THE EVOLUTION OF PORTFOLIO THEORY AND THE DEVELOPMENT OF THE ARBITRAGE PRICING THEORY:

A REVIEW OF THE LITERATURE

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Introduction

In the study of financial economics, the issue of the *value* of an asset has received a great deal of attention in the last few decades. As financial markets become more complex and international in scope, there is an increasing practical and theoretical need for effective asset pricing models. As an extension of Modern Portfolio Theory, the traditional Capital Asset Pricing Model (CAPM) has long servedas the standard for valuing assets in the context of a financial portfolio, and extensive research has been devoted over the past few years to improvements in the model and its applications. Nevertheless, the weaknesses of the model have inspired an increasingly popular alternative: the Arbitrage Pricing Theory (APT). This new approach to equilibrium asset pricing is still in its infancy both in terms of theoretical development and practical applicability. The growing debate regarding its validity, however, has resulted in much confusion over its true strengths and weaknesses. The following study will attempt to clarify the essential arguments for and against the Arbitrage Pricing Model.

The first part of the study outlines the basic evolution of Portfolio Theory from the basic CAPM models to the APT, highlighting the key characteristics of the competing models. With this, a clearer understanding can be made of the current debate over the superiority of the APT which occupies the second part of the study. By clarifying the large amounts of literature in the area through such an approach, conclusions can tentatively be drawn as to whether or not the APT is indeed a superior model and a usable alternative to the CAPM. Conclusions in support of the APT should further encourage both theoretical development and practical applications.

PART I Evolution of Portfolio Theory

Modern Portfolio Theory and the Capital Asset Pricing Model

In traditional financial research, expected returns have been the primary focus in asset pricing, and risk, on its own, was evaluated rather subjectively or intuitively. The development of what is today referred to as Modern Portfolio Theory introduced the concept of evaluating assets on the basis of both return *and* risk. This approach made the following basics of Portfolio Theory considerably more attractive. By combining asset risks and returns with investor preferences, Modern Portfolio Theory lay the groundwork for many future advances in financial research.

Modern Portfolio Theory

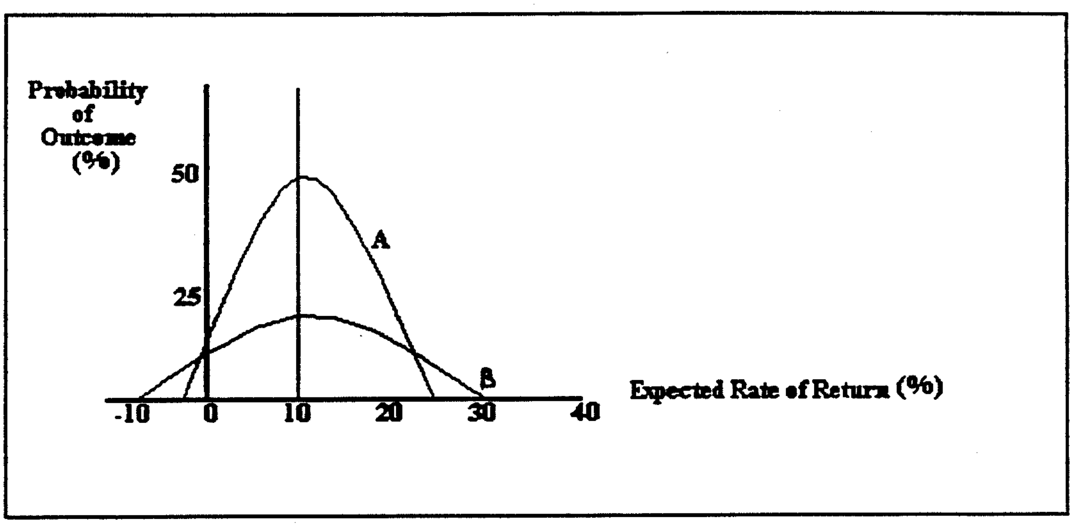
Modern Portfolio Theory (MPT) relies on relatively simple and basic concepts. Returns are simply the expected returns from an investment. Put in equation form, the expected returns for a stock over a single period t are:

$$\text{Expected Total Return} = \frac{\text{Dividend} + (\text{Market Price}_t - \text{Market Price}_{t-1})}{\text{Market Price}_{t-1}}$$

Since these forecasts are rarely perfectly accurate, there is potential that returns may be above or below estimates. A commonly accepted measure of such forecast uncertainty is called the <u>variance</u>. The variance measures the extent by which returns deviate from their mean value, or the breadth of the distribution of expected returns from an investment (squared deviations from the mean forecast). The squared root of the variance, or the Standard Deviation, is a more widely used version of forecast uncertainty.

Variance or Standard Deviation are used in MPT for estimating the potential risk of making an investment. Generally speaking, in the context of MPT, "risk" is defined as forecast uncertainty, or the potential for forecast error. Figure 1 shows the possible outcomes for two independent investments.

<u>Figure 1</u>



The above graph describes the distribution of returns that might be expected for two investments, A and B with the same mean. Investment B is obviously more risky, since the variance of A is less than the variance of B. Since in MPT returns are by assumption normally distributed, only mean and variance are needed to describe the entire distribution.

When moving from a single asset in isolation to evaluating an entire portfolio of assets, some of the elements differ. Expected return is now the weighted average of expected returns of each of the assets in the portfolio, the weights being the relative proportion of each asset held in the portfolio; i.e. if 20% of the portfolio is in asset i, then $w_i = .2$. The expected return of the portfolio is:

$$E(X_p) = \sum_{i=1}^{n} w_i \, E(X_i) \qquad \text{for n different assets}$$

where $E(X_p)$ = the expected return of portfolio p;

$E(X_i)$ = the expected return of asset i; and

w_i = the proportional weight of asset i in portfolio p.

Risk is the variance of the expected return of the portfolio. However, this risk (of the entire portfolio) is viewed quite differently from the risk of the portfolio's components, that is the risk of the individual assets considered on their own. The covariance measures the correlation between the two stocks. For a portfolio containing any number of assets:

$$\sigma^2 \, (X_p) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \, \rho_{ij} \, \sigma_i \, \sigma_j$$

where ρ_{ij} = the correlation coefficient between assets i and j;

σ_i = the variance of asset i's returns;

σ_j = the variance of asset j's returns; and

$\rho_{ij}\, \sigma_i\, \sigma_j = cov_{ij}$ = covariance between assets i and j.

The risk of the portfolio is a combination of the risks of each of the individual stocks, thus, in general for a given expected return, risk can be reduced by diversification-i.e. by increasing the number of assets in the portfolio. The greater the number of assets in the portfolio, the lower the risk, for a given level of expected return. The intuitive notion here is that the greater the number of assets one adds to a portfolio, the closer one comes to having the true "market" portfolio. Thus, since diversification reduces risk, the key issue to any investor, individual or institutional, is not how risky an asset is on its own, but rather how that asset affects the riskiness of the portfolio. Thus, MPT defines risk as the variability of efficient portfolio returns, rather than the variability of returns of individual assets.

Given this basic introduction to the concept of risk, attention can now turn to one of the first models to deal explicitly with the concept of risk in the context of a portfolio- the Markowitz Model, as developed by Harry Markowitz in 1952.[1] While derivations and extrapolations of the model can become quite complex, the basic ideas are relatively simple, relying on common economic principles. The Markowitz Model is commonly considered as the foundation for Modern Portfolio Theory.

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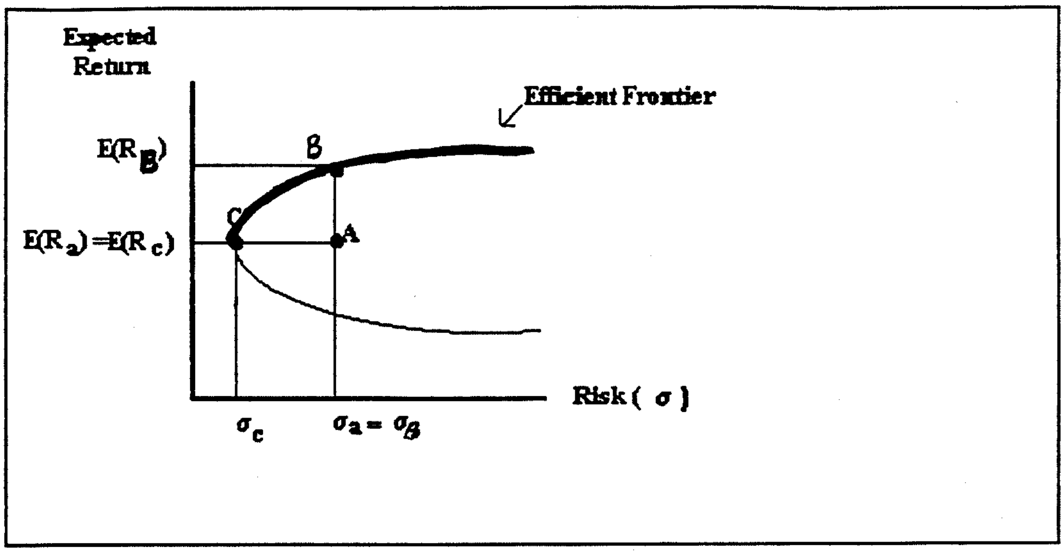
[1] Markowitz, Harry,"Portfolio Selection", Journal of Finance. March 1952, pp. 77-91.

