

A phenomenological approach to understand the experiences of teachers
who value listening to student mathematical reasoning

Tatiana Peres Toledo

Faculty of Education

University of Ottawa

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of
MA (Ed.) in Teaching and Learning.

© Tatiana Peres Toledo, Ottawa, Canada, 2017

Acknowledgements

I would first like to thank my participants for their contributions to this project. They were open and generously shared their time, thoughts, and stories in the interviews.

I am deeply grateful to my thesis supervisor, Dr. Barbara Graves, for the encouragement and patience throughout this project, and, for welcoming me into the world of mathematics despite my lack of experience teaching mathematics. I greatly appreciated this wonderful opportunity to expand my perspectives.

My gratitude extends to my committee members, Dr. Christine Suurtamm, Dr. Angus McMurtry, and Dr. Richard Barwell for their valuable time reading my proposal and thesis. Your feedback and contributions were essential to better reflect on my assumptions and question my beliefs.

Lastly, I would like to thank my partner, friends, and family for all the patience and support provided during the completion of this Master of Arts.

Abstract

Listening to students' mathematical reasoning is commonly highlighted as one of the main features of progressive mathematics teaching, yet, little is known about how teachers enact listening in the classroom. This project explores the experiences of teachers who value a listening orientation in their teaching practice. My research question is: How do teachers who profess to have a listening orientation experience listening and responding to student mathematical thinking? I will draw from enactivism (Varela, Thompson, & Rosch, 1991) as a theoretical framework to understand the world of significance brought forth by teachers. A phenomenological approach will ground this project on participants' experiences. I conducted interviews with five elementary school teachers from grades three to eight. In my data analysis I have identified stories and observations which reveal teachers' patterns of interaction with students and the type of relationships established with the students. The findings suggest similarities and uniqueness among the world brought forth by the teachers interviewed. The relevance of establishing relationships and encouraging students to express their mathematical reasoning seems to be central to maintaining a listening environment in teaching.

Table of Contents

Abstract.....	iii
Introduction.....	1
Literature Review	5
Enacting a listening orientation for mathematics teaching.....	5
Obstacles for developing a listening orientation	9
Circumstances for developing a listening orientation.....	12
Conclusion	15
Theoretical framework.....	18
Methodology	21
Research Design	22
Participant selection.....	22
Data collection	23
Interpretative approach and analysis	25
Credibility and trustworthiness.....	26
Participants’ perspectives	27
Kate: Let’s keep it real	28
Abby: Rediscovering Mathematics.....	35
Julie: A relaxed perspective	39
Carol: Teacher as artist.....	44
Monica: Taking a positive stance	49
Summary.....	54
Description of classroom moments.....	56
Kate	56
Abby	58
Julie	60
Carol.....	63
Monica.....	66
Summary.....	69
Contrasting participants’ stories and literature	70
Enacting a listening orientation.....	70
Challenges and supporting circumstances.....	75
What are teachers listening for?.....	81

Making space for the other	82
Conclusion	86
New roads	87
References.....	89
Appendix A: Recruitment text.....	93
Appendix B: Interview Guide	94

Introduction

Jameson (1991) suggests that we live in a society in which cultural products have become commodities and social relations work on the basis of trade. Considering teachers and students as producers of culture, we could say that schooling is also affected by this process of commodification. In the context of mathematics, Brown and McNamara (2011) have claimed that “mathematics manifests itself as many commodities exchanged in the educational marketplace” (p. 131). These authors understand commodification as a process through which social-historical forms substitute a desire, for example, society’s desire for mathematics might be substituted by a set of skills, or a curriculum (Brown & McNamara, 2011). Illich (1970) has described the same phenomenon of commodification using the term “schooling of society”. Schooling of society means that different types of institutions mediate people’s desires for knowledge, health, and other basic needs. With a slightly different focus, Kohn (2010) has described how grades and standardized testing stand in for pupils’ desire to learn and to succeed at school. The overall consequence of commodification is the habit of relying on external approval from authorities to assess one’s competencies (Illich, 1970).

This process of commodification of schooling engenders negative consequences for teachers and students. Teachers usually subject their personal understandings of teaching to the discourse of employability, adhering to a set of skills, procedures, competencies, administrative language and standards of practice (Brown & McNamara, 2011). The risk embedded in this process is that teaching may be reduced to following a set of rules. When teachers’ sense of being professionals is diminished, their professional satisfaction might also be lessened (Brown & McNamara, 2011; Ohanian, 1992). If teachers do not feel as professionals capable of promoting student learning, levels of stress tend to increase and possibly lead to teacher burnout (Thompson, 1985).

The process of commodification of mathematics teaching also affects students’ development. Following a set of rules could reduce the attention given to students’ mathematical reasoning (Brown & McNamara, 2011; Levin, Hammer & Coffey, 2009) failing to contribute to the promotion of rich mathematical investigations (Brown & McNamara, 2011; Ohanian, 1992; Schifter & Fosnot, 1993). Standardized testing also has negative consequences on students’ learning. It diminishes students’ desire to learn since pupils become too concerned with their scores to enjoy learning for the sake of learning (Kohn, 2010) and reduces the time for open-

ended mathematical exploration since teachers tend to teach to the test (Kohn, 2011; Suurtamm, Lawson & Koch, 2008). As a consequence, students may have the curiosity and passion for learning schooled out of them (Illich, 1970). While there are many illustrations of how students and teachers might become dispirited, Tompkins (1996), a prominent literary scholar, describes how she overcame this process by listening to her students without pressure to achieve a certain outcome. Teachers listening to students is the focus of the research study which follows.

In my own experience as a language teacher, listening to students' opinions, ideas, and suggestions contributed to my teaching practices. It enabled me to better understand students' thinking as well as not to rely as much on norms of practice which I had initially followed like a recipe. Brown and McNamara (2011) have alerted us to the risk of treating norms of practice as fixed rules. However, despite feeling that I was listening to students and providing space for them to investigate concepts, some students did not seem to remember the topics discussed in class. I wanted to find a way for students to engage even more with learning for the sake of learning. I have encountered moments in which students really engaged with their own learning. One of these moments came out of need as much as desire – Grade 5 students prepared a treasure hunt for their colleagues in class saving preparation time. With this activity, a student who was never much excited about class, participated fully and used language in a very meaningful way. I was really satisfied with the student's attitude because it relates to my own passion for learning.

My desire to help students engage with their learning processes stems from my own childhood learning experiences. As a child, I rejoiced in learning moments. I remember one of these moments about probability – more specifically, the possible outcomes of rolling two dice. I was playing a board game with my family when my father said:

Father: Ok. My turn, I think I might win because I need a seven!

“My father is going crazy”, is what crossed my mind.

Tatiana: Why would it be easier to get a seven?

F: There are more combinations to get a seven.

T: (puzzled and now suspecting that my father was joking) What?

F: We'll talk about it later.

From this moment on I could not help but wonder why a seven would be easier. I cannot remember if my father won the game, or not. I could only think of the seven. At the end of the game, my father got the dice, paper, and pencils and I was very curious.

T: Why is it easier to get a seven?

F: What are all the possible outcomes when you roll two dice?

We wrote down all the possible combinations together, for $2=1+1$, for $3=2+1$, for $4=2+2$ and $3+1$, and, henceforth until we had all the combinations and I noticed that for seven, there were more options than for the other numbers. I remember I had to actually organize the dice in the possible combinations. Next, we decided to prove it. We made a graph with the possible outcomes as the horizontal axis and the vertical axis as the number of times we got that result. Then, we rolled the dice and registered the results on the graph. After a while the graph looked like a mountain with the number 7 as the peak and numbers 2 and 12 at the base of the mountain. I do not remember how many times we rolled the dice but at some point he said it was good enough and claimed that the graph would look sharper, if we did it a hundred times. Much later in high school, I learned about probability and, it might have been just a coincidence, but probabilities sounded very logical. This discovery moment made me feel as some kind of scientist; I could see the world with other colours.

This project stems from a personal reflection about what makes teaching into significant learning. Learning has always been a pleasurable experience for me and I think people are all capable of learning depending on their interests. Yet, going to school is not always a pleasurable experience for students. While there is still resistance to incorporating experiences like the one from my childhood, there are alternatives to the commodified notion of education. One of those alternatives involves creating a listening environment in the classroom. Listening is said to promote caring and sharing relationships and more meaningful learning contexts for students (Davis, 1996; Schifter & Fosnot, 1993; Towers, 1998). Listening is an essential element for a progressive perspective on mathematics teaching since students' thinking can be enhanced when teachers listen to students (Crespo, 2000; Graves, 2011; Suurtamm & Vézina, 2010). Furthermore, listening to students' thinking promotes a deeper understanding of the mathematical concepts for teachers and students as well as improvement in students' achievement (Brown & McNamara, 2011; Empson & Jacobs, 2008; Graves, 2011; Wisheart, 2004). One of the first remarks about the importance of listening to students' mathematical reasoning came from NCTM (National Council of Teachers of Mathematics). The NCTM Professional Standards for teaching mathematics (1991) posited the importance of listening to students' mathematical reasoning as an essential feature of a progressive perspective on

mathematics teaching. In order to establish a community of practice in which students create and share their mathematical ideas, a listening orientation on the part of teacher and students is necessary. In fact, research suggests that when teachers' listen to students' mathematical reasoning, students tend to pay more attention to their own ideas (Empson & Jacobs, 2008; Levin et al., 2009).

In this dissertation, I start by discussing the literature review related to listening in the mathematics classroom with special attention to the enactment of a listening orientation and the challenges and enabling factors of a listening orientation. Next, I present my interests and research question as well as the theoretical framework and methodological choices adopted. Then, in the methodology I discuss how I recruited participants and my experiences adapting interviewing guidelines. Ensuing, I describe the perspectives and the classroom moments shared by my five participants. Finally, I conclude by contrasting my participants' experiences and the literature. I focus on how participants brought forth elements of listening to student mathematical reasoning that were not present in the literature.

Literature Review

This review explores the literature regarding a listening orientation towards student thinking, especially mathematical thinking. Three main topics were identified in the literature: (1) enacting a listening orientation for mathematics teaching; (2) obstacles for the development of a listening orientation, and (3) circumstances for the development of a listening orientation.

Enacting a listening orientation for mathematics teaching

In his book, Davis (1996) defines listening as embodied, intentional, and selective. For him, listening does not mean the same as hearing, nor is it a technique to be applied; listening is an activity to be done with the whole body and all the senses. Moreover, listening is intentional and selective. Our senses receive a mass of information, which is selectively interpreted (Davis, 1996). This selection is enabled and constrained by the sociocultural historical environment in which we are immersed. Davis (1996) defines three types of listening. The first type he calls evaluative listening. It is highly selective and limited to identifying right and wrong answers to specific pre-planned questions. The teacher only listens to students to identify the answers he expects to receive for the question asked. In this type of listening, students' contributions are only welcome if they match exactly what the teacher wants. The second is interpretive listening, in which the teacher accepts and interprets students' contributions to understand how students make sense of the subject matter. However, the teacher only interprets students' contributions to direct students' thinking back to the lesson plan or goal of the lesson. The third type is hermeneutic listening, which is a shared conversation in which meaning is negotiated and perspectives are deconstructed and reconstructed collectively. The teacher interprets students' reasoning and allows the class to be directed by students' questions and interests. In hermeneutic listening, control and responsibility for the classroom are shared among students and teachers. Neither side of the dialogue knows how the conversation will unfold since both contribute to the learning experience. However, for hermeneutical conversations to take place, it is important to start the conversation with the premise that the interlocutor has something of value or importance to contribute (Jardine, personal communication, June, 2015).

Davis's (1996) framework has been adopted by many researchers such as Coles (2002), Crespo (2000), Empson and Jacobs (2008), and Mhlolo and Schafer (2012). In some cases, these authors have made changes to the names given to each category of listening, but the substance of the categories remained the same. In the field of noticing, research has described teachers' actions when listening and responding to students (Jacobs, Lamb & Philipp, 2010). Noticing

seems to be similar to listening to students but noticing is concerned with decomposing teaching practices to make them learnable (Sherin, Jacobs & Philipp, 2011), whereas listening seems to be discussing classroom interaction in a broader way. Jacobs and colleagues (2010) describe three different actions to interact with student mathematical thinking: (1) notice; (2) interpret; and (3) respond. Notice is the ability to realize that students are performing valuable mathematical thinking. Interpret is the ability to understand and make sense of students' mathematical thinking. Respond is the ability to respond to students' mathematical investigations in a supportive way allowing students to further explore their hypotheses. It is important to understand that responding should be done as a way to foster the reasoning of students and not as a way to direct students to the teacher's goal. Jacobs and colleagues (2010) seem to place listening in a continuum of actions or steps for teaching.

Empson and Jacobs (2008), Davis (1996), Ohanian (1992) among other researchers depict short vignettes or anecdotes as evidence of how a listening orientation takes place in the classroom. These researchers seem to have included the vignettes or anecdotes to better demonstrate their arguments. Yet, these short accounts are limited to moments and do not describe the overall structure of the classroom. Graves (2011) presented a more detailed description of how one teacher creates and maintains conversational spaces which facilitate listening to students. Therefore, the description of the listening orientation goes beyond a sequence of teaching moves. In this study Graves (2011) describes how attentive listening is enacted in the complexity of a classroom. The researcher observed a teacher, Judith, in a multi-age classroom for two months and found that students were encouraged to be autonomous and independent allowing the teacher to focus on listening to students. Judith encouraged students to be more independent through shared control of the classroom – students could write down what they wanted to do on a planning board which also contained the activities planned for the day. Judith did not have to direct students to the activities, the board allowed students to self-organize. It is also important to point out that Graves (2011) proposes a different relationship with her participant. This relationship seems like a partnership between researcher and participant since the real name of the teacher is used. Moreover, this teacher seems to be proud and confident to be sharing her thoughts about how to structure classes.

Researchers have described varied ways of listening attentively to student reasoning in the classroom depending on the particularities of contextual aspects, such as students and the

topic being studied (Davis, 1996; Empson & Jacobs, 2008; Ohanian, 1992; Schifter & Fosnot, 1993). Nonetheless, it is valuable to describe some similarities of the listening orientations that have been described by these researchers. These similarities are: (1) not controlling outcomes; (2) asking appropriate questions; and (3) having a common repertoire. The first similarity, not controlling outcomes is necessary when teachers listen to students, since a listening classroom would be “laying down a path in walking” (Varela et al., 1991, p.237). The voice of the teacher should have the same value as the voices of the students – “a teacher is not guiding a sight-seeing tour through a thoroughly mapped-out region, but is dwelling in, with, and through the complexity and ambiguity of emergent knowings” (Davis, 1996, p. 264).

Ohanian (1992) describes a way through which teachers can share responsibility for the classroom with the students. In her book, she tells an anecdote about Hakim, one of the grade 3 teachers she observed while traveling around the US. This teacher had pumpkins and she invited her students to suggest how to use them. The teacher noticed that students commented about size and weight over and over, so she developed a lesson plan for the next day about size, but “full of “ifs” and “mights” (p.73). The attitude of this teacher validates students’ thoughts and ideas creating an environment of mutual respect and caring in the classroom. In this teachers’ classroom, children would self-organize and new activities would emerge, for example, from measuring pumpkins into measuring objects around the classroom. Ohanian (1992) points out that learning how to dig deeper into one activity is a significant sign that teachers are listening to students’ exploration and making the most out of their investigations. Another possible way of sharing control is for teachers to show their students how they find it difficult to resolve certain problems. Seeing the teacher making mistakes and struggling to reach an answer allows students to feel more at ease with their own difficulties (Kohn, 2011).

Research has found that asking appropriate questions to elicit students’ thinking is essential to establish clear communication. It is very hard to define what an appropriate question sounds like but it should be open enough to allow students to investigate diverse solutions (Davis, 1996; Empson & Jacobs, 2008; Suurtamm & Vézina, 2010). Empson and Jacobs (2008) have analyzed a sequence of interactions of a teacher, who values listening, asking questions to help a student understand a problem. The problem was the following: *Melanie has 4 pockets and in each pocket, she has 5 rocks. How many rocks does Melanie have?* The authors point out that the teacher rephrases the problem using the student’s outfit as an example. By using a familiar

context, the teacher assures that the context of the problem is clearly understood by the student. Moreover, the teacher inquires on how the student reached the correct answer and proposes similar problems to confirm if the reasoning is accurate. This teacher did not plan the questions ahead of time and allowed the child's contributions to lead the questions on the spot. Empson and Jacobs (2008) remark that this teacher's questions aim at creating clear communication and discussing important mathematical concepts. Additionally, Graves (2011) also discusses the nature of the questions asked by the teacher. Judith, the teacher, takes advantage of students' comments to ask questions that will foster students' understanding. For example, while discussing the solution proposed by a group of students, she proposes that they rewrite the problem to make the problem fit the solution offered by the students.

Finally, establishing a listening environment in a classroom does not happen overnight and Davis (1996) reminds us of the importance of having a common repertoire. Common repertoire is analogous to a common language which allows actors in a context to comprehend one another. In the same way that a language has redundancies, students and teachers need to understand the structure of interaction to make sense of one another's contributions. Davis (1996) describes a lesson guided by Tom Kieren in which students use fraction kits to explore fractions. This lesson cited by Davis is based on an activity created by two grade 7 students during a unit whose "emphasis was on presenting opportunities for students to develop a repertoire of "fraction-ing" experiences from which various mathematics concepts could emerge" (Kieren, Davis & Mason, 1996). The students had plenty of opportunities to play with paper-manipulating activities to become familiar with fractions and ended up developing the idea of these fraction kits. The kits seem to have worked as common repertoire allowing students to communicate their mathematical understandings through the concrete experiences with fractions kits.

This section of the literature review describes the nature of listening, classroom practices, and different frameworks to understand listening. Davis's (1996) categories of listening (evaluative, interpretative, and hermeneutic) are the most common in the literature. In the field of noticing Jacobs et al. (2010) decompose teachers' actions when they are listening to students (notice, interpret, and respond). Furthermore, there are similarities among the studies which describe an open and responsive way of listening. These similarities are: (1) not controlling

procedures and outcomes; (2) asking thoughtful questions; (3) developing a common repertoire in the classroom.

Obstacles for developing a listening orientation

Establishing a listening orientation in the classroom is not an easy process since there are obstacles related to school context (Illich, 1970; Liston, 2000; Noddings, 2007) and to teachers' beliefs, habits, and assumptions (Crespo, 2000; Empson & Jacobs, 2008; Schifter & Fosnot, 1993). First, I will discuss the challenges related to the school context, and, secondly the challenges about teachers' beliefs.

Numerous researchers have shown how certain aspects of school environment, such as standardized testing and the demand to cover the curriculum, might reduce teachers' opportunities to make mathematics teaching more progressive (Davis, 1996; Levin et al., 2009; Mhlolo & Schafer, 2012; Noddings, 2007; Suurtamm et al., 2008). Standardized testing has proven to be an obstacle for the establishment of a listening orientation as well as ineffective to indicate learning and school quality (Kohn, 2010; Noddings, 2007; Ohanian, 1992). Suurtamm and colleagues (2008) analyzed the Ontario grade 9 standardized tests against the progressive curriculum of the province in order to comprehend how teachers keep their practice progressive within large-scale assessment. They have found that, although large-scale assessment in Ontario is aligned with many features of the curriculum, there are still improvements to be made. Teachers tend to focus on the content of the exam, hence, the time and effort reserved for investigation in the classroom could be reduced.

Disciplinary measures of control and authority as well as the demand to cover the curriculum might lead teachers to reduce the number of opportunities for students to listen to one another (Brown & McNamara, 2011; Davis, 1996; Levin et al., 2009). In her book about challenges within school reform, Noddings (2007) draws from her own experience participating in a meeting with school principals in the US to give an example of aimless authority. One of the principals reported being proud to have achieved silent halls in his school - teachers had to stand in front of the classrooms to remind students to be quiet. This anecdote shows how school leaders might consider control as part of a teacher's role. Gallagher (2010) demonstrated how the phenomenon of surveillance takes place in schools and is regarded as one of the roles of students, teachers, and school staff. In South Africa, Mhlolo and Schafer (2012) analyzed video recordings of four grade 11 teachers to observe if the listening practices of those teachers was

conducive and respectful of learners' thinking. Their findings have shown that the four teachers overlooked students' contributions without interpreting students' reasoning. Mhlolo and Schafer (2012) concluded that the teachers did not share control of the class and excluded students' contributions.

Levin and colleagues (2009) wanted to understand how novice teachers are shaped by the school context in which they are immersed. They observed, interviewed, and collected assignments from nine beginning high school science teachers who were doing their master's program and working as part-time teachers in a local school district. The article focused on two of those teachers: one who demonstrated a listening orientation to student reasoning, but had difficulties with class management and curricular objectives, and, another who controlled the group well and followed the curriculum, but whose fixed class structure excluded opportunities for students to share their thinking. Levin and colleagues (2009) have found that the school administration was only concerned with the performance of the first teacher and believed the second teacher to have adequate practices. It is also important to highlight that Levin and colleagues (2009) use *framing* as a theoretical framework to understand how teachers understand the importance of listening to students. Framing (McLachan & Reid as cited in Levin et al., 2009) is a theory about how an individual or group understands a phenomenon in a certain context. Teachers might frame teaching as covering curricular objectives or fostering student thinking. Having framing as a theoretical framework allowed the authors to understand the teachers' attitude not as personal traits of these teachers but as the understanding of what is relevant in a given context moulded by the context itself. Therefore, the authors conclude that novice teachers might be able to attend to students' reasoning as long as they frame teaching as attending to students' thinking instead of thinking about keeping control of students or covering the curriculum.

