Effects of Additive Outliers on Granger-Causality Tests: A Monte-Carlo Simulation Study

by

Thierno Aliou Baldé

Major Paper presented to the Department of Economics of the University of Ottawa in partial fulfillment of the requirements of the M.A. Degree

Supervisor

Professor Gabriel Rodríguez

ECO 7997

Ottawa, Canada
December 2002

Acknowledgments

I wish to express my gratitude to Professor Gabriel Rodríguez for introducing me to the fascinating field of time series analysis and for his constant support throughout this work. He suggested to me the subject of this study and followed the progress in its minute details. I am also indebted to Statistics Canada, particularly BSMD, for its financial support and for allowing me to pursue my economic studies during working hours. My special thanks go to the Department of Economics of Ottawa University for their support and for the high quality of the teaching I received. Finally, great thanks to my family and friends for their constant support.

Abstract

Finite samples of the coefficients associated to Granger causality and size and power of Granger causality test are analyzed using Monte-Carlo simulations. We established that additive outliers have not asymptotic effects on size and power of the Granger causality test. However, important bias and MSE are observed in finite samples. Results indicate that the presence of additive outliers in the variable y_t is source of size and power distortions. Empirical applications are proportionated to illustrate effects of additive outliers on Granger causality test.

Keywords: Additive Outliers, Granger Causality, Bias, MSE, Finite Sample Size.

JEL Classification: C2, C3, C5.

Contents

1 Introduction

Many observed time series have some points that are strangely different from the other observations. These atypical values, often called outliers, can be produced by non-systematic changes in the variables that are driving the series or affecting them. They are often associated with identifiable events such as wars, strikes or changes in policies. Many economic time series are often contaminated by such events. For example, Balke and Fomby [1] examined 15 macroeconomic time series and found outliers in almost every series.

Since the forecasts from any time series model are based on the extrapolation of the historical patterns, if the parameters of the model are very dependent on some atypical observations, then the quality of the forecasts can be very poor. When these parameters have physical or economic interpretations, the presence of undetected aberrant observations can mislead the properties of the model.

Neglecting (not deleting) aberrant observations can have major effects in time series modelling. One such effect could be to generate wrong estimates for the parameters (see Denby and Martin [7] , Ledolter [17]), hence producing forecasting bias. Even when the aberrant observation does not substantially affect the parameter estimates of the underlying model, it affects generally the variance of the estimates. A second major effect, especially when the aberrant observations are near the forecasting origin, is that the forecasts can be very inaccurate. Finally, third major effect is that the estimated variance of the innovations ($\hat{\sigma}^2$) from the contaminated data can be much larger than the true variance (σ^2), which implies that the corresponding forecasting intervals will be much wider.

In this paper, we examine the implications for Granger causality tests, in a VAR context, of the presence of additive outliers. For reasons of presentation, we focus on the bivariate case. Motivation for this study comes in part from an empirical analysis of the "causal" relations between the growth rate of the money supply (M_2) and the inflation rate in Peru. During the past twenty years, Peru's economy experienced high rates of inflation, particularly in September 1988, and July and August 1990 (see Figure 1). Traditionally, it is widely acknowledged in economic analysis that the total money supply is related to the inflation rate in a causal manner. That is, one of this variable can be predicted using both its past values and values of the other variable. However, simple application of a standard Granger causality test resulted in a non rejection of the null hypothesis. We think that this result, which is contrary to both intuition and economic theory, is caused by the presence of large additive outliers. We will return to this claim in the empirical section of this paper.

The remainder of the paper is organized as follows. Section 2 deals with a literature review on outlier modelling in time series analysis. Section 3 defines the VAR model with additive outliers and considers the implications for the parameter estimates when the outliers are omitted. Section 4 presents the properties of F-tests for Granger causality in the presence of outliers. A simulation study is conducted to outline the empirical implications of the results obtained in the theoretical results. Section 5 presents a brief empirical application. Section 6 summarizes and concludes. At the end of this paper, an appendix contains technical details.

2 Outlier Modelling in Time Series Analysis

The two common approaches widely used in modelling time series with aberrant observations are the *additive outlier* (AO) approach and the *innovation oulier* (IO) approach. Fox [8] first addressed outlier problems in time series by using this classification. Later, Tsay [31] extended this classification to allow for structural changes as well and considers *level changes* (LC), and *variance changes* (VC).

A general definition of the different types of outliers involves a series z_t without any disturbances (for the AO and LC models) and a disturbance term $f(t)$ so that the observed series y_t is the sum of z_t and $f(t)$, that is, $y_t = z_t + f(t)$. The functional form of $f(t)$ depends on the nature and type of the outlier.

An additive outlier can be viewed as an observation which is the genuine data point plus or minus some value. This latter value can be caused by a recording error or also by misinterpreting sudden "news", which can cause a stock market return to take unexpectedly large values, while such news may appear not important in the next period. In other words, the data point is then aberrant because of some cause outside the intrinsic economic environment that generates the time series data at hand.

For an additive outlier, $f(t) = \delta I_t[t = T_{AO}]$ and the model is:

$$y_t = z_t + \delta I_t[t = T_{AO}], \tag{1}$$

where $I_t[.]$ is an indicator dummy variable observed for $t = 1, 2, ..., T$. It takes a value of 1 when $t = T_{AO}$ and a value of zero otherwise. The time series z_t is the uncontaminated but unobserved time series, while y_t corresponds to the observed variable. The size of the outlier is denoted by δ.

In the autoregressive case AR(1) with drift, an additive outlier can be specified as:

$$\begin{aligned} y_t &= \mu + \delta I_t[t = T_{AO}] + z_t \\ z_t &= \phi z_{t-1} + \epsilon_t. \end{aligned} \tag{2}$$

The innovation outlier (IO) is a type of observation where the outlier occurs in the underlying noise process. In the autoregressive AR(1) case with drift, for example, the functional form of $f(t)$ is given by:

$$f(t) = \frac{1}{1-\rho L}\delta I_t[t = T_{IO}]. \tag{3}$$

which leads to specification for the innovational outlier model as:

$$\begin{aligned} y_t &= \mu + z_t + \frac{1}{1-\rho L}\delta I_t[t = T_{IO}] \\ z_t &= \rho z_{t-1} + \epsilon_t. \end{aligned} \tag{4}$$

In this case, the behavior of the series y_t depends on the autoregressive structure of z_t which, for simplicity, has been assumed here to be an AR(1) model. The innovational outlier affects the T_{IO}th observation by δ and affects the subsequent observations by $\rho^{t-T_{IO}}\delta$. Thus its impact on the subsequent observations decay at the rate ρ.

Level changes (LC) and variance changes (VC) fall into a category called structural changes. Level change represents a change in the mean of the series, while variance change represents changes in the variance of innovations. An example of variance change is when the exchange rate switches from the fixed exchange rate regime to the flexible exchange rate regime. In the AR(1) case, the level changes (LC) model is defined by:

$$f(t) = \frac{1}{1-L}\delta I_t[t = T_{LC}] \tag{5}$$

where, as in the AO model, the behavior of the y_t series under LC models does not depend on the structure of z_t. In the case of $\rho = 1$ the pattern of the impact of the IO will coincide with that of the LC.

The variance changes(VC) model is defined

$$f(t) = \frac{1}{1-\rho L}\delta S_t[t = T_{VC}] \tag{6}$$

where $S_t[t = T_{VC}]$ is the jump function that takes a value of 1 when $t \geq T_{VC}$ and a value of zero otherwise. In this case the variance of the innovations driving the series y_t changes at time $t = T_{VC}$ from σ^2 to $(1+\delta)^2\sigma^2$, so the subsequent values of y_t are affected.

In a study of the effect of additive outliers (AO) on ARIMA models, Ledolter [18] noted a resulting increase in the mean square of the l-step-ahead forecast error. He showed that this increase is due to a carry-over effect of the outlier on the forecast and a bias in the estimates of the autoregressive and moving average coefficients. Also, he showed that this increase can be rather small when the outlier does not occur too close to the forecast origin. But the conclusion is different for the width of the prediction intervals. In fact these intervals are quite sensitive to AO since outliers inflate the estimated variance ($\hat{\sigma}^2$) of the innovations.

The impact of outliers in unit root tests and cointegration have also been studied in the literature. Franses and Haldrup [9] and Shin et al. [28] established that the presence of outliers in a univariate time series affects the limiting distribution of Dickey-Fuller unit root tests. In particular, the presence of additive outliers shifts the distribution of both Dickey-Fuller statistics to the left, leading to tests with exact size greater than the asymptotic nominal size. Thus by using the usual critical values, one tends to over reject the null hypothesis of unit root in the presence of AO. See also Vogelsang [32] and Perron and Rodríguez [25].

In the IO and LC case, there is a tendency to under reject the unit root hypothesis when using the usual critical values. But for the VC case, the situation is more complicated [see Yin and Maddala [33]]. In fact, whether there is a tendency to over reject or under reject depends on some intrinsic characteristics of the series at hand.

To deal with the problems induced by outliers in unit root testing, a class of robust tests have been proposed in the literature (see Martin and Yohai [23], Lucas [19], Hoek et al. [12] , Rothenberg and Stock [27], etc.). These tests are based on the class of M-estimators introduced by Huber ([13], [14]) in the area of robust estimation theory. From a simulation point of view, these robust tests are more powerful than those based on OLS estimators if the errors come from fat-tailed distributions. But they are less powerful if the errors are normally distributed. For a recent empirical application of this kind of method, see Rodríguez [26].

In the cointegration case, Franses and Haldrup [9] showed that additive outliers can seriously affect empirical cointegration analysis. They calculated empirical fractiles of the Johansen cointegration test (see Johansen [15], [16]) based on many Monte Carlo replications of time series with various sizes of additive outliers. These fractiles markedly exceed those for the no-outlier case. Hence, using the standard critical values in the case of outliers may lead to spurious cointegration.

To reduce the effect of outliers in cointegration analysis, Lucas [20] proposed a Johansen-type testing procedure based on non-Gaussian pseudo likelihood. Franses and Lucas [10] extended these results to the case of a multivariate cointegrated vector autoregressive model. They proposed a robust estimator that signal the positions of aberrant observations and automatically assign smaller weights to them. These weights may also be used to suggest model improvements.

2.1 Outlier Detection

If the date and type of the outlier is known, the intervention analysis introduced by Box and Tiao [4] allows the irregular component of the time series to be modeled in such a way that estimates of the size δ of the outlier can be obtained from the coefficients of "intervention dummies". In this way, it is possible to remove the outlier effect from the noise function and introduce it in the deterministic part of the series. The noise function can then be analyzed without the outlier.

Outlier detection in ARMA models has been given a great deal of attention in the literature (see Tsay [30], [31], Chang et al. [5], Chen and Liu [6], Shin et al. [28]). The standard approach is to estimate a fully parameterized $ARMA$ model and construct a t-statistic for the presence of an outlier. The statistic is constructed for all possible dates ($t = 1, \dots, T$) and a supremum is taken. The value of the supremum is then compared to a critical value to determine whether an outlier is present.

Let us consider the parameterized model described in Tsay [31]

$$y_t = \Psi_\delta(L)\delta I_t + \Psi_e(L)e_t \tag{7}$$

where $\Psi_e(L)e_t$ is a moving average representation of an $ARMA(p,q)$ and $\Psi_e(L) = \phi^{-1}(L)\theta(L)$. Also, $\Psi_\delta(L)$ represents the dynamic that the outlier has on y_t. If $\Psi_\delta(L) = 1$, then δ is an additive outlier. If $\Psi_\delta(L) = \Psi_e(L)$ then δ is an innovation outlier. The t-statistic is defined for each period as a ratio where the numerator is the estimate of the size of the outlier and the denominator is its standard error. A supremum is then taken, and if this statistic exceeds a given critical value, then an outlier has occurred. A general sequential algorithm for identifying outliers is as follows:

- Estimate an ARMA model and extract the residuals and the residual variance;

- Search for outliers in the residuals using the supremum statistic;
- If an outlier is found, remove the effect of the outlier and recalculate the residuals and the residual variance;
- Continue searching and adjusting until no more outliers are indicated;
- Re-estimates the ARMA model using the adjusted series and extract the residuals. Once again, search for outliers;
- Stop the algorithm when no additional outliers are found.

An important problem with this class of procedures which is noted by Balke and Fomby [1] and Balke [2] is the fact that the estimation of the parameter from models based on the data with outliers left in, may produce a bias in the parameter estimates and thus affect the efficiency of outlier detection. Chen and Liu [6] address this problem.

Other outlier detection procedures have been proposed in the literature under a unit root framework (see Vogelsang [32] and Perron and Rodríguez [25]). This framework offers a distinct advantage, namely that one can work under the null hypothesis that a unit root is present with two useful features. First, it does not require a fully parametric model of the errors and is valid for general forms of dynamic structure. Second, an asymptotic distribution can be obtained and critical values tabulated even without having to make specific distributional assumption about the errors.

2.2 Granger Causality

A question that frequently arises in time series analysis is whether or not one economic variable can help forecast another economic variable, or simply is there a way of establishing if one variable has been a leading indicator of another over the past? For instance it has been well documented that nearly all the postwar economic recessions have been preceded by large increases in the price of oil. Does this imply that oil shocks cause recessions?

Granger [11] first proposed an approach to address this kind of question and Sims [29] popularized it. The idea of Granger causality can be expressed as follows. A variable X Granger-causes Y if Y can be better predicted using the histories of both X and Y than it can using the history of Y alone. Conceptually, the idea has several components:

- Temporality: only past values of X can "cause" Y;

- Exogeneity: Sims [29] pointed out that a necessary condition for X to be exogenous to Y is that X fails to Granger-cause Y; and
- Independence: Similarly, variables X and Y are only independent if both fail to Granger-cause the other.

Wald tests are standard tools in testing the null hypothesis of non Granger causality. These tests are mainly tests for 'zero restrictions' on parameter estimates obtained by fitting a linear regression model (involving lagged values) to the data. Like most statistical tests, Granger causality tests require that the relationship between the variables remains stable over the sample period being tested. In the VAR time series representation, one of the basic assumption in Granger causality testing is stationarity, that is the absence of trends, seasonal components and structural instabilities in the sample period. In a simulation study, Lütkepohl [22] has shown that Granger causality tests may provide quite incorrect inference about causality relations in the presence of structural changes. He showed that the sizes of the tests in finite samples are excessive when mean shifts are ignored. However, these pitfalls of Granger causality tests may be avoided if the number and the dating of the breaks are known, as the tests could be safely applied in those sub samples where no structural instabilities are detected. Lütkepohl [22], [21]) described an approach for the detection of structural breaks based on predictions tests. In another paper, Bianchi [3] proposed a Bayesian model which can make Granger causality tests robust to the presence of structural instabilities. Recently, in the VAR context, Ng and Vogelsang [24] proposed a strategy which removes the breaks and estimates simultaneously the breaks with the autoregressive parameters. It is the model which is adapted in the following section allowing for additive outliers.