The Markowitz Model basically starts by assuming that investors choose among their investment alternatives on the basis of risk and return. These two criteria, or characteristics, can be combined in graphic form to show all of the possible combinations of the individual investment alternatives. As shown in Figure 2, each point can either be an asset or a group of assets (a portfolio), and represents a possible investment.

Figure 2



where $E(R_A)$ = expected return of portfolio A;

$E(R_B)$ = expected return of portfolio B;

$E(R_C)$ = expected return of portfolio C; and

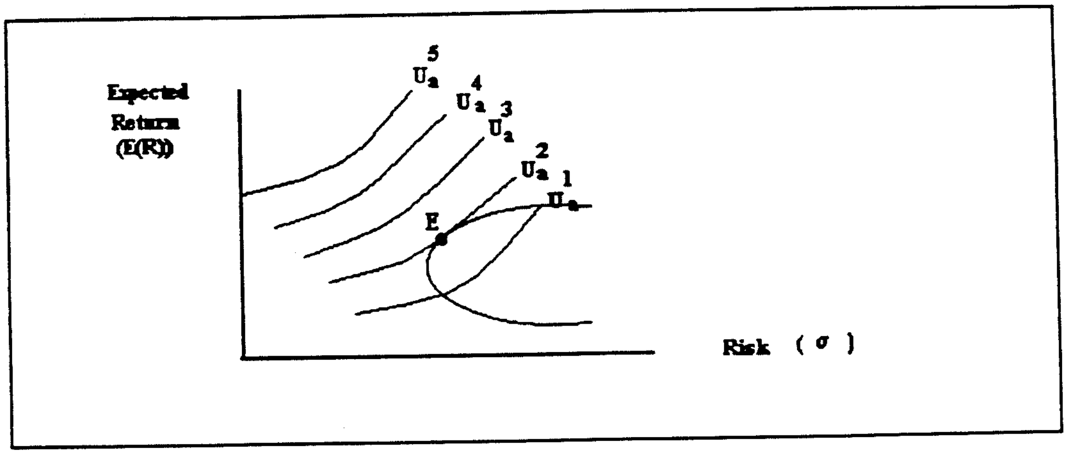
σ_A ,σ_B, and σ_C are their respective variances.

Given all of the possible choices of portfolios, the investor will choose investments with the highest level of expected return for a given level of risk, or conversely that with the lowest level of risk for a given level of expected return. These maximum expected return (or minimum variance) portfolios can be deemed "efficient" and by linking all such efficient portfolios together, an "efficient frontier" can be traced out.

Notice that rationality of the investor dictates that he would never choose points inside the frontier, for any given level of risk or expected return. For example, point A would never be chosen over B, since, for the same level of risk (var A = var B), B gives a higher expected return. Similarly, A would never be chosen over C, since for the same level of expected return ($E(R_a) = E(R_c)$), C has a lower level of risk.

The efficient frontier has many interesting implications when combined with the notion of investor preferences. Very risk-averse investors can chose portfolios low on the efficient frontier (lower risk), and those that are relatively less risk-averse can choose higher points (greater risk). Using utility functions defined over the first two moments (expected return and standard deviation), or more general utility functions with normally distributed returns, indifference curves can be derived to plot each investor's trade-offs between risk and return, and thus his relative risk-aversion. Notice that as risk increases, the required (expected) rate of return increases- to compensate for the extra risk. For risk-averse agents, the graph would look as follows:

Figure 3



where U^1_a ... U^5_a are investor a's indifference curves.

The higher and farther to the left the indifference curve, the more satisfied the investor. The efficient frontier, however, serves as a constraint to such investor goals, so that the highest attainable level of utility is the one just tangent to the efficient frontier. E is the optimal point for the investor "a", assuming his inability to borrow or lend. Of course each investor's utility curves (attitudes/preferences) are different, and the results depend on the particular investor's risk aversion.

The complexity and difficulty in empirically testing and using the Markowitz portfolio model kept practitioners from applying it for a considerably long time. Academics, however, have capitalized on its intuitive appeal to develop increasingly practical, albeit simplified, versions of the basic model. The most commonly used of these over the last thirty years is the Standard "Capital Asset Pricing Model" (CAPM), as developed independently by W. Sharpe, J. Lintner, J. Mossin, and often referred to as the "Sharpe-Lintner-Mossin CAPM".[2]

The Capital Asset Pricing Model

The Sharpe-Lintner-Mossin adaptation of the MPT is widely recognized as the standard Capital Asset Pricing Model, and is, in essence, an even further simplification of the above mentioned process. Empirical applications of the standard CAPM use two fundamental (and controversial) adaptations of the Markowitz Model. The first is with the "market portfolio" and the second is the addition of the "risk-free" asset. These two concepts are the essential characteristics of CAPM and should be dealt with in greater detail.

When every asset that exists in the "universe" of assets is in the portfolio, in some given proportion, the investor has created a Market Portfolio. The variability of the Market portfolio cannot be eliminated, and is often referred to as "systematic" risk inherent in the system.[3] Such risk is the result of changes in economic activity that have market-wide effects. Systematic, or undiversifiable, risk is inherent in any

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[2] Lintner, J.,"The Valuation of Risk Assets and the selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics. Vol. 47, Feb 1965, pp.13-37.

Mossin, J., "Equilibrium in a Capital Asset Market", Econometrica. Vol 34, Oct.1966, pp.768-783.

Sharpe, W. ,"Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", Journal of Finance. Vol 19, Sept 1964, pp. 425-442.

[3] Elton, E. and Gruber, M., Modern Portfolio Theory and Investment Analysis, Toronto: John Wiley and Sons, 1987, p.149

portfolio, as compared to unsystematic, or "firm-specific" risk, which can be eliminated through diversification.

The CAPM uses an index of stocks, using a weighted average technique, as a proxy for the market portfolio. Early CAPM development did not involve a lot of concern with the choice of index. Indexes were highly correlated, and therefore assumed to be rather interchangeable and a good proxy for the "true" market portfolio.[4] If the proxy for the market was not fully diversified (not a good representation of the market for all risky assets), then the model would not have an effective "market return" element, and thus would not properly differentiate between systematic and unsystematic risk. The original rationalizations of this index choice as a proxy involved the use of correlation analysis; however, further tests showed that two indices could be highly correlated with each other, but not correlated with the underlying "true" market for all risky assets. Thus many have concluded that finding a true proxy would be impossible, and using an incorrect index would be useless.[5] Supporters for the use of indices argue that just the fact that two different indices result in similar Betas implies some sort of confidence in the results, and since indices are the common benchmark used by investors in practice, they do provide significant practical information.[6]

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[4] Elton, E. and Gruber, M., p.288

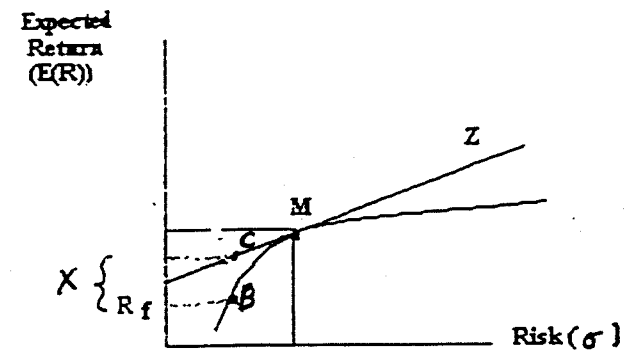
[5] Ibid, p. 289.

[6] Ibid, p. 289.

The second major adaptation of the basic Markowitz Model is the addition of the risk-free rate. The risk-free rate is, in theory, the asset that has zero variance and zero covariance with any other asset- in effect, no risk. Again, a proxy has been used to represent the risk-free rate, a most common substitute being the 90-day Treasury Bill rate which is government-issued and thus, for practical purposes, assumed to be free of risk. Since it is a proxy, and thus an estimate, any errors in the risk free rate (or the chance that such a rate doesn't even exist) would result in an incorrect model and thus a poor estimate of the excess expected rate of return on an asset. Like the market proxy, the risk-free rate is also an area of great controversy, putting to question the practical applicability of CAPM. For the moment, the risk-free rate will be assumed to exist, and it will also be assumed that its proxies are effective representations, so that a study of the equilibrium models may be done.

