CAHIER DE RECHERCHE #2501E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER #2501E
Department of Economics
Faculty of Social Sciences
University of Ottawa

Stationarity of the Optimal Enforcement Contract in the Complete Information Case

Aggey Simons (Semenov)*

January 2025

---

* Department of Economics, University of Ottawa, 9039-120 University Private, Ottawa, Ontario, Canada, K1N 6N5;
e-mail: aggey.simons@uottawa.ca.

Abstract

This paper examines the stationarity of optimal contracts in infinitely repeated principal–agent relationships under complete information and enforcement constraints. We demonstrate that stationarity emerges as a robust feature of optimal contracts when agent types and actions are fully observable, and contract enforcement is supported by both public remedies and private termination threats. Under complete information, the trade-offs between enforcement costs and relational value become significantly simplified, resulting in stationary outcomes even when enforcement constraints are binding. These findings offer insights into contract design in environments where non-stationary profiles are either impractical or prohibitively costly.

Key words: *dynamic contracts, contract enforcement, stationarity, complete information*.

JEL Classification: D82, D86.

1 Introduction

Our model examines a dynamic contracting setup between a buyer and a seller where compliance is ensured through enforcement mechanisms. The buyer offers a long-term contract for the delivery of a good or service, with payments structured both before and after delivery. Both parties can breach the contract, which results in immediate termination (the private enforcement mechanism) and penalties imposed by a court (the public enforcement mechanism). We find that, under complete information, these enforcement mechanisms are sufficient to ensure that contracts remain stationary over time.

This result has practical applications. In many industries, formal long-term contracts are stationary. For example, franchise agreements are typically long-term, with both sides face fines or termination of the agreement for breaches, such as failing to meet quality or branding standards. As documented by Lafontaine and Shaw (1999), franchise fees and royalty rates are fixed for the duration of the contract. Similarly, Power Purchase Agreements (PPAs) and commercial real estate leases are examples of formal contracts enforceable by courts and the threat of termination. These contracts are typically are stationary but may include adjustments for inflation.

This paper addresses a gap in the dynamic contracting literature. In the case of private information, Baron and Besanko (1984) demonstrate that in regulatory environments without enforcement frictions, the optimal long term contract is stationary, replicating the static solution in each period. Their work underscores that stationarity naturally arises when full commitment is possible, a result that aligns with our findings in complete information settings.

In contrast, Martimort et al. (2017) analyze environments with asymmetric information and limited enforcement, showing how dynamic distortions sustain contracts when there is persistent private information and enforcement frictions. In this paper, we focus on the simpler but essential case of complete information. We show that, in this setting, the optimal contract is stationary, in contrast to the non-stationary contracts required under asymmetric information.

Finally, our paper is related to the relational contracting literature. In this field, contracts are self-enforced and typically informal, relying not on courts but on the promise of continued profitable interaction. Levin (2003) showed that in situations close to complete information, the optimal relational contract is stationary. In contrast, Halac (2012) analyzes relational contracts under persistent private information about the value of the relationship or the principal's outside option. In such cases, the optimal relational contract is non-stationary, with non-stationarity arising from the need to balance incentives with learning about one party's private information. We show that even when there are limits on liability, the best contracts in dynamic settings remain stationary.

2 Model

A seller (the agent) delivers a divisible good to a buyer (the principal) in exchange for payment at each period. Time is infinite, $t = 0, 1, 2, \ldots$, and it is discounted by the common discount factor $\delta \in (0, 1)$. The per-period payoffs for the buyer and the seller are given by:

$$V_t = B(q_t) - p_t \quad \text{and} \quad U_t = p_t - cq_t,$$

where p_t and q_t denote the payment and output in each period, respectively.[1]

The contract is a triplet $\mathcal{C} = \{(p_{1,t}, p_{2,t}, q_t)\}_{t=0}^{\infty}$, where $p_{1,t}$ and $p_{2,t}$ represent pre-delivery and post-delivery payments, respectively. The total payment is given by

$$p_t = p_{1,t} + p_{2,t}.$$

The timing proceeds as follows

1. The buyer offers a contract $\mathcal{C}$ for the whole duration of the relationship. The seller either accepts or rejects the offer. If the seller rejects, the reservation value is normalized to zero.
2. At each $t \geq 0$:
   - The buyer makes the pre-delivery payment $p_{1,t}$;
   - The seller either produces q_t or breaches the contract, incurring a penalty Π_s;
   - If q_t is produced, the buyer pays $p_{2,t}$ or breaches the contract, incurring a penalty Π_b;
   - If either party breaches the contract, it is terminated immediately.

