Technological change for a two-level CES production function

An empirical study for Canada

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Abstract

This paper estimates the parameters of two-level CES production functions with capital, labour and energy as inputs for Canada. Using the industry-level data from ten Canadian industries, we find that the nesting structure where labour and energy are combined first, then this labour-energy composite is combined with capital, fits the Canadian data best, and for all industries we can reject that all three inputs can be put into one single nest, thus proved the effectiveness of the nesting structure of the technology. We also find evidence, though limited, of Leontief technology at the first level of nesting (the top level), and of Cobb-Douglas technology at the second level of nesting (the low level), contrary to the somewhat commonly agreed observation of limited empirical support for Cobb-Douglas technology, a.k.a. unitary elasticities of substitution. Still, for the majority of the industries in our study, our findings are in favour of elasticity values considerably different from one. Finally, we find evidence for TFP growth for the majority of industries, and evidence for labour-saving and capital- and energy-saving technology for the other industries.

Keywords

Two-level CES function; elasticities of substitution; production structures; technological change

1 Introduction

Substitution between factors and factor-augmenting technology progress are critical parameters in many areas of economics; they have been linked to many important economic phenomena, such as differences in domestic or international factor returns and convergence (e.g., Klump and Preissler, 2000), change in income distribution (Blanchard, 1997), trade and development patterns (Jones, 1965; Duffy and Papageorgiou, 2000), economic growth (Klump and de La Grandville, 2000), and global Greenhouse Gas (GHG) mitigation policies (Beckman and Hertel, 2009). Given this, though, limited consensus has emerged on the sign or value of the substitution elasticities between factors of interest, or the nature of technological progress. Many reasons could give rise to this situation. First, most of the previous studies on estimating the elasticity of substitution and biased technical progress have only focused on two classical factors, i.e., capital and labour, and it was not until recently that energy and non-energy intermediate inputs were included in the estimation, as in an increasing number of studies as early as Hudson and Jorgenson (1974) and Berndt and Wood (1975), and as recent as Ma et al. (2008), Dissou et al. (2010), and Smyth et al. (2011). The inclusion of energy and non-energy intermediate inputs reflects people's concern on sustainability, namely the relationship between economic growth, scarcity of resource, and capacity of environment, but it may change previous estimation of elasticity of substitution between the classical factors as well.

Functional forms also matter. Several functional forms have been proposed and utilized to estimate these critical substitutions. The functional forms on this list include Cobb-Douglas function, constant elasticity of substitution (CES) function, Trans-log function and other flexible functional forms. Cobb-Douglas function, the most famous and least complicated functional form, has been most widely applied in empirical studies; however, its dominance in modeling aggregated production has been challenged. Some recent studies revealed its failure to support empirical analysis of biased technical change because much empirical data do not exhibit properties that are compatible with the standard neo-classical growth model with Cobb-Douglas production function (Klump et al., 2007b), and others argue that empirical estimation in favour of Cobb-Douglas function might be a bias from restrictive a priori assumptions on the nature of technology progress (Antràs, 2004). In addition, the trans-log function (Christensen et al., 1971, 1973), one of the flexible form functions, an even more popular tool for estimating the elasticities of substitutions among three or more inputs and technical progress, has its deficiencies as well. For example, some studies based on Monte Carlo experiments, such as Guilkey and Lovell (1980), Guilkey et al. (1983), and Despotakis (1986), found that trans-log-function-based estimations are not reliable when the real value of elasticity of substitution is not close to unity (Prywes, 1986).

In comparison, the constant elasticity of substitution (CES) production function, proposed by Arrow et al. (1961) and generalized and extended by Uzawa (1962), McFadden (1963), Mukerji (1963), and Sato (1967), is a powerful tool for analyzing substitution between factors and factor-augmenting technology progress, and has been adopted by many researchers in their estimation of elasticity of substitution between value added, energy and non-energy inputs (Prywes, 1986; Chang, 1994; Khan, 1994; Kemfert, 1998; Kemfert and Heinz, 2000; Kuper and van Soest, 2003; Markandya and Suzette, 2007) because it more closely mimics the substitution possibilities among factors (Beckman and Hertel, 2009) and formalizes a correspondence between these substitution possibilities and growth (de La Grandville, 1989; Klump and de La Grandville, 2000; de La Grandville, 2009).

The methods for estimating the CES function generally fall within two categories: parameters are estimated either in single equations (mostly linear equations) or in a multi-equation system, which involves usually two to three equations (León-Ledesma et al., 2010). The linear single equations are obtained from the first order conditions (FOCs) of a profit-maximizing or cost-minimizing problem, or by using the Kmenta approximation (Kmenta, 1967), which is a special kind of trans-log function (Christensen et al., 1973). Most of the above-mentioned studies, which adopt CES functions to estimate energy-related substitution possibilities, have been single-equation-based. Still, single equations can be nonlinear, as in several studies such as Kemfert (1998), Kemfert and Heinz (2000), and Markandya and Suzette (2007), that directly estimated the nonlinear CES production function for the parameter of interest. However, single equation estimates could be systematically biased, since they are based on factor demand functions, and factor inputs demand is dependent on relative factor prices which again depend on the relative factor inputs actually used (David and van de Klundert, 1965; Willman, 2002).

The system approach estimates the key parameters of the CES function by estimating a set of simultaneous equations, which usually include both the linear equations of input demand derived from the FOCs (two-equation system) or both those input demand equations plus the CES function itself (three-equation system). Many studies, such as Marschak and Andrews (1944), Bodkin and Klein (1967) and Berthold, Fehn and Thode (2002), adopt two-equation systems to alleviate such a systematic simultaneous equation bias as in single equation estimates by estimating demand functions for both input factors.

In addition, the production structure is critical for estimating elasticities of substitution possibilities. Many of the studies on estimating elasticities of substitution possibilities between factors carry a priori assumption on production structure (e.g., Bosetti et al., 2006; Kemfert, 2002; Sue Wing, 2003; etc). The choice of the production structure may affect the value and the significance of the estimated elasticities between factors in the same pair. Moreover, the way in which technological change enters the production function differs as well. In light of this, some authors set out to validate popular CES-function-based production structure and technological nature by comparing the significance of estimated parameters in each of them. For example, van der Werf (2008) compared between three popular nesting structures using industrial data for seven industries of twelve OECD countries from 1978-1996. His findings are in favour of the *(KL)E* nesting structure, non-unitary elasticities and factor-specific technology trends.

This study attempts to offer the following improvements to earlier studies. First, it builds a three equation system for estimating capital-labour-energy substitution elasticities with biased technical progress. This should be able to better capture production and technical parameters than two equations approach. Second, it makes no a priori assumption on the production structure; rather, within the two-level CES production function framework proposed by Sato (1967), it compares three popular production structures, which are the *(KL)E* structure, the *K(LE)* structure and the *(KE)L* structure.[1] Our findings for Canadian industries support the *K(LE)* nesting structure, favour TFP growth for most of the industries under study and find evidence for the presence of Leontief and Cobb-Douglas technology, the former at the top level and the latter at the low level, for certain industries.

The paper proceeds as follows. Section 2 reviews some relevant technical concepts of the CES function with technical change, two-level CES production function, and briefly appraises existing CES-relevant empirical studies and their apparent lack of robustness. The subsequent section describes the model. Section 4 describes the data and presents the results. Section 5 offers corresponding discussions. Lastly, section 6 provides conclusive remarks.

2 Review: CES function, one- and two-level

2.1 One-level CES function

The traditional form of CES function is developed by Arrow et al. (1961) as: $Y = A\left[\alpha K^{-\rho} + (1-\alpha) L^{-\rho}\right]^{-1/\rho}$, where *Y*, *K* and *L* representing gross output, capital and labour respectively, *A* being the efficiency parameter representing technology level, $\alpha (0 < \alpha < 1)$ being the distribution coefficient representing the capital income share, and $\rho = (1-\sigma)/\sigma$ being the substitution parameter (σ being the elasticity of substitution). The CES production function is a linear homogeneous function that encompasses both Cobb-Douglas form ($\sigma = 1$) and Leontief form ($\sigma = 0$). This specification, which implies neutral technical progress and indicates equal levels of efficiency of capital and labour or constant ratio of marginal output of capital to that of labour, can be combined with the specification suggested by David and van de Klundert (1965). The specification of David and van de Klundert (1965): $Y = \left[(A_K K)^{-\rho} + (A_L L)^{-\rho}\right]^{-1/\rho}$, reflects the biased technical progress by adding to the form A_K and A_L, levels of efficiency of capital and

[1] A *(KL)E* structure means that capital and labour are combined first using CES technology, and this composite is later combined with energy also using CES technology. The definitions for a *K(LE)* structure and a *(KE)L* structure are likewise.

labour respectively. Thus, the combined form: $Y=\theta\left[\alpha\left(A_K K\right)^{-\rho}+\left(1-\alpha\right)\left(A_L L\right)^{-\rho}\right]^{-1/\rho}$, where θ is an efficiency coefficient (León-Ledesma et al., 2010), is able to capture different types of technical change, i.e., neutral, capital- and labour-augmenting.

