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Investigation of Transfer Pricing Models Among a Network Firm for a Distributed Product

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Investigation of Transfer Pricing Models Among a Network Firm for a Distributed Product

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Abstract

This work investigates transfer pricing models among network-manufacturing firms for a distributed product.

The particularities of the models are: instead of each company trying to maximize individually the value added to its supply chain in which it is embedded, The models propose to maximize the value added by the network-companies in the global supply chain. Under specific assumptions on the nature of production, cost and value functions in typical production/distribution companies, it optimizes the supply chain structure for network-companies, distributed in one economic region. We calculate the transfer price and share the value added between the networked firms by two approaches: resource-based approach and efficiency-based approach. The models are formally defined, optimally formulated, and solved.

Keywords

Optimization, Mixed Integer Programming, Network, Transfer Price

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List of Acronyms

Initials	Meanings
MIP	Mixed Integer Programming
LP	Linear Programming
ERP	Enterprise Resource Planning
MNF	Multinational Firms

1. Introduction

1.1 Network and transfer price

Companies often build partnerships with other firms to achieve informational and organizational efficiencies. This management aspect is generally called **network-company**. The networked companies [Figure 1] are bound together and the product value added is distributed among the network.

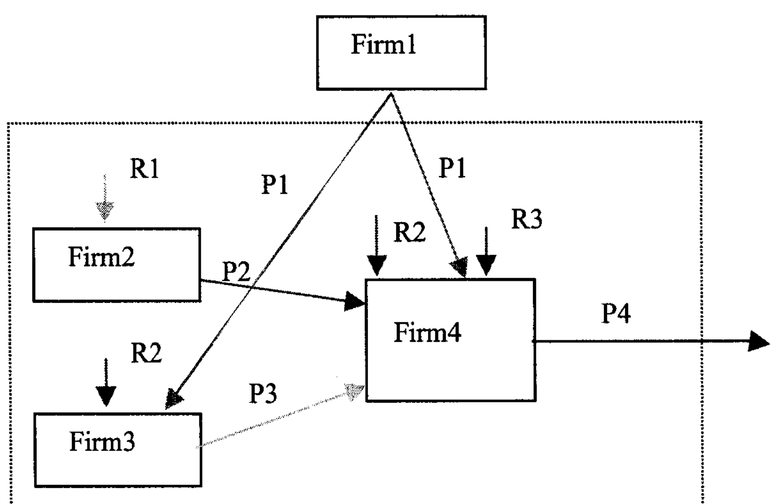

Figure 1 Example of a network

To make all individual companies agree to be bound together to shape a network-company and maximize the network-company's profit, a method for fairly distributing the profit generated by the network-company is needed and accepted by all participating companies. A common solution to this problem is to set prices for intermediate goods which are transferred from one networked firm to another. These prices are known as transfer prices. How to determine the transfer prices is a complicated problem. Some approaches have been proposed to solve this problem.

1.2 Context of the research

Two approaches have been widely used to derive transfer pricing in complex organizations (Pleiffer, 1999). They are the economic approach and the mathematical approach. The economic approach, which is used for problems without capacity constraints, uses marginal analysis to derive transfer prices (Madan 2000). The mathematical approach, which overcomes the limitation of the economic approach, is based on the dual decomposition principle (Shi, 1998, Marcos and Ronen 1999).

Among previous research about transfer prices, the paper of Lakhal and Lashine (2001) proposes to maximize the value added by the network-company in the global supply chain. It is most reasonable and realizable.

Nevertheless, one of the main issues, or primary remaining problem in most previous work, is that research has not provided a mechanism to incite companies to improve efficiency and minimize cost. In particular, the issues still to be addressed are:

1) to modify the above-mentioned model by including new parameters in order to distribute the profits to networked companies in a more reasonable and fair way to all the chained companies;

2) to introduce an efficient mechanism which can incite companies to minimize the cost and maximize their own profits.

Our goal is to distribute the profit among the network-company more fairly and reasonably to all the chained companies. The method of distributing profit will punish low efficient companies and reward high performance companies, so as to encourage all internal companies to improve management and productivity. It will enhance the ability of a network-company to compete with other companies.

In our research, based on the analysis of existing models, we present new models for network-companies. In detail we focus on the development of the Mixed Integer

Programming models which are used to improve the efficiency as well as to optimize costs and profits. Also we will illustrate this idea with a network-company case. Finally, a comparative analysis between the modified and existing models will be given.

2. Literature Review

The Transfer Pricing (TP) problem was first formalized in economic terms by Hirshleifer (1956). Hirshleifer described the TP problem with full information as a maximization problem. He considered two firms in a decentralized organization. Firm 1 produces an intermediate good and Firm 2 transforms it into a final good and sells it in the market. Firm 1 sells a quantity of the intermediate good to Firm 2 at a certain price. This price (the transfer price) is used to place a value on the transaction between the two firms. The total transaction value is considered as income by the selling firm and as expense by the buying firm. This allows the net profits of the two firms to be determined. The organization's objective is to maximize its total profit. The optimal solution is achieved when the marginal cost of the selling firm equals the marginal gross profit of the buying firm. This approach is economic approach.

Although simple and insightful, Hirshleifer's analysis does not have sufficient fidelity for practical use at operational levels of the firm; firms may face capacity constraints; the product may have to be manufactured by a chain of more than two firms etc. These conditions are considered in our research.

The revelation principle (Gibbard, 1973) proposed a "simple" mechanisms that are direct. A direct mechanism was one in which every firm manager reported his information. Central management then dictated an outcome and every firm manager was compensated an amount (positive or negative) that depended on the reports of all the firm managers. But this mechanism did not consider the case that the firm manager reported false information. We considered this in our efficiency-based transfer pricing approach.

Brown and Weida (1991) introduced a new approach to the TP problem whereby central management decides which firm (buying or selling) will be the planner. It takes the decision of who is to be the leader after comparing the expected profit obtained when the buying firm is the leader to that when the selling firm is the leader. The practical motivation for this approach is that central management, being well aware of the inferiority of its information, prefers to leave all the decisions to the firms themselves through negotiation. For this mechanism is balanced, it is not necessarily optimal for the network-company. But our research ensured the network-company would gain maximal profit.

The literature on transfer pricing which focused on decentralized multinational firms (MNF) (Elliot and Clive, 2000, Madan, 2000) emphasized the interactions between governments and MNF, and between branches of the MNF. According to Zhao (2000), the interactions between firms have been largely neglected.

Our research is based on the system modeling mechanism in Lakhal and Lashine (2001). In this paper, Lakhal and Lashine constructed a system model, which maximizes the network-company's profit, and derived the transfer-price. Nevertheless, like most other previous work, the research has not provided a mechanism to incite companies to improve efficiency and minimize cost either.

Here the costs reported by each company are supposed to be true. The profit could be distributed unfairly to the network-company, the firm with lowest efficiency (compared with similar firms) could get more profits than those firms with higher efficiency, the result could punish the firms with higher performance, and hurt their enthusiasm to improve performance, so they would lose interest to improve efficiency and make the cost higher. If this really happens, it will hurt the network-company and the network-company might break or disappear because it cannot compete with external companies due to its low efficiency.

3. System Model and Transfer Pricing Model

3.1 General Network System Model

As we know, it is very interesting to find methods to maximize profits of a whole network. The system model is from the research of Lakhal and Lashine (2000). The basic components of the model are firms, resources and products. The set of firms associated with a network-company is denoted by A_N. It can be partitioned into the set A_N^I of its internal companies and the set of A_N^E of its external companies (business partners: raw material suppliers and clients). A_N is the union of A_N^I and A_N^E ($A_N = A_N^I \cup A_N^E$). It is clear that a firm cannot be both internal and external at the same time. We denote the set of all the distinct durable resources of the network N by R_N. The amount, under some appropriate metric, of durable resource r available during the considered planning horizon is denoted by x_r, and the column vector of these amounts by x. The set of the durable resources which are used by a firm f is denoted by $R_f \subset R_N$. The amount of durable resource r used by an internal firm f during the considered time horizon is denoted by x_{rf}, the column vector of these amounts by x_f and the matrix of all the x_{rf}'s by X. The set of resources considered include all the resources used at the beginning of the planning horizon, plus any new durable resources that could be needed to improve the performance of the network.

In a network-company, a firm takes immediately preceding firms' (according to the manufacturing process) outputs as its inputs, and supplies its outputs to the immediately succeeding firms as their inputs. The inputs of the network are mostly raw materials and basic components, and the outputs of the network are the products supplied to markets. The index used for products is p and the set of products associated with a network-company N is denoted by P_N. For a given firm, an input product can come from several internal or external suppliers, and an output product can be delivered to several destinations. To track this properly, we construct four sets associate with a firm $f \in A_N{}^I$:

P_f^i: the set of its input products,

A_f^i: the set of its immediately preceding firms,

P_f^o: the set of its output products,

A_f^o: the set of its immediately succeeding firms.

For the horizon considered here, to a firm f, the quantity of its input product $p \in P_f{}^i$, which comes from its predecessor $i \in A_f{}^i$, is denoted by y_{ifp}, and the quantity of its output product $q \in P_f{}^o$ provided for its successor $j \in A_f{}^o$ by y_{fjq}. The set of all the products associated with a firm f is $P_f = P_f{}^i \cup P_f{}^o$. For a given network N, the set of all products supplied by external firms is P_N^i; the set of all products delivered to external firms is P_N^o; and the set of all products, which come from and go to internal firms, is P_N^I.

An internal firm f is thus characterized by its inputs, its resources and its outputs, as well as by a method m_f, which specifies how the resources are used to transform input products into outputs. For internal firm f, we use y_f^i to denote the vector of the input quantities, y_f^o to denote the vectors of the output quantities, and x_f to denote durable resource quantities. Then we can use the following function to model the method:

$$fm_f(y_f^{\,i}, x_f, y_f^{\,o}) = 0 \qquad f \in A_N^{\,I} \tag{F 3.1}$$

For external firms, we do not need to model the resources and methods as we do for internal firms. Instead, we assume that the output vector y_a^o for an external source $a \in A_f^i$ is restricted to a predetermined domain Y_a^o that characterizes its capacity, and the input vector y_a^i for an external destination $a \in A_f^o$ is restricted to a predetermined domain Y_a^i that characterizes its demand.

With these constructs, a network can be modeled as a directed network-topology of activities (A_N, S_N), where S_N is the set of directed arcs indicating the sequence of the activities required to get input products from external suppliers, to manufacture/transport internally, and deliver output products to specific markets. A Link (i, j, p) $\in S_N^o$ corresponds to a product p being passed from firm i to firm j, and it is associated with the flow variable y_{ijp}. For firm f, the set of input arcs is denoted by S_f^i, and the set of its output arcs is denoted by S_f^o. For the network-company N, the sets of the internal arcs, of the arcs with an external source and of the arcs with an external destination are denoted by S_N^I, S_N^i and S_N^o, respectively.

In order to profit from collaboration decisions, an enterprise must be able to adjust its available resources from its initial status of durable resource base. We therefore assume that the available resources at the beginning of the planning horizon, x^o, a matrix of elements x_r^o, which are defined as the quantity of available resource r at the beginning of the planning horizon, can be adjusted within certain limits (hiring or laying off employees, buying or selling equipment or facilities, etc). Decisions must be taken as to the amount of resources $x^+ \in X^+$ to be added or $x^- \in X$ to be disposed of, where X^+ and X^- are the sets of feasible resource addition and subtraction, respectively. For a given resource $r \in R_N^I$, the amount of resource available during the planning horizon is therefore

$$x_r = x_r^o - x_r^- + x_r^+, \qquad x^- \in X^-, \quad x^+ \in X^+$$

and, depending on the context, X^- and X^+ can be discrete or continuous, bounded or unbounded. Clearly, the variables x_r^- and x_r^+ cannot be positive simultaneously. Since some resources can be shared by several activities and it is possible that not all the resources of the enterprise will be used all the time, the following resource utilization constraints must be satisfied:

$$\sum_{f \in A_N^I} x_{rf} + x_r^- - x_r^+ \le x_{ro}, \qquad r \in R_N^I \qquad \text{(F3.2)}$$

Resources are not free. The cost of using or owning x_r units of resource r during the considered time horizon, can take various forms represented here by the cost function $c_r(x_r)$, the vector of these functions is denoted by $c(x^+, x^-)$. We assume that the price u_f paid by a firm f for the use of these resources is based on a cost sharing mechanism described by the implicit composite vector valued function C_f

$$C_f(c(x^+, x^-), X_f, u_f) = 0 \qquad \text{(F 3.3)}$$

The nature of this function will be discussed in a later section.

The product p associated with any link of the network-company has a certain value. The price paid for product p depends on the extent to which producer firm i is able to provide the attributes client firm j wants, as well as on the quantity y_{ijp} of product from firm i to firm j. The resulting revenues associated with link (i, j, p) are modeled by the value function $v_{ijp}(y_{ijp})$ and hence the average unit price paid for this product is $v_{ijp}(y_{ijp}) / y_{ijp}$. The function $v_{ijp}(y_{ijp})$ can take several forms dependent on the nature of the raw material suppliers and the markets. If a link starts with an external supplier or ends with an external demand destination, for example, its value function may increase up to a certain threshold after which it starts declining. The quantities on arcs involving external demand destination may be limited by market conditions. The determination of the value of the products on internal arcs is somewhat arbitrary. Fortunately, as we can see, the internal value functions are not required in our models.

Our objective is to find a networking strategy, which can maximize the value added by the internal activities of the network, which is equivalent to maximizing profits. The value added, va_f, by an internal firm f is obtained by subtracting the costs of the input products, and the cost of the resources used, from the revenue generated by output products, which is given by the following function

$$va_f = \sum_{(f,j,p)\in S_f o} v_{fjp}(y_{fjp}) - \sum_{(i,f,p)\in S_f i} v_{ifp}(y_{ifp}) - \sum_{(f,j,p)\in S_f{}^o} u_r g_{rfp} y_{fjp} \quad \text{(F3.4)}$$

Here g_{rfp} is the quantity of resources used by firm f to produce one unit of product p.

The value added by network N, is obtained simply by adding the value added by all its internal firms, which yields:

$$va_N = \sum_{f\in A_N{}^I} va_f = \sum_{(f,j,p)\in S_N{}^O} v_{fjp}(y_{fjp}) - \sum_{(i,f,p)\in S_N{}^i} v_{ifp}(y_{ifp}) - \sum_{(f,j,p)\in S_n{}^I} u_r g_{rfp} y_{fjp} \quad \text{(F3.5)}$$

Note that since the prices of products on the internal arcs are both the cost of a firm and the revenue of another firm, their value functions are cancelled when the value added by the networked companies is computed. Therefore these functions are not necessary to compute the network-company profits. This avoids the difficult problem of computing internal transfer prices when we consider maximizing the profit of the network

Taking all these into account, we can see that to maximize the value added by all internal firms, the network must solve a mathematical program having the following form:

$$\text{Max} \quad va_N = \sum_{f\in A_{NI}} va_f = \sum_{(f,j,p)\in S_{No}} v_{fjp}(y_{fjp}) - \sum_{(i,f,p)\in S_N{}^i} v_{ifp}(y_{ifp}) - \sum_{(f,j,p)\in S_n{}^I} u_r g_{rfp} y_{fjp} \quad \text{(GM)}$$

subject to

$$f_{mf}(y_f^i, x_f, y_f^o) = 0 \qquad \forall f \in A_N{}^i$$

$$y_f^i \in Y_f^{\,i} \quad \forall f \in A_N^{\,o},$$

$$y_f^o \in Y_f^{\,o} \quad \forall f \in A_N^{\,o},$$

$$y_{ifp} \geq 0 \quad \forall (i,f,p) \in S_N^{\,I}$$

$$\sum_{f \in A_N^{\,I}} x_{rf} + x_r^{\,-} - x_r^{\,+} < x_r^{\,o} \qquad \forall r \in R_N^{\,I}$$

$$x_r^{\,+} \in X_r^{\,+}, \quad x_r^{\,-} \in X_r^{\,-}$$

$$C_N\left(c\left(x^{-}, x^{+}\right), X, u\right) = 0 \qquad u \geq 0, \quad X \geq 0$$

where as previously stated :

$A_N^{\,i}$: the set of sources of the network-company,

$A_N^{\,o}$: the set clients of the network-company,

$A_N^{\,I}$: the set of internal firms of the network-company,

$S_N^{\,i}$: the set of input arcs of the network-company,

$S_N^{\,o}$: the set of output arcs of the network-company,

$S_N^{\,I}$: the set of internal arcs of the network-company,

x_r^{+} : the set of feasible resources addition,

x_r^{-} : the set of feasible resources subtraction,

x^o : the resources available at the beginning of the planning horizon,

$Y_f^{\,i}$: the set of input products quantities of firm f,

$Y_f^{\,o}$: the set of output products quantities of firm f,

u_f : the vector of the unit price of resources paid by firm f,

va_N: the profit of the network company,

va_f: the profit of an internal firm of the network company,

v_{fjp} : the value of product p transferring from firm f to firm j,

x_f : the amount of durable resources quantities of firm f,

x_{rf} : the amount of durable resource r used by internal firm f, Since the durable resources are shared within the network, we can split the firm f into several independent activities according to its production, then this value-added-maximization model is simplified to the classical mathematical programming model of a multi-product, multi-factor firm (Naylor and Vernon, 1969). Under this classical theory of the firm paradigm, input products are variable factors and durable resources are fixed factors. Program GM (general model) can thus be seen as a generalization of the classical model of the firm for the case of a multi-activity network-company.