Thus, given the two basic adaptations of the Markowitz Model, a simple derivation and explanation of the CAPM can now be made. The ability to borrow or lend at the risk-free rate changes considerably the investor's opportunities on the efficient frontier. The Risk-free rate, R_f, presents the investor with a *new* efficient frontier. This new efficient frontier with borrowing and lending, the R_fZ line displays the risk-return trade-offs available to all investors. Since R_f is, by definition, a zero variance (no risk) asset, it is located on the vertical intercept on Figure 4 below.

Figure 4



The tangency point of the R_fZ line and the efficient frontier of all risky assets gives us the market portfolio. Recalling the argument of an efficient frontier using MPT, the R_fZ line presents a better investment opportunity than would previously have been chosen under MPT. Assuming a given investor's indifference curve was tangent at point B using MPT, the investor would improve his return by X with no additional risk. This can be seen by the fact that portfolio C generates a higher level of expected return with no additional risk (the variance of portfolios B and C are the same). In general, investors would simply have to purchase M, the market portfolio, and lend at R_f in a combination that suited their tolerance for risk. Some investors would only hold the market portfolio (M), while others would be somewhere along MZ, (essentially borrowing money at R_f to buy more of M than his initial wealth allowed, thus investing in the (risky) market portfolio with leverage (debt).

The element that makes CAPM so practical is its definition of risk, namely the volatility of a security's returns relative to the volatility of the market portfolio's returns- essentially the co-variability of the security's returns with that of the market. This is the systematic (market) risk, i.e. the risk inherent in the asset that cannot be diversified away. Since CAPM indicates that only this systematic risk is rewarded (the reward being higher expected returns) investors who choose not to be fully diversified will not achieve the best risk-return combination.

Systematic, or market-related risk of an asset j is designated by Beta (β_j)

$$\beta_j = \frac{\text{cov}(R_m, R_j)}{\text{var } R_m}$$

or
$$\beta_j = \frac{(\text{corr } j,m)(\text{st.dev } R_m)(\text{st.dev } R_j)}{\text{var } R_m}$$

where
corr j,m = the correlation coefficient of assets j and m;
st. dev R_m = the standard deviation of the returns on asset m;
st. dev R_j = the standard deviation of the returns on asset j;
var R_m = variance of returns on asset m.

R_m, R_j are stochastic returns, where R_m is the return of the market portfolio (M). As can be seen, the market β is obviously one. Assets with less risk (volatility) than the market would have $\beta<1$, while those with more risk, $\beta>1$. It can be shown that the expected rate of return of asset j is equal to the risk-free interest rate plus the expected excess market portfolio rate of return multiplied by the systematic risk of asset j (β_j).

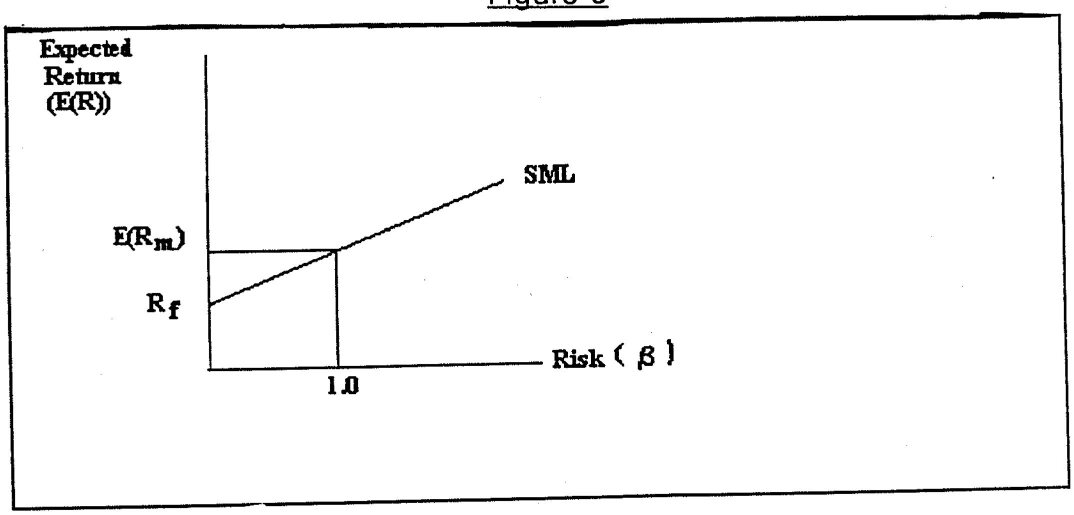
Equation 1: For an asset j $\quad E(R_j) = (R_f) + \beta_j(E(R_m) - R_f)$

The extra compensation for taking on systematic risk (rather than holding only R_f) is $(E(R_j) - R_f)$, which is the excess expected return of asset j over the risk-free rate. This is equal to the market price of risk, or the risk premium:

$$\beta_j\ (E(R_m) - R_f)$$

We can plot the expected risk-return relationship for individual assets that make up the market portfolio. The line created is called the Security Market Line (SML), and represents the trade-off between systematic risk and expected return for each asset, as shown in Figure 5:

Figure 5



If we assume investors only hold fully diversified portfolios (that is, the market portfolio M with no unsystematic risk), β replaces the standard deviation as the measure of risk for each individual asset.

1. that the assumptions are too unrealistic;
2. that tests of CAPM prove that it does not describe what has really occurred; and
3. that since it is impossible to agree on the best forecast for R_f and R_m, or even the existence of an R_f, CAPM is essentially useless.

Of the three general areas of criticism of CAPM, the rigidity of the assumptions has been the primary focus, as the second two arguments are far more debatable. The CAPM can arguably be said to rely on eight basic assumptions, the first five of which underlie the Efficient Markets Hypothesis (EMH), while the last three are essential to MPT and subsequently CAPM. Given this distinction, they will be divided as such.

The EMH Assumptions

1. *the investor's objective is to maximize terminal wealth*
   - "maximize terminal wealth" makes no distinction among the forms that wealth can take. If the investor preferred capital gains over dividends, the simple CAPM risk measure would no longer be adequate.

2. *investors make choices on the basis of risk and return, and these are measured by the mean and variance of a portfolio's return*

   - the distribution of returns is normal by assumption. These first two assumptions define the risk-return trade-off facing investors.

3. *investors have homogeneous expectations of risk and return*

   - simply stated, all investors' estimates of risk and return are the same, and thus the single efficient frontier of MPT exists. This assumption is essential to an equilibrium model. Relaxations of the homogeneous expectations assumption have been developed, the most common of which are Lintner's heterogeneous expectations model, the "state preference models" and Miller's "Theory of Investor Optimism". In all three cases, the model becomes considerably more complex.[7]

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[7] Lintner, John ,"The Aggregation of Investor's Diverse Judgements and Preferences in Purely Competitive Security Markets",Journal of Financial and Quantitative Analysis, Vol. IV, No. 4, Dec. 1969, pp. 347-400.

Miller, Edward, "Risk, Uncertainty and Divergences of Opinion", Journal of Finance. 32, Sept 1977, pp.1151-68.

For "state-preference models", see:

Copeland, T. and Weston, T., Financial Theory and Corporate Policy, 2nd Ed. Boston: Addison-Wesley, 1985.

4. *investors have identical time horizons*

   - investors form portfolios to attain wealth at a single, common terminal date. This allows for the construction of a single-period model. Since actual behaviour differs (investors do not make single period buy-and-hold decisions), continuous-time models have been developed, but are considerably more complex. To keep single-period models as strong representations of multi-period behaviour, returns are assumed to be independent over time, and expectations are independent of past or current information.

5. *information is freely and simultaneously available to all investors*

   - this is one interpretation of market efficiency, allowing a set of common forecasts and a single efficient frontier. Arguments in favour of this condition typically cite the "semi-strong form" of market efficiency, that most of the market value of the stock market consists of stocks of the most carefully analyzed firms, and the information on such stocks is readily available to large portfolio managers.