Table 1: Empirical studies of aggregate elasticity of substitution and technological change in the US using one-level CES function

				Estimated annual rate of efficiency change		
Study	Sample	Assumption on technological change	Estimated elasticity of substitution $\hat{\sigma}$	Hicks neutral: $\hat{\gamma}_L=\hat{\gamma}_K$	Labor augmenting: $\hat{\gamma}_L$	Capital augmenting: $\hat{\gamma}_K$
Arrow et al. (1961)[a]	1909-1949	Hicks neutral	0.57	1.8	-	-
Kendrick and Sato (1963)[b]	1919-1960	Hicks neutral	0.58	2.1	-	-
Brown and De Cani (1963)[a]	1890-1918	Factor augmenting	0.35	Labor saving ($\gamma_L-\gamma_K=0.48$)		
	1919-1937		0.08	Labor saving ($\gamma_L-\gamma_K=0.62$)		
	1938-1958		0.11	Labor saving ($\gamma_L-\gamma_K=0.36$)		
	1890-1958		0.44	?		
David and van de Klundert (1965)[a]	1899-1960	Factor augmenting	0.32	-	2.2	1.5
Bodkin and Klein (1967)[c]	1909-1949	Hicks neutral	0.5-0.7	1.4-1.5		
Wilkinson (1968)[d]	1899-1953	Factor augmenting	0.5	Labor saving ($\gamma_L-\gamma_K=0.51$)		
Sato (1970)[a]	1909-1960	Factor augmenting	0.5-0.7	-	2.0	1.0
Panik (1976)[a]	1929-1966	Factor augmenting	0.76	Labor saving ($\gamma_L-\gamma_K=0.27$)		
Berndt (1976)[e]	1929-1968	Hicks neutral	0.96-1.25	?	-	-
Kalt (1978)[e]	1929-1967	Factor augmenting	0.76	-	2.2	0.01
Antras (2004)[e]	1948-1998	Hicks neutral	0.94-1.02	1.14	-	-
		Factor augmenting	0.80	Labor saving ($\gamma_L-\gamma_K=3.15$)		
Klump et al. (2007a)[f]	1953-1998	Factor augmenting	0.51	-	1.7	0.4

Notes: a=linear single equation; b=implicit; c=linear and nonlinear single equation; d=linear system; e=linear single equation and linear system; f=nonlinear system (Klump et al. 2007a report both constant and time-varying factor-augmenting growth cases; the results reported in this table are from the former case.) This table is updated from Kalt (1978) and Klump et al. (2007b). A full source of references to the above studies may be found therein.

Despite the centrality of the substitution elasticity and technical biases in many areas of economics, and the huge efforts devoted to their identification, there seems to be little empirical consensus on their values and nature. Moreover, despite its pervasive use, we observe at the sectoral level limited support for Cobb-Douglas and for above-unitary substitution elasticities in general. Referring to Kalt (1978) and Klump et al. (2007b), Table 1 summarizes some well-known empirical studies on estimating elasticities using one-level CES functions for the United States.

In Table 1, we observe a variety in the augmented forms and in the elasticity values. This is perhaps partially due to the variance in data quality and lack of data consistency arising from problems endemic to production function estimation, e.g., the correct measurement of the user cost and capital income, the possible use of quality-adjusted measures for factor inputs, neglect of capital depreciation and the aggregate mark-up, the treatment of indirect taxes, assumptions about self-employed labour income, measurement of capacity utilization rates, etc (León-Ledesma et al., 2010). Other reasons that may explain the divergence in the estimated parameters may stem from the variation of choices of estimating equation, and the estimation method, such as the application of OLS to the FOCs or linearized variants of the production function, nonlinear methods to the CES function itself, instrumental variable (IV) or full-information approaches to the system, etc.

3.2 Two-level CES function

After the seminal work of Arrow et al. (1961), several forms of the CES function, which are capable of depicting the relationship among more than two inputs, occurred, thanks to the creative work of Uzawa (1962), McFadden (1963), Mukerji (1963) and Sato (1967). Among those forms, the two-level CES production function developed by Sato (1967) not only has a close relationship with other forms, but also has the advantages of intuitive economic meaning, simple and flexible form, thus convenient for empirical analysis, and has been widely applied in exploring the relationship between energy and non-energy inputs, despite its lack of emphasis on biased technical change, which, when included, could give us a generalized two-level capital-labour-energy CES production function as

$$Z = \theta_Z \left[\alpha_1 \left(A_1 X_1 \right)^{\frac{\sigma_1 - 1}{\sigma_1}} + \left(1 - \alpha_1 \right) \left(A_2 X_2 \right)^{\frac{\sigma_1 - 1}{\sigma_1}} \right]^{\frac{\sigma_1}{\sigma_1 - 1}} \tag{1}$$

$$Y=\theta_Y\left[\alpha_2\left(A_Z Z\right)^{\frac{\sigma_2-1}{\sigma_2}}+\left(1-\alpha_2\right)\left(A_3 X_3\right)^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}} \tag{2}$$

Eq. (1) is the first level function (going from the bottom) and Eq. (2) is the second level function, where Z can be regarded as the "intermediate output" (Chang, 1994) of inputs at the first level, X_i represents input i, θ_Z and θ_Y are the efficiency parameters for the first and second levels respectively, A_i ($i=1,2,3,Z$) represents the levels of efficiency of input i, α_1, α_2, σ_1 and σ_2 are the distribution parameters and elasticities of substitution for the first level and second level, respectively.[2] Input X_i could naturally have different meanings in different forms of CES function. In the case of three inputs, it could either be capital, labour or energy and, consequently, Z could be the intermediate (or index) output, produced from either capital and labour, capital and energy, or labour and energy, hence three potential nesting structures of the aforementioned two-level CES function: *(KL)E*, *(KE)L* and *K(LE)*.

In the studies using two-level CES functions, no more consensuses have been reached than in the case of studies using one-level CES functions. Disagreements mainly originate from the choice of nesting structure, as well as assumptions on the nature of technology progress. Table 2 presents an overview of the production structures, elasticities of substitution and types of technological change of some applied studies that are based on CES function models. Looking at the elasticities of substitution in Table 2, we can see that different values for the elasticities of substitution were used even for the same nesting structure. Inconsistency therein lies largely in the general lack of empirical foundation when authors refer to other papers – that are not empirically validated themselves – for the chosen nesting structures and elasticities, and in the way that technology is modeled – that the modeller finds suited best. The sensitivity of the result of such studies to their assumptions on the production structure and their chosen values of parameters thus necessitates careful scrutiny.

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[2] The levels of output, inputs and the technology parameters are time and possibly country or industry-dependent, but we suppressed the subscripts to ease notation.

Table 2 Nesting structure and elasticities of substitution for several models

Author(s)	Nesting Structure [a]	Elasticities [b]	Techn. Change [c]
Bosetti et al. (2006)	*(KL)E*	$\sigma_{K,L}$=1; $\sigma_{KL,E}$=0.5	TFP; Energy-specific
Burniaux et al. (1992)[d]	*(KE)L*	$\sigma_{K,E}$=0 or 0.8; $\sigma_{KE,L}$=0 or 0.12 or 1	TFP
Edenhofer et al. (2005)	*KLE*	$\sigma_{K,L,E}$=0.4	Factor-specific
Gerlagh and Van der Zwaan (2003)	*(KL)E*	$\sigma_{K,L}$=1; $\sigma_{K,L,E}$=0.4	Energy-specific
Goulder and Schneider (1999)	*KLEM*	$\sigma_{K,L,E,M}$=1	TFP
Kemfert (2002)	*(KLM)E*	$\sigma_{KLM,E}$=0.5	Energy-specific
Manne et al. (1995)	*(KL)E*	$\sigma_{K,L}$=1; $\sigma_{KL,E}$=0.4	TFP
Paltsev et al. (2005)	*(KL)E*	$\sigma_{K,L}$=1; $\sigma_{KL,E}$=0.4-0.5	TFP
Popp (2004)	*KLE*	$\sigma_{K,L,E}$=1	Energy-specific
Sue Wing (2003)[e]	*(KL)(EM)*	$\sigma_{K,L}$=0.68-0.94; $\sigma_{E,M}$=0.7; $\sigma_{KL,EM}$=0.7	TFP

Notes: This table is updated from van der Werf (2008). A full source of references to the above studies may be found therein.

[a] *(KL)E* means a nesting structure in which capital and labour are combined first, and then this composite is combined with energy with a different elasticity of substitution. *KLE* means that all inputs are in a single-level CES function.

[b] $\sigma_{i,j}$ is the elasticity of substitution between inputs *i* and *j* and $\sigma_{ij,k}$ is the elasticity of substitution between the composite of inputs *i* and *j* on the one hand, and input k on the other.

[c] TFP=Total Factor Productivity growth.

[d] Lower elasticities for old capital, higher elasticities for new capital.

[e] Elasticities taken from Cruz and Goulder (1992).

