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Characteristics of Unregulated Power System Networks**

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MODELING, SIMULATION AND NUMERICAL ANALYSIS OF
TRANSIENT CHARACTERISTICS OF UNREGULATED
POWER SYSTEM NETWORKS

By
MOHAMED ABUL BASHER

A thesis submitted to the Faculty of Graduate and Postdoctoral Studies
University of Ottawa

Under the direction of Dr. N U Ahmed, Professor Emeritus of the School of Information
Technology and Engineering (SITE), in partial fulfillment of the requirements for the
degree of M. Sc., Systems Science.

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ABSTRACT

Design and operations of electrical distribution-transmission networks are analyzed mathematically, implemented numerically and validated by simulation. A dynamic model of a three node network with capacitors, inductors, load current controllers and regulators is proposed and cast in a general model of differential state-space equations in canonical form. The model is implemented via a Runge-Kutta algorithm. Realistic values of distribution systems are chosen as input and validated interactively so as to avoid instabilities and maintain reasonable characteristics. Typically cases are analyzed and the behavior of state variable is represented graphically. The software used and mode of representation aim at providing a robust environment to help power managers in their daily control of load balancing. The analysis also opens directions for the design of power distribution network.

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CHAPTER 1

1.1 INTRODUCTION

This thesis sets the basis for the development of a power system model, its simulation, analysis and an aggregation decomposition solution procedure. This will aid managers in proposing future remedies to the systemic problem of energy loss reduction. It is based on two interrelated efforts:

- the formulation of the dynamics of power systems network losses in the form of differential equations and in particular the development of a mathematical model with state-space equations in canonical form.
- the solution of the model by a Runge-Kutta algorithm, imbedded in software for its simulation and validation via graphical representation.

It is important to note here that in the dynamic model the controls (load currents) are time constant or varying parameters. As a result, it is essential to consider line currents and node voltage, or the charge stored in the capacitors as the state variables rather than the voltages across the buses.

Outline of Thesis

Chapter 1 gives a survey of power system network problems, different types of management models and offers a technical analysis of different equivalent circuits (equivalent 'T' and 'Pi' circuits) used in the power systems. Here, general economic, managerial and engineering issues are analyzed and common modeling strategies are presented, based on a selective review of the state of the art. Possible choices of power systems modeling are then assessed with an operational management point of view.

Chapter 2, addresses technical details on the design of dynamic models for the power distribution system (lines and loads). The transmission line model also includes the controllers for load current control. (Park's Transformation is also used and applied into this model). The complete set of non-linear differential equations representing the entire system in state space

form is also presented; the generalized power system model coupled with the transmission line and loads is developed for analysis. A complete set of linear differential equations representing the entire system in state space form also the power load and loss factors are presented here. Presented details of different models used for practical simulation and numerical analysis for regulation of the power network and computation enabling to determine factors as state variables like load, line currents and node voltages. The chapter highlights certain load conditions under which the stability and state of the system can be regulated through the system voltage, either directly by controlling load currents or through the capacitor bank. The chapter also provides information on a per unit representation of systems parameters, variables, in a T and or Pi equivalent circuits

Chapter 3, an analysis on the results of numerical tests data, their plots and graphs have been included, based on the D.C. or A.C. supply voltage as well as different load conditions or controls, and frequencies. The output data are obtained by writing a program to solve the system model. This includes numerical results on state variables and operational power loss with the changing load demand and consumption, load types (or conditions) and characteristic behavioral changes of state variables of the unregulated power system.

Chapter 4 presents a discussion of the results obtained numerically and their plotted graphs, such as their impact on phase symmetry management, load characteristics. In addition, some conclusions are drawn about the simulated results derived.

Appendix section provides practical details about the Runge-Kutta method used in the numerical computation and on the use of the software in solving differential equations. Appendices contain detailed information on relevant statistical data, text copy of the developed Fortran programs, some numerical data from experiments done, analysis (sample procedure), relevant statistical operational data, power system losses report from the Internet and resonance frequency.

Contribution of the thesis

The thesis proposes a mathematical approach to the design of some components of a power distribution network. It offers a complete dynamic model of a power system of transmission and distribution lines, its implementation and verification by simulation. The

model encompasses line as well as leakage and grounding resistance, line inductance, capacitance installed in parallel at the nodes and loads. The results indicate that with the variation of loads, the desired voltage regulation can be achieved by proper choice of load currents (by regulating load resistance and or load inductance), or using control capacitors placed at the nodes.

In the context of any power systems (e.g. of Ontario Hydro, Ottawa Hydro), the results of this thesis broadly support the modeling, simulation and numerical analysis for reduction of power system losses for operational management (using necessary computer software facilities. This can hopefully enable those types of organizations concerned in effectively managing and controlling their system losses.

1.2 REVIEW OF MODELS OF POWER DISTRIBUTION

The chapter sets a general framework of reference for the thesis, in which one can address the regulation of load currents from load centers, using any control system node voltages or all of the methods mentioned below. More specifically, general, economic, managerial and engineering issues are analyzed and common modeling strategies are presented. T or Pi –equivalent networks and responses of inductance, and capacitance with the changes of state variables, their impacts on node voltages, energy are discussed.

1.3 THE DRIVERS FOR ENHANCED POWER DISTRIBUTION

With increasing worldwide demand of power energy, especially electrical, much attention is given to the economical transfer of electricity from its place of production to consumer premises. In particular, every public utility authority and company tries to achieve proper returns and manage energy efficiently by seeking a reduction of system losses. We first review themes that underpin our effort to rationalize power distribution.

1.3.1 Economic context

Thus, this thesis is developed with a view of managing the systems loss reduction problem of any large Canadian corporations such as Ontario Hydro, or Ottawa Hydro company or a public statutory body, where yearly decisions are made about what sets of alternative projects or operations may be initiated to reduce technical as well as non- technical losses in a particular sector.

It has been observed that a huge percentage of electrical power generated throughout the system is being lost directly or indirectly in absence of a control system for operational management. As a remedial measure, some technical as well as administrative and financial management steps may be taken to improve the existing systems and thereby greatly reduce this unwanted loss.

1.3.2 Sources of Power Systems losses

A first focus is placed on the sources of losses, their remedy, and a framework to analyze electrical distribution. The structure of an electric power system is very large and complex. Its main components (or sub-systems) can be identified as:

- generation,
- transmission,
- distribution,

These three inter-related sub-systems form the basis of the electric power system. The model developed in this thesis addresses on the well-known issues that follow:

- the distribution system is faced with large energy losses,
- these losses can be controlled or minimized and computed through proper regulation after analysis of their characteristic behaviors.

The occurrence of losses can be divided into two broad categories, e.g. technical losses and non-technical losses, which may be detailed as below.

1.3.2.1 *Technical losses*

- (i) line loss that include not only energy dissipation but also distribution along non-optimal routes,
- (ii) leakage loss through grounding and earth fault and or insulators,
- (iii) auxiliary equipment losses (such as from transformers, circuit breakers and other sub-station equipment),
- (iv) loss due to local distribution to (e.g. due to faulty energy meters or connections bypassing (unintentionally) of the meters at) consumer premises,
- (v) abrupt load demand fluctuation.

All of above losses can be controlled or minimized, often simply by adopting appropriate technical measures [Anderson and Fouad, 1997].

1.3.2.2 *Non - technical losses*

Those appear under the form of revenue losses, they may effectively distort the pattern of demand and consumption and practically arise from commercial, socio-political factors such as:

- (i) using service connection bypassing the usual energy meter, meter tempering, tendency of the consumer to run their facilities at overload bypassing the power factor correction (PFC) meter to avoid PFC charges, or avoiding maximum demand indicator (MDI), damaging the peak and off- peak load recording timer;
- (ii) wrong entry of meter reading or wrong billing incorrect supply of services and improper accounting;
- (iii) lack of conservation in premises;
- (iv) peak-hour usage, whether official or not.

These losses can be controlled by:

- socio-political motivation, regulation and enforcement, vigilance and
- taking appropriate management control through accurate computation, optimal operation, regular monitoring and control.

Both these broad types of losses are inter-related in some way either at the locus of demand or at an organizational level as well. This calls for a system-wide response to minimize system loss in power distribution. Here, considering the vast of the field, I have concentrated my research only on the analysis of the characteristic behavior for technical load and losses of equivalent power network systems for their analysis and optimal management.

This analysis of the characteristic behavior of load and loss may then be generalized to the entire system for reduction of loss for the entire power network. An overview of a practical system load balancing is given in Section 1.4

1.4 BALANCING AN UNSYMMETRICAL RESISTIVE LOAD (3-phase):

The aim of this section is to briefly demonstrate the application of load reduction to a real case of power management. A typical power distribution system comprises 3-phase sources with three impedances as a symmetrical set of 3 equal resistors.

As the power factor (P. F.) of the “symmetrical load” equals one, the source neither delivers nor receives any reactive power. The explanation is, of course, that with the LC making current equation (magnitude version) as $|I_L| = |I_C| = 1/\sqrt{3} |I_R|$, the exact reactive power needed by the reactor is delivered by the capacitor. It should be mentioned that if the phase sequence was reversed or the two reactive elements changed place, the above symmetrical features would not result.

1.4.1 Load characteristics

The term load here refers to a device or a conglomeration of devices that tap energy from the network. In general, load devices may range from a few Watts to a multi-Megawatt heating equipment, induction motor or to a diverse electronic gear. Figure 1.1 shows the phasor sequence of three-phased power supply network.

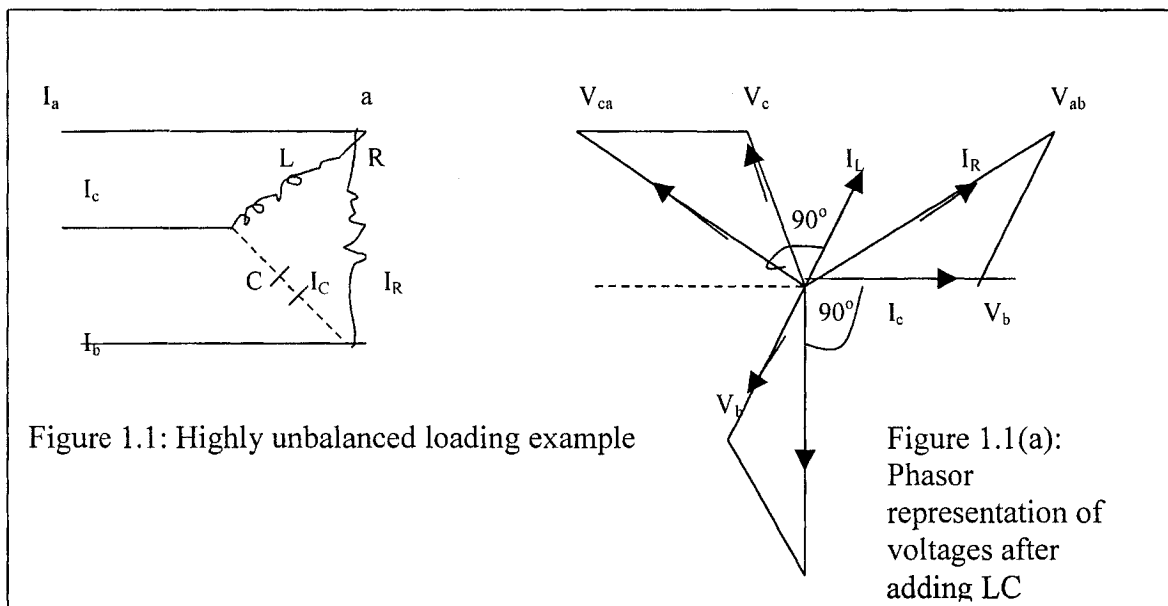
In practice, to supply this load, the electrical distributors use a multitude of devices characterized by vast differences in regard to:

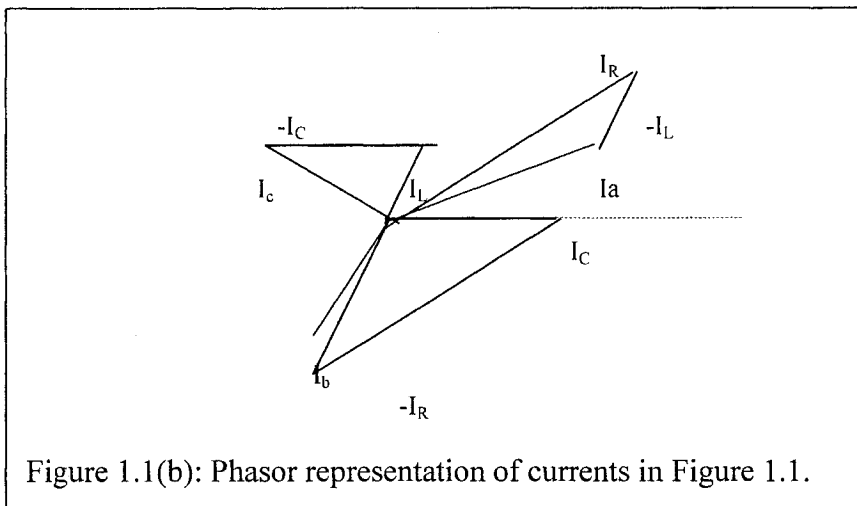
- (1) size,
- (2) symmetry (single or 3-phase),
- (3) load constancy (in respect of time, Frequency, and voltage),
- (4) use of cycle (regular or random use).

However, the effect of such variation is dampened by its past load demand and distribution statistics.

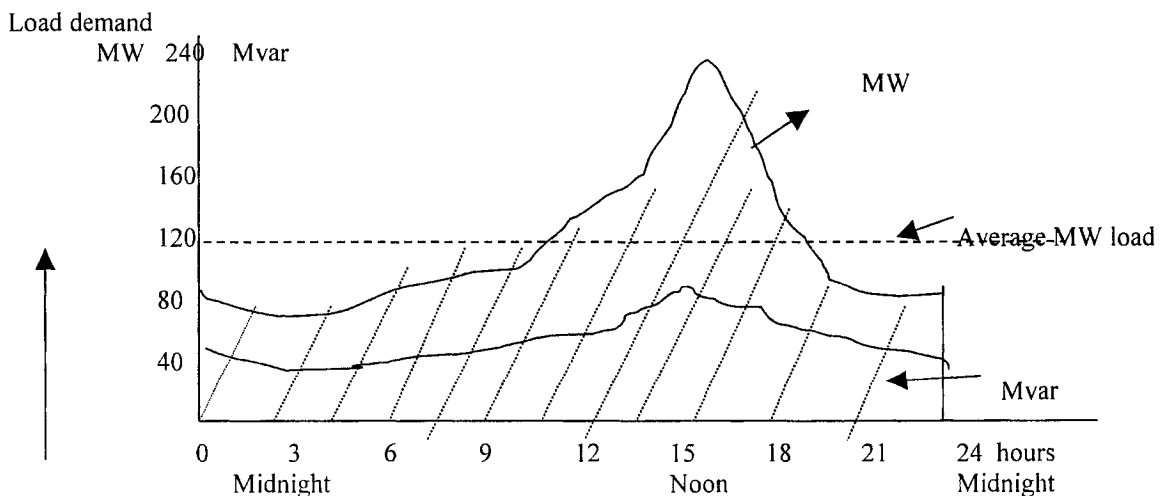
In summary, we can take the following rules characterizing the typical systems loads.

