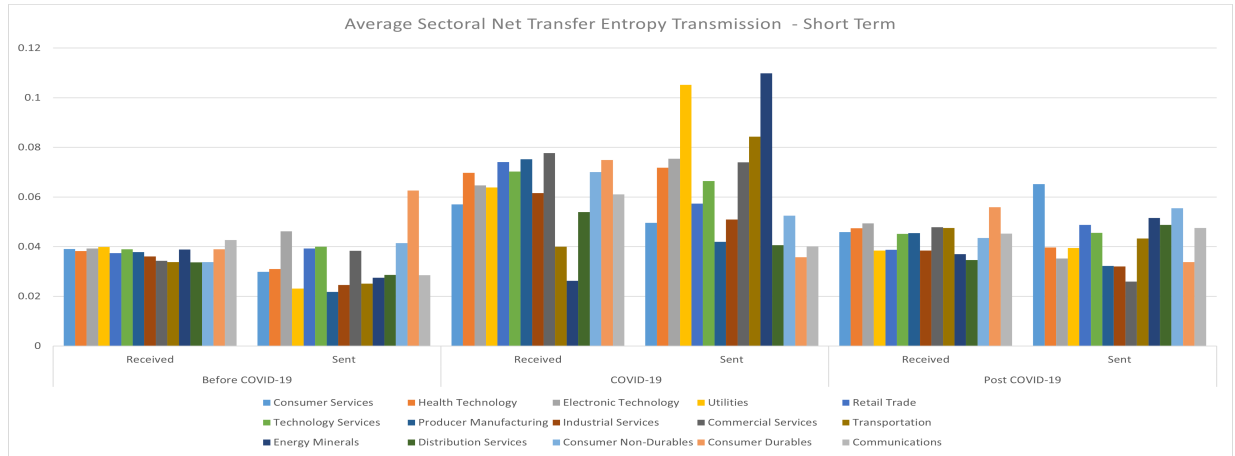
Figure 2.19 does not provide us with a full picture of the information transfer because the position of a sector within the network is also a crucial factor that can either amplify or reduce its influence on the network. Therefore, we compute the in-degree and out-degree sectoral centralities for the Nasdaq 100 net transfer entropy network. Table 2.16 shows the short-term in-degree/out-degree centrality. The technology services sector plays a pivotal role across periods in the short term, as it is a top sector by both out-degree centrality and in-degree centrality. This result highlights the fact that the technology services sector makes up approximately 43% of the market capitalization of Nasdaq 100, making its performance a barometer for the overall performance of Nasdaq 100. The health technology sector has the top out-degree centrality during the pre COVID-19 period, but since the onset of the global pandemic, it has become a critical node in the network with a high rank in both in-degree and out-degree measurements. The post crisis market volatility is also influenced by the consumer services, retail trade, and consumer non-durables sectors, as evidenced

by their robust top out-degree ranking.<sup>16</sup> Table 2.17 provides a long-term measurement. Regardless of sample periods (pre-, during-, and post- COVID-19), the technology services and electronic technology sectors have a large out-degree centrality and act as key net transfer entropy senders, that provide essential information for predicting long-term market volatility. Furthermore, the health technology sector has shown a considerable long-term influence due to the large out-degree centrality since the onset of the global pandemic. Additionally, the retail trade sector also has become a key sector with high long-term out-degree centrality in the aftermath of the global pandemic. It is worth noting that the energy minerals sector has a relatively low and weak out-degree centrality across periods, although it has sent a considerable information to the other sectors (see Figure 2.19).<sup>17</sup> It supports the evidence that the performance of the Nasdaq 100 is less sensitive to the global energy market volatility. The above analysis also highlights the significance of a node's network position, as a pivotal location can markedly amplify the volatility spillover impacts.

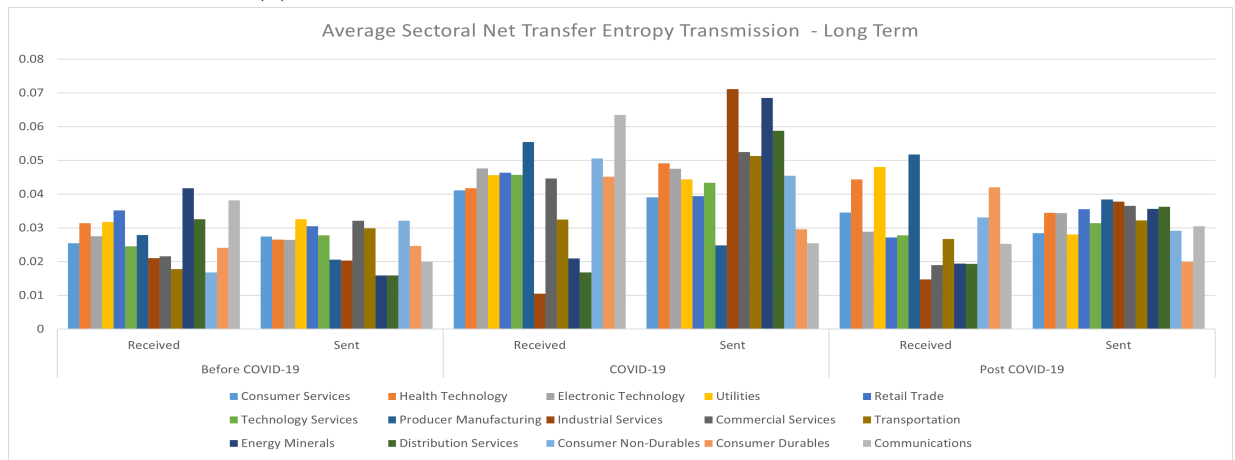
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<sup>16</sup>The consumer services, retail trade and consumer non-durables sectors kept a high out-degree centrality under different threshold values.

<sup>17</sup>Table 2.16 and Table 2.17 show the in-degree/out-degree centrality under different threshold values. Note that the energy minerals sector centrality decreases as the threshold value increases.



(a) Short-Term Frequency Data Sector-wise Transmission



(b) Long-Term Frequency Data Sector-wise Transmission

Figure 2.19: Nasdaq 100 Index Constituents Average Net Transfer Entropy - Sub-periods

## 2.5.2 Diebold-Yilmaz Spillover Analysis

The Diebold-Yilmaz (DY) spillover framework differs conceptually from the measurement of transfer entropy. The transfer entropy is a non-parametric approach to analyze the probability density of annualized daily volatility where  $NT_{x \rightarrow y}$  measures the net information flow from stock  $x$  to stock  $y$ . This information is interpreted as the amount of information that is stored in the annualized daily volatility of stock  $x$  to predict the annualized daily volatility of stock  $y$ . On the other hand, the DY framework uses H-step ahead forecast error variance decomposition based on a conventional VAR model. Essentially, the DY framework investigates the linear relationship in the variance of the error term. For example,  $\mathbf{Nd}_{xy}^g(5)$  measures the proportion of the 5-step-ahead error variances in forecasting

the daily volatility of stock  $y$  due to innovations in the daily volatility of stock  $x$ .

**TSX 60** This section applies the DY spillover framework to assess the daily volatility of the TSX 60 Index constituents. Figure 2.20 summarizes the information from Tables 2.7 and 2.8. The left panel of Figure 2.20 shows the DY spillover index based on the entire dataset (all three periods). The diagonal blocks have a much lighter color than the rest of the blocks, indicating that shocks to the stock itself explain most of the variance in the forecast error. The right panel shows the net pairwise volatility spillovers and eliminates the self impacts (i.e. Diagonal blocks are now zero). Table 2.18 shows the average of the volatility spillover for each sector. The top five sectors with high contribution (sending) to the volatility are the high dividend sectors. The real estate and consumer cyclical sectors are top volatility receivers.

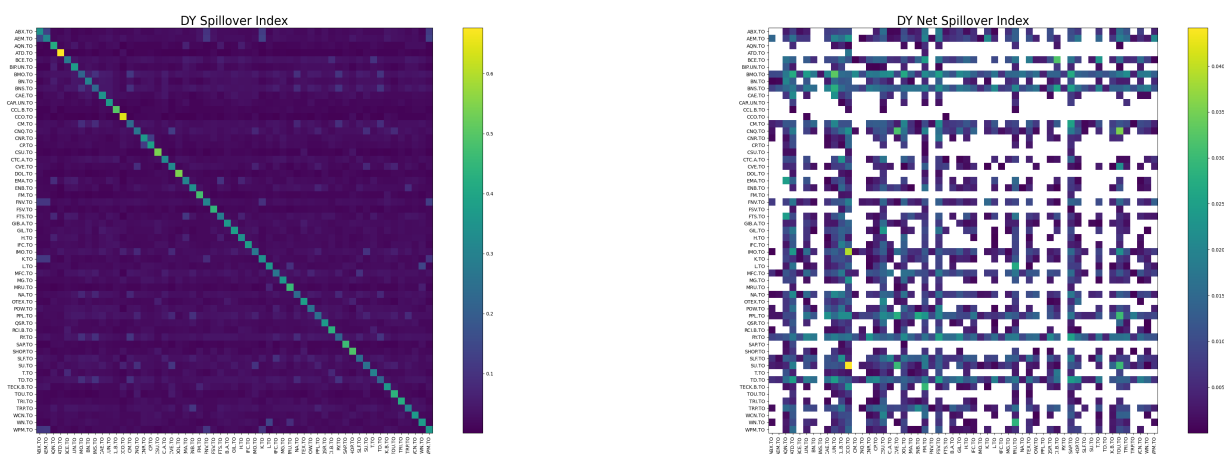
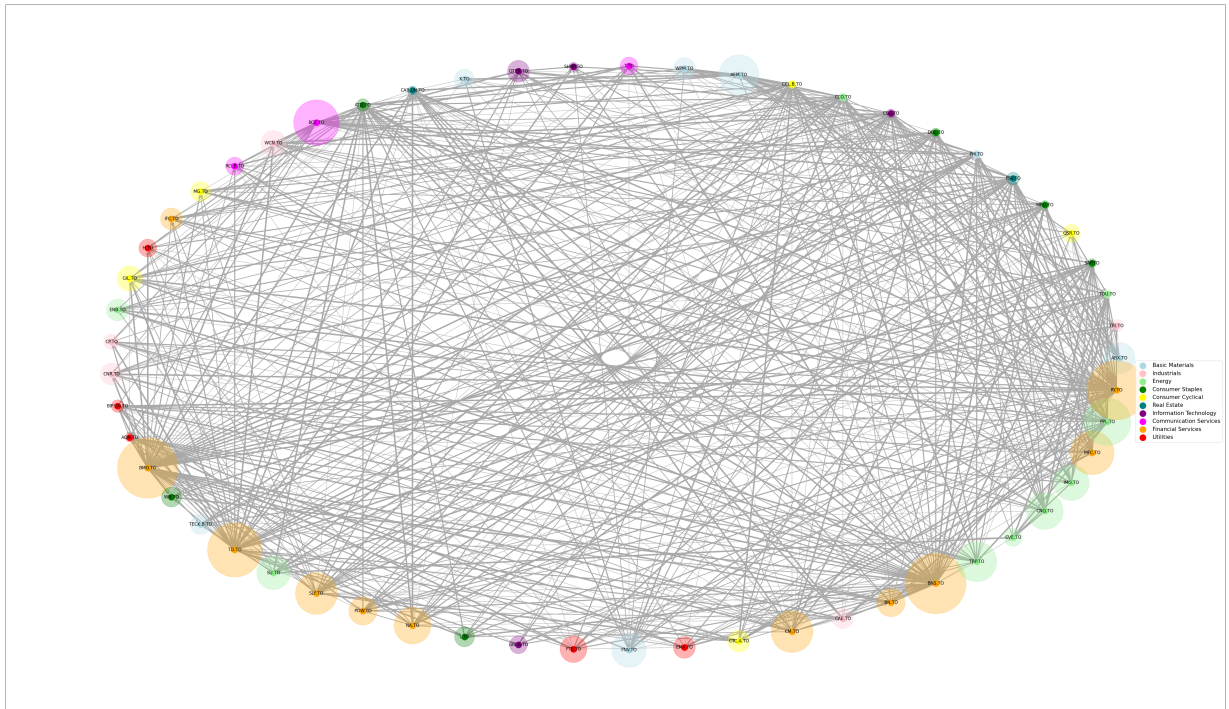
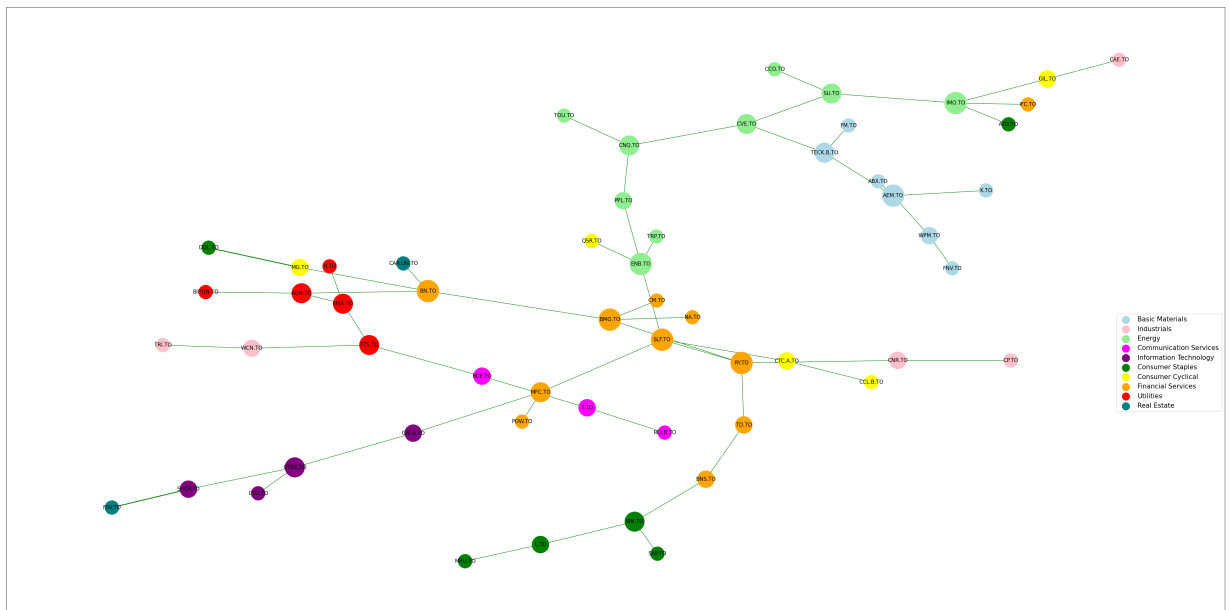


Figure 2.20: DY Spillover Index Analysis - TSX 60

Figure 2.21a presents a net DY spillover index network with a threshold of 0.01. The high dividend sectors have strong connections with the rest of the sectors. To reflect this, the size of node in the figure is proportional to the number of outflow connections. Figure 2.21b illustrates the MST for the DY spillover index based on the left panel of Figure 2.20. It clearly shows that the financial service, energy, utility and communication services sectors are key bridges in the maximum volatility flow. Additionally, the sector-wise clusters indicate that the volatility spillover is mainly intra-sectoral.



(a) TSX 60 Index Constituents Net DY Spillover Index - Threshold 0.01



(b) TSX 60 Index Constituents DY Spillover Index MST

Figure 2.21: TSX 60 Index Constituents DY Spillover Index

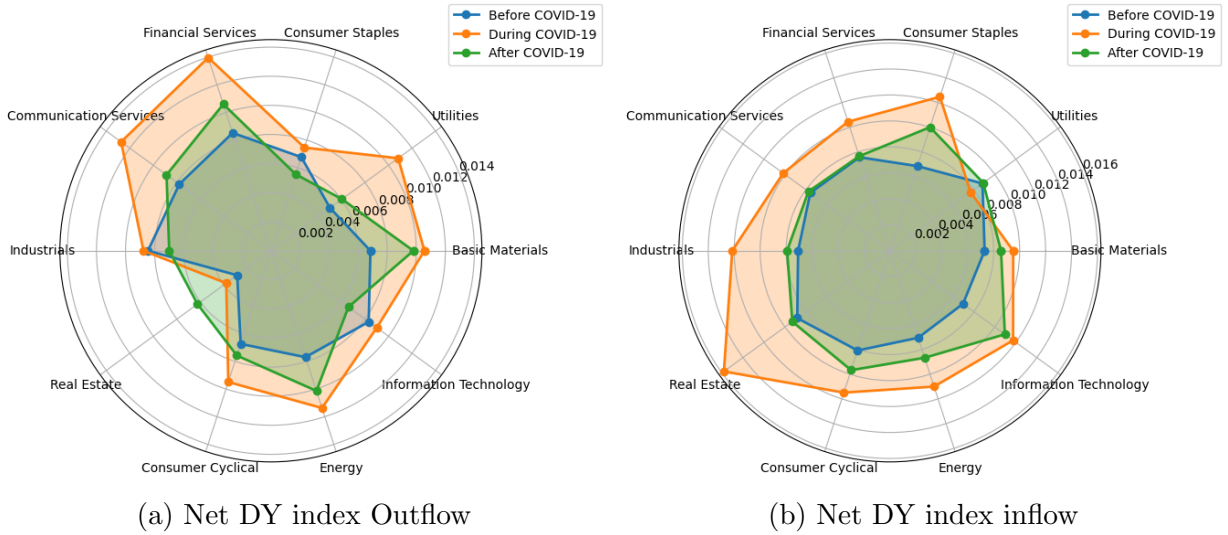
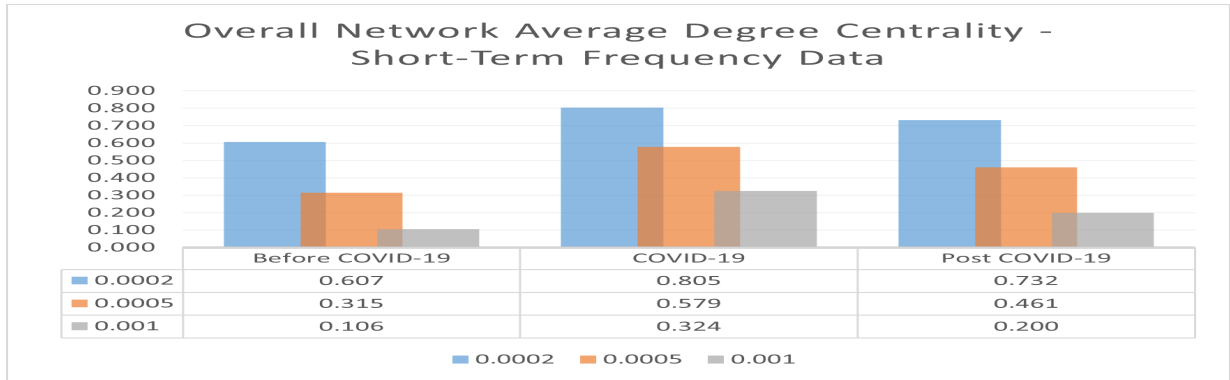


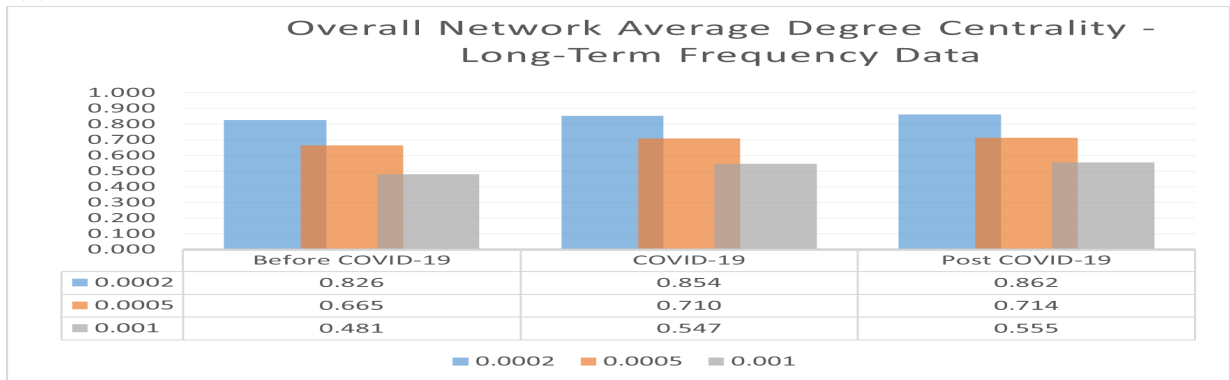
Figure 2.22: Net DY Index Spillover - TSX 60

In addition, we have applied the DY framework to the sub-period samples. Figure 2.22 illustrates the sectoral average of the net DY index spillover in the TSX 60 network across three distinct time periods. Notably, the sectoral averages of net DY index transmission are significantly higher during the global pandemic, suggesting that stock volatility intensified during this period. Furthermore, the Post COVID-19 spillover is stronger than the pre-pandemic level but weaker than the level observed during the pandemic. Figure 2.22a indicates that the financial services, energy, communication services, and basic materials sectors show a strong influence on the volatility of other sectors in all periods. The utilities sector exhibited a notably larger volatility outflow during the pandemic compared to other periods. Additionally, the real estate sector had little influence on the volatility of other sectors prior to the onset of the pandemic, but it demonstrated a small gradual increase in influence on market volatility since the pandemic. Figure 2.22b shows that the majority of data points are situated in relatively close proximity across sectors, suggesting a notable degree of consistency. In other words, the volatility of the TSX 60 constituents is mutually connected, as each sector demonstrates a relatively equal volatility inflow from other sectors. Moreover, the radar chart indicates that each sector was significantly impacted by the global pandemic because the orange contour dominates the other contours. It is worth noting that the utilities and basic materials sectors exhibited a relatively stable volatility inflow across all periods, whereas the consumer cyclical, consumer staples and real estate

sectors demonstrated an increased volatility inflow. Intuitively, the implementation of lockdown policies and the disruption to global supply chains caused by the pandemic had a significant impact on the consumer cyclical, consumer staples and real estate sectors.



(a) TSX 60 Index Constituents Net DY Spillover Index Network - Short-Term Frequency Data



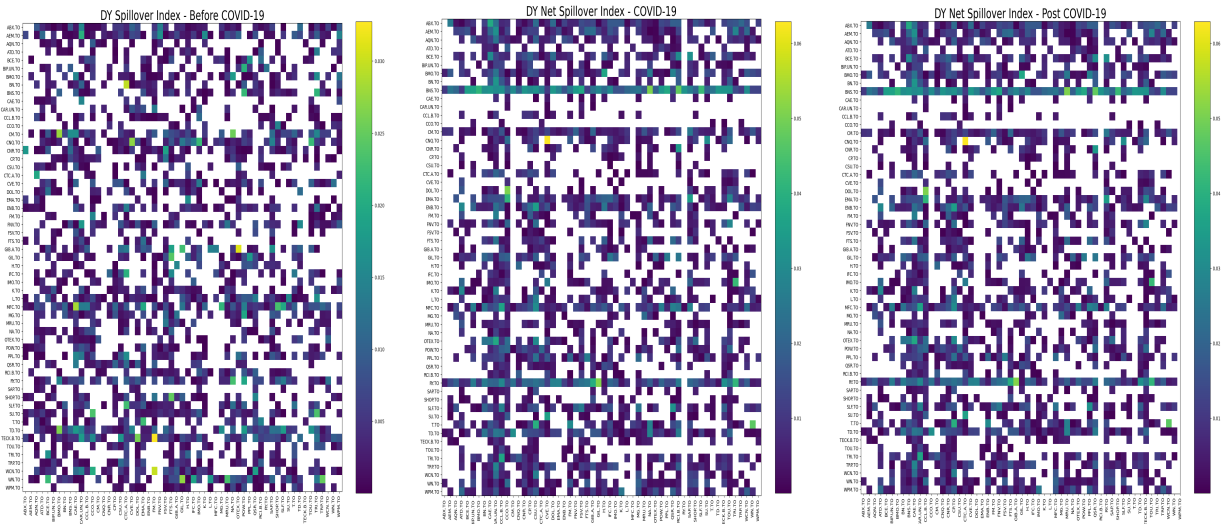
(b) TSX 60 Index Constituents Net DY Spillover Index Network - Long-Term Frequency Data

Figure 2.23: TSX 60 Index Constituents Overall Sector Degree Centrality - Sub-periods. Note: Node degree centrality measures the number of nodes that a node is connected to. The measurement is typically normalized by dividing the maximum possible degree in a simple graph, which is  $2(n - 1)$ , where  $n$  represents the number of nodes in a network. The average centrality of a network is the sum of the individual node centrality divided by  $n$ . The maximum average network degree centrality is 1. [0.0002, 0.0005, 0.001] is a set of threshold values.

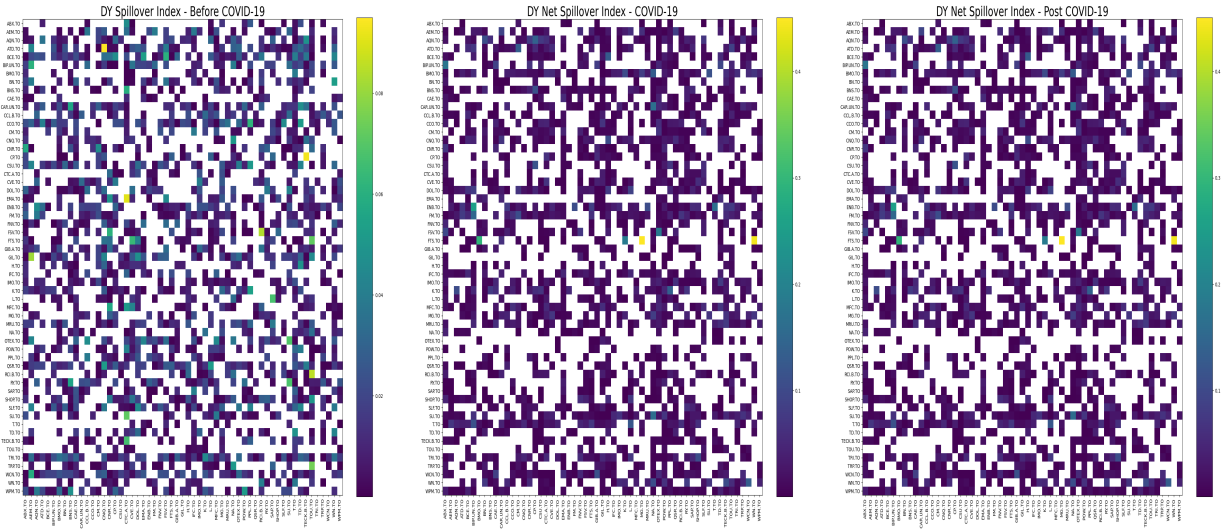
We now investigate the overall average centrality of the TSX 60 volatility network in different data frequencies (i.e., short-term versus long-term). Figure 2.23 summarizes the average network degree centrality across different periods under various thresholds. Figure 2.23a shows that the DY spillover index intensified during the crisis, while maintaining its high value in the post-COVID-19 period. Furthermore, Figure 2.23b indicates that the average degree centrality of the net DY index spillover is slightly higher in long-term frequency data than in the short run in all three periods. This result indicates that, in the short run, the TSX 60 volatility network is relatively sparse, reflecting weak interconnectedness among stocks, whereas in the long run the network becomes substantially

denser, indicating a more systematic and pervasive pattern of volatility spillovers across all three periods. A comparison of the net transfer entropy analysis and the net DY index network reveals a stronger long-run volatility spillover for the DY index and a stronger short-run volatility transmission for the net transfer entropy. Intuitively, this discrepancy can be attributed to the differing methodologies employed. Transfer entropy essentially relies on data distribution whereby a smoothed data pattern, as is the case for long-run data, results in fewer identified correlations. In contrast, the DY spillover framework is not influenced by data smoothness and can detect connections between stocks as long as there is correlation in the error term.

Furthermore, we calculate the in-degree/out-degree sectoral centrality for the net DY spillover network in both short-term and long-term. Table 2.19 and Table 2.20 summarize the results and include a robustness check using various threshold values to filter out weak connections. The results align with the net transfer entropy analysis, indicating that high dividend sectors have a notable network presence, as evidenced by their high in-degree and/or out-degree centrality values in different frequencies. It is worth noting that the real estate sector in short run has a zero out-degree centrality and an in-degree centrality of 0.56 during the COVID-19 period under a threshold of 0.001. This indicates that the real estate sector does not have any volatility spillover impact on other sectors, but has received considerable impacts from other sectors. Figure 2.24 provides a set of heatmaps to illustrate the net DY spillover index across different periods and frequencies. Figure 2.24b confirms a denser network in the long-term (as it has more colored blocks than Figure 2.24a). Additionally, the magnitude of the DY spillover index is higher in the long-term. Furthermore, the middle panel of 2.24a highlights that Scotiabank (BNS) had a notable net DY volatility index outflow during the period of the global pandemic, with no net DY volatility index inflow from other stocks in the short run. This indicates that the volatility impact from Scotiabank was considerably more significant than the influence of other stocks on Scotiabank during the same period.



(a) TSX 60 Index Constituents Net DY Spillover Index Short-Term Frequency Data



(b) TSX 60 Index Constituents Net DY Spillover Index Long-Term Frequency Data

Figure 2.24: TSX 60 Index Constituents DY Spillover Index Heatmap

**Nasdaq 100** The DY spillover index, based on Nasdaq 100 data, also yields interesting results. The left panel of Figure 2.25 displays a heatmap for Table 2.9, while the right panel visualizes the net pairwise DY spillover index from Table 2.10. The right panel displays only one light yellow block (circled for easier detection), which measures the volatility spillover from PepsiCo Inc to Kraft Heinz Co. This finding is also supported by their stock performance, as these two companies exhibit a very similar volatility pattern. As illustrated in Table 2.21, the technology services, commercial services, consumer non-durables, and electronic technology sectors have strong volatility spillover contribution. Conversely, the consumer durable, industrial services, and communications sectors appear to be net

receivers' sectors suggesting that these sectors are influenced by others.

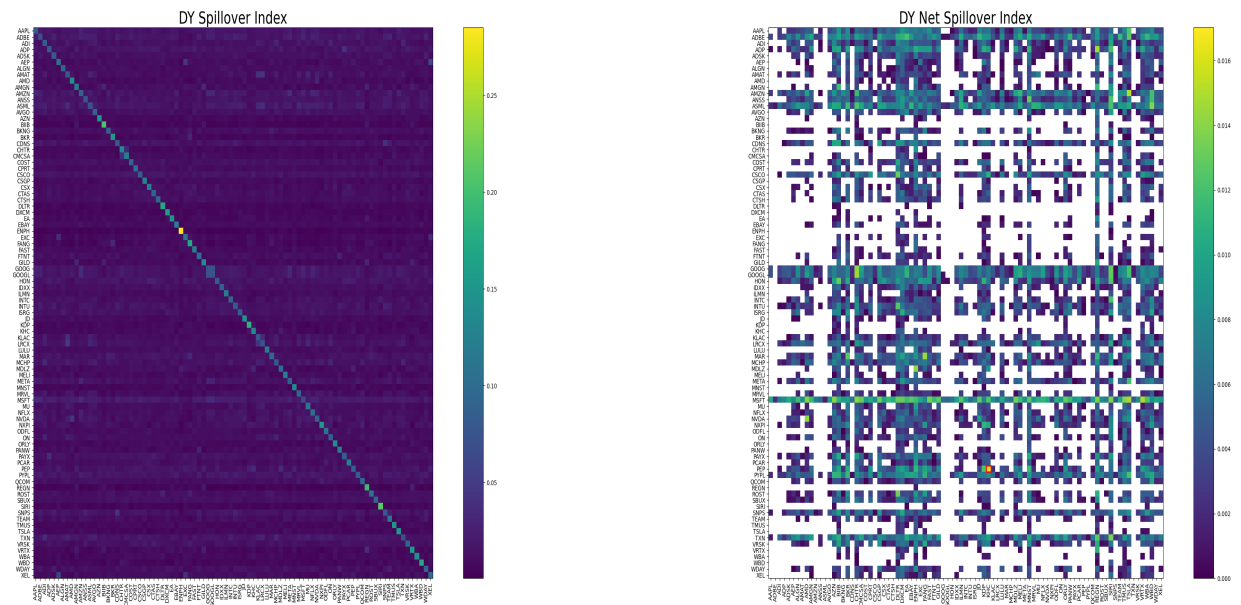
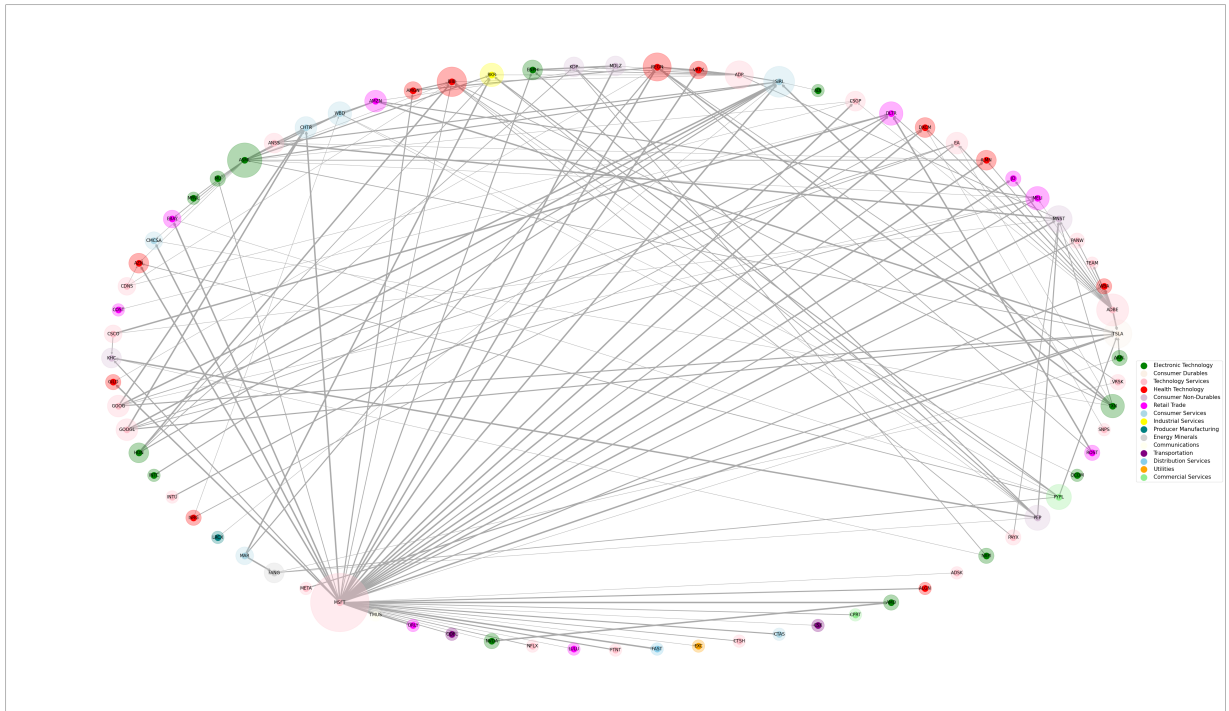
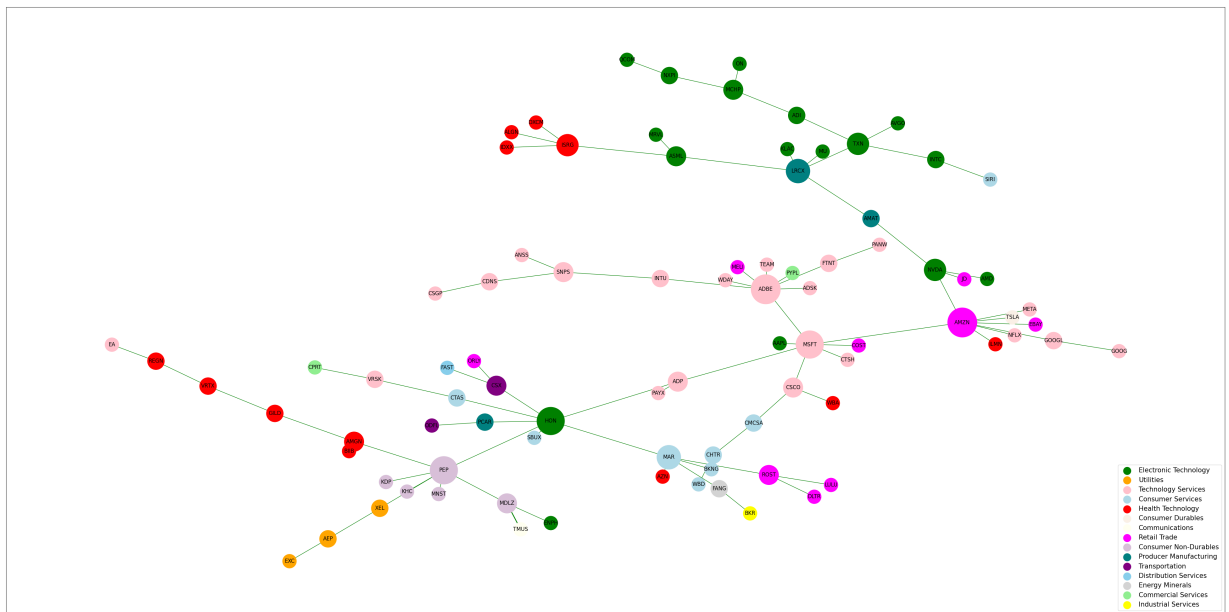


Figure 2.25: DY Spillover Index Analysis - Nasdaq 100

Figure 2.26 provides an overview of Nasdaq 100 DY spillover index network. Figure 2.26a is a net DY spillover index network for values greater than 0.01. The technology services and electronic technology sectors have many large inter-sectoral connections. There is no doubt that these two sectors are key sectors in the Nasdaq 100 network because of their high volatility spillover contribution and numerous network connections. In addition, Figure 2.26a indicates that the health technology sector has a considerable influx of connections, substantiating its high ranking in terms of volatility spillover receiver as illustrated in Table 2.21. The MST in Figure 2.26b shows that the volatility transmission is mainly intra-sectoral.



(a) Nasdaq 100 Index Constituents DY Spillover Index - Threshold 0.01

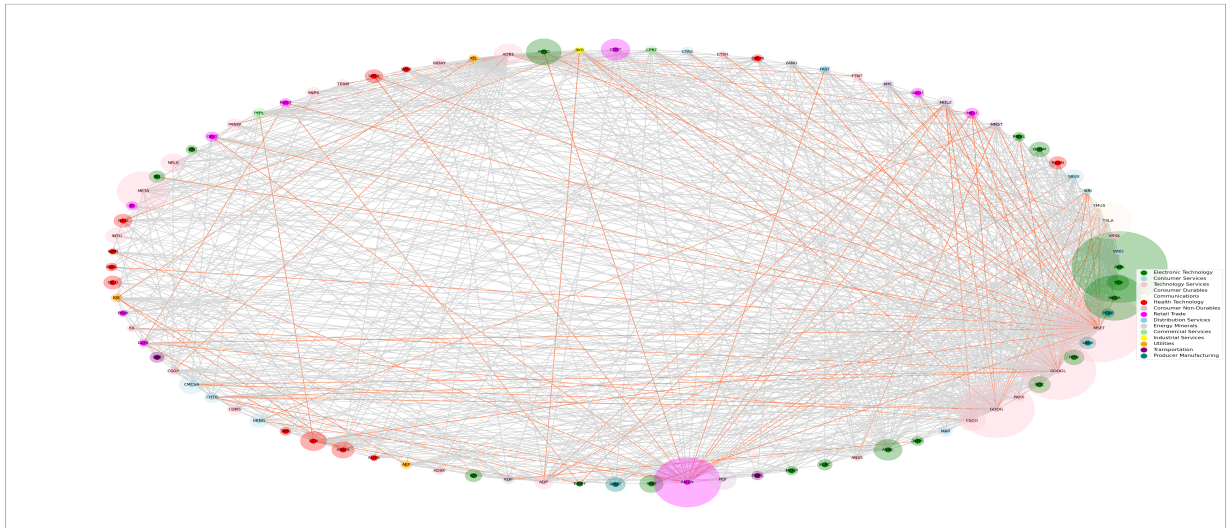


(b) Nasdaq 100 Index Constituents DY Spillover Index MST

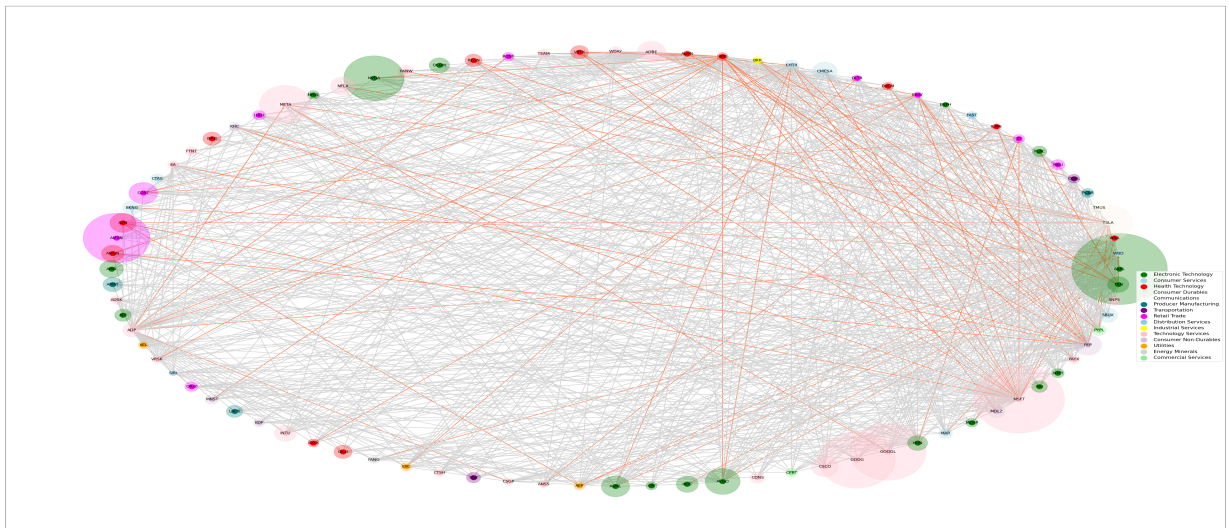
Figure 2.26: Nasdaq 100 Index Constituents DY Spillover Index

In the sub-periods analysis, Figure 2.27 illustrates three sample periods of volatility networks, with a particular emphasis on the most significant edges. All edges in each network are filtered by retaining only those with edge weights exceeding the 80th percentile threshold. The edges colored orange represent weights greater than the 90% percentile threshold within the top 20% of data points (i.e., as a result of the 80th percentile thresh-

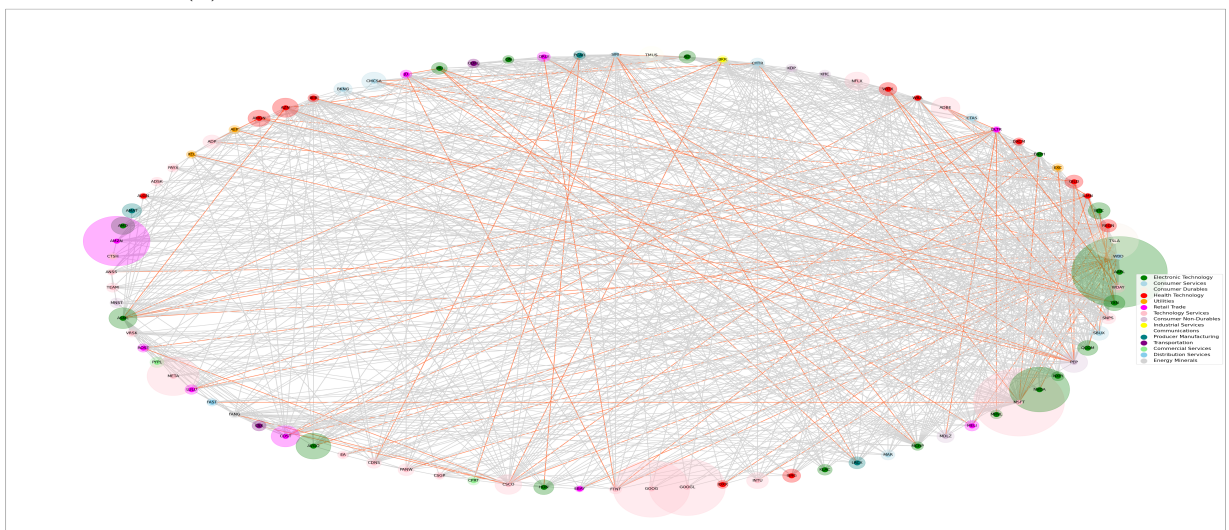
old). The size of the nodes is indicative of the market capitalization within the Nasdaq 100 in 2023. Figure 2.27a illustrates that the larger technical giants, such as Microsoft, Google, and Amazon, exerted a considerable volatility outflow prior to the crisis. The network suggests that larger companies possess a more substantial market influence. During the COVID-19 period, Amazon lost all of its orange-colored outflow edges, while the health technology sector received numerous significant inflow edges, as depicted in Figure 2.27b. This evidence supports the view that the health technology sector gained prominence as a key area of focus throughout the global pandemic. Another interesting observation is that PepsiCo Inc had a considerable number of orange-colored outflow edges during the crisis, which exerted a notable impact on the volatility of other stocks, despite its relatively modest capitalization compared to the technical giants. In the post COVID-19 period, Figure 2.27c shows that numerous orange-colored edges link relatively small-sized companies, suggesting that these relatively small capitalized firms may partially take on the role of larger tech giants, exerting considerable influence on market volatility. Additionally, the companies in the semiconductor industry, such as ASML Holding N.V., Lam Research Corporation (Producer Manufacturing), Microchip Technology Inc (Electronic Technology) and NVIDIA Corp (Electronic Technology), also have a strong influence on the volatility of other stocks in the post COVID-19 period. The evolution of the Nasdaq 100 volatility network shows its ability to dynamically capture changes in the hub of market volatility.



(a) Nasdaq 100 Index Constituents DY Spillover Index - Before COVID-19



(b) Nasdaq 100 Index Constituents DY Spillover Index - COVID-19



(c) Nasdaq 100 Index Constituents DY Spillover Index - After COVID-19

Figure 2.27: Nasdaq 100 Index Constituents DY Spillover Index Network - Sub-periods

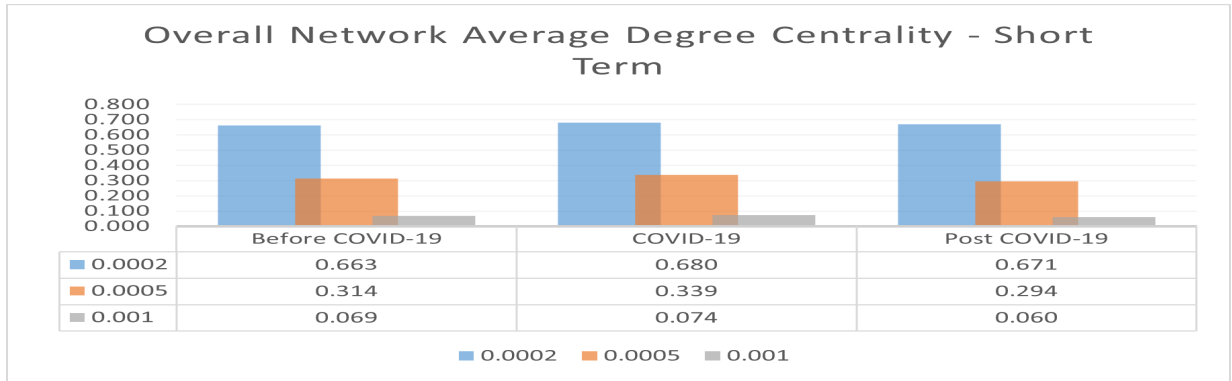
Figure 2.28 shows that the average network centrality is stable across periods in both short term and long term as it remains around 0.7 for a threshold of 0.0002. Moreover, the long term connections are more robust than the short term as the average network centrality is less sensitive to threshold values.<sup>18</sup> Table 2.22 ranks sector-wise in-degree/out-degree centralities in the short term. The health technology, consumer services and retail trade sectors are consistently ranked as the top in-degree sectors across all periods, while the technology service and electronic technology sectors are consistently ranked as the top out-degree centrality sectors. Intuitively, high out-degree centrality implies a greater influence on the volatility of other sectors. Thus, the technology service and electronic technology sectors play a pivotal role in spreading the uncertainty. A possible explanation could be the weight of these two sectors in the Nasdaq 100. On the other hand, the volatility of health technology, consumer services and retail trade sectors is significantly influenced by other sectors. It is worth noting that we have experienced a semiconductor supply shortage in the post COVID-19 period. Since two-thirds of companies in the manufacturing sector are semiconductor manufacturers, and the technology sector has a significant weight in the composition of the Nasdaq 100, the manufacturing sector has become one of the most influential sectors affecting the volatility of other sectors in the short term.<sup>19</sup> This is evidenced by the high out-degree centrality in the post COVID-19 period. Table 2.23 summarizes the long-term in-degree/out-degree centrality. It provides a straightforward message that the volatility of health technology, technology service and electronic technology underpins the Nasdaq 100 net volatility spillover in all periods because they demonstrates a high ranking in both in-degree and out-degree centrality. Figure 2.29 provides stock-pairwise net DY spillover index heatmaps. It indicates that the long-term volatility spillover (Figure 2.29b) is more robust than in the short period (Figure 2.29a). This is evidenced by the reduced number of white blocks (i.e., no connection) in the long-term heatmap in comparison to the short-term heatmap. Figure 2.28 corroborates this observation, as evidenced by the higher average network centrality in the long-term than in the short-term. In addition, Figure 2.29a shows that Microsoft Corp is not a volatility receiver in the “before COVID-19”

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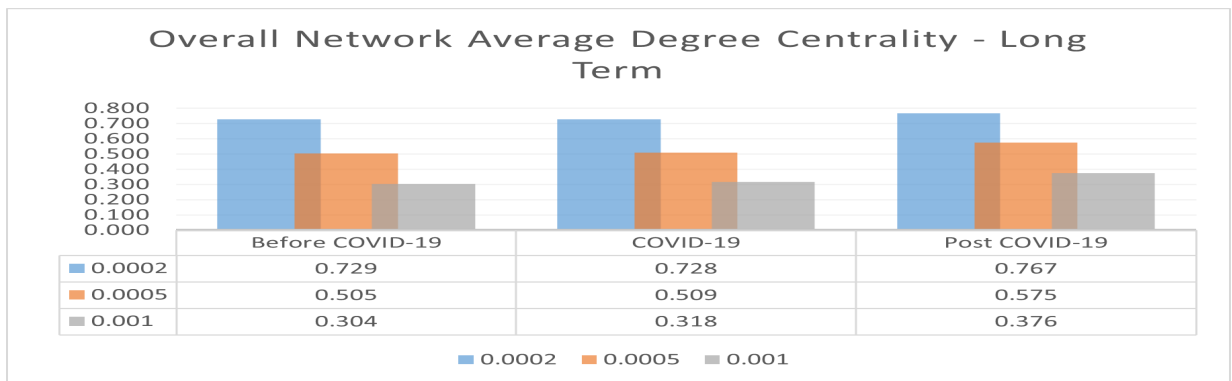
<sup>18</sup>Appendix G.2 provides a visualization of net DY spillover network in both short and long term.

