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Stochastic Analysis of Standby Robot-Safety Systems

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STOCHASTIC ANALYSIS OF STANDBY ROBOT-SAFETY SYSTEMS

Shen Cheng

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ABSTRACT

Nowadays, the application of robots covers almost all aspects of our daily life. They are used to perform increasing complex and critical operations. The increased critical applications have led to various reliability and safety problems, especially many people are injured and killed every year.

This study presents reliability and availability analyses of six different standby robot-safety systems: one robot and (n-1) standby safety units with a perfect switch, one robot and (n-1) standby safety units with an imperfect switch, (n-1) standby robots and one safety unit with a perfect switch, (n-1) standby robots and one safety unit with an imperfect switch, n parallel robots and (m-1) standby unit with a perfect switch, and two parallel robots and one standby unit with an imperfect switch.

With the aid of Markov and supplementary variable methods, general expressions for system state probabilities, system availability, reliability and mean time to failure were obtained. Plots of some of these expressions are shown to demonstrate the effect of varying failure rates or repair rates of the safety unit, and other parameters.

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Chapter 1

INTRODUCTION

1.1 Introduction:

In 2004, 20th Century Fox presented us a Hollywood movie, "I, robot", which described the relationship between human and robot in the future. It maybe not the truth, but it mentioned the important "Three Laws of Robotics" [75] first appeared from a short robot story, "Runaround", wrote by Asimov in 1942:

1. A robot may not injure a human being, or, through inaction, allow a human being to come to harm.
2. A robot must obey orders given to it by human beings except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

Actually it was until 1923, the word, "robot", first appeared in English as the translation of the Czech word, "worker" [76]. The word "robotics" also comes from the short robot story, "Runaround" in 1942. From the Egyptians starting to build water-powered clock and the Chinese and Greeks starting to build water and steam powered toys and robots have become highly complex and precise [15]. Today the application of robots covers nearly every aspect of our life including industry, exploration, medicine, the military and police, entertainment, toys, and so on. As a result, the population of robot is increasing at an impressive rate in recent years. According to the statistic data from the United Nations

Economic Commission for Europe (UN/ECE) and the International Federation of Robotics (IFR) [77, 78], worldwide investment in industrial robots has increased 19% in 2003, 17% in 2004 and orders for robots were up by 13% in the first half of 2005. A forecast is about 6% annual increase for the world wide investment increase in the robotic area for 2005 – 2008 [78], which means the industrial robots' installations will be projected to 121,000 in 2008 [78]. The projected distribution of the world industrial robot population for the period 2003-2008 is shown in Figure 1.1 [78].

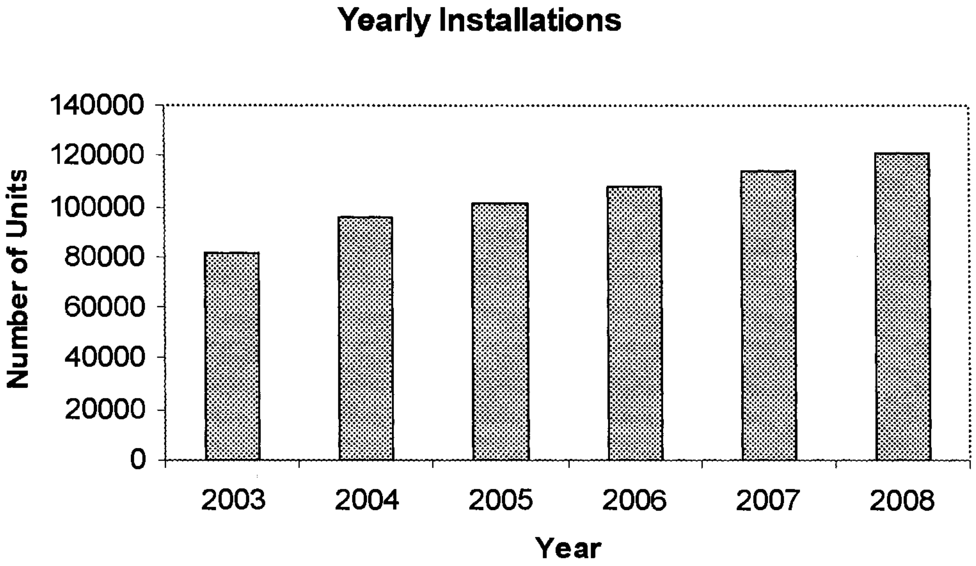


Figure 1.1 The projected distribution of the world industrial robot population

A pie chart, depicting the use of robots world wide in 2004, is shown in Figure 1.3 [78]. The threat to human safety from robots has to be taken into consideration serious since every year many people are injured and killed [78-81].

A comprehensive review of publications on robot reliability and safety for the period 1973 – 2001 is presented in Reference [17].

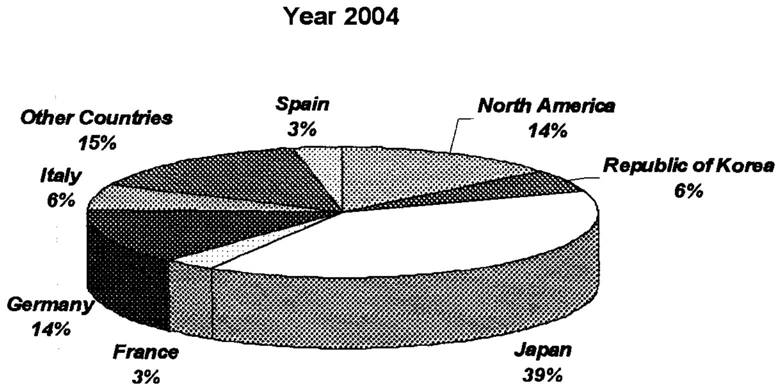


Figure 1.2 A pie chart depicting the use of robots worldwide for the year 2004

1.2 Literature Review

This section presents a review of published literatures on robot reliability and safety for the period (2000-2005). Table 1.1 lists the reviewed publications.

Table 1.1: The sources of collected publications

1. Journal of Robotics Today
2. Proceedings of the International Workshop on Bio-Robotics and Teleoperation, 2001
3. Journal of the Japan Society of Precision Engineering
4. Reliability Engineering & System Safety
5. Proceedings of the 7th International Conference on Optimization of Electrical and Electronic Equipments, 2000
6. High Technology Letters
7. Proceedings of the International Symposium on Product Quality and Integrity, 2000

8. Journal of the Harbin Institute of Technology
9. Die Casting Engineer
10. Robot
11. ABB Review
12. CIRP Annals - Manufacturing Technology
13. Systems Engineering and Electronics
14. Proceedings of the IEEE International Conference on Robotics and Automation, 2003
15. Journal of Shanghai Jiaotong University
16. Shanghai Jiaotong Daxue Xuebao
17. Proceedings of the NASA/DoD Conference on Evolvable Hardware, 2003
18. Proceedings of the Fifth World Congress on Intelligent Control and Automation,2004
19. Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2004
20. Proceedings of the Eighth IASTED International Conference on Artificial Intelligence and Soft Computing, 2004
21. Proceedings of the IEEE International Conference on Robotics and Automation,2004
22. Solid State Technology
23. Proceedings of the International Symposium on Micromechatronics and Human Science, 2000
24. Operations Research Letters
25. Robotics World
26. Proceedings of the 17th IAARC/CIB/IEEE/IFR International Symposium on Automation and Robotics in Construction, 2000

27. International Journal of Manufacturing Technology and Management
28. Bulletin of Aichi Institute of Technology
29. International Journal of Robotics Research
30. Journal of Intelligent Manufacturing
31. Proceedings of the ICRA. Millennium Conference. IEEE International Conference on Robotics and Automation, 2000
32. Experimental Robotics VI
33. Proceedings. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, 2000
34. Welding Design and Fabrication
35. IEEE Transactions on Systems, Man and Cybernetics Part C (Applications and Reviews)
36. Proceedings of the IASTED International Conference. Robotics and Applications, 2001
37. Proceedings of the International Symposium on Micromechatronics and Human Science, 2001
38. Advanced Robotics
39. Prace Naukowe Instytutu Cybernetyki Technicznej Politechniki Wroclawskiej
40. Proceedings of the ICRA. IEEE International Conference on Robotics and Automation, 2001
41. Proceedings of the International Conference on Multisensor Fusion and Integration for Intelligent Systems, 2001
42. Proceedings of the IASTED International Conference Automation, Control, and

Information Technology, 2002

43. Proceedings of the IEEE International Conference on Robotics and Automation, 2002
44. Industrial Robot
45. Expert Systems
46. Proceedings of the 6th World Multiconference on Systemics, Cybernetics and Informatics, 2002
47. Dynamic Systems and Control Division
48. Managing Automation
49. Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, 2003
50. Control Theory & Applications
51. Automatisierungstechnik
52. WSEAS Transactions on Systems
53. China Mechanical Engineering
54. Computing & Control Engineering
55. Proceedings of the 10th IEEE Real-Time and Embedded Technology and Applications Symposium, 2004
56. Journal of Aerospace Computing, Information and Communication
57. Automatisierungstechnische Praxis
58. Journal of Quality in Maintenance Engineering

1.2.1 Publications Classification

The collected Literature on Robot reliability and safety is classified under three main categories:

- Robot Reliability
- Robot Safety
- Miscellaneous

Under robot reliability category, publications are classified under three different areas of publications: methods, evaluation and models. Similarly under robot safety category, publications are classified four categories: safety general, safety standard, safety systems/methods, and human factors. The miscellaneous category contains all the publications. The following Table 1.2 presents the above three categories and their corresponding references.

Table 1.2 Classification of publications on robot reliability and safety

A. Robot Reliability

- Method: [8] [9] [10] [12] [14] [23] [24] [35] [39] [43] [44] [46] [62] [63] [69]
- Evaluation: [7] [29] [40]
- Model: [4] [13] [55] [68]

B. Robot Safety

- General [1] [16] [22] [27] [30] [47] [50] [53]
- Human factors: [19] [26] [28] [31] [34] [38] [41] [48] [49] [57] [58] [66]
- System: [36] [37] [54] [56] [60] [64]
- Method: [5] [6] [25] [32] [33] [45] [51] [52] [59] [65] [67] [70]

- Standard: [3] [21]

C. Miscellaneous [2] [11] [15] [17] [18] [20] [42] [61]

Publications falling under each of the Table 1.2 category are reviewed below.

1.2.2 Robot reliability

The published literatures on robot reliability classified under three categories are reviewed as the followed:

- **Robot method**

In 2000, Chen et al. [12] presented two disintegrating-coordinating strategies: objective coordinating method and mixed coordinating method as effective tools to overcome the obstacles of reliability optimizing distribution for a portable arc-welding robot system. In the same year Carreras and Walker proposed and discussed novel interval-based methods [9] via fault trees to obtain significantly improved robot reliability estimates [8].

In 2001, a total four publications appeared. Zhou et al. [69] described a multisensor data fusion method based on Bayesian theory to assess the robot reliability accurately. At the same year, Cheboxarov and Vasin [10] presented a new solution to increase the accuracy and reliability of the different robotic machining processes based on a force unloading of long flexible manipulator links. LaRussa and David [39] discussed that flexible robotic automation can provide the die casting industry with new levels of flexibility, reliability and accuracy. Also in the same year, Nakamura and Sakai [46] applied statistical methods, designing a reliability line model for robot mean time between failures (MTBF) and developing an estimation method for the mechanical parts' life, to improve robot reliability.

In 2002, Chen et al. [14] presented an effective two-level reliability distribution method, which can offer a reliability distribution result for portable robot systems. In the same year, Gu and Zhou [24] developed the method of group-control on the multiped robot, which provided more reliable control. Also in the same year Yamada and Takata [63] presented an effective method, for optimizing operation plans based on deterioration evaluation to improve the reliability of industrial robots.

In 2003, Jackson et al. [35] utilized a novel method, the embryonic array, which provided inherent fault-tolerance, to improve standard techniques.

In 2004, a total of four publications appeared. Xu and Deng [62] developed the reliability apportionment method for X-ray inspection real time imaging pipeline robot based on fuzzy synthesis. In the same year Morales, Chinellato et al. [43] proposed an active learning approach for assessing robot grasp reliability and Sanz and Andrew [44] presented a two-part practical method to predict grasp reliability for a multifinger robot hand by using visual features. Finally, Gao and Zhang [23] proposed a more practical reliability optimal allocation method of a control system, since the whole system reliability model included the factor of the fault diagnostic device.

- **Robot evaluation**

In 2000, Hou et al. [29] proposed a novel quantification evaluation for reliability distribution of industrial robot system. In the following year, Leuschen et al. [40] presented and developed a new approach, a logical extension of the underlying concepts of fuzzy sets and Markov models, to analyze fault tolerant designs. In 2003 Carlson and Murphy [3] found the reliability is low and presented the analysis about it.

- **Robot Model**

In 2000, Chen et al. [13] proposed a two-level reliability optimizing distribution and principle. In the same year, Baban et al. [4] presented a global and structural reliability analysis of industrial robots using a reliability model. Also in the same year Savsar [55] developed mathematical models to analyze the reliability of a flexible manufacturing cell (FMC). In 2003, Zhang and Lu [68] utilized Fuzzy fault tree analysis to meet the requirement of the demands of high reliability of cable painting robot.

1.2.3 Robot Safety

The published literature on robot safety classified under five categories is reviewed below.

- **General**

In 2000, Pace and Seward [50] presented the concerns and application of safety for a robotic excavator. In the following year, Hamada and Fujie [27] discussed some aspects of the application of robotics for social society. In 2002, Dhillon [16] discussed the importance of robot safety, the factors which influence robot safety and various methods for robot safety. In 2003, Neil [47] described the approach to prevent human worker from harm, by improving the standard and integration of vision systems and ancillary equipment into robotic workcells. In the same year Adamy and Bechtel [1] presented specific hazards, possible safety solutions and requirements for the applications for different types of mobile robots. In 2004 [30] Hu described the development process of robot safety engineering and the methods to improve robot safety including design, manufacture, installation, use and maintenance. In the same year Fryman [22] discussed the importance and the reasons for a culture of safety. In 2005, Piggin [53] discussed the safety technology, standards, interaction between them, and industrial robotic safety functionality.

- **Human factors**

In 2000, Ikeda et al. [31] presented a safety control mechanism by integrating a monitoring system and a safe actuator to allow human worker and robot co-operate safely. In the same year Lim and Tanie [41] presented a new type of passively movable human-friendly robot to avoid harm to human in human-robot collisions and Zelinsky et al. [66] presented a vision-based interface and a control scheme to guarantee human safety. In 2001, O'Laughlin [49] presented a new safeguarding device, the safety light curtain, and explained their working principles. In the same year, Ebert and Henrich [19] proposed a system equipped with cameras and applied a look-up-table-based fusion algorithm to back-project distance images to manipulator's configuration space. In 2002, Nokata et al. [48] presented a safety-optimizing method of human-care robot design and control by utilizing danger evaluation approach to increase robot safety. Also in 2002 Sunita and Mishra [58] discussed medical robot safety and proposed safety specific guidelines, safety driven design and safety reinforcement strategy. In 2003, a total of four publications appeared. Guiochet et al. [26] applied Unified Modeling Language (UML) to analyze human factors for safety of a medical robot for tele-echography. Heinzmann and Zelinsky [28] discussed two important issues, nature interaction interfaces and safety guarantees for users, and presented a control scheme of robot manipulators and developed an interactive pick-and-place system by integrating a visual control interface and the safety-controlled robot. Ikuta et al. [34] presented the first general method of evaluating safety for human-care robots that can be applied to evaluate the contribution of each safety strategy to the over-all safety performance. Kulic et al. proposed [38] an approach, minimizing a danger criterion including two formulations during the planning stage, to improve the safety of human-robot interaction. In 2005, Stopp et al. [57] presented a new safety concept, the current state of the DaimlerChrysler manufacturing assistant, for robot assistants in manufacturing.

- **System**

In 2000, Vanderperre [60] presented a twin robot safety device system and applied the Sokhotski-Plemelj formulae to solve the long-run availability. Also in 2000, Karlsson et al. [36] proposed a dynamic safety system based on sensor fusion and the system that can automatically adjust to changes in the guarded area according to the sensor information to cut down the risk of incidents. In 2001, Siemiatkowska and Kosinski [56] proposed a new vision system to help robot to classify the dangerous degree and reacted to prevent incidents. In 2002, Yan et al. [64] discussed and analyzed variable structure control technology of safety system dynamics for a wall-climbing robot. In 2003, Kosinski [37] presented an intelligent neural system which can analyze images from the camera and react by situation in a real time. In 2004, Roderick et al. [54] presented an autonomous software-based hazard control system for a dexterous space robot examined with fault trees and developed the sensitivity analysis.

- **Method**

In 2000, Yang et al. [65] applied quality function deployment (QFD) to evaluate robot safety, and discussed merits and defects. In the same year, Banno et al. [5] proposed a new type of safety area sensor with distance-voltage transfer method, which is cheaper, instead of the laser distance meter. Also in 2000, Musliner et al. [45] applied CIRCA (cooperative intelligent real-time control architecture)'s automatically-generated control plans to guarantee safety and avoided mission-critical failures. In 2001, Zurada et al. [70] proposed a new method, which consisted of an integrated sensing architecture and a new detection and decision logic for robot system safety. In the same year, Guiochet [25] presented a new deductive method, including the combination of safety analysis and the development of process and computer control systems analysis. Also in 2001, Ikuta et al. [32] presented a new type of 3-D robot simulator for danger evaluation and in Reference [33] proposed a general safety evaluation method, which can be used to assess the contribution of human-care robot control. In 2002, Piggin [51] [62] discussed the SafetyBUS, a can-based technology and implement at BMW. Tunstel [59] described rover safety and health issues and proposed rover safety module for safe off-road robot mobility. In 2003, Zhang and Xi [67] presented a new type of path planning method for a mobile robot in dynamic uncertain environment. In 2005, Bredereke and Lankenau [6] presented a new classification of mode confusions by cause, which was helpful to reduce the confusion problems.

- **Standard**

In 2000, Fryman [21] presented a new robot standard, ANSI/RIA R15.06-1999, which includes many new features in the design and installation of industrial robots. In 2001, Anderson [3] introduced 1999 version necessitates design changes to robot safety and the reason need to be considered.

1.2.4 Miscellaneous

In 2000, Dhillon and Aleem [15] presented the results of a survey of robot reliability and safety in Canada In 2001, Vanderperre [61] applied a stochastic process with time dependent probability measures to solve the point availability of a robot safety system attended by two repairmen, one of them in charge of the safety unit and another taking care of robots. In the same year, Chen et al. [11] described the development level and the development obstacles of the reliability/safety/maintainability (RSM) of space robots and presented a basic measure for it. Also in the same year, Ferreira [20] presented the model of a man/manipulator cooperation scheme through a master-slave teleoperated system to increase safety and reliability. In 2002, Dhillon et al. [17] presented a review of robot systems reliability and safety from different three aspects for the period 1973-2001: robot reliability, robot safety and miscellaneous. In the same year, Adamyan and He [2] developed methods for combining Petri net with fault free analysis to determine system failure rates. In 2004 Dhillon and Li [18] presented time dependent and steady state availability analysis of a maintainable robot-safety system with common-cause failures. Also in 2004, Long et al. [42] analyzed the suspension system of the CMS-3 type electromagnetic suspension sample train and presented the viewpoints and methods to improve reliability and safety.

1.3 Thesis Objectives

Three objectives of thesis are as follows:

- To study standby robot-safety systems with perfect switch and imperfect switch.
- To develop mathematical models for standby robot-systems, obtain the general expressions for state probability, availability, reliability and mean time to failure, and show plots for special cases.
- Tailored the above analysis to predict the reliability of standby robot-safety system and system maintenance in real life.

1.4 Thesis Structure

This study is composed of eight Chapters:

Chapter 1 presents a brief introduction to robot reliability and safety and a literature review.

Chapter 2 presents stochastic analysis of a system containing one robot and (n-1) standby safety units with a perfect switch.

Chapter 3 presents stochastic analysis of a system containing one robot and (n-1) standby safety units with a imperfect switch.

Chapter 4 presents stochastic analysis of a system containing (n-1) standby robots and one safety unit with a perfect switch.

Chapter 5 presents stochastic analysis of a system containing (n-1) standby robots and one safety unit with an imperfect switch.

Chapter 6 presents stochastic analysis of a system containing n parallel robots and (m-1) standby safety units and a perfect switch.

Chapter 7 presents stochastic analysis of a system containing two parallel robots and one
standby safety unit with an imperfect switch.

Chapter 8 presents conclusions and recommendations for future study.

In Chapter 2-7, the failure rates and repair rates of robot, safety unit and switch are assumed constant and the failed system repair times are assumed arbitrarily distributed. The Markov method and the supplementary variable method are utilized to perform mathematical analysis for the six robot-safety systems. The Markov method is used to perform mathematical analysis for systems with constant failure and repair rate. Similarly, the supplementary method is employed to handle the systems with constant failure rate and non-constant system repair rate. General expressions for state probabilities, system availability, reliability and mean time to failure are obtained for special cases.

Stochastic Analysis of a System containing One Robot and (N-1) Standby Safety Units with a Perfect Switch

2.1 Introduction:

In this chapter, a robot safety system is presented, consisted of three parts: one robot, (n-1) standby safety units, and a switch in mechanism that cannot fail. More specifically, the robot-safety system is composed of one robot, n identical safety units and a switch to replace a failed safety unit.

At time t =0, robot, one safety unit and the switch to replace a failed safety unit start operating and n-1 safety units are on standby. The overall robot-safety system can fail the following three ways:

- The robot-safety system fails with an incident.
- The robot-safety system fails safely.
- The robot-safety system fails due to the malfunction of the robot.

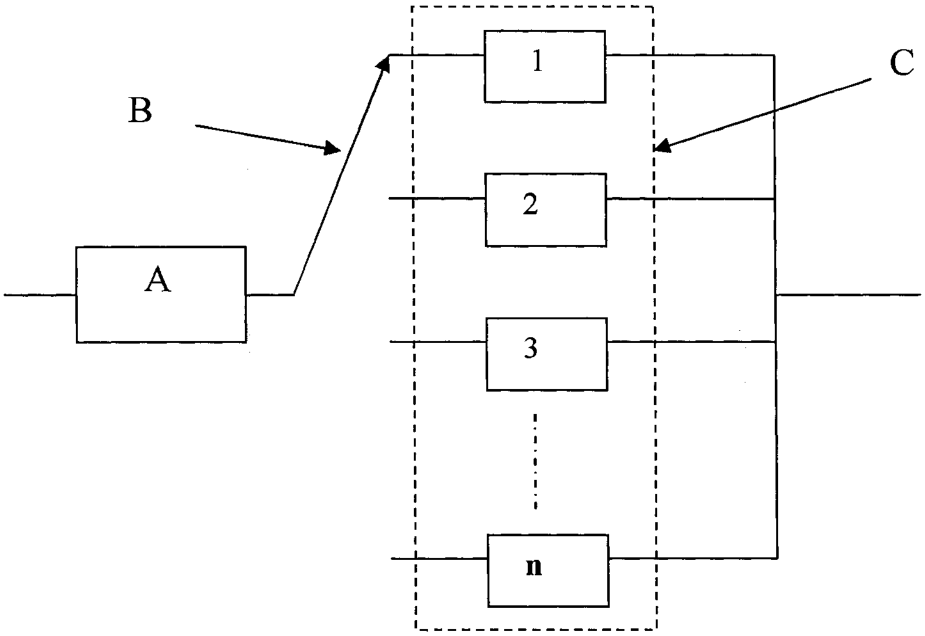
The partly or fully failed robot-safety system is repaired.

The following assumptions are associated with this model:

- The robot-safety system is composed of one robot, n identical safety units (only one operates and the rest remain on standby) and a switch.
- Robot, switch and one safety unit start operating simultaneously.
- The perfect switch means it never fail or the failure rate is too small to be considered compared with other parts such as robot and safety unit.

- The completely failed robot-safety system and its individually failed units (i.e. robot, and safety unit) can be repaired. Failure and repair rates of robot and safety units are constant.
- The failure robot-safety system repair rates can be constant or non-constant.
- All failures are statistically independent.
- A repaired safety unit, robot or the total robot-safety system is as good as new.

The block diagram of the robot system is shown in Figure2.1 and its corresponding state space diagram is presented in Figure2.2. The numerals and letter n in the boxes of Figure2.2 denote system state.



A : Robot

B : Switch for replacing a failed safety unit and it cannot fail.

C: n identical Safety Units (one operating and n-1 on standby)

Figure 2.1 The block diagram of the robot-safety system containing one robot and (n-1) standby safety units with a perfect switch

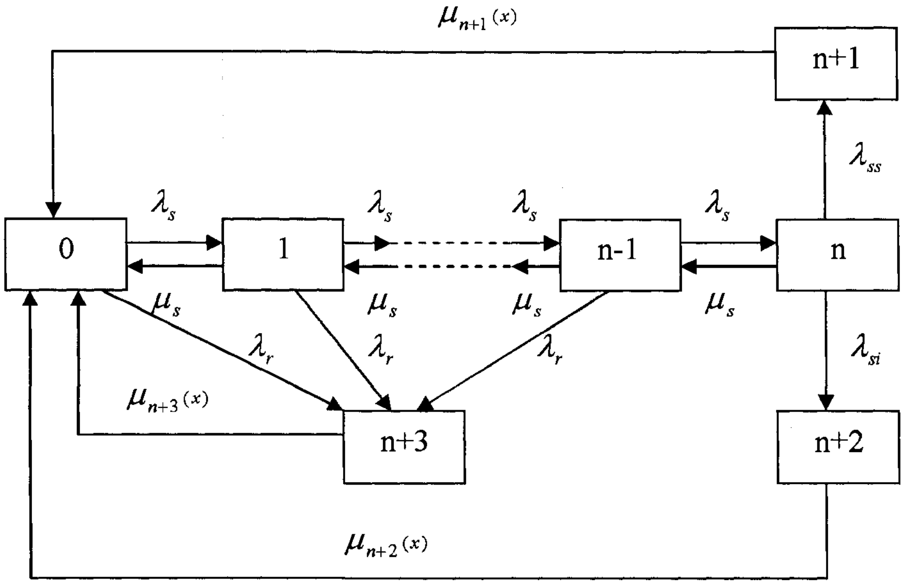


Figure 2.2 The state space diagram of the robot-safety system containing one robot and (n-1) standby safety units with a perfect switch

2.1.1 Notation:

The following symbols are associated with the model:

i i^{th} state of the robot-safety system:

for $i = 0$, means the robot, the switch and one safety unit are working normally;

for $i = 1$, means the robot, the switch, one safety unit are working normally and one safety unit has failed;

for $i = k$, means the robot, the switch, one safety unit are working normally and k safety units have failed; (i.e., k = 2,3……n-1);

for $i = n$, means the robot, the switch are working normally and all the safety units have failed.

j	j^{th} state of the robot–safety system: for j = n + 1, the robot-safety system failed safely; for j = n + 2, the robot-safety system failed with an incident; for j = n + 3, the robot-safety system failed due to the malfunction of the robot;
t	Time.
λ_s	Constant failure rate of the safety unit.
λ_r	Constant failure rate of the robot.
λ_{ss}	Constant failure rate of the robot-safety system failing safely.
λ_{si}	Constant failure rate of the robot-safety system failing with an incident.
μ_s	Constant repair rate of the safety unit.
Δ**x**:	Finite repair time interval.
$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in State j and has an elapsed repair time of x; for j = n+1, n+2, n+3.
$P_j(x,t)\ \Delta$x	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x, x+Δx]; for j = n+1, n+2, n+3 .
pdf	Probability density function.
w_j(x)	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j = n+1, n+2, n+3.
P_j(t)	Probability that the robot-safety system is in state j at time t; for j = n+1, n+2, n+3.

$P_i(t)$	Probability that the robot safety system is in state i at time t; for $i = 0, 1, 2 \ldots n$.
P_i	Steady state probability that the robot safety system is in state i; for $i = 0, 1 \ldots n$.
P_j	Steady state probability that robot-safety system is in state j; for $j = n+1, n+2, n+3$.
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i; for $i = 0,1,2\ldots n$.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for $j = n+1, n+2, n+3$.
AVrs(s)	Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot.
AVr(s)	Laplace transform of the robot-safety system availability with or without a normally working safety unit.
AVrs (t)	Robot-safety system time dependent availability with one normally working safety unit, the switch and the robot.
AVr(t)	Robot-safety system time dependent availability with or without one normally working safety units.
SSAVrs	Robot-safety system steady state availability with one normally working safety unit.
SSAVr	Robot-safety system steady state availability with or without one normally working safety unit.

Rrs(s)	Laplace transform of the robot-safety system reliability with one normally working safety unit.
Rr(s)	Laplace transform of the robot-safety system reliability with or without one normally working safety unit.
MTTFrs	Robot-safety system mean time to failure with one normally working safety unit.
MTTFr	Robot-safety system mean time to failure with or without one normally working safety unit.

2.2 Generalized Robot-Safety System Analysis

By using the supplementary method [71, 72], we write the following equations for the Figure 2.2 diagram:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_s P_1(t) + \sum_{j=n+1}^{n+3} P_j(x,t)\mu_j(x)dx \tag{2.1}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \mu_s P_{i+1}(t) + \lambda_s P_{i-1}(t) \tag{2.2}$$

(for $i = 1,2,\ldots\ldots,n-1$)

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) \tag{2.3}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \tag{2.4}$$

(for $j = n+1, n+2, n+3$)

where

$$a_0 = \lambda_s + \lambda_r$$

$a_i = \lambda_s + \lambda_r + \mu_s$ (for i = 1,2,…….,n-1)

$a_n = \lambda_{ss} + \lambda_{si} + \mu_s$

The associated boundary conditions are as follows:

$$P_{n+1}(0,t) = \lambda_{ss} P_n(t) \tag{2.5}$$

$$P_{n+2}(0,t) = \lambda_{si} P_n(t) \tag{2.6}$$

$$P_{n+3}(0,t) = \lambda_r \sum_{i=0}^{n-1} P_i(t) \tag{2.7}$$

At time t = 0, $P_0(0) = 1$, and all other state probabilities are equal to zero.

2.3 Generalized Robot-Safety System Time Dependent Analysis

By solving Equations (2.1) – (2.7) with the Laplace transform method *, we get the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1+\sum_{i=1}^{n} Y_i(s)) + \sum_{j=n+1}^{n+3} a_j(s)\frac{1-W_j(s)}{s}]^{-1} = \frac{1}{G(s)} \tag{2.8}$$

$$P_i(s) = Y_i(s)P_0(s) \tag{2.9}$$

(for i = 1,2,………n)

$$P_j(s) = a_j(s)\frac{1-W_j(s)}{s} P_0(s) \quad (\text{for } j = n+1, n+2, n+3) \tag{2.10}$$

where

$$Y_i(s) = \prod_{k=1}^{i} L_k(s) \qquad (\text{for } i = 1,2,\ldots\ldots\ldots n)$$

---

* : Please see Appendix A1.

$$a_{n+1}(s) = \lambda_{ss} Y_n(s)$$

$$a_{n+2}(s) = \lambda_{si} Y_n(s)$$

$$a_{n+3}(s) = \lambda_r [1+\sum_{i=1}^{n-1} Y_i(s)]$$

$$L_n(s) = \frac{\lambda_s}{s+a_n}$$

$$L_i(s) = \frac{\lambda_s}{(s+a_i)-\mu_s L_{i+1}(s)} \qquad \text{(for } i = 1, 2.........\text{n-1)}$$

$$G(s) = s[1+\sum_{i=1}^{n} Y_i(s) + \sum_{j=n+1}^{n+3} a_j(s)\frac{1-W_j(s)}{s}]^{-1} \tag{2.11}$$

$$W_j(s) = \int_0^{\infty} e^{-sx} w_j(x)dx \qquad \text{(for } j = \text{ n+1,n+2,n+3)} \tag{2.12}$$

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta\,]\ \mu_j(x)$$

The Laplace transform of the robot-safety system availability with one normally working safety unit, the switch, and the robot is given by:

$$AV_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{1+\sum_{i=1}^{n-1} Y_i(s)}{G(s)} \tag{2.13}$$

The Laplace transform of the robot-safety system availability with or without one normally working safety unit is given by:

$$AV_r(s) = \sum_{i=0}^{n} P_i(s) = \frac{1+\sum_{i=1}^{n} Y_i(s)}{G(s)} \tag{2.14}$$

Taking the inverse Laplace transforms of the above equations, we can get the time dependent state probabilities, $P_i(t)$and $P_j(t)$, and robot-safety system availabilities, AVrs(t) and AVr(t).

2.3.1 Robot-Safety System Time Dependent Analysis for a Special Case

For three safety units (i.e. one working, others on standby), by substituting n=3 into Equations (2.8)-(2.14), we obtain:

$$P_0(s) = \frac{1}{s[1+\sum_{i=1}^{2} Y_i(s) + \sum_{j=3}^{5} a_j(s)\frac{1-W_j(s)}{s}]} \qquad (2.15)$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad \text{(for } i = 1, 2,3) \qquad (2.16)$$

$$P_j(s) = a_j(s)\frac{1-W_j(s)}{s}\, P_0(s) \qquad \text{(for } j = 4, 5, 6) \qquad (2.17)$$

where

$$Y_i(s) = \prod_{k=1}^{i} L_k(s) \qquad \text{(for } i = 1, 2, 3)$$

$$a_4(s) = \lambda_{ss}\, Y_n(s)$$

$$a_5(s) = \lambda_{si}\, Y_n(s)$$

$$a_6(s) = \lambda_r[1+\sum_{i=1}^{2} Y_i(s)]$$

$$L_3(s) = \frac{\lambda_s}{s+a_3}$$

$$L_i(s) = \frac{\lambda_s}{(s+a_i)-\mu_s L_{i+1}(s)} \qquad \text{(for } i = 1, 2)$$

$$G(s) = s[1+\sum_{i=1}^{3} Y_i(s) + \sum_{j=4}^{6} a_j(s)\frac{1-W_j(s)}{s}] \qquad (2.18)$$

The Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot is given by:

$$\mathrm{AV}_{rs}(\mathrm{s}) = \sum_{i=0}^{2} P_i(s) = \frac{1+Y_1(s)}{G(s)} \tag{2.19}$$

The Laplace transform of the robot-safety system availability with or without a normally working safety unit is given by:

$$\mathrm{AV}_{r}(\mathrm{s}) = \sum_{i=0}^{3} P_i(s) = \frac{1+\sum_{i=1}^{3} Y_i(s)}{G(s)} \tag{2.20}$$

Taking the inverse Laplace transforms of the above equations, we can obtain the time dependent state probabilities, P_i(t)and P_j(t), and robot-safety system availabilities, AVrs(t) and AVr(t) respectively.

Thus, for the failed robot-safety system exponentially distributed repair times, the probability function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \mu_j \, e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 4,5,6) \tag{2.21}$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Substituting Equation (2.21) into Equation (2.12), we yield

$$\mathrm{W}_j(\mathrm{s}) = \frac{\mu_j}{s+\mu_j} \qquad (\mu_j > 0, j = 4,\ 5,\ 6) \tag{2.22}$$

By inserting Equation (2.22) into Equations (2.15) – (2.20), setting

λ_s= 0.001, λ_r= 0.0009, λ_{ss}=0.0035, λ_{si}= 0.0015, μ_s= 0.0002, μ_4= 0.0003, μ_5= 0.0001, μ_6= 0.00035; and using Matlab computer program [74], the Figure 2.3 and Figure2.4

were obtained. These plots show that state probabilities and system availabilities decrease and increase with varying time t.

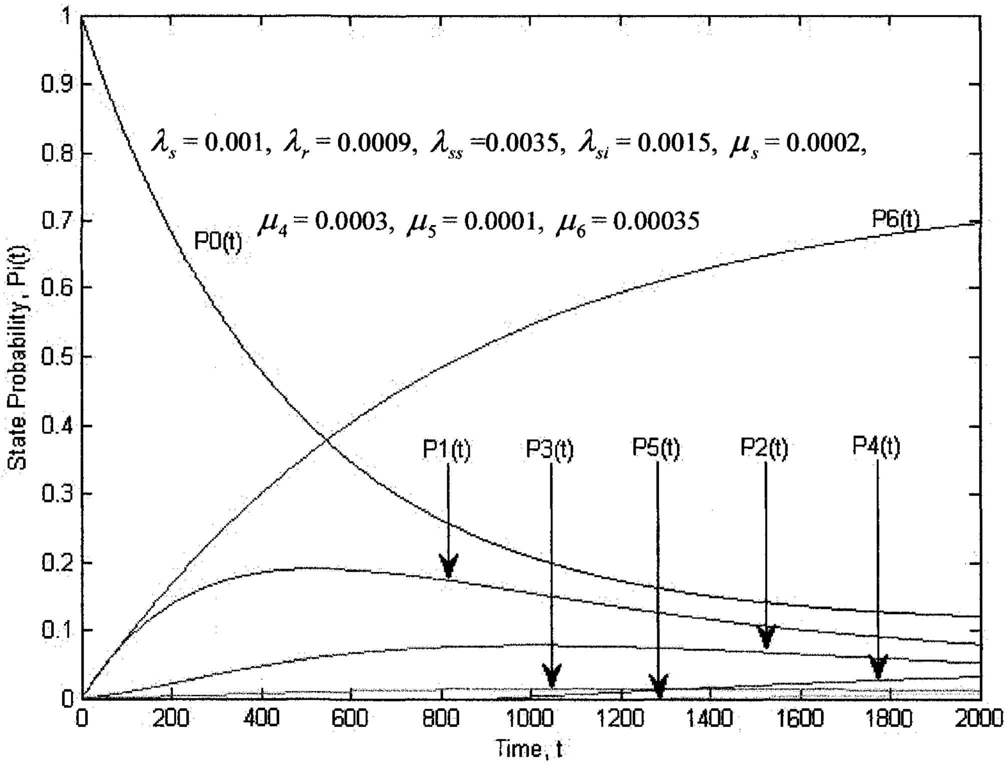


Figure 2.3 Time-dependent state probability plots for a robot-safety system with exponentially distributed failed system repair times.

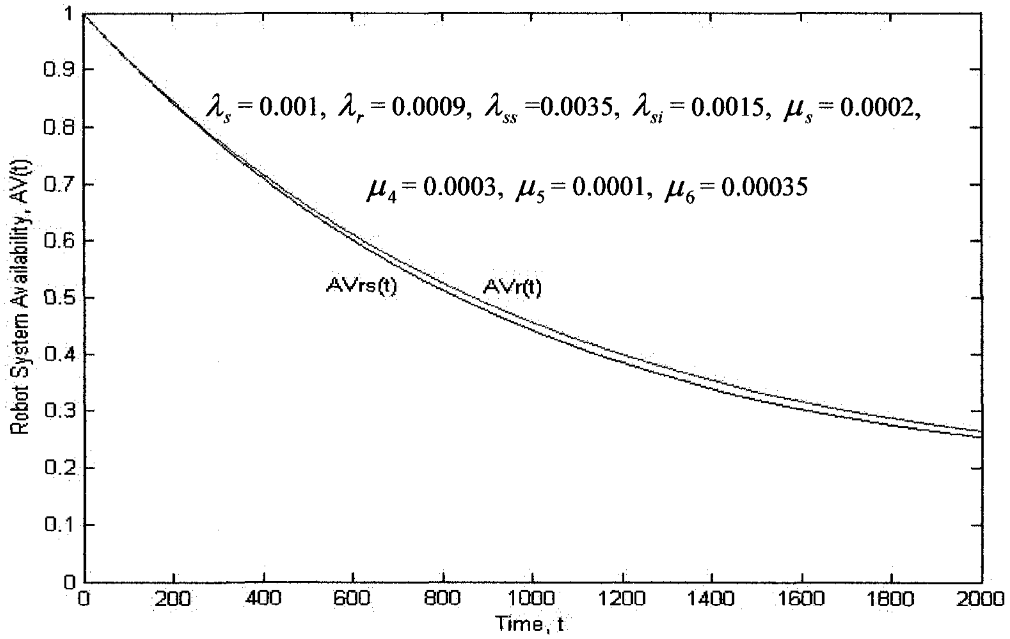


Figure 2.4 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair times.

2.4 Generalized Robot-Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (2.1)-(2.7) we get:

$$a_0 P_0 = \mu_s P_1 + \sum_{j=n+1}^{n+3} \int_0^\infty \mu_j(x) P_j(x)dx \qquad (2.23)$$

$$a_i P_i = \mu_s P_{i+1} + \lambda_s P_{i-1} \qquad (\text{for } i = 1,2,3\ldots\ldots,\text{n-1}) \qquad (2.24)$$

$$a_n P_n = \lambda_s P_{n-1} \qquad (2.25)$$

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \qquad (\text{ for } j = \text{n+1,n+2,n+3}) \qquad (2.26)$$

The associated boundary conditions are as follows:

$$P_{n+1}(0) = \lambda_{ss} P_n \qquad (2.27)$$

$$P_{n+2}\ (0) = \lambda_{si}\ P_n \tag{2.28}$$

$$P_{n+3}\ (0) = \lambda_r \sum_{i=0}^{n-1} P_i \tag{2.29}$$

Solving Equations (2.23) – (2.26) *, together with

$$\sum_{i=0}^{n} P_i + \sum_{j=n+1}^{n+3} P_j = 1 \tag{2.30}$$

We obtain:

$$P_0 = [1+\sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j E_j[x]\,]^{-1} = \frac{1}{G} \tag{2.31}$$

$$P_i = Y_i P_0 \tag{2.32}$$

(for $i = 1, 2 \ldots\ldots\ldots$ n)

$$P_j(s) = a_j E_j[x]\ P_0(s) \quad (\text{for } j = n+1,\ n+2,\ n+3) \tag{2.33}$$

where

$$Y_i = \lim_{s\to 0} Y_i(s) = \prod_{k=1}^{i} L_k \qquad (\text{for } i = 1, 2 \ldots\ldots\ldots n)$$

$$a_{n+1} = \lambda_{ss} Y_n$$

$$a_{n+2} = \lambda_{si} Y_n$$

$$a_{n+3} = \lambda_r [1+\sum_{i=0}^{n-1} Y_i\ ]$$

$$L_n = \frac{\lambda_s}{a_n}$$

$$L_i = \frac{\lambda_s}{a_i - \mu_s L_{i+1}}$$

* : Please see Appendix A.2.

$$G = 1+\sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j E_j[x] \tag{2.34}$$

$$E_j[x] = \int_0^{\infty} \exp[-\int_0^{x} \mu_j(\delta)d\delta]dx \tag{2.35}$$

$$= \int_0^{\infty} x w_j(x)dx \qquad (\text{ for } j = \text{n+1, n+2, n+3})$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function.

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

The generalized steady state availability of the robot safety system with one normally working normally safety unit, the switch and the robot is given by:

$$SSAVrs = \sum_{i=0}^{n-1} P_i = \frac{1+\sum_{i=1}^{n-1} Y_i}{G} \tag{2.36}$$

Similarly, the generalized steady state availability of the robot safety system with or without a working safety unit is:

$$SSAVr = \sum_{i=0}^{n} P_i = \frac{1+\sum_{i=1}^{n} Y_i}{G} \tag{2.37}$$

For different failed robot-safety system repair time distributions, we get different expressions for G as follows:

i) For the failed robot-safety system gamma distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{\mu_j^{\beta} x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \qquad (\beta > 0,\ j = n+1,\ n+2,\ n+3) \qquad (2.38)$$

where

x is the repair time variable, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^{\infty} x w_j(x) dx = \frac{\beta}{\mu_j} \qquad (\beta > 0,\ j = n+1,\ n+2,\ n+3) \qquad (2.39)$$

Substituting Equation (2.39) into Equation (2.34), we get

$$G = 1 + \sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j \frac{\beta}{\mu_j} \qquad (2.35)$$

ii) For the failed robot-safety system Weibull distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j (x)^{\beta}} \qquad (\beta > 0,\ j = n+1,\ n+2,\ n+3) \qquad (2.36)$$

where

x is the repair time variable, μ_j is the scale parameter, and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^{\infty} x W_j(x) dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \qquad (\beta > 0,\ j = n+1,\ n+2,\ n+3) \qquad (2.37)$$

Substituting (2.37) into Equation (2.34), we obtain

$$G = 1 + \sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \qquad (2.38)$$

iii) For the failed robot-safety system Rayleigh distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j x e^{-\mu_j x^2/2} \qquad (\mu_j > 0,\ j = n+1, n+2, n+3) \tag{2.39}$$

where

x is the repair time variable, μ_j is the scale parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty x W_j(x)dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (\mu_j > 0,\ j = n+1,\ n+2,\ n+3) \tag{2.40}$$

Substituting Equation(2.40) into Equation (2.34), we get

$$G = 1 + \sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j \sqrt{\frac{\pi}{2\mu_j}} \tag{2.41}$$

iv) For the failed robot system Lognormal distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{1}{\sqrt{2\pi x \sigma_{y_j}}} \; e^{[\frac{-(Inx-\mu_{y_j})^2}{2\sigma_{y_j}^2}} \qquad (\text{for } j = n+1,\ n+2,\ n+3) \tag{2.42}$$

where

x is the repair time variable, Inx is the natural logarithm of x with a mean μ and variance σ^2. The conditions on parameters are:

$$\sigma_{y_j} = \mathrm{In}\sqrt{1+(\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \tag{2.43}$$

and

$$\mu_{y_j} = \ln \sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \qquad (2.44)$$

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (\text{for } j = n+1,\ n+2,\ n+3) \qquad (2.45)$$

Substituting Equation(2.45) into Equation (2.34), we get

$$G = 1 + \sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (2.46)$$

v) For the failed robot system exponentially distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = n+1,\ n+2,\ n+3) \qquad (2.47)$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^{\infty} x w_j(x) dx = \frac{1}{\mu_j} \qquad (\beta > 0,\ j = n+1,\ n+2,\ n+3) \qquad (2.48)$$

Substituting Equation (2.48) into Equation (2.34), we get

$$G = 1 + \sum_{i=1}^{n} Y_i + \sum_{j=n+1}^{n+3} a_j \frac{1}{\mu_j} \qquad (2.49)$$

2.4.1 Robot-Safety System Steady State Analysis for a Special Case

For three safety units (i.e. one working, others on standby) by substituting n=3 into Equations (2.31)-(2.37), we obtain:

$$P_0 = \frac{1}{1+\sum_{i=1}^{3} Y_i(s) + \sum_{j=4}^{6} a_j E_j[x]} \qquad (2.50)$$

$$P_i = Y_i P_0 \qquad (\text{for } i = 1, 2, 3) \qquad (2.51)$$

$$P_j(s) = a_j E_j[x] P_0 \qquad (\text{for } j = 4, 5, 6) \qquad (2.52)$$

where

$$Y_i = \prod_{k=1}^{i} L_k \qquad (\text{for } i = 1, 2, 3)$$

$$a_4 = \lambda_{ss} Y_3$$

$$a_5 = \lambda_{si} Y_3$$

$$a_6 = \lambda_r [1+\sum_{i=1}^{2} Y_i]$$

$$L_3 = \frac{\lambda_s}{a_3}$$

$$L_i = \frac{\lambda_s}{a_i - \mu_s L_{i+1}} \qquad (\text{for } i = 1, 2)$$

$$G = 1+\sum_{i=1}^{3} Y_i + \sum_{j=4}^{6} a_j E_j[x] \qquad (2.53)$$

The steady state availability of the robot-safety system with one normally working safety unit, the switch and the robot is given by:

$$SSAV_{rs} = \sum_{i=0}^{2} P_i = \frac{1+\sum_{i=1}^{2} Y_i}{G} \qquad (2.54)$$

The steady state availability of the robot-safety system with or without a normally working safety unit is given by:

$$\mathrm{SSAV}_r = \sum_{i=0}^{3} P_i = \frac{1+\sum_{i=1}^{3} Y_i}{G} \tag{2.55}$$

1) For exponentially distributed failed robot-safety system repair time, substituting Equation (2.48) into Equations (2.50)-(2.53), setting:

$\lambda_s = 0.001$, $\lambda_r = 0.0009$, $\lambda_{ss} = 0.0035$, $\lambda_{si} = 0.0015$, $\mu_4 = 0.0003$, $\mu_5 = 0.0001$, $\mu_6 = 0.00035$ and using Matlab computer program[74], the Figures 2.5 and 2.6 plots were obtained.

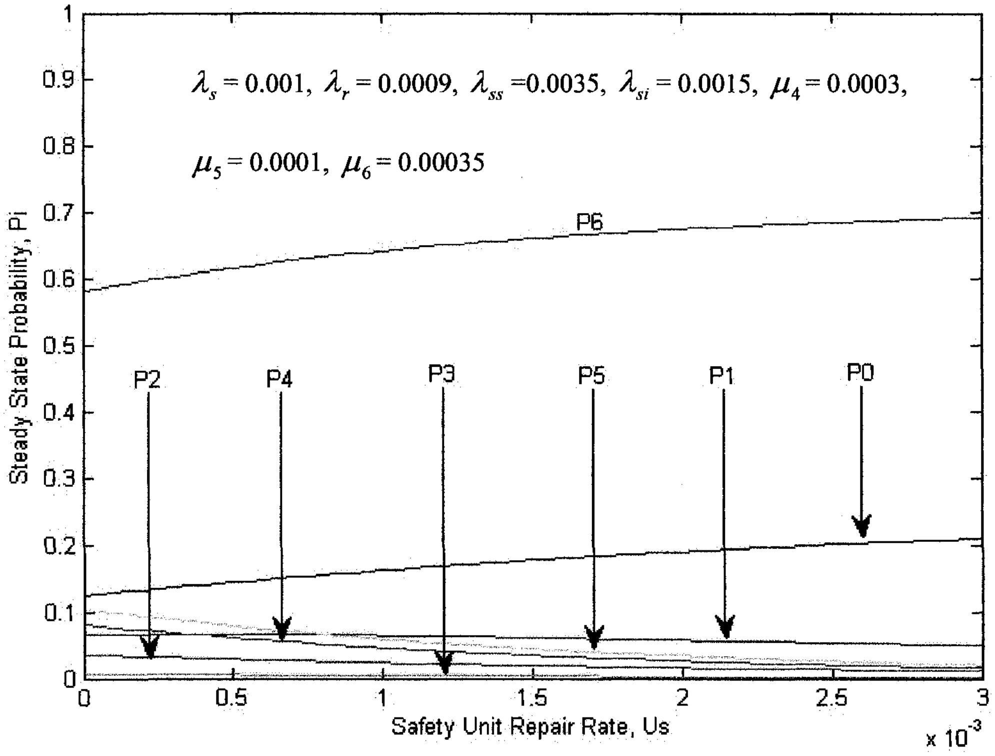


Figure 2.5 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots for Exponentially distributed failed system repair times.

