ABSTRACT

A continuous liquid flow system consisting of an inflow pipe, an open vessel, and an outflow pipe containing an orifice can be represented by a first order nonlinear differential equation. The changes in the linearized dynamic properties; - steady-state gain and time constant -, of the nonlinear system with three mathematically expressible pulses and step forcing have been studied. It was found that the width of the forcing pulse was the best criterion for estimating the linear model of the physical system and that the linearized dynamic properties varied linearly with pulse strength. This behavior was represented by the equation $Y = A + BX$, where A and B were polynomial functions of the pulse width, Y was the dynamic property and X was pulse strength. It was observed that the optimal pulse width, i.e., the pulse width that yielded the correct linearized dynamic properties, was about twice that of the time constant of the systems in this study. For the step testing, the correct linearized dynamic properties were obtained by extrapolating to the steady-state operating condition to zero step height. All the calculations were carried out on an IBM model 360/65 computer.

ACKNOWLEDGEMENT

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Appreciation is also due Dr. S. D. Chang and Dr. H. Sugie for their valuable suggestions in computer programming.

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NOMENCLATURE

a_k , b_k	Fourier series coefficients.
A	Constant in pulse reduction formulas, or cross-sectional area of systems.
AO, AI	Coefficients of polynomial .
A2, A3	Coefficients of polynomial.
A_n	Fourier integral coefficient.
B	Constant in pulse reduction formulas.
C	Constant in pulse reduction formulas, or arbitraryconstant (see Equation (4.1 - 2)).
$CO(T_p)$	Intercept, function of pulse width.
$CI(T_p)$	Slope, function of pulse width.
D	Constant in pulse reduction formulas.
e	Base of nature logarithm.
f (t)	Arbitraryfunction of time.
F( )	Denotes Fourier transformation.
G	Steady-state gain.
G (jw)	Transfer function in frequency domain.
G (s)	Transfer function in s-domain.
h	Interval of the Runge-Kutta numerical integration.
IM (w)	Imaginary part of G (jw).
j	$\sqrt{-1}$
k	Arbitrary index.

L	Level height of system.
$L\{\ \}$	Denotes Laplace transformation.
m	Arbitrary index.
m_i	Influent flow rate of system.
m_0	Effluent flow rate of system.
M	Input magnitude of sinusoidal wave applied to linear device.
N	Output magnitude of sinusoidal wave from linear device.
n	Arbitrary index.
$O(h^5)$	Truncation error of fourth order Runge-Kutta formula.
p	The first term of fourth order Runge-Kutta formula.
P	Steady-state value of step forcing, defined in Figure 22.
q	The second term of fourth order Runge-Kutta formula.
Q	Steady-state value of step response, defined in Figure 22.
r	The third term of fourth order Runge-Kutta formula.
R	Resistance of the discharge orifice.
$R(w)$	Magnitude ratio.
$Re(w)$	Real part of $G(jw)$.
s	Complex variable of the Laplace transform.
$s(w)$	Frequency content of a pulse function.
$s(w)_n$	Normalized frequency content, $= s(w)/s(0)$.
$s(w)_0$	Zero frequency content, or pulse strength of input pulse.
$s(w)_x$	Frequency content of input pulse.
$s(w)_y$	Frequency content of output pulse.

t	Real time.
T	Time constant, or period of a periodic function.
T_p	Pulse width.
T_x	Width of system input pulse.
T_y	Width of system output pulse.
Δt_x	Interval width on forcing function.
Δt_y	Interval width on response function.
v	The fourth term of fourth order Runge-Kutta formula.
w	Frequency.
w_f	Fundamental frequency of a periodic function.
$x(s)$	Laplace transform of forcing function.
$x(t)$	Input or forcing function.
$x^*(t)$	Defined in Equation (B. 2 - 1).
$y(s)$	Laplace transform of response function.
$y(t)$	Output or response function.
$y^*(t)$	Defined in Equation (B. 2 - 2).
$\rho(w)$	Coefficient density of a periodic function.
$\phi(w)$	Phase lag, $= \varphi$.

SUBSCRIPTS

ss	Steady state.
n	Normalized.
x	For input, defined at point of use.
y	For output, defined at point of use.

o At zero frequency, or center of Taylor series expansion.

i Arbitrary index.

ABBREVIATIONS

Rec. Rectangular pulse function.

Tri. Triangular pulse function.

H-S. Half-Sine pulse function.

L.D.P. Linearized dynamic property.

CHAPTER 1

INTRODUCTION

It is possible to derive the dynamic model of the physical system from the application of appropriate mass, energy and momentum balances. The mathematical expression is said to be linear if the dependent variable and it's derivatives are of the first order power, otherwise, it is nonlinear. Also, if any of the coefficients vary as a function of the dependent variable, the differential equation is considered as nonlinear. However, systems of chemical engineering interest are almost universally nonlinear in nature and one of the most important problems that confronts the chemical engineer is the study of the dynamic characteristics of nonlinear systems.

In its most general interpretation, knowledge of the process dynamics implies cognizance of the functional relation between the input and output of a system. In practice, this functional relationship is determined by testing the existing system and representing the results as a linearized model of the system. This approach has certain advantages because a vast sources of analytical and control system design methods have been developed using linear differential equations. The problems associated with testing nonlinear systems to obtain linearized models of the system have not been studied. This study represents a preliminary investigation into these problems using a simple nonlinear system and two of the common test methods; pulse testing and step testing. Both test methods are based on the assumption that the system behaves as a linear system during the test. The objective was to determine the effects of the amplitude, width,and shape of various forcing pulses on the resulting steady-state gain and time constant of the system which were computed from the test results. The results were compared with the true steady-state gain and time constant computed analytically from the " linearized " equation representing the system.

CHAPTER 2

REVIEW OF CURRENT LITERATURE

Most current control system design methods are based on the use of a linear model of the physical systems. The study of the behavior of nonlinear system dynamic properties obtained by dynamic testing has not been reported in the literature. Thus, a study of the problem of determining a linear model of a nonlinear system and the relationship between the system dynamic properties and the forcing functions appears to be a worthwhile project. In order to meet this end, a review of some testing techniques, which are pertinent to this study, seems essential.

The most fruitful techniques for process control and analysis are based on the frequency response method (1, 2) and thus are valid only for the linear system. In order to obtain enough points to define a frequency response curve adequately, it is necessary to make a number of sinusoidal tests at various frequencies. This is a significant disadvantage in many cases. Furthermore, the sinusoidal method is imcompatible with many systems, for instance, one involving a fast chemical reaction.

The pulse testing method was developed (3,– 7) to circumvent difficulties encountered in the frequency response method. It is very simple in principle and in application. The system input is given an arbitrary closed pulse and the resultant output recorded. If certain precautions (described later) are observed, the entire useful portion of the frequency response curve can be mathematically derived from these data alone. Several workers (4, 8 - 17) recognized the value of pulse techniques and used them to determine the dynamic response around the operating conditions for chemical processes including heat exchangers, reactors, even the whole plant (18, 19) . Recently , Marino et al (20, 21) and Thusoo (22) both showed that the pulse technique was a reliable method for obtaining dynamic response data of the distillation column.

Another forcing which is easy to generate experimentally is a step-forcing function. This method consists of forcing a step change on the input and recording the transient of the output. This technique is valid for the linear system only (1, 23, 24) . However, if the magnitude of the step change is small compared with the steady-state operating condition, the technique can be applied to nonlinear systems to obtain the dynamic response. Several investigators (25, 26, 27, 28) have published detailed discussions and applied this technique to some practical systems.

CHAPTER 3

THEORETICAL BACKGROUND

In general, the theoretical background used in this study is based on linear system analysis. The response of linear systems is well known. When nonlinear systems are tested using linear system testing technique to obtain dynamic information, one must assume that the system behaves linearly during the test. Whereas linear systems respond linearly with forcing functions nonlinear systems do not. The output response from a nonlinear system is a function of the input forcing shape and magnitude. Small deviations from linearity can be ignored if the magnitude of the test disturbance is small. If a sinusoidal testing technique is used, these deviations can easily be detected by observation of the distortion of the sinusoidal output signal. However, nonlinearities cannot readily be detected in the output signal from pulse and step tests. Thus, if these testing techniques are used, there is a strong possibility that errors will be introduced in the parameters describing the linearized model of the system. The magnitude of these errors and their behavior are the subject of this thesis. The analysis of the results of this study are based on linear system theory; in particular, the frequency response technique. The theory concerning the various testing techniques is considered essential for this study and is now reviewed.

3 . 1 SINUSOIDAL TESTING

The frequency response of a system is the response of the system to steady-state sinusoidal forcing over a wide range of frequency. Consider the linear system of Figure 1, which is forced by a sustained sinusoidal signal at constant amplitude M and constant frequency w . The response signal will be another sine wave having the same frequency but with a different amplitude and a displacement of the output with respect to the input wave. In the figure this response is identified as N SIN (wt + ψ), where ψ is the angular displacement of the output wave from the input wave.

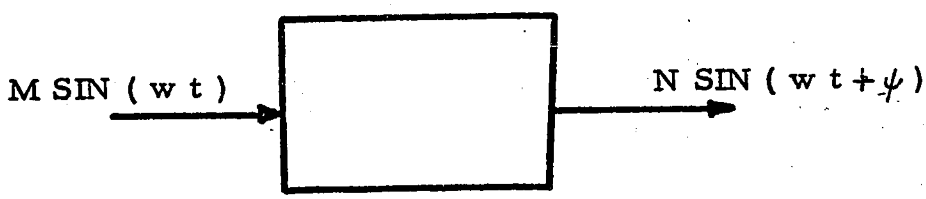


Figure 1. Sinusoidally forced element

The frequency-response characteristics of a linear system are given at each frequency by the ratio of the amplitude of the response sinusoid to the amplitude of the forcing sinusoid, and the phase shift between the two sinusoids. Thus the frequency-response characteristics are

Magnitude Ratio $$R(w) = \frac{N}{M}$$

Phase Angle $$\phi(w) = \psi$$

then $$G(jw) = R(w)\, e^{j\phi(w)}$$

The complex quantity, G (j w), where amplitude is R (w) and whose phase angle is ϕ (w) will then be defined as the frequency response function of the system (1).

3 . 2 PULSE TESTING

The transfer function, G (s), of a single-input and single-output linear system may be defined as the ratio of the Laplace transform of the output and input. Thus, using the definition of the Laplace transform,

$$G(s) = \frac{L\{y(t)\}}{L\{x(t)\}} = \frac{\int_0^\infty y(t)\, e^{-st}\, dt}{\int_0^\infty x(t)\, e^{-st}\, dt} \qquad (3.2-1)$$

where x(t) and y(t) are the input and output functions respectively, and s is the complex variable of the Laplace transform. It has been shown (29) that the frequency response can be derived analytically from the transfer function by substituting jw for s. In this form, the transfer function

$$G(jw) = \frac{\int_0^\infty y(t)\, e^{-jwt}\, dt}{\int_0^\infty x(t)\, e^{-jwt}\, dt}$$

$$= R(w)\, e^{j\phi(w)} \qquad (3.2-2)$$

becomes a Fourier transform.

Consider the case when the system input x (t) is a closed pulse, i. e., it is zero at t = 0, assumes a finite value for a finite time interval T_x, and returns to zero at the end of the interval. It is possible to make the assumption, which is valid for most systems, that the output resulting from the pulsed input is a pulse of the same type, existing only for $0 \leqslant t \leqslant T_y$, after which interval y(t) has returned to its steady-state value so closely that any difference can no longer be distinguished.

Then Equation (3.2 - 2) becomes

$$G(jw) = \frac{\int_0^{T_y} y(t)\, e^{-jwt}\, dt}{\int_0^{T_x} x(t)\, e^{-jwt}\, dt} \qquad (3.2 - 3)$$

It can be shown (see Appendix A) that the frequency content, s(w), of a nonperiodic function is given by its Fourier transform

$$s(w) = F\left[f(t)\right] = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt \qquad (3.2 - 4)$$

If only the pulse function is considered, it follows that

$$s(w) = \int_0^{T_p} f(t)\, e^{-jwt}\, dt \qquad (3.2 - 5)$$

where T_p is pulse width. The frequency content of the input pulse, x(t), is given by the absolute magnitude of its Fourier transform. Therefore, the magnitude at zero frequency is given by

$$s(w)_0 = \left| \int_0^{T_p} x(t)\, dt \right| \qquad (3.2 - 6)$$

This integral is just the area under the curve x (t). For this study, this value is referred to as the input pulse strength.

Making use of the properties of complex numbers, the magnitude ratio is the ratio of output frequency content to input frequency content, and the phase angle is the phase angle between $s(w)_y$ and $s(w)_x$, where the subscripts indicate frequency content of system output and input, respectively. Thus from Equations (3.2 - 3) and (3.2 - 5), it follows that

$$R(w) = \frac{|s(w)_y|}{|s(w)_x|}$$

$$= \frac{\left| \int_0^{T_y} y(t)\, e^{-jwt}\, dt \right|}{\left| \int_0^{T_x} x(t)\, e^{-jwt}\, dt \right|} \qquad (3.2-7)$$

$$\phi(w) = \text{angle}\left[s(w)_y\right] - \text{angle}\left[s(w)_x\right] \qquad (3.2-8)$$

If the Fourier transform integrals of Equation (3.2 - 3) are evaluated numerically, frequency response of the system with the use of Equations (3.2 - 7) and (3.2 - 8) can be obtained. The pulse reduction procedure is presented in Appendix B.1 .

The frequency content of a non-periodic function is a continuous function of frequency (Appendix A) and contains a continuous frequency spectrum. If such a pulse is applied as the input to a physical system, the system will be excited by all frequencies simultaneously, and the system output will consist of the combined responses to these frequencies.

The close relationship between the frequency content, especially of the input pulse, and the frequency response derived from a pulse test is of paramount importance to the efficient recovery of frequency response data from the pulse test. For purpose of comparing pulses of different shape with regard to frequency content, it is a common practice to plot a normalized frequency content, $s(w)_n$, rather than $s(w)$ itself. $s(w)_n$ is given by

$$s(w)_n = \frac{s(w)}{s(0)} \qquad (3.2-9)$$

i.e., the ratio of the frequency content at w to the content at w = 0. Behavior of frequency content depends on both pulse shape and pulse width, T_p, If the normalized frequency content is plotted against $w \cdot T_p$, it will depend on pulse shape only. In Figure 2, plots for frequency content of some simple pulses are given (4, 30). Theoretically, with the pulse test, frequency response can be obtained over the frequency range except where $s(w)_n$ is zero. But practical limitations, like the accuracy of the numerical evaluation of the Fourier transforms, data reading errors etc. limit the valuable results to the value of $s(w)_n$ going down to 0.3 before first zero occurs (4, 6).

3 . 3 STEP FUNCTION TESTING

The step function testing consists of changing the input variable that is initially at a constant value up to time zero to a different constant value (higher or lower) after time zero. The corresponding transient output is then obtained with a different magnitude. In this study, the forcing strength of the step function has been defined as its magnitude.

The conversion of transient response to frequency response is of considerable practical importance, because, theoretically, a single response curve is sufficient to generate the response to the entire frequency spectrum. The data reduction principle is presented in Appendix B.2 .

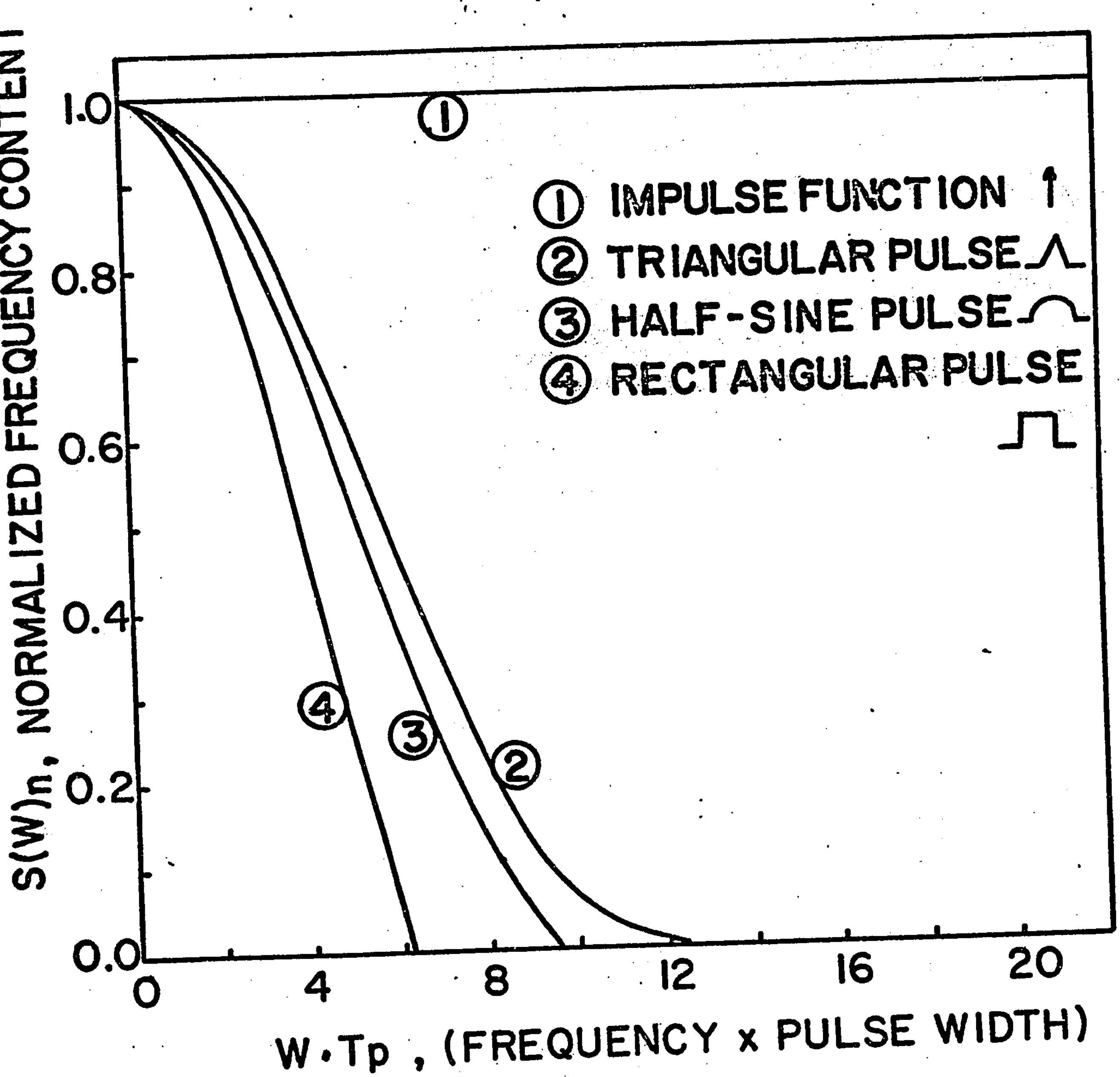


Figure 2. Normalized Frequency Content of Some Mathematically Expressible Pulses.

CHAPTER 4

DESCRIPTION OF PHYSICAL SYSTEM AND TESTING PROCEDURE

4 . 1 DESCRIPTION OF PHYSICAL SYSTEM

The system studied in this investigation was a liquid-level system consisting of a cylindrical tank, an inflow pipe, and an outflow pipe that contained an orifice. If the volumetric rate of flow through the discharge orifice has a square root relationship to the liquid level, then by material balance, the system can be described using the following nonlinear differential equation:

$$-A\frac{dL}{dt} = m_0 - m_i \qquad (4.1 - 1)$$

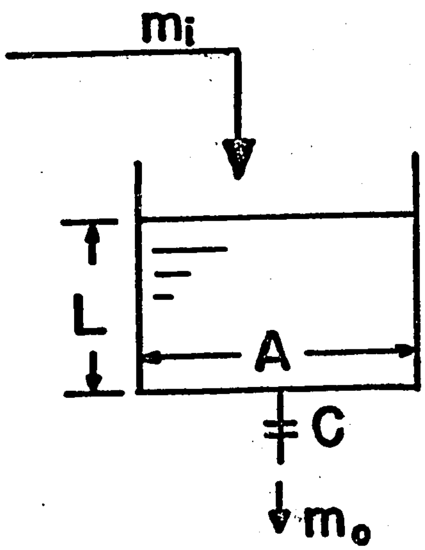
Since $m_0 = C L^{1/2}$ (4.1 - 2)

so $A\frac{dL}{dt} + C L^{1/2} = m_i$ (4.1 - 3)

where A is the cross-sectional area of the vessel and C is the coefficient of the orifice. The values of A and the coefficient C for the different cases are tabulated in Table 1 . The systems are shown in Figures 3 and 4.

In case IV the nonlinearity was a cube root relationship and case V consisted of two first order noninteracting tanks in series. In the latter case, two vessels both with the square root nonlinearities were arranged so that the outlet flow from vessel 1 discharged directly into the atmosphere before spilling into vessel 2 (see Figure 4). Thus, the variation in L_2 did not affect the transient response occurring in vessel 1.