<sup>19</sup>The chip companies under the technology sector rely on the stable supply of semiconductor.

heatmap, Adobe Inc is not a volatility receiver in the “COVID-19” heatmap, and ASML Holding NV is not a volatility receiver in the “ post COVID-19” heatmap. In other words, each company demonstrates considerable influence on other stocks’ volatility in different periods.



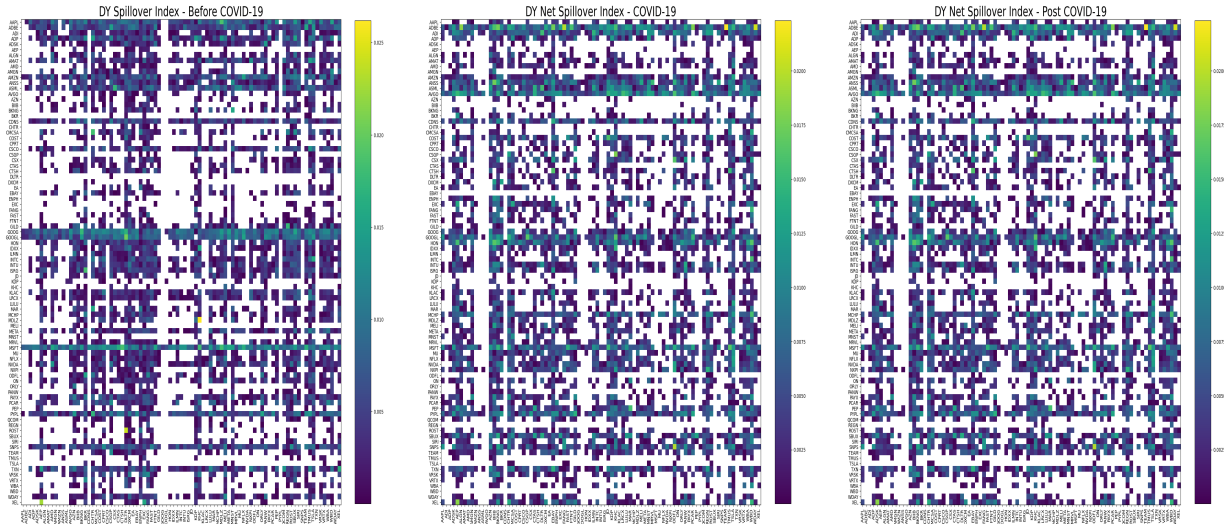
(a) Nasdaq 100 Index Constituents Net DY Spillover Index Network - Short-Term Frequency Data



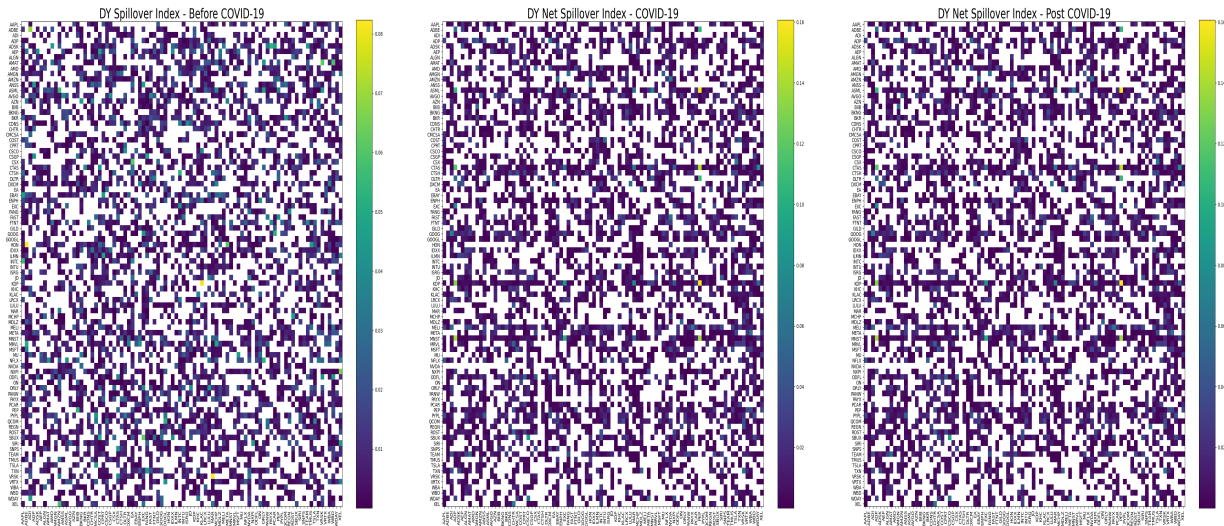
(b) Nasdaq 100 Index Constituents Net DY Spillover Index Network - Long-Term Frequency Data

Figure 2.28: Nasdaq 100 Index Constituents Overall Sector Degree Centrality - Sub-periods.

Note:Node degree centrality measures the number of nodes that a node is connected to. The measurement is typically normalized by dividing the maximum possible degree in a simple graph, which is  $2(n - 1)$ , where  $n$  represents the number of nodes in a network. The average centrality of a network is the sum of the individual node centrality divided by  $n$ .The maximum average network degree centrality is 1. [0.0002,0.0005,0.001] is a set of threshold values.



(a) Nasdaq 100 Index Constituents Net DY Spillover Index Short Term Heatmap



(b) Nasdaq 100 Index Constituents Net DY Spillover Index Long Term Heatmap

Figure 2.29: Nasdaq 100 Index Constituents DY Spillover Index

## 2.6 Conclusion

This chapter uses network analysis to study the development of directed networks in the US and Canadian markets over the past six years, including the COVID-19 pandemic period. In general, both markets demonstrated substantial volatility spillover connections during the crisis. Furthermore, the overall connectedness of the networks showed a change in the post-crisis period, as evidenced by an increase in the average network centrality. For the Canadian stock market, our investigation revealed that dividend-driven sectors exerted a

considerable influence on market volatility across all sample periods. These sectors typically have a strong history of capital accumulation, regular dividend payouts, and robust cash flow to mitigate market risk and stabilize the market during periods of market turbulence. The analysis also highlights that the explosive growth of AI has increased the influence of the information technology sector in the post COVID-19 period. In the US market, we examine the evolution of the Nasdaq 100 network. While the technology service and electronic technology sectors are the foundation of the network, the Nasdaq 100 volatility network has demonstrated its ability to recover from market distress and adapt to changes over time. The development of technology and artificial intelligence during the sample period has undoubtedly contributed to the growth of the Nasdaq 100. The global pandemic has stimulated growth in the health technology sector, making it a key volatility spillover contributor in the late stage of the sample period.

This chapter compares two popular methodologies for volatility analysis: net transfer entropy and the Diebold-Yilmaz spillover index. The net transfer entropy method is a non-parametric approach used to explore the correlation among stocks' daily volatility. On the other hand, the Diebold-Yilmaz spillover framework constructs volatility connections based on the conventional VAR model. Although both methods draw very similar conclusions regarding the TSX 60 and Nasdaq 100 network analysis, the different edge identification strategies reveal some additional mutually exclusive information. The transfer entropy method demonstrates the ability to capture non-linear relationships by examining the pattern between the empirical probability density of daily stock volatility. The MST analysis reveals some inter-sectoral connections that are not revealed by the covariance analysis. However, the effectiveness of this method is limited when examining a relatively smooth dataset, such as with long-term frequency data. In contrast, the Diebold-Yilmaz spillover framework compensates for the inherent weaknesses of the transfer entropy methodology. It emphasizes long-term volatility spillovers. Finally, this chapter demonstrates that the use of EMD not only relaxes the stationarity requirement typically imposed on financial time series but also enriches the dimensional structure of the underlying data. Compared with traditional moving average techniques, EMD more effectively disentangles short-term

and long-term dynamics at each point in time, without introducing artificial temporal lags. While current regulatory frameworks largely rely on low-frequency information, such as quarterly balance sheet data, to evaluate systemic risk, our EMD-based analysis shows that more than 60 percent of contagion spillovers originate in the high-frequency domain associated with short-term scales. This finding suggests that conventional macroprudential policies may be subject to a substantial detection lag in identifying emerging systemic risks. To conclude, this chapter introduces a novel tool for conducting systemic risk management analysis. Both the net transfer entropy and DY spillover index show interesting results in identifying key influential sectors. However, knowledge of volatility spillover alone is insufficient for managing or hedging risk. The position of a sector within the network is a crucial factor that can either amplify or reduce its influence on the overall network. Therefore, we incorporate conventional network analysis techniques into the volatility analysis to reveal the pivotal sectors in a network. The combination of network analysis and volatility analysis sheds light on identifying the hub of systemic risk.

## 2.7 Tables

Table 2.1: TSX 60 Constituents' Annualized daily Volatility Descriptive Statistics

| Stock  | Sector                 | Ave.   | Std Deviation | Min.  | Max.    | ADF Statistics | p value |
|--------|------------------------|--------|---------------|-------|---------|----------------|---------|
| ABX    | Basic Materials        | 30.308 | 18.656        | 6.797 | 360.611 | -6.819         | 0.000   |
| AEM    | Basic Materials        | 31.348 | 19.326        | 5.016 | 310.074 | -7.320         | 0.000   |
| AQN    | Utilities              | 20.772 | 15.522        | 3.610 | 185.920 | -4.723         | 0.000   |
| ATD    | Consumer Staples       | 21.571 | 14.171        | 0     | 132.124 | -6.908         | 0.000   |
| BCE    | Communication Services | 13.184 | 9.6612        | 2.834 | 133.061 | -6.876         | 0.000   |
| BIP-UN | Utilities              | 21.594 | 16.831        | 3.247 | 262.809 | -6.838         | 0.000   |
| BMO    | Financial Services     | 15.521 | 11.719        | 3.870 | 145.613 | -6.001         | 0.000   |
| BN     | Financial Services     | 22.548 | 16.126        | 4.876 | 275.169 | -6.375         | 0.000   |
| BNS    | Financial Services     | 14.724 | 10.973        | 3.460 | 137.891 | -5.551         | 0.000   |
| CAE    | Industrials            | 27.848 | 22.279        | 3.842 | 319.247 | -5.300         | 0.000   |
| CAR-UN | Real Estate            | 19.589 | 13.193        | 2.702 | 192.254 | -5.352         | 0.000   |
| CCL-B  | Consumer Cyclical      | 24.020 | 14.461        | 4.979 | 176.385 | -5.996         | 0.000   |
| CCO    | Energy                 | 39.793 | 22.878        | 6.537 | 222.262 | -7.708         | 0.000   |
| CM     | Financial Services     | 14.847 | 11.205        | 3.607 | 172.341 | -6.335         | 0.000   |
| CNQ    | Energy                 | 32.068 | 22.421        | 6.088 | 274.549 | -5.227         | 0.000   |
| CNR    | Industrials            | 19.310 | 10.444        | 0     | 128.712 | -6.873         | 0.000   |
| CP     | Industrials            | 21.632 | 12.297        | 5.812 | 165.549 | -8.593         | 0.000   |
| CSU    | Information Technology | 27.679 | 14.141        | 3.846 | 189.833 | -6.161         | 0.000   |
| CTC-A  | Consumer Cyclical      | 21.990 | 17.354        | 5.100 | 235.335 | -5.771         | 0.000   |
| CVE    | Energy                 | 42.785 | 27.510        | 9.714 | 346.956 | -4.758         | 0.000   |
| DOL    | Consumer Staples       | 23.622 | 14.163        | 4.989 | 148.944 | -6.998         | 0.000   |
| EMA    | Utilities              | 16.408 | 11.892        | 4.457 | 144.905 | -6.406         | 0.000   |
| ENB    | Energy                 | 19.078 | 13.755        | 4.577 | 214.130 | -7.401         | 0.000   |
| FM     | Basic Materials        | 51.885 | 27.710        | 8.273 | 351.973 | -7.080         | 0.000   |
| FNV    | Basic Materials        | 26.199 | 16.823        | 5.399 | 315.690 | -6.218         | 0.000   |
| FSV    | Real Estate            | 25.027 | 16.349        | 3.415 | 282.801 | -6.422         | 0.000   |
| FTS    | Utilities              | 15.729 | 11.382        | 4.321 | 171.449 | -7.325         | 0.000   |
| GIB-A  | Information Technology | 18.852 | 10.648        | 4.538 | 152.041 | -8.139         | 0.000   |
| GIL    | Consumer Cyclical      | 27.257 | 19.163        | 4.597 | 225.167 | -5.390         | 0.000   |
| H      | Utilities              | 16.391 | 11.834        | 2.244 | 208.718 | -5.716         | 0.000   |
| IFC    | Financial Services     | 17.819 | 12.990        | 2.837 | 223.399 | -6.043         | 0.000   |
| IMO    | Energy                 | 30.272 | 21.305        | 6.762 | 291.030 | -4.343         | 0.000   |
| K      | Basic Materials        | 40.661 | 24.582        | 7.126 | 436.169 | -7.005         | 0.000   |
| L      | Consumer Staples       | 18.458 | 10.992        | 3.844 | 150.996 | -7.398         | 0.000   |
| MFC    | Financial Services     | 20.467 | 13.458        | 5.231 | 167.272 | -6.389         | 0.000   |
| MG     | Consumer Cyclical      | 27.441 | 16.999        | 7.470 | 297.942 | -5.793         | 0.000   |
| MRU    | Consumer Staples       | 17.887 | 9.5338        | 4.314 | 122.695 | -7.084         | 0.000   |
| NA     | Financial Services     | 16.138 | 13.768        | 3.698 | 190.052 | -5.925         | 0.000   |
| OTEX   | Information Technology | 21.307 | 12.139        | 4.596 | 111.598 | -8.218         | 0.000   |
| POW    | Financial Services     | 18.804 | 14.645        | 2.519 | 164.153 | -6.211         | 0.000   |
| PPL    | Energy                 | 23.265 | 25.937        | 3.893 | 437.065 | -5.332         | 0.000   |
| QSR    | Consumer Cyclical      | 23.008 | 16.255        | 5.598 | 235.976 | -5.586         | 0.000   |
| RCI-B  | Communication Services | 18.651 | 13.021        | 3.437 | 211.777 | -7.568         | 0.000   |
| RY     | Financial Services     | 13.930 | 9.7544        | 3.430 | 115.224 | -6.648         | 0.000   |
| SAP    | Consumer Staples       | 21.238 | 12.995        | 4.747 | 130.652 | -8.859         | 0.000   |
| SHOP   | Information Technology | 51.507 | 28.195        | 8.056 | 250.305 | -4.284         | 0.000   |
| SLF    | Financial Services     | 17.078 | 11.043        | 3.641 | 187.192 | -6.237         | 0.000   |
| SU     | Energy                 | 29.459 | 21.363        | 5.883 | 256.355 | -4.815         | 0.000   |
| T      | Communication Services | 14.978 | 10.598        | 0     | 168.841 | -7.022         | 0.000   |
| TD     | Financial Services     | 14.884 | 10.957        | 0     | 157.321 | -6.175         | 0.000   |
| TECK-B | Basic Materials        | 39.646 | 22.378        | 9.438 | 287.411 | -5.274         | 0.000   |
| TOU    | Energy                 | 39.240 | 22.513        | 7.788 | 339.461 | -5.965         | 0.000   |
| TRI    | Industrials            | 17.478 | 10.341        | 4.080 | 104.206 | -7.341         | 0.000   |
| TRP    | Energy                 | 20.339 | 15.265        | 4.380 | 209.369 | -7.810         | 0.000   |
| WCN    | Industrials            | 18.080 | 10.496        | 4.903 | 116.213 | -7.190         | 0.000   |
| WN     | Consumer Staples       | 18.282 | 11.418        | 4.356 | 156.119 | -6.645         | 0.000   |
| WPM    | Basic Materials        | 30.841 | 19.185        | 0     | 324.751 | -7.678         | 0.000   |

The total number of observations for each selected stock/index is 1693 (i.e., from 2016-12-31 to 2023-09-30). The data is retrieved from Yahoo Finance as of 2024-03.

Table 2.2: Nasdaq 100 Constituents' Annualized daily Volatility Descriptive Statistics

| Stock | Sector                 | Ave.   | Std Deviation | Min.  | Max.    | ADF Statistics | p value |
|-------|------------------------|--------|---------------|-------|---------|----------------|---------|
| AAPL  | Electronic Technology  | 24.008 | 14.106        | 5.030 | 125.431 | -6.269         | 0.000   |
| ADBE  | Technology Services    | 28.410 | 16.764        | 5.728 | 173.850 | -5.958         | 0.000   |
| ADI   | Electronic Technology  | 27.097 | 14.941        | 7.143 | 187.096 | -5.993         | 0.000   |
| ADP   | Technology Services    | 21.566 | 14.416        | 5.190 | 198.551 | -7.622         | 0.000   |
| ADSK  | Technology Services    | 32.713 | 17.987        | 7.127 | 238.731 | -6.278         | 0.000   |
| AEP   | Utilities              | 20.405 | 14.152        | 4.308 | 179.339 | -5.764         | 0.000   |
| ALGN  | Health Technology      | 40.598 | 22.719        | 8.233 | 222.008 | -5.901         | 0.000   |
| AMAT  | Producer Manufacturing | 34.986 | 18.863        | 7.507 | 168.719 | -5.912         | 0.000   |
| AMD   | Electronic Technology  | 48.737 | 23.742        | 12.12 | 162.544 | -5.852         | 0.000   |
| AMGN  | Health Technology      | 22.780 | 12.534        | 5.989 | 146.847 | -7.273         | 0.000   |
| AMZN  | Retail Trade           | 27.545 | 16.334        | 4.294 | 124.080 | -4.209         | 0.000   |
| ANSS  | Technology Services    | 28.256 | 16.966        | 6.366 | 233.211 | -6.400         | 0.000   |
| ASML  | Electronic Technology  | 24.829 | 15.363        | 5.591 | 193.881 | -4.768         | 0.000   |
| AVGO  | Electronic Technology  | 28.568 | 17.625        | 5.990 | 262.795 | -9.490         | 0.000   |
| AZN   | Health Technology      | 16.818 | 9.7297        | 4.142 | 136.621 | -6.680         | 0.000   |
| BIIB  | Health Technology      | 29.737 | 23.540        | 0     | 566.400 | -10.54         | 0.000   |
| BKNG  | Consumer Services      | 27.857 | 16.777        | 3.923 | 172.684 | -5.968         | 0.000   |
| BKR   | Industrial Services    | 38.754 | 21.963        | 0     | 257.631 | -5.750         | 0.000   |
| CDNS  | Technology Services    | 27.879 | 15.759        | 4.767 | 135.737 | -6.487         | 0.000   |
| CHTR  | Consumer Services      | 28.492 | 15.300        | 6.384 | 140.648 | -6.624         | 0.000   |
| CMCSA | Consumer Services      | 24.557 | 14.394        | 5.218 | 162.324 | -7.449         | 0.000   |
| COST  | Retail Trade           | 19.825 | 11.224        | 4.255 | 101.603 | -6.004         | 0.000   |
| CPRT  | Commercial Services    | 25.357 | 16.185        | 6.086 | 237.170 | -5.896         | 0.000   |
| CSCO  | Technology Services    | 21.209 | 12.920        | 5.812 | 152.014 | -5.989         | 0.000   |
| CSGP  | Technology Services    | 29.461 | 17.544        | 6.263 | 219.099 | -5.683         | 0.000   |
| CSX   | Transportation         | 24.700 | 13.745        | 5.191 | 162.228 | -8.582         | 0.000   |
| CTAS  | Consumer Services      | 22.580 | 14.890        | 5.664 | 197.397 | -6.255         | 0.000   |
| CTSH  | Technology Services    | 22.958 | 14.132        | 4.754 | 170.129 | -6.271         | 0.000   |
| DLTR  | Retail Trade           | 28.176 | 18.322        | 7.562 | 287.460 | -8.643         | 0.000   |
| DXCM  | Health Technology      | 42.269 | 22.537        | 10.22 | 212.254 | -5.155         | 0.000   |
| EA    | Technology Services    | 27.731 | 15.429        | 6.924 | 159.184 | -8.447         | 0.000   |
| EBAY  | Retail Trade           | 27.709 | 14.649        | 5.706 | 153.669 | -5.868         | 0.000   |
| ENPH  | Electronic Technology  | 74.613 | 38.978        | 20.11 | 370.696 | -8.233         | 0.000   |
| EXC   | Utilities              | 22.112 | 14.718        | 5.988 | 239.530 | -5.776         | 0.000   |

Continued on next page

**Table 2.2 – continued from previous page**

| <b>Stock</b> | <b>Sector</b>          | <b>Ave.</b> | <b>Std Deviation</b> | <b>Min.</b> | <b>Max.</b> | <b>ADF Statistics</b> | <b>p value</b> |
|--------------|------------------------|-------------|----------------------|-------------|-------------|-----------------------|----------------|
| FANG         | Energy Minerals        | 44.435      | 29.007               | 7.343       | 342.482     | -4.337                | 0.000          |
| FAST         | Distribution Services  | 25.514      | 14.469               | 6.390       | 147.655     | -7.636                | 0.000          |
| FTNT         | Technology Services    | 34.582      | 20.475               | 6.906       | 212.137     | -5.387                | 0.000          |
| GILD         | Health Technology      | 23.715      | 13.532               | 6.141       | 140.902     | -5.686                | 0.000          |
| GOOG         | Technology Services    | 23.847      | 13.284               | 4.468       | 111.059     | -5.838                | 0.000          |
| GOOGL        | Technology Services    | 24.176      | 13.275               | 4.376       | 110.298     | -6.335                | 0.000          |
| HON          | Electronic Technology  | 19.951      | 13.843               | 4.449       | 235.462     | -5.780                | 0.000          |
| IDXX         | Health Technology      | 30.401      | 16.827               | 7.613       | 146.916     | -7.258                | 0.000          |
| ILMN         | Health Technology      | 34.151      | 20.148               | 8.024       | 236.292     | -6.798                | 0.000          |
| INTC         | Electronic Technology  | 27.630      | 15.984               | 6.189       | 206.865     | -6.357                | 0.000          |
| INTU         | Technology Services    | 28.089      | 16.747               | 4.504       | 163.133     | -5.463                | 0.000          |
| ISRG         | Health Technology      | 27.881      | 16.275               | 5.068       | 176.577     | -6.256                | 0.000          |
| JD           | Retail Trade           | 39.512      | 20.403               | 6.233       | 199.795     | -8.068                | 0.000          |
| KDP          | Consumer Non-Durables  | 19.940      | 14.293               | 3.517       | 245.137     | -7.024                | 0.000          |
| KHC          | Consumer Non-Durables  | 23.363      | 15.239               | 5.307       | 201.006     | -6.660                | 0.000          |
| KLAC         | Electronic Technology  | 33.623      | 18.678               | 6.905       | 206.691     | -5.746                | 0.000          |
| LRCX         | Producer Manufacturing | 35.965      | 19.578               | 7.115       | 207.185     | -6.893                | 0.000          |
| LULU         | Retail Trade           | 32.377      | 17.374               | 7.949       | 208.584     | -6.778                | 0.000          |
| MAR          | Consumer Services      | 29.614      | 24.651               | 4.750       | 420.323     | -5.601                | 0.000          |
| MCHP         | Electronic Technology  | 32.529      | 17.365               | 8.113       | 195.800     | -5.282                | 0.000          |
| MDLZ         | Consumer Non-Durables  | 18.753      | 11.015               | 5.093       | 124.018     | -6.899                | 0.000          |
| MELI         | Retail Trade           | 46.891      | 24.987               | 5.117       | 217.167     | -5.720                | 0.000          |
| META         | Technology Services    | 30.105      | 16.585               | 4.262       | 121.825     | -5.778                | 0.000          |
| MNST         | Consumer Non-Durables  | 23.665      | 13.592               | 6.578       | 169.380     | -8.298                | 0.000          |
| MRVL         | Electronic Technology  | 38.675      | 20.387               | 6.188       | 166.045     | -4.834                | 0.000          |
| MSFT         | Technology Services    | 22.955      | 14.024               | 4.300       | 160.858     | -6.052                | 0.000          |
| MU           | Electronic Technology  | 38.970      | 18.813               | 10.91       | 184.477     | -5.470                | 0.000          |
| NFLX         | Technology Services    | 36.612      | 19.577               | 6.987       | 180.433     | -5.733                | 0.000          |
| NVDA         | Electronic Technology  | 41.560      | 22.816               | 9.698       | 190.280     | -5.141                | 0.000          |
| NXPI         | Electronic Technology  | 30.616      | 21.375               | 1.288       | 313.756     | -4.483                | 0.000          |
| ODFL         | Transportation         | 30.477      | 16.079               | 7.081       | 172.296     | -5.915                | 0.000          |
| ON           | Electronic Technology  | 41.121      | 23.566               | 10.14       | 339.170     | -6.275                | 0.000          |
| ORLY         | Retail Trade           | 25.324      | 14.913               | 6.596       | 160.510     | -7.201                | 0.000          |
| PANW         | Technology Services    | 31.669      | 17.926               | 7.239       | 319.202     | -6.053                | 0.000          |
| PAYX         | Technology Services    | 22.011      | 16.869               | 5.210       | 331.036     | -8.212                | 0.000          |

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**Table 2.2 – continued from previous page**

| <b>Stock</b> | <b>Sector</b>          | <b>Ave.</b> | <b>Std Deviation</b> | <b>Min.</b> | <b>Max.</b> | <b>ADF Statistics</b> | <b>p value</b> |
|--------------|------------------------|-------------|----------------------|-------------|-------------|-----------------------|----------------|
| PCAR         | Producer Manufacturing | 24.815      | 13.457               | 5.494       | 135.080     | -8.326                | 0.000          |
| PEP          | Consumer Non-Durables  | 17.226      | 11.536               | 3.192       | 178.095     | -5.657                | 0.000          |
| PYPL         | Commercial Services    | 32.493      | 17.862               | 7.495       | 140.953     | -4.694                | 0.000          |
| QCOM         | Electronic Technology  | 30.284      | 18.268               | 4.429       | 246.657     | -7.213                | 0.000          |
| REGN         | Health Technology      | 31.287      | 15.458               | 7.472       | 158.610     | -9.973                | 0.000          |
| ROST         | Retail Trade           | 27.944      | 17.383               | 7.610       | 211.277     | -5.581                | 0.000          |
| SBUX         | Consumer Services      | 22.052      | 14.235               | 5.361       | 193.007     | -5.916                | 0.000          |
| SIRI         | Consumer Services      | 28.975      | 22.228               | 4.810       | 437.415     | -4.840                | 0.000          |
| SNPS         | Technology Services    | 26.679      | 15.529               | 4.134       | 159.265     | -5.322                | 0.000          |
| TEAM         | Technology Services    | 46.147      | 24.710               | 8.702       | 221.979     | -4.538                | 0.000          |
| TMUS         | Communications         | 24.115      | 14.037               | 3.408       | 207.683     | -8.192                | 0.000          |
| TSLA         | Consumer Durables      | 49.667      | 28.104               | 9.692       | 266.035     | -5.624                | 0.000          |
| TXN          | Electronic Technology  | 24.689      | 13.445               | 5.168       | 141.996     | -5.835                | 0.000          |
| VRSK         | Technology Services    | 21.168      | 13.334               | 4.946       | 146.117     | -6.522                | 0.000          |
| VRTX         | Health Technology      | 29.428      | 15.166               | 7.307       | 149.696     | -6.872                | 0.000          |
| WBA          | Health Technology      | 27.303      | 16.308               | 4.953       | 174.472     | -6.598                | 0.000          |
| WBD          | Consumer Services      | 43.941      | 28.314               | 7.757       | 596.866     | -5.250                | 0.000          |
| WDAY         | Technology Services    | 35.510      | 17.751               | 9.358       | 197.203     | -5.438                | 0.000          |
| XEL          | Utilities              | 20.826      | 13.463               | 0           | 192.131     | -5.800                | 0.000          |

The total number of observations for each selected stock/index is 1694 (i.e., 6 years and 9 months). The data is retrieved from Yahoo

Finance as of 2024-03.

Table 2.3: Transfer Entropy - TSX 60

|        | ABX   | AEM   | AQN   | ATD   | BCE   | BIP-UN | BMO   | BN    | BNS   | CAE   | CAR-UN | CCL-B | CCO   | CM    | CNQ   | CNR   | CP    | CSU   | CTC-A |
|--------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| ABX    | 0     | 1.46  | 1.604 | 1.72  | 1.637 | 1.648  | 1.6   | 1.676 | 1.603 | 1.576 | 1.702  | 1.719 | 1.62  | 1.594 | 1.632 | 1.743 | 1.706 | 1.658 | 1.616 |
| AEM    | 1.457 | 0     | 1.613 | 1.72  | 1.632 | 1.653  | 1.535 | 1.621 | 1.601 | 1.574 | 1.675  | 1.717 | 1.65  | 1.593 | 1.669 | 1.71  | 1.691 | 1.673 | 1.649 |
| AQN    | 1.702 | 1.712 | 0     | 1.698 | 1.637 | 1.575  | 1.58  | 1.581 | 1.609 | 1.58  | 1.673  | 1.707 | 1.661 | 1.584 | 1.667 | 1.732 | 1.765 | 1.708 | 1.652 |
| ATD    | 1.689 | 1.711 | 1.624 | 0     | 1.67  | 1.65   | 1.58  | 1.656 | 1.617 | 1.556 | 1.664  | 1.686 | 1.661 | 1.619 | 1.66  | 1.706 | 1.708 | 1.696 | 1.667 |
| BCE    | 1.686 | 1.692 | 1.585 | 1.732 | 0     | 1.639  | 1.538 | 1.597 | 1.6   | 1.567 | 1.652  | 1.711 | 1.643 | 1.543 | 1.664 | 1.653 | 1.712 | 1.693 | 1.636 |
| BIP-UN | 1.711 | 1.685 | 1.57  | 1.709 | 1.649 | 0      | 1.548 | 1.59  | 1.563 | 1.554 | 1.66   | 1.734 | 1.628 | 1.591 | 1.694 | 1.719 | 1.711 | 1.678 | 1.679 |
| BMO    | 1.683 | 1.627 | 1.581 | 1.706 | 1.659 | 1.595  | 0     | 1.549 | 1.373 | 1.52  | 1.649  | 1.67  | 1.637 | 1.345 | 1.583 | 1.663 | 1.675 | 1.671 | 1.601 |
| BN     | 1.682 | 1.713 | 1.591 | 1.713 | 1.645 | 1.625  | 1.502 | 0     | 1.542 | 1.478 | 1.63   | 1.701 | 1.612 | 1.541 | 1.664 | 1.675 | 1.661 | 1.651 | 1.634 |
| BNS    | 1.705 | 1.678 | 1.579 | 1.726 | 1.611 | 1.627  | 1.355 | 1.577 | 0     | 1.532 | 1.648  | 1.68  | 1.658 | 1.378 | 1.603 | 1.668 | 1.696 | 1.652 | 1.619 |
| CAE    | 1.727 | 1.698 | 1.612 | 1.711 | 1.634 | 1.643  | 1.549 | 1.574 | 1.589 | 0     | 1.653  | 1.681 | 1.627 | 1.56  | 1.65  | 1.704 | 1.711 | 1.683 | 1.611 |
| CAR-UN | 1.729 | 1.673 | 1.616 | 1.7   | 1.623 | 1.611  | 1.559 | 1.576 | 1.592 | 1.569 | 0      | 1.712 | 1.641 | 1.561 | 1.669 | 1.682 | 1.656 | 1.692 | 1.643 |
| CCL-B  | 1.708 | 1.714 | 1.613 | 1.733 | 1.646 | 1.639  | 1.605 | 1.615 | 1.585 | 1.554 | 1.663  | 0     | 1.645 | 1.606 | 1.712 | 1.697 | 1.721 | 1.694 | 1.633 |
| CCO    | 1.691 | 1.694 | 1.617 | 1.705 | 1.699 | 1.654  | 1.59  | 1.591 | 1.633 | 1.555 | 1.673  | 1.734 | 0     | 1.592 | 1.661 | 1.636 | 1.69  | 1.692 | 1.68  |
| CM     | 1.706 | 1.686 | 1.591 | 1.698 | 1.615 | 1.594  | 1.323 | 1.56  | 1.415 | 1.577 | 1.636  | 1.708 | 1.633 | 0     | 1.619 | 1.684 | 1.662 | 1.675 | 1.638 |
| CNQ    | 1.705 | 1.679 | 1.587 | 1.674 | 1.61  | 1.636  | 1.513 | 1.627 | 1.54  | 1.558 | 1.668  | 1.711 | 1.661 | 1.564 | 0     | 1.72  | 1.668 | 1.67  | 1.653 |
| CNR    | 1.738 | 1.703 | 1.621 | 1.687 | 1.622 | 1.639  | 1.54  | 1.616 | 1.571 | 1.56  | 1.69   | 1.688 | 1.608 | 1.573 | 1.66  | 0     | 1.519 | 1.693 | 1.672 |
| CP     | 1.727 | 1.703 | 1.66  | 1.676 | 1.657 | 1.638  | 1.562 | 1.609 | 1.584 | 1.552 | 1.661  | 1.7   | 1.637 | 1.562 | 1.643 | 1.54  | 0     | 1.684 | 1.662 |
| CSU    | 1.706 | 1.713 | 1.616 | 1.699 | 1.702 | 1.612  | 1.572 | 1.635 | 1.596 | 1.573 | 1.659  | 1.713 | 1.62  | 1.608 | 1.665 | 1.723 | 1.692 | 0     | 1.641 |
| CTC-A  | 1.703 | 1.702 | 1.601 | 1.695 | 1.656 | 1.636  | 1.566 | 1.597 | 1.583 | 1.557 | 1.641  | 1.701 | 1.631 | 1.543 | 1.668 | 1.717 | 1.716 | 1.69  | 0     |

Note: The above table provides an example of transfer entropy among TSX 60 constituents by using the entire dataset. The matrix represents the first 19 stocks in the TSX 60 list.

Table 2.4: Net Transfer Entropy - TSX 60

|        | ABX   | AEM   | AQN   | ATD   | BCE   | BIP-UN | BMO   | BN    | BNS   | CAE   | CAR-UN | CCL-B | CCO   | CM    | CNQ   | CNR   | CP    | CSU   | CTC-A |
|--------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| ABX    | 0     | 0.003 | 0     | 0.031 | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0.012 | 0     | 0     | 0     | 0.005 | 0     | 0     | 0     |
| AEM    | 0     | 0     | 0     | 0.009 | 0     | 0      | 0     | 0     | 0     | 0     | 0.002  | 0.003 | 0     | 0     | 0     | 0.007 | 0     | 0     | 0     |
| AQN    | 0.098 | 0.099 | 0     | 0.074 | 0.052 | 0.006  | 0     | 0     | 0.031 | 0     | 0.057  | 0.093 | 0.044 | 0     | 0.08  | 0.111 | 0.105 | 0.092 | 0.051 |
| ATD    | 0     | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0.019 | 0.031 | 0     | 0     |
| BCE    | 0.049 | 0.06  | 0     | 0.062 | 0     | 0      | 0     | 0     | 0     | 0     | 0.03   | 0.065 | 0     | 0     | 0.054 | 0.031 | 0.055 | 0     | 0     |
| BIP-UN | 0.062 | 0.032 | 0     | 0.058 | 0.01  | 0      | 0     | 0     | 0     | 0     | 0.049  | 0.095 | 0     | 0     | 0.058 | 0.081 | 0.072 | 0.065 | 0.043 |
| BMO    | 0.083 | 0.093 | 0     | 0.127 | 0.121 | 0.047  | 0     | 0.047 | 0.018 | 0     | 0.091  | 0.065 | 0.047 | 0.023 | 0.069 | 0.123 | 0.113 | 0.1   | 0.034 |
| BN     | 0.005 | 0.093 | 0.011 | 0.056 | 0.048 | 0.035  | 0     | 0     | 0     | 0     | 0.054  | 0.086 | 0.021 | 0     | 0.036 | 0.059 | 0.053 | 0.016 | 0.037 |
| BNS    | 0.102 | 0.077 | 0     | 0.108 | 0.011 | 0.064  | 0     | 0.035 | 0     | 0     | 0.056  | 0.095 | 0.025 | 0     | 0.063 | 0.097 | 0.112 | 0.056 | 0.037 |
| CAE    | 0.151 | 0.123 | 0.033 | 0.155 | 0.067 | 0.089  | 0.029 | 0.096 | 0.056 | 0     | 0.085  | 0.127 | 0.072 | 0     | 0.093 | 0.144 | 0.159 | 0.11  | 0.054 |
| CAR-UN | 0.027 | 0     | 0     | 0.036 | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0.049 | 0     | 0     | 0.001 | 0     | 0     | 0.033 | 0.002 |
| CCL-B  | 0     | 0     | 0     | 0.047 | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0.001 | 0.009 | 0.02  | 0     | 0     |
| CCO    | 0.071 | 0.044 | 0     | 0.044 | 0.056 | 0.026  | 0     | 0     | 0     | 0     | 0.032  | 0.089 | 0     | 0     | 0     | 0.028 | 0.053 | 0.071 | 0.049 |
| CM     | 0.112 | 0.093 | 0.006 | 0.078 | 0.073 | 0.003  | 0     | 0.02  | 0.037 | 0.017 | 0.075  | 0.102 | 0.042 | 0     | 0.055 | 0.111 | 0.1   | 0.067 | 0.096 |
| CNQ    | 0.073 | 0.01  | 0     | 0.014 | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0.06  | 0.026 | 0.005 | 0     |
| CNR    | 0     | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0.009  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| CP     | 0.021 | 0.012 | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0.006  | 0     | 0     | 0     | 0     | 0.021 | 0     | 0     | 0     |
| CSU    | 0.048 | 0.04  | 0     | 0.002 | 0.008 | 0      | 0     | 0     | 0     | 0     | 0      | 0.019 | 0     | 0     | 0     | 0.03  | 0.008 | 0     | 0     |
| CTC-A  | 0.087 | 0.053 | 0     | 0.028 | 0.02  | 0      | 0     | 0     | 0     | 0     | 0      | 0.068 | 0     | 0     | 0.015 | 0.045 | 0.054 | 0.05  | 0     |

Note: The above table provides an example of net transfer entropy among TSX 60 constituents by using the entire dataset. The matrix represents the first 19 stocks in the TSX 60 list. For example: The net transfer entropy from ABX to AEM is 0.003, which is calculated as  $1.46(T_{ABX \rightarrow AEM}) - 1.457(T_{AEM \rightarrow ABX})$  in Table 2.3.

Table 2.5: Transfer Entropy - Nasdaq 100

|      | AAPL  | ADBE  | ADI   | ADP   | ADSK  | AEP   | ALGN  | AMAT  | AMD   | AMGN  | AMZN  | ANSS  | ASML  | AVGO  | AZN   | BIIB  | BKNG  | BKR   | CDNS  |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| AAPL | 0     | 1.434 | 1.584 | 1.531 | 1.567 | 1.589 | 1.564 | 1.555 | 1.591 | 1.637 | 1.328 | 1.48  | 1.398 | 1.609 | 1.649 | 1.632 | 1.435 | 1.624 | 1.495 |
| ADBE | 1.452 | 0     | 1.623 | 1.532 | 1.458 | 1.59  | 1.546 | 1.529 | 1.575 | 1.57  | 1.312 | 1.419 | 1.415 | 1.551 | 1.728 | 1.67  | 1.417 | 1.64  | 1.467 |
| ADI  | 1.491 | 1.531 | 0     | 1.553 | 1.585 | 1.618 | 1.586 | 1.477 | 1.603 | 1.61  | 1.449 | 1.508 | 1.402 | 1.552 | 1.658 | 1.674 | 1.499 | 1.623 | 1.513 |
| ADP  | 1.464 | 1.506 | 1.611 | 0     | 1.577 | 1.583 | 1.555 | 1.597 | 1.605 | 1.596 | 1.428 | 1.508 | 1.463 | 1.631 | 1.662 | 1.641 | 1.469 | 1.628 | 1.528 |
| ADSK | 1.497 | 1.397 | 1.56  | 1.586 | 0     | 1.628 | 1.572 | 1.576 | 1.588 | 1.609 | 1.404 | 1.478 | 1.457 | 1.605 | 1.689 | 1.708 | 1.449 | 1.648 | 1.474 |
| AEP  | 1.561 | 1.534 | 1.657 | 1.586 | 1.643 | 0     | 1.587 | 1.649 | 1.659 | 1.599 | 1.463 | 1.57  | 1.545 | 1.7   | 1.682 | 1.658 | 1.508 | 1.636 | 1.571 |
| ALGN | 1.519 | 1.47  | 1.608 | 1.586 | 1.584 | 1.595 | 0     | 1.551 | 1.605 | 1.624 | 1.42  | 1.557 | 1.479 | 1.62  | 1.66  | 1.674 | 1.475 | 1.594 | 1.534 |
| AMAT | 1.483 | 1.509 | 1.517 | 1.593 | 1.576 | 1.64  | 1.584 | 0     | 1.606 | 1.654 | 1.401 | 1.551 | 1.287 | 1.539 | 1.671 | 1.646 | 1.472 | 1.645 | 1.519 |
| AMD  | 1.531 | 1.497 | 1.646 | 1.644 | 1.599 | 1.647 | 1.598 | 1.595 | 0     | 1.613 | 1.433 | 1.555 | 1.523 | 1.643 | 1.701 | 1.692 | 1.524 | 1.638 | 1.603 |
| AMGN | 1.543 | 1.536 | 1.657 | 1.582 | 1.608 | 1.611 | 1.624 | 1.623 | 1.653 | 0     | 1.459 | 1.592 | 1.581 | 1.661 | 1.659 | 1.581 | 1.494 | 1.639 | 1.616 |
| AMZN | 1.385 | 1.389 | 1.579 | 1.536 | 1.556 | 1.59  | 1.524 | 1.559 | 1.569 | 1.617 | 0     | 1.515 | 1.38  | 1.608 | 1.676 | 1.653 | 1.418 | 1.587 | 1.51  |
| ANSS | 1.425 | 1.398 | 1.545 | 1.527 | 1.524 | 1.573 | 1.547 | 1.549 | 1.545 | 1.578 | 1.394 | 0     | 1.462 | 1.608 | 1.67  | 1.663 | 1.419 | 1.613 | 1.412 |
| ASML | 1.41  | 1.419 | 1.533 | 1.569 | 1.557 | 1.584 | 1.53  | 1.379 | 1.559 | 1.65  | 1.368 | 1.456 | 0     | 1.55  | 1.63  | 1.649 | 1.458 | 1.608 | 1.477 |
| AVGO | 1.498 | 1.5   | 1.49  | 1.606 | 1.573 | 1.654 | 1.595 | 1.492 | 1.597 | 1.638 | 1.458 | 1.523 | 1.434 | 0     | 1.669 | 1.691 | 1.474 | 1.674 | 1.527 |
| AZN  | 1.557 | 1.591 | 1.653 | 1.605 | 1.652 | 1.613 | 1.602 | 1.625 | 1.648 | 1.599 | 1.476 | 1.643 | 1.523 | 1.662 | 0     | 1.669 | 1.461 | 1.638 | 1.579 |
| BIIB | 1.525 | 1.575 | 1.65  | 1.64  | 1.657 | 1.625 | 1.624 | 1.645 | 1.65  | 1.57  | 1.496 | 1.584 | 1.544 | 1.668 | 1.678 | 0     | 1.516 | 1.644 | 1.601 |
| BKNG | 1.476 | 1.47  | 1.586 | 1.562 | 1.611 | 1.591 | 1.583 | 1.575 | 1.643 | 1.616 | 1.361 | 1.536 | 1.477 | 1.626 | 1.642 | 1.653 | 0     | 1.63  | 1.551 |
| BKR  | 1.524 | 1.58  | 1.646 | 1.641 | 1.615 | 1.613 | 1.62  | 1.618 | 1.651 | 1.659 | 1.493 | 1.6   | 1.533 | 1.714 | 1.703 | 1.658 | 1.53  | 0     | 1.606 |
| CDNS | 1.464 | 1.395 | 1.526 | 1.547 | 1.515 | 1.615 | 1.539 | 1.509 | 1.629 | 1.609 | 1.385 | 1.37  | 1.401 | 1.577 | 1.669 | 1.665 | 1.438 | 1.623 | 0     |

Note: The above table provides an example of transfer entropy among Nasdaq 100 constituents by using the entire dataset. The matrix represents the first 19 stocks in the Nasdaq 100 list.