3 The Model

The AO and IO approach in univariate time series models can be easily extended to multivariate time series representing jointly evolving processes. Multivariate analysis investigates temporal dependence and interactions among a set of variables in vector valued processes. It has a wide range of applications in economics and finance. In finance for example, portfolios generating returns on a set of assets form vector valued processes. Financial portfolios contain various quantities of risky assets, which have to be regularly adjusted according to the expected future changes of asset prices. Such strategic allocation updating can be tuned to forecasts from a Vector Autoregressive model of returns.

This paper provides a tentative analysis of the properties of VARs in the presence of additive outliers. Stationarity of the series is assumed for the whole analysis. We focus on whether additive outliers in one or more series will induce bias in the OLS estimates of the VAR. Also, we examine the consistency properties of estimates and the implications on Granger causality tests in the associated regressions.

A bivariate additive outlier can be specified as:

$$\begin{aligned} y_t &= \mu + D_t\delta + z_t \\ z_t &= Az_{t-1} + e_t \end{aligned} \tag{8}$$

where $y_t = (y_{1t}, y_{2t})'$ for $t = 1, \ldots, T$, $z_t = (z_{1t}, z_{2t})'$, $e_t = (e_{1t}, e_{2t})'$, $\mu = (\mu_1, \mu_2)'$, $\delta = (\delta_1, \delta_2)'$, D_t is a 2 by 2 diagonal matrix with $diag(D_t) = (D_{1t}, D_{2t})$, $D_{it} = 1(t = T_{AOi})$ $i = 1, 2$ where T_{AOi} is the outlier date for the series y_i and A is a 2 by 2 matrix with elements a_{ij}. Also,

$$D_{it} = \begin{cases} 1 & if \quad t = T_{AOi} \\ 0 & \quad\quad otherwise. \end{cases}$$

The Additive outlier model (8) can be rewritten as

$$y_t = \mu^* + D_t^*\delta + A\Delta D_t\delta + Ay_{t-1} + e_t \tag{9}$$

where

$$D_t = \begin{bmatrix} D_{1t} & 0 \\ 0 & D_{2t} \end{bmatrix}$$

$$\Delta D_{it} = D_{it} - D_{it-1} = \begin{cases} 1 & if \quad t = T_{AOi} \\ -1 & if \quad t = T_{AOi} + 1 \\ 0 & \quad\quad otherwise \end{cases}$$

$$\Delta D_t = D_t - D_{t-1} = \begin{bmatrix} \Delta D_{1t} & 0 \\ 0 & \Delta D_{2t} \end{bmatrix}$$

$$\mu^* = (I_2 - A)\mu = \begin{bmatrix} (1 - a_{11})\mu_1 - a_{12}\mu_2 \\ -a_{21}\mu_1 + (1 - a_{22})\mu_2 \end{bmatrix}$$

$$D_t^* = (I_2 - A)D_t = \begin{bmatrix} (1-a_{11})D_{1t} & -a_{12}D_{2t} \\ -a_{21}D_{1t} & (1-a_{22})D_{2t} \end{bmatrix}$$

Equation (9) is derived as follows. It is possible to see, from the model (8), that

$$z_t = y_t - \mu - D_t\delta = A[y_{t-1} - \mu - D_{t-1}\delta] + e_t \tag{10}$$

which can be written as

$$y_t - Ay_{t-1} = \mu - A\mu + D_t\delta - AD_{t-1}\delta + e_t \tag{11}$$

and finally, we have

$$\begin{aligned} y_t &= (I_2 - A)\mu + Ay_{t-1} + D_t\delta - AD_{t-1}\delta + AD_t\delta - AD_t\delta + e_t \\ &= (I_2 - A)\mu + Ay_{t-1} + (I_2 - A)D_t\delta + A[D_t - D_{t-1}]\delta + e_t \\ &= \mu^* + Ay_{t-1} + D_t^*\delta + A\Delta D_t\delta + e_t. \end{aligned} \tag{12}$$

For more detail, the components of the last system can be expressed as

$$\begin{aligned} y_{1t} &= \mu_1^* + \delta_1(1-a_{11})D_{1t} - \delta_2 a_{12}D_{2t} + \delta_1 a_{11}\Delta D_{1t} \\ &\quad + \delta_2 a_{12}\Delta D_{2t} + a_{11}y_{1t-1} + a_{12}y_{2t-1} + e_{1t} \end{aligned} \tag{13}$$

$$\begin{aligned} y_{2t} &= \mu_2^* - \delta_1 a_{21}D_{1t} + \delta_2(1-a_{22})D_{2t} + \delta_1 a_{21}\Delta D_{1t} \\ &\quad + \delta_2 a_{22}\Delta D_{2t} + a_{21}y_{1t-1} + a_{22}y_{2t-1} + e_{2t}. \end{aligned} \tag{14}$$

Notice that when the dates of the outliers coincide, that is when $D_{1t} = D_{2t} \equiv D_{0t}$, we can rewrite y_{1t} and y_{2t} as:

$$\begin{aligned} y_{1t} &= \mu_1^* + [\delta_1(1-a_{11}) - \delta_2 a_{12}]D_{0t} + [\delta_1 a_{11} + \delta_2 a_{12}]\Delta D_{0t} \\ &\quad + a_{11}y_{1t-1} + a_{12}y_{2t-1} + e_{1t} \end{aligned} \tag{15}$$

$$\begin{aligned} y_{2t} &= \mu_2^* + [-\delta_1 a_{21} + \delta_2(1-a_{22})]D_{0t} + [\delta_1 a_{21} + \delta_2 a_{22}]\Delta D_{0t} \\ &\quad + a_{21}y_{1t-1} + a_{22}y_{2t-1} + e_{2t}. \end{aligned} \tag{16}$$

The effects of AO on the VAR can be seen by considering one of the equations, say, y_{2t}. This equation reveals that the effects of AO on y_{2t} will depend on whether or not $a_{21} = 0$. If y_1 does not Granger cause y_2, that is $a_{21} = 0$, y_1 is weakly exogenous for y_2. When $a_{21} = 0$, the coefficients of D_{0t} and ΔD_{0t} do not depend on δ_1, the size of the AO in y_1. So an AO on y_1 will not appear in the equation for y_2. Under Granger non-causality, an AO dummy will appear in the equation for y_2 only if there is an AO in the function for y_2 itself, that is if $\delta_2 \neq 0$. However, when y_1 Granger cause y_2, the AO in y_1 will, in general, appear in the conditional model for y_2 whether we have AO in y_1, y_2, or both.

Now let us consider the regression equation for y_2 when the outliers are omitted, that is

$$y_{2t} = \mu_2^* + a_{21}y_{1t-1} + a_{22}y_{2t-1} + e_{2t}, \tag{17}$$

where it is clear that regression equation (17) is a misspecified model. This misspecification is obvious upon rewriting (16) as:

$$y_{2t} = \mu_2^* + a_{21}y_{1t-1} + a_{22}y_{2t-1} + e_{2t}^*$$

where $e_{2t}^* = e_{2t} + [-\delta_1 a_{21} + \delta_2(1 - a_{22})]D_{0t} + [\delta_1 a_{21} + \delta_2 a_{22}]\Delta D_{0t}$. Consequently, regressions based upon (17) suffer from omitted variables bias since the regressors D_{0t} and ΔD_{0t} are excluded from the AO model although $\delta_2 \neq 0$ and/or $\delta_1 \neq 0$ and $a_{21} \neq 0$. The following theorem presents the asymptotic effects of omitted additive outliers.

Theorem 1 *Let $y_t = (y_{1t}, y_{2t})'$, $t = 1, \ldots, T$ be generated by the AO model (8). Let the parameters $a_2 = (a_{21}, a_{22})'$ be estimated from the misspecified model (17) by OLS to yield $\hat{a}_2 = (\hat{a}_{21}, \hat{a}_{22})'$. Then*

1. The limiting distribution of $T^{\frac{1}{2}}(\hat{a}_2 - a_2)$ is a normal distribution with variance $\sigma_2^2\Gamma$, that is,

$$T^{\frac{1}{2}}(\hat{a}_2 - a_2) \Longrightarrow N[0, \sigma_2^2\Gamma]$$

where this variance is also a function of the parameter matrix A.

2. $P\lim(\hat{a}_2 - a_2) = 0$.

The theorem establishes that when there are AO in the series y_1 and y_2, the estimates associated with the autoregressions which omits the AO will depends on the parameters a_{ij} for finite samples. But in general these estimates remain consistent. The proof of theorem 1 is provided in the Appendix section.

4 Properties of the F-Tests for Granger Causality

Now let us consider the effects of omitted additive outliers on Granger causality test. Specifically, we are interested in testing whether y_1 Granger causes

y_2. The statistic is the F-statistic for testing the hypothesis that $a_{21} = 0$ in the misspecified model (17). Then, we have

$$\begin{aligned} GC &= \frac{(\hat{a}_{21} - a_{21})^2}{s^2(\tilde{X}'\tilde{X})^{-1}_{11}} \\ &= \frac{T(\hat{a}_{21} - a_{21})^2}{s^2(T^{-1}\tilde{X}'\tilde{X})^{-1}_{11}} \end{aligned} \tag{18}$$

where $\tilde{X}$ is the $(T-1) \times 2$ matrix of demeaned (y_{1t-1}, y_{2t-1}).

Under the null hypothesis of no Granger causality, $a_{21} = 0$ and the test statistic has asymptotically a χ^2_1 limiting distribution in the absence of AO. When $\delta_2 \neq 0$ and the AO is omitted, we have in Theorem 1 that $T(\hat{a}_{21} - a_{21})^2$ converges to a normal distribution and that $(T^{-1}\tilde{X}'\tilde{X})^{-1}$ converges to Γ^{-1}. That is $(T^{-1}\tilde{X}'\tilde{X})^{-1}_{11}$ converges to Γ^{-1}_{11}. Since $s^2 = T^{-1}\hat{e}_2'\hat{e}_2$ we have that the GC statistic converges too. These results are presented in the next Theorem.

Theorem 2 *Suppose the data are generated by the AO model (8), and the misspecified model (17) is used to test whether y_1 Granger causes y_2. Then under the null hypothesis of no Granger causality as $T \longrightarrow \infty$, we have*

1. *If $\delta_2 = 0$ then $GC \Rightarrow \chi^2_1$*
2. *If $\delta_2 \neq 0$ then the limiting distribution of GC is a χ^2 that is dependent on the parameters of the matrix A.*

The theorem states that AO have effects on the Granger causality test in that the limiting distribution of the Granger causality statistic is dependent on the parameters of the matrix A. As we will see in the simulations, the parameters contaminate the sizes of the F-test in the finite sample case. The proof of theorem 2 is provided in the Appendix section.

4.1 Simulations

In this subsection a simulation study is used to outline the empirical implications of the theorems in finite samples. Bivariate series (precisely a VAR(1) series)are generated according to model (8) using different values for the sample size T and the additive outlier size δ. The errors e_t are *i.i.d.* draws from a standard bivariate normal distribution. The AO date, T_{AO}, is chosen to occur at the mid-point $(0.5 * T)$ of the sample and it is the same for both series.

Eight possible combinations of δ were considered: $(0,0), (5,5), (10,10)$, $(50,50)$, $(10,0), (50,0), (0,10), (0,50)$. This allows for the possibility that an additive outlier occurs in none, one, or both series. Three values are considered for the causality parameter a_{21} $(-0.3, 0.0, 0.3)$ and two values for the parameter a_{11} $(-0.5, 0.5)$. Values for a_{12} and a_{22} are taken from the parameter set $\{-0.6, -0.4, -0.2, 0.0, 0.2, 0.3, 0.4\}$ for each value of a_{11}. Without loss of generality, we report results for $a_{11} = 0.5$. All set of parameters have eigenvalues of the matrix A that lie inside the unit circle with unequal roots. This rules out explosive models. In all cases, 1000 replications were considered. For each replication the misspecified model (17) was estimated by OLS and the value of the parameter of interest $(\hat{a}_{21})$ is calculated. The values reported in the tables are the means (averages) of these estimates over simulations. The results are presented in tables 1-25.

Tables 1-2 show the mean estimates of the coefficient $\hat{a}_{21}$ when there are no additive outliers in both series. for small sample sizes ($T = 50$ or $T = 100$), some distorsions still affect the estimates, but for $T = 500$, the mean estimates are much closer to the true parameter. That is, for large samples, the coefficients are almost precisely estimated. When $a_{21} = 0$ (no Granger causality) and at least y_2 has additive outliers (see tables 5, 11 and 17), the estimated coefficient $\hat{a}_{21}$ is affected in a complex way. Actually, the bias follows some patterns, as for example, combinations of negative values of a_{12} with negative values of a_{22} tend to cause larger values for the bias. The same results are observed with combinations of negative values for a_{12} with positive values of a_{22}.

When $a_{21} \neq 0$, that is y_1 Granger causes y_2, additive outliers in y_1 and/or y_2 will always affect the estimates because of the feedback between the two series. When $a_{21} = -0.30$ and $\delta_2 \neq 0$ (see tables 6, 12 and 18), the largest values of the bias are obtained with the combination of negative values for a_{12} with negative values of a_{22}. The bias values tend to diminish as a_{22} goes to zero. When $a_{21} = 0.30$ and $\delta_2 \neq 0$ (see tables 7, 13 and 19), the largest values of the bias are obtained with the combination of positive values for a_{12} with positive values of a_{22}. Similarly, these bias values tend to diminish as a_{22} goes to zero.

Up to now the results show that additive outliers do have an impact on the parameter estimates. Let us now look at the implications of part ii) of theorem 1. The theorem states that as the sample size T goes to infinity, the mean estimates are closer to the true parameter estimates. Intuitively, one might expect this kind of result as long as the number of outliers doesn't increase with the sample size. When $a_{21} = 0$ and $\delta_2 \neq 0$ (see tables 5, 11 and 17), the bias tend to diminish as we move from $T = 50$ to $T = 500$, although some significant discrepencies remain even for $T = 500$, the largest being equal to 0.56. When $a_{21} = -0.30$ and $\delta_2 \neq 0$ (see tables 6, 12 and 18), here too, the bias tend to diminish as we move from $T = 50$ to $T = 500$. The same result is observed when $a_{21} = 0.30$ and $\delta_2 \neq 0$. So overall there is evidence that the effects of AO on the parameter estimates become less severe as the sample size increases.