1. Although individually of random type, the lumped or composite loads as we encounter them at sub-transmission or transmission levels, are highly of predictable character.
2. These lumped loads vary in predictable fashion with time. Figure 1.2 shows some typical load variations for a small company. In general there is considerable variation, not only throughout the hours of the day, but also between weekdays and weekends / holidays and between seasons.





3. Although the loads are time variant, the variations are relatively slow. From minute to minute, we have an almost constant load. A minute is a long time compared to the electrical time constants of the power system. This permits us to consider the system operating in steady state, a steady state that slowly shifts through the hours of the day (quasi-static operation).
4. The typical load always consumes reactive power. The reason for this is that motor load is an important (actually the most important) ingredient in most cases. Motors are *always* (with the exception of over-excited synchronous machines) inductive.



5. The typical load is always symmetric. In the case of large motors (greater than a few horse power), this symmetry is automatic since they are always designed for balanced 3-phase operation.

In the case of single-phase devices, the symmetry comes about by statistical effects and intentional distribution between phases.

1.4.2 Voltage and Frequency Load Dependency

An important feature characterizing all loads is their dependency on voltage and frequency. During faults and other abnormal situations, the former may vary greatly, resulting in major load fluctuations. Even minor changes in voltage and frequency can cause load changes of practical significance (like for impedance and motor loads).

Impedance type loads:

Impedance type load comprises of lighting, heaters, ovens, and similar load objects.

Defining P and Q respectively as real and reactive power load, V as the voltage, X as the reactance, Y as the admittance, we can relate them as below-

$$\text{We know that } P + jQ = |V|^2 Y^* = |V|^2 1/(R-jX) = |V|^2 (R+jX)/(R^2+X^2)$$

Clearly, the real and reactive loads both are proportional to the square of voltage |V|. For small voltage perturbation Δ|V|, we have for the real power,

$$\Delta P / \Delta |V| \approx \partial P / \partial |V| = 2 |V| R / (R^2+X^2) = (2 / |V|) P$$

$$\text{Or, } \Delta P / P \approx 2 \Delta |V| / |V| \quad \dots \quad \dots \quad \dots \quad (1.1)$$

The result tells that a small relative change in voltage results in twice the relative change in megawatts.

The reactance X depends upon frequency f according to $X = 2\pi fL$.

Thus we have,

$$\begin{aligned} \Delta P / \Delta f &\cong \delta P / \delta f = - |V|^2 (4RX\pi L) / (R^2+X^2) \\ &= - |V|^2 (2RX^2) / (R^2+X^2) = -P (2X^2) / f (R^2+X^2) \end{aligned}$$

$$\text{Or, } \Delta P / P \cong -\{(2X^2)/(R^2+X^2)\} (\Delta f/f) \quad \dots \quad \dots \quad (1.2)$$

For a power factor i.e. $\cos\phi = 0.8$ we have

$$X^2/(R^2+X^2) = \sin^2\phi = 0.36, \therefore \Delta P / P \cong 0.72 (\Delta f/f)$$

Thus a one percent frequency *drop* results in 0.72 percent load *increase*.

Motor load:

Induction motor load dominates this group. Its dependence upon voltage and frequency is somewhat more complicated to analyze.

In conclusion, compared to 2% reduction in the case of an impedance load, a motor will reduce its power drain by only 0.2% for a voltage drop of 1%.. Power companies in times of generation shortage, will resort to load shedding (“brownouts”) in order to match their load to available generation. They can accomplish this by lowering slightly the operating voltage. The above teaches us that impedance loads give a considerably better power reduction than motor loads under such conditions.

1.4.3 Load Management

Power companies may consider that they have the “holy” duty to plan their generating and or transmission capacity. This is done to meet every whim of their customers keeping the system operation economically as well as technically viable and taking into account the peaks and valleys in the demand curve (Figure 1.2).

It can be implemented by timer-controlled switching of load objects of selected low priority loads and or by specially designed tariff or rate structures, which encourage the individual customers to re-adjust their electric use schedules.

1.5 OVERVIEW OF THE METHODOLOGY

With the preceding general background, we can now focus on the methods used in the thesis along two directions. As a preamble, we focus on the main technique of simulation used in Chapters 3 and 4. Then we review the classical network reduction technique, Pi and T – equivalence, the former of which is used throughout Chapters 2 and 3.

1.5.1 Simulation of Networks

Computer simulation is the discipline of *designing and modeling* an actual or theoretically physical system, executing the model on a digital computer and analyzing the executed output. Simulation is the principle of “learning by doing” – to learn about the system one may ‘first build a model and make it run’ [Fishwick, 1995].

Hence, with a view to set a basis for the optimized management of the entire array of power system losses, we may develop models, simulate and test them to find out their optimal parameters. The engineers in their decision-making processes may use this.

The next section reviews classical methods that characterize intrinsic properties of electrical networks [Gonen, 1988]; in particular, they allow for negative variables and implicitly address time-phased flows via impedances.

1.5.2 Equivalent ‘T’ and ‘Pi’ - Circuits

For the purpose of calculating technical losses (described above) or analyzing the behavior of any power system network, we may use the benefit of equivalent T and Pi circuits. This can be summarized in general for n number of sections connecting different nodes or load centers constituting the power system under consideration.

Note however that the impedance and admittance of T or Pi networks do not have the same values as those of which would be measured in the equipment. For instance, a particular equipment having general network constants A, B, C, and D is to be replaced by an equivalent T -circuit, that is a circuit composed of two impedances Z_1 and Z_2 (not necessarily equal as nominal T) and admittance Y_3 connected between their junction and the opposite line, as shown in Figure 1.5.a.

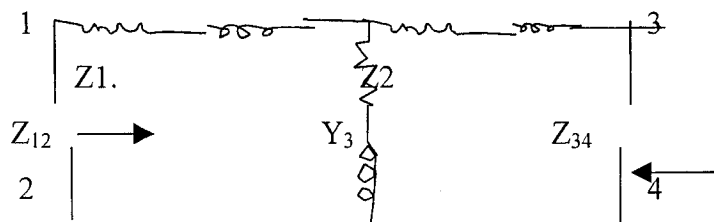


Figure 1.5.a - T network

We obtain the values of Z_1 , Z_2 and Y_3 by equating the elements of the impedance matrix of the T network, the general constants of the actual equipment. The impedance matrix of the T network of Figure 5.a is

$$\begin{vmatrix} 1 + Z_1 Y_3 & Z_1 + Z_2(1 + Z_1 Y_3) \\ Y_3 & 1 + Z_2 Y_3 \end{vmatrix} \quad \dots \quad (1.3)$$

$$A = 1 + Z_1 Y_3$$

Equating the elements of this matrix to the given constants, $C = Y_3$ (1.4)

$$D = 1 + Z_2 Y_3$$

Gives the values for the impedance and admittance of the equivalent T -network as,

$$Z_1 = (A-1)/C, Z_2 = (D-1)/C, Y_3 = C. \quad \dots \quad (1.5)$$

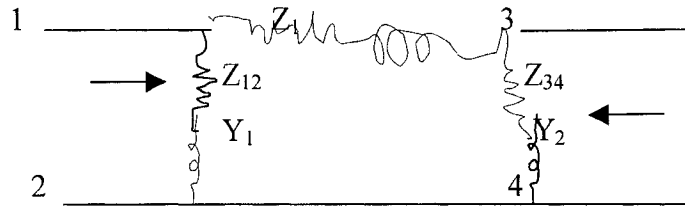


Figure 1.5.b - Pi network.

In similar manner, the values of the Pi –equivalent network of Figure 5.b, Y_1 , Y_2 , and Z_3 may be determined; the results being,

$$Y_1 = (D - 1)/B, Y_2 = (A-1)/B, Z_3 = B. \quad \dots \quad (1.6)$$

A direct determination of the equivalent T-impedance and admittance from test is possible by measuring input impedance of a network with the other end first open-circuited, then short circuited. In Figure 1.5.a and b, with supply terminals 1 and 2, the input current and voltage are measured to obtain the input impedance under two conditions:

- (1) On open - circuit; and
- (2) On short - circuit.

From Figure 1.5a and b, it will be seen that input impedance of the circuit, in the two cases, has the following values:

$$(1) \text{ With terminal 3 and 4 open - circuited, } Z_{12oc} = Z_1 + 1/Y_3. \quad \dots \quad (1.7)$$

$$(2) \text{ With terminal 3 and 4 short - circuited, } Z_{12sc} = Z_1 + Z_2 / (1 + Z_2 Y_3) \quad \dots \quad (1.8)$$

With supply from terminal 3 and 4 and an open - circuit on terminals 1 and 2, the supply voltage and current are measured, as before to obtain the input impedance.

$$Z_{34oc} = Z_2 + 1/Y_3, \quad \dots \quad \dots \quad \dots \quad (1.9)$$

By subtraction, we get

$$Z_{12oc} - Z_{12sc} = (1 / Y_3 + Z_2 / Y_3 - Z_2 / Y_3) / \{(1 / Y_3) + Z_2\}, \quad \dots \quad (1.10)$$

Or, by substitution,

$$Y_3 = 1 / \pm \sqrt{\{Z_{34oc} (Z_{12oc} - Z_{12sc})\}} \quad \dots \quad \dots \quad (1.11)$$

Substituting the values of Y_3 we get impedance Z_1 and Z_2 ,

$$Z_1 = Z_{12oc} \pm \sqrt{\{Z_{34oc} (Z_{12oc} - Z_{12sc})\}}, \quad \text{and} \quad \dots \quad \dots \quad (1.12)$$

$$Z_2 = Z_{34oc} \pm \sqrt{\{Z_{34oc} (Z_{12oc} - Z_{12sc})\}}, \quad \dots \quad \dots \quad (1.13)$$

If the network is replaced by a symmetrical equivalent T circuit, the third measurement for Z_{34oc} is unnecessary since $Z_2 = Z_1$.

The equivalent Pi -circuit of Figure 5.b may also have its impedance and admittance determined by open and short circuit tests.

The expressions are,

$$Z_3 = Z_{34oc} \cdot Z_{12sc} / \pm \sqrt{\{Z_{34oc} (Z_{12oc} - Z_{12sc})\}}, \quad \dots \quad \dots \quad (1.14)$$

$$Y_1 = [Z_{34oc} \pm \sqrt{\{Z_{34oc} (Z_{12oc} - Z_{12sc})\}}] / (Z_{34oc} \cdot Z_{12sc}) \quad \dots \quad (1.15)$$

$$Y_2 = [Z_{12oc} \pm \sqrt{\{Z_{34oc} (Z_{12oc} - Z_{12sc})\}}] / (Z_{34oc} \cdot Z_{12sc}) \quad \dots \quad (1.16)$$

And, we may obtain the similar expressions for the equivalent T circuit above.

CHAPTER 2

MODELING AND SIMULATION OF THE NETWORK SYSTEM

Extending the classical analysis of electric power network presented at the end of Chapter 1, this chapter will cast it as a complete model to be used for simulation and numerical analysis, also making provisions for future optimization. Section 2.1 is a description of the general methodology used to describe electric networks, using state equations in differential form. Section 2.2 casts it as a consistent system in differential equation form. Section 2.3 adapts the system to one more easily used by power engineers, which will in turn enables us to select practical energy distribution characteristics. In this spirit, Section 2.4 illustrates the benefit of the previous transformation to power engineering, using a specific example.

2.1 MODELING ELECTRIC NETWORKS

This section sets the methodological foundations for the simulation. It also introduces the notation specific to the case analyzed. It derives the differential equations necessary to build the overall system. Using the equations, it calculates the quantities of practical interest to power managers. Formulation of suitable dynamic (mathematical) model describing the relationships between state variables such as voltages, currents, and power flow in the interconnected system.

A general analysis of electrical power systems can be divided as follows:

1. Specification of the power and voltage-current constraints that must apply to the various buses of the network.
2. Numerical computation of the power-flow equations subject to the above constraints: these computations give us the values of all bus voltages line and load current.
3. When all bus voltages have been determined, one must finally compute the actual power-flow (and thereby the energy) in all transmission-distribution lines.
4. When all bus voltages and line currents have been determined, one must finally compute the actual load demand factor and loss factor of the power distribution system.

Such calculations aim at steady-state behavior of the system. It is well-known that the related transient behavior may be unacceptable. A controlled theoretical framework enables us to address both transient and steady state conditions. Actually, it may allow designers to satisfy additional conditions (maximum deviations of node voltages, limits on load and loss factors, etc) even though such conditions are handled only implicitly in this thesis. A complete dynamic model for a transmission and distribution line and loads is derived. It is important to note that in the dynamic model the controls (capacitors or load currents or load resistance and inductance) are time-varying parameters.

2.1.1 Modeling Methodology

Additionally, the mathematical model could further be developed to minimize certain functional related to system losses; for example one that decrease the magnitude of the oscillations of the transient phase, or its duration. Such optimization is not attempted directly in this thesis because of its difficulty. It is addressed indirectly however by the selection of proper parameters that yield solutions satisfying the targets delineated above. In this sense, it is an attempt to optimize distribution systems iteratively. We also need to apply more detailed rules for the transmission line which in this study, is assumed to be long and connected to an infinite bus. Here, circuits are limited to three nodes from which load currents can be drawn. A single-line diagram of the transmission and its loads is shown in Figure 2.1.

Generally, the term ‘load’ refer to a device or to a conglomeration of devices that tap energy from the network. Load management is an important part of the power industry. In a power distribution system, the load varies widely over the period of twenty-four hours and consequently, the voltage may also change. Small deviation of voltages from a fixed reference is regulated by automatic voltage regulators located at generating terminals. But this process cannot necessarily control the voltages at distant buses. However, the voltages at the distant buses can be controlled in the following ways:

- a) either controlling load current from distribution load centers with the help of regulators (like rheostats), or changing both load resistance and inductance; or,
- b) a proper application of capacitors placed at the nodes; the normal practice in this regard, is to apply either:

- i) synchronous capacitors (varied continuously) or
- ii) static capacitors (switched in steps).

The preceding list of stages will enable us to develop, a complete dynamic model for a transmission-distribution line and loads, taking the above options into consideration. It is important to note in the dynamic model that, even though the controls are time varying parameters; we simplify the assumptions by considering line currents and voltages across the bus capacitors as the state variables instead of essentially considering the charge stored in the capacitor.

The following assumptions are made:

- i. line parameters are of per unit values (of line lengths) in all the cases of resistance, inductance, capacitance,
- ii. eddy current losses are neglected,
- iii. leakage currents are normally negligible (though incorporated here in the program for numerical analysis),
- iv. the transmission line distances between all 3 -phases as well as from the ground are uniform at safe distance. 3-phase sources experiences the 3 impedance as a symmetrical set of 3 equal resistors.
- v. the generator and source supply voltage remain more or less constant.

Considering only analog networks, flows can occur across a component in either direction, so we must base our dynamics on conservation of energy flow and potential.

1. **Power Relation:** keeping track of two variables, current and voltage for a circuit, $V = RI$ defines the voltage drop across a resistor = resistance times current flowing through the resistor. This is linear model - as relationship involves a linear combination of variables, [certain resistors can be defined by more complex non-linear relationships such as FETs' (field emitting transistors) that contain a region of approximate linear (I.V) behavior. A variable load would cause varying currents and voltages in the transmission lines and nodes respectively. This requires a dynamic representation of the transmission system.
2. **Kirchoff s' current law (KCL):** sum value of all current entering a node equals sum value of all current exiting that node. Since currents are denoted positive or negative depending on

whether they flow to or from node; the law is more precisely defined so that all currents for any node add to zero. KCL is a node law, since it focuses on the conservation of flow where links meet (at nodes).