Table 2.6: Net Transfer Entropy - Nasdaq 100

|      | AAPL  | ADBE  | ADI   | ADP   | ADSK  | AEP   | ALGN  | AMAT  | AMD   | AMGN  | AMZN | ANSS  | ASML  | AVGO  | AZN   | BIIB  | BKNG  | BKR   | CDNS  |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| AAPL | 0     | 0     | 0.093 | 0.067 | 0.07  | 0.028 | 0.045 | 0.072 | 0.061 | 0.094 | 0    | 0.055 | 0     | 0.111 | 0.092 | 0.107 | 0     | 0.1   | 0.031 |
| ADBE | 0.018 | 0     | 0.092 | 0.025 | 0.061 | 0.056 | 0.077 | 0.019 | 0.078 | 0.034 | 0    | 0.021 | 0     | 0.05  | 0.137 | 0.095 | 0     | 0.06  | 0.072 |
| ADI  | 0     | 0     | 0     | 0     | 0.025 | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0.063 | 0.005 | 0.025 | 0     | 0     | 0     |
| ADP  | 0     | 0     | 0.058 | 0     | 0     | 0     | 0     | 0.004 | 0     | 0.013 | 0    | 0     | 0     | 0.025 | 0.057 | 0.002 | 0     | 0     | 0     |
| ADSK | 0     | 0     | 0     | 0.009 | 0     | 0     | 0     | 0     | 0     | 0.001 | 0    | 0     | 0     | 0.032 | 0.037 | 0.05  | 0     | 0.033 | 0     |
| AEP  | 0     | 0     | 0.039 | 0.003 | 0.015 | 0     | 0     | 0.009 | 0.011 | 0     | 0    | 0     | 0     | 0.046 | 0.068 | 0.033 | 0     | 0.022 | 0     |
| ALGN | 0     | 0     | 0.022 | 0.03  | 0.011 | 0.008 | 0     | 0     | 0.007 | 0     | 0    | 0.01  | 0     | 0.025 | 0.059 | 0.05  | 0     | 0     | 0     |
| AMAT | 0     | 0     | 0.041 | 0     | 0     | 0     | 0.033 | 0     | 0.011 | 0.031 | 0    | 0.001 | 0     | 0.048 | 0.046 | 0.001 | 0     | 0.028 | 0.009 |
| AMD  | 0     | 0     | 0.042 | 0.039 | 0.01  | 0     | 0     | 0     | 0     | 0     | 0    | 0.01  | 0     | 0.046 | 0.053 | 0.042 | 0     | 0     | 0     |
| AMGN | 0     | 0     | 0.048 | 0     | 0     | 0.012 | 0     | 0     | 0.04  | 0     | 0    | 0.014 | 0     | 0.023 | 0.06  | 0.011 | 0     | 0     | 0.007 |
| AMZN | 0.057 | 0.078 | 0.13  | 0.108 | 0.151 | 0.126 | 0.105 | 0.158 | 0.136 | 0.159 | 0    | 0.121 | 0.012 | 0.15  | 0.2   | 0.157 | 0.057 | 0.094 | 0.125 |
| ANSS | 0     | 0     | 0.037 | 0.019 | 0.046 | 0.002 | 0     | 0     | 0     | 0     | 0    | 0     | 0.006 | 0.086 | 0.027 | 0.079 | 0     | 0.013 | 0.042 |
| ASML | 0.013 | 0.004 | 0.131 | 0.106 | 0.1   | 0.038 | 0.052 | 0.092 | 0.036 | 0.07  | 0    | 0     | 0     | 0.116 | 0.107 | 0.105 | 0     | 0.075 | 0.076 |
| AVGO | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0     | 0.007 | 0.023 | 0     | 0     | 0     |
| AZN  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| BIIB | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0     | 0.009 | 0     | 0     | 0     | 0     |
| BKNG | 0.041 | 0.052 | 0.087 | 0.093 | 0.163 | 0.082 | 0.108 | 0.103 | 0.119 | 0.122 | 0    | 0.117 | 0.019 | 0.152 | 0.181 | 0.168 | 0     | 0.1   | 0.113 |
| BKR  | 0     | 0     | 0.023 | 0.012 | 0     | 0     | 0.026 | 0     | 0.012 | 0.02  | 0    | 0     | 0     | 0.04  | 0.065 | 0.014 | 0     | 0     | 0     |
| CDNS | 0     | 0     | 0.013 | 0.019 | 0.041 | 0.044 | 0.004 | 0     | 0.027 | 0     | 0    | 0     | 0     | 0.05  | 0.09  | 0.063 | 0     | 0.017 | 0     |

Note: The above table provides an example of net transfer entropy among Nasdaq 100 constituents by using the entire dataset. The matrix represents the first 19 stocks in the Nasdaq 100 list.

Table 2.7: Diebold-Yilmaz Spillover Index - TSX 60

|        | ABX   | AEM   | AQN   | ATD   | BCE   | BIP.UN | BMO   | BN    | BNS   | CAE   | CAR.UN | CCL.B | CCO   | CM    | CNQ   | CNR   | CP    | CSU   | CTC.A |
|--------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| ABX    | 0.345 | 0.133 | 0.014 | 0.013 | 0.027 | 0.011  | 0.016 | 0.01  | 0.019 | 0.017 | 0.027  | 0.019 | 0.017 | 0.019 | 0.028 | 0.015 | 0.015 | 0.025 | 0.022 |
| AEM    | 0.147 | 0.309 | 0.014 | 0.024 | 0.029 | 0.014  | 0.022 | 0.02  | 0.021 | 0.029 | 0.035  | 0.023 | 0.035 | 0.021 | 0.026 | 0.025 | 0.024 | 0.02  | 0.022 |
| AQN    | 0.011 | 0.006 | 0.417 | 0.012 | 0.024 | 0.042  | 0.015 | 0.032 | 0.02  | 0.013 | 0.024  | 0.025 | 0.001 | 0.015 | 0.009 | 0.01  | 0.011 | 0.01  | 0.02  |
| ATD    | 0.005 | 0.008 | 0.007 | 0.676 | 0.008 | 0.006  | 0.007 | 0.013 | 0.005 | 0.012 | 0.012  | 0.004 | 0.028 | 0.006 | 0.009 | 0.011 | 0.013 | 0.008 | 0.005 |
| BCE    | 0.025 | 0.022 | 0.04  | 0.022 | 0.28  | 0.039  | 0.027 | 0.027 | 0.034 | 0.022 | 0.026  | 0.029 | 0.016 | 0.028 | 0.03  | 0.036 | 0.033 | 0.023 | 0.027 |
| BIP.UN | 0.008 | 0.007 | 0.054 | 0.009 | 0.026 | 0.376  | 0.021 | 0.03  | 0.023 | 0.015 | 0.025  | 0.021 | 0.004 | 0.023 | 0.014 | 0.013 | 0.017 | 0.015 | 0.017 |
| BMO    | 0.026 | 0.032 | 0.033 | 0.034 | 0.033 | 0.045  | 0.218 | 0.058 | 0.097 | 0.052 | 0.05   | 0.028 | 0.018 | 0.111 | 0.045 | 0.039 | 0.038 | 0.028 | 0.053 |
| BN     | 0.011 | 0.02  | 0.052 | 0.027 | 0.022 | 0.042  | 0.04  | 0.3   | 0.033 | 0.045 | 0.041  | 0.025 | 0.019 | 0.033 | 0.017 | 0.03  | 0.035 | 0.019 | 0.041 |
| BNS    | 0.027 | 0.033 | 0.035 | 0.022 | 0.043 | 0.041  | 0.094 | 0.049 | 0.226 | 0.041 | 0.042  | 0.026 | 0.015 | 0.096 | 0.037 | 0.039 | 0.033 | 0.022 | 0.043 |
| CAE    | 0.009 | 0.019 | 0.02  | 0.033 | 0.013 | 0.019  | 0.032 | 0.041 | 0.021 | 0.348 | 0.043  | 0.027 | 0.025 | 0.021 | 0.019 | 0.016 | 0.017 | 0.028 | 0.042 |
| CAR.UN | 0.018 | 0.016 | 0.022 | 0.02  | 0.015 | 0.021  | 0.017 | 0.026 | 0.014 | 0.021 | 0.368  | 0.017 | 0.01  | 0.015 | 0.016 | 0.007 | 0.01  | 0.017 | 0.024 |
| CCL.B  | 0.008 | 0.007 | 0.02  | 0.004 | 0.014 | 0.012  | 0.013 | 0.013 | 0.01  | 0.02  | 0.017  | 0.501 | 0.003 | 0.01  | 0.008 | 0.014 | 0.015 | 0.017 | 0.021 |
| CCO    | 0.006 | 0.012 | 0.001 | 0.011 | 0.005 | 0.005  | 0.004 | 0.006 | 0.003 | 0.015 | 0.005  | 0.001 | 0.646 | 0.004 | 0.012 | 0.014 | 0.016 | 0.016 | 0.003 |
| CM     | 0.023 | 0.027 | 0.027 | 0.017 | 0.028 | 0.035  | 0.09  | 0.048 | 0.084 | 0.032 | 0.035  | 0.021 | 0.015 | 0.231 | 0.028 | 0.036 | 0.034 | 0.02  | 0.035 |
| CNQ    | 0.03  | 0.026 | 0.016 | 0.027 | 0.026 | 0.023  | 0.035 | 0.019 | 0.029 | 0.024 | 0.026  | 0.024 | 0.034 | 0.03  | 0.264 | 0.022 | 0.025 | 0.02  | 0.026 |
| CNR    | 0.016 | 0.021 | 0.012 | 0.021 | 0.024 | 0.013  | 0.021 | 0.024 | 0.021 | 0.017 | 0.011  | 0.017 | 0.028 | 0.023 | 0.018 | 0.357 | 0.125 | 0.017 | 0.014 |
| CP     | 0.011 | 0.016 | 0.015 | 0.023 | 0.018 | 0.017  | 0.02  | 0.025 | 0.016 | 0.018 | 0.015  | 0.016 | 0.03  | 0.021 | 0.015 | 0.119 | 0.384 | 0.018 | 0.013 |
| CSU    | 0.009 | 0.007 | 0.012 | 0.01  | 0.008 | 0.013  | 0.007 | 0.013 | 0.007 | 0.019 | 0.015  | 0.016 | 0.018 | 0.007 | 0.008 | 0.007 | 0.012 | 0.54  | 0.012 |
| CTC.A  | 0.022 | 0.02  | 0.03  | 0.014 | 0.021 | 0.027  | 0.031 | 0.038 | 0.029 | 0.037 | 0.034  | 0.043 | 0.01  | 0.026 | 0.02  | 0.016 | 0.02  | 0.029 | 0.323 |

Table summarizes the first 19 stocks' Diebold-Yilmaz spillover index. The VAR model uses the lag 1 and the variance decomposition uses 5 steps. The value of diagonal elements are greater than the rest elements' value, indicating that shocks to the stock itself explain most of the variance in the forecast error.

Table 2.8: Net Diebold-Yilmaz Spillover Index - TSX 60

|        | ABX   | AEM   | AQN   | ATD   | BCE   | BIP.UN | BN    | BNS   | CAE   | CAR.UN | CCL.B | CCO   | CM    | CNQ   | CNR   | CP    | CSU   | CTC.A | CVE   |
|--------|-------|-------|-------|-------|-------|--------|-------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| ABX    | 0     | 0     | 0.003 | 0.008 | 0.002 | 0.003  | 0     | 0     | 0.008 | 0.009  | 0.01  | 0.01  | 0     | 0     | 0     | 0.004 | 0.016 | 0.001 | 0.001 |
| AEM    | 0.015 | 0     | 0.008 | 0.016 | 0.007 | 0.007  | 0     | 0     | 0.009 | 0.019  | 0.016 | 0.023 | 0     | 0     | 0.004 | 0.008 | 0.012 | 0.002 | 0.003 |
| AQN    | 0     | 0     | 0     | 0.005 | 0     | 0      | 0     | 0     | 0     | 0.002  | 0.004 | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| ATD    | 0     | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0      | 0     | 0.017 | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| BCE    | 0     | 0     | 0.017 | 0.014 | 0     | 0.012  | 0.006 | 0     | 0.01  | 0.01   | 0.015 | 0.012 | 0     | 0.004 | 0.012 | 0.014 | 0.014 | 0.006 | 0.005 |
| BIP.UN | 0     | 0     | 0.012 | 0.003 | 0     | 0      | 0     | 0     | 0     | 0.004  | 0.009 | 0     | 0     | 0     | 0     | 0     | 0.001 | 0     | 0     |
| BMO    | 0.01  | 0.01  | 0.018 | 0.027 | 0.006 | 0.024  | 0.018 | 0.002 | 0.02  | 0.033  | 0.016 | 0.015 | 0.021 | 0.01  | 0.018 | 0.017 | 0.021 | 0.022 | 0.009 |
| BN     | 0     | 0.001 | 0.02  | 0.013 | 0     | 0.012  | 0     | 0     | 0.004 | 0.015  | 0.011 | 0.013 | 0     | 0     | 0.006 | 0.01  | 0.006 | 0.003 | 0     |
| BNS    | 0.008 | 0.012 | 0.015 | 0.017 | 0.009 | 0.018  | 0.016 | 0     | 0.02  | 0.028  | 0.016 | 0.012 | 0.012 | 0.008 | 0.018 | 0.016 | 0.014 | 0.013 | 0.012 |
| CAE    | 0     | 0     | 0.007 | 0.021 | 0     | 0.003  | 0     | 0     | 0     | 0.022  | 0.006 | 0.01  | 0     | 0     | 0     | 0     | 0.009 | 0.005 | 0     |
| CAR.UN | 0     | 0     | 0     | 0.008 | 0     | 0      | 0     | 0     | 0     | 0      | 0     | 0.005 | 0     | 0     | 0     | 0     | 0.002 | 0     | 0     |
| CCL.B  | 0     | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0      | 0     | 0.002 | 0     | 0     | 0     | 0     | 0.001 | 0     | 0     |
| CCO    | 0     | 0     | 0     | 0     | 0     | 0.001  | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| CM     | 0.004 | 0.006 | 0.012 | 0.012 | 0     | 0.011  | 0.015 | 0     | 0.01  | 0.02   | 0.011 | 0.011 | 0     | 0     | 0.013 | 0.013 | 0.013 | 0.009 | 0.003 |
| CNQ    | 0.002 | 0     | 0.007 | 0.018 | 0     | 0.008  | 0.002 | 0     | 0.005 | 0.01   | 0.016 | 0.022 | 0.001 | 0     | 0.004 | 0.011 | 0.012 | 0.007 | 0.032 |
| CNR    | 0.001 | 0     | 0.002 | 0.011 | 0     | 0      | 0     | 0     | 0.001 | 0.004  | 0.003 | 0.014 | 0     | 0     | 0     | 0.006 | 0.011 | 0     | 0.002 |
| CP     | 0     | 0     | 0.004 | 0.011 | 0     | 0      | 0     | 0     | 0.001 | 0.005  | 0.001 | 0.014 | 0     | 0     | 0     | 0     | 0.006 | 0     | 0     |
| CSU    | 0     | 0     | 0.002 | 0.002 | 0     | 0      | 0     | 0     | 0     | 0      | 0     | 0.002 | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| CTC.A  | 0     | 0     | 0.01  | 0.01  | 0     | 0.01   | 0     | 0     | 0     | 0.01   | 0.022 | 0.007 | 0     | 0     | 0.002 | 0.007 | 0.017 | 0     | 0     |

Table summarizes the first 19 stocks' net Diebold-Yilmaz spillover index. The VAR model uses the lag 1 and the variance decomposition uses 5 steps.

Table 2.9: Diebold-Yilmaz Spillover Index - Nasdaq 100

|      | AAPL  | ADBE  | ADI   | ADP   | ADSK  | AEP   | ALGN  | AMAT  | AMD   | AMGN  | AMZN  | ANSS  | ASML  | AVGO  | AZN   | BIIB  | BKNG  | BKR   | CDNS  |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| AAPL | 0.084 | 0.023 | 0.013 | 0.019 | 0.012 | 0.009 | 0.012 | 0.013 | 0.013 | 0.01  | 0.03  | 0.016 | 0.021 | 0.014 | 0.008 | 0.004 | 0.014 | 0.005 | 0.015 |
| ADBE | 0.019 | 0.07  | 0.012 | 0.014 | 0.021 | 0.009 | 0.012 | 0.012 | 0.012 | 0.011 | 0.025 | 0.021 | 0.023 | 0.015 | 0.006 | 0.005 | 0.012 | 0.004 | 0.021 |
| ADI  | 0.015 | 0.015 | 0.085 | 0.013 | 0.012 | 0.009 | 0.008 | 0.024 | 0.01  | 0.009 | 0.014 | 0.014 | 0.026 | 0.02  | 0.006 | 0.004 | 0.012 | 0.006 | 0.015 |
| ADP  | 0.018 | 0.017 | 0.012 | 0.083 | 0.012 | 0.013 | 0.012 | 0.01  | 0.006 | 0.013 | 0.014 | 0.017 | 0.015 | 0.012 | 0.008 | 0.006 | 0.012 | 0.009 | 0.016 |
| ADSK | 0.014 | 0.028 | 0.017 | 0.014 | 0.082 | 0.007 | 0.012 | 0.013 | 0.013 | 0.01  | 0.019 | 0.021 | 0.019 | 0.016 | 0.006 | 0.004 | 0.013 | 0.004 | 0.02  |
| AEP  | 0.011 | 0.014 | 0.011 | 0.019 | 0.01  | 0.105 | 0.008 | 0.01  | 0.006 | 0.015 | 0.012 | 0.014 | 0.015 | 0.009 | 0.011 | 0.005 | 0.013 | 0.01  | 0.012 |
| ALGN | 0.015 | 0.02  | 0.012 | 0.016 | 0.015 | 0.008 | 0.098 | 0.013 | 0.009 | 0.01  | 0.019 | 0.015 | 0.021 | 0.013 | 0.008 | 0.007 | 0.011 | 0.008 | 0.017 |
| AMAT | 0.017 | 0.016 | 0.022 | 0.012 | 0.011 | 0.006 | 0.011 | 0.08  | 0.011 | 0.007 | 0.017 | 0.014 | 0.035 | 0.018 | 0.006 | 0.005 | 0.012 | 0.006 | 0.016 |
| AMD  | 0.018 | 0.021 | 0.014 | 0.009 | 0.017 | 0.006 | 0.01  | 0.016 | 0.143 | 0.008 | 0.025 | 0.016 | 0.018 | 0.018 | 0.008 | 0.003 | 0.009 | 0.002 | 0.012 |
| AMGN | 0.015 | 0.021 | 0.014 | 0.022 | 0.012 | 0.014 | 0.01  | 0.008 | 0.006 | 0.121 | 0.011 | 0.014 | 0.012 | 0.012 | 0.012 | 0.018 | 0.011 | 0.007 | 0.014 |
| AMZN | 0.027 | 0.028 | 0.01  | 0.011 | 0.014 | 0.008 | 0.012 | 0.013 | 0.015 | 0.007 | 0.091 | 0.018 | 0.023 | 0.009 | 0.007 | 0.003 | 0.015 | 0.004 | 0.016 |
| ANSS | 0.017 | 0.025 | 0.014 | 0.017 | 0.018 | 0.01  | 0.012 | 0.012 | 0.011 | 0.009 | 0.018 | 0.078 | 0.02  | 0.014 | 0.006 | 0.004 | 0.013 | 0.006 | 0.023 |
| ASML | 0.019 | 0.022 | 0.02  | 0.011 | 0.014 | 0.008 | 0.012 | 0.027 | 0.01  | 0.007 | 0.022 | 0.016 | 0.073 | 0.015 | 0.007 | 0.005 | 0.013 | 0.006 | 0.018 |
| AVGO | 0.017 | 0.022 | 0.023 | 0.014 | 0.014 | 0.007 | 0.011 | 0.02  | 0.012 | 0.01  | 0.014 | 0.017 | 0.023 | 0.09  | 0.006 | 0.005 | 0.01  | 0.004 | 0.016 |
| AZN  | 0.013 | 0.013 | 0.014 | 0.018 | 0.009 | 0.017 | 0.012 | 0.008 | 0.008 | 0.016 | 0.016 | 0.011 | 0.017 | 0.01  | 0.15  | 0.006 | 0.014 | 0.007 | 0.017 |
| BIIB | 0.013 | 0.014 | 0.009 | 0.016 | 0.009 | 0.007 | 0.014 | 0.01  | 0.004 | 0.03  | 0.008 | 0.01  | 0.015 | 0.009 | 0.008 | 0.206 | 0.012 | 0.011 | 0.016 |
| BKNG | 0.019 | 0.019 | 0.014 | 0.015 | 0.011 | 0.013 | 0.01  | 0.012 | 0.007 | 0.01  | 0.023 | 0.018 | 0.018 | 0.01  | 0.009 | 0.004 | 0.087 | 0.009 | 0.015 |
| BKR  | 0.013 | 0.012 | 0.011 | 0.019 | 0.01  | 0.013 | 0.013 | 0.009 | 0.005 | 0.013 | 0.011 | 0.012 | 0.017 | 0.009 | 0.01  | 0.008 | 0.016 | 0.156 | 0.012 |
| CDNS | 0.016 | 0.026 | 0.016 | 0.015 | 0.018 | 0.008 | 0.012 | 0.014 | 0.007 | 0.009 | 0.016 | 0.024 | 0.022 | 0.013 | 0.007 | 0.005 | 0.012 | 0.005 | 0.079 |

Table summarizes the first 19 stocks' Diebold-Yilmaz spillover index. The VAR model uses the lag 1 and the variance decomposition uses 5 steps.

Table 2.10: Net Diebold-Yilmaz Spillover Index - Nasdaq 100

|      | AAPL  | ADBE  | ADI   | ADP   | ADSK  | AEP   | ALGN  | AMAT  | AMD   | AMGN  | AMZN  | ANSS  | ASML | AVGO  | AZN   | BIIB  | BKNG  | BKR   | CDNS  |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|-------|-------|-------|-------|-------|-------|
| AAPL | 0     | 0     | 0.003 | 0     | 0.002 | 0.003 | 0.003 | 0.004 | 0.005 | 0.005 | 0     | 0.001 | 0    | 0.004 | 0.005 | 0.009 | 0.004 | 0.008 | 0.001 |
| ADBE | 0.004 | 0     | 0.003 | 0.004 | 0.008 | 0.005 | 0.009 | 0.005 | 0.009 | 0.01  | 0.003 | 0.004 | 0    | 0.007 | 0.008 | 0.01  | 0.007 | 0.008 | 0.004 |
| ADI  | 0     | 0     | 0     | 0     | 0.005 | 0.003 | 0.004 | 0     | 0.004 | 0.004 | 0     | 0     | 0    | 0.004 | 0.008 | 0.004 | 0.002 | 0.005 | 0.001 |
| ADP  | 0     | 0     | 0.001 | 0     | 0.002 | 0.006 | 0.005 | 0.002 | 0.003 | 0.009 | 0     | 0.001 | 0    | 0.001 | 0.01  | 0.01  | 0.003 | 0.01  | 0     |
| ADSK | 0     | 0     | 0     | 0     | 0     | 0.002 | 0.003 | 0     | 0.004 | 0.002 | 0     | 0     | 0    | 0     | 0.003 | 0.005 | 0     | 0.006 | 0     |
| AEP  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0.006 | 0.003 | 0     | 0.003 | 0     |
| ALGN | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.001 | 0     | 0     | 0     | 0    | 0     | 0.003 | 0.007 | 0     | 0.004 | 0     |
| AMAT | 0     | 0     | 0.001 | 0     | 0.001 | 0.004 | 0.002 | 0     | 0.005 | 0.001 | 0     | 0     | 0    | 0.002 | 0.003 | 0.005 | 0     | 0.003 | 0     |
| AMD  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0.001 | 0     | 0.003 | 0     |
| AMGN | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.002 | 0     | 0     | 0     | 0    | 0     | 0.004 | 0.012 | 0     | 0.006 | 0     |
| AMZN | 0.003 | 0     | 0.005 | 0.003 | 0.005 | 0.004 | 0.006 | 0.004 | 0.01  | 0.005 | 0     | 0     | 0    | 0.005 | 0.009 | 0.005 | 0.008 | 0.007 | 0.001 |
| ANSS | 0     | 0     | 0     | 0     | 0.003 | 0.004 | 0.003 | 0.002 | 0.005 | 0.005 | 0     | 0     | 0    | 0.004 | 0.005 | 0.006 | 0.005 | 0.006 | 0.001 |
| ASML | 0.002 | 0.001 | 0.006 | 0.004 | 0.005 | 0.007 | 0.009 | 0.008 | 0.008 | 0.005 | 0.002 | 0.003 | 0    | 0.008 | 0.01  | 0.01  | 0.005 | 0.011 | 0.004 |
| AVGO | 0     | 0     | 0     | 0     | 0.001 | 0.003 | 0.002 | 0     | 0.006 | 0.002 | 0     | 0     | 0    | 0     | 0.004 | 0.005 | 0     | 0.005 | 0     |
| AZN  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0.001 | 0     | 0.003 | 0     |
| BIIB | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0     | 0     | 0     | 0     | 0     |
| BKNG | 0     | 0     | 0     | 0     | 0.001 | 0     | 0.001 | 0     | 0.001 | 0.001 | 0     | 0     | 0    | 0     | 0.005 | 0.008 | 0     | 0.007 | 0     |
| BKR  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 0.004 | 0     | 0     | 0     | 0     |
| CDNS | 0     | 0     | 0     | 0     | 0.002 | 0.003 | 0.005 | 0.002 | 0.005 | 0.005 | 0     | 0     | 0    | 0.003 | 0.01  | 0.01  | 0.002 | 0.007 | 0     |

Table summarizes the first 19 stocks' net Diebold-Yilmaz spillover index. The VAR model uses the lag 1 and the variance decomposition uses 5 steps.

Table 2.11: Average Net Transfer Entropy (NTE) by Sectors - TSX 60

| <i>Sector</i>          | <i>Average NTE Sent</i> | <i>Rank</i> | <i>Average NTE Received</i> | <i>Rank</i> |
|------------------------|-------------------------|-------------|-----------------------------|-------------|
| Basic Materials        | 0.018                   | 10          | 0.053                       | 1           |
| Information Technology | 0.027                   | 7           | 0.051                       | 2           |
| Industrials            | 0.056                   | 1           | 0.05                        | 3           |
| Consumer Staples       | 0.023                   | 9           | 0.048                       | 4           |
| Consumer Cyclical      | 0.036                   | 5           | 0.044                       | 5           |
| Real Estate            | 0.025                   | 8           | 0.043                       | 6           |
| Communication Services | 0.028                   | 6           | 0.042                       | 7           |
| Energy                 | 0.051                   | 3           | 0.038                       | 8           |
| Financial Services     | 0.055                   | 2           | 0.033                       | 9           |
| Utilities              | 0.051                   | 4           | 0.032                       | 10          |

The net transfer matrix (i.e. Table 2.4) is used to calculate the average transfer entropy sent and received by each sector.

Table 2.12: The In/Out Degree Centrality of TSX 60 Short Term Network by Periods

| <i>Thresholds</i> | <i>Sector</i>          | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|-------------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                   |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0                 | Basic Materials        | 1               | 0.9        | 1         | 0.86       | 1             | 0.96       |
|                   | Utilities              | 0.962           | 0.962      | 0.731     | 0.962      | 0.942         | 1          |
|                   | Consumer Staples       | 0.941           | 0.941      | 0.902     | 0.725      | 0.98          | 0.941      |
|                   | Financial Services     | 1               | 1          | 0.587     | 0.957      | 1             | 0.978      |
|                   | Communication Services | 0.815           | 0.907      | 0.407     | 0.963      | 0.852         | 0.852      |
|                   | Industrials            | 0.885           | 0.981      | 0.865     | 0.712      | 0.942         | 0.846      |
|                   | Real Estate            | 0.927           | 0.545      | 0.945     | 0.2        | 0.818         | 0.527      |
|                   | Consumer Cyclical      | 0.846           | 1          | 0.788     | 0.904      | 0.942         | 0.942      |
|                   | Energy                 | 1               | 0.958      | 0.958     | 1          | 0.958         | 1          |
|                   | Information Technology | 0.906           | 0.906      | 0.962     | 0.321      | 0.943         | 0.83       |
| 0.002             | Basic Materials        | 1               | 0.7        | 0.96      | 0.78       | 0.92          | 0.92       |
|                   | Utilities              | 0.808           | 0.75       | 0.538     | 0.962      | 0.673         | 0.962      |
|                   | Consumer Staples       | 0.784           | 0.843      | 0.843     | 0.627      | 0.941         | 0.745      |
|                   | Financial Services     | 0.957           | 0.935      | 0.413     | 0.913      | 0.978         | 0.87       |
|                   | Communication Services | 0.537           | 0.704      | 0.241     | 0.907      | 0.685         | 0.759      |
|                   | Industrials            | 0.615           | 0.885      | 0.827     | 0.596      | 0.846         | 0.769      |
|                   | Real Estate            | 0.727           | 0.255      | 0.927     | 0.127      | 0.691         | 0.236      |
|                   | Consumer Cyclical      | 0.596           | 0.981      | 0.731     | 0.827      | 0.788         | 0.827      |
|                   | Energy                 | 0.938           | 0.854      | 0.938     | 1          | 0.833         | 0.958      |
|                   | Information Technology | 0.736           | 0.755      | 0.962     | 0.226      | 0.717         | 0.698      |
| 0.005             | Basic Materials        | 0.72            | 0.36       | 0.92      | 0.7        | 0.58          | 0.68       |
|                   | Utilities              | 0.385           | 0.346      | 0.423     | 0.827      | 0.5           | 0.827      |
|                   | Consumer Staples       | 0.412           | 0.431      | 0.765     | 0.569      | 0.686         | 0.392      |
|                   | Financial Services     | 0.609           | 0.652      | 0.37      | 0.826      | 0.783         | 0.63       |
|                   | Communication Services | 0.148           | 0.37       | 0.148     | 0.796      | 0.278         | 0.426      |
|                   | Industrials            | 0.25            | 0.596      | 0.692     | 0.462      | 0.538         | 0.327      |
|                   | Real Estate            | 0.364           | 0.055      | 0.873     | 0.036      | 0.491         | 0.073      |
|                   | Consumer Cyclical      | 0.308           | 0.808      | 0.577     | 0.731      | 0.519         | 0.654      |
|                   | Energy                 | 0.625           | 0.438      | 0.875     | 0.875      | 0.5           | 0.771      |
|                   | Information Technology | 0.358           | 0.34       | 0.868     | 0.132      | 0.472         | 0.34       |
| 0.01              | Basic Materials        | 0.2             | 0          | 0.66      | 0.46       | 0.13          | 0.196      |
|                   | Utilities              | 0.13            | 0.087      | 0.192     | 0.615      | 0.041         | 0.571      |
|                   | Consumer Staples       | 0.04            | 0.04       | 0.627     | 0.373      | 0.277         | 0.064      |
|                   | Financial Services     | 0.136           | 0.091      | 0.174     | 0.674      | 0.364         | 0.091      |
|                   | Communication Services | 0               | 0.038      | 0.074     | 0.481      | 0.04          | 0.16       |
|                   | Industrials            | 0.042           | 0.167      | 0.538     | 0.308      | 0.102         | 0.02       |
|                   | Real Estate            | 0               | 0          | 0.655     | 0          | 0.078         | 0          |
|                   | Consumer Cyclical      | 0.043           | 0.391      | 0.481     | 0.481      | 0.083         | 0.188      |
|                   | Energy                 | 0.167           | 0          | 0.625     | 0.771      | 0.068         | 0.295      |
|                   | Information Technology | 0.042           | 0.125      | 0.736     | 0.075      | 0.122         | 0.061      |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.13: The In/Out Degree Centrality of TSX 60 Long Term Network by Periods

| <i>Thresholds</i> | <i>Sector</i>          | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|-------------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                   |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0                 | Basic Materials        | 0.96            | 0.92       | 0.92      | 0.92       | 0.84          | 0.98       |
|                   | Utilities              | 0.885           | 0.788      | 0.981     | 0.904      | 0.942         | 0.846      |
|                   | Consumer Staples       | 0.902           | 0.804      | 0.941     | 0.882      | 0.902         | 0.941      |
|                   | Financial Services     | 1               | 1          | 0.913     | 1          | 0.978         | 0.826      |
|                   | Communication Services | 0.741           | 0.759      | 0.704     | 0.778      | 0.907         | 0.648      |
|                   | Industrials            | 0.788           | 0.962      | 0.942     | 0.923      | 0.827         | 0.962      |
|                   | Real Estate            | 0.891           | 0.855      | 0.964     | 0.291      | 0.327         | 0.891      |
|                   | Consumer Cyclical      | 1               | 0.904      | 0.923     | 0.923      | 0.885         | 0.885      |
|                   | Energy                 | 0.896           | 0.979      | 0.958     | 0.979      | 1             | 0.958      |
|                   | Information Technology | 0.849           | 0.887      | 0.887     | 0.642      | 0.868         | 0.962      |
| 0.002             | Basic Materials        | 0.84            | 0.66       | 0.82      | 0.84       | 0.66          | 0.9        |
|                   | Utilities              | 0.692           | 0.596      | 0.962     | 0.885      | 0.846         | 0.692      |
|                   | Consumer Staples       | 0.843           | 0.667      | 0.863     | 0.765      | 0.745         | 0.843      |
|                   | Financial Services     | 0.913           | 0.87       | 0.804     | 0.891      | 0.935         | 0.63       |
|                   | Communication Services | 0.5             | 0.556      | 0.5       | 0.704      | 0.796         | 0.37       |
|                   | Industrials            | 0.519           | 0.865      | 0.846     | 0.827      | 0.596         | 0.846      |
|                   | Real Estate            | 0.709           | 0.673      | 0.891     | 0.182      | 0.2           | 0.745      |
|                   | Consumer Cyclical      | 0.962           | 0.75       | 0.846     | 0.904      | 0.827         | 0.654      |
|                   | Energy                 | 0.792           | 0.896      | 0.854     | 0.917      | 1             | 0.792      |
|                   | Information Technology | 0.623           | 0.679      | 0.774     | 0.547      | 0.849         | 0.774      |
| 0.005             | Basic Materials        | 0.48            | 0.34       | 0.54      | 0.68       | 0.22          | 0.76       |
|                   | Utilities              | 0.308           | 0.269      | 0.827     | 0.769      | 0.577         | 0.423      |
|                   | Consumer Staples       | 0.49            | 0.412      | 0.647     | 0.667      | 0.471         | 0.431      |
|                   | Financial Services     | 0.63            | 0.565      | 0.522     | 0.783      | 0.804         | 0.326      |
|                   | Communication Services | 0.185           | 0.167      | 0.259     | 0.407      | 0.463         | 0.204      |
|                   | Industrials            | 0.192           | 0.519      | 0.635     | 0.712      | 0.231         | 0.577      |
|                   | Real Estate            | 0.418           | 0.382      | 0.636     | 0.036      | 0.055         | 0.418      |
|                   | Consumer Cyclical      | 0.577           | 0.327      | 0.654     | 0.712      | 0.519         | 0.308      |
|                   | Energy                 | 0.417           | 0.604      | 0.542     | 0.708      | 0.958         | 0.458      |
|                   | Information Technology | 0.377           | 0.415      | 0.491     | 0.245      | 0.623         | 0.472      |
| 0.01              | Basic Materials        | 0.167           | 0.111      | 0.239     | 0.304      | 0             | 0.231      |
|                   | Utilities              | 0               | 0          | 0.417     | 0.417      | 0.095         | 0.143      |
|                   | Consumer Staples       | 0.105           | 0.105      | 0.327     | 0.184      | 0.077         | 0.205      |
|                   | Financial Services     | 0.222           | 0.278      | 0.167     | 0.524      | 0.27          | 0.027      |
|                   | Communication Services | 0               | 0          | 0         | 0.157      | 0.14          | 0          |
|                   | Industrials            | 0               | 0.3        | 0.292     | 0.208      | 0             | 0.167      |
|                   | Real Estate            | 0.2             | 0.1        | 0.294     | 0          | 0             | 0.093      |
|                   | Consumer Cyclical      | 0.158           | 0.105      | 0.25      | 0.271      | 0.049         | 0.049      |
|                   | Energy                 | 0.2             | 0          | 0.178     | 0.356      | 0.553         | 0.105      |
|                   | Information Technology | 0.05            | 0.1        | 0.163     | 0          | 0.317         | 0.244      |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.14: Average Net Transfer Entropy (NTE) by Sectors - Nasdaq 100

| <i>Sector</i>          | <i>Average NTE Sent</i> | <i>Rank</i> | <i>Average NTE Received</i> | <i>Rank</i> |
|------------------------|-------------------------|-------------|-----------------------------|-------------|
| Electronic Technology  | 0.071                   | 1           | 0.053                       | 7           |
| Energy Minerals        | 0.066                   | 2           | 0.055                       | 4           |
| Commercial Services    | 0.063                   | 3           | 0.054                       | 6           |
| Consumer Services      | 0.062                   | 4           | 0.047                       | 14          |
| Retail Trade           | 0.057                   | 5           | 0.052                       | 9           |
| Technology Services    | 0.053                   | 6           | 0.049                       | 11          |
| Consumer Durables      | 0.052                   | 7           | 0.045                       | 15          |
| Consumer Non-Durables  | 0.037                   | 8           | 0.055                       | 5           |
| Health Technology      | 0.035                   | 9           | 0.064                       | 2           |
| Producer Manufacturing | 0.034                   | 10          | 0.053                       | 8           |
| Industrial Services    | 0.031                   | 11          | 0.049                       | 12          |
| Utilities              | 0.03                    | 12          | 0.048                       | 13          |
| Communications         | 0.026                   | 13          | 0.059                       | 3           |
| Transportation         | 0.025                   | 14          | 0.05                        | 10          |
| Distribution Services  | 0.024                   | 15          | 0.074                       | 1           |

The net transfer matrix (i.e. Table 2.6) is used to calculate the average transfer entropy sent and received by each sector.

Table 2.15: Average Net Transfer Entropy (NTE) by Sectors & Periods - Nasdaq 100

| <i>Sector</i>          | Before COVID-19 |       | COVID-19 |       | Post COVID-19 |       |
|------------------------|-----------------|-------|----------|-------|---------------|-------|
|                        | Received        | Sent  | Received | Sent  | Received      | Sent  |
| Consumer Services      | 0.039           | 0.03  | 0.057    | 0.05  | 0.046         | 0.065 |
| Health Technology      | 0.038           | 0.031 | 0.07     | 0.072 | 0.047         | 0.04  |
| Electronic Technology  | 0.039           | 0.046 | 0.065    | 0.075 | 0.049         | 0.035 |
| Utilities              | 0.04            | 0.023 | 0.064    | 0.105 | 0.038         | 0.039 |
| Retail Trade           | 0.037           | 0.039 | 0.074    | 0.057 | 0.039         | 0.049 |
| Technology Services    | 0.039           | 0.04  | 0.07     | 0.066 | 0.045         | 0.046 |
| Producer Manufacturing | 0.038           | 0.022 | 0.075    | 0.042 | 0.045         | 0.032 |
| Industrial Services    | 0.036           | 0.025 | 0.062    | 0.051 | 0.038         | 0.032 |
| Commercial Services    | 0.034           | 0.038 | 0.078    | 0.074 | 0.048         | 0.026 |
| Transportation         | 0.034           | 0.025 | 0.04     | 0.084 | 0.048         | 0.043 |
| Energy Minerals        | 0.039           | 0.027 | 0.026    | 0.11  | 0.037         | 0.051 |
| Distribution Services  | 0.034           | 0.029 | 0.054    | 0.041 | 0.035         | 0.049 |
| Consumer Non-Durables  | 0.034           | 0.041 | 0.07     | 0.052 | 0.044         | 0.055 |
| Consumer Durables      | 0.039           | 0.063 | 0.075    | 0.036 | 0.056         | 0.034 |
| Communications         | 0.043           | 0.029 | 0.061    | 0.04  | 0.045         | 0.048 |

The sector-wise net transfer entropy is the average of net transfer entropy of each sector's stocks.

Table 2.16: The In/Out Degree Centrality of Nasdaq 100 Short Term Network by Periods

| Thresholds        | Sector                 | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|-------------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                   |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0                 | Consumer Services      | 0.988           | 0.925      | 0.938     | 0.838      | 0.938         | 1          |
|                   | Health Technology      | 1               | 0.934      | 0.947     | 0.974      | 0.987         | 0.974      |
|                   | Electronic Technology  | 1               | 1          | 0.986     | 0.986      | 1             | 0.944      |
|                   | Utilities              | 0.941           | 0.612      | 0.788     | 0.965      | 0.741         | 0.847      |
|                   | Retail Trade           | 0.987           | 0.975      | 0.962     | 0.886      | 0.949         | 0.975      |
|                   | Technology Services    | 1               | 0.985      | 1         | 0.985      | 1             | 0.97       |
|                   | Producer Manufacturing | 0.941           | 0.647      | 0.918     | 0.682      | 0.941         | 0.659      |
|                   | Industrial Services    | 0.632           | 0.368      | 0.586     | 0.414      | 0.575         | 0.425      |
|                   | Commercial Services    | 0.581           | 0.872      | 0.837     | 0.802      | 0.884         | 0.372      |
|                   | Transportation         | 0.837           | 0.605      | 0.302     | 0.872      | 0.709         | 0.791      |
|                   | Energy Minerals        | 0.724           | 0.276      | 0.034     | 0.966      | 0.23          | 0.77       |
|                   | Distribution Services  | 0.506           | 0.494      | 0.506     | 0.494      | 0.414         | 0.586      |
|                   | Consumer Non-Durables  | 0.88            | 0.976      | 0.892     | 0.783      | 0.771         | 0.976      |
| Consumer Durables | 0.138                  | 0.862           | 0.632      | 0.368     | 0.69       | 0.31          |            |
| Communications    | 0.621                  | 0.379           | 0.586      | 0.414     | 0.494      | 0.506         |            |
| 0.002             | Consumer Services      | 0.938           | 0.838      | 0.813     | 0.7        | 0.888         | 1          |
|                   | Health Technology      | 0.974           | 0.829      | 0.908     | 0.947      | 0.987         | 0.895      |
|                   | Electronic Technology  | 0.972           | 0.986      | 0.944     | 0.972      | 1             | 0.845      |
|                   | Utilities              | 0.718           | 0.376      | 0.612     | 0.941      | 0.576         | 0.753      |
|                   | Retail Trade           | 0.911           | 0.937      | 0.899     | 0.886      | 0.797         | 0.962      |
|                   | Technology Services    | 1               | 0.97       | 1         | 0.985      | 0.97          | 0.97       |
|                   | Producer Manufacturing | 0.694           | 0.388      | 0.835     | 0.553      | 0.812         | 0.435      |
|                   | Industrial Services    | 0.437           | 0.195      | 0.448     | 0.31       | 0.379         | 0.299      |
|                   | Commercial Services    | 0.442           | 0.651      | 0.709     | 0.744      | 0.744         | 0.209      |
|                   | Transportation         | 0.535           | 0.326      | 0.198     | 0.837      | 0.57          | 0.616      |
|                   | Energy Minerals        | 0.506           | 0.138      | 0.023     | 0.966      | 0.126         | 0.609      |
|                   | Distribution Services  | 0.322           | 0.333      | 0.379     | 0.356      | 0.276         | 0.448      |
|                   | Consumer Non-Durables  | 0.602           | 0.904      | 0.735     | 0.687      | 0.566         | 0.916      |
| Consumer Durables | 0.08                   | 0.747           | 0.563      | 0.23      | 0.575      | 0.184         |            |
| Communications    | 0.483                  | 0.195           | 0.471      | 0.287     | 0.379      | 0.391         |            |
| 0.005             | Consumer Services      | 0.638           | 0.475      | 0.625     | 0.613      | 0.588         | 0.988      |
|                   | Health Technology      | 0.671           | 0.566      | 0.803     | 0.908      | 0.829         | 0.724      |
|                   | Electronic Technology  | 0.761           | 0.972      | 0.817     | 0.93       | 0.958         | 0.662      |
|                   | Utilities              | 0.4             | 0.071      | 0.435     | 0.918      | 0.259         | 0.341      |
|                   | Retail Trade           | 0.544           | 0.772      | 0.81      | 0.734      | 0.481         | 0.81       |
|                   | Technology Services    | 0.712           | 0.909      | 0.909     | 0.924      | 0.894         | 0.924      |
|                   | Producer Manufacturing | 0.318           | 0.047      | 0.635     | 0.353      | 0.553         | 0.212      |
|                   | Industrial Services    | 0.126           | 0.046      | 0.287     | 0.184      | 0.161         | 0.057      |
|                   | Commercial Services    | 0.151           | 0.337      | 0.57      | 0.581      | 0.488         | 0.081      |
|                   | Transportation         | 0.244           | 0.105      | 0.105     | 0.686      | 0.337         | 0.349      |
|                   | Energy Minerals        | 0.218           | 0.034      | 0         | 0.839      | 0.069         | 0.379      |
|                   | Distribution Services  | 0.115           | 0.057      | 0.207     | 0.172      | 0.08          | 0.253      |
|                   | Consumer Non-Durables  | 0.289           | 0.699      | 0.59      | 0.566      | 0.361         | 0.711      |
| Consumer Durables | 0.046                  | 0.517           | 0.448      | 0.115     | 0.333      | 0.057         |            |
| Communications    | 0.184                  | 0.092           | 0.299      | 0.138     | 0.172      | 0.161         |            |
| 0.01              | Consumer Services      | 0.143           | 0          | 0.25      | 0.275      | 0.175         | 0.85       |
|                   | Health Technology      | 0.179           | 0.015      | 0.434     | 0.737      | 0.316         | 0.197      |
|                   | Electronic Technology  | 0.186           | 0.797      | 0.451     | 0.859      | 0.479         | 0.099      |
|                   | Utilities              | 0.11            | 0          | 0.2       | 0.8        | 0.024         | 0.082      |
|                   | Retail Trade           | 0.101           | 0.261      | 0.544     | 0.38       | 0.076         | 0.392      |
|                   | Technology Services    | 0.158           | 0.386      | 0.636     | 0.773      | 0.303         | 0.47       |
|                   | Producer Manufacturing | 0.068           | 0          | 0.365     | 0.071      | 0.106         | 0.012      |
|                   | Industrial Services    | 0.04            | 0          | 0.126     | 0.057      | 0.034         | 0          |
|                   | Commercial Services    | 0               | 0.08       | 0.314     | 0.302      | 0.116         | 0          |
|                   | Transportation         | 0.054           | 0          | 0.023     | 0.384      | 0.058         | 0.023      |
|                   | Energy Minerals        | 0.013           | 0          | 0         | 0.563      | 0.011         | 0.069      |
|                   | Distribution Services  | 0.027           | 0          | 0.092     | 0          | 0.011         | 0.057      |
|                   | Consumer Non-Durables  | 0.069           | 0.167      | 0.325     | 0.205      | 0.096         | 0.325      |
| Consumer Durables | 0                      | 0.147           | 0.172      | 0         | 0.069      | 0.023         |            |
| Communications    | 0.04                   | 0               | 0.115      | 0.011     | 0.023      | 0.069         |            |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.17: The In/Out Degree Centrality of Nasdaq 100 Long Term Network by Periods

| Thresholds     | Sector                 | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|----------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0              | Consumer Services      | 0.925           | 0.988      | 0.975     | 0.838      | 0.95          | 0.925      |
|                | Health Technology      | 0.987           | 0.961      | 0.961     | 0.974      | 0.974         | 0.987      |
|                | Electronic Technology  | 0.986           | 0.986      | 1         | 0.986      | 0.972         | 0.986      |
|                | Utilities              | 0.835           | 0.918      | 0.859     | 0.847      | 0.988         | 0.706      |
|                | Retail Trade           | 0.987           | 0.949      | 0.975     | 0.911      | 0.873         | 1          |
|                | Technology Services    | 0.985           | 1          | 0.985     | 1          | 0.97          | 0.985      |
|                | Producer Manufacturing | 0.929           | 0.706      | 0.941     | 0.447      | 0.965         | 0.929      |
|                | Industrial Services    | 0.517           | 0.483      | 0.046     | 0.954      | 0.264         | 0.736      |
|                | Commercial Services    | 0.57            | 0.895      | 0.744     | 0.849      | 0.57          | 0.826      |
|                | Transportation         | 0.64            | 0.919      | 0.465     | 0.802      | 0.814         | 0.674      |
|                | Energy Minerals        | 0.793           | 0.207      | 0.126     | 0.874      | 0.23          | 0.77       |
|                | Distribution Services  | 0.69            | 0.31       | 0.184     | 0.816      | 0.425         | 0.575      |
|                | Consumer Non-Durables  | 0.771           | 0.988      | 0.94      | 0.831      | 0.952         | 0.855      |
|                | Consumer Durables      | 0.552           | 0.448      | 0.586     | 0.414      | 0.816         | 0.184      |
| Communications | 0.828                  | 0.172           | 0.701      | 0.299     | 0.517      | 0.483         |            |
| 0.002          | Consumer Services      | 0.75            | 0.863      | 0.875     | 0.738      | 0.863         | 0.638      |
|                | Health Technology      | 0.934           | 0.789      | 0.789     | 0.908      | 0.974         | 0.895      |
|                | Electronic Technology  | 0.901           | 0.93       | 0.972     | 0.944      | 0.915         | 0.915      |
|                | Utilities              | 0.694           | 0.706      | 0.694     | 0.718      | 0.929         | 0.4        |
|                | Retail Trade           | 0.962           | 0.848      | 0.899     | 0.797      | 0.759         | 0.987      |
|                | Technology Services    | 0.894           | 0.894      | 0.924     | 0.909      | 0.924         | 0.939      |
|                | Producer Manufacturing | 0.659           | 0.329      | 0.882     | 0.259      | 0.906         | 0.718      |
|                | Industrial Services    | 0.23            | 0.218      | 0         | 0.862      | 0.069         | 0.552      |
|                | Commercial Services    | 0.291           | 0.593      | 0.558     | 0.744      | 0.256         | 0.593      |
|                | Transportation         | 0.233           | 0.663      | 0.302     | 0.733      | 0.5           | 0.442      |
|                | Energy Minerals        | 0.621           | 0.057      | 0.069     | 0.77       | 0.08          | 0.494      |
|                | Distribution Services  | 0.414           | 0.092      | 0.057     | 0.736      | 0.172         | 0.402      |
|                | Consumer Non-Durables  | 0.373           | 0.819      | 0.843     | 0.759      | 0.735         | 0.602      |
|                | Consumer Durables      | 0.322           | 0.195      | 0.448     | 0.241      | 0.609         | 0.057      |
| Communications | 0.563                  | 0.046           | 0.632      | 0.161     | 0.276      | 0.264         |            |
| 0.005          | Consumer Services      | 0.338           | 0.4        | 0.563     | 0.563      | 0.588         | 0.263      |
|                | Health Technology      | 0.605           | 0.395      | 0.5       | 0.776      | 0.882         | 0.5        |
|                | Electronic Technology  | 0.563           | 0.408      | 0.761     | 0.732      | 0.648         | 0.662      |
|                | Utilities              | 0.247           | 0.329      | 0.447     | 0.471      | 0.753         | 0.118      |
|                | Retail Trade           | 0.785           | 0.405      | 0.62      | 0.608      | 0.342         | 0.532      |
|                | Technology Services    | 0.591           | 0.545      | 0.742     | 0.742      | 0.606         | 0.576      |
|                | Producer Manufacturing | 0.271           | 0.071      | 0.671     | 0.071      | 0.682         | 0.353      |
|                | Industrial Services    | 0.023           | 0.034      | 0         | 0.667      | 0             | 0.161      |
|                | Commercial Services    | 0.058           | 0.279      | 0.407     | 0.535      | 0.035         | 0.314      |
|                | Transportation         | 0.035           | 0.174      | 0.174     | 0.5        | 0.116         | 0.14       |
|                | Energy Minerals        | 0.287           | 0.011      | 0         | 0.609      | 0.011         | 0.172      |
|                | Distribution Services  | 0.161           | 0          | 0.011     | 0.448      | 0.034         | 0.138      |
|                | Consumer Non-Durables  | 0.036           | 0.422      | 0.566     | 0.602      | 0.41          | 0.253      |
|                | Consumer Durables      | 0.057           | 0.023      | 0.241     | 0.069      | 0.31          | 0.011      |
| Communications | 0.264                  | 0.011           | 0.391      | 0.034     | 0.046      | 0.103         |            |
| 0.01           | Consumer Services      | 0.065           | 0.065      | 0.133     | 0.067      | 0.136         | 0.051      |
|                | Health Technology      | 0.1             | 0.067      | 0.159     | 0.42       | 0.698         | 0.057      |
|                | Electronic Technology  | 0.125           | 0.063      | 0.288     | 0.379      | 0.148         | 0.204      |
|                | Utilities              | 0               | 0.059      | 0.167     | 0.103      | 0.27          | 0          |
|                | Retail Trade           | 0.679           | 0.071      | 0.315     | 0.205      | 0             | 0.121      |
|                | Technology Services    | 0.037           | 0.148      | 0.322     | 0.339      | 0.04          | 0.16       |
|                | Producer Manufacturing | 0.059           | 0          | 0.244     | 0          | 0.213         | 0.082      |
|                | Industrial Services    | 0               | 0          | 0         | 0.25       | 0             | 0.048      |
|                | Commercial Services    | 0               | 0.029      | 0.038     | 0.101      | 0             | 0.032      |
|                | Transportation         | 0               | 0          | 0         | 0.113      | 0.016         | 0.048      |
|                | Energy Minerals        | 0               | 0          | 0         | 0.213      | 0             | 0.063      |
|                | Distribution Services  | 0               | 0          | 0         | 0.075      | 0             | 0.016      |
|                | Consumer Non-Durables  | 0               | 0.129      | 0.237     | 0.145      | 0.017         | 0.085      |
|                | Consumer Durables      | 0               | 0.029      | 0         | 0          | 0.016         | 0          |
| Communications | 0                      | 0               | 0.138      | 0         | 0          | 0.032         |            |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by diving the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.18: Simple Average Diebold-Yilmaz (DY) Index by Sectors - TSX 60

| <i>Sector</i>          | <i>Average Sent</i> | <i>Rank</i> | <i>Average Received</i> | <i>Rank</i> |
|------------------------|---------------------|-------------|-------------------------|-------------|
| Financial Services     | 0.011               | 1           | 0.0066                  | 10          |
| Energy                 | 0.0085              | 2           | 0.0085                  | 4           |
| Communication Services | 0.0077              | 3           | 0.0072                  | 8           |
| Basic Materials        | 0.0072              | 4           | 0.0072                  | 7           |
| Consumer Cyclical      | 0.0063              | 5           | 0.0084                  | 5           |
| Utilities              | 0.0057              | 6           | 0.0078                  | 6           |
| Industrials            | 0.0057              | 7           | 0.007                   | 9           |
| Consumer Staples       | 0.0056              | 8           | 0.0094                  | 2           |
| Information Technology | 0.0047              | 9           | 0.0086                  | 3           |
| Real Estate            | 0.0042              | 10          | 0.0108                  | 1           |

The net DY Spillover matrix (i.e. Table 2.8) is used to calculate the average transfer entropy sent and received by each sector.

