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MEASUREMENT OF INTERACTION PROPERTIES IN THE CANADIAN METROPOLITAN SYSTEM

by

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A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirements for the Ph.D. degree in Geography

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ABSTRACT

The purpose of this study is to introduce a new technique for the analysis of urban systems and to test it on the Canadian Metropolitan System. In the past considerable attention has been paid to the themes of urban hierarchy and urban structure and very little work has centred on inter-urban interaction. However, with the increasing interdependency of the urban centres in Western countries much more attention is being given to the identification and measurement of the interaction characteristics of urban systems.

This thesis reviews the characteristics of existing spatial interaction techniques and identifies the need for a systems oriented technique capable of measuring feedback circuits. Essentially the technique presented in this work is founded on flow-graph theory as it applies to the analysis of system properties that are inherent to an origin-destination interaction matrix of any system. The technique identifies four system properties: interactance ratios, interactance multipliers, interactance sensitivities, and an index of total system sensitivity.

The interactance ratios compare the total interaction of each secondary node to that of the main node in the system. The interactance multipliers identify the sub-networks binding any node to the main node and compare their relative importance. The interactance sensitivities probe deeper into the system by measuring the sensitivities of any node to fluctuations on any of the links in the system. Finally the index of total system sensitivity measures a node's overall system sensitivity.

A technique is also developed for evaluating the performance of the system by comparing the relative importance of the activity on the sub-networks with their relative structural importance within the system.

The model developed in this study is applied to two different data sets,

intermetropolitan air-flows and intermetropolitan migrations. The results of the air-flow analyses show that some change has occurred in the Canadian metropolitan system in the 1963-1973 period. Firstly, the set of Canadian metropolitan centres now forms a more interdependent national system with Toronto as its main node. Secondly, the smaller semi-independent networks within the country are now incorporated into two large regional sub-systems, one in the East and another in the West. The core cities of the eastern sub-system are Toronto and Montreal, and to a lesser degree Ottawa-Hull and London. Quebec, Windsor, Sudbury, Thunder Bay, and the Atlantic cities constitute the hinterland of this sub-system. The western regional system has Vancouver, Calgary, and Edmonton as its core cities and Winnipeg, Victoria, Regina, and Saskatoon as its hinterland centres.

The analyses show that the core centres of both systems have a number of common traits. They have strong ties with other core centres of their own sub-system, with the national centre Toronto, and with major centres outside their own sub-system. The core centres also serve as the focal points for the hinterland centres of their own sub-systems. On the other hand, hinterland centres have interactions that are almost entirely restricted to their own core centres.

The results of the migration analyses are similar in several respects to those of the air-flow analyses. They reveal the national role of Toronto, the existence of two regional sub-systems and patterns of interaction of core and hinterland centres similar to those outlined above. In addition, the migration results indicate that Hamilton has not the strong interaction patterns of cities of similar size, e.g. Edmonton and Calgary; that Kitchener-Waterloo and St. Catharines-Niagara have ties that are predominantly with Toronto; and that Chicoutimi-Jonquiere is nothin more than a regional hinterland centre. A comparison of the results of both analyses reveals two major differences:

a) Vancouver has stronger migration but weaker air-flow networks than Montreal's;

b) all migration networks are generally much more limited in extent than the air-flow ones.

Whereas the gross flow and factor analyses were adequate for the determination of broad patterns of interactions, they were incapable of differentiating sub-network structures and operations and it is essentially at this level that system evolution occurs. The flow-graph technique of this study, by contrast, was specifically designed to identify and measure sub-system structures and operations.

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CHAPTER 1

URBAN SYSTEMS AND SPATIAL INTEGRATION: INTRODUCTION AND PURPOSE

Introduction:

The growth and urbanization of the population that accompanied the 19th century industrial revolution in North America and Western Europe are well documented (Deevey, 1960; Davis, 1965). A shift of the population from rural to urban settlements along with a movement from agricultural to manufacturing activities were two of the most significant characteristics of the early stages of this development. The more recent stage is characterized by a concentration of the population within metropolitan centres (Ray et al., 1976, p. 24), by the rapid development of tertiary activities, and by the importance of the role played by communications and information flows both within and between these new urban structures (Goddard, 1975 p. 142; Pred, 1973, p. 1). So great have been the effects of this urban revolution that it has led social scientists to study in depth the urbanization process to determine the events which launched it, the factors which control it, and its economic, social, and cultural consequences. A considerable literature has rapidly developed on urban systems and in most countries there has been debate over the problems and prospects of formulating and implementing urban systems' policies (Bourne, 1975).

Purpose:

The purpose of this research is to introduce a new technique for the analysis of urban systems and to apply it to the Canadian Metropolitan System.

The technique is founded on flow-graph theory and may be used for the analysis of various properties that are inherent to an origin-destination interaction matrix of any system defined as a set of highly interdependent nodes. The system properties defined and measured through this technique are different from those obtained by other methods of interaction analysis. The most distinctive feature of the flow-graph approach is that it focuses on a system's feedback circuits and nodal sensitivities. The conceptual and mathematical bases of the flow-graph model that was created for the purposes of this research, the A.S.T. model[1], are presented in detail in Chapter Three. A suitable technical and empirical context for the analysis is provided in the first two chapters. Chapter 1 will review some literature on structural change and the Canadian metropolitan system, some problems of systems analysis in geography and various characteristics of other interaction models and approaches. The second chapter will determine some of the recent organizational characteristics of the Canadian metropolitan system through the use of gross flow and factorial analyses.

Structural Changes in the Canadian Metropolitan System:

The Canadian metropolitan system comprises a distinctive urban structure. While it is implied that the composition or the structure of the system indicates some constancy in the proportions of activities attributable to each element in the system (Kuznets, 1966, p. 86) it is not intended here to convey a meaning of secular arrangement of the elements as opposed to short-term fluctuations in the structure (Machlup, 1958). Any change in proportions of the activities within the network becomes a structural change.

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[1] "Analyse et Simulation Topologiques"; R. Lamarche et J. Francon (1975), "Analyse et simulation topologiques en géographie, application a l'étude des migrations interrégionales." Cahiers de Géographie de Québec, Vol. 19, No. 46, pp. 189-207.

The term "structure" may refer to the internal composition of each of the centres in the system, or to the hierarchy of the centres within the system, or again it may refer to the nature and intensity of the bonds linking centres together. These three different aspects of urban structure are not independent of one another and although this dissertation is concerned principally with the interaction structures of the Canadian system, attention will first focus on the factors that force changes in the internal structures of cities which in turn have an impact on the interaction patterns between the cities.

Because modern urban systems are inextricably tied to the process of the growth and development of the national economic system (Lithwick, 1970, p. 32) structural changes within the urban system can be measured, in part at least, by the shifts in the economic internal structure of cities (Ray and Brewis, 1976). The factors contributing to structural shifts have been a matter of some dispute, but a useful distinction between factors may be made according to impact time. Thompson (1965) lists three factors which in a short-run perspective could promote urban changes: changing per capita income, relative price changes, and sudden shifts in consumption patterns. Rising per capita income will lead consumers to allocate more of their income to luxury goods and services and force a new mix of basic and non-basic activities within the cities.

In the long-run perspective the focus shifts towards the supply side rather than demand (Thompson, 1965). The ability of an urban centre to adjust its supply functions to changes in demand determines the extent of the structural changes it can undergo to maintain or promote its position within the network. Areas that are endowed with skilled labour, creative entrepreneurship, and capital resources probably have a much greater capacity for adjustment through structural change.

Lithwick (1970, p. 47) has summarized the forces of structural change in much the same manner as Thompson but has stressed two key elements of structural change: changing consumer spending habits and changing technology to meet these changes in demand.

The evolution of the internal structures of the urban centres has led to a number of changes in the urban system as a whole, not the least of which is metropolitanization.

Metropolitanization, A Structural Change:

Urbanization until very recently in North America has been characterized by an increasing concentration of population within metropolitan centres (centres having 100,000 or more inhabitants). There has also been a growing degree of interdependency among these centres. These two elements acting together define the metropolitanization process. It is easy enough to show that metropolitan areas now contain a larger share of the population than in the years preceding the Second World War, but the data for documenting the increased interdependency is not so abundant. Migration flows are probably the most striking form of intermetropolitan interaction. Intermetropolitan migrant flows are now more important in volume and in impact than the rural-to-urban flows of old (Stone, 1969; Alonso, 1971). Lampard (1968) sees the new population mobility as one of the major forces of urban-economic structural change and Madden (1956) recognizes that this has led to greater interdependency between the metropolitan areas. The end result has been the integration of large urban centres into regional, national or even international systems.

Metropolitan development has been accompanied by increases in the mobility of the population, changes in the demographic profiles, and by changes in the labour force composition, the most striking being the increased importance of the service sector. All these factors have played a role in altering the composition of the urban system and it can be justifiably stated that metropolitanization is the process by which urban centres have been welded into strongly integrated systems, a fact which is particularly relevant to the purpose of the present research since the essential prerequisite to the use of the A.S.T. model is the existence of just such a system.

There is considerable evidence that the urban transformations, leading to the greater system integration which Borchert (1972) and others had predicted for countries with advanced economies, have been occurring in the Canadian Metropolitan System. Several studies help indicate the evolution of the Canadian urban network towards greater integration and interdependency. The Canadian population has become largely concentrated within a set of 22 metropolitan areas (Table 1-1). This concentration is likely to continue for some time (Table 1-2) and it is a matter of both significance and concern to Canadian planners, politicians, and researchers (Bourne, 1975). Population concentration in Canada is associated with increased intermetropolitan interaction and there are two types of interaction measures that are of special importance to this research: intermetropolitan migration and intermetropolitan information flows as measured by air-passenger traffic. The forces that generate these interactions and the nature of their action on the urban centres differ. Both play major roles in shaping the urban system.

The importance of intermetropolitan migration in the development of a modern urban system has been discussed in detail elsewhere (Stone, 1969;

Alonso, 1971). The very powerful impact these flows have on the dynamism of an urban centre stems, in a large part, from the socioeconomic characteristics of the migrants, as a high percentage of them are usually young and educated.

Stone reports that the 1956-61 intermetropolitan migrant streams had unusually high percentages of persons with higher-level education and occupational skills (Stone, 1969, p. 128). This is equally true for the 1966-71 streams as well. The 1966-71 intermetropolitan migration ratios are lower than those of 1956-61. The increase in population has not been accompanied by a similar increase in internal migration.

A modern well-integrated urban system is characterized by a second type of interaction: information flows. The operation and management of modern economies require and generate huge volumes of information flows. Domestic air passenger traffic is thought to provide a very good measure of the type of information flows that are likely to have some impact on the key decisions made within the economic system.[2] In Canada, the intermetropolitan passenger traffic more than tripled during the 1963-73 period (Table 1-4). A very large percentage of these flows was reportedly of a business nature (Canadian Transport Commission, 1970), so such flows may be regarded as fair measures of information exchanges and high-level personnel contacts, an essential feature of the diffusion of information and innovation in modern systems (Tornqvist, 1970).

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2 See the section on Data in Chapter Two regarding the use of air passenger traffic as a surrogate variable for information flows.

Table 1-1

URBAN CANADA

	1921	1931	1941	1951	1961	1971
Total Population *	8776	10363	11490	13623	17743	21568
Urban Population % **	47.4	52.7	55.7	62.9	70.2	76.6
Metropolitan Population %	31.9	35.9	36.3	39.2	43.8	47.1

* in thousands

** the values reported do not include Newfoundland

Source: Ray, D. M., 1976, Canadian Urban Trends, Tables 1.6, 1.7, pp. 18-19

Table 1-2

METROPOLITAN CONCENTRATION IN

CANADA AND MAJOR REGIONS, 2001

Census Year	Canada	British Columbia	Prairies	Ontario	Quebec	Atlantic Provinces
1961	44%	53%	37%	48%	50%	15%
1966	48	53	43	55	52	14
1971	55	58	48	66	56	22
2001A	57	58	64	59	61	22
2001B	65	48	75	69	78	26

Source: Ray, D. M., 1976, Canadian Urban Trends, Table 1.0, p. 23.

Table 1-3

INTERMETROPOLITAN MIGRATION RATIOS, CANADA

	1956-61[1]	1966-71[2]
In-migration ratio	9.9%	5.4%
Out-migration ratio	7.8	6.0
Net migration ratio	2.2	-0.7
Gross migration ratio	17.7	11.4

[1] Source: Stone, L.O., 1969, Migration in Canada, Tables 4.2, 4.4, and 4.5.

[2] Source: Table A4-79 of the Appendix

(The definitions of the ratios are given in Stone (1969) p. 28.)

Table 1-4

INTERMETROPOLITAN AIR PASSENGER TRAFFIC

CMA	1963	1973	1973/1963
Calgary	105,890	520,485	4.9
Edmonton	101,885	456,870	4.5
Halifax	68,065	231,660	3.4
London	28,095	85,385	3.0
Montreal	341,485	852,575	2.5
Ottawa-Hull	119,720	469,640	3.9
Quebec	67,205	122,010	1.8
Regina	44,780	152,540	3.4
St. John's	25,200	88,415	3.5
Saint John	25,315	60,145	2.4
Saskatoon	29,435	122,610	4.2
Sudbury	10,145	60,265	5.9
Thunder Bay	29,010	107,400	3.7
Toronto	466,645	1,406,215	3.0
Vancouver	145,170	636,005	4.4
Victoria	37,725	123,975	3.3
Windsor	57,345	114,960	2.0
Winnipeg	120,780	402,865	3.3
Total	1,823,895	6,014,020	3.3

Three recent empirical studies provide insight into the changing nature of the Canadian urban network. Yeates, in his analysis of the Windsor-Quebec Axis, applied a gravity model to 1961 and 1970 metropolitan airline passenger flows as a means of measuring the distance-decay effect on flows as one moves away from the Windsor-Quebec Axis. A weak but significant distance effect was present for the 1961 flows but not for the 1970 traffic and Yeates concludes that "the urban village is at hand, for these metropolitan areas are now the links that tie the country to the axis market place" (Yeates, 1975, p. 224).

Hartwick and Crowley applied a clustering algorithm to the Canadian cities as a technique for isolating autarkic units in the national set of urban centres (Hartwick and Crowley, 1973). The algorithm calculated deviations from predicted values of labour forces normalized with respect to city size. They noted some marked differences between the 1951 and 1961 measurements of autarky. In general they observed that the Canadian clusters did not tend to be strongly independent, already an indication of the systemic nature of the national urban structure. However, Winnipeg, Vancouver, and Halifax did present in 1951 a certain degree of autarky. By 1961, Winnipeg and Vancouver had become integrated into larger Western system while Halifax had changed but little. Montreal and Toronto presented some interesting contrasts. The 1961 results show that the Montreal cluster, when it included Toronto, had become an almost autarkic unit whereas the Toronto cluster, even when it did include Montreal, failed to show signs of closure and the authors conclude that "the Toronto complex is presumably integrated into a larger inter-cluster network" and that in general "the

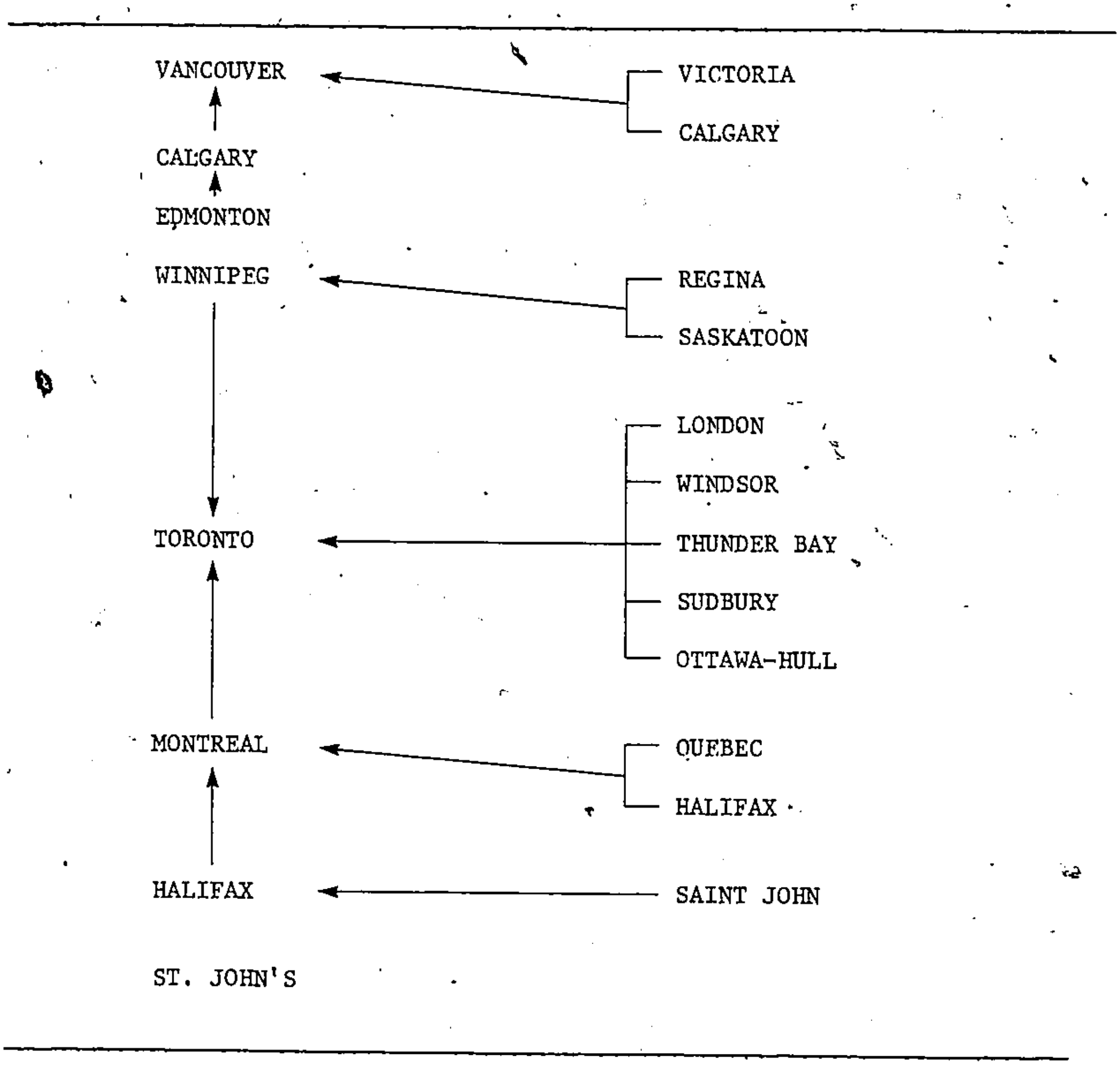
Canadian clusters act like single cities within a much larger system of clusters "(Hartwick and Crowley, 1973).

Storey and Bulker (1975) measured Canadian spatial integration by means of transactional flow analysis for air passenger traffic in the 1960-1973 period. Their results indicate that for the 1960-1965 period the salient flows showed clear regional concentration. In 1970 the Prairie Provinces were linked to the Ontario and Quebec centres. By 1973, all of the separate regional sub-systems had joined to form a single system: national integration was achieved. They conclude that "over time, transition towards a single integrated unit is apparent with increasing polarization of flows focusing particularly on Toronto" (Storey and Bulker, 1975, p. 2). Figures 1-2 and 1-3 illustrate these changes in spatial organization.

Thus, metropolitan concentration and other structural changes that have occurred in Canada since the 1950's have generated greater interdependency among the nation's metropolitan centres and promoted the development of a more integrated urban system. However, integration of the urban system would likely be greater, were it not for the factors of distance and external economic control (Simmons, 1976).

FIGURE 1-1

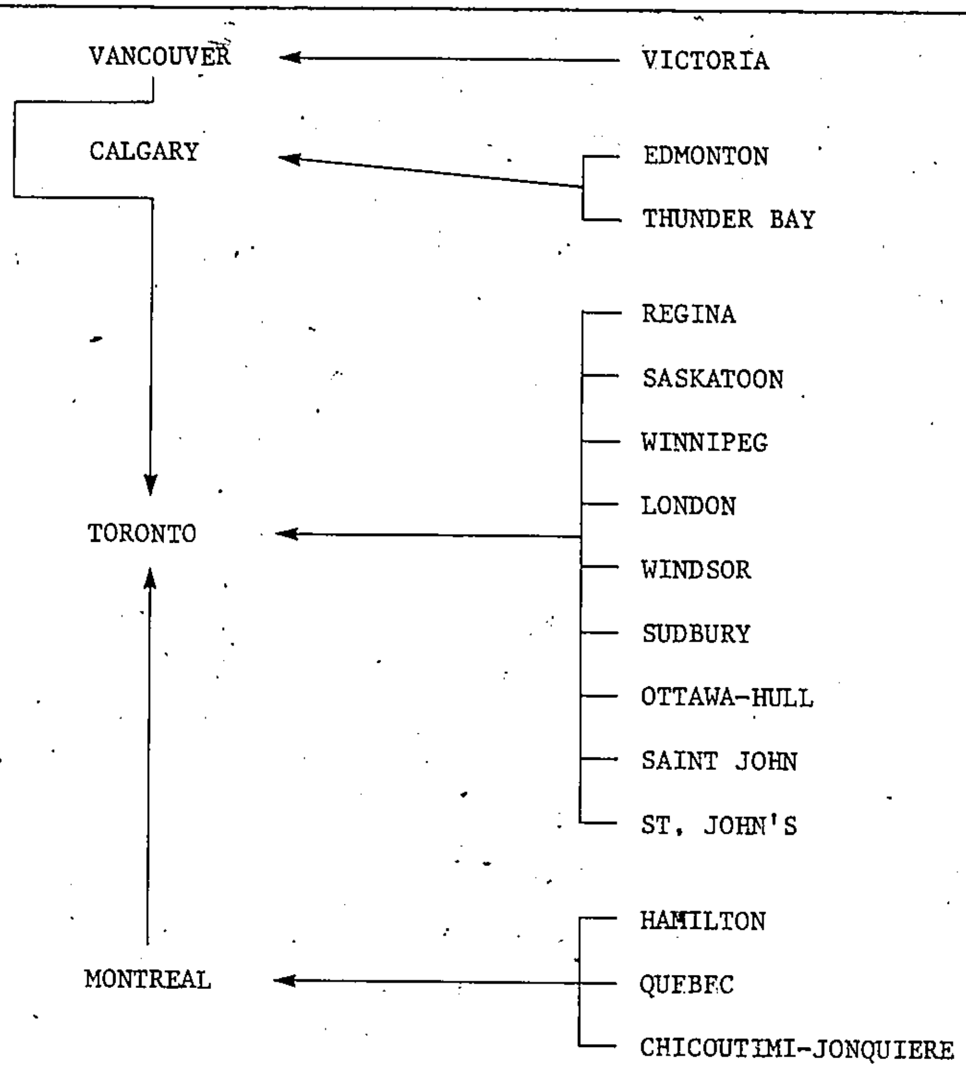
DOMINANT AIR PASSENGER FLOWS, 1960



(Adapted from Storey and Bulker, 1975)

FIGURE 1-2

DOMINANT AIR PASSENGER FLOWS, 1973



(Adapted from Storey and Bulker, 1975)

Systems: -- Definition and Problems in Application:

The use of systems concepts and approaches has become of growing interest and debate in geography. A system is commonly defined as a set of interrelated elements (Harvey, 1970, p. 450). The elements of a system have at least a few properties in common and the state of any one element is conditioned by that of the others through a network of relationships (Langton, 1972, p. 128). A system implies complexity of bonds between the elements so that "the essential characteristic of any system is that everything is related to everything else" (Lowe and Moryadas, 1975, p. 197). The concept also suggests that the elements in a system are distinguishable from those outside it and that the elements are working towards some common goal (Johnson et al. 1963, in Langton, 1972, p. 129).

Systems may be divided into two major categories: open and closed. A closed system is one in which the elements of the system have no ties outside the system and consequently it receives no energy inputs from outside its boundaries nor delivers any energy to points beyond its limits. The field of physics, thermodynamics in particular, provides a few examples of closed systems. In such systems the end result is complete homogeneity or entropy, such as might be the case of two fluids at different temperatures enclosed in a perfectly insulated container. As the container can neither absorb nor dissipate energy, energy flows will be limited to the exchanges between the fluids. When both fluids attain the same temperature, the system will be in a state of entropy. However, as closed systems are a rare occurrence generally and are non-existent in a social context, only open systems will be considered henceforth.

Open systems have a distinct set of properties which sets them apart from closed systems, properties which make them difficult to use as models (Katz and Kahn, 1966, in Langton, p. 103). Firstly, open systems are subject to energy exchanges with the environments that surround them. Inputs from outside the system flow into it, are transformed by it, and flow out as some finished product. As long as these transfers with the environment continue, the system can be maintained in a state of negentropy, a state where different parts of the system have different levels of energy and are in a position to interact. Secondly, open systems are also characterized by the fact that with time they tend toward greater complexity of structure and operation. Their sub-networks become more differentiated as their functions become more specialized and as these more intricate structures must be serviced by a more sophisticated linkage network, the whole system increases in complexity. Thirdly, the presence of numerous feedback circuits is also characteristic of open systems. A feedback circuit consists of detectors, feedback loops, and regulators. A detector monitors the output operation of a system and feeds back to the input unit information about any deviation from normal operations by means of a feedback loop. The regulator modifies the input to correct for the deviation. A feedback circuit may be negative or positive according to its purpose. Negative feedback circuits are designed to maintain a system in its present state by counteracting tendencies to deviate from the present state of the system. Positive circuits seek to amplify the deviations by causing the system to move in the direction of the deviation. Myrdal's (1957) cumulative-causation model illustrates the working of a positive feedback circuit while Janelle's (1969) spatial reorganization model contains examples of both types of feedback circuits. A system is said to be homeostatic if its feedback circuits maintain it in its present state and morphogenetic if the feedback circuits seek to modify it by accentuating any of its deviations. Katz and Kahn (in Emery, 1969, p. 103) describe open social systems as being of a dynamic homeostatic nature. The combination of positive and negative feedback circuits in these systems causes them to evolve slowly towards more complex structures. Finally, there is a fourth property of open systems which explains much of the difficulty encountered in their use in social research: equifinality. Equifinality defines a situation whereby the same goal may be reached by a variety of ways. The difficulty resides in establishing a cause-effect relationship with acceptable certainty.

The use of the concept of system as an organizing theme in geographical research is beset with numerous difficulties. In some instances the problems are generated by the confusion surrounding system terminology. The special characteristics of open systems and the very restrictive assumptions of the system models used to depict complex spatial organizations create a second set of problems which are much more difficult to resolve.

There is considerable ambiguity surrounding the use of the term "system". "System" sometimes defines a set of loosely related objects, e.g. the international economic system of Europe in the Middle Ages, and in other instances "system" refers to a group of elements forming a highly interdependent structure operating within a set of confining rules, e.g. a closed system such as one might come across in electronics or thermodynamics. Thus "system" covers a wide range of situations, from the lowest level of interaction where the inter-element linkages are minimal, to the highest level where the bonds are the controlling factor, and any situation in between. While "system" may refer to any level of interaction, terms such as "systems analysis", "systems techniques", and similar combinations usually refer to situations where the linkages are numerous and strong, and the role of interaction predominant.

Ambiguity about the meaning of system, system structure, and system properties also stems from the variety of objectives pursued in the use of the concepts. If the aim is to determine the structural characteristics of an urban network rather than its interaction patterns, the set of urban centres in the network zone are taken as elements of the system, some demographic and socioeconomic variables are recorded and analyzed and the system is described as having such and such structural characteristics. In these analyses, transaction flows play a minor role. The models used are often borrowed from some other social science or from statistics, e.g. clustering algorithms, regression models, or factor analysis.

But if the aim of the study is to determine interaction characteristics, the models used are often borrowed from the field of physics or mathematics, e.g. gravity, entropy, and graph theory models. Here the main objective is to measure the effects of inter-nodal flows through the identification of feedback circuits and system sensitivities. In this type of work it is essential to ensure that the basic assumptions of the interaction models are satisfied, often a more difficult task than in the preceeding case. In interaction analyses, a set of urban centres are considered to form a system only if they satisfy these additional conditions of interaction models.

Confusion also comes from the fact that some researchers see in the system concept a means of providing geography with some much needed theoretical background (Goddard, 1975, p. 162; Greer-Wootten, 1972, p. 269) while others, particularly those engaged in the planning field (Forrester, 1969; A.J. Scott, 1971; Langton, 1972) see the concept as a useful empirical technique either for forecasting network development, measuring system properties, or evaluating system performance. The divergent views add to the confusion about the definition of system, for what might be satisfactory in an operational sense might be insufficient in a theoretical one.

The greatest difficulties in applying the system concept spring from the characteristic properties of open systems and the basic assumptions of the models used to analyze them. The first difficulty consists in delimiting the boundaries of an open system, the problem of closure. Determining the configuration of the system is difficult because, by definition, the system is open, that is, imbedded within a larger network. The problem of closure is a major one. If it is not satisfactorily resolved there is the danger of ascribing some traits of system structure or behavior to variables and centres within the system when in fact they are caused by factors outside the improperly defined system. Closure in geographical analysis is made more difficult because it becomes necessary to close the system with respect to the set of urban centres involved as well as with respect to the variables to be used in the analysis. If the system is not properly closed, analysis is impossible. It must be stressed that the system closure referred to here is not as restrictive as that of closed systems in the electronic or thermodynamic sense where a system must be perfectly insulated from its surroundings.

The issue of closure as it applies to the Canadian Metropolitan System is discussed in Chapter Two in the section dealing with the data used in the analyses of this thesis.

The problem of closure is not restricted to the systems approach. Langton sees it as a "fundamental problem of geography" (Langton, 1972, p. 134). Thus closure is also a problem in regional analysis and in studies based on central place theory. The difficulties relating to closure do not necessarily invalidate the findings of studies based on regional or central place concepts nor do they destroy the usefulness of the systems concept. However, the systems approach might be more vulnerable than others in this respect because of the sensitivity of the systems models. How severe will be the disruptive effects of the failure to resolve the closure problem satisfactorily will depend in a large measure on the nature of the model used in the analysis. If a study calls for the measurement of delicate interaction properties, e.g. determining nodal sensitivities to flow variations, a sophisticated type of model is usually required, a model which will likely be quite sensitive to any failure in meeting one or more of its basic requirements. On the other hand, if the analysis calls for a more primitive or robust model, the results will not be so sensitive to the closure issue.

Determining the proper scale at which the study should be done presents another obstacle to the implementation of the systems concept. The importance of choosing the proper scale is well illustrated by Ray and Villeneuve (1975, p. 99), who found that the slope patterns of the national components of Canadian urban growth are not at all similar to those of the regional and local components. Depending upon the nature of the problem, a national, regional, or local set of centres will constitute a system suitable for analysis. What might appear as a random pattern of system behavior at one scale might show up as some recognizable pattern at another. The problem of scale is also indirectly related to that of system closure.

There is also a problem caused by the disparity between the complexity of geographical systems and the simplicity of the models used to depict them. Many factors influence the evolution of an urban system but system models can handle only a few factors. Urban systems are nonlinear, but many of the models used in systems analysis are linear. Social systems are adaptive rather than mechanistic and are subject to considerable interchange of parts and functions (Emery and Trist, in Bourne, 1975, p. 16) and this tends to place social systems beyond the scope of most models borrowed from the field of physics. Thus the flexibility of social systems can make the use of rigid systems techniques hazardous or even thoroughly misleading.

However, system models are only "convenient abstractions" (Harvey, 1970, p. 448) that help in the search for hypotheses concerning the structure and operation of networks. The simplicity and elegance of a model can be one of its attractive assets in the study of structural and interactional characteristics. If the purpose of the researcher is to simulate urban behavior, simple models might be found too lacking and he might want to construct more elaborate models where the complexity of the model approximates the real situation, e.g. Forrester's (1969) dynamic urban model. In this approach the model serves somewhat as a "black box" model in which case the intermediate transformations within the system are of secondary importance, the attention being centred on the output product: the approximation of the real world situation.

Finally, the determination of the lag time between action and reaction in social systems is one of the main stumbling blocks to the use of the systems approach in urban research. Social systems operate slowly (Langton, 1972,

p. 141) and the combination of this slow response coupled with the openness of such systems makes it extremely difficult to establish the relationships between the input forces and the system's response (Bannister, 1976). Work on the problem of lag time has only recently begun and has been confined principally to diffusion-type problems (Langton, 1972, p. 139). The reaction time is conditioned by numerous factors such as the magnitude of the input force, the political or economic clout of the decision-makers, or the urgency of the moment. The solution to this problem is not to be expected in the near future.

In spite of all these difficulties, the system concept can play a useful role in geographical research largely because it is oriented towards interaction analysis, and interaction is an important element of spatial processes. Providing that the operational definition is in harmony with the basic assumptions of the model employed in a given study and that the conclusions do not stretch beyond that which is allowed by the limitations of the model and the data, there is much to be gained by the systems approach to geographical analysis.

Urban Systems:

When the term system is taken in its broadest sense -- a set of related objects -- it is fair to say that the system concept has been in use in geography for quite some time. What is new to the discipline is the much narrower definition of the concept which is implied in "systems analysis" where the principal objective is the measurement of interaction circuits and properties, usually for the purpose of exercising some control over the system (Beer, in McDaniel and Hurst, 1968, p. 1). Geographers have found the concept appealing because complex systems always present some strong hierarchical features such as is readily perceivable in most urban and economic organizations. More recently, a few authors have seen the system concept as a means of unifying the numerous sub-disciplines within the field of geography (Johnston, 1971, p. 2). This attempt seems to have met with little success so far presumably because the theoretical developments of the different branches within the discipline are still in their infancy or else the divergences between them are too great to be covered by a single organizing theme.

Economic geographers have shown considerable interest in urban system analysis (Berry et al., 1976). Originally analyses of economic and urban systems were largely taken independently, each requiring its own conceptual and theoretical background. The economic approach elaborated the role of the factors of capital, labour, resources, and markets, while the urban system emphasized changes in urban structures, in land-use, in levels of income of the urban dwellers, and on the new demands for goods and services these changes entailed. In geography the separation between the two fields of endeavour was bridged at least in one respect when geographers established a link between city size and economic functions by means of the central place theory (Berry and Pred, 1961). Only recently has the development of both systems been explicitly tied together. The importance of intra-urban structures on the national and regional economic system has been brought out by Thompson (1965), Mills (1972), and Richardson (1973). Lampard's paper "The Evolving System of Cities in the United States: Urbanization and Economic Development" (1968, p. 99) suggests that both systems are so strongly interconnected that evolution of the spatial economy is directly related to both systems in a morphogenetic fashion; that is, any development in either of these systems reinforces its own present tendency and that of the other system. In a recent paper, Lasuen (1973, p. 181) argues that urbanization in most countries is strongly influenced by the innovation diffusion process at the international scale, a process which is characterized by two main traits, a) a decreasing time-lag between successive innovations, and b) increased complementarity among innovations. The net effect on urban development has been the promotion of greater metropolitan concentration of population and economic activites and thus greater integration of both systems. This obviously raises important issues concerning planning policy and firm organizational structures (Goddard, 1975). Another good example of the need to study both systems simultaneously is Perrin's (1975) model of the relationships between industrial and urban developments in a regional context.

It seems then that more attention is being paid now and likely will be in the immediate future to the problem of sorting out the interrelationships between economic and urban systems (Richardson, 1973, p. 74). Because of the complexities involved in this type of interaction, it is also likely that the concept of system will be of considerable service in this respect.

The system concept has been used in a dual purpose in geography, first for structural analysis, and more recently for interaction studies. Among the earlier applications of the system concept to urban geographical research, the principal objective was usually to outline the structural features of urban systems. Whether the researcher was considering a regional set of urban centres or a single centre, the first aim was to identify its principal components and establish the hierarchical relationships among them. That structural analysis held precedence over interactional analysis in the earlier uses of the concept is understandable for many reasons. Firstly, it was necessary to sort out the components of an urban system before trying to picture the interaction patterns between them. Secondly, the rapid urbanization that followed the industrial revolution had resulted in the growth in number and size of urban centres as well as in the differentiation of their functions and structures. As centres grew, their internal structures became more differentiated, more complex, and more structured (Berry, 1973). It was also apparent that at the regional and national levels, cities could be differentiated according to their functions, e.g. some centres had developed into manufacturing centres while others became mainly resource centres. To evaluate the effects of the intra-urban and regional organizational changes on the urban system it was necessary to identify the new hierarchies that had been created. Thirdly, because complex organizations do form hierarchies, satisfactory results were obtained from models based on the structural aspect of the system concept. This promoted still further studies along the same lines, a process that explains the numerous studies that grew out of the Christaller central place theory.

In many instances, system terminology was not explicitly used as researchers preferred to label their models as urban location models. However, Berry and Horton (1970, p. V) in an exhaustive review of urban location theory and techniques specifically refer to them as belonging to the field of urban systems.

There is an extensive body of literature on the subject of urban system structure. While it is beyond the scope of this work to review all the concepts and techniques developed in these studies, it is possible to review briefly the leading concepts, particularly those relating to interregional analysis as they are directly related to this thesis. It might seem, in the following paragraphs on structural analysis, that the system concept has been stretched to the point where it is made to cover all studies reported in the literature. However, this section is justified on three grounds: first, since geographers have long taken a global view in spatial analysis, it is only normal to find that the system concept underlies a great many of the concepts developed; secondly, the systemic element in all the studies reviewed in the following section has been explicitly recognized (Berry and Horton, 1970); and thirdly, because of the recurrence of the system concept in so many structural-type analyses, researchers have begun to deal with the more formal aspect of system analysis, e.g. nodal interdependencies and interactions, and to apply systems analysis to urban studies.

There are several important concepts in national and regional analysis that are founded on the notion of system, but it is probably the central place theory (Christaller, 1933) that best exemplifies the use of the concept in geographical research. Central place theory has been important because a) it has given geographers a conceptual framework for the study of city size and location, and b) it has stimulated some much-needed theoretical development. The theory uses a deductive approach to study the size and location of cities on a uniform plane by means of the three principles of marketing, administration, and transportation. Central place theory has been widely used to account for the distribution of trade and service institutions. More recently its limitations have been revealed and it has been found necessary to introduce several new developments in keeping with a changing urban system.

Berry and Horton (1970, p. 173) note that central place theory as originally formulated is not so well suited to a modern industrial system whose spatial form can often be better described by some simpler concepts, some of them derived from central place theory, such as the concepts of threshold and of range of a good.

Central place theory played a strong role in promoting both the deductive approach and the system concept within geography. Numerous efforts have been made to verify Christaller's theory empirically, giving rise to a great number of studies having the system concept as a basic assumption: e.g. consumer travel analysis and market area boundaries (Ray, 1967; Berry et al., 1962), changes in trade centre viability with changing transportation technology (Hodge, 1965), the creation of models of location based on probability functions and their comparison with the central place theory model (Dacey, 1966). The number of studies based on central place theory or on concepts derived from it is great indeed (Berry and Pred, 1961; Greer-Wootten, 1972).

Another major system concept developed to explain urban organization is the heartland-hinterland concept (Harris, 1954). Harris' concern was to relate industrial growth to market accessibility. He described urban organization in terms of a heartland and a hinterland with the former being the zone of high population density, the principal market area, and the location of industrial growth and the latter, the hinterland, a zone of lesser population and slower growth, often a zone of primary resources. However, the heartland-hinterland concept failed to account satisfactorily for the spatial organization that followed the coming of post-industrial technologies such as petrochemistry, avionics, electronics, and telecommunications. The concept was incapable also of accounting for the growing propensity of people and industries to locate in more amenable surroundings. Harris developed the concept within a United States context, but it has been applied to the Canadian system as well: Ray (1972) recognizes a heartland-hinterland component in the location of population growth, Bourne et al. (1974, p. 4) in a series of studies on forecasting the urban future of central Canada explicitly refers to it as the heartland of the country, and the concept is implicitly recognized in a recent comprehensive analysis of the Windsor-Quebec corridor (Yeates, 1975, p. 3).

The concept of system is also fundamental to the use of the rank-size rule (Stewart, 1947), a technique of urban development analysis that had considerable vogue in spatial research (Berry, 1970; Mills, 1972). The rank-size rule is an index of urban system development. Essentially it states that a well developed national urban system should give a straight-line graph when populations of the centres are plotted against their ranks, both values being expressed in logarithmic form. The rank-size rule implies a regularity between the size of a city and its rank in the system. If the largest city has a population of one million the next one in rank should have approximately one half million and so on down the line. When urban systems gave graphs in accordance with this pattern, they were thought to be fully developed systems; if not it was assumed that planning policy should be directed at developing this hierarchy. The concept is interesting in that it is descriptive of so many developed areas but it has one major failing: no suitable theoretical basis has been found to support it and it remains a purely empirical technique (Mills, 1972, p. 106).

The system concept also underlies the two principal forms of the concept of region developed in geographical research. The earlier form of the region concept stressed the interplay of human and physical factors in defining the landscapes of human organization, principally within rural autarkic economic systems. Vidal de la Blache (Wrigley, 1970) was the leading proponent of this approach, one which would generate a large number of ad hoc studies. The notion of system was present in this approach, even though only in a primitive form. Although some authors find that this concept of region has not been productive of empirical generalizations, laws, and theories, it has nonetheless been a valuable one because it has emphasized the interrelationships and hierarchies within small closed systems and it has been the source of considerable inductive research (Amedeo and Golledge, 1975, p. 136).

Later, the concept of region was modified to model the urban-economic systems that grew out of the Industrial Revolution (Wrigley, 1970). In these new structures the regions became highly interdependent and linked together by means of transportation networks. Regional analysis then centred around such themes as regional growth, regional disparity, system hierarchy and interdependence. Within this context, concepts and techniques of systems analysis found numerous applications in regional analysis. In his review of the state of the art of regional analysis Richardson (1973) discussed a number of regional models, e.g. the export-base, cumulative-causation, and input-output models. One could add to this list the growth pole model as Perroux (1955) first proposed it in which the role of space was a key factor, a consideration which faded out of the limelight as the early proponents of the growth pole theory became almost exclusively concerned with the analysis of the of the multiplier effects of various economic sectors within a single growth pole. The role of space would be re-emphasized later by Lasuen (1972) and Hansen (1972) as they stressed the necessity of considering the spatial aspects of networks of growth poles forming regional and national systems. It is quite clear, even from a cursory review of the literature, that the regional analysis concept has led to the development of a large number of models, theories and techniques and that the concept of system structure is fundamental to them all. That all the regions within a country form the elements of the national urban system is the basic assumption under which two recent voluminous studies of urban development were carried out indicates the extent to which the system concept has become implanted in regional analysis (Berry, 1973; Hall, 1973). Because regional analysis is concerned with the role of interaction between the elements rather than the structural characteristics within each of the regions, further discussion of the use of these models will be pursued in the next section of this chapter.

Finally, studies on classification of cities are also based on the system concept. The Harris (1943) and Nelson (1955) classifications are two of the most cited in geographical circles (Yeates and Garner, 1971; Berry and Horton, 1970) but there are numerous others based on a variety of clustering algorithms, e.g. correlation coefficients as in Linkage Analysis (Racine, 1972), on distance as in CONGROUP (Berry, 1967), and HGROUP algorithms (Veldman, 1967), on discriminant functions (King, 1970), and on factor analysis (King, 1966). Classification is an important aspect of scientific research but it seems that more effort was devoted to the introduction of new clustering algorithms than to the analysis of the classification problem itself (Berry and Horton, 1970, p. 111).

Interaction Models, (Table 1-5):

The formulation of generalizations, laws, and theories about urban spatial forms depends on a thorough understanding of the processes operating in systems. A process is "a collection of interrelated activities that operate on a set of events and, consequently, produce (or in some cases, prevent) changes in the characteristics of these events through time and space" (Amedeo and Golledge, 1975, p. 177). The assumption behind process research on urban systems is that it is possible to relate spatial patterns to certain stages in the development of a process operating within the system. While there are no "pure" spatial processes (Amedeo and Golledge, 1975, p. 172) there are some that have direct effects on spatial organization, e.g. the urbanization, diffusion, marketing, and growth processes.

Process implies interaction. The interest geographers have shown in developing concepts and techniques for interaction analysis comes from a realization that processes generate or regulate urban growth and form. In Meyer's (1973) view urban locational research should be completely reoriented so as to focus on urban change and not on urban structure as it has in the past. Underlying all the interaction concepts and techniques used in urban geography is the assumption that different components within an urban structure form part of an interacting system. This explains why system concepts and techniques, including systems analysis, have found wide applications in interaction research.

Because this thesis is about system sensitivites and feedback circuits in national and regional urban systems, the following review of interaction models will focus on interregional and not intraurban models. Diffusion models will not be reviewed either, as their primary concern is to establish

Table 1 - 5 INTERACTION MODELS

MODEL	OBJECTIVES	DATA REQUIREMENTS	ASSUMPTIONS	APPLICATIONS
Gravity Model	- explain interaction among urban centres in terms of their masses and the distances separating them; - forecast changes in interaction levels between centres.	- a measure of the masses of the centres, e.g., total population, active population total income; - a measure of the distance between centres, e.g., distance measured in miles, in time, or cost of travel; - a measure of interaction, e.g., migrant, commuter, information, or commodity flows.	- the interaction between centres varies directly as a function of their masses and inversely as a function of the distance separating them; - the network configuration has no impact on the values of the parameters.	- the analysis of migration, information, and commodity flows; - the study of consumer behaviour, of commuter and leisure travel; - the differentiation between centres of greater than expected complementarity from those with levels of interaction lower than expected (the Ullman concept); - the evaluation of the strength of barriers to interaction: political barriers, intervening centres, the frictional effect of distance for different systems.
Graph - Theory Models				
A) Structural Models	- measure network characteristics: diameter, shape, and cyclomatic number; - measure nodal characteristics: connectivity, centrality, and accessibility; - identify nodal hierarchies and sub-network structures; - relate network characteristics to socioeconomic variables.	- a binary matrix of internodal linkages or a matrix of the distances separating the nodes.	- an urban system is taken as an abstract set of points linked by a set of straight lines that can be subjected to topological analysis; - all nodes are characterized solely by their linkages; - interaction between nodes is expressed by the presence or absence of a link between them or by the distance separating them.	- the study of highway, railway, and airway network characteristics; - the study of relationships between network characteristics and economic development; - the identification of urban hierarchies and sub-regional stucture.
B) Flow Models	- identify nodal hierarchies and sub-network structures by the analysis of dominant or salient flows.	- a matrix of internodal flows.	- the urban system in these models are subject to the same abstractions and treatment as in the above models; - the flows are the measure of nodal interaction; - in the Nystuen and Dacey model a centre is assumed independent if its largest flow is to a smaller centre and dependent of it is towards a larger centre; - in the transactional model it is assumed that the centres are independent of each other and draw flows only in proportion to their rank in the system.	- the identification of urban hierarchies and sub-regional stuctures.
Factor Analysis Model	- identify, within large interaction matrices, common patterns of activity that form independent fields of interaction; - measure the size of each field.	- origin-destination matrices of interaction, e.g., migrant flows, telephone calls, air-passenger and commodity traffic.	- the relationships between the flow vectors are linear; - each factor is independent of the others; - only the direct contacts between the nodes are significant in defining the interaction fields.	- the identification of fields of attraction and emission within regional and urban systems based on volumes of flows exchanged.

Table 1 - 5 continued

MODEL	OBJECTIVES	DATA REQUIREMENTS	ASSUMPTIONS	APPLICATIONS
Linear Programming Model	- the optimal allocation of resources through the maximization or minimization of a linear function of variables subject to a number of constraints.	- data specifying the amounts of goods to be allocated and the data detailing the constraints under which they are to be allocated.	- the relationships between the variables in the objective equation are linear; - the constraints can be written as linear inequalities; - the variables have nonnegative values.	- the optimal allocation of transportation, water, industrial, commercial, and financial resources.
Markov Chain Model	- evaluate stochastically the configuration a system will have in the future if present trends hold; - measure the rate at which the system is evolving; - simulate and project system changes by incorporating provisions for transforming the transition probability matrix of the model.	- a vector describing the initial state of the system; - a transition matrix giving the probabilities of movement in the system; the probabilities may be calculated from empirical data or directly derived from a theoretical formulation of the probable relationships.	- the process is markovian, i.e. the present state is dependent on the one immediately preceding; - the transition probablites are constant; - the probabilities represent the sequence of events through time; - the parameters of the system remain constant, an assumption of stationarity; - the system is a closed one.	- the forecasting of migrations of populations, firms, and industrial plants; - the analysis of filtering processes in residential mobility.
Input - Output Model	- measure the interdependency of national or regional economic sectors; - predict the impact of changes in final demand on production outputs of individual sectors and on total output; - forecast regional impacts of changes in final demand in the analysis of interregional systems.	- for a single region model an interindustry transaction table; - a vector or vectors of final demand; - for an interregional model, the transactions must be broken down by region as well as by economic sector.	- constant and linear production functions; - no substitution of inputs; - no maximization of profits, no optimal allocations, and no economies of scale; - the level of input into a sector depends only on the sector's output; - in interregional models, the exchanges between regions are constant.	- to analyze the impact on the various sectors of the economy of changes in final demand due to changes in taxation, subsidization, and investment policies, etc.
Signal Flow - Graph Model	- measure feedback subnetworks in transmission systems; - measure nodal sensitivity to variations in arc activity.	- an origin-destination matrix of interaction data: signal flows, an input-output transaction table, high-level administrative information flows.	- an urban system is taken as an abstract set of points linked by a set of straight lines that can be subjected to topological analysis; - the output of a node is linearly related its inputs; - the links between the nodes represent causal relationships, an input generates an output; - the urban system as represented by its flow-graph can be subjected to topological analysis.	- the analysis of feedback effects in electrical networks; - the study of feedback circuits in economic systems; - the analysis of feedback effects in information networks in urban systems.

one-way patterns of innovation adoption. As the flow-graph model developed in the thesis is based on linear matrix algebra, the review will be confined almost exclusively to this type of model with the exception of the gravity model which warrants consideration in any review of interaction research.

Regional interaction analysis can be performed on a variety of types of flows. A great number of studies have been done on commodity flows although Smith (1970) notes that it was not until the mid-fifties when Ullman introduced his spatial interaction model was any significant development in concepts and methods of commodity flow analysis achieved. Some studies have focused on a single commodity such as steel (Nelson, 1971; Black, 1972), aluminium (Cox, 1965), or grain (Morrill and Garrison, 1960) while others concerned whole categories of commodities such as manufactured goods (Black, 1972, Britton, 1972), agricultural products (Smith, 1970), or both (Berry, 1966). In some instances the purpose of the analysis was to define the urban structure generating the flows (Berry, 1967; Suzuki, 1971; Chisholm and Sullivan, 1973) and in others the aim was to optimize flow patterns (Cox, 1965; Nelson, 1971).

Other studies have analyzed interregional interdependencies and hierarchies through the analysis of flows and linkages between banks (Borchert, 1972; Conzen, 1977), or flows of interregional telephone calls. In most of these cases, the main purpose was to establish the urban system structures revealed by the flow patterns (Illeris and Pedersen, 1968; Simmons, 1972; Clark, 1973; Clayton, 1974; Langdale, 1975; Yeates, 1975). Studies have also been carried out on inter-urban airline passenger flows. Here again the purpose has been to identify urban system structure (Reed, 1970; Yeates, 1975; Storey and Bulker, 1975; Soot, 1976). Another productive line of flow analysis has focussed on inter-urban migrations. In some instances the testing of techniques has been the aim (Schwind, 1975; Slater, 1976; MacKinnon and Skarke, 1977). In other cases the characteristics of the interregional migrants (Stone, 1969) or the factors promoting the flows and the effects of the flows on the urban systems have been the principal interest of the research (Alonso, 1971).

Inter-urban road traffic has been extensively analyzed, particularly by the government agencies responsible for highway traffic and maintenance such as the Canadian provincial ministries of transport, the Canadian Transport Commission (1975), or Transport Canada (1975). The effects on the urban system of changes in highway connectivity have been researched by Garrison (1960), Whebell (1969), and Janelle (1969).

Finally, following the pioneering works of Tornqvist (1968), several authors including Pred (1973) and Goddard (1975), have focussed on the effect of information flows on the development of urban systems. This type of approach could well become the subject of intensive research if appropriate measures of information exchange can be established. For the moment the surrogate variables are still unsatisfactory (Pred, 1976).

Researchers have developed a variety of techniques for the analysis of flow matrices. The purpose of the following paragraphs is to review a number of the most important of these techniques and to show that the development of signal flow-graph analysis is an important contribution to this battery of analytical techniques because the signal flow-graph model is the only one that has been created for the specific purpose of analyzing feedback circuits and system sensitivity. The techniques that will be considered are the following: gravity models, graph theory, factor analysis, Markov chain models, linear programming, input-output analysis, and signal flow-graph theory. As each of these techniques has been amply presented in the literature on interaction analysis, no attempt will be made here to repeat the detailed mathematical and theoretical developments on which they rest. Attention will focus on the specific objectives of the various models, their data requirements, their underlying assumptions, and their applications in geographical research.

Gravity models:

Of all the models used in interaction research, the gravity model is the best known. Although its introduction in human interaction analysis dates back to Carey in 1858 (Carrothers, 1956), it was first used in inter-regional studies by Pallin in 1920 (Finney, 1972, p. 85). The two most common objectives of the use of gravity models have been, a) to explain interaction between centres in terms of their respective masses and the distances separating them, and b) to forecast changes in levels of interaction resulting from network transformations and changes in the mass variables. The gravity model has data requirements that are easy to satisfy: a measure of the masses of the urban centres, such as total population or total income, a measure of separation, such as distance in miles, in time, or in cost, and a measure of interaction such as passenger or commodity flows. In many instances other variables are included in the model to account for various other factors influencing interaction, factors such as income or political barriers (Canadian Transportation Commission, 1970).

The basic assumption underlying the model is that interaction varies directly as a function of the product of the centres' masses and inversely as a function of the distances between centres. It is assumed also that the geometry of the system does not influence the parameter values. When the model comprises additional variables, there is at least one additional assumption per new variable. Thus, the inclusion of a variable related to income implies a relationship between income and interaction.

Ullman made an important contribution to the gravity model technique when he introduced his concept of spatial interaction. The Ullman model hypothesized that interaction between two centres should be greater than expected if the two centres exhibit complementarity and that it should be less than expected if there are intervening opportunities located between the two centres. In terms of the gravity model, this could be taken to mean that positive residuals are to be explained as complementarity interaction while negative residuals reveal the existence of intervening opportunities. Thus instead of searching for a better fit of the gravity model line, research could be directed towards the analysis of complementarity factors and intervening opportunities (Smith, 1964).

There are several difficulties associated with the use of the gravity model. Firstly, it lacks a theoretical basis, which leaves the model open to a variety of interpretations of its parameters (Olsson, 1970; Curry, 1972; Johnston, 1976). Niedercorn and Bechdolt (1969) have proposed an economic derivation for the model but so far it has not proved completely satisfactory (Mathur, 1970; Ewing, 1974).

Secondly, it is difficult to separate the effect of the system's geometry from the frictional effect of distance on the exponent value of the distance variable, a problem which Johnston (1976) investigates thoroughly. Both Johnson and Ewing propose, as one solution, that the distance variable be expressed as a ratio of the system's total distance, a solution which resembles the empirical formulation generally used by transportation planning researchers (Finney, 1972, p. 87; Black, 1972). Black (1972) has also attempted to evaluate the distance exponent directly from a set of variables rather than through empirical adjustments on a sample data set. For the present, this difficulty is still to be resolved.

Another difficulty consists in the fact that the gravity model is deterministic rather than probabilistic. Olsson (1970) saw this as a major obstacle to the development of interaction theory from this model. However, Wilson (1970) has reformulated the gravity model within a general systems framework. In this entropy-maximization model, the flows are distributed so as to maximize entropy according to a probability function regulated by a number of constraints. In this fashion Wilson converted the gravity model into a probabilistic one, an important step in interaction research (Curry, 1972; Lowe and Moryadas, 1975, p. 195). An interesting alternative to Wilson's entropy maximization model is Beckmann's exponential model. The model is probabilistic, dispenses with the "questionable assumption of 'equally likely trips' that is essential to an entropy model" (Beckmann, 1976), and is easy to implement. Whether it is as effective as the entropy model is still to be demonstrated.

The gravity model has been used extensively in geographical research principally for two reasons: firstly, the empirical results have been found to be genrally satisfactory, particularly in short-run forecasting; and secondly, appropriate data are relatively easy to assemble. There have been two main types of application of this model in geography. It is probably the most common technique of forecasting interurban and interregional flows of all types, whether they be passenger movements (Canadian Transport

Commission, 1970), migrants (Stone, 1969), or commodity flows (Black, 1972). The model has also been used to evaluate complementarity of regions (Yeates, 1975), and to measure intervening barriers and intervening opportunities (Mackay, in Lowe and Maryadas, 1975, p. 186). In this second application of the model interpretation of the parameters, particularly the exponent value of the distance variable, has not been satisfactory because of the difficulty in separating the frictional effect of distance from the influence of system configuration on this exponent value. The difficulties associated with the gravity model have prompted the search for more appropriate models.

Graph-theory Models:

Another set of interaction models have come from graph-theory. The models can be sub-divided into two categories; a) structural-type models which are concerned with internodal connectivity structures, and b) flow-type models which deal with internodal flows.

The first objective of the structural-type models is to measure a number of network and nodal characteristics and through them identify nodal hierarchies and sub-network structures. The second objective consists in relating network characteristics to socioeconomic variables in an effort to account for system development. The objective of the flow-type models is also to identify nodal hierarchies and sub-networks but in terms of volumes of flows instead of nodal connections. The data requirements of these models are simple: for the first type, a binary matrix of internodal linkages or a matrix of the distances separating the nodes will suffice. For flow-type models, a matrix of internodal flows is required.

There are a number of basic assumptions underlying graph-theory models. They imply that an urban system can be taken as a set of dimensionless points linked by a set of straight lines that can be subjected to topological analysis. They also imply that nodes are characterized only by their linkages or their flows and the links or flows adequately represent nodal interaction.

An additional assumption of independence is implied in the Nystuen-Dacey and transactional analysis graph-theory models. In the Nystuen and Dacey (1961) model a centre is assumed independent if its largest flow is to a smaller centre and dependent it is towards a larger centre. In the transactional analysis model (Storey and Bulker, 1975) it is implied that centres are independent of each other and draw flows only in proportion to their rank in the system. The model makes no provision for complementarity of interaction or intervening opportunities.

Graph-theory models have found a number of applications in geographical analysis. Garrison (1960) first introduced the graph-theory concepts of network connectivity, diameter, and accessibility to describe the American interstate highway system. Garrison and Marble (1962) later developed other graph indices and attempted to relate them to socioeconomic variables by means of regression models, but with little success. Kansky (1963) created a graph model based on these same indices to predict network of Sicilian railroad growth closely approximated the real network. However, these early graph-theory models had one major limitation as they relied mainly on network connectivity features only and not on actual interaction.

Nystuen and Dacey (1961) developed a technique of flow matrix analysis based on graph theory concepts. Their purpose was to create indices to measure the strength of direct and indirect linkages between centres as revealed by the dominant flows within the system. The indices would then allow them to identify network sub-systems and the hierarchy of centres within each sub-system.

In graph theory a network can be represented by a graph or its adjacency matrix, a binary matrix that indicates the presence or absence of links between the centres in a network. The interest of the Nystuen and Dacey model lies in their method of calculating the adjacency matrix, a method which accounts for the indirect as well as the direct flows and which assigns the lowest values to the most extended channels of interaction under the assumption that interaction decreases with the length of the channel.

Usually the adjacency matrix of a graph is set up by inscribing a value of 1 in the cell a_{ij} if there is a direct link between centres i and j, and a value of 0 if no direct connection exists. This then is the matrix of direct linkage. When this matrix is raised to the second power it indicates all the channels made of two paths and, when raised to the third power, it will show all the three-path channels in the system and so on up the expansion ladder. The summation of all the power matrices will give the total number of direct and indirect interaction linkages between all the centres. Nystuen and Dacey transformed their original origin-destination matrix so as to assign smaller linkage values to the longer channels. They achieved this by dividing every element in the matrix by its largest column total, a standardization procedure. This produced a matrix wherein all the elements had values less than 1. The higher the power expansion of this new matrix, the smaller its elements became and the lesser its impact on the summation. However, there has been some debate over the question of using the maximum column total as the weighting factor (Britton, 1971; Simmons, 1972, p. 212).

The next step was to divide the matrix into a number of sub-graphs using dominant flows as the criterion. The cities were ranked according to their total in-coming flows, that is according to their column totals. Hierarchic relations between centres were measured by the largest out-bound flow to a higher-order centre. Thus the centres were grouped into subsets on the basis of their dominant flows. When a city's largest outgoing flow was towards a centre of higher rank, the city formed part of this subset. In this manner all the centres are clustered into sub-graphs that indicate the limits of each region and the hierarchy of centres within each one.

It is worthy of note that the Nystuen and Dacey technique performs much the same task as factor analysis, that is, it delineates the interaction fields and shows the relative importance of the centres within each field (Britton, 1972). However, there is one important difference in that the Nystuen and Dacey matrix is a probabilistic model. Raising the direct flow matrix to some power is, in a sense, a measure of the probability of indirect contacts occurring. Whether or not this is the best approach to establishing the probabilities of interaction is a point of some controversy (Stephenson, 1974; Tinkler, 1976).

Reed's (1970) graph theory model is somewhat similar to the Nystuen and Dacey model. From the powered adjacency matrix he calculated the average graph distances of shortest paths in a network. In a second step, each of the nodes in turn is removed and the distances are recalculated. If the node omitted is a central one, removal of its links from the matrix will increase the average graph distances, a feature which Reed uses to establish system linkage structure and hierarchy. Reed suggests that it is possible to use either links, as in the example he provides, or flows, as in the Nystuen and Dacey approach. However, it should be noted that if links are used, the emphasis is on the connectivity patterns but if flows are used, interaction is emphasized.

Storey and Bulker (1975) compared the Nystuen and Dacey model to a Transactional Flow technique developed by Goodman (1963). Whereas the graph technique determines system structure from dominant flows, the transaction model determines structure on the basis of deviations about a standardized expected norm of flows. The norm is founded on the hypothesis that a centre should receive from other centres a proportion of their flows equivalent to its own rank within the system, regardless of economic ties and spatial location. This form of standardization is superior (Britton, 1971) to the one used in the Nystuen and Dacey model where each cell is divided by the maximum column total. However, the transactional model introduces new problems of its own in flow analysis such as the question of determining deviation thresholds and the validity of the hypothesis of indifference on which the standardization procedure is based. The results of Storey and Bulker's analyses using the graph-theory and transactional technique show that the interdependency between the Canadian urban centres and sub-systems has increased considerably over the 1960-1973 period.

Factor Analysis Models:

Factor analysis is another technique which has been applied to interaction research. The technique is specifically designed to reduce data matrices to a few basic dimensions (King, 1969) by regrouping into clusters called factors, those variables which are highly intercorrelated. Factor analysis was first introduced to geography as a method of outlining the underlying structure of a set of geographical units as exhibited by a number of socioeconomic variables. Because of the similarity of form between structural matrices and origin-destination tables, factor analysis became a tool of interaction research as well (Berry, 1966).

When an origin-destination table is factor analyzed, the destinations that are correlated are regrouped into factors and the flow matrix is reduced to a few basic dimensions. Each of the factors can be interpreted as forming an interaction field within the broader system. As each of the factors explains as much as possible of the variance still unaccounted for, the interaction fields will be presented in the decreasing order of their importance. Another important feature of the factor analysis model is that each factor is statistically independent of the others so that the original flow matrix is broken down into a number of statistically independent sub-systems (Illeris and Pedersen, 1968). Analysis of the fields over a period of time reveals the evolution of the fields, e.g. growth of one field at the expense of others or integration of a number of fields into a single one (Berry, 1966). Thus, the purpose of the technique is to identify common patterns of exchanges that form sub-networks within the system. Origin-destination matrices are the only data required by the model.

There are three assumptions underlying the model. Firstly, it is assumed that the relationships between the flow vectors are linear. Secondly, the fields of interaction as represented by the factors are assumed to be independent of each other, in statistical terms at least. These two first assumptions result from the fact that the flow matrix is processed as a multi-linear vector space by means of matrix algebra. Thirdly, the use of this model implies that only direct flows between centres are of significance in delimiting the fields.

In comparison to other techniques, factor analysis has a number of good features. Britton (1971) found that in comparison to the Nystuen and Dacey and the transactional flow techniques, factor analysis was remarkably easy to interpret, that it was free of built-in problems relating to the standardization of the data, and that it dealt with common relationships rather than individual arcs. Because the factor model searches out independent fields of interaction, it is useful in regional analysis as a method for identifying the most appropriate locations for regional activities (Illeris and Pedersen, 1968). Black (1973) saw it as an effective and objective method of flow analysis in spite of the problems that might be caused by some skewness of the data.

There are a number of difficulties associated with the use of factor analysis in interaction research. Firstly, the results may be biased by the skewness of the data (Simmons, 1972). Secondly, the technique focuses on the matrix of direct flows only, and this may become a serious limitation (Simmons, 1972, p. 211). Thirdly, it is often difficult to determine the lower cut-off values in evaluating the extent of each field. Fourthly, it can be difficult to delimit the various zones of interaction within a given field, e.g. determining the zones of higher, intermediate, and lower interaction. Finally, if two closely related centres of approximately equal strength perform as the two-centre nucleus of a given field, the factor model may identify two lower-order fields instead of a single strong one and this may present a problem of interpretation.

Notwithstanding the limitations outlined above, factor analysis can be helpful either by itself in taxonomic studies or in conjunction with other techniques such as canonical correlation or multiple regression where it can serve to relate interaction patterns and structural characteristics (Berry, 1966).

The model has been applied principally to the identification of regional interaction networks (Illeris and Pedersen, 1968; Britton, 1971; Simmons, 1972).

Linear Programming Models:

Linear programming is another commonly used technique of interaction analysis. As a technique for achieving optimal solutions to allocation problems, linear programming has been a basic tool of operational research since the 1950's when it was first developed by Dantzig and others (Campbell, 1971; Scott, 1971). The objective in the use of the model is to optimize allocation of resources through the maximization or minimization of a linear function of variables subject to a number of constraints. Mathematically it consists in solving a set of simultaneous linear equations made up of the objective function to be optimized and the constraint equations. Geometrically, one can visualize the solutions to the set of simultaneous linear equations as forming a region of feasible solutions in n-dimensions where "n" is equal to the number of variables in the objective function. Each constraint equation acts as a straight-line boundary to the region of feasible solutions and each of the intersections of the constraint equations, the corners of the solution figure, forms a basic solution to the allocation problem. The corner which maximizes (or minimizes as the case may be) the objective function will optimize the allocation problem, subject to the constraints. The model requires the data specifying the types and quantities of the resources to be allocated, and the constraint conditions under which they are to be allocated.

The assumptions under which these models operate are simple. The relationships between the variables in the objective function must be linear, the constraints can be written as linear inequalities, and all variables must have nonnegative values.

One of the first applications of linear programming in geography was done by Morrill (1967). Because he considered too unrealistic the optimal solutions provided by the model commonly referred to as the "Transportation Problem", Morrill sought to formulate a model of linear programming that would provide a variety of acceptable non-optimal solutions for the transport problem which is itself a simplified version of a linear programming model. Morrill found that when dealing with aggregate data, potential or gravity models provided better estimates than linear programming but for single commodity analysis, the latter was superior (Morrill, 1967; Black, 1972). Cassetti (1966) applied linear programming to the problem of allocating steel plants and iron ore sources within the Ontario-Quebec area. In spite of the difficulty in establishing the necessary constraint equations, Cassetti described the technique as useful in this type of venture.

A much more complex set of allocation models for interregional analysis was developed by Nelson (1971). As his purpose was to reproduce as closely as possible the essential features of the U.S. steel industry, Nelson used a linear programming model coupled with a variety of other models in such a manner as to be able to account for variations in technological achievements and the effects of feedback circuits. The set of models operated sequentially somewhat like a simulation model and the results seemed to have been quite satisfactory.

Linear programming, as a forecasting technique, was tested on U.K. road and rail flows by O'Sullivan (1972). Besides noting that the model worked better when a single commodity was involved, O'Sullivan found that when the emitting and receiving zones were somewhat homogeneous in size and type of activity, the model performed much better. O'Sullivan recognized the objections raised in the literature against the assumption of optimality, but he found it more satisfying than the assumption of stability of past trends on which are founded regression models used for the same purpose. Granholm and Ohlsson (1975) reemphasize the point that the technique is good for single commodity analysis, but if a more complex set of variables are used, as is often the case in geographical research, the quality of the results drops off rapidly because too many unrealistic simplifications have to be made. Finally, Lowe and Moryadas (1975, p. 284), in their review of a variety of studies, e.g. Morrill and Garrison (1960), Yeates (1963), and Gould and Leinback (1966), find that the technique works best when the decision-making process lies within a single body such as a large corporation, the military, or the government in times of crises.

Linear programming is a technique for optimizing an allocation function: it does not attempt to search out system structures and feedback circuits, nor is it capable of measuring nodal sensitivity to flow fluctuations. It is essentially an operational technique and although it is an extremely important tool of operational research, it cannot provide the type of information about system operations which is sought through the use of flow-graph models.

<u>Markov Chain Models</u>:

Markov chain models provide another approach to the analysis of interaction. They are particularly useful in evaluating the general direction of movement within a system and in assessing the rate at which the system is changing. Because these are two important characteristics of system development, Markov chain models have generated considerable interest in geography (Collins et al., 1974). The data required by the model are a vector or matrix describing the initial state of the system and a transition matrix giving the probabilities of movements within the system. The initial state is usually described by giving the number of events or cases in each of the system's categories or cells. The transition matrix may be calculated from empirical observations or derived directly from a theoretical formulation of most likely patterns of movements.

The Markov model is subject to a number of restrictive assumptions which have hindered its use in geographical research. The model assumes that the process is markovian, i.e. the present state is dependent on the one immediately preceding it. It also implies that the system is a closed one. There are two other important assumptions basic to the model, the assumptions of stationarity and uniform probability. Stationarity implies that the parameters remain constant during the evolutionary period, and the assumption of uniform probability assumes that the probability functions will be the same for all the elements in the system. It is quite unlikely that either of these assumptions hold true, particularly the second one for it is assuming a lot to suppose that small cities or small plants behave in the same manner as their larger counterparts.

The Markov chain model, like the Nystuen and Dacey model, is a probabilistic model. Both are based upon matrix power series expansion (Tinkler, 1975, compared both techniques in great detail). Rogers (1966) presented in considerable detail the mathematics of the Markov chain technique and demonstrated how it could be used in interregional migration analysis.

The models can serve in two capacities, either as descriptive tools to indicate the configuration the present system will have if current movement trends continue, or as predictive models, in which case a number of refinements have to be incorporated into the model to offset some of the difficulties associated with the rigid assumptions on which the model is based. Collins (1975, p. 28) presented two possible modifications which can be made to the transition probability matrix when Markov models are to be used in forecasting techniques. However, he noted that in this respect, Markov models are still in the exploratory stage.

Recognizing the difficulties raised by the rigid assumptions of the model, particularly the assumption of uniform probability, Rogers (1966) suggested that the problem could be partially overcome by doing a series of short term analyses on continuously updated data such as one might find in the automobile license registrations for migration studies. Brown and Horton (1970) found a Markov chain model satisfactory in defining hierarchies in a system by interpreting the matrix of Mean First Passage of migration flows as relative measures of functional distances. Lindsay and Barr (1972) compared the Monte Carlo and Markov chain techniques in interregional migration analysis. They preferred the Markov chain model because they found it more realistic and the operational procedures simpler and more elegant. The simplicity of the matrix operations makes the Markov model a better analytical tool than the simulation procedures of the Monte Carlo technique. Hirst (1976) used the technique in a Ugandan migration study. Although he was aware of the limitations of the model, he favoured it principally because it is a probabilistic model and it considers both direct and indirect flows.

Input-Output Models:

The input-output technique is one of general equilibrium analysis of economic activity (see Richardson, 1972, pp. 7-30). It was created to analyze the interdependencies among industrial sectors as exhibited in a transactions table where the destination of the outputs of each industrial sector is compiled. Its specific purpose is to determine the effects of change in final demand on gross output. In input-output analysis it is assumed that the gross output of each industrial sector can be broken down into two or more types of components, that which is destined to final demand and those which serve as direct inputs into the other industrial sectors. This information, which is contained in the transactions table, can be written as a set of simultaneous equations. In matrix algebra notation the equation of the system would be:

$$X - AX = Y$$

where

X = the gross output vector

Y = the final demand vector

A = the n by n matrix of direct input coefficients

The system can be transformed to isolate X:

$$(I - A) X = Y$$

and

$$X = (I - A)^{-1} Y$$

where

X = the gross output vector

Y = the final demand vector

$(I - A)^{-1}$ = the Leontief inverse matrix of direct and indirect requirements.

The effects on the gross output X for any change in the final demand Y are thus easily calculated. The inverse matrix, the Leontief matrix, is the key instrument of input-output analysis. It indicates the direct and indirect requirements from each industrial sector to produce a unit change in final demand. The inverse matrix has also provided the basis for the computation of a number of multiplier indices which assess the various impacts on industrial production through changes in final demand, such as might be generated by public and private investments.

The input-output models are based on a number of assumptions. It is assumed that the production functions are linear and constant. As production functions are rarely linear or constant input-output analysis has come under severe criticism. The technique has difficulty in accounting for economies of scale and factor substitution and it is not suited to deal with production functions which are not linear. However, in Richardson's view, input-output analysis has not been overly handicapped by these problems (Richardson, 1972, p. 9).

The input-output technique has been adapted to interregional analysis by expanding the input-output transaction tables (Isard and Kavesh, 1954). The output of each industrial sector for each geographical region is taken as a separate entity. Although the manipulation of these enlarged matrices creates no problems because of the availability of large scale computers, the introduction of space and distance to the models has caused a number of serious problems. The literature on these problems and their proposed solutions has been reviewed by Meyers (1963), Goode and Tolley (1966), and more recently by Richardson (1972).

Isard and Kavesh (1954) discuss the difficulty in setting up suitable categories of industrial activity because of the practice in government classifications of grouping together local-oriented and region-oriented industries. They also discuss the effects of interregional transportation costs on the model. Isard (1960) commented on the types of industries which should be included in the analyses, how the sectors should be aggregated, and the type of regions which can be included. He stated that the input-output technique can be used as a predictive tool only if the assumptions of constant interregional input coefficients and constant production coefficients and constant production coefficients hold true; or as Harrison put it: "the model implies that the proportion of a given input (i) imported from a particular region (r) for use by a specific purchasing sector (j) located in a specific region (s) should also be constant. This assumption, in effect, freezes the spatial structure...as well as the technological structure" (Harrison, 1972, p. 58). Of these two assumptions, the first one, the one relating to the stability of the interregional trade coefficients, seems to cause the greater difficulty (Moses, 1955). Goode and Tolley (1966) stressed the point that input-output models are powerful tools of regional analysis, but they are nothing more than models and should be treated as such, and that it is unfair to expect more of them. Richardson (1973, p. 43) believed that these models do not deserve the bad press they have received as they are good flexible models that work best in combination with other models such as gravity and linear programming models. Perrin (1975) saw interregional input-output models as a means of linking the two processes of industrialization and urbanization, a feature he considered too little understood in regional analysis.