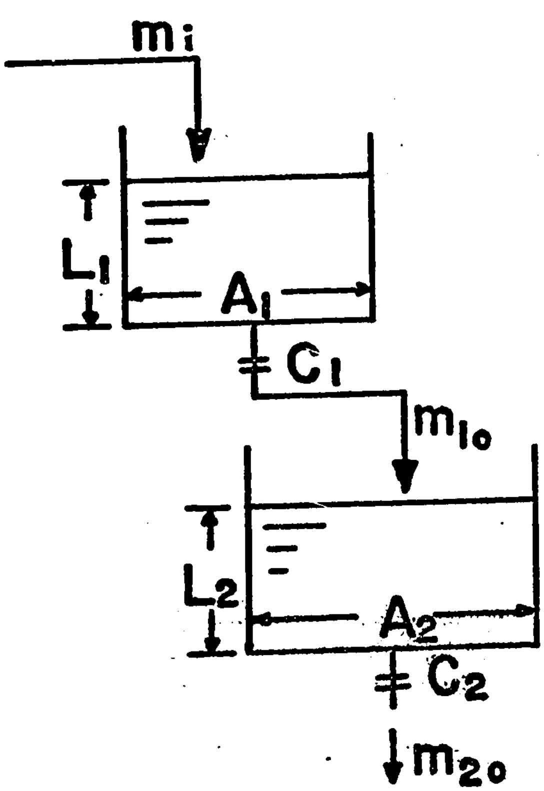
At steady state, the term dL/dt in Equation (4.1 - 3) is equal to zero,



$$A\frac{dL}{dt} + C\,L^{1/2} = m_i$$

where $m_0 = C\,L^{1/2}$

Figure 3. First Order continuous Liquid Flow System.



$$A_1\frac{dL_1}{dt} + C_1 L_1^{1/2} = m_i$$

$$A_2\frac{dL_2}{dt} + C_2 L_2^{1/2} = m_{10}$$

where $m_{10} = C_1 L_1^{1/2}$

$m_{20} = C_2 L_2^{1/2}$

Figure 4. Second Order Noninteracting Flow System.

then the Equation reduces to $L_{ss}^{1/2} = m_i/C$. The steady-state operating characteristics of inflow rate and steady-state level of the systems are also tabulated in Table 1. By means of a Taylor-series expansion, the systems were linearized by expanding the nonlinear variable, $m_0(L)$, around the steady-state value L_{ss} ; thus

$$m_0 = m_0(L_{ss}) + m_0'(L_{ss})(L-L_{ss}) + \frac{m_0''(L_{ss})(L-L_{ss})}{2!} + \cdots \qquad (4.1-4)$$

where $m_0'(L_{ss})$ is the first derivative of m_0 evaluated at L_{ss}, $m_0''(L_{ss})$ the second derivative, etc. If we keep only the linear term, the result is

$$m_0 = m_0(L_{ss}) + m_0'(L_{ss})(L-L_{ss}) \qquad (4.1-5)$$

Introducing Equation (4.1 - 2) into Equation (4.1 - 5) and applying the technique of deviation variable (23), the linearized system could be expressed as

$$A\frac{dL}{dt} + L/R = m_i \qquad (4.1-6)$$

where $R^{-1} = C\,L_{ss}^{-1/2}/2$. Rearranging Equation (4.1 - 6), the time constant of the system can be expressed as $T = A\,R$, which is the product of a resistance and a capacitance and has the unit of time, and the steady-state gain is equal to R . The term R is simply the conversion factor which relates $L(t)$ to $m_i(t)$ when the system is at steady state.

The desired dynamic properties for this study were the steady-state (zero frequency) gain and the time constant of the system. These properties of the linearized systems are listed in Table 1.

The means of obtaining the transient response in this study was the Runge - Kutta method (31, 32, 33) . The concept of this method first assumes that the function can be approximated by a truncated Taylor series expansion and then makes use of a numerical procedure to approximate the truncated Taylor series. The fourth order Runge - Kutta

Table 1. Characteristics of the systems studied.

System	Nonlinearity	Cross-sectional area, A, ft^2 (Diameter, ft)	Steady-state inflow rate m_i, ft^3/min	Steady-state level height L, ft.	Constant C $ft^3/min\text{-}ft^{1/2}$	Linearized system T (min)	Linearized system G $(ft\text{-}min/ft^3)$
I	$L^{1/2}$	19.635 (5)	250	2.233	167.3	0.35076	0.017864
II	$L^{1/2}$	9.8175 (2.5)	250	4.466	118.2989	0.35076	0.035728
III	$L^{1/2}$	19.635 (5)	250	4.466	118.2989	0.70152	0.035728
IV	$L^{1/3}$	19.635 (5)	250	2.233	191.28	0.526	0.02679
V	$L^{1/2}$, $L^{1/2}$	19.635 (5)	250	2.233	167.3	0.35076	0 017864
		19.635 (5)		2.233	167.3	0.35076	

formula is comparatively simple and is by far the most popular. This formula is given without proof:

$$y_{i+1} = y_i + (p + 2q + 2r + v)/6 + O (h^5) \qquad (4.1 - 7)$$

where

$$p = h f (x_i , y_i) \qquad (4.1 - 8)$$

$$q = h f (x_i + h/2 , y_i + p/2) \qquad (4.1 - 9)$$

$$r = h f (x_i + h/2 , y_i + q/2) \qquad (4.1 - 10)$$

and

$$v = h f (x_i + h, y_i + r) \qquad (4.1 - 11)$$

When using these equations one must find the terms p, q, r and v for each point in the solution. This means that the function $f (x, y)$ must in general be computed for four different sets of arguments. The increment on y for second interval is computed by the same formulas, with (x_i , y_i) replaced by (x_{i+1} , y_{i+1}). Thus all intervals are computed in the same manner, using for the initial values the values at the beginning of each interval. The method does not need any special formulas to get the solution started and it adapts itself very nicely to a computational form.

4 . 2 TESTING PROCEDURE

In applying the pulse technique to investigate the particular non-linear systems, one varied the influent liquid flow rate as the input variable in a pulse-like manner. This meant that the variable was displaced from its equilibrium position for only a finite time. Before and after the duration of the pulse, this variable was held constant.

The output, change of level, of the system would also behave in a pulse-like shape.

In this investigation, three pulses; rectangular, triangular and half-sine, in mathematical form were varied in width from 0.3 to 1.5 minutes for systems I and II, from 0.7 to 2.8 minutes for system III. At each pulse duration, the amplitudes of the three input pulses were conducted by 4%, 10% and 16% deviation from the steady-state value. For the system of case I, IV and V, step forcing was done on the influent flow rate.

Recording both the time histories of input and output, which were simulated by using the Runge-Kutta method, frequency response of the system was calculated as the ratio of the output and input Fourier transform. The time constant was then calculated by taking the reciprocal of corner frequency, that is, the frequency at -45 degrees phase shift. The steady-state gain was the amplitude ratio at zero frequency.

To facilitate the computational requirement, the author has written a program based on Equations (4.1 - 7) through (4.1 - 11) for an IBM 360/65 computer which carried out the pulse traces generation. Also, computer programs which carried out the pulse evaluation using the trapezoidal rule (Equations (B.1 - 3) through (B.1 - 15) and Equations (B. 2 - 7) through (B. 2 - 12)) have been written.

The programs calculate numerically the area under product curves as specified by Equations (B.1 - 16) through (B.1 - 20) and Equations (B. 2 - 7) through (B. 2 - 10). These areas are calculated separately for the interval Δt, being set to 0.01 in both input and output traces, and then summed to yield the total values of A, B, C and D. Magnitude ratio and phase angle are then computed using Equations (B.1 - 10), (B.1 - 11), (B. 2 - 11) and (B. 2 - 12), beginning at a specified value of w and incrementing it by a specified amount for each additional point until the highest value desired has been reached. Also calculated and punched simultaneously is $s(w)_x$, the frequency content of the input pulse at the w-value in question. It would be more meaningful for $s(w)_x$ when w was equal to zero (that is, the strengh of the input pulse). The effect of the zero frequency content of input pulse on the dynamic property will be discussed in detail later.

CHAPTER 5

RESULTS

This chapter presents all the test results. For the nonlinear system, the dynamic properties differed with different strengh of forcings applied on the systems. The results tested by pulse forcings of the systems I, II and III are shown plotted on Figures 5 through 10 and are tabulated in Appendix C.1. These are followed by the step-testing results of systems I, IV and V shown in Figures 11 through 13, also tabulated in Appendix C.2. The test results of the dynamic properties of the linearized systems are tabulated in Table 2.

Table 2. The Dynamic Properties of the Linearized Systems

System	Time Constant (min)	Steady-state Gain (ft/gpm)
I	0.35076	0.01784
II	0.35076	0.03571
III	0.70152	0.03566
IV	0.52600	0.02679
V	0.54841 0.14560	0.01786

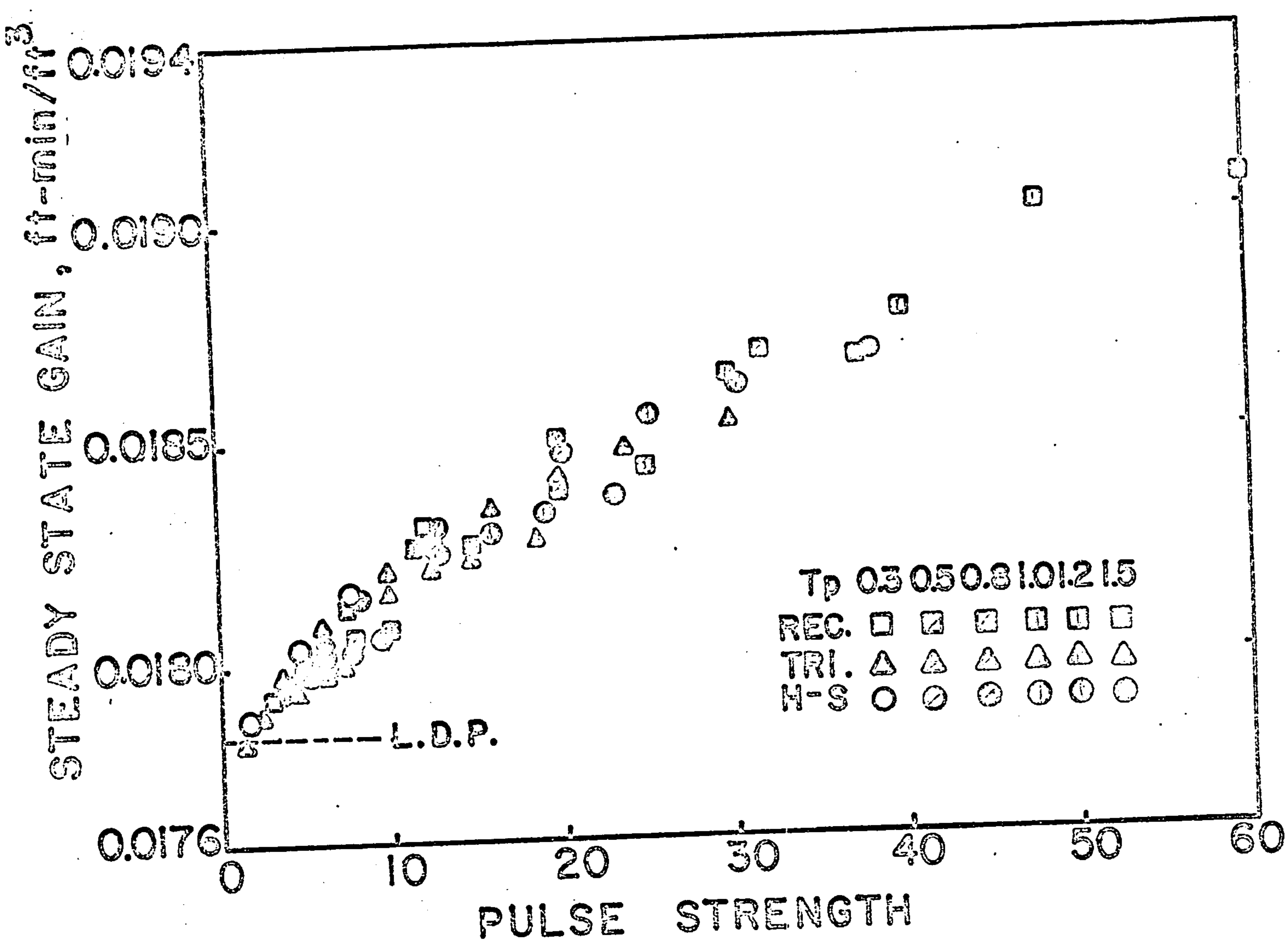


Figure 5. Variation of Steady-state Gain With Pulse Strength of System I.

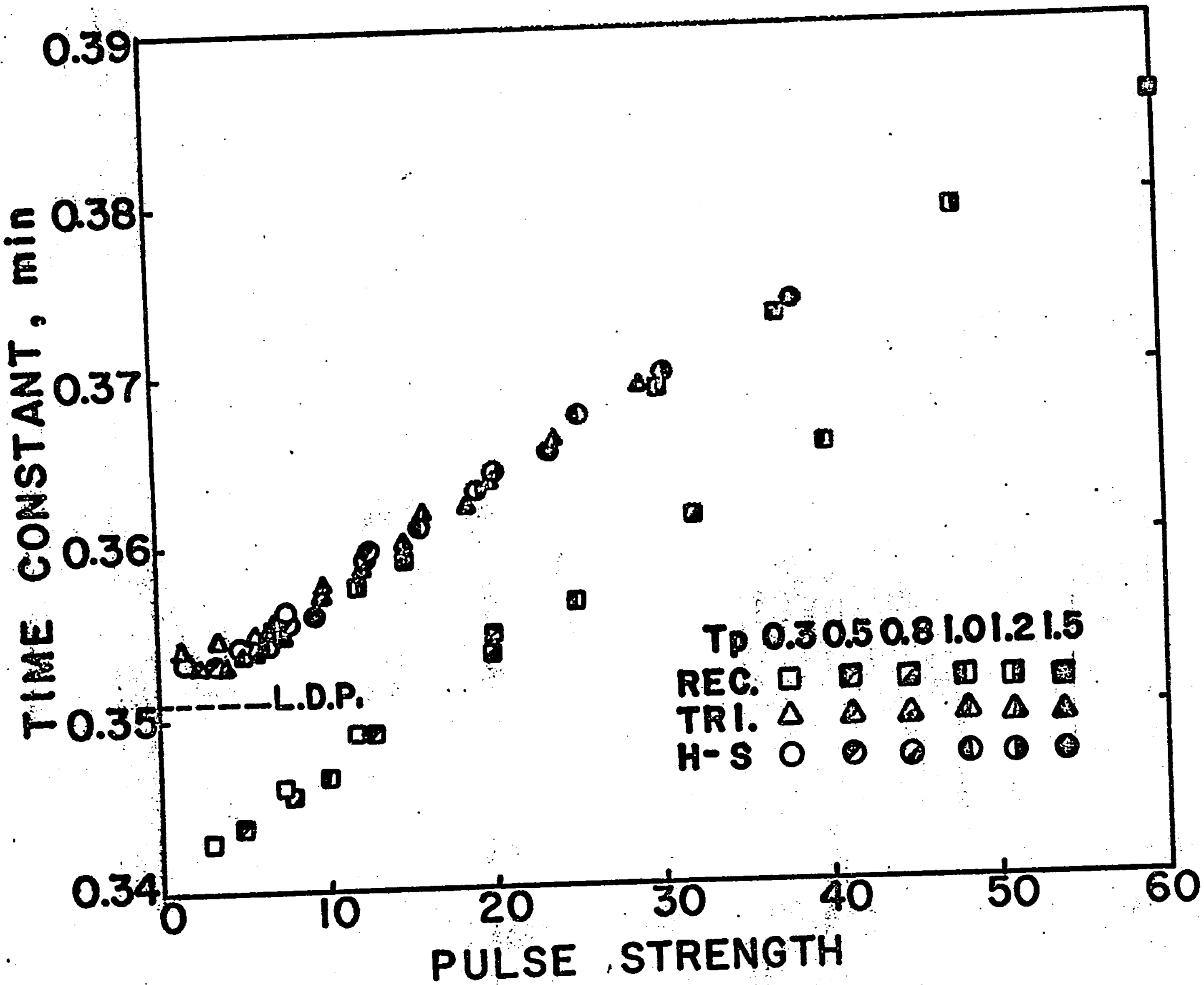


Figure 6. Variation of Time Constant with Pulse Strength of System I.

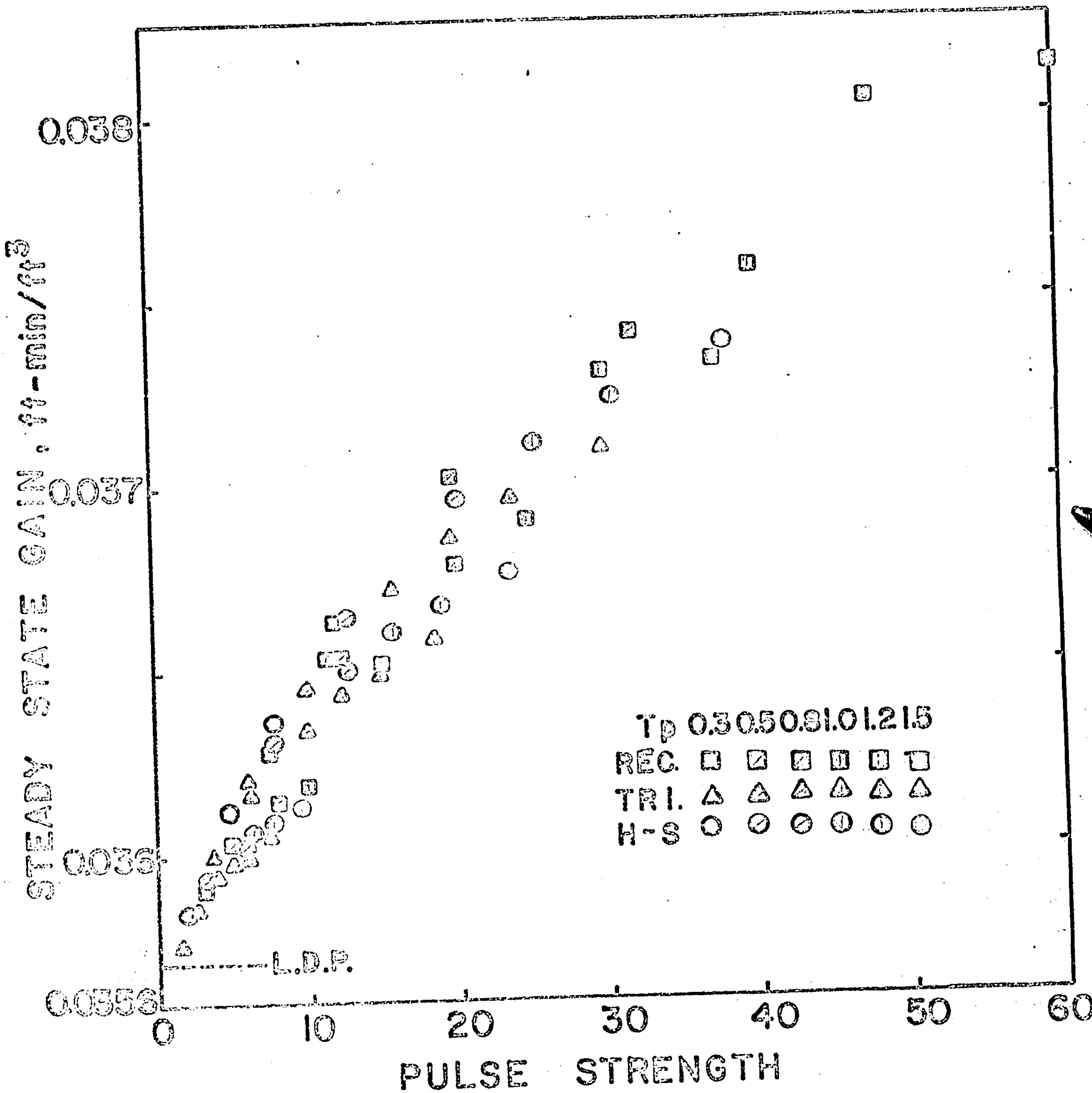


Figure 7. Variation of Steady-state Gain With Pulse Strength of System II.

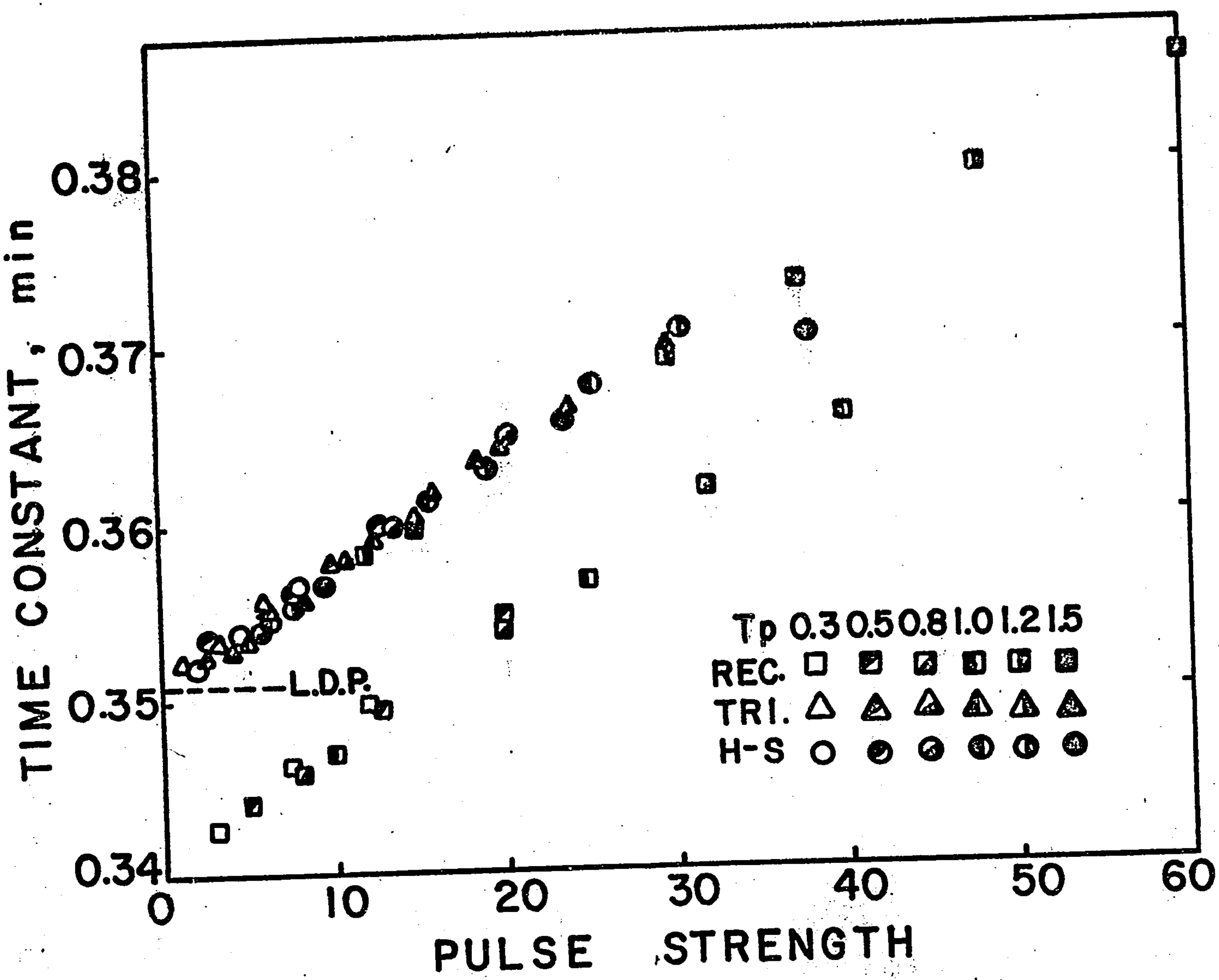


Figure 8. Variation of Time Constant With Pulse Strength of System II.