3.2 Typical Production/Distribution Network System Model

Model GM is a generic model. To solve a specific network, the nature of its production functions, cost sharing function, value function and products flow/resource variable domains must be provided. In Lakhal and Lashine (2001), the nature and domains of the production functions are well discussed. We will present them in detail in the following section.

3.2.1 Production functions and flow variable domains

Most production/distribution companies nowadays use ERP (Enterprise Resource Planning) systems to help analyze and manage their production and distribution activities. These software systems enable the enterprise to effectively maintain the bill of materials and the bill of resources, which characterize their material and resource requirements. These bills of materials/resources are inherently linear so that, in our framework, the production function $f_{m_a}(y_a{}^i, x_a, y_a{}^o) = 0$ of an activity $a \in A_f{}^I$ reduces to the following linear equations:

$$\begin{cases} E_a^{p^i} y_a^{\ i} - B_a^{\ po} y_a^{\ o} = 0 \\ x_a - G_a E_a^{\ po} y_a^{\ o} = 0 \end{cases} \qquad a \in A_f^{\ I}, \qquad \text{(F 3.6)}$$

where, B_a is a matrix of elements b_{qp}, which are defined as the quantity of input product q used by activity a to produce one unit of output product p; G_a is a matrix of elements g_{rp}, which are defined as the quantity of durable resource r required by activity a for per unit of output product p, and $E_a^{\ pi}$, $E_a^{\ po}$ are 0-1 matrices specifying, for activity a, the arcs (i,a,p) providing input product p and the arcs (a,j,q) associated with output product q, respectively. More specifically, $E_a^{\ pi}$ is a $|P_a^{\ I}| \times |S_a^{\ I}|$ matrix and its element

$$e_p, (i, a, q) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{otherwise} \end{cases}$$

Similarly, $E_a^{\ po}$ is a $|P_a^{\ o}| \times |S_a^{\ o}|$ matrix and its element

$$e_p, (a, j, q) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{otherwise} \end{cases}$$

The flow variables for the internal activities are described by the linear system (F 3.6). For the external source, $a \in A_f^{\ i}$, we assume the values of the flow variables are restricted by the capacity domain Y_a^o, which can be provided by the supplier. For external destination, $a \in A_f^o$, we assume the flow variables are restricted by the domains Y_a^i, which can be needed by the market.

The capacity restrictions of source domain $Y_a^{\ o}, a \in A_f^{\mathrm{i}}$ are modeled by the bounds

$$0 \le y_{ajp} \le y_{ajp}^{\max} \qquad (a, j, p) \in S_f^{\mathrm{i}} \qquad \text{(F 3.7)}$$

where y_{ajp}^{max} is the maximum quantity of product p the external activity a can provide to internal activity j. Customer demand, is independent of the activity which is responsible for delivery end products, and depends on the quantities of the end products required by market area. So outbound flows (from networked company to market) are constrained by lower and upper quantity limits. The domains, $Y_a^o, a \in A_f^o$ can thus be modeled by the constraints

$$\underline{d}_{pa} \leq \sum_{(i,a,p)\in S_f^o} y_{iap} \leq \overline{d}_{pa} \quad a \in A_f^o, p \in P_f^o \tag{F 3.8}$$

where $\underline{d}_{pa}$ is the minimal market penetration target set by the network-company and $\overline{d}_{pa}$ is the possible maximum demand for product p in market a, which can be considered as a external firm of the network-company.

3.2.2 Resource cost functions and domains

Several cost sharing mechanisms are possible but the simplest, and probably the most commonly used in practice, is proportional allocation. With this mechanism, the partition of the total cost $c_r(x_r)$ of resources shared by firm a is directly proportional to the quantity x_{ra} of resource r it uses, i.e., the unit cost charged for resource r is $c_r(x_r)/x_r^*$, where x_r^* is the total amount of resource r used by the network-company. Under this assumption, the resource cost functions can be written as follows:

$$C_f = \sum_{r\in R_f} \left(\frac{x_{ra}}{x_r^*} c_r(x_r) \right) - u_a = 0 \ , \quad a \in A_f^I \ ; \qquad x_r^* = \sum_{a\in A_f^I} x_{ra} \tag{F3.9}$$

The cost functions of resources can be quite complex. To make things simple, we assume that the resources are divisible, and that the resource amount x_r^o and the resource cost c_r^o at the beginning of the planning horizon are both known, and can be adjusted within certain limits as a linear cost. The function considered is illustrated in Figure 2. Under this assumption, the domains X_r^+ and X_r^- reduce to the intervals

$$0 \le x_r^- \le X_r^- \quad \text{and} \quad 0 \le x_r^+ \le X_r^+ \qquad r \in R_f^I$$

where X_r^- is the maximal possible resource subtraction and X_r^+ is the maximal possible resource addition, and the cost of resource r can thus be modeled as follows:

$$c_r(x_r) = c_r^0 - \gamma_r^- x_r^- + \gamma_r^+ x_r^+ \qquad \text{(F 3.10)}$$

where γ_r^+ and γ_r^- are the unit cost increase or decrease of a resource respectively. It is clear that x_r^+ and x_r^- cannot be both positive simultaneously since the firm can not decrease and increase the same resource simultaneously.

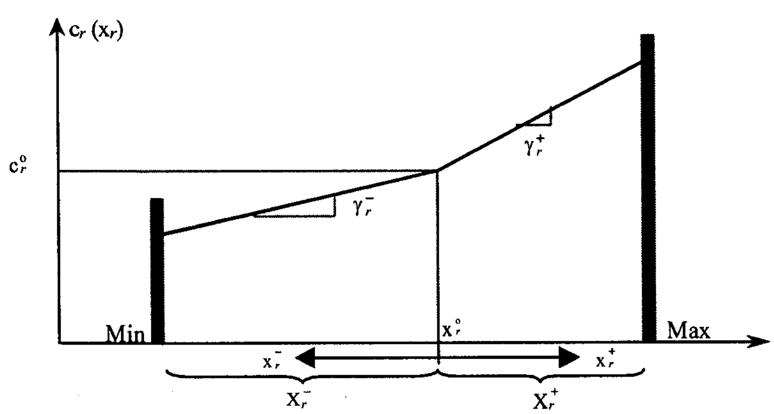

Figure 2 Resource cost function

These cost modeling assumptions are reasonable when the network-company wants to make only marginal changes to its current resource base. When the network-company considers getting new types of resources (related to the implementation of a new technology or a new facility, for example), then more complex cost functions, reflecting fixed costs and economies of scale, must be used. The approach remains the same, but the resulting model is more complex.

3.2.3 Product value functions

Porter (1985) defined value as "the amount buyers are willing to pay for what a firm provides them". He added: "Value is measured by total revenue, a reflection of the price a firm's product commands and the units it can sell". As indicated earlier, we are concerned here only with products, which come from external suppliers and subcontracted activities or sell to markets. The total revenue generated by these transactions is the result of negotiations between the parties or of market forces. In practice, negotiations between parties often result in pricing schemes including volume discounts and discontinuities, but generally piecewise linearity is preserved to simplify implementation. Here we assume that piecewise linear functions of the type illustrated in Figure 3 are sufficiently general to capture most practical situations.

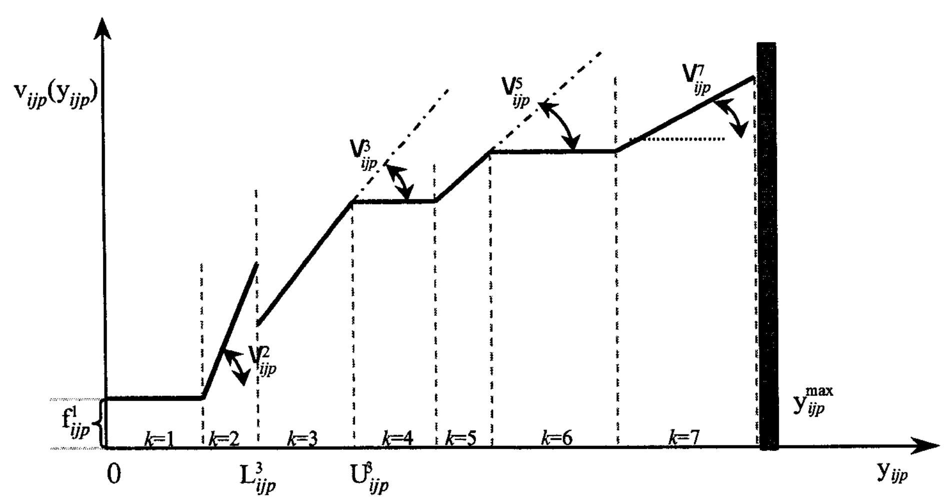

Figure 3 Product value function

With the modeling approach introduced by Kettani (1988), we can write the value function associated with external link (i,j,p) as

$$v_{ijp}(y_{ijp}) = \sum_{k\in K_{ijp}} \alpha_{ijp}^k (f_{ijp}^k + V_{ijp}^k y_{ijp}), \qquad \sum_{k\in K_{ijp}} \alpha_{ijp}^k = 1 \qquad \text{(F 3.11)}$$

where K_{ijp} is the set of intervals on the y-axis of the value function, f_{ijp}^k and V_{ijp}^k are the value at the origin and the slope of the linear function in interval $k\in K_{ijp}$ and α_{ijp}^k is the binary variable

$$\alpha_{ijp}^k = \begin{cases} 1 \text{ if } L_{ijp}^k \le y_{ijp} < U_{ijp}^k \\ 0 \text{ otherwise} \end{cases}, \qquad \forall\, k \in K_{ijp},$$

where L_{ijp}^k and U_{ijp}^k are, respectively, the lower and the upper bounds of interval k on the y-axis.

The term $\alpha_{ijp}^k V_{ijp}^k y_{ijp}$ in (F3.11) is quadratic but, as suggested by Oral and Kettani (1992), it can be linearized by replacing it with a non-negative continuous variable L_{ijp}^k. When the value function v_{ijp} (y_{ijp}) must be minimized, which is the case for external input arcs, then (F3.11) can be replaced by the following relations:

$$v_{ijp}(y_{ijp}) = \sum_{k\in K_{ijp}} \left(f_{ijp}^k \alpha_{ijp}^k + \mathsf{L}_{ijp}^k\right), \quad \sum_{k\in K_{ijp}} \alpha_{ijp}^k = 1, \quad \alpha_{ijp}^k \in \{0,1\}, \qquad \text{(F 3.12)}$$

$$\mathsf{L}_{ijp}^k \ge V_{ijp}^k y_{ijp} - V_{ijp}^k y_{ijp}^{max}(1-\alpha_{ijp}^k), \qquad \mathsf{L}_{ijp}^k \ge 0,\ \forall\, k \in K_{ijp} \qquad \text{(F 3.13)}$$

$$\sum_{k\in K_{ijp}} L_{ijp}^k \alpha_{ijp}^k \le y_{ijp} \le \sum_{k\in K_{ijp}} U_{ijp}^k \alpha_{ijp}^k \qquad \text{(F 3.14)}$$

where y_{ijp}^{max} is the maximum value of y_{ijp}, as previously defined. Note that since y_{ijp}^{max} corresponds to the upper bound of the last interval, as shown in Figure 3, the constraints (F3.14) automatically ensures that $0 \le y_{ijp} \le y_{ijp}^{max}$, so the bounds (3.7) are redundant.

When the value function must be maximized, which is the case for external output arcs, then (F 3.12) and (F 3.13) are replaced by:

$$v_{ijp}(y_{ijp}) = \sum_{k\in K_{ajp}} \left[\left(f_{ajp}^{k} + V_{ajp}^{k} y_{ajp}^{\max}\right)\alpha_{ajp}^{k} - L_{ajp}^{k}\right], \quad \sum_{k\in K_{ijp}} \alpha_{ijp}^{k} = 1,\ \alpha_{ijp}^{k} \in \{0,1\}, \qquad \text{(F 3.15)}$$

$$L_{ijp}^{k} \geq V_{ijp}^{k} y_{ijp}^{\max}\alpha_{ijp}^{k} - V_{ijp}^{k} y_{ijp}, \qquad L_{ijp}^{k} \geq 0, \qquad \forall\, k \in K_{ijp} \qquad \text{(F 3.16)}$$

3.2.4 System model

Taking the previous discussions into account, the model of the optimal networking strategy of a typical production-distribution company would be:

$$\text{Max } va_f = \sum_{(a,j,p)\in S_f^{o}} \sum_{k\in K_{ajp}} \left[\left(f_{ajp}^{k} + V_{ajp}^{k} y_{ajp}^{\max}\right)\alpha_{ajp}^{k} - L_{ajp}^{k}\right]$$

$$- \sum_{(i,a,p)\in S_f^{i}} \sum_{k\in K_{ajp}} \left(f_{iap}^{k}\alpha_{iap}^{k} + L_{iap}^{k}\right) - \sum_{r\in R_f}\left(c_r^{0} - \gamma_r^{-} x_r^{-} + \gamma_r^{+} x_r^{+}\right) \qquad \textbf{(MIP)}$$

subject to

- The production and demand constraints (MIP-1)

$$\begin{cases} E_a^{pi} y_a^{i} - B_a E_a^{po} y_a^{o} = 0 \\ x_a - G_a E_a^{po} y_a^{o} = 0 \end{cases} \qquad \forall\, a \in A_f^{I}$$

$$\underline{d}_{pa} \leq \sum_{(i,a,p)\in S_a^{i}} y_{iap} \leq \overline{d}_{pa} \qquad \forall a \in A_f^{o},\ p \in P_f^{o}$$

$$y_{ijp} \geq 0 \qquad \forall\, (i,j,p) \in S_f$$

- The value function constraints (MIP-2)

$$\sum_{k\in K_{ijp}} L_{ijp}^{k}\alpha_{ijp}^{k} \leq y_{ijp} \leq \sum_{k\in K_{ijp}} U_{ijp}^{k}\alpha_{ijp}^{k} \quad \forall (i,j,p) \in S_f^{o} \cup S_f^{i}$$

$$\sum_{k\in K_{ijp}} \alpha_{ijp}^{k} = 1 \qquad \forall (i,j,p) \in S_f^{o} \cup S_f^{i}$$

$$L_{ajp}^{k} \geq -V_{ajp}^{k} y_{ajp} + V_{ajp}^{k} y_{ajp}^{\max}\alpha_{ajp}^{k} \qquad \forall (a,j,p) \in S_f^{o},\ k \in K_{ajp}$$

$$L^k_{iap} \geq V^k_{iap} y_{iap} - V^k_{iap} y^{\max}_{iap} (1 - \alpha^k_{iap}) \qquad \forall (i,a,p) \in S^{\mathrm{i}}_f,\ k \in K_{iap}$$

$$\alpha^k_{ijp} \in \{0,1\},\ L^k_{ijp} \geq 0 \qquad \forall (i,j,p) \in S^{\mathrm{o}}_f \cup S^{\mathrm{i}}_f,\ k \in K_{ijp}$$

- The capacity constraints (MIP-3)

$$\sum_{a \in A^{\mathrm{I}}_f} x_{ra} + x^-_r - x^+_r \leq x^0_r \qquad \forall r \in R^{\mathrm{I}}_f$$

$$0 \leq x^-_r \leq X^-_r \ ;\ 0 \leq x^+_r \leq X^+_r \qquad \forall r \in R^{\mathrm{I}}_f$$

$$x_{ra} \geq 0 \qquad \forall a \in A^{\mathrm{I}}_f, r \in R^{\mathrm{I}}_f$$

We can see that this is a large scale mixed integer programming (MIP) problem. Its objective and constraint functions are linear, but they involve a large number of 0-1 variables.

3.2.5 Solution Method

Since it is difficult to use program **MIP** to solve realistic problems optimally, we propose a simple method, which can be used to solve it effectively. The method has two phases: in the first phase, we reduce the number of 0-1 variables as much as possible by eliminating regions of the value functions which are not likely to be used in an optimal solution, because of capacity and demand constraints propagation effects. For example, if the quantity of products are more than the maximal need of the market, then we do not consider this quantity; in the second phase, we gradually fix 0-1 variables by obtaining a series of improved feasible solutions and by inspecting them to determine if the flows of products on external arcs fall in the segments of the value functions associated with the product considered at the lowest cost or with the highest profit. This second phase is based on a well known idea used by Kuzdrall and Britney (1982) to solve single-item lot-sizing problems with quantity discounts. (Lakahal, Martel, Kettani, and Oral, 2000)

Let V^{*}_{ap} be the smallest unit cost which can be paid for the external supply of product p to activity a, i.e. let

$$V^{*}_{ap} = \min_{i,k}\left(V^{k}_{iap} > 0, \quad (i,a,p) \in S^{i}_{f} \right) \qquad \text{(F 3.17)}$$

and let V^{o}_{ap} be the largest unit price which can be obtained on the market for product p coming from activity a, i.e. let

$$V^{o}_{ap} = \max_{j,k}\left(V^{k}_{ajp}, \quad (a,j,p) \in S^{o}_{f} \right) \qquad \text{(F3.18)}$$

Also, let $S^{V}_{f} \subset S_{f}$ be the set of the arcs on which the best value segments are found, k_{ijp} be the index of the selected segment for link $(i,j,p) \in S^{V}_{f}$ and $L^{k_{ijp}}_{ijp}$ and $U^{k_{ijp}}_{ijp}$ be the lower and upper bounds of segment k_{ijp}, respectively.