The school context itself also has a strong influence on teachers since it is common for teachers to incorporate school practices and habits, often in an unconscious way (Brown & McNamara, 2011; Davis & Sumara, 1997; Rogoff, 2003). Evidence of this process of acquisition of habits is demonstrated in Davis and Sumara's (1997) teaching experiment. They had semi-monthly sessions about enactivism and education with teachers and parents of a small inner-city school in Canada. Enactivism (Varela, Thompson & Rosch, 1991) is a theoretical perspective based on the idea that individual and world co-emerge. Therefore, enactivism in education

assumes that teacher and students build lessons through interaction and that teachers cannot control students' learning outcomes. As the sessions passed, some parents and teachers of the school suggested that Davis and Sumara teach a study unit with the class teacher according to the principles of enactivism. Davis co-taught an introduction to fractions to grades 3-4 and Sumara co-taught a language arts unit about a popular novel to grades 5-6. The most interesting finding for the researchers is how the classroom teachers and the researchers influenced each other's teaching practices. The researchers inadvertently adopted habits from the school context which they advised against, such as asking questions expecting a specific answer. At the same time, the teachers adopted some practices from the researchers, such as allowing lessons to be shaped by students. Davis and Sumara (1997), with an enactivist lens, conclude that teachers' practices are shaped not only by their individual characteristics but by the environment in which teachers are immersed – school context presupposes certain roles to be played and to see what is taken-for-granted requires constant vigilance. In the literature review, the obstacles were categorized as related to school environment or teachers' personal characteristics for organization. Nevertheless, school environment and teachers' practices constantly shape each other. Now, I will discuss obstacles related to teachers' beliefs, assumptions, and habits.

Some of the challenges for the creation of a listening orientation are related to teachers' personal characteristics. Habits and beliefs have been engraved in teachers by years of mandatory schooling. Therefore, changing these habits requires great effort on the part of teachers (Brown & McNamara, 2011; Suurtamm & Vézina, 2010). In their book, Schifter and Fosnot (1993) discuss the processes that elementary teachers go through to transform their teaching practices from traditional to constructivist. Traditional teaching practices are based on the transmission model of education in which teachers explain and students listen, whereas, constructivist practices recognize that students build their own knowledge through “meaningful problems designed to encourage and facilitate the constructive process” (p. 9). Schifter and Fosnot (1993) conducted a long intervention for professional development, in which teachers underwent six months of training and one year of weekly trainers' visits to their classrooms. The teachers kept research journals and were interviewed. During this process, the authors identified some challenges related to the teachers' experiences. The first challenge is the repeated belief that learning happens through a transmission model. For example, some teachers viewed the content from the professional development sessions as a set of rules to be followed. Not

surprisingly, teachers reported feeling constrained by these sets of rules and could only transform their teaching practices after denying these rules. The second challenge is teachers' familiarity and confidence with their own mathematical knowledge (Schifter & Fosnot, 1993). Some teachers might not feel comfortable with their own mathematical thinking, hence, a feeling of being unprepared to support students' exploration might set in. In the interviews conducted by Schifter and Fosnot (1993), they found that some teachers feel insecure about listening and eliciting students' mathematical thinking because they fear not being able to understand students' reasoning. The third challenge is related to teachers' assumptions about student abilities to reason mathematically. There is a tendency "to under- and to overestimate children's mathematical understanding. On the one hand, [teachers] do not believe that their students can figure something out on their own...But on the other hand, they must come to appreciate that severe misunderstandings can lie behind correct answers." (Schifter & Fosnot, 1993, p. 31). In addition, some teachers misread their students' difficulties as lack of interest or effort (D'Ambrosio, 1995).

In summary, the obstacles are presented in 2 different groups: (1) obstacles stemming from the school environment and (2) obstacles related to teachers' beliefs. These two groups are intertwined since teachers may feel the need to adhere to values promoted by the school environment (Brown & McNamara, 2011) and teachers might also influence the school environment (Davis & Sumara, 1997). The challenges related to school environment are standardized testing (Kohn, 2011), control and curriculum (Davis, 1996; Levin et al., 2009), and the school environment itself (Davis & Sumara, 1997). As for teachers' beliefs, the first obstacle is related to the beliefs acquired from long years of traditional schooling in a transmission model, the second is lack of confidence with mathematical knowledge, and the third regards teachers' assumptions of students' capabilities (Schifter & Fosnot, 1993). It is relevant for this project that most of the articles dealing with obstacles do not discuss the relations between school environment and teachers' beliefs. Moreover, there is no description in the literature of how teachers who value a listening orientation perceive these so-called obstacles.

Circumstances for developing a listening orientation

Researchers have investigated how to support the development of a listening orientation since teachers seem to face numerous challenges to listen openly and non-judgmentally to student reasoning. Researchers seem to agree that listening to students' thinking rarely happens

naturally. In fact, most studies show that it takes years of teaching practice and/or professional development for teachers to learn how to listen effectively to students' mathematical reasoning, especially, to respond to students' understandings (Empson & Jacobs, 2008; Jacobs et al., 2010; Sherin, Jacobs & Philipp, 2011).

Jacobs and colleagues (2010) have measured the ability of teachers to notice, interpret, and respond to students' mathematical thinking through a scenario methodology. In a scenario methodology, participants are presented with a situation - in this case, 131 teachers looked at written work or video recording of students solving mathematics problems. Participants were asked to describe the reasoning of the students and to propose a follow-up activity to support student understanding. The authors' goal was to test prospective and practicing teachers' abilities to notice, interpret, and respond to students' thinking. Teachers were separated in 4 different groups according to their years of teaching experience and professional development. Professional development included 5 full days per year of training focused on students' thinking. Data were treated qualitatively aligning the years of experience of the participants with their abilities to notice, interpret, and respond. Their findings demonstrate that teaching experience increased teachers' ability to notice students' thinking, but responding seemed to emerge only after 2 years of professional development. Only one beginning teacher was able to notice, interpret, and respond to students' thinking (Jacobs et al., 2010). This study makes a case for the importance of professional development to develop the ability to notice students' reasoning. Noticing was also studied by Mason (2002) as a way to research one's own practice. Mason (2002) discusses how being conscious of actions, imagining different outcomes, and pausing before actions might support change of habitual reactions.

A number of studies have shown that professional development opportunities for reflecting on one's own mathematical reasoning and on children's mathematical thinking may foster teachers' disposition to listen and support students' thinking (Davis, 1996; Empson & Jacobs, 2008; Shifter & Fosnot, 1993). Empson & Jacobs (2008) have drawn from a synthesis of research to create benchmarks for teacher educators who wish to support the development of a listening orientation in teachers. The authors highlight that listening to students' reasoning is a complex task which cannot be reduced to a list of discrete steps. The three benchmarks described in the article are: (1) reviewing students' written work, (2) watching video recordings of students' reasoning, and (3) face-to-face interaction with children. Empson and Jacobs (2008)

also point out that face-to-face interaction allows the teacher to experience the complexity of listening to students in the moment of teaching.

Suurtamm & Vezina, (2010) have also discussed how to support teachers' development of a listening orientation through a case study on a professional development initiative for francophone schools in Ontario. The authors conducted interviews, questionnaires, observed lessons, and collected training materials of the second year of the project with teachers from grades 4-5. Teachers participated in professional development meetings during the school year. Therefore, these teachers were in contact with students in class and could reflect on students' reasoning. The authors concluded that the professional development initiative made a great difference in teachers' classroom strategies. One strategy, in particular, referred to the practice of asking for two different solutions for the same mathematical problem. In this way, students often used the algorithm as the first solution and were encouraged to reflect on the subject matter to develop a more personalized solution. Teachers noticed some differences in their own practices: (1) they felt more confident with mathematics, (2) they could listen and appreciate students' thinking, (3) the support offered by trainers and other teachers generated the habit of teachers relying on one another. This study concludes that an important aspect of developing a listening orientation can be fostered by having a supportive group of teachers.

Teacher education courses are another means of offering opportunities for the development of a listening orientation. A number of researchers have pointed out that teacher education courses should focus more on students' mathematical thinking (Crespo, 2000; Davis and Sumara, 1997; Towers, 1998). Putting this idea into practice, Towers (1998) describes how she used video to attract the attention of pre-service teachers to students' thinking in her own practice as a teacher educator. Moreover, she believes that reflecting on students' mathematical thinking is more important than discussing all the mathematical topics that teachers might have to cover. Crespo (2000) also found a way to bring schools closer to university through a letter exchange experiment between pre-service teachers and elementary students in British Columbia. She has also found that teachers learn to appreciate the variety of possible solutions when they are presented with unfamiliar problems. Moreover, she posits that having time to reflect on the thinking of one or a group of students without the pressure and distraction of the fast paced environment of a classroom contributes to develop teachers' ability to listen to students' mathematical thinking (Crespo, 2000).

Teachers can also develop a listening orientation through interacting with students in their own classrooms (Ohanian, 1992; Paley, 1986). For example, Paley (1986) has developed a listening orientation through her own experience as a kindergarten teacher. She claims that curiosity enables teachers to listen to students. Ohanian (1992) also describes how a teacher called Stone developed a listening orientation. Ohanian's (1992) book reports on lessons that she observed and Grade 3 teachers whom she interviewed around the US for a year. Ohanian's goal was to look for practices that reflected a constructivist approach to mathematics teaching. She tells inspiring anecdotes of teachers and schools with notable practices. Stone was a grade 3 teacher who had a moment of awakening while talking to a student. A student used Stone's intonation as a communicational cue to infer if a response was right or wrong. Stone realized how her teaching practices were encouraging that student to look for right and wrong answers in the interaction instead of reflecting on the subject matter. Sfard (2001) has described the same phenomenon, that is, how students use communicational cues as a strategy to find correct answers.

To sum up, listening to students' mathematical thinking does not seem to happen early in the teaching practice. Research evidence shows that the main conditions for the development of a listening orientation are teaching experience and professional development. In professional development courses, reflecting on their own reasoning as well as students' seems to improve teachers' listening skills (Empson & Jacobs, 2008; Jacobs et al., 2010). Furthermore, it is important to have a good support system with consultants and colleagues to provide conditions for this change (Suurtamm & Vezina, 2010). Teacher educators have tried different ways of exposing teachers to students' reasoning in graduate courses, such as watching and discussing videos of students' thinking (Towers, 1998) and exchanging letters with problems between children and pre-service teachers (Crespo, 2000). In their practice, teachers might be able to develop this listening orientation through contact with students or colleagues (Ohanian, 1992; Paley, 1986).

Conclusion

Listening as an attitude for mathematics teaching was approached by many authors as one of the key elements of progressive teaching. One of the most complete works on listening for mathematics teaching is *Teaching Mathematics: Toward a sound alternative* by Davis (1996). A significant part of the literature on listening is in the field of professional development with a

progressive lens on mathematics teaching. The methodology of choice seems to be qualitative with the exception of Jacobs and colleagues (2010). Many of the qualitative studies employ interventions, case studies, or teaching experiments which show how teachers transform their practices, usually as a result of exposure to their own and children's mathematical reasoning. Most of the studies discuss teachers' learning processes and give some examples of how classroom practice is affected. Davis (1996) points out the importance of discussing how listening takes place in the context of classrooms. However, his discussion of listening in classrooms is limited to short vignettes which suggest but do not present a more detailed picture of hermeneutic listening in the classroom. Many of the other studies also present snapshots of a small group or one student interacting with the teacher as illustrations of a listening community. Therefore, evidence of how teachers, who value a listening orientation, maintain this listening environment in their everyday teaching practice is sparse.

While there are many studies on the obstacles to the development of a listening orientation, especially, from the perspective of teachers who do not value a listening orientation, there is no account of how teachers who value listening to student reasoning perceive apparent obstacles. The experiences of elementary teachers who value listening to students' opinions, feelings, and mathematical understandings could be further discussed in the literature. These teachers' perceptions of possible obstacles to enacting a listening orientation and how they cope with these obstacles could be a specific focus of interest. Most of the research related to listening focused only on the school context or on teachers' beliefs, habits, and assumptions. The focus seems to have been on how teachers develop a listening orientation regardless of the classroom environment. It seems to be important to learn how teachers enact this listening orientation within the classroom and the school (Jacobs et al., 2010). Incorporating stories and descriptions of how teachers perceive the listening environment in their classrooms might point towards key aspects of listening to students possibly disregarded when observing specific interventions. A lens that posits that teachers' practices and school context shape each other and are constantly being constructed and reconstructed through interactions in the doing of the classroom could support understanding of how the ins and outs of listening to student thinking is shaped by classroom practices. Therefore, I drew from enactivism to explore teachers' accounts of their classroom listening practices as constructed and reconstructed through the constant interactions among the variety of factors influencing a classroom environment. As teachers describe what

listening means to them and how it is instantiated in their classrooms, a picture of their classroom environment can be painted. As I aimed to explore participants' experiences, my methodology borrows from phenomenology. By accessing the stories and experiences of elementary teachers in the interviews I relived their classroom experiences to understand what listening to students means to teachers in the context of their practices. Thus, my research question is: How do teachers who profess to have a listening orientation **experience listening and responding** to student mathematical thinking? Through this question, I accessed teachers' perceptions of listening in the daily practice of teaching in the classroom.

Theoretical framework

This study is informed by enactivism (Varela et al., 1991). Enactivist theory seems to rely on an understanding of the world informed by systems theory, that is, living happens through the interactions of varied nested systems. Thus, when we look at education through enactivism, each teacher, student, classroom, and/or school can be seen as a different system. For example, a student is a system composed of a body, thoughts, feelings, among other things. According to enactivism, each system is self-organized in a constantly changing structure. This structure determines how a system perceives and responds to environmental stimuli (Varela et al., 1991). If we take a song as an example of environmental stimuli, a rock and person might perceive the sound waves differently depending on how they are structured physically. In addition, two individuals could also perceive the same song differently depending on their cultural experiences influenced their structures. The environment of a system can be comprised of other systems. For example, the environment of a teacher is composed of other systems: students. These systems repeatedly interact with their environments, and system and environment co-emerge through a process of structural coupling (Reid & Mgombelo, 2015). Structural coupling is the process of connection between systems and their environments through a recurrent history of interactions (Varela et al., 1991). Every interaction changes the structures of all systems involved in the process. The structures of the systems involved do not necessarily change in the same way since reactions depend on the structure of each system. An example of structural coupling is how wolves and pre-historical humans have co-evolved into humans and domesticated dogs as we know them. For enactivism, every action is a cognitive action. Cognition is perceptually guided action, hence, knowing is doing and doing is knowing. This means that the bodily doing of our daily lives are all contributing to our knowing. Structural coupling is an active process which determines how we think, feel, or respond in our praxis of living. Praxis of living is the experience of being, acting, and feeling in the world (Maturana & Varela, 1987).

A recurrent history of interactions between individual and environment create the niche of a system. The niche of a system is all the effective interactions of a system that compose its reality. Reid and Mgombelo (2015) refer to these effective interactions of a system as its world of significance. The niche or world of significance might influence future actions of a system. For example, a teacher might listen to students according to his or her relations to previous students and classroom elements. Every teacher couples differently with the environment, hence,

different teachers might attribute different relevance to classroom elements. Some teachers might think that definite lesson plans are important to listen to students, whereas other teachers might feel that classroom spatial organization promotes listening. Enactivism as a theoretical framework posits every structure determined system conveys a unique world of significance (Reid & Mgombelo, 2015). In an attempt to welcome these varied worlds of significance, I have not specified the teaching practices that I wished to analyse. Enactivism will allow me to explore teachers' worlds of significance without predetermined ideas of what should be relevant for listening to students. When the teachers bring forth their world of significance in an interview, they demonstrate how they couple structurally with their environments. By describing how interaction takes place in the classroom, teachers might be shaping their own and students' structures. In the interviews, I explored which classroom elements participants highlight or overlook. As a fictional example, a teacher who perceives a noisy classroom as a challenge to successful learning might reduce students' opportunities to speak, whereas a teacher who believes noise could be beneficial to education might encourage students' interaction. These teachers are not only changing their classroom practices; the physical and cultural structure of these teachers might also change in the process. Maturana and Varela (1987) describe how enactivism proposes to connect the divide between internal dynamics and environment:

It is the observer who correlates them from his outside perspective. It is he who recognizes that the structure of the system determines its interaction by specifying which configurations of the environment can trigger structural changes in it. It is he who recognizes that the environment does not specify or direct the structural changes of a system (p. 135).

Maturana and Varela (1987) describe how the observer is a bridge that connects environment and internal changes. They provide the example of a submarine in which the person in control is not aware of the whole structure, nor the external world. An observer from the outside might look at the submarine emerge and congratulate the controller on the maneuver. The observer is making the connection between the movement of the submarine and the imagined actions of the controller since the outside observer did not see the controller in action. In the analysis, I assume a similar role to the outside observer. However, I did not see the teachers while teaching. In my interpretation of the submarine example, the observer does not have to be a visual spectator, the observer is a reflective standpoint from where one can ponder on the relations between

environment and internal changes – the observer could even be the same person who is performing the actions. I observe their descriptions of classroom moments. I make connections between the experiences described and the imagined internal dynamics of teachers, in other words, between the descriptions of classroom events and the meaning teachers attribute to listening.

Another relevant feature of enactivism is that systems act in a proscriptive (not prescriptive) logic; that is, whatever is not forbidden, is permitted (Varela et al., 1991). Therefore, there are many possible actions which can be effective in keeping the integrity of the system (Varela et al., 1991). The challenge is that after successful recurrent interactions with the environment, systems tend to repeat the same actions. Varela and colleagues (1991) present a notion of reflection capable of changing the usual responses of a system. This notion draws from Buddhism to “cut the chain of habitual thought patterns and preconceptions” (p. 27). The mindful reflection is an embodied experience of reflection and is described as mindful and open-ended (Varela et al., 1991). Through this way of reflecting, the structural coupling of an individual and the environment can be transformed and what seems like obstacles might become opportunities. I intend to make the interviews in this project into opportunities for teachers to notice their habitual patterns while describing them during the interview. This attention to their actions might lead teachers to insights about their own teaching practices. However, it is beyond my reach to impose this reflective attitude on participants. I can only hope that describing stories and events during the interview will increase self-awareness and reflectivity in participants.

Methodology

Each methodology brings within it a view of the world and by choosing your methodology, you might be inadvertently setting the philosophical perspective of your inquiry (Englander, 2012; Heath, Hindmarsh & Luff, 2010). My choice of phenomenology is not only a choice of methodology, it is a personal, professional, political choice. The use of phenomenology is a statement of what I feel we need to pay attention to – our experiences and their meanings. Our experiences and the meanings we attribute to them constantly influence our choices and actions. Yet, we hardly ever take the time to observe these experiences and how we relate to them. For example, teachers often act and react in classrooms according to habitual patterns of interaction accumulated through their own experiences as students without reflecting on their actions. Merleau-Ponty (1945) has stated that: “All my knowledge of the world...is gained from my own particular point of view, or from some experience of the world without which the symbols of science would be meaningless” (p. ix). Phenomenology demands us to observe and explore experiences without the preconceived notions of habitual patterns of interaction (Van Manen, 1990). Phenomenology moves us to turn that which is known into unknown; familiar into foreign in order to gain new perspectives (Sheets-Johnstone, personal communication, July, 2016). Personally, by reflecting on my actions, I realized how some of my assumptions about children’s thinking have limited the nature of my listening, and, I could identify my experiences in the literature about listening to student mathematical thinking. Therefore, in this project I made every effort to keep a reflective look at my experiences as a student, teacher, and researcher to identify my personal perspectives and review assumptions.

In order to keep this reflective look, I have employed the basic method of phenomenology - reduction, bracketing, or *époche*. This method is approached differently by the many currents of phenomenology. Van Manen (2014) approaches the reduction as a phenomenological attitude of “attentive awareness to the things of the world as we live them rather than as we conceptualize or theorize them” (p. 41). Merleau-Ponty (1945) describes that “the most important lesson which reduction teaches us is the impossibility of a complete reduction” (p. xv). Drawing from the authors above, a complete reduction is impossible, so reduction was always an unattainable goal in this project. I kept a journal to keep track of my preconceptions and to register the research process through which I was going. By trying to suspend my personal perspectives, the unattainability of the task became more noticeable. As I

became more aware of my assumptions, I became more aware of how they influence my judgement and actions. This tension forced me to be vigilant of my own preconceptions during the research process as well as more open to welcome participants' perspectives since I was not blindly attached to my own perspectives. Moreover, keeping track of my perspectives allowed me to observe my own assumptions. Interestingly, this phenomenological attitude seems to resemble the Buddhist mindfulness of Varela and colleagues (1991). These two methods seem to share the common trait of detachment from goals, habits, or interpretations to reflect on the "here and now" of embodied actions. Buddhism changes unnecessary habits, and the phenomenological attitude silences preconceptions. Another similarity shared by phenomenology (Van Manen, 1990) and enactivism (Varela et al., 1991) is how the experience of reflecting on one's actions might transform an individual. Van Manen (1990) calls this process the *Bildung* (Education): "in bringing to reflective awareness the nature of the events experienced in our natural attitude, we are able to form or remake ourselves" (p. 7). Varela and colleagues (1991) borrow from Buddhism to transform habitual patterns of actions through observation. Therefore, talking about their actions, participants might realize or see their teaching practices from another perspective. In addition, Mason (2002) describes the practice of noticing with techniques and practices to foster the ability to notice and reshape our actions.