Given these efficient markets assumptions, several additional assumptions are required to develop CAPM:

The CAPM Assumptions:

1. *there is a risk-free asset, and investors can borrow and lend unlimited amounts at this risk-free rate;*
2. *there are no taxes, transaction costs, restrictions on short sales, or other market imperfections; and*
3. *total asset quantity is fixed, and all assets are marketable and divisible.*

Arguments in support of or against the Efficient Markets Assumptions have been well documented and are beyond the scope of this study. With regards to the CAPM specific assumptions, the most troubling has been the notion of a risk-free asset. Two fundamental questions can be identified about this assumption. The first is whether or not a purely risk-free asset actually exists, and the second is that if it does, can all investors borrow and lend at the risk-free rate?

There have been, to date, no definitive proofs of the existence of an asset that has absolutely no covariance with the market. However, one of the most popular responses has been by Black, Jensen, and Scholes in 1972, who argued that although there is no such thing as a riskless asset, a minimum risk portfolio (Z) can still be created, which has no covariance with the market portfolio. This can be done by selling assets short, and results in the following relationship.[8]

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[8] Black, F., Jensen, M. and Scholes, M., "The Capital Asset pricing Model: Some Empirical Tests", <u>Studies in the Theory of Capital Markets</u>. Ed. Jensen, M. New York: Praeger, 1972, pp. 79-121.

$$E(R_j) = (1-\beta_j)\, E(R_z) + \beta_j\, E(R_m)$$

where $E(R_z) < E(R_m)$, and R_z holdings are in equilibrium (shortsellers = lenders of securities)

Black et al. essentially replaced R_f with a portfolio that has no covariance with the market portfolio. Black did not, however, come to a decision as to what a zero Beta portfolio might be; he just stated that it exists. The CAPM has been adapted to deal with the effect of inflation; however, the models become more complex and lose their practical attractiveness. In summary, inflation alone makes the existence of a truly riskless asset unlikely, and makes the covariance of the least-risk asset with other assets very likely. In such a case, the underpinnings of any such asset pricing model would be flawed.

The other main issue with the riskless rate is whether or not all investors can borrow and lend at the same risk-free rate. While most, if not all, investors can lend at the risk-free rate, only a few can borrow at the same rate. CAPM models have been developed that relax this assumption; however complexity arises, and the Security Market Line becomes broken, as its linearity, slope and intercept are all affected.

Given the rigidities of many of the assumptions of the CAPM, many have raised the issue of its practicality, and even its underlying testability. Many have defended the model, by testing to see whether it was able to describe or predict if some or all of the assumptions were relaxed or excluded. In all cases, the CAPM becomes more complex, and in most cases, the slopes and intercepts of the SML change, or multiple market lines exist rather than a simple homogeneous efficient frontier. This has led to further criticisms, as it was the simplicity of the original model that made it so attractive in the first place.

While CAPM has been defended, often relatively successfully albeit with more complex models, the need for a more realistic and testable model has persisted. Stephen Ross' 1976 article "The Arbitrage Pricing Theory of Capital Asset Pricing" attempted to respond to these demands, and consequently lay the groundwork for a new alternative to the CAPM.[9]

The Arbitrage Pricing Theory

Ross' 1976 article introduced the possibility of a new approach to pricing assets, using an equilibrium model markedly different from CAPM and CAPM-derived models. Ross developed a mechanism that derives assets prices from arbitrage arguments that are similar to those that apply to the CAPM. This mechanism derives such prices, given a particular process that generates security returns. The Arbitrage Pricing Theory (APT) is generally an extension of the simple one-factor-return generating process of the CAPM, where, in the CAPM, this one factor is the market risk premium. However, the underlying motivation behind the development of APT

[9] Ross, Stephen,"The Arbitrage Theory of Capital Asset Pricing", Journal of Economic Theory. Vol 13, Dec 1976, pp. 341-360

is that returns are NOT simply functions of the market risk premium, but rather have a number of significant factors that affect them, and thus the APT is a multi-factor *equilibrium* model. Equilibrium is stressed here because multi-factor-return generating models have been developed earlier; however none have explicitly put them in an equilibrium framework.

The APT arbitrage argument centres on the "law of one price"- two identical items cannot have different prices, at least not for any measurable length of time. The standard economic rationale applies: the consumer (investor) will simultaneously buy the lower priced asset and sell the higher priced one, and similar market wide actions will continue until the two prices converge to equality. This process of buying and selling simultaneously is generally referred to as "zero risk, zero investment arbitrage".

The APT has gained considerable popularity at even the most superficial levels, for it is indeed a more general account of asset equilibrium than CAPM-oriented models, and has more intuitive appeal. Asset pricing is not seen to be affected simply by means and variances, but rather by a whole array of influences, so that the mean-variance perspective of CAPM is replace by an assumption about the *process* generating an asset's returns. (Some of the elements of MPT are still maintained, however, such as homogeneous investor expectations).

APT states that the return on any asset is linearly related to a set of indices:

$$R_j = a_j + b_{j1}I_1 + b_{j2}I_2 + \dots + b_{jn}I_n + e_j$$

for n number of indices, where

a_j = expected level of return on asset j if all indices have a value of 0 (this can be interpreted as the risk-free rate R_f, although not necessarily)

I_n = the value of the n^{th} index that impacts the return on asset j

b_{jn} = the sensitivity of asset j's return to the n^{th} index

e_j = a random term with mean zero, and variance σ_j^2

This random element, of course, explains all movements in returns that cannot be otherwise accounted for. By definition, its value is small, its average zero, and there is no pattern in the error term for one asset compared to another (firm-specific risk in a sense), and no relationship between it and any of the formally stated indices.

The inspiration for the development of APT traces back to the earlier arguments about the validity of CAPM. One of the key weaknesses of the CAPM is the fact that it accounts for one single source of relevant risk- the market. That the market is the only factor determining the excess expected return on an asset has been an issue brought up since CAPM's inception. Various analyses have been done regarding the possibility of other factors, such as the industry or sector in which the firm is located, or other general economic indicators or elements, but since such multi-factor models say nothing about market efficiency or equilibrium, they only served to describe the factors that affected an asset or a group of assets at a particular point in time. What was needed was an equilibrium theory that defined the relationship between the expected returns of a security and the common factors affecting that security- in essence, to describe what will influence the prices of assets. APT appears to have introduced a method by which such a multi-factor equilibrium pricing model can be attained. A simple, intuitive derivation of APT equilibrium follows.

In what follows, a two-index model will be used for simplicity, to demonstrate the expected returns that must arise from the APT. (A mathematical proof would show that such an demonstration would hold for a model with any number of indices).[10] Assume the following model describing returns:

$$R_j = a_j + b_{j1}I_1 + b_{j2}I_2 + e_j \quad \text{for any asset } j$$

If the investor holds a diversified portfolio, only systematic risk will matter, and the only terms that affect such risk are the b's. Given the assumption that the investor is concerned only with expected return and risk, he is only concerned with three attributes of his portfolio: R_p, b_{p1}, and b_{p2} , where the return generating process for the portfolio (any portfolio) is by definition, the same as for the asset j:

$$R_p = a_p + b_{p1}I_1 + b_{p2}I_2 + e_p$$

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[10] A straightforward and rigorous mathematical proof can be found in: Elton, E. and Gruber, M., p.340.

For the purposes of demonstration, we assume the existence of three well diversified portfolios:

Portfolio	Expected Return	b_{x1}	b_{x2}
A	.20	1.0	0.8
B	.18	0.8	0.7
C	.10	0.3	0.3

Consider a portfolio E, with $b_{E1} = 0.7$ and $b_{E2} = 0.6$ and an expected return of 18%. Consider another portfolio D constructed by placing 1/3 of the funds in portfolio A, 1/3 in B, and 1/3 in C. The b's and expected returns on this new portfolio are weighted according to the amount in each of the original three, A, B, and C.

So $b_{D1} = 1/3\ b_{A1} + 1/3\ b_{B1} + 1/3\ b_{C1} = 0.7$

$$b_{D2} = 1/3\ b_{A2} + 1/3\ b_{B2} + 1/3\ b_{C2} = 0.6$$

$$E(R_D) = 1/3\ E(R_A) + 1/3\ E(R_B) + 1/3\ E(R_C) = 0.16$$

Notice that the b's for portfolio's D and E are the same, which implies that their risks are identical. Notice, however, that while risks are the same, the expected returns differ, that of portfolio E being higher. Since D's expected return matches that of the expected return from the equation of the plane ($R_p = 0.16$), the investment E (portfolio E) does not lie on the equation of the plane, so the opportunity for riskless arbitrage exists. By the law of one price, two portfolios with the same risk cannot sell at a different expected return (a different "price"). The arbitrageur would obviously buy portfolio E while simultaneously selling an equal amount of D short (financing the purchase of E by selling D short), guaranteeing a profit with no investment and no risk. The arbitrage portfolio would gain an extra 2% return, with zero b's (zero systematic risk). This would of course continue until portfolio E lies on the same plane as the other four.