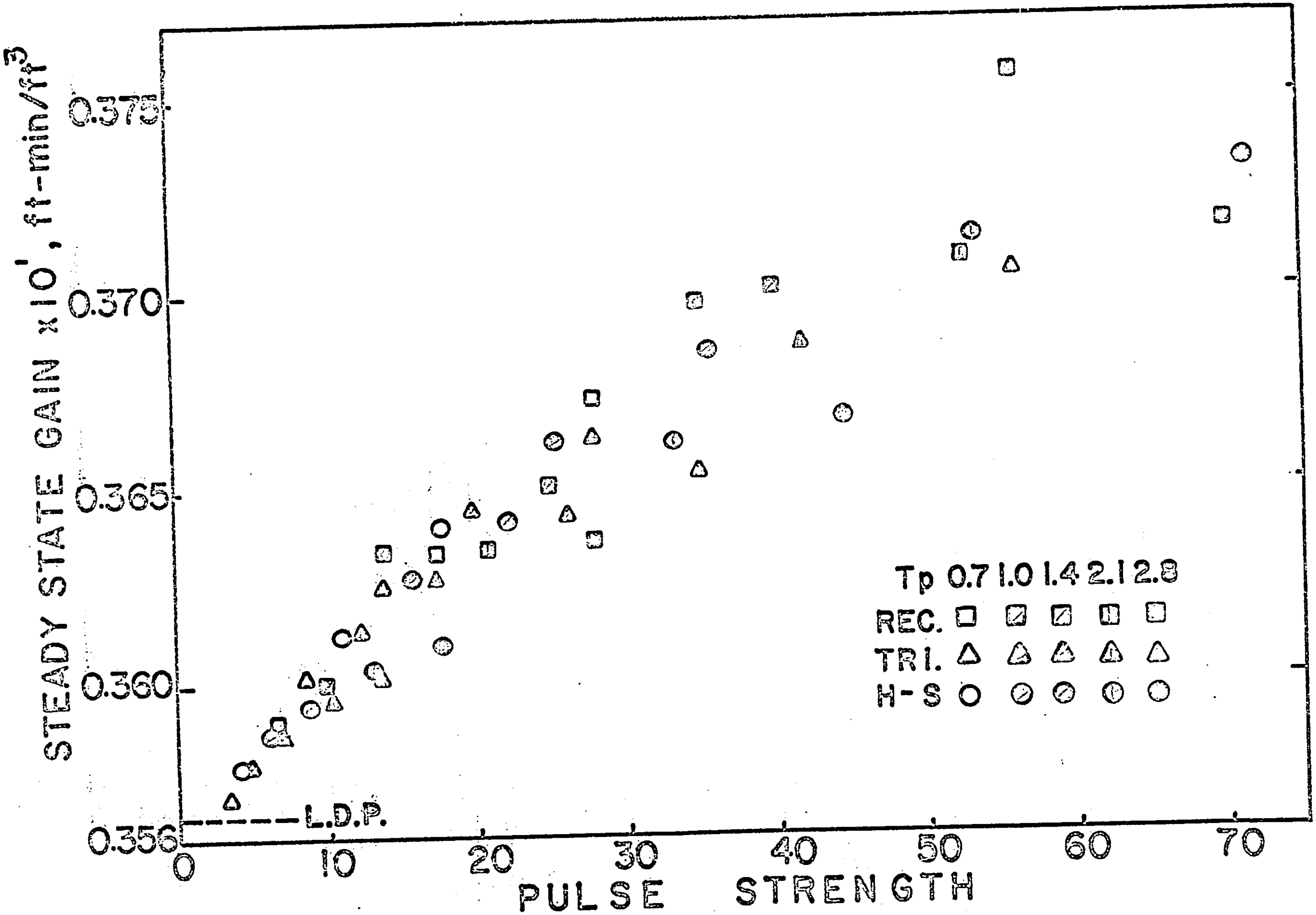


Figure 9. Variation of Steady-state Gain With Pulse Strength of System III.

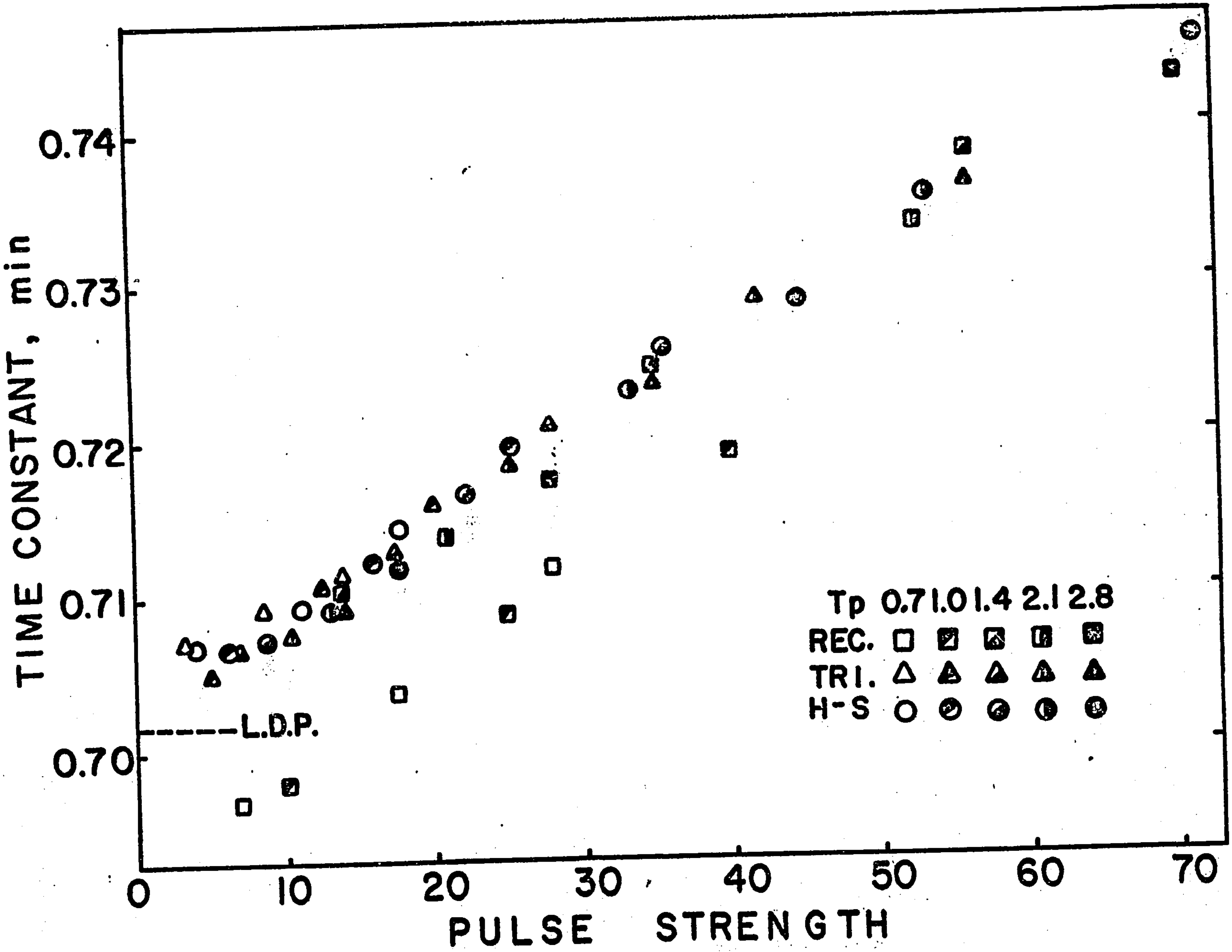


Figure 10. Variation of Time Constant With Pulse Strength of System III.

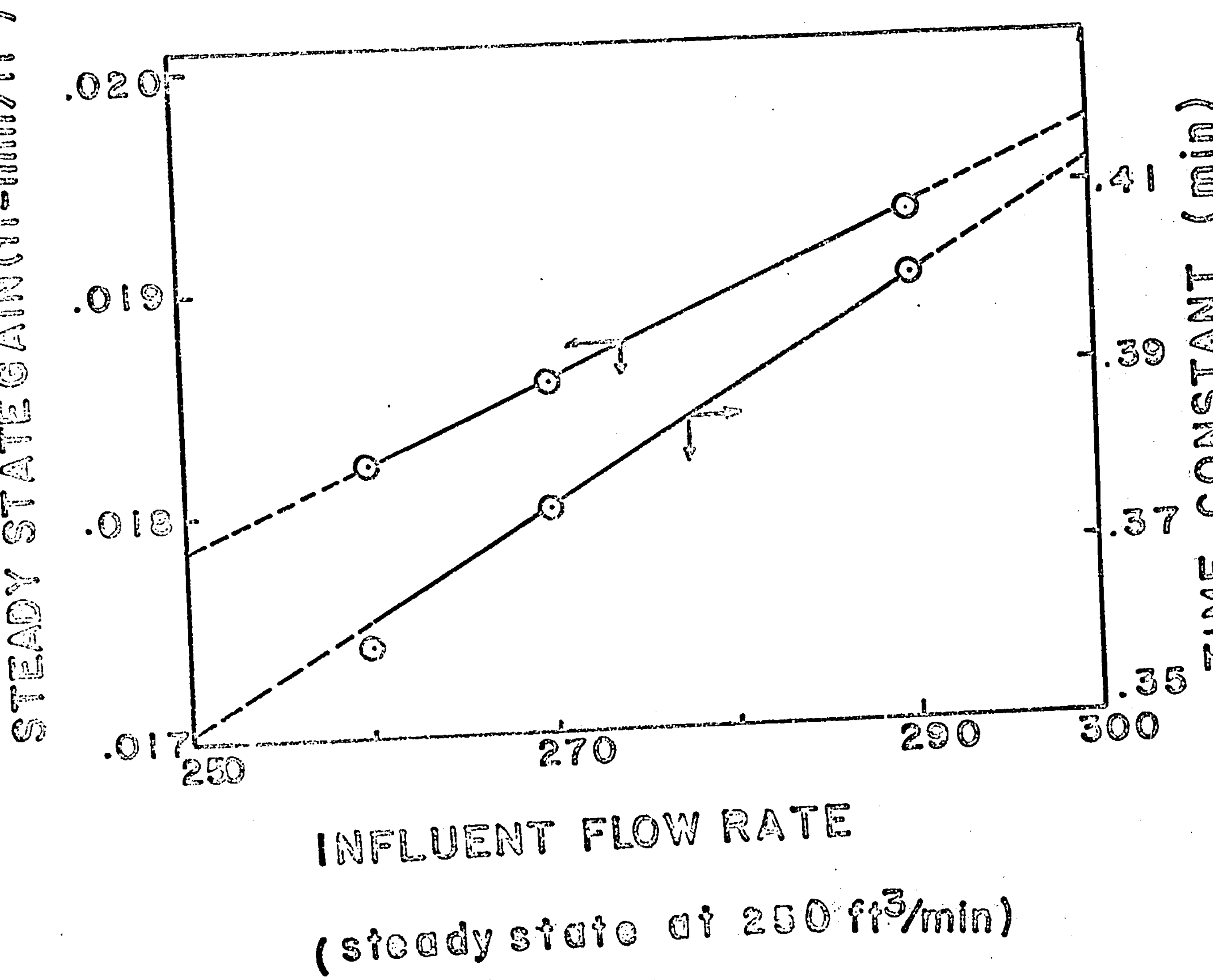


Figure 11. The Step Response of Steady-state Gain and Time Constant of System I.

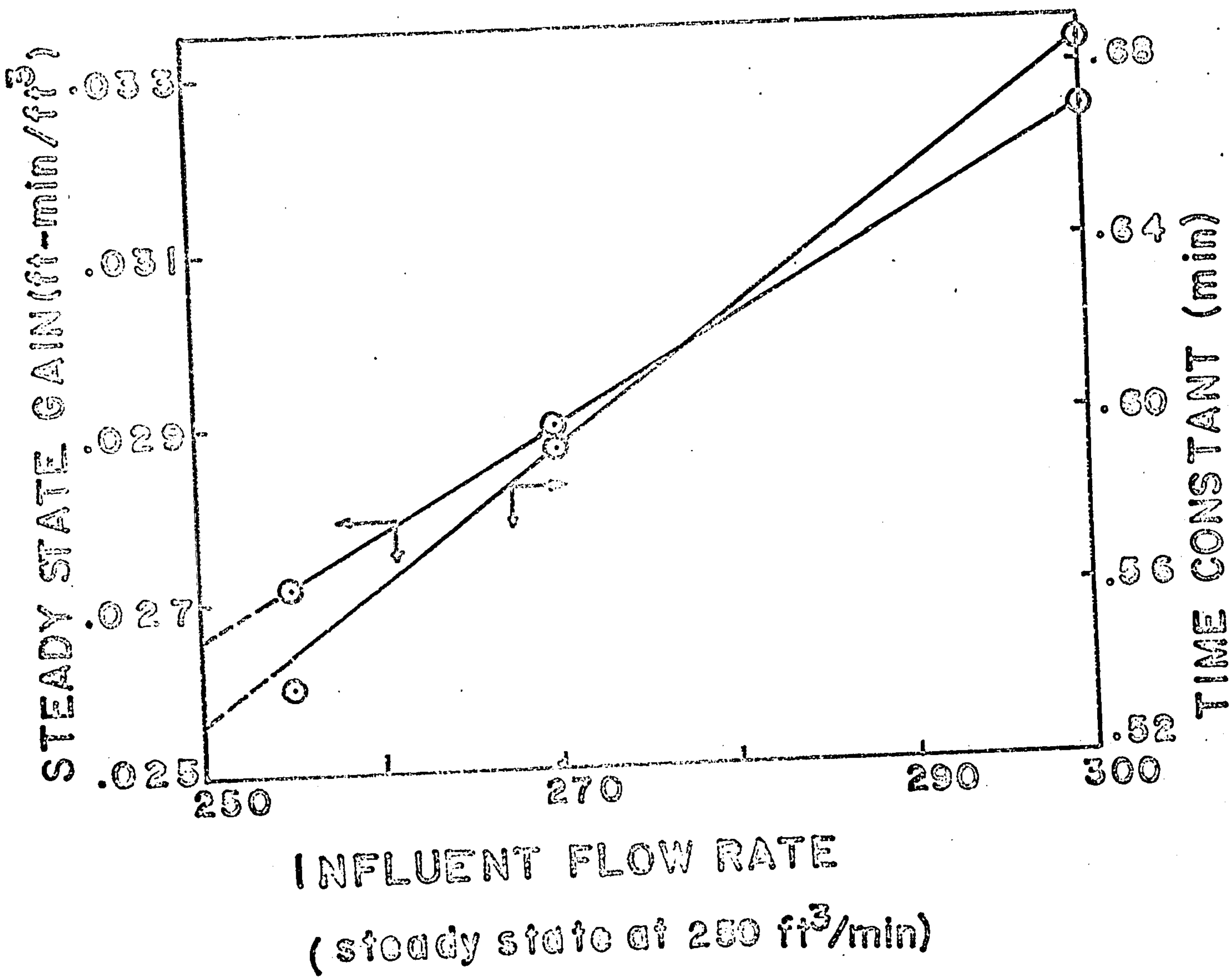


Figure 12. The Step Response of Steady-state Gain and Time constant of System IV.

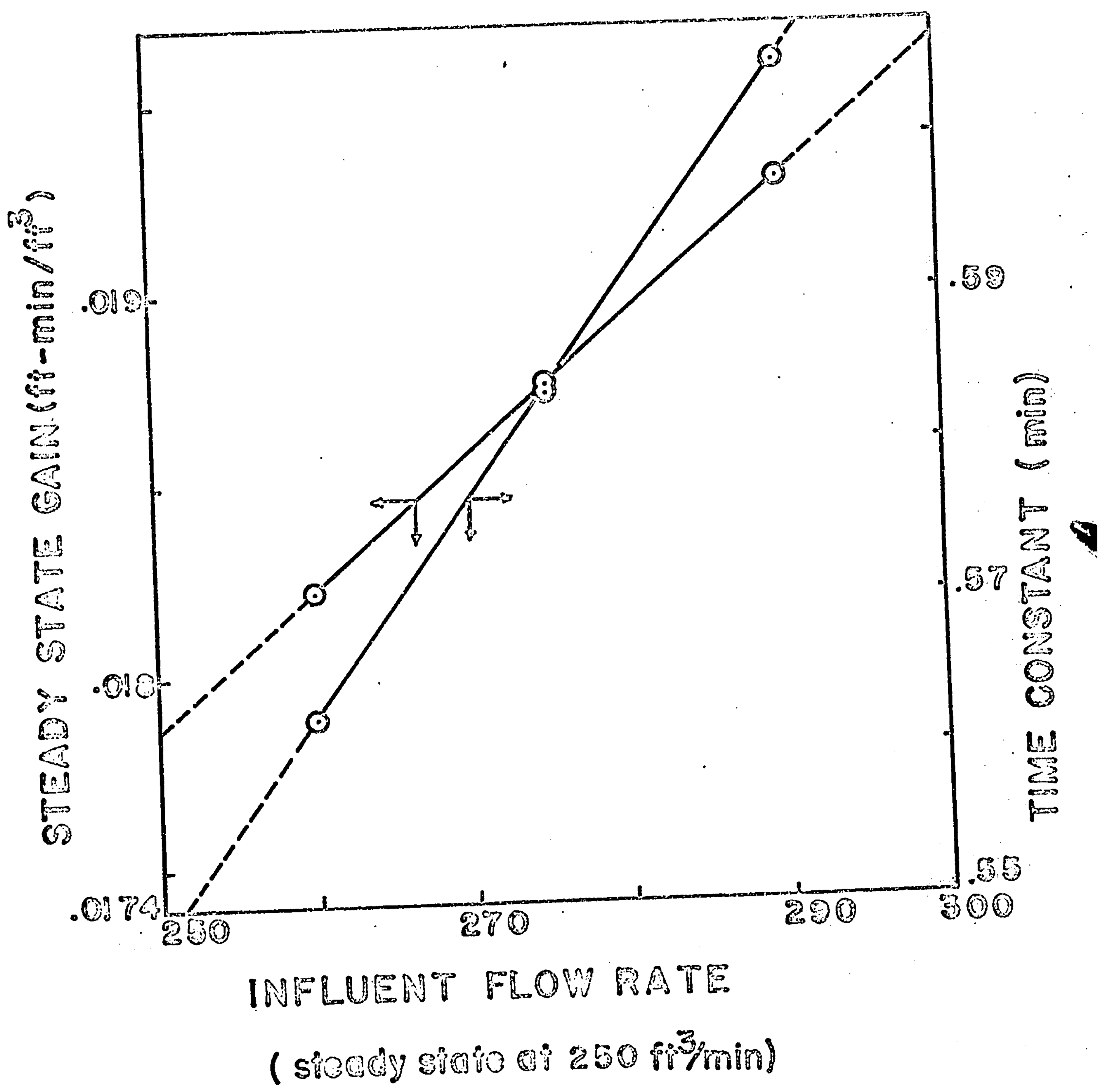


Figure 13. The Step Response of Steady-state Gain and Time Constant of System V.

CHAPTER 6

DISCUSSION

It is interesting to note (Figures 11 to 13) that the changes of the steady-state gain with a few step forcings formed a linear relationship and by extrapolating to the steady-state operating condition (zero step height), the steady-state gains were identical to that of linearized system listed in Table 2. A linear relationship also held for the time constant except at a low magnitude of step height.

Pulse testing results are shown in Figures 5 through 10. The linear relationship still existed between the system dynamic properties and the input pulse strength. However, these lines did not extrapolate to give the true linearized system properties. Separate curves were obtained for each pulse shape and width, i. e., with different pulse widths, the slopes and the intercepts of the lines changed.

In Figures 6, 8 and 10, at small values of T_p, the intercepts tested by the rectangular pulse are much lower than that of linearized system. This situation, in general, agrees with that obtained by Hennig (19) who has pointed out that a square-wave input in most cases causes large error. This type of signal must therefore be avoided where a nonlinearity is suspected.