3 The Model

Under the assumption of constant returns to scale, which is a normal practice in most applied studies on CES functions, the two-level three-input CES production, illustrated with the *(KL)E* structure[3], looks as follows:

$$Z=\left[\pi_1\left(A_K K\right)^{\frac{\sigma_{K,L}-1}{\sigma_{K,L}}}+\left(1-\pi_1\right)\left(A_L L\right)^{\frac{\sigma_{K,L}-1}{\sigma_{K,L}}}\right]^{\frac{\sigma_{K,L}}{\sigma_{K,L}-1}} \tag{3}$$

$$Y=\left[\pi_2\left(A_E E\right)^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}}+\left(1-\pi_2\right)\left(Z\right)^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}}\right]^{\frac{\sigma_{KL,E}}{\sigma_{KL,E}-1}} \tag{4}$$

[3] *(KE)L* and *K(LE)* structures are dealt with likewise.

When Eq. (3) is substituted into Eq. (4), we can find that if $\sigma_{KL,E} = \sigma_{K,L}$, the nested function will reduce to a one-level CES production function where all three inputs are equally easy to substitute for each other. In contrast, if these two elasticities are not equal, the production function will be a two-level nested function. Different nesting structures imply different values for the substitution elasticities. A natural investigation would then amount to testing whether $\sigma_{KL,E} = \sigma_{K,L}$.[4] Also we can test whether the production function in either level of nesting is indeed Cobb-Douglas, by testing $\sigma_{KL,E} = 1$ or $\sigma_{K,L} = 1$. Further, using the system of equations derived from cost-minimization in the remainder of the paper, we can also test for technological progress in each of the ten industries under scrutiny thereof.

With a two-level CES production function, the cost-minimization problem of a firm can be represented as a two-stage problem, starting from either the top or the bottom level of nesting.[5] In the following case of the *(KL)E* nesting structure, we first find the optimal demand for *E* and *Z* per unit of *Y* produced, given the input prices and technology, and then use the optimal demand for *Z* in the top level of nest to solve for the optimal demand for *K* and *L*.[6] Presented below, the cost-minimizing problem in the bottom level of the *(KL)E* structure is as follows:

$$\min_{E,Z} P_E E + P_Z Z, s.t.$$

$$\overline{Y} = \left(\pi_2 (A_E E)^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}} + (1-\pi_2)(Z)^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}} \right)^{\frac{\sigma_{KL,E}}{\sigma_{KL,E}-1}}$$

where $\overline{Y}$ is a given output level. From the first-order conditions we can derive the cost function. By applying Shephard's lemma to the cost function, we find the conditional

[4] This test is for *(KL)E* structure. For *(KE)L* structure and *K(LE)* structure, we test whether $\sigma_{KE,L}=\sigma_{K,E}$ and $\sigma_{LE,K}=\sigma_{L,E}$, respectively.

[5] It is important to note that with constant returns to scale, both profit-maximizing and cost-minimizing behaviour lead to the equation of marginal cost with market price. Hence, we will focus our analysis on cost-minimization in this study to solve for optimal demand for input.

[6] Price for *Z* is the dual price, or the marginal price of *Z*, obtained from cost-minimizing at the bottom nest. This price is unobservable in the data. We will apply a procedure to deal with this problem in the following text.

factor demands for each input. Under the assumption of price-taking behaviour by firms, the price of output is equal to its marginal cost, thus the conditional factor demand function of E, for example, is a function of the price of the factor, the price of output P_Y and the level of output, written as $E=E(P_E,P_Y,Y)$. We then take logarithms of this function, rearrange it to obtain the following equation for E (the equation for Z is analogous):

$$\ln\left(\frac{E}{Y}\right)=\sigma_{KL,E}\ln\pi_1+\left(\sigma_{KL,E}-1\right)\ln A_E+\sigma_{KL,E}\ln\left(\frac{P_Y}{P_E}\right) \tag{5}$$

After taking first differences of both side of Eq. (5) (i.e. for each variable X we take X(t)-X(t-1)), to get percentage changes in Eq. (5), in order to drop out the first term on the right hand side so that the technology parameter A_E can be identified, and by applying this procedure to equations for input Z and the lower nest, we get the following system of equations:[7]

$$e-y=\left(\sigma_{KL,E}-1\right)a_E+\sigma_{KL,E}\left(p_Y-p_E\right) \tag{6}$$

$$z-y=\sigma_{KL,E}\left(p_Y-p_Z\right) \tag{7}$$

$$k-z=\left(\sigma_{K,L}-1\right)a_K+\sigma_{K,L}\left(p_Z-p_K\right) \tag{8}$$

$$l-z=\left(\sigma_{K,L}-1\right)a_L+\sigma_{K,L}\left(p_Z-p_L\right) \tag{9}$$

Next, we apply the method used by van der Werf (2008) to circumvent the problem related with unobservable variables (in this case, z and p_Z). First, we add $p_K-p_Y-\left(p_Y-p_E\right)$ to both sides of Eq. (8), which gives us the growth rate of the share of capital costs in the costs of the intermediate input on the left-hand side of the following Eq. (10):

$$p_K+k-\left(p_Z+z\right)=\left(\sigma_{K,L}-1\right)a_K+\left(\sigma_{K,L}-1\right)\left(p_Z-p_Y-\left(p_K-p_Y\right)\right) \tag{10}$$

[7] $x\equiv\ln X(t)-\ln X(t-1)=d\ln X(t)$ is the discrete time approximation of $d\ln X(t)/dt$, the growth rate or percentage change of X in continuous time. The same procedure is applied to all variables.

Then, by adding $p_Z - p_Y$ to both sides of Eq. (7), and dividing both sides by $\sigma_{KL,E} - 1$, we get:

$$p_Y - p_Z = \frac{p_Z + z - (p_Y + y)}{\sigma_{KL,E} - 1}$$

which we will then substitute into the right-hand side of Eq. (10) to find

$$p_K + k - (p_Z + z) = (\sigma_{K,L} - 1)a_K + (\sigma_{K,L} - 1)\left(\frac{p_Z + z - (p_Y + y)}{1 - \sigma_{KL,E}} - (p_K - p_Y)\right) \qquad (11)$$

Having solved the problem of z and p_Z being unobservable now that $p_K + k - (p_Z + z)$ and $p_Z + z - (p_Y + y)$ are observable changes in cost-shares, we go on applying the same procedure to Eq. (9), and using a new notation, to obtain the following system of equations:

$$e - y = (\sigma_{KL,E} - 1)a_E + \sigma_{KL,E}(p_Y - p_E) \qquad (12)$$

$$\delta_{KZ} = (\sigma_{K,L} - 1)a_K + \frac{\sigma_{K,L} - 1}{1 - \sigma_{KL,E}}\delta_{ZQ} + (1 - \sigma_{K,L})(p_K - p_Y) \qquad (13)$$

$$\delta_{LZ} = (\sigma_{K,L} - 1)a_L + \frac{\sigma_{K,L} - 1}{1 - \sigma_{KL,E}}\delta_{ZQ} + (1 - \sigma_{K,L})(p_L - p_Y) \qquad (14)$$

where $\delta_{mn} \equiv p_m + m - (p_n + n)$ is the percentage change of the cost-share of input M in the costs of producing N. For the case of the *(KL)E* nesting structure, this leads to the following model to be estimated:

$$y_1 = \alpha_1 + \beta_1 x_1 + \varepsilon_1 \qquad (15)$$

$$y_2 = \alpha_2 + \beta_{21} x_{21} + \beta_{22} x_{22} + \varepsilon_2 \qquad (16)$$

$$y_3 = \alpha_3 + \beta_{31} x_{31} + \beta_{32} x_{32} + \varepsilon_3 \qquad (17)$$

where the εs are error terms and the dependent variables are: $y_1 = e - y$, $y_2 = p_K + k - (p_Z + z) = p_K + k - d\ln(P_K K + P_L L)$, and $y_2 = p_L + l - d\ln(P_K K + P_L L)$, and the independent variables are: $x_1 = p_Y - p_E$, $x_{21} = x_{31} = d\ln(P_K K + P_L L) - p_Y - y$, $x_{22} = p_K - p_Y$ and $x_{32} = p_L - p_Y$. Referring to Eqs. (13) and (14) the following cross-equation restrictions have to be imposed during the estimation process:

$\beta_{22} = \beta_{32}$ and $\beta_{21} = \beta_{31} = \beta_{22} / (\beta_1 - 1)$.

Our parameters of interest can be derived from the estimated coefficients as follows:

$\sigma_{KL,E} = \beta_1$, $\sigma_{KL} = 1 - \beta_{22}$, $a_E = \alpha_1 / (\beta_1 - 1)$, $a_K = -\alpha_2 / \beta_{22}$ and $a_L = -\alpha_3 / \beta_{32}$.

The model in Eqs. (15)-(17) enables us to test for a Cobb-Douglas function for one of the two levels by testing $\beta_1 = 1$ and $\beta_{22} = \beta_{32} = 0$, respectively; for whether the production function is a one-level, non-nested CES function by testing $\beta_{22} / (1 - \beta_1) = 1$.