3. Kirchoff s' voltage law (KVL): For all loops in a circuit. The voltage drops around each loop add to zero. KVL can be thought of as a loop law.

More specifically, the above three types of rules serve to characterize the lumped dynamics of all electrical circuits. First, note that the sum of all currents at a node is zero and that of all voltages around a closed circuit is also zero; components in this type of network are current flow, electrical potential.

Using a typical configuration given in Figure 2.1, the mathematical description of each system will be derived by considering a case of power distribution and systematically applying the above rules.

2.1.2 Description of components and variables

The general methodology is now applied to the specific case of Figure 2.1, which depicts an electrical transmission-distribution network with π -equivalent circuit sections connected to an infinite bus. We focus on the special case of equivalent 3 bus (out of an n-th bus equivalent circuits) network with additional constraints at the nodes and links.

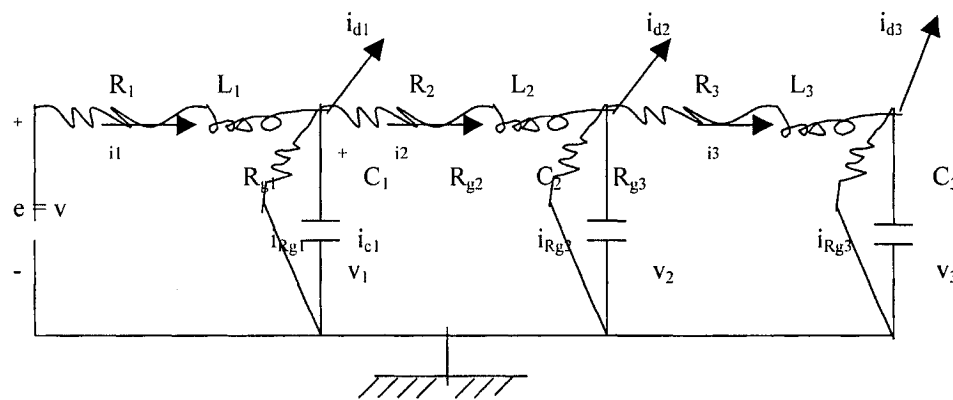


Figure 2.1 Typical Power Distribution System

Its elements and parameters are as listed below.

A supply source and input voltage, $v(t) = V_{k-1} \sin(2\pi ft)$

Line resistors at different bus and section $(R_1, R_2, \dots) = R_k$, where $k=1, 2, 3$

Some line inductance at different buses and sections $(L_1, L_2, \dots) = L_k$,

Grounding wire and insulator leakage resistances at different bus sections $(R_{g1}, R_{g2}, \dots) = R_{gk}$

Load current flows at different bus section or tapped from nodes (i_{d1}, i_{d2}, \dots) with values:

$$i_{dk}(t) = I_{dk} \sin(2\pi ft) \text{ and}$$

Compensating (Control) capacitors installed at different bus sections with the capacitance value

$$C_k = (C_1, C_2, \dots)$$

The power demand (or loads consumed) from different nodes with value P_{ck} ,

where, $k = 1, 2, 3$.

Given these characteristics, we state additional conditions that will help us produce workable solutions.

2.1.3 Basic Derivation of the Model

In this section, the state equations are derived from the previous of inter-relationships of the state variables. Consider Figure 2.1, where the types of *systems components* are the voltage source, line and leakage resistance, line inductance, node capacitance and load resistance, load inductance, and state variables are line currents, node voltages, control is load current or load parameters. Applying the KVL, KCL, and Power relations cited above normally (without load) we get:

$$v = V_{in} = iR + (1/C)idt + L(di/dt) = iR + C(dv/dt) + L(di/dt). \quad (2.1)$$

Considering our systems of power network (Figure 2.1), with an A.C. supply at any time t , *state variables* are defined as:

Line current of k -th loop = $i_k(t) = I_k \sin(\omega t)$ amps, where:

$$I_k(t) = \text{max amplitude of line current of } k\text{-th loop};$$

Voltage at the k -th node = $v_k(t) = V_k \sin(\omega t)$ volts, where:

$$V_k(t) = \text{max amplitude of voltage at the } k\text{-th node};$$

Controlled Load demand in terms of current from the k -th node and load center is:

$$i_{dk}(t) = I_{dk} \sin(\omega t), \text{ where:}$$

$I_{dk}(t)$ = max amplitude of load current demand from the k-th node; and

$\omega = 2\pi f$, where, f = systems supply frequency;

Line parameters at the k-th loop (like line resistance, inductance, capacitance and the Insulator and grounding resistance taken per unit length) are respectively R_k, L_k, C_k, R_{gk} .

Applying the KVL and KCL in our system at node 1 and loop 1 we get,

$$\begin{aligned} L_1 di_1/dt + R_1 i_1 + v_1 &= v \\ C_1 dv_1/dt - v_1/R_{g1} - i_2 - i_{d1} &= i_1 \end{aligned} \quad (2.2)$$

Applying the KVL and KCL in our system at node 2 and loop 2 we get,

$$\begin{aligned} L_2 di_2/dt + R_2 i_2 + v_2 &= v_1 \\ C_2 dv_2/dt - v_2/R_{g2} - i_3 - i_{d2} &= i_1 \end{aligned} \quad (2.3)$$

Applying the KVL and KCL in our system at node 3 and loop 3 we get,

$$\begin{aligned} L_3 di_3/dt + R_3 i_3 + v_3 &= v_2 \\ C_3 dv_3/dt - v_3/R_{g3} - i_4 - i_{d3} &= i_3 \end{aligned} \quad (2.4)$$

Similarly, applying the KVL and KCL in our system at nth node and nth loop we get,

$$\begin{aligned} L_n di_n/dt + R_n i_n + v_n &= v_{n-1} \\ C_n dv_n/dt - v_n/R_{g2} - i_{n+1} - i_{dn} &= i_n \end{aligned} \quad (2.5)$$

2.1.4 A model in differential equation form

The various state equations are now grouped as a complete system of differential equations. A varying load would cause varying currents and voltages in the transmission lines and at the nodes, respectively. This requires a dynamic representation of the power transmission systems network. By applying Kirchoff's laws of currents and voltages (as shown in Equations 2.1 – 2.5) and considering the systems parameters R_k, L_k, C_k, R_{gk} . (as constants or varying pu parameters) and the loads (unknown resistive and inductive in series), we can develop and write the mathematical dynamics of the model in the form of a set of following differential equations.

$$\begin{aligned} di_1/dt &= -(R_1 / L_1)i_1 + v / L_1 - v_1 / L_1 \\ dv_1/dt &= -v_1/(C_1 R_{g1}) + i_1/C_1 - i_2/C_1 - i_{d1}/C_1 \\ di_2/dt &= -(R_2/L_2)i_2 + v_1/L_2 - v_2/L_2 \\ dv_2/dt &= -(1/C_2 R_{g2})v_2 + i_2/C_2 - i_3/C_2 - i_{d2}/C_2 \\ di_3/dt &= -(R_3/L_3)i_3 + v_2/L_3 - v_3/L_3 \end{aligned}$$

$$\begin{aligned}
dv_3/dt &= -(1/C_3 R_{g3})v_3 + i_3/C_3 - i_4/C_3 - i_{d3}/C_3 & \dots & \dots & (2.6) \\
\dots & \dots & \dots & \dots & \\
di_n/dt &= -(R_n/L_n)i_n + v_{n-1}/L_n - v_n/L_n \\
dv_n/dt &= -(1/C_n R_{gn})v_n + i_n/C_n - i_{n+1}/C_n - i_{dn}/C_n
\end{aligned}$$

Angular velocity $\omega = 2\pi f = 2 \times 3.14 f$, generally we can simulate taking different values of systems frequency $f =$ e.g. 60 cps, 30 cps, 10 cps, 5 cps, 1cps, 0.5 cps, 0.1 cps etc.

Incorporating the practical sinusoidal nature of power system voltages and currents as defined above, enables one to develop a program that will numerically solve and analyze the characteristic behavior of the power systems network, using the above set of differential equations.

2.1.5 Power systems loss factor and the load factor

The system described in section 2.1.4 enables us to find the practical relationships that enable distribution managers to compute load and loss factors, which will be analyzed in chapter 3. For our distribution system Figure 2.1 and its set of Equations (2.1- 2.6), the power balances at different nodes at any time t , are shown below.

Sending end power = receiving end power consumed + losses (i.e. line losses and leakage losses).

At node and load center 1,

$$v_1 i_1 = v_1 i_{d1} + i_1^2 R_1 + (v_1/R_{g1})^2 R_{g1} + v_1 i_2 \quad \dots \quad (2.7)$$

At node and load center 2,

$$v_1 i_2 = v_2 i_{d2} + i_2^2 R_2 + (v_2/R_{g2})^2 R_{g2} + v_2 i_3 \quad \dots \quad (2.8)$$

At node and load center 3,

$$v_2 i_3 = v_3 i_{d3} + i_3^2 R_3 + (v_3/R_{g3})^2 R_{g3} + v_3 i_4 \quad \dots \quad (2.9)$$

At node and load center 4,

$$v_3 i_4 = v_4 i_{d4} + i_4^2 R_4 + (v_4/R_{g4})^2 R_{g4} + v_4 i_5 \quad \dots \quad (2.10)$$

At node and load center n ,

$$v_{n-1} i_n = v_n i_{dn} + i_n^2 R_n + (v_n/R_{gn})^2 R_{gn} \quad \dots \quad (2.11)$$

Taking account of only the real power (summing them all of above equations 2.7 - 2.11) we get,

$$\begin{aligned}
v i_1 &= v_1 i_{d1} + i_1^2 R_1 + (v_1)^2 / R_{g1} + v_2 i_{d2} + i_2^2 R_2 + (v_2)^2 / R_{g2} + v_3 i_{d3} + i_3^2 R_3 + (v_3)^2 / R_{g3} + \\
&\dots + v_n i_{dn} + i_n^2 R_n + (v_n)^2 / R_{gn} + v_n i_{n+1} \\
&= \sum_{k=1}^n v_k i_{dk} + \sum_{k=1}^n i_k^2 R_k + \sum_{k=1}^n v_k^2 / R_{gk} + v_k i_{n+1} \\
&= \left\{ \sum_{k=1}^n V_k I_{dk} + \sum_{k=1}^n I_k^2 R_k + \sum_{k=1}^n V_k^2 / R_{gk} + V_k I_{n+1} \right\} (\sin(\omega t))^2
\end{aligned}$$

$$\text{or } P_{input}(t) = \sum_{k=1}^n P_{dk}(t) + \sum_{k=1}^n P_{lossk}(t) = P_c(t) + P_{ls}(t) \quad \dots \quad (2.12) \text{ where,}$$

$P_{input}(t)$ is the power supplied to the system at any time t,

$P_c(t)$ is the power load consumed from the system at time t, and

$P_{ls}(t)$ is the power loss from the system at time t.

Or Power intensity of loss at any time t:

$$\begin{aligned}
P_{loss} &= \left[\sum_{k=1}^n P_{k-1}(t) + \sum_{k=1}^n P_{dk}(t) \right] = v^* i_1 - \sum_{k=1}^n v_k i_{dk} \\
&= \sum_{k=1}^n i_k^2 R_k + \sum_{k=1}^n v_k^2 / R_{gk} = \left\{ \sum_{k=1}^n I_k^2 R_k + \sum_{k=1}^n V_k^2 / R_{gk} \right\} (\sin(\omega t))^2 \quad \dots \quad (2.13)
\end{aligned}$$

Or Total energy loss throughout the period (T),

$$\begin{aligned}
E_{ls}(T) &= \int_0^T [v x_1 - \sum_{k=1}^n x_{2k} u_k] (\sin(\omega t))^2 \\
&= \int_0^T \left[\sum_{k=1}^n x_{2k-1}^2 R_k + \sum_{k=1}^n x_{2k}^2 R_{gk} \right] (\sin(\omega t))^2 dt \quad \dots \quad (2.14)
\end{aligned}$$

Average of energy loss (power per unit of the total time period of the day or simulation time T),

$$\begin{aligned}
E_{lsa} = P_{lsa} &= (1/T) \int_0^T P_{ls}(t) dt = (1/T) \int_0^T \left[\sum_{k=1}^n P_{k-1}(t) - \sum_{k=1}^n P_{dk}(t) \right] dt \\
&= (1/T) \int_0^T \left[\sum_{k=1}^n v^* i_1(t) - \sum_{k=1}^n v_k i_{dk}(t) \right] dt \\
&= (1/T) \int_0^T \left[\sum_{k=1}^n R_k * I_k^2(t) + \sum_{k=1}^n (1/R_{gk}) v_k^2(t) \right] (\sin(\omega t))^2 dt \quad \dots \quad (2.15) \\
&= (1/T) \int_0^T \left[\sum_{k=1}^n R_k * x_k^2(t) + \sum_{k=1}^n (1/R_{gk}) x_{2k}^2(t) \right] (\sin(\omega t))^2 dt
\end{aligned}$$

Power intensity of loss at peak load demand,

$$\begin{aligned}
 P_{LossPeak} &= P_{ls}(t_p) = \left[\sum_{k=1}^n R_k i_k^2 + \sum_{k=1}^n (1/R_{gk}) v_k^2 \right]^{peak} \\
 &= \left[\sum_{k=1}^n R_k I_k^2 + \sum_{k=1}^n (1/R_{gk}) V_k^2 \right] (\sin(\omega t))^2 \dots \quad (2.16) \\
 &= \left\{ \sum_{k=1}^n R_k x_k^2(t_p) + \sum_{k=1}^n (1/R_{gk}) x_{2k}^2 \right\} (\sin(\omega t))^2 \dots
 \end{aligned}$$

When magnitude of i_{dk} is peak, i.e. at time of peak load demand t_p ; [P_{lsP} can be found from the $P_{Loss}(t)$ vs t graph].

Loss factor, $F_{ls} = \frac{E_{lsa}}{P_{ls}(t_p)} = \frac{P_{lsa}}{P_{ls}(t_p)} \dots \quad (2.17)$

or $F_{Ls} = 1/T \left[\left\{ \sum_{k=1}^n \int_0^T i_k^2 R_k dt + \sum_{k=1}^n \int_0^T (v_k)^2 / R_{gk} dt \right\} / \left[\sum_{k=1}^n i_k^2 R_k + \sum_{k=1}^n (v_k)^2 / R_{gk} \right]^{Peak} \right.$
 $= 1/T \left[\sum_{k=1}^n \int_0^T x_{2k-1}^2 R_k dt + \sum_{k=1}^n \int_0^T (x_{2k})^2 / R_{gk} dt \right] / \left[\sum_{k=1}^n x_{2k-1}^2 R_k + \sum_{k=1}^n (x_{2k})^2 / R_{gk} \right]^{Peak} \dots \quad (2.18)$

Which is expected to be (preferably be approx. zero) $\equiv \leq 1$ for a certain period of time considered.