Research Design

I am interested in the experiences of elementary school teachers who claim to listen and respond conscientiously to students' mathematical thinking. I chose to interview five elementary teachers from grades three to eight about their experiences listening to student mathematical thinking. I have chosen to collect data through interviews to access how teachers attribute meaning to their practices in their descriptions rather than directly observing their classroom practices. I preferred to focus on what teachers find important rather than selecting what seems important through classroom observations. In addition, the time limitations of a master's program might not allow me to obtain the ethics approval from the varied institutions involved: schools and school boards.

Participant selection

I selected participants from varied elementary grades because listening is a valuable aspect of the teaching practice regardless of the grade level and to expand my possibilities of recruitment. The variety of teachers who responded also contributed to shape this research project. I have not included the kindergarten level because kindergarten teachers rely on listening

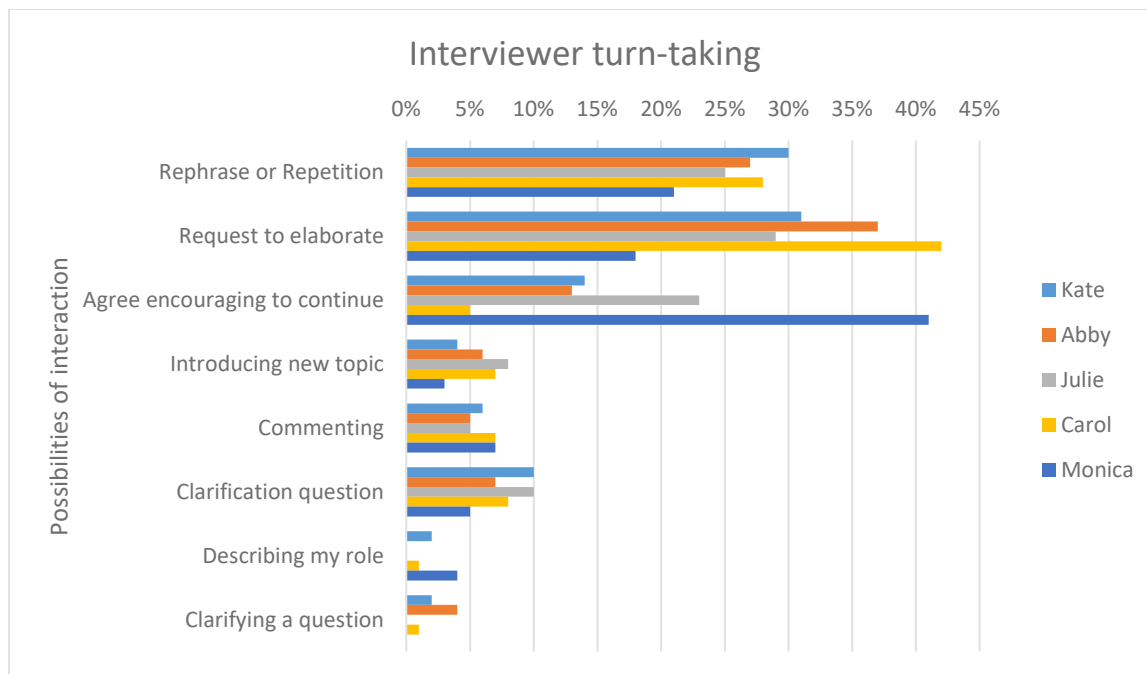
to students as their main method of interaction and assessment. I interviewed five teachers to explore a variety of experiences and perspectives. Four of the participants worked in the Ontario school system, while one worked in the Quebec school system. One of the participants was participating in a professional development initiative outside of the classroom. Another was doing a special assignment with grades one to four. The other participants were teaching grades three to eight. With the intention to be aware of my assumptions about these teachers, I kept a research journal to acknowledge my expectations and judgements related to the interviews. Participants were recruited through an advertisement (see Appendix A for a copy of advertisement) posted on a Facebook community for Ontario teachers, one visit to a graduate course at the University of Ottawa with permission from the instructor, and through my own and the participants' personal contacts. During the visit I handed out the recruitment advertisement. I also sent the recruitment advertisement by email. Participants were selected on a first come, first served basis.

Data collection

I conducted one semi-structured interview with each of the five participants (see Appendix B for a copy of interview protocol). I have chosen the following pseudonyms for my participants: Kate, Abby, Julie, Carol, and Monica. The interviews happened at a place and time convenient for the participants, and, were audio recorded. The phenomenological interview aims at “exploring and gathering experiential narrative material, stories, or anecdotes” (Van Manen, 2014, p. 315) and tries to establish a connection of equality between participant and researcher (Englander, 2012). In order to try to establish an empathic connection with participants, I disclosed my interest in the value of listening as a language teacher and my own limited experience as a mathematics teacher. In this way, participants seemed comfortable to share their experiences without fear of judgement. I have explored protocols and techniques to foster a reflective interview focused on participants' concrete experiences from the literature. Researchers have been observing phenomena through this lens more and more frequently. Laurinda Brown and Alf Coles (n.d.) adapted the phenomenological interview from Petitmengin (2006) to an enactivist perspective and she performs what can be called a process of interviewing by staying with the details. She proposes techniques that could allow for the interviewing process to stay with details. The interview is focused on encouraging participants to remember details of a specific event. In Petitmengin's (2006) study, she used this protocol to interview patients and

support their descriptions of moments of crisis. The techniques are: (1) stabilising attention, (2) asking how, instead of why or what, (3) repeating descriptive parts of what the person has just said.

In order to practice using this protocol, I decided to interview friends according to the techniques of the protocol. As I was doing these interviews, I felt that the interaction seemed artificial. Selecting one moment from years of experience is challenging due to the fact that memories become mixed with time. My friends seemed uneasy when asked to remember specific situations in teaching. Noticing their possible discomfort, I became anxious and told them not to worry making the situation even more uncomfortable. I owe my difficulty to the fact that I was “implementing a technique”. I focused on the techniques as prescriptive instructions, instead of focusing on the here and now of the interaction with participants. After this experience, my new goal became to create space for participants to share their experiences. The interviews took shape as participants were asked to elaborate on their teaching practices. My focus was to stay in the here and now of the interview and to listen actively to participants. During my data analysis, I became interested in also observing how I had acted in the interviews by analysing my turn taking during data collection. In order to observe how I had interacted with participants, I organize my interventions into eight different types of turn taking. These eight types of turn taking are: (1) rephrasing or repeating; (2) asking to elaborate; (3) agreeing; (4) introducing a new topic; (5) commenting; (6) clarification question; (7) describing role; (8) clarifying a question. With this data, I created a graph to observe how I intervened in the conversation.



The three most common types of intervention were (1) requests to elaborate on a topic; (2) repetition or rephrasing participants' utterances; (3) agreeing to encourage participants to continue. It seems that despite not having focused on the techniques, I ended up acting similarly to the techniques suggested by the protocols. I directed participants' accounts to their experiences by asking them to elaborate, for more details or examples. I adopted a position of curiosity similar to the one described by Paley (1986) as the main ingredient of a good listener. Paley (1986) describes a good listener as someone who echoes the voices of speakers and allows their understandings to come forth. I would often repeat or rephrase what participants had said. With the objective of being aware of my preconceptions, I kept a research journal in which I explored how each interview resonated with me and how I have been transformed by the encounters with participants. In this project, the findings describe how I would like to continue my own process of constant transformation.

Interpretative approach and analysis

I transcribed the interviews without software support. I read the transcripts numerous times with the phenomenological attitude (Van Manen, 2004) and with the research questions in mind to identify stories or anecdotes of listening and responding to student mathematical thinking. For me, the phenomenological attitude means that I will be reading the transcript paying attention to my own actions. Noticing my interactions with the interview allowed me to understand what I see in the world of significance of each participant and how my lens affects

my views. I kept my attention open to any new aspects that might prove relevant to the practice of listening to student thinking. I made notes in my journal about the specific moments which identify teachers' actions and understandings of classroom elements. Next, I isolated these moments which resonate with participants' lived experiences of listening and responding to students in a word document organized in two columns. One column had a title to identify the excerpt and the other column contained the excerpt. I reflected on these specific parts without imposing an interpretation, that is, I played with possible relations between the excerpts as well as with connections among participants and to me by reading and reorganizing the excerpts in groups of similar themes. Examples of the themes were descriptions of classroom moments, how to listen to students, what listen to students entails, or why to listen to students. In order to make these relations more visual, I used a whiteboard and markers that allowed me to reflect on the characteristics of each participant. I created a visual interpretation of how each participant interacted within her environment. I wrote the pseudonym of the teacher and distributed the different groups with their characteristics around the pseudonym of the teacher. I took a picture of the portrait of each participant and contrasted these portraits to observe similarities and differences among the participants.

Once I discerned similarities and differences among the participants, I contrasted my ideas with the literature. Gallagher and Varela (2001) have proposed that findings from phenomenology and the sciences could be contrasted to broaden the knowledge of a phenomenon. Therefore, I explored links between previous research findings and the experiences of the teachers in this project. I used enactivism to understand how each teacher brings forth his or her own world of significance. Enactivism allowed me to discuss the dynamics of interaction proposed by teachers and what type of knowing seems to be fostered by the doing in these teachers' classrooms. I observed redundancies or variations among the experiences of participants exploring connections in the data until saturation. Saturation was reached when I was not able to see any new connecting points in the data.

Credibility and trustworthiness

A discussion on credibility and trustworthiness is relevant to attest for the soundness of this research project. Marshall and Rossman (2016) describe how credibility and trustworthiness are terms originated to align with the terms of reliability and validity from quantitative research. Credibility refers to the confidence placed on the researcher and the rigour of methods, whereas

trustworthiness refers to what makes data reliable. Some of the ways to establish credibility and trustworthiness in qualitative research are collaboration with participants, engaging in reflexivity, and searching for disconfirming evidence (Marshall & Rossman, 2016). I have attempted to incorporate these procedures in my study. As I am a teacher, I am familiar with the environment of teaching and find it natural to talk about teaching experiences. In addition, I have purposefully practiced interviewing friends in order to make sure I would allow enough space for my participants. I increased awareness of my habitual actions that could make participants uncomfortable and learned to avoid them. I have often returned to the data throughout the analysis and interpretation processes to verify how participants' statements related to the whole of the interaction during the interview. I would often return to the data to make sure that my interpretation of participants' words was not an exaggeration or influenced by my assumptions. As this research project is a phenomenological inquiry I kept a journal and often checked my own assumptions and preconceived notions to ensure that my interpretation of data was solid. I also provided enough verbatim data so that the reader can observe my interpretations and make his or her own conclusions. As participants in this project describe classroom moments and their interpretations of classroom moments, the stories and interpretations could be contrasted to observe similarities or differences. The similarities and differences among the five teachers interviewed incorporate multiple perspectives into this study.

Participants' perspectives

I have separated my data into two categories for the data chapters. The first category describes how teachers attribute meaning to their experiences and the second category describes actual classroom moments described by teachers. The data was separated in these two groups to allow observation of the correspondence between the actual experiences and the meaning attributed to these experiences. This separation resembles the submarine example described by Maturana and Varela (1987) in which the observer bridges the gap between environment and internal changes. The environment would be the teachers' descriptions of classroom moments in interaction with an environment, whereas internal changes would be the discourses chosen by teachers to identify their actions. Teachers are constantly surrounded by diverse discourses promoted by school boards, research, colleagues. These discourses are interpreted and become a part of how teachers describe their practices (Brown & McNamara, 2011). The first chapter discusses the internal changes of the system, that is, how participants see themselves regardless

of the actual interaction with the environment. The first chapter describes the background and context of the teachers interviewed as well as how these teachers attribute meaning to teaching and listening to students. The background and context of the teachers allow the reader to become familiar with the discourses and interests that constitute these participants. I have decided to present the teachers individually to provide a clearer idea of each one.

Kate: Let's keep it real

Story and context. Kate describes herself as a single woman who loves travelling, music, and tap dance. Kate studied music at university, but she has never been a music teacher. Kate is a mathematics and English teacher in a grade 8 French immersion program. She has been working at the same school for ten years. Kate teaches. She also works as a resource specialist at the school and visits other teachers' classrooms weekly. Kate's school is in a rural region of Ontario and Kate's Grade 8 group is a little unusual since it is located in a high school building. This arrangement creates a connection between the intermediate and secondary levels which might not be possible in other scenarios. Teachers have always been an important part of Kate's life. During high school, Kate's family went through a tough period and Kate's teachers helped take care of her and her four siblings. In this process, Kate became very close with her teachers. Kate actually had the opportunity to travel to New Mexico as a volunteer with one of her favorite teachers. Kate and her teacher built houses and worked on other initiatives in New Mexico. During this trip, Kate and her teacher talked about Kate's intent to become a lawyer. After the conversation, Kate realized that she actually wanted to become a teacher instead of a lawyer.

Kate's trajectory into becoming a mathematics teacher was quite fortuitous. She started as a French as a second language teacher, but she did not do it for long. Her principal at the time observed one of her lessons and noticed something interesting in her way of working with students. Kate describes what her principal observed in her lesson:

[The principal and I] just realized that teaching French was not really my thing because what I was doing was getting a new group of kids every 40 minutes and so I knew all the kids and had lots of relationships. [The principal] said "I do not think that that is really effective for you. I think you are actually good at working with kids" so [the principal] wanted me to be working with the same kids longer than 40 minutes. In order to do that, you have to be a classroom teacher.

The principal seems to have noticed how Kate established relationships with students and believed this ability would be useful for a homeroom teacher. Kate's move into teaching mathematics was unplanned, but she has fallen in love with it. At first, Kate was not sure she could do it, but the principal provided her with enough confidence and opportunities for development. She describes it in her own words:

I think I had a good administrator for a while, I still have good ones, but I had a particularly good one and he just gave me the leadership to be able to do what I wanted and he supported whatever ideas I came up with. (Kate follows with a hypothetical dialogue and then finishes her point):

Kate: There is a resource that I really think is good.

Principal: Did you check it out? Did you ask other people?

Kate: Yes.

Principal: Then buy it.

So, that is good because then it gives me, first of all, it gives me more resources, but it also gave me a bit of confidence to be like "ok, if he thinks that what I think is important enough to do or to get, then it must be". I think that was a big confidence thing too that he was like, "Yeah. I trust you, go ahead."

Kate's principal was very supportive and sent her to every workshop and initiative related to mathematics in the school board. Moreover, Kate's principal also allowed her to select and ask for specific training that she found relevant. Even though Kate did not have the academic background, she had access to resources and support. Kate turned an apparent challenge into a strength. When Kate started teaching mathematics for intermediate grades, she did not have any credits in mathematics or English in her academic background. Due to the lack of familiarity with the material, Kate's first year teaching was quite traditional – she followed the textbook. As a way to stay one topic ahead of the students, Kate would do the textbook material herself before working with students. She believes that it was useful because she asked herself lots of the questions that the students asked her in class. Kate believes that having gone through the textbook herself, she could identify with students' questions. In the beginning of her career, Kate was concerned with her lack of previous experience, whereas now she perceives her initial lack of exposure to mathematics as a strength rather than a weakness. This experience even led Kate to wonder if the best teachers might be the ones who have themselves struggled a bit with

mathematics. Kate is not sure about it, but it seems possible to her that teachers who struggled with mathematics and overcame their challenges would find it easier to identify with students' struggles.

The current school administration still offers financial and professional support for Kate to develop her teaching style and acquire resources. Moreover, Kate describes how the school remodeled Grade 9 Applied Mathematics classes to look more like Kate's classroom space and teaching style. Kate explained that administrators, teachers, and consultants observed her lessons in order to reshape Grade 9 Applied for the 2014-2015 school year. After this reshaping, the 2015 Education Quality and Accountability Office (EQAO) results from Grade 9 Applied were higher than in previous years. The 2015-2016 school year will be the second time that the remodeled Grade 9 Applied will do EQAO exams. Thus, if the stronger results repeat themselves, Kate feels that there will be strong evidence that the remodeling of Grade 9 Applied inspired by Kate's teaching style occasioned the increase in EQAO results. Kate explains in her own words:

It is interesting because there is a difference in how [2015 Grade 9 Applied students] have learnt and performed in EQAO. So our Grade 9 Applied results from [2014] compared to [2015], [our school was] one of the lower schools in our board for kids who were achieving at level 3 and 4 and then, [in 2015] we jumped up by forty-five percent. We completely revamped the way Grade 9 applied got taught, we completely transformed it to look like [my classroom] so what [school administrators] noticed [is that] it was way more hands-on, more choice, more interactive. It was way less paper business and the scores actually on a paper and pencil test, the scores still jumped.

Kate feels that Grade 9 applied students seem to need more time to explore, investigate, and discuss mathematical ideas with concrete activities rather than work with paper and pencil exercises. Kate is very excited to see 2016 EQAO results because it would be a pleasing confirmation to see if the Grade 9 Applied students will consistently achieve higher marks in EQAO exams. Kate would feel more confident that what she has been doing with students has helped them learn and engage with mathematics at a more meaningful level.

Meaning of listening. For Kate, listening to her students seems to require transparency as well as genuine interest and respect for one another's opinions and values. Kate shares stories of her own personal life with students, such as the time when she got stranded on an island in

Australia and she allows students to also share personal aspects of their lives. Kate thinks it is important to listen not only to her students' mathematical thinking, but also to their stories and interests. She believes that talking to students about everyday aspects of their lives might make students feel more comfortable to talk about their mathematical reasoning since a connection between them is being built. Moreover, Kate uses personal content shared in class to prepare the problems that students solve in class. All the problems in Kate's classroom involve contexts which are familiar for students, such as school staff or students and their relatives. Kate gives an example of a possible type of question:

So, it could be something as simple as one of my teaching partners and I are both single, we like to spend money and we like to go on trips and the kids know this. We are pretty transparent with the kids about how we are and we both work two jobs. I might put together a question like I did the other day about how [my teaching partner] and I want to save money to go to Greece this summer, which is true, and the kids know. So, we put all this money in our bank account but then Miss Bell had to buy new shoes so she went to buy new shoes and I had to go pay for my dance lessons. It was integers, deposit, withdrawing, but the kids think it is hysterical because they read this question and they say "This is so what would happen."

Kate believes that students seem to relate differently to problems involving people they know. According to Kate, when her students are working on mathematics problems, they identify characters in the questions by name. Kate says that: "The fact that they actually identify the people in the problem might make it more real because it is somebody they know, somebody they have an interest in." Kate believes that if students did not know the characters, they might only focus on the numbers and not the whole context of the problem.

Kate's problems could also be born out of conversations in class. Kate remembers a problem about how the US was trying to steal Canada's beaver tail recipe and the Canadian military had to protect it by patrolling the diameter and perimeter of a circle:

The whole defending Ontario from the Americans stemmed out of a conversation that [I and the students] had because I used to work at the duty-free on the border. And I would tell [the students] all these crazy stories about the stuff that would happen.

Even though, this problem does not refer to a real military menace between the US and Canada, this problem seems to be real for students. This problem relates to conversations that took place

in the classroom, hence, the relevance and proximity of the topic seem to make these problems real and significant for students. Kate believes that student engagement with problems that are real for them might lead to better understanding of mathematical concepts:

...their learning cannot progress if you do not actually listen to what each kid needs. Also if [the problem] is not real to them, they are not going to engage. You cannot make it real unless you talk with them and find out what is real, unless you give those opportunities where you can talk about things, not just math things, but where you talk about things. Then, I feel like you can reach [students] a lot easier, a lot faster, and a lot more effectively. If they buy it the first time, you do not have to do it a third, a fourth, and a fifth time, if you tab in what they are into the first time, then you can move forward.

Kate believes that if you find something that is relevant for students you do not need to practice the same concept numerous times, this means that she prefers to have students work on one problem for the whole class than to do thirty problems about the same concept. Her goal is to make the one problem they are studying relevant for students. In accordance with this practice, Kate assigns no homework for students. Kate believes that as a teacher she appreciates having a break from school and she sees no reason why students would not also appreciate a break. In addition, Kate explains that if students decide to do mathematics outside of mathematics class, it should be due to their own will.

Kate often uses the word real with the meaning of relevant and meaningful. Kate also seems to look at schooling and her students' learning processes in a realistic way:

I am very realistic to the fact that a lot of [the students] are not going to need some of this math that I teach them, but they are not going to ever forget how we learned it. They are not going to forget all the times that we spent laughing, they are not going to forget all the times that we were doing some crazy assignment, and they are not going to forget how hard they worked. I just feel like math is important, but not as important as the relationship that I have with the kids so if they are not feeling good then they are not going to learn their math.

Kate's main goal in education seems to be establishing relationships with her students. She describes how she needs to trust her students and to trust that her students will go talk to her if they have questions or problems. In addition, Kate also extends the need to establish

relationships to parents and school administrators. Parents and school administrators need to trust the work being done by teachers and students in the classroom:

I think establishing relationships is the most important thing. I think if you do not have relationships with any of these people (students, teacher, parents, and administrators) or if one of those is missing, then there is no point. We are not teaching math, we are teaching kids and if that is not our goal, myself, my administrators, or even the kids, then there is no point in anything.

Kate feels that she would not run as many risks if she was new to the school and did not have relationships with the parents and the community. She is aware that she would not have tried changing the classroom structure as much if she did not have support from the community:

We got some good feedback from the community as well. I think that that was helpful. Parents in a small town are pretty vocal. So, they do not hide how they feel, it is blatantly clear when the parents in this community are happy and when they are not. If parents in this community are happy, then administrators are obviously much more willing to take risks and willing to let [teachers] take risks in [their] classrooms.