Now, let us consider results for the size of the F-test for Granger causality. When there are no additive outliers in both series, the size depends on whether or not there is Granger causality. In the case of Granger non causality, the size is very close to the nominal size of 5 % (see table 3). However the size is near unity in the case of Granger causality (see table 4). When $a_{21} = 0$ (null hypothesis of Granger non causality) The size of the GC statistic depends on whether or not y_2 has outliers. When y_2 has no outliers (see tables 8, 14, 20 and 25), the GC statistic has an exact size close to the nominal size of 5 % as specified in Theorem 1. When y_2 has outliers (that is $\delta_2 \neq 0$), size distortions are larger and follows complex patterns according to the values of the parameters a_{12} and a_{22}.
When $a_{21} = -0.30$ and $\delta_2 \neq 0$ (see tables 9, 15 and 21), the largest values of the size of the F-test are obtained with the combination of negative values for a_{12} with negative values of a_{22}. The same results are observed with combinations of positive values for a_{12} with positive values of a_{22}. When $a_{21} = 0.30$ and $\delta_2 \neq 0$ (see tables 10, 16 and 22), the largest values of the size of the F-test are obtained for positive values of a_{22}.

The case of multiple outliers was also considered, always with $\delta_1 = 0$, for sample sizes $T = 50$, 100 and 200. In this case, the additive outliers (δ_2) are located at $0.20T$, $0.40T$, $0.60T$ and $0.80T$. The results are presented in Tables 26-28. We observe that in the case of no Granger causality ($a_{21} = 0$), size distortions are smaller the closer a_{22} is to zero. When $a_{21} = -0.30$ and $\delta_2 \neq 0$ the largest values of the size of the F-test are obtained with negative values for a_{12}. The same results are observed with combinations of positive values for a_{12} with positive values of a_{22}. When $a_{21} = 0.30$ and $\delta_2 \neq 0$ the larger values of the size of the F-test are obtained for positive values of a_{22}.

5 Empirical Application

As it was mentioned in the introduction of this paper, the standard F-test for Granger causality test was applied to two time series of Peru. The variables are the inflation rate and the growth rate of total liquidity in the economy, defined also as M2 (source: Central Bank of Peru). The period analyzed is 1980 until 1997 with a monthly frequency. Both series are shown in Figure 1, where is easy to observe the presence of huge additive outliers. Overall, these observations are related to well know stabilization programs applied by government of Peru with the goal of stopping high inflation rates. Given that a simple visual observation is not sufficient to statistically identify additive outliers, we used the procedure of Perron and Rodríguez [25] to identify precisely the observations qualified as additive outliers. See also Rodríguez [26] for a similar approach.

Table 29 presents the results obtained from the F-test applied to both series before and after correcting for the presence of additive outliers. Before correcting for outliers, the results show that we cannot reject the null hypothesis that the growth of money does not cause inflation rate. Results in the opposite direction are different. Apparently, we can reject the null hypothesis that inflation rate does not cause growth of money. While this last result is acceptable (possibly because the feedback between both variables), the first result is contrary to intuition, especially in an economy where higher inflation and high growth rates of money were evident in the period analyzed.

The results change when we apply the same F-test after eliminating the influence of the additive outliers. After the additive outliers were identified, dummy variables were created and a regression of both variables against them was performed. Then, the residuals from both regressions are free of the influence of the additive outliers. These two residual time series are used to apply the standard F-test to verify the null hypothesis of no Granger causality. Results indicate that there is a strong rejection of the null hypothesis of no Granger causality from the growth rate of money to the inflation rate. The analysis in the opposite direction confirms the original result but with a lower p-value.

6 Conclusion

In this paper, we analyzed the implications for Granger causality tests, in the VAR context, of the presence of additive outliers. We showed that although the least squares estimates of the VAR are consistent, they will depend on the parameter Matrix in finite samples. That is, additive outliers contaminate these estimates. Furthermore the theoretical results showed that the limiting distribution of the Granger causality statistic is dependent on the parameter matrix . The simulation results confirmed this fact and showed that the parameters contaminate the sizes of the F-tests. Hence conducting hypothesis testing in a VAR without adjusting for additive outliers can lead to incorrect inference. The results obtained in analyzing the inflation rate and the total money supply in Peru can now be explained fully by this study. It is worth to mention however, that the results of this study are obtained in a VAR(1) set up. In practice however, VAR models are often of higher order. We intend to study, in a future research, the implications of AO in higher order VARs setups.

References

[1] E. M. L. Balke and A. Fomby. Large shocks, small shocks, and economic fluctuations: Outliers in macroeconomic time series. *Journal of Applied Econometrics*, 9:181–200, 1994.

[2] N. Balke. Detecting level shifts in time series. *Journal of Business and Economic Statistics*, 11:81–92, 1993.

[3] M. Bianchi. Granger causality in the presence of structural changes. Technical report, Working Paper Series No 33, Bank of England, United Kingdom, 1995.

[4] G. Box and G. Tiao. Intervention analysis with applications to economic and environmental problems. *Journal of American statistical Association*, 70:70–79, 1975.

[5] I. Chang, G. Tiao, and C. Chen. Estimation of time series parameters in the presence of outliers. *Technometrics*, 30:193–204, 1988.

[6] C. Chen and L. Liu. Joint estimation of model parameters and outlier effects in time series. *Journal of American statistical Association*, 88:284–297, 1993.

[7] L. Denby and R. Martin. Robust estimation of the first-order autoregressive parameter. *Journal of American statistical Association*, 74:140–146, 1979.

[8] A. J. Fox. Outliers in time series. *Journal of the Royal Statistical Society*, 74, Series B:350–363, 1972.

[9] P. H. Franses and N. Haldrup. The effects of additive outliers on tests for unit roots and cointegration. *Journal of Business and Economic Statistics*, 12:471–478, 1994.

[10] P. H. Franses and A. Lucas. Outliers robust cointegration analysis. Technical report, Report 9529A, Erasmus University, Rotterdam, 1995.

[11] C. Granger. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, 37:424–438, 1969.

[12] H. Hoek, A. Lucas, and H. Van Dijk. Classical and bayesian aspect of robust unit root inference. *Journal of Econometrics*, 69:27–59, 1995.

[13] P. Huber. Robust estimation of a local parameter. *Annals of Mathematical Statistics*, 35:73–101, 1964.

[14] P. Huber. Robust regression: Asymptotics, conjectures and montecarlo. *Annals of Statistics*, 14:781–818, 1973.

[15] S. Johansen. Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12:231–254, 1988.

[16] S. Johansen. Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59:1551–1580, 1991.

[17] J. Ledolter. The effect of outliers on the estimates in and the forecasts from arima time series models. In *Proceedings of the Business and Economic Statistics Section on the American Statistical Association*, pages 453–458, 1987.

[18] J. Ledolter. The effect of additive outliers on the forecasts from arima models. *International Journal of Forecasting*, 5:231–240, 1989.

[19] A. Lucas. Unit root tests based on m-estimators. *Econometric Theory*, 11:331–346, 1995.

[20] A. Lucas. Cointegration testing using pseudo likelihood ratio tests. *Econometric Theory*, 13:149–169, 1997.

[21] H. Lütkepohl. The stability assumption in tests of causality between money and income. pages 75–86. 1989.

[22] H. Lütkepohl. *Introduction to Multiple Time Series Analysis.* Springer Verlag, Berlin, 1991.

[23] R. Martin and V. Yohai. Influence functionals for time series. *Annals of statistics*, 14:781–818, 1986.

[24] S. Ng and T. J. Vogelsang. Analysis of vector autoregressions in the presence of shifts in mean. *Econometric Reviews*, 21:353–381, 2002.

[25] P. Perron and G. Rodríguez. Searching for additive outliers in nonstationary time series. *Forthcoming in the Journal of Time Series Analysis*, 2003.

[26] G. Rodríguez. An empirical note about additive outliers and nonstationarity in latin american inflation series. *Forthcoming in Empirical Economics*, 2003.

[27] T. Rothenberg and J. Stock. Inference in a nearly integrated autoregressive model with nonnormal innovations. *Journal of Econometrics*, 80:269–286, 1997.

[28] D. Shin, S. Sarkar, and J. Lee. Unit root tests for time series with outliers. *Stat. prob. lett.*, 30:189–197, 1996.

[29] C. Sims. Money, income and causality. *American Economic Review*, 62:540–542, 1972.

[30] R. Tsay. Time series model specification in the presence of outliers. *Journal of American statistical Association*, 81:132–141, 1986.

[31] R. Tsay. Outliers, level shifts, and variance changes in time series. *Journal of Forecasting*, 7:1–20, 1988.

[32] T. J. Vogelsang. Two simple procedures for testing for a unit root when there are additive outliers. *Journal of Time Series Analysis*, 20:237–252, 1999.

[33] Y. Yin and G. Maddala. The effects of different types of outliers on unit root tests. In T. Fomby and R. Hill, editors, *Advance in Econometrics*, volume 13. JAI Press, Greenwich, Connecticut, 1997.

7 Appendix

Proof of theorem 1. Let $\gamma_0 = -\delta_1 a_{21} + \delta_2(1 - a_{22})$ and $\gamma_{00} = \delta_1 a_{21} + \delta_2 a_{22}$. We can rewrite the DGP as:

$$y_{2t} = \mu_2^* + \gamma_0 D_{0t} + \gamma_{00}\Delta D_{0t} + a_{21} y_{1t-1} + a_{22} y_{2t-1} + e_{2t}. \tag{A.1}$$

Let us also consider the $(T-1)\times 1$ vectors $\tilde{Y}_2$, $\tilde{D}_0$, $\Delta\tilde{D}_0$ and $\tilde{e}_{2t}$ corresponding to the demeaned y_{2t}, D_{0t} and ΔD_0. Let $\tilde{X}$ be the $(T-1)\times 2$ matrix of demeaned (y_{1t-1}, y_{2t-1}). The DGP model can be rewritten as

$$\tilde{Y}_2 = \gamma_0 \tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + a_2\tilde{X} + \tilde{e}_{2t}. \tag{A.2}$$

The misspecified model (17) can also be rewritten as:

$$\tilde{Y}_2 = \hat{a}_2\tilde{X} + \hat{e}_2 \tag{A.3}$$

where $\hat{a}_2 = (\hat{a}_{21}, \hat{a}_{22}) = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{Y}_2$. Plugging the values of $\tilde{Y}_2$ from the DGP model (A.2) yields

$$\hat{a}_2 - a_2 = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\left[\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_2\right]. \tag{A.4}$$

But, we know that

$$\tilde{X}'\tilde{X} = \begin{bmatrix} \sum_{t=1}^T \tilde{y}_{1t-1}^2 & \sum_{t=1}^T \tilde{y}_{1t-1}\tilde{y}_{2t-1} \\ \sum_{t=1}^T \tilde{y}_{2t-1}\tilde{y}_{1t-1} & \sum_{t=1}^T \tilde{y}_{2t-1}^2 \end{bmatrix}.$$

Also from (8) we have

$$\begin{aligned} \tilde{y}_{1t-1} &= \delta_1\tilde{D}_{0t-1} + \tilde{Z}_{1t-1} \\ \tilde{y}_{2t-1} &= \delta_2\tilde{D}_{0t-1} + \tilde{Z}_{2t-1} \end{aligned}$$

Consequently,

$$\begin{aligned} \sum_{t=1}^T \tilde{y}_{1t-1}^2 &= \sum_{t=1}^T \left[\delta_1\tilde{D}_{0t-1} + \tilde{Z}_{1t-1}\right]^2 \\ &= \delta_1^2\sum_{t=1}^T \tilde{D}_{0t-1}^2 + 2\delta_1\sum_{t=1}^T \tilde{D}_{0t-1}\tilde{Z}_{1t-1} + \sum_{t=1}^T \tilde{Z}_{1t-1}^2. \end{aligned} \tag{A.5}$$

However, $\tilde{D}_{0t}$ can be estimated by OLS residuals from the regression equation

$$D_{0t} = \theta + \tilde{D}_{0t}.$$

By the standard formulae of OLS, we have that $\hat{\theta} = (T+1)^{-1}\sum_{t=1}^{T} D_{0t} = (T+1)^{-1}$ and $\tilde{D}_{0t} = D_{0t} - \hat{\theta} = D_{0t} - (T+1)^{-1}$. Similarly $\tilde{D}_{0t-1} = D_{0t-1} - T^{-1}$ and $\sum_{t=1}^{T} \tilde{D}_{0t-1}^2 = \sum_{t=1}^{T}\left[D_{0t-1} - T^{-1}\right]^2 = \sum_{t=1}^{T} D_{0t-1}^2 - T^{-1} = 1 - T^{-1}$. Also, we have

$$\begin{aligned}
\sum_{t=1}^{T} \tilde{D}_{0t-1}\tilde{Z}_{1t-1} &= \sum_{t=1}^{T}\left[(D_{0t-1} - T^{-1})(Z_{1t-1} - T^{-1}\sum_{t=1}^{T} Z_{1t-1})\right] \\
&= \begin{array}{l} \sum_{t=1}^{T} D_{0t-1}Z_{1t-1} - T^{-1}\sum_{t=1}^{T} Z_{1t-1} \\ -T^{-1}\sum_{t=1}^{T} Z_{1t-1}\sum_{t=1}^{T} D_{0t-1} + T^{-1}\sum_{t=1}^{T} Z_{1t-1} \end{array} \\
&= \sum_{t=1}^{T} D_{0t-1}Z_{1t-1} - T^{-1}\sum_{t=1}^{T} Z_{1t-1} \\
&= Z_{1T_{AO}} - T^{-1}\sum_{t=1}^{T} Z_{1t-1} = Z_{1T_{AO}} - \bar{Z}_1.
\end{aligned}$$

Because $\sum_{t=1}^{T} \tilde{Z}_{1t-1}^2$ is a sum of squares of a demeaned vector, we can say that

$$T^{-1}\sum_{t=1}^{T} \tilde{Z}_{1t-1}^2 \Rightarrow \Gamma_{11}.$$

where Γ_{11} is the first element of the covariance matrix Γ of the z's. We can now derive the final expression for $\sum \tilde{y}_{1t-1}^2$:

$$\sum_{t=1}^{T} \tilde{y}_{1t-1}^2 = \delta_1^2(1 - T^{-1}) + 2\delta_1\left(Z_{1T_{AO}} - \bar{Z}_1\right) + \sum_{t=1}^{T} \tilde{Z}_{1t-1}^2 \tag{A.6}$$

Multiplying both sides by T^{-1} we get

$$T^{-1}\sum_{t=1}^{T} \tilde{y}_{1t-1}^2 \Rightarrow \Gamma_{11}. \tag{A.7}$$

Similarly we have

$$T^{-1}\sum_{t=1}^{T} \tilde{y}_{2t-1}^2 \Rightarrow \Gamma_{22}, \tag{A.8}$$

$$T^{-1}\sum_{t=1}^{T} \tilde{y}_{1t-1}\tilde{y}_{2t-1} \Rightarrow \Gamma_{12}, \tag{A.9}$$

$$T^{-1}\sum_{t=1}^{T} \tilde{y}_{2t-1}\tilde{y}_{1t-1} \Rightarrow \Gamma_{21}. \tag{A.10}$$