Power intensity of load demand or consumption (instantaneous),

$P_c(t) = P_d(t) = \sum_{k=1}^n R_{dk} i_{dk}^2 = R_{d1} i_{d1} + R_{d2} i_{d2} + R_{d3} i_{d3} \quad \text{for } n = 3 \dots \quad (2.19)$

Peak of Power intensity of Load demand or consumption,

$P_{cpeak}(t_p) = P_{dpeak}(t_p) = \sum_{k=1}^n R_{dk} i_{dk}^2 = [R_{d1} i_{d1} + R_{d2} i_{d2} + R_{d3} i_{d3}]^{peak} \dots \quad (2.20)$

where, t_p = is the time of peak demand

Total **Energy demand or consumed** at the end of period T, is

$E_c(T) = \int_0^T P_c(t) dt \dots \quad (2.21)$

Average of Energy demand or consumed during the period T, is

$E_{ca}(T) = (1/T) \int_0^T P_c(t) dt \dots \quad (2.22)$

Load demand Factor, $F_c = F_d = E_{ca}(T) / P_c(t_p)$

= Average load demand or consumed / Peak load demand or consumed

$= 1/T \left[\sum \int_0^T v_k i_{dk} dt \right] / \left[\sum v_k i_{dkPeak} \right] = 1/T \left[\sum \int_0^T v_k i_{dk} dt \right] / \left[\sum v_k i_{dkPeak} \right],$

$$= 1/T [\Sigma \int_0^T x_{2k} u_k dt] / [\Sigma (x_{2k} u_k)^{Peak}], \quad \dots \quad (2.23)$$

Clearly, the above expression is no greater than 1. Actually, values close to 1 are preferable for the period of time considered. We can numerically find out this $P_c(t_p)$ or from their $P_c(t)$ versus t plots.

2.2 STATE EQUATIONS OF THE SYSTEM

Modern control theory offers integrated methods of finding the behavior of a differential system and controlling it, while optimizing some objective selected by decision makers. This is applicable to the system that we are analyzing, a particular case of the general multiple-input, multiple-output systems that modern control theory can address. These systems may be linear or non-linear, time-invariant or time varying, Such systems can be described by sets of differential equations, which give mathematical relations between the input and output. In general, these equations may be written in the form,

$$\dot{x}(t) = f(t, x(t), u(t)), \quad \text{for } 0 < t \leq T$$

$$x(0) = x_0$$

Where, we define

$x(t) \in R^n$ denotes the state vector at time t and $x_0 \in R^n$ is the initial state;

$u(t) \in R^n$ is the control vector; T is the terminal time and

$\dot{x}(t)$ is the derivative of x with respect to time t .

While this framework of Section 2.1 is simple, we find it useful to recast it in a more general formulation so as to harness it to the methodology described immediately above, which in particular will enable us to perform simulation and analyze it in Chapter 3 and 4.

2.2.1 Canonical form of state equations of the power systems network

In our case, the state equations for the power distribution system (possibly a continuous, time varying system with control) may be derived from Equations (2.6), and is re-written as:

$$\dot{x} = \frac{d}{dt} x = Ax + Bu + Gv \quad \dots \quad (2.24)$$

where, $x(t)$ the state vector. A is the line parameter matrix, B is the load parameter matrix, G is the supply matrix. And α is the line parameter co-efficient. Selecting line parameters per unit of length we can ignore α for the sake of modeling and simulation. A solution of the same is given by:

$$x(t) = e^{tA} X_0 + \int_0^t e^{(t-\theta)A} B(\theta) u(\theta) d\theta + \int_0^t e^{(t-\theta)A} G(\theta) V(\theta) d\theta \quad \dots \quad (2.25)$$

In our system, we may take the line currents $i_k = x(t)_{2k-1}$, the node voltages $v_k = x(t)_{2k}$, Load currents $i_d = u(t) = [u_1(t), u_2(t), \dots, u_k(t)]$ E U as the controls, for $k=1, 2, 3, \dots, n$ = the number of nodes.

Given the main system supply voltage = $v(t)$ of the system for any non-negative value of t [i.e. $t \geq 0$]; for our distribution systems we may take $v(t)$ to be at constant amplitude which is generally being maintained by the Generation and or Transmission authority.

Neglecting the reactive loads we can model the power distribution systems as a distinct one and adding them up to n th-nodes and load centers (as shown above in equation 2.26). Normalizing the line parameters by considering their per unit values (i.e. Resistance, Inductance, and Capacitance per unit of line lengths) we can analyze the system numerically.

Now taking, $i_1 = x_1$, $v_1 = x_2$, $i_2 = x_3$, $v_2 = x_4$, $i_3 = x_5$, $v_3 = x_6$, and $i_{d1} = u_1$, $i_{d2} = u_2$, $i_{d3} = u_3$, we can re-write Equation (3.5) in canonical form as:

$$\begin{aligned} \frac{dx_1}{dt} &= (1/L_1)(v - x_2 - R_1 x_1) \\ \frac{dx_2}{dt} &= (1/(C_1 + C_{d1}))(x_1 - (1/R_{g1})x_2 - x_3 - u_1) \\ \frac{dx_3}{dt} &= (1/L_2)(x_2 - x_4 - R_2 x_3) \\ &\dots \dots \dots (2.26) \\ \frac{dx_4}{dt} &= (1/(C_2 + C_{d2}))(x_3 - (1/R_{g2})x_4 - x_5 - u_2) \\ \frac{dx_5}{dt} &= (1/L_3)(x_4 - x_6 - R_3 x_5) \\ \frac{dx_6}{dt} &= (1/(C_3 + C_{d3}))(x_5 - (1/R_{g3})x_6 - u_6) \end{aligned}$$

Rewriting our systems state equations we get matrix form as shown below:

$$\begin{aligned}
x \bullet &= \begin{pmatrix} d/dt \\ \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \dots \\ x_{2n-1} \\ x_{2n} \end{bmatrix} = \begin{pmatrix} d/dt \\ \end{pmatrix} \begin{bmatrix} v_1 \\ i_2 \\ \dots \\ i_n \\ v_n \end{bmatrix} = \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix} (v) + \\
& \begin{bmatrix} -R_1/L_1 & -1/L_1 & 0 & 0 & \dots & 0 & 0 \\ 1/(C_1+Cd_1) & 1/((C_1+Cd_1)*R_{g1}) & -1/(C_1+Cd_1) & 0 & \dots & 0 & 0 \\ 0 & 1/L_2 & -R_2/L_2 & -1/L_2 & \dots & 0 & 0 \\ 0 & 1/(C_2+Cd_2) & 1/((C_2+Cd_2)*R_{g2}) & -1/(C_2+Cd_2) & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1/L_n & -R_n/L_n & -1/L_n & 0 \\ 0 & 0 & \dots & 0 & 1/(C_n+Cd_n) & 1/((C_1+Cd_1)*R_{gn}) & -1/(C_n+Cd_n) \end{bmatrix} \\
& \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_2 \\ \dots \\ i_n \\ v_n \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1/(C_1+Cd_1) & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1/(C_2+Cd_2) & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 1/(C_n+Cd_n) & 0 \end{bmatrix} \begin{bmatrix} 0 \\ id_1 \\ 0 \\ id_2 \\ \dots \\ 0 \\ id_n \end{bmatrix}
\end{aligned}$$

Matrix form of the state equations for power distribution network system with n-nodes and load centers.

2.2.2 Defining the initial conditions, physical constants and parameters:

Our initial time $t = t_0 = 0$, final period of simulation time $t = T = (N 2\pi f)$, $f = 60$ cps or else (taken to be constant) at certain time period, $N = \#$ of time discrete step;

The load current $i_{dk}(0) = 0 = I_{dk}(0) \sin(\omega \cdot 0) = U_k(0) \sin(\omega \cdot 0)$; thereby pursuing for initial respective loop line current $i_k(0) = 0 = I_k \cdot \sin(\omega \cdot 0) = X_{2k-1} \cdot \sin(\omega \cdot 0)$, and node voltage $v_k(0) = V_k \cdot \sin(\omega \cdot 0) = x_{2k} \cdot \sin(\omega \cdot 0)$, main supply voltage $= v(0) = V \sin(\omega \cdot 0) = 0$;

2.2.3 System power and energy

Also, the state equations are given below for system power and energy parameters:--

Instantaneous power demanded or consumed Load (time intensity),

$$P_c(t) = dE_c(t)/dt, \quad \text{where, } E_c(t) \text{ is the energy consumed up to time } t$$

$$x_{11} = R_{d1} u_1^2 + R_{d2} u_2^2 + R_{d3} u_3^2 \quad \dots \quad (2.27)$$

Instantaneous power loss (time intensity) of the system,

$$P_{ls}(t) = dE_{ls}(t)/dt, \quad \text{where, } E_{ls}(t) \text{ is the energy loss up to time } t$$

$$x_{12} = R_1 x_1^2 + R_2 x_3^2 + R_3 x_5^2 + x_2^2/R_{g1} + x_4^2/R_{g2} + x_6^2/R_{g3} \quad \dots \quad (2.28)$$

Energy consumed or demand during the period T of simulation, $E_c(t) = \int_0^T P_c(t) dt$

$$E_c(T) = x_{13} = \int_0^T x_{10} dt = x_{13}(T) \quad \dots \quad (2.29)$$

Energy loss during the period T of simulation,

$$E_{ls}(t) = \int_0^T P_{ls}(t) dt$$

$$E_{ls} = x_{14} = \int_0^T x_{12} dt = x_{14}(T) \quad \dots \quad (2.30)$$

Average of energy consumed or demand during the period T of simulation

$$E_{ca}(T) = x_{15} = (1/T)x_{13}(T) = (1/T) \int_0^T x_{10} dt$$

$$= (1/T) \int_0^T (R_{d1} u_1^2 + R_{d2} u_2^2 + R_{d3} u_3^2) dt \quad \dots \quad (2.31)$$

Average of energy loss during the period T of simulation

$$E_{lsa}(T) = E_{ls}(T)(1/T)$$

$$= (1/T)x_{14}(T) = (1/T) \int_0^T x_{12} dt$$

$$= (1/T) \int_0^T (R_1 x_1^2 + R_2 x_3^2 + R_3 x_5^2 + x_2^2/R_{g1} + x_4^2/R_{g2} + x_6^2/R_{g3}) dt \quad \dots \quad (2.32)$$

2.3 PER UNIT CONVERSION

The systems and line parameters used in the equations of the preceding sections are not in convenient form for engineering use. One difficulty is the numerically awkward values, for example expressing the generator and grid supply voltage in kilovolt range and expressing the field node voltage at a much lower level. To solve this problem, Section 2.3.1 normalizes the parameters to convenient base values and expresses them in per unit (pu) form.

In Section 2.3.2, a practical solution to this concern also yields a methodological one, i.e. to clarify relationships that hold between practical network characteristics considered by distribution engineers and to select the basic quantities chosen for power distribution network systems design or their operation . For example, all basic quantities can be derived from the rms voltage V_b and the rms current I_b , which would constitute the basis for the design. They could alternatively be derived from the base power S_b and circuit resistance R_b . Then all other basic values of interest (V_b , I_b , but also Z_b) could be derived from them and conversely. Section 2.3.3 illustrates the derivation and Section 2.3.4 explains how the impedance and frequency need to be scaled to adapt to the per unit approach.

2.3.1 Per Unit representation of current, voltages and powers

The electric power engineers often prefer to express impedance, currents, voltages, and powers in per-unit (pu) values rather than in Ohms, Amperes, kilovolts, and megawatts or megavars. Several advantages can be gained by this practice.

1. Per-unit data representation yields valuable relative magnitude information.
2. Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.
3. Circuit parameters tend to fall in relatively narrow numerical ranges making erroneous data easy to spot. [Alvarez, 1978]

Definitions:

The basic features of per-unit method are as described below.

Considering a system link transmitting the complex power the phasors V and I represent S , the voltage and current respectively. Arbitrarily, we now define two values $|V_b|$ and $|I_b|$ expressed in rms voltage and current respectively. Per-unit voltage and current phasors are then defined by the dimensionless and complex ratios

$$\begin{aligned} V_{pu} &\equiv V / |V_b| = |V| / |V_b| \angle V \text{ pu volts} \\ I_{pu} &\equiv I / |I_b| = |I| / |I_b| \angle I \text{ pu amps} \quad \dots \end{aligned} \quad (2.33)$$

This per unit value thus immediately tells us that the actual voltage exceeds the standard level by some percent. We next define base value of power and impedance in terms of already chosen voltage and current bases:

$$\begin{aligned} |S_b| &\equiv |V_b| |I_b| \text{ VA} \\ |Z_b| &\equiv |V_b| / |I_b| \quad \Omega \quad \dots \quad \dots \end{aligned} \quad (2.34)$$

A complex power S and an impedance Z can then be expressed in dimensionless per-unit values, according to the following equations.

$$\begin{aligned} S_{pu} &\equiv S / |S_b| = (P + jQ) / |S_b| = P / |S_b| + jQ / |S_b| = P_{pu} + jQ_{pu} \\ Z_{pu} &\equiv Z / |Z_b| = (R + jX) / |Z_b| = R / |Z_b| + jX / |Z_b| = R_{pu} + jX_{pu} \end{aligned} \quad (2.35)$$

2.3.2 Choosing a base for transmission quantities

Pre-specified variables such as E_{k-1} , i_{dk} , L_{dk} , $i_k(t)$, $v_k(t)$, for some node values k , can express the transmission systems quantities per phase (phase to neutral) because they relate directly to the systems parameters through Park's Transformation. Using the subscript 'b' to identify base value, and we normalize every quantity, dividing it by the base quantity of the same dimension.

Power and formulas in Per-Unit values:

It is important to realize that the general (complex) power formula $S = VI^*$... (2.36) where, conjugate current $I^* \cong |I| e^{-j\angle I}$, and phasor voltage $V = |V| e^{j\angle V}$, phasor current $I = |I| e^{j\angle I}$ holds true if all quantities are expressed in per-unit values.

By dividing Equation. (2.36) by $|S_b|$ and making use of the 1st of Equation. (2.34) we have,

$$S_{pu} \equiv S / |S_b| = VI^* / |S_b| = (V / |V_b|)(I^* / |I_b|) = V_{pu} I_{pu}^* \quad \dots \quad (2.37)$$

Of equal importance is the fact that per-unit values still satisfy the general Ohm's law. We prove this as follows:

$$V_{pu} \equiv V / |V_b| = ZI / |V_b| = Z / (|V_b| / |I_b|) (I / |I_b|) = Z_{pu} I_{pu} \quad \dots \quad (2.38)$$

Of the four base values $|V_b|$, $|I_b|$, $|S_b|$, and $|Z_b|$ only two can be arbitrarily chosen. In practice, it is most common to choose the pair $|V_b|$ and $|S_b|$. The remaining two are then computed from

$$|I_b| = |S_b| / |V_b| \quad \dots \quad \dots \quad \dots \quad (2.39)$$

$$\text{And } |Z_b| = |V_b| / |I_b| = (|V_b|^2) / (|I_b| |V_b|) = |V_b|^2 / |S_b| \quad \dots \quad (2.40)$$

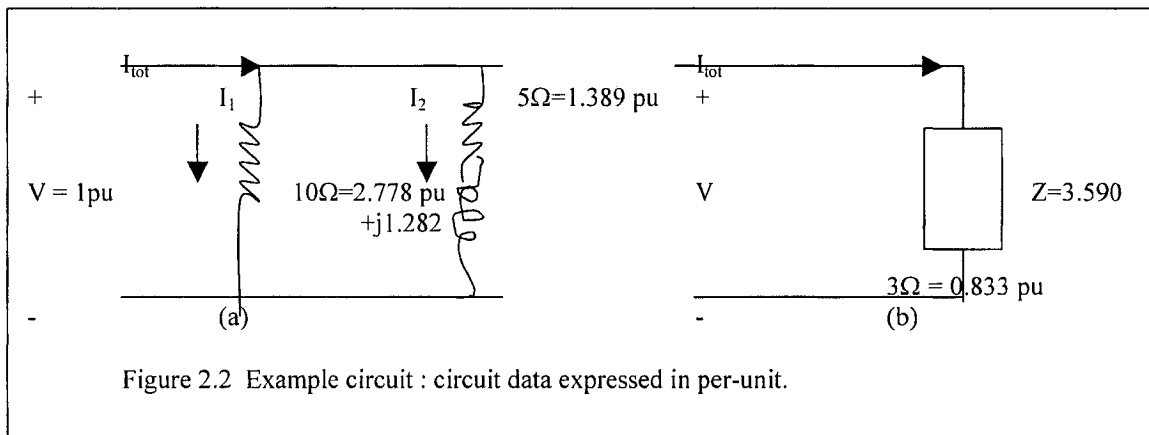
It remains to prescribe how such identities relate in a way of letting designers calculate circuit effectively. The method is illustrated here by way of an example.