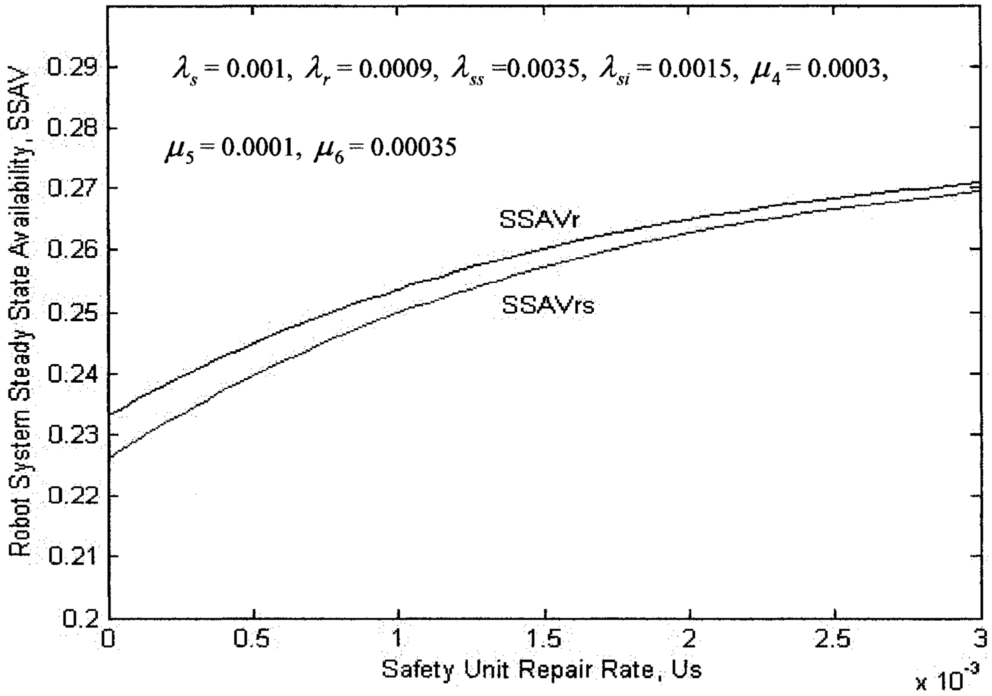


Figure 2.6 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots for exponentially distributed failed system repair times.

2) For Rayleigh distributed failed robot-safety system repair time, substituting Equation (2.40) into Equations (2.50)-(2.53), setting:

$\lambda_s = 0.001$, $\lambda_r = 0.0009$, $\lambda_{ss} = 0.0035$, $\lambda_{si} = 0.0015$, $\mu_4 = 0.0000003$, $\mu_5 = 0.0000001$, $\mu_6 = 0.00000035$ and using Matlab computer program[74], the Figure 2.7 and 2.8 were obtained.

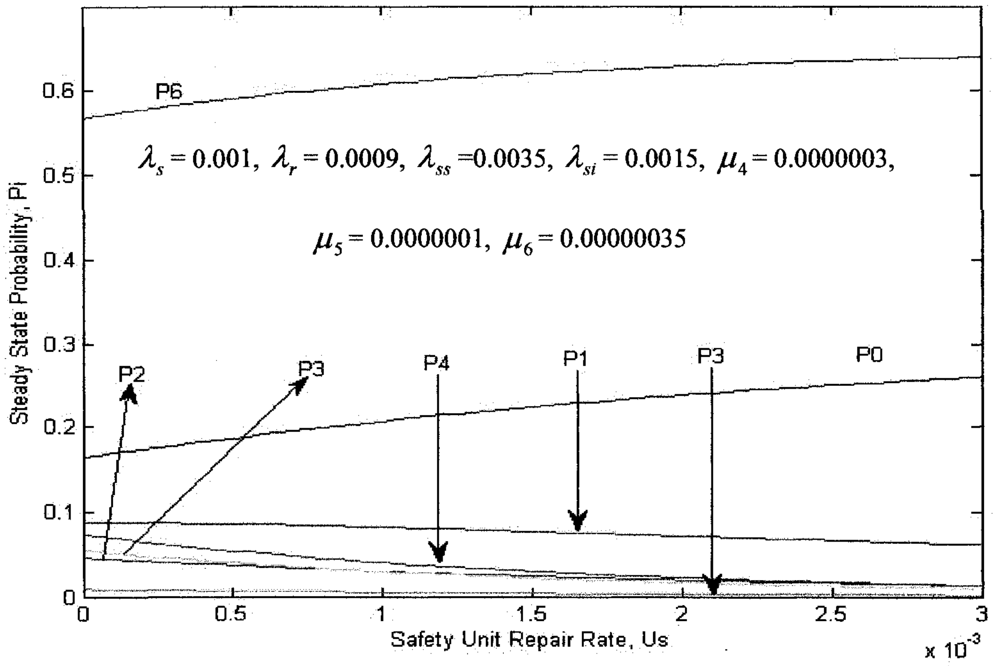


Figure 2.7 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots for Rayleigh distributed failed system repair times.

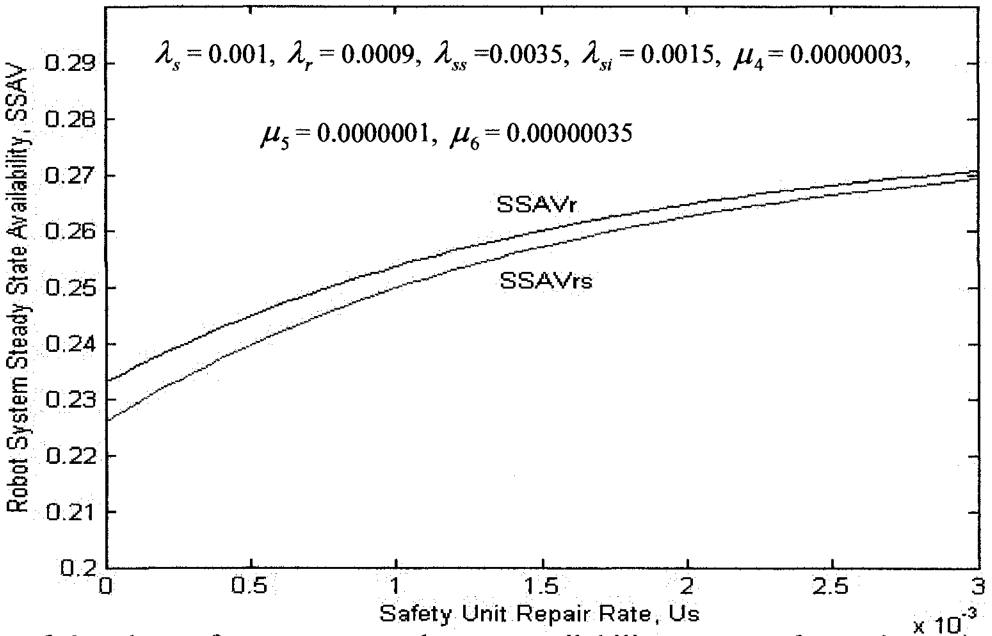


Figure 2.8 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots for Rayleigh distributed failed system repair times.

3) For lognormally distributed failed robot-safety system repair time, substituting Equation (2.45) into Equations (2.50)-(2.53), setting:

λ_s = 0.001, λ_r = 0.0009, λ_{ss} =0.0035, λ_{si} = 0.0015, μ_4 = 0.0009, μ_5 = 0.0006, μ_6 = 0.0007, σ = 0.5 and using Matlab computer program [74], the Figure 2.9 and Figure2.10 plots were obtained.

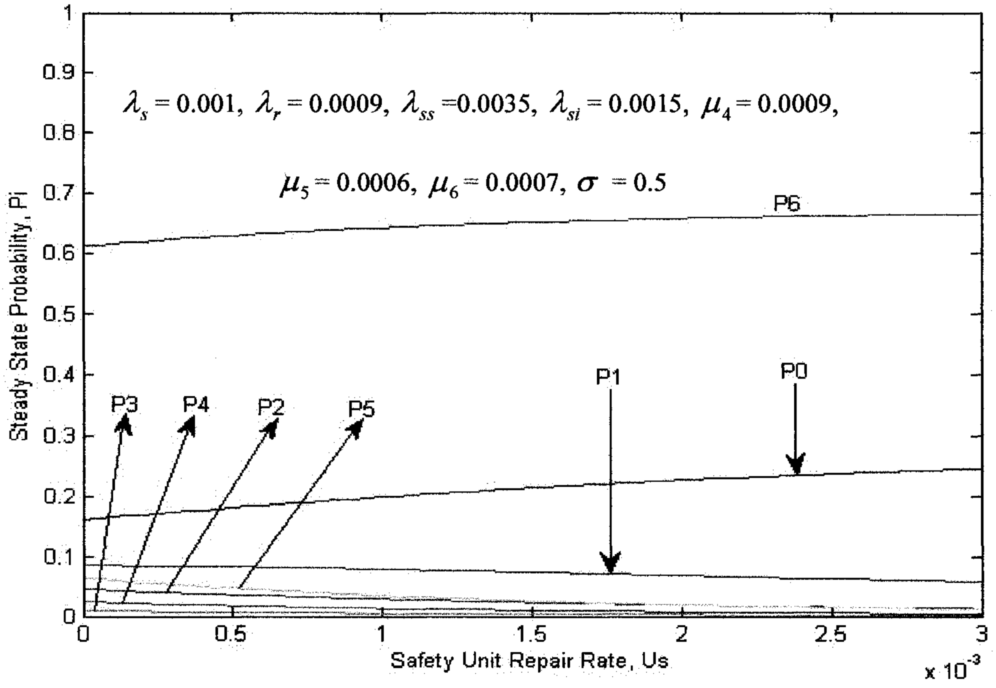


Figure 2.9 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots for lognormally distributed failed system repair times.

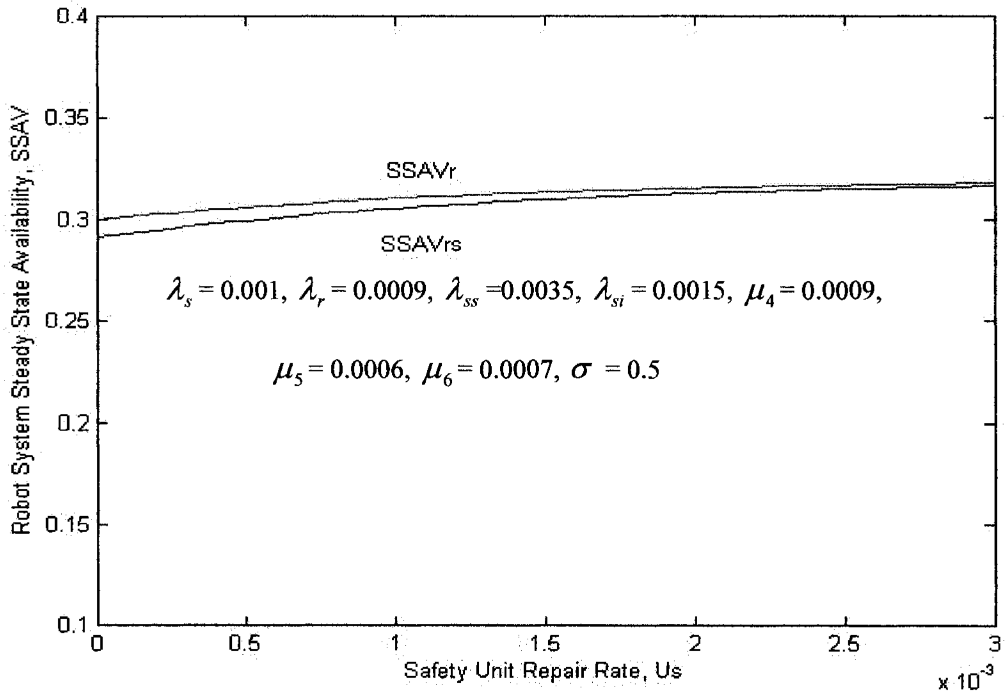


Figure 2.10 Robot- safety system steady state availability versus safety unit repair rate Us (means μ_s) plots for lognormally distributed failed system repair times.

2.5 Robot Safety System Reliability and Mean Time to Failure Analysis

Setting $\mu_j = 0$, (for j = n+1, n+2, n+3), in Figure 2.2 and using the Markov method [73], we obtain the following set of differential equations:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_s P_1(t) \tag{2.56}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \mu_s P_{i+1}(t) + \lambda_s P_{i-1}(t) \tag{2.57}$$

(for i = 1,2,.......,n-1)

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) \tag{2.58}$$

where

$$a_0 = \lambda_s + \lambda_r$$

$$a_i = \lambda_s + \lambda_r + \mu_s \qquad (\text{ for } i = 1,2,\ldots\ldots.,\text{n-1})$$

$$a_n = \lambda_{ss} + \lambda_{si} + \mu_s$$

$$\frac{dP_{n+1}(t)}{dt} = \lambda_{ss} P_n(t) \tag{2.59}$$

$$\frac{dP_{n+2}(t)}{dt} = \lambda_{si} P_n(t) \tag{2.60}$$

$$\frac{dP_{n+3}(t)}{dt} = \lambda_r \sum_{i=0}^{n-1} P_i(t) \tag{2.61}$$

At time t = 0, $P_0(0) = 1$, and other initial condition probabilities are equal to zero.

By solving Equations (2.56)-(2.61) with the Laplace transform method *, together with

$$\sum_{i=0}^{n} P_i(s) + \sum_{j=n+1}^{n+3} P_j(s) = \frac{1}{s}, \tag{2.62}$$

we get the following general Laplace transform Equations of state probabilities:

$$P_0(s) = [s(1+\sum_{i=1}^{n} Y_i(s) + \sum_{j=n+1}^{n+3} \frac{a_j(s)}{s}]^{-1} = \frac{1}{G(s)} \tag{2.63}$$

$$P_i(s) = Y_i(s)P_0(s) \qquad (\text{for } i = 1,2,\ldots\ldots\ldots\text{n}) \tag{2.64}$$

$$P_j(s) = \frac{a_j(s)}{s} P_0(s) \qquad (\text{for } j = \text{n+1,n+2,n+3}) \tag{2.65}$$

where

$$G(s) = s[1+\sum_{i=1}^{n} Y_i(s) + \sum_{j=n+1}^{n+3} \frac{a_j(s)}{s}] \tag{2.66}$$

* : Please see Appendix A.3.

The Laplace transform of the robot-safety system reliability with one normally working safety unit, the switch and the robot is given by:

$$\mathrm{R}_{rs}(\mathrm{s}) = \sum_{i=0}^{n-1} P_i(s) = \frac{1+\sum_{i=1}^{n-1} Y_i(s)}{G(s)} \tag{2.67}$$

Similarly, the Laplace transform of the robot safety system reliability with or without a working safety unit is given by:

$$\mathrm{R}(\mathrm{s}) = \sum_{i=0}^{n} P_i(s) = \frac{1+\sum_{i=1}^{n} Y_i(s)}{G(s)} \tag{2.68}$$

Using Equation (2.67) and Reference [73], the robot-safety system mean time to failure with one normally working safety unit, the switch and the robot is given by:

$$\mathrm{MTTF}_{rs} = \lim_{s\to 0} \mathrm{R}_{rs}(\mathrm{s}) = \frac{1+\sum_{i=1}^{n-1} Y_i}{\sum_{j=n+1}^{n+3} a_j} \tag{2.69}$$

Similarly, using Equation (2.68) and Reference [73], the robot safety system mean time to failure with or without a working safety unit is given by:

$$\mathrm{MTTF}_{r} = \lim_{s\to 0} \mathrm{R}_{r}(\mathrm{s}) = \frac{1+\sum_{i=1}^{n} Y_i}{\sum_{j=n+1}^{n+3} a_j} \tag{2.70}$$

2.5.1 Robot-Safety System Mean Time to Failure Analysis for a Special Case

Substituting n = 3 into Equations (2.69) and (2.70), we get:

The robot-safety system mean time to failure with one normally working safety unit, the switch and the robot is given by

$$\mathrm{MTTF}_{rs} = \lim_{s\to 0} \mathrm{R}_{rs}(\mathrm{s}) = \frac{1+\sum_{i=1}^{2} Y_i}{\sum_{j=4}^{6} a_j} \tag{2.71}$$

The robot safety system mean time to failure with or without a working safety unit is

$$\mathrm{MTTF}_{r} = \lim_{s\to 0} \mathrm{R}_{r}(\mathrm{s}) = \frac{1+\sum_{i=1}^{3} Y_i}{\sum_{j=4}^{6} a_j} \tag{2.72}$$

For $\lambda_r = 0.0009$, $\lambda_{ss} = 0.0035$, $\lambda_{si} = 0.0015$, $\lambda_s = 0.001$; and using Equations (2.71) and (2.72), the Figure 2.11 plots were obtained.

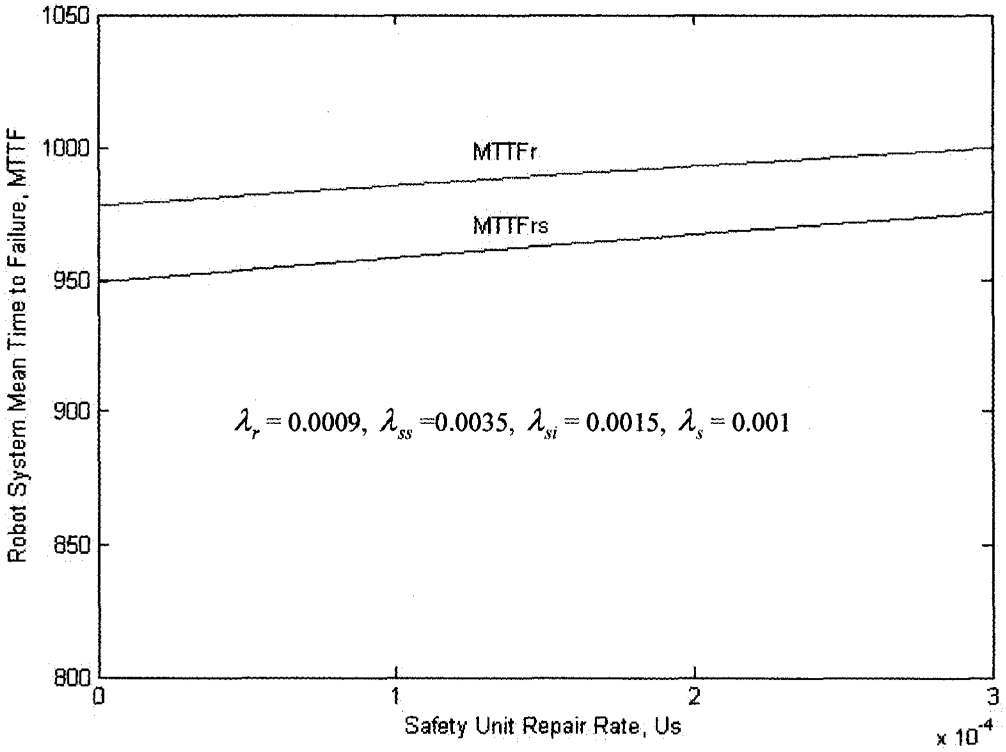


Figure 2.11 The robot-safety system mean time to failure plots for the increasing Us (means μ_s) values of the safety unit repair rate.

2.6 Conclusions

- For exponentially distributed failed robot system repair time x, the system time dependent availability decreases with the increasing time.
- For exponentially, Rayleigh or lognormally distributed failed robot system repair time x, the system steady state availability increases with the increasing safety unit repair rate.
- For exponentially distributed failed robot system repair time x, the robot-safety system mean time to failure (MTTF) increases with the increasing safety unit repair rate.

Stochastic Analysis of a System containing One Robot and (N-1) Standby Safety Units with an Imperfect Switch

3.1 Introduction

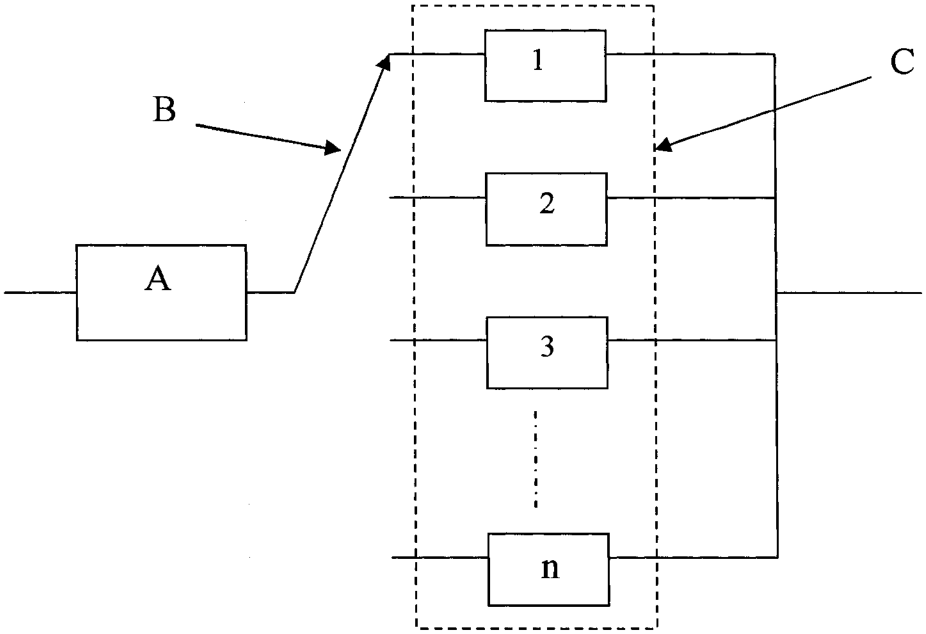
In previous chapter, the switch was assumed perfect. In this chapter, the failure of switch is taken into account which means imperfect.

Thus, this chapter presents a mathematical model for performing reliability and availability analyses of a system containing one robot and (n-1) standby safety units with a switch that can fail. More specifically, the robot system is composed of one robot, n identical safety units and a switch to replace a failed safety unit.

The block diagram of the robot system is shown in Figure 3.1 and its corresponding state space diagram is presented in Figure 3.2. The numerals and letter n in the boxes of Figure 3.2 denote system states.

At time t = 0, robot, one safety unit, and the switch to replace a failed safety unit start operating and n-1 safety units are on standby. The overall robot-safety system can fail the following two ways:

- The robot fails with a normally working safety unit and the switch. In addition, zero or more safety units are on standby.
- The robot fails with one or more safety units failed or considered failed and the switch is either working or failed.



A : Robot

B : Switch for replacing a failed safety unit and it can fail.

C: n identical Safety Units (one operating and n-1 on standby)

Figure 3.1 The block diagram of the robot-safety system containing one robot and (n-1) standby units with an imperfect switch

The following assumptions are associated with this model:

- The robot-safety system is composed of one robot, n identical safety units (only one operates and the rest remain on standby) and a switch.
- Robot, switch and one safety unit start operating simultaneously.
- The completely failed robot-safety system and its individually failed units (i.e. robot, switch and safety unit) can be repaired. Failure and repair rates of robot, switch and safety units are constant.

- The failed robot-safety system repair rates can be constant or non-constant.
- All failures are statistically independent.
- A repaired safety unit, robot, switch or the total robot-safety system is as good as new.

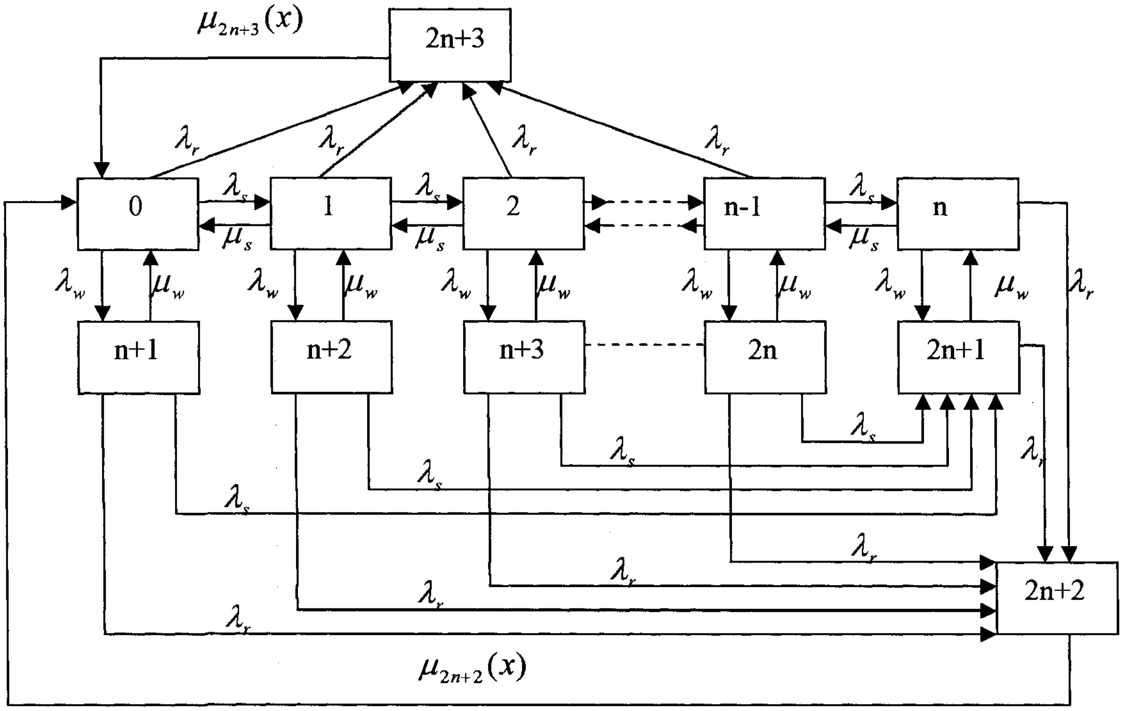


Figure 3.2 The state space diagram of the robot-safety system containing one robot and (n-1) standby safety units with an imperfect switch

3.1.1 Notation:

The following symbols are associated with the model:

i $\quad$ i^{th} state of the robot-safety system:

for $i = 0$, means the robot, the switch and one safety unit are working normally;

for $i = 1$, means the robot, the switch, one safety unit are working normally and one safety unit has failed;

for i = k, means the robot, the switch, one safety unit are working normally and k safety units have failed (i.e., k = 2,3……n-1);

for i = n, means the robot work, the switch are working normally and all safety units have failed;

for i = h, means the robot, one safety unit still work normally and h-n safety units and the witch have failed (i.e., h = n+1, n+2,…… 2n);

for i = 2n+1, means the robot work normally and all the safety units and the switch have failed;

j j^{th} state of the robot–safety system:

for j = 2n+2, means the total robot-safety system has failed (i.e., the robot , one or more safety units have failed or considered failed and the switch is either working or failed);

for j = 2n+ 3, means the robot-safety system has failed (i.e., the robot has failed while a safety unit and the switch are working normally. In addition, zero or more safety units are on standby);

t	Time.
λ_s	Constant failure rate of a safety unit.
λ_r	Constant failure rate of the robot.
λ_w	Constant failure rate of the switch.
μ_s	Constant repair rate of a safety unit.
μ_w	Constant repair rate of the switch.
$\Delta \mathbf{x}$:	Finite repair time interval.

$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in state j, and has an elapsed repair time of x, for j = 2n+2, 2n+3.
$P_j(x,t)\,\Delta x$	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x, x+ Δx]; for j = 2n+2, 2n+3 .
pdf	Probability density function.
$w_j(x)$	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j = 2n+2, 2n+3 .
$P_j(t)$	Probability that the robot safety system is in state j at time t; for j = 2n+2, 2n+3.
$P_i(t)$	Probability that the robot-safety system is in state i at time t; for i = 0, 1,...2n+1.
P_i	Steady state probability that the robot-safety system is in state i; for i=0, 1..2n+1.
P_j	Steady state probability that robot-safety system is in state j; for j = 2n+2, 2n+3.
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i; for i = 0,1,2...2n+1.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for j = 2n+2, 2n+3.
AVrs(s)	Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot.
AVr(s)	Laplace transform of the robot-safety system availability with or without a normally safety unit.
AVrs (t)	Robot-safety system time dependent availability with one normally working safety unit, the switch and the robot.

AVr(t)	Robot-safety system time dependent availability with or without a normally working safety unit.
SSAVrs	Robot-safety system steady state availability with one normally working safety unit, the switch and the robot.
SSAVr	Robot-safety system steady state availability with or without a normally working safety unit.
Rrs(s)	Laplace transform of the robot-safety system reliability with one normally working safety unit, the switch and the robot.
Rr(s)	Laplace transform of the robot safety system reliability with or without a normally working safety unit.
MTTFrs	Robot-safety system mean time to failure when the robot working normally with one normally working safety unit.
MTTFr	Robot-safety system mean time to failure with or without a normally working safety unit.

3.2 Generalized Robot-Safety System Analysis

Using the supplementary method [71, 72], we write down the following equations for the Figure 3.2 diagram:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_s P_1(t) + \mu_w P_{n+1}(t) + \sum_{j=2n+2}^{2n+3} P_j(x,t)\mu_j(x)dx \tag{3.1}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_s P_{i-1}(t) + \mu_s P_{i+1}(t) + \mu_w P_{i+n+1}(t) \tag{3.2}$$

(for $i = 1, 2, \ldots\ldots, n-1$)

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) + \mu_w P_{2n+1}(t) \qquad (3.3)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_w P_{i-n-1}(t) \quad (\text{ for } i = n+1, n+2, \ldots\ldots, 2n) \qquad (3.4)$$

$$\frac{dP_{2n+1}(t)}{dt} + a_{2n+1} P_{2n+1}(t) = \lambda_s \sum_{i=n+1}^{2n} P_i(t) + \lambda_w P_n(t) \qquad (3.5)$$

where

$$a_0 = \lambda_s + \lambda_w + \lambda_r$$

$$a_i = \lambda_s + \lambda_w + \lambda_r + \mu_s \qquad (\text{for } i = 1, 2, \ldots\ldots, n-1)$$

$$a_n = \lambda_w + \lambda_r + \mu_s$$

$$a_i = \lambda_s + \lambda_r + \mu_w \qquad (\text{for } i = n+1, n+2, \ldots\ldots, 2n)$$

$$a_{2n+1} = \lambda_r + \mu_w$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \quad (\text{ for } j = 2n+2, 2n+3) \qquad (3.6)$$

The associated boundary conditions are as follows:

$$P_{2n+2}(0,t) = \lambda_r \sum_{i=n}^{2n+1} P_i(t) \qquad (3.7)$$

$$P_{2n+3}(0,t) = \lambda_r \sum_{i=0}^{n-1} P_i(t) \qquad (3.8)$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

3.3 Generalized Robot-Safety System Laplace Transforms of State Probabilities

By solving Equations (1)-(8) with the Laplace transform method *, we get the following

* : Please see Appendix B.1.

Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} a_j(s)\frac{1 - W_j(s)}{s})]^{-1} = \frac{1}{G(s)} \quad (3.9)$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad (\text{for } i = 1,2,\ldots\ldots,n) \qquad (3.10)$$

$$P_i(s) = V_i(s)\, P_0(s) \qquad (\text{for } i = n+2,n+2,\ldots\ldots,2n+1) \qquad (3.11)$$

$$P_{n+1}(s) = \frac{\lambda_w}{s + a_{n+1}}\, P_0(s) \qquad (3.12)$$

$$P_j(s) = a_j(s)\frac{1 - W_j(s)}{s}\, P_0(s) \qquad (\text{for } j = 2n+2,\ 2n+3) \qquad (3.13)$$

where

$$L_i(s) = (s + a_i) - \frac{\lambda_w \mu_w}{s + a_{i+n+1}} \qquad (\text{for } i = 1,2,\ldots\ldots,n)$$

$$D_1(s) = L_1(s)$$

$$D_i(s) = L_i(s) - \frac{\lambda_s \mu_s}{D_{i-1}(s)} \qquad (\text{for } i = 2,\ldots\ldots,n)$$

$$A_i(s) = \frac{\lambda_s^i}{\prod_{h=1}^{i} D_h(s)} \qquad (\text{for } i = 1,2,\ldots\ldots,n-1)$$

$$B_i(s) = \frac{\mu_s}{D_i(s)} \qquad (\text{for } i = 1,2,\ldots\ldots,n-1)$$

$$Y_i(s) = \sum_{h=i}^{n-1} A_h(s) \prod_{k=i}^{h-1} B_k(s) + \prod_{h=i}^{n-1} B_h(s) Y_n(s)$$

$$(\text{for } i = 1,2,\ldots\ldots,n-1)$$

$$V_i(s) = \frac{\lambda_w}{s + a_i}\, Y_{i-n-1}(s) \qquad (\text{for } i = n+2,\ldots\ldots 2n)$$

$$\mathrm{V}_{2n+1}(\mathrm{s}) = \frac{\lambda_s \lambda_w}{(s+a_{n+1})(s+a_{2n+1})} + \frac{\lambda_s}{s+a_{2n+1}} \sum_{i=1}^{n-1} \frac{\lambda_w}{s+a_{i+n+1}} \mathrm{Y}_i(\mathrm{s}) + \frac{\lambda_w}{s+a_{2n+1}} \mathrm{Y}_n(\mathrm{s})$$

$\mathrm{Y}_n(\mathrm{s}) =$

$$\frac{\lambda_s A_{n-1}(s) + \dfrac{\lambda_s \lambda_w \mu_w}{(s+a_{n+1})(s+a_{2n+1})} + \dfrac{\lambda_s \mu_w}{s+a_{2n+1}} \sum_{i=1}^{n-1} \dfrac{\lambda_w}{s+a_{i+n+1}} \sum_{h=i}^{n-1} [A_h(s) \prod_{k=i}^{h-1} B_k(s)]}{L_n(s) - \lambda_s B_{n-1}(s) - \dfrac{\lambda_s \mu_w}{s+a_{2n+1}} \sum_{i=1}^{n-1} \dfrac{\lambda_w}{s+a_{i+n+1}} \prod_{h=i}^{n-1} B_h(s)}$$

$$\mathrm{a}_{2n+2}(\mathrm{s}) = \lambda_r [\, \mathrm{Y}_n(\mathrm{s}) + \frac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s)\,]$$

$$\mathrm{a}_{2n+3}(\mathrm{s}) = \lambda_r [1 + \sum_{i=1}^{n-1} Y_i(s)\,]$$

$$\mathrm{G(s)} = \mathrm{s}(1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} a_j(s) \frac{1-W_j(s)}{s}) \qquad (3.14)$$

$$\mathrm{W}_j(\mathrm{s}) = \int_0^\infty e^{-sx} \mathrm{w}_j(\mathrm{x})\mathrm{dx} \qquad \text{(for } j = \text{2n+2, 2n+3)} \qquad (3.15)$$

$$\mathrm{w}_j(\mathrm{x}) = \exp[-\int_0^x \mu_j(\delta) d\delta\,]\; \mu_j(x)$$

where

$\mathrm{w}_j(\mathrm{x})$ is the failed robot safety system repair time probability density function.

The Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot is given by:

$$\mathrm{AV}_{rs}(\mathrm{s}) = \sum_{i=0}^{n-1} P_i(s) + \sum_{i=n+1}^{2n} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s) + \dfrac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n} V_i(s)}{G(s)} \qquad (3.16)$$

The Laplace transform of the robot-safety system availability with or without a normally working safety unit is given by:

$$\mathrm{AV}_r(\mathrm{s}) = \sum_{i=0}^{2n+1} P_i(s) = \frac{1 + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=1}^{n} Y_i(s) + \sum_{i=n+2}^{2n+1} V_i(s)}{G(s)} \tag{3.17}$$

Taking the inverse Laplace transforms of the above equations, we can obtain the time dependent state probabilities, P_i (t)and P_j (t), and robot-safety system availabilities, AVrs(t) and AVr(t).

3.3.1 Robot Safety System Time Dependent Analysis for a Special Case

For two safety units (i.e., one working, other one on standby), by substituting n=2 into Equations (9)-(16), we get

$$\mathrm{P}_0(\mathrm{s}) = \frac{1}{s[1 + \sum_{i=1}^{2} Y_i(s) + \frac{\lambda_w}{s + a_3} + \sum_{i=4}^{5} V_i(s) + \sum_{j=6}^{7} a_j(s) \frac{1 - W_j(s)}{s}]} = \frac{1}{G(s)} \tag{3.18}$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{Y}_i(\mathrm{s})\, \mathrm{P}_0(\mathrm{s}) \qquad (\text{for } i = 1, 2) \tag{3.19}$$

$$\mathrm{P}_3(\mathrm{s}) = \frac{\lambda_w}{s + a_3} \mathrm{P}_0(\mathrm{s}) \tag{3.20}$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{V}_i(\mathrm{s})\mathrm{P}_0(\mathrm{s}) \qquad (\text{for } i = 4, 5) \tag{3.21}$$

$$\mathrm{P}_j(\mathrm{s}) = \mathrm{a}_j(\mathrm{s}) \frac{1 - W_j(s)}{s} \mathrm{P}_0(\mathrm{s}) \tag{3.22}$$

where

$$\mathrm{Y}_2(\mathrm{s}) = \frac{\lambda_s \frac{\lambda_s}{L_1(s)} + \frac{\lambda_s \lambda_w \mu_w}{(s + a_3)(s + a_5)} + \frac{\lambda_s \lambda_w \mu_w}{(s + a_4)(s + a_5)} \frac{\mu_s}{L_1(s)}}{L_2(s) - \lambda_s \frac{\mu_s}{L_1(s)} - \frac{\lambda_s \lambda_w \mu_w}{(s + a_4)(s + a_5)} \frac{\mu_s}{L_1(s)}}$$

$$\mathrm{Y}_1(\mathrm{s}) = \frac{\lambda_s}{L_1(s)} + \frac{\mu_s}{L_1(s)} \mathrm{Y}_2(\mathrm{s})$$

$$V_5(s) = \frac{\lambda_s \lambda_w}{(s+a_3)(s+a_5)} + \frac{\lambda_s}{s+a_5} \frac{\lambda_w}{s+a_4} Y_1(s) + \frac{\lambda_w}{s+a_5} Y_2(s)$$

$$V_4(s) = \frac{\lambda_w}{s+a_4} Y_1(s)$$

$$a_6(s) = \lambda_r [\, Y_2(s) + \frac{\lambda_w}{s+a_3} + \sum_{i=4}^{5} V_i(s) \,]$$

$$a_7(s) = \lambda_r [\, 1 + Y_1(s)]$$

$$L_1(s) = (s+a_1) - \frac{\lambda_w \mu_w}{s+a_4}$$

$$L_2\ (s) = (s+a_2) - \frac{\lambda_w \mu_w}{s+a_5}$$

$$G(s) = s[1+\sum_{i=1}^{2} Y_i(s) + \frac{\lambda_w}{s+a_3} + \sum_{i=4}^{5} V_i(s) + \sum_{j=6}^{7} a_j(s) \frac{1-W_j(s)}{s}] \tag{3.23}$$

The Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot is given by:

$$AV_{rs}(s) = \sum_{i=0}^{1} P_i(s) + \sum_{i=3}^{4} P_i(s) = \frac{1 + Y_1(s) + \frac{\lambda_w}{s+a_3} + V_4(s)}{G(s)} \tag{3.24}$$

Similarly, the Laplace transform of the robot-safety system availability with or without a normally working safety unit is given by:

$$AV_r(s) = \sum_{i=0}^{5} P_i(s) = \frac{1 + \sum_{i=1}^{2} Y_i(s) + \frac{\lambda_w}{s+a_3} + \sum_{i=4}^{5} V_i(s)}{G(s)} \tag{3.25}$$

Taking the inverse Laplace transforms of the above Equations, we can obtain the time dependent state probabilities, P_i(t)and P_j(t), and robot-safety system availabilities, AVrs(t) and AVr(t).

For the failed robot-safety system exponentially distributed repair time x, the probability function is expressed by:

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 6, 7) \qquad (3.26)$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Substituting Equation (3.26) into Equation (3.15), we can get:

$$W_j(s) = \frac{\mu_j}{s + \mu_j} \qquad (\mu_j > 0, j = 6, 7) \qquad (3.27)$$

By inserting Equation (3.27) into Equations (3.9)-(3.25), setting λ_s=0.002, μ_s= 0.00015, λ_w=0.001, μ_w= 0.0003, λ_r= 0.00009, μ_6 = 0.0001, μ_7= 0.00015; and using Matlab computer program [74], the Figure 3.3 and Figure 3.4 plots were obtained. These plots show that state probabilities and system availability decrease and increase with varying time t.

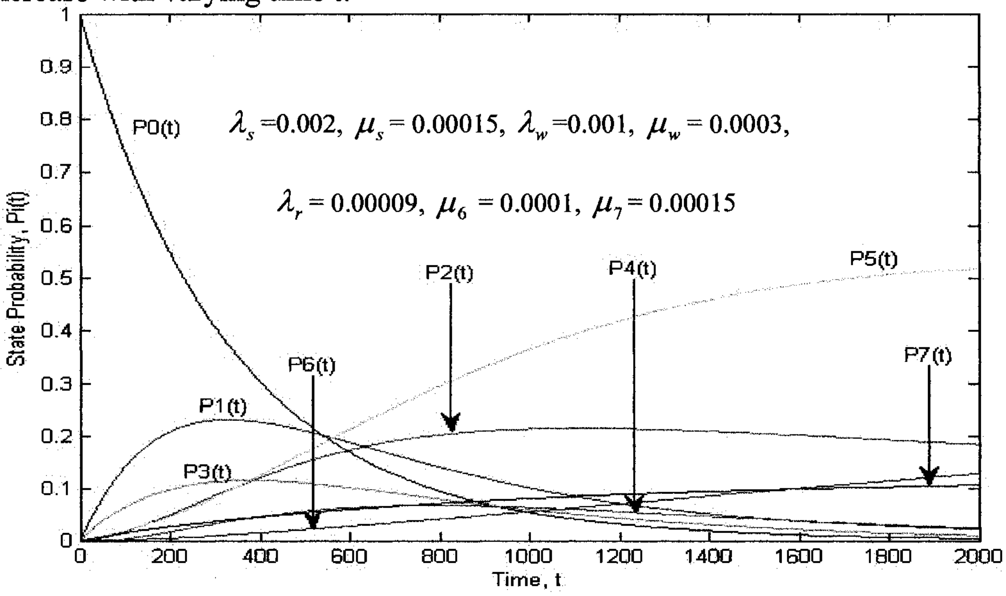


Figure 3.3 Time-dependent probability plots for a robot-safety system with exponentially distributed failed system repair times.

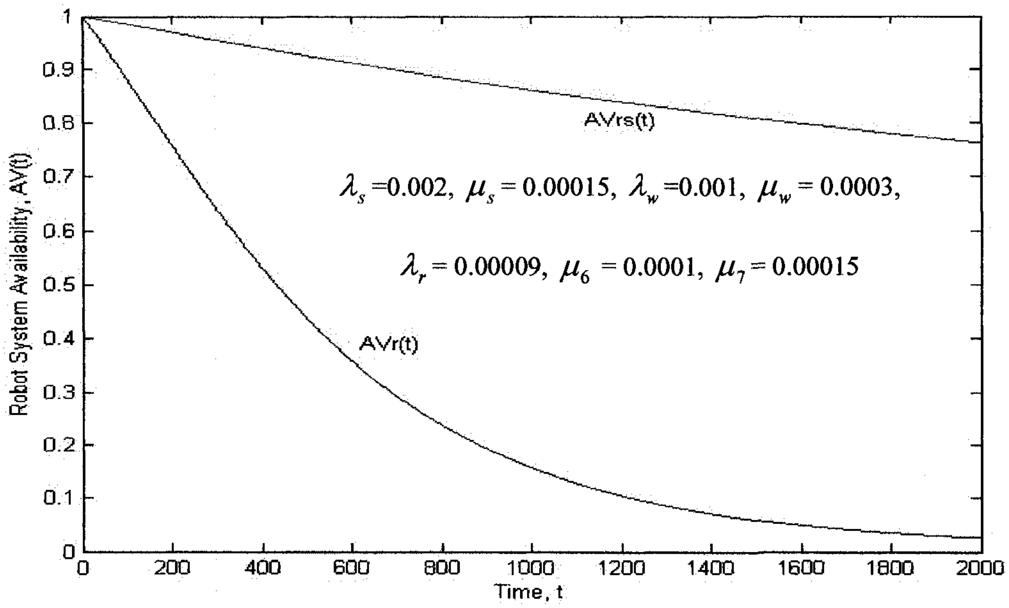


Figure 3.4 Time-dependent state availability plots for a robot-safety system with exponentially distributed failed system repair times.

3.4 Generalized Robot Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (3.1)-(3.8) we get:

$$a_0 P_0 = \mu_s P_1 + \mu_w P_n + \sum_{j=2n+2}^{2n+3} P_j(x)\mu_j(x)dx \tag{3.28}$$

$$a_i P_i = \lambda_s P_{i-1} + \mu_s P_{i+1} + \mu_w P_{i+n+1} \tag{3.29}$$

(for i = 1,2,........,n-1)

$$a_n P_n = \lambda_s P_{n-1} + \mu_w P_{2n+1} \tag{3.30}$$

$$a_i P_i = \lambda_w P_{i-n-1} \tag{3.31}$$

(for i = n+1,n+2,........,2n-k-1)

$$a_{2n+1} P_{2n+1} = \lambda_s \sum_{i=n+1}^{2n} P_i + \lambda_w P_n \tag{3.32}$$

where

$$a_0 = \lambda_s + \lambda_w + \lambda_r$$

$$a_i = \lambda_s + \lambda_w + \lambda_r + \mu_s \qquad \text{(for } i = 1, 2\text{........n-1)}$$

$$a_n = \lambda_w + \lambda_r + \mu_s$$

$$a_i = \lambda_s + \lambda_r + \mu_w \qquad \text{(for } i = \text{n+1, n+2........2n)}$$

$$a_{2n+1} = \lambda_r + \mu_w$$

$$\frac{dP_j(x)}{dx} + \mu_j(\mathrm{x})\mathrm{P}_j(\mathrm{x}) = 0 \qquad \text{(for } j = \text{2n+2, 2n+3)} \tag{3.33}$$

The associated boundary conditions are as follows:

$$\mathrm{P}_{2n+2}\,(0) = \lambda_r \sum_{i=n}^{2n+1} P_i \tag{3.34}$$

$$\mathrm{P}_{2n+3}\,(0) = \lambda_r \sum_{i=0}^{n-1} P_i \tag{3.35}$$

Solving Equations (3.28) - (3.33) *, together with

$$\sum_{i=0}^{2n+1} P_i + \sum_{j=2n+2}^{2n+3} P_j = 1 \tag{3.36}$$

we get:

$$\mathrm{P}_0 = (1 + \sum_{i=1}^{n} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n} V_i + \sum_{j=2n+2}^{2n+3} a_j E_j[x])^{-1} = \frac{1}{G} \tag{3.37}$$

$$\mathrm{P}_i = \mathrm{Y}_i\,\mathrm{P}_0 \qquad \text{(for } i = 1, 2\text{........n)} \tag{3.38}$$

$$\mathrm{P}_i = \mathrm{V}_i\,\mathrm{P}_0 \qquad \text{(for } i = \text{n+2........2n+1)} \tag{3.39}$$

$$\mathrm{P}_{n+1} = \frac{\lambda_w}{a_n}\,\mathrm{P}_0 \tag{3.40}$$

* : Please see Appendix B.2.

$$P_j = a_j E_j[x] P_0 \qquad \text{(for } j = 2n+2, 2n+3) \qquad (3.41)$$

where

$$L_i = \lim_{s \to 0} L_i(s) \qquad \text{(for } i = 1, 2 \ldots\ldots n)$$

$$D_1 = L_1$$

$$D_i = L_i - \frac{\lambda_s \mu_s}{D_{i-1}} \qquad \text{(for } i = 2, 3 \ldots\ldots n)$$

$$A_i = \frac{\lambda_s^i}{\prod_{h=1}^{i} D_h} \qquad \text{(for } i = 1, 2 \ldots\ldots n-1)$$

$$B_i = \frac{\mu_s}{D_i} \qquad \text{(for } i = 1, 2 \ldots\ldots n-1)$$

$$Y_i = \sum_{h=i}^{n-1} A_h \prod_{k=i}^{h-1} B_k + \prod_{h=i}^{n-1} B_h Y_n$$

$$\text{(for } i = 1, 2, \ldots\ldots n-1)$$

$$V_i = \frac{\lambda_w}{a_i} Y_{i-n-1} \qquad \text{(for } i = n+2 \ldots\ldots 2n)$$

$$V_{2n+1} = \frac{\lambda_s \lambda_w}{a_{n+1} a_{2n+1}} + \frac{\lambda_s}{a_{2n+1}} \sum_{i=1}^{n-1} \frac{\lambda_w}{a_{i+n+1}} Y_i + \frac{\lambda_w}{a_{2n+1}} Y_{n-k}$$

$$Y_n = \frac{\lambda_s A_{n-1} + \dfrac{\lambda_s \lambda_w \mu_w}{a_n a_{2n+1}} + \dfrac{\lambda_s \mu_w}{a_{2n+1}} \displaystyle\sum_{i=1}^{n-1} \frac{\lambda_w}{a_{i+n+1}} \sum_{h=i}^{n-1} A_h \prod_{k=i}^{h-1} B_k}{L_n - \lambda_s B_{n-1} - \dfrac{\lambda_s \mu_w}{a_{2n+1}} \displaystyle\sum_{i=1}^{n-1} \frac{\lambda_w}{a_{i+n+1}} \prod_{h=i}^{n-1} B_h}$$

$$a_{2n+2} = \lambda_r \left(Y_n + \sum_{i=n+2}^{2n+1} V_i + \frac{\lambda_w}{a_{n+1}}\right)$$

$$a_{2n+3} = \lambda_r \left(1 + \sum_{i=1}^{n-1} Y_i\right)$$

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j E_j[x] \tag{3.42}$$

$$E_j[x] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta) d\delta] dx \tag{3.43}$$

$$= \int_0^\infty x w_j(x) dx \qquad (\text{for } j = 2n+2, 2n+3)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

The generalized steady state availability of the robot safety system with one normally working normally safety unit, the switch and the robot is given by

$$SSAVrs = \sum_{i=0}^{n-1} P_i + \sum_{i=n+1}^{2n} P_i = \frac{1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n} V_i}{G} \tag{3.44}$$

Similarly, the generalized steady state availability of the robot safety system with or without a working safety unit is given by:

$$SSAVr = \sum_{i=0}^{2n+1} P_i = \frac{1 + \sum_{i=1}^{n} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i}{G} \tag{3.45}$$

For different failed robot-safety system repair time distributions, we get different expressions for G as follows:

i) For the failed robot-safety system gamma distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{\mu_j^\beta x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \qquad (\beta > 0,\ j = 2n+2, 2n+3) \tag{3.46}$$

where

x is the repair time variable, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty x w_j(x)dx = \frac{\beta}{\mu_j} \qquad (\beta > 0, j = 2n+2, 2n+3) \tag{3.47}$$

Substituting Equation (47) into Equation (42), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \frac{\beta}{\mu_j} E_j[x] \tag{3.48}$$

ii) For the failed robot-safety system Weibull distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j(x)^\beta} \qquad (\beta > 0, \ j = 2n+2, 2n+3) \tag{3.49}$$

where

x is the repair time variable, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^\infty x W_j(x)dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right) \qquad (\beta > 0, \ j = 2n+2, 2n+3) \tag{3.50}$$

Substituting Equation (3.50) into Equation (3.42), we can get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right) \tag{3.51}$$

iii) For the failed robot-safety system Rayleigh distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \mu_j x e^{-\mu_j x^2/2} \qquad (\mu_j > 0,\ j = 2n+2, 2n+3) \qquad (3.52)$$

where

x is the repair time variable, μ_j is the scale parameter.