Overall we have established that $(T^{-1}\tilde{X}'\tilde{X}) \Rightarrow \Gamma$. It follows that

$$(T^{-1}\tilde{X}'\tilde{X})^{-1} \Rightarrow \Gamma^{-1}.$$

Now, we need to evaluate the expression

$$\tilde{X}'\left[\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_2\right] = \gamma_0\tilde{X}'\tilde{D}_0 + \gamma_{00}\tilde{X}'\Delta\tilde{D}_0 + \tilde{X}'\tilde{e}_2 \qquad \text{(A.11)}$$

for which we analyze each term:

1. The first term is

$$\tilde{X}'\tilde{D}_0 = \left(\sum_{t=1}^{T}\tilde{y}_{1t-1}\tilde{D}_{0t}, \sum_{t=1}^{T}\tilde{y}_{2t-1}\tilde{D}_{0t}\right)',$$

where, developing the first sub-term we have

$$\begin{aligned}
\sum_{t=1}^{T}\tilde{y}_{1t-1}\tilde{D}_{0t} &= \sum_{t=1}^{T}\left[\delta_1\tilde{D}_{0t-1} + \tilde{Z}_{1t-1}\right]\tilde{D}_{0t} \\
&= \sum_{t=1}^{T}\left[\delta_1\tilde{D}_{0t-1} + \tilde{Z}_{1t-1}\right]\left[D_{0t} - T^{-1}\right] \\
&= \delta_1\sum_{t=1}^{T}\tilde{D}_{0t-1}D_{0t} - \delta_1 T^{-1}\sum_{t=1}^{T}\tilde{D}_{0t-1} + \sum_{t=1}^{T}\tilde{Z}_{1t-1}D_{0t} \\
&\quad - T^{-1}\sum_{t=1}^{T}\tilde{Z}_{1t-1} \\
&= \delta_1\sum_{t=1}^{T}\left[(D_{0t-1} - T^{-1})D_{0t}\right] - \delta_1 T^{-1}\sum_{t=1}^{T}\left[D_{0t-1} - T^{-1}\right] \\
&\quad + \sum_{t=1}^{T}\tilde{Z}_{1t-1}D_{0t} - T^{-1}\sum_{t=1}^{T}\tilde{Z}_{1t-1} \\
&= \delta_1\sum_{t=1}^{T}D_{0t-1}D_{0t} - \delta_1 T^{-1}\sum_{t=1}^{T}D_{0t} - \delta_1 T^{-1}\sum_{t=1}^{T}D_{0t-1} + \delta_1 T^{-1} \\
&\quad + \sum_{t=1}^{T}\left[Z_{1t-1} - T^{-1}\sum_{t=1}^{T}Z_{1t-1}\right]D_{0t} \\
&\quad - T^{-1}\sum_{t=1}^{T}\left[Z_{1t-1} - T^{-1}\sum_{t=1}^{T}Z_{1t-1}\right] \\
&= Z_{1T_{AO}-1} - \delta_1 T^{-1} - \bar{Z}_1.
\end{aligned}$$

In a similar way we have that

$$\sum_{t=1}^{T} \tilde{y}_{2t-1}\tilde{D}_{0t} = Z_{2T_{AO}-1} - \delta_2 T^{-1} - \bar{Z}_2.$$

With last two terms, we have $\tilde{X}'\tilde{D}_0 = Z_{T_{AO}-1} - T^{-1}\delta - \bar{Z}$, which is equivalent to say that

$$\gamma_0\tilde{X}'\tilde{D}_0 = \gamma_0 Z_{T_{AO}-1} - \gamma_0 T^{-1}\delta - \gamma_0\bar{Z}.$$

2. The second term is

$$\tilde{X}'\Delta\tilde{D}_0 = \left(\sum_{t=1}^{T}\tilde{y}_{1t-1}\Delta\tilde{D}_{0t}, \sum_{t=1}^{T}\tilde{y}_{2t-1}\Delta\tilde{D}_{0t}\right)'$$

where, developing the first sub-term we have

$$\begin{aligned}
\sum_{t=1}^{T}\tilde{y}_{1t-1}\Delta\tilde{D}_{0t} &= \sum_{t=1}^{T}\left[\delta_1\tilde{D}_{0t-1} + \tilde{Z}_{1t-1}\right]\Delta\tilde{D}_{0t} \\
&= \delta_1\sum_{t=1}^{T}\tilde{D}_{0t-1}\Delta\tilde{D}_{0t} + \sum_{t=1}^{T}\tilde{Z}_{1t-1}\Delta\tilde{D}_{0t} \\
&= \delta_1\sum_{t=1}^{T}\left[D_{0t-1} - T^{-1}\right]\Delta\tilde{D}_{0t} \\
&\quad + \sum_{t=1}^{T}\tilde{Z}_{1t-1}\left[\Delta D_{0t} - T^{-1}\sum_{t=1}^{T}\Delta D_{0t}\right] \\
&= \delta_1\sum_{t=1}^{T}\left[D_{0t-1} - T^{-1}][\Delta D_{0t} - T^{-1}\sum_{t=1}^{T}\Delta D_{0t}\right] \\
&\quad + \sum_{t=1}^{T}\left[Z_{1t-1} - T^{-1}\sum_{t=1}^{T}Z_{1t-1}\right]\left[\Delta D_{0t} - T^{-1}\sum_{t=1}^{T}\Delta D_{0t}\right] \\
&= \delta_1\sum_{t=1}^{T}D_{0t-1}\Delta D_{0t} - \delta_1 T^{-1}\sum_{t=1}^{T}\Delta D_{0t} + \sum_{t=1}^{T}Z_{1t-1}\Delta D_{0t} \\
&\quad - T^{-1}\sum_{t=1}^{T}Z_{1t-1}\sum_{t=1}^{T}\Delta D_{0t} \\
&= -\delta_1 + (Z_{1T_{AO-1}} - Z_{1T_{AO}}) = -\delta_1 - \Delta Z_{1T_{AO}}.
\end{aligned}$$

In a similar way we have

$$\sum_{t=1}^{T} \tilde{y}_{2t-1} \Delta \tilde{D}_{0t} = -\delta_2 - \Delta Z_{2T_{AO}}.$$

With last two terms, we have $\tilde{X}' \Delta \tilde{D}_0 - \delta - \Delta Z_{T_{AO}}$, which is equivalent to say that

$$\gamma_{00} \tilde{X}' \Delta \tilde{D}_0 = \gamma_{00} \left(-\delta - \Delta Z_{T_{AO}}\right).$$

3. The third term is

$$\tilde{X}' \tilde{e}_2 = \left(\sum_{t=1}^{T} \tilde{y}_{1t-1} \tilde{e}_{2t}, \sum_{t=1}^{T} \tilde{y}_{2t-1} \tilde{e}_{2t} \right)'$$

where $\tilde{e}_{2t} \approx e_{2t} - T^{-1} \sum e_{2t} = e_{2t}$. So we have

$$\begin{aligned}
\sum_{t=1}^{T} \tilde{y}_{1t-1} \tilde{e}_{2t} &= \sum_{t=1}^{T} \left[\delta_1 \tilde{D}_{0t-1} + \tilde{Z}_{1t-1} \right] e_{2t} \\
&= \delta_1 \sum_{t=1}^{T} \tilde{D}_{0t-1} e_{2t} + \sum_{t=1}^{T} \tilde{Z}_{1t-1} e_{2t} \\
&= \delta_1 \sum_{t=1}^{T} \left[D_{0t-1} - T^{-1} \right] e_{2t} \\
&\quad + \sum_{t=1}^{T} \left[Z_{1t-1} - T^{-1} \sum_{t=1}^{T} Z_{1t-1} \right] e_{2t} \\
&= \delta_1 \sum_{t=1}^{T} D_{0t-1} e_{2t} - \delta_1 T^{-1} \sum_{t=1}^{T} e_{2t} + \sum_{t=1}^{T} Z_{1t-1} e_{2t} \\
&\quad - T^{-1} \sum_{t=1}^{T} Z_{1t-1} \sum_{t=1}^{T} e_{2t} \\
&= \delta_1 e_{2T_{AO}+1} + \sum_{t=1}^{T} Z_{1t-1} e_{2t}
\end{aligned}$$

In a similar way, we have that

$$\sum_{t=1}^{T} \tilde{y}_{2t-1} \tilde{e}_{2t} = \delta_2 e_{2T_{AO}+1} + \sum_{t=1}^{T} Z_{2t-1} e_{2t}.$$

which implies that

$$\tilde{X}'\tilde{e}_2 = \delta e_{2T_{AO}+1} + Z'e_2.$$

With last terms we have

$$\tilde{X}'\left[\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_2\right] = \gamma_0 Z_{T_{AO}-1} - \gamma_0 T^{-1}\delta - \gamma_0\bar{Z} - \gamma_{00}\delta - \gamma_{00}\Delta Z_{T_{AO}} \\ +\delta e_{2T_{AO}+1} + Z'e_2.$$

We conclude that

$$plim(\hat{a}_2 - a_2) = plim\left[(T\tilde{X}'\tilde{X})^{-1}T^{-1}\tilde{X}'[\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_2]\right] = 0.$$

Hence, the proof of first part of the Theorem is completed.
Now let us consider

$$\begin{aligned} T^{\frac{1}{2}}(\hat{a}_2 - a_2) &\approx \Gamma^{-1}T^{-\frac{1}{2}}\tilde{X}'\left[\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_2\right] \\ &\approx \Gamma^{-1}\left[T^{-\frac{1}{2}}\gamma_0 Z_{T_{AO}-1} - T^{-\frac{1}{2}}\gamma_0\bar{Z} - T^{-\frac{1}{2}}\gamma_{00}\delta - T^{-\frac{1}{2}}\gamma_{00}\Delta Z_{T_{AO}}\right] \\ &\quad +\Gamma^{-1}\left[T^{-\frac{1}{2}}\delta e_{2T_{AO}+1} + T^{-\frac{1}{2}}Z'e_2\right]. \end{aligned}$$

From last expression, we have that $P\lim\left[T^{\frac{1}{2}}(\hat{a}_2 - a_2)\right] = plim(T^{-\frac{1}{2}}Z'e_2)$.
We also know that $E\left[Z'e_2\right] = 0$ and

$$\begin{aligned} Var\left[Z'e_2\right] &= E\left[Z'e_2e_2'Z\right] \\ &= Z'E\left[e_2e_2'\right]Z \\ &= Z'\sigma_2^2 Z \\ &= \sigma_2^2\left[Z'Z\right] \\ &= \sigma_2^2\Gamma. \end{aligned}$$

we conclude that the limiting distribution of $T^{-\frac{1}{2}}Z'e_2$ is normal with variance $\sigma_2^2\Gamma$. So overall we have

$$T^{\frac{1}{2}}(\hat{a}_2 - a_2) \Rightarrow N[0, \sigma_2^2\Gamma].$$

It remains to prove that the variance matrix Γ of the Z's depends on the coefficient matrix A. From the DGP model $Z_t = AZ_{t-1} + e_t$. Then, it is possible to write that as $Z_t = A^t Z_0 + \sum_i^{t-1} A^i e_{t-i}$. Taking $Z_0 = 0$, we get $Z_t = \sum_i^{t-1} A^i e_{t-i}$. Also, $E[Z_t = 0]$ and

$$\begin{aligned} V[Z_t] &= V[\sum_i^{t-1} A^i e_{t-i}] \\ &= E\left[(\sum_i^{t-1} A^i e_{t-i})(\sum_i^{t-1} A^i e_{t-i})'\right] \\ &= \sum_i^{t-1} A^i \Omega A^{i\prime}. \end{aligned} \tag{A.12}$$

Hence, we conclude that the variance of the Z's depends on the parameter matrix A. The proof of Theorem 1 is thus completed.■

Proof of theorem 2. When $\delta_2 = 0$ the AO model (16) reduces to $y_{2t} = \mu_2^* + a_{21}y_{1t-1} + a_{22}y_{2t-1} + e_{2t}$. Therefore regression (17) is correctly specified and standard OLS results apply from which, part 1 of the theorem is derived. When $\delta_2 \neq 0$, we have from Theorem 3 that $T(\hat{a}_{21} - a_{21})^2 = [T^{\frac{1}{2}}(\hat{a}_2 - a_2)]^2$ converges to a χ^2 distribution and that $(T^{-1}\tilde{X}'\tilde{X})_{11}^{-1}$ converges to Γ_{11}^{-1}. We need to evaluate $s^2 = T^{-1}\hat{e}_2'\hat{e}_2$. Let $M = I - \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'$. We have

$$s^2 = T^{-1}\hat{e}_2'\hat{e}_2 = T^{-1}\tilde{Y}_2'M\tilde{Y}_2$$

where $\tilde{Y}_2 = \gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + a_2\tilde{X} + \tilde{e}_2$. Also, we have $\tilde{X}'M = M\tilde{X} = 0$, so that

$$\begin{aligned} s^2 &= T^{-1}\left[(\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_{2t})'M(\gamma_0\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0 + \tilde{e}_2)\right] \\ &= T^{-1}\left[\gamma_0^2\tilde{D}_0'M\tilde{D}_0 + \gamma_0\gamma_{00}\tilde{D}_0'M\Delta\tilde{D}_0 + \gamma_0\tilde{D}_0'M\tilde{e}_2\right] \\ &\quad + T^{-1}\left[\gamma_0\gamma_{00}\Delta\tilde{D}_0'M\tilde{D}_0 + \gamma_{00}^2\Delta\tilde{D}_0'M\Delta\tilde{D}_0 + \gamma_{00}\Delta\tilde{D}_0'M\tilde{e}_2\right] \\ &\quad + T^{-1}\left[\gamma_0\tilde{e}_2'M\tilde{D}_0 + \gamma_{00}\tilde{e}_2'M\Delta\tilde{D}_0 + \tilde{e}_2'M\tilde{e}_2\right]. \end{aligned}$$

From the results in Theorem 1, only the term $\tilde{e}_2'M\tilde{e}_2$ is non zero. So we have $s^2 = \tilde{e}_2'M\tilde{e}_2 \Rightarrow \sigma^2$, That is the denominator of GC converges to $\sigma^2\Gamma_{11}^{-1}$. Also, the numerator of GC converges to a χ^2 distribution. So overall $GC \Rightarrow \chi^2$. But this distribution is dependent to the covariance matrix Γ. So the limiting distribution is dependent to the parameter matrix A. The proof is completed.■