In addition, following the analysis above, we see that if we assume that technological progress is not factor specific, but assume instead a neutral technological progress by applying the above procedure, for the *(KL)E* structure for instance, we will get the following system:

$$e - y = (\sigma_{KL,E} - 1) a_Y + \sigma_{KL,E} (p_Y - p_E) \tag{18}$$

$$\delta_{KZ} = \frac{\sigma_{K,L} - 1}{1 - \sigma_{KL,E}} \delta_{ZQ} + (1 - \sigma_{K,L})(p_K - p_Y) \tag{19}$$

$$\delta_{LZ} = \frac{\sigma_{K,L} - 1}{1 - \sigma_{KL,E}} \delta_{ZQ} + (1 - \sigma_{K,L})(p_L - p_Y) \tag{20}$$

Note that Eqs. (18)-(20) are Eqs. (12)-(14) with $a_K = a_L = 0$. Hence, we can test for biased technological change in energy by testing $\alpha_1 / (\beta_1 - 1) = 0$; for biased technological change in capital by testing $\alpha_2 / \beta_{22} = 0$; for biased technological change in labour by testing $\alpha_3 / \beta_{32} = 0$; and for TFP growth by testing $\alpha_2 / \beta_{22} = \alpha_3 / \beta_{32} = 0$.[8]

4 Data and empirical results

We estimated the system in Eqs. (15)-(17) for each of our three nesting structures, with a panel of ten Canadian industries to estimate industry-specific elasticities, using seemingly unrelated regressions (SUR). The data are obtained from the Productivity Program Database of Statistics Canada, for prices and quantities of capital, labour, energy and gross output plus capital compensation, labour compensation and cost of energy during the time period 1962-1997. All prices are in 1992 chained Fisher index of Canadian dollars. Industry output is the Fisher indexed sum of value-added and the value of energy, which are all Törnqvist index quantity indices. The industry set comprises of four energy intensive industries: primary metal industry, paper and allied products industry, chemical and chemical products industry, and non-metallic mineral products industry; and of six energy non-intensive industries: transportation industry, food industry, transportation equipment industry, fabricated metal products industry, textile products industry, and machinery industry (except electrical machinery). All the industries that comprise our set are sub-industries of the manufacturing industries because data on all the variables in our system are available for these industries. All data have passed the augmented Dickey-Fuller (ADF) test for unit root in level, thus are stationary and appropriate for the following analysis.[9]

As noted in the introduction, the literature on CES production parameter estimation generally lacks a systematic comparison of the empirical relevance of the nesting structures *(KL)E*, *K(LE)* and *(KE)L*, with only a few exceptions such as van der Werf (2008). Thus we present the goodness of fit of the three nesting structures in terms of determinant residual covariance (DRC) in Table 3.

[8] We can also test for TFP growth by testing $\alpha_2 = \alpha_3 = 0$, which is also statistically correct but may give different results from testing $\alpha_2/\beta_{22} = \alpha_3/\beta_{32} = 0$. Our conclusions, however, are qualitatively unaffected when using this alternative test.

[9] The results of ADF test are reported in the Appendix A.

Table 3: Goodness of fit

	(KL)E	*(KE)L*	*K(LE)*
Primary metal	3.64E-07	1.11E-07	7.22E-11
Paper etc.	6.08E-08	7.22E-08	8.56E-11
Chemicals	1.77E-09	2.67E-09	9.86E-10
Non-metal. mineral.	2.12E-10	3.06E-10	9.84E-11
Transportation	1.29E-10	1.06E-10	3.42E-11
Food	1.30E-10	1.05E-10	3.54E-11
Trans. equip.	1.31E-08	1.92E-08	4.91E-11
Fabricated metal	7.93E-10	1.00E-09	1.72E-10
Textile	8.14E-09	5.41E-09	3.50E-10
Machinery	2.99E-09	3.69E-09	5.12E-11

Note: Determinant residual covariance (DRC).

Table 4: Estimated elasticities of substitution

	(KL)E		*(KE)L*		*K(LE)*	
	$\sigma_{KL,E}$	$\sigma_{K,L}$	$\sigma_{KE,L}$	$\sigma_{K,E}$	$\sigma_{LE,K}$	$\sigma_{L,E}$
Primary metal	0.1026* (0.0285)	0.1549* (0.0151)	0.0858* (0.0160)	0.3034* (0.0164)	0.0707** (0.0317)	0.9969*** (0.0017)
Paper etc.	0.1529* (0.0286)	0.4389* (0.0102)	0.0868* (0.0187)	0.4743* (0.0171)	0.1281* (0.0463)	1.0008 (0.0038)
Chemicals	0.4833* (0.0406)	0.2514* (0.0301)	0.2739* (0.0446)	0.4315* (0.0521)	0.1485* (0.0369)	0.6492* (0.0324)
Non-metal. mineral.	0.3061* (0.0699)	0.4395* (0.0177)	0.3582* (0.0297)	0.4667* (0.0413)	0.4247* (0.0257)	0.6631* (0.0524)
Transportation	0.5409* (0.0500)	0.3605* (0.0317)	0.1692* (0.0407)	0.4475* (0.0471)	0.3712* (0.0372)	0.6094* (0.0362)
Food	0.7435* (0.0427)	0.3522* (0.0364)	0.2900* (0.0468)	0.6621* (0.0630)	0.1028 (0.0664)	0.7463* (0.0313)
Trans. equip.	0.3560* (0.0590)	0.3860* (0.0282)	0.2662* (0.0320)	0.4328* (0.0302)	0.2630* (0.0705)	0.9754*** (0.0151)
Fabricated metal	0.2436* (0.0899)	0.4567* (0.0284)	0.3144* (0.0327)	0.4384* (0.0478)	0.4037* (0.0492)	0.7803* (0.0604)
Textile	0.3397* (0.0673)	0.3678* (0.0301)	0.2272* (0.0297)	0.4598* (0.0387)	0.2611* (0.0698)	0.9008* (0.0333)
Machinery	0.2042** (0.1068)	0.4650* (0.0217)	0.3714* (0.0277)	0.4401* (0.0431)	0.3771* (0.0515)	0.9519 (0.0445)

Note: Standard errors in parentheses. */**/***1%/5%/10% level of significance.

Table 3 shows substantial differences in how well each nesting structure fits the data. We find that *K(LE)* structure fits the data best, given its lowest DRC among all three structures for all industries. As for the other two structures, we find that *(KL)E* structure fits the data better than *(KE)L* structure for six out of ten industries.

Table 4 presents our estimation results for the elasticities of substitution.[10] We'll discuss them by nesting structures.

4.1 The *(KL)E* nesting structure

Several dynamic climate policy models use the *(KL)E* or *(KL)(EM)* nesting structure. That is, they first combine capital and labour, and this composite is subsequently combined with energy (or an energy-materials composite) using a different elasticity of substitution. The first column of Table 4 shows our estimates for the elasticity of substitution between energy and the capital-labour composite. We see a considerable amount of variation over industries. The industry estimates range from 0.10 to 0.74. Note that we cannot reject perfect complementarity (i.e., elasticity equal to zero) between energy and the capital-labour composite for machinery industry at 1% level of significance.

The elasticities for capital and labour are reported in the second column of Table 4 and show quite some variation as well, with estimates ranging from 0.15 to 0.47. Note that for the elasticities for capital and labour, we can reject perfect complementarity for all industries. Table 5 presents the probability values for the two-sided tests for perfect complementarity.

Table 5: Tests for perfect complementarity: Two-sided p-values for H_0: elasticity equal to 0

	(KL)E		*(KE)L*		*K(LE)*	
	$\sigma_{KL,E}$	$\sigma_{K,L}$	$\sigma_{KE,L}$	$\sigma_{K,E}$	$\sigma_{LE,K}$	$\sigma_{L,E}$
Primary metal	0.0003	0.0000	0.0000	0.0000	0.0255	0.0000
Paper etc.	0.0000	0.0000	0.0000	0.0000	0.0056	0.0000
Chemicals	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
Non-metal. mineral.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Transportation	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Food	0.0000	0.0000	0.0000	0.0000	0.1214	0.0000
Trans. equip.	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000
Fabricated metal	0.0068	0.0000	0.0000	0.0000	0.0000	0.0000
Textile	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000
Machinery	0.0559	0.0000	0.0000	0.0000	0.0000	0.0000

[a] Two-sided *p*-values for H_0: elasticity equal to 0. A *p*-value smaller than 0.01 implies that we can reject the null-hypothesis at the 1% significance level.

[10] Confidence intervals for the estimated substitution elasticities are reported in Appendix B.

Table 6: Tests for Cobb-Douglas function: Two-sided p-values for H_0: elasticity equal to 1

	(KL)E		*(KE)L*		*K(LE)*	
	$\sigma_{KL,E}$	$\sigma_{K,L}$	$\sigma_{KE,L}$	$\sigma_{K,E}$	$\sigma_{LE,K}$	$\sigma_{L,E}$
Primary metal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0686
Paper etc.	0.0000	0.0000	0.0000	0.0000	0.0000	0.8422
Chemicals	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Non-metal. mineral.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Transportation	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Food	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Trans. equip.	0.0000	0.0000	0.0000	0.0000	0.0000	0.1028
Fabricated metal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003
Textile	0.0000	0.0000	0.0000	0.0000	0.0000	0.0029
Machinery	0.0000	0.0000	0.0000	0.0000	0.0000	0.2804

[a] Two-sided p-values for H_0: elasticity equal to 1. A p-value smaller than 0.01 implies that we can reject the null-hypothesis at the 1% significance level.

