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and Code Division Multiple Access Systems

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Parity Bit Selected Spreading Sequences for Spread Spectrum and Code Division Multiple Access Systems

by

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A thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
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for the degree of
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Abstract

Systematic block codes compute $(n-k)$ parity bits for an information block of k bits, and append the parity bits to the end of the information bits to form an n -bit codeword. This redundancy provides certain ability for the receiver to detect or correct a limited number of errors caused by noise and interference on the channel. In direct sequence spread spectrum (DS-SS) communication systems, the information is spread over a large bandwidth by multiplying the information bits with a high rate pseudonoise (PN) sequence. In this thesis, these two techniques are combined to create a new technique, named parity bit selected direct sequence code division multiple access (parity bit selected DS-CDMA). The proposed scheme uses the $(n-k)$ parity bits to select one of $2^{(n-k)}$ mutually orthogonal PN sequences. Then the selected PN sequence is used to spread each bit in the information block of k bits. But the $(n-k)$ parity bits are not appended to the end of the information bits, and are not transmitted to the receiver. The receiver can obtain the parity bits by using $2^{(n-k)}$ matched filters to determine which PN sequence is used for an information block of k bits. Then the errors can be corrected by employing a decoding algorithm. Block codes with different lengths are considered in our simulation in order to find which one is best for the proposed scheme at a given E_b/N_o . The simulation results in a multi-user environment show that the proposed parity bit selected DS-CDMA system can provide better BER performance than both the uncoded DS-CDMA and the block coded DS-CDMA systems.

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LIST OF ACRONYMS

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
BSC	Binary Symmetric Channel
CDMA	Code Division Multiple Access
DS-SS	Direct Sequence Spread Spectrum
FEC	Forward Error Correction
FH-SS	Frequency Hopping Spread Spectrum
GPS	Global Positioning System
IS	Interim Standard
LPI	Low Probability of Intercept
MAI	Multiple Access Interference
MLD	Maximum Likelihood Detection
PCS	Personal Communication Systems
PN	Pseudonoise
QPSK	Quadrature Phase Shift Keying
TIA	Telecommunications Industry Association

LIST OF SYMBOLS

A	Amplitude of signal
b	Information bit in bipolar format
$\hat{b}$	Flip version of information bit b
c	Output vector of an encoder
c_i	i -th codeword of a code
$\tilde{c}_i$	Bipolar format of codeword c_i
$\hat{c}_i$	Evaluation of codeword c_i
$c(t)$	Spreading waveform
d_{ij}	Hamming distance between codewords c_i and c_j
d_{min}	Minimum distance of a block code
e	Error pattern
$\hat{e}_i$	Evaluation of error pattern e
E	Energy of signal
E_b	Energy per information bit
E_c	Energy per bit in a codeword
f_c	Frequency of carrier
$f(t)$	Basic function
$\mathbf{G}$	Generator matrix
$g(t)$	Rectangular signal pulse
$\mathbf{H}$	Parity check matrix

$I_i^{(0)}, I_i^{(1)}$	Interferences from other users to the i -th user
k	Number of the information bits in a code
K	Length of an m -sequence
L	Processing gain
$\mathbf{m}$	Input vector of an encoder
M	Number of the codewords of a block code
M_I	Number of different information sequences carried by a given spreading sequence
$\mathbf{M}_n$	An $n \times n$ Hadamard matrix
$m(t)$	Information signal
n	Length of a code
N	Number of the users in a system
$N^{(j)}$	Output of the matched filter corresponding to the input noise
N_o	Single-sided noise power spectral density
N'_o	Equivalent single-sided noise power spectral density of the noise plus interference
$n(t)$	A sample function of the AWGN process with double-sided power spectral density $N_o/2$
ρ	Cross correlation coefficient of two PN sequences
$\mathbf{P}$	Parity matrix
p_a	Probability of bit error at the input of hard decision decoder
p_b	Information bit error rate

$p_{b,1}$	Bit error probability when the receiver correctly determines the spreading sequence employed
p_e	Probability of selecting the incorrect PN sequence used
p_m	Probability of a codeword error
q_i	i -th group of information words
Δq	Difference between groups q_h and q_1
$\mathbf{r}$	Output vector of a matched filter (or correlator)
r_c	Code rate of a code
r_l	Run length
$r(t)$	Received signal
$\mathbf{R}$	Received codeword
$R(\mathbf{r}, \tilde{\mathbf{c}}_i)$	Cross correlation of $\mathbf{r}$ and $\tilde{\mathbf{c}}_i$
R_b	Information bit rate
R_c	Chip rate
$\mathbf{S}$	Error syndrome
$s(t)$	Transmitted signal
t	Maximum number of errors that the block code can correct
T	Symbol interval
T_b	Bit interval
T_c	Chip interval
U	Decision variable at the output of a matched filter
$\hat{U}$	Flip version of decision variable U
w_i	Hamming weight of a codeword

Chapter 1

1. Introduction

1.1 Background and Motivation

In direct sequence spread spectrum (DS-SS) communication systems, the transmitted information is spread over a large bandwidth by multiplying the information bits with a high rate pseudonoise (PN) sequence (spreading sequence) [1], [4]. Since the signal is spread over the entire bandwidth of the channel, it looks similar to the noise, making the signal difficult to be detected by the unintended receivers. As each user employs different PN sequence to spread its signal, only the receivers who know the right PN sequence can correctly despread the received signal. But with the increase of users in a direct sequence code division multiple access (DS-CDMA) system, the bit error rate (BER) performance of each user degrades due to multiple access interference (MAI). It is well known that employing forward error control (FEC) can be highly effective in increasing the achievable information bit rate, improving the BER performance and system capacity of CDMA systems. The DS-CDMA system using block codes is called block coded DS-CDMA system.

In the block coded DS-CDMA system, an information block (i.e., information word) of k bits is encoded by a linear (n, k) block code to obtain an n -bit codeword, which consists of k information bits and $(n-k)$ parity bits. If the block code is systematic, then the first k bits of the codeword are identical to the information bits and the following $(n-k)$ bits are parity bits [2]. Let d_{min} represents the minimum distance of a block code. In order to correct t random bit errors in a codeword, the minimum distance should be $d_{min} \geq 2t+1$. Block coded DS-CDMA systems employ the error correction ability of block codes to improve the BER performance. We can trade off this improved BER performance for increased spectral efficiency.

In this thesis, a new technique is discussed just based on the two techniques mentioned above. The name of this new technique is “parity bit selected spreading sequences”, which was first proposed in [1], [7]. At the transmitter, we use $(n-k)$ parity bits to select one spreading sequence from a set of $2^{(n-k)}$ mutually orthogonal spreading sequences of length $2^{(n-k)}$. But we do not append them to the end of the information sequence. This selected spreading sequence is employed to spread all of the k bits in the information block. The receiver employs $2^{(n-k)}$ matched filters to despread the received signal. Each filter is designed to match one of the $2^{(n-k)}$ mutually orthogonal spreading sequences. Based on the sum of the squares of the magnitudes of the decision variables over an information block of k bits at the output of each matched filter, the receiver can determine the spreading sequence employed by the transmitter. Each spreading sequence corresponds to a unique block of parity bits, which in turn corresponds to a subset of the

2^k possible information sequences. Assuming the k decision variables at the output of the h -th matched filter have the maximum sum of squares of magnitudes, then these k decision variables and the parity bits corresponding to the h -th matched filter are sent to the decoder to recover the original information bits.

In order to show the performance enhancement achieved by parity bit selected DS-CDMA system, we compare it with other two conventional DS-CDMA systems, i.e., uncoded DS-CDMA system and coded DS-CDMA system. For parity bit selected DS-CDMA system and coded DS-CDMA system, both hard decision decoding and soft decision decoding are considered in this thesis.

1.2 Objectives of this Thesis

In this thesis, we observe the performance of the proposed technique on the additive white Gaussian noise (AWGN) channel. This is shown by performing the following tasks:

- Simulation of parity bit selected DS-CDMA system using block codes of length 10 bits through 16 bits in single user system. We find which block code provides the best BER performance for a given E_b/N_o .
- Investigation of the effect of MAI on the performance of parity bit selected DS-CDMA system, since MAI may have a significant effect on the detection of the transmitted spreading sequence.
- Modeling and simulation of the three DS-CDMA systems in order to see whether parity bit selected DS-CDMA system can provide better BER

performance than the other two conventional DS-CDMA systems. Single user, two-user and five-user asynchronous systems are considered.

- Studying BER performance of parity bit selected DS-CDMA systems, which employs two kinds of block codes respectively. One is for the block codes with the same length of codeword but different number of parity bits. The other is for block codes with the same length of information word but different number of parity bits.

1.3 Thesis Contributions

The following are the main contributions of this thesis:

- Theoretical analysis of BER performance in multi-user parity bit selected DS-CDMA system is provided. Approximations of bit error probabilities for both hard decision decoding and soft decision decoding are given respectively.
- Based on the simulation results of parity bit selected DS-CDMA system using block codes with different length in single user system, we can see that when E_b/N_o is small, there is not one block code that always provides the best BER performance for both soft decision decoding and hard decision decoding.
- Performance evaluation of parity bit selected DS-CDMA system through simulations. The performance was compared with the uncoded DS-CDMA system and coded DS-CDMA system. From the simulation results for the situations considered, we show that parity bit selected DS-CDMA system provides better BER performance than the

other two DS-CDMA systems for both soft decision decoding and hard decision decoding.

- Performance comparison of parity bit selected DS-CDMA systems using block codes with the same length of codeword but different number of parity bits, or with the same length of information word but different number of parity bits. Simulation results show that the system using block codes with 4 parity bits provides better BER performance than that using block codes with 3 parity bits when E_b/N_o is larger than a certain threshold value. The threshold value decreases as the number of users increases. For soft and hard decision decoding, the threshold values are different. As $(n-k)$ decreases, the number of information sequences per spreading code increases, decreasing d_{min} in each coset. Thus, the probability of incorrectly determining PN sequence used by the transmitter decreases linearly, but the probability of incorrectly identifying the information word when the receiver correctly determines the PN sequence employed increases exponentially. As a result, when $(n-k)$ decreases, the probability of bit error will increase.

1.4 Thesis Outline

The rest of this thesis is organized as follows:

Chapter 2 introduces the knowledge of block codes, pseudonoise (PN) sequences, uncoded and coded direct sequence spread spectrum (DS-SS) signal, and conventional DS-CDMA systems.

Chapter 3 presents our proposed parity bit selected DS-CDMA scheme in multi-user system. The transmitter and receiver of the proposed parity bit selected DS-CDMA system are introduced in detail. The probabilities of bit error are discussed for both soft decision decoding and hard decision decoding.

Chapter 4 shows the simulation results and the corresponding discussion. The BER performance of parity bit selected DS-CDMA systems is compared with both the uncoded DS-CDMA system and coded DS-CDMA system. The BER performance of parity bit selected DS-CDMA systems using block codes with different number of parity bits are also simulated and discussed.

Chapter 5 concludes the thesis and discusses the probability of further research.

Chapter 2

2. Block Codes and Conventional DS-CDMA systems

2.1 Block codes

2.1.1 Digital communication channel

Figure 2-1 illustrates the functional diagram and the basic elements of a digital communication system. The information source provides information, in either an analog signal or a digital signal to the system. The source encoder generates a binary signal that gives an efficient representation of the source information. As a result, there is little or no redundancy in the output message of source encoder. Source decoder is used to uniquely reconstruct the message at the receiver side [2], [4].

The sequence of binary digits output from the source encoder is called information sequence, which is sent to the channel encoder. The channel encoder employs error-control coding to introduce some redundancy in the information sequence in order to overcome the effects of noise and interference encountered in the transmission of the signal through the channel. This is achieved by including additional information such that

the channel decoder is able to recover the source information sequence with high probability when the data is corrupted by noise and interference [2], [4].

The binary sequence at the output of the channel encoder is sent to the modulator, which maps the binary information sequence into signal waveforms. Then the signal waveforms are transmitted through the channel. At the receiver side, the demodulator carries out the inverse operation of the modulator to process the channel-corrupted transmitted waveform and produces a stream of bits from the received signal. The destination of the information transmission is the information user, which hopefully accurately receives the original signal from the source. But due to the channel decoding errors and possible distortion introduced by the source encoder and source decoder, the signal at the input of information user is an approximation to the original source output [2], [4].

The communication channel is the physical medium that is used to send the signal from the transmitter to the receiver. It could be wire lines, optical fiber cables, or a wireless channel. One important channel model is the binary symmetric channel (BSC) in which the probabilities of transmitted bits 0 and 1 incurring errors are equal. BSC is the simplest model of a channel with errors, but it captures many of the complexities of the general problem. As shown in Figure 2-2, when an error occurs, a 0 is received as a 1 and vice versa. The probability of a bit incurring an error is p and is called the bit-error probability. Therefore the probability of a bit being received correctly is $1 - p$. Errors

occur randomly in the BSC, so whether a bit incurs an error is independent of other bits [2], [4], [16].

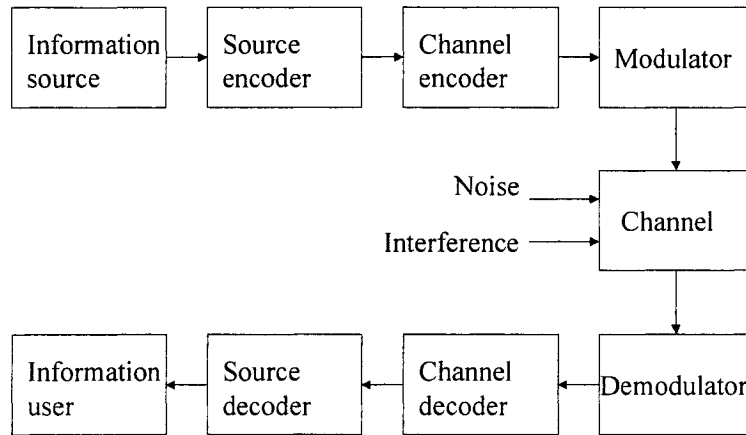


Figure 2-1: Block diagram of a communication system

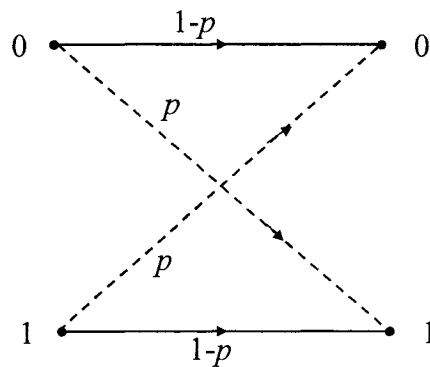


Figure 2-2: Binary symmetric channel

2.1.2 Introduction to block codes

A block code is a set of fixed-length words that has a well-defined structure [2]. Figure 2-3 shows a block diagram of block code encoder. The function of the encoder is that it segments the input bit stream into information words of length k bits first, then

adds $(n-k)$ parity bits (also referred to as parity check bits or check bits) to each information word according to a certain mathematical rule. The output of the encoder is an n -bit codeword.

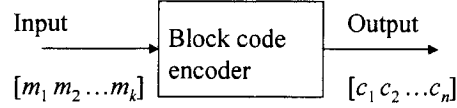


Figure 2-3: Block code encoder

For n bits, there are 2^n possible codewords. From them, we can select 2^k codewords to form a code, which is referred to as an (n, k) block code whose codewords have k information bits and $(n-k)$ parity bits. The ratio k/n denotes the code rate. It is a useful parameter to measure the redundancy of a block code. High code rate means low level of redundancy of the code and high efficiency of the information transmission.

In the error-control theory, other two important parameters are Hamming distance and Hamming weight. Hamming distance is the number of corresponding elements or positions in which they are different. Given two codewords, c_i and c_j , their Hamming distance can be represented by d_{ij} , which satisfies the condition $0 < d_{ij} \leq n$. In an (n, k) block code, any two codewords have a certain distance between them. For a given code, the smallest value of d_{ij} is referred to as the minimum Hamming distance, d_{min} . So we have [9]

$$d_{min} = \min_{c_i, c_j \in (n, k)} \{d_{ij} \text{ for } i, j = 1, 2, \dots, 2^k \text{ and } i \neq j\} \quad (2-1)$$

Suppose c_i is a codeword in an (n, k) block code, its Hamming weight denoted as w_i is the number of nonzero elements in it.

If the information bits are placed together in a codeword, then the codeword is said to be systematic. Otherwise, if the information bits are dispersed within the codeword, then the codeword is referred to as non-systematic [2]. We can also describe a code as either linear or non-linear. Suppose c_i and c_j are two codewords in an (n, k) block code and let a_1 and a_2 be any two elements selected from the alphabet of $\{0, 1\}$. Then the code is said to be linear if and only if $a_1c_i + a_2c_j$ is also a codeword of the same block code. When $a_1 = a_2 = 1$ and $c_i = c_j$, $a_1c_i + a_2c_j$ is the all-zero codeword. This means that the all-zero codeword is always included in a linear code [4]. In this thesis, all the block codes used are linear. For a linear block code, its minimum distance is equal to the minimum weight of the nonzero codewords. In other words, if we suppose c_1 represents the all-zero codeword, the minimum distance of a linear block code is

$$d_{\min} = \min_{c_i \in (n, k)} \{w_i \text{ for } i = 2, 3, \dots, 2^k\} \quad (2-2)$$

2.1.3 Generator matrix and parity check matrix

In Figure 2-3, the relationship between the input vector and output vector of the block code encoder can be expressed as

$$c = mG \quad (2-3)$$

where $\mathbf{m}=[m_1 \ m_2 \ \dots \ m_k]$ is the input vector, $\mathbf{c}=[c_1 \ c_2 \ \dots \ c_n]$ is the output vector, and $\mathbf{G}$ is the generator matrix of the (n, k) linear block code. The generator matrix $\mathbf{G}$ is a $k \times n$ matrix, which has the following form

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & & \vdots \\ g_{k1} & g_{k2} & \cdots & g_{kn} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \vdots \\ \mathbf{g}_k \end{bmatrix} \quad (2-4)$$

where $\mathbf{g}_i = [g_{i1} \ g_{i2} \ \dots \ g_{in}]$, $i=1, 2, \dots, k$. And $\mathbf{g}_i$ and $\mathbf{g}_j$ are linearly independent for $i \neq j$. Since the 2^k codewords of a linear (n, k) block code form a k -dimension subspace of the n -dimension space, the set of $\{\mathbf{g}_i\}$ forms a basis for the (n, k) code [4]. A linear space may have more than one set of basis vectors, so the generator matrix of the code is not unique.

Using row operations and column permutations, any generator matrix of a linear (n, k) code can be changed to the systematic form. In other words, a k by n generator matrix $\mathbf{G}$ can be expressed as [2]

$$\mathbf{G} = [\mathbf{I}_k | \mathbf{P}] \quad (2-5)$$

where $\mathbf{I}_k$ is a k by k identity matrix and $\mathbf{P}$ is a k by $n-k$ parity matrix. Then from (2-3) and (2-5), we can get the output vector (i.e., codeword) of the encoder as follows

$$\mathbf{c} = \mathbf{m} \bullet [\mathbf{I} | \mathbf{P}] = [\mathbf{m} | \mathbf{p}] \quad (2-6)$$

where

$$\mathbf{p} = \mathbf{mP} \quad (2-7)$$

is the parity vector consisting of parity check bits. From (2-6) we can see that the codewords generated from a systematic generator matrix consist of two parts. In other words, the first k bits of each codeword are exactly the same as the k information bits to be coded, and the last $(n-k)$ bits are linear combinations of the information bits. Note that, for simplicity, only the systematic linear block codes are considered in this thesis.

From a k by n generator matrix $\mathbf{G}$, we can construct the parity check matrix as follows

$$\mathbf{H} = [\mathbf{P}^T | \mathbf{I}_{n-k}] \quad (2-8)$$

where $\mathbf{I}_{n-k}$ is $n-k$ by $n-k$ identity matrix, and $\mathbf{P}^T$ is the transpose of parity matrix $\mathbf{P}$. The parity check matrix $\mathbf{H}$ is employed in hard decision decoding to calculate the error syndrome, which will be introduced in section 2.1.5. The parity check matrix of a linear block code has some fixed relationships with the code and its generator matrix. That is to say, from (2-5) and (2-8), we can get [2]

$$\begin{aligned} \mathbf{GH}^T &= [\mathbf{I}_k | \mathbf{P}] \begin{bmatrix} \mathbf{P} \\ \mathbf{I}_{n-k} \end{bmatrix} \\ &= \mathbf{I}_k \mathbf{P} + \mathbf{P} \mathbf{I}_{n-k} \\ &= \mathbf{P} + \mathbf{P} \\ &= \mathbf{0} \end{aligned} \quad (2-9)$$

and from (2-3) and (2-8), we can get

$$\begin{aligned} \mathbf{cH}^T &= (\mathbf{mG})\mathbf{H}^T \\ &= \mathbf{m}(\mathbf{GH}^T) \\ &= \mathbf{m}\mathbf{0} \\ &= \mathbf{0} \end{aligned} \quad (2-10)$$

2.1.4 Soft decision decoding of linear block code

As one of the most efficient modulation methods, binary phase-shift keying (BPSK) has two signal waveforms, which are represented as [4]

$$s_1(t) = g(t)\cos(2\pi f_c t), \quad 0 \leq t \leq T \quad (2-11)$$

$$s_2(t) = -g(t)\cos(2\pi f_c t), \quad 0 \leq t \leq T \quad (2-12)$$

where T is the symbol interval, and $g(t)$ is the rectangular signal pulse shown in Figure 2-

4. $g(t)$ can be expressed as

$$g(t) = \begin{cases} A, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (2-13)$$

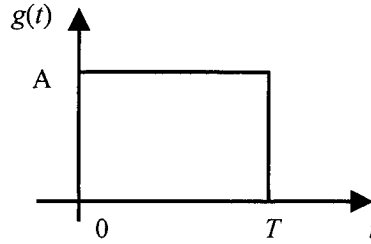


Figure 2-4: Signal pulse shape of $g(t)$

The two signal waveforms, $s_1(t)$ and $s_2(t)$, have the same energy, which is calculated as follows

$$\begin{aligned} E &= \int_0^T s_i^2(t) dt \\ &= \int_0^T A^2 \cos^2(2\pi f_c t) dt \\ &= \frac{1}{2} A^2 T, \quad \text{for } i = 1 \text{ or } 2 \end{aligned} \quad (2-14)$$

From (2-11) and (2-12), we know that these two signals are one-dimensional, and the basis function is

$$\begin{aligned} f(t) &= \frac{s_1(t)}{\sqrt{E}} = \frac{g(t) \cos(2\pi f_c t)}{\sqrt{\frac{1}{2} A^2 T}} \\ &= \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \leq t \leq T \end{aligned} \quad (2-15)$$

We assume the transmitted signal $s_i(t)$ is passed through the AWGN channel, which is shown in Figure 2-5. The received signal $r(t)$ is the sum of $s(t)$ and $n(t)$, that is to say

$$r(t) = s_i(t) + n(t), \quad \text{for } i=1 \text{ or } 2 \quad (2-16)$$

where $n(t)$ is an AWGN process with 0 mean and double sided noise spectral density $N_0/2$.