Table 1. Mean Estimates for $\hat{a}_{21}$, $\delta = (0,0)$, $a_{21} = 0$

a_{12}	a_{22}			
	-0.6	-0.2	0.2	0.6
	$T = 50$			
-0.6	-0.18	-0.75	0.20	0.06
-0.2	-0.10	-0.15	-0.59	0.15
0.2	-0.07	-0.08	-0.12	-0.45
0.6	-0.05	-0.06	-0.07	-0.09
	$T = 100$			
-0.6	-0.09	-0.37	0.09	0.03
-0.2	-0.05	-0.07	-0.27	0.06
0.2	-0.04	-0.04	-0.05	-0.19
0.6	-0.03	-0.03	-0.03	-0.04
	$T = 500$			
-0.6	-0.02	-0.07	0.02	0.00
-0.2	-0.01	-0.01	-0.05	0.01
0.2	-0.01	-0.01	-0.01	-0.03
0.6	-0.00	-0.01	-0.01	-0.01

Table 2. Mean Estimates of $\hat{a}_{21}$, $\delta = (0, 0)$, $a_{21} = 0.30$

a_{12}	a_{22}			
	-0.6	-0.2	0.2	0.6
		$T = 50$		
-0.6	0.04	0.53	0.35	0.31
-0.2	0.17	0.01	0.50	0.34
0.2	0.21	0.18	0.04	0.48
0.6	0.23	0.22	0.19	0.11
		$T = 100$		
-0.6	0.13	0.42	0.32	0.30
-0.2	0.23	0.16	0.39	0.32
0.2	0.26	0.24	0.18	0.37
0.6	0.26	0.26	0.25	0.21
		$T = 500$		
-0.6	0.27	0.32	0.31	0.30
-0.2	0.29	0.27	0.32	0.30
0.2	0.29	0.29	0.27	0.32
0.6	0.29	0.29	0.29	0.28

Table 3. Size of the F-Test, $\delta = (0, 0)$, $a_{21} = 0$

a_{12}	a_{22}			
	-0.6	-0.2	0.2	0.6
	$T = 50$			
-0.6	0.07	0.06	0.07	0.07
-0.2	0.07	0.06	0.07	0.07
0.2	0.07	0.06	0.07	0.07
0.6	0.07	0.06	0.07	0.07
	$T = 100$			
-0.6	0.07	0.07	0.06	0.05
-0.2	0.07	0.07	0.06	0.05
0.2	0.07	0.07	0.06	0.05
0.6	0.07	0.07	0.06	0.05
	$T = 500$			
-0.6	0.04	0.05	0.06	0.06
-0.2	0.04	0.05	0.06	0.06
0.2	0.04	0.05	0.06	0.06
0.6	0.04	0.05	0.06	0.06

Table 4. Power of the F-Test, $\delta = (0,0)$, $a_{21} = 0.30$

a_{12}	a_{22}			
	-0.6	-0.2	0.2	0.6
	$T = 50$			
-0.6	0.04	0.11	0.35	0.76
-0.2	0.12	0.04	0.11	0.40
0.2	0.31	0.11	0.04	0.13
0.6	0.57	0.35	0.13	0.04
	$T = 100$			
-0.6	0.06	0.14	0.57	0.95
-0.2	0.28	0.06	0.13	0.64
0.2	0.70	0.33	0.07	0.14
0.6	0.94	0.77	0.39	0.07
	$T = 500$			
-0.6	0.20	0.32	1.00	1.00
-0.2	0.96	0.24	0.36	1.00
0.2	1.00	0.98	0.28	0.43
0.6	1.00	1.00	1.00	0.33

Table 5. Mean Estimates of $\hat{a}_{21}$; $a_{21} = 0, T = 50$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5, 5)$			
-0.6	-0.29	-0.19	-0.07	0.05	0.12	0.12	0.08
-0.4	-0.21	-0.23	-0.16	0.02	0.19	0.23	0.13
-0.2	-0.05	-0.13	-0.16	-0.11	0.24	0.62	0.25
0.0	0.05	-0.02	-0.08	-0.13	-0.18	-0.27	-0.10
0.2	0.09	0.03	-0.02	-0.08	-0.16	-0.36	-1.02
0.4	0.10	0.05	0.00	-0.05	-0.12	-0.24	-0.45
0.6	0.10	0.06	0.01	-0.04	-0.10	-0.18	-0.29
				$\delta = (10, 10)$			
-0.6	-0.21	-0.10	-0.00	0.06	0.09	0.07	0.02
-0.4	-0.21	-0.17	-0.06	0.07	0.15	0.16	0.08
-0.2	0.01	-0.11	-0.13	-0.00	0.29	0.47	0.24
0.0	0.18	0.05	-0.05	-0.12	-0.16	-0.01	-0.69
0.2	0.24	0.13	0.03	-0.07	-0.21	-0.49	-0.99
0.4	0.25	0.16	0.07	-0.04	-0.16	-0.33	-0.54
0.6	0.23	0.16	0.07	-0.02	-0.13	-0.25	-0.38
				$\delta = (10, 0)$			
-0.6	-0.05	-0.03	0.01	0.05	0.06	0.05	0.04
-0.4	-0.04	-0.04	-0.03	-0.01	0.02	0.04	0.04
-0.2	-0.03	-0.03	-0.02	-0.02	-0.01	0.01	0.02
0.0	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.00
0.2	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01
0.4	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
0.6	-0.02	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
				$\delta = (0, 10)$			
-0.6	-0.33	-0.21	-0.09	0.02	0.13	0.22	0.26
-0.4	-0.38	-0.25	-0.12	0.01	0.15	0.28	0.35
-0.2	-0.37	-0.27	-0.14	-0.00	0.16	0.34	0.49
0.0	-0.28	-0.24	-0.15	-0.03	0.14	0.36	0.62
0.2	-0.14	-0.17	-0.14	-0.06	0.07	0.27	0.58
0.4	-0.04	-0.09	-0.10	-0.07	0.01	0.14	0.37
0.6	0.02	-0.04	-0.07	-0.07	-0.03	0.05	0.19

Table 6. Mean Estimates of $\hat{a}_{21}$ under Granger Causality, $a_{21} = -0.30$, $T = 50$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
	$\delta = (5,5)$						
-0.6	-0.41	-0.50	-0.52	-0.43	-0.25	-0.12	-0.15
-0.4	-0.18	-0.29	-0.39	-0.49	-0.56	0.02	0.12
-0.2	-0.09	-0.17	-0.24	-0.33	-0.44	-0.66	-0.94
0.0	-0.07	-0.13	-0.20	-0.26	-0.34	-0.44	-0.59
0.2	-0.07	-0.13	-0.18	-0.24	-0.31	-0.39	-0.48
0.4	-0.08	-0.13	-0.19	-0.24	-0.30	-0.36	-0.43
0.6	-0.10	-0.15	-0.19	-0.24	-0.29	-0.34	-0.39
	$\delta = (10,10)$						
-0.6	-0.33	-0.39	-0.34	-0.26	-0.20	-0.17	-0.23
-0.4	0.08	-0.13	-0.29	-0.38	-0.31	0.01	0.08
-0.2	0.17	0.03	-0.11	-0.24	-0.41	-0.66	-0.85
0.0	0.15	0.04	-0.07	-0.18	-0.30	-0.43	-0.57
0.2	0.12	0.03	-0.07	-0.17	-0.27	-0.38	-0.50
0.4	0.09	0.01	-0.08	-0.17	-0.26	-0.36	-0.45
0.6	0.06	-0.02	-0.10	-0.18	-0.27	-0.35	-0.42
	$\delta = (10,0)$						
-0.6	-0.05	-0.03	0.00	0.03	0.03	-0.02	-0.10
-0.4	-0.08	-0.07	-0.05	-0.02	0.00	0.02	0.01
-0.2	-0.10	-0.10	-0.09	-0.08	-0.06	-0.05	-0.03
0.0	-0.13	-0.13	-0.12	-0.12	-0.11	-0.10	-0.09
0.2	-0.15	-0.16	-0.16	-0.16	-0.15	-0.15	-0.14
0.4	-0.19	-0.20	-0.20	-0.20	-0.20	-0.19	-0.18
0.6	-0.24	-0.24	-0.25	-0.24	-0.24	-0.23	-0.22
	$\delta = (0,10)$						
-0.6	-0.72	-0.56	-0.42	-0.28	-0.13	-0.01	-0.00
-0.4	-0.72	-0.59	-0.46	-0.30	-0.12	0.08	0.22
-0.2	-0.57	-0.54	-0.47	-0.35	-0.19	0.04	0.32
0.0	-0.37	-0.42	-0.42	-0.38	-0.30	-0.16	0.05
0.2	-0.24	-0.32	-0.36	-0.38	-0.36	-0.32	-0.25
0.4	-0.18	-0.26	-0.32	-0.36	-0.38	-0.39	-0.38
0.6	-0.16	-0.24	-0.30	-0.34	-0.38	-0.40	-0.41

Table 7. Mean Estimates of $\hat{a}_{21}$ under Granger Causality, $a_{21} = 0.30$, $T = 50$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	0.18	0.29	0.37	0.43	0.44	0.41	0.36
-0.4	0.13	0.25	0.39	0.49	0.53	0.49	0.40
-0.2	0.13	0.20	0.37	0.59	0.72	0.66	0.47
0.0	0.18	0.18	0.28	0.56	1.07	1.13	0.58
0.2	0.24	0.20	0.21	0.31	0.76	3.52	0.55
0.4	0.28	0.23	0.20	0.18	0.15	-0.29	-3.83
0.6	0.30	0.26	0.22	0.16	0.06	-0.22	-0.95
				$\delta = (10,10)$			
-0.6	0.21	0.31	0.38	0.42	0.42	0.39	0.33
-0.4	0.18	0.30	0.41	0.48	0.50	0.47	0.38
-0.2	0.16	0.27	0.43	0.59	0.68	0.65	0.47
0.0	0.20	0.21	0.36	0.66	1.06	1.17	0.69
0.2	0.26	0.21	0.24	0.43	1.08	3.45	0.60
0.4	0.33	0.25	0.20	0.18	0.15	-0.73	-3.77
0.6	0.38	0.29	0.21	0.11	-0.10	-0.62	-1.29
				$\delta = (10,0)$			
-0.6	-0.39	0.45	0.40	0.31	0.26	0.24	0.23
-0.4	-0.16	-0.45	-0.40	0.05	0.15	0.17	0.19
-0.2	-0.04	-0.13	-0.27	-0.25	-0.08	0.04	0.11
0.0	0.02	-0.02	-0.08	-0.13	-0.11	-0.05	0.02
0.2	0.06	0.04	0.01	-0.02	-0.03	-0.03	-0.01
0.4	0.10	0.08	0.07	0.05	0.04	0.03	0.02
0.6	0.13	0.12	0.11	0.10	0.09	0.08	0.07
				$\delta = (0,10)$			
-0.6	0.02	0.12	0.22	0.31	0.41	0.48	0.52
-0.4	-0.02	0.09	0.20	0.31	0.43	0.53	0.59
-0.2	-0.05	0.07	0.18	0.31	0.45	0.58	0.69
0.0	-0.04	0.05	0.16	0.30	0.45	0.63	0.80
0.2	0.01	0.06	0.15	0.27	0.44	0.64	0.90
0.4	0.10	0.09	0.15	0.25	0.39	0.60	0.89
0.6	0.19	0.14	0.15	0.22	0.34	0.53	0.80

Table 8. Size of the F-Test, $a_{21} = 0, T = 50$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5, 5)$			
-0.6	0.11	0.05	0.02	0.02	0.03	0.06	0.06
-0.4	0.10	0.09	0.05	0.02	0.04	0.09	0.07
-0.2	0.05	0.07	0.07	0.05	0.05	0.13	0.09
0.0	0.05	0.04	0.06	0.06	0.06	0.05	0.04
0.2	0.07	0.04	0.03	0.05	0.10	0.15	0.45
0.4	0.14	0.06	0.03	0.05	0.12	0.21	0.48
0.6	0.21	0.08	0.03	0.06	0.13	0.28	0.55
				$\delta = (10, 10)$			
-0.6	0.03	0.01	0.00	0.00	0.01	0.01	0.01
-0.4	0.06	0.04	0.02	0.01	0.01	0.02	0.02
-0.2	0.05	0.05	0.05	0.04	0.03	0.04	0.04
0.0	0.11	0.04	0.05	0.06	0.06	0.04	0.06
0.2	0.31	0.11	0.04	0.05	0.08	0.20	0.60
0.4	0.65	0.28	0.06	0.05	0.15	0.47	0.83
0.6	0.90	0.54	0.14	0.05	0.26	0.72	0.97
				$\delta = (10, 0)$			
-0.6	0.02	0.04	0.03	0.03	0.03	0.03	0.04
-0.4	0.04	0.03	0.04	0.04	0.04	0.04	0.04
-0.2	0.05	0.05	0.04	0.04	0.04	0.05	0.05
0.0	0.05	0.05	0.05	0.04	0.04	0.04	0.05
0.2	0.05	0.05	0.05	0.04	0.04	0.04	0.05
0.4	0.05	0.04	0.05	0.04	0.04	0.04	0.03
0.6	0.04	0.04	0.04	0.04	0.04	0.04	0.03
				$\delta = (0, 10)$			
-0.6	0.42	0.16	0.05	0.03	0.07	0.20	0.31
-0.4	0.40	0.17	0.06	0.03	0.07	0.22	0.38
-0.2	0.32	0.17	0.07	0.03	0.08	0.23	0.48
0.0	0.20	0.15	0.08	0.03	0.06	0.21	0.53
0.2	0.11	0.11	0.07	0.04	0.04	0.12	0.44
0.4	0.04	0.07	0.06	0.04	0.02	0.06	0.24
0.6	0.04	0.04	0.06	0.05	0.03	0.03	0.10

Table 9. Power of the F-Test, $a_{21} = -0.30$, $T = 50$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5, 5)$			
-0.6	0.25	0.29	0.25	0.15	0.06	0.04	0.24
-0.4	0.09	0.15	0.20	0.19	0.11	0.02	0.06
-0.2	0.04	0.10	0.15	0.19	0.21	0.21	0.21
0.0	0.04	0.10	0.20	0.30	0.42	0.52	0.63
0.2	0.06	0.15	0.31	0.50	0.68	0.82	0.92
0.4	0.11	0.26	0.50	0.72	0.90	0.97	0.99
0.6	0.19	0.42	0.68	0.90	0.97	0.99	1.00
				$\delta = (10, 10)$			
-0.6	0.09	0.09	0.05	0.03	0.01	0.01	0.32
-0.4	0.05	0.06	0.08	0.07	0.04	0.01	0.03
-0.2	0.11	0.03	0.05	0.09	0.13	0.16	0.26
0.0	0.17	0.04	0.04	0.13	0.35	0.66	0.91
0.2	0.23	0.03	0.07	0.35	0.77	0.98	1.00
0.4	0.23	0.03	0.20	0.71	0.98	1.00	1.00
0.6	0.14	0.06	0.46	0.95	1.00	1.00	1.00
				$\delta = (10, 0)$			
-0.6	0.06	0.04	0.04	0.04	0.05	0.04	0.25
-0.4	0.13	0.09	0.07	0.06	0.05	0.06	0.05
-0.2	0.21	0.19	0.17	0.14	0.10	0.08	0.06
0.0	0.27	0.27	0.26	0.24	0.23	0.20	0.17
0.2	0.30	0.33	0.34	0.36	0.35	0.34	0.31
0.4	0.34	0.40	0.43	0.46	0.47	0.48	0.48
0.6	0.42	0.47	0.54	0.58	0.61	0.64	0.70
				$\delta = (0, 10)$			
-0.6	0.88	0.70	0.44	0.18	0.06	0.03	0.06
-0.4	0.74	0.60	0.38	0.17	0.05	0.04	0.11
-0.2	0.51	0.47	0.35	0.18	0.06	0.03	0.12
0.0	0.29	0.35	0.33	0.24	0.12	0.03	0.01
0.2	0.18	0.29	0.34	0.34	0.26	0.15	0.05
0.4	0.14	0.28	0.40	0.46	0.49	0.46	0.36
0.6	0.15	0.33	0.49	0.62	0.72	0.78	0.79