From Figures 5 through 10, it is obvious that the width of the pulse is the parameter of the sets of lines. An attempt was made to correlate the slopes and intercepts of these lines as a function of pulse strength and width. The following relationship were used,

$$G = CO(T_p) + CI(T_p) \cdot s(w)_0 \qquad (6.1 - 1)$$

$$T = CO'(T_p) + CI'(T_p) \cdot s(w)_0 \qquad (6.1 - 2)$$

where $s(w)_0$ is the pulse strength or the zero frequency content of forcing pulse, and T_p is the pulse width.

Taking the trend of half-sine pulse as the basis (data shown in Table 3 and plotted on Figure 14), it was found that $CO(T_p)$ and $CI(T_p)$ could approximately be expressed as a third order polynomial as follows:

$$CO(T_p) = AO + AI \cdot T_p + A2 \cdot T_p^2 + A3 \cdot T_p^3 \qquad (6.1-3)$$

$$CI(T_p) = AO' + AI' \cdot T_p + A2' \cdot T_p^2 + A3' \cdot T_p^3 \qquad (6.1-4)$$

Applying the least square method (computer program is presented in Appendix D), the correlated constants AO, AI, A2 and A3 were obtained and tabulated in Table 4. The polynomial expressions of intercept and slope are shown graphically as the solid lines in Figures 14 through 19 .

It was found that the Equations (6.1 - 3) and (6.1 - 4) developed for the half-sine pulse also predicted the steady-state gains and time constants obtained using rectangular and triangular pulses. Comparison of the system dynamic properties calculated from test and that from Equations (6.1 - 1) and (6.1 - 2) for the various pulse shapes with known values of $s(w)_0$ and T_p are shown on Figures 20 and 21 and tabulated in Appendix E. Deviation between these two sets of values was within $\pm$ 1% except for the time constants obtained by using shorter rectangular pulse duration.

When the pulse strength $s(w)_0$ in Equations (6.1 - 1) and (6.1 - 2) is zero, the intercepts $CO(T_p)$ and $CO'(T_p)$ yield the dynamic properties G and T of the linear system. Thus, the value

Table 3. Intercepts and slopes of lines caused by half-sine forcing.

For steady-state gain of system I

T_p(min)	0.3	0.5	0.8	1.0	1.2	1.5
slope	0.50663×10^{-4}	0.38759×10^{-4}	0.31421×10^{-4}	0.27754×10^{-4}	0.25309×10^{-4}	0.21991×10^{-4}
intercept	0.17788×10^{-1}	0.17822×10^{-1}	0.17833×10^{-1}	0.17845×10^{-1}	0.17840×10^{-1}	0.17852×10^{-1}

For time constant of system I

slope	0.46118×10^{-3}	0.66533×10^{-3}	0.67556×10^{-3}	0.66877×10^{-3}	0.63058×10^{-3}	0.62246×10^{-3}
intercept	0.35250	0.35102	0.35052	0.35036	0.35059	0.35016

For steady-state gain of system II

slope	0.90851×10^{-4}	0.74381×10^{-4}	0.61540×10^{-4}	0.54988×10^{-4}	0.49747×10^{-4}	0.43635×10^{-4}
intercept	0.35673×10^{-1}	0.35698×10^{-1}	0.35710×10^{-1}	0.35715×10^{-1}	0.35713×10^{-1}	0.35715×10^{-1}

For time constant of system II

slope	0.76881×10^{-3}	0.67893×10^{-3}	0.69257×10^{-3}	0.67563×10^{-3}	0.66027×10^{-3}	0.47408×10^{-3}
intercept	0.35053	0.35108	0.35047	0.35038	0.35021	0.35272

For steady-state gain of system III

T_p(min)	0.7	1.0	1.4	2.1	2.8
slope	0.46394×10^{-4}	0.39277×10^{-4}	0.34039×10^{-4}	0.27431×10^{-4}	0.23002×10^{-4}
intercept	0.35583×10^{-1}	0.35628×10^{-1}	0.35642×10^{-1}	0.35680×10^{-1}	0.35695×10^{-1}

For time constant of system III

slope	0.56724×10^{-3}	0.66567×10^{-3}	0.69122×10^{-3}	0.66377×10^{-3}	0.62498×10^{-3}
intercept	0.70375	0.70220	0.70101	0.70052	0.70073

Table 4. Correlated Constants of Polynomials in Equations 6.1 - 3 and 6.1 - 4.

System	Dynamic property	Equation	A O	A I	A 2	A 3
I	G	6.1 - 3	0.17702×10^{-1}	0.38860×10^{-3}	-0.36570×10^{-3}	0.11555×10^{-3}
I	G	6.1 - 4	0.76980×10^{-4}	-0.11353×10^{-3}	0.90465×10^{-4}	-0.26183×10^{-4}
I	T	6.1 - 3	0.35670×10^{0}	-0.19314×10^{-1}	0.19281×10^{-1}	-0.62066×10^{-4}
I	T	6.1 - 4	-0.94458×10^{-4}	0.26231×10^{-2}	-0.27331×10^{-2}	0.86970×10^{-3}
II	G	6.1 - 3	0.35609×10^{-1}	0.28132×10^{-3}	-0.25010×10^{-3}	0.73001×10^{-4}
II	G	6.1 - 4	0.12505×10^{-3}	-0.14250×10^{-3}	0.99309×10^{-4}	-0.27047×10^{-4}
II	T	6.1 - 3	0.34722×10^{0}	0.17731×10^{-1}	-0.25525×10^{-1}	0.10758×10^{-1}
II	T	6.1 - 4	0.10814×10^{-2}	-0.15741×10^{-2}	0.19756×10^{-2}	-0.79758×10^{-3}
III	G	6.1 - 3	0.35456×10^{-1}	0.24792×10^{-3}	-0.98182×10^{-4}	0.14373×10^{-4}
III	G	6.1 - 4	0.70301×10^{-4}	-0.44998×10^{-4}	0.16853×10^{-4}	-0.24354×10^{-5}
III	T	6.1 - 3	0.71034×10^{0}	-0.12902×10^{-1}	0.55264×10^{-2}	-0.76560×10^{-3}
III	T	6.1 - 4	0.10706×10^{-3}	0.98216×10^{-3}	-0.51451×10^{-3}	0.82096×10^{-4}

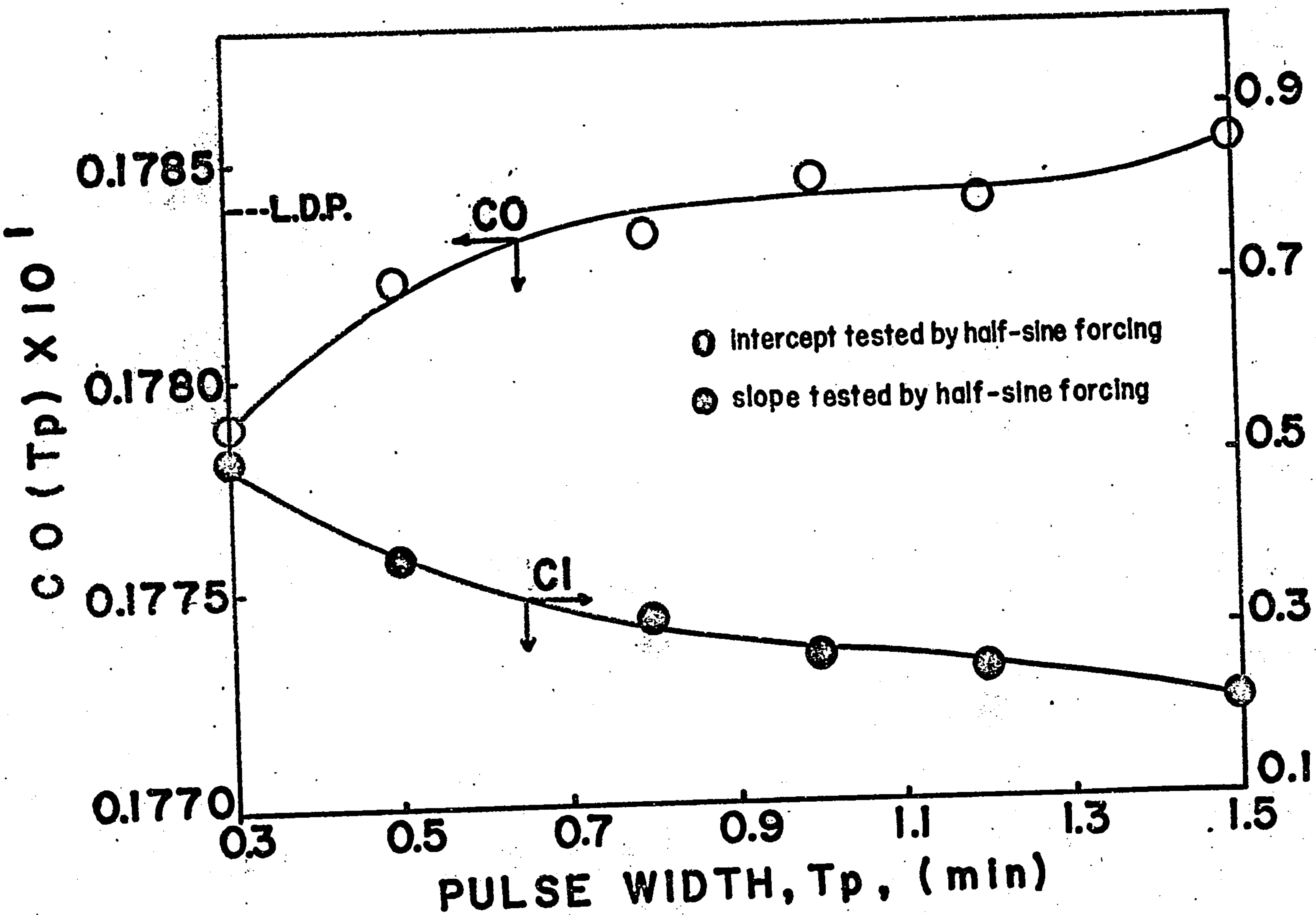


Figure 14. Intercept (CO (T_p)) and slope (CI(T_p)) Polynomial Expressions versus Pulse Width for Steady-state Gain of System I.

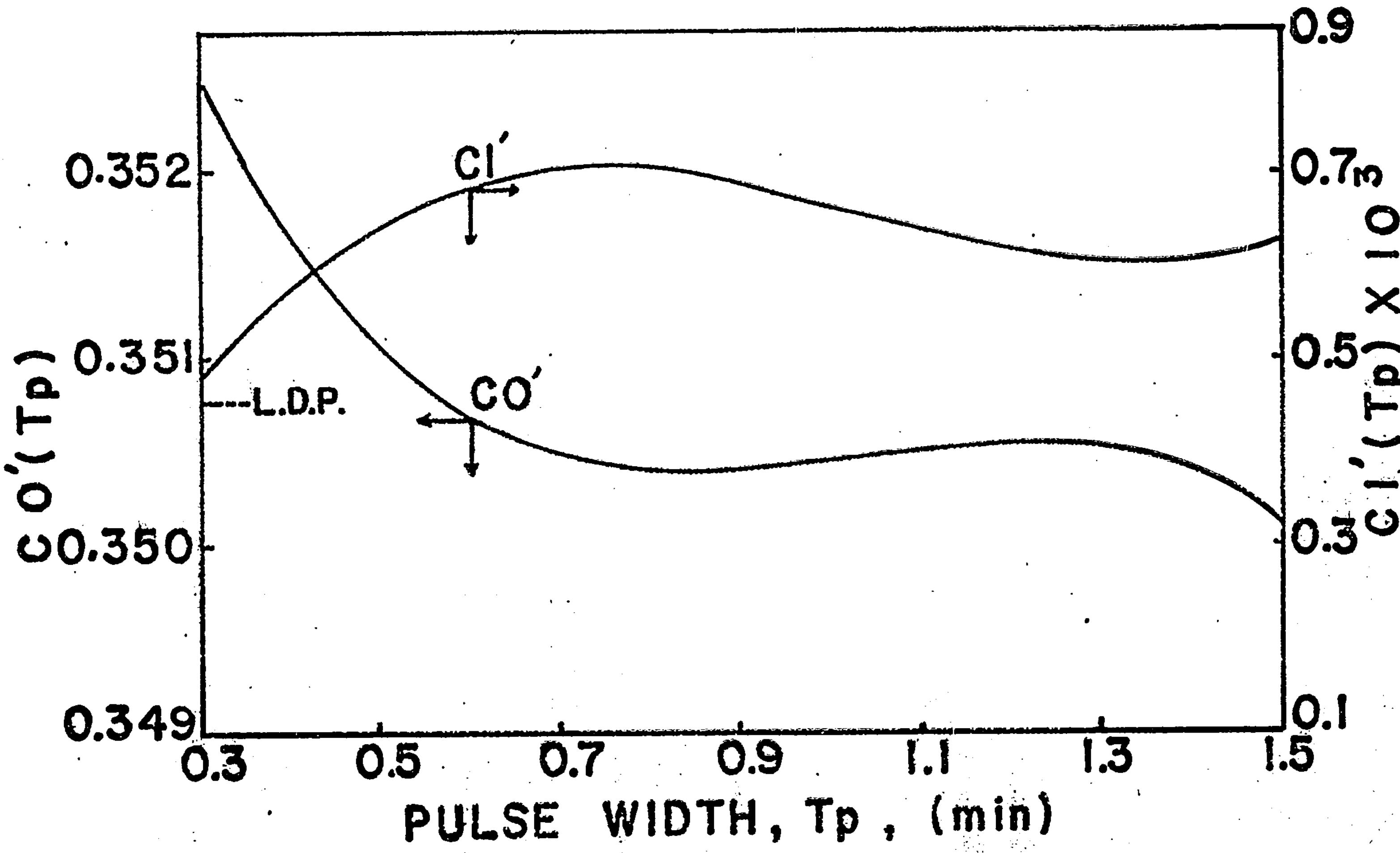


Figure 15. Intercept ($CO(T_p')$) and slope ($CI(T_p')$) Polynomial Expressions versus Pulse Width for Time Constant of System I.

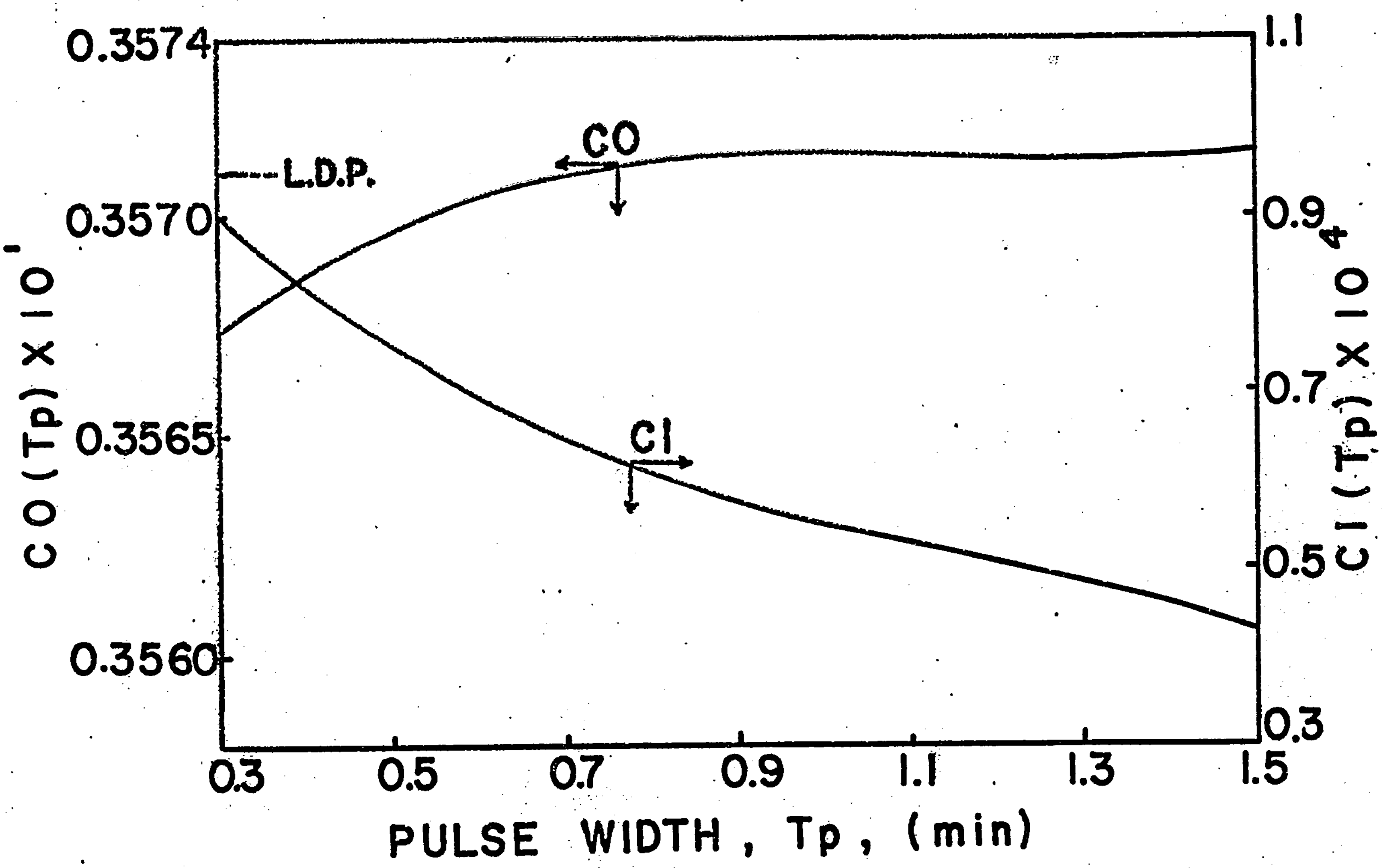


Figure 16. Intercept ($CO(T_p)$) and Slope ($CI(T_p)$) Polynomial Expressions versus Pulse Width for Steady-state Gain of System II.

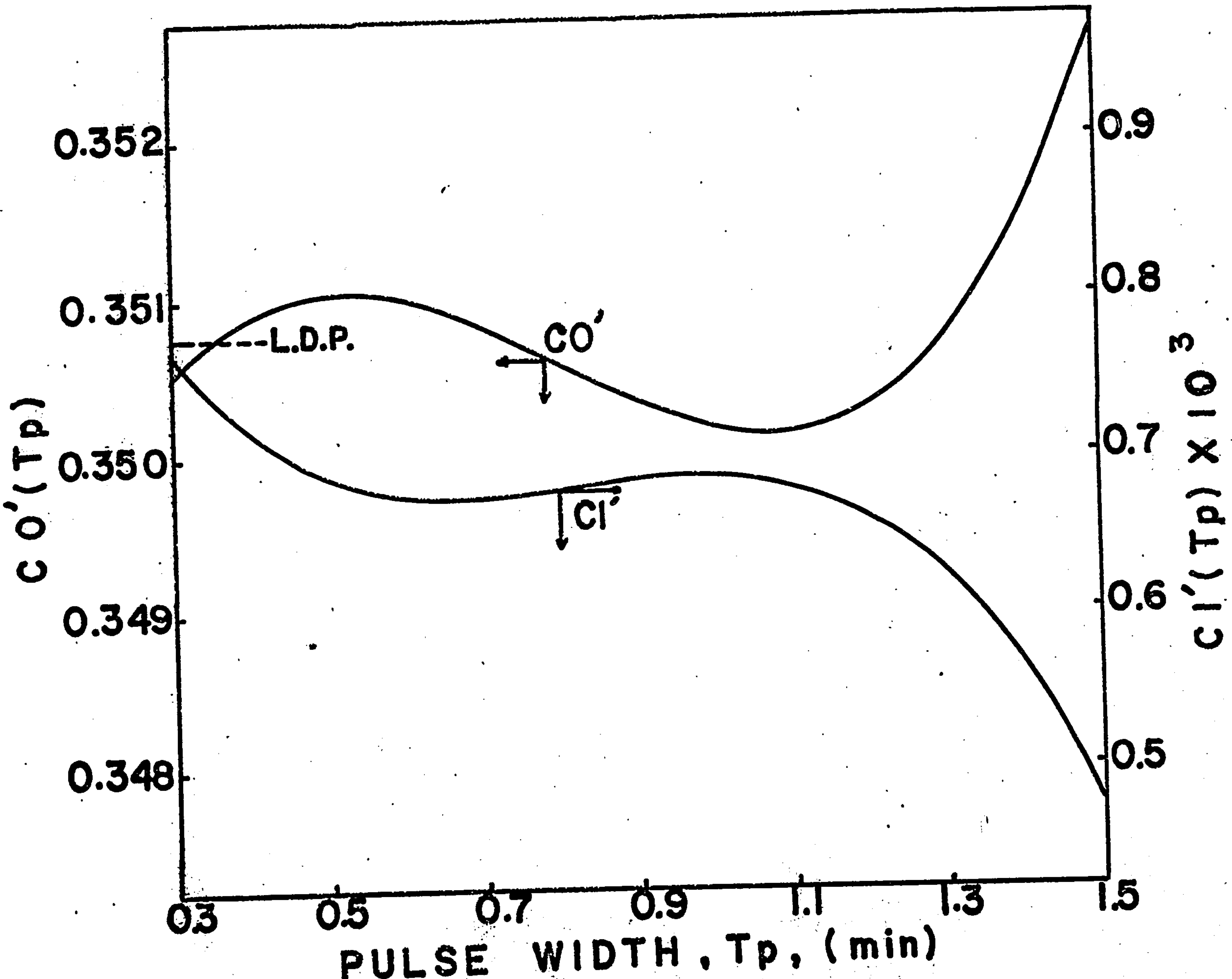


Figure 17. Intercept ($CO(T_p)$) and Slope ($CI(T_p)$) Polynomial Expressions versus Pulse Width for Time Constant of System II.