The input-output model presented in the earlier paragraphs is simplistic. There have been a great many mathematical and theoretical developments made within input-output analysis. These refinements have made the technique more reliable as a means of measuring the interdependencies among economic sectors and geographical regions, though the lack of proper data still constitutes a major handicap to its use.

Signal Flow-graph Models:

Since the A.S.T. flow-graph model is central to this study, the conceptual basis of flow-graph models will be discussed in great detail in this section. The detailed mathematical formulations of the A.S.T. model will be given in Chapter Three.

Researchers have sought for some time now for a method of tracing economic impulses through strongly interdependent systems of cities (Ponsard, 1967; King et al., 1969). The interest in impulse and feedback analysis may be attributed mainly to three reasons: a) a growing interest in understanding the behaviour of the system through the analysis of network interaction patterns (Simmons, 1978, p. 61), b) the concerted effort to develop a theory to account for growth in modern economics in terms of urban interdependencies (Tornqvist, 1968; Pred, 1973); and c) a need to complement some of the inadequacies of the interregional input-output models (Ponsard, 1969; Bannister, 1976).

The objective of the flow-graph model is to measure feedback effects in strongly interdependent networks. The model indicates how a node's output will be influenced by its various sub-networks and individual arcs for changes in outputs of the other nodes. The data required by the model are difficult to obtain as they must be in the form of a transaction table depicting the interaction between centres of highly interdependent networks. Input-output transaction tables and origin-destination matrices of information exchanges between high-level administrative and executive personnel are two examples of the type of data suited to the model.

There are two major assumptions which underlie the application of flow-graph models to urban systems, an assumption of causality in the linkages and an assumption of linearity.

Since the A.S.T. flow-graph model is concerned with the analysis of feedback effects in urban systems it is important to define the nature of the bonds implied in the modelling of urban networks as flow-graphs. It will be recalled that flow-graph theory is based on that branch of mathematics called topology and that directed graphs in topology do not imply causal linkages between the nodes. Topology is only concerned with a number of properties of geometric figures, the most important one, in the present case, being that of connectedness between the nodes of a graph. Thus, topological transformations of a network will ascertain that the connectivity of the network remains the same even though it is subjected to various transformations, but there is no implication as to the nature of the bonds between the nodes.

Mason developed signal flow-graph theory in order to measure feedback effects in electrical networks by subjecting them to a number of transformations without, however, altering their connectivity properties. Thus, he resorted to a topological approach. But in modelling networks as flow-graphs Mason assumed that a signal transmission network could be interpreted as a signal flow-graph where each node receives signals from various sources, combines them, and emits resultant signals towards other nodes in the system. Since the size of a given node's output is related to its inputs, a causal relationship is therefore implied. Chen (1971, p. 169) notes that Mason's approach has been useful in solving a number of difficult problems and that the assumption of causality has not caused any serious difficulty. Thus, Mason was able to create a number of algorithms for measuring feedback effects through topological transformations of networks.

When urban systems are modelled as flow-graphs, causal linkages between the nodes are also implied. This is apparent in the manner in which the linear equations and flow-graphs are set up (Figure 3-2A, 3-2B, and

3-2C of Chapter Three). Consider an urban system formulated in the following flow-graph equation system : (Chapter Three, Equation System No. 4) :

$$x_1 = c_{10}x_0 + t_{11}x_1 + t_{12}x_2 + ... + t_{1n}x_n$$
$$x_2 = c_{20}x_0 + t_{21}x_1 + t_{22}x_2 + ... + t_{2n}x_n$$
$$....$$
$$x_n = c_{n0}x_0 + t_{n1}x_1 + t_{n2}x_2 + ... + t_{nn}x_n$$

The total output x_i of each node is expressed as the sum of the individual outputs from the node to all the others in the system. Since the coefficients c_{io} and t_{ij} give the ratios of inputs into nodes x_o and x_{ij} to their own total output, the equation implies that there exists a relationship between a node's output and its inputs.

There is an explicit recognition of this type of interdependence in studies devoted to the tracing of flow impulses through urban systems (King, Cassetti, and Jeffrey, 1969; Bannister, 1976). Even though the problem of lag-time between action and reaction in urban system remains unsolved, it has not deterred researchers away from pursuing this type of research because of the potential benefits that would come from the development of an applicable feedback analysis model. Of the various types of flows exhibiting causal linkages, information flows are among those having received the most attention in the recent literature (Tornqvist, 1968, 1973; Pred, 1973, 1976; Goddard, 1975). Dunn (1970), in discussing the use of graph theory in urban analysis, notes that for reasons of economies of scale most urban interaction networks serve a multitude of functions and are not restricted to a single purpose. There is an implication here that if it is possible to identify the network structure and feedback patterns of one type of interaction, the results apply, in varying degrees no doubt, to other types of interaction.

In summary then, the A.S.T. model in its application of flow-graph theory to urban systems implies causal linkages between the nodes and that in spite of the difficulty in fulfilling the data requirements for this type of research, development of a flow-graph model and its application to urban systems is a valid and potentially useful endeavour.

There is assumption of linearity, in addition to that of the causality of linkages, implied in the flow-graph model of Chapter Three.

In the following discussion on linearity, a distinction is made between two very different applications of the model. As it now stands the A.S.T. model is set up to analyze internodal activity covering a single time period. The model is a static one in that the lag-times in reaction between the nodes are considered nil. However, there are indications that the model could be adapted to take into account the delays in reaction time, in which case the model would be a dynamic one. Although the development of such a dynamic model would prove interesting, there are still so many major difficulties to overcome that its development is still far off in the future. The implications of the linearity assumption are different for these two approaches and they will be discussed separately.

In the case of the static model, the objective is to determine feedback effects and circuits in a network at a given moment. The data of the internodal activity is therefore contained within the origin-destination table. The assumption of linearity in this case is quite straightforward. It implies that, for a given matrix, each of the entries into region j coming from the other regions i is proportional to the total output of region j.[3] Linearity is implied in the equation system associated with the network's flow-graph. The equations define the total output of each node x_i in terms of the flows it sends to the nodes x_o and x_{ij}. The coefficients c_{io} and t_{ij} express these inputs as ratios of the total outputs of nodes x_o and x_{ij}. Since the equations contain no products of variables nor do any of the variables have any exponential indices, the equations are linear. The equations imply that the output of a node is proportional to the inputs it receives from the nodes to which it is connected. If the relationships between the nodes are not linear but curvilinear, the static model then describes the characteristics of the network at a given point in time in terms of the tangent to the curve at this point.

The procedure followed in the flow-graph analyses of Chapters Four and Five has been to apply the static model separately to a series of origin-destination tables covering a period of years and to observe the network characteristics revealed in these static analyses of the system. Because the feedback characteristics and nodal sensitivities are always measured with respect to a unit output from the source-node, it was possible to compare the results of the separate analyses without resorting to any other standardization procedure. As a result, it was possible to detect changes experienced by the system over this short period of time.

In the case of the dynamic model, the approach is totally different. The flow-graph and its equation system are much more complex because the

[3]The formula for calculating the coefficients of linearity c_{ij} is : $c_{ij} = f_{ij} / \sum^{i} f_{ji}$, f being the inter-urban flows.

must take into account the lag-times in reaction between the centres. Some suggestions as to the manner in which such a dynamic model could be set up have been put forward by Ponsard (1969). He proposes a flow-graph in which a single centre is represented by as many nodes as the centre has different reaction lag-times. Thus, if centre A has different lag-times for its interactions with centres B and C, centre A would be represented by two different nodes, one for each reaction. As each centre may have different delay-times for all of its interactions, the flow-graph can quickly be blown into gigantic proportions. However, once the flow-graph is established, the equation system can be written and solved. The major difficulty lies in that the lag-times must be known before either the flow-graph or the equations can be established. Determining the lag-time coefficients is in itself a major difficulty and establishing the linearity of the relationships in the equations is another in such dynamic models.

Applying a flow-graph model to an urban system requires that one urban centre be chosen as the system's source-node. How this choice should be made merits a few additional comments. In any flow-graph the nodes represent output activity of the centres and the arcs symbolize the relationships between the centres. All the nodes, with the exception of the source-node, have inbound and outbound arcs with the arrows pointing in the direction of the on-coming flows. But the source-node has only out-going arrows, that is only the individual inflows to the source-node are identified and not the outflows. This implies that the whole system is analyzed with respect to a unit flow out of the source-node. The source-node is considered the independent node and the whole network's behaviour is analyzed with respect to a unit output emitted by the source node. There is a reason of mathematical necessity, which is thoroughly discussed on page 95 in the second section of Chapter Three, for assigning a unit value to the total output of the source-node. If the flow-graph technique were applied to an inter-industry input-output problem, total final demand would be considered as the independent variable, the source-node, and the system's feedback characteristics would be analyzed with respect to a unit change in total final demand. In applying the flow-graph technique to an urban system, any node may arbitrarily be chosen as the source-node. In the studies of Chapters Four and Five, the nodes with the largest outputs were taken as the independent node, the source-nodes. The feedback circuits and nodal sensitivities of the other nodes were examined with respect to a unit output of these major centres. This is implying that the activity in the major centre is an independent variable with the rest of the system adjusting to its behaviour. Since the largest centre usually contains the elements with the strongest influences in the decision-making processes this choice was deemed justifiable.

If any node other than the major node is taken as the source-node the analysis can still be carried out. The results will show how the system behaves with respect to a unit output from this newly chosen source-node. The results will differ considerably from those of the original analysis. In general, the system's sub-networks should report higher feedback circuit values, (these transmission sub-networks are defined in Chapter Three under the label of transmission gains), since the largest node with presumably the best overall network connections, is now included in the dependent set of nodes. Its strong network linkages should increase the values of the various feedback circuits. The nodal sensitivity values should also differ. In general, the sensitivity values should bear an inverse relationship to the magnitude of the source-node output. This is accounted for by the fact that sensitivity is measured with respect to a unit output of whatever source-node has been chosen. While a unit output might be but a small share of large centre's activity, it represents a substantially larger share of a much smaller node's output. Therefore, an increase of output of one unit from a large centre will not alter the sensitivity results as much as a unit output from a smaller centre.

Signal flow-graph models have but a short history and as of yet they have not been widely applied to network research.

Ponsard (1967) was not the first to realize the advantage of treating a graph and its associated matrix within a topological context. Mason (1953) had already modelled an electronic signal network as a flow-graph and developed a set of topological transformations of the network graph for the purpose of identifying feedback circuits and network sensitivities. Ponsard generalized some of Mason's formulas and applied them to the analysis of industrial intersector transactions. It is important to note that Ponsard was not adapting a model of electronic analysis to economic research. Both he and Mason were applying a mathematical theory in its "pure form" (Amedeo and Golledge, 1975, p. 61) to two different situations and, therefore, there is no need to establish a correlation between the concepts of economic analysis and those of signal transmission systems. The flow-graph theory is a logical system much in the same way as probability theory is and, providing the flow-graph model can be adjusted to reflect the economic system without violating its own basic assumptions, it is possible to apply it to the analysis of an economic system and to interpret the results within a topological context. (Rota,1971; pp. 40-51) related mathematically the input-output model to the flow-graph one. He demonstrated that the final demand vector, usually labelled Y in input-output is the equivalent of the source-node activity vector $x_o C$ of the system of equations (2) of Chapter Three, that the Leontief inverse matrix and the matrix of t_{ij} coefficients of system (2) are also related, and that the X vector of input-output is the equivalent of the dependent nodes of the flow-graph model of system (2), the dependent nodes being all the nodes in the system with the exception of the source-node. Vuez (1971) applied a flow-graph model to a seventeen sector input-output table (data taken from French national transaction tables of 1959) and tested the utility of the model in detecting the impact of variations in the technological coefficients on other points in the system. He found the performance of the model superior to that of the input-output model in this respect because the flow-graph model was more flexible through its topological algorithms. Bannister (1976) introduced a signal flow-graph model to measure system feedback and sensitivities. He discussed various approaches to the problem of measuring transmission functions and time lags in the transmission of impulses.

As all the models discussed in this Chapter deal with interaction analysis it is appropriate here to point out the specific differences between the A.S.T. model and the others.

The A.S.T. flow-graph model is fundamentally different in its approach and in its objectives from the models derived from graph-theoretic concepts outlined earlier. The approach is different in that the flow-graph model rests on the analysis of the system of equations associated with a flow-graph and not on the creation of a number of indices pulled from a connectivity matrix as in other graph-theory approach. The topological transformations in the A.S.T. model are performed on the equation system and not on the connectivity matrix. There is also a fundamental difference in the objectives. The A.S.T. model deals with feedback effects and nodal sensitivities and not with nodal connectivity per se. In the flow-graph model the gain at a node is always discussed with respect to its response to changes in outputs in other nodes. The three properties of the flow-graph model, system gain, transmission gain, and system sensitivity have been developed for the sole purpose of measuring feedback effects in systems. System feedback analysis is so complex that without an appropriate technique it is almost a hopeless task.

Factor analysis, Markov-chain models and linear programming are similar to the flow-graph model in so far as they are all formulated as linear equation systems but they differ in their objectives. Factor analysis identifies fields of interaction by clustering nodes with similar patterns of activity. Markov-chain models attempt to project network evolution and measure the rate at which the transformations are occurring. Linear programming models aim at determining interaction patterns to optimize allocation of resources. Thus, none of these models attempt to sort out feedback networks in systems and are therefore basically different from the A.S.T. flow-graph model.

Of all the models outlined earlier, the input-output model is the closest to the flow-graph model because they both deal with system interdependencies. The input-output model analyzes the interdependencies between the economic sectors of a transaction table and provides a number of multiplier indices but it does not provide a method of identifying and measuring feedback circuits which is the specific objective of the flow-graph approach.

Signal flow-graph theory, therefore, has its own specific contribution to make to interaction analysis. It pursues objectives which are distinctly different from the other techniques outlined in the preceding paragraphs. Certainly, it is incapable of solving a number of problems that the other techniques can, e.g..optimizing a distribution or predicting the major trends of change within a system, but as a technique for identifying feedback circuits and nodal sensitivity to flow fluctuations, it is the best model devised so far. As a systems technique it is particularly appealing in that it is founded on a theory in its pure form rather than on concepts borrowed from some other social science or from the field of physics.

CHAPTER 2

THE CANADIAN METROPOLITAN URBAN SYSTEM: SELECTED ANALYSES

The Canadian urban system has recently been the object of considerable study. Ray (1972) has analyzed the dimensions of urban growth in Canada, Yeates (1975) has researched the evolution of Canada's Heartland over the 1961-71 period, Lithwick (1970) and Bourne (1975) have stressed the necessity of establishing a national strategy of urban development, and Simmons (1974) has formulated a conceptual framework of systems analysis applicable to Canada's urban network. Others, such as Hartwick and Crowley (1973), Storey and Bulker (1975), have studied some traits of Canada's urban evolution throughout the 1951-1973 period.

The analyses presented in this chapter add to our knowledge of the structural characteristics of the Canadian system by revealing through gross flow and factorial analyses certain main features of the intermetropolitan air passenger flows over the 1963-1973 period.

Gross flow and factorial analyses of intermetropolitan airflows provide convenient methods for determining some of the organizational characteristics of the Canadian urban network. As these analyses cover a decade of activities, they may provide some initial understanding of the metropolitanization process that has been transforming the Canadian urban system. The structural characteristics revealed by these analyses may later be compared with the system properties discovered through graph-theory and presented in Chapters Four and Five.

Measuring system properties and forecasting system behaviour require interaction data of one kind or another, usually in the form of origin-destination matrices. As there exists many sets of spatial linkages -- Simmons (1976) speaks of no less than some 100,000 such networks -- it is important to determine which of these spatial interactions are most relevant to the purpose at hand. A review of the recent literature on urban systems shows that the most commonly used data fall within one of the following categories: telephone calls, financial flows, commodity exchanges, air passenger and internal migration flows.

Two recent studies have used telephone calls as their main data base, one by Simmons (1972) on Central Canada and another by Clark (1973) on Wales. Simmons noted that telephone matrices served his purpose better than air passenger flows because several cities in his study were too close together to warrant air travel or some of them did not have major airport facilities. Clark ended his study with an important remark about the usefulness of telecommunication data in urban systems. While he had hoped that call patterns would provide some insight into future trends of the Wales urban network, he found that they adopted "a lag rather than a lead role in determining spatial form" (Clark, 1973, p. 321). This supports Tornqvist's (1973) and Goddard's (1975) argument that operational information flows (e.g. telecommunications) have little impact on system evolution; their main function is to keep the present one operational not to generate system changes. It is the person-to-person contacts in their view that provide the forces of system change. Since the present study is one of system evolution, telecommunications data would be of limited value. Financial and commodity flows are also of limited value. Both these linkage patterns reflect past or actual performance of the system (Goddard, 1974). In brief then, while all types of interaction data would project some light on urban system operation, telecommunications, commodity, and financial flows are not as helpful as personal contact information flows in projecting system behavior (Storey and Bulker, 1975, p. 1). However, data sources regarding personal contacts are extremely rare, and it has been necessary to resort to the use of a surrogate variable, air-passenger flows.

Inter-urban air passenger flows have been central to a large sector of recent urban systems research. Pred (1973) reports that the Swedish researchers Sahlberg and Hedberg relied heavily on this type of flow in their analyses of the Swedish urban system. The importance of this type of interaction was strongly reinforced by Tornqvist's (1973) discovery that of the two types of information flows, personal contacts and telecommunication exchanges, the first was the decisive one in determining urban system development. Tornqvist's insistance on the importance of face-to-face contacts is now widely accepted, (Dunn, 1970; Pred, 1973; Yeates, 1974; Goddard, 1975; and Storey and Bulker, 1975). If one considers that over 65% of inter-urban air passenger flows are company business contacts (Canadian Transport Commission, 1970), it is not surprising that air passenger flows have been used widely in urban systems analysis.

The data on intermetropolitan flows used in the following chapters were compiled by Statistics Canada (Cat. 51-204, annual) and cover a period of eleven years, 1963-1973, which is "about the maximum period over which the role, relative size of cities, and their hierarchical relationships can be taken unchanged", (Simmons, 1976). Since 1959, the origin-destination matrices published by Statistics Canada have been calculated from a continuous sampling survey of passenger coupons for all the major national and regional airlines in Canada. The two largest airlines, Air Canada and C.P. Air, take a 10% sample of all their domestic flights, while the smaller airlines provide a

20% sample. This last group of airlines includes Eastern Provincial Airways, Pacific Western Airlines, Quebecair, Transair, Nordair, and Norcanair. All of the above airlines participated in the surveys throughout the 1968-1973 period with one exception: Norcanair ceased to report its flight figures in 1973. Only domestic flows are reported in the air passenger matrices analyzed in this chapter. Through traffic is excluded as only journeys whose origin and destination were Canadian cities were included in the sampling survey. Some caution must be exercised when comparing the origin-destination matrices covering a number of years because factors other than person-to-person contacts also influence airline flows. Some factors promoted increases in passenger traffic, such as the opening of new routes or improvements in airport facilities. Thus Transair's new Winnipeg-Thunder Bay-Toronto service in 1970 and Nordair's new Montreal-Hamilton route in 1971 are very likely to have been partially responsible for increased interaction between these metropolitan centres. On the other hand, some factors exerted a negative influence on these airline flows such as the strikes by Canadian pilots and air traffic controllers in 1972 and 1973. The elimination of airline routes, such as C.P. Air's decision to abandon part of its regional services in British Columbia in 1970, must have had a negative effect on airline traffic.

The fact that this research into the Canadian urban system is limited to the set of metropolitan centres when origin-destination data is available for all centres with airports in Canada may demand some justification. Conceptually, the A.S.T. model implies that there is a high degree of interdependence among the nodes in the system to which it is applied. There is ample evidence in the literature -- Garrison, (1960), Borchert (1972), Pred (1973a), Lasuen (1973), and Goddard (1975) -- that these strong linkages exist among the large urban centres in a modern economy. This is not particularly surprising considering the concentration of demographic, economic, financial, and political forces in the metropolitan areas. Limiting the data set to metropolitan centres implies that the system is closed with respect to the centres involved in determining the functioning of the system. It must be recognized, however, that the system is not truly closed. Its boundaries are arbitrary and cannot be wished away. Nevertheless, with so many of the decisive factors of urban growth now concentrated within these large centres, limiting the system in this manner is assumed to be justifiable.

Gross Flow Analysis of Intermetropolitan Air-Passenger Traffic:

To provide some initial sense of proportion and scale this section will describe and compare the destinations of the largest flows of the metropolitan centres for 1963 and 1973, the first and last years of the study period. The flows are given as percentages of the total system outbound traffic (tables A4-67 and A4-77 of the appendix). Table 2-1 and Figure 2-1 summarize the results of the analyses.

Total intermetropolitan traffic increased from 1,823,895 to 6,014,020, a growth of 229% over the eleven year period. A notable feature of the period was the drop in the relative importance of the exchanges between the two largest centres, Toronto and Montreal. In 1963, the largest single flow was from Montreal to Toronto, which represented 10.6% of the system's total output, whereas in 1973, the value of that flow dropped to 7.6%. This suggests the development of a more evenly distributed pattern of activity. The new distribution pattern may be a reflection of such factors as expanded airline facilities throughout the system, or new travelling patterns of the public in general, or again it may be a confirmation of Pred's (1973) theory that

Table 2-1
Canadian Intermetropolitan Domestic Air-Passenger Traffic
Largest Flows as a Percentage of Total System Outbound Traffic

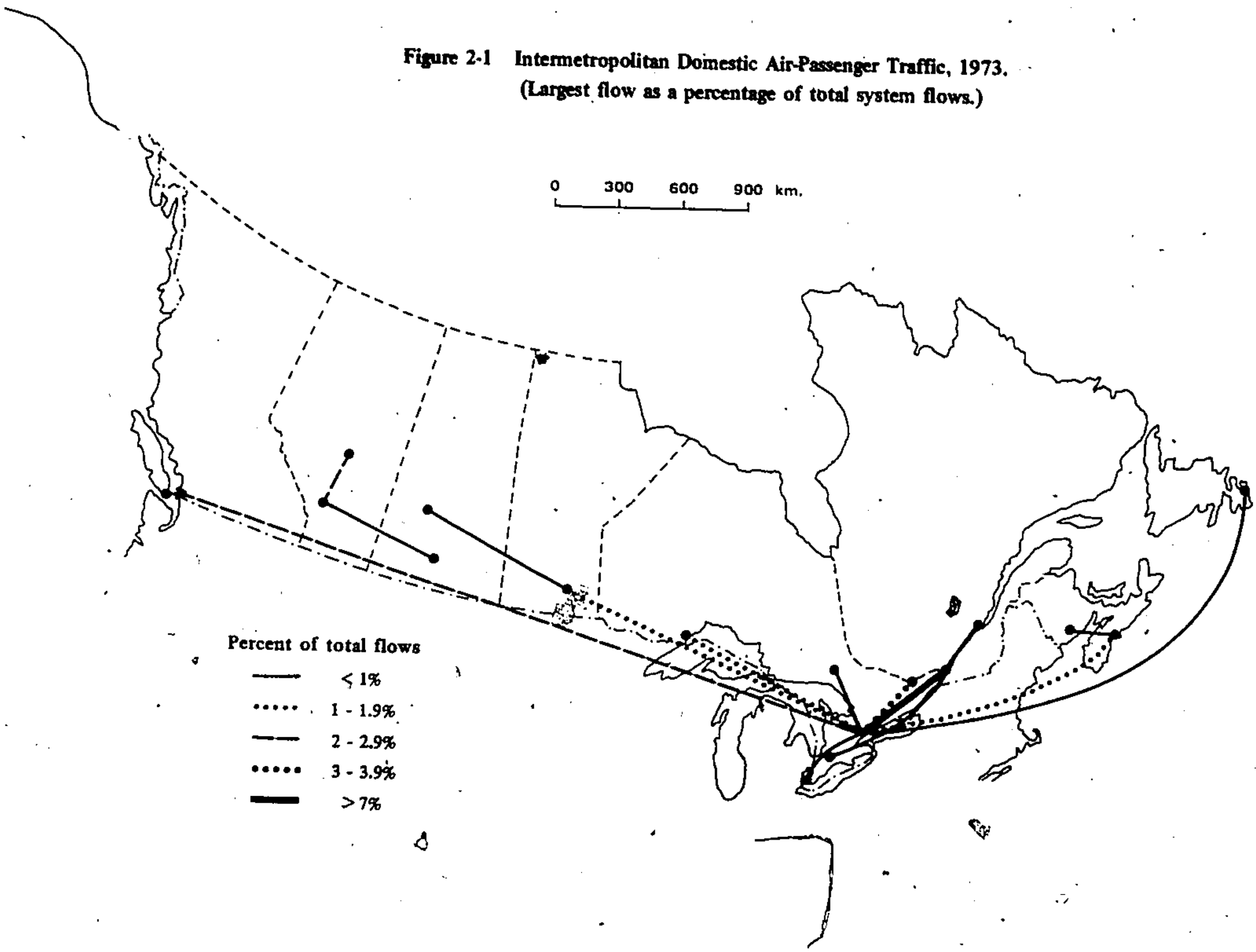
1963
(t = total system outbound flows: 1,823,895)

RANK	LINKS	LARGEST FLOW	PERCENT OF t
	Toronto bound:		
2	Montreal – Toronto	193,355	10.6
3	Ottawa – Toronto	66,470	3.6
5	Winnipeg – Toronto	36,415	2.0
6	Windsor – Toronto	36,385	2.0
9	Vancouver – Toronto	29,520	1.6
11	Halifax – Toronto	19,135	1.0
14	Sudbury – Toronto	8,015	0.44
15	London – Toronto	7,905	0.43
	Other destinations:		
1	Toronto – Montreal	193,195	10.6
4	Quebec – Montreal	45,375	2.5
7	Edmonton – Calgary	32,655	1.8
13	Regina – Calgary	10,175	0.6
8	Calgary – Edmonton	31,420	1.7
10	Victoria – Vancouver	20,260	1.1
12	Saint John – Halifax	10,200	0.6
16	St. John's – Halifax	7,640	0.42
17	Saskatoon – Winnipeg	6,725	0.37

1973
(t = 6,014,020)

RANK	LINKS	LARGEST FLOW	PERCENT OF t
	Toronto bound:		
1	Montreal – Toronto	459,120	7.6
3	Ottawa – Toronto	216,225	3.6
6	Vancouver – Toronto	138,185	2.3
7	Winnipeg – Toronto	106,200	1.8
8	Halifax – Toronto	74,185	1.2
9	Thunder Bay – Toronto	60,535	1.0
11	Windsor – Toronto	53,095	0.9
12	Sudbury – Toronto	43,120	0.7
14	St. John's – Toronto	29,955	0.5
	Other destinations:		
2	Toronto – Montreal	456,455	7.6
10	Quebec – Montreal	53,500	0.9
17	London – Montreal	19,450	0.3
4	Edmonton – Calgary	165,565	2.8
15	Regina – Calgary	27,815	0.5
5	Calgary – Edmonton	166,645	2.8
13	Victoria – Vancouver	33,085	0.6
16	Saskatoon – Winnipeg	24,700	0.4
18	Saint John – Halifax	16,975	0.3

Figure 2-1 Intermetropolitan Domestic Air-Passenger Traffic, 1973.
(Largest flow as a percentage of total system flows.)



development of modern economies is accompanied by greater horizontal as well as vertical interdependency of large urban centres. The case of Regina may be an illustration of Pred's argument. In 1963, Regina's output was principally towards two centres, Calgary (0.56%) and Winnipeg (0.51%)*. In 1973, its flows were more evenly balanced between four centres, Calgary (0.46%), Winnipeg (0.46%), Vancouver (0.45%) and Toronto (0.43%).

Table 2-1 indicates that Toronto was the dominant centre with strong links to all the major centres across the country, several of which themselves served as poles for regional sub-systems. By 1973, Toronto had extended its field by the addition of Thunder Bay and St. John's. That London did not appear in the Toronto field likely reflects the convenience of other modes of travel over the short distance between the cities. The strength of Toronto's field is even more apparent if the number of cases where Toronto received from centres outside its field almost as many flows as these centres sent to their own primary pole, e.g., Regina, Saskatoon, and Saint John. Toronto dominate flows East and West. There was an increase in relative and absolute terms in the number of flows into Toronto from Vancouver, Thunder Bay, Halifax, and St. John's. There was a loss in relative terms in the flows from Montreal, Windsor, Winnipeg and London.

Montreal was second in importance within the national system. It was the primary pole for Toronto and Quebec. Ottawa was probably also primarily oriented toward Montreal.although the short distance between the centres has favoured other modes of travel. Montreal, like Toronto, had strong ties with all the centres across the country. Its ties in the East were comparable to Toronto's but its links to the Western centres were much weaker.

The other important centres within the national system are Vancouver,

* Regina's output as a percentage of the system's total output.

Ottawa, Calgary, Edmonton, Winnipeg and to a lesser degree, Halifax. Vancouver, in addition to being the primary pole for Victoria received large flows from Toronto, Montreal, Ottawa, and all the Western centres. Its influence over the smaller Western centres of Regina and Saskatoon had greatly increased by 1973. In 1963 Regina's output towards Vancouver was only 0.27% of the total system traffic while in 1973 it had increased to 0.45%. Saskatoon's share had gone from 0.19% to 0.39% over the same period. Vancouver's ties with the Maritime centres remained weak, however. The growth in volume of traffic of the three centres of Ottawa, Calgary, and Edmonton place them within the mainstream of the national network but only Ottawa had strong ties in the Maritimes as well as in the Western cities by 1973. In 1963 Halifax's output towards Ottawa was only 0.27% of the system's total activity, but by 1973 it had grown to 0.47%. Vancouver's share into Ottawa had gone from 0.26% to 0.50%; Winnipeg's had increased from 0.30% to 0.45%; and Calgary and Edmonton's share had increased from an average of 0.16% to one of 0.26%. The growth experienced by Calgary was principally from flows coming from Vancouver, 1.3% in 1963 to 2.1% in 1973, and to a lesser degree through the additional flows from Toronto, 0.77% in 1963 to 0.93% in 1973. Edmonton's growth was similar in these respects to Calgary's. Winnipeg sustained losses from Regina and Saskatoon, the two centres having increased their flows into Vancouver and Toronto. Halifax developed as a Maritime regional centre. It sent to, and received from, Toronto and Montreal considerably more flows than did either Saint John or St. John's. Of the three Maritime centres it is the one that had the strongest links with the Western centres. In the map showing the largest flows in the system, the predominance of Toronto stands out. Only the largest of the two flows between pairs are shown as the matrix is quite symmetrical (Figure 2-1). Since the largest flow patterns were almost identical for both 1963 and 1973 only the latter have been mapped. The frictional effect of distance on the flow patterns appears to be strong except in the case of Toronto.

The description of gross flows has simply revealed a number of the main features of Canada's intermetropolitan air flow activity. Restricting the description to the exchanges of largest flows, of course, imposes severe limitations. To complement the preceding gross flow analysis, the air-passenger traffic data will be studied by means of factor analysis, which searches for similar patterns of activity based on all the flows in a matrix.

Factor Analyses of Intermetropolitan Air Passenger Flows for the 1963-1973 Period:

A series of factor analyses was performed on air passenger origin-destination matrices for the 1963-1973 years. These analyses revealed some major structural features of the Canadian interaction system and supported arguments, based on other analytical techniques, concerning the existence of a trend towards greater integration of the metropolitan system with Toronto as the national focus (Hartwick and Crowley, 1973; Storey and Bulker, 1975).

Factor analysis has been used by several authors for the study of origin-destination matrices. Illeris and Pedersen (1968) used the technique to determine regional functional structures within Denmark; Simmons (1972) used it to analyze structures within Central Canada; and Tinkler (1972) found the technique more satisfactory than Gould's "direct analyses" in describing Ugandan road networks.

The principal interest of the technique lies in the fact that each factor in turn extracts as much of the variance of the matrix as possible. Each factor describes a major sub-system within the network by grouping the urban centres that strongly interact together. In the present study, this property of the factor analysis model is particularly useful. As the technique is applied to a set of matrices covering a decade, it is possible to determine whether the sub-systems are expanding by the addition of new centres or regressing through the loss of some of their elements.

The analyses were first performed on the matrices of air flows in absolute values to determine the main features of the system's structure and their relative importance within the system for the 11-year period. There was some difficulty in identifying the finer traits of the system with the data in absolute values as the influence of the strong elements in the system, e.g. Toronto, Montreal, and Vancouver, tended to obliterate some of the weaker system structures. To do away with this problem of size the analyses were repeated with the data expressed as percentages of the emitting cities' total output. This led to the discovery of some of the finer traits of network structure.

The data can be factor analyzed in two ways, either as an origin-destination matrix or as its transpose, a destination-origin matrix. In the first instance the factor loadings identify centres with similar receiving patterns, e.g. attraction fields, and the factor scores list the centres emitting flows towards these fields in order of their importance. In the second case the factor loadings outline the emission fields and the factor scores give the list of receiving centres. The interaction fields outlined in this chapter have been derived from the results of the first type of analysis and confirmed through the second type.

Defining the Main Features of System Structure:

The results of the factor analyses of the data in absolute values show that the interaction patterns have changed over the 1963-1973 period. They also show that there has been two distinct periods of structural change in the network during this time, with the first extending from 1963 to 1968 and the second from 1969 to 1973. A comparison of the 1963 and 1973 results bring out the principal changes that have taken place during these years (Tables 2-2, 2-3, 2-4, 2-5).

The 1963-1968 Period:

In 1963 there were four major fields of interaction in the Canadian Metropolitan System and they remained fairly stable until 1968. The Toronto and Montreal fields were the main ones in the east while Vancouver and Calgary were the central poles of the two western major fields.

The Toronto field was the largest of the four comprising over 35% of the flow matrix. The 35% value defines the eigenvalue/trace ratio in percentage form of the Toronto factor and as such it measures Toronto's influence within the metropolitan network. The extent of the Toronto field of attraction at the beginning of the study period can be seen from the factor loadings of the metropolitan centres on the Toronto factor (Table 2-2), Toronto's influence covered central Canada, (Montreal, Sudbury, Ottawa, Windsor, Thunder Bay, London), extended as far west as Winnipeg, and included part of the Maritimes (Halifax, St. John's).

The cut-off value of factor loading in the field delimitations that was tentatively chosen was 0.45. A loading of 0.45 is equivalent to a coefficient of determination of 0.20, or in percentage form 20%. Thus factor loadings equal to or greater than 0.45 indicate that the factor accounts for 20% or or more of the interaction of the given metropolitan centre. What matters are the changes which have occurred within the interaction fields over the 11-year period.

The cities listed as being under the influence of Toronto in the preceding paragraphs do not represent the limits of Toronto's influence within the metropolitan system in 1963. They represent only the primary field of its influence, for Toronto's flows were strong on the other three fields as well as the factor scores clearly show (Table 2-3). Even in 1963 Toronto already had a national dimension in its interaction patterns.

The Montreal field (22%) in 1963 was the second in importance in the system and considerably weaker than Toronto's. The field consisted essentially of two parts: ties with Quebec, Toronto, and London in central Canada, and ties with all the Maritime centres.

Unlike Toronto, Montreal's influence was essentially limited to this factor though its flows into the centres of the Toronto field were not insignificant (0.8783, Table 2-3). Thus Montreal was essentially only a regional pole in 1963.

The Calgary field (17%) was third in importance and included Regina, Saskatoon, Vancouver, and Edmonton. The factor scores reveal an important point about this field. Though Calgary has the highest scores (1.5760) it is followed closely by Winnipeg (1.4046) and Vancouver (0.9769, Table 2-3) and to label this factor as the Calgary factor is an over-simplification. It would probably be more appropriate to label it the Calgary-Winnipeg field.

The Vancouver factor (12%) is the smallest field as it includes only Victoria, Calgary, Edmonton and Winnipeg. The fact that the Vancouver factor is independent of the Winnipeg-Calgary one shows that the Western networks

Table 2-2 Factor Loadings: Intermetropolitan Air Flows in Absolute Values (1963)

(Origin-destination Matrix)

Toronto Factor (35%)	
Montreal	0.9748
Sudbury	0.9688
Ottawa	0.9529
Windsor	0.9343
Thunder Bay	0.8883
Winnipeg	0.7270
Halifax	0.5896
London	0.5890
St. John's	0.4991
Vancouver	0.3901
Saint John	0.3837
Saskatoon	0.2361
Regina	0.1528
Calgary	0.1383
Quebec	0.1257
Edmonton	0.0526
Victoria	0.0181

Montreal Factor (22%)	
Quebec	0.9531
Toronto	0.9386
London	0.7411
St. John's	0.6970
Saint John	0.6773
Halifax	0.6397
Windsor	0.2684
Winnipeg	0.2471
Ottawa	0.2095
Sudbury	0.1550
Thunder Bay	0.1290
Vancouver	0.1044
Saskatoon	0.0236
Victoria	0.0160
Montreal	0.0046
Edmonton	-0.0151
Calgary	-0.0347
Regina	-0.0365

Calgary Factor (17%)	
Regina	0.8896
Saskatoon	0.8330
Vancouver	0.7991
Edmonton	0.6322
Thunder Bay	0.3573
Calgary	0.3318
Winnipeg	0.2614
London	0.2465
Windsor	0.2043
Ottawa	0.1748
Sudbury	0.1504
Victoria	0.1293
Toronto	0.1035
Montreal	0.0961
Quebec	0.0931
Halifax	0.0600
St. John's	-0.1682
Saint John	-0.2241

Vancouver Factor (12%)	
Victoria	0.9641
Calgary	0.6550
Edmonton	0.5348
Winnipeg	0.4724
Saskatoon	0.3747
Regina	0.3007
Ottawa	0.0890
London	0.0738
Montreal	0.0725
Windsor	0.0640
Sudbury	0.0599
St. John's	0.0423
Saint John	0.0398
Thunder Bay	0.0336
Toronto	0.0109
Quebec	-0.0372
Vancouver	-0.0702
Halifax	-0.0867

Table 2-3 Standardized Factor Scores for the Factor Loadings of Table 2-2

	Toronto Factor	Calgary Factor	Montreal Factor	Vancouver Factor
Toronto	3.88544	2.48425	2.21228	1.86279
Montreal	0.87826	0.33728	3.11970	0.05828
Ottawa	0.14230	-0.25284	0.80109	-0.30280
London	-0.46824	-0.68537	-0.54447	-0.65280
Quebec	-0.35851	-0.78303	-0.49479	-0.74123
Windsor	-0.48634	-0.68428	-0.40349	-0.66242
Sudbury	-0.60786	-0.85877	-0.62287	-0.81546
Thunder Bay	-0.50253	-0.73412	-0.51901	-0.63365
Halifax	-0.00683	-1.03443	0.57533	-0.51868
Saint John	-0.41376	-0.82622	-0.33636	-0.85471
St. John's	-0.45427	-0.83323	-0.39889	-0.83975
Winnipeg	0.10975	1.40457	-0.18948	0.57615
Regina	-0.34994	-0.21966	-0.51525	-0.03143
Saskatoon	-0.45072	-0.45965	-0.56885	-0.37920
Calgary	-0.10054	1.57604	-0.41483	0.94666
Edmonton	-0.07496	0.81115	-0.37933	0.84379
Victoria	-0.41565	-0.21860	-0.55215	-0.64022
Vancouver	-0.04099	0.97691	-0.26862	2.78470

Table 2-4 Factor Loadings: Intermetropolitan Air Flows in Absolute Values (1973)

(Origin-destination Matrix)

Toronto Factor (38%)	
Sudbury	0.9720
Montreal	0.9640
Ottawa	0.9450
Thunder Bay	0.9180
Windsor	0.8356
St. John's	0.7390
Halifax	0.7194
Winnipeg	0.6616
Saint John	0.5744
London	0.4750
Vancouver	0.4628
Quebec	0.3749
Saskatoon	0.2426
Regina	0.2178
Calgary	0.0796
Victoria	0.0767
Edmonton	0.0256

Montreal Factor (21%)	
Toronto	0.9606
Quebec	0.9041
London	0.8019
Saint John	0.6499
Halifax	0.5863
St. John's	0.4972
Windsor	0.4828
Winnipeg	0.2297
Ottawa	0.1664
Vancouver	0.1574
Sudbury	0.1039
Thunder Bay	0.0729
Regina	0.0712
Saskatoon	0.0656
Victoria	0.0489
Edmonton	0.0193
Calgary	0.0155

Vancouver - Calgary Factor (27%)

Victoria	0.9538
Regina	0.9384
Saskatoon	0.9018
Edmonton	0.8189
Calgary	0.6742
Vancouver	0.6185
Thunder Bay	0.2839
London	0.2456
Ottawa	0.2362
Montreal	0.2213
Windsor	0.2189
Sudbury	0.1663
Toronto	0.1055
Halifax	0.0750
Quebec	0.0377
St. John's	-0.0255
Saint John	-0.0911

Table 2-5 Standardized Factor Scores for the Factor Loadings of Table 2-4

	Toronto Factor	Montreal Factor	Vancouver-Calgary Factor
Toronto	3.8082	2.4485	2.3854
Montreal	0.9253	2.7085	0.2212
Ottawa	0.2139	0.9542	0.0304
London	-0.4804	-0.6587	-0.7287
Quebec	-0.4766	-0.6167	-0.7752
Windsor	-0.5371	-0.6154	-0.7511
Sudbury	-0.6073	-0.6171	-0.8333
Thunder Bay	-0.5217	-0.5277	-0.6955
Halifax	0.0519	0.2577	-0.7081
Saint John	-0.0524	-0.5926	-0.8406
St. John's	-0.4648	-0.4864	-0.8250
Winnipeg	0.0399	-0.0851	0.7697
Regina	-0.4487	-0.5828	-0.4433
Saskatoon	-0.4852	-0.6160	-0.5283
Calgary	0.0042	-0.1610	1.5125
Edmonton	-0.1100	-0.2743	0.8979
Victoria	-0.5046	-0.6426	-0.4565
Vancouver	0.1132	0.1077	1.7685

were not thoroughly integrated at this stage.

The 1969-1973 Period:

Throughout these years strong forces had been progressively altering the interaction fields of the Canadian network and by 1973 the principal elements of the new structure were well established. By 1973 there were only three major fields, two in the East and one in the West. The existence of only three fields where there had been four is some indication of the system integration that had taken place during this period.

The Toronto field had increased considerably (38%, Table 2-4). It covered central Canada, all of the Maritimes, and included Vancouver as well as Winnipeg. The ties with Vancouver can be taken as an indication of Toronto's growing importance within the national system. Toronto is the only centre which displays such an extensive influence.

Vancouver and Montreal formed the nuclei of two strong regional fields. All the centres in the western provinces formed a single field with Vancouver (1.7685) as its main pole (27%). Calgary (1.5125) and Edmonton (0.8979) were important secondary poles (Tables 2-4 and 2-5). Integration of the western cities into one large regional field was a major change within the period.

Finally, there was the Montreal field (21%). There is some indication that this field had stronger ties with southern Ontario in 1973 than in 1963 for both Toronto and Windsor have stronger loadings (Table 2-4). Another significant transformation is revealed by the changes in the factor scores. Ottawa and Halifax have switched places in the rankings (Tables 2-3 and 2-5). By 1973 Ottawa was an important node within this field and Halifax was relegated to a much smaller role within this eastern regional network.

Additional Features of the Canadian System:

The analyses of the matrices with the flows expressed as percentages revealed two other interesting system changes within the networks.

a) The Saint John - St. John's Field:

This field had been thoroughly blotted out by the analyses on the absolute values and yet it presents some interesting features. Throughout the 11 years the two cities of Saint John and St. John's formed a small attraction field for the flows out of Halifax. Even though this factor remained largely unchanged over the period there are some small but significant changes in the factor scores reported for this field. Halifax experienced a slight but continuous loss in scores, dropping from 3.7866 to 3.4047 while Montreal and Toronto gained slightly (Tables 2-6, 2-7). By themselves these changes are too small to lead to any definite conclusion about system evolution, but in the light of other research on the national system they may be taken as an indication of the tendency of the Maritime sub-system to be absorbed by the great eastern network centred on Toronto and Montreal.

b) The Halifax-Winnipeg Sub-Systems:

The percentage analyses revealed another interesting feature of evolution within the national network: changes within the western component of the Halifax-Winnipeg dipole (Tables 2-8 and 2-9).

The analyses show the existence of a bipolar factor which opposed the behavior of two regional sub-systems, one in the East and one in the West. In the East the network has Halifax as its primary node and Montreal as its secondary one. The dependent centres revolving around these nodes are Saint John, St. John's, and Quebec City. In the West Winnipeg and Calgary serve as the principal and secondary nodes. The fact that this is a bipolar factor

Table 2-6 Factor Loadings and Factor Scores of the Saint John - St. John's Factor

(1963-64-65)

Loadings 1963		Loadings 1964		Loadings 1965	
Winnipeg	0.2154	Winnipeg	0.2329	Winnipeg	0.2383
Toronto	0.1556	Halifax	0.1571	Halifax	0.2022
Saskatoon	0.1375	Calgary	0.1387	Saskatoon	0.1765
Regina	0.1294	Saskatoon	0.1300	Calgary	0.1515
Halifax	0.1161	Regina	0.1224	Regina	0.1362
Thunder Bay	0.1048	Toronto	0.1042	Vancouver	0.0983
Edmonton	0.0955	Vancouver	0.0943	Edmonton	0.0943
Calgary	0.0841	Thunder Bay	0.0842	Toronto	0.0520
Vancouver	0.0825	Sudbury	0.0737	Thunder Bay	0.0508
Windsor	0.0085	Edmonton	0.0698	Victoria	0.0493
Sudbury	-0.0020	Victoria	0.0479	Sudbury	-0.0021
Quebec City	-0.0548	Windsor	-0.0276	Windsor	-0.0207
Victoria	-0.0587	Quebec	-0.0623	Quebec	-0.0623
Montreal	-0.1672	Montreal	-0.1813	London	-0.1459
Ottawa	-0.1914	London	-0.1878	Montreal	-0.1913
London	-0.2281	Ottawa	-0.2115	Ottawa	-0.1924
St. John's	-0.9584	St. John's	-0.9560	St. John's	-0.9590
Saint John	-0.9694	Saint John	-0.9695	Saint John	-0.9650

	Scores 1963	Scores 1964	Scores 1965
Toronto	-0.47892	-0.42268	-0.47400
Montreal	-0.42801	-0.45865	-0.51670
Ottawa	-0.08080	-0.15426	-0.19121
London	0.02670	-0.04345	-0.14032
Quebec	-0.32033	-0.32980	-0.36911
Windsor	0.30390	0.26195	0.13899
Sudbury	0.28069	0.06747	0.04437
Thunder Bay	0.52367	0.49113	0.41151
Halifax	-3.78655	-3.74844	-3.67534
Saint John	-0.15231	-0.11389	-0.05475
St. John's	-0.12355	-0.14462	-0.10195
Winnipeg	0.57702	0.56795	0.59213
Regina	0.70851	0.73919	0.89985
Saskatoon	0.61184	0.74005	0.72523
Calgary	0.81735	0.72811	0.83202
Edmonton	0.57767	0.71596	0.77790
Victoria	0.46928	0.48611	0.46850
Vancouver	0.47385	0.61786	0.63288

Table 2-7 Factor Loadings and Factor Scores of the Saint John - St. John's Factor

(1971-72-73)

Loadings 1971		Loadings 1972		Loadings 1973	
Winnipeg	0.2303	Winnipeg	0.2741	Winnipeg	0.2863
Vancouver	0.1742	Vancouver	0.1601	Vancouver	0.1758
Saskatoon	0.1573	Saskatoon	0.1509	Calgary	0.1573
Halifax	0.1442	Calgary	0.1486	Saskatoon	0.1392
Regina	0.1329	Halifax	0.1402	Regina	0.1213
Calgary	0.1304	Victoria	0.1176	Thunder Bay	0.1169
Thunder Bay	0.1087	Regina	0.1151	Victoria	0.1126
Toronto	0.0945	Edmonton	0.1150	Edmonton	0.1085
Edmonton	0.0805	Thunder Bay	0.1056	Halifax	0.1085
Victoria	0.0361	Toronto	0.0741	Sudbury	0.0718
Sudbury	0.0238	Sudbury	0.0465	Toronto	0.0718
Windsor	-0.0728	London	-0.0012	London	-0.0384
London	-0.1008	Windsor	-0.0712	Windsor	-0.0723
Quebec	-0.1632	Quebec	-0.2244	Quebec	-0.1231
Ottawa	-0.2419	Montreal	-0.2596	Montreal	-0.2716
Montreal	-0.2630	Ottawa	-0.2645	Ottawa	-0.2962
Saint John	-0.9394	Saint John	-0.9512	Saint John	-0.9428
St. John's	-0.9447	St. John's	-0.9552	St. John's	-0.9456

	Scores 1971	Scores 1972	Scores 1973
Toronto	-0.53820	-0.55902	-0.52920
Montreal	-0.66530	-0.67826	-0.56630
Ottawa	-0.15988	-0.09864	-0.07288
London	-0.22352	-0.24434	-0.22389
Quebec	-0.60623	-0.68981	-0.69316
Windsor	0.12808	0.08013	0.06107
Sudbury	0.04474	0.10864	0.03473
Thunder Bay	0.46454	0.54281	0.55076
Halifax	-3.45808	-3.42156	-3.40473
Saint John	-0.28727	-0.32237	-0.40989
St. John's	-0.27313	-0.28768	-0.40888
Winnipeg	0.75857	0.70081	0.70323
Regina	0.79228	0.73786	0.76063
Saskatoon	0.80877	0.80646	0.80623
Calgary	0.91400	0.95196	0.94685
Edmonton	0.86814	0.84917	0.90327
Victoria	0.75297	0.73688	0.80994
Vancouver	0.67954	0.78697	0.73224

Table 2-8 Factor Loadings and Factor Scores of the Halifax-Winnipeg Dipole

(1963-64-65)

Loadings 1963		Loadings 1964		Loadings 1965	
Winnipeg	0.8134	Winnipeg	0.8048	Halifax	0.6116
Calgary	0.7674	Calgary	0.6185	Montreal	0.4610
Victoria	0.3558	Edmonton	0.4089	Ottawa	0.2041
Saskatoon	0.2777	Victoria	0.3162	Toronto	0.1947
Vancouver	0.1909	Vancouver	0.2984	Quebec	0.1382
Edmonton	0.1538	Saskatoon	0.2105	St. John's S.	0.1099
Windsor	0.1088	Regina	0.1967	Thunder Bay	0.1022
Regina	0.0939	Windsor	0.0834	St. John's N.	0.0878
London	0.0636	Sudbury	0.0182	Sudbury	0.0855
Sudbury	-0.0023	London	0.0158	London	-0.0150
Saint John	-0.0785	Quebec	-0.0400	Windsor	-0.1021
St. John's	-0.1031	Saint John	-0.0852	Regina	-0.1045
Quebec City	-0.1153	St. John's	-0.1129	Edmonton	-0.1395
Ottawa	-0.1335	Toronto	-0.2033	Vancouver	-0.2424
Thunder Bay	-0.1628	Thunder Bay	-0.2090	Victoria	-0.3159
Toronto	-0.1936	Ottawa	-0.2407	Saskatoon	-0.4327
Montreal	-0.4232	Montreal	-0.5534	Winnipeg	-0.7531
Halifax	-0.5453	Halifax	-0.6360	Calgary	-0.7661

	Scores 1963	Scores 1964	Scores 1965
Toronto	-0.48182	-0.61931	0.58680
Montreal	-0.56047	-0.42331	0.59374
Ottawa	-0.29424	-0.19871	0.26027
London	-0.46342	-0.49754	0.46802
Quebec	-1.17964	-1.23845	1.10741
Windsor	-0.58813	-0.49942	0.48962
Sudbury	-0.54052	-0.57583	0.51873
Thunder Bay	0.49334	0.43528	-0.39504
Halifax	-0.86912	-0.97132	0.99178
Saint John	-1.51878	-1.52464	1.54428
St. John's	-1.32586	-1.36125	1.38069
Winnipeg	0.07029	0.02419	-0.34950
Regina	1.73976	1.56012	-1.81990
Saskatoon	1.31693	1.39168	-1.13559
Calgary	0.77001	1.21370	-0.86123
Edmonton	1.54357	1.30171	-1.58035
Victoria	0.42661	0.66002	-1.53389
Vancouver	1.46147	1.32308	-1.26587

Table 2-9 Factor Loadings and Factor Scores of the Halifax-Winnipeg Dipole

(1971-72-73)

Loadings 1971		Loadings 1972		Loadings 1973	
Halifax	0.5402	Halifax	0.6954	Winnipeg	0.7398
Montreal	0.3981	Montreal	0.6158	Victoria	0.6959
Thunder Bay	0.2239	Ottawa	0.3836	Vancouver	0.6722
Toronto	0.1831	Toronto	0.3548	Calgary	0.6020
Ottawa	0.1632	Quebec City	0.1777	Edmonton	0.5575
Saint John	0.1038	Saint John	0.1497	Regina	0.3014
Saskatoon	0.0951	St. John's	0.1045	Saskatoon	0.2506
St. John's	0.0822	London	0.0670	Windsor	0.0402
London	0.0671	Thunder Bay	0.0630	Thunder Bay	0.0017
Regina	0.0528	Sudbury	0.0622	Sudbury	-0.0215
Quebec	0.0223	Windsor	-0.0837	St. John's	-0.0968
Windsor	-0.0801	Saskatoon	-0.2690	Quebec	-0.1333
Sudbury	-0.0932	Regina	-0.3078	Saint John	-0.1455
Edmonton	-0.2238	Edmonton	-0.5680	London	-0.1498
Vancouver	-0.2353	Calgary	-0.6009	Ottawa	-0.3282
Victoria	-0.3797	Vancouver	-0.6302	Toronto	-0.4283
Calgary	-0.4189	Victoria	-0.6775	Montreal	-0.5652
Winnipeg	-0.9000	Winnipeg	-0.7672	Halifax	-0.6905

	Scores 1971	Scores 1972	Scores 1973
Toronto	0.34420	0.44514	-0.34702
Montreal	0.32987	0.56275	-0.61813
Ottawa	0.23089	0.26792	-0.36800
London	0.48586	0.60794	-0.42599
Quebec	1.07032	1.18176	-1.08915
Windsor	0.47774	0.58340	-0.62745
Sudbury	0.75842	0.75483	-0.88279
Thunder Bay	-0.63703	-0.05628	-0.01785
Halifax	0.94608	1.00069	-0.98342
Saint John	1.46115	1.40997	-1.39534
St. John	1.39463	1.32611	-1.36697
Winnipeg	0.51119	-0.53150	0.57114
Regina	-1.64339	-1.15084	1.17125
Saskatoon	-1.72091	-1.26366	1.21618
Calgary	-0.99133	-1.58144	1.60018
Edmonton	-1.17406	-1.33288	1.39495
Victoria	-0.63092	-0.98598	1.04100
Vancouver	-1.21272	-1.23795	1.12741

stresses the regional nature of both sub-systems. The centres revolving around the nucleus of network have little interaction with the network at the opposite end of the factor.

It is the Western component of this factor which presents the most noticeable changes. In the early years only Winnipeg and Calgary served as the attraction poles of this regional network (Table 2-8, Loadings for 1963-64-65). By 1971, the attraction field of the Western component had expanded with the inclusion of Vancouver, Victoria, and Edmonton (Table 2-8, Loadings for 1971-72-73). The small regional sub-system clustered around Winnipeg and Calgary had been superseded by a larger regional network that included all the major Western cities. There is no evidence that there had been a comparable expansion of the Maritime regional network. In the East the integration of most centres about the Toronto-Montreal nodes had not allowed for the expansion of the Maritime regional network.

Conclusion:

The purposes of this Chapter have been to bring out the main structural features of the Canadian metropolitan system and assess whether there have been some system transformations over the 1963-73 period.

It is not overly surprising in the light of recent research (Borchert, 1972, Pred, 1973) to find that the factor analyses have detected some evolution of the metropolitan system towards greater integration of its structures as this is a characteristic trait of the urbanization process presently operating in modern economies.

A notable feature of the evolution of the Canadian system is the fact that there have been changes in two different directions. On the one hand there has been change in a vertical direction with a partial hierarchical restructuring through a series of newly created dependency relationships between the urban centres, and on the other, there has been a horizontal movement towards a greater direct interdependency among the metropolitan centres.

The hierarchical restructuring of the 1963-1973 period has resulted in the creation in Canada of a truly national urban field centered on Toronto. Within this large national field there are two sub-systems, one in the West centred on Vancouver and Calgary and covering the whole of Western Canada, and another covering Central and Atlantic Canada with Toronto and Montreal as central nodes for Toronto has both a national and a regional role. These large networks have replaced the numerous small semi-independent sub-systems still predominant in the early 1960's (Simmons, 1973; Bourne, 1975) and have given Canada a stronger hierarchical organization.

The move towards greater metropolitan interdependency is not as clearly demonstrated by these analyses as the hierarchical feature for the factor analyses do not specifically provide a means of measuring this type of relationship. However, greater interdependency can be inferred from the increase in the size and the decrease in the number of interaction fields over the period.

Useful as these techniques may be they are incapable of going beyond these broad generalizations of interaction field delimitation. They cannot determine the intra-system structures formed by clusters of bonds performing as network sub-structures nor evaluate their performances. They cannot measure the system's feedback patterns nor the sensitivity of the nodes to flow fluctuations on the individual arcs.

For a better understanding of the urbanization process, it becomes necessary to develop techniques conceived specifically for the purpose of analyzing the detailed interaction patterns of urban systems and measuring system interaction properties at the system, sub-system, and individual arc level. Chapter 3 is an attempt at providing just such a technique.

CHAPTER 3

MEASURING SYSTEM CHARACTERISTICS: THE A.S.T. MODEL[4]

The first chapter has shown that there exists a growing concentration of population in metropolitan centres and an increasing interdependency between the metropolitan centres of an urban system. The second chapter has attempted to demonstrate some structural features and recent transformations within the Canadian Metropolitan System. This third chapter presents a model for measuring system interaction properties.

The flow-graph model developed in this chapter has several interesting features.[5] Any urban system that can be expressed as a set of simultaneous linear equations can be formulated within this flow-graph theory framework and its system characteristics measured. The direct and indirect internodal activity can be studied in depth because the flow-graph model can be subjected to topological transformations for the purpose of identifying the feedback circuits within the system which, in turn, provide a measure of nodal sensitivities. Conceptually the flow-graph model is an elegant analytical technique. It provides a three level approach to the analysis of a system. System gain at any

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[4] Lamarche, R. et Francon, J. (1975), "Analyse et Simulation Topologique en Geographie", Cahiers de Geographie de Quebec, Vol. 19, No. 46, pp. 189-207.

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[5] This model was the outcome of a series of discussions with Professor H. Reymond, Department de Geographie, Universite Louis Pasteur, Strasbourg. Because the derivation of the sensitivity formulae in particular presented some difficulties, Professor J. Francon was consulted and this led to the publication of the above mentioned paper.

node is analyzed first with respect to the activity in the system as a whole, then with regards to the activity on each of the system's sub-networks, and finally with respect to fluctuations on each individual arc. Operationally, it is an easy task to programme the model because of its matrix algebra formulation.

However, there is a problem in the application of a flow-graph model to urban analysis. In all likelihood, the bonds between urban centres, such as intermetropolitan migration movements or air-passenger exchanges, are not as tight as the assumptions of the model demand. However, the analogy between an urban system and a flow-graph model has been justified in Chapters One and Two. The basic assumptions of the flow-graph model, rigorous as they are, are no more so than those of the input-output model, a model it closely resembles. Just as the input-output model, in spite of its restrictive assumptions, has led to a better understanding of economic systems, it is hoped that the flow-graph model will shed considerable light on urban systems.

There are two different approaches to the derivations of the formulae that measure system properties in flow-graph models, the combinatorial (Mason, 1953; Henley and Williams, 1973) and the matrix algebra approach (Ponsard, 1969; Lamarche et Francon, 1975). The combinatorial algorithms are usually a reformulation of the original Mason algorithms which are based on the progressive reduction of the flow-graph down to a single arc. Mason developed a number of algebraic rules for the combination and addition of arcs in graph reduction. This approach was developed essentially for small networks though Henley and Williams later created a combinatorial computer programme for large networks. However, this thesis rests on the formulae presented by Lamarche et Francon (1975) as they are suited to the analysis of large systems, are easy to programme, and constitute a more elegant analytical technique.

Chapter Three is divided into three parts. The first section presents some basic concepts of flow-graph theory, the second details the mathematical formulations of A.S.T. model, and the third section gives a geographical interpretation of flow-graph system characteristics. The third section also includes an example of the technique as it applies to a small urban system with the calculations worked out in detail.

Basic Concepts:

Definitions -

There is within the wider field of flow-graph theory a particular branch which deals with flow-graphs that can be expressed as a system of simultaneous linear equations. Graphs of this type are properly called signal flow-graphs (Mayeda, 1972). This thesis deals exclusively with signal flow-graphs. But, for brevity's sake, the term flow-graph rather than signal flow-graph will henceforth be used.

A flow-graph consists of a set of nodes interconnected by a set of arcs or branches. In the context of urban systems analysis the nodes represent the centres of an urban system and the arcs their interrelationships. One node is defined as the independent variable, labelled x_o and is called the major or source-node of the system. The other nodes are the dependent variables and labelled x_i (Figure 3-1).

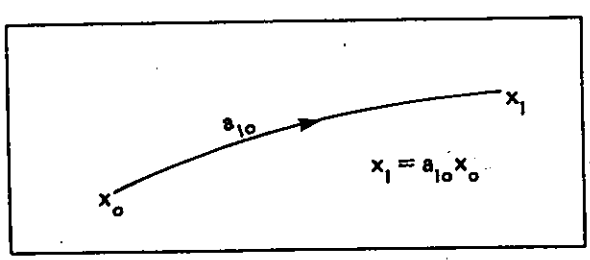


Figure 3 - 1 A Flow-graph relationship

The arrow of Figure 3-1 indicates the dependency of x_i on x_o and not the direction of the flow. The flows go from x_i to x_o. The coefficient a_{io} describes x_i in terms of x_o, that is it maps x_o into x_i. Thus in Figure 3-1 the flows into x_o from x_i are a fraction of x_o's total flows. If Figure 3-1 were expanded to include several dependent nodes, then x_i would be defined as a sum of interactions, a sum which can be calculated by simply summing the flows which the node x_i sends to all the other nodes. In flow-graph theory a node is simply the sum of interactions a centre has with the other centres, a point which is illustrated in the equation in the following paragraph.

Any node can be expressed in terms of the other nodes. For example, in a system of "n" nodes, the dependent node x_1 is defined as:

$$x_1 = a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n$$

Any other dependent node can be formulated in a similar fashion. Evidently the above equation implies that the nodes belong to the same system as each node is a function of all the others. When the system is written with each node in turn, starting with the first, isolated on the one side of the equation, the system is said to be in its canonical form (Ponsard, 1969). This implies that x_1 is isolated in the first equation, x_2 in the second and so on down to the last node. The canonical form suggests how a diagram of the network could be drawn from the complete set of a system's equations. This will be illustrated in the following sub-section.

The whole system can be written in summation notation as:

(1)

$$x_i = \sum_{j=0}^{n} a_{ij}x_j \quad (i = 1, \dots, n)$$

In system (1) each of the nodes is shown to be a function of all the others. System (1) is in its canonical form. If the equations are modified slightly so that the left-hand side is made equal to zero, the system is said to be homogeneous and would take the general form of:

(2) $$0 = \sum_{j=0}^{n} a_{ij} x_j \qquad (i = 1, \ldots, n)$$

Drawing a Flow-graph Diagram -

This section demonstrates how a flow-graph diagram can be drawn for any origin-destination matrix of interaction.

Assume that Figure 3-2A represents the exchanges between the nodes of a small system.

		Destination			
	node	x_0	x_1	x_2	total
Origin	x_0	0	50	60	110
	x_1	30	0	40	70
	x_2	10	20	0	30

Figure 3-2A : O-D Matrix of a Three - Node System.

From the O-D matrix, the arc coefficients are calculated according to the assumption of linearity which states that each entry into a node j coming from region i, is proportional to the total output of region j. This is achieved by dividing each row by the total column, the first cell in the row being divided by the first element in the total column, the second member in the row by the second member in the total column and so on. Figure 3-2B summarizes these calculations.

Node	x_0	x_1	x_2
x_0	—	—	—
x_1	.27	0	1.3
x_2	.09	.29	0

Figure 3 -2B : Arc Coefficient Matrix.

Note that in Figure 3-2B the coefficients for the x_o node have not been calculated and the figure provides the data for a two-equation system only:

$$x_1 = 0.27\ x_o + 1.3\ x_2$$
$$x_2 = 0.09\ x_o + 0.29\ x_1$$

There are three variables and only two equations and the system cannot be solved as it now stands. Even if the coefficients for the first row of Figure 3-2B were added to define x_o as:

$$x_o = .71\ x_1 + 2.0\ \ x_2$$

the system would still be unsolvable. Since the system is a homogeneous system in which the constants for all three equations are 0, the augmented matrix is the same as the coefficient matrix. The determinant of the augmented matrix will not be equal to 0 and the system remains unsolvable. One of the variables must be assigned a value, unsually a unit value, before the system can be solved. The node which is arbitrarily assigned a unit value is labelled the source-node in flow-graph theory. Any variable may be assigned the value of one and therefore any node may be taken as the source-node. A comparison with the input-output model should serve to clarify the notion of source-node.

The Leontief equation system of the input-output model gives the distribution of the total output of each sector:

$$\begin{aligned} x_1 - a_{11}x_1 - \dots - a_{1n}x_n &= y_1 \\ \dots \quad \dots \quad & \dots \\ x_i - a_{i1}x_1 - \dots - a_{in}x_n &= y_i \\ \dots \quad \dots \quad & \dots \\ x_n - a_{n1}x_1 - \dots - a_{nn}x_n &= y_n \end{aligned}$$

the x values represent the total output of the n sectors, the y values give the final demand sectors, and the coefficients a_{ij} are the direct intersector requirements. By a slight modification, the system can be changed to:

$$\left[\begin{array}{l} x_1 - a_{11}x_1 - \dots - a_{1n}x_n = a_{1o}y_o \\ \dots \quad \dots \quad \dots \quad \dots \\ x_i - a_{i1}x_1 - \dots - a_{in}x_n = a_{io}y_o \\ \qquad\qquad\quad \dots \quad \dots \\ x_n - a_{n1}x_1 - \dots - a_{nn}x_n = a_{no}y_o \end{array}\right] \begin{array}{l} = y_1 \\ \quad \dots \\ = y_i \\ \quad \dots \\ = y_n \end{array}$$

in this system y_o represents the total final demand sector. The bracketed system is a flow-graph formulation of interaction. The right-hand side of the equation system expresses what each sector x_i sends into the final demand as a ratio of total final demand $a_{io}y_o$ instead of in absolute values y_i. There is no equation in the above system showing how total final demand is distributed among the sectors but the right-hand of the system shows the proportions, $a_{io}y_o$, that each sector sends to the demand sector. The flow-graph of this system would show y_o as a node with arcs oriented in one direction only whereas all the other nodes would have incoming and outgoing arcs. In keeping with flow-graph theory requiring that the arrows point in the direction of the oncoming flows, the y_0 node would have arcs oriented away from itself and this is by definition of the theory a source-node. In such a system every other sector would be analyzed with respect to a unit output of total final demand. The above input-output system could be transformed slightly to isolate any sector as the source-node in order to analyze the rest of the system with respect to a unit output of this new source-node. In a similar fashion any system of equations depicting inter-urban interaction can be written with any of the centres serving as the source-node.

Remembering that the arrows are pointing in the direction of on-coming flows, the flow-graph diagram of Figure 3-2B would give the diagram of Figure 3-2C.

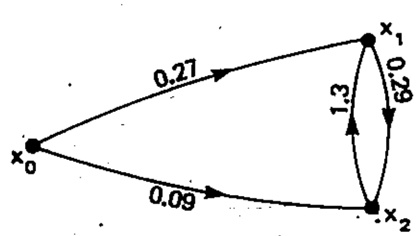


Figure 3 - 2C : Flowgraph Diagram.

The flow-graph diagram suggests a method for writing the canonical equations of the system. In a canonical equation a node is expressed in terms of all the flows it emits. For any node in the system, the diagram indicates these flows by the number of arcs whose arrows are pointing to the node in question. Each equation contains as many variables on the right-hand side of the equality sign as there are arrows leading into the node described by the equation. The canonical set of equations of the system in Figure 3-2C is as follows:

$$x_1 = 0.27x_0 + 1.3x_2$$
(3)
$$x_2 = 0.09x_0 + 0.29x_1$$

The system represented by the flow-graph of Figure 3-2C and its associated system of equations (3) can now be subjected to a number of topological transformations for the dual purpose of identifying its feedback circuits and measuring the sensitivity of nodes to fluctuations in the system.

Mathematical Formulation of the A.S.T. Model:[6]

A system of "n" interrelated variables can be expressed in its canonical form as:

(4)
$$\begin{aligned} x_1 &= c_{10}x_0 + t_{11}x_1 + t_{12}x_2 + \dots + t_{1n}x_n \\ x_2 &= c_{20}x_0 + t_{21}x_1 + t_{22}x_2 + \dots + t_{2n}x_n \\ &\dots\dots \\ x_n &= c_{n0}x_0 + t_{n1}x_1 + t_{n2}x_2 + \dots + t_{nn}x_n \end{aligned}$$

The major node x_o has its arcs labelled C_{io} while all other arcs are labelled t_{ij}. The C_{io} values bind the major node, the independent variable, to all the other nodes while the t_{ij} coefficients show the bonds between the dependent variables x_i and x_j where $i = j = 1, \dots, n$. In flow-graph theory the major node is the independent variable while the other nodes are called dependent variables. System (4) is transformed to (5a) by isolating the major node x_o on the right-hand side:

[6]The definitions of the system properties developed in this section are summarized in Figure 3-6A.

$$(5a)\qquad \begin{aligned} &(t_{11}-1)x_1 + t_{12}x_2 + \ldots + t_{1n}x_n = -c_{10}x_0 \\ &t_{21}x_1 + (t_{22}-1)x_2 + \ldots + t_{2n}x_n = -c_{20}x_0 \\ &\ldots. \\ &t_{n1}x_1 + t_{n2}x_2 + \ldots + (t_{nn}-1)x_n = -c_{n0}x_0 \end{aligned}$$

To simplify the writing of the equations, for values of i = 1, 2, ..., n and j = 1, 2, ..., n, let

$$b_i = -c_{i0}$$
$$a_{ij} = t_{ij}\ ,\ i \neq j$$
$$a_{ii} = t_{ii} - 1$$

Then system (5a) becomes (5b):

$$(5b)\qquad \begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1x_0 \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2x_0 \\ &\ldots. \\ &a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_nx_0 \end{aligned}$$

Let X be the vector of x_1, x_2, ..., x_n, B the vector of b_1, b_2, ..., b_n and A the matrix of the a_{ij} elements. System (5b) can then be written:

$$(6a)\qquad [A][X] = x_0[B]$$

or if the vector C contains C_{10}, C_{20}, ..., C_{no}, as (6b):

(6b) $[A][X] = x_0[-C]$

The diagram of Figure 3-3 outlines system (6a). The enclosed area symbolizes the complete set of arcs linking the dependent variables.

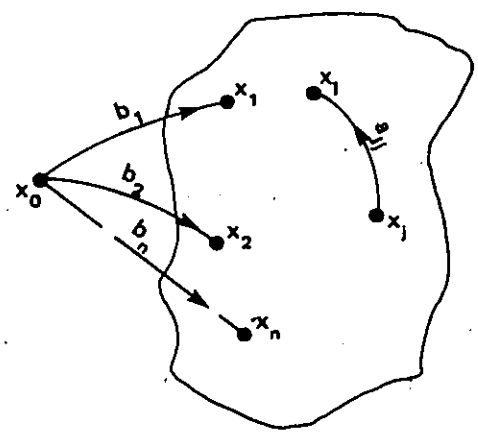


Figure 3 - 3 Generalized Flowgraph Diagram

As this second section of the chapter deals with the derivation of the formulae, flow-graph terminology of system properties will be used, namely, system gain, transmission gain, and sensitivity. However, in the third section of the chapter where the geographical meaning of these properties will be examined, terms more appropriate to urban systems will be suggested.

Before proceeding to the derivation of the formulae it would likely be useful to outline in diagrammatic form the specific manner in which each of the three flow-graph properties helps to describe a system's operational structure (Figures 3-4A, 3-4B, 3-4C).

System gain (Figure 3-4A) is the equivalent of reducing the whole graph to a single arc extended between the source-node x_o and the study node x_s whose system gain is being measured. In effect, system gain then compares the output generated at x_s with the output of the source-node.

Transmission gain (Figure 3-4B) partitions the system into sub-systems and measures their ability to transmit activity from the major node to the secondary node.

Suppose that the coefficients C_{10}, C_{20}, ..., C_{no} of Figure 3-4B were equal in value and yet twice as much of the system gain at x_s was transmitted through sub-network T.G.$_1$ than through any other sub-network. This would indicate that the bonds between x_1 and x_s through T.G.$_1$ were twice as strong as any other sub-network, that is the transmission gain, T.G.$_1$, would be responsible for most of the high system gain at x_s. Close study of the transmission gains reveals much about the internal structure of a system.