In the *first* phase of the proposed method, in order to determine the sub-domain of each value function we are interested in, we replace the value function of every external link by the best possible value V^{*}_{ap} or V^{o}_{ap} of its product, and obtain the following function:

$$\text{Max va}_f = \sum_{(a,j,p)\in S^{o}_{f}} V^{o}_{ap} y_{ajp} - \sum_{(i,a,p)\in S^{i}_{f}} V^{*}_{ap} y_{iap} - \sum_{r \in R_f} \left(c^{0}_{r} - \gamma^{-}_{r} x^{-}_{r} + \gamma^{+}_{r} x^{+}_{r} \right) \qquad \textbf{(LP-1)}$$

subject to

- production/demand constraints MIP-1

- supply constraints

$$y_{ajp} \leq y_{ajp}^{\max} \qquad \forall (i,j,p) \in S_f^{\mathrm{i}} \qquad \text{(F3.19)}$$

- capacity constraints MIP-3

Let y_{ijp}^{*} denote the optimal flows obtained by solving **LP-1**. The rational behind the first phase is the following: the maximum quantity of product p which should be required by an activity a at the end of a sourcing link is $\Sigma_{i \in A_a^{\mathrm{i}}} y_{iap}^{*}$; hence, any segment of the value functions on the arcs $(i,a,p) \in S_a^{\mathrm{i}}$ which applies to flow quantities higher than this total should not be relevant. Consequently, for all the sourcing arcs, we redefine the value functions by removing any irrelevant intervals from the sets K_{iap}, and by setting

$$y_{iap}^{\max} = U_{iap}^{|K_{iap}|} = \min\left(y_{iap}^{\max}, \Sigma_{j \in A_a^{\mathrm{i}}} y_{jap}^{*}\right), \quad (i,a,p) \in S_f^{\mathrm{i}} \qquad \text{(F 3.20)}$$

We use MIP_o to denote the revised MIP obtained by revising the value functions in this way.

Now the second phase can be started. The second phase is iterative and it starts with the partial solution of $\mathbf{MIP_s}$, a version of $\mathbf{MIP_o}$ in which some of the 0-1 variables have been fixed. At the first iteration, the problem to solve is $\mathbf{MIP_o}$. The problem is partially solved with AMPL code. Let y_{ijp}^{s} denote the flows of the new solution obtained at iteration s. If for some arcs $(i,a,p) \in S_f^{\mathrm{V}}$, the flow obtained falls in the best value interval, i.e., if $\left(L_{iap}^{k_{iap}} \leq y_{iap}^{s} \leq U_{iap}^{k_{iap}}\right)$, then we set $\alpha_{iap}^{k_{iap}} = 1$ and $\alpha_{iap}^{k} = 0, \forall k \neq k_{iap}$. This defines $\mathbf{MIP_{s+1}}$, the next program to solve. The procedure continues like this until an optimal solution for the current MIP is found.

The complete algorithm is summarized in Figure 4.

Phase I

(1) Compute the best unit values V^{*}_{ap} and V^{o}_{ap} with (3.17) and (3.18), and specify the corresponding set S^{V}_{f} and parameters k_{ijp}, $L^{k_{ijp}}_{ijp}$ and $U^{k_{ijp}}_{ijp}$

(2) Solve **LP-1** and let y^{*}_{ijp} denote the optimal flows obtained

(3) For each activity a and product p such that $(i,a,p) \in S^{i}_{f}$,

If $\Sigma_{j \in A^{i}_{a}} y^{*}_{jap} < y^{max}_{iap}$,

Then redefine K_{iap} and set $y^{max}_{iap} = U^{|K_{iap}|}_{iap} = \Sigma_{j \in A^{i}_{a}} y^{*}_{jap}$

Let **MIP**$_{o}$ denote the revised mixed integer program

Set s = 0

Phase II

(4) Start solving **MIP**$_{s}$ with AMPL code

Interrupt the solution procedure when the optimal solution of **MIP**$_{s}$ is found

Otherwise, let y^{s}_{ijp} denote the flows of the best solution found

(5) For all arcs $(i,a,p) \in S^{V}_{f}$,

If $\left(L^{k_{iap}}_{iap} \le y^{s}_{iap} \le U^{k_{iap}}_{iap} \right)$ and $\left(p \notin P^{i}_{f} \cap P^{I}_{f} \right)$,

Then set $\alpha^{k_{iap}}_{iap} = 1$ and $\alpha^{k}_{iap} = 0, \forall k \neq k_{iap}$

Let **MIP**$_{s+1}$ denote the revised mixed integer program

Set s = s+1 and go back to (4)

Figure 4 Solution method

3.3 The transfer pricing models

3.3.1 Introduction

The solution of the model (GM) provides an optimum added value for the products manufactured by the network-manufacturing venture and the value of the decision variables (inputs, outputs, durable resources). This optimal solution will be used to derive models to calculate the transfer prices.

The outputs of a network-company are the final products to a market. The inputs are raw materials and components needed for manufacturing the products. The durable resources are those used by the network-company for the production. In concrete terms, the solution of the model GM provides the quantities on the arcs (y_{ijp}) and the modification on the resources capacities (x_r^+ and x_r^-). These results, the bills of the materials, and the bills of the resources will be used to determine the cost prices and transfer prices between the networked companies.

3.3.2 Determination of the cost price

Let:

- w_{ijp} : unit cost price on the link (i,j) for product *p*,
- v_{ijp} : unit transfer price on the link (i, j) for product *p*,
- $w_{\text{-}fp}$: the average unit cost price for an input product *p* to firm *f*, no matter the origin of the product,
- $v_{\text{-}jp}$: the average unit transfer price for an input product *p* to firm *f*, no matter its origin,
- $w_{f\text{-}p}$: the average unit cost price for an output product *p* from firm *f*, no matter its destination,
- $v_{f\text{-}p}$: the average unit transfer price for an output product *p* from firm *f*, no matter its destination,

w^o_f: the average unit cost price vector for the output products to a firm *f*,

v^o_f: the average unit transfer price vector for the output products from a firm *f*,

w^i_f: the average unit cost price vector for the input products from a firm *f*,

v^i_f: the average unit transfer price vector for the input products to a firm *f*,

u_r: the unit cost of a resource *r* used by any firm considering the available resource $u_r = \frac{c_r(x_r)}{x_r *}$,

u_f : unit cost vector of the resources used by firm f.

If we assume linearity, the cost price of the output is given by

$$w_f{}^o = w_f^{i'} B_f + u_f' G_f \qquad \text{(F 3.21)}$$

As we defined in previous sections, B_f is a matrix of elements b_{qp}, which are defined as the quantity of input product *q* used by activity *f* to produce one unit of output product *p*, G_f is a matrix of elements g_{rp}, which are defined as the quantity of durable resource *r* required by activity *f* per unit of output product *p*.

$w_f^{i'}$ and u_f' are the transposed vectors of average costs of the inputs and resources used by firm *f* respectively.

The input cost price of product *p* for firm *f* can be obtained:

$$w_{ifp} = \frac{v_{ifp}(y_{ifp})}{y_{ifp}} \qquad \forall(i,f,p) \in Y_{P_N^i} \qquad \text{(F 3.22)}$$

Hence, for a product provided by many origins, we have

$$w_{-fp} = \frac{\sum_{(i,f,p)\in Y_{P_f^i}} w_{ifp}(y_{ifp})}{y_{\bullet fp}} \qquad \text{(F 3.23)}$$

It is possible to calculate the cost price w_{fjp} for a quantity y_{fjp} of product *p* delivered from firm *f* to another firm *j*, using the following relation:

$$w_{f-p} = \left[\left(\sum_{\Sigma q \in P_f^i} b_{qp} \frac{\sum_{(i,f,q) \in Y_{P_f^i}} v_{ifq}(y_{ifq})}{\sum_{(i,f,q) \in Y_{P_f^i}} y_{ifq}} \right) \right] + \left(\sum_{r \in R_f} g_{rp} \frac{c_r^o - r_r^- x_r^- + r_r^+ x_r^+}{x_r^*} \right) \qquad \text{(F 3.24)}$$

where r_r^- and r_r^+ are the unit cost of a resource decrease and increase respectively. It is easy to understand that usually $r_r^+ > r_r^-$ and the firm does not pay to increase and decrease the amount of resources simultaneously.

The first part of this equation indicates the costs of the inputs and the second part illustrates the costs of the resources in the unit cost price w_{fjp}. To calculate the cost price of output of a firm *f* having inputs from other firms inside the network, we should first calculate the transfer price of its inputs. Assuming that the network is a graph without a circuit, after a topological sorting, we can calculate w_{fjp} level by level starting from the beginning of the manufacturing process.

3.3.3 Approaches to transfer pricing

The resolution of the system model allows the calculation of the network profit. Two approaches to distribute the profits of network will be discussed in this section. The first approach is resource-based approach and the other is efficiency-based approach.

3.3.3.1 The resource-based approach

The resource-based approach is constructed as follows:

Assuming proportional repartition among the firms of the network according to the assets used to manufacture the products (cost of durable resources), the contribution of va_r is:

$$va_r = \frac{c_r^{o} - r_r^{-} x_r^{-} + r_r^{+} x_r^{+}}{\sum_{r \in R_N} (c_r^{o} - r_r^{-} x_r^{-} + r_r^{+} x_r^{+})} va_N \qquad \text{(F 3.25)}$$

In this formula, the numerator of function (F3.25) is the cost of a specific durable resource r used by the network and the denominator is the total cost of all durable resources used by the network. Let b_{qp} denote the technical parameter indicating the corresponding relations between the output products and the input products usage, and let g_{rp} denote the relations between the output products and resources consumption. The transfer price $v_{f\text{-}p}$ for product q forwarded from firm i to another firm f is defined as follow:

$$v_{f\text{-}p} = \left[\left(\sum_{q \in P_f^{i}} b_{qp} \frac{\sum_{(i,f,q) \in Y_{Pf}^{i}} v_{ifq}(y_{ifq})}{\sum_{(i,f,q) \in Y_{Pf}} y_{ifq}} \right) + \left(\sum_{r \in R_f} g_{rp} \frac{c_r^{o} - r_r^{-} x_r^{-} + r_r^{+} x_r^{+} + va_r}{x_r^{*}} \right) \right] \qquad \text{(F3.26)}$$

The first part indicates the average costs of inputs to produce one unit of product p , and the second part illustrates the average costs of resources to produce one unit of product p and the profit share due to using the resources.

Let $v_{ijp} = \dfrac{v_{ijp}(y_{ijp})}{y_{ijp}}$ $\quad (i,j,p) \in Y_{P_N i} \cup Y_{P_N o}$

For a firm f, the profit is

$$va_f = v_f^{o} y_f^{o} - v_f^{i} y_f^{i} - u_f \qquad u_f = \sum_{r \in R_f} \left(\frac{x_{rf}}{x_r^{*}} C_r(x_r) \right) \qquad \text{(F3.27)}$$

The first part calculates the value of output products made, the second part calculates the cost of input, and the third part calculates the cost of resource.

The profit for the whole network is:

$$va_N = \sum_{fjp \in Y_{P_N o}} (v_{fjp} - w_{fjp}) y_{fjp} \qquad \text{(F3.28)}$$

3.3.3.2 The efficiency-based approach

In this approach, we propose that, when distributing profit, the profits are distributed to the networked firms not only according to the reported resource quantities the firm consumed, but also according to the average market resource consumptions for producing one unit of product. For example, there are three companies *A*, *B*, *C* grouped together to provide a market with product *P3*(Figure 5) . *A* provides product *P1* to *B*, *B* provides product *P2* to *C*, *C* sells product *P3* to the market. To maximize their profits from selling product *P3*, they must negotiate a way to decide how the profits will be distributed among them, that means how the transfer price will be set between *A* and *B*, and between *B* and *C*. If the resource costs claimed by *A* are higher than the average resource costs of the industry, no matter the firm manager over claimed the resource cost on purpose to get more profit or the firm is just low efficiency, are *B* and *C* willing to share the profits created by the network-company just according to what *A* claimed? If the low efficient company (like *A*) can get more profits, it could incite the other companies to do the same (maybe lower their efficiency on purpose), so the network-company loses competitiveness. In order to solve this problem, an efficient mechanism, which can incite companies to minimize cost and maximize their own profits, must be introduced.

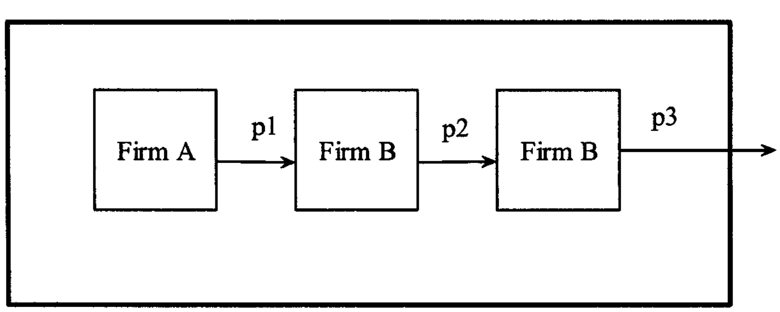

Figure 5 Network in efficiency based approach

Therefore we construct the following functions to derive transfer price:

First, we distribute profit among the internal firms of the network-company according to its product:

$$\frac{\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p}}{\sum_{f \in N^I} \sum_{p \in P_f^o} \left(\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p} \right)} va_n = va_{fp} \tag{F3.29}$$

where:

va_{fp} : the profit of firm f for product p

uc_{fp} : the actual (or claimed) quantity of resource consumption by firm f for producing unit product p

$\overline{uc_p}$: the average market resource consumption for producing unit product p

Note: $\overline{uc_p}$ can be calculated from the cost of all selling firms or from market prices if the market prices of the resources are available.

P_f^o: the set of output product of firm f

v_a^n : the overall profit of the network

v_a^f : the profit of firm f

Then we get the profit of firm f:

$$\mathrm{va_f} = \sum_{p \in P_f^o} va_{fp} = \sum_{p \in P_f^o} \frac{\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p}}{\sum_{f \in N^I} \sum_{p \in P_f^o} \left(\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p} \right)} va_n \tag{F3.30}$$

If we add each firm's profit, the overall profit of the network is still $\mathrm{va_n}$

$$\sum_{f \in N^I} va_f = \sum_{f \in N^I} \sum_{p \in P_f^o} \frac{\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p}}{\sum_{f \in N^I} \sum_{p \in P_f^o} \left(\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p} \right)} va_n = \mathrm{va_n} \tag{F3.31}$$

Finally, we get the transfer price as:

$$v_{f-p} = \left[\sum_{q \in P_f^i} b_{qp} \frac{\sum_{(i,f,q) \in Y_{Pf}^i} v_{ifq}(y_{ifq})}{\sum_{(i,f,q) \in Y_{Pf}^i} y_{ifq}} \right] + \sum_{r \in R_f} g_{rfp} \frac{c_r^o - r_r^- x_r^- + r_r^+ x_r^+}{x_r^*}$$

$$+ \frac{\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p}}{\sum_{f \in N^I} \sum_{p \in P_f^o} \left(\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p} \right)} va_n \qquad \text{(F 3.32)}$$

There may be a special case that all $\frac{\overline{uc_p}}{uc_{fp}}=1$; in this case the results from the two approaches will be same.

3.3.4 Comparison of the two approaches

The advantages of the efficiency-based approach are as follows:

Advantages:

It is a more comprehensive approach than the resource-based approach. The performance of firms on resource saving is considered; the resource-saving firm can get more profit than the resource-wasting firm. Since, in the resource-based approach, it is possible for the lowest efficient company to take more profit than the most efficient company, this can undermine the production of the network-company and diminish the development of the network-company in the future.

It attracts more companies to join a network-company. Since every internal company will earn a portion of the profit from a sale/contract, and no firm will lose money, it encourages companies to participate in a network-company.

It helps lower the interest of internal firm managers to overstate their resource cost for getting more profit.

It is fairer to all the chained firms since it is not focused on only the cost reported by managers.

It helps the network company to increase market percentage and enhance competition ability.

Further comparisons of these two approaches will be given in the next chapter by numerical examples.

4. Application of the Model: Case Study

Our objective in this section is, first, to show that our networking model can be solved efficiently, with the proposed method, for realistic problems, and second, to illustrate how the model can be used for strategic planning. To do this, we use a simplified case problem based on the context of an air conditioner manufacturer operating in China. We obtained detailed information about the company through our interview with the engineer in that company. The details of the case studied are presented in Appendix A.I. The supply chain of the network-company includes 8 internal primary firms and 31 potential external firms. At the time the problem was solved, some products can be purchased from external firms and transferred from internal firms. In our case, product filter can be purchased from subcontractor firm *a37*, and can be transferred from internal firm *a32*; product circuit control can be purchased from *a39* externally, and transferred from *a30*; product fan can be purchased from *a29*, and transferred from *a34*. Several alternative suppliers are also available for raw materials. The network-company would like to investigate the impact of certain marketing strategies on its networking decisions. Also, a new labor agreement needs to be negotiated soon and the company would like to investigate the interest of the subcontracting opportunities available under various labor conditions.

In order to solve **MIP** for this case, AMPL is used. AMPL is a strong optimization software. It will be introduced in detail in Appendix A.III.

4.1 Networking strategy

In this section, we will explore how to form a network-company. To illustrate the potential of the approach for strategy formulation, five potential scenarios are elaborated, each concerned with a particular issue. The optimal values of the flow variables directly related to the activities, which can be externalized, are reported in Table 1. Corresponding networking strategies are presented for each scenario. The first issue considered is the impact of the adopted marketing strategy to the network structure. The first three scenarios are related to this issue. The details of the scenarios are described as follows.

Scenario 1. Assume that the firm does not impose any minimal market penetration targets ($\underline{d}_{pa} = 0$) and that the demand is unbounded ($\overline{d}_{pa} = \infty$). In other words, facing an unlimited market, given the air conditioner pricing policy of the company and the resource redeployment and outsourcing opportunities available, what networking strategy should the network-company adopt? As indicated by the optimal networking solution in Table 1, in such a context, the strategy which maximizes profit is to produce as much as possible, which leads to using external partners to supplement the internal capacity available: one circuit control subcontractor (firm 37) is used in addition to the internal circuit control producing activity (firm 32), and one filter subcontractor (firm 39) is used in addition to the internal filter producing activity (firm 30). Production capacities of the internal activities are fully used while outsourcing opportunities are considered.