Thus, the mean time to robot-safety system repair is given by:

$$\mathrm{E}_j(\mathrm{x}) = \int_0^{\infty} x W_j(x)dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (\mu_j > 0, j = 2n+2, 2n+3) \qquad (3.53)$$

Substituting Equation (3.53) into Equation (3.42), we get:

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \sqrt{\frac{\pi}{2\mu_j}} \qquad (3.54)$$

iv) For the failed robot system Lognormal distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \frac{1}{\sqrt{2\pi x \sigma_{y_j}}} e^{[\frac{-(Inx-\mu_{y_j})^2}{2\sigma_{y_j}^2}} \qquad (\text{for } j = 2n+2,\ 2n+3) \qquad (3.55)$$

where

x is the repair time variable, Inx is the natural logarithm of x with a mean μ and variance σ^2. The conditions on parameters are:

$$\sigma_{y_j} = \mathrm{In}\sqrt{1 + (\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \qquad (3.56)$$

$$\mu_{y_j} = \text{In}\sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \tag{3.57}$$

Thus, the mean time to robot-safety system repair is given by

$$\text{E}_j(\text{x}) = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (\text{for } j = 2\text{n}+2,\ 2\text{n}+3) \tag{3.58}$$

Substituting Equation (3.58) into Equation (3.42), we get

$$\text{G} = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (\text{for } j = 2\text{n}+2, 2\text{n}+3) \tag{3.59}$$

v) For the failed robot system exponentially distributed repair time x, the probability density function is expressed by

$$\text{w}_j(\text{x}) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 2\text{n}+2,\ 2\text{n}+3) \tag{3.60}$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Thus, the mean time to robot-safety system repair is given by

$$\text{E}_j(\text{x}) = \int_0^\infty x w_j(x) dx = \frac{1}{\mu_j} \qquad (\beta > 0,\ j = 2\text{n}+2,\ 2\text{n}+3) \tag{3.61}$$

Substituting Equation (61) into Equation (42), we get

$$\text{G} = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \frac{1}{\mu_j} \tag{3.62}$$

3.4.1 The Robot-Safety System Steady State Analysis for a Special Case

For n = 2, from Equations (3.37)-(3.45), we get

$$\mathrm{P}_0 = \frac{1}{1+\sum_{i=1}^{2} Y_i + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i + \sum_{j=6}^{7} a_j E_j[x]} \qquad (3.63)$$

$$\mathrm{P}_i = \mathrm{Y}_i \ \mathrm{P}_0 \qquad \text{(for } i = 1, 2) \qquad (3.64)$$

$$\mathrm{P}_3 = \frac{\lambda_w}{a_3} \ \mathrm{P}_0 \qquad (3.65)$$

$$\mathrm{P}_i = \mathrm{V}_i \mathrm{P}_0 \qquad \text{(for } i = 4, 5) \qquad (3.66)$$

$$\mathrm{P}_j = \mathrm{a}_j \ \mathrm{E}_j[\mathrm{x}] \, \mathrm{P}_0 \qquad (3.67)$$

where

$$\mathrm{Y}_2(\mathrm{s}) = \frac{\lambda_s \frac{\lambda_s}{L_1} + \frac{\lambda_s \lambda_w \mu_w}{a_3 a_5} + \frac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \frac{\mu_s}{L_1}}{L_2 - \lambda_s \frac{\mu_s}{L_1} - \frac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \frac{\mu_s}{L_1}}$$

$$\mathrm{Y}_1 = \frac{\lambda_s}{L_1} + \frac{\mu_s}{L_1} \mathrm{Y}_2$$

$$\mathrm{V}_5 = \frac{\lambda_s \lambda_w}{a_3 a_5} + \frac{\lambda_s \lambda_w}{a_4 a_5} \mathrm{Y}_1 + \frac{\lambda_w}{a_5} \mathrm{Y}_2$$

$$\mathrm{V}_4 = \frac{\lambda_w}{a_4} \ \mathrm{Y}_1$$

$$\mathrm{a}_6 = \lambda_r [\mathrm{Y}_2 + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i \,]$$

$$\mathrm{a}_7 = \lambda_r [1 + \mathrm{Y}_1]$$

$$\mathrm{L}_1 = \mathrm{a}_1 - \frac{\lambda_w \mu_w}{a_4}$$

$$\mathrm{L}_2 = \mathrm{a}_2 - \frac{\lambda_w \mu_w}{a_5}$$

$$G = 1 + \sum_{i=1}^{2} Y_i + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i + a_j E_j[x] \tag{3.68}$$

$$\text{SSAVrs} = \sum_{i=0}^{1} P_i + \sum_{i=3}^{4} P_i = \frac{1 + Y_1 + \dfrac{\lambda_w}{a_3} + a_4}{G} \tag{3.69}$$

$$\text{SSAVr} = \sum_{i=0}^{5} P_i = \frac{1 + \sum_{i=1}^{2} Y_i + \dfrac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i}{G} \tag{3.70}$$

1) For exponentially distributed failed robot-safety system repair time, substituting Equation (3.61) into Equations (3.63) - (3.70), setting:

λ_s=0.0002, λ_w=0.001, μ_w = 0.0003, λ_r = 0.00009, μ_6 = 0.0001, μ_7 = 0.00015;

and using Matlab computer program [74], Figure 3.5 and Figure 3.6 plots were obtained.

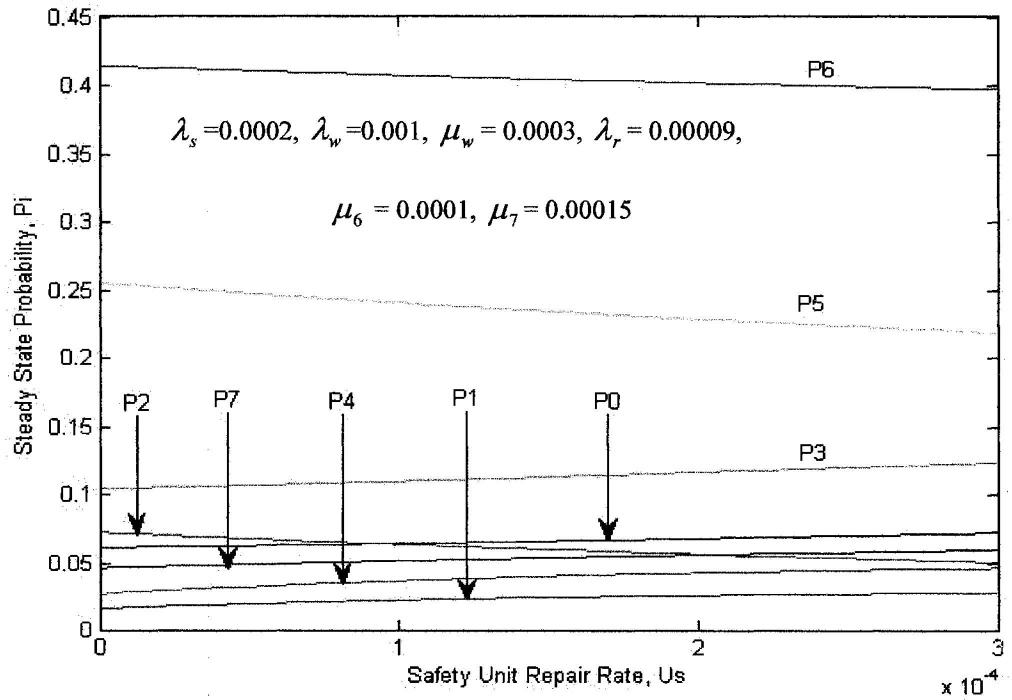


Figure 3.5 Robot-safety system steady state probabilities versus safety unit repair rate Us (means μ_s) plots for exponentially distributed failed system repair times.

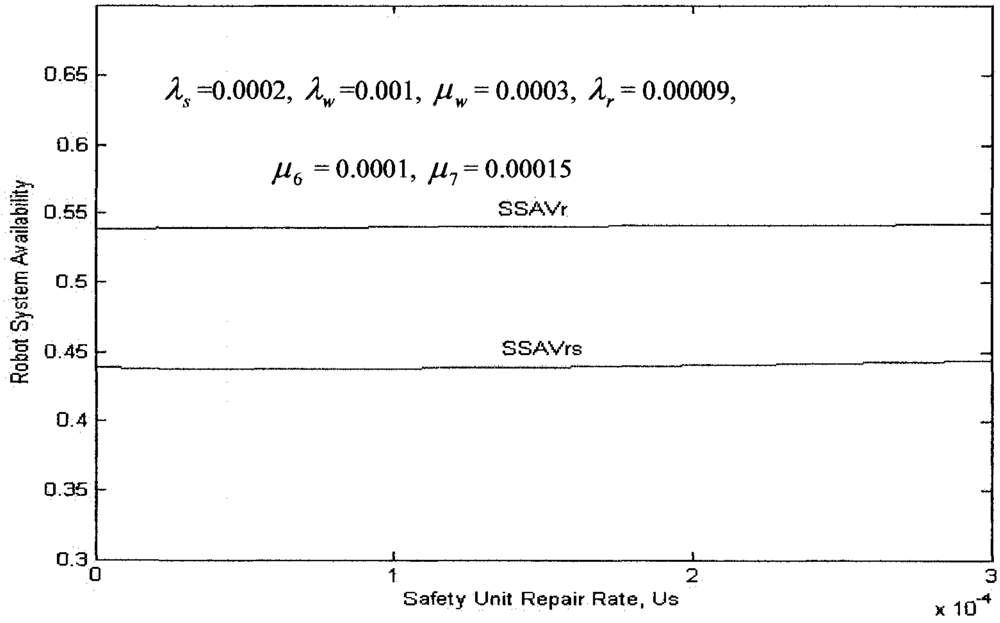


Figure 3.6 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots for exponentially distributed failed system repair times.

2) For Rayleigh distributed failed robot-safety system repair time, substituting Equation (3.53) into Equations (3.63) - (3.70), setting:

λ_s=0.0002, λ_w=0.001, μ_w= 0.0003, λ_r= 0.00009, μ_6 = 0.0001, μ_7= 0.00015;

and using Matlab computer program [74], Figure 3.7 and Figure 3.8 plots were obtained.

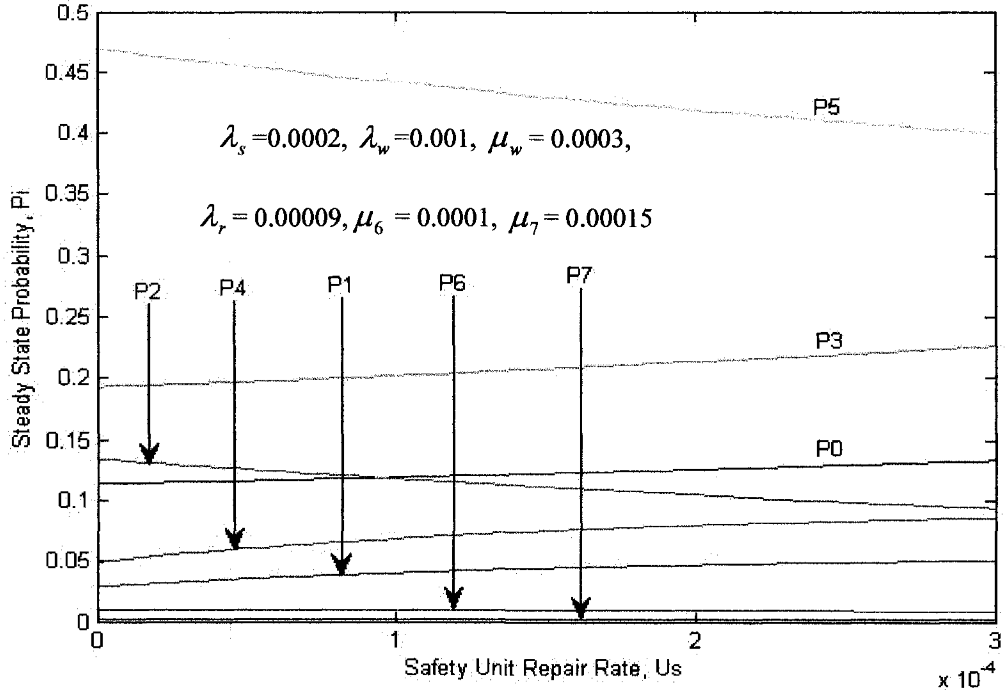


Figure 3.7 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots for Rayleigh distributed failed system repair times.

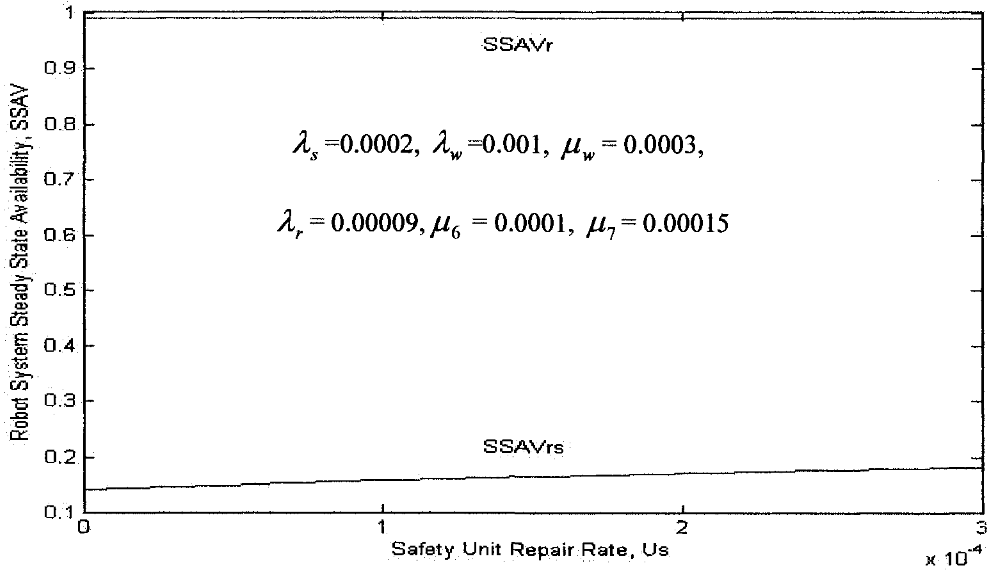


Figure 3.8 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s)plots with Rayleigh distributed failed system repair times.

3) For lognormally distributed failed robot-safety system repair time, substituting Equation(3.58) into Equations (3.63) - (3.70), setting:

λ_s=0.0002, λ_w=0.001, μ_w= 0.0003, λ_r= 0.00009, μ_6 = 0.0001, μ_7= 0.00015, σ =4; and using matlab computer program [10], Figure 3.9 and Figure 3.10 plots were obtained.

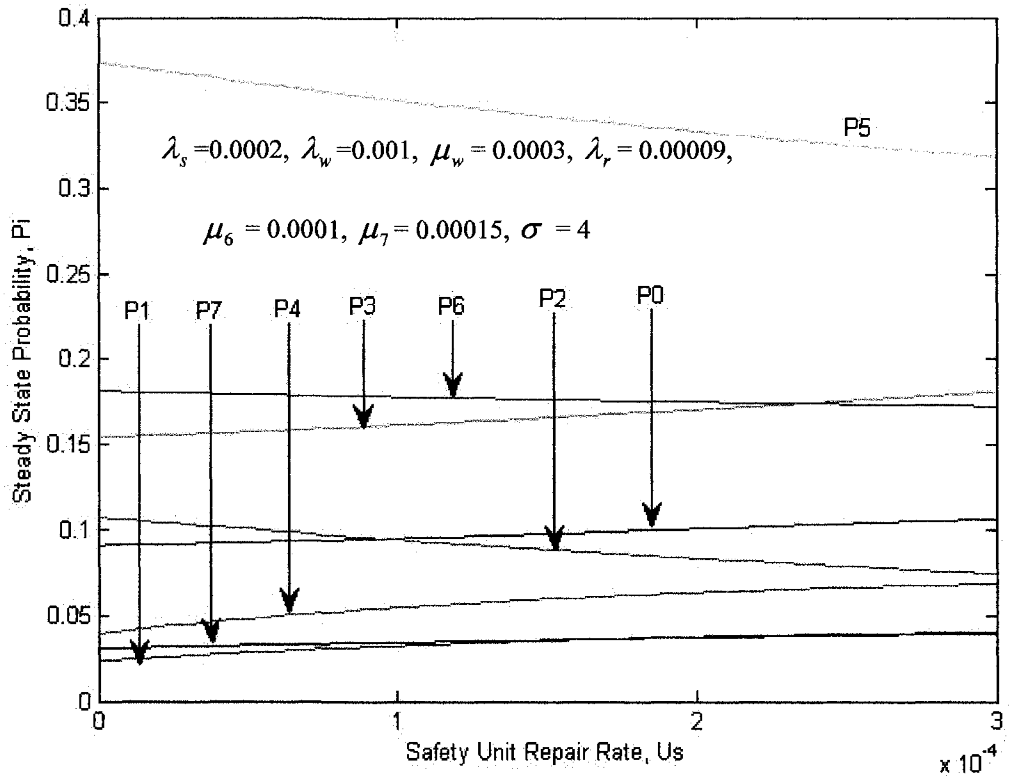


Figure 3.9 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots with lognormally distributed failed system repair times.

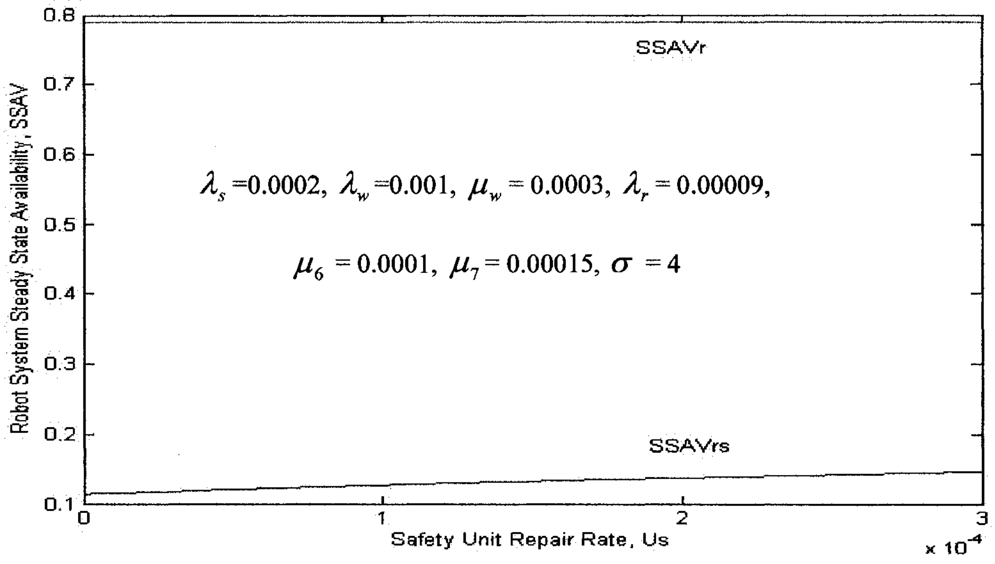


Figure3.10 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots for lognormally distributed failed system repair times.

3.5 Robot-Safety System Reliability and Mean Time to Failure Analysis

Setting $\mu_j = 0$, (for j = 2n+2, 2n+3), in Figure 3.2 and using the Markov method [73], we write the following equations for the modified figure:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_s P_1(t) + \mu_w P_{n+1}(t) \tag{3.71}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_s P_{i-1}(t) + \mu_s P_{i+1}(t) + \mu_w P_{i+n+1}(t) \tag{3.72}$$

(for i = 1, 2........n-1)

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) + \mu_w P_{2n+1}(t) \tag{3.73}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_w P_{i-n-1}(t) \qquad \text{(for } i = \text{n+1, n+2........2n)} \tag{3.74}$$

$$\frac{dP_{2n+1}(t)}{dt} + \mathrm{a}_{2n+1}\mathrm{P}_{2n+1}(\mathrm{t}) = \lambda_s \sum_{i=n+1}^{2n} P_i(t) + \lambda_w \mathrm{P}_n(\mathrm{t}) \tag{3.75}$$

$$\frac{dP_{2n+2}(t)}{dt} = \lambda_r \sum_{i=n}^{2n+1} P_i(t) \tag{3.76}$$

$$\frac{dP_{2n+3}(t)}{dt} = \lambda_r \sum_{i=0}^{n-1} P_i(t) \tag{3.77}$$

where

$$\mathrm{a}_0 = \lambda_s + \lambda_w + \lambda_r$$

$$\mathrm{a}_i = \lambda_s + \lambda_w + \lambda_r + \mu_s \qquad \text{(for } i = 1, 2\text{........n-1)}$$

$$\mathrm{a}_n = \lambda_w + \lambda_r + \mu_s$$

$$\mathrm{a}_i = \lambda_s + \lambda_r + \mu_w \qquad \text{(for } i = \text{n+1, n+2........2n)}$$

$$\mathrm{a}_{2n+1} = \lambda_r + \mu_w$$

At time t = 0, $\mathrm{P}_0(0) = 1$ and all other initial conditions state probabilities are equal to zero.

By solving Equations (3.71) – (3.77) with the aid of Laplace transforms *, we get:

$$\mathrm{P}_0(\mathrm{s}) = \mathrm{P}_0(\mathrm{s}) = [\mathrm{s}(1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} \frac{a_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{3.78}$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{Y}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad \text{(for } i = 1, 2\text{........n)} \tag{3.79}$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{V}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad \text{(for } i = \text{n+2, n+2........2n+1)} \tag{3.80}$$

$$\mathrm{P}_{n+1}(\mathrm{s}) = \frac{\lambda_w}{s + a_{n+1}}\,\mathrm{P}_0(\mathrm{s}) \tag{3.81}$$

$$\mathrm{P}_j(\mathrm{s}) = \frac{a_j(s)}{s}\,\mathrm{P}_0(\mathrm{s}) \qquad \text{(for } j = \text{ 2n+2, 2n+3)} \tag{3.82}$$

* : Please see Appendix B.3.

$$G(s) = s[1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} \frac{a_j(s)}{s}] \tag{3.83}$$

The Laplace transform of the robot-safety system reliability with one normally working safety unit, the switch and the robot is given by:

$$R_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) + \sum_{i=n+1}^{2n} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n} V_i(s)}{G(s)} \tag{3.84}$$

Similarly, the Laplace transform of the robot safety system reliability with or without a normally working safety unit is

$$R_r(s) = \sum_{i=0}^{2n+1} P_i(s) = \frac{1 + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=1}^{n} Y_i(s) + \sum_{i=n+2}^{2n+1} V_i(s)}{G(s)} \tag{3.85}$$

Using Equation (3.84) and Reference [73], the robot-safety system mean time to failure with one normally working safety unit, the switch and the robot is given by

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n} V_i}{\sum_{j=2n+2}^{2n+3} a_j} \tag{3.86}$$

Similarly, using Equation (3.85) and Reference [73], the robot safety system mean time to failure with or without a working safety unit is

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{1}{\lambda_r} \tag{3.87}$$

3.5.1 Robot-Safety System Mean Time to Failure Analysis for a Special Case

Substituting n = 2 into Equation (3.86) and (3.87), we get

$$\mathrm{MTTF}_{rs} = \frac{1 + Y_1 + \dfrac{\lambda_w}{a_3} + V_4}{\sum_{j=6}^{7} a_j} \tag{3.88}$$

$$\mathrm{MTTF}_{r} = \frac{1}{\lambda_r} \tag{3.89}$$

where

$$\mathrm{Y}_2 = \frac{\lambda_s \dfrac{\lambda_s}{L_1} + \dfrac{\lambda_s \lambda_w \mu_w}{a_3 a_5} + \dfrac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \dfrac{\mu_s}{L_1}}{L_2 - \lambda_s \dfrac{\mu_s}{L_1} - \dfrac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \dfrac{\mu_s}{L_1}}$$

$$\mathrm{Y}_1 = \frac{\lambda_s}{L_1} + \frac{\mu_s}{L_1} \mathrm{Y}_2$$

$$\mathrm{V}_5 = \frac{\lambda_s \lambda_w}{a_3 a_5} + \frac{\lambda_s \lambda_w}{a_4 a_5} \mathrm{Y}_1 + \frac{\lambda_w}{a_5} \mathrm{Y}_2$$

$$\mathrm{V}_4 = \frac{\lambda_w}{a_4} \mathrm{Y}_1$$

$$\mathrm{a}_6 = \lambda_r [\mathrm{Y}_2 + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i]$$

$$\mathrm{a}_7 = \lambda_r [1 + \mathrm{Y}_1]$$

$$\mathrm{L}_1 = \mathrm{a}_1 - \frac{\lambda_w \mu_w}{a_4}$$

$$\mathrm{L}_2 = \mathrm{a}_2 - \frac{\lambda_w \mu_w}{a_5}$$

For λ_s=0.0002, λ_w=0.001, μ_w= 0.0003, λ_r= 0.00009, and using Equations (3.88)-(3.89) and Matlab computer program [74], in Figure 3.11 MTTF_{rs} and MTTF_{r} plots were obtained.

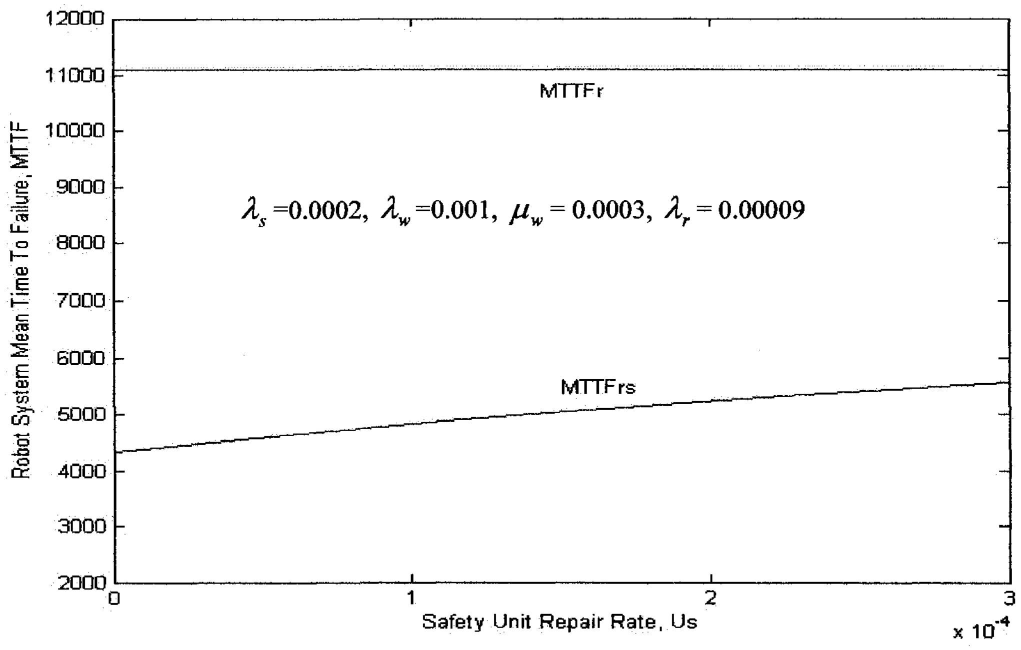


Figure 3.11 The robot-safety system mean time to failure plots for the increasing value of the safety unit repair rate Us (means μ_s).

3.6 Conclusions

- For exponentially distributed failed robot system repair time x, the system time dependent availability decreases with the increasing time.
- For exponentially, Rayleigh or lognormally distributed failed robot system repair time x, the system steady state availability increases with the increasing safety unit repair rate.
- For exponentially distributed failed robot system repair time x, the robot-safety system mean time to failure with one normally working safety unit increases with the increasing safety unit repair rate.

Chapter 4

Stochastic Analysis of a System Containing (N-1) Standby Robots and One Safety Unit with a Perfect Switch

4.1 Introduction

In previous two chapters, the models of redundancy of safety units for a robot-safety system were presented. In this Chapter, the model of redundancy of robot with a perfect switch is considered.

Thus, a mathematical model is introduced to perform reliability and availability analysis for the robot-safety system containing n identical robots, one safety unit, and a perfect switch which cannot fail. The switch is used to replace a failed robot.

The block diagram of the robot system is shown in Figure 4.1 and its corresponding state space diagram is presented in Figure 4.2. The numerals and letter n in the boxes of Figure 4.2 denote system states.

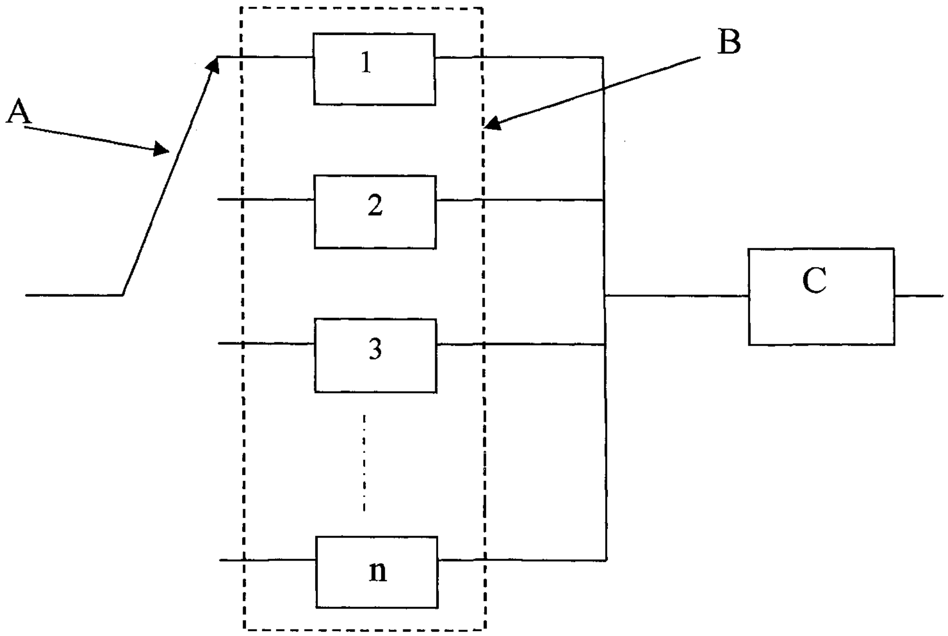
At time $t = 0$, one robot, safety unit and the switch to replace a failed robot start operating and n-1 robots are on standby. The overall robot-safety system can fail the following two ways:

- All the robots fail with the normally working safety unit.
- All the robots fail with the failed safety unit.

The following assumptions are associated with this model:

- The robot-safety system is composed of n identical robots (only one operates and the rest remain on standby), one safety unit and a switch.
- One robot, switch and a safety unit start operating simultaneously.

- The completely failed robot-safety system and its individually failed units (i.e. robots) can be repaired. Failure and repair rates of robots and the safety unit are constant.
- The failed robot-safety system repair rates can be constant or non-constant.
- All failures are statistically independent.
- A repaired robot, safety unit or the total robot-safety system is as good as new.



A : Switch for replacing a failed robot

B : n identical Robots (one operating and n-1 on standby)

C: Safety Unit

Figure 4.1 The block diagram of the robot-safety system containing (n-1) standby robots and one safety unit with a perfect switch

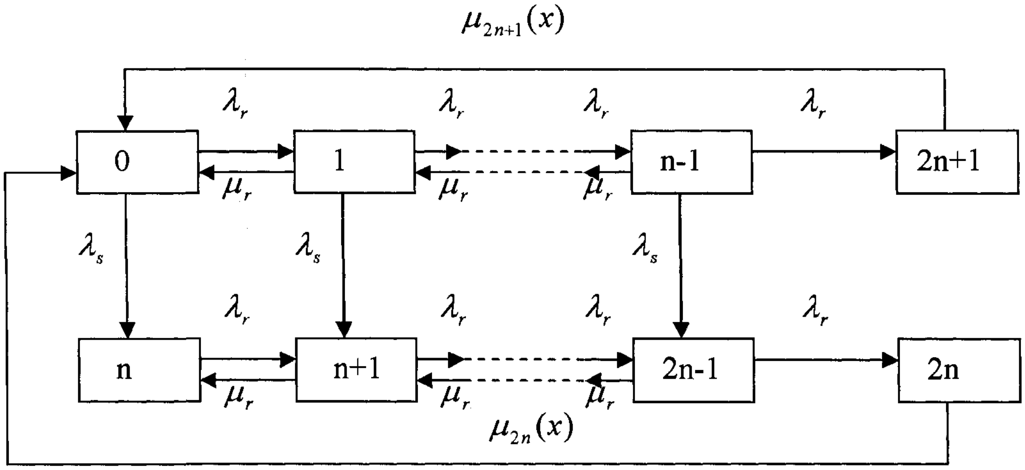


Figure 4.2 The state space diagram of the robot-safety system containing (n-1) robots and one safety unit with a perfect switch

4.1.1 Notation:

The following symbols are associated with the model:

i $\quad$ i^{th} state of the robot-safety system:

for $i = 0$, means one robot, the switch and the safety unit are working normally;

for $i = 1$, means one robot, the switch, the safety unit are working normally and one robot has failed;

for i = k, means one robot, the switch, the safety unit are working normally and k robots have failed; (i.e., k = 2,3......n-1);

for i = n, means one robot work and the switch are working normally and the safety unit has failed;

for i = h, means one robot and the switch are working normally while h-n robots and the safety unit have failed; (i.e., h = n+1, n+2...... 2n-1);

j	j^{th} state of the robot–safety system: for j = 2n, means the total robot-safety system has failed (i.e., all the robots and the safety unit have failed while the switch is working normally.); for j = 2n+ 1, means the robot-safety system has failed (i.e., all the robots have failed while the safety unit and the switch are working normally.)
t	Time.
λ_s	Constant failure rate of a safety unit.
λ_r	Constant failure rate of the robot.
μ_r	Constant repair rate of the robot.
Δ**x**:	Finite repair time interval.
$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in state j: and has an elapsed repair time of x; for j = 2n, 2n+1.
$P_j(x,t)\ \Delta x$	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x, x+Δx]; for j = 2n, 2n+1 .
pdf	Probability density function.
$w_j(x)$	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j = 2n, 2n+1.
$P_j(t)$	Probability that the robot safety system is in state j at time t; for j = 2n, 2n+1.
$P_i(t)$	Probability that the robot-safety system is in state i at time t; for i = 0, 1...2n-1.
P_i	Steady state probability that the robot-safety system is in state i; for i=0, 1...2n-1.
P_j	Steady state probability that robot-safety system is in state j; for j = 2n, 2n+1.

s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state *i*; for *i* = 0, 1, 2…2n-1.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state *j*; for *j* = 2n, 2n+1.
AVrs(s)	Laplace transform of the robot-safety system availability with the normally working safety unit.
AVr(s)	Laplace transform of the robot-safety system availability with or without the normally working safety unit.
AVrs (t)	Robot-safety system time dependent availability with the normally working safety unit.
AVr(t)	Robot-safety system time dependent availability with or without the normally working safety unit.
SSAVrs	Robot-safety system steady state availability with the normally working safety unit, the switch and the robot.
SSAVr	Robot-safety system steady state availability with or without the normally working safety unit.
Rrs(s)	Laplace transform of the robot-safety system reliability with the normally working safety unit.
Rr(s)	Laplace transform of the robot safety system reliability with or without the normally working safety unit.
MTTFrs	Robot-safety system mean time to failure with the normally working safety unit.

MTTFr Robot-safety system mean time to failure with or without the normally working safety unit.

4.2 Generalized Robot-Safety System Analysis

By using the supplementary method [71, 72], we write the following Equations for the Figure 4.2 diagram:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_r P_1(t) + \sum_{j=2n}^{2n+1} P_j(x,t)\mu_j(x)dx \tag{4.1}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_r P_{i-1}(t) + \mu_r P_{i+1}(t) \tag{4.2}$$

(for $i = 1, 2 \ldots\ldots$ n-2)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = \lambda_r P_{n-2}(t) \tag{4.3}$$

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_0(t) + \mu_r P_{n+1}(t) \tag{4.4}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) + \mu_r P_{i+1}(t) \tag{4.5}$$

(for i = n+1,n+2....,2n-2)

$$\frac{dP_{2n-1}(t)}{dt} + a_{2n-1} P_{2n-1}(t) = \lambda_r P_{2n-2}(t) + \lambda_s P_{n-1}(t) \tag{4.6}$$

where

$a_0 = \lambda_r + \lambda_s$

$a_i = \lambda_r + \lambda_s + \mu_r$ (for $i = 1, 2 \ldots\ldots$ n-1)

$a_n = \lambda_r$

$a_i = \lambda_r + \mu_r$ (for i = n+1, n+2........2n-1)

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x)\, P_j(x,t) = 0 \qquad (\text{for } j = 2n, 2n+1) \tag{4.7}$$

The associated boundary conditions are as follows:

$$P_{2n}(0,t) = \lambda_r P_{2n-1}(t) \tag{4.8}$$

$$P_{2n+1}(0,t) = \lambda_r P_{n-1}(t) \tag{4.9}$$

At time t = 0, $P_0(0) = 1$, and other initial state probabilities are equal to zero.

4.3 Generalized Robot-Safety System Time Dependent Analysis

By solving Equations (4.1) – (4.9) with the Laplace transform method *, we get the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{j=2n}^{2n+1} a_j(s)\frac{1-W_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{4.10}$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad \text{for } i = 1, 2 \ldots\ldots\ldots \text{n-1}) \tag{4.11}$$

$$P_i(s) = V_i(s)\, P_0(s) \qquad (\text{for } i = n, n+1 \ldots\ldots\ldots 2n-1) \tag{4.12}$$

$$P_j(s) = a_j(s)\frac{1-W_j(s)}{s}\, P_0(s) \qquad (\text{for } j = 2n, 2n+1) \tag{4.13}$$

where

$$Y_i(s) = \prod_{k=1}^{i} L_k(s) \qquad (\text{for } i = 1, 2 \ldots\ldots\ldots n-1)$$

$$L_{n-1}(s) = \frac{\lambda_r}{s + a_{n-1}}$$

$$L_i(s) = \frac{\lambda_r}{(s + a_i) - \mu_r L_{i+1}(s)} \qquad (\text{for } i = 1, 2 \ldots\ldots\ldots n-2)$$

* : Please see Appendix C.1.

$$\mathrm{V}_n(\mathrm{s}) = \frac{\lambda_s}{L_n(s)} + \sum_{i=1}^{n-1} \mu_r^i \frac{\lambda_s}{\prod_{k=n}^{n+i} L_k(s)} Y_i(s)$$

$$\mathrm{V}_i(\mathrm{s}) = \frac{\lambda_r}{L_i(s)} \mathrm{V}_{i-1}(\mathrm{s}) + \sum_{h=i-n}^{n-1} \mu_r^{h-i+n} \frac{\lambda_s}{\prod_{k=i}^{h+n} L_k(s)} Y_h(s)$$

(for i = n+1,n+2.......2n-1)

$$\mathrm{L}_{2n-1}(\mathrm{s}) = \mathrm{s} + \mathrm{a}_{2n-1}$$

$$\mathrm{L}_i(\mathrm{s}) = (\mathrm{s} + \mathrm{a}_i) - \mu_r \frac{\lambda_r}{L_{i+1}(s)} \qquad \text{(for } i = \text{n, n+1........2n-2)}$$

$$\mathrm{a}_{2n}(\mathrm{s}) = \lambda_r \mathrm{V}_{2n-1}(\mathrm{s})$$

$$\mathrm{a}_{2n+1}(\mathrm{s}) = \lambda_r \mathrm{Y}_{n-1}(\mathrm{s})$$

$$\mathrm{G}(\mathrm{s}) = \mathrm{s}[1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{j=2n}^{2n+1} a_j(s) \frac{1 - W_j(s)}{s}] \qquad (4.14)$$

$$\mathrm{W}_j(\mathrm{s}) = \int_0^\infty e^{-sx} \mathrm{w}_j(\mathrm{x})\mathrm{dx} \qquad \text{(for } j = \text{n+1,n+2,n+3)} \qquad (4.15)$$

$$\mathrm{w}_j(\mathrm{x}) = \exp[-\int_0^x \mu_j(\delta) d\delta\,]\ \mu_j(x)$$

The Laplace transform of the robot-safety system availability with the normally working safety unit is given by:

$$\mathrm{AV}_{rs}(\mathrm{s}) = \sum_{i=0}^{n-1} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s)}{G(s)} \qquad (4.16)$$

The Laplace transform of the robot-safety system availability with or without the working safety unit is given by:

$$\mathrm{AV}_r(s)=\sum_{i=0}^{2n-1}P_i(s)=\frac{1+\sum_{i=1}^{n-1}Y_i(s)+\sum_{i=n}^{2n-1}V_i(s)}{G(s)} \tag{4.17}$$

Taking the inverse Laplace transforms of the above equations, we can get the time dependent state probabilities, P_i (t)and P_j (t), and system availabilities, AVrs(t) and AVr(t).

4.3.1 Robot-Safety System Time Dependent Analysis for a Special Case

For two robots (i.e., one working, another one on standby) by substituting n=2 into Equations (4.10)-(4.17), we obtain

$$P_0(s)=\frac{1}{s[1+Y_1(s)+\sum_{i=2}^{3}V_i(s)+\sum_{j=4}^{5}a_j(s)\frac{1-W_j(s)}{s}]} \tag{4.18}$$

$$P_1(s)=Y_1P_0(s) \tag{4.19}$$

$$P_i(s)=V_i(s)P_0(s) \qquad (\text{ for } i=2,\ 3) \tag{4.20}$$

$$P_j(s)=a_j(s)\frac{1-W_j(s)}{s} \qquad (\text{for } j=\ 4,\ 5) \tag{4.21}$$

where

$$Y_1(s)=\frac{\lambda_r}{s+a_1}$$

$$V_2(s)=\frac{\lambda_s}{L_2(s)}+\mu_r\frac{\lambda_s}{L_2(s)L_3(s)}Y_1(s)$$

$$V_3(s)=\frac{\lambda_s}{L_3(s)}Y_1(s)+\frac{\lambda_r}{L_3(s)}V_2(s)$$

$$L_3(s)=s+a_3$$

$$L_2(s) = (s + a_2) - \mu_r \frac{\lambda_r}{L_3(s)}$$

$$a_4(s) = \lambda_r V_3(s)$$

$$a_5(s) = \lambda_r Y_1(s)$$

$$G(s) = s[\, 1 + Y_1(s) + \sum_{i=2}^{3} V_i(s) + \sum_{j=4}^{5} a_j(s) \frac{1 - W_j(s)}{s} \,] \tag{4.22}$$

The Laplace transform of the robot-safety system availability with the normally working safety unit is given by:

$$AV_{rs}(s) = \sum_{i=0}^{1} P_i(s) = \frac{1 + Y_1(s)}{G(s)} \tag{4.23}$$

The Laplace transform of the robot-safety system availability when the robot working normally with or without the normally working safety unit is expressed by:

$$AV_r(s) = \sum_{i=0}^{3} P_i(s) = \frac{1 + Y_1(s) + \sum_{i=2}^{3} V_i(s)}{G(s)} \tag{4.24}$$

Taking the inverse Laplace transforms of the above equations, we can get the time dependent state probabilities, $P_i(t)$and $P_j(t)$, and system availabilities, AVrs(t) and AVr(t).

Thus, for the failed robot-safety system exponentially distributed repair time, the probability density function is expressed by

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0, j = 4,5) \tag{4.25}$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Substituting Equation (4.25) into Equation (4.15), we get

$$W_j(s) = \frac{\mu_j}{s+\mu_j} \qquad (\mu_j > 0, j = 4, 5) \tag{4.26}$$

By inserting Equation (4.26) into Equations (4.18) – (4.25), setting $\lambda_s = 0.0009$, $\lambda_r = 0.002$, $\mu_r = 0.001$, $\mu_3 = 0.00015$, $\mu_4 = 0.0002$; and using Matlab computer program [74], the Figure 4.3 and Figure 4.4 plots were obtained. These plots show that state probabilities and system availabilities decrease and increase with varying time t.

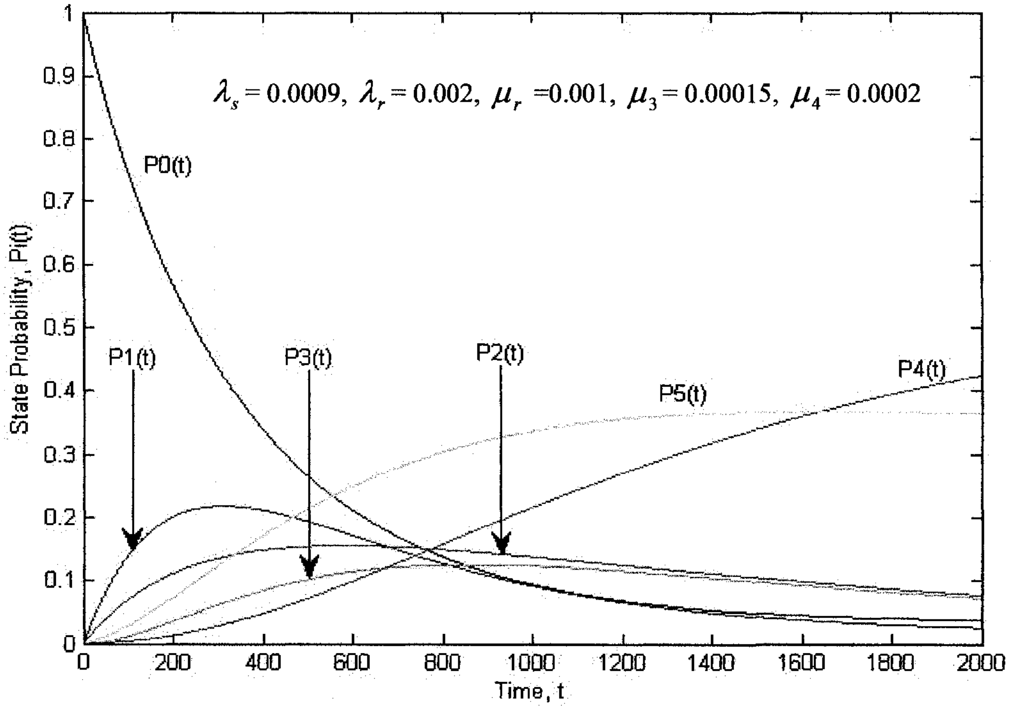


Figure 4.3 Time-dependent state probability plots for a robot-safety system with exponentially distributed failed system repair times.

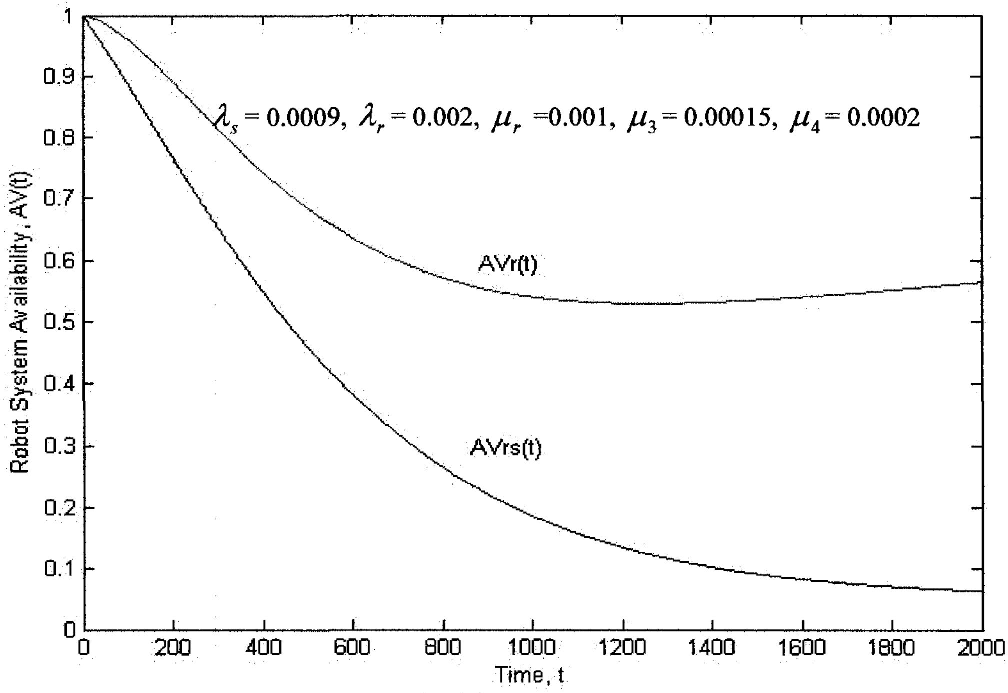


Figure 4.4 Time-dependent availability plots for a robot safety system with exponentially distributed failed system repair times.