Transmission gain sub-networks are not as simple to outline as Figure 3-4B seems to imply. Each transmission gain is one element of the inverse matrix in equation (11). (This will be demonstrated in detail later.) The T.G.$_1$, T.G.$_2$ and T.G.$_n$ sub-networks may have, and are likely to have, numerous nodes in common, a point which is not conveyed by Figure 3-4B because the diagram is a greatly over-simplified representation of network structure. We can say that the transmission gain T.G.$_1$, represented by a single value of the inverse matrix, measures the capacity of the sub-network between x_1 and x_s of Figure 3-4B to transmit activity generated at C_{10} to x_s through the intervening network T.G.$_1$. The diagram is an over-simplification but

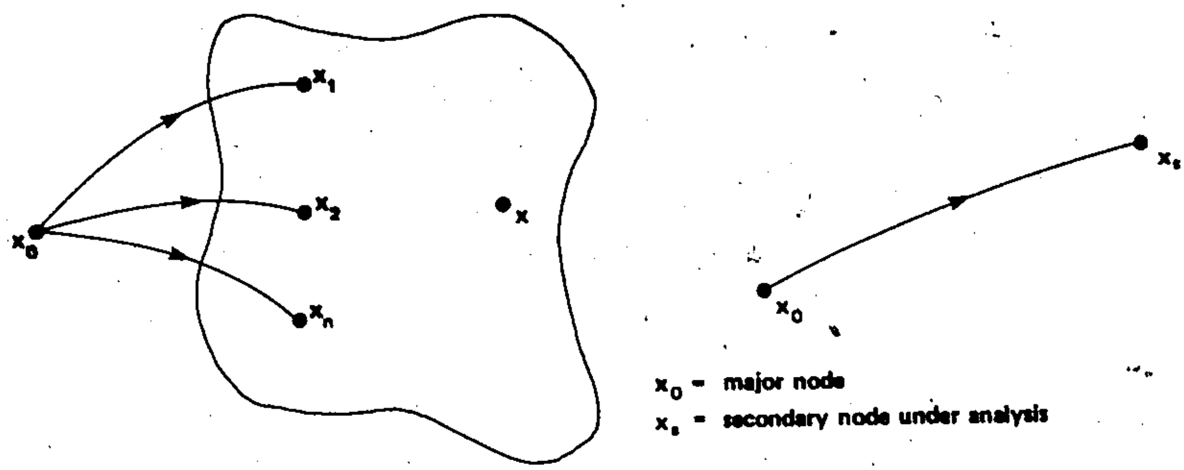


Figure 3 - 4A : System Gain

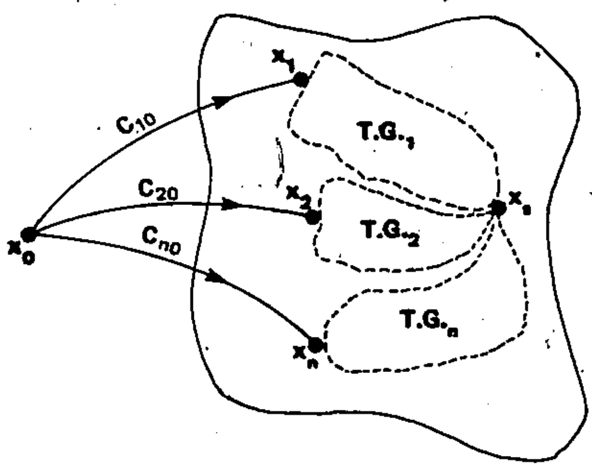


Figure 3 - 4B : Tramsmission Gain

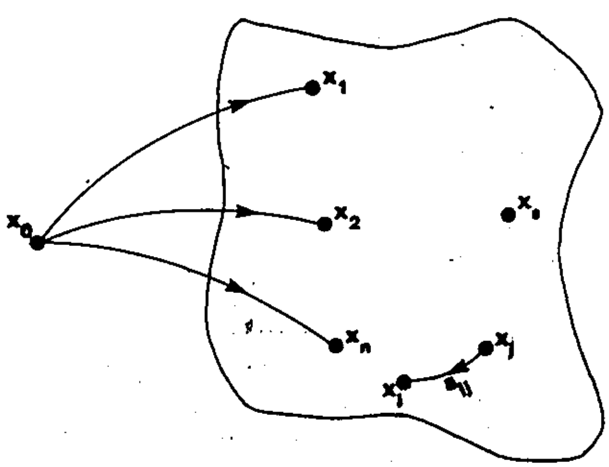


Figure 3 - 4C : Sensitivity

the fact remains that $T.G._1$ measures the operational structure of the subsystem linking x_1 and x_s.

To measure the influence any individual arc might have on the secondary node, flow-graph theorists have developed the sensitivity measure (Figure 3-4C). Sensitivity measures the change in system gain for any change occurring on an arc. A node which is insensitive to a majority of the arcs would be poorly integrated in the system while a node which is highly sensitive to most of the arcs must have an extensive network. Sensitivity carries systems analysis down to the level of the individual arc. The three measures of system gain, transmission gain and sensitivity provide considerable knowledge about the operation of a system.

System Gain:

In flow-graph theory system gain for node x_i with respect to node x_o is defined as:

(7) $$g_i = \frac{x_i}{x_0}$$

System gain establishes the relative importance of the dependent node x_i to the independent node x_o.

The equations of system (5b) can be written as (8) if the definition of (7) is taken into account:

(8)
$$a_{11}\frac{x_1}{x_0} + a_{12}\frac{x_2}{x_0} + \ldots + a_{1n}\frac{x_n}{x_0} = b_1$$

$$a_{21}\frac{x_1}{x_0} + a_{22}\frac{x_2}{x_0} + \ldots + a_{2n}\frac{x_n}{x_0} = b_2$$

....

$$a_{n1}\frac{x_1}{x_0} + a_{n2}\frac{x_2}{x_0} + \ldots + a_{nn}\frac{x_n}{x_0} = b_n$$

or again as (9):

$$
(9)\quad
\begin{aligned}
&a_{11}g_1 + a_{12}g_2 + \dots + a_{1n}g_n = b_1 \\
&a_{21}g_1 + a_{22}g_2 + \dots + a_{2n}g_n = b_2 \\
&\dots\dots \\
&a_{n1}g_1 + a_{n2}g_2 + \dots + a_{nn}g_n = b_n
\end{aligned}
$$

and in matrix algebra as (10):

$$(10)\qquad [A][G]=[B]$$

The vector of system gains for the dependent nodes is obtained by solving for G as in (11):

$$(11)\qquad [G]=[A^{-1}][B]$$

In equations (10) and (11) G and B are vectors, A and A^{-1} are matrices.

In the analysis of a system's operation it is usually more helpful to solve for the system gain vector G of (11) than it is to work out the actual flow values of vector X of (6a) because the G vector describes the performance of the nodes relative to the major node for as long as the system's structure remains unchanged, whereas the X vector simply states the volume of flows at each node, given a matrix of node interaction. Equation (11) requires one equation less than equation (6a) since the division by x_o in (11) is equivalent to attributing a value of 1 to x_o, $\frac{x_o}{x_o} = 1$. Since the system presented in (11) is a flow-graph system of equations it is necessarily a homogeneous system. One variable must be chosen as the source-node and given a value of 1 otherwise the system remains unsolvable.

Although system gain is but a ratio of two outputs, flow-graph theory provides a means of determining how the gain is influenced by each of the other elements in the system. The flow-graph model brings out the successive dependencies and interdependencies in the system and it becomes possible to follow the path of the interactions generating a node's system gain.

Transmission Gain:

Equation (11) shows that system gains at the nodes result from the matrix multiplication of the A^{-1} matrix by the B vector, operation of equation (11). Using summation notation equation (11) can be written:

(12) $$g_i = \sum_{j=1}^{n} [A^{-1}]_{ij} \cdot b_j$$

Since $b_j = -C_{jo}$ Equations (5b) and (12) can be expressed as (13) and (13a) if A^{-1} and $-C$ are both multiplied by -1:

(13) $$g_i = \sum_{j=1}^{n} [-A^{-1}]_{ij} \cdot c_j$$

(13a) $$[G] = [-A^{-1}]\ [C]$$

In equation (13) system gain is expressed in terms of the x_o coefficients and in terms of the $-A^{-1}$ matrix. The matrix contains the actual transmission gains for all the sub-networks in the system. The value a_{ij} in the $-A^{-1}$ matrix multiplies the corresponding C_{jo} coefficient in the calculation of system gain at node x_i. System gain at x_i is simply the dot product of row i of the matrix by the C vector. Thus the magnitude of the gain registered at x_i is a function of two factors: the coefficients in the C vector and values in the i-row of the $-A^{-1}$ matrix. The C coefficients measure the relative importance of the flows a node sends to the major node. The matrix coefficients weigh the bonds between the secondary node x_s and the other nodes.

Thus if all the coefficients C_{jo} were equal in value, the system gain at the secondary node x^i would depend entirely on the a_{ij} values of the $-A^{-1}$ matrix. Thus the a_{ij} values measure the strengths of the sub-networks connecting the secondary node x_s to each of the other dependent nodes. Each value in the $-A^{-1}$ matrix represents one sub-network. The value itself summarizes in a single number the strength of the sub-network in transmitting activity from the source node to the secondary node through a given sub-set of linkages.

Determining which of a sub-network's arcs are most influential in generating feedback effects presents a serious problem. Knowing that the transmission gain is one single value of the inverse matrix is not very helpful in tracing interaction through a network. Even for a small 3 x 3 matrix, evaluating which of the arcs influence a given value of an inverse matrix is difficult as Figure 3-5 illustrates.

The inverse of a 3x3 matrix:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

then the determinant of A = $\left[(aei + bfg + cdh) - (gec + hfa + idb) \right]$

and the inverse of A is:

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} (ei - hf) & -(bi - hc) & (bf - ec) \\ -(di - gh) & (ai - gc) & -(af - dc) \\ (dh - ge) & -(ah - gb) & (ae - db) \end{bmatrix}$$

Figure 3 - 5 The Sub-Network Structure and the Inverse Matrix

There is another manner by which this problem can be tackled and that is by the gradual reduction of the graph. Mason divised two rules by which a graph can be reduced to a single arc. If at each step the new sub-network structures are measured it becomes feasible to estimate which arcs in a given sub-network are most effective in transmitting interaction. Mason's first rule is one of addition of arcs. When two arcs are in parallel and oriented in the same way, they can be replaced by a single arc bearing a coefficient equal to the sum of the coefficients of the two arcs it replaces. His second rule is one of multiplication. When two arcs are in series, they may be replaced by a single one carrying a coefficient equal to the product of the two separate coefficients. Note that in this case, one node will be eliminated. To illustrate this last point assume that a region A's exports are directly dependent on the inputs into region B from region C, the multiplication rule will evaluate the total impact of regions B and C on region A. By repeatedly applying both rules of reduction, it becomes possible to estimate the strongest links in a sub-network.

A third approach to the problem of identifying critical arcs in the feedback systems consists in a close examination of the sensitivity characteristics of nodes. If the sensitivity of a node is calculated with respect to every arc in the system it is then possible to determine the most influential arcs of the feedback sub-networks. It is to this problem that we now turn.

Sensitivity:

System sensitivity measures the change in system gain at a node x_1 for any change in the arc coefficient linking nodes x_k and x_1.

$$(14) \qquad {}_{l}S_{kl} = \frac{\partial [G_l]}{\partial a_{kl}}$$

However, sensitivity is more often expressed in relative terms such that it measures the relative change in system gain at x_i for a relative change in the arc coefficient, and (14) becomes (15):

$$(15) \qquad {}_iS_{kl} = \frac{\partial[G_i]}{\partial a_{kl}} \cdot \frac{a_{kl}}{g_i} = \frac{\partial[G_i]}{g_i} \bigg/ \frac{\partial a_{kl}}{a_{kl}}$$

The notation ${}_iS_{kl}$ refers to the sensitivity at node x_i for any fluctuations on the arc joining x_k to x_l; The symbol ∂ refers to the partial differentiation of the vectors. Since the derivation of the formula of sensitivity involves the derivatives of matrices and vectors, the derivation of the formula will be presented in detail in the appendix (A-30 to A-38). Formula (15) defines sensitivity in general terms. In practice other formulas derived from it are used. If sensitivity of a node is to be measured with respect to an arc linked to the source-node, formulas (16) and (16a) are used. The sensitivity of node x_i to change on the arc linking a node x_k to the source-node is, in absolute terms:

$$(16) \qquad \frac{\partial g_i}{\partial c_k} = [-A^{-1}]_{ik}$$

and in relative terms:

$$(16a) \qquad {}_iS_{k\,o} = [-A^{-1}]_{ik} \frac{c_k}{g_i}$$

Sensitivity is given in terms of the inverse matrix, the coefficient C_k on the source-node arc, and the system gain at node x_i.

If the sensitivity of node x_i is to be measured with respect to any arc other than those originating in the source-node formulas (17) and (17a) are used. The sensitivity of node x_i to change on an arc going from node x_k to node x_l is, in absolute terms:

$$(17) \quad \frac{\partial g_i}{\partial a_{kl}} = -g_l [A^{-1}]_{ik}$$

and in relative terms:

$$(17a) \quad {}_iS_{kl} = [-A^{-1}]_{ik} \cdot g_l \frac{a_{kl}}{g_i}$$

Sensitivity is here expressed in terms of the inverse matrix, the system gains at the nodes x_i and x_l and the coefficient of the arc between nodes x_k and x_l. For any given node, there is a matrix of sensitivity values, one for each arc in the system.

Sensitivity analysis of flow-graph theory corresponds to elasticity analysis of economic theory. It can be particularly useful for a number of purposes. Thus in input-output or interregional analysis, sensitivity indicates the response of one sector or one region to variations of the intersectoral or interregional coefficients. Sensitivity can serve to distinguish between stable and non-stable elements in the system. It also provides a means of differentiating between the central and peripheral nodes. Finally, as noted earlier in the discussion on transmission gain, sensitivity analysis can be used to identify the critical arcs in the sub-networks of the system.

There are two other aspects of system analysis which have not been covered in flow-graph theory but which have interesting applications. The first is the calculation of an Index of Total System Sensitivity (Lamarche et Francon, 1975) and the second is the development of a technique for evaluating system performance, a technique which has been developed by the author in the hope that it might have some practical utility.

An Index of Total System Sensitivity:

The concept of system sensitivity described in the preceding paragraphs applies to a particular node for fluctuations in a given single arc. However, it is useful to define an index of sensitivity which measures a node's over-all sensitivity to changes on all the arcs: the Index of Total System Sensitivity for node x_i:

$$(18) \qquad I_i = \left[\frac{1}{n} \sum_{k=1}^{n} \left(\frac{\partial g_i}{\partial c_k} \right)^2 + \frac{1}{m} \sum_{k=1}^{n} \sum_{l=1}^{n} \left(\frac{\partial g_i}{\partial a_{kl}} \right)^2 \right]^{1/2}$$

where n equals the number of arcs tied to the source node and m equals the number of arcs binding the other nodes.

The index is in effect a measure of the average sensitivity of a node to fluctuations in the arc coefficients. Note that the index gives sensitivity in absolute values, having been derived from equations (16) and (17). For medium or large networks a high index value suggests that the node is sensitive to fluctuations on a large number of arcs. A low index value signifies that the node is isolated with respect to most of the nodes within the system. Even though an isolated node might have a few strong ties, its overall average index value will be low because of the many weak ties it has. The evolution of the index values over a prolonged period is a clear indication of a node's ability or inability to develop intra-network ties.

The chief advantage of this index over other well known measures of centrality and dispersion lies in that first it is truly a system characteristic and secondly, it is founded on the principle of sensitivity rather than on distance between the nodes in a system.

For any given node x_i, the role of row i of the $-A^{-1}$ matrix is essential in the calculation of the node's system gain (Equation 13), of its average transmission gain, and of its index of total system sensitivity (Equation 16, 17, 18). It is not surprising then that the rankings of the nodes on all three measures might be similar since the largest networks tend to have the best developed sub-networks. However, the mean value of transmission gain of the sub-networks (a measure that will be discussed further in Chapter Four) and the Index of Total System Sensitivity are not introduced as additional ranking measures but for other purposes. The mean value of transmission gains of a network along with other related descriptive statistics are used to study the distribution curve of the strengths of its various sub-networks, and the index sensitivity values serves no other purpose but to indicate if nodes are sensitive to a wide diversity of arcs or if their sensitivity is limited to a few arcs only.

Figure 3-6 summarized the principal characteristics of the system properties discussed in the preceding paragraphs.

Evaluating System Performance, A Performance Index:

If measuring system properties is of prime importance in urban and regional research, in the end it is the ability to evaluate a system's performance which really matters to planners and decision-makers. The key to the understanding of a system's reaction to policy implementation lies in performance analysis. In planning new urban development policies it is important to determine whether or not the existing system sub-networks are being used to their full capacity. Policies aimed at strongly developed under-utilized networks are likely to have greater impact than those directed at networks of restricted development. Performance and sensitivity analyses

Figure 3-6 . continued

SYSTEM PROPERTIES

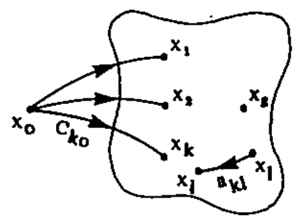
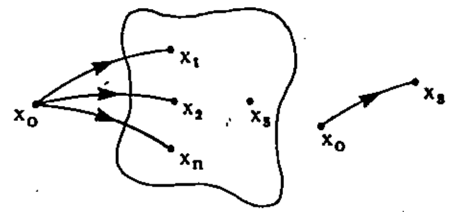
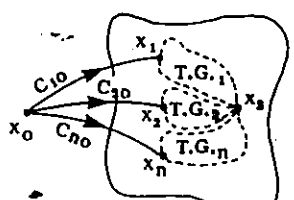
	FLOWGRAPH DIAGRAM *	MATHEMATICAL DEFINITION	GEOGRAPHICAL MEANING
C) System Sensitivity	 System sensitivity measures the change in system gain at any node x_s for any fluctuation in activity on any arc in the system. Sensitivity can be expressed in absolute or relative terms.	A) Relative System Sensitivity to fluctuations on the arcs linked to the source-node x_o: $${}_sS_{ko} = [-A^{-1}]_{sk} \cdot \frac{c_{ko}}{g_s}$$ B) Relative System Sensitivity to fluctuations on the arcs between dependent nodes: $${}_sS_{il} = [-A^{-1}]_{si} \cdot g_l\ \frac{a_{il}}{g_s}$$ S : sensitivity s : the node whose system gain is influenced by arc fluctuations k,i : the nodes emitting the flows, the nodes to which the arrows are pointing l,o : the node receiving the flows c_{ko} : the arc linking the source-node x_o to node x_k g_s : system gain at node x_s g_l : system gain at node x_l a_{kl}: the arc extending from node x_l to node x_k $[-A^{-1}]_{sk}$: row s column k of the transmission gain matrix	Since system sensitivity measures the sensitivity of a node's interaction ratio to fluctuations on the system's arcs it can be labelled as Interactance Sensitivity.
D) Index of Total System Sensitivity		$$I_l = \left(\frac{1}{n} \sum_{j=1}^{n} \left(\frac{\partial g_l}{\partial c_{ko}}\right)^2 + \frac{1}{m} \sum_{k=1}^{n} \sum_{l=1}^{n} \left(\frac{\partial g_l}{\partial a_{kl}} \right)^2 \right)^{\frac{1}{2}}$$ The equation calculates the average sensitivity in absolute terms of node x_l to fluctuations on all the arcs in the system. I_l : Index of Total System Sensitivity for node x_l n : the number of nodes in the system m : the number of arcs extended between the dependent nodes g_l : the system gain at node x_l c_{ko}: the arcs extended from source-node x_o to node x_k a_{kl}: the arc extended from node x_l to node x_k ∂ : the partial differentiation symbol	A high index value indicates strong node interdependency and a low value points to a weakly developed network. This is particularly true if the system is a large one. A node with a low index value might have one or two strong ties but its overall average value will be low. The Index of Total System Sensitivity should be analyzed in conjuction with System Sensitivity measures of section C above.

Figure 3-6A **SYSTEM PROPERTIES**

	FLOWGRAPH DIAGRAM *	MATHEMATICAL DEFINITION	GEOGRAPHICAL MEANING
A) System Gain	 In flowgraph theory, the calculation of system gain at a node x_s is the equivalent of reducing the whole graph to a single arc extended between the source-node x_o and the study node x_s. The algebraic rules governing graph reduction are developed in Mason (1953).	$g_s = \frac{x_s}{x_o}$ The system of equations is solved for all values of x. System gain for a dependent node x_s is defined as the ratio x_s/x_o.	$g_s = \frac{\text{Total Flows out of Dependent Node } x_s}{\text{Total Flows out of Independent Node } x_o}$ Reducing the flowgraph to a single arc $x_o \rightarrow x_s$, or determining the ratio x_s/x_o is in effect comparing the total flows of the dependent and independent nodes. Thus system gain can properly be called an Interactance Ratio.
B) Transmission Gain	 Activity on the arc C_{10} linking the source-node x_o to x_1 can have an impact on the study node x_s only through the network linking x_1 to x_s. In the diagram this intervening network is symbolized by T.G.. Thus transmission gains measure the strength of networks intervening between the dependent nodes in a system. The T.G. networks can have, and are likely to have, many nodes in common even though the diagram seems to imply complete separation of the T.G. networks. The diagram is meant to represent nothing more than the fact that x_s and x_o are tied by an intervening network T.G. of known strength.	Transmission gains are the values in the inverse matrix $[-A^{-1}]$ used to calculate system gains at the dependent nodes. Since system gain is defined as: $[G] = [-A^{-1}]\,[C]$ where G is the vector of system gains for all the dependent nodes, C, the vector of coefficients for all the arcs tied to the source-node x_o, it follows that the strength of each of the networks linking the dependent nodes is represented by a single value of the matrix $[-A^{-1}]$.	Transmission gain is an interactance multiplier coefficient. It describes how effectively activity on an arc linking the source-node x_o and dependent node x_i can be amplified at the study node x_s through the network linking x_i and x_s. Thus transmission gain defines an Interactance Transmission Multiplier.

* *a) By signal flowgraph convention, the arrows point in the direction of the on-coming flows.*

b) Any node in the system can be taken as the system's source-node, but there can be no more than one source-node at any given time. This follows from the fact that all the system characteristics are defined in terms of ratios of the dependent nodes to the one source-node.

c) The system is an open one as there only n-1 equations for n nodes. The system is solved by assigning a value of 1 to the source-node's total output.

d) The system describes the dependent node's internodal activity in terms of the source-node's total output. Thus the diagram may have arcs showing the origins and destinations of all the flows of the dependent nodes but it cannot have arcs indicating the individual outputs from the source-node.

together should help in determining the areas where the greatest beneficial effects are likely to accur. This becomes especially significant when investment funds are limited.

This thesis provides a technique for measuring system performance. The technique is simple enough in that it rests on the comparison of two system properties presented earlier, System Gain and Transmission Gain.

It will be recalled that the transmission gain is a measure of the internal sub-networks of a system (Figure 3-4B).

Assume for a moment that the arc coefficients C_{10}, C_{20}, ..., C_{no} of Figure 3-4B are all equal to 1, then the share of the study node's output transmitted through each of the internal networks will be proportional to their transmission gains. The relative shares of the transmission gains measure the importance of the different sub-networks that link the secondary node to the major node. However, it is highly unlikely that all the arcs emanating from the major node have coefficients of equal value.

The technique of evaluating a system's performance consists in comparing the actual share of system gain carried by a sub-network to a node with the relative importance of the sub-network in the system. The relative value of a sub-network is simply its transmission gain expressed as a percentage of the total vector of transmission gains. The actual share of total system gain carried by each sub-network is calculated quite easily. Since the system gain at node x_s is the product of the Transmission gain vector by the C vector of arc coefficients, the percentage of system gain at node x_s attributable to each sub-network is:

(%S.G.). % of System Gain through $T.G._{i} = \frac{(T.G._{i})(C_{io})}{\text{System Gain at } x_s} \times 100$

$T.G._{i}$ and C_{io} refer to Figures 3-7A and 3-7B.

Let the coefficients C_{10}, C_{20}, and C_{30} of Figure 3-7B be 0.2, 0.4, and 0.8 respectively. Then the proportional shares of system gain through each sub-network T.G. are those of Figure 3-8. Note that the highest share of transmission gain is found in sub-network $T.G._{1}$ and yet the traffic is greater on C_{20} and C_{30}. The % values of the T.G. row vector depict the real network structure, the % values in the C vector of Figure 3-8 show the actual use that is made of each sub-network in generating the system gain at the study node. This is in effect a comparison of a system's operations versus its structure and that is by definition a measure of performance (Figure 3-7).

In the example of Figure 3-8, the performance index values for the three sub-networks are:

$$P.I._{1} = \frac{27.3\%}{50.0\%} = 0.55$$

$$P.I._{2} = \frac{36.4\%}{33.3\%} = 1.09$$

$$P.I._{3} = \frac{36.4\%}{16.7\%} = 2.18$$

A performance index (P.I.) of 1, shows that the structure and its use are in perfect harmony, a P.I. less than 1, indicates that the structure is not being used to its full potential, and a P.I. value greater than 1, shows that there is greater use of the network than the structure warrants. In the example, the structure between X_1 and X_s has the greatest development since its interaction generating ability is equal to 50% of the whole system's, and yet it generates only 27.3% of the activity of X_s because the activity on the arc

$$P.I._i = \frac{\left[(T.G._i)\, C_{io}/G_s\right] \times 100}{\left(T.G._i / \sum_{i=1}^{n} T.G._i\right) \times 100}$$

$P.I._i$ = Performance index of sub-network i.

$T.G._i$ = Transmission gain of sub-network i.

G_s = System gain at node x_s.

C_{io} = Arc coefficient linking major node x_o to node x_i.

Figure 3-7 System Performance Index

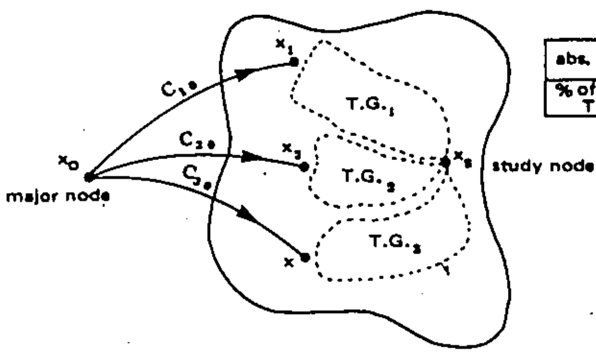


Figure 3-7A System Performance (A)

	T.G.1	T.G.2	T.G.3
abs. value	3	2	1
% of total T.G.	50.0	33.3	16.7

[T.G.]

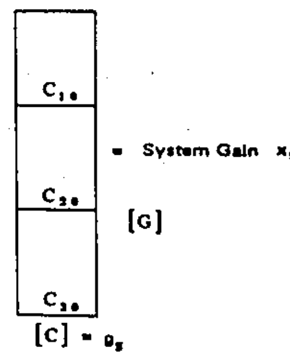
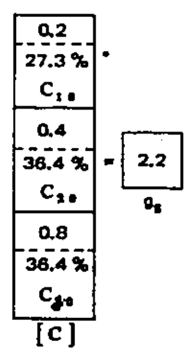


Figure 3-7B System Performance (B)

	T.G.1	T.G.2	T.G.3
abs. value	3	2	1
% of total T.G.	50.0	33.3	16.7

[T.G.]



* The % values in the C vector indicate the % of total system gain transmitted through C_{io} and $T.G._i$ as calculated by equation of Figure 3-7.

$$P.I.^*_1 = \frac{27.3\%}{50.0\%} = 0.55$$

$$P.I._2 = \frac{36.4\%}{33.3\%} = 1.09$$

$$P.I._3 = \frac{36.4\%}{16.7\%} = 2.18$$

*Performance Indicies of Sub-networks $T.G._1$, $T.G._2$, $T.G._3$.

Figure 3-8 Evaluating System Performance

C_{10} is very weak in comparison to that of the other two arcs. Increasing the activity of the arc C_{10}, that is generating more flows from x_1 to x_o, would have rather widespread effects in the general direction of x_s, as this is the system's best internal network with regards to the structure between the major node and the study node.

Again referring to the example, the T.G.$_3$ network has a high P.I. value, 2.18. This implies that the study node's interactions are due mainly to the very strong ties between x_3 and x_o rather than to the T.G.$_3$ intervening network. Increasing the activity on C_{30} would have repercussions in x_s, but the system effects to the other nodes in the sub-networks would be very limited.

As it was demonstrated in the preceding paragraphs, evaluating the performance of different parts of a system is fairly simple once the system's characteristic properties are known. The performance index does not specify which of the arcs are responsible for the strong (or weak) ties between the nodes in question, nor does it give any indication as to the nature of the socioeconomic factors generating the interaction, but it does outline the system's operational features. Once sufficient research has been done in the application of the technique to urban systems, it could become in conjunction with other techniques a useful tool in urban systems research.

Geographical Interpretation of Flow-graph Characteristics:

The three properties of system gain, transmission gain, and sensitivity developed in flow-graph theory and presented in the preceding section can find wide application in urban systems analysis. The only two major factors which would preclude their use would be the unavailability of interaction data, on the one hand, and the low level of interdependency between the elements of some systems on the other.

Because the terms system gain and transmission gain are somewhat misleading as they imply a "gain" while the model is in effect measuring the internal response of the network to a unit output by the major node, Lamarche and Ray (1976) have proposed a new terminology for the three network properties, one which is more descriptive of their nature in the geographical context. In their paper, system gain, transmission gain, and sensitivity are labelled interactance ratio, interactance multiplier, and interactance sensitivity respectively.

The purpose of this section is to redefine these three properties in geographical terms and to illustrate the calculations involved in the application of flow-graph to a small urban network. The calculations are worked out in detail.

In this example, the data matrix represents information flows that have a bearing on the transmission of growth in the system. Person-to-person contacts among high-level administrators are judged to be of this type. The assumptions of the model are taken to be respected. The system is assumed closed with respect to the problem under study. Because the interaction occurs at the decision-making level, it is assumed also that the contacts result in decisions affecting the growth patterns of the organizations thereby influencing the development of the centres in which the firms are located. Since the matrix covers a single time period, linearity presents no particular problem.

The example is based on three cities in Atlantic Canada. Halifax is defined as the source-node since it is the largest of the three. The whole system is studied with respect to a unit output from Halifax. This does not imply that all the growth impulses do or must originate in this centre. The model is designed primarily to measure interdependency rather than hierarchy. (As Pred (1976) has shown in a number of case studies, growth does not filter down according to some hierarchical diffusion process but is transmitted through a network of interdependency relationships). This example has been inserted simply to illustrate the flow-graph technique.

Flow-graph Analysis of Atlantic Canada, 1963:

Figure 3-9 presents the interaction data base for the analysis.

	Halifax	St. John's	Saint John	Total
Halifax, N.S.	0	7465	9650	17,115
St. John's, Nfld.	7640	0	600	8240
Saint John, N.B.	10,200	610	0	10,810

Figure 3 - 9 : Air Passenger Traffic in Atlantic Canada : 1963.

From Figure 3-9 the arc coefficients are calculated in accordance with the linearity hypothesis.

	Halifax	St. John's	Saint John
Halifax	——	——	——
St. John.s	0.446	0	0.056
Saint John	0.596	0.0740	0

Figure 3 - 10 : Arc Cofficient Matrix.

The Graph of the system can be drawn as in Figure 3-11.

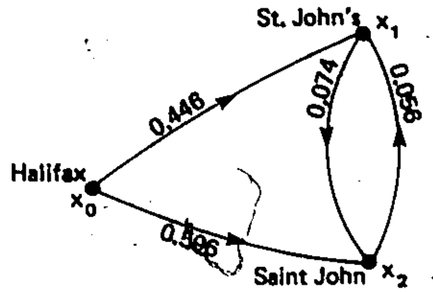


Figure 3-11: Flow - Graph diagram.

The canonical equations of the system are written as in (19):

$$(19)\quad \begin{aligned} x_1 &= 0.446x_0 + 0.056x_2 \\ x_2 &= 0.596x_0 + 0.074x_1 \end{aligned}$$

In (19) each node x_i is described in terms of the flows it emits to the other nodes per unit flow of their own respective outputs.

<u>System Gain = Interactance Ratio</u>:

From (19) isolate x_o on the right:

$$(20)\quad \begin{aligned} -x_1 + 0.056x_2 &= -0.446x_0 \\ 0.074x_1 - x_2 &= -0.596x_0 \end{aligned}$$

Dividing by x_o:

$$(21)\quad \begin{aligned} \frac{-x_1}{x_0} + 0.056\frac{x_2}{x_0} &= -0.446 \\ 0.074\frac{x_1}{x_0} - \frac{x_2}{x_0} &= -0.596 \end{aligned}$$

Since $g_i = \frac{x_i}{x_0}$ (21) becomes (22)

$$(22)\quad \begin{aligned} -g_1 + 0.056g_2 &= -0.446 \\ 0.074g_1 - g_2 &= -0.596 \end{aligned}$$

In matrix notation:

$$[A][G] = [-C]$$

$$\begin{bmatrix} -1 & 0.056 \\ 0.074 & -1 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} -0.446 \\ -0.596 \end{bmatrix}$$

where A is the coefficient matrix, G the gain vector, and C the vector of coefficients of the arcs out of x_o.

Solving for G:

$$G = [A^{-1}][-C]$$

or $$G = [-A^{-1}][C]$$

Since $[A][A^{-1}] = [I]$

$$A^{-1} = \begin{bmatrix} -1.004 & -0.0562 \\ -0.0743 & -1.004 \end{bmatrix}$$

$$[G] = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 1.004 & 0.0562 \\ 0.0743 & 1.004 \end{bmatrix} \begin{bmatrix} 0.446 \\ 0.596 \end{bmatrix} = \begin{bmatrix} 0.481 \\ 0.631 \end{bmatrix}$$

The system gain at g_1 equals 0.481.

But

(23) $$g_1 = \frac{x_1}{x_0} = \frac{\text{St. John's total output}}{\text{Halifax's total output}} = \frac{8{,}240}{17{,}115} = 0.481$$

System Gain at a node is in fact an INTERACTANCE RATIO. It compares a node's output to the major node's output. While the term system gain misleadingly implies some gain, interactance ratio more appropriately describes what is actually being measured. It is obvious from equation (23) that if interactance ratio were the only system property to be measured there would be no need to resort to matrix algebra for its calculation. However, all other system properties and indices of the A.S.T. model require as a first step, the calculation of interactance ratios in this manner.

Transmission Gain = Interactance Multiplier:

Since $[G] = [-A^{-1}][C]$

the Interactance ratio at node x_1 comes from the dot product of the $[-A^{-1}]$ matrix and the $[C]$ vector.

$$\begin{bmatrix} 1.004 & 0.0562 \\ & \end{bmatrix} \begin{bmatrix} 0.446 \\ 0.596 \end{bmatrix} = 0.481$$

The values in the $-A^{-1}$ matrix multiply the interactance coefficients of the arcs tied to the x_o node. The transmission gain values 1.004 and 0.0562 are thus better described as INTERACTANCE MULTIPLIERS. Each interactance multiplier represents the way in which the arc coefficients 0.446 and 0.596 out of the source node x_o are amplified within the system to produce the interactance ratio at node x_i.

Sensitivity = Interactance Sensitivity:

In this example, the sensitivity of the St. John's node will be measured for changes on all the arcs in the system. Since the equations involve the arc coefficients and the interactance ratio, it is appropriate to speak of the INTERACTANCE SENSITIVITY of the node.

Interactance sensitivity to x_o arcs are calculated as in formula (16a):

(16a) $${}_iS_{k,o} = [-A^{-1}]_{ik} \cdot \frac{c_{ko}}{g_i}$$

i: the node whose sensitivity is being measured:

k; the node from which the flow are originating;

o: refers to the source-node;

C_{ko}: the arc linking nodes x_o and x_k;

g_i: system gain or interactance ratio at node x_i.

The interactance sensitivity of St. John's to flow on the Halifax-St.-John's arc is:

$${}_1S_{1,0} = [-A^{-1}]_{11} \cdot \frac{c_{10}}{g_1}$$
$$= 1.004 \cdot \frac{0.446}{0.481} = 0.931$$

The sensitivity of St. John's to flows on the Halifax-Saint John would be:

$${}_1S_{2,0} = [-A^{-1}]_{12} \cdot \frac{c_{20}}{g_1}$$
$$= \left[\begin{array}{c|c} & 0.0562 \\ \hline & \end{array}\right] \frac{0.596}{0.481} = 0.0694$$

Interactance Sensitivity to arcs not linked to the x_o node are calculated as in (17a):

(17a) $${}_iS_{k,l} = [-A^{-1}]_{ik} \cdot g_l \frac{a_{kl}}{g_i}$$

where i refers to the node whose sensitivity is being measured, k refers to the node to which the arc is pointing and l to the node from which the arrow originates.

The interactance sensitivity of St. John's to flows on the St. John's-Saint John arc is given by:

$${}_1S_{2,1} = [-A^{-1}]_{12} \cdot g_1 \frac{a_{21}}{g_1}$$
$$= \left[\begin{array}{c|c} & 0.0562 \\ \hline & \end{array}\right] 0.481 \cdot \frac{0.074}{0.481} = 0.004$$

Figure 3-12 lists all the interactance sensitivities for the St. John's node.

Interactance Sensitivity of St. John's

Halifax – St. John's Arc	0.931
Saint John – St. John's Arc	0.073 *
Halifax – Saint John Arc	0.069
St. John's – Saint John Arc	0.004

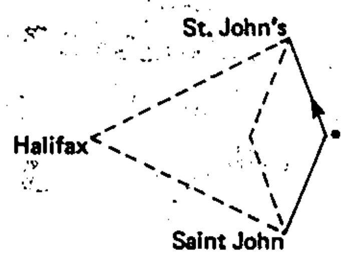


Figure 3 - 12: Interactance Sensitivity.

* The value 0.073 refers to St. John's interactance sensitivity to flows going from St. John's to Saint John, the Saint John - St. John's arc.

Index of Total System Sensitivity:

The index of total system sensitivity is not calculated from the relative values of interactance sensitivities as they are listed in Figure 3-11 for the St. John's node. The index is calculated from the absolute values of interactance ratio and arc coefficients as equation (18) shows:

(18)

$$I_i = \left[\frac{1}{n} \sum_{k=1}^{n} \left(\frac{\partial g_i}{\partial c_k} \right)^2 + \frac{1}{m} \sum_{k=1}^{m} \left(\frac{\partial g_i}{\partial a_{ki}} \right)^2 \right]^{1/2}$$

where n and m stand for the number of arcs involved in each case. Use of the relative instead of absolute values tends to decrease the importance of the arcs carrying the largest flows.

Equations (16) and (17) define the sensitivities in absolute values for node x_i to changes on source-node arcs and non-source node arcs respectively.

$$(16) \qquad \frac{\partial g_i}{\partial c_k} = [-A^{-1}]_{ik}$$

$$(17) \qquad \frac{\partial g_i}{\partial c_k} = -g_i \cdot [A^{-1}]_{ik}$$

In (16) and (17) i identifies the source-node whose sensitivity is being measured, k the node at which the arrow is pointing (the emitter node) and 1 the node from which the arrow originates (the receiving node).

In equation (18) the values are first squared to offset the influence of signs and added. The square root of the sum gives the value of total sensitivity in absolute numbers.

<u>The Index of Total System Sensitivity of St. John's</u>:

St. John's sensitivity to flows on source node arcs: First sensitivity at St. John's to flows between Halifax (x_o) and St. John's (x_1):

$$(T.S._1) \qquad \frac{\partial g_i}{\partial c_k} = \frac{\partial g_i}{\partial c_{ko}} = \frac{\partial g_i}{\partial c_{10}} = [-A^{-1}]_{11} = 1.004$$

($T.S._1$ = Sensitivity to flows in absolute values)

Secondly, sensitivity to flows on arc between Halifax (x_o) and Saint John:

$$\frac{\partial g_1}{\partial c_{20}} = [-A^{-1}]_{12} = 0.0562$$

Thirdly, sensitivity of St. John's to flows coming into St. John's from Saint John:[7]

$$\frac{\partial g_l}{\partial a_{kl}} = -g_l \cdot [A^{-1}]_{lk}$$

$$(T.S._3) \qquad \frac{\partial g_1}{\partial a_{21}} = -g_1 \cdot [A^{-1}]_{12} = (-0.481)(-0.0562) = 0.027$$

Fourthly, sensitivity of St. John's to flows coming into Saint John from St. John's:

$$(T.S._4) \qquad \frac{\partial g_1}{\partial a_{12}} = -g_2 \cdot [A^{-1}]_{11} = (-0.631)(-1.004) = 0.633$$

$$\text{The Index} \quad = \sqrt{\frac{(T.S._1)^2 + (T.S._2)^2}{2} + \frac{(T.S._3)^2 + (T.S._4)^2}{2}}$$

$$= \sqrt{\frac{(1.004)^2 + (0.056)^2}{2} + \frac{(0.027)^2 + (0.633)^2}{2}}$$

$$= \sqrt{0.506 + 0.200}$$

$$I_1 = 0.841$$

The Index of Total System Sensitivity is 0.841. The value has meaning when compared to values of other nodes of the same system or if the value of a single node is calculated for several successive years. Higher values mean greater total sensitivity, therefore, greater extension of the node's network because its interactance ratio is more sensitive to flow changes on a greater number of arcs.

---

[7] The k in A_{kl} refers to the node to which the arrow is pointing, the node emitting the flows.

System Performance of the Atlantic Network:

System performance evaluates system use versus system structure in a network's sub-systems.

In the Atlantic network St. John's interactance ratio is dependent on two sub-systems as Figure 3-13 illustrates. I.R. defines the interactance ratio or system gain; I.M., the interactance multiplier or transmission gain; and C_{io}, an arc from node x_i to the source-node.

$$[-A^{-1}][C] = [G]$$

1.004	0.056
Interactance Multiplier No. 1	I.M.$_2$

0.446
C_{10}
0.596
C_{20}

= 0.481 I.R.

C_{10}, C_{20} are the arcs tied to the source node.

Figure 3-13 : Interactance Ratio of St. John's.

Since the total value of interactance multipliers in Figure 3-13 is 1.078, the relative interactance multipliers can be written as:

94.7%	5.3%
I.M.$_1$	I.M.$_2$

If the arc coefficients C_{10} and C_{20} were equal in value, the I.M.$_1$, or the first sub-system would carry 94.7% of the interactions registered by St. John's. However, the actual percentage of interactance ratio actually carried by the first sub-network is calculated as:

$$\frac{(\text{I.M.}_1)(C_{10})}{\text{I.R.}} \times 100\% = \frac{(1.004)(0.446)}{0.481} \times 100\% = 93.01\%$$

Since the performance index is:

$$\text{P.I.} = \frac{\text{Use}}{\text{Structure}} = \frac{\dfrac{(\text{I.M.}_1)\,(C_{10})\%}{\text{I.R.}}}{\text{I.M.}_1\,\%} = \frac{93.01\%}{94.7\%} = 0.98$$

Thus structure and operational use are in harmony since the P.I. is approximately equal to 1.

Obviously this is not the best example to illustrate the concept of system performance. Better examples are provided later on in the analysis of the Canadian Metropolitan System. An interesting case occurs in the Quebec City system where Quebec shows a low system performance because so much of its interactance ratio is related to its Montreal arc flows in spite of the complex networks that bind Quebec to the other Canadian metropolitan centres.

Conclusion:

Urban systems like any other system have operational characteristics. The A.S.T. model provides a technique for measuring some of these characteristics. The assumptions of interdependency and linearity on which the model is based are important and must not be overlooked.

The validity of the model will be tested in Chapters Four and Five where the A.S.T. technique will be applied to the analysis of intermotropolitan air-passenger traffic and intermetropolitan migrations. These analyses will indicate more concretely the model's conceptual and potential practical contributions to urban research: conceptually by providing a new theoretical framework for the analysis of urban systems, and practically by introducing a technique for the measurement of system properties and performance. At this stage in the development of the A.S.T. model, it is possible that the values provided in these analyses are of limited use as further research must be done before the hypotheses on which the model is based can be fully tested. However, the A.S.T. model as it is presented here measures the order of magnitude of the direct and indirect system interactions and thereby outlines an urban system's operational structure in its essential features. By simulating changes in the interaction patterns, the A.S.T. model could be used also as a simulation model. However, no work has been undertaken in this direction as yet.

CHAPTER 4

INTERMETROPOLITAN AIR-PASSENGER FLOWS: A.S.T. APPLIED

Introduction:

In this chapter the A.S.T. model is applied to the Canadian national and regional metropolitan networks. The purpose is to measure system properties of the networks and to reveal the direction and pace of their evolution through time. The interactance ratios, multipliers, and sensitivities are precise measurements of inter-urban activity and help reveal the nature and the form of inter-urban bonds. Changes in these characteristics from 1963 to 1973 will show some features of the evolution of Canada's metropolitan system which other techniques of interaction analysis cannot reveal.

The interactance ratios show the relative growth of the activity of the dependent nodes to that of the main node. Of particular interest are the situations of those centres that have sustained either a steady increase or decrease in interactance ratios over the period, reflecting differential population growth.

The interactance multipliers describe the activity on the sub-networks linking the study node to all the other dependent nodes. The increase in value of an interactance multiplier indicates either an actual expansion of a sub-network to include more nodes or an intensification of activity on the existing structure. Note that the first case does not apply in the present context since all the centres in the Canadian Metropolitan System are already linked directly to each other as the origin-destination matrices show. The interactance multipliers do not specify whether the increase in activity is occurring throughout the whole sub-network or is limited to a few arcs. This is a severe limitation of interactance multiplier analysis. If, for a given study node, an increase over time in the average value of its interactance multipliers is accompanied by a decrease in the standard deviation, it can be assumed that the node has experienced some degree of network development since it is now less dependent on just a few of its sub-networks for its internodal activity, the activity being more evenly distributed over the set of networks. It is this kind of network development which is of interest in this chapter.

Of the three system properties, sensitivity is the most useful in determining if system change is occurring or not. Because it is a relative measure, it establishes a ratio of change in nodal interaction per unit change in arc activity. Thus, sensitivity is unaffected by the factor of the size of the interacting urban centres. A careful examination of the sensitivity values through time will indicate if the status of a centre within the system is changing. If a centre becomes increasingly more sensitive to only a few arcs in its immediate surroundings the centre is becoming more isolated from the rest of the network. If a centre shows greater sensitivity to a greater number of arcs outside its immediate environment as well as within it, it is a sign of greater diversity of activity within the system, e.g. the case of Ottawa.

The Index of Total System Sensitivity also provides useful indications concerning system change. Since it defines an average value of sensitivity, any substantial increase in the index value reveals some measure of network development. Thus in a completely interconnected system of 18 nodes with over 300 arcs such as the Canadian Metropolitan System, the index value will not change substantially if a node has experienced greater sensitivity to only a few additional nodes. There must be greater sensitivity to a fairly large number of arcs before the index shows a significantly higher value.

The A.S.T. analyses will be performed on the set of eleven matrices reporting the intermetropolitan air-passenger flows for the 1963-1973 period. The results of these analyses should be viewed within the limitations of the data as noted in Chapter Two. Of course, the changes in system patterns revealed by this set of data might not be the same if the model were applied to other types of flows, and one problem with the use of air traffic flows is that they are not a good measure of interaction when two centres are rather close together. In such a situation it can be assumed that some of the flows are diverted to other forms of transportation, so the results are likely to be somewhat distorted. Nevertheless, it is felt, despite these deficiencies, that the analyses provide a good deal of insight into inter-regional linkages and reveal several interesting changes in the Canadian Metropolitan System during the 1963-1973 period.

Chapter Four consists of four parts. The national system of the eighteen Canadian metropolitan centres having a major airport is analyzed in the first section. The second section deals with a sub-system of eleven of the above metropolitan centres located in the East and forming the Eastern Sub-System. The Western Sub-System, consisting of seven cities, is studied in the third part. The last section of this chapter is an evaluation of the performance of the National System. The performance indices of the various sub-networks within the national system will be presented and analyzed.

Part I: The National System 1963-1973:

Toronto is assumed to be the source-node of the national system since it has the highest intermetropolitan air-passenger traffic in Canada (Tables A4-67 to A4-77 of the Appendix). The complete list of centres in the national system appears in Table 4-1.

Tables 4-1 and 4-2 summarize the changes in the system parameters over the 1963-1973 period. A number of observations regarding the behaviour of the system in general can be drawn from these tables. Firstly, on the average, percentage growth in flows has been over ten times greater than population growth. Secondly, growth in interactance ratio, that is flow growth in relation to Toronto's output, has not been equally strong for all the centres. The combination of a low average (19.1%) with a high standard deviation (37%) suggests the existence of two sub-groups within the system. Sudbury, Calgary, Thunder Bay, Vancouver, and Edmonton show great increases in interactance ratios while Quebec, Windsor, and Saint John have sustained losses. Thirdly, the same observations hold true for the interactance multipliers and indices of system sensitivity. In both cases a number of centres have experienced strong growth, e.g. Ottawa-Hull, Calgary, and Edmonton while other have shown losses, e.g. Quebec, Montreal, and Saint John. Fourthly, the performance indices show an average loss of -3.6% and a standard deviation of 22.3%. For a number of centres such as Calgary, Edmonton, Vancouver, and Sudbury, the decrease in performance value is largely attributable to higher values of interactance multipliers. The better performance values reported for Windsor and Quebec result from losses in interactance ratio.

The correlation coefficients of table 4-3 also warrant commentary. The correlation between growth in population and growth in flows has a value of only 0.43. While some centres have experienced pronounced growth on both variables, e.g. Calgary and Edmonton, and other centres have had low increases on both variables, e.g. Windsor and Saint John, there is a third group of

Table 4-1 : System Parameters of the Cities for 1963 and 1973.

(* Cities are ranked according to total outbound flows in 1963)

RANK*	CITY	INTER METROPOLITAN TOTAL OUTBOUND FLOWS		INTERACTANCE RATIO		AVERAGE INTERACTANCE MULTIPLIER		INDEX OF TOTAL SYSTEM SENSITIVITY		AVERAGE PERFORMANCE INDEX	
		1963	1973	1963	1973	1963	1973	1963	1973	1963	1973
1	Toronto	466,645	1,406,215	–	–	–	–	–	–	–	–
2	Montreal	341,485	852,575	0.731	0.606	0.441	0.394	0.511	0.453	0.62	0.66
3	Vancouver	145,170	636,005	0.311	0.452	0.395	0.465	0.535	0.570	1.31	1.05
4	Winnipeg	120,780	402,865	0.258	0.286	0.290	0.299	0.393	0.392	1.16	1.06
5	Ottawa-Hull	119,720	469,640	0.256	0.334	0.188	0.265	0.290	0.344	0.83	0.81
6	Calgary	105,890	520,485	0.226	0.370	0.325	0.430	0.468	0.558	1.48	1.18
7	Edmonton	101,885	456,870	0.218	0.324	0.308	0.381	0.466	0.508	1.46	1.19
8	Halifax	68,065	231,660	0.145	0.164	0.184	0.182	0.329	0.310	1.19	1.12
9	Quebec	67,205	122,010	0.144	0.086	0.141	0.111	0.286	0.262	1.00	1.30
10	Windsor	57,345	114,960	0.122	0.081	0.102	0.099	0.251	0.252	0.91	1.21
11	Regina	44,780	152,540	0.096	0.108	0.163	0.157	0.303	0.292	1.76	1.47
12	Victoria	37,725	123,975	0.080	0.088	0.148	0.144	0.302	0.286	1.90	1.66
13	Saskatoon	29,435	122,610	0.063	0.087	0.125	0.136	0.274	0.279	2.04	1.59
14	Thunder Bay	29,010	107,400	0.049	0.076	0.089	0.095	0.250	0.251	0.88	1.26
15	London	28,095	85,385	0.060	0.060	0.102	0.105	0.254	0.257	1.72	1.76
16	Saint John	25,315	60,145	0.054	0.042	0.104	0.089	0.271	0.255	1.98	2.11
17	St. John's	25,200	88,415	0.054	0.062	0.097	0.100	0.262	0.260	1.86	1.61
18	Sudbury	10,145	60,265	0.021	0.042	0.063	0.072	0.243	0.244	2.93	1.66

Table 4 - 2 : Growth in System Parameters and Other Attributes.

– Cities are ranked as in table 4 - 1

– Parameter Growth = $\frac{P_{1973} - P_{1963}}{1963} \times 100$

← 1961 population adjusted for 1971 C.M.A. boundaries
source: Ray et al. (1976), Canadian Urban Trends, Vol. 1, Toronto; Copp Clark Pub., p.30.

– Population Growth = $\frac{Pop_{1971} - Pop_{1961}}{Pop_{1961}} \times 100$

– Accessibility Index: $I = \frac{\sum_{j}^{n} d_{ij}}{\sum_{i}^{n}\sum_{j}^{n} d_{ij}}$ $i, j = 1, 18$

d_{ij} = distance between centres i and j as given in Table A5 - 18.

RANK	CITY	POPULATION		PERCENTAGE GROWTH						ACCESSIBILITY INDEX
		1961	1971	Population	Flows	Interactance Ratio	Average Interactance Multiplier	Index of Total System Sensitivity	Average Performance Index	
1	Toronto	1,919,409	2,628,043	36.9	201	–	–	–	–	0.045
2	Montreal	2,215,627	2,743,208	23.8	150	-17.1	-10.7	-11.4	6.5	0.047
3	Vancouver	826,798	1,082,352	30.1	338	45.3	17.7	6.5	-19.8	0.082
4	Winnipeg	476,543	540,262	13.4	234	10.9	3.1	-0.3	-8.6	0.048
5	Ottawa-Hull	457,038	602,510	31.8	292	30.0	41.0	18.6	-2.4	0.045
6	Calgary	279,062	403,319	44.5	392	63.7	32.3	19.2	-20.3	0.063
7	Edmonton	359,821	495,702	37.8	348	48.6	23.7	13.9	-18.5	0.064
8	Halifax	193,353	222,637	15.1	240	8.7	-1.1	-5.8	-5.9	0.067
9	Quebec	379,067	480,502	26.8	82	-40.3	-21.3	-8.4	30.0	0.051
10	Windsor	217,215	252,643	19.1	100	-33.6	-26.9	0.4	33.0	0.047
11	Regina	113,749	140,734	23.7	240	12.5	-3.7	-3.6	-16.5	0.053
12	Victoria	155,763	195,800	25.7	229	10.0	-2.7	-5.3	-12.6	0.085
13	Saskatoon	95,564	126,449	32.3	317	38.1	8.8	1.8	-22.1	0.056
14	Thunder Bay	105,085	112,093	9.8	270	55.1	6.7	0.4	43.2	0.045
15	London	226,669	286,011	26.2	204	0.0	2.9	1.2	2.3	0.046
16	Saint John	98,083	106,744	8.8	138	-22.2	-14.4	-5.9	6.6	0.061
17	St. John's	106,666	131,814	23.6	251	14.8	3.1	-0.8	-13.4	0.096
18	Sudbury	127,446	155,424	22.0	494	100.0	14.3	0.4	-43.3	0.043
	Average			24.4	254	19.1	5.0	1.2	-3.6	
	Standard Deviation			9.5	105	37.0	16.2	8.8	22.3	

% Growth in	Population	Flows	Interactance Ratio	Average I.M.	Index of Total System Sensitivity	Average P.I.	Accessibility
Population	1.00	0.43	0.34	0.60	0.66	-0.45	0.19
Flows		1.00	0.98	0.75	0.52	-0.75	0.09
Interactance Ratio			1.00	0.73	0.57	-0.61	0.03
Average Interactance Multiplier				1.00	0.94	-0.45	0.01
Index of Total System Sensitivity					1.00	-0.31	0.002
Average Performance Index						1.00	-0.33
Accessibility							1.00

Table 4-3: Correlations between the System Parameters

centres that have shown increases on the one variable alone, centres such as Sudbury, Montreal, and Thunder Bay. The correlation between flows and interactance ratios is high (0.98) since the ratios are directly related to the flow variable. Table 4-3 also shows that the relationship between growth in flows and growth in interactance multipliers is fairly strong (0.75), indicating that the centres that have had the greatest increases in flows are also those that have developed the strongest sub-network structures, e.g. Calgary, Edmonton and Ottawa-Hull. Regina, Victoria and Halifax are exceptions to this rule; although their flows have increased greatly, their interactance multipliers have decreased.

Because the index of total system sensitivity and the average interactance multipliers describe the influence of the arcs on interactance ratios, both properties are strongly correlated (0.94). Note that the performance indices are negatively correlated with the flows (-0.75), with the interactance ratios (-0.61), and with the interactance multipliers (-0.45). A detailed examination of the various parameters that enter into the calculation of the performance index will reveal the meaning of their negative correlations in terms of network evolution. This will be done in the detailed analyses of a subsequent section. Finally, the correlations between the accessibility index and the other parameters are all low valued. There are centres with high accessibility indices that exhibit strong values for their other parameters, e.g. Vancouver and Saskatoon, and there are other centres with similarly high accessibility indices that do not report high values for their other system characteristics, e.g. Quebec and Saint John.

A) Interactance Ratios:

Table 4-4 shows the interactance ratio of every metropolitan area for

TABLE 4-4

CMA INTERACTANCE RATIOS, NATIONAL SYSTEM, TORONTO AS THE MAJOR NODE, 1963-1973

CMA	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
MONTREAL	0.731	0.729	0.706	0.682	0.777	0.664	0.649	0.637	0.622	0.614	0.606
VANCOUVER	0.311	0.318	0.329	0.360	0.374	0.403	0.424	0.415	0.422	0.430	0.452
WINNIPEG	0.258	0.236	0.257	0.289	0.300	0.303	0.311	0.315	0.289	0.291	0.286
OTTAWA-HULL	0.256	0.255	0.252	0.266	0.272	0.284	0.287	0.307	0.325	0.322	0.334
CALGARY	0.226	0.255	0.271	0.297	0.311	0.344	0.368	0.357	0.364	0.373	0.370
EDMONTON	0.218	0.226	0.241	0.271	0.284	0.293	0.313	0.304	0.309	0.325	0.324
HALIFAX	0.145	0.150	0.148	0.155	0.158	0.158	0.162	0.160	0.162	0.163	0.164
QUEBEC	0.144	0.135	0.112	0.109	0.094	0.086	0.085	0.087	0.091	0.082	0.086
WINDSOR	0.122	0.117	0.113	0.112	0.105	0.092	0.083	0.090	0.083	0.078	0.081
REGINA	0.096	0.097	0.103	0.109	0.113	0.109	0.109	0.103	0.105	0.105	0.108
VICTORIA	0.080	0.081	0.083	0.082	0.094	0.103	0.086	0.087	0.083	0.081	0.088
SASKATOON	0.063	0.059	0.066	0.071	0.081	0.083	0.083	0.085	0.086	0.085	0.087
LONDON	0.060	0.054	0.054	0.056	0.050	0.051	0.054	0.056	0.055	0.057	0.060
SAINT JOHN	0.054	0.053	0.052	0.051	0.054	0.052	0.045	0.046	0.043	0.041	0.042
ST. JOHN'S	0.054	0.060	0.058	0.063	0.063	0.059	0.059	0.056	0.060	0.064	0.062
THUNDER BAY	0.049	0.061	0.067	0.073	0.074	0.075	0.074	0.074	0.081	0.077	0.076
SUDBURY	0.021	0.019	0.022	0.027	0.030	0.031	0.034	0.041	0.045	0.041	0.042
MEAN	0.170	0.171	0.173	0.181	0.190	0.188	0.190	0.189	0.190	0.190	0.192
STANDARD DEVIATION	0.169	0.170	0.168	0.166	0.187	0.174	0.170	0.170	0.170	0.170	0.170
SKEWNESS	2.041	2.009	1.851	1.558	1.762	1.280	1.138	1.114	1.063	1.003	0.975
KURTOSIS	4.202	4.053	3.342	2.097	2.906	0.923	0.314	0.242	0.032	-0.196	-0.329

the 1963-1973 period and Figure 4-1 represents graphically the results of table 4-4. Throughout this period all the CMAs have had increased inter-metropolitan interaction, as shown by the origin-destination matrices (Table A4-67 to A4-77 of the Appendix, but there have also been important changes in the relative proportions of system interaction. Some centres, e.g. Vancouver, Calgary, and Ottawa-Hull, have increased consistently in relation to Toronto's interaction during this period, an indication that they have become progressively more active within the national system. The increased metropolitan interaction may result from greater activity of one centre with most of the other nodes -- this would be a case of widespread system interdependency -- or the increase might result from greater activity between a few centres only, such as greater interaction between the major pole of a system and a peripheral centre, without increased interaction of the peripheral centre with the other nodes in the system. This second case would represent very little in terms of system development. In most cases analyzed here the increased interactance ratios have resulted from a general increase in inter-metropolitan activity. This raises the question as to whether or not the greater metropolitan interdependency described for the United States by Borchert (1972) is also occurring in Canada.

The most important interactance ratio gains have been achieved by Vancouver, Calgary, Ottawa-Hull, and Edmonton. Vancouver's interactance ratios increased from 0.311 in 1963 to 0.452 in 1973. Thus while Vancouver's inter-metropolitan activity was less than a third of Toronto's in 1963, by 1973 it had grown to almost half of Toronto's. Calgary, Ottawa-Hull, and Edmonton form a second set of CMAs whose interactions have increased very substantially during this time, their interactance ratios averaging 0.234 in 1963 to 0.343

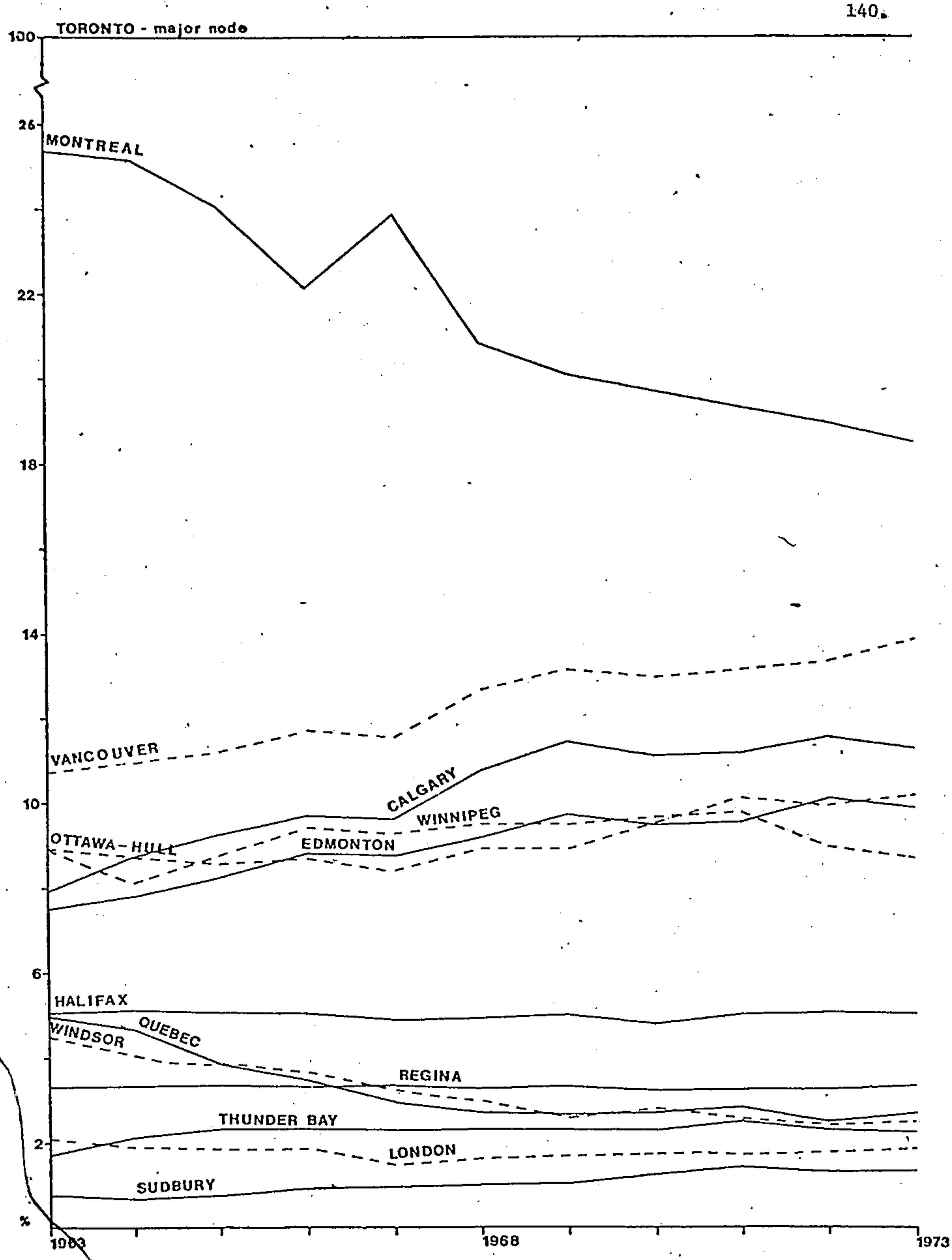


FIGURE 4-1 CMA INTERACTANCE RATIOS AS A % OF THE NATIONAL SYSTEM TOTAL INTERACTANCE RATIO

in 1973. The growth of several of the higher order Western metropoli has been combined with a relatively greater degree of intermetropolitan interaction.

There are four centres whose interactance ratios have dropped, some gradually, others more precipitously throughout this period. They are Montreal, Quebec, Windsor, and Saint John. Montreal's interactance ratio dropped by 17 percent from 0.732 in 1963 to 0.606 in 1973. Even though this still leaves Montreal with the highest level of intermetropolitan exchanges after Toronto, the drop is large and perhaps significant in that it implies that Montreal is rapidly moving into a position relatively much lower than Toronto, at least in terms of volume of intermetropolitan activity. Quebec and Windsor dropped even more rapidly than Montreal, from values of 0.144 and 0.123 to 0.087 and 0.082 respectively. This, too, could be a significant change because Quebec and Windsor, the extreme poles of the Heartland, have become weaker elements on the fringe of the Canadian Manufacturing Heartland.

Seven centres have had a period of interactance-ratio growth followed by either a period of stabilization or one of slight decline. In this group we find Halifax, St. John's, Winnipeg, Saskatoon, Regina, Thunder Bay and Sudbury.

Victoria and London are special cases. Victoria has experienced a rise in its interactance ratio followed by a period of stabilization. It seems plausible that as Vancouver was asserting itself as the major pole of the western field (Storey and Bulker, 1975), Victoria's intermetropolitan activity benefited from Vancouver's proximity and growing importance. Once it had achieved a certain degree of interdependency with the centres of the Vancouver sub-system its own level of activity stabilized and continued to keep pace with that of the national pole, Toronto.

London, by contrast, had a period of decline followed by a constant increase in interactance ratio. Whatever the reason for this, it would seem that up to 1967 its interaction was mostly limited to the Windsor-Quebec corridor and since then it has gradually, as Toronto has continued to expand its field, become a much more active and dynamic element of the national system, not only keeping pace with it but attaining a more significant role within this structure. The growth of London's interactance ratio, along with Ottawa-Hull's accompanied as it has been by a decrease in gain for Windsor and Quebec, could signal a restructuring of the Canadian Heartland into a central core consisting of the cities of Toronto, Montreal, Ottawa-Hull, and London, surrounded by a fringe area made up essentially of Windsor, Thunder Bay, Sudbury, Quebec, and Chicoutimi-Jonquiere. This would be a considerable change from what had been essentially a two sub-system arrangement of the Heartland, an eastern section centred on Montreal and a western section centred on Toronto. More evidence of this restructuring will be presented later.[8]

The smaller CMAs or those that are less suitably located, show indication that the size of the country and the weakness of some regional economies make it difficult for these centres to interact more fully within the larger national metropolitan network. In most cases, these cities have been following, it would seem, a trend of greater system interaction, but not at the national scale, for their activity is still limited mostly to their own sub-systems. The sheer size of this country is still important (Simmons, 1976) as a limiting

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[8] Note that Central Canada CMAs that do not have a major airport, such as Hamilton and Kitchener, are not considered here.

factor of interaction even though great progress has been made in transport technology. A detailed analysis of the sub-systems' sensitivities will show that, indeed, most CMAs are undergoing a process of network development but, for several centres, it has been at the regional level rather than at the national level, where development is limited to the country's major nodes. Again it is stressed that the interactance ratios do not reveal the factors responsible for these changes.

B). <u>Interactance Multipliers</u>:

The interactance ratio says little about the system's internal operations, for it fails to measure the strengths of the sub-networks that link the study node to the main node. The Interactance Multiplier is specifically designed to measure the most important set of these internal networks and thereby monitor the system's internal structure. The interactance multipliers of a system measure the set of networks located between the study node and each of the system's nodes that have a direct link to the main node. The derivation of the interactance multiplier, is based on the partitioning of the arcs of a system into two sets. The first set includes all the arcs emanating directly from the main node, and the second set all the other arcs.

<u>The National System, 1963-1973</u>:

The interactance multiplier technique is used here to measure and compare the national system's internal sub-network structure. The interaction characteristics in this section refer to the air passenger traffic between the 18 major airports of Canada with Toronto taken as the major node as it has the greatest total interaction. Since there are 17 direct links from each of the 17 CMAs to Toronto, the system has 17 interactance multipliers to be considered in the analysis of each study node. In effect then, there are

17 different internal sub-networks tying a given city to all of the others of the national system. To identify the interactance multipliers the original data matrix of direct linkages has been transformed into a matrix depicting both the direct and indirect linkages between the centres. Consider for instance, the study node Ottawa-Hull which is tied to Toronto directly and indirectly through the other 16 nodes, and hence by 17 sub-networks. The interactance multipliers summarize in a single vector of 17 values, each of the networks involved in the generation of flows at the study node.

If the average value of the interactance multipliers for a given study node is low, the node is isolated from the rest of the system. In poorly developed networks, one interactance multiplier is very strong and all of the others are low valued. The strong interactance multiplier is the direct arc between the main node and the study node, while the rest of the nodes have very little impact on the study node. In the case of well developed networks, the average interactance multiplier is high and no single interactance multiplier dominates all the others.

A close scrutiny of the evolution of the system's interactance multiplier vectors over the eleven year period, from 1963 to 1973, will show the changing patterns of interactions and indicate the sub-networks showing either greater or lesser activity over the period (Tables A4-1 to A4-17 of the Appendix).

<u>The Western CMAs of the National System</u>:

<u>Vancouver</u> (Table A4-14):

Table A4-14 indicates the changes in Vancouver's interaction patterns. Vancouver has, over this time period, greatly intensified its network activity with the other metropolitan areas. Its average interactance multiplier value has increased by almost 18%. As the drop in standard deviation shows, the network increase has been broadly spread out and not limited to a few CMA sub-networks. Vancouver relies less on its Toronto-Vancouver link in 1973 than it did in 1963 and its ties with the Prairie cities have risen considerably. Vancouver's role within the western sub-system is much greater in 1973 as the networks intervening between it and the other CMAs are more equally balanced. Vancouver has, over this short period, also developed its ties with each of the eastern and Atlantic cities. It has strong sub-networks with centres across the country as well as with its immediate western neighbours.

Calgary (Table A4-1):

Calgary has become less dependent on Toronto for its interactions and its network ties with the western CMAs have not changed substantially. Even in 1963 Calgary had good regional ties. By 1973 it had developed stronger sub-networks with the Heartland cities of Toronto, London, Ottawa-Hull, Montreal, and Quebec.

Edmonton (Table A4-2):

Edmonton's interaction patterns are much the same as Calgary's. Throughout this period, its networks with the Heartland cities have grown while its networks within the western sub-system have been maintained.

Regina and Saskatoon (Tables A4-8, A4-11):

The cities of Regina and Saskatoon have altered their patterns considerably. Even though Regina and Saskatoon remain well entrenched within the Western System, Edmonton and Calgary play a relatively less important role in their total interurban activity. Their Vancouver sub-networks are responsible for more of their activity but their dependency on Toronto as a generator of interaction still remains. It is striking that all networks linking both Regina and Saskatoon to Central and Atlantic Canada have increased.

Winnipeg (Table A4-17):

Winnipeg received in 1973 a smaller share of its interactions from all the Western cities and an increasingly larger share from the east with major gains coming from the Heartland cities of Ottawa-Hull, Windsor, and London.

Victoria (Table A4-15):

Victoria did not rely so heavily on Vancouver in 1973. It has stronger western networks and has developed to some extent its network ties with the east.

The Central-Canada CMAs:

Montreal (Table A4-5):

There has been a consistent drop, though not a large one, in Montreal's average interactance multipliers over the eleven-year period, (Table 4-5). The standard deviations have remained much lower in value than the average interactance multiplier because Montreal's intermetropolitan activity is not limited to a few major arcs but is national in scope. The networks linking Montreal to all the other CMAs are very well developed so that no single one is responsible for an exceedingly large share of its intermetropolitan interaction.

Montreal's level of interaction has become more dependent on its direct Toronto-Montreal arc. Its ties with the dynamic western cities, Vancouver, Calgary, Edmonton, Winnipeg, and even Saskatoon and Regina have grown and it has also developed greater ties with Ottawa-Hull and Windsor. There has been a decrease of intervening network activity with Quebec, Saint John, Halifax,

Table 4-5

MONTREAL CMA INTERACTANCE MULTIPLIER
EXPRESSED AS A PERCENTAGE OF THE NATIONAL SYSTEM'S
TOTAL INTERACTANCE MULTIPLIER, TORONTO AS THE MAJOR NODE

ARC	1963	1973
Toronto-Montreal	16.50%	17.60%
Toronto-Quebec	11.60	9.10
Toronto-Saint John	7.90	7.30
Toronto-Halifax	7.30	6.50
Toronto-London	6.80	6.70
Toronto-St. John's	6.80	5.90
Toronto-Victoria	4.70	4.60
Toronto-Vancouver	4.50	4.80
Toronto-Calgary	4.50	4.80
Toronto-Winnipeg	4.40	4.50
Toronto-Edmonton	4.30	4.70
Toronto-Saskatoon	4.30	4.40
Toronto-Regina	4.00	4.40
Toronto-Ottawa-Hull	3.90	4.30
Toronto-Windsor	3.70	5.70
Toronto-Thunder Bay	2.70	2.60
Toronto-Sudbury	2.20	2.30
Average Interactance Multiplier	0.441	0.394
Standard Deviation	0.265	0.230

The Average and Standard Deviation are in absolute values.

St. John's, and Thunder Bay.

In short, Montreal has stronger networks within the Canadian national system mainly because it has increased its interdependency ties with western CMAs.

Toronto, with Montreal as the Major Node (Table 4-6):

It is instructive to reverse for a moment the roles of Montreal and Toronto within the national system and to measure intervening network structure when Montreal serves as the major node and Toronto as the study node. The question that can be resolved by this operation is how well developed are the networks linking Toronto to the CMAs when Montreal serves as the major node?

Table 4-6 shows that Toronto's intermetropolitan networks are more developed than Montreal's as all of Toronto's intervening networks contribute fairly equally well to its total interactions without any one sub-network dominating the others. It is indicative of Toronto's national role that the Central Canada CMAs, although they are much closer to Toronto than the western cities, provide only slightly larger shares of Toronto's intermetropolitan activity. It is striking also that the Central Canada CMAs which in 1963 did have a slim margin over the rest of Canada CMAs, have been the ones to see their shares of Toronto's total interaction drop in importance by 1973. The Atlantic Region cities and Quebec, on the other hand, became responsible by 1973 for a larger proportion of Toronto's activity.