The model assumes that the agreements between parties and penalties Π_b and Π_s are enforceable by courts. Contracts in infrastructure projects often include provisions for penalties or remedies if a party defaults. For example, if a private firm fails to meet the agreed standards in construction or operation, it may face financial penalties. In contrast, in relational contracting the agreements between parties are self-enforcing and generally do not rely on enforcement by courts.[2]

---

[1] The function B is differentiable, increasing, and concave, with $B(0) = 0$ and $B'(0)$ sufficiently large.

[2] See Malcomson, 2013, for an excellent survey on relational contracts.

The contract C is individually rational when

$$\widetilde{U}_0 = \sum_{s=0}^{\infty} \delta^s U_s \geq 0. \tag{1}$$

The buyer must pay her due payment to the seller in each period. If she deviates and fails to pay for delivery, she incurs a penalty Π_b (public side of enforcement) and the relationship ends (the private side). Therefore, the contract $\mathcal{C}$ is buyer-enforceable if

$$\delta \sum_{s=0}^{\infty} \delta^s V_{t+1+s} \geq p_{2,t} - \Pi_b, \quad \forall t. \tag{2}$$

The buyer's discounted payoff from period $t+1$ onwards on the equilibrium path should be higher than the deviation payoff for the current period. It accounts for the fact that trade never occurs from date t onward following a breach by the buyer.

The contract $\mathcal{C}$ is seller-enforceable if

$$-cq_t + p_{2,t} + \delta \sum_{s=0}^{\infty} \delta^s U_{t+1+s} \geq -\Pi_s, \quad \forall t. \tag{3}$$

At each t, after $p_{1,t}$ is paid, this payment becomes sunk and the enforcement of the contract depends on the interaction between continuation value and the penalty. The enforcement constraint (3) ensures that the seller adheres to the targeted contract rather than breaching it. The seller must either comply with the contract or face penalties for breach.

Definition 2.1. The quantity profile $\{q_t\}_{t=0}^{\infty}$ is enforceable if there exists a system of payments $(p_{1,t}, p_{2,t})_{t=0}^{\infty}$ such that the contract $\mathcal{C} = \{(p_{1,t}, p_{2,t}, q_t)\}_{t=0}^{\infty}$ is individually rational and both buyer and seller enforceable.

Let $\Pi = \Pi_b + \Pi_s$ and $\mathbf{q}_t = \{q_s\}_{s=t}^{\infty}$. We define the enforcement surplus as

$$\Psi(\mathbf{q}_t) = \delta \sum_{s=0}^{\infty} \delta^s (S(q_{t+1+s}) - cq_{t+1+s}) - cq_t + \Pi. \tag{4}$$

Pooling the individual enforcement constraints (2) and (3) yields the necessity of the following feasibility condition.

Lemma 2.2. *The quantity profile $\mathbf{q} = \{q_t\}_{t=0}^{\infty}$ is* ***enforceable*** *if and only if*

$$\Psi(\mathbf{q}_t) \geq 0, \quad \forall t. \tag{5}$$

Proof. We need to show the sufficiency of (5) for enforceability of the contract. Define

$$p_{1,t} = \Pi_s \quad \forall t$$

and the payments $p_{2,t}$ are defined by binding constraints (3) for all t. It is clear by construction and inequality (5) that $\mathcal{C} = \{(p_{1,t}, p_{2,t}, q_t)\}_{t=0}^{\infty}$ is individually rational and both buyer- and seller-enforceable.[3]

□

The enforcement surplus (4) reflects the net gain from continuing the contract from date t onward. It incorporates future trade benefits and the avoidance of penalties from deviation at date t.

Note that Ψ depends only on the aggregate penalty $\Pi = \Pi_b + \Pi_s$; therefore, the distribution of remedies between the buyer and seller is irrelevant. The buyer, having full bargaining power, can adjust payments to internalize the consequences of a breach. Thus, the effective threat simplifies to the possibility of a buyer's breach, with a corresponding penalty of Π.

