Three essays on Environmental Economics and Industrial Organization:

Tradable Permits, Environmental R&D and Taxation

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Abstract

Chapter 1: Tradable Permits under Environmental and Cost-reducing R&D: This chapter models simultaneous investments in both environmental and cost-reducing R&D by asymmetric Cournot duopolist. Pollution rights (emission permits) are allocated by the regulator and can be traded between firms. Both R&D competition and cooperation are considered. In a three-stage game, firms first invest in R&D, then trade permits, and then compete in output. The strategic interaction between different types of R&D investments is analyzed. It is found that giving more permits to one firm induces it to conduct more cost-reducing but less environmental R&D. The second-best optimal allocation of pollution rights is also analyzed. This allocation matters for social welfare under R&D competition, but is irrelevant under R&D cooperation. Moreover, the optimal allocation depends on R&D spillovers. This paper also studies the grandfathering of permits based on historical output. Compared with the second-best optimal allocation, the higher the emissions reduction level, the more likely that grandfathering allocates too few permits to the large firm and too many permits to the small firm. Adding an R&D budget constraint leads firms to under-invest in cost-reducing R&D relative to environmental R&D.

Chapter 2: Tradable Permits under Environmental R&D between Upstream and Downstream Industries: This chapter models the simultaneous investments in environmental R&D by both downstream and upstream industries, with two symmetric firms within each industry competing à la Cournot. Pollution rights are allocated by the regulator, and firms can trade permits. R&D competition, intra-industry (horizontal), inter-industry (vertical) and both intra- and inter-industry (generalized) R&D cooperations are

considered. In a four-stage game, firms first invest in R&D, then trade permits, then upstream firms compete in intermediate good production, and finally downstream firms compete in final food production. The strategic interactions between R&D investments are analyzed. It is found that an increase in either vertical or horizontal R&D spillovers reduce the permit price but increase production, but the spillover effects on R&D investments are ambiguous and they depend on the number of permits that a firm receives from the government. However, firms undertake more R&D under generalized cooperation than vertical cooperation, irrespective of spillovers and the allocation of permits, and this results in higher social welfare under generalized cooperation than vertical cooperation. The optimal allocation of pollution rights by the regulator is also considered. This allocation matters for social welfare under R&D competition and horizontal cooperation, but is irrelevant under vertical and generalized cooperations.

Chapter 3: Is There a Principle of Targeting in Environmental Taxation?: This chapter studies whether the "principle of targeting", which is referred to by Dixit (1985) as the tax formulae for dirty goods have "additivity property" (Sandmo 1975) and externality-generating sources should be directly targeted (Bhagwati and Johnson 1960), can be applicable in the presence of a uniform commodity tax with an additional emissions tax. We consider three perfectly competitive markets, one of them produces a non-polluting good and the other two produce polluting goods. The regulator chooses optimal taxes on all three markets to maximize social welfare and finances an exogenous public expenditure. First all, it is found that the additivity property does not hold under differentiated taxes, and is even further weakened with a uniform commodity tax. It is also shown that the Pigouvian tax is unlikely to apply on the top of the uniform commodity tax. Furthermore, if there is only tax instrument available -- *i.e.* either the uniform commodity tax or the emissions tax -- then the uniform commodity tax (emissions tax) induces higher social welfare when marginal social damage is low (high).

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CONTENTS

General Introduction

My doctoral dissertation includes three chapters that study two important market-based environmental regulatory tools: permits and taxes. The first chapter, titled "Tradable Permits under Environmental and Cost-reducing R&D" and the second chapter, titled "Tradable Permits under Environmental R&D between Upstream and Downstream Industries" explore tradable emissions permits, and the third chapter, titled "Is There a Principle of Targeting in Environmental Taxation" focuses on environmental taxation.

The theme of the first chapter is the introduction of two types of R&D: the cost-reducing R&D that reduces the production cost, and the environmental R&D that reduces both the emissions and the compliance costs due to government's environmental regulation. It is well-known that tradable emissions permits provide emitters incentives to develop clean technologies through research and development (Downing and White 1986). However, once we begin considering R&D, then many other issues come up as well: (i) R&D has spillover effects which affect the optimal allocation of pollution permits, (ii) firms also conduct production related (cost-reducing) R&D, and (iii) firms either compete or cooperate in R&D. To date, few studies have analyzed the interaction between all of these issues concerning R&D and tradable emissions permits. The first chapter endeavors to do so.

To incorporate those issues, I model the imperfection of the output market with asymmetric Cournot duopoly. Furthermore, firms take permit price as given in the permit market. As I assume emissions in a fixed proportion to output quantities, firms' interaction in the permit market is indirectly determined through their strategic behavior in the output market. Hence, the permit price is endogenously set by firms' derived demand and supply of permits. In a three-stage game, firms first compete or cooperate in R&D investments, then trade permits, and then compete in the output market.

It is shown that R&D competition generates more (less) aggregate investments of both types than R&D cooperation when spillovers are low (high), even though R&D cooperation internalizes the spillover externality. This result echoes the R&D literature with only cost-reducing R&D (d'Aspremont and Jacquemin 1988).

In terms of the interdependency of R&D choices, regardless whether firms compete or cooperate in R&D, and regardless whether spillovers are low or high, an increase in cost-reducing R&D by one firm always reduces the other firm's return from cost-reducing R&D but always increases the other firm's return from environmental R&D. an increase in R&D by one firm reduces (increases) the other firm's return from R&D when spillovers are low (high). This result is different from previous literature (d'Aspremont and Jacquemin 1998, Liao 2007, Leahy and Neary 2005) that, in the context of only one type of R&D (cost-reducing), an increase in R&D by one firm reduces (increases) the other firm's return from R&D when spillovers are low (high).

As for the effect of the initial permit allocation on R&D choices, the comparative statics of the model reveal that the more permits a firm receives, the more it conducts cost-reducing R&D but less environmental R&D. This implies that government needs to take into account which type of R&D it tends to trigger when allocating permits to firms.

The first chapter also reexamines the grandfathering of permits based on historical output level. In the literature, grandfathering permits is often criticized because, by giving a large proportion of permits to a small number of firms, it will likely lead market power to large established firms. However, by comparing the optimal allocation of permits and the allocation via grandfathering, it is shown that, in the presence of these two types of R&D, grandfathering does not always allocate too many permits to the large firm. In particular, grandfathering allocates too many permits to the large firm when the R&D spillover rate is high; in contrast, when the emission reduction level is high, grandfathering allocates too few permits to the large firm.

This chapter also studies how firms allocate limited financial resources between the two types of R&D investments when facing an R&D budget constraint. This issue is particularly relevant during economic downturn when firms are financially constrained to undertake R&D. By adding an R&D budget, it is found that firms will reduce R&D investments for both types, compared with unconstrained situation. Furthermore, the reduction in cost-reducing R&D investment is more considerable than the reduction in environmental R&D.

In the first chapter, permits are traded within the same industry, but in the real world, permits are often traded between different industries since they may emit the same pollutant (for example, both petroleum and plastic industries emit CO_2). The second chapter extends the study in the first chapter by allowing permits to be traded between two vertically related industries. For example, within the European Union Emissions Trading Scheme (EU ETS), permit trading takes place among energy producers and energy-intensive industries, such as power generators, steel & iron and combustion activities. For tractability, this chapter only considers that firms undertake environmental R&D.

In this chapter, I model two vertically related industries. The upstream industry produces intermediate good, and the downstream industry purchases the intermediate good as input to produce final good. Both industries emit the same pollutant as by-product. There are symmetric Cournot duopolists in each industry, but firms in different industries are asymmetric (with different marginal costs). Furthermore, I assume that firms in the same industry receive equal number of permits due to symmetry. Therefore, permits are traded only between industries. Moreover, firms take permit price as giving when trading permits. To reduce abatement costs, firms undertake environmental R&D which spills between all firms. Hence, firms face two types of R&D spillovers, *i.e.* the horizontal R&D spillover when technology spills within the same industry, and the vertical R&D spillover when technology spills between different industries.

The model includes 4 stages: first of all, firms either compete or cooperate in R&D investments and then they trade permits, after that, upstream firms compete in intermediate good and downstream firms purchase intermediate good as input to produce final good, and finally downstream firms compete in the final good market. When firms cooperate in R&D, they can either cooperate within the same industry (horizontal cooperation), between industries (vertical cooperation) or both within and between industries (generalized cooperation).

It is found that higher horizontal or vertical R&D spillovers reduce the permit price but increase production from both industries. In terms of spillover effects on R&D

investments, even though the effects are ambiguous, firms invest more under generalized cooperation than vertical cooperation, irrespective of spillovers. Moreover, it is shown that generalized cooperation leads to more investment than vertical cooperation, since the former internalizes both the horizontal and vertical externalities while the latter only internalizes the vertical externality.

This chapter also considers the optimal allocation of permits. The allocation matters for social welfare under R&D competition and horizontal cooperation, but is irrelevant under either vertical or generalized cooperation. Furthermore, generalized cooperation leads to higher social welfare than vertical cooperation.

Another market-based environmental regulatory tool is the emissions tax. Pigou (1920) demonstrates that the optimal emissions tax should equal the marginal social damage. However, this first-best Pigouvian tax is subject to strict assumptions such as perfect competition and low transaction cost. In the real world, market power may exist and governments use distortionary taxes to pay for public goods. In the first best policy, the regulator has dedicated instruments to deal with each distortion directly and lump-sum taxes to raise revenue. In this case, the Pigouvian rule remains the optimal choice for an emission tax. In contrast, in second best policy, the regulator does not have enough instruments at hand, which does not in general eliminate all distortions completely. In this case, the optimal emission tax does not usually follow the Pigouvian rule (Buchanan 1969, Barnette 1980, Lavin 1985, Shaffer 1995, Goulder 1995, Bevenberg and de Mooij 1994).

However, several authors argue that there is still a role for the Pigouvian rule in the second best setting. Sandmo (1975) illustrates that the pollution externality only appears in the tax formulae for the dirty goods, and such tax formulae can be decomposed into two weighted average of two parts: the efficiency term resembles the theory of optimal taxation and the term with marginal social damage. This result is defined as the "additivity property". Bhagwati and Johnson (1960) argue that sources of distortions should be directly addressed by a tax instrument. Dixit (1985) refers both Sandmo's additivity property and Bhagwati-Johnson's targeting theory as the "principle of targeting". Kopczuk (2003) further generalizes the principle of targeting by establishing that the tax formulae for the pollution generating goods are composed by the Pigouvian tax plus a correction tax / subsidy.

Sandmo's additivity property is based on differentiated commodity taxes. However, this framework seems unrealistic. In the real world, most commodity taxes are uniform across goods. Therefore, the third chapter studies whether the additivity principle holds with a uniform commodity tax. It first examines the demonstration of the additivity property. Surprisingly, Sandmo's conclusion holds only without explicitly solving for the marginal cost of public funds (MCPF). Once MCPF is precisely solved, the externality will appear in the tax formulae for both clean and dirty goods. In addition, the externality cannot be additively separable in the tax formulae for dirty goods. As a result, the principle of targeting does not hold even under differentiated taxes.

The chapter then studies the case where a uniform commodity tax is imposed on all goods, and an emissions tax is applied to dirty goods on top of the uniform commodity tax. It is found that when the revenue requirement is funded by both taxes, Sandmo's additivity property is further weakened, as the emissions externality appears in the tax formulae for both the commodity tax and the emission tax

The chapter also examines two cases where only one tax – i.e. either the uniform commodity tax or the emissions tax – is used to finance government spending. It is found that the uniform commodity tax induces higher social welfare than the emissions tax when the marginal social damage is low, and the result is reversed with high marginal social damage. In other words, in this same-numbers exercise, it is not true that it is always better to address the pollution externality directly through a dedicated emissions tax. Therefore, we conclude that there does not exist an environmental principle of targeting which is distinct from the benefit of adding an additional tax instrument.

Chapter 1

Tradable Permits under Environmental and Cost-reducing R&D

1 Introduction

Economists have long advocated the use of tradable emissions permits to control pollution. There is currently a debate among economists about which approach is better for the initial allocations of permits – either free allocations or auctioning (e.g. Parry 2002, Cropper et al. 1992, Stavins 1998 and Simshauser 2008). In practice, most systems have used a free allocation and this paper takes this approach as well.

It is well-known that tradable emissions permits provide emitters incentives to develop clean technologies through research and development (Downing and White 1986). However, once we begin considering R&D, then many other issues come up as well: (*i*) R&D has spillover effects which affect the optimal allocation of pollution permits, (*ii*) firms also conduct production related (cost-reducing) R&D, and (*iii*) firms either compete or cooperate in R&D. To date, few studies have analyzed the interaction between all of these issues concerning R&D and tradable emissions permits. The present paper endeavours to do so.

The paper also considers market power in the output market. The early literature focuses on perfect competition in both the output and the permit markets (Montgomery 1972). However, more recent research has shown that market power in either of these markets affects the efficiency of the final equilibrium (Hahn 1984, Hintermann 2009, Sartzetakis and McFetridge 1999). To account for this problem, this paper assumes market power in the output market.

In contrast, firms take permit price as given in the permit market. Since emissions are assumed to be in fixed proportion to output quantities, firms' interaction in the permit market is indirectly determined through their strategic behavior in the output market. Hence, the permit price is endogenously set by firms' derived demand and supply of permits.[1]

[1] In principle, strategic behaviour may also occur in the permit market. However, the core of the bilateral bargaining game which characterizes the permit market may be large, and therefore precise results may not be possible. For tractability, price taking is assumed. Sanin and Zanaj (2007) use this

We develop a model with asymmetric Cournot duopolists that trade permits and invest in both cost-reducing and environmental R&D. In a three-stage game, firms first invest in R&D, then trade permits and then compete in output. Both R&D competition and cooperation are considered, and firms face R&D spillovers in each type of R&D investment.[2] In this framework, we compare (*i*) the effects of competition and cooperation on total R&D investment, and we investigate (*ii*) the interdependency between firms' R&D choices, (*iii*) the effect of the initial permit allocation on each firm's R&D choices, (*iv*) the optimal initial allocation of permits between large and small firms, and (*v*) the effect of an R&D budget constraint on firms' R&D decisions.

It is shown that R&D competition generates more (less) aggregate investments of both types than R&D cooperation when spillovers are low (high), even though R&D cooperation internalizes the spillover externality. This result echoes the R&D literature (d'Aspremont and Jacquemin 1988).

In terms of the interdependency of R&D choices, previous authors (d'Aspremont and Jacquemin 1998, Liao 2007, Leahy and Neary 2005) have shown that, in the context on only one type of R&D (cost-reducing), an increase in R&D by one firm reduces (increases) the other firm's return from R&D when spillovers are low (high). This effect holds both when firms compete in R&D and when they cooperate.[3] In the first case (competition in R&D), this effect is referred to as strategic substitutability (complementarity), whereas in the second case (cooperation in R&D), this effect is referred to as cooperative substitutability (complementarity).

In contrast, the current model, with two types of R&D, shows that, regardless whether firms compete or cooperate in R&D, and regardless whether spillovers are low or high, R&D investments of the same type are substitutes, but R&D investments of different

approach as well.

[2] R&D cooperation can be realized either through private channels or through government coordination. For example, the government of Israel has established the Global Enterprise R&D Cooperation Framework to encourage cooperation in industrial R&D between Israel and MultiNational Companies.

[3] When they cooperate, the effect is on the joint rather than individual profitability.

types are complements. For example, an increase in cost-reducing R&D by one firm always reduces the other firm's return from cost-reducing R&D but always increases the other firm's return from environmental R&D.

As for the effect of the initial permit allocation on R&D choices, the comparative statics of the model reveal that the more permits a firm receives, the more it conducts cost-reducing R&D but less environmental R&D. This implies that government needs to take into account which type of R&D it tends to trigger when allocating permits to firms. The socially optimal allocation of permits is then analyzed. Much of the existing literature focuses on the role of market power in determining the optimal allocation of permits (e.g. Hahn 1984). In contrast, the present paper shows that the optimal allocation of permits between large and small firms also depends on the strength of R&D spillovers and on the stringency of the emissions cap.

When the stringency of emissions cap is held constant, higher spillovers lower the share of permits large firm receives in the optimum, i.e. the large firm receives fewer permits compared with grandfathering. Because the large firm has lower marginal production cost and hence larger market share, it is able to generate greater social benefit from a given expenditure on cost-reducing R&D. Therefore, when R&D spillovers are low, it is socially desirable to focus cost-reducing R&D on the large firm. As noted above, this outcome is achieved by allocating more pollution permits to the large firm. However, this effect is relaxed as spillovers increase, because the large firm is better able to access the benefit of cost-reducing R&D undertaken by the small firm. Therefore, in general, assuming an exogenous emissions cap, the greater the rate of spillover, the fewer permits are allocated to the large firm in the optimum.

In contrast, when the rate of spillover is held constant, then increasing the stringency of the emissions cap increases the share of permits that the large firm receives in the optimum, i.e. the large firm receives even more permits compared with grandfathering. This result follows from the fact that total production depends on the emissions cap.

The more stringent the emissions cap, i.e. the fewer permits the regulator issues towards firms, the lower total production will be in the economy. It therefore becomes increasingly important that production be concentrated where costs are lowest – the large firm. Achieving this concentration of production is achieved by allocating proportionately more pollution permits to the large firm. This result appears to contradict the common notion that it is undesirable to skew the allocation of permits toward large firms, for example through grandfathering.

We also study the effect of an R&D budget constraint on firms' R&D decisions. This issue is especially relevant during an economic downturn, where firms face reduced profits and growth. In this context, firms will cut R&D investments of both types. However, since the less environmental R&D firms undertake, the more pollution rights they will need to hold, and hence the more permits they have to purchase from their competitor, they will cut more cost-reducing R&D investment relative to environmental R&D investment.

The paper is organized as follows. Section 2 introduces the basic framework. Sections 3 and 4 present the R&D competition and cooperation models, and provide some comparative statics results. Section 5 compares the competitive and cooperative equilibrium results. Section 6 studies the second-best allocations of permits with exogenous and endogenous permits caps. Section 7 analyzes the difference between the grandfathering of permits based on historical quantity-ratio and the second-best allocations of permits. Section 8 examines the impacts on R&D outcomes in the presence of the R&D budget constraint. Section 9 concludes.

2 The basic framework

Consider two Cournot asymmetric firms, 1 and 2, producing a homogeneous good in quantities q_1 and q_2 by adopting a constant returns to scale technology. The inverse market demand is defined as $P = a - Q$, where $Q = q_1 + q_2$ denotes the aggregate production. Without loss of generality, we assume the marginal production cost to be $c + k$ for firm 1 and $c - k$ for firm 2, with $k \in [0, c)$. Hence, firm 1 (the small firm) has a strictly higher marginal cost than firm 2 (the large firm). Each firm invests in cost-reducing R&D to reduce production cost, and environmental R&D to reduce pollution. Firms face R&D spillovers $\beta \in [0, 1]$, which allow them to benefit from each other's technologies without payment. Thus, spillovers reduce both costs and emissions. For simplicity, we assume the spillover rates of the two types of R&D are identical.

With cost-reducing R&D, firm 1's production cost can be reduced from $c + k$ to $c + k - x_1 - \beta x_2$, where x_1 represents the R&D output of firm 1 and x_2 of firm 2. The cost of R&D is γx_1^2. Similarly, firm 2's production cost is reduced from $c - k$ to $c - k - x_2 - \beta x_1$, and the R&D cost is γx_2^2.

Production causes pollution. If there is no regulation restraining pollution, then firms have no incentives to abate emissions. Without installing any clean technology, the emission level for each firm just equals output, *i.e.* $f_1 = q_1$ and $f_2 = q_2$. Thus, the environmental regulation not only targets to reduce this negative externality, but also promotes clean innovations. Through investing in environmental R&D, the emission levels are cut to $f_1 = q_1 - w_1 - \beta w_2$ for firm 1 and $f_2 = q_2 - w_2 - \beta w_1$ for firm 2, where w_1 and w_2 represent the individual environmental R&D levels and δw_1^2 and δw_2^2 the costs of environmental R&D respectively. Hence, firm 1 (2) needs f_1 (f_2) permits to pollute and produce. In addition, we assume $\gamma \geqslant 1$ and $\delta \geqslant 1$ so that the convex R&D cost functions ensure the second order conditions for R&D maximization problems hold (Banerjee and Lin, 2003). Furthermore, γ and δ also need to satisfy the condition $-9 + 4\gamma + 4(-3 + 4\gamma)\delta > 0$.

Firms can either compete or cooperate in R&D activities but remain competitive in the output market. Under R&D cooperation, we only consider the case that firms cooperate in both types of R&D.

Government regulates emissions by freely allocating pollution rights – emission permits, namely e_1 and e_2, to each firm. The total number of permits is $L = e_1 + e_2$. In addition, $f_1 + f_2 = e_1 + e_2$; in other words, firms are not allowed to bank permits. Nevertheless, in general, the initial allocations of permits are not equivalent to firms' exact needs, thus permit trading will take place. The equilibrium unit permit price σ is obtained such that the demand for permits equals the supply of them. Each firm determines how many permits it wants to buy or sell, taking the permit price as given. If firm 1 gets fewer (more) permits than what it needs, *i.e.* $e_1 < f_1$ ($e_1 > f_1$), then it then will buy (sell) $e_1 - f_1$ permits from (to) firm 2. Meanwhile, firm 2 will need to sell (buy) $e_2 - f_2$ permits to (from) firm 1 as it gets more permits than it needs, *i.e.* $e_2 > f_2$ ($e_2 < f_2$) . Moreover, because of the duopolistic structure of the industry, each firm's decisions on permit trading and R&D investments will have a considerable impact on the number of traded permits and their price.

Let us first consider the situation where firms compete in investing in both cost-reducing and environmental R&D.

3 R&D competition model

Firms play a three-stage game: they first invest in both types of R&D, then trade permits, and finally compete in production. The game is analyzed by using backward induction.

3.1 Third stage: quantity competition

The total costs of production and R&D investments for firm 1 and 2 are:

$$C_1 = (c + k - x_1 - \beta x_2)\, q_1 + \gamma x_1^2 + \delta w_1^2 \tag{1}$$

$$C_2 = (c - k - x_2 - \beta x_1)\, q_2 + \gamma x_2^2 + \delta w_2^2 \tag{2}$$

If a firm is a permit buyer (seller), it pays (receives) $\sigma(f_i - e_i)$ in permits trading. The profit function for firm i is:

$$\begin{aligned} \pi_i &= Pq_i - C_i - \sigma(f_i - e_i) \\ &= Pq_i - C_i - \sigma(q_i - w_i - \beta w_j - e_i) \end{aligned} \tag{3}$$

In this stage, firm i's profit is maximized by choosing quantity. The two first-order necessary conditions with respect to q_1 and q_2 are:

$$\frac{\partial \pi_1}{\partial q_1} = (a - 2q_1 - q_2) - (c + k - x_1 - \beta x_2) - \sigma = 0 \tag{4}$$

$$\frac{\partial \pi_2}{\partial q_2} = (a - 2q_2 - q_1) - (c - k - x_2 - \beta x_1) - \sigma = 0 \tag{5}$$

Rewriting (4) and (5), we get:

$$(a - 2q_1 - q_2) - (c + k - x_1 - \beta x_2) = \sigma \tag{6}$$

$$(a - 2q_2 - q_1) - (c - k - x_2 - \beta x_1) = \sigma \tag{7}$$

The left hand sides of (6) and (7) establish marginal revenue ($MR_i = P - q_i$) minus marginal cost, which can be considered as marginal abatement cost[4] through reducing quantities q_i; the right hand sides are the unit price of permits. (6) and (7) demonstrate that firms set marginal abatement cost equal to the unit permit price to maximize profits: the permit price is just the forgone net profit (Mansur, 2007). However, Sartzetakis and McFetridge (1999) indicate that "equalization of marginal abatement cost across firms yields the efficient distribution of abatement effort, but due to the oligopolistic product market structure, it cannot achieve the efficient production

[4]Mckitrick (1999) defines two terminologies of abatement: (1) costly undertaking which reduces emissions subject to diminishing returns, and (2) simply emission reductions from individual pollution level without environmental regulation. In this chapter, I focus on the second terminology.

allocation.... Trading of permits does not necessarily yield the first-best allocation of resources when product markets are imperfectly competitive" (Sartzetakis and McFetridge, 1999:49).

Solving (4) and (5) for interior solutions[5] yields:

$$q_1 = \frac{1}{3}[a - c - 3k - \sigma + (2 - \beta)x_1 - (1 - 2\beta)x_2] \tag{8}$$

$$q_2 = \frac{1}{3}[a - c + 3k - \sigma - (1 - 2\beta)x_1 + (2 - \beta)x_2] \tag{9}$$

3.2 Second stage: permits trading

Firms are price takers in the permit market, and the equilibrium price is such that the demand for permits equals their supply. Moreover, firms are not allowed to bank permits. Substituting (8) and (9) into the permit market-clearing condition $f_1 + f_2 = e_1 + e_2$ yields:

$$\sigma = \frac{1}{2}[2(a - c) - 3(e_1 + e_2) - 3(1 + \beta)(w_1 + w_2) + (1 + \beta)(x_1 + x_2)] \tag{10}$$

(10) clearly shows that the impact of environmental R&D on the permit price is negative. Environmental R&D reduces the demand for permits by a permit buyer, and increases the supply of permits by a seller. This is consistent with Montero (2002a), who employs a permits-Cournot model. Besides, the permit price positively depends upon the cost-reducing R&D, as lower marginal cost increases production and pollution and hence the need and value for pollution rights. Substituting (10) into (8) and (9) provides

$$q_1 = \frac{1}{2}[e_1 + e_2 - 2k + (1 + \beta)(w_1 + w_2) + (1 - \beta)(x_1 - x_2)] \tag{11}$$

$$q_2 = \frac{1}{2}[e_1 + e_2 + 2k + (1 + \beta)(w_1 + w_2) - (1 - \beta)(x_1 - x_2)] \tag{12}$$

[5] The second-order sufficient conditions $\frac{\partial^2 \pi_1}{\partial q_1^2} = \frac{\partial^2 \pi_2}{\partial q_2^2} = -2 < 0$ assure unique maximum solutions.

(11) and (12) demonstrate that an increase in w_i increases both q_i and q_j, even if technologies do not spill, *i.e.* $\beta = 0$: higher w_i leads firm i to produce more with less need of permits, thus firm j can keep more permits and hence increase production. The higher the spillovers, the higher both q_i and q_j. Moreover, through the interaction in the permit market, an increase in x_i will reduce q_j, despite of β. This is different from the standard R&D literature (d'Aspremont and Jacquemin 1988), where one firm's cost-reducing R&D investment can either increase or decrease its rival's production, depending on the critical value of β. In the current model, the lower production cost induces more production from firm i and higher permit price. If firm j is a permit buyer (seller), it will reduce production so that it needs to buy (sell) fewer (more) permits from firm i. Moreover, the production expansion of firm i drives up its demand for permits, this leaves firm j to keep fewer permits and produce even less. However, x_i and x_j do not affect either q_i or q_j in the case with full spillovers, *i.e.* $\beta = 1$.

3.3 First stage: R&D competition

In the first stage, both firms choose R&D investments (x_i, w_i) simultaneously to maximize their own profits by substituting (11) and (12) into (3):

$$(\overline{x}_i, \overline{w}_i) = \arg\max_{x_i, w_i} \ \pi_i \tag{13}$$

We want to investigate the strategic interactions between R&D investments before explicitly solving for $(\overline{x}_i, \overline{w}_i)$.

Proposition 1 *The R&D investments of the same type (environmental or cost-reducing) are strategic substitutes, but there is strategic complementarity across different types of R&D.*

Proof. The second order partial derivatives of (13) are

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} &= -\frac{1}{2}(1-\beta)^2 \leq 0 & \frac{\partial^2 \pi_i}{\partial w_i \partial w_j} &= -(1+\beta)^2 < 0 \\
\frac{\partial^2 \pi_i}{\partial w_i \partial x_j} &= \frac{1}{2}\beta(1+\beta) > 0 & \frac{\partial^2 \pi_i}{\partial x_i \partial w_j} &= \frac{1}{2}(1+\beta) > 0 \\
\frac{\partial^2 \pi_i}{\partial x_i \partial w_i} &= \frac{1}{2}(2+\beta-\beta^2) > 0
\end{aligned}$$

The first inequality holds strictly when $\beta < 1$. ■

Let us first study the strategic substitutability between the same types of R&D. Following Liao (2007), the strategic substitutability depends on two factors: (i) the spillover effect, and (ii) the rivalry effect. Without loss of generality, assume firm 1 to be a permit buyer and thus firm 2 to be a permit seller. (4) and (10) shows that when x_1 increases by 1 unit, the marginal cost from both producing and purchasing permits of firm 1 decrease by $\frac{1}{2}(1-\beta)$. Hence, q_1 needs to increase by $\frac{1}{4}(1-\beta)$ so that the marginal revenue equals marginal cost.[6]The increase in x_1 lowers the marginal cost of firm 2 by β (5) but increases its marginal revenue from selling permits by $\frac{1}{2}(1+\beta)$ (10); also, the increase in q_1 decreases its marginal revenue from the output market by $\frac{1}{4}(1-\beta)$.[7]Altogether, the aggregate marginal revenue increases, which encourages firm 2 to invest more in x_2 so that higher q_2 will drive down the marginal revenue and increase the marginal cost, and the profit maximization (*i.e.* marginal revenue = marginal cost) condition will be maintained. This is the negative spillover effect since the increase in x_1 increases x_2.

At the same time, the increase in x_2 decreases q_1 (11), which increases the prices of both output and permits, as well as the aggregate marginal revenue of firm 2. Thus, firm 2 is willing to reduce q_2[8] and hence reduce x_2. This is positive rivalry effect since the increase in x_1 decreases x_2. Overall, the positive rivalry effect dominates the negative spillover effect, as a result, an increase in x_1 decreases x_2, whence the strategic

[6] $\frac{\partial MR_1}{\partial q_1} = -2$.

[7] $\frac{\partial MR_2}{\partial q_1} = -1$.

[8] Recall that the total revenue is concave in quantity.

substitutability between x_1 and x_2. With β increasing, the difference between the two countervailing effects becomes smaller, which alleviates the substitutability. When $\beta = 1$, the strategic substitutability between x_1 and x_2 diminishes; in other words, how much firm 2 invests in x_2 does not affect firm 1's decision on x_1 and hence on q_1 (11).

The same logic explains the strategic substitutability between w_1 and w_2. However, higher β reinforces the strategic substitutability by increasing the difference between the two opposite effects.

Intuitively, when firm 1 increases its cost-reducing R&D, it needs more permits to produce, which increases (decreases) its demand (supply) for permits, thus firm 2 can hold fewer permits in hands, which lower its incentive to conduct cost-reducing R&D. This interaction through the permit market further reduces firm 2's investment on cost-reducing R&D, whence the strategic substitutability between x_1 and x_2. Likewise, when firm 1 invests more in environmental R&D, it needs fewer permits, which decreases (increases) the demand (supply) for permits. Thus, firm 2 can hold more permits in hand and further reduces its investment on environmental R&D, whence the strategic substitutability between w_1 and w_2.

Next, let us examine the strategic complementarity between different types of R&D. It is difficult to follow the spillover effect and the rivalry effect. Instead, taking derivatives of (13) with respect to x_i and w_i yields

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial x_i \partial w_i} &= P' \frac{\partial q_j}{\partial x_i} \frac{\partial q_i}{\partial w_i} + \frac{\partial q_i}{\partial w_i} - \frac{\partial \sigma}{\partial x_i} (\frac{\partial q_i}{\partial w_i} - 1) \\ &= \frac{1}{4}(1-\beta^2) + \frac{1}{2}(1+\beta)[2 - \frac{1}{2}(1+\beta)] = \frac{1}{2}(2+\beta-\beta^2) \end{aligned} \tag{14}$$

The first term of (14) represents the impact of w_i on firm i's marginal revenue, and rest indicates the impact of w_i on firm i's marginal cost. Higher w_i increases firm i's marginal revenue but decreases its marginal cost, which makes firm i more profitable at the margin. Therefore, firm i is willing to invest more in x_i to produce more so that the profit maximization condition will be satisfied, whence the strategic complementarity between x_i and w_i. The same reason demonstrates the strategic complementarity between w_i and x_j, as well as x_i and w_j.

Intuitively, when firm i invests more in w_i, the same production generates less pollution, thus firm i increases x_i for more production and to maintain the same pollution level, whence the strategic complementarity between x_i and w_i. When firm j conducts more x_j (w_j), it needs more (fewer) pollution rights and hence increases (decreases) its demand (supply) for permits, thus firm i tends to invest more in w_i (x_i) to produce since it holds fewer (more) permits in hand, whence the strategic complementarity between $x_j(w_j)$ and $w_i(x_i)$.