Scenario 2. Now, at the opposite, suppose that the company wants to know what would happen if it bases its decisions on a specific demand forecast per product-market for the planning horizon. In other words, if the network-company produces a fixed number of products, what networking strategy should the network-company adopt? Assume that

10,000 air conditioners can be sold during the planning horizon. Note that, under this assumption, the revenues earned are constant, and MIP is simplified to minimize the total cost of the input products and resources. Naturally, the optimal profit generated under this scenario is much lower than in scenario 1 ($1.34 millions instead of $1.99). The solution suggests to keep the fan producing activity inside, but to completely outsource the production of filter. It also suggests externalizing part of the production of circuit control.

Scenario 3. In most cases, companies would rather adopt a marketing strategy which falls in-between the approaches taken in the first two scenarios: for each product-market, a minimum penetration target is fixed, and potential sales are limited by an upper bound based on the total market potential and on historical market shares. The company then has to decide how much of each finished product it should make, and how to organize to supply them, in order to maximize profits. This is the assumption in Scenario 3. The solution obtained under this scenario suggests that the network company produces 15,000 air conditioners, which is the same as in Scenario 1, but the profit made under this marketing policy is slightly lower ($1.98 millions instead of $1.99). It also suggests that the network company adopts the same production strategy as in Scenario 1.

The first conclusion of the study of these three scenarios is that the best networking strategy for the company depends on product-market potential and on the marketing strategy. Different marketing contexts lead to different networking and resource deployment decisions, and added value cannot be maximized if this is not exploited. Several other issues can be explored with the model.

Scenario 4. Starting with the assumptions of Scenario 2, suppose in addition that, following a forthcoming new labor convention, wages are likely to increase from $10.00 to $12.00 per hour and the cost of additional personnel from $12.00 to $14.00 per hour. What could be the economic consequences of such an event? As shown by the results in Table 1, an obvious consequence is that, with all other things being equal, the profit made would decrease from \$1.34 to \$1.25 millions. The networking strategy is the same as the one used for Scenario 2.

What would happen if internal resources were completely disposable without any penalty? We will discuss that in the following scenario.

Scenario 5. We investigate this by setting the bounds on the subtraction of resources equal to current resource levels ($X_r^- = x_r^0 , \forall r \in \boldsymbol{R}_f$), and by setting the amounts recovered equal to the current resource unit costs ($\gamma_r^- = c_r^0 / x_r^0 , \forall r \in \boldsymbol{R}_f$). As could be expected, the first consequence of this additional flexibility is an increase in profits (from \$1.34 to \$1.38millions). The interesting aspect of this scenario is that it shows which internal activities are not productive when compared with concurrent external activities. In this case, for example, it is clear that the internal filter production activity is not competitive and that the network-company must act to correct the situation either by externalizing it, or by urging the firm to take steps to improve its production methods. It also highlights the competitive internal activities. In this case, for example, the fan production activity is kept completely inside (see Table 1), which indicates that it performs very well.

Table 1 Value of the solution obtained for various scenarios

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Market Demand					
$(\underline{d}_{22,36}, \bar{d}_{22,36})$	(0, ∞)	(6000,6000)	(600,9000)	(6000,6000)	(6000,6000)
$(\underline{d}_{22,38}, \bar{d}_{22,38})$	(0, ∞)	(4000,4000)	(400,6000)	(4000,4000)	(4000,4000)

Optimal Flows					
y30,33,12 (internal link)	3 000	0	3 000	0	0
y39,33,12 (external link)	12 000	10 000	12 000	10 000	10 000
y32,35,18 (internal link)	9 000	9 000	9 000	9 000	9 000
y37,35,18 (external link)	6 000	1 000	6 000	1 000	1 000
y34,35,21 (internal link)	15 000	10 000	15000	10 000	10 000
y29,35,21 (external link)	0	0	0	0	0
Tot. Prod,	15 000	10 000	15 000	10 000	10 000
Profit($)	1 991 836	1 336 258	1 979 836	1 249 738	1 380 501

4.2 Transfer Pricing Case Study

Our objective in this section is to show by numerical examples that it is efficient to use the efficiency-based approach to distribute profit within the network. To do this, we use Scenario 2 in Section 4.2. The details of how to derive the transfer price are presented in Appendix A.II. Transfer prices derived from two approaches are listed in Table 2.

Table 2 Transfer price derived from two approaches

Products	Transfer price from resource-based approach	Transfer price from efficiency-based approach
P18	4.03	3.67
P21	34.18	35.24

Here we derive transfer prices by considering the performance of internal firms on using the resources. We balance the transfer price by introducing the average resource cost. The transfer price of products produced by a resource saving firm can have higher transfer price in the efficiency-based approach than in the resource-based approach. For example, if firm *a34* uses less resource than the average : *a34* uses 0.02 *r2* and 0.01 *r3* (see Table 12), while average resource cost is 0.01 *r2* and 0.008 *r3*(see Table 3) , then it is assigned more profit in the efficiency-based approach than in the resource-based approach (see Appendix), thus the product *p21* produced by this firm has a higher transfer price (see Table2). On the opposite, the transfer price of the product, produced by a resource-wasting firm would have lower transfer price in the efficiency approach than in the resource-based approach. For example, firm *a32* uses more resource than the average (see Table3 and Table12) and it is assigned less profit (see appendix), thus the product p18 by this firm would have lower transfer price in the efficiency-based approach (see Table2).

The efficiency-based approach can help avoid firm manager overstating their costs for getting more profit. Since if he did that, he could only get less profit. This approach is much fairer to all the internal firms since it is not only according to the cost reported by firm managers. This approach incites a firm to take steps to produce more profit with less resources. If this model is adopted, the network-company will earn more market percentage and be more competitive.

Table 3 Average unit resource cost directly related to the transfer prices

	Resources								
Product	R1	R2	R3	R4	R5	R6	R7	R8	R9
P18	0	0.01	0.008	0	0	0	0	0	0
P21	0	0	0.12	0	0	0	0	0.11	0

5. Conclusion and Future Work

5.1 Summary and conclusion

An effective system model and two approaches to the transfer price have been constructed and applied to a case study. The proposed methods are applied to a Chinese air conditioner network-company, and five scenarios, each with specific meaning in mind, are elaborated, and corresponding networking strategies are found for each scenario. The results show that the models can effectively support the elaboration of the networking and resource deployment strategy of a network-company. The efficiency-based approach can effectively encourage firms to take steps to produce more profit with less resources consumption.

In conclusion, we have presented effective system models and transfer pricing models for maximizing the profit of a network and distributing the profit among the network more fairly and reasonably to all chained firms. It could enhance the ability of a network-company to compete with other companies. We believe this research gives the basis required to develop a good strategic decision support system.

5.2 Future work

One important limitation of our model is that it is static. Supply chains are living organisms: the resources used and the activities performed by companies change in time as a result of the decisions made to compete better. The dynamics of supply chains may be taken into consideration in the future. Another limitation of our research is that we assume the network-company is in the same economic region, without tax rate differentials. This limitation can be overcome in the future.

Bibliography

1. Brown, Palmer, Clark, and Nancy, Weida. 1991. The Role of Corporate Management in Negotiated Transfer Pricing, Working Paper, Parnell Hall, University of Delaware.
2. Elliot, Jamie and Clive, Emmanuel. 2000. International Transfer Pricing: Searching for Patterns. European Management Journal. Vol. 18(2): 216-222.
3. Gibbard, Allan. 1973. Manipulation of Voting Schemes, Econometrica, 41 : 587-601.
4. Hirshleifer, Jack. 1956. On the economics of transfer pricing. Journal of Business (July): 172-184.
5. Kettani, Ossama. 1988. Nouvelles formulations des problèmes de choix multicritère basées sur la programmation mathématique. Ph.D. Thesis, Faculté des sciences de l'administration, Université Laval, Québec, Canada.
6. Kusdrall, Paul J., and Britney, Rrobert, R. 1982. Total Setup Lot Sizing with Quantity Discounts. Decision Sciences, 13: 101-112.
7. Lakhal, Salem and Lashine, Sherif. 2001. A Transfer Pricing Model Among a Network-Manufacturing. Conference Gulf Cooperation Council (GCC) Economics 21st Century Opportunities, Saudi-Arabia: February 13-15, 2001: Vol. 2, 220-243.
8. Lakhal, Salem, Martel, Alainand , and Oral, Muhittin. 2000. Network Companies and Competitiveness: A framework for Analysis. European Journal of Operation Research. Vol. 118(2): 278-294.
9. Madan, Vibhas. 2000. Transfer Prices The Structure of Intra-Firm Trade. The Canadian Journal of Economics, Vol. 33(l): 53-68.
10. Marcos, Avila and Ronen, Joshua. 1999. Transfer-Pricing Mechanisms: An Experimental Investigation. European Journal of Operational Research. Vol. 17(5): 689-715.

11. Naylor, Thomas and Vernon, John. 1969. Microeconomics and Decision Models of the Firm. Harcourt, Brace & World.

12. Oral, Muhittin and Kettani, Ossama. 1992, A Linearization Procedure for Quadratic and Cubic Mixed-Integer Problems. Operations Research, 40-1:109-116.

13. Pleiffer Thomas. 1999. Transfer Pricing and Decentralized Dynamic Lot-Sizing in Multistage, Multi-Product Production Process. European Journal of Operational Research. Vol. 116(2): 319-330.

14. Porter, Michael. 1985. Competitive Advantage: Creating and Sustaining Superior Performance. The Free Press, New York.

15. Shi, Yong. 1998. Optimal System Design with MC2 Linear Programming: A Dual Contingency Plan Approach. European Journal of Operational Research. Vol.107(3): 692-709.

16. Zhao, Laixum. 2000. Decentralization and Transfer Pricing Under Oligopoly. Southern Economic Journal. Vol. 67(2): 414-426.

17. AMPL® A Modeling Language for Mathematical Programming http://www.ampl.com/

APPENDICES

A.I Infrastructure of a Network-Company in a Case Study

This appendix briefly describes the air conditioner manufacturing company, which is used to test the models proposed in this paper. The outputs of the network are a quantity of products. The inputs are raw materials and components needed to manufacture the products. The durable resources are those used by each company for the production needs. The activities of the network are presented in Figure 6. The supply chain of the network-company includes 26 external activities (activity *a1*— activity *a26*) which supply the input to the network, 5 internal activities for production level I (activity *a27*, activity *a28*, activity *a30*, activity *a31* and activity *a32*) which use the inputs from external activities to produce Level I products, 2 internal activities (activity *a33* and activity *a34*) which use the Level I products as inputs to product level II products, 1 internal activity (activity *a35*) which use level II products as inputs to produce final product, 1 external activity (activity *a39*) which can produce Level I product , 2 external activities (activity *a29* and activity *a27*) which can produce level II products, and 2 markets (activity *a36* and activity *a38*). At the time the problem is solved, opportunities exist to externalize filter-producing activity (external activity *a39* vs. internal activity *a30*), circuit control producing activity (internal activity *a32* vs. external activity *a37*) and fan producing activity (external activity *a29* vs. internal activity *a34*). Several alternative suppliers are also available for raw materials. The internal are listed in Table 4. The external activities are listed in Table 5. The durable resources and products used in the analysis are listed in Tables 6 and Table 7, respectively.

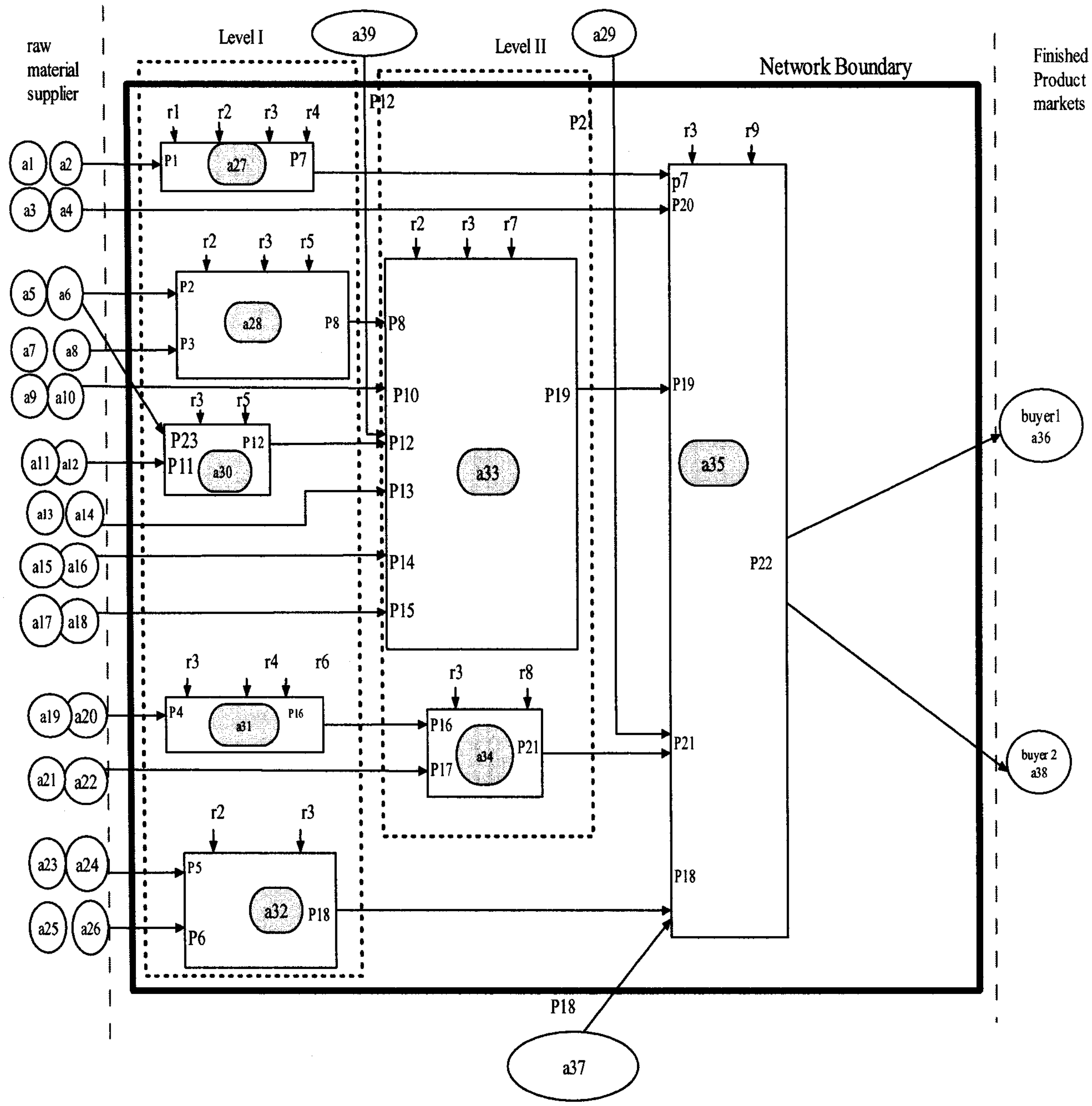

Figure 6 An air conditioner manufacturing network-company

Table 4 List of Internal activities

Index	Description
a27	Internal firm producing shell
a28	Internal firm producing heat exchanger
a30	Internal firm producing filter
a31	Internal firm producing blade
a32	Internal firm producing circuit control
a33	Internal firm producing cold producing unit
a34	Internal firm producing fan
a35	Internal firm assembling air conditioner

Table 5 List of External activities

Index	Description
a1	Iron sheet supplier
a2	Iron sheet supplier
a3	Motor supplier
a4	Motor supplier
a5	Pipe supplier
a6	Pipe supplier
a7	Fin supplier
a8	Fin supplier

a9	Refrigerant supplier
a10	Refrigerant supplier
a11	Silicon supplier
a12	Silicon supplier
a13	Compressor supplier
a14	Compressor supplier
a15	Compressor staring device supplier
a16	Compressor staring device supplier
a17	Metering device supplier
a18	Metering device supplier
a19	Plastic supplier
a20	Plastic supplier
a21	Motor for fan
a22	Motor for fan
a23	Temperature sensor supplier
a24	Temperature sensor supplier
a25	Wires supplier
a26	Wires supplier
a29	External supplier of fan
a36	Consumer market 1

a37	External supplier of Circuit control
a38	Consumer market 2
a39	External supplier of filter

Table 6 List of resources

Index	Description
r1	presses
r2	solders
r3	workers
r4	iron sheet cutter
r5	bender
r6	thermal forming machines
r7	cold producing unit assembly line
r8	fan assembly line
r9	air conditioner assembly line

Table 7 List of products

Index	Designation	Units
$p1$	Iron sheet for air conditioner shell	sheet

$p2$	Pipe	feet
$p3$	Fin	piece
$p4$	Plastic for fan blade	sheet
$p5$	Temperature sensor	unit
$p6$	Wires	feet
$p7$	Shell for air conditioner	unit
$p8$	Heat exchanger	unit
p9	Pipe for filter	feet
$p10$	Refrigerant	kg
$p11$	Silicon	kg
$p12$	Filter	unit
$p13$	Compressor	unit
$p14$	Compressor starting device	unit
$p15$	Metering device	unit
p16	Blade	unit
$P17$	Motor in fan	unit
$P18$	Circuit control	unit
$P19$	Cold producing unit	unit
$P20$	Motor in air conditioner	unit
$P21$	Fan	unit
$P22$	Air conditioner (Final Product)	unit

A.II Two approaches to derive transfer price

Our objective in this section is to show how to derive transfer price with resource-based approach and efficiency-based approach. We use Scenario 2 in Chapter 4 to show this.