Table 7: Tests for common elasticities (no nesting): two-sided p-values for H_0: $\sigma_{ij,k}=\sigma_{i,j}$

	(KL)E	*(KE)L*	*K(LE)*
Primary metal	0.1147	0.0000	0.0000
Paper etc.	0.0000	0.0000	0.0000
Chemicals	0.0000	0.0409	0.0000
Non-metal. mineral.	0.0434	0.0604	0.0001
Transportation	0.0004	0.0000	0.0000
Food	0.0000	0.0000	0.0000
Trans. equip.	0.6062	0.0000	0.0000
Fabricated metal	0.0113	0.0196	0.0000
Textile	0.6757	0.0000	0.0000
Machinery	0.0109	0.2119	0.0000

[a] Two-sided p-values for H_0: $\sigma_{ij,k}=\sigma_{i,j}$. A p-value smaller than 0.01 implies that we can reject the null-hypothesis at the 1% significance level.

Table 6 presents the probability values for the two-sided tests for Cobb-Douglas function, that is, whether each elasticity is equal to one. For all industries and for both nesting levels, the null-hypothesis of a unitary elasticity is rejected.

In addition, we tested for common elasticities over the two nests (i.e. $\sigma_{KL,E}=\sigma_{K,L}$). That is, we tested whether the production function could have a single elasticity of substitution and hence could be non-nested. As is shown in the second column of Table 7, we cannot reject a non-nested production function for six out of ten industries at 1% significance level.

4.2 The *(KE)L* nesting structure

Next, we test for *(KE)L* nesting structure, which is first to combine capital and energy, and this composite is subsequently combined with labour, using a different elasticity of substitution. This structure has seen a wide application in GHG mitigation policies analysis, though it is least empirically favoured in this study. The third column of Table 4 shows our estimates for the elasticity of substitution between labour and the capital-energy composite. Again, we see an amount of variation over industries, but the range is smaller than in the *(KL)E* structure. The industry estimates range from 0.09 to 0.37. We can reject perfect complementarity between labour and the capital-energy composite for all industries at 1% level of significance. The elasticities for capital and energy are reported in the fourth column of Table 4 and the estimates range from 0.30 to 0.66. For the elasticities for capital and energy, we can reject perfect complementarity for all industries as well. Also, we can reject the hypothesis of Cobb-Douglas function for all industries and for both levels of nesting. When testing for common elasticities over the two nests (i.e. $\sigma_{KE,L}=\sigma_{K,E}$), we cannot reject a non-nested production function for four out of ten industries at 1% significance level.

4.3 The *K(LE)* nesting structure

For the *K(LE)* structure, at the top level, the elasticities of substitution differ significantly from zero for eight industries and their values range between 0.13 and 0.42. The exceptions are primary metal industry and food industry. For primary metal industry, we cannot reject the hypothesis of perfect complementarity, but we can reject the hypothesis of a unitary elasticity. This implies that at the top level, the production function is very likely Leontief. This may have stemmed from the nature of the industry. Production of primary metal products uses specialized, usually highly automatic machines which need only a fixed number of human operators apiece. More man hands will not prove more efficient with these machines. Also the amount of energy each machine uses during functioning is more likely to be fixed than in other industries, because, for example, the temperature which it takes to melt different metal is a fixed parameter, hence the energy used to maintain this temperature is fixed as well. These may contribute to the fixed ratio of machines (a main source of capital investment expenditure) to labour-energy composite, though other factors peculiar to this industry may be in play as well. For food industry, the situation is similar. We cannot reject perfect complementarity but we can reject unitary elasticity. This implies that the technology at the top level for food industry is Leontief as well.

At the low level, the elasticities of substitution are statistically significant for six industries and their values range between 0.61 and 0.90. For the other four industries whose substitution elasticities are not statistically significant, we can reject perfect complementarity but cannot reject unitary elasticity; this implies that for these four industries: paper industry, primary metal, transportation equipment industry and machinery industry, production technology at the low level is most likely to be Cobb-Douglas.

For the test for common elasticities over the two nests for the *K(LE)* structure (i.e. $\sigma_{LE,K}=\sigma_{L,E}$), we can reject a non-nested production function for all industries at 1% significance level.

4.4 Technological change

The models in Table 1 and Table 2 not only differ in the sizes of substitution elasticities and nesting structure, but also in the way productivity improvements enter the production function. Most models use total factor productivity for technological change. We test this hypothesis in this section.

4.4.1 The *(KL)E* nesting structure

Table 8 shows the (constant) factor-specific technology trends for the *(KL)E* structure. At 1% significance level, the rates of energy-augmenting technological change are not significantly different from zero for all industries; the rates of capital-augmenting technological change are not significantly different from zero for nine industries and significantly different from zero for one; the rates of labour-augmenting technological change are not significantly different from zero for eight industries and significantly different from zero for two industries. For industries that have rates of capital-augmenting technological change that are significantly different from zero, all rates of change are negative; and for labour-augmenting technological rates, all significant rates are positive. Since a positive factor-augmenting technological change reduces the cost-share of that factor and is hence factor-saving, and a negative factor-augmenting technological change increases the cost-share of that factor and is hence factor-using, we can conclude that if we assume production is a *(KL)E* structure, then the technological change has been energy-neutral, capital-using, and labour-saving. However, for most industries, there's no proof of factor-augmenting technological change, which implies that these industries, which present no trend of factor-saving, have not been afflicted by the rising cost of production factors such as in other countries in the same period. This is reasonable, in light of the relatively low and constant inflation rate in Canada for the past decades. Moreover, the energy-neutral behaviour of industries, among which is a traditionally considered energy intensive industry (chemicals industry), indicates that energy cost is not as important a component of total cost as we may have imagined.

Table 8: Rates of factor-specific technological change, *(KL)E* structure

	Energy	Capital	Labour
Primary metal	-0.0167 (0.0106)	-0.0180 (0.0450)	0.0048 (0.0376)
Paper etc.	-0.0182 (0.0118)	-0.0179 (0.0643)	0.0142 (0.0204)
Chemicals	-0.0429*** (0.0226)	-0.0187** (0.0089)	0.0128* (0.0038)
Non-metal. mineral.	-0.0079 (0.0116)	-0.0044 (0.0092)	0.0013 (0.0038)
Transportation	-0.0274** (0.0140)	-0.0076 (0.0099)	0.0057 (0.1276)
Food	0.0022 (0.0267)	-0.0233* (0.0082)	0.0100* (0.0031)
Trans. equip.	-0.0107 (0.0128)	-0.0398 (0.0336)	0.0057 (0.0093)
Fabricated metal	-0.0117 (0.0107)	-0.0092 (0.0168)	-0.0011 (0.0057)
Textile	-0.0234*** (0.0128)	-0.0275 (0.0219)	-0.0021 (0.0102)
Machinery	-0.0091 (0.0154)	-0.0232 (0.0207)	0.0031 (0.0054)

Note: Standard errors in parentheses. */**/***1%/5%/10% level of significance.

For our purpose it is interesting to see whether the technology trends for the three inputs differ from each other. Table 9 presents, for each industry and for *(KL)E* structure, tests whether the technology trends are equal. We cannot reject that the rate of energy-augmenting technological change and the rate of labour-augmenting technological change are equal, or that the rate of energy-augmenting technological change and the rate of capital-augmenting technological change are equal for all industries. We therefore conclude that rates of factor-specific technological change tend not to differ over factors in Canada.

Table 9: Tests for a_i=a_j, for *(KL)E* structure

	$a_E=a_L$	$a_E=a_K$	$a_K=a_L$
Primary metal	0.9759	0.5459	0.1438
Paper etc.	0.9966	0.1183	0.0204
Chemicals	0.2850	0.0105	0.0007
Non-metal. mineral.	0.8088	0.4067	0.7938
Transportation	0.5565	0.0362	0.0612
Food	0.3532	0.7562	0.0001
Trans. equip.	0.4123	0.2330	0.1542
Fabricated metal	0.8990	0.2694	0.8603
Textile	0.8705	0.1331	0.3527
Machinery	0.5599	0.3943	0.3065

Note: Two-sided *p*-values for H_0: $a_i=a_j$.

Table 10: Rates of factor-specific technological change, *(KE)L* structure

	Labour	Capital	Energy
Primary metal	0.0061 (0.0113)	-0.0163 (0.0558)	-0.0196 (0.0285)
Paper etc.	0.0127 (0.0092)	-0.0153 (0.0546)	-0.0271*** (0.0144)
Chemicals	0.0168* (0.0055)	-0.0195*** (0.0107)	-0.0427 (0.0267)
Non-metal. mineral.	0.0025 (0.0045)	-0.0035 (0.0086)	-0.0085 (0.0189)
Transportation	0.0056** (0.0025)	-0.0069 (0.0097)	-0.0241** (0.0118)
Food	0.0086* (0.0033)	-0.0237*** (0.0127)	0.0155 (0.0108)
Trans. equip.	0.0093 (0.0068)	0.0190 (0.0245)	-0.0097 (0.0213)
Fabricated metal	0.0028 (0.0038)	-0.0069 (0.0133)	-0.0171 (0.0184)
Textile	0.0046 (0.0060)	-0.0226 (0.0184)	-0.0266 (0.0198)
Machinery	0.0067 (0.0047)	-0.0198 (0.0180)	-0.0099 (0.0267)

Note: Standard errors in parentheses. */**/***1%/5%/10% level of significance.