4.4 Generalized Robot-Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (4.1)-(4.9) we get:

$$a_0 P_0 = \mu_r P_1 + \sum_{j=2n}^{2n+1} P_j(x)\mu_j(x)dx \tag{4.27}$$

$$a_i P_i = \lambda_r P_{i-1}(s) + \mu_r P_{i+1} \tag{4.28}$$

(for i = 1,2,........,n-2)

$$a_{n-1} P_{n-1} = \lambda_r P_{n-2} \tag{4.29}$$

$$a_n P_n = \lambda_s P_0 + \mu_r P_{n+1} \tag{4.30}$$

$$a_i P_i = \lambda_r P_{i-1} + \lambda_s P_{i-n} + \mu_r P_{i+1} \tag{4.31}$$

(for i = n+1, n+2....2n-2)

$$a_{2n-1}P_{2n-1} = \lambda_r P_{2n-2} + \lambda_s P_{n-1} \qquad (4.32)$$

$$\frac{dP_j(x)}{dx} + \mu_j(x)P_j(x) = 0 \qquad (\text{ for } j = 2n, 2n+1) \qquad (4.33)$$

The associated boundary conditions are as follows:

$$P_{2n}(0) = \lambda_r P_{2n-1} \qquad (4.34)$$

$$P_{2n+1}(0) = \lambda_r P_{n-1} \qquad (4.35)$$

Solving equation (4.27) – (4.33) *, together with

$$\sum_{i=0}^{2n-1} P_i + \sum_{j=2n}^{2n+1} P_j = 1 \qquad (4.36)$$

we obtain:

$$P_0 = (1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j E_j[x])^{-1} = \frac{1}{G} \qquad (4.37)$$

$$P_i = Y_i\, P_0 \qquad (\text{for } i = 1, 2.........n\text{-}1) \qquad (4.38)$$

$$P_i = V_i\, P_0 \qquad (\text{for } i = n, n+1......2n\text{-}1) \qquad (4.39)$$

$$P_j = a_j E_j[x]\, P_0 \qquad (\text{for } j = 2n, 2n+1) \qquad (4.40)$$

where

$$Y_i = \prod_{k=1}^{i} L_k \qquad (\text{for } i = 1, 2.........n\text{-}1)$$

$$L_{n-1} = \lim_{s \to 0} L_{n-1}(s) = \frac{\lambda_r}{a_{n-1}}$$

$$L_i = \lim_{s \to 0} L_i(s) = \frac{\lambda_r}{a_i - \mu_r L_{i+1}} \qquad (\text{for } i = 1, 2........n\text{-}2)$$

* : Please see Appendix C.2.

$$\mathrm{V}_n = \frac{\lambda_s}{L_n} + \sum_{i=1}^{n-1} \mu_r^i \frac{\lambda_s}{\prod_{k=n}^{n+i} L_k} Y_i$$

$$\mathrm{V}_i = \frac{\lambda_r}{L_i} \mathrm{V}_{i-1} + \sum_{h=i-n}^{n-1} \mu_r^{h-i+n} \frac{\lambda_s}{\prod_{k=i}^{h+n} L_k} Y_h$$

(for i = n+1, n+2.......2n-1)

$$\mathrm{L}_{2n-1} = \lim_{s \to 0} \mathrm{L}_{2n-1}(\mathrm{s}) = \mathrm{a}_{2n-1}$$

$$\mathrm{L}_i = \lim_{s \to 0} \mathrm{L}_i(\mathrm{s}) = \mathrm{a}_i - \mu_r \frac{\lambda_r}{L_{i+1}}$$

(for i = n, n+1........2n-2)

$$\mathrm{a}_{2n} = \lambda_r \mathrm{V}_{2n-1}$$

$$\mathrm{a}_{2n+1} = \lambda_r \mathrm{Y}_{n-1}$$

$$\mathrm{G} = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j E_j[x] \tag{4.41}$$

The generalized steady state availability of the robot safety system with the normally working safety unit is given by

$$\mathrm{SSAVrs} = \sum_{i=0}^{n-1} P_i = \frac{1 + \sum_{i=1}^{n-1} Y_i}{G} \tag{4.42}$$

The generalized steady state availability of the robot safety system with or without the working safety units is given by

$$\mathrm{SSAVr} = \sum_{i=0}^{2n-1} P_i = \frac{1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i}{G} \tag{4.43}$$

For different failed robot-safety system repair time distributions, we get different expressions for G as follows:

i) For the failed robot-safety system gamma distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{\mu_j^{\beta} x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \qquad (\beta > 0,\ j = 2n,\ 2n+1) \tag{4.44}$$

where

x is the repair time variable, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^{\infty} x w_j(x) dx = \frac{\beta}{\mu_j} \qquad (\beta > 0,\ j = 2n,\ 2n+1) \tag{4.45}$$

Substituting Equation (4.45) into Equation (4.41), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j \frac{\beta}{\mu_j} \tag{4.46}$$

ii) For the failed robot-safety system Weibull distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j (x)^{\beta}} \qquad (\beta > 0,\ j = 2n,\ 2n+1) \tag{4.47}$$

where

x is the repair time variable, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^{\infty} x W_j(x) dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \quad (\beta > 0, j = 2n,\ 2n+1) \tag{4.48}$$

Substituting Equation (4.48) into Equation (4.41), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j (\frac{1}{\mu_j})^{1/\beta} \frac{1}{\beta} \Gamma(\frac{1}{\beta}) \qquad (4.49)$$

iii) For the failed robot-safety system Rayleigh distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j x e^{-\mu_j x^2/2} \qquad (\mu_j > 0,\ j = 2n, 2n+1) \qquad (4.50)$$

where

x is the repair time variable, μ_j is the scale parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^\infty x W_j(x) dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (\mu_j > 0,\ j = 2n, 2n+1) \qquad (4.51)$$

Substituting Equation (4.51) into Equation (4.41), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j \sqrt{\frac{\pi}{2\mu_j}} \qquad (4.52)$$

iv) For the failed robot system lognormal distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{1}{\sqrt{2\pi} x \sigma_{y_j}} e^{[\frac{-(Inx - \mu_{y_j})^2}{2\sigma_{y_j}^2}} \qquad (\text{for } j = 2n, 2n+1) \qquad (4.53)$$

where

x is the repair time variable, Inx is the natural logarithm of x with a mean μ and variance σ^2. The conditions on parameters are:

$$\sigma_{y_j} = \text{In} \sqrt{1 + (\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \qquad (4.54)$$

$$\mu_{y_j} = \text{In}\sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \qquad (4.55)$$

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (\text{for } j = 2n,\ 2n+1) \qquad (4.56)$$

Substituting Equation (4.56) into Equation (4.41), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (4.57)$$

v) For the failed robot system exponentially distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 2n,\ 2n+1) \qquad (4.58)$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^\infty x w_j(x) dx = \frac{1}{\mu_j} \qquad (\beta > 0,\ j = 2n,\ 2n+1) \qquad (4.59)$$

Substituting Equation (4.59) into Equation (4.41), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{j=2n}^{2n+1} a_j \frac{1}{\mu_j} \qquad (4.60)$$

4.4.1 Robot-Safety System Steady State Analysis for a Special Case

For two robots (i.e. one working, another one on standby) by substituting n=2 into Equations (4.37)-(4.43), we obtain:

$$P_0 = \frac{1}{s[1 + Y_1 + \sum_{i=2}^{3} V_i + \sum_{j=4}^{5} a_j E_j[x]]} \tag{4.61}$$

$$P_1 = Y_1 P_0 \tag{4.62}$$

$$P_i = V_i P_0 \qquad \text{(for } i = 2, 3) \tag{4.63}$$

$$P_j = a_j E_j [x] P_0 \qquad \text{(for } j = 4, 5) \tag{4.64}$$

where

$$Y_1 = \frac{\lambda_r}{a_1}$$

$$V_2 = \frac{\lambda_s}{L_2} + \mu_r \frac{\lambda_s}{L_2 L_3} Y_1$$

$$V_3 = \frac{\lambda_s}{L_3} Y_1 + \frac{\lambda_r}{L_3} V_2$$

$$L_3 = a_3$$

$$L_2 = a_2 - \mu_r \frac{\lambda_r}{L_3}$$

$$a_4 = \lambda_r V_3$$

$$a_5 = \lambda_r Y_1$$

$$G(s) = 1 + Y_1 + \sum_{i=2}^{3} V_i + \sum_{j=4}^{5} a_j E_j[x] \tag{4.65}$$

The steady state availability of the robot-safety system with the normally working safety unit is given by:

$$AV_{rs} = \sum_{i=0}^{1} P_i = \frac{1 + Y_1}{G} \tag{4.66}$$

The steady state availability of the robot-safety system with or without a normally working safety unit is given by:

$$AV_r(s) = \sum_{i=0}^{3} P_i(s) = \frac{1 + Y_1 + \sum_{i=2}^{3} V_i}{G} \tag{4.67}$$

1) For exponentially distributed failed robot-safety system repair time, substituting Equation (4.60) into Equations (4.61)-(4.67), setting:

$\lambda_r = 0.002$, $\mu_r = 0.001$, $\mu_3 = 0.00015$, $\mu_4 = 0.0002$; and using Matlab computer program [74], the Figures 4.5 and 4.6 plots were obtained.

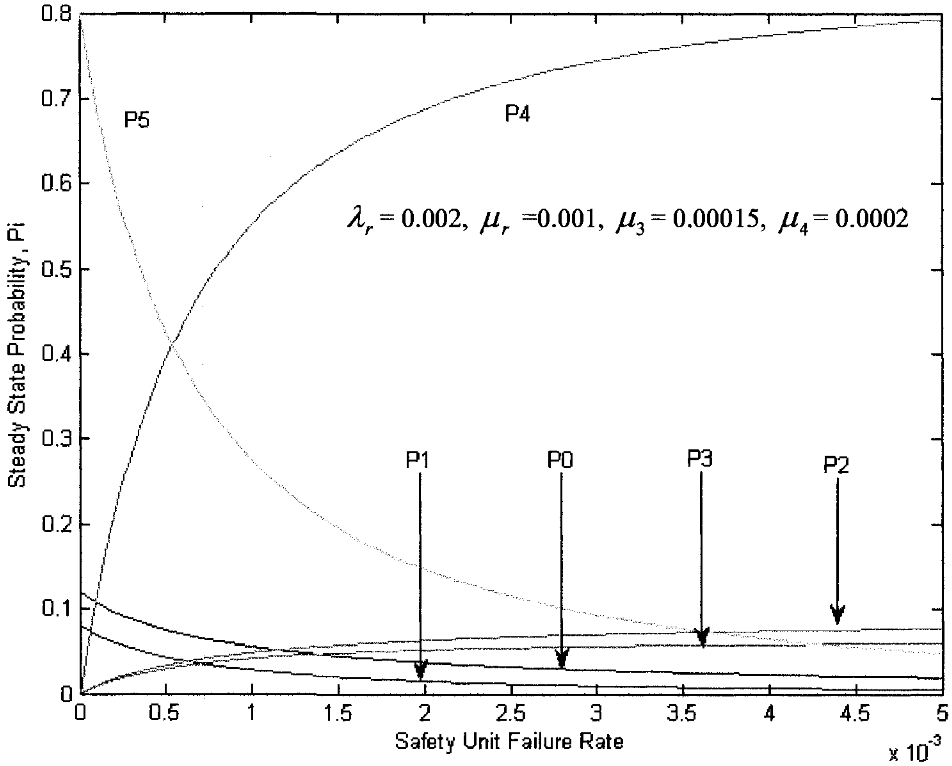


Figure 4.5 Robot-safety system steady state probability versus safety unit failure rate (λ_s) plots with exponentially distributed failed system repair times.

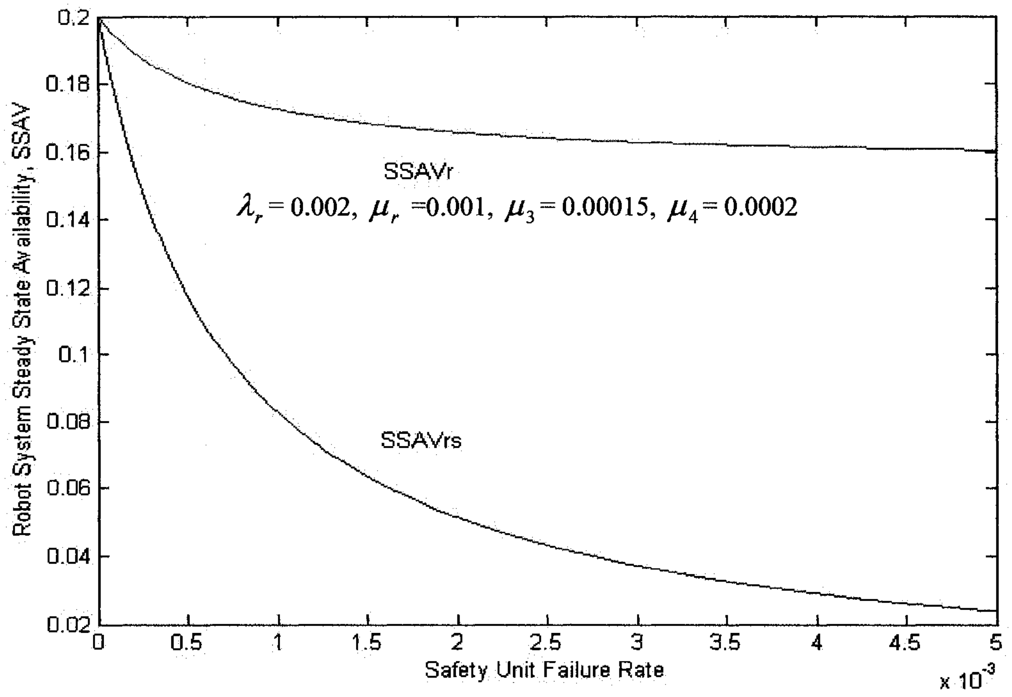


Figure 4.6 Robot system steady state availability versus safety unit failure rate(λ_s) plots with exponentially distributed failed system repair times.

2) For Rayleigh distributed failed robot-safety system repair time, substituting Equation (4.52) into Equations (4.61)-(4.67), setting:

$\lambda_r = 0.002$, $\mu_r = 0.001$, $\mu_3 = 0.00015$, $\mu_4 = 0.0002$; and using Matlab computer program [74], the Figure 4.7 and 4.8 plots were obtained.

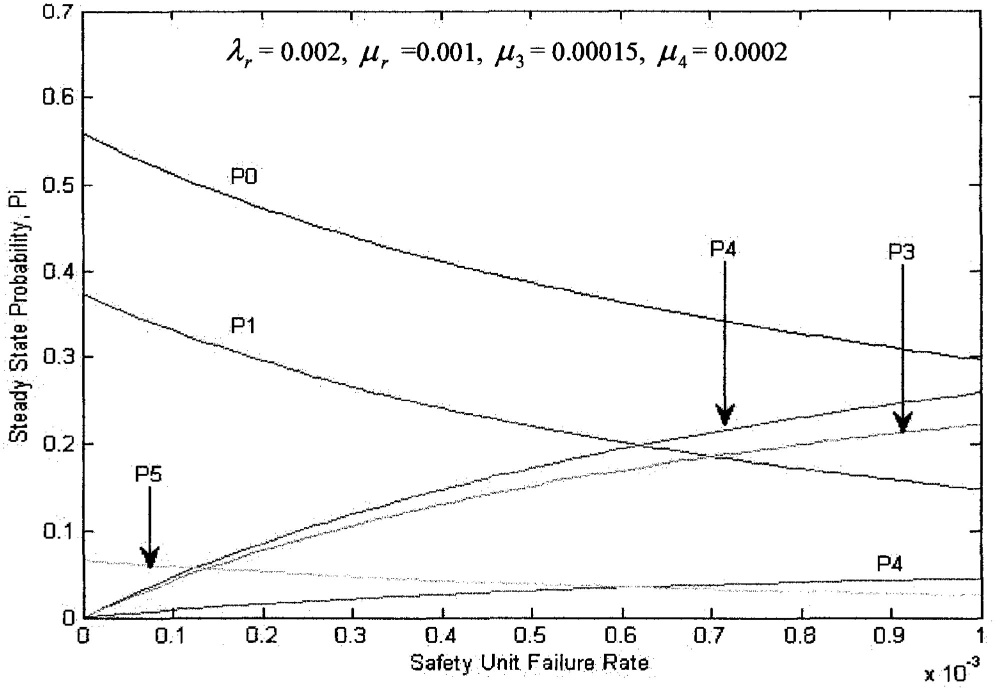


Figure 4.7 Robot-safety system steady state probability versus safety unit failure rate(λ_s) plots with Rayleigh distributed failed system repair times.

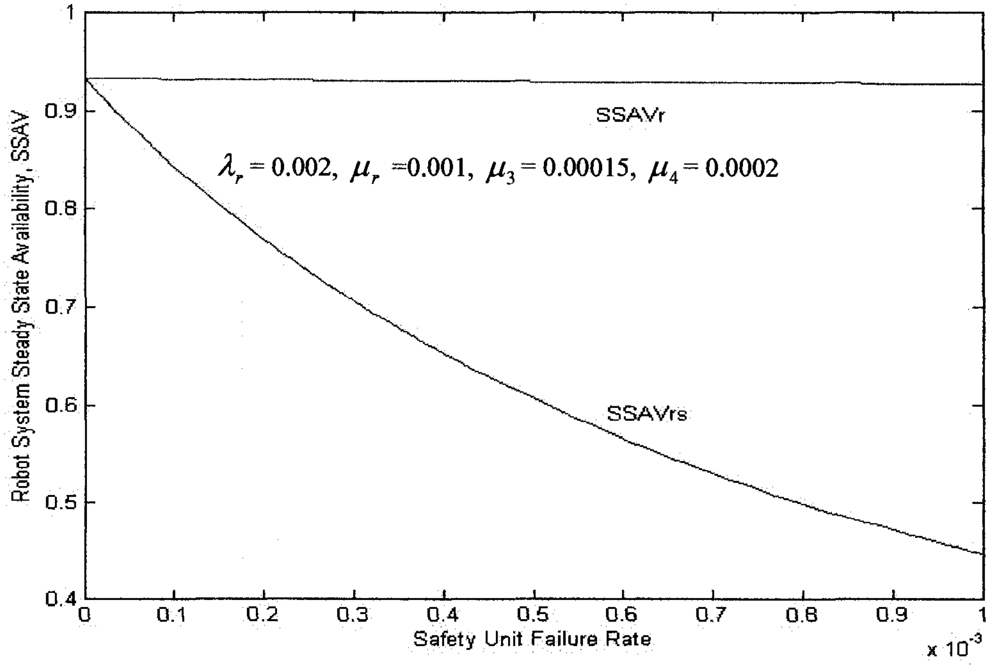


Figure 4.8 Robot-safety system steady state availability versus safety unit failure rate (λ_s) plots with Rayleigh distributed failed system repair times.

3) For lognormally distributed failed robot-safety system repair time, substituting Equation (4.57) into Equations (4.61)-(4.67), setting:

$\lambda_r = 0.002$, $\mu_r = 0.001$, $\mu_3 = 0.00015$, $\mu_4 = 0.0002$, $\sigma = 4$; and using Matlab computer program [74], we obtained the following plots (Figure 4.9, Figure 4.10):

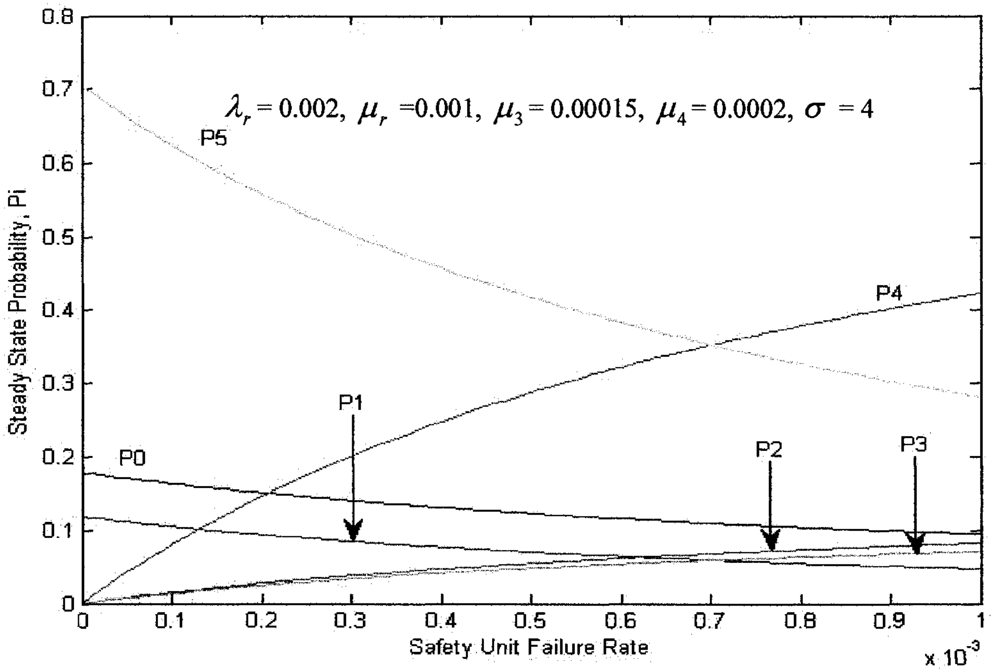


Figure 4.9 Robot-safety system steady state probability versus safety unit failure rate (λ_s) plots with lognormally distributed failed system repair times.

4.5 Robot Safety System Reliability and Mean Time to Failure Analysis

Setting $\mu_j = 0$, (for $j = 2n, 2n+1$), in Figure 4.2 and using the Markov method [73], we get the following equations:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_r P_1(t) \qquad (4.68)$$

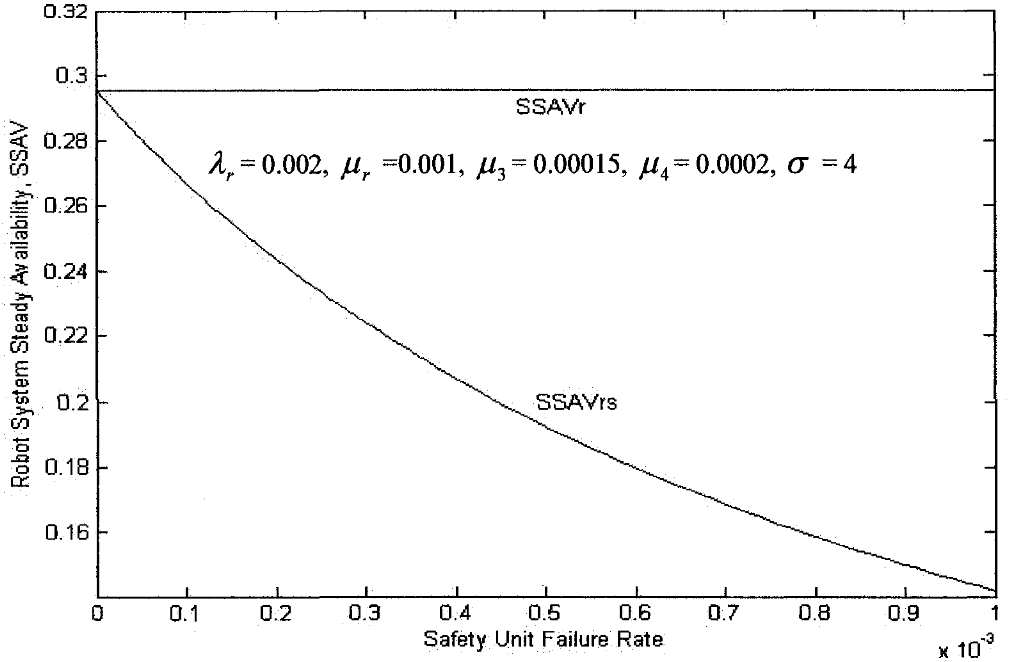


Figure 4.10 Robot-safety system steady state availability versus safety unit failure rate (λ_s) plots with lognormally distributed failed system repair times.

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_r P_{i-1}(t) + \mu_r P_{i+1}(t) \tag{4.69}$$

(for $i = 1, 2 \ldots\ldots n-2$)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = \lambda_r P_{n-2}(t) \tag{4.70}$$

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_0(t) + \mu_r P_{n+1}(t) \tag{4.71}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) + \mu_r P_{i+1}(t) \tag{4.72}$$

(for $i = n+1, n+2 \ldots, 2n-1$)

$$\frac{dP_{2n-1}(t)}{dt} + a_{2n-1} P_{2n-1}(t) = \lambda_r P_{2n-2}(t) + \lambda_s P_{n-1}(t) \tag{4.73}$$

where

$$a_0 = \lambda_r + \lambda_s$$

$$a_i = \lambda_r + \lambda_s + \mu_r \quad \text{(for } i = 1, 2........\text{n-1)}$$

$$a_n = \lambda_r$$

$$a_i = \lambda_r + \mu_r \quad \text{(for } i = \text{n, n+1........2n-1)}$$

$$\frac{dP_{2n}(t)}{dt} = \lambda_r \mathrm{P}_{2n-1}(\mathrm{t}) \tag{4.74}$$

$$\frac{dP_{2n+1}(t)}{dt} = \lambda_r \mathrm{P}_{n-1}(\mathrm{t}) \tag{4.75}$$

At time t = 0, $\mathrm{P}_0(0) = 1$ and other initial conditions are equal to zero.

By solving Equations (4.68)-(4.75) with the Laplace transform method * , together with

$$\sum_{i=0}^{2n-1} P_i(s) + \sum_{j=2n}^{2n+1} P_j(s) = \frac{1}{s}, \tag{4.76}$$

We obtain the general Laplace transforms of state probabilities:

$$\mathrm{P}_0(\mathrm{s}) = [\mathrm{s}(1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{j=2n}^{2n+1} \frac{a_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{4.77}$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{Y}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad \text{(for } i = 1,2.........\text{n-1)} \tag{4.78}$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{V}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad \text{(for } i = \text{n, n+1......2n-1)} \tag{4.79}$$

$$\mathrm{P}_j(\mathrm{s}) = \frac{a_j(s)}{s}\mathrm{P}_0(\mathrm{s}) \qquad \text{(for } j = \text{2n, 2n+1)} \tag{4.80}$$

where

$$\mathrm{Y}_i(\mathrm{s}) = \prod_{k=1}^{i} L_k(s) \qquad \text{(for } i = 1, 2.........\text{n-1)}$$

---

* : Please see Appendix C.3.

$$L_{n-1}(s) = \frac{\lambda_r}{s + a_{n-1}}$$

$$L_i(s) = \frac{\lambda_r}{(s + a_i) - \mu_r L_{i+1}(s)} \qquad (\text{for } i = 1, 2 \ldots\ldots\ldots n\text{-}2)$$

$$V_n(s) = \frac{\lambda_s}{L_n(s)} + \sum_{i=1}^{n-1} \mu_r^i \frac{\lambda_s}{\prod_{k=n}^{n+i} L_k(s)} Y_i(s)$$

$$V_i(s) = \frac{\lambda_r}{L_i(s)} V_{i-1}(s) + \sum_{h=i-n}^{n-1} \mu_r^{h-i+n} \frac{\lambda_s}{\prod_{k=i}^{h+n} L_k(s)} Y_h(s)$$

$$(\text{for } i = n+1, n+2 \ldots\ldots\ldots 2n\text{-}1)$$

$$L_{2n-1}(s) = s + a_{2n-1}$$

$$L_i(s) = (s + a_i) - \mu_r \frac{\lambda_r}{L_{i+1}(s)} \qquad (\text{for } i = n, n+1 \ldots\ldots\ldots 2n\text{-}2)$$

$$a_{2n}(s) = \lambda_r V_{2n-1}(s)$$

$$a_{2n+1}(s) = \lambda_r Y_{n-1}(s)$$

$$G(s) = s[1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{j=2n}^{2n+1} \frac{a_j(s)}{s}] \tag{4.81}$$

The Laplace transform of the robot-safety system reliability with the normally working safety unit is given by:

$$R_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s)}{G(s)} \tag{4.82}$$

The Laplace transform of the robot safety system reliability with or without the working safety unit is expressed by:

$$\mathrm{R}_r\ (\mathrm{s}) = \sum_{i=0}^{2n-1} P_i(s) = \frac{1+\sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s)}{G(s)} \qquad (4.83)$$

Using Equation (4.81), the robot-safety system mean time to failure with the normally working safety unit is obtained:

$$\mathrm{MTTF}_{rs} = \lim_{s \to 0} \mathrm{R}_{rs}(\mathrm{s}) = \frac{1+\sum_{i=1}^{n-1} Y_i}{\sum_{j=2n}^{2n+1} a_j} \qquad (4.84)$$

Similarly, using Equation (4.83), the robot-safety system mean time to failure with or without the working safety unit is given by:

$$\mathrm{MTTF}_r = \lim_{s \to 0} \mathrm{R}_r(\mathrm{s}) = \frac{1+\sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i}{\sum_{j=2n}^{2n+1} a_j} \qquad (4.85)$$

4.5.1 Robot-Safety System Mean Time to Failure Analysis for a Special case

Substituting n = 2 into Equations (4.84) and (4.85), we get the followings:

The robot-safety system mean time to failure with the normally working safety unit is

$$\mathrm{MTTF}_{rs} = \lim_{s \to 0} R_{rs} = \frac{1+Y_1}{\sum_{j=4}^{5} a_j} \qquad (4.86)$$

The robot safety system mean time to failure with or without the working safety unit is

$$\mathrm{MTTF}_r = \lim_{s \to 0} R_r = \frac{1+Y_1+\sum_{i=2}^{3} V_i}{\sum_{j=4}^{5} a_j} \qquad (4.87)$$

For λ_r= 0.002, μ_r =0.001; and using Equations (4.86) and (4.87), the Figure 4.11 was obtained.

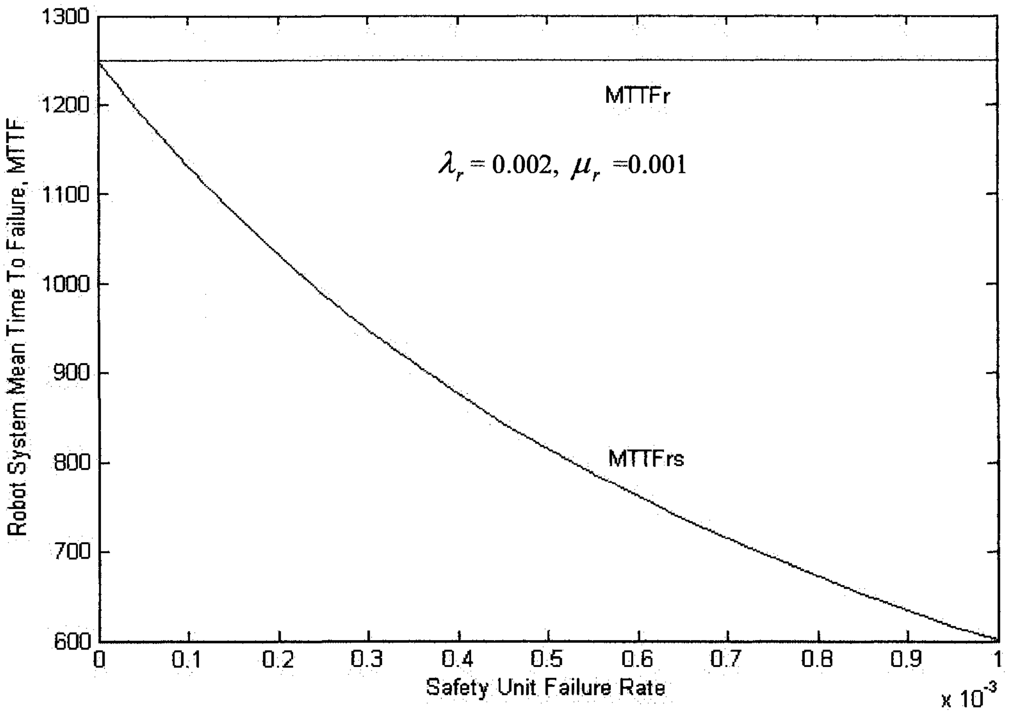


Figure 4.11 The robot-safety system mean time to failure plots for the increasing values of the safety unit failure rate(λ_s).

4.6 Conclusions

- For exponentially distributed failed robot system repair time x, system time dependent availability with the normally working safety unit decreases as the time increases.
- For exponentially, Rayleigh or lognormally distributed failed robot system repair time x, the system steady state availability decreases with the increasing safety unit failure rate.
- For exponentially distributed failed robot system repair time x, the robot-safety system mean time to failure (MTTF) decreases with the increasing values of the safety unit failure rate.

Chapter 5

Stochastic Analysis of a System containing (N-1) Standby Robots and One Safety Unit with an Imperfect Switch

5.1 Introduction

In the previous chapter, the model of a robot-safety system with a perfect switch is presented. This chapter presents a mathematical model to perform state probabilities, system availability and mean time to failure for the robot-safety system containing (n-1) standby robots, one safety unit and an imperfect switch. More specifically, the robot-safety system is composed of n identical robots, one safety unit and a switch to replace a failed safety unit.

The block diagram of the robot system is shown in Figure 5.1 and its corresponding state space diagram is presented in Figure5.2. The numerals and letter n in the boxes of Figure5.2 denotes system state.

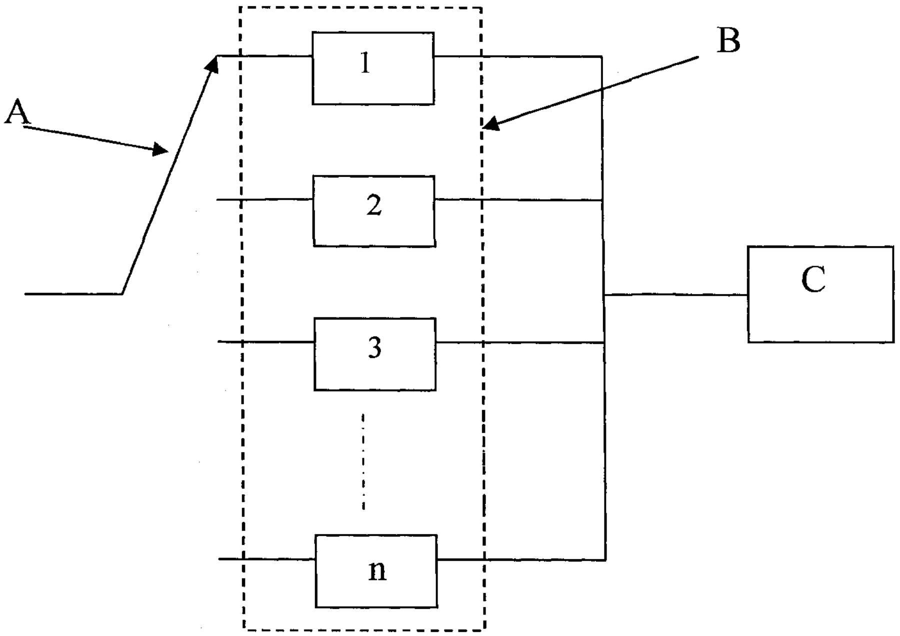
At time t =0, one robot, safety unit and the switch to replace a failed robot start operating and n-1 robots are on standby. The overall robot-safety system can fail the following two ways:

- All the robots fail with the normally working safety unit.
- All the robots fail with the failed safety unit.

The following assumptions are associated with this model:

- The robot-safety system is composed of n identical robots (only one operates and the rest remain on standby), one safety unit and a switch.
- One robot, switch, and safety unit start operating simultaneously.

- The completely failed robot-safety system and its individually failed units (i.e. robot, switch, and safety unit) can be repaired. Failure and repair rates of robots, switch and safety units are constant.
- The failed robot-safety system repair rates can be constant or non-constant.
- The switch has the priority to be repaired when the switch and the safety unit /or anyone robot failed simultaneously.



A : Switch for replacing a failed robot

B : n identical Robots (one operating and n-1 on standby)

C: Safety Unit

Figure 5.1 The block diagram of the robot-safety system containing (n-1) standby robots and one safety unit with an imperfect switch

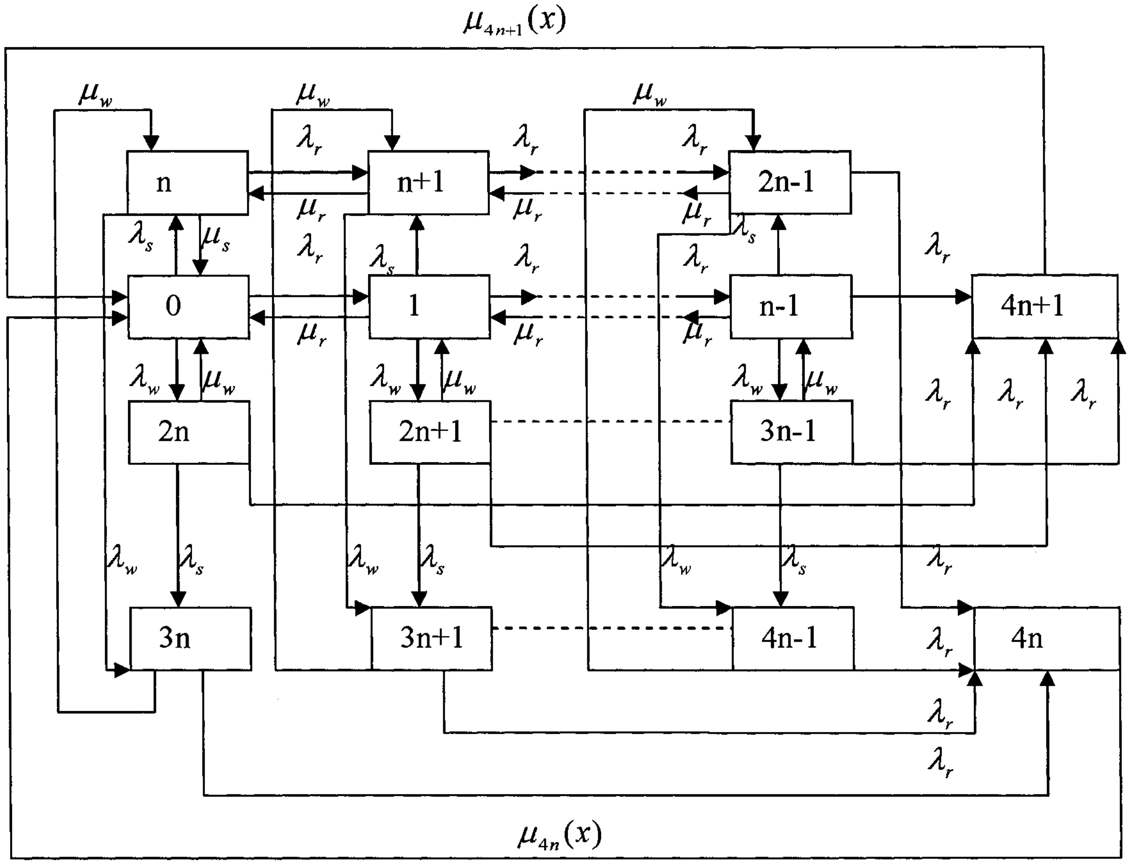


Figure 5.2 The state space diagram of the robot-safety system containing (n-1) standby robots and one robot with an imperfect switch

- A robot has the priority to be repaired when anyone robot and the safety unit failed simultaneously.
- All failures are statistically independent.
- A repaired robot, safety unit or the total robot-safety system is as good as new.

5.1.1 Notation:

The following symbols are associated with the model:

i i^{th} state of the robot-safety system:

for $i = 0$, means one robot, the switch and the safety unit are working normally;

for $i = 1$, means one robot, the switch, the safety unit are working normally and one robot has failed;

for $i = k$, means one robot, the switch, the safety unit are working normally and k robots have failed (i.e., k = 2,3......n-1);

for $i = n$, means one robot work, the switch are working normally and the safety unit has failed;

for $i = k$, means one robot and the switch are working normally while h-n robots and the safety unit have failed (i.e., k = n+1, n+2,...... 2n-1);

for $i = 2n$, means one robot and the safety unit are working normally while the switch has failed;

for $i = f$, means one robot, and the safety unit are working normally while f-2n robots and the switch have failed (i.e., f = 2n+1, 2n+2......3n-1);

for $i = 3n$, means one robot is working normally while the safety unit and the switch have failed;

for $i = g$, means one robot is working normally while g-3n robots, the switch and the safety unit have failed (i.e., g = 3n+1, 3n+2........4n-1).

j j^{th} state of the robot–safety system:

for $j = 4n$, means the total robot-safety system has failed (i.e., all the robots and the safety unit have failed);

for $j = 4n+1$, means the robot-safety system has failed (i.e., all the robot has failed while the safety is still working normally.)

t	Time.
λ_s	Constant failure rate of the safety unit.
λ_r	Constant failure rate of a robot.
λ_w	Constant failure rate of the switch
μ_s	Constant repair rate of the safety unit.
μ_r	Constant repair rate of a robot
μ_w	Constant repair rate of the switch
$\Delta \mathbf{x}$:	Finite repair time interval.
$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in state j and has an elapsed repair time of x; for j = 2n, 2n+1.
$P_j(x,t)\ \Delta x$	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x, x+Δx]; for j = 2n, 2n+1 .
pdf	Probability density function.
w_j(x)	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j = 4n, 4n+1.
P_j(t)	Probability that the robot safety system is in state j at time t; for j = 4n, 4n+1.
P_i(t)	Probability that the robot-safety system is in state i at time t; for i = 0,1,2...4n-1.
P_i	Steady state probability that the robot-safety system is in state i; for i=0,1,..4n-1.
P_j	Steady state probability that robot-safety system is in state j; for j = 4n, 4n+1.
s	Laplace transform variable.
P_i(s)	Laplace transform of the probability that the robot-safety system is in state i;

	for $i = 0, 1, 2 \ldots 4n-1$.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for $j = 4n, 4n+1$.
AVrs(s)	Laplace transform of the robot-safety system availability with the normally working safety unit.
AVr(s)	Laplace transform of the robot-safety system availability with or without the normally working safety unit.
AVrs (t)	Robot-safety system time dependent availability with the normally working safety unit.
AVr(t)	Robot-safety system time dependent availability with or without the normally working safety unit.
SSAVrs	Robot-safety system steady state availability with the normally working safety unit, the switch and the robot.
SSAVr	Robot-safety system steady state availability with or without the normally working safety unit.
Rrs(s)	Laplace transform of the robot-safety system reliability with the normally working safety unit.
Rr(s)	Laplace transform of the robot safety system reliability with or without the normally working safety unit.
MTTFrs	Robot-safety system mean time to failure with the normally working safety unit.
MTTFr	Robot-safety system mean time to failure with or without the normally working safety unit.

5.2 Generalized Robot-Safety System Analysis

By using the supplementary method [71, 72], we write down the following equations for the Figure 5.2 diagram:

$$\frac{dP_0(t)}{dt} + \mathrm{a}_0\mathrm{P}_0(\mathrm{t}) = \mu_r\mathrm{P}_1(\mathrm{t}) + \mu_s\mathrm{P}_n(\mathrm{t}) + \mu_w\mathrm{P}_{2n}(\mathrm{t}) + \sum_{j=4n}^{4n+1} P_j(x,t)\mu_j(x)dx \tag{5.1}$$

$$\frac{dP_i(t)}{dt} + \mathrm{a}_i\mathrm{P}_i(\mathrm{t}) = \lambda_r\mathrm{P}_{i-1}(\mathrm{t}) + \mu_r\mathrm{P}_{i+1}(\mathrm{t}) + \mu_w\mathrm{P}_{2n+i}(\mathrm{t}) \tag{5.2}$$

(for i = 1,2.........n-2)

$$\frac{dP_{n-1}(t)}{dt} + \mathrm{a}_{n-1}\mathrm{P}_{n-1}(\mathrm{t}) = \lambda_r\mathrm{P}_{n-2}(\mathrm{t}) + \mu_w\mathrm{P}_{3n-1}(\mathrm{t}) \tag{5.3}$$

$$\frac{dP_n(t)}{dt} + \mathrm{a}_n\mathrm{P}_n(\mathrm{t}) = \lambda_s\mathrm{P}_0(\mathrm{t}) + \mu_r\mathrm{P}_{n+1}(\mathrm{t}) + \mu_w\mathrm{P}_{3n}(\mathrm{t}) \tag{5.4}$$

$$\frac{dP_i(t)}{dt} + \mathrm{a}_i\mathrm{P}_i(\mathrm{t}) = \lambda_r\mathrm{P}_{i-1}(\mathrm{t}) + \lambda_s\mathrm{P}_{i-n}(\mathrm{t}) + \mu_r\mathrm{P}_{i+1}(\mathrm{t}) + \mu_w\mathrm{P}_{2n+i}(\mathrm{t}) \tag{5.5}$$

(for i = n+1, n+2....2n-2)

$$\frac{dP_{2n-1}(t)}{dt} + \mathrm{a}_{2n-1}\mathrm{P}_{2n-1}(\mathrm{t}) = \lambda_r\mathrm{P}_{2n-2}(\mathrm{t}) + \lambda_s\mathrm{P}_{n-1}(\mathrm{t}) + \mu_w\mathrm{P}_{4n-1}(\mathrm{t}) \tag{5.6}$$

$$\frac{dP_i(t)}{dt} + \mathrm{a}_i\mathrm{P}_i(\mathrm{t}) = \lambda_w\mathrm{P}_{i-2n}(\mathrm{t}) \tag{5.7}$$

(for i = 2n, 2n+1....3n-1)

$$\frac{dP_i(t)}{dt} + \mathrm{a}_i\mathrm{P}_i(\mathrm{t}) = \lambda_w\mathrm{P}_{i-2n}(\mathrm{t}) + \lambda_s\mathrm{P}_{i-n}(\mathrm{t}) \tag{5.8}$$

(for i = 3n, 3n+1....4n-1)

where

$\mathrm{a}_0 = \lambda_r + \lambda_s + \lambda_w$

$\mathrm{a}_i = \lambda_r + \lambda_s + \lambda_w + \mu_r$ (for i = 1,2,........,n-1)

$$a_n = \lambda_r + \mu_s + \lambda_w$$

$$a_i = \lambda_r + \mu_r + \lambda_w \qquad \text{(for } i = \text{n+1, n+2........,2n-1)}$$

$$a_i = \lambda_r + \lambda_s + \mu_w \qquad \text{(for } i = \text{2n, 2n+1..........3n-1)}$$

$$a_i = \lambda_r + \mu_w \qquad \text{(for } i = \text{3n, 3n+1..........4n-1)}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x)\, P_j(x,t) = 0 \qquad (\text{ for } j = 4n, 4n+1) \tag{5.9}$$

The associated boundary conditions as follows:

$$P_{4n}(0,t) = \lambda_r [\, P_{2n-1}(t) + \sum_{i=3n}^{4n-1} P_i(t) \,] \tag{5.10}$$

$$P_{4n+1}(0,t) = \lambda_r [\, P_{n-1}(t) + \sum_{i=2n}^{3n-1} P_i(t) \,] \tag{5.11}$$

At time t = 0, $P_0(0) = 1$, and other initial state probabilities are equal to zero.

5.3 Generalized Robot-Safety System Time Dependent Analysis

By solving Equations (5.1) – (5.11) with the Laplace transform method *, we get the following Laplace transforms of state probabilities:

$$P_0(s) = [\, s(1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{i=2n}^{3n-1} M_i(s) + \sum_{i=3n}^{4n-1} O_i(s) + \sum_{j=4n}^{4n+1} a_j(s) \frac{1 - W_j(s)}{s})]^{-1}$$

$$= \frac{1}{G(s)} \tag{5.12}$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad \text{(for } i = \text{1,2.........n-1)} \tag{5.13}$$

$$P_i(s) = V_i(s)\, P_0(s) \qquad \text{(for } i = \text{n, n+1......2n-1)} \tag{5.14}$$

---

* : Please see Appendix D.1.