The solution of the system model provides the value of the maximum profit and the value of the decision variables (inputs, outputs, durable resource). This optimal solution will be used to calculate the transfer prices.

As mentioned in Chapter 3, the unit cost price of output product includes the use of inputs and the use of resource. We will show how to derive the cost of inputs and the cost of resources in the following context.

First, we calculated the cost of inputs level by level from the beginning of the manufacturing process of the network-company. We take firm 28 as an example; this firm uses two input products p2 and p3 to produce product p8. According to the optimal result of Scenario 2, firm 5 supplies p2 at \$0.6, firm 7 supplies p3 at \$1.6. The optimal input quantity of product p2 $y^*_{5,28,2}$ and p3 $y^*_{7,28.3}$ equal 100,000 and 10,000 respectively. According to the first part of function (F3.24) and referring to Table 10, which provides the value of b_{qp}, which are defined as the quantity of input product q used by activity a to produce one unit of output product p; and referring to Table 12 which provides the value of g_{rp}, which are defined as the quantity of durable resource r required by activity a for per unit of output product p.

$$w_{f-p} = \left[\left(\sum_{\Sigma q \in P_f^i} b_{qp} \frac{\sum_{(i,f,q)\in Y_{P_f^i}} v_{ifq}(y_{ifq})}{\sum_{(i,f,q)\in Y_{P_f^i}} y_{ifq}}\right)\right] + \left(\sum_{r\in R_f} g_{rp} \frac{c_r^o - r_r^- x_r^- + r_r^+ x_r^+}{x_r^*}\right) \qquad \text{(F3.24)}$$

We can get the cost of inputs for product p8 as:

$$\frac{0.6*100000+1.6*100000}{20000}=11$$

where 20,000 is the optimal quantity of output product p8.

Similarly, we can get the input cost of unit of outputs at level I as:

Table 8 Material cost of unit of level I product

Products	Cost of inputs
P7	3.00
P8	11.00
P16	2.00
P18	3.50

Secondly, we calculate the cost of inputs for level II products. The calculation procedure is very similar to level 1, the only difference is that for level 1, the input product price is given and for level 2, the input product price is the transfer price calculated from level 1 (see Table 14). Then we can get the material cost of unit level II product as follows:

Table 9 Material cost of unit of level II product

Products	Cost of inputs of unit products
p19	216.89
p21	29.99

Table 10 Bill of material for manufacturing

	Output products							
Input products	P8	P12	P19	P18	P7	P16	P21	P22
P23	0	0.5	0	0	0	0	0	0
P2	5	0	0	0	0	0	0	0
P3	5	0	0	0	0	0	0	0
P11	0	0.10	0	0	0	0	0	0

P14	0	0	1	0	0	0	0	0
P13	0	0	1	0	0	0	0	0
P8	0	0	2	0	0	0	0	0
P12	0	0	1	0	0	0	0	0
P10	0	0	0.5	0	0	0	0	0
P15	0	0	1	0	0	0	0	0
P5	0	0	0	1	0	0	0	0
P6	0	0	0	0.50	0	0	0	0
P1	0	0	0	0	1	0	0	0
P4	0	0	0	0	0	0.10	0	0
P16	0	0	0	0	0	0	2	0
P20	0	0	0	0	0	0	0	1
P17	0	0	0	0	0	0	1	0
P7	0	0	0	0	0	0	0	1
P21	0	0	1	0	0	0	0	1
P18	0	0	1	0	0	0	0	1
P19	0	0	0	0	0	0	0	1

After calculating the cost of material, we calculate the cost of resource. We also take firm 28 as an example. The resources used by this firm are r2, r3 and r5. From the result of the optimal solution of the model, the quantities of resources r2, r3 and r5 used by firm 28 are 2000, 4000 and 2000, respectively; while the quantity of resources r2, r3 and r5 used by the whole network are 5,380, 13290 and 2000, respectively. According to the function $\sum_{r \in R_f} g_{rp} \frac{c_r^{\,o} - r_r^{\,-} x^- + r_r^{\,+} x^+}{x_r^{\,*}}$ and referring to Table 10, which provides the value of g_{rp}, and Table 12, the cost of resources for one unit of product p8 is:

$$
\begin{aligned}
& 0.1 * \frac{3650.4 + (5380 - 2160) * 1.82}{5380} \\
& + 1 * \frac{220500 + (43290 - 22050) * 12}{43290} \\
& + 0.1 * \frac{2562 + (2000 - 1050) * 2.74}{2000} \\
& = 11.41631965(\$)
\end{aligned}
$$

Similarly, we can get the resource cost of a unit product as follows:

Table 11 Resource costs of unit product

Products	Resource cost of unit product
P7	3.33
p8	11.42
p16	2.208
p18	0.15
p19	11.47
p21	1.193

Table 12 Bill of resources

	Product							
Resources	P8	P12	P19	P18	P7	P16	P21	P22
r1	0	0	0	0	0.12	0	0	0

r2	0.1	0	0.2	0.02	0.12	0	0	0
r3	1	0.1	1	0.01	0.2	0.1	0.1	1
r4	0	0	0	0	0.12	0.1	0	0
r5	0.1	0.02	0	0	0	0	0	0
r6	0	0	0	0	0	0.5	0	0
r7	0	0	0.3	0	0	0	0	0
r8	0	0	0	0	0	0	0.1	0
r9	0	0	0	0	0	0	0	0.5

Table 13 Price of resources

	X_0	C_R($)	R_R^-($)	R_R^+($)
r1	600	498	0.6	1.02
r2	2160	3650.4	1.42	1.82
r3	22050	220500	8	12
r4	650	4244.5	5.2	6.9
r5	1050	2562	2.05	2.74
r6	250	200	0.65	0.9
r7	1500	600	0.31	0.51
r8	500	440	0.75	1.02
r9	2500	2825	0.8	1.3

A.II.I Resource-based approach to derive transfer prices

Assuming proportional repartition of profit among the firms of the network according to the resources used to manufacture the products, the contribution va_r of resource r is obtained as follows:

$$va_r = \frac{c_r^{\,o} - r_r^{\,-} x_r^{\,-} + r_r^{\,+} x_r^{\,+}}{\sum_{r \in R_N} (c_r^{\,o} - r_r^{\,-} x_r^{\,-} + r_r^{\,+} x_r^{\,+})} va_N \qquad \text{(F 3.25)}$$

The numerator of function (F3.25) is the cost of a specific durable resource used by the network-company; the denominator is the cost of all durable resources by network-company that equals the sum of each durable resource's cost. According to function (F3.25) and referring to Table 13,

where the denominator --- the cost of all durable resources is :

$$\sum_{r \in R_N} (c_r^{\,o} - r_r^{\,-} x_r^{\,-} + r_r^{\,+} x_r^{\,+})$$

= 498+(1200-600)*1.02

+3650.4+(5380-2160)*1.82

+220500+(43290-22050)*12

+4244.5+(1400-650)*6.9

+2562+(2000-1050)*2.74

+200+(1000-250)*0.9

+600+(3000-1500)*0.51

+440+(1000-500)*1.02

+2825+(5000-2500)*1.3

=509850.30 ($)

we can get :

$$
\begin{aligned}
va_{r2} &= \frac{cr_2^{\,o} - r_{r2}^{\,-} x_{r2}^{\,-} + r_{r2}^{\,+} x_{r2}^{\,+}}{\sum_{r \in R_N} (c_{r2}^{\,o} - r_{r2}^{\,-} x_{r2}^{\,-} + r_{r2}^{\,+} x_{r2}^{\,+})} va_N \\
&= \frac{3650.4 + 3220 * 1.82}{509850.3} * 1336258.9 \\
&= 24926.71
\end{aligned}
$$

where 1336258.9 is the maximal profit of the whole network-company, which comes from the solution of the system model. 3220 = (5380-2160) is the value of X_{r2}^{+}.

Similarly, we can get the profits distributed to the resource r3 and r5 as follows:

$$
\begin{aligned}
va_{r3} &= \frac{220500 + (43290 - 22050) * 12}{509850.3} * 1336258.9 \\
&= 1245916.22
\end{aligned}
$$

$$
\begin{aligned}
va_{r5} &= \frac{2562 + (2000 - 1050) * 2.74}{509850.3} * 1336258.9 \\
&= 13536.87
\end{aligned}
$$

According to function (F 3.26) and since we already got the input cost and resource cost of unit output product p8, we can get the transfer price of p8 as follows:

$$
\begin{aligned}
v_{28-8} &= 11 + 11.4163 + 0.1 * \frac{24926.71}{5380} + 1 * \frac{1245916.22}{43290} + 0.1 * \frac{13536.87}{2000} \\
&= 52.34
\end{aligned}
$$

Similarly, we can get the transfer prices of other products at level I. These transfer prices are listed in Table 14:

Table 14 Transfer prices of product in resource-based approach

Product	Transfer price
P7	15.05

P8	52.34
P16	9.99
P18	4.03
P19	258.43
P21	34.18

A.II.II Efficiency-based approach to derive transfer prices

In this section, we will show how to use an efficiency-based approach to derive transfer price.

According to function (F3.29) and referring to Table 3 and Table 12,

$$\frac{\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p}}{\sum_{f \in N^I} \sum_{p \in P_f^o} \left(\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p} \right)} va_n = va_{fp} \qquad \text{(F3.29)}$$

We can get $\overline{uc_{18}} = 0.01 * \frac{3650.4}{2160} + 0.008 * \frac{220500}{22050} = 0.0969$

And $uc_{32,18} = 0.02 * \frac{3650.4}{2160} + 0.01 * \frac{220500}{22050} = 0.1338$

$$va_{32,18} = -\frac{\frac{0.0969}{0.1338} * 0.0969 \cdot 9000}{509850.3} 1336258.9 = 1655.32$$

According to function (F.3.32),

$$v_{f-p} = \left[\sum_{q \in P_f^i} b_{qp} \frac{\sum_{(i,f,q) \in Y_{Pf}^i} v_{ifq}(y_{ifq})}{\sum_{(i,f,q) \in Y_{Pf}^i} y_{ifq}} \right] + \sum_{r \in R_f} g_{rfp} \frac{c_r^{\,o} - r_r^{\,-} x_r^{\,-} + r_r^{\,+} x_r^{\,+}}{x_r^{\,*}}$$

$$+ \frac{\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p}}{\sum_{f \in N^I} \sum_{p \in P_f^o} \left(\frac{\overline{uc_p}}{uc_{fp}} \cdot \overline{uc_p} \cdot y_{f-p} \right)} va_n \qquad \text{(F 3.32)}$$

and since the result of the first two part of this function has been calculated in previous context (see Table 8, Table 9 and Table 11), we can get the transfer price of p18 as:

$$v_{32-18} = 3.6451 + \frac{\frac{0.0969}{0.1338} * 0.0969}{509850.3} * 1336258.9 = 3.67 \qquad \text{(F 3.32)}$$

Similarly, we can get the transfer price of product p21 as follows:

$$\overline{uc_{21}} = 0.1 * \frac{220500}{22050} + 0.1 * \frac{440}{500} = 1.088$$

$$uc_{21} = 0.12 * \frac{220500}{22050} + 0.11 * \frac{440}{500} = 1.2968$$

$$v_{34-21} = 31.8621 + \frac{\frac{1.2968}{1.088} * 1.2968}{509850.3} * 1336258.9 = 35.24$$

A.III AMPL

AMPL is the optimization software used to solve the MIP. AMPL is a Modeling Language for Mathematical Programming. It has a special syntax to create the optimization instance. It allows model file and separates data files, that give more flexibility.

The AMPL Template is as follows:

- Define Sets
- Define Parameters
- Define Variables ¦ Also can define variable bound constraints in this section
- Define Objective ¦ Define Constraints

A.IV Numerical Code and Results

A.IV.I Model File

mnet3.mod

```
### set ###
set setInActivities;
set setOutActivities;
set setInternalActivities;
set setActivity := setInActivities union setOutActivities union setInternalActivities;
# sets for products
set setInProduct;
set setOutProduct;
set setInternalProduct;
set setProduct := setInProduct union setOutProduct union setInternalProduct;
set setResource;
    #resources set.
```

```
set InProdlink within {i in setActivity, j in setActivity, p in setProduct: i<>j};
    #i as supplier, j as current activity, p as product produced by the activity
set setInputOutput within {pi in setProduct, po in setProduct};
    #pi as input product, po as output product, it is the relationship between input and output.
set setResOutput within {i in setActivity, p in setProduct, res in setResource};
    #set represent the relationship between resource and product


param maxQuatity >=0 integer;
        #maximum quatity for possible price changes.
set setQuatity := 1..maxQuatity;

set inAP within { i in setInActivities, p in setProduct };
    # input activity and input product relationship
set outAP within { i in setOutActivities, p in setProduct };

###papameter###
param IOProdRelation {setInputOutput} > 0;
    #how many input for one output product.
param RPARelation {setResOutput} > 0;
    #how many durable resource used for output product
param x0{i in setResource}>0 integer;
    #original amount of resource
param rd{i in setResource};
    #unit resource save for decreasing resource
param ri{i in setResource};
    #unit resource spend for increasing resource
param c0{i in setResource};
```

```
    #original cost of the resource

#upper limit for resources, input products, and output products to the market.
param xMax{i in setResource} >= 0;
param xMin{i in setResource} >= 0;
    #maximum available resource
param yInMax{(i,p ) in inAP } >= 0 integer;
    #maximum available input products including raw material
param yOutMax{(i,p ) in outAP } >= 0 integer ;
    #maximum market capacity.
param Price{(i,j,p) in InProdlink} > 0;

#param outPrice{setOutProduct} > 0;
param rPrice{(i,p,r) in setResOutput};

###variables###
    #increased resource
var xr {(i,p,r) in setResOutput} >=0   integer ;
var cr {i in setResource} >= 0;
#amount of product p transfered between activities.
var y{InProdlink} >= 0  integer ;

###objective###
maximize profit:
    sum {(i,j,p) in InProdlink: j in setOutActivities} Price[i,j,p]*y[i,j,p] -
    sum {(i,j,p) in InProdlink: i in setInActivities} Price[i,j,p]*y[i,j,p]
    - sum{r in setResource} cr[r];

### constraints###
```

```
yinmax { (i,p ) in inAP } : sum { (i,j,p) in InProdlink } y[i,j,p] <= yInMax[i,p]
;               #Input products restriction.
youtmax { (j,p ) in outAP } : sum { (i,j,p) in InProdlink } y[i,j,p] <= yOutMax[j,p] ;
            #output products restriction.
yIOrelation { i in setInternalActivities,pi in setProduct, po in setProduct: (pi,po) in
setInputOutput
}:  sum { (k,i,pi) in InProdlink } y[k,i,pi] = sum {(i,j,po) in InProdlink} y[i,j,po] *
IOProdRelation[pi,po];

rRelation{ i in setActivity, p in setProduct, r in setResource:
(i,p,r) in setResOutput}:
  xr[i,p,r] =  sum {(i,j,p) in InProdlink} y[i,j,p] * RPARelation[i,p,r] ;
#xr[i,p,r] = max ( sum {(i,j,p) in InProdlink} y[i,j,p] * RPARelation[i,p,r], 10 ) ;

rCost{r in setResource}:
cr[r] = if (sum{(i,p,r) in setResOutput} xr[i,p,r] > x0[r])
then (c0[r] +( sum{(i,p,r) in setResOutput} xr[i,p,r] - x0[r])* ri[r])
else (c0[r] - ( x0[r] -sum{(i,p,r) in setResOutput} xr[i,p,r])* rd[r]);

rlim{  r in setResource }:
xMin[r] <=sum { i in setActivity, j in setActivity,p in setProduct: (i,j,p) in
InProdlink and (i,p,r) in setResOutput }
 y[i,j,p] * RPARelation[i,p,r] <= xMax[r];
```