Table 2.19: The In/Out Degree Centrality of TSX 60 Short Term Network by Periods - DY Spillover Index

| <i>Thresholds</i>      | <i>Sector</i>          | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|------------------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                        |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0                      | Basic Materials        | 0.9             | 0.98       | 0.92      | 0.9        | 0.92          | 1          |
|                        | Utilities              | 1               | 0.846      | 0.904     | 0.885      | 0.923         | 0.846      |
|                        | Consumer Staples       | 1               | 0.98       | 0.941     | 0.804      | 1             | 0.784      |
|                        | Financial Services     | 0.957           | 0.978      | 0.87      | 1          | 0.891         | 0.913      |
|                        | Communication Services | 0.852           | 0.778      | 0.796     | 0.796      | 0.759         | 0.815      |
|                        | Industrials            | 0.981           | 0.923      | 0.923     | 0.808      | 0.865         | 0.788      |
|                        | Real Estate            | 0.836           | 0.436      | 0.982     | 0.309      | 0.8           | 0.564      |
|                        | Consumer Cyclical      | 0.923           | 0.904      | 0.942     | 0.808      | 0.981         | 0.673      |
|                        | Energy                 | 1               | 1          | 1         | 0.854      | 0.938         | 0.979      |
| Information Technology | 0.906                  | 0.906           | 0.925      | 0.774     | 0.943      | 0.717         |            |
| 0.0002                 | Basic Materials        | 0.66            | 0.9        | 0.8       | 0.88       | 0.82          | 0.98       |
|                        | Utilities              | 0.885           | 0.519      | 0.788     | 0.885      | 0.788         | 0.769      |
|                        | Consumer Staples       | 0.824           | 0.706      | 0.882     | 0.765      | 0.922         | 0.588      |
|                        | Financial Services     | 0.717           | 0.913      | 0.761     | 1          | 0.783         | 0.848      |
|                        | Communication Services | 0.667           | 0.556      | 0.63      | 0.759      | 0.63          | 0.759      |
|                        | Industrials            | 0.846           | 0.75       | 0.865     | 0.712      | 0.788         | 0.673      |
|                        | Real Estate            | 0.636           | 0.273      | 0.945     | 0.164      | 0.636         | 0.455      |
|                        | Consumer Cyclical      | 0.808           | 0.673      | 0.904     | 0.75       | 0.865         | 0.558      |
|                        | Energy                 | 0.875           | 0.938      | 0.938     | 0.833      | 0.917         | 0.979      |
| Information Technology | 0.755                  | 0.755           | 0.868      | 0.642     | 0.83       | 0.547         |            |
| 0.0005                 | Basic Materials        | 0.38            | 0.72       | 0.6       | 0.86       | 0.54          | 0.9        |
|                        | Utilities              | 0.615           | 0.269      | 0.615     | 0.731      | 0.635         | 0.519      |
|                        | Consumer Staples       | 0.431           | 0.471      | 0.804     | 0.569      | 0.765         | 0.471      |
|                        | Financial Services     | 0.543           | 0.826      | 0.674     | 1          | 0.63          | 0.783      |
|                        | Communication Services | 0.444           | 0.278      | 0.5       | 0.63       | 0.444         | 0.556      |
|                        | Industrials            | 0.538           | 0.462      | 0.75      | 0.615      | 0.615         | 0.538      |
|                        | Real Estate            | 0.4             | 0.145      | 0.764     | 0.055      | 0.436         | 0.255      |
|                        | Consumer Cyclical      | 0.615           | 0.365      | 0.75      | 0.635      | 0.712         | 0.404      |
|                        | Energy                 | 0.583           | 0.729      | 0.792     | 0.771      | 0.688         | 0.917      |
| Information Technology | 0.453                  | 0.472           | 0.736      | 0.434     | 0.642      | 0.283         |            |
| 0.001                  | Basic Materials        | 0.14            | 0.22       | 0.34      | 0.7        | 0.32          | 0.64       |
|                        | Utilities              | 0.192           | 0.058      | 0.404     | 0.481      | 0.404         | 0.154      |
|                        | Consumer Staples       | 0.176           | 0.196      | 0.569     | 0.294      | 0.51          | 0.216      |
|                        | Financial Services     | 0.217           | 0.478      | 0.37      | 1          | 0.348         | 0.5        |
|                        | Communication Services | 0.13            | 0.037      | 0.333     | 0.407      | 0.204         | 0.352      |
|                        | Industrials            | 0.327           | 0.173      | 0.538     | 0.269      | 0.423         | 0.25       |
|                        | Real Estate            | 0.291           | 0.018      | 0.564     | 0          | 0.182         | 0.091      |
|                        | Consumer Cyclical      | 0.288           | 0.231      | 0.5       | 0.481      | 0.365         | 0.135      |
|                        | Energy                 | 0.229           | 0.25       | 0.479     | 0.542      | 0.417         | 0.542      |
| Information Technology | 0.17                   | 0.189           | 0.509      | 0.245     | 0.358      | 0.151         |            |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.20: The In/Out Degree Centrality of TSX 60 Long Term Network by Periods - DY Spillover Index

| <i>Thresholds</i>      | <i>Sector</i>          | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|------------------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                        |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0                      | Basic Materials        | 1               | 0.98       | 0.98      | 0.98       | 0.94          | 1          |
|                        | Utilities              | 0.904           | 0.904      | 0.981     | 0.827      | 0.962         | 0.942      |
|                        | Consumer Staples       | 0.98            | 0.922      | 0.941     | 0.961      | 0.941         | 1          |
|                        | Financial Services     | 0.978           | 1          | 1         | 1          | 1             | 1          |
|                        | Communication Services | 0.963           | 0.944      | 0.778     | 0.907      | 0.963         | 0.815      |
|                        | Industrials            | 0.923           | 0.981      | 0.827     | 0.923      | 1             | 0.981      |
|                        | Real Estate            | 0.655           | 0.782      | 0.636     | 0.764      | 0.709         | 0.673      |
|                        | Consumer Cyclical      | 0.962           | 1          | 1         | 0.923      | 0.962         | 0.923      |
|                        | Energy                 | 1               | 1          | 0.979     | 0.979      | 1             | 0.979      |
| Information Technology | 0.811                  | 0.962           | 0.962      | 0.849     | 0.868      | 0.962         |            |
| 0.0002                 | Basic Materials        | 0.96            | 0.94       | 0.96      | 0.98       | 0.92          | 1          |
|                        | Utilities              | 0.846           | 0.865      | 0.962     | 0.827      | 0.923         | 0.923      |
|                        | Consumer Staples       | 0.961           | 0.922      | 0.824     | 0.922      | 0.882         | 1          |
|                        | Financial Services     | 0.978           | 0.978      | 0.978     | 1          | 1             | 1          |
|                        | Communication Services | 0.907           | 0.796      | 0.722     | 0.852      | 0.889         | 0.778      |
|                        | Industrials            | 0.904           | 0.904      | 0.75      | 0.865      | 1             | 0.885      |
|                        | Real Estate            | 0.6             | 0.764      | 0.545     | 0.709      | 0.582         | 0.6        |
|                        | Consumer Cyclical      | 0.904           | 0.962      | 0.962     | 0.865      | 0.962         | 0.865      |
|                        | Energy                 | 0.979           | 1          | 0.896     | 0.979      | 1             | 0.938      |
| Information Technology | 0.679                  | 0.925           | 0.925      | 0.774     | 0.83       | 0.943         |            |
| 0.0005                 | Basic Materials        | 0.94            | 0.88       | 0.9       | 0.98       | 0.8           | 1          |
|                        | Utilities              | 0.75            | 0.827      | 0.923     | 0.808      | 0.904         | 0.885      |
|                        | Consumer Staples       | 0.922           | 0.843      | 0.765     | 0.922      | 0.843         | 0.98       |
|                        | Financial Services     | 0.978           | 0.957      | 0.978     | 1          | 0.978         | 1          |
|                        | Communication Services | 0.833           | 0.722      | 0.611     | 0.778      | 0.815         | 0.648      |
|                        | Industrials            | 0.788           | 0.846      | 0.731     | 0.808      | 0.981         | 0.808      |
|                        | Real Estate            | 0.491           | 0.655      | 0.473     | 0.618      | 0.545         | 0.545      |
|                        | Consumer Cyclical      | 0.846           | 0.962      | 0.904     | 0.788      | 0.885         | 0.846      |
|                        | Energy                 | 0.979           | 0.979      | 0.875     | 0.958      | 1             | 0.917      |
| Information Technology | 0.547                  | 0.868           | 0.83       | 0.698     | 0.698      | 0.943         |            |
| 0.001                  | Basic Materials        | 0.76            | 0.78       | 0.8       | 0.92       | 0.76          | 0.96       |
|                        | Utilities              | 0.615           | 0.731      | 0.788     | 0.75       | 0.769         | 0.769      |
|                        | Consumer Staples       | 0.725           | 0.706      | 0.706     | 0.863      | 0.745         | 0.922      |
|                        | Financial Services     | 0.913           | 0.848      | 0.891     | 0.957      | 0.978         | 0.978      |
|                        | Communication Services | 0.667           | 0.574      | 0.444     | 0.685      | 0.648         | 0.556      |
|                        | Industrials            | 0.577           | 0.788      | 0.635     | 0.712      | 0.846         | 0.731      |
|                        | Real Estate            | 0.418           | 0.473      | 0.345     | 0.473      | 0.455         | 0.509      |
|                        | Consumer Cyclical      | 0.731           | 0.923      | 0.769     | 0.615      | 0.846         | 0.769      |
|                        | Energy                 | 0.896           | 0.917      | 0.854     | 0.938      | 0.917         | 0.854      |
| Information Technology | 0.415                  | 0.792           | 0.698      | 0.528     | 0.642      | 0.774         |            |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.21: Simple Average Diebold-Yilmaz (DY) Index by Sectors - Nasdaq 100

| <i>Sector</i>          | <i>Average Sent</i> | <i>Rank</i> | <i>Average Received</i> | <i>Rank</i> |
|------------------------|---------------------|-------------|-------------------------|-------------|
| Commercial Services    | 0.0051              | 1           | 0.0036                  | 10          |
| Technology Services    | 0.0047              | 2           | 0.0034                  | 14          |
| Consumer Non-Durables  | 0.0041              | 3           | 0.0044                  | 5           |
| Electronic Technology  | 0.0041              | 4           | 0.0036                  | 12          |
| Producer Manufacturing | 0.0038              | 5           | 0.0031                  | 15          |
| Retail Trade           | 0.0035              | 6           | 0.0041                  | 8           |
| Consumer Services      | 0.0034              | 7           | 0.0042                  | 7           |
| Transportation         | 0.0031              | 8           | 0.0034                  | 13          |
| Health Technology      | 0.0028              | 9           | 0.0045                  | 4           |
| Energy Minerals        | 0.0027              | 10          | 0.0042                  | 6           |
| Utilities              | 0.0025              | 11          | 0.0036                  | 11          |
| Distribution Services  | 0.002               | 12          | 0.0041                  | 9           |
| Consumer Durables      | 0.0018              | 13          | 0.0049                  | 2           |
| Communications         | 0.0017              | 14          | 0.0048                  | 3           |
| Industrial Services    | 0.0015              | 15          | 0.0055                  | 1           |

The net DY Spillover matrix (i.e. Table 2.10) is used to calculate the average transfer entropy sent and received by each sector.

Table 2.22: The In/Out Degree Centrality of Nasdaq 100 Short Term Network by Periods - DY Spillover Index

| Thresholds     | Sector                 | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|----------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0              | Consumer Services      | 0.975           | 0.713      | 1         | 0.9        | 0.988         | 0.7        |
|                | Health Technology      | 0.947           | 0.816      | 0.987     | 0.829      | 1             | 0.697      |
|                | Electronic Technology  | 0.901           | 0.93       | 0.831     | 0.986      | 0.789         | 1          |
|                | Utilities              | 0.941           | 0.353      | 0.906     | 0.776      | 0.871         | 0.353      |
|                | Retail Trade           | 1               | 0.924      | 0.987     | 0.911      | 0.975         | 0.911      |
|                | Technology Services    | 0.939           | 1          | 0.939     | 1          | 0.818         | 0.955      |
|                | Producer Manufacturing | 0.376           | 0.847      | 0.812     | 0.647      | 0.812         | 0.812      |
|                | Industrial Services    | 0.92            | 0.08       | 0.678     | 0.322      | 0.759         | 0.241      |
|                | Commercial Services    | 0.837           | 0.953      | 0.477     | 0.907      | 0.64          | 0.895      |
|                | Transportation         | 0.605           | 0.605      | 0.86      | 0.616      | 0.826         | 0.372      |
|                | Energy Minerals        | 0.862           | 0.138      | 0.644     | 0.356      | 0.954         | 0.046      |
|                | Distribution Services  | 0.713           | 0.287      | 0.621     | 0.379      | 0.333         | 0.667      |
|                | Consumer Non-Durables  | 0.988           | 0.542      | 0.928     | 0.807      | 0.952         | 0.627      |
|                | Consumer Durables      | 0.885           | 0.115      | 0.908     | 0.092      | 0.747         | 0.253      |
| Communications | 0.851                  | 0.149           | 0.839      | 0.161     | 0.575      | 0.425         |            |
| 0.002          | Consumer Services      | 0.825           | 0.488      | 0.9       | 0.725      | 0.863         | 0.588      |
|                | Health Technology      | 0.868           | 0.605      | 0.947     | 0.566      | 0.961         | 0.421      |
|                | Electronic Technology  | 0.718           | 0.901      | 0.62      | 0.972      | 0.606         | 1          |
|                | Utilities              | 0.694           | 0.176      | 0.647     | 0.588      | 0.729         | 0.153      |
|                | Retail Trade           | 0.848           | 0.848      | 0.848     | 0.747      | 0.823         | 0.772      |
|                | Technology Services    | 0.636           | 1          | 0.818     | 1          | 0.606         | 0.803      |
|                | Producer Manufacturing | 0.2             | 0.765      | 0.635     | 0.4        | 0.671         | 0.541      |
|                | Industrial Services    | 0.69            | 0          | 0.46      | 0.126      | 0.552         | 0.046      |
|                | Commercial Services    | 0.547           | 0.884      | 0.209     | 0.826      | 0.384         | 0.791      |
|                | Transportation         | 0.372           | 0.395      | 0.663     | 0.488      | 0.64          | 0.233      |
|                | Energy Minerals        | 0.701           | 0.057      | 0.437     | 0.161      | 0.793         | 0          |
|                | Distribution Services  | 0.552           | 0.092      | 0.483     | 0.207      | 0.161         | 0.402      |
|                | Consumer Non-Durables  | 0.892           | 0.349      | 0.783     | 0.711      | 0.795         | 0.41       |
|                | Consumer Durables      | 0.655           | 0.034      | 0.701     | 0          | 0.494         | 0.172      |
| Communications | 0.655                  | 0.023           | 0.701      | 0.023     | 0.402      | 0.253         |            |
| 0.005          | Consumer Services      | 0.538           | 0.288      | 0.75      | 0.438      | 0.613         | 0.35       |
|                | Health Technology      | 0.487           | 0.355      | 0.724     | 0.263      | 0.803         | 0.132      |
|                | Electronic Technology  | 0.423           | 0.775      | 0.324     | 0.873      | 0.451         | 0.859      |
|                | Utilities              | 0.447           | 0.035      | 0.329     | 0.353      | 0.376         | 0.071      |
|                | Retail Trade           | 0.696           | 0.646      | 0.671     | 0.468      | 0.608         | 0.519      |
|                | Technology Services    | 0.333           | 1          | 0.379     | 0.939      | 0.379         | 0.667      |
|                | Producer Manufacturing | 0.082           | 0.471      | 0.388     | 0.153      | 0.329         | 0.271      |
|                | Industrial Services    | 0.379           | 0          | 0.264     | 0.046      | 0.299         | 0.023      |
|                | Commercial Services    | 0.233           | 0.593      | 0.081     | 0.628      | 0.174         | 0.442      |
|                | Transportation         | 0.186           | 0.128      | 0.372     | 0.291      | 0.372         | 0.047      |
|                | Energy Minerals        | 0.287           | 0.023      | 0.23      | 0.08       | 0.471         | 0          |
|                | Distribution Services  | 0.23            | 0.011      | 0.253     | 0.103      | 0.011         | 0.161      |
|                | Consumer Non-Durables  | 0.542           | 0.133      | 0.578     | 0.325      | 0.446         | 0.253      |
|                | Consumer Durables      | 0.391           | 0          | 0.46      | 0          | 0.172         | 0.034      |
| Communications | 0.333                  | 0               | 0.471      | 0         | 0.195      | 0.046         |            |
| 0.01           | Consumer Services      | 0.139           | 0.038      | 0.342     | 0.101      | 0.278         | 0.083      |
|                | Health Technology      | 0.2             | 0.04       | 0.293     | 0.08       | 0.529         | 0.014      |
|                | Electronic Technology  | 0.114           | 0.343      | 0.113     | 0.549      | 0.159         | 0.413      |
|                | Utilities              | 0.048           | 0          | 0.083     | 0.071      | 0.078         | 0.026      |
|                | Retail Trade           | 0.19            | 0.139      | 0.269     | 0.064      | 0.167         | 0.194      |
|                | Technology Services    | 0.077           | 0.738      | 0.077     | 0.615      | 0.066         | 0.311      |
|                | Producer Manufacturing | 0.024           | 0.095      | 0.107     | 0.012      | 0.065         | 0.039      |
|                | Industrial Services    | 0.081           | 0          | 0.047     | 0.012      | 0.114         | 0.013      |
|                | Commercial Services    | 0.047           | 0.188      | 0.012     | 0.141      | 0             | 0.063      |
|                | Transportation         | 0.047           | 0.047      | 0.118     | 0.059      | 0.051         | 0.013      |
|                | Energy Minerals        | 0.035           | 0          | 0.035     | 0.012      | 0.215         | 0          |
|                | Distribution Services  | 0.023           | 0          | 0.047     | 0          | 0.013         | 0.051      |
|                | Consumer Non-Durables  | 0.159           | 0          | 0.146     | 0.049      | 0.107         | 0.093      |
|                | Consumer Durables      | 0.105           | 0          | 0.174     | 0          | 0.038         | 0.013      |
| Communications | 0.151                  | 0               | 0.081      | 0         | 0          | 0             |            |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by diving the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

Table 2.23: The In/Out Degree Centrality of Nasdaq 10 Long Term Network by Periods - DY Spillover Index

| Thresholds     | Sector                 | Before COVID-19 |            | COVID-19  |            | Post COVID-19 |            |
|----------------|------------------------|-----------------|------------|-----------|------------|---------------|------------|
|                |                        | In Degree       | Out Degree | In Degree | Out Degree | In Degree     | Out Degree |
| 0              | Consumer Services      | 1               | 1          | 0.988     | 1          | 1             | 0.988      |
|                | Health Technology      | 1               | 1          | 1         | 0.987      | 1             | 1          |
|                | Electronic Technology  | 1               | 1          | 1         | 1          | 1             | 1          |
|                | Utilities              | 0.882           | 0.882      | 0.835     | 0.871      | 0.882         | 0.871      |
|                | Retail Trade           | 1               | 1          | 1         | 1          | 1             | 1          |
|                | Technology Services    | 1               | 1          | 1         | 1          | 1             | 1          |
|                | Producer Manufacturing | 0.906           | 0.859      | 0.894     | 0.8        | 0.894         | 0.918      |
|                | Industrial Services    | 0.391           | 0.609      | 0.494     | 0.506      | 0.483         | 0.517      |
|                | Commercial Services    | 0.779           | 0.744      | 0.744     | 0.802      | 0.686         | 0.733      |
|                | Transportation         | 0.756           | 0.767      | 0.651     | 0.709      | 0.709         | 0.767      |
|                | Energy Minerals        | 0.609           | 0.391      | 0.69      | 0.31       | 0.494         | 0.506      |
|                | Distribution Services  | 0.586           | 0.414      | 0.391     | 0.609      | 0.494         | 0.506      |
|                | Consumer Non-Durables  | 0.964           | 0.976      | 0.819     | 0.988      | 0.964         | 0.952      |
|                | Consumer Durables      | 0.598           | 0.402      | 0.598     | 0.402      | 0.46          | 0.54       |
| Communications | 0.609                  | 0.391           | 0.667      | 0.333     | 0.494      | 0.506         |            |
| 0.002          | Consumer Services      | 0.988           | 0.988      | 0.95      | 0.913      | 0.963         | 0.975      |
|                | Health Technology      | 1               | 1          | 1         | 0.895      | 1             | 1          |
|                | industrial Services    | 1               | 1          | 1         | 1          | 1             | 1          |
|                | Utilities              | 0.729           | 0.741      | 0.718     | 0.729      | 0.8           | 0.753      |
|                | Retail Trade           | 0.975           | 1          | 0.937     | 1          | 0.987         | 1          |
|                | Technology Services    | 1               | 1          | 1         | 1          | 1             | 1          |
|                | Producer Manufacturing | 0.776           | 0.706      | 0.824     | 0.647      | 0.788         | 0.894      |
|                | Industrial Services    | 0.264           | 0.471      | 0.368     | 0.322      | 0.333         | 0.391      |
|                | Commercial Services    | 0.64            | 0.605      | 0.64      | 0.721      | 0.628         | 0.663      |
|                | Transportation         | 0.605           | 0.512      | 0.523     | 0.605      | 0.616         | 0.663      |
|                | Energy Minerals        | 0.448           | 0.264      | 0.517     | 0.184      | 0.356         | 0.391      |
|                | Distribution Services  | 0.471           | 0.333      | 0.241     | 0.506      | 0.333         | 0.402      |
|                | Consumer Non-Durables  | 0.94            | 0.904      | 0.699     | 0.928      | 0.892         | 0.855      |
|                | Consumer Durables      | 0.414           | 0.276      | 0.483     | 0.253      | 0.379         | 0.437      |
| Communications | 0.483                  | 0.276           | 0.483      | 0.264     | 0.345      | 0.391         |            |
| 0.005          | Consumer Services      | 0.9             | 0.875      | 0.888     | 0.825      | 0.9           | 0.938      |
|                | Health Technology      | 0.934           | 0.987      | 0.947     | 0.829      | 1             | 1          |
|                | Electronic Technology  | 0.986           | 1          | 1         | 1          | 1             | 1          |
|                | Utilities              | 0.541           | 0.659      | 0.518     | 0.588      | 0.694         | 0.671      |
|                | Retail Trade           | 0.911           | 0.975      | 0.899     | 0.937      | 0.924         | 0.962      |
|                | Technology Services    | 1               | 1          | 1         | 1          | 1             | 1          |
|                | Producer Manufacturing | 0.612           | 0.553      | 0.624     | 0.459      | 0.624         | 0.741      |
|                | Industrial Services    | 0.23            | 0.402      | 0.253     | 0.241      | 0.264         | 0.333      |
|                | Commercial Services    | 0.5             | 0.395      | 0.453     | 0.581      | 0.523         | 0.558      |
|                | Transportation         | 0.512           | 0.372      | 0.349     | 0.512      | 0.488         | 0.5        |
|                | Energy Minerals        | 0.287           | 0.161      | 0.345     | 0.103      | 0.264         | 0.31       |
|                | Distribution Services  | 0.333           | 0.184      | 0.126     | 0.402      | 0.253         | 0.299      |
|                | Consumer Non-Durables  | 0.843           | 0.747      | 0.59      | 0.807      | 0.759         | 0.723      |
|                | Consumer Durables      | 0.322           | 0.149      | 0.379     | 0.103      | 0.299         | 0.287      |
| Communications | 0.31                   | 0.195           | 0.345      | 0.149     | 0.276      | 0.333         |            |
| 0.01           | Consumer Services      | 0.688           | 0.675      | 0.688     | 0.638      | 0.675         | 0.825      |
|                | Health Technology      | 0.829           | 0.882      | 0.75      | 0.724      | 0.961         | 0.882      |
|                | Electronic Technology  | 0.873           | 0.986      | 0.944     | 0.901      | 0.986         | 1          |
|                | Utilities              | 0.353           | 0.459      | 0.294     | 0.388      | 0.518         | 0.471      |
|                | Retail Trade           | 0.772           | 0.823      | 0.696     | 0.835      | 0.772         | 0.924      |
|                | Technology Services    | 0.939           | 0.985      | 0.909     | 0.985      | 0.985         | 0.985      |
|                | Producer Manufacturing | 0.388           | 0.412      | 0.447     | 0.224      | 0.471         | 0.518      |
|                | Industrial Services    | 0.149           | 0.23       | 0.161     | 0.184      | 0.184         | 0.218      |
|                | Commercial Services    | 0.291           | 0.279      | 0.267     | 0.43       | 0.279         | 0.407      |
|                | Transportation         | 0.337           | 0.326      | 0.244     | 0.395      | 0.291         | 0.36       |
|                | Energy Minerals        | 0.195           | 0.115      | 0.253     | 0.034      | 0.138         | 0.138      |
|                | Distribution Services  | 0.23            | 0.092      | 0.057     | 0.287      | 0.115         | 0.23       |
|                | Consumer Non-Durables  | 0.627           | 0.518      | 0.482     | 0.602      | 0.59          | 0.542      |
|                | Consumer Durables      | 0.23            | 0.069      | 0.195     | 0.034      | 0.184         | 0.126      |
| Communications | 0.184                  | 0.092           | 0.23       | 0.08      | 0.149      | 0.276         |            |

In-degree/Out-degree centrality calculation is implemented by NetworkX python library. The group degree centrality is normalized by diving the group degree by the number of non-group nodes. The short-term and long-term data are generated from the complete empirical mode decomposition based on the adaptive noise method.

# Chapter 3

## Network-based SIR Analysis

Chapter 2 directed network analysis revealed asymmetric spillover patterns, showing that certain sectors consistently transmit risk while others predominantly absorb it. While these findings shed light on the structure and intensity of directional dependencies, they do not explicitly model the dynamic process of how shocks evolve and spread through the market over time. In other words, the analysis so far has identified where and in which direction volatility flows, but not how fast or under what conditions these flows can lead to widespread contagion.

To address this, Chapter 3 integrates the directed network framework with an epidemiological modeling approach, the Susceptible, Infected and Recovered (SIR) model. By calibrating the SIR model parameters ( $\beta$  and  $\gamma$ ) to market data and embedding them within the network structure, this chapter simulates the temporal spread of financial distress. The combined network - SIR framework enables scenarios based analysis of contagion pathways, highlights the role of network topology in crisis amplification, and provides practical insights for policymakers aiming to contain systemic risk before it escalates.

### 3.1 Introduction

The increasing interconnectedness of financial markets has amplified their vulnerability to systemic crises, where localized shocks can rapidly propagate through a complex network.

Recent events, such as the financial disruptions triggered by the COVID-19 pandemic, have underscored the need for robust analytical frameworks to better understand the mechanisms of financial contagion. During the pandemic, market shocks spread across sectors and regions with unprecedented speed and severity, exposing weaknesses in traditional risk assessment tools and emphasizing the critical importance of mitigating systemic risks to safeguard market stability.

This study addresses this challenge by developing a novel framework that integrates financial network analysis with the Susceptible-Infected-Recovered (SIR) model, originally derived from epidemiology, to analyze the transmission of shocks across stock market networks. We primarily focus on the TSX 60. The study constructs directed, weighted networks based on the Diebold-Yilmaz spillover index, which quantifies the strength of interconnections between stocks. By applying the SIR model, the framework captures the temporal dynamics of contagion, enabling the identification of key factors that drive both the spread and containment of financial shocks.

The results contribute to the literature in three significant ways. First, it constructs financial networks for the TSX 60 using a methodology that captures directional spillovers between stocks, providing a nuanced view of market interconnectedness. Second, it proposes an analytical framework that enables the examination of contagion dynamics at multiple levels, including system-wide and sector-specific perspectives. Third, the research investigates the role of sectoral shocks, with a particular focus on the financial services sector in the TSX 60, which are shown to play a central role in amplifying or mitigating systemic risk.

The findings reveal how network topology, particularly edge weights and centrality measures, influences the pathways of contagion propagation. The insights provided by this study are of practical relevance to policymakers and market regulators, offering strategies for enhancing market resilience by identifying critical sectors and connections within financial networks.

The remainder of the chapter is organized as follows: Section 3.2 reviews the relevant literature on financial networks, epidemic modeling, and contagion analysis. Section 3.3

briefly discusses the data used in the study. Section 3.4 outlines the identification strategy and application of the SIR model to financial networks. Section 3.5 presents the empirical results, including insights from sector-specific shock simulations. Section 3.6 concludes with policy implications and recommendations for future research.

## **3.2 Literature Review**

Systemic risk captures that the potential for widespread financial instability arising from the failure of a single institution, triggering cascading effects across the financial system. In contrast to idiosyncratic risk, which impacts individual institutions in isolation, systemic risk emerges from structural interdependencies within financial markets, amplifying shock transmission and exacerbating market-wide disruptions. The 2008 global financial crisis serves as an example of systemic risk, wherein the collapse of Lehman Brothers initiated a chain reaction that destabilized global financial markets, demonstrating the far-reaching consequences of interconnected financial fragility. The literature on systemic risk in macro-finance has expanded considerably since the global financial crisis, with particular focus on how shocks propagate through interconnected institutions. From a theoretical perspective, Brunnermeier and Sannikov (2014) developed a dynamic stochastic general equilibrium (DSGE) model showing that systemic risk arises endogenously as a result of financial sector leverage and risk-taking behaviour. Their framework highlights that lower levels of exogenous risk can paradoxically lead to greater systemic fragility, as reduced perceived risk encourages higher equilibrium leverage. Tobias and Brunnermeier (2016) introduced the CoVaR (Conditional Value at Risk) methodology to quantify an individual institution's contribution to system-wide risk, with a particular focus on joint tail events. By measuring the value at risk of the financial system conditional on institutions being under distress, CoVaR provides a robust framework for assessing spillover risk. From a network-theoretic perspective, a growing body of research highlights the critical role of network topology in shaping systemic risk, emphasizing that the interconnectedness of financial institutions creates complex networks where shocks propagate and amplify losses across the system.

Financial markets are inherently structured as networks, where the degree of centrality and connectivity determines the extent to which systemic risk spreads. For instance, Diem et al. (2020) demonstrated that systemic risk in interbank loan networks can be mitigated by reorganizing the cross-sectional structure of financial contract networks. Their findings underscore that systemically important banks, characterized by their high levels of interconnectedness, contribute disproportionately to systemic fragility by accelerating financial contagion.

Network analysis (NA) is a methodological framework designed to examine the structural properties of systems through the representation of networks. These networks consist of two fundamental components: nodes and edges. A node symbolizes an analytic entity within the system under study; for instance, an individual may serve as a node within a social network, while a stock may function as a node within a stock market network. Edges, on the other hand, denote the presence of connections or relationships between nodes. Depending on the nature of these relationships, graphs can be categorized as either directed or undirected (Freeman, 2004).

In the context of a directed network, edges represent not only the connections between nodes but also the directionality of these relationships. The measurement of directional dependence is a crucial component in constructing directed networks. One potential approach to achieving this involves the use of econometric models, such as vector autoregression (VAR), to construct connectedness matrices. Diebold and Yilmaz (2009) introduced a forecast error variance decomposition (FEVD) method, enabling the application of VAR-based identifications in directed network analysis. However, a notable limitation of their original framework is its dependence on the ordering of variables. To address this, Diebold and Yilmaz (2012) and Diebold and Yilmaz (2014) adopted the generalized variance decomposition method, initially proposed by Sims (1980). This method is invariant to variable ordering, thus overcoming the shortcomings of the 2009 model. This refinement has significantly enhanced the applicability of the Diebold-Yilmaz (DY) framework in financial spillover analysis. Since its introduction, the DY framework has been widely utilized to study financial networks and spillover effects. For instance, Demiret et al. (2018) developed

a global bank network using FEVDs techniques and introduced the LASSO method in a high-dimensional network setting. Other notable contributions include Brunner and Hipp (2021), Demirer et al. (2018), Balcilar and Bekun (2020), and Meglioli and Gauci (2021), who have leveraged the DY framework to explore various dimensions of financial spillovers, demonstrating its versatility and robustness in the analysis of financial interconnectedness.

Analyzing stock market networks provides critical insights into the importance of individual sectors or stocks and the relationships that exist among them. However, tracking risk spillover during market shocks remains challenging. To better understand risk transmission and observe the response of specific sectors or stocks during a crisis, simulations of shocks on stock networks become essential.

One sound theoretical framework relevant to this domain is the SIR model (Susceptible, Infectious, Recovered), introduced by Kermack and McKendrick (1927) to study disease propagation. The SIR model monitors the temporal dynamics of susceptible, infectious, and recovered entities. Extensions to the SIR model, including the addition of immunity and exposure stages, have been comprehensively reviewed by Hethcote (2000), who also elaborates on the significance of the basic reproduction number,  $R_0$ . This parameter quantifies the average number of secondary infections caused by a single infected individual in an otherwise fully susceptible population. The SIR model has been successfully applied to simulate real-world epidemics, such as foot-and-mouth disease, SARS, and COVID-19 (see Ferguson et al. 2001, Ng et al. 2003, and Kudryashov et al. 2021). More recently, the SIR model has been integrated with network analysis to explore complex systems. Sottile et al. (2024) used the SIR model on a static, scale-free random network to simulate epidemic dynamics under varying initial conditions. Their findings underscore the interplay between disease contagiousness and network connectivity in shaping epidemic outcomes. Yuan et al. (2022) adapted the SIR model to study the spread and polarization of public opinion within dynamic social networks, highlighting the influence of structural network properties and dynamic interactions on the evolution of public opinion. In the context of financial systems, Bozhidarova et al. (2024) proposed an innovative application of the SIR model to simulate the propagation of financial crises within complex stock market networks. Their

model incorporates a multi-layer network structure, where each layer captures a specific type of stock connection. Their findings demonstrate that the interactions among companies within these networks significantly influence the spread of financial distress.

Both Chapter 1 and 2 were dedicated to the examination of undirected and directed networks and directed networks. In this chapter, we employ an epidemic model, the SIR model, in a stock network to investigate the crisis spillover. While Bozhidarova et al. (2024) utilize a multi-layer network structure to model various types of stock connections, this chapter constructs a single-layer weighted, directed network based specifically on the Diebold-Yilmaz spillover index to capture directional volatility transmission. Furthermore, the study explicitly investigates the role of sector-specific shocks and demonstrates how network centrality drives the asymmetry of financial contagion.

## **3.3 Data**

### **3.3.1 Data Collection**

Returns and volatility of stocks are potential metrics to capture relevant stock connections because they reflect forward-looking assessments of all available information (e.g., see Diebold and Yilmaz 2014). In this chapter, we collected the daily high, low and close stock prices with adjustment from January 2016 to September 2023, for the TSX 60 constituents. The data for Nutrien Ltd (NTR), Brookfield Asset Management Ltd (BAM-A), and Shaw Communications Inc (SJR-B) are excluded from the TSX 60 dataset due to incomplete observations over the sample period. For the network construction itself, the dataset spans from August 1, 2018, to February 29, 2020, ensuring a comprehensive representation of pre-pandemic market conditions. By identifying the edges of the network in its pre-pandemic market, we want to filter out noise and distortions introduced by the extreme volatility observed during the crisis, as many of these changes are transitory. It is generally recognized that a stock market crash unfolds too rapidly to induce lasting alterations in the underlying network structure, although the long-term impacts of such crises may eventually lead to

more permanent structural changes in the network. Therefore, the connections among stock returns during more stable periods are deemed to offer a more accurate representation of the systemic linkages that exist at the onset of a market collapse. The stock network is constructed based on logarithmic returns. The logarithmic return, commonly used to assess stock performance, is defined as  $R_{t,i} = \log P_{t+1,i} - \log P_{t,i}$  where  $P_{t,i}$  is the daily adjusted closing price of the stock  $i$  at time  $t$ . In addition, we calculate the Parkinson volatility for each stock. The annualized daily volatility of stock  $i$  at time  $t$  is defined as:

$$\sigma_{i,t} = 100 * \sqrt{365 * \underbrace{(0.361 * [\ln(P_{i,t}^{Max}) - \ln(P_{i,t}^{Min})])^2}_{\text{Part 1}}} \quad (3.1)$$

where  $\ln(P_{i,t}^{Max})$  is the log of the highest price of stock  $i$  on day  $t$ ;  $\ln(P_{i,t}^{Min})$  is the log of lowest price of stock  $i$  on day  $t$ ; Part 1 in Equation 3.1 is an estimator of the daily volatility that is originally proposed by Parkinson, where  $\frac{1}{4 \ln 2} = 0.361$  captures the relationship between the diffusion constant and the square of stock daily variation.<sup>1</sup> In addition, we use the log of annualized daily volatility for the construction of the DY network to ensure the validity of the normality assumption.

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<sup>1</sup>It is generally accepted that a random walk process provides a good approximation of the movement of the log of stock price. Parkinson (1980) investigates the relationship between the movement of the log stock price and the diffusion constant in a continuous random walk. The diffusion constant is the variance of the displacement of stock price after a unit time interval. The study shows that the extreme value estimate for the diffusion constant ( $D$ ) can be written as  $D = \frac{0.361}{n} \sum_{i=1}^n (\ln \frac{\ln(P_i^{Max})}{\ln(P_i^{Min})})$ , where  $n$  is the unit time interval (i.e.  $n = 1$  in our case).

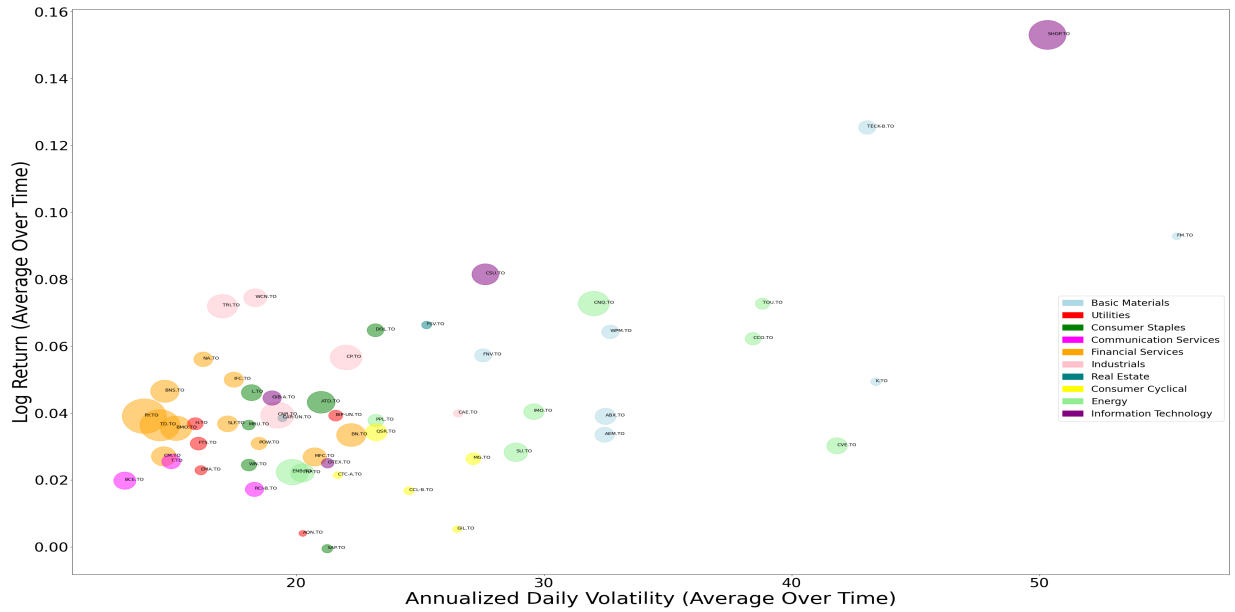


Figure 3.1: Annualized Daily Stock Volatility v.s. Stock Log Return The size of scatter represents the market capitalization size of entity. The time horizon: January, 2016 to September, 2023

Figure 3.1 illustrates the relationship between the average daily log return and volatility of stocks over the sample period. Stocks in the financial services sector (orange) exhibit relatively low volatility and similarly low returns, suggesting a more stable risk-return profile. In contrast, stocks in the energy sector (green) and basic materials sector (light blue) display greater dispersion, indicating higher variability in both risk and return. However, it is noteworthy that higher volatility does not always correspond to higher returns, particularly in the energy and basic materials sectors. These findings emphasize the importance of jointly analyzing return and volatility when evaluating a stock’s performance, as excessive volatility may not necessarily lead to superior returns.

### 3.3.2 Data Processing

In order to simulate the emergence and spread of a stock market crisis using the SIR model, it is essential to define its three key categories: Susceptible (S), Infected (I), and Recovered (R). In this context, an infected stock is one experiencing a period of intense financial distress, typically characterized by heightened volatility and exceptionally low returns (see Bozhidarova et al. 2024). When an infected stock exits this category, it is then labelled as recovered. Stocks that have not yet been infected are classified as susceptible.

Since a point-in-time value cannot accurately reflect stock performance trends or delineate periods of infection, we apply a moving average approach to capture their performance fluctuations. Specifically, a stock is identified as “infected” on day  $t$  if its moving average of daily volatility exceeds the 90th percentile of its historical volatility, while its moving average of log returns falls below the 10th percentile of its historical log returns over the entire sample period. This criterion ensures that a stock is identified as infected only when it experiences both heightened volatility and significant negative returns, capturing extreme market distress relative to its own historical performance.<sup>2</sup> A 14-trading-day window was selected to calculate the moving average, as this duration effectively captures key milestones in market reactions, providing insight into the progression and impact of the crisis.<sup>3</sup>

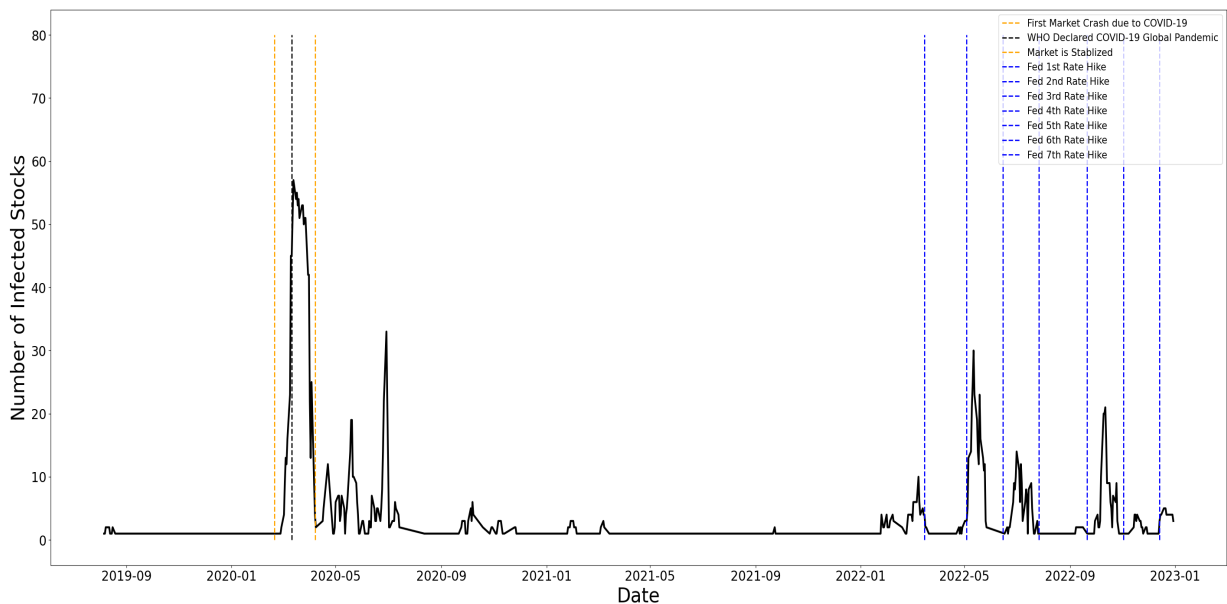


Figure 3.2: Number of Infected Stocks in TSX 60

Figure 3.2 illustrates the number of infected stocks over time within the TSX 60 index.<sup>4</sup> Vertical lines on the graph highlight key event dates, including significant milestones such as the onset of the global pandemic and the subsequent interest rate hikes. The graph

<sup>2</sup>Since the 10th percentile is computed from the volatility distribution over the period from January 1, 2016, to September 30, 2023, which includes stressed market conditions, the 10th percentile of log returns corresponds to negative returns.