In d'Aspremont and Jacquemin (1998) where there is only cost-reducing R&D, the strategic interactions depend on β: $\beta < \frac{1}{2}$ leads to strategic substitutability and $\beta > \frac{1}{2}$ generates strategic complementarity. Liao (2007) confirms this conclusion with a more general critical value of β. However, the current model demonstrates that in the presence of environmental R&D and tradable permits, the substitutability and complementarity stand irrespective of β.

Then, solving the four first-order necessary conditions for interior solutions yields the equilibrium R&D investments $(\overline{x}_i, \overline{w}_i)$.[9]

3.4 Comparative statics

We now present comparative statics results to show how changes in cost and permit allocation affect the equilibrium R&D investments, output levels and the needs of permit. Since the competitive results are algebraically complex, we focus on two extreme values of spillovers: $\beta = 0$ and $\beta = 1$. Table 1 displays the comparative statics results. Proofs are shown in Appendix 1.

[9] The second-order Hessian matrix is negative definite. Details are provided in Appendix 1.

Table 1: Comparative Statics of Competitive R&D Results

$\beta = 0$			$\beta = 1$		
$\frac{\partial \overline{x}_1}{\partial k} < 0$	$\frac{\partial \overline{x}_1}{\partial e_1} > 0$	$\frac{\partial \overline{x}_1}{\partial e_2} > 0$	$\frac{\partial \overline{x}_1}{\partial k} = 0$	$\frac{\partial \overline{x}_1}{\partial e_1} > 0$	$\frac{\partial \overline{x}_1}{\partial e_2} < 0$
$\frac{\partial \overline{x}_2}{\partial k} > 0$	$\frac{\partial \overline{x}_2}{\partial e_1} > 0$	$\frac{\partial \overline{x}_2}{\partial e_2} > 0$	$\frac{\partial \overline{x}_2}{\partial k} = 0$	$\frac{\partial \overline{x}_2}{\partial e_1} < 0$	$\frac{\partial \overline{x}_2}{\partial e_2} > 0$
$\frac{\partial \overline{w}_1}{\partial k} < 0$	$\frac{\partial \overline{w}_1}{\partial e_1} < 0$	$\frac{\partial \overline{w}_1}{\partial e_2} < 0$	$\frac{\partial \overline{w}_1}{\partial k} < 0$	$\frac{\partial \overline{w}_1}{\partial e_1} < 0$	$\frac{\partial \overline{w}_1}{\partial e_2} > 0$
$\frac{\partial \overline{w}_2}{\partial k} > 0$	$\frac{\partial \overline{w}_2}{\partial e_1} < 0$	$\frac{\partial \overline{w}_2}{\partial e_2} < 0$	$\frac{\partial \overline{w}_2}{\partial k} > 0$	$\frac{\partial \overline{w}_2}{\partial e_1} > 0$	$\frac{\partial \overline{w}_2}{\partial e_2} < 0$
$\frac{\partial \overline{f}_1}{\partial k} < 0$	$\frac{\partial \overline{f}_1}{\partial e_1} > 0$	$\frac{\partial \overline{f}_1}{\partial e_2} < 0$	$\frac{\partial \overline{f}_1}{\partial k} < 0$	$\frac{\partial \overline{f}_1}{\partial e_1} = 0$	$\frac{\partial \overline{f}_1}{\partial e_2} = 0$
$\frac{\partial \overline{f}_2}{\partial k} > 0$	$\frac{\partial \overline{f}_2}{\partial e_1} < 0$	$\frac{\partial \overline{f}_2}{\partial e_2} > 0$	$\frac{\partial \overline{f}_2}{\partial k} > 0$	$\frac{\partial \overline{f}_2}{\partial e_1} = 0$	$\frac{\partial \overline{f}_2}{\partial e_2} = 0$
$\frac{\partial \overline{q}_1}{\partial k} < 0$	$\frac{\partial \overline{q}_1}{\partial e_1} > 0$	$\frac{\partial \overline{q}_1}{\partial e_2} < 0$	$\frac{\partial \overline{q}_1}{\partial k} < 0$	$\frac{\partial \overline{q}_1}{\partial e_1} = 0$	$\frac{\partial \overline{q}_1}{\partial e_2} = 0$
$\frac{\partial \overline{q}_2}{\partial k} > 0$	$\frac{\partial \overline{q}_2}{\partial e_1} < 0$	$\frac{\partial \overline{q}_2}{\partial e_2} > 0$	$\frac{\partial \overline{q}_2}{\partial k} > 0$	$\frac{\partial \overline{q}_2}{\partial e_1} = 0$	$\frac{\partial \overline{q}_2}{\partial e_2} = 0$

First consider the effect of an increase in the cost difference k. Higher k means that the marginal cost of firm 2 becomes lower and the marginal cost of firm 1 becomes higher. Despite R&D spillovers, an increase in k reduces production and pollution ($\frac{\partial \overline{q}_1}{\partial k} < 0$) by firm 1 so that it has less need to invest in environmental R&D ($\frac{\partial \overline{w}_1}{\partial k} < 0$); meanwhile, firm 2 produces more and pollutes more ($\frac{\partial \overline{q}_2}{\partial k} > 0$) and hence needs to invest more in environmental R&D ($\frac{\partial \overline{w}_2}{\partial k} > 0$). As a result, firm 1 needs fewer permits ($\frac{\partial \overline{f}_1}{\partial k} < 0$) but firm 2 needs more permits ($\frac{\partial \overline{f}_2}{\partial k} > 0$).

However, the effect of k on cost-reducing R&D depends upon the spillovers. When $\beta = 0$, the increase in k reduces $\overline{x}_1$ due to the reduction of $\overline{q}_1$, but increases $\overline{x}_2$ since $\overline{q}_2$ increases. In contrast, when $\beta = 1$, $\overline{q}_i$ is independent of $\overline{x}_i$ and $\overline{x}_j$ ((11) and (12)),therefore, k does not affect output, even though it affects $\overline{x}_i$ and $\overline{x}_j$.

Paradoxically, reducing the output of the small polluting firm will also result in a reduction in investment in environmental R&D, decreasing the net social benefit of the reduction in pollution. This is in addition to the negative effect of production reduction on cost-reducing R&D (with low spillovers).

The following two propositions study the effects of the allocation of permits on both R&D investments. Recall that the total number of permits is fixed, thus giving one more permit to one firm implies giving one fewer permits to the other.

Proposition 2 $\frac{\partial \overline{x}_i}{\partial e_i} > 0, \frac{\partial \overline{w}_i}{\partial e_i} < 0$*: the more permits a firm receives, the more it invests in cost-reducing R&D but less in environmental R&D.*

The more permits firm i receives from government, the more it produces for any given environmental R&D level, which increases the value of cost reduction and the investment in cost-reducing R&D. This is the direct effect – *i.e.* the effect of e_i on $\overline{x}_i$, which is valid despite spillovers. In contrast, the indirect effect – *i.e.* the effect of e_j on $\overline{x}_i$ – depends on spillovers. When $\beta = 0$, $\frac{\partial \overline{x}_i}{\partial e_j} > 0$. Since the increase in $\overline{x}_i$ decreases $\overline{q}_j$ and thus reduces $\overline{x}_j$, firm j needs fewer permits. This decreases the demand (supply) for permits so that firm i can hold more permits for production, which further reduces $\overline{x}_i$. When $\beta = 1$, $\frac{\partial \overline{x}_i}{\partial e_j} < 0$. In this context, firm j gets full benefit from $\overline{x}_i$ and hence reduces $\overline{x}_j$; moreover, the fewer permits it receives further reduces $\overline{x}_j$. As a result, $\overline{x}_i$ also decreases because the net benefit of increasing in $\overline{x}_i$ is low. Overall, the direct effect dominates the indirect effect, and hence giving more permits to firm i increases $\overline{x}_i$.

Similarly, the more permits firm i receives, the less it invests in environmental R&D, given the production and pollution levels. This direct effect is independent of spillovers. In contrast, the indirect effect varies with spillovers. When $\beta = 0$, $\frac{\partial \overline{w}_i}{\partial e_j} < 0$. Since there is no R&D benefit, an increase in $\overline{w}_i$ decreases $\overline{q}_j$ and thus reduces $\overline{w}_j$, which gives firm i more incentive to conduct $\overline{w}_i$. In contrast, when $\beta = 1$, $\frac{\partial \overline{w}_i}{\partial e_j} > 0$. Since firms receive R&D benefit to the maximum extent, when $\overline{w}_j$ decreases, firm i has no inentive to conduct $\overline{w}_i$ because the net benefit is limite, therefore $\overline{w}_i$ decreases as well. Overall, the direct effect dominates the indirect effect, and hence giving more permits to firm i reduces $\overline{w}_i$.

Moreover, productions and firms' needs of permits are also affected by the allocation of permits.

Proposition 3 *When* $\beta = 0, \frac{\partial q_i}{\partial e_i} > 0, \frac{\partial f_i}{\partial e_i} > 0$*: the more permits a firm receives, the more it produces and hence the more permits it needs. When* $\beta = 1$*, the allocation of permits does not affect firms' productions and needs of permits.*

Equations (11) and (12) demonstrate that $\overline{q}_i$ depends on $(w_i + w_j)$ and $(x_i - x_j)$. Since $(w_i + w_j)$ depends on the total number of permits,[10] we only need to consider how the allocation of permits affects $(x_i - x_j)$. When $\beta = 0$, giving more permits to firm i increases $\overline{x}_i$ but reduces $\overline{x}_j$. Hence, the increase in $(x_i - x_j)$ increases $\overline{q}_i$. As a result, firm i needs more permits to meet the production expansion. Intuitively, since $\overline{x}_i$ increases (Proposition 2), firm i is able to produce more.

In contrast, when $\beta = 1$, the difference between x_i and x_j does not affect production, hence, even though changing the allocation of permits affects x_i and x_j, the production is not affected and so be firm's need of permits.[11]

4 R&D cooperation model

The R&D cooperation model is practically the same as the competition model except that firms cooperate in both types of R&D in the first stage. Instead of maximizing individual profits, firms pursue joint profit maximization with respect to R&D investments $(\widetilde{x}_i, \widetilde{x}_j, \widetilde{w}_i, \widetilde{w}_j)$:

$$(\widetilde{x}_i, \widetilde{x}_j, \widetilde{w}_i, \widetilde{w}_j) = \arg \max_{x_i, x_j, w_i, w_j} \quad \pi_c = \pi_i + \pi_j \tag{15}$$

The concept of cooperative substitutability / complementarity is introduced by Leahy and Neary (2005). The main difference between cooperative substitutability / complementarity and the strategic substitutability / complementarity is that the former is "the effect of one firm's R&D on the marginal contribution of another firm's R&D to industry profits" while the latter "refers to the cross-effect of one firm's R&D on the marginal

[10] See $(A3)$ in Appendix 1.

[11] $(A42)$ and $(A43)$ in Appendix 1 show that $\overline{f}_1$ and $\overline{f}_2$ depend on the total number of permits.

profits of another firm" (Leahy and Neary, 2005:383). In d'Aspremont and Jacquemin (1988), corresponding to strategic substitutes / complements, the cost-reducing R&D investments are cooperative substitutes when $\beta < \frac{1}{2}$ and cooperative complements when $\beta > \frac{1}{2}$.

Once more, let us examine the cooperative interactions before explicitly solving for $(\widetilde{x}_i, \widetilde{x}_j, \widetilde{w}_i, \widetilde{w}_j)$.

Proposition 4 *Under R&D cooperation, the R&D investments of the same type (environmental or cost-reducing) are cooperative substitutes, but there is cooperative complementarity across different types of R&D.*

Proof. The second order partial derivatives of (15) yield:

$$\begin{array}{ll} \frac{\partial^2 \pi_c}{\partial x_i \partial x_j} = -(1-\beta)^2 \leq 0 & \frac{\partial^2 \pi_c}{\partial w_i \partial w_j} = -2(1+\beta)^2 < 0 \\ \frac{\partial^2 \pi_c}{\partial x_i \partial w_i} = \frac{1}{2}(1+\beta)^2 > 0 & \frac{\partial^2 \pi_c}{\partial x_i \partial w_j} = \frac{1}{2}(1+\beta)^2 > 0 \end{array}$$

The first inequality holds strictly when $\beta < 1$. ■

Comparing Propositions 1 and 4, we see that the analytical results for cooperative substitutability and complementarity are (qualitatively) similar to the strategic interactions.

Let us first study the cooperative substitutability between cost-reducing R&D. Taking derivatives of π_c with respect to x_i and x_j provides

$$\begin{aligned} \frac{\partial^2 \pi_c}{\partial x_i \partial x_j} &= P'(Q)\left(\frac{\partial q_j}{\partial x_i}\frac{\partial q_i}{\partial x_j} + \frac{\partial q_i}{\partial x_i}\frac{\partial q_j}{\partial x_j}\right) + \frac{\partial q_i}{\partial x_j} + \beta\frac{\partial q_j}{\partial x_j} \\ &= -\frac{1}{2}(1-\beta)^2 - \frac{1}{2}(1-\beta) + \frac{\beta}{2}(1-\beta) = -(1-\beta)^2 \end{aligned} \tag{16}$$

The first and last two terms of (16) represent how changes in x_j affect the joint marginal revenue and the joint marginal cost. Clearly, an increase in x_j reduces the joint marginal revenue but increases the joint marginal costs, thus, overall, the increase in x_j leads to lower joint profits. As a result, firm i reduces x_i to produce less, whence the cooperative substitutability between x_i and x_j. Furthermore, higher β relaxes the cooperative substitutability. When $\beta = 1$, neither the joint marginal revenue nor the marginal cost is affected by x_j, thus cooperative substitutability vanishes. The same logic explains the cooperative substitutability between w_i and w_j. In addition, a higher β reinforces cooperative substitutability, as the joint marginal cost increases in β.

Next, consider the cooperative complementarity between different types of R&D. Taking derivative of π_c with respect to x_i and w_i yields

$$\begin{aligned}
\frac{\partial^2 \pi_c}{\partial x_i \partial w_i} &= P'(Q)(\frac{\partial q_j}{\partial x_i}\frac{\partial q_i}{\partial w_i} + \frac{\partial q_i}{\partial x_i}\frac{\partial q_j}{\partial w_i}) + \frac{\partial q_i}{\partial w_i} + \beta\frac{\partial q_j}{\partial w_i} \qquad (17)\\
&= -[-\frac{1}{4}(1-\beta)(1+\beta) + \frac{1}{4}(1-\beta)(1+\beta)] + \frac{1}{2}(1+\beta)^2 = \frac{1}{2}(1+\beta)^2
\end{aligned}$$

The first terms of (17) represent the impact of w_i on the joint marginal revenue from x_i. Obviously, changes in w_i do not affect the joint marginal revenue. The rest of (17) indicates the impact of w_j on the joint marginal cost, where higher w_j reduces the joint marginal cost. Overall, the increase in w_i leads to higher joint profits, hence firm i increases x_i to produce more, whence the cooperative complementarity between x_i and w_i. The same logic explains the cooperative complementarity between x_i and w_j. Furthermore, as the marginal cost decreases in β, higher β reinforces the cooperative complementarity.

The equilibrium R&D levels $(\widetilde{x}_i, \widetilde{x}_j, \widetilde{w}_i, \widetilde{w}_j)$ are derived through solving for the four first-order necessary conditions.[12]

The following comparative statics results reveal the impacts from the changes of costs and the allocation of permits on cooperative R&D investments, the production level and the actual needs of permits amounts of permits. Appendix 2 provides proof details.

[12] See Appendix 2 for details.

4.1 Comparative Statics

Table 2: Comparative Statics of Cooperative R&D Results

$\frac{\partial \widetilde{x}_1}{\partial k} < 0$	$\frac{\partial \widetilde{x}_1}{\partial (e_1+e_2)} > 0$
$\frac{\partial \widetilde{x}_2}{\partial k} > 0$	$\frac{\partial \widetilde{x}_2}{\partial (e_1+e_2)} > 0$
$\frac{\partial \widetilde{w}_1}{\partial k} = 0$	$\frac{\partial \widetilde{w}_1}{\partial (e_1+e_2)} < 0$
$\frac{\partial \widetilde{w}_2}{\partial k} = 0$	$\frac{\partial \widetilde{w}_2}{\partial (e_1+e_2)} < 0$
$\frac{\partial \widetilde{f}_1}{\partial k} < 0$	$\frac{\partial \widetilde{f}_1}{\partial (e_1+e_2)} > 0$
$\frac{\partial \widetilde{f}_2}{\partial k} > 0$	$\frac{\partial \widetilde{f}_2}{\partial (e_1+e_2)} > 0$
$\frac{\partial \widetilde{q}_1}{\partial k} < 0$	$\frac{\partial \widetilde{q}_1}{\partial (e_1+e_2)} > 0$
$\frac{\partial \widetilde{q}_2}{\partial k} > 0$	$\frac{\partial \widetilde{q}_2}{\partial (e_1+e_2)} > 0$

Table 2 shows that the comparative statics from changing in k under R&D cooperation is qualitatively similar to competitive results, except that the cost difference k does not affect the environmental R&D investments, which is due to the fact that firms equalize the environmental R&D investments to maximize joint profits. Moreover, all cooperative equilibria depend on the total number of permits.[13] Given the aggregate emission cap, the changes of individual allocations do not affect the equilibria because the positive effects are cancelled out by the exact amount of negative effects. However, if the level of pollution ceiling changes, for example, the government revises its emission reduction target, then the equilibrium outcomes will be affected. If the government aims at reducing more pollution, it will release fewer permits in the market. With less right to pollute, firms have to reduce production and hence cost-reducing R&D, even though they invest more in environmental R&D and need fewer permits to produce.

Until now, we have obtained competitive and cooperative equilibria. One may wonder which type of R&D activity induces more R&D investments and production. In the

[13] See ($A66$) to ($A68$) and ($A70$) to ($A72$) in Appendix 2.

literature with only cost-reducing R&D (d'Aspremont and Jacquemin 1988, Kamien et. al 1992), the R&D cooperation does not always generate more R&D than competition, even though it internalizes the R&D spillover externality. In fact, the R&D competition (cooperation) induces more R&D investments when spillovers are low (high). We then want to examine if this result still hold in the presence of two types of R&D and tradable permits.

5 Comparisons of competitive and cooperative equilibria

Let us first compare the aggregate R&D investments. Given all the exogenous parameters, the comparisons depend on spillovers. To simplify notations, we let the competitive aggregate cost-reducing R&D be $\overline{X} = \overline{x}_1 + \overline{x}_2$, environmental R&D be $\overline{W} = \overline{w}_1 + \overline{w}_2$ and production be $\overline{Q} = \overline{q}_1 + \overline{q}_2$. Similarly, let the cooperative aggregate cost-reducing R&D be $\widetilde{X} = \widetilde{x}_1 + \widetilde{x}_2$, environmental R&D be $\widetilde{W} = \widetilde{w}_1 + \widetilde{w}_2$ and production be $\widetilde{Q} = \widetilde{q}_1 + \widetilde{q}_2$.

Define β_x as the critical value of spillovers such that $\overline{X}(\beta_x) = \widetilde{X}(\beta_x)$, *i.e.* firms invest the same level of cost-reducing R&D under competition and cooperation, and β_w as the critical value of spillovers such that $\overline{W}(\beta_w) = \widetilde{W}(\beta_w)$, *i.e.* firms invest the same level of environmental R&D under competition and cooperation. De facto, β_x and β_w are not necessarily identical, as they depend upon the choices of other parameters.

Proposition 5 *With* $\beta \in [0, \min\{\beta_x, \beta_w\})$, $\overline{X} > \widetilde{X}$ *and* $\overline{W} > \widetilde{W}$*; with* $\beta \in (\max\{\beta_x, \beta_w\}, 1]$, $\overline{X} < \widetilde{X}$ *and* $\overline{W} < \widetilde{W}$*.*[14]

Proposition 5 reveals when spillovers are low, *i.e.* $\beta < \min\{\beta_x, \beta_w\}$, the aggregate competitive R&D level of either type is higher than cooperative level; the result is versed

[14] By doing numerical simulation with the benchmark $a-c = 300, d = 30, \gamma = 2, \delta = 1, e_1 = 30, e_2 = 70$, we obtain $\beta_x = 0.36, \beta_w = 0.49$. This tells that when $0 \leqslant \beta < 0.36$, the aggregate competitive R&D is higher; when $0.49 < \beta \leqslant 1$, the aggregate cooperative R&D is higher. When $0.36 < \beta < 0.49$, the aggregate competitive environmental R&D is higher, but cost-reducing levels are lower.

when spillovers are high, *i.e.* $\beta > \max\{\beta_x, \beta_w\}$. This proposition echoes the R&D literature with only cost-reducing R&D.

When R&D spillovers are low, firms receive limited benefits from each other, but the net benefits of their own R&D investments are large. Hence, under R&D competition, firms tend to invest more in both types of R&D. In contrast, when firms cooperate in R&D with low spillovers, since the R&D spillover externality is internalized, firms receive more benefits from each other than under competition, therefore, them invest less in either type of R&D, compared with competitive investments.

When spillovers are high, firms receive large benefits from each other, but the net benefits of their own R&D investments are low. Therefore, when firms compete in R&D, they are reluctant to invest in either type of R&D. In contrast, with R&D cooperation and high spillovers, firms invest even more in R&D to increase such already large benefits, compared with competitive investments.

In addition, (11) and (12) demonstrate

$$\overline{Q} - \widetilde{Q} = (1+\beta)(\overline{W} - \widetilde{W}) \tag{18}$$

Obviously, the comparison of aggregate production is reduced to the comparison of aggregate environmental R&D, which follows Proposition 5.

Then, let us compare the individual R&D investments and production levels. When $\beta < 1$, it is difficult to obtain tractable analytical results. Furthermore, numerical simulations show that the comparisons vary with the value choices of parameters.

However, $\beta = 1$ provides clear result. Since R&D cooperation fully internalizes the spillover externality, firms invest more in cost-reducing R&D and hence produce more than under competition. Ideally, firms also invest more in environmental R&D under cooperation, but it can only be supported by numerical simulation.[15,16]

[15] These qualitative results are robust to parameter changes.

[16] Appendix 3 analytically establishes that when $\beta = 0$, $\overline{X} > \widetilde{X}$, $\overline{W} > \widetilde{W}, \overline{Q} > \widetilde{Q}$ and $\overline{\sigma} > \widetilde{\sigma}$; when $\beta = 1$, $\overline{X} < \widetilde{X}$, $\overline{W} < \widetilde{W}, \overline{Q} < \widetilde{Q}, \overline{\sigma} > \widetilde{\sigma}, \widetilde{x}_i > \overline{x}_i$ and $\widetilde{q}_i > \overline{q}_i$.

6 The second-best optimal allocation of permits

When firms pursue profit maximization, they do not consider pollution damage. For the regulator, this negative externality must be taken into account to maximize social welfare. Thus, the regulator chooses only the initial allocation of permits, but leaves the choices of R&D investment and production to firms. To incorporate this dimension, the game is extended to four stages. In the first stage, the government maximizes social welfare by choosing the allocations of permits. The other three stages remain the same as before.

Social welfare (SW) is defined as total benefit (TB) net of total costs and total damage (TD). Trading permits is only a transfer between firms and hence does not appear in the social welfare function. Total damage $D(e_1 + e_2)$ is assumed to be convex in pollution (total permits): $D'(e_1 + e_2) > 0, D''(e_1 + e_2) > 0$ (*i.e.* damages increase with pollution at an increasing rate).

Total benefit is given by

$$TB = \int_0^Q (a - x)dx \tag{19}$$

The social damage is simply defined as

$$TD = D(e_1 + e_2) = (e_1 + e_2)^2 \tag{20}$$

Therefore, the social welfare function becomes

$$\begin{aligned} SW &= \int_0^Q (a - x)dx - C_1 - C_2 - TD \qquad (21) \\ &= (a - c)Q - \frac{1}{2}Q^2 - k(q_1 - q_2) + x_1(q_1 + \beta q_2) + x_2(q_2 + \beta q_1) \\ &\quad -\gamma(x_1^2 + x_2^2) - \delta(w_1^2 + w_2^2) - (e_1 + e_2)^2 \end{aligned}$$

Recall that the regulator is obliged to meet the emissions cap L, which can be chosen either exogenously (for example, through international negotiation) or endogenously

($i.e.$maximizing social welfare). Therefore, combining with different type of R&D collaboration, we develop 4 cases for the following analysis: (i) case 1: R&D competition with exogenous L, (ii) case 2: R&D competition with endogenous L, (iii) case 3: R&D cooperation with exogenous L, and (iv) case 4: R&D cooperation with endogenous L.

6.1 Case 1: R&D competition with exogenous L

In this case, the regulator maximizes social welfare by choosing the initial allocation of permits e_1 and e_2, subject to the exogenous permits constraint L:

$$\max_{e_1,\, e_2} SW \tag{22}$$
$$s.t.\ e_1 + e_2 = L$$

Substituting $e_2 = L - e_1$ into (22) and solving for the first-order necessary condition with respect to e_1,[17] we obtain the optimal allocation:

$$e_{1L}^{n} = \frac{k\gamma[4(-1+\beta)\delta - 6(1+\beta)(-1+\beta+2\gamma)]}{2(1+\beta)\{[2+(-3+\beta)\beta-3\gamma]^2+\delta[-(-1+\beta)^2+\gamma]\}} + \frac{L}{2} \tag{23}$$

and thus

$$\begin{aligned} e_{2L}^{n} &= L - e_{1L}^{n} \\ &= \frac{k\gamma[3(1+\beta)(-1+\beta+2\gamma) - 2\delta(-1+\beta)]}{(1+\beta)\{[2+(-3+\beta)\beta-3\gamma]^2+\delta[-(-1+\beta)^2+\gamma]\}} + \frac{L}{2} \end{aligned} \tag{24}$$

Given all the exogenous parameters, (23) and (24) show that, with an exogenous emissions cap, the optimal allocation of permits depends on spillovers.

Proposition 6 *With R&D competition and an exogenous emissions cap, the higher the R&D spillovers, the more permits the small firm receives and the fewer permits the large firm receives.*

[17] The second-order sufficient condition is $\frac{\partial^2 SW}{\partial e_1^2} = \frac{4(1+\beta)^2\{-[2+(-3+\beta)\beta-3\gamma]^2+[(-1+\beta)^2-\gamma]\delta\}}{\{(-1+\beta^2)[2+(-3+\beta)\beta-3\gamma]^2-2\delta[(-1+\beta)^2-2\gamma]\}^2} < 0$.

Proof. See Appendix 4. ∎

When spillovers are low, firms are willing to invest more in R&D as the net benefits (from their own investment) are high. It is shown that,[18] giving one extra unit of permit to either firm generates the same level of cost-reducing R&D, but it is socially desirable to let the large firm produce, given the fact that its marginal cost is lower.

When spillovers are high, they (i) generate more productions and hence more pollution from both firms due to the reduction in marginal costs, and (ii) give firms less incentives to invest in R&D since the net benefits are low. But the reduction in environmental R&D is more critical because the regulator has to meet the emissions cap. Therefore, she would reduce permits to both firms, but it is not practical due to the exogenous permit constraint. Hence, the large firm will need to undertake more environmental R&D due to its large pollution base, which is achieved through receiving fewer permits.

6.2 Case 2: R&D competition with endogenous L

In contrast, if the emissions cap is endogenous, the initial allocation of permits is chosen to maximize social welfare without the constraint:

$$\max_{e_1,\, e_2} SW \tag{25}$$

Solving for the two first-order necessary conditions with respect to e_1 and e_2 yields the optimal permit allocation (e_1^n, e_2^n):

$$e_1^n = \gamma \frac{VM - kN[2(1+\beta)^2R^2 + \delta RS + 4\gamma\delta^2 T]}{(1+\beta)M[(2(1+\beta)^2R^2) + \delta RS + 4\gamma\delta^2 T]} \tag{26}$$

$$e_2^n = \gamma \frac{VM + kN[2(1+\beta)^2R^2 + \delta RS + 4\gamma\delta^2 T]}{(1+\beta)M[(2(1+\beta)^2R^2) + \delta RS + 4\gamma\delta^2 T]} \tag{27}$$

[18] See ($A17$).

where $M = [2+(-3+\beta)\beta-3\gamma]^2+[-(-1+\beta)^2+\gamma]\delta$, $N = 3(1+\beta)(-1+\beta+2\gamma)-2(-1+\beta)\delta$, $R = -3-2\beta+\beta^2+2(7+\beta)\gamma$, $S = -3+46\gamma+\beta(-2+\beta+34\gamma)$, $T = -3+\beta(-14+5\beta)+48\gamma$ and $V = -2\delta(a-c)(1+\beta)\{9+3\beta[-1+(-3+\beta)]-40\gamma-4\gamma\beta(5+\beta)-16\gamma\delta\}$.

Even though (26) and (27) also establish that the allocation of permits depends on spillovers, it is difficult to examine the spillover effect analytically. Nevertheless, numerical simulations show $\frac{\partial e_1^n}{\partial \beta} < 0$ and $\frac{\partial e_2^n}{\partial \beta} < 0$. As β increases, the value of environmental R&D increases. In addition, because of the endogenous emissions cap, the regulator does not need to choose *which* firm gets fewer permits. Therefore, the regulator reduces permits to both firms.

6.3 Case 3: R&D cooperation with exogenous L

Case 3 is practically the same as case except that firms cooperation in R&D. Thus, we compute the optimal permit allocation (e_{1L}^c, e_{2L}^c) by following the same steps as in case 1. However, there is no explicit solution since the cooperative equilibria depend only on L.[19] Therefore, social welfare also depends on L only and does not vary with the distribution of permits between firms.

6.4 Case 4: R&D cooperation with endogenous L

Case 4 differs from case 2 only in the way that firms cooperate in R&D. Hence, following the same steps in case 2 yields:

$$\begin{aligned} & e_1^c + e_2^c \\ = & \frac{4(a-c)\gamma\delta[-(1+\beta)^4+12\gamma(1+\beta)^2+4\gamma\delta]}{2(1+\beta)^4[(1+\beta)^2-8\gamma]^2+(1+\beta)^2\delta[(1+\beta)^4-32(1+\beta)^2+192\gamma^2]-4\gamma\delta^2[(1+\beta)^2-12\gamma]} \end{aligned} \tag{28}$$

Similarly as case 3, since the equilibria depend on L, the allocation of permits cannot be solved explicitly.

[19] See Appendix 2.

Furthermore, Appendix 2 and (28) tell that under R&D cooperation, the regulator need not worry too much about the transaction cost which occurs when she collects information from emitters, as all the equilibria depend only on the total number of permits; in other words, the regulator does not need much of emitters' information to distribute permits, as the individual allocation will have no impact on equilibria.

In the next two sections, we consider two extensions of the model. First, we consider grandfathering allocating permits based on historical output-ratio under R&D competition, and then compare it with the optimal allocation derived in Section 6.1. Second, we study how a budget constraint distorts firms' investments in both types of R&D investments.

7 Grandfathering of permits basis on historical output

In practice, one common way of distributing permits is in proportion to (pre-permit) output,[20] which is known as grandfathering permits. In the literature, economists often criticize grandfathering which allocates too many permits to large established firms, but they rarely take into account of the fact that firms undertake R&D. Therefore, it remains questionable whether the critique on grandfathering is still valid under the current model. Thus, in this section, we will compare the optimal allocation of permits with the grandfathering allocation of permits. Since social welfare only depends on the total number of permits when firms cooperate in R&D, we only consider the case with the R&D competition and an exogenous emissions cap.

To start with, we first define the business-as-usual total emission level as $q_1 + q_2$, where q_1 and q_2 are equilibrium productions (emissions) without introducing tradable

[20] In our model, allocating permits in proportion to output or to pollution is equivalent. This is because of the one-to-one relationship between production and pollution.

permits.[21] If the government commits to reducing $1-\eta$ percent emissions of business-as-usual level, it then issues $L = e_1 + e_2 = \eta(q_1 + q_2)$ permits to both firms. Thus, η represents the stringency of emissions cap. Since grandfathering permits is based on firms' historical output levels (or market shares), grandfathering allocation of permits is obtained as

$$e_1^g = \frac{q_1}{q_1+q_2}L = 3\eta\gamma\frac{(-2+\beta)[(-1+\beta)s+(1+\beta)k]+3\gamma(-s+3k)}{[2+(-3+\beta)\beta-3\gamma][-2+(-1+\beta)\beta+9\gamma]} \tag{29}$$

$$e_2^g = \frac{q_2}{q_1+q_2}L = 3\gamma\eta\frac{(-2+\beta)[(-1+\beta)s-(1+\beta)k]-3\gamma(-s+3k)}{[2+(-3+\beta)\beta-3\gamma][-2+(-1+\beta)\beta+9\gamma]} \tag{30}$$

Comparing the optimal and grandfathering allocations of permits from (23), (24), (29) and (30) by substituting $L = \eta(q_1+q_2)$ into (23) and (24) yields

$$\begin{aligned} & e_{1L}^n - e_1^g \qquad (31)\\ = \;& k\gamma\left[\frac{-3[(1+\beta)(-1+\beta+2\gamma)+2\delta(-1+\beta)]}{(1+\beta)\{[2+(-3+\beta)\beta-3\gamma]^2+[-(-1+\beta)^2+\gamma]\delta\}} - \frac{3\eta}{2+(-3+\beta)\beta-3\gamma}\right] \end{aligned}$$

$$\begin{aligned} & e_{2L}^n - e_2^g \qquad (32)\\ = \;& k\gamma\left[\frac{3[(1+\beta)(-1+\beta+2\gamma)-2\delta(-1+\beta)]}{(1+\beta)\{[2+(-3+\beta)\beta-3\gamma]^2+[-(-1+\beta)^2+\gamma]\delta\}} + \frac{3\eta}{2+(-3+\beta)\beta-3\gamma}\right] \end{aligned}$$

Obviously, (31) and (32) depend on the stringency of emissions cap η and R&D spillovers β, given all other exogenous parameters. To get the sign of (31) and (32), we need to use numerical simulations.[22]

[21] See Appendix 5 for details.