Kate seems to promote classroom dynamics that allow her to listen to students and to establish these relationships. In order to encourage a listening environment in the classroom, Kate believes that letting go of classroom control is essential. Kate feels that her classroom dynamics only work because she does not need to have control of what students are doing all the time in class. Kate states that she encourages students to look for answers among themselves and to make their own decisions while working. Furthermore, she also encourages students to look at their own work and ask themselves if their answers make sense:

A lot of [the students] never did that before, they just think “I got an answer. Perfect” and hand it in. They would not stop and say “Does this make sense?” so the group [work] is good for that too because the other kids will say things like “Wait! That doesn’t make sense” to somebody else’s work. Therefore, it takes this responsibility out of the teacher’s shoulders and encourages the group to take agency.

Kate also states that she gives students as much power as she can to make their own decisions and find their own mathematical solutions. Kate believes that each student is unique, therefore, each student might learn and express mathematical thinking differently. In Kate’s classroom, she welcomes diversity of thinking:

I try to give [students] choice, as much as possible, so for learning integers there is not one way, there are three [ways] or if we are calculating circumferences, there is not one way, there are two or four, and then they pick the one they like the best.

For Kate, offering choice and giving voice to students seem to relate to her description of how she tries and makes the lessons real for students. Kate wishes lessons to be relevant for the students by welcoming their perspectives.

Kate describes the quality of listening in her interactions with students. Kate feels that listening includes more than just the sense of hearing. It seems to be a combination of seeing, doing, talking, and listening. Kate feels that listening to a student requires an effort to be attentive and engaged. Moreover, it also seems to demand an effort not to focus on other things that happen in the hectic environment of a classroom:

If I am not actively telling myself to focus on something, I will focus everywhere. Part of that is just being a teacher, I guess. I think that you are constantly managing situations so I am looking somewhere, but if I want to focus on a kid I have to actually force myself to ignore whatever else is happening. I will say, just in my head, “do not worry about what is happening over there, those guys are drawing some kind of happy face beside the map, I do not care right now, I need to be here.” Otherwise there is no point in you being there and having that conversation with that kid because it is not going to be meaningful if you are not actively engaged in it.

Nevertheless, Kate says that she is not capable of actively listening to the thinking of the thirty-two students in class every lesson. Thus, Kate sometimes videotapes or photographs her students in the process of solving and/or the product of the solution. In this way, she can go back and understand how students reached a certain solution to be able to provide support for them if necessary. Thus, Kate uses pedagogical documentation as a way to assess student work.

For Kate, confidence is a fundamental aspect of a teacher’s work, be it her confidence or her students’ confidence. Kate describes how she feels more confident when she notices that students are engaged and more confident of their mathematical skills. Kate presents examples of moments that she believes demonstrate students’ engagement, such as, hearing a student say “Now, I get it” in a group discussion or seeing three students labeled as weak build on each other’s understandings to solve a problem. Kate believes that her confidence in students augments students’ confidence in themselves. As a cycle, students’ confidence on themselves

will feed back to Kate's confidence. Kate did not seem completely sure to be doing the right thing, but she seemed confident enough to keep doing it. Actually, not being completely certain seems to keep her open to new ideas, suggestions, and willing to change.

Abby: Rediscovering Mathematics

Story and context. Abby began her career as a teacher outside of Canada. As her qualifications were not accepted by the Ontario College of Teachers, she did her undergrad and her teacher education program in Ontario. After Abby's teacher education program, she started a master's at the same time that she started working as a substitute teacher in Ontario. When Abby finished her master's, she also obtained a teaching job and since then, she has worked in different positions. Abby has taught grades 2 to 5 and worked with reading programs and special education. In one of the schools where Abby worked, she was lucky enough to be responsible for attending workshops on problem solving and sharing the content with the school team for a year:

A couple of years ago, there was a collaborative inquiry group in the school that I was at because I was in the leadership team. At that particular school not many teachers valued learning. Whenever there were learning initiatives, the principal would ask me to go because no one would volunteer for it.

In one of these workshops, Abby was introduced to the three part problem solving technique. Abby employed the three part problem solving technique in the 2015-2016 school year. The technique will be described when discussing Abby's classroom moments. In the 2015-2016 school year, Abby was supposed to teach grade 4, but she ended up taking over a project about measurement and problem solving. Abby says that it was hard for the school to find a teacher for the measurement and problem solving project because most teachers found it to seem like heavy work. Abby found the assignment challenging at the beginning, but she feels grateful that it strengthened her love for mathematics. Another advantage of the project was the positive relationships that Abby believes to have developed with students and homeroom teachers. She feels that she has learnt a great deal through this project. Due to the success of the measurement and problem solving unit, Abby was given a new assignment to teach a digital literacy program in the school year of 2016-2017. This digital literacy program will combine computational thinking and mathematics. Grade 1 students will learn how to program Lego pieces to move using an iPad. Abby took workshops on scratch coding during the summer to be able to tackle her new challenge.

Meaning of listening. Abby's one year assignment about measuring and problem solving has affected the way she listens to students and what she understands as listening to student thinking. Abby feels that her ability to listen to student mathematical reasoning has improved during her year in the problem solving project. Abby notices that small actions in how she listens to students have changed:

I know that I am doing it better because during questioning, not only do I pause to allow the kids to think but while they are talking I make sure I am recording some of the things that they say so that when I have to give them feedback, I can look at what they said to see if it matches what we are talking about and the end product that I am looking for.

We can see the two specific actions that Abby seems to be doing differently. She allows more time for students and keeps track of what they are thinking to verify if she has reached the goal of the lesson. However, Abby was the most excited when describing how the assignment developed her own and the students' confidence with mathematics. It seems that encouraging young students to express their mathematical thinking allowed Abby to realize how mathematics is a discipline to be explored by everyone:

...some students see math and they do not even try to want to learn it, they see numbers and the numbers become a barrier to their learning and one of the things that I showed them is that "You know it, you can do it. Go ahead and do it. Do not allow the numbers to be barriers. You can solve it, they are just numbers. If you try, you will get it. Go ahead, give it a try." I think the kids realized they were young mathematicians themselves because when we do the reflection at the end, the words they used to describe were "interesting, fun, different, embracing math"

Listening to students and seeing student confidence increase might have allowed Abby to see mathematics from another perspective.

Abby states that another important consequence of the problem solving assignment was the relationship that Abby developed with the students and the teachers. Abby feels that the homeroom teachers and students became much closer to her and demonstrated appreciation for the positive results achieved by the assignment:

I had my final session in the classroom and the teacher sat and she listened and after [the students said one word about their experiences] she said she could not let me go without having an Ipi ipi uha. So we did an Ipi ipi uha and they clapped and everybody came and

gave me a group hug. It was nice because she said she saw the change that has happened to them.

In order to create an environment in which students could express their reasoning confidently, Abby found it important to define the success criteria with students. Abby believes the success criteria encouraged students to share their thinking and to give students a fair chance to succeed. During the school year, the success criteria were revisited and students included ideas, such as everyone needs to participate. Some examples of what is part of the success criteria are: students need to write what they know about the problem and what they need to find out, students need to write solutions clearly so that anyone can understand the answer. During the interview, Abby emphasised that students developed the success criteria by themselves, but she also made sure that the language expected of students was part of the success criteria:

Based on the examples I did, we did as a class, they came up with the success criteria after I showed them the strategies and tools. I used a problem to show them the different strategies and tools and based on that I worked with them. They came up with the success criteria and they came up with [the criteria] on their own...I would rephrase to have the language that is needed but they came up with the ideas.

When asked about the meaning of listening, Abby referred to how important it is for students to listen to one another and to the teacher. She describes that as an essential element of a good classroom:

When I did the strategies and taught them how to do the problem solving, one of the things that I pointed to was to listen to each other and listen to what your classmates are saying, if they were not listening to their classmates, they would not be fully able to communicate their mathematical thinking at all, so they have to listen to what their classmates are saying, when they are communicating their thinking, then you are better able to give descriptive feedback to the classmates and by listening it is enhancing their growth, as well as their classmates growth, they have to be listening to me and they have to be listening to their classmates. If they are not listening, then, they will not learn.

When asked about how she attributed meaning to her own listening, Abby described listening to students as an essential way to identify if the students have reached the end product which was the goal of the lesson:

I have to be listening to what they say, what it is that they have said, if I am not listening I am not able to take them along the growth continuum because I would not know if they have learned if I am not listening, I have to be listening to the different presentation, listening to their interpretation of the problem, to know that it was not the correct interpretation and to take a different path so I can teach them the correct path they need to take to be successful.

Abby also highlights how recording students' comments is important to be able to refer to them later when providing feedback, supporting the understanding of other students, identifying weaker groups which need more attention, or checking if the goal of the lesson was achieved. Abby also describes how clarifying students' comments is important and how listening is the only way to be able to clarify students' comments. Moreover, Abby talks about how her attentive listening is also a way to model how students should act in the classroom.

Abby also describes how she listens and asks questions to help groups that seem to need more help:

Based on [students'] discussion, I would pose problems or guiding questions that would put them back on the path. Let's say I was doing area and their discussion was based on finding the perimeter. I would say "Remember now, if you are going on the outside that is the perimeter, the things that you are measuring is the inside, and sometimes even do some teaching in some of the groups that finding the area you look at the inside, whereas if you are finding the perimeter, you would have to count all the sides to get the perimeter of the object." Some groups that you visit you have to pose guiding questions, for example, the level one you really have to go and work with that group and put things differently "What do you think I would get if I did this?" "What do you think about this?" you really have to be channelling their thinking because they are not going to be having the mathematical communicating that the level four or three would have so you have to listen to what they say and take what they say and turn it around and pose it back to them and build their self-esteem by putting what they have just said in a different way to help them move along the growth continuum.

Abby seems to find it important to direct student thinking to the definitions that were taught in class so that students are in the right path. The right path in this case seems to be the path presented by the teacher. In addition, Abby seems to have preconceived ideas of the abilities and

skills of different grades. However, Abby talks about how she was surprised with the abilities of the lower grades:

I taught the strategies to the Grades 3 and 4 and that was easy for them and then I introduced it to Grade 2 and I thought I would not get the level of mathematical thinking from Grade 2. That is what I was expecting, but I was blown away because as you brought it, the Grade 2, they got into expressing their thoughts and how they thought mathematically. I enjoyed that totally because I thought that it was such a hard concept that problem solving to Grade 2. They found a pattern, they could use a model, and they could guess and check and just the way they expressed their thoughts.

It seems that Abby was attentive enough to realize that students were capable of more than her expectations and to revisit her expectations of what Grade 2 students. Abby was capable of adjusting her expectations to the reality of students' abilities.

Julie: A relaxed perspective

Story and context. Julie sees teaching as a mix of different roles: teacher, learner, and researcher. Julie had an opportunity to become an observer in mathematics lessons for three years. Julie's impression is that while she was an itinerant teacher, she was closer to the researcher and learner aspects of the profession. When she describes classroom practices, she does it choosing words commonly used in research such as data. In her speech, it is clear that she identifies the roles of a teacher and a researcher as being very close. Julie comes from a small city in southern Ontario and moved to Ottawa to attend university. After concluding her studies in education, she obtained a full-time job as a teacher and decided to stay in Ottawa. Julie has taught mathematics as well as language arts for middle grades for 15 years with her first year being in grade six. She is fond of and feels comfortable doing mathematics, despite not considering herself the fastest person at doing mathematics. The interview for this project was Julie's first experience as a participant in academic research. Yet, she has taken advantage of every professional development opportunity and appreciates the varied and frequent development opportunities offered by her school board. She has also pursued a variety of additional qualifications including vice-principal training and special education. In 2014 Julie had the opportunity to join a new initiative for professional development on which she is still

involved. This project is and has been very important in shaping her understanding of listening to students and mathematics teaching¹.

It is a three-year-pilot project supported by the school board. The official name for Julie's role is an intermediate itinerant teacher. The project consists of a group of seven teachers who are exempted from their teaching responsibilities and each one of them individually observes intermediate mathematics lessons of seven different teachers across the school board. Consequently, a total of 49 teachers having a partner to support professional development in the classroom. These teachers go into intermediate mathematics classrooms and work in a co-learning stance with the host teacher. They focus on observing students' experiences and sharing these observations with host teachers. Julie prefers to take notes of her observations and strives to do it in a way that is non-judgemental. For example, she would write that a student has his back to the teacher, or is tapping his pencil instead of writing down an interpretation of the action - that the student is not paying attention. As she said, she tries and writes down only what she sees and not what she thinks she sees. From her observations, she looks for patterns or themes that might emerge, either for individual students or for the group as a whole. Julie's observations might have a specific purpose depending on the challenges of the group, previous observations, or a suggestion from the teacher. Julie also tries to be mindful of what she is failing to document and her reason not to focus on something. The sharing of observations with teachers is done in the most convenient way for the teacher. It might be right after the class or throughout the class depending on the teacher's schedule. They engage with teachers in a conversation about what they noticed and what they are wondering from their observations. Consequently, the teachers start talking about possibilities and what next steps to take to support students' understanding. As Julie said: "we start to explore and pick that kind of right inch to go deeper at impacting student learning." Currently, the seven teachers visiting the classrooms are gathering evidence to observe which themes emerge across the school board. The goal is to see what can be learned about student learning and pedagogy from these observations of almost fifty teachers in the school board. So far, they seem to agree that teaching is highly dependent on contexts and individuals.

Meaning of listening. Julie feels that she started listening to students through a gut instinct to do things differently. She mentions how her reflection on her teaching practices and

¹ This description of the project is my interpretation of the participant's account. It is not an official description approved by the school board.

her interaction with colleagues seem to have all influenced this feeling that she had to change the traditional way of teaching. She describes two specific moments in class that allowed her to value listening to student thinking. One moment in particular allowed Julie to realize how students could be more independent than she would have expected:

One year, I was sitting with a small group of students because I thought this group needed my guide and support. Later, I realized that I had been engaged with them for a while. I thought, “What are the rest of them doing?” I looked around and they were working independently, some were helping another person, some were saying, “I don’t get it, maybe, I will go see so and so.” I could have left the room and they still would progress and build this understanding together.

Julie seems to have realized that students can keep working successfully without her gaze and felt more comfortable with the idea of allowing students to work on their own. She noticed the benefit of having students discuss their different answers to figure out which answer is right without the intervention of the teacher. Therefore, she states that her first conscious attitude to promote student engagement and classroom discussion was to bite her tongue. Now Julie realizes that she needs to refrain from talking to allow students to show their thinking. She feels that by not speaking, she creates room for students to speak. Through listening, Julie feels that she can learn so much more about what students are capable of doing and how they construct their understandings. She describes a moment in which she bit her tongue and noticed how the situation unfolded:

Two students were rolling the dice and making fractions, so they roll the dice and they have to construct a fraction with whatever the dice said. Then, they have to decide whose fraction is bigger. One boy had $\frac{4}{1}$ and one boy had $\frac{5}{5}$ and they came up to me and said, “Well, we don’t know which one is bigger.” If I had been teaching traditionally I would probably have just told the students how to do it or how to figure out and I might never have noticed that they would come across that. It would have been like, do this, do this, do this. I thought how surprising it was that they could see the difference. I just said, “Well, can you draw it?” So they went back, sat down, and worked on their drawings. They came back and said, “We drew it and we talked about it and we both agree that $\frac{5}{5}$ is the biggest one.” I never thought that a kid might think like that, so I asked, “Why do you think that?” to which they said, “Well, because the numbers are bigger.”

Julie was really surprised that the students would have this misconception because that is not the way she sees fractions. Julie realized how much she had been missing about her students' thinking so she decided to have more of those discussions and opportunities for students to debate and explore misconceptions.

Another important experience that allowed Julie to start the process of shifting her teaching practices was related to the school culture. In Julie's career, she had some challenging students with special needs and for a period, she had a special education teacher to aid with those students. This special education teacher would not take the students out of the classroom, she would participate in the class. Julie noticed a great difference between the type of relation she had with this teaching aide and the other teachers.

I think I felt that the culture of the school I was in, whenever we had opportunities to talk to each other, which was usually on recess duty or lunch duty, we would sit and we would just vent, we would just complain about everything and that became the culture. Near the end when I was in the classroom, I was fortunate enough to have a special education support teacher.

The environment of the teachers' room did not seem very positive. Julie began to have lunch with the teacher aid instead of the teachers' room. Julie and the teacher aid would talk about the students in a more positive way with the goal of finding a way to reach out for students: "...our chats over lunch were now more about the kids and the challenges we were having, but not in a venting complaining way, in a what-are-we-going-to-do way?" The goal seems to be the improvement of classroom experience for teachers and students.

Julie believes that her three years as an itinerant teacher have also made her more capable of listening to student mathematical thinking. Julie feels that as a teacher she was too involved in the immediate demands of a classroom to be relaxed enough to have insights and review her assumptions in the way she did during these three years of professional development.

[As an itinerant teacher] I am in a relaxed position and I get to choose what I want to focus on. When I am the teacher, there are 30 kids and I am trying to focus on all of them at the same time and on the content knowledge that you are trying not to mess up and there is a behavior going over there, this phone ringing over there, so and so is out of his seat again.

Julie is curious to see how her teaching practices will be different after these three years observing lessons and reflecting on student voices from a more relaxed position. In Julie's words: "I feel sometimes far removed from the actual classroom experience and I'd be interested to see how I can take my learning and apply it when I go back". Julie wonders how she will deal with certain challenges, such as time constraints: "The reality of classrooms, there is only so much time in the day". When Julie returns to her school, she would like to encourage a shift in the culture of her school by promoting more encounters and exchanges among teachers. Julie would like to encourage teachers to share their successes and exchange activities as well as sharing their difficulties and doubts. Such an exchange could help teachers feel more comfortable trying new things.

Sharing the successes, but also being open to sharing challenges like: "I did do this, I did do that, it did not work, and I do not know why." Because I think sometimes as teachers we need to be perceived as being knowledgeable, and having all the right answers, but we also need to model that it is ok to make mistakes and that we can always learn and we can always get better and I am hoping that as a culture we do that too.

For Julie, the new understandings of teaching that she wishes to take into the classroom seem to be closely related to listening to student thinking. Student thinking seems to be the focus of what teaching means to her at the moment. Julie describes the new place from which she speaks:

You do not start with the textbook, you start with your learner. I think it is just a total shift, going from talking to listening, from teaching to learning. It is positioning myself more as a co-learner with my students. I think that is really kind of where I always felt I wanted to be and I think I am getting closer to doing that. I think I could never pull myself away from that anymore. [Seeing] other teachers that have sort of the same mindset, I think it helped me look at my preconceived judgements of things and students.

In order to achieve the goal to start with the learner, Julie feels that it is important to create conditions in which students feel comfortable to speak, especially intermediate students who might prefer not to expose themselves. These conditions include having enough time for students to think about the problems before asking for an answer and allowing for a variety of students to provide answers. Julie also thinks that listening to student reasoning is important because she has access to student misperceptions. Teachers might never know how students

think mathematically if they do not take the time to listen to student thinking. As an example, the story of students who believed $5/5$ is bigger than $4/1$ because 5 is bigger than 1.

Julie seems to be at a stage in her own professional development in which she has changed her thinking and her idea of classroom practices. Julie has done some teaching using these new perspectives. However, Julie is not completely sure how to enact her insights in the classroom and she wonders how to incorporate all her new experiences. Her biggest concern is time to be able to do everything that she considers relevant. Therefore, the shift from telling to listening has been done in her thinking, but she might find other challenges and make new discoveries in the process of enacting her ideas when she goes back into the classroom. In the school year of 2016-2017, she will probably go back in to the classroom and she is eager to try the new things that she has experienced.

Carol: Teacher as artist

Story and context. Carol feels that teaching is an art. A painter knows the colors, the design, the materials and how to combine them on the canvas, whereas a teacher knows about assessment, management, learning theories and how to relate these in a manner relevant for students:

If you are an artist you understand your basic elements, you understand design, you understand color, you understand those things, and as the artist, you put them together in different ways to create different artifacts. I guess, that is what my days are like, they are different artifacts of my artistry with the children.

Carol studied political science and moved to northern Canada to foster community development. She ended up staying and teaching in the Northwest Territories for two and a half years. Carol's work with education in northern Canada motivated her to enroll in and complete a teacher education degree at the University of Ottawa. In Ottawa, Carol started teaching French and English as a second language for middle grades. Carol always took advantage of every professional development opportunity available. Not long after, Carol started facilitating professional development sessions for language teaching. Carol came back to the classroom in 2010 to teach mathematics and English. Carol's interest with mathematics teaching was unexpected, but she quickly developed a passion for mathematics teaching. After facilitating training sessions related to language, she was assigned to facilitate a training session about mathematics without any warning or indication of why she was chosen to facilitate a

mathematics session. Carol found it to be a strange choice since she had no experience with mathematics. Carol had no doubts that she would be honest with the teachers at the session about her lack of experience teaching mathematics. Carol proposed that teachers develop their thinking together as a group in the sessions. Parallel to the work as a facilitator, Carol began attending any professional development opportunity related to mathematics. She would take the activities done in professional development sessions outside of the school board into the session that she was facilitating.

Carol learned mathematics with the traditional transmission model; consequently, she saw mathematics as rote memorization. Through professional development courses, Carol developed a new understanding of mathematics and how to teach it. Mathematics became more attractive to her and she relished opportunities to engage with all the different facets of thinking mathematically. Mathematics became exploratory and exciting in a way that it had never been for her. This experience was so life-changing that Carol decided to go back into the classroom with the goal of modifying the way students experience mathematics. In the last five years Carol has been teaching mathematics and Language Arts at an inner-city school in Ottawa. Carol usually teaches grades seven and eight, but, in the 2015-2016 school year, she decided to teach grade three. Carol's goal was to become familiar with the continuum of mathematical development in the elementary grades.