<sup>3</sup>We tested 7, 14, and 24 trading-day windows. The 14 trading-day window demonstrated superior alignment with the identified key milestones. The threshold values are derived based on data collected from January 1, 2016, to September 30, 2023. The results are presented in Appendix H

<sup>4</sup>Figure 3.2 presents a snapshot of the evolution of infected stocks from June 30, 2019, to January 1, 2023. The threshold values are derived based on data collected from January 1, 2016, to September 30, 2023.

reveals that the number of infected stocks (as defined in our analysis) tended to increase around these pivotal events, reflecting the heightened market stress associated in these periods. To focus on capturing the rapid spread of the crisis over a condensed time-frame, the data period selected for this analysis is deliberately narrower than in previous chapters. Specifically, for the calibration of SIR model parameters, the data spans ten months of data, from January 1, 2020, to October 31, 2020, which allows us to isolate the crisis dynamics without the confounding effects of interest rate hikes in the post COVID-19 period. In addition, the infection period for an individual stock is defined as the longest series of consecutive days during which the stock maintains its infected status. Figure 3.3 illustrates the number of infected stocks over the selected time period, January to October 2020.<sup>5</sup>

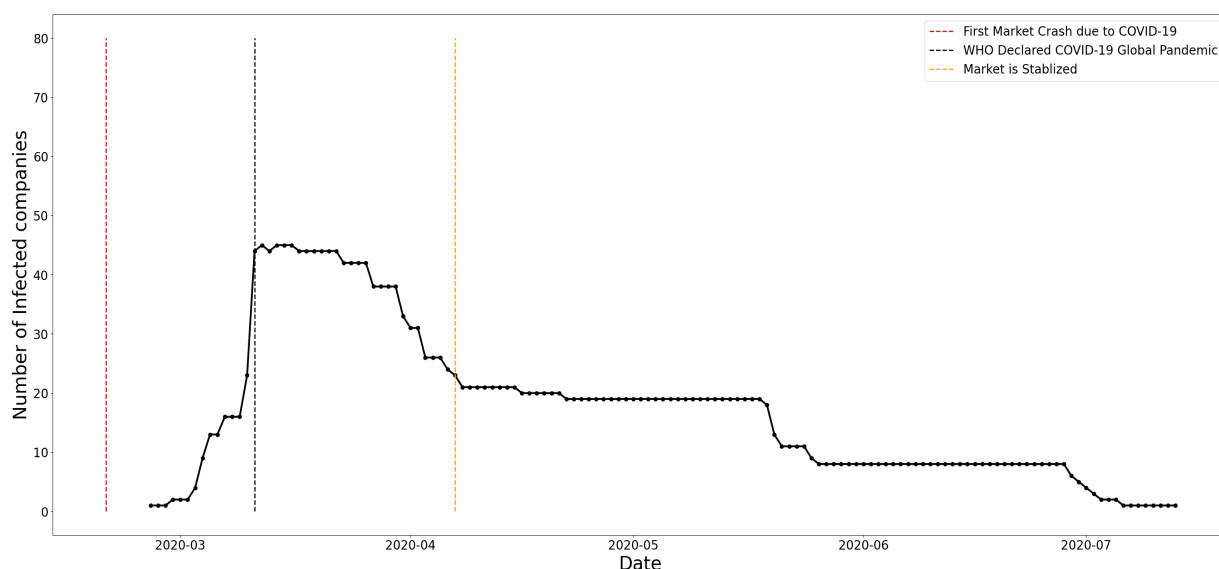


Figure 3.3: Number of Infected Stocks in TSX 60 - Jan to Oct 2020

<sup>5</sup>Figure 3.3 adopts the definition of infection periods as the longest sequence of consecutive days during which a stock remains in an infected state, which results in slight variations in the observed peaks compared with Figure 3.2.

## 3.4 Identification Strategy

### 3.4.1 SIR Model - without network

The SIR model is a type of compartmental model which consists of three compartments, Susceptible ( $S$ ), Infected ( $I$ ), and Recovered ( $R$ ), respectively. The model uses a set of differential equations to understand the spread of infectious diseases. In this chapter, we employ the SIR model to explore the propagation of a financial contagion in a stock market from initially infected stocks (as defined in Section 3.3.2).<sup>6</sup> The population is a fixed number ( $N$ ) of stocks in the TSX 60 index. The model assumes that  $N = S(t) + I(t) + R(t)$ , which means that a stock is in one of these three categories in any time ( $t$ ). Figure 3.4 illustrates the transition of stock status within the SIR model framework. The parameter  $\beta$  represents the transmission rate, while parameter  $\gamma$  denotes the reciprocal of the average duration (in days) that a stock remains in the infected status.<sup>7</sup> The reproduction number  $R_0$  is defined as  $\frac{\beta}{\gamma}$ , which measures the expected number of new “infected” stocks (secondary cases) from a single “infected” stock in an otherwise completely susceptible population (i.e., at the outset of the financial contagion). A higher  $R_0$  indicates a greater potential for risk propagation within the market, reflecting the extent to which financial distress spreads through the stock network.

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<sup>6</sup>Each stock is assumed to occupy one of three mutually exclusive states, namely susceptible, infected, or recovered, where infection is characterized by extreme downside returns accompanied by elevated volatility and recovery reflects a return to relatively stable market conditions. The stock population is treated as fixed over the simulation horizon, abstracting from entry, exit, and delisting, which is reasonable given the short crisis window used for parameter calibration.

<sup>7</sup>For example, if a stock takes 5 days to be recovered, then  $\gamma$  is  $\frac{1}{5} = 0.2$ .



Figure 3.4: SIR Model Transition Flow Chart

To derive the ordinary differential equations (ODE), we first examine the daily transition dynamics within the SIR framework. The probability that a stock becomes “infected” on any given day is the product of the transmission rate ( $\beta$ ) and the likelihood that a contact is infected, expressed as:

$$\beta \frac{I}{N},$$

where  $I$  denotes the number of “infected” stocks, and  $N$  represents the total number of stocks. The number of new infections per day, among all susceptible stocks, is :

$$\beta \frac{I}{N} S = \frac{\beta}{N} IS,$$

where  $S$  is the number of susceptible stocks. The infection process occurs when a susceptible stock “encounters” an infected stock and becomes infected with probability  $\beta$ , which determines the rate of contagion within the market.<sup>8</sup> On the other side, the number of infected stocks decreases as stocks recover over time, with the daily recovery rate given by  $\gamma I$ , where  $\gamma = 1 / \text{number of infectious days}$ , represents the probability of recovery per infected stock. This dynamic interaction between infection and recovery governs the

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<sup>8</sup>In the context of financial contagion, the strict application of the SIR model to simulate the transmission of crises presents certain limitations in terms of robustness. A key challenge is the definition of “encounters” within the SIR framework without a stock network. While it is theoretically possible for the failure of one stock to precipitate the collapse of another, such transmission should be predicated on the existence of a strong and meaningful connection between the two entities. In the absence of such linkages, the impact of a stock’s failure on an unrelated or weakly connected stock is minimal. Therefore, capturing the connectedness among stocks is critical to improving the reliability of contagion simulations. In Section 3.4.2, we provide a detailed discussion on the methodology used to measure the strength and directionality of connections between stocks.

evolution of financial contagion, shaping the spread and resolution of market distress. For each state, the ODE can be written as the difference between inflow and outflow, such as :

$$\frac{dS}{dt} = -\frac{\beta}{N}IS, \quad (3.2)$$

$$\frac{dI}{dt} = \frac{\beta}{N}IS - \gamma I, \quad (3.3)$$

$$\frac{dR}{dt} = \gamma I. \quad (3.4)$$

Furthermore, since  $N = S + I + R$  and  $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$ , then we have  $N = S(t) + R(t) + I(t)$ . A financial contagion occurs when the number of infected stocks increases over time, which is expressed from Equation 3.3 as:  $\frac{\beta IS}{N\gamma} > I$ . At the outset of an epidemic, nearly all stocks are susceptible, meaning that  $N \simeq S$ . Substituting  $S = N$  in the inequality above, we have that:  $\frac{\beta}{\gamma} = R_0 > 1$ , where  $R_0$  is the reproduction number. When  $R_0 > 1$ , financial contagion is self-sustaining, leading to the continued spread of market distress. Conversely, if  $R_0 < 1$ , the contagion eventually dies out as the infection fails to propagate effectively.

The calibration of parameters  $\beta$  and  $\gamma$  is informed by empirical data, ensuring that the SIR model accurately reflects observed market dynamics. In Section 3.3.2, we outlined the criteria for defining an infectious stock. Parameter optimization is conducted by minimizing the discrepancy between empirical observations and the analytical predictions generated by the ODE of the SIR model.<sup>9</sup> Specifically, Figure 3.5 displays the fitting plot for the TSX 60 market, where the calibrated values are  $\beta = 0.3613$  and  $\gamma = 0.0225$ . Based on these estimates, the implied reproduction number is calculated as:

$$R_0 = \frac{\beta}{\gamma} = \frac{0.3613}{0.0225} = 16.06. \quad (3.5)$$

This estimate implies that, at the initial stage of contagion, the average infected stock induces financial distress in roughly 16 additional stocks under baseline conditions, indicating a pronounced capacity for systemic propagation within the TSX 60 network over the sample period.

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<sup>9</sup>A detailed calibration approach is presented in Appendix H.1

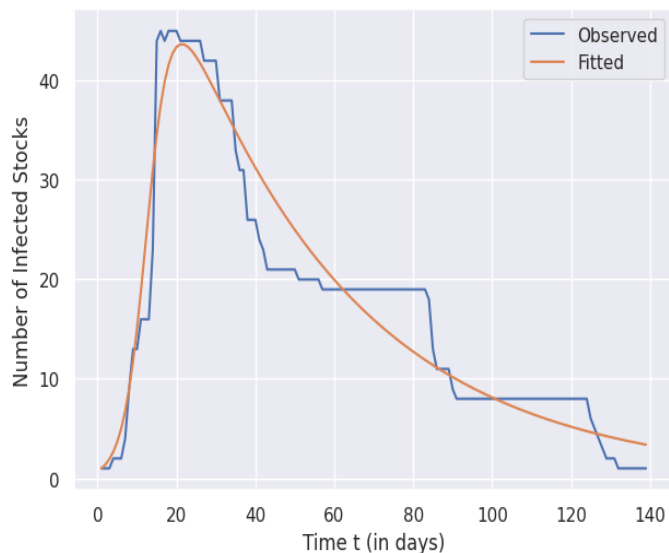


Figure 3.5: SIR Model Parameters Calibration - TSX 60 ( $\beta = 0.3613$  and  $\gamma = 0.0225$ ) The first stock infection was identified on 2020-02-26, and the last infection was on 2020-07-13.

### 3.4.2 Network Construction

#### Edge Identification

In this chapter, we examine the dynamics of financial contagion propagation within directed networks. The methodology for edge identification closely aligns with the approach outlined in Chapter 2. To construct the stock network, we utilize the Diebold-Yilmaz spillover index, which provides complementary measures of directional influence and interconnectedness.

**Diebold-Yilmaz (DY) Spillover Index** Vector Autoregression (VAR) is a conventional econometric model in which each variable is described by an equation that incorporates its own lagged values, the lagged values of other variables, and an error term. The  $p$ th-order VAR (VAR(P)) model is written as:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t. \quad (3.6)$$

Here  $c$  denotes an  $(n \times 1)$  vector of constants, and  $A_i$  a time-invariant  $(n \times n)$  matrix for  $j = 1, 2, \dots, p$ . The  $(n \times 1)$  vector  $e_t$  is a vector generalization of white noise:

$$E(e_t) = 0 \quad (3.7)$$

$$E(e_t e'_\tau) = \begin{cases} \Sigma & \text{for } t = \tau \\ 0 & \text{otherwise.} \end{cases} \quad (3.8)$$

For example, the VAR(1) two-variable model is defined as:

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + e_t, \quad (3.9)$$

where  $\mathbf{y}_t = (y_{1,t}, y_{2,t})$  and  $\mathbf{A}$  is a  $2 \times 2$  coefficients matrix. Assume the data covariance is stationary, the moving average representation of VAR exists and is defined as:

$$\mathbf{y}_t = \Theta(L)e_t,$$

where  $\Theta(L) = (\mathbf{I} - \Phi L)^{-1}$ . Let  $\mathbf{Z}(L) = \Theta(L)\mathbf{Q}_t^{-1}$ ,  $\epsilon_t = \mathbf{Q}_t e_t$ ,  $E(\epsilon_t \epsilon'_t) = \mathbf{I}$ , and  $\mathbf{Q}_t^{-1}$  be the lower-triangular Cholesky factor of the covariance matrix of  $e_t$ . Then we have  $\mathbf{y}_t = \mathbf{Z}(L)\epsilon_t$ .

Now consider the 1-step ahead forecast. Based on Equation 3.9, it can be written as:

$$\mathbf{y}_{t+1} - \hat{\mathbf{y}}_{t+1|t} = \mathbf{Z}_0 \epsilon_{t+1} = \begin{bmatrix} z_{0,11} & z_{0,12} \\ z_{0,21} & z_{0,22} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \end{bmatrix},$$

where the covariance matrix is:

$$E(\mathbf{e}_{t+1,t} \mathbf{e}'_{t+1,t}) = \mathbf{Z}_0 \mathbf{Z}'_0.$$

Diebold and Yilmaz (2009) define that  $z_{0,21}^2$  quantifies the contribution from the  $\mathbf{y}_{1,t}$  shocks to the forecast error variance of  $\mathbf{y}_{2,t}$ . Similarly,  $z_{0,12}^2$  measures the contribution from the  $\mathbf{y}_{2,t}$  shocks to the forecast error variance of  $\mathbf{y}_{1,t}$ .

Although Cholesky factorization ensures orthogonality, the variance decomposition is

sensitive to the ordering of variables in the VAR model. To address this limitation, Diebold and Yilmaz (2012) and Diebold and Yilmaz (2014) adopt the generalized VAR framework, as proposed by Pesaran and Shin (1998) and Koop et al. (1996), to compute an order-invariant variance decomposition. This approach leverages the historically realized error distribution to effectively account for correlated shocks. The H-step ahead forecast error variance decomposition ( $\mathbf{d}_{ij}^g(H)$ ) under the generalized VAR framework for  $N$  stocks is written as:

$$\mathbf{d}_{ij}^g(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (i'_i \theta_h \Sigma i_j)^2}{\sum_{h=0}^{H-1} (i'_i \theta_h \Sigma \theta'_h i_i)}, \quad (3.10)$$

where  $\Sigma$  is the variance matrix for the error vector ( $e_t$ ),  $\sigma_{jj}$  is the standard deviation of the error term for the  $j$ th variable equation, and  $i_j$  is a selection vector to select the  $j$ th element. Since  $\sum_{j=1}^N \mathbf{d}_{ij}^g(H) \neq 1$ , Diebold-Yilmaz (DY) framework normalizes the variance decomposition matrix as:

$$\tilde{\mathbf{d}}_{ij}^g(H) = \frac{\mathbf{d}_{ij}^g(H)}{\sum_{j=1}^N \mathbf{d}_{ij}^g(H)},$$

where  $\sum_{j=1}^N \tilde{\mathbf{d}}_{ij}^g(H) = 1$  and  $\sum_{i,j=1}^N \tilde{\mathbf{d}}_{ij}^g(H) = N$ .

In this chapter, we utilize both directional values for the DY spillover index, rather than the net values employed in Chapter 2. This approach enables capturing the mutual influences between pairs of stocks, thereby providing a more comprehensive representation of risk transmission within the network. The VAR model uses a lag of 1, as we assume that the daily stock return is mainly affected by previous day's return. Table 3.1 summarizes the DY spillover index for the alphabetically first 19 constituents of the TSX 60, providing insights into the connection dynamics within the network. Figure 3.6 illustrates the DY spillover relationship between ABX (Barrick Gold) and AEM (Agnico Eagle Mines) based on the information in Table 3.1. Both ABX and AEM exhibit strong self-influence, with more than 50% of their forecast error variance explained by shocks originating from themselves, indicating a high degree of idiosyncratic volatility. The Diebold-Yilmaz (DY) spillover index reveals that 21.5% of AEM's forecast error variance is attributable to shocks originating from ABX, while 21.9% of ABX's forecast error variance is driven by shocks from AEM. This bidirectional spillover effect suggests that ABX and AEM are intercon-

nected, likely due to their similar exposure to commodity price fluctuations, macroeconomic conditions, and sector-specific factors.

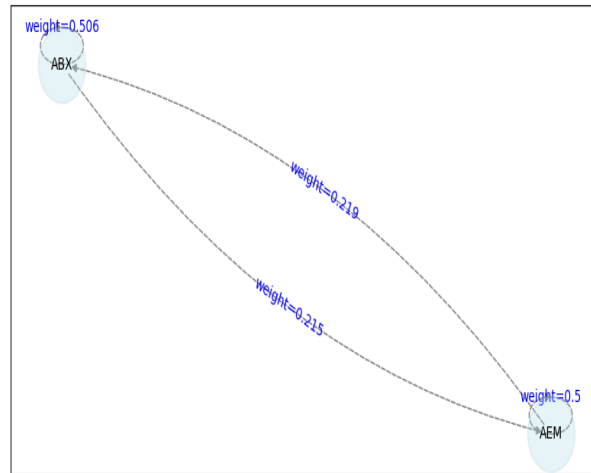


Figure 3.6: Table 3.1 DY Spillover Index Illustration

### SIR Network Integration

To investigate the propagation of financial contagion within directed networks, it is necessary to integrate the SIR model with the network constructed using the aforementioned edge identification strategy. Kiss et al. (2017) provide a comprehensive review of the application of epidemic models in network studies. In this chapter, we adopt an event-driven approach, as discussed in their work, to enhance simulation efficiency.

Rather than determining the next event at each time step by evaluating all possible events, this approach streamlines the process by calculating the timing of key events when a node becomes infected. A fundamental characteristic of the infection process is that, once a node is infected, its recovery time, the selection of nodes to which it transmits the infection, and the timing of those transmissions are independent of any subsequent actions taken by its neighboring nodes. Consequently, upon infection, both the recovery time and the transmission schedule can be deterministically computed and stored as discrete events in a priority queue, which processes events in chronological order. During each iteration of the simulation, the event with the earliest timestamp is dequeued and executed. If the event corresponds to a transmission, additional events are generated and added to the

queue, representing both the recovery of the infected node and subsequent transmission events affecting its neighboring nodes. This iterative process continues until either a predefined simulation time is reached or the priority queue is emptied, indicating that no further infections persist within the system. By adopting this event-driven simulation approach, computational efficiency is significantly enhanced while maintaining an accurate representation of the temporal evolution of financial contagion within the network (Kiss et al., 2017). In addition, the simulation is based on two key assumptions: (i) transmission from an infected node to a susceptible node occurs along an edge as a Poisson process with rate  $\beta$ , and (ii) the recovery of an infected node is also modeled as a Poisson process with rate  $\gamma$ . Furthermore, since infection and recovery events are modeled as Poisson processes, their inter-arrival times are independent and follow an exponential distribution. This implies that both the time to infection and the time to recovery are exponentially distributed, ensuring a memoryless process, where the probability of an event occurring depends only on the present state and not on past events.<sup>10</sup> These assumptions provide a tractable mathematical framework for modeling financial contagion dynamics. The auxiliary functions for the event-driven simulation is defined in Algorithm 2:

---

<sup>10</sup>Although the trigger of an event follows a memoryless process, the temporal dependencies, such as autocorrelation and volatility clustering, are implicitly captured through the network structure itself, particularly because the Diebold-Yilmaz (DY) spillover index is derived from a VAR(1) process, which accounts for lagged relationships in return dynamics. Thus, while the model assumes that the infection process is memoryless at the individual node level, the actual transmission of contagion is governed by the topology of the network.

---

**Algorithm 2:** Transition - SIR model single node ( $u$ )

---

**input** : Network ( $G$ ), Event Node ( $u$ ), Event time ( $t$ ), transmission rate ( $\beta$ ),  
recovery rate ( $\gamma$ ),  $times$ , S, I, R, Q, maximum time ( $t_{max}$ )

```
1 Function transition(*):
2    $u(status) \leftarrow infected$ 
3    $u.rec(t) \leftarrow t + exponential\ variate(\gamma)$ 
4   if  $u.rec(t) < t_{max}$  then
5     event  $\leftarrow \{node : u, time : u.rec(t), recovered\}$  ;
6     add event to  $Q$  ;
7   end
8   for  $v \in G.neighbors(u)$  do
9     if  $v(status) = susceptible$  then
10       $infected\ time \leftarrow t + exponential\ variate(node.weight * \beta)$ 
11      if  $infected\ time < \min(u.rec(t), v(pred\ infected\ time), t_{max})$  then
12        event  $\leftarrow \{node : v, time : infected\ time, infected\}$  ;
13        add event to  $Q$  ;
14         $v(pred\ infected\ time) \leftarrow infected\ time$  ;
15      end
16    end
end
```

---

Then the event-driven simulation pseudocode is designed as:

---

**Algorithm 3:** Event-driven Simulation - SIR model

---

```

input : Network  $G$ , transmission rate  $\beta$ , recovery rate  $\gamma$ , set of initial node(s)  $I_0$ ,
        maximum time  $t_{max}$ 
output:  $S, I, R$ , time
1 Function SIR simulation(*):
2    $times, S, I, R \leftarrow [0], [G.nodes], [0], [0]$  ;
3    $Q \leftarrow Empty$  ;
4   for  $u$  in  $G.nodes$  do
5      $u(status) \leftarrow susceptible$  ;
6      $u(pred\ infected\ time) \leftarrow \infty$  ;
7   end
8   for  $u$  in  $I_0$  do
9     event  $\leftarrow \{node : u, time : 0, infected\}$  ;
10     $u(pred\ infected\ time) \leftarrow 0$  ;
11    add event to  $Q$  (ordered by time) ;
12  end
13  while  $Q$  is not empty do
14    Event  $\leftarrow$  earliest remaining event in  $Q$  ;
15    if Event(action) = Infected then
16      if Event.node.status = susceptible then
17        transition( $G$ , Event.node, Event.time,  $\beta, \gamma, times, S, I, R, Q, t_{max}$ )
18        (Outlined in algorithm 2)
19      end
20    else
21      append  $times, S, I, R$  with Event.time,  $S.last, I.last-1$ , and  $R.last+1$  ;
22      Event.node(status)  $\leftarrow$  recovered
23    end
24  end
return  $times, S, I, R$ 

```

---

To recap, the DY spillover index is used to construct a TSX 60 stock network, capturing both relationships and directional dependencies among its constituents. This network serves as the foundation for implementing the SIR model to examine the transmission of financial contagion within the market. To approximate real-world conditions, the directed network is constructed using stock return data from the pre-COVID-19 period (August 1, 2018, to March 1, 2020). This time frame represents a relatively stable economic environment, providing a baseline for analyzing contagion dynamics and serving as a benchmark for comparison with crisis periods. Figure 3.7 presents a heatmap illustrating the network

structure of TSX 60 stocks based on the DY spillover index. The left panel depicts the full network, displaying all observed connections, while the right panel presents the network where a threshold has been applied to filter out insignificant connections.<sup>11</sup> This threshold, set at the 50th percentile of DY spillover index values (0.016), ensures that only meaningful interactions are retained in the analysis. After applying the threshold, the network is reduced to 1,596 significant connections, effectively isolating the dominant spillovers within the system. Table 3.2 provides a detailed summary of the first 19 stocks already mentioned in previous section, displaying only those edges with attributes exceeding the threshold value, thereby offering a clearer representation of key contagion pathways within the TSX 60 network. A detailed analysis of financial contagion transmission within the TSX 60 network, integrating both the SIR model and the DY spillover index framework, is presented in Section 3.5.

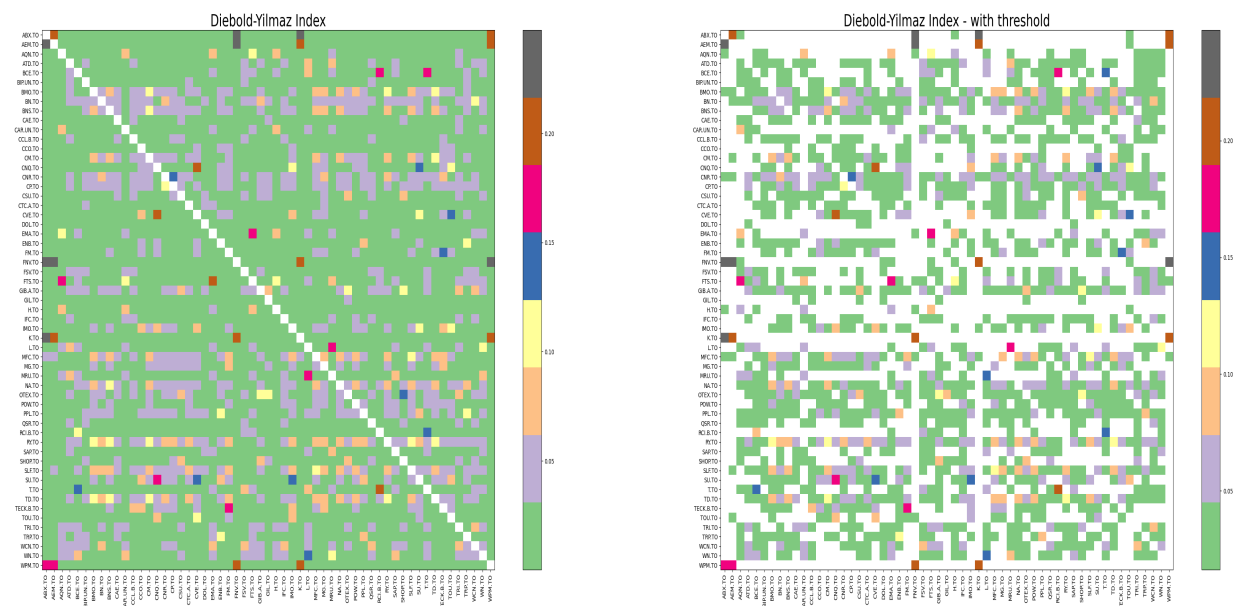


Figure 3.7: DY Spillover Index Heatmap (Prior to the Crisis) Note: the threshold of the Left Panel is 0.016.

<sup>11</sup>The self connections are removed from the analysis, as secondary infection in the SIR model is only transmitted through the connections between infectious and susceptible stocks. Once a stock becomes infected and subsequently recovers, it cannot be infected again within the framework of the SIR model. Therefore, including self-connections would not provide meaningful insights into the contagion dynamics being modeled.

## 3.5 Empirical Results

### 3.5.1 Network Simulation

As shown in Figure 3.3, the contagion originates from a single stock and propagates over a span of 139 days. Accordingly, we simulate the transmission of financial contagion by initiating the process on the first day of the 139-day horizon with a randomly selected infectious stock. To evaluate the propagation of crises within the TSX 60 network, a series of 10,000 simulations was conducted assuming the network-based SIR model described in Section 3.4.2. The simulations utilize the calibrated parameter values of  $\beta = 0.3613$  and  $\gamma = 0.0225$  (derived in Section 3.4.1). These simulations, which require significant computational resources, aim to model the dynamics of crisis propagation through a weighted, directed network. Both the edge weights and the total number of connections are critical factors influencing the spread of crises. Therefore, prior to initiating the simulation process, the network is assessed to determine whether it could be further simplified by modifying the threshold, as introduced in the discussion surrounding Figure 3.7, without sacrificing generality.

Figure 3.8 presents the average results of 10,000 simulations in various scenarios in which the TSX 60 network is constructed using alternative threshold values beyond the 50th percentile. As the threshold increases, the total number of edges and connections decreases, and in some cases, bilateral connections are replaced by unidirectional links due to differences in edge weights (i.e., when the edge from stock  $i \rightarrow j$  is above the threshold, while the edge from  $j \rightarrow i$  is below it). Three specific thresholds are examined: the 20th percentile threshold, which retains a larger number of connections and preserves weaker relationships; the 50th percentile threshold, which strikes a balance between filtering out weaker links while maintaining key interactions (with simulation results summarized in Table 3.2); and the 80th percentile threshold, which retains only the strongest connections, significantly reducing network density but potentially omitting important spillover effects. Figure 3.8a illustrates the average number of susceptible stocks over time for these scenar-

ios, where the solid purple line represents the empirical data, serving as a benchmark for comparison. The yellow dotted line, corresponding to the 80th percentile threshold (retaining only 20% of edges), deviates significantly from the empirical data. This divergence is primarily due to the high threshold value, which eliminates a substantial number of connections and frequently converts previously bidirectional links into unidirectional ones, thereby limiting the propagation of financial contagion within the network. In contrast, the blue dotted line, representing the full network (No threshold as in Figure 3.7, left panel), aligns closely with the empirical observation. Similarly, the red dotted line, corresponding to the 20th percentile threshold, follows a pattern nearly identical to the full network, which is expected given that this lower threshold retains the majority of edges, ensuring that most spillover effects remain intact. The observations in Figure 3.8a are consistent with those in Figures 3.8b and 3.8c, which illustrate the evolution of infection and recovery cases. Among the three thresholds, the green dotted line, representing the 50th percentile threshold, achieves the best alignment with the empirical data in Figures 3.8a and 3.8c. Although the peak of infections is slightly underestimated compared to the full network and the 20th percentile threshold in Figure 3.8b, the 50th percentile threshold strikes an effective balance between accuracy and efficiency. By halving the number of weak connections, this threshold significantly reduces the computational burden while preserving simulation results that closely resemble those of the full network, making it an optimal choice for modelling financial contagion dynamics. In addition, we summarize the root mean squared error (RMSE) and coefficient of determination ( $R^2$ ) between the empirical infection and simulation results in Table 3.3. The results indicate that the network constructed using the 50th percentile threshold, with 50% of the total number of edges, achieves a comparable fit to that of the network retaining 80% of edges (i.e., the 20th percentile threshold). In contrast, the network constructed with only 20% of total edges (i.e., the 80th percentile threshold) performs significantly worse, exhibiting a noticeably higher RMSE and lower  $R^2$ . These goodness-of-fit metrics underscore that the 50th percentile threshold strikes an effective balance between model accuracy and computational efficiency, preserving key contagion pathways while eliminating redundant or weak connections that contribute little

to the overall dynamics.

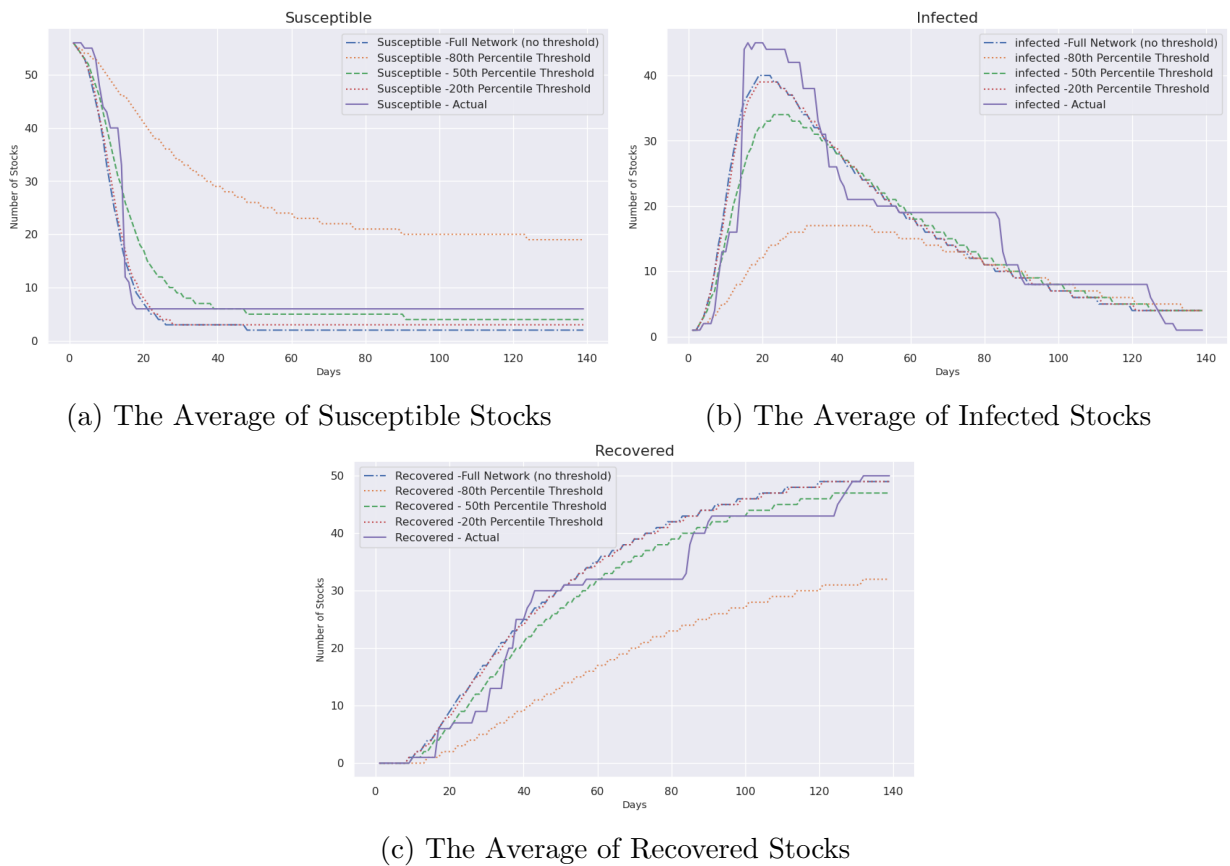


Figure 3.8: TSX 60 Network Selection ( $\beta = 0.3613$  and  $\gamma = 0.0225$ )

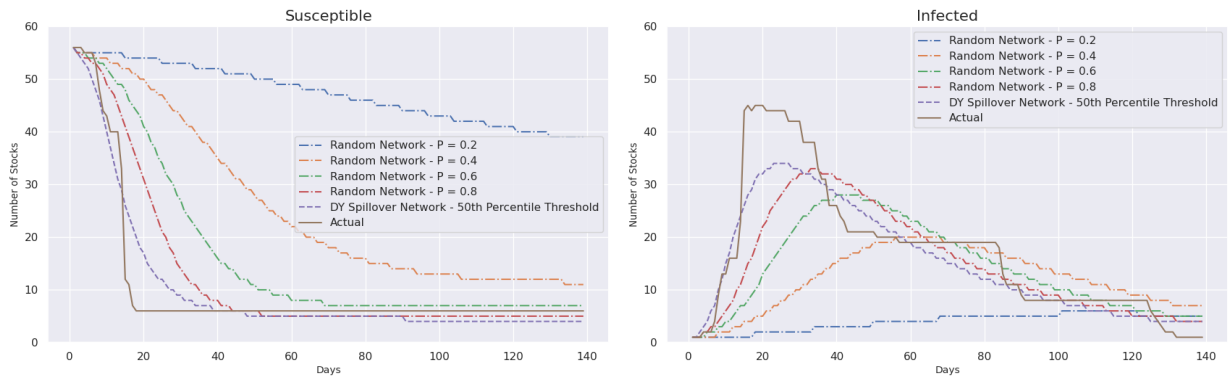
To further demonstrate that the DY spillover index contains accurate information on the strength of connections among the TSX 60 stocks, we also simulate the SIR model on a random graph, also known as Erdős-Rényi graph. The number of nodes ( $n$ ) in the random graph matches the number of stocks in the TSX 60, while the number of edges is given by a predetermined probability ( $p$ ). A directed random graph algorithm chooses the  $n(n - 1)$  possible edges with probability  $p$  and the edge weights are randomly generated from a scaled standard lognormal distribution.<sup>12</sup> Figure 3.9 illustrates the simulation results obtained from various network configurations. The green dotted line, representing a random network with  $p = 0.6$ , represents the propagation of the crisis within a network containing a comparable number of edges (i.e. 1885 edges) to the DY spillover network at the 50th percentile threshold (i.e. 1596 edges). However, the average of 10,000 simulations

<sup>12</sup>The lognormal distribution is selected because the weight must be positive. Furthermore, the scale parameter is calibrated to replicate the statistical characteristics of the DY spillover index, aligning with its observed weight distribution.

for the random network fails to replicate the actual empirical patterns. This discrepancy underscores the critical role of the DY spillover index, which encapsulates essential information about the strength of connections among the TSX 60 stocks. Although the random network with  $p = 0.8$  shows an improved performance compared to the random network with lower values of  $p$ , the DY spillover network with a 50th percentile threshold remains superior. This is due to its greater efficiency in capturing real stock connections, as it contains fewer edges than the random network with  $p = 0.8$  while still accurately reflecting market dependencies. Additionally, the 50th percentile DY spillover network more precisely captures the infection peak, whereas the random network exhibits a delayed peak, as illustrated in Figure 3.9b.

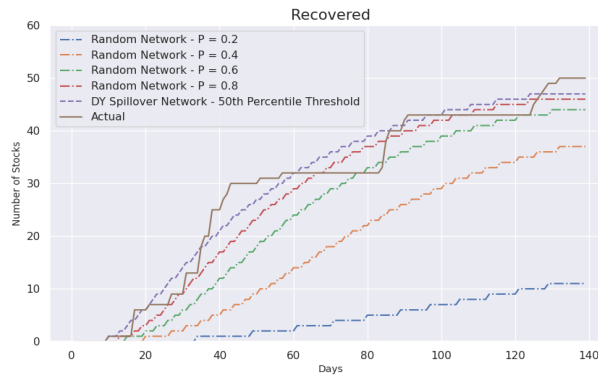
Even if an increase in  $p$  (i.e., an increase in the number of edges) in the Erdős-Rényi networks enhances the performance of simulations, these random networks consistently fail to replicate empirical patterns with accuracy. A plausible explanation for this discrepancy is that a higher edge density introduces additional potential infection pathways, leading to an overly generalized propagation dynamic that does not adequately capture the complexities of real-world financial connections. As a result, the random graph remains a simplistic approximation, incapable of emulating the nuanced contagion dynamics observed in empirical data. Table 3.4 summarizes the goodness-of-fit metrics used to evaluate the performance of various network configurations. The results shows that the DY spillover network constructed using the 50th percentile threshold yields the highest  $R^2$  and lowest RMSE, outperforming all other random networks. Notably, random networks with connection probabilities ( $p$ ) below 0.6 exhibit negative  $R^2$  values, suggesting that their simulated outcomes are less accurate than a naive model, such as a horizontal line representing the mean of the observed data. These findings highlight the importance of utilizing the DY spillover index measures in the stock network rather than relying on purely random connectivity assumptions when modeling financial contagion dynamics. Incorporating data-driven network structures ensures that the model more accurately reflects real-world market dependencies, improving its ability to capture systemic risk and contagion transmission with greater precision. This approach is both intuitive and effective, as the DY

spillover index encapsulates the strength of connections, enabling the simplified network to retain the most significant relationships. The 50th percentile threshold provides a practical and robust method for optimizing simulations of crisis propagation while maintaining close alignment with observed patterns. Therefore, in the subsequent analysis, this threshold is adopted for the SIR—DY spillover network.



(a) The Average of Susceptible Stocks

(b) The Average of Infected Stocks



(c) The Average of Recovered Stocks

Figure 3.9: TSX 60 Network v.s. Random Network ( $\beta = 0.3613$  and  $\gamma = 0.0225$ )

### 3.5.2 Sensitivity Analysis

In addition, we investigate the sensitivity of average simulation paths to variations in the infection rate  $\beta$  and recovery rate  $\gamma$ . In the first scenario, we assume  $\beta$  values ranging from 20% below to 20% above the initial calibrated rate ( $\beta = 0.3613$ ) while keeping  $\gamma$  fixed. Figure 3.10 illustrates the sensitivity results. The purple line (highest infection rate) corresponds to  $\beta = 0.43355$ , while the blue line (lowest infection rate) represents the results for  $\beta = 0.28904$ . Although the spread between the highest and lowest  $\beta$  values is

approximately 50%, the peak infection rate differs by only 9% point.<sup>13</sup> This indicates that an increase in the propagation rate does not result in a proportional change in the number of infections. In all scenarios, the calibrated recovery rate ( $\gamma$ ) is set at 0.0225, implying that an infected stock takes an average of 44 days to recover. As depicted in Figure 3.10, the number of infections is initially higher when  $\beta$  is high during the first 44 days. However, once stocks begin recovering after day 44, the trend reverses. A higher  $\beta$  results in a more substantial accumulation of infectious stocks prior to the recovery phase, leading to a sharper decline in infections once recovery begins. Since the SIR model assumes that recovered stocks cannot be reinfected, the recovery dynamics influence the shape of the infection curve. Under the same recovery rate, a lower infection rate produces a flatter tail in Figure 3.10, reflecting a more gradual decline in the number of infections over time.

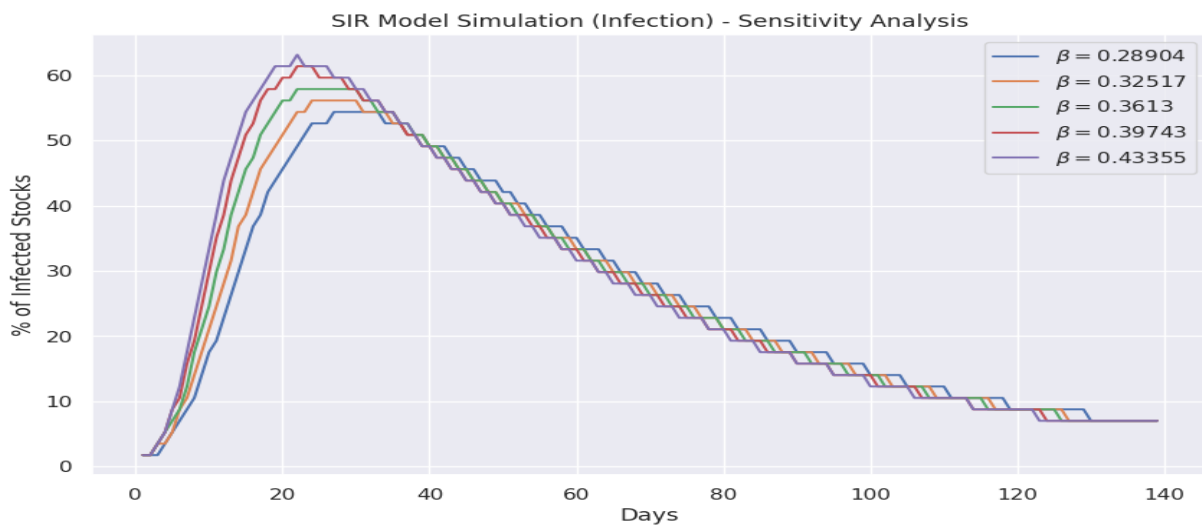


Figure 3.10: TSX 60 DY Spillover Index Network -  $\beta$  Sensitivity Analysis ( $\gamma = 0.0225$ )

In the second sensitivity scenario, we assess the sensitivity of simulation results to both  $\beta$  and  $\gamma$  jointly within the TSX 60 network. Figure 3.11 presents the simulation results, illustrating how varying these parameters influences the percentage of infected stocks over time. The green line represents the baseline scenario :  $\beta = 0.3613$  and  $\gamma = 0.0225$ . To examine the effect of an increased recovery rate, we raise  $\gamma$  to 0.04 (red line). The results indicate, as expected, a notable decrease in peak infection levels compared to the green line, reinforcing the conclusion that a faster recovery rate significantly mitigates peak contagion

<sup>13</sup>The peak infection rate for  $\beta = 0.28904$  is 54%, and the peak infection rate for  $\beta = 0.43355$  is 63%

intensity. Additionally, the results highlight a distinct relationship between recovery rates and infection dynamics. Regardless of the infection rate, the number of infections declines rapidly as the recovery rate increases. This is evidenced by the pink and red lines in Figure 3.11, which both feature high recovery rates. Despite variations in infection rates, these curves exhibit a sharp decline in infection levels, converging toward zero within a relatively short period. This finding underscores the dominant role of recovery rates in shaping the trajectory of financial contagion, suggesting that enhanced resilience mechanisms, such as improved liquidity measures or policy interventions, could significantly curb systemic risk in financial markets.

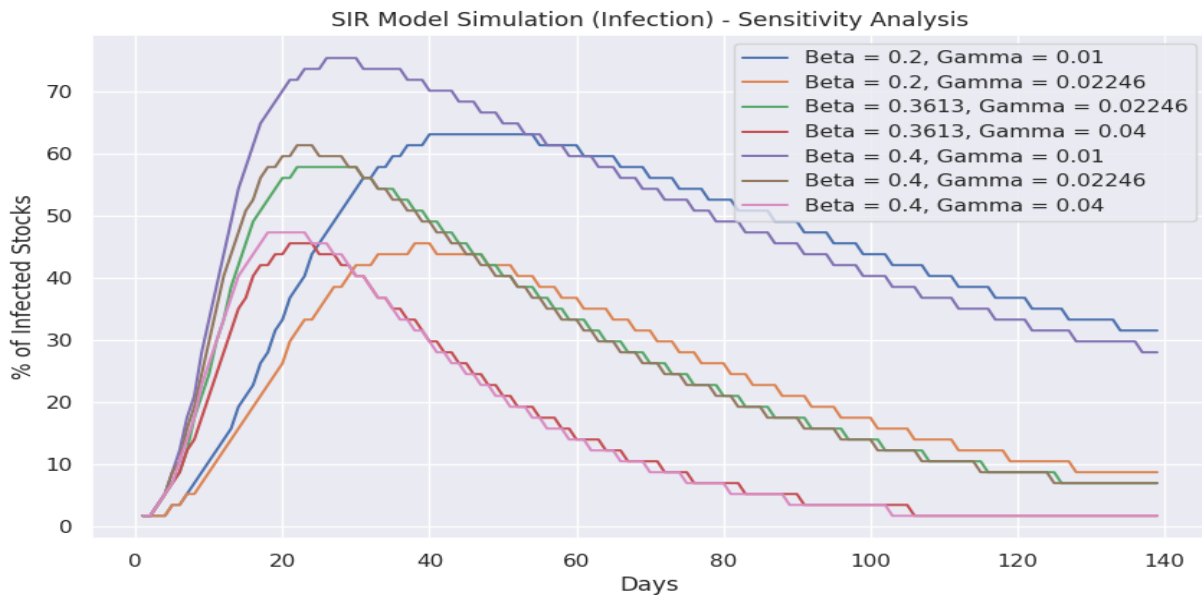


Figure 3.11: TSX 60 DY Spillover Index Network -  $\beta$  &  $\gamma$  Sensitivity Analysis

### 3.5.3 Calibration versus Ex Post Fitted Values

Figure 3.12 reconsolidates Panels (a) - (c) of Figure 3.8 and compares the simulation results for S, I, and R, based on the DY network with 50th percentile threshold and calibrated values,  $\beta = 0.3613$  and  $\gamma = 0.0225$ , with original empirical observations at the aggregated level. Each round of simulations randomly selects one financial “crisis-triggering” stock on day 1. As there are 10,000 rounds, this means that 10,000 initial triggers are randomly chosen across the TSX 60, covering all sectors. The model then simulates financial contagion (conditional on the initial trigger) over a 139-day period, incorporating both

the connections among stocks and the strength of these connections. The dashed lines in Figure 3.12 represent the average values across 10,000 simulations, closely replicating the susceptible, infection, and recovery dynamics observed in the empirical data at the aggregate level. Nonetheless, it is important to note that the dashed red line in Figure 3.12 does not perfectly capture the peak infection point, whereas Figure 3.5 shows that the calibrated parameters (i.e.  $\beta = 0.3613$  and  $\gamma = 0.0225$ ) are able to accurately reproduce the infection pattern, including the peak point when no network structure is imposed. This discrepancy highlights the critical role of network structure in financial contagion analysis. In other words, the results in Figure 3.5 are based on the SIR model without stock connections, and this implies that an infected stock can infect any other stock with probability  $\beta = 0.3613$ . In contrast, Figure 3.12 assumes that financial contagion evolves according to the stock network connections, and that the severity of contagion is conditional on the initial trigger. In other words, the propagation of financial crises is now constrained by the underlying structure of connections. Specifically, stocks that are strongly connected to infected stocks face an elevated probability of contagion (i.e.  $edge\ weight * \beta$ ), reflecting the topology-dependent nature of contagion. In addition, the impact of network structure in the SIR simulation analysis can be quantified through the reproduction ratio ( $R_0 = \frac{\beta}{\gamma}$ ). Based on the SIR ordinary differential equation (ODE) alone, the actual data patterns suggests a reproduction number of  $\frac{0.3613}{0.0225} = 16.06$  (see Equation 3.5). However, when the stock network structure is incorporated into the SIR simulation analysis, the simulated results (dashed lines in Figure 3.12) yield slightly different ex post fitted values, for  $\beta$  (0.303),  $\gamma$  (0.0262) and, a lower reproduction number of  $R_0 = 11.56$ .<sup>14</sup> Thus, when the SIR parameters  $\beta$  and  $\gamma$ , calibrated without network constraints, are used when simulating the network-based SIR model, then the simulations result in a less severe contagion dynamics ( $R_0 = 11.56 < 16.06$ ).