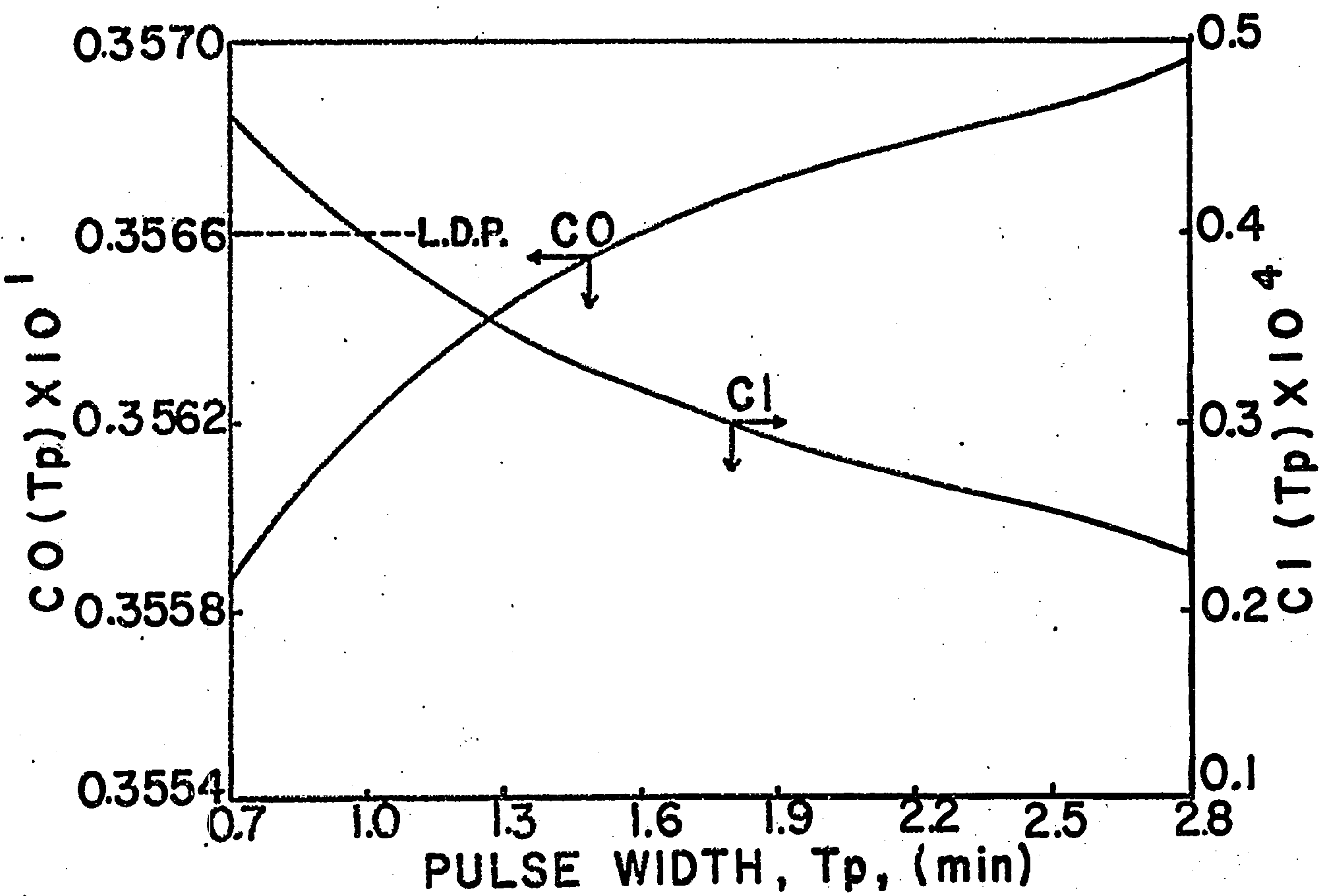


Figure 18. Intercept ($CO(T_p)$) and Slope ($CI(T_p)$) Polynomial Expressions versus Pulse Width for Steady-state Gain of System III.

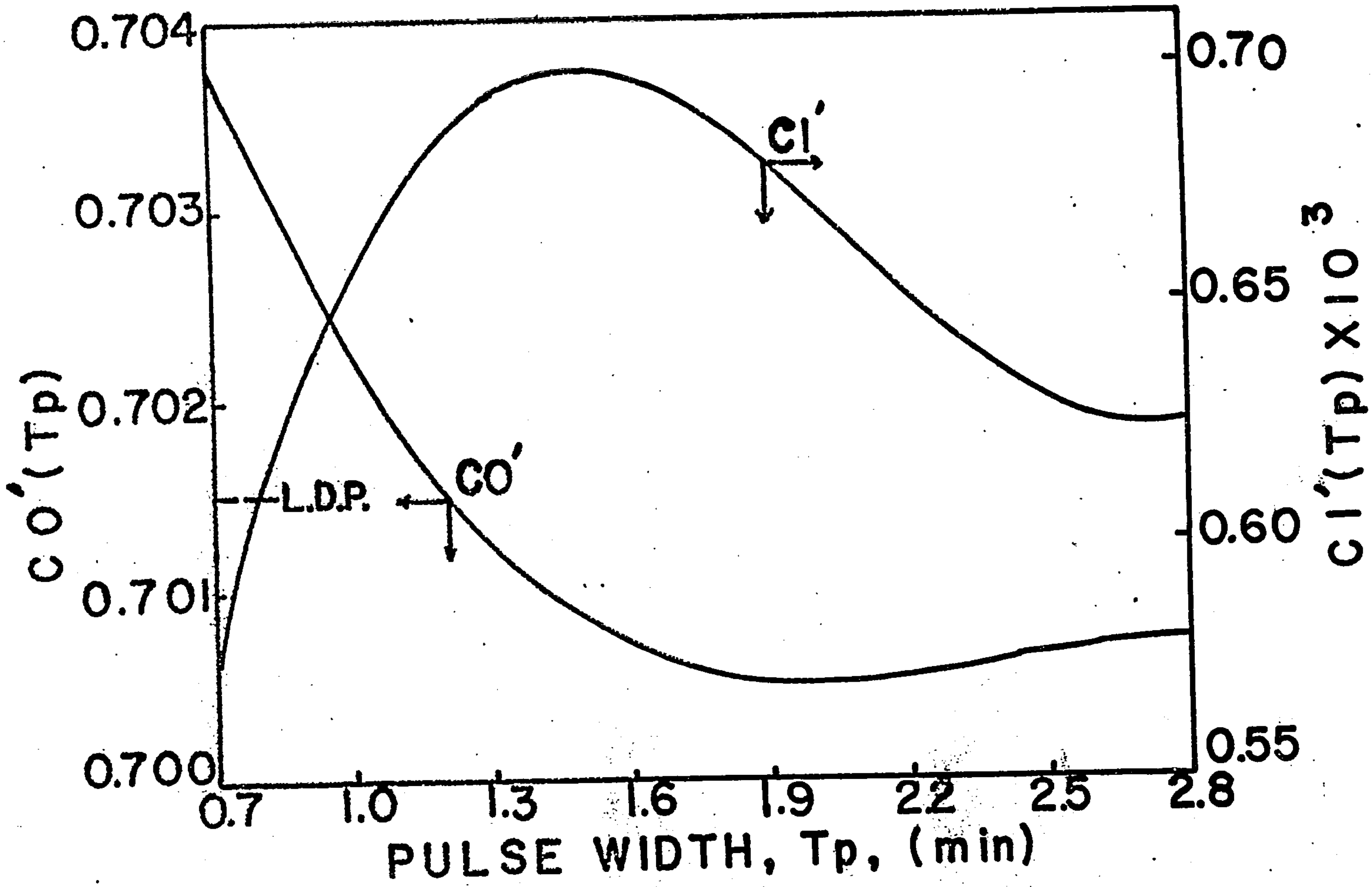


Figure 19. Intercept ($CO(T_p)$) and Slope ($CI(T_p)$) Polynomial Expressions versus Pulse Width for Time Constant of System III.

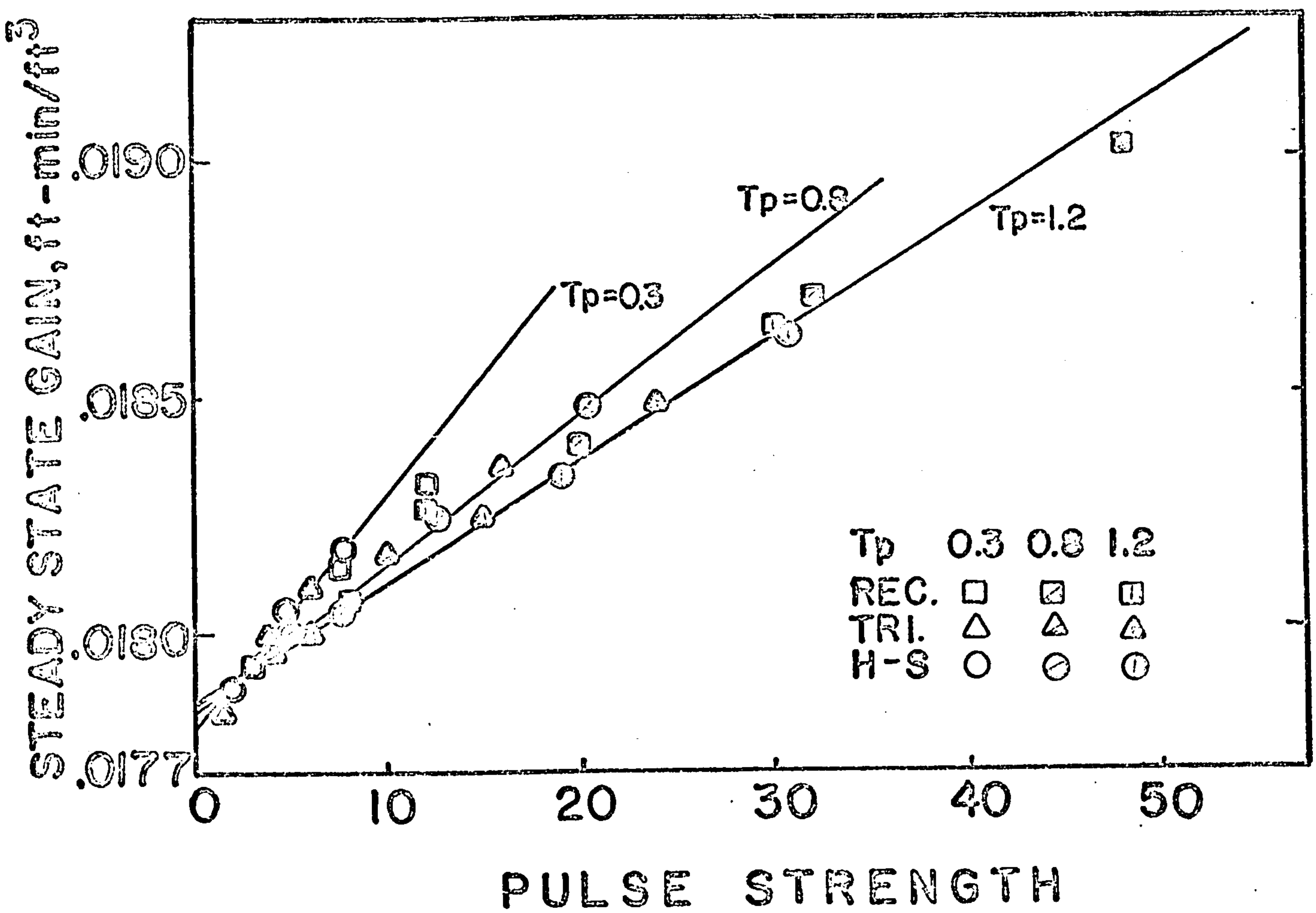


Figure 20. Steady-state Gain versus Pulse Strength.

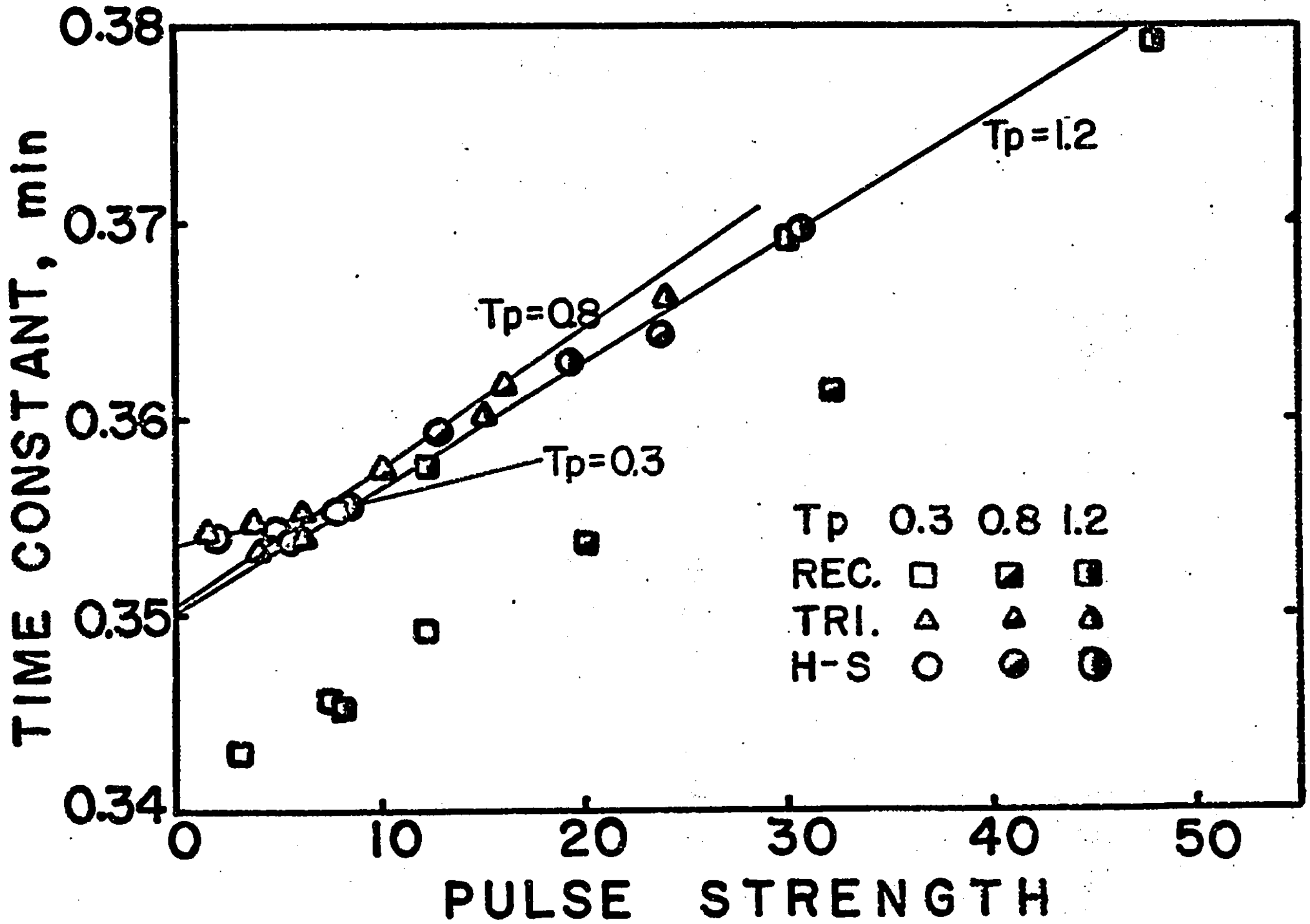


Figure 21. Time Constant versus Pulse Strength.

of pulse width T_p (in Equation (6.1 - 3)) that gives the correct value of intercept CO (T_p) identical to G or T in Equations(6.1 - 1) and (6.1 - 2) is taken as the optimal pulse width. The optimal pulse width was read from the CO or CO' line in Figures14 through 19. The results are tabulated in Table 5. It was found that this value of T_p was approximately 1.6 to 2.6 times the time constant of the linearized system. A further observation resulting from Figures14 through 19 shows that the lower the applied pulse width, the higher the error of the linear dynamic property.

The error effects contributed by the calculation procedure will be discussed here. The fourth order Runge-Kutta formula (see Equation (4.1 - 7)) gives y_{i+1} with a fixed truncation error of order h^5. that is, the right hand side of Equation (4.1 - 7) agrees with the first four terms of the Taylor series of y_{i+1}. The integration interval, h, used in the computer program was set to 0.01. Thus, the effect of the truncation error on the transient response calculation was undistinguishable. Furthermore, Walsh (34) pointed out that the Runge-Kutta method is limited by stability consideration to a minimum interval of 10^{-4}. This means that the integration procedure is unstable when the interval is less than 10^{-4}. Therefore, the interval that used in this study was reasonable.

In this study, since the pulse traces generated from the computer program based on Runge-Kutta theory were first stored in machine memory and then called to the program subroutine for the frequency response calculation, the artificial reading and data truncating error were eliminated.

A frequency response which is evaluated by using the Fourier transform of input pulse and response will yield exact information. Although the analytically mathematical expression of the tested records were not known, the Fourier transform calculated by the trapezoidal numerical approximation and choice of integration step size were very precise in this study. This can be seen by comparison of the results given in Table 2 with the analytical results given in Table 1.

Table 5. The Optimal Pulse Duration Estimated From Figures 14 Through 19.

System	Dynamic Property	Optimal Pulse Width (min)
I	G	0.93
I	T	0.58
II	G	0.74
II	T	0.74
III	G	1.60
III	T	1.18

CHAPTER 7

CONCLUSIONS

For the systems studied, it was found that the distortion of dynamic properties due to nonlinearity could be determined by applying different forcing strengths on the systems. The stronger the applied strength, the higher the distortion. However, there existed a linear relationship between the variation of linearized dynamic properties and the forcing strength. The slopes and intercepts of these lines varied with respect to the shape and width of the forcings. From the correlated results, regardless of the applied shape, the variation of dynamic properties was a linear function of the strength of forcing with specified pulse width as a parameter, except for the time constant obtained by using shorter rectangular pulse duration. The pulse width that predicted the true value of the linearized system dynamic property was taken as the optimal pulse width. It was found that the optimal pulse width was about twice the value of time constant of the system. In other words, applying a few pulses of any shape with the optimal width on the existing system, the linearized system characteristics could be evaluated by extrapolating to the zero pulse strength. The techniques presented are considered useful for determining the linear model of nonlinear system.

CHAPTER 8

RECOMMENDATIONS

The pulse testing and correlation techniques presented are easy to undertake and practical for the design purposes. It is advisable to apply these techniques to more complicated systems experimentally for determining the linear model of physical systems.

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APPENDIX A

DERIVATION OF FREQUENCY CONTENT OF THE NON-PERIODIC FUNCTION

DERIVATION OF FREQUENCY CONTENT OF THE NON-PERIODIC FUNCTION

Consider a periodic function, f (t), of period T with the Fourier series expansion, it can be expressed as a summation of individual trignometric components, each possessing definite frequency and magnitude. Thus f (t) can be expressed as the series,

$$f(t) = \sum_{k=0}^{\infty} \left[a_k \, \mathrm{SIN}\,(kwt/T) + b_k \, \mathrm{COS}\,(kwt/T) \right] \qquad (A-1)$$

where $\sqrt{a_k^{\,2} + b_k^{\,2}}$ represents the total amplitude of the component of angular frequency kw/T . The identically equivalent complex exponential form of Equation (A - 1) can be expressed as

$$f(t) = \sum_{n=-\infty}^{\infty} A_n \, e^{jnwt} \qquad (A-2)$$

where

$$A_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, e^{-jnwt} \, dt \qquad (A-3)$$

By inspection of Equation (A - 3), it can be seen that the Fourier coefficients A_n approach zero as the period T becomes infinitely large — that is, for the case of a non-periodic function like a non-recurrent pulse. To avoid this difficulty, it is convenient to define the coefficient density,

$$\rho(w) = T A_n = \int_{-T/2}^{T/2} f(t)\, e^{-jnwt} \, dt \qquad (A-4)$$

and to regard ρ (w) as defining the " frequency content " of a periodic function (30). For such a function, ρ (w) exists only at discrete frequencies that are integral multiples of the fundamental frequency ($w_f = 2\pi/T$), corresponding to discrete terms in the Fourier series expansion.

For the case of total non-periodic pulse, $T \rightarrow \infty$, the fundamental frequency, w_f , becomes infinitesmal, and Equation (A - 4) can be written as

$$s(w) = \lim_{T \rightarrow \infty} \rho(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt \qquad (A - 5)$$

which is the Fourier transform of f (t) .

APPENDIX B

THE DATA REDUCTION PROCEDURE

B . 1 THE DATA REDUCTION FOR PULSE TESTING

Recalling Equation (3.2 - 3) and making the identity, $e^{-jwt} =$ COS wt -jSIN wt , one obtains,

$$G(jw) = \frac{\int_0^{T_y} y(t)\, e^{-jwt}\, dt}{\int_0^{T_x} x(t)\, e^{-jwt}\, dt}$$

$$= \frac{\int_0^{T_y} y(t) \cos wt\, dt - j\int_0^{T_y} y(t) \sin wt\, dt}{\int_0^{T_x} x(t) \cos wt\, dt - j\int_0^{T_x} x(t) \sin wt\, dt} \qquad (B.1 - 1)$$

where the integration limits are extended only over the actual pulse widths T_x and T_y .