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{M}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad (\text{for } i = 2\mathrm{n}, 2\mathrm{n}+1\ldots\ldots 3\mathrm{n}-1) \qquad (5.15)$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{O}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad (\text{for } i = 3\mathrm{n}, 3\mathrm{n}+1\ldots\ldots 4\mathrm{n}-1) \qquad (5.16)$$

$$\mathrm{P}_j(\mathrm{s}) = \mathrm{a}_j(\mathrm{s})\,\frac{1-W_j(s)}{s}\,\mathrm{P}_0(\mathrm{s}) \qquad (\text{for } j = 4\mathrm{n}, 4\mathrm{n}+1) \qquad (5.17)$$

where

$$\mathrm{Y}_i(\mathrm{s}) = \prod_{l=1}^{i} b_l(s) \qquad (\text{for } i = 1,2\ldots\ldots\ldots \mathrm{n}-1)$$

$$\mathrm{b}_{n-1}(\mathrm{s}) = \frac{\lambda_r}{(s+a_{n-1}) - \mu_w \dfrac{\lambda_w}{s+a_{3n-1}}}$$

$$\mathrm{b}_i(\mathrm{s}) = \frac{\lambda_r}{(s+a_i) - \mu_r b_{i+1}(s) - \mu_w \dfrac{\lambda_w}{s+a_{i+2n}}} \qquad (\text{ for } i = 1,2\ldots\ldots\ldots \mathrm{n}-1)$$

$$\mathrm{M}_{2n}(\mathrm{s}) = \frac{\lambda_w}{s+a_{2n}}$$

$$\mathrm{M}_i(\mathrm{s}) = \frac{\lambda_w}{s+a_i}\,\mathrm{Y}_{i-2n}(\mathrm{s}) \qquad (\text{for } i = 2\mathrm{n}+1, 2\mathrm{n}+2\ldots\ldots 3\mathrm{n}-1)$$

$$\mathrm{O}_i(\mathrm{s}) = \frac{\lambda_s}{s+a_i}\,\mathrm{M}_{i-n}(\mathrm{s}) + \frac{\lambda_w}{s+a_i}\,\mathrm{V}_{i-2n}(\mathrm{s})$$

$$\mathrm{V}_n(\mathrm{s}) = \frac{\lambda_s}{L_n(s)} + \sum_{i=1}^{n-1} \frac{\mu_r^i \lambda_s}{\prod_{h=n}^{i+n} L_h(s)} Y_i(s) + \sum_{i=0}^{n-1} \frac{\mu_r^i}{\prod_{k=n}^{i+n} L_k(s)} \frac{\mu_w \lambda_s}{s+a_{i+3n}} M_{i+2n}(s)$$

$$\mathrm{V}_i(\mathrm{s}) = \frac{\lambda_r}{L_i(s)}\,\mathrm{V}_{i-1}(\mathrm{s}) + \sum_{k=i}^{2n-1} \frac{\mu_r^{k-i} \lambda_s}{\prod_{h=i}^{k} L_h(s)} Y_{k-n}(s) + \sum_{k=i}^{2n-1} \frac{\mu_r^{k-i} \mu_w}{\prod_{h=i}^{k} L_h(s)} \frac{\lambda_s}{s+a_{k+2n}} M_{k+n}(s)$$

$$(\text{for } i = \mathrm{n}+1,\ldots\ldots 2\mathrm{n}-1)$$

$$L_{2n-1}(s) = (s + a_{2n-1}) - \mu_w \frac{\lambda_w}{s + a_{4n-1}}$$

$$L_i(s) = (s + a_i) - \mu_w \frac{\lambda_w}{s + a_{i+2n}} - \mu_r \frac{\lambda_r}{L_{i+1}(s)}$$

(for i = n, n+1,......2n-2)

$$a_{4n}(s) = \lambda_r [\, V_{2n-1}(s) + \sum_{i=3n}^{4n-1} O_i(s) \,]$$

$$a_{4n+1}(s) = \lambda_r [\, Y_{n-1}(s) + \sum_{i=2n}^{3n-1} M_i(s) \,]$$

$$G(s) = s(1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{i=2n}^{3n-1} M_i(s) + \sum_{i=3n}^{4n-1} O_i(s)$$

$$+ \sum_{j=4n}^{4n+1} a_j(s) \frac{1 - W_j(s)}{s}) \tag{5.18}$$

$$W_j(s) = \int_0^\infty e^{-sx} w_j(x)dx \qquad \text{(for } j = \text{ n+1,n+2,n+3)} \tag{5.19}$$

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta \,]\, \mu_j(x)$$

The Laplace transform of the robot-safety system availability with the normally working safety unit is given by:

$$AV_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) + \sum_{i=2n}^{3n-1} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=2n}^{3n-1} M_i(s)}{G(s)} \tag{5.20}$$

The Laplace transform of the robot-safety system availability with or without the working safety unit is given by:

$$AV_r(s) = \sum_{i=0}^{4n-1} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{i=2n}^{3n-1} M_i(s) + \sum_{i=3n}^{4n-1} O_i(s)}{G(s)} \tag{5.21}$$

Taking the inverse Laplace transforms of the above equations, we can get the time dependent state probabilities, $P_i(t)$and $P_j(t)$, and system availabilities, AVrs(t) and AVr(t).

5.3.1 Robot-Safety System Time Dependent Analysis for a Special Case

For two robots (i.e., one working, another one on standby) by substituting n=2 into Equations (5.12)-(5.21), we obtain:

$$P_0(s) = [s(Y_1(s) + \sum_{i=2}^{3} V_i(s) + \sum_{i=4}^{5} M_i(s) + \sum_{i=6}^{7} O_i(s) + \sum_{j=8}^{9} a_j(s) \frac{1-W_j}{s})]^{-1} \qquad (5.22)$$

$$P_1(s) = Y_1(s)\, P_0(s) \qquad (5.23)$$

$$P_i(s) = V_i(s)\, P_0(s) \qquad \text{(for } i = 2, 3) \qquad (5.24)$$

$$P_i(s) = M_i(s)\, P_0(s) \qquad \text{(for } i = 4, 5) \qquad (5.25)$$

$$P_i(s) = O_i(s)\, P_0(s) \qquad \text{(for } i = 6, 7) \qquad (5.26)$$

$$P_j(s) = a_j(s)\, \frac{1-W_j}{s} P_0(s) \qquad \text{(for } j = 8, 9) \qquad (5.27)$$

where

$$Y_1(s) = \frac{\lambda_r}{(s+a_1) - \mu_w \dfrac{\lambda_w}{s+a_5}}$$

$$M_4(s) = \frac{\lambda_w}{s+a_4}$$

$$M_5(s) = \frac{\lambda_w}{s+a_5}\, Y_1(s)$$

$$V_2(s) = \frac{\lambda_s}{L_2(s)} + \mu_r \frac{\lambda_s}{L_3(s)L_2(s)} Y_1(s) + \frac{\mu_w \lambda_s}{(s+a_6)L_2(s)} M_4(s) +$$

$$V_3(s) = \frac{\lambda_r}{L_3(s)}V_2(s) + \frac{\lambda_s}{L_3(s)}Y_1(s) + \frac{\mu_w\lambda_s}{(s+a_7)L_3(s)}M_5(s)$$

$$O_6(s) = \frac{\lambda_w}{s+a_6}V_2(s) + \frac{\lambda_s}{s+a_6}M_4(s)$$

$$O_7(s) = \frac{\lambda_w}{s+a_7}V_3(s) + \frac{\lambda_s}{s+a_7}M_5(s)$$

$$a_8(s) = \lambda_r[\, V_3(s) + \sum_{i=6}^{7} O_i(s)\,]$$

$$a_9(s) = \lambda_r[\, Y_1(s) + \sum_{i=4}^{5} M_i(s)\,]$$

$$L_3(s) = (s+a_3) - \mu_w \frac{\lambda_w}{s+a_7}$$

$$L_2(s) = (s+a_2) - \mu_w \frac{\lambda_w}{s+a_6} - \mu_r \frac{\lambda_r}{L_3(s)}$$

$$G(s) = s[\, 1 + Y_1(s) + \sum_{i=2}^{3} V_i(s) + \sum_{i=4}^{5} M_i(s) + \sum_{i=6}^{7} O_i(s) + \sum_{j=8}^{9} a_j(s)\, \frac{1-W_j}{s}\,] \tag{5.28}$$

The Laplace transform of the robot-safety system availability with the normally working safety unit is given by:

$$AV_{rs}(s) = \sum_{i=0}^{1} P_i(s) + \sum_{i=4}^{5} P_i(s) = \frac{1 + Y_1(s) + \sum_{i=4}^{5} M_i(s)}{G(s)} \tag{5.29}$$

The Laplace transform of the robot-safety system availability when the robot is working normally with or without working safety units is

$$AV_r(s) = \sum_{i=0}^{7} P_i(s) = \frac{1 + Y_1(s) + \sum_{i=1}^{2} V_i(s) + \sum_{i=4}^{5} M_i(s) + \sum_{i=6}^{7} O_i(s)}{G(s)} \tag{5.30}$$

For the failed robot-safety system exponentially distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0, j = 8, 9) \tag{5.31}$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Substituting Equation (5.31) into Equation (5.19), we get:

$$W_j(s) = \frac{\mu_j}{s + \mu_j} \qquad (\mu_j > 0, j = 8, 9) \tag{5.32}$$

By inserting Equation (5.32) into Equations (5.22) – (5.30), setting $\lambda_s = 0.002$, $\lambda_r = 0.0015$, $\lambda_w = 0.001$, $\mu_s = 0.0001$, $\mu_w = 0.0002$, $\mu_r = 0.0003$, $\mu_8 = 0.0004$, $\mu_9 = 0.0005$; and using Matlab computer program [74], the Figure 4.3 and Figure 4.4 plots were obtained. These plots show that state probabilities and system availabilities decrease and increase with varying time t.

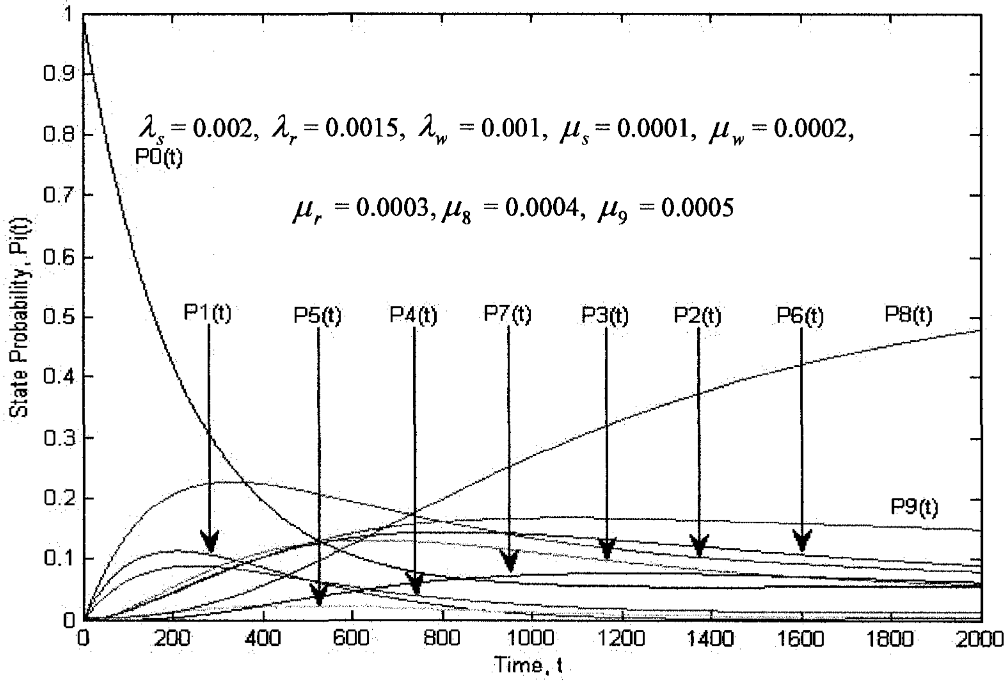


Figure 5.3 Time-dependent state probability plots for a robot-safety system with exponentially distributed failed system repair times.

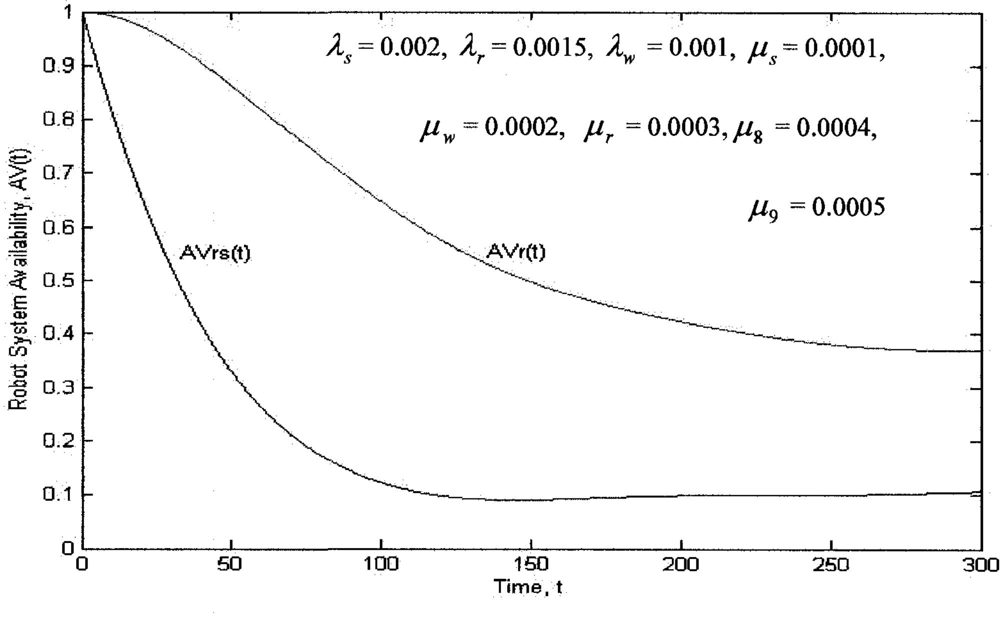


Figure 5.4 Time-dependent availability plots for a robot safety system with exponentially distributed failed system repair times.

5.4 Generalized Robot-Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (5.1)-(5.11) we get:

$$a_0 P_0 = \mu_r P_1 + \mu_s P_n + \mu_w P_{2n} + \sum_{j=4n}^{4n+1} P_j(x)\mu_j(x)dx \tag{5.33}$$

$$a_i P_i = \lambda_r P_{i-1} + \mu_r P_{i+1} + \mu_w P_{2n+i} \tag{5.34}$$

(for i = 1,2.........n-2)

$$a_{n-1} P_{n-1} = \lambda_r P_{n-2} + \mu_w P_{3n-1} \tag{5.35}$$

$$a_n P_n = \lambda_r P_0 + \mu_r P_{n+1} + \mu_w P_{3n} \tag{5.36}$$

$$a_i P_i = \lambda_r P_{i-1} + \mu_r P_{i+1} + \lambda_s P_{i-n} + \mu_w P_{2n+i} \tag{5.37}$$

(for i = n+1,n+2......2n-2)

$$a_{2n-1} P_{2n-1} = \lambda_r P_{2n-2} + \lambda_s P_{n-1} + \mu_w P_{4n-1} \tag{5.38}$$

$$a_i P_i = \lambda_w P_{i-2n} \tag{5.39}$$

(for i = 2n, 2n+1....,3n-1)

$$a_i P_i = \lambda_w P_{i-2n} + \lambda_s P_{i-n} \tag{5.40}$$

(for i = 3n, 3n+1....,4n-1)

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \quad (\text{for } j = 4n, 4n+1) \tag{5.41}$$

The associated boundary conditions are as follows:

$$P_{4n}(0) = \lambda_r [P_{2n-1} + \sum_{i=3n}^{4n-1} P_i] \tag{5.42}$$

$$P_{4n+1}(0) = \lambda_r [P_{n-1} + \sum_{i=2n}^{3n-1} P_i] \tag{5.43}$$

Solving equation (5.33) – (5.41) * , together with

$$\sum_{i=0}^{2n-1} P_i + \sum_{j=2n}^{2n+1} P_j = 1 \tag{5.44}$$

we obtain:

$$P_0(s) = (1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i + \sum_{j=4n}^{4n+1} a_j E_j[x])^{-1} = \frac{1}{G} \tag{5.45}$$

$$P_i = Y_i P_0 \qquad \text{(for } i = 1, 2\text{.........n-1)} \tag{5.46}$$

$$P_i = V_i P_0 \qquad \text{(for } i = \text{n, n+1......2n-1)} \tag{5.47}$$

$$P_i = M_i P_0 \qquad \text{(for } i = \text{2n, 2n+1......3n-1)} \tag{5.48}$$

$$P_i = O_i P_0 \qquad \text{(for } i = \text{3n, 3n+1......4n-1)} \tag{5.49}$$

$$P_j(s) = a_j E_j[x] P_0 \qquad \text{(for } j = \text{4n, 4n+1)} \tag{5.50}$$

where

$$Y_i = \prod_{l=1}^{i} b_l \qquad \text{(for } i = 1, 2\text{.........n-1)}$$

$$b_{n-1} = \lim_{s \to 0} b_{n-1}(s) = \frac{\lambda_r}{a_{n-1} - \mu_w \dfrac{\lambda_w}{a_{3n-1}}}$$

$$b_i = \lim_{s \to 0} b_i(s) = \frac{\lambda_r}{a_i - \mu_r b_{i+1} - \mu_w \dfrac{\lambda_w}{a_{i+2n}}} \qquad \text{(for } i = 1, 2\text{.........n-1)}$$

$$M_{2n} = \frac{\lambda_w}{a_{2n}}$$

$$M_i = \frac{\lambda_w}{a_i} Y_{i-2n} \qquad \text{(for } i = \text{2n+1, 2n+2......3n-1)}$$

* : Please see Appendix D.2.

$$\mathrm{O}_i = \frac{\lambda_s}{a_i}\mathrm{M}_{i-n} + \frac{\lambda_w}{a_i}\mathrm{V}_{i-2n}$$

$$\mathrm{V}_n = \frac{\lambda_s}{L_n} + \sum_{i=1}^{n-1}\frac{\mu_r^i\lambda_s}{\prod_{h=n}^{i+n}L_h}Y_i + \sum_{i=0}^{n-1}\frac{\mu_r^i}{\prod_{k=n}^{i+n}L_k}\frac{\mu_w\lambda_s}{a_{i+3n}}M_{i+2n}$$

$$\mathrm{V}_i = \frac{\lambda_r}{L_i}\mathrm{V}_{i-1} + \sum_{k=i}^{2n-1}\frac{\mu_r^{k-i}\lambda_s}{\prod_{h=i}^{k}L_h}Y_{k-n} + \sum_{k=i}^{2n-1}\frac{\mu_r^{k-i}\mu_w}{\prod_{h=i}^{k}L_h}\frac{\lambda_s}{a_{k+2n}}M_{k+n}$$

(for i = n+1,……2n-1)

$$\mathrm{L}_{2n-1} = \lim_{s\to 0}L_{2n-1}(s) = \mathrm{a}_{2n-1} - \mu_w\frac{\lambda_w}{a_{4n-1}}$$

$$\mathrm{L}_i = \lim_{s\to 0}L_i(s) = \mathrm{a}_i - \mu_w\frac{\lambda_w}{a_{i+2n}} - \mu_r\frac{\lambda_r}{L_{i+1}}$$

(for i = n, n+1,……2n-2)

$$\mathrm{a}_{4n} = \lambda_r[\mathrm{V}_{2n-1} + \sum_{i=3n}^{4n-1}O_i]$$

$$\mathrm{a}_{4n+1} = \lambda_r[\mathrm{Y}_{n-1} + \sum_{i=2n}^{3n-1}M_i]$$

$$\mathrm{G} = (1 + \sum_{i=1}^{n-1}Y_i + \sum_{i=n}^{2n-1}V_i + \sum_{i=2n}^{3n-1}M_i + \sum_{i=3n}^{4n-1}O_i + \sum_{j=4n}^{4n+1}a_jE_j[x]) \tag{5.51}$$

The generalized steady state availability of the robot safety system with the normally working safety unit is presented by:

$$\mathrm{SSAV}_{rs} = \sum_{i=0}^{n-1}P_i + \sum_{i=2n}^{3n-1}P_i = \frac{1+\sum_{i=1}^{n-1}Y_i + \sum_{i=2n}^{3n-1}M_i}{G} \tag{5.52}$$

The generalized steady state availability of the robot safety system with or without the working safety units is presented:

$$\mathrm{SSAV}_r = \sum_{i=0}^{4n-1} P_i = \frac{1+\sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i}{G} \tag{5.53}$$

For different failed system repair time distributions, we can get different value of G respectively as follows:

i) For the failed robot-safety system gamma distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \frac{\mu_j^{\beta} x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \qquad (\beta > 0,\ j = 4\mathrm{n},\ 4\mathrm{n}+1) \tag{5.54}$$

where

x is the repair time variable, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$\mathrm{E}_j[\mathrm{x}] = \int_0^{\infty} x w_j(x) dx = \frac{\beta}{\mu_j} \qquad (\beta > 0,\ j = 4\mathrm{n},\ 4\mathrm{n}+1) \tag{5.55}$$

Substituting Equation (5.55) into Equation (5.51), we get

$$\mathrm{G} = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i + \sum_{j=4n}^{4n+1} a_j \frac{\beta}{\mu_j} \tag{5.56}$$

ii) For the failed robot-safety system Weibull distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \mu_j \beta x^{\beta-1} e^{-\mu_j (x)^{\beta}} \qquad (\beta > 0,\ j = 4\mathrm{n},\ 4\mathrm{n}+1) \tag{5.57}$$

where

x is the repair time variable, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$\mathrm{E}_j[\mathrm{x}] = \int_0^\infty xW_j(x)dx = (\frac{1}{\mu_j})^{1/\beta}\frac{1}{\beta}\Gamma(\frac{1}{\beta}) \quad (\beta > 0, j = 4\mathrm{n}, 4\mathrm{n}+1) \tag{5.58}$$

Substituting Equation (5.58) into Equation (5.51), we get

$$\mathrm{G} = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i + \sum_{j=4n}^{4n+1} a_j(\frac{1}{\mu_j})^{1/\beta}\frac{1}{\beta}\Gamma(\frac{1}{\beta}) \tag{5.59}$$

iii) For the failed robot-safety system Rayleigh distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \mu_j x e^{-\mu_j x^2/2} \qquad (\mu_j > 0,\ j = 4\mathrm{n}, 4\mathrm{n}+1) \tag{5.60}$$

where

x is the repair time variable, μ_j is the scale parameter.

Thus, the mean time to robot-safety system repair is given by

$$\mathrm{E}_j[\mathrm{x}] = \int_0^\infty xW_j(x)dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (\mu_j > 0,\ j = 4\mathrm{n}, 4\mathrm{n}+1) \tag{5.61}$$

Substituting Equation (5.61) into Equation (5.51), we get

$$\mathrm{G} = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i + \sum_{j=4n}^{4n+1} a_j\sqrt{\frac{\pi}{2\mu_j}} \tag{5.62}$$

iv) For the failed robot system lognormally distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \frac{1}{\sqrt{2\pi x\sigma_{y_j}}}\ e^{[\frac{-(Inx-\mu_{y_j})^2}{2\sigma_{y_j}^2}} \qquad (\text{for } j = 4\mathrm{n}, 4\mathrm{n}+1) \tag{5.63}$$

where

x is the repair time variable, Inx is the natural logarithm of x with a mean μ and variance σ^2 . The conditions on parameters are:

$$\sigma_{y_j} = \text{In}\sqrt{1+(\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \tag{5.64}$$

$$\mu_{y_j} = \text{In}\sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2+\sigma_{x_j}^2}} \tag{5.65}$$

Thus, the mean time to robot-safety system repair is given by

$$\text{E}_j\,[\text{x}] = e^{(\mu_{y_j}+\frac{\sigma_{y_j}^2}{2})} \qquad (\text{for } j = 4\text{n, }4\text{n}+1) \tag{5.66}$$

Substituting Equation (5.66) into Equation (5.51), we get

$$\text{G} = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i + \sum_{j=4n}^{4n+1} a_j\, e^{(\mu_{y_j}+\frac{\sigma_{y_j}^2}{2})} \tag{5.67}$$

v) For the failed robot system exponentially distributed repair time x, the probability density function is expressed by

$$\text{w}_j\,(\text{x}) = \mu_j\, e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 4\text{n, }4\text{n}+1) \tag{5.68}$$

where

x is the repair time variable and μ_j is the constant repair rate of state j.

Thus, the mean time to robot-safety system repair is given by

$$\text{E}_j\,[\text{x}] = \int_0^\infty x w_j(x) dx = \frac{1}{\mu_j} \qquad (\beta > 0,\ j = 4\text{n, }4\text{n}+1) \tag{5.69}$$

Substituting Equation (5.69) into Equation (5.51), we get

$$\text{G} = 1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i + \sum_{j=4n}^{4n+1} a_j \frac{1}{\mu_j} \tag{5.70}$$

5.4.1 Robot-Safety System Steady State Analysis for a Special Case

For two robots (i.e. one working, another one on standby) by substituting n = 2 into Equations (5.45)-(5.51), we obtain:

$$P_0 = (1+Y_1 + \sum_{i=2}^{3} V_i + \sum_{i=4}^{5} M_i + \sum_{i=6}^{7} O_i + \sum_{j=8}^{9} a_j E_j[x])^{-1} \tag{5.71}$$

$$P_1 = Y_1\ P_0 \tag{5.72}$$

$$P_i = V_i\ P_0 \qquad \text{(for } i = 2, 3) \tag{5.73}$$

$$P_i = M_i\ P_0 \qquad \text{(for } i = 4, 5) \tag{5.74}$$

$$P_i = O_i\ P_0 \qquad \text{(for } i = 6, 7) \tag{5.75}$$

$$P_j = a_j E_j[x]\ P_0 \qquad \text{(for } j = 8, 9) \tag{5.76}$$

where

$$Y_1 = \frac{\lambda_r}{a_1 - \mu_w \dfrac{\lambda_w}{a_5}}$$

$$M_4 = \frac{\lambda_w}{a_4}$$

$$M_5 = \frac{\lambda_w}{a_5}\ Y_1$$

$$V_2 = \frac{\lambda_s}{L_2} + \mu_r \frac{\lambda_s}{L_3 L_2} Y_1 + \frac{\mu_w \lambda_s}{a_6 L_2} M_4 + \frac{\mu_w \mu_r \lambda_s}{a_7 L_2 L_3} M_5$$

$$V_3 = \frac{\lambda_r}{L_3} V_2 + \frac{\lambda_s}{L_3} Y_1 + \frac{\mu_w \lambda_s}{a_7 L_3} M_5$$

$$O_6 = \frac{\lambda_w}{a_6} V_2 + \frac{\lambda_s}{a_6} M_4$$

$$O_7 = \frac{\lambda_w}{a_7} V_3 + \frac{\lambda_s}{a_7} M_5$$

$$a_8 = \lambda_r [V_3 + \sum_{i=6}^{7} O_i]$$

$$a_9 = \lambda_r [Y_1 + \sum_{i=4}^{5} M_i]$$

$$L_3(s) = a_3 - \mu_w \frac{\lambda_w}{a_7}$$

$$L_2(s) = a_2 - \mu_w \frac{\lambda_w}{a_6} - \mu_r \frac{\lambda_r}{L_3}$$

$$G = 1 + Y_1 + \sum_{i=2}^{3} V_i + \sum_{i=4}^{5} M_i + \sum_{i=6}^{7} O_i + \sum_{j=8}^{9} a_j E_j[x] \qquad (5.77)$$

The steady state availability of the robot-safety system with the normally working safety unit is given by:

$$SSAVrs = \sum_{i=0}^{1} P_i + \sum_{i=4}^{5} P_i = \frac{1 + Y_1 + \sum_{i=4}^{5} M_i}{G} \qquad (5.78)$$

The steady state availability of the robot-safety system with or without a normally working safety unit is given by:

$$SSAVr = \sum_{i=0}^{7} P_i = \frac{1 + \sum_{i=2}^{3} V_i + \sum_{i=4}^{5} M_i + \sum_{i=4}^{5} O_i}{G} \qquad (5.79)$$

1) For exponentially distributed failed robot-safety system repair time, substituting Equation (5.70) into Equations (5.71)-(5.79), setting:

$\lambda_s = 0.0004$, $\lambda_r = 0.00045$, $\lambda_w = 0.0003$, $\mu_w = 0.0002$, $\mu_r = 0.0003$, $\mu_8 = 0.0004$, $\mu_9 = 0.0005$; and using Matlab computer program[74], the Figure 4.5 and 4.6 plots were obtained.

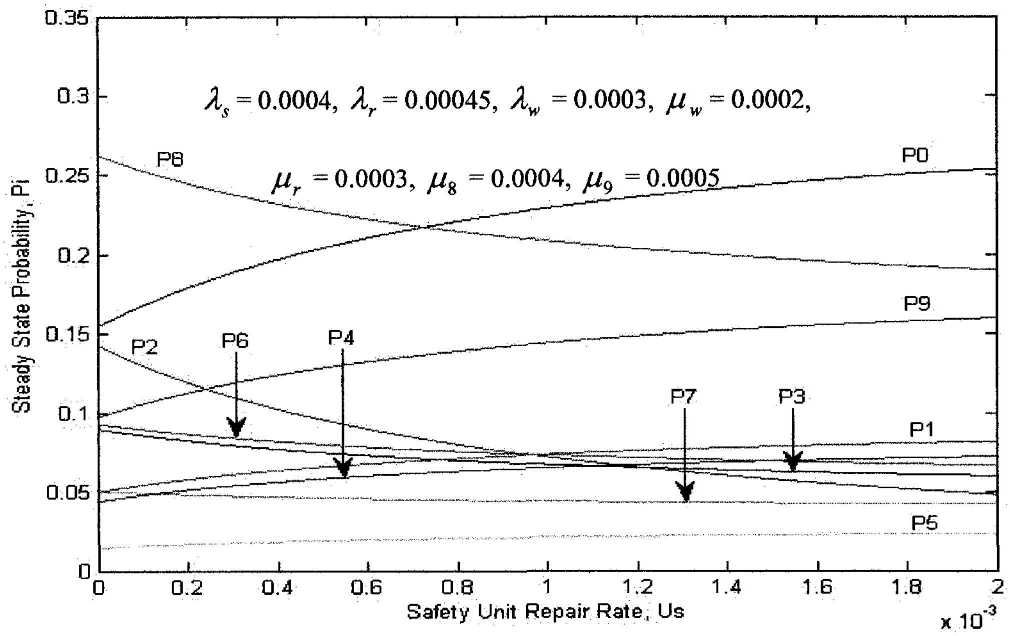


Figure 5.5 Robot-safety system steady state probability versus safety unit repair rate Us(means μ_s)plots with exponentially distributed failed system repair times.

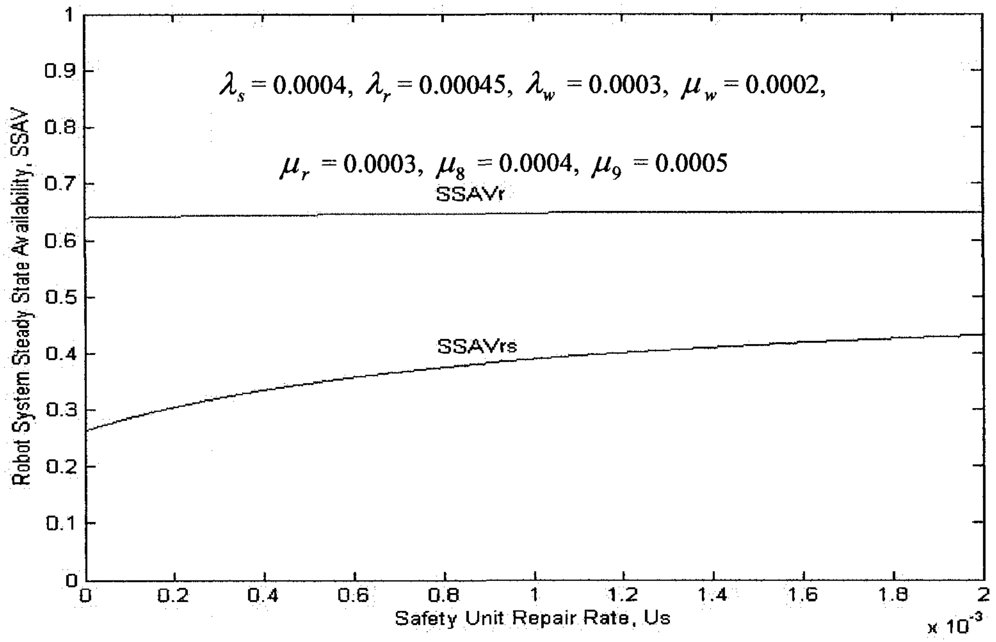


Figure 5.6 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots with exponentially distributed failed system repair times.

2) For Rayleigh distributed failed robot-safety system repair time, substituting Equation (5.62) into Equations (5.71)-(5.79), setting:

$\lambda_s = 0.0032$, $\lambda_r = 0.002$, $\lambda_w = 0.004$, $\mu_w = 0.0002$, $\mu_r = 0.0003$, $\mu_8 = 0.00001$, $\mu_9 = 0.00002$; and using Matlab computer program [74], the Figure 4.7 and 4.8 plots were obtained.

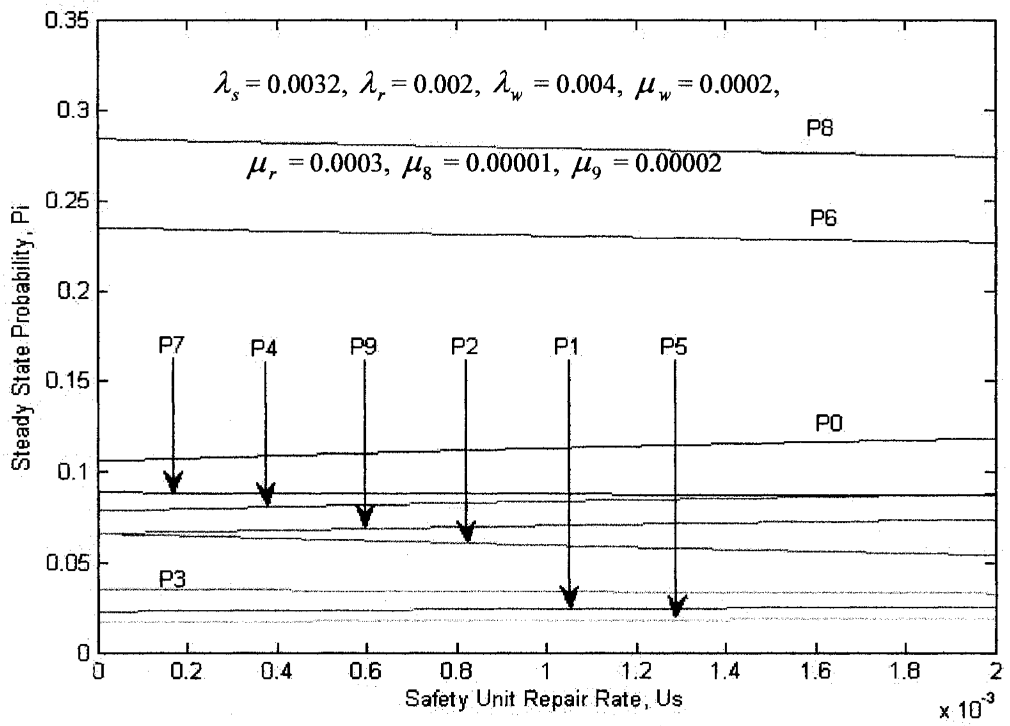


Figure 5.7 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots with Rayleigh distributed failed system repair times.

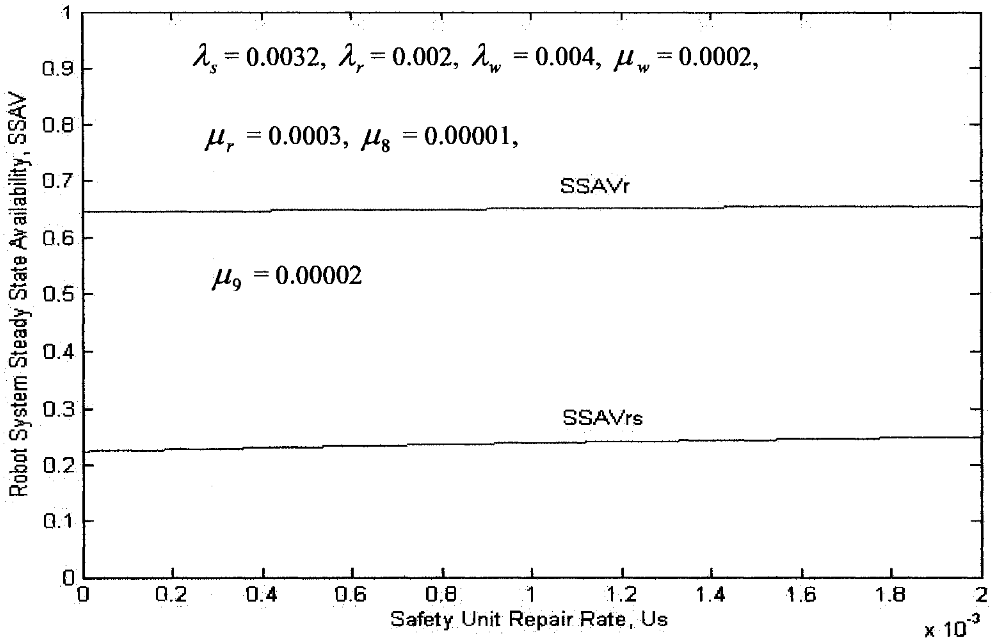


Figure 5.8 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots with Rayleigh distributed failed system repair times.

3) For lognormally distributed failed robot-safety system repair time, substituting Equation (5.67) into Equations (5.71)-(5.79), setting:

$\lambda_s = 0.0004$, $\lambda_r = 0.00045$, $\lambda_w = 0.0003$, $\mu_w = 0.0002$, $\mu_r = 0.0003$, $\mu_8 = 0.0004$, $\mu_9 = 0.0005$, $\sigma = 3.4$; and using Matlab computer program [74], the Figure 5.9 and 5.10 plots were obtained.

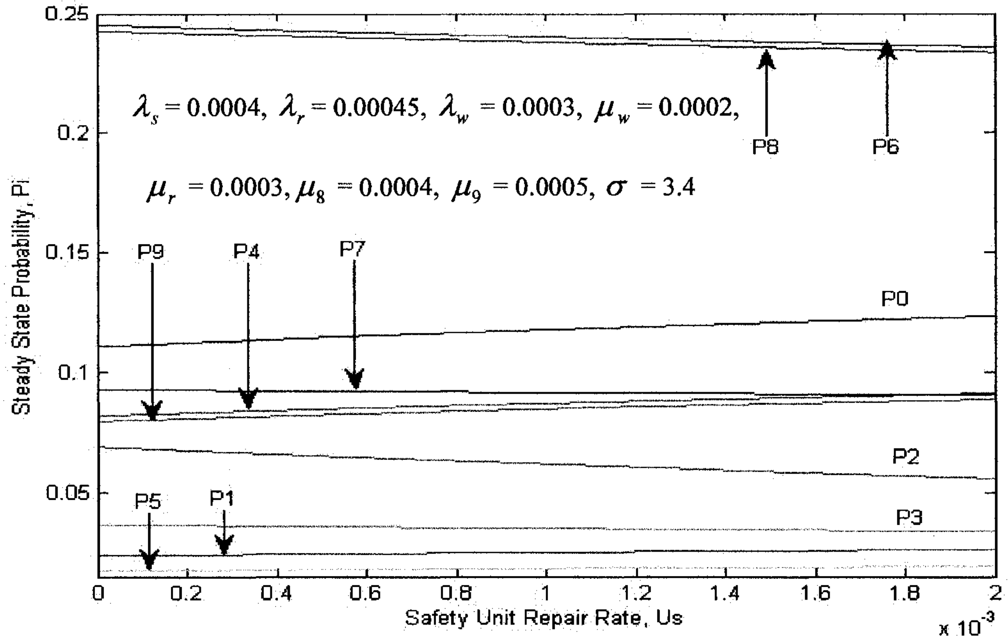


Figure 5.9 Robot-safety system steady state probability versus safety unit repair rate Us (means μ_s) plots with lognormally distributed failed system repair times.

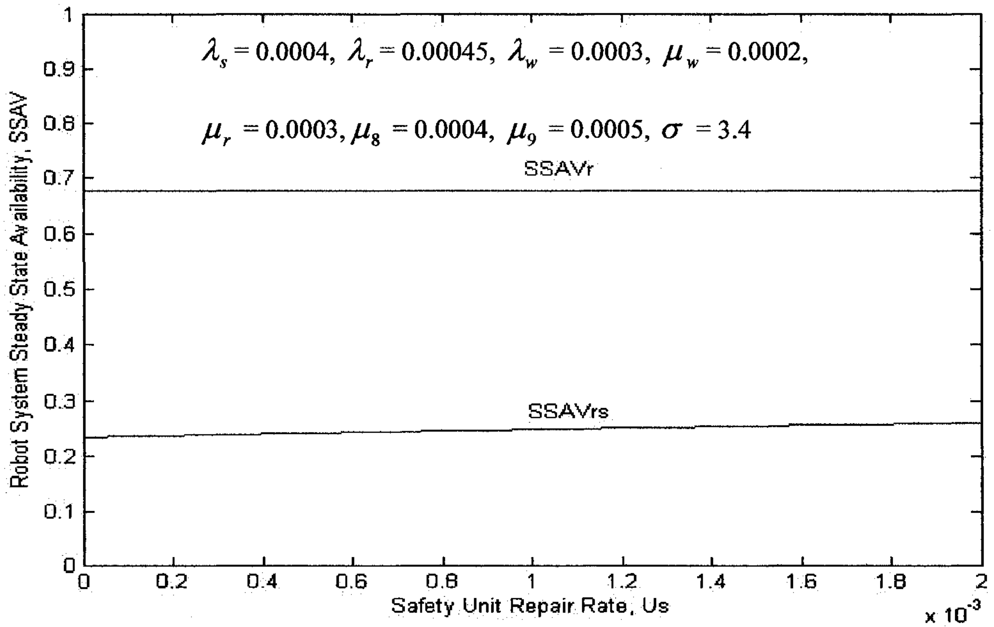


Figure5.10 Robot-safety system steady state availability versus safety unit repair rate Us (means μ_s) plots with lognormally distributed failed system repair times.

5.5 Robot-Safety System Reliability and Mean Time to Failure Analysis

Setting $\mu_j = 0$, (for j = 4n, 4n+1), in Figure 5.2 and using the Markov method [73], we get the following equations:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_r P_1(t) + \mu_s P_n(t) + \mu_w P_{2n}(t) \tag{5.80}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_r P_{i-1}(t) + \mu_r P_{i+1}(t) + \mu_w P_{2n+i}(t) \tag{5.81}$$

(for i = 1, 2.........n-2)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = \lambda_r P_{n-2}(t) + \mu_w P_{3n-1}(t) \tag{5.82}$$

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_0(t) + \mu_r P_{n+1}(t) + \mu_w P_{3n}(t) \tag{5.83}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) + \mu_r P_{i+1}(t) + \mu_w P_{2n+i}(t) \tag{5.84}$$

(for i = n+1, n+2......2n-2)

$$\frac{dP_{2n-1}(t)}{dt} + a_{2n-1} P_{2n-1}(t) = \lambda_r P_{2n-2}(t) + \lambda_s P_{n-1}(t) + \mu_w P_{4n-1}(t) \tag{5.85}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_w P_{i-2n}(t) \qquad \text{(for } i = 2n, 2n+1.....3n-1) \tag{5.86}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_w P_{i-2n}(t) + \lambda_s P_{i-n}(t) \tag{5.87}$$

(for i = 3n, 3n+1...., 4n-1)

$$\frac{dP_{4n}(t)}{dt} = \lambda_r [P_{2n-1}(t) + \sum_{i=3n}^{4n-1} P_i(t)] \tag{5.88}$$

$$\frac{dP_{4n+1}(t)}{dt} = \lambda_r [P_{n-1}(t) + \sum_{i=2n}^{3n-1} P_i(t)] \tag{5.89}$$

where

$$a_0 = \lambda_r + \lambda_s + \lambda_w$$

$$a_i = \lambda_r + \lambda_s + \lambda_w + \mu_r \qquad \text{(for } i = 1, 2 \ldots\ldots\ldots n\text{-}1)$$

$$a_n = \lambda_r + \mu_s + \lambda_w$$

$$a_i = \lambda_r + \mu_r + \lambda_w \qquad \text{(for } i = n+1, n+2 \ldots\ldots\ldots 2n\text{-}1)$$

$$a_i = \lambda_r + \lambda_s + \mu_w \qquad \text{(for } i = 2n, 2n+1 \ldots\ldots\ldots 3n\text{-}1)$$

$$a_i = \lambda_r + \mu_w \qquad \text{(for } i = 3n, 3n+1 \ldots\ldots\ldots 4n\text{-}1)$$

At time t = 0, $P_0(0) = 1$ and other initial conditions are equal to zero.

By solving Equations (5.80)-(5.89) with the Laplace transform method *, together with

$$\sum_{i=0}^{2n-1} P_i(t) + \sum_{j=2n}^{2n+1} P_j(t) = 1, \tag{5.90}$$

we obtain the general Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{i=2n}^{3n-1} M_i(s) + \sum_{i=3n}^{4n-1} O_i(s) + \sum_{j=4n}^{4n+1} \frac{a_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{5.91}$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad \text{(for } i = 1, 2 \ldots\ldots\ldots n\text{-}1) \tag{5.92}$$

$$P_i(s) = V_i(s)\, P_0(s) \qquad \text{(for } i = n, n+1 \ldots\ldots\ldots 2n\text{-}1) \tag{5.93}$$

$$P_i(s) = M_i(s)\, P_0(s) \qquad \text{(for } i = 2n, 2n+1 \ldots\ldots 3n\text{-}1) \tag{5.94}$$

$$P_i(s) = O_i(s)\, P_0(s) \qquad \text{(for } i = 3n, 3n+1 \ldots\ldots 4n\text{-}1) \tag{5.95}$$

$$P_j(s) = a_j(s)\, \frac{1}{s} P_0(s) \qquad \text{(for } j = 4n, 4n+1) \tag{5.96}$$

* : Please Appendix D.3.

where

$$Y_i(s) = \prod_{l=1}^{i} b_l(s) \qquad \text{(for } i = 1, 2 \ldots\ldots\ldots \text{n-1)}$$

$$b_{n-1}(s) = \frac{\lambda_r}{(s+a_{n-1}) - \mu_w \dfrac{\lambda_w}{s+a_{3n-1}}}$$

$$b_i(s) = \frac{\lambda_r}{(s+a_i) - \mu_r b_{i+1}(s) - \mu_w \dfrac{\lambda_w}{s+a_{i+2n}}} \qquad \text{(for } i = 1,2 \ldots\ldots\ldots \text{n-1)}$$

$$M_{2n}(s) = \frac{\lambda_w}{s+a_{2n}}$$

$$M_i(s) = \frac{\lambda_w}{s+a_i} Y_{i-2n}(s) \qquad \text{(for } i = \text{2n+1, 2n+2} \ldots\ldots \text{3n-1)}$$

$$O_i(s) = \frac{\lambda_s}{s+a_i} M_{i-n}(s) + \frac{\lambda_w}{s+a_i} V_{i-2n}(s)$$

$$V_n(s) = \frac{\lambda_s}{L_n(s)} + \sum_{i=1}^{n-1} \frac{\mu_r^i \lambda_s}{\prod_{h=n}^{i+n} L_h(s)} Y_i(s) + \sum_{i=0}^{n-1} \frac{\mu_r^i}{\prod_{k=n}^{i+n} L_k(s)} \frac{\mu_w \lambda_s}{s+a_{i+3n}} M_{i+2n}(s)$$

$$V_i(s) = \frac{\lambda_r}{L_i(s)} V_{i-1}(s) + \sum_{k=i}^{2n-1} \frac{\mu_r^{k-i} \lambda_s}{\prod_{h=i}^{k} L_h(s)} Y_{k-n}(s) + \sum_{k=i}^{2n-1} \frac{\mu_r^{k-i} \mu_w}{\prod_{h=i}^{k} L_h(s)} \frac{\lambda_s}{s+a_{k+2n}} M_{k+n}(s)$$

(for i = n+1,……2n-1)

$$L_{2n-1}(s) = (s + a_{2n-1}) - \mu_w \frac{\lambda_w}{s+a_{4n-1}}$$

$$L_i(s) = (s + a_i) - \mu_w \frac{\lambda_w}{s+a_{i+2n}} - \mu_r \frac{\lambda_r}{L_{i+1}(s)}$$

(for i = n, n+1,……2n-2)

$$a_{4n}(s) = \lambda_r [\, V_{2n-1}(s) + \sum_{i=3n}^{4n-1} O_i(s) \,]$$

$$a_{4n+1}(s) = \lambda_r [\, Y_{n-1}(s) + \sum_{i=2n}^{3n-1} M_i(s) \,]$$

$$G(s) = s(1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{i=2n}^{3n-1} M_i(s) + \sum_{i=3n}^{4n-1} O_i(s) + \sum_{j=4n}^{4n+1} \frac{a_j(s)}{s}) \tag{5.97}$$

The Laplace transform of the robot-safety system reliability with the normally working safety unit is given by:

$$R_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) + \sum_{i=2n}^{3n-1} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=2n}^{3n-1} M_i(s)}{G(s)} \tag{5.98}$$

The Laplace transform of the robot safety system reliability with or without the working safety unit is expressed by:

$$R_r(s) = \sum_{i=0}^{4n-1} P_i(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i(s) + \sum_{i=n}^{2n-1} V_i(s) + \sum_{i=2n}^{3n-1} M_i(s) + \sum_{i=3n}^{4n-1} O_i(s)}{G(s)} \tag{5.99}$$

Using Equation (5.98), the robot-safety system mean time to failure with the normally working safety unit is obtained:

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=2n}^{3n-1} M_i}{\sum_{j=4n}^{4n+1} a_j} \tag{5.100}$$

Similarly, using Equation (4.99), the robot-safety system mean time to failure with or without the working safety unit is given by:

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{1 + \sum_{i=1}^{n-1} Y_i + \sum_{i=n}^{2n-1} V_i + \sum_{i=2n}^{3n-1} M_i + \sum_{i=3n}^{4n-1} O_i}{\sum_{j=4n}^{4n+1} a_j} \tag{5.101}$$

5.5.1 Robot-Safety System Mean Time to Failure Analysis for a Special Case

Substituting n = 2 into Equations (5.100) and (5.101), we get:

The robot-safety system mean time to failure with the normally working safety unit is

$$\mathrm{MTTF}_{rs} = \lim_{s\to 0} \mathrm{R}_{rs}(\mathrm{s}) = \frac{1+Y_1+\sum_{i=4}^{5} M_i}{\sum_{j=8}^{9} a_j} \tag{5.102}$$

The robot safety system mean time to failure with or without the working safety unit is

$$\mathrm{MTTF}_{r} = \lim_{s\to 0} \mathrm{R}_{r}(\mathrm{s}) = \frac{1+Y_1+\sum_{i=2}^{3} V_i+\sum_{i=4}^{5} M_i+\sum_{i=6}^{7} O_i}{\sum_{j=2n}^{2n+1} a_j} \tag{5.103}$$

For λ_s = 0.0006, λ_r = 0.0005, λ_w = 0.0007, μ_w = 0.0002, μ_r = 0.0003, and using Equations (4.86) and (4.87), the Figure 5.11 plot was obtained.