[22] These qualitative results are robust to changes in parameters.

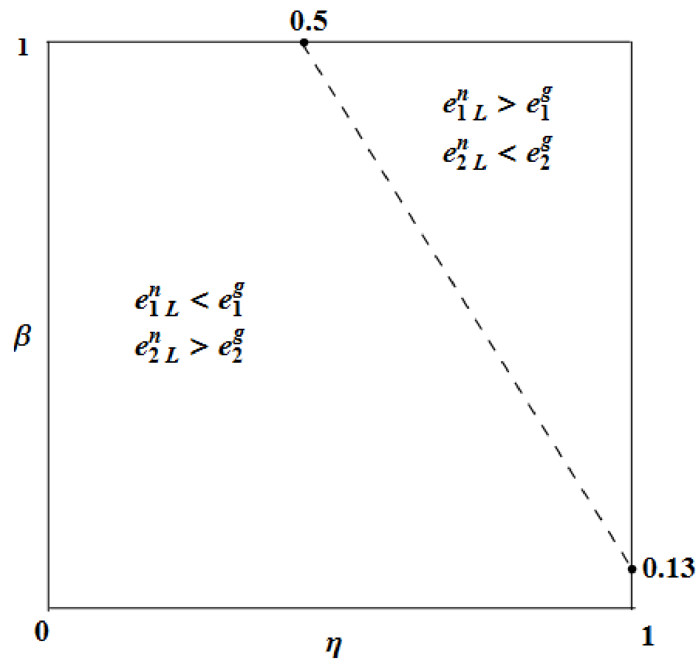


Figure 1: Comparison between optimal and grandfathering allocations of permits

Figure 1 shows the comparison between grandfathering and optimal allocations of permits. The upper right region represents the situation where grandfathering allocates too many permits to the large firm, while the lower left region represents the case that grandfathering allocates too few permits to the large firm. On the dashed line, the two allocations of permits coincide. For example, if the regulator keeps the business-as-usual emission level with $\eta = 1$, the two types of allocations coincide at $\beta = 0.13$. She then gives more (fewer) permits to the large (small) firm compared with grandfathering ($e^n_{1L} < e^g_1, e^n_{2L} > e^g_2$) when $\beta < 0.13$, and the result is reversed when $\beta > 0.13$.[23] When η reaches 0.5, grandfathering always allocates too few permits to the large firm, irrespective

[23] When $\eta = 0.9$, the break-even value of β is 0.23; when $\eta = 0.7$, the break-even value of β is 0.63, etc.

of β.

To see why, we first fix η and let β vary. The analysis is then reduced to Section 6.1: the higher the R&D spillovers, the fewer permits the large firm receives at the optimum. On the graph, we see that the higher β, the more likely that the comparison falls into the region on the upper right region, where grandfathering allocates too many permits to the large firm.

Then, we fix β and let η vary. In contrast, when the emissions cap reduces, it is optimal to give more permits to the large firm. The reduction of emissions cap also reduces the production of each firm, but the production reduction from the large firm is more than the reduction from the small firm. This is not optimal since the large firm's marginal cost is lower. Thus, it is optimal to give more permits proportionately to the large firm to induce more cost-reducing R&D and hence more production, compared with grandfathering permits.

For example, suppose the large firm's market share is 70% and the market share of the small firm is thus 30%. Initially the regulator commits to reducing 20% emissions of business-as-usual level, so she distributes $80(q_1 + q_2)\%$ permits to both firm. In other words, the large firm obtains $56(q_1+q_2)\%$ permits and the small firm receives $24\%(q_1+q_2)$ permits. If the emissions cap is reduced by 10%, then the total number of permits is $70(q_1+q_2)\%$, hence, the large firm obtains $49(q_1+q_2)\%$ permits and the small firm receives $21(q_1 + q_2)\%$ permits. In other words, the large firm receives $7(q_1 + q_2)\%$ fewer permits but the small firm receives $3(q_1 + q_2)\%$ fewer permits. Since one extra unit of permits generates the same level of production from each firm, thus the production reduction from the large firm is more than the reduction from the small firm, which is not socially desirable because the large firm's marginal cost is lower. As a result, at the optimum, the large firm receives more permits proportionately, compared with the grandfathering allocation.

When η decreases, the regulator distributes fewer permits to firms, which reduces the aggregate production. Giving one more unit of permit to either firm generates the same amount of cost-reducing R&D,[24]but giving such a permit to the large firm is socially desirable as it is applied to a larger output with lower marginal cost. Also, facing the entire output reduction, it is sensible to let the lower cost firm conduct more production. At the same time, the small firm's high investment on clean technology contributes to the increase in the aggregate environmental R&D level. Recall that fewer permits increases the price of permits. If the small firm is a permit buyer, then it needs to invest more in environmental R&D so that it can buy fewer permits from the large firm; if it is a permit seller, then more environmental innovation leaves it more permits to sell to the large firm. In both cases, it is profitable for the small firm to invest more in clean technology. As a consequence, the regulator gives more permits to the large firm to compensate the production loss; meanwhile, the small firm conducts environmental R&D to fulfill the emission reduction commitment by receiving fewer permits.

When both η and β vary, the regulator faces a trade-off between the spillover effect and her commitment on environment. When the emission reduction is low, the aggregate production is not reduced by a large scale. Thus, she will weigh more upon the spillover effect and only give more permits to the large firm if the critical value of β is low. When she becomes more environmentally aggressive, since the aggregate production is reduced more heavily and the social welfare is hurt more badly, she will assign more permits to the large firm to counteract the production reduction and weigh less on the spillover effect. Therefore, the critical value of β increases in response to the decrease in η. When the emission reduction target is very ambitious, *i.e.* $\eta \leqslant 0.5$, the aggregate output and social welfare is squeezed so massively that the regulator will ignore the spillover effect and always give more permits to the large firm.

[24] See $(A22)$ and $(A23)$ in Appendix 1.

8 R&D budget constraint

Until now, we have not taken into account of the fact that firms are financially constrained in how much they can invest in innovation. Such a constraint will distort firms' investments in both cost-reducing and environmental technologies from their unconstrained profit-maximizing levels. This issue becomes particularly relevant in an economic downturn, where firms face declining profits, reduced demand, lower employment and reduced growth prospect. This section incorporates such a constraint. The goal is to see how financially constrained firms allocate their limited resources between cost-reducing and environmental R&D. Given the difficulty of incorporating this constraint into the model with asymmetric firms, we limit the analysis here to symmetric firms with $k = 0$.

To begin with, we examine the case under R&D competition.

8.1 Constrained competitive R&D investments

The game is the same as Section 3 except for the budget constraint in the first stage (R&D competition). Let $B_i > 0$ be the total amount that can be invested in R&D by a firm. Firm i and j maximizes profits by choosing R&D investments simultaneously subject to the limitation on R&D expenditures:

$$\max_{x_i,\, w_i} \quad \pi_i = Pq_i - (c - x_i - \beta x_j)\, q_i + \gamma x_i^2 + \delta w_i^2 - \sigma(q_i - w_i - \beta w_j - e_i) \tag{33}$$

$$s.t \;\; \gamma x_i^2 + \delta w_i^2 \leq B_i$$

If the budget is binding, then solving for $\gamma x_i^2 + \delta w_i^2 = B_i$ yields $w_i = \frac{\sqrt{B_i - \gamma x_i^2}}{\sqrt{\delta}}$.[25] Substituting w_i into (33) and imposing symmetries $x_i = x_j$ and $w_i = w_j$, we can solve for (financially) constrained R&D investments x_i^{nb}, and w_i^{nb}. Table 3 displays the numerical

[25] We drop the negative root $w_i = -\frac{\sqrt{B_i - \gamma x_i^2}}{\sqrt{\delta}}$.

simulation results[26] for the ratios of constrained to non-constrained R&D investments by allowing β to vary:

Table 3: The ratios of constrained to unconstrained competitive R&D investment

	$\beta = 0.3$				$\beta = 0.5$				$\beta = 0.9$			
B_i, B_j	$0.1\overline{E}$	$0.3\overline{E}$	$0.5\overline{E}$	$0.7\overline{E}$	$0.1\overline{E}$	$0.3\overline{E}$	$0.5\overline{E}$	$0.7\overline{E}$	$0.1\overline{E}$	$0.3\overline{E}$	$0.5\overline{E}$	$0.7\overline{E}$
$\frac{x_i^{nb}}{\overline{x}_i}, \frac{x_j^{nb}}{\overline{x}_j}$	0.23	0.48	0.66	0.81	0.24	0.48	0.66	0.81	0.24	0.49	0.67	0.82
$\frac{w_i^{nb}}{\overline{w}_i}, \frac{w_j^{nb}}{\overline{w}_j}$	0.48	0.70	0.82	0.91	0.49	0.71	0.83	0.91	0.52	0.73	0.84	0.91

Without being constrained, firms would invest $(\overline{x}_i, \overline{w}_i)$ which were calculated in Section 2. $\overline{E}$ in Table 3 denotes the *endogenous* total R&D expense, *i.e.* $\gamma\overline{x}^2 + \delta\overline{w}^2 = \overline{E}$ (due to symmetry, both firms invest in the same levels of R&D, and hence have the same R&D budget). Thus, any budget beyond $\overline{E}$ means that firms do not have financial constraint and leads them to invest $(\overline{x}_i, \overline{w}_i)$, while any budget below $\overline{E}$ means that firms face a tight financial constraint and they have to reduce either type or both types of R&D investments from unconstrained levels $(\overline{x}_i, \overline{w}_i)$. obviously, when the budget constraint lower than $\overline{E}$ ($\gamma x_i^2 + \delta w_i^2 = B_i < \overline{E}$, for example, $B_i = 0.1\overline{E}_i, 0.3\overline{E}$ etc.), investments in both types of R&D are reduced, but more so (proportionally) for cost-reducing R&D. While distorting either cost-reducing or environmental R&D from their unconstrained levels reduces profits, reducing environmental R&D is particularly costly, since it forces firms to buy more permits. Hence, the higher the permit price, the less severe will be the negative effect of a financial constraint on investments in environmental innovation.

Next, let us analyze the effect of the constraint under R&D cooperation.

[26] The benchmark configuration is $(a - c) = 300, \gamma = 2, \delta = 1, e_1 = e_2 = 50$. As the two firms are symmetric, it is realistic to assume that they get equal amount of permits.

8.2 Constrained cooperative R&D investments

The game is the same as in Section 4 except for the budget constraint in the first stage. Firm i and j maximizes profits by choosing R&D investments cooperatively subject to the financial constraints of both:

$$\max_{x_i,\, x_j,\, w_i,\, w_j} \quad \pi_c = \pi_i + \pi_j \tag{34}$$

$$s.t.\ \gamma x_i^2 + \delta w_i^2 \leq B_i \text{ and } \gamma x_j^2 + \delta w_j^2 \leq B_j$$

Following the same logic as in Section 8.1, we obtain the constrained R&D investments $(x_i^{cb}, x_j^{cb}, w_i^{cb}, w_j^{cb})$. Table 4 shows the numerical simulation results for the ratios of constrained to non-constrained R&D investments $(\widetilde{x}_i, \widetilde{x}_j, \widetilde{w}_i, \widetilde{w}_j)$ by allowing β to vary. Similarly, $\widetilde{E}$ denote the endogenous total costs from both R&D investments, *i.e.* $\gamma \widetilde{x}_i^2 + \delta \widetilde{w}_i^2 = \widetilde{E}$ (due to symmetry, both firms have the same R&D budget).

Table 4: The ratios of constrained to unconstrained cooperative R&D investment

	$\beta = 0.3$				$\beta = 0.5$				$\beta = 0.9$			
B_i, B_j	$0.1\widetilde{E}$	$0.3\widetilde{E}$	$0.5\widetilde{E}$	$0.7\widetilde{E}$	$0.1\widetilde{E}$	$0.3\widetilde{E}$	$0.5\widetilde{E}$	$0.7\widetilde{E}$	$0.1\widetilde{E}$	$0.3\widetilde{E}$	$0.5\widetilde{E}$	$0.7\widetilde{E}$
$\frac{x_i^{cb}}{\widetilde{x}_i}, \frac{x_j^{cb}}{\widetilde{x}_j}$	0.23	0.48	0.66	0.81	0.24	0.50	0.67	0.82	0.28	0.52	0.69	0.83
$\frac{w_i^{cb}}{\widetilde{w}_i}, \frac{w_j^{cb}}{\widetilde{w}_j}$	0.50	0.73	0.84	0.91	0.53	0.74	0.84	0.91	0.55	0.72	0.83	0.91

Obviously, we obtain (qualitatively) similar results to those with R&D competition.

9 Conclusion

In this paper, we analyze emission permit trading in the presence of both cost-reducing and environmental R&D performed by asymmetric Cournot duopolist. Firms can choose to either cooperate or compete in both types of R&D activities. In each type of R&D

investments, technologies spill between firms for free. It is found that, through the interaction in the permit market, an increase in one firm's cost-reducing R&D always reduces its rival's production, in spite of R&D spillovers. It is also shown that, irrespective of R&D spillovers, the R&D investments of the same type (the cost-reducing or environmental R&D) are strategic (cooperative) substitutes, while there is strategic (cooperative) complementarity across different types. Even though R&D cooperation internalizes the spillover externality, it does not necessarily lead to higher R&D outputs than R&D competition. In fact, when the spillovers are low, firms invest more R&D in total under competition than under cooperation, and the result is reversed with high spillovers. Nevertheless, the individual investments depend upon the market size, the cost difference and the R&D marginal costs. When technologies fully spill between firms, both aggregate and individual R&D investments are higher under cooperation than under competition. We also consider the second-best allocation of permits, where the regulator only chooses how to distribute permits to maximize social welfare, but leaves the R&D investments and production decisions to firms. Both endogenous and exogenous emissions cap are analyzed. Under R&D competition, despite the fact that aggregate production and R&D levels depend on permits cap, since the equilibria vary with how permits are distributed, the allocation of permits matters for social welfare. Furthermore, under R&D competition, the regulator tends to assign fewer permits to both firms when R&D spillovers increases if the emission cap is endogenously chosen, but with exogenous permits cap, she will issue more permits to the small firm and fewer permits to the large firm, in response to an increase in spillovers. In contrast, under R&D cooperation, both the aggregate and individual results depend on the cap since firms equalize marginal production costs and marginal R&D costs to maximize joint profits, and the allocation of permits does not matter for social welfare. This implies that, under R&D cooperation, the transaction cost of permits distribution will not be enormous.

We also introduce two extensions of the model. One extension is the grandfathering of permits based on historical output (emission) ratio. Compared with the second-best optimal allocations of permits, we find that grandfathering of permits does not always allocate too many permits to large established firms. More specifically, the comparison depends on the emission reduction level and the spillover effect: the more emissions the regulator commits to reducing, the more permits the large firm receives proportionally, compared to grandfathering allocation. When the emission reduction level is high, the aggregate output and hence the social welfare will be massively reduced. Therefore, the regulator needs to give more permits to the large firm to compensate the production loss, and forces the small firm to conduct more environmental R&D to meet the reduction target by giving it fewer permits. In other words, grandfathering of permits not always distorts efficiency because it gives too many permits to large firms; on the contrary, they need to be assigned even more permits to improve efficiency under tight environmental regulation. In contrast, the higher the spillovers, the fewer permits the large firm receives at the optimum. The other extension studies how an R&D budget affects firms' R&D investments. This issue is particularly important during an economic slowdown. When firms are financially constrained, they underinvest in both types of R&D, but underinvest more in cost-reducing R&D relative to environmental R&D, as reducing the latter causes higher costs from buying more permits.

There are several possible extensions that can be incorporated into this paper. First, we model firms to be price takers in the permit market. However, as there are only two firms compete in quantity in the product market, it is likely that they will exercise market power in the permit market as well (Sarzetakis and McFedrige, 1999). With the possibility of applying the positioning strategy (raising rival's cost), firms will have different incentives to conduct both types of R&D. As a result, the comparison between grandfathering of permits on output and the second-best optimal allocations of permits will need to be redefined. Another fact is that firms have been conducting cost-reducing R&D long before obeying environmental regulation and hence innovate to reduce pollution, so the two R&D activities actually take place sequentially instead of simultaneously. The change of timing may also bring different results. The third possible extension is to include government's commitment on emission reduction. For example, in period 1 and 2, the regulator announces the emission cap, which drives firms to conduct both types of R&D. Observing firms' R&D activities, in period 3, the regulator adapts to another pollution standard. Moreover, permits in period 1 and 2 can not be used in period 3. For example, in EU ETS, permits holders are allowed to bank permits and trade them within a given time, such as in Phase I between 2005 and 2007, and in Phase II between 2008 and 2012, but permits expire after that period. Also, the total emission levels between Phase I and Phase II are not the same. As a result, bankable permits and government's commitment will also affect firms' trading and R&D decisions. Last but not least, permits are traded not only within industries, but also among industries. It would be interesting to incorporate permit trading between vertically related markets.

Chapter 2

Tradable Permits under Environmental R&D between Upstream and Downstream Industries

1 Introduction

Environmental regulation is the focus of ever increasing attention from economists, climate change scientists and policy makers. Governments can use market-based tradable emissions permits to achieve certain emission reduction targets. Emissions permits are usually distributed either through auctioning or free of charge (Atkinson and Tietenberg 1984, Lyon 1982, Lai 2008). Since in practice most systems have used a free allocation, this current paper adopts this approach as well.

It is well-known that tradable emissions permits provide emitters incentives to develop clean technologies through research and development (Downing and White, 1986). However, once we begin considering environmental R&D, several other issues come up as well: (i) R&D has spillover effects which affect the optimal allocation of pollution permits, (ii) different industries, especially vertically related industries, may emit the same pollutant and trade pollution rights when they face the same environmental regulation,[1,2] and (iii) upstream and downstream firms may either cooperate or compete in their environmental R&D efforts. To date, most research focuses on environmental R&D within one industry (Montero 2002a, 2002b); few studies have analyzed the interaction between all of these issues concerning R&D and tradable permits. The present paper endeavours to do so.

This paper also considers market power in the output market. The early literature focuses on perfect competition in both the output and the permit markets (Montgomery 1972). However, more recent research has shown that market power in either of these markets affects the efficiency of the final equilibrium (Hahn 1984, Tietenberg 1985). To incorporate this problem, this paper assumes market power in the output market.

In contrast, firms take permit price as given in the permit market. Since emissions are

[1] For example, both Petroleum and Plastic industries emit CO_2.

[2] Within the European Union Emissions Trading System (EU ETS), emission trading takes place among energy producers and energy-intensive industries, such as power generators, steel & iron and combustion activities.

assumed to be in fixed proportion to output quantities, firms' interaction in the permit market is indirectly determined through their strategic behavior in the output market. Hence, the permit price is endogenously set by firms' derived demand and supply of permits. For tractability, the permit market is assumed to be perfectly competitive, although it would be worth in later research to consider the effect of market power in this market as well.

We develop a model with symmetric Cournot duopolists in both upstream and downstream industries that trade permits and undertake environmental R&D. In a four-stage game, firms first invest in environmental R&D, then trade permits between industries (firms within the same industry receive the same number of permits due to symmetry and do not need to trade permits). After that, upstream firms compete in intermediate good production and sell them to downstream firms, and finally downstream firms compete in the final good market. Both R&D competition and cooperation are considered. When firms cooperate in R&D, they can either cooperate within the same industries (horizontal cooperation), between industries (vertical cooperation), or both within and between industries (generalized cooperation). In each type of R&D investment, firms face R&D spillovers from both within the industry (horizontal spillovers) and between industries (vertical spillovers). In this framework, we investigate (i) the interdependency between firm's R&D choices; (ii) the R&D spillover effect on permit price, production and R&D investments, (iii) the optimal allocation of permits between all firms, and we compare (iv) the effects of competition and cooperation on R&D investments and social welfare.

In terms of the interdependency of R&D choices, Banerjee and Lin (2003) have shown that, in the context of cost-reducing R&D, an increase in R&D by a downstream firm induces the upstream firms to invest more in R&D when firms compete in R&D. This effect is referred to as strategic complementarity. In contrast, the current model, with environmental R&D and tradable permits, shows that, an increase in R&D by a downstream firm reduces the R&D investments from its competitor and from upstream firms. This effect is referred to as strategic substitutability.

As for the spillover effect on permit price, a higher horizontal or vertical spillover rate reduces pollution for all firms, hence they need fewer permits, which drives up the supply or drives down the demand for permits and thus reduces permit price. In contrast, higher horizontal or vertical spillover rate increases productions of both intermediate and final goods, because all firms benefit from their competitors and the other industry so that the same levels of production generate less pollution, thus firms are willing to produce more, given the permits they hold and R&D investments.

The R&D investment is determined by both the allocation of permits and spillovers. If a firm receives more permits from the government, it then has less need to undertake R&D. In contrast, the spillover effects on R&D are ambiguous and they depend on the number of permits a firm receives from government. If a downstream (upstream) firm receives more permits than an upstream (downstream) firm, then higher horizontal or vertical spillovers will increase (decrease) R&D investments from the upstream (downstream) firm.

We also consider which type of R&D activities, either competition or cooperation, generates more R&D investments. In the cost-reducing R&D literature with two vertically related industries, Inkmann (1999) shows that vertical R&D cooperation tends to generate more R&D investments than horizontal R&D cooperation, but Atallah (2002) concludes that no setting of R&D cooperation uniformly dominates the others. In contrast, the current model shows that generalized cooperation always induces more R&D output than vertical cooperation, since the former internalizes both the horizontal and vertical spillover externalities, while the latter only incorporate vertical externality. Moreover, under both R&D competition and horizontal cooperation, upstream firms invest more (less) in R&D than downstream firms if downstream (upstream) firms receive more permits.

The last part the paper studies the optimal allocation of permits from the regulator. It is shown that the allocation of permits does not matter for social welfare under either vertical or generalized cooperation. These types of collaboration internalizes vertical externality, so the regulator can equalize the marginal R&D cost and hence the R&D investments across industries to maximize social welfare. Moreover, social welfare under generalized cooperation is always higher than under vertical cooperation. Therefore, in contrast to Satzetakis and McFedrige (1988), the limited information of allocating permits does not matter with optimal permit allocations under both vertical and generalized R&D cooperations.

In contrast, the allocation of permits matters for social welfare under both R&D competition and horizontal cooperation. Furthermore, upstream industry always receive more permits than downstream industry at the optimum. Upstream firms make higher (total) profits and hence invest more in R&D. Due to convex R&D costs, the regulator would prefer industries to invest the same amount of R&D. Therefore, she equalizes R&D marginal costs across industries to maximize social welfare by giving the downstream firms fewer permits to encourage more R&D, and allocating upstream firms more permits to reduce their R&D investments.

The paper is organized as follows. Section 2 introduces the basic framework. Sections 3 and 4 present the R&D competition and cooperation models. Section 5 compares R&D levels under different types of R&D collaboration. Section 6 studies the second-best optimal allocation of permits under either exogenous or endogenous emissions cap. Section 7 concludes.

2 The basic framework

There are symmetric Cournot duopolists producing intermediate good in the upstream industry, and symmetric Cournot duopolists transforming the intermediate good into final product in the downstream industry. Upstream suppliers face a constant marginal cost c_u to produce intermediate good y, with $Y = y_m + y_n$, $m \neq n$, $m, n = 1, 2$ stands for total production of intermediate good. Downstream buyers have constant marginal cost c_d and purchase intermediate good at the unit price t to produce final good q. The final good market demand is $P = a - Q$, where $Q = q_i + q_j$, $i \neq j$, $i, j = 1, 2$ represents the aggregate final good. In general, $c_u \neq c_d$. Furthermore, we assume a fixed-coefficient technology that one unit of intermediate good is needed for producing one unit of final good, therefore, $Q = Y$.

Production causes pollution. Without clean technology, we assume a one-to-one relationship between production and emissions, *i.e.* $y_m = f_m$, $q_i = f_i$. The environmental regulation gives each firm incentives to undertake environmental R&D, which reduces emissions and the abatement costs. Since R&D spills between firms, they benefit from each other's investment without payment. Moreover, technology can spill in the same industry with horizontal spillover rate $h \in [0, 1]$, and between different industries with vertical spillover rate $v \in [0, 1]$. Hence, through undertaking R&D, an upstream firm's emissions can be reduced to $f_m = y_m - w_{um} - hw_{un} - v(w_{di} + w_{dj})$, where w_{um} and w_{un} represent the individual R&D investment from upstream firms respectively, and w_{di} and w_{dj} represent the individual R&D investment from downstream firms respectively. Note that upstream firm m benefits hw_{un} from its competitor's technology, and $v(w_{di} + w_{dj})$ from downstream firms' innovation. Similarly, a downstream firm's emissions can be reduced to $f_i = q_i - w_{di} - hw_{dj} - v(w_{um} + w_{un})$. Again, downstream firm i benefits from its competitor's R&D by hw_{di} and from downstream firms by $v(w_{um} + w_{un})$. The costs of R&D are γw_{um}^2 and γw_{di}^2 respectively, and we assume $\gamma \geqslant 1$ to ensure the maximum R&D output.

Firms can either compete or cooperate in R&D but remain competitive in the output market. If firms cooperate in innovation, they cooperate either within the same industries (intra–industry or horizontal cooperation), between industries (inter-industry or vertical cooperation) or both within and between industries (generalized cooperation).[3] Horizontal technological collaboration stimulates firms to do further research and yields more revenue (Gilsing *et.al.*, 2008), while vertical collaboration strengthens firms' competitiveness in the core domains (Belderbos *et.al.*, 2004).

To regulate emissions, the regulator freely allocates permits to all firms: an upstream firm receives e_{um} permits and a downstream firm receives e_{di} permits. Due to symmetry, firms in the same industry receive the same number of permits, but firms in different industries receive different initial permit endowments. Hence, permits will only be traded between industries.[4] To simplify notations, we define $e_d \equiv e_{di} = e_{dj}$ and $e_u \equiv e_{um} = e_{un}$. The total number of permits (emissions cap) is thus defined as $L = 2e_d + 2e_u$. The unit permit price σ is determined by the equilibrium between the derived demand and supply of permits, and each firm decides how many permits it wants to buy or sell, taking the permit price as given. Furthermore, we assume that firms use up all permits, which means that banking permits is not allowed.

Then, we can derive the total cost for downstream firm i as

$$C_{di} = (c_d + t)q_i + \gamma w_{di}^2 \tag{1}$$

which is composed of total production cost and the cost of R&D. If firm i buys (sells) permits, it will pay (receive) $\sigma(f_{di} - e_d)$ from trading. Thus, firm i's profit function becomes:

$$\pi_{di} = P(Q)q_i - C_{di} - \sigma(f_{di} - e_d) \tag{2}$$

Similarly, the total cost function for an upstream firm m is defined as

$$C_{um} = c_u y_m + \gamma w_{um}^2 \tag{3}$$

[3]The types of R&D cooperation are similar to Atallah (2002).

[4]A more general model would allow both intra-industry and inter-industry permit trading. This can be done by considering asymmetric firms within industries, and/or asymmetric permit allocations within industries.

and the profit function becomes

$$\pi_{um} = t(Y)\ y_m - C_{um} - \sigma(f_{um} - e_u) \tag{4}$$

3 R&D competition model

Firms play a four-stage game. They first choose R&D investments simultaneously and then trade permits; after that, upstream firms compete in intermediate good production, and finally downstream firms compete in final good à la Cournot, given the price of the intermediate good. In the R&D stages, four types of R&D activities will be considered: R&D competition, horizontal cooperation, vertical cooperation and generalized cooperation.

We start with the R&D competition by using backward induction.

3.1 Fourth stage: downstream final good quantity competition

In this stage, downstream firms maximize their profits with respect to final good production. The first-order necessary condition becomes

$$\frac{\partial \pi_{di}}{\partial q_i} = (a - 2q_i - q_j) - (c_d + t) - \sigma = 0 \tag{5}$$

Rewriting (5) yields

$$(a - 2q_i - q_j) - (c_d + t) = \sigma \tag{6}$$

The left hand side of (6) is the marginal revenue net of the marginal production cost, which could be seen as the marginal abatement cost through quantity reduction; the right hand side is the unit permit price. Equation (6) implies that firms set marginal abatement cost equal to unit permit price when maximizing profit (Mansur, 2007), which resembles the first-best condition. However, as Sartzetakis and McFetridge (1999) point out, this result is still second-best because of output market imperfection.

The optimal quantity is solved by imposing symmetry:[5]

$$q_i = q_j = q = \frac{1}{3}(a - t - c_d - \sigma) \tag{7}$$

and hence we obtain the final good price

$$P = \frac{1}{3}[a + 2(t + \sigma) + 2c_d] \tag{8}$$

3.2 Third stage: upstream intermediate good quantity competition

Given $Y = Q$ and equation (7), we can derive the upstream inverse demand function:

$$t = a - \sigma - c_d - \frac{3}{2}(y_m + y_n) \tag{9}$$

In this stage, upstream firms choose the production of intermediate good to maximize profits. The first-order necessary condition is obtained by using (4)

$$\frac{\partial \pi_{um}}{\partial y_m} = a - c_d - c_u - 2\sigma - 3y_m - \frac{3}{2}y_n = 0 \tag{10}$$

Rearranging (10) yields

$$(a - 3y_m - \frac{3}{2}y_n) - (c_d + c_u) = 2\sigma \tag{11}$$

Different from downstream profit maximization condition in (6), (11) shows that for upstream firms, the marginal abatement cost through quantity reduction equals to two

[5]The second-order sufficient condition $\frac{\partial^2 \pi_{di}}{\partial q_i^2} = -2 < 0$ garantees the maximum solution.

times of unit permit price. In other words, the upstream marginal abatement cost is twice higher than the downstream marginal abatement cost.

The optimal quantity is obtained by imposing symmetry:[6]

$$y_m = y_n = y = \frac{2}{9}(a - c_d - c_u - 2\sigma) \tag{12}$$

and the price of intermediate good becomes

$$t = \frac{1}{3}(a + \sigma - c_d + 2c_u) \tag{13}$$

3.3 Second stage: permits trading

The equilibrium price of permits is such that the demand for permits equals the supply of permits:

$$f_{di} + f_{dj} + f_{um} + f_{un} = 2e_d + 2e_u \tag{14}$$

Substituting $(7), (12)$ and (13) into (14) solves for the equilibrium permit price

$$\sigma = \frac{1}{2}[(a - c_d - c_u)] - \frac{9}{16}[2(e_d + e_u) + (1 + h + 2v)(w_{di} + w_{dj} + w_{um} + w_{un})] \tag{15}$$

Equation (15) shows that R&D investments negatively affect the permit price: R&D reduces the demand for permits by a permit buyer, and increases the supply of permits by a permit seller. This finding is consistent with Montero (2002a), as well as the first chapter of the thesis.

Substituting (15) into (7) and (12) generates

$$q = y = \frac{1}{4}[2(e_d + e_u) + (1 + h + 2v)(w_{di} + w_{dj} + w_{um} + w_{un})] \tag{16}$$

Substituting (16) into (2) and (4) yields profit functions $\pi_{di}\ (a, c_d, c_u, w, e, h, v, \gamma)$

[6] The second-order sufficient condition $\frac{\partial^2 \pi_{um}}{\partial y_m^2} = -3 < 0$ garantees the maximum solution.

of downstream firm i and π_{um} $(a, c_d, c_u, w, e, h, v, \gamma)$ of upstream firm m, where $w = w(w_{di}, w_{dj}, w_{um}, w_{un})$ is the vector of R&D outputs from both upstream and downstream firms, and $e = e(e_d, e_u)$ is the vector of emission permits. Before solving for optimal R&D levels, let us consider the strategic interactions between R&D investments.