Example:

Consider the A.C load shown in Figure 2.2. It is fed from a 600 volt source. Compute various power absorbed in two load branches; expressing all quantities in per-unit. Use following base values: $|V_b| = 600 \text{ v}$, $|S_b| = 100 \text{ kVA}$. $X_L = \omega L = 2\pi fL = 3\Omega$

Solution:

As the voltage phasor V is common for both branches we choose it as our reference phasor in our analysis.



We obtain for the branch currents

$$I_1 = V/10 = 600/10 = 60.0 \angle 0^\circ \text{ A}$$

$$I_2 = 600/(5+j3) = 102.9 \angle -31.0^\circ \text{ A}$$

The resistive load branch thus absorbs

$$P_1 = 600 \cdot 60 \cdot \cos 0^\circ = 36.0 \text{ kW}$$

$$Q_1 = 600 \cdot 60 \cdot \sin 0^\circ = 0 \text{ kvar}$$

The inductive load branch thus absorbs

$$P_2 = 600 \cdot 102.9 \cdot \cos 31.0^\circ = 52.9 \text{ kW}$$

$$Q_1 = 600 \cdot 102.9 \cdot \sin 31.0^\circ = 31.8 \text{ kvar}$$

Were we interested in the load powers absorbed by both load branches we could have first replaced the actual circuit by its equivalent impedance Z :

$$Z = 10(5+j3)/(10+5+j3) = 3.590+j1.282\Omega$$

For the load current we then have

$$I_{\text{tot}} = 600/(3.590+j1.282) = 157.4 \angle -19.7^\circ \text{ A}$$

The total powers will thus be

$$P_{\text{tot}} = 600 \cdot 157.4 \cdot \cos 19.7^\circ = 88.9 \text{ kW} = P_1 + P_2$$

$$O_{\text{tot}} = 600 \cdot 157.4 \cdot \sin 19.7^\circ = 31.8 \text{ kvar} = Q_1 + Q_2$$

i.e. the total real and reactive powers can be obtained as the algebraic sums of the individual branch powers.

Apparent powers in two load branches

$$\text{kVA}_1 = 0.6 \cdot 60 = 36 \text{ kVA}, \quad \text{kVA}_2 = 0.6 \cdot 102.9 = 61.7 \text{ kVA}$$

the apparent power equals

$$\text{kVA}_{\text{tot}} = 0.6 \cdot 157.4 = 94.4 \text{ kVA}$$

Note that, $\text{kVA}_{\text{tot}} < \text{kVA}_1 + \text{kVA}_2$

Here, algebraic summation rule does not apply for parallel circuits. Using formula (8) we first compute the base impedance

$$|Z_b| = 600^2/100,000 = 3.600 \Omega$$

For the base current we get

$$|I_b| = 100,000/600 = 166.7 \text{ A}$$

In terms of the above base voltage and base impedance we then obtain the per-unit circuit (shown in above diagram). From this diagram we first determine the per-unit currents in the two load branches

$$I_1 = 1.00/2.778 = 0.360 \angle 0^\circ \text{ pu}$$

$$I_2 = 1.00/(1.389+j0.833) = 0.529-j0.318 = 0.617 \angle -31.0^\circ \text{ pu}$$

For the load powers we have

$$P_1 + jQ_1 = VI_1^* = 1.00(0.36 + j0) = 0.36 + j0 \text{ pu}$$

$$P_2 + jQ_2 = VI_2^* = 1.00(0.529 + j0.318) = 0.529 + j0.318 \text{ pu}$$

As a check we then compute the following actual current and power values, (which were earlier computed in the preceding paragraph.

$$|I_1| = 0.36 \cdot 166.7 = 60.0 \text{ A}$$

$$|I_2| = 0.617 \cdot 166.7 = 102.9 \text{ A}$$

$$P_1 = 0.360 \cdot 100 = 36.0 \text{ kW,}$$

$$Q_1 = 0 \text{ kVar,}$$

$$P_2 = 0.529 \cdot 100 = 52.9 \text{ kW,}$$

$$Q_2 = 0.318 \cdot 100 = 31.8 \text{ kVar.}$$

2.3.3 Procedure for the computation of PU values

In Section 2.3.2, the values of state variables were calculated, using two per unit values: the rms voltage V_b and the rms current I_b . Other basic values could have been chosen, such as the base power S_b and circuit resistance R_b . Next, we illustrate the relationships that hold between each set of such basic per unit characteristics.

Assume for example the following standard values of the system (of the per phase transmission-distribution):

Rated system power transmitting (per phase) = 10 MVA

Rated system voltage = 6.6KV phase to phase

Power factor, $\cos \theta = 0.85$

$L =$ Henry (H) per km, $R =$ Ohms (Ω) per km

$R_g =$ Ohms (Ω) per km, $C =$ Farads (F) per km

Selecting these base values, we can follow the procedure illustrated in Sections 2.3.1 and 2.3.2 and determine sequentially other pu parameter values for use in the simulation experiment.

With the previous quantities selected, we can then calculate:

$$S_b = 10 \text{ MVA,} \quad V_b = 6600/\sqrt{3} \text{ v,} \quad I_b = S_b / V_b \text{ A,}$$

$$R_b = V_b / I_b \text{ } \Omega, \quad L_b = V_b / \omega I_b = V_b / 2\pi f I_b \text{ H}$$

$$C_b = (V_b - R_b I_b) / V_b \text{ F,} \quad t_b \text{ sec}$$

In an open circuit, the mutual inductance will be zero and can also be found in a similar way, but without considering any reactive impedance.

2.3.4 Per unit values of line parameters

In practice, the per unit values of line parameters are directly taken from reference sources of parameter per unit of line length. If α is the constant line length and b is the base input voltage for an electrical circuit, the state equation for the systems variables may be written in normalized [pu] form as:

$$\dot{X} = \frac{d}{dt} X = \tilde{A}X + \tilde{B}U + \tilde{G}V \quad \text{or} \quad \alpha \left(\frac{\dot{X}}{b} \right) = \alpha A \left(\frac{X}{b} \right) + \alpha B \left(\frac{U}{b} \right) + \alpha G \left(\frac{V}{b} \right)$$

Or, $\dot{x} = \frac{d}{dt} x = Ax + Bu + Gv \quad \dots \quad \dots \quad (2.41)$

where, $X = bx$, $U = bu$, $V = bv$; and

$$\tilde{A} = \alpha A, \quad \tilde{B} = \alpha B \quad \tilde{G} = \alpha G$$

The methodology presented above, is then applied for different types of networks and load conditions as described in the following section.

Impedance scaling:

Using time realistic elements (e.g 10,000 Ω , 1000 μF , 500 mH) and un-realistic elements (e.g. 1 Ω , 1 F, 100 H).

Mathematically it does not matter what numerical values are used; Analysis techniques remain the same.

	Real circuit N	Un-realistic (simulated) circuit N'
Resistor	$R = r$ Ohms	$R' = K r$
Inductor	$L = l$ Henrys	$L' = K l$ Henrys
Capacitor	$C = c$ Farads	$C' = c/K$ Farads
Voltage controlled current source	$J_D = g_D V$	$J_D' = (g_D/K) V'$
Current controlled voltage source	$V_D = r_D I$	$V_D' = K r_D I'$

Let the determinant of the equations of N' be Δ' and the co-factor be Δ'_{ij} . Then it is clear that

$$\Delta' = K^M \Delta, \quad \Delta'_{ij} = K^{M-1} \Delta_{ij}. \quad \dots \quad (2.42) \text{ Let}$$

$V(j\omega)$ be the voltage across Z in loop 2 with currents I_2 , and let voltage transfer function be

$$H(j\omega) = V(j\omega)/E, \text{ where } E \text{ is the excitation in loop 1. Then,}$$

$$H(j\omega) = \frac{I_2 Z}{E} = \frac{Z \Delta_{12}}{\Delta}$$

The corresponding transfer function in N' is function of N' is now

$$H'(j\omega) = \frac{Z' \Delta'_{12}}{\Delta'} = \frac{K Z K^{M-1} \Delta_{12}}{K^M \Delta} = \frac{Z \Delta_{12}}{\Delta} H(j\omega)$$

Thus, the voltage transfer function of N' is the same as that of N. On the other hand, the input

$$Z'_{in} = \frac{\Delta'}{\Delta'_{11}} = \frac{K^M \Delta}{K^{M-1} \Delta_{11}} = K \frac{\Delta}{\Delta_{11}} = K Z_{in}$$

impedance or any impedance function of N' is now,

The impedance of N' is K times that of N.

Frequency scaling:

Suppose we construct a circuit N'' from N by following scheme:

	Circuit N	Simulated circuit N''
Resistor	R = r Ohms	R'' = r Ohms
Inductor	L = l Henrys	L'' = F l Henrys
Capacitor	C = c Farads	C'' = F c Farads

Let frequency variable is ω ; the corresponding inductor L'' in N'' will have same impedance as at

$$\omega'' = \omega/F$$

$$\text{Since impedance of inductor} = j\omega L = j\omega'' F L = j\omega'' L''$$

Similarly, impedance of capacitor in N'' will have the same numerical value as that of its corresponding capacitor in N if $\omega'' = \omega/F$.

Since the value of the determinant of the loop equations depends entirely on the impedance of

$$H''(j\omega'') = H(jF\omega'')$$

$$Z''(j\omega'') = Z(jF\omega'')$$

the elements, we have,

In words, the network functions of N'' can be obtained from the corresponding functions of N by replacing the variable ω in the latter by $F\omega''$.

[Omar Wing, 1978]

2.4 BASE CASE FOR SIMULATION AND ANALYSIS

We now can apply the general model developed in Section 2.1, 2.2 and 2.3 to focus on the development and the testing of a specific type of power network systems and its different parameter values in view of an analysis of their characteristic and operational behavior. The description starts with a detailed mathematical specification of each component, i.e.:

- RLC branches with leakage (or grounding) resistance in parallel to capacitor and loads (Resistive load with self-inductance in series and or stay capacitor across load)
- Pi-section lines
- Mutual inductances (used per unit [pu] values of line parameters)
- Distributed parameter of lines (used pu values of line parameters)
- AC voltage and current sources (used pu values of base parameters)
- DC voltage sources (used pu values of base parameters).

After a derivation of the precise state equations of the physical model, we present a sample of load parameter variations; such values are obtained, using the steps described in Section 2.1.

Three-node power RLC distribution network system having loads (with resistance, self-inductance connected in series plus stay capacitor across load)

Here in this section, we particularly consider a three-loop and three-node RLC circuit with connected load (R_d in series with L_d , and stay capacitor C_d across load) . Here, Line R and L in series, leakage resistance R_g in parallel with node capacitor C , Loads (with time varying

resistance $R_d(t)$ and self-inductance $L_d(t)$ connected in series and stray capacitor C_d . placed across the load) is tapped from the node as shown in Figure 2.3.

Here, $\omega = 2\pi f$, f = system supply frequency, $v = V \sin(2\pi ft)$ = input voltage

i = current through the line, $i_c = C dv_c/dt$ = current through the capacitor,

i_L = current through the inductor, v_L = voltage across the inductor = $L di/dt$,

Leakage current $i_g = v_c / R_g = v_1 / R_{g1}$, $i_d = u$ = load current = $I_d \sin(2\pi ft)$

We know that for L = constant (for a particular conductor) voltage across an inductor $V_L = L di_L/dt$; but for a time varying load resistance $R_d(t)$ and inductance $L_d(t)$ voltage drops due to load inductance.

Which can be shown as,

$$v_{Ld} = d(L_d i_d)/dt = L_d d(i_d)/dt + i_d d(L_d)/dt \quad \dots \quad (2.43)$$

Using KVL and KCL we have the dynamics for the power system network,

At node1 and loop1, $i_1 = i_{g1} + i_{c1} + i_2 + i_{d1}$. But $i_d = i_{Rd} + i_{Cd} = i_{Rd} + C_d \cdot dv_c/dt$

Or $i_1 = v_{c1} / R_{g1} + C_1 dv_{c1}/dt + i_2 + i_{d1}$,
 $= v_1 / R_{g1} + C_1 dv_1/dt + i_2 + i_{Rd1} + C_{d1} \cdot dv_1/dt$,

Or $dv_1/dt = (1/(C_1 + C_d))(i_1 - v_{c1} / R_{g1} - i_2 - i_{Rd1})$, ... (2.44)

$R_1 i_1 + L_1 di_1/dt + v_{c1} = v$,

Or $di_1/dt = (v - v_{c1} - R_1 i_1) / L_1$, ... (2.45)

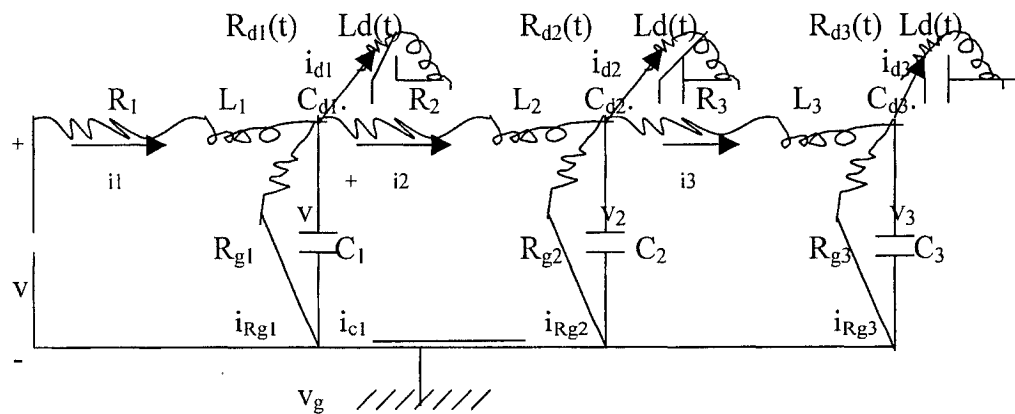


Figure 2.3- Typical 3-nodes Power Distribution Network system having Loads (with time varying resistance $R_d(t)$ and self-inductance $L_d(t)$ connected in series, plus stray capacitor C_d . across load) tapped from the nodes.