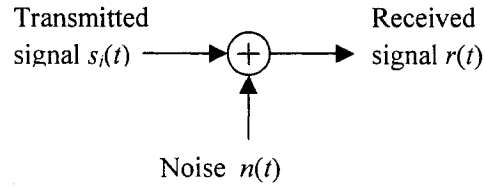


Figure 2-5: AWGN channel model

The demodulation of $r(t)$ can be done by using correlators or matched filters. A block diagram of the correlation-type demodulator for BPSK signal is given in Figure 2-6, where

$$r = s_i + n, \quad \text{for } i = 1 \text{ or } 2 \quad (2-17)$$

and

$$s_i = \int_0^T s_i(t) f(t) dt, \quad \text{for } i = 1 \text{ or } 2 \quad (2-18)$$

$$n = \int_0^T n(t) f(t) dt \quad (2-19)$$

where $f(t)$ is the basis function given by (2-15).

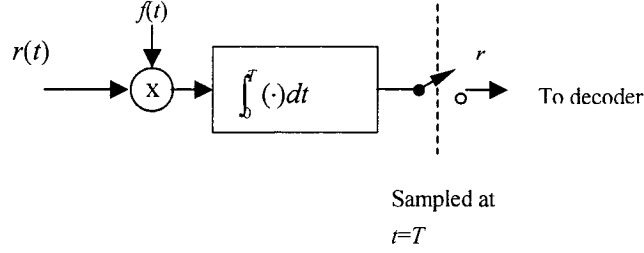


Figure 2-6: Correlation-type demodulator for BPSK signal

The codewords of a block code have the fixed length of n bits. For soft decision decoding, all the n bits of a codeword should be considered together by the decoder to achieve the output, which is an evaluation of the transmitted original codeword. We denote the n sampled outputs of the correlator in Figure 2-6 as $r_j, j= 1, 2, \dots, n$, where these n outputs correspond to a certain codeword. Based on (2-17), r_i can be expressed as follows

$$r_j = \begin{cases} \sqrt{E_c} + n_j, & \text{if the } j\text{-th bit of the codeword is 1} \\ -\sqrt{E_c} + n_j, & \text{if the } j\text{-th bit of the codeword is 0} \end{cases} \quad (2-20)$$

where E_c is the energy per bit in the codeword, and n_j is the sampled noise output of the correlator (or matched filter). n_j is a AWGN variable with 0 mean and variance $N_o/2$ [4].

For a codeword with code rate $r_c=k/n$, the energy per bit in the codeword E_c and the energy per information bit E_b has the following relationship

$$E_c = \frac{k}{n} E_b = r_c E_b \quad (2-21)$$

For soft decision decoding, the cross correlation of $\mathbf{r} = [r_1, r_2, \dots, r_n]$ and each of the 2^k codewords should be calculated. At first, the codeword is transformed from unipolar format (i.e., 0 and 1) into the bipolar format (i.e., -1 and 1) by the following equation

$$\tilde{c}_i = 2c_i - \mathbf{1}, \quad i = 1, 2, \dots, 2^k \quad (2-22)$$

where $\mathbf{1}$ means the $1 \times n$ all-one vector, c_i and $\tilde{c}_i$ are the unipolar and bipolar formats of the i -th codeword respectively. Then the cross correlation of $\mathbf{r}$ and each of the 2^k codewords is given by

$$R(\mathbf{r}, \tilde{c}_i) = \mathbf{r} \tilde{c}_i^T, \quad i = 1, 2, \dots, 2^k \quad (2-23)$$

where T means the transpose operation. Based on (2-23), we can show that the cross correlation corresponding to the actual transmitted codeword will have a mean value $n\sqrt{E_c}$, while the other cross correlations should have smaller mean values than $n\sqrt{E_c}$ [4]. In this case, if we select codeword corresponding to the largest cross correlation as the output of the decoder, we could get the right answer. But this is not always true when there is noise and interference present on the channel. The decoder will output a wrong result when the largest value does not correspond to the actual transmitted codeword.

When a linear block code is employed on a binary-input, symmetric channel such as the AWGN channel with optimum soft decision decoding, the error probability for the transmission of the j -th codeword is the same for all j . Therefore, for simplicity, we can assume without loss of generality that a n -bit all-zero codeword named c_1 is to be transmitted [4] [11]. Both a tight upper bound and a loose upper bound of the probability of a codeword error are given in [4]. These two upper bounds are expressed as follows

$$p_m \leq \sum_{i=2}^M Q\left(\sqrt{\frac{2E_b}{N_o} r_c w_i}\right) \quad (\text{tight bound}) \quad (2-24)$$

$$\leq (M-1)Q\left(\sqrt{\frac{2E_b}{N_o} r_c d_{\min}}\right) \quad (\text{loose bound}) \quad (2-25)$$

where $M=2^k$ is the number of the codewords for a block code, r_c is the code rate, w_i denotes the weight of the i -th codeword, and $d_{\min}$ denotes the minimum distance between the codewords. Since the result of $Q(x)$ function will increase when x decreases, so from (2-2) we know that (2-25) is greater than (2-24).

We can show that for soft decision decoding, the information bit error rate is upper bounded by [1], [3]:

$$p_b < \frac{d_{\min} N_{d_{\min}}}{n} Q\left(\sqrt{\frac{2E_b}{N_o} r_c d_{\min}}\right) \quad (2-26)$$

where $N_{d_{\min}}$ is the number of codewords of weight $d_{\min}$.

2.1.5 Hard decision decoding of linear block code

From the discussion above, we know that the sampled outputs of the cross correlator or matched filter are not quantized. In other words, in Figure 2-6, the output $\mathbf{r}$ is not quantized before it is sent to the decoder. Based on this processing, the soft decision decoding can yield the best performance. However, the soft decision decoding is time-consuming, its computational complexity becomes huge when the value of k is large, where k is the length of information bits in a codeword of the (n, k) block code [4].

In some cases, we can sacrifice some performance to reduce the computational

complexity. This is achieved by performing a threshold decision on the output $\mathbf{r}$ before it is sent to the decoder. The decision result is referred to as $\mathbf{R}$. Then the decoder uses the parity check matrix $\mathbf{H}$ to decode $\mathbf{R}$. This decoding scheme is called hard decision decoding.

Assuming the channel is a binary symmetric AWGN channel, if the coherent demodulation is used, the probability of bit error at the input of decoder is given by [4]

$$p_a = Q\left(\sqrt{\frac{2E_c}{N_o}}\right) = Q\left(\sqrt{\frac{2r_c E_b}{N_o}}\right). \quad (2-27)$$

Suppose the transmitted codeword is $\mathbf{c}$, and it is sent to the receiver through an AWGN channel. Then at the input of the decoder on the receiver side, the received codeword may be expressed as

$$\mathbf{R} = \mathbf{c} + \mathbf{e} \quad (2-28)$$

where $\mathbf{e}$ is a vector, which denotes the error pattern caused by the channel. For binary data transmission and hard decision decoding, the elements in $\mathbf{c}$, $\mathbf{e}$, and $\mathbf{R}$ are all binary digits. The purpose of the decoder is to obtain $\hat{\mathbf{e}}$, the estimate of $\mathbf{e}$, then to obtain $\hat{\mathbf{c}}$, the estimate of $\mathbf{c}$, by the following equation

$$\hat{\mathbf{c}} = \mathbf{R} - \hat{\mathbf{e}} \quad (2-29)$$

If $\hat{\mathbf{c}} = \mathbf{c}$, the decoder has correctly decoded the received codeword. Otherwise, the decoded information has errors.

As we have discussed before, each codeword of an (n, k) block code should satisfy the equation (2-10). So we can use (2-10) to check the received codeword $\mathbf{R}$ as follows

$$\begin{aligned}
\mathbf{RH}^T &= (\mathbf{c} + \mathbf{e})\mathbf{H}^T \\
&= \mathbf{cH}^T + \mathbf{eH}^T \\
&= \mathbf{eH}^T
\end{aligned} \tag{2-30}$$

If $\mathbf{e} = \mathbf{0}$, then $\mathbf{RH}^T = \mathbf{0}$, and $\mathbf{R}$ must be a codeword. If $\mathbf{e} \neq \mathbf{0}$, then $\mathbf{RH}^T \neq \mathbf{0}$ and $\mathbf{R}$ isn't a codeword. Equation (2-30) shows that $\mathbf{RH}^T$ is related to the error pattern, and has no relationship with the codeword being transmitted. We define

$$\mathbf{S} = \mathbf{RH}^T = \mathbf{eH}^T \tag{2-31}$$

as the error syndrome of $\mathbf{R}$. $\mathbf{S}$ is an $1 \times (n-k)$ dimensional vector. If $\mathbf{R}$ contains no errors, i.e. $\mathbf{e} = \mathbf{0}$, then the error syndrome $\mathbf{S}$ is zero. We know that for an (n, k) block code there are 2^n possible error patterns and only 2^{n-k} error syndromes [4]. In other words, each syndrome may correspond to several different error patterns.

There are two ways of decoding linear block codes, one using the standard array and the other using the syndrome table. Table 2-1 shows the standard array for an (n, k) block code [2], [4], [9]. All 2^k codewords are listed in the first row starting with the all-zero codeword c_1 . The rows of the standard array are called cosets; each coset has the same error pattern. From (2-31), we know that all elements in the same coset must have the same error syndrome. We also know that different cosets have the different error syndromes. The elements in the first column, $c_1, e_2, e_3, \dots$, and $e_{2^{n-k}}$ are called the coset leaders. If we suppose the all-zero codeword c_1 represents the all-zero error pattern, then the coset leaders should be the words that have the least weight in the remaining words other than codewords.

For a received codeword R , the decoder checks the standard array to find the position of R in it. Then the decoder output the codeword which locates in the same column as R . This kind of decoding is called minimum distance decoding [4].

Using standard array to decode received codeword R , the decoder need to store 2^n elements of the standard array. The complexity of decoder will exponentially increase as n increases. It is not a practical decoding scheme for the codes where n and k are large.

Table 2-1: Standard array for an (n, k) block code

Codewords →	Coset leader ↓	c_1	c_2	c_3	...	c_{2^k}
		e_2	$c_2 + e_2$	$c_3 + e_2$		$c_{2^k} + e_2$
		e_3	$c_2 + e_3$	$c_3 + e_3$		$c_{2^k} + e_3$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$
		$e_{2^{n-k}}$	$c_2 + e_{2^{n-k}}$	$c_3 + e_{2^{n-k}}$		$c_{2^k} + e_{2^{n-k}}$

From (2-31), we know that the error pattern e decides the value of syndrome S . Although there may be several error patterns corresponding to a given syndrome, we can select the most possible error pattern to correspond to that syndrome. In this way, we can make the probability of decoding errors to be least. The most possible error pattern is the one which has the least weight. In Table 2-1, the coset leaders, $c_1, e_2, e_3, \dots$, and $e_{2^{n-k}}$, are the most possible error patterns. Based on the discussion above, the standard array can be

simplified to a syndrome table, which is shown in Table 2-2. The first row in Table 2-2 consists of the coset leaders, and the second row consists of the corresponding error syndromes obtained by (2-31).

Table 2-2: Error syndromes for an (n, k) block code

Error patterns (Coset leaders)	c_1	e_2	e_3	...	$e_{2^{n-k}}$
Error syndromes	S_1	S_2	S_3	...	$S_{2^{n-k}}$

When the decoder receives word R_i , it multiplies R_i with the parity check matrix H according to (2-31) to get the error syndrome S_i . Then it looks up Table 2-2 to find $\hat{e}_i$, the evaluation of error pattern, corresponding to S_i . Finally, it obtains $\hat{c}_i$, the evaluation of the transmitted codeword, by (2-29). In this way, the decoder only needs to store 2^{n-k} coset leaders and 2^{n-k} error syndromes. The size of memory is much less than the case in which standard array is used. Since this method can achieve the least probability of error for hard decision decoding, and calculating S and finding $\hat{e}$ are easy and fast, so it has been popularly used in many fields.

The upper bound of the probability of codeword error is given by [3], [4]:

$$\begin{aligned}
 P_m &\leq 1 - \sum_{i=0}^t \binom{n}{i} p_a^i (1-p_a)^{n-i} \\
 &= \sum_{i=t+1}^n \binom{n}{i} p_a^i (1-p_a)^{n-i}
 \end{aligned} \tag{2-32}$$

where $t = \left\lfloor \frac{1}{2}(d_{\min} - 1) \right\rfloor$ is referred to as the maximum number of errors that the block code can correct, and p_a is the probability of bit error at the input of decoder. p_a is given by (2-27).

In [13], an equation to approximate the BER was given by

$$p_b \approx \frac{2t+1}{n} \binom{n}{t+1} p_a^{t+1} (1-p_a)^{n-t-1} \quad (2-33)$$

The equation above is approximated by considering only the dominant case, in which there are $(t+1)$ errors occur. This approximation is good for large values of E_b/N_o , but it is inaccurate for the small values of E_b/N_o . A more accurate and tighter upper bound on the probability of bit error was presented in [11]. This bound considers all the uncorrectable errors other than only the most common case, i.e. $(t+1)$ errors. The bound may be expressed as follows

$$\begin{aligned} p_b &\leq \frac{1}{n} \sum_{i=t+1}^{n-t} \binom{n}{i} p_a^i (1-p_a)^{n-i} (i+t) + \frac{1}{n} \sum_{i=n-t+1}^n \binom{n}{i} p_a^i (1-p_a)^{n-i} n \\ &= \frac{1}{n} \sum_{i=t+1}^{n-t} \binom{n}{i} p_a^i (1-p_a)^{n-i} (i+t) + \sum_{i=n-t+1}^n \binom{n}{i} p_a^i (1-p_a)^{n-i} \end{aligned} \quad (2-34)$$

The first term in above equation denotes the probability of bit error when $t < i \leq (n-t)$, where i is the number of errors in a codeword. In the first term, the decoded bit stream is assumed to have $(i+t)$ errors, which is the worst case. The second term in (2-34) denotes the probability of bit error when $(n-t) < i \leq n$, and the decoded bit stream is assumed to have n errors, which is the worst case [11].

2.2 Conventional DS-CDMA systems

2.2.1 Direct sequence spread spectrum (DS-SS)

A spread spectrum system is one in which the transmitted signal is spread over a frequency band whose bandwidth is much greater than the minimum bandwidth required to transmit the information being sent [4], [17], [22]. There are two main types of spread spectrum techniques. One is direct sequence spread spectrum (DS-SS), another is frequency hopping spread spectrum (FH-SS). If the information being sent is spread by multiplying it with a wideband pseudonoise (PN) sequence, then this kind of spread spectrum is called DS-SS. If the carrier frequency rapidly hops from one frequency to another according to the output of a PN sequence generator, and the hopping frequency band is much wider than the bandwidth occupied by the information being sent, then this kind of spread spectrum is denoted as FH-SS. In this thesis, we will focus our discussion only on DS-SS.

PN sequences are also called pseudorandom sequences, which are employed in a spread spectrum system to spread the information data. They can make the transmitted signals appear like random noise and make it difficult for any unintended receiver to detect or demodulate the information being sent. Normally, the spread spectrum technique is employed in the following fields

- Code division multiple access (CDMA) communication systems

In a CDMA system, there may be a number of users who transmit information simultaneously over a common channel at any given time. So each user suffers from interference caused by the other users in the same system. If each user employs a different PN sequence, the receiver can extract the desired signal from the receiver signal using the same PN sequence, even in the presence of interference and noise [4].

- Low probability of intercept (LPI) signal

The information being sent is spread over a wide frequency band, making it appear to be a random noise. In this way, the important information can be hidden in the background noise and is difficult to be detected by unintended receivers who have no knowledge of the PN sequence. This kind of transmitted signal is called LPI signal [4].

- Distance measurement in global positioning system (GPS)

Suppose a transmitter transmits a signal $x(t)$ spread with a PN sequence. The transmitted signal is reflected off a target back to the transmitter. The reflected signal received at the transmitter is denoted as $y(t)$. The autocorrelation of $x(t)$, named $R_{xx}(t)$, is calculated. And the cross correlation between $x(t)$ and $y(t)$, named $R_{xy}(t)$, is also calculated. By measuring the time difference between the two peaks of $R_{xx}(t)$ and $R_{xy}(t)$, we can compute the distance between the transmitter and target. This technique has an important application in the global positioning system, which can determine a person's or an object's latitude, longitude, and altitude on the earth [18].

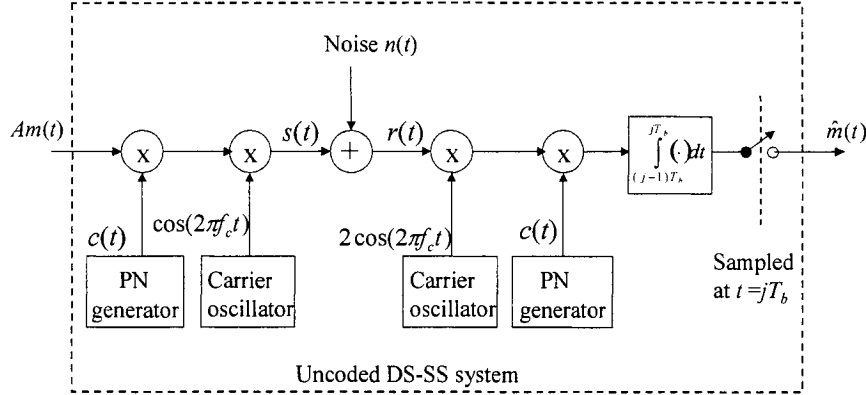


Figure 2-7: Uncoded direct sequence spread spectrum (DS-SS) system

Figure 2-7 illustrates the block diagram of an uncoded DS-SS system [4], [10], [18]. On the transmitter side, the information being sent, $m(t)$, is multiplied by a PN sequence $c(t)$ first, then the multiplication result is sent to the modulator. There are a number of modulation methods can be used for the modulator. In this thesis, for simplicity, the BPSK signal is considered. The transmitted signal $s(t)$ can be expressed as

$$s(t) = Am(t)c(t)\cos(2\pi f_c t) \quad (2-35)$$

where A and f_c are amplitude and frequency of the carrier respectively.

Suppose the rate of PN sequence $c(t)$ is R_c (chips/s). The reciprocal of R_c , denoted by T_c , is called the chip interval, which defines the duration of a chip. If we assume the rate of information $m(t)$ is R_b (bits/s). Then the bit interval $T_b = 1/R_b$. Thus we can define the processing gain (also called bandwidth expansion factor) as [4], [10]

$$L = \frac{R_c}{R_b} = \frac{T_b}{T_c} \quad (2-36)$$

Normally, $R_c \gg R_b$, i.e., $T_c \ll T_b$, thus we have $L \gg 1$. In practical systems, R_c is a multiple of R_b . In other words, the processing gain L is an integer, so that there is integer number of chips for each information bit [4].

Assuming $s(t)$ is sent through an AWGN channel, then the received signal is given by

$$r(t) = s(t) + n(t) \quad (2-37)$$

where $n(t)$ is an AWGN process with zero mean and double sided noise spectral density $N_0/2$. Suppose the intended receiver achieves perfect carrier synchronization and PN sequence synchronization with the transmitter. Then the decision variable at the output of the matched filter on the signalling interval $(j-1)T_b \leq t \leq jT_b$ is

$$\begin{aligned} U^{(j)} &= \int_{(j-1)T_b}^{jT_b} 2r(t)c(t) \cos(2\pi f_c t) dt \\ &= \int_{(j-1)T_b}^{jT_b} 2Am(t) dt + \int_{(j-1)T_b}^{jT_b} 2n(t)c(t) \cos(2\pi f_c t) dt \end{aligned} \quad (2-38)$$

For BPSK, on the time interval $(j-1)T_b \leq t \leq jT_b$, $m(t)$ can be expressed as

$$m(t) = b_j = \begin{cases} 1 & \text{if the informatin bit is 1} \\ -1 & \text{if the informatin bit is 0} \end{cases} \quad (2-39)$$

Then (2-38) becomes

$$U^{(j)} = Ab_j T_b + N^{(j)} \quad (2-40)$$

where $N^{(j)} = \int_{(j-1)T_b}^{jT_b} 2n(t)c(t) \cos(2\pi f_c t) dt$ is the output of the matched filter corresponding to

the input noise, and it is a Gaussian random variable with 0 mean and variance $N_0 T_b$.

In an AWGN channel, the bit error probability of uncoded BPSK DS-SS system is given as follows [4], [21]

$$P_a = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) \quad (2-41)$$

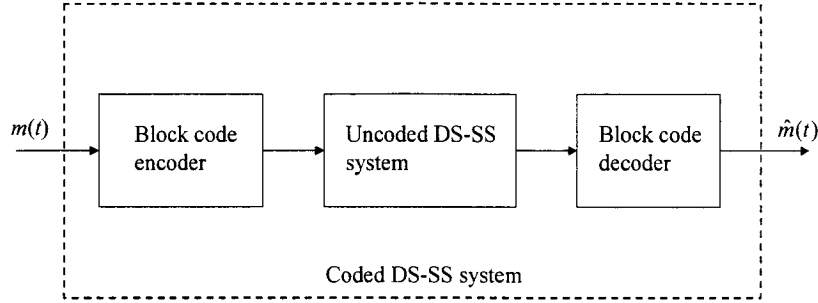


Figure 2-8: Coded direct sequence spread spectrum (DS-SS) system

The block diagram of a block coded DS-SS system is shown in Figure 2-8. The codeword error probabilities of a block coded DS-SS system are identical to those of a linear binary block code in an AWGN channel for both soft and hard decision decoding. In other word, for soft decision decoding, the upper bounds of codeword error probability are given by (2-24) and (2-25), and the upper bound of probability of bit error is given by (2-26) [4], [21]. For hard decision decoding, the upper bound of codeword error probability is given by (2-32), and the upper bound of probability of bit error is given by (2-34) [3], [4].

2.2.2 Pseudonoise (PN) sequences

PN sequences are very important for the spread spectrum systems. Their properties, such as autocorrelation and cross-correlation, may greatly influence the performance of systems. Therefore, the design of a set of PN sequences to meet certain performance requirement is always a significant issue in a spread spectrum system.

Usually, in practical applications, PN sequences consist of binary elements $\{0, 1\}$. Binary PN sequences can be generated using a feedback shift-register whose contents in one time interval is some linear or nonlinear combination of the contents in the previous time interval [19]. Two kinds of PN sequences will be introduced below.

- *m*-sequence

The maximum length linear shift-register sequence, also named *m*-sequence for simplicity, is one of the most widely known binary PN sequences. Figure 2-9 illustrates a general n stages shift-register with linear feedback [4], [10], whose feedback function is expressed as follows

$$a_k = c_1 a_{k-1} + c_2 a_{k-2} + \cdots + c_{n-1} a_{k-n+1} + c_n a_{k-n} \quad (2-42)$$

where ‘+’ means module-2 addition, a_{k-i} ($i = 1, 2, \dots, n$) denotes the state of the i -th memory unit of the register, c_i ($i = 1, 2, \dots, n$) denotes the feedback coefficient of the i -th memory unit, $c_i = 1$ means that the state of the i -th memory unit is used to calculate the feedback value, $c_i = 0$ means that the state of the i -th memory unit is not used. Note that c_0 and c_n must be 1. This is because that if $c_0 = 0$, the shift-register has no feedback, and if $c_n = 0$, the n stages shift-register is reduced to a $(n-1)$ stages shift-register.

An *m*-sequence is periodic with period $2^n - 1$. In each period of the *m*-sequence, there are 2^{n-1} ones and $(2^{n-1} - 1)$ zeros. This is called ‘0’, ‘1’ balanced property.

Run length r_l means the all-ones of length r_l or the all-zeros of length r_l in one *m*-sequence of length $2^n - 1$. Obviously, we have $r_l \leq n$. The relative frequency of run length

r_l (zeros or ones) is $1/2^n$ for all $r_l \leq n-1$, $1/2^{(n-1)}$ for $r_l = n$, with no run lengths possible for $r_l > n$ [20]. Therefore, the distribution of run length of m -sequence is similar to that of a random sequence.