In short, Toronto has well developed networks, and its ties with Central Canada, the Western and Maritime centres are stronger in 1973 than in 1963.

We now go back to the analysis of the cities in the national system

Table 4-6

TORONTO CMA INTERACTANCE MULTIPLIER
EXPRESSED AS A PERCENTAGE OF THE NATIONAL SYSTEM'S
TOTAL INTERACTANCE MULTIPLIER, MONTREAL AS THE MAJOR NODE

ARC	1963	1973
Montreal-Toronto	9.00%	8.30%
Montreal-Thunder Bay	8.40	6.80
Montreal-Sudbury	7.20	7.10
Montreal-Windsor	6.90	5.70
Montreal-Ottawa-Hull	6.70	6.20
Montreal-Calgary	6.00	6.00
Montreal-Victoria	5.90	6.00
Montreal-Vancouver	5.80	5.80
Montreal-Edmonton	5.80	6.00
Montreal-Saskatoon	5.80	6.00
Montreal-Winnipeg	5.70	6.00
Montreal-London	5.30	5.30
Montreal-Halifax	4.80	5.20
Montreal-Saint John	4.40	4.70
Montreal-St. John's	4.20	5.10
Montreal-Quebec	2.40	3.80
Average Interactance Multiplier	1.120	1.388
Standard Deviation	0.296	0.235

The Average and Standard Deviation are in absolute values.

with Toronto as its major node.

Ottawa-Hull (Table A4-6):

The networks between Ottawa-Hull and all the other CMAs have grown consistently throughout the period. Through increased intermetropolitan inter-dependency Ottawa-Hull has increased its role within the national metropolitan system and reduced its dependency on Toronto for interurban activity. By 1973 it was a much more important element of the national system.

Quebec (Table A4-7):

As Quebec's average network structure has not changed and its ties with Toronto have increased, there must have been some reduction in Quebec's ties with other CMAs. Its ties with Saint John, Halifax, London and St. John's have decreased considerably. Quebec's ties with Montreal have also decreased proportionately. (This might be due to greater highway inter-city travel through highway improvement. The results presented here are, of course, based on air passenger traffic.) From these results it seems that Quebec also is being drawn into the Toronto sphere of influence on the one hand and being relegated to the fringe of the Central Canada Heartland on the other. Sensitivity analyses should shed light on Quebec's evolution throughout this period.

London, Windsor, Thunder Bay, and Sudbury (Tables A4-4, A4-12, A4-13 and A4-16):

London and Windsor rely on Toronto for a very large share of their total interactance response, in the range of 60%. However, while Windsor has experienced a slight increase of dependency on Toronto, London seems to have lowered, if only slightly, its dependency on Toronto and to have developed its networks with the other Heartland cities. Both cities had, in 1973, less influence on the interactions of the western cities than they had in 1963. It seems also that London has developed slightly greater network ties with the Atlantic cities.

Thunder Bay and Sudbury were heavily dependent in 1963 on Toronto with the Toronto sub-networks accounting for 67.7% and 92.9% of the structure respectively. By 1973 both cities had developed other sub-networks to a certain degree, Toronto's share of their interaction having dropped to 63.5% and 82.5% respectively. Thunder Bay had developed a few links with the eastern cities and reduced its dependency on the west. Sudbury had developed all of its intermetropolitan networks during the period although it is still strongly Toronto oriented.

The Atlantic Region CMAs:

Halifax has a much lower dependency on Toronto (36%) than Saint John (69%) or St. John's (62%). Its links with the Atlantic and the Heartland cities of London, Ottawa-Hull, Montreal, and Quebec are twice as intense as those of the other two. Throughout the period, Saint John experienced very little network development with the exception of its ties with Toronto. St. John's, on the other hand, decreased its dependency on Toronto and increased its ties with other CMAs, Ottawa-Hull in particular. It also experienced slight increases with the western cities.

During the 1963-1973 period most of the intervening sub-networks between the dependent nodes have experienced growth. This is not surprising as this reflects the increase in intermetropolitan interaction at all these nodes. What is of interest is the fact that several nodes have a new pattern of sub-networks activity. This serves as a first indication of system restructuring.

The four western cities of Vancouver, Calgary, Edmonton, and Saskatoon along with the eastern city of Ottawa-Hull have developed much broader and more balanced intermetropolitan networks with a reduced dependency on Toronto as a generator of interactions. The cities of London, Thunder Bay, and Sudbury have followed the same pattern of evolution but at a much lower scale. The cities of Winnipeg, Victoria, Halifax, and St. John's have undergone some slight changes also. In general their dependency on Toronto has decreased and although the average values of their interactance multipliers have remained almost constant, the standard deviations have dropped as the cities generally improved their network links with the rest of Canada.

During this same period, Montreal, Quebec, Saint John, and Regina have experienced a drop in their average interactance multiplier values and a reduction in standard deviations as well, an indication that these centres now have networks which are better developed, that is networks with a greater number of stronger links. For all four centres Toronto plays an increasingly important role.

In general, then, it seems that several changes have occurred in the sub-network structures of the Canadian Urban System over the 1963-1973 period. The sub-networks show an expected increase in their role in generating nodal interaction. Some have also shifted in their orientation. The emergence of Toronto as the national pole is also amply demonstrated. The western semi-independent sub-systems have evolved into a more interdependent system, with Vancouver as the major pole and Calgary-Edmonton as the secondary one. Nevertheless, Toronto is the whole western system's national pole.

C) Interactance Sensitivities:

Interactance sensitivities go a step further than interactance multipliers in defining system structure. While the multiplier measures the study node's dependency on whole sub-networks of the system, e.g., the intervening networks, interactance sensitivity measures the node's response to flow fluctuations on any given arc of the system. It probes more deeply into system operations. Just as any other elasticity measure, interactance sensitivity is a relative measure:

$$\text{SENSITIVITY} = \frac{\partial \text{ interactance ratio}}{\text{interactance ratio}} \Big/ \frac{\partial \text{ arc activity}}{\text{arc activity}}$$

The interactive sensitivity results are used here to determine if the nodes have extended their interaction networks. If a node is more sensitive to a growing number of arcs over the period it has in effect diversified its ties within the system and in this sense at least it has undergone network development.

As there are as many sensitivity measurements for each node as there are arcs in the system, it is important to define an Index of Total System Sensitivity which summarizes in a single index a node's reaction to overall system change. (Lamarche and Francon, 1975):

$$I_i = \left[\frac{1}{n} \sum_{k=1}^{n} \left(\frac{\partial g_i}{\partial c_{ko}} \right)^2 + \frac{1}{m} \sum_{k=1}^{n} \sum_{l=1}^{n} \left(\frac{\partial g_i}{\partial a_{kl}} \right)^2 \right]^{1/2}$$

In essence the index is the average value of arc sensitivity expressed in absolute terms. A high index value indicates strong node interdependency and a low value points to weak integration network development.

In the analyses to follow both the index and the interactance sensitivities will serve to depict system behavior and system structure.

The results for the index of total system sensitivity are presented in Table 4-7 and Figure 4-2. The values show that there have been some important changes within the national system over the 1963-1973 period. The most important changes have been the losses sustained by Montreal and Quebec and the gains made by Vancouver, Calgary, and Edmonton.

Because Montreal holds such an important position within the Canadian hierarchy it is most important to understand the nature of Montreal's losses, whether they refer namely to arcs within the eastern system or outside it, for it is essential to know whether the role of Montreal within the national system is changing or whether it is only its role within the eastern subsystem that is changing, or both. The interactance sensitivities should clarify these points.

The rather stable values for the other cities in the system are interesting in that they show that the relative positions of these cities within the system have not changed. However, this apparent stability could be misleading. Some centres such as Winnipeg, might have experienced some losses in one sub-system and gains in another. Here again a look at the interactance sensitivities will show whether or not there have been significant changes in network structure.

TABLE 4–7

INDEX OF TOTAL SYSTEM SENSITIVITY, NATIONAL SYSTEM, TORONTO AS THE MAJOR NODE, 1963-1973

CMA	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
Montreal	0.522	0.523	0.522	0.509	0.599	0.490	0.505	0.497	0.482	0.472	0.465
Ottawa-Hull	0.297	0.300	0.301	0.310	0.319	0.314	0.320	0.332	0.344	0.346	0.353
London	0.261	0.260	0.261	0.261	0.262	0.261	0.263	0.263	0.263	0.264	0.264
Quebec	0.294	0.291	0.283	0.281	0.276	0.271	0.272	0.272	0.272	0.269	0.269
Windsor	0.257	0.257	0.257	0.259	0.263	0.258	0.258	0.258	0.258	0.258	0.259
Sudbury	0.249	0.249	0.249	0.250	0.251	0.250	0.251	0.251	0.251	0.251	0.251
Thunder Bay	0.256	0.256	0.257	0.258	0.260	0.258	0.258	0.257	0.258	0.258	0.258
Halifax	0.338	0.332	0.329	0.329	0.329	0.329	0.326	0.324	0.321	0.323	0.319
Saint John	0.278	0.274	0.273	0.270	0.272	0.271	0.266	0.266	0.263	0.263	0.263
St. John's	0.268	0.269	0.268	0.270	0.271	0.268	0.268	0.267	0.268	0.269	0.267
Winnipeg	0.403	0.383	0.403	0.419	0.424	0.419	0.421	0.432	0.432	0.405	0.403

TABLE 4-5 (Cont'd)

INDEX OF TOTAL SYSTEM SENSITIVITY, NATIONAL SYSTEM, TORONTO AS THE MAJOR NODE, 1963-73

CMA	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
Regina	0.311	0.312	0.315	0.313	0.315	0.309	0.307	0.305	0.303	0.299	0.301
Saskatoon	0.281	0.280	0.283	0.285	0.291	0.291	0.291	0.293	0.291	0.288	0.287
Calgary	0.479	0.520	0.534	0.534	0.545	0.565	0.584	0.587	0.593	0.583	0.573
Edmonton	0.458	0.478	0.491	0.502	0.511	0.510	0.522	0.524	0.528	0.527	0.522
Victoria	0.309	0.315	0.308	0.298	0.308	0.310	0.290	0.294	0.292	0.291	0.295
Vancouver	0.548	0.565	0.552	0.550	0.564	0.579	0.577	0.586	0.583	0.575	0.586

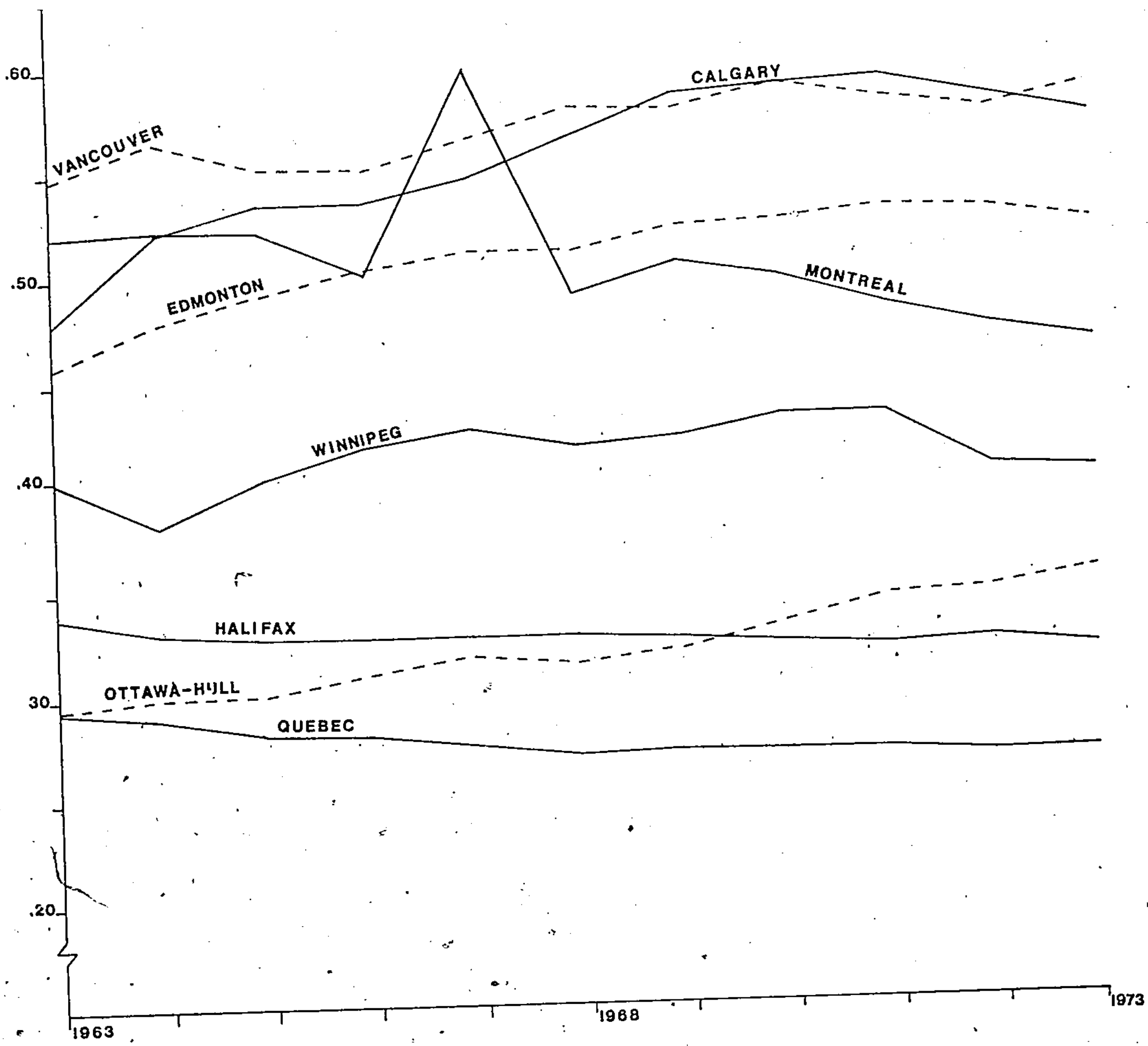

FIGURE 4-2 INDEX OF TOTAL SYSTEM SENSITIVITY, NATIONAL SYSTEM

<u>Montreal and Quebec</u>
(Tables 4-2, A4-38, A4-40):

Both cities are less sensitive to changes in the arc flows as the indices show, Montreal's going from 0.522 in 1963 to 0.456 in 1973, and Quebec's dropping from 0.294 to 0.269 in the same period.

Analysis of the individual interactance sensitivities for both these cities indicates important changes within the Canadian system, particularly in the Windsor-Quebec axis. By 1973 Montreal was much less sensitive to fluctuations on the Montreal-to-Quebec and Toronto-to-Montreal arcs. As these are among Montreal's most important arcs, the losses greatly influence the index value. It must be noted that Montreal has become more sensitive to flows on other arcs such as those between Halifax and Montreal, Toronto and Ottawa-Hull, Ottawa-Hull and Montreal, Winnipeg and Montreal, and Vancouver and Montreal but these gains are not important enough to compensate for the losses. It is surprising to find that Montreal's index remains lower than Vancouver's throughout the period, 1967 excepted. Vancouver's higher values reflect the greater diversity of its network.

Quebec's evolution is simpler. It is less sensitive to changes on its arcs with Montreal and much more influenced by flows on the Toronto-to-Quebec, Toronto-to-Ottawa-Hull, and the Ottawa-to-Quebec arcs. A graphical display of Quebec's sensitivity, Figure 4-3 summarizes in part the evolution in the eastern system structure.

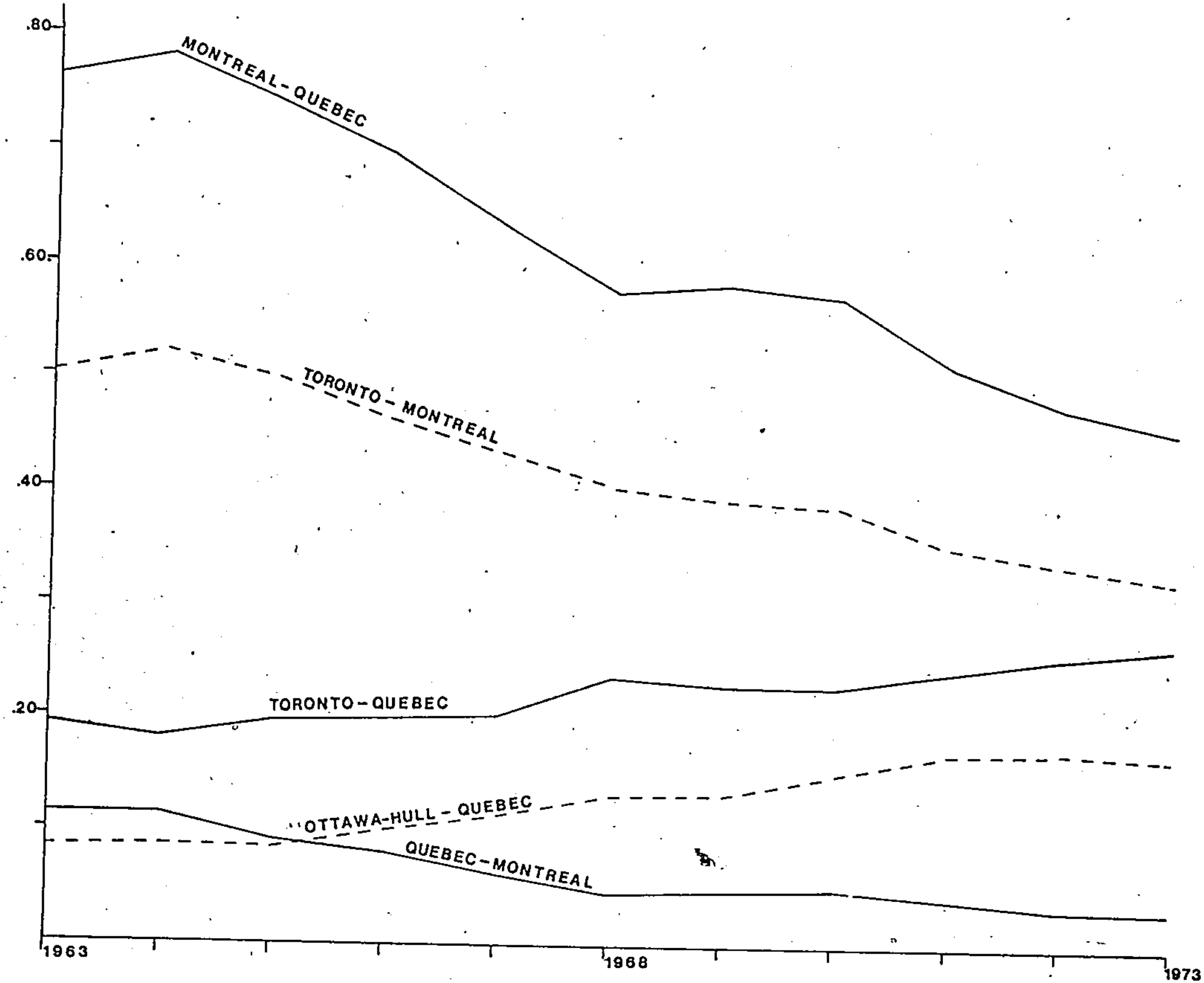

FIGURE 4-3 INTERACTANCE SENSITIVITY, QUEBEC, NATIONAL SYSTEM

The major changes within the interaction patterns are two-fold. First, there is the increasingly dominant role of Toronto within the eastern system along with a growing Ottawa-Hull role, with an accompanying loss of importance for Montreal. Secondly, the gains made by Montreal show an increased interdependency of the national system as Montreal became more responsive to fluctuations on arcs originating in what had been in 1963 more distant sub-systems.

London, Windsor, Thunder Bay, and Sudbury
(Tables A4-37, A4-45, A4-46, A4-49)

The average value of the index of total system sensitivity has remained practically unchanged, 0.256 in 1963, and 0.258 in 1973. The individual interactance sensitivities for Sudbury and Thunder Bay, confirm that there has been little change in sensitivity patterns for both these cities, but London and Windsor have altered their patterns considerably. London is less sensitive to flows on the Toronto-to-Montreal arc and more sensitive to flows on the Ottawa-Hull-to-London, Toronto-to-Ottawa-Hull, and on flows between Calgary and Edmonton. Windsor is less sensitive to its arc with Toronto and more to the arcs linking it to Montreal and to the Toronto-to-Montreal arc.

In brief, of the four cities, London is the only one that has become national in scope, to a certain degree at least. The other three centres are strongly dependent on the Heartland, but with little interaction outside it.

Halifax, Saint John, and St. John's
(Tables A4-36, A-4-42, A4-43):

The changes in interactance sensitivities for these cities are important even though the index values have changed but little. Halifax is less sensitive to fluctuations on arcs linking it to Montreal and Saint John and much more to changes on its links with Toronto and Ottawa-Hull. St. John's is also more sensitive to its links with Toronto and less with its ties to Montreal and Halifax. Saint John's patterns have evolved in much the same way, its links with Toronto and Ottawa-Hull becoming more important and its ties with Montreal and Halifax decreasing in importance. The behavior of Saint John and Quebec is very similar as their graphs show, Figures A4-2 and 4-3.

The evolution of the interaction patterns of the three cities shows the increased role of Toronto in the eastern system, the extinction of the Atlantic network as a separate entity by 1973, and the decreasing role of Montreal in the east.

Winnipeg, Regina, Saskatoon, and Victoria
(Tables A4-50, A4-41, A4-44, A4-48):

The index values for these four cities have not changed substantially throughout the period. Winnipeg, in part no doubt because of its location at the junction of the eastern and western sub-systems, has a high level of sensitivity to arcs in both sub-systems. Over the period Winnipeg has become increasingly sensitive to variations on the arcs linking it to Vancouver, Edmonton, and Calgary on the one hand, and to arcs linking Toronto to Ottawa-Hull and Toronto to Vancouver on the other. The city is less sensitive in 1973 to arcs linking it to Montreal and to the Toronto-Montreal arc. In short, Winnipeg has become more integrated within the western sub-system and has strong ties with Toronto and Ottawa-Hull, both dynamic centres of the eastern system. Its ties with Montreal are still strong but have sustained nonetheless a loss throughout the period.

Regina's evolution in the west is similar to Quebec's in the east. It is increasingly more sensitive to the arc linking it to the western system's new pole, Vancouver, and less to its own regional centre of Winnipeg. Regina is also more sensitive to changes in the arcs between Vancouver, Calgary, and Edmonton.

Saskatoon's pattern is not as clearly defined as Regina's but it is similar in that it is now more oriented towards Vancouver, Calgary, and Edmonton, and less towards Winnipeg.

Victoria, much like Regina, Saskatoon, and Quebec has reduced its ties with its strong neighbour, Vancouver in this case, and has become more influenced by flows between the western system's other main centres of Calgary, and Edmonton. Like Regina and Saskatoon, Victoria has experienced greater interaction with other western centres.

In summary, the small semi-independent network of Regina, Saskatoon, and Winnipeg, with this last centre serving as the major node, has ceased to exist, and a wider and more interdependent network covering the whole western part of Canada has come into existence with Vancouver as its main centre.

Vancouver, Calgary, and Edmonton
(Tables A4-47, A4-34, A4-35):

The index values of Vancouver have increased from 0.548 in 1963 to 0.586 in 1973, due primarily to Vancouver's increasingly dominant role in the western system. Its role in the west is greater than Montreal's in the east where Montreal must compete with Toronto. Vancouver has also increased in sensitivity to flows on the Vancouver-to-Toronto, the Vancouver-to-Montreal, and the Montreal-to-Toronto arcs, a net indication of greater national system interdependency.

Both Calgary and Edmonton have also undergone great increases in interactance sensitivities. In their role as secondary poles within the western network they have become increasingly more influenced by flows on the western network as it became progressively more unified into a highly interdependent network.

Ottawa-Hull (Table A4-39):

Over the period, Ottawa-Hull's role in the national urban system has become more apparent, not so much in the scale of its interactance sensitivity response as in the widespread nature of the network structure to which it responds. Its index value has gone from 0.297 in 1963 to 0.353 in 1973, a net indication of network extension. What is most striking in Ottawa-Hull's case is its passage from a regional city to a national city and nowhere is this more apparent than in the maps outlining the network arcs to which it responds (Figure 4-4a, 4-4b). In 1963 Ottawa-Hull was an element of the eastern system. In 1973 it has become an important element of the national system. Its sensitivity to flows on the arcs linking it to Toronto and Montreal has dropped but it is influenced by a network of arcs covering the nation.

Conclusion:

Interactance sensitivity analysis may be considered a valuable tool of system analysis principally because it allows the researcher to study at the individual arc level the behavior of a node's response to fluctuations on it.

The analyses have brought out some of the important changes that the Canadian national system has undergone: a generally greater dependency of the metropolitan centres on Toronto, the partial restructuring of the eastern and western sub-systems, and the differentiation within these sub-systems of the major and minor elements.

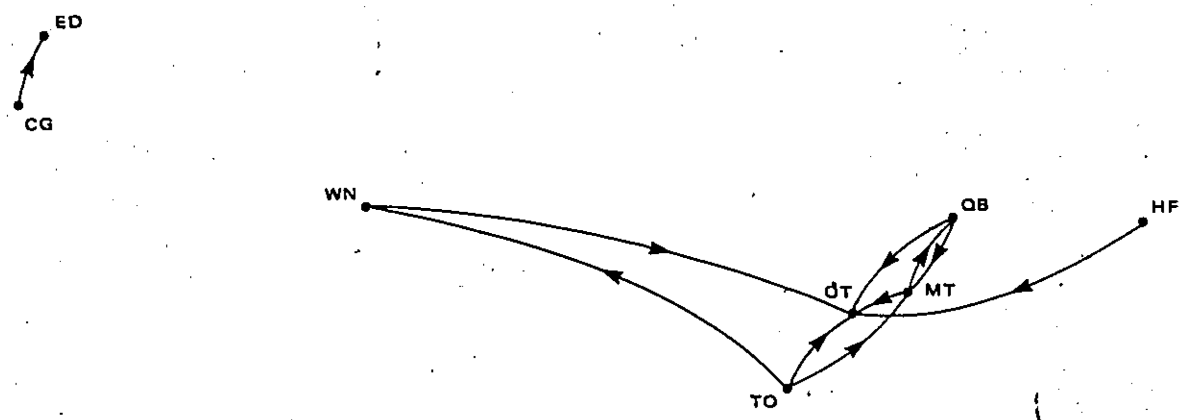

Figure 4-4b High Sensitivity Arcs - Ottawa 1963

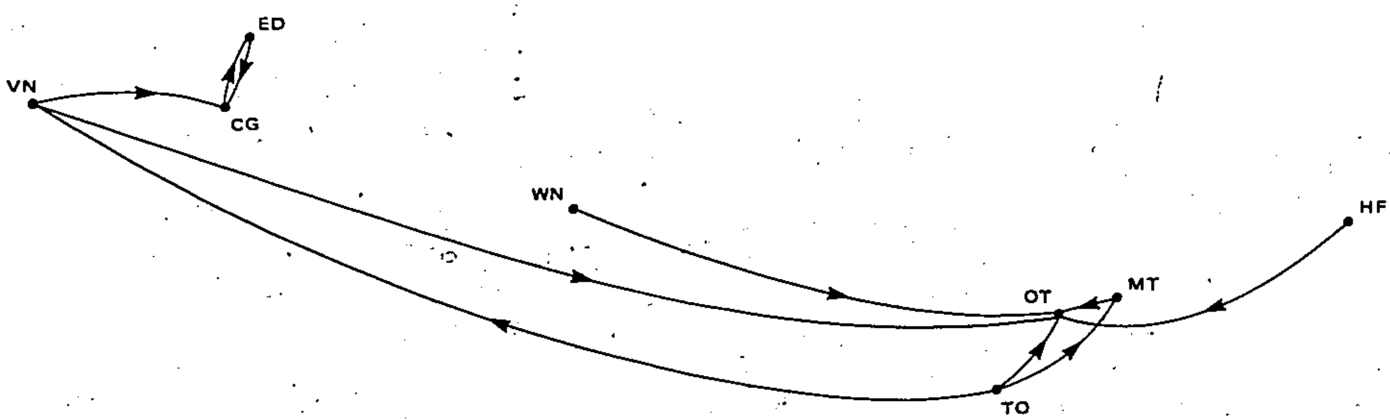

Figure 4-4b High Sensitivity Arcs - Ottawa 1973

Part II: The Eastern Sub-System, 1963-1973:

A) Interactance Ratios:

This sub-system consists of eleven metropolitan areas with Toronto taken as the major node. Table 4-8 and Figure 4-5 present the major changes which have occurred in the system during this period.

A striking feature here is the contrast between the role of Montreal in this regional field as opposed to its role at the national level. At the national level Montreal's intermetropolitan activity dropped from 75% of Toronto's in 1963 to 61% by 1973 (Table 4-4). Within the Eastern Sub-System, Montreal's activity was 83% of Toronto's in 1963 and dropped slowly to 71% in 1973 (Table 4-8). Within the more limited space of the Eastern Field Montreal has managed to keep abreast of Toronto whereas at the national scale the dynamism of Toronto has more readily overcome the frictional effect of distance and increased interactions with the western cities.

Ottawa's share of the total interactance ratio has gone from a value of 25.6% in 1963 to one of 33.4% in 1973. The results reported in Table 4-8 suggest that the Eastern system will become progressively more centred on the three CMAs of Toronto, Montreal, and Ottawa while the cities of Quebec and Windsor decline. The combination of high interactance ratio in the central CMAs of the Axis, along with the decreasing gains in the marginal cities, again suggest that present evolution trends could lead to a well-defined two-element Canadian Heartland, a central core of highly interactive CMAs surrounded by a peripheral zone of centres of less importance.

London's intermetropolitan activities have slightly increased in relation to Toronto's and then stabilized. Sudbury's interactance has increased,

TABLE 4-8

CMA INTERACTANCE RATIOS, EASTERN SUB-SYSTEM, TORONTO AS THE MAJOR NODE,1963-1973

CMA	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
HALIFAX	0.171	0.181	0.181	0.192	0.195	0.203	0.209	0.204	0.204	0.207	0.282
LONDON	0.059	0.053	0.053	0.056	0.049	0.052	0.057	0.058	0.059	0.062	0.064
MONTREAL	0.825	0.827	0.803	0.780	0.832	0.769	0.748	0.736	0.720	0.714	0.709
OTTAWA-HULL	0.275	0.273	0.273	0.284	0.284	0.314	0.321	0.337	0.354	0.348	0.363
QUEBEC	0.178	0.168	0.141	0.141	0.118	0.114	0.115	0.115	0.118	0.108	0.112
SAINT JOHN	0.066	0.067	0.065	0.066	0.069	0.069	0.061	0.061	0.056	0.056	0.057
ST. JOHN'S	0.066	0.075	0.073	0.082	0.080	0.078	0.079	0.075	0.078	0.085	0.084
SUDBURY	0.026	0.023	0.028	0.035	0.038	0.042	0.045	0.054	0.060	0.055	0.057
THUNDER BAY	0.055	0.056	0.063	0.068	0.070	0.074	0.076	0.076	0.084	0.078	0.078
WINDSOR	0.139	0.134	0.128	0.128	0.121	0.108	0.099	0.106	0.099	0.092	0.097
MEAN	0.186	0.186	0.181	0.183	0.182	0.182	0.181	0.182	0.183	0.189	0.183
SKEWNESS	1.870	1.877	0.886	1.843	1.904	1.773	1.727	1.686	1.630	1.631	1.587
KURTOSIS	2.319	2.342	2.350	2.202	2.382	1.915	1.752	1.588	1.372	1.375	1.210

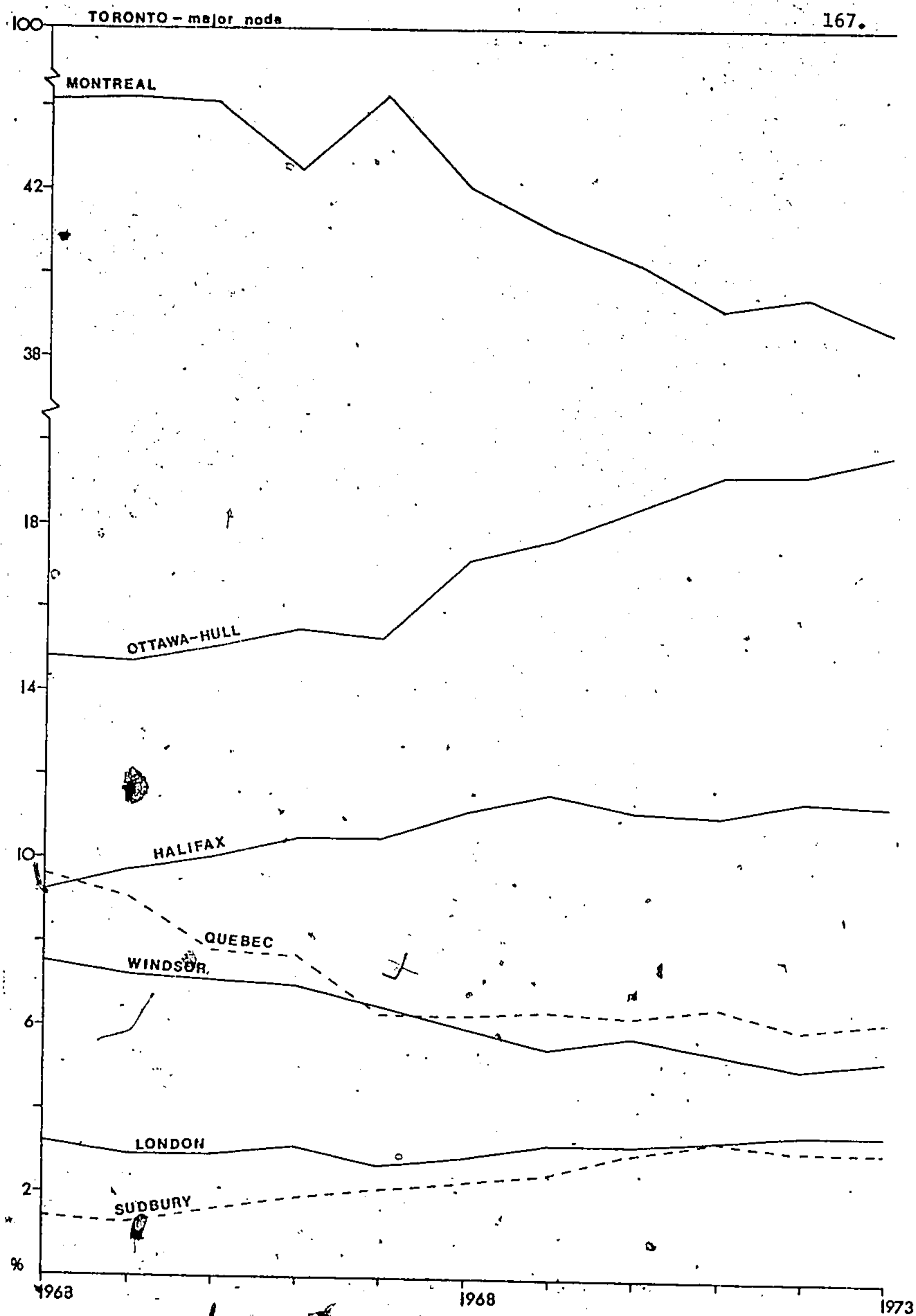

FIGURE 4-5. CMA INTERACTANCE RATIOS AS A % OF THE EASTERN SUB-SYSTEM TOTAL INTERACTANCE RATIO

(Tables A4-67 to A4-77) but this increase has been very heavily Toronto centred. Sudbury is still an outlier in terms of system interaction.

Table 4-8 reveals a striking increasing interactance ratio for Thunder Bay. Any suggestion that the distance effect has been largely reduced and that Thunder Bay has become, in this age of improved transit, an integrated part of the Eastern system, is not supported, however, by the origin-destination matrices. In 1963, 78% of Thunder Bay's exchanges were with Toronto. In 1973 the percent has not changed. The interactance ratio through the years increased at a greater rate than Toronto's but as most of the flows have been Toronto centred, Thunder Bay has remained in the hinterland of the Eastern Field, caught mid-way between the Eastern and Western networks.

Halifax and St. John's have successfully increased their share of inter-metropolitan activity in relation to Toronto's. Halifax has gone from 17.1% to 28.2% and St. John's from 6.6% to 8.4%. Their increase is diversified and not solely Toronto oriented.

In contrast, Saint John has experienced a relative drop in metropolitan interaction going from 3.6% to 3.1% of the total interactance ratio. From 1971 on, the value held steady. Saint John has not developed its ties with all the CMAs of the Eastern sub-system.

The interactance ratios show that most of the metropolitan centres of the Eastern sub-system have experienced greater inter-urban activity in relation to Toronto's total output. Is all this new activity attributable to the one factor of population growth or are some of these exchanges indication of system integration as recent literature on urban systems suggests (Borchert, 1973; Pred, 1977)? An examination of the changes in network structures through interactance multiplier and sensitivity analyses should shed some light on this question.

B) Interactance Multipliers:

The analysis of interactance multipliers over the 1963-1973 period will show what evolution there has been, if any, in the interaction networks linking the centres of the Eastern Sub-System (Tables A4-24 to A4-33 of the Appendix).

Montreal and Quebec:

Both Montreal and Quebec have experienced a net reduction in the average values of their interactance multipliers as the inter-urban networks linking them to the other centres have become less capable of generating interactions in response to flows on the arcs linking these other centres to Toronto. Both centres exhibit a greater dependency upon Toronto as the flows from Montreal and Quebec have progressively formed a larger share of Toronto's total output.

Montreal has seen several of its sub-networks regress, in particular, the network structure linking it to Quebec, to the Atlantic cities, and to Thunder Bay. On the other hand, it has developed its network ties with Ottawa-Hull and Windsor. Montreal still has an important role within the eastern system but a reduced one. Its ties with the Heartland cities have increased but it has lost some of its responsiveness to interactions occurring outside the Heartland core.

Quebec has become progressively more isolated from the centres in the eastern system with the exception of its network ties with Toronto, Ottawa-Hull, and Windsor. Its networks have become more limited but more responsive to the interactions generated in the core. It has done less well with the hinterland cities of the eastern system.

Ottawa-Hull, London, and Windsor:

During the 1963-1973 period Ottawa-Hull has extensively developed its intermetropolitan network, the average value of I.M. has increased from 0.212 to 0.287 while the standard deviation has dropped slightly. This increase applies to all the CMAs with the single exception of Toronto, indicative of the greater national rôle of Ottawa. Its greatest network developments have occurred within the Windsor-Quebec axis, especially with London and Quebec.

London has experienced a slight increase in network growth. Its ties with Ottawa-Hull, Sudbury, and Thunder Bay have increased while those with Quebec, Montreal and the Atlantic cities have stagnated.

Finally Windsor has shown very little progress in intermetropolitan system development. Throughout the period its proportional shares of I.M. with the CMAs have not changed. It remains highly dependent on Toronto and fairly well linked to the Heartland cities of Montreal, Quebec, and Ottawa-Hull.

Thunder Bay and Sudbury:

Both cities are extremely Toronto oriented though this dependency has decreased over the decade, the average value of interaction dropping from 93% in 1963 to 85% in 1973. Sudbury has intensified all of its intermetropolitan links, Toronto excepted, with the largest growth involving Heartland cities of London, Ottawa-Hull, Windsor, and Thunder Bay. However, all of these network developments remain slight in regard to Sudbury's interdependent ties with Toronto. Thunder Bay has increased its links with Sudbury and London in particular. Because Thunder Bay lies on the fringe of the Western system, a good part of its network development has involved the western cities. Vancouver and Victoria in particular.

Halifax, Saint John, and St. John's:

In general, network structure of the three Atlantic centres of Halifax, Saint John and St. John's has changed slightly during the period. Halifax has increased its ties with all of the Central Canada centres and has undergone some reduction in interdependencies with Saint John and St. John's. Saint John has experienced a reduction in I.M. with all of the centres with the exception of Toronto. Saint John in 1973 is much more isolated within the eastern system and St. John's has remained fairly stable, its share of losses in interaction with Toronto being compensated by its growing ties with Ottawa-Hull.

In summary, the evolution of interaction networks of the Eastern system reveal some important if not dramatic changes. In most cases the reduced dependency on Toronto has been slight and almost always in favour of one or several of the other Central Canada CMAs, Ottawa-Hull and London showing the greatest gains. Halifax is the one exception, for in spite of its distant location from Central Canada, it has become an integral part of the eastern system. Cities that remain highly dependent on Toronto are centres that fringe the Canadian Heartland, such as Thunder Bay, Sudbury, Quebec, St. John's, Saint John, and Windsor. It would appear that the eastern system is evolving towards a two-components system: a core containing Toronto, Montreal, Ottawa-Hull and London, and surrounded by a fringe area, the hinterland consisting of Windsor, Thunder Bay, Sudbury, Quebec, Saint John, and St. John's.

This reorganization of the eastern system's interurban internal networks departs from the interaction patterns that existed in the early 1960's. At the beginning of the period the data suggest three different system components, two major and one minor. The minor consisted of an Atlantic sub-system centred on Halifax and including the cities of Saint John and St. John's. The two major components consisted of an Ontario sub-system centred on Toronto, and a Quebec sub-system centred on Montreal. Although both interacted they were largely two separate entities. If further research confirms the findings presented here, the Eastern Sub-System has undergone an important restructuring within the short period extending between 1963 and 1973.

C) Interactance Sensitivities:

Analysis of the tables of individual interactance sensitivities, Tables A4-34 to A4-50 as well as the indices of total system sensitivities, Table 4-9 and Figure 4-6, largely confirm the changes outlined in the analysis of the evolution of the national system.

By 1973 the Atlantic cities of Halifax, Saint John and St. John's form an integral part of Toronto's network structure and this has been accompanied by a reduced Montreal influence in the Atlantic region. More symbolic of Toronto's growing influence is its capability of drawing even Quebec within its orbit. The role of Montreal in the Eastern sub-system is still very important for the flows on the Toronto to Montreal arc strongly influence Windsor, London, Ottawa-Hull, and the Atlantic cities, but Montreal in 1973 cannot be described as the only major node of what was in 1963 a Quebec-Atlantic sub-system. The sub-system has been absorbed into the Eastern System and Toronto has assumed the role of central node.

Another trait of the Eastern network that comes out here as well as in the national system analysis, is the increasing importance of Ottawa-Hull in Canada. Most of the cities in the system are sensitive to flows in the arcs emanating from Ottawa-Hull, but Ottawa-Hull itself has reduced sensitivity to the arcs within the eastern system. It has passed from a centre of regional importance to one of national significance. London's behavior within the

Table 4-9

INDEX OF TOTAL SYSTEM SENSITIVITY,
EASTERN SUB-SYSTEM, TORONTO AS THE MAJOR NODE

CMA	1963	1966	1969	1971	1973
Montreal	0.627	0.599	0.583	0.564	0.539
Ottawa-Hull	0.365	0.371	0.384	0.403	0.409
London	0.340	0.338	0.340	0.341	0.342
Quebec	0.392	0.373	0.360	0.359	0.355
Windsor	0.335	0.335	0.334	0.335	0.336
Sudbury	0.329	0.329	0.328	0.329	0.329
Thunder Bay	0.330	0.329	0.329	0.330	0.330
Halifax	0.452	0.439	0.434	0.426	0.421
Saint John	0.373	0.359	0.353	0.348	0.347
St. John's	0.358	0.358	0.355	0.354	0.354

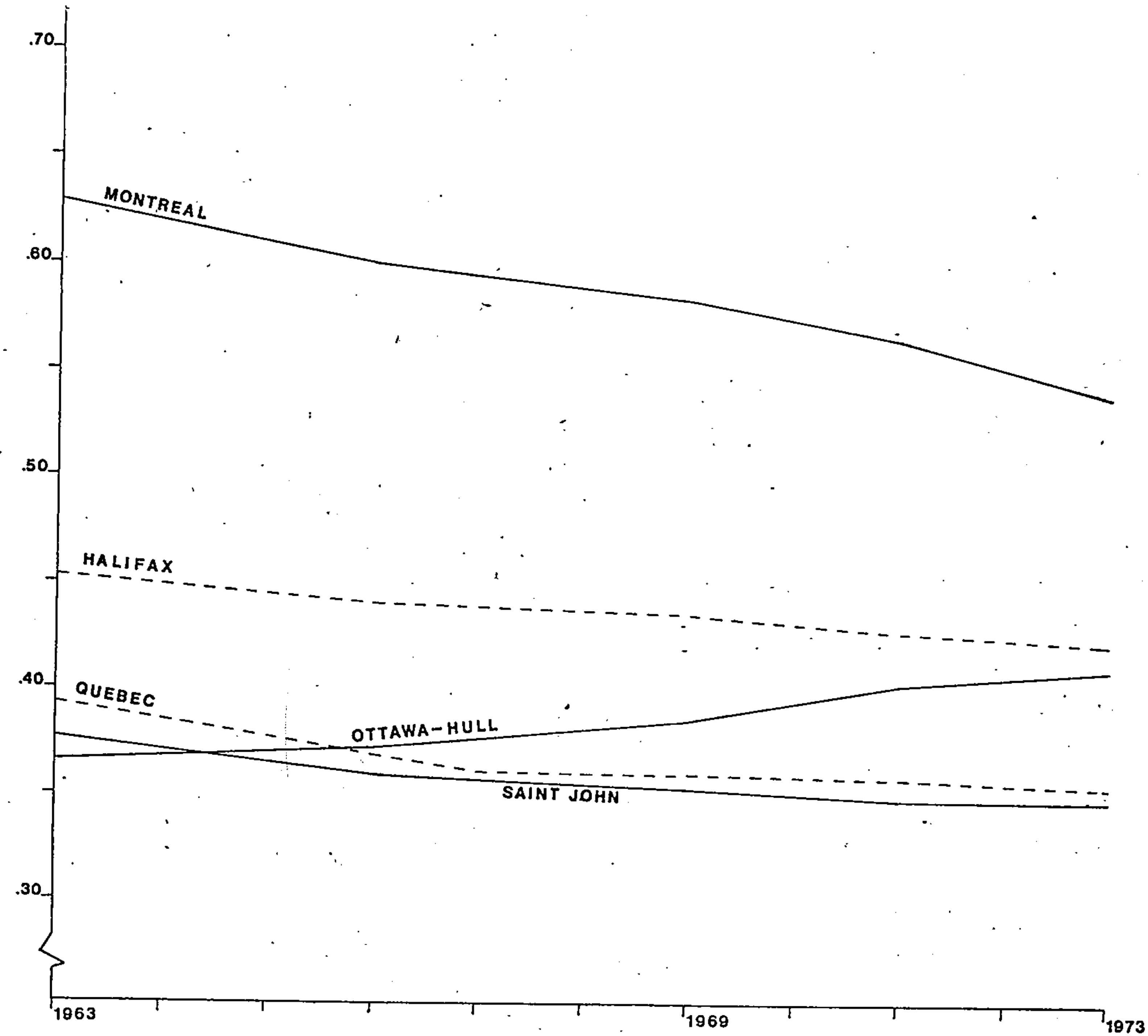

FIGURE 4-6 INDEX OF TOTAL SYSTEM SENSITIVITY, EASTERN SUB-SYSTEM.

eastern system is similar to Ottawa's in that it is also less sensitive to flows in the Eastern Sub-System for it too has spread its network outside the limits of the eastern field, but not to the same extent as Ottawa-Hull.

Part III: The Western Sub-System, 1963-1973:

The Western CMAs of Canada form a well defined sub-system of the Canadian metropolitan system. Through the 1963-1973 period these cities have experienced more interurban activity. What effect has this growth in interaction had on the network structures of the western sub-system? The behavior of the interactance ratios, multipliers, and sensitivities of the sub-system should reveal some of the network changes that have occurred. The cities in the sub-system are Vancouver, Victoria, Calgary, Edmonton, Regina, Saskatoon, and Winnipeg with Vancouver acting as the major node of the sub-system.

A) Interactance Ratios:

The western segment of the Canadian national metropolitan system has changed to a much lesser extent than its eastern counterpart. In 1963 the Western region consisted of several semi-independent sub-systems. The presence of physical barriers, the extensive distances and the absence of a very large urban centre capable of dominating the smaller systems probably help explain the slow rate of evolution of the western urban network.

The results of Table 4-10 and Figure 4-7 stress two points. The interactance ratios of the two largest centres of Calgary and Edmonton have increased consistently over the entire period while the smaller centres have experienced either decline or stagnation. The Western sub-system may be described as a two component network with an inner core consisting of the large centres of Vancouver, Calgary, and Edmonton, and an outer peripheral zone made up of the remaining centres. The high level of intermetropolitan activity of the core centres is

TABLE 4-10

CMA INTERACTANCE RATIOS, WESTERN SUB-SYSTEM, VANCOUVER AS THE MAJOR NODE, 1963-1973

CMA	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
CALGARY	0.873	0.967	1.032	1.031	1.044	1.035	1.079	1.074	1.096	1.095	1.054
EDMONTON	0.814	0.862	0.904	0.948	0.954	0.903	0.930	0.918	0.937	0.952	0.930
REGINA	0.347	0.348	0.365	0.354	0.350	0.313	0.299	0.289	0.289	0.273	0.277
SASKATOON	0.219	0.207	0.224	0.234	0.247	0.239	0.232	0.243	0.236	0.230	0.244
VICTORIA	0.309	0.319	0.308	0.265	0.303	0.296	0.225	0.236	0.233	0.228	0.221
WINNIPEG	0.560	0.461	0.553	0.576	0.557	0.532	0.540	0.560	0.516	0.511	0.504
MEAN	0.520	0.527	0.564	0.568	0.576	0.553	0.551	0.553	0.551	0.548	0.538
SKEWNESS	0.218	0.406	0.347	0.311	0.360	0.394	0.383	0.366	0.423	0.417	0.408
KURTOSIS	-2.004	-1.819	-1.914	-1.987	-1.964	-1.921	-1.926	-1.931	-1.901	-1.924	-1.937

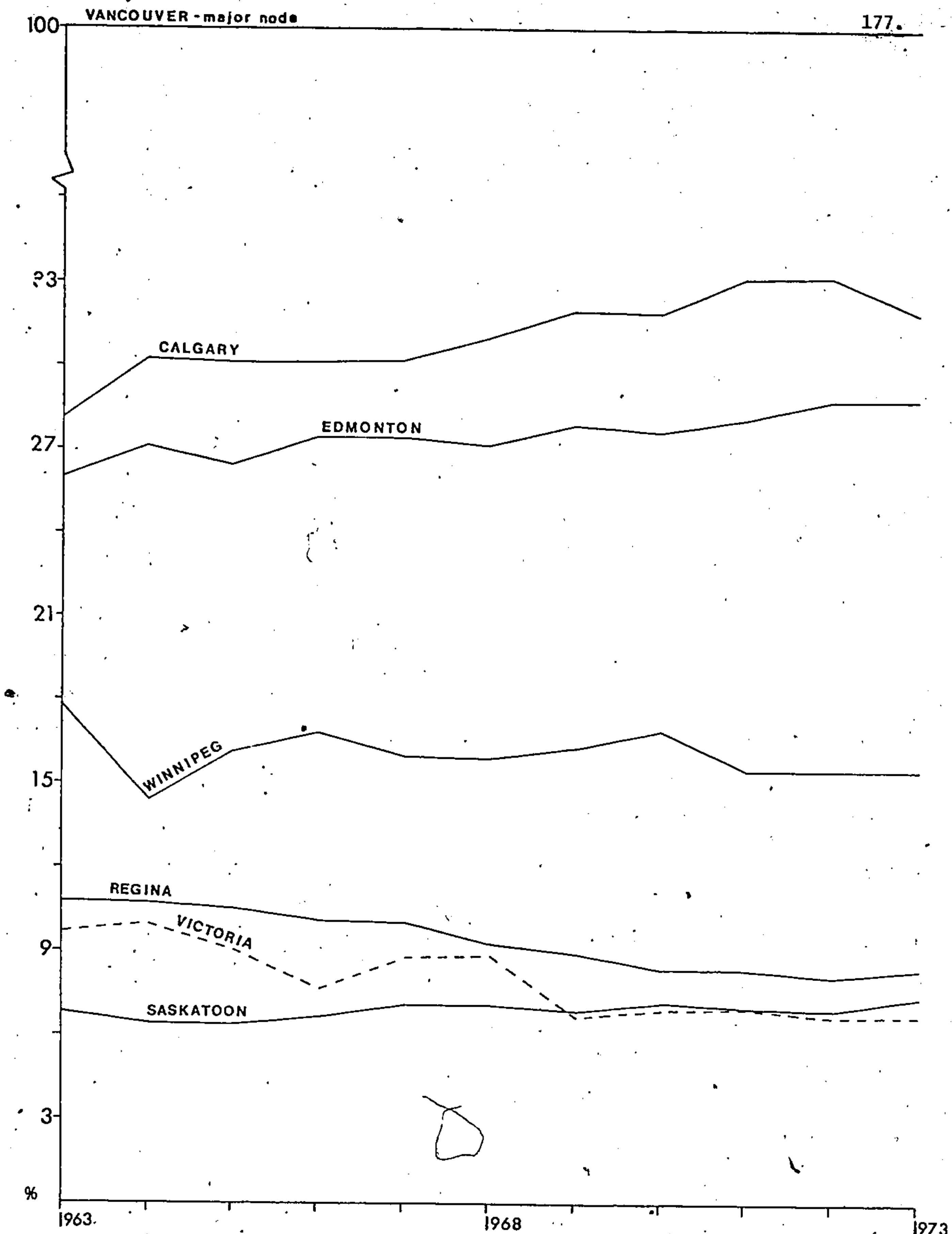

FIGURE 4-7 CMA INTERACTANCE RATIOS AS A % OF THE WESTERN SUB-SYSTEM TOTAL INTERACTANCE RATIO

indicative of their dominant positions in the western hierarchy.

The Prairie cities of Winnipeg and Regina have experienced a decreasing interactance ratio. A comparison of their behavior in the Western Sub-System with that of the national system confirms that these two centres have become progressively drawn into the Toronto field of activity and this has served to cut, relatively, some of their ties with the Western Field.

B) Interactance Multipliers:

Calgary and Edmonton:

The network structures tying these two cities to the others in the Vancouver centred western sub-system were already strong in 1963 and by 1973 they have developed still further. Their average interactance multiplier was 0.80 in 1963 and 0.85 in 1973. As the average standard deviation value of the interactance multipliers has dropped from 0.405 to 0.349, it is clear that all the cities within the system have experienced a greater interdependency. The greatest developments were in the networks linking Calgary and Edmonton to Victoria. The interactance multipliers for the direct links Vancouver-Calgary and Vancouver-Edmonton have remained practically unchanged. These arcs were responsible for approximately 30% of the total interactance multiplier for both Alberta cities and again confirms that both cities have widespread networks with a limited dependency on Vancouver as a source of interaction.

Winnipeg, Regina, and Saskatoon:

The average values for the I.M. and their standard deviations have changed only slightly throughout this period. The only changes have been in the increasing importance that the direct links Vancouver-Winnipeg, Vancouver-Regina, and Vancouver-Saskatoon have taken, as these arcs in 1963 accounted for 45.3% of the total I.M. and in 1973 the percentage has risen to 50.2%, a clear indication of Vancouver's rising dominance of the whole western system. The networks linking Victoria to all three of these cities have developed considerably as the response of the three cities to flows on the Vancouver-Victoria arc have almost doubled over the period, from a value of 4.5% to 8.5% of these three cities' total I.M.

Victoria:

Victoria's average value of I.M. has increased from 0.262 to 0.319 over the period and its standard deviation has remained stable at a value of 0.38, an indication that the city has increased its intermetropolitan activity. This is also confirmed by the decrease in Vancouver's share of Victoria's total interaction from 65.7% of I.M. in 1963 to 57.7% in 1973. By 1973 Victoria has developed its networks linking it to all the other western systems making it much more responsive to changes in their metropolitan interactions.

C) Interactance Sensitivities:

Interactance sensitivity analysis of the seven western cities taken as a separate sub-system simply confirm the findings recorded in the previous section. The indices of individual CMA total sensitivity, Table 4-11 and Figure 4-8, as well as the individual interactance sensitivities of the different nodes show that Vancouver, Calgary, and Edmonton form the core of the western network with Vancouver gradually taking a more dominant role. Surrounding the core is a secondary zone consisting of Winnipeg, Regina, Saskatoon, and Victoria. This is quite a different pattern of interaction from what existed in the early 1960's with its three semi-independent networks, one for the Pacific, one in Alberta, and one in the other Prairie provinces. The new two-component structure is suggested by the sensitivity patterns recorded in Tables A4-51 to A4-66. The centres are sensitive to changing flows on the arcs linking

Table 4-11

INDEX OF TOTAL SYSTEM SENSITIVITY
WESTERN SUB-SYSTEM, VANCOUVER AS THE MAJOR NODE

CMA	1963	1966	1969	1971	1973
Winnipeg	0.808	0.859	0.791	0.780	0.745
Regina	0.691	0.723	0.654	0.628	0.600
Saskatoon	0.593	0.628	0.613	0.601	0.570
Calgary	1.023	1.161	1.150	1.143	1.084
Edmonton	0.944	1.067	1.018	1.025	0.982
Victoria	0.496	0.513	0.517	0.540	0.558

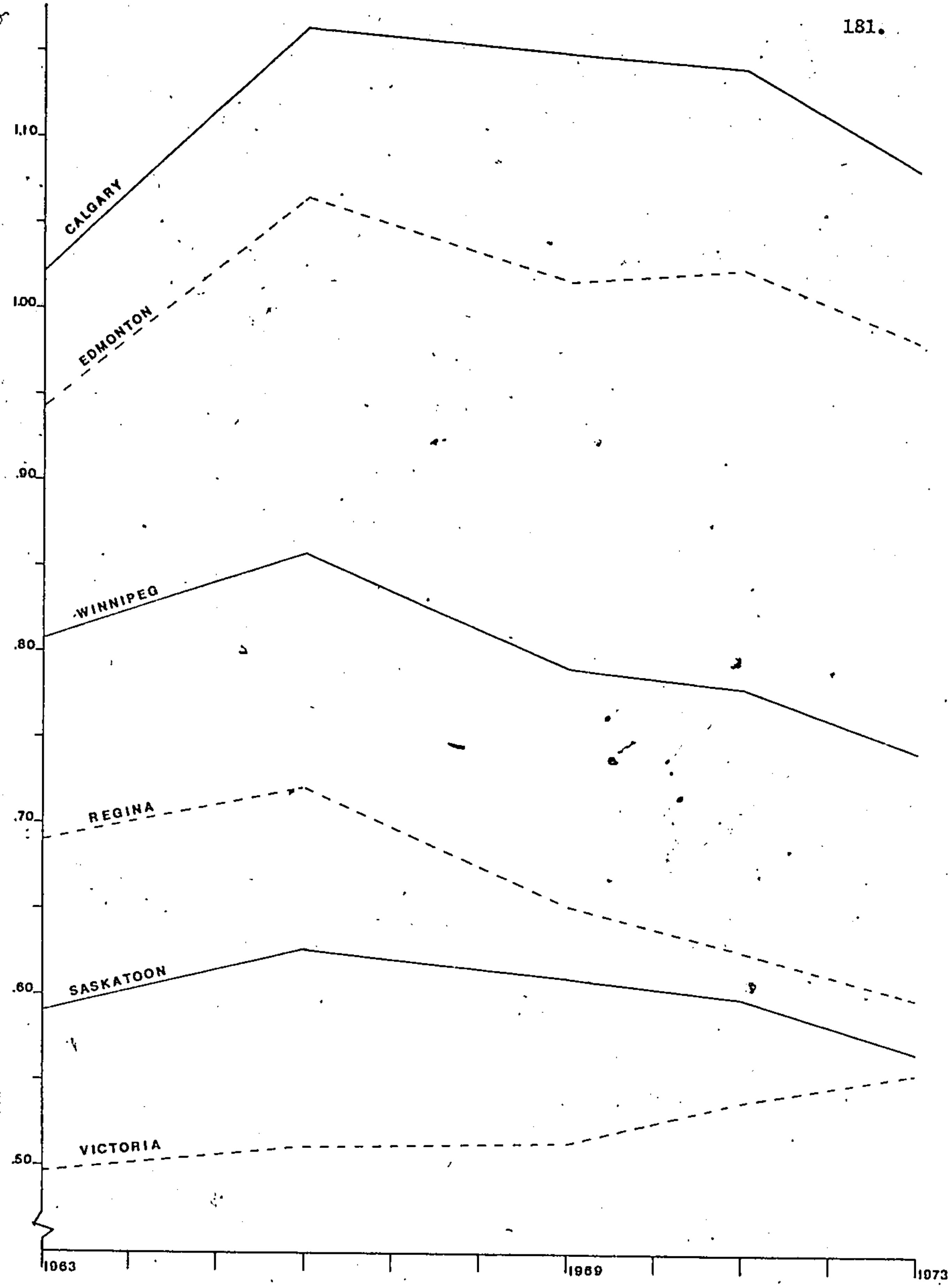

FIGURE 4-8 INDEX OF TOTAL SYSTEM SENSITIVITY, WESTERN SYSTEM

them to the core cities, or the arcs linking two core cities, but they are influenced very little by any flow changes on the arcs between the secondary elements outside this western Heartland. This is quite evident in particular in the dwindling influence of Winnipeg on Regina and Saskatoon. Both cities have much stronger bonds in 1973 with Vancouver, Calgary, and Edmonton. Victoria finds itself in 1973 well entrenched within the western system whereas it was previously but an extension of Vancouver, in its shadow as it were.

Part IV: Performance of the Canadian Metropolitan System:

Air-Passenger Traffic, 1963-1973

The performance indices (P.I.) have been calculated for the main sub-networks surrounding each of the Canadian CMAs for the 1963-1973 period.

If the P.I. value remains constant throughout the period, then either there has been no change within the network structure and its use, or both the structure and the use made of it have evolved in the same fashion at the same rate. If the P.I. values increase with time, either the relative share of interactance ratio transmitted through the sub-network has increased while the relative share of interactance multiplier has remained stable, or the share of interactance ratio has remained constant while the multiplier share has decreased, or again both might be decreasing with the multiplier decreasing at a greater rate than its interactance ratio counterpart. If the P.I. values drop over time, it may be due to a reduction of activity on the arc linking the sub-network to the major centre, or to an increase in the sub-network's relative share of interactance multiplier (I.M.). Finally, if both values are dropping, the lower P.I. values show that the losses of the interactance ratios have been more substantial than those of the interactance multipliers.

The Montreal and Vancouver Internal Networks:

Along with Toronto, Montreal and Vancouver are the strongest elements of the Canadian Metropolitan System and their sub-networks are extensive. The term "internal network" as used in this section refers to the interactance multiplier sub-networks illustrated in Figure 3-6A. The interactance multiplier is itself a coefficient related to the magnitude of the interactions of these sub-networks, but the term "internal network" refers to the set of centres and arcs that link the study node to another dependent node. Thus in Table 4-12, the Calgary-Montreal internal network refers to the network that binds Montreal to Toronto through Calgary.

In general, the performance indices of the Montreal sub-networks increased over the 1963-1973 period. The P.I. of the network linking Montreal directly to Toronto dropped from 4.5 to 3.6. The drop is attributable to a decrease in the relative importance of Montreal's flows into Toronto and to a decrease in the I.M. value of this network. A larger share of Montreal's interaction has been through its other networks, namely, Ottawa-Hull, Vancouver, and Halifax.

Vancouver's networks underwent a different evolution (Table 4-13). The P.I. of Toronto-Vancouver increased from 1.4 to 1.7 in spite of a decrease in the I.M. of this sub-network. The increase is attributable to Vancouver's greater flows into Toronto. The lower I.M. value of this sub-network is counterbalanced by increases in the I.M. values of its Ottawa-Hull and Montreal networks. The table also reports increases in the I.R. values of the Ottawa-Hull, Calgary, and Edmonton sub-networks.

The maps of Figures 4-9 and 4-10 compare the complete set of 1963 and 1973 performance indices of the two cities. In Montreal's case the 1973 values

Table 4-12 Performance of the Montreal Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Montreal	16.5	70.1	4.3	16.7	67.7	4.1	16.6	64.5	3.9	17.0	63.8	3.7	17.6	63.5	3.6
Ottawa-Hull-Montreal	3.9	5.7	1.5	3.9	5.8	1.5	3.8	6.4	1.7	4.1	7.4	1.8	4.3	7.3	1.7
Vancouver-Montreal	4.5	1.4	0.3	4.6	3.7	0.8	4.7	4.7	1.0	4.7	4.7	1.0	4.8	5.2	1.1
Winnipeg-Montreal	4.4	3.5	0.8	4.4	3.8	0.9	4.5	4.5	1.0	4.5	4.0	0.9	4.5	3.8	0.8
Calgary-Montreal	4.5	1.4	0.3	4.6	1.9	0.4	4.7	2.4	0.5	4.7	2.3	0.5	4.8	2.4	0.5
Halifax-Montreal	7.3	3.1	0.4	7.2	3.4	0.5	7.1	4.0	0.6	6.8	3.9	0.6	6.5	3.8	0.6
Quebec-Montreal	11.6	2.9	0.3	10.9	2.3	0.2	9.8	2.1	0.2	9.4	2.2	0.2	9.1	2.2	0.2

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-13 Performance of the Vancouver Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Vancouver	21.6	29.5	1.4	20.4	29.7	1.5	19.3	29.9	1.5	19.1	31.6	1.7	18.5	31.7	1.7
Montreal-Vancouver	2.2	20.0	9.1	2.8	20.5	7.3	3.1	19.5	6.3	2.9	17.6	6.1	3.1	17.8	5.7
Ottawa-Hull-Vancouver	2.6	8.1	3.1	3.1	8.4	2.7	3.3	8.7	2.6	3.3	9.6	2.9	3.6	9.5	2.6
Calgary-Vancouver	10.1	6.7	0.7	9.7	7.1	0.7	9.6	7.8	0.8	9.9	7.8	0.8	9.5	7.6	0.8
Edmonton-Vancouver	9.9	6.4	0.7	9.7	6.2	0.6	9.8	6.7	0.7	10.0	6.5	0.7	9.6	6.7	0.7
Winnipeg-Vancouver	6.4	10.8	1.7	6.2	9.8	1.6	6.5	10.2	1.6	6.5	9.4	1.5	6.5	8.5	1.3

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

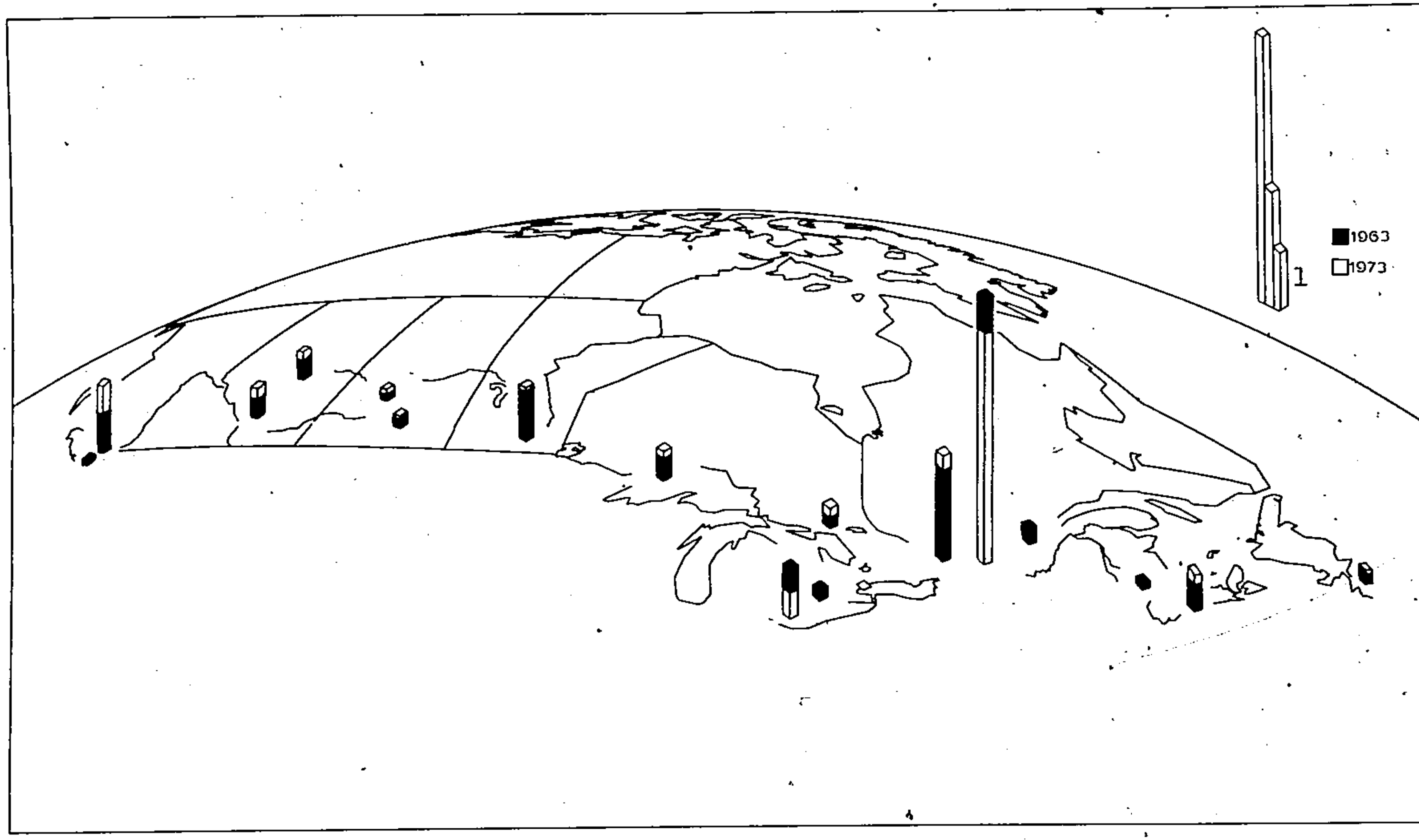

Figure 4-9 Performance of the Montreal Internal Networks, Air-Passenger Flows

Note: The Performance Maps were drawn by means of computer programmes developed by Professor D. Douglas of the Department of Geography, University of Ottawa.

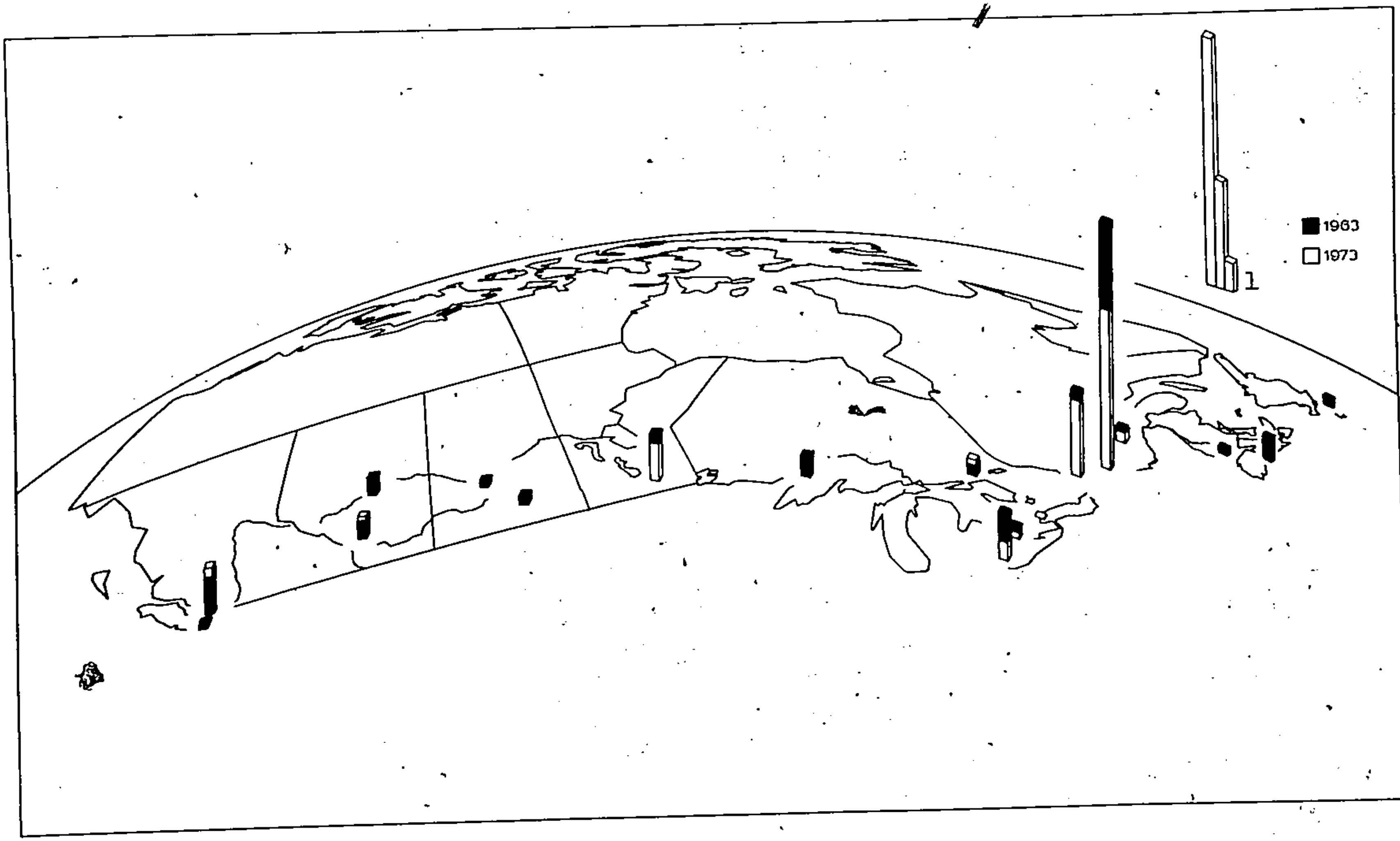

Figure 4-10 Performance of the Vancouver Internal Networks, Air-Passenger Flows

are generally higher while in Vancouver's case the opposite holds true.

The Ottawa-Hull, Calgary, and Edmonton Internal Networks:

The three CMAs of Ottawa-Hull, Calgary, and Edmonton form a very dynamic set of urban centres within the Canadian Metropolitan System. Ottawa-Hull has made remarkable changes throughout the decade. In almost every instances its networks show higher performance indices (Table 4-14). The higher values are due in part to a more evenly distributed pattern of interactance multipliers and in part to increased activity on the arcs directly linked to Toronto. Ottawa-Hull received a smaller share of its interaction from Toronto and Montreal and a greater share from most of its other networks in 1973.