Table 10. Power of the F-Test, $a_{21} = 0.30, T = 50$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5, 5)$			
-0.6	0.05	0.13	0.27	0.44	0.56	0.61	0.63
-0.4	0.05	0.08	0.18	0.34	0.48	0.54	0.51
-0.2	0.07	0.08	0.13	0.27	0.44	0.51	0.41
0.0	0.11	0.09	0.10	0.19	0.42	0.52	0.29
0.2	0.25	0.15	0.09	0.11	0.20	0.65	0.08
0.4	0.50	0.30	0.17	0.10	0.06	0.06	0.86
0.6	0.78	0.56	0.32	0.14	0.06	0.09	0.60
				$\delta = (10, 10)$			
-0.6	0.01	0.06	0.17	0.31	0.41	0.42	0.37
-0.4	0.02	0.05	0.11	0.21	0.30	0.33	0.24
-0.2	0.05	0.07	0.11	0.19	0.28	0.32	0.21
0.0	0.07	0.07	0.10	0.18	0.34	0.43	0.20
0.2	0.16	0.10	0.09	0.12	0.26	0.66	0.06
0.4	0.46	0.20	0.11	0.07	0.06	0.08	0.96
0.6	0.89	0.59	0.26	0.07	0.06	0.39	0.97
				$\delta = (10, 0)$			
-0.6	0.04	0.02	0.06	0.12	0.22	0.39	0.58
-0.4	0.04	0.11	0.07	0.01	0.04	0.13	0.29
-0.2	0.03	0.08	0.13	0.10	0.04	0.03	0.09
0.0	0.05	0.04	0.07	0.10	0.10	0.05	0.04
0.2	0.11	0.06	0.04	0.04	0.06	0.06	0.04
0.4	0.25	0.19	0.13	0.09	0.07	0.06	0.06
0.6	0.42	0.37	0.32	0.26	0.19	0.14	0.11
				$\delta = (0, 10)$			
-0.6	0.02	0.08	0.28	0.58	0.82	0.94	0.98
-0.4	0.03	0.04	0.18	0.43	0.73	0.89	0.96
-0.2	0.04	0.04	0.12	0.32	0.61	0.86	0.94
0.0	0.05	0.04	0.10	0.24	0.53	0.81	0.94
0.2	0.06	0.06	0.09	0.21	0.45	0.75	0.94
0.4	0.08	0.07	0.10	0.17	0.40	0.71	0.93
0.6	0.18	0.11	0.12	0.19	0.37	0.68	0.92

Table 11. Mean Estimates of $\hat{a}_{21}$, $a_{21} = 0, T = 100$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	-0.27	-0.22	-0.11	0.02	0.10	0.09	0.04
-0.4	-0.15	-0.19	-0.16	-0.02	0.17	0.18	0.07
-0.2	-0.03	-0.08	-0.11	-0.09	0.19	0.54	0.12
0.0	0.03	-0.01	-0.04	-0.07	-0.09	-0.08	-0.16
0.2	0.05	0.02	-0.01	-0.04	-0.09	-0.20	-0.82
0.4	0.06	0.03	0.00	-0.03	-0.07	-0.14	-0.30
0.6	0.06	0.03	0.01	-0.02	-0.06	-0.11	-0.18
				$\delta = (10,10)$			
-0.6	-0.28	-0.18	-0.06	0.03	0.07	0.07	0.02
-0.4	-0.22	-0.23	-0.14	0.01	0.14	0.16	0.06
-0.2	0.00	-0.10	-0.13	-0.05	0.25	0.50	0.16
0.0	0.13	0.04	-0.03	-0.08	-0.08	0.16	-0.71
0.2	0.18	0.10	0.03	-0.04	-0.14	-0.36	-0.99
0.4	0.19	0.12	0.05	-0.02	-0.12	-0.26	-0.48
0.6	0.18	0.12	0.06	-0.01	-0.10	-0.20	-0.32
				$\delta = (10,0)$			
-0.6	-0.03	-0.03	0.00	0.04	0.04	0.03	0.02
-0.4	-0.03	-0.02	-0.02	-0.00	0.02	0.03	0.03
-0.2	-0.02	-0.02	-0.02	-0.01	-0.00	0.01	0.02
0.0	-0.02	-0.02	-0.01	-0.01	-0.01	-0.00	-0.01
0.2	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.00
0.4	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
0.6	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
				$\delta = (0,10)$			
-0.6	-0.35	-0.23	-0.11	0.00	0.11	0.19	0.20
-0.4	-0.36	-0.25	-0.14	-0.00	0.14	0.26	0.29
-0.2	-0.31	-0.24	-0.15	-0.02	0.15	0.33	0.44
0.0	- 0.20	-0.19	-0.13	-0.03	0.12	0.35	0.61
0.2	-0.09	-0.12	-0.10	-0.04	0.07	0.26	0.59
0.4	-0.02	-0.06	-0.07	-0.04	0.02	0.15	0.38
0.6	0.03	-0.02	-0.04	-0.04	-0.00	0.07	0.21

Table 12. Mean Estimates of $\hat{a}_{21}$ under Granger Causality, $a_{21} = -0.30, T = 100$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	-0.39	-0.46	-0.50	-0.48	-0.32	-0.19	-0.22
-0.4	-0.24	-0.30	-0.36	-0.42	-0.49	-0.08	-0.11
-0.2	-0.18	-0.23	-0.27	-0.31	-0.37	-0.50	-0.87
0.0	-0.16	-0.20	-0.24	-0.28	-0.32	-0.39	-0.51
0.2	-0.16	-0.19	-0.23	-0.27	-0.31	-0.35	-0.42
0.4	-0.16	-0.20	-0.23	-0.26	-0.30	-0.34	-0.38
0.6	-0.17	-0.20	-0.23	-0.26	-0.30	-0.33	-0.36
				$\delta = (10,10)$			
-0.6	-0.39	-0.47	-0.46	-0.38	-0.28	-0.23	-0.26
-0.4	-0.04	-0.19	-0.31	-0.41	-0.42	-0.11	-0.11
-0.2	0.06	-0.05	-0.15	-0.24	-0.34	-0.52	-0.79
0.0	0.06	-0.03	-0.11	-0.19	-0.29	-0.40	-0.54
0.2	0.04	-0.04	-0.11	-0.19	-0.27	-0.37	-0.47
0.4	0.01	-0.05	-0.12	-0.19	-0.27	-0.35	-0.43
0.6	-0.01	-0.07	-0.14	-0.20	-0.27	-0.34	-0.40
				$\delta = (10,0)$			
-0.6	-0.10	-0.07	-0.03	0.02	0.03	-0.05	-0.15
-0.4	-0.13	-0.11	-0.08	-0.05	-0.01	0.03	-0.02
-0.2	-0.15	-0.14	-0.13	-0.11	-0.08	-0.05	-0.03
0.0	-0.17	-0.16	-0.16	-0.15	-0.13	-0.12	-0.10
0.2	-0.19	-0.19	-0.19	-0.18	-0.17	-0.16	-0.15
0.4	-0.22	-0.22	-0.21	-0.21	-0.21	-0.20	-0.19
0.6	-0.25	-0.25	-0.24	-0.24	-0.23	-0.23	-0.22
				$\delta = (0,10)$			
-0.6	-0.68	-0.56	-0.44	-0.30	-0.16	-0.04	-0.11
-0.4	-0.62	-0.55	-0.45	-0.32	-0.15	0.06	0.15
-0.2	-0.48	-0.47	-0.43	-0.34	-0.20	0.02	0.32
0.0	-0.34	-0.37	-0.38	-0.35	-0.28	-0.15	0.08
0.2	-0.25	-0.31	-0.34	-0.34	-0.33	-0.28	-0.19
0.4	-0.21	-0.27	-0.31	-0.33	-0.34	-0.34	-0.31
0.6	-0.19	-0.25	-0.29	-0.32	-0.35	-0.36	-0.35

Table 13. Mean Estimates of $\hat{a}_{21}$ under Granger Causality, $a_{21} = 0.30, T = 100$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	0.15	0.26	0.35	0.41	0.42	0.39	0.34
-0.4	0.13	0.22	0.36	0.48	0.50	0.45	0.36
-0.2	0.17	0.20	0.34	0.57	0.69	0.57	0.39
0.0	0.23	0.22	0.28	0.52	1.06	0.94	0.44
0.2	0.27	0.25	0.25	0.32	0.65	3.55	0.36
0.4	0.29	0.26	0.25	0.25	0.24	0.04	-3.93
0.6	0.30	0.27	0.25	0.23	0.18	0.02	-0.62
				$\delta = (10,10)$			
-0.6	0.17	0.27	0.35	0.41	0.42	0.40	0.34
-0.4	0.13	0.25	0.37	0.47	0.50	0.47	0.38
-0.2	0.14	0.21	0.37	0.56	0.69	0.65	0.45
0.0	0.21	0.21	0.33	0.60	1.04	1.16	0.58
0.2	0.28	0.24	0.27	0.43	1.00	3.50	0.31
0.4	0.33	0.27	0.25	0.25	0.26	-0.25	-3.91
0.6	0.35	0.30	0.25	0.18	0.05	-0.35	-1.17
				$\delta = (10,0)$			
0.6	-0.26	0.12	0.32	0.28	0.26	0.25	0.25
-0.4	-0.06	-0.34	-0.44	0.02	0.15	0.19	0.21
-0.2	0.04	-0.06	-0.02	-0.25	-0.07	0.07	0.14
0.0	0.09	0.04	-0.03	-0.10	-0.11	-0.04	0.05
0.2	0.13	0.10	0.06	0.02	-0.02	-0.03	0.00
0.4	0.16	0.14	0.12	0.09	0.06	0.04	0.03
0.6	0.19	0.17	0.16	0.14	0.12	0.10	0.08
				$\delta = (0,10)$			
-0.6	-0.00	0.10	0.20	0.30	0.39	0.46	0.48
-0.4	-0.03	0.07	0.19	0.30	0.41	0.50	0.54
-0.2	-0.04	0.06	0.17	0.30	0.43	0.56	0.63
0.0	-0.00	0.06	0.16	0.29	0.44	0.62	0.76
0.2	0.07	0.10	0.16	0.27	0.43	3.64	0.89
0.4	0.16	0.14	0.18	0.26	0.40	0.62	0.93
0.6	0.23	0.19	0.20	0.26	0.37	0.55	0.85

Table 14. Size of the F-Test, $a_{21} = 0$, $T = 100$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	0.21	0.13	0.04	0.01	0.06	0.09	0.06
-0.4	0.12	0.13	0.07	0.02	0.06	0.12	0.07
-0.2	0.05	0.07	0.08	0.05	0.06	0.19	0.07
0.0	0.06	0.05	0.06	0.06	0.06	0.04	0.05
0.2	0.10	0.07	0.06	0.06	0.07	0.12	0.45
0.4	0.12	0.08	0.05	0.06	0.09	0.17	0.39
0.6	0.15	0.08	0.05	0.06	0.10	0.21	0.40
				$\delta = (10,10)$			
-0.6	0.16	0.05	0.01	0.00	0.02	0.03	0.03
-0.4	0.13	0.10	0.03	0.01	0.03	0.07	0.05
-0.2	0.06	0.07	0.06	0.04	0.05	0.17	0.08
0.0	0.17	0.07	0.05	0.06	0.06	0.05	0.14
0.2	0.40	0.15	0.05	0.06	0.12	0.29	0.85
0.4	0.72	0.31	0.09	0.06	0.20	0.52	0.92
0.6	0.90	0.55	0.14	0.06	0.28	0.71	0.97
				$\delta = (10,0)$			
-0.6	0.04	0.04	0.03	0.03	0.03	0.04	0.04
-0.4	0.05	0.05	0.05	0.04	0.04	0.05	0.05
-0.2	0.06	0.06	0.06	0.06	0.05	0.05	0.05
0.0	0.06	0.06	0.06	0.06	0.06	0.05	0.05
0.2	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.4	0.05	0.04	0.04	0.05	0.05	0.04	0.05
0.6	0.05	0.05	0.04	0.04	0.03	0.04	0.04
				$\delta = (0,10)$			
-0.6	0.84	0.45	0.11	0.02	0.14	0.41	0.50
-0.4	0.74	0.43	0.12	0.02	0.14	0.47	0.63
-0.2	0.55	0.35	0.14	0.03	0.12	0.50	0.78
0.0	0.29	0.24	0.12	0.04	0.10	0.45	0.90
0.2	0.12	0.13	0.10	0.05	0.06	0.27	0.85
0.4	0.06	0.08	0.08	0.05	0.06	0.15	0.56
0.6	0.07	0.05	0.06	0.05	0.05	0.08	0.30