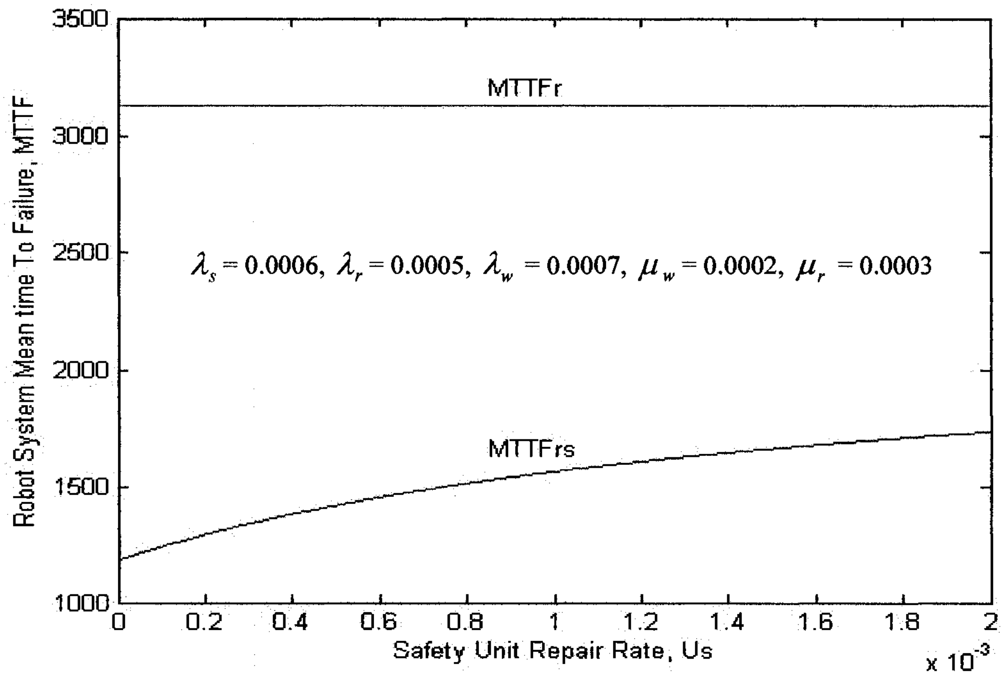


Figure 5.11 The robot-safety system mean time to failure plots for the increasing values of the safety unit Repair rate Us (means μ_s).

5.6 Conclusions

- For exponentially distributed failed robot system repair time x, the system time dependent availability decreases with the increasing time t.
- For exponentially, Rayleigh or lognormally distributed failed robot system repair time x, the system steady state availability increases as the safety unit repair rate increases.
- For exponentially distributed failed robot system repair time x, the robot-safety system mean time to failure (MTTF) increases with the increasing values of the safety unit repair rate.

Chapter 6

Stochastic Analysis of a System Containing N Parallel Robots and (M-1) Standby Safety Units with a Perfect Switch

6.1 Introduction

In previous chapters, either redundancy of robots or of safety units were considered, but not of both cases. This chapter presents a robot-safety system, which contains n robots and (m-1) standby safety units and a perfect switch. More specifically the system is composed of n identical robots and m identical safety units with a perfect switch.

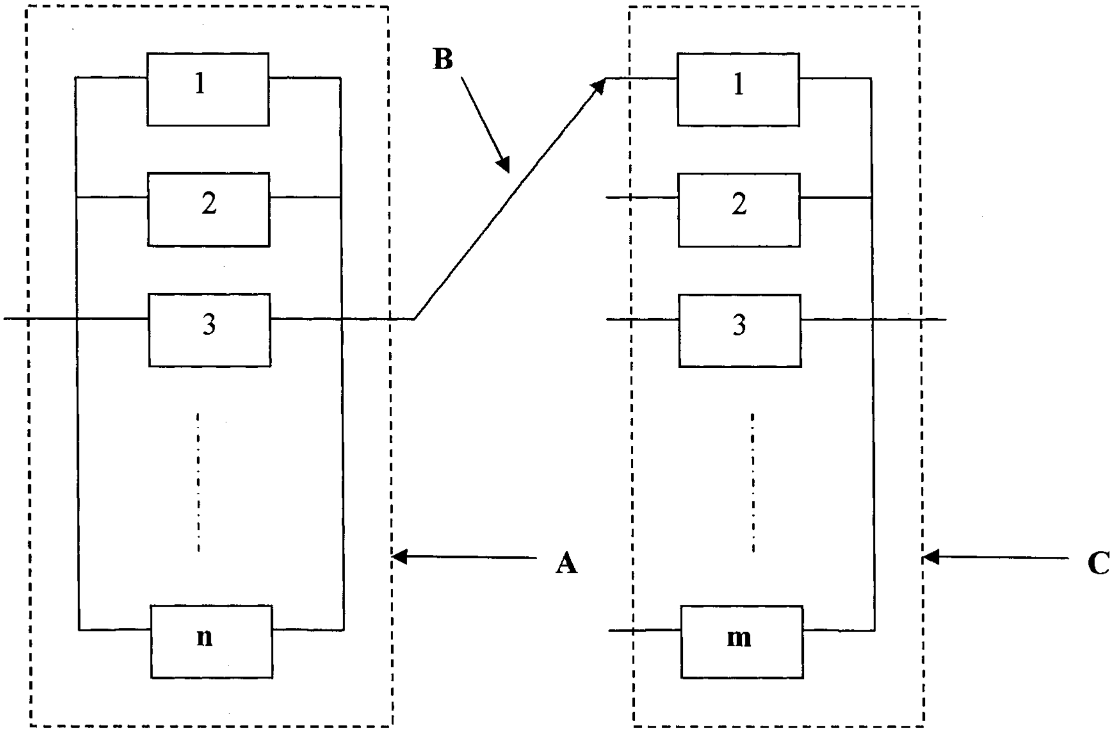
The block diagram of the robot system is shown in Figure6.1 and its corresponding state space diagram is presented in Figure 6.2. The numerals and letters n and m in the boxes of Figure 6.2 denote system states.

At time t =0, n robots, one safety unit and the switch to replace a failed safety unit start operating and m-1 safety units are on standby. The overall robot-safety system can fail only if all the robots fail.

The following assumptions are associated with this model:

- The robot-safety system is composed of n identical robots, m identical safety units (only one operates and the rest remain on standby) and a switch.
- N robots, the switch and one safety unit start operating simultaneously.
- The completely failed robot-safety system and its individually failed units (i.e. robot and safety unit) can be repaired. Failure and repair rates of robots and safety units are constant.
- The failed robot-safety system repair rates can be constant or non-constant.

- A robot has the priority to be repaired when safety unit and robot failed simultaneously.
- All failures are statistically independent.
- A repaired robot, safety unit, or the total robot-safety system is as good as new.



A : n identical Robots

B : Switch for replacing a failed safety unit and it cannot fail.

C: n identical Safety Units (one operating and n-1 on standby)

Figure 6.1 The block diagram of the robot-safety system containing n parallel robots and (m-1) standby safety units and a perfect switch

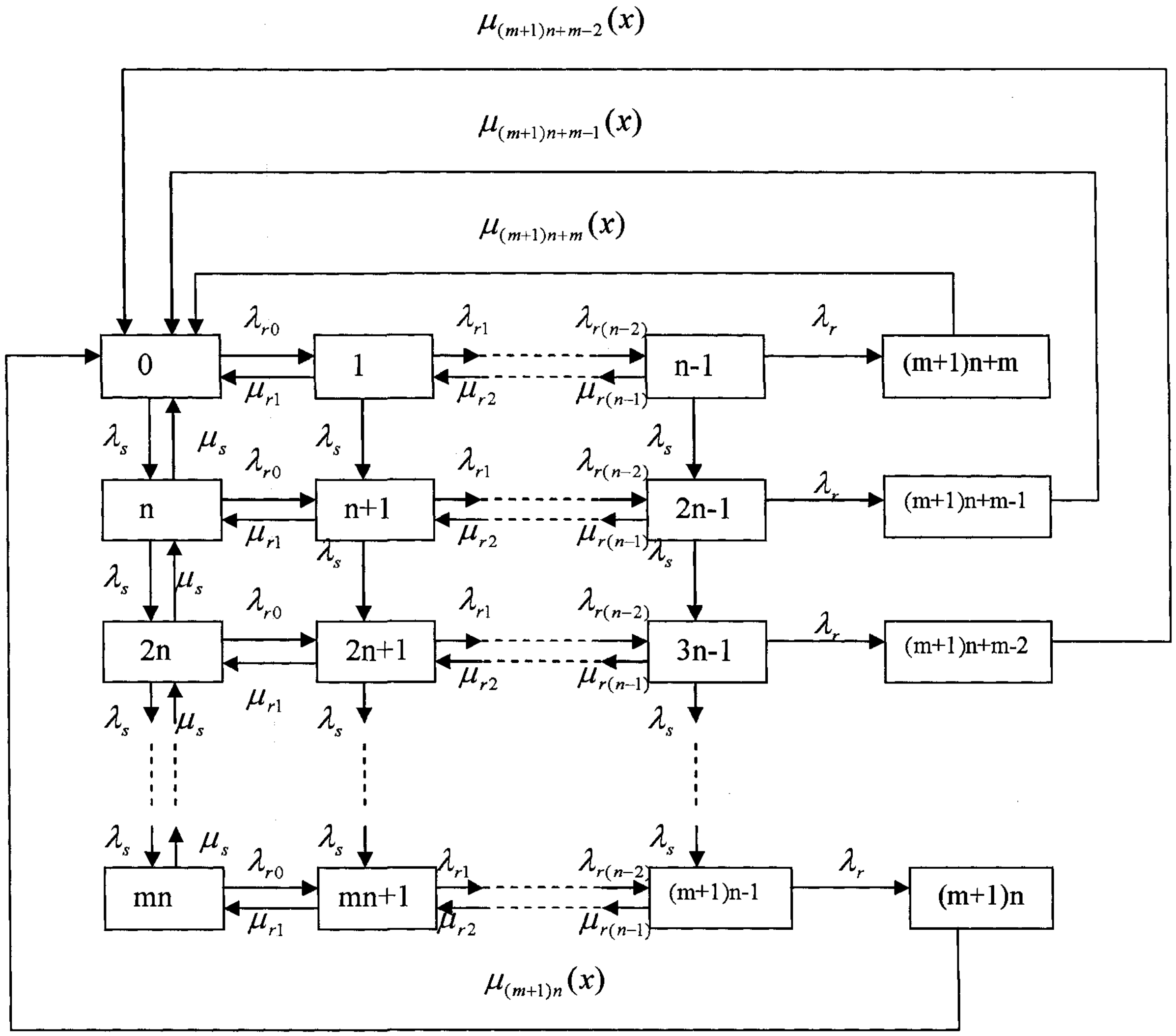


Figure 6.2 The state space diagram of the robot-safety system containing n parallel robots and (m-1) standby safety units with a perfect switch

6.1.1 Notation:

The following symbols are associated with the model:

i $\quad$ ith state of the robot-safety system:

for i = 0, means n robots, the switch and one safety unit are working normally;

	for i = kn+q, means (n-q) robots, the switch and one safety units are working normally while q robots and k safety units have failed; (i.e., k = 0,1, 2…m-1; q = 0, 1, 2…n) for i = mn+q, means (n-q) robots, the switch are working normally while all m safety units and q robots have failed; (i.e., q = 0, 1, 2…n)
j	j^{th} state of the robot–safety system: for j = (m+1)n+k, means the total robot-safety system has failed (i.e., all the robots and (m-k) safety units have failed. (i.e., k = 0, 1, 2…m)
t	Time.
λ_s	Constant failure rate of a safety unit.
λ_r	Constant failure rate of a robot.
μ_r	Constant repair rate of a robot.
μ_s	Constant repair rate of a safety unit.
Δ**x**:	Finite repair time interval.
$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in state j: and has an elapsed repair time of x; for j = (m+1)n, (m+1)n+1…(m+1)n+m.
$P_j(x,t)\ \Delta$x	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x, x+Δx]; for j = (m+1)n, (m+1)n+1…(m+1)n+m
pdf	Probability density function.
w_j(x)	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j = (m+1)n, (m+1)n+1…(m+1)n+m.

$P_j(t)$	Probability that the robot safety system is in state j at time t; for $j = (m+1)n, (m+1)n+1 \ldots (m+1)n+m$.
$P_i(t)$	Probability that the robot-safety system is in state i at time t; for $i = 0, 1, 2 \ldots (m+1)n-1$.
P_i	Steady state probability that the robot-safety system is in state i; for $i=0, 1, 2 \ldots (m+1)n-1$.
P_j	Steady state probability that robot-safety system is in state j; for $j = (m+1)n, (m+1)n+1 \ldots (m+1)n+m$.
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i; for $i = 0,1,2 \ldots (m+1)n-1$.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for $j = (m+1)n, (m+1)n+1 \ldots (m+1)n+m$.
AVrs(s)	Laplace transform of the robot-safety system availability with one normally working safety unit.
AVr(s)	Laplace transform of the robot-safety system availability with or without one normally safety unit.
AVrs (t)	Robot-safety system time dependent availability with one normally working safety unit.
AVr(t)	Robot-safety system time dependent availability with or without one normally working safety unit.
SSAVrs	Robot-safety system steady state availability with one normally working

	safety unit, the switch and the robot.
SSAVr	Robot-safety system steady state availability with or without one normally working safety unit.
Rrs(s)	Laplace transform of the robot-safety system reliability with one normally working safety unit.
Rr(s)	Laplace transform of the robot safety system reliability with or without one normally working safety unit.
MTTFrs	Robot-safety system mean time to failure with one normally working safety unit.
MTTFr	Robot-safety system mean time to failure with or without one normally working safety unit.

6.2 Generalized Robot-Safety System Analysis

By using the supplementary method [71, 72] and setting $\mu_s=0$, we writ down the equations for the Figure 6.2 diagram:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \sum_{j=(m+1)n}^{(m+1)n+m} P_j(x,t)\mu_j(x)dx \tag{6.1}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_r P_{i-1}(t) + \mu_{ri+1} P_{i+1}(t) \tag{6.2}$$

(for $i = 1,2\ldots\ldots\ldots n-2$)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) \tag{6.3}$$

$$\frac{dP_{kn}(t)}{dt} + a_{kn} P_{kn}(t) = \lambda_s P_{(k-1)n}(t) + \mu_{r1} P_{kn+1}(t) \tag{6.4}$$

(for $k = 1,2\ldots\ldots\ldots m-1$)

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(k+1)n-i+1]\lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) + \mu_{rq+1} P_{i+1}(t) \tag{6.5}$$

(for $i = kn+q$: $k = 1,2.......,m$; $q = 1,2......,n-2$)

$$\frac{dP_{(k+1)n-1}(t)}{dt} + a_{(k+1)n-1} P_{(k+1)n-1}(t) = \lambda_s P_{kn-1}(t) + 2\lambda_r P_{(k+1)n-2}(t) \tag{6.6}$$

(for $k = 1,2.........m$)

$$\frac{dP_{mn}(t)}{dt} + a_{mn} P_{mn}(t) = \lambda_s P_{(m-1)n}(t) + \mu_{r1} P_{mn+1}(t) \tag{6.7}$$

where

$a_0 = n\lambda_r + \lambda_s$

$a_{kn} = n\lambda_r + \lambda_s$ (for $k = 1,2.........m-1$)

$a_{mn} = n\lambda_r$

$a_i = [(k+1)n - i]\lambda_r + \lambda_s + \mu_{rq}$

(for $i = kn+q$: $k = 0,1,2.......,m-1$; $q = 1,2......,n-1$)

$a_i = [(k+1)n - i]\lambda_r + \mu_{rq}$

(for $i = kn+q$: $k = m$; $q = 1,2......,n-1$)

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \tag{6.8}$$

(for $j = (m+1)n, (m+1)n+1,....(m+1)n+m$)

The associated boundary conditions as follows:

$$P_j(0,t) = \lambda_r P_{[(m+1)(n+1)-j]n-1}(t) \tag{6.9}$$

(for $j = (m+1)n, (m+1)n+1,....(m+1)n+m$)

At time $t = 0$, $P_0(0) = 1$, and other initial state probabilities are equal to zero.

6.3 Generalized Robot-Safety System Time Dependent Analysis

By solving Equations (6.1) – (6.9) with the Laplace transform method *, we get the following Laplace transforms of state probabilities:

$$\mathrm{P}_0(\mathrm{s}) = [\mathrm{s}(1 + \sum_{i=1}^{(m+1)n-1} Y_i(s) + \sum_{j=(m+1)n}^{(m+1)n+m} a_j(s)\frac{1-W_j(s)}{s})]^{-1} = \frac{1}{G(s)} \qquad (6.10)$$

$$\mathrm{P}_i(\mathrm{s}) = \mathrm{Y}_i(\mathrm{s})\,\mathrm{P}_0(\mathrm{s}) \qquad (6.11)$$

(for $i = kn+q$, $k = 0,1,2.........\mathrm{m}$, $q = 0, 1.....\mathrm{n}-1$)

The letter k denotes the number of failed safety units

The letter q denotes the number of failed robots

$$\mathrm{P}_j(\mathrm{s}) = \mathrm{a}_j(\mathrm{s})\,\frac{1-W_j(s)}{s}\,\mathrm{P}_0(\mathrm{s}) \qquad (6.12)$$

(for $j = (\mathrm{m}+1)\mathrm{n}, (\mathrm{m}+1)\mathrm{n}+1,....(\mathrm{m}+1)\mathrm{n}+\mathrm{m}$)

where

$$\mathrm{Y}_0(\mathrm{s}) = 1$$

$$\mathrm{Y}_i(\mathrm{s}) = \prod_{l=1}^{i} b_l(s) \qquad \text{(for } i = kn+q,\ k = 0,\ q = 1,2.....\mathrm{n}-1 \text{)}$$

$$\mathrm{Y}_i(\mathrm{s}) = \sum_{h=0}^{n-1} \frac{\lambda_s \prod_{g=0}^{h} \mu_{rg}}{\prod_{v=i}^{i+h} L_v(s)} Y_{i-n+h}(s) \qquad \text{(for } i = kn,\ k = 1,2.....\mathrm{m} \text{)}$$

$$\mathrm{Y}_i(\mathrm{s}) = \frac{\lambda_s}{L_i(s)} Y_{i-n}(s) + \frac{[(k+1)n-i+1]\lambda_r}{L_i(s)} Y_{i-1}(s) + \sum_{h=i-n+1}^{kn-1} \frac{\prod_{g=q+1}^{h-(k-1)n} \mu_{rg}}{\prod_{f=i}^{h+n} L_f(s)} Y_h(s)$$

(for $i = kn+q$, $k = 1,2....\mathrm{m}$, $q = 1,2.....\mathrm{n}-1$)

* : Please see Appendix E.1.

$$a_j(s) = \lambda_r P_{[(m+1)(n+1)-j]n-1}(s)$$

(for j = (m+1)n, (m+1)n+1,....(m+1)n+m)

$$b_l(s) = \frac{[(k+1)n-i+1]\lambda_r}{s+a_i} \qquad (\text{ for } i = kn+q,\ k = 0,\ q = \text{n-1})$$

$$b_l(s) = \frac{[(k+1)n-i+1]\lambda_r}{(s+a_i)-\mu_{ri+1}b_{i+1}(s)} \qquad (\text{ for } i = kn+q,\ k = 0,\ q = 1{,}2.....\text{n-2})$$

$$\mu_{r0} = 1$$

$$L_i(s) = s + a_i \qquad (\text{for } i = kn+q,\ k = 1{,}2......\text{m},\ q = \text{n-1})$$

$$L_i(s) = (s + a_i) - \mu_{rq+1}\frac{[(k+1)n-i]\lambda_r}{L_{i+1}(s)}$$

(for $i = kn+q$, k = 1,2......m, q = n-2)

$$G(s) = s(1 + \sum_{i=1}^{(m+1)n-1} Y_i(s) + \sum_{j=(m+1)n}^{(m+1)n+m} a_j(s)\frac{1-W_j(s)}{s}) \tag{6.13}$$

$$W_j(s) = \int_0^{\infty} e^{-sx} w_j(x)dx \tag{6.14}$$

(for j = (m+1)n, (m+1)n+1...(m+1)n+m)

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta]\ \mu_j(x)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

The Laplace transform of the robot-safety system availability with one normally working safety unit is given by:

$$\mathrm{AV}_{rs}(s) = \sum_{i=0}^{mn-1} P_i(s) = \frac{1 + \sum_{i=1}^{mn-1} Y_i(s)}{G(s)} \tag{6.15}$$

The Laplace transform of the robot-safety system availability with or without the working safety unit is expressed by:

$$\mathrm{AV}_{r}(s) = \sum_{i=0}^{(m+1)n-1} P_i(s) = \frac{1 + \sum_{i=1}^{(m+1)n-1} Y_i(s)}{G(s)} \tag{6.16}$$

Taking the inverse Laplace transforms of the above equations, we can get the time dependent state probabilities, P_i(t)and P_j(t), and system availabilities, AVrs(t) and AVr(t).

6.4 Generalized Robot-Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (6.1)-(6.9) we get:

$$\mathrm{a}_0\mathrm{P}_0 = \mu_{r1}\mathrm{P}_1 + \sum_{j=(m+1)n}^{(m+1)n+m} P_j(x)\mu_j(x)dx \tag{6.17}$$

$$\mathrm{a}_i\mathrm{P}_i = (\mathrm{n\text{-}i+1})\lambda_r \mathrm{P}_{i-1} + \mu_{ri+1}\mathrm{P}_{i+1} \tag{6.18}$$

(for i = 1,2.........n-2)

$$\mathrm{a}_{n-1}\mathrm{P}_{n-1} = 2\lambda_r \mathrm{P}_{n-2} \tag{6.19}$$

$$\mathrm{a}_{kn}\mathrm{P}_n = \lambda_s \mathrm{P}_{(k-1)n} + \mu_{r1}\mathrm{P}_{kn+1} \tag{6.20}$$

(for k = 1,2.........m-1)

$$\mathrm{a}_i\mathrm{P}_i = [(\mathrm{k+1})\mathrm{n\text{-}i+1}]\lambda_r \mathrm{P}_{i-1} + \lambda_s \mathrm{P}_{i-n} + \mu_{rq+1}\mathrm{P}_{i+1} \tag{6.21}$$

(for $i = kn+q$: k= 1,2.......,m ; q = 1,2......,n-2)

$$a_{(k+1)n-1} P_{(k+1)n-1} = \lambda_s P_{kn-1} + 2\lambda_r P_{(k+1)n-2} \tag{6.22}$$

(for k = 1,2.........m)

$$a_{mn} P_{mn} = \lambda_s P_{(m-1)n} + \mu_{r1} P_{mn+1} \tag{6.23}$$

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \qquad (\text{for } j = (m+1)n, (m+1)n+1, \ldots .(m+1)n+m) \tag{6.24}$$

The associated conditions as follows:

$$P_j(0) = \lambda_r P_{[(m+1)(n+1)-j]n-1} \qquad (\text{ for } j = (m+1)n, (m+1)n+1, \ldots .(m+1)n+m) \tag{6.25}$$

Solving Equation (6.17) – (6.24) * , together with

$$\sum_{i=0}^{(m+1)n-1} P_i + \sum_{j=(m+1)n}^{(m+1)n+m} P_j = 1 \tag{6.26}$$

we get:

$$P_0 = (1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)n+m} a_j E_j[x])^{-1} = \frac{1}{G} \tag{6.27}$$

$$P_i = Y_i \, P_0 \qquad (\text{for } i = kn+q, \ k = 0,1,2 \ldots\ldots\ldots m, \ q = 0, 1 \ldots.. n-1) \tag{6.28}$$

The letter k denotes the number of failed safety units

The letter q denotes the number of failed robots

$$P_j = a_j E_j[x] \, P_0 \qquad (\text{for } j = (m+1)n, (m+1)n+1, \ldots .(m+1)n+m) \tag{6.29}$$

where

$$Y_0 = 1$$

$$Y_i = \prod_{l=1}^{i} b_l \qquad (\text{for } i = kn+q, \ k = 0, \ q = 1,2 \ldots.. n-1)$$

$$\mu_{r0} = 1$$

* : Please see Appendix E.2.

$$\mathrm{Y}_i = \sum_{h=0}^{n-1} \frac{\lambda_s \prod_{g=0}^{h} \mu_{rg}}{\prod_{v=i}^{i+h} L_v} Y_{i-n+h} \qquad \text{(for } i = kn,\ k = 1,2\ldots\ldots\text{m)}$$

$$\mathrm{Y}_i = \frac{\lambda_s}{L_i} Y_{i-n} + \frac{[(k+1)n-i+1]\lambda_r}{L_i} Y_{i-1} + \sum_{h=i-n+1}^{kn-1} \frac{\prod_{g=q+1}^{h-(k-1)n} \mu_{rg}}{\prod_{f=i}^{h+n} L_f} Y_h$$

(for $i = kn+q$, $k = 1,2\ldots.\text{m}$, $q = 1,2\ldots..\text{n-1}$)

$$\mathrm{a}_j = \lim_{s\to 0} a_j(s) = \lambda_r \mathrm{P}_{[(m+1)(n+1)-j]n-1}$$

(for j = (m+1)n, (m+1)n+1,….(m+1)n+m)

$$\mathrm{b}_l = \lim_{s\to 0} b_l(s) = \frac{[(k+1)n-i+1]\lambda_r}{a_i}$$

(for $i = kn+q$, $k = 0$, $q = \text{n-1}$)

$$\mathrm{b}_l = \lim_{s\to 0} b_l(s) = \frac{[(k+1)n-i+1]\lambda_r}{a_i - \mu_{ri+1} b_{i+1}}$$

(for $i = kn+q$, $k = 0$, $q = 1,2\ldots..\text{n-2}$)

$$\mathrm{L}_i = \lim_{s\to 0} L_i(s) = \mathrm{a}_i \qquad \text{(for } i = kn+q,\ k = 1,2\ldots\ldots\text{m},\ q = \text{n-1)}$$

$$\mathrm{L}_i = \lim_{s\to 0} L_i(s) = \mathrm{a}_i - \mu_{rq+1} \frac{[(k+1)n-i]\lambda_r}{L_{i+1}}$$

(for $i = kn+q$, $k = 1,2\ldots\ldots\text{m}$, $q = \text{n-2}$)

$$\mathrm{G} = 1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)n+m} a_j E_j[x] \tag{6.30}$$

$$\mathrm{E}_j\,[\mathrm{x}] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta)d\delta]dx \tag{6.31}$$

$$= \int_0^\infty x w_j(x)dx \qquad \text{(for } j = \text{(m+1)n, (m+1)n+1…(m+1)n+m)}$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

The generalized steady state availability of the robot safety system with one normally working safety unit is given by:

$$\mathrm{SSAV}_{rs} = \sum_{i=0}^{mn-1} P_i = \frac{1 + \sum_{i=1}^{mn-1} Y_i}{G} \tag{6.32}$$

The generalized steady state availability of the robot safety system with or without one working safety unit is expressed by:

$$\mathrm{SSAV}_{r} = \sum_{i=0}^{(m+1)n-1} P_i = \frac{1 + \sum_{i=1}^{(m+1)n-1} Y_i}{G} \tag{6.33}$$

For different failed system repair time distributions, we get different G as follows:

i) For the failed robot-safety system gamma distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{\mu_j^{\beta} x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \quad (\beta > 0,\ \mu_j > 0, j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m)$$

(6.34)

where

x is the repair time, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β

is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$\mathrm{E}_j(\mathrm{x}) = \int_0^\infty x w_j(x)dx = \frac{\beta}{\mu_j} \tag{6.35}$$

Substituting Equation (6.35) into Equation (6.30), we get:

$$\mathrm{G} = 1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)n+m} a_j \frac{\beta}{\mu_j} \tag{6.36}$$

ii) For the failed robot-safety system Weibull distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \mu_j \beta x^{\beta-1} e^{-\mu_j (x)^\beta} \tag{6.37}$$

$$(\beta > 0,\ \mu_j > 0 j = (\mathrm{m}+1)\mathrm{n},\ (\mathrm{m}+1)\mathrm{n}+1, \ldots .(\mathrm{m}+1)\mathrm{n}+\mathrm{m}\ )$$

where

x is the repair time, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$\mathrm{E}_j[\mathrm{x}] = \int_0^\infty x W_j(x)dx = (\frac{1}{\mu_j})^{1/\beta} \frac{1}{\beta} \Gamma(\frac{1}{\beta}) \tag{6.38}$$

Substituting Equation (6.38) into Equation (6.30), we get:

$$\mathrm{G} = 1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)n+m} a_j (\frac{1}{\mu_j})^{1/\beta} \frac{1}{\beta} \Gamma(\frac{1}{\beta}) \tag{6.39}$$

iii) For the failed robot-safety system Rayleigh distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j x e^{-\mu_j x^2/2} \qquad (\mu_j > 0, j = (m+1)n, (m+1)n+1, \ldots .(m+1)n+m) \qquad (6.40)$$

where

x is the repair time, μ_j is the scale parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty x W_j(x) dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (6.41)$$

Substituting Equation (6.41) into Equation (6.30), we get:

$$G = 1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)n+m} a_j \sqrt{\frac{\pi}{2\mu_j}} \qquad (6.42)$$

iv) For the failed robot system lognormally distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{1}{\sqrt{2\pi} x \sigma_{y_j}} e^{[\frac{-(Inx-\mu_{y_j})^2}{2\sigma_{y_j}^2}} \qquad (6.43)$$

$$(\mu_j > 0, j = (m+1)n, (m+1)n+1, \ldots .(m+1)n+m)$$

where

x is the repair time, Inx is the natural logarithm of x with a mean μ and variance σ^2. The conditions on parameters are:

$$\sigma_{y_j} = \text{In}\sqrt{1 + (\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \qquad (6.44)$$

$$\mu_{y_j} = \text{In}\sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \qquad (6.45)$$

Thus, the mean time to robot-safety system repair is given by:

$$\mathrm{E}_j(\mathrm{x}) = e^{(\mu_{yj}+\frac{\sigma_{yj}^2}{2})} \tag{6.46}$$

Substituting Equation (6.46) into Equation (6.30), we get:

$$\mathrm{G} = 1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)n+m} a_j\, e^{(\mu_{yj}+\frac{\sigma_{yj}^2}{2})} \tag{6.47}$$

v) For the failed robot system exponentially distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j(\mathrm{x}) = \mu_j\, e^{-\mu_j x} \qquad (\mu_j > 0, j = (\mathrm{m+1})\mathrm{n}, (\mathrm{m+1})\mathrm{n+1}, \ldots .(\mathrm{m+1})\mathrm{n+m}) \tag{6.48}$$

where

x is the repair time, μ_j is the constant repair rate of state *j*.

Thus, the mean time to robot-safety system repair is given by:

$$\mathrm{E}_j(\mathrm{x}) = \int_0^{\infty} x w_j(x) dx = \frac{1}{\mu_j} \tag{6.49}$$

Substituting Equation (6.49) into Equation (6.30), we get:

$$\mathrm{G} = 1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n4n}^{(m+1)n+m} a_j \frac{1}{\mu_j} \tag{6.50}$$

6.5 Robot Safety System Reliability and Mean Time to Failure Analysis

Setting $\mu_j = 0$, (for j = (m+1)n, (m+1)n+1...(m+1)n+m), in Figure 6.2 and using the Markov method [73], we get the following differential equations:

$$\frac{dP_0(t)}{dt} + \mathrm{a}_0 \mathrm{P}_0(\mathrm{t}) = \mu_{r1} \mathrm{P}_1(\mathrm{t}) \tag{6.51}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_r P_{i-1}(t) + \mu_{ri+1} P_{i+1}(t) \tag{6.52}$$

(for i = 1, 2.........n-2)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) \tag{6.53}$$

$$\frac{dP_{kn}(t)}{dt} + a_{kn} P_{kn}(t) = \lambda_s P_{(k-1)n}(t) + \mu_{r1} P_{kn+1}(t) \tag{6.54}$$

(for k = 1, 2.........m-1)

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(k+1)n-i+1]\lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) + \mu_{rq+1} P_{i+1}(t) \tag{6.55}$$

(for $i = kn+q$: k= 1,2.......,m ; q = 1,2......,n-2)

$$\frac{dP_{(k+1)n-1}(t)}{dt} + a_{(k+1)n-1} P_{(k+1)n-1}(t) = \lambda_s P_{kn-1}(t) + 2\lambda_r P_{(k+1)n-2}(t) \tag{6.56}$$

(for k = 1,2.........m)

$$\frac{dP_{mn}(t)}{dt} + a_{mn} P_{mn}(t) = \lambda_s P_{(m-1)n}(t) + \mu_{r1} P_{mn+1}(t) \tag{6.57}$$

$$\frac{dP_j(t)}{dt} = \lambda_r P_{[(m+1)(n+1)-j]n-1}(t) \qquad \text{(for } j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m) \tag{6.58}$$

At time t = 0, $P_0(0)$ =1 and all other initial state probabilities are equal to zero.

By solving Equations (6.51) – (6.58) with the aid of Laplace transforms *, we get:

$$P_0(s) = [s(1 + \sum_{i=1}^{(m+1)n-1} Y_i(s) + \sum_{j=(m+1)n}^{(m+1)n+m} \frac{a_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{6.59}$$

$$P_i(s) = Y_i(s) P_0(s) \qquad \text{(for } i = kn+q,\ k = 0,1,2 \ldots m,\ q = 0, 1 \ldots n-1) \tag{6.60}$$

The letter k denotes the number of failed safety units

* : Please see Appendix E.3.

The letter q denotes the number of failed robots

$$P_j(s) = \frac{a_j(s)}{s} P_0(s) \quad (\text{for } j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m) \tag{6.61}$$

$$G(s) = s[1 + \sum_{i=1}^{(m+1)n-1} Y_i(s) + \sum_{j=(m+1)n}^{(m+1)n+m} \frac{a_j(s)}{s}] \tag{6.62}$$

The Laplace transform of the robot-safety system reliability with one normally working safety unit is given by:

$$R_{rs}(s) = \sum_{i=0}^{mn-1} P_i(s) = [1 + \sum_{i=1}^{mn-1} Y_i(s)]P_0(s) \tag{6.63}$$

The Laplace transform of the robot safety system reliability with or without one working safety unit is expressed by:

$$R_r(s) = \sum_{i=0}^{(m+1)n-1} P_i(s) = [1 + \sum_{i=1}^{(m+1)n-1} Y_i(s)]P_0(s) \tag{6.64}$$

Using Equation (4.63) and Reference [73], the following robot-safety system mean time to failure expression with one normally working safety unit was obtained:

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{1 + \sum_{i=1}^{mn-1} Y_i}{\sum_{j=(m+1)n}^{(m+1)n+m} a_j} \tag{6.65}$$

Similarly, using equation (6.64) and Reference [73], the robot-safety system mean time to failure expression with or without one working safety unit was obtained:

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{1 + \sum_{i=1}^{(m+1)n-1} Y_i}{\sum_{j=(m+1)n}^{(m+1)n+m} a_j} \tag{6.66}$$

6.6 Special Case Analysis for Robot-Safety System

Setting: m =2, n = 2 and $\mu_s \neq 0$, in Figure 6.2 we obtain the following equations:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_2(t) + \sum_{j=6}^{8} P_j(x,t)\mu_j(x)dx \tag{6.67}$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) \tag{6.68}$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_s P_0(t) + \mu_{r1} P_3(t) + \mu_s P_4(t) \tag{6.69}$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = \lambda_s P_1(t) + 2\lambda_r P_2(t) \tag{6.70}$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = \lambda_s P_2(t) + \mu_{r1} P_5(t) \tag{6.71}$$

$$\frac{dP_5(t)}{dt} + a_5 P_5(t) = \lambda_s P_3(t) + 2\lambda_r P_4(t) \tag{6.72}$$

where

$$a_0 = 2\lambda_r + \lambda_s$$

$$a_i = \lambda_r + \mu_{r1} + \lambda_s \qquad \text{(for } i = 1, 3)$$

$$a_2 = 2\lambda_r + \lambda_s + \mu_s$$

$$a_4 = 2\lambda_r + \mu_s$$

$$a_5 = \lambda_r + \mu_{r1}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \qquad \text{(for } j = 6, 7, 8\text{)} \tag{6.73}$$

The associated boundary conditions as follows:

$$P_j(0,t) = \lambda_r P_{[9-j]2-1}(t) \qquad \text{(for } j = 6, 7, 8\text{)} \tag{6.74}$$

At time t = 0, $P_0(0) = 1$, and other initial state probabilities are equal to zero.

6.6.1 Time Dependent Analysis for the Special Case

By solving Equations (6.67) – (6.74) with the Laplace transform method *, we get the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{5} Y_i(s) + \sum_{j=6}^{8} a_j(s)\frac{1-W_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{6.75}$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad \text{(for } i = 1, 2, 3, 4, 5\text{)} \tag{6.76}$$

$$P_j(s) = a_j(s)\, \frac{1-W_j(s)}{s} P_0(s) \qquad \text{(for } j = 6, 7, 8\text{)} \tag{6.77}$$

where

$$Y_1(s) = \frac{2\lambda_r}{L_1(s)}$$

$$Y_2(s) = \frac{\lambda_s}{L_2(s)} + (\mu_{r1}\frac{\lambda_s}{L_2(s)L_3(s)} + \mu_{r1}\,\mu_s\frac{\lambda_s^2}{L_2(s)L_3(s)L_4(s)L_5(s)})Y_1(s)$$

$$Y_3(s) = \frac{2\lambda_r}{L_3(s)}Y_2(s) + \frac{\lambda_s}{L_3(s)}Y_1(s)$$

$$Y_4(s) = \frac{\lambda_s}{L_4(s)}Y_2(s) + \mu_{r1}\frac{\lambda_s}{L_4(s)L_5(s)}Y_3(s)$$

$$Y_5(s) = \frac{2\lambda_r}{L_5(s)}Y_4(s) + \frac{\lambda_s}{L_5(s)}Y_3(s)$$

$$a_6(s) = \lambda_r Y_5(s)$$

$$a_7(s) = \lambda_r Y_3(s)$$

* : Please see Appendix E.4.1.

$$a_8(s) = \lambda_r Y_1(s)$$

$$L_1(s) = s + a_1$$

$$L_2(s) = (s+a_2) - \mu_{r1}\frac{2\lambda_r}{L_3(s)} - \mu_s\frac{\lambda_s}{L_4(s)} - \mu_s\,\mu_{r1}\frac{2\lambda_r\lambda_s}{L_3(s)L_4(s)L_5(s)}$$

$$L_3(s) = s + a_3$$

$$L_4(s) = (s+a_4) - \mu_{r1}\frac{2\lambda_r}{L_5(s)}$$

$$L_5(s) = s + a_5$$

$$G(s) = s[1 + \sum_{i=1}^{5} Y_i(s) + \sum_{j=6}^{8} a_j(s)\frac{1-W_j(s)}{s}] \tag{6.78}$$

$$W_j(s) = \int_0^{\infty} e^{-sx} w_j(x)dx \qquad (\text{for } j = 6, 7, 8) \tag{6.79}$$

$$w_j(x) = \exp[-\int_0^{x}\mu_j(\delta)d\delta]\,\mu_j(x)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

The Laplace transform of the robot-safety system availability with one normally working safety unit is given by:

$$AV_{rs}(s) = \sum_{i=0}^{3} P_i(s) = \frac{1+\sum_{i=1}^{3} Y_i(s)}{G(s)} \tag{6.80}$$

The Laplace transform of the robot-safety system availability with or without a normally working safety unit is given by:

$$AV_r(s) = \sum_{i=0}^{5} P_i(s) = \frac{1+\sum_{i=1}^{5} Y_i(s)}{G(s)} \tag{6.81}$$

Taking the inverse Laplace transforms of the above equations, we can obtain the time dependent state probabilities, $P_i(t)$and $P_j(t)$, and robot-safety system availabilities, AVrs(t) and AVr(t).

Thus, for the exponentially distributed failed robot-safety system repair time x, the probability function is expressed by

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 6, 7, 8) \qquad (6.82)$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Substituting Equation (6.82) into Equation (6.79), we get

$$W_j(s) = \frac{\mu_j}{s+\mu_j} \qquad (\mu_j > 0, j = 6, 7, 8) \qquad (6.83)$$

By inserting Equation (6.83) into Equations (6.75)-(6.81), setting:

$\lambda_s = 0.002$, $\lambda_r = 0.003$, $\mu_s = 0.002$, $\mu_{r1} = 0.002$, $\mu_6 = 0.003$, $\mu_7 = 0.004$, $\mu_8 = 0.005$; and using Matlab computer program [74], the Figure 6.3 and Figure 6.4 plots were obtained. These plots show that state probabilities and system availability decrease and increase with varying time t.

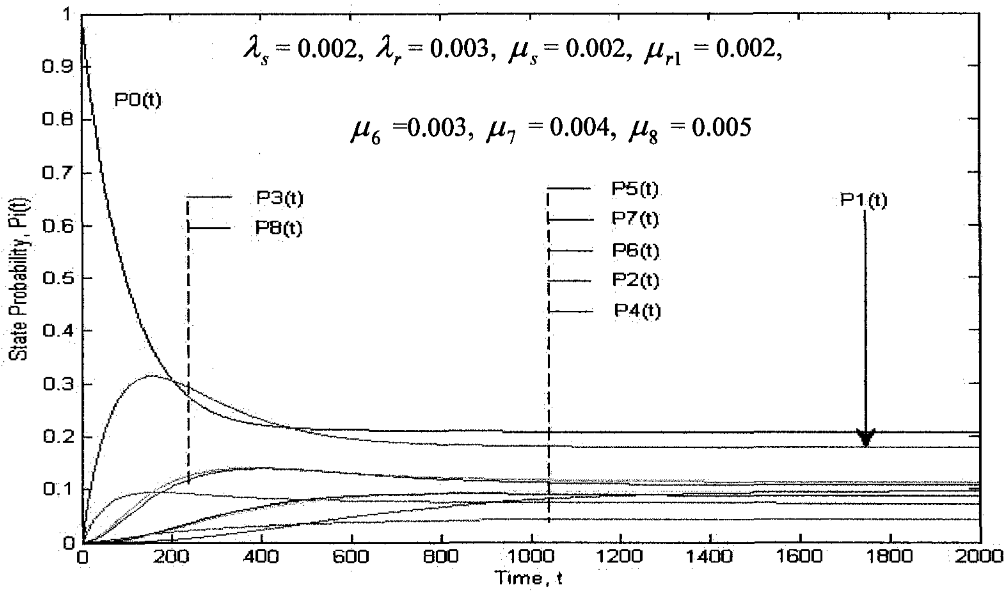


Figure 6.3 Time-dependent state probability plots for a robot-safety system with exponentially distributed failed system repair times.

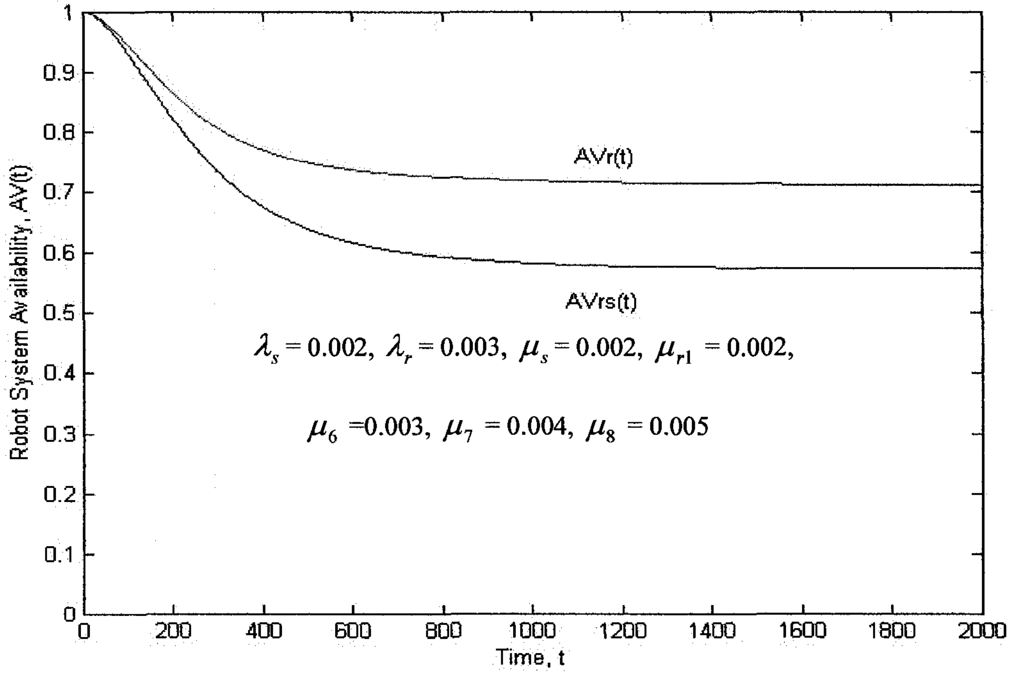


Figure 6.4 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair times.

6.6.2 Steady State Analysis for the Special Case

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (6.67)-(6.74) we get:

$$a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_2\ (t) + \sum_{j=6}^{8} P_j(x,t)\mu_j(x)dx \tag{6.84}$$

$$a_1 P_1\ (t) = 2\lambda_r P_0(t) \tag{6.85}$$

$$a_2 P_2(t) = \lambda_s P_0(t) + \mu_{r1} P_3\ (t) + \mu_s P_4(t) \tag{6.86}$$

$$a_3 P_3\ (t) = \lambda_s P_1\ (t) + 2\lambda_r P_2\ (t) \tag{6.87}$$

$$a_4 P_4(t) = \lambda_s P_2(t) + \mu_{r1} P_5\ (t) \tag{6.88}$$

$$a_5 P_5(t) = \lambda_s P_3\ (t) + 2\lambda_r P_4\ (t) \tag{6.89}$$

$$\frac{\partial P_j(x)}{\partial x} + \mu_j(x)\, P_j(x) = 0 \qquad \text{(for } j = 6, 7, 8) \tag{6.90}$$

The associated boundary conditions as follows:

$$P_j(0) = \lambda_r P_{[9-j]2-1} \qquad \text{(for } j = 6, 7, 8) \tag{6.91}$$

Solving Equation (6.84) – (6.91) *, together with

$$\sum_{i=0}^{5} P_i + \sum_{j=6}^{8} P_j = 1 \tag{6.92}$$

we get:

$$P_0 = [(1 + \sum_{i=1}^{5} Y_i + \sum_{j=6}^{8} a_j E_j[x])]^{-1} = \frac{1}{G} \tag{6.93}$$

$$P_i = Y_i P_0 \qquad \text{(for } i = 1, 2, 3, 4, 5) \tag{6.94}$$

$$P_j = a_j E_j\, [x]\, P_0 \qquad \text{(for } j = 6, 7, 8) \tag{6.95}$$

* : Please see Appendix E.4.2.

where

$$Y_1 = \frac{2\lambda_r}{L_1}$$

$$Y_2 = \frac{\lambda_s}{L_2} + (\mu_r \frac{\lambda_s}{L_2 L_3} + \mu_{r1}\, \mu_s \frac{\lambda_s^2}{L_2 L_3 L_4 L_5})Y_1$$

$$Y_3 = \frac{2\lambda_r}{L_3} Y_2 + \frac{\lambda_s}{L_3} Y_1$$

$$Y_4 = \frac{\lambda_s}{L_4} Y_2 + \mu_{r1} \frac{\lambda_s}{L_4 L_5} Y_3$$

$$Y_5 = \frac{2\lambda_r}{L_5} Y_4 + \frac{\lambda_s}{L_5(s)} Y_3$$

$$a_6 = \lambda_r Y_5$$

$$a_7 = \lambda_r Y_3$$

$$a_8 = \lambda_r Y_1$$

$$L_1 = a_1$$

$$L_2 = a_2 - \mu_{r1} \frac{2\lambda_r}{L_3} - \mu_s \frac{\lambda_s}{L_4} - \mu_s\, \mu_{r1} \frac{2\lambda_r \lambda_s}{L_3 L_4 L_5}$$

$$L_3 = a_3$$

$$L_4 = a_4 - \mu_{r1} \frac{2\lambda_r}{L_5}$$

$$L_5 = a_5$$

$$G = 1 + \sum_{i=1}^{5} Y_i + \sum_{j=6}^{8} a_j E_j[x] \tag{6.96}$$

$$E_j[x] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta) d\delta] dx \tag{6.97}$$

$$= \int_0^{\infty} x w_j(x) dx \qquad \text{(for } j = 6, 7, 8\text{)}$$

where

w_j (x) is the failed robot safety system repair time probability density function

E_j [x] is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

The steady state availability of the robot-safety system with one normally working safety unit is given by:

$$\text{SSAV}_{rs} = \sum_{i=0}^{3} P_i = \frac{1 + \sum_{i=1}^{3} Y_i}{G} \tag{6.98}$$

The steady state availability of the robot-safety system with or without one working safety unit is expressed by:

$$\text{SSAV}_r = \sum_{i=0}^{5} P_i = \frac{1 + \sum_{i=1}^{5} Y_i}{G} \tag{6.99}$$

1) For exponentially distributed failed robot-safety system repair time substituting Equation (6.49) into Equations (6.93) - (6.99), setting:

$\lambda_s = 0.002$, $\lambda_r = 0.003$, $\mu_{r1} = 0.002$, $\mu_6 = 0.003$, $\mu_7 = 0.004$, $\mu_8 = 0.005$; and using Matlab computer program [74], the Figure 6.5 and 6.6 plots were obtained.