Proposition 1 *The intra-industry and inter-industry R&D investments are strategic substitutes.*

Proof. The second order partial derivatives of equations (2) and (4) yield:

$$
\begin{aligned}
\frac{\partial^2 \pi_{di}}{\partial w_{dj} \partial w_{di}} &= -\frac{1}{16}(7 + 7h - 4v)(1 + h + 2v) < 0, \\
\frac{\partial^2 \pi_{um}}{\partial w_{un} \partial w_{um}} &= -\frac{3}{8}(1 + h - v)(1 + h + 2v) < 0, \\
\frac{\partial^2 \pi_{di}}{\partial w_{di} \partial w_{um}} &= \frac{\partial^2 \pi_{di}}{\partial w_{di} \partial w_{un}} = \frac{1}{16}(-7 + 2h - 5v)(1 + h + 2v) < 0, \\
\frac{\partial^2 \pi_{um}}{\partial w_{um} \partial w_{di}} &= \frac{\partial^2 \pi_{um}}{\partial w_{um} \partial w_{di}} = \frac{3}{16}(-2 + h - v)(1 + h + 2v) < 0.
\end{aligned}
$$

■

Banerjee and Lin (2003) establish that the cost-reducing R&D investments by vertically related industries are strategic complements, while Atallah (2002) shows that the R&D investments can be either strategic substitutes or complements, depending on horizontal and vertical R&D spillovers. In contrast, Proposition 1 shows that in the presence of environmental R&D and tradable emissions permits, R&D investments are always strategic substitutes, irrelevant of R&D spillovers.

For the intra-industry strategic substitutability, when downstream (upstream) firm i (m) invests more in R&D, it will buy fewer or sell more permits to other firms. This will make downstream (upstream) firm j (n) benefit more through horizontal spillover and hence reduce R&D investments. At the same time, firm j (n) can hold more permits and therefore further is able to further reduce its R&D investment. The same logic explains inter-industry strategic substitutability.

Moreover, the intra-industry strategic substitutability reinforces when horizontal R&D spillover h increases,[7] as a firm benefits more from the competitor and hence it is reluctant to invest in R&D. Similarly, higher vertical spillover v makes the inter-industry strategic substitutability stronger,[8] as firms in the other industry receive more benefit and have less need to undertake R&D. Therefore, there are two sources for the strategic substitutability: changes in the demand or supply of permits, and R&D spillovers.

3.4 First stage: R&D competition

In this stage, both downstream and upstream firms invest in R&D simultaneously to maximize their profits:

$$w_{um}^{NC} = \arg\max_{w_{um}} \ \pi_{um} \ (a, c_d, c_u, w, e, h, v, \gamma) \tag{17}$$

$$w_{di}^{NC} = \arg\max_{w_{di}} \ \pi_{di} \ (a, c_d, c_u, w, e, h, v, \gamma) \tag{18}$$

Define the aggregate R&D output as $W^{NC} = w_{di}^{NC} + w_{dj}^{NC} + w_{um}^{NC} + w_{un}^{NC}$. Substituting $w_{di}^{NC}, w_{dj}^{NC}, w_{um}^{NC}$ and w_{un}^{NC}[9] into (15) yields the permit price σ

$$\sigma^{NC} = \frac{[(1+h+2v)^2 - 32\gamma](a - c_d - c_u) + 72\gamma(e_d + e_u)}{2[-35 + h^2 + 4(-17+v)v + h(-34+4v) - 32\gamma]} \tag{19}$$

and we obtain the intermediate and final outputs by substituting (19) into (7) and (12)

$$q^{NC} = y^{NC} = -\frac{8[(1+h+2v)(a - c_d - c_u) + 2\gamma(e_d + e_u)]}{-35 + h^2 + 4(-17+v)v + h(-34+4v) - 32\gamma} \tag{20}$$

Notice that the levels of intermediate and final outputs are equalized due to the fixed-

[7] $\frac{\partial(-\frac{1}{16}(7+7h-4v)(1+h+2v))}{\partial h} = \frac{1}{8}(-7-7h-5v) < 0$, $\frac{\partial(-\frac{3}{8}(1+h-v)(1+h+2v))}{\partial h} = -\frac{3}{8}(2+2h+v) < 0$.

[8] $\frac{\partial(\frac{1}{16}(-7+2h-5v)(1+h+2v))}{\partial v} = \frac{1}{16}(-19-h-20v)$, $\frac{\partial(\frac{3}{16}(-2+h-v)(1+h+2v))}{\partial v} = \frac{3}{16}(-5+h-4v)$.

[9] See Appendix 1 for details.

coefficient technology. Moreover, $w_{di}^{NC} = w_{dj}^{NC}$ and $w_{um}^{NC} = w_{un}^{NC}$[10] due to symmetry.

Montgomery (1972) shows that the permit price is independent of the initial allocation of the permits, given both the permit and output markets are perfectly competitive permit. This result still holds in Sanin and Zanaj (2007, 2009), who study a perfectly competitive permit market and a symmetric Cournot output market, and has been extended by Sartzetakis (1997), who set up permit trading between industries. Equation (19) shows that this conclusion can also be extended to inter-industry permit trading with environmental R&D.

3.5 Comparative statics

Now we present comparative statics results to show how changes in costs, allocation of permits and R&D spillovers affect the permit price, R&D investments, output levels and firms' needs of permits. Appendix 2 provides proofs.

Let $w_d^{NC} \equiv w_{d1}^{NC} = w_{d2}^{NC}$ and $w_u^{NC} \equiv w_{u1}^{NC} = w_{u2}^{NC}$. In general, the higher the marginal costs of one industry or the other industry, the lower the productions of both intermediate and final goods ($\frac{\partial q^{NC}}{\partial c_d} < 0$, $\frac{\partial y^{NC}}{\partial c_u} < 0$, $\frac{\partial q^{NC}}{\partial c_u} < 0$ and $\frac{\partial y^{NC}}{\partial c_d} < 0$).[11] Moreover, the higher the marginal cost of either industry, the lower the permit price ($\frac{\partial \sigma^{NC}}{\partial c_d} < 0$, $\frac{\partial \sigma^{NC}}{\partial c_u} < 0$), since an increase in marginal cost of either industry decreases production and pollution of both intermediate and final goods, which results in more supply or less demand for permits, which drives down the permit price.

Proposition 2 establishes how R&D spillovers affect the permit price.

Proposition 2 $\frac{\partial \sigma^{NC}}{\partial h} < 0$, $\frac{\partial \sigma^{NC}}{\partial v} < 0$: *higher (horizontal or vertical) spillovers reduce the permit price.*

Given R&D investments, an increase in h or v reduces pollution for all firms, hence they need fewer permits, which drives up the supply or drives down the demand for

[10] See Appendix 1 for details and proof.

[11] While this result is standard in a Cournot model, it is useful to state it here to understand the effect of costs on permit price.

permits and thus reduces permit price. Therefore, R&D spillovers also have a negative effect on permit price, in addition to their roles in strengthening strategic substitutability between R&D investments (noted in Section 3.3).

Moreover, even though there is positive relationship between σ^{NC} and the final good price P^{NC} (8), equation ($A7$) shows $\frac{\partial P^{NC}}{\partial c_d} = \frac{\partial P^{NC}}{\partial c_u} > 0$; in other word, an increase in marginal costs decreases σ^{NC} but increases P^{NC}. c_d and c_u affect P both directly and indirectly through σ^{NC}, but the direct effect dominates the indirect effect, thus an increase in marginal cost ultimately increases the price of final product.[12]

Proposition 3 studies the how allocation of permits and spillovers affect the production.

Proposition 3 *The productions of intermediate and final goods positively depend on the emissions cap. Furthermore,* $\frac{\partial q^{NC}}{\partial h} = \frac{\partial y^{NC}}{\partial h} > 0$, $\frac{\partial q^{NC}}{\partial v} = \frac{\partial y^{NC}}{\partial v} > 0$*: the higher the (horizontal or vertical) spillover rate, the more the productions.*

Equation (16) reveals that the productions of both intermediate and final goods depend on the aggregate R&D investments, W^{NC}, which is determined by the emissions cap ($A10$). Therefore, the production depend on the total number of permits (20), given all other exogenous parameters. As a result, a lower emissions cap results in lower production.

With higher h or v, all firms benefit from their competitors and the other industry so that the same levels of production generate less pollution. Therefore, firms are willing to produce more, given the permits they hold and R&D investments.

The effect of the allocation of permits on R&D investments is analyzed in Proposition 4.

Proposition 4 $\frac{\partial w_d^{NC}}{\partial e_d} < 0, \frac{\partial f_d^{NC}}{\partial e_d} > 0, \frac{\partial w_u^{NC}}{\partial e_u} < 0, \frac{\partial f_u^{NC}}{\partial e_u} > 0$*: the more permits a firm receives, the less R&D it undertakes, but the more permits it needs to hold for production.*

[12] This is also consistent with $\frac{\partial q^{NC}}{\partial c_d} < 0$: given $P = a - Q$, the reduction in Q increases price.

Since there is a permits cap, the more permits downstream (upstream) firms receive, the fewer permits upstream (downstream) firms receive. Appendix 2 shows that giving more permits to a downstream firm reduces its R&D investment (the direct effect), but the impact on upstream firms' R&D investments (the indirect effect) remains ambiguous. However, the direct effect ultimately dominates the indirect effect and hence giving more permits to a downstream firm reduces its R&D investment. The same logic explains that giving more permits to an upstream firm also reduces its R&D investment.

Thus, when a firm reduces R&D investment, it has to hold more permits to keep the same level of production (since production is independent of the allocation of permits (Proposition 3)).

3.5.1 R&D Spillover effects on R&D investments

Atallah (2002) shows that, in the presence of cost-reducing R&D, vertical spillover always increase R&D, while horizontal spillover may increase or decrease it. In this section, we want to reexamine this result under the current model. Since spillover effects on R&D are analytically intractable, we need to resort to numerical simulations. The basic configuration is set as $a = 1500, c_d = 30, c_u = 50$[13] and $\gamma = 40$. Let there be 400 permits in the market. We consider three cases for the allocation of permits: downstream firms hold all permits ($e_d = 200$, $e_u = 0$), permits are symmetrically allocated ($e_d = e_u = 100$) between industries, and upstream firms hold all permits ($e_d = 0$, $e_u = 200$). The only parameters that we do not fix are spillover rates h and v.

Result 1 With $(e_d, e_u) \in \{(200, 0), (100, 100)\}$, $\frac{\partial w_d^{NC}}{\partial h} < 0$, $\frac{\partial w_d^{NC}}{\partial v} < 0$, $\frac{\partial w_u^{NC}}{\partial h} > 0$, $\frac{\partial w_u^{NC}}{\partial v} > 0$: when downstream firms receive more permits than upstream firms, or when both industries receive the same number of permits, the higher the (horizontal or vertical) spillovers, the less R&D the downstream firms undertake, but the more

[13] Appendix 1 shows that equilibria are functions of $c_d + c_u$, thus the results will hold as long as $c_d + c_u = 80$.

R&D the upstream firms undertake. The result is reversed if upstream firms receive more permits than downstream firms.

The following three tables based on simulations help us understand this result. Table 1 illustrates the case when downstream firms hold all permits. In this context, downstream firms have the least incentive to invest in R&D and hence w_d is expected to be the lowest among all the three cases. Since upstream firms receive no permits, they must invest high w_u to reduce abatement costs.

When h increases, it is profitable for upstream firms to invest more in R&D, despite the facts that (i) permits price becomes lower (Proposition 2), and (ii) they compete in R&D and higher h leads the competitor to receive more benefits. Therefore, they are willing to enlarge the R&D benefits from each other, expand production and need fewer permits from downstream firms. As a result, downstream firms can produce more and reduce their R&D investments due to (i) the larger R&D benefit from upstream firms through vertical spillover v, and (ii) hold more permits in hands. When v increases, it is even more profitable for upstream firms to undertake more R&D, as well as buy more permits from downstream firms since the permits price falls considerably. Therefore, downstream firms receive even larger R&D benefit from increases in both v and aggregate upstream R&D investments. As a result, they have even less need to undertake R&D, even though they now hold fewer permits. Notice that an increase in v increases downstream profits significantly than an equivalent increase in h.

The same logic explains that when upstream firms receive more permits, higher (horizontal or vertical) spillovers reduce R&D investments from upstream industry, but increase R&D investments from downstream industry (Table 3). Moreover, an increase in v significantly increases upstream profits than an equivalent increase in h.

Table 2 illustrates the case when both industries hold the same number of permits. The simulation results are analytically similarly to the case when downstream firms hold all permits, so we can use the same analysis.

$(e_d, e_u) = (200, 0)$

	$(0.2, 0.1)$	$(0.2, 0.5)$	$(0.2, 0.8)$	$(0.1, 0.2)$	$(0.5, 0.2)$	$(0.8, 0.2)$
$\boldsymbol{(h, v)}$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
	$(0.8, 0.1)$	$(0.8, 0.5)$	$(0.8, 0.8)$	$(0.1, 0.8)$	$(0.5, 0.8)$	$(0.8, 0.8)$
$\boldsymbol{\Delta q^{NC}(\Delta y^{NC})}$	3.48	3.43	3.4	6.92	6.87	6.84
$\boldsymbol{\Delta w_d^{NC}}$	-0.55	-0.55	-0.55	-1.19	-1.19	-1.19
$\boldsymbol{\Delta w_u^{NC}}$	0.47	0.47	0.48	1.03	1.04	1.05
$\boldsymbol{\Delta B_d^{NC}}$	2.5	2.41	2.35	10.22	10.14	10.07
$\boldsymbol{\Delta B_u^{NC}}$	4.56	4.53	4.52	3.77	3.74	3.74
$\boldsymbol{\Delta f_d^{NC}}$	1.54	1.57	1.60	-2.12	-2.08	-2.05
$\boldsymbol{\Delta f_u^{NC}}$	-1.54	-1.57	-1.57	2.12	2.08	2.05
$\boldsymbol{\Delta \sigma}$	-7.84	-7.73	-7.66	-15.56	-15.46	-15.39
$\boldsymbol{\Delta t}$	-2.61	-2.58	-2.55	-5.19	-5.15	-5.13
$\boldsymbol{\Delta \pi_d}$	231.81	143.62	67.93	2760.71	2664.16	2593.6
$\boldsymbol{\Delta \pi_u}$	3102.87	3013.4	2961.0	3696.83	3614.76	3558.39

Table 1: Spillover effects when downstream industry holds all permits

$\Delta q^{NC}(\Delta y^{NC})$: change of final good (intermediate good) quantities when spillover changes (e.g. (h,v) changes from (0.2,0.1) to (0.8,0.1))

Δw_d^{NC}: change of downstream R&D investment when spillover changes

Δw_u^{NC}: change of upstream R&D investment when spillover changes

ΔB_d^{NC}: change of R&D benefits a downstream firms gets through spillovers[14]

ΔB_u^{NC}: change of R&D benefits an upstream firms gets through spillovers[15]

Δf_d^{NC}: change of permits a downstream firm's need of permits due to spillover changes

Δf_u^{NC}: change of permits an upstream firm's need of permits due to spillover changes

$\Delta\sigma$: change of permit price due to spillover changes

[14] The R&D benefit is $B_d^{NC} = hw_d + 2vw_u$. The two benefit functions tell us that firms benefit more easily from the other industry's R&D investment.

[15] The R&D benefit is: $B_u^{NC} = hw_u + 2vw_d$

Δt: change of intermediate good price due to spillover changes

$\Delta \pi_d$: change of downstream firms' profits when spillover changes

$\Delta \pi_u$: change of upstream firms' profits when spillover changes

$(e_d, e_u) = (100, 100)$

	$(0.2, 0.1)$	$(0.2, 0.5)$	$(0.2, 0.8)$	$(0.1, 0.2)$	$(0.5, 0.2)$	$(0.8, 0.2)$
$\boldsymbol{(h, v)}$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
	$(0.8, 0.1)$	$(0.8, 0.5)$	$(0.8, 0.8)$	$(0.1, 0.8)$	$(0.5, 0.8)$	$(0.8, 0.8)$
$\boldsymbol{\Delta q^{NC}(\Delta y^{NC})}$	3.48	3.43	3.4	6.92	6.87	6.84
$\boldsymbol{\Delta w_d^{NC}}$	-0.15	-0.15	-0.15	-0.32	-0.32	-0.32
$\boldsymbol{\Delta w_u^{NC}}$	0.07	0.08	0.09	0.16	0.18	0.19
$\boldsymbol{\Delta B_d^{NC}}$	3.32	3.26	3.22	7.66	7.61	7.57
$\boldsymbol{\Delta B_u^{NC}}$	3.73	3.68	3.65	6.33	6.3	6.25
$\boldsymbol{\Delta f_d^{NC}}$	0.31	0.32	0.33	-0.42	-0.42	-0.41
$\boldsymbol{\Delta f_u^{NC}}$	-0.31	-0.32	-0.33	0.42	0.42	0.41
$\boldsymbol{\Delta \sigma}$	-7.84	-7.73	-7.66	-15.56	-15.46	-15.39
$\boldsymbol{\Delta \pi_d}$	1441.08	1374.54	1328.3	3301.9	3237.28	3190.03
$\boldsymbol{\Delta \pi_u}$	2007.5	1952.44	1916.64	3470.66	3417.13	3380.48

Table 2: Spillover effects when permits are equally distributed between industries

$(e_d, e_u) = (0, 200)$

	$(0.2, 0.1)$	$(0.2, 0.5)$	$(0.2, 0.8)$	$(0.1, 0.2)$	$(0.5, 0.2)$	$(0.8, 0.2)$
$\boldsymbol{(h, v)}$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
	$(0.8, 0.1)$	$(0.8, 0.5)$	$(0.8, 0.8)$	$(0.1, 0.8)$	$(0.5, 0.8)$	$(0.8, 0.8)$
$\boldsymbol{\Delta q^{NC}(\Delta y^{NC})}$	3.48	3.43	3.4	6.92	6.87	6.84
$\boldsymbol{\Delta w_d^{NC}}$	0.25	0.25	0.24	0.55	0.55	0.54
$\boldsymbol{\Delta w_u^{NC}}$	-0.34	-0.32	-0.31	-0.71	-0.69	-0.68
$\boldsymbol{\Delta B_d^{NC}}$	4.15	4.11	4.1	5.1	5.07	5.06
$\boldsymbol{\Delta B_u^{NC}}$	2.91	2.83	2.77	8.89	8.81	8.76
$\boldsymbol{\Delta f_d^{NC}}$	-0.91	-0.92	-0.94	1.27	1.25	1.24
$\boldsymbol{\Delta f_u^{NC}}$	1.16	0.92	0.94	-1.27	-1.25	-1.24
$\boldsymbol{\Delta \sigma}$	-7.84	-7.73	-7.66	-15.56	-15.46	-15.39
$\boldsymbol{\Delta \pi_d}$	2575.05	2495.14	2450.83	3636.9	3566.74	3517
$\boldsymbol{\Delta \pi_u}$	836.82	781.48	734.426	2976.43	2976.43	2933.1

Table 3: Spillover effects when upstream industry holds all permits

4 R&D cooperation model

In this section, we will study R&D cooperation model. This model is practically similar to R&D competition model except that firms cooperate in R&D in the first R&D stage to maximize their joint profits. There are three types of R&D cooperation: horizontal cooperation, vertical cooperation and generalized cooperation. We start with horizontal cooperation.

4.1 Horizontal cooperation

Under horizontal cooperation, firms in the same industry maximize the their joint profits with respect to R&D investments:

$$(w_{d1}^{HC},\ w_{d2}^{HC}) = \arg\max_{w_{d1},\, w_{d2}} \pi_{d1} + \pi_{d2} \tag{21}$$

$$(w_{u1}^{HC},\ w_{u2}^{HC}) = \arg\max_{w_{u1},\, w_{u2}} \pi_{u1} + \pi_{u2} \tag{22}$$

Solving for the four first-order necessary conditions of (21) and (22) yields[16]

$$\begin{aligned} w_d^{HC} &= w_{d1}^{HC} = w_{d2}^{HC} \qquad (23)\\ &= \{4(1+h)[(1+h+2v)(11+11h-14v)+16\gamma](a-c_d-c_u) - [9(1+h)(1+h+2v)(17+17h-2v)\\ &+ 32\gamma(7+7h+5v)]e_d + 2[153(1+h)^2v + 288(1+h)v^2 - 36v^3 - 40(1+h)\gamma + 64v\gamma]e_u\}/\\ &\{[(1+h+2v)(17+17h-2v)+16\gamma][9((1+h)^2-4v^2)+16\gamma]\} \end{aligned}$$

$$\begin{aligned} w_u^{HC} &= w_{u1}^{HC} = w_{u2}^{HC} \qquad (24)\\ &= \{4(1+h)[(1+h+2v)(11+11h-14v)+16\gamma](a-c_d-c_u) + 6[51(1+h)^2v + 96(1+h)v^2\\ &- 12v^3 - 8(1+h)\gamma + 32v\gamma]e_d - 3[3(1+h)[(1+h+2v)(17+17h-2v) + 32(2+2h+v)\gamma]e_u\}/\\ &\{[(1+h+2v)(17+17h-2v)+16\gamma][9((1+h)^2-4v^2)+16\gamma]\} \end{aligned}$$

Under horizontal cooperation, firms equalize R&D investments within the same industry by equalizing the marginal R&D costs ($2\gamma w_d$ and $2\gamma w_u$ respectively) to minimize the total cost of R&D. Therefore, both downstream / upstream firms invest in the same level of R&D.

[16] The second-order Hessian matrix is negative definite, which ensures the maximum solutions.

4.2 Vertical cooperation

Under vertical cooperation, firms cooperate in R&D across industry boundaries, *i.e.* one upstream firm cooperate with one downstream firm to maximize joint profits with respect to R&D investments:

$$(w_{d1}^{VC},\ w_{u1}^{VC}) = \arg \max_{w_{d1},\ w_{u1}} \pi_{d1} + \pi_{u1} \tag{25}$$

$$(w_{d2}^{VC},\ w_{u2}^{VC}) = \arg \max_{w_{d2},\ w_{u2}} \pi_{d2} + \pi_{u2} \tag{26}$$

Solving for the four first-order necessary conditions of (25) and (26) generates[17]

$$\begin{aligned} w_d^{VC} &\equiv w_{d1}^{VC} = w_{d2}^{VC} = w_u^{VC} \equiv w_{u1}^{VC} = w_{u2}^{VC} \\ &= \frac{8(1+v)(-a+c_d+c_u)+(17-h+16v)(e_d+e_u)}{2[(1+h+2v)(-17+h-16v)-16\gamma]} \end{aligned} \tag{27}$$

and therefore,

$$q^{vc} = y^{vc} = \frac{4(1+v)(1+h+2v)(-a+c_d+c_u)-8\gamma(e_d+e_u)}{(1+h+2v)(-17+h-16v)-16\gamma} \tag{28}$$

Note that under vertical cooperation, firms equalize R&D investments between industries by equalizing marginal R&D costs ($2\gamma w$) to minimize the total costs of R&D. Therefore, all firms invest in the same level of R&D.

4.3 Generalized cooperation

Under generalized cooperation, all four firms cooperate in R&D to maximize the joint profit:

$$(w_{d1}^{GC},\ w_{d2}^{GC},\ w_{u1}^{GC},\ w_{u2}^{GC}) = \arg \max_{w_{d1},\ w_{d2},\ w_{u1},\ w_{u2}} \pi_{d1} + \pi_{d2} + \pi_{u1} + \pi_{u2} \tag{29}$$

[17] The second-order Hessian matrix is negative definite, which ensures the maximum solutions.

Solving for the four first-order necessary conditions of (29) yields[18]

$$w^{GC} = w_{d1}^{GC} = w_{d2}^{GC} = w_{u1}^{GC} = w_{u2}^{GC} = \frac{(1+h+2v)(a-c_d-c_u-2e_d-2e_u)}{4[(1+h+2v)^2+\gamma]} \tag{30}$$

and then

$$q_1^{GC} = q_2^{GC} = y_1^{GC} = y_2^{GC} = \frac{4(1+h)(1+h+2v)(a-c_d-c_u)+8\gamma(e_d+e_u)}{(1+h+2v)(17+17h-2v)+16\gamma} \tag{31}$$

Similarly, under generalized cooperation, firms equalize R&D investments to achieve cost minimization and hence undertake the same level of R&D.

In the next section, we will compare the competitive and cooperative R&D equilibria.

5 Comparison of competitive and cooperative R&D equilibria

In the cost-reducing R&D literature with two vertically related industries, Inkmann (1999) shows that vertical R&D cooperation tends to generate more R&D investments than horizontal R&D cooperation, but Atallah (2002) concludes that no setting of R&D cooperation uniformly dominates the others. In the current paper, we want to reexamine which type of R&D activity will induce more investments. Since it is difficult to get analytically tractable comparisons, the same numerical simulation as in Section 3.5.1 is used to rank R&D investments under different type of R&D collaboration. Again, we consider three cases regarding the allocation of permits – *i.e.* $(e_d, e_u) \in \{(200,0),(100,100),(0,200)\}$. To get tractable results, we need to focus on two extreme spillover values – *i.e.* $h, v \in \{0,1\}$. Appendix 3 provides details. Tables 4 to 6 summarize the simulation results.

Result 2 In all cases, firms invest more in R&D under generalized cooperation than

[18] The second-order Hessian matrix is negative definite, which ensures the maximum solutions.

under vertical cooperation.

R&D generalized cooperation internalizes both horizontal and vertical R&D spillovers, while vertical cooperation internalizes only the vertical spillover. Thus, ceteris paribus, R&D investments under generalized cooperation are higher than under vertical cooperation. Moreover, the allocation of permits does not matter for R&D investments under these two types of cooperations, as shown by (27) and (30) that R&D investments depend only on the aggregate permits cap. However, the allocation of permits affects the R&D outputs under R&D competition and horizontal cooperation.

Result 3 Under both R&D competition and horizontal cooperation, with $(e_d, e_u) \in \{(200, 0), (100, 100)\}$, upstream firms invest in more R&D than downstream firms. The result is reversed with $(e_d, e_u) \in (0, 200)$.

As explained in Proposition 4, the more permits downstream firms receive, the less R&D they undertake. Hence, upstream firms have to invest more in R&D, which results from receiving limited R&D benefits from the downstream industry and receiving fewer permits from the government. Similarly, when downstream firms get fewer permits than upstream firms, they will need to conduct more R&D.

Notice that Results 2 and 3 hold irrelevant of spillovers. However, spillovers indeed affect firms' R&D decisions. Tables 4 to 6 illustrate the spillover effects on R&D.

Result 4 When $v = 1$, generalized cooperation generates the highest R&D output.

R&D generalized cooperation internalizes both horizontal and vertical spillovers. Thus, with perfect vertical spillovers, firms get maximum benefits from both industries, and they are willing to invest even more in R&D to enlarge such already large benefits.

Result 5 When $v = 0$, w_u^{HC} is the highest with $(e_d, e_u) \in \{(200, 0), (100, 100)\}$, and so be w_d^{HC} with $(e_d, e_u) \in (0, 200)$.

When there is no vertical R&D spillover, firms only benefit from their competitors within the same industry. When downstream firms receive all permits, upstream firms do not benefit from downstream firms' R&D investments at all despite the fact that downstream firms only invest in minimum amount of R&D, so they need to undertake as much R&D as they can to minimize the abatement costs (buying permits from downstream firms). As horizontal cooperation induces firms to internalize the external horizontal benefits of their R&D, upstream firms will invest the maximum amount of R&D. Therefore, there are two factors making w_u^{HC} the highest: the lack of permits and the internalization of horizontal spillover through cooperation. When upstream firms receive all permits, the result is reversed. When both industries receive the same number of permits, upstream firms still benefit from increasing R&D than trading permits with the downstream industry.

Result 6 When $h = 0$, w_d^{HC} is the lowest with $(e_d, e_u) \in \{(200, 0), (100, 100)\}$, and so be w_u^{HC} with $(e_d, e_u) \in (0, 200)$. When $h = 1$, w_d^{NC} is the lowest with $(e_d, e_u) \in \{(200, 0), (100, 100)\}$, and so be w_u^{NC} with $(e_d, e_u) \in (0, 200)$.

Similarly to Result 5, when $h = 0$ with $(e_d, e_u) \in \{(200, 0), (100, 100)\}$, downstream firms invest the minimum level of R&D under horizontal cooperation if they receive all permits, as this type of cooperation does not lead to the internalization of any benefit, in addition to the fact that downstream firms always have more permits than what they need. The same reason explains w_u^{HC} to be the lowest if upstream firms get all permits.

When $h = 1$, firms get maximum benefit from their competitors in the same industry. Hence, they are willing to conduct more R&D to enlarge such benefit if they cooperate in R&D, but are reluctant to invest in innovation if they compete in R&D. In such context, if downstream firms receive all permits or both industries receive equal number of permits, then w_d^{NC} is the lowest, and so be w_u^{NC} if upstream firms obtain all permits.

$(e_d = 200, e_u = 0)$

$(h = 0, v = 0)$	$w_u^{HC} > w_u^{NC} > w^{GC} > w^{VC} > w_d^{NC} > w_d^{HC}$
$(h = 0, v = 1)$	$w^{GC} > w_u^{HC} > w^{VC} > w_u^{NC} > w_d^{NC} > w_d^{HC}$
$(h = 1, v = 0)$	$w_u^{HC} > w^{GC} > w_d^{HC} > w_u^{NC} > w^{VC} > w_d^{NC}$
$(h = 1, v = 1)$	$w^{GC} > w_u^{HC} > w^{VC} > w_u^{NC} > w_d^{HC} > w_d^{NC}$

Table 4: R&D ranking with $e_d = 200, e_u = 0$

$(e_d = 100, e_u = 100)$

$(h = 0, v = 0)$	$w_u^{HC} > w^{GC} > w_u^{NC} > w^{VC} > w_d^{NC} > w_d^{HC}$
$(h = 0, v = 1)$	$w^{GC} > w^{VC} > w_u^{HC} > w_u^{NC} > w_d^{NC} > w_d^{HC}$
$(h = 1, v = 0)$	$w_u^{HC} > w^{GC} > w_d^{HC} > w_u^{NC} > w^{VC} > w_d^{NC}$
$(h = 1, v = 1)$	$w^{GC} > w_u^{HC} > w^{VC} > w_d^{HC} > w_u^{NC} > w_d^{NC}$

Table 5: R&D ranking with $e_d = 100, e_u = 100$

$(e_d = 0, e_u = 200)$

$(h = 0, v = 0)$	$w_d^{HC} > w_d^{NC} > w^{GC} > w^{VC} > w_u^{NC} > w_u^{HC}$
$(h = 0, v = 1)$	$w^{GC} > w^{VC} > w_d^{HC} > w_d^{NC} > w_u^{NC} > w_u^{HC}$
$(h = 1, v = 0)$	$w_d^{HC} > w^{GC} > w_u^{HC} > w_d^{NC} > w^{VC} > w_u^{NC}$
$(h = 1, v = 1)$	$w^{GC} > w_d^{HC} > w^{VC} > w_d^{NC} > w_u^{HC} > w_u^{NC}$

Table 6: R&D ranking with $e_d = 0, e_u = 200$

6 The optimal allocation of permits

When firms pursue profit maximization, they do not consider pollution damage. For the regulator, this negative externality must be incorporated in order to maximize social welfare. To incorporate that dimension, the game is extended to five stages. In the first stage, the regulator maximizes social welfare by choosing the allocation of permits. The other four stages remain the same as before. Hence, government only controls the initial allocation of permits, but leaves the choice of R&D and production to firms.

It is well known that social welfare (SW) is the sum of consumer surplus (CS) and total profits (TP), net of total damage (TD). Total damage $D(L)$ is defined as a function of total permits, where $D'(L) > 0$, $D''(L) > 0$, *i.e.* damages increase with pollution at an increasing rate.

Consumer surplus is derived as

$$CS = \frac{1}{2}Q^2 \tag{32}$$

and total profits are

$$TP = \pi_{di} + \pi_{dj} + \pi_{um} + \pi_{un} \tag{33}$$

Also, we define the total damage as

$$TD = D(L) = 4(e_d + e_u)^2 \tag{34}$$

The regulator can determine the emissions cap either exogenously (for example, through international negotiation) or endogenously (based on social welfare maximization). In particular, endogenous emissions cap is a special case of exogenous cap. We start with exogenous permits constraint.

6.1 Exogenous permits constraint

The regulator maximizes social welfare by choosing the initial permit allocations e_d and e_u, subject to the exogenous permits constraint L:

$$\max_{e_d,\, e_u} \; SW = CS + TP - TD \tag{35}$$

$$s.t. \; 2(e_d + e_u) = L$$

Substituting $e_d = \frac{L}{2} - e_u$ into (35) and solving for the first-order necessary condition with respect to e_u yield the optimal allocations of permits (e_{sd}^{NC}, e_{su}^{NC}) under R&D competition and (e_{sd}^{HC}, e_{su}^{HC}) under R&D horizontal cooperation:

$$e_{sd}^{NC} = \frac{64(1+h+2v)(a-c_d-c_u) + 9(-35+h+2v)(1+h+2v)L - 224\gamma L}{36[(-35+h+2v)(1+h+2v) - 32\gamma]} \tag{36}$$

$$e_{su}^{NC} = \frac{-64(1+h+2v)(a-c_d-c_u) + 9(-35+h+2v)(1+h+2v)L - 224\gamma L}{36[(-35+h+2v)(1+h+2v) - 32\gamma]} \tag{37}$$

$$e_{sd}^{HC} = \frac{-32(1+h)(1+h+2v)(a-c_d-c_u) + [153(1+h)(1+h+2v) + 18v]L}{36[(1+h+2v)(17+17h-2v) + 16\gamma]} \tag{38}$$

$$e_{su}^{HC} = \frac{32(1+h)(1+h+2v)(a-c_d-c_u) + [153(1+h)(1+h+2v) - 18v]L}{36[(1+h+2v)(17+17h-2v) + 16\gamma]} \tag{39}$$

Under both vertical and generalized cooperations, as R&D investments and outputs are functions of total number of permits, the initial allocation of permits is irrelevant for the social welfare as well. This is because these two types of cooperation involve vertical coordination, which induces firms to choose equal R&D investment across industries (to equalize marginal R&D costs).