And $d(L_{d1} i_{d1})/dt + R_{d1} i_{d1} = v_{c1}$,

Or $(1/L_{d1})di_{d1}/dt = v_{c1} - R_{d1} i_{d1} - i_{d1} d(L_{d1})/dt$

Or $di_{d1}/dt = (1/L_{d1})(v_{c1} - R_{d1} i_{d1} - i_{d1} d(L_{d1})/dt) \dots$ (2.46)

Similarly at node2 and loop2,

$di_2/dt = (v_{c1} - v_{c2} - R_2 i_2) / L_2, \dots$ (2.47)

$dv_{c2} /dt = (i_2 - v_{c2} / R_{g2} - i_3 - i_{d2}) / (C_2 + C_{d2}). \dots$ (2.48)

$di_{d2}/dt = (v_{c2} - R_{d2} i_{d2} - i_{d2} d(L_{d2})/dt) / L_{d2} \dots$ (2.49)

Similarly at node2 and loop2,

$di_3/dt = (v_{c2} - v_{c3} - R_3 i_3) / L_3, \dots$ (2.50)

$dv_{c3} /dt = (i_3 - v_{c3} / R_{g3} - i_4 - i_{d3}) / (C_3 + C_{d3}). \dots$ (2.51)

$di_{d3}/dt = (v_{c3} - R_{d3} i_{d3} - i_{d3}d(L_{d3})/dt) / L_{d3} \dots$ (2.52)

[But as at last node (here node3) $i_4 = 0$.]

Instantaneous Power demand and consumed Load (time intensity)

$dE_c(t)/dt = P_c(t) = id_1 R_{d1} + id_2 R_{d2} + id_3 R_{d3}, \dots$ (2.53)

where, $E_c(t)$ = energy consumed up to time t.

Average Energy (Power) demand and consumed Load during the period T,

$E_{cavg.} = (1/T) \int_0^T P_c(t) dt = (1/T) \int_0^T (id_1 R_{d1} + id_2 R_{d2} + id_3 R_{d3}) dt. \dots$ (2.54)

Load demand Factor during the period T,

$F_d = F_c$, (Ave rage Power demand) / (Peak of Power demand). \dots (2.55)

Instantaneous Power loss (time intensity) of the systems $P_{ls}(t) = dE_{ls}(t)/dt$

Where, $E_{ls}(t)$ = energy loss up to time t.

$P_{ls}(t) = \text{Line loss} + \text{leakage loss}$

$= R_1 i_1^2 + R_2 i_2^2 + R_3 i_3^2 + v_{c1}^2 / R_{g1} + v_{c2}^2 / R_{g2} + v_{c3}^2 / R_{g3}; \dots$ (2.56)

Ave rage Energy loss of the systems during the period T,

$E_{lsavg.} = (1/T) \int_0^T P_{ls}(t) dt = (1/T) E_{ls}(T)$
 $= (1/T) \int_0^T (R_1 i_1^2 + R_2 i_2^2 + R_3 i_3^2 + v_{c1}^2 / R_{g1} + v_{c2}^2 / R_{g2} + v_{c3}^2 / R_{g3}) dt \dots$ (2.57)

Power systems loss Factor during the period T,

$F_{ls} = (\text{Average Power loss}) / (\text{Peak of Power loss intensity}).$

Now, taking the state variables as,

$y_1 = i_1, y_2 = v_{c1}, y_3 = i_{d1}, y_4 = i_2, y_5 = v_{c2}, y_6 = i_{d2}, y_7 = i_3, y_8 = v_{c3}, y_9 = i_{d3},$

[Controlled load current $i_d = u = U \sin(2\pi ft)$ with variable load resistance and inductance.]

State equations in canonical form may then be re-written as,

$$\begin{aligned}
 dy_1/dt &= (1/L_1) (v - y_2 - R_1 y_1), \text{ and} \\
 dy_2/dt &= (1/(C_1+C_{d1.}))(y_1 - y_2/ R_{g1} - y_3 - y_4), \\
 dy_3/dt &= (1/L_{d1})(y_2 - R_{d1} y_3 - y_3 dL_{d1}/dt) \\
 dy_4/dt &= (1/L_2) (y_2 - y_5 - R_2 y_4), \text{ and} \\
 dy_5/dt &= (1/(C_2+C_{d2.}))(y_4 - y_5/ R_{g2} - y_6 - y_7), \\
 dy_6/dt &= (1/L_{d2})(y_5 - R_{d2} y_6 - y_6 dL_{d2}/dt) \\
 dy_7/dt &= (1/L_3) (y_5 - y_8 - R_3 y_7), \text{ and} \\
 dy_8/dt &= (1/(C_3+C_{d3.}))(y_7 - y_8/ R_{g3} - y_9), \\
 dy_9/dt &= (1/L_{d3})(y_8 - R_{d3} y_9 - y_9 dL_{d3}/dt) \quad \dots \quad (2.58)
 \end{aligned}$$

Thus, we obtain the state equations below.

Instantaneous Power demanded and consumed Load (time intensity)

$$y_{10} = R_{d1} y_3^2 + R_{d2} y_6^2 + R_{d3} y_9^2. \quad \dots \quad (2.59)$$

Average Energy consumed and demand during the period T of simulation

$$E_c(T) = (1/T) \int_0^T y_{10} dt = (1/T) \int_0^T (R_{d1} y_3^2 + R_{d2} y_6^2 + R_{d3} y_9^2) dt. \quad \dots \quad (2.60)$$

Instantaneous Power loss (time intensity) of the systems,

$$y_{11} = R_1 y_1^2 + R_2 y_4^2 + R_3 y_7^2 + y_2^2/R_{g1} + y_5^2/R_{g2} + y_8^2/R_{g3}. \quad \dots \quad (2.61)$$

Average of Energy loss during the period T of simulation

$$E_{1s}(T) = (1/T) \int_0^T y_{11} dt \quad \dots \quad (2.62)$$

Now, after experimenting with different parameter values, and iteratively taking a workable value of these parameters, if total time period of simulation is T, we choose load parameters:

$$L_{d1} = L_{d2} = L_{d3} \cdot \text{pu farads}, L_d = 10.00 = L_{d1}, C_{d1} = 0.2, C_{d2} = 0.8 \times C_{d1}, C_{d3} = 0.6 \times C_{d1},$$

for $0 \leq t \leq t_1 (=0.4T)$, $L_d(t_{01}) = 10.00$

for $0.4T \leq t \leq t_2 (=0.42T)$,

$$\begin{aligned}
 L_d(t_{12}) &= L_d(t_{01}) + (L_{d1}(t_{23}) - L_{d1}(t_{01}))(t-t_1)(1/(t_2-t_1)) \\
 &= L_{d1}(t_{01}) + (0.8 L_{d1}(t_{01}) - L_{d1}(t_{01}))(t-0.4 T)(1/(0.42 T -0.4 T)) \\
 &= (5T-10t)(1/T) \times 10.0
 \end{aligned}$$

$$\text{for } 0.4T \leq t \leq 0.42T \quad R_d(t_{12}) = 0.6 \times 1.36,$$

$$\text{for } 0.42T \leq t \leq 0.8T \quad R_d(t_{14}) = 0.6 \times 1.36,$$

$$\text{for } 0.8T \leq t \leq 0.82T \quad R_d(t_{14}) = 1.2 \times 1.36,$$

$$\text{for } 0.8T \leq t \leq T \quad R_d(t_{45}) = 1.2 \times 1.36,$$

The line parameters chosen are shown next.

$$R_1=0.2 R_3, R_2=0.6 R_3, R_3=0.136 ; L_1=L_2=L_3=0.124 ; R_g=99.9, C_1=0.2, C_2=1.2 C_1, C_3=1.4 C_1.$$

$$\text{for } 0.42T \leq t \leq t_3(=0.8T), \quad L_{d1}(t_{23}) = 0.8 \times 10.0$$

for $0.8T \leq t \leq t_4(=0.82T)$,

$$\begin{aligned} L_d(t_{34}) &= L_{d1}(t_{23}) + (L_{d1}(t_{45}) - L_{d1}(t_{23}))(t - t_3)(1/(t_4 - t_3)) \\ &= 0.8 L_{d1}(t_{01}) + (0.6 L_{d1}(t_{01}) - 0.8 L_{d1}(t_{01}))(t - 0.8 T)(1/(0.82 T - 0.8 T)) \\ &= (7.2 T - 10 t)(1/T)10.0 \end{aligned}$$

$$\text{for } 0.82T \leq t \leq t_5 (=T), \quad L_d(t_{45}) = 0.6 \times 10.0$$

Taking, $R_{d1}=R_d = 1.36$, $R_{d2}=1.2 R_d$, and $R_{d3} = 1.1 R_d$. Pu Ohms,

$$\text{for } 0 \leq t \leq 0.4T \quad R_d(t_{01}) = 1.36,$$

This model will be validated by simulation in Chapter 3.

CHAPTER 3

VALIDATION: SIMULATED OUTPUTS AND ANALYSIS

This chapter applies simulation to the network system shown in Figure 2.3. A standard strategy would be to select some design factors and apply them systematically through a full experimental design. However, this ideal scheme encounters practical difficulties arising from two major sources:

- The dynamic behaviour of the network may create instabilities and invalidate a particular choice of parameters. Since no optimization criterion was integrated in the simulation, we apply control interactively and thereby select a small number of meaningful experiments.
- Those are further carried out selectively to satisfy iteratively additional conditions, such as the following operating characteristics:
 - maintain the node voltages within $\pm 5\%$ of rated voltage,
 - maintain the stability of the system with changing load,
 - each systems operation satisfies the two constraints: e.g.
 - a) load factor, $F_c \leq 1$ or preferably be 1, so that the plant capacity does not remain idle for longer time period and thereby minimize capital establishment) cost;
 - b) loss factor $F_{ls} \leq 1$ or preferably be as low as zero, so that there may be loss which is not continued for longer duration of time.

By satisfying these constraints, the simulation validates the mathematical foundations of thesis by providing some empirical benchmark:

- for each basic system configuration, it yields an example of stable power distribution,
- it achieves some limited form of control of state variables.

The major merit of the simulation is therefore to aid an interactive exploration of feasible power distribution configurations. This exploration is best illustrated qualitatively, in particular by representing the behaviour of the state variables graphically. This furnishes expressive characterization of feasible state responses, both understandable to practitioners and giving tangible examples upon which optimizing criteria would base in future research.

Overview of the experimental design

The simulation uses a 4-factor experimental design:

Factor 1 - A.C Vs D.C power supply (maintained at a constant level).

Factor 2 - 1, 2, 3-node network.

Factor 3 - Constant, variable load (this case is divided as):

Factor 4 (a)- load with variable resistance,

(b)- load with variable resistance and inductance,

(c)-load with variable resistance, inductance and capacitance.

The details of the design are given at the beginning of Section 3.2. In practice, the following results of each case of simulation actually entail an experimentation with different values of system frequencies, line parameters, single and multiple nodes, with load (simple resistive load or resistive with inductive load combined in series) or without load; and also coupled with or without stay capacitor placed across the load. Sufficient care had been taken (in choosing inductance, capacitance, and frequency values) so that the system does not get bust out (i.e. blow out) because of resonance. This was obtained each time by calculating the resonance frequency, that calculate the systems models, [i.e. $\omega=\sqrt{LC}$ or $\omega=\sqrt{L}$] and of the line or load parameters before running the programs of Section 3.1. More details are given at the Appendix 3 on resonance frequency.

The outcome of the simulation experiment is a series of figures showing the time response of the system state variables, using a particular set of network parameters and load types or load conditions (see output graphs of Figures 3.3 to 3.8 and 3.11 to 3.16). The type of graphs is chosen to:

- show and help us compare the system in time response for its transient versus steady state conditions, (see Figures 3.3 to 3.19);
- help us to compare and show constancy of the system node voltages (maintained within desired voltage level through iterative regulation i.e. control on node capacitor or load parameters); (see Figures 3.7 and 3.15);
- enable us to compare the power density as well as energy load consumed and loss; (see Figures 3.9, 3.19, 3.17, and 3.18) based on the load types or conditions and make decision on time method of controlling load and

- help us to compute load and loss factors and act according to desire for future operational management of the system with specific load types or load conditions. (Figures are not shown here, but some data are given in Appendix 4).

3.1 ALGORITHMIC DESCRIPTION

Figure 3.1 gives a flowchart of the simulated computation process. Here, 'FCT' denotes the functions subroutine, 'RK4' denotes the 4th order Runge-Kutta method subroutine (detailed in Section 3.1.1), which is detailed in the following section, 't' (here X) is the instantaneous time and 'T' is the total time period (XT) of simulation.

3.1.1 Algorithms for the Main program

1. Define State variables (Y(I), for I← 1, N)
2. Open file 'FLOW' for writing computed output data
3. Write column headings (Time (X(t)) and State variables (Y(I)))
4. Read
 - 'Time displacement' (DT),
 - 'Initial value of print time' (DXPR),
 - 'Terminal Time' (XT)
5. Read state variables (Y(I), for I← 1, N)
6. Initialize time & print time X← 0.0, XPR← DXPR
7. Write Time (X), State variables Y(I)
8. Call RK4 (N, Y, X, DT)
9. If time to print (X – XPR) < 0.00001 then
10. Write (X, Y(I))
11. XPR ← XPR +DT
12. end if
13. Check if terminal time (XT) has been reached: (X – XT) < 0.00001 then stop processing
14. Go to step 8

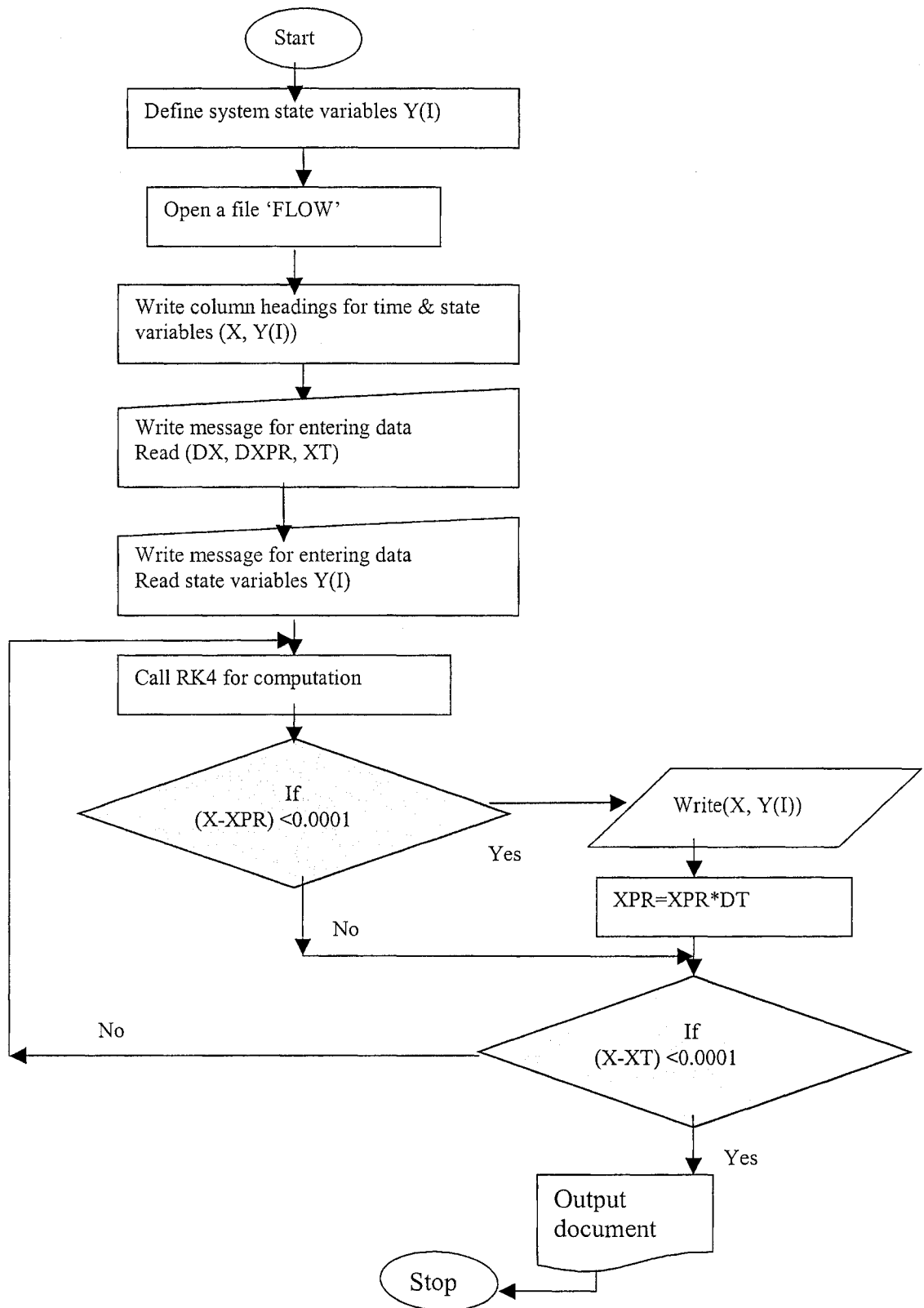


Figure 3.1: Flow Chart for Simulation (Main Program)