Table 15. Power of the F-Test, $a_{21} = -0.30, T = 100$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5, 5)$			
-0.6	0.51	0.56	0.53	0.36	0.15	0.19	0.66
-0.4	0.30	0.39	0.41	0.35	0.19	0.02	0.13
-0.2	0.28	0.38	0.43	0.44	0.38	0.29	0.22
0.0	0.33	0.48	0.59	0.66	0.70	0.70	0.71
0.2	0.47	0.65	0.79	0.88	0.92	0.94	0.96
0.4	0.64	0.83	0.94	0.97	0.99	1.00	1.00
0.6	0.82	0.95	0.98	0.99	1.00	1.00	1.00
				$\delta = (10, 10)$			
-0.6	0.34	0.46	0.41	0.26	0.14	0.24	0.79
-0.4	0.06	0.13	0.22	0.23	0.11	0.01	0.12
-0.2	0.05	0.04	0.11	0.20	0.26	0.28	0.39
0.0	0.06	0.03	0.14	0.36	0.63	0.86	0.98
0.2	0.04	0.05	0.28	0.71	0.96	1.00	1.00
0.4	0.03	0.12	0.58	0.95	1.00	1.00	1.00
0.6	0.04	0.33	0.86	1.00	1.00	1.00	1.00
				$\delta = (10, 0)$			
-0.6	0.14	0.08	0.06	0.05	0.05	0.07	0.64
-0.4	0.28	0.21	0.13	0.08	0.06	0.06	0.06
-0.2	0.48	0.41	0.33	0.23	0.15	0.09	0.07
0.0	0.64	0.60	0.55	0.48	0.41	0.31	0.20
0.2	0.75	0.73	0.71	0.69	0.66	0.59	0.51
0.4	0.82	0.83	0.83	0.82	0.82	0.81	0.78
0.6	0.88	0.90	0.92	0.93	0.93	0.94	0.94
				$\delta = (0, 10)$			
-0.6	1.00	0.97	0.87	0.50	0.14	0.03	0.25
-0.4	0.95	0.92	0.79	0.45	0.09	0.03	0.12
-0.2	0.82	0.81	0.74	0.49	0.15	0.03	0.29
0.0	0.64	0.73	0.72	0.62	0.36	0.07	0.03
0.2	0.55	0.70	0.76	0.75	0.67	0.42	0.12
0.4	0.54	0.75	0.84	0.88	0.89	0.84	0.70
0.6	0.59	0.83	0.91	0.96	0.98	0.98	0.98

Table 16. Power of the F-Test, $a_{21} = 0.30$, $T = 100$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	0.08	0.24	0.51	0.71	0.84	0.89	0.90
-0.4	0.08	0.14	0.33	0.59	0.75	0.80	0.82
-0.2	0.14	0.14	0.21	0.47	0.68	0.71	0.65
0.0	0.32	0.22	0.21	0.32	0.65	0.67	0.44
0.2	0.60	0.41	0.29	0.26	0.30	0.85	0.10
0.4	0.86	0.67	0.48	0.30	0.16	0.05	0.96
0.6	0.98	0.90	0.73	0.49	0.20	0.05	0.45
				$\delta = (10,10)$			
-0.6	0.05	0.23	0.52	0.70	0.82	0.86	0.82
-0.4	0.04	0.12	0.34	0.59	0.73	0.77	0.68
-0.2	0.08	0.11	0.22	0.48	0.71	0.73	0.57
0.0	0.19	0.14	0.19	0.40	0.76	0.83	0.43
0.2	0.45	0.27	0.23	0.29	0.55	0.97	0.07
0.4	0.85	0.55	0.34	0.23	0.15	0.07	1.00
0.6	1.00	0.92	0.64	0.28	0.06	0.33	1.00
				$\delta = (10,0)$			
-0.6	0.05	0.02	0.10	0.27	0.49	0.73	0.89
-0.4	0.05	0.13	0.13	0.03	0.11	0.33	0.65
-0.2	0.05	0.06	0.14	0.15	0.06	0.06	0.26
0.0	0.16	0.07	0.06	0.10	0.12	0.06	0.07
0.2	0.41	0.22	0.09	0.06	0.06	0.07	0.05
0.4	0.68	0.53	0.35	0.20	0.11	0.07	0.06
0.6	0.84	0.79	0.71	0.56	0.40	0.26	0.15
				$\delta = (0,10)$			
-0.6	0.02	0.13	0.53	0.87	0.98	1.00	1.00
-0.4	0.04	0.06	0.34	0.75	0.96	0.99	1.00
-0.2	0.04	0.05	0.23	0.62	0.92	0.99	1.00
0.0	0.05	0.06	0.19	0.51	0.89	0.99	1.00
0.2	0.10	0.11	0.20	0.46	0.83	0.98	1.00
0.4	0.30	0.23	0.29	0.48	0.80	0.98	1.00
0.6	0.56	0.41	0.41	0.57	0.81	0.98	1.00

Table 17. Mean Estimates of $\hat{a}_{21}$ when $a_{21} = 0$, $T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (10, 10)$			
-0.6	-0.22	-0.20	-0.11	0.01	0.07	0.06	0.01
-0.4	-0.09	-0.13	-0.12	-0.00	0.15	0.12	0.03
-0.2	0.00	-0.03	-0.04	-0.02	0.24	0.44	0.04
0.0	0.05	0.02	-0.00	-0.01	-0.00	0.12	-0.29
0.2	0.06	0.03	0.01	-0.01	-0.04	-0.11	-0.67
0.4	0.06	0.04	0.02	-0.01	-0.04	-0.09	-0.22
0.6	0.06	0.04	0.02	-0.00	-0.03	-0.07	-0.14
				$\delta = (10, 0)$			
-0.6	-0.01	-0.02	-0.01	0.01	0.01	0.01	0.00
-0.4	-0.01	-0.01	-0.01	-0.01	0.01	0.01	0.01
-0.2	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.01
0.0	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00
0.2	-0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.00
0.4	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
0.6	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
				$\delta = (0, 10)$			
-0.6	-0.27	-0.21	-0.12	0.00	0.10	0.11	0.07
-0.4	-0.20	-0.17	-0.11	-0.00	0.14	0.19	0.13
-0.2	-0.12	-0.12	-0.08	-0.01	0.15	0.32	0.25
0.0	-0.06	-0.07	-0.05	-0.01	0.10	0.34	0.56
0.2	-0.02	-0.03	-0.03	-0.01	0.05	0.18	0.55
0.4	-0.00	-0.01	-0.02	-0.01	0.02	0.08	0.23
0.6	0.01	-0.00	-0.01	-0.01	-0.01	0.14	0.10

Table 18. Mean Estimates of $\hat{a}_{21}$ under Granger Causality, $a_{21} = -0.30, T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (10, 10)$			
-0.6	-0.34	-0.39	-0.43	-0.44	-0.34	-0.27	-0.30
-0.4	-0.22	-0.26	-0.30	-0.33	-0.35	0.13	-0.27
-0.2	-0.18	-0.21	-0.24	-0.27	-0.30	-0.36	-0.82
0.0	-0.17	-0.20	-0.23	-0.26	-0.29	-0.33	-0.43
0.2	-0.17	-0.20	-0.23	-0.26	-0.29	-0.33	-0.38
0.4	-0.18	-0.20	-0.23	-0.26	-0.29	-0.32	-0.36
0.6	-0.19	-0.21	-0.24	-0.26	-0.29	-0.32	-0.34
				$\delta = (10, 0)$			
-0.6	-0.22	-0.20	-0.16	-0.07	0.02	-0.16	-0.26
-0.4	-0.24	-0.22	-0.20	-0.17	-0.09	0.02	-0.16
-0.2	-0.25	-0.24	-0.23	-0.21	-0.18	-0.12	-0.03
0.0	-0.26	-0.25	-0.25	-0.24	-0.22	-0.20	-0.15
0.2	-0.27	-0.26	-0.26	-0.25	-0.24	-0.23	-0.21
0.4	-0.27	-0.27	-0.27	-0.26	-0.26	-0.25	-0.25
0.6	-0.28	-0.28	-0.28	-0.27	-0.27	-0.27	-0.26
				$\delta = (0, 10)$			
-0.6	-0.48	-0.45	-0.40	-0.30	-0.16	-0.11	-0.27
-0.4	-0.40	-0.39	-0.37	-0.31	-0.17	0.06	-0.06
-0.2	-0.34	-0.35	-0.34	-0.31	-0.24	-0.06	0.35
0.0	-0.31	-0.32	-0.32	-0.31	-0.28	-0.22	-0.03
0.2	-0.28	-0.30	-0.31	-0.31	-0.30	-0.28	-0.23
0.4	-0.27	-0.29	-0.30	-0.31	-0.31	-0.30	-0.29
0.6	-0.26	-0.28	-0.29	-0.30	-0.31	-0.31	-0.31

Table 19. Mean Estimates of $\hat{a}_{21}$ under Granger Causality, $a_{21} = 0.30$, $T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (10, 10)$			
-0.6	0.15	0.25	0.35	0.40	0.40	0.37	0.33
-0.4	0.16	0.24	0.37	0.47	0.48	0.41	0.34
-0.2	0.22	0.25	0.37	0.58	0.66	0.51	0.36
0.0	0.27	0.27	0.33	0.55	1.09	0.84	0.38
0.2	0.30	0.29	0.30	0.37	0.68	3.64	0.26
0.4	0.31	0.30	0.29	0.29	0.31	0.19	-4.01
0.6	0.32	0.30	0.29	0.27	0.24	0.12	-0.48
				$\delta = (10, 0)$			
-0.6	0.09	-0.28	0.24	0.27	0.27	0.28	0.28
-0.4	0.17	-0.01	-0.51	0.09	0.22	0.25	0.27
-0.2	0.21	0.15	-0.03	-0.28	0.04	0.19	0.24
0.0	0.23	0.20	0.14	-0.00	-0.13	0.04	0.18
0.2	0.25	0.23	0.20	0.14	0.04	-0.03	0.06
0.4	0.26	0.25	0.23	0.20	0.15	0.08	0.03
0.6	0.27	0.26	0.24	0.23	0.20	0.16	0.11
				$\delta = (0, 10)$			
-0.6	-0.00	0.09	0.21	0.30	0.36	0.38	0.36
-0.4	0.02	0.08	0.18	0.30	0.39	0.41	0.39
-0.2	0.09	0.11	0.18	0.30	0.43	0.48	0.45
0.0	0.16	0.16	0.20	0.30	0.45	0.58	0.56
0.2	0.22	0.21	0.23	0.29	0.43	0.66	0.78
0.4	0.26	0.25	0.25	0.29	0.38	0.59	0.96
0.6	0.28	0.27	0.27	0.29	0.35	0.47	0.76

Table 20: Size of the F-Test when $a_{21} = 0$ and $T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (50, 50)$			
-0.6	0.62	0.04	0.00	0.00	0.00	0.00	0.18
-0.4	0.28	0.10	0.01	0.00	0.00	0.00	0.06
-0.2	0.05	0.06	0.05	0.01	0.02	0.04	0.01
0.0	0.80	0.16	0.04	0.04	0.04	0.18	0.79
0.2	1.00	0.85	0.20	0.03	0.21	0.85	1.00
0.4	1.00	1.00	0.75	0.03	0.79	1.00	1.00
0.6	1.00	1.00	1.00	0.03	1.00	1.00	1.00
				$\delta = (50, 0)$			
-0.6	0.01	0.01	0.01	0.00	0.00	0.01	0.01
-0.4	0.02	0.02	0.02	0.02	0.01	0.01	0.02
-0.2	0.03	0.03	0.02	0.02	0.03	0.04	0.04
0.0	0.05	0.05	0.04	0.04	0.04	0.05	0.05
0.2	0.04	0.03	0.03	0.03	0.03	0.03	0.03
0.4	0.02	0.02	0.02	0.02	0.02	0.01	0.01
0.6	0.03	0.03	0.03	0.02	0.01	0.01	0.01
				$\delta = (0, 50)$			
-0.6	1.00	0.91	0.29	0.02	0.28	0.82	0.97
-0.4	1.00	0.89	0.32	0.02	0.33	0.91	1.00
-0.2	0.99	0.82	0.33	0.03	0.35	0.94	1.00
0.0	0.82	0.62	0.26	0.04	0.33	0.95	1.00
0.2	0.20	0.28	0.15	0.03	0.19	0.89	1.00
0.4	0.04	0.06	0.07	0.02	0.09	0.67	1.00
0.6	0.20	0.02	0.04	0.02	0.04	0.34	0.97

Table 21. Power of the F-Test, $a_{21} = -0.30, T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (50, 50)$			
-0.6	0.83	0.95	0.95	0.97	1.00	1.00	1.00
-0.4	0.35	0.04	0.22	0.31	0.17	0.08	0.86
-0.2	0.99	0.33	0.02	0.14	0.39	0.74	1.00
0.0	1.00	0.77	0.01	0.58	1.00	1.00	1.00
0.2	1.00	0.95	0.04	1.00	1.00	1.00	1.00
0.4	1.00	0.98	0.54	1.00	1.00	1.00	1.00
0.6	1.00	0.68	1.00	1.00	1.00	1.00	1.00
				$\delta = (50, 0)$			
-0.6	0.03	0.10	0.16	0.11	0.02	0.88	1.00
-0.4	0.19	0.10	0.04	0.07	0.14	0.18	0.06
-0.2	0.80	0.77	0.70	0.62	0.52	0.40	0.33
0.0	0.93	0.96	0.97	0.98	0.98	0.98	0.96
0.2	0.92	0.97	0.98	1.00	1.00	1.00	1.00
0.4	0.91	0.97	0.99	1.00	1.00	1.00	1.00
0.6	0.96	0.99	1.00	1.00	1.00	1.00	1.00
				$\delta = (0, 50)$			
-0.6	1.00	1.00	1.00	0.96	0.63	0.27	0.98
-0.4	1.00	1.00	1.00	0.83	0.23	0.05	0.29
-0.2	1.00	1.00	0.98	0.78	0.23	0.04	0.69
0.0	0.83	0.96	0.96	0.91	0.66	0.16	0.01
0.2	0.41	0.89	0.98	0.99	0.99	0.99	0.96
0.4	0.33	0.93	1.00	1.00	1.00	1.00	1.00
0.6	0.49	0.99	1.00	1.00	1.00	1.00	1.00

Table 22. Power of the F-Test, $a_{21} = 0.30, T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (50, 50)$			
-0.6	0.33	0.99	1.00	1.00	1.00	1.00	1.00
-0.4	0.03	0.51	0.96	1.00	1.00	1.00	0.99
-0.2	0.06	0.20	0.63	0.94	0.99	0.99	0.84
0.0	0.36	0.25	0.47	0.85	0.99	1.00	0.67
0.2	0.85	0.47	0.49	0.79	1.00	1.00	0.04
0.4	1.00	0.94	0.69	0.55	0.48	0.23	1.00
0.6	1.00	1.00	1.00	0.44	0.23	1.00	1.00
				$\delta = (50, 0)$			
-0.6	0.18	0.55	0.98	1.00	1.00	1.00	1.00
-0.4	0.25	0.74	0.31	0.02	0.37	0.91	1.00
-0.2	0.04	0.41	0.89	0.87	0.29	0.01	0.22
0.0	0.04	0.12	0.65	0.90	0.90	0.59	0.09
0.2	0.53	0.11	0.05	0.17	0.29	0.28	0.14
0.4	0.99	0.96	0.93	0.82	0.65	0.43	0.29
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00
				$\delta = (0, 50)$			
-0.6	0.01	0.64	1.00	1.00	1.00	1.00	1.00
-0.4	0.02	0.28	0.94	1.00	1.00	1.00	1.00
-0.2	0.07	0.12	0.75	1.00	1.00	1.00	1.00
0.0	0.08	0.08	0.57	0.97	1.00	1.00	1.00
0.2	0.04	0.11	0.50	0.95	1.00	1.00	1.00
0.4	0.28	0.30	0.62	0.95	1.00	1.00	1.00
0.6	0.88	0.66	0.82	0.98	1.00	1.00	1.00