The equilibrium model produced by APT would simply be a general equation of a plane in $E(R_j)$, b_{j1}, b_{j2} space:

$$E(R_j) = \lambda_0 + \lambda_1 b_{j1} + \lambda_2 b_{j2}$$

This is the equilibrium APT when returns are generated by a two-index model. The λ_1 is the increase in expected return for a one-unit increase in b_{j1}. So in equilibrium, λ_1 and λ_2 are the extra returns awarded for taking on the extra risk associated with I_1 and I_2 respectively, in essence, risk premia. Notice that if both b's equal zero, the expected return is λ_0, and this could be a zero b_j portfolio. This expected return can be interpreted as the R_f described in CAPM. Here, it will be referred to as R_z. Additionally, in general, if we assign a value of 1 to b_{j1}, and zero to the other b's, we have $\lambda_1 = R(R_j) - R_z$, or simply the excess return on a security that is only subject to the risk of index 1, and having a unit measure of this risk.

The above approach can of course be generalized to the N-index case, where:

$$R_j = a_j + b_{j1}I_1 + b_{j2}I_2 + \ldots + b_{jN}I_N + e_j$$

and in equilibrium, all securities and all portfolios have expected returns described by this same N dimensional hyperplane:

$$E(R_j) = \lambda_0 + \lambda_1 b_{j1} + \lambda_2 b_{j2} + \ldots + \lambda_N b_{jN}$$

where $\lambda_0 = E(R_z)$ (the return if all other factors were zero); and

$\lambda_1 = E(R_j) - E(R_z)$, (the increase in expected return on asset j due to the first factor I_1 when all other b's are zero).

This form holds for all investments and all portfolios. If an investment were to lie above or below this plane, an opportunity would exist for riskless arbitrage.

So, from the original, multi-factor return generating process:

$$1) \qquad R_j = a_j + \sum_{i=1}^{n} b_{ji}I_i + e_j$$

the equilibrium equation below was obtained:

$$2) \qquad E(R_j) = E(R_z) + \sum_{i=1}^{n} b_{ji}\,\lambda_i$$

Before continuing, a brief discussion of the meaning of the three variables b, I, and λ is required. From equation 1, it can be seen that each security j has a unique sensitivity to the factor I, and that any I factor has the same value for all securities. If I was unique to security j, then it would be represented in the residual (error) term e_j. These factors I are also the source of covariance between the securities. The b_{ji}'s of course are unique to each security, and represent an attribute of the security, in a loose sense, analogous to an individual firm's "Beta"; only here there are many "Betas", one for each factor influencing its return, rather than a single Beta for the only important factor in CAPM, the market. The attribute b_{ji} can either be the sensitivity of the asset to the specific factor I_i, or it can be a characteristic of the asset such as dividend yield, that is affected by factor I_i. From equation 2, the λ_i is the excess return required due to an asset's sensitivity to the i^{th} attribute of the asset.

The specific methods and approaches to testing and using the APT will be referred to often in the second part of the study, and thus a simple generalized description of the most common testing methodologies should be pointed out here.

To test the APT, equation 2, the equilibrium equation, must be tested. However, since this requires estimates of the b_{ji}'s, the common approach is to use equation 1 to estimate the b_{ji}'s. Consequently, in order to estimate the b_{ji}'s, definitions of the relevant factors, I_i's are required. The most common approach is to estimate simultaneously both the factors (I_i's) and firm attributes (b_{ij}'s) for equation 1. Once this simultaneous estimation technique has been accomplished, a test of the equilibrium model, equation 2, can be done. This approach can never exactly achieve the desired result (exactly specified factors and attributes (I_i's and b_{ji}'s), so that the covariance between residual returns (e_j's) is zero). However, it can approximate the result, essentially minimize the size of the residual terms, through a statistical technique known as factor analysis.[11]

While factor analysis is the most widely used approach for testing the APT, two alternative approaches that have grown in popularity should be pointed out. One method is to hypothesize *a priori* (before actually beginning the test) a set of influences, (factors I_i) that might affect returns, and use equation 1 to estimate the b_{ji}'s. These influences might include such things as the rate of inflation, or the interest rate. The second alternative method is to specify the set of attributes (firm characteristics, or b_{ji}'s) that might affect expected return, such as dividend yield or earnings growth. Instead of specifying the factors, this approach would directly specify the b_{ji}'s, and proceed to test equation 2, the equilibrium model.

In either of the alternative approaches, the test actually becomes a joint test of 1) the APT, and 2) the significance of the hypothesized factors or characteristics as

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[11] For an example of the use of factor analysis, see:

Fama, E. and MacBeth, J., "Risk, Return, and Equilibrium: Empirical Tests", Journal of Political Economy. Vol 81, No. 3, May 1973, pp. 607-636.

determinants of equilibrium. While factor analysis is still the most common approach, the methods discussed above have become increasingly popular alternatives, (in particular the former) as will become more evident in the second part of the study.

The Arbitrage Pricing Theory is, like its CAPM counterpart, very general. And, like CAPM, this generality is what makes it very attractive and at the same time susceptible to a lot of criticism. As with the CAPM, the APT's generality can indeed be viewed as its weakness. What remains to be seen, however, is whether this criticism holds up. A review of the significant arguments against APT, and the corresponding advances made in favour of the model, will perhaps shed some light on this issue. While ultimate conclusions can certainly not be drawn at this level of analysis, an understanding of the key criticisms and developments made thus far will certainly reveal to some extent whether or not APT is indeed a testable and ultimately usable alternative to the CAPM.

PART II Criticisms of the APT & New Developments in Support of the Model

The Arbitrage Pricing Theory has allowed for an alternative asset pricing model to the CAPM; however in its initial development it gave very little, if any, evidence as to what an appropriate model might be, only that such a model existed. Furthermore, the basic development of the model specified nothing about the size or the signs of the factors, or, for that matter, even what the factors might be. This has been the key issue for both those that attacked it as well as those that supported the model. Indeed, even to the casual reader, this seems to be the most pressing issue surrounding the model. While APT *may seem* more intuitive than the CAPM in terms of what it assumes about the process that generates returns on assets; how can it be tested and ultimately used if we have no idea of what the specific factors of the model might be? It is on the basis of this argument that tentative conclusions can be drawn as to both the theoretical and practical usefulness of the model.

Criticisms of the APT

The critics of APT have, since its inception, questioned the models testability and ability to estimate returns. APT theory focuses on specifying a structure for asset pricing; it does not specify the economic or firm characteristics that should affect the expected return. The main argument, then, against APT is that despite sophisticated methods of testing and analysis, the model still cannot properly identify the components or elements that affect returns that would subsequently become relevant to asset pricing.

To simplify the extensive body of literature that criticises the APT, the following arguments against the theory have several things in common. The first is that, in most cases, the argument put forth by proponents of APT that "the CAPM is not testable unless the *true* market portfolio of *all* assets is used" has been accepted. Having admitted to this fundamental flaw of CAPM, those who still argue against the APT base their critiques on the grounds that, although CAPM may not be truly testable, this is not a reason to discard the model altogether in favour of a model that is equally untestable for different reasons. Additionally, even if the APT theory cannot be rejected after testing, it still does not imply that it is preferable to CAPM. In this sense, the critiques are narrowed down to those that either argue against the testability of the APT altogether, or to those that tentatively accept the APT tests, but reject any conclusions that it is preferable to the CAPM. In either approach, the

CAPM itself may either be omitted from discussion, or defended on theoretical grounds or through testing and practical applications. With respect to the extent of the existing literature, and the scope of this study, only the key general arguments against APT will be highlighted.

The critiques that centre on the testability of the APT initially focused on the ability of the model to develop an exactly linear risk-return relationship. In the case of models with both finite and infinite numbers of assets, almost all tests did not produce exact linear relationships, which is essentially a fundamental part of the theory, and puts to question the validity of the theory on a general level. More sophisticated critiques of APT testing focus on the empirical methods, and specifically on factor analysis as a useful methodology. The factors method, as this approach is called, can be manipulated arbitrarily, by essentially repackaging a given set of securities, producing a new set of returns, with virtually any pre-specified random variables as the factors.[12] Thus, the argument concludes that factor analysis is not an ideal tool for identifying the components that affect an asset's returns.