Meaning of listening. Carol seems to be making an effort to listen to students since the beginning of her teaching career. She started her career with the goal of fostering development in northern Canada and to give voice to underprivileged demographics. Evidence of Carol's willingness to listen to students is reflected in her attitude in the mathematics professional development workshops that she facilitated. She felt at ease making room for the teachers and inviting them to develop together. Nevertheless, Carol emphasised a specific experience that she believes to have contributed to her listening to student reasoning. Carol had teaching partners in her classroom for two years and she believes that having a teaching partner led her to improve her ability to listen to student mathematical thinking. Carol was working with the intermediate itinerant teachers. She was one of the teachers observed by Julie in her role as an itinerant teacher. Carol had a different itinerant teacher for each year:

My listening has changed especially in the past four years after working with a collaborator. It is about having two skilled people in the room, one of them is teaching

and trying to be responsive to students, while the other one is documenting. So one would focus and document during that lesson, or that class, what students said and did. Then, we would talk about it and figure out where [students] are, what the next step for them is. And so I think starting with that very intentional partnership, it has now become part of my habits.

Carol feels that the process of registering student thinking to reflect on how to follow-up with students allowed her to incorporate listening to student thinking as a habit of her practice. Carol states that she now writes down students' comments and questions to be able to remember and address those comments. Carol would take notes of observations, make sense of them with her teaching partner, and respond to students. This shared process, with reflections from another skilled person, allowed Carol to see from a different perspective. It allowed Carol to see things that she might not have seen while teaching and responding to students.

Carol also feels that what has changed in her actions is her understanding of what she is listening for when interacting with students. Supporting student mathematical thinking requires an ability to listen for something that is not right or wrong answers. Carol describes what she developed as a focus to be able to look past traditional classroom interaction. Most teachers only experienced traditional classroom interaction as students and developing a new structure for interaction is challenging but should not be overlooked. Carol seems to have perfected her approach to support student thinking:

I think it is the skill of what you are listening for that gets better as you pay more attention to it, what I am listening for in terms of their thinking. For example, "Do they have a logical answer?" "Does the reasoning make sense?" "Do they have a misconception about something?" You could figure those things out by listening to what kids are saying about the math that they are doing. You have to get them to talk about it first or show you. That is how you can ask more questions if they do something that does not make sense to you. You can ask them about it because maybe it actually works, you just never thought of it. There could also be some kind of misconception, sometimes things happen, like, they are just lucky.

Carol seems to strive to listen for the logic in students' answers. She seems to be listening for how students make sense and express their reasoning. She also describes how she has learned to listen to big mathematical ideas, such as commutative property. She has learned about

mathematics through reading the curriculum, research studies, and Ministry of Education monographs. Carol feels that these readings allowed her to have a deeper perspective of the mathematics.

Once Carol started teaching in a way more responsive to students, she felt that student engagement was higher. Carol attributes this change to the fact that students were learning about what was meaningful to them. Carol reports that improved student engagement encouraged her to further develop her listening and responding to student thinking:

It was a seven/eight split so I had a couple of students that I taught for two years.

Students with math anxiety, students who would just shut down when it was time for math - head on the desk saying: "I cannot do it, I am not doing it." I saw those kids turn around to the point that at the end of the year, my teaching partner and I could sit down and they could teach the class. Seeing that happen was powerful enough for me, having kids that say "I hate math, I cannot do it", turn around and say "I get it. I feel so good about it that I am going to teach everyone else about it" I do not know what more I could want.

When asked to describe how she listens to students, Carol talked about the need to establish relationships with students. In Carol's experience, she develops trust relationships to create an environment where students feel safe. It seems like Carol wishes to establish an environment of complicity between herself, students, and parents. When asked about how she established a relationship with this student in particular, Carol describes her attitude and actions:

I think I established a relationship just by listening to her, trying not to judge her, and having a ton of patience with her. I would let her get mad at me, never turn my back on her. I tried to realize some of the things in class that she was struggling with and then trying to help her with them. I do not really know what it exactly was. A lot of praise, tons of praise, because I also think she had the experience of being told that she was wrong, or telling herself that. So, there was lots of filling that cup with positivity, and calling her mom and saying she is doing really great, like "this is what she did today, this was really great" so that she was getting some positives.

Carol says that she finds it important to extend this relationship to include parents. Carol emphasises the importance of having parents as partners.

Carol uses a metaphor to describe the process of establishing relationships with students - *building real estate*. Carol says she would prefer to have a less capitalist metaphor, but she could not think of a different one:

I build a lot of real estate with my students, I call it real estate. I try to build them up as much as I can because I know some day I am going to have to take something away, but when I take it away or when I get angry, I have money in the bank. You make deposits, so then you are not coming down to brand new, you always got something in the bank to work with.

When Carol says that she builds them up as much as she can, she means that students are allowed to get angry, but they are not allowed to say mean things or hit anyone in class. Carol describes how she respects the fact that students might have bad days as anybody else. In addition, students can choose how they prefer to do an activity or if they prefer to do a different activity on that day. Carol explains that in her classroom, there are mandatory activities, but students are not forced to do everything that she proposes. This metaphor of real estate seems to indicate that Carol sees the relationship of teacher and students as a relationship of equality. Students seem to have the same rights as the teacher to have a say in what happens in class. Carol describes how she finds it important to be the authority to keep students safe, but she allows them make choices about how they prefer to work. Carol's goal seems to be to create a welcoming environment where students feel safe to express their feelings. Carol strives to make it clear for students that she will always be there to listen to them.

Carol also believes that listening to student thinking fosters students' confidence and sense of self-worth. She seems to be very focused on making the lessons about student reasoning rather than her own reasoning. Carol says that she tries to remind herself of a quote about the one who does the work, does the learning. From Carol's perspective the learning is the thinking. Carol values student reasoning in hope that students will also see this value. Carol has been working on shaping instruction based on student thinking. She is constantly trying to be responsive to students by being attentive to what they are saying and doing:

Each of the students are individuals and they come to class with different knowledges, different ideas, and even different goals. It is listening to know that whole kid, it is listening to ensure that the students feel like they have value. Students should know that

their ideas are valuable. A teacher should ask students why and talk about what they think, even though you might find student thinking to be wrong or unsophisticated. Carol's need to listen to students seems to relate to a certain respect for diversity since she believes that each student is different and this diversity should be valued. The respect for diversity also reflects on how Carol assesses student work. Children are developing their mathematical understandings and every type of solution should be encouraged. Carol seems to want the time spent in the classroom to be pleasurable. Carol herself loves playing with mathematical solutions and she wants to encourage students to delight in the pleasure of mathematical thinking. The unpredictable interaction with students and the possibility to integrate different ideas and perspectives in class seem to be the most pleasurable aspect of teaching for Carol. She believes that sewing all the different ideas of students is the artistry in teaching.

There is the craft and that is knowing what to do and all that kind of stuff and assessment and knowing that I have to make a judgement at a certain point and I have to give feedback that kids have to know how they are doing and what they need to do to get better. That is my job, to give them that feedback and to be that assessor, so those are the things that make it when you develop that craft to be able to do that and to know how to make decisions, what decisions to make next. But the creativity in how things look different all the time, that is like art to me, and what I love about it.

Monica: Taking a positive stance

Story and context. In her teaching career, Monica taught many students that were considered to be challenging, such as students in complicated social economical situations or students with disabilities. Monica saw all these experiences as great opportunities for professional and personal growth. In her stories, Monica seems to celebrate every challenge in her path. Monica's career started in 1991 and other than a number of years off for having children, she has been teaching ever since. Monica has taught everything from kindergarten through to the end of grade 8. Monica has also taught teenagers first aid and driving lessons. Monica feels like she has been a teacher for life since she was the oldest child and cousin in her family. Currently, Monica teaches in an Anglophone school in Quebec. In the 2015-2016 school year, Monica taught mathematics to a grade 3/4 split and a 4/5 split. The 4/5 split was an immersion group and she taught mathematics in English. Monica perceived the 4/5 group as

more academically inclined, whereas the 3/4 group seemed to struggle more. Monica also teaches English to the 3/4 group. In the 2016-2017 school year, Monica will teach the same sort of 3/4 class and she will keep working with the 4/5 split as a 5/6 group. Monica is very excited about following the 4/5 group because Monica thinks that the students will be able to start the year with the frame of mind required to discuss mathematics since the students have already adapted to the classroom dynamics.

Monica decided to become a teacher because her high school guidance counselor suggested that she might not be as happy as a nurse:

[My guidance counselor] took me aside one day and said: “Monica, if you want to work in the medicine field, have you considered being a doctor?” and I said, “No, that is not what I want to do. I want to work with the people, I want to be right involved actively in people’s lives.” In the 70s and 80s, doctors were removed from people, they were not doing family care the way they are now. She also pointed out to me that she felt I would be very frustrated in the position of nurse in the 70s and 80s. She felt I would know a lot more than some of the young doctors coming on staff and I would be frustrated that I would have to hold my mouth.

Monica’s desire to become a nurse demonstrates an interest in supporting the development of people and by being involved in people’s lives. Throughout the interview, this passion seemed to come through in her stories of teaching.

Monica seems to be very passionate about mathematics. Monica’s passion for mathematics started when she was in grade 11 and she says she was blessed with a horrible mathematics teacher:

Mr. Smith was a lousy math teacher and he did not teach math but I had a good textbook and I taught myself using the textbook and not only that, I taught all the girls in the class using the textbook so that was probably the start.

Monica says that for many years she relied on her memory to do well at mathematics. However, there was a point when she could not rely on rote memorization anymore. At that point she had to learn how to be flexible with mathematics.

Monica remembers that she was not a good student during her teacher education program. She had little intention to teach because she was not very fond of what she saw at school as a student and as a student teacher at university. She decided to go into teaching after

being a driving instructor for teenagers. Monica felt like she had something to contribute to the teaching profession so she got a job in northern Manitoba on an indigenous reserve and stayed there for a year.

Monica feels that her experience teaching in the north of Manitoba has shaped who she is as a teacher and made her into a listening teacher. There were two things that pushed Monica towards becoming a teacher with a listening orientation. The most important of them is the fact that students would not go to class, she could not take for granted that students would be in class:

I had thirty people on my roll, five who came to school every day, ten whom I saw at least, probably twice a week and the others would come whenever they felt like it. They used to come in, check in for homeroom, and, then run in the school, the school was a circle and they just ran.

Monica realized that she had to find a way to make the class engaging for students and to meet students' learning needs. Monica realized how important it was to learn about the students in the classroom and to pay attention to their feelings. She describes how she observed the effect of her own background:

I also learned the importance of recognizing that my culture and my views and my biases are not necessarily useful and could in fact be harmful to the learning of children. So, the importance of really looking at who that kid is and where they are and what it is they need and today may not be a learning.

Monica also created expectations for students to meet. She believes that it is important to have realistic goals for students without serious consequences if the goal is not reached. Moreover, she made kids responsible for their learning goals. For example, the goal of a worksheet is to show mastery of something, hence, her students never have to do a whole worksheet. The goal is to do as many as one needs to achieve mastery:

I encourage [students] to do two or three questions only and then check their answers. If they are right, then they keep going and if they are wrong, it is their job to work out what went wrong and if that does not work, they come and see me. And once they have worked out what did not go right, when they finish the sheet, I never ask people to do a whole sheet, I am sorry, I ask them to show mastery, whatever that is, if the sheet has got 30 questions, for that kid, it might be 10 so they feel they have mastered, they come and

show it to me and we talk about: what did they learn from the sheet? What went right? What did they have to overcome?

Monica says that students started increasing their attendance. With a positive look, Monica celebrated the higher attendance. Monica thinks that the second main aspect of how teaching in northern Manitoba shaped her teaching is the lack of heavy supervision that might dictate how the teaching job should be done. Monica feels that she had the chance to develop herself as a teacher: “I did not have any mentor teachers trying to tell me what I was doing was wrong. I had a principal who told me it was my job to entertain the children and I felt he was wrong.”

Meaning of listening. Throughout her career, Monica has observed that her practices in the classroom seem to foster positive attitudes in students and an environment of trust. Monica says that other teachers would often ask how she achieved these results, especially with challenging students.

I remember one day, there was a sports day going on and anybody who was not managing the sports day had to come to me and I had Lego out and these two boys from grade 5 ended up in my room. They had gone off property and their teacher was furious with them. [I was] just casually talking about it, “Why do you think your teacher was mad?”, “Oh, because she is a grumpy lady”, they said. “Let’s look at it from her point of view”, I said. I laid out the fears that [the teacher] might have been feeling and [the students] stopped, “What do you think you should do?” [The boys] wrote letters of apology to that teacher and that teacher came in with these letters in her hands and asked, “How did you get this to happen?” I said, “I did not do anything. I just listened to the kids” so by the time I finished there, there were people looking at it thinking, “oh, this is a good thing to do.”

Monica seems to have encouraged the boys to reflect on the event from the perspective of the teacher.

Monica found it hard to translate her way of establishing relationships into words: I had lots of teachers who would come in and watch me teach and they would say, “How come these kids behave for you? How come they are learning? What is going on?” and all I could ever say to them was: I treat them like human beings, I did not have the words

to describe what was happening and whereas Gordon's theories provided some of the words for me.

Gordon Neufeld (2014) talks about attachment theory and establishing relationships as parents and teachers. Monica was somewhat able to describe through Gordon Neufeld how she was establishing relationships with her students. According to Monica's description of Gordon Neufeld's theory, she seems to play the role of a figure of authority who supports children in dealing with constraints and obstacles that cannot be overcome by direct confrontation.

Monica was trained in what she calls *active listening* by a researcher who visited a school where she worked in the 90s. According to Monica, active listening is the practice of rephrasing what has been said by another person to verify if the interpretation of the utterance was appropriate and allowing the other people in the interaction to correct any misunderstanding. Monica thinks that her habit of reflecting out loud might come from this practice of active listening. Monica talks about how in class, this also allows other students to have more time to reflect and continue the conversation and the collective process of investigation.

Monica describes how providing examples instead of telling students that they are wrong, listening, honouring all of students' ideas, and honouring the linkage of ideas promotes a rich environment and ownership of knowledge. Monica realizes that honouring the linkage of ideas might be the biggest aspect of encouraging students' exploration:

The most exciting time in the classroom for me happens about April, maybe May, when kids start saying, "Hey, this is like this and what about this and is this the same as this? And miss, is it like this? And they get so excited and once they start making those links, that probability really is only fractions and fractions really is decimals and decimals really is the same thing as whole numbers and percent, oh my goodness, that is tied in too, oh my goodness it is all the same stuff and it all follows the same set of rules and it all has the same meaning but it is a slightly different way of looking at it which may or may not be more useful at the situation. Once they see that, oh my goodness, it is game over because they now are flexible and I think it is that flexibility of playing with numbers that makes them math students as opposed to students in a math class.

When Monica talks about teaching, she focuses on how learning is a natural desire for most people and how the role of the teacher is to support this learning. She believes that once students actually feel, see, and determine their own learning, the teacher intervention becomes

unnecessary and children can become responsible for their own learning. Monica feels that if there are expectations of what students are going to achieve and the process of learning is meaningful and useful to them, most discipline and behavior problems fall by the wayside: “Once they start making their own goals, they are self-driven, it is not my job to drive children, it is my job to provide opportunities for kids to see what they feel they need and give them a bit of a kick at times too.” Monica believes that most people want to learn and it is just a matter of accessing the learning and making it possible for them. Yet, she still makes it mandatory for students to memorize certain things. Monica’s goal is to combine the memorization with understanding of the concepts. Monica says that she provides a structure of management for the class and selects goals in terms of learning. She believes that the children’s conversations dictate the paths that they take in class: “the journey of how we get there comes from students’ understanding and their discussions.”

Still, Monica seems to see some conflict between her personal values as a teacher and the educational system:

In my heart of hearts, I feel like giving them all this time to explore and listen to each other and talk is beneficial and will make them better math understanders and math consumers and math life-long users but our school success written right into the documents of whether our principal does his job well is our end of year exam results.

Scary, huh?

Monica seems to try and keep her classes as exploratory as she can, but she is aware of how the school system might prevent certain learning practices to take place.

Summary

The teachers interviewed for this project agree that listening to student thinking is an important and complex task. Each teacher described her way of creating a listening environment and all the perspectives and attitudes that seemed to direct them towards turning the classroom into a listening environment. There seemed to be room for these teachers to explore and reflect on new perspectives and ideas. Moreover, these teachers were open to exchange ideas and reflect on their practices. None of these teachers described their teaching practices as final and unchangeable. All these teachers seemed to see their practices as continuous development. Each teacher gave her own colour to the practice of listening to student thinking. Kate’s desire to make classroom experiences real for students, Abby’s discovery that mathematics can be exciting,

Julie's epiphanies, Carol's artistry, and Monica's celebration of small victories. All these elements give unique colours to these teachers. The next chapter will describe how these teachers enact listening in the classroom. My access to the classroom is through the stories described by participants. Nevertheless, there are similarities uniting most of these teachers. In their experiences, there seems to be an important element of connecting and caring about students. These teachers seem to be teaching kids, not mathematics. This primary concern with the kids instead of the mathematics seems to turn these teachers into listening teachers. The relationships between teachers and students seems to be a constant in the process of listening to students.

Description of classroom moments

In the previous chapter, I described how these teachers feel that they listen to students. In this chapter, I describe how these teachers describe their actions and interactions with students with the support of enactivist theory. For enactivism, the world we know is brought forth through our interactions with others (Maturana & Varela, 1987, p. 244). The doing of the classroom is a valuable part of knowing. These teachers' descriptions of classroom moments demonstrate how their worlds are organized and the relations established with their environments. I am aware that teachers' experiences are limited to their descriptions, and therefore, amenable to interpretation. Nevertheless, the classroom moments described can be analysed assuming that the actions took place as described. I can observe how the classroom moments relate or not to how teachers attribute meaning to their experiences being that all of them come from descriptions provided by the teachers. By looking at teachers' classroom moments, their environments come forth and the relations to the internal dynamics can be explored.

Kate

Kate's classroom does not look like a traditional classroom. Kate has whiteboards and windows all around the walls and student desks organized in trios. Each desk has a card number assigned to it. Students know their places for the lesson according to the cards. Kate describes the typical classroom routine: she hands out cards to assign places as students arrive. Class usually starts with a problem and each trio goes to one of the vertical surfaces to solve the problem. Kate says that she walks around to listen to student thinking and identify misconceptions that should be addressed in the whole group discussion. Kate believes that the sitting arrangement has transformed interaction in the classroom:

They are so used to every day having to go through that process that now they have all talked to each other so now the conversation is much less hesitant. It is like they are way more willing to disagree, they are way more willing to say "Hey, I think you made a mistake here" "I think you need to do this" whereas before they would just be standing there, looking at each other because grade 8 is hard too. I mean, I helped them initially, by teaching them how to talk to each other, good questions to ask, such as "I do not understand what you did here. Can you show me?", "Can you explain how to do this?" or "Can you draw this for me?" So they have those little catch phrases that I taught them, but they are way more willing to use them.

Kate explains that she taught the students some phrases to help discuss mathematical work efficiently. Kate seems to conclude that the sitting arrangement made students more willing to use these phrases. Once students have found a solution to the problem, some groups present their solutions and they discuss as a whole class until they reach an agreement. If there is an error being repeated many times, Kate might address this misconception directly as a whole class. If Kate notices a misconception done by one student in particular, she will speak one-to-one with the student or group of students.

Kate described one classroom moment in which she realized how students engage deeply with the tasks. This is an example of a moment that provided confidence for Kate that she must have been doing something right to get Grade 8 students to care about a mathematical discussion, especially, if you consider that Kate's group is diverse and not every student felt naturally comfortable about their mathematics skills at the beginning of the school year. Kate describes the moment in which her principal brought to her attention the students' engagement:

This happened when my principal was here actually. [The students] had a big task that day, so they were all working on the boards. [My principal] was sitting on my desk. I walked by and he goes "You know they have been on these boards for twenty-five minutes." And I am like "Really?" I picked up the clock, and [my principal] said "They are all still working. You had no idea 25 minutes had passed" and I am like "No, no idea." That moment it felt right because I [figured] engagement must be high, but again it was a task that was real. [The students] were saving an apartment building, this villain was attacking an apartment building and integer girl had to save the day. Sometimes listening to student voice means you get a bit ridiculous. Superman versus Batman would just come out.

Kate also seemed to be very engaged with the class since she also did not realize that twenty-five minutes had passed.

After the mathematics period, Kate explained that they have recess, but students have a classroom snack instead of going outside. Kate said that students usually approach her to talk about mathematics questions or personal problems during the snack:

Everyone is just eating or chatting. They do not have to stay at their desks, they can wonder around, so nobody is noticing if I am having a conversation with two or three of them because I have a conversation with two or three of them at all times. That is not

weird, so, that is usually when they will say, “I don’t really understand what we are doing” or “Can you clarify this?”

Kate feels comfortable having students walking around the class, talking to one another, measuring things to solve problems. Kate enjoys giving students as much freedom as possible to explore the mathematics. An activity that shows the autonomy fostered on students is the work with fractions through cooking:

I have a couple of kids who are bakers, so I thought fractions plus baking. Perfect! So, [students] had to do all this stuff with my old family recipes. Then I said, (participant described this part as a conversation):

Teacher: Ok, let’s go.

Students: Where are we going?

Teacher: We are going to go see if you did this right.