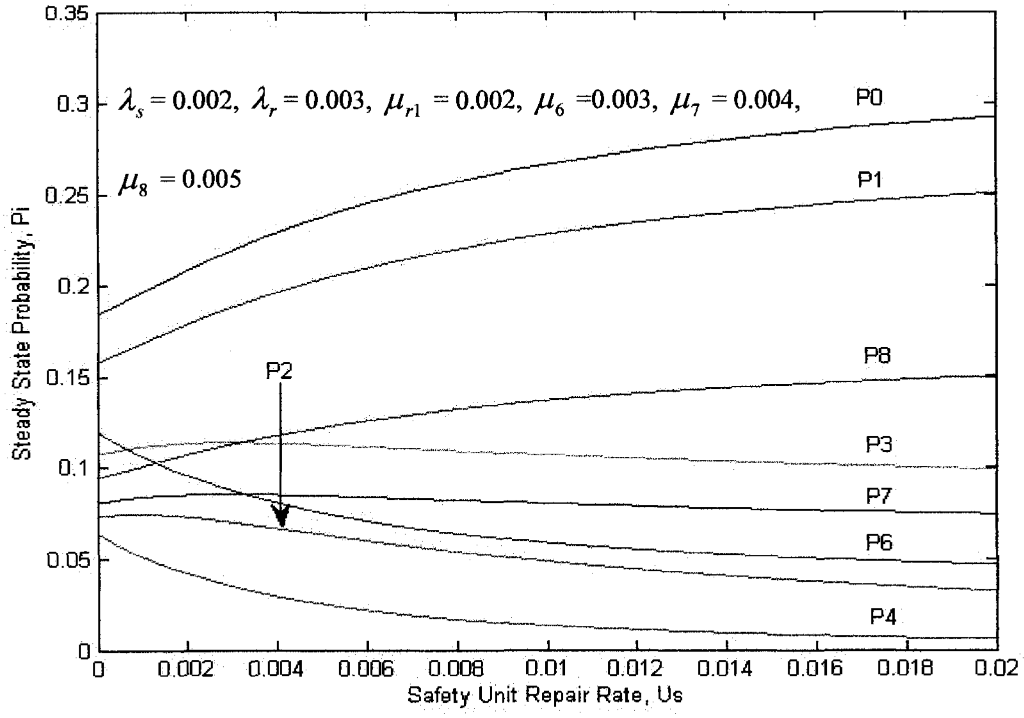


Figure 6.5 Robot-safety system steady state probability versus safety unit repair rate Us(means μ_s) plots with exponentially distributed failed system repair times.

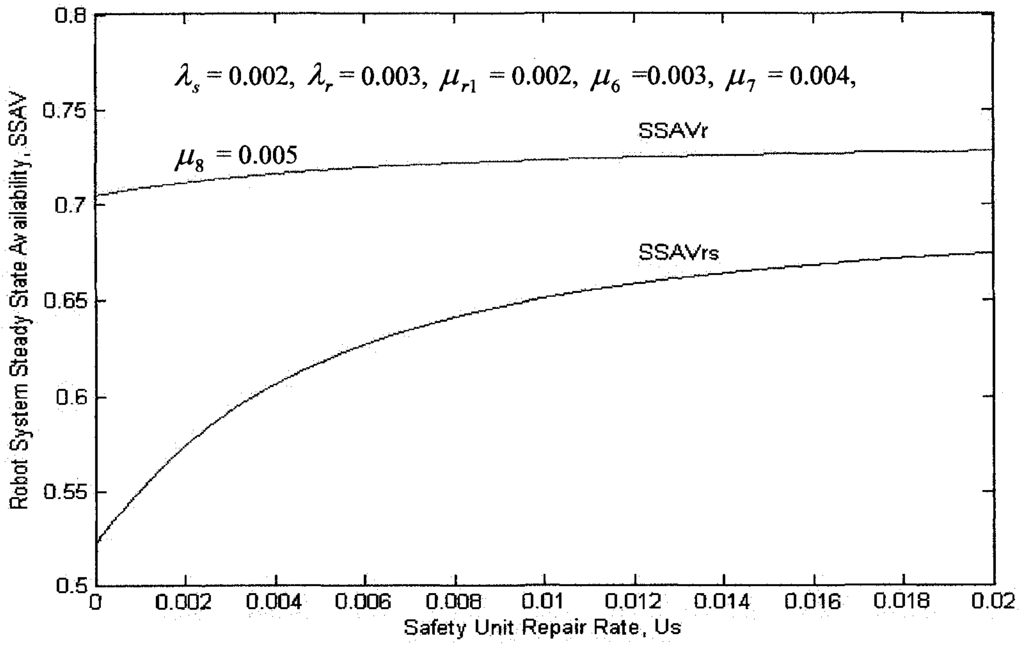


Figure 6.6 Robot system steady state availability versus safety unit repair rate plots Us(means μ_s) with exponentially distributed failed system repair times.

2) For Rayleigh distributed failed robot-safety system repair time, substituting Equation (6.41) into Equations (6.93)-(6.99), setting:

$\lambda_s = 0.002$, $\lambda_r = 0.003$, $\mu_{r1} = 0.002$, $\mu_6 = 0.003$, $\mu_7 = 0.004$, $\mu_8 = 0.005$; and using Matlab computer program [74], the Figure 6.7 and 6.8 plots were obtained.

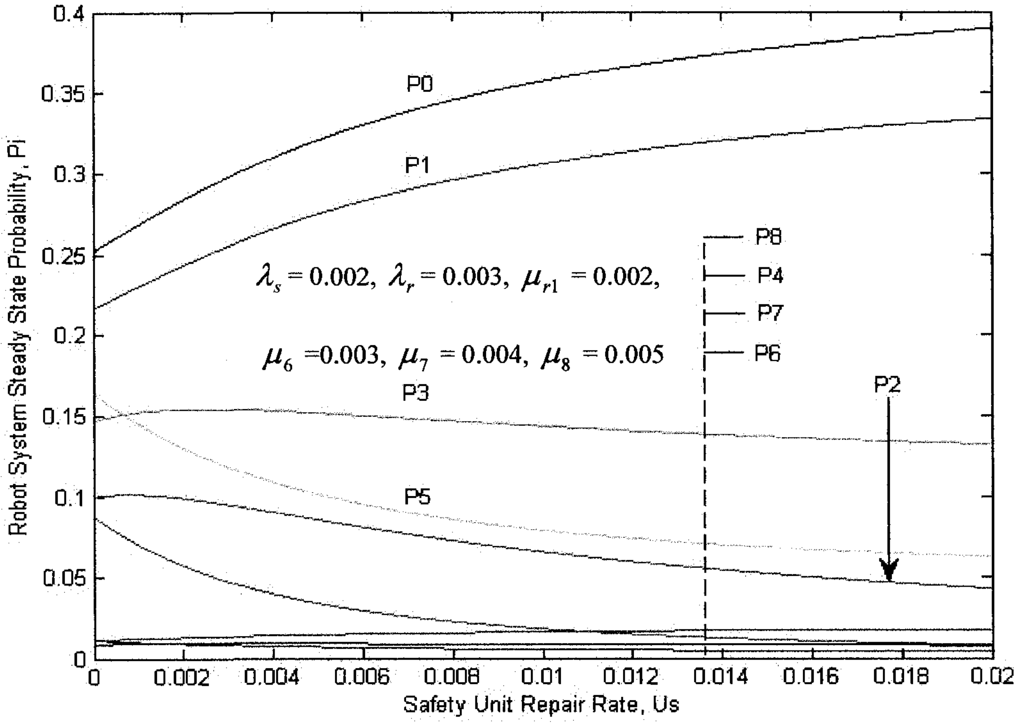


Figure 6.7 Robot-safety system steady state probability versus safety unit repair rate Us(means μ_s) plots with Rayleigh distributed failed system repair times

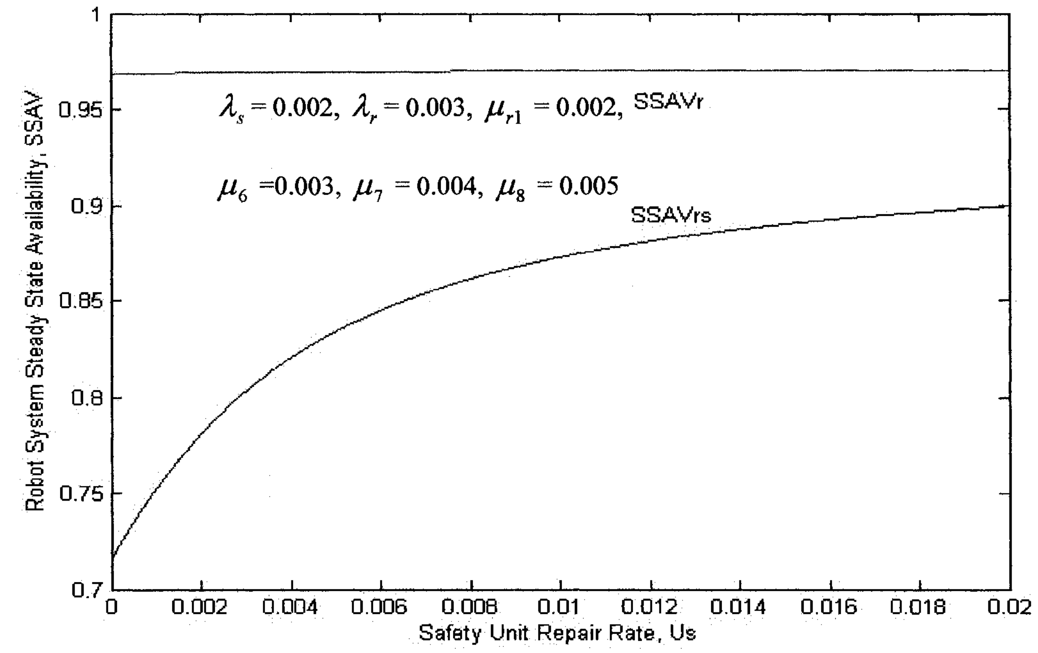


Figure 6.8 Robot-safety system steady state availability versus safety unit repair rate Us(means μ_s) plots with Rayleigh distributed failed system repair times.

3) For lognormally distributed failed robot-safety system repair time, substituting Equation (6.46) into Equations (6.93)-(6.99), setting:

$\lambda_s = 0.002$, $\lambda_r = 0.003$, $\mu_{r1} = 0.002$, $\mu_6 = 0.003$, $\mu_7 = 0.004$, $\mu_8 = 0.005$, $\sigma = 3$; and using Matlab computer program [74], the Figure 6.9 and 6.10 plots were obtained.

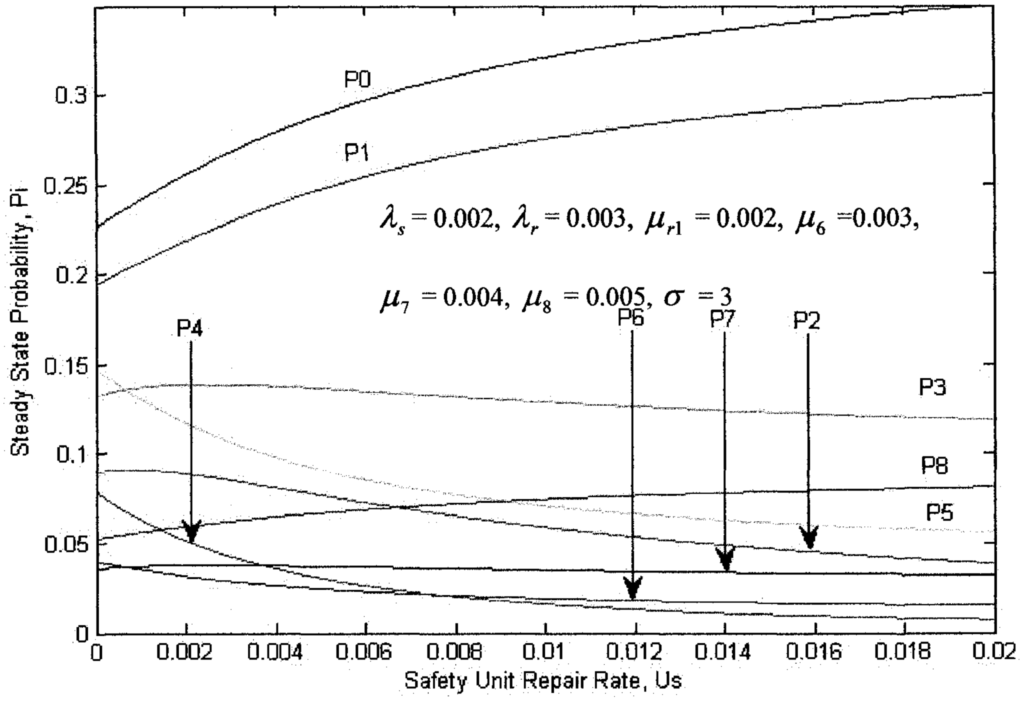


Figure 6.9 Robot-safety system steady state probability versus safety unit repair rate Us(means μ_s) plots with lognormally distributed failed system repair times.

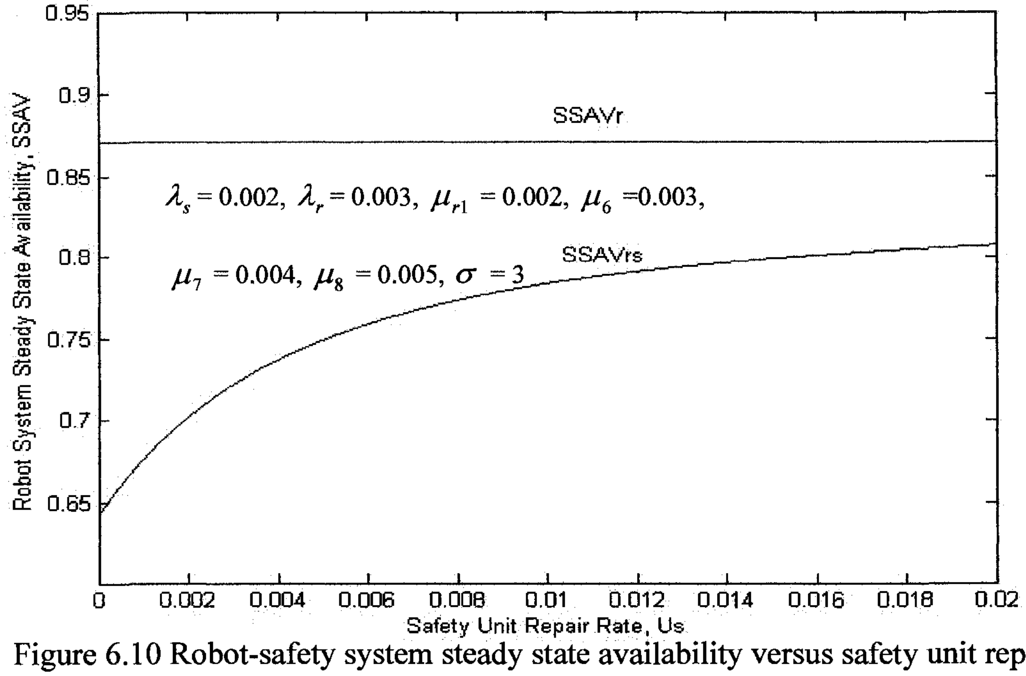


Figure 6.10 Robot-safety system steady state availability versus safety unit repair rate Us(means μ_s) plots for lognormally distributed failed system repair times.

6.6.3 Reliability and Mean Time to Failure Analysis for the Special Case

Setting $\mu_j = 0$, (for $j = 6, 7, 8$), in Figure 6.2 and using the Markov method [73], we get the following equations:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_2(t) \qquad (6.100)$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) \qquad (6.101)$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_s P_0(t) + \mu_{r1} P_3(t) + \mu_s P_4(t) \qquad (6.102)$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = \lambda_s P_1(t) + 2\lambda_r P_2(t) \qquad (6.103)$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = \lambda_s P_2(t) + \mu_{r1} P_5(t) \qquad (6.104)$$

$$\frac{dP_5(t)}{dt} + a_5 P_5(t) = \lambda_s P_3(t) + 2\lambda_r P_4(t) \qquad (6.105)$$

$$\frac{dP_j(t)}{dt} = \lambda_r P_{(9-j)2-1}(t) \qquad \text{(for } j = 6, 7, 8) \qquad (6.106)$$

At time t = 0, $P_0(0) = 1$ and all other initial conditions state probabilities are equal to zero.

By solving Equations (6.100) – (6.106) with the aid of Laplace transforms *, we get:

$$P_0(s) = [s(1 + \sum_{i=1}^{5} Y_i(s) + \sum_{j=6}^{8} \frac{a_j(s)}{s})]^{-1} = \frac{1}{G(s)} \qquad (6.107)$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad \text{(for } i = 1, 2, 3, 4, 5) \qquad (6.108)$$

$$P_j(s) = \frac{a_j(s)}{s} P_0(s) \qquad \text{(for } j = 6, 7, 8) \qquad (6.109)$$

* : Please see Appendix E.4.3.

$$G(s) = s\left[1+\sum_{i=1}^{5} Y_i(s) + \sum_{j=6}^{8} \frac{a_j(s)}{s} \right] \tag{6.110}$$

The Laplace transform of the robot-safety system reliability with one normally working safety unit is given by:

$$R_{rs}(s) = \sum_{i=0}^{3} P_i(s) = \frac{1+\sum_{i=1}^{3} Y_i(s)}{G(s)} \tag{6.111}$$

The Laplace transform of the robot-safety system reliability with or without one normally working safety unit is given by:

$$R_r(s) = \sum_{i=0}^{5} P_i(s) = \frac{1+\sum_{i=1}^{5} Y_i(s)}{G(s)} \tag{6.112}$$

Using Equation (6.111) and Reference [73], the robot safety system mean time to failure with one normally working safety unit is given by

$$MTTF_{rs} = \lim_{s\to 0} R_{rs}(s) = \frac{1+\sum_{i=1}^{3} Y_i}{\sum_{j=6}^{8} a_j} \tag{6.113}$$

Similarly, using equation (6.112) and Reference [73], the robot-safety system mean time to failure with or without one normally working safety unit is given by:

$$MTTF_r = \lim_{s\to 0} R_r(s) = \frac{1+\sum_{i=1}^{5} Y_i}{\sum_{j=6}^{8} a_j} \tag{6.114}$$

For $\lambda_s = 0.002$, $\lambda_r = 0.003$, $\mu_{r1} = 0.002$, and using Equations (6.113) and (6.114), the Figure 6.11 plots were obtained.

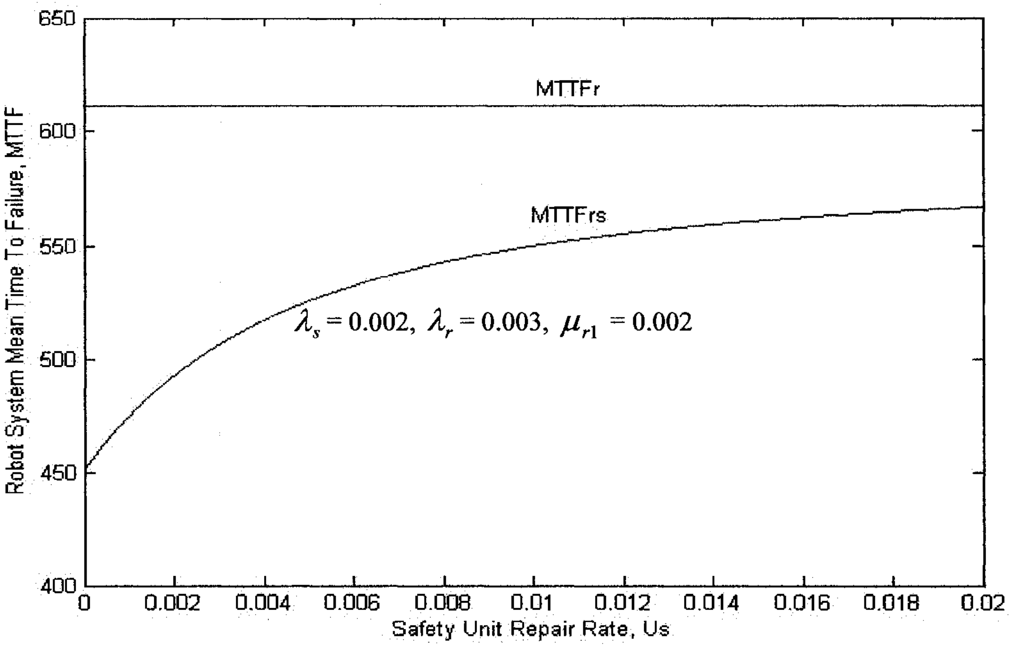


Figure 6.11 The robot-safety system mean time to failure plots for the increasing values of the safety unit repair rate Us(means μ_s).

6.7 Conclusions

- For exponentially distributed failed robot system repair time x, the system time dependent availability decreases with the increasing time t.
- For exponentially, Rayleigh or lognormally distributed failed robot system repair time x, the system steady state availability increases as the safety unit repair rate increases.
- For exponentially distributed failed robot system repair time x, the robot-safety system mean time to failure (MTTF) increases with the increasing value of the safety unit repair rate.

Stochastic Analysis of a System containing Two Parallel Robots and One Standby Safety Unit with an Imperfect Switch

7.1 Introduction

This chapter presents a mathematical model of a robot-safety system which containing two robots and one standby safety unit and an imperfect switch. More specifically, the robot system is composed of two identical robots in parallel, two identical safety units, and a switch to replace a failed safety unit.

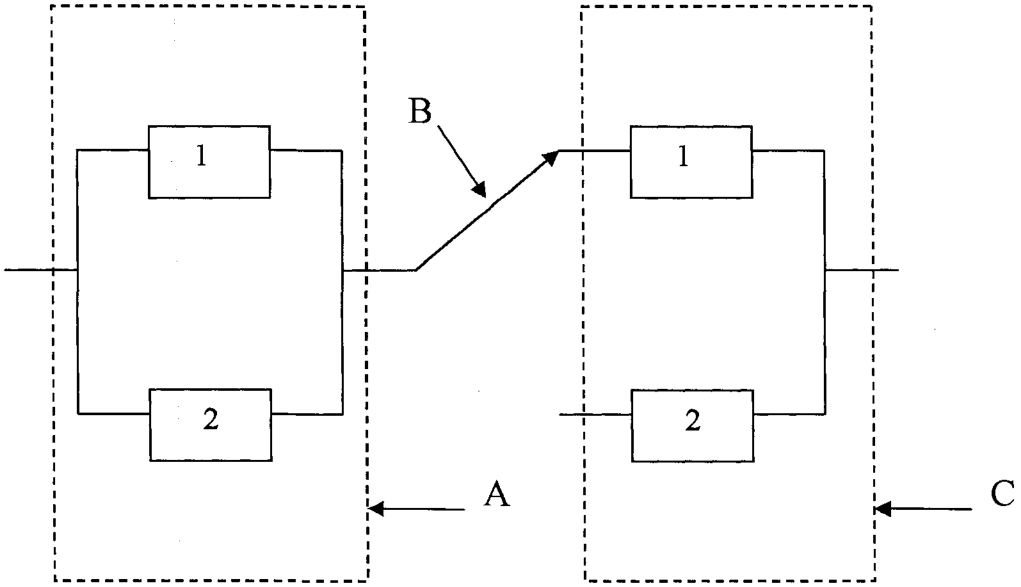
The block diagram of the robot system is shown in Figure7.1 and its corresponding state space diagram is presented in Figure7.2. The numerals in the boxes of Figure7.2 denote system states.

At time t =0, two robots, one safety unit and the switch to replace a failed safety unit start operating and another safety unit is on standby. The overall robot-safety system can fail only if both robots fail.

The following assumptions are associated with this model:

- The robot-safety system is composed of 2 identical robots in parallel, 2 identical safety units (only one operates and the other remains on standby) and a switch which can fail.
- Two robots, the switch and one safety unit start operating simultaneously.
- Only the completely failed robot-safety system can be repaired. Failure rates of robots, safety units and switch are constant.
- The failed robot-safety system repair rates can be constant or non-constant.
- All failures are statistically independent.

- The repaired total robot-safety system is as good as new.



A: two identical Robots

B: Switch for replacing a failed safety unit and it cannot fail.

C: n identical Safety Units (one operating and n-1 on standby)

Figure 7.1 The block diagram of the robot-safety system containing two parallel robots and one standby safety unit with an imperfect switch

7.1.1 Notation:

The following symbols are associated with the model:

i i^{th} state of the robot-safety system:

for i = q, means q robots, the switch and one safety unit are working normally while (2-q) robots have failed. (i.e. q = 0,1)

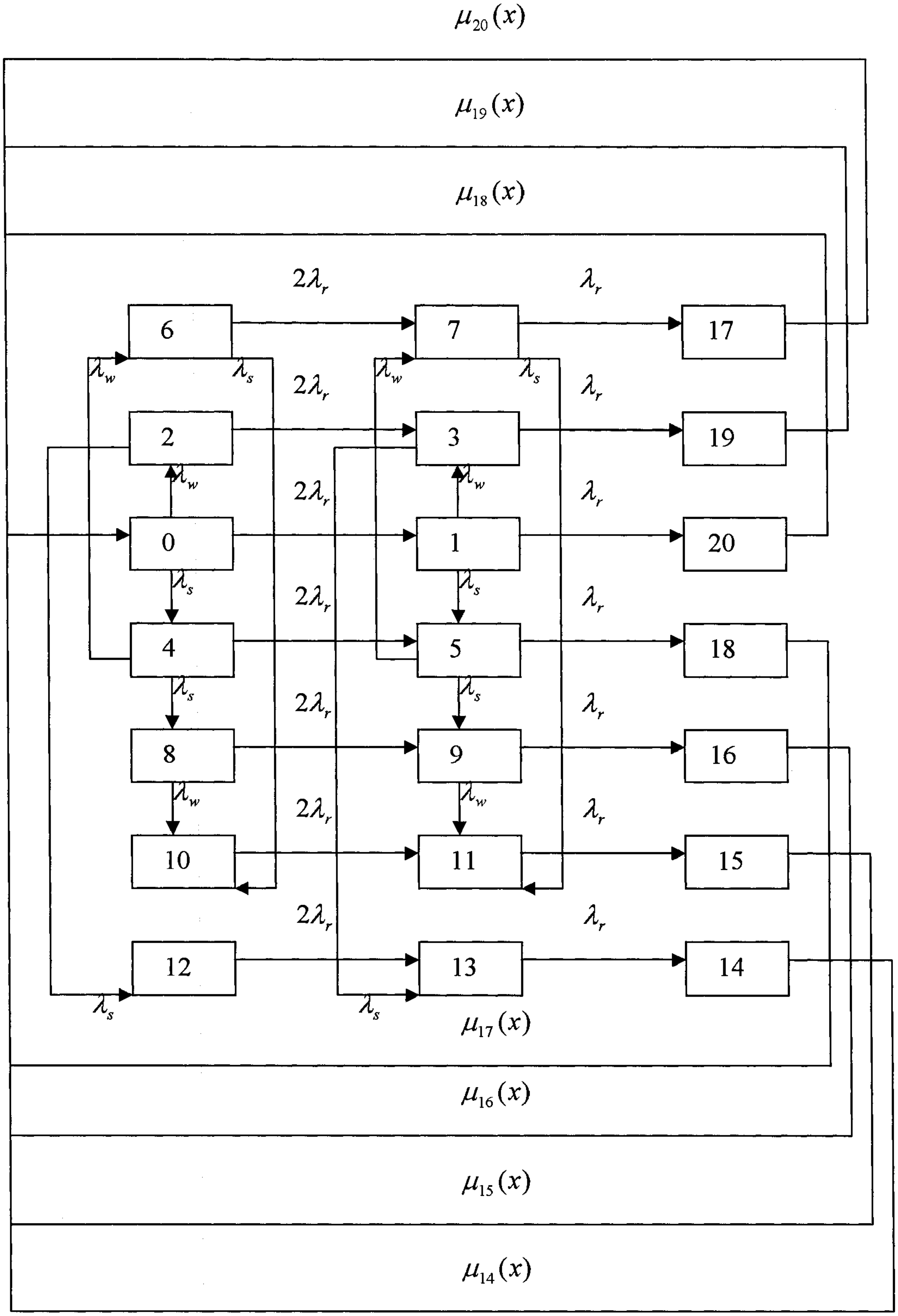


Figure 7.2 The state space diagram of the robot-safety system containing two parallel robots, and one standby safety unit with an imperfect switch

for i = 2+q, means q robots and one safety are working normally while the switch has failed and another safety unit is considered failed. (i.e. q = 0,1)

for i = 4+q, means q robots, the switch and one safety unit are working normally while another safety unit has failed. (i.e. q = 0,1)

for i = 6+q, means q robots and one safety are working normally while another safety unit and the switch have failed. (i.e. q = 0,1)

for i = 8+q, means q robots and the switch are working normally while both the two safety units have failed. (i.e. q = 0,1)

for i = 10+q, means q robots are working normally while the switch and the two safety units have failed. (i.e. q = 0,1)

for i = 12+q, means q robots are working normally while one safety unit has failed, another safety unit is considered failed and the switch has failed. (i.e. q = 0,1)

j j^{th} state of the robot–safety system:

for j = 14, means the total robot-safety system has failed. (i.e., both the robots, one safety unit and the switch have failed and another safety unit is considered failed.)

for j = 15, means the total robot-safety system has failed. (i.e., all the robots, all the safety units and the switch have failed.)

for j = 16, means the total robot-safety system has failed. (i.e., all the robots and all the safety units have failed.)

for j = 17, means the total robot-safety system has failed. (i.e., all the robots, one safety unit and the switch have failed.)

	for $j = 18$, means the total robot-safety system has failed. (i.e., all the robots and one safety unit have failed.)
	for $j = 19$, means the total robot-safety system has failed. (i.e., all the robots and the switch have failed and one safety unit is considered failed.)
	for $j = 20$, means the total robot-safety system has failed. (i.e., all the robots have failed.)
t	Time.
λ_s	Constant failure rate of a safety unit.
λ_r	Constant failure rate of a robot.
λ_w	Constant failure rate of the switch
$\Delta \mathbf{x}$:	Finite repair time interval.
$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in state j: and has an elapsed repair time of x; for $j = 14, 15 \ldots 20$.
$P_j(x,t)\ \Delta x$	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval $[x, x+\Delta x]$; for $j = 14, 15 \ldots 20$.
pdf	Probability density function.
$w_j(x)$	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for $j = 14, 15 \ldots 20$.
$P_j(t)$	Probability that the robot safety system is in state j at time t; for $j = 14, 15 \ldots 20$.
$P_i(t)$	Probability that the robot-safety system is in state i at time t; for $i = 0, 1, 2 \ldots 13$.
P_i	Steady state probability that the robot-safety system is in state i; for $i=0, 1, 2\ldots$

	13.
P_j	Steady state probability that robot-safety system is in state j; for j = 14, 15...20.
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i; for i = 0,1,2...13.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for j = 14, 15...20.
AVrs(s)	Laplace transform of the robot-safety system availability with one normally working safety unit.
AVr(s)	Laplace transform of the robot-safety system availability with or without one normally safety unit.
AVrs (t)	Robot-safety system time dependent availability with one normally working safety unit.
AVr(t)	Robot-safety system time dependent availability with or without one normally working safety unit.
SSAVrs	Robot-safety system steady state availability with one normally working safety unit, the switch and the robot.
SSAVr	Robot-safety system steady state availability with or without one normally working safety unit.
Rrs(s)	Laplace transform of the robot-safety system reliability with one normally working safety unit.
Rr(s)	Laplace transform of the robot safety system reliability with or without one normally working safety unit.

MTTFrs	Robot-safety system mean time to failure with one normally working safety unit.
MTTFr	Robot-safety system mean time to failure with or without one normally working safety unit.

7.2 Generalized Robot-Safety System Analysis

By using the supplementary method [71, 72], we writ down the following equations for the Figure 7.2 diagram:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \sum_{j=14}^{20} P_j(x,t)\mu_j(x)dx \tag{7.1}$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) \tag{7.2}$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_w P_0(t) \tag{7.3}$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = \lambda_w P_1(t) + 2\lambda_r P_2(t) \tag{7.4}$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = \lambda_s P_0(t) \tag{7.5}$$

$$\frac{dP_5(t)}{dt} + a_5 P_5(t) = \lambda_w P_1(t) + 2\lambda_r P_4(t) \tag{7.6}$$

$$\frac{dP_6(t)}{dt} + a_4 P_6(t) = \lambda_w P_4(t) \tag{7.7}$$

$$\frac{dP_7(t)}{dt} + a_7 P_7(t) = \lambda_w P_4(t) + 2\lambda_r P_6(t) \tag{7.8}$$

$$\frac{dP_8(t)}{dt} + a_8 P_8(t) = \lambda_s P_4(t) \tag{7.9}$$

$$\frac{dP_9(t)}{dt} + a_9 P_9(t) = \lambda_s P_5(t) + 2\lambda_r P_8(t) \tag{7.10}$$

$$\frac{dP_{10}(t)}{dt} + a_{10} P_{10}(t) = \lambda_w P_8(t) + \lambda_s P_6(t) \tag{7.11}$$

$$\frac{dP_{11}(t)}{dt} + a_{11} P_{11}(t) = \lambda_w P_9(t) + \lambda_s P_7(t) + 2\lambda_r P_{10}(t) \tag{7.12}$$

$$\frac{dP_{12}(t)}{dt} + a_{12} P_{12}(t) = \lambda_s P_2(t) \tag{7.13}$$

$$\frac{dP_{13}(t)}{dt} + a_{13} P_{13}(t) = \lambda_s P_3(t) + 2\lambda_r P_{12}(t) \tag{7.14}$$

where

$a_i = 2\lambda_r + \lambda_s + \lambda_w$ (for $i = 0, 4$)

$a_i = 2\lambda_r + \lambda_s$ (for $i = 2, 6$)

$a_8 = 2\lambda_r + \lambda_w$

$a_i = 2\lambda_r$ (for $i = 10, 12$)

$a_i = \lambda_r + \lambda_s + \lambda_w$ (for $i = 1, 5$)

$a_i = \lambda_r + \lambda_s$ (for $i = 3, 7$)

$a_9 = \lambda_w + \lambda_s$

$a_i = \lambda_r$ (for $i = 11, 13$)

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x)\, P_j(x,t) = 0 \qquad (\text{for } j = 14, 15 \ldots\ldots\ldots 20) \tag{7.15}$$

The associated boundary conditions are as follows:

$$P_j(0,t) = \lambda_r P_{(21-j)2-1}(t) \qquad (\text{for } j = 14, 15 \ldots\ldots\ldots 20) \tag{7.16}$$

At time t = 0, $P_0(0) = 1$, and other initial state probabilities are equal to zero.

7.3 Robot-Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from Equations (7.1)-(7.16) we get:

$$a_0 P_0 = \sum_{j=14}^{20} P_j(x)\mu_j(x)dx \tag{7.17}$$

$$a_1 P_1 = 2\lambda_r P_0 \tag{7.18}$$

$$a_2 P_2 = \lambda_w P_0 \tag{7.19}$$

$$a_3 P_3 = \lambda_w P_1 + 2\lambda_r P_2 \tag{7.20}$$

$$a_4 P_4 = \lambda_s P_0 \tag{7.21}$$

$$a_5 P_5 = \lambda_w P_1 + 2\lambda_r P_4 \tag{7.22}$$

$$a_4 P_6 = \lambda_w P_4 \tag{7.23}$$

$$a_7 P_7 = \lambda_w P_4 + 2\lambda_r P_6 \tag{7.24}$$

$$a_8 P_8 = \lambda_s P_4 \tag{7.25}$$

$$a_9 P_9 = \lambda_s P_5 + 2\lambda_r P_8 \tag{7.26}$$

$$a_{10} P_{10} = \lambda_w P_8 + \lambda_s P_6 \tag{7.27}$$

$$a_{11} P_{11} = \lambda_w P_9 + \lambda_s P_7 + 2\lambda_r P_{10} \tag{7.28}$$

$$a_{12} P_{12} = \lambda_s P_2 \tag{7.29}$$

$$a_{13} P_{13} = \lambda_s P_3 + 2\lambda_r P_{12} \tag{7.30}$$

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \qquad \text{(for } j = 14, 15 \ldots\ldots\ldots 20) \tag{7.31}$$

The associated boundary conditions are as follows:

$$P_j(0) = \lambda_r P_{(21-j)2-1} \qquad \text{(for } j = 14, 15 \ldots\ldots\ldots 20) \tag{7.32}$$

Solving Equations (7.17) - (7.32) *, together with

$$\sum_{i=0}^{13} P_i + \sum_{j=14}^{20} P_j = 1 \qquad (7.33)$$

we get:

$$P_0 = (1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j E_j[x])^{-1} = \frac{1}{G} \qquad (7.34)$$

$$P_i = Y_i P_0 \qquad \text{(for } i = 1, 2 \ldots\ldots 13) \qquad (7.35)$$

$$P_j = a_j E_j[x]\, P_0 \qquad \text{(for } j = 14, 15 \ldots\ldots\ldots\ldots 20) \qquad (7.36)$$

where

$$Y_1 = \frac{2\lambda_r}{a_1}$$

$$Y_2 = \frac{\lambda_w}{a_2}$$

$$Y_3 = \frac{\lambda_w}{a_3}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_3}\frac{\lambda_w}{a_2}$$

$$Y_4 = \frac{\lambda_s}{a_4}$$

$$Y_5 = \frac{\lambda_s}{a_5}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_5}\frac{\lambda_s}{a_4}$$

$$Y_6 = \frac{\lambda_w}{a_6}\frac{\lambda_s}{a_4}$$

$$Y_7 = \frac{\lambda_w}{a_7}(\frac{\lambda_s}{a_5}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_5}\frac{\lambda_s}{a_4}) + \frac{2\lambda_r}{a_7}\frac{\lambda_w}{a_6}\frac{\lambda_s}{a_4}$$

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* : Please see Appendix F1.

$$Y_8 = \frac{\lambda_s}{a_8}\frac{\lambda_s}{a_4}$$

$$Y_9 = \frac{\lambda_s}{a_9}\left(\frac{\lambda_s}{a_5}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_5}\frac{\lambda_s}{a_4}\right) + \frac{2\lambda_r}{a_9}\frac{\lambda_s}{a_8}\frac{\lambda_s}{a_4}$$

$$Y_{10} = \frac{\lambda_s}{a_{10}}\frac{\lambda_w}{a_6}\frac{\lambda_s}{a_4} + \frac{\lambda_w}{a_{10}}\frac{\lambda_s}{a_8}\frac{\lambda_s}{a_4}$$

$$Y_{11} = \frac{\lambda_s}{a_{11}}\left[\frac{\lambda_w}{a_7}\left(\frac{\lambda_s}{a_5}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_5}\frac{\lambda_s}{a_4}\right) + \frac{2\lambda_r}{a_7}\frac{\lambda_w}{a_6}\frac{\lambda_s}{a_4}\right] + \frac{\lambda_w}{a_{11}}\left[\frac{\lambda_s}{a_9}\left(\frac{\lambda_s}{a_5}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_5}\frac{\lambda_s}{a_4}\right) + \frac{2\lambda_r}{a_9}\frac{\lambda_s}{a_8}\frac{\lambda_s}{a_4}\right] + \frac{2\lambda_r}{a_{11}}\left(\frac{\lambda_s}{a_{10}}\frac{\lambda_w}{a_6}\frac{\lambda_s}{a_4} + \frac{\lambda_w}{a_{10}}\frac{\lambda_s}{a_8}\frac{\lambda_s}{a_4}\right)$$

$$Y_{12} = \frac{\lambda_s}{a_{12}}\frac{\lambda_w}{a_2}$$

$$Y_{13} = \frac{\lambda_s}{a_{13}}\left(\frac{\lambda_w}{a_3}\frac{2\lambda_r}{a_1} + \frac{2\lambda_r}{a_3}\frac{\lambda_w}{a_2}\right) + \frac{2\lambda_r}{a_{13}}\frac{\lambda_s}{a_{12}}\frac{\lambda_w}{a_2}$$

$$a_j = \lambda_r Y_{(21-j)2-1} \qquad (\text{for } j = 14, 15 \ldots\ldots 20)$$

$$G = 1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j E_j[x] \tag{7.37}$$

$$E_j[x] = \int_0^\infty \exp\left[-\int_0^x \mu_j(\delta)d\delta\right]dx \tag{7.38}$$

$$= \int_0^\infty x w_j(x)dx \qquad (\text{for } j = 14, 15 \ldots\ldots 20)$$

where

w_j (x) is the failed robot safety system repair time probability density function.

E_j [x] is the mean time to robot safety system repair when the failed robot safety system is in state *j* and has an elapsed repair time x.

The steady state availability of the robot safety system with one normally working normally safety unit, the switch and the robot is given by:

$$\mathrm{SSAV}_{rs} = \sum_{i=0}^{7} P_i = \frac{1+\sum_{i=1}^{7} Y_i}{G} \tag{7.39}$$

Similarly, the steady state availability of the robot safety system with or without a normally working safety unit is:

$$\mathrm{SSAV}_{r} = \sum_{i=0}^{13} P_i = \frac{1+\sum_{i=1}^{13} Y_i}{G} \tag{7.40}$$

For different failed robot-safety system repair time distributions, we get different expressions for G as follows:

i) For the failed robot-safety system gamma distributed repair time x, the probability density function is expressed by

$$\mathrm{w}_j\,(\mathrm{x}) = \frac{\mu_j^{\beta} x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \qquad (\beta > 0,\ j = 14, 15 \ldots 20) \tag{7.41}$$

where

x is the repair time variable, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$\mathrm{E}_j\,(\mathrm{x}) = \int_0^{\infty} x w_j(x) dx = \frac{\beta}{\mu_j} \qquad (\beta > 0,\ j = 14, 15 \ldots 20) \tag{7.42}$$

Substituting Equation (7.42) into Equation (7.37), we get

$$G = 1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j \frac{\beta}{\mu_j} \tag{7.43}$$

ii) For the failed robot-safety system Weibull distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j (x)^\beta} \qquad (\beta > 0, j = 14, 15 \ldots 20) \tag{7.44}$$

where

x is the repair time variable, μ_j is the scale parameter and β is the shape parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^\infty x W_j(x) dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \qquad (\beta > 0, j = 14, 15 \ldots 20) \tag{7.45}$$

Substituting Equation (7.45) into Equation (7.37), we get

$$G = G = 1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \tag{7.46}$$

iii) For the failed robot-safety system Rayleigh distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j x e^{-\mu_j x^2 / 2} \qquad (\mu_j > 0,\ j = 14, 15 \ldots 20) \tag{7.47}$$

where

x is the repair time variable, μ_j is the scale parameter.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty xW_j(x)dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (\mu_j > 0, j = 14, 15 \ldots 20) \tag{7.48}$$

Substituting Equation (7.48) into Equation (7.37), we get

$$G = 1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j \sqrt{\frac{\pi}{2\mu_j}} \tag{7.49}$$

iv) For the failed robot system lognormally distributed repair time x, the probability density function is expressed by

$$w_j(x) = \frac{1}{\sqrt{2\pi x \sigma_{y_j}}} e^{[\frac{-(Inx-\mu_{yj})^2}{2\sigma^2_{yj}}} \qquad (\text{for } j = 14, 15 \ldots 20) \tag{7.50}$$

where

x is the repair time variable, Inx is the natural logarithm of x with a mean μ and variance σ^2. The conditions on parameters are:

$$\sigma_{y_j} = \text{In}\sqrt{1 + (\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \tag{7.51}$$

$$\mu_{y_j} = \text{In}\sqrt{\frac{\mu^4_{x_j}}{\mu^2_{x_j} + \sigma^2_{x_j}}} \tag{7.52}$$

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = e^{(\mu_{y_j} + \frac{\sigma^2_{yj}}{2})} \qquad (\text{for } j = 14, 15 \ldots 20) \tag{7.53}$$

Substituting Equation (7.53) into Equation (7.37), we get

$$G = 1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j e^{(\mu_{y_j} + \frac{\sigma^2_{yj}}{2})} \tag{7.54}$$

v) For the failed robot system exponentially distributed repair time x, the probability density function is expressed by

$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 14, 15 \ldots 20) \tag{7.55}$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*.

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^{\infty} x w_j(x) dx = \frac{1}{\mu_j} \qquad (\beta > 0, j = 14, 15 \ldots 20) \tag{7.56}$$

Substituting Equation (7.56) into Equation (7.37), we get

$$G = 1 + \sum_{i=1}^{13} Y_i + \sum_{j=14}^{20} a_j \frac{1}{\mu_j} \tag{7.57}$$

7.3.1 Robot-Safety System Steady State Analysis for a Special Case

1) For exponentially distributed failed robot-safety system repair times, substituting Equation (7.57) into Equations (7.39) - (7.40), setting:

$\lambda_r = 0.001$, $\lambda_w = 0.002$, μ_{14}=0.0001, $\mu_{15} = 0.00015$, $\mu_{16} = 0.0002$, $\mu_{17} = 0.00025$; $\mu_{18} = 0.0003$; $\mu_{19} = 0.00035$, $\mu_{20} = 0.0004$;

and using Matlab computer program [74], Figure 7.3 plots were obtained.

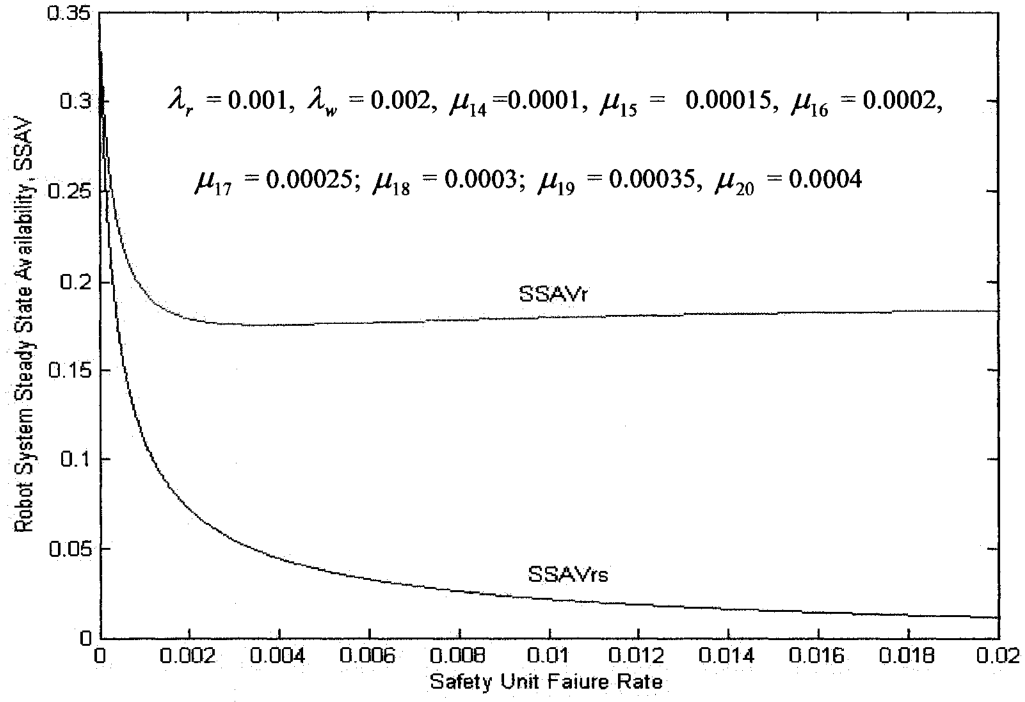

Figure 7.3 Robot-safety system steady state availability versus safety unit failure rate (λ_s) plots for exponentially distributed failed system repair times.

2) For Rayleigh distributed failed robot-safety system repair time Equation (7.49) into Equations (7.39) - (7.40), setting:

λ_r = 0.001, λ_w = 0.002, μ_{14}=0.0001, μ_{15} = 0.00015, μ_{16} = 0.0002, μ_{17} = 0.00025; μ_{18} = 0.0003; μ_{19} = 0.00035, μ_{20} = 0.0004; and using Matlab computer program [74], Figure 7.4 plots were obtained.

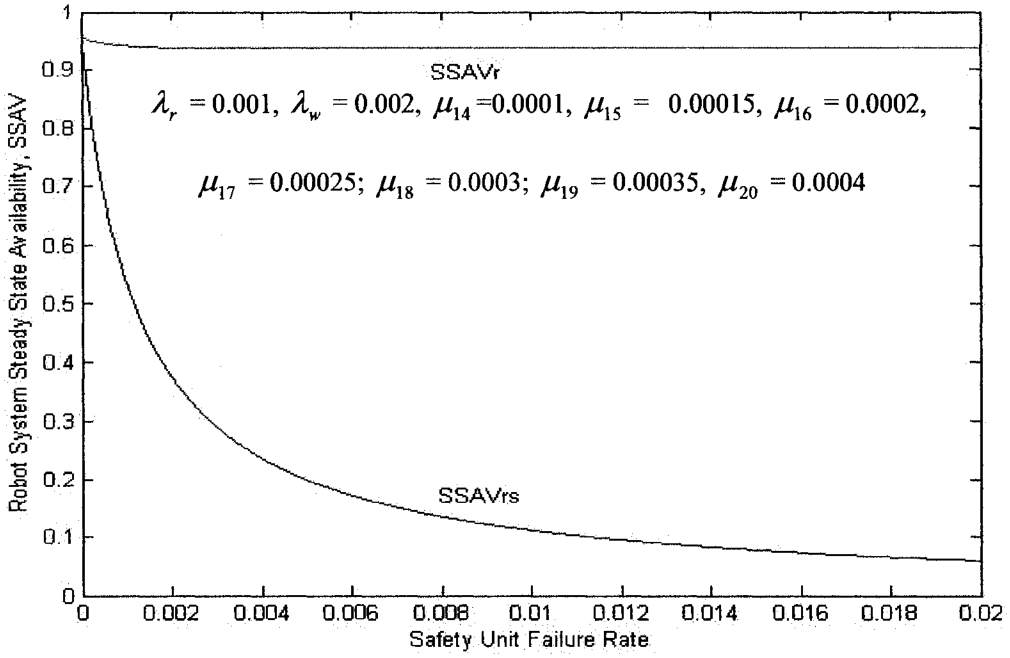

Figure 7.4 Robot-safety system steady state availability versus safety unit failure rate (λ_s) plots with Rayleigh distributed failed system repair times.

3) For lognormally distributed failed robot-safety system repair time, substituting Equation (7.54) into Equations (7.39) - (7.40), setting:

λ_r = 0.01, λ_w = 0.02, μ_{14}=0.01, μ_{15} = 0.015, μ_{16} = 0.02, μ_{17} = 0.025, μ_{18} = 0.03; μ_{19} = 0.035, μ_{20} = 0.04, $\sigma = 3$; and using Matlab computer program [74], Figure 7.5 were obtained.