As noted in section 3, we can test for *(KL)E* structure for input-neutral total factor productivity growth by testing $a_K=a_L=0$. As can be inferred from Table 9, we can reject $a_K=a_L=0$ for chemicals industry and food industry at 1% level of significance, but we cannot reject it for the other eight industries. Hence, we can conclude that for the *(KL)E* structure, technological change is very likely to be TFP growth for Canada.

4.4.2 The *(KE)L* nesting structure

Table 10 shows the (constant) factor-specific technology trends for the *(KE)L* structure. At 1% significance level, the rates of labour-augmenting technological change are significantly different from zero for two industries, but the rates of capital- or energy-augmenting technological change are not significantly different from zero for all industries. This finding suggests that for the *(KE)L* structure, technology is labour-saving (positive rates of labour-augmenting technological change) and capital- and energy-neutral.

Table 11: Tests for $a_i=a_j$, for *(KE)L* structure

	$a_L=a_K$	$a_L=a_E$	$a_K=a_E$
Primary metal	0.6271	0.1819	0.5715
Paper etc.	0.5732	0.0061	0.1558
Chemicals	0.0046	0.0473	0.0871
Non-metal. mineral.	0.5474	0.6143	0.8368
Transportation	0.1901	0.0212	0.0219
Food	0.0204	0.8338	0.1426
Trans. equip.	0.1002	0.4361	0.3634
Fabricated metal	0.5113	0.3016	0.6066
Textile	0.1148	0.1384	0.1644
Machinery	0.1921	0.5693	0.5421

Note: Two-sided *p*-values for H_0: $a_i=a_j$.

In Table 11, we cannot reject the hypothesis that the rate of labour-augmenting technological change and the rate of energy-augmenting technological change are equal for nine industries, and we cannot reject the hypothesis that the rate of capital-augmenting technological change and the rate of labour-augmenting technological change are equal for nine industries.

In Table 11, we also test for the *(KE)L* structure for input-neutral total factor productivity growth by testing $a_K=a_E=0$. We cannot reject $a_K=a_E=0$ for all industries at 1% level of significance. Hence, we conclude that for the *(KE)L* structure, technological change is also very likely of Hicks neutral type for Canada.

4.4.3 The *K(LE)* nesting structure

Table 12 shows the (constant) factor-specific technology trends for the *K(LE)* structure. The rates of capital-augmenting technological change are significantly different from zero for only one industry (food industry) at 1% significance level. For all the other industries, the rates of capital-augmenting technological change are not significantly different from zero.

Table 12: Rates of factor-specific technological change, *K(LE)* structure

	Capital	Energy	Labour
Primary metal	-0.0150 (0.0108)	-2.7878 (3.3198)	0.0895 (0.9922)
Paper etc.	-0.0220** (0.0089)	0.1809 (0.9165)	-0.0480 (0.2436)
Chemicals	-0.0147*** (0.0082)	-0.0778*** (0.0406)	0.0200* (0.0058)
Non-metal. mineral.	-0.0019 (0.0080)	-0.0139 (0.0302)	0.0019 (0.0055)
Transportation	-0.0069 (0.0078)	-0.0334** (0.0165)	0.0070*** (0.0037)
Food	-0.0192* (0.0056)	-0.0130 (0.0376)	0.0082** (0.0038)
Trans. equip.	-0.0280 (0.0234)	-0.3134 (0.4414)	0.0131 (0.0317)
Fabricated metal	-0.0027 (0.0114)	-0.0433 (0.0467)	-0.0004 (0.0091)
Textile	-0.0140 (0.0138)	-0.1641 (0.1227)	0.0026 (0.0188)
Machinery	-0.0146 (0.0140)	-0.0910 (0.3069)	0.0036 (0.0295)

Note: Standard errors in parentheses. */**/***1%/5%/10% level of significance.

Table 13: Tests for $a_i = a_j$, for *K(LE)* structure

	$a_K = a_E$	$a_K = a_L$	$a_E = a_L$
Primary metal	0.4036	0.3591	0.6399
Paper etc.	0.8434	0.8446	0.9807
Chemicals	0.1251	0.0008	0.0027
Non-metal. mineral.	0.7217	0.6934	0.8532
Transportation	0.1902	0.0690	0.0374
Food	0.8709	0.0001	0.0227
Trans. equip.	0.5134	0.2102	0.7384
Fabricated metal	0.3919	0.8693	0.6021
Textile	0.2222	0.4056	0.3642
Machinery	0.8030	0.5542	0.9465

Note: Two-sided *p*-values for H_0: $a_i = a_j$.

Another interesting finding is that for all industries, we find no support for energy-augmenting technological change; rates of energy-augmenting technological change do not differ significantly from zero at 1% level of significance. As for labour-augmenting technological change, we find that except for chemicals industry, the rates of labour-augmenting technological change are not significantly different from zero. For the food industry, the capital-augmenting technological change rate is -1.92%, implying more capital has been used in this industry. For chemicals industry, the labour-augmenting technological change rate is 2%, meaning more labour has been used. This finding, that technological change has been labour-saving and capital-using, and that the magnitude of labour-augmenting technological change is larger than capital-augmenting technological change, is in line with van der Werf (2008), though he also reports energy-augmenting technological change in Canada overall from 1978-1996.

In Table 13, we cannot reject that the rate of energy-augmenting technological change and the rate of labour-augmenting technological change are equal for all industries. This is not surprising, given that neither of these two rates is significantly different from zero. We cannot reject the hypothesis that the rate of capital-augmenting technological change and the rate of labour-augmenting technological change are equal for all industries, but chemicals industry and food industry. This is also consistent with Table 12, since chemicals industry presents significant rate of labour-augmenting technological change while food industry presents significant rate of capital-augmenting technological change. We therefore conclude that rates of factor-specific technological change tend not to differ over factors in Canada. This conclusion is the same as for the *(KL)E* structure and the *(KE)L* structure.

Also from Table 13, we cannot reject $a_E=a_L=0$ for all industries but chemicals industry at 1% level of significance. Hence, we can conclude that for the *K(LE)* structure, technological change is also very likely to be TFP growth for Canada.

5 Discussion

Comparing the results of the previous section with the models summarized in Table 1 and Table 2, we can draw three conclusions. The first conclusion refers to the nesting structure as in Table 2. Nearly all models in Table 2 have capital and labour in the same nest, and this nesting structure is supported by the results of van der Werf (2008). On the other hand, there is another type of model that is also widely used by computable general equilibrium (CGE) modellers in their studies on climate policy.[11] In this model, capital and energy are in the same nest. However, our result for Canada support the third nesting structure, the *K(LE)* structure, in which labour and energy are in the same nest. Moreover, our results suggest that the *(KE)L* nesting structure fits the data poorly. The argument is that the demand for capital and energy may not be determined jointly since energy cost is a variable cost while physical capital comprises fixed investment which usually takes up a large portion of capital stock. This fixed investment is less mobile than labour and energy, hence the elasticity of substitution between capital and labour or energy should be different from the substitution elasticity between energy and labour, as the *K(LE)* structure suggests. Note that the assumption of *K(LE)* structure performs better in terms of theoretical consistency: for all our ten industries we can reject the null-hypothesis of non-nested structure, whereas for the *(KL)E* structure and the *(KE)L* structure, this ratio is four out of ten and six out of ten, respectively. This is evidentially in support of the *K(LE)* nesting structure.

Table 14: Size of elasticities of substitution for top and low level for the *(KL)E* structure

	Top level	Low level
Primary metal*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Paper etc.	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Chemicals	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Non-metal. mineral.*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Transportation	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Food	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Trans. equip.*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Fabricated metal*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Textile*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Machinery*	Leontief	non-zero, non-unitary elasticity

* We cannot reject the non-nested structure hypothesis.

[11] This is the widely used GTAP-E model, outlined in Burinaux and Truong (2002), subsequently streamlined and improved by McGougall and Golub (2007), and utilized by many, such as Beckman and Hertel (2009), Ronneberger et al (2006), Rosen (2003), Bosello et al (2007), Nijkamp et al (2005), Kemfert et al (2006), Banse et al (2007), Birur et al (2007), Taheripour et al (2008), Gan and Smith (2005) and Berrittella et al (2005). A supposed merit of this model, according to Beckman and Hertel (2009), is that it more closely mimics the ability of firms to substitute among alternative fuels as well as between labour, capital and energy. It also incorporates CO2 emissions from the combustion of fossil fuels as well as a mechanism to trade these emissions internationally. See Beckman and Hertel (2009) for a full source of references to the above studies.