And so they brought their assignments because if they had done it right, the recipe would work, if they had done it wrong, it would not. We all went down to the kitchen and I said “Figure it out.” For some of them it was a disaster and they had to go back and fix the recipe.

Abby

In the classroom, Abby relies on the three part problem solving to foster students to express their mathematical reasoning and, consequently, listen to student thinking. The first part is called Minds-on: this part introduces the concept that will be the focus of the lesson. The minds-on does not have to be a problem, it could be a video or a topic of conversation. Abby does not identify the second part by name, but she says that it is a mathematical problem for students to solve in groups. The third part is called Gallery Walk. In the gallery walk, students’ work is posted on the walls and the teacher and the students walk around the classroom to see and discuss the possible solutions to the problem. In the gallery walk, groups also present their solutions to the class. Abby describes how she usually ends the lesson with a reflective moment. This reflective moment is when Abby asks students what they learned in the lesson. The goal of this moment is to confirm if what students learned matches the goal of the lesson:

When the lesson is completed you call them back to the carpet and you find out what they have learned and if they did not fully grasp what they should have learned, the concept is reintroduced the next day to make sure that they understand what they are doing.

Abby explained that before she started working with problem solving with the students, she presented strategies and tools to solve problems. Abby presented the strategies and tools to solve problems through examples, one example for each strategy and tool. Some examples of the strategies are building a model, finding a pattern, logical reasoning, guessing and checking, or working backwards. Some of the tools are using friendly numbers, building an array, and using an algorithm. Next, Abby created an anchor chart with all the possible strategies and tools to help her students use the expected language in class. Abby reminds her students that they are expected to express their thinking using the language related to the strategies and tools. Abby's students are supposed to choose two strategies and two tools that they will use to solve the problem. Abby seems to believe that primary students would not be able to come up with these strategies and tools if they had not been presented previously.

Abby gave some examples of activities that she had done with the students on the week of the interview. At the time, Abby was working with the concepts of time and elapsed time. In the description of the activities, we can see how the listening takes place in the classroom. This problem was done with Grade 1:

The problem was simple, I gave them a graph and I said your teacher, she left school at 3pm and I showed them the graph and it went up and down, and then the graph went up and down again, and then it showed that she got home. Then the interaction followed as below (Abby described this interaction in a dialogue):

Teacher: What do you think that could be happening in the periods when the graph went down?

Students: She stopped.

Teacher: What did she do when she stopped?

There was a pause and the graph went down for 1 minute.

Teacher: What do you think she went to get in that one minutes?

Students: She went to go get us candies.

Teacher: That could be it, but that was two minutes. Could it be that she stopped because she is in the traffic and the traffic light stopped for that one minute?

Students: Oh, yeah. That could have been it.

Teacher: She stopped for another three minutes.

Students: She went to get candy.

Teacher: Maybe she ran out of gas, so she made a stop at the gas station.

And then it showed that she went on the highway.

Teacher: Look at this, she is going at 80km per hour. Do you think she could be driving in your neighborhood?

Students: No. we would be playing on the street.

Teacher: Where would she drive that she has to drive...

Students: She is on the 401!

You know what I mean, you have to put it in simple terms so that you help them to maneuver the graph because I know that reading the graph could be difficult for them, but asking questions, this is a graph and I am telling them what the graph shows, but what do you think it was happening here and what did you learn. And based on that, they would write what they know and the best way to solve the problem from it and even when they present you have to be helping, guiding.

Julie

In her three years as an observer, Julie feels that she had many opportunities to develop her thinking about the meaning of teaching and her practices. Julie reflected on her own assumptions about teaching through the observation of colleagues. Seeing herself in what other teachers did in class seems to have brought a new perspective to Julie's teaching choices. By reviewing her assumptions, Julie had insights about teaching. Julie's insights are described as moments of epiphany when she realizes how certain teaching practices are not conducive to listening to student thinking. In the interview, she sometimes refers to these insights as aha moments.

When I see other teachers that have sort of the same mindset, I think it helped me look at my biases and my preconceived judgements of things and students. I told you about the one where I probably just assumed that if I call on the kids with their hands up and they get it, everybody gets it. Now I know that that is not true and I had another big aha [moment] watching a teacher who was teaching fractions...

Julie would talk about her insights as she remembered them in the flow of our conversation.

The first epiphany that she had was the relief of not having to know everything. Julie describes her relief:

I thought I needed to have all the answers and do everything right. Now, the more I learn, the more I learn I do not know. I do know that we will find it out together and I want the kids to feel that too.

One of the topics about student voice that emerged from the evidence collected by the itinerant teachers is about students' sense of self-worth. Teachers' attempts to efficiently portray perfect understanding of all mathematics concepts and problems did not seem to emulate the trial and error process that students undertake as they are exploring their own mathematical solutions. Students might create an idea that they are not capable of doing mathematics because teachers seem to do it so easily. It is important for students to realize that sometimes solutions to mathematics take time to rationalize and work out. Some teachers might feel pressured into being the source of knowledge in a classroom. Julie realized how she was also feeling pressured to know everything and how this pressure influences her attitude in class. Julie also reflected on how a teacher's positioning as the "holder of knowledge" might contribute to students' low sense of self-worth: "...the way I was taught was the teacher stands up in the front, the teacher is the holder of the knowledge, the students listen to the teacher and I think it should be reversed".

Julie also thinks that teaching procedurally might prevent the creation of a listening environment. She gives an example of one of the teachers she observed:

[He was] trying to teach how to use fraction representations to perform operations. I cannot remember if it was addition, subtraction, multiplication, or division, but it was one of those and I do remember that he was using fraction strips, and number lines, and good models. I think he was trying to teach the kids, that is my judgement, how to procedurally use the different representations. Whereas representations, I feel you have to understand the concept of them. We are trying to get away from teaching "ok, this is how you add fractions, step 1, step 2, step 3, step 4" and move to what are the big ideas behind fractions. However, I think still as a system and myself included I was still teaching the representations in a procedural manner so that is another shift. I thought I was so cool because I am pulling out all these manipulatives but if I do not understand the concept, the underlying concept, those manipulatives, I don't even know if they are really helping the kids.

The realization that procedures might curb thinking allowed Julie to reconsider her perspectives of what good teaching means. Moreover, it allowed Julie to see how procedures

might not be connected to the conceptual understanding. This procedural way of teaching is so ingrained in the way that Julie was used to teaching that she could only realize how harmful it could be by observing other teachers. The opportunity to realize how assumptions can be dangerous led Julie to ask herself more questions about her choices of teaching practices:

I am really starting to question everything I do. So, this is a problem and a blessing. I always start with "Why am I doing this right now with these kids?" Is it really relevant or am I turning the page in the textbook?

Julie seems to be making an effort to be more conscious of her own choices when teaching. It seems like she is also trying to reflect on how her choices relate within her teaching context.

Julie describes how her third epiphany happened while observing the lesson of a teacher who moved too fast through material. In fact, Julie realized that the teacher was moving too fast because she herself did not have enough time to find an answer. Nevertheless, when the same teacher pulled out the white boards for students to work on, the pace of the lesson would slow down and more students could participate. This insight led Julie to reflect on how teachers focus on covering the curriculum: "I think it is teachers again in this culture, everything has to happen fast, we have got this curriculum that we have to cover like a checklist and I got get it done". For Julie, students need time to think and asking if there are any questions to the whole class does not seem to be enough to invite participation. Now Julie feels that to get engagement and relevant thinking from the students, teachers need to ask thoughtful questions and allow plenty of time. Consequently, students will solve fewer problems but Julie feels that deep thinking and learning does not require students to solve fifty problems in a lesson.

Another insight revealed in her observations of lessons is that directing students' actions is beyond her control. Julie wishes to develop a new habit: "I cannot always control what [students] are doing right now, but I can control what I am doing. So, what can I do to change whatever it is that I am trying to change?" Asking herself this question shows that Julie is taking responsibility over the challenges that happen in class and making an effort to act on these problems. Julie's insights seem to relate to these four topics in which they were organized: (1) relief of not having to know everything; (2) teaching procedurally does not promote student thinking; (3) slowing down the pace of the lesson; and (4) using one's own actions to occasion change. All these insights seem to propose a shift from teacher to learning partner, from speaking to listening.

Carol

Carol describes examples of how listening takes place in her classroom and some of the conditions to create an environment in which students feel comfortable to share their thinking. She also shares the story of one student in particular. This student overcame difficulties through a lot of support and the creation of a genuine relationship. Carol describes a moment in her grade seven/eight split when students were working with integers. There was a long strip of chart paper on the wall and students were placing the numbers on that strip. While working with the number lines, many concepts were brought up by students. The teachers allowed the conversation to be guided by the students' questions and took notes of students' questions. Carol and her teaching partner observed and did not say anything in those conversations. One of the conversations that Carol remembers was about "Does zero have value?"

Somebody said "Does zero have value?" and then somebody else said, "Oh, yeah" and somebody else said "No, it doesn't" and somebody else said something else and I think there were five people within the whole class of twenty talking about "Does zero have value?" At the end of the conversation somebody said "Ok, well, I guess zero is kind of a balancing point" I think that was either acceptable enough for people or they thought "Oh, I don't even know what to say about that so we are going to stop the conversation now"

This conversation rose from students' questions and students themselves created and disagreed with their hypothesis. Carol's goal seems to be that students come up with their own questions and have ownership of their mathematical work. Neither the choice of topic, nor the path of this discussion about zero were controlled by the teacher. Students were given space to explore and develop their own questions. The role of the teacher was to provide the environment and resources for students to explore integers.

Carol says that she allows herself to make mistakes in front of students to encourage them to see mistakes as part of the mathematical thinking. She also celebrates students' mistakes to encourage students to learn from their mistakes. Carol sometimes selects her favorite mistake to be discussed in class:

I would not do it without asking the student if everyone could look at it first, if we could talk about it, make sure that [the student] is ok with that. I could either point out the mistake or I could say, "There is a mistake here, can you find it?" so that you are

engaging other people in being able to find it and the person who made the mistake to see if he can find his own mistake. Then someone says, “It was a calculation” and then [the students] fix it and see how that changes the result.

Carol values mistakes to stimulate students to reflect about their own work. Moreover, Carol also thinks that looking at mistakes done by students who are considered strong might increase other students’ feelings of self-efficacy and confidence. Self-efficacy is the feeling that you are capable of doing something. Carol believes that weaker students will feel more confident and capable of doing mathematics when they see that strong students also make mistakes.

Carol believes that her teaching practices are in accordance with the Ontario curriculum. The curriculum proposes that teachers investigate and explore with students. Carol feels like she is taking advantage of these curriculum suggestions to make learning meaningful to her students’ needs: “the curriculum does say things like investigate and explore so we were doing those things, it was not just memorizing.” Carol has recently read the curriculum of another Canadian province and was surprised to find that the curriculum defines when each topic should be taught. Therefore, she feels grateful for the flexibility afforded by the Ontario curriculum. For example, she likes to start the year with data management doing a survey so that she and the students can get to know one another. She also appreciates keeping her lesson plan flexible. Even though, at the beginning of her career, she would stick to her lesson plan. Carol now feels that she acquired flexibility through years of experience. Carol has a goal for her lessons and an idea of how to get to her goal. Yet, she is attentive and ready to change her plan depending on the interaction with students.

My student teacher and I have laughed at how [the lesson] does not look any different than when I plan it because things always change depending on what the students do so no matter even if I plan [the lesson], things often happen that I did not expect and so we go in different directions anyways.

Carol also described how the student teacher is amazed that teachers can improvise in lessons, especially when improvised lessons proceed as smoothly as if they had been planned.

Carol posits that her interactions with students are built on constant feedback valuing students’ work. Carol asks questions to understand the logic in student thinking and directs students’ attention to their own work so that they review assumptions and keep developing their thinking as in the following account:

We have been talking about location and movement as part of the geometry curriculum. I just said “How do you get from our class to your French class?” That was the question and they used the square tiles to show how to get there. Some students were making really long ones, some left big spaces, other were iterating. We could look at the different ways that four different people did it and “ok, you’ve got left and right, that makes sense, but this one and this one are different. Are they both good? Does one need to change?” I try not to be the one to tell then that, I try to get them to think of it, and get into conversation about “Does it matter that they are spaced apart? Does it matter if they are equal? What does each of those squares actually mean? Is it a step? Is it a meter? What is it? What were you thinking about to decide? Those are the kind of conversations we have, that gets ones into some deeper mathematics than just build me a grid, “Yes, that’s right” or “No, that’s wrong” and do it this exact way.

Carol’s questions seem to stimulate student reflection on their own work as well as interaction with the other students. Moreover, Carol seems to encourage discussion on big mathematical ideas that will allow for a greater understanding of mathematics.

Another way in which Carol provides feedback are notebooks called maths-on. Students write on their maths-on almost every day when they solve problems in class. Carol provides feedback for students on those notebooks. The feedback is usually done through questions or comments about the students’ mathematical thinking and the way chosen to express the mathematical thinking. These notebooks are another avenue to listen and support student thinking.

Carol’s practices of assessment have also been influenced by listening to student thinking. Carol’s formative assessment is done through the notebooks as well as pictures of students’ work. Carol collects pedagogical documentation as a way to provide evidence for assignments which are not done on paper. The goal of the formative assessment is to inform Carol’s next steps in teaching. Carol also does a summative assessment as an independent assignment. Her summative assessment has no deadline, students can ask any questions, and use any resources. For Carol, the goal of the summative assessment is to see what students can do by themselves since in classroom students always work collectively.

Carol remembers one student in particular that initially refused to participate and ended up becoming confident enough to share her solution to a problem with the class. Carol describes

how listening to this grade eight student in particular allowed her to realize the student's difficulties:

This kid had extremely strong personality and to most people seemed angry and scary but it is probably super vulnerable. At least, that is what I saw – (describing the feeling of the student) if I am angry or mean, nobody needs to know how fragile I am. I do not know exactly when in the year, maybe half way through the year. It took me that long and I felt terrible but I realized she could not add two numbers to make ten. She did not have a learning disability, there was no issue like that. It was just something instructionally she missed whether she was not there or she was not paying attention. She missed it and she needed that to be filled in and once those things were filled in for her. I mean, she was no super star by the end but it was the confidence that she had at the end.

Carol was open to the student and saw more than the student's attitude. Carol's willingness to approach this student allowed her to establish a relationship of trust. The trust was essential for Carol to see the student's mathematical reasoning. Carol worked with this and other students using number talks which fostered students' confidence. At the end of the year, this child felt confident enough to show the class how she solved a mathematics problem. Carol described this student's story in a very emotional tone and I could see how she values listening, understanding, and building students' confidence.

Monica

Monica has been working to create conditions for student listening since she started her career in northern Manitoba and she does remember some moments in her teaching experience that seem to demonstrate the different quality of her relationships with students. One of these moments occurred when she was working with a multiage classroom which originated from an enclosed K-3 class and a 4/5 class. Monica's multiage group had fifteen students ranging in age from grade 2 to grade 6. When Monica took over this group, she was told that the school did not think the group would work:

At the beginning the staff did not appreciate what I was doing because it was very different than what had been done so I would have kids running into the room followed by the teacher who had them the year before:

Teacher from the year before: Did you know that Phillipe was outside without his snow pants?

Monica: Ok. Thank you very much.

Then I would turn to Phillipe who was in grade 2.

Monica: What were you doing outside without your snow pants?

Now, the teacher has left.

Phillipe: Well, I had a French detention. By the time, French was over if I had stopped to put my snow pants on, I would not have gotten outside.

Monica: Ok, you understand that once you are outside you cannot come back in if you get cold.

Phillipe: I know. That is why I was staying on the path and not jumping in the snow.

Monica: Sounds to me like you made a very good decision, Phillipe. Well done.

Whereas the teacher was assuming I would give him detention for not being smart enough to put on his snow pants. There was a lot of teacher flak because I was not doing what they were expecting but by the end of the second year, teachers were saying to me, “How did you get that kid to do that? How did that happen?”

Monica also describes how she creates opportunities to listen to student mathematical reasoning. She says that she is not completely aware of how she fosters students’ abilities to express their thinking. Yet, she has had researchers visit her room and make observations with which Monica agrees:

The researcher did comment that she was amazed at that level of reflective thought that was just a natural part of my teaching and the children’s learning. The kids were doing it too. Kids were putting up their hands and saying, “I was thinking this, but now after so and so said this, now I am thinking this and this and this.” And someone else would say, “well...” and jump right into it, so there is a collective understanding being built in the class and I do not think it was anything I was doing other than providing an accepting environment.

Monica describes how understanding is constantly built as a collective in her classroom. She explained this understanding using a metaphor of construction in which each student gives one block and slowly kids can build a house. As an example, Monica described the story of Sally who is a 63% - 65% student. Monica sounds very fond of her students and offered parents the opportunity to make an appointment and see all the exams to better understand at which point each student is. Sally and her father decided to see all the exams. Monica and the father agreed

that there is no common error through any of the exams and their hypothesis is that Sally can evaluate her answers in the collective because she can contrast with her colleagues' answers. However, Monica thinks Sally cannot independently evaluate her answers.

Monica describes one activity for each of the groups she was teaching during the last school year. According to Monica, these activities are typical examples of how the listening environment takes place in her classroom. In the 3/4 class, Monica was working with square numbers. Kids had little cubes to build squares and students had to validate their colleagues' answers. Monica describes the classroom moment:

[The kids] did not know what I was doing and I introduced the terminology 2 by 2, or 6 by 6 which they very quickly worked out. "Look that answers 6x6", but that came from them. My words were 6 by 6 and I think the critical piece there was to make sure that we had two or three groups validate the answer because it meant everybody had lots of experiences with lots of different numbers, no one sat back and said, "they are all done, I am finished" so that encouraged the multiple work and they were working in partners or groups of three or four or whatever they were doing. And in the process, we develop the numbers 4, 9, 16. All the way up to 100, I think one group got to 144, they just get into what they are doing...We were in a class discussion at this point and someone says, "Hey, hey, hey, just a second, what if we just have one square?" and so they worked out that and then somebody else wanted to throw in a zero. "Well, what about zero? Zero times zero, that is zero, Miss" so I put that on the board and so I stop, "What do you think about this, guys?" So, they are looking at it, they are exploring it, they are discussing it among their groups, and someone puts up their hand up and says, "How can you have a square that has zero in it? I cannot make a zero square" and that was very much the case of where this whole progression of squared numbers came from the kids and their discussions and listening to them and having them explore and validate each other's answers.

Monica did not seem to control the outcome of the activity. She requested students to build squares with different sizes. Through exploration, students wondered about different sizes and decided to keep building squares. Monica seems to have provided enough space for students to explore and express their mathematical ideas.

In the 4/5 group the lesson was about multiplication. The design of the activity was to

present the questions 63×2 , 63×7 , 63×17 , 63×1.7 and ask the kids to do any two of them. Students would write their solutions on the board and the whole class discusses and contrasts different ways of solving the computations. Monica's goal was to explore diverse ways of solving the two computations in the middle, which are the focus of the class. For the curriculum, solving 63×7 is a Grade 4 skill, whereas 63×17 is a Grade 5 skill. Monica's second goal was raise awareness of differences such as 17 and 1.7. Monica says that the 4/5 group came from a very procedural background. Thus, in the first week of class students were dissatisfied and complained that Monica had not organized their mathematics notebooks in sections for vocabulary and notes. Monica acquiesced, organized the notebook in sections, and presented formal notes so that students could feel at ease. This procedural way of presenting the ways to multiply was challenging for Monica because she prefers to have notes built from the work in the classroom. Thus, she did it for a while to make students feel safe, but quit as soon as students seemed confident:

So I actually had them write formal notes, against my will, but formal notes on six different ways of doing this so then after we had done that and now we are going into more chaotic and I say, "I do not care which way you use. They all work. It does not matter to me. Which one talks to you?" But I [gave them formal notes] because they were not in a position where they could explore and so once they had a number to play with, "Ok. Go ahead and do this. Do it anyway you want, if you are very comfortable with one of these methods, do it a different way and so now when I got my four methods up on the board, they knew that those methods worked. That was the security to start an exploration, I had not thought of that before, but that would be how that happened.

Summary

The classroom moments of these teachers describe different ways of listening to student mathematical thinking and varied classroom dynamics and types of interaction. However, there are similarities which seem to stand out in the descriptions. Kate, Carol, and Monica describe classroom situations in which students interact among themselves without much teacher direction. These three teachers tend to ask questions rather than make statements about student work. Julie seems to share the goal to encourage students to interact without too much direction since she describes the relief of not having to know everything. Julie describes classroom moments that led to epiphanies of how Julie does not want to teach. In the classroom moments,

Abby's description comes across somewhat differently from the other participants' descriptions. Abby describes an interaction that takes place between teacher and students instead of among students. In addition, Abby adjusts students' answers directly rather than posing questions to further student reflection.

Contrasting participants' stories and literature

In this chapter, I address my research question about how my group of participating teachers who profess to have a listening orientation experience listening and responding to student thinking. The perspectives and classroom moments described in the previous chapters provide access to the experiences of listening and responding to student mathematical reasoning. Frequent consultation of the data and a reflectivity posture of the researcher provide credibility to this research project. In addition, all the participants' quotes provide a trustworthy interpretation of the data. Now, I discuss how the perspectives of my participants relate to the literature on listening and what aspects of the participants' experiences were not included in the literature on listening. I also discuss similarities and differences in these teachers' experiences.