In fact, the systemic evolution of a simulated financial crisis is significantly influenced by the sector that absorbs the initial shock. If the initial shock occurs in a central sector within the network, the systemic impact is likely to be much stronger than if the shock

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<sup>14</sup>These results are obtained from fitting the simulated infection results into the SIR ODE questions. These new (ex post) fitted values,  $\beta$  and  $\gamma$ , capture the pattern of simulated infection results.

originates in a peripheral sector with fewer connections. For example, a shock originating in the basic materials sector may not directly infect the real estate sector, though secondary effects or broader economic connections could induce volatility.<sup>15</sup> To gain a more comprehensive understanding of how a financial contagion is conditional on its triggering element, the subsequent analysis investigates the propagation effects associated with alternative scenarios on the sectoral origin of the shock.

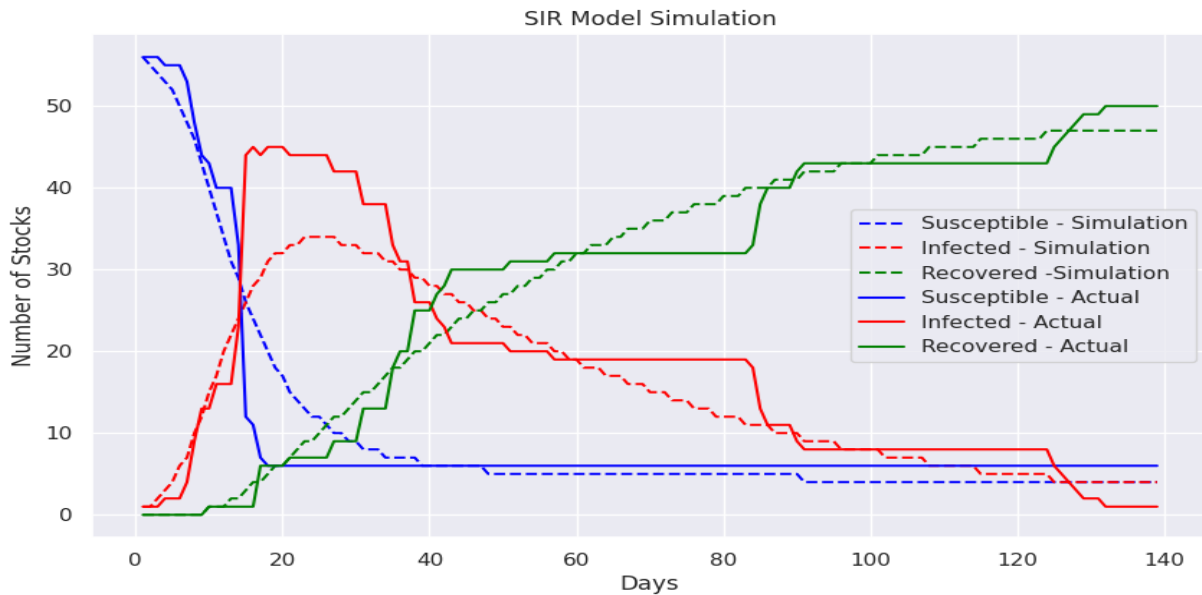


Figure 3.12: SIR Model Simulation Results - TSX 60

### 3.5.4 Sectoral Analysis

To investigate this point further, this section illustrates multiple financial contagion scenarios, each assuming a different triggering sector. We use the strategy outlined in Section 3.4.1 to determine the ex post fitted values of  $\beta$  and  $\gamma$  based on the observed simulation results.<sup>16</sup> These ex post values serve as the basis for quantifying the severity of the financial contagion in each scenario conditional on the triggering sector. Table 3.5 summarizes the fitted values for  $\beta$  and  $\gamma$  from 10,000 simulations for six scenarios, each corresponding to a crisis originating from different sectoral shocks. The financial services sector is the largest

<sup>15</sup>The TSX 60 real estate sector is characterized by its concentration on rental and building management activities, while the basic materials sector is predominantly influenced by the mineral material industry.

<sup>16</sup>For the sectoral analysis, the network-based simulations use the SIR parameters ( $\beta = 0.3613$  and  $\gamma = 0.0225$  without network structure) to initiate the simulation process. This approach ensures control over model variance and allows for a consistent basis of comparison across different scenarios.

sector in the TSX 60, so that we further categorize the types of shocks within this sector. First, we simulate a shock to the four largest banks. As shown in Table 3.5, the ex post  $\beta$  is 0.681, a significantly higher value than the initial calibrated value ( $\beta = 0.3613$ ), and with a corresponding ex post reproduction number  $R_0 = 29.459$ . Second, if the financial contagion is triggered by a more diversified set of financial services companies, such as banks, insurance firms, and investment firms, then the ex post infection rate increases to  $\beta = 0.713$ , with an ex post reproduction number of  $R_0 = 30.662$ . Third, the energy sector, ranked as the second largest sector (in terms of market capitalization) in the network, also induces a considerable impact on the market, with an ex post  $R_0$  of 23.408. Finally, shocks to the basic materials, communication services, and real estate sectors result in significantly lower ex post reproduction numbers, especially for the basic materials sector. As previously noted, the reproduction number measures the expected number of new infected stocks from a single infected stock in the susceptible population. We conclude from these observations that the financial services sector shock lead to the most severe scenario, followed by the energy sector. The intuition behind these observations is that shocks to the more central sectors of the network are more likely to induce widespread stock market turmoil compared to shocks originating in peripheral sectors. These findings underscore the importance of maintaining stability within the financial services and energy sectors to safeguard overall market stability. It is worth noting that the ex post fitted value for the recovery rate ( $\gamma$ ) remains stable across all scenarios illustrated in Table 3.5. This consistency arises from the assumption that each infected stock recovers independently, so that the strength of connections does not influence the recovery time. This ensures controlled comparisons of ex post reproduction numbers ( $R_0$ ) across different sectoral shock scenarios (see Algorithm 2).

The performance of the basic materials sector warrants special attention due to its lowest ex post reproduction number among all scenarios. Four stocks were selected from the basic materials sector for the analysis: ABX.TO, FNV.TO, WPM.TO, and AEM.TO. AEM.TO and WPM.TO are ranked in the stock network with the lowest betweenness centrality and out-degree centrality measurements. This finding suggests that the low ex post

reproduction number observed when the shock originate from the basic materials sector is largely attributable to stocks with peripheral influence within the network. To validate this observation, additional simulations are conducted by selecting stocks based on centrality measurements, irrespective of their sectoral classification. The results, summarized in Table 3.6, provide further corroboration. The group of stocks with the lowest out-degree centrality produced the lowest impacts, followed by the group with the lowest betweenness centrality. Conversely, the group of stocks with the highest out-degree centrality had the most severe impact on the market, with the highest ex post  $R_0$ . In addition, three of the four stocks in the highest out-degree centrality group are banks, reinforcing the critical role of the financial services sector in driving systemic risk. These findings align with the earlier results, highlighting the importance of network centrality measurements in determining the severity of financial contagion and supporting the conclusion that stocks with high betweenness centrality and high out-degree centrality, particularly in influential sectors like financial services, play a disproportionate role in propagating crises.

The simulations we have discussed spans a total of 139 days, capturing the progression of financial contagion over time. Understanding the evolution of propagation over time is crucial for uncovering the dynamics of shock spreading within the network. Figure 3.13 illustrates this evolution by categorizing infected stocks based on their respective sectors. The figure measures the percentage of infected stocks within each sector relative to the total number of TSX 60 constituents ( $N$ ), providing insights into sector-specific vulnerabilities and the broader contagion patterns within the market.<sup>17</sup> For instance, Figure 3.13a illustrates the propagation dynamics of the financial contagion triggered by an initial shock originating in the financial services sector, specifically within the banking subgroup. The blue line represents the sectoral distribution of infections during the first week of contagion spread. Notably, the energy sector exhibits an 11% infection rate in the first week. This indicates that, on average across 10,000 simulation runs, 11% of the total

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<sup>17</sup>The percentage value represents the average outcome derived from 10,000 simulation iterations. Specifically, in the first week, the simulation generates 10,000 independent infection scenarios. Each iteration calculates the proportion of infected stocks relative to the total number of stocks within each sector. The final reported percentage is obtained by averaging these individual proportions across all simulations.

TSX 60 constituents infected in the initial week are firms from the energy sector.<sup>18</sup> We also observe that the infection speed begins to stabilize by the fourth week following the initial round of infection.

However, the propagation dynamics exhibit significant variation depending on the sector in which the initial infection originates. The sectoral origin affects the magnitude and the amplification or containment of the financial contagion. The results in Figure 3.13 a-c reveal that infection rates are consistently high across most sectors during the first two weeks, as shown by the blue and orange lines extending farther from the origin in the radar charts, compared to other scenarios in Figure 3.13 d-f. This indicates that shocks originating from the financial services or energy sectors induce broader and more widespread contagion effects compared to other sectors.

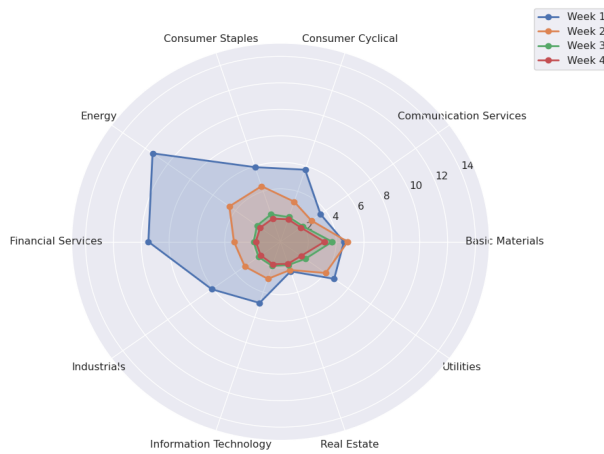
In addition, shocks in the financial services and energy sectors exhibit the most severe impact during the first week, as shown by the blue circle (first week results) being larger than the orange circle (second week results). This highlights the rapid speed of propagation from these sectors, which far exceeds the speed observed in other scenarios. For shocks originating in other sectors, the first-week infection rates are comparatively lower, with contagion becoming more noticeable during the second week. This suggests that the contagion in these scenarios is driven by secondary infections originating from stocks initially impacted during the first week. Another notable observation is the strong interconnection between the financial services and energy sectors. These two sectors consistently exhibit the highest cumulative number of infections across all scenarios. The key distinction lies in the timing of their responses to shocks. For instance, shocks originating in the basic materials, communication services, and real estate sectors begin to noticeably influence the financial services and energy sectors during the second and third weeks. In contrast, shocks directly affecting the financial services or energy sectors result in immediate and severe impacts within the first two weeks. Figure 3.13c also suggests that shocks originating in the energy sector are often amplified by subsequent contagion spreading from the

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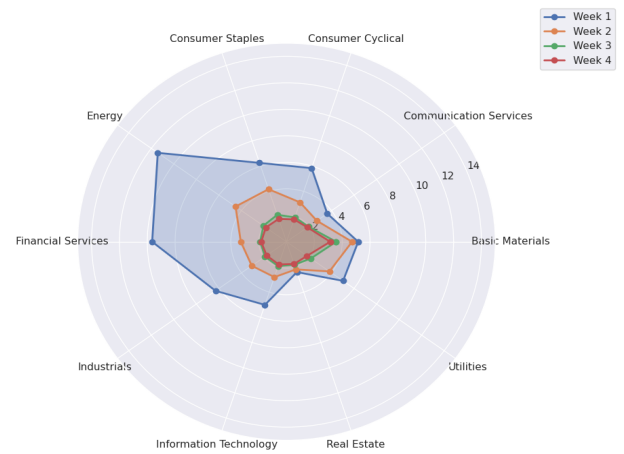
<sup>18</sup>For standardization purposes, the total number of TSX 60 constituents ( $N$ ) is used as the denominator when calculating the proportion of infections in each round of simulation. This approach allows for a consistent comparison of infection rates across different sectors and simulation scenarios.

financial services sector, as demonstrated by the financial services sector experiencing a sharp increase in first-week infections (outermost blue contour).

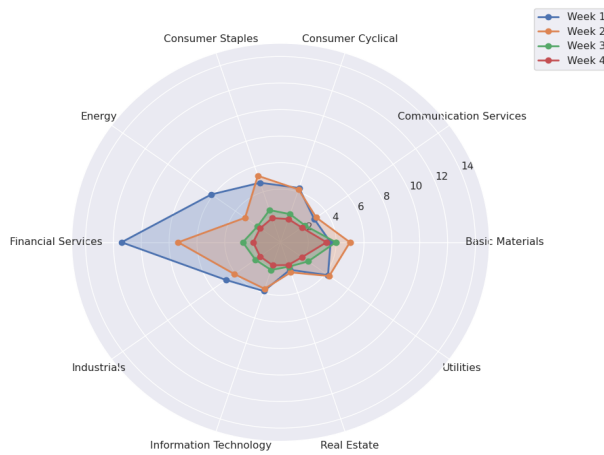
These results underscore the critical importance of maintaining stability within the financial services sector for overall market resilience. Ensuring the robustness of this sector is essential for safeguarding the development and stability of the stock market as a whole. In times of crisis, policymakers should prioritize mitigating the initial impacts on the financial services sector to reduce the cascading effects and stabilize the broader market.



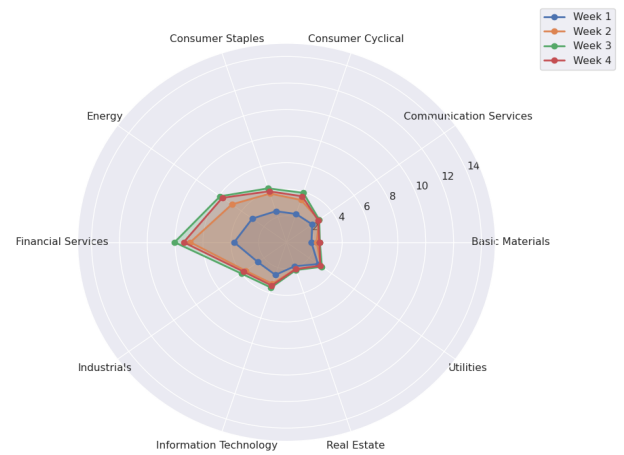
(a) Financial Services Sector (Banks Only) The total number of infections in the first week was 29.



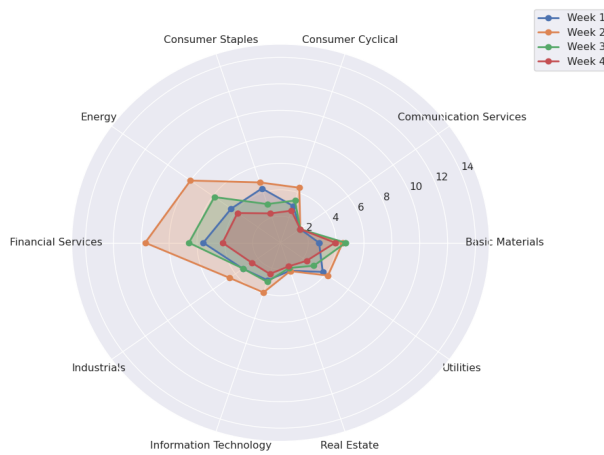
(b) Financial Services Sector (Mixed) The total number of infections in the first week was 30.



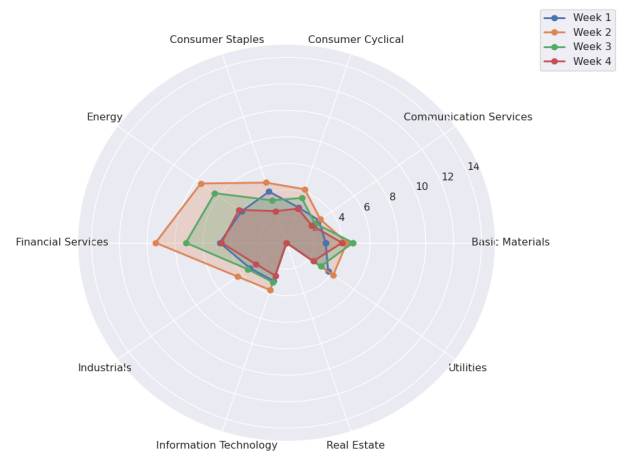
(c) Energy Sector The total number of infections in the first week was 22.



(d) Basic Materials Sector The total number of infections in the first week was 5.



(e) Communication Services Sector The total number of infections in the first week was 10.



(f) Real Estate Sector The total number of infections in the first week was 8.

Figure 3.13: Sector Level Analysis - TSX 60

## 3.6 Conclusion

This chapter proposes a framework for studying the dynamics of financial contagion in stock networks, focusing on the TSX 60 index, using a weighted, directed network-based SIR model. By simulating sector-specific shocks and studying their propagation across the network, it provides key insights into the systemic importance of various sectors and the factors driving the contagion. The findings highlight the critical roles of the financial services sector within the TSX 60 in stabilizing or destabilizing the stock market and market resilience. Their prominence within the network underscores the importance of targeted efforts to safeguard their stability as a means of mitigating overall market contagion. From a technical perspective, the study demonstrates the utility of network topology in financial contagion analysis. The edge weights between stocks quantify the strength of interconnections, with higher weights corresponding to a greater likelihood of shock propagation. Incorporating the network structure into the SIR model enables not only high-level crisis severity measurements but also a more granular understanding of contagion dynamics, particularly at the sectoral level.<sup>19</sup> This dual perspective provides a valuable tool for analyzing financial crises and assessing systemic risks.

Nevertheless, it is important to recognize the limitations of the framework. The model relies on historical stock price data, which provides only a partial view of a company's financial stability. Moreover, the network-based SIR framework operates under several basic assumptions. First, companies are categorized into either susceptible, infected, or recovered over the whole period. This could be relaxed by doing a SIRS model or SEIRS Model.<sup>20</sup> Additionally, the recovery rate is assumed independent and identical for each company. In reality, recovery rates are likely to depend on factors such as a company's

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<sup>19</sup>A network-based SIR framework not only captures the aggregate dynamics of susceptible, infected, and recovered entities over time but also provides insights into the individual contributions and impacts of each node within the network. By incorporating the underlying structure of interconnections, this approach facilitates a more granular analysis of contagion propagation, highlighting the systemic importance and vulnerability of specific entities within the financial system.

<sup>20</sup>The SIRS model assumes that recovered individuals/stocks can lose "immunity" after some time and become susceptible again. The SEIRS model further extends this framework by introducing an exposed (latent) period, during which individuals/stocks are infected but not yet infectious.

financial health, balance sheet strength, and capital position. These simplifications, while necessary for tractability, may limit the model's ability to fully capture the complexities of real-world contagion dynamics.

The primary goal of this research is to provide a conceptual framework for studying the spread of financial risk in networks, rather than to make precise predictions on forecasting about specific crises. Despite its limitations, the integration of network structures into the SIR model proves effective in capturing the interdependencies among stocks and offers a powerful tool for understanding financial contagion dynamics. This research opens several avenues for further exploration. Incorporating alternative data sources, such as balance sheet information or macroeconomic indicators, could improve the model's accuracy and applicability. Studying networks that evolve over time, reflecting real-world changes in company relationships and market conditions, could lead to a more realistic understanding of contagion dynamics. Extending the framework to include inter-market linkages or cross-sector dependencies would offer a more comprehensive view of systemic risk. By addressing these areas, future research might build upon the foundation established here so as to develop more robust tools for analyzing and mitigating financial contagion.

## 3.7 Tables

Table 3.1: Diebold-Yilmaz Spillover Index - TSX 60

|        | ABX   | AEM   | AQN   | ATD   | BCE   | BIP.UN | BMO   | BN    | BNS   | CAE   | CAR.UN | CCL.B | CCO   | CM    | CNQ   | CNR   | CP    | CSU   | CTC.A |
|--------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| ABX    | 0.506 | 0.215 | 0.022 | 0.002 | 0.005 | 0.006  | 0.008 | 0     | 0.004 | 0.001 | 0.011  | 0.002 | 0.003 | 0.006 | 0.003 | 0.006 | 0.004 | 0.001 | 0     |
| AEM    | 0.219 | 0.5   | 0.012 | 0.001 | 0.001 | 0.005  | 0.012 | 0.006 | 0.006 | 0     | 0.004  | 0.002 | 0.001 | 0.01  | 0.003 | 0.01  | 0.005 | 0.001 | 0.012 |
| AQN    | 0.024 | 0.012 | 0.493 | 0.008 | 0.021 | 0.02   | 0.003 | 0.012 | 0.003 | 0.014 | 0.09   | 0.002 | 0.004 | 0.004 | 0.001 | 0.002 | 0.004 | 0.008 | 0.01  |
| ATD    | 0.008 | 0.002 | 0.017 | 0.438 | 0.028 | 0.045  | 0.018 | 0.022 | 0.012 | 0.029 | 0.027  | 0.036 | 0.001 | 0.004 | 0.004 | 0.03  | 0.029 | 0.023 | 0.023 |
| BCE    | 0.005 | 0.004 | 0.025 | 0.033 | 0.362 | 0.031  | 0.016 | 0.026 | 0.017 | 0.013 | 0.029  | 0.011 | 0.006 | 0.012 | 0.004 | 0.011 | 0.003 | 0.007 | 0.024 |
| BIP.UN | 0.002 | 0     | 0.019 | 0.034 | 0.026 | 0.532  | 0.008 | 0.024 | 0.017 | 0.012 | 0.001  | 0.023 | 0.017 | 0.01  | 0.012 | 0.017 | 0.018 | 0.016 | 0.032 |
| BMO    | 0.018 | 0.029 | 0.016 | 0.032 | 0.03  | 0.02   | 0.216 | 0.062 | 0.07  | 0.04  | 0.002  | 0.045 | 0.034 | 0.104 | 0.044 | 0.061 | 0.054 | 0.021 | 0.054 |
| BN     | 0.002 | 0.018 | 0.026 | 0.039 | 0.045 | 0.054  | 0.056 | 0.239 | 0.049 | 0.038 | 0.028  | 0.034 | 0.032 | 0.049 | 0.014 | 0.074 | 0.053 | 0.048 | 0.027 |
| BNS    | 0.008 | 0.014 | 0.009 | 0.016 | 0.027 | 0.032  | 0.064 | 0.051 | 0.226 | 0.038 | 0.003  | 0.048 | 0.046 | 0.094 | 0.033 | 0.052 | 0.047 | 0.04  | 0.05  |
| CAE    | 0.002 | 0.004 | 0.018 | 0.024 | 0.01  | 0.016  | 0.016 | 0.018 | 0.017 | 0.463 | 0.001  | 0.041 | 0.009 | 0.021 | 0.003 | 0.023 | 0.028 | 0.034 | 0.02  |
| CAR.UN | 0.009 | 0.005 | 0.075 | 0.025 | 0.022 | 0.003  | 0.002 | 0.013 | 0.003 | 0.001 | 0.653  | 0.012 | 0.005 | 0     | 0.001 | 0.003 | 0.004 | 0.013 | 0.001 |
| CCL.B  | 0.005 | 0.002 | 0.008 | 0.035 | 0.012 | 0.029  | 0.025 | 0.02  | 0.028 | 0.043 | 0.015  | 0.444 | 0.022 | 0.019 | 0.016 | 0.029 | 0.035 | 0.033 | 0.01  |
| CCO    | 0.008 | 0.002 | 0.001 | 0.007 | 0.005 | 0.014  | 0.012 | 0.011 | 0.019 | 0.007 | 0.005  | 0.017 | 0.585 | 0.013 | 0.031 | 0.018 | 0.019 | 0.014 | 0.011 |
| CM     | 0.013 | 0.018 | 0.007 | 0.004 | 0.019 | 0.015  | 0.088 | 0.046 | 0.087 | 0.033 | 0.002  | 0.033 | 0.036 | 0.252 | 0.04  | 0.044 | 0.034 | 0.02  | 0.053 |
| CNQ    | 0.003 | 0.006 | 0.003 | 0.001 | 0.003 | 0.02   | 0.029 | 0.01  | 0.023 | 0.007 | 0.001  | 0.013 | 0.056 | 0.031 | 0.321 | 0.022 | 0.024 | 0.016 | 0.021 |
| CNR    | 0.02  | 0.019 | 0.008 | 0.058 | 0.017 | 0.047  | 0.06  | 0.079 | 0.056 | 0.055 | 0.006  | 0.055 | 0.048 | 0.051 | 0.031 | 0.221 | 0.149 | 0.05  | 0.038 |
| CP     | 0.011 | 0.01  | 0.009 | 0.047 | 0.007 | 0.04   | 0.043 | 0.047 | 0.041 | 0.059 | 0.01   | 0.058 | 0.041 | 0.032 | 0.029 | 0.123 | 0.268 | 0.06  | 0.021 |
| CSU    | 0.014 | 0.003 | 0.014 | 0.025 | 0.011 | 0.02   | 0.011 | 0.029 | 0.027 | 0.042 | 0.021  | 0.035 | 0.021 | 0.014 | 0.012 | 0.028 | 0.04  | 0.412 | 0.008 |
| CTC.A  | 0     | 0.013 | 0.013 | 0.014 | 0.023 | 0.037  | 0.021 | 0.015 | 0.025 | 0.025 | 0      | 0.019 | 0.015 | 0.029 | 0.017 | 0.017 | 0.01  | 0.004 | 0.443 |

Table summarizes the Diebold-Yilmaz spillover index for the first 19 stocks by using the data from 2018-08-01 to 2020-03-01. The VAR model uses the lag 1 and the variance decomposition uses 5 steps. Note: Table 2.7 uses data from January 1, 2017, to September 30, 2023. Therefore, the Diebold-Yilmaz spillover index in Table 2 differs from that in Table 3.1.

Table 3.2: Diebold-Yilmaz Spillover Index - TSX 60 (With threshold)

|        | ABX   | AEM   | AQN   | ATD   | BCE   | BIP.UN | BMO   | BN    | BNS   | CAE   | CAR.UN | CCLB  | CCO   | CM    | CNQ   | CNR   | CP    | CSU   | CTC.A |
|--------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| ABX    | 0     | 0.215 | 0.022 | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| AEM    | 0.219 | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| AQN    | 0.024 | 0     | 0     | 0.021 | 0.028 | 0.045  | 0.018 | 0.022 | 0     | 0.029 | 0.027  | 0.036 | 0     | 0     | 0     | 0.03  | 0.029 | 0.023 | 0.023 |
| ATD    | 0     | 0     | 0.017 | 0     | 0.028 | 0.031  | 0     | 0.026 | 0.017 | 0     | 0.029  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.024 |
| BCE    | 0     | 0     | 0.025 | 0.033 | 0     | 0.031  | 0     | 0.026 | 0.017 | 0     | 0.029  | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.032 |
| BIP.UN | 0     | 0     | 0.019 | 0.034 | 0.026 | 0      | 0     | 0.024 | 0.017 | 0     | 0.029  | 0.023 | 0.017 | 0     | 0     | 0.017 | 0.018 | 0     | 0.032 |
| BMO    | 0.018 | 0.029 | 0     | 0.032 | 0.03  | 0.02   | 0     | 0.062 | 0.07  | 0.04  | 0      | 0.045 | 0.034 | 0.104 | 0.044 | 0.061 | 0.054 | 0.021 | 0.054 |
| BN     | 0     | 0.018 | 0.026 | 0.039 | 0.045 | 0.054  | 0.056 | 0     | 0.049 | 0.038 | 0.028  | 0.034 | 0.032 | 0.049 | 0     | 0.074 | 0.053 | 0.048 | 0.027 |
| BNS    | 0     | 0     | 0     | 0     | 0.027 | 0.032  | 0.064 | 0.051 | 0     | 0.038 | 0      | 0.048 | 0.046 | 0.094 | 0.033 | 0.052 | 0.047 | 0.04  | 0.05  |
| CAE    | 0     | 0     | 0.018 | 0.024 | 0     | 0      | 0.016 | 0.018 | 0.017 | 0     | 0      | 0.041 | 0     | 0.021 | 0     | 0.023 | 0.028 | 0.034 | 0.02  |
| CAR.UN | 0     | 0     | 0.075 | 0.025 | 0.022 | 0      | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| CCLB   | 0     | 0     | 0     | 0.035 | 0     | 0.029  | 0.025 | 0.02  | 0.028 | 0.043 | 0      | 0     | 0.022 | 0.019 | 0     | 0.029 | 0.035 | 0.033 | 0     |
| CCO    | 0     | 0     | 0     | 0     | 0     | 0      | 0     | 0     | 0.019 | 0     | 0      | 0.017 | 0     | 0     | 0.031 | 0.018 | 0.019 | 0     | 0     |
| CM     | 0     | 0.018 | 0     | 0     | 0.019 | 0      | 0.088 | 0.046 | 0.087 | 0.033 | 0      | 0.033 | 0.036 | 0     | 0.04  | 0.044 | 0.034 | 0.02  | 0.053 |
| CNQ    | 0     | 0     | 0     | 0     | 0     | 0.02   | 0.029 | 0     | 0.023 | 0     | 0      | 0     | 0.056 | 0.031 | 0     | 0.022 | 0.024 | 0     | 0.021 |
| CNR    | 0.02  | 0.019 | 0     | 0.058 | 0.017 | 0.047  | 0.06  | 0.079 | 0.056 | 0.055 | 0      | 0.055 | 0.048 | 0.051 | 0.031 | 0     | 0.149 | 0.05  | 0.038 |
| CP     | 0     | 0     | 0     | 0.047 | 0     | 0.04   | 0.043 | 0.047 | 0.041 | 0.059 | 0      | 0.058 | 0.041 | 0.032 | 0.029 | 0.123 | 0     | 0.06  | 0.021 |
| CSU    | 0     | 0     | 0     | 0.025 | 0     | 0.02   | 0     | 0.029 | 0.027 | 0.042 | 0.021  | 0.035 | 0.021 | 0     | 0     | 0.028 | 0.04  | 0     | 0     |
| CTC.A  | 0     | 0     | 0     | 0     | 0.023 | 0.037  | 0.021 | 0     | 0.025 | 0.025 | 0      | 0.019 | 0     | 0.029 | 0.017 | 0.017 | 0     | 0     | 0     |

Table summarizes the Diebold-Yilmaz spillover index for the first 19 stocks by using the data from 2018-08-01 to 2020-03-01. The VAR model uses the lag 1 and the variance decomposition uses 5 steps.

Table 3.3: Summary of Goodness-of-Fit Metrics ( $R^2$  and RMSE) Comparing Simulated Infection with Empirical Data - Figure 3.8

| TSX 60 Network Selection  | Coefficient of Determination ( $R^2$ ) | Root Mean Squared Error (RMSE) |
|---------------------------|--|--------------------------------|
| Full Network              | 0.882                                  | 4.381                          |
| 80th Percentile Threshold | 0.124                                  | 11.929                         |
| 50th Percentile Threshold | 0.841                                  | 5.089                          |
| 20th Percentile Threshold | 0.885                                  | 4.318                          |

Table 3.4: Summary of Goodness-of-Fit Metrics ( $R^2$  and RMSE) Comparing Simulated Infection with Empirical Data - Figure 3.9

| TSX 60 Network v.s. Random Network | Coefficient of Determination ( $R^2$ ) | Root Mean Squared Error (RMSE) |
|------------------------------------|--|--------------------------------|
| 50th Percentile Threshold          | 0.841                                  | 5.089                          |
| Random Network (P=0.2)             | -1.222                                 | 18.996                         |
| Random Network (P=0.4)             | -0.287                                 | 14.445                         |
| Random Network (P=0.6)             | 0.271                                  | 10.879                         |
| Random Network (P=0.8)             | 0.607                                  | 7.991                          |

Table 3.5: TSX 60 Network - Sectoral Shock Analysis

| Initial Infected Sector         | Number of Infected Stocks | Ex Post $\beta$ | Ex Post $\gamma$ | Ex Post $R_0$ |
|---------------------------------|---------------------------|-----------------|------------------|---------------|
| Financial Services (Banks only) | 4                         | 0.681           | 0.023            | 29.459        |
| Financial Services (Mixed)      | 4                         | 0.713           | 0.023            | 30.662        |
| Energy                          | 4                         | 0.544           | 0.023            | 23.408        |
| Basic Materials                 | 4                         | 0.158           | 0.03             | 5.225         |
| Communication Service           | 2                         | 0.353           | 0.023            | 15.039        |
| Real Estate                     | 2                         | 0.326           | 0.024            | 13.818        |

The financial services (Banks only) scenario contains RY.TO, BMO.TO, TD.TO, CM.TO; The financial services (Mixed) scenario contains RY.TO, BMO.TO, MFC.TO, BN.TO; The energy scenario contains CNQ.TO, ENB.TO, SU.TO, TRP.TO; The basic materials scenario contains ABX.TO, FNV.TO, WPM.TO, AEM.TO. The communication service scenario contains BCE.TO, T.TO; and the real estate scenario contains CAR.UN.TO, FSV.TO.

Table 3.6: TSX 60 Network - Centrality Shock Analysis

| Initial Infected Criteria      | Number of Infected Stocks | Ex Post $\beta$ | Ex Post $\gamma$ | Ex Post $R_0$ |
|--------------------------------|---------------------------|-----------------|------------------|---------------|
| Lowest Betweenness Centrality  | 4                         | 0.394           | 0.023            | 16.651        |
| Highest Betweenness Centrality | 4                         | 0.477           | 0.023            | 20.499        |
| Lowest Out-degree Centrality   | 4                         | 0.176           | 0.027            | 6.634         |
| Highest Out-degree Centrality  | 4                         | 0.719           | 0.023            | 30.962        |

The centrality measurement is based on the DY spillover index network with 50th percentile threshold. The top four lowest betweenness group includes AEM.TO, K.TO, WPM.TO, CNQ.TO; The top four highest betweenness group includes TOU.TO, CTC.A.TO, ENB.TO, BIP.UN.TO; The top four lowest out-degree group includes K.TO, WPM.TO, DOL.TO, AEM.TO; The top four highest out-degree group includes CNR.TO, BN.TO, RY.TO, NA.TO.

# General Conclusion

This dissertation has explored the application of network theory to the analysis of North American stock markets, providing a comprehensive framework for understanding market structure, systemic risk, and contagion dynamics. By integrating undirected and directed network approaches with advanced analytical techniques, including graphical LASSO, net transfer entropy, and the Diebold-Yilmaz spillover index, this research has shed light on the complex and evolving interdependencies within the TSX 60 and Nasdaq 100.

The study's findings demonstrate that stock networks undergo significant structural changes in response to major economic events, such as the COVID-19 pandemic and subsequent monetary policy shifts. In particular, the undirected network analysis highlighted shifts in connectivity and systemic importance among market participants, while the directed network analysis revealed asymmetric volatility spillovers and directional risk transmissions that vary across time horizons. Another contribution of this dissertation is the development and application of an integrated SIR-network model to simulate the spread of financial contagion within the TSX 60. The model underscores the influence of network topology, including edge weights and centrality, in determining the speed of contagion propagation. The insights gained from this research offer practical implications for enhancing systemic risk and market risk management. By identifying critical nodes and connections within stock networks, regulators and policymakers can develop more targeted interventions to prevent or mitigate the spread of financial crises. Moreover, the network-based connectedness insights provided by this research can assist investors in developing more robust hedging and diversification strategies to mitigate market risk.

In conclusion, this dissertation advances the understanding of stock market dynam-

ics through the lens of network analysis. It provides valuable tools and perspectives for navigating the complexities of modern financial systems, fostering greater resilience and stability in the face of future economic uncertainties.

# Appendix A

## Partial Correlation Example

### A.1 The relationship between partial correlation and precision matrix

Suppose the covariance matrix  $\Sigma = (\sigma_{ij})$  is positive definite and invertible. If the precision matrix is defined as  $\Omega = \Sigma^{-1}$ , then the partial correlation coefficient between returns of stocks (e.g.  $R_i$  and  $R_j$ ), given all other returns  $V \setminus \{(R_i, R_j)\}$  is:

$$\rho_{(R_i, R_j | V \setminus \{(R_i, R_j)\})} = -\frac{[\Omega_{11}]_{12}}{\sqrt{[\Omega_{11}]_{22} [\Omega_{11}]_{11}}}.$$

*Proof: Equation 1.1.* Assume  $V \setminus (R_i, R_j)$  is  $Z$ , then the partial correlation can be written as  $\rho_{(R_i, R_j | Z)}$ . The covariance matrix is written as:

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where  $\Sigma_{11}$  is the covariance matrix of  $(R_i, R_j)$ ,  $\Sigma_{22}$  is the covariance matrix of  $Z$ , and  $\Sigma_{12}$ ,  $\Sigma_{21}$  are the covariance matrices of  $(R_i, R_j)$  and  $Z$ .

The precision matrix can be defined similarly as:

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix},$$

where  $\Omega_{11}$  is the precision matrix of  $(R_i, R_j)$ ,  $\Omega_{22}$  is the precision matrix of  $Z$ , and  $\Omega_{12}$ ,  $\Omega_{21}$  are the precision matrices of  $(R_i, R_j)$  and  $Z$ .

Since the precision matrix is the inverse of covariance matrix, that is  $\Omega = \Sigma^{-1}$ , we can apply Schur's formula for block-matrix inversion to find that  $\Omega_{11}^{-1} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$ , which is also the covariance matrix of  $(R_i, R_j) | Z$ . Then the partial correlation can be defined as:

$$\rho_{(R_i, R_j | Z)} = \frac{[\Omega_{11}^{-1}]_{12}}{\sqrt{[\Omega_{11}^{-1}]_{11} [\Omega_{11}^{-1}]_{22}}} \quad (\text{A.1})$$

where  $[\Omega_{11}^{-1}]_{ij}$  represents the  $(i, j)$ th element of matrix  $\Omega_{11}^{-1}$ .

The difference between Equation 1.1 and Equation A.1 is that Equation A.1 is a function of the inverse of precision matrix, while Equation 1.1 is a function of precision matrix. Therefore, Equation A.1 could be further simplified by using the covariance concept. Since  $\Omega_{11}^{-1} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$  is the covariance matrix of  $(R_i, R_j) | Z$ , then:

$$\begin{aligned} \Omega_{11}^{-1} &= \begin{bmatrix} \text{Cov}(R_i, R_i | Z) & \text{Cov}(R_i, R_j | Z) \\ \text{Cov}(R_j, R_i | Z) & \text{Cov}(R_j, R_j | Z) \end{bmatrix} \\ &= \begin{pmatrix} [\Omega_{11}]_{11} & [\Omega_{11}]_{12} \\ [\Omega_{11}]_{21} & [\Omega_{11}]_{22} \end{pmatrix}^{-1} \\ &= \frac{1}{\det \Omega_{11}} \begin{pmatrix} [\Omega_{11}]_{22} & -[\Omega_{11}]_{12} \\ -[\Omega_{11}]_{21} & [\Omega_{11}]_{11} \end{pmatrix} \end{aligned} \quad (\text{A.2})$$

Then,

$$\begin{aligned}
\rho_{(R_i, R_j | Z)} &= \frac{[\Omega_{11}^{-1}]_{12}}{\sqrt{[\Omega_{11}^{-1}]_{11} [\Omega_{11}^{-1}]_{22}}} \\
&= \frac{\text{Cov}(R_1, R_2 | Z)}{\sqrt{\text{Cov}(R_1, R_1 | Z) \text{Cov}(R_2, R_2 | Z)}} \\
&= \frac{-\frac{1}{\det \Omega_{11}} [\Omega_{11}]_{12}}{\sqrt{\frac{1}{\det \Omega_{11}} [\Omega_{11}]_{22} \frac{1}{\det \Omega_{11}} [\Omega_{11}]_{11}}} \\
&= \frac{-[\Omega_{11}]_{12}}{\sqrt{[\Omega_{11}]_{22} [\Omega_{11}]_{11}}}
\end{aligned} \tag{A.3}$$

□

## A.2 Partial Correlation Example

Section 1.4.1 illustrates a numerical simulation example. The following is the covariance matrix,  $\Sigma$ , derived from the simulated sample:

$$\Sigma = \begin{pmatrix} X & Y & Z \\ 16.038 & 13.907 & 16.278 \\ 13.907 & 24.126 & 14.342 \\ 16.278 & 14.342 & 29.8 \end{pmatrix} \begin{matrix} X \\ Y \\ Z \end{matrix}$$

Given this covariance matrix  $\Sigma$ , we can compute the precision matrix  $\Sigma_{-1} = \Omega$  as follows:

Step 1: Calculate the determinant of  $\Sigma$ :

$$\det(\Sigma) = 16.038 \begin{vmatrix} 24.126 & 14.342 \\ 14.342 & 29.8 \end{vmatrix} - 13.907 \begin{vmatrix} 13.907 & 14.342 \\ 16.278 & 29.8 \end{vmatrix} + 16.278 \begin{vmatrix} 13.907 & 24.126 \\ 16.278 & 14.342 \end{vmatrix}$$

$$\begin{vmatrix} 24.126 & 14.342 \\ 14.342 & 29.8 \end{vmatrix} = 718.9548 - 205.701164 = 513.253636$$

$$\begin{vmatrix} 13.907 & 14.342 \\ 16.278 & 29.8 \end{vmatrix} = 414.3786 - 233.498276 = 180.880324$$

$$\begin{vmatrix} 13.907 & 24.126 \\ 16.278 & 14.342 \end{vmatrix} = 199.403694 - 392.697528 = -193.293834$$

$$\det(\Sigma) = 16.038 \cdot 513.253636 - 13.907 \cdot 180.880324 + 16.278 \cdot (-193.293834) = 2567.58554$$

Step 2: Calculate the cofactor Matrix:

$$\text{Cof}(\Sigma) = \begin{pmatrix} 513.253636 & -180.880324 & -193.293834 \\ -180.880324 & 426.566604 & -175.093076 \\ -193.293834 & -175.093076 & 418.421804 \end{pmatrix}$$

Step 3: Transpose the cofactor Matrix :

$$\text{Adj}(\Sigma) = \begin{pmatrix} 513.253636 & -180.880324 & -193.293834 \\ -180.880324 & 426.566604 & -175.093076 \\ -193.293834 & -175.093076 & 418.421804 \end{pmatrix}$$

Step 4: Compute the inverse using the formula  $\Sigma^{-1} = \frac{1}{\det(\Sigma)} \text{Adj}(\Sigma)$ :

$$\begin{aligned}
\Omega &= \Sigma^{-1} \\
&= \frac{1}{2567.58554} \begin{pmatrix} 513.253636 & -180.880324 & -193.293834 \\ -180.880324 & 426.566604 & -175.093076 \\ -193.293834 & -175.093076 & 418.421804 \end{pmatrix} \\
&= \begin{pmatrix} 0.1998 & -0.0704 & -0.0752 \\ -0.0704 & 0.0829 & -0.0014 \\ -0.0752 & -0.0014 & 0.0753 \end{pmatrix}
\end{aligned}$$

Then we apply Equation 1.1 to derive the partial correlation from the inverse covariance matrix  $\Sigma^{-11}$ :

$$\begin{array}{ccc}
& X & Y & Z \\
\begin{pmatrix} 1 & 0.547 & 0.613 \\ 0.547 & 1 & 0.018 \\ 0.613 & 0.018 & 1 \end{pmatrix} & X \\
& Y \\
& Z
\end{array}$$

---

<sup>1</sup>For example, the partial correlation of X and Y (i.e.  $\rho_{(X,Y|Z)}$ ) is calculated as:  $\frac{-(-0.0704)}{\sqrt{0.0829*0.1998}} = 0.547$ .

# Appendix B

## TVGL Bayesian optimization

The Skopt Python library is employed to implement the Bayesian optimization procedure, which is utilized to identify the optimized parameters in Equation 1.5 of Section 1.4.1. This procedure obviates the need for a priori specification of a list of hyper-parameters to be tested, as the model will automatically identify these parameters.

The evaluation and partial dependence plots indicate the parameters selected at locations that minimize the objective function. Figure B.1 illustrates the evaluation plots for optimization results based on the TSX 60 dataset. The diagonal histograms show the distribution of samples for each parameter dimension. The scatter plots below the histograms indicate the position of the optimized parameters. The color of each point represents the evaluation order, with darker/purple colors referring to earlier samples and lighter/yellow colors referring to later samples. The lighter/yellow colored points cluster around the red star which indicates the location of the minimum found by the optimization process. Figure B.2 shows the partial dependence plots of each parameter in the TSX 60 network analysis. The partial dependence is used to measure the dependence between the target response and a set of input features of interest, whereby the values of all other input features are marginalized. The diagonal shows the impact of a single parameter on the minimization problem in the parameter searching space. The flat curve may indicate that the parameter affects the minimization mainly through interactions with other parameters. The off-diagonal heatmap panels show the joint influence on the objective function when

varying two parameters. A lighter color indicates a greater joint influence between the penalty parameters and the objective function.

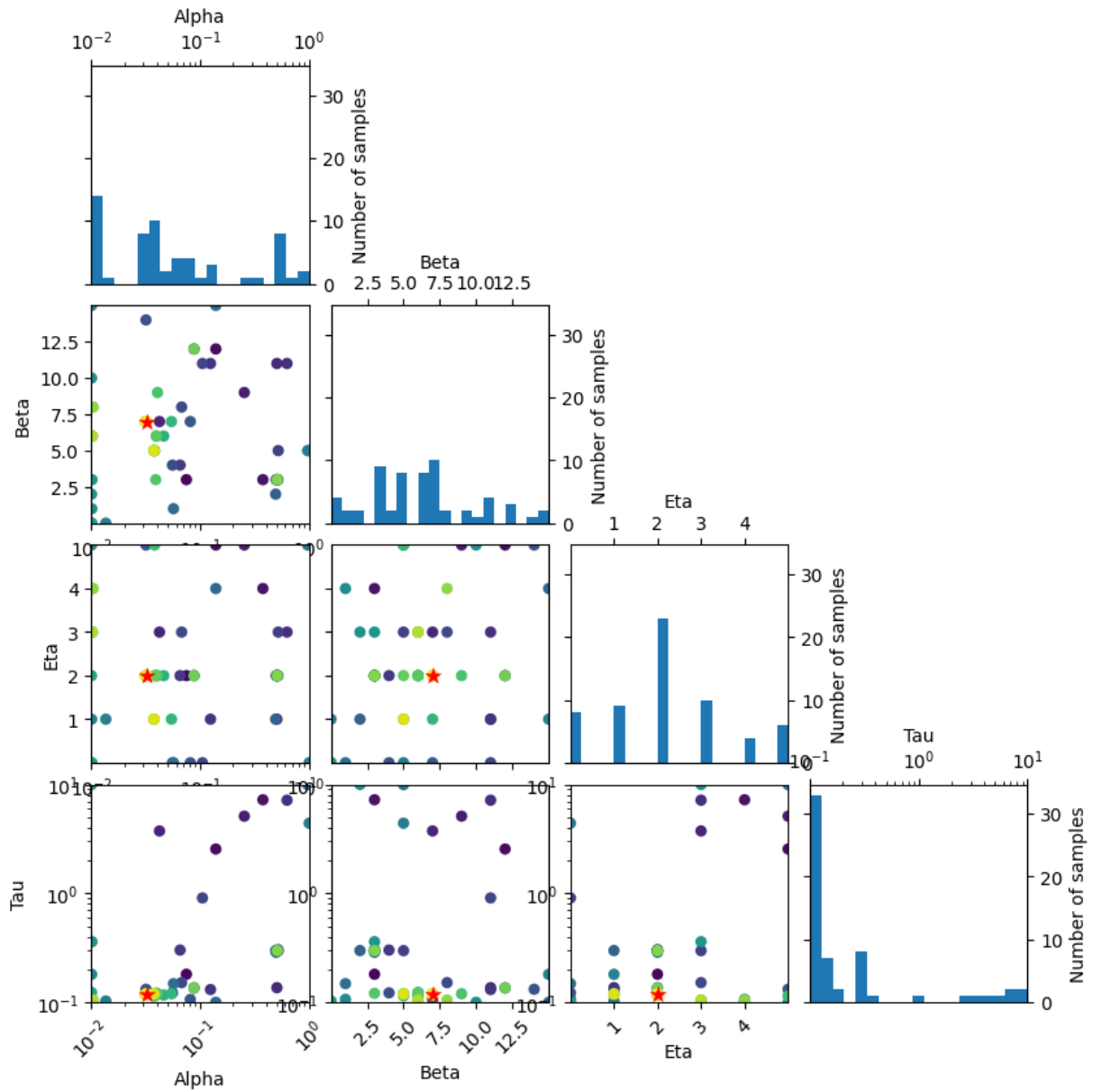


Figure B.1: TSX 60: LTGL Parameters Optimization - Evaluation Plot

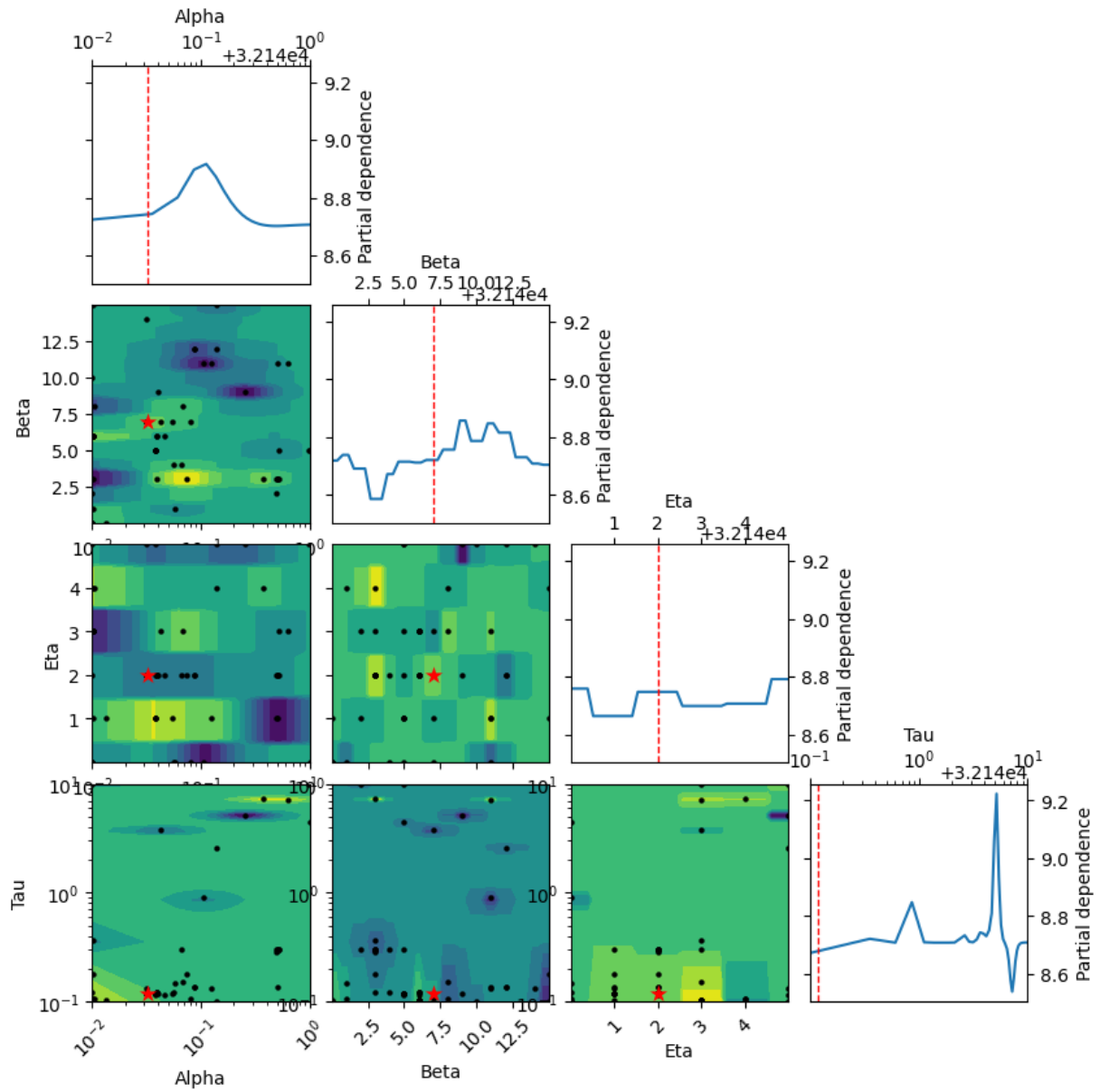


Figure B.2: TSX 60: LTGL Parameters Optimization - Partial Dependence Plot

Figure B.3 and Figure B.4 demonstrate optimization results on the Nasdaq 100 dataset. It is worth noting that the penalty parameters for the observed variable and latent variable (i.e.  $\alpha$  and  $\tau$ ) show little marginal effect on the objective function because of a flat partial dependence curve.