Moreover, substituting (36) to (39) into (A5), (A6), (23) and (24) generates the optimal R&D outputs:

$$w_{sd}^{NC} = w_{su}^{NC} = \frac{32(-a+c_d+c_u) - (-35+h+2v)L}{4[(-35+h+2v)(1+h+2v) - 32\gamma]} \tag{40}$$

$$w_{sd}^{HC} = w_{su}^{HC} = \frac{16(1+h)(a-c_d-c_u) - (17+17h-2v)L}{4[(-35+h+2v)(1+h+2v) - 32\gamma]} \tag{41}$$

Furthermore, equations (36) to (39) indicate

$$e_{sd}^{NC} - e_{su}^{NC} = \frac{32[(1+h+2v)(a-c_d-c_u) + \gamma L]}{36[(-35+h+2v)(1+h+2v) - 32\gamma]} < 0 \tag{42}$$

$$e_{sd}^{HC} - e_{su}^{HC} = -\frac{16[(1+h)(1+h+2v)(a-c_d-c_u) + \gamma L]}{9[(1+h+2v)(17+17h-2v) + 16\gamma]} < 0 \tag{43}$$

When the permits cap is exogenously chosen, under either R&D competition or horizontal cooperation, upstream firms always receive more permits than downstream firms. Upstream firms make higher (total) profits and hence invest more in R&D. Due to convex R&D costs, the regulator would prefer industries to invest the same amount of R&D. Therefore, she equalizes R&D marginal costs across industries to maximize social welfare by giving the downstream firms fewer permits to encourage more R&D, and gives upstream firms more permits to reduce their R&D investments.

Proposition 5 *With exogenous permits constraint, R&D investments between industries under both generalized cooperation and vertical cooperation are equalized irrespective of* (e_d, e_u)*. Furthermore, generalized cooperation always generates higher welfare than vertical cooperation.*

Proof. From (27), (28), (30) and (31), it is easy to see that output and R&D levels under both vertical and generalized cooperations depend on total permits only. Figure 1 shows that social welfare under generalized cooperation is always higher than under vertical cooperation.[19] ∎

[19] This graph is drawn based on the same simulation as in Section 3.6, and the result is very robust to changes in parameters.

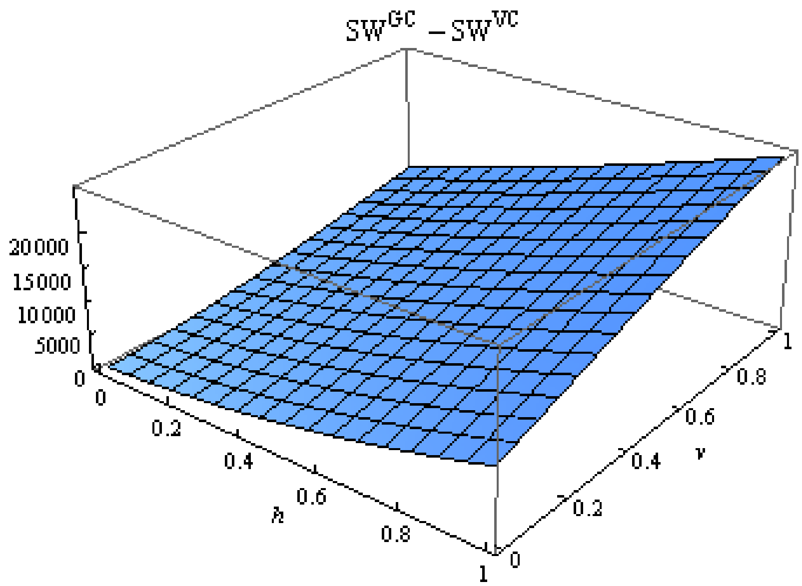


Figure 1: Comparison of social welfare under generalized and vertical cooperations

Satzetakis and McFedrige (1998) demonstrate that with product differentiation, as long as one industry is imperfectly competitive, social optimum is hard to achieve even if the permit market is competitive. They explain that "removal of an environmental externality while leaving an offsetting product market distortion in place may make matters worse" (Sarzetakis and McFedrige, 1998:63). Here, we see that with two vertically related industries with environmental R&D investment, if firms cooperate in R&D vertically or generally, the optimum would not be affected by (e_d, e_u), even if firms have market power in both industries. Thus, the regulator only needs information about how to set the emission "ceiling", but not about individual firms.

6.2 Endogenous permits constraint

When the permits constraint is endogenously settled, we obtain qualitatively similarly results as those with exogenous permits cap. See Appendix 4 for details.

7 Conclusion

In this paper, we address permit trading between two vertically related industries, *i.e.* an upstream industry that produces intermediate good and a downstream industry which purchases the intermediate good as input to produce final good. In each industry, there are two symmetric firms competing à la Cournot and investing in environmental R&D to reduce emissions and abatement costs. Technology spills between firms that makes them benefit from R&D without payment within and between industries. We consider four types of R&D collaboration: competition, horizontal cooperation, vertical cooperation and generalized cooperation.

It is first shown that both intra-industry and inter-industry R&D levels are strategic substitutes. In other words, an increase in one firm's environmental R&D investment reduces the return of R&D of its competitor in the same industry, as well as the return of firms in the other industry. This is different from Banerjee and Lin (2003) who argue that production cost-reducing R&D investments between vertically related industries are strategic complements, *i.e.* an increase in one firm's R&D investment increases the return of R&D of both its competitor and firms in the other industry. Even though firms are price takers in the permit market, the R&D spillovers affect the permit price. More specifically, higher horizontal or vertical spillovers reduce the permit price, as they reduce the demand (and increase the supply) of permits. In contrast, higher spillovers increase production from both industries, because all firms benefit from their competitors and the other industry so that the same levels of production generate less pollution. As a result, firms are willing to produce more, given the permits they hold and R&D investments.

In terms of R&D investment, it depends on both the spillovers and the allocation of permits. If a downstream (upstream) firm receives more permits than an upstream (downstream) firm, then higher horizontal or vertical spillovers will increase (decrease) R&D investments from the downstream(upstream) firm. However, irrespective of spillovers and the allocation of permits, generalized cooperation induces more R&D than vertical cooperation. This is because the generalized cooperation internalizes both the horizontal and vertical spillover externalities, while the vertical cooperation only internalizes the vertical externality. This result is in contrast to Inkmann (1999), who shows that vertical R&D cooperation tends to generate more R&D investments than horizontal R&D cooperation, and Atallah (2002) who concludes that no setting of R&D cooperation uniformly dominates the others.

We also consider the optimal allocation of permits by the regulator. It is shown that under both exogenous or endogenous permits cap, the allocation matters for social welfare under R&D competition and horizontal cooperation, but is irrelevant under generalized cooperation and vertical cooperation. Moreover, generalized cooperation induces higher social welfare than vertical cooperation. Hepburn *et al.* (2006) argue that firms benefit from grandfathering permits in three ways: first, consumers proportionally share the increased marginal costs, which are due to permit trading; second, the increased marginal costs give firms incentives to adopt abatement technologies, which will reduce their costs, and third, tradable permits are distributed freely, which can be seen as a lump-sum transfer to firms. But, Hung and Sartzetakis (1997) and Sartzetakis and McFedrige (1998) show that market structures and enough information are key requirements to achieve efficient permit allocations between industries. However, when regulated industries are vertically related, such as fossil-fuel-supplying and fossil-fuel-using industries, our results imply that the regulator does not need to worry too much about firms' private information. She can induce firms to either inter-industrially cooperate or both intra- and inter-industrially cooperate in environmental R&D to reach the optimal output and R&D levels, given an emission "ceiling".

Chapter 3

Is There a Principle of Targeting in Environmental Taxation?*

* This chapter is based on joint work with Leslie Shiell. We both contributed equally.

1 Introduction

Like optimal tax theory in general, the literature on environmental taxation makes an important distinction between first-best and second-best. In the first best, as expressed by Pigou (1920), the optimal environmental tax is equal to the marginal social damage of emissions of the pollutant in question. In the second best, the optimal environmental tax usually differs from this Pigouvian level due to other distortions. For example, Buchanan (1969), Barnette (1980), Lavin (1985), and Shaffer (1995) consider the effect of market power in the goods market. Their conclusions all point to a value of the optimal environmental tax that is lower than the marginal social damage of emissions. Similarly, in the context of fixed government expenditures and pre-existing distortionary taxes, Bovenberg and de Mooij (1994) and Goulder (1995) conclude that the optimal emissions tax is lower than the Pigouvian level.

However, several papers have argued that the Pigouvian rule still has a role to play in the second best. Sandmo (1975) shows that, in the presence of differentiated commodity taxes, the pollution externality only appears in the tax formula for the pollution-generating good. Moreover, this tax formula can be decomposed into a weighted average of two parts: the efficiency term, related to the inverse elasticity of demand, resembles the theory of optimal commodity taxation, and the environmental term, based upon the marginal social damage of the pollutants. Sandmo refers to this result as the "additivity property".

Dixit (1985) argues that this additivity property represents a particular case of Bhagwati and Johnson's (1960) principle of targeting, according to which a distortion should be directly addressed by a dedicated tax instrument rather than indirectly addressed through adjustments to other taxes. Recently, Kopczuk (2003) claims to generalize this environmental principle of targeting by establishing that the optimal tax formula for pollution-generating goods can be decomposed into the Pigouvian tax plus a correction tax / subsidy.

The present paper considers the relevance of these claims for environmental tax policy. Of particular note, we observe that Sandmo considers the rather specialized case of differentiated commodity taxes. In contrast, in the real world, most products are subject to a uniform commodity tax, and an emissions tax would then be applied to pollution-generating goods on top of the uniform commodity tax. Given this tax structure, we question whether the appearance of an environmental principle of targeting does not in fact arise simply from the increase in the number of tax instruments, with the addition of the emissions tax. Instead, if the principle of targeting is to be meaningful, it would have to hold in a same-numbers exercise – *i.e.* in a comparison where the number of instruments was not changed.

To study these issues, we consider three perfectly competitive markets, one of which produces a "clean" good without pollution by-product, and the other two produce "dirty" goods with pollution by-product. The government collects tax revenues from all three markets to finance an exogenous public expenditure. The optimal taxation is determined by maximizing social welfare.

In Section 2 of the paper, we re-examine the additivity property in the context of differentiated commodity taxes. Sandmo obtains the results by leaving the marginal cost of public funds (MCPF) unsolved in the tax formula. However, he overlooks the fact that the MCPF also depends on the pollution externality. Once the MCPF is precisely solved, the externality will appear in the tax formulae for both clean and dirty goods. In addition, the externality cannot be additively separable in the tax formula for dirty goods. Therefore, Sandmo's additivity property is not valid even in the presence of differentiated taxes.

In Section 3 of the paper, we study whether the principle of targeting is valid in the case with a uniform commodity tax on all goods, and an emissions tax applied only to the dirty goods on the top of the uniform commodity tax. It is shown that, when the government revenue is funded by both taxes, Sandmo's additivity property is further weakened as the emissions externality appears in the tax formulae for both the commodity tax and the emission tax, even if the MCPF remains unsolved. Furthermore, the emissions tax is unlikely to follow the first-best Pigouvian form (marginal social damage).

In Section 4, we consider a same-numbers exercise where only one tax – *i.e.* either the uniform commodity tax or the emissions tax – finances government spending and corrects for pollution. It is found that the uniform commodity tax will induce higher social welfare than the emissions tax when the marginal social damage of the pollutant is low, and the result is reversed when the marginal social damage is high. In other words, in this same-numbers exercise, it is not true that it is always better to address the pollution externality directly through a dedicated emissions tax. Therefore we conclude that there does not exist an environmental principle of targeting which is distinct from the benefit of adding an additional tax instrument.

The paper is organized as follows. Sections 2 and 3 introduce models with three differentiated output taxes, and a uniform commodity tax with an additional emission tax respectively. Section 4 compares social welfare when only the uniform commodity tax or the emissions tax is available. Section 5 concludes.

2 Three differentiated output taxes $(\theta_1, \theta_2, \theta_3)$

Suppose there are three perfectly competitive industries producing one clean good q_1 and two dirty goods q_2 and q_3, where $q_i, i = 1, 2, 3$ is the quantity. The productions of dirty goods generate pollution while the clean good does not. Each good is sold at the price p_i and produced at the constant marginal cost c_i; furthermore, each good is subject to a unit output tax θ_i. The demand for each good is assumed to be linear: $p_i = a_i - q_i$. Each dirty good industry's emission level is assumed to be in proportion to output: $E_2 = e_2 q_2$, $E_3 = e_3 q_3$, where $e_2 > 0$, $e_3 > 0$ are emission intensities. The total social damage from pollution is defined to be $D = \xi(E_2 + E_3)$, where $\xi > 0$ is the marginal social damage.

Perfect competition requires price equal to marginal cost plus the tax rate:

$$p_i = c_i + \theta_i \tag{1}$$

Then, the equilibrium quantities are found to be:

$$\begin{aligned} q_i &= a_i - p_i \\ &= a_i - c_i - \theta_i \end{aligned} \tag{2}$$

For simplicity, we define the market size $a_i - c_i$ to be s_i.[1] Rewriting (2), we have

$$q_i = s_i - \theta_i \tag{3}$$

Note that in order to have an interior solution, it must be the case that $\theta_i < s_i$.

The goal of the government is to choose the output taxes θ_i to maximize social welfare, subject to the government budget constraint. Social welfare is composed of consumer surpluses plus firms' profits (which are zero under perfect competition and constant returns to scale) and tax revenues, net of pollution damage.

The consumer surplus and tax revenue in each industry are given by

$$CS_i = \frac{1}{2}q_i^2 \tag{4}$$

$$TR_i = \theta_i q_i \tag{5}$$

Then, social welfare can be expressed as

$$SW = \sum_{i=1}^{3}(CS_i + TR_i) - D \tag{6}$$

[1] The market size is given by q_i, but since the slope of the inverse demand is (-1), we have $q_i = a_i - c_i$ (in the absence of taxation).

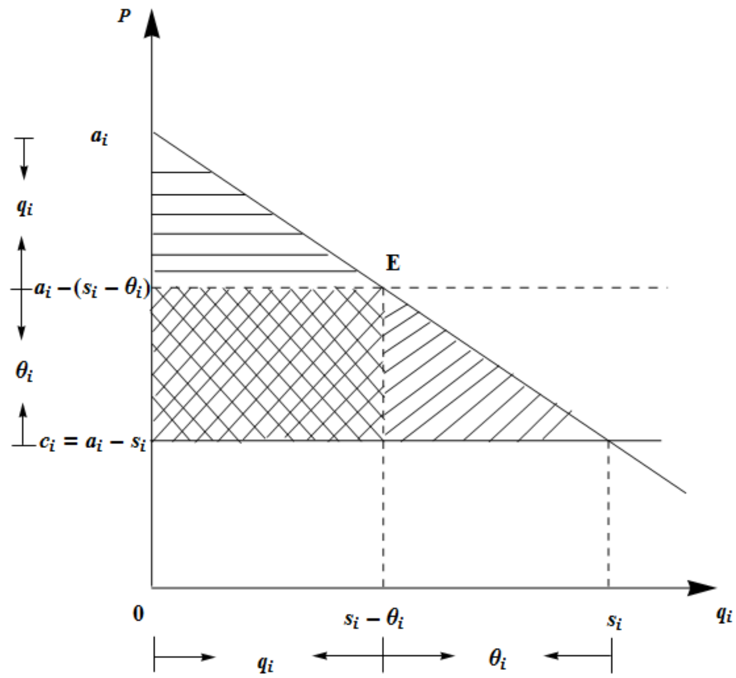


Figure 1: Social welfare for the i^{th} good

Figure 1 illustrates the solution in the market for the i^{th} good. Equilibrium in the presence of the tax occurs at point E on the inverse demand curve. The upper shaded triangle represents consumer surplus derived from the good. The hatched rectangle represents the revenue raised from taxation of the i^{th} good, and the lower shaded triangle represents the corresponding deadweight loss of the tax. Algebraically, the deadweight loss is equal to $\frac{1}{2}\theta_i^2$.

The government needs to generate revenues to cover a budget equal to B:

$$\sum_{i=1}^{3} TR_i = B \tag{7}$$

which (in general) will induce it to choose strictly positive output taxes. In the absence of a budget constraint, the government would impose an output tax on the dirty goods to internalize the environmental externality, but, it would not impose tax on the clean good, since this tax generates deadweight loss which the government wishes to avoid. The total deadweight loss in this problem equals the aggregate across the three markets, *i.e.* $\frac{1}{2}(\theta_1^2 + \theta_2^2 + \theta_3^2)$.

The existence of a solution to (7) depends on the revenue requirement, B, being not too big. The maximum amount of tax revenue that can be raised in market i is $\frac{s_i^2}{4}$.[2] Therefore, the existence of a solution to (7) requires that $B \leqslant \sum_{i=1}^{3} \frac{s_i^2}{4}$.

The optimal taxes are chosen by the government to maximize social welfare subject to the budget constraint:

$$\max_{\theta_i} \; SW \tag{8}$$
$$s.t. \sum_{i=1}^{3} TR_i = B$$

The Lagrangian is

$$\begin{aligned} L &= \sum_{i=1}^{3}(CS_i + TR_i) - D + \alpha(\sum_{i=1}^{3} TR_i - B) \qquad (9)\\ &= \sum_{i=1}^{3}[\frac{1}{2}(s_i - \theta_i)^2 + \theta_i(s_i - \theta_i)] - \xi[e_2(s_2 - \theta_2) + e_3(s_3 - \theta_3)] + \alpha[\sum_{i=1}^{3} \theta_i(s_i - \theta_i) - B] \end{aligned}$$

Focusing on interior solution for θ_i, the four first-order necessary conditions are

$$\frac{\partial L}{\partial \theta_1} = \alpha s_1 - (1 + 2\alpha)\theta_1 = 0 \tag{10}$$

$$\frac{\partial L}{\partial \theta_2} = \alpha s_2 - (1 + 2\alpha)\theta_2 + e_2\xi = 0 \tag{11}$$

$$\frac{\partial L}{\partial \theta_3} = \alpha s_3 - (1 + 2\alpha)\theta_3 + e_3\xi = 0 \tag{12}$$

[2] This result is obtained by substituting (3) into (5), maiximizing with respect to θ_i, and then evaluating (5) at the resulting tax rate.

$$\frac{\partial L}{\partial \alpha} = \sum_{i=1}^{3} TR_i - B = 0 \tag{13}$$

Rewriting (10) to (12), we get

$$\theta_1 = \frac{s_1}{1+2\alpha}\alpha \tag{14}$$

$$\theta_2 = \frac{s_2}{1+2\alpha}\alpha + \frac{e_2\xi}{1+2\alpha} \tag{15}$$

$$\theta_3 = \frac{s_3}{1+2\alpha}\alpha + \frac{e_3\xi}{1+2\alpha} \tag{16}$$

Equations (14) to (16) resemble the tax expressions in Sandmo (1975). In the absence of a pollution externality – *i.e.* with $\xi = 0$ – the commodity taxes for all goods exhibit a similar structure, which only depend on individual market sizes. In the presence of a pollution externality – *i.e.* with $\xi > 0$ – the marginal damage and emission intensities only appear in the tax formulae for dirty goods, and they (in the form of products) are additively separable from the commodity tax portions of the expressions (*i.e.* the first terms). Thus, these results suggest an additivity property, as in Sandmo.

Furthermore, the elasticity of demand for good i can be derived from the demand function $p_i = a_i - q_i$:

$$\varepsilon_i = -\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} = \frac{c_i + \theta_i}{s_i - \theta_i} \tag{17}$$

which gives

$$s_i = \frac{c_i + (1+\varepsilon_i)\theta_i}{\varepsilon_i} \tag{18}$$

Substituting (18) into (14) to (16) demonstrates

$$\theta_1 = \frac{c_1}{(1+\alpha)\varepsilon_1 - \alpha}\alpha \tag{19}$$

$$\theta_2 = \frac{c_2}{(1+\alpha)\varepsilon_2 - \alpha}\alpha + \frac{e_2\xi}{1+2\alpha} \tag{20}$$

$$\theta_3 = \frac{c_3}{(1+\alpha)\varepsilon_3 - \alpha}\alpha + \frac{e_3\xi}{1+2\alpha} \tag{21}$$

Clearly, (19) to (21) shows that the differentiated output taxes follow the Ramsey inverse elasticity rule: the optimal tax rates and the elasticity of demand should be inversely related. Since the price elasticity of linear demand diminishes as we move down the inverse curve, the Ramsey rule entails that the optimal tax rate varies directly with market sizes s_i, as shown in (14) to (16).

However, (14) to (16) do not represent the final solutions for θ_i, as the Lagrangian multiplier α is also an endogenous variable. Substituting (2), (14), (15) and (16) into (13) and solving yields[3]

$$\alpha = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}}{2\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}} - \frac{1}{2} \tag{22}$$

Equation (22) illustrates that the marginal cost of public funds (Lagarangian multiplier α) is a function of the pollution externality. Sandmo exaggerates the additivity property by ignoring this connection. Thus, the optimal taxes indeed depend on the externality, even for the clean good, notwithstanding the appearance of additivity in (14) to (16). More precisely, we explicitly solve θ_i by substituting (22) into (14) to (16):

$$\theta_1^* = \frac{s_1}{2}\left(1 - \frac{\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}}{\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}}\right) \tag{23}$$

$$\theta_2^* = \frac{s_2}{2} - \frac{(s_2 - 2e_2\xi)\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}}{2\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}} \tag{24}$$

$$\theta_3^* = \frac{s_3}{2} - \frac{(s_3 - 2e_3\xi)\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}}{2\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}} \tag{25}$$

The appearance of (ξ, e_2, e_3) in (23) to (25) lead to the following result.

Proposition 1 *Sandmo's additivity property does not hold under differentiated commod-*

[3] (22) is the positive root of a quadratic expression. Since α represents the marginal cost of public funds (in welfare units), it must be positive.

ity taxes, as the pollution externality appears in the tax formulae for both the clean and the dirty goods, and it is not additively separable in the tax formulae for the dirty goods.

It is possible to decompose the differentiated taxes on the dirty goods in order to isolate a role for the first-best pollution tax on the dirty goods. First, define $\widehat{\theta}_2 \equiv \theta_2^* - e_2\xi$, $\widehat{\theta}_3 \equiv \theta_3^* - e_3\xi$. Then, θ_2^* and θ_3^* can be decomposed as follows:

$$\theta_2^* = \widehat{\theta}_2 + e_2\xi \tag{26}$$

$$\theta_3^* = \widehat{\theta}_3 + e_3\xi \tag{27}$$

These expressions indicate that one government agency (*e.g.* the environment ministry) could apply the first-best pollution tax on the dirty goods (*i.e.* the Pigouvian tax ξ multiplied by e_2 and e_3) without jeopardizing the optimality of the tax system, provided there was another agency (*e.g.* the finance ministry) who could apply a corrective tax or subsidy, $\widehat{\theta}_2$ and $\widehat{\theta}_3$. This result echoes Kopczuk (2003) and follows directly from the additivity of taxes on the dirty goods.

However, there is less to this result than it appears. First, any linear decomposition of θ_2^* and θ_3^* is possible, not just one based on the Pigouvian tax. Second, in practical terms, this result amounts to little more than saying that one department can choose any dirty-goods tax it wants, including one based on the Pigouvian rule, as long as there is a second department which will apply the necessary correction. This is hardly a serious recommendation for policy.

3 A uniform commodity tax τ with an additional emissions tax t

We now turn to the model where a uniform commodity tax is applicable. The model is under the same setting except that all goods face a uniform per-unit output tax τ, and emissions from the two dirty goods are charged a tax t.[4]

Perfect competition requires price to be equal to marginal cost plus the tax burden(s)

$$p_1 = c_1 + \tau \tag{28}$$

$$p_j = c_j + \tau + te_j,\ \ j = 2, 3 \tag{29}$$

Consequently, the equilibrium quantities are

$$q_1 = s_1 - \tau \tag{30}$$

$$q_j = s_j - \tau - te_j \tag{31}$$

Furthermore, for an interior solution, (30) and (31) indicate that $s_1 > \tau$ and $s_j > \tau + te_j$. It then follows that $s_j > \tau$ and $s_j > te_j$.

Define the output tax revenue to be TR_q and emissions tax revenue to be TR_e:

$$TR_q = \tau \sum_{i=1}^{3} q_i \tag{32}$$

$$TR_e = t \sum_{j=2}^{3} E_j \tag{33}$$

Note that the commodity tax is applied in all three markets, while the emissions tax is applied only in the dirty goods markets. The social welfare function is defined as

$$SW(\tau, t) = \sum_{i=1}^{3} CS_i + TR_q + TR_e - D \tag{34}$$

The optimal taxes are chosen to maximize social welfare. We consider both the constrained optimization, where the total tax revenue must equal the budget requirement

[4] With 3 goods (1 clean and 2 dirty goods) and 2 taxes, we can see the difference between the 2 tax systems. If there are only two goods (1 clean and 1 dirty goods) and 2 taxes, then the two tax systems are identical.

B, as well as unconstrained optimization. We also consider the possibility of corner solutions for the tax rates, *i.e.* $t = 0$ or $\tau = 0$. Formally, in the case of the constrained optimization, the problem is

$$\max_{\tau, t} \; SW(\tau, t) \tag{35}$$

$$s.t.(i) \; TR_q + TR_e = B, (ii) \; \tau \geqslant 0, \text{ and } (iii) \; t \geqslant 0$$

Then, the Lagrangian function for this problem is

$$\begin{aligned} L &= \sum_{i=1}^{3} CS_i + TR_q + TR_e - D + \lambda(TR_q + TR_e - B) \qquad (36 \\ &= \frac{1}{2}[(s_1-\tau)^2 + \sum_{j=2}^{3}(s_j - \tau - te_j)^2] + \tau[(s_1-\tau) + \sum_{j=2}^{3}(s_j-\tau-te_j)] + (t-\xi)\sum_{j=2}^{3}[e_j(s_j-\tau-te_j) \\ &\quad +\lambda\{\tau[(s_1-\tau) + \sum_{j=2}^{3}(s_j-\tau-te_j)] + t\sum_{j=2}^{3}[e_j(s_j-\tau-te_j)] - B\} \end{aligned}$$

The unconstrained optimization (i.e. no revenue constraint) is a special case of this problem for which $\lambda = 0$.

The possibility of zero and non-zero values for all three variables λ, τ and t yields eight different cases, of which only four are of practical interest. In particular, we consider (i) $\lambda = 0, \tau = 0$ and $t > 0$, (ii) $\lambda > 0, \tau > 0$ and $t > 0$, (iii) $\lambda > 0, \tau > 0$ and $t = 0$, and (iv) $\lambda > 0, \tau = 0$ and $t > 0$. The Kuhn-Tucker conditions for the problem are

$$\frac{\partial L}{\partial \tau} = -3\tau + (-6\tau + s_1 + s_2 + s_3)\lambda + (e_2 + e_3)(t - \xi + 2t\lambda) \leqslant 0 \tag{37}$$

$$\text{and } \frac{\partial L}{\partial \tau} * \tau = 0$$

$$\frac{\partial L}{\partial t} = (e_2^2 + e_3^2)(-t + \xi - 2t\lambda) - e_2[\tau + (2\tau - s_2)\lambda] - e_3[\tau + (2\tau - s_3)\lambda] \leqslant 0 \tag{38}$$

$$\text{and } \frac{\partial L}{\partial t} * t = 0$$

$$\frac{\partial L}{\partial \lambda} = \tau[(s_1 - \tau) + \sum_{j=2}^{3}(s_j - \tau - te_j)] + t\sum_{j=2}^{3}[e_j(s_j - \tau - te_j)] - B = 0 \tag{39}$$

We consider now the four cases.[5]

3.1 Case 1: $\lambda = 0, \tau = 0$ and $t > 0$

In this case where only the emissions tax corrects the externality and there is no revenue requirement, the only first-order necessary condition (derived from 38) is

$$\frac{\partial L}{\partial t} = (e_2^2 + e_3^2)(-t + \xi) = 0 \tag{40}$$

Given e_2, $e_3 > 0$, (40) yields

$$t = \xi \tag{41}$$

Equation (41) is just the standard first-best solution, where the optimal emissions tax equals the Pigouvian tax (marginal social damage rate).

3.2 Case 2: $\lambda > 0, \tau > 0$ and $t > 0$

In this case, both uniform commodity tax and emissions tax contribute to government expenditure. The existence of a solution to $TR_q + TR_e = B$ depends on the revenue requirement, B, being not too big. The maximum amount of tax revenue that can be raised under present assumptions is $\frac{1}{36}[9s_1^2 - \sum_{j=2}^{3} \frac{4(1+(-3+e_j)e_j)(1+e_j^2)s_j^2}{e_j^2}]$.[6] Therefore, it is

[5] The condition (39) is an equality since the budget constraint, when there is one, must hold with equality.

[6] This result is obtained by substituting (30) and (31) into (32) and (33), maximizing with respect to τ and t, and then evaluating (32) and (33) at the resulting tax rate.

necessary that $B \leqslant \frac{1}{36}[9s_1^2 - \sum_{j=2}^{3} \frac{4(1+(-3+e_j)e_j)(1+e_j^2)s_j^2}{e_j^2}]$.

From (37) and (38), we get

$$\tau^* = \frac{(s_1 + s_3)e_2^2 - (s_2 + s_3)e_2e_3 + (s_1 + s_2)e_3^2}{2(1 + 2\lambda)(e_2^2 - e_2e_3 + e_3^2)}\lambda \tag{42}$$

$$t^* = \frac{(2s_2 - s_1 - s_3)e_2 + (2s_3 - s_1 - s_2)e_3}{2(1 + 2\lambda)(e_2^2 - e_2e_3 + e_3^2)}\lambda + \frac{\xi}{1 + 2\lambda} \tag{43}$$

(42) and (43) establish that Sandmo's additivity property is even further weakened in the presence of the uniform commodity tax even without solving for the marginal cost of public funds λ. Different from differentiated taxes, even in the absence of marginal social damage – *i.e.* $\xi = 0$, the emissions intensities e_2 and e_3 emerge in the expressions of the commodity tax and the emissions tax. Therefore, the externality affects both optimal taxes. Additionally, (43) illustrates that the externality is not additively separable in that the emissions intensities appear in the first term and the social damage ξ appears in the second term (as in (15) and (16)). Moreover, λ can be obtained by substituting (42) and (43) into (39)[7]

$$\lambda = \frac{1}{2}(-1 + \frac{\sqrt{A + C}}{2\sqrt{C}}) \tag{44}$$

where $A = 8(e_2^2 - e_2e_3 + e_3^2)\{B + \xi[(\xi e_2 - s_2)e_2 + (\xi e_3 - s_3)e_3]\}$, $B = [(s_1 + s_3)^2 + 2(s_2^2 - 4B)]e_2^2 - 2[s_2^2 - s_2s_3 + s_3^2 + s_1(s_2 + s_3) - 4B]e_2e_3 + [(s_1 + s_2)^2 + 2(s_3^2 - 4B)]e_3^2$. Obviously, the marginal cost of public funds is a function of the externality, as in the previous case.

Moreover, (43) and (44) demonstrate that in general the optimal emissions tax does not follow the Pigouvian tax (tax equal to marginal social damage). The only special case where it does follow the Pigouvian tax is when

$$\xi = \frac{(2s_2 - s_1 - s_3)e_2 + (2s_3 - s_1 - s_2)e_3}{4(e_2^2 - e_2e_3 + e_3^2)} \tag{45}$$

by substituting (44) into (43), and equating t^* with ξ.

[7] Again, we drop the negative root.

(42) to (45) lead to the following result.

Proposition 2 *(i) Sandmo's additivity property is further weakened under the combination of a uniform commodity tax and an emissions tax, as the emissions intensities appear in the tax formulae for both the commodity tax and the emission tax. (ii) The Pigouvian tax is unlikely to apply on the dirty goods.*

The next two cases involve the use of a single tax instrument – either the commodity tax, τ or the emissions tax, t – to meet the revenue requirement. The discussion of these cases will be taken up in the subsequent section.