Enacting a listening orientation

The literature review proposes certain classroom practices that are usually common in listening environments. These similarities are: (1) not controlling procedures and outcomes; (2) asking thoughtful questions; and (3) developing a common repertoire in the classroom (Davis, 1996; Empson & Jacobs, 2008; Ohanian, 1992; Schifter & Fosnot, 1993). There is a great variety in how the teachers in this project have enacted a listening orientation. In order to discuss this variety, I will borrow from enactivist theory. I will reflect on what type of world seems to be brought forth in the actions of these teachers. Maturana and Varela (1987) state that "doing is knowing". That is, the actions and interactions in a classroom are part of the learning/cognition taking place in schools. In this discussion, my goal is to make explicit what underlies these actions and the nuances of the worlds brought forth by these teachers.

Striving not to control outcomes. The teachers in this project seem to promote classroom dynamics whose outcomes are minimally controlled by the teacher. Yet, there are differences on how much these teachers control outcomes. All teachers seem to allow activities to last for as long as necessary within the limiting constraints of a school schedule. In some cases the activities are proposed by the teachers. For example, Kate proposed a cooking activity that offered freedom for students to make their own choices. In the cooking class, students can make

mistakes and correct themselves without interference from the teacher. Kate does not give students the right answer, nor tell them to do things in a specific way. Furthermore, Carol and Monica told stories of how students took ownership of certain activities and raised their own questionings. Carol describes an example of how students raised and engaged in a discussion about the value of zero spontaneously. Carol did not direct students to the topic of zero, nor did she interfere in the outcome of the discussion. Monica's grade 3/4 classroom spontaneously decided to explore square numbers further. Monica did not plan that students would go up to 144, nor that they would wonder about the minimum number of units on the sides of a square. Carol and Monica seem to have provided environments in which students could express their mathematical ideas and the discussions were proposed and developed by students. In addition, Julie seems to be following a similar path when she describes how non-procedural teaching allows for better understanding. Julie realized how prescriptive activities might limit student exploration of mathematical concepts. These teachers seem to make an effort to keep their preferences and answers to themselves, whereas students express their mathematical ideas and delve in a collective reflection process. These teachers' worlds of significance seem to value diversity and variety of answers over unity and standardization. Carol, Kate, Monica, and Julie bring forth a world in which teachers are attentive and quiet, while students express their thinking processes and gain a feeling of agency. Since doing is knowing, the students' physical act of exploring a mathematical concept is a cognitive act of learning. Students are not only learning about the mathematical concept itself, students are learning how to look for solutions. This learning might not happen consciously, but students are living through the process of finding solutions in a community by listening and respecting diverse contributions. By not having controlled outcomes and steps to follow, students might take agency that could reflect in their lives in the future. Students' experiences of going through the process of reflecting critically might foster critical behavior in the future.

Abby also seems to strive not to control outcomes as she encourages student participation and invites students to express their reasoning. However, Abby's classroom stories seem to bring forth a slightly different world. For example, Abby invites students' opinions in the process of developing success criteria, but she altered the language of the success criteria to reflect the strategies and tools presented by herself. In Abby's time graph activity, students offer their hypotheses, but they do not discuss the inadequacy of their suggestions. Abby seems to direct

students to her original plan for the graph. The interaction seems to be: teacher invites students to participate, hears their answers, explains why students' suggestions are not plausible, and offers the answer that had been planned previously. Abby also explained how she found it important to intervene in a group of students who was talking about perimeter when they should find the area. Abby seems to believe that students should express their reasoning so that it can be corrected, whereas the other teachers believe students should express their thoughts to correct and find the correct answers in the collective process of the classroom. The cognitive effort of doing in this interaction seems to be limited to guessing the answers that the teacher has in mind.

Asking thoughtful questions. The second aspect of listening classrooms is the practice of asking thoughtful questions. Thoughtful questions seem to allow teachers to look deeper into student thinking while keeping pre-conceived assumptions from interfering in the questions. The teachers in this project gave examples of usual question and answer exchanges with students. These examples were only accessed through interviews, hence, these accounts are limited to these teachers' perceptions and recollections. Kate, Julie, Carol, and Monica try to ask questions that do not direct student thinking. Some examples of these questions are: "Does this make sense?" or "Does how many squares you use matter?" These questions seem to focus on promoting student thinking rather than obtaining pre-established answers. Kate says that she encourages students to check their own and one another's answers. Carol's questions about location and movement encourage students to express the choices made when creating their maps. Monica's active listening and out loud reflection seem to work as thoughtful questions. Monica explains how she rephrases what students say to confirm that she understands students' utterances correctly. This seems to give time for students to reflect on what they said and invite more students to reflect on another student's contribution. In addition, these teachers seem to prefer to let students ask questions than to ask questions themselves. In fact, Kate and Carol reported that they explicitly taught phrases and questions for students to use while discussing. Kate, Carol, and Monica would like students to take ownership of the mathematical discussions in class. Related to questions but not directly about the questions is the idea of doing fewer problems and exploring each problem in depth. All the teachers seemed to agree that the quality in exploring problems is more beneficial than the quantity of problems explored. All these teachers seemed to slow down the pace of the lesson to invite more participation from students.

Julie's account of how Julie herself was not able to think of an answer while observing a class shows how lessons might be too fast to engage students in mathematical reasoning.

On the other hand, Abby seems to find it important to direct student thinking to pre-established instructions and definitions. In Abby's example of the exercise about perimeter and area, Abby did not ask students why they were reflecting on perimeter when the exercise asked for area. Abby reminded the group of the definitions of perimeter and area in hope that students would direct their thinking only to area. Yet, talking about perimeter might have been valuable for students to make sense of the relations between perimeter and area.

In the interaction brought forth by Kate, Carol, and Monica, students might improve their confidence on their abilities to reflect and find valuable solutions. In addition, students might feel more responsible for their own learning. In contrast, redirecting students' actions might teach students to rely on the definitions.

Common repertoire. The third aspect from the literature is that listening communities have a common repertoire. They share a common language that allows communication to be effective. The fact that Kate and Carol taught appropriate language to communicate in classroom seems to be an effort towards creating common repertoire among students. Kate also used students' contexts to create problems for her lessons. These problems might provide a common repertoire allowing students to communicate effectively about the problems. In Monica's class, students requested to have formal notes on multiplication. Despite her own preferences, Monica accepted students' requests. These formal notes provided some safety for students to start experimenting with multiplication. These formal notes might have worked as a common repertoire to support students' communication. In addition, Monica's choice to acquiesce to students' desire demonstrates how she values students' opinions. Abby seems to be the only teacher that purposefully developed a common repertoire for the classroom. She seems to make a conscious effort to create a common language for students. Abby presented and created an anchor chart with strategies and tools for problem solving. Abby's goal was to encourage use of proper language related to problem solving. However, there is some risk in emphasising specific terms. The risk is that students might focus more on the discursive aspect of problem solving than the experience of problem solving (Sfard, personal communication, July 24, 2016). Correlating students' successes to their abilities to employ specific terms might lead students to focus more on the terms than on the mathematics and the problem solving process. Thus,

students might be capable of using mathematical discourse without experiencing mathematical concepts (Sfard, personal communication, July 24, 2016).

Assessment and pedagogical documentation. During the interviews, the participants in this project brought up the topic of assessment. Teaching by listening to student mathematical thinking seems to affect how these participating teachers assess students. Most participants seemed dissatisfied with traditional testing. Monica asks students to assess her throughout the year. By having students assess the teacher, Monica is again encouraging students to value their opinions and to reflect on their experiences. In addition, Monica creates an avenue for students to express their thoughts about the classroom. Kate and Carol feel that assessment should respect the individual characteristics of students. If students learn differently, they should also be assessed differently. Kate and Carol promote more flexibility for assessment. Students can choose when to do their assignments and they are also free to retake tests. Carol adamantly denies the need to use traditional tests and she allows her students to do tests in groups or to use any type of manipulatives for individual assignments. In addition, students are not obliged to finish their individual assignments in one class period. Students are permitted to take as long as they need to complete their exams. In order to provide evidence of how students completed their work, Carol takes pictures or collects written explanations of how students solved the problems.

Another interesting aspect of assessment is that Kate and Carol make use of pedagogical documentation to expand the number of possible solutions and to accommodate different learning styles, strengths, and weaknesses. Carol takes pictures of student work and keeps them as evidence to support marks. Kate not only takes pictures, but she also videotapes students solving problems. Carol and Kate have found similar solutions to keep a record of student work that was not done on paper. Coincidentally, Julie collected pedagogical documentation for her work as an itinerant teacher. Moreover, Julie was involved in preliminary discussions about pedagogical documentation and ethical considerations in the school board. Some participants in this research project reported to be employing pedagogical documentation as evidence of student achievement meanwhile the school board is still debating risks and consequences of pedagogical documentation.

Conclusion. For most of the teachers in this project, the enactment of a listening orientation in the classroom seems to be more connected to thoughtful questions and to the goal of not controlling outcomes than to the relevance of a common repertoire. Yet, these teachers

talked about the time required to have students adapt to the style of interaction proposed in the classroom. In the descriptions of these teachers, developing a listening environment required some months of shaping the language and attitudes of students. The type of interaction that these teachers propose is unfamiliar to students at the beginning of the school year. Structural coupling takes place through repeated interactions until actions become habitual. As structural coupling happens the structure of teacher and students become more adapted and ready to act in the classroom context. A change in the classroom environment means that changes in the structures of systems need to take place, hence, creating a listening community seems to require time. Listening to students does not seem to happen overnight, nor produce immediate results. A classroom is composed of systems and its dynamics are established through interactions. Therefore, a classroom will always go through a process of structural coupling regardless of the teacher's awareness of the process. Enactivism sheds light on aspects of teaching, such as the time required to become a community, that are not often part of the literature on listening. The doing of classrooms is a constant cognitive process that shapes teachers and students' structural beings. Careful observation of what and how students are doing and coupling with their environments might give hints to what doing they are learning. Moreover, constant communication with students to understand how they perceive classroom practices could shed light on better practices.

Challenges and supporting circumstances

Two of the relevant themes of the literature on listening are challenges and supporting factors for developing a listening community in the classroom. Now, I will discuss how challenges and supporting factors from the literature have played out in the praxis of these teachers. The challenges discussed in the literature review are standardized testing (Kohn, 2011), control and curriculum (Davis, 1996; Levin et al., 2009), the school environment itself (Davis & Sumara, 1997), years of traditional schooling in a transmission model, lack of confidence with mathematical knowledge, and teachers' assumptions of students' capabilities (Schifter & Fosnot, 1993). According to the literature, circumstances that might foster a listening orientation are: professional development sessions, support from colleagues, teaching experience, reflection on one's own mathematical reasoning, and teacher education courses (Crespo, 2000; Empson & Jacobs, 2008; Jacobs et al., 2010; Ohanian, 1992; Paley, 1986; Suurtamm & Vezina, 2010; Towers, 1998).

Standardized testing. Monica talked about standardized tests as hindrances and support for students' development. Kate talked about standardized testing as a supportive of the Grade 9 Applied remodelling and her teaching practices. Rather than being a hindrance, the EQAO exams demonstrate positive results for Kate's teaching style corroborating her listening practices. Monica's students have two weeks of standardized testing in Quebec. Monica's school board uses the results of these standardized exams to measure the quality of teachers' work. Monica makes it clear that she does not appreciate the value attributed to the standardized exams; however, Monica makes the most out of the exams by inviting parents to come see the exams. Monica took advantage of students' work to listen to their reasoning and to deepen her relationship with parents. These two teachers believe that students who are used to thinking and reflecting do better in standardized exams. Furthermore, these are the results received from exams done in previous years. The standardized exams became evidence to support their teaching practices, hence, the exams moved from a challenge to a circumstance to promote listening. Looking through the lens of enactivism, the same phenomena in our praxis of living might be perceived as a challenge or a supporting factor depending on the world of significance brought forth by the individual. These teachers' attitudes seem to demonstrate how their world of significance proposes a positive perspective on how to make the most out of challenges. These teachers seem to make an effort to turn apparent hindrances into positive circumstances.

Curriculum. It is common to hear teachers complain that they do not have enough time to cover the curriculum. Yet, none of the participants in this project sees the curriculum as an obstacle. Carol, Kate, Julie, and Abby all use the Ontario curriculum. Carol describes how she appreciates the flexibility afforded by the Ontario Elementary Mathematics Curriculum. Kate takes advantage of the space that the curriculum leaves for investigation. Julie describes how rushing through content to cover the curriculum does not promote listening practices. Julie thinks that the focus could be on teaching the students, not the curriculum. Carol, Kate, and Julie seem to share similar views about the curriculum. Monica teaches in Quebec and she does not criticize the curriculum either. These common complaints about covering the curriculum might be related to an interpretation or a different view of the role of a curriculum in the case of Ontario. Some teachers might view the curriculum as a list of tasks to be accomplished. However, not every curriculum necessarily presents itself as a to-do list. Carol mentions how the curriculum in other

Canadian provinces might restrict teachers' freedom to organize different topics in the school year.

School culture and structure. The school culture came up in all interviews, but each teacher put forward a different experience with the culture of the school. By contrasting the accounts of Julie and Kate, we can see how the school culture can influence teachers' work. Julie describes how the culture of her school discouraged teachers to listen and value student reasoning. For Julie, the school culture encouraged teachers to complain about student behavior without changing teaching practices. Kate describes how her previous principal encouraged her to pursue a more progressive teaching style and how the current administration is also supportive. Kate's school seems to be interested in maintaining positive relationships among students, parents, teachers, and administrators. The drive to discuss and solve problems as a community rather than blame students seems much bigger in Kate's school than Julie's school. These different environments seem to have fostered Kate and Julie's professional development in very different ways. For Julie, the school culture represented a challenge, whereas, for Kate it was a supporting circumstance. Abby feels that the school culture was a supporting circumstance for her professional development. The other teachers in Abby's school did not value professional development opportunities so the principal asked Abby to attend the professional development sessions. Thus, an apparently more conservative school culture with teachers who are not interested in professional development fostered Abby's search for more progressive teaching. Abby's reaction to her context turned a challenge into a supporting circumstance. Abby did not assimilate the characteristics of the school environment, but distinguished herself from the culture of the school. In fact, Abby ended up moving to another school. Monica appreciates that in her first year teaching, she was on a reserve in northern Manitoba and did not have a school culture that forced her to teach in a prescribed way. The freedom and challenges of working on a reserve allowed Monica to experiment and explore different ways of teaching. Monica feels that it would be hard to oppose an environment with a very fixed structure. School culture seems to have a major role in how teachers are encouraged or discouraged to explore diverse teaching practices. Even though the environment cannot determine the reactions of a system, it seems to be able to limit or initiate the reactions of a system. For example, Julie's school culture has limited how much exploration and innovation she could bring to class, whereas Kate's school culture stimulated her to listen to students and turn the classroom over to them. Monica

flourished in a less fixed environment as a beginning teacher. Nevertheless, other teachers might have reacted differently to the lack of a community environment and structured rules.

In addition, Monica and Carol seem to share a common concern related to school structure. Monica and Carol seem to feel restrained by the structure of the classes. These two teachers report frustration with the fact that sometimes the lesson is finished but the conversation is still developing. Carol and Monica wish they could keep pursuing student thinking instead of starting the next class period. Monica and Carol seem to see this as a challenge within the school structure. They are looking for ideas and suggestions that could allow students to pursue certain questions despite the class structure. Monica has been thinking of allowing students to separate from the big group and develop their ideas. Carol feels that she needs to find a better way to wrap-up the class.

Lack of confidence with mathematical knowledge. Kate was the only teacher to mention how her initial lack of confidence with mathematical knowledge played in her journey to develop a listening orientation. Kate had not been trained as a mathematics teacher so in her first year she studied the content before the students using the same material. Kate's perspective on teaching turned this apparent challenge into a supporting circumstance of her ability to listen to student reasoning. Having gone through the material herself, Kate could relate to students' difficulties and questions. Instead of perceiving her lack of knowledge of mathematics as a weakness, Kate kept confident and learned from her experience. It might have been Kate's confidence that allowed her to perceive her studies as a supporting circumstance of her development. An initial lack of mathematical knowledge does not seem to prevent teachers from listening to student reasoning, yet, perceiving a lack of mathematical knowledge as a weakness might affect confidence and prevent a teacher from exploring student thinking. Kate might have been able to keep confident despite not having the mathematical background because of the school culture that supported her teaching. It might not be the lack of mathematical knowledge but the perception of this lack as a weakness that leads to teachers who are not confident and prefer to avoid interaction with mathematical reasoning.

Professional development and colleagues' support. All the teachers mentioned professional development initiatives as part of the process of moving from telling to listening. Professional development can support listening to students in many ways. The teacher could learn about listening to students in professional development courses. Abby feels that the

collaborative inquiry group taught her to listen to students. Kate, Julie, Carol, and Monica have mentioned how professional development sessions have provided components to support their classroom practices. Moreover, professional development might also provide lexicon and arguments for teachers to make a case for the pedagogical choices they make. For example, Monica feels that her process of establishing relationships with students is instinctive and found it difficult to describe her actions, but she found a way to express her ideas through the work of Gordon Neufeld. Carol always experiments with ideas from professional development. Moreover, she learned how to perceive and teach mathematics differently with professional development. Another important aspect of professional development is the opportunity to congregate teachers. Professional development sessions allow teachers to meet peers and establish a supportive network. Kate emphasised how professional development opportunities allowed her to connect with peers who were interested in the same teaching style as she was. Julie and Carol also mentioned how the support of colleagues was a valuable condition to promote a listening orientation. Professional development might offer an opportunity for teachers who come from schools with more conservative cultures to see and connect with different ways of teaching. A change in environment might occasion transformation in the systems involved in the interaction.

Teaching experiences. Monica, Julie, and Abby described how teaching experiences have shaped their ways of listening to student reasoning. Monica believes that the experience of teaching a challenging group of students in northern Manitoba helped her to learn how to make the learning about the students. For Monica, this experience was a supporting factor for her development as a teacher. The environment of a school in northern Manitoba is the context that triggered Monica's listening to students, but the reaction of focusing on students is determined by Monica's structural organization. Monica seems to see challenging experiences as opportunities for improvement. Abby also seems to take advantage of challenges to foster development. Abby was willing to take up on challenging assignments that were refused by other teachers. Abby believes to have learned a lot about student thinking from doing the problem solving assignment. Through interaction with the students that are part of the environment, Abby learned how Grade 2 students are capable of understanding and using more problem solving strategies than she would have expected. In this case, Abby's environment triggered a reaction to adjust her expectations. In Julie's case, students' incorrect answers about fractions caused her to

bite her tongue and wait for students to express their reasoning. Julie seems to be in the process of becoming aware and transforming her habitual actions to create space for students to explore mathematical concepts.

Teacher education courses. None of the participants mentioned that their teacher education courses contributed to their practices of listening to student thinking. It might be that these teachers chose not to mention their teacher education courses because there were more recent experiences to be discussed. There was one mention to teacher education courses having a negative influence in a more progressive perspective of teaching. Monica mentioned how her teacher education course pushed her away from teaching. Monica reacted to the environment of the teacher education course by excluding herself from the teaching profession. Monica seems to have reacted by denying the experience of the teacher education course and looking for other jobs in which she could use her educational academic background.

Reflective nature. An interesting feature that seems to be present in most of the teachers in this project is a reflective nature and an openness to invite different perspectives. These teachers are willing to reflect and appreciate having varied input on their teaching practices. For example, almost all these teachers invited me to observe their lessons at the end of the project. This openness and reflective nature might encourage teachers to be constantly looking for more meaningful ways to teach instead of being accommodated with repeating patterns of actions from their days as students. Varela and colleagues (1991) describe how mindfulness might support transformation of habitual actions. These teachers seem to make an effort to be mindful of their practices. In addition to this reflective nature, these teachers seem to focus on personal and social development as personal goals. Carol and Monica have taught in reserves in northern Canada with the intention of supporting a marginalized group. Kate travelled to work as a volunteer building houses and Abby often travels to non-developed countries to work as a volunteer.

Conclusion. The teachers participating in this study have acknowledged some challenges that might inhibit the development of a listening orientation. Yet, these teachers do not seem to feel like these challenges prevent the development of a listening orientation. These teachers' perceptions of challenges seem to be more influential than the actual challenge. Their structural couplings with their environments seem to hold a sense of agency. These teachers do not seem to feel that they are at the mercy of the school system. They seem to have brought their personal perspectives on their teaching practices. Challenges and supporting circumstances depend on

how individual structures react to the environment. Perceptions of the environment seem to influence teachers' actions rather than the environment directly. Enactivism allows us to notice that challenges and supporting circumstances are dependent on the environment and structure of each individual system. Thus, developing a listening orientation in the classroom does not seem to be a matter of providing same training and similar opportunities, but of fostering teachers to be agents in their environments and to observe their teaching practices and the effects on students.

What are teachers listening for?