A.IV.II Data Files and Programming Results

Scenario 1 Unlimited Market

Dnet902s1.dat (scenario 1 $0 \le y_{35,36,22} \le \infty$, $0 \le y_{35,38,22} \le \infty$) :

```
data;

### set ###

##set activity##

set setInActivities:= a3 a4  a29  a23 a24 a25 a26 a37 a1 a2 a19 a20 a21 a22 a39
a9  a10  a13  a14  a15  a16 a17  a18  a5 a7   a11 a6 a8 a12  ;

set setOutActivities := a36 a38;

set setInternalActivities := a32 a35 a27 a34 a31 a33 a28 a30;

# sets products####
set setInProduct := p1 p20 p19 p5 p6 p4 p17 p21   p10  p13 p14 p15 p2 p3 p23 p11;

set setOutProduct := p22;

set setInternalProduct := p18 p7 p21 p16 p12 p8 ;

## set resources##

set setResource :=  r3  r9 r2 r1 r4 r6 r8 r7 r5 ;

## set of relationships

set InProdlink :=   a1 a27 p1          a2 a27 p1            a27 a35 p7
```

```
a3 a35 p20      a4 a35 p20      a33 a35 p19
a31 a34 p16     a19 a31 p4      a20 a31 p4
a21 a34 p17     a22 a34 p17     a34 a35 p21
a32 a35 p18     a29 a35 p21     a37 a35 p18
a35 a36 p22     a35 a38 p22     a23 a32 p5
a24 a32 p5      a25 a32 p6      a26 a32 p6
a39 a33 p12     a28 a33 p8      a30  a33  p12
a9 a33 p10      a10 a33 p10     a13 a33 p13
a14 a33 p13     a15 a33 p14     a16 a33 p14
a17 a33 p15     a18 a33 p15     a5 a28 p2
a7 a28 p3       a6 a28 p2       a8 a28  p3
a11 a30 p11     a5 a30 p23      a12 a30 p11
a6 a30 p23 ;

## sets associated with inactivity##

set inAP :=  a1 p1     a2 p1     a19 p4    a20 p4    a21 p17
             a22 p17   a3 p20    a4 p20    a37 p18   a23 p5
             a24 p5    a25 p6    a26 p6    a29 p21   a39 p12
             a9 p10    a13 p13   a15 p14   a17 p15   a5 p2
             a7 p3     a5 p23    a11 p11   a6 p2     a8 p3
             a6 p23    a12 p11 ;

## set associated with out activity

set outAP := a36 p22  a38 p22;


## set associated with resources
```

```
set setResOutput := a35 p22 r9      qa35 p22 r3  qa32 p18 r2       a32 p18 r3
                    a34 p21 r8      a34 p21 r3   a31 p16 r6        a31 p16 r4
                    a31 p16 r3      a27 p7 r1    a27 p7 r2         a27 p7 r3
                    a27 p7 r4       a33 p19 r7   a33 p19 r2        a33 p19 r3
                    a28 p8 r5       a28 p8 r2    a28 p8 r3         a30 p12 r5
                    a30 p12 r3;
```

```
set setInputOutput :=  p7  p22      p20 p22      p21 p22      p18 p22
                       p19 p22      p5 p18       p6 p18       p1 p7
                       p16 p21      p17 p21      p4 p16       p8 p19
                       p10 p19      p12 p19      p13 p19      p14 p19
                       p15 p19      p2 p8        p3 p8        p23 p12
                       p11 p12 ;
```

```
##papameter###
```

```
param IOProdRelation :=  p7  p22 1      p20 p22 1      p21 p22 1
                         p18 p22 1      p19 p22 1      p5  p18 1
                         p6 p18 0.5     p1 p7 1        p16 p21 2
                         p17 p21 1      p4 p16 0.1     p8 p19 2
                         p10 p19 1      p12 p19 1      p13 p19 1
                         p14 p19  1     p15 p19 1      p2  p8  5
                         p3 p8 5        p23 p12 0.5    p11 p12  0.1  ;
```

```
param RPARelation := a35 p22 r9 0.5     a35 p22 r3 1        a32 p18 r2 0.02
                     a32 p18 r3 0.01    a27 p7 r1 0.12      a27 p7 r2 0.12
                     a27 p7 r3 0.2      a27 p7 r4 0.12      a34 p21 r8 0.1
                     a34 p21 r3 0.1     a31 p16 r6 0.05     a31 p16 r4 0.01
                     a31 p16 r3 0.01    a33 p19 r7 0.3      a33 p19 r2 0.2
                     a33 p19 r3 1       a28 p8 r5 0.1       a28 p8 r2 0.1
                     a28 p8 r3 1        a30 p12 r5 0.02     a30 p12 r3 0.1;

param x0:= r9 2500    r3 22050    r2 2160    r1 600    r4 650
          r6 250     r8 500      r7 1500    r5 1050;

param rd:= r9 0.8   r3 8   r2 1.42  r1 0.6   r4 5.2   r6 0.65  r8 0.75
          r7 0.31  r5 2.05;

param c0:= r9 2825    r3 220500    r2 3650.4    r1 498   r4 4244.5

          r6 200  r8 440    r7 600    r5 2562;

param ri:= r9 1.3   r3 12   r2 1.82    r1 1.02    r4 6.9   r6 0.9  r8 1.02
          r7 0.51  r5 2.74;

param xMax := r9 23000  r3 80000    r2 25000    r1 24000
              r4 22000  r6 24000    r8 22000    r7 22000
              r5 25000;

param xMin := r9 25  r3 69   r2 22   r1 6     r4 7    r6 3
              r8 5   r7  15  r5 11 ;
```

```
param yInMax :=     a1 p1 15000       a2 p1 5000       a3 p20 22000
                    a4 p20 10000      a19 p4 11000     a20 p4 8000
                    a21 p17 8000      a22 p17 7000     a23 p5 6000
                    a24 p5 3000       a25 p6 8000      a26 p6 7000
                    a37  p18 6000     a29 p21 12000    a39 p12 12000
                    a9 p10 30000      a13 p13 32000    a15 p14 33000
                    a17 p15 22000     a5 p2 220000     a7 p3 240000
                    a5 p23 15000      a11 p11 22000    a6 p2 200000
                    a8  p3 200000     a6 p23 11000     a12  p11 12000;

param yOutMax:= a36 p22 100000     a38 p22 100000 ;

param Price :=   a29 a35 p21 34.50     a1 a27 p1  3         a2 a27 p1 3.05
                 a3 a35 p20 50         a4 a35 p20 51        a19 a31 p4 20
                 a20 a31 p4 20.05      a21 a34 p17 10       a22  a34  p17  10.08
                 a23 a32 p5 3          a24 a32  p5 3.2      a25 a32 p6 1
                 a26 a32  p6 1.1       a37 a35 p18 3.8      a35   a38   p22   380
                 a35 a36 p22 382       a39 a33 p12 1.5      a9 a33  p10 0.5
                 a10 a33  p10 0.51     a13 a33 p13 100      a14 a33 p13 100.03
                 a15 a33 p14 1         a16 a33 p14 1.01     a17 a33 p15 1
                 a18 a33 p15 1.01      a5 a28 p2 0.6        a7 a28 p3 1.6
                 a5 a30 p23 0.8        a11 a30 p11 1        a6 a28 p2 0.62
                 a8 a28 p3 1.63        a6 a30 p23 0.82      a12 a30 p11 1.04;
```

Result of scenario 1 :

MINOS 5.5: optimal solution found.
44 iterations, objective 1991836.3
Nonlin evals: constrs = 160, Jac = 159.

```
ampl: display y;
y [*,a27,*]
:       p1        :=
a1   15000
a2      6.34981e-11

 [*,a28,*]
:    p2      p3     :=
a5   150000     .
a6       0    .
a7    .     150000
a8    .         0

 [*,a30,*]
:    p11   p23    :=
a11   300    .
a12     0   .
a5     .   1500
a6     .      0

 [*,a31,*]
:    p4    :=
a19   3000
a20      0

 [*,a32,*]
:    p5    p6    :=
a23   6000    .
a24   3000    .
```

```
a25   .   4500
a26   .      0

[*,a33,*]
:     p10    p12    p13    p14    p15    p8    :=
a10     0      .      .      .      .      .
a13     .      .  15000      .      .      .
a14     .      .      0      .      .      .
a15     .      .      .  15000      .      .
a16     .      .      .      0      .      .
a17     .      .      .      .  15000      .
a18     .      .      .      .      0      .
a28     .      .      .      .      .  30000
a30     .   3000      .      .      .      .
a39     .  12000      .      .      .      .
a9  15000      .      .      .      .      .

[*,a34,*]
:     p16    p17    :=
a21     .   8000
a22     .   7000
a31 30000      .

[*,a35,*]
:     p18    p19    p20    p21    p7    :=
a27     .      .      .      .  15000
a29     .      .      .      0      .
a3      .      .  15000      .      .
a32  9000      .      .      .      .
```

```
a33   .    15000    .     .     .
a34   .     .     .    15000    .
a37  6000    .     .     .     .
a4    .     .      0     .     .

 [a35,a36,p22]  15000

 [a35,a38,p22]  0
;

ampl: display xr;
xr [a27,*,*] (tr)
:   p7    :=
r1  1800
r2  1800
r3  3000
r4  1800

 [a28,*,*] (tr)
:   p8    :=
r2   3000
r3  30000
r5   3000

 [a30,*,*] (tr)
:   p12   :=
r3  300
r5   60
```

```
[a31,*,*] (tr)
:   p16    :=
r3   300
r4   300
r6  1500

[a32,*,*] (tr)
:   p18   :=
r2  180
r3   90

[a33,*,*] (tr)
:    p19    :=
r2   3000
r3  15000
r7   4500

[a34,*,*] (tr)
:   p21     :=
r3  1500
r8  1500

[a35,*,*] (tr)
:    p22    :=
r3  15000
r9   7500
;

ampl: display cr;
```

cr [*] :=
r1 1722
r2 14242.8
r3 738180
r4 14249.5
r5 8069.4
r6 1325
r7 2130
r8 1460
r9 9325
;

ampl:

Scenario 2 Limited Fixed Market

$(\bar{d}_{35,36} = 6000 \ , \ \bar{d}_{35,38} = 4000 \ )$

dnet902s2.dat

data;

set

##set activity##

set setInActivities:= a3 a4 a29 a23 a24 a25 a26 a37 a1 a2 a19 a20 a21 a22 a39
a9 a10 a13 a14 a15 a16 a17 a18 a5 a7 a11 a6 a8 a12 ;

set setOutActivities := a36 a38;

```
set setInternalActivities := a32 a35 a27 a34 a31 a33 a28 a30;


# sets products####
set setInProduct := p1 p20 p19 p5 p6 p4 p17 p21   p10  p13 p14 p15 p2 p3 p23 p11;

set setOutProduct := p22;

set setInternalProduct := p18 p7 p21 p16 p12 p8 ;



## set resources##

set setResource :=  r3  r9 r2 r1 r4 r6 r8 r7 r5 ;

## set of relationships


set InProdlink :=  a1 a27 p1        a2 a27 p1        a27 a35 p7
                   a3 a35 p20       a4 a35 p20       a33 a35 p19
                   a31 a34 p16      a19 a31 p4       a20 a31 p4
                   a21 a34 p17      a22 a34 p17      a34 a35 p21
                   a32 a35 p18      a29 a35 p21      a37 a35 p18
                   a35 a36 p22      a35 a38 p22      a23 a32 p5
                   a24 a32 p5       a25 a32 p6       a26 a32 p6
                   a39 a33 p12      a28 a33 p8       a30  a33  p12
                   a9 a33 p10       a10 a33 p10      a13 a33 p13
```

```
        a14 a33 p13     a15 a33 p14     a16 a33 p14
        a17 a33 p15     a18 a33 p15     a5 a28 p2
        a7 a28 p3       a6 a28 p2       a8 a28  p3
        a11 a30 p11     a5 a30 p23      a12 a30 p11
        a6 a30 p23 ;

## sets associated with inactivity##

set inAP :=   a1 p1     a2 p1     a19 p4    a20 p4
              a21 p17   a22 p17   a3 p20    a4 p20
              a37 p18   a23 p5    a24 p5    a25 p6
              a26 p6    a29 p21   a39 p12   a9 p10
              a13 p13   a15 p14   a17 p15   a5 p2
              a7 p3     a5 p23    a11 p11   a6 p2
              a8 p3     a6 p23    a12 p11 ;

## set associated with out activity

set outAP := a36 p22  a38 p22;

## set associated with resources

set setResOutput :=   a35 p22 r9   a35 p22 r3   a32 p18 r2       a32 p18 r3
                      a34 p21 r8   a34 p21 r3   a31 p16 r6       a31 p16 r4
                      a31 p16 r3   a27 p7 r1    a27 p7 r2 a27    p7 r3 a27 p7 r4
                      a33 p19 r7   a33 p19 r2   a33 p19 r3       a28 p8 r5
                      a28 p8 r2    a28 p8 r3    a30 p12 r5       a30 p12 r3;
```

```
set setInputOutput :=  p7  p22      p20 p22      p21 p22      p18 p22
                       p19 p22      p5 p18       p6 p18       p1 p7
                       p16 p21      p17 p21      p4 p16       p8 p19
                       p10 p19      p12 p19      p13 p19      p14 p19
                       p15 p19      p2 p8        p3 p8        p23 p12
                       p11 p12 ;


##papameter###


param IOProdRelation :=  p7  p22 1      p20 p22 1      p21 p22 1
                         p18 p22 1      p19 p22 1      p5  p18 1
                         p6 p18 0.5     p1 p7 1        p16 p21 2
                         p17 p21 1      p4 p16 0.1     p8 p19 2
                         p10 p19 1      p12 p19 1      p13 p19 1
                         p14 p19  1     p15 p19 1      p2 p8  5
                         p3 p8 5        p23 p12 0.5    p11 p12  0.1  ;

param RPARelation :=  a35 p22 r9 0.5       a35 p22 r3 1        a32 p18 r2 0.02
                      a32 p18 r3 0.01      a27 p7 r1 0.12      a27 p7 r2 0.12
                      a27 p7 r3 0.2        a27 p7 r4 0.12      a34 p21 r8 0.1
                      a34 p21 r3 0.1       a31 p16 r6 0.05     a31 p16 r4 0.01
                      a31 p16 r3 0.01      a33 p19 r7 0.3      a33 p19 r2 0.2
                      a33 p19 r3 1         a28 p8 r5 0.1       a28 p8 r2 0.1
                      a28 p8 r3 1          a30 p12 r5 0.02     a30 p12 r3 0.1;


param x0:=   r9 2500      r3 22050      r2 2160  r1 600   r4 650   r6 250
```

```
             r8 500        r7 1500       r5 1050;

param rd:=   r9 0.8    r3 8      r2 1.42   r1 0.6    r4 5.2    r6 0.65
             r8 0.75   r7 0.31   r5 2.05;

param c0:=   r9 2825   r3 220500    r2 3650.4    r1 498    r4 4244.5
             r6 200    r8 440       r7 600       r5 2562;

param ri:=   r9 1.3    r3 12     r2 1.82   r1 1.02   r4 6.9    r6 0.9
             r8 1.02   r7 0.51   r5 2.74;

param xMax :=    r9 23000    r3 80000       r2 25000       r1 24000
                 r4 22000    r6 24000       r8 22000       r7 22000
                 r5 25000;
param xMin :=    r9 25       r3 69       r2 22       r1 6
                 r4 7        r6 3        r8 5        r7  15
                 r5 11 ;

param yInMax :=      a1 p1 15000        a2 p1 5000        a3 p20 22000
                     a4 p20 10000       a19 p4 11000      a20 p4 8000
                     a21 p17 8000       a22 p17 7000      a23 p5 6000
                     a24 p5 3000        a25 p6 8000       a26 p6 7000
                     a37  p18 6000      a29 p21 12000     a39 p12 12000
                     a9 p10 30000       a13 p13 32000     a15 p14 33000
                     a17 p15 22000      a5 p2 220000      a7 p3 240000
                     a5 p23 15000       a11 p11 22000     a6 p2 200000
                     a8  p3 200000      a6 p23 11000      a12  p11 12000;

param yOutMax:= a36 p22 6000  a38 p22 4000 ;
```

param Price :=	a29 a35 p21 34.5	a1 a27 p1 3	a2 a27 p1 3.05
	a3 a35 p20 50	a4 a35 p20 51	a19 a31 p4 20.00
	a20 a31 p4 20.05	a21 a34 p17 10	a22 a34 p17 10.08
	a23 a32 p5 3	a24 a32 p5 3.2	a25 a32 p6 1
	a26 a32 p6 1.1	a37 a35 p18 3.8	a35 a38 p22 380
	a35 a36 p22 382	a39 a33 p12 1.5	a9 a33 p10 0.5
	a10 a33 p10 0.51	a13 a33 p13 100	a14 a33 p13 100.03
	a15 a33 p14 1	a16 a33 p14 1.01	a17 a33 p15 1
	a18 a33 p15 1.01	a5 a28 p2 0.6	a7 a28 p3 1.6
	a5 a30 p23 0.8	a11 a30 p11 1	a6 a28 p2 0.62
	a8 a28 p3 1.63	a6 a30 p23 0.82	a12 a30 p11 1.04;

result of scenario 2:

```
sw: ampl
ampl: model mnet3.mod;
ampl: data dnet902s2.dat;
ampl: solve;
MINOS 5.5: ignoring integrality of 61 variables
MINOS 5.5: optimal solution found.
45 iterations, objective 1336258.9
Nonlin evals: constrs = 149, Jac = 148.
ampl:
ampl: display y;
y [*,a27,*]
:   p1    :=
a1  10000
```

a2 0

[*,a28,*]
: p2 p3 :=
a5 1e+05 .
a6 0 .
a7 . 1e+05
a8 . 0

[*,a30,*]
: p11 p23 :=
a11 2.04144e-14 .
a12 0 .
a5 . 0
a6 . 0

[*,a31,*]
: p4 :=
a19 2000
a20 0

[*,a32,*]
: p5 p6 :=
a23 6000 .
a24 3000 .
a25 . 4500
a26 . 0

[*,a33,*]

:	p10	p12	p13	p14	p15	p8	:=
a10	0	.	.	.	.	.	
a13	.	.	10000	.	.	.	
a14	.	.	0	.	.	.	
a15	.	.	.	10000	.	.	
a16	.	.	.	0	.	.	
a17	.	.	.	.	10000	.	
a18	.	.	.	.	0	.	
a28	.	.	.	.	.	20000	
a30	.	0	.	.	.	.	
a39	.	10000	.	.	.	.	
a9	10000	.	.	.	.	.	

[*,a34,*]

:	p16	p17	:=
a21	.	8000	
a22	.	2000	
a31	20000	.	