Servomechanism theory (29) tells that the magnitude and phase of G (jw) are the magnitude ratio and phase angle of the frequency response of the system whose transfer function is G (s) . For brevity, Equation (B.1 - 1) can be written as

$$G(jw) = \frac{A - jB}{C - jD} \qquad (B.1 - 2)$$

where

$$A = \int_0^{T_y} y(t) \cos wt\, dt \qquad (B.1 - 3)$$

$$B = \int_0^{T_y} y(t) \sin wt\, dt \qquad (B.1 - 4)$$

$$C = \int_0^{T_x} x(t) \cos wt\, dt \qquad (B.1 - 5)$$

and

$$D = \int_0^{T_x} x(t) \, \text{SIN} \, wt \, dt \qquad (B.1-6)$$

Then, the complex function becomes

$$G(jw) = Re(w) + IM(w) \qquad (B.1-7)$$

where

$$Re(w) = \frac{AC + BD}{C^2 + D^2} \qquad (B.1-8)$$

$$IM(w) = \frac{AD - BC}{C^2 + D^2} \qquad (B.1-9)$$

From the rules of complex algebra, the magnitude ratio and the phase shift are given by

$$R(w) = \sqrt{Re^2(w) + IM^2(w)} \qquad (B.1-10)$$

$$\phi(w) = \tan^{-1} \frac{IM(w)}{Re(w)} \qquad (B.1-11)$$

respectively, The frequency content, s (w), of the input pulse, x (t), is given by the absolute magnitude of its Fourier transform

$$s(w)_x = \left| \int_0^{T_x} x(t) \, e^{-jwt} \, dt \right| \qquad (B.1-12)$$

$$= \left| C - jD \right|$$

$$= \sqrt{C^2 + D^2} \qquad (B.1-13)$$

The normalized frequency content is defined as $s(w)_x / s(0)_x$. Setting $w = 0$ in Equation (B.1 - 12), one gets

$$s(0)_x = \int_0^{T_x} x(t)\,dt \qquad (B.1 - 14)$$

so

$$s(w)_n = \frac{\sqrt{C^2 + D^2}}{\int_0^{T_x} x(t)\,dt} \qquad (B.1 - 15)$$

The computational problem is the evaluation of A, B, C and D from experimental data. The most direct method is the application of a quadrature formula, such as the trapezoidal rule,

$$\int_a^b f(t)\,dt \approx \Delta t\,(f_0/2 + f_1 + f_2 + \cdots + f_{n-1} + f_n/2) \qquad (B.1 - 16)$$

into the trignometric integrals in Equations (B.1 - 3) through (B.1 - 6). Thus, Equations (B.1 - 3) through (B.1 - 6) become

$$A = \Delta t_y \sum_{k=1}^{n-1} y(k\Delta t_y)\,\mathrm{COS}(wk\Delta t_y) \qquad (B.1 - 17)$$

$$B = \Delta t_y \sum_{k=1}^{n-1} y(k\Delta t_y)\,\mathrm{SIN}(wk\Delta t_y) \qquad (B.1 - 18)$$

$$C = \Delta t_x \sum_{i=1}^{m-1} x(i\Delta t_x)\,\mathrm{COS}(wi\Delta t_x) \qquad (B.1 - 19)$$

$$D = \Delta t_x \sum_{i=1}^{m-1} x(i\Delta t_x)\,\mathrm{SIN}(wi\Delta t_x) \qquad (B.1 - 20)$$

where (excluding zero end points), n - 1 points are read on the y (t) curve at intervals Δt_y and m - 1 points are read on the x (t) curve at intervals Δt_x . Substitution of a particular value of w in Equations (B.1 - 17) through (B.1 - 20) and performing the indicated summations using experimental data for x (t) and y (t) will yield A, B, C and D. R (w), ϕ (w) and s $(w)_n$ then follow from Equations (B.1 - 8) through (B.1 - 15).

It should be noted that the approximation formula for the Fourier transform using trapezoidal rule was carried out by Draper et al (1). Their expression is

$$G(jw) = \frac{\Delta t_y \left[\frac{\text{SIN}(w\Delta t_y/2)}{(w\Delta t_y/2)} \right]^2 \sum_{k=1}^{n} y(k\Delta t_y)\, e^{-jwk\Delta t_y}}{\Delta t_x \left[\frac{\text{SIN}(w\Delta t_x/2)}{(w\Delta t_x/2)} \right]^2 \sum_{\ell=1}^{m} x(\ell\Delta t_x)\, e^{-jw\ell\Delta t_x}} \qquad (B.1 - 21)$$

It shows that the numerical approximation for the Fourier transform factors into two terms. The first depends only on the polynomial form of the approximation function; whereas the second depends only on the endpoints of the intervals, the data points. The first term is defined as the correction factor which is independent of the function values.

Hougen and Walsh (4) pointed out that Equation (B.1 - 21) gives the exact Fourier transform when applies to a pulse composed of straight line segments, if the segment boundaries coincide with Δ t-boundaries.

When the same correction factor and integral interval are used for both input and output, then Equation (B.1 - 21) leads to Equation (B.1 - 1).

B . 2 THE DATA REDUCTION PROCEDURE FOR STEP TESTING

Fourier transformation is valid only when output and input traces are closed curves. However, if the input is a step change — that is, neither x (t) nor y (t) returns to its original value — the integrals in Equation (3. 2 - 3) do not exist. Figure 22 shows a typical step input, x (t) starting at zero (the steady-state value for $t < 0$) and ending at the value P , and a typical response to this input, y (t), starting at zero and reaching the value Q .

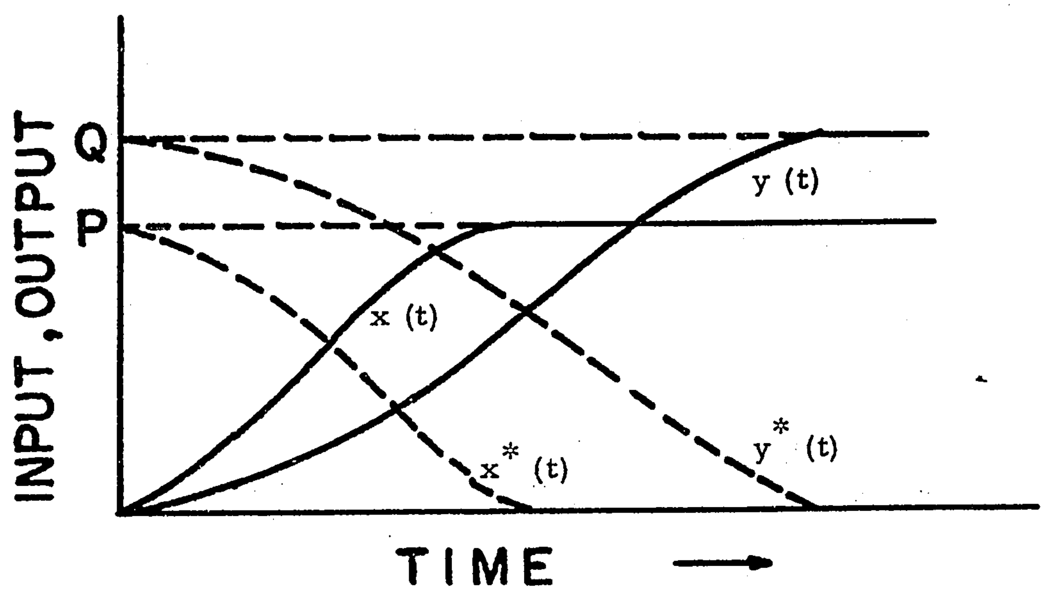

Figure 22. Typical step-response curves for a system.

The ratio Q/P is the steady-state (or zero frequency) gain of the system and includes a conversion factor if input and output are not in the same units. The following functions, also shown in Figure 22 ,

$$x^{*}(t) = P - x(t) \qquad (B.2 - 1)$$

$$y^{*}(t) = Q - y(t) \qquad (B.2 - 2)$$

are used as the basis of the theory for the development.

Multiplying the Laplace transform of a function by s yields the transform of the first derivative of that function (provided the function has a zero initial value). If the numerator and denominator of Equation (B. 2 - 1) are multiplied by s , therefore, the result may be written

$$G(s) = \frac{s\,y(s)}{s\,x(s)} = \frac{\int_0^{\infty} \frac{d\,y(t)}{d\,t}\, e^{-st}\, d\,t}{\int_0^{\infty} \frac{d\,x(t)}{d\,t}\, e^{-st}\, d\,t}$$

$$= \frac{-\int_0^{\infty} \frac{d\,y^*(t)}{d\,t}\, e^{-st}\, d\,t}{-\int_0^{\infty} \frac{d\,x^*(t)}{d\,t}\, e^{-st}\, d\,t} \qquad (B.2-3)$$

where the last equality is obtained from the definitions of Equations (B. 2 - 1) and (B. 2 - 2) of $y^*(t)$ and $x^*(t)$. If Euler's relation is applied to Equation (B. 2 - 3) it is seen that G (jw) becomes

$$G(jw) = \frac{-\int_0^{\infty} \frac{d\,y^*(t)}{d\,t}\, \text{COS}(wt)\, d\,t + j\int_0^{\infty} \frac{d\,y^*(t)}{d\,t}\, \text{SIN}(wt)\, d\,t}{-\int_0^{\infty} \frac{d\,x^*(t)}{d\,t}\, \text{COS}(wt)\, d\,t + j\int_0^{\infty} \frac{d\,x^*(t)}{d\,t}\, \text{SIN}(wt)\, d\,t}$$

(B. 2 - 4)

Rather than evaluate $d\,y^*(t)/d\,t$ and $d\,x^*(t)/d\,t$ from the experimental response curve, the integrals in Equation (B. 2 - 4) can first be evaluated by parts. For example, the first integral in the numerator becomes

$$-\int_0^{\infty} \frac{d\,y^*(t)}{d\,t} \cos(wt)\,d\,t = Q - w\int_0^{\infty} y^*(t) \sin(wt)\,d\,t \qquad (B.2-5)$$

taking into account the fact that $y^*(0) = Q$. This process, repeated for the other three integrals, yields the final result

$$G(jw) = \frac{A + jB}{C + jD} \qquad (B.2-6)$$

where

$$A = Q - w\int_0^{\infty} y^*(t) \sin(wt)\,d\,t \qquad (B.2-7)$$

$$B = -w\int_0^{\infty} y^*(t) \cos(wt)\,d\,t \qquad (B.2-8)$$

$$C = P - w\int_0^{\infty} x^*(t) \sin(wt)\,d\,t \qquad (B.2-9)$$

and

$$D = -w\int_0^{\infty} x^*(t) \cos(wt)\,d\,t \qquad (B.2-10)$$

Since $x^*(t)$ and $y^*(t)$ eventually reach zero, the integrals in Equations (B. 2 - 7) through (B. 2 - 10) may be approximated by numerical techniques.

Reduction of Equation (B. 2 - 6) to frequency response is accomplished by the Equations

$$R(w) = \left| G(jw) \right| = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \qquad (B.2-11)$$

and

$$\phi (w) = \angle G (w) = \tan^{-1} \frac{B}{A} - \tan^{-1} \frac{D}{C} \qquad (B. 2 - 12)$$

It should be noted that in the special case where the input is an instantaneous step change from zero to P , the denominator of Equation (B.2 - 6) reduces to the value P , and only two integrals (B. 2 - 7) and (B. 2 - 8) need be evaluated in order to obtain the frequency response.

B . 3 COMPUTER PROGRAM FOR REDUCTION OF TRANSIENT RESPONSE TO FREQUENCY RESPONSE

The following is the computer program based on the trapezoidal rule for the evaluation of Equations (B.1 - 3) through (B.1 - 15).

```
C
C
C     FREQUENCY RESPONSE OF A NON-LINEAR SYSTEM USING TRIANGULAR PULSE
C
C     RUNGE KUTTA METHOD
C
      DIMENSICN DEL(999),QI(999)
C     THE MAIN PROGRAM IS ONLY FOR THE EVALUATION OF TRANSIENT DATA
C     FCLLOWING IS NOTATICN OF MAIN SYMBOLS USED IN THE PROGRAM
C     DE(I) IS THE TRANSIENT RESPONSE OF LEVEL HEIGHT
C     QI(I) IS THE VARIATION OF THE INFLUENT FLOW
C     D IS TIME INTERVAL
C     TXX IS THE PULSE WIDTH
C     XNN IS THE INITIAL TIME
C     YNN IS THE STEADY STATE LEVEL HEIGHT
C     A AND C ARE CROSS-SECTIONAL AREA AND CAPACITY OF VESSEL
C     AMP IS AMPLITUDE OF PULSE
C     Q AND QQ ARE THE INFLUENT AND EFLUENT FLOW RATE
      DATA D,TXX,XNN,YNN,A,C/0.01,0.4,0.0,4.466,19.635,118.2989/
      AMP=10.00
      WRITE(3,20)
      WRITE(3,21) AMP
      ITER=1
      DD=0.5*D
   40 IIT=1
      TX=TXX
      XN=XNN
      YN=YNN
      DEL(IIT)=0.00000
      QI(IIT)=0.00000
    1 X=XN
      IIT=IIT+1
      Y=YN
      N=1
    2 DEPUL=AMP*((2.*X)/TX)
      Q=250.+DEPUL
      QQ=C*SQRT(Y)
      EQU=(Q-QQ)/A
      Z=EQU
C
C
C
```

```
C
C
      GO TO (3,4,5,6),N
    3 C1=D*Z
      X=XN+DD
      Y=YN+C1/2.
      N=2
      GO TO 2
    4 C2=D*Z
      Y=YN+C2/2.
      N=3
      GO TO 2
    5 C3=D*Z
      X=XN+D
      Y=YN+C3
      N=4
      GO TO 2
    6 XN=X
      C4=D*Z
      YN=YN+(C1+2.*(C2+C3)+C4)/6.
      DEL(IIT)=YN-4.466
      QI(IIT)=DEPUL
  100 IF(ABS(XN-TX/2.) - 0.00001) 7,7,90
   90 IF(XN - TX/2.) 1,7,7
    7 X=XN
      IIT=IIT+1
      Y=YN
      N=1
    9 DEPUL=AMP*(2.-(2.*X)/TX)
      Q=250.+DEPUL
      QQ=C*SQRT(Y)
      EQU1=(Q-QQ)/A
      Z1=EQU1
      GO TO (10,11,12,13),N
   10 D1=D*Z1
      X=XN+DD
      Y=YN+D1/2.
      N=2
      GO TO 9
C
C
C
```

```
C
C
   11 D2=D*Z1
      Y=YN+D2/2.
      N=3
      GO TO 9
   12 D3=D*Z1
      X=XN+D
      Y=YN+D3
      N=4
      GO TO 9
   13 XN=X
      D4=D*Z1
      YN=YN+(D1+2.*(D2+D3)+D4)/6.
      DEL(IIT)=YN-4.466
      QI(IIT)=DEPUL
  101 IF(ABS(XN-TX) - 0.00001) 31,31,91
   91 IF(XN - TX) 7,31,31
   31 INDEX=0
   14 X=XN
      IIT=IIT+1
      Y=YN
      N=1
   15 Q=250.
      QQ=C*SQRT(Y)
      EQU2=(Q-QQ)/A
      Z2=EQU2
      GO TO (16,17,18,19),N
   16 E1=D*Z2
      X=XN+DD
      Y=YN+E1/2.
      N=2
      GO TO 15
   17 E2=D*Z2
      Y=YN+E2/2.
      N=3
      GO TO 15
   18 E3=D*Z2
      X=XN+D
C
C
C
```

```
C
C
      Y=YN+E3
      N=4
      GO TO 15
   19 XN=X
      E4=D*Z2
      YN=YN+(E1+2.*(E2+E3)+E4)/6.
      DEL(IIT)=YN-4.466
  102 IF(INDEX .EQ. 0) GO TO 29
      IF(ABS(YN-YNOLD) - 0.00001) 30,30,29
   29 YNOLD=YN
      INDEX=1
      GO TO 14
   30 IF((IIT-(IIT/2)*2) .EQ. 0) GO TO 103
      IIT=IIT+1
      DEL(IIT)=0.00000
      IT=IIT
      GO TO 104
  103 IT=IIT
      DEL(IIT)=0.00000
  104 QI(41)=0.00000
      WRITE(3,23) QI(1)
      WRITE(3,22) (QI(IIT),IIT=2,41)
      WRITE(3,24)
      WRITE(3,23) DEL(1)
      WRITE(3,22) (DEL(IIT),IIT=2,IT)
      Z11=41.00
      Z22=IT
C     FREQUENCY CONTENT OF INPUT SIGNAL
      CALL FRCONT(QI,Z11)
C     FREQUENCY CONTENT OF OUTPUT SIGNAL
      CALL FRCONT(DEL,Z22)
C     CALCULATE THE FREQUENCY RESPONSE OF THE PULSE TESTING
      CALL FREDAT(DEL,QI,Z11,Z22)
      IF(ITER - 2) 44,45,46
   44 ITER=ITER+1
      AMP=25.00
      WRITE(3,21) AMP
C
C
C
```

```
C
C
      GO TO 40
   45 ITER=ITER+1
      AMP=40.00
      WRITE(3,21) AMP
      GO TO 40
   20 FORMAT(1H1,'TESTING FROM STEADY STATE OF FLOW RATE 250 GAL./MIN.',
     1'BY TRIANGULAR PULSE (WIDTH=0.50,AMP.=20) H(1/3)')
   21 FORMAT(////3X,'DEVIATION OF FORCING PULSE TIME INTERVAL IS 0.01 AM
     1PLITUDE EQUAL TO ',F10.3)
   22 FORMAT(1H0,1X,10(F10.5,2X))
   23 FORMAT(//2X,F10.5)
   24 FORMAT(////3X,'DEVIATION OF THE LEVEL TIME INTERVAL IS 0.01')
   46 RETURN
      END
```

```
C
C
      SUBROUTINE FRCONT(Z,ZZ1)
C     DETERMINATION OF FREQUENCY CONTENT VS FREQUENCY.PULSE WIDTH
C     Z(I) IS TRANSIENT VARIATION OF INPUT PULSE
C     W(I) IS FREQUENCY
C     SWN(I) IS FREQUENCY CONTENT OF INPUT PULSE
C     WW(I) IS NORMALIZED FREQUENCY CONTENT
      DIMENSICN Z(999),W(999),SWN(999),WW(999)
      DW=0.05
      DT1=C.C1
      WMAX=3C.00
      INDEX=0
      K1=ZZ1+0.05
      KK=K1-1
      II=1
      C=0.00
      DO 12 I=1,K1
   12 C=C+Z(I)
      SO=C*DT1
      W(II)=0.000
      SWN(II)=SO
      GO TO 40
    5 RAD=W(II)*DT1/2.
      CORFC=(SIN(RAD)/RAD)**2
      DO 14 I=2,KK
      T=T+DT1
      C=C+Z(I)*COS(W(II)*T)
   14 D=D+Z(I)*SIN(W(II)*T)
      CS=C*DT1*CORFC
      DS=D*DT1*CORFC
      Q=CS*CS+DS*DS
      SWN(II)=SCRT(Q)
   40 WW(II)=SWN(II)/SO
      IF(W(II) .LT. 5.) GO TO 22
      IF(INDEX .EQ. 0) GO TO 41
      GO TO 42
   41 IF(WOLD-SWN(II)) 20,20,22
   42 IF(WMAX-W(II)) 221,221,11
C
C
C
```

```
C
C
   22 WOLD=SWN(II)
   11 AW=W(II)
      II=II+1
      W(II)=AW+DW
      T=0.00
      C=0.00
      D=0.00
      GO TO 5
   20 NN=II-1
      N=NN-1
      E3=W(NN)
      INDEX=1
      GO TO 11
  221 NNN=II
  220 WRITE(3,199)
      WRITE(3,197)
      WRITE(3,30) (W(I),SWN(I),WW(I),I=1,NNN)
      TH=0.00
      DO 300 I=1,N
  300 TH=TH+(SWN(I)+SWN(I+1))*(W(I+1)-W(I))/2.
      TH=TH-SWN(NN)*E3
      AVH=2.*TH/E3
      WRITE(3,210) AVH
      WRITE(3,211) TH,E3
  199 FORMAT(1H1,3X,'THE DATA OF FREQUENCY CONTENT VS FREQUENCY ')
  197 FORMAT(////2X,' W ',2X,'S(W)'3X,'S(W)N')
   30 FORMAT(1H0,5(F9.3,F8.5,F8.5))
  211 FORMAT(//5X,'TOTAL AREA = ',F9.5,10X,'INTERSECT POINT AT ',F9.5)
  210 FORMAT(1H1,5X,'*** THE AVERAGE HEIGHT OF FREQUENCY CONTENT EQUAL T
     1O ',F7.3,' ***')
      RETURN
      END
```

```
                C
                C
0001                  SUBROUTINE FREDAT (Y,X,ZZ1,ZZ2)
                C
                C     DETERMINATION OF M.R. AND PHASE ANGLE OF A NON-LINEAR SYSTEM
                C     TRAPEZOIDAL RULE
                C     XMR1 IS MAGNITUDE RATIO
                C     RE1 IS REAL PART VALUE
                C     XIM IS IMAGINARY PART VALUE
                C     XMR IS NORMALIZED M. R.
                C     ANGLE IS PHASE LAG
                C     SWN IS NORMALIZED FREQUENCY CONTENT OF INPUT PULSE
                C     SW IS ZERO FREQUENCY CONTENT OR PULSE STRENGTH AT W=0
                C
0002                  DIMENSION Y(999),X(999)
0003                  WRITE(3,40)
0004                  K1=ZZ1+0.05
0005                  K2=ZZ2+0.05
0006                  WO=0.00
0007                  DW=0.01
0008                  DDW=1.00
0009                  WMAX=2.00
0010                  DT1=0.01
0011                  DT2=0.01
0012                  C=0.0
0013                  DO 12 I=1,K1
0014               12 C=C+X(I)
0015                  SO=C*DT1
0016                  INDEX=0
0017                  T=0.0
0018                  W=WO
0019                  C=0.0
0020                  D=0.0
0021                5 DO 14 I=2,K1
0022                  T=T+DT1
0023                  C=C+X(I)*COS(W*T)
0024               14 D=D+X(I)*SIN(W*T)
0025                  CS=C*DT1
0026                  DS=D*DT1
                C
                C
                C
```

```
C
C
      T=0.0
      A=0.0
      B=0.0
      DO 16 J=2,K2
      T=T+DT2
      A=A+Y(J)*COS(W*T)
   16 B=B+Y(J)*SIN(W*T)
      AS=A*DT2
      BS=B*DT2
      QQ=SQRT(AS*AS+BS*BS)
      Q=CS*CS+DS*DS
      RE1=(AS*CS+BS*DS)/Q
      XIM=(AS*DS-BS*CS)/Q
      XMR1=SQRT(RE1*RE1+XIM*XIM)
      IF(INDEX .EQ. 0) GO TO 60
      GO TO 61
   60 REOLD=RE1
      XMROLD=XMR1
      WRITE(3,70) XMROLD
   61 RE=RE1/REOLD
      XMR=XMR1/XMROLD
      ARG=XIM/RE1
      IF(ARG) 20,21,20
   21 ANGLE=0.0
      GO TO 32
   20 ABARG=ABS(ARG)
      IF(ABARG-1.0) 22,22,23
   22 ANGL=57.295779*ATAN(ABARG)
      GO TO 25
   23 ANGL=90.0-57.295779*ATAN(1.0/ABARG)
   25 IF(ARG)26,26,27
   27 ANGLX=ANGL
      GO TO 28
   26 ANGLX=-1.0*ANGL
   28 IF(RE) 30,30,31
   31 ANGLE=ANGLX
      GO TO 32
C
C
C
```

```
      C
      C
   30 ANGLE=ANGLX-180.0
   32 SW=SQRT(Q)
      SWN=SW/SO
      WRITE(3,41) W,XMR1,ANGLE,RE1,XIM,XMR,SWN,SW
      IF(W) 80,80,81
   80 W=W+DDW
      GO TO 35
   81 W=W+DW
      IF(WMAX-W)34,35,35
   35 T=0.0
      INDEX=1
      A=0.0
      B=0.0
      C=0.0
      D=0.0
      GO TO 5
   40 FORMAT(////5X,'W,RAD./SEC.',10X,'M.R.',10X,'ANGLE',11X,'RE(W)',
     110X,'IM(W)',10X,'MR(N)',10X,'S(W)NORMOLIZED',3X,'AMP. OF INPUT')
   70 FORMAT(//5X,'ZERO FREQUENCY GAIN = ',F13.8)
   41 FORMAT(//3X,F13.8,7(3X,F13.8))
   34 RETURN
      END
```