Table 23. Power of the F-Test, $a_{21} = -0.30$, $T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	1.00	0.99	0.95	0.74	0.37	0.79	1.00
-0.4	1.00	1.00	0.99	0.93	0.54	0.04	0.85
-0.2	1.00	1.00	1.00	1.00	0.97	0.69	0.28
0.0	1.00	1.00	1.00	1.00	1.00	1.00	0.93
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00
				$\delta = (10,10)$			
-0.6	0.99	1.00	0.99	0.95	0.81	0.92	1.00
-0.4	0.94	0.97	0.96	0.90	0.57	0.04	0.81
-0.2	0.96	0.99	1.00	0.99	0.94	0.74	0.69
0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00
				$\delta = (10,0)$			
-0.6	0.98	0.85	0.46	0.08	0.04	0.55	1.00
-0.4	1.00	1.00	0.95	0.67	0.14	0.05	0.65
-0.2	1.00	1.00	0.99	0.85	0.30	0.06	1.00
0.0	1.00	1.00	1.00	1.00	1.00	0.97	0.66
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00
				$\delta = (0,10)$			
-0.6	1.00	1.00	1.00	0.96	0.43	0.33	1.00
-0.4	1.00	1.00	1.00	0.98	0.45	0.07	0.15
-0.2	1.00	1.00	1.00	1.00	0.89	0.07	0.77
0.0	1.00	1.00	1.00	1.00	1.00	0.87	0.04
0.2	1.00	1.00	1.00	1.00	1.00	1.00	0.96
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 24. Power of the F-Test, $a_{21} = 0.30$, $T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	0.27	0.49	0.89	0.99	1.00	1.00	1.00
-0.4	0.56	0.43	0.64	0.92	0.98	1.00	1.00
-0.2	0.92	0.73	0.58	0.79	0.92	0.97	1.00
0.0	1.00	0.96	0.82	0.66	0.88	0.82	0.92
0.2	1.00	1.00	0.98	0.84	0.60	0.95	0.36
0.4	1.00	1.00	1.00	0.99	0.81	0.25	0.98
0.6	1.00	1.00	1.00	1.00	0.99	0.72	0.05
				$\delta = (10,10)$			
-0.6	0.38	0.84	1.00	1.00	1.00	1.00	1.00
-0.4	0.39	0.64	0.95	1.00	1.00	1.00	1.00
-0.2	0.77	0.71	0.87	0.99	1.00	1.00	1.00
0.0	0.99	0.93	0.89	0.97	1.00	1.00	0.94
0.2	1.00	1.00	0.97	0.92	0.94	1.00	0.24
0.4	1.00	1.00	1.00	0.98	0.80	0.14	1.00
0.6	1.00	1.00	1.00	1.00	0.93	0.20	0.89
				$\delta = (10,0)$			
-0.6	0.05	0.07	0.24	0.82	0.99	1.00	1.00
-0.4	0.31	0.04	0.32	0.09	0.68	0.98	1.00
-0.2	0.83	0.31	0.05	0.27	0.07	0.63	0.98
0.0	0.99	0.87	0.35	0.05	0.14	0.08	0.67
0.2	1.00	1.00	0.90	0.45	0.06	0.06	0.11
0.4	1.00	1.00	1.00	0.95	0.62	0.14	0.06
0.6	1.00	1.00	1.00	1.00	0.99	0.80	0.35
				$\delta = (0,10)$			
-0.6	0.04	0.32	0.92	1.00	1.00	1.00	1.00
-0.4	0.06	0.22	0.76	1.00	1.00	1.00	1.00
-0.2	0.28	0.36	0.68	0.98	1.00	1.00	1.00
0.0	0.83	0.76	0.82	0.96	1.00	1.00	1.00
0.2	1.00	0.97	0.97	0.98	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 25. Size of the F-Test, $a_{21} = 0$, $T = 500$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$\delta = (5,5)$			
-0.6	0.19	0.20	0.10	0.04	0.08	0.08	0.06
-0.4	0.07	0.08	0.07	0.04	0.10	0.10	0.06
-0.2	0.04	0.04	0.05	0.05	0.08	0.15	0.06
0.0	0.05	0.04	0.04	0.05	0.06	0.05	0.07
0.2	0.06	0.03	0.04	0.05	0.07	0.08	0.24
0.4	0.07	0.04	0.04	0.05	0.07	0.09	0.16
0.6	0.07	0.04	0.03	0.05	0.07	0.10	0.15
				$\delta = (10,10)$			
-0.6	0.68	0.51	0.16	0.02	0.11	0.14	0.06
-0.4	0.18	0.23	0.14	0.03	0.18	0.24	0.07
-0.2	0.04	0.05	0.07	0.04	0.19	0.51	0.08
0.0	0.13	0.05	0.04	0.05	0.05	0.07	0.17
0.2	0.30	0.11	0.04	0.04	0.09	0.19	0.91
0.4	0.48	0.18	0.05	0.05	0.12	0.30	0.80
0.6	0.58	0.26	0.07	0.05	0.15	0.41	0.80
				$\delta = (10,0)$			
-0.6	0.04	0.03	0.04	0.04	0.06	0.06	0.06
-0.4	0.04	0.04	0.04	0.04	0.05	0.06	0.06
-0.2	0.04	0.04	0.04	0.05	0.04	0.06	0.06
0.0	0.04	0.04	0.04	0.04	0.05	0.04	0.06
0.2	0.04	0.04	0.04	0.04	0.05	0.04	0.04
0.4	0.04	0.04	0.04	0.04	0.04	0.04	0.04
0.6	0.04	0.04	0.04	0.04	0.03	0.04	0.03
				$\delta = (0,10)$			
-0.6	0.98	0.85	0.35	0.03	0.29	0.57	0.45
-0.4	0.85	0.71	0.31	0.03	0.38	0.75	0.64
-0.2	0.54	0.43	0.20	0.04	0.34	0.91	0.91
0.0	0.21	0.21	0.13	0.05	0.21	0.89	1.00
0.2	0.07	0.10	0.09	0.05	0.12	0.58	1.00
0.4	0.05	0.05	0.06	0.05	0.07	0.28	0.81
0.6	0.05	0.04	0.05	0.05	0.05	0.14	0.46

Table 26. Size of the F-Test, $a_{21} = 0$, Multiple outliers

$(\delta_1, \delta_2) = (0,10), (0,5), (0,5), (0,5)$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$T = 50$			
-0.6	0.18	0.05	0.03	0.06	0.16	0.30	0.41
-0.4	0.20	0.08	0.04	0.06	0.14	0.32	0.47
-0.2	0.18	0.10	0.05	0.05	0.12	0.30	0.55
0.0	0.14	0.10	0.07	0.04	0.07	0.22	0.53
0.2	0.09	0.09	0.07	0.04	0.03	0.11	0.37
0.4	0.04	0.07	0.06	0.04	0.02	0.03	0.16
0.6	0.03	0.05	0.07	0.06	0.03	0.01	0.04
				$T = 100$			
-0.6	0.73	0.30	0.06	0.03	0.17	0.47	0.60
-0.4	0.67	0.30	0.07	0.03	0.17	0.54	0.73
-0.2	0.52	0.28	0.09	0.03	0.16	0.56	0.86
0.0	0.28	0.21	0.10	0.04	0.11	0.51	0.91
0.2	0.12	0.14	0.10	0.04	0.07	0.32	0.86
0.4	0.05	0.09	0.09	0.05	0.03	0.15	0.63
0.6	0.05	0.05	0.07	0.05	0.03	0.07	0.34
				$T = 200$			
-0.6	0.98	0.73	0.18	0.02	0.29	0.70	0.75
-0.4	0.93	0.68	0.19	0.03	0.29	0.80	0.89
-0.2	0.75	0.54	0.18	0.03	0.27	0.88	0.99
0.0	0.40	0.34	0.16	0.03	0.20	0.85	1.00
0.2	0.14	0.17	0.12	0.04	0.11	0.64	1.00
0.4	0.05	0.09	0.09	0.05	0.06	0.36	0.93
0.6	0.06	0.05	0.07	0.05	0.04	0.17	0.65

Table 27. Power of the F-Test, $a_{21} = -0.30$, Multiple outliers
$(\delta_1, \delta_2) = (0,10), (0,5), (0,5), (0,5)$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$T = 50$			
-0.6	0.68	0.37	0.18	0.07	0.04	0.06	0.05
-0.4	0.57	0.38	0.20	0.09	0.04	0.07	0.14
-0.2	0.44	0.37	0.25	0.13	0.06	0.03	0.10
0.0	0.28	0.34	0.32	0.24	0.13	0.04	0.01
0.2	0.20	0.32	0.36	0.38	0.32	0.21	0.09
0.4	0.18	0.33	0.44	0.53	0.57	0.57	0.52
0.6	0.20	0.39	0.57	0.71	0.81	0.87	0.92
				$T = 100$			
-0.6	0.99	0.94	0.73	0.30	0.06	0.02	0.15
-0.4	0.94	0.88	0.67	0.29	0.06	0.06	0.19
-0.2	0.81	0.77	0.62	0.36	0.09	0.03	0.27
0.0	0.55	0.65	0.63	0.51	0.27	0.04	0.02
0.2	0.41	0.59	0.67	0.67	0.59	0.36	0.11
0.4	0.38	0.63	0.78	0.84	0.86	0.84	0.75
0.6	0.45	0.73	0.88	0.95	0.98	0.99	0.99
				$T = 200$			
-0.6	1.00	1.00	0.98	0.79	0.20	0.02	0.63
-0.4	1.00	1.00	0.96	0.72	0.13	0.06	0.18
-0.2	0.98	0.98	0.94	0.78	0.27	0.03	0.60
0.0	0.93	0.96	0.95	0.90	0.64	0.13	0.04
0.2	0.89	0.97	0.98	0.98	0.94	0.76	0.25
0.4	0.91	0.98	0.99	1.00	1.00	1.00	0.96
0.6	0.96	0.99	1.00	1.00	1.00	1.00	1.00

Table 28. Power of the F-Test, $a_{21} = 0.30$, Multiple outliers
$(\delta_1, \delta_2) = (0,10), (0,5), (0,5), (0,5)$

a_{12}	a_{22}						
	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6
				$T = 50$			
-0.6	0.05	0.18	0.42	0.70	0.88	0.94	0.98
-0.4	0.04	0.12	0.31	0.56	0.80	0.91	0.96
-0.2	0.04	0.09	0.22	0.44	0.70	0.87	0.95
0.0	0.04	0.08	0.16	0.33	0.59	0.81	0.94
0.2	0.05	0.07	0.11	0.25	0.47	0.75	0.93
0.4	0.06	0.05	0.09	0.18	0.38	0.68	0.91
0.6	0.09	0.06	0.07	0.14	0.32	0.64	0.92
				$T = 100$			
-0.6	0.02	0.20	0.62	0.92	0.99	1.00	1.00
-0.4	0.02	0.10	0.43	0.81	0.97	1.00	1.00
-0.2	0.03	0.07	0.30	0.71	0.94	1.00	1.00
0.0	0.04	0.07	0.24	0.58	0.90	1.00	1.00
0.2	0.07	0.09	0.22	0.52	0.85	0.99	1.00
0.4	0.19	0.17	0.26	0.50	0.83	0.98	1.00
0.6	0.44	0.30	0.34	0.52	0.82	0.98	1.00
				$T = 200$			
-0.6	0.02	0.28	0.87	1.00	1.00	1.00	1.00
-0.4	0.03	0.14	0.70	0.98	1.00	1.00	1.00
-0.2	0.04	0.10	0.52	0.95	1.00	1.00	1.00
0.0	0.06	0.14	0.47	0.90	1.00	1.00	1.00
0.2	0.25	0.29	0.53	0.87	1.00	1.00	1.00
0.4	0.66	0.56	0.68	0.89	0.99	1.00	1.00
0.6	0.93	0.82	0.83	0.93	1.00	1.00	1.00

Table 29. P-values for Granger-Causality F-Test; Peru series before and after adjustments for outliers*

X	Y	
	Inflation	Liquidity
	Before	
Inflation		0.028
Liquidity	0.710	
	After	
Inflation		0.000
Liqudity	0.000	

* The null hypothesis is: X does not Granger cause Y.

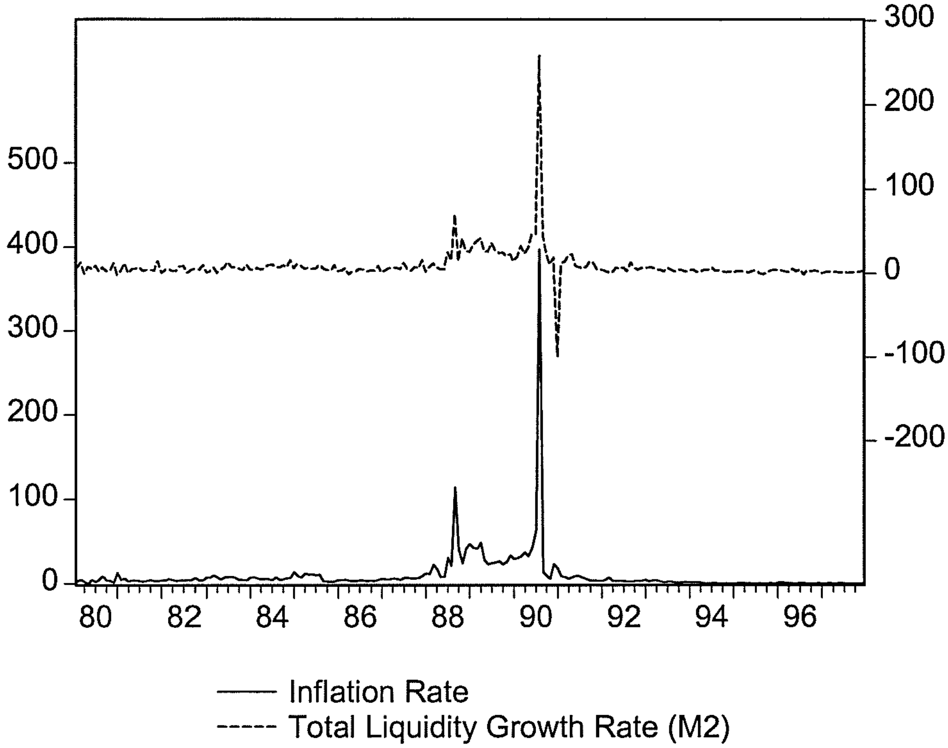

Figure 1: Inflation Rate versus Total Liquidity Growth Rate in Peru