3.3 case 3: $\lambda > 0, \tau > 0$ and $t = 0$

In this case, the regulator only uses the uniform commodity tax to collect revenue and control pollution. Once again, the existence of a solution to $TR_q = B$ depends on the revenue requirement, B, being not too big. The maximum amount of tax revenue that can be raised under present assumptions is $\frac{(s_1+s_2+s_3)^2}{12}$.[8] Therefore, the existence of a solution for $TR_q = B$ requires that $B \leqslant \frac{(s_1+s_2+s_3)^2}{12}$.

The two first-order necessary conditions from (37) and (39) become

$$\frac{\partial L}{\partial \tau} = -3\tau + \xi(e_2 + e_3) + (-6\tau + s_1 + s_2 + s_3)\lambda = 0 \tag{46}$$

$$\frac{\partial L}{\partial \lambda} = \tau[(s_1 - \tau) + \sum_{j=2}^{3}(s_j - \tau)] - B = 0 \tag{47}$$

The revenue constraint (47) determines τ and condition (46) then determines λ. Hence τ is obtained directly from (47)

$$\tau = \frac{1}{6}(s_1 + s_2 + s_3 - \sqrt{(s_1 + s_2 + s_3)^2 - 12B}) \tag{48}$$

[8] This result is obtained by substituting (30) and (31) into (32), maiximizing with respect to τ, and then evaluating (32) at the resulting tax rate.

and thus λ from (46)

$$\lambda = \frac{1}{2}[-1 + \frac{\sqrt{[-2\xi(e_2+e_3)+s_1+s_2+s_3]^2}}{\sqrt{(s_1+s_2+s_3)^2-12B}}] \tag{49}$$

We note that $(s_1+s_2+s_3)^2 - 12B \geqslant 0$ in both these expressions by virtue of the assumption that $B \leqslant \frac{(s_1+s_2+s_3)^2}{12}$.

In this case, the commodity tax is used to indirectly control pollution.

3.4 Case 4: $\lambda > 0, \tau = 0$ and $t > 0$

In this case, only the emissions tax contributes to the government revenue and corrects the externality. The maximum amount of tax revenue that can be raised under present assumptions is $\frac{(e_2s_2+e_3s_3)^2}{4(e_2^2+e_3^2)^2}$.[9] Therefore, the existence of a solution for $TR_e = B$ requires that $B \leqslant \frac{(e_2s_2+e_3s_3)^2}{4(e_2^2+e_3^2)^2}$.

The two first-order necessary conditions developed from (38) and (39) become

$$\frac{\partial L}{\partial t} = (e_2^2+e_3^2)(-t+\xi) + [e_2s_2+e_3s_3-2t(e_2^2+e_3^2)]\lambda = 0 \tag{50}$$

$$\frac{\partial L}{\partial \lambda} = t\sum_{j=2}^{3}[e_j(s_j-te_j)] - B = 0 \tag{51}$$

As in the previous case, the revenue constraint (51) determines the optimal tax value, t. Condition (50) then determines λ. Hence, t is obtained directly from (51)

$$t = \frac{e_2s_2+e_3s_3-\sqrt{(e_2s_2+e_3s_3)^2-4B(e_2^2+e_3^2)}}{2(e_2^2+e_3^2)} \tag{52}$$

and thus λ from (50)

[9]This result is obtained by substituting (31) into (33), maiximizing with respect to t, and then evaluating (33) at the resulting tax rate.

$$\lambda = \frac{1}{2}\left(\frac{\sqrt{|2\xi(e_2^2+e_3^2)-(e_2s_2+e_3s_3)|}}{\sqrt{(e_2s_2+e_3s_3)^2-4B(e_2^2+e_3^2)}}-1\right) \tag{53}$$

We note that $(e_2s_2+e_3s_3)^2-4B(e_2^2+e_3^2)>0$ in both these expressions by virtue of the assumption that $B \leqslant \frac{(e_2s_2+e_3s_3)^2}{4(e_2^2+e_3^2)^2}$.

(52) illustrates that the second-best emissions tax depends upon the emissions intensities but not on pollution damage (*i.e.* ξ does not appear). This follows from the budget constraint, since the government still has to finance its expenditure target B.

4 Comparison between $SW(\tau, 0)$ and $SW(0, t)$

Proposition 2 indicates that there does not exist a principle of targeting in environmental taxation. Nonetheless, the idea of using an emissions tax to directly address a pollution externality remains appealing. There are at least two reasons which may explain this appeal. First, the logic of the first best solution (the Pigouvian tax) is so compelling and transparent that we tend to transform it into a rule of thumb, which we then apply universally, even in cases when it is not appropriate.

Second, and perhaps more important, is the observation that two tax instruments can never be worse than one and in many cases will prove to be better, in terms of social welfare. Since a uniform commodity tax already exists in most jurisdictions, the choice is whether to adjust the commodity tax to reflect pollution damages or to add an emissions tax to address pollution emissions directly, while maintaining the commodity tax. Since two instruments are better than one, the second choice is preferable. Thus, we arrive at a result which looks rather like a principle of targeting but which in fact is due to the additional degree of freedom which follows from adding another tax instrument.

It follows from this discussion that an environmental principle of targeting would only be meaningful when (i) we are in a second best setting, and (ii) we must choose between the same number of different instruments rather than between say n instruments and

$n+1$ instruments, where the $n+1^{th}$ instrument is an emissions tax. To illustrate, we consider the same distortions as before, *i.e.* the pollution externality and the lack of a lump-sum tax to meet the revenue requirement. In terms of the instruments, we consider a choice between using the commodity tax on its own and the emissions tax on its own (*i.e.* the same number of different instruments). These choices correspond with Case 3 and Case 4 respectively, in the previous section.

To demonstrate the absence of a principle of targeting, we must be able to show that, for some parameter values, the commodity tax will be preferred to the emissions tax, despite the presence of the pollution externality. Substituting (48) into (34) and setting $t=0$ yields the value of social welfare using the commodity tax only, *i.e.*

$$\begin{aligned} & SW(\tau,0) \\ = \; & \frac{1}{2}(s_1-\tau^*)^2+\frac{1}{2}(s_2-\tau^*)^2+\frac{1}{2}(s_3-\tau^*)^2+B-\xi[e_2(s_2-\tau^*)+e_3(s_3-\tau^*)] \end{aligned} \tag{54}$$

Then substituting (52) into (34) and setting $\tau=0$ yields the value of social welfare using the emissions tax only, *i.e.*

$$\begin{aligned} & SW(0,t) \\ = \; & \frac{1}{2}(s_1)^2+\frac{1}{2}(s_2-\widehat{t}_2)^2+\frac{1}{2}(s_3-\widehat{t}_3)^2+B-\xi[e_2(s_2-\widehat{t}_2)+e_3(s_3-\widehat{t}_3)] \end{aligned} \tag{55}$$

where $\widehat{t}_2 \equiv te_2$ and $\widehat{t}_3 \equiv te_3$. Taking the difference between (54) and (55) yields

$$\begin{aligned} & SW(\tau,0)-SW(0,t) \\ = \; & \frac{1}{2}(\widehat{t}_2^2+\widehat{t}_3^2)-\frac{3}{2}\tau^2-\xi[e_2(\widehat{t}_2-\tau)+e_3(\widehat{t}_3-\tau)] \end{aligned} \tag{56}$$

where we have exploited the fact that $TR_q=TR_e=B$.

Note that the first two terms in (56), *i.e.* $\frac{1}{2}(\widehat{t}_2^2+\widehat{t}_3^2)-\frac{3}{2}\tau^2$, represent the difference in deadweight loss between the emissions tax and the commodity tax (see the discussion of the calculation of deadweight loss in section 2). Conventional wisdom suggests that the deadweight loss under the emissions tax will be larger than under the commodity tax, since the emissions tax raises the same revenue from a narrower tax base, *e.g.* two markets rather than three, under present assumptions. For the same reason, we should expect the emissions tax rate to be higher than the commodity tax rate, *i.e.* $\widehat{t}_2 > \tau$ and $\widehat{t}_3 > \tau$.

If true, then inspection of (56) indicates that $SW(\tau, 0) - SW(0, t) > 0$ and the commodity tax will be preferred, for a sufficiently small value of pollution damage, ξ. In contrast, for a high value of ξ, $SW(\tau, 0) - SW(0, t) < 0$ and the pollution tax will be preferred. We summarize as follows.

Proposition 3 *If* $\frac{1}{2}(\widehat{t}_2^2 + \widehat{t}_3^2) > \frac{3}{2}\tau^2$ *and* $\widehat{t}_2, \widehat{t}_3 > \tau$, *then there exists a value of pollution damage* $\overline{\xi}$ *such that* $SW(\tau, 0) - SW(0, t) > 0$ *for* $0 < \xi < \overline{\xi}$ *and* $SW(\tau, 0) - SW(0, t) < 0$ *for* $\xi > \overline{\xi}$.

The practical significance of this result is that there is no presumption in favour of the emissions tax, notwithstanding the existence of a pollution externality. Rather, when faced with a choice between the emissions tax alone and the commodity tax alone, the preferred choice depends upon parameter values. Stated differently, there does not exist any principle of targeting in environmental taxation which holds that we should always prefer to target the pollution externality directly, when choosing between the same number of different instruments.

Proving that the deadweight loss is greater under the emission tax (*i.e.* $\frac{1}{2}(\widehat{t}_2^2 + \widehat{t}_3^2) > \frac{3}{2}\tau^2$) and that emission tax rates are greater than the commodity tax rate (*i.e.* $\widehat{t}_2, \widehat{t}_3 > \tau$) is complicated by the fact that, for a uniform emissions tax, t, the corresponding dirty-good taxes, $\widehat{t}_2$ and $\widehat{t}_3$, are differentiated by exogenous differences in the emission intensities, e_2 and e_3. Nonetheless, in order to show the failure of the principle of targeting, we need to find only one case where the commodity tax is preferable to the emissions tax.

The simplest case which can be proven analytically involves only one dirty good and markets of equal size. Consider then the case where goods 1 and 2 are clean goods and good 3 is the only dirty good (*i.e.* $e_2 = 0, e_3 > 0$), and where all three markets are of equal size ($s_1 = s_2 = s_3 = s$). The revenue requirement under the emissions tax is then

$$\widehat{t}_3(s - \widehat{t}_3) = B \tag{57}$$

and under the commodity tax it is

$$3\tau(s - \tau) = B \tag{58}$$

The deadweight loss of the emissions tax is now $\frac{1}{2}\widehat{t}_3^2$, and the welfare differential is

$$\begin{aligned} & SW(\tau, 0) - SW(0, t) \\ = \ & \frac{1}{2}\widehat{t}_3^2 - \frac{3}{2}\tau^2 - \xi e_3(\widehat{t}_3 - \tau) \end{aligned} \tag{59}$$

The existence of solutions to (57) and (58) depends on B being not too big. The maximum amount of tax revenue that can be raised by the emission tax is $\frac{s^2}{4}$, while the maximum amount of revenue that can be raised by the commodity tax is $\frac{3s^2}{4}$.[10] Since the first amount is lower, it provides the upper bound on the permissible value of B, *i.e.* $B \leqslant \frac{s^2}{4}$.

First, we will prove that the emission tax rate $\widehat{t}_3$ exceeds the commodity tax, *i.e.* $\widehat{t}_3 > \tau$. Rearranging (57) and (58) yields quadratic equations

$$\widehat{t}_3^2 - s\widehat{t}_3 + B = 0 \tag{60}$$

[10] See above for the details of caculating the maximum obtainable tax revenue.

and

$$\tau^2 - s\tau + \frac{B}{3} = 0 \tag{61}$$

Solving for the smallest roots (*i.e.* the left-hand side of the Laffer curve), we obtain

$$\widehat{t}_3 = \frac{s - \sqrt{s^2 - 4B}}{2} \tag{62}$$

and

$$\tau = \frac{s - \sqrt{s^2 - \frac{4B}{3}}}{2}$$

These expressions are well defined given the assumed upper-bound on B (*i.e.* $B \leqslant \frac{s^2}{4}$). It follows that $\widehat{t}_3 > \tau$, since $s^2 - 4B < s^2 - \frac{4B}{3}$.

To prove that the deadweight loss is greater under the emissions tax in this case, we consider a thought experiment in which an emissions tax, $\widehat{t}$, is imposed on the one polluting good, while at the same time a uniform commodity tax, $\widehat{\tau}$, is imposed on the two clean goods but not on the dirty good. In this case, the revenue constraint is

$$2\widehat{\tau}(s - \widehat{\tau}) + \widehat{t}(s - \widehat{t}) = B \tag{63}$$

The case of an emissions tax alone which we have studied above corresponds with values $\widehat{\tau} = 0$ and $\widehat{t} = \widehat{t}_3$.

We now consider tradeoffs between $\widehat{\tau}$ and $\widehat{t}$ which keep the revenue constraint satisfied. Taking the differential of (63) yields

$$2(s - 2\widehat{\tau})\ d\widehat{\tau} + (s - 2\widehat{t})\ d\widehat{t} = 0 \tag{64}$$

which, upon rearranging, gives the tradeoff

$$d\widehat{\tau} = -\frac{s - 2\widehat{t}}{2(s - 2\widehat{\tau})} d\widehat{t} \tag{65}$$

The deadweight loss of taxation in this case is

$$DWL = \widehat{\tau}^2 + \frac{1}{2}\widehat{t}^2 \tag{66}$$

The change in deadweight loss which can obtained by shifting the tax burdens between $\widehat{\tau}$ and $\widehat{t}$ is given by

$$dDWL = 2\widehat{\tau}\ d\widehat{\tau} + \widehat{t}\ d\widehat{t} \tag{67}$$

Starting with $\widehat{\tau} = 0$ and $\widehat{t} = \widehat{t}_3$, we have

$$dDWL = \widehat{t}\ d\widehat{t} \tag{68}$$

which is negative for a tax shift from $\widehat{t}$ to $\widehat{\tau}$, *i.e.* for $d\widehat{t} < 0$ and $d\widehat{\tau} > 0$. This result verifies the conventional wisdom that expanding from a narrow tax base to a broader tax base, given B, will decrease aggregated deadweight loss, since the measure of deadweight loss is quadratic in the tax rate.

We wish to characterize the combination of $\widehat{t}$ and $\widehat{\tau}$ which minimize deadweight loss. Setting $dDWL = 0$ in (67) and substituting for $d\widehat{\tau}$ from (65), we obtain

$$\widehat{t}\ d\widehat{t} = 2\widehat{\tau}\frac{s - 2\widehat{t}}{2(s - 2\widehat{\tau})}d\widehat{t} \tag{69}$$

which reduces to $\widehat{t} = \widehat{\tau}$. In other words, starting from a position of the emissions tax alone ($\widehat{t} = \widehat{t}_3$ and $\widehat{\tau} = 0$), we can reduce the deadweight loss by reducing $\widehat{t}$ and increasing $\widehat{\tau}$, following (65), until we reach a position of equality, $\widehat{t} = \widehat{\tau}$, at which point the deadweight loss is minimized. But this outcome is none other than the uniform commodity tax on all three goods, *i.e.* $\widehat{t} = \widehat{\tau} = \tau$. It follows that the deadweight loss of the tax system is smaller under the uniform commodity tax than under the emissions tax alone. This result, combined with $\widehat{t}_3 > \tau$, verifies the assumption required for Proposition 3 in the simple case of one dirty good and markets of equal size.

This exercise quickly becomes intractable for more complex cases involving differentiated market sizes and emissions intensities. Nonetheless, we can easily test numerical parameters values to verify the applicability of Proposition 3. For example, if we assume the parameters are uniformly distributed over the intervals $100 \leqslant s_1 \leqslant 400$, $200 \leqslant s_2 \leqslant 500$, $50 \leqslant s_3 \leqslant 300$, $0 \leqslant e_2 \leqslant 10$, $0 \leqslant e_3 \leqslant 5$ and let ξ vary, then the results from running Monte Carlo experiments 1000 times by randomly choosing the values are shown in Table 1.[11] Obviously, the higher the value of ξ, the more likely that $SW(\tau, 0) - SW(0, t) < 0$, which echoes Proposition 3.

Table 1: Monte Carlo Experiment Results

	$\xi = 1$		$\xi = 3$		$\xi = 5$		$\xi = 10$		$\xi = 20$		$\xi = 50$	
$SW(\tau, 0) - SW(0, t)$	P	N	P	N	P	N	P	N	P	N	P	N
Number of results	801	199	646	354	588	412	408	592	244	756	92	908

5 Conclusion

The "principle of targeting" in environmental taxation provides policy makers an easily implemented rule to curb pollution. Dixit (1985) first refers to this principle, based on Bhagwati and Johnson (1960), that externality-generating sources should be directly targeted, and Sandmo (1975) that the externality only appears in the tax formulae for polluting goods and it is additively separable. Kopczuk (2003) further generalizes this principle to state that the first-best Pigouvian tax should be directly imposed on polluting sources in the second best, provided a corrective tax or subsidy is also applied. However, Sandmo's additivity property is based upon differentiated taxes. In reality, governments do not always have enough instruments to correct each distortion. More precisely, consumption goods are usually subject to a uniform commodity tax, and an emissions tax would be charged on top of that. Thus, it would be important to investigate

[11] P denotes $SW(\tau, 0) - SW(0, t) > 0$ and N represents $SW(\tau, 0) - SW(0, t) < 0$.

whether the principle of targeting still remains valid under such a tax system.

The current paper establishes that the principle of targeting is unlikely to be valid under either differentiated taxes or a uniform commodity tax with an additional emissions tax. We first reexamine Sandmo's additivity property under differentiated taxes. It is found that Sandmo exaggerates such property by overlooking the explicit value of the marginal cost of public funds (MCPF). Given that the MCPF depends on pollution damage, the externality also appears in the tax formulae for the non-polluting goods. Furthermore, the externality is no long additively separable in the tax formulae for the polluting goods. The additivity property is further weakened under a uniform commodity tax since the externality appears in the tax formulae for the non-polluting goods even without solving for the MCPF, since the uniform tax is jointly chosen from all three markets.

We also compare the social welfare when only one tax instrument – either the uniform commodity tax or the emissions tax – is available to the regulator. It is demonstrated that the emissions tax should only be employed when the marginal social damage is relatively high; otherwise, the uniform commodity tax needs to be imposed to generate higher social welfare. Therefore, it is not always true that a direct approach to controlling emissions is preferable when the same number of different instruments is compared.

Finally, linear decomposition of the optimal tax on dirty goods and the optimal tax on emissions are possible, as argued by Kopczuk (2003), but they hardly present a meaningful role for the first-best Pigouvian rule in the second best. We conclude that a principle of targeting does not exist in environmental taxation.

Conclusion

This dissertation explores two important market-based environmental regulatory tools: permits and taxes. The first chapter, titled "Tradable Permits under Environmental and Cost-reducing R&D", and the second chapter, titled "Tradable Permits under Environmental R&D between Upstream and Downstream Industries", study tradable emissions permits, and the third chapter, titled "Is There a Principle of Targeting in Environmental Taxation", focuses on environmental taxation.

In *Tradable Permits under Environmental and Cost-reducing R&D*, I model asymmetric Cournot duopolistes who trade emissions permits and undertake two types of R&D: the cost-reducing R&D which reduces the production cost and the environmental R&D which reduces emissions and abatement cost. Firms can either compete or cooperate in R&D, but remain competitive in production. On one hand, the allocation of permits influences firms' R&D decision; on the other hand, R&D spillovers also influence the optimal allocation of permits.

The main results show that the more permits a firm receives, the more it conducts cost-reducing R&D but less environmental R&D. Moreover, competition (cooperation) induces more R&D investment when R&D spillovers are low (high). Comparing the optimal allocation of permits and allocation via grandfathering in the presence of exogenous emissions cap and R&D competition, it is shown that the comparison depends on two factors: the R&D spillovers and the level of emission reduction. At optimum, the large firm receives fewer permits when R&D spillovers are high and receive more permits when the emission reduction is high.

The second chapter, *Tradable Permits under Environmental R&D between Upstream and Downstream Industries*, extends study in the first chapter to two vertically related industries that invest in environmental R&D and trade permits. Four types of R&D activities are considered: competition, horizontal, cooperation, vertical cooperation and generalized cooperation.

The main results show that higher horizontal or vertical R&D spillovers reduce the permit price. In terms of the R&D investments, it is shown that generalized cooperation leads to more investment than vertical cooperation, since the former internalizes both the horizontal and vertical externalities. As for the optimal allocation of permits, it matters for social welfare under R&D competition and horizontal cooperation, but is irrelevant under either vertical or generalized cooperation.

The third chapter, *Is There a Principle of Targeting in Environmental Taxation*, coauthored with Leslie Shiell, reexamines the principle of targeting (Dixit 1980, Kopczuk 2003) that refers to Sandmo (1975)'s "additivity property" and Bhagwati and Johnson (1960)'s targeting theory.

The main results show that the Sandmo's additivity does not hold under either differentiated taxes or a uniform commodity tax with an emissions tax on the top of it. Furthermore, this chapter also examines two cases where only one tax – i.e. either the uniform commodity tax or the emissions tax – is used to finance government spending. It is not true that it is always better to address the pollution externality directly through a dedicated emissions tax. Therefore, we conclude that there does not exist an environmental principle of targeting which is distinct from the benefit of adding an additional tax instrument.

Appendix of Chapter 1

Appendix 1: Competitive R&D equilibrium results and proofs for comparative statics with $\beta = 0$ and $\beta = 1$

The second-order Hessian matrix of (13) is

$$H = \begin{bmatrix} \frac{1}{2}(-1+\beta)^2 - 2\gamma & \frac{1}{2}(2+\beta+\beta^2) \\ \frac{1}{2}(2+\beta+\beta^2) & \frac{1}{2}[-5+(-4+\beta)\beta - 4\delta] \end{bmatrix} \tag{A1}$$

Given $\beta \in [0,1]$, $\gamma \geqslant 1$ and $\delta \geqslant 1$, the second-order sufficient conditions exhibit $|H_1| < 0$, $|H_2| > 0$, thus the negative definite Hessian matrix guarantees maximum solutions.

The equilibria are obtained by solving the four first-order conditions for interior solutions. Without imposing restrictions on β, the aggregate R&D investments, quantity level and the permit price are

$$\overline{x}_1 + \overline{x}_2 = \frac{2(3-\beta)[s(1+\beta)+\delta(e_1+e_2)]}{(1+\beta)[-3-2\beta+\beta^2+2(7+\beta)\gamma]+8\gamma\delta} \tag{A2}$$

$$\overline{w}_1 + \overline{w}_2 = \frac{8s\gamma - [-3-2\beta+\beta^2+2(7+\beta)\gamma](e_1+e_2)]}{(1+\beta)[-3-2\beta+\beta^2+2(7+\beta)\gamma]+8\gamma\delta} \tag{A3}$$

$$\overline{q}_1 + \overline{q}_2 = \frac{8\gamma[s(1+\beta)+\delta(e_1+e_2)]}{(1+\beta)[-3-2\beta+\beta^2+2(7+\beta)\gamma]+8\gamma\delta} \tag{A4}$$

$$\overline{\sigma} = \frac{2s\gamma[(1+\beta)^2+4\delta] - [-3-2\beta+\beta^2+12\gamma](e_1+e_2)]}{(1+\beta)[-3-2\beta+\beta^2+2(7+\beta)\gamma]+8\gamma\delta} \tag{A5}$$

(1) Equilibria results with $\beta = 0$

Define $A = \gamma(3+4\delta) - 2(1+\delta)$, $B = -3 + 2\gamma(7+4\delta)$, and $s = a - c$. Given $\gamma \geqslant 1$ and $\delta \geqslant 1$, $A > 0$ and $B > 0$.

When $\beta = 0$, solving the four first-order conditions for interior solutions generates

$$\overline{x}_1 = \frac{6As - 4Bk(1+\delta) + \delta[32\gamma(1+\delta) - 3(5+4\delta)]e_1 + \delta[-9+4\gamma+4(-3+4\gamma)\delta]e_2}{2AB} \tag{A6}$$

$$\overline{x}_2 = \frac{6As + 4Bk(1+\delta) + \delta[-9+4\gamma+4(-3+4\gamma)\delta]e_1 + \delta[32\gamma(1+\delta) - 3(5+4\delta)]e_2}{2AB} \tag{A7}$$

$$\overline{w}_1 = \frac{(4As - 2Bk)\gamma - [3(2+\delta) + \gamma(-37+42\gamma-28\delta+40\gamma\delta)]e_1 + \delta[-3+4(3-4\gamma)\gamma]e_2}{2AB} \tag{A8}$$

$$\overline{w}_2 = \frac{(4As + 2Bk)\gamma + \delta[-3+4(3-4\gamma)\gamma]e_1 - [3(2+\delta) + \gamma(-37+42\gamma-28\delta+40\gamma\delta)]e_2}{2AB} \tag{A9}$$

$$\overline{q}_1 = \frac{[8As - 2Bk(3+4\delta)]\gamma}{2AB} + \frac{\delta[-3+2\gamma(-1-4\delta+4\gamma(3+4\delta))]e_1 + \delta[3+6\gamma(-5+4\gamma)+8\gamma(-3+4\gamma)\delta]e_2}{2AB} \tag{A10}$$

$$\overline{q}_2 = \frac{[8As + 2Bk(3+4\delta)]\gamma +}{2AB} + \frac{\delta[3+6\gamma(-5+4\gamma)+8\gamma(-3+4\gamma)\delta]e_1 + \delta[-3+2\gamma(-1-4\delta+4\gamma(3+4\delta))]e_2}{2AB} \tag{A11}$$

$$\overline{f}_1 = \frac{-2k\gamma(1+4\delta) + (-4+6\gamma-\delta+4\gamma\delta)e_1 + (-3+4\gamma)\delta e_2}{-4(1+\delta)+\gamma(6+8\delta)} \tag{A12}$$

$$\overline{f}_2 = \frac{2k\gamma(1+4\delta) + (-3+4\gamma)\delta e_1 + (-4+6\gamma-\delta+4\gamma\delta)e_2}{-4(1+\delta)+\gamma(6+8\delta)} \tag{A13}$$

$$\overline{\sigma} = \frac{2s\gamma(1+4\delta) - 3(4\gamma-1)\delta(e_1+e_2)}{B} \tag{A14}$$

(2) Proofs of competitive R&D comparative statics results with $\beta = 0$

In theory, higher marginal production cost results in lower production. Thus, an increase in k increases the marginal cost of firm 1 but decreases the marginal cost of firm 2.

Therefore, taking derivative of $\overline{q}_i$ with respect to k by using $(A10)$ and $(A11)$ yields

$$\frac{\partial \overline{q}_1}{\partial k} = \frac{\gamma(3+4\delta)}{-A} < 0 \tag{A15}$$

$$\frac{\partial \overline{q}_2}{\partial k} = \frac{\gamma(3+4\delta)}{A} > 0 \tag{A16}$$

given $A > 0$. Furthermore, taking derivative of $\overline{q}_i$ with respect to e_1 and e_2 gives

$$\frac{\partial \overline{q}_1}{\partial e_1} = \frac{\partial \overline{q}_2}{\partial e_2} = \frac{\delta[-3+2\gamma(-1-4\delta+4\gamma(3+4\delta))]}{2AB} > 0 \tag{A17}$$

$$\frac{\partial \overline{q}_1}{\partial e_2} = \frac{\partial \overline{q}_2}{\partial e_1} = \frac{\delta[3+8\gamma^2(3+4\delta)-6\gamma(5+4\delta)]}{2AB} > 0 \tag{A18}$$

The denominators $2AB$ of $(A17)$ and $(A18)$ are positive. Also, the numerator of $(A17) >$ $-3+2[-1-4\delta+4(3+4\delta)] = 19+24\delta$ and the numerator of $(A18) > 3+8(3+4\delta)-6(5+4\delta) = 8\delta-3$ are positive. Therefore, $\frac{\partial \overline{q}_1}{\partial e_1} = \frac{\partial \overline{q}_2}{\partial e_2} > 0$ and $\frac{\partial \overline{q}_1}{\partial e_2} = \frac{\partial \overline{q}_2}{\partial e_1} > 0$. In addition, $(A17)$ and $(A18)$ indicate

$$\frac{\partial \overline{q}_1}{\partial e_1} - \frac{\partial \overline{q}_1}{\partial e_2} = \frac{\partial \overline{q}_2}{\partial e_2} - \frac{\partial \overline{q}_2}{\partial e_1} = \frac{8\gamma[2(3\gamma-2)+4\delta(2\gamma-1)]}{2AB} > 0 \tag{A19}$$

Equation $(A19)$ establishes that the direct effect of initial permits on production $\frac{\partial \overline{q}_i}{\partial e_i}$ dominates the indirect effect $\frac{\partial \overline{q}_i}{\partial e_j}$.

Next, taking derivatives of $\overline{x}_i$ with respect to k by using $(A6)$ and $(A7)$ generates

$$\frac{\partial \overline{x}_1}{\partial k} = \frac{2+2\delta}{-A} < 0 \tag{A20}$$

$$\frac{\partial \overline{x}_2}{\partial k} = \frac{2+2\delta}{A} > 0 \tag{A21}$$

Also, taking derivatives of $\overline{x}_i$ with respect to e_1 and e_2 demonstrates

$$\frac{\partial \overline{x}_1}{\partial e_1} = \frac{\partial \overline{x}_2}{\partial e_2} = \frac{\delta[32\gamma(1+\delta) - 3(5+4\delta)]}{2AB} > 0 \tag{A22}$$

$$\frac{\partial \overline{x}_1}{\partial e_2} = \frac{\partial \overline{x}_2}{\partial e_1} = \frac{\delta[-9+4\gamma+4(-3+4\gamma)\delta]}{2AB} > 0 \tag{A23}$$

The numerator of $(A22) > 32(1+\delta) - 3(5+4\delta) = 17 + 20\delta > 0$, thus $\frac{\partial \overline{x}_1}{\partial e_1} = \frac{\partial \overline{x}_2}{\partial e_2} > 0$. Moreover, the assumption $-9+4\gamma+4(-3+4\gamma)\delta > 0$ implies $\frac{\partial \overline{x}_1}{\partial e_2} = \frac{\partial \overline{x}_2}{\partial e_1} > 0$.

$(A22)$ and $(A23)$ also exemplify

$$\frac{\partial \overline{x}_1}{\partial e_1} - \frac{\partial \overline{x}_1}{\partial e_2} = \frac{\partial \overline{x}_2}{\partial e_2} - \frac{\partial \overline{x}_2}{\partial e_1} = \frac{\delta(28\gamma - 6 + 16\gamma\delta)}{2AB} > 0 \tag{A24}$$

where the direct effect of initial permits on cost-reducing R&D $\frac{\partial \overline{x}_i}{\partial e_i}$ dominates the indirect effect $\frac{\partial \overline{x}_i}{\partial e_j}$.

And then taking derivatives of $\overline{w}_i$ with respect to k by using $(A8)$ and $(A9)$ illustrates

$$\frac{\partial \overline{w}_1}{\partial k} = \frac{2\gamma}{-A} < 0 \tag{A25}$$

$$\frac{\partial \overline{w}_2}{\partial k} = \frac{2\gamma}{A} > 0 \tag{A26}$$

Besides, taking derivatives of $\overline{w}_i$ with respect to e_1 and e_2 yields

$$\frac{\partial \overline{w}_1}{\partial e_1} = \frac{\partial \overline{w}_2}{\partial e_2} = \frac{1}{2}[\frac{2-3\gamma}{A} + \frac{3-14\gamma}{B}] < 0 \tag{A27}$$

$$\frac{\partial \overline{w}_1}{\partial e_2} = \frac{\partial \overline{w}_2}{\partial e_1} = \frac{\delta[-3+4(3-4\gamma)\gamma]}{AB} < 0 \tag{A28}$$

$2 - 3\gamma < 0$ and $3 - 14\gamma < 0$ prove $(A27)$ is negative, thus $\frac{\partial \overline{w}_1}{\partial e_1} = \frac{\partial \overline{w}_2}{\partial e_2} < 0$. Since the numerator of $(A28)$ is negative, $\frac{\partial \overline{w}_1}{\partial e_2} = \frac{\partial \overline{w}_2}{\partial e_1} < 0$.

$(A27)$ and $(A28)$ also reveal

$$\frac{\partial \overline{w}_1}{\partial e_1} - \frac{\partial \overline{w}_1}{\partial e_2} = \frac{\partial \overline{w}_2}{\partial e_2} - \frac{\partial \overline{w}_2}{\partial e_1} = \frac{2-3\gamma}{A} < 0 \tag{A29}$$

where the direct effect of initial permits on environmental R&D $\frac{\partial \overline{w}_i}{\partial e_i}$ dominates the indirect effect $\frac{\partial \overline{w}_i}{\partial e_j}$.