3.1.2 Algorithms for the Function sub-routine: FCT

1. Define State Variables Y(15), Derivatives of System state variables YPRIME(11)

2. Initialize total time span, T=100.00

3. Calculate System Load parameters ($L_d(X)$, $C_d(X)$, $R_d(X)$)

i) Calculate load inductance $A_{Ld}(t)$ = time varying, pu HENRY/Km

if (X.GE.0.0.AND.X.LT.0.4*T) then

$$A_{Ld} \leftarrow 1.0D+01$$

$$A_{Ld}PRIME \leftarrow 0$$

else if (X.GE.0.40*T.AND.X.LT.0.42*T) then

$$A_{Ld} \leftarrow (5*T-10*X)*(1./T)*1.0D01$$

$$A_{Ld}PRIME \leftarrow -(10./T)*1.0D+01$$

else if (X.GE.0.42*T.AND.X.LT.0.8*T) then

$$A_{Ld} \leftarrow 0.8*1.0D+01$$

$$A_{Ld}PRIME \leftarrow 0$$

else IF (X.GE.0.80*T.AND.X.LT.0.82*T) then

$$A_{Ld} \leftarrow (8.8*T-10*X)*(1./T)*1.0D+01$$

$$A_{Ld}PRIME \leftarrow -(10./T)*1.0D01$$

else If (X.GE.0.82*T.AND.X.LE.1.0*T) then

$$A_{Ld} \leftarrow 0.6*1.0D+01$$

$$A_{Ld}PRIME \leftarrow 0$$

else

$$A_{Ld1} \leftarrow A_{Ld}$$

$$A_{Ld2} \leftarrow A_{Ld}$$

$$A_{Ld3} \leftarrow A_{Ld}$$

End if

ii) Calculate Load Resistance $R_d(t)$ =time varying pu Ohms/Km

if (X.GE.0.0.AND.X.LT.0.4*T) then

$$R_d \leftarrow 1.36D+00$$

else IF (X.GE.0.4*T.AND.X.LT.0.8*T) then

$$R_d \leftarrow 0.6*1.36D+00$$

else If (X.GE.0.8*T.AND.X.LE.1.0*T) then

$$R_d \leftarrow 1.2*1.36D+00$$

else

$$R_{d1} \leftarrow 1.2*R_d$$

$$R_{d2} \leftarrow 1.1*R_d$$

$$R_{d3} \leftarrow R_d$$

iii) C_d =Time constant stay capacitance across loads (e.g. motors)

$$C_d \leftarrow 2.0$$

$$C_{d1} \leftarrow C_d$$

$$C_{d2} \leftarrow 0.8*C_d$$

$$C_{d3} \leftarrow 0.6*C_d$$

- iv) A_L = time constant, Line Inductances pu Henry/Km
 $A_{L1} \leftarrow 1.24D-01$
 $A_{L2} \leftarrow AL1$
 $A_{L3} \leftarrow AL1$
- v) C = time constant, Line Capacitors capacitance pu Farads/Km
 $C_1 \leftarrow 2.00D-01$
 $C_2 \leftarrow 1.2 * C_1$
 $C_3 \leftarrow 1.4 * C_1$
- vi) R = time constant, Line Resistance pu Ohms/Km
 $R_3 \leftarrow 1.36D-01$
 $R_2 \leftarrow 0.6 * R_3$
 $R_1 \leftarrow 0.2 * R_3$
- vii) R_{G1} = time constant, Insulator/grounding Leakage Resistance pu Ohms/Km
 $R_{G1} \leftarrow 9.99D+01$
 $R_{G2} \leftarrow R_{G1}$
 $R_{G3} \leftarrow R_{G1}$
- viii) at terminal time T take $Y(12) = Y(12, T) = Y12T$, $Y(13) = Y(13, T) = Y13T$
 $Y12T \leftarrow 0.3251E-04$
 $Y13T \leftarrow 0.1084E-01$
- ix) v = Systems Supply Voltage in pu volts A.C.
 $V \leftarrow 1.32D+00$
 $PI \leftarrow 3.14$
- x) f = Systems Frequency in cps (i.e. Herz)
 $f \leftarrow 0.10$

4. System Equations

```

v ← V*Sin(2*PI*f*x)
PRIME(1) ← (1.0/AL1)*(v-Y(2)-R1*Y(1))
YPRIME(2) ← (1.0/(C1+Cd1))*(Y(1)-(1./RG1)*Y(2)-Y(3)-Y(4))
YPRIME(3) ← (1.0/ALd1)*(Y(2)-Rd1*Y(3)-Y(3)*ALd1PRIME)
YPRIME(4) ← (1.0/AL2)*(Y(2)-Y(5)-R2*Y(4))
YPRIME(5) ← (1.0/(C2+Cd2))*(Y(4)-(1./RG2)*Y(5)-Y(6)-Y(7))
YPRIME(6) ← (1.0/ALd2)*(Y(5)-Rd2*Y(6)-Y(6)*ALd2PRIME)
YPRIME(7) ← (1.0/AL3)*(Y(5)-Y(8)-R3*Y(7))
YPRIME(8) ← (1.0/(C3+Cd3))*(Y(7)-(1./RG3)*Y(8)-Y(9))
YPRIME(9) ← (1.0/ALd3)*(Y(8)-Rd3*Y(9)-Y(9)*ALd3PRIME)
Y(10) ← Rd1*Y(3)**2 + Rd2*Y(6)**2 + Rd3*Y(9)**2
Y(11) ← R1*Y(1)**2 + R2*Y(4)**2 + R3*Y(7)**2
+(1./RG1)*Y(2)**2 + (1./RG2)*Y(5)**2 + (1./RG3)*Y(8)**2
YPRIME(12) ← (1./2)*Y(10)
YPRIME(13) ← (1./2)*Y(11)
Y(14) ← (1./T)*Y12T
Y(15) ← (1./T)*Y13T
RETURN
END

```

FCT: Function Sub-Routine:

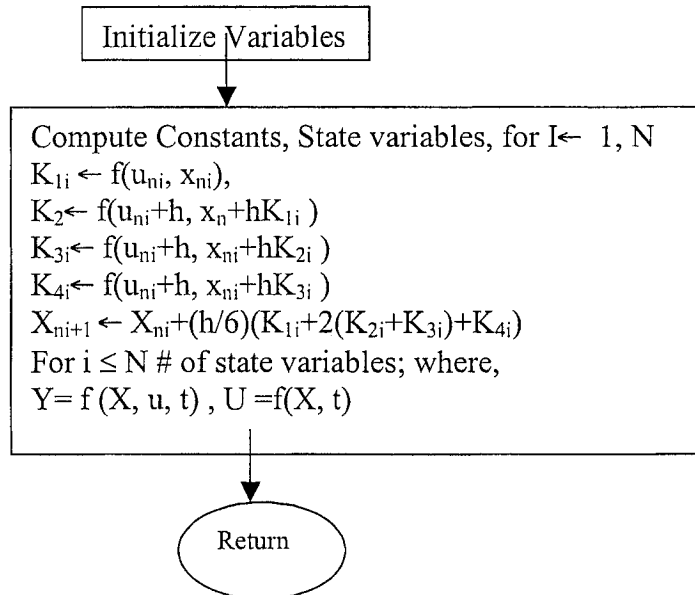


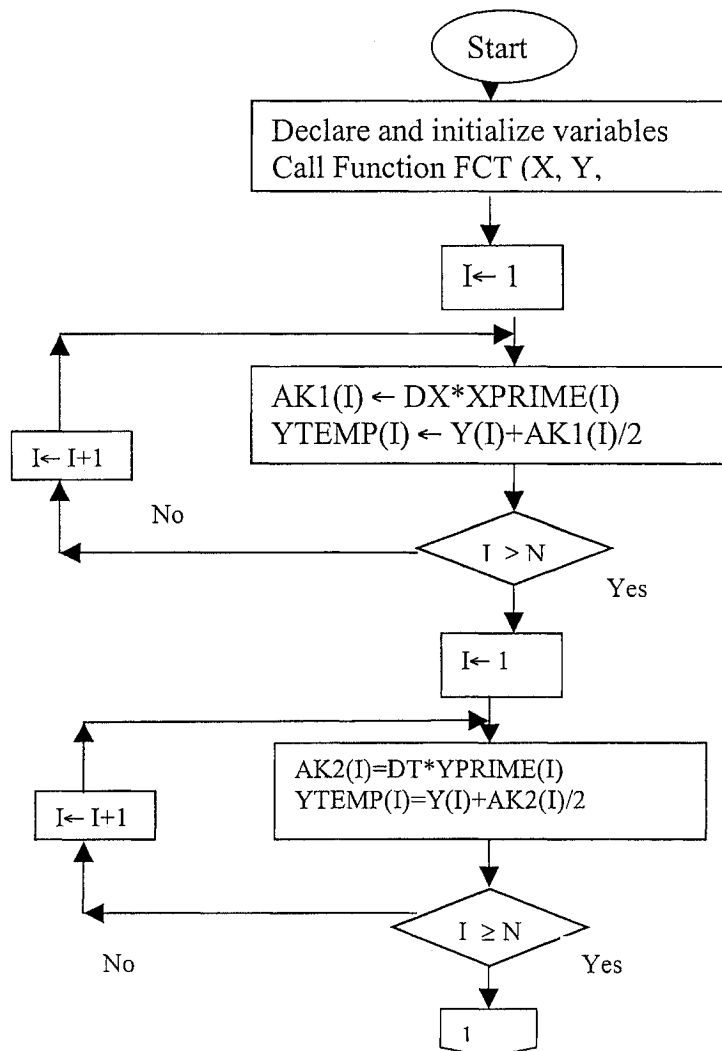
Figure 3.2 The Algorithm for simulated computation of functional variables

3.1.3 Algorithms for the RK4 Sub routine

1. Declare variables Y(11), YTEMP(20), AK1(20), AK2(20), AK3(20), AK4(20)
2. Call Fun (Time(X), State variables (Y(I)), Derivatives(YPRIME(I)))
3. Calculate AK1(I) & determine values of Y called YTEMP for AK2(I)
 - for I ← 1 to N by 1 do
 - AK1(I) ← DT*YPRIME(I),
 - YTEMP(I) ← Y(I) + AK1(I) /2
 - repeat
4. Calculate AK2(I) for AK3(I)
 - for I ← 1 to N by 1 do
 - AK2(I) ← DT*YPRIME(I),
 - YTEMP(I) ← Y(I) + AK2(I) /2
 - repeat
5. Call Fun (X + DT/2, YTEMP, YPRIME)
6. Calculate AK3(I) for AK4(I)
 - for I ← 1 to N by 1 do
 - AK3(I) ← DT*YPRIME(I),
 - YTEMP(I) ← Y(I) + AK3(I) /2
 - repeat

7. Call Fun (X + DT, YTEMP, YPRIME)
8. Calculate AK4(I)
 - for I ← 1 to N by 1 do
 - AK4(I) ← DT*YPRIME(I),
 - repeat
9. Calculate Y(I) at the end of the step of magnitude DT
 - for I ← 1 to N by 1 do
 - Y(I) ← Y(I) + (AK1(I) + 2*AK2(I) + 2*AK3(I) + AK4(I))/6
 - repeat
10. Increment X to its end step:
 - X ← X+ DT
 - Return
 - End.

The Runge-Kutta algorithm: RK4: Flow diagram.



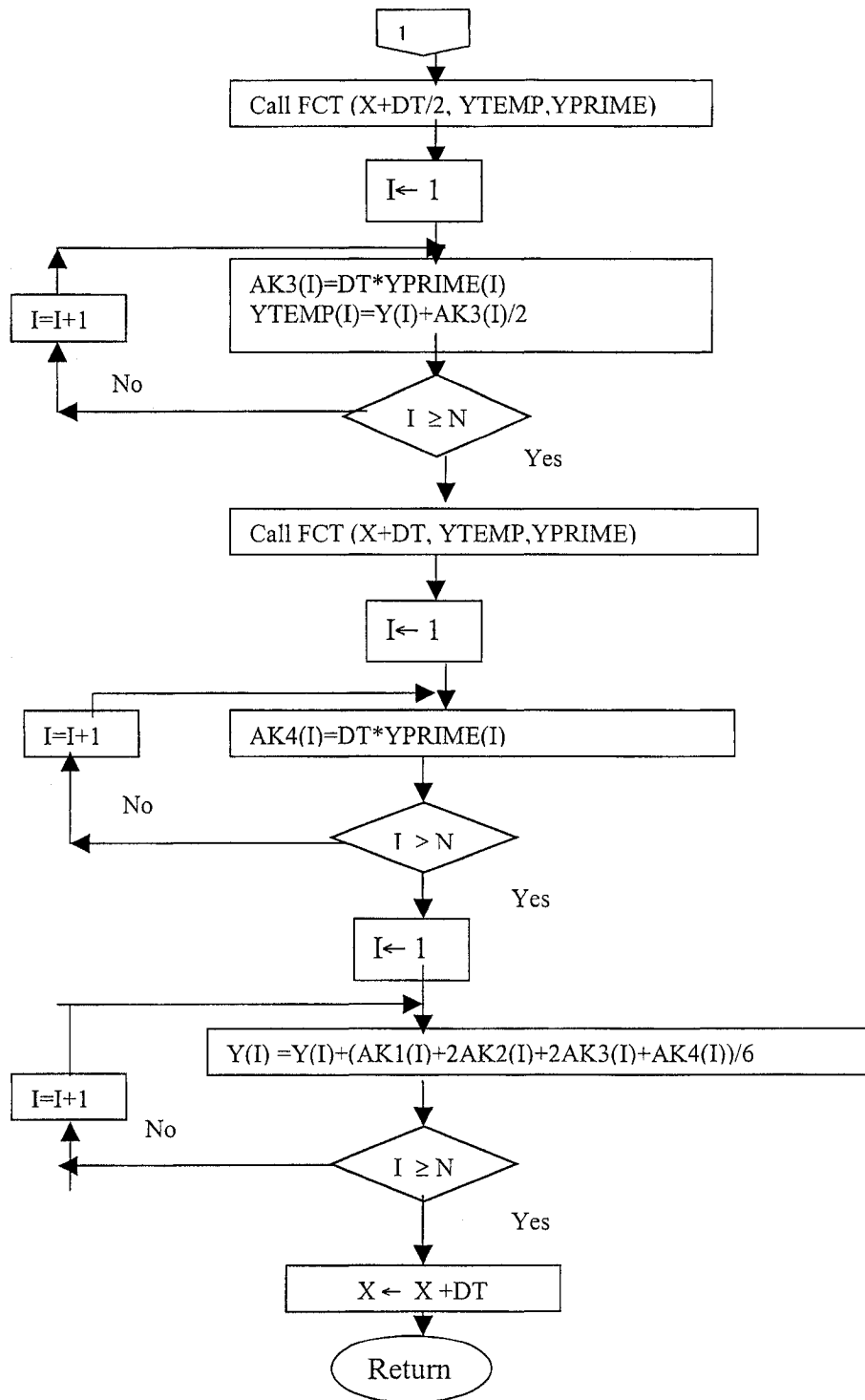


Figure 3.2a: RK4's core update.