Calgary and Edmonton also report higher P.I. values. Both centres received a larger share of their interaction from Vancouver, Ottawa-Hull, and each other as the I.R. values show. The interactance multipliers of their Ottawa-Hull, Montreal, and Winnipeg sub-networks are higher but the I.M. of the Toronto and Vancouver networks are lower. These network changes show a greater interdependency of these centres within the national system.

The London, Windsor, Quebec, Sudbury and Thunder Bay Internal Networks (Tables 4-17, 4-18, 4-19, 4-20, 4-21):

London, Windsor, and Quebec show higher P.I. values for their Ottawa-Hull sub-network in 1973. In all three cases, there has been a large increase in the I.R. values and a slight increase in the I.M. values. Therefore, the higher indices are the outcome of the greater share of flows Ottawa sends to Toronto and to the fact that the Ottawa-Hull sub-network carries a larger share of the interaction of the three cities. Quebec also shows higher I.M., I.R. and P.I. values for its Toronto sub-network as a result of its increased interaction with this centre.

Table 4-14 Performance of the Ottawa-Hull Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Ottawa-Hull	33.2	58.9	1.8	30.7	56.5	1.8	29.0	55.9	1.9	25.8	54.0	2.1	24.7	51.4	2.1
Montreal-Ottawa-Hull	3.3	16.9	5.2	3.1	15.5	5.0	3.3	15.0	4.5	2.4	14.9	4.4	3.5	15.3	4.4
Vancouver-Ottawa-Hull	4.3	3.4	0.8	4.3	4.3	1.0	4.3	4.9	1.1	4.5	5.4	1.2	4.6	6.1	1.3
Winnipeg-Ottawa-Hull	4.2	4.2	1.0	4.5	4.7	1.1	4.4	5.0	1.1	4.7	4.9	1.1	4.7	4.7	1.0
Calgary-Ottawa-Hull	4.6	1.7	0.4	4.5	2.2	0.5	4.4	2.6	0.6	4.7	2.1	0.6	4.7	2.9	0.6
Quebec-Ottawa-Hull	5.0	1.5	0.3	5.1	1.3	0.3	5.9	1.4	0.2	6.2	1.7	0.3	6.3	1.9	0.3
Halifax-Ottawa-Hull	5.1	2.6	0.5	4.9	2.9	0.6	4.9	3.2	0.6	5.3	3.5	0.7	5.4	3.9	0.7

*Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-15 Performance of the Calgary Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Calgary	26.0	19.0	0.7	23.5	19.0	0.8	21.5	19.5	0.9	21.5	18.9	0.9	21.2	19.3	0.9
Vancouver-Calgary	8.8	13.6	1.6	8.6	13.9	1.6	8.6	14.8	1.7	8.7	16.1	1.9	8.3	16.2	2.0
Winnipeg-Calgary	6.0	11.4	1.9	6.0	10.5	1.8	6.2	10.8	1.7	6.2	10.2	1.6	6.1	9.2	1.5
Edmonton-Calgary	11.8	8.6	0.7	12.3	8.8	0.7	12.0	9.0	0.8	12.0	8.8	0.7	11.0	8.9	0.8
Montreal-Calgary	1.9	19.1	10.1	2.4	19.8	8.3	2.7	18.9	7.0	2.5	17.3	6.8	2.7	17.2	6.4
Ottawa-Hull-Calgary	2.4	8.0	3.5	2.8	8.5	3.0	3.0	8.8	2.9	3.0	9.8	3.3	3.2	9.8	3.1

*Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-16 Performance of the Edmonton Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Edmonton	26.6	19.0	0.7	24.9	18.0	0.7	24.0	18.0	18.0	24.0	18.0	0.7	23.0	18.0	0.8
Vancouver-Edmonton	8.9	14.0	1.5	8.7	14.0	1.6	8.6	15.0	1.7	8.5	16.0	1.9	8.0	16.0	2.0
Winnipeg-Edmonton	6.0	11.3	1.9	5.9	10.0	1.8	6.0	10.0	1.7	6.0	10.0	1.7	6.0	9.0	0.9
Calgary-Edmonton	12.0	9.0	0.7	12.0	10.0	0.8	11.9	11.0	0.9	12.0	11.0	0.9	11.0	10.0	0.9
Montreal-Edmonton	1.9	19.0	10.0	2.4	18.7	8.0	2.7	18.6	7.0	2.5	17.0	6.9	2.6	17.1	6.5
Ottawa-Hull-Edmonton	2.5	8.5	3.4	2.9	8.9	3.0	3.0	9.0	3.0	3.1	10.0	3.3	3.3	10.0	3.0

* Toronto as the Major Node
IN -TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-17 Performance of the London Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-London	59.7	29.1	0.5	60.7	27.0	0.4	61.0	21.9	0.4	60.4	20.1	0.3	58.1	20.9	0.4
Montreal-London	2.4	28.2	11.9	2.3	26.0	11.3	2.4	26.0	10.8	2.5	25.4	10.3	2.5	24.0	9.6
Ottawa-Hull-London	2.7	11.1	4.1	2.9	12.1	4.2	3.1	14.3	4.6	3.3	16.0	4.9	3.3	15.2	4.5

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-18 Performance of the Windsor Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Windsor	59.0	64.6	1.1	57.0	57.0	1.0	62.5	53.1	0.9	61.5	50.6	0.8	61.3	47.4	0.8
Montreal-Windsor	2.7	15.6	5.8	2.9	17.4	6.0	2.6	17.8	6.8	2.8	18.0	6.6	2.9	19.6	6.7
Ottawa-Hull-Windsor	2.5	5.1	2.0	2.9	6.4	2.2	2.5	7.4	2.9	2.7	8.4	3.2	2.7	8.4	3.2

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-19 Performance of, the Quebec Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Quebec	47.0	19.5	0.4	52.0	20.0	0.4	56.8	23.0	0.4	55.0	24.5	0.5	56.0	26.8	0.5
Montreal-Quebec	7.3	50.4	6.9	6.4	46.6	7.3	5.3	39.8	7.5	5.0	35.8	7.2	4.7	33.0	7.0
Ottawa-Hull-Quebec	3.8	9.0	2.4	3.6	9.7	2.7	3.6	11.6	3.2	4.0	13.6	3.4	3.9	13.0	3.3

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-20 Performance of the Sudbury Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Sudbury	93.0	79.0	0.8	90.0	75.0	0.8	88.0	77.0	0.9	82.0	72.0	0.9	83.0	72.0	0.9
Montreal-Sudbury	0.4	8.0	20.0	0.6	10.0	15.0	0.7	8.0	11.6	1.0	9.4	9.4	0.9	8.4	9.0
Ottawa-Hull-Sudbury	0.4	2.5	7.0	0.7	4.0	5.7	0.8	4.0	5.0	1.2	5.0	4.3	1.2	5.0	4.3

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-21 Performance of the Thunder Bay Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Thunder Bay	67.7	57.0	0.8	65.0	56.0	0.9	65.5	58.0	0.9	63.0	60.0	0.9	64.0	58.0	0.9
Montreal-Thunder Bay	1.3	13.0	10.0	1.5	12.6	8.2	1.5	11.0	7.3	1.5	10.0	6.6	1.4	9.7	6.9
Winnipeg-Thunder Bay	5.1	10.0	1.9	5.2	9.2	1.8	4.6	8.5	1.8	4.8	7.6	1.6	4.7	7.4	1.6

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Sudbury received a slightly larger share of its interaction through its Ottawa-Hull network, but it remained highly dependent on Toronto right through the period. Thunder Bay showed a small decrease in its dependency on its Winnipeg sub-network and a comparable increase in its ties with Toronto. Both cities are regional centres highly dependent on Toronto.

The Winnipeg, Victoria, Regina, and Saskatoon Internal Networks (Tables 4-22, 4-23, 4-24, 4-25):

Winnipeg, Victoria, Regina, and Saskatoon reported higher P.I. values for their Vancouver sub-networks in 1973. Winnipeg, Regina, and Saskatoon received a larger share of their interaction through this network principally as a result of the increased importance of Vancouver's flows to Toronto. Regina and Saskatoon also reported increases in their I.M., increases which suggest their greater dependency on their Vancouver sub-networks. The P.I. values for the Calgary sub-networks of all four cities have increased. For Winnipeg, Victoria, and Saskatoon these increases in P.I. are accompanied by increases in the I.R. and I.M. values. The cities received a larger share of their interaction through their Calgary sub-networks. Victoria showed increases for all three parameters of its Ottawa-Hull sub-network and Winnipeg exhibited almost similar patterns. Both cities show greater interdependency with the national capital. Regina and Saskatoon report decreases in P.I. values and increases in I.R. and I.M. values for their Ottawa-Hull sub-networks. They too show a relative increase in their ties with Ottawa over the period.

The Halifax, Saint John, and St. John's Internal Networks (Tables 4-26, 4-27, 4-28):

The three Atlantic centres of Halifax, Saint John and St. John's have undergone a number of similar changes in their networks over the decade. In general, they show relative increases in interaction with Toronto and decreases

Table 4-22 Performance of the Winnipeg Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Winnipeg	24.6	36.6	1.5	23.4	34.3	1.5	23.0	35.0	1.5	24.0	34.0	1.4	24.0	32.0	1.3
Vancouver-Winnipeg	7.0	8.6	1.2	6.6	7.0	1.1	6.7	10.0	1.5	6.9	11.0	1.6	6.4	11.0	1.7
Calgary-Winnipeg	7.8	4.5	0.6	7.2	4.9	0.7	7.1	5.5	0.8	7.2	5.5	0.8	6.9	5.6	0.8
Montreal-Winnipeg	2.4	19.0	8.0	2.8	19.1	7.0	3.0	18.0	6.0	2.8	17.0	6.0	2.9	16.5	5.8
Ottawa-Hull-Winnipeg	2.9	7.9	2.7	3.4	8.6	2.5	3.4	8.6	2.5	3.5	9.7	2.8	3.6	9.8	2.7

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-23 Performance of the Victoria Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Victoria	46.0	14.0	0.3	47.8	15.0	0.3	47.6	18.0	0.4	47.0	16.0	0.3	46.0	15.0	0.3
Vancouver-Victoria	9.6	19.2	2.0	8.0	18.0	2.0	7.0	16.5	2.3	6.6	17.0	2.6	6.0	16.4	2.7
Calgary-Victoria	6.7	6.4	0.9	6.1	8.9	1.5	5.9	7.2	1.2	6.6	8.0	1.2	6.7	8.5	1.3
Montreal-Victoria	1.5	19.7	13.0	1.9	20.0	11.0	2.0	18.7	9.0	1.8	17.0	9.4	1.9	17.0	9.0
Ottawa-Hull-Victoria	2.0	9.0	4.5	2.4	9.9	4.0	2.4	9.5	4.0	2.3	10.0	4.5	2.4	10.0	4.3

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-24. Performance of the Regina Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Regina	41.0	18.0	0.4	40.0	18.0	0.5	41.0	18.0	0.4	42.0	18.0	0.4	42.0	19.0	0.5
Vancouver-Regina	6.0	11.0	1.8	5.6	11.0	2.0	5.6	12.0	2.2	5.8	13.0	2.3	5.7	14.0	2.4
Calgary-Regina	8.7	7.7	0.9	7.8	7.7	1.0	7.0	8.1	1.1	7.1	7.8	1.1	6.9	7.7	1.1
Montreal-Regina	1.5	18.0	12.0	1.9	19.0	10.0	2.0	17.8	8.7	1.9	16.0	8.5	2.0	16.0	8.0
Ottawa-Hull-Regina	2.1	8.4	4.0	2.4	8.9	3.7	2.5	9.2	3.8	2.6	10.5	4.1	2.8	10.5	3.8

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-25. Performance of the Saskatoon Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Saskatoon	51.0	18.5	0.4	49.0	17.0	0.3	47.0	16.0	0.3	46.7	18.5	0.4	47.0	20.0	0.4
Vancouver-Saskatoon	4.9	10.4	2.1	4.7	11.0	2.3	5.0	12.0	2.4	5.0	13.0	2.5	5.0	14.0	2.6
Calgary-Saskatoon	6.4	6.5	1.0	5.9	6.9	1.2	6.0	7.9	1.3	6.2	7.3	1.2	6.0	7.0	1.2
Montreal-Saskatoon	1.4	19.0	14.0	1.6	19.0	12.0	1.9	18.0	9.8	1.8	16.0	9.0	1.9	16.0	8.6
Ottawa-Hull-Saskatoon	1.8	8.7	4.8	2.1	9.0	4.3	2.2	9.0	4.0	2.3	10.0	4.4	2.5	10.0	4.0

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-26 Performance of the Halifax Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Halifax	37.0	33.0	0.9	37.0	33.0	0.9	37.0	35.0	0.9	36.0	35.0	0.9	36.2	36.0	1.0
Saint John-Halifax	16.0	3.8	0.2	14.5	3.4	0.2	13.4	2.8	0.2	12.0	2.6	0.2	11.9	2.6	0.2
St. John's-Halifax	15.7	3.8	0.2	14.5	3.4	0.2	13.4	2.8	0.2	12.0	2.6	0.2	11.9	2.6	0.2
Montreal-Halifax	3.5	30.9	8.9	4.0	30.3	7.5	4.3	28.2	6.6	4.0	25.4	6.4	3.8	23.6	6.0
Ottawa-Hull-Halifax	2.9	9.0	3.1	3.4	9.6	2.8	3.4	9.1	2.8	3.7	11.0	3.0	3.9	11.3	3.0

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

Table 4-27 Performance of the Saint John Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-Saint John	57.5	23.0	0.4	64.0	24.0	0.4	67.0	25.0	0.4	69.0	27.0	0.4	69.0	28.0	0.4
Halifax-Saint John	10.9	15.0	1.3	8.6	12.0	1.4	7.5	12.7	1.7	6.6	11.7	1.8	6.4	12.0	1.9
Montreal-Saint John	2.5	33.7	13.5	2.8	33.0	12.0	2.6	30.8	11.2	2.4	28.0	11.7	2.3	26.0	11.5
Ottawa-Hull-Saint John	1.8	8.3	4.6	2.0	8.8	4.5	1.7	8.7	5.0	1.8	10.0	5.6	1.9	10.0	5.4

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : Interactance Multiplier
IR : Interactance Ratio
PI : Performance Index

Table 4-28 Performance of the St. John's Internal Networks, Air Traffic Data *

IN - TN	1963			1966			1969			1971			1973		
	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI	IM	IR	PI
Toronto-St. John's	64.0	31.0	0.5	61.0	33.0	0.6	62.8	33.0	0.5	62.0	35.0	0.6	62.0	36.0	0.6
Halifax-St. John's	9.1	11.0	1.3	8.9	11.0	1.2	8.0	11.0	1.4	7.9	11.0	1.4	8.1	11.5	1.4
Montreal-St. John's	2.4	30.8	12.7	2.7	28.0	10.0	2.8	26.9	9.8	2.5	23.0	9.3	2.5	22.3	8.8
Ottawa-Hull-St. John's	1.9	8.2	4.4	2.3	8.9	3.8	2.1	8.7	4.0	2.3	10.3	4.4	2.4	9.8	4.2

* Toronto as the Major Node
IN - TN : Initial and Terminal Nodes of the Internal Networks
IM : % Interactance Multiplier
IR : % Interactance Ratio
PI : Performance Index

with regards to Montreal. The three cities also report higher parameter values for their Ottawa-Hull sub-networks. The sub-networks linking these three cities have decreased relatively in importance as their interaction with Toronto and Ottawa-Hull have grown over the decade.

The objective of this section has been to apply a technique for revealing system performance through the comparative analysis of the structure and operation of a system's component parts and, in particular to indicate the performance of the Canadian metropolitan system. The technique has distinguished between performance fluctuations that are due to network development from those that are simply the result of variations in flow size of the system's main arcs. The analysis has revealed the pattern of linkage development of the metropolitan areas forming the national system. It has shown the higher and more widespread interaction resulting from network growth and operation.

Conclusion:

The major features of the Canadian Metropolitan System brought out in the analyses of air passenger flows are as follows:

a) Toronto is the major node in the whole national system and it has grown in importance over the period;

b) after Toronto, Montreal and Vancouver exert the greatest influence;

c) another set of less important nodes that have national as well as regional roles are Ottawa-Hull, Calgary, and Edmonton;

d) in the east, the key heartland cities comprise Toronto, Montreal Ottawa-Hull and London. The other centres forming the eastern hinterland are: Quebec, Windsor, Sudbury, Thunder Bay, and the Atlantic cities;

e) Winnipeg, Victoria, Regina, and Saskatoon form a western hinterland around the core cities of Vancouver, Calgary and Edmonton.

Although restructuring of the eastern and western sub-systems into two large entities each with its own core and hinterland comes out clearly in all the analyses, the positions of Quebec and Windsor may seem anomalous. Further research will be required before those centres can properly be located within this centre-periphery type classification (Stone, L.O., 1976, p. 310).

The performance analyses show that the linkages of Ottawa-Hull, Calgary, and Edmonton have attained prominence at a national scale. The increase in their volume of flows as well as the greater diversification of their linkages support this argument.

At the regional level the analyses reveal several important developments that have led to the two large regional sub-systems now in place. Most pertinent has been the growth of the Vancouver networks in the western sub-system and a slight regression of the Montreal networks in the east. Although Vancouver dominates the western network, Calgary and Edmonton also exert strong influence. In the east, Toronto and Montreal dominate. In the west greater use of the existing network has been largely responsible for the creation of the new large western system. In the east, the analyses show that the establishment of a single large sub-system has resulted from the absorption within the Toronto sub-system of the Atlantic networks.

Application of the A.S.T. technique has enabled us to carry interaction analysis down to the sub-network and individual arc level, an approach that is not feasible with either gross flow or factorial analysis. The A.S.T. technique has thus provided an additional insight into urban system operation. The analyses reported in this chapter have outlined the changing interaction patterns of intermetropolitan air-passenger flows in the Canadian Metropolitan System. They have shown the relative importance of the various metropolitan networks and the changes in the patterns over the 1963-1973 period. They have measured the strength of the numerous sub-networks by which a centre interacts with others and contrasted the evolution of the sub-systems over the period. The analyses have also measured changes in the sensitivity of the nodes to movement on the arcs over this same period. Finally, they have discussed the performance of the various sub-networks of each of the nodes used in transmitting activity through the system.

The A.S.T. analyses have simply identified and measured these patterns and the nature of the changes they have undergone. The interaction patterns are, of course, only the external shape of the urban system's activity, the symptoms of the interaction processes as it were. For a more thorough understanding of urban systems it is obviously essential to arrive at some understanding of the demographic, economic, and political processes by which the interaction patterns are generated and transformed and this the A.S.T. model cannot do.

Nonetheless, the A.S.T. technique may be expected to serve a useful purpose in urban analysis in two respects at least. Firstly, a more detailed knowledge of a system's interaction patterns can help in the formulation of a number of relevant hypotheses about the processes generating interaction. A thorough analysis of the interactance ratios, of the evolution of a node's sub-networks, or of the system's sensitivity patterns may prove to be of some assistance in the formulation of a number of detailed hypotheses about their causes. Secondly, because the A.S.T. technique provides quantitative measures of system interaction properties, it provides by the same token the possibility of relating interaction patterns to structural characteristics. This could possibly be achieved either through multiple regression or canonical correlation analyses.

CHAPTER 5

CANADIAN INTERMETROPOLITAN MIGRATION: A.S.T. ANALYSIS

Introduction

This chapter applies the A.S.T. model to intermetropolitan migration data. The system characteristics revealed with this data are then compared and contrasted with the results reported for the air passenger studies of Chapter Four. The specific objectives of these migration analyses are a) to outline the principal characteristics of the Canadian metropolitan system with respect to migration interaction, b) to determine the position within the system of those metropolitan centres which were not included in the air passenger analyses, and c) to compare the interaction patterns of migration and air passenger networks. Little attempt will be made here to undertake the complex task of evaluating the nature and strength of the factors promoting or inhibiting migration flows as this would be beyond the scope of this research.

The origin-destination matrix of migrants used in this chapter covers a single period from 1966-1971, (table A4-78). Though this is a short period, the data is sufficient for two purposes: first, it is sufficient for the measurement of system properties, and second, it provides results that can be compared with those from the air passenger analyses for the same period.

It must be pointed out that geographical units for the compilation of the migration data do not necessarily coincide with those of the air passenger data. In the case of migration, the units are the Census Metropolitan Areas of the 1971 Canadian Census. There are problems associated with the use of the census definitions (Simmons,1976,p.73) but there is also an advantage in that it becomes possible to correlate the migration flows with other variables reported for the same areas.

The airport catchment areas have no such well defined boundaries, a fact which creates some difficulty when it is desirable to compare migration and air-flow systems. The difficulty arises in particular with respect to the Toronto, Montreal, and perhaps Vancouver airports,for in all other instances it is common practice to equate the Census Metropolitan Area and the Airport Market Area (see the Airport Operations analysis Reports prepared for the Ministry of Transport by the Airports and Fields Operations Branch, Department of Transport Library, Ottawa).

There are at least two air-passenger survey reports for the Toronto Malton airport: The Study of Airport Utilization vs Accessibility (1972) and the Originating-Terminating Passenger Forecasts for the Toronto Airports System, Volume II,Appendices (1974). The first of these two reports analyzes data collected in 1968. It shows that over 80% of the domestic flows originated within 45 miles of the airport and 95% within 75 miles. The second report (Exhibit A-4 of the report) estimates on the basis of a sample survey taken at Malton in 1971 that 82% of the domestic flows originated within the Toronto CMA. For the Montreal Dorval Airport an air-passenger survey (NMIAPO Planning Group, 1973) shows that 93% of the flows origninated within the Greater Montreal Region. This area includes Montreal Island and Laval, the north shore as far as Joliette and St. Jerome, the south shore from Beauharnois to St.Hyacinthe (19 miles) and the western centres as far as Vaudreuil and Soulanges (23 miles). Though no such survey is available for the Vancouver airport, it Is likely that a large percentage of the flows are from the Vancouver CMA. It seems reasonable to conclude that a comparison of the migration and air-passenger systems would not be overly distorted because of the discrepancies in the definition of the geographical units.

From the viewpoint of grasping the evolution of the metropolitan system, the importance of intermetropolitan migration stems from two facts (Alonso, 1971). Firstly, internal migration is becoming a major element of city population growth as birth rates decline and foreign immigration is curtailed. Internal migration is a major contributor, therefore, to the redistribution of our human resources. As migrants are generally young, educated, and perhaps even enthusiastic, this redistribution favours some cities at the expense of others. Secondly, the sheer size of the migration volumes has repercussions in time on the social and economic structure of cities (Tables 5-1 and 5-2).

Close to 50% of the Canadian population changed residence between 1966 and 1971 and for every two changes in residence, one entailed a change of municipality, which is defined as a migration. The tables show that urban people are more mobile than others and that inter-provincial migration had become more widespread in the 1966-71 period than in the previous period of 1956-61. Close to 15% of the population of the Canadian Metropolitan Areas were in-migrants in 1971.

Migration ratios are systematically associated with a network of economic and social indicators (Stone,1969, p.18). The mean gross migration ratio increased from 17.7% in the earlier period to 26% in the latter, that is, one person in four living in the CMAs was either entering into or emerging from the migration streams during this period. The net migration ratios for the individual metropolitan centres are even more revealing. They vary tremendously, going from -4.2% for Chicoutimi-Jonquiere to 7% for Calgary. The centres with the highest net ratios are Calgary, Ottawa-Hull, Victoria, Quebec, and Vancouver. The lowest centres are Chicoutimi-Jonquiere, Halifax, Winnipeg, and Regina. A few centres, such as St. John's, Quebec, and Montreal draw a very large percentage of their internal migrants from non-CMA centres.

The figures reported in these tables illustrate the growing extent of mobility of the Canadian population, presumably a partial response to changing economic activity patterns within the country. A better knowledge of the characteristics of these flow patterns should enhance our understanding of the Canadian urban system.

A.S.T. Analysis of Migration Flows, 1966-1971:

The migration data matrix is assumed linear, transformed into a set of flow-graph equations and subjected to flow-graph analysis for three purposes. Firstly, it will reveal the system's structure at a given moment, for no matter how curvilinear is the urban system's curve, any point on the curve may be taken as constituting a linear function, though of course no projections can be made from the results obtained from this approach. Secondly, the analysis will provide some indication of the roles played within the Canadian urban system by the CMAs not included in the previous analyses because they lacked a major airport. Finally, the analyses may reveal similar or perhaps contrasting characteristics to those featured in the preceding air traffic studies.

Before the migrant flow matrix is run through the A.S.T. programme for analysis, the matrix is transposed to provide a destination-origin matrix, rather than an origin-destination one as was the case in the previous analyses. The logic behind the transformation is quite straightforward. In migrant flow analysis the fundamental system characteristics that are sought are tied to the process by which a centre is capable of drawing migrants out of other centres, and not to its own emission of migrants (Alonso, 1971 p. 328). The essential features are not linked to the emission field of a centre but to its attraction field and the network that extends throughout this field and supports it. In the case of information flows (e.g. air passenger traffic) the main properties of the system have been related to the process of information emission on the assumption that dynamism of a centre is better measured in terms of its ability to generate outbound interaction flows than through its information drawing power. Thus a destination-origin matrix is required in the migrant flow case, and an origin-destination one in the analysis of information flows. This is the conclusion that has been reached in this study after a careful consideration of the results of factor and A.S.T. analyses performed on the Canadian intermetropolitan migration data.

In the analyses of interactance ratios, multipliers, and sensitivities that are to follow, Toronto has been taken as the major node since it has received the greatest number of intermetropolitan migrants and each of the other CMAs will in turn serve as study nodes.

Table 5-1

IN-MIGRANTS, 5 YEARS AND OVER AS A % OF
1971 TOTAL POPULATION, 5 YEARS AND OVER

A) Origin of Internal Migrants:

	1956-61	1966-71	Dif.
Urban centres	21.5%	24.9%	3.4%
Rural non-farm	22.6%	25.0%	2.4%
Rural farm	8.6%	9.7%	1.1%

B) Inter and Intra Provincial Migration:

	Intra		Inter	
	1956-61	66-71	1956-61	66-71
Urban centres	13.5%	13.8%	3.9%	4.7%
Rural non-farm	17.5%	17.9%	3.4%	3.9%
Rural farm	6.9%	7.3%	1.0%	1.1%

C) CMA Internal Migration:

	1956-61	1966-71
In-migration ratio	9.9%	13.7%
Out-migration ratio	7.8%	12.4%
Net in-migration ratio	2.2%	1.4%
Gross migration ratio	17.7%	26.0%

Source: Stone, L. O., Migration In Canada
Canada: 1971 Census, Cat. 92-746, vol. 1, part 5.

Table 5-2

IN-MIGRANTS, 5 YEARS AND OVER
AS A % OF 1971 CMA POPULATION, 5 YEARS AND OVER

	R_1	R_2	R_3	R_4
Calgary	21.4	15.9	7.0	36.2
Chicoutimi-Jonquiere	8.5	11.8	-4.2	21.3
Edmonton	17.9	15.5	3.2	32.6
Halifax	16.1	17.3	-1.8	34.0
Hamilton	10.0	8.6	1.4	18.6
Kitchener-Waterloo	15.9	13.1	3.7	28.2
London	16.2	13.8	2.9	29.5
Montreal	6.3	6.5	-0.3	13.0
Ottawa-Hull	15.5	10.9	5.1	25.9
Quebec	11.8	7.7	4.2	19.4
Regina	20.0	20.8	-1.7	41.5
St. Catherines-Niagara	8.3	8.7	-0.6	17.2
St. John's	12.2	11.0	1.2	23.1
Saint John	10.1	10.0	-0.6	20.9
Saskatoon	23.6	24.3	-0.7	48.0
Sudbury	16.2	14.0	2.6	29.9
Thunder Bay	10.3	9.7	0.2	20.5
Toronto	7.7	9.0	-0.8	16.2
Vancouver	13.1	9.1	4.6	21.6
Victoria	19.5	13.1	6.9	32.0
Windsor	7.9	8.2	-0.4	16.1
Winnipeg	11.8	13.3	-1.8	25.4
Mean	13.7	12.4	1.4	26.0

R_1 = In-migration ratio = (Total internal migrants - (intra-CMA movers + unspecified) / population 1971) x 100.

R_2 = out-migration ratio = (Total migrants out of the CMA / population 1966) x 100

R_3 = Net migration ratio = (In-migrants - out-migrants / population 1971) x 100

R_4 = Gross migration ratio = (In-migrants + out-migrants / population 1971 x 100

Source: 1971 Census, Canada, Cat. 92-746, Vol. I, Part 5.

A) Interactance Ratio:

The interactance ratios do nothing more than establish ratios of inter-metropolitan migrant attraction; and the more detailed features of the migration interaction system will not become apparent until the interactance multipliers and sensitivities are analyzed. However, a few comments can be made concerning these ratios.

Table 5-3 presents the set of interactance ratios for the 21 Census Metropolitan Areas. The average value of the set is 0.203 and its standard deviation is 0.178. The vector can be subdivided into three groups with the first including cities having interactance ratios greater than one standard deviation above the mean, that is, greater than 0.381, the second group, those that have an I.R. greater than 0.203 and less than 0.381, and the third, the cities whose I.R. value is below the average.

The first set includes Vancouver, Montreal, and Ottawa-Hull. Vancouver has almost 75% of Toronto's attraction power, while Montreal has less than 50% of Toronto's migrant attraction. That Vancouver displays a migrant attraction so much stronger than Montreal's raises a few interesting questions regarding the interaction patterns of their networks. Are there any differences in the extent of the networks of both these centres? Is the Vancouver network more national in scope than Montreal's? To what degree, if any, is the Montreal network restricted to exchanges with other centres in the province of Quebec? None of these questions can be answered through the use of interactance ratios but the analysis of the interactance multipliers and sensitivities will clarify these and other points regarding these network structures. Considering its size, Ottawa-Hull has a very large drawing power, 44% of Toronto's. Since Ottawa is the nation's capital it might be reasonably expected that its migrant network extends right across the country with its strongest links being with the major administrative and economic centres of the country. These and other points will also be clarified in the subsequent sections.

The second set includes Calgary, Edmonton, Hamilton, London, Winnipeg, and Victoria. The number of migrants they draw ranges from 38% of Toronto's number of migrants in the case of Calgary to 21% in the case of Victoria. Most of the cities of this group along with the cities of the previous group form the cores of the eastern and western sub-systems. Thus, it might be expected that the important roles these centres play within their own sub-systems will be reflected in their migration networks. These network structures should also reflect a number of other features of these centres such as : a) their location within the country, e.g. migration streams of western centres are likely different from those of the Atlantic centres; b) the urban environment in which the centres are located; the higher metropolitan density of southern Ontario will create networks that differ from those of more isolated centres such as Winnipeg; c) and finally, other factors such as economic vitality and the development of new resources that could create linkages which extend beyond the immediate sub-systems of some of these centres. Here again answers to these queries will be provided through the analysis of interactance multipliers and sensitivities.

The third group includes small cities that form the hinterlands of the western and eastern sub-systems, namely: Regina, Saskatoon, Quebec, Windsor, Sudbury, Thunder Bay, Chicoutimi-Jonquiere, Halifax, Saint John, and St. John's. It also contains the centres that are within the eastern heartland but that do not exert much influence beyond their immediate surroundings: Kitchener-Waterloo, and St. Catharines-Niagara. The centres of this third group that draw the most migrants are the larger ones: Quebec, Kitchener-Waterloo, St. Catharines-Niagara, Halifax, and Windsor. The remaining centres are smaller and more peripherally located. The size and location of these centres are surely important factors in determining the number of migrants they attract and the strength and configuration of the networks they have. The principal characteristics of these interaction patterns will also be studied in the following sections.

Table 5-3 lists all the CMAs according to their interactance ratios, including the centres which were not part of the air-traffic studies. The importance of Hamilton within the urban system stands out in Table 5-3 where it is ranked immediately after Edmonton. Kitchener-Waterloo and St. Catharines-Niagara are almost as strong as Quebec while Chicoutimi-Jonquiere fares no better than Saint John. The ratios also serve to point the strong role of London within the system, a point not apparent in the air traffic analyses, because of the proximity of Toronto and its airport. The ratios also bring out the strong position of Halifax within the system as opposed to the much weaker positions held by Saint John and St. John's.

Table 5-3

CMA INTERACTANCE RATIOS,
NATIONAL SYSTEM, 1966-1971

Arc		Interactance Ratio
Vancouver	/Toronto	0.7262
Montreal	/Toronto	0.4712
Ottawa-Hull	/Toronto	0.4351
Calgary	/Toronto	0.3789
Edmonton	/Toronto	0.2982
Hamilton	/Toronto	0.2715
London	/Toronto	0.2261
Winnipeg	/Toronto	0.2081
Victoria	/Toronto	0.2074
Quebec	/Toronto	0.1601
Kitchener-Waterloo	/Toronto	0.1596
St.Catharines-Niagara	/Toronto	0.1294
Halifax	/Toronto	0.1128
Windsor	/Toronto	0.1037
Sudbury	/Toronto	0.0871
Regina	/Toronto	0.0735
Saskatoon	/Toronto	0.0698
Thunder Bay	/Toronto	0.0425
Chicoutimi-Jonquiere	/Toronto	0.0380
Saint John	/Toronto	0.0338
St. John's	/Toronto	0.0286

Mean Value: 0.203 Standard Deviation: 0.178

B) <u>Interactance Multipliers and Sensitivities</u>:

1 - <u>Vancouver, Montreal, and Ottawa</u>

(<u>Tables 5-4-i, 5-4-ii, 5-4-iii</u>)

The sub-networks of Vancouver have interactance multiplier values which are considerably greater than those of Montreal and Ottawa. Vancouver's strength stems first from its position within the well developed western sub-system which provides it with 56% of its migrants, and secondly from its position within the national system through its ties with Toronto, Montreal, and Ottawa-Hull, which combined provide 34% of its migrants. Montreal receives no less than 34% of its intermetropolitan migrants from the cities of Quebec and Chicoutimi-Jonquiere, an indication of restraining cultural factors on its migration attraction (Table A4-79). Ottawa-Hull receives 53% of its migrants from Toronto and Montreal; the others come from all sections of the country in keeping with Ottawa's role as the nation's capital. An examination of the mean values, standard deviation values, and the individual interactance multipliers reveals the principal characteristics of the network structures serving these centres.

The mean value of interactance multiplier for Vancouver is the highest recorded in the analyses and this indicates the centre's strong sub-network structure. However, the high standard deviation value suggests that Vancouver's sub-networks can be classified into two sub-groups with the first having high multiplier values and the second sub-group having very low values. The high multiplier values for the sub-networks of Saskatoon, Regina, and Winnipeg are indicative of their dependency on Vancouver and its networks for their migration patterns. The three centres, Saskatoon, Regina, and Winnipeg, send to

Vancouver and the other western centres that are strongly related to it more migrants than they receive. Any migration activity occurring in these centres should have some impact on Vancouver. The case of Chicoutimi is more difficult to explain. The fact that Chicoutimi has strong ties with Montreal and Ottawa which in turn strongly interact with Vancouver is probably responsible for Chicoutimi's high multiplier value. It is emphasized again that the interactance multipliers only describe the internodal structures and not their actual usage in determining interactance ratios such that, though the sub-networks discussed above might have the highest multiplier values, they do not necessarily have the greatest impact on Vancouver's interactance ratio. Vancouver's intermetropolitan activity is not controlled solely by these sub-networks but also by the flows on the arcs binding these sub-systems to Toronto. Since Saskatoon and Regina have weak flows to Toronto, their sub-networks have only a small effect on Vancouver's migration patterns in spite of their high interactance multiplier values.

Montreal has sub-networks which are stronger on the average than Ottawa-Hull's but are weaker than Vancouver's. The high standard deviation for the Montreal sub-networks indicates they also form two distinct sub-groups. Montreal has strong sub-networks with Chicoutimi, Quebec and Toronto. The first two probably reflect cultural aspects of migration patterns while the latter is presumably a question of economic bonds. The Ottawa-Hull sub-networks have a standard deviation which is somewhat smaller than the mean value which implies that the sub-networks are more evenly distributed, as might be expected in the case of the nation's capital. The Chicoutimi, Toronto, and Quebec sub-networks have the strongest multiplier values. This is due to

Table 5-4-i

MIGRATION INTERACTANCE MULTIPLIERS OF VANCOUVER

Mean Value: 1.169 Standard Deviation: 0.835

	I.M.	% of Total I.M.
Toronto-Saskatoon	2.9608	12.07%
Toronto-Regina	2.9535	12.03%
Toronto-Chicoutimi	2.7760	11.31%
Toronto-Winnipeg	1.9195	7.85%
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Toronto-Windsor	0.4306	1.75%
Toronto-Hamilton	0.3731	1.52%
Toronto-London	0.3676	1.50%
Toronto-Kitchener	0.2997	1.22%

Table 5-4-ii

MIGRATION INTERACTANCE MULTIPLIERS OF MONTREAL

Mean Value: 0.8010 Standard Deviation: 0.8912

	I.M.	% of Total I.M.
Toronto-Chicoutimi	3.9711	13.61%
Toronto-Montreal	1.9356	11.51%
Toronto-Quebec	1.9035	11.32%
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Toronto-Victoria	0.2050	1.22%
Toronto-Vancouver	0.1893	1.13%
Toronto-Kitchener	0.1863	1.11%

Table 5-4-iii

MIGRATION INTERACTANCE MULTIPLIERS OF OTTAWA-HULL

Mean Value: 0.6579 Standard Deviation: 0.5102

	I.M.	% of Total I.M.
Toronto-Chicoutimi	2.2337	16.17%
Toronto-Ottawa-Hull	1.3031	9.43%
Toronto-Quebec	1.0937	7.92%
Toronto-Victoria	0.2197	1.59%
Toronto-Kitchener	0.1865	1.35%
Toronto-Vancouver	0.1792	1.25%

the strong ties that Montreal has with Ottawa-Hull and with each of these other centres.

The interactance sensitivity analyses for the three centres of Vancouver, Montreal, and Ottawa-Hull are interesting in several respects (Tables 5-4-iv; 5-4-v; 5-4-vi). Vancouver has the highest system index although the largest of its individual arc sensitivity values is about half that of Montreal's and Ottawa-Hull's highest values. The Vancouver network is more diversified than either Montreal's or Ottawa-Hull's. The sets of arcs to which each of the centres respond are also revealing. Vancouver is highly sensitive to fluctuations on the arcs linking it to the major poles of the western network and to those that bind it to the centres in the east that have national stature, Toronto, Montreal and Ottawa-Hull. It is also quite sensitive to flows on the secondary arcs within the western networks, e.g. the Regina-Saskatoon arc, and to flows on the arcs that bind Vancouver to the centres on the periphery of the Canadian Heartland, Thunder Bay, Quebec and Chicoutimi-Jonquiere. It shows very little sensitivity to flows on the arcs linking the urban centres of southwestern Ontario.

Montreal shows sensitivity to fluctuations on the arcs within its own provincial territory and to changing flows on the arcs that link it to Toronto, Vancouver, and Ottawa-Hull. It is also sensitive to the arcs between Edmonton and Calgary. Finally it shows some sensitivity to fluctuations on the arcs linking the other Prairie cities.

Ottawa-Hull is sensitive to the network of arcs linking it to Toronto, Montreal, Quebec and Chicoutimi-Jonquiere, an indication that Ottawa-Hull is an important node of the eastern Canada network. It is also sensitive to the

Table 5-4-iv

MIGRATION SENSITIVITIES OF VANCOUVER

Index of Total System Sensitivity: 1.4245

Individual Arc Sensitivities:

Toronto - Vancouver:	0.227
Quebec - Montreal:	0.206
Winnipeg - Vancouver:	0.203
Edmonton - Vancouver:	0.183
Montreal - Quebec:	0.180
Montreal - Vancouver:	0.175
Toronto - Montreal:	0.165
Calgary - Vancouver:	0.147
Calgary - Edmonton:	0.129
Edmonton - Calgary:	0.129
Victoria - Vancouver:	0.117
Montreal - Ottawa:	0.106
Ottawa - Montreal:	0.105

Table 5-4-v

MIGRATION SENSITIVITIES OF MONTREAL

Index of Total System Sensitivity: 1.1824

Individual Arc Sensitivities:

Quebec - Montreal:	0.506
Toronto - Montreal:	0.406
Montreal - Quebec:	0.402
Ottawa - Montreal	0.259
Chicoutimi - Montreal:	0.172
Montreal - Chicoutimi:	0.160
Montreal - Ottawa	0.140
Chicoutimi - Quebec:	0.140
Winnipeg - Montreal:	0.105
Quebec - Chicoutimi:	0.105
Vancouver - Montreal:	0.097
Toronto - Ottawa:	0.096

Table 5-4-vi

MIGRATION SENSITIVITIES OF OTTAWA-HULL

Index of Total System Sensitivities: 0.8251

Individual Arc Sensitivities:

Montreal - Ottawa:	0.406
Toronto - Ottawa:	0.279
Quebec - Montreal:	0.261
Montreal - Quebec:	0.250
Toronto - Montreal:	0.209
Ottawa - Montreal:	0.133
Montreal - Chicoutimi:	0.098
Chicoutimi - Montreal:	0.089
Chicoutimi - Quebec:	0.084

arcs linking the Prairie cities to itself and to Montreal and Toronto.

2 - Calgary, Edmonton, Hamilton, London, Winnipeg, and Victoria (Tables 5-5-i, 5-5-ii, 5-5-iii, 5-5-iv, 5-5-v)

Calgary and Edmonton have stronger sub-network structures than Hamilton, London, Winnipeg or Victoria. Because their standard deviation values are somewhat lower than their average multiplier values it can be assumed that the strengths of the sub-networks of Calgary and Edmonton are more evenly distributed than those of Hamilton, London, and Winnipeg. The low average multiplier values for Hamilton and London suggest that their migration patterns are weak by comparison to those of Calgary, and dominated by a few centres only. Without being as important as Calgary and Edmonton within the western sub-system, Winnipeg and Victoria have fairly high average multiplier values that are indicative of their position within the western structure.

The Total System Sensitivity indices of Calgary and Edmonton show that they are sensitive to fluctuations on a large number of arcs. Both centres are strongly sensitive to two separate groups of arcs. Firstly, they are very sensitive to arcs extended between all of the western centres. Secondly, Calgary and Edmonton are also sensitive to the arcs binding them to the national centres of Toronto, Montreal, Vancouver, and to a lesser degree Ottawa-Hull, and to the arcs between these national centres. On the other hand, both CMAs are rather untouched by the flows on arcs binding the secondary centres of the eastern system, such as arcs linking Hamilton, Sudbury, Windsor and Thunder Bay.

Winnipeg and Victoria have fairly high indices of total system sensitivity. Much like Calgary and Edmonton, both these centres are sensitive to

Table 5-5-i

MIGRATION INTERACTANCE MULTIPLIERS OF CALGARY

Mean Value: 0.718 Standard Deviation: 0.603

	I.M.	% of Total I.M.
Regina-Calgary	2.1398	14.19%
Saskatoon-Calgary	2.1067	13.97%
Chicoutimi-Calgary	1.4632	9.70%
Toronto-Calgary	1.4221	9.43%

Table 5-5-ii

MIGRATION INTERACTANCE MULTIPLIERS OF EDMONTON

Mean Value: 0.569 Standard Deviation: 0.493

	I.M.	% of Total I.M.
Saskatoon-Edmonton	1.6658	13.94%
Regina-Edmonton	1.6146	13.51%
Toronto-Edmonton	1.4462	12.10%

Table 5-5-iii

MIGRATION INTERACTANCE MULTIPLIERS AND SENSITIVITIES

OF HAMILTON, LONDON, WINNIPEG, AND VICTORIA

INTERACTANCE MULTIPLIER

CMA	Mean	Standard Deviation	Index of Total System Sensitivity
Hamilton	0.277	0.227	0.3548
London	0.258	0.221	0.3361
Winnipeg	0.378	0.357	0.5143
Victoria	0.394	0.307	0.4951

Table 5-5-iv

MIGRATION SENSITIVITIES OF CALGARY

Index of Total System Sensitivity: 0.9287

Individual Arc Sensitivities:

Edmonton - Calgary:	0.417
Quebec - Montreal:	0.203
Calgary - Edmonton:	0.181
Winnipeg - Calgary:	0.181
Montreal - Quebec:	0.180
Toronto - Montreal:	0.163
Vancouver - Calgary:	0.142

Table 5-5-v

MIGRATION SENSITIVITIES OF EDMONTON

Index of Total System Sensitivity: 0.7449

Individual Arc Sensitivities:

Calgary - Edmonton:	0.385
Quebec - Montreal:	0.204
Vancouver - Edmonton:	0.195
Edmonton - Calgary:	0.184
Montreal - Quebec:	0.182
Winnipeg - Edmonton:	0.175
Toronto - Montreal:	0.164
Saskatoon - Edmonton:	0.134
Toronto - Edmonton:	0.125

fluctuations on the arcs linking them directly to centres of national importance and to other centres in the western system, and again much like Calgary and Edmonton, they are insensitive to flows on the arcs within southwestern Ontario. Hamilton and London are both strongly influenced by the arcs that bind them directly to Toronto. They are both sensitive, but to a lesser degree, to fluctuations on the arcs binding them to the other centres of central Canada, but they are both indifferent to arc activity between the centres in western Canada.

3 - Southwestern Ontario: Windsor, Kitchener-Waterloo and St. Catharines - Niagara. (Table 5-6-i)

All three of these southwestern Ontario centres have limited migration sub-networks. The average values of interactance multiplier are very low. The only sub-networks that demonstrate any vigour are those which tie each of them directly to Toronto. The Toronto sub-networks alone account for approximately 30% of the total multiplier value and all other sub-networks are much less significant. Apart from their direct link with Toronto all three centres exhibit sensitivity principally to those arcs that link them directly to other centres within central Canada, and remain largely unaffected by arcs extending outside this field.

4 - Eastern and Western Urban Hinterlands: Quebec, Chicoutimi-Jonquiere; Sudbury, Thunder Bay, Regina, and Saskatoon (Table 5-6-ii):

The six urban centres of this group display the same limited characteristics but in varying degrees. In general their interaction behaviour is characterized by a high degree of interdependency with the large nodes of

Table 5-6-i

SYSTEM CHARACTERISTICS

CMAs	Interactance Multiplier		Index of Total System Sensitivity
	Mean	Standard Deviation	
Windsor	0.150	0.215	0.267
Kitchener-Waterloo	0.190	0.212	0.291
St. Catharines-Niagara	0.168	0.219	0.281

Table 5-6-ii

SYSTEM CHARACTERISTICS

CMAs	Interactance Multiplier		Index of Total System Sensitivity
	Mean	Standard Deviation	
Quebec	0.359	0.532	0.651
Chicoutimi-Jonquiere	0.117	0.287	0.313
Sudbury	0.141	0.215	0.262
Thunder Bay	0.098	0.221	0.245
Regina	0.186	0.305	0.363
Saskatoon	0.179	0.300	0.355

their respective sub-systems, some interaction with the hinterland centres of both sub-systems and very little interaction with the centres of southwestern Ontario. The great disparity between the sub-networks is shown by the very high values of standard deviations of the interactance multipliers.

Of the group, Quebec has the strongest sub-networks but they are largely confined in two directions, a small but strong Quebec province network, and a weaker secondary network that extends in two directions, towards the Prairie centres on the one hand and towards the Atlantic centres on the other. Its relatively high index is due to Quebec's close ties with Montreal, a major centre that responds strongly to changes in a fairly large number of centres, such as Toronto and Ottawa-Hull in particular.

If it is difficult to determine whether Quebec City belongs to Central Canada's heartland or to its hinterland, no such difficulty occurs for defining Chicoutimi-Jonquiere's position within the system. It clearly belongs to the eastern system's hinterland. Chicoutimi-Jonquiere is strictly a regional town. Its sensitivity is mainly limited to fluctuations on the arcs that link it to Montreal and Quebec though it exhibits some sensitivity to the arcs linking Montreal to Toronto and Ottawa-Hull.

Sudbury and Thunder Bay both belong to this group as both centres have very limited interaction networks. Sudbury and Thunder Bay are both dominated by the arcs that bind them to Toronto. Both centres are also mildly sensitive to changes on other arcs as well. Sudbury reacts to changes on the arcs between Toronto, Montreal, Ottawa-Hull, Quebec and Sudbury, while Thunder Bay responds to fluctuations in the arcs linking it to Toronto, Montreal, Winnipeg, and Ottawa-Hull. Southwestern Ontario and the Atlantic centres exert very little influence on the two cities.

Regina and Saskatoon have nearly identical migration interaction networks. Both centres are very sensitive to fluctuations on the arcs linking the three Prairie centres of Saskatoon, Regina, and Winnipeg. They both react quite strongly to changes on the arcs linking them to Calgary, Edmonton, and Vancouver. They react also but more mildly to changes on the arcs linking Toronto, Montreal, and Ottawa-Hull. Strangely enough both centres react to fluctuations on the Montreal-Quebec and the Quebec-Montreal arcs. In short, these centres belong to the hinterland areas of the western sub-system.

5 - The Atlantic Centres: Halifax, Saint John and St. John's (Table 5-6-iii):

Halifax, Saint John, and St. John's have similar interaction networks. The main links in their networks extend to the other centres in eastern Canada: Toronto, Montreal, Ottawa-Hull, Quebec, Chicoutimi-Jonquiere and to the other two Atlantic cities. The three networks have some links with the western centres of Calgary and Edmonton. Halifax distinguishes itself by having its interaction network more widespread than the other two and as such it is somewhat less dependent on the major node Toronto for its interaction patterns.

In spite of the limitations of the migration data, the preceding analyses raise a number of interesting points concerning the centres which were not included in the previous analyses. They have, on the one hand, indicated the importance of Hamilton within the system, and on the other, they have shown that its sub-networks and sensitivity patterns are quite limited when compared to those of other centres of comparable size. Presumably, the proximity of Toronto is largely responsible for this state of affairs as migrants moving

Table 5-6-iii

SYSTEM CHARACTERISTICS

CMAs	Interactance Multiplier		Index of Total System Sensitivity
	Mean	Standard Deviation	
Halifax	0.214	0.252	0.337
Saint John	0.093	0.221	0.243
St. John's	0.080	0.222	0.239

to southern Ontario would likely be drawn to Toronto where the greatest job market exists. Kitchener-Waterloo and St. Catharines-Niagara appear to be affected in much the same way as Hamilton by their proximity to Toronto. Their sub-networks and sensitivity patterns are limited to southern Ontario. Finally, Chicoutimi-Jonquiere is very regionally oriented with ties limited to Montreal and Quebec.

System Characteristics of Migration and Air Passenger Flows: A Comparison

Since the migration and air passenger flows are generated by the same urban system, it might be expected that there would be a number of similar traits between the system characteristics reported here and those found in Chapter Four.

Interactance Ratios:

A comparison of the 1966-71 migration and 1971 air-traffic interactance ratios reveals a number of interesting features of the Canadian metropolitan system (Table 5-7). The most striking point is the powerful role of Toronto within the system. The average interaction of the other centres for both types of activity is only 20% that of Toronto's. Toronto draws flows, not only from its own sub-system but from centres right across the country. The high standard deviation for the ratio suggests that the centres can be classified into one of three categories according to their migration ratios.

The first group consists of the largest centres in the system: Toronto, Vancouver, Montreal, and Ottawa-Hull. Vancouver ranks first after Toronto with almost 75% of Toronto's migrant attraction while Montreal attains only 47%. In air-traffic activity the ranks are reversed, Montreal having 62% of Toronto's output and Vancouver 42%. Vancouver draws its migrant

Table 5 - 7 : System Parameters of 1971 Air-Traffic and 1966 - 1971 In - Migration Interaction.

(* Cities are ranked as in Table 5-3)

CMAs*	Total Outbound Air Traffic 1971	Total In - Migration 1966-1971	Air Traffic Interactance Ratio	Migration Interactance Ratio	Percentage Difference	Air Traffic Average Interactance Multiplier	Migration Average Interactance Multiplier	Air Traffic Index of Total System Sensitivity	Migration Index of Total System Sensitivity
Toronto	1,031,240	95,335	–	–	–	–	–	–	–
Vancouver	435,695	69,230	0.422	0.726	0.304	0.45	1.17	0.583	1.425
Montreal	641,685	44,920	0.622	0.471	-0.151	0.41	0.80	0.482	0.182
Ottawa-Hull	335,520	41,480	0.325	0.435	0.110	0.25	0.66	0.344	0.825
Calgary	375,795	36,125	0.364	0.379	0.015	0.44	0.72	0.593	0.929
Edmonton	319,295	28,430	0.309	0.298	-0.011	0.38	0.57	0.528	0.745
Hamilton	–	25,885	–	0.272	–	–	0.28	–	0.355
London	57,260	21,560	0.055	0.226	0.171	0.10	0.26	0.263	0.366
Winnipeg	298,620	19,835	0.289	0.208	-0.081	0.30	0.38	0.432	0.514
Victoria	85,460	19,770	0.083	0.207	0.124	0.14	0.39	0.292	0.495
Quebec	93,795	15,260	0.091	0.160	0.069	0.11	0.36	0.272	0.651
Kitchener-Waterloo	–	15,215	–	0.160	–	–	0.19	–	0.291
St. Catharines-Niagara Falls	–	12,340	–	0.129	–	–	0.17	–	0.281
Halifax	167,150	10,750	0.162	0.113	-0.049	0.18	0.21	0.321	0.337
Windsor	86,460	9,890	0.083	0.104	0.021	0.10	0.15	0.258	0.267
Sudbury	46,770	8,305	0.045	0.087	0.042	0.07	0.14	0.251	0.262
Regina	108,775	7,005	0.105	0.074	-0.031	0.11	0.19	0.303	0.363
Saskatoon	89,195	6,655	0.086	0.070	-0.016	0.14	0.18	0.291	0.355
Thunder Bay	84,445	4,050	0.081	0.043	-0.038	0.09	0.10	0.258	0.245
Chicoutimi-Jonquière	–	3,620	–	0.038	–	–	0.12	–	0.313
Saint John	44,505	3,225	0.043	0.034	-0.009	0.09	0.09	0.263	0.243
St. John's	62,285	2,730	0.060	0.029	-0.031	0.10	0.08	0.267	0.239
Average	242,442	22,801	0.190	0.203	0.013	0.204	0.343	0.353	0.460
Standard Deviation	259,152	23,151	0.170	0.178	0.008	0.138	0.287	0.121	0.304

and air-traffic flows principally from the other western centres. It also has strong interaction with the national system's major centres of Toronto, Montreal, and to a lesser degree Ottawa-Hull. It appears that Vancouver serves as the major pole for the western sub-system for both types of interaction. Montreal's patterns are somewhat different. Montreal's air-traffic interaction extends to all the major centres in the country but its migration activity is principally limited to the eastern centres of Toronto, Quebec, Ottawa-Hull, and Chicoutimi-Jonquiere. Ottawa-Hull has a migration ratio of 0.435, a figure substantially higher than its air-traffic ratio of 0.325. This last figure might be somewhat underestimated as a result of the short distance separating Ottawa and Montreal. For a centre that has a population less than 25% of Toronto's, Ottawa-Hull has high interactance ratios. It interacts strongly with all the major centres in the East and West in keeping with its role as the nation's capital.

The second group includes Calgary, Edmonton, Hamilton, Winnipeg, and Victoria. Calgary has air-traffic and migration ratios that are approximately equal in value. It is an active centre for both types of flows and its activity is primarily oriented towards the other western centres. Edmonton shows similar patterns, but at a slightly lower scale. Hamilton's migration activity is dominated by Toronto (47%). Victoria's migration ratio is much higher than its air-traffic one. On the other hand, Winnipeg's migration ratio is lower. Winnipeg is shown in this analysis not to serve as a regional pole for the prairie cities of Regina and Saskatoon. Both these centres have stronger ties with Calgary, Edmonton, and Vancouver than with Winnipeg.

The third group consists of all the smaller centres in the system. The

Atlantic cities report migration ratios lower than their air-traffic ratios. All three centres have stronger migration interaction with Toronto and Montreal than they do among themselves. Regina, Saskatoon and Thunder Bay also have lower migration values. The competition from the Alberta and B.C. centres is probably responsible for their weak attraction. In this third group, only Quebec, Windsor, and Sudbury have migration values higher than their air-traffic ratios. The location of these centres at the periphery of the Windsor-Quebec axis could be the reason (Simmons; 1978a). In general, air traffic is strongly related to city size (r =0.91) but the correlation between migration and population is not so strong (r = 0.81).

Sub-network Structures, Interactance Multipliers:

Vancouver's migration sub-networks are on the average much stronger than its air-traffic ones (Table 5-7). For both data sets the standard deviations of the interactance multipliers are high, an indication that a small number of sub-networks strongly influence Vancouver's interaction. For the most part the breakdowns of the sub-networks into two categories vary considerably for the two data sets (Tables A4-14 and 5-4-i). The migration sub-networks are much stronger for those centres that emit more migrants than they receive. The air-passenger networks provide a different illustration of Vancouver's position within the metropolitan system simply because they emphasize the bonds between Vancouver and the other major western centres. The migration networks on the other hand, stress the Vancouver ties with the minor western centres.

A comparison of the Montreal sub-networks for both types of flows presents some distinct contrasts. As in the Vancouver case and probably for the same reason, the average migration interactance multiplier is almost twice that of the air flows, but, while the standard deviation for the air flows is just about half the mean, for the migration data the standard deviation is actually larger than the mean. Although in terms of economic activity Montreal has strong networks right across the country, its migration attraction power has proved limited to only a few widely dispersed centres, namely Quebec City, Chicoutimi-Jonquiere, Toronto, Ottawa-Hull, Vancouver, and Winnipeg.

The migration and air passenger sub-networks of Ottawa-Hull, Calgary, and Edmonton have one point in common: In all three cases the standard deviations are substantially smaller than the mean values of the interactance multipliers. In this respect both types of sub-networks are similar in that all three centres draw flows from a great number of centres right across the country (Tables A4-6; 5-4-iii; 5-5-ii). The important migration sub-networks for each centre seem much more restricted in extent, however, than do the air traffic ones.

London, Winnipeg, and Victoria have migration sub-networks for which the standard deviation values of the interactance multipliers are only slightly smaller than their mean values. Thus, the centres have a number of strong sub-networks and a comparable number of poor ones. In this respect Victoria has a migration sub-network structure which is more evenly distributed than its air passenger one (Table A4-15). The Toronto-Victoria sub-network of air flows accounts for 47% of Victoria's total multiplier value while the same sub-network accounts for only 13% of the migrant value. Winnipeg has similar sub-network patterns for both types of data. It has strong ties with all western cities on both counts. The case of London is hard to assess. Its air flow multiplier standard deviation is much greater than its mean value while both values are approximately the same in the case of migration patterns. Because of London's location within the southern Ontario peninsula it is less meaningful perhaps to argue the air passenger figures are valid measures of person-to-person contacts and a comparison of the migration and air passenger patterns of interaction has less value.

For the remainder of the metropolitan centres, the migration subnetworks are similar to their air flow structures. In both instances these centres have but one or two strong sub-networks that bind them to their regional centres. Their ties with the other centres are very weak on both accounts (Tables 5-6-i; 5-6-ii; and Tables A4-3 to A4-17).

Interactance Sensitivities:

Sensitivity analysis in the A.S.T. technique relies on two distinct measures, the index of Total System Sensitivity and the individual arc sensitivities. Since the indices are calculated in absolute values they are of no use when comparing the results of analyses carried out on different matrices as is the case here. On the other hand, the individual arc sensitivities are relative measures and can therefore be used in a comparative analysis. Only the individual arc sensitivities will be considered in the brief discussion to follow.

Two points of interest stand out in the comparison of the sensitivities of metropolitan centres to migration and air passenger activity. Firstly, in almost every instance the centres will exhibit greater sensitivity to the air passenger activity on their main arcs than to the migration activity on these arcs. Secondly, the sensitivity figures for the migration data, though not as strong, show a more even distribution pattern over a greater number of arcs. It is probably the case that the strong air passenger sensitivities of centres to activity on a few of their principal arcs is a consequence of the hierarchical structure within the urban system. As for the weaker and more evenly distributed migration sensitivities they may reflect the interplay of a number of migration forces which might or might not follow the hierarchical network structures, forces such as the existence of job opportunities, the attraction exerted by the larger metropolitan centres, or the need for corporate and government organizations to extend or restructure their administrative functions. Beyond these few remarks, and the fact that both types of data outline in broad terms the same system structures, very little can, for the present, be drawn from a comparison of the results of the analyses carried out on migration and air passenger data.

Conclusion:

The migration analyses have identified a number of features of the migration patterns within the Canadian Metropolitan System. The results show that Toronto is the system's major centre, that the metropolitan system is made up of two sub-systems, one in the west centred on Vancouver and Calgary, and another in the east with Toronto, Montreal, and to a lesser degree, Ottawa-Hull as its main centres. The analyses of the interactance multipliers and sensitivities show that the core centres of both sub-systems have a number of common traits. The core centres have strong ties with other core centres of their own sub-system, with the national centre Toronto, and with the major core centres outside their own sub-system. The core centres also serve as the focal point for the networks of the minor centres of their own sub-system.

On the other hand, the minor or hinterland centres of both sub-systems have networks that are almost entirely restricted to the major nodes within their own sub-system. The analyses also show that the migration patterns of the Atlantic centres are dominated by a few centres in the east, namely Toronto, Montreal, Ottawa-Hull, and Halifax. Finally, the centres of southwestern Ontario, Windsor in particular, have very restricted migration patterns dominated for the most part by Toronto.

The migration analyses have made it possible to identify a number of characteristics of the networks of centres which were not included in the air passenger analyses. The centres referred to here are Hamilton, Kitchener-Waterloo, St. Catharines-Niagara, and Chicoutimi-Jonquière. Hamilton has an interactance ratio which ranks it after Calgary and Edmonton. Thus Hamilton draws a considerable number of migrants. But its interaction networks differ widely from those of the two western cities. Its average interactance multiplier value is much lower than theirs and therefore its networks are much weaker. Its sensitivity to arc fluctuations differs widely from those of Calgary and Edmonton as well. Hamilton's networks are dominated by the arcs that link it to Toronto and Montreal whereas the western centres have more diversified networks. Unlike these two western centres, Hamilton has but weak bonds outside its own sub-system.

Kitchener-Waterloo and St. Catharines-Niagara have interactance ratios which rank them slightly lower than Quebec and somewhat higher than Windsor. Their average interactance multiplier values are very low which implies that their network structures are weak. Both centres are dominated by their links with Toronto. Both centres are slightly sensitive to other centres of southern Ontario.

Chicoutimi-Jonquiere appears from these analyses as little more than a regional centre much like Saint John. The only network ties that really matter are those that bind it to Montreal, Quebec, Ottawa-Hull, and Toronto.

In an earlier section of this Chapter a comparison between the migration and air passenger networks was made. It is appropriate to recall here the principal observations that were made. The most striking differences noted concerned Vancouver and Montreal. Vancouver's migration networks were stronger and more extensive than Montreal's. But in air passenger network structures, Montreal's position was stronger on both counts. Ottawa-Hull, Calgary, and Edmonton had migration networks that were more limited than their air passenger ones. Winnipeg has similar network patterns for both types of data. Finally, for the remaining centres, a comparison of the two series of analyses failed to bring out any strong differences. On the whole the main traits of network structures for these centres were similar.

Thus the analyses of intermetropolitan migration have provided additional insight into the interaction structures of the Canadian Metropolitan System. It will become possible to compare the evolution of network patterns of both the air passenger and migration networks when more migration data is made available. A comparison of the evolutionary processes of these two networks could prove to be of considerable help in the analysis of the transformation of the Canadian System.

CHAPTER 6

SUMMARY AND CONCLUSION

The main purpose of this dissertation has been to develop a new technique for the analysis of interaction within urban systems and to apply it to the Canadian Metropolitan System. The final Chapter will consist of three parts: a) an evaluation of the merits and limitations of the A.S.T. flow-graph technique, b) a review of the main characteristics of the Canadian Metropolitan System brought out in the analyses of Chapters Four and Five, and c) a discussion of the most promising directions for further research in the use of flow-graph techniques in urban research.

The principal merit of the A.S.T. flow-graph model is that it is a technique for the analysis of urban interactions within a systems framework. It is an approach which emphasizes the interdependency of the centres within a system whose constituent parts are complex, strongly differentiated, very specialized and highly interdependent. In the analysis of the interaction flows of such a system the more well-known techniques such as regression and factorial analyses can determine broad patterns of field attraction and emission as well as the basic structural characteristics of the system, but they are only capable of determining network structure at a rudimentary level. They are incapable of differentiating sub-network structures and operations, and it is most likely at these finer levels that system evolution first occurs. Urban systems do not, in general, change in drastic fashion, but slowly, progressively, as certain of their sub-networks respond to new regional dynamic forces. Even before changes become apparent in the overall system structure, the evolutionary process is already discernible in the sub-network operations.

The definition of sub-system structure and operation is mainly a question of inter-node activity patterns. The A.S.T. flow-graph model was developed to measure both internal sub-network structures and the impact of individual arc activity on various elements of the system. The A.S.T. model is specifically designed to measure these interaction properties directly. The interactance ratio shows a centre's overall response to the exchanges occurring in the system to which it belongs. The interactance multipliers define the different sub-networks binding a city to all the others in its system. The multipliers also measure the absolute and relative importance of a city's sub-networks in generating its interaction output. The interactance sensitivities determine a centre's sensitivity to changes in flows on any arc in the system. Finally, the A.S.T. model provides a means of evaluating the performance of a system's sub-networks, a feature which in itself suffices to mark the difference separating the A.S.T. model from other interaction models presently in use. Performance analysis is made possible because the flow-graph technique provides the means of comparing the relative structural importance of a system's sub-networks with the proportions of flows they are actually carrying. This comparison of structure and operation is unique to the A.S.T. model, by no means a small feature in systems analysis, It may be appropriate to make a few comments here to refute any objections that might be raised concerning the existence of similarities between the signal flow-graph model as presented in this study and the input-output technique, a point that was raised earlier in Chapter One. This thesis demonstrates conclusively that there is an advantage in examining a given data set in the light of the well developed theoretical framework of signal flow-graph theory. Because the interaction matrices were examined within this theoretical framework it became possible to distinguish sub-network structures, to measure their carrying capacities, and to evaluate their performance. The discovery of this approach is certainly a key point of originality of the contribution of this thesis to interaction analysis.

There are a number of difficulties associated with the use of the A.S.T. model in the analysis of interaction patterns, difficulties which are not entirely insurmountable, however. The first limitation derives from the assumption of linearity on which the A.S.T. is founded. The assumption implies that the inputs into a node are proportional to its total output. When the model is applied to a matrix covering a single time period, the linearity assumption is not overly restrictive. In this case, if the relationships are curvilinear rather than linear, the results of the analysis describe the characteristics of the tangent to the system at that point in time. Linearity in this case is no more an impediment to the use of the A.S.T. model than it is to any other linear model commonly in use in various disciplines, e.g., factor analysis, linear programming, regression models, and Markov chain models.

Linearity is more difficult to establish for dynamic models where lag-times are incorporated into the equation system. These dynamic models are

are more complex because each centre is represented by several different nodes in the flow-graphs and in the equations. A centre will have as many different nodes as it has different reaction lag-times with other centres. This leads to a multiplication of the variables in the equations which makes it more difficult to establish the linearity assumption of the model. Since the lag-times must be known beforehand, setting up the equation systems of the dynamic models is even more difficult than testing for linearity. Of course, this thesis does not attempt to treat a series of consecutive matrices simultaneously.

A second limitation of the A.S.T. model lies in the fact that it simply identifies and measures interaction patterns and characteristics. A better understanding of the functioning of an urban system must go beyond this and be founded on the analysis of the forces generating these network patterns. However, while the A.S.T. model does not go this far, it does provide the means of achieving this objective, a feature which is not found in other models. Because the A.S.T. model provides vectors and matrices of interaction measurements it makes it possible, through regression and canonical correlation analyses to relate interaction patterns to structural characteristics. This may prove an important step towards the better understanding of the interaction processes at work in urban and regional systems.

A third limitation to the use of the model as it applies to the analysis of metropolitan systems is a deficiency in theoretical development. The model implies that strong interdependencies exist between metropolitan centres, that nodes are sensitive to feedback effects and to arc variations in activity. The failure of growth-pole, central-place, and similar types of models to account for transmission of growth in modern urban systems, (Pred, 1976), has led to attempt to develop a number of alternative concepts that may provide a better theoretical basis for the introduction of flow-graph models, such as the A.S.T., in urban research (Bannister, 1976). These concepts relate transmission of growth in modern economies to the person-to-person contacts by corporate and government decision-makers, (Tornqvist, 1968), to the patterns of interurban information flows. (Pred, 1973, Lasuen, 1973), and to the influence of spatial structures of corporate organizations, (Goddard, 1973). These new approaches stress the high degree of interdependency among cities and the powerful influence of such interdependencies in the evolution of modern urban systems. These interdependency networks stretch inter and intraregionally and are often viewed in terms of heartland-hinterland relationships. The networks extend horizontally between cities of similar size as well as vertically up and down the urban hierarchy. Flow-graph models are admirably suited for the analysis of such highly interdependent systems.

The A.S.T. analyses of Chapters Four and Five identified a number of system characteristics and interaction patterns of the Canadian Metropolitan System. In many respects, the A.S.T. analyses reiterate the findings of other researchers regarding the evolution of the Canadian system. The merit of the A.S.T. approach has been that it has pinpointed the finer network structural changes through which the system has recently evolved.

The analyses show that there has been a number of important changes within the system. This is not too surprising considering the growth of metropolitan population in general and in air traffic in particular. The dominant role of Toronto as the national system's central node has increased over the 1963-1973 period, at least in terms of air passenger traffic. Centres which in the early 1960 years had only minor links with Toronto have developed their ties with the national centre to the point where their Toronto sub-network is either their strongest or second strongest one. All metropolitan centres in Canada are sensitive to the activity on the links that bind Toronto to themselves. They are also sensitive to the links that bind Toronto to other major nodes in the system.

The A.S.T. analyses revealed the pattern of emergence of two broad sub-systems within Canada, one in the west and another in the east. Over the study period the diversity of the bonds within both sub-systems has increased, the interdependency between the centres has grown, and this has resulted in the absorption of the earlier smaller semi-independent sub-systems by their larger counterparts in the east and west. The western sub-system consists of a core and a hinterland. Vancouver, Calgary and Edmonton are the core cities of this sub-system while the other western cities form the hinterland centres. In the east, the core centres are Toronto and Montreal, and Ottawa-Hull to a lesser degree. The Atlantic centres have been pulled into the Toronto-Montreal field and form part of the eastern hinterland.

Within each of these sub-systems there are a number of interesting features. A major feature of the western sub-system is the strong and diversified bonds that link all the centres into a tight web. Even the hinterland centres of Regina and Saskatoon experience strong linkages with all the other western centres. The western sub-system would be a good testing ground for Pred's (1973) argument that modern systems are more characterized by their indernodal activity than by their hierarchical structures. This is not to say that the hierarchical relationships between these core centres and their hinterland areas are negligible. The growing importance of Vancouver within this sub-system is another major feature. Vancouver has become the dominant centre of this sub-system. The other centres are very sensitive to the arcs linking them to Vancouver and to the Vancouver-Toronto and Vancouver-Calgary arcs as well.

The eastern sub-system has demonstrated a number of interesting traits as well. The major centres are Toronto and Montreal with Toronto assuming the dominant role within the sub-system. Ottawa-Hull has grown rapidly to become the most important centre in this network after Toronto and Montreal. The centres of Hamilton, Kitchener-Waterloo, St. Catharines-Niagara, Windsor, and to a lesser degree, London, are highly dependent on Toronto. Their ties with Toronto surpass by far all their other linkages, although they still have important bonds with Montreal and with each other. London, in particular, seems to have attained a degree of national importance. The centres of northern Ontario, Thunder Bay and Sudbury, have developed very little breadth in their intermetropolitan interdependencies. They remain as regional centres whose links are almost exclusively dominated by Toronto. Of the two secondary centres of the Province of Quebec, Quebec City has apparently undergone the greatest changes in its network structures. By 1973, it appears relatively much more dependent on its Toronto sub-networks than previously, with a subsequent reduction in its ties with Montreal. However, the apparent decrease in flows with Montreal might simply reflect a change in the relative importance of the modes of travel between Montreal and Quebec. Chicoutimi-Jonquiere appears as nothing more than a regional centre on the periphery of the eastern sub-system. Finally, the Atlantic centres form an integral part of the eastern system rather than an independent network of their own in 1973.

These analyses have revealed a number of changes in the network structures over the period. But the analyses are only as good as the data on which they are based. Air passenger traffic as a surrogate variable for person-to-person information flows within a decision-making context is not so satisfactory that it can be relied on to depict all of the finer points of the network structures of the Canadian Metropolitan System.

The A.S.T. technique offers some interesting possibilities as a research tool for interaction analysis but only further research will determine its real value. Further research should focus on two themes for the present. Some work should be done on developing a better understanding of the flow-graph model itself. This implies determining the relationships between nodal sensitivity and network configuration as well as the relationships between the strength of a sub-network and its structure. This could be achieved in two ways. The first approach would consist in experimenting on a number of small networks and establishing the desired relationships. These experimental systems could be made to evolve systematically and the results tabulated and graphed. A second approach would be to investigate the manner in which the a_{ij} values of the inverse matrix $-A^{-1}$ vary with different structures. These a_{ij} coefficients represent sub-network structures and are of great importance in the calculation of nodal sensitivity and other flow-graph measures. This approach involves the calculation of the inverse of a matrix by the cofactor expansion method. This ultimately leads into the combinatorial approach in calculating flow-graph properties (Rota, 1971). This type of investigation is feasible with small systems. Either approach should lead to a better understanding of the likely impact on the system of network transformations.

Finally, research should be undertaken to establish the relationships between network patterns and the forces that generate and control them. The ability of the researcher to forecast changes in network structures in response to new forces acting on the system rests on this type of investigation. Two methods can serve in this endeavour. The use of factor analysis in combination with multiple regression is one approach. Factor analysis could determine the system's structural characteristics and multiple regression would serve to weigh the impact of each factor in determining nodal sensitivity and sub-network strength. Another approach would be to relate directly through canonical correlation the interactional and structural matrices of a system. Either of these methods would, to some extent, shed light on the manner in which interaction systems respond to various forces at work in the system.