An additional criticism of APT through factor analysis is that the simplification of testing a subset of the universe of assets to test the APT is also flawed. This argument centres on the notion that economy-wide factors or influences do not transcend to apply to smaller sets of securities, despite a priori intuition. Simply put,

[12] Shanken, Jay, "The Arbitrage Pricing Theory: Is it Testable?", The Journal of Finance, Vol. XLVII, No. 4, Sept. 1992, p. 1137.

the factors that generate returns for a particular asset or set of assets are not the same, in either size or number, as those that affect the returns of all assets in the economy.[13]

In conclusion, the assumption that all assets' returns conform to a strict common factor structure is wrong, and as such any statistical methods based on this strict factor model assumption are inappropriate for testing APT. Since virtually *any* variables could serve as factors in a well-defined strict factor model, any conclusions drawn from such tests would be relatively useless-especially in the greater context of supporting a new asset pricing model.

The most common alternative to refuting the testability of APT is to argue it in terms of its inferiority to CAPM both on theoretical and practical grounds. These arguments essentially attempt to demonstrate that even if APT is testable and usable, CAPM still does a better job of describing reality. These studies essentially find that when assumptions of CAPM are effectively relaxed (and in some cases APT has yet to consider relaxing such assumptions as taxes and transaction costs), they serve as a better estimator of asset prices. Since there is a large amount of applied examples showing the superiority of CAPM in estimating expected returns, the following will concern itself with theoretical support of the market model.

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[13] Ibid, p.1137.

Modern CAPM models have responded to the problems of the "market portfolio" by further justifying the proxies such as market indices. For example, as a weighted average holdings of N number of investors, any market proxy that is held by the N investors can be employed in the model with a relative degree of success. Those who defend the merits of CAPM cite that while their model may not always properly explain the economy-wide market for all assets, they still do so better than APT. More significantly for the practising investor, proponents of CAPM argue that such market proxies are of great use, since not only do they reflect the popularly held indices, but since these proxies are typically highly correlated with other large portfolios, they have a great use and appeal for a large number of investors, who, in essence, represent the market. Another interpretation of this argument is that since neither model works in *truly* explaining economy-wide asset pricing equilibrium, the proper conclusion should be that the better model is that which has more practical use for the typical investor, namely the CAPM.

As a final area of criticism of the APT, the issue of errors in pricing assets should be considered. This area of critique falls much along the lines of the aforementioned arguments that accept the testability of APT, but support CAPM as a preferable model. The issue of pricing errors has received relatively little attention in APT critiques, and most of the work has focused on questioning the testability of the model and its ability to identify the relevant factors or supporting CAPM as a preferred model despite the existence of a testable APT. By definition, APT requires that the resulting pricing errors be small if the model is to hold. The distribution and significance of pricing errors had initially been ignored- the pricing relationship has been treated as a strict equality. However, tests of factor analysis have shown that these pricing errors tended to be very large, indicating that even if it is testable, this serious weakness in the APT's practical results for decision-makers puts to question its usability in an applied environment.[14] While building pricing errors into the APT pricing model may to some extent counteract this issue, the underlying problem of large errors in the model's pricing results puts to question the practical use of APT as compared to CAPM.

While this brief review of the main criticisms of APT is by no means exhaustive, it serves to highlight the key areas of concern raised by both academics and practitioners regarding the testability and usefulness of the model. Many improvements and dramatic developments have been made as a reaction to such critiques. For the remainder of this study, the arguments in support of the APT and improvements over the original model will be reviewed. Such a review will help the reader to reach some tentative conclusions as to whether or not the APT is or promises to be a viable and usable alternative to the CAPM.

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[14] Robin, A. and Shukla, R. ,"The Magnitude of Pricing Errors in the Arbitrage Pricing Theory", The Journal of Financial Research, Vol. XIV, No. 1, Spring 1992, p.81.

New Developments in Support of the APT

The arguments in support of APT are similar in approach to those who favour CAPM. There are those that defend the testability of APT, others that refer to actual applied work that display the predictive superiority of APT, and finally there are those that stress the greater weaknesses of CAPM- even if APT is not accurate in "describing reality". The final type of argument will not be explicitly dealt with here, but there are large amounts of literature that criticize the CAPM and support APT in light of this, despite the newer model's drawbacks. The first two approaches to defending APT are very useful, in that they help clarify the theoretical and empirical grounds on which the theory is based, as well as offer large amount of practical applied examples which seem to support the superiority of the APT as a usable model.

The criticisms of CAPM supporters of the APT's inherent weaknesses most often centre on its inability to be tested and more fundamentally, on the inability to determine accurately the factors that generate returns on any assets in a market. Supporters of APT have defended their theory on a number of grounds, the strongest and most effective of which has been the clarification of the relationship between APT theory and its empirical tests. This argumentation centres on the notion that APT is indeed testable on subsets (smaller samples of a larger "universe" of assets). The notion of testability on subsets is intuitively analogous to the CAPM argument that indices or other smaller portfolios are good proxies for the true market portfolio. APT proponents argue that their model is indeed testable on subsets, while that of CAPM is not. The arguments that the CAPM market proxies are ineffective substitutes is well known; however, the APT testability on subsets needs clarification.

The notion of testability of subsets for APT is characterised by the fact that *any* subset, drawn at random from the original population of assets, can be tested, since by assumption the factor structure applies to any sub-grouping of the universe of assets. In order for the APT factor structure to make sense, it is necessary for each factor to be present in a large number of assets, so that it would not be mistaken for noise in that particular asset's return generating process. In a universe with a large number of assets, this essentially leads to the ability to approximately duplicate the factors with linear combinations of the assets. If this condition holds, then APT is testable on subsets, and in fact it is testable on any subset drawn randomly from the universe of assets.[15]

This argumentation supports the factor analysis approach in its ability to extract factor structure from asset returns. It also underlies the notion that expected returns are indeed linear in factor sensitivities, counter to criticisms, since this is a direct consequence of the absence of arbitrage; portfolios with identical factor sensitivities are perfect substitutes. In addition, if the factors were to be a priori determined from exogenous economic data, the conclusions are even stronger- APT is always testable

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[15] Dybvig, P. and Ross, S.,"Yes, the Apt is Testable", The Journal of Finance, Vol. XL, No. 4, Sept. 1985, p.1182.

on subsets without exception, and even pooling assets will still yield an unbiased test using this method. Furthermore and finally, even where testability on subsets is biased, the bias is always towards rejection, so there is no danger of "spurious acceptance" of the APT.[16] The conclusion to this argument is that APT is indeed testable, and holds.

The remaining arguments in favour of the APT centre on the notion of using macroeconomic factors instead of the more empirically susceptible factor analysis approach. Two particular studies have been isolated to display the superiority of the macro factors approach, either when compared with the factor analysis method, or in direct comparison with the CAPM. If the macro factors approach to using APT proves to be superior, then it can be argued that those who criticize APT based on the factor analysis approach are criticising the statistical implementation of the model, and not the validity of the model itself. In such a case, the Arbitrage Pricing Theory would indeed be a useable alternative to the market model.

Much of the recent literature on empirical tests in the APT have strived to show the benefits of using macrovariables rather than derived factors as a methodology. One of the most successful in this regard has been Chen and Jordan's study, "Some

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[16] Dybvig, P. and Ross, S., "Yes, the APT is Testable", The Journal of Finance, Vol. XL, No. 4, Sept. 1985, p.1183.

empirical tests in the Arbitrage Pricing Theory: Macrovariable vs. derived factors".[17] The study investigates the ability of the two models to predict portfolio returns over the period 1971 to 1986. Two significant advantages of the macro factors approach are highlighted. The first is that it is not subject to the statistical problems of the factor analysis approach. The second is that, as a result of specifying the factors as macrovariables, research using this method "may provide a more direct link between corporate strategic policies (e.g. capital structure and dividend policy) and changing economic events"[18] Their approach lies on the implicit assumption that interpreting factors in asset markets as a set of macroeconomic variables can fit returns data as well, if not better than, some sample specific variables produced by factor analysis.

The study uses equally weighted portfolios of firms based on three digit Standard Industrial Classification (SIC) codes, this is done in order to focus on major inter-industry differences in risk and return. The monthly returns on 69 such industry portfolios are then used to derive the factors. The significant factors (or factor loadings, as the literature refers to them) were derived, using the standard "maximum likelihood factor analysis". Following the approach of previous research, five factors

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[17] Chen, Su-Jane and Jordan, Bradford, "Some Empirical tests in the Arbitrage Pricing Theory: Macrovariables vs. derived factors", Journal of Banking and Finance, Vol. 17, 1993, pp. 65-89.

[18] Ibid, pg. 65.

were pre-specified as the relevant number.[19] The macroeconomic variables are chosen on the basis of theory and intuition. Chen uses the rationale that the value of a financial asset is equal to the sum of its discounted expected future cash flows. "Any economic phenomenon that systematically influences either the expected cash flows or the discount factor will have an effect on prices and observed returns."[20] On this basis, Chen selects *a priori* the following five variables:

1. the unexpected change in the term structure,
2. the unexpected change in risk premiums,
3. the change in expected inflation,
4. the unexpected inflation rate,
5. the unexpected change in the growth rate in industrial production.