Structural coupling and structural changes happen regardless of the type of changes or the awareness of the systems involved. Listening also takes place regardless of how listeners interpret utterances. Varela and colleagues (1991) remark how structural coupling leads to the creation of habitual actions. Systems tend to repeat habitual actions to the point that the actions are repeated automatically. These habitual actions might change depending on the stimulus from the environment. The act of listening is always selective and might become habitual such as any other action. Teachers become accustomed to listen and focus their attention on certain things. The main difference for teachers who profess to listen to student reasoning seems to be what their habitual actions are. The question becomes: What are these teachers listening for? When Davis (1996) describes three types of listening, the quality of the listening seems to be different in each one because of what teachers are listening for. In the evaluative listening, teachers seem to listen for right answers without any focus on interpreting how students reason. In the interpretive listening, teachers seem to listen for student reasoning that coincides with the teacher's reasoning and will ultimately arrive at the teacher's goal. For hermeneutic listening, teachers seem to listen for student reasoning in itself. The teacher seems to listen for student reasoning and for the establishment of relations among the reasoning of students. Regarding the participants in this research project, one could say that Abby seems to be listen for students' interpretations so that she can direct students to the interpretation that she deems appropriate as exemplified in the two classroom moments described: calculating area and analysing a graph. The other four teachers, Kate, Julie, Carol, and Monica seem to make an effort towards establishing hermeneutic listening with their students. As my access is limited to these teachers' descriptions of their classroom practices, I can only go so far as to say that they seem to encourage hermeneutic listening in the classroom. Julie wishes to be a listener when she goes

back into the classroom by sharing responsibility and encouraging students to explore their thinking. Kate allows students to look for their own correct answers, such as in the baking activity. The other two teachers Carol and Monica described classroom moments in which student reasoning drove the discussion in class into new questionings not anticipated by the teacher. Carol described how students started debating the value of zero and Monica described how students wondered about what would be the biggest and smallest squares possible to be built. Therefore, the main difference for these teachers does not seem to be how they listen to students, but what they are listening for. Teaching by listening seems to be listening for an attitude, a way of expressing ideas rather than for fixed ideas. When teaching by listening, students' contributions vary greatly and teachers might have to keep their attitude of attentive awareness to be able to make sense of students' contributions. It might be that to listen for students' ideas and reasoning, teachers need to make sense of students' ideas. Teachers might be required to have a constant attitude of attentive awareness. In this process of being aware, the teachers in this project also noticed students' mathematical misconceptions. Most teachers in this project talked about students' misconceptions and how important it was to listen to students to be able to see their misconceptions. Moreover, these teachers were focused on adjusting students' misconceptions through a variety of possibilities such as thoughtful questions and exit cards. None of these teachers would have ignored students' misconceptions.

Making space for the other

In the findings of this research study, establishing relationships seems to be an important piece to create a listening environment in the classroom. Each teacher described a particular way of developing relationships. From an enactivist perspective, learning is doing and happens in every moment of our existence. Humans and other animals are cognitive beings, we learn with every action. Most of the time we might not even be aware that we are learning. However, this proposition does not explain why we tend to remember certain things we learn while we forget others. Our learning and our doing seem to be selective. It might be possible that the relationships we establish define what we learn; the people which we consider relevant in our interactions teach us by their presence and attitudes (Wessels, 2015). This realization makes me wonder about the roles of establishing relationships in education. The goals of education could be focused on attitudes and actions of caring, rather than results of specific disciplines (Noddings, 1992). Making space for the other seems to relate to the goals of social justice. The

teachers in this project seemed to have a concern with promoting an environment of acceptance to all students in their groups. Listening to students and believing in students' abilities to achieve success builds on self-esteem and encourages students to be active participants of lessons. Freire (1996) describes how social justice takes place through being open and welcoming all the different perspectives into a discussion.

Relationships among teachers. All of the teachers have reported stories about colleagues with whom they shared their questions and thoughts about students and teaching. The participants commented on experiences of how colleagues fostered their development. All of the teachers in this project highlighted how relationships were a valuable part of their professional development. Julie and Carol know each other and both mentioned how their interactions were beneficial to learn how to listen to students. Julie wishes to create a community of teachers in her school with whom she can share her successes and challenges. Carol consults with colleagues when she cannot make sense of how a student has solved a problem. Kate is constantly exchanging ideas with her administrators and keeps a healthy relationship in which teacher and administrators support each other. Abby felt that one of the greatest benefits of teaching problem solving was the relationship that she developed with the homeroom teachers. Monica seems to keep a solid relationship with her school principal.

Relationships with students. Establishing relationships with students seemed to be fundamental for most teachers in this study. All teachers seem to agree that to have a listening environment students need to feel confident to participate in the dialogues and express their mathematical thinking. Yet, each teacher saw the nuances of how to create this environment differently. The type of interactions required to create the listening environment was also different for each teacher. Julie describes the need to develop conditions in which students will feel comfortable to express their reasoning, but she did not mention establishing relationships directly. Julie is not teaching her own group at the moment and establishing relationships seems to become relevant in the classroom context. Abby realized how the relationships she established with students in the school year of 2015-2016 were one of the most significant aspects of her experiences with the problem solving assignment. In Abby's case, a classroom community might have been developed through the practice of solving problems together regularly. Carol, Kate, and Monica reported how establishing relationships with students was a key aspect of

encouraging students to express their mathematical thinking. Furthermore, these three teachers seem to understand the idea of establishing relationships in similar ways.

Carol, Kate, and Monica describe how it takes them some months to create the proper environment in the classroom and to develop relationships. Carol makes an effort to make students feel safe and respected. For her, building this environment in which they feel safe and respected means to accept that students might not be at their best every day and might not want to do something on a certain day. Creating a safe environment is about guaranteeing that they will not be laughed at, that she will never yell at them. Her effort is to make them feel secure enough and to trust on her to share them. For example, if one of the students is not feeling well and does not want to participate on that day, she respects that student's needs and allows him or her to stay out of an activity. Carol seems not to demand students to meet her where she expects them to be, she is open to go meet students where they are mathematically and personally. Students are not obliged to act in a certain way, she listens to them and negotiates outcomes with students. She is not the only person in control of the classroom, she is one individual in the classroom. She strives to create a classroom in which students and teacher have the same value. In the co-emergence of systems, Carol and the students seem to couple structurally through respectful interactions.

Kate wants to get to know students and wants them to feel comfortable to share their thinking whether it is their mathematics reasoning or their personal problems. She believes that to have students feel comfortable in class and to establish relations with them, she needs to talk to them about apparently irrelevant things. Kate is willing to welcome the topics that students enjoy talking about into the classroom. Talking allows Kate to know students as well as to gather material to create problems for lessons. These talks have different purposes which are connected to make the lessons work. Her goal in the classroom is to teach mathematics because she is part of a school board and she is focused on doing the job she is paid for. However, she aims at teaching them that they are capable and strong enough to find a solution for mathematics problems or for life problems. She feels like her goal is to make her students believe that they can solve any problem. Kate's descriptions seem to indicate that students are more important than the mathematics in her classroom.

Monica is aware that she establishes relationships with students, but she finds it hard to put into words the ways in which she builds these relationships. Monica feels that the work of

Gordon Neufeld is a useful reference to help her with words. Gordon Neufeld (2014) describes how it is important for the child to be known, heard, and accepted. Moreover, Neufeld talks about how it is important to connect before directing the child – recognizing the child’s presence and asking questions about his or her day instead of starting the interaction with a request. Monica’s story about Philippe, the boy who did not put on his snow pants, demonstrates how Monica respects and creates space for her students. Furthermore, Philippe’s story indicates how Monica respects and encourages children’s autonomy and authority to make decisions on their own. Monica would not discipline Philippe for not putting on his snow pants without understanding his reasons not to put on snow pants. For enactivism, doing is knowing and Philippe is learning with every experience in his actions. From Philippe’s perspective, being punished for not putting on his snow pants might have seemed unfair since he made conscious choices of how to protect himself from the cold and the snow. Moreover, punishing Philippe without listening to the situation might have taught Philippe that his opinions or choices are irrelevant in face of school rules. Philippe might have distanced himself from a teacher who did not seem to connect with him. The structural changes taking place through punishment might be Philippe not valuing his own choices. On the other hand, Monica’s attitude builds Philippe’s confidence in his ability to make right choices.

Relations with parents. Carol described how she would call the mother of her student to provide positive feedback. Parents’ support is really important to be able to connect to students and to improve students’ self-esteem. Carol told the story of a girl who overcame great difficulties in her group. Carol called the girl’s mom to strength the relationship with the student as well as the parent. This attitude shows that Carol is aware that students have lives outside of school and that these lives also affect their actions in class. Carol demonstrates a concern with the whole world of significance of the student. Calling the student’s mom allowed Carol to establish a connection between teacher and parent. Calling a parent to tell a positive attitude in class demonstrates how Carol reflects on her actions and wishes to rethink habitual actions such as calling to give bad news all the time. Kate is teaching in the countryside in a community which is much closer than big city schools. The school where Kate works has a very strong parents association and parents are part of the school decision-making processes. Kate considers herself to be lucky to have had parental support for her teaching style. Kate seems to realize how parents are an important aspect of establishing positive relationships with students. Parents’

concerns and questions should be valued. If parents do not appreciate the work of teachers, students might not be willing to connect and enjoy the classroom. These teachers wish to have parents as partners and to invite collaboration in the joint effort of raising children.

Conclusion

These teachers seem to have genuine and caring relations with their colleagues, students, and students' parents. These teachers' experiences of listening and responding to student mathematical thinking seems influenced by an ethical position of establishing respectful relationships with students as well as with other actors in the educational system. The most relevant answer to the research question seems to be that these teachers experience listening and responding to student mathematical reasoning as a mutual respectful effort to interact with another valuable human being. The goal of these teachers seems to be to increase their students' confidence. Teachers and parents' confidence of children might increase kids' confidence of themselves. Maturana and Varela (1987) talk about making space for the Other in interaction:

Biology also shows us that we can expand our cognitive domain. This arises through a novel experience brought forth through reasoning, through the encounter with a stranger, or, more directly, through the expression of a biological interpersonal congruence that lets us see the other person and open up for him room for existence beside us. This act is called *love*, or, if we prefer a milder expression, the acceptance of the other person beside us in our daily living (p. 246).

Interaction with the Other is based on love. Love as the possibility of legitimating and recognizing the existence of the Other. In their practice of listening to students, these teachers seem to resonate Maturana's idea of making space for the other. These teachers try to respect students' individuality and are establishing relations based on love and caring. Their goal is to make students aware of how capable they are. "We think that the task of education as an artificial relational and operational space of coexistence, should allow, facilitate, and guide the growth of our children so that they can become human beings that live and act in self-respect and respect for other" (Maturana & Rezepka, 1997). There is no listening if the student does not feel safe and comfortable to speak.

The importance of establishing relationships does not seem to be included in the literature on listening. This might be attributed to the fact that the literature on listening is mostly related to out of the classroom environments. In the classroom environment, the whole child is present,

whereas in experiments only limited aspects of interaction are observed. The teachers in this project need to make room for genuine interaction to take place and communities for mathematics discussion to be created. However, there is a vast literature on establishing relationships and caring education. Researchers have described how relationships are essential to learning processes (Ellerbrock, Abbas, DiCicco, Denmon, Sabella & Hart, 2015; Noddings, 1992; Wessels, 2015). Furthermore, studies on caring relationships describe a teacher attitude and a classroom environment which seem similar to the ones described in the literature on listening. Noddings (1992) mentions how listening and being responsive is an essential aspect of developing caring relationships. Moreover, Noddings (1992) discusses that caring relationships should be the focus of education. “The living other is more important than any theory” (Noddings, 1992, p. xix). Education should be accountable not for results, but for being responsive to students. The relationship between teacher and students should allow for equal contributions. Caring is not a virtue of the carer, it is an act of establishing a caring relationship (Noddings, 1992). This thesis has brought closer two important aspects of mathematics education. Establishing relationships and listening to student thinking are two pillars of mathematics education that seem to be connected in the experiences of the teachers interviewed for this project. However, establishing relationships did not seem to be part of the body of literature on listening to student thinking. Therefore, the main contribution of this thesis to the literature is attracting attention to how establishing relationships is a significant part of listening to student mathematical reasoning.

New roads

As I interviewed the participants in this project, I also learned about my own assumptions and misconceptions of teaching. In addition, I reflected on the relation between research and practice and how the push for progressive educational reforms might have occasioned different results than expected (Noddings, 2007). I wonder how reform attempts are capable of giving a new outfit to the educational system, but hardly ever change the basic structure of the educational system. In addition, how reform is interpreted by school leadership and teachers and how the goals might be enacted in a diverse way than how they were intended to be. During this master’s project I have reflected on how teaching by presenting steps for students to follow limits students’ possibilities of posing their own questions and choosing their paths for exploring content. Julie referred to this insight as teaching procedurally. As a teacher I planned careful

trajectories for students to follow in hope that they would make the same discoveries as me. I was often disappointed and now I understand the reason for the disappointment. Teaching could be more about exposing and allowing students to interact with a discipline in a meaningful way. If the goal of a student is to answer questions in class because the teacher requested those questions to be answered, student engagement might be low and students' attention will not last as long.

During this master's project I have gone back to teaching and I look forward to the challenge of making teaching different in my practice. I hope that I can engage students and support their learning in meaningful ways. I look forward to being a researcher teacher and to be constantly amazed at what students can do. The stories that these teachers have generously shared with me will help me keep focused on how much difference we can make in children's lives depending on the "world brought forth through our interactions with others" (Maturana & Varela, 1987, p. 244).

References

- Brown, L. & Coles, A. (n.d.). Reciprocal narrative interviewing. doi: <http://www.inter-disciplinary.net/probing-the-boundaries/wp-content/uploads/2015/08/LBrown-story8-dpaper.pdf>
- Brown, T. & McNamara, O. (2011). *Becoming a mathematics teacher*. London, NY: Springer.
- Coles, A. (2002). Teaching strategies related to listening and hearing in two secondary classrooms. *Research in Mathematics Education*, 4(1), 21-34.
- Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education*, 3, 155-181.
- D'Ambrosio, B. S. (1995). Highlighting the humanistic dimensions of mathematics activity through classroom discourse. *The Mathematics Teacher*, 88(9), 770-772.
- Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York, NY: Garland Publishing Inc.
- Davis, B. & Sumara, D. J. (1997). Cognition, complexity, and teacher education. *Harvard Educational Review*, 67 (1), 105-125.
- Ellerbrock, C., Abbas, B., DiCicco, M., Denmon, J., Sabella, L. & Hart, J. (2015). Relationships: The fundamental R in education. *Kappan Magazine*, 96(8), 48-51.
- Empson, S. & Jacobs, V. R. (2008). Learning to listen to children's mathematics. In D. Tirosh & T. Wood (Eds.). *Tools and processes in mathematics teacher education* (pp. 257-281), Rotterdam, Taipei: Sense Publishers.
- Englander, M. (2012). The interview: Data collection in descriptive phenomenological human scientific research. *Journal of Phenomenological Psychology*, 43, 13-35.
- Freire, P. (1996). *Pedagogia da Autonomia: Saberes necessários à prática educativa*. São Paulo, SP: Paz e Terra
- Gallagher, M. (2010). Are schools panoptic? *Surveillance and Society*, 7 (3/4), 262-272.
- Gallagher, S. & Varela, F. (2001). Redrawing the map and resetting the time: Phenomenology and the cognitive sciences. In S. Crowell, L. Embree, S. J. Julian (Eds.). *The reach of reflection: Issues for phenomenology's second century* (pp. 17-43), Center for advanced research in phenomenology, Inc.

- Graves, B. (2011). Creating spaces for children's mathematical reasoning. *Teaching Children Mathematics*, Focus Issue. National Council of Teachers of Mathematics (NCTM), 152-161.
- Heath, C., Hindmarsh, J. & Luff, P. (2010). *Video in qualitative research: Analysing social interaction in everyday life*. London, England: Sage Publications.
- Illich, I. (1970). *Deschooling society*. New York, NY: Marion Boyars.
- Jacobs, V., Lamb, L. & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169-202.
- Jameson, F. (1991). *Postmodernism, or the cultural logic of late capitalism*. Durham, North Carolina: Duke University Press.
- Kieren, T., Davis, B. & Mason, R. (1996). Fraction flags: Learning from children to help children learn. *Mathematics Teaching in the Middle School*, 2(1), 14-19.
- Kohn, A. (2010). "No" to threats and bribes. *The Phi Delta Kappan*, 92(1), 18.
- Kohn, A. (2011). *Feel-bad education and other contrarian essays on children and schooling*. Boston, Massachusetts: Beacon Press.
- Levin, D. M., Hammer, H. & Coffey, J. E. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education*, 60(2), 142-154.
- Liston, D. P. (2000). Love and despair in teaching. *Educational Theory*, 50(1), 81-102.
- Marshall, C., & Rossman, Gretchen B., author. (2016). *Designing qualitative research* (Sixth ed.).
- Mason, J. (2002). *Researching your own practice*. London, UK: RoutledgeFalmer.
- Maturana, H. R. & Varela, F. J. (1987). *The tree of knowledge*. Boston, MA: Shambhala
- Maturana, H. R. & Rezepka, S. N. (1997). Human awareness: Understanding the biological basis of knowledge and love in education. *Proceedings of the 6th Conference of the International Association for Cognitive Education*. 1-28.
doi:http://www.sympoetic.net/Courses/502_Outline_2014_files/Jane%20Cull%20Maturana%20and%20Nisis.pdf
- Merleau-Ponty, M. (1945). *Phenomenology of perception*. New York, NY: Routledge.
- Mhlolo, M. K. & Schafer, M. (2012). Towards empowering learners in a democratic mathematics classroom: To what extent are teachers' listening orientations conducive to and respectful of learners' thinking? *Pythagoras*, 33(2), 1-9. doi:

<http://dx.doi.org/10.4102/pythagoras.v33i2.166>

- National Council of Teachers of Mathematics. (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.
- Noddings, N. (1992). *The challenge to care in schools: An alternative approach to education*. New York: Teachers College Press.
- Noddings, N. (2007). *When school reform goes wrong*. New York, NY: Teachers College Press.
- Ohanian, S. (1992). *Garbage pizza, patchwork quilts, and math magic: Stories about teachers who love to teach and children who love to learn*. New York, NY: Freeman and Company.
- Paley, V. G. (1986). On listening to what the children say. *Harvard Educational Review*, 56(2), 122-131.
- ParentMap. (2014, April 8). *Dr. Gordon Neufeld on the attachment-based model of child development* [Video file]. Retrieved from <https://www.youtube.com/watch?v=C4hbAGaTAGs>.
- Petitmengin, C. (2006). Describing one's subjective experience in the second person: An interview method for the science of consciousness. *Phenom Cogn Sci*, 9 (229-269)
- Reid, D. A. & Mgombelo, J. (2015). Survey of key concepts in enactivist theory and methodology. *ZDM Mathematics Education*, 47, 171-183.
- Rogoff, B. (2003). *Cultural nature of human development*. Oxford, NY: Oxford University Press.
- Schifter, D. & Fosnot, C. T. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform*. New York, NY: Teachers College Press.
- Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46, 13-57.
- Sherin, G., Jacobs, V. R. & Philipp, R.A. (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York, NY: Routledge.
- Suurttamm, C.; Lawson, A.; Koch, M. (2008). The challenge of maintaining the integrity of reform mathematics in large-scale assessment. *Studies in Educational Evaluation*, 34, 31-43.

- Suurtamm, C. & Vézina, N. (2010) Transforming pedagogical practice in mathematics: Moving from telling to listening. *International Journal for Mathematics Teaching and Learning*. <http://www.cimt.plymouth.ac.uk/journal/default.html>.
- Thompson, C. L. (1985). *A comparison of nonburned out and burned out teachers* (Unpublished PhD thesis). The Pennsylvania State University, Pennsylvania, United States.
- Tompkins, J. (1996). *A life in school: What the teacher learned*. Cambridge, Massachusetts: Perseus Books.
- Towers, J. (1998). *Close encounters of the complex kind: Using videotape in teacher education*. Paper presented at the Annual Meeting of the American Educational Research Association. San Diego, CA, US. Retrieved from <http://files.eric.ed.gov/fulltext/ED420625.pdf>.
- Van Manen, M. (1990). *Researching lived experience: Human science for an action sensitive pedagogy*. New York, New York: State University of New York Press.
- Van Manen, M. (2014). *Phenomenology of practice: Meaning-giving methods in phenomenological research and writing*. Walnut Creek, California: Left Coast Press.
- Varela, F., Thompson, E. & Rosch, E. (1991). *The embodied mind*. Cambridge, Massachusetts: MIT Press.
- Wessels, F. (2015). The critical role of relationship in education. *Theological Studies* 71(3), 1-8
Retrieved from: <http://dx.doi.org/10.4102/hts.v71i3.2702>
- Wisehart, R. (2004). Nurturing passionate teachers. *Teacher Education Quarterly*, 31(4), 45-53.

Appendix A: Recruitment text

If you are an elementary teacher (Grades 1-8) who:

- values listening to students' mathematical thinking and
- allows lessons to be shaped by students' contributions.

I invite you to participate in a research study. The purpose of the study is to understand how elementary teachers (Grades 1-8) experience listening and responding to student mathematical thinking.

The study is my Master's project in education at the University of Ottawa. Participants will be selected on a first come, first served basis.

If you are interested or would like further information please contact me,
Tatiana Peres Toledo.

Appendix B: Interview Guide

Demographic questions:

1. When did you start teaching?
2. What is your academic background?
3. What grades do you teach or have taught?
4. What subjects do you teach or have taught?
5. How often do you take part in professional development?

Possible questions:

1. What does listening to students mean to you?
2. Have you always seen listening in this way?
3. How does your understanding of listening reflect on your practice?
4. When do you feel like you most listen to students?
5. Why have you chosen to listen to students this way?
6. How do you use your knowledge of what students said?
7. Why have you chosen to listen the way you do?
8. Is there anything that you would like to talk about that we have not touched on?

Support questions (the gaps are supposed to be completed with participants' words):

1. Could you give me an example?
2. What did you do?
3. How did you react?
4. How do you think the students found it?
5. What is it _____? What do you mean by _____?
6. I am not sure I understand what you mean by _____. Could you elaborate on it?
7. How did the students react to it?
8. You were saying that you _____. What does it mean to you?