[*,a35,*]

:	p18	p19	p20	p21	p7	:=
a27	.	.	.	.	10000	
a29	.	.	.	0	.	
a3	.	.	10000	.	.	
a32	9000	.	.	.	.	
a33	.	10000	.	.	.	
a34	.	.	.	10000	.	
a37	1000	.	.	.	.	
a4	.	.	0	.	.	

```
[a35,a36,p22]   6000

[a35,a38,p22]   4000
;

ampl: display xr;
xr [a27,*,*] (tr)
:   p7    :=
r1  1200
r2  1200
r3  2000
r4  1200

 [a28,*,*] (tr)
:   p8    :=
r2   2000
r3  20000
r5   2000

 [a30,*,*] (tr)
:      p12       :=
r3  1.13687e-13
r5  2.84217e-14

 [a31,*,*] (tr)
:  p16    :=
r3   200
r4   200
```

```
r6  1000

 [a32,*,*] (tr)
:   p18   :=
r2  180
r3   90

 [a33,*,*] (tr)
:    p19    :=
r2   2000
r3  10000
r7   3000

 [a34,*,*] (tr)
:   p21    :=
r3  1000
r8  1000

 [a35,*,*] (tr)
:    p22    :=
r3  10000
r9   5000
;

ampl: display cr;
cr [*] :=
r1    1110
r2    9510.8
r3  475380
```

```
r4   9419.5
r5   5165
r6    875
r7   1365
r8    950
r9   6075
;
```

ampl:

7.2.4 Scenario 3 Variable Market

$y_{35,36,22} \in (600,9000)$, $y_{35,38,22} \in (400,6000)$)

Model file :

m.mod

```
### set ###

set setInActivities;
set setOutActivities;
set setInternalActivities;

set setActivity := setInActivities union setOutActivities union setInternalActivities;


# sets for products
set setInProduct;
set setOutProduct;
```

```
set setInternalProduct;

set setProduct := setInProduct union setOutProduct union setInternalProduct;

set setResource;
    #resources set.

set InProdlink within {i in setActivity, j in setActivity, p in setProduct: i<>j};
    #i as supplier, j as current activity, p as product produced by the activity

set setInputOutput within {pi in setProduct, po in setProduct};
    #pi as input product, po as output product, it is the relationship between input and
output.
set setResOutput within {i in setActivity, p in setProduct, res in setResource};
    #set represent the relationship between resource and product


param maxQuatity >=0 integer;
        #maximum quatity for possible price changes.
set setQuatity := 1..maxQuatity;

set inAP within { i in setInActivities, p in setProduct };
    # input activity and input product relationship
set outAP within { i in setOutActivities, p in setProduct };

###papameter###
param IOProdRelation {setInputOutput} > 0;
    #how many input for one output product.
param RPARelation {setResOutput} > 0;
```

```
    #how many durable resource used for output product

param x0{i in setResource}>0 integer;
    #original amount of resource
param rd{i in setResource};
    #unit resource save for decreasing resource
param ri{i in setResource};
    #unit resource spend for increasing resource
param c0{i in setResource};
    #original cost of the resource

#upper limit for resources, input products, and output products to the market.
param xMax{i in setResource} >= 0;
param xMin{i in setResource} >= 0;
    #maximum available resource
param yInMax{(i,p ) in inAP } >= 0 integer;
    #maximum available input products including raw material

param yOutMax{(i,p ) in outAP } >= 0 integer ;
#maximum market capacity.


param yOutMin{(i,p ) in outAP } >= 0 integer ;


param Price{(i,j,p) in InProdlink} > 0;

#param outPrice{setOutProduct} > 0;
```

```
param rPrice{(i,p,r) in setResOutput};


###variables###

    #increased resource
var xr {(i,p,r) in setResOutput} >=0   integer ;
var cr {i in setResource} >= 0;

#amount of product p transfered between activities.
var y{InProdlink} >= 0  integer ;



###objective###
maximize profit:
    sum {(i,j,p) in InProdlink: j in setOutActivities} Price[i,j,p]*y[i,j,p] -
    sum {(i,j,p) in InProdlink: i in setInActivities} Price[i,j,p]*y[i,j,p]
    - sum{r in setResource} cr[r];



### constraints###

    yinmax { (i,p ) in inAP } : sum { (i,j,p) in InProdlink } y[i,j,p] <= yInMax[i,p] ;
        #Input products restriction.

    youtmax { (j,p ) in outAP } : sum { (i,j,p) in InProdlink } y[i,j,p] <= yOutMax[j,p] ;
```

```
#output products restriction.

youtmin { (j,p ) in outAP } : sum { (i,j,p) in InProdlink } y[i,j,p] >= yOutMin[j,p] ;

#output products restriction.


yIOrelation { i in setInternalActivities,pi in setProduct, po in setProduct: (pi,po) in setInputOutput
}: sum { (k,i,pi) in InProdlink } y[k,i,pi] = sum {(i,j,po) in InProdlink} y[i,j,po] * IOProdRelation[pi,po];


rRelation{ i in setActivity, p in setProduct, r in setResource:
(i,p,r) in setResOutput}:
  xr[i,p,r] = sum {(i,j,p) in InProdlink} y[i,j,p] * RPARelation[i,p,r] ;
#xr[i,p,r] = max ( sum {(i,j,p) in InProdlink} y[i,j,p] * RPARelation[i,p,r], 10 ) ;

rCost{r in setResource}:
cr[r] = if (sum{(i,p,r) in setResOutput} xr[i,p,r] > x0[r])
then (c0[r] +( sum{(i,p,r) in setResOutput} xr[i,p,r] - x0[r])* ri[r])
else (c0[r] - ( x0[r] -sum{(i,p,r) in setResOutput} xr[i,p,r])* rd[r]);


rlim{ r in setResource }:
xMin[r] <=sum { i in setActivity, j in setActivity,p in setProduct: (i,j,p) in InProdlink and (i,p,r) in setResOutput }
 y[i,j,p] * RPARelation[i,p,r] <= xMax[r];
```

```
data file
ds3.dat;
### set ###

##set activity##

set setInActivities:= a3    a4   a29      a23  a24  a25  a26  a37  a1    a2    a19  a20  a21
                      a22  a39  a9    a10      a13      a14      a15      a16  a17      a18
                      a5    a7    a11
                      a6    a8    a12  ;

set setOutActivities :=      a36  a38;

set setInternalActivities :=      a32  a35  a27  a34  a31  a33  a28  a30;


# sets products####
set setInProduct :=     p1  p20   p19  p5   p6  p4     p17  p21       p10   p13
                        p14  p15  p2  p3     p23  p11;

set setOutProduct := p22;

set setInternalProduct := p18    p7    p21  p16  p12  p8 ;
```

```
## set resources##

set setResource := r3      r9   r2   r1   r4   r6   r8   r7   r5 ;

## set of relationships

set InProdlink :=  a1 a27 p1     a2 a27 p1     a27 a35 p7     a3 a35 p20
                   a4 a35 p20    a33 a35 p19   a31 a34 p16    a19 a31 p4
                   a20 a31 p4    a21 a34 p17   a22 a34 p17    a34 a35 p21
                   a32 a35 p18   a29 a35 p21   a37 a35 p18    a35 a36 p22
                   a35 a38 p22   a23 a32 p5    a24 a32 p5     a25 a32 p6
                   a26 a32 p6    a39 a33 p12   a28 a33 p8     a30  a33  p12
                   a9 a33 p10    a10 a33 p10   a13 a33 p13    a14 a33 p13
                   a15 a33 p14   a16 a33 p14   a17 a33 p15    a18 a33 p15
                   a5 a28 p2     a7 a28 p3     a6 a28 p2      a8 a28  p3
                   a11 a30 p11   a5 a30 p23    a12 a30 p11    a6 a30 p23 ;
## sets associated with inactivity##

set inAP :=  a1 p1    a2 p1     a19 p4    a20 p4    a21 p17   a22 p17
             a3 p20   a4 p20    a37 p18   a23 p5    a24 p5    a25 p6
             a26 p6   a29 p21   a39 p12   a9 p10    a13 p13   a15 p14
             a17 p15  a5 p2     a7 p3     a5 p23    a11 p11   a6 p2
             a8 p3    a6 p23    a12 p11 ;

## set associated with out activity

set outAP := a36 p22  a38 p22;
```

```
## set associated with resources

set setResOutput :=   a35 p22 r9   a35 p22 r3   a32 p18 r2   a32 p18 r3
                      a34 p21 r8   a34 p21 r3   a31 p16 r6   a31 p16 r4
                      a31 p16 r3   a27 p7 r1    a27 p7 r2    a27 p7 r3
                      a27 p7 r4    a33 p19 r7   a33 p19 r2   a33 p19 r3
                      a28 p8 r5    a28 p8 r2    a28 p8 r3        a30 p12 r5
                      a30 p12 r3;


set setInputOutput := p7  p22        p20 p22    p21 p22    p18 p22
                      p19 p22        p5 p18     p6 p18     p1 p7
                      p16 p21        p17 p21    p4 p16     p8 p19
                      p10 p19        p12 p19    p13 p19    p14 p19
                      p15 p19        p2 p8      p3 p8      p23 p12
                      p11 p12 ;


##papameter###


param IOProdRelation := p7  p22 1      p20 p22 1      p21 p22 1
                        p18 p22 1      p19 p22 1      p5  p18 1
                        p6 p18 0.5     p1 p7 1        p16 p21 2
                        p17 p21 1      p4 p16 0.1     p8 p19 2
                        p10 p19 1      p12 p19 1      p13 p19 1
                        p14 p19  1     p15 p19 1      p2 p8  5
                        p3 p8 5        p23 p12 0.5    p11 p12  0.1  ;
```

```
param RPARelation :=    a35 p22 r9 0.5    a35 p22 r3 1      a32 p18 r2 0.02
                        a32 p18 r3 0.01   a27 p7 r1 0.12    a27 p7 r2 0.12
                        a27 p7 r3 0.2     a27 p7 r4 0.12    a34 p21 r8 0.1
                        a34 p21 r3 0.1    a31 p16 r6 0.05   a31 p16 r4 0.01
                        a31 p16 r3 0.01   a33 p19 r7 0.3    a33 p19 r2 0.2
                        a33 p19 r3 1      a28 p8 r5 0.1     a28 p8 r2 0.1
                        a28 p8 r3 1       a30 p12 r5 0.02   a30 p12 r3 0.1;


param x0:=  r9 2500   r3 22050    r2 2160  r1 600  r4 650  r6 250
            r8 500    r7 1500     r5 1050;

param rd:=  r9 0.8   r3 8   r2 1.42  r1 0.6   r4 5.2   r6 0.65  r8 0.75
            r7 0.31  r5 2.05;

param c0:=  r9 2825  r3 220500   r2 3650.4   r1 498   r4 4244.5
            r6 200   r8 440      r7 600      r5 2562;

param ri:=  r9 1.3   r3 12   r2 1.82  r1 1.02  r4 6.9   r6 0.9
            r8 1.02  r7 0.51  r5 2.74;

param xMax :=  r9 23000    r3 80000    r2 25000    r1 24000
               r4 22000    r6 24000    r8 22000    r7 22000
               r5 25000;
param xMin :=  r9 25   r3 69   r2 22   r1 6      r4 7    r6 3
               r8 5    r7  15  r5 11 ;

param yInMax := a1 p1 15000    a2 p1 5000     a3 p20 22000
                a4 p20 10000   a19 p4 11000   a20 p4 8000
```

```
a21 p17 8000          a22 p17 7000          a23 p5 6000
a24 p5 3000           a25 p6 8000           a26 p6 7000
a37  p18 6000         a29 p21 12000         a39 p12 12000
a9 p10 30000          a13 p13 32000         a15 p14 33000
a17 p15 22000         a5 p2 220000          a7 p3 240000
a5 p23 15000          a11 p11 22000         a6 p2 200000
a8  p3 200000         a6 p23 11000          a12  p11 12000;

param yOutMax:= a36 p22 9000    a38 p22 6000 ;
param yOutMin:= a36 p22 600     a38 p22 400 ;

param Price :=  a29 a35 p21 34.50    a1 a27 p1  3          a2 a27 p1 3.05
                a3 a35 p20 50        a4 a35 p20 51         a19 a31 p4 20
                a20 a31 p4 20.05     a21 a34 p17 10        a22 a34 p17 10.08
                a23 a32 p5 3         a24 a32  p5 3.2       a25 a32 p6 1
                a26 a32  p6 1.1      a37 a35 p18 3.8       a35 a38 p22 380
                a35 a36 p22 382      a39 a33 p12 1.5       a9 a33  p10 0.5
                a10 a33  p10 0.51    a13 a33 p13 100       a14 a33 p13 100.03
                a15 a33 p14 1        a16 a33 p14 1.01      a17 a33 p15 1
                a18 a33 p15 1.01     a5 a28 p2 0.6         a7 a28 p3 1.6
                a5 a30 p23 0.8       a11 a30 p11 1         a6 a28 p2 0.62
                a8 a28 p3 1.63       a6 a30 p23 0.82       a12 a30 p11 1.04;
```

Results of scenario 3

sw: ampl

```
ampl: model m.mod;
ampl: data ds3.dat;
ampl: solve;
MINOS 5.5: ignoring integrality of 61 variables
MINOS 5.5: optimal solution found.
35 iterations, objective 1979836.3
Nonlin evals: constrs = 110, Jac = 109.
ampl: display y;
y [*,a27,*]
:   p1    :=
a1  15000
a2      0

 [*,a28,*]
:    p2      p3     :=
a5  150000    .
a6      0     .
a7     .    150000
a8     .       0

 [*,a30,*]
:   p11  p23    :=
a11  300    .
a12    0    .
a5     .  1500
a6     .     0

 [*,a31,*]
:    p4    :=
```

```
a19  3000
a20     0

 [*,a32,*]
:    p5    p6    :=
a23  6000   .
a24  3000   .
a25   .    4500
a26   .      0

 [*,a33,*]
:    p10    p12    p13    p14    p15    p8    :=
a10     0    .     .     .     .     .
a13     .    .   15000   .     .     .
a14     .    .      0    .     .     .
a15     .    .     .   15000   .     .
a16     .    .     .      0    .     .
a17     .    .     .     .   15000   .
a18     .    .     .     .      0    .
a28     .    .     .     .     .   30000
a30     .   3000   .     .     .     .
a39     .  12000   .     .     .     .
a9  15000    .     .     .     .     .

 [*,a34,*]
:    p16    p17    :=
a21    .    8000
a22    .    7000
a31  30000    .
```

```
[*,a35,*]
:   p18   p19   p20   p21   p7    :=
a27   .     .     .     .   15000
a29   .     .     .     0     .
a3    .     .   15000   .     .
a32  9000   .     .     .     .
a33   .   15000   .     .     .
a34   .     .     .   15000   .
a37  6000   .     .     .     .
a4    .     .     0     .     .

 [a35,a36,p22]  9000

 [a35,a38,p22]  6000
;

ampl: display xr;
xr [a27,*,*] (tr)
:   p7    :=
r1  1800
r2  1800
r3  3000
r4  1800

 [a28,*,*] (tr)
:   p8    :=
r2   3000
r3  30000
```

r5 3000

[a30,*,*] (tr)
: p12 :=
r3 300
r5 60

[a31,*,*] (tr)
: p16 :=
r3 300
r4 300
r6 1500

[a32,*,*] (tr)
: p18 :=
r2 180
r3 90

[a33,*,*] (tr)
: p19 :=
r2 3000
r3 15000
r7 4500

[a34,*,*] (tr)
: p21 :=
r3 1500
r8 1500

```
[a35,*,*] (tr)
:    p22    :=
r3   15000
r9    7500
;

ampl: display cr;
cr [*] :=
r1    1722
r2   14242.8
r3  738180
r4   14249.5
r5    8069.4
r6    1325
r7    2130
r8    1460
r9    9325
;

ampl:

ampl:
```