Chen also adds another two variables:

6. a market index return and;
7. the unexpected change in oil prices.

The market index is used to see if the study supports previous APT findings that "even though a stock market explains a significant portion of time variability of stock returns, it has an insignificant influence on pricing (on expected returns) when

[19] As has been the case since its inception, there is great debate as to the number of factors relevant in factor analysis. There is yet to be any significant success in solving this problem, so most researchers resort to convention when deciding on the number to be used. A good foundation for the factor analysis approach can be had by referencing Fama and MacBeth ,"Risk, return and equilibrium: Empirical Tests",Journal of Political Economy, May, 1973, pp.607-636.

[20] Chen and Jordan, pg. 69.

compared against the economic state variables."[21] The "oil risk" is also used to test its overall lack of effect on asset pricing. The study uses the following data series:

1. inflation,
2. Treasury-bill rate,
3. Long-term government bonds,
4. Industrial production,
5. Oil prices,
6. Yield on new industrials rated Aaa,
7. Yield on new industrials rated Baa,
8. Stock market index.

While correlation between some of the variables existed as expected (such as that between the market index and some of the other variables), no two were perfectly correlated, and none of the variables could readily have been replaced with another. The expectations series were formed using a first-order autoregressive model.

Since both approaches are linear factor models, they differ only in the nature and number of factors, and thus the "two-pass regression approach" was used to fit

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[21] Ibid, pg. 70.

the models.[22] The tests used show that the factor model is only marginally statistically superior to the macro factor approach, and when "out-of-sample" assets are used, the macro model actually proves superior in explaining cross-sectional variation in monthly returns. The macro model also proves superior in a test of the two models' ability to predict *ex ante* security returns over a test period.

The key conclusion to this study is that nothing is significantly lost by using the predetermined macro factors approach rather than conventional factor analysis. Additionally, the macro factor model may turn out to be a better model when the two are tested against a "holdout" sample or a test period. The Chen and Jordan study proves to be very promising for APT, since the macro approach has economically interpretable factors, while the factors approach never specifies what the factors might be. Additionally, the authors suggest that the macro factor model's performance may even be understated, since the study made no explicit attempt to determine the best set of macro variables, or even how to best measure the ones selected.

The next case to be considered is that of electric utilities and using APT to develop cost of capital estimates. This example serves two purposes: it supports the APT as a superior model over CAPM, and it shows the advantage of using

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[22] The first pass is a time-series fit, to estimate the sensitivity of the industry portfolio returns to the factors. These can be interpreted as factor "betas", using CAPM terminology. The second pass is a cross-sectional fit, to estimate the sensitivity of the excess industry portfolio returns (actual return less the risk free rate) to the factor "betas" estimated from the first pass.

predetermined macroeconomic factors over the traditional statistical factors (factor analysis) method of testing.[23] Since estimates of cost of capital are used as measures of a regulated utility's allowable rate of return, it has been a popular testing ground for alternative CAPM methods. The following article, "The Arbitrage Pricing Theory and the Cost of Capital Estimation: The Case of Electric Utilities", shows the APT application's superior estimating ability, specifically with the macroeconomic factors method.

The electric utility sample uses 31 continuously traded New York Stock Exchange (NYSE) firms from 1972 to 1982. The firms' equity size (number of shares multiplied by share price), average annual security returns, and the standard deviation of monthly security returns are used. The utility portfolio is the weighted average of each of the 31 firms.

The authors use the standard CAPM approach to estimating cost of capital, as well as two APT methodologies. The first is the standard statistical factors approach, where factors are extracted using factor analysis, while the second uses predetermined macroeconomic variables in lieu of the statistical factors. The paper studies the impact of these different methodologies, by comparing the cost of capital estimates obtained by using both 5 and 10 statistical factors, as well as macroeconomic factors. Three sets of sample firms are used to provide the statistical

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[23] Goldenberg, D. and Robin, A.,"The Arbitrage Pricing Theory and Cost of Capital Estimation: The Case of Electric Utilities", The Journal of Financial Research, Vol. XIV, No. 3, Fall 1991, p. 181.

factors for the former, and for the latter simply for the calculation of factor risk premiums. The study shows that the different methodological choices within the statistical factors approach (in terms of number of factors as well as factor providing firms) makes for a more drawn-out and complex process, leading to significantly different estimates. Compared with the CAPM model, these estimates can be higher or lower, depending on both the number of factors employed (5 or 10), and the sample firms chosen to extract the factors. This method is thus highly sample-specific, and leads to the conclusion that the traditional statistical factors approach leads to a lack of robustness of the APT with respect to the choice of factor providing firms.[24]

The aforementioned lack of robustness does not question the validity of the APT, but it only raises doubts concerning the statistical method of implementing the APT. To demonstrate this point, the authors used the macroeconomic factors APT approach. The macro factors approach relies on the firms sample only for factor risk premiums, but otherwise the variables are pre-specified. The authors used the following four macroeconomic factors:

1. a measure of the <u>bond default premium</u> (difference in monthly returns on twenty year corporate and government bond portfolios)
2. a measure of <u>bond maturity premiums</u> (difference in monthly returns on a twenty year government bond portfolio and monthly returns on T-Bills)

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[24] <u>Ibid</u>. p.184.

3. an <u>unexpected inflation series</u> (difference between expected and actual inflation rate, using Kalman Filtering methods for the former, and the deflator for non-durable consumption goods for the latter)

4. a measure of the <u>unexpected growth in sales</u> (difference between current and future expectations of the growth rate in sales; assuming expectations are formed using an autoregressive model with lagged values for growth rate and real disposable income).

The following table shows their results for cost of capital estimates, using the primary sample of firms. Alternate firm samples differ in degree; however the comparative results between the three approaches are similar.

<u>Utility Cost of Capital</u>

Portfolio	Market (CAPM)	APT 10 Factors	APT 5 Factors	APT Macro Factors	Actual
k_e	0.1246	0.1131	0.0866	0.1626	13.72*
β	0.7621				
R^2	0.4939	0.7107	0.7073	0.5537	

k_e = cost of capital estimate ; R^2 = adjusted R^2 of the time-series regressions.

* average yield for Aaa public utility bonds, 1980-82.

For the statistical factors method, the cost of capital estimate, k_e, was determined as follows:

$$k_e = E(R_j) = E(R_z) + \sum_{n=1}^{N} b_{jn}\lambda_n$$

A time series of the risk premia (λ's) are extracted, from an independent set of factor providing firms. A time series regression is performed on

$$R_j^E = b_o + \sum_{n=1}^{N} b_{jn}\lambda_n$$

where R^E_j = utility returns over the estimating period. The average λ's over the forecasting period are then used (λ_n). Utility returns for the first year after the forecasting period are forecast by:

$$E(R) = b_o + \sum_{n=1}^{N} b_{jn}\lambda_n$$

For the macro factors, the procedure is similar, but we substitute factor analysis derived λ's with predetermined macro factors. The CAPM method involves the usual procedure of estimating the k_e using the utility portfolio's beta and the R_f. The results of the test seem to support the macro factors approach, which explained more in-sample variation than the CAPM approach.

Although the APT Factors approach has relatively high R^2 values in comparison to the Macro Factors approach or CAPM, two subsequent tests using different sets of sample firms reveal considerable variability in the results. Using two other sample sets, cost of capital estimates 0.1024 and 0.1535 for 10 Factors, and 0.0947 and 0.1156 for 5 Factors, compared with 0.1131 and 0.0866 respectively in the above table. When all three methods were compared to realized cost of capital figures for the period in question, the Macro Factors approach was most accurate.

The results of a comparison of the macro factors APT estimates with those of CAPM and statistical factors APT weigh heavily in favour of the macro factors APT. While the authors concede that the macro factors cost of capital estimate is not the most accurate in the initial testing, they stress the statistical strengths of the macro factors approach. This alternative approach was shown to explain more in-sample return variation than the market model and avoids the potential problems associated with the traditional factor analysis method.

In summary, this important article not only strongly supports the general validity of the APT, but also introduces macro factors APT to cost of capital estimating techniques. It has very significant implications for the future use of statistical factors APT in this area, and makes evident the importance for further study into the macro factors approach, which evidently has some considerable advantages over other asset pricing methods.