Our second conclusion refers to the sizes of the elasticities of substitution in Tables 14-16. As noted in the introduction and in section 2, Cobb-Doulas function, despite its pervasive use in many areas of economics, is not empirically supported by many studies. This is also confirmed by our results. In Table 14, we can see that for the *(KL)E* structure, for as much as nine out of ten industries, elasticities are non-zero and non-unitary for both the outer and low level. For machinery industry, production function is Leontief at the top level, and at the low level, it also has non-zero and non-unitary substitution elasticities. In Table 15, for *(KE)L* structure, we can see that for all industries, substitution elasticities are non-zero, non-unitary for both nests. In Table 16, which presents the case for the *K(LE)* structure, however, we see more evidence for Leontief and Cobb-Douglas functions. For the primary metal industry, technology is Leontief at the top level, and Cobb-Douglas at the low level. For food industry, technology is Leontief at the top level, but the substitution elasticity is non-zero, non-unitary at the low level. For paper industry, transportation equipment industry and machinery industry, technology is Cobb-Douglas at the low level, but the substitution elasticities are non-zero, non-unitary at the top level. And for the other industries, the substitution elasticities are non-zero, non-unitary at both the outer and the low level.

Our third conclusion refers to technological change in Tables 17-19. For the *(KL)E* structure, eight industries exhibit TFP growth. The exceptions are chemicals industry and food industry. They both exhibit labour-saving and capital-using technology. For the *(KE)L* structure, also eight industries exhibit TFP growth. The exceptions are chemical industry and transportation industry. The former exhibits labour-saving and capital-using technology and the latter exhibits labour-saving and energy-using technology. For the *K(LE)* structure, seven industries exhibit TFP growth. The exceptions are chemical industry, food industry and transportation industry. Chemicals industry exhibits labour-saving, capital-using and energy-using technology, whereas transportation industry exhibits labour-saving and energy-using technology and food industry exhibits labour-saving and capital-using industry.

To summarize, in all nesting structures, the primary metal industry, paper industry, non-metallic mineral products industry, transportation equipment industry, fabricated metal industry, textile industry and machinery industry exhibit TFP growth. In all structures, chemicals industry exhibits labour-saving and capital-using technology. In both the *(KE)L* structure and the *K(LE)* structure, transportation industry exhibits labour-saving and energy-using technology, and in both the *(KL)E* structure and the *K(LE)* structure, food industry exhibits labour-saving and capital-using technology.

Table 15: Size of elasticities of substitution for top and low level for the *(KE)L* structure

	Top level	Low level
Primary metal	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Paper etc.	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Chemicals*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Non-metal. mineral.*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Transportation	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Food	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Trans. equip.	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Fabricated metal*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Textile	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Machinery*	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity

* We cannot reject the non-nested structure hypothesis.

Table 16: Size of elasticities of substitution for top and low level for the *K(LE)* structure

	Top level	Low level
Primary metal	Leontief	Cobb-Douglas
Paper etc.	non-zero, non-unitary elasticity	Cobb-Douglas
Chemicals	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Non-metal. mineral.	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Transportation	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Food	Leontief	non-zero, non-unitary elasticity
Trans. equip.	non-zero, non-unitary elasticity	Cobb-Douglas
Fabricated metal	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Textile	non-zero, non-unitary elasticity	non-zero, non-unitary elasticity
Machinery	non-zero, non-unitary elasticity	Cobb-Douglas

Table 17 Technological change for the *(KL)E* structure

	Labour-augmenting	Capital-augmenting	Energy-augmenting	TFP growth
Primary metal	-	-	-	Yes
Paper etc.	-	-	-	Yes
Chemicals	Labour-saving	Capital-using	-	-
Non-metal. mineral.	-	-	-	Yes
Transportation	-	-	-	Yes
Food	Labour-saving	Capital-using	-	-
Trans. equip.	-	-	-	Yes
Fabricated metal	-	-	-	Yes
Textile	-	-	-	Yes
Machinery	-	-	-	Yes

Table 18 Technological change for the *(KE)L* structure

	Labour-augmenting	Capital-augmenting	Energy-augmenting	TFP growth
Primary metal	-	-	-	Yes
Paper etc.	-	-	-	Yes
Chemicals	Labour-saving	Capital-using	-	-
Non-metal. mineral.	-	-	-	Yes
Transportation	Labour-saving	-	Energy-using	-
Food	-	-	-	Yes
Trans. equip.	-	-	-	Yes
Fabricated metal	-	-	-	Yes
Textile	-	-	-	Yes
Machinery	-	-	-	Yes

Table 19 Technological change for the *K(LE)* structure

	Labour-augmenting	Capital-augmenting	Energy-augmenting	TFP growth
Primary metal	-	-	-	Yes
Paper etc.	-	-	-	Yes
Chemicals	Labour-saving	Capital-using	Energy-using	-
Non-metal. mineral.	-	-	-	Yes
Transportation	Labour-saving	-	Energy-using	-
Food	Labour-saving	Capital-using	-	-
Trans. equip.	-	-	-	Yes
Fabricated metal	-	-	-	Yes
Textile	-	-	-	Yes
Machinery	-	-	-	Yes

6 Summary and conclusions

This paper contributes to the literature on CES production function by estimating nested CES production functions using capital, labour and energy as inputs. We find that the nesting structure, *K(LE)*, in which energy and labour are first combined using a CES function, and then this composite of energy and labour is combined with capital in a second CES function, fits the Canadian data best. For this structure we were, for all industries in our panel, able to reject the hypothesis that the elasticities are equal for both nests. Hence, our nesting assumption cannot be rejected. At the top level, we find evidence of Leontief technology for two industries, and at the low level, we find evidence of Cobb-Douglas technology for four industries. Regarding technological change, we find evidence for TFP growth for more than half of our studied industries. Additionally, we found that the factor-specific technological changes are labour-saving, and capital- and energy-using in Canada.

For *K(LE)* structure, our estimates for the elasticities of substitution are substantially low for the top level (the average is lower than 0.5) and higher for the low level (the average is higher than 0.6). Given that lower elasticities at the top level imply that it becomes harder to substitute away from capital, and the higher elasticities at the low level imply that it is easier to substitute towards energy, the role of endogenous technological change in reducing the costs of energy may be bigger in giving rise to more emissions related to energy use in Canada. Whether this claim holds should of course be tested by models adapted to our empirical findings, and comparing the additional effect of endogenous technological change in the original model with that from the adapted model.

Appendix A: Augmented Dickey-Fuller (ADF) test results

1. *(KL)E* structure

Variables	t-statistics				
	Prim.metal	Transport.	Food	Transport. eq.	Fabric. metal
p_y-p_e	-5.029784* (-3.632900)	-4.659405* (-3.632900)	-6.704435* (-3.632900)	-6.732815* (-3.632900)	-5.026681* (-3.632900)
$\delta_{kl,y}$	-4.121700* (-3.632900)	-4.414822* (-3.632900)	-4.217317* (-3.632900)	-4.762675* (-3.632900)	-4.055839* (-3.632900)
p_k-p_y	-6.375689* (-3.632900)	-4.016058* (-3.632900)	-4.960506* (-3.632900)	-5.094454* (-3.646342)	-4.722179* (-3.639407)
p_l-p_y	-4.960577* (-3.632900)	-5.597935* (-3.632900)	-7.104088* (-3.632900)	-7.632516* (-3.632900)	-6.186329* (-3.632900)
e-y	-5.654000* (-3.632900)	-5.310902* (-3.632900)	-7.626522* (-3.632900)	-6.200622* (-3.632900)	-5.729320* (-3.639407)
$\delta_{k,kl}$	-6.481636* (-3.632900)	-4.452615* (-3.632900)	-7.264906* (-3.632900)	-6.607773* (-3.632900)	-5.659755* (-3.632900)
$\delta_{l,kl}$	-4.891864* (-3.639407)	-4.246406* (-3.632900)	-6.839905* (-3.632900)	-6.235509* (-3.632900)	-5.633082* (-3.632900)

Variables	t-statistics				
	Paper	Textile	Chemicals	Non-metal.	Machinery
p_y-p_e	-4.919058* (-3.646342)	-8.098118* (-3.632900)	-4.516100* (-3.632900)	-4.177186* (-3.632900)	-5.482516* (-3.646342)
$\delta_{kl,y}$	-5.442888* (-3.653730)	-4.746283* (-3.632900)	-3.875063* (-3.632900)	-3.929960* (-3.632900)	-5.249576* (-3.632900)
p_k-p_y	-5.327341* (-3.639407)	-6.579911* (-3.632900)	-3.822248* (-3.632900)	-3.682743* (-3.632900)	-5.311278* (-3.639407)
p_l-p_y	-4.881860* (-3.639407)	-9.292672* (-3.632900)	-6.142793* (-3.632900)	-5.146705* (-3.632900)	-5.249011* (-3.646342)
e-y	-6.731516* (-3.632900)	-6.203395* (-3.632900)	-6.510915* (-3.632900)	-6.214025* (-3.632900)	-6.760757* (-3.632900)
$\delta_{k,kl}$	-5.641810* (-3.639407)	-9.526377* (-3.632900)	-5.763634* (-3.632900)	-5.024561* (-3.632900)	-5.756804* (-3.632900)
$\delta_{l,kl}$	-4.937979* (-3.653730)	-9.275971* (-3.632900)	-5.045426* (-3.632900)	-4.579311* (-3.632900)	-5.779656* (-3.632900)

Note: * 1% significance level. In parentheses are critical values for the significance level. A t-statistics smaller than the critical value means that data series are stationary.