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APPENDIX

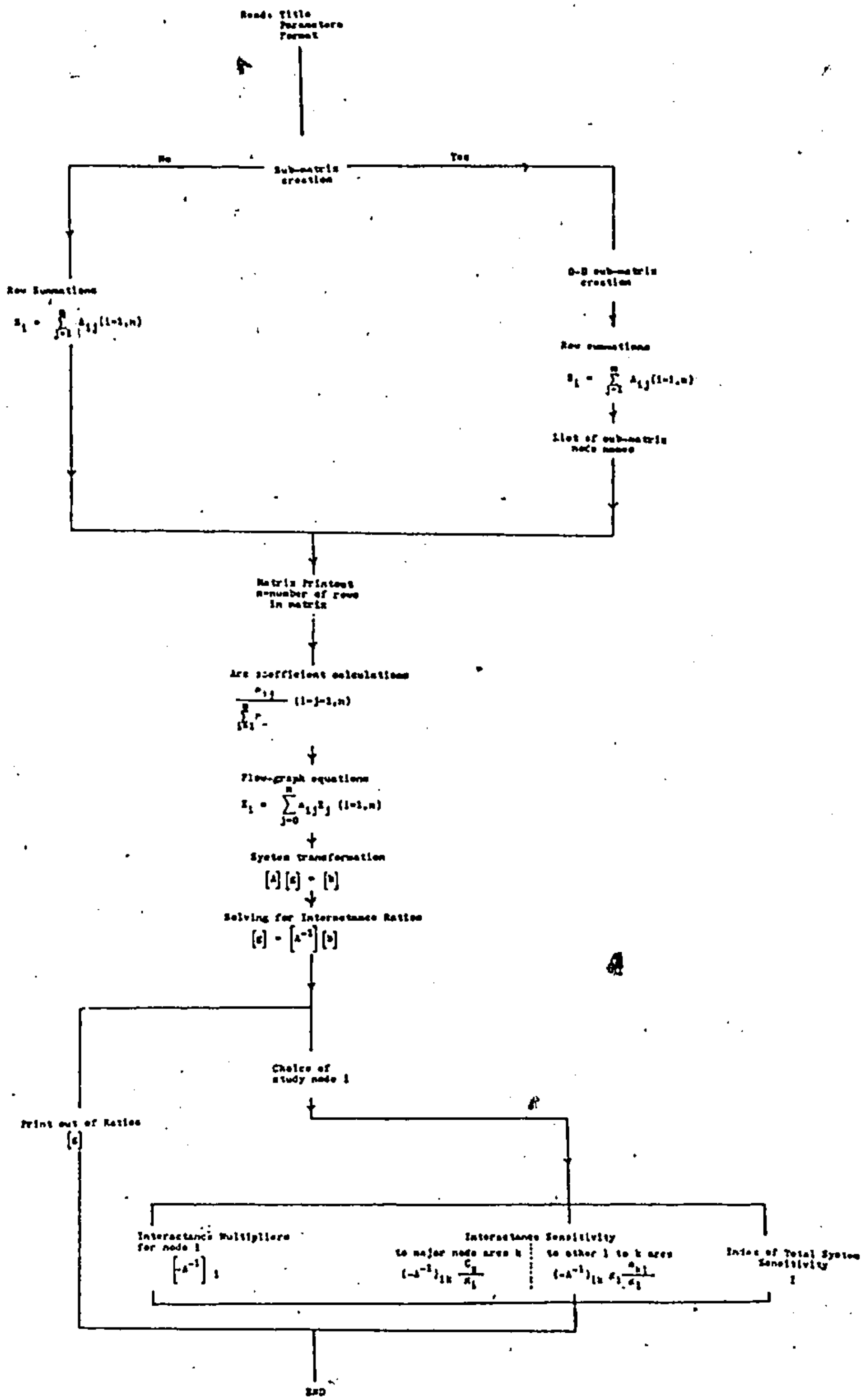
A.S.T. PROGRAMME: GENERAL OUTLINE AND DECK SETUP

The A.S.T. programme operations are summarized in the simplified flow diagram of Figure A3-1. The programme consists of a two-step procedure. The first step reads-in the required data and performs the necessary operations of data manipulations in the cases where only a sub-system taken from the main matrix is required for the second step analyses. The second step applies the A.S.T. algorithms, performs the required operations and prints out the results.

The A.S.T. deck set-up required is illustrated in Figure A3-2. The first data card to be read is an appropriate title card (80 columns). The second card is a parameter card that indicates the order of the matrix, the major node and the study node, and whether a sub-matrix is to be pulled from the matrix that has been read in. The third card is the format card of the original O-D matrix that follows. The names of the nodes are then listed on separate cards, a maximum of 12 characters is allowed for each name (3A4 format). If a sub-matric is to be generated, a parameter card for its generation follows immediately after the last node name. The sub-matrix parameter card gives the number of rows of the sub-matrix, its major node, the chosen study node followed by a list of all the nodes to be included with the major node heading the list.

An example of the control cards required for the creation of a sub-system consisting of three Canadian Atlantic CMAs taken from the 22 X 22 matrix of the total Canadian CMA system is provided in Figure A3-3.

FIGURE A3-1 GENERAL OUTLINE OF A.S.T. PROGRAMME



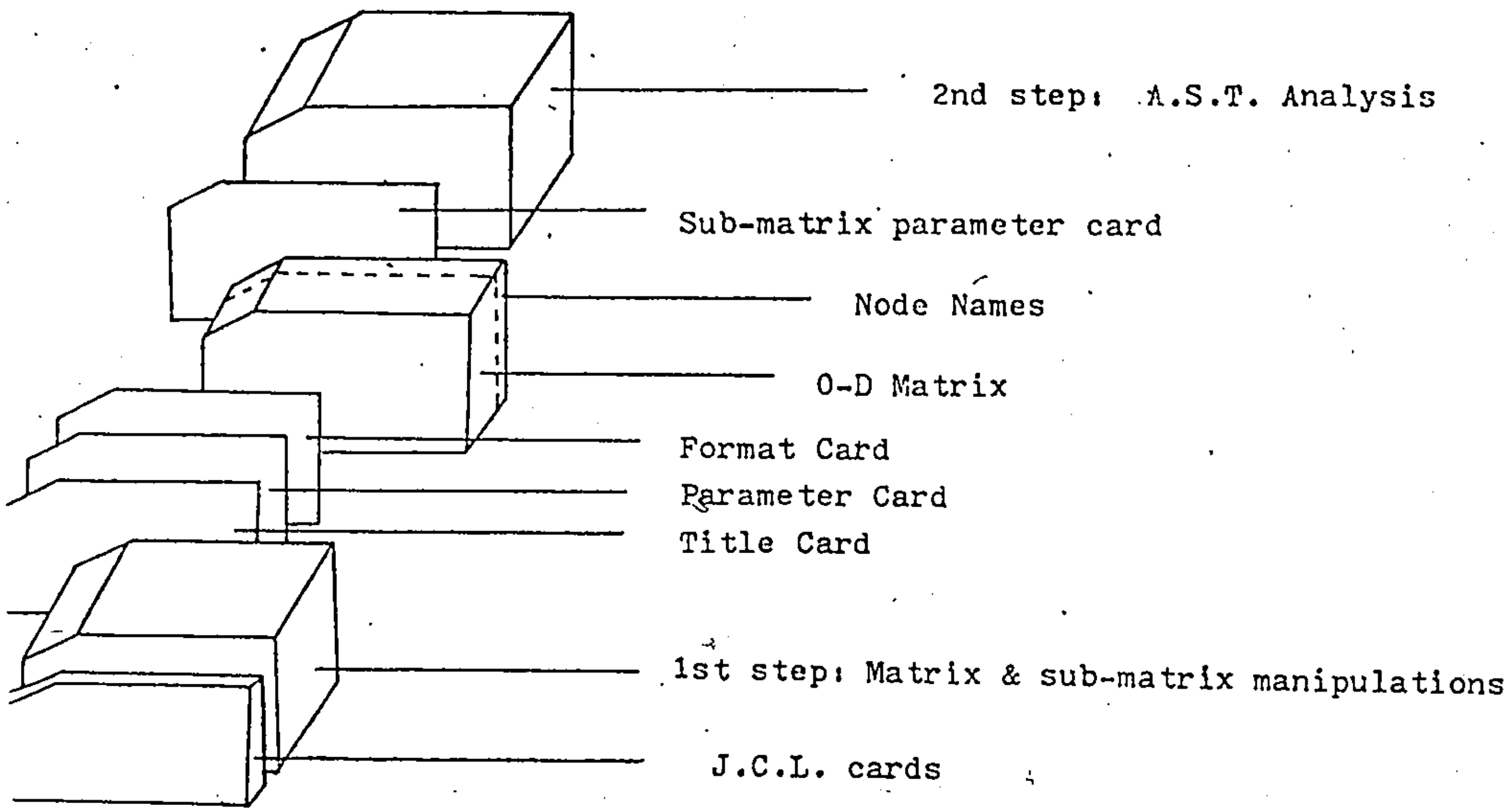
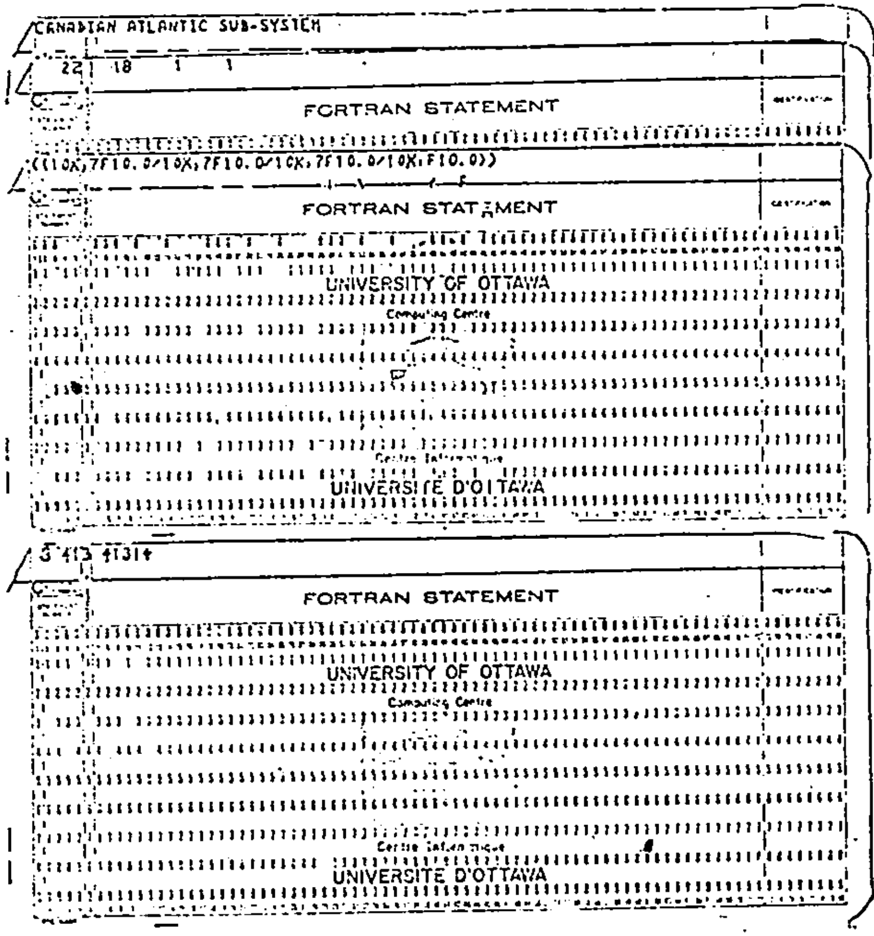


FIGURE A3-2 A.S.T. DECK SET-UP

FIGURE A3-3



TITLE CARD

Parameter Card

col. 1-5 22 a 22 x 22 matrix

col. 6-10 18 Node 18 (Toronto) is the major node

col. 11-15 1 Node 1 (Calgary) the study node

col. 16-20 1 1-a sub-matrix is to be pulled from the matrix read in originally.

FORMAT CARD of the O-D Matrix

DATA FOLLOWS : O-D Matrix
22 names

Sub-matrix parameter card

col. 1-2 : 3 a 3 x 3 matrix required

col. 3-4 : 4 the 4th city (Halifax) is the major node of the sub-system.

col. 5-6 : 13 the 13th city (St. John's) is the study node.

col. 7-8 : 4 Halifax

col. 9-10 : 13 St. John's

col. 11-12 : 14 Saint John

POOR COPY.

A.S.T. ANALYSES: TABLES A4-1 to A4-66; FIGURES A4-1 to A4-7.

TABLE A4-1 INTERACTANCE MULTIPLIERS, CALGARY, NATIONAL SYSTEM, 1963-1973*

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-CALGARY	26.03	24.36	24.15	23.51	22.74	22.13	21.50	21.53	21.53	21.11	21.23
TORONTO-EDMONTON	11.78	12.18	12.90	12.25	12.01	12.27	12.01	11.98	12.11	11.45	11.33
TORONTO-REGINA	9.92	9.61	9.65	9.29	8.76	9.17	8.94	9.31	8.65	8.36	8.50
TORONTO-VANCOUVER	8.82	9.03	8.83	8.56	8.47	8.75	8.60	8.76	8.69	8.64	8.32
TORONTO-VICTORIA	8.64	8.97	8.47	8.34	7.89	8.09	7.80	8.12	9.03	9.35	9.33
TORONTO-SASKATOON	8.45	8.76	8.56	8.17	8.15	8.71	8.81	8.93	8.58	8.31	7.97
TORONTO-WINNIPEG	6.03	6.35	6.20	5.99	5.82	6.17	6.17	6.30	6.22	6.19	6.14
TORONTO-LONDON	3.48	3.80	3.88	4.38	4.42	4.39	4.68	4.18	4.05	4.17	4.27
TORONTO-THUNDER BAY	2.39	2.38	2.47	2.52	2.49	2.52	2.58	2.43	2.32	2.50	2.63
TORONTO-OTTAWA-HULL	2.38	2.41	2.43	2.79	2.98	2.81	2.98	2.98	3.03	3.21	3.21
TORONTO-HALIFAX	2.12	2.02	2.04	2.22	2.60	2.38	2.49	2.51	2.57	2.70	2.75
TORONTO-WINDSOR	1.96	1.93	2.04	2.37	2.54	2.42	2.66	2.51	2.44	2.72	2.99
TORONTO-MONTREAL	1.90	1.97	2.04	2.40	2.82	2.49	2.73	2.58	2.54	2.58	2.67
TORONTO-SAINT JOHN	1.89	1.90	1.94	2.23	2.48	2.22	2.33	2.34	2.42	2.51	2.49
TORONTO-QUEBEC	1.85	1.87	2.01	2.21	2.76	2.32	2.53	2.43	2.50	2.53	2.67
TORONTO-ST. JOHN'S	1.62	1.56	1.66	1.86	2.22	2.12	2.14	2.08	2.26	2.23	2.24
TORONTO-SUDBURY	0.77	0.90	0.71	0.92	0.84	1.01	1.06	1.03	1.07	1.31	1.27
MEAN	0.32	0.36	0.37	0.38	0.40	0.41	0.43	0.43	0.44	0.44	0.43
SKEWNESS	1.87	1.67	1.66	1.74	1.78	1.60	1.60	1.54	1.55	1.59	1.65
KURTOSIS	3.37	2.55	2.47	2.81	2.99	2.23	2.25	2.02	2.06	2.21	2.46
STANDARD DEVIATION	0.35	0.37	0.38	0.36	0.36	0.38	0.38	0.38	0.39	0.37	0.36

TABLE A4-2 INTERACTANCE MULTIPLIERS, EDMONTON, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-EDMONTON	26.56	25.91	26.09	24.87	24.07	24.59	23.83	23.86	23.81	22.99	23.07
TORONTO-CALGARY	12.32	12.45	12.72	12.48	12.07	11.87	11.86	11.78	11.96	11.28	11.23
TORONTO-VANCOUVER	8.90	9.07	8.85	8.65	8.49	8.83	8.58	8.66	8.51	8.52	8.29
TORONTO-VICTORIA	8.57	9.06	8.35	8.24	7.89	8.01	7.59	7.91	8.53	8.93	8.97
TORONTO-REGINA	8.36	8.41	8.32	8.18	7.86	8.33	8.24	8.53	7.95	7.84	7.95
TORONTO-SASKATOON	8.16	8.12	8.25	7.99	7.99	8.84	8.35	8.44	8.35	8.00	7.65
TORONTO-WINNIPEG	6.03	6.24	6.04	5.88	5.74	5.91	5.88	6.10	6.04	5.99	5.97
TORONTO-LONDON	3.70	3.73	4.00	4.29	4.29	4.29	4.66	4.12	4.01	4.14	4.26
TORONTO-OTTAWA-HULL	2.47	2.52	2.55	2.86	3.04	2.84	2.97	2.99	3.05	3.24	3.26
TORONTO-THUNDER BAY	2.37	2.34	2.43	2.48	2.45	2.43	2.44	2.33	2.23	2.50	2.59
TORONTO-HALIFAX	2.14	2.02	2.06	2.20	2.59	2.31	2.42	2.49	2.52	2.72	2.70
TORONTO-WINDSOR	2.10	1.91	2.02	2.29	2.57	2.39	2.60	2.45	2.40	2.65	2.81
TORONTO-SAINT JOHN	1.95	1.88	1.92	2.20	2.45	2.15	2.26	2.30	2.38	2.54	2.46
TORONTO-MONTREAL	1.94	1.97	2.05	2.38	2.82	2.39	2.65	2.52	2.48	2.55	2.62
TORONTO-QUEBEC	1.91	1.90	2.04	2.22	2.80	2.25	2.50	2.42	2.48	2.55	2.69
TORONTO-ST. JOHN'S	1.70	1.56	1.64	1.87	2.19	2.08	2.12	2.08	2.22	2.26	2.21
TORONTO-SUDBURY	0.82	0.93	0.69	0.93	0.86	0.93	1.06	0.98	1.07	1.28	1.25
MEAN	0.31	0.32	0.33	0.35	0.36	0.36	0.37	0.37	0.38	0.38	0.38
SKEWNESS	1.98	1.88	1.93	1.92	1.97	1.91	1.91	1.87	1.87	1.87	1.91
KURTOSIS	3.80	3.37	3.55	3.51	3.75	3.53	3.52	3.35	3.34	3.38	3.54
STANDARD DEVIATION	0.33	0.34	0.35	0.35	0.35	0.35	0.35	0.36	0.36	0.35	0.34

TABLE A4-3 INTERACTANCE MULTIPLIERS, HALIFAX, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-HALIFAX	37.10	37.96	38.01	37.02	36.49	37.20	36.49	36.62	36.17	35.95	36.16
TORONTO-SAINT JOHN	15.66	15.75	15.33	14.48	13.38	14.47	13.37	12.74	12.13	12.22	11.88
TORONTO-ST. JOHN'S	12.48	12.34	11.94	11.86	11.12	11.94	11.72	11.94	11.74	11.79	11.07
TORONTO-LONDON	3.87	4.12	4.08	3.89	4.06	4.08	4.28	4.11	4.45	4.18	4.35
TORONTO-MONTREAL	3.46	3.83	3.87	4.03	4.09	4.09	4.27	4.14	3.97	3.85	3.84
TORONTO-QUEBEC	3.36	3.61	3.60	3.74	3.91	3.71	3.85	3.74	3.69	3.82	3.53
TORONTO-VICTORIA	3.20	2.33	2.45	2.81	2.83	2.49	2.59	2.74	2.87	2.95	3.02
TORONTO-OTTAWA-HULL	2.93	3.22	3.20	3.44	3.46	3.35	3.38	3.57	3.72	3.79	3.50
TORONTO-CALGARY	2.46	2.16	2.28	2.43	2.72	2.39	2.56	2.63	2.81	2.80	2.94
TORONTO-VANCOUVER	2.37	2.20	2.33	2.41	2.63	2.41	2.52	2.57	2.59	2.69	2.79
TORONTO-EDMONTON	2.35	2.12	2.31	2.41	2.71	2.42	2.54	2.66	2.79	2.82	2.92
TORONTO-SASKATOON	2.25	2.11	2.12	2.26	2.47	2.23	2.40	2.54	2.69	2.62	2.70
TORONTO-REGINA	2.18	2.07	2.05	2.27	2.55	2.25	2.42	2.65	2.68	2.78	2.80
TORONTO-WINNIPEG	2.14	2.19	2.08	2.29	2.52	2.24	2.41	2.51	2.56	2.56	2.62
TORONTO-WINDSOR	1.80	1.78	1.85	1.90	2.47	2.18	2.50	2.23	2.41	2.39	2.65
TORONTO-THUNDER BAY	1.26	1.27	1.44	1.43	1.55	1.37	1.43	1.34	1.37	1.44	1.51
TORONTO-SUDBURY	1.12	0.95	1.08	1.36	1.03	1.19	1.21	1.27	1.36	1.35	1.30
MEAN	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
SKEWNESS	2.54	2.55	2.60	2.63	2.72	2.63	2.68	2.72	2.75	2.74	2.81
KURTOSIS	5.80	5.91	6.16	6.32	6.81	6.32	6.63	6.80	6.59	6.93	7.29
STANDARD DEVIATION	0.28	0.28	0.27	0.27	0.27	0.27	0.26	0.26	0.26	0.26	0.26

* In table A 4-1 to A 4-33, the interactance multipliers are expressed as a percentage of the system's total interactance - multiplier value. The descriptive statistics are in absolute values.

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TABLE A4-4 INTERACTANCE MULTIPLIERS. LONDON. NATIONAL SYSTEM. 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-LONDON	59.69	61.64	61.07	60.70	62.66	62.93	61.00	60.23	60.43	59.38	58.14
TORONTO-SASKATOON	3.19	2.77	0.0	2.82	2.65	2.42	2.66	2.65	2.59	2.59	2.47
TORONTO-VICTORIA	3.07	2.87	3.07	3.14	2.64	2.67	2.74	2.91	2.59	2.70	2.91
TORONTO-EDMONTON	3.01	2.63	2.86	2.84	2.64	2.62	2.72	2.78	2.65	2.63	2.84
TORONTO-CALGARY	2.57	2.74	2.74	2.91	2.69	2.59	2.65	2.75	2.64	2.58	2.81
TORONTO-REGINA	2.88	2.60	2.66	2.75	2.58	2.50	2.66	2.80	2.53	2.72	2.75
TORONTO-VANCOUVER	2.77	2.59	2.69	2.68	2.54	2.53	2.53	2.65	2.50	2.54	2.63
TORONTO-HALIFAX	2.73	2.58	2.59	2.41	2.32	2.48	2.40	2.55	2.57	2.68	2.66
TORONTO-WINNIPEG	2.73	2.70	2.69	2.70	2.64	2.52	2.54	2.65	2.56	2.55	2.78
TORONTO-OTTAWA-HULL	2.71	2.78	2.98	2.89	2.98	2.90	3.10	3.14	3.25	3.31	3.34
TORONTO-SAINT JOHN	2.56	2.50	2.49	2.46	2.22	2.33	2.38	0.0	2.42	2.57	2.61
TORONTO-QUEBEC	2.40	2.14	2.22	2.19	2.32	2.19	2.46	2.32	2.50	2.38	2.43
TORONTO-MONTREAL	2.37	2.19	2.27	2.30	2.19	2.12	2.42	2.39	2.40	2.45	2.49
TORONTO-ST. JOHN'S	2.22	2.14	2.21	2.06	2.07	2.06	2.03	2.14	2.31	2.21	2.17
TORONTO-THUNDER BAY	1.75	1.74	1.89	1.93	1.92	2.13	2.11	2.11	2.17	2.43	2.52
TORONTO-SUDBURY	1.70	2.20	1.45	1.77	1.40	1.67	2.13	2.07	2.36	2.66	2.67
TORONTO-WINDSOR	1.27	1.19	1.30	1.44	1.53	1.34	1.47	1.42	1.48	1.54	1.59
MEAN	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
SKEWNESS	3.41	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42
KURTOSIS	10.30	10.32	10.31	10.31	10.32	10.32	10.32	10.32	10.33	10.33	10.32
STANDARD DEVIATION	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

TABLE A4-5 INTERACTANCE MULTIPLIERS. MONTREAL. NATIONAL SYSTEM. 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-MONTREAL	16.51	16.67	16.44	16.67	14.06	16.92	16.55	16.74	17.03	17.26	17.60
TORONTO-QUEBEC	11.55	11.96	11.60	10.87	9.26	9.92	9.82	10.01	9.40	9.28	9.06
TORONTO-SAINT JOHN	7.88	8.45	8.64	8.07	7.72	7.85	7.76	7.76	7.69	7.70	7.20
TORONTO-HALIFAX	7.26	7.44	7.54	7.19	7.15	7.40	7.14	7.16	6.80	6.72	6.49
TORONTO-LONDON	6.82	6.87	6.97	6.59	6.53	6.51	6.94	6.60	6.86	6.73	6.68
TORONTO-ST. JOHN'S	6.75	6.70	6.62	6.47	6.65	7.42	6.74	6.69	6.54	6.24	5.88
TORONTO-VICTORIA	4.65	4.44	4.43	4.74	4.93	4.53	4.46	4.53	4.50	4.70	4.63
TORONTO-VANCOUVER	4.53	4.43	4.51	4.59	5.15	4.72	4.78	4.77	4.66	4.77	4.82
TORONTO-CALGARY	4.50	4.21	4.33	4.57	5.05	4.46	4.66	4.59	4.67	4.64	4.75
TORONTO-WINNIPEG	4.35	4.45	4.32	4.38	4.91	4.46	4.49	4.48	4.48	4.51	4.50
TORONTO-EDMONTON	4.33	4.04	4.39	4.48	5.05	4.45	4.59	4.61	4.65	4.62	4.68
TORONTO-SASKATOON	4.25	4.12	4.11	4.18	4.84	4.23	4.45	4.37	4.44	4.42	4.37
TORONTO-REGINA	4.03	3.93	3.99	4.17	4.70	4.19	4.37	4.44	4.32	4.35	4.42
TORONTO-OTTAWA-HULL	3.92	3.81	3.62	3.87	3.70	3.70	3.82	3.96	4.14	4.20	4.30
TORONTO-WINDSOR	3.72	3.59	3.79	4.09	5.42	4.47	4.47	4.57	4.95	4.88	5.60
TORONTO-THUNDER BAY	2.70	2.92	3.00	2.82	3.38	2.66	2.63	2.53	2.39	2.55	2.58
TORONTO-SUDBURY	2.21	1.99	1.72	2.25	1.73	2.04	2.08	2.18	2.46	2.44	2.28
MEAN	0.44	0.44	0.44	0.43	0.53	0.41	0.43	0.42	0.41	0.40	0.39
SKEWNESS	1.68	1.61	1.55	1.79	1.37	1.81	1.83	1.87	2.03	2.13	2.24
KURTOSIS	2.30	1.91	1.83	2.86	2.16	3.21	3.33	3.45	4.18	4.59	5.16
STANDARD DEVIATION	0.26	0.27	0.27	0.25	0.25	0.24	0.24	0.24	0.23	0.23	0.23

TABLE A4-6 INTERACTANCE MULTIPLIERS. OTTAWA-HULL. NATIONAL SYSTEM. 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-OTTAWA-HULL	33.10	32.42	32.30	30.66	29.32	29.80	28.50	27.07	25.76	25.59	24.74
TORONTO-LONDON	6.46	7.05	7.40	6.78	7.46	7.50	7.76	7.46	7.63	7.47	7.31
TORONTO-HALIFAX	5.06	5.03	5.12	4.90	4.96	5.16	4.92	5.20	5.25	5.25	5.44
TORONTO-QUEBEC	4.56	4.86	4.87	5.07	5.45	5.78	5.91	6.10	6.23	6.34	6.25
TORONTO-VICTORIA	4.91	4.68	4.79	5.33	4.96	4.60	4.66	4.82	4.85	5.06	4.98
TORONTO-SASKATOON	4.63	4.71	4.41	4.71	4.69	4.41	4.57	4.78	4.91	4.93	4.82
TORONTO-CALGARY	4.56	4.29	4.21	4.49	4.53	4.34	4.30	4.58	4.70	4.60	4.74
TORONTO-EDMONTON	4.52	4.31	4.48	4.58	4.65	4.46	4.44	4.67	4.80	4.80	4.80
TORONTO-SAINT JOHN	4.42	4.54	4.48	4.63	4.54	4.49	4.58	4.90	4.77	4.90	5.06
TORONTO-REGINA	4.42	4.28	4.25	4.46	4.70	4.40	4.56	4.85	4.92	4.86	5.06
TORONTO-VANCOUVER	4.27	4.31	4.24	4.33	4.44	4.35	4.31	4.40	4.50	4.63	4.55
TORONTO-ST. JOHN'S	4.24	4.48	4.67	4.39	4.28	4.52	4.50	4.45	4.80	4.34	4.51
TORONTO-WINNIPEG	4.24	4.36	4.22	4.45	4.59	4.37	4.36	4.59	4.67	4.61	4.65
TORONTO-MONTREAL	3.28	3.10	3.02	3.12	3.12	3.12	3.34	3.39	3.38	3.35	3.47
TORONTO-WINDSOR	2.93	3.33	3.43	3.57	3.56	4.60	3.81	3.84	3.91	4.02	4.23
TORONTO-THUNDER BAY	2.57	2.53	2.64	2.64	2.86	2.80	2.96	2.79	2.71	2.77	2.90
TORONTO-SUDBURY	1.34	1.73	1.57	1.87	1.81	2.06	1.97	2.01	2.21	2.30	2.47
MEAN	0.19	0.19	0.19	0.21	0.22	0.21	0.22	0.24	0.25	0.25	0.26
SKEWNESS	3.27	3.27	3.25	3.26	3.21	3.23	3.20	3.16	3.12	3.13	3.14
KURTOSIS	9.68	9.64	9.55	9.60	9.37	9.43	9.32	9.16	8.97	9.02	9.07
STANDARD DEVIATION	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.22

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TABLE A4-7 INTERACTANCE MULTIPLIERS, QUEBEC, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-QUEBEC	47.10	49.13	52.58	52.34	53.10	57.80	56.77	56.83	55.00	57.59	56.12
TORONTO-MONTREAL	7.32	7.36	6.65	6.44	5.07	5.13	5.25	5.30	5.04	4.62	4.67
TORONTO-SAINT JOHN	5.16	4.89	4.82	4.27	4.03	3.93	3.91	3.83	4.03	3.85	3.73
TORONTO-HALIFAX	4.50	4.36	4.00	3.93	3.74	3.57	3.41	3.40	3.38	3.18	3.11
TORONTO-LONDON	4.16	4.02	3.84	3.63	3.75	3.38	3.74	3.31	3.66	3.31	3.41
TORONTO-ST. JOHN'S	3.90	3.86	3.46	3.35	3.37	3.46	3.07	2.95	2.90	2.73	2.55
TORONTO-OTTAWA-HULL	3.78	3.77	3.32	3.65	3.52	3.52	3.57	3.75	4.03	3.88	3.90
TORONTO-VICTORIA	2.81	2.61	2.48	2.75	2.63	2.18	2.15	2.31	2.41	2.38	2.48
TORONTO-CALGARY	2.76	2.46	2.35	2.50	2.64	2.13	2.21	2.25	2.45	2.28	2.47
TORONTO-VANCOUVER	2.69	2.55	2.43	2.49	2.64	2.16	2.25	2.29	2.38	2.29	2.43
TORONTO-EDMONTON	2.69	2.40	2.43	2.47	2.67	2.12	2.21	2.28	2.48	2.33	2.51
TORONTO-SASKATOON	2.65	2.44	2.35	2.31	2.58	2.07	2.16	2.24	2.44	2.23	2.37
TORONTO-WINNIPEG	2.60	2.60	2.38	2.37	2.63	2.12	2.19	2.27	2.38	2.25	2.43
TORONTO-REGINA	2.46	2.32	2.21	2.31	2.52	2.04	2.16	2.25	2.35	2.28	2.51
TORONTO-WINDSOR	2.35	2.25	2.09	2.45	2.59	2.13	2.43	2.35	2.56	2.42	2.68
TORONTO-THUNDER BAY	1.72	1.77	1.66	1.54	1.57	1.33	1.33	1.32	1.23	1.23	1.38
TORONTO-SUDBURY	1.35	1.22	0.97	1.22	0.95	0.94	1.88	1.07	1.28	1.17	1.24
MEAN	0.14	0.13	0.12	0.12	0.12	0.11	0.11	0.11	0.11	0.11	0.11
SKEWNESS	3.32	3.33	3.36	3.37	3.39	3.39	3.39	3.39	3.39	3.40	3.40
KURTOSIS	9.87	9.90	10.02	10.08	10.18	10.18	10.18	10.19	10.18	10.22	10.23
STANDARD DEVIATION	0.26	0.26	0.25	0.25	0.25	0.25	0.25	0.25	0.24	0.24	0.24

TABLE A4-8 INTERACTANCE MULTIPLIERS, REGINA, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-REGINA	41.30	40.95	40.16	39.91	38.59	40.95	41.17	42.13	41.89	42.52	42.15
TORONTO-CALGARY	8.71	8.22	8.37	7.82	7.49	7.31	7.03	7.15	7.08	6.70	6.85
TORONTO-SASKATOON	7.42	7.50	7.90	7.83	7.72	7.76	7.58	6.80	6.15	4.88	4.57
TORONTO-EDMONTON	7.15	6.88	7.42	6.93	6.76	6.96	6.61	6.80	6.75	6.41	6.53
TORONTO-WINNIPEG	6.23	6.94	6.59	6.41	6.20	6.30	6.10	6.08	6.10	5.84	5.90
TORONTO-VANCOUVER	5.93	5.89	5.86	5.59	5.53	5.69	5.55	5.69	5.77	5.67	5.71
TORONTO-VICTORIA	5.79	5.82	5.58	5.40	5.24	5.22	5.10	5.37	6.06	6.32	6.29
TORONTO-LONDON	3.03	3.21	3.24	3.58	3.68	3.34	3.66	3.21	3.21	3.38	3.36
TORONTO-THUNDER BAY	2.31	2.36	2.41	2.47	2.54	2.44	2.35	2.20	2.03	2.35	2.38
TORONTO-OTTAWA-HULL	2.05	2.05	2.12	2.44	2.69	2.30	2.45	2.45	2.59	2.70	2.77
TORONTO-WINDSOR	1.75	1.77	1.95	2.03	2.36	2.15	2.19	2.08	2.10	2.38	2.41
TORONTO-HALIFAX	1.68	1.62	1.63	1.80	2.11	1.81	1.91	1.97	1.98	2.14	2.16
TORONTO-SAINT JOHN	1.55	1.54	1.58	1.77	2.30	1.65	1.82	1.80	1.88	1.95	2.00
TORONTO-MONTREAL	1.54	1.61	1.66	1.91	2.29	1.86	2.04	1.94	1.90	1.94	2.03
TORONTO-QUEBEC	1.51	1.55	1.63	1.79	2.23	1.82	1.95	1.86	1.97	1.99	2.12
TORONTO-ST. JOHN'S	1.35	1.35	1.31	1.52	1.80	1.64	1.67	1.61	1.70	1.76	1.72
TORONTO-SUDBURY	0.68	0.73	0.59	0.77	0.70	0.78	0.79	0.65	0.83	1.07	1.06
MEAN	0.16	0.16	0.17	0.17	0.17	0.16	0.16	0.16	0.16	0.15	0.16
SKEWNESS	3.01	3.01	2.98	3.04	3.07	3.08	3.12	3.13	3.14	3.20	3.19
KURTOSIS	8.43	8.84	8.29	8.58	8.71	8.75	8.94	9.02	9.03	9.30	9.28
STANDARD DEVIATION	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.25	0.25

TABLE A4-9 INTERACTANCE MULTIPLIERS, SAINT JOHN, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-SAINT JOHN	61.91	63.89	63.98	64.48	62.51	64.83	67.06	66.48	68.83	65.25	68.99
TORONTO-HALIFAX	10.89	9.71	9.57	8.60	8.72	8.84	7.45	7.46	6.61	6.64	6.42
TORONTO-ST. JOHN'S	5.50	5.06	4.70	4.59	4.64	4.49	4.33	4.17	3.95	3.75	3.65
TORONTO-QUEBEC	2.54	2.64	2.75	2.53	2.54	2.46	2.45	2.43	2.22	2.26	2.16
TORONTO-MONTREAL	2.50	2.81	2.77	2.75	2.85	2.72	2.59	2.60	2.41	2.28	2.28
TORONTO-LONDON	2.37	2.36	2.47	2.45	2.46	2.30	2.39	2.35	2.16	2.17	2.25
TORONTO-OTTAWA-HULL	1.80	1.82	1.83	1.96	1.97	1.85	1.70	1.91	1.79	1.84	1.88
TORONTO-VICTORIA	1.66	1.43	1.41	1.62	1.64	1.47	1.36	1.52	1.43	1.39	1.43
TORONTO-CALGARY	1.45	1.20	1.37	1.44	1.64	1.38	1.37	1.42	1.39	1.33	1.41
TORONTO-EDMONTON	1.44	1.24	1.38	1.41	1.63	1.37	1.36	1.43	1.40	1.35	1.40
TORONTO-VANCOUVER	1.41	1.32	1.41	1.43	1.62	1.43	1.35	1.42	1.36	1.34	1.41
TORONTO-SASKATOON	1.39	1.28	1.30	1.33	1.50	1.29	1.30	1.36	1.29	1.31	1.30
TORONTO-WINNIPEG	1.33	1.36	1.26	1.36	1.55	1.31	1.29	1.35	1.27	1.25	1.30
TORONTO-REGINA	1.32	1.23	1.29	1.34	1.51	1.30	1.33	1.42	1.29	1.29	1.35
TORONTO-WINDSOR	1.06	1.07	1.09	1.12	1.59	1.36	1.32	1.27	1.26	1.18	1.34
TORONTO-THUNDER BAY	0.76	0.83	0.88	0.85	0.99	0.81	0.75	0.78	0.67	0.72	0.78
TORONTO-SUDBURY	0.68	0.64	0.56	0.74	0.63	0.78	0.60	0.63	0.67	0.64	0.66
MEAN	0.10	0.10	0.10	0.10	0.10	0.10	0.09	0.09	0.09	0.09	0.09
SKEWNESS	3.27	3.31	3.32	3.34	3.33	3.34	3.36	3.36	3.38	3.32	3.38
KURTOSIS	9.61	9.74	9.82	9.94	9.92	9.93	10.06	10.07	10.14	10.14	10.16
STANDARD DEVIATION	0.26	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

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TABLE A4-10 INTERACTANCE MULTIPLIERS, ST. JOHN'S, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-ST. JOHN'S	64.17	62.38	63.15	61.02	60.08	63.40	62.82	63.76	62.49	61.16	61.97
TORONTO-HALIFAX	9.05	9.16	8.80	8.89	8.33	8.03	8.01	7.89	7.94	8.52	8.07
TORONTO-SAINT JOHN	5.80	6.05	5.77	[illegible]	5.44	5.49	5.57	5.27	5.26	5.38	5.28
TORONTO-MONTREAL	2.43	2.84	2.51	2.73	2.84	2.81	2.76	2.54	2.45	2.57	2.53
TORONTO-LONDON	2.33	2.71	2.50	2.51	2.83	2.34	2.50	2.42	2.67	2.62	2.59
TORONTO-QUEBEC	2.19	2.50	2.30	2.36	2.76	2.56	2.43	2.21	2.14	2.47	2.17
TORONTO-OTTAWA-HULL	1.87	2.20	2.19	2.33	2.33	2.09	2.07	2.08	2.31	2.29	2.36
TORONTO-VICTORIA	1.66	1.45	1.48	1.71	1.76	1.53	1.51	1.54	1.63	1.82	1.72
TORONTO-VANCOUVER	1.41	1.40	1.41	1.50	1.71	1.52	1.53	1.45	1.59	1.66	1.67
TORONTO-CALGARY	1.40	1.35	1.39	1.59	1.73	1.47	1.54	1.50	1.67	1.71	1.71
TORONTO-EDMONTON	1.39	1.34	1.41	1.54	1.72	1.46	1.53	1.53	1.68	1.75	1.72
TORONTO-SASKATOON	1.30	1.32	1.30	1.41	1.61	1.38	1.52	1.44	1.66	1.62	1.55
TORONTO-REGINA	1.29	1.35	1.27	1.47	1.63	1.44	1.54	1.54	1.56	1.66	1.60
TORONTO-WINNIPEG	1.29	1.42	1.27	1.44	1.64	1.39	1.44	1.43	1.56	1.54	1.56
TORONTO-WINDSOR	1.11	1.17	1.23	1.25	1.71	1.38	1.54	1.32	1.38	1.39	1.61
TORONTO-THUNDER BAY	0.72	0.83	0.89	0.93	1.04	0.86	0.80	0.77	0.84	0.90	0.91
TORONTO-SUDBURY	0.59	0.72	1.16	1.41	0.86	0.85	0.89	1.29	1.14	0.94	0.95
MEAN	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
SKEWNESS	3.32	3.30	3.32	3.32	3.33	3.34	3.34	3.35	3.35	3.33	3.34
KURTOSIS	9.84	9.79	9.82	9.83	9.90	9.95	9.94	9.98	9.97	9.90	9.96
STANDARD DEVIATION	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

TABLE A4-11 INTERACTANCE MULTIPLIERS, SASKATOON, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-SASKATOON	51.24	52.45	50.29	49.22	46.49	47.15	47.08	46.33	46.70	47.21	47.37
TORONTO-REGINA	6.40	6.45	6.82	7.02	6.82	7.03	6.72	[illegible]	5.62	4.41	4.20
TORONTO-CALGARY	6.36	6.09	6.21	5.88	6.15	6.20	6.13	[illegible]	6.20	5.99	5.96
TORONTO-EDMONTON	5.79	5.44	6.06	5.67	5.86	6.12	5.89	[illegible]	6.15	5.88	5.76
TORONTO-WINNIPEG	5.46	5.84	5.53	5.73	5.62	5.93	5.75	5.06	5.92	5.71	5.66
TORONTO-VANCOUVER	4.90	4.65	4.79	4.74	4.83	5.07	4.99	5.37	5.31	5.74	5.30
TORONTO-VICTORIA	4.84	4.39	4.52	4.66	4.51	4.64	4.60	5.01	5.42	5.78	5.77
TORONTO-LONDON	2.74	2.63	2.52	3.06	3.34	3.00	3.33	2.95	2.93	3.06	3.11
TORONTO-THUNDER BAY	1.90	2.09	2.23	2.32	2.14	2.32	2.14	2.03	2.02	2.16	2.18
TORONTO-OTTAWA-HULL	1.82	1.78	1.83	2.11	2.32	2.11	2.19	2.26	2.33	2.46	2.46
TORONTO-HALIFAX	1.47	1.36	1.42	1.48	1.83	1.61	1.72	1.77	1.77	1.85	1.92
TORONTO-WINDSOR	1.40	1.33	1.56	1.67	2.01	1.92	1.97	1.91	1.89	2.13	2.11
TORONTO-SAINT JOHN	1.36	1.27	1.37	1.49	1.73	1.50	1.62	1.64	1.74	1.76	1.80
TORONTO-MONTREAL	1.36	1.33	1.40	1.59	2.04	1.68	1.85	1.79	1.77	1.79	1.86
TORONTO-QUEBEC	1.35	1.29	1.42	1.51	2.07	1.64	1.79	1.73	1.81	1.79	1.96
TORONTO-ST. JOHN'S	1.11	1.02	1.21	1.21	1.59	1.42	1.41	1.50	1.59	1.53	1.56
TORONTO-SUDBURY	0.49	0.59	0.52	0.63	0.66	0.68	0.80	0.76	0.84	1.03	1.00
MEAN	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14
SKEWNESS	3.25	3.26	3.24	3.24	3.24	3.22	3.24	3.21	3.24	3.26	3.27
KURTOSIS	9.54	9.61	9.51	9.52	9.51	9.40	9.50	9.39	9.49	9.61	9.66
STANDARD DEVIATION	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

TABLE A4-12 INTERACTANCE MULTIPLIERS, SUDBURY, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-SUDBURY	92.91	93.22	92.77	89.97	89.76	88.74	88.13	85.63	82.44	82.58	82.45
TORONTO-LONDON	0.86	1.07	1.14	1.65	1.62	1.63	1.86	2.14	2.83	2.85	2.86
TORONTO-WINDSOR	0.59	0.43	0.46	0.73	0.77	0.86	0.94	1.18	1.45	1.48	1.45
TORONTO-VICTORIA	0.51	0.36	0.45	0.50	0.50	0.55	0.60	0.76	0.84	0.84	0.89
TORONTO-HALIFAX	0.50	0.39	0.40	0.68	0.60	0.69	0.67	0.83	0.99	0.95	0.93
TORONTO-SAINT JOHN	0.45	0.38	0.41	0.66	0.63	0.70	0.65	0.79	0.96	0.83	0.87
TORONTO-REGINA	0.45	0.34	0.36	0.45	0.49	0.52	0.56	0.79	0.80	0.91	0.90
TORONTO-WINNIPEG	0.44	0.37	0.40	0.52	0.51	0.54	0.63	0.74	0.84	0.88	0.91
TORONTO-VANCOUVER	0.43	0.36	0.39	0.49	0.52	0.58	0.59	0.74	0.84	0.87	0.90
TORONTO-EDMONTON	0.42	0.33	0.39	0.48	0.54	0.57	0.58	0.73	0.87	0.87	0.89
TORONTO-CALGARY	0.42	0.36	0.38	0.46	0.51	0.58	0.59	0.74	0.86	0.87	0.90
TORONTO-MONTREAL	0.39	0.46	0.43	0.64	0.63	0.64	0.69	0.88	1.04	0.92	0.90
TORONTO-QUEBEC	0.36	0.44	0.43	0.60	0.56	0.63	0.71	0.80	0.98	0.87	1.04
TORONTO-SASKATOON	0.35	0.34	0.44	0.47	0.56	0.60	0.61	0.71	0.91	0.94	1.00
TORONTO-OTTAWA-HULL	0.35	0.44	0.44	0.69	0.65	0.78	0.80	0.93	1.15	1.15	1.19
TORONTO-ST. JOHN'S	0.29	0.34	0.42	0.57	0.65	0.81	0.78	0.94	1.02	0.90	0.86
TORONTO-THUNDER BAY	0.27	0.36	0.29	0.44	0.51	0.55	0.66	0.71	1.16	1.29	1.07
MEAN	0.06	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.07
SKEWNESS	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42
KURTOSIS	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.33	10.33	10.33	10.33
STANDARD DEVIATION	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

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TABLE A4-13 INTERACTANCE MULTIPLIERS, THUNDER BAY, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-THUNDER BAY	67.71	68.60	66.33	64.55	63.79	65.10	65.52	65.20	63.22	63.16	63.53
TORONTO-WINNIPEG	5.10	5.27	5.28	5.24	4.98	5.05	4.63	4.51	4.83	4.65	4.66
TORONTO-SASKATOON	2.92	2.96	3.07	3.21	2.88	3.10	2.91	2.81	3.04	2.84	2.03
TORONTO-REGINA	2.84	2.64	2.79	3.09	2.96	2.90	2.73	2.86	2.80	2.79	2.79
TORONTO-CALGARY	2.87	2.27	2.49	2.63	2.57	2.43	2.42	2.48	2.59	2.54	2.53
TORONTO-EDMONTON	2.54	1.99	2.52	2.58	2.52	2.44	2.35	2.43	2.55	2.51	2.51
TORONTO-VANCOUVER	2.43	2.23	2.43	2.55	2.48	2.40	2.35	2.40	2.43	2.44	2.46
TORONTO-VICTORIA	2.36	2.13	2.28	2.46	2.33	2.20	2.21	2.37	2.51	2.60	2.59
TORONTO-LONDON	2.29	2.37	2.83	2.93	3.05	3.42	3.40	3.44	3.62	3.90	3.67
TORONTO-OTTAWA-HULL	1.44	1.37	1.52	1.70	1.94	1.79	1.91	1.85	1.99	1.92	1.91
TORONTO-MONTREAL	1.27	1.31	1.50	1.54	1.76	1.44	1.51	1.48	1.48	1.41	1.42
TORONTO-HALIFAX	1.26	1.15	1.30	1.30	1.48	1.34	1.38	1.32	1.43	1.39	1.36
TORONTO-WINDSOR	1.25	1.30	1.22	1.39	1.77	1.15	1.59	1.75	1.87	1.80	1.89
TORONTO-QUEBEC	1.24	1.37	1.49	1.45	1.70	1.39	1.50	1.50	1.42	1.38	1.44
TORONTO-SAINT JOHN.	1.22	1.13	1.44	1.34	1.49	1.23	1.32	1.30	1.34	1.35	1.29
TORONTO-ST. JOHN'S	0.96	0.95	1.07	1.06	1.29	1.21	1.13	1.22	1.31	1.17	1.14
TORONTO-SUDBURY	0.51	0.92	0.47	0.96	0.97	0.96	1.12	1.06	1.57	2.15	1.98
MEAN	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
SKEWNESS	3.40	3.40	3.39	3.39	3.40	3.40	3.40	3.40	3.40	3.40	3.40
KURTOSIS	10.22	10.23	10.21	10.21	10.24	10.22	10.25	10.25	10.24	10.24	10.24
STANDARD DEVIATION	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

TABLE A4-14 INTERACTANCE MULTIPLIERS, VANCOUVER, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-VANCOUVER	21.60	21.34	21.19	20.43	19.96	19.89	19.28	19.29	19.14	19.12	18.50
TORONTO-VICTORIA	14.00	14.66	13.47	12.57	12.19	12.08	10.48	10.57	10.48	10.11	9.71
TORONTO-CALGARY	10.13	9.91	9.96	9.66	9.37	9.46	9.62	9.74	9.92	9.59	9.46
TORONTO-EDMONTON	9.57	9.79	10.15	9.68	9.44	9.90	9.79	9.95	10.01	9.63	9.57
TORONTO-REGINA	7.86	7.63	7.61	7.49	7.08	7.66	7.89	8.24	8.11	7.91	8.08
TORONTO-SASKATOON	7.70	7.55	7.49	7.54	7.18	7.78	7.87	8.22	8.27	8.28	8.07
TORONTO-WINNIPEG	6.42	6.51	6.32	6.24	6.01	6.30	6.53	6.51	6.48	6.48	6.48
TORONTO-LONDON	3.74	4.02	4.31	4.54	4.63	4.69	4.84	4.52	4.41	4.48	4.64
TORONTO-OTTAWA-HULL	2.83	2.65	2.73	3.07	3.25	3.08	3.28	3.22	3.32	3.54	3.55
TORONTO-THUNDER BAY	2.55	2.58	2.67	2.79	2.73	2.69	2.76	2.62	2.47	2.77	2.91
TORONTO-HALIFAX	2.20	2.19	2.31	2.50	2.87	2.60	2.74	2.72	2.73	2.66	3.01
TORONTO-MONTREAL	2.24	2.25	2.41	2.78	3.22	2.82	3.14	2.98	2.90	2.93	3.11
TORONTO-QUEBEC	2.13	2.12	2.33	2.56	3.11	2.56	2.83	2.75	2.78	2.82	3.03
TORONTO-SAINT JOHN	2.11	2.06	2.27	2.53	2.82	2.49	2.58	2.57	2.60	2.76	2.85
TORONTO-WINDSOR	1.98	1.95	2.09	2.50	2.71	2.55	2.80	2.66	2.63	2.87	3.13
TORONTO-ST. JOHN'S	1.83	1.74	1.89	2.09	2.45	2.38	2.35	2.28	2.45	2.38	2.47
TORONTO-SUDBURY	0.93	1.05	0.81	1.04	0.98	1.08	1.24	1.14	1.22	1.46	1.43
MEAN	0.39	0.41	0.40	0.41	0.43	0.44	0.45	0.45	0.45	0.45	0.46
SKEWNESS	1.40	1.38	1.40	1.44	1.52	1.39	1.39	1.33	1.31	1.41	1.38
KURTOSIS	1.25	1.11	1.29	1.53	1.84	1.39	1.55	1.28	1.22	1.61	1.55
STANDARD DEVIATION	0.39	0.38	0.37	0.35	0.35	0.36	0.35	0.36	0.35	0.34	0.34

TABLE A4-15 INTERACTANCE MULTIPLIERS, VICTORIA, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-VICTORIA	46.44	46.25	45.96	47.74	44.16	43.71	47.56	46.54	47.26	47.71	45.79
TORONTO-VANCOUVER	9.61	9.97	9.32	8.27	8.79	8.89	7.10	7.29	6.61	6.18	6.02
TORONTO-CALGARY	6.74	6.70	6.62	6.13	6.26	6.42	5.93	6.19	6.61	6.38	6.66
TORONTO-EDMONTON	6.39	6.74	6.65	6.02	6.29	6.62	5.95	6.20	6.44	6.19	6.50
TORONTO-REGINA	5.20	5.06	5.06	4.67	4.75	5.22	4.85	5.29	5.34	5.39	5.61
TORONTO-SASKATOON	5.14	4.56	5.09	4.79	4.80	5.25	4.90	5.31	5.43	5.32	5.56
TORONTO-WINNIPEG	4.09	4.26	4.06	3.83	3.94	4.16	3.95	4.07	4.05	4.10	4.27
TORONTO-LONDON	2.78	2.88	3.27	3.25	3.57	3.60	3.59	3.25	2.94	3.01	3.12
TORONTO-OTTAWA-HULL	2.01	1.98	2.16	2.44	2.59	2.43	2.38	2.36	2.28	2.37	2.40
TORONTO-HALIFAX	1.70	1.68	1.72	1.86	2.20	1.94	1.99	2.02	1.89	1.92	2.05
TORONTO-THUNDER BAY	1.71	1.69	1.76	1.75	1.82	1.83	1.75	1.64	1.60	1.74	1.91
TORONTO-SAINT JOHN	1.58	1.51	1.59	1.88	2.05	1.86	1.75	1.88	1.79	1.80	1.82
TORONTO-MONTREAL	1.52	1.54	1.64	1.85	2.20	1.98	2.01	1.94	1.78	1.78	1.87
TORONTO-QUEBEC	1.47	1.48	1.68	1.77	2.18	1.88	1.89	1.86	1.81	1.78	1.51
TORONTO-ST. JOHN'S	1.37	1.26	1.39	1.46	1.89	1.73	1.60	1.65	1.64	1.57	1.62
TORONTO-WINDSOR	1.37	1.37	1.42	1.64	1.82	1.77	1.93	1.75	1.74	1.81	1.93
TORONTO-SUDBURY	0.69	0.78	0.61	0.64	0.70	0.71	0.86	0.77	0.78	0.95	0.94
MEAN	0.15	0.15	0.15	0.14	0.15	0.15	0.14	0.14	0.14	0.14	0.14
SKEWNESS	3.13	3.11	3.13	3.22	3.18	3.14	3.26	3.22	3.23	3.25	3.23
KURTOSIS	8.96	8.87	9.00	9.41	9.21	9.03	9.58	9.43	9.44	9.55	9.44
STANDARD DEVIATION	0.27	0.27	0.27	0.26	0.26	0.26	0.25	0.25	0.25	0.25	0.25

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TABLE A4-16 INTERACTANCE MULTIPLIERS, WINDSOR, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-WINDSOR	59.00	60.06	58.73	57.21	55.97	61.77	62.46	60.79	61.48	62.21	61.26
TORONTO-CALGARY	3.02	2.63	2.89	3.02	2.92	2.52	2.50	2.61	2.48	2.51	2.71
TORONTO-WINNIPEG	2.99	3.10	3.10	3.17	3.05	2.78	2.58	2.87	2.69	2.74	2.70
TORONTO-REGINA	2.96	2.96	3.12	3.01	3.06	2.73	2.58	2.73	2.60	2.67	2.66
TORONTO-SASKATOON	2.89	2.78	3.00	2.94	3.09	2.85	2.65	2.71	2.57	2.67	2.59
TORONTO-EDMONTON	2.87	2.50	2.93	2.96	2.96	2.56	2.51	2.63	2.51	2.49	2.60
TORONTO-VICTORIA	2.74	2.62	2.70	2.95	2.76	2.34	2.37	2.71	2.36	2.41	2.43
TORONTO-MONTREAL	2.68	2.51	2.61	2.90	3.45	2.66	2.62	2.72	2.75	2.60	2.94
TORONTO-VANCOUVER	2.63	2.52	2.66	2.85	2.78	2.46	2.36	2.52	2.34	2.35	2.45
TORONTO-HALIFAX	2.61	2.24	2.35	2.41	2.74	2.27	2.32	2.25	2.32	2.21	2.35
TORONTO-LONDON	2.61	2.68	2.71	2.85	2.89	2.46	2.40	2.45	2.43	2.28	2.22
TORONTO-QUEBEC	2.58	2.44	2.59	2.83	2.96	2.48	2.62	2.62	2.68	2.61	2.64
TORONTO-OTTAWA-HULL	2.54	2.75	2.79	2.86	2.70	2.57	2.53	2.60	2.65	2.56	2.66
TORONTO-SAINT JOHN	2.31	2.19	2.38	2.47	2.73	2.22	2.26	2.24	2.21	2.13	2.18
TORONTO-ST. JOHN'S	2.21	1.87	2.09	1.97	2.52	2.07	2.03	1.93	2.04	1.81	1.86
TORONTO-THUNDER BAY	1.86	2.01	1.01	1.71	1.99	1.70	1.63	1.87	1.82	1.65	1.78
TORONTO-SUDBURY	1.51	2.16	1.55	1.89	1.42	1.54	1.61	1.74	2.07	2.10	1.97
MEAN	0.10	0.10	0.10	0.10	0.11	0.10	0.10	0.10	0.10	0.10	0.10
SKEWNESS	3.42	3.49	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42
KURTOSIS	10.32	10.33	10.31	10.31	10.31	10.33	10.33	10.33	10.33	10.33	10.33
STANDARD DEVIATION	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

TABLE A4-17 INTERACTANCE MULTIPLIERS, WINNIPEG, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-WINNIPEG	24.64	26.55	24.69	23.37	22.70	23.40	23.08	22.58	23.96	23.92	24.00
TORONTO-SASKATOON	9.18	9.77	9.06	9.47	8.98	9.58	9.30	9.34	9.14	8.85	8.66
TORONTO-REGINA	9.08	9.46	9.39	9.09	8.54	9.02	8.78	9.07	8.64	8.09	8.10
TORONTO-CALGARY	7.82	7.06	7.51	7.16	6.97	7.09	7.08	7.32	7.20	6.95	6.92
TORONTO-EDMONTON	7.45	5.67	7.51	7.08	6.88	6.98	6.84	7.26	7.14	6.82	6.82
TORONTO-VANCOUVER	7.14	6.62	6.82	6.64	6.48	6.69	6.74	6.81	6.58	6.55	6.39
TORONTO-VICTORIA	6.73	6.31	6.30	6.27	5.99	5.98	6.01	6.26	6.53	6.79	6.75
TORONTO-THUNDER BAY	5.89	6.56	6.30	6.20	5.93	6.12	5.76	5.41	5.09	5.56	5.56
TORONTO-LONDON	4.36	4.37	4.57	4.91	5.17	4.95	5.17	4.76	4.56	4.64	4.76
TORONTO-OTTAWA-HULL	2.95	2.89	2.91	3.36	3.63	3.29	3.38	3.48	3.48	3.60	3.58
TORONTO-WINDSOR	2.48	2.54	2.64	2.87	3.18	3.04	3.19	3.19	3.06	3.38	3.38
TORONTO-HALIFAX	2.44	2.32	2.32	2.51	2.84	2.56	2.66	2.77	2.74	2.81	2.83
TORONTO-MONTREAL	2.43	2.43	2.49	2.77	3.25	2.84	3.04	2.90	2.81	2.84	2.86
TORONTO-QUEBEC	2.31	2.31	2.43	2.56	3.19	2.68	2.87	2.79	2.78	2.78	2.96
TORONTO-SAINT JOHN	2.27	2.23	2.26	2.52	2.78	2.41	2.53	2.55	2.57	2.63	2.58
TORONTO-ST. JOHN'S	1.56	1.85	1.91	2.11	2.49	2.27	2.29	2.31	2.43	2.31	2.31
TORONTO-SUDBURY	1.00	1.06	0.91	1.12	1.00	1.09	1.28	1.20	1.30	1.48	1.55
MEAN	0.29	0.26	0.29	0.31	0.32	0.31	0.31	0.32	0.30	0.30	0.30
SKEWNESS	2.15	2.35	2.17	2.15	2.27	2.18	2.23	2.13	2.32	2.40	2.44
KURTOSIS	4.77	5.54	4.85	4.76	5.32	4.87	5.12	4.66	5.47	5.85	6.02
STANDARD DEVIATION	0.24	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.26	0.26

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TABLE A4-18 INTERACTANCE MULTIPLIERS, CALGARY, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-CALGARY	31.6	29.7	29.0	28.7	29.1
VANCOUVER-REGINA	16.8	16.8	15.8	14.6	14.5
VANCOUVER-EDMONTON	16.6	17.5	17.5	17.4	16.7
VANCOUVER-SASKATCON	15.0	14.8	15.7	14.7	13.6
VANCOUVER-WINNIPEG	13.6	13.7	13.4	13.4	12.9
VANCOUVER-VICTORIA	6.5	7.5	8.7	11.3	13.2
MEAN	0.837	0.942	0.945	0.948	0.904
SKEWNESS	0.650	0.597	0.719	1.063	1.225
KURTOSIS	-0.817	-0.896	-0.827	-0.551	-0.329
STANDARD DEVIATION	0.409	0.413	0.384	0.354	0.339

TABLE A4-19 INTERACTANCE MULTIPLIERS, EDMONTON, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-EDMONTON	32.6	31.3	32.0	31.6	31.6
VANCOUVER-CALGARY	17.6	18.2	17.6	17.2	16.7
VANCOUVER-REGINA	14.8	15.2	14.6	13.4	13.5
VANCOUVER-SASKATOON	14.8	14.6	14.9	14.3	13.0
VANCOUVER-WINNIPEG	13.7	13.4	12.6	13.0	12.5
VANCOUVER-VICTORIA	6.5	7.3	8.4	10.5	12.6
MEAN	0.764	0.854	0.814	0.826	0.799
SKEWNESS	0.759	0.733	0.918	1.126	1.238
KURTOSIS	-0.762	-0.833	0.681	-0.466	0.311
STANDARD DEVIATION	0.400	0.411	0.395	0.378	0.359

TABLE A4-20 INTERACTANCE MULTIPLIERS, REGINA, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-REGINA	45.2	45.1	49.2	50.5	53.2
VANCOUVER-WINNIPEG	14.1	14.5	13.2	13.0	12.3
VANCOUVER-SASKATOON	13.4	14.0	13.1	10.5	7.7
VANCOUVER-CALGARY	12.4	11.4	9.9	9.6	9.5
VANCOUVER-EDMONTON	10.5	10.1	9.2	9.1	9.0
VANCOUVER-VICTORIA	4.5	4.8	5.5	7.2	8.4
MEAN	0.476	0.482	0.415	0.392	0.366
SKEWNESS	1.176	1.177	1.265	1.322	1.337
KURTOSIS	-0.337	-0.347	-0.232	-0.148	-0.124
STANDARD DEVIATION	0.411	0.416	0.402	0.393	0.394

TABLE A4-21 INTERACTANCE MULTIPLIERS, SASKATOON, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-SASKATCON	54.6	53.2	54.5	55.2	58.5
VANCOUVER-WINNIPEG	12.4	12.9	12.3	12.6	11.7
VANCOUVER-REGINA	11.3	12.6	11.5	9.4	6.7
VANCOUVER-CALGARY	9.4	8.8	8.6	8.3	8.0
VANCOUVER-EDMONTON	8.6	8.3	8.1	8.2	7.6
VANCOUVER-VICTORIA	3.0	4.2	5.0	6.4	7.5
MEAN	0.360	0.375	0.362	0.352	0.323
SKEWNESS	1.283	1.265	1.300	1.326	1.340
KURTOSIS	-0.201	-0.232	-0.180	-0.143	-0.120
STANDARD DEVIATION	0.407	0.401	0.401	0.401	0.398

TABLE A4-22 INTERACTANCE MULTIPLIERS, VICTORIA, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-VICTORIA	65.7	67.5	66.7	61.2	57.7
VANCOUVER-CALGARY	7.3	6.8	6.8	8.1	8.8
VANCOUVER-SASKATOON	6.9	6.5	6.8	7.9	8.5
VANCOUVER-REGINA	6.8	6.4	6.7	7.7	8.6
VANCOUVER-EDMONTON	6.7	6.5	6.6	7.8	8.4
VANCOUVER-WINNIPEG	6.6	6.2	6.5	7.3	8.1
MEAN	0.262	0.258	0.261	0.292	0.319
SKEWNESS	1.361	1.361	1.361	1.360	1.360
KURTOSIS	-0.084	-0.084	-0.083	-0.084	-0.084
STANDARD DEVIATION	0.378	0.382	0.383	0.383	0.384

TABLE A4-23 INTERACTANCE MULTIPLIERS, WINNIPEG, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-WINNIPEG	36.8	36.2	37.9	38.3	38.9
VANCOUVER-SASKATOON	17.4	18.1	17.8	16.9	16.2
VANCOUVER-REGINA	16.8	17.7	16.7	15.7	14.8
VANCOUVER-CALGARY	12.2	11.3	10.8	10.5	10.4
VANCOUVER-EDMONTON	11.5	11.0	10.1	10.4	10.1
VANCOUVER-VICTORIA	5.2	5.7	6.8	8.3	9.7
MEAN	0.614	0.635	0.576	0.569	0.546
SKEWNESS	0.852	0.802	0.963	1.100	1.192
KURTOSIS	-0.765	-0.896	-0.716	-0.515	-0.372
STANDARD DEVIATION	0.398	0.405	0.387	0.378	0.367

TABLE A4-24 INTERACTANCE MULTIPLIERS, HALIFAX, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-HALIFAX	43.7	43.2	42.6	43.0	43.5
TORONTO-SAINT JOHN	19.1	17.7	16.5	15.3	15.2
TORONTO-ST. JOHN'S	15.3	14.5	14.5	14.8	14.1
TORONTO-LONDON	5.2	5.2	5.9	6.2	6.1
TORONTO-MONTREAL	4.3	5.1	5.6	5.2	5.0
TORONTO-QUEBEC	4.2	4.7	4.9	4.8	4.6
TORONTO-OTTAWA-HULL	3.6	4.4	4.3	4.9	5.3
TORONTO-WINDSOR	2.0	2.2	3.0	3.0	3.3
TORONTO-SUDBURY	1.3	1.6	1.4	1.7	1.6
TORONTO-THUNDER BAY	1.2	1.3	1.3	1.3	1.4
MEAN	0.270	0.268	0.270	0.265	0.261
SKEWNESS	1.557	1.633	1.664	1.710	1.753
KURTOSIS	1.216	1.495	1.621	1.782	1.920
STANDARD DEVIATION	0.359	0.346	0.339	0.333	0.331

TABLE A4-25 INTERACTANCE MULTIPLIERS, LONDON, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-LONDON	76.4	77.1	75.5	73.8	72.5
TORONTO-HALIFAX	3.4	2.9	2.9	3.2	3.3
TORONTO-OTTAWA-HULL	3.3	3.6	3.9	4.2	4.4
TORONTO-SAINT JOHN	3.2	3.0	2.9	3.0	3.2
TORONTO-QUEBEC	2.9	2.6	3.0	3.1	3.0
TORONTO-MONTREAL	2.9	2.7	2.9	3.0	3.0
TORONTO-ST. JOHN'S	2.8	2.5	2.5	2.8	2.7
TORONTO-SUDBURY	2.1	2.2	2.6	2.9	3.3
TORONTO-THUNDER BAY	1.8	2.0	2.4	2.6	3.0
TORONTO-WINDSOR	1.2	1.4	1.5	1.6	1.6
MEAN	0.134	0.133	0.137	0.140	0.143
SKEWNESS	2.273	2.274	2.274	2.273	2.273
KURTOSIS	3.559	3.562	3.561	3.560	3.558
STANDARD DEVIATION	0.314	0.314	0.315	0.315	0.314

TABLE A4-26 INTERACTANCE MULTIPLIERS, MONTREAL, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-MONTREAL	23.5	24.0	24.1	24.6	25.6
TORONTO-QUEBEC	16.7	16.0	14.6	14.0	13.8
TORONTO-SAINT JOHN	11.6	12.0	11.7	11.5	11.0
TORONTO-HALIFAX	10.8	10.8	10.6	10.2	9.9
TORONTO-LONDON	10.7	10.3	10.9	10.8	10.6
TORONTO-ST. JOHN'S	9.9	9.6	10.2	9.8	8.8
TORONTO-OTTAWA-HULL	5.5	5.4	5.3	5.9	6.2
TORONTO-WINDSOR	5.1	5.7	6.8	7.2	8.4
TORONTO-THUNDER BAY	3.1	3.2	2.8	2.6	2.7
TORONTO-SUDBURY	3.1	3.1	2.8	3.4	3.1
MEAN	0.521	0.499	0.488	0.474	0.450
SKEWNESS	0.714	0.816	0.800	0.959	1.124
KURTOSIS	-0.606	-0.339	-0.134	0.256	0.656
STANDARD DEVIATION	0.335	0.321	0.310	0.297	0.291

TABLE A4-27 INTERACTANCE MULTIPLIERS, OTTAWA-HULL, EASTERN SUB-SYSTEM 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-OTTAWA-HULL	49.1	46.5	42.6	39.0	37.7
TORONTO-LONDON	10.3	11.1	12.6	12.6	12.2
TORONTO-HALIFAX	7.5	7.3	7.2	8.0	8.3
TORONTO-QUEBEC	7.2	7.4	8.5	9.4	9.6
TORONTO-SAINT JOHN	6.5	6.8	6.7	7.1	7.4
TORONTO-ST. JOHN'S	6.3	6.5	6.6	7.2	6.8
TORONTO-MONTREAL	4.6	4.2	4.5	4.7	4.9
TORONTO-WINDSOR	3.9	5.0	5.2	5.6	6.0
TORONTO-THUNDER BAY	3.0	2.8	3.5	3.2	3.6
TORONTO-SUDBURY	1.8	2.6	2.7	3.2	3.5
MEAN	0.212	0.226	0.248	0.276	0.287
SKEWNESS	2.145	2.127	2.050	1.980	1.989
KURTOSIS	3.181	3.123	2.872	2.677	2.709
STANDARD DEVIATION	0.296	0.295	0.293	0.292	0.290

TABLE A4-20 INTERACTANCE MULTIPLIERS, QUEBEC, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-QUEBEC	58.7	60.8	64.2	63.5	65.5
TORONTO-MONTREAL	9.5	8.4	6.8	6.6	6.1
TORONTO-SAINT JOHN	6.7	5.6	5.1	5.2	4.9
TORONTO-HALIFAX	6.0	5.2	4.5	4.4	4.1
TORONTO-LONDON	5.9	5.0	5.2	5.1	4.7
TORONTO-ST. JOHN'S	5.1	4.4	4.0	3.8	3.3
TORONTO-OTTAWA-HULL	4.9	4.7	4.6	5.4	5.3
TORONTO-WINDSOR	2.8	3.1	3.0	3.2	3.4
TORONTO-THUNDER BAY	1.8	1.5	1.3	1.1	1.3
TORONTO-SUDBURY	1.7	1.5	1.4	1.6	1.5
MEAN	0.205	0.182	0.168	0.169	0.163
SKEWNESS	2.187	2.224	2.242	2.242	2.250
KURTOSIS	3.305	3.412	3.470	3.469	3.491
STANDARD DEVIATION	0.333	0.327	0.321	0.320	0.319

TABLE A4-29 INTERACTANCE MULTIPLIERS, SAINT JOHN, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-SAINT JOHN	66.8	69.6	71.8	73.8	74.0
TORONTO-HALIFAX	12.8	10.1	8.8	7.9	7.8
TORONTO-ST. JOHN'S	6.5	5.5	5.2	4.8	4.4
TORONTO-LONDON	3.1	3.2	3.2	2.8	3.0
TORONTO-QUEBEC	3.0	3.1	3.0	2.7	2.7
TORONTO-MONTREAL	3.0	3.3	3.2	3.0	2.8
TORONTO-OTTAWA-HULL	2.1	2.4	2.1	2.2	2.4
TORONTO-WINDSOR	1.2	1.3	1.5	1.5	1.6
TORONTO-SUDBURY	0.8	0.8	0.7	0.8	0.7
TORONTO-THUNDER BAY	0.7	0.8	0.7	0.6	0.7
MEAN	0.165	0.155	0.148	0.143	0.142
SKEWNESS	2.149	2.207	2.227	2.240	2.243
KURTOSIS	3.164	3.355	3.416	3.457	3.466
STANDARD DEVIATION	0.335	0.327	0.323	0.321	0.320

TABLE A4-30 INTERACTANCE MULTIPLIERS, ST. JOHN'S, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-ST. JOHN'S	69.2	66.2	68.0	68.2	67.5
TORONTO-HALIFAX	10.7	10.5	9.6	9.6	9.9
TORONTO-SAINT JOHN	6.9	7.1	6.7	6.4	6.5
TORONTO-LONDON	3.0	3.3	3.3	3.5	3.4
TORONTO-MONTREAL	2.9	3.3	3.4	3.1	3.1
TORONTO-QUEBEC	2.6	2.9	3.0	2.7	2.7
TORONTO-OTTAWA-HULL	2.2	2.9	2.6	2.9	3.0
TORONTO-WINDSOR	1.2	1.4	1.8	1.6	1.9
TORONTO-SUDBURY	0.7	1.6	1.0	1.3	1.1
TORONTO-THUNDER BAY	0.6	0.9	0.7	0.7	0.8
MEAN	0.154	0.161	0.156	0.156	0.157
SKEWNESS	2.188	2.190	2.206	2.210	2.204
KURTOSIS	3.293	3.299	3.353	3.363	3.346
STANDARD DEVIATION	0.324	0.322	0.321	0.321	0.320

TABLE A4-32 INTERACTANCE MULTIPLIERS, THUNDER BAY, EASTERN SUB-SYSTEM,1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-THUNDER BAY	90.2	87.8	85.6	83.6	83.9
TORONTO-LONDON	2.2	3.1	4.0	4.5	4.6
TORONTO-OTTAWA-HULL	1.2	1.4	1.9	2.0	1.9
TORONTO-MONTREAL	1.1	1.3	1.3	1.3	1.2
TORONTO-QUEBEC	1.1	1.3	1.3	1.3	1.2
TORONTO-HALIFAX	1.1	1.1	1.2	1.3	1.1
TORONTO-SAINT JOHN	1.0	1.1	1.2	1.2	1.1
TORONTO-WINDSOR	1.0	1.1	1.4	1.9	1.8
TORONTO-ST. JOHN'S	0.8	0.9	1.0	1.2	1.0
TORONTO-SUDBURY	0.4	1.0	1.2	1.9	2.4
MEAN	0.111	0.114	0.117	0.120	0.120
SKEWNESS	2.276	2.275	2.272	2.271	2.270
KURTOSIS	3.567	3.563	3.557	3.552	3.546
STANDARD DEVIATION	0.313	0.313	0.312	0.311	0.311