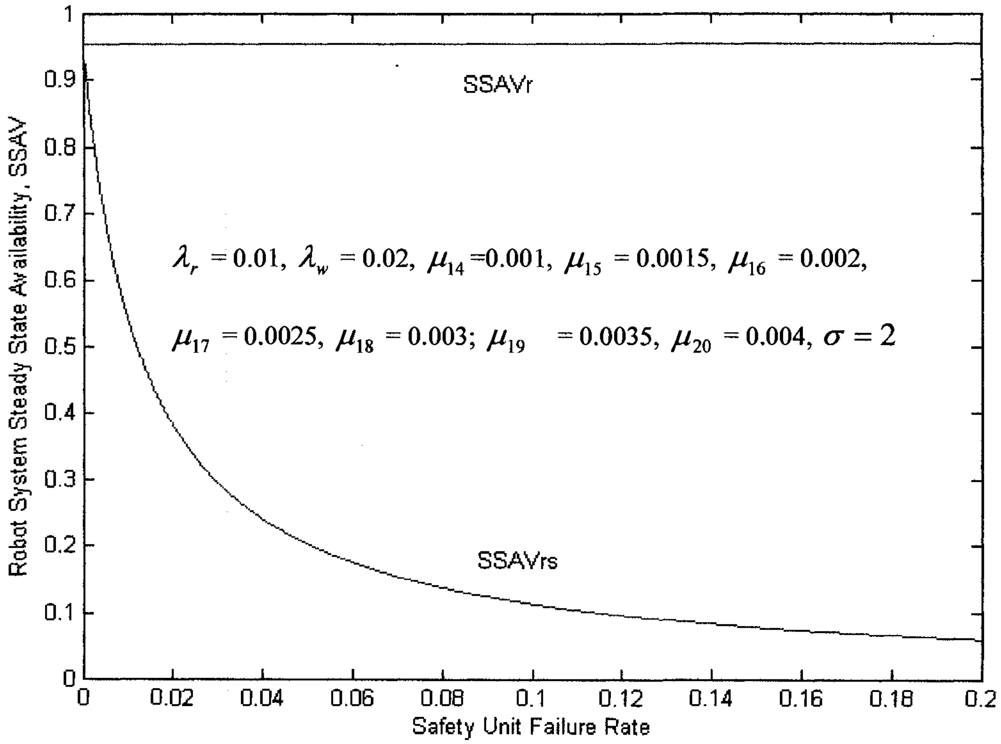

Figure7.5 Robot-safety system steady state availability versus safety unit failure rate (λ_s) plots for lognormally distributed failed system repair times.

7.4 Robot-Safety System Reliability and Mean Time to Failure Analysis

Setting $\mu_j = 0$, (for $j = 14, 15 \ldots 20$), in Figure 6.2 and using the Markov method [73], we write the following equations for the modified figure:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = 0 \tag{7.58}$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) \tag{7.59}$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_w P_0(t) \tag{7.60}$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = \lambda_w P_1(t) + 2\lambda_r P_2(t) \tag{7.61}$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = \lambda_s P_0(t) \tag{7.62}$$

$$\frac{dP_5(t)}{dt} + a_5 P_5(t) = \lambda_w P_1(t) + 2\lambda_r P_4(t) \tag{7.63}$$

$$\frac{dP_6(t)}{dt} + a_4 P_6(t) = \lambda_w P_4(t) \tag{7.64}$$

$$\frac{dP_7(t)}{dt} + a_7 P_7(t) = \lambda_w P_4(t) + 2\lambda_r P_6(t) \tag{7.65}$$

$$\frac{dP_8(t)}{dt} + a_8 P_8(t) = \lambda_s P_4(t) \tag{7.66}$$

$$\frac{dP_9(t)}{dt} + a_9 P_9(t) = \lambda_s P_5(t) + 2\lambda_r P_8(t) \tag{7.67}$$

$$\frac{dP_{10}(t)}{dt} + a_{10} P_{10}(t) = \lambda_w P_8(t) + \lambda_s P_6(t) \tag{7.68}$$

$$\frac{dP_{11}(t)}{dt} + a_{11} P_{11}(t) = \lambda_w P_9(t) + \lambda_s P_7(t) + 2\lambda_r P_{10}(t) \tag{7.69}$$

$$\frac{dP_{12}(t)}{dt} + a_{12} P_{12}(t) = \lambda_s P_2(t) \tag{7.70}$$

$$\frac{dP_{13}(t)}{dt} + a_{13} P_{13}(t) = \lambda_s P_3(t) + 2\lambda_r P_{12}(t) \tag{7.71}$$

$$\frac{dP_j(t)}{dt} = \lambda_r P_{(21-j)2-1}(t) \qquad \text{(for } j = 14, 15.......20) \tag{7.72}$$

At time t = 0, $P_0(0) = 1$ and all other initial conditions state probabilities are equal to zero.

By solving Equations (7.58) – (7.72) with the aid of Laplace transforms *, we get:

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* : Please see Appendix F.2.

$$P_0(s) = (1 + \sum_{i=1}^{13} Y_i(s) + \sum_{j=14}^{20} \frac{a_j(s)}{s})^{-1} = \frac{1}{G(s)} \tag{7.73}$$

$$P_i(s) = Y_i P_0(s) \qquad (\text{for } i = 1, 2\ldots\ldots13) \tag{7.74}$$

$$P_j = \frac{a_j(s)}{s} E_j[x]\, P_0 \qquad (\text{for } j = 14, 15\ldots\ldots\ldots\ldots20) \tag{7.75}$$

where

$$Y_1(s) = \frac{2\lambda_r}{s+a_1}$$

$$Y_2(s) = \frac{\lambda_w}{s+a_2}$$

$$Y_3(s) = \frac{\lambda_w}{s+a_3}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_3}\frac{\lambda_w}{s+a_2}$$

$$Y_4(s) = \frac{\lambda_s}{s+a_4}$$

$$Y_5(s) = \frac{\lambda_s}{s+a_5}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_5}\frac{\lambda_s}{s+a_4}$$

$$Y_6(s) = \frac{\lambda_w}{s+a_6}\frac{\lambda_s}{s+a_4}$$

$$Y_7(s) = \frac{\lambda_w}{s+a_7}\left(\frac{\lambda_s}{s+a_5}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_5}\frac{\lambda_s}{s+a_4}\right) + \frac{2\lambda_r}{s+a_7}\frac{\lambda_w}{s+a_6}\frac{\lambda_s}{s+a_4}$$

$$Y_8(s) = \frac{\lambda_s}{s+a_8}\frac{\lambda_s}{s+a_4}$$

$$Y_9(s) = \frac{\lambda_s}{s+a_9}\left(\frac{\lambda_s}{s+a_5}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_5}\frac{\lambda_s}{s+a_4}\right) + \frac{2\lambda_r}{s+a_9}\frac{\lambda_s}{s+a_8}\frac{\lambda_s}{s+a_4}$$

$$Y_{10}(s) = \frac{\lambda_s}{s+a_{10}}\frac{\lambda_w}{s+a_6}\frac{\lambda_s}{s+a_4} + \frac{\lambda_w}{s+a_{10}}\frac{\lambda_s}{s+a_8}\frac{\lambda_s}{s+a_4}$$

$$\mathrm{Y}_{11}(\mathrm{s}) = \frac{\lambda_s}{s+a_{11}}\left[\frac{\lambda_w}{s+a_7}\left(\frac{\lambda_s}{s+a_5}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_5}\frac{\lambda_s}{s+a_4}\right) + \frac{2\lambda_r}{s+a_7}\frac{\lambda_w}{s+a_6}\frac{\lambda_s}{s+a_4}\right]$$

$$+ \frac{\lambda_w}{s+a_{11}}\left[\frac{\lambda_s}{s+a_9}\left(\frac{\lambda_s}{s+a_5}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_5}\frac{\lambda_s}{s+a_4}\right) + \frac{\lambda_s}{s+a_8}\frac{\lambda_s}{s+a_4}\right] +$$

$$\frac{2\lambda_r}{s+a_{11}}\left(\frac{\lambda_s}{s+a_{10}}\frac{\lambda_w}{s+a_6}\frac{\lambda_s}{s+a_4} + \frac{\lambda_w}{s+a_{10}}\frac{\lambda_s}{s+a_8}\frac{\lambda_s}{s+a_4}\right)$$

$$\mathrm{Y}_{12}(\mathrm{s}) = \frac{\lambda_s}{s+a_{12}}\frac{\lambda_w}{s+a_2}$$

$$\mathrm{Y}_{13}(\mathrm{s}) = \frac{\lambda_s}{s+a_{13}}\left(\frac{\lambda_w}{s+a_3}\frac{2\lambda_r}{s+a_1} + \frac{2\lambda_r}{s+a_3}\frac{\lambda_w}{s+a_2}\right) + \frac{2\lambda_r}{s+a_{13}}\frac{\lambda_s}{s+a_{12}}\frac{\lambda_w}{s+a_2}$$

$$\mathrm{a}_j\ (\mathrm{s}) = \lambda_r \mathrm{Y}_{(21-j)2-1}(\mathrm{s}) \qquad (\text{for } j = 14,15......20)$$

$$\mathrm{G(s)} = 1 + \sum_{i=1}^{13} Y_i(s) + \sum_{j=14}^{20} \frac{a_j(s)}{s} \tag{7.76}$$

The Laplace transform of the robot-safety system reliability with one normally working safety unit is given by:

$$\mathrm{R}_{rs}(\mathrm{s}) = \sum_{i=0}^{7} P_i(s) = \frac{1+\sum_{i=1}^{7} Y_i(s)}{G(s)} \tag{7.77}$$

Similarly, the Laplace transform of the robot safety system reliability with or without a normally working safety unit is

$$\mathrm{R}_r\ (\mathrm{s}) = \sum_{i=0}^{13} P_i(s) = \frac{1+\sum_{i=1}^{13} Y_i(s)}{G(s)} \tag{7.78}$$

Using Equation (7.77) and Reference [73], the robot-safety system mean time to failure with one normally working safety unit is given by:

$$\mathrm{MTTF}_{rs} = \lim_{s\to 0} \mathrm{R}_{rs}(s) = \frac{1+\sum_{i=1}^{7} Y_i}{\sum_{j=14}^{20} a_j} \tag{7.79}$$

Similarly, using Equation (7.78) and Reference [73], the robot safety system mean time to failure with or without a normally working safety unit is

$$\mathrm{MTTF}_{r} = \lim_{s\to 0} \mathrm{R}_{r}(s) = \frac{1+\sum_{i=1}^{13} Y_i}{\sum_{j=14}^{20} a_j} \tag{7.80}$$

7.4.1 Robot-Safety System MTTF Analysis for a Special Case

For $\lambda_r = 0.001$, $\lambda_w = 0.002$ and using Equations (7.79)-(7.80) and Matlab computer program [74], in Figure 7.6 MTTF_{rs} and MTTF_r plots were obtained.

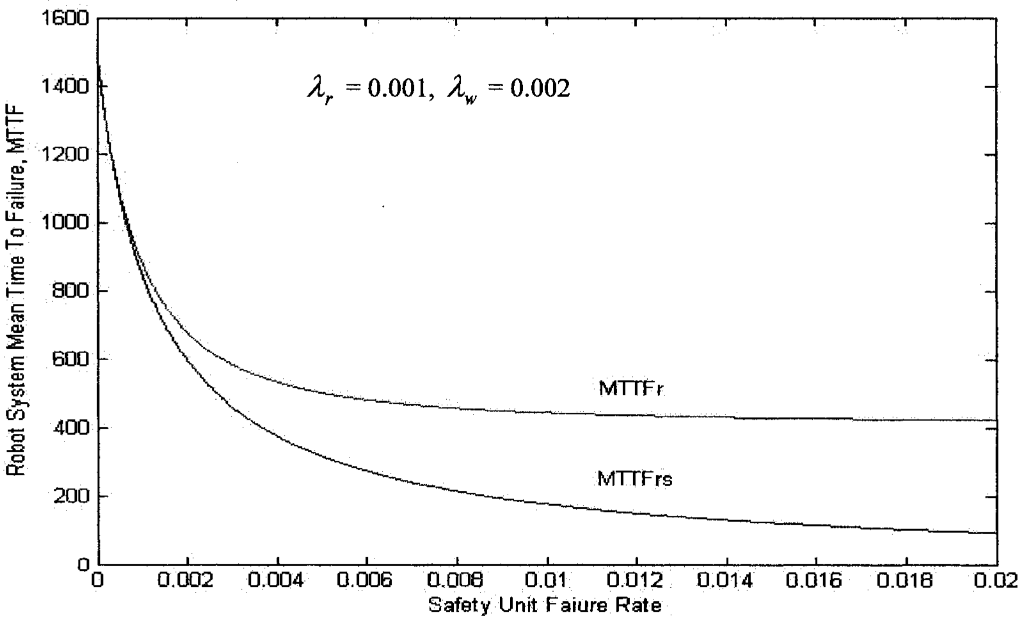

Figure 7.6 The robot-safety system mean time to failure plots for the increasing value of the safety unit failure rate (λ_s).

7.5 Conclusions

- For the exponentially, Rayleigh or lognormally distributed failed robot system repair time x, the system steady state availability decreases as the safety unit failure rate increases.
- For exponentially distributed failed robot system repair time x, the robot-safety system mean time to failure (MTTF) decreases with the increasing values of the safety unit failure rate.

Chapter 8

Discussions,

Conclusions

And Future Study

8.1 Discussion

This thesis studied standby robot-safety systems and reviewed related published literature for the period 2000-2005.

In Chapters 2, 4, and 6, analyses were conducted in a relationship to systems containing a perfect switch. Furthermore, for the models presented in those chapters, it was assumed that the repair of robot has the priority over the repair of safety unit when the overall system is in the partially operational state.

In Chapters 3, 5, and 7, this study performed analysis of systems containing an imperfect switch. Moreover, all the models in these chapters are subject to the assumption that the switch has the priority to be repaired when switch and safety unit/or robot failed simultaneously, and however robot has the priority to be repair when robot and safety unit failed simultaneously.

8.2 Conclusions

The main results from this study can be summarized as follows:

- In this study, the Markov and supplementary variable methods were successfully employed to perform state probability, system availability, reliability and mean time to failure analyses of robot-safety systems when the failed robot-safety system repair time is governed by such distributions as exponential, gamma, Weibull, Rayleigh or lognormal.

- From plots of the special cases, it is easily observed that the system steady state availability increases with the increment of safety unit repair rate.
- The generalized formulas developed in this study can be utilized to obtain robot-safety system reliability, availability and mean time to failure. Thus, they can be useful to design and maintain robot-safety systems effectively.

8.3 Future Study

- The future study can be further expanded to analyse such a robot-safety system containing n parallel robots and (m-1) standby safety units with an imperfect switch.
- Based on the assumptions, the studied robot-safety systems are composed of identical safety units and identical robots with a switch. The next step recommended is to analyse standby robot-safety systems that include non-identical robots and safety units with switch.
- In this study, all failures were assumed statistically independent. Future study should perhaps consider the failures of robots, safety units and switch which are correlated.
- The models studied the influence of robot, safety unit and switch on the robot-safety system. It may be more practical for the models to consider additional issues, which are critical to the robot-safety system, such as human error, common cause failures and so on.

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APPENDIX A

A System Containing One Robot and (n-1) Standby Safety Units with a Perfect Switch

A.1 Robot-Safety System Time Dependent Analysis

Using Laplace transforms and solving Equations (2.1) – (2.7), we get:

$$(s + a_0)\, P_0(s) = 1 + \mu_1 P_1(s) + \sum_{j=n+1}^{n+3} \int_0^{\infty} P_j(x,s)\mu_j(x)dx \tag{A.1}$$

$$(s + a_i)\, P_i(s) = \mu_{i+1} P_{i+1}(s) + \lambda_i P_{i-1}(s) \qquad \text{(for } i = 1,2 \ldots\ldots\ldots \text{n-1)} \tag{A.2}$$

$$(s + a_n)\, P_n(s) = \lambda_n P_{n-1}(s) \tag{A.3}$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)\, P_j(x,s) = 0 \qquad \text{(for } j = \text{n+1,n+2,n+3)} \tag{A.4}$$

The boundary conditions are as follows:

$$P_{n+1}(0, s) = \lambda_{ss}\, P_n(s) \tag{A.5}$$

$$P_{n+2}(0, s) = \lambda_{si}\, P_n(s) \tag{A.6}$$

$$P_{n+3}(0, s) = \lambda_r \sum_{i=0}^{n-1} P_i(s) \tag{A.7}$$

Solving Equation (A.4), we get:

$$P_j(s) = P_j(0,s)e^{-sx} \exp[- \int_0^{x} \mu_j(\delta)d\delta\,] \qquad \text{(for } j = \text{n+1,n+2,n+3)} \tag{A.8}$$

Together with

$$P_j(s) = \int_0^{\infty} P_j(x,s)dx \qquad \text{(for } j = \text{n+1,n+2,n+3)} \tag{A.9}$$

we obtain:

$$\mathrm{P}_j(\mathrm{s}) = \mathrm{P}_j(0,\mathrm{s})\,\frac{1-W_j(s)}{s} \qquad \text{(for } j = \text{n+1,n+2,n+3)} \qquad \text{(A.10)}$$

where

$$\frac{1-W_j(s)}{s} = \mathrm{P}_j(0,\mathrm{s})\int_0^{\infty} e^{-sx}\ \exp[-\int_0^{x}\mu_j(\delta)d\delta\,] \qquad \text{(A.11)}$$

$$\text{(for } j = \text{n+1,n+2,n+3)}$$

or

$$\mathrm{W}_j(\mathrm{s}) = \int_0^{\infty} e^{-sx}\,\mathrm{w}_j(\mathrm{x})\mathrm{dx} \qquad \text{(for } j = \text{n+1,n+2,n+3)} \qquad \text{(A.12)}$$

$$\mathrm{w}_j(\mathrm{x}) = \exp[-\int_0^{x}\mu_j(\delta)d\delta\,]\ \mu_j(x) \qquad \text{(A.13)}$$

where

$\mathrm{w}_j(\mathrm{x})$ is the failed robot safety system repair time probability density function

A.2 Robot-Safety System Steady State Analysis

By solving Equation (2.26), we get:

$$\mathrm{P}_j(\mathrm{x}) = \mathrm{P}_j(0)\ \exp[-\int_0^{x}\mu_j(\delta)d\delta\,] \qquad \text{(A.14)}$$

Together with:

$$\mathrm{P}_j = \int_0^{\infty} P_j(x)dx \qquad \text{(for } j = \text{n+1, n+2, n+3)} \qquad \text{(A.15)}$$

We get:

$$\mathrm{P}_j = \mathrm{P}_j(0)\mathrm{E}_j[\mathrm{x}] \qquad \text{(for } j = \text{n+1, n+2, n+3)} \qquad \text{(A.16)}$$

where

$$\mathrm{E}_j[\mathrm{x}] = \int_0^{\infty} \exp[-\int_0^{x}\mu_j(\delta)d\delta]dx$$

$$= \int_0^{\infty} x w_j(x) dx \quad (\text{for } j = \text{n+1, n+2, n+3}) \tag{A.17}$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

A.3 Robot-Safety System Reliability and Mean Time to Failure Analysis

Using Laplace transforms and solving Equations (2.56) – (2.61), we get:

$$sP_0(s) + a_0 P_0(s) = \mu_s P_1(s) + 1 \tag{A.18}$$

$$sP_i(s) + a_i P_i(s) = \mu_s P_{i+1}(s) + \lambda_s P_{i-1}(s) \qquad (\text{ for } i = 1,2 \ldots\ldots\text{n-1}) \tag{A.19}$$

$$(s + a_n) P_n(s) = \lambda_n P_{n-1}(s) \tag{A.20}$$

$$sP_{n+1}(s) = \lambda_{ss} P_n(s) \tag{A.21}$$

$$sP_{n+2}(s) = \lambda_{si} P_n(s) \tag{A.22}$$

$$sP_{n+3}(s) = \lambda_r \sum_{i=0}^{n-1} P_i(s) \tag{A.23}$$

APPENDIX B

A System Containing One Robot and (n-1) Standby Safety Units with an Imperfect Switch

B.1 Robot-Safety System Time Dependent Analysis

Using Laplace transforms and solving Equations (3.1) – (3.8), we get:

$$(s + a_0)\, P_0(s) = 1 + \mu_s P_1(s) + \mu_w P_{n+1}(s) + \sum_{j=2n+2}^{2n+3} P_j(x,s)\mu_j(x)dx \qquad (B1)$$

$$(s + a_i)\, P_i(s) = \lambda_s P_{i-1}(s) + \mu_s P_{i+1}(s) + \mu_w P_{i+n+1}(s) \qquad (B2)$$

(for $i = 1,2,\ldots\ldots\ldots,n-1$)

$$(s + a_n)P_n(s) = \lambda_s P_{n-1}(s) + \mu_w P_{2n+1}(s) \qquad (B3)$$

$$(s + a_i)\, P_i(s) = \lambda_w P_{i-n-1}(s) \qquad (B4)$$

(for $i = n+1,n+2,\ldots\ldots\ldots,2n$)

$$(s + a_{2n+1})P_{2n+1}(s) = \lambda_s \sum_{i=n+1}^{2n} P_i(s) + \lambda_w P_n(s) \qquad (B5)$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)\, P_j(x,s) = 0 \qquad (B6)$$

(for $j = 2n+2,\ 2n+3$)

$$P_{2n+2}(0,s) = \lambda_r \sum_{i=n}^{2n+1} P_i(s) \qquad (B7)$$

$$P_{2n+3}(0,t) = \lambda_r \sum_{i=0}^{n-1} P_i(s) \qquad (B8)$$

Solving Equation (B6), we get:

$$P_j(s) = P_j(0,s) e^{-sx} \exp[-\int_0^x \mu_j(\delta) d\delta] \quad (\text{for } j = 2n+2, 2n+3) \quad \text{(B9)}$$

Together with

$$P_j(s) = \int_0^\infty P_j(x,s) dx \quad (\text{for } j = 2n+2, 2n+3) \quad \text{(B10)}$$

we can get:

$$P_j(s) = P_j(0,s) \frac{1 - W_j(s)}{s} \quad (\text{for } j = 2n+2, 2n+3) \quad \text{(B11)}$$

where

$$\frac{1 - W_j(s)}{s} = P_j(0,s) \int_0^\infty e^{-sx} \exp[-\int_0^x \mu_j(\delta) d\delta] \quad \text{(B12)}$$

$$(\text{for } j = 2n+2, 2n+3)$$

or

$$W_j(s) = \int_0^\infty e^{-sx} w_j(x) dx \quad (\text{for } j = 2n+2, 2n+3) \quad \text{(B13)}$$

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta) d\delta] \, \mu_j(x) \quad \text{(B14)}$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

B.2 Robot-Safety System Steady State Analysis

By solving Equation (3.33), we get:

$$P_j(x) = P_j(0) \exp[-\int_0^x \mu_j(\delta) d\delta] \quad \text{(B.15)}$$

Together with:

$$\mathrm{P}_j = \int_0^{\infty} P_j(x)dx \qquad (\text{for } j = 2n+2,\ 2n+3) \qquad (B.16)$$

We get:

$$\mathrm{P}_j = \mathrm{P}_j(0)\mathrm{E}_j[\mathrm{x}] \qquad (\text{for } j = 2n+2,\ 2n+3) \qquad (B.17)$$

where

$$\mathrm{E}_j[\mathrm{x}] = \int_0^{\infty} \exp[-\int_0^{x} \mu_j(\delta)d\delta]dx$$

$$= \int_0^{\infty} x w_j(x)dx \quad (\text{for } j = 2n+2,\ 2n+3) \qquad (B.18)$$

where

$\mathrm{w}_j(\mathrm{x})$ is the failed robot safety system repair time probability density function

$\mathrm{E}_j[\mathrm{x}]$ is the mean time to robot safety system repair when the failed robot safety system is in state *j* and has an elapsed repair time x.

B.3 Robot-Safety System Reliability and Mean Time to Failure Analysis

Using Laplace transforms and solving Equations (3.71) – (3.77), we get:

$$\mathrm{sP}_0(\mathrm{s}) + \mathrm{a}_0\mathrm{P}_0(\mathrm{s}) = \mu_s\mathrm{P}_1(\mathrm{s}) + 1 + \mu_w\mathrm{P}_{n+1}(\mathrm{s}) \qquad (B.19)$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = \lambda_s\mathrm{P}_{i-1}(\mathrm{s}) + \mu_s\mathrm{P}_{i+1}(\mathrm{s}) + \mu_w\mathrm{P}_{i+n+1}(\mathrm{s}) \qquad (B.20)$$

(for $i = 1,2,\ldots\ldots,n-1$)

$$(\mathrm{s} + \mathrm{a}_n)\mathrm{P}_n(\mathrm{s}) = \lambda_s\mathrm{P}_{n-1}(\mathrm{s}) + \mu_w\mathrm{P}_{2n+1}(\mathrm{s}) \qquad (B.21)$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = \lambda_w\mathrm{P}_{i-n-1}(\mathrm{s}) \qquad (\text{for } i = n+1, n+2,\ldots\ldots,2n) \qquad (B.22)$$

$$(\mathrm{s} + \mathrm{a}_{2n+1})\mathrm{P}_{2n+1}(\mathrm{s}) = \lambda_s \sum_{i=n+1}^{2n} P_i(s) + \lambda_w\mathrm{P}_n(\mathrm{s}) \qquad (B.23)$$

$$\mathrm{sP}_{2n+2}(\mathrm{s}) = \lambda_r \sum_{i=n}^{2n+1} P_i(s) \tag{B.24}$$

$$\mathrm{sP}_{2n+3}(\mathrm{s}) = \lambda_r \sum_{i=0}^{n-1} P_i(s) \tag{B.25}$$

APPENDIX C

A System Containing (n-1) Standby Robots and One Units with a Perfect Switch

C.1 Robot-Safety System Time Dependent Analysis

Using Laplace transforms and solving Equations (4.1) – (4.9), we get:

$$(\mathrm{s} + \mathrm{a}_0)\, \mathrm{P}_0(\mathrm{s}) = 1 + \mu_r \mathrm{P}_1(\mathrm{s}) + \sum_{j=2n}^{2n+1} P_j(x,s)\mu_j(x)dx \tag{C.1}$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = \lambda_r \mathrm{P}_{i-1}(\mathrm{s}) + \mu_r \mathrm{P}_{i+1}(\mathrm{s}) \qquad \text{(for } i = 1, 2..\text{n-2)} \tag{C.2}$$

$$(\mathrm{s} + \mathrm{a}_{n-1})\mathrm{P}_{n-1}(\mathrm{s}) = \lambda_r \mathrm{P}_{n-2}(\mathrm{s}) \tag{C.3}$$

$$(\mathrm{s} + \mathrm{a}_n)\mathrm{P}_n(\mathrm{s}) = \lambda_r \mathrm{P}_0(\mathrm{s}) + \mu_r \mathrm{P}_{n+1}(\mathrm{s}) \tag{C.4}$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = \lambda_r \mathrm{P}_{i-1}(\mathrm{s}) + \mu_r \mathrm{P}_{i+1}(\mathrm{s}) + \lambda_s \mathrm{P}_{i-n}(\mathrm{s}) \tag{C.5}$$

(for i = n+1,n+2….,2n-2)

$$(\mathrm{s} + \mathrm{a}_{2n-1})\mathrm{P}_{2n-1}(\mathrm{s}) = \lambda_r \mathrm{P}_{2n-2}(\mathrm{s}) + \lambda_s \mathrm{P}_{n-1}(\mathrm{s}) \tag{C.6}$$

$$\mathrm{sP}_j(\mathrm{x,s}) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(\mathrm{x})\, \mathrm{P}_j(\mathrm{x,s}) = 0 \qquad \text{(for } j = \text{2n, 2n+1)} \tag{C.7}$$

The associated boundary conditions are as follows:

$$\mathrm{P}_{2n}(0,\mathrm{s}) = \lambda_r \mathrm{P}_{2n-1}(\mathrm{s}) \tag{C.8}$$

$$\mathrm{P}_{2n+1}(0,\mathrm{s}) = \lambda_r \mathrm{P}_{n-1}(\mathrm{s}) \tag{C.9}$$

Solving Equation (C.7), we get:

$$\mathrm{P}_j(\mathrm{s}) = \mathrm{P}_j(0,\mathrm{s})\, e^{-sx} \exp[-\int_0^x \mu_j(\delta)d\delta\,] \qquad \text{(for } j = \text{2n, 2n+1)} \tag{C.10}$$

Together with

$$P_j(s) = \int_0^{\infty} P_j(x,s)dx \qquad \text{(for } j = 2n, 2n+1) \qquad (C.11)$$

We obtain:

$$P_j(s) = P_j(0,s)\,\frac{1-W_j(s)}{s} \qquad \text{(for } j = 2n, 2n+1) \qquad (C.12)$$

where

$$\frac{1-W_j(s)}{s} = P_j(0,s)\int_0^{\infty} e^{-sx} \exp[-\int_0^x \mu_j(\delta)d\delta\,] \qquad (C.13)$$

$$\text{(for } j = 2n, 2n+1)$$

or

$$W_j(s) = \int_0^{\infty} e^{-sx} w_j(x)dx \qquad \text{(for } j = 2n, 2n+1) \qquad (C.14)$$

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta\,]\ \mu_j(x) \qquad (C.15)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

A.2 Robot-Safety System Steady State Analysis

By solving Equation (4.33), we get:

$$P_j(x) = P_j(0)\exp[-\int_0^x \mu_j(\delta)d\delta\,] \qquad (C.16)$$

Together with:

$$P_j = \int_0^{\infty} P_j(x)dx \qquad \text{(for } j = 2n, 2n+1) \qquad (C.17)$$

We get:

$$P_j = P_j(0)E_j[x] \qquad \text{(for } j = 2n, 2n+1) \qquad (C.18)$$

where

$$E_j[x] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta)d\delta]dx$$

$$= \int_0^\infty x w_j(x)dx \qquad (\text{for } j = 2n, 2n+1) \qquad (C.19)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

C.3 Robot-Safety System Reliability and Mean Time to Failure Analysis

Using Laplace transforms and solving Equations (4.58) – (4.75), we get:

$$(s + a_0)P_0(s) = 1 + \mu_r P_1(s) \qquad (C.20)$$

$$(s + a_i)P_i(s) = \lambda_r P_{i-1}(s) + \mu_r P_{i+1}(s) \qquad (C.21)$$

(for i = 1,2,........,n-2)

$$(s + a_{n-1})P_{n-1}(s) = \lambda_r P_{n-2}(s) \qquad (C.22)$$

$$(s + a_n)P_n(s) = \lambda_r P_0(s) + \mu_r P_{n+1}(s) \qquad (C.23)$$

$$(s + a_i)P_i(s) = \lambda_r P_{i-1}(s) + \mu_r P_{i+1}(s) + \lambda_s P_{i-n}(s) \qquad (C.24)$$

(for i = n+1,n+2....,2n-2)

$$(s + a_{2n-1})P_{2n-1}(s) = \lambda_r P_{2n-2}(s) + \lambda_s P_{n-1}(s) \qquad (C.25)$$

$$sP_{2n}(s) = \lambda_r P_{2n-1}(s) \qquad (C.26)$$

$$sP_{2n+1}(s) = \lambda_r P_{n-1}(s) \qquad (C.27)$$

APPENDIX D

A System Containing (n-1) Standby Robots and One Safety Units with an Imperfect Switch

D.1 Robot-Safety System Time Dependent Analysis

Using Laplace transforms and solving Equations (5.1) – (5.11), we get:

$$(s + a_0)\, P_0(s) = 1 + \mu_r P_1(s) + \mu_s P_n(s) + \mu_w P_{2n}(s) + \sum_{j=4n}^{4n+1} P_j(x,s)\mu_j(x)dx \tag{D.1}$$

$$(s + a_i)P_i(s) = \lambda_r P_{i-1}(s) + \mu_r P_{i+1}(s) + \mu_w P_{2n+i}(s) \tag{D.2}$$

(for i = 1, 2.........n-2)

$$(s + a_{n-1})P_{n-1}(s) = \lambda_r P_{n-2}(s) + \mu_w P_{3n-1}(s) \tag{D.3}$$

$$(s + a_n)P_n(s) = \lambda_r P_0(s) + \mu_r P_{n+1}(s) + \mu_w P_{3n}(s) \tag{D.4}$$

$$(s + a_i)P_i(s) = \lambda_r P_{i-1}(s) + \mu_r P_{i+1}(s) + \lambda_s P_{i-n}(s) + \mu_w P_{2n+i}(s) \tag{D.5}$$

(for i = n+1,n+2....,2n-2)

$$(s + a_{2n-1})P_{2n-1}(s) = \lambda_r P_{2n-2}(s) + \lambda_s P_{n-1}(s) + \mu_w P_{4n-1}(s) \tag{D.6}$$

$$(s + a_i)P_i(s) = \lambda_w P_{i-2n}(s) \qquad \text{(for } i = 2n, 2n+1...., 3n-1) \tag{D.7}$$

$$(s + a_i)P_i(s) = \lambda_w P_{i-2n}(s) + \lambda_s P_{i-n}(s) \tag{D.8}$$

(for i = 3n, 3n+1....,4n-1)

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)\, P_j(x,s) = 0 \qquad \text{(for } j = 4n, 4n+1) \tag{D.9}$$

The associated boundary conditions are as follows:

$$P_{4n}(0,s) = \lambda_r [\, P_{2n-1}(s) + \sum_{i=3n}^{4n-1} P_i(s)\,] \tag{D.10}$$

$$P_{4n+1}(0,s) = \lambda_r [P_{n-1}(s) + \sum_{i=2n}^{3n-1} P_i(s)] \tag{D.11}$$

Solving equation (D.9), we get:

$$P_j(s) = P_j(0,s) e^{-sx} \exp[- \int_0^x \mu_j(\delta) d\delta] \qquad (\text{for } j = 4n, 4n+1) \tag{D.12}$$

Together with

$$P_j(s) = \int_0^\infty P_j(x,s) dx \qquad (\text{for } j = 4n, 4n+1) \tag{D.13}$$

We obtain:

$$P_j(s) = P_j(0,s) \frac{1-W_j(s)}{s} \qquad (\text{for } j = 4n, 4n+1) \tag{D.14}$$

where

$$\frac{1-W_j(s)}{s} = P_j(0,s) \int_0^\infty e^{-sx} \exp[- \int_0^x \mu_j(\delta) d\delta] \tag{D.15}$$

(for j = 4n, 4n+1)

or

$$W_j(s) = \int_0^\infty e^{-sx} w_j(x) dx \qquad (\text{for } j = 4n, 4n+1) \tag{D.16}$$

$$w_j(x) = \exp[- \int_0^x \mu_j(\delta) d\delta] \; \mu_j(x) \qquad (\text{for } j = 4n, 4n+1) \tag{D.17}$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

D.2 Robot-Safety System Steady State Analysis

By solving Equation (5.41), we get:

$$P_j(x) = P_j(0) \exp[- \int_0^x \mu_j(\delta) d\delta] \tag{D.18}$$

Together with:

$$P_j = \int_0^\infty P_j(x)dx \qquad \text{(for } j = 4n, 4n+1) \qquad (D.19)$$

We get:

$$P_j = P_j(0)E_j[x] \qquad \text{(for } j = 4n, 4n+1) \qquad (D.20)$$

where

$$E_j[x] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta)d\delta]dx$$

$$= \int_0^\infty xw_j(x)dx \qquad \text{(for } j = 4n, 4n+1) \qquad (D.21)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

D.3 Robot-Safety System Reliability and Mean Time to Failure Analysis

Using Laplace transforms and solving Equations (5.80) – (5.90), we get:

$$(s + a_0)P_0(s) = 1 + \mu_r P_1(s) + \mu_s P_n(s) + \mu_w P_{2n}(s) \qquad (D.22)$$

$$(s + a_i)P_i(s) = \lambda_r P_{i-1}(s) + \mu_r P_{i+1}(s) + \mu_w P_{2n+i}(s) \qquad (D.23)$$

(for $i = 1,2.........n-2$)

$$(s + a_{n-1})P_{n-1}(s) = \lambda_r P_{n-2}(s) + \mu_w P_{3n-1}(s) \qquad (D.24)$$

$$(s + a_n)P_n(s) = \lambda_r P_0(s) + \mu_r P_{n+1}(s) + \mu_w P_{3n}(s) \qquad (D.25)$$

$$(s + a_i)P_i(s) = \lambda_r P_{i-1}(s) + \mu_r P_{i+1}(s) + \lambda_s P_{i-n}(s) + \mu_w P_{2n+i}(s) \qquad (D.26)$$

(for $i = n+1,n+2....,2n-2$)

$$(\mathrm{s} + \mathrm{a}_{2n-1})\mathrm{P}_{2n-1}(\mathrm{s}) = \lambda_r \mathrm{P}_{2n-2}(\mathrm{s}) + \lambda_s \mathrm{P}_{n-1}(\mathrm{s}) + \mu_w \mathrm{P}_{4n-1}(\mathrm{s}) \tag{D.27}$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = \lambda_w \mathrm{P}_{i-2n}(\mathrm{s}) \qquad \text{(for } i = 2\text{n, 2n+1…., 3n-1)} \tag{D.28}$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = \lambda_w \mathrm{P}_{i-2n}(\mathrm{s}) + \lambda_s \mathrm{P}_{i-n}(\mathrm{s}) \qquad \text{(for } i = 3\text{n, 3n+1…., 4n-1)} \tag{D.29}$$

$$\mathrm{sP}_{4n}(\mathrm{s}) = \lambda_r [\mathrm{P}_{2n-1}(\mathrm{s}) + \sum_{i=3n}^{4n-1} P_i(s)] \tag{D.30}$$

$$\mathrm{sP}_{4n+1}(\mathrm{s}) = \lambda_r [\mathrm{P}_{n-1}(\mathrm{s}) + \sum_{i=2n}^{3n-1} P_i(s)] \tag{D.31}$$

APPENDIX E

A System Containing n Parallel Robots and (m-1) Standby Safety Units with a Perfect Switch

E.1 Robot-Safety System Time Dependent Analysis

Using Laplace transforms and solving Equations (6.1) – (6.9), we get:

$$(s + a_0)\, P_0(s) = 1 + \mu_{r1} P_1(s) + \sum_{j=(m+1)n}^{(m+1)n+m} P_j(x,s)\mu_j(s)dx \qquad (E.1)$$

$$(s + a_i)P_i(s) = (n\text{-}i\text{+}1)\lambda_r P_{i-1}(s) + \mu_{ri+1} P_{i+1}(s) \qquad (E.2)$$

(for i = 1,2.........n-2)

$$(s + a_{n-1})P_{n-1}(s) = 2\lambda_r P_{n-2}(s) \qquad (E.3)$$

$$(s + a_{kn})P_n(s) = \lambda_s P_{(k-1)n}(s) + \mu_{r1} P_{kn+1}(s) \qquad (E.4)$$

(for k = 1,2.........m-1)

$$(s + a_i)P_i(s) = [(k\text{+}1)n\text{-}i\text{+}1]\lambda_r P_{i-1}(s) + \lambda_s P_{i-n}(s) + \mu_{rq+1} P_{i+1}(t) \qquad (E.5)$$

(for $i = kn+q$: k= 1,2.......,m ; q = 1,2......,n-2)

$$(s + a_{(k+1)n-1})P_n(s) = \lambda_s P_{kn-1}(s) + 2\lambda_r P_{(k+1)n-2}(s) \qquad (E.6)$$

(for k = 1,2.........m)

$$(s + a_{mn})P_{mn}(s) = \lambda_s P_{(m-1)n}(s) + \mu_{r1} P_{mn+1}(s) \qquad (E.7)$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)\, P_j(x,s) = 0 \qquad (E.8)$$

(for j = (m+1)n, (m+1)n+1,....(m+1)n+m)

The associated boundary conditions are as follows:

$$P_j(0,s) = \lambda_r P_{[(m+1)(n+1)-j]n-1}(s) \tag{E.9}$$

(for $j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m$)

Solving equation (E.9), we get:

$$P_j(s) = P_j(0,s) e^{-sx} \exp[-\int_0^x \mu_j(\delta)d\delta] \tag{E.10}$$

(for $j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m$)

Together with

$$P_j(s) = \int_0^\infty P_j(x,s)dx \qquad (\text{for } j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m) \tag{E.11}$$

We obtain:

$$P_j(s) = P_j(0,s) \frac{1-W_j(s)}{s} \qquad (\text{for } j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m) \tag{E.12}$$

where

$$\frac{1-W_j(s)}{s} = P_j(0,s)\int_0^\infty e^{-sx} \exp[-\int_0^x \mu_j(\delta)d\delta] \tag{E.13}$$

(for $j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m$)

or

$$W_j(s) = \int_0^\infty e^{-sx} w_j(x)dx \qquad (\text{for } j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m) \tag{E.14}$$

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta]\ \mu_j(x) \tag{E.15}$$

(for $j = (m+1)n, (m+1)n+1, \ldots (m+1)n+m$)

where

$w_j(x)$ is the failed robot safety system repair time probability density function

E.2 Robot-Safety System Steady State Analysis

By solving Equation (6.24), we get:

$$\mathrm{P}_j(\mathrm{x}) = \mathrm{P}_j(0)\exp[-\int_0^x \mu_j(\delta)d\delta\,] \tag{E.16}$$

Together with:

$$\mathrm{P}_j = \int_0^\infty P_j(x)dx \qquad (\text{for } j = (\mathrm{m}+1)\mathrm{n}, (\mathrm{m}+1)\mathrm{n}+1,\ldots.(\mathrm{m}+1)\mathrm{n}+\mathrm{m}) \tag{E.17}$$

We get:

$$\mathrm{P}_j = \mathrm{P}_j(0)\mathrm{E}_j[\mathrm{x}] \qquad (\text{for } j = (\mathrm{m}+1)\mathrm{n}, (\mathrm{m}+1)\mathrm{n}+1,\ldots.(\mathrm{m}+1)\mathrm{n}+\mathrm{m}) \tag{E.18}$$

where

$$\mathrm{E}_j[\mathrm{x}] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta)d\delta]dx$$

$$= \int_0^\infty x w_j(x)dx \qquad (\text{for } j = (\mathrm{m}+1)\mathrm{n}, (\mathrm{m}+1)\mathrm{n}+1,\ldots.(\mathrm{m}+1)\mathrm{n}+\mathrm{m}) \tag{E.19}$$

where

$\mathrm{w}_j(\mathrm{x})$ is the failed robot safety system repair time probability density function

$\mathrm{E}_j[\mathrm{x}]$ is the mean time to robot safety system repair when the failed robot safety system is in state *j* and has an elapsed repair time x.

E.3 Robot-Safety System Reliability and Mean Time to Failure Analysis

Using Laplace transforms and solving Equations (6.51) – (6.58), we get:

$$(\mathrm{s} + \mathrm{a}_0)\,\mathrm{P}_0(\mathrm{s}) = 1 + \mu_{r1}\mathrm{P}_1(\mathrm{s}) \tag{E.20}$$

$$(\mathrm{s} + \mathrm{a}_i)\mathrm{P}_i(\mathrm{s}) = (\mathrm{n}\text{-}\mathrm{i}+1)\lambda_r\mathrm{P}_{i-1}(\mathrm{s}) + \mu_{ri+1}\mathrm{P}_{i+1}(\mathrm{s}) \tag{E.21}$$

$$(\text{for } i = 1,2\ldots\ldots\ldots\mathrm{n}\text{-}2)$$

$$(\mathrm{s} + \mathrm{a}_{n-1})\mathrm{P}_{n-1}(\mathrm{s}) = 2\lambda_r\mathrm{P}_{n-2}(\mathrm{s}) \tag{E.22}$$

$$(s + a_{kn})P_n(s) = \lambda_s P_{(k-1)n}(s) + \mu_{r1} P_{kn+1}(s) \tag{E.23}$$

(for $k = 1,2 \ldots\ldots\ldots m-1$)

$$(s + a_i)P_i(s) = [(k+1)n-i+1]\,\lambda_r P_{i-1}(s) + \lambda_s P_{i-n}(s) + \mu_{rq+1} P_{i+1}(t) \tag{E.24}$$

(for $i = kn+q$: $k= 1,2\ldots\ldots,m$; $q = 1,2\ldots\ldots,n-2$)

$$(s + a_{(k+1)n-1})P_n(s) = \lambda_s P_{kn-1}(s) + 2\lambda_r P_{(k+1)n-2}(s) \tag{E.25}$$

(for $k = 1,2 \ldots\ldots\ldots m$)

$$(s + a_{mn})P_{mn}(s) = \lambda_s P_{(m-1)n}(s) + \mu_{r1} P_{mn+1}(s) \tag{E.26}$$

$$sP_j(s) = \lambda_r P_{[(m+1)(n+1)-j]n-1}(s) \tag{E.27}$$

(for $j = (m+1)n,\ (m+1)n+1, \ldots.(m+1)n+m$)

E.4.1 Time Dependent Analysis for the Special Case

Using Laplace transforms and solving Equations (6.67) – (6.74), we get:

$$(s + a_0)\, P_0(s) = 1 + \mu_{r1} P_1(s) + \mu_s P_2\ (s) + \sum_{j=6}^{8} P_j(x,s)\mu_j(x)dx \tag{E.28}$$

$$(s + a_1)\, P_1(s) = 2\lambda_r P_0(s) \tag{E.29}$$

$$(s + a_2)P_2(s) = \lambda_s P_0(s) + \mu_{r1} P_3(s) + \mu_s P_4(s) \tag{E.30}$$

$$(s + a_i)\, P_i(s) = 2\lambda_r P_{i-1}(s) + \lambda_s P_{i-2}(s) \qquad \text{(for } i = 3,5\text{)} \tag{E.31}$$

$$(s + a_4)P_4(s) = \lambda_s P_2(s) + \mu_{r1} P_5(s) \tag{E.32}$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)\, P_j(x,s) = 0 \qquad \text{(for } j = 6,\ 7,\ 8\text{)} \tag{E.33}$$

The associated boundary conditions are as follows:

$$P_j(0,t) = \lambda_r P_{[9-j]2-1}(t) \qquad \text{(for } j = 6,\ 7,\ 8\text{)} \tag{E.34}$$

E.4.2 Steady State Analysis for the Special Case

By solving Equation (6.90), we get:

$$P_j(x) = P_j(0) \exp[- \int_0^x \mu_j(\delta) d\delta] \qquad (E.35)$$

Together with:

$$P_j = \int_0^\infty P_j(x) dx \qquad (\text{for } j = 6, 7, 8) \qquad (E.36)$$

We get:

$$P_j = P_j(0) E_j[x] \qquad (\text{for } j = 6, 7, 8) \qquad (E.37)$$

where

$$E_j[x] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta) d\delta] dx$$

$$= \int_0^\infty x w_j(x) dx \qquad (\text{for } j = 6, 7, 8) \qquad (E.38)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

E.4.3 Reliability and Mean Time to Failure Analysis for the Special Case

Using Laplace transforms and solving Equations (6.100) – (6.106), we get:

$$(s + a_0) P_0(s) = 1 + \mu_{r1} P_1(s) + \mu_s P_2(s) \qquad (E.39)$$

$$(s + a_1) P_1(s) = 2\lambda_r P_0(s) \qquad (E.40)$$

$$(s + a_2) P_2(s) = \lambda_s P_0(s) + \mu_{r1} P_3(s) + \mu_s P_4(s) \qquad (E.41)$$

$$(s + a_i) P_i(s) = 2\lambda_r P_{i-1}(s) + \lambda_s P_{i-2}(s) \qquad (\text{for } i = 3, 5) \qquad (E.42)$$

$$(s + a_4)P_4(s) = \lambda_s P_2(s) + \mu_{r1} P_5(s) \qquad \text{(E.43)}$$

$$sP_j(s) = \lambda_r P_{(9-j)2-1}(s) \qquad (\text{for } j = 6, 7, 8) \qquad \text{(E.44)}$$

APPENDIX F

A System Containing Two Parallel Robots and One Standby Safety Units with an Imperfect Switch

F.1 Robot-Safety System Steady State Analysis

By solving Equation (7.31), we get:

$$P_j(x) = P_j(0) \exp[-\int_0^x \mu_j(\delta) d\delta] \tag{F.1}$$

Together with:

$$P_j = \int_0^\infty P_j(x) dx \qquad \text{(for } j = 14, 15 \ldots 20) \tag{F.2}$$

We get:

$$P_j = P_j(0) E_j[x] \qquad \text{(for } j = 14, 15 \ldots 20) \tag{F.3}$$

where

$$E_j[x] = \int_0^\infty \exp[-\int_0^x \mu_j(\delta) d\delta] dx$$

$$= \int_0^\infty x w_j(x) dx \qquad \text{(f14, 15} \ldots 20) \tag{F.4}$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function

$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.

F.2 Robot-Safety System Reliability and Mean Time to Failure Analysis

Using Laplace transforms and solving Equations (7.58) – (7.72), we get:

$$(s+a_1)P_1(s) = 2\lambda_r P_0(s) \tag{F.5}$$

$$(s+a_2)P_2(s) = \lambda_w P_0(s) \tag{F.6}$$

$$(s+a_3)P_3(s) = \lambda_w P_1(s) + 2\lambda_r P_2(s) \tag{F.7}$$

$$(s+a_4)P_4(s) = \lambda_s P_0(s) \tag{F.8}$$

$$(s+a_5)P_5(s) = \lambda_w P_1(s) + 2\lambda_r P_4(s) \tag{F.9}$$

$$(s+a_4)P_6(s) = \lambda_w P_4(s) \tag{F.10}$$

$$(s+a_7)P_7(s) = \lambda_w P_4(s) + 2\lambda_r P_6(s) \tag{F.11}$$

$$(s+a_8)P_8(s) = \lambda_s P_4(s) \tag{F.12}$$

$$(s+a_9)P_9(s) = \lambda_s P_5(s) + 2\lambda_r P_8(s) \tag{F.13}$$

$$(s+a_{10})P_{10}(s) = \lambda_w P_8(s) + \lambda_s P_6(s) \tag{F.14}$$

$$(s+a_{11})P_{11}(s) = \lambda_w P_9(s) + \lambda_s P_7(s) + 2\lambda_r P_{10}(s) \tag{F.15}$$

$$(s+a_{12})P_{12}(s) = \lambda_s P_2(s) \tag{F.16}$$

$$(s+a_{13})P_{13}(s) = \lambda_s P_3(s) + 2\lambda_r P_{12}(s) \tag{F.17}$$

$$sP_j(s) = \lambda_r P_{(21-j)2-1}(s) \qquad (\text{ for } j = 14,15\ldots\ldots\ldots\ldots 20) \tag{F.18}$$