2. *K(LE)* structure

Variables	t-statistics				
	Prim.metal	Transport.	Food	Transport. eq.	Fabric. metal
p_y-p_k	-6.375689* (-3.632900)	-4.016058* (-3.632900)	-4.960506* (-3.632900)	-5.094454* (-3.646342)	-4.722179* (-3.639407)
$\delta_{el,y}$	-4.892746* (-3.639407)	-4.120817* (-3.632900)	-6.871050* (-3.632900)	-6.269020* (-3.632900)	-5.607389* (-3.632900)
p_e-p_y	-5.029784* (-3.632900)	-4.659405* (-3.632900)	-6.704435* (-3.632900)	-6.732815* (-3.632900)	-5.026681* (-3.632900)
p_l-p_y	-4.960577* (-3.632900)	-5.597935* (-3.632900)	-7.104088* (-3.632900)	-7.632516* (-3.632900)	-6.186329* (-3.632900)
k-y	-4.676327* (-3.632900)	-5.076663* (-3.632900)	-4.886748* (-3.632900)	-4.616051* (-3.639407)	-4.429461* (-3.632900)
$\delta_{e,el}$	-4.460346* (-3.632900)	-4.775681* (-3.632900)	-4.283080* (-3.632900)	-4.234771* (-3.632900)	-4.198047* (-3.632900)
$\delta_{l,el}$	-4.309604* (-3.632900)	-4.916264* (-3.632900)	-3.892141* (-3.632900)	-4.170605* (-3.632900)	-4.162359* (-3.632900)

Variables	t-statistics				
	Paper	Textile	Chemicals	Non-metal.	Machinery
p_y-p_k	-5.327341* (-3.639407)	-6.579911* (-3.632900)	-3.822248* (-3.632900)	-3.682743* (-3.632900)	-5.311278* (-3.639407)
$\delta_{el,y}$	-4.938145* (-3.653730)	-9.212695* (-3.632900)	-4.939938* (-3.632900)	-4.435182* (-3.632900)	-5.771680* (-3.632900)
p_e-p_y	-4.919058* (-3.646342)	-8.098118* (-3.632900)	-4.516100* (-3.632900)	-4.177186* (-3.632900)	-5.482516* (-3.646342)
p_l-p_y	-4.881860* (-3.639407)	-9.292672* (-3.632900)	-6.142793* (-3.632900)	-5.146705* (-3.632900)	-5.249011* (-3.646342)
k-y	-4.544756* (-3.632900)	-6.011857* (-3.632900)	-4.745564* (-3.632900)	-6.300675* (-3.632900)	-5.297104* (-3.653730)
$\delta_{e,el}$	-5.349437* (-3.632900)	-3.655279* (-3.632900)	-4.518361* (-3.632900)	-4.841574* (-3.632900)	-5.070233* (-3.632900)
$\delta_{l,el}$	-4.914859* (-3.632900)	-7.742683* (-3.632900)	-4.243698* (-3.632900)	-4.763804* (-3.632900)	-4.993601* (-3.632900)

Note: * 1% significance level. In parentheses are critical values for the significance level. A t-statistics smaller than the critical value means that data series are stationary.

3. *(KE)L* structure

Variables	t-statistics				
	Prim.metal	Transport.	Food	Transport. eq.	Fabric. metal
p_y-p_e	-4.960577* (-3.632900)	-5.597935* (-3.632900)	-7.104088* (-3.632900)	-7.632516* (-3.632900)	-6.186329* (-3.632900)
$\delta_{ke,y}$	-4.830559* (-3.632900)	-4.765364* (-3.632900)	-7.199560* (-3.632900)	-6.687749* (-3.632900)	-5.761596* (-3.632900)
p_k-p_y	-6.375689* (-3.632900)	-4.016058* (-3.632900)	-4.960506* (-3.632900)	-5.094454* (-3.646342)	-4.722179* (-3.639407)
p_e-p_y	-5.029784* (-3.632900)	-4.659405* (-3.632900)	-6.704435* (-3.632900)	-6.732815* (-3.632900)	-5.026681* (-3.632900)
l-y	-5.863377* (-3.639407)	-6.927501* (-3.632900)	-8.273523* (-3.632900)	-8.581757* (-3.632900)	-5.057869* (-3.653730)
$\delta_{k,ke}$	-6.936451* (-3.632900)	-4.233743* (-3.632900)	-5.664909* (-3.632900)	-6.334204* (-3.632900)	-4.943512* (-3.632900)
$\delta_{e,ke}$	-4.514318* (-3.632900)	-4.148662* (-3.632900)	-6.164156* (-3.632900)	-6.465364* (-3.632900)	-4.751632* (-3.632900)

Variables	t-statistics				
	Paper	Textile	Chemicals	Non-metal.	Machinery
p_y-p_e	-4.881860* (-3.639407)	-9.292672* (-3.632900)	-6.142793* (-3.632900)	-5.146705* (-3.632900)	-5.249011* (-3.646342)
$\delta_{ke,y}$	-5.172612* (-3.639407)	-9.453753* (-3.632900)	-7.270289* (-3.661661)	-5.248862* (-3.632900)	-5.777730* (-3.632900)
p_k-p_y	-5.327341* (-3.639407)	-6.579911* (-3.632900)	-3.822248* (-3.632900)	-3.682743* (-3.632900)	-5.311278* (-3.639407)
p_e-p_y	-4.919058* (-3.646342)	-8.098118* (-3.632900)	-4.516100* (-3.632900)	-4.177186* (-3.632900)	-5.482516* (-3.646342)
l-y	-5.034200* (-3.632900)	-9.364804* (-3.632900)	-5.672136* (-3.632900)	-4.725311* (-3.632900)	-5.646932* (-3.646342)
$\delta_{k,ke}$	-5.438620* (-3.689194)	-8.623066* (-3.632900)	-4.473337* (-3.632900)	-4.363544* (-3.632900)	-5.537207* (-3.632900)
$\delta_{e,ke}$	-4.787760* (-3.653730)	-7.672902* (-3.632900)	-4.672369* (-3.632900)	-4.324985* (-3.632900)	-5.529808* (-3.646342)

Note: * 1% significance level. In parentheses are critical values for the significance level. A t-statistics smaller than the critical value means that data series are stationary.

Appendix B: Confidence Intervals for Estimated Substitution Elasticities

	$\sigma_{KL,E}$		$\sigma_{K,L}$		$\sigma_{KE,L}$		$\sigma_{K,E}$		$\sigma_{LE,K}$		$\sigma_{L,E}$	
	Lower bound	**Upper bound**	**Lower bound**	**Upper bound**	**Lower bound**	**Upper bound**	**Lower bound**	**Upper bound**	**Lower bound**	**Upper bound**	**Lower bound**	**Upper bound**
1	0.0904	0.1148	0.1484	0.1614	0.0789	0.0927	0.2964	0.3104	0.0571	0.0843	0.9962	0.9976
2	0.1406	0.1652	0.4345	0.4433	0.0788	0.0948	0.4670	0.4816	0.1082	0.1480	0.9992	1.0024
3	0.4659	0.5007	0.2385	0.2643	0.2548	0.2930	0.4091	0.4539	0.1327	0.1643	0.6353	0.6631
4	0.2761	0.3361	0.4319	0.4471	0.3455	0.3709	0.4490	0.4844	0.4137	0.4357	0.6406	0.6856
5	0.5194	0.5624	0.3469	0.3741	0.1517	0.1867	0.4273	0.4677	0.3552	0.3872	0.5939	0.6249
6	0.7252	0.7618	0.3366	0.3678	0.2699	0.3101	0.6351	0.6891	0.0743	0.1313	0.7329	0.7597
7	0.3307	0.3813	0.3739	0.3981	0.2525	0.2799	0.4198	0.4458	0.2327	0.2933	0.9689	0.9819
8	0.2050	0.2822	0.4445	0.4689	0.3004	0.3284	0.4179	0.4589	0.3826	0.4248	0.7544	0.8062
9	0.3108	0.3686	0.3549	0.3807	0.2145	0.2399	0.4432	0.4764	0.2311	0.2911	0.8865	0.9151
10	0.1584	0.2500	0.4557	0.4743	0.3595	0.3833	0.4216	0.4586	0.3550	0.3992	0.9328	0.9710

Note: 1% significance level.
Industry index is as follows:
1. Primary metal industries.
2. Transportation industries.
3. Food industry.
4. Transportation equipment industries.
5. Fabricated metal products industries.
6. Paper and allied products industries.
7. Textile products industries.
8. Chemical and chemical products industries.
9. Non-metalic mineral products industries.
10. Machinery industry (except electrical machinery).

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