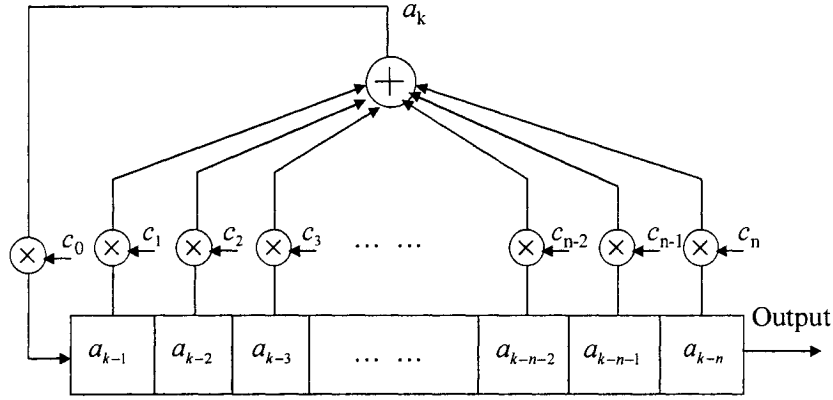


Figure 2-9 General n stages shift register with linear feedback

The auto-correlation function of one m -sequence of length $K=2^n-1$ is given by [10]

$$R(\tau) = \begin{cases} 1 - \left(\frac{K+1}{K} \right) \frac{|\tau|}{T_c} & |\tau| \leq T_c \\ -\frac{1}{K} & |\tau| > T_c \end{cases} \quad (2-43)$$

where T_c denotes the chip interval of the m -sequence. $R(\tau)$ is an even function, i.e.,

$R(\tau) = R(-\tau)$. It has peak value of 1 at $\tau = 0$. It equals to $-1/K$ at $\tau = T_c, 2T_c, \dots, KT_c$.

This shows that m -sequence has good property of auto-correlation.

However, for most m -sequences, the peak values of their cross-correlation function are big, and can't be neglected compared with the peak values of their auto-correlation functions. Furthermore, the number of m -sequences generated from a given n -stages shift

register is limited. It is difficult to find enough m -sequences for one practical CDMA system.

- Hadamard codes

A Hadamard code is generated from a Hadamard matrix, whose rows form a set of orthogonal codes. Hadamard matrices are $n \times n$ matrices (n is an even positive integer) whose entries are +1 or -1, and whose rows and columns are mutually orthogonal. That is to say, an $n \times n$ Hadamard matrix $\mathbf{M}_n$ has the following property

$$\mathbf{M}_n \mathbf{M}_n^T = n \mathbf{I}_n \quad (2-44)$$

where $\mathbf{M}_n^T$ is the transpose of $\mathbf{M}_n$, and $\mathbf{I}_n$ is the n by n identity matrix.

In a Hadamard matrix, there is one row that contains all ones. And each of the other rows consists of $n/2$ ones and $n/2$ minus ones. Any two rows in a Hadamard matrix are different in exactly $n/2$ positions. So are two columns. Therefore, the minimum distance of the code is $d=n/2$. We define $\mathbf{M}_1=[1]$, and suppose $\mathbf{M}_n$ is an n by n Hadamard matrix, where n is a nonnegative power of 2. Then, the $2n$ by $2n$ Hadamard matrix, denoted as $\mathbf{M}_{2n}$, can be generated by the following equation [4], [32]

$$\mathbf{M}_{2n} = \begin{bmatrix} \mathbf{M}_n & \mathbf{M}_n \\ \mathbf{M}_n & -\mathbf{M}_n \end{bmatrix} \quad (2-45)$$

Above we introduced a method to generate the Hadamard codes whose length n is a nonnegative power of 2. Besides these codes, generating Hadamard codes of other lengths is also possible. But in this case, the Hadamard codes generated are nonlinear.

2.2.3 Direct sequence code division multiple access (DS-CDMA)

Since DS-SS signals can enhance the performance of a system due to the processing gain, it is possible to allow a number of users to transmit DS-SS signals simultaneously over a wide frequency band, which is much wider than the minimum bandwidth required to transmit the information. In this kind of digital communication system, each user is assigned a distinct PN sequence, which is used to spread the information being sent. The transmitted signal can only be recovered by the intended receiver, who has exactly the same PN sequence as the transmitter, but not by any other receivers who have no prior knowledge of the PN sequence [4]. This communication system is called direct sequence code division multiple access (DS-CDMA) system. If error control codes are employed, then these kinds of systems are coded DS-CDMA systems. Otherwise, they are called uncoded DS-CDMA systems.

For simplicity, we assume that all users transmit signal with the same data rate, and all signals reach the receivers at identical average power. We also assume that PN sequences are mutually orthogonal and the interference from other users adds on a power basis only. In one DS-CDMA system having N simultaneous users, if the interference from other users can be treated as a random process, the power ratio of signal to interference at the input of a given receiver is expressed by [4]

$$\frac{P_{av}}{J_{av}} = \frac{P_{av}}{(N-1)P_{av}} = \frac{1}{N-1} \quad (2-46)$$

where P_{av} denotes the power of received signal, J_{av} denotes the power of interference from other users, N is the number of simultaneous users in the system.

Suppose the variance of noise, σ_n^2 , is much smaller than the variance of MAI, σ_I^2 , i.e., $\sigma_n^2 \ll \sigma_I^2$. Thus, the upper bound of codeword error probability for soft decision decoding at the given receiver is expressed by [4]

$$p_m \leq \sum_{i=2}^M Q\left(\sqrt{\frac{2R_c/R}{N-1}} r_c w_i\right) \leq (M-1)Q\left(\sqrt{\frac{2R_c/R}{N-1}} r_c d_{\min}\right) \quad (2-47)$$

In the last decades, different methods have been used to calculate the probability of bit error for DS-CDMA systems. The method of the Gaussian approximation for the MAI was proposed and the average BER performance rather than worst-case performance was analyzed in [26]. An improved Gaussian approximation to the probability of data bit error is performed by calculating the distribution of the MAI variance over all possible delay and phase values, taking a Gaussian approximation to the probability of data bit error over the support of this distribution, and averaging the results [25]. The moment spaces method was employed in [27] and [28], and the characteristic function method was employed in [29] and [30]. Although these two methods can achieve accurate results, the computation is complex. The BER performance for asynchronous DS-CDMA over multipath fading channels was discussed in [23], [25] and [31]. Using these models, [23] developed the probability of bit error by modeling the multiple access interference (MAI) as a Gaussian process, as well as an improved Gaussian process.

- Applications of DS-CDMA technology

North American DS-CDMA digital cellular system (IS-95) is one practical example that adopts DS-CDMA as multiple-access method [4], [19]. In June 1989, Qualcomm Inc. proposed the use of DS-CDMA technology as a means of overcoming the expected capacity limits of analog cellular systems. After several years of development, the ideal of a DS-CDMA digital cellular system became an actual implementation, which was standardized and adopted as an Interim Standard (IS-95) by the Telecommunications Industry Association (TIA) for digital cellular communications at 800 MHz and for personal communication systems (PCS) in the frequency band of 1850 to 1970 MHz [19].

In the IS-95 system, for the forward link (base-to-mobile), the data from speech coder is encoded by a rate $1/2$, constraint length 9 convolutional code. Each user employs Walsh sequence of length 64 to multiply the data sequence. Then the resulting binary sequence is spread by multiplication with two PN sequences of length 2^{15} to create Quadrature Phase Shift Keying (QPSK) signal. A RAKE demodulator and Viterbi soft-decision decoder are employed at the receiver. For the reverse link (mobile-to-base), a constraint length 9, rate $1/3$ convolutional code is used. The data is modulated using a set of 64-ary orthogonal Walsh sequences of length 64. The resulting binary sequence is then further multiplied by two PN sequences of length 2^{15} , whose rate is 1.2288Mchips/s. At the receiver, a fast Hadamard transform and Viterbi detector are used to recover the transmitted data [4].

Chapter 3

3 Parity Bit Selected DS-CDMA System

3.1 Introduction

The technique of parity bit selected spreading sequences was first proposed in [1] and [7]. It is based on two common techniques – block coding and direct sequence spread spectrum. At the transmitter side, instead of appending the $(n-k)$ parity bits at the end of the information sequence, we use them to select a spreading sequence from a set of $2^{(n-k)}$ mutually orthogonal spreading sequences. Then this selected spreading sequence is used to spread all k bits of the information block. At the receiver, the received signal is sent to $2^{(n-k)}$ matched filters to be despread. Here, each filter is matched to one of the $2^{(n-k)}$ orthogonal spreading sequences. According to the sum of the squares of the magnitudes of the decision variables over a k -bit information block at the output of each matched filter, we can determine which spreading sequence is employed to spread the k -bit information block. Then block code decoding is employed to recover the transmitted information bits. In this thesis, both hard decision decoding and soft decision decoding are used to decode the information block. For soft decision decoding, the maximum likelihood detection (MLD) is employed [34]. For hard decision decoding, the syndrome decoding method is used [4].

3.2 Transmitter for Parity Bit Selected DS-CDMA System

In Figure 3-1, a block diagram of the transmitter of our proposed parity bit selected DS-CDMA system is given [1]. The input bit stream is segmented into information blocks. Each block has k bits and is sent to the BPSK modulator and parity bit calculator, respectively.

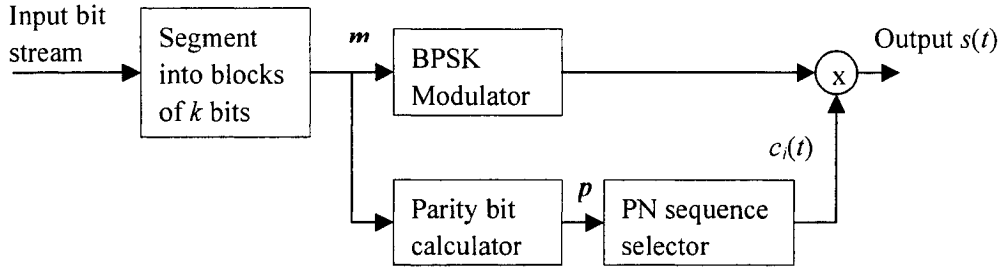


Figure 3-1: Transmitter of parity bit selected DS-CDMA system

Suppose vector $\mathbf{m}=[m_1 \ m_2 \ \dots \ m_k]$ represents the information block of k bits and vector $\mathbf{b}=[b_1 \ b_2 \ \dots \ b_k]$ represents the bipolar format of vector $\mathbf{m}$, then we have

$$b_i = 2m_i - 1 \quad (3-1)$$

where $m_i = 0$ or 1 , $b_i = -1$ or $+1$, and $i = 1, 2, \dots, k$. Parity bit calculator is used to calculate the parity vector $\mathbf{p}$ by multiplying $\mathbf{m}$ and the parity matrix $\mathbf{P}$, i.e.,

$$\mathbf{p} = \mathbf{mP} \quad (3-2)$$

where $\mathbf{P}$ is the k by $n-k$ parity matrix that is responsible for the way in which the parity-check bits are constructed from the information bits [2]. For a block code, its generator matrix is in the form of [2], [3], [9],[15]

$$\mathbf{G} = [\mathbf{I}_k | \mathbf{P}] \quad (3-3)$$

where $\mathbf{I}_k$ is a k by k identity matrix. Here, $\mathbf{G}$ is said to be in a systematic form as it generates systematic codewords.

In this thesis, we consider several block codes whose lengths are 10 bits through 16 bits including 4 or 3 parity bits for each block code. The 4 parity bits case will be discussed first. Then the 3 parity bits case will be discussed later.

The parity matrices for these block codes with 4 parity bits are shown as follows

$$\begin{aligned}
 \mathbf{P}_{10,6} &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{P}_{11,7} &= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{P}_{12,8} &= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{P}_{13,9} &= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 \mathbf{P}_{14,10} &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{P}_{15,11} &= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{P}_{16,12} &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \quad (3-4)$$

In the case of 4 parity bits, there are $2^4 = 16$ parity vectors. Each parity vector corresponds to $2^k / 2^4 = 2^{(k-4)}$ information words of the block code, where k is the length of information words. For example, for (12, 8) block code $k = 8$ and there are 16 information words of length 8 correspond to each parity vector.

Table 3-1: Information words group q_1 and q_2 for (12, 8) block code

Information words group q_1 corresponding to p_1	Information words group q_2 corresponding to p_2	$q_2 - q_1$
00000000	00000111	00000111
00011011	00011100	
00101110	00101001	
00110101	00110010	
01001101	01001010	
01010110	01010001	
01100011	01100100	
01111000	01111111	
10000111	10000000	
10011100	10011011	
10101001	10101110	
10110010	10110101	
11001010	11001101	
11010001	11010110	
11100100	11100011	
11111111	11111000	

Table 3-1 gives the information words group q_1 associated with $p_1 = 0000$ and information words group q_2 associated with $p_2 = 0001$. From Table 3-1 we can find that $q_2 - q_1 = 00000111$ is a constant vector. This means the group q_2 of information words that produce p_2 can be calculated by adding the group q_1 of information words that produce p_1 and the constant vector 00000111. In the same way, we can get all the

differences between information words group q_i and q_1 , which are shown in Table 3-2, where $i = 2, 3, \dots, 16$ for (12, 8) block code. Another example of (10, 6) block code is given in [1]. The results show that each parity vector is associated with a set of information words that is linearly related to the set of information words that produces p_1 .

Table 3-2 : Linear relationship between q_1 and q_i for (12, 8) block code

$q_2 - q_1$	0	0	0	0	0	1	1	1
$q_3 - q_1$	0	0	0	0	1	1	0	1
$q_4 - q_1$	0	0	0	0	1	0	1	0
$q_5 - q_1$	0	0	0	0	1	1	1	0
$q_6 - q_1$	0	0	0	0	1	0	0	1
$q_7 - q_1$	0	0	0	0	0	0	1	1
$q_8 - q_1$	0	0	0	0	0	1	0	0
$q_9 - q_1$	0	0	0	0	1	1	0	0
$q_{10} - q_1$	0	0	0	0	1	0	1	1
$q_{11} - q_1$	0	0	0	0	0	0	0	1
$q_{12} - q_1$	0	0	0	0	0	1	1	0
$q_{13} - q_1$	0	0	0	0	0	0	1	0
$q_{14} - q_1$	0	0	0	0	0	1	0	1
$q_{15} - q_1$	0	0	0	0	1	1	1	1
$q_{16} - q_1$	0	0	0	0	1	0	0	0

By observing the information words group q_i , we can see that q_1 is a 4 dimensional closed set, which can be used as the codewords of the (8, 4) block code. However, $q_2, q_3, \dots, q_{16}$ are not closed sets. The generator matrix of this (8, 4) block code can be found just by selecting four special codewords from q_1 , that is to say

$$\mathbf{G}_{8,4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} = [\mathbf{I}_4 \mid \mathbf{P}_{8,4}] \quad (3-5)$$

where

$$\mathbf{P}_{8,4} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad (3-6)$$

is the parity matrix of the new created (8, 4) block code, and $\mathbf{I}_4$ is the 4 by 4 identity matrix. This (8, 4) code is obtained from (12, 8) code, so it may be called shortened block code of (12, 8) code.

Above we discussed some block codes with 4 parity bits. Now we try to find what the situation will be for the block codes with 3 parity bits. In this case, (10, 7) block code is used as an example. The parity matrix used for (10, 7) block code is shown below:

$$\mathbf{P}_{10,7} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad (3-7)$$

In the same way as the 4 parity bits case, the information words group q_1 and q_2 , and linear relationship between q_i and q_l are shown in Table 3-3 and Table 3-4 respectively. And the generator matrix of the new created (7, 4) block code can be achieved and shown below:

$$\mathbf{G}_{7,4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = [\mathbf{I}_4 \mid \mathbf{P}_{7,4}] \quad (3-8)$$

where

$$\mathbf{P}_{7,4} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (3-9)$$

is the parity matrix of the new created (7, 4) block code.

According to the discussion above, we can draw some conclusions as follows:

- q_i is a coset of q_1 , i.e., the difference between the members of q_i and members of q_1 is a constant vector for a given i .
- $q_i - q_1 \neq q_j - q_1$, if $i \neq j$.
- q_1 is a closed set, which can be used as the complete codewords of a shortened block code, i.e., an $(k, 2k - n)$ block code.

Table 3-3: Information words group q_1 and q_2 for (10, 7) block code

Information words group q_1 corresponding to p_1	Information words group q_2 corresponding to p_2	$q_2 - q_1$
0000000	0000100	0000100
0001111	0001011	
0010101	0010001	
0011010	0011110	
0100011	0100111	
0101100	0101000	
0110110	0110010	
0111001	0111101	
1000110	1000010	
1001001	1001101	
1010011	1010111	
1011100	1011000	
1100101	1100001	
1101010	1101110	
1110000	1110100	
1111111	1111011	

Table 3-4: Linear relationship between q_1 and q_i for (10, 7) block code

$q_2 - q_1$	0	0	0	0	1	0	0
$q_3 - q_1$	0	0	0	0	1	1	1
$q_4 - q_1$	0	0	0	0	0	1	1
$q_5 - q_1$	0	0	0	0	0	1	0
$q_6 - q_1$	0	0	0	0	1	1	0
$q_7 - q_1$	0	0	0	0	1	0	1
$q_8 - q_1$	0	0	0	0	0	0	1

For a given k -bit input information data named $\mathbf{m}$, parity bit calculator calculates its parity bits that can form a parity vector $\mathbf{p}_i$, where i is the decimal value of the binary sequence $\mathbf{p}$ plus 1. That is to say, $\mathbf{p}_1 = [0\ 0\ 0\ 0]$, $\mathbf{p}_2 = [0\ 0\ 0\ 1]$, and $\mathbf{p}_{16} = [1\ 1\ 1\ 1]$ for the (12, 8) block code case, while $\mathbf{p}_1 = [0\ 0\ 0]$, $\mathbf{p}_2 = [0\ 0\ 1]$, and $\mathbf{p}_8 = [1\ 1\ 1]$ for the (10, 7) block code case [7]. Then the parity vector $\mathbf{p}_i$ is sent to the PN sequence selector, which selects a PN sequence from a set of mutually orthogonal PN sequences. The selected PN sequence named $c_i(t)$ corresponds to the input $\mathbf{p}_i$. For example, $c_1(t)$ will be selected when input is $\mathbf{p}_1$. Orthogonal PN sequences have the following property on the signalling interval $(j-1)T_b \leq jT_b$:

$$\frac{1}{T_b} \int_{(j-1)T_b}^{jT_b} c_{i_1}(t) c_{i_2}(t) dt = 0 \quad \text{if } i_1 \neq i_2 \quad (3-10)$$

where T_b is the time interval. It is also called bit interval for BPSK modulation.

In this thesis, we will use Hadamard sequences to be the orthogonal PN sequences. The number of bits in the parity vector $\mathbf{p}_i$ is $(n-k)$. So there are $2^{(n-k)}$ parity vectors. This means that the number of orthogonal PN sequences is $2^{(n-k)}$. In order to get $2^{(n-k)}$ PN sequences, the length of the Hadamard sequences must be greater or equal to $2^{(n-k)}$ [7].

Then the selected PN sequence is used as the spreading sequence to multiply the uncoded BPSK signal to get the transmitted signal $s(t)$. The k bits in the same information block are spread by the same PN sequence. Note that the spreading sequence is selected every information block of k bits and determined by the parity vector calculated by the parity bit calculator. Note that the PN sequence is fixed for each user in the conventional DS-CDMA systems. On the time interval $(j-1)T_b \leq t \leq jT_b$, the i -th user's transmitted signal $s_i(t)$ is given by [1] [5] [23] [24]:

$$s_i(t) = A_i b_i^{(j)} c_i(t) \cos(2\pi f_c t + \phi_i) \quad (3-11)$$

where $j=1, 2, \dots, k$, $b_i^{(j)}$ denotes the i -th user's bit on the time interval $(j-1)T_b \leq t \leq jT_b$ and given by (3-1), f_c is the carrier frequency, A_i and ϕ_i are the amplitude and phase of the i -th carrier respectively, and $c_i(t)$ is the PN sequence selected by the parity bits. As we have discussed before, the PN sequence $c_i(t)$ is used to spread all k bits of the information block.

3.3 Receiver for Parity Bit Selected DS-CDMA System

In this thesis, we focus our research on asynchronous DS-CDMA systems. For simplicity, all users are assumed to transmit signal with the same data rate, and all signals are received at the same power. The message signal for user 1 on the time interval $(j-1)T_b \leq t \leq jT_b$ is $s_1(t)$ given by (3-11) in the previous section. For the other users, they transmit similar signals to that of user 1 in the same bandwidth. On the time interval

$(j-1)T_b \leq t \leq jT_b$, for the asynchronous DS-CDMA system, the received signal named $r(t)$ can be expressed as follows [5] [24]

$$\begin{aligned} r(t) &= \sum_{i=1}^N s_i(t - \tau_i) + n(t) \\ &= \sum_{i=1}^N A_i b_i^{(j)} c_i(t - \tau_i) \cos[2\pi f_c(t - \tau_i) + \phi_i] + n(t) \end{aligned} \quad (3-12)$$

where j is referred to as the time interval index, N is the number of users in the system, τ_i is the i -th user's transmission delay, and $n(t)$ is additive white Gaussian noise with 0 mean and double sided noise spectral density $N_0/2$. A block diagram of the proposed receiver for our parity bit selected DS-CDMA system is given in Figure 3-2, where $U_{i,h}^{(j)}$ is the decision variable for the i -th user at the output of the h -th matched filter ($h=1, 2, \dots, 2^{(n-k)}$) at the sample time jT_b , and $U_{i,h_{\max}}$ consists of k decision variables whose sum of squares of the magnitudes is the maximum. The comparator in Figure 3-2 is used to sum the squares of the magnitudes of decision variables over an information block of k bits and find the maximum.