Another area of support for the Arbitrage Pricing Theory lies in the international capital asset environment. The development of an *International* Arbitrage Pricing Theory (IAPT) by Bruno Solnik has led to increased support for the model.[25] The CAPM, Solnik argues, requires the identification of all the securities in the world in order to construct the world market portfolio and derive the theoretical pricing relationship. Moreover, since effective indices or other proxies for the international market portfolio are even harder to justify than their domestic counterparts, the CAPM becomes even less valid in an international context. Solnik suggests an international APT model, arguing that, since it can, by definition, be applied to a subset of investments, it doesn't need to consider every single world asset. Using a standard linear factor model, Solnik develops a very simple four factor model. A description of the proposed factors would be as follows: 1) *World Economic Activity*, affecting all firms but to different degrees. This helps explain, for example, why stock prices in the world tend to be positively correlated, 2) *Domestic Real Growth* relative to world real growth would be a country-specific factor, 3) *Currency Movements*, depending on its import-export structure and the currency denomination of its financing, a firm would have a positive or negative "beta" with respect to this particular factor, and finally 4) *Industry-wide Conditions* would be factors that are industry-specific and independent of national boundaries.[26] Solnik gives an intuitive example of how the IAPT approach would work: "supposing a manager believed in the following economic

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[25] Solnik, Bruno, International Investments (Toronto: Addison-Wesley Publishing Company, 1991), pg.144.

[26] Ibid, pg. 145.

scenarios: worldwide slowdown in real growth and inflation, and better relative economic performance of Japan. He or she will then invest in companies that are less sensitive to changes in real growth, most favourably influenced by the reduction in inflation, and directly affected by the better performance of the Japanese economy."[27]

Before any conclusions can be made as to whether or not the APT is a superior and usable alternative to CAPM, consideration needs to be given to the possibility of a theoretical compromise between the two models, or what William Sharpe referred to as a "Synthesis" of the two models- essentially incorporating the attractive features of both.[28] Since the APT and CAPM are not inconsistent, Sharpe considers a world in which 1) returns are generated by a factor model, 2) the remaining assumptions of the APT hold, and 3) the assumptions of the CAPM hold. Can significant conclusions be reached by making assumptions about both the return-generating process and investor preferences? Sharpe develops a relationship between the "beta" of a security and its sensitivities to the factors:

$$\beta_i = b_{j1}\beta_{F1} + b_{j2}\beta_{F2} + \ldots + b_{jM}\beta_{FM}$$

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[27] Solnik, Bruno, International Investments, (Toronto: Addison-Wesley Publishing Company, 1991), pg. 146.

[28] Sharpe, William F., "Factor Models, CAPMs, and the APT", The Journal of Portfolio Management, Vol. 11, Number 1, Fall 1984 pp.21-25.

The new β's are the beta values of the *factors* to changes in the return on the market portfolio (i.e. β_{F1} is the change in factor 1 associated with a change in the market, and so on). If the assumptions of the CAPM hold, expected returns must conform to the standard market risk premium equation. Similarly, if the assumptions of the APT hold, then if returns are generated by an M factor model, the beta values must conform to the M factor APT equation. Sharpe combines these two results:

$$E_j = R_f + (E_m - R_f) * (b_{j1}\beta_{F1} + \ldots + b_{jM}\beta_{FM})$$

thus, $E_j = R_f + b_{j1}((E_m - R_f)\beta_{F1}) + b_{j1}((E_M - R_f)\beta_{F2}) + \ldots + b_{jM}((E_m - R_f)\beta_{FM})$

Comparing this to the equilibrium conditions of the APT, it follows that

$$\begin{aligned}
\lambda_0 &= R_f \\
\lambda_1 &= (E_m - R_f)\beta_{F1} \\
\lambda_2 &= (E_m - R_f)\beta_{F2} \\
&\vdots \\
\lambda_M &= (E_m - R_f)\beta_{FM}
\end{aligned}$$

Since in this synthesis, assumptions were also made about investor preferences, the factor-expected return premiums can be interpreted slightly differently: for example, factors that move with the market will provide higher expected return premiums, factors that move against (provide hedges for) the market will provide negative expected return premiums, etc. By including additional sources of investor utility, Sharpe argues that both CAPM and APT can be extended in a combined framework.

Governed by the belief that *neither* model is truly testable (CAPM models are clear about the major item to be measured, but it is unmeasurable, APT deals with items that might easily be measured, but cannot define what these items are), Sharpe argues for the combined use of *both* approaches: "It is undoubtedly true that relatively few key factors account for the most important aspects of portfolio risk and return. It is also undoubtedly true that most investors like high expected returns and dislike risk. It is undoubtedly true that the average investor prefers liquid assets to illiquid ones."[29] Sharpe considers both models to serve as a valuable framework within which to approach investment decisions methodically, and refutes the notion that *either* model should be hastily abandoned. Sharpe likens the present debate over the two models with an analogy to the historical argument over the downward sloping demand curve: "Since Alfred Marshall, economists have approached the world secure in the belief that demand curves are downward-sloping. They continue to do so, even though one can never definitively establish that the theorem holds for a specific good, let alone identify the precise location of a relevant curve on a piece of graph paper."[30]

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[29] Ibid, pg. 24.

[30] Ibid, pg. 24.

The notion of combining the two models to some extent is indeed an appealing one. The Sharpe synthesis, however, still relies on the existence of a usable and effective proxy for the market portfolio. If this existence is refuted, then so too should the synthesis of the two models- which again brings to question the superiority, or lack of it, of the Arbitrage Pricing Theory. Given a general review of the major arguments in support of or against the model, some tentative conclusions can now be reached.

Conclusion

The attempt to reach conclusions regarding the superiority of the Arbitrage Pricing Theory is certainly a premature one, and thus tentative on what further research work and practical applications reveal of the model's robustness and usability. The APT was introduced as an equilibrium model in 1976, and only over the course of the last ten years has attention to it intensified, as compared to the nearly three decades of work done on the CAPMs. However, as the above study has shown, support of the model is becoming more and more widespread, and there is an increasing amount of literature being produced almost on a monthly basis about the merits and applicability of the APT. This interest has not been unique to academia: many large Wall Street firms have developed customized APT models to serve their asset pricing requirements. Given the practitioner's attention and the depth of study done on the model, some form of tentative conclusion can be reached.

In the context of increasingly complex and global financial markets, any faith in a model that describes the equilibrium expected returns of an asset using a *proxy* for the true global asset market seems unqualified. On this basis alone, APT seems to be a more realistic alternative. Given this, however, the issue of methodology is again raised regarding the APT's use of the highly susceptible statistical factors method. Even if the equilibrium assumptions about the model and its various factors hold true, there is much debate over how these factors are, in essence, *created* by the nature of the factor analysis approach. As the criticisms have shown, *any* combination or manipulation of factors can be shown to be valid. Furthermore, even if there are a given number of factors, or *risk premia*, that hold true for all assets, the statistical factors approach indeed says nothing about what these might be, but only that they exist. This however, does <u>not</u> prove that the APT is an inferior model, but only that the method by which it is tested and used is susceptible to errors and a general lack of robustness. In recognition of this, the APT can still be defended as a usable alternative to CAPM through the increasingly popular macroeconomic factors method.

The macro factors method not only makes the APT a more usable model, but also highlights the many attractive features of the APT approach. By predetermining, albeit rather informally, the important risk premia that determine an asset's expected returns, the macro factors APT not only avoids the empirical problems of the statistical factors method, but has also been proven superior to both the original APT approach and, more importantly, to the CAPM in explaining variation in security returns. Once determined, the factors are relatively easily measurable, rendering the task of determining an equilibrium expected return rather mechanical. This approach is gaining greater popularity and acceptance in both academic and practitioner circles, promising further refinements to the methodology. While predetermining the relevant factors may *seem* to be cause for concern, the approach is validated by the fact that the factors are chosen on the basis of sound economic and financial principles: the value of *any* financial asset is based on the present value of all its future cash flows. Choosing the relevant factors for asset pricing in equilibrium depends simply on which factors are key to influencing this discounted cash flow. Whatever the key factors are that influence cash flow and the discount rate are those that apply to the APT model. As mentioned in the study, most applications centre on four or five relevant factors, such as the term structure of interest rates, unexpected inflation, etc. The validity of the macro factors cannot realistically be disputed; what can be questioned is the data series or source being used- this becomes a question of measurement and not approach.

In conclusion, if a *true* market portfolio could be measured, the need for the macro factors APT would be eliminated- the market portfolio would reflect all of the relevant information in the macro factors, giving the same results with far less computation, as only one "beta" would be needed. Since it is widely agreed that this is nearly impossible, the APT appears to be an effective alternative to CAPM, and deserves even further study and development so that predetermined factors can be as precisely measured and effective as possible.

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