From $(A14)$, it is obvious that $\overline{\sigma}$ depends on the total number of permits. Taking derivatives of $\overline{\sigma}$ with respect to $(e_1 + e_2)$ by using gives

$$\frac{\partial \overline{\sigma}}{\partial (e_1 + e_2)} = \frac{-3(4\gamma - 1)\delta}{B} < 0 \tag{A30}$$

As $-3(4\gamma - 1)\delta < 0$, $\frac{\partial \overline{\sigma}}{\partial e_1} = \frac{\partial \overline{\sigma}}{\partial e_2} < 0$. Consequently, taking derivatives of $\overline{f}_i$ with respect to k by using $(A12)$ and $(A13)$ demonstrates

$$\frac{\partial \overline{f}_1}{\partial k} = \frac{\gamma(1+4\delta)}{-A} < 0 \tag{A31}$$

$$\frac{\partial \overline{f}_2}{\partial k} = \frac{\gamma(1+4\delta)}{A} > 0 \tag{A32}$$

Additionally, taking derivatives of $\overline{f}_i$ with respect to e_1 and e_2 illustrates

$$\frac{\partial \overline{f}_1}{\partial e_1} = \frac{\partial \overline{f}_2}{\partial e_2} = \frac{2(3\gamma - 2) + \delta(4\gamma - 1)}{2A} > 0 \tag{A33}$$

$$\frac{\partial \overline{f}_1}{\partial e_2} = \frac{\partial \overline{f}_2}{\partial e_1} = \frac{(4\gamma - 3)\delta}{2A} > 0 \tag{A34}$$

Both the denominators and numerators of $(A33)$ and $(A34)$ are positive, so $\frac{\partial \overline{f}_1}{\partial e_1} = \frac{\partial \overline{f}_2}{\partial e_2} > 0$, $\frac{\partial \overline{f}_1}{\partial e_2} = \frac{\partial \overline{f}_2}{\partial e_1} > 0$. Furthermore, $(A33)$ and $(A34)$ show

$$\frac{\partial \overline{f}_1}{\partial e_1} - \frac{\partial \overline{f}_2}{\partial e_2} = \frac{\partial \overline{f}_2}{\partial e_2} - \frac{\partial \overline{f}_2}{\partial e_1} = \frac{2(3\gamma - 2) + 2\delta}{2(3\gamma - 2) + 4\delta(2\gamma - 1)} > 0 \tag{A35}$$

where the direct effect of initial permits on the needs of permits $\frac{\partial \overline{f}_i}{\partial e_i}$ dominates the indirect effect $\frac{\partial \overline{f}_i}{\partial e_j}$.

(3) Equilibria results with $\beta = 1$

Define $G = -2 + 2\gamma(4+\delta)$. When $\beta = 1$, solving the four first-order conditions for interior solutions provide

$$\overline{x}_1 = \frac{2\gamma s + [2\gamma(2+\delta) - 1]e_1 - (4\gamma - 1)e_2}{2\gamma G} \tag{A36}$$

$$\overline{x}_2 = \frac{2\gamma s - (4\gamma - 1)e_1[2\gamma(2+\delta) - 1]e_2}{2\gamma G} \tag{A37}$$

$$\overline{w}_1 = \frac{2\gamma s - 4k[\gamma(4+\delta) - 1] + (3 - 12\gamma + \delta - 7\gamma\delta)e_1 + (-3 + 12\gamma + \delta - \gamma\delta)e_2}{2\delta G} \tag{A38}$$

$$\overline{w}_2 = \frac{2\gamma s + 4k[\gamma(4+\delta) - 1] + (-3 + 12\gamma + \delta - \gamma\delta)e_1 + (3 - 12\gamma + \delta - 7\gamma\delta)e_2}{2\delta G} \tag{A39}$$

$$\overline{q}_1 = \frac{2\gamma s - 2k[\gamma(4+\delta) - 1] + \delta\gamma(e_1 + e_2)}{G} \tag{A40}$$

$$\overline{q}_2 = \frac{2\gamma s + 2k[\gamma(4+\delta) - 1] + \delta\gamma(e_1 + e_2)}{G} \tag{A41}$$

$$\overline{f}_1 = \frac{1}{2}(-2k + e_1 + e_2) \tag{A42}$$

$$\overline{f}_2 = \frac{1}{2}(2k + e_1 + e_2) \tag{A43}$$

$$\overline{\sigma} = \frac{2s\gamma(1+\delta) - 3(3\gamma - 1)(e_1 + e_2)}{G} \tag{A44}$$

(4) Proofs of competitive R&D comparative statics with $\beta = 1$

Given $\gamma \geqslant 1$ and $\delta \geqslant 1$, $G = -2 + 2\gamma(4+\delta) > -2 + 2(4+\delta) = 6 + 2\delta > 0$. First of all, taking derivatives of $\overline{q}_i$ with respect to k by using $(A40)$ and $(A41)$ yields

$$\frac{\partial \overline{q}_1}{\partial k} = \frac{-2[\gamma(4+\delta) - 1]}{G} < 0 \tag{A45}$$

$$\frac{\partial \overline{q}_2}{\partial k} = \frac{2[\gamma(4+\delta) - 1]}{G} > 0 \tag{A46}$$

Furthermore, $\overline{q}_i$ depends on the total number of permits, which indicates that the allocation of permits does not affect production, as the direct effect is offset by the indirect effect. Hence, taking derivative of $(A40)$ and $(A41)$ with respect to $(e_1 + e_2)$ illustrates

$$\frac{\partial \overline{q}_1}{\partial (e_1 + e_2)} = \frac{\partial \overline{q}_2}{\partial (e_1 + e_2)} = \frac{\delta \gamma}{G} > 0 \tag{A47}$$

Then, taking derivatives of $\overline{x}_i$ with respect to k by using $(A36)$ and $(A37)$ generates

$$\frac{\partial \overline{x}_1}{\partial k} = \frac{\partial \overline{x}_2}{\partial k} = 0 \tag{A48}$$

The change of marginal production cost does not affect firms' cost-reducing R&D investments. Besides, taking derivatives of $\overline{x}_i$ with respect to e_1 and e_2 gives

$$\frac{\partial \overline{x}_1}{\partial e_1} = \frac{\partial \overline{x}_2}{\partial e_2} = \frac{2\gamma(2+\delta) - 1}{2\gamma G} > 0 \tag{A49}$$

$$\frac{\partial \overline{x}_1}{\partial e_2} = \frac{\partial \overline{x}_2}{\partial e_1} = \frac{1 - 4\gamma}{2\gamma G} < 0 \tag{A50}$$

The denominators of $(A49)$ and $(A50)$ are positive. The numerator of $(A49) > 0$ leads $\frac{\partial \overline{x}_1}{\partial e_1} = \frac{\partial \overline{x}_2}{\partial e_2} > 0$. Given that the numerator of $(A50)$ is negative, therefore $\frac{\partial \overline{x}_1}{\partial e_2} = \frac{\partial \overline{x}_2}{\partial e_1} < 0$. In addition, $(A49)$ and $(A50)$ reveal

$$\frac{\partial \overline{x}_1}{\partial e_1} + \frac{\partial \overline{x}_1}{\partial e_2} = \frac{\partial \overline{x}_2}{\partial e_2} + \frac{\partial \overline{x}_2}{\partial e_1} = \frac{2\gamma\delta}{2\gamma G} > 0 \tag{A51}$$

where the direct effect of initial permits on the cost-reducing R&D $\frac{\partial \overline{x}_i}{\partial e_i}$ dominates the indirect effect $\frac{\partial \overline{x}_i}{\partial e_j}$. After that, taking derivatives of $\overline{w}_i$ with respect to k by using $(A38)$ and $(A39)$ demonstrates

$$\frac{\partial \overline{w}_1}{\partial k} = -\frac{1}{\delta} < 0 \tag{A52}$$

$$\frac{\partial \overline{w}_2}{\partial k} = \frac{1}{\delta} > 0 \tag{A53}$$

Furthermore, taking derivatives of $\overline{w}_i$ with respect to e_1 and e_2 illustrates

$$\frac{\partial \overline{w}_1}{\partial e_1} = \frac{\partial \overline{w}_2}{\partial e_2} = \frac{3 - 12\gamma + \delta - 7\gamma\delta}{2\delta G} < 0 \tag{A54}$$

$$\frac{\partial \overline{w}_1}{\partial e_2} = \frac{\partial \overline{w}_2}{\partial e_1} = \frac{-3 + 12\gamma + \delta - \gamma\delta}{2\delta G} > 0 \tag{A55}$$

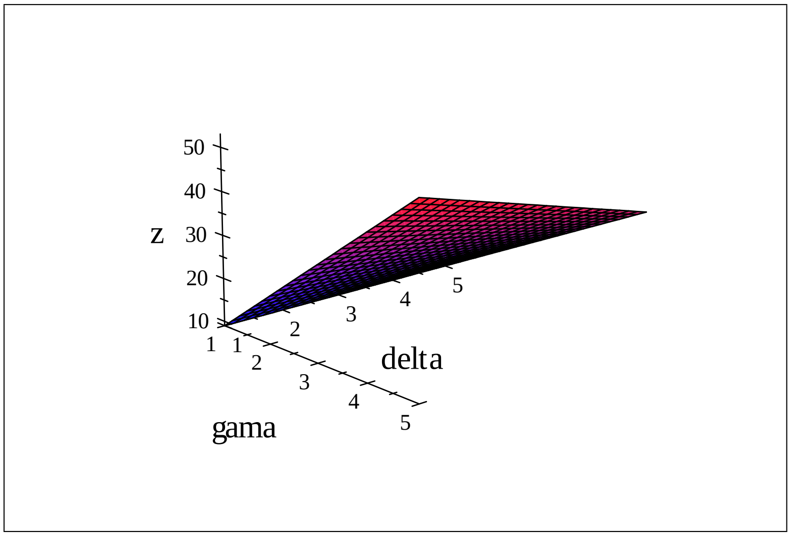


Figure $A1$: The value of $-3 + 12\gamma + \delta - \gamma\delta$ given $\gamma \geqslant 1$ and $\delta \geqslant 1$

The numerator of $(A54) < 0$ leads$\frac{\partial \overline{w}_1}{\partial e_1} = \frac{\partial \overline{w}_2}{\partial e_2} < 0$. Figure $A1$ shows that given $\gamma \geqslant 1$ and $\delta \geqslant 1$, the numerator of $(A55) > 0$, hence $\frac{\partial \overline{w}_1}{\partial e_2} = \frac{\partial \overline{w}_2}{\partial e_1} > 0$. Moreover, $(A54)$ and $(A55)$ illustrate

$$\frac{\partial \overline{w}_1}{\partial e_1} + \frac{\partial \overline{w}_1}{\partial e_2} = \frac{\partial \overline{w}_2}{\partial e_2} + \frac{\partial \overline{w}_2}{\partial e_1} = \frac{-6 + 2\delta(1 - 4\gamma)}{2\delta G} < 0 \tag{A56}$$

where the direct effect of initial permits on the environmental R&D $\frac{\partial \overline{w}_i}{\partial e_i}$ dominates the indirect effect $\frac{\partial \overline{w}_i}{\partial e_j}$. Consequently, taking derivatives of $\overline{f}_i$ with respect to k by using $(A42)$ and $(A43)$ shows

$$\frac{\partial \overline{f}_1}{\partial k} = -1 < 0 \tag{A57}$$

$$\frac{\partial \overline{f}_2}{\partial k} = 1 > 0 \tag{A58}$$

In addition, $\overline{f}_i$ and $\overline{\sigma}$ depend on the total number of permits, irrespective of the allocation of permits

$$\frac{\partial \overline{f}_1}{\partial(e_1+e_2)} = \frac{\partial \overline{f}_2}{\partial(e_1+e_2)} = \frac{1}{2} \tag{A59}$$

$$\frac{\partial \overline{\sigma}}{\partial(e_1+e_2)} = \frac{\partial \overline{\sigma}}{\partial(e_1+e_2)} = \frac{-3(3\gamma-1)}{G} < 0 \tag{A60}$$

Appendix 2: Cooperative R&D equilibrium results

The 4 first-order necessary conditions of (15) with respect to (x_i, w_i, x_j, w_j) are:

$$\begin{aligned} &\frac{\partial \pi_c}{\partial x_1} \\ &= \frac{1}{2}\{4k(-1+\beta)+(1+\beta)(e_1+e_2)+(1+\beta)^2(w_1+w_2)+2[(-1+\beta)^2-2\gamma]x_1 \\ &-2(-1+\beta)^2x_2\}=0 \end{aligned} \tag{A61}$$

$$\begin{aligned} &\frac{\partial \pi_c}{\partial x_2} \\ &= \frac{1}{2}\{-4k(-1+\beta)+(1+\beta)(e_1+e_2)+(1+\beta)^2(w_1+w_2)-2(-1+\beta)^2x_1 \\ &+2[(-1+\beta)^2-2\gamma]x_2\}=0 \end{aligned} \tag{A62}$$

$$\begin{aligned} &\frac{\partial \pi_c}{\partial w_1} \\ &= \frac{1}{2}\{2s(1+\beta)-4(1+\beta)[e_1+e_2+(1+\beta)w_2]-4[(1+\beta)^2+\delta]w_1 \\ &+(1+\beta)^2(x_1+x_2)\}=0 \end{aligned} \tag{A63}$$

$$\begin{aligned} &\frac{\partial \pi_c}{\partial w_2} \\ &= \frac{1}{2}\{2s(1+\beta)-4(1+\beta)[e_1+e_2+(1+\beta)w_1]-4[(1+\beta)^2+\delta]w_2 \\ &+(1+\beta)^2(x_1+x_2)\}=0 \end{aligned} \tag{A64}$$

The Hessian matrix of the second derivatives is:

$$H = \begin{bmatrix} (1-\beta)^2 - 2\gamma & -(1-\beta)^2 & \frac{1}{2}(1+\beta)^2 & \frac{1}{2}(1+\beta)^2 \\ -(1-\beta)^2 & (1-\beta)^2 - 2\gamma & \frac{1}{2}(1+\beta)^2 & \frac{1}{2}(1+\beta)^2 \\ \frac{1}{2}(1+\beta)^2 & \frac{1}{2}(1+\beta)^2 & -2[(1+\beta)^2+\delta] & -2(1+\beta)^2 \\ \frac{1}{2}(1+\beta)^2 & \frac{1}{2}(1+\beta)^2 & -2(1+\beta)^2 & -2[(1+\beta)^2+\delta] \end{bmatrix} \tag{A65}$$

Given $\beta \in [0,1]$, $\gamma \geqslant 1$ and $\delta \geqslant 1$, the second-order sufficient conditions lead $|H_1| < 0$, $|H_2| > 0, |H_3| < 0, |H_4| > 0$, which assure global maximum solutions.

Solving the four system equations $(A61)$ to $(A64)$, we obtain the equilibrium R&D outputs $(\widetilde{x}_i, \widetilde{w}_i, \widetilde{x}_j, \widetilde{w}_j)$:

$$\widetilde{x}_1 = \frac{k(-1+\beta)[-(1+\beta)^4 + 8\gamma(1+\beta)^2 + 4\gamma\delta] - (1+\beta)[(1-\beta)^2-\gamma][s(1+\beta)^2 + \delta(e_1+e_2)]}{[(1-\beta)^2-\gamma][(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta]} \tag{A66}$$

$$\widetilde{x}_2 = \frac{k(-1+\beta)[(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta] - (1+\beta)[(1-\beta)^2-\gamma][s(1+\beta)^2 + \delta(e_1+e_2)]}{[(1-\beta)^2-\gamma][(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta]} \tag{A67}$$

$$\widetilde{w}_1 = \widetilde{w}_2 = \frac{(1+\beta)\{4s\gamma + [(1+\beta)^2 - 8\gamma](e_1+e_2)\}}{2(1+\beta)^2[(1+\beta)^2 - 8\gamma] - 8\gamma\delta} \tag{A68}$$

Additionally, when $\beta = 1$, $(A68)$ becomes

$$\widetilde{w}_1 = \widetilde{w}_2 = \frac{s\gamma - (2\gamma-1)(e_1+e_2)}{-4 + \gamma(8+\delta)} \tag{A69}$$

Since $\widetilde{w}_i > 0$, $(A69)$ requires $s > \frac{2\gamma-1}{\gamma}(e_1+e_2)$. As the maximum of $\frac{2\gamma-1}{\gamma}$ is 2, thus $s > (e_1+e_2)$ assures positive environmental R&D outputs.

Substituting $(A66)$ to $(A68)$ into (10) to (12) yields:

$$\widetilde{q}_1 = \gamma \frac{k[(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta] - 2[(1-\beta)^2-\gamma][s(1+\beta)^2 + \delta(e_1+e_2)]}{[(1-\beta)^2-\gamma][(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta]} \tag{A70}$$

$$\widetilde{q}_2 = \gamma \frac{k[-(1+\beta)^4 + 8\gamma(1+\beta)^2 + 4\gamma\delta] - 2[(1-\beta)^2 - \gamma][s(1+\beta)^2 + \delta(e_1+e_2)]}{[(1-\beta)^2 - \gamma][(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta]} \quad \text{(A71)}$$

$$\widetilde{\sigma} = \frac{2s\gamma[(1+\beta)^2 + 2\delta] + [(1+\beta)^2 - 6\gamma]\delta(e_1+e_2)}{(1+\beta)^4 - 8\gamma(1+\beta)^2 - 4\gamma\delta} \quad \text{(A72)}$$

$(A69), (A70)$ and $(A71)$ also imply the numbers of permits that firms need to produce are

$$\widetilde{f}_1 = \frac{2k\gamma + [(1-\beta)^2 - \gamma](e_1+e_2)}{2[(1-\beta)^2 - \gamma]} \quad \text{(A73)}$$

$$\widetilde{f}_2 = \frac{-2k\gamma + [(1-\beta)^2 - \gamma](e_1+e_2)}{2[(1-\beta)^2 - \gamma]} \quad \text{(A74)}$$

Thus, the aggregate R&D investments are output level are

$$\widetilde{x}_1 + \widetilde{x}_2 = \frac{2(1+\beta)[s(1+\beta)^2 + \delta(e_1+e_2)]}{-(1+\beta)^4 + 8\gamma(1+\beta)^2 + 4\gamma\delta} \quad \text{(A75)}$$

$$\widetilde{w}_1 + \widetilde{w}_2 = \frac{(1+\beta)\{4s\gamma + [(1+\beta)^2 - 8\gamma]\}(e_1+e_2)}{-(1+\beta)^4 + 8\gamma(1+\beta)^2 + 4\gamma\delta} \quad \text{(A76)}$$

$$\widetilde{q}_1 + \widetilde{q}_2 = \frac{4\gamma[s(1+\beta)^2 + \delta(e_1+e_2)]}{-(1+\beta)^4 + 8\gamma(1+\beta)^2 + 4\gamma\delta} \quad \text{(A77)}$$

Proof of cooperative R&D comparative statics results

Taking derivative of $(A66)$ to $(A68), (A70), (A71), (A73)$ and $(A74)$ with respect to k generates

$$\frac{\partial \widetilde{x}_1}{\partial k} = \frac{-1+\beta}{-(1-\beta)^2 + \gamma} < 0 \quad \text{(A78)}$$

$$\frac{\partial \widetilde{x}_2}{\partial k} = \frac{-1+\beta}{(1-\beta)^2 - \gamma} > 0 \quad \text{(A79)}$$

$$\frac{\partial \widetilde{w}_1}{\partial k} = \frac{\partial \widetilde{w}_1}{\partial k} = 0 \quad \text{(A80)}$$

$$\frac{\partial \widetilde{q}_1}{\partial k} = \frac{\gamma}{(1-\beta)^2 - \gamma} < 0 \quad \text{(A81)}$$

$$\frac{\partial \widetilde{q}_2}{\partial k} = -\frac{\gamma}{(1-\beta)^2-\gamma} > 0 \tag{A82}$$

$$\frac{\partial \widetilde{f}_1}{\partial k} = \frac{\gamma}{(1-\beta)^2-\gamma} < 0 \tag{A83}$$

$$\frac{\partial \widetilde{f}_2}{\partial k} = -\frac{\gamma}{(1-\beta)^2-\gamma} > 0 \tag{A84}$$

Notice that $(A66)$ to $(A68), (A70), (A71), (A73)$ and $(A74)$ and demonstrate that the cooperative equilibrium results depend on the total number of permits. It also imply that given the total number of available permits, the allocation of permits does not affect the equilibrium outcomes, as the direct effect will be offset by the indirect effect. Moreover, taking derivative of $(A66)$ to $(A68), (A70), (A71), (A73)$ and $(A74)$ with respect to e_1+e_2 gives

$$\frac{\partial \widetilde{x}_1}{\partial (e_1+e_2)} = \frac{\partial \widetilde{x}_2}{\partial (e_1+e_2)} = -\frac{(1+\beta)\delta}{(1+\beta)^4-8\gamma(1+\beta)^2-4\gamma\delta} > 0 \tag{A85}$$

$$\frac{\partial \widetilde{w}_1}{\partial (e_1+e_2)} = \frac{\partial \widetilde{w}_2}{\partial (e_1+e_2)} = -\frac{(1+\beta)[(1+\beta)^2-8\gamma]}{2(1+\beta)^4-16\gamma(1+\beta)^2-8\gamma\delta} < 0 \tag{A86}$$

$$\frac{\partial \widetilde{f}_1}{\partial (e_1+e_2)} = \frac{\partial \widetilde{f}_2}{\partial (e_1+e_2)} = \frac{1}{2} > 0 \tag{A87}$$

$$\frac{\partial \widetilde{q}_1}{\partial (e_1+e_2)} = \frac{\partial \widetilde{q}_2}{\partial (e_1+e_2)} = -\frac{2\gamma}{(1+\beta)^4-8\gamma(1+\beta)^2-4\gamma\delta} > 0 \tag{A88}$$

$$\frac{\partial \widetilde{\sigma}}{\partial (e_1+e_2)} = \frac{[(1+\beta)^2-6\gamma]\delta}{-(1+\beta)^4+8\gamma(1+\beta)^2+4\gamma\delta} < 0 \tag{A89}$$

Appendix 3: proof of Section 5 with $\beta = 0$ and $\beta = 1$

(1) Proof of comparison between aggregate R&D and output levels with $\beta = 0$ and $\beta = 1$

When $\beta = 0$, subtracting $(A2)$ to $(A5)$ from $(A75)$ to $(A77)$ and $(A72)$ generates

$$\overline{x}_1 + \overline{x}_2 - (\widetilde{x}_1 + \widetilde{x}_2) = \frac{4\gamma(5+2\delta)[s+\delta(e_1+e_2)]}{[-1+4\gamma(2+\delta)][-3+2\gamma(7+4\delta)]} > 0 \tag{A90}$$

$$\overline{w}_1+\overline{w}_2-(\widetilde{w}_1+\widetilde{w}_2)=\frac{4\gamma(1+2\gamma)[s+\delta(e_1+e_2)]}{[-1+4\gamma(2+\delta)][-3+2\gamma(7+4\delta)]}>0 \tag{A91}$$

$$\overline{q}_1+\overline{q}_2-(\widetilde{q}_1+\widetilde{q}_2)=\frac{4\gamma(1+2\gamma)[s+\delta(e_1+e_2)]}{[-1+4\gamma(2+\delta)][-3+2\gamma(7+4\delta)]}>0 \tag{A92}$$

$$\overline{\sigma}-\widetilde{\sigma}=-\frac{4\gamma(-1+3\gamma-\delta)[s+\delta(e_1+e_2)]}{[-1+4\gamma(2+\delta)][-3+2\gamma(7+4\delta)]}<0 \tag{A93}$$

When $\beta=1$, subtracting $(A2)$ to $(A45)$ from $(A75)$ to $(A77)$ and $(A72)$ yields

$$\overline{x}_1+\overline{x}_2-(\widetilde{x}_1+\widetilde{x}_2)=-\frac{2s\gamma(8+3\delta)+\delta(2+\gamma\delta)(e_1+e_2)}{2[-1+\gamma(4+\delta)][-4+\gamma(8+\delta)]}<0 \tag{A94}$$

$$\overline{w}_1+\overline{w}_2-(\widetilde{w}_1+\widetilde{w}_2)=-\frac{\gamma[2s(2+\gamma\delta)-(4\gamma-3)\delta(e_1+e_2)]}{2[-1+\gamma(4+\delta)][-4+\gamma(8+\delta)]}<0 \tag{A95}$$

$$\overline{q}_1+\overline{q}_2-(\widetilde{q}_1+\widetilde{q}_2)=-\frac{\gamma[2s(2+\gamma\delta)-(4\gamma-3)\delta(e_1+e_2)]}{[-1+\gamma(4+\delta)][-4+\gamma(8+\delta)]}<0 \tag{A96}$$

$$\overline{\sigma}-\widetilde{\sigma}=\frac{2s\gamma[-2+3(-1+\gamma)\delta]-\delta[2+\gamma(-9+12\gamma+\delta)](e_1+e_2)}{2[-1+\gamma(4+\delta)][-4+\gamma(8+\delta)]}<0 \tag{A97}$$

The numerator of $(A95)>4(2+\gamma\delta)(e_1+e_2)-(4\gamma-3)\delta(e_1+e_2)=(8+3\delta)(e_1+e_2)>0$, thus $\overline{w}_1+\overline{w}_2-(\widetilde{w}_1+\widetilde{w}_2)<0$ and hence $\overline{q}_1+\overline{q}_2-(\widetilde{q}_1+\widetilde{q}_2)<0$. The numerator of $(A96)<4(e_1+e_2)\gamma[-2+3(-1+\gamma)\delta]-\delta[2+\gamma(-9+12\gamma+\delta)](e_1+e_2)=-[2\delta+\gamma(8+\delta(3+\delta))](e_1+e_2)<0$, therefore $\overline{\sigma}-\widetilde{\sigma}<0$.

(2) Proof of comparison between individual cost-reducing R&D levels with $\beta=1$

subtracting $(A66)$ and $(A67)$ by $(A36)$ and $(A37)$ and imposing $\beta=1$ illustrates

$$\begin{aligned}&\widetilde{x}_1-\overline{x}_1 \\ =\ &\frac{2s\gamma^2(8+3\delta)+\{-4+\gamma[24+7\delta-4\gamma(8+3\delta)]\}e_1+\{4+\gamma[-3(8+\delta)+2\gamma(16+\delta(6+\delta))]\}e_2}{2\gamma G[-4+\gamma(8+\delta)]}\end{aligned} \tag{A98}$$

$$\widetilde{x}_2 - \overline{x}_2 = \frac{2s\gamma^2(8+3\delta) + \{4+\gamma[-3(8+\delta)+2\gamma(16+\delta(6+\delta))]\}e_1 + \{-4+\gamma[24+7\delta-4\gamma(8+3\delta)]]\}e_2}{2\gamma G[-4+\gamma(8+\delta)]} \quad \text{(A99)}$$

The denominators of $(A98)$ and $(A99)$ are positive. In the numerator of $(A91)$, the first and the last terms are positive, while the second term is negative. Comparing the two coefficients of e_1 and e_2 yields $4+\gamma[-3(8+\delta)+2\gamma(16+\delta(6+\delta))] > |-4+\gamma[24+7\delta-4\gamma(8+3\delta)]|$; moreover, to have all results non-negative requires $e_2 > e_1$. Therefore the sum of the last two terms is positive and hence the numerator of $(A98)$, so $\widetilde{x}_1 - \overline{x}_1 > 0$. Following the proof of $\widetilde{x}_1 - \overline{x}_1 > 0$, the last two terms in the numerator of $(A99)$ are negative. However, $\widetilde{w}_i > 0$ requires $s > (2-\frac{1}{\gamma})(e_1+e_2)$ (from $(A69)$ if $\beta = 1$), which indicates $s > 2(e_1+e_2)$. Consider the case where firm 2 is given all permits, where the negativity from e_2 is not offset by the positivity from e_1 at all. Then $s > 2(e_1+e_2)$ becomes $s > 4e_2$, thus the numerator is $2s\gamma^2(8+3\delta)+2\{-4+\gamma[24+7\delta-4\gamma(8+3\delta)]]\}e_2 > 8\gamma^2(8+3\delta)e_2+2\{-4+\gamma[24+7\delta-4\gamma(8+3\delta)]]\}e_2 = 2[-4+\gamma(24+7\delta)]e_2 > 0$. As a result, the numerator of $(A99)$ is always positive and hence $\widetilde{x}_2 - \overline{x}_2 > 0$.

(3) Proof of comparison individual quantity outputs with $\beta = 1$

$$\widetilde{q}_1 - \overline{q}_1 = \widetilde{q}_2 - \overline{q}_2 = \frac{\gamma[2s(2+\gamma\delta) - (-3+4\gamma)\delta(e_1+e_2)]}{2G[-4+\gamma(8+\delta)]} \quad \text{(A100)}$$

The numerator of $(A100) > 4(2+\gamma\delta)(e_1+e_2) - (-3+4\gamma)\delta(e_1+e_2) = 8+4\gamma\delta+3\delta > 0$, thus $\widetilde{q}_1 - \overline{q}_1 = \widetilde{q}_2 - \overline{q}_2 > 0$.

Appendix 4: proof of Proposition 6

The derivative of (24) with respect to β is

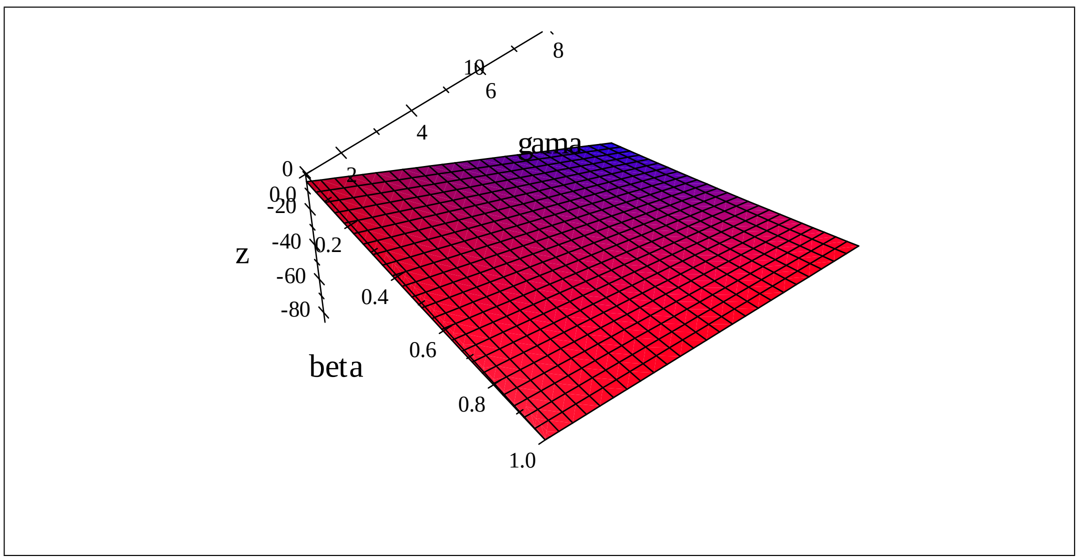
$$\frac{\partial e_2}{\partial \beta} = \frac{k\gamma(H+I-J)}{\{(1+\beta)\{[2+(-3+\beta)\beta-3\gamma]^2+\delta[-(-1+\beta)^2+\gamma]\}\}^2} < 0 \quad \text{(A101)}$$

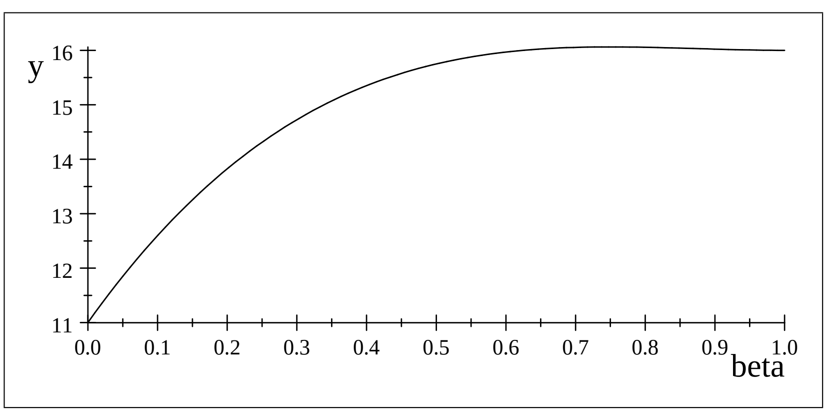
where $H = -3(1+\beta)^2[2+(-3+\beta)\beta-3\gamma][4-9\gamma+\beta(-7+3\beta+8\gamma)]$, $I = \delta\{(-1+\beta)^2[11+\beta(18+\beta(-21+8\beta))]+3[1+\beta(-18+(25-4\beta)\beta)\gamma-36\gamma^2]\}$, $J = 4\delta^2[(-1+\beta)^2+\gamma]$.

In H, $-3(1+\beta)^2 < 0$, $2+(-3+\beta)\beta-3\gamma < 0$ given $\gamma \geqslant 1$ and $\beta \in [0,1]$. Figure $A2$ shows $4-9\gamma+\beta(-7+3\beta+8\gamma) < 0$, therefore $H < 0$.