7.2.5 scenario 4 Cost Increased Human Resource

d902s4.dat

```
data;
```

```
### set ###

##set activity##

set setInActivities:=  a3    a4    a29   a23   a24   a25   a26
                       a37   a1    a2    a19   a20   a21   a22
                       a39   a9    a10   a13   a14   a15   a16
                       a7    a18   a5    a7    a11   a6    a8
                       a12  ;

set setOutActivities :=     a36 a38;

set setInternalActivities := a32    a35    a27    a34    a31    a33    a28
                       a30;


# sets products####
set setInProduct :=    p1  p20 p19 p5  p6  p4  p17 p21  p10 p13   p14 p15 p2  p3
                       p23 p11;

set setOutProduct := p22;

set setInternalProduct := p18   p7  p21 p16 p12 p8 ;



## set resources##

set setResource := r3 r9  r2  r1  r4  r6  r8  r7  r5 ;
```

set of relationships

```
set InProdlink :=  a1 a27 p1    a2 a27 p1     a27 a35 p7
                   a3 a35 p20   a4 a35 p20    a33 a35 p19
                   a31 a34 p16  a19 a31 p4    a20 a31 p4
                   a21 a34 p17  a22 a34 p17   a34 a35 p21
                   a32 a35 p18  a29 a35 p21   a37 a35 p18
                   a35 a36 p22  a35 a38 p22   a23 a32 p5
                   a24 a32 p5   a25 a32 p6    a26 a32 p6
                   a39 a33 p12  a28 a33 p8    a30  a33  p12
                   a9 a33 p10   a10 a33 p10   a13 a33 p13
                   a14 a33 p13  a15 a33 p14   16 a33 p14
                   a17 a33 p15  a18 a33 p15   a5 a28 p2
                   a7 a28 p3    a6 a28 p2     a8 a28  p3
                   a11 a30 p11  a5 a30 p23    a12 a30 p11
                   a6 a30 p23 ;
```

sets associated with inactivity##

```
set inAP :=  a1 p1    a2 p1    a19 p4   a20 p4    21 p17
             a22 p17  a3 p20   a4 p20   a37 p18   a23 p5
             a24 p5   a25 p6   a26 p6   a29 p21   a39 p12
             a9 p10   a13 p13  a15 p14  a17 p15   5 p2
             a7 p3    a5 p23   a11 p11  a6 p2     8 p3
             a6 p23   a12 p11 ;
```

set associated with out activity

```
set outAP := a36 p22  a38 p22;


## set associated with resources

set setResOutput :=   35 p22 r9    a35 p22 r3    a32 p18 r2
                      a32 p18 r3   34 p21 r8     a34 p21 r3
                      a31 p16 r6   a31 p16 r4    a31 p16 r3
                      a27 p7 r1    a27 p7 r2     a27 p7 r3
                      a27 p7 r4    a33 p19 r7    a33 p19 r2
                      a33 p19 r3   a28 p8 r5     a28 p8 r2
                      a28 p8 r3    a30 p12 r5    a30 p12 r3;

set setInputOutput :=  p7  p22      p20 p22      p21 p22      p18 p22
                       p19 p22      p5 p18       p6 p18       p1 p7
                       p16 p21      p17 p21      p4 p16       p8 p19
                       p10 p19      p12 p19      p13 p19      p14 p19
                       p15 p19      p2 p8        p3 p8        p23 p12
                       p11 p12 ;


##papameter###


param IOProdRelation :=  p7  p22 1     p20 p22 1     p21 p22 1
                         p18 p22 1     p19 p22 1     p5  p18 1
                         p6 p18 0.5    p1 p7 1       p16 p21 2
                         p17 p21 1     p4 p16 0.1    p8 p19 2
                         p10 p19 1     p12 p19 1     p13 p19 1
```

```
                  p14 p19  1    p15 p19 1     p2 p8  5
                  p3 p8 5       p23 p12 0.5   p11 p12  0.1  ;

param RPARelation := a35 p22 r9 0.5     a35 p22 r3 1        a32 p18 r2 0.02
                     a32 p18 r3 0.01    a27 p7 r1 0.12      a27 p7 r2 0.12
                     a27 p7 r3 0.2      a27 p7 r4 0.12      a34 p21 r8 0.1
                     a34 p21 r3 0.1     a31 p16 r6 0.05     a31 p16 r4 0.01
                     a31 p16 r3 0.01    a33 p19 r7 0.3      a33 p19 r2 0.2
                     a33 p19 r3 1       a28 p8 r5 0.1       a28 p8 r2 0.1
                     a28 p8 r3 1        a30 p12 r5 0.02     a30 p12 r3 0.1;


param x0:=  r9   2500    r3   22050     r2   2160    r1 600
            r4   650     r6   250       r8   500     r7 1500
            r5 1050;

param rd:=  r9 0.8    r3 10    r2 1.42  r1 0.6    r4 5.2    r6 0.65  r8 0.75
            r7 0.31   r5 2.05;

param c0:=  r9 2825  r3 264600   r2 3650.4   r1 498   r4 4244.5
            r6 200   r8 440      r7 600      r5 2562;

param ri:=  r9 1.3   r3 14       r2 1.82     r1 1.02       r4 6.9
            r6 0.9   r8 1.02     r7 0.51     r5 2.74;

param xMax :=  r9 23000      r3 80000      r2 25000      r1 24000
               r4 22000      r6 24000      r8 22000      r7 22000
               r5 25000;
```

```
param xMin :=   r9   25      r3 69    r2 22    r1 6        r4 7
                r6 3         r8 5     r7  15   r5 11 ;

param yInMax := a1 p1 15000         a2 p1 5000       a3 p20 22000
                a4 p20 10000        a19 p4 11000     a20 p4 8000
                a21 p17 8000        a22 p17 7000     a23 p5 6000
                a24 p5 3000         a25 p6 8000      a26 p6 7000
                a37  p18 6000       a29 p21 12000    a39 p12 12000
                a9 p10 30000        a13 p13 32000    a15 p14 33000
                a17 p15 22000       a5 p2 220000     a7 p3 240000
                a5 p23 15000        a11 p11 22000    a6 p2 200000
                a8  p3 200000       a6 p23 11000     a12  p11 12000
;

param yOutMax:=    a36 p22 6000          a38 p22 4000 ;

param Price :=  a29 a35 p21 34.5       a1 a27 p1  3         a2 a27 p1 3.05
                a3 a35 p20 50          a4 a35 p20 51        a19 a31 p4 20
                a20 a31 p4 20.005      a21 a34 p17 10       a22 a34 p17 10.08
                a23 a32 p5 3           a24 a32  p5 3.2      a25 a32 p6 1
                a26 a32  p6 1.1        a37 a35 p18 3.8      a35 a38 p22 380
                a35 a36 p22 382        a39 a33 p12 1.5      a9 a33  p10 0.5
                a10 a33  p10 0.51      a13 a33 p13 100      a14 a33 p13 100.03
                a15 a33 p14 1          a16 a33 p14 1.01     a17 a33 p15 1
                a18 a33 p15 1.01       a5 a28 p2 0.6        a7 a28 p3 1.6
                a5 a30 p23 0.8         a11 a30 p11 1        a6 a28 p2 0.62
                a8 a28 p3 1.63         a6 a30 p23 0.82      a12 a30 p11 1.04;
```

result of scenario 4 :

```
sw: ampl
ampl: model mnet902.mod;
ampl: data d902s4.dat;
ampl: solve;
MINOS 5.5: optimal solution found.
26 iterations, objective 1249738.9
Nonlin evals: constrs = 85, Jac = 84.
ampl: display y;
y [*,a27,*]
:   p1     :=
a1  10000
a2      0

 [*,a28,*]
:   p2     p3    :=
a5  1e+05   .
a6     0    .
a7   .    1e+05
a8   .       0

 [*,a30,*]
:  p11 p23   :=
a11  0  .
a12  0  .
a5   .  0
a6   .  0

 [*,a31,*]
```

```
:    p4    :=
a19  2000
a20     0

 [*,a32,*]
:    p5    p6    :=
a23  6000   .
a24  3000   .
a25    .   4500
a26    .     0

 [*,a33,*]
:    p10    p12    p13    p14    p15    p8    :=
a10     0   .    .    .    .    .
a13     .    .   10000   .    .    .
a14     .    .      0   .    .    .
a15     .    .    .   10000   .    .
a16     .    .    .      0   .    .
a17     .    .    .    .   10000   .
a18     .    .    .    .      0   .
a28     .    .    .    .    .   20000
a30     .      0   .    .    .    .
a39     .  10000   .    .    .    .
a9   10000   .    .    .    .    .

 [*,a34,*]
:    p16   p17    :=
a21    .   8000
a22    .   2000
```

```
a31   20000   .

 [*,a35,*]
:    p18    p19    p20    p21    p7     :=
a27   .     .     .     .    10000
a29   .     .     .       0    .
a3    .     .   10000     .    .
a32  9000    .     .     .     .
a33   .   10000    .     .     .
a34   .     .     .   10000    .
a37  1000    .     .     .     .
a4    .     .      0     .     .

 [a35,a36,p22]   6000

 [a35,a38,p22]   4000
;

ampl: display xr;
xr [a27,*,*] (tr)
:    p7     :=
r1   1200
r2   1200
r3   2000
r4   1200

 [a28,*,*] (tr)
:    p8     :=
r2    2000
```

r3 20000

r5 2000

[a30,*,*] (tr)

: p12 :=

r3 1.5959e-13

r5 0

[a31,*,*] (tr)

: p16 :=

r3 200

r4 200

r6 1000

[a32,*,*] (tr)

: p18 :=

r2 180

r3 90

[a33,*,*] (tr)

: p19 :=

r2 2000

r3 10000

r7 3000

[a34,*,*] (tr)

: p21 :=

r3 1000

r8 1000

```
[a35,*,*] (tr)
:   p22    :=
r3  10000
r9   5000
;

ampl: display cr;
cr [*] :=
r1    1110
r2    9510.8
r3  561960
r4    9419.5
r5    5165
r6     875
r7    1365
r8     950
r9    6075
;

ampl:
```

7.2.6.Scenario 5 disposable without penalty

based on scenario 2 , resource completely disposable without penalty

dnet902s5.dat

set

```
##set activity##

set setInActivities:= a3    a4   a29     a23 a24 a25 a26 a37 a1   a2   a19
                a20 a21 a22 a39 a9   a10  a13     a14 a15     a16 a17
                a18     a5   a7   a11 a6   a8   a12  ;

set setOutActivities := a36      a38;

set setInternalActivities := a32 a35  a27 a34 a31 a33 a28 a30;


# sets products####
set setInProduct := p1      p20 p19 p5   p6   p4   p17 p21   p10     p13 p14 p15 p2   p3
                p23
                p11;

set setOutProduct := p22;

set setInternalProduct := p18    p7   p21 p16 p12 p8 ;



## set resources##

set setResource :=  r3  r9 r2 r1 r4 r6 r8 r7 r5 ;

## set of relationships
```

set InProdlink := a1 a27 p1 a2 a27 p1 a27 a35 p7
a3 a35 p20 a4 a35 p20 a33 a35 p19
a31 a34 p16 a19 a31 p4 a20 a31 p4
a21 a34 p17 a22 a34 p17 a34 a35 p21
a32 a35 p18 a29 a35 p21 a37 a35 p18
a35 a36 p22 a35 a38 p22 a23 a32 p5
a24 a32 p5 a25 a32 p6 a26 a32 p6
a39 a33 p12 a28 a33 p8 a30 a33 p12
a9 a33 p10 a10 a33 p10 a13 a33 p13
a14 a33 p13 a15 a33 p14 a16 a33 p14
a17 a33 p15 a18 a33 p15 a5 a28 p2
a7 a28 p3 a6 a28 p2 a8 a28 p3
a11 a30 p11 a5 a30 p23 a12 a30 p11
a6 a30 p23 ;

sets associated with inactivity##

set inAP := a1 p1 a2 p1 a19 p4 a20 p4 a21 p17
a22 p17 a3 p20 a4 p20 a37 p18 a23 p5
a24 p5 a25 p6 a26 p6 a29 p21 a39 p12
a9 p10 a13 p13 a15 p14 a17 p15 a5 p2
a7 p3 a5 p23 a11 p11 a6 p2 a8 p3
a6 p23 a12 p11 ;

set associated with out activity

set outAP := a36 p22 a38 p22;

set associated with resources

```
set setResOutput :=    a35 p22 r9    a35 p22 r3    a32 p18 r2    a32 p18 r3
                       a34 p21 r8    a34 p21 r3    a31 p16 r6    a31 p16 r4
                       a31 p16 r3    a27 p7 r1     a27 p7 r2     a27 p7 r3
                       a27 p7 r4     a33 p19 r7    a33 p19 r2    a33 p19 r3
                       a28 p8 r5     a28 p8 r2     a28 p8 r3     a30 p12 r5
                       a30 p12 r3;


set setInputOutput :=  p7  p22     p20 p22     p21 p22     p18 p22
                       p19 p22     p5 p18      p6 p18      p1 p7
                       p16 p21     p17 p21     p4 p16      p8 p19
                       p10 p19     p12 p19     p13 p19     p14 p19
                       p15 p19     p2 p8       p3 p8       p23 p12
                       p11 p12 ;


##papameter###


param IOProdRelation :=  p7  p22 1      p20 p22 1      p21 p22 1
                         p18 p22 1      p19 p22 1      p5  p18 1
                         p6 p18 0.5     p1 p7 1        p16 p21 2
                         p17 p21 1      p4 p16 0.1     p8 p19 2
                         p10 p19 1      p12 p19 1      p13 p19 1
                         p14 p19  1     p15 p19 1      p2 p8  5
                         p3 p8 5        p23 p12 0.5    p11 p12  0.1  ;
```

```
param RPARelation :=    a35 p22 r9 0.5      a35 p22 r3 1       a32 p18 r2 0.02
                        a32 p18 r3 0.01     a27 p7 r1 0.12     a27 p7 r2 0.12
                        a27 p7 r3 0.2       a27 p7 r4 0.12     a34 p21 r8 0.1
                        a34 p21 r3 0.1      a31 p16 r6 0.05    a31 p16 r4 0.01
                        a31 p16 r3 0.01     a33 p19 r7 0.3     a33 p19 r2 0.2
                        a33 p19 r3 1        a28 p8 r5 0.1      a28 p8 r2 0.1
                        a28 p8 r3 1         a30 p12 r5 0.02    a30 p12 r3 0.1;

param x0:=   r9 2500  r3 22050   r2 2160  r1 600   r4 650   r6 250   r8 500
             r7 1500  r5 1050;

param rd:=   r9 1.13     r3 10      r2 1.69    r1 0.83    r4 6.53
             r6 0.8      r8 0.88    r7 0.4     r5 2.44;

param c0:=   r9 2825        r3 220500   r2 3650.4   r1 498
             r4 4244.5      r6 200      r8 440      r7 600
             r5 2562;

param ri:=   r9 1.13   r3 10    r2 1.69   r1 0.83   r4 6.53
             r6 0.8    r8 0.88  r7 0.4    r5 2.44;

param xMax := r9 23000     r3 80000    r2 25000       r1 24000
              r4 22000     r6 24000    r8 22000       r7 22000
              r5 25000;

param xMin :=   r9 0    r3 0    r2 0    r1 0    r4 0
```

```
r6 0     r8 0     r7  0     r5 0 ;

param yInMax :=     a1 p1 15000        a2 p1 5000         a3 p20 22000
                    a4 p20 10000       a19 p4 11000       a20 p4 8000
                    a21 p17 8000       a22 p17 7000       a23 p5 6000
                    a24 p5 3000        a25 p6 8000        a26 p6 7000
                    a37  p18 6000      a29 p21 12000      a39 p12 12000
                    a9 p10 30000       a13 p13 32000      a15 p14 33000
                    a17 p15 22000      a5 p2 220000       a7 p3 240000
                    a5 p23 15000       a11 p11 22000      a6 p2 200000
                    a8  p3 200000      a6 p23 11000       a12  p11 12000;

param yOutMax:= a36 p22 6000       a38 p22 4000 ;

param Price :=  a29 a35 p21 34.5     a1 a27 p1  3         a2 a27 p1 3.05
                a3 a35 p20 50        a4 a35 p20 51        a19 a31 p4 20.00
                a20 a31 p4 20.05     a21 a34 p17 10       a22 a34 p17 10.08
                a23 a32 p5 3         a24 a32  p5 3.2      a25 a32 p6 1
                a26 a32  p6 1.1      a37 a35 p18 3.8      a35 a38 p22 380
                a35 a36 p22 382      a39 a33 p12 1.5      a9 a33  p10 0.5
                a10 a33  p10 0.51    a13 a33 p13 100      a14 a33 p13 100.03
                a15 a33 p14 1        a16 a33 p14 1.01     a17 a33 p15 1
                a18 a33 p15 1.01     a5 a28 p2 0.6        a7 a28 p3 1.6
                a5 a30 p23 0.8       a11 a30 p11 1        a6 a28 p2 0.62
                a8 a28 p3 1.63       a6 a30 p23 0.82      a12 a30 p11 1.04;
```

result of scenario 5 :

```
sw: ampl
ampl: model mnet902.mod;
ampl: data dnet902s5.dat;
ampl: solve;
MINOS 5.5: ignoring integrality of 61 variables
MINOS 5.5: optimal solution found.
9 iterations, objective 1380501.2
Nonlin evals: constrs = 40, Jac = 39.
ampl: display y;
y [*,a27,*]
:   p1    :=
a1  10000
a2      0

 [*,a28,*]
:   p2    p3    :=
a5  1e+05    .
a6      0    .
a7      .  1e+05
a8      .      0

 [*,a30,*]
:  p11 p23   :=
a11  0  .
a12  0  .
a5   .  0
a6   .  0

 [*,a31,*]
```

```
:     p4    :=
a19   2000
a20      0

[*,a32,*]
:     p5     p6    :=
a23   6000   .
a24   3000   .
a25   .      4500
a26   .      0

[*,a33,*]
:     p10     p12     p13     p14     p15     p8     :=
a10   0       .       .       .       .       .
a13   .       .       10000   .       .       .
a14   .       .       0       .       .       .
a15   .       .       .       10000   .       .
a16   .       .       .       0       .       .
a17   .       .       .       .       10000   .
a18   .       .       .       .       0       .
a28   .       .       .       .       .       20000
a30   .       0       .       .       .       .
a39   .       10000   .       .       .       .
a9    10000   .       .       .       .       .

[*,a34,*]
:     p16    p17    :=
a21   .      8000
a22   .      2000
```

```
a31   20000    .

 [*,a35,*]
:   p18    p19    p20    p21    p7     :=
a27   .      .      .      .    10000
a29   .      .      .      0     .
a3    .      .    10000    .     .
a32  9000    .      .      .     .
a33   .    10000    .      .     .
a34   .      .      .    10000   .
a37  1000    .      .      .     .
a4    .      .      0      .     .

 [a35,a36,p22]  6000

 [a35,a38,p22]  4000
;

ampl: display xr;
xr [a27,*,*] (tr)
:    p7     :=
r1   1200
r2   1200
r3   2000
r4   1200

 [a28,*,*] (tr)
:    p8     :=
r2   2000
```

r3 20000

r5 2000

[a30,*,*] (tr)

: p12 :=

r3 0

r5 0

[a31,*,*] (tr)

: p16 :=

r3 200

r4 200

r6 1000

[a32,*,*] (tr)

: p18 :=

r2 180

r3 90

[a33,*,*] (tr)

: p19 :=

r2 2000

r3 10000

r7 3000

[a34,*,*] (tr)

: p21 :=

r3 1000

r8 1000

```
[a35,*,*] (tr)
:    p22    :=
r3   10000
r9    5000
;

ampl: display cr;
cr [*] :=
r1     996
r2    9092.2
r3  432900
r4    9142
r5    4880
r6     800
r7    1200
r8     880
r9    5650
;

ampl:
```