APPENDIX C

C . 1 DATA OF DYNAMIC PROPERTY TESTED BY PULSE FORCINGS (Tables 6 Through 8)

C . 2 DATA OF DYNAMIC PROPERTY TESTED BY STEP FORCING (Table 9)

TABLE 6-1. TESTED DYNAMIC PROPERTIES AND PULSE STRENGTH OF SYSTEM CASE I

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	STEADY STATE GAIN($ft\text{-}min/ft^3$)	TIME CONSTANT (MIN)
0.3	REC.	4	3.00000	0.01793	0.34294
		10	7.50000	0.01813	0.34590
		16	12.00000	0.01831	0.34928
0.3	TRI.	4	1.50000	0.01783	0.35411
		10	3.75000	0.01798	0.35461
		16	6.00000	0.01809	0.35499
0.3	H-S.	4	1.90811	0.01788	0.35361
		10	4.77027	0.01804	0.35423
		16	7.63244	0.01817	0.35625
0.5	REC.	4	5.00000	0.01800	0.34362
		10	12.50000	0.01826	0.34928
		16	20.00000	0.01850	0.35461
0.5	TRI.	4	2.50000	0.01780	0.35336
		10	6.25000	0.01807	0.35499
		16	10.00000	0.01821	0.35778
0.5	H-S.	4	3.18204	0.01794	0.35336
		10	7.95511	0.01814	0.35587
		16	12.72818	0.01831	0.35971
0.8	REC.	4	8.00000	0.01806	0.34542
		10	20.00000	0.01839	0.35373
		16	32.00000	0.01870	0.36140
0.8	TRI.	4	4.00000	0.01795	0.35336
		10	10.00000	0.01816	0.35740
		16	16.00000	0.01835	0.36166
0.8	H-S.	4	5.09227	0.01799	0.35398
		10	12.73072	0.01824	0.35907
		16	20.36916	0.01847	0.36430

TABLE 6-2. TESTED DYNAMIC PROPERTIES AND PULSE STRENGTH OF SYSTEM CASE I

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	STEADY STATE GAIN(ft-min/ft^3)	TIME CONSTANT (MIN)
1.0	REC.	4	10.00000	0.01808	0.34650
		10	25.00000	0.01844	0.35638
		16	40.00000	0.01879	0.36550
1.0	TRI.	4	5.00000	0.01798	0.35373
		10	12.50000	0.01821	0.35881
		16	20.00000	0.01842	0.36390
1.0	H-S.	4	6.36561	0.01802	0.35461
		10	15.91413	0.01829	0.36101
		16	25.46262	0.01855	0.36738
1.2	REC.	4	12.00000	0.01826	0.35765
		10	30.00000	0.01864	0.36900
		16	48.00000	0.01902	0.37922
1.2	TRI.	4	6.00000	0.01799	0.35398
		10	15.00000	0.01824	0.36010
		16	24.00000	0.01848	0.36603
1.2	H-S.	4	7.63896	0.01803	0.35524
		10	19.09749	0.01833	0.36298
		16	30.55608	0.01861	0.36969
1.5	REC.	4	15.00000	0.01825	0.35907
		10	37.50000	0.01866	0.37313
		16	60.00000	0.01907	0.38521
1.5	TRI.	4	7.50000	0.01801	0.35486
		10	18.75000	0.01828	0.36232
		16	30.00000	0.01854	0.36941
1.5	H-S.	4	9.54901	0.01806	0.35600
		10	23.87267	0.01838	0.36523
		16	38.19637	0.01869	0.37383

TABLE 7-1. TESTED DYNAMIC PROPERTIES AND PULSE STRENGTH OF SYSTEM CASE II

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	STEADY STATE GAIN(ft-min/ft^3)	TIME CONSTANT (MIN)
		4	3.00000	0.03591	0.34247
0.3	REC.	10	7.50000	0.03628	0.34602
		16	12.00000	0.03663	0.34965
		4	1.50000	0.03576	0.35211
0.3	TRI.	10	3.75000	0.03600	0.35336
		16	6.00000	0.03620	0.35524
		4	1.90811	0.03584	0.35211
0.3	H-S.	10	4.77027	0.03612	0.35398
		16	7.63244	0.03636	0.35651
		4	5.00000	0.03603	0.34400
0.5	REC.	10	12.50000	0.03653	0.34065
		16	20.00000	0.03702	0.35486
		4	2.50000	0.03585	0.35236
0.5	TRI.	10	6.25000	0.03617	0.35500
		16	10.00000	0.03645	0.35778
		4	3.18204	0.03593	0.35336
0.5	H-S.	10	7.95511	0.03630	0.35625
		16	12.72818	0.03664	0.35984
		4	8.00000	0.03614	0.34566
0.8	REC.	10	20.00000	0.03678	0.35398
		16	32.00000	0.03741	0.36166
		4	4.00000	0.03594	0.35323
0.8	TRI.	10	10.00000	0.03634	0.35765
		16	16.00000	0.03672	0.36166
		4	5.09227	0.03602	0.35398
0.8	H-S.	10	12.73072	0.03650	0.35932
		16	20.36916	0.03696	0.36456

TABLE 7-2. TESTED DYNAMIC PROPERTIES AND PULSE STRENGTH OF SYSTEM CASE II

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	STEADY STATE GAIN(ft-min/ft^3)	TIME CONSTANT (MIN)
1.0	REC.	4	10.00000	0.03619	0.34686
		10	25.00000	0.03690	0.35651
		16	40.00000	0.03759	0.36563
1.0	TRI.	4	5.00000	0.03598	0.35398
		10	12.50000	0.03643	0.35907
		16	20.00000	0.03685	0.36430
1.0	H-S.	4	6.36561	0.03606	0.35461
		10	15.91413	0.03660	0.36127
		16	25.46262	0.03711	0.36751
1.2	REC.	4	12.00000	0.03653	0.35804
		10	30.00000	0.03730	0.36900
		16	48.00000	0.03804	0.37951
1.2	TRI.	4	6.00000	0.03601	0.35411
		10	15.00000	0.03649	0.36036
		16	24.00000	0.03696	0.36630
1.2	H-S.	4	7.63896	0.03609	0.35524
		10	19.09749	0.03667	0.36284
		16	30.55608	0.03723	0.37037
1.5	REC.	4	15.00000	0.03651	0.35997
		10	37.50000	0.03733	0.37327
		16	60.00000	0.03813	0.38551
1.5	TRI.	4	7.50000	0.03605	0.35524
		10	18.75000	0.03658	0.36271
		16	30.00000	0.03709	0.36955
1.5	H-S.	4	9.54901	0.03613	0.35651
		10	23.87267	0.03676	0.36550
		16	38.19637	0.03738	0.37009

TABLE 8-1. TESTED DYNAMIC PROPERTIES AND PULSE STRENGTH OF SYSTEM CASE III

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	STEADY STATE GAIN(ft-min/ft^3)	TIME CONSTANT (MIN)
		4	7.00000	0.03590	0.69686
0.7	REC.	10	17.50000	0.03634	0.70373
		16	28.00000	0.03673	0.71174
		4	3.50000	0.03570	0.70771
0.7	TRI.	10	8.75000	0.03601	0.70022
		16	14.00000	0.03625	0.71124
		4	4.45556	0.03578	0.70671
0.7	H-S.	10	11.13893	0.03612	0.70922
		16	17.82233	0.03640	0.71429
		4	10.00000	0.03600	0.69784
1.0	REC.	10	25.00000	0.03652	0.70872
		16	40.00000	0.03701	0.71891
		4	5.00000	0.03579	0.70175
1.0	TRI.	10	12.50000	0.03614	0.71073
		16	20.00000	0.03643	0.71582
		4	6.36561	0.03587	0.70671
1.0	H-S.	10	15.91413	0.03627	0.71225
		16	25.46262	0.03662	0.71942
		4	14.00000	0.03634	0.71023
1.4	REC.	10	35.00000	0.03697	0.72464
		16	56.00000	0.03756	0.73801
		4	7.00000	0.03587	0.70671
1.4	TRI.	10	17.50000	0.03627	0.71276
		16	28.00000	0.03662	0.72098
		4	8.91231	0.03594	0.70721
1.4	H-S.	10	22.28093	0.03641	0.71633
		16	35.64957	0.03685	0.72569

TABLE 3-2. TESTED DYNAMIC PROPERTIES AND PULSE STRENGTH OF SYSTEM CASE III

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	STEADY STATE GAIN(ft-min/ft^3)	TIME CONSTANT (MIN)
2.1	REC.	4	21.00000	0.03635	0.71378
		10	52.50000	0.03709	0.73368
		16	84.00000	0.03780	0.75245
2.1	TRI.	4	10.50000	0.03596	0.70771
		10	26.25000	0.03643	0.71839
		16	42.00000	0.03687	0.72886
2.1	H-S.	4	13.36912	0.03604	0.70922
		10	33.42302	0.03661	0.72307
		16	53.47556	0.03714	0.73584
2.8	REC.	4	28.00000	0.03637	0.71736
		10	70.00000	0.03717	0.74294
		16	112.00000	0.03796	0.76628
2.8	TRI.	4	14.00000	0.03601	0.70922
		10	35.00000	0.03654	0.72359
		16	56.00000	0.03705	0.73638
2.8	H-S.	4	17.82584	0.03610	0.71174
		10	44.56403	0.03673	0.72886
		16	71.30179	0.03733	0.74516

Table 9. Dynamic Properties Tested by Step Function

System	Input Magnitude Deviation (%)	Steady State Gain(ft-min/ft^3)	Time Constant (min)
I	+ 4	0.01822	0.360
I	+ 8	0.01857	0.375
I	+ 16	0.01929	0.400
IV	+ 2	0.02714	0.540
IV	+ 8	0.02894	0.587
IV	+ 20	0.03248	0.685
V	+ 4	0.01821	* 0.562 0.151
V	+ 10	0.01875	0.584 0.159
V	+ 16	0.01929	0.605 0.168

* These values are calculated by using the method proposed by Harriott(24), P. 48.

APPENDIX D

THE CORRELATION COMPUTER PROGRAM

```
C
C
C
C     THE CORRELATION FOR THE GAIN AND THE TIME CONSTANT AGAINST PULSE STRENGTH
C
C     FOLLOWING IS NOTATION OF MAIN SYMBOLS USED IN THE PROGRAM
C     AO(I)   INTERCEPT OF EQUATION 5.1-1 IN TEXT
C     A1(I)   SLOPE OF EQUATION 5.1-1 IN TEXT
C     PT(I),TP   PULSE WIDTH
C     S(I)    PULSE STRENGTH
C     G(I)    THE VALUE OF CORRELATED GAIN OR TIME CONSTANT
C     GG(I)   THE VALUE OF GAIN OR TIME CONSTANT CALCULATED FROM TESTING
C     ER(I)   PERCENTAGE DEVIATION OF CORRELATED VALUE FROM TESTING VALUE
C     GAIN(I) IS TESTING VALUE OF GAIN OR TIME CONSTANT
C     ZFCIP(I) IS ZERO FREQUENCY CONTENT OF INPUT PULSE OR PULSE STRENGTH
C     MM(I)   INPUT PULSE DEVIATION (%)
C     AO1,AO2,AO3 AND AO4 ARE THE CORRELATED CONSTANTS OF THE THIRC CRDER
C     POLYNOMIAL FOR THE INTERCEPT OF EQUATION 5.1-1
C     A11,A12,A13 AND A14 ARE THE CORRELATED CONSTANTS OF THE THIRC ORDER
C     POLYNOMIAL FOR THE SLOPE OF EQUATION 5.1-1
      CIMENSICN AO(30),A1(30),PT(30),S(30),G(30),GG(30),ER(30)
      DIMENSICN H(10),H1(10),GAIN(30),ZFCIP(30),H2(30),MM(3)
      III=0
      KK=3
      NN=1
      READ(1,50) (PT(I),I=1,6)
   70 L=0
      DO 150 I=1,6
      L=L+1
      READ(1,30) (ZFCIP(J),J=1,KK)
      READ(1,30) (GAIN(J),J=1,KK)
      WRITE(3,39)
      WRITE(3,40) (GAIN(J),J=1,KK)
      WRITE(3,41) (ZFCIP(J),J=1,KK)
C     DETERMINING THE INTERCEPT AND SLOPE OF THE TESTING DATA
      CALL KURVE (ZFCIP,GAIN,NN,KK,H2)
      AO(L)=H2(1)
      A1(L)=H2(2)
  150 CONTINUE
      WRITE(3,200) (AO(I),I=1,6)
C
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```
      C
      C
      C
            WRITE(3,201) (A1(I),I=1,6)
            WRITE(3,202) (PT(I),I=1,6)
            IIN=0
      C     THIRD ORDER POLYNOMIAL CORRELATION
            CALL KURVE (PT,A0,3,6,H)
            A01=H(1)
            A02=H(2)
            A03=H(3)
            A04=H(4)
            CALL LTK (A01,A02,A03,A04)
            CALL KURVE (PT,A1,3,6,H1)
            A11=H1(1)
            A12=H1(2)
            A13=H1(3)
            A14=H1(4)
            CALL LTK (A11,A12,A13,A14)
            NI=0
            N=3
            TP=0.3
            NA=0
            MM(1)=4
            MM(2)=10
            MM(3)=16
            INDEX=0
            WRITE(3,90)
            WRITE(3,91)
            WRITE(3,92)
            WRITE(3,93)
         10 READ(1,19) (S(I),I=1,N)
            READ(1,19) (GG(I),I=1,N)
            DO 5 I=1,N
            G(I)=(A01+A02*TP+A03*TP**2+A04*TP**3)+(A11+A12*TP+A13*TP**2+A14*TP
           1**3)*S(I)
            ER(I)=100.*(G(I)-GG(I))/GG(I)
          5 CONTINUE
            NA=NA+1
            IF(NA-2) 25,26,27
      C
      C
      C
```

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C
C
C
   25 WRITE(3,20) (MM(1),S(1),GG(1),G(1),ER(1))
      WRITE(3,21) (TP,MM(2),S(2),GG(2),G(2),ER(2))
      WRITE(3,20) (MM(3),S(3),GG(3),G(3),ER(3))
      GO TO 28
   26 WRITE(3,20) (MM(1),S(1),GG(1),G(1),ER(1))
      WRITE(3,22) (TP,MM(2),S(2),GG(2),G(2),ER(2))
      WRITE(3,20) (MM(3),S(3),GG(3),G(3),ER(3))
      GO TO 28
   27 WRITE(3,20) (MM(1),S(1),GG(1),G(1),ER(1))
      WRITE(3,23) (TP,MM(2),S(2),GG(2),G(2),ER(2))
      WRITE(3,20) (MM(3),S(3),GG(3),G(3),ER(3))
   28 IF(NA-3) 10,11,11
   11 IF(IIN-1) 100,101,102
  100 IF(INDEX - 1) 6,7,8
    6 TP=0.5
      INDEX=1
      NA=0
      GO TO 10
    7 TP=0.8
      INDEX=2
      NA=0
      GO TO 10
    8 TP=1.0
      IIN=1
      WRITE(3,90)
      WRITE(3,91)
      WRITE(3,92)
      WRITE(3,93)
      NA=0
      GO TO 10
  101 TP=1.2
      IIN=2
      NA=0
      GO TO 10
  102 IF(NI) 105,105,106
  105 TP=1.5
      NI=1
C
C
C
```

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C
C
C
      NA=0
      GO TO 10
   50 FORMAT(6E12.5)
   30 FORMAT(3F10.5)
   39 FORMAT(1H1,17X,'CORRELATED RESULT')
   40 FORMAT(1H0,5X,'TESTED DATA',8X,3(E12.5,5X))
   41 FORMAT(1H0,5X,'PULSE STRENGTH',5X,3(E12.5,5X))
  200 FORMAT(///5X,'INTERCEPTS',5X,6E12.5)
  201 FORMAT(1H0,5X,'SLOPES',9X,6E12.5)
  202 FORMAT(1H0,5X,'PULSE WIDTH',4X,6E12.5)
   90 FORMAT(1H1,/////12X,'TABLE     THE COMPARISON BETWEEN THE TIME CON
     1STANTS CALCULATED')
   91 FORMAT(/22X,'FROM TESTING AND THAT FROM EQUATION 5.1-2  FOR SYSTEM
     1 I')
   92 FORMAT(///11X,'PULSE WIDTH',2X,'PULSE',2X,'INPUT PULSE',4X,'PULSE'
     1,5X,'FROM',5X,'CAL. FROM',2X,'ERROR (%)')
   93 FORMAT(14X,'(MIN)',5X,'SHAPE',2X,'DEVIATION (%)',2X,'STRENGTH',2X,
     1'TESTING',2X,'EQ. 5.1-2',/)
   19 FORMAT(3F10.5)
   20 FORMAT(36X,I2,8X,F8.5,2X,F7.5,2X,F7.5,4X,F5.2)
   21 FORMAT(15X,F3.1,6X,'REC.',8X,I2,7X,F9.5,2X,F7.5,2X,F7.5,4X,F5.2)
   22 FORMAT(15X,F3.1,6X,'TRI.',8X,I2,7X,F9.5,2X,F7.5,2X,F7.5,4X,F5.2)
   23 FORMAT(15X,F3.1,6X,'H-S',9X,I2,7X,F9.5,2X,F7.5,2X,F7.5,4X,F5.2)
  106 RETURN
      END
```

```
C
C
C
      SUBROUTINE KURVE (X,Y,N,M,HH)
C     LEAST SQUARE METHOD
C     CASE OF Y=SIGMA(A(I)*X**I),(I=0,N)
C     M=NUMBER OF DATA
C     N=DEGREE OF THE FUNCTION FOR CURVE FITTING
C     Z IS WEIGHTED DEVIATION
      DIMENSION X(30),Y(30),A(30),B(30),S(30),T(30),H(10,11),HH(10)
      N1=2*N+1
      DO 20 I=1,M
      A(I)=X(I)
   20 B(I)=Y(I)
      S(1)=M
      DO 10 K=2,N1
      S(K)=0.
      DO 10 I=1,M
      S(K)=S(K)+A(I)
   10 A(I)=X(I)*A(I)
      N2=N+1
      DO 30 K=1,N2
      T(K)=0.0
      DO 30 I=1,M
      T(K)=T(K)+B(I)
   30 B(I)=X(I)*B(I)
      N3=N+2
      DO 50 I=1,N2
      DO 40 J=1,N2
      K=I+J-1
   40 H(I,J)=S(K)
   50 H(I,N3)=T(I)
      DO 60 K=1,N2
      K1=K+1
      DO 60 J=K1,N3
      H(K,J)=H(K,J)/H(K,K)
      DO 60 I=1,N2
      IF(I-K) 17,60,17
   17 H(I,J)=H(I,J)-H(I,K)*H(K,J)
   60 CONTINUE
C
C
C
```

```
C
C
C
      DO 70 I=1,N2
   70 HH(I)=H(I,N3)
      DO 91 K=1,M
      S(K)=H(1,N3)
      DO 90 I=2,N2
   90 S(K)=S(K)+H(I,N3)*X(K)**(I-1)
   91 T(K)=Y(K)-S(K)
      DEV=0.0
      DO 92 I=1,M
   92 DEV=DEV+ABS(T(I))
      W=M
      Z=DEV/W
      WRITE(3,100) Z,(HH(I),I=1,N2)
  100 FORMAT(1H0,10X,E12.5,4(5X,E12.5))
      RETURN
      END
```

```
C
C
C
C
C
C
      SUBROUTINE LTK (B01,B02,B03,B04)
C     SUBROUTINE TO COMPUTE EQUATIONS 5.1-3 AND 5.1-4 IN TEXT
C     CO      THE CORRELATED CONSTANT
C     CT      PULSE WIDTH
      DIMENSICN CO(300),CT(300)
      K=1
      CT(K)=0.0
   95 CO(K)=B01+B02*CT(K)+B03*CT(K)**2+B04*CT(K)**3
      IF(K - 150) 90,90,91
   90 CC=CT(K)
      K=K+1
      CT(K)=CC+0.01
      GO TO 55
   91 KK=K
      WRITE(3,79) CT(1),CO(1)
      WRITE(3,80) (CT(K),CO(K),K=2,KK)
   79 FORMAT(///2E12.5)
   80 FORMAT(5(2E12.5,1X))
      RETURN
      END
```