This section details the content of the boxes called “RK4” subroutine and other as shown in Figure 3.1.

The guessed parameter is initially chosen arbitrarily, but as the simulation progresses, more acceptable values are used as initial guesses. The results of the corresponding state variables are saved in the file 'FLOW'.

The goal of RK4 is to solve the state equations (2.26 to 2.32), for given line parameters. At the start of the computing operation, load parameter at each node is guessed and given (inserted) for RK4 when the main program reads them.

After being given the initial values of the state variables, RK4 also needs the initial specification of the time discrete value, time interval, and total period of simulated computations.

The core of RK4 is an incremental calculation of state variables, as shown in the loop at the Figure 3.2a. The search stops when a test of convergence succeeds (see "Yes" "No" at the bottom of the figure). Within the loop, an iteration process updates the value of state variables along a step of the search direction.

The program subroutine RK4 calls the subroutine FCT for continuous incremental computation and final solutions of the state variables until the end of the simulation. The algorithm is explained in Figure 3.2a above.

3.1.2 Data Input

For working on the models before simulation, I arbitrarily selected initial set of line and load parameters from [Beaty, 2001] and calibrated them iteratively. Through a continuous process of simulation steps, I obtained an acceptable set of parameter values, which give me no resonance with least energy loss, keeping node voltages within $\pm 5\%$ of desired level.

(1) For the line parameters: $R_1=0.2R_3$, $R_2=0.6R_3$, $R_3=0.136$, at all loops & node $R_g=99.9$, $L=0.124$, $C_1=0.2$, $C_2=1.2 C_1$, $C_3=1.4 C_1$, time constant D.C input voltage amplitude $V=1.32$ pu volts, [or frequency $f=0.1$ v = $V \sin (2\pi ft)$ for A.C supply]

For the load parameters: $R_d=1.36=R_{d3}$, $R_{d1}=1.2 R_d$, $R_{d2}=1.1 R_d$, $L_d=10.0$; and stay capacitors $C_{d1}=C_d$, $C_{d2}=0.8 C_d$, $C_{d3}=0.6 C_d$.

(i) In case of time constant load, $R_d(t)=\text{constant}$ over time

(ii) In case of time varying load, $R_d=1.36$ for $t=[0, 0.4T]$, $R_d=0.8 \cdot 1.36$ for $t=[0.4T, 0.8T]$, $R_d=1.2 \times 1.36$ for $t=[0.8T, T]$, but $C_d=0.2$ & $L_d=10.0$ for $t=[0, T]$

Or,

(2) For the line parameters: $R_1=0.2R_3$, $R_2=0.6R_3$, $R_3=0.03$, at all loops and node $R_g=99.9$,

$L=0.12$, $C_1=0.2$, $C_2=0.6 C_1$, $C_3=0.4 C_1$, time constant D.C input voltage amplitude

$V=1.32$ pu volts, [or frequency $f=0.1$

$v = V \sin (2\pi ft)$ for A.C supply]

For the load parameters: $R_{d1}=R_d$, $R_{d3}=0.5 R_d$, $R_{d2}=0.7 R_d$,

$L_d=L_{d1}=L_{d2}=L_{d3}$; & stay capacitors $C_{d1}=C_d$, $C_{d2}=0.8 C_d$, $C_{d3}=0.6 C_d$.

(i) In the time constant load case, $R_d(t)=\text{constant}$ over time

(ii) In the time varying case of load,

$R_d=1.2$ for $t=[0, 0.4T]$, $R_d=0.8 \times 1.2$ for $t=[0.4T, 0.8T]$,

$R_d=1.2 \times 1.2$ for $t=[0.8T, T]$, but $C_d=2.0$ & $L_d=2.0$ for $t=[0, T]$

3.1.3 Data Output

Test output data were generated setting all the initial values of load current, node voltages and line currents at zero. I tested my models through simulation for analysis of the **characteristic behaviors** of an un-regulated power system. These data are used in the numerical plots given in the subsequent sections. Some sample numerical outputs (data) are given in the tables of Appendix 4 for reference.

3.2 EXPERIMENTAL DESIGN AND ANALYSIS

The simulation relies on the experimental design summarized at the beginning of Section 3.1. More specifically, it applies a variation of time discretization that used different step sizes and total simulation times, initial values of state variables. Three stages of analysis are adopted here:

a) a transient (and dynamic) response analysis to determine the response of a power system to commanded inputs and disturbance inputs; a final test is to verify whether the resulting characteristics are satisfactory or not;

b) a steady-state analysis to determine the response after transient response has disappeared, concluded by a verification of the adequacy of the resulting response characteristics via the plotting of output data of state variables against time;

c) an adjustment of line parameters with suitable values (per unit values) for the system until the preceding characteristics is found satisfactory.

The adjustment typically calls for a number of iterations of the preceding steps, using different time discretization, step-size, total interval (e.g., time period) and initial values of state variables, across the following dimensions:

- a) D.C. and A.C (with different frequencies) with constant supply (input) voltage and constant load current amplitude (as a form of control).
- b) D.C. and A.C (with different frequencies) with constant supply (input) voltage but time varying load current amplitude (as a form of control).
- c) D.C, and A.C (with different frequencies) with or without load tapped from the system node.
- d) A systematic variation of circuit layout, i.e.
 - (i) single loop and node elementary RLC circuit. (Line resistance 'R' is in series with the resultant impedance of inductance L and capacitance C that are parallel with each other);
 - (ii) single loop and node RLC circuit (line resistance R and inductance L in series including grounding resistance R_g which is in series with capacitance C and is parallel to load) with load current tapping from its node;
 - (iii) 3 loops and nodes RLC circuit (line resistance R and inductance L in series including grounding resistance R_g which is in parallel with capacitance C and is parallel to load) with only resistive load tapping from its nodes;
 - (iv) single loop and node RLC circuit (line resistance R and inductance L in series including grounding resistance, which is, parallel to capacitance C and is in parallel to load) with resistive load R_d (having its self inductance in series) tapping from its node;
 - (v) 3 loops and nodes RLC circuit (line resistance R and inductance L in series including grounding resistance which is parallel to capacitance C and is in parallel to load) with a resistive load (having its self-inductance in series) tapping from its nodes;
 - (vi) single as well as three loop and nodes RLC circuit: line resistance R and inductance L in series, including grounding resistance R_g which is parallel to

capacitance C and is in parallel to load (with time varying resistive load having its time varying self-inductance in series) tapping from its nodes; by taking the load current as one of the state variables and load resistance and inductance as controls (regulating tools);

(vi) Single as well as three loop and nodes RLC circuit: line resistance R and inductance L in series, grounding resistance R_g is parallel to node capacitance C , which is parallel to load [considered time invariant as well as time varying resistance $R_d(t)$, its time constant or varying self-inductance $L_d(t)$ in series and time constant stray capacitance C_d . placed across loads] tapping from its nodes.

By taking the load current as one of the state variables and load resistance and inductance as controls (regulating tools).

We summarize above all experimental designs in the following table.

Distribution line network	Source voltage	Ckt # of nodes & loops	Load types and conditions		
			Resistive (R_d)	Resistive R_d , inductive L_d	Resistive (R_d), Inductive (L_d), capacitive (C_d)
Line R in series with L & C	constant	ok	constant	Nil	Nil
Line R in series with L but leakage R_g is parallel to node C	constant	ok	Constant, variable	Constant, variable R_d , and L_d ,	Constant, Variable R_d , L_d , but only constant C_d
Distribution time variation steps used	constant	ok	0 to 0.4T 0.4T to 0.8T 0.8T to T	0 to 0.4T 0.4T to 0.8T 0.8T to T	0 to 0.4T 0.4T to 0.8T 0.8T to T
Computation discrete time (dt) steps used	0.01		0.01	0.01	0.01

Used constant D.C input voltage as well as time constant amplitude A.C. supply (i.e. input) voltage for test simulation with different frequencies:

In all the above cases: f were = 0.1, 0.5, 1.0, 5.0, 10.0, 20.0, 30.0, and 60.0 pu Hertz

In every case above, the verification entails checking whether response characteristics are satisfactory or not by plotting output data of state variables with time and analyzed.

3.2.1 Transient Analysis

The analysis of the graphs of the state variables (i.e., line currents and voltages at nodes, load currents) versus time contained in Section 3.3, gives us occasion to make the following observations.

Initially, line currents in loops increased gradually from zero to their peak (and fluctuated abruptly with bigger amplitudes for a short while) then dropped down slightly and became steady for the rest of the period. Based on the load current through application of their controls (i.e. load resistance or inductance, stay capacitors) or node capacitance, voltages at nodes initially incurred a drastic fall, then increased gradually or sharply to the peak value because of the effect of capacitor placed across the node or due to the load inductance and then become gradually steady at certain values for the rest of the period.

While using a time-constant load current, line currents shoot up and fluctuate during the initial transition time, showing an unstable behavior (sharp fluctuations with unusually bigger amplitudes), but they gradually become stable during rest of the time. On the other hand while using a time-varying load current, they show some sort of unstable fluctuations and shoot up at each of the change of load currents, then gradually become stable for rest of the period. By varying the load current or the load resistance and inductance, we can find out nominal values at which the operation of the system would cause least energy loss during the period of the day. This can be seen from the calculations of the load factor and loss factor of the corresponding period, as analyzed in this chapter. Such variations can help system managers in taking their operational decisions.

Referring to Figure 3.10, which display instantaneous as well as total average energy consumed and loss versus time (here in the three nodes case), it is observed that initially the energy loss is more than the energy consumed. This can be explained by the fact that initially, it takes time for the node capacitors to get charged and help boost node voltage. Hence, due to the lower node voltage, initially the energy loss exceeds energy load consumed.

But as the system become stable, the steady state condition of the system brings

node voltages to a desired level and reduce loss compared to load consumed; this has been observed from the steady-state case of power flow [Fig 3.3 to 3.18]. The transient analysis is in fact amalgamated with steady-state analysis in the graphical representation of Section 3.2.

3.2.2 Steady-state analysis

The steady-state analysis focuses on the same cases as the transient analysis, but practically provides a different guide to take account of the steady-state part of the behaviors of the variables with respect to time.

3.2.2.1 Steady-state analysis with D.C supply

The analysis is for a 3-node electrical network, with line resistance (R) and inductance (L) in series, capacitance (C) at the nodes are placed in parallel with grounding resistance (R_g), initially setting conditions of state variables at zero.

For both D.C. and A.C. supply, from the numerical output data (samples given in Table 1 and 2 of Appendix 4) obtained through simulation of the system, it is observed that we can maintain the node voltages within our desired level by manipulating the systems load parameters as well as the node capacitors. This can help us operate the power network systems keeping load factor nearest to but less than 1; while the loss factor less than 1 (nearest to but more than zero) during that period.

The characteristic behavior of the systems state variables, power intensity of load consumed and loss, total energy consumed and lost, plus reactive power during the period of operation can be seen below from their following numerical plots in Figures 3.3 to Fig 3.10 with time.

3.2.2.2 Steady-state analysis with A.C supply

The next stage of the analysis shifts to a case of A.C supply in a 3-node electrical network with line R and L in series, C placed at the node is in parallel with R_g , initially setting conditions of the state variables to zero.

The observations of the preceding section apply generally; they are repeated here with slight variations. From the numerical data (samples given in Table 2 of Appendix-

4) obtained through simulation of the system, it is observed that we can maintain the node voltages within our desired level by manipulating the systems load parameters as well as the node capacitors. This can help us operate the power network systems keeping load factor nearest to but less than 1; while the loss factor less than 1 (and nearest to but more than zero) during that period. The characteristic behavior of the systems state variables, power intensity of load consumed and loss, total energy consumed and lost as well as their operating factors during the period of operation (with A.C. supply) can be seen below from their numerical plots in Figure 3.11 to 3.18 with time.

3.3 GRAPHICAL REPRESENTATION OF EXPERIMENTAL RESULTS

Among the factors hindering a systematic comparison of experiments is the interactive control that we apply to avoid resonance, long transient states or peak values. Thus, conditions of operation are not standardized and comparisons can only remain essentially qualitative. It is therefore decided to summarize them via numerical plots of state variables with time. The graphical representation spans several experiment, using a systematic order of presentation. For each experiment, the first set of graphs represent state variables gathered in one figure for each node and loop of the network, so as to ease the comparison of their behaviour. In a second set, the same information is synthesized in the subsequent figures that show line currents, node voltages, load currents, power density, and energy of the network, thus permitting inter-node or loop comparisons. Each of the figures introduced so far displays transient and steady state currents and voltages in the line or at the node.

3.3.1 Analysis with D.C. supply and constant load

The following sub-sections demonstrate the behavioral changes of different state variables with respect to time and loads for analysis. Here, variations of line currents, node voltages, load currents, power density or energy (of load consumed and loss), with the time as well as with different load types (or load conditions) are analyzed.

3.3.1.1 At different Nodes, behavioral changes of D.C line currents, node voltages, and load currents with time.

The simulation first focuses on the case of a 3 loop and node RLC power network *circuit with constant load* resistance and inductance (in series) plus stray capacitors across load (i.e. motors) as in Figure 2.3. This yield some numerical values of state variables with time (plots are not shown here). We note in this as well as following experiments a similarity between the behavior of currents and voltages of different nodes i.e. a gradual transition to steady state. Naturally, they differ in amplitude because of the applied loads from node to node. In this case, all of the state variables initially show temporary instability, followed by a gradual stabilization, then continue with constant amplitude because of the constant loads over time. The transient behavior is approximately similar at nodes 1, 2 and 3 for all state variables. As Node 3 is the last node, the line and the load current become equal after the system has become steady and stable.

Here, differences in voltage amplitudes from node to node also appear because of the loads. However, their amplitudes are managed to vary only within $\pm 5\%$ of the desired level through application of suitable controls.

3.3.1.2 Behavioral changes of D.C power (consumed and loss) intensity and energy load (consumed and losses) with time.

Note that loss is less than load consumed; both show initial transient unstable behavior but become stable and steady after a while and continue like that.

The results of the power density are integrated to show the changes of energy (of load consumed, loss) with time for the whole system. One can see that the systems energy loss is much lower than that consumed though at the start of operation (before the system pickup load and the line inductance, plus node capacitor get energized i.e. during transient stage), energy loss is more than loads consumed.

3.3.2 Analysis with D.C. supply and with time-varying load resistance

The analysis is now performed for a 3-loop and 3-node RLC circuit with resistance (time varying) in series, load inductance (time constant) and stay capacitors C_d . (time constant) across load (e.g. motor). (Network shown in Figure 2.3). The analyses of Section

3.3.1 