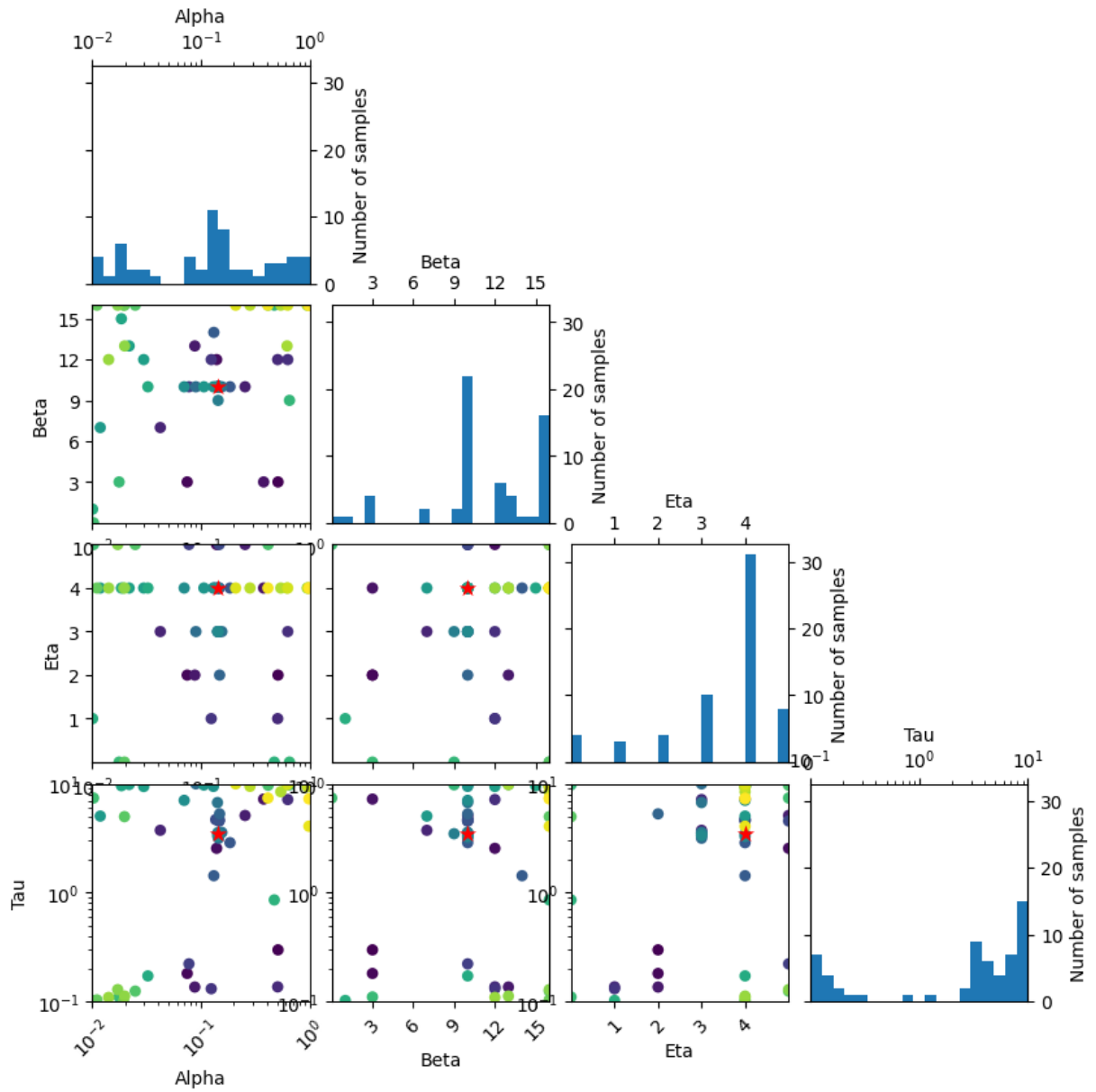


Figure B.3: Nasdaq 100: LTGL Parameters Optimization - Evaluation Plot

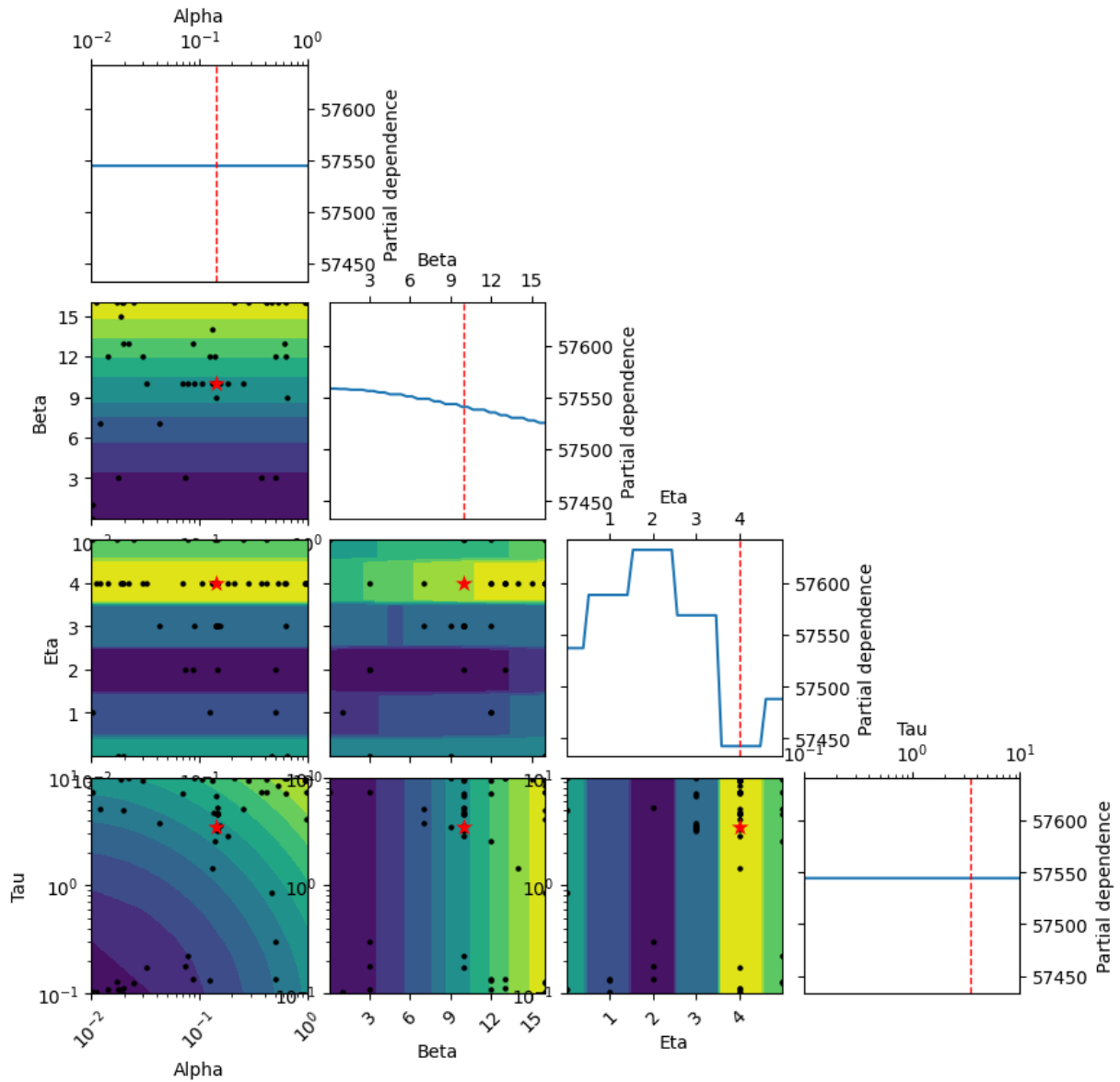


Figure B.4: Nasdaq 100: LTGL Parameters Optimization - Partial Dependence Plot

The evaluation plots for both the TSX 60 and Nasdaq 100 (Figure B.1 and B.3) depict the progress of minimizing the objective function. These figures indicate that the initial samples are distributed throughout the entire parameter space, while the subsequent samples begin to cluster around the minimum. Also, the optimized parameters are captured where the distribution of samples is concentrated. This demonstrates that the optimization process is searching for the location of the minimum. The partial dependence plots (Figures B.2 and B.4) demonstrate the marginal effects of optimized parameters. These figures illustrate the partial dependence of the optimized models on their respective parameters.

The left-bottom plot indicates a clear joint impact of the observed variable penalty and latent variable penalty, as evidenced by the substitution effect observed in the plot.

# Appendix C

## Minimum Spanning Tree

The Kruskal's algorithm is a widely used algorithms that permits to determine the edges of a minimum spanning tree. The algorithm 4 depicts the pseudocode of Kruskal's algorithm:

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**Algorithm 4:** Kruskal's Algorithm Pseudocode

---

- 1 Let  $G$  be a network, such as  $G(V, E)$ , where  $V$  is a set of nodes;  $E$  is a set of edge with weights. Let  $S$  be an empty set.
  - 2 Initialize a priority queue  $Q$  by sorting all the edges  $E$  of  $G(V, E)$  in non-decreasing order of their weights.
  - 3 Initialize a disjoint-set data structure to keep track of which vertices are in which sets.
  - 4 For each node  $v$  (i.e.  $v \in V$  in  $G(V, E)$ ): Make a set containing only  $v$ .
  - 5 While  $S$  does not form a spanning tree and  $Q$  is not empty: Remove the edge with the smallest weight from  $Q$ . Let this edge be  $(u, v)$ .
  - 6 If  $u$  and  $v$  are not already in the same set: Add  $(u, v)$  to  $S$ . Merge the sets containing  $u$  and  $v$ .
  - 7 End While (point 5)
  - 8 Return  $S$ .
-

The following is an example of the implementation of Kruskal algorithm.

Let  $G$  be a graph with four nodes  $A, B, C, D$  and edges with weights as follows:

- $A \rightarrow B:1.5$
- $A \rightarrow C:3$
- $B \rightarrow C:4$
- $B \rightarrow D:6$
- $C \rightarrow D:2.2$
- $A \rightarrow D:3.2$

Figure C.1 shows a visualization of the above network settings.

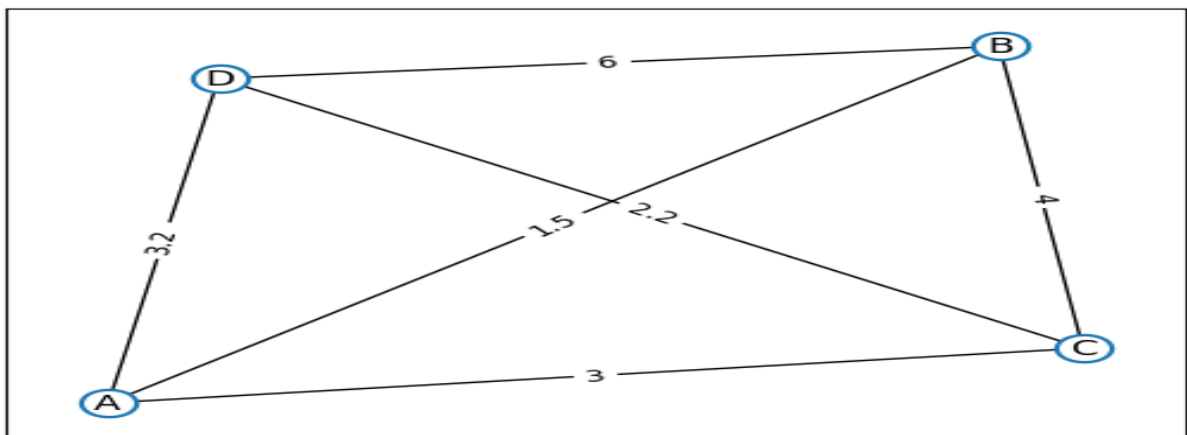
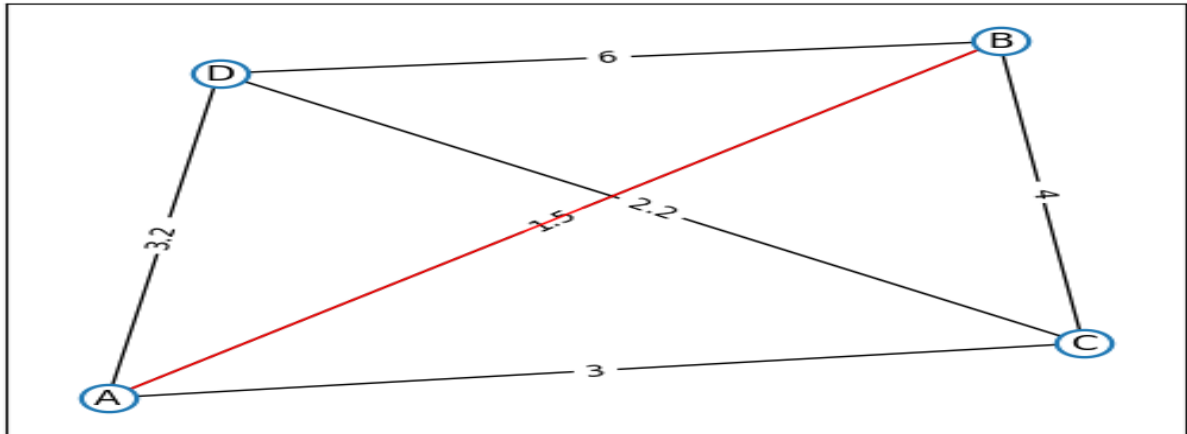


Figure C.1: Weighted Undirected Network (Four Nodes)

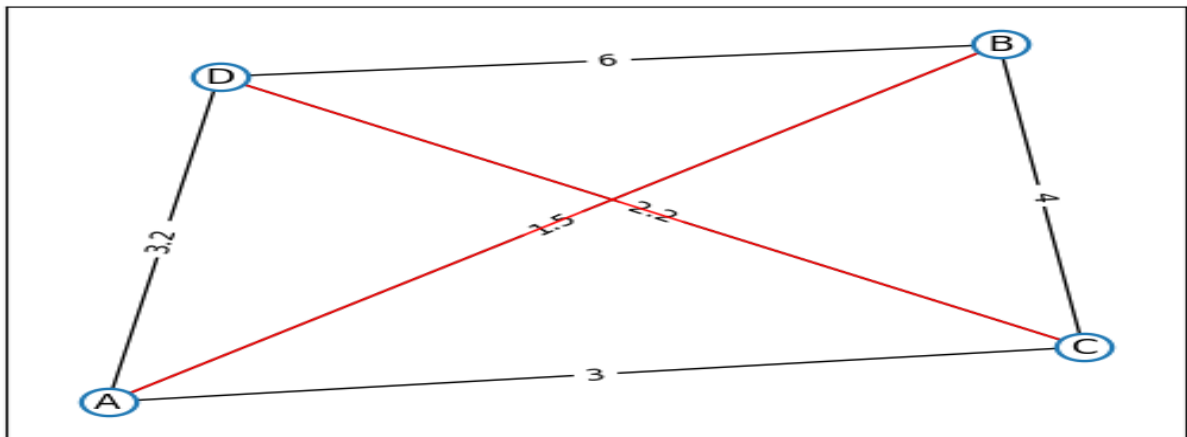
Using Kruskal's Algorithm:

1. Sort edges by ascending weights:  $A \rightarrow B:1.5$ ;  $C \rightarrow D:2.2$ ;  $A \rightarrow C:3$ ;  $A \rightarrow D:3.2$ ;  
 $B \rightarrow C:4$ ;  $B \rightarrow D:6$ ;
2. Add  $A \rightarrow B$ : no cycle.



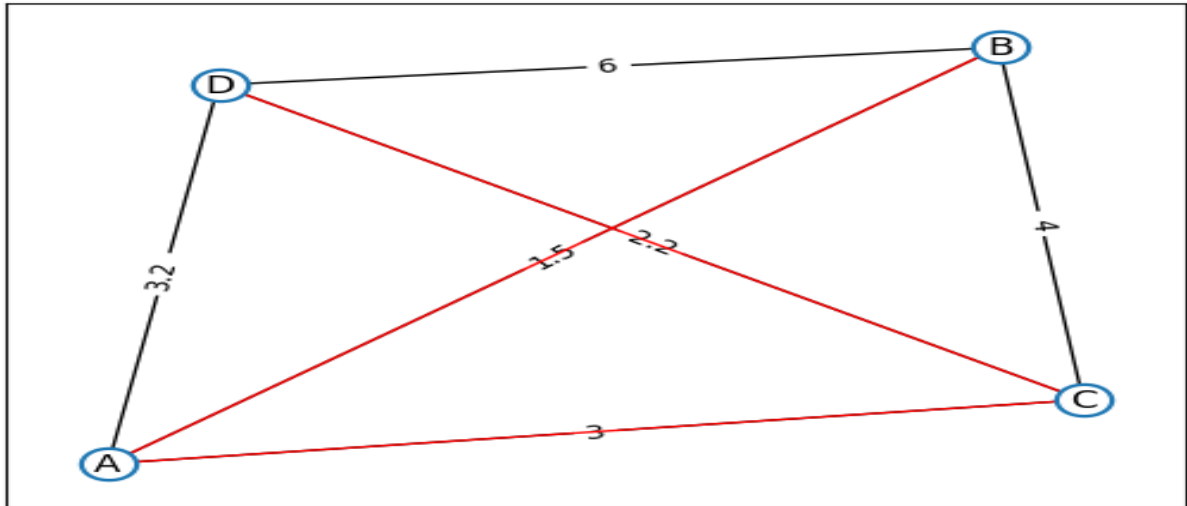
Kruskal's Algorithm- Step 1

3. Add  $C \rightarrow D$ : no cycle.



Kruskal's Algorithm- Step 2

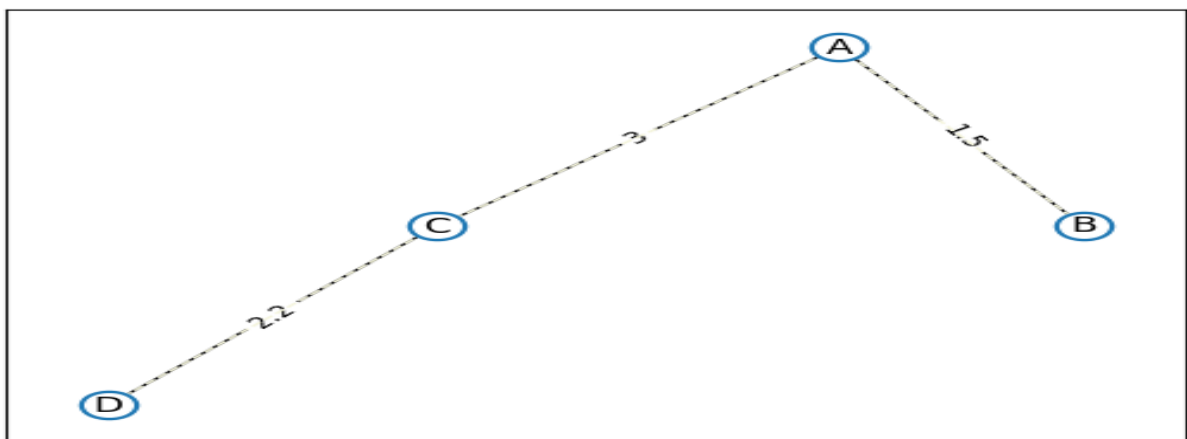
4. Add  $A \rightarrow C$ : no cycle.



Kruskal's Algorithm- Step 3

5. Skip  $A \rightarrow D$ : cycle ADC.
6. Skip  $B \rightarrow C$ : cycle BCA.
7. Skip  $B \rightarrow D$ : cycle BDA.

Therefore, the MST edges are  $A \rightarrow B$ ,  $C \rightarrow D$ ,  $A \rightarrow C$ , and the MST can be represented more conventionally as a tree. It is notable that with  $N = 4$  nodes, the tree has  $N - 1 = 3$  edges.



Kruskal's Algorithm Result

# Appendix D

## Canada and US Policy Indexes

Equity markets are forward-looking markets where investors quickly consume news and reflect their sentiment in the market. Baker et al. (2020) and Qureshi et al. (2021) discuss the impact of economic policy uncertainty during the COVID-19 period in their recent studies. In this section, we include an economic policy uncertainty index and combine the TSX 60 and Nasdaq 100 sector returns in our network analysis to investigate potential connections. The “Economic Uncertainty Policy Index” gauges policy-related economic uncertainty based on newspaper articles.<sup>1</sup> It quantifies the uncertainty related to economic policy by analyzing the frequency of references to policy uncertainty in major newspapers. The rate of change in the index measures the economic policy shift. Figure D.1 depicts the average rate of change in the Canadian and the US monthly economic uncertainty indexes and the US daily economic uncertainty index. While the daily index exhibits a volatile pattern, the two monthly indexes demonstrate a certain degree of positive correlation, indicating a moderate policy synergy between the two countries.

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<sup>1</sup>The Economic Uncertainty Policy Index is retrieved from <https://www.policyuncertainty.com/> on 2024-2-21.

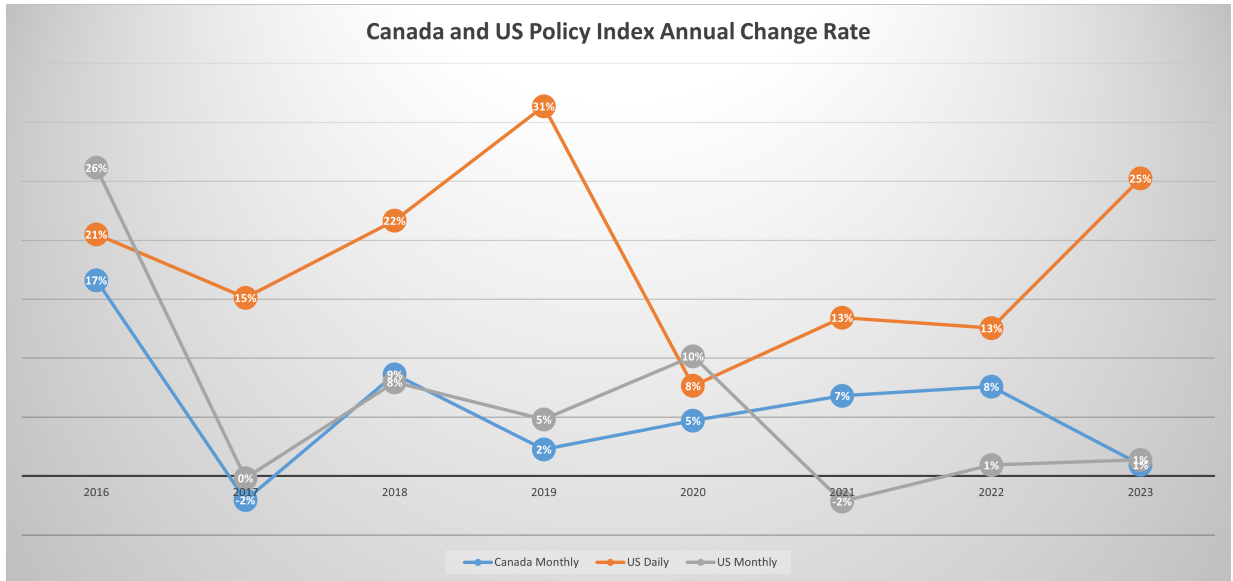


Figure D.1: Canada and U.S. Policy Index Annual Change Rate

Eventually, the US daily news index was selected as it has the same frequency as the US stock data. The US daily news index is a news-based economic policy uncertainty index based on news articles from US newspapers. With regard to the Canadian policy index, it should be noted that only the Canada monthly index is available, which has a different frequency than that of the stock return. Therefore, we convert the monthly growth rate to a daily rate in order to maintain consistency in data frequency. Additionally, we account for potential lags in the impact of news by including index change rates with a 1-day, 6-day, and 30-day lags for the US index and 1-month lag for the Canadian index. As for stocks, the sector logarithmic return is calculated by grouping each individual stock return and weighting it by the market capitalization of each stock. Figure D.2 illustrates the interconnections between policy indexes and the stock network, with the use of GLASSO optimization. The correlation between the policy indexes and the sectoral returns reflects the relationship between industry performance and policy uncertainty. The rate of change of the policy index serves to quantify the degree of policy uncertainty. If a correlation exists between policy index and sector return, then it can be inferred that the sector was also influenced by policy decisions. The right panel of Figure D.2 demonstrates that when the partial correlation threshold is set to 0.1, the policy indexes are exclusively linked within the domestic context, exhibiting no correlation with stock returns. Table D.1 measures

the group degree centrality at varying thresholds. The Canada policy index consistently exhibits the smallest possible influence within the network, as evidenced by its zero group degree centrality. Despite the US policy index's notable connections with the markets, the group degree centrality markedly declines to 0.11 from its initial high level of 0.96 upon the application of a threshold.

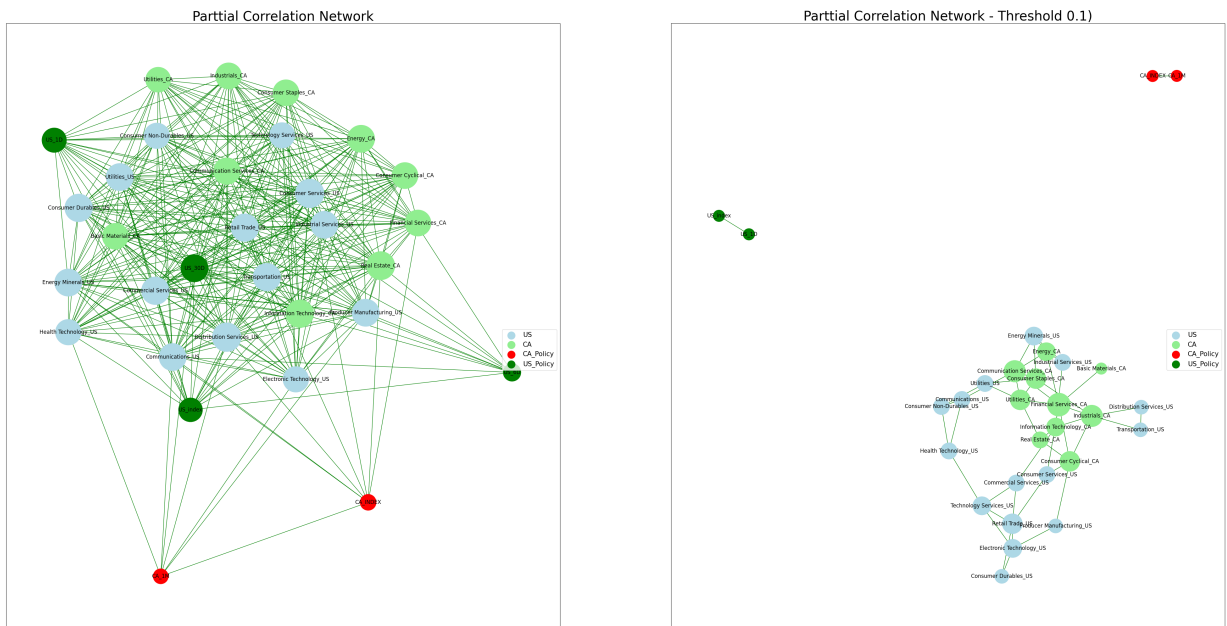


Figure D.2: TSX 60 and Nasdaq 100 Sector Returns and Policy Indexes Partial Correlation Network. Note:  $\alpha$  is 0.01.

Table D.1: Degree Centrality of International and Policy Network and Groups

| Sector        | Partial Correlation Thresholds |      |      |
|---------------|--------------------------------|------|------|
|               | 0                              | 0.05 | 0.1  |
| Nasdaq 100    | 1                              | 0.73 | 0.73 |
| TSX 60        | 0.95                           | 0.75 | 0.75 |
| Canada Policy | 0.38                           | 0    | 0    |
| US Policy     | 0.96                           | 0.11 | 0.11 |

Degree centrality measures the fraction of nodes that a node is connected to. The degree centrality values are normalized by dividing them by the maximum possible degree in a simple graph, which is  $n-1$ , where  $n$  represents the number of nodes in a network. The group degree centrality is normalized by dividing the group degree by the number of non-group nodes. The maximum degree centrality is 1. Note: GLASSO uses the optimized  $\alpha$  of 0.01.

# Appendix E

## Entropy

Information theory studies the information quantification, storage and communication in a mathematical way. In late 1940s, Claude Shannon proposed Shannon Entropy ( $H$ ) in his seminal work, A Mathematical Theory of Communication (Shannon, 1948). This measurement is derived from the frequency of an event.

Lets start with a single outcome event ( $x$ ), the Shannon information of  $x$  is defined as:

$$\begin{aligned} \text{Info}(x) &= \log_2\left(\frac{1}{P(x)}\right) \\ &= \log_2(1) - \log_2(P(x)) \\ &= -\log_2(P(x)), \end{aligned} \tag{E.1}$$

where  $\text{Info}(x)$  is called Shannon Information that represents the amount of information obtained if an event  $x$  happens, and  $P(x)$  is the probability of event  $x$ .<sup>1</sup> The Shannon Information shows that information exists only when beliefs are challenged. To illustrate, when the probability of an event ( $x$ ) is equal to one, the associated information is zero because there is no uncertainty regarding the occurrence of this event. However, if one observes an event whose probability is small, it is surprising and information increases, thereby establishing the inverse relation,  $\frac{1}{P(x)}$ . The log of  $\frac{1}{P(x)}$  is taken to ensure that when

---

<sup>1</sup>Note that we will typically use base 2 log in this section. For the sake of simplicity, we will omit the subscript of 2 (i.e.  $\log()$  refers to  $\log_2()$ )

$P(x) = 1$ , information is exactly equal to zero.<sup>2</sup>

Now consider a set of outcomes ( $x$ ) for an event ( $X$ ) with a probability distribution  $\mathbf{P}$ , the expected amount of information is defined as  $E[Info(X)]$ , and is called Shannon Entropy and noted as  $H(X)$ . It quantifies how surprising the random variable is, on average. If  $X$  is discrete, then:

$$H(X) = - \sum_{x \in X} p_i \log(p_i), \quad (\text{E.2})$$

where  $p_i \in \mathbf{P}$ . If  $X$  is continuous, then:

$$H(X) = - \int_x p(x) \log(p(x)) dx.$$

We previously defined entropy for a single variable  $X$  (or  $Y$ ), but what information is contained in  $X$  and  $Y$  collectively as opposed to  $X$  and  $Y$  individually? The amount of information contained in two random variables is known as joint entropy. For a bivariate discrete case, suppose the joint distribution of  $x, y$  is  $P(x, y), x \in X$  and  $y \in Y$ . Then  $H(X, Y)$  (Shannon Entropy) is defined as:

$$H(X, Y) = - \sum_x \sum_y p_{X,Y}(x, y) \log(p_{X,Y}(x, y)).$$

Similarly, the continuous case is written as:

$$H(X, Y) = - \int_{x,y} p_{X,Y}(x, y) \log(p_{X,Y}(x, y)) dx dy.$$

The conditional entropy (i.e.  $H(Y|X)$ ) measures the amount information in a random variable  $Y$  given the value of another random variable  $X$ . In other words, it quantifies the remaining uncertainty about  $Y$  when  $X$  is known. The conditional entropy satisfies  $H(Y|X) = H(X, Y) - H(X)$ . The mathematical expression for the discrete distribution is

---

<sup>2</sup>if the probability of an event ( $x$ ) is equal to zero, the associated information is not defined, as this event will never occur.

:

$$\begin{aligned}
 H(Y|X) &= -E_{(x,y)}(\log p(y|x)) \\
 &= -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) \log p(y|x) \\
 &= -\sum_{x \in X} \sum_{y \in Y} p(x,y) \log p(y|x).
 \end{aligned} \tag{E.3}$$

Figure E.1 shows a Ven diagram to depict the relationship among joint entropy  $H(X, Y)$ , conditional entropies  $H(X|Y), H(Y|X)$  and entropies for  $X$  and  $Y$ ,  $H(X), H(Y)$ . It also shows graphically how much information is exchanged between  $X$  and  $Y$ ,  $I(X, Y)$ , or mutual dependence between these two variables. From Figure E.1, we can also see that entropy for two random variable  $X$  and  $Y$  satisfies the following inequality:

$$H(X), H(Y) \leq H(X, Y) \leq H(X) + H(Y). \tag{E.4}$$

Furthermore, if two random variables  $X$  and  $Y$  are identical, then the information in the  $H(*)$  is the same as the information in each of them, and we get  $H(X, Y) = H(X) = H(Y)$ . On the other hand, if  $X$  and  $Y$  are independent, then  $H(X, Y) = H(X) + H(Y)$ .

## Venn Diagram for Mutual Information

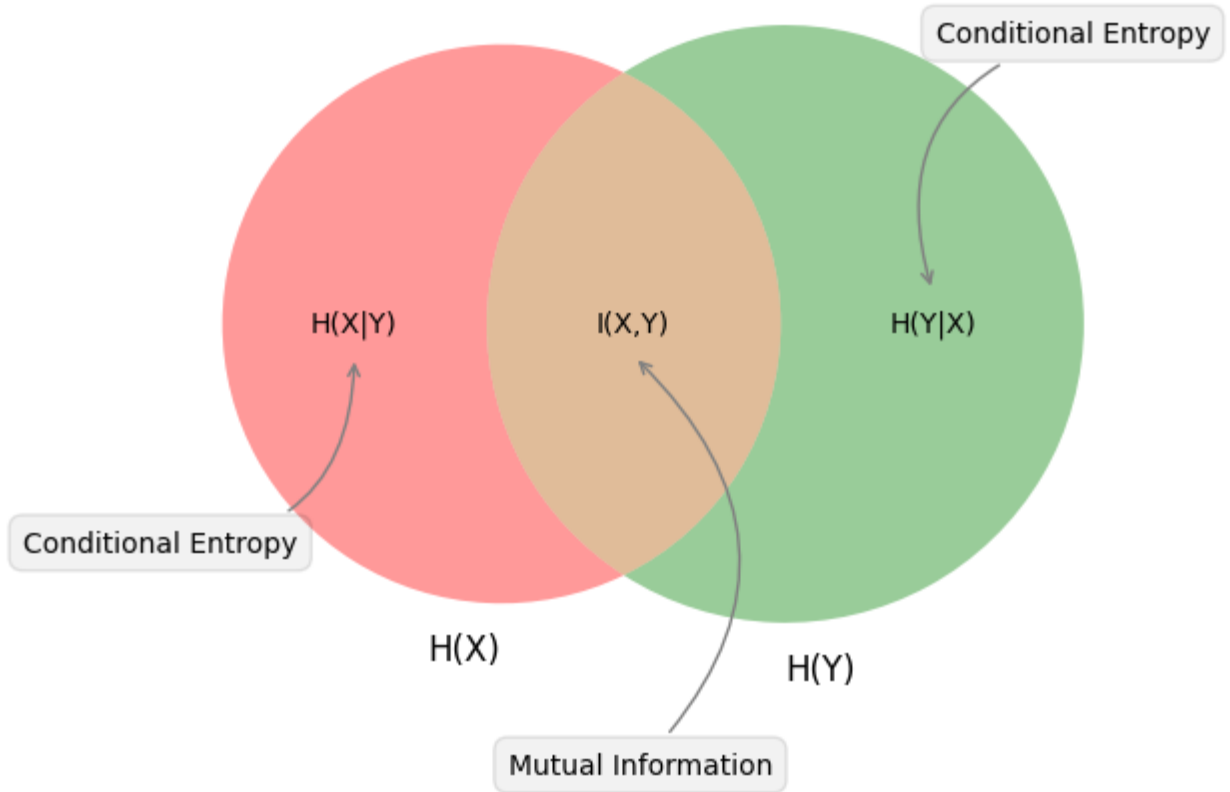


Figure E.1: Entropy Ven Diagram

Mathematical note:

$$I(X, Y) = \sum_{x \in X} \sum_{y \in Y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)} = H(X) - H(X|Y). \quad (\text{E.5})$$

*Proof.*

$$I(X, Y) = H(X) - H(X|Y)$$

Substituting  $H(X|Y)$  in Equation E.5:

$$\begin{aligned} I(X, Y) &= H(X) - \left( - \sum_{x \in X} \sum_{y \in Y} p(x, y) \log \frac{p(x, y)}{p(y)} \right) \\ &= H(X) + \sum_{x \in X} \sum_{y \in Y} p(x, y) \log \frac{p(x, y)}{p(y)} \end{aligned} \quad (\text{E.6})$$

We now want to determine  $H(X)$  given its expression in Equation E.2 and  $p(x) =$

$\sum_{y \in \mathcal{Y}} p(x, y)$ :

$$\begin{aligned}
H(X) &= - \sum_{x \in \mathcal{X}} p(x) \log p(x) \\
&= - \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(x) \\
&= \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \frac{1}{\log p(x)}
\end{aligned} \tag{E.7}$$

Substituting Equation E.8 into Equation E.6:

$$\begin{aligned}
I(X, Y) &= - \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(x) + \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log \frac{p(x, y)}{p(y)} \\
&= \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \left( \log \frac{p(x, y)}{p(x)p(y)} \right)
\end{aligned} \tag{E.8}$$

Thus, we have derived the mutual information expression in Equation E.5 □

Schreiber (2000) proposes the transfer entropy  $T_{y \rightarrow x}$  by using the concept of the Kullback-Leibler divergence (Kullback and Leibler, 1951). The Kullback-Leibler divergence between two probability distributions  $P$  and  $Q$  is defined as:

$$D_{\text{KL}}(P||Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)} \tag{E.9}$$

In the bivariate case, the information flow from process  $Y$  to process  $X$  is quantified by calculating the discrepancy from the generalized Markov property. For transfer entropy, we consider the KL divergence between two conditional distributions:

- $p(x_{t+1} | x_t^{(k)}, y_t^{(l)})$ : The probability of  $X$  at time  $t + 1$  given its own past  $x_t^{(k)}$  (i.e.  $x_t^{(k)} = (x_t, \dots, x_{t-k+1})$ ) and the past of  $y_t^{(l)}$  (i.e.  $x_t^{(k)} = (x_t, \dots, x_{t-k+1})$ ).
- $p(x_{t+1} | x_t^{(k)})$ : The probability of  $X$  at time  $t + 1$  given only its own past  $x_t^{(k)}$ .

Applying the above two conditional distributions into Equation E.9:

$$D_{\text{KL}} \left( p(x_{t+1} | x_t^{(k)}, y_t^{(l)}) \parallel p(x_{t+1} | x_t^{(k)}) \right) = \sum_{x_{t+1}} p(x_{t+1} | x_t^{(k)}, y_t^{(l)}) \log \frac{p(x_{t+1} | x_t^{(k)}, y_t^{(l)})}{p(x_{t+1} | x_t^{(k)})}. \quad (\text{E.10})$$

Since transfer entropy is concerned with how information transfer depends on the joint distribution of past and future states, we average the KL divergence over the joint distribution  $p(x_{t+1}, x_t^{(k)}, y_t^{(l)})$ , then:

$$T_{y \rightarrow x} = \sum_{x_{t+1}, x_t^{(k)}, y_t^{(l)}} p(x_{t+1}, x_t^{(k)}, y_t^{(l)}) \log \frac{p(x_{t+1} | x_t^{(k)}, y_t^{(l)})}{p(x_{t+1} | x_t^{(k)})}. \quad (\text{E.11})$$

In addition, the transfer entropy can be interpreted as the conditional mutual information between  $x_{t+1}$  and  $y_t^{(l)}$  given  $x_t^{(k)}$  :

$$\begin{aligned} T_{y \rightarrow x} &= I(x_{t+1}, y_t^{(l)} | x_t^{(k)}) \\ &= \sum_{x_{t+1}, x_t^{(k)}, y_t^{(l)}} p(x_{t+1}, x_t^{(k)}, y_t^{(l)}) \log \frac{p(x_{t+1}, y_t^{(l)} | x_t^{(k)})}{p(x_{t+1} | x_t^{(k)}) p(y_t^{(l)} | x_t^{(k)})} \\ &= \sum_{x_{t+1}, x_t^{(k)}, y_t^{(l)}} p(x_{t+1}, x_t^{(k)}, y_t^{(l)}) \log \frac{p(x_{t+1} | x_t^{(k)}, y_t^{(l)}) p(y_t^{(l)} | x_t^{(k)})}{p(x_{t+1} | x_t^{(k)}) p(y_t^{(l)} | x_t^{(k)})} \\ &\quad \sum_{x_{t+1}, x_t^{(k)}, y_t^{(l)}} p(x_{t+1}, x_t^{(k)}, y_t^{(l)}) \log \frac{p(x_{t+1} | x_t^{(k)}, y_t^{(l)})}{p(x_{t+1} | x_t^{(k)})}, \end{aligned} \quad (\text{E.12})$$

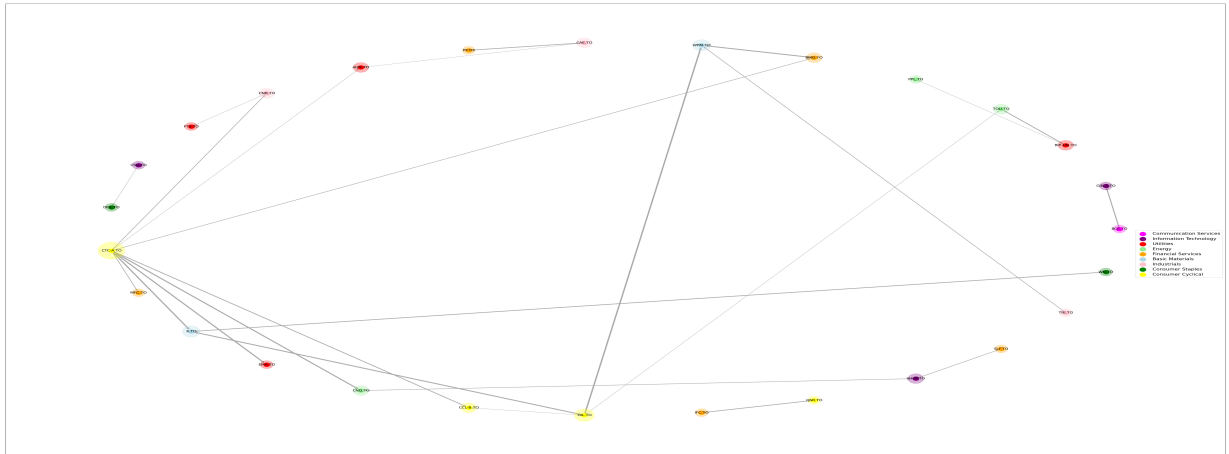
where  $p(x_{t+1}, y_t^{(l)} | x_t^{(k)}) = p(x_{t+1} | x_t^{(k)}, y_t^{(l)}) p(y_t^{(l)} | x_t^{(k)})$ .