TABLE A4-31 INTERACTANCE MULTIPLIERS, SUDBURY, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-SUDBURY	95.8	92.4	91.1	86.3	86.8
TORONTO-LONDON	1.0	2.1	2.4	3.7	3.8
TORONTO-WINDSOR	0.6	0.8	1.0	1.7	1.7
TORONTO-HALIFAX	0.5	0.8	0.7	1.1	1.0
TORONTO-SAINT JOHN	0.5	0.7	0.7	1.1	1.0
TORONTO-MONTREAL	0.4	0.7	0.8	1.2	1.0
TORONTO-QUEBEC	0.4	0.7	0.8	1.1	1.2
TORONTO-OTTAWA-HULL	0.3	0.8	0.9	1.4	1.4
TORONTO-ST. JOHN'S	0.3	0.6	0.8	1.2	1.0
TORONTO-THUNDER BAY	0.2	0.4	0.7	1.4	1.2
MEAN	0.105	0.109	0.110	0.117	0.116
SKEWNESS	2.277	2.276	2.276	2.273	2.273
KURTOSIS	3.569	3.567	3.566	3.560	3.558
STANDARD DEVIATION	0.315	0.314	0.314	0.312	0.312

TABLE A4-33 INTERACTANCE MULTIPLIERS, WINDSOR, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-WINDSOR	74.8	73.8	76.4	74.6	75.6
TORONTO-MONTREAL	3.3	3.6	3.2	3.4	3.7
TORONTO-HALIFAX	3.2	3.0	2.8	2.8	2.9
TORONTO-QUEBEC	3.2	3.5	3.2	3.3	3.3
TORONTO-LONDON	3.1	3.4	2.7	2.9	2.5
TORONTO-OTTAWA-HULL	3.1	3.6	3.1	3.3	3.3
TORONTO-SAINT JOHN	2.8	3.0	2.8	2.7	2.7
TORONTO-ST. JOHN'S	2.7	2.4	2.5	2.5	2.2
TORONTO-THUNDER BAY	1.9	1.4	1.5	2.0	1.8
TORONTO-SUDBURY	1.9	2.3	1.9	2.5	2.4
MEAN	0.135	0.138	0.133	0.137	0.136
SKEWNESS	2.275	2.273	2.275	2.275	2.274
KURTOSIS	3.563	3.558	3.563	3.565	3.562
STANDARD DEVIATION	0.308	0.309	0.310	0.310	0.311

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TABLE A4-34 INTERACTANCE SENSITIVITY, CALGARY, NATIONAL SYSTEM , 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
EDMONTON-CALGARY	0.426	0.482	0.526	0.522	0.530	0.522	0.536	0.521	0.537	0.508	0.493
VANCOUVER-CALGARY	0.317	0.331	0.320	0.320	0.321	0.336	0.359	0.374	0.383	0.374	0.370
CALGARY-EDMONTON	0.200	0.245	0.279	0.282	0.285	0.285	0.302	0.293	0.308	0.275	0.265
TORONTO-CALGARY	0.192	0.198	0.198	0.192	0.177	0.195	0.194	0.186	0.189	0.196	0.192
TORONTO-MONTREAL	0.191	0.197	0.182	0.198	0.245	0.189	0.188	0.181	0.173	0.171	0.172
VANCOUVER-EDMONTON	0.141	0.149	0.155	0.157	0.154	0.169	0.170	0.169	0.168	0.169	0.171
TORONTO-VANCOUVER	0.135	0.141	0.142	0.138	0.126	0.143	0.148	0.151	0.161	0.159	0.161
REGINA-CALGARY	0.133	0.123	0.127	0.118	0.109	0.101	0.092	0.095	0.085	0.079	0.083
WINNIPEG-CALGARY	0.122	0.116	0.123	0.118	0.113	0.128	0.127	0.136	0.121	0.121	0.115
TORONTO-WINNIPEG	0.114	0.114	0.106	0.105	0.096	0.107	0.107	0.112	0.101	0.098	0.091
CALGARY-VANCOUVER	0.107	0.120	0.117	0.116	0.117	0.130	0.143	0.149	0.153	0.151	0.144

TABLE A4-35 INTERACTANCE SENSITIVITY, EDMONTON, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
CALGARY-EDMONTON	0.446	0.527	0.567	0.577	0.572	0.583	0.609	0.591	0.614	0.560	0.545
VANCOUVER-EDMONTON	0.315	0.321	0.315	0.322	0.312	0.344	0.344	0.341	0.336	0.345	0.353
EDMONTON-CALGARY	0.199	0.249	0.278	0.279	0.283	0.285	0.300	0.289	0.303	0.274	0.263
TORONTO-MONTREAL	0.192	0.200	0.193	0.197	0.246	0.185	0.185	0.180	0.171	0.171	0.171
TORONTO-EDMONTON	0.190	0.171	0.187	0.179	0.162	0.182	0.183	0.176	0.176	0.182	0.183
VANCOUVER-CALGARY	0.148	0.171	0.169	0.171	0.171	0.183	0.201	0.207	0.216	0.202	0.198
TORONTO-VANCOUVER	0.135	0.143	0.142	0.141	0.127	0.147	0.150	0.151	0.160	0.159	0.162
WINNIPEG-EDMONTON	0.127	0.112	0.112	0.119	0.108	0.099	0.092	0.114	0.103	0.100	0.097
TORONTO-WINNIPEG	0.112	0.113	0.104	0.104	0.096	0.104	0.104	0.110	0.100	0.096	0.089
CALGARY-VANCOUVER	0.107	0.121	0.118	0.119	0.118	0.133	0.145	0.149	0.152	0.151	0.144
EDMONTON-VANCOUVER	0.106	0.112	0.109	0.111	0.111	0.124	0.126	0.129	0.127	0.128	0.128

TABLE A4-36 INTERACTANCE SENSITIVITY, HALIFAX, NATIONAL SYSTEM 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-HALIFAX	0.327	0.332	0.337	0.327	0.317	0.339	0.347	0.350	0.349	0.350	0.358
TORONTO-MONTREAL	0.308	0.323	0.316	0.302	0.325	0.294	0.282	0.271	0.253	0.246	0.235
MONTREAL-HALIFAX	0.304	0.327	0.335	0.326	0.369	0.326	0.331	0.315	0.289	0.272	0.258
SAINT JOHN-HALIFAX	0.164	0.152	0.143	0.130	0.121	0.128	0.099	0.095	0.083	0.082	0.079
ST. JOHN'S-HALIFAX	0.127	0.132	0.122	0.129	0.115	0.116	0.113	0.111	0.116	0.127	0.113
TORONTO-OTTAWA-HULL	0.089	0.091	0.090	0.095	0.096	0.097	0.095	0.105	0.112	0.110	0.112
OTTAWA-HULL-HALIFAX	0.085	0.092	0.091	0.099	0.098	0.102	0.099	0.115	0.126	0.129	0.136

TABLE A4-37 INTERACTANCE SENSITIVITY, LONDON, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-LONDON	0.290	0.283	0.244	0.271	0.233	0.241	0.218	0.223	0.200	0.207	0.208
TORONTO-MONTREAL	0.292	0.280	0.286	0.260	0.288	0.251	0.260	0.249	0.253	0.248	0.239
MONTREAL-LONDON	0.275	0.264	0.275	0.246	0.269	0.229	0.259	0.244	0.259	0.250	0.237
OTTAWA-HULL-LONDON	0.120	0.139	0.155	0.140	0.165	0.168	0.176	0.185	0.205	0.201	0.200
TORONTO-OTTAWA-HULL	0.110	0.119	0.129	0.121	0.136	0.139	0.142	0.147	0.159	0.153	0.151

TABLE A4-38 INTERACTANCE SENSITIVITY, MONTREAL, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-MONTREAL	0.709	0.709	0.699	0.677	0.656	0.670	0.645	0.643	0.637	0.640	0.634
QUEBEC-MONTREAL	0.159	0.155	0.128	0.117	0.089	0.081	0.084	0.087	0.082	0.071	0.073
MONTREAL-QUEBEC	0.114	0.114	0.092	0.081	0.060	0.049	0.052	0.053	0.046	0.038	0.030

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TABLE A4-39 INTERACTANCE SENSITIVITY, OTTAWA-HULL, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-OTTAWA-HULL	0.589	0.584	0.584	0.565	0.557	0.572	0.550	0.546	0.539	0.526	0.513
TORONTO-MONTREAL	0.169	0.166	0.158	0.155	0.169	0.147	0.149	0.152	0.148	0.150	0.152
MONTREAL-OTTAWA-HULL	0.130	0.113	0.106	0.098	0.101	0.096	0.104	0.108	0.107	0.109	0.112

TABLE A4-40 INTERACTANCE SENSITIVITY, QUEBEC, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
MONTREAL-QUEBEC	0.761	0.781	0.743	0.696	0.636	0.579	0.586	0.577	0.516	0.485	0.463
TORONTO-MONTREAL	0.504	0.521	0.500	0.465	0.437	0.408	0.397	0.390	0.357	0.345	0.330
TORONTO-QUEBEC	0.154	0.182	0.198	0.200	0.202	0.239	0.230	0.233	0.245	0.260	0.267
QUEBEC-MONTREAL	0.144	0.114	0.092	0.081	0.062	0.049	0.051	0.052	0.046	0.038	0.038
TORONTO-OTTAWA-HULL	0.089	0.090	0.085	0.097	0.105	0.113	0.115	0.124	0.136	0.132	0.130
OTTAWA-QUEBEC	0.085	0.089	0.084	0.101	0.115	0.132	0.135	0.152	0.173	0.174	0.171

TABLE A4-41 INTERACTANCE SENSITIVITY, REGINA, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
CALGARY-REGINA	0.260	0.261	0.262	0.240	0.227	0.229	0.219	0.224	0.210	0.200	0.204
WINNIPEG-REGINA	0.239	0.256	0.253	0.257	0.248	0.249	0.239	0.244	0.223	0.206	0.202
TORONTO-MONTREAL	0.184	0.192	0.186	0.188	0.240	0.176	0.177	0.172	0.162	0.161	0.162
TORONTO-REGINA	0.182	0.179	0.179	0.180	0.150	0.179	0.175	0.170	0.180	0.196	0.189
EDMONTON-CALGARY	0.169	0.193	0.217	0.208	0.216	0.216	0.222	0.219	0.221	0.201	0.198
CALGARY-EDMONTON	0.144	0.165	0.192	0.192	0.193	0.202	0.211	0.210	0.214	0.193	0.190
TORONTO-WINNIPEG	0.140	0.148	0.135	0.135	0.124	0.136	0.134	0.137	0.124	0.116	0.109
VANCOUVER-REGINA	0.128	0.125	0.123	0.121	0.119	0.137	0.151	0.155	0.178	0.178	0.198
VANCOUVER-CALGARY	0.126	0.133	0.132	0.128	0.127	0.138	0.148	0.157	0.157	0.148	0.168
EDMONTON-REGINA	0.123	0.121	0.121	0.121	0.120	0.132	0.126	0.133	0.126	0.131	0.132

TABLE A4-42 INTERACTANCE SENSITIVITY, SAINT JOHN, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
HALIFAX-SAINT JOHN	0.439	0.377	0.378	0.345	0.345	0.362	0.330	0.325	0.295	0.309	0.294
TORONTO-MONTREAL	0.336	0.369	0.360	0.332	0.360	0.320	0.307	0.302	0.282	0.274	0.263
MONTREAL-SAINT JOHN	0.275	0.342	0.333	0.316	0.362	0.302	0.300	0.300	0.288	0.270	0.258
TORONTO-SAINT JOHN	0.226	0.226	0.222	0.242	0.215	0.240	0.252	0.241	0.273	0.269	0.277
TORONTO-HALIFAX	0.145	0.132	0.134	0.122	0.120	0.131	0.127	0.127	0.117	0.121	0.119
MONTREAL-HALIFAX	0.135	0.130	0.133	0.121	0.140	0.126	0.121	0.114	0.097	0.094	0.086

TABLE A4-43 INTERACTANCE SENSITIVITY, ST. JOHN'S, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
HALIFAX-ST. JOHN'S	0.321	0.306	0.298	0.300	0.282	0.277	0.289	0.295	0.290	0.305	0.290
TORONTO-ST. JOHN'S	0.312	0.320	0.335	0.332	0.290	0.313	0.331	0.336	0.348	0.356	0.356
TORONTO-MONTREAL	0.307	0.308	0.291	0.291	0.310	0.295	0.269	0.253	0.231	0.230	0.222
MONTREAL-ST. JOHN'S	0.274	0.265	0.251	0.252	0.307	0.209	0.258	0.234	0.208	0.208	0.203
TORONTO-HALIFAX	0.113	0.111	0.110	0.107	0.102	0.106	0.112	0.115	0.111	0.116	0.114

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TABLE A4-44 INTERACTANCE SENSITIVITY, SASKATOON, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
WINNIPEG-SASKATOON	0.248	0.261	0.237	0.273	0.249	0.266	0.258	0.261	0.245	0.233	0.220
TORONTO-MONTREAL	0.180	0.193	0.184	0.185	0.239	0.176	0.180	0.171	0.161	0.163	0.161
TORONTO-SASKATOON	0.184	0.172	0.187	0.169	0.162	0.173	0.159	0.162	0.184	0.174	0.197
CALGARY-SASKATOON	0.158	0.178	0.162	0.145	0.164	0.177	0.187	0.187	0.166	0.170	0.167
EDMONTON-CALGARY	0.143	0.174	0.189	0.185	0.195	0.201	0.216	0.211	0.207	0.198	0.186
TORONTO-WINNIPEG	0.143	0.152	0.133	0.143	0.125	0.142	0.141	0.143	0.129	0.125	0.113
CALGARY-EDMONTON	0.136	0.158	0.184	0.185	0.187	0.196	0.210	0.207	0.210	0.194	0.101
VANCOUVER-SASKATOON	0.125	0.115	0.120	0.127	0.117	0.132	0.144	0.168	0.178	0.210	0.211
EDMONTON-SASKATOON	0.118	0.118	0.125	0.116	0.119	0.126	0.122	0.125	0.125	0.127	0.115
VANCOUVER-CALGARY	0.107	0.120	0.115	0.114	0.117	0.130	0.144	0.151	0.148	0.146	0.139
TORONTO-VANCOUVER	0.104	0.105	0.108	0.109	0.097	0.114	0.122	0.127	0.132	0.138	0.138

TABLE A4-45 INTERACTANCE SENSITIVITY, SUDBURY, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-SUDBURY	0.790	0.756	0.795	0.749	0.777	0.759	0.767	0.756	0.723	0.711	0.718
TORONTO-MONTREAL	0.079	0.106	0.083	0.097	0.093	0.085	0.080	0.085	0.093	0.089	0.083
MONTREAL-SUDBURY	0.071	0.109	0.080	0.094	0.094	0.078	0.075	0.086	0.092	0.080	0.068

TABLE A4-46 INTERACTANCE SENSITIVITY, THUNDER BAY, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-THUNDER BAY	0.565	-0.574	0.569	0.563	0.547	0.578	0.580	0.587	0.595	0.575	0.576
WINNIPEG-THUNDER BAY	0.229	0.221	0.222	0.230	0.219	0.223	0.203	0.195	0.184	0.188	0.185
TORONTO-MONTREAL	0.127	0.133	0.137	0.126	0.153	0.112	0.109	0.105	0.097	0.099	0.097
TORONTO-WINNIPEG	0.096	0.095	0.098	0.091	0.082	0.089	0.084	0.081	0.076	0.078	0.074

TABLE A4-47 INTERACTANCE SENSITIVITY, VANCOUVER, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-VANCOUVER	0.294	0.301	0.304	0.296	0.267	0.296	0.298	0.259	0.316	0.316	0.317
CALGARY-VANCOUVER	0.233	0.256	0.251	0.249	0.249	0.268	0.289	0.294	0.300	0.300	0.283
EDMONTON-VANCOUVER	0.232	0.237	0.232	0.234	0.234	0.249	0.251	0.255	0.250	0.255	0.251
VICTORIA-VANCOUVER	0.201	0.218	0.193	0.159	0.180	0.184	0.116	0.118	0.098	0.081	0.079
TORONTO-MONTREAL	0.200	0.204	0.202	0.204	0.253	0.194	0.194	0.187	0.175	0.173	0.177
WINNIPEG-VANCOUVER	0.169	0.149	0.149	0.155	0.145	0.153	0.166	0.161	0.143	0.143	0.138
CALGARY-EDMONTON	0.149	0.178	0.193	0.199	0.202	0.209	0.221	0.218	0.226	0.207	0.197
EDMONTON-CALGARY	0.147	0.177	0.193	0.192	0.197	0.202	0.215	0.255	0.220	0.206	0.194
VANCOUVER-VICTORIA	0.131	0.148	0.123	0.096	0.113	0.113	0.064	0.067	0.052	0.042	0.039
MONTREAL-VANCOUVER	0.129	0.130	0.137	0.143	0.194	0.137	0.148	0.141	0.130	0.124	0.137
VANCOUVER-CALGARY	0.109	0.121	0.117	0.117	0.119	0.130	0.144	0.151	0.157	0.152	0.146
TORONTO-WINNIPEG	0.108	0.105	0.097	0.098	0.090	0.099	0.102	0.104	0.094	0.092	0.085
VANCOUVER-EDMONTON	0.105	0.108	0.109	0.111	0.109	0.123	0.124	0.125	0.123	0.127	0.128

TABLE A4-48 INTERACTANCE SENSITIVITY, VICTORIA, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
VANCOUVER-VICTORIA	0.628	0.680	0.610	0.543	0.580	0.569	0.437	0.442	0.370	0.321	0.298
TORONTO-MONTREAL	0.196	0.203	0.199	0.202	0.244	0.189	0.186	0.182	0.169	0.170	0.168
TORONTO-VANCOUVER	0.189	0.295	0.194	0.178	0.167	0.183	0.165	0.168	0.171	0.164	0.163
CALGARY-VANCOUVER	0.150	0.174	0.160	0.150	0.155	0.165	0.157	0.166	0.162	0.156	0.146
EDMONTON-VANCOUVER	0.149	0.161	[illegible]	0.141	0.146	0.154	0.138	0.144	0.135	0.132	0.129
EDMONTON-CALGARY	0.141	0.174	[illegible]	0.180	0.186	0.190	0.199	0.200	0.230	0.220	0.217
TORONTO-VICTORIA	0.141	0.120	0.139	0.151	0.138	0.165	0.181	0.167	0.157	0.150	0.148
CALGARY-EDMONTON	0.139	0.178	0.186	0.184	0.190	0.193	0.201	0.202	0.228	0.214	0.218
VICTORIA-VANCOUVER	0.129	0.148	0.123	0.095	0.112	0.113	0.064	0.066	0.053	0.042	0.040
VANCOUVER-CALGARY	0.105	0.119	0.113	0.111	0.112	0.122	0.133	0.143	0.164	0.162	0.162
VANCOUVER-EDMONTON	0.098	0.108	0.103	0.103	0.103	0.114	0.113	0.117	0.124	0.132	0.138
EDMONTON-VICTORIA	0.078	0.103	0.084	0.084	0.089	0.088	0.098	0.101	0.176	0.142	0.129
CALGARY-VICTORIA	0.097	0.096	0.105	0.105	0.097	0.109	0.126	0.135	0.191	0.204	0.214

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TABLE A4-49 INTERACTANCE SENSITIVITY, WINDSOR, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-WINDSOR	0.646	0.649	0.615	0.574	0.492	0.553	0.530	0.536	0.505	0.496	0.473
MONTREAL-WINDSOR	0.163	0.153	0.166	0.184	0.295	0.193	0.194	0.192	0.208	0.201	0.230
TORONTO-MONTREAL	0.156	0.151	0.161	0.174	0.242	0.177	0.177	0.172	0.182	0.181	0.196

TABLE A4-50 INTERACTANCE SENSITIVITY, WINNIPEG, NATIONAL SYSTEM, 1963-1973

ARC	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
TORONTO-WINNIPEG	0.366	0.380	0.349	0.343	0.316	0.346	0.347	0.343	0.338	0.336	0.321
TORONTO-MONTREAL	0.192	0.193	0.192	0.190	0.237	0.183	0.180	0.173	0.165	0.166	0.165
VANCOUVER-WINNIPEG	0.166	0.159	0.160	0.165	0.154	0.169	0.192	0.180	0.174	0.174	0.182
MONTREAL-WINNIPEG	0.138	0.141	0.134	0.134	0.186	0.135	0.132	0.121	0.115	0.116	0.108
EDMONTON-CALGARY	0.100	0.111	0.134	0.132	0.136	0.142	0.152	0.151	0.155	0.147	0.144
CALGARY-EDMONTON	0.099	0.090	0.133	0.136	0.137	0.138	0.148	0.151	0.157	0.144	0.143
CALGARY-WINNIPEG	0.095	0.105	0.101	0.094	0.095	0.115	0.117	0.121	0.117	0.120	0.118
REGINA-WINNIPEG	0.090	0.108	0.106	0.102	0.095	0.094	0.088	0.087	0.086	0.079	0.080

TABLE A4-51 INTERACTANCE SENSIVITY, CALGARY, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
EDMONTON-CALGARY	0.610	0.749	0.723	0.702	0.655
VANCOUVER-CALGARY	0.455	0.460	0.484	0.501	0.492
CALGARY-EDMONTON	0.336	0.457	0.439	0.432	0.378
VANCOUVER-EDMONTON	0.237	0.255	0.247	0.236	0.245
REGINA-CALGARY	0.192	0.169	0.124	0.111	0.111
WINNIPEG-CALGARY	0.176	0.170	0.172	0.158	0.153
VANCOUVER-WINNIPEG	0.139	0.132	0.129	0.110	0.106
CALGARY-REGINA	0.106	0.095	0.066	0.057	0.054

TABLE A4-52 INTERACTANCE SENSITIVITY, EDMONTON, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
CALGARY-EDMONTON	0.645	0.807	0.803	0.802	0.718
VANCOUVER-EDMONTON	0.456	0.451	0.453	0.438	0.465
EDMONTON-CALGARY	0.336	0.452	0.438	0.429	0.377
VANCOUVER-CALGARY	0.250	0.278	0.293	0.306	0.293
WINNIPEG-EDMONTON	0.184	0.153	0.122	0.134	0.128
VANCOUVER-WINNIPEG	0.137	0.127	0.121	0.108	0.102
REGINA-CALGARY	0.105	0.102	0.075	0.068	0.063

TABLE A4-53 INTERACTANCE SENSITIVITY, REGINA, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
CALGARY-REGINA	0.406	0.382	0.327	0.311	0.308
WINNIPEG-REGINA	0.374	0.410	0.356	0.332	0.305
EDMONTON-CALGARY	0.345	0.428	0.391	0.368	0.329
CALGARY-EDMONTON	0.302	0.395	0.365	0.356	0.311
VANCOUVER-CALGARY	0.257	0.263	0.262	0.262	0.247
VANCOUVER-EDMONTON	0.213	0.221	0.206	0.194	0.202
VANCOUVER-WINNIPEG	0.206	0.208	0.201	0.168	0.155
VANCOUVER-REGINA	0.201	0.193	0.225	0.265	0.299
EDMONTON-REGINA	0.193	0.193	0.187	0.186	0.199

TABLE A4-54 INTERACTANCE SENSITIVITY, SASKATOON, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
WINNIPEG-SASKATOON	0.389	0.415	0.375	0.365	0.333
EDMONTON-CALGARY	0.312	0.387	0.381	0.349	0.305
CALGARY-EDMONTON	0.296	0.382	0.362	0.351	0.293
CALGARY-SASKATOON	0.248	0.221	0.272	0.246	0.252
VANCOUVER-CALGARY	0.232	0.238	0.255	0.249	0.229
VANCOUVER-WINNIPEG	0.217	0.219	0.211	0.178	0.162
VANCOUVER-EDMONTON	0.209	0.213	0.204	0.192	0.190
VANCOUVER-SASKATOON	0.197	0.193	0.209	0.265	0.318
EDMONTON-SASKATOON	0.186	0.176	0.177	0.186	0.173
REGINA-SASKATOON	0.127	0.165	0.121	0.061	0.0

TABLE A4-55 INTERACTANCE SENSITIVITY, VICTORIA, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-VICTORIA	0.729	0.692	0.599	0.487	0.395
EDMONTON-CALGARY	0.125	0.180	0.222	0.284	0.300
CALGARY-EDMONTON	0.120	0.178	0.219	0.279	0.291
CALGARY-VICTORIA	0.113	0.134	0.173	0.251	0.285
VANCOUVER-CALGARY	0.093	0.110	0.148	0.203	0.225
EDMONTON-VICTORIA	0.090	0.107	0.134	0.166	0.209
VANCOUVER-EDMONTON	0.085	0.099	0.123	0.152	0.188

TABLE A4-56 INTERACTANCE SENSITIVITY, WINNIPEG, WESTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
VANCOUVER-WINNIPEG	0.431	0.421	0.444	0.400	0.402
EDMONTON-CALGARY	0.271	0.344	0.326	0.328	0.295
CALGARY-EDMONTON	0.266	0.345	0.309	0.329	0.288
CALGARY-WINNIPEG	0.247	0.240	0.271	0.269	0.260
EDMONTON-WINNIPEG	0.237	0.228	0.173	0.205	0.202
REGINA-WINNIPEG	0.233	0.260	0.203	0.199	0.177
VANCOUVER-CALGARY	0.202	0.211	0.218	0.234	0.221
VANCOUVER-EDMONTON	0.188	0.193	0.174	0.180	0.186
SASKATOON-WINNIPEG	0.161	0.188	0.173	0.180	0.170
CALGARY-REGINA	0.121	0.121	0.085	0.078	0.070
WINNIPEG-REGINA	0.111	0.130	0.092	0.083	0.069

TABLE A4-57 INTERACTANCE SENSITIVITY, HALIFAX, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-HALIFAX	0.360	0.358	0.384	0.390	0.408
TORONTO-MONTREAL	0.360	0.359	0.347	0.310	0.290
MONTREAL-HALIFAX	0.335	0.356	0.366	0.323	0.294
SAINT JOHN-HALIFAX	0.181	0.141	0.110	0.093	0.091
ST. JOHN'S-HALIFAX	0.140	0.142	0.125	0.130	0.120
TORONTO-OTTAWA-HULL	0.104	0.115	0.116	0.140	0.143
OTTAWA-HALIFAX	0.093	0.108	0.110	0.141	0.155

TABLE A4-58 INTERACTANCE SENSITIVITY, LONDON, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-LONDON	0.374	0.364	0.293	0.259	0.278
MONTREAL-LONDON	0.355	0.331	0.349	0.335	0.316
TORONTO-MONTREAL	0.346	0.322	0.338	0.326	0.311
OTTAWA-HULL-LONDON	0.155	0.187	0.237	0.266	0.267
TORONTO-OTTAWA-HULL	0.137	0.159	0.196	0.218	0.212

TABLE A4-59 INTERACTANCE SENSITIVITY, MONTREAL, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-MONTREAL	0.785	0.778	0.768	0.750	0.752
QUEBEC-MONTREAL	0.179	0.135	0.100	0.057	0.086
MONTREAL-QUEBEC	0.131	0.095	0.063	0.056	0.047
HALIFAX-MONTREAL	0.072	0.088	0.104	0.093	0.086

TABLE A4-60 INTERACTANCE SENSITIVITY,OTTAWA-HULL,EASTERN SUB-SYSTEM,1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-OTTAWA-HULL	0.686	0.690	0.686	0.671	0.651
TORONTO-MONTREAL	0.186	0.168	0.169	0.171	0.177
MONTREAL-OTTAWA-HULL	0.151	0.120	0.128	0.133	0.143

TABLE A4-61 INTERACTANCE SENSITIVITY, QUEBEC, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
MONTREAL-QUEBEC	0.794	0.731	0.621	0.556	0.510
TORONTO-MONTREAL	0.575	0.546	0.485	0.436	0.410
TORONTO-QUEBEC	0.203	0.210	0.244	0.264	0.295
QUEBEC-MONTREAL	0.131	0.095	0.063	0.056	0.047
TORONTO-OTTAWA-HULL	0.101	0.114	0.140	0.169	0.165
OTTAWA-HULL-QUEBEC	0.089	0.106	0.143	0.186	0.189

TABLE A4-62 INTERACTANCE SENSIT.VITY, SAINT JOHN, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
HALIFAX-SAINT JOHN	0.459	0.363	0.349	0.314	0.315
TORONTO-MONTREAL	0.389	0.392	0.377	0.343	0.322
MONTREAL-SAINT JOHN	0.288	0.332	0.319	0.307	0.278
TORONTO-SAINT JOHN	0.236	0.254	0.267	0.291	0.298
TORONTO-HALIFAX	0.165	0.140	0.148	0.138	0.145
MONTREAL-HALIFAX	0.154	0.139	0.142	0.115	0.104
TORONTO-OTTAWA-HULL	0.095	0.104	0.104	0.122	0.128

TABLE A4-63 INTERACTANCE SENSITIVITY, ST. JOHN'S, EASTERN SUB-SYSTEM,1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-MONTREAL	0.355	0.330	0.329	0.281	0.271
HALIFAX-ST. JOHN'S	0.335	0.315	0.307	0.311	0.312
TORONTTO-ST. JOHN'S	0.326	0.349	0.352	0.373	0.382
MONTREAL-ST. JOHN'S	0.287	0.265	0.274	0.223	0.218
TORONTO-HALIFAX	0.129	0.123	0.131	0.132	0.138
MONTREAL-HALIFAX	0.120	0.122	0.125	0.110	0.100
TORONTO-OTTAWA-HULL	0.093	0.106	0.104	0.127	0.123

TABLE A4-64 INTERACTANCE SENSITIVITY, SUDBURY, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-SUDBURY	0.835	0.777	0.805	0.757	0.765
TORONTO-MONTREAL	0.082	0.108	0.089	0.106	0.093
MONTREAL-SUDBURY	0.075	0.098	0.079	0.097	0.073

TABLE A4-65 INTERACTANCE SENSITIVITY, THUNDER BAY, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-THUNDER BAY	0.789	0.789	0.786	0.788	0.786
TORONTO-MONTREAL	0.112	0.112	0.097	0.087	0.083
MONTREAL-THUNDER BAY	0.108	0.113	0.088	0.075	0.067

TABLE A4-66 INTERACTANCE SENSITIVITY, WINDSOR, EASTERN SUB-SYSTEM, 1963-1973

ARC	1963	1966	1969	1971	1973
TORONTO-WINDSOR	0.721	0.669	0.625	0.589	0.564
MONTREAL-WINDSOR	0.182	0.214	0.228	0.242	0.275
TORONTO-MONTREAL	0.170	0.195	0.209	0.216	0.238
OTTAWA-WINDSOR	0.056	0.076	0.091	0.105	0.108

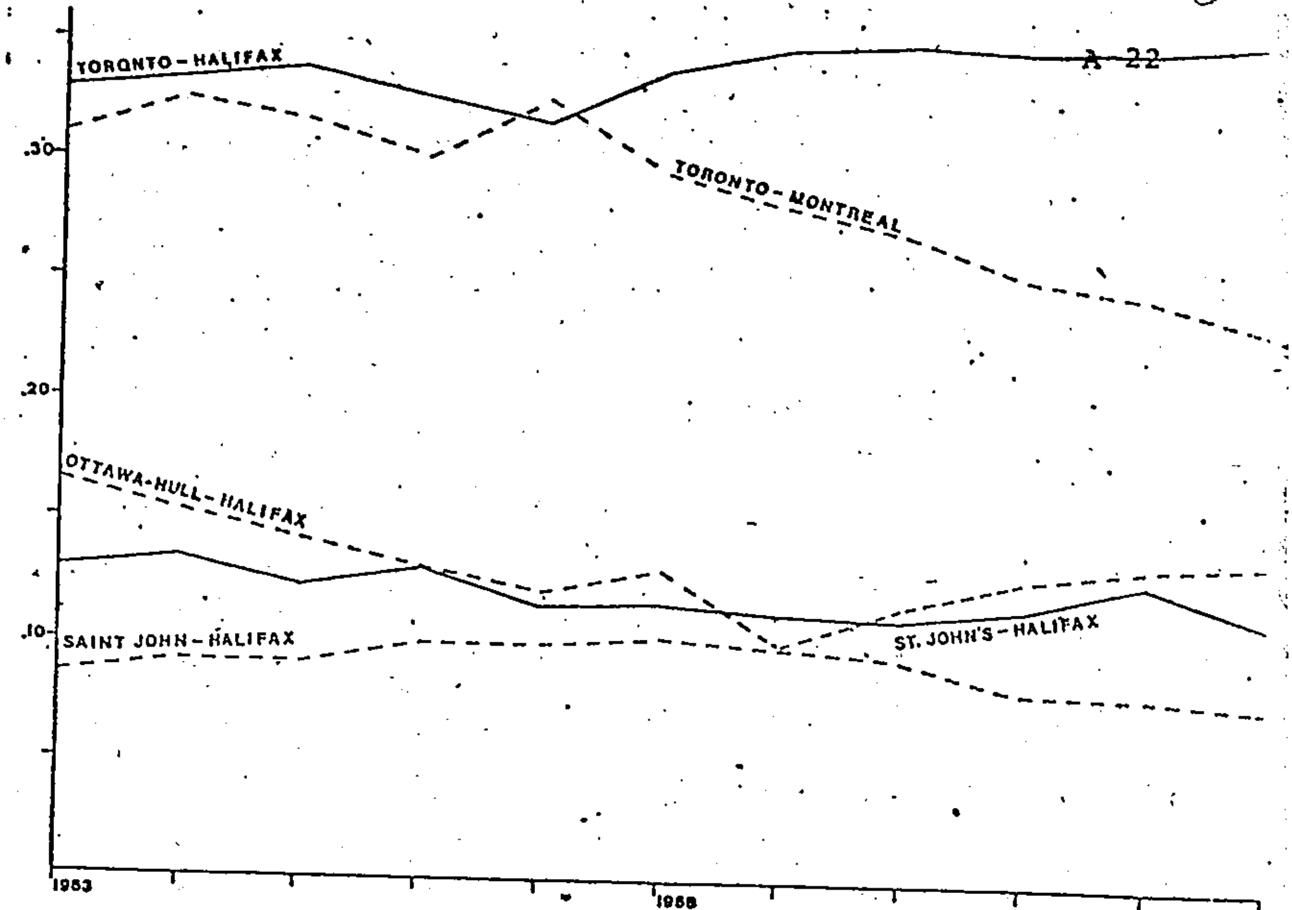

FIGURE A4-1 INTERACTANCE SENSITIVITY, HALIFAX, NATIONAL SYSTEM

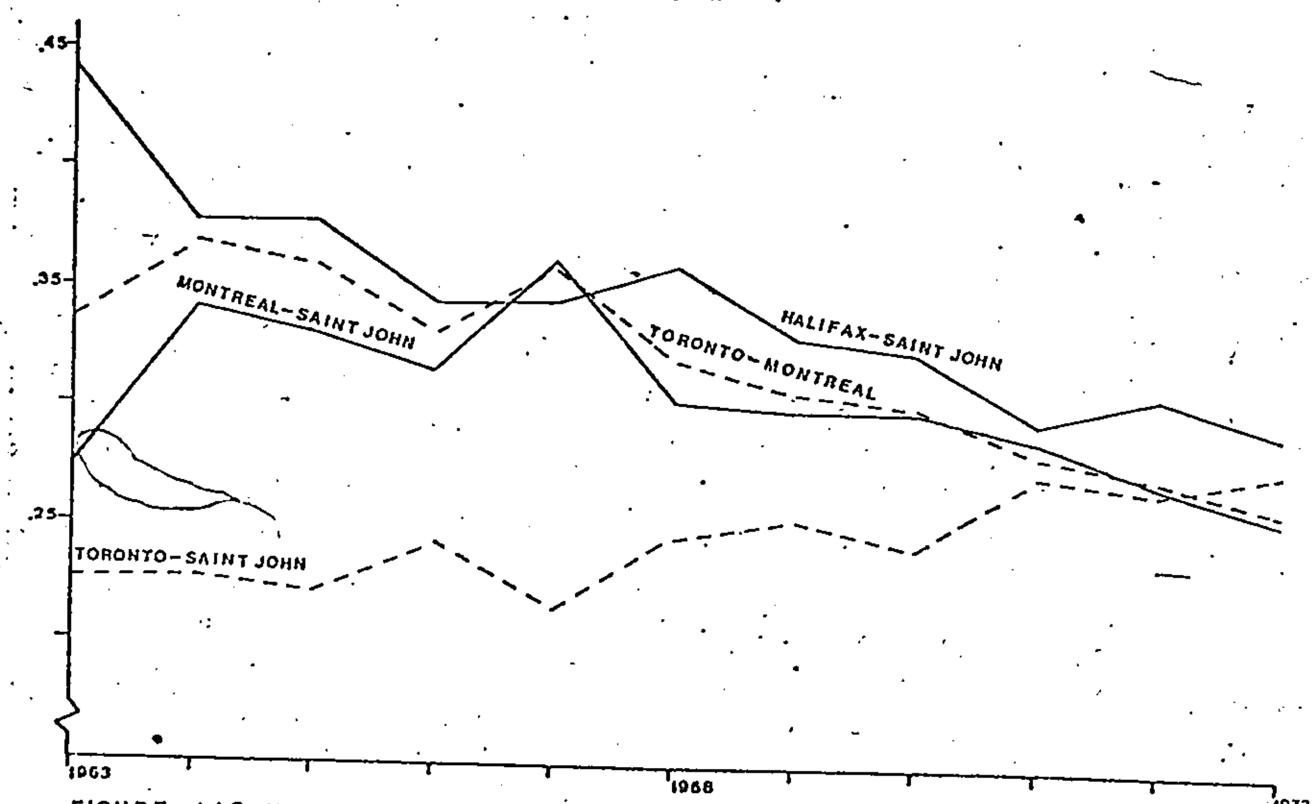

FIGURE A4-2 INTERACTANCE SENSITIVITY, SAINT JOHN, NATIONAL SYSTEM

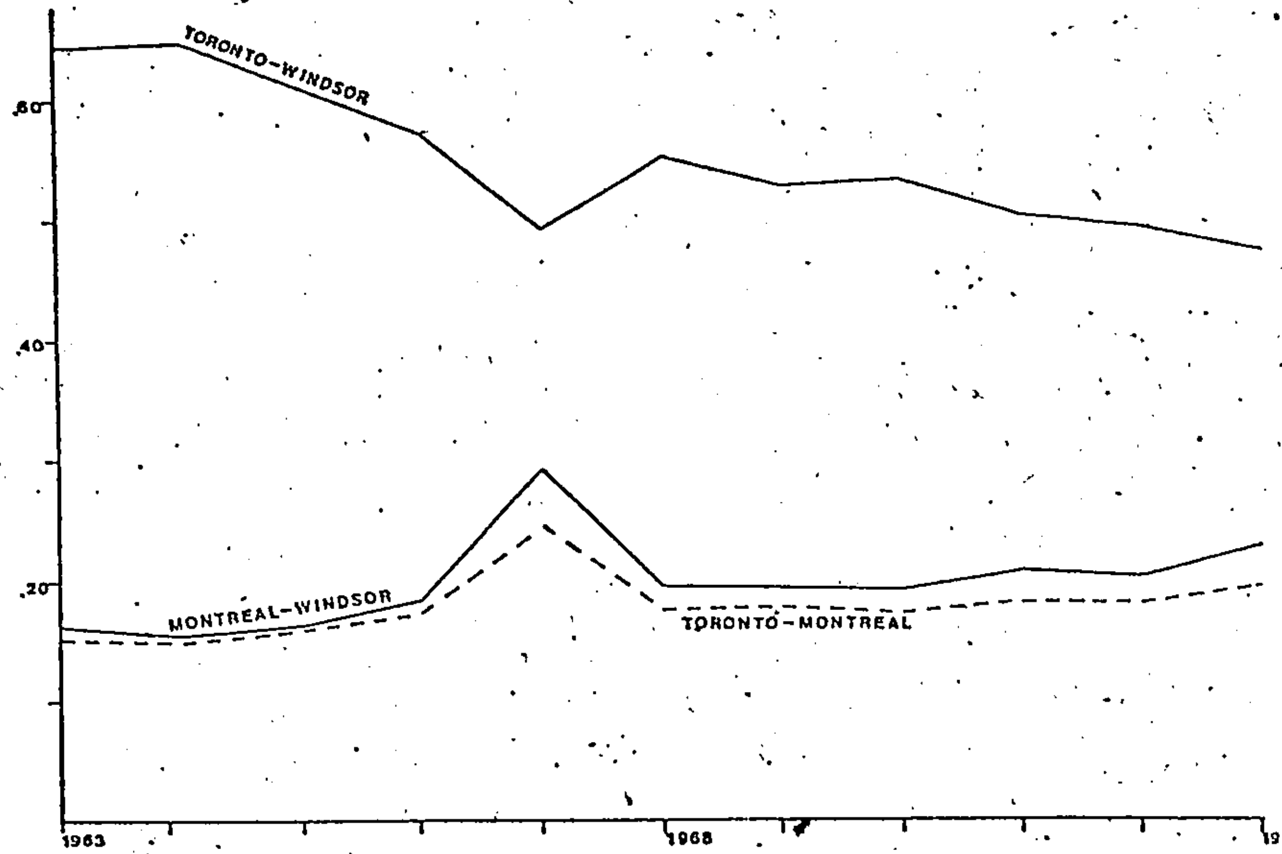

FIGURE A4-3 INTERACTANCE SENSITIVITY, WINDSOR, NATIONAL SYSTEM

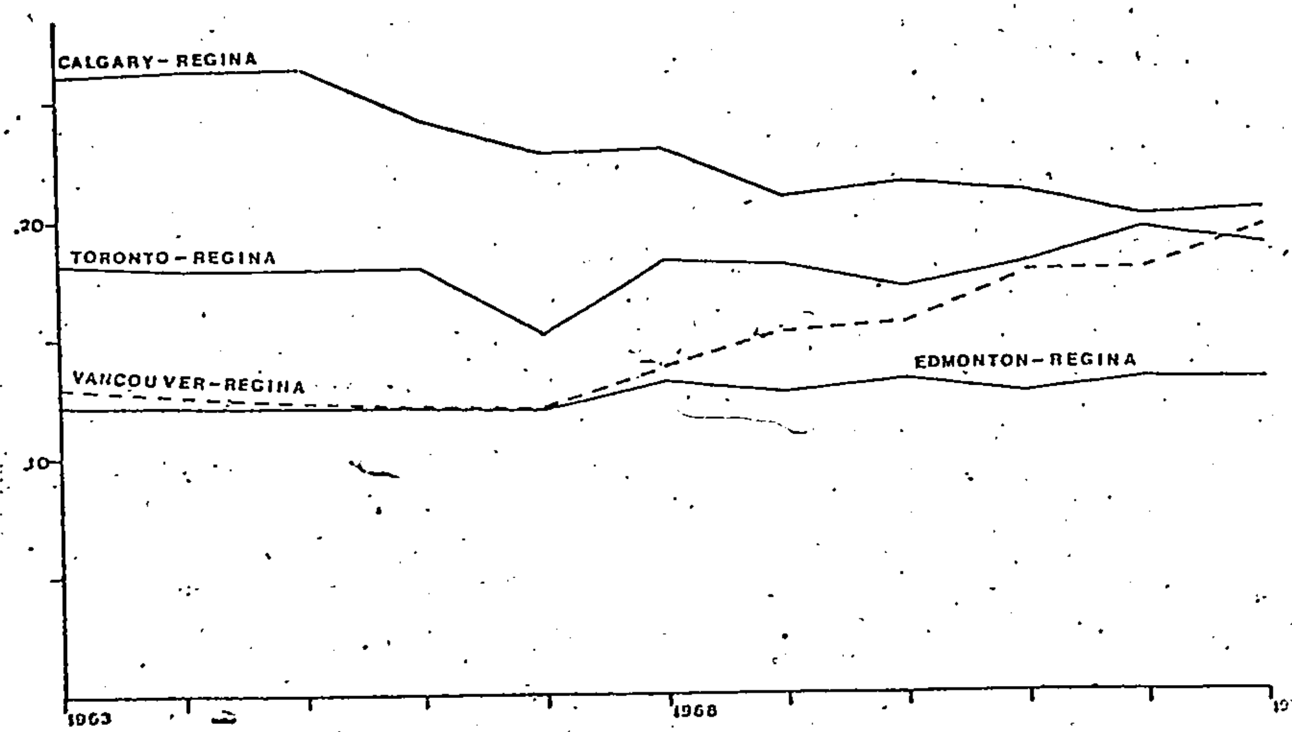

FIGURE A4-4 INTERACTANCE SENSITIVITY, REGINA, NATIONAL SYSTEM

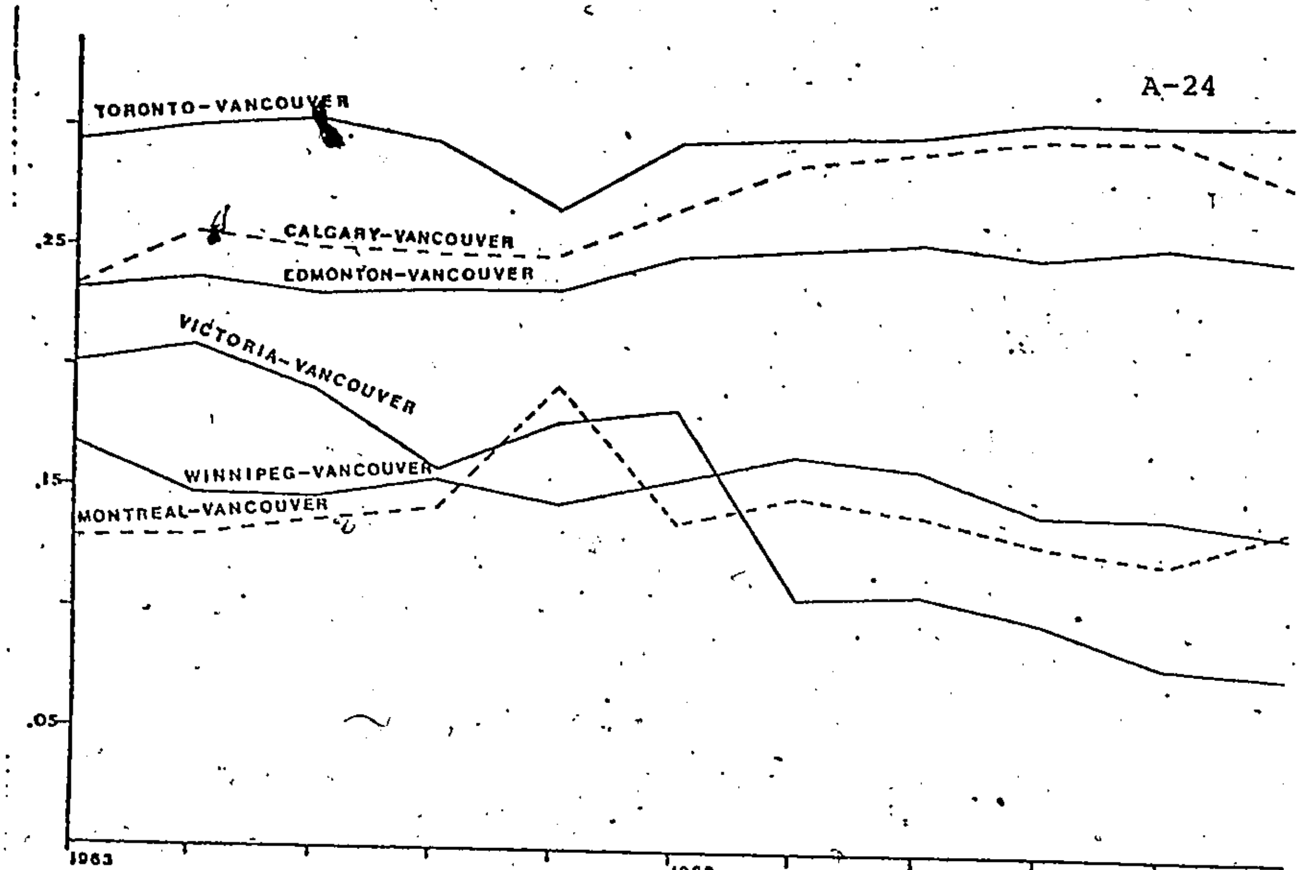

FIGURE A4-5 INTERACTANCE SENSITIVITY, VANCOUVER, NATIONAL SYSTEM

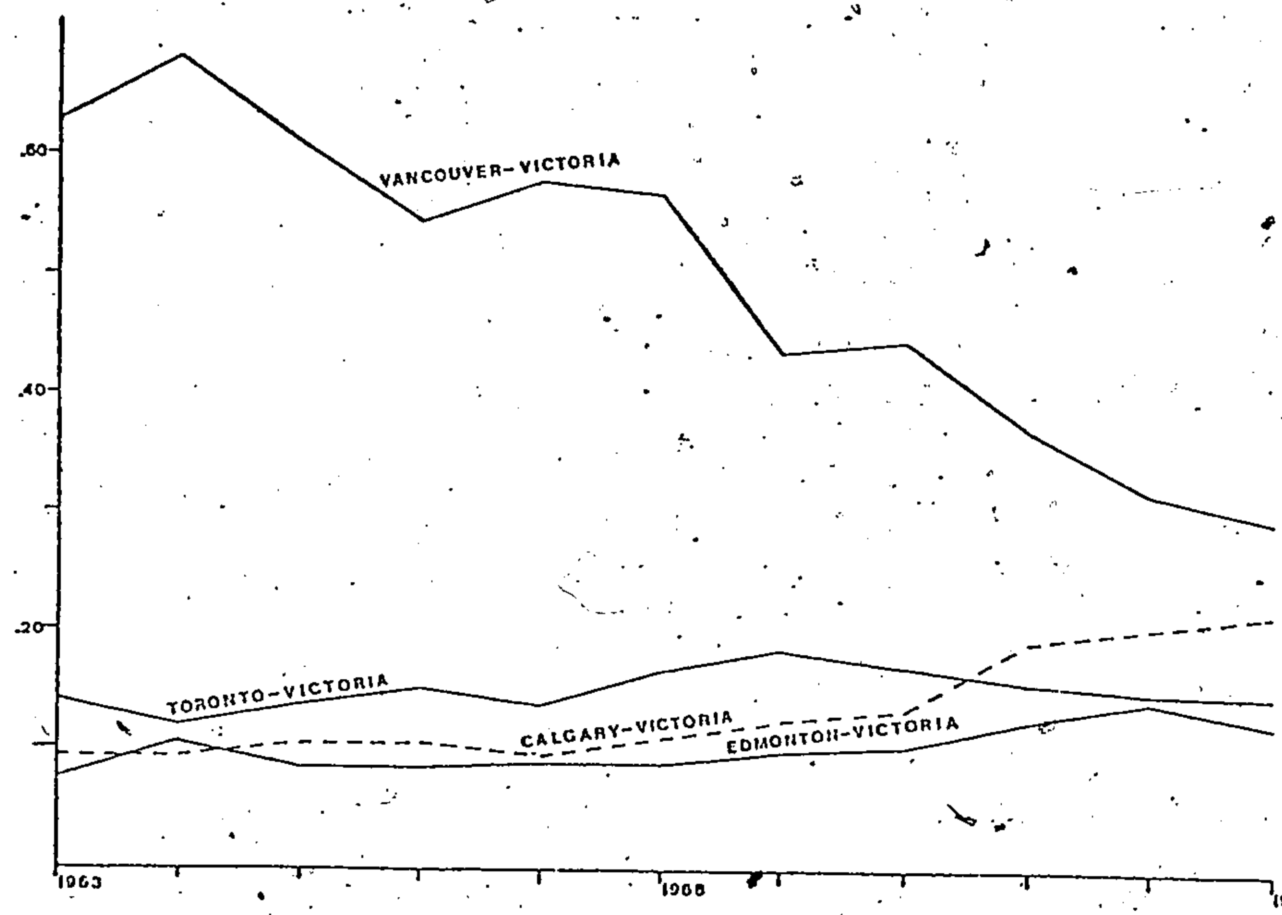

FIGURE A4-6 INTERACTANCE SENSITIVITY, VICTORIA, NATIONAL SYSTEM

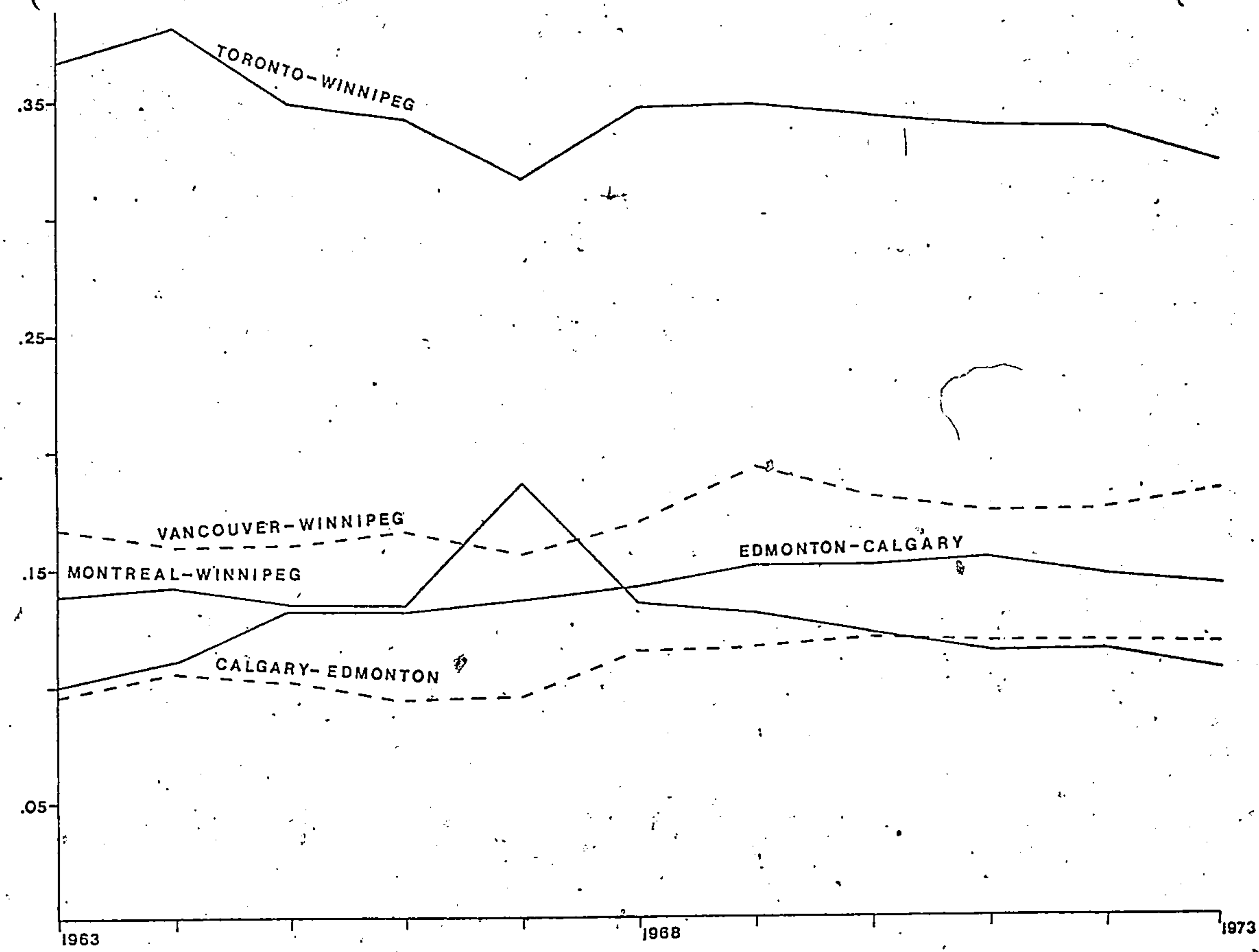

FIGURE A4-7 INTERACTANCE SENSITIVITY, WINNIPEG, NATIONAL SYSTEM

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INTERMETROPOLITAN AIR TRAFFIC MATRICES 1963-1973: TABLES A4-67 to A4-77

TABLE A4-67 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1963

	TO	MT	OT	LO	OB	WO	SO	TB	HF	SN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	193195	65925	7915	11165	35578	7300	15035	18580	8135	5760	34248	6000	4260	13250	12175	3475	37028
MT	193355	0	14100	7425	44105	9065	945	2015	17845	4960	5440	13720	1410	1525	6000	4440	1665	12200
OT	68410	14670	0	3450	6000	2740	170	840	5110	900	1220	5670	1345	1130	2455	2545	1340	4145
LO	7405	7495	2780	0	930	270	210	340	1130	250	210	1600	480	400	810	1135	300	1370
OB	11615	43175	5105	470	0	430	60	170	1125	630	210	485	80	120	310	355	70	445
WO	36365	9230	2630	310	810	0	170	360	940	120	210	2140	570	300	890	875	200	1655
SO	8015	720	190	170	30	270	0	30	150	30	0	105	50	0	40	70	50	140
TB	16040	2210	420	360	140	340	20	0	220	60	20	6530	250	200	460	460	60	440
HF	19115	17020	4480	1260	1065	1010	130	185	0	9050	7465	1140	255	200	665	650	1060	1135
SN	8755	6600	1040	240	550	170	80	20	10200	0	610	240	50	50	80	190	30	190
SS	7410	6515	1180	200	700	320	10	10	7640	600	0	245	60	30	60	170	70	390
WN	36415	13760	5670	2085	505	2645	130	5695	1210	255	190	0	6965	6210	9400	9120	1615	16560
RG	7135	1830	1515	500	120	700	30	280	240	70	70	9380	0	2140	10175	4850	660	5045
SK	5005	1540	1225	430	120	340	0	130	220	50	10	4725	2200	0	4290	3210	310	3410
CG	14210	4615	2515	770	245	1020	30	400	865	100	70	9055	9475	4030	0	31420	3210	23440
ED	13940	4805	3055	1140	370	1515	40	330	765	100	170	9310	4420	3350	32655	0	2740	23070
VC	4555	1500	1390	350	70	210	30	110	660	40	40	1570	680	510	3155	2515	0	20260
VN	29320	12950	4750	1275	515	1195	130	655	995	220	210	16965	8205	3645	23365	23300	20225	0

SOURCE AIR TRANSPORT BOARD, JUNE 1965. "ORIGIN AND DESTINATION STATISTICS, MAINLINE REVENUE PASSENGERS, DOMESTIC SURVEY, 1963-1964." PREPARED BY THE STATISTICS SECTION, DEPARTMENT OF TRANSPORT ON BEHALF OF THE AIR TRANSPORT BOARD, OTTAWA, CANADA.

TABLE A4-68 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1964

	TO	MT	OT	LO	OB	WO	SO	TB	HF	SN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	198315	65800	7330	10270	34805	6665	15915	20095	8135	7130	33600	6665	4135	15435	11035	3745	28710
MT	198500	0	12520	7120	43520	8195	675	2405	20150	7455	6655	12250	1705	1350	8205	4665	1490	12770
OT	66755	12905	0	3760	4870	3620	260	810	8630	1070	1770	3515	1345	1225	2665	3210	1130	6650
LO	7115	6640	3500	0	310	230	260	330	1110	250	270	1690	370	300	555	890	385	1295
OB	10425	44540	5140	270	0	840	40	170	965	360	275	455	60	50	190	260	70	365
WO	35405	8630	3380	310	410	0	250	410	670	130	150	2360	620	260	750	700	210	1115
SO	4695	995	310	210	50	140	0	60	70	10	20	100	20	10	60	30	10	110
TB	14430	2530	610	360	230	430	70	0	180	30	50	6335	170	220	330	240	60	735
HF	20585	20280	5725	1180	680	640	50	110	0	9410	6175	1065	250	140	610	640	240	1105
SN	8355	8110	1030	180	305	140	20	60	6930	0	660	230	40	30	90	70	70	180
SS	6725	7220	1025	340	315	240	30	30	8365	690	0	310	90	20	90	150	40	240
WN	35150	13260	5915	1730	490	2780	100	6335	1190	230	200	0	10170	6580	9650	1730	1345	14875
RG	7265	2215	1420	500	110	710	20	180	250	60	110	10605	0	1980	10395	4925	630	5625
SK	4485	1475	1155	300	75	270	10	190	210	30	0	6785	2305	0	6640	3080	190	3000
CG	14000	6720	2485	1035	180	1060	50	280	780	160	90	9435	10005	4585	0	39120	3370	26875
ED	12950	5075	3350	820	280	930	60	230	690	90	80	8450	4775	2880	39790	0	3140	24365
VC	3405	1500	1185	280	90	220	30	60	430	40	40	1455	928	220	3145	2155	0	22105
VN	30885	13380	6120	1620	390	1280	140	700	1065	160	215	15330	4945	3230	26275	24335	22385	0

SOURCE AIR TRANSPORT BOARD, JUNE 1965. "ORIGIN AND DESTINATION STATISTICS, MAINLINE REVENUE PASSENGERS, DOMESTIC SURVEY, 1963-1964." PREPARED BY THE STATISTICS SECTION, DEPARTMENT OF TRANSPORT ON BEHALF OF THE AIR TRANSPORT BOARD, OTTAWA, CANADA.

TABLE A4-69 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1965

	TO	MT	OT	LO	OB	WO	SO	TB	HF	SN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	223240	74430	6875	11265	36615	8090	19778	23505	5870	8605	38695	7630	5540	18785	14040	5570	35285
MT	221510	0	11135	8365	40600	10055	810	3205	24315	9075	7475	15045	2410	1725	6360	5665	2010	16020
OT	74165	13830	0	4570	4830	4160	350	1140	6780	1130	2165	6870	1870	1300	2440	3580	1825	5840
LO	7025	7915	480	0	310	275	210	470	1210	250	315	2100	460	420	910	1090	500	1705
OB	11080	41545	4700	310	0	410	40	140	875	520	225	595	90	130	180	270	130	435
WO	37495	10145	3440	300	480	0	230	340	790	190	250	2940	780	430	1065	1000	270	1415
SO	4870	995	360	260	50	190	0	40	80	20	50	165	20	45	50	50	50	130
TB	20725	3600	1110	550	100	310	20	0	200	110	40	8100	200	330	350	350	100	640
HF	24020	23855	6500	1240	800	900	130	280	0	10175	6725	1050	190	160	855	680	390	1670
SN	5970	8880	1280	200	540	130	10	50	10090	0	610	145	90	50	140	110	30	345
SS	10155	7625	2300	295	220	310	140	70	9040	750	0	200	80	40	200	150	80	340
WN	40560	15580	6855	2110	610	3130	155	8175	1190	215	245	0	12400	7400	11770	10575	1745	18575
RG	6610	2520	1020	490	100	970	20	260	260	40	30	12420	0	3145	12855	5650	770	6080
SK	4765	5490	1340	390	125	510	20	350	230	50	90	7915	3310	0	8435	4190	450	4010
CG	19305	6350	2620	990	290	1220	50	400	790	130	190	12085	12455	8350	0	51320	4160	31500
ED	16845	6010	3790	1155	330	1040	30	340	800	50	80	10115	5290	4180	51030	0	3415	28375
VC	8515	2115	1970	520	200	200	30	110	440	30	75	1770	785	640	4160	3368	0	24135
VN	37905	17075	5680	1800	515	1490	118	935	1360	375	395	18650	5860	4015	31355	26925	24140	0

SOURCE AIR TRANSPORT BOARD, MAY 1966. "AIRLINE PASSENGER ORIGIN AND DESTINATION STATISTICS, DOMESTIC REPORT, 1965." PREPARED BY THE STATISTICS SECTION, DEPARTMENT OF TRANSPORT ON BEHALF OF THE AIR TRANSPORT BOARD, OTTAWA, CANADA.

TABLE A4-70 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1966

	TO	MT	OT	LO	OB	WO	SO	TB	HF	EN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	229380	84120	7080	11180	38435	11330	23045	27170	6840	30700	47015	9875	6580	22100	18690	7045	42070
MT	278895	0	15655	8470	39615	11400	1460	3120	24150	9015	8650	27455	2685	1740	6940	7345	2425	19265
OT	84700	14740	0	4410	4030	4770	560	1320	7660	1600	2460	9840	2415	1785	3945	4730	2775	7225
LO	9060	8230	4470	0	370	370	340	410	1210	310	320	2525	570	430	1570	1250	640	2005
OB	12125	42110	6140	355	0	820	60	140	1335	370	340	405	120	70	385	310	245	665
WO	38310	12270	4340	390	705	0	370	330	955	220	370	3725	490	380	1440	1200	350	2225
SO	13330	1570	680	460	70	350	0	90	250	50	60	265	20	20	10	80	20	185
TB	24310	3490	1330	650	100	410	130	0	530	60	30	9450	470	340	400	470	140	1250
HF	24555	26025	8150	1240	1290	850	210	280	0	10640	10720	1795	370	230	450	920	670	1800
SN	7110	9275	1680	360	440	130	30	50	10135	0	820	350	70	40	200	130	110	400
SS	12130	9200	2705	350	220	240	220	90	10960	950	0	370	125	30	340	220	110	340
WN	48745	19005	10180	2470	650	3715	205	9950	1730	340	410	0	14505	10550	13500	12790	3200	23615
RG	10445	3308	2535	700	150	850	40	360	440	40	110	14930	0	3745	13915	7040	810	7640
SK	6675	1910	1795	480	100	670	20	400	200	50	70	10870	4335	0	5790	4630	700	5055
CG	22745	9375	3680	1730	275	1755	80	600	1000	220	220	14055	13970	5700	0	41760	4885	37940
ED	19885	7750	4740	1380	380	1170	90	420	810	140	250	12200	6530	4755	64075	0	3800	35815
VC	6600	2885	2750	510	160	300	0	140	590	150	60	2070	730	663	4640	3620	0	23850
VC	[illegible]	21900	7510	2240	815	2475	195	1340	1815	440	445	23600	7120	8345	37960	35720	24210	0

SOURCE AIR TRANSPORT BOARD. "AIRLINE PASSENGER ORIGIN AND DESTINATION STATISTICS, DOMESTIC REPORT, 1966." PREPARED BY THE AVIATION STATISTICS CENTRE ON BEHALF OF THE AIR TRANSPORT BOARD, OTTAWA, CANADA.

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TABLE A4-71 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1967

	TO	MT	OT	LO	QB	WD	SD	TB	WP	SN	SS	HX	RG	SK	CG	ED	VC	VN
TO	0	289940	101440	8450	12705	35085	16185	28000	41540	7565	11665	51780	10225	6105	26100	22120	8155	[illegible]
MT	290345	0	16550	10035	39770	22705	1665	5850	36030	13375	13205	31875	5345	4560	14945	13315	4240	[illegible]
OT	101245	16475	0	6060	7130	4570	690	1440	9550	2000	2440	12955	3640	2240	4035	8530	3190	[illegible]
LO	8220	9505	5830	0	500	240	330	700	1170	250	350	2965	405	445	1330	1170	455	[illegible]
QB	13420	40365	7300	525	0	615	70	110	1500	540	435	1165	215	170	910	560	195	[illegible]
WD	36705	21740	4450	280	410	0	330	420	1040	270	140	3420	910	450	1300	1340	340	[illegible]
SD	14990	2075	710	460	40	370	0	150	160	40	110	190	30	70	70	160	20	[illegible]
TB	20680	5500	2155	680	140	460	140	0	270	70	50	11665	650	250	760	630	140	[illegible]
WP	31135	16505	4670	1260	1450	1010	170	280	0	12100	11555	1965	540	150	1450	1300	750	[illegible]
SN	7925	13305	1430	270	330	250	60	60	17690	0	1000	415	60	20	240	100	70	[illegible]
SS	12925	13245	3160	470	640	370	170	90	12125	1070	0	490	110	50	310	320	130	[illegible]
HX	83455	32445	13950	3175	6190	3585	190	11555	1745	340	815	0	14025	11390	16660	14960	2010	[illegible]
RG	10775	6165	3765	690	320	1165	40	670	550	70	100	17765	0	5270	16250	8645	1230	[illegible]
SK	8675	4680	2375	590	290	460	60	255	315	20	60	13245	5310	0	8720	6330	755	[illegible]
CG	26040	15435	6150	1490	540	1310	70	870	1490	260	340	16575	14045	6515	0	77855	5790	[illegible]
ED	22400	16335	6070	1170	700	1500	110	460	1220	130	210	15015	8355	4750	70270	0	8210	[illegible]
VC	8195	4410	3320	460	200	310	40	140	670	110	140	2700	1070	790	5760	5315	0	[illegible]
VN	49630	36045	9415	2180	1205	2870	265	1505	2165	845	460	26955	6075	6345	46320	43475	33635	

SOURCE AIR TRANSPORT COMMITTEE. "AIRLINE PASSENGER ORIGIN AND DESTINATION STATISTICS, DOMESTIC REPORT, 1967." PREPARED BY THE AVIATION STATISTICS CENTRE ON BEHALF OF THE AIR TRANSPORT COMMITTEE. OTTAWA, CANADA.

TABLE A4-72 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1968

	TO	MT	OT	LO	QB	WD	SD	TB	WP	SN	SS	HX	RG	SK	CG	ED	VC	VN
TO	0	266140	113420	8540	14200	36640	16900	31840	35150	8960	12710	61615	12190	9330	32060	25230	11570	[illegible]
MT	281170	0	19100	9000	34050	12610	1610	3260	34870	10340	12120	24235	3150	2350	17780	8265	3940	[illegible]
OT	113900	19165	0	6570	7975	3020	1040	2140	11030	1050	2730	17645	2660	1985	4060	6130	3130	[illegible]
LO	9110	8660	6160	0	610	240	450	1010	1710	340	300	2925	500	310	1600	1470	210	[illegible]
QB	34415	39930	6110	465	0	555	60	170	1530	650	815	870	170	140	545	325	200	[illegible]
WD	37690	13170	4990	280	560	0	390	540	1050	250	260	3770	810	700	1390	1210	300	[illegible]
SD	18150	1675	1110	490	90	440	0	180	260	80	170	550	30	60	230	120	60	[illegible]
TB	32000	3620	2300	1050	160	650	190	0	350	50	190	12370	490	470	670	590	170	[illegible]
WP	35620	34165	10720	1670	1365	1160	250	310	0	13430	18190	2030	620	270	1500	1430	650	[illegible]
SN	9100	11175	2030	290	400	310	90	70	13340	0	750	380	80	40	250	130	160	[illegible]
SS	13745	12205	2910	780	520	340	110	90	11720	1105	0	410	170	60	370	170	140	[illegible]
HX	64210	25100	12970	3700	1015	1940	250	12530	2000	370	370	0	17595	18140	21315	14105	3040	[illegible]
RG	17945	3105	2695	545	230	1010	70	820	510	50	150	18035	0	4975	12560	5615	1255	[illegible]
SK	9860	2565	2225	370	170	700	40	490	340	50	50	15125	8175	0	10060	7165	940	[illegible]
CG	32410	13530	6330	1420	520	1620	240	860	1640	200	320	21425	16950	10065	0	67160	7565	[illegible]
ED	26920	8840	5900	1470	360	1395	90	490	1700	100	300	14720	9030	7340	66040	0	6730	[illegible]
VC	11180	4070	3320	760	260	310	10	190	400	150	120	3135	1330	1600	7365	6940	0	[illegible]
VN	60675	20205	10650	2660	775	2330	340	1440	2340	600	740	31410	9350	7635	84495	91085	37720	[illegible]

SOURCE DOMINION BUREAU OF STATISTICS, JULY 1971. AIRLINE PASSENGER ORIGIN AND DESTINATION STATISTICS, DOMESTIC REPORT, 1968. OTTAWA, THE QUEEN'S PRINTER.

TABLE A4-73 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1969

	TO	MT	OT	LO	QB	WD	SD	TB	WP	SN	SS	HX	RG	SK	CG	ED	VC	VN
TO	0	293065	125455	10050	15190	35215	21795	34125	47095	[illegible]	15060	72275	14160	9415	37260	31165	12165	69700
MT	293010	0	23155	11830	38260	14090	2165	3585	34820	[illegible]	12220	27310	4020	3160	16095	11000	3910	33645
OT	175455	23610	0	6170	9045	6170	1000	2670	11900	[illegible]	3360	14260	3475	2650	7210	6645	3420	13155
LO	9775	11625	7910	0	650	220	760	1110	1670	[illegible]	310	3170	770	540	1900	1990	610	2030
QB	15745	40810	9235	700	0	755	170	150	1605	[illegible]	360	1095	190	100	640	455	120	950
WD	36770	13640	5360	230	790	0	490	890	1450	260	320	3540	760	610	1750	1510	460	2310
SD	22145	2180	1210	480	130	480	0	260	270	70	160	480	40	60	185	120	70	310
TB	35810	4030	3030	1170	220	370	280	0	470	80	40	12550	675	460	1040	460	370	1615
WP	42115	40725	12870	1615	1365	1570	290	390	0	12085	13010	2070	460	440	2160	1530	885	3040
SN	9770	11045	1920	410	430	230	30	50	12135	0	1090	480	140	50	350	240	130	[illegible]
SS	15880	12355	3180	390	430	300	150	20	13840	1260	0	570	250	125	410	340	110	745
HX	73490	26170	14335	3820	1240	4110	460	12965	2560	340	520	0	18805	16045	79010	14555	3450	40490
RG	14795	4150	3575	850	210	660	30	545	665	130	180	19440	0	5345	17605	10255	1500	17210
SK	10070	3760	2585	660	215	625	80	340	525	80	20	14440	8325	0	11940	7765	1190	9205
CG	30475	16340	7680	2440	525	1950	150	1080	2040	310	400	25725	18185	12435	0	105960	8345	70510
ED	32770	11390	7090	2210	625	1570	170	580	1400	100	430	14795	10160	8190	100455	0	6705	40250
VC	12010	4160	3440	910	190	550	100	230	1040	70	90	9010	1470	1120	8390	6500	0	22005
VN	73780	36625	14090	2860	965	2660	810	1740	3120	440	570	40775	12365	8805	71000	61690	28650	0

SOURCE DOMINION BUREAU OF STATISTICS, DECEMBER 1970. AIR PASSENGER ORIGIN AND DESTINATION, 1969, DOMESTIC REPORT. OTTAWA: THE QUEEN'S PRINTER.