Suppose that the buyer offers a stationary output profile $\mathbf{q} = \{q\}_{t=0}^{\infty}$. The enforcement surplus becomes

$$\Psi(\mathbf{q}) = \delta S(q) - cq + (1 - \delta)\Pi.$$

Note that $\Psi(\mathbf{q})$ is strictly concave in q and admits a zero at some positive q^m, provided that Π is not too large (an assumption we maintain henceforth)

$$\delta S(q^m) - cq^m + (1 - \delta)\Pi = 0. \tag{6}$$

It follows that q^m is the maximal enforceable output.

3 Optimal Dynamic Contract

With the change of variables, the buyer's objective is to maximize the discounted net surplus obtained from trade, subject to the seller's participation and enforcement constraints

$$\max_{(\mathbf{q}, \widetilde{U}_0)} \sum_{t=0}^{\infty} \delta^t \left(S(q_t) - cq_t\right) - \widetilde{U}_0$$

$$\text{subject to } \widetilde{U}_0 \geq 0 \quad \text{and} \quad \Psi(\mathbf{q}_\tau) \geq 0, \quad \forall t.$$

---

[3] Note that by construction the constraint 1 is binding.

This program can be re-written as

$$(\mathcal{P}) : \max_{\{q_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t (S(q_t) - cq_t)$$

$$\text{subject to } cq_t \leq \sum_{s=1}^{\infty} \delta^s (S(q_{t+s}) - cq_{t+s}) + \Pi, \quad \forall t. \tag{7}$$

Denote by q^* the first best level of output: $q^* = \arg\max_q S(q) - cq$. Note that the social surplus is a strictly concave function of q.

Suppose now that $\{q_t\}_{t=0}^{\infty}$ is a solution of the program (P).

Lemma 3.1. *If there exist t' such that $q_{t'} = q^*$, then for all $t, q_t = q^*$.*

Proof. For t' we have

$$cq_{t'} = cq^* \leq \sum_{s=1}^{\infty} \delta^s (S(q_{t'+s}) - cq_{t'+s}) + \Pi \leq \sum_{s=1}^{\infty} \delta^s (S(q^*) - cq^*) + \Pi.$$

By this property, if there is $q_t \neq q^*$ then the stationary contract $\{q^*\}$ strictly dominates $\{q_t\}_{t=0}^{\infty}$. A contradiction. Therefore, it must be that for all t we have $q_t = q^*$. □

Lemma 3.2. *If $\{q_t\}_{t=0}^{\infty}$ is optimal but not the first best stationary contract then all constraints (7) are satisfied with equality.*

Proof. Suppose there exist t' such that

$$cq_{t'} < \sum_{s=1}^{\infty} \delta^s (S(q_{t'+s}) - cq_{t'+s}) + \Pi.$$

Since $q_{t'} \neq q^*$ there exist $\widetilde{q}$ such that

$$c\widetilde{q} < \sum_{s=1}^{\infty} \delta^s (S(q_{t'+s}) - cq_{t'+s}) + \Pi \tag{8}$$

and

$$S(\widetilde{q}) - c\widetilde{q} > S(q_{t'}) - cq_{t'}. \tag{9}$$

Consider a new contract $\{\widetilde{q}_t\}_{t=0}^{\infty}$ where $\widetilde{q}_t = q_t$ for all $t \neq t'$ and $q_{t'} = \widetilde{q}$. We check the constraint (7). If $t > t'$ then it is unaffected. If $t < t'$, then because of (9)

$$cq_t \leq \sum_{s=1}^{\infty} \delta^s (S(q_{t+s}) - cq_{t+s}) + \Pi < \sum_{s=1}^{\infty} \delta^s (S(\widetilde{q}_{t+s}) - c\widetilde{q}_{t+s}) + \Pi.$$

Indeed, for all $t \neq t'$ the right-hand side of the inequality is the same and for $t = t'$ by (9) there is a strict inequality. Finally by (8) the constraint (7) is satisfied for $t = t'$.