# Appendix F

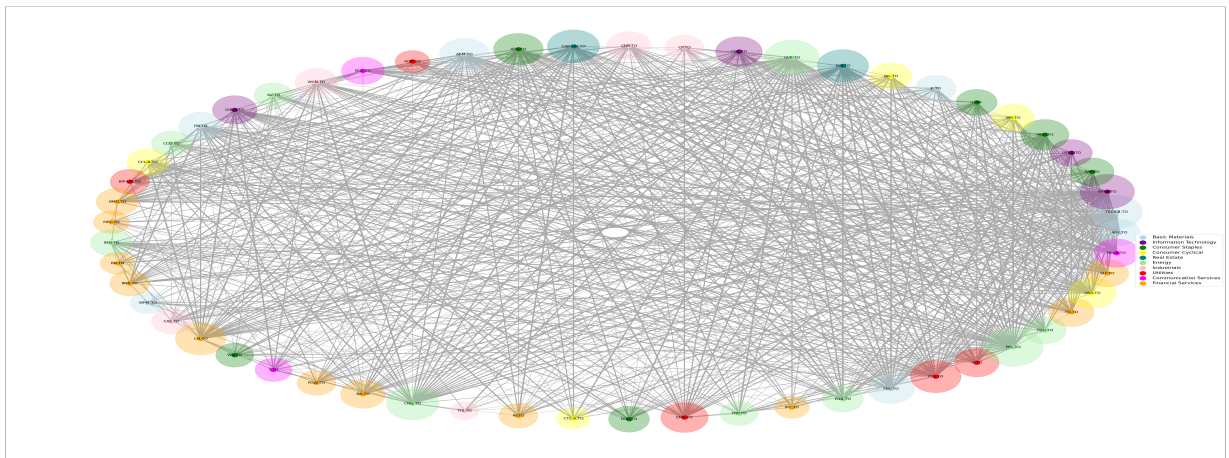
## Net Transfer Entropy Analysis

### F.1 TSX 60

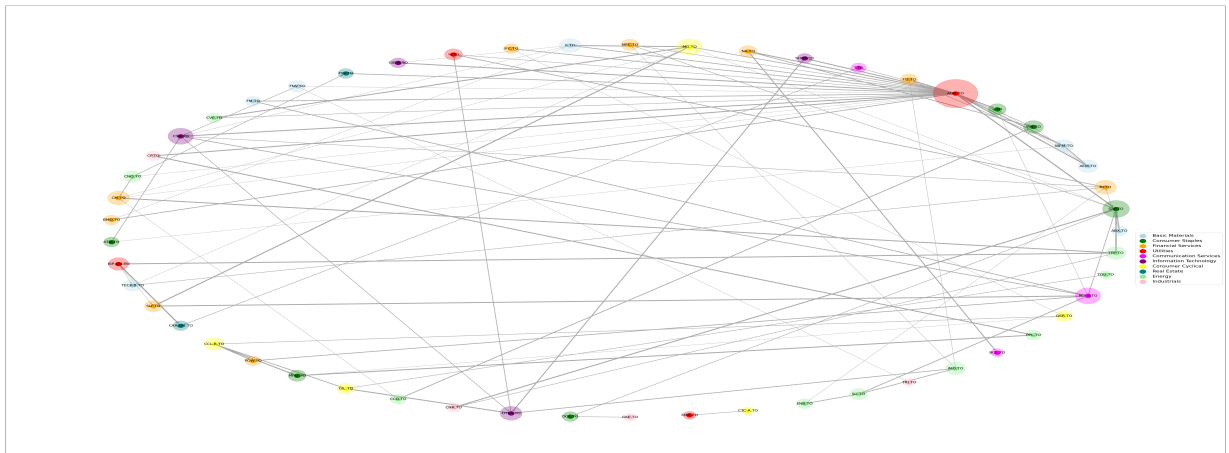
Additional plots for Section 2.5.1 TSX 60 analysis:



(a) Before COVID-19

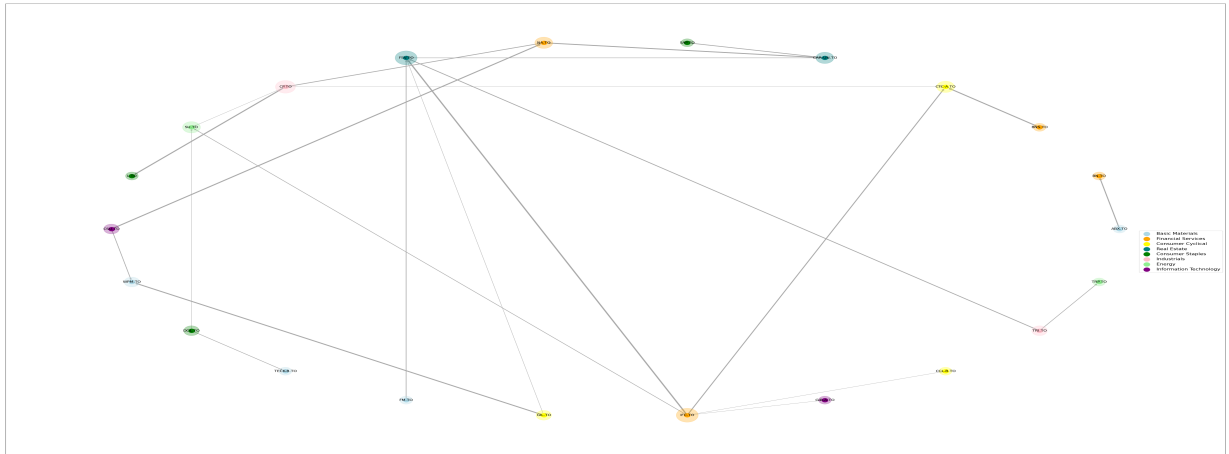


(b) COVID-19 Period

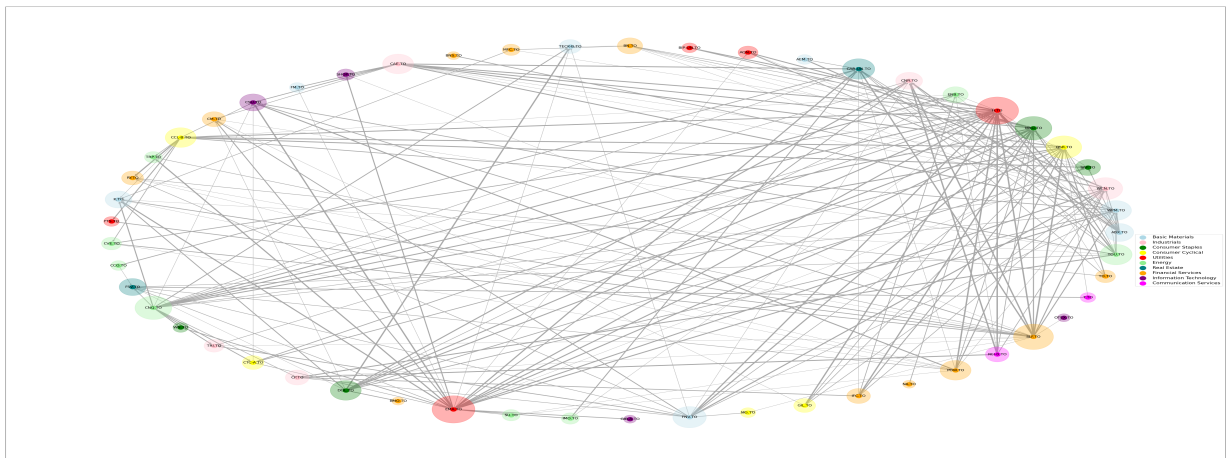


(c) Post COVID-19

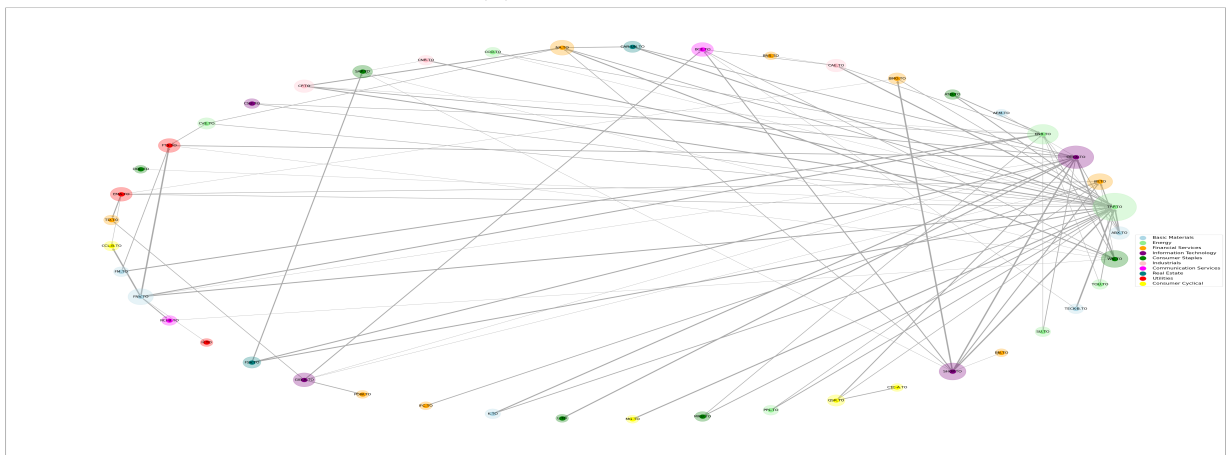
Figure F.1: TSX 60 Index Constituents Short Term Net Transfer Entropy Network (Threshold 0.1) - Sub-periods



(a) Before COVID-19



(b) COVID-19 Period

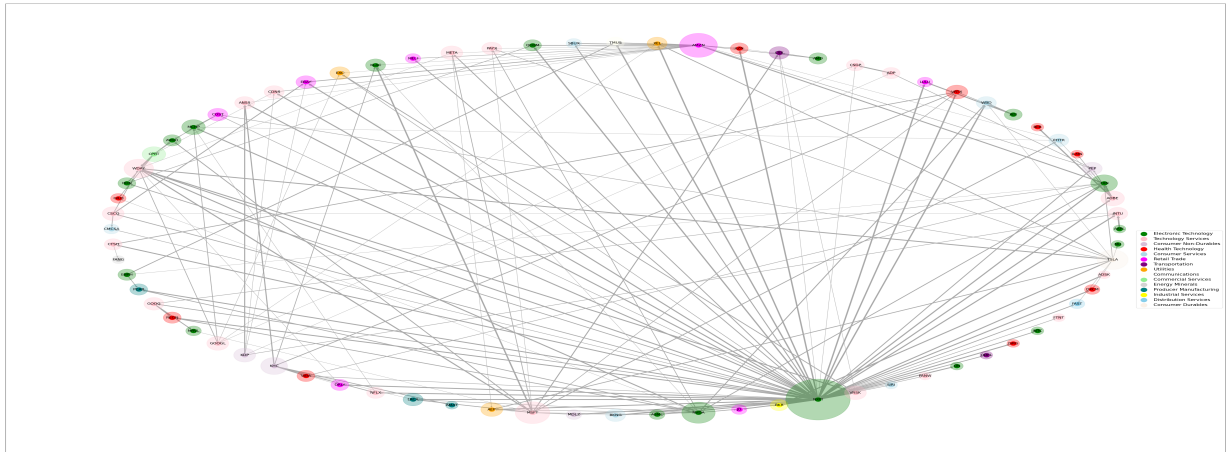


(c) Post COVID-19

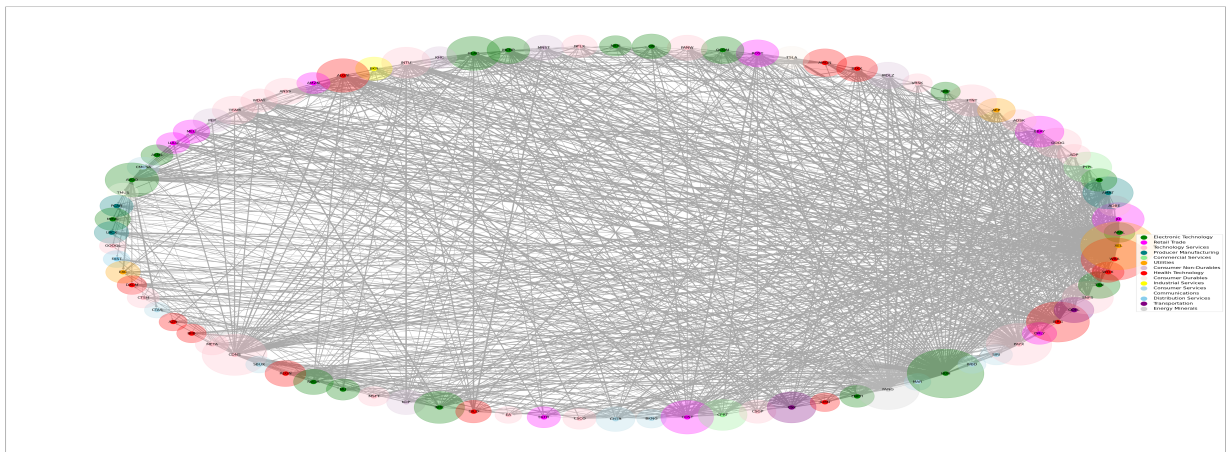
Figure F.2: TSX 60 Index Constituents Long Term Net Transfer Entropy Network (Threshold 0.1) - Sub-periods

## F.2 Nasdaq 100

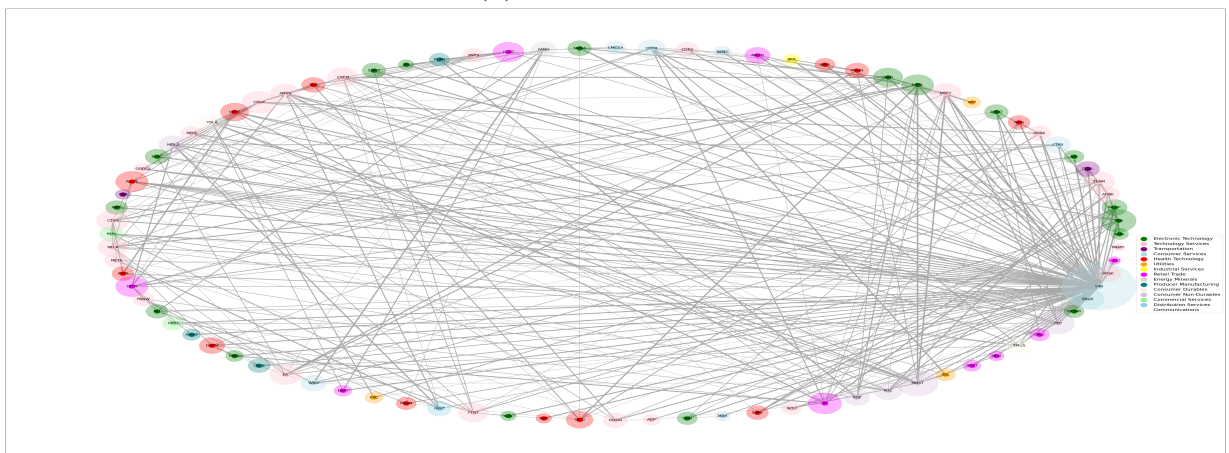
Additional plots for Section 2.5.1 Nasdaq 100 analysis:



(a) Before COVID-19

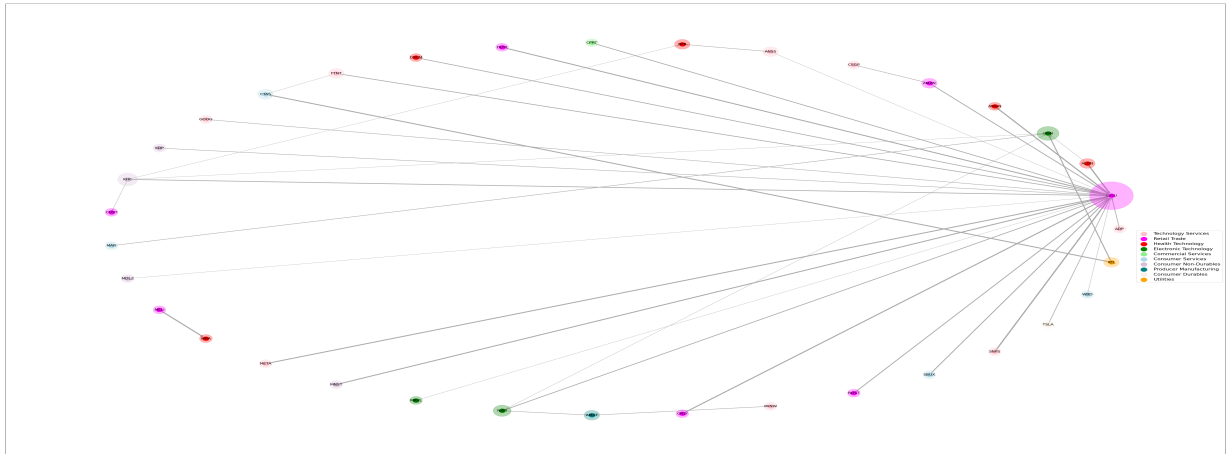


(b) COVID-19 Period

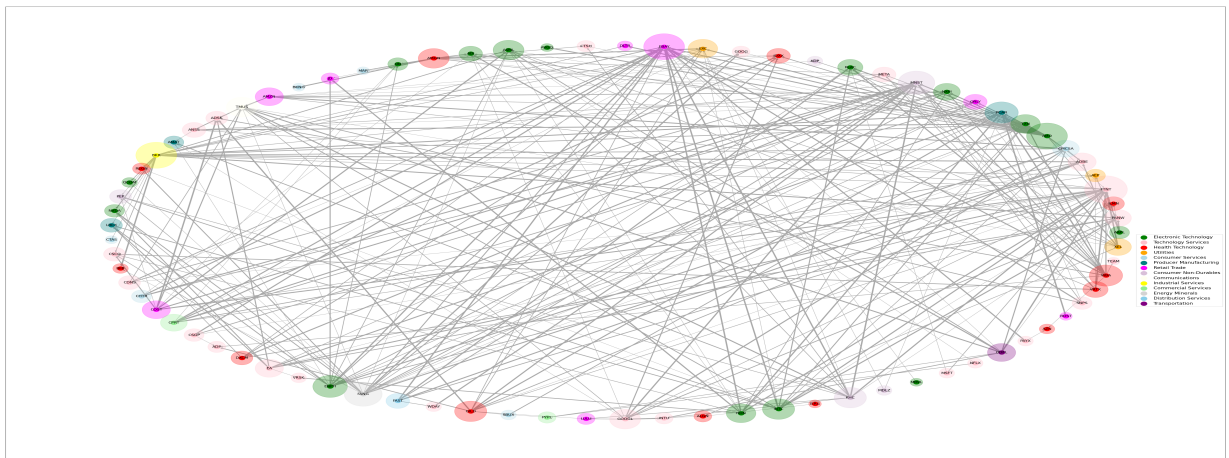


(c) Post COVID-19

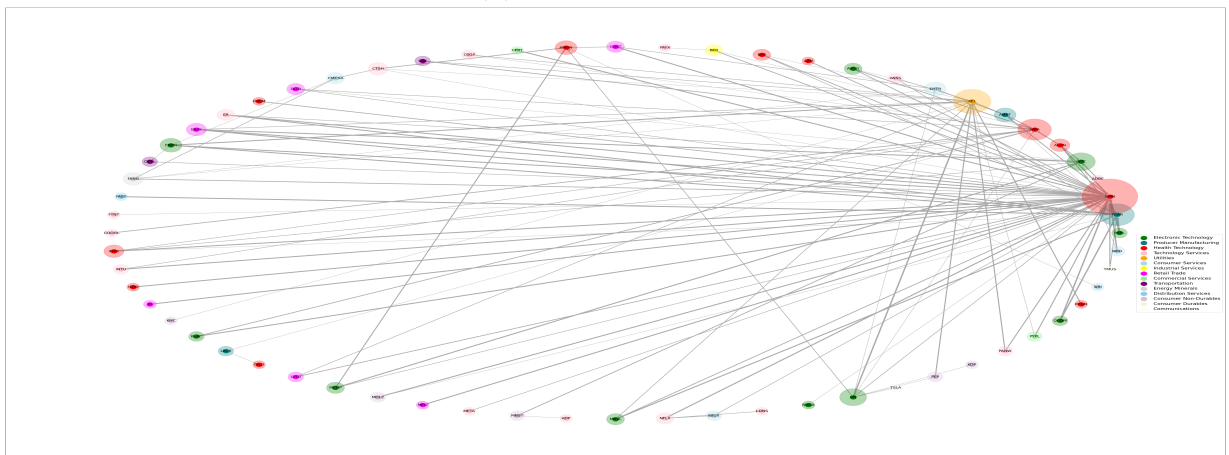
Figure F.3: Nasdaq 100 Index Constituents Short Term Net Transfer Entropy Network (Threshold 0.1) - Sub-periods



(a) Before COVID-19



(b) COVID-19 Period



(c) Post COVID-19

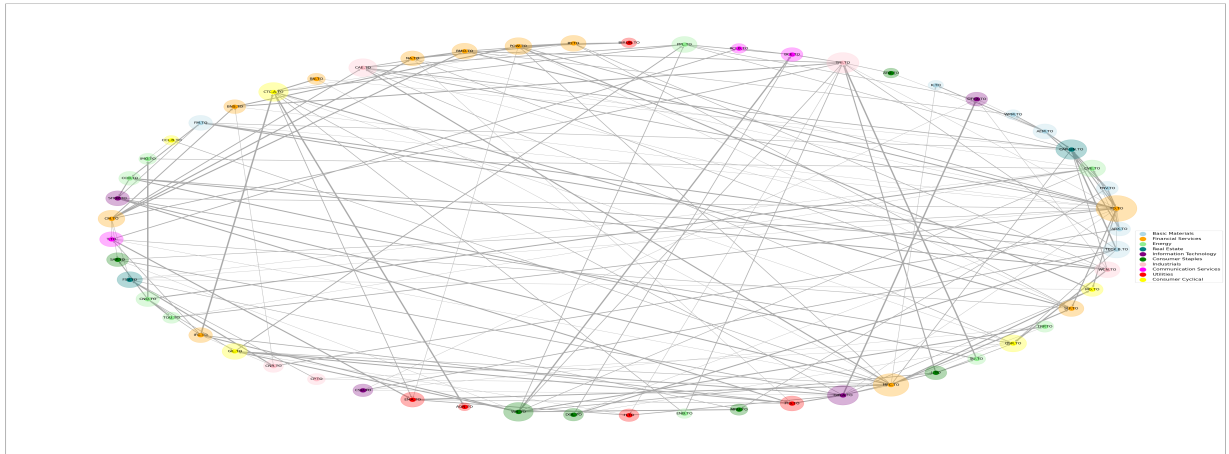
Figure F.4: Nasdaq 100 Index Constituents Long Term Net Transfer Entropy Network (Threshold 0.1) - Sub-periods

# Appendix G

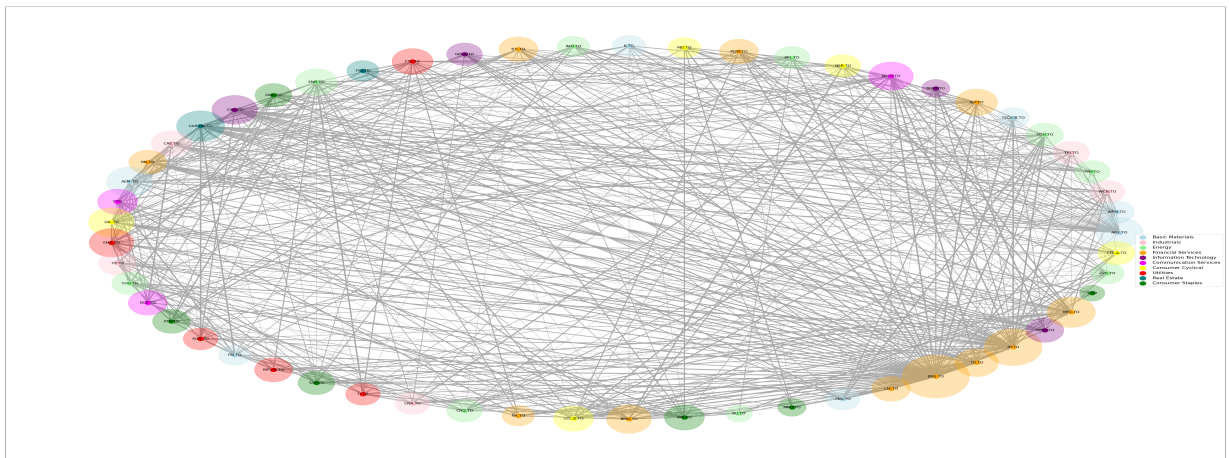
## Diebold-Yilmaz Spillover Analysis

### G.1 TSX 60

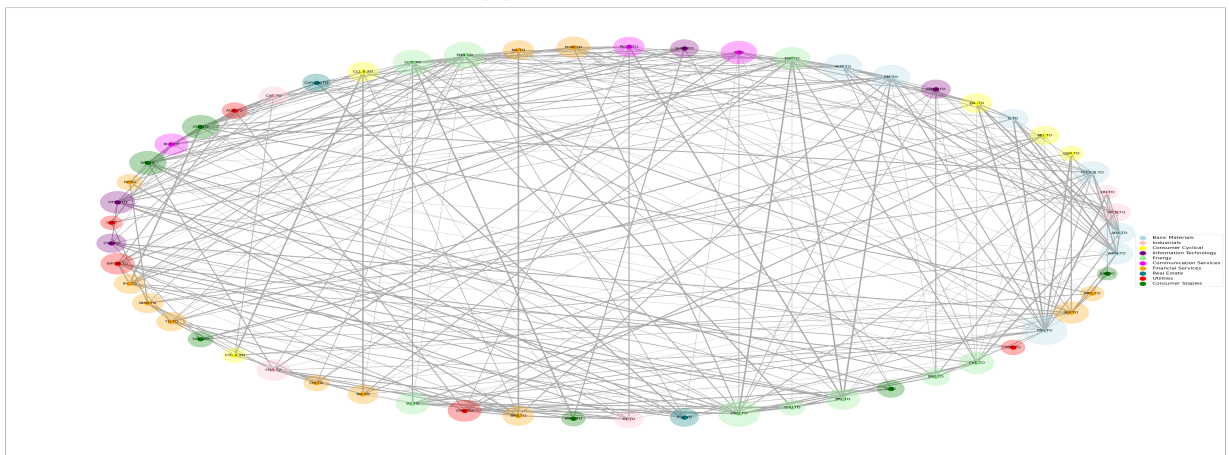
Additional plots for Section 2.5.2 TSX 60 analysis:



(a) Before COVID-19

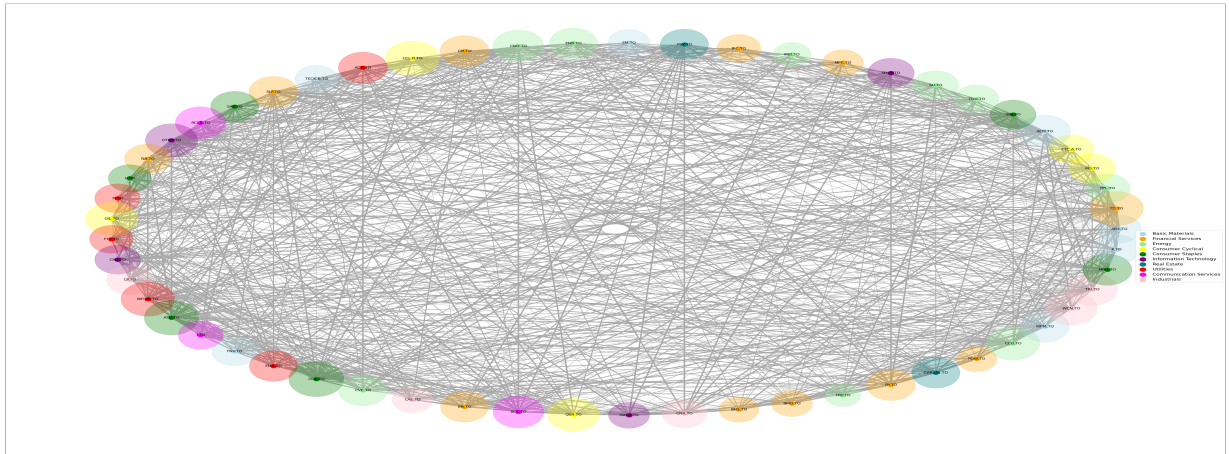


(b) COVID-19 Period

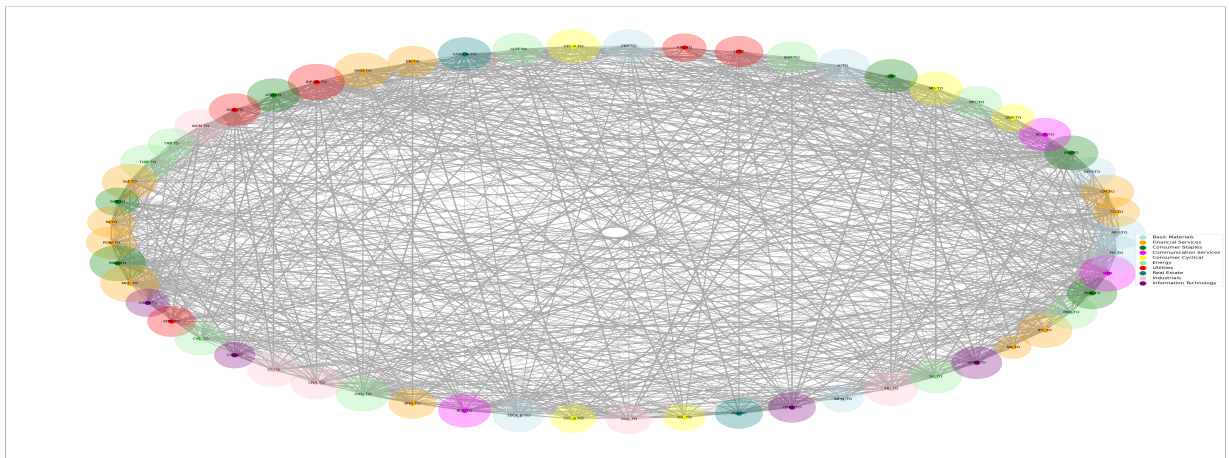


(c) Post COVID-19

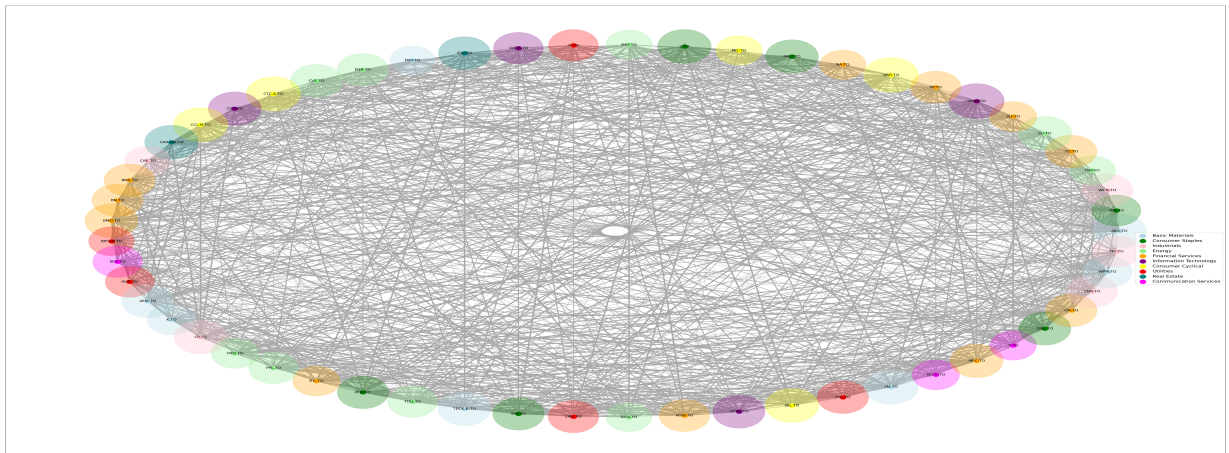
Figure G.1: TSX 60 Index Constituents Short Term DY Spillover Index Entropy Network (Threshold 0.01) - Sub-periods



(a) Before COVID-19



(b) COVID-19 Period

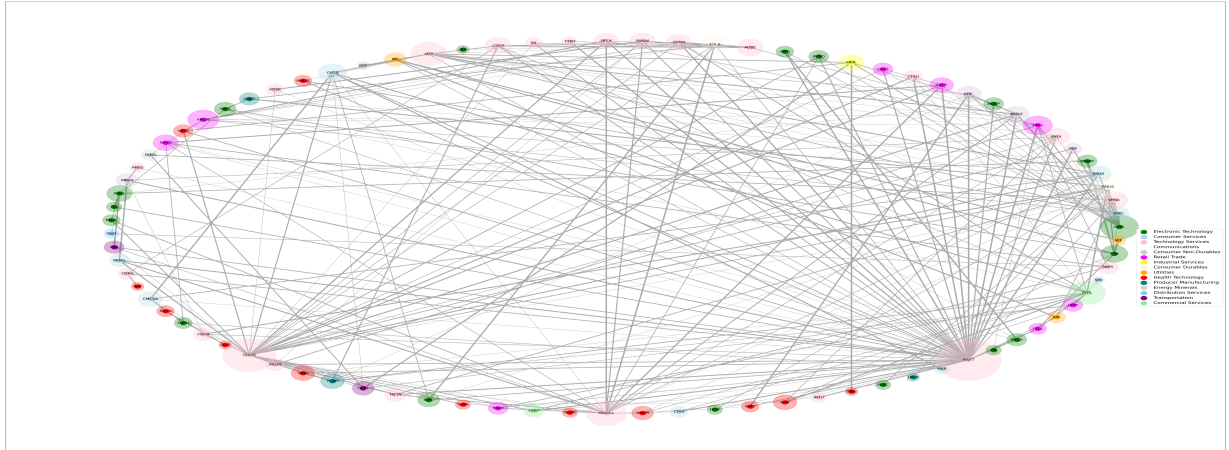


(c) Post COVID-19

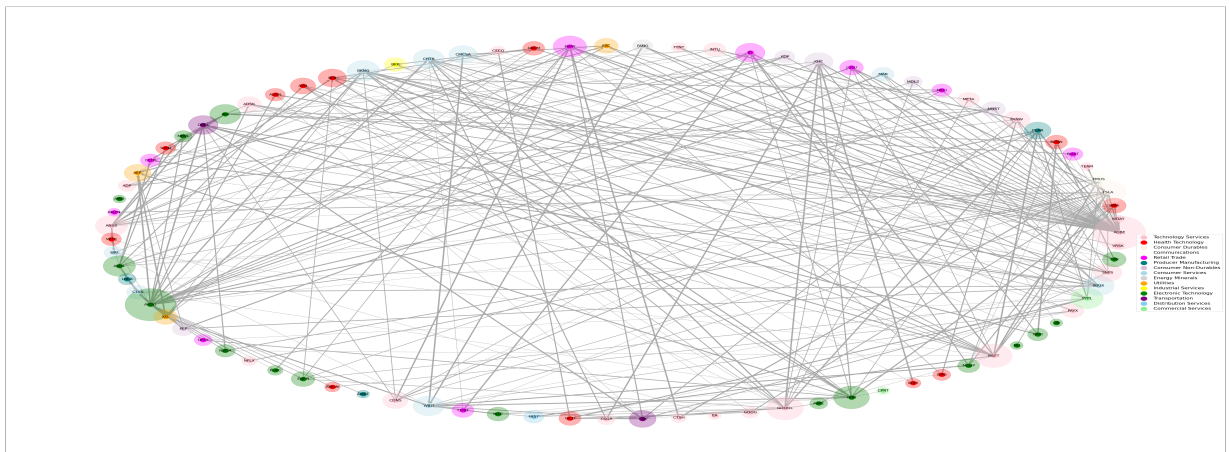
Figure G.2: TSX 60 Index Constituents Long Term DY Spillover Index Network (Threshold 0.01) - Sub-periods

## G.2 Nasdaq 100

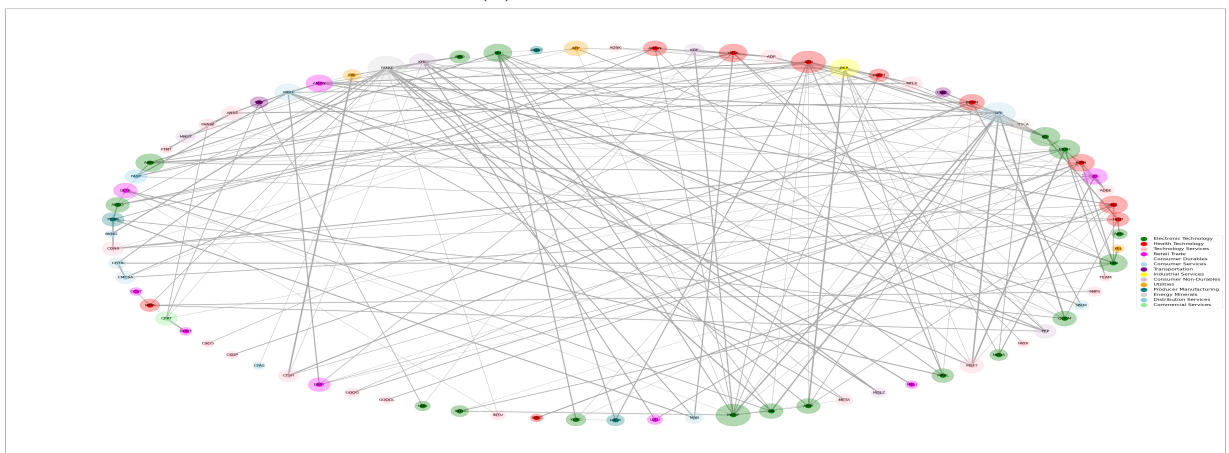
Additional plots for Section 2.5.2 Nasdaq 100 analysis:



(a) Before COVID-19

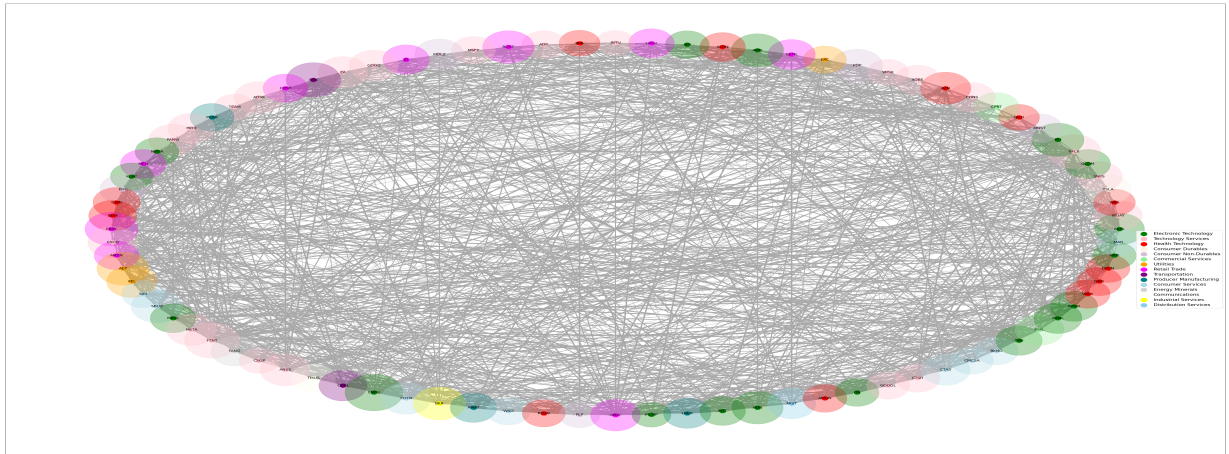


(b) COVID-19 Period

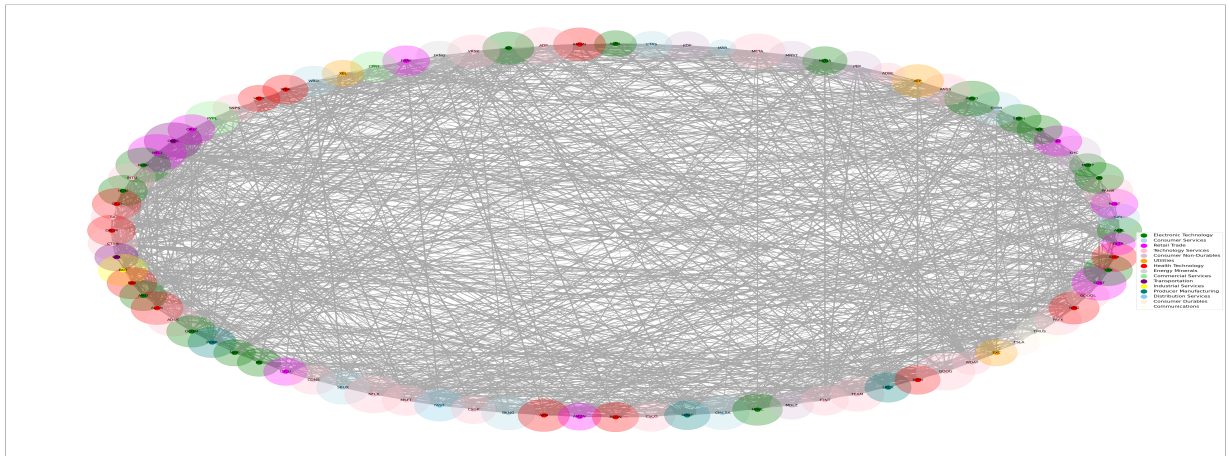


(c) Post COVID-19

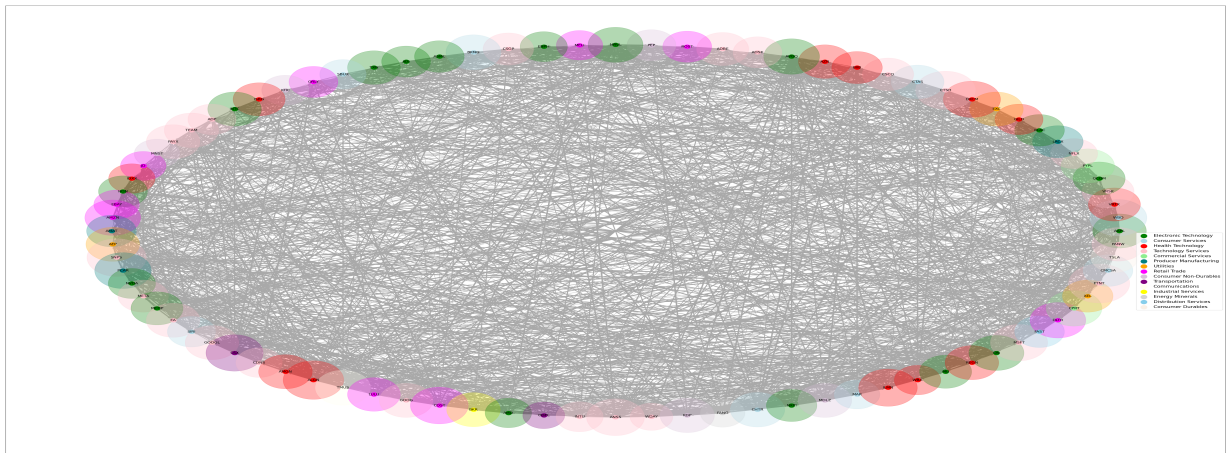
Figure G.3: Nasdaq 100 Index Constituents Short Term DY Spillover Index Entropy Network (Threshold 0.01) - Sub-periods



(a) Before COVID-19



(b) COVID-19 Period



(c) Post COVID-19

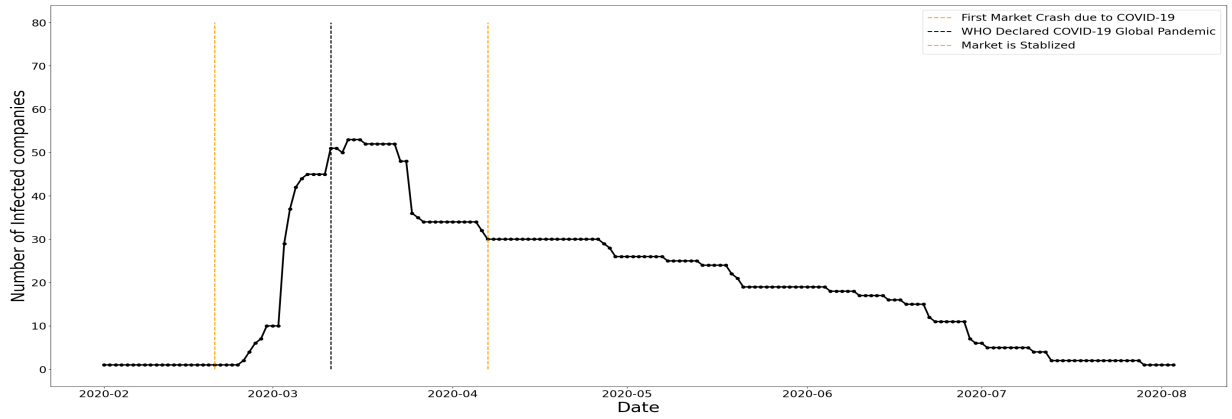
Figure G.4: Nasdaq 100 Index Constituents Long Term DY Spillover Index Entropy Network (Threshold 0.01) - Sub-periods

# Appendix H

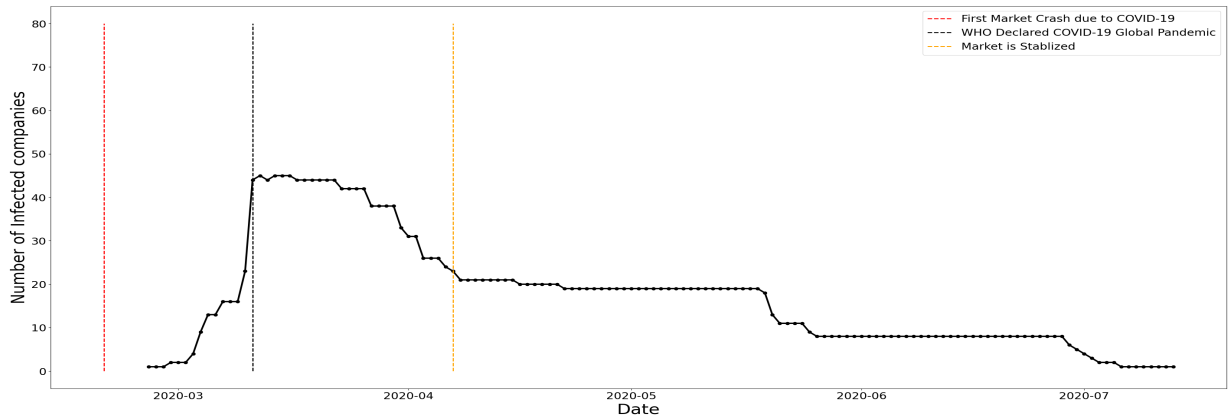
## Moving Average Window Selection

Figure H.1 illustrates the results of different moving average window selections. The first yellow dashed vertical line marks February 20, 2020, representing the first notable market reaction to COVID-19. The middle black dashed line, positioned on March 11, 2020, corresponds to the World Health Organization's (WHO) declaration of COVID-19 as a global pandemic - a point at which the number of infected stocks in the TSX 60 peaked. The second yellow dashed line corresponds to April 7, 2020, signaling market stabilization.

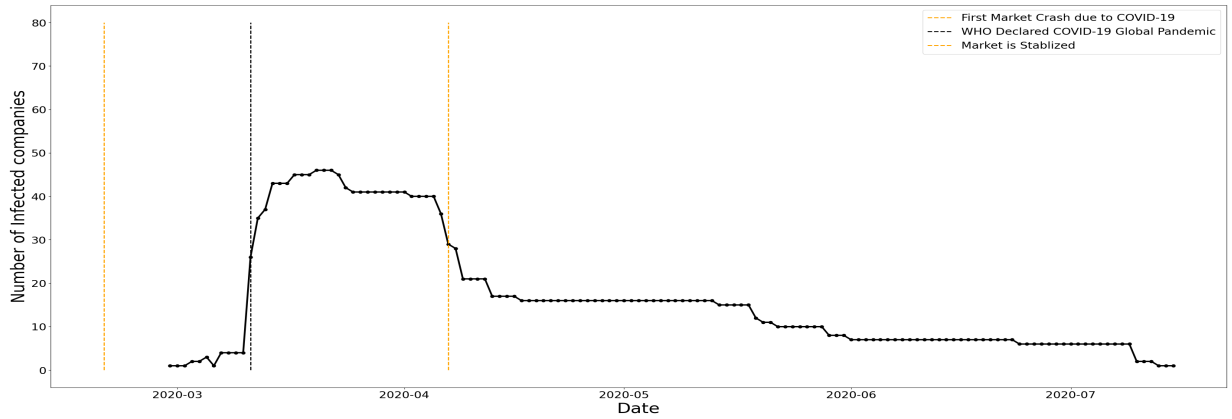
To examine stock infection patterns, we calculate moving averages for daily volatility and log returns over 7-, 14-, and 21-trading-day windows. Figures H.1a and H.1c fail to capture market stabilization, as they show a significant number of infected stocks after the observed stabilization date. In contrast, Figure H.1b aligns well with observed market trends, demonstrating a peak in infections around the pandemic declaration and a substantial decrease in infected stocks as markets stabilized. Based on this alignment, the 14-day moving average window is selected for subsequent analyses.



(a) Number of Infected Stocks - 7 trading-day window



(b) Number of Infected Stocks - 14 trading-day window



(c) Number of Infected Stocks - 21 trading-day window

Figure H.1: Trading Window Selection - TSX 60

## H.1 SIR Model Parameters Calibration: Transmission Rate ( $\beta$ ) and Recovery Rate ( $\gamma$ )

This section outlines the methodological approach employed to calibrate the key parameters of the Susceptible-Infected-Recovered (SIR) model, the transmission rate ( $\beta$ ) and the recovery rate ( $\gamma$ ). These parameters are fundamental in determining the dynamics of SIR model and are estimated by fitting the theoretical model to observed data. The SIR model is a type of compartmental model which consists of three compartments, Susceptible (S), Infected (I), and Recovered (R), respectively.

The dynamics of these compartments are described by a system of ordinary differential equations (ODEs), representing the rates of change over time. The model assumes a closed population, where the total number of stocks remains constant over the period of study. The calibration of  $\beta$  and  $\gamma$  follows the following steps:

### 1. Data and Initial Conditions

**Data** The calibration process requires time series data representing the number of infected stocks over time. In this analysis, the observed data consists of the recorded number of active infected stocks within the TSX 60, measured at daily intervals.

**Initial Conditions** Initial conditions must be specified to solve the SIR system numerically. These include:  $S_0 = 57$ ,  $I_0 = 1$ , and  $R_0 = 0$ .

### 2. Parameter Calibration Approach

**Objective Function** An objective function is formulated to quantify the difference between the model predictions and the observed data. In this case, the objective function calculates the sum of squared errors between the predicted number of infected individuals and the corresponding empirical data points.

**Model Simulation** The SIR model is solved numerically over the 139-day time horizon, given a initial guess and initial conditions. This generates model-based

predictions for the number of susceptible, infected, and recovered individuals at each time point.

**Optimization Procedure** An optimization algorithm searches for the parameter values that minimize the objective function. This iterative process adjusts  $\beta$  and  $\gamma$  systematically to achieve the best fit between the model and the data. We used the SciPy minimize function to implement this optimization process.

The calibration approach outlined in this appendix is also used in Section 3.5.3.

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