TABLE A4-74 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1970

	TO	MT	OT	LO	QB	WD	SD	TB	WP	SN	SS	HX	RG	SK	CG	ED	VC	VN
TO	0	336120	153070	11710	18005	43030	28365	41695	49150	10085	17325	84660	16025	11530	41115	34885	13310	[illegible]
MT	336645	0	30230	13490	45795	17000	3280	4075	45785	12995	13165	30730	4105	3165	15925	12175	4340	[illegible]
OT	157490	30230	0	10230	12175	7300	1640	3160	16165	3100	3685	19435	4265	3340	5170	8510	4870	[illegible]
LO	12065	13195	9900	0	540	270	1030	1350	2210	440	430	4060	640	510	2305	1970	1025	[illegible]
QB	14895	46730	12370	475	0	640	125	300	1730	670	240	1450	190	125	470	490	240	[illegible]
WD	47145	16070	6870	350	810	0	700	475	1340	290	260	5305	780	430	1850	1600	630	[illegible]
SD	30765	3500	1770	970	110	410	0	320	390	70	240	545	150	65	290	160	110	[illegible]
TB	42805	4665	3370	1560	245	990	340	0	310	40	155	14215	625	405	1325	655	420	[illegible]
WP	49570	44620	16305	2210	1440	1340	390	245	0	13475	16810	3600	470	550	2365	2000	1015	[illegible]
SN	10540	13130	3120	510	440	290	40	110	14230	0	1035	510	130	70	370	300	230	[illegible]
SS	17975	12510	3095	530	365	365	480	60	15745	1415	0	740	240	95	470	520	200	[illegible]
HX	86760	30445	19010	4240	1310	5730	630	14680	3740	475	630	0	21930	19660	30340	22800	4935	[illegible]
RG	15470	4380	4165	705	160	960	140	570	930	120	130	22155	0	4240	20330	12070	1955	14070
SK	12345	3405	3195	515	165	720	95	365	530	80	140	19950	6645	0	14760	5565	1625	[illegible]
CG	41860	16225	9215	2390	405	2155	310	1050	2340	390	380	30615	21330	15165	0	116895	10325	[illegible]
ED	35170	12575	9130	1950	240	1650	135	600	2130	290	420	22920	11560	9255	117950	0	7575	[illegible]
VC	17565	4715	4075	890	210	440	80	195	1210	200	180	4685	1990	1765	16445	7645	0	[illegible]
VN	83535	33510	15955	3810	1160	3665	540	2685	3230	670	560	45055	14725	12545	87240	71255	33050	0

SOURCE DOMINION BUREAU OF STATISTICS, JUNE 1971. AIR PASSENGER ORIGIN AND DESTINATION, 1970, DOMESTIC REPORT. OTTAWA, INFORMATION CANADA.

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TABLE A4-75 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1971

	TO	MT	OT	LD	QB	WD	SD	TB	HF	SN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	241615	162800	10845	20190	43385	32805	47301	81130	10920	20010	80680	16940	13965	42265	35490	11740	88015
MT	344190	0	35040	14165	44645	17935	4210	4055	43060	12420	13708	28565	4370	3770	17175	17505	3540	34195
OT	163740	32635	0	11350	14665	7420	2210	3140	14520	2950	8070	20130	8660	4170	10245	9990	3725	19255
LD	11080	14295	11160	0	850	270	1350	1705	2370	380	580	3530	710	600	2230	1465	670	3970
QB	21340	65455	15275	770	0	1070	240	125	2200	915	290	1655	370	340	870	500	250	1540
WD	42775	17640	7640	310	1060	0	1090	1165	1695	240	350	4665	950	650	1740	1590	370	2730
SD	33720	4310	2410	1440	208	990	0	790	630	100	240	660	70	160	300	290	80	680
TB	40725	4715	4180	1765	165	1090	755	0	575	75	165	18245	600	710	1420	535	430	2405
HF	51985	43085	18880	2775	2190	1670	485	925	0	12445	17405	3550	1040	750	3340	2400	1160	3625
SN	11650	12795	2910	380	690	270	65	50	12590	0	1165	480	90	40	410	340	110	890
SS	20570	12115	5240	650	345	275	385	125	17145	1440	0	960	160	245	430	875	130	1055
WN	87595	28115	19650	3650	1535	4615	815	14545	3715	475	630	0	31165	19170	24575	21760	5130	42460
RG	17460	4215	9730	795	370	1030	60	570	995	140	120	21600	0	3195	20285	12160	2465	17245
SK	14910	3675	4115	610	240	710	130	615	550	190	160	19830	3365	0	13415	10145	2060	14425
CG	44430	17180	9910	2195	885	1610	210	1330	3250	410	610	20510	20100	14355	0	124220	14270	80120
ED	34910	12580	10140	1910	870	1460	270	845	2355	350	550	21550	10745	10655	128500	0	8810	70495
VC	12110	3695	3120	615	305	505	80	355	1025	160	105	4765	2750	2245	14680	9700	0	20455
VN	83900	38690	19090	2730	1575	2660	660	2160	3515	830	990	42890	17680	14520	89250	74420	20235	0

SOURCE STATISTICS CANADA, JULY 1972. AIR PASSENGER ORIGIN AND DESTINATION, DOMESTIC REPORT, 1971. OTTAWA, INFORMATION CANADA.

TABLE A4-76 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1972

	TO	MT	OT	LD	QB	WD	SD	TB	HF	SN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	379100	174270	12470	21365	42870	32845	47680	55835	12370	23920	87440	20390	15380	31030	43835	12540	100380
MT	379390	0	38215	13455	47550	17300	3860	4230	45180	13035	14865	35510	5155	4105	19490	15200	4185	43335
OT	173350	36215	0	12660	10245	7875	2420	3050	40545	3435	4795	21545	6200	4950	12305	12775	4280	23250
LD	13025	15790	12700	0	620	250	1580	2065	2950	490	670	3790	1260	700	2260	2425	745	4035
QB	22945	47815	15165	700	0	1025	170	130	2240	930	445	1730	470	220	895	1070	281	1645
WD	43445	17615	7610	290	1100	0	1100	840	1750	265	305	6340	1250	405	2465	1440	360	2003
SD	31410	3780	2610	1440	150	1070	0	920	600	50	240	460	225	210	430	350	35	610
TB	49240	4470	3995	2350	140	970	1145	0	550	110	135	16120	905	400	1615	1370	455	2535
HF	57710	44475	21285	2725	2675	1695	490	610	0	12515	21065	4015	1525	630	3905	3240	1120	4005
SN	12270	12120	3425	530	790	210	50	85	14065	0	1205	840	130	150	320	[illegible]	160	500
SS	24690	14410	5040	715	750	245	210	145	21165	1400	0	755	275	180	790	955	250	1170
WN	91215	31710	21005	4190	1400	5570	625	16405	4095	510	760	0	21290	20300	32760	24240	6365	47676
RG	21150	5000	6600	1140	360	1420	190	975	1460	150	310	22200	0	485	21565	14110	3235	19140
SK	15400	4030	4465	615	210	980	210	655	680	120	150	20560	460	0	15010	11225	2615	18470
CG	53215	19155	11360	2730	610	2465	455	1605	3735	415	750	32645	21580	15025	0	137625	17455	101525
ED	44880	15080	12590	2150	1050	1440	318	1170	3700	530	880	24730	13670	11320	137710	0	11050	84615
VC	12565	4240	4300	610	220	405	140	330	1035	150	190	6295	3695	2560	17020	11840	0	20755
VN	105645	41525	23600	4170	1695	3305	865	2745	4385	870	955	47960	19580	17740	100420	85245	27265	0

SOURCE STATISTICS CANADA, DECEMBER 1973. AIR PASSENGER ORIGIN AND DESTINATION DOMESTIC REPORT, 1972. OTTAWA, INFORMATION CANADA.

TABLE A4-77 INTERMETROPOLITAN AIR TRAFFIC STATISTICS, OUTBOUND PASSENGERS, 1973

	TO	MT	OT	LD	QB	WD	SD	TB	HF	SN	SS	WN	RG	SK	CG	ED	VC	VN
TO	0	454455	216280	17760	28175	51235	42370	89055	73150	15110	24985	104265	24025	20280	64155	55195	16730	133230
MT	459170	0	50290	19140	52865	26770	4120	4755	52770	15010	14425	35335	6050	4910	24335	17400	4445	18335
OT	216725	47605	0	16250	20045	10620	3465	5290	28410	4560	4875	26080	9100	6210	14345	17295	8585	29740
LD	17130	19450	16430	0	950	90	2470	2405	3570	725	760	5430	1240	890	3470	3360	1165	5410
QB	30890	53500	15410	1045	0	1385	325	305	2735	1205	465	2740	825	440	1405	1695	410	2780
WD	51075	25880	10160	120	1215	0	1280	1200	2425	285	360	5740	1290	440	4005	2275	370	4340
SD	43120	4140	3550	2160	440	3270	0	815	700	110	260	900	210	340	450	340	100	1220
TB	40535	6225	8055	2705	310	1310	1310	0	580	110	165	19526	1080	815	1950	1675	865	4454
HF	74185	67495	28240	1900	2600	2135	830	610	0	16550	23450	5135	1525	1020	5165	4015	1550	7285
SN	18035	14550	4305	770	495	330	90	150	16975	0	1610	720	180	320	580	470	110	1205
SS	79955	17100	7125	800	345	515	300	165	24460	2000	0	1040	200	205	915	845	220	1255
WN	104200	35720	27280	5905	2710	6310	1130	10725	5200	600	1000	0	24465	23590	39135	30410	8765	60320
RG	25890	6420	8920	1210	535	1470	230	1065	1610	270	230	27460	0	0	27815	17930	4785	26560
SK	22050	5100	6260	1070	510	960	230	770	1030	210	210	24700	0	0	18730	12880	4020	23420
CG	64550	27965	15095	3705	1345	4765	470	1010	4875	610	1020	38605	28035	18405	0	145565	23775	124303
ED	58075	18430	16435	3400	1645	2205	340	1605	3640	440	650	29715	17430	13000	166445	0	17040	107625
VC	16505	4965	5368	1000	370	450	170	610	1665	170	245	8700	5040	4155	23850	17440	0	33045
VN	138185	59905	30035	6135	2920	5020	1050	4160	7240	1470	1645	60420	27330	23355	123315	100375	34445	0

SOURCE STATISTICS CANADA, JULY 1974. AIR PASSENGER ORIGIN AND DESTINATION, DOMESTIC REPORT, 1973. OTTAWA, INFORMATION CANADA.

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INTERMETROPOLITAN MIGRATION TABLES; TABLES A4-78 and A4-79

TABLE A4-78 ORIGIN-DESTINATION INTERMETROPOLITAN MIGRANT FLOW MATRIX, 1966-71

	CG	CJ	ED	HF	HT	KW	LD	MT	OT	QU	RG	SC	SS	SN	SK	SD	TB	TO	VN	VC	WD	WN
CG	0	0	7575	255	340	205	320	850	890	85	725	170	86	50	680	110	65	2300	7565	2065	105	1620
CJ	50	0	100	55	5	25	10	4005	465	3200	5	30	10	15	10	20	5	225	75	5	35	150
ED	10605	10	0	350	280	140	405	835	1270	60	440	125	70	50	820	65	115	2365	9330	2470	176	1700
HF	650	30	445	0	580	320	500	1415	2840	175	55	210	370	715	85	245	60	3510	1055	1340	290	595
HT	435	20	365	100	0	1630	1700	1000	1045	15	45	1840	65	165	40	460	225	6715	1370	245	765	545
KW	340	10	165	100	1220	0	1275	340	590	30	35	810	110	10	5	285	110	4535	440	45	280	285
LD	455	10	330	255	1240	1450	0	790	1105	40	75	635	40	40	65	370	185	6775	1025	200	1515	460
MT	2400	1815	1390	2240	2670	1270	1700	0	12915	9475	210	1200	485	600	255	990	290	25315	2670	1065	1100	2010
OT	1100	270	1105	1100	1020	970	1105	6005	0	1125	225	540	235	140	165	890	190	6440	3075	900	400	1015
QU	130	1165	152	260	110	60	85	11765	2025	0	10	75	10	70	0	45	0	760	230	105	20	75
RG	3340	5	2235	75	235	115	160	190	735	10	0	85	40	25	2035	75	65	990	2615	655	80	2015
SC	295	10	145	240	2445	1030	1210	456	745	30	45	0	18	100	45	610	135	5555	450	85	490	335
SS	115	10	110	660	205	330	210	430	455	100	50	65	0	100	25	130	0	2405	230	150	65	145
SN	140	0	65	855	135	35	115	340	270	5	15	75	30	0	15	75	20	1170	250	60	48	75
SK	2910	5	2430	45	135	75	125	350	500	20	2135	50	15	5	0	115	135	920	2590	830	75	1355
SD	155	15	165	60	640	310	490	350	1040	40	25	780	45	10	75	0	135	3355	335	95	370	230
TB	375	0	215	45	215	190	190	140	165	5	30	210	20	30	85	215	0	1240	710	240	175	1065
TO	3050	85	2465	2245	12100	6245	8560	9435	8870	340	330	4695	920	780	485	2730	1185	0	11635	1565	4420	3345
VN	3610	35	3410	370	510	245	345	2245	1665	70	470	245	65	90	850	165	140	4030	0	8310	145	2180
VC	965	15	610	605	70	45	115	245	910	30	205	25	40	35	185	30	30	875	4005	0	45	390
WD	175	0	140	105	730	455	2085	306	615	40	20	385	40	35	15	335	120	4005	520	60	0	185
WN	4590	90	3445	490	890	400	655	2410	2515	335	1350	345	15	90	955	145	820	5670	10615	2055	275	0

SOURCE D. M. RAY, ED., 1976. CANADIAN URBAN TRENDS, VOLUME 2, METROPOLITAN PERSPECTIVE. TORONTO: COPP CLARK PUBLISHING, IN ASSOCIATION WITH THE MINISTRY OF STATE FOR URBAN AFFAIRS AND PRINTING AND PUBLISHING, SUPPLY AND SERVICES CANADA.

TABLE A4-79 DESTINATION-ORIGIN INTERMETROPOLITAN MIGRANT FLOW MATRIX, 1966-71

	CG	CJ	ED	HF	HT	KW	LD	MT	OT	QU	RG	SC	SS	SN	SK	SD	TB	TO	VN	VC	WD	WN
CG	0	90	10605	650	455	340	455	2400	1100	130	3340	295	115	140	2910	155	375	3050	3610	965	175	4590
CJ	0	0	10	30	20	10	10	1815	270	1165	5	10	10	0	5	15	0	85	35	15	0	90
ED	7575	100	0	445	365	165	330	1390	1300	155	2235	145	110	85	2430	165	215	2465	3410	610	140	3445
HF	255	55	350	0	300	100	255	2240	1100	260	75	240	660	855	45	60	45	2245	370	605	105	490
HT	340	5	280	580	0	1220	1240	2670	1020	110	235	2445	205	135	135	640	235	12100	510	70	730	890
KW	205	25	140	320	1630	0	1450	1270	970	60	115	1030	330	35	75	330	190	6245	245	45	455	400
LD	320	10	405	500	1700	1275	0	1700	1105	85	160	1210	210	115	125	450	190	8560	345	115	2085	655
MT	850	4005	835	1415	1030	340	790	0	6005	11765	190	455	430	340	350	350	140	9435	2245	245	305	2410
OT	890	465	1270	2840	1045	590	1105	12915	0	2025	735	745	455	270	500	1040	155	8870	1665	910	615	2515
QU	85	3200	50	175	15	30	40	9475	1125	0	10	30	100	5	20	40	5	340	70	30	40	335
RG	725	5	440	55	45	25	75	210	225	10	0	45	40	15	2135	25	35	330	470	205	20	1350
SC	120	30	125	210	1840	815	635	1200	540	75	85	0	65	75	50	780	210	4695	245	25	385	345
SS	85	10	70	370	65	110	40	485	235	10	40	10	0	30	15	45	20	920	65	40	40	15
SN	50	15	50	715	165	10	60	660	140	70	25	100	100	0	5	10	30	780	90	35	35	90
SK	680	10	820	85	40	5	65	255	165	0	2035	45	25	15	0	75	85	485	850	185	15	955
SD	110	20	65	245	460	285	370	990	890	45	75	610	130	75	115	0	215	2730	165	30	335	145
TB	65	5	115	60	225	110	185	290	190	0	65	135	0	20	135	135	0	1185	140	30	120	820
TO	2300	225	2365	3510	6775	4535	6775	25315	6440	740	990	5555	2405	1170	920	3355	1240	0	4030	875	4005	5670
VN	7565	75	9330	1055	1370	440	1025	2670	3075	230	2615	650	230	250	2590	335	770	11635	0	4005	520	10615
VC	2065	5	2470	1340	245	45	200	1065	900	100	655	85	150	60	830	95	240	1565	8310	0	60	2055
WD	105	35	170	290	765	280	1515	1100	400	20	80	490	65	45	75	370	170	3420	145	45	0	275
WN	1620	150	1700	595	545	285	460	2010	1015	75	2015	335	145	75	1355	230	1065	3345	2180	390	185	0

SOURCE D. M. RAY, ED., 1976. CANADIAN URBAN TRENDS, VOLUME 2, METROPOLITAN PERSPECTIVE. TORONTO: COPP CLARK PUBLISHING, IN ASSOCIATION WITH THE MINISTRY OF STATE FOR URBAN AFFAIRS AND PRINTING AND PUBLISHING, SUPPLY AND SERVICES CANADA.

DERIVATION OF THE SENSITIVITY FORMULAS

OF

CHAPTER THREE (p. 107)

INTRODUCTION:

System sensitivity measures the change in system gain at a node x_i for any change in the arc coefficient linking the nodes x_k and x_l. For any given dependent node there are as many sensitivity measures as there are coefficients in the origin-destination matrix, that is for each node, the formulas generate a matrix of sensitivity measures.

(1) $${}_iS_{kl} = \frac{\partial[G_i]}{\partial a_{kl}}$$

However, sensitivity is more often expressed in relative terms such that it measures the relative change in system gain at x_i for a relative change in the arc coefficient, and (1) becomes (2):

(2) $${}_iS_{kl} = \frac{\partial[G_i]}{\partial a_{kl}} \cdot \frac{a_{kl}}{g_i} = \frac{\partial[G_i]}{g_i} \bigg/ \frac{\partial a_{kl}}{a_{kl}}$$

The notation ${}_iS_{kl}$ refers to the sensitivity at node x_i for any fluctuations on the arc joining x_k to x_l. The symbol ∂ refers to the partial differentiation of the vectors. Since the derivation of the formula of sensitivity involves the derivatives of matrices and vectors, the derivation of the formula will be presented in detail.

It is recalled here that in general, if V is a vector composed of the elements v_1, v_2, ..., v_n that are functions of a variable x, then the derivative of V, $\frac{\partial[V]}{\partial x}$ has as its elements $\frac{\partial v_1}{\partial x}$, $\frac{\partial v_2}{\partial x}$, ..., $\frac{\partial v_n}{\partial x}$. If M is a matrix with elements m_{ij} that are functions of x, then $\frac{\partial[M]}{\partial x}$ defines the matrix whose elements are $\frac{\partial m_{ij}}{\partial x}$.

In calculating sensitivities of a node with respect to fluctuations on the arcs, there are two situations that must be dealt with, because the derivation of sensitivity to an arc tied to the major node is different from that which measures sensitivity to fluctuation on the arcs extended between the dependent nodes.

First Case: Sensitivity to Change on the Source-node Arcs:

Equation (3) specifies the gain at node x_i as:

(3) $$g_i = \sum_{j=1}^{n} [-A^{-1}]_{ij} \cdot c_j$$

Then sensitivity at node x_i for a change on major-node arc linking x_o to x_i whose coefficient is C_{ko} would be described as:

(4) $$\frac{\partial g_i}{\partial c_k} = \sum_{j=1}^{n} [-A^{-1}]_{ij} \cdot \frac{\partial c_j}{\partial c_k} \quad *$$

Note that the inverse matrix contains no "C" terms, therefore it acts

*Since only those arcs that originate in the x_o are labelled C, the abbreviated form ∂C_k instead of ∂C_{ko} is used in equation (4). The abbreviated form is also used in most of the subsequent equations dealing with C coefficients since there is no possibility of confusion and the writing of the equations is thereby simplified.

as a constant in the equation. (Equations 6b and 13 of Chapter Three).

The general rules of differentiation apply to matrices and vectors and it seems advisable to recall here those that will be used in the sensitivity formulae.

First, for clarity's sake equation (3) will be expressed in its matrix form as (5):

(5) $$[G] = [-A^{-1}][C]$$

The first rule that is of interest deals with the role of the constant in an expression such as 2x. The differentiation of 2x with respect to x is given in (6):

(6) $$\frac{\partial 2x}{\partial x} = 2\frac{\partial x}{\partial x} = (2)(1) = 2$$

This rule applies as well to matrices and vectors as is illustrated in equations (7) and (8).

Let

(7) $$[X] = [A][Y]$$

where X and Y are vectors and A is a matrix. If Y is a function of u and A is not, then,

(8) $$\frac{\partial[X]}{\partial u} = [A]\frac{\partial[Y]}{\partial u}$$

In the same manner the differentiation of (5) gives (9):

(9) $$\frac{\partial[G_i]}{\partial c_k} = [-A^{-1}]\frac{\partial[C]}{\partial c_k}$$

In (9) the $-A^{-1}$ matrix is considered a constant since it contains no C coefficients. The sensitivity at node x_i is found with respect to arc C_k, Consider a vector C whose derivatives with respect to C_k are sought. In general,

$$(10) \qquad \frac{\partial[C]}{\partial c_k} = \begin{bmatrix} \frac{\partial c_1}{\partial c_k} \\ \frac{\partial c_2}{\partial c_k} \\ \cdots \\ \frac{\partial c_n}{\partial c_k} \end{bmatrix}$$

The right-hand side of equation (10) will give a vector of zeros in all positions save a single one which will have a value of 1. Element C_k in the vector will have a value of 1 and all others will be zero.

For example:

$$(11) \qquad \frac{\partial[C]}{\partial c_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

This results from the fact that in equation (11) differentiation of vector C is made with respect to C_1, C_1 then acting as a variable of the first degree. Just as the derivative of x with respect to x equals 1, the derivative of $\frac{\partial[C]}{\partial c_1}$ will give a value of 1 for the first element in the vector. All other elements are not functions of C_1 but of C_2, C_3, ..., C_n. They are therefore regarded as constants with respect to C_1 and their derivatives are equal to zero.

Because (10) provides a vector containing a single value of 1, the sensitivity of node x_i to changes on the arc linking x_o and x_k whose coefficient is C_k, equation (9) becomes (12).

$$\frac{\partial g_i}{\partial c_k} = [-A^{-1}]_{ik} \tag{12}$$

Equation (12) states that sensitivity of node x_i to changes on arc C_k is equal to the value of element in cell a_{ik} of the $-A^{-1}$ matrix (Figure 1).

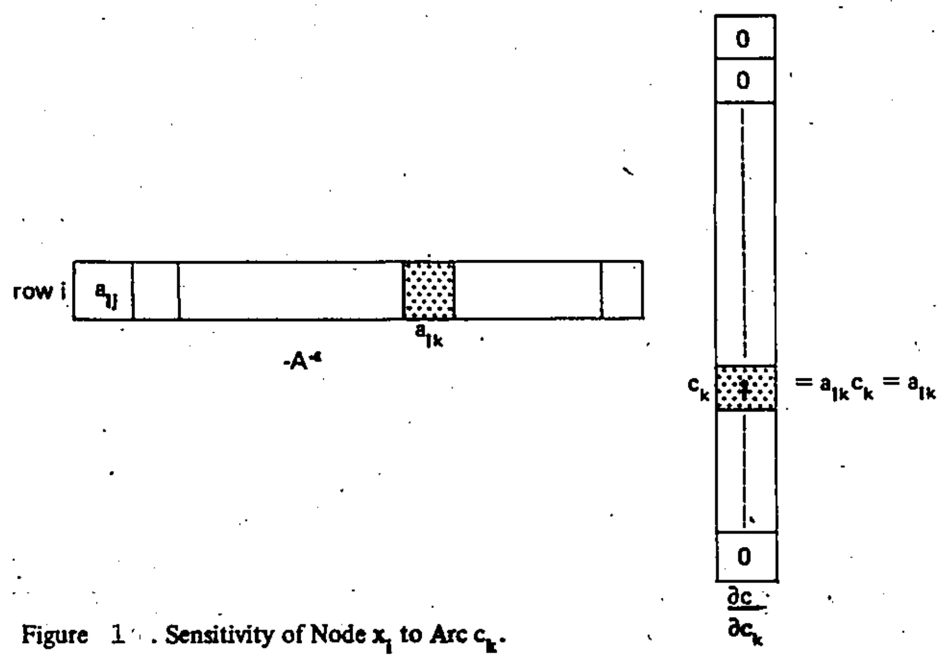

Figure 1 . Sensitivity of Node x_i to Arc c_k.

In relative terms sensitivity of (2) can be written as (13):

$${}_iS_{k\,o} = [-A^{-1}]_{ik}\frac{c_k}{g_i} \tag{13}$$

Second Case: <u>Sensitivity of a Node to Change in Arcs not Linked to the Major Node</u>

Equation (1) defines in absolute terms the sensitivity of node x_i to fluctuations on arc a_{kl} joining node x_k and x_l as being:

(1) $${}_iS_{kl} = \frac{\partial[G_i]}{\partial a_{kl}} \quad *$$

Therefore the derivative of equation (11)** must be determined for node x_i with respect to arc a_{kl}. This gives equation (14).

Since:

(11)** $$[G] = [A^{-1}][B]$$

then:

(14) $$\frac{\partial[G_i]}{\partial a_{kl}} = \frac{\partial[A^{-1}]}{\partial a_{kl}}[B]$$

Since vector B is not a function of a_{kl} the differentiation of (11)** follows the general rule which is illustrated in (15) and (16).

Let X and Y be vectors and A a matrix in equation (15).

(15) $$[X] = [A][Y]$$

If matrix A is a function of u and vector Y is not, the derivative of X with respect to u is:

(16) $$\frac{\partial[X]}{\partial u} = \frac{\partial[A]}{\partial u}[Y]$$

The derivatives of the matrix $\frac{\partial[A^{-1}]}{\partial a_{kl}}$ are not difficult to calculate in matrix algebra. Since $[A][A^{-1}] = [I]$ the identity matrix, the differentiation of $[A][A^{-1}]$ will give (17):

(17) $$\frac{\partial[A][A^{-1}]}{\partial a_{kl}} = [0]$$

---

* In equation (14) the k subscript in ${}_iS_{kl}$ and a_{kl} refers to the node to which the arrow of the arc is pointing.

** Equation (11) from Chapter Three.

which can be written as (18):

(18) $$\frac{\partial[A\,A^{-1}]}{\partial a_{kl}} = [0]$$

However since equation (18) involves the differentiation of a product of two matrices, both of which are a function of a_{kl}, equation (18) can be written as (19) following the general rule of differentiation of a product.

(19) $$\frac{\partial[A]}{\partial a_{kl}}[A^{-1}] + [A]\frac{\partial[A^{-1}]}{\partial a_{kl}} = [0]$$

which gives

(20) $$\frac{\partial[A^{-1}]}{\partial a_{kl}} = [-A^{-1}]\frac{\partial[A]}{\partial a_{kl}}[A^{-1}]$$

Note that the right-hand side is a three-matrix product.

Substituting in (14):

(21) $$\frac{\partial[G]}{\partial a_{kl}} = [-A^{-1}]\frac{\partial[A]}{\partial a_{kl}}[A^{-1}][B]$$

The right-hand side involves the multiplication of three matrices and one vector.

Since (11)** give $[G] = [A^{-1}][B]$, (21) simplifies to:

(22) $$\frac{\partial[G]}{\partial a_{kl}} = [-A^{-1}]\frac{\partial[A]}{\partial a_{kl}}[G]$$

The differentiation of matrix A with respect to A_{kl} will give a matrix whose cells are all equal to zero with the single exception of A_{kl} in row k, column l where the value of the derivative will be 1. This follows the same logic that was outlined earlier for the differentiation of vectors in the discussion on sensitivity with respect to arcs tied to the major node.

**Equation (11) from Chapter Three.

In (22)

$$\frac{\partial [A]}{\partial a_{kl}} = \begin{matrix} 1 \\ \\ k \\ \\ n \end{matrix}\begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 1 & \cdots & \vdots \\ \vdots & & \vdots & & \vdots \\ 0 & & 0 & & 0 \end{bmatrix} = [A]_{kl}$$

Then

(23)

$$\frac{\partial [A]}{\partial a_{kl}} [G] = k \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 1 & \cdots & \vdots \\ \vdots & & \vdots & & \vdots \\ 0 & & 0 & & 0 \end{bmatrix} \begin{bmatrix} g_1 \\ \vdots \\ g_l \\ \vdots \\ g_n \end{bmatrix} = g_l = [G]_l$$

Every coefficient of the $\frac{\partial A}{\partial a_{kl}}$ matrix is equal to zero with the exception of the coefficient in row k colum l which is equal to one. Therefore $\frac{\partial A}{\partial a_{kl}}$ G is a column vector whose coefficients are all equal to zero except for the k one which has a value equal to G_l. Then the vector $\frac{\partial G}{\partial a_{kl}}$ has values equal to the product of G_l by the k column of the $-A^{-1}$ matrix.

Expression (23) simplifies to $[G]_l$ or g_l which is the system gain at node x_l.

Equation (22) can now be written as:

(24) $$\frac{\partial [G]}{\partial a_{kl}} = [-A^{-1}][G]_l$$

or

(25) $$\frac{\partial [G]}{\partial a_{kl}} = [-A^{-1}] \cdot g_l$$

Equation (25) becomes (26) when the sensitivity of the single node x_i is measured with respect to arc a_{kl}:

(26) $$\frac{\partial g_i}{\partial a_{kl}} = -g_l [A^{-1}]_{ik}$$

Equation (26) gives the sensitivity in absolute value. In relative terms, sensitivity is given by equation (27).

(27) $${}_iS_{kl} = [-A^{-1}]_{ik} \cdot g_l \frac{a_{kl}}{g_i}$$

Equations (13) and (27) are the formulas for the calculation of sensitivity measures for flow change in all the arcs of a system.

TABLE A5-18 INTERMETROPOLITAN DISTANCES

	CG	CJ	ED	MF	HT	KG	LD	MT	OT	OB	RG	SC	SS	SN	SK	SD	TB	TO	VN	VC	WD	WN
CG		2622	186	3090	2176	2065	2020	2326	2208	2494	475	[illegible]	3936	2898	385	1893	1274	2134	657	722	2014	830
CJ	2622		2635	607	673	697	750	296	414	128	2147	703	1453	403	2307	735	1348	631	3279	3344	858	1792
ED	186	2635		3115	2189	2078	2033	2339	2221	2507	400	2149	3956	2923	328	1906	1287	2147	773	838	2027	843
MF	3090	607	3115		1153	1177	1230	776	894	610	2627	1183	934	192	2787	1209	1828	1111	3759	3824	1338	2272
HT	2176	673	2189	1153		36	78	377	290	545	1701	35	1994	961	1861	276	902	42	2833	2898	198	1346
KG	2065	697	2078	1177	36		65	401	309	569	1590	71	2018	985	1750	286	906	69	2722	2787	178	1336
LD	2020	750	2033	1230	78	65		454	362	622	1545	113	2071	1038	1705	340	960	114	2677	2742	118	1391
MT	2326	296	2339	776	377	401	454		118	168	1851	407	1617	584	2011	433	1052	335	2983	3048	562	1496
OT	2208	414	2221	894	290	309	362	118		286	1733	315	1735	702	1893	307	934	248	2865	2930	475	1378
OB	2494	128	2507	610	545	569	622	168	286		2019	575	1468	418	2179	601	1220	503	3151	3216	730	1664
RG	475	2147	408	2627	1701	1590	1545	1851	1733	2019		1661	3468	2435	160	1418	799	1659	1132	1197	1539	355
SC	2065	703	2149	1183	35	71	113	407	315	575	1661		2024	991	1821	303	923	69	2793	2858	226	1354
SS	3936	1453	3956	934	1994	2018	2071	1617	1735	1468	3468	2024		1073	3628	2050	2669	1952	4600	4725	2184	3113
SN	2898	403	2923	192	961	985	1038	504	702	418	2435	991	1073		2595	1017	1636	919	3569	3632	1146	2000
SK	385	2307	328	2787	1861	1750	1705	2011	1893	2179	160	1821	3628	2595		1578	959	1819	1042	1107	1699	515
SD	1893	735	1906	1209	276	386	340	433	307	601	1418	303	2050	1017	1578		622	241	2550	2615	457	1052
TB	1274	1348	1287	1828	902	906	960	1052	934	1220	799	923	2669	1636	959	622		860	1931	1996	801	444
TO	2134	631	2147	1111	42	69	114	335	248	503	1659	69	1952	919	1819	241	860		2791	2856	227	1304
VN	657	3279	773	3759	2833	2722	2677	2983	2865	3151	1132	2793	4600	3567	1042	2550	1931	2791		65	2671	1387
VC	722	3344	838	3824	2898	2787	2742	3048	2930	3216	1197	2858	4725	3632	1107	2615	1996	2856	65		2736	1452
WD	2014	858	2027	1338	198	178	118	562	475	730	1539	226	2184	1146	1699	457	801	227	2671	2736		1184
WN	830	1792	843	2272	1346	1336	1391	1496	1378	1664	355	1354	3113	2080	515	1052	444	1304	1387	1452	1184	

SOURCES
1- CANADIAN GOVERNMENT TRAVEL BUREAU, CANADA HIGHWAY MAP & NORTHERN US. CANADA, OTTAWA, 1971.
2- MINISTRY OF TRANSPORTATION & COMMUNICATIONS, CANADA OFFICAL ROAD MAP ONTARIO, 1971.
3- QUEBEC DEPARTMENT OF TRANSPORT, LE QUEBEC 1975, 1975.

POOR COPY

A.S.T. PROGRAMME LISTING

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FORTRAN IV G LEVEL  21                MAIN              DATE = 76144        14/55/00              PAGE 0001
           C
           C   CREATION D'UNE SOUS-MATRICE TIREE DE LA MATRICE PRINCIPALE
           C
           C LA CARTE PARAMETRE DE LA SOUS-MATRICE VIENT IMMEDIATEMENT APRES LES NOMS
           C  SI ON DESIRE UNE SOUS-MATRICE SMAT=1
           C   SI SMAT=0 ON NE TRAITE QUE LA MATRICE INITIALE
           C
           C
           C   LA CARTE-PARAMETRE POUR LA CREATION DE LA SOUS-MATRICE:
           C   NEQ=NOMBRE DE RANGEES,COL.1 ET 2
           C   NEWSP=LE NOUVEAU NOEUD-SOURCE,COL.3-4
           C   NEWTP=LE NOUVEAU NOEUD-TEST,COL. 5-6
           C   NTOWN(I) = LE NUMERO DES VILLES FAISANT PARTIE DE LA SOUS-MATRICE
           C   ATTENTION, ATTENTION: LE NOUVEAU NOEUD-SOURCE  EST INSCRIT LE PREMIER
           C      SUR CETTE LISTE   (FORMAT I2 POUR CES NOMBRES)
           C
                INTEGER TEMP, TNOM
                INTEGER SMAT
                INTEGER*4 TITRE(20)
                DIMENSION NTOWN(30)
                 DIMENSION FMT(20),F(30,33),           NOM(30,5)
                DIMENSION TNOM(1,5)
                NAMELIST/INPUT/ NT,NTEST,F,TITRE,NOM
                READ(5,3) (TITRE(J),J=1,20)
              3 FORMAT(20A4)
                WRITE(6,605)(TITRE(J),J=1,20)
            605 FORMAT('1',25X,20A4/1X,25X,80('*')//)
                READ(5,16) NT,NSRCE,NTEST,SMAT
             16 FORMAT(4I5)
                N=NT-1
                NU = N+2
                 READ(5,3) FMT
                WRITE(6,911)
            911 FORMAT(///,45X,'LA MATRICE D''ENTREE'//)
                 DO 77 I=1,NT
                READ(5,FMT) (F(I,J),J=1,NT)
                SOM=0.0
                DO 78 K=1,NT
             78 SOM=SOM + F(I,K)
                F(I,NU) = SOM
                WRITE(6,912)(F(I,J),J=1,NU)
            912 FORMAT('0',12F10.1)
             77 CONTINUE
                DO 607 I=1,NT
                READ(5,606)(NOM(I,J),J=1,5)
            606  FORMAT(5A4)
            607 CONTINUE
                WRITE(6,608) NSRCE
            608 FORMAT(///,35X,' LA MATRICE AVEC SON NOUVEAU NOEUD-SOURCE, LE NOEU
               1D : ',I5,///)
                DO 680 K=1,NT
            680 WRITE(6,17) K,(NOM(K,J),J=1,5)
           C
                IF(SMAT .EQ. 1) GO TO 650
           C
           C
           C   REMANIEMENT DES COLONNES
           C
```

```
  683 DO 10 L=1,NT
      TEMP = F(L,1)
      F(L,1) = F(L,NSRCE)
   10 F(L,NSRCE) = TEMP
C
C REMANIEMENT DES RANGEES
C
      DO 11 L=1,NU
      TEMP= F(1,L)
      F(1,L) = F(NSRCE,L)
   11 F(NSRCE,L) = TEMP
C
C REMANIEMENT DES NOMS
C
      DO 12 K=1,5
      TEMP = NOM(1,K)
      NOM(1,K)= NOM(NSRCE,K)
   12 NOM(NSRCE,K) = TEMP
      DO 20 KK=1,5
   20 TNOM(1,KK) = NOM(1,KK)
      DO 21 LK=1,N
      NN = LK+1
      DO 21 LL=1,5
   21 NOM(LK,LL)= NOM(NN,LL)
      DO 22 K=1,5
   22 NOM(NT,K)= TNOM(1,K)
C
      DO 14 I=1,NT
      WRITE(6,912)(F(I,J),J=1,NU)
   14 CONTINUE
      WRITE(6,19)
   19 FORMAT('0',5X,'LES NOMS DES NOEUDS,LE NOEUD-SOURCE EST LE DERNIER'
     A )
      DO 18 K=1,NT
      WRITE(6,17) K,(NOM(K,J),J=1,5)
   17 FORMAT('0',3X,I5,3X,5A4)
   18 CONTINUE
  666 WRITE(9,INPUT)
      ENDFILE 9
      GO TO 665
  650 READ(5,651) NEQU,NEWSR,NEWTP,(NTOWN(I),I=1,NEQU)
  651 FORMAT(40I2)
C
C LE NOEUD-SOURCE EST PLACE EN PREMIER POSITION EN NTOWN
C
      DO 691 I=2,NEQU
      IF(NEWTP .EQ. NTOWN(I)) NEWTP = I
  691 CONTINUE
      WRITE(6,695) NEQU,NEWSR,NEWTP
  695 FORMAT('0',10X,'LE NOMBRE DE VILLES DANS LA SOUS-MATRICE:',2X,I2/
     B '0',10X,'LE NOEUD-SOURCE:',2X,I2/
     C '0',10X,'LE NOEUD-TEST:',2X,I2//)
      WRITE(6,698)
  698 FORMAT('0',10X,'LES VILLES DE LA SOUS-MATRICE:',/1X,10X,30('-'))
      DO 696 I=1,NEQU
      WRITE(6,697) I,(NOM(NTOWN(I),J),J=1,5)
  697 FORMAT('0',2X,I5,5X,5A4)
  696 CONTINUE
```

```
C
C     FORMATION DE LA SOUS-MATRICE
C
          DO 652 I=1,NEQU
          DO 652 J=1,NEQU
      652 F(I,J) = F(NTOWN(I),NTOWN(J))
          DO 653 I=1,NEQU
          NTT=NTOWN(I)
          DO 653 J=1,5
      653 NOM(I,J)=NOM(NTT,J)
C
C     REMANIEMENT DES NOMS, LE NOM-SOURCE A LA FIN DE LA LISTE
          DO 692 J=1,5
      692 TNOM(1,J)=NOM(1,J)
          NEQB = NEQU-1
          DO 690 LZ=1,NEQB
          LV=LZ+1
          DO 690 J=1,5
      690 NOM(LZ,J)=NOM(LV,J)
          DO 694 J=1,5
      694 NOM(NEQU,J)=TNOM(1,J)
C     ON FAIT LA SOMMATION DES RANGEES
          NO=NEQU+1
          DO 658 I=1,NEQU
          SOM=0.0
          DO 657 K=1,NEQU
      657 SOM=SOM + F(I,K)
          F(I,NO)= SOM
      658 CONTINUE
          WRITE(6,659) NEWSR
      659 FORMAT(///,35X,'LA SOUS-MATRICE AVEC SON NOEUD-SOURCE, LE NOEUD: '
         8,I5,///)
          DO 660 I=1,NEQU
      660 WRITE(6,912) (F(I,J),J=1,NO)
          WRITE(6,19)
          NT=NEQU
          NTEST=NEWTP - 1
          WRITE(9,INPUT)
          ENDFILE 9
      665 STOP
          END
```

```
C         LE MODELE A.S.T.                                                C
C                                                                         C
C         A.S.T. A ETE CREE PAR R. LAMARCHE DE L'UNIVERSITE D'OTTAWA      C
C                       OTTAWA,CANADA                                     C
C                                                                         C
C      MATHEMATICIEN-CONSULTANT:J. FRANCON DU C. DE RECHERCHES NUCLEAIRES C
C                       STRASBOURG, FRANCE                                C
C                                                                         C
C
C
C
C      NT= # DE VARIABLES, INDEPENDANTE COMPRISE
C      NU= # DE COLONNES, TOTAUX DES LIGNES COMPRIS
C
C    1-ERE CARTE: UN TITRE , 80 COLONNES
C    2-E CARTE: NT=LE NOMBRE D'EQUATIONS ,COL 1 A 5,I5,
C    3-E CARTE LE FORMAT INSERE ENTRE 2 PARENTHESES , COL 1 A 80
C         LES DONNEES:
C                   LA REGION INDEPENDANTE  D'ABORD
C
C
C      LE NOM DE L'INDEPENDANTE D'ABORD
C
C
C
C
C
C    LA SIMULATION:
C
C    SIMULATION EN COL 1 A 10,NTEST, LE NOEUD-TEST EN COL 16 A 20,I5
C         NPAR= NOMBRE DE PAR. A EVOLUER COL 21 A 25,I5
C         NFCIS= NOMBRE DE FOIS QU''IL FAUT APPLIQUER L''INCREMENT
C
C    PARAMETRE COL 1 A 9,LGNE=LA LIGNE MATRICIELLE DE CE PARA. COL21 A25, I5
C         CLONNE= LA COLONNE MATRICIELLE DE CELUI-C-, 26 A 30,I5
C          PCT= LE MULTIPLICATEUR DE LA VALEUR DU COEFFICIENT, COL 31 A 40,F10.2
0001       NAMELIST/INPUT/ NT,NTEST,F,TITRE,NOM
0002       INTEGER*4 TITRE(20)
0003       DIMENSION          FMT(20),A(30,30),B(30,30)
0004       DIMENSION LGNE(30), PCT(30)
0005       DIMENSION NOM(30,5)
0006       DIMENSION F(30,33)
0007        DIMENSION P(30),RR(30),RRP(30)
0008       INTEGER CLONNE(30)
0009       DATA VA/'OPTI'/,SIM/'SIMU'/,PAR/'PARA'/
0010       WRITE(6,2)
0011     2 FORMAT('1',/////,45X,'LE MODELE DE SIMULATION TOPOLOGIQUE',/45X,35
          1('*'),///10X,'MODELE ET PROGRAMME CREES'//10X,'PAR RODOLPHE LAMARC
          2HE'//10X,'DEPARTEMENT DE GEOGRAPHIE'// 10X,'UNIVERSITE D''OTTAWA',
          3//10X,'1974'///)
C
C
0012       REWIND 9
0013       READ(9,INPUT)
```

```
    3 FORMAT(20A4)
      N=NT-1
      NU = N+2
      IF(N .EQ. 0 .OR. NTEST .EQ. 0) WRITE(6,73)
   73 FORMAT(//,'0',5('*** ERREUR ***')//
     A20X,'ATTENTION VOUS AVEZ OUBLIE LA 2-E CARTE DES DONNEES'/
     B 20X,'CELLE QUI INDIQUE LE NOMBRE D''EQUATIONS ET LE NO DU NOEUD-
     CTEST'//)
      IF(N .EQ. 0 .OR. NTEST .EQ. 0) GO TO 500
   16 FORMAT(2I5)
    5 FORMAT(20A4)
      NS= N+5
      DO 9 I=1,NS
      DO 9 J=1,NT
      A(I,J)=0.0
      B(I,J) = 0.0
    9 CONTINUE
      DO 607 I=1,NT
  606  FORMAT(5A4)
  607 CONTINUE
      WRITE(6,605) (TITRE(J),J=1,20)
  605 FORMAT('1',25X,20A4/1X,25X,80('*')//)
C
C  ON INSCRIT CES DONNEES POUR  FINS DE VERIFICATIONS
      WRITE(6,911)
  911 FORMAT(///,45X,'LA MATRICE D''ENTREE'//)
  912 FORMAT('0',12F10.1)
      DO 688 I=1,NT
      WRITE(6,912)(F(I,J),J=1,NU)
  688 CONTINUE
C
C
C    POUR LE CALCUL DES COEFFICIENTS VOIR SPATIAL ORGANISATION, BY
C     ABLER, GOULD ET AL. P. 207  PRENTICE-HALL
C
      DO 910 I=1,NT
      DO 910 J=1,NT
      A(I,J)=F(I,J)/(F(J,NU))
  910  CONTINUE
C
      K=0
      DO 152 I=2,NT
      K=K+1
      DO 152 J=1,NT
       A(K,J) = A(I,J)
  152 CONTINUE
      WRITE(6,203)
  203 FORMAT(///,45X,'LA MATRICE DES EQUATIONS CANONIQUES'//)
      DO 7 I=1,N
      WRITE(6,8)(A(I,J),J=1,NT)
    8 FORMAT('0',15F8.4)
    7 CONTINUE
C
C TRANSFORMATION DU SYSTEME D'EQUATIONS
C
      DO 10 I=1,N
      B(I,NT)= -A(I,1)
   10  CONTINUE
```

```
          NC = 0
          DO 11 J=2,NT
          NC = NC+1
          DO 17 I=1,N
          B(I,NC)= A(I,J)
       17 CONTINUE
       11  CONTINUE
C
C  ON REDUIT LA DIAGONALE DES COEFFICIENTS
C
          DO 12 I=1,N
          DO 12 J=1,N
          IF(I .EQ. J) B(I,J) = B(I,J)-1.
       12 CONTINUE
          WRITE(6,202)
      202 FORMAT(///,45X,'LA MATRICE DES EQUATIONS TRANSFORMEES'//)
          DO 18 I=1,N
           WRITE(6,14)(B(I,J),J=1,NT)
       18 CONTINUE
       14 FORMAT('0',12F8.4)
C
          DO 19 I=1,N
          R(I) = B(I,NT)
          RR(I) =A(I,1)
          RRR(I)=A(I,1)
       19 CONTINUE
C
           CALL GASEN(N,B,R,RR,RRR,NTEST,A,NCM)
C
C
C
C          LA SIMULATION : ON FOIT EVOLUER LES  ARC A VOLONTE
C
C
C         SIMULATION
C
C         NTEST : LE NOEUD-TEST A L'ETUDE
C         NPAR: LE NOMBRE DE PARAMETRES SINGULIERS QUI EVOLUENT EN MEME TEMPS
C         NFOIS LE NOMBRE DE VARIATIONS IMPRIMEES A CHAQUE PARAMETRE
C         PARAMETRE: LE MOT PARAMETRE
C         LIGNE: L. DE LA MATRICE CANONIQUE OU SE TROUVE LE PARAMETRE
C         COLONNE: LA COLONNE DE LA MATRICE CANONIQUE OU EST LE PARAMETRE
C         PCT:  UN MULTIPLICATEUR  DE LA VALEUR DU PARAMETRE, EN PLUS OU EN MOINS
C
          IRD=0
      200 READ(5,90,END=500) SIMU,NTEST,NPAR,NFOIS
          IRD=IRD+1
          WRITE(6,124) IRD,(NOM(NTEST,JX),JX=1,5),NPAR, NFOIS
      124 FORMAT(///,56X,'SIMULATION NC ',I3/ 1X,56X,17('*')//
         A1X,55X,'LE NOEUD-TEST :',5A4//
         B1X,48X,'LE NOMBRE DE PARAMETRES IMPLIQUES :',I3//
         C1X,53X,'LE NOMBRE D''ITERATIONS :',I3//)
          IF (SIM .NE. SIMU) WRITE(6,122)
          IF(SIM .NE. SIMU) GO TO 503
      122 FORMAT(//'0',5('*** ERREUR ***')/
         A'0',10X,'ATTENTION VOUS AVEZ OUBLIEZ D''INCLURE LA CARTE SIMULATIO
         BN',//)
       90 FORMAT(A4,11X,3I5)
```

```
0090          WRITE(6,517)
0091     517  FORMAT(//,'0',38X,'LE(S) PARAMETRE(S) IMPLIQUE(S) DANS CETTE SIMU
             ALATION'/1X,38X,52('-')/)
0092          DO 91 IZZ=1,NPAR
0093          READ(5,92) PARA,LGNE(IZZ),CLONNE(IZZ) , PCT(IZZ)
0094          IF(PAR .NE. PARA) WRITE(6,123)
0095          IF(PAR .NE. PARA)GO TO 503
0096     123  FORMAT(//'0',5('*** ERREUR ***')/
             A'0',10X,'ATTENTION VOUS AVEZ OUBLIEZ D''INCLURE LA CARTE PARAMETRE
             B'//)
0097      92  FORMAT(A4,11X,2I5,F10.2)
0098          WRITE(6,516) LGNE(IZZ),CLONNE(IZZ),PCT(IZZ)
0099     516  FORMAT('0',10X,'LE PARAMETRE DE LA LIGNE',I3,' ET DE LA COLONNE',
             AI3,': UN COEFF. MULTIPLICATEUR DE ',F10.2)
0100      91  CONTINUE
        C
0101          DO 100 IVV=1,NFOIS
        C     ON CREE LA NOUVELLE MATRICE
0102          DO 95 K=1,NPAR
0103          LL=LGNE(K)
0104          KK=CLONNE(K)
        C
        C
        C         ON PEUT MODIFIER  LA NATURE DE L'OPERATION A EFFECTUER
        C         SUR LE PARAMETRE EN MODIFIANT LA FONCTION SUIVANTE
0105          F(LL,KK)=F(LL,KK)*PCT(K)
        C
        C
0106      95   CONTINUE
0107          WRITE(6,550) IVV
0108     550  FORMAT(//,56X,'ITERATION NO :',I4/1X,55X,18('*')//)
0109          WRITE(6,911)
0110          DO 894 I=1,NT
0111          WRITE(6,912)(F(I,J),J=1,NU)
0112     894  CONTINUE
0113          DO 921 I=1,N
0114          DO 921 J=1,NT
0115          A(I,J)=F(I,J)/(F(J,NU))
0116     921  CONTINUE
0117          K=0
0118          DO 153 I=2,NT
0119          K=K+1
0120           DO 153 J=1,NT
0121           A(K,J) = A(I,J)
0122     153  CONTINUE
0123          WRITE(6,205)
0124     205  FORMAT(///,45X,'LA MATRICE DES EQUATIONS CANONIQUES'//)
0125          DO 920 I=1,N
0126          WRITE(6,8)(A(I,J),J=1,NT)
0127     920  CONTINUE
        C
        C TRANSFORMATION DU SYSTEME D'EQUATIONS
        C
0128          DO 209 I=1,N
0129          B(I,NT)= -A(I,1)
0130     209   CONTINUE
0131          NC = 0
0132          DO 211 J=2,NT
```

```
              NC = NC+1
              DO 212 I=1,N
              B(I,NC)= A(I,J)
          212 CONTINUF
          211 CONTINUE
      C
      C   ON REDUIT LA DIAGONALE DES COEFFICIENTS
      C
              DO 213 I=1,N
               DO 213 J=1,N
              IF(I .EQ. J) B(I,J) = B(I,J)-1.
          213 CONTINUE
              WRITE(6,204)
          204 FORMAT(///,45X,'LA MATRICE DES EQUATIONS TRANSFORMEES'//)
              DO 215 I=1,N
               WRITE(6,14)(B(I,J),J=1,NT)
          215 CONTINUE
      C
      C
              DO 216 I=1,N
              R(I) = B(I,NT)
              RRR(I)=A(I,1)
          216  CONTINUE
      C
               CALL GASEN(N,B,R,RR,RRR,NTEST,A,NOM)
      C
      C
      C
          100 CONTINUE
          503  GO TO 200
          500  STOP
              END
```

```
C
      SUBROUTINE GALANT(N,D)
C
C
C      A. GALANT DE L'UNIVERSITE D'OTTAWA, CANADA
C
      DIMENSION D(100),DT(100),MO(100),FR(100),FC(100),PF(100)
      DATA ET/'*'/
      DO 43 I=1,100
   43 PF(I)=ET
C     CALCUL DE LA MOYENNE
      SQ=0
      SS=0.
      DO 25 I=1,N
   25 SS=SS+D(I)
      DM=SS/N
C     CALCUL DE L'ECART-TYPE
      DO 26 I=1,N
   26 SQ=(D(I)-DM)**2+SQ
C     LA VARIANCE
      VAR=SQ/(N-1)
C     L'ECART-TYPE
      ECT=SQRT(VAR)
C     L'ERREUR-TYPE
      TN=N
      ES=ECT/(SQRT(TN))
C     LA MISE EN ORDRE
      I=1
C   ATTENTION ICI
    4 CONTINUE
      IF(D(I)-D(I+1)) 27,27,28
   28  L=I
    5 CONTINUE
      A=D(L)
      D(L)=D(L+1)
      D(L+1)=A
      IF(L .EQ. 1) GO TO 27
      L=L-1
      IF(D(L) .GT. D(L+1)) GO TO 5
   27  I=I+1
      IF(I .LE. (N-1)) GO TO 4
C  LE CALCUL DE LA MEDIANE
      NN=N/2
      N1=NN*2
      IF(N1 .EQ. N) GO TO 7
C  LA MEDIANE SI N EST IMPAIR
      DMEU = D(NN+1)
      GO TO 8
C LA MEDIANE SI N EST PAIR
    7 DMEU=(D(NN) +D(NN+1))/2
C     LE CALCUL DU MODE
    8 L=1
      DT(L)=D(1)
C     ON CHERCHE LES VALEURS SEMBLABLES
      N2=N-1
      DO 29 I=1,N2
      IF(D(I) .EQ. D(I+1)).GO TO 29
      L=L+1
```

```
      DT(L)=D(I+1)
   29 CONTINUE
   68 N3=L
C     LE CALCUL DE LA FREQUENCE
      DO 30 L=1,N3
      MO(L) = 0
      DO 30 I=1,N
      IF(D(I) .NE. DT(L)) GO TO 30
      MO(L)=MO(L)+1
   30 CONTINUE
      K=1
      IT= -9999999999.
      DO 31 L=1,N3
   31 IT= MAX0(MO(L),IT)
      DO 61 L=1,N3
      IF(IT .EQ. MO(L)) GO TO 32
   61 CONTINUE
C     CALCUL DU MODE
   32 DMODE=DT(L)
C     LA KURTOSE
      DK=0
      DO 33 I=1,N
   33 DK=((D(I)-DM)/ECT)**4+DK
C     LA KURTOSE
      DK=(DK/N)-3
C     LE CALCUL DE LA DISSYMETRIE
      DS=0
      DO 34 I=1,N
   34 DS=((D(I)-DM)/ECT)**3+DS
C     LA DISSYMETRIE
      DS=DS/N
C     L'ECART ENTRE LES VALEURS MAXIMUM ET MINIMUM
      EC=D(N)-D(1)
C     LA FREQUENCE RELATIVE ET CUMULATIVE
      F=0
      DO 35 I=1,N3
      TMO=MO(I)
      FR(I) = (TMO/N)*100
      FC(I) = FR(I) + F
   35 F=FC(I)
C     IMPRESSION DES RESULTATS
      WRITE(6,10)
   10 FORMAT(2X,// T18,'VALEUR   FREQUENCE  FREQUENCE  FREQUENCE'/
     1 T27,'ABSOLUE   RELATIVE   CUMULATIVE'/1X, T18,40('-')/)
      DO 62 I=1,N3
   62 WRITE(6,11) DT(I),MO(I),FR(I),FC(I)
   11 FORMAT(T14,F10.2,T28,I3,T38,F6.2,T49,F6.2/)
      TFR=0
      IFA=0
      DO 12 I=1,N3
      IFA=MO(I) + IFA
   12 TFR=FR(I) +TFR
      WRITE(6,13) IFA,TFR,TFR
   13 FORMAT(/T28,'---',T38,6('-'),T49,6('-')/T18,'TOTAL',T28,I3,T38,F6.
     12,T49,F6.2)
      WRITE(6,14) DM,ES,DMED,DMODE,ECT,VAR,DK,DS,EC,D(1),D(N)
   14 FORMAT(//,2X,'STATISTIQUES DESCRIPTIVES'/1X, 2X,23('-')//2X,
     1'MOYENNE',2X,F10.4,T32,'ERREUR STANDARD',1X,F10.4,T68,'MEDIANE',3X
```

```
              A,F10.4//2X,'MODE',T12,F10.4,T32,'ECART-TYPE',T48,F10.4,T68,'VARIAN
              BCE', 2X,F10.4//2X,'KURTOSE',2X,F10.4,T32,'DISSYMETRIE',T48,F10.4,T
              C68, 'ECART', T77,F10.4//2X,'MINIMUM',2X,F10.4,T32,
              D'MAXMINUM',T48,F10.4///)
0048       41  RETURN
0049           END
```

```
C
      SUBROUTINE GASEN(N,B,R,RR,RRR,NTEST,A,NCM)
C
C
      DIMENSION RLR(30)
      DIMENSION DATR(30)
      DIMENSION AA(30,30),BB(30,30)
      DIMENSION IDD(900,10)
      DIMENSION SMINX(30)
      DIMENSION GATRP(30)
      DIMENSION ID(30,5),RL(30)
      DIMENSION SOMB(30)
      DIMENSION NCM(30,5)
      DIMENSION GATR(30)
      DIMENSION RPR(30)
      DIMENSION R(30),B(30,30),BINV(30,30)
      DIMENSION PR(30),GNTEST(30), A(30,30),DSEN(900) ,SENSIB(900)
      DATA S/'SENS'/
C
C      CETTE SUBROUTINE CALCULE LES GAINS DE SYSTEMES ET LES SENSIBILITES
C     DES GAINS AUX VARIATIONS DE CERTAINS ARCS.
C     N : LE NOMBRE D'EQUATIONS DANS LE SYSTEME
C     B : LA MATRICE DES COEFFICIENTS DES INDEPENDANTES
C     LE VECTEUR DES DEPENDANTES
C
C
C
      NT=N+1
      DO 715 IM=1,N
      SOM=0.0
      DO 716 JM=1,N
      BGLOB=ABS(B(IM,JM))
      SOM=SOM+BGLOB
  716 CONTINUE
      SOMB(IM)=SOM
  715 CONTINUE
      XMAX=-999999999.
      DO 105 IM=1,N
      IF(XMAX .LT. SOMB(IM)) XMAX=SOMB(IM)
  105 CONTINUE
      DO 22 I=1,N
      DO 22 J=1,N
   22 BINV(I,J)=B(I,J)
C
      CALL MATINV(B,N,R,1,DET)
C
      PG=R(NTEST)
C   U = MATRICE INVERSE APRES LA SOLUTION DU PROBLEME.
C   N = L'ORDRE DE LA MATRICE
C   C = LE VECTEUR DE SOLUTION.
C   M= 0 SI ON DESIRE L'INVERSION SEULEMENT.
C   SI = 1 POUR LA SOLUTION DU PROBLEME
C   DET = DETERMINANT.
C
C
C   TEST DE L'INVERSION DE LA MATRICE
C
      DO 802 I=1,N
```

```
      DO 802 J=1,N
      AA(I,J)=0.0
      DO 802 L=1,N
      AA(I,J)=AA(I,J) + BINV(I,L)*B(L,J)
  802 CONTINUE
      WRITE(6,805)
  805 FORMAT('0',///,10X,'TEST DE L''INVERSION DE LA MATRICEF A*A-INV =
     1 MATRICE IDENTITE'///)
      DO 803 I=1,N
  803 WRITE(6,804) (AA(I,J),J=1,N)
  804 FORMAT('0',12F10.2)
   29 WRITE(6,25)
   25 FORMAT('1',/////,45X,'LE GAIN DE SYSTEME'/45X,18('*')//,
     1 40X,'LE VECTEUR DE GAINS COMPREND TOUS LES GAINS'/,
     1 40X,'DU SOMMET X=0,(LE NOEUD-SOURCE,L''INDEPENDANTE,)'/
     1 40X, 'A TOUS LES AUTRES SOMMETS DU SYSTEME',/////)
C
C     ECRIRE LA VALEUR DU DETERMINANT
C
      WRITE(6,600) DET
  600 FORMAT('0',10X,'LE DETERMINANT = ',E20.10////)
      WRITE(6,740)
  740 FORMAT(1X,40X,'DEPENDANTE',20X,'INDEPENDANTE',8X,'GAIN',//)
      DO 744 K=1,NT
      DO 745 J=1,5
      ID(K,J)=NOM(K,J)
  745 CONTINUE
  744 CONTINUE
      DO 782 JT=1,N
  782 RL(JT)=R(JT)
C     LA SOMMATION DES GAINS DE SYSTEME
C
      SOMS = 0.0
      DO 1011 IX=1,N
      SOMS = SOMS + RL(IX)
 1011 CONTINUE
      DO 1012 IX=1,N
      RLR(IX)=(RL(IX)/SOMS)*100.0
 1012 CONTINUE
      NI=N-1
      DO 742 I=1,NI
      IP=I+1
      DO 742 J=IP,N
      IF(RL(I) .GE. RL(J)) GO TO 742
      DO 743 L=1,5
      ITEM=ID(I,L)
      ID(I,L)=ID(J,L)
  743 ID(J,L)=ITEM
      TM=RL(I)
      RL(I)=RL(J)
      RL(J)=TM
      SMM=RLR(I)
      RLR(I)=RLR(J)
      RLR(J)=SMM
  742 CONTINUE
      DO 20 I=1,N
      GNTEST(I)=R(I)
      NT=N+1
```

```
      WRITE(6,26)I, ( ID(I,J),J=1,5),( ID(NT,J),J=1,5),RL(I), RLR(I)
   26 FORMAT(35X,80('-'),/35X,'I',78X,'I'/
     A 35X,'I',I6,2X,5A4,' / ',5A4,' : ',F12.4,2X,F6.2,1X,'X',' I')
   20 CONTINUE
      WRITE(6,27)
   27 FORMAT(35X,80('-')/////)
      WRITE(6,28)
   28 FORMAT('0',40X,' STATISTIQUES DESCRIPTIVES  DU GAIN DE SYSTEME'/
     1 42X, 44('-')//)
C
      CALL GALANT(N,R)
C
C     L'INVERSION DE LA MATRICE DES COEFFICIENTS
      DO 809 I=1,N
      DO 809 J=1,N
      BB(I,J)=BINV(I,J)
  809  CONTINUE
C
      CALL MATINV(BINV,N,C,0,DETT)
C
      WRITE(6,805)
      DO 806 I=1,N
      DO 806 J=1,N
      AA(I,J)=0.0
      DO 806 L=1,N
      AA(I,J)=AA(I,J)+BB(I,L)*BINV(L,J)
  806  CONTINUE
      DO 810 I=1,N
  810 WRITE(6,804) (AA(I,J),J=1,N)
C
C     LES GAINS DE TRANSMISSION
C
      WRITE(6,50) (NOM(NTEST,J),J=1,5)
   50   FORMAT(//,'0',42X,'LES GAINS DE TRANSMISSION',/1X,42X,25('*')//
     A1X,42X,'POUR LE NOEUD-TEST : ',5A4/1X,42X,25('*')//
     B40X,'LE GAIN DE TRANSMISSION REPRESENTE POUR CHAQUE NOEUD',/
     C40X,'LA FACON DONT CHAQUE ARC PARTANT DU NOEUD-SOURCE',/
     D40X,'EST AMPLIFIE AU NOEUD-TEST.',/
     E40X,'IL Y A DONC UN GAIN DE TRANSMISSION POUR TOUS LES',/
     F40X,'NOEUDS MEME S''IL N''Y A PAS D''ARC EMANANT DU',/
     G40X,'NOEUD-SOURCE VERS TOUS LES NOEUDS.'//)
C
      WRITE(6,801) DETT
  801 FORMAT('0',10X,'LE DETERMINANT = ',E20.10////)
C        LES GAINS DE TRANSMISSION
C
      DO 46 IM=1,N
      GATR(IM)=-BINV(NTEST,IM)
      DO 770 J=1,5
  770 IDD(IM,J)=NOM(NT,J)
      KL=6
      DO 771 J=1,5
      IDD(IM,KL)=NOM(IM,J)
      KL=KL+1
  771 CONTINUE
   46 CONTINUE
      DO 550 IU=1,N
      FTN=GATR(IU)*RPR(IU)
```

```
      RRA(IU)=RTN*100./RG
  850 CONTINUE
C
C   LA SOMMATION DES GAINS DE TRANSMISSION
      SOMG=0.0
      DO 1002 IK=1,N
      SOMG=SOMG  + GATR(IK)
 1002 CONTINUE
      DO 1003 IK=1,N
      GATRP(IK)=(GATR(IK)/SOMG)*100.0
 1003 CONTINUE
      NI=N-1
      DO 772 I=1,NI
      IP=I+1
      DO 772 J=IP,N
      IF(GATR(I) .GE. GATR(J)) GO TO 772
       DO 773 L=1,5
      ITEM=IDD(I,L)
      IDD(I,L)=IDD(J,L)
  773 IDD(J,L)=ITEM
      DO 774 LL=6,10
      ITEM=IDD(I,LL)
      IDD(I,LL)=IDD(J,LL)
  774 IDD(J,LL)=ITEM
      TW=GATR(I)
      GATR(I)=GATR(J)
      GATR(J)=TW
      TR=RRR(I)
      RRR(I)=RRR(J)
      RRR(J)=TR
      TSOM=GATRP(I)
      GATRP(I)=GATRP(J)
      GATRP(J)=TSOM
  772 CONTINUE
      WRITE(6,1004)
 1004 FORMAT('0',94X,'GT',8X,'%GT',7X,'%GS'/)
      DO 777 IM=1,N
  777 WRITE(6,47) IM,(IDD(IM,J),J=1,5),(IDD(IM,K),K=6,10),
     A GATR(IM),GATRP(IM),RRF(IM)
   47  FORMAT(26X,96('-')/26X,'I',94X,'I'/
     A26X,'I',I6,2X,'SUP L''ARC ',5A4,' -> ',5A4,2X,F8.4,
     B2X,F6.2,1X,'%',2X,F6.2,1X,'%',2X,'I')
        WRITE(6,49)
   49 FORMAT(26X,96('-')/////)
C
      WRITE(6,889)
  889 FORMAT('0',10X,'ANALYSE DES GAINS DE TRANSMISSION'//)
       DO 893 I=1,N
  893 DATR(I)=GATR(I)
C
      CALL GALANT(N,DATR)
C
      WRITE(6,853)
  853 FORMAT('0',/// '0',11X,'MISE EN ORDRE DECROISSANT DE POURCENTAGE',
     *////)
      NI=N-1
      DO 872 I=1,NI
      IP=I+1
```

```
0171            DO 872 J=IP,N
0172             IF(RRR(I) .GE. RRR(J)) GO TO 872
0173            DO 873 L=1,5
0174            ITEM=IDD(I,L)
0175            IDD(I,L)=IDD(J,L)
0176      873   IDD(J,L)=ITEM
0177            DO 874 LL=6,10
0178            ITEM=IDD(I,LL)
0179            IDD(I,LL)=IDD(J,LL)
0180      874   IDD(J,LL)=ITEM
0181            TX=GATR(I)
0182            GATR(I)= GATR(J)
0183            GATR(J)=TX
0184            TR=RRR(I)
0185            RRR(I)=RRR(J)
0186            RRR(J)=TR
0187            TSOM=GATRP(I)
0188            GATRP(I)=GATRP(J)
0189            GATRP(J)=TSOM
0190      872   CONTINUE
0191            WRITE(6,1004)
0192            DO 877 IM=1,N
0193      877   WRITE(6,47) IM,(IDD(IM,J),J=1,5),(IDD(IM,K),K=6,10),
               A GATR(IM),GATRP(IM),RRR(IM)
0194            WRITE(6,49)
0195            WRITE(6,889)
0196      889   FORMAT('0',10X,'ANALYSE  DES POURCENTAGES',//)
            C
0197            CALL  GALANT(N,RRR)
            C
0198            DO 715 IM=1,N
0199            SOM=0.0
0200            DO 719 JM=1,N
0201            BINGO=ABS(BINV(IM,JM))
0202            SOM=SOM+ BINGO
0203      719   CONTINUE
0204            SOMB(IM)=SOM
0205      718   CONTINUE
0206            BINMAX=-999999999.
0207            DO 720 IM=1,N
0208            IF(BINMAX .LT.SOMB(IM))BINMAX=SOMB(IM)
0209      720   CONTINUE
            C
            C      LA SENSIBILITE GLOBALE DU RESEAU
            C
0210            SENGLO=XMAX*BINMAX
0211            WRITE(6,107) SENGLO
0212      107   FORMAT(///,1X,37X,'LA SENSIBILITE GLOBALE DU RESEAU :',F20.4/
               A1X,37X,54('*')///)
0213            WRITE(6,78)(NOM(NTEST,J),J=1,5)
0214       78   FORMAT(//,'0',57X,'LA SENSIBILITE'/1X,57X,14('*')///
               A'0',15X,'LES SENSIBILITES DU GAIN AUX VARIATIONS DES ARCS'/'0',15X
               B,'N.B.: ON MESURE ICI LES SENSIBILITES DU GAIN AU NOEUD-TEST : ',
               C 5A4//)
            C
            C      ON DETERMINE LA SENSIBILITE  DES ARCS EMANANT DU NOEUD-SOURCE
            C
0215            IPI=0
```

```
      DO 75 I=1,N
      IF(RR(I) .EQ. 0.) GO TO 75
      IRI=IRI+1
      DSEN(IRI) = -BINV(NTEST,I)
      SENSIB(IRI)=DSEN(IRI)*RR(I)/GNTEST(NTEST)
      DO 751 J=1,5
  751 IDD(IRI,J)=NOM(NT,J)
      KL=6
      DO 752 J=1,5
      IDD(IRI,KL)=NOM(I,J)
      KL=KL+1
  752 CONTINUE
   77 FORMAT(27X,81('-')/27X,'I',79X,'I'/
     A27X,'I',I6,2X,'L''ARC ',5A4, ' -> ',5A4,F19.6,'   I')
   75 CONTINUE
      IVI=IRI
       SOMT=0.0
      DO 510 IL=1,IVI
  510 SOMT=SOMT + (DSEN(IL)**2)
      SOMC = SOMT/N
C
C     LES SENSIBILITES POUR CE MEME NOEUD-TEST AVEC LES VARIATIONS DES
C       AUTRES ARCS
C
C     ON BALAIE LA MATRICE DES COEFFICIENTS, LA MAT B
      NT=N + 1
      DO 80 K=1,N
      DO 80 L=2,NT
      IF(A(K,L) .EQ. 0.) GO TO 80
      IRI=IRI+1
C     D'ABORD  LA DERIVEE, ENSUITE LA SENSIBILITE
      LXL=L-1
       DSEN(IRI)= -GNTEST(LXL)*BINV(NTEST,K)
      SENSIB(IRI)=DSEN(IRI)*A(K,L)/GNTEST(NTEST)
      DO 753 J=1,5
  753 IDD(IRI,J)=NOM(LXL,J)
      KZ=6
      DO 754 J=1,5
      IDD(IRI,KZ)=NOM(K,J)
      KZ=KZ+1
  754 CONTINUE
   80 CONTINUE
      NI=IRI-1
      DO 755 I=1,NI
      IP=I+1
      DO 755 J=IP,IRI
      IF(SENSIB(I) .GE. SENSIB(J)) GO TO 755
      DO 756 L=1,10
      ITEM=IDD(I,L)
      IDD(I,L) =IDD(J,L)
  756 IDD(J,L) = ITEM
      TW=SENSIB(I)
      SENSIB(I)=SENSIB(J)
      SENSIB(J)=TW
  755 CONTINUE
      DO 759 I=1,IRI
      WRITE(6,77) I,(IDD(I,J),J=1,5),(IDD(I,M),M=6,10),SENSIB(I)
  759  CONTINUE
```

```
0267            WRITE(6,82)
0268         82 FORMAT(27X,81('-'),/////)
           C
           C UNE MESURE DU DEGRE DE SENSIBILITE DU RESEAU  TOUT ENTIER AU NOEUD-TEST
           C    ON FAIT LA SOMMATION DES CARRES DES DERIVEES ET ON PREND
           C      LA RACINE CARREE DE CE NOMBRE,UN GENRE DE SENSIBILITE GLOBALE
0269            SOM=0.0
0270             IB=IVI +1
0271             SOMT=0.0
0272            DO 511  IA=IB,IRI
0273        511 SOMT = (DSEN(IA)**2) + SOMT
0274            SCMD = SOMT/(N**2-N)
0275             ECART = SQRT(SOMC + SCMD)
0276            WRITE(6,84)(NOM(NTEST,J),J=1,5),ECART
0277         84 FORMAT(//,'0',20X,'POUR LE NOEUD-TEST :',23X,5A4/
               A1X,20X,62('*')//
               B1X,20X,'L''INDICE DE  VARIATION  GLOBALE :',7X,F10.4/
               C1X,20X,45('*')//)
           C
           C
0278        500  RETURN
0279            END
```

```
      SUBROUTINE MATINV(A,N,B,M,DET)
      DIMENSION A(30,30)
      DIMENSION     IPIVOT( 30),B( 30,1),INDEX( 30,2),PIVOT( 30)
      COMMON PIVOT,INDEX,IPIVOT
      EQUIVALENCE(IROW,IROW),(ICOLUM),(AMAX,T,SWAP)
C     INITIALIZATION
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C     SEARCH FOR PIVOT ELEMENT.
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF(IPIVOT(J)-1) 60,105,60
   60 DO 100 K=1,N
   70 IF(IPIVOT(K)-1) 80,100,740
   80 IF(ABS(AMAX)-ABS(A(J,K))) 85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C     INTERCHANGE ROWS TO PIVOT ELEMENT ON DIAGONAL
  130 IF(IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170  A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205  IF(M) 260,260,210
  210 DO 250 L=1,M
  220 SWAP=B(IROW,L)
  230  B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260  INDEX(I,1)=IROW
  270  INDEX(I,2)=ICOLUM
  310 PIVOT(I)=A(ICOLUM,ICOLUM)
  320  DETERM=DETERM*PIVOT(I)
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT.
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C     REDUCE NON-PIVOT ELEMENTS
  380  DO 550 L1=1,N
  390  IF(L1-ICOLUM)400,550,400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M)550,550,460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550  CONTINUE
C     INTERCHANGE COLUMS
```

```
600 DO 710 I=1,N
610 L=N+1-I
620 IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
630  JROW=INDEX(L,1)
640 JCOLUM=INDEX(L,2)
650  DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUM)
700 A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
    END
```