Since we are concerned with relative time delays and phases, without loss of generality, we can assume that $\tau_1 = 0, \phi_1 = 0$ and $kT_b > \tau_i > \tau_1, 2\pi > \phi_i > \phi_1$ for any $i > 1$ [23], [24], [26]. The decision variable for the first user at the output of the h -th matched filter on the signaling interval $0 \leq t \leq T_b$ is given below [1], [5], [24]

$$U_{1,h}^{(1)} = \int_0^{T_b} 2r(t)c_{1,h}(t) \cos(2\pi f_c t) dt \quad (3-13)$$

$$= \begin{cases} b_1^{(1)} A_1 T_b + \sum_{i=2}^N b_i^{(0)} A_i T_b \rho_{1,h,i}^{(0)} \cos \theta_{1,i} + \sum_{i=2}^N b_i^{(1)} A_i T_b \rho_{1,h,i}^{(1)} \cos \theta_{1,i} + N_{1,h}^{(1)} & \text{if } c_{1,h}(t) = c_1^{(1)}(t) \\ \sum_{i=2}^N b_i^{(0)} A_i T_b \rho_{1,h,i}^{(0)} \cos \theta_{1,i} + \sum_{i=2}^N b_i^{(1)} A_i T_b \rho_{1,h,i}^{(1)} \cos \theta_{1,i} + N_{1,h}^{(1)} & \text{if } c_{1,h}(t) \neq c_1^{(1)}(t) \end{cases}$$

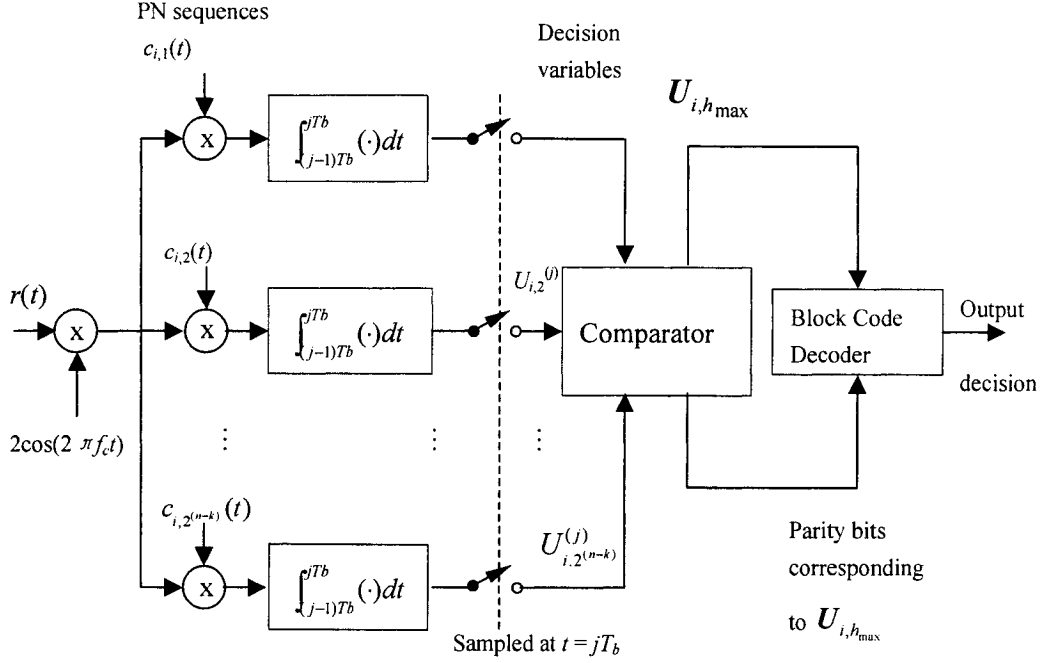


Figure 3-2: Receiver of parity bit selected DS-CDMA system

In (3-13), $c_{1,h}(t)$ is the h -th PN sequence for user 1, $c_1^{(1)}(t)$ is the actual PN sequence used by user 1 on interval $0 \leq t \leq T_b$, $b_i^{(0)}$ is the i -th user's bit on the interval $\tau_i - T_b \leq t \leq \tau_i$, $b_i^{(1)}$ is i -th user's bit on the interval $\tau_i \leq t \leq \tau_i + T_b$, $\theta_{1,i}$ is the phase difference between the first and i -th users' carriers. $N_{1,h}^{(1)}$ is the output of the first user's h -th matched filter corresponding to the input noise. $\theta_{1,i}$ is uniformly distributed on $[0, 2\pi]$, so $E[\cos \theta_{1,i}] = 0$ and $E[\cos^2 \theta_{1,i}] = 1/2$ for asynchronous transmission system. $N_{1,h}^{(1)}$ is a Gaussian random variable with 0 mean and variance $N_o T_b$, where N_o is the

single-sided noise spectral density. $\rho_{1,h,i}^{(0)}$ is the cross-correlation coefficient of $c_{1,h}(t)$ and $c_i^{(0)}(t)$, and $\rho_{1,h,i}^{(1)}$ is the cross-correlation coefficient of $c_{1,h}(t)$ and $c_i^{(1)}(t)$, where $i=2, 3, \dots, N$, $c_i^{(0)}(t)$ and $c_i^{(1)}(t)$ are the PN sequences of i -th user on interval $\tau_i - T_b \leq t \leq \tau_i$ and interval $\tau_i \leq t \leq \tau_i + T_b$ respectively. $\rho_{1,h,i}^{(0)}$ and $\rho_{1,h,i}^{(1)}$ can be calculated by using the following equations [5]

$$\rho_{1,h,i}^{(0)} = \frac{1}{T_b} \int_0^{\tau_i} c_{1,h}(t) c_i^{(0)}(t - \tau_i) dt \quad (3-14)$$

$$\rho_{1,h,i}^{(1)} = \frac{1}{T_b} \int_{\tau_i}^{\tau_i + T_b} c_{1,h}(t) c_i^{(1)}(t - \tau_i) dt \quad (3-15)$$

In (3-13), when we calculate the decision variable $U_{1,h}^{(1)}$, the double frequency component of $r(t) \cos(2\pi f_c t)$ is removed by the integration since $f_c \gg T_b^{-1}$, which is always satisfied in a practical DS-CDMA communication system.

Note that in our proposed parity bit selected DS-CDMA system, the PN sequence used as the spreading sequence may change every k information bits depending on the parity bits. So if interval $\tau_i - T_b \leq t \leq \tau_i$ and interval $\tau_i \leq t \leq \tau_i + T_b$ belong to the same information block of k bits, then $c_i^{(0)}(t)$ and $c_i^{(1)}(t)$ are the same. Otherwise, they are different.

For asynchronous transmission, the expectation of delay τ_i is $T_b/2$, i.e., $E[\tau_i] = T_b/2$.

Based on this, it is easy to show that [5]

$$E[\rho_{1,h,i}^{(0)}] = E[\rho_{1,h,i}^{(1)}] = 0 \quad (3-16)$$

$$E[(\rho_{1,h,i}^{(0)})^2] = \frac{2}{3} E[\tau_i] T_c / T_b^2 = \frac{T_c}{3T_b} = \frac{R_b}{3R_c} \quad (3-17)$$

$$E[(\rho_{1,h,i}^{(1)})^2] = \frac{2}{3} E[T_b - \tau_i] T_c / T_b^2 = \frac{T_c}{3T_b} = \frac{R_b}{3R_c} \quad (3-18)$$

where T_b is the bit interval that is the inverse of the bit rate R_b , T_c is the chip interval that is the inverse of the chip rate R_c .

Using the same method as [5], we can rewrite (3-13) as follows

$$U_{1,h}^{(1)} = \begin{cases} b_1^{(1)} A_1 T_b + I_1^{(0)} + I_1^{(1)} + N_{1,h}^{(1)} & \text{if } c_{1,h}(t) = c_1^{(1)}(t) \\ I_1^{(0)} + I_1^{(1)} + N_{1,h}^{(1)} & \text{if } c_{1,h}(t) \neq c_1^{(1)}(t) \end{cases} \quad (3-19)$$

and

$$\begin{cases} E[I_1^{(0)}] = E[I_1^{(1)}] = 0 \\ E[(I_1^{(0)})^2] = E[(I_1^{(1)})^2] = \sum_{i=2}^N \left(\frac{A_i T_b}{2} \right)^2 \frac{2R_b}{3R_c} \end{cases} \quad (3-20)$$

where $I_1^{(0)}$ and $I_1^{(1)}$ are referred to as the interferences from other users to user 1 on interval $\tau_i - T_b \leq t \leq \tau_i$ and interval $\tau_i \leq t \leq \tau_i + T_b$, respectively.

According to the Central Limit Theorem in [4], the distribution of a sum of a large number of random variables tends towards a Gaussian distribution. We can use a Gaussian approximation for the multiple access interference components. In other words, if we assume $N_{1,h}^{(1)}$ to be the sum of interference $I_1^{(0)}$ and $I_1^{(1)}$ plus noise $N_{1,h}^{(1)}$, then we can regard $N_{1,h}^{(1)}$ as a single Gaussian variable whose mean is 0 and variance is [5]

$$\begin{aligned} \sigma_{N+I}^2 &= N_o T_b + E[(I_1^{(0)})^2] + E[(I_1^{(1)})^2] \\ &= N_o T_b + 2 \sum_{i=2}^N \left(\frac{A_i T_b}{2} \right)^2 \frac{2R_b}{3R_c} \\ &= N_o T_b \left[1 + \sum_{i=2}^N \frac{(A_i T_b)^2}{2N_o} \frac{2R_b}{3R_c} \right] \end{aligned}$$

$$= N_o T_b \left[1 + \sum_{i=2}^N \frac{E_{b,i}}{N_o} \frac{2R_b}{3R_c} \right] \quad (3-21)$$

where $E_{b,i} = A_i^2 T_b / 2$ is the i -th user's energy-per-bit. Therefore, $E_{b,i} / N_o$ is the ratio of energy-per-bit to noise spectral density for the i -th user. For ease of analysis and to simplify the simulations that are presented later on, we assume that all users transmit signal with the same data rate, and all signals are received at the same power at the receiver side (by perfect power control) [5]. Based on this assumption, then $E_{b,i} / N_o = E_b / N_o$, for $i=1, 2, \dots, N$. Therefore, (3-21) becomes

$$\begin{aligned} \sigma_{N+I}^2 &= N_o T_b \left[1 + (N-1) \frac{E_b}{N_o} \frac{2R_b}{3R_c} \right] \\ &= N_o' T_b \end{aligned} \quad (3-22)$$

where $N_o' = N_o \left[1 + (N-1) \frac{E_b}{N_o} \frac{2R_b}{3R_c} \right]$ is the equivalent single-sided noise spectral density of the noise plus interference term $N_{1,h}^{(1)}$.

For single user system, the variance of the noise is given by

$$\sigma_N^2 = N_o T_b \quad (3-23)$$

From (3-22) and (3-23), we can get the relationship between E_b / N_o' and E_b / N_o as follows [4], [7]

$$\frac{E_b}{N_o'} = \frac{E_b / N_o}{1 + (N-1) \frac{E_b}{N_o} \frac{2R_b}{3R_c}} \quad (3-24)$$

Above we discussed how to calculate decision variable $U_1^{(1)}$ for user 1 on signalling interval $0 \leq t \leq T_b$. Using the same method and assuming $\tau_1 < \tau_2 < \dots < \tau_N < kT_b$ in (3-12),

we can get the decision variable for any arbitrary user at the output of the h -th matched filter ($h=1, 2, \dots, 2^{(n-k)}$) on an arbitrary signalling interval $(j-1)T_b \leq t \leq jT_b$, which is given below [5]

$$\begin{aligned}
 U_{i,h}^{(j)} &= \begin{cases} b_i^{(j)} A_i T_b + \sum_{g=i+1}^N b_g^{(j-1)} A_g T_b \rho_{i,h,g}^{(j-1)} \cos \theta_{i,g} + \sum_{\substack{g=1 \\ g \neq i}}^N b_g^{(j)} A_g T_b \rho_{i,h,g}^{(j)} \cos \theta_{i,g} \\ \quad + \sum_{g=1}^{i-1} b_g^{(j+1)} A_g T_b \rho_{i,h,g}^{(j+1)} \cos \theta_{i,g} + N_{i,h}^{(j)} & \text{if } c_{i,h}(t) = c_i^{(j)}(t) \\ \\ \sum_{g=i+1}^N b_g^{(j-1)} A_g T_b \rho_{i,h,g}^{(j-1)} \cos \theta_{i,g} + \sum_{\substack{g=1 \\ g \neq i}}^N b_g^{(j)} A_g T_b \rho_{i,h,g}^{(j)} \cos \theta_{i,g} \\ \quad + \sum_{g=1}^{i-1} b_g^{(j+1)} A_g T_b \rho_{i,h,g}^{(j+1)} \cos \theta_{i,g} + N_{i,h}^{(j)} & \text{if } c_{i,h}(t) \neq c_i^{(j)}(t) \end{cases} \\
 &= \begin{cases} b_i^{(j)} A_i T_b + I_i^{(j-1)} + I_i^{(j)} + I_i^{(j+1)} + N_{i,h}^{(j)} & \text{if } c_{i,h}(t) = c_i^{(j)}(t) \\ I_i^{(j-1)} + I_i^{(j)} + I_i^{(j+1)} N_{i,h}^{(j)} & \text{if } c_{i,h}(t) \neq c_i^{(j)}(t) \end{cases} \\
 &= \begin{cases} b_i^{(j)} A_i T_b + N_{i,h}^{(j)} & \text{if } c_{i,h}(t) = c_i^{(j)}(t) \\ N_{i,h}^{(j)} & \text{if } c_{i,h}(t) \neq c_i^{(j)}(t) \end{cases}
 \end{aligned} \tag{3-25}$$

where $c_{i,h}(t)$ is the h -th PN sequence for the i -th user, $c_i^{(j)}(t)$ is the actual PN sequence used by i -th user on interval $(j-1)T_b \leq t \leq jT_b$, $b_g^{(j-1)}$ is the g -th user's bit on the interval $\tau_i + (j-2)T_b \leq t \leq \tau_i + (j-1)T_b$, $b_g^{(j)}$ is g -th user's bit on the interval $\tau_i + (j-1)T_b \leq t \leq \tau_i + jT_b$, $\theta_{i,g}$ is the phase difference between the i -th and g -th users' carriers. $N_{i,h}^{(j)}$ is the output of the i -th user's h -th matched filter corresponding to the input noise. $\rho_{i,h,g}^{(j-1)}$ is the cross-correlation coefficients of $c_{i,h}(t)$ and $c_g^{(j-1)}(t)$ where $c_g^{(j-1)}(t)$ is the

actual PN sequence used by g -th user on the interval $\tau_i + (j-2)T_b \leq t \leq \tau_i + (j-1)T_b$, $\rho_{i,h,g}^{(j)}$ is the cross-correlation coefficients of $c_{i,h}(t)$ and $c_g^{(j)}(t)$ where $c_g^{(j)}(t)$ is the actual PN sequence used by g -th user on the interval $\tau_i + (j-1)T_b \leq t \leq \tau_i + jT_b$, and $\rho_{i,h,g}^{(j+1)}$ is the cross-correlation coefficients of $c_{i,h}(t)$ and $c_g^{(j+1)}(t)$ where $c_g^{(j+1)}(t)$ is the actual PN sequence used by g -th user on the interval $\tau_i + jT_b \leq t \leq \tau_i + (j+1)T_b$. $\rho_{i,h,g}^{(j-1)}$, $\rho_{i,h,g}^{(j)}$ and $\rho_{i,h,g}^{(j+1)}$ can be calculated by using the following equations:

$$\rho_{i,h,g}^{(j-1)} = \frac{1}{T_b} \int_{(j-1)T_b}^{(j-1)T_b + \tau_g - \tau_i} c_{i,h}(t) c_g^{(j-1)}(t - \tau_g + \tau_i) dt \quad \text{for } i+1 \leq g \leq N \quad (3-26)$$

$$\rho_{i,h,g}^{(j)} = \begin{cases} \frac{1}{T_b} \int_{(j-1)T_b}^{jT_b + \tau_g - \tau_i} c_{i,h}(t) c_g^{(j)}(t - \tau_g + \tau_i) dt & \text{for } 1 \leq g \leq i-1 \\ \frac{1}{T_b} \int_{(j-1)T_b + \tau_g - \tau_i}^{jT_b} c_{i,h}(t) c_g^{(j)}(t - \tau_g + \tau_i) dt & \text{for } i+1 \leq g \leq N \end{cases} \quad (3-27)$$

$$\rho_{i,h,g}^{(j+1)} = \frac{1}{T_b} \int_{jT_b + \tau_g - \tau_i}^{jT_b} c_{i,h}(t) c_g^{(j+1)}(t - \tau_g + \tau_i) dt \quad \text{for } 1 \leq g \leq i-1 \quad (3-28)$$

In (3-25), $I_i^{(j-1)}$, $I_i^{(j)}$, and $I_i^{(j+1)}$ represent MAI components for the i -th user, and $N_{i,h}^{(j)}$ is the noise output from the i -th user's h -th matched filter corresponding to the input noise. $N'_{i,h}^{(j)}$ is the sum of the three MAI components and $N_{i,h}^{(j)}$. It can be simply regarded as a Gaussian variable with zero mean and variance given by (3-22). Since the $2^{(n-k)}$ spreading sequences for a given user are mutually orthogonal, there is no correlation between the samples of MAI and noise output from the different matched filters. That is to say, $E[N'_{i,h_1}^{(j)} N'_{i,h_2}^{(j)}] = 0$ for $h_1 \neq h_2$ [1][5][14].

In this thesis, a two-step method is implemented to detect the received signal. In the first step, the spreading sequence used to carry the information block must be determined,

and its corresponding parity bits can also be obtained. In the second step, only the sequence of decision variables at the output of the matched filter corresponding to that spreading sequence should be considered. They are sent to the Block Code Decoder to recover the transmitted original data. There are two ways to decode the data, i.e., hard decision decoding and soft decision decoding.

For hard decision decoding, the decision variables must be changed to be 0 or 1 before they are multiplied by the transpose of parity check matrix to get error syndrome, then the transmitted original data can be recovered by adding the error pattern corresponding to that error syndrome to the $\{0,1\}$ sequence of decision variables. For soft decision decoding, the sequence of decision variables is compared to all the possible information blocks that may be transmitted by the spreading sequence, which is obtained in the first step. To do so, the maximum likelihood detection (MLD) is used in this thesis and [1], [7]. Normally, soft decision decoding provides a 3dB gain over the hard decision decoding. But it is more complex, especially when the block codes are long.

3.3.1 Determination of the spreading sequence used

The proposed receiver has $2^{(n-k)}$ matched filters. At the time jT_b , they output $2^{(n-k)}$ decision variables $U_{i,h}^{(j)}$, where $h = 1, 2, \dots, 2^{(n-k)}$. The matched filter that output the maximum decision variable is thought to be the one using the same spreading sequence as that of the expected user on the signaling interval $(j-1)T_b \leq t \leq jT_b$. However, this is not always true due to the existence of noise and MAI in the channel. We know that the k

bits in each information block are spread by the same spreading sequence selected by parity bits corresponding to the information block. So, in order to decrease the error probability of determination of the spreading sequence employed, we consider decision variables over the k consequent signaling intervals instead of only one decision variable. The receiver decides that $c_{i,h_1}(t)$ is the most likely employed spreading sequence by the i -th user if the following inequation is true

$$\sum_{j=1}^k |U_{i,h_1}^{(j)}|^2 > \sum_{j=1}^k |U_{i,h_2}^{(j)}|^2, \text{ for all } h_2 = 1, 2, \dots, 2^{(n-k)} \text{ and } h_2 \neq h_1 \quad (3-29)$$

where $U_{i,h_1}^{(j)}$ and $U_{i,h_2}^{(j)}$ are decision variables at the output of the i -th user's h_1 -th and h_2 -th matched filters respectively. They are given by (3-25). In [1] it states that the decision rule in (3-29) is the same as the decision rule for noncoherent M -ary orthogonal signaling over k channels. Therefore the probability of selecting the incorrect PN sequence, P_e , is equal to the probability of symbol error of noncoherent M -ary orthogonal signaling over k different AWGN channels where $M=2^{(n-k)}$. Based on the discussion above, P_e can be upper bounded by [1] [4]

$$P_e < \frac{2^{(n-k)} - 1}{2^{2k-1}} e^{-\frac{kE_b}{2N_o}} \sum_{m=0}^{k-1} c_m \left(\frac{kE_b}{N_o} \right)^m \quad (3-30)$$

where E_b / N_o is given by (3-24), and c_m is given by

$$c_m = \frac{1}{m!} \sum_{l=0}^{k-1-m} \binom{2k-1}{l} \quad (3-31)$$

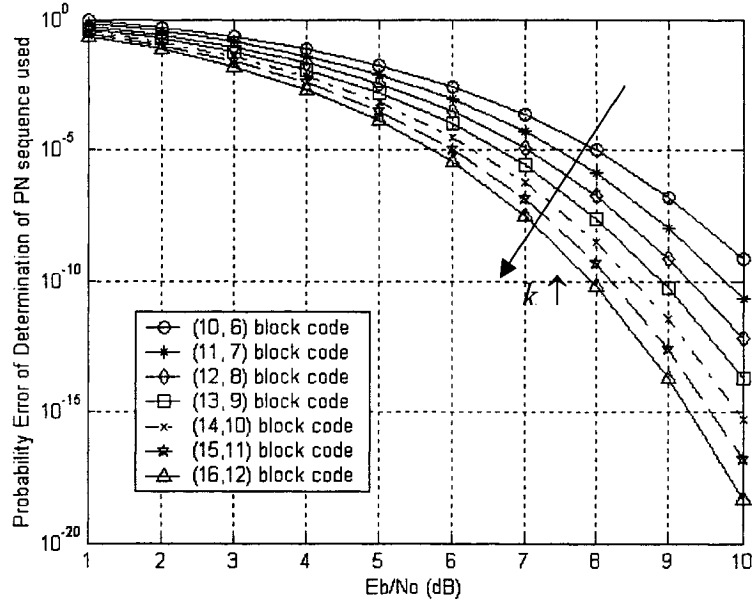


Figure 3-3: The probability of selecting the incorrect PN sequence used

In Figure 3-3, some calculation results of the upper bound of p_e for different length of block codes in single user system are provided, where the value of k is 6 through 12. We can see that the probability of incorrectly determining which spreading sequence was used becomes very small when k or E_b / N_o become large. Therefore, in order to increase the probability of determining the correct spreading sequence, we should use long block codes. But the complexity of receiver will be increased.

3.3.2 Detection of the transmitted information block

As we have discussed before, for the purpose of comparison, both hard decision decoding and soft decision decoding are used to decode the decision variables in this section.

Suppose the receiver has determined that $c_{i,h}(t)$ is the spreading sequence used by the i -th transmitter. $c_{i,h}(t)$ corresponds to the information words group q_h . Then we can check the Table 3-2 to get the difference Δq between q_h and q_1 . According to Δq , we flip the sequence of decision variables $U_{i,h}^{(1)} U_{i,h}^{(2)} \dots U_{i,h}^{(k)}$ like this

$$\hat{U}_{i,h}^{(j)} = \begin{cases} -U_{i,h}^{(j)}, & \text{if the } j\text{-th bit in } \Delta q \text{ is } 0 \\ U_{i,h}^{(j)}, & \text{if the } j\text{-th bit in } \Delta q \text{ is } 1 \end{cases} \quad (3-32)$$

The new sequence of decision variables $\hat{U}_{i,h}^{(1)} \hat{U}_{i,h}^{(2)} \dots \hat{U}_{i,h}^{(k)}$ corresponds to the information words in group q_1 . The set of words in group q_1 form a complete linear block code of $(k, 2k-n)$. We can use this new block code to decode the sequence of $\hat{U}_{i,h}^{(1)} \hat{U}_{i,h}^{(2)} \dots \hat{U}_{i,h}^{(k)}$ to get $\hat{b}^{(1)} \hat{b}^{(2)} \dots \hat{b}^{(k)}$. Then we flip $\hat{b}^{(1)} \hat{b}^{(2)} \dots \hat{b}^{(k)}$ according to Δq to get the $b^{(1)} b^{(2)} \dots b^{(k)}$ using the same method as (3-32), i.e.,

$$b^{(j)} = \begin{cases} -\hat{b}^{(j)}, & \text{if the } j\text{-th bit in } \Delta q \text{ is } 0 \\ \hat{b}^{(j)}, & \text{if the } j\text{-th bit in } \Delta q \text{ is } 1 \end{cases} \quad (3-33)$$

where $b^{(1)} b^{(2)} \dots b^{(k)}$ is the decoded k -bit information block corresponding to the sequence of decision variables $U_{i,h}^{(1)} U_{i,h}^{(2)} \dots U_{i,h}^{(k)}$.

In order to decode the sequence of $\hat{U}_{i,h}^{(1)} \hat{U}_{i,h}^{(2)} \dots \hat{U}_{i,h}^{(k)}$ to get $\hat{b}^{(1)} \hat{b}^{(2)} \dots \hat{b}^{(k)}$, two ways will be considered. One is soft decision decoding, and another is hard decision decoding.

- Soft Decision Decoding

When soft decision decoding is employed, the decision variable for a k -bit information block of the i -th user is given by

$$U_{i,B_l} = \sum_{j=1}^k (-1)^{(b_l^{(j)}+1)j} \hat{U}_{i,h}^{(j)}, \text{ for } i = 1, 2, \dots, N \quad (3-34)$$

where

$$(-1)^{(b_l^{(j)}+1)j} = \begin{cases} -1, & \text{for } b_l^{(j)} = 0 \\ +1, & \text{for } b_l^{(j)} = 1 \end{cases} \quad (3-35)$$

In (3-34), $\hat{U}_{i,h}^{(j)}$ is the flip version of the j -th decision variable and is given by (3-32), the subscript B_l represents the l -th information word in the information words group q_1 , and $b_l^{(j)}$ is the j -th bit in the l -th information word B_l , where $l=1, 2, \dots, 2^{(k-4)}$. Using (3-34) we can get $2^{(k-4)}$ decision variables and find the maximum decision variable, which is considered to correspond to the most likely transmitted information word.

Now we use the (12, 8) block code as an example to show the process introduced above. The (12, 8) block code has 16 parity vectors. Each parity vector corresponds to 16 information words of length 8. Two such information words groups, q_1 and q_2 , are given in Table 3-1. Suppose $c_{i,1}(t)$ is determined to be the spreading sequence used, according to the equation (3-34) above we can get the decision variable for information block 00000000 as follows

$$U_{i,00000000} = -U_{i,1}^{(1)} - U_{i,1}^{(2)} - U_{i,1}^{(3)} - U_{i,1}^{(4)} - U_{i,1}^{(5)} - U_{i,1}^{(6)} - U_{i,1}^{(7)} - U_{i,1}^{(8)} \quad (3-36)$$

Using the same way, we can get the decision variable for information block 00011011 as follows

$$U_{i,00011011} = -U_{i,1}^{(1)} - U_{i,1}^{(2)} - U_{i,1}^{(3)} + U_{i,1}^{(4)} + U_{i,1}^{(5)} - U_{i,1}^{(6)} + U_{i,1}^{(7)} + U_{i,1}^{(8)} \quad (3-37)$$

Continue to do the same thing, until we get 16 decision variables for the whole information words group q_1 . From these 16 decision variables we can find the maximum value, which is supposed to correspond to the most likely transmitted information word. During the decoding process, some errors may exist due to the noise and MAI. The bit error probability will be discussed under two different conditions, i.e., the spreading sequence used by transmitter is determined correctly or incorrectly. Then we will combine those two cases to get the final result.

Assuming that the receiver correctly determines the spreading sequence employed, the probability that it incorrectly identifies the information word carried by it is upper-bounded by the expression [1] [7]

$$p_m \leq (M_I - 1) Q \left(\sqrt{\frac{2d_{\min} E_b}{N'_o}} \right) \quad (3-38)$$

where E_b / N'_o is given by (3-24), $M_I = 2^{2k-n}$ is the number of different information words carried by a given spreading sequence. Actually, M_I also denotes the number of codewords of the shortened block code. $d_{\min}$ is the minimum distance between codewords of the shortened block code. For (12, 8) block code case, $M_I = 16$ and $d_{\min} = 4$. For (10, 6) block code case in [1], $M_I = 4$ and $d_{\min} = 4$.

Then the bit error probability when the receiver correctly determines the spreading sequence employed is given by [1]

$$p_{b,1} < \frac{d_{\min} N_{d_{\min}}}{k} Q \left(\sqrt{\frac{2E_b}{N'_o} d_{\min}} \right) \quad (3-39)$$

where $N_{d_{min}}$ is the number of codewords of weight d_{min} for the shortened block code.

Since the codewords of the shortened block code is the information words of the original block code, there is no coding rate loss in (3-39).

If the receiver incorrectly determines the spreading sequence employed, the receiver will select any information word from groups other than the one including the expected word with equal probability [7]. In this case, the bit error probability when the receiver incorrectly determines the spreading sequence employed can be calculated by the following equation

$$p_{b,2} = \frac{\overline{d_2}}{k} \quad (3-40)$$

where $\overline{d_2}$ is the average distance between the expected information word and each information word in any other groups other than the one to which the expected information word belongs. In other words, if we suppose the expected information word belongs to the l -th information words group, then $\overline{d_2}$ is given by

$$\overline{d_2} = \frac{1}{2^{n-k}-1} \sum_{\substack{j=1 \\ j \neq l}}^{2^{n-k}} \left(\frac{1}{2^{2k-n}} \sum_{i=1}^{2^{2k-n}} d_{j,i} \right) \quad (3-41)$$

where $d_{j,i}$ means the distance between expected information word and i -th information word in j -th group, where $j \neq l$. For linear block codes, the average distance $\overline{d_2}$ is the same for all codewords. It can be shown that when the spreading sequence is incorrectly detected, $\overline{d_2}/k = 1/2$ [1]. Thus, (3-40) becomes

$$p_{b,2} = \frac{1}{2} \quad (3-42)$$

Above we discussed the calculation of bit error probabilities when the spreading sequence used by transmitter was determined correctly or incorrectly, now we consider these two conditions together to get the approximation of the overall bit error probability as follows

$$p_b \approx p_{b,1}(1 - p_e) + \frac{1}{2}p_e \quad (3-43)$$

where p_e and $p_{b,1}$ are given by (3-30) and (3-39), respectively.

- Hard Decision Decoding

Unlike the soft decision decoding case, the decision variables for hard decision decoding must be changed to be 0 or 1 before they are multiplied by the transpose of parity check matrix to get the error syndrome. In other words, the first step is to get

$$R_{i,h}^{(j)} = \begin{cases} 0 & \text{if } \hat{U}_{i,h}^{(j)} < 0 \\ 1 & \text{if } \hat{U}_{i,h}^{(j)} > 0 \end{cases}, \text{ for } j=1,2, \dots, k \quad (3-44)$$

$$\mathbf{R} = \begin{bmatrix} R_{i,h}^{(1)} R_{i,h}^{(2)} & \dots & R_{i,h}^{(k-1)} R_{i,h}^{(k)} \end{bmatrix} \quad (3-45)$$

where $\hat{U}_{i,h}^{(j)}$ is the j -th decision variable in the sequence of decision variables $\hat{U}_{i,h}^{(1)} \hat{U}_{i,h}^{(2)} \dots \hat{U}_{i,h}^{(k)}$ given by (3-32), $\mathbf{R}$ is the vector representing the information word corresponding to that decision variables sequence, and $R_{i,h}^{(j)}$ is the j -th bit in $\mathbf{R}$. The size of $\mathbf{R}$ is $1 \times k$. The decision rule in (3-44) above is the same as the decision rule for the binary PAM signals. Therefore the probability of bit error at the input of decoder may be expressed by [4]

$$p_a = Q\left(\sqrt{\frac{2E_b}{N'_o}}\right) \quad (3-46)$$

where E_b / N'_o is given by (3-24).

The second step is to get the syndrome S by

$$S = RH^T \quad (3-47)$$

where H^T is the transpose of the parity check matrix of the block code.

The third step is to check the syndrome table to find the error pattern $\hat{e}$ corresponding to the syndrome S obtained by (3-47). The last step is to calculate $\hat{m}$, which is the decoder's estimate of the transmitted original data m . By simply adding the error pattern $\hat{e}$ to vector R , we can get $\hat{m}$ as follows:

$$\hat{m} = R + \hat{e} \quad (3-48)$$

Note that '+' means modulo-2 addition.

The bit error probability will be discussed under two different conditions, i.e., the spreading sequence employed by transmitter is determined correctly or incorrectly. Then the overall bit error probability will be obtained by combining the results of two cases.

Assuming that the receiver correctly determines the spreading sequence employed, the upper bound of the probability that it incorrectly identifies the information word carried by it can be obtained as follows [3], [9]

$$p_m \leq \sum_{i=t+1}^k \binom{k}{i} p_a^i (1 - p_a)^{k-i} \quad (3-49)$$

where p_a is given by (3-46).

Then the upper bound of bit error probability when the receiver correctly determines the spreading sequence employed is given by

$$p_{b,1} \leq \sum_{t=t+1}^{k-t} \binom{k}{i} p_a^i (1-p_a)^{k-i} \frac{i+t}{k} + \sum_{i=k-t+1}^k \binom{k}{i} p_a^i (1-p_a)^{k-i} . \quad (3-50)$$

If the receiver incorrectly determines the spreading sequence employed, the bit error probability for hard decision decoding case is exactly the same as that for soft decision decoding case. That is to say, equations (3-40) through (3-42) are also suitable for the hard decision decoding case.

Up to now, the bit error probabilities have been obtained when the spreading sequence used by transmitter is determined correctly and incorrectly respectively. Considering these two conditions we can get the approximation of the overall bit error probability as follows:

$$p_b \approx p_{b,1}(1-p_e) + \frac{1}{2} p_e \quad (3-51)$$

where p_e and $p_{b,1}$ are given by (3-30) and (3-50), respectively. Note that (3-51) has the same form as (3-43). The difference between (3-51) and (3-43) is the calculation for $p_{b,1}$. In other words, for soft decision decoding case, (3-39) is used to calculate $p_{b,1}$, and for hard decision decoding case, (3-50) is used.

3.4 Performance Analysis of Parity Bit Selected DS-CDMA System

From (3-43) and (3-51), we know that the bit error probability of parity bit selected DS-CDMA system is calculated using the same equation for soft and hard decision

decoding, except that $p_{b,1}$ is obtained using (3-39) and (3-50) respectively. For convenience, we rewrite (3-43) and (3-51), and divide it into two parts as follows

$$p_b \approx \underbrace{p_{b,1}(1-p_e)}_{\text{part 1}} + \underbrace{\frac{1}{2}p_e}_{\text{part 2}} \quad (3-52)$$

where part 1 and part 2 represent the bit error rate when PN sequences used by the transmitter are correctly and incorrectly determined by the receiver respectively.

We separate the block codes of length 10 through 16 bits including 4 parity bits into two groups, i.e., (10,6), (11,7) and (12, 8) in the first group, (13,9), (14,10), (15,11) and (16,12) in the second group. The shortened block codes of the first group have minimum distance $d_{min}=4$. And the shortened block codes of the second group have $d_{min}=3$.

Based on (3-52), the approximation of BER of single user parity bit selected DS-SS-CDMA system for soft and hard decision decoding are calculated and shown in Figure 3-4 and Figure 3-5 respectively.

Figure 3-4 shows BER for seven block codes of length 10 through 16 bits in which soft decision decoding is used. In order to see more clearly, the data corresponding to Figure 3-4 are also given. Table 3-5, Table 3-6 and Table 3-7 show the results of part 1, part2, and overall result of (3-52) respectively. From Table 3-5 and Table 3-6, we know that part 1 is negligible compared to part 2 for the block codes in the first group. But for the block codes in the second group, part 2 is more important than part 1 at low values of E_b/N_o , and it is negligible compared to part 1 only when $E_b/N_o \geq 11\text{dB}$. According to

these figures and tables, we can find which block code has the best BER performance for a given E_b/N_o . The results are shown in Table 3-10.

Figure 3-5 shows the BER for seven block codes of length 10 through 16 bits in which hard decision decoding is used. Since part 2 in (3-52) is the same for both hard and soft decision decoding, so Table 3-6 is still suitable for here. Table 3-8 and 3-9 show part 1 and overall result of (3-52) respectively. From Table 3-8 and 3-6, we can get that, for the block codes that have small value of k , such as (10, 6) code, part 1 is negligible compared to part 2 at low values of E_b/N_o , and part 2 is negligible compared to part 1 at high values of E_b/N_o . For the block codes that have large value of k , such as (16, 12) code, part 2 is as important as part 1 at low values of E_b/N_o , but it can be neglected at high values of E_b/N_o . Based on the figures and tables, the block code that has the best BER performance for a given E_b/N_o are found and shown in Table 3-11.

From Table 3-10 and Table 3-11, we note that, in single user system, there is not one block code that always provides the best BER performance when E_b/N_o is small for both soft and hard decision decoding. But when $E_b / N_o \geq 13\text{dB}$, (12, 8) block code can always provide the best BER performance for soft decision decoding. For hard decision decoding, it is (10, 6) block code that always provides the best BER performance when $E_b / N_o \geq 12\text{dB}$.

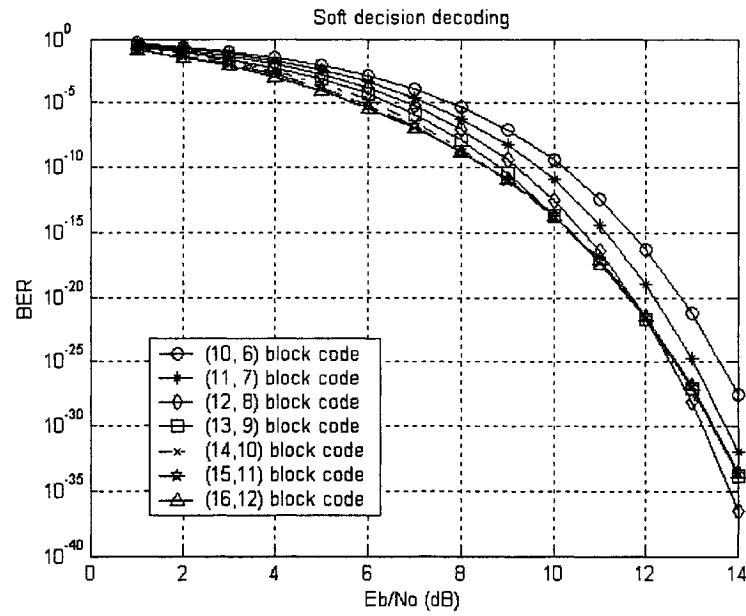


Figure 3-4: BER of seven block codes with 1 user and soft decision decoding

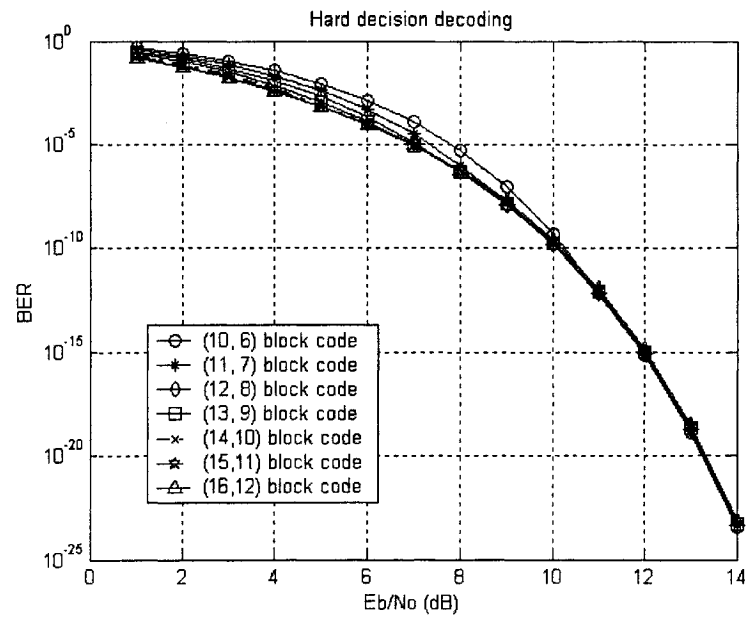


Figure 3-5: BER of seven block codes with 1 user and hard decision decoding

Table 3-5: Part 1 of (3-52) for soft decision decoding

E_b/N_o Block codes	1dB	2 dB	3 dB	4 dB	5 dB	6 dB	7 dB	8 dB	9 dB	10 dB	11 dB	12 dB	13 dB	14 dB
(10, 6)	1.14e-4	1.88e-4	5.08e-5	6.84e-6	4.83e-7	1.66e-8	2.4e-10	1.2e-12	1.6e-15	3.7e-19	1.1e-23	2.1e-29	1.4e-36	1.3e-45
(11, 7)	8.43e-4	4.80e-4	1.11e-4	1.42e-5	9.75e-7	3.33e-8	4.8e-10	2.4e-12	3.1e-15	7.5e-19	2.1e-23	4.1e-29	2.7e-36	2.6e-45
(12, 8)	2.30e-3	9.68e-4	2.07e-4	2.52e-5	1.71e-6	5.83e-8	8.5e-10	4.2e-12	5.5e-15	1.3e-18	3.7e-23	7.2e-29	4.8e-36	4.5e-45
(13, 9)	3.89e-3	1.95e-3	5.95e-4	1.19e-4	1.54e-5	1.19e-6	4.86e-8	8.9e-10	5.9e-12	1.1e-14	4.2e-18	2.1e-22	8.5e-28	1.4e-34
(14, 10)	5.25e-3	2.40e-3	7.03e-4	1.39e-4	1.79e-5	1.38e-6	5.62e-8	1.03e-9	6.8e-12	1.3e-14	4.8e-18	2.5e-22	9.9e-28	1.6e-34
(15, 11)	7.67e-3	3.28e-3	9.35e-4	1.83e-4	2.35e-5	1.81e-6	7.38e-8	1.35e-9	9.0e-12	1.7e-14	6.4e-18	3.2e-22	1.3e-27	2.1e-34
(16, 12)	9.92e-3	4.04e-3	1.13e-3	2.20e-4	2.82e-5	2.17e-6	8.85e-8	1.62e-9	1.1e-11	2.0e-14	7.6e-18	3.8e-22	1.6e-27	2.6e-34

Table 3-6: Part 2 of (3-52) for soft decision decoding

E_b/N_o Block codes	1dB	2 dB	3 dB	4 dB	5 dB	6 dB	7 dB	8 dB	9 dB	10 dB	11 dB	12 dB	13 dB	14 dB
(10, 6)	0.462	0.246	0.107	3.60e-2	8.62e-3	1.32e-3	1.14e-4	4.70e-6	7.48e-8	3.5e-10	3.5e-13	5.0e-17	5.8e-22	2.9e-28
(11, 7)	0.360	0.175	6.79e-2	1.94e-2	3.75e-3	4.31e-4	2.54e-5	6.31e-7	5.18e-9	1.0e-11	3.4e-15	1.1e-19	2.0e-25	9.2e-33
(12, 8)	0.282	0.126	4.33e-2	1.06e-2	1.64e-3	1.42e-4	5.69e-6	8.55e-8	3.6e-10	3.0e-13	3.2e-17	2.5e-22	7.1e-29	3.0e-37
(13, 9)	0.435	9.09e-2	2.78e-2	5.78e-3	7.24e-4	4.69e-5	1.29e-6	1.17e-8	2.6e-11	9.1e-15	3.2e-19	5.8e-25	2.5e-32	9.8e-42
(14, 10)	0.175	6.59e-2	1.79e-2	3.17e-3	3.21e-4	1.56e-5	2.92e-7	1.60e-9	1.8e-12	2.7e-16	3.1e-21	1.3e-27	9.0e-36	3.2e-46
(15, 11)	0.139	4.79e-2	1.16e-2	1.75e-3	1.43e-4	5.20e-6	6.66e-8	2.2e-10	1.3e-13	8.1e-18	3.0e-23	3.1e-30	3.2e-39	1.1e-50
(16, 12)	0.110	3.50e-2	7.51e-3	9.68e-3	6.38e-5	1.74e-6	1.52e-8	3.1e-11	9.2e-15	2.4e-19	3.0e-25	7.1e-33	1.2e-42	3.5e-55

Table 3-7: Probability of bit error for soft decision decoding

E_b/N_o Block codes	1dB	2 dB	3 dB	4 dB	5 dB	6 dB	7 dB	8 dB	9 dB	10 dB	11 dB	12 dB	13 dB	14 dB
(10, 6)	0.462	0.246	0.107	3.60e-2	8.62e-3	1.32e-3	1.14e-4	4.70e-6	7.48e-8	3.5e-10	3.5e-13	5.0e-17	5.8e-22	2.9e-28
(11, 7)	0.361	0.176	6.81e-2	1.95e-2	3.75e-3	4.31e-4	2.54e-5	6.31e-7	5.18e-9	1.0e-11	3.4e-15	1.1e-19	2.0e-25	9.2e-33
(12, 8)	0.284	0.127	4.35e-2	1.06e-2	1.64e-3	1.42e-4	5.70 e-6	8.55e-8	3.6e-10	3.0e-13	3.3e-17	2.5e-22	7.1e-29	3.0e-37
(13, 9)	0.226	9.29e-2	2.84e-2	5.90e-3	7.40e-4	4.80e-5	1.33e-6	1.26e-8	3.1e-11	2.0e-14	4.5e-18	2.1e-22	8.5e-28	1.4e-34
(14, 10)	0.181	6.83e-2	1.86e-2	3.31e-3	3.39e-4	1.70e-5	3.48e-7	2.63e-9	8.7e-12	1.3e-14	4.8e-18	2.5e-22	9.9e-28	1.6e-34
(15, 11)	0.147	5.12e-2	1.25e-2	1.93e-3	1.66e-4	7.02e-6	1.40e-7	1.57e-9	9.1e-12	1.7e-14	6.4e-18	3.2e-22	1.3e-27	2.1e-34
(16, 12)	0.120	3.90e-2	8.64e-3	1.19e-3	9.19e-5	3.92e-6	1.04e-7	1.65e-9	1.1e-11	2.0e-14	7.6e-18	3.9e-22	1.6e-27	2.6e-34

Table 3-8: Part 1 of (3-52) for hard decision decoding

E_b/N_o Block codes	1dB	2 dB	3 dB	4 dB	5 dB	6 dB	7 dB	8 dB	9 dB	10 dB	11 dB	12 dB	13 dB	14 dB
(10, 6)	1.59e-3	4.94e-3	2.93e-3	1.06e-3	2.58e-4	4.24e-5	4.47e-6	2.73e-7	8.48e-9	1.1e-10	5.1e-13	6.1e-16	1.3e-19	3.5e-24
(11, 7)	6.83e-3	7.41e-3	3.82e-3	1.31e-3	3.11e-4	5.10e-5	5.36e-6	3.28e-7	1.02e-8	1.3e-10	6.1e-13	7.3e-16	1.6e-19	4.2e-24
(12, 8)	1.20e-2	9.75e-3	4.65e-3	1.54e-3	3.64e-4	5.94e-5	6.25e-6	3.82e-7	1.19e-7	1.6e-10	7.2e-13	8.5e-16	1.9e-19	4.9e-24
(13, 9)	1.71e-2	1.19e-2	5.42e-3	1.77e-3	4.15e-4	6.78e-5	7.14e-6	4.37e-7	1.36e-8	1.8e-10	8.2e-13	9.7e-16	2.1e-19	5.6e-24
(14, 10)	2.17e-2	1.40e-2	6.16e-3	1.98e-3	4.66e-4	7.62e-5	8.03e-6	4.92e-7	1.53e-8	2.0e-10	9.2e-13	1.1e-15	2.4e-19	6.3e-24
(15, 11)	2.60e-2	1.59e-2	6.85e-3	2.19e-3	5.16e-4	8.45e-5	8.92e-6	5.46e-7	1.70e-8	2.2e-10	1.0e-12	1.2e-15	2.7e-19	7.0e-24
(16, 12)	3.00e-2	1.76e-2	7.50e-3	2.40e-3	5.66e-4	9.29e-5	9.81e-6	6.01e-7	1.87e-8	2.5e-10	1.1e-12	1.3e-15	2.9e-19	7.7e-24

Table 3-9: Probability of bit error for hard decision decoding

E_b/N_o Block codes	1dB	2 dB	3 dB	4 dB	5 dB	6 dB	7 dB	8 dB	9 dB	10 dB	11 dB	12 dB	13 dB	14 dB
(10, 6)	0.464	0.251	0.110	3.71e-2	8.87e-3	1.36e-3	1.19e-4	4.98e-6	8.33e-8	4.7e-10	8.7e-13	6.6e-16	1.3e-19	3.5e-24
(11, 7)	0.367	0.183	7.18e-2	2.08e-2	4.06e-3	4.82e-4	3.08e-5	9.60e-7	1.54e-8	1.5e-10	6.2e-13	7.3e-16	1.6e-19	4.2e-24
(12, 8)	0.294	0.136	4.80e-2	1.21e-2	2.00e-3	2.01e-4	1.19e-5	4.68e-7	1.22e-8	1.6e-10	7.2e-13	8.5e-16	1.9e-19	4.9e-24
(13, 9)	0.239	0.103	3.32e-2	7.55e-3	1.14e-4	1.15e-5	6.43e-6	4.49e-7	1.36e-8	1.8e-10	8.2e-13	9.7e-16	2.1e-19	5.6e-24
(14, 10)	0.197	7.99e-2	2.41e-2	5.16e-3	7.9e-4	9.18e-5	8.32e-6	4.93e-7	1.53e-8	2.0e-10	9.2e-13	1.1e-15	2.4e-19	6.3e-24
(15, 11)	0.165	6.37e-2	1.84e-2	3.95e-3	6.59e-4	8.97e-5	8.99e-6	5.46e-7	1.70e-8	2.2e-10	1.0e-12	1.2e-15	2.7e-19	7.0e-24
(16, 12)	0.140	5.25e-2	1.50e-2	3.37e-3	6.30e-4	9.46e-5	9.82e-6	6.01e-7	1.87e-8	2.5e-10	1.1e-12	1.3e-15	2.9e-19	7.7e-24

Table 3-10: Block codes having the best BER for a given E_b/N_o with 1 user and soft decision decoding

E_b/N_o (dB)	1	2	3	4	5	6	7	8	9	10	11	12	≥ 13
Block code having the best BER	(16, 12)												(12, 8)
	(15, 11)												(14, 10)
	(13, 9)												(12, 8)

Table 3-11: Block codes having the best BER for a given E_b/N_o with 1 user and hard decision decoding

E_b/N_o (dB)	1	2	3	4	5	6	7	8	9	10	11	≥ 12
Block code having the best BER	(16, 12)					(15, 11)	(14, 10)	(13, 9)	(12, 8)	(11, 7)	(10, 6)	

Chapter 4

4 Simulation Results

In this chapter, we will give some simulation results for uncoded, coded and parity bit selected DS-CDMA systems on an AWGN channel. Based on these results, some discussion will be also provided.

4.1 Models and Conditions for system simulation

Table 4-1 shows the parameters that are considered in simulation for three types of DS-CDMA systems. Some explanations about this table are given below:

- Number of users

In this thesis, we show the BER performance of three DS-CDMA systems that suffer from multiple access interference and additive white Gaussian noise, and compare the results with those of the single user system. The number of users used in our simulation is at most 5 due to excessive running time at higher number of users.

- Length of block codes

The block coded DS-CDMA system uses block codes with length of 6 through 12 bits. And the parity bit selected DS-CDMA system uses block codes with length of 10 through 16. Using these codes, the subset of information sequence that corresponds to each spreading waveform has some properties of the codes used in coded CDMA system.

- Length of parity bits

The length of the block codes' parity bits considered in this thesis is 3 or 4. For example, for block code of length 10, we will consider (10, 7) and (10, 6) codes. The length ($n-k$) of parity bits determines the number (M) of matched filters used at the receiver side, i.e., $M = 2^{(n-k)}$.

Table 4-1: Parameters considered in simulation for three types of DS-CDMA systems

System type Parameter	Uncoded DS-CDMA	Block coded DS-CDMA	Parity bit selected DS-CDMA
Number of users	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5
Length of block codes	N/A	6, 7, 8, 9, 10, 11, 12	10, 11, 12, 13, 14, 15, 16
Length of parity bits	N/A	4, 3	4, 3
Hard decision decoding	N/A	Yes	Yes
Soft decision decoding	N/A	Yes	Yes
Number of PN sequences	1	1	16, 8
Length of PN sequence	16, 8	16, 8	16, 8
Number of samples/chip	4	4	4
Data type for users	Random data	Random data	Random data

- Hard and soft decision decoding

For coded DS-CDMA system and proposed DS-CDMA system that employs parity bit selected spreading waveforms, we will investigate their performance for both hard and soft decision decoding.

- Number of PN sequences

In the uncoded DS-CDMA and the block coded DS-CDMA systems, each user has been assigned a fixed PN sequence as the spreading sequence. However, in the parity bit

selected DS-CDMA system, each user has been assigned a set of mutually orthogonal PN sequences. The input bit stream is segmented into k -bit information blocks. For each k -bit information block, their parity bits are calculated first. But these parity bits are not sent to the receiver. They are just used to select a PN sequence from a set of mutually orthogonal PN sequences to be the spreading sequence for the given information block. This means the spreading sequence may change every k -bit information block. Each user is assigned a unique set of spreading sequences. In our simulation, Hadamard codes are employed to create the set of mutually orthogonal PN sequences of the desired user. There are two different sets of Hadamard codes used. One set includes 16 orthogonal PN sequences of length 16. Another set includes 8 orthogonal PN sequences of length 8.

- Length of PN sequence

As we introduced above, the lengths of PN sequences used in our simulation are 16 and 8.

- Number of samples/chip

The three DS-CDMA systems in our simulation are assumed to be asynchronous systems. This means that the chips of the spreading sequences for different users may have different delays. The delays are uniformly distributed over a bit interval [6]. In our simulation, we have resolved the delays to multiples of one quarter of a chip. Consequently, there are 4 samples/chip in our simulation. The higher the resolution is, the more accurate the delays are.

- Receiver synchronization

We assume that the receiver's PN sequences are perfectly synchronized to the desired transmitter's PN sequences.

Table 4-2 shows some results of the variance of the cross-correlation coefficients of PN sequences with different number of samples in one chip. To get Table 4-2, parity bit selected DS-CDMA system is observed. We consider two types of data, fixed data or random data, as the source information bits for user 1 respectively. And we assume all the other users employ random data as their source information bits. The set of mutually orthogonal PN sequences for user 1 consists of 16 Hadamard codes of length 16. For any other user, its set of mutually orthogonal PN sequences is obtained by adding a 16-chip random sequence to the set of PN sequences for user 1. It is important that the 16-chip random sequence is not a PN sequence from any other user's set, otherwise their sets will be identical. Since the length of each PN sequence is 16 chips, so we get that R_b/R_c equals 1/16, where R_b is the bit rate and R_c is the chip rate of PN sequences. For the purpose of comparison, both the chip asynchronous and chip synchronous are shown in Table 4-2. All the results were obtained by simulation in Matlab.

According to (3-17) and (3-18), we know that for asynchronous transmission, the theoretical variance of $\rho_{1,h,i}^{(0)}$ or $\rho_{1,h,i}^{(1)}$ should be $(1/3)(R_b/R_c) = 1/48 = 0.0208$, where R_b/R_c equals 1/16. From Table 4-2, we can get the following:

- If user 1 uses the fixed information bits (all zeros here), the variance of the cross-correlation coefficients of PN sequences using 4 or 10 samples/chip has little difference from that of the synchronous case. In other words, we can't get the expected MAI simulation effect for asynchronous system when user 1 uses the fixed information bits, since the results are far from the theoretical values to which they should approach.

- If user 1 uses random data, the variance of the cross-correlation coefficients of PN sequences closely matches the theoretical one when we consider 4 or 10 samples/chip. Note that there is only little difference between the result of 4 samples/chip case and the result of 10 samples/chip case. Therefore, for simplicity, 4 samples/chip will be employed in our simulation.

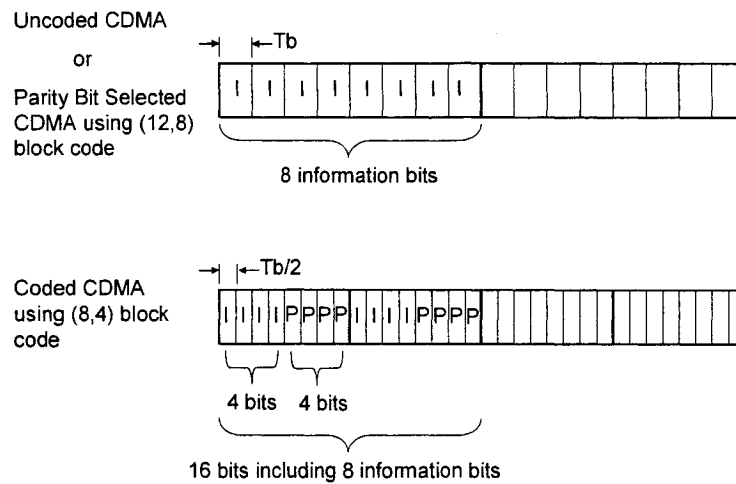
- Data type for users

Based on the discussion above, we know that all users should use random data as their source information bits.

Table 4-2: Variance of the cross-correlation coefficients of PN sequences with different samples/chip

Variance of the cross-correlation coefficients of PN sequences		Samples/chip		
		chip <i>synchronous</i>	chip <i>asynchronous</i>	
		1	4	10
Information bits for user 1	All zeros	$\rho_{1,h,i}^{(0)} = 0.0300$	$\rho_{1,h,i}^{(0)} = 0.0282$	$\rho_{1,h,i}^{(0)} = 0.0292$
		$\rho_{1,h,i}^{(1)} = 0.0320$	$\rho_{1,h,i}^{(1)} = 0.0339$	$\rho_{1,h,i}^{(1)} = 0.0324$
	Random data	$\rho_{1,h,i}^{(0)} = 0.0297$	$\rho_{1,h,i}^{(0)} = 0.0200$	$\rho_{1,h,i}^{(0)} = 0.0210$
		$\rho_{1,h,i}^{(1)} = 0.0335$	$\rho_{1,h,i}^{(1)} = 0.0217$	$\rho_{1,h,i}^{(1)} = 0.0216$

For the purpose of comparison, coded DS-CDMA system should use the same information bit rate and chip rate as uncoded DS-CDMA system and parity bit selected DS-CDMA system. Since redundancy encoding of block code reduces the information bit rate relative to the other two systems, the length of PN sequence in the coded DS-CDMA system must be reduced in order to maintain the same channel bandwidth of its corresponding uncoded DS-CDMA system and parity bit selected DS-CDMA system [6].



Note:

'I' and 'P' are referred to as information bit and parity bit, respectively.

Figure 4-1: Frame structure for three DS-CDMA systems using the same information bit rate

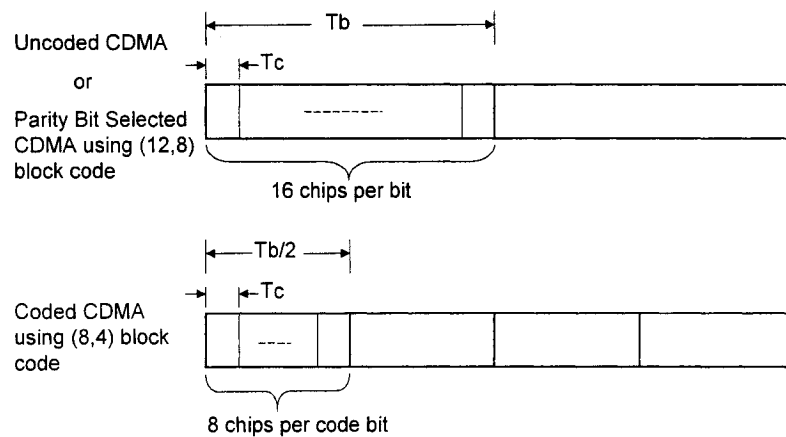


Figure 4-2: Chip structure for three DS-CDMA systems using the same chip rate

Now we use (8, 4) block code as an example for the coded DS-CDMA system. Correspondingly, (12, 8) block code should be used for the parity bit selected DS-CDMA system. Figure 4-1 shows that in order to keep the same information bit, the bit interval of coded DS-CDMA system is half of that of uncoded DS-CDMA system or parity bit selected DS-CDMA system. T_b is referred to as the bit interval of the uncoded DS-CDMA system or the parity bit selected DS-CDMA system. From Figure 4-2, we can see that in order to have the same chip rate, the PN sequence of length 8 chips must be used for the coded DS-CDMA system, whereas the PN sequence of length 16 chips is used for the other two systems.

4.2 Simulation results

4.2.1 Simulation results of the conventional DS-CDMA systems

In this section, the BER performance of two conventional DS-CDMA systems is considered. One is uncoded DS-CDMA system and another is coded DS-CDMA system.

The simulation results for 1 user through 5 users uncoded DS-CDMA systems are given in Figure 4-3. The results show that the BER performance of the uncoded DS-CDMA system degrades as the number of users in the system increases. The reason for this is that the MAI increases as the number of simultaneously transmitting users increases.

In order to improve BER performance of uncoded DS-CDMA system, some techniques have been studied and implemented, such as Forward Error Correction (FEC). It is well known that employing FEC can be highly effective in increasing the achievable information bit rate, improving the BER performance and system capacity of CDMA systems. Block codes can be used due to their simplicity and good performance. Here,

some simulation results of BER performance for coded DS-CDMA system using (8, 4) block code with different number of users are provided in Figure 4-4 and Figure 4-5, which correspond to soft decision decoding and hard decision decoding respectively. From the results we can see that soft decision decoding case has better performance than hard decision decoding case. For example, in the 2 users system, the soft decision decoding case has about 2 dB gain over the hard decision decoding case at a BER of 10^{-3} . But for soft and hard decision decoding, their BER performance also degrades as we increase the number of users.

The simulation results of these two DS-CDMA systems show that the existence of MAI decreases the quality of service and limits the number of users in the system. So it is necessary and valuable to find a better technique that can improve the BER performance and provide reasonable quality of service to allow active users on DS-CDMA systems.

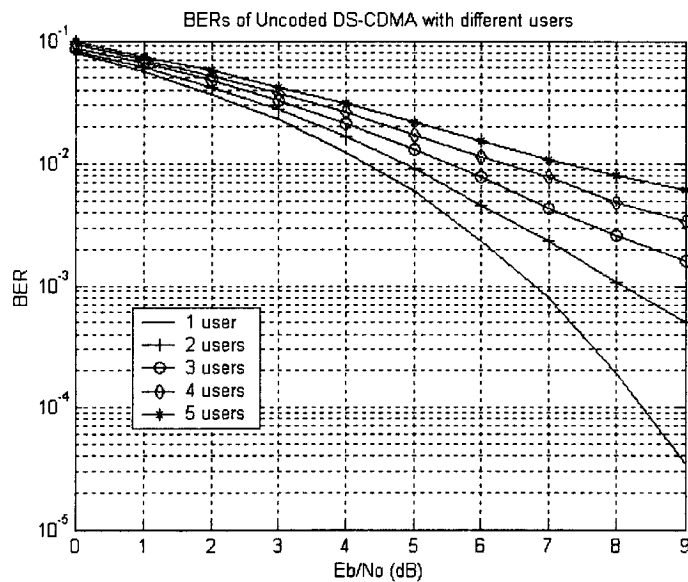


Figure 4-3: BER performance of the uncoded DS-CDMA with a spreading factor of 16 for different users

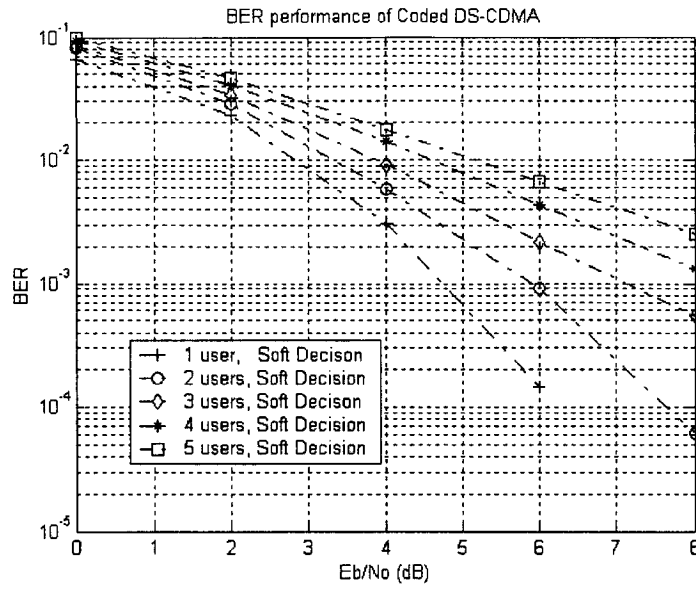


Figure 4-4: BER comparison of coded DS-CDMA for different number of users using (8, 4) block code with a spreading factor of 8 and soft decision decoding

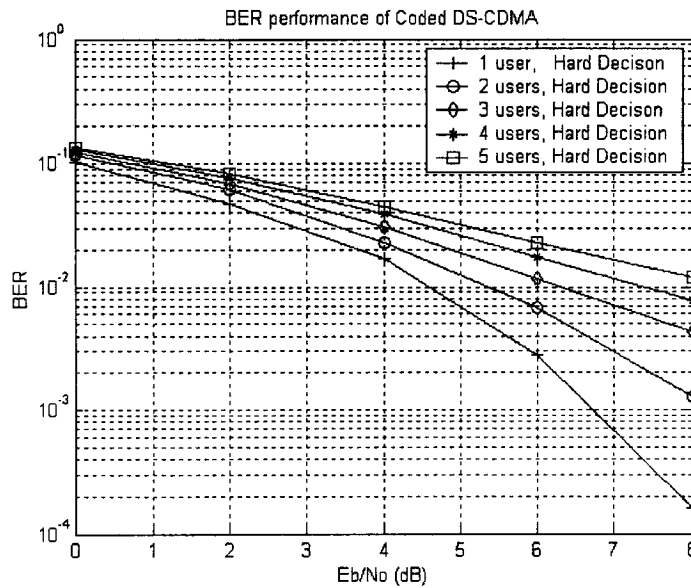


Figure 4-5: BER comparison of coded DS-CDMA for different number of users using (8, 4) block code with a spreading factor of 8 and hard decision decoding

4.2.2 Simulation results of the parity bit selected DS-CDMA system

- BER comparison of seven block codes with single user

From (3-30), we know that the probability of incorrectly determining the PN sequence employed in the transmitter decreases as the information bits block length k increases for a given E_b/N_o . For two block codes (n_1, k_1) and (n_2, k_2) , we assume $k_1 < k_2$ (i.e., $n_1 < n_2$) and $(n_1 - k_1) = (n_2 - k_2)$, which means they have the same length of parity bits. On the one hand, according to (3-30), we have $P_{e,(n_1,k_1)} < P_{e,(n_2,k_2)}$, where p_e is the probability of incorrectly determining the PN sequence. On the other hand, we know that different block codes have different ability of error correction. Since code rate k_1/n_1 is smaller than code rate k_2/n_2 and $(n_1 - k_1) = (n_2 - k_2)$, block code (n_1, k_1) has better error correction ability than block code (n_2, k_2) if they have the same minimum distance. Based on the discussion above, we know that there is a trade off between the ability of determining the correct PN sequence and the ability of correcting errors when the receiver correctly determines the PN sequence employed in the transmitter.

Now we try to find which block code is the best one for a given E_b/N_o in the parity bit selected DS-CDMA system. In our simulation, both hard decision decoding and soft decision decoding are used. The BER results of block codes of length 10 bits through 16 bits are obtained by simulation. For soft decision decoding, the results are shown in Figure 4-6. For hard decision decoding, the results are shown in Figure 4-7. Since the BER performance of different block codes are very close to each other, especially when hard decision decoding is employed, we show the BER results in Table 4-3 and Table 4-4 along with the figures in order to distinguish them more clearly. In our simulation, the BER is calculated when E_b/N_o equals 0 dB, 2dB, 4dB, and 6dB. It will take long time to

get an accurate BER by simulation when E_b/N_o is larger than 6dB due to the low error probabilities that are obtained when $E_b/N_o > 6\text{dB}$. So the largest value of E_b/N_o considered here is 6dB.

Table 4-3: BER for soft decision decoding with single user and a spreading factor of 16

E_b/N_o Block codes	0	2	4	6
(10, 6)	0.15634	0.053952	0.0073146	0.0002203
(11, 7)	0.13301	0.038884	0.0038205	5.19e-005
(12, 8)	0.1155	0.027544	0.0019707	1.51e-005
(13, 9)	0.10368	0.019553	0.0009958	4.1e-006
(14, 10)	0.088285	0.014794	0.0005888	2.7e-006
(15, 11)	0.081886	0.011944	0.0003608	1.4e-006
(16, 12)	0.076863	0.010953	0.0003453	3.1e-006

Table 4-4: BER for hard decision decoding with single user and a spreading factor of 16

E_b/N_o Block codes	0	2	4	6
(10, 6)	0.16798	0.058362	0.0080677	0.0002395
(11, 7)	0.14744	0.044752	0.0047554	7.92e-005
(12, 8)	0.13212	0.034584	0.002927	4.97e-005
(13, 9)	0.12431	0.028232	0.0023806	4.87e-005
(14, 10)	0.1116	0.02507	0.002175	5.05e-005
(15, 11)	0.10666	0.022641	0.0020912	6.8e-005
(16, 12)	0.10185	0.023293	0.0024516	6.89e-005

From Figure 4-6, Figure 4-7, Table 4-3 and Table 4-4, we can find the block codes that have the best BER performance when E_b/N_o equals 0dB, 2dB, 4dB and 6dB respectively. The results are shown in Table 4-5 and Table 4-6, which correspond to soft decision decoding and hard decision decoding respectively. We note that there is not one block code that always provides the best BER performance when E_b/N_o is small for either soft decision decoding or hard decision decoding. It is also true for hard decision decoding, but it is a different block code that provides the best BER performance from that of soft decision decoding for a given E_b/N_o .

Table 4-5: Block codes having the best BER performance for a given E_b/N_o with single user, a spreading factor of 16 and soft decision decoding

$E_b/N_o(\text{dB})$	0	2	4	6
Block codes having the best BER performance	(16, 12)	(16, 12)	(16, 12)	(15, 11)

Table 4-6: Block codes having the best BER performance for a given E_b/N_o with single user, a spreading factor of 16, and hard decision decoding

$E_b/N_o(\text{dB})$	0	2	4	6
Block codes having the best BER performance	(16, 12)	(15, 11)	(15, 11)	(13, 9)

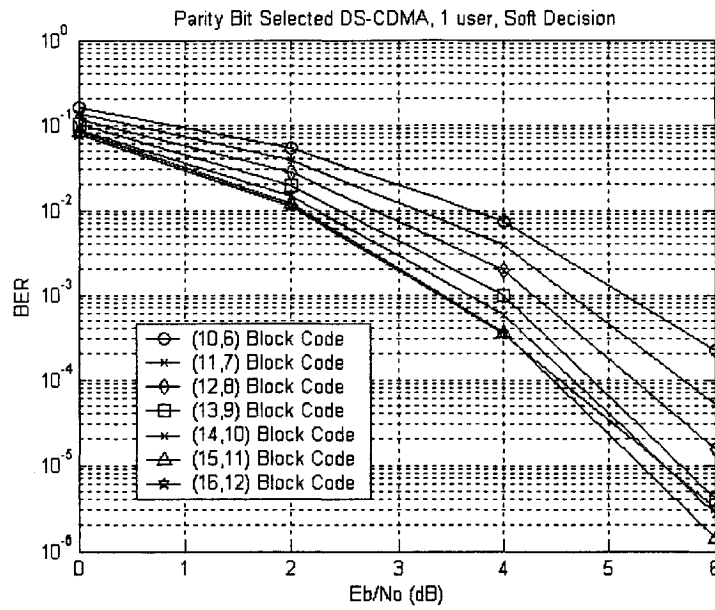


Figure 4-6: BER comparison of parity bit selected DS-CDMA for different block codes with 1 user, a spreading factor of 16, and soft decision decoding

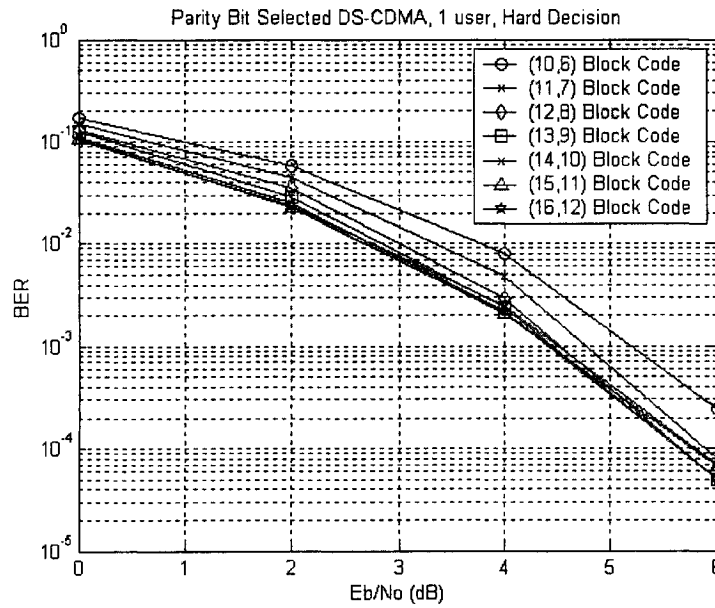


Figure 4-7: BER comparison of parity bit selected DS-CDMA for different block codes with 1 user, a spreading factor of 16, and hard decision decoding

- Results of 4 parity bits block code (12, 8) with different number of users

Now we use (12, 8) block code as an example to see if our proposed parity bit selected DS-CDMA scheme has better BER performance than other two schemes, i.e., uncoded DS-CDMA and coded DS-CDMA. First, we will give some simulation results of the (12, 8) based parity bit selected DS-CDMA system for 1 user through 5 users respectively. Then, we will compare them with their corresponding results of uncoded DS-CDMA and (8, 4) based coded DS-CDMA systems.

Figure 4-8 and Figure 4-9 correspond to soft decision decoding and hard decision decoding respectively, which show some simulation results of BER performance for the (12, 8) based parity bit selected DS-CDMA system with different number of users. In these two figures, there are 1 user, 2 users, 3 users, 4 users and 5 users are considered. From the results we get that the BER performance becomes worse with the increase of the number of users. This shows the influence of MAI. We can also see that soft decision decoding case has better performance than hard decision decoding case. For instance, in the single user case, the soft decision decoding has about 0.5 dB gain over the hard decision decoding at a BER of 10^{-4} . Note that the gain that soft decision over hard decision decoding case is not too much when $E_b/N_o < 5\text{dB}$. This means that we can use hard decision decoding instead of soft decision decoding if we care much about the complexity of implementation when the value of E_b/N_o is small. It is well known that hard decision decoding is easier to implement.

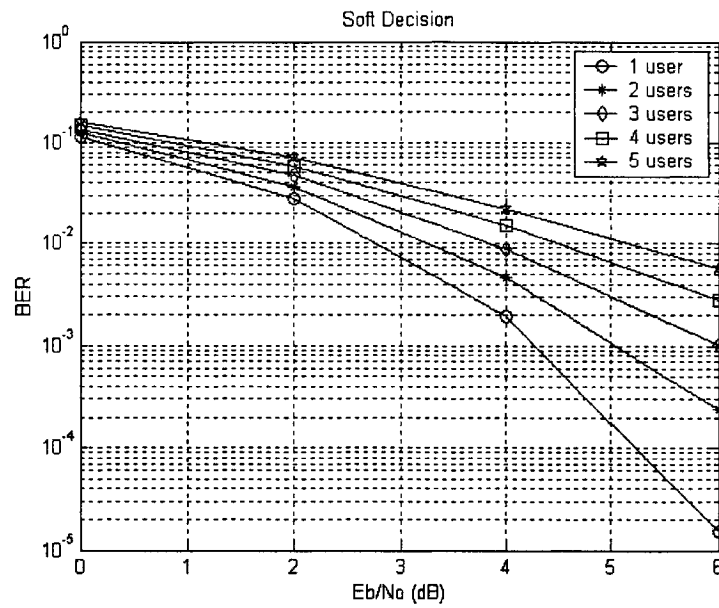


Figure 4-8: BER comparison for different number of users in parity bit selected DS-SS-CDMA system with a spreading factor of 16 and soft decision decoding

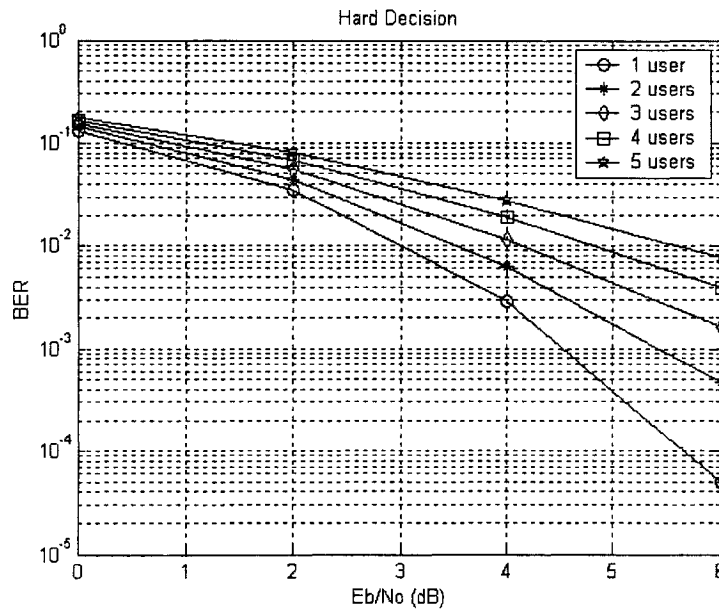


Figure 4-9: BER comparison for different number of users in parity bit selected DS-SS-CDMA system with a spreading factor of 16 and hard decision decoding

In order to see the advantage of our proposed parity bit selected DS-CDMA system, we will compare its simulation results with those of uncoded DS-CDMA and coded DS-CDMA systems. Here, we will compare the three systems for 1 user, 2 users, and 5 users systems, respectively. For clearness, the simulation results of soft and hard decision decoding are plotted in different figures. And the results for different number of users are also plotted in different figures. Figure 4-10, 4-12, and 4-14 give the BER comparison of the three DS-CDMA systems when soft decision decoding is used. Figure 4-11, 4-13, and 4-15 give the BER comparison of the three systems when hard decision decoding is used.

The results in these figures show that the parity bit selected DS-CDMA system performs very well compared to the other two DS-CDMA systems. In single user system, at a BER of 10^{-4} , the (12, 8) based parity bit selected DS-CDMA system has a 1.0 dB gain over (8, 4) coded DS-CDMA system and a 3.2 dB gain over uncoded DS-CDMA system when soft decision decoding is used. For hard decision decoding, the two corresponding gains are 2.6 dB and 2.7 dB at a BER of 10^{-4} . In two users system, at a BER of 10^{-4} , the (12, 8) based parity bit selected DS-CDMA system has a 1.2 dB gain over (8, 4) coded DS-CDMA system and a 4.5 dB gain over uncoded DS-CDMA system when soft decision decoding is used. For hard decision case, the two corresponding gains are 3.75 dB and 3.8 dB at a BER of 10^{-4} .

We can see that the crossover points of BER curves of parity bit selected DS-CDMA and coded DS-CDMA systems move to right as the number of users increases. The E_b/N_o values corresponding these crossover points are 2.5 dB, 3 dB, and 5.2 dB for 1 user, 2 users, and 5 users respectively in soft decision decoding case, and 0.8 dB, 0.8 dB, and 2 dB for 1 user, 2 users, and 5 users respectively in hard decision decoding case.

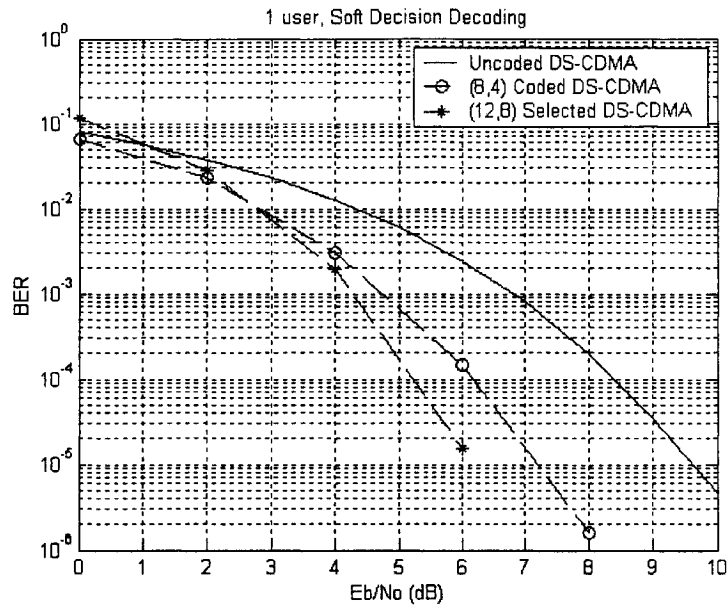


Figure 4-10: BER comparison of three DS-CDMA systems with 1 user and soft decision decoding

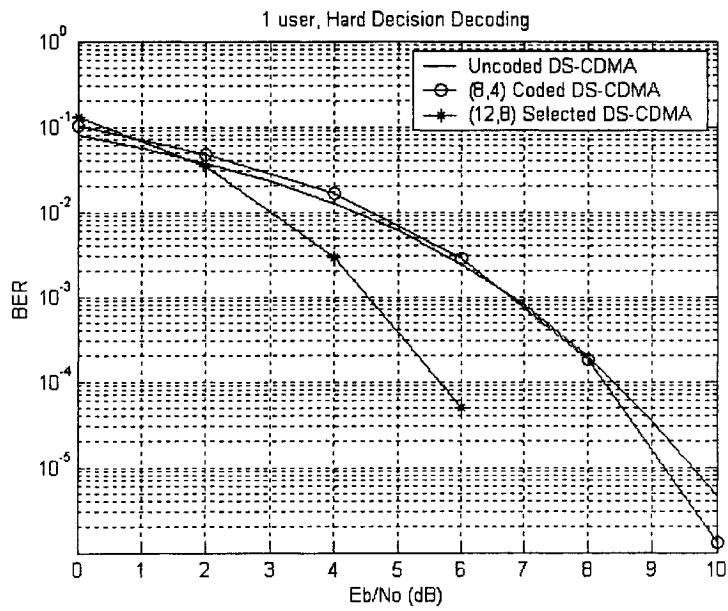


Figure 4-11: BER comparison of three DS-CDMA systems with 1 user and hard decision decoding

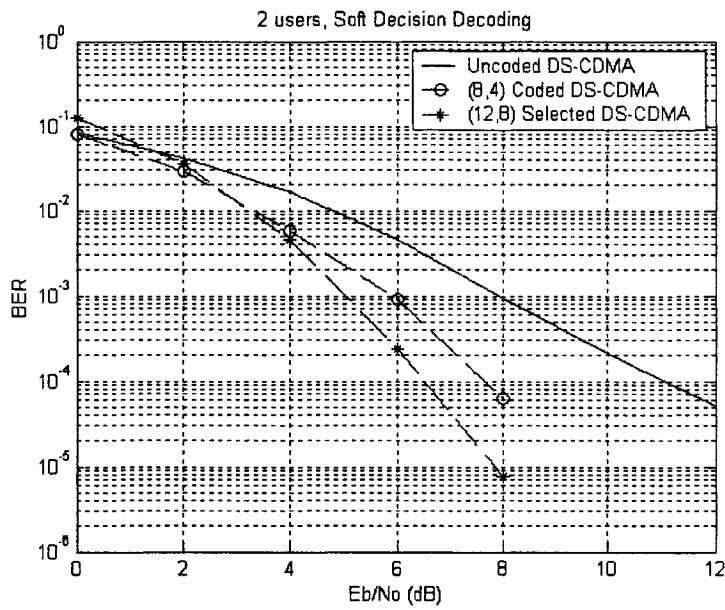


Figure 4-12: BER comparison of three DS-CDMA systems with 2 users and soft decision decoding

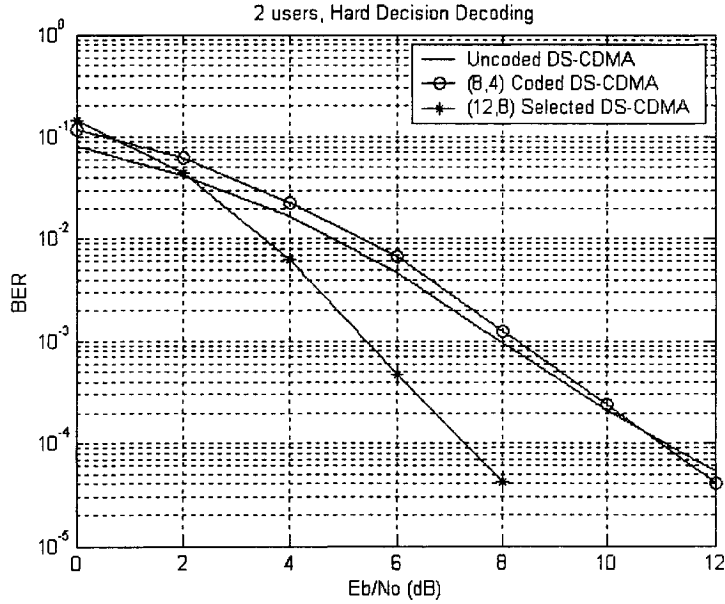


Figure 4-13: BER comparison of three DS-CDMA systems with 2 users and hard decision

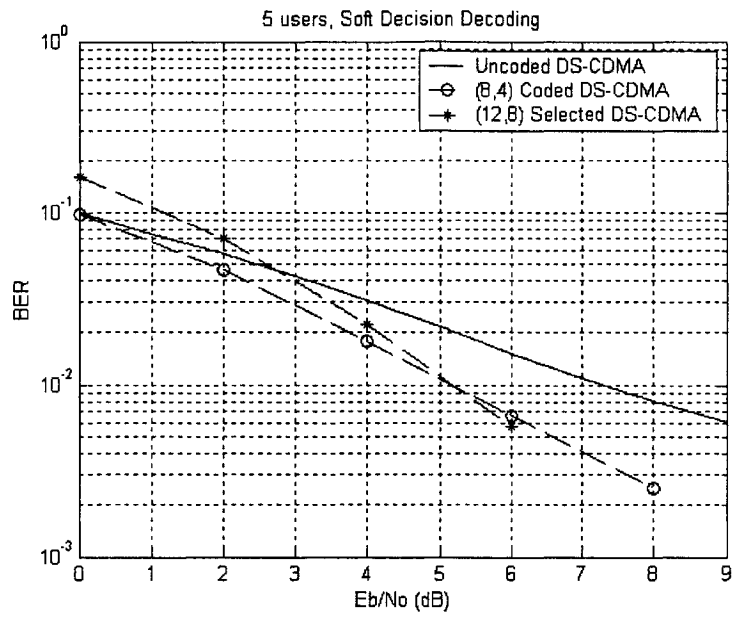


Figure 4-14: BER comparison of three DS-CDMA systems with 5 users and soft decision decoding

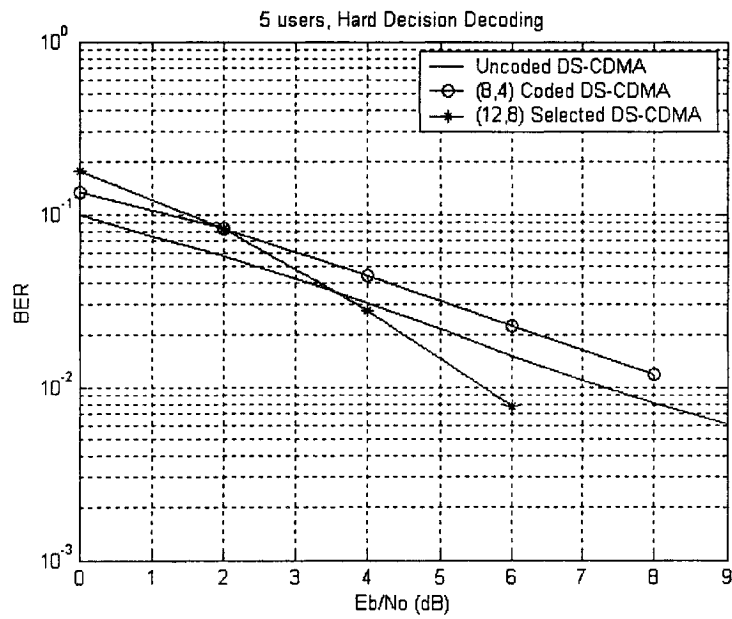


Figure 4-15: BER comparison of three DS-CDMA systems with 5 users and hard decision

- Results of 3 parity bits block code (10, 7)

Now we will use (10, 7) based parity bit selected DS-CDMA system to observe what the situation will be for the block codes with 3 parity bits. First, we will compare its BER performance with those of uncoded DS-CDMA system and (7, 4) based coded DS-CDMA system with 1 user, 2 users and 5 users respectively. We choose (7, 4) coded DS-CDMA system to compare against (10, 7) based parity bit selected DS-CDMA system since the latter has the former inherently built into it. Then, we will compare its BER performance with other three block codes, (10, 6), (11, 7), and (12, 8), which have 4 parity bits. Note that (10, 7) and (10, 6) block codes have the same length of codeword, i.e., 10. And (10, 7) block code has the same length of information block as that of (11, 7) block code, i.e., 7. For (12, 8) block code, it is used as a reference for comparison.

The simulation results of soft and hard decision decoding are plotted in different figures. And the results for 1 user, 2 users and 5 users systems are also plotted in different figures. Figure 4-16, 4-18, and 4-20 give the BER comparison of three DS-CDMA systems when soft decision decoding is used. Figure 4-17, 4-19, and 4-21 give the BER comparison of three DS-CDMA systems when hard decision decoding is used. The results in these figures show that the parity bit selected DS-CDMA system performs very well compared to the other two DS-CDMA systems. In single user system, at a BER of 10^{-3} , the (10, 7) based parity bit selected DS-CDMA system has a 0.8 dB gain over (7, 4) coded DS-CDMA system and a 2.2 dB gain over uncoded DS-CDMA system when soft decision decoding is used. For hard decision case, the two corresponding gains are 1.8 dB and 2.0 dB at a BER of 10^{-3} . In two users system, at a BER of 10^{-3} , the (10, 7) based parity bit selected DS-CDMA system has a 1.5 dB gain over (7, 4) coded DS-CDMA

system when soft decision decoding is used. It is also much better than uncoded DS-CDMA system. For hard decision case, the two corresponding gains are 1.7 dB and 1.6 dB at a BER of 10^{-2} . The crossover points of BER curves of parity bit selected DS-CDMA and coded DS-CDMA systems move to right as the number of users increases. The E_b/N_o values corresponding these crossover points are 1 dB, 3 dB, and 6 dB for 1 user, 2 users, and 5 users respectively for soft decision decoding, and 1.6 dB, 2.1 dB, and 4.8 dB for 1 user, 2 users, and 5 users respectively for hard decision decoding.

Figure 4-22 through 4-27 show the results of four block codes, (10, 7), (10, 6), (11, 7) and (12, 8) in parity bit selected DS-CDMA system. From Figure 4-22 and 4-23 which show the results for 1 user system with soft decision decoding and hard decision decoding respectively, we can see that, for both soft and hard decision decoding, the (10, 7) based system has better BER performance than the (10, 6) based system and the (11, 7) based system when E_b/N_o is in the range of [0dB, 6dB]. The (10, 6) based system is even better than the (12, 8) based system when $E_b/N_o < 3$ dB. But when $E_b/N_o > 3$ dB, the (12, 8) based system has the best BER performance for both soft and hard decision decoding.

Figure 4-24 and 4-25 show the results for the two users system with soft decision decoding and hard decision decoding respectively. From Figure 4-24, we can see that (10, 7) based system is worse than (10, 6) based system when $E_b/N_o > 5.5$ dB and worse than (11, 7) when $E_b/N_o > 3$ dB when soft decision decoding is used. From Figure 4-25, we can get that (10, 7) based system is worse than (10, 6) based system when $E_b/N_o > 4.7$ dB and worse than (11, 7) when $E_b/N_o > 2.4$ dB when hard decision decoding is used. The (12, 8) based system always has the best BER performance in this two users system.

Figure 4-26 and 4-27 show the results for the five users system with soft decision decoding and hard decision decoding respectively. We can see that (10, 7) based system is worse than other three block codes, except that it is better than (10, 6) based system when $E_b/N_o < 2\text{dB}$ for soft decision decoding and $E_b/N_o < 1.5\text{ dB}$ for hard decision decoding, respectively.

The reason that the system using block codes with 4 parity bits are better than those using block codes with 3 parity bits is because as $(n-k)$ decreases, the number of information sequences per spreading code increases, decreasing d_{min} in each coset. Thus, the probability of incorrectly determining PN sequence used by the transmitter decreases linearly, but the probability of incorrectly identifying the information word when the receiver correctly determines the PN sequence employed increases exponentially. As a result, when $(n-k)$ decreases, the probability of bit error will increase.

4.3 Summary of Simulation Results

According to the simulation results and corresponding discussions in the previous sections, we can make some conclusions as follows:

- For both soft and hard decision decoding, the BER performance of the three DS-SS-CDMA systems degrades as the number of users in the systems increases. This is because that the MAI increases as the number of simultaneously transmitting users increases.
- In parity bit selected DS-SS-CDMA system, when E_b/N_o is small, there is not one block code that always provides the best BER performance for either soft decision decoding or hard decision decoding. The reason is that, for a given block code, there is a tradeoff between the ability of determining the correct PN

sequences and the ability of correcting errors when the receiver correctly determines the PN sequences employed by transmitter.

- For both block codes with 3 parity bits and block codes with 4 parity bits, our proposed parity bit selected DS-CDMA system always provides the best BER performance in the three DS-CDMA systems.
- For parity bit selected DS-CDMA systems using block codes with the same length of codeword but different number of parity bits, such as (10, 7) and (10, 6) codes, the system using block code with 4 parity bits provides better BER performance than that using block code with 3 parity bits when E_b/N_o is larger than a certain threshold value. The threshold value decreases as the number of users increases. For soft and hard decision decoding, the threshold values are different. This conclusion is also true for parity bit selected DS-CDMA systems using block codes with the same length of information word but different number of parity bits, such as (11, 7) and (10, 7) codes. The reason that the system using block codes with 4 parity bits are better than those using block codes with 3 parity bits is because as $(n-k)$ decreases, the probability of bit error of the proposed parity bit selected DS-CDMA system will increase.
- In uncoded DS-CDMA and block coded DS-CDMA systems, there is only one matched filter at each receiver. In parity bit selected DS-CDMA system, there are $2^{(n-k)}$ matched filters at each receiver. Therefore, although parity bit selected DS-CDMA system has the best BER performance in these three systems, it is more complex than other two DS-CDMA systems.

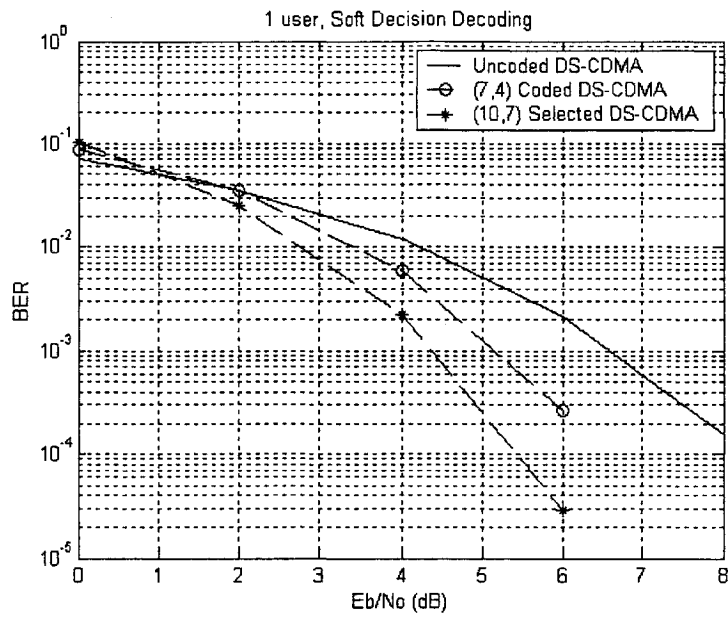


Figure 4-16: BER comparison of three DS-CDMA systems with 1 users and soft decision decoding

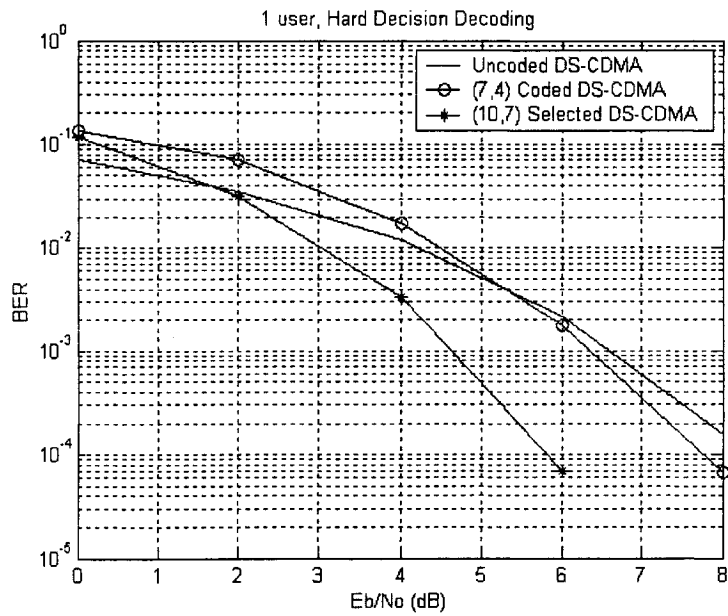


Figure 4-17: BER comparison of three DS-CDMA systems with 1 users and hard decision decoding

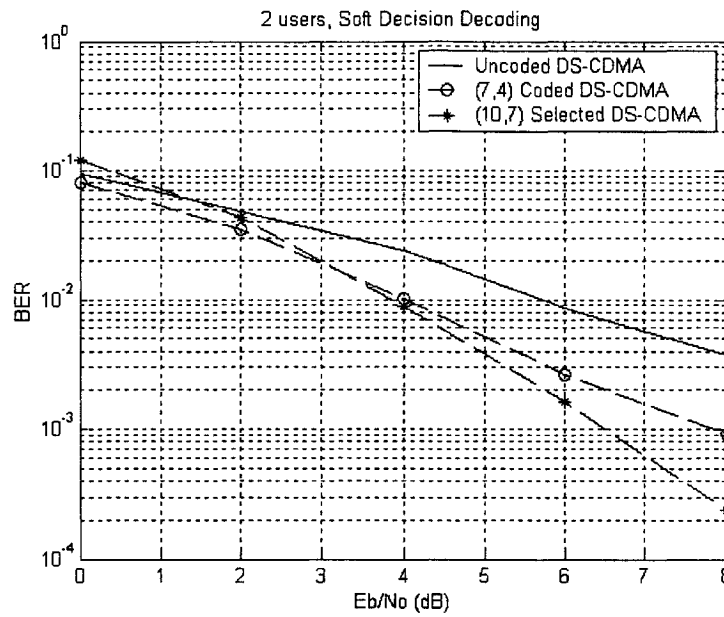


Figure 4-18: BER comparison of three DS-CDMA systems with 2 users and soft decision decoding

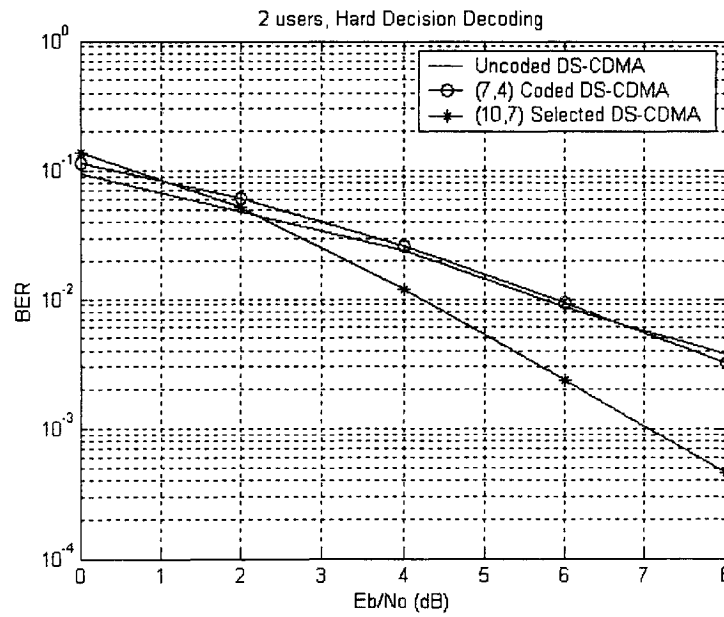


Figure 4-19: BER comparison of three DS-CDMA systems with 2 users and hard decision decoding

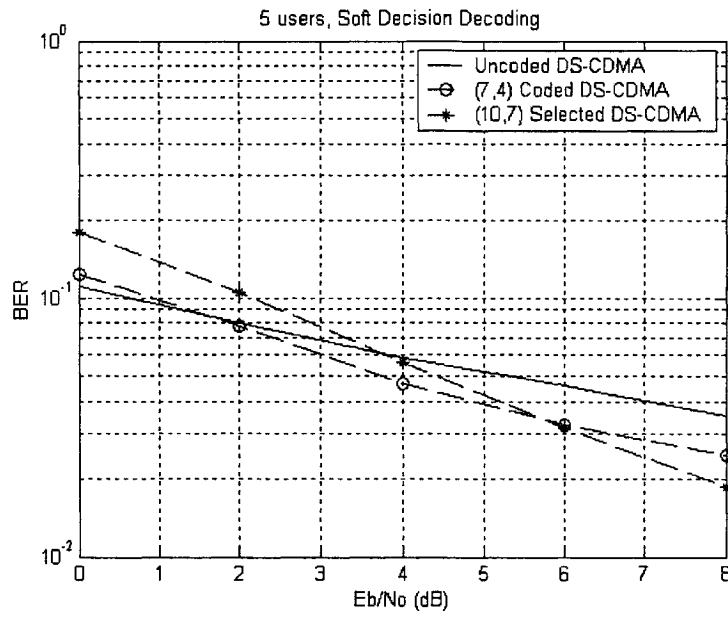


Figure 4-20: BER comparison of three DS-CDMA systems with 5 users and soft decision decoding

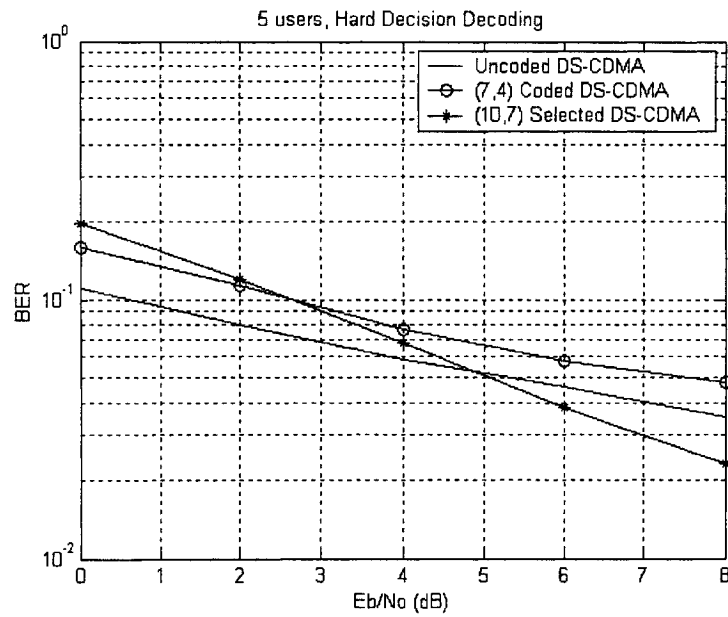


Figure 4-21: BER comparison of three DS-CDMA systems with 5 users and hard decision decoding

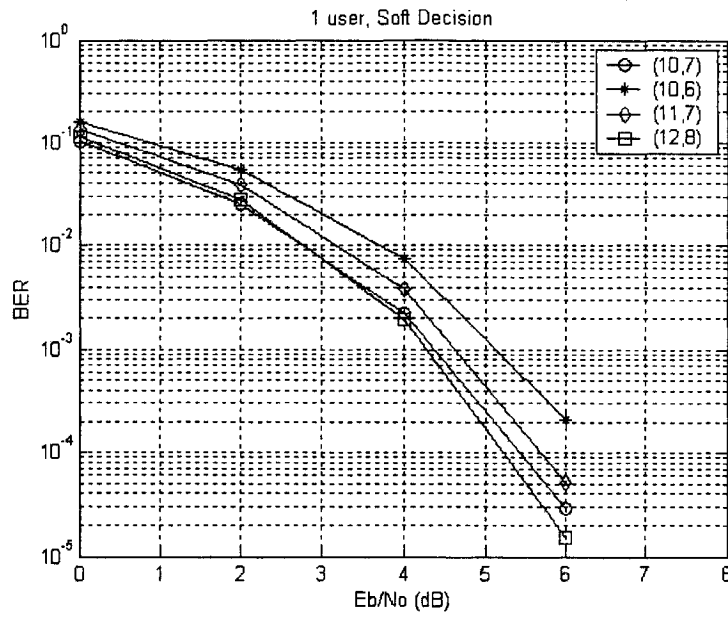


Figure 4-22: BER comparison of (10,7) with (10,6), (11,7) and (12,8) block codes with 1 user and soft decision decoding in parity bit selected DS-CDMA system

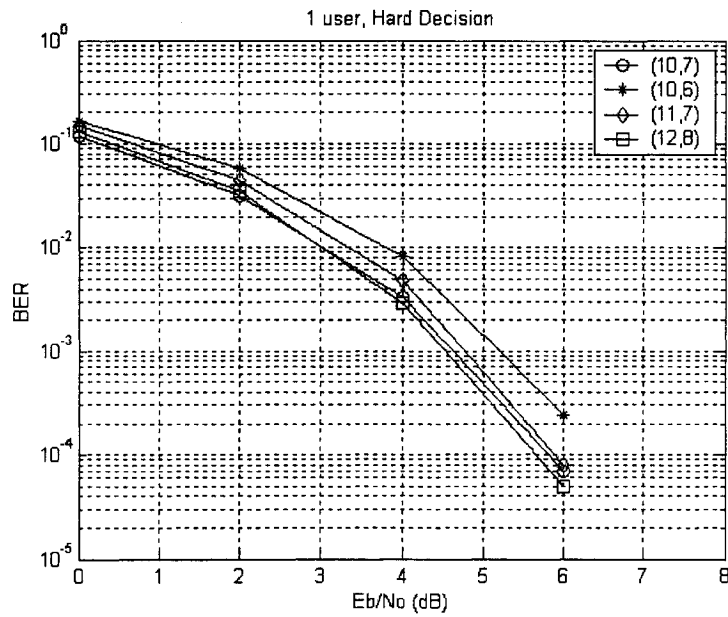


Figure 4-23: BER comparison of (10,7) with (10,6), (11,7) and (12,8) block codes with 1 user and hard decision decoding in parity bit selected DS-CDMA system

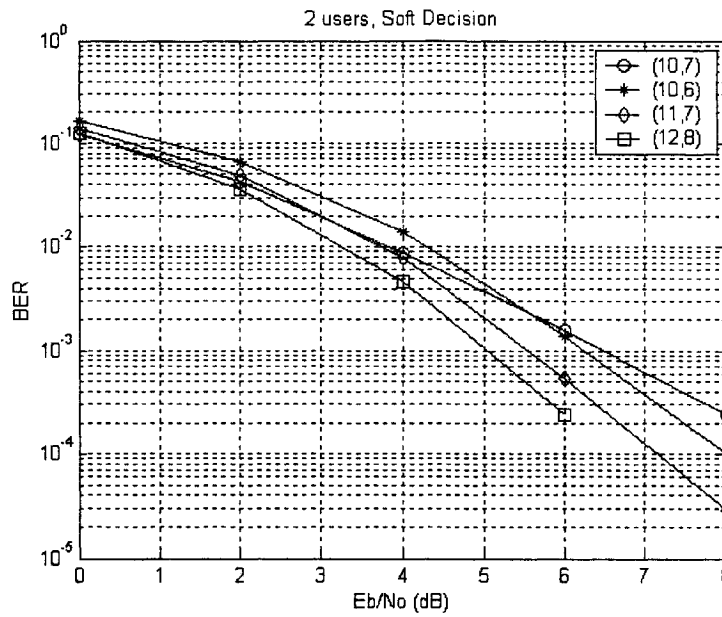


Figure 4-24: BER comparison of (10,7) with (10,6), (11,7) and (12,8) block codes with 2 users and soft decision decoding in parity bit selected DS-CDMA system

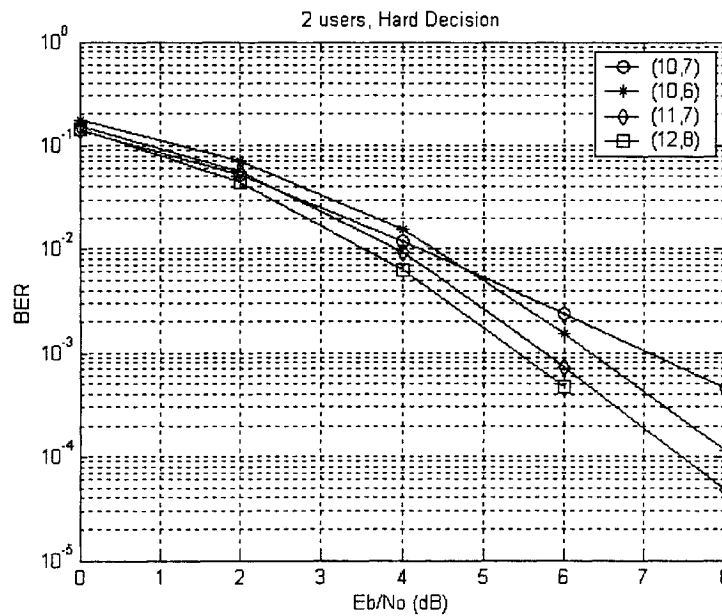


Figure 4-25: BER comparison of (10,7) with (10,6), (11,7) and (12,8) block codes with 2 users and hard decision decoding in parity bit selected DS-CDMA system

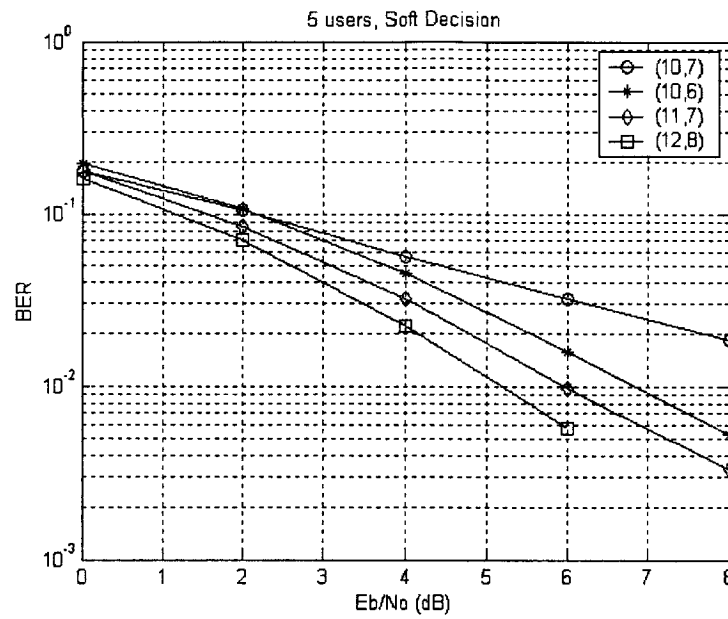


Figure 4-26: BER comparison of (10,7) with (10,6), (11,7) and (12,8) block codes with 5 users and soft decision decoding in parity bit selected DS-CDMA system

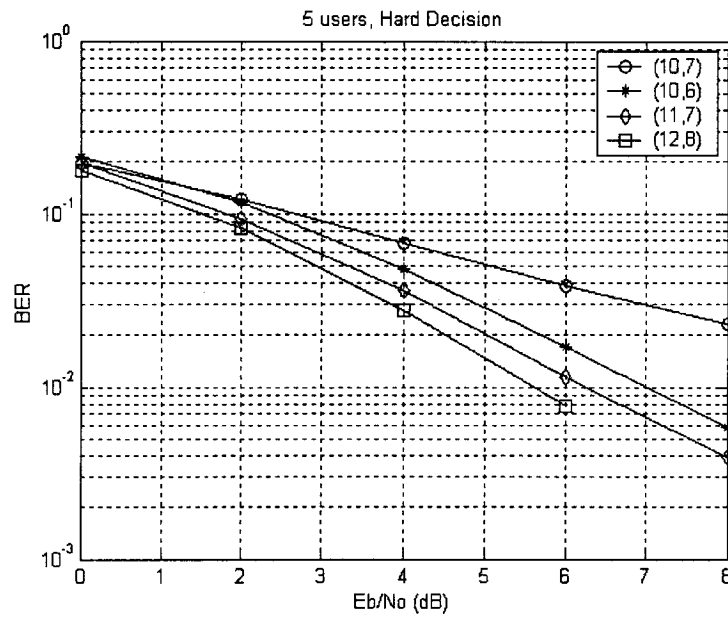


Figure 4-27: BER comparison of (10,7) with (10,6), (11,7) and (12,8) block codes with 5 users and hard decision decoding in parity bit selected DS-CDMA system

Chapter 5

5. Conclusions and Future Research

5.1 Conclusions

Parity bit selected DS-CDMA system discussed in this thesis is based on the parity bit selected spreading sequences for spread spectrum communication, which is first proposed in [1] and [7].

In parity bit selected DS-CDMA system, we combine two common techniques, block codes and DS-SS. At the transmitter, for a given information block of k bits, the corresponding $(n-k)$ parity bits are obtained according to the block code algorithm. The $(n-k)$ parity bits are used to select one spreading sequence from a set of $2^{(n-k)}$ mutually orthogonal spreading sequences. Note that in our proposed scheme, the parity bits are not appended to the end of the information block. The selected spreading sequence is used to spread all of the k bits in the information block. The spreading sequence may change every k -bit information block.

The transmitted signals are sent through an AWGN channel. In the multi-user system, for a given user, its signal is subjected to interference from other users. The interference is called MAI, which degrades the performance of the system.

At the receiver side, $2^{(n-k)}$ matched filters are employed to despread the received signal. Each filter is matched one of the $2^{(n-k)}$ mutually orthogonal spreading sequences,

which are uniquely assigned to a transmitter-receiver pair. By observing the sum of squares of the magnitudes of decision variables over the information block of k bits at the output of each matched filter, the spreading sequence employed by the transmitter can be determined by the receiver. Then the receiver decodes the k decision variables at the output of the determined matched filter to get the estimate of the original information block.

According to the principle introduced above, this thesis provided the approximations of probability of bit error in parity bit selected DS-CDMA system for both soft and hard decision decoding. The approximations of probability of bit error provide a tool for us to analyze the BER performance of the system in theory.

The simulation results of three DS-CDMA systems with different number of users showed that the existence of MAI degrades the quality of service and limits the number of users in the system.

In parity bit selected DS-CDMA system, there is not one block code that always provides the best BER performance for either soft decision decoding both hard decision decoding when E_b/N_o is small. These results are proved by theoretical analysis as well as simulation. When E_b/N_o is high, there are two block codes which always provide the best BER performance for soft decision decoding and hard decision decoding respectively. These results are proved by theoretical analysis.

The simulation results proved that, in the three DS-CDMA systems, our proposed parity bit selected DS-CDMA system always provides a better BER performance than uncoded DS-CDMA systems and block coded DS-CDMA systems employing block codes of similar complexity to our system. In our simulation, both block codes with 3

parity bits and block codes with 4 parity bits are employed. Single user, two users, and five users systems are considered.

In parity bit selected DS-CDMA systems, for block codes with the same length of codeword, the system using block code with 4 parity bits provides better BER performance than that using block code with 3 parity bits when E_b/N_o is larger than a certain threshold value. The same result is obtained for the systems using block codes with the same length of information word but different number of parity bits. The reason for this is that, as $(n-k)$ decreases, the number of information sequences per spreading code increases, and d_{min} in each coset is decreased. Thus, the probability of incorrectly determining PN sequence used by the transmitter decreases linearly, but the probability of incorrectly identifying the information word when the receiver correctly determines the PN sequence employed increases exponentially. As a result, the probability of bit error will increase as $(n-k)$ decreases.

In parity bit selected DS-CDMA system, each receiver uses $2^{(n-k)}$ matched filters to get the decision variables. However, in uncoded DS-CDMA and block coded DS-CDMA systems, each receiver only needs one matched filter. That is to say, parity bit selected DS-CDMA system is more complex than the other two DS-CDMA systems.

5.2 Future Research

In this thesis, we simulated and discussed performance of DS-CDMA systems on an AWGN channel. We can further exploit the performance of parity bit selected DS-CDMA systems in the fading multipath channels which are more appropriate for wireless communication [4].

In our simulation, most of probabilities of bit error are obtained when $E_b/N_o < 10\text{dB}$. The BER performance of parity bit selected DS-CDMA system at high E_b/N_o need to be simulated in the future research. Based on the results, we will be able to observe which block codes provide the best BER performance for soft and hard decision decoding in our proposed parity bit selected DS-CDMA system.

We can use our system in concatenation with other coding methods. Iterative decoding can be used to improve the receiver's ability of correctly identify which PN sequence was used over a given block.

The number of users employed in our simulation is at most 5 due to excessive running time at higher number of users. We need to exploit efficient decoding algorithms which approach the performance of MLD. As one of iterative decoding methods, the message passing algorithm is a good choice [32], [33]. If the receiver has incorrectly determined the spreading sequence, the message passing algorithm will most probably require many iterations without arriving at a solution. Thus, the receiver knows that the incorrect spreading sequence is obtained and it will try to decode the message with a different spreading sequence. If we find some more efficient decoding algorithms, we could easily simulate and observe the performance of more users systems.

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