In I, Figure $A3$ indicates $[11+\beta(18+\beta(-21+8\beta))] > 0$, thus $(-1+\beta)^2[11+\beta(18+\beta(-21+8\beta))] > 0$. $3[1+\beta(-18+(25-4\beta)\beta)\gamma-36\gamma^2] < 0$, but its absolute value is greater than $(-1+\beta)^2[11+\beta(18+\beta(-21+8\beta))]$, so $I < 0$.

Given both $H < 0, I > 0$ and $J > 0$, the numerator of $(A101)$ is negative, as a result, $\frac{\partial e_2}{\partial \beta} < 0$. With the constraint $e_1 + e_2 = L$, higher β results in lower e_2 and hence higher e_1.





with $\beta \in [0,1]$

Appendix 5: Pre-permit trading quantities under R&D noncooperation

Compared with the game under R&D noncooperation, the only difference is that there is no permit trading. Then, firms will not conduct any environmental R&D. We still solve the game by backward induction. The profit functions become

$$\pi_i = pq_i - C_i - \gamma x_i^2 \tag{A102}$$

The two first-order necessary conditions are

$$\frac{\partial \pi_1}{\partial q_1} = s - k - 2q_1 - q_2 + x_1 + \beta x_2 = 0 \tag{A103}$$

$$\frac{\partial \pi_2}{\partial q_2} = s + k - q_1 - 2q_2 + \beta x_1 + x_2 = 0 \tag{A104}$$

Solving for q_1 and q_2, we get

$$q_1 = \frac{1}{3}[s - 3k + (2-\beta)x_1 + (2\beta - 1)x_2] \tag{A105}$$

$$q_2 = \frac{1}{3}[s + 3k + (2\beta - 1)x_1 + (2-\beta)x_2] \tag{A106}$$

Substituting $(A105)$ and $(A106)$ into $(A102)$, and then taking derivatives with respect to (x_i, w_i), we obtain

$$x_1 = (-2+\beta)[\frac{-k}{2 + (-3+\beta)\beta - 3\gamma} + \frac{-s}{-2 + (-1+\beta)\beta + 9\gamma}] \tag{A107}$$

$$x_2 = (-2+\beta)[\frac{k}{2 + (-3+\beta)\beta - 3\gamma} + \frac{-s}{-2 + (-1+\beta)\beta + 9\gamma}] \tag{A108}$$

Substituting $(A107)$ and $(A108)$ into $(A105)$ and $(A106)$ yields:

$$q_1 = 3\gamma \frac{(-2+\beta)[(-1+\beta)s + (1+\beta)k] + 3\gamma(-s+3k)}{[2+(-3+\beta)\beta - 3\gamma][-2+(-1+\beta)\beta + 9\gamma]} \tag{A109}$$

$$q_2 = 3\gamma \frac{(-2+\beta)[(-1+\beta)s - (1+\beta)k] - 3\gamma(-s+3k)}{[2+(-3+\beta)\beta - 3\gamma][-2+(-1+\beta)\beta + 9\gamma]} \tag{A110}$$

Thus, the business-as-usual emission level is

$$q_1 + q_2 = \frac{6\gamma(-2+\beta)(-1+\beta)s}{[2+(-3+\beta)\beta - 3\gamma][-2+(-1+\beta)\beta + 9\gamma]} \tag{A111}$$

Thus, the market shares of both firms are defined as

$$\frac{q_1}{q_1+q_2} = \frac{(-2+\beta)[(-1+\beta)s + (1+\beta)k] + 3\gamma(-s+3k)}{2s[2+(-3+\beta)\beta - 3\gamma]} \tag{A112}$$

$$\frac{q_2}{q_1+q_2} = \frac{(-2+\beta)[(-1+\beta)s - (1+\beta)k] - 3\gamma(-s+3k)}{2s[2+(-3+\beta)\beta - 3\gamma]} \tag{A113}$$

Appendix of Chapter 2

Appendix 1

The first-order necessary conditions and the second-order sufficient conditions with respect to w_{di} are:

$$\frac{\partial \pi_{di}}{\partial w_{di}} = \frac{1}{16}\{8(a - c_d - c_u) - (23 + 5h + 10v)e_d + 2(-7 + 2h + 4v)e_u + (1 + h + 2v)$$
$$[(-16 + 2h + 4v - 32\gamma)w_{di} - (7 + 7h - 4v)w_{dj} + (-7 + 2h - 5v)(w_{u1} + w_{u2})]\} = 0 \quad \text{(A1)}$$

$$\frac{\partial^2 \pi_{di}}{\partial w_{di}^2} = \frac{1}{8}[(-7 + h)h + 4hv + 4v^2 - 2(4 + 7v + 8\gamma)] < 0 \quad \text{(A2)}$$

The first-order necessary conditions and the second-order sufficient conditions with respect to w_{ui} are:

$$\frac{\partial \pi_{ui}}{\partial w_{ui}} = \frac{1}{16}\{8(a - c_d - c_u) + 6(-2 + h + 2v)e_d - 3(7 + h + 2v)e_u$$
$$+ (1 + h + 2v)[3(-2 + h - v)(w_{d1} + w_{d2}) + 3(-5 + h + 2v)w_{ui} - 6(1 + h - v)w_{uj} - 32\gamma w_{ui}]\} = 0$$
$$\text{(A3)}$$

$$\frac{\partial^2 \pi_{ui}}{\partial w_{ui}^2} = \frac{3}{16}(-5 + h + 2v)(1 + h + 2v) - 2\gamma < 0 \quad \text{(A4)}$$

Solving the four first-order conditions simultaneously and imposing symmetry yield:

$$w_d^{NC} = w_{d1}^{NC} = w_{d2}^{NC} \quad \text{(A5)}$$
$$= \{(-a + c_d + c_u)[8(7 + 7h - 22v)(1 + h + 2v) + 256\gamma] + [-9(1 + h)(-35 + h + 2v)(1 + h + 2v)$$
$$+ 32\gamma(23 + 5h + 10v)]e_d + 2[9v(-35 + h + 2v)(1 + h + 2v) - 32(-7 + 2h + 4v)]e_u\}/$$
$$\{[(-35 + h + 2v)(1 + h + 2v) - 32\gamma][9((1 + h)^2 - 4v^2) + 32\gamma]\}$$

$$w_u^{NC} = w_{u1}^{NC} = w_{u2}^{NC} \tag{A6}$$

$$= \{(-a + c_d + c_u)[(11 + 11h - 14v)(1 + h + 2v) + 32\gamma] + 6[3v(-35 + h + 2v)(1 + h + 2v)$$

$$- 32(-2 + h + 2v)]e_d + 3[-3(1 + h)(-35 + h + 2v)(1 + h + 2v) + 32\gamma(7 + h + 2v)]e_u\}/$$

$$\{[(-35 + h + 2v)(1 + h + 2v) - 32\gamma][9((1 + h)^2 - 4v^2) + 32\gamma]\}$$

Substituting $(A5)$ and $(A6)$ into (8), (14) and (15) generates:

$$p^{NC} = a + \frac{16[(1 + h + 2v)(a - c_d - c_u) + 2\gamma(e_d + e_u)]}{-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma} \tag{A7}$$

$$\sigma^{NC} = \frac{[(1 + h + 2v)^2 - 32\gamma](a - c_d - c_u) + 72\gamma(e_d + e_u)}{2[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]} \tag{A8}$$

$$q^{NC} = y^{NC} = -\frac{8[(1 + h + 2v)(a - c_d - c_u) + 2\gamma(e_d + e_u)]}{-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma} \tag{A9}$$

Adding $(A5)$ and $(A6)$ giving the total R&D level $W^{NC} = 2w_d^{NC} + 2w_u^{NC}$:

$$W^{NC} = \frac{-32(a - c_d - c_u) + 2(35 - h - 2v)(e_d + e_u)}{-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma} \tag{A10}$$

Appendix 2

1. Proof of $\frac{\partial q^{NC}}{\partial c_d} < 0$**,** $\frac{\partial y^{NC}}{\partial c_u} < 0$, $\frac{\partial \sigma^{NC}}{\partial c_d} < 0$ **and** $\frac{\partial \sigma^{NC}}{\partial c_u} < 0$.

Taking derivative of $(A9)$ with respect to c_d and c_u generates:

$$\frac{\partial q^{NC}}{\partial c_d} = \frac{\partial y^{NC}}{\partial c_u} = \frac{8(1 + h + 2v)}{-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma} < 0 \tag{A11}$$

Given $h, v \in [0, 1]$, $(-17 + v) < 0$, $(-34 + 4v) < 0$, thus the denominator of $(A11)$ is negative and hence $\frac{\partial q^{NC}}{\partial c_d} = \frac{\partial y^{NC}}{\partial c_u} < 0$.

Taking derivative of $(A8)$ with respect to c_d and c_u yields:

$$\frac{\partial \sigma^{NC}}{\partial c_d} = \frac{\partial \sigma^{NC}}{\partial c_u} = \frac{-(1 + h + 2v)^2 + 32\gamma}{2[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]} < 0 \tag{A12}$$

Given $\gamma \geqslant 1$, the numerator of $(A12)$ is positive, therefore $\frac{\partial \sigma^{NC}}{\partial c_d} = \frac{\partial \sigma^{NC}}{\partial c_u} < 0$.

Also, equations (9) and (11) indicate

$$P = \frac{1}{9}(5a + 8\sigma + 4c_d + 4c_u) \tag{A13}$$

An increase in c_d or c_u affects P both directly and indirectly (through σ), and the net effect is derived as

$$\begin{aligned} \frac{\partial P}{\partial c_d} &= \frac{\partial P}{\partial c_u} = \frac{4}{9}(2\frac{\partial \sigma^{NC}}{\partial c_d} + 1) \qquad \text{(A14)} \\ &= -\frac{144(1 + h + 2v)}{9[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]} > 0 \end{aligned}$$

2. Proof of Proposition 2

Taking derivatives of $(A8)$ with respect to h and v yields:

$$\frac{\partial \sigma^{NC}}{\partial h} = -\frac{18\{[(1 + h + 2v)^2 + 32\gamma](a - c_d - c_u) + 4\gamma(-17 + h + 2v)(e_d + e_u)\}}{[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]^2} \tag{A15}$$

$$\frac{\partial \sigma^{NC}}{\partial v} = -\frac{36\{[(1 + h + 2v)^2 + 32\gamma](a - c_d - c_u) + 4\gamma(-17 + h + 2v)(e_d + e_u)\}}{[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]^2} \tag{A16}$$

Since the total R&D level has to be positive, $(A10) > 0$. As the denominator of $(A10)$ is negative (from $(A11)$), the numerator of $(A10)$ must be negative, which requires

$$32(a - c_d - c_u) > (35 - h - 2v)(e_{d1} + e_{d2} + e_{u1} + e_{u2}) \tag{A17}$$

Given $h, v \in [0, 1]$, the range of the right hand side of $(A17)$ is derived as

$$64(e_d + e_u) < 2(35 - h - 2v)(e_d + e_u) < 70(e_d + e_u) \tag{A18}$$

Thus, the validation of $(A17)$ needs

$$32(a - c_d - c_u) > 70(e_d + e_u) \tag{A19}$$

which is equivalent to

$$(a - c_d - c_u) > \frac{35}{16}(e_d + e_u) \tag{A20}$$

Multiply $[(1 + h + 2v)^2 + 32\gamma]$ to both sides of $(A20)$ yields:

$$[(1 + h + 2v)^2 + 32\gamma](a - c_d - c_u) > \frac{35}{16}[(1 + h + 2v)^2 + 32\gamma](e_d + e_u) \tag{A21}$$

We now need to compare $\frac{35}{16}[(1+h+2v)^2+32\gamma](e_d+e_u)$ with $|4\gamma(-17 + h + 2v)(e_d + e_u)|$ from $(A16)$.

$$\frac{35}{16}[(1+h+2v)^2+32\gamma]-[-4\gamma(-17+h+2v)] = \frac{35}{16}(1+h+2v)^2+4\gamma(h+2v)+2\gamma > 0 \tag{A22}$$

Thus, $\frac{35}{16}[(1 + h + 2v)^2 + 32\gamma](e_d + e_u) > |4\gamma(-17 + h + 2v)(e_d + e_u)|$, and hence $[(1 + h + 2v)^2 + 32\gamma](a - c_d - c_u) > |4\gamma(-17 + h + 2v)(e_d + e_u)|$.

Therefore, the numerators and denominators of both $(A15)$ and $(A16)$ are positive, and hence $\frac{\partial \sigma^{NC}}{\partial h} < 0$ and $\frac{\partial \sigma^{NC}}{\partial v} < 0$.

3. Proof of Proposition 3

Taking derivatives of $(A9)$ with respect to h and v yields

$$\frac{\partial q^{NC}}{\partial h} = \frac{\partial y^{NC}}{\partial h} = \frac{8\{[(1 + h + 2v)^2 + 32\gamma](a - c_d - c_u) + 4\gamma(-17 + h + 2v)(e_d + e_u)\}}{[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]^2} > 0 \tag{A23}$$

$$\frac{\partial q^{NC}}{\partial v} = \frac{\partial y^{NC}}{\partial v} = \frac{16\{[(1 + h + 2v)^2 + 32\gamma](a - c_d - c_u) + 4\gamma(-17 + h + 2v)(e_d + e_u)\}}{[-35 + h^2 + 4(-17 + v)v + h(-34 + 4v) - 32\gamma]^2} > 0 \tag{A24}$$

4. Proof of Proposition 4

Taking derivatives of $(A5)$ with respect to e_d and e_u generates

$$\frac{\partial w_d^{NC}}{\partial e_d} = \frac{-9(1+h)(-35+h+2v)(1+h+2v)+32\gamma(23+5h+10v)}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} \tag{A25}$$

which is the direct effect of e_d on w_d^{NC}, and

$$\frac{\partial w_d^{NC}}{\partial e_u} = \frac{2[9v(-35+h+2v)(1+h+2v)-32\gamma(-7+2h+4v)]}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} \tag{A26}$$

which denotes the indirect of e_u on w_d^{NC}.

Figure $A1$ shows the value of the numerator of $(A25)$ with $\gamma = 1$. Given $h, v \in [0, 1]$, the numerator is positive at the minimum γ, thus $\frac{\partial w_d^{NC}}{\partial e_d} < 0$. Intuitively, $\frac{\partial w_d^{NC}}{\partial e_u} < 0$, but it is valid only when h and v are not too small.

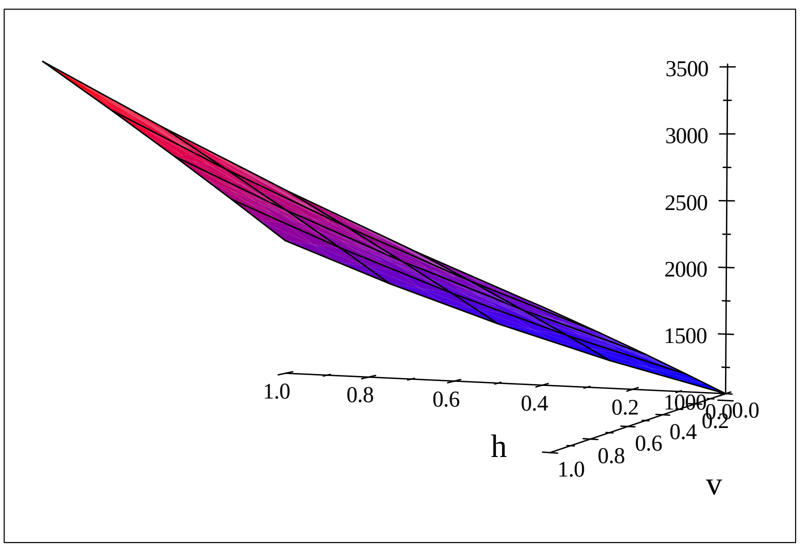


Figure $A1$: value of $-9(1+h)(-35+h+2v)(1+h+2v)+32(23+5h+10v)$

Adding $(A25)$ and $(A26)$ yields

$$\begin{aligned} &\frac{\partial w_d^{NC}}{\partial e_d} + \frac{\partial w_d^{NC}}{\partial e_u} \\ =\ &\frac{-9(1+h-2v)(-35+h+2v)(1+h+2v)+32\gamma(37+h+2v)}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} \end{aligned} \tag{A27}$$

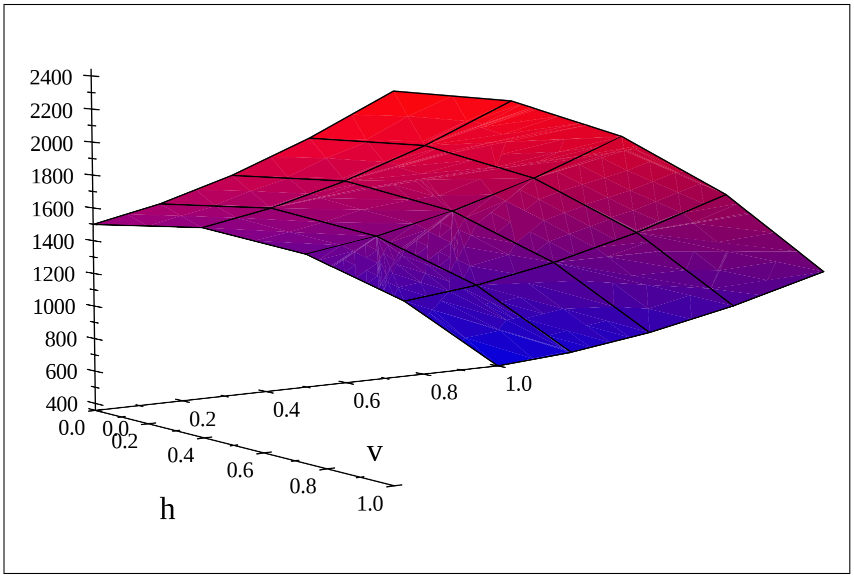


Figure $A2$: value of $-9(1+h-2v)(-35+h+2v)(1+h+2v)+32(37+h+2v)$

Figure $A2$ establishes that the numerator of $(A27)$ is negative, and hence $\frac{\partial w_d^{NC}}{\partial e_d} + \frac{\partial w_d^{NC}}{\partial e_u} < 0$. Since the direct effect dominates the indirect effect, giving more permits to an downstream firm will reduce its R&D investment.

Similarly

$$\frac{\partial w_u^{NC}}{\partial e_u} = \frac{3[-3(1+h)(-35+h+2v)(1+h+2v)+32\gamma(7+h+2v)]}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} < 0 \tag{A28}$$

$$\frac{\partial w_u^{NC}}{\partial e_d} = \frac{18v(-35+h+2v)(1+h+2v)-192\gamma(-2+h+2v)]}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} \lesseqgtr 0 \tag{A29}$$

$$\frac{\partial w_u^{NC}}{\partial e_u} + \frac{\partial w_u^{NC}}{\partial e_d} \tag{A30}$$
$$= \frac{-9(1+h-2v)(-35+h+2v)(1+h+2v)-96\gamma(-11+h+2v)]}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} < 0$$

f_d is obtained by using $f_d = q - w_d - hw_d - v(w_u + w_u)$ and f_u is obtained by using

$f_u = y - w_u - hw_u - v(w_d + w_d)$. Then we get:

$$\frac{\partial f_d^{NC}}{\partial e_d} \tag{A31}$$
$$= \frac{9(-35+h+2v)(1+h-2v)(1+h+2v)^2 - 16(55+19h-42v)(1+h+2v)\gamma - 512\gamma^2}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} > 0$$

$$\frac{\partial f_d^{NC}}{\partial e_u} = \frac{-16\gamma[37+h^2+h(38-4v)+4(17-3v)v+32\gamma]}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} > 0 \tag{A32}$$

$$\frac{\partial f_d^{NC}}{\partial e_d} + \frac{\partial f_u^{NC}}{\partial e_u} \tag{A33}$$
$$= \frac{9(-35+h+2v)(1+h-2v)(1+h+2v) - 64\gamma(23+5h-12v)(1+h+2v) - 1024\gamma^2}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} > 0$$

Figure $A3$ illustrates that the numerator of $(A33)$ is negative, hence $\frac{\partial f_d^{NC}}{\partial e_d} + \frac{\partial f_u^{NC}}{\partial e_u} > 0$.

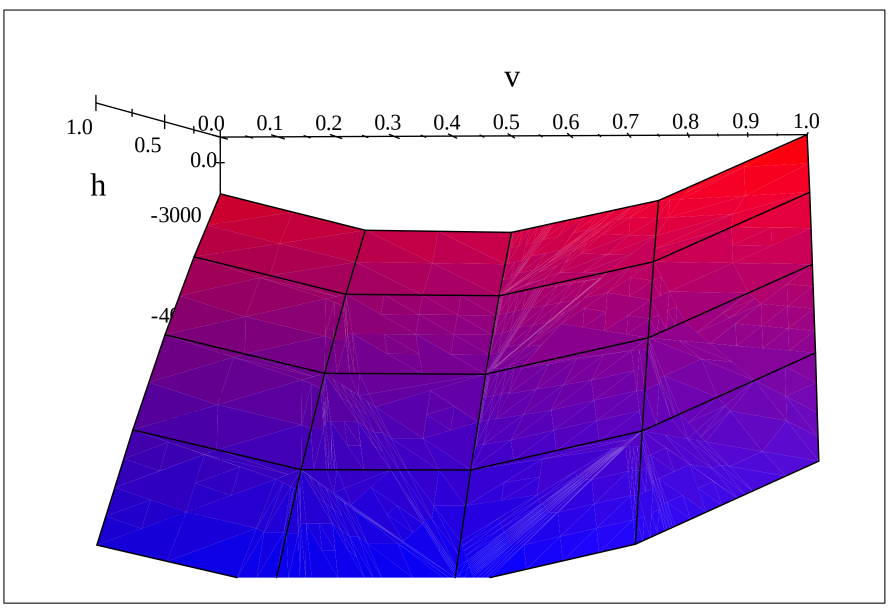


Figure $A3$: value of

$$9(-35+h+2v)(1+h-2v)(1+h+2v) - 64(23+5h-12v)(1+h+2v) - 1024$$

Similarly,

$$\frac{\partial f_u^{NC}}{\partial e_u} \tag{A34}$$
$$= \frac{9(-35+h+2v)(1+h-2v)(1+h+2v)^2 - 16(51+15h-34v)(1+h+2v)\gamma - 512\gamma^2}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} > 0$$

$$\frac{\partial f_u^{NC}}{\partial e_d} = \frac{16\gamma[-33+3h^2-4v(17+v)+h(-30+4v)-32\gamma]}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} > 0 \tag{A35}$$

$$\frac{\partial f_u^{NC}}{\partial e_u} + \frac{\partial f_u^{NC}}{\partial e_d} \tag{A36}$$
$$= \frac{9(-35+h+2v)(1+h-2v)(1+h+2v) - 64\gamma(21+3h-8v)(1+h+2v) - 1024\gamma^2}{[-35+h^2+4(-17+v)v+h(-34+4v)-32\gamma]\{9[(1+h)^2-4v^2]+32\gamma\}} > 0$$

Appendix 3

By using the same numerical simulations, we get the ranks of R&D levels under competition (NC), horizontal cooperation (HC), vertical cooperation (VC) and generalized cooperation (GC) under extreme spillover rates ($h, v \in \{0, 1\}$) and $(e_d, e_u) \in \{(200, 0), (100, 100), (0, 200)\}$:

$(e_d = 200, e_u = 0)$	w_d^{HC}	w_u^{HC}	w^{VC}	w^{GC}	w_d^{NC}	w_u^{NC}
$(h = 0, v = 0)$	4.34	7.77	6.06	6.22	5.11	6.84
$(h = 0, v = 1)$	0.54	11.66	10.91	15.61	3.13	8.56
$(h = 1, v = 0)$	7.86	14.63	6.07	11.59	4.2	7.61
$(h = 1, v = 1)$	3.22	18.03	10.63	18.21	2.21	9.38

$(e_d = 100, e_u = 100)$	w_d^{HC}	w_u^{HC}	w^{VC}	w^{GC}	w_d^{NC}	w_u^{NC}
$(h = 0, v = 0)$	5.73	6.38	6.06	6.22	5.81	6.14
$(h = 0, v = 1)$	4.94	7.26	10.91	15.61	5.28	6.41
$(h = 1, v = 0)$	10.52	11.97	6.07	11.59	5.57	6.24
$(h = 1, v = 1)$	8.84	12.41	10.63	18.21	5.03	6.57

$(e_d = 0, e_u = 200)$	w_d^{HC}	w_u^{HC}	w^{VC}	w^{GC}	w_d^{NC}	w_u^{NC}
$(h = 0, v = 0)$	7.12	5.00	6.06	6.22	6.51	5.44
$(h = 0, v = 1)$	9.35	2.86	10.91	15.61	7.44	4.25
$(h = 1, v = 0)$	13.18	9.30	6.07	11.59	6.93	4.88
$(h = 1, v = 1)$	14.47	6.78	10.63	18.21	7.84	3.75

Thus,

$(e_d = 200, e_u = 0)$	
$(h = 0, v = 0)$	$w_u^{HC} > w_u^{NC} > w^{GC} > w^{VC} > w_d^{NC} > w_d^{HC}$
$(h = 0, v = 1)$	$w^{GC} > w_u^{HC} > w^{VC} > w_u^{NC} > w_d^{NC} > w_d^{HC}$
$(h = 1, v = 0)$	$w_u^{HC} > w^{GC} > w_d^{HC} > w_u^{NC} > w^{VC} > w_d^{NC}$
$(h = 1, v = 1)$	$w^{GC} > w_u^{HC} > w^{VC} > w_u^{NC} > w_d^{HC} > w_d^{NC}$

$(e_d = 100, e_u = 100)$	
$(h = 0, v = 0)$	$w_u^{HC} > w^{GC} > w_u^{NC} > w^{VC} > w_d^{NC} > w_d^{HC}$
$(h = 0, v = 1)$	$w^{GC} > w^{VC} > w_u^{HC} > w_u^{NC} > w_d^{NC} > w_d^{HC}$
$(h = 1, v = 0)$	$w_u^{HC} > w^{GC} > w_d^{HC} > w_u^{NC} > w^{VC} > w_d^{NC}$
$(h = 1, v = 1)$	$w^{GC} > w_u^{HC} > w^{VC} > w_d^{HC} > w_u^{NC} > w_d^{NC}$

$(e_d = 0, e_u = 200)$	
$(h = 0, v = 0)$	$w_d^{HC} > w_d^{NC} > w^{GC} > w^{VC} > w_u^{NC} > w_u^{HC}$
$(h = 0, v = 1)$	$w^{GC} > w^{VC} > w_d^{HC} > w_d^{NC} > w_u^{NC} > w_u^{HC}$
$(h = 1, v = 0)$	$w_d^{HC} > w^{GC} > w_u^{HC} > w_d^{NC} > w^{VC} > w_u^{NC}$
$(h = 1, v = 1)$	$w^{GC} > w_d^{HC} > w^{VC} > w_d^{NC} > w_u^{HC} > w_u^{NC}$

Appendix 4

When emissions cap is endogenously settled, the regulator maximizes social welfare to determine the allocation of permits, as well as the permits constraint L:

$$\max_{e_d,\, e_u} SW = CS + TP - TD \tag{A35}$$

Solving for the two first-order necessary conditions with respect to e_d and e_u yields the optimal allocations of permits (e_{sd}^{nNC}, e_{su}^{nNC}), (e_{sd}^{nHC}, e_{su}^{nHC}), (e_{sd}^{nVC}, e_{su}^{nVC}) and (e_{sd}^{nGC}, e_{su}^{nGC}) under different types of R&D activities:

$$e_{sd}^{nNC} = \frac{\{8(-35+h+2v)(1+h+2v)^2 - [-160+h(101+9h)+202v+36hv+36v^2]\gamma + 224\gamma^2\}}{\frac{1}{8(a-c_d-c_u)}9[4(-35+h+2v)^2(1+h+2v)^2 - 3(-35+h+2v)(97+85h+170v)\gamma + 4608\gamma^2]} \tag{A36}$$

$$e_{su}^{nNC} = -\frac{\{8(-35+h+2v)(1+h+2v)^2 + [-812+h(-407+9h)-814v+36hv+36v^2]\gamma - 352\gamma^2\}}{\frac{1}{8(a-c_d-c_u)}9[4(-35+h+2v)^2(1+h+2v)^2 - 3(-35+h+2v)(97+85h+170v)\gamma + 4608\gamma^2]} \tag{A37}$$

$$e_{sd}^{nHC} = \frac{-8(a-c_d-c_u)[4(1+h)(17+17h-2v)(1+h+2v)^2 - 9\gamma(1+h-2v)(4+4h+v) - 56\gamma^2]}{9\{4(17+17h-2v)^2(1+h+2v)^2 + (17+17h-2v)[145(1+h)+254v]\gamma + 1152\gamma^2\}} \tag{A38}$$

$$e_{su}^{nHC} = \frac{8(a-c_d-c_u)\{4(1+h)(17+17h-2v)(1+h+2v)^2 + 9\gamma[22(1+h)^2 + 21(1+h)v - 2v^2] + 88}{9\{4(17+17h-2v)^2(1+h+2v)^2 + (17+17h-2v)[145(1+h)+254v]\gamma + 1152\gamma^2\}} \tag{A39}$$

Again, social welfare is independent from the initial permit allocation under vertical cooperation and generalized cooperation.

In addition,

$$\begin{aligned} L &= e_{sd}^{nNC} + e_{su}^{nNC} \qquad \text{(A40)} \\ &= -\frac{16[-54+h^2-34v+4v^2+h(-17+4v)-32\gamma]\gamma(a-c_d-c_u)}{9[4(-35+h+2v)^2(1+h+2v)^2 - 3(-35+h+2v)(97+85h+170v)\gamma + 4608\gamma^2]} \end{aligned}$$

$$
\begin{aligned}
L &= e_{sd}^{nHC} + e_{su}^{nHC} \qquad \text{(A41)}\\
&= \frac{16\gamma[13(1+h)^2 + 7(1+h)v - 2v^2 + 8\gamma](a - c_d - c_u)}{4(17+17h-2v)^2(1+h+2v)^2 + (17+17h-2v)[145(1+h)+254v]\gamma + 1152\gamma^2}
\end{aligned}
$$

$L = e_{sd}^{nGC} + e_{su}^{nGC}$

$= \{\gamma(a - c_d - c_u)[(1+h+2v)^2(325(1+h)^2 - 72(1+h)v - 4v^2) + 576(1+h)(1+h+2v)\gamma + 256\gamma^2]\}$

$8\{(17+17h-2v)^2(1+h+2v)^4 + 6(17+17h-2v)(1+h+2v)^2(71+71h+82v)\gamma + 72(1+h+2v)$

$[103(1+h)+50v]\gamma^2 + 2496\gamma^3\}$ (A42)

Furthermore,

$$
\begin{aligned}
w_{sd}^{nNC} &= w_{sd}^{nNC} \qquad \text{(A43)}\\
&= \frac{8[4(-35+h+2v)(1+h+2v) - (109+h+2v)\gamma](a - c_d - c_u)}{4(-35+h+2v)^2(1+h+2v)^2 - 3(-35+h+2v)(97+85h+170v)\gamma + 4608\gamma^2}
\end{aligned}
$$

$$
\begin{aligned}
w_{sd}^{nHC} &= w_{sd}^{nHC} \qquad \text{(A44)}\\
&= \frac{4[4(1+h)(17+17h-2v)(1+h+2v) + (61+61h+2v)\gamma](a - c_d - c_u)}{4(17+17h-2v)^2(1+h+2v)^2 + (17+17h-2v)[145(1+h)+254v]\gamma + 1248\gamma^2}
\end{aligned}
$$

which are constant, given all exogenous parameters.

Moreover,

$$
e_{sd}^{nNC} - e_{su}^{nNC} = \frac{32[4(-35+h+2v)(1+h+2v)^2 - (163+127h+254v)\gamma - 32\gamma^2](a - c_d - c_u)}{9[4(-35+h+2v)^2(1+h+2v)^2 - 3(-35+h+2v)(97+85h+170v)\gamma + 4608\gamma^2]} < 0 \qquad \text{(A45)}
$$

$$
\begin{aligned}
& e_{sd}^{nHC} - e_{su}^{nHC} \qquad \text{(A46)}\\
= & \frac{8(a - c_d - c_u)\{-8(1+h)(17+17h-2v)(1+h+2v)^2 - 18\gamma(1+h)(9+9h+14v) - 32\gamma^2\}}{91\{4(17+17h-2v)^2(1+h+2v)^2 + (17+17h-2v)[145(1+h)+254v]\gamma + 1248\gamma^2\}} < 0
\end{aligned}
$$

Again, from (A43) and (A44), under both R&D competition and horizontal cooperation, upstream and downstream firms invest the same amount of R&D. The regulator maximizes social welfare by equalizing R&D marginal costs, giving fewer permits to downstream firms to induce them to invest more in R&D.

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