We established that $\{\widetilde{q}_t\}_{t=0}^{\infty}$ is an admissible contract. Now by (9) it delivers strictly larger payoff than $\{q_t\}_{t=0}^{\infty}$. A contradiction. Therefore all constraints are satisfied with equality. ☐

Note now that since all (7) are satisfied with equality we have the following set of equalities:

$$\delta S(q_{t+1}) - cq_t = -(1-\delta)\Pi \quad \forall t. \tag{10}$$

We subtract (10) for t from the corresponding equality for $t+1$ to obtain

$$c\,(q_{t+1} - q_t) = \delta(S(q_{t+2}) - S(q_{t+1})) \quad \forall t.$$

From this we obtain the following

Lemma 3.3. *If $\{q_t\}_{t=0}^{\infty}$ is optimal, then it is strictly monotonic or constant.*

The next important Lemma says that the optimal contract cannot surpass the first-best.

Lemma 3.4. *If there exists t such that $q_\tau < q^* < q_{t+1}$ then $\{q_t\}_{t=0}^{\infty}$ cannot be optimal.*

Proof. For $t+1$ we have

$$cq^* < cq_{t+1} \leq \sum_{s=1}^{\infty} \delta^s (S(q_{t+1+s}) - cq_{t+1+s}) + \Pi < \sum_{s=1}^{\infty} \delta^s (S(q^*) - cq^*) + \Pi.$$

Hence, the first-best contract is optimal. A contradiction. ☐

With this Lemma we have the following five cases for a monotonic optimal contract.

1. $q_0 < q_1 < .... < q^*$
2. $q^* < q_0 < q_1 < ....$
3. $q_0 > q_1 > ... > q^*$
4. $q^* > q_0 > q_1 > ...$
5. The contract is stationary.

We prove now the following

Lemma 3.5. *The optimal contract is stationary.*

Proof. Assume that the contract $\{q_t\}_{t=0}^{\infty}$ is optimal.

Consider case 1 above. It is clear that the contract $\{q_{t+1}\}_{t=0}^{\infty}$ strictly dominates $\{q_t\}_{t=0}^{\infty}$. A contradiction.

Case 2. In this case the stationary first-best contract is optimal: $\theta q^* < \sum_{s=1}^{\infty} \delta^s (S(q^*) - \theta q^*) + \Pi$.

Case 3. The stationary first-best contract is optimal.

Case 4. Consider a stationary contract $\{q_0\}$. We have

$$\theta q_o < \sum_{s=1}^{\infty} \delta^s (S(q_s) - \theta q_s) + \Pi < \sum_{s=1}^{\infty} \delta^s (S(q_0) - \theta q_0) + \Pi.$$

Therefore $\{q_0\}$ is admissible and since it is closer to the first-best, by concavity of the surplus function, we have that it dominates $\{q_t\}_{t=0}^{\infty}$. A contradiction. □

Finally we have the following

Proposition 3.6. *The optimal contract in the complete information case is stationary; the corresponding optimal output is given by*

$$q = \min\{q^m, q^*\}, \tag{11}$$

where q^* *is the first-best output, and* q^m *is characterized by the following equation*

$$\delta S(q^m) - \theta q^m + (1 - \delta)\Pi = 0.$$

Note that payments can by chosen using the proof of Lemma 2.2, i.e.

$$p_{1,t} = \Pi_s \quad \forall t$$

and the payments $p_{2,t}$ are defined by binding constraints (3) for all t.

References

[1] Baron, D.P. and Besanko, D., 1984. Regulation and information in a continuing relationship. Information Economics and policy, 1(3), pp.267-302.

[2] Halac, M., 2012. Relational contracts and the value of relationships. American Economic Review, 102(2), pp.750-779.

[3] Lafontaine, F. and Shaw, K.L., 1999. The dynamics of franchise contracting: Evidence from panel data. Journal of political Economy, 107(5), pp.1041-1080.

[4] Levin, J., 2003. Relational incentive contracts. American Economic Review, 93(3), pp.835-857.

[5] Malcomson, J.M., 2013. Relational incentive contracts. The handbook of organizational economics, pp.1014-1065.

[6] Martimort, D., Semenov, A. and Stole, L., 2017. Optimal stationary contract with two-sided imperfect enforcement and persistent adverse selection. Economics Letters, 159, pp.18-22.