APPENDIX E

COMPARISON OF THE TESTED AND CORRELATED DYNAMIC PROPERTIES OF SYSTEMS

TABLE 10-1. COMPARISON OF STEADY STATE GAINS CALCULATED FROM TESTING AND FROM EQUATION 6.1-1 FOR SYSTEM I

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-1	ERROR (%)
0.3	REC.	4	3.00000	0.01793	0.01794	0.05
		10	7.50000	0.01813	0.01817	0.20
		16	12.00000	0.01831	0.01839	0.45
0.3	TRI.	4	1.50000	0.01783	0.01786	0.19
		10	3.75000	0.01798	0.01798	-0.01
		16	6.00000	0.01809	0.01809	0.00
0.3	H-S	4	1.90811	0.01788	0.01788	0.03
		10	4.77027	0.01804	0.01803	-0.06
		16	7.63244	0.01817	0.01817	0.02
0.5	REC.	4	5.00000	0.01800	0.01802	0.10
		10	12.50000	0.01826	0.01831	0.30
		16	20.00000	0.01850	0.01861	0.60
0.5	TRI.	4	2.50000	0.01789	0.01792	0.16
		10	6.25000	0.01807	0.01807	-0.02
		16	10.00000	0.01821	0.01822	0.03
0.5	H-S	4	3.18204	0.01794	0.01795	0.03
		10	7.95511	0.01814	0.01813	-0.03
		16	12.72818	0.01831	0.01832	0.07
0.8	REC.	4	8.00000	0.01806	0.01808	0.13
		10	20.00000	0.01839	0.01845	0.33
		16	32.00000	0.01870	0.01882	0.64
0.8	TRI.	4	4.00000	0.01795	0.01796	0.06
		10	10.00000	0.01816	0.01814	-0.08
		16	16.00000	0.01835	0.01833	-0.12
0.8	H-S	4	5.09227	0.01799	0.01799	0.02
		10	12.73072	0.01824	0.01823	-0.06
		16	20.36916	0.01847	0.01846	-0.04

TABLE 10-2. COMPARISON OF STEADY STATE GAINS CALCULATED FROM TESTING AND FROM EQUATION 6.1-1 FOR SYSTEM I

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-1	ERROR (%)
1.0	REC.	4	10.00000	0.01808	0.01812	0.21
		10	25.00000	0.01844	0.01853	0.51
		16	40.00000	0.01879	0.01895	0.85
1.0	TRI.	4	5.00000	0.01798	0.01798	-0.01
		10	12.50000	0.01821	0.01819	-0.13
		16	20.00000	0.01842	0.01839	-0.14
1.0	H-S	4	6.36561	0.01802	0.01802	-0.02
		10	15.91413	0.01829	0.01828	-0.05
		16	25.46262	0.01855	0.01855	-0.02
1.2	REC.	4	12.00000	0.01826	0.01815	-0.60
		10	30.00000	0.01864	0.01861	-0.14
		16	48.00000	0.01902	0.01908	0.30
1.2	TRI.	4	6.00000	0.01799	0.01800	0.03
		10	15.00000	0.01824	0.01823	-0.07
		16	24.00000	0.01848	0.01846	-0.11
1.2	H-S	4	7.63896	0.01803	0.01804	0.04
		10	19.09749	0.01833	0.01833	0.02
		16	30.55608	0.01861	0.01863	0.10
1.5	REC.	4	15.00000	0.01825	0.01818	-0.39
		10	37.50000	0.01866	0.01867	0.06
		16	60.00000	0.01907	0.01916	0.49
1.5	TRI.	4	7.50000	0.01801	0.01802	0.03
		10	18.75000	0.01828	0.01826	-0.10
		16	30.00000	0.01854	0.01851	-0.18
1.5	H-S	4	9.54901	0.01806	0.01806	0.00
		10	23.87267	0.01838	0.01837	-0.04
		16	38.19637	0.01869	0.01869	-0.02

TABLE 11-1 COMPARISON OF TIME CONSTANTS CALCULATED FROM TESTING AND FROM EQUATION 6.1-2 FOR SYSTEM I

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-2	ERROR (%)
0.3	REC.	4	3.00000	0.34294	0.35388	3.19
		10	7.50000	0.34590	0.35599	2.92
		16	12.00000	0.34928	0.35811	2.53
0.3	TRI.	4	1.50000	0.35411	0.35318	-0.26
		10	3.75000	0.35461	0.35423	-0.11
		16	6.00000	0.35499	0.35529	0.08
0.3	H-S	4	1.90811	0.35361	0.35337	-0.07
		10	4.77027	0.35423	0.35471	0.14
		16	7.63244	0.35625	0.35606	-0.05
0.5	REC.	4	5.00000	0.34362	0.35430	3.11
		10	12.50000	0.34928	0.35912	2.82
		16	20.00000	0.35461	0.36393	2.63
0.5	TRI.	4	2.50000	0.35336	0.35269	-0.19
		10	6.25000	0.35499	0.35510	0.03
		16	10.00000	0.35778	0.35751	-0.08
0.5	H-S	4	3.18204	0.35336	0.35313	-0.07
		10	7.95511	0.35587	0.35620	0.09
		16	12.72818	0.35971	0.35926	-0.12
0.8	REC.	4	8.00000	0.34542	0.35601	3.07
		10	20.00000	0.35373	0.36441	3.02
		16	32.00000	0.36140	0.37281	3.16
0.8	TRI.	4	4.00000	0.35336	0.35321	-0.04
		10	10.00000	0.35740	0.35741	0.00
		16	16.00000	0.36166	0.36161	-0.01
0.8	H-S	4	5.09227	0.35398	0.35397	-0.00
		10	12.73072	0.35907	0.35932	0.07
		16	20.36916	0.36430	0.36467	0.10

TABLE II-2. COMPARISON OF TIME CONSTANTS CALCULATED FROM TESTING AND FROM EQUATION 6.1-2 FOR SYSTEM I

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-2	ERROR (%)
1.0	REC.	4	10.00000	0.34650	0.35711	3.06
		10	25.00000	0.35638	0.36709	3.00
		16	40.00000	0.36550	0.37706	3.16
1.0	TRI.	4	5.00000	0.35373	0.35378	0.02
		10	12.50000	0.35881	0.35877	-0.01
		16	20.00000	0.36390	0.36376	-0.04
1.0	H-S	4	6.36561	0.35461	0.35469	0.02
		10	15.91413	0.36101	0.36104	0.01
		16	25.46262	0.36738	0.36739	0.00
1.2	REC.	4	12.00000	0.35765	0.35800	0.10
		10	30.00000	0.36900	0.36917	0.05
		16	48.00000	0.37922	0.38034	0.29
1.2	TRI.	4	6.00000	0.35398	0.35428	0.09
		10	15.00000	0.36010	0.35987	-0.07
		16	24.00000	0.36603	0.36545	-0.16
1.2	H-S	4	7.63896	0.35524	0.35530	0.02
		10	19.09749	0.36298	0.36241	-0.16
		16	30.55608	0.36969	0.36952	-0.05
1.5	REC.	4	15.00000	0.35907	0.35955	0.13
		10	37.50000	0.37313	0.37363	0.13
		16	60.00000	0.38521	0.38771	0.65
1.5	TRI.	4	7.50000	0.35486	0.35486	-0.00
		10	18.75000	0.36232	0.36190	-0.12
		16	30.00000	0.36941	0.36894	-0.13
1.5	H-S	4	9.54901	0.35600	0.35614	0.04
		10	23.87267	0.36523	0.36510	-0.04
		16	38.19637	0.37383	0.37407	0.06

TABLE 12-1. COMPARISON OF STEADY STATE GAINS CALCULATED FROM TESTING AND FROM EQUATION 6.1-1 FOR SYSTEM II

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-1	ERROR (%)
0.3	REC.	4	3.00000	0.03591	0.03595	0.10
		10	7.50000	0.03628	0.03635	0.20
		16	12.00000	0.03663	0.03676	0.36
0.3	TRI.	4	1.50000	0.03576	0.03581	0.14
		10	3.75000	0.03600	0.03601	0.04
		16	6.00000	0.03620	0.03622	0.05
0.3	H-S	4	1.90811	0.03584	0.03585	0.02
		10	4.77027	0.03612	0.03611	-0.04
		16	7.63244	0.03636	0.03637	0.01
0.5	REC.	4	5.00000	0.03603	0.03607	0.12
		10	12.50000	0.03653	0.03664	0.29
		16	20.00000	0.03702	0.03720	0.49
0.5	TRI.	4	2.50000	0.03585	0.03588	0.10
		10	6.25000	0.03617	0.03617	-0.01
		16	10.00000	0.03645	0.03645	-0.00
0.5	H-S	4	3.18204	0.03593	0.03594	0.02
		10	7.95511	0.03630	0.03629	-0.01
		16	12.72818	0.03664	0.03665	0.04
0.8	REC.	4	8.00000	0.03614	0.03620	0.16
		10	20.00000	0.03678	0.03693	0.40
		16	32.00000	0.03741	0.03766	0.66
0.8	TRI.	4	4.00000	0.03594	0.03595	0.04
		10	10.00000	0.03634	0.03632	-0.06
		16	16.00000	0.03672	0.03668	-0.10
0.8	H-S	4	5.09227	0.03602	0.03602	0.00
		10	12.73072	0.03650	0.03648	-0.04
		16	20.36916	0.03696	0.03695	-0.03

TABLE 12-2. COMPARISON OF STEADY STATE GAINS CALCULATED FROM TESTING AND FROM EQUATION 6.1-1 FOR SYSTEM II

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-1	ERROR (%)
		4	10.00000	0.03619	0.03626	0.20
1.0	REC.	10	25.00000	0.03690	0.03708	0.50
		16	40.00000	0.03759	0.03791	0.84
		4	5.00000	0.03598	0.03599	0.02
1.0	TRI.	10	12.50000	0.03643	0.03640	-0.08
		16	20.00000	0.03685	0.03681	-0.11
		4	6.36561	0.03606	0.03606	0.01
1.0	H-S	10	15.91413	0.03660	0.03659	-0.04
		16	25.46262	0.03711	0.03711	-0.00
		4	12.00000	0.03653	0.03632	-0.58
1.2	REC.	10	30.00000	0.03730	0.03722	-0.20
		16	48.00000	0.03804	0.03813	0.24
		4	6.00000	0.03601	0.03602	0.02
1.2	TRI.	10	15.00000	0.03649	0.03647	-0.06
		16	24.00000	0.03696	0.03692	-0.10
		4	7.63896	0.03609	0.03610	0.02
1.2	H-S	10	19.09749	0.03667	0.03668	0.01
		16	30.55608	0.03723	0.03725	0.06
		4	15.00000	0.03651	0.03637	-0.39
1.5	REC.	10	37.50000	0.03733	0.03735	0.04
		16	60.00000	0.03813	0.03832	0.51
		4	7.50000	0.03605	0.03604	-0.03
1.5	TRI.	10	18.75000	0.03658	0.03653	-0.14
		16	30.00000	0.03709	0.03702	-0.19
		4	9.54901	0.03613	0.03613	-0.00
1.5	H-S	10	23.87267	0.03676	0.03675	-0.02
		16	38.19637	0.03738	0.03738	-0.01

TABLE 13.1 COMPARISON OF TIME CONSTANTS CALCULATED FROM TESTING AND FROM EQUATION 6.1-2 FOR SYSTEM II

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-2	ERROR (%)
		4	3.00000	0.34247	0.35280	3.02
0.3	REC.	10	7.50000	0.34602	0.35625	2.96
		16	12.00000	0.34965	0.35969	2.87
		4	1.50000	0.35211	0.35166	-0.13
0.3	TRI.	10	3.75000	0.35336	0.35338	0.01
		16	6.00000	0.35524	0.35510	-0.04
		4	1.90811	0.35211	0.35197	-0.04
0.3	H-S	10	4.77027	0.35398	0.35416	0.05
		16	7.63244	0.35651	0.35635	-0.05
		4	5.00000	0.34400	0.35453	3.06
0.5	REC.	10	12.50000	0.34965	0.35969	2.87
		16	20.00000	0.35486	0.36486	2.82
		4	2.50000	0.35236	0.35281	0.13
0.5	TRI.	10	6.25000	0.35500	0.35539	0.11
		16	10.00000	0.35778	0.35797	0.05
		4	3.18204	0.35336	0.35328	-0.02
0.5	H-S	10	7.95511	0.35625	0.35656	0.09
		16	12.72818	0.35984	0.35985	0.00
		4	8.00000	0.34566	0.35603	3.00
0.8	REC.	10	20.00000	0.35398	0.36417	2.88
		16	32.00000	0.36166	0.37231	2.94
		4	4.00000	0.35323	0.35332	0.02
0.8	TRI.	10	10.00000	0.35765	0.35739	-0.07
		16	16.00000	0.36166	0.36145	-0.06
		4	5.09227	0.35398	0.35406	0.02
0.8	H-S	10	12.73072	0.35932	0.35924	-0.02
		16	20.36916	0.36456	0.36442	-0.04

TABLE 13-2. COMPARISON OF TIME CONSTANTS CALCULATED FROM TESTING AND FROM EQUATION 6.1-2 FOR SYSTEM II

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-2	ERROR (%)
1.0	REC.	4	10.00000	0.34686	0.35703	2.93
		10	25.00000	0.35651	0.36731	3.03
		16	40.00000	0.36563	0.37759	3.27
1.0	TRI.	4	5.00000	0.35398	0.35360	-0.11
		10	12.50000	0.35907	0.35875	-0.09
		16	20.00000	0.36430	0.36389	-0.11
1.0	H-S	4	6.36561	0.35461	0.35454	-0.02
		10	15.91413	0.36127	0.36109	-0.05
		16	25.46262	0.36751	0.36763	0.03
1.2	REC.	4	12.00000	0.35804	0.35821	0.05
		10	30.00000	0.36900	0.37008	0.29
		16	48.00000	0.37951	0.38194	0.64
1.2	TRI.	4	6.00000	0.35411	0.35426	0.04
		10	15.00000	0.36036	0.36019	-0.05
		16	24.00000	0.36630	0.36612	-0.05
1.2	H-S	4	7.63896	0.35524	0.35534	0.03
		10	19.09749	0.36284	0.36289	0.01
		16	30.55608	0.37037	0.37044	0.02
1.5	REC.	4	15.00000	0.35997	0.35981	-0.04
		10	37.50000	0.37327	0.37047	-0.75
		16	60.00000	0.38551	0.38112	-1.14
1.5	TRI.	4	7.50000	0.35524	0.35626	0.29
		10	18.75000	0.36271	0.36159	-0.31
		16	30.00000	0.36955	0.36691	-0.71
1.5	H-S	4	9.54901	0.35651	0.35723	0.20
		10	23.87267	0.36550	0.36401	-0.41
		16	38.19637	0.37009	0.37080	0.19

TABLE 14-1. COMPARISON OF STEADY STATE GAINS CALCULATED FROM TESTING AND FROM EQUATION 6.1-1 FOR SYSTEM III

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-1	ERROR (%)
		4	7.00000	0.03590	0.03591	0.03
0.7	REC.	10	17.50000	0.03634	0.03639	0.15
		16	28.00000	0.03673	0.03688	0.41
		4	3.50000	0.03570	0.03575	0.13
0.7	TRI.	10	8.75000	0.03601	0.03599	-0.05
		16	14.00000	0.03625	0.03623	-0.05
		4	4.45556	0.03578	0.03579	0.03
0.7	H-S	10	11.13893	0.03612	0.03610	-0.05
		16	17.82233	0.03640	0.03641	0.03
		4	10.00000	0.03600	0.03602	0.05
1.0	REC.	10	25.00000	0.03652	0.03661	0.25
		16	40.00000	0.03701	0.03721	0.54
		4	5.00000	0.03579	0.03582	0.08
1.0	TRI.	10	12.50000	0.03614	0.03612	-0.07
		16	20.00000	0.03643	0.03641	-0.04
		4	6.36561	0.03587	0.03587	0.01
1.0	H-S	10	15.91413	0.03627	0.03625	-0.05
		16	25.46262	0.03662	0.03663	0.03
		4	14.00000	0.03634	0.03612	-0.60
1.4	REC.	10	35.00000	0.03697	0.03683	-0.39
		16	56.00000	0.03756	0.03753	-0.07
		4	7.00000	0.03587	0.03589	0.04
1.4	TRI.	10	17.50000	0.03627	0.03624	-0.09
		16	28.00000	0.03662	0.03659	-0.08
		4	8.91231	0.03594	0.03595	0.03
1.4	H-S	10	22.28093	0.03641	0.03640	-0.03
		16	35.64957	0.03685	0.03685	-0.00

TABLE 14-2. COMPARISON OF STEADY STATE GAINS CALCULATED FROM TESTING AND FROM EQUATION 6.1-1 FOR SYSTEM III

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-1	ERROR (%)
2.1	REC.	4	21.00000	0.03635	0.03626	-0.26
		10	52.50000	0.03709	0.03712	0.09
		16	84.00000	0.03780	0.03799	0.51
2.1	TRI.	4	10.50000	0.03596	0.03597	0.02
		10	26.25000	0.03643	0.03640	-0.08
		16	42.00000	0.03687	0.03683	-0.10
2.1	H-S	4	13.36912	0.03604	0.03605	0.01
		10	33.42302	0.03661	0.03660	-0.03
		16	53.47556	0.03714	0.03715	0.03
2.8	REC.	4	28.00000	0.03637	0.03634	-0.09
		10	70.00000	0.03717	0.03730	0.36
		16	112.00000	0.03796	0.03827	0.81
2.8	TRI.	4	14.00000	0.03601	0.03602	0.02
		10	35.00000	0.03654	0.03650	-0.11
		16	56.00000	0.03705	0.03698	-0.18
2.8	H-S	4	17.82584	0.03610	0.03611	0.01
		10	44.56403	0.03673	0.03672	-0.03
		16	71.30179	0.03733	0.03733	0.01

TABLE 15-1. COMPARISON OF TIME CONSTANTS CALCULATED FROM TESTING AND FROM EQUATION 6.1-2 FOR SYSTEM III

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-2	ERROR (%)
0.7	REC.	4	7.00000	0.69686	0.70775	1.56
		10	17.50000	0.70373	0.71374	1.42
		16	28.00000	0.71174	0.71973	1.12
0.7	TRI.	4	3.50000	0.70771	0.70575	-0.28
		10	8.75000	0.70922	0.70875	-0.07
		16	14.00000	0.71124	0.71174	0.07
0.7	H-S	4	4.45556	0.70671	0.70630	-0.06
		10	11.13893	0.70922	0.71011	0.13
		16	17.82233	0.71429	0.71392	-0.05
1.0	REC.	4	10.00000	0.69784	0.70877	1.57
		10	25.00000	0.70872	0.71862	1.40
		16	40.00000	0.71891	0.72847	1.33
1.0	TRI.	4	5.00000	0.70175	0.70548	0.53
		10	12.50000	0.71073	0.71041	-0.05
		16	20.00000	0.71582	0.71533	-0.07
1.0	H-S	4	6.36561	0.70671	0.70638	-0.05
		10	15.91413	0.71225	0.71265	0.06
		16	25.46262	0.71942	0.71892	-0.07
1.4	REC.	4	14.00000	0.71023	0.71079	0.08
		10	35.00000	0.72464	0.72547	0.11
		16	56.00000	0.73801	0.74015	0.29
1.4	TRI.	4	7.00000	0.70671	0.70590	-0.11
		10	17.50000	0.71276	0.71324	0.07
		16	28.00000	0.72098	0.72058	-0.06
1.4	H-S	4	8.91231	0.70721	0.70724	0.00
		10	22.28093	0.71633	0.71658	0.03
		16	35.64957	0.72569	0.72592	0.03

TABLE 15-2. COMPARISON OF TIME CONSTANTS CALCULATED FROM TESTING AND FROM EQUATION 6.1-2 FOR SYSTEM III

PULSE WIDTH (MIN)	PULSE SHAPE	INPUT PULSE DEVIATION (%)	PULSE STRENGTH	FROM TESTING	CAL. FROM EQ. 6.1-2	ERROR (%)
2.1	REC.	4	21.00000	0.71378	0.71441	0.09
		10	52.50000	0.73368	0.73522	0.21
		16	84.00000	0.75245	0.75604	0.48
2.1	TRI.	4	10.50000	0.70771	0.70747	-0.03
		10	26.25000	0.71839	0.71788	-0.07
		16	42.00000	0.72886	0.72829	-0.08
2.1	H-S	4	13.36912	0.70922	0.70936	0.02
		10	33.42302	0.72307	0.72262	-0.06
		16	53.47556	0.73584	0.73587	0.00
2.8	REC.	4	28.00000	0.71736	0.71825	0.12
		10	70.00000	0.74294	0.74452	0.21
		16	112.00000	0.76628	0.77080	0.59
2.8	TRI.	4	14.00000	0.70922	0.70949	0.04
		10	35.00000	0.72359	0.72263	-0.13
		16	56.00000	0.73638	0.73577	-0.08
2.8	H-S	4	17.82584	0.71174	0.71189	0.02
		10	44.56403	0.72886	0.72861	-0.03
		16	71.30179	0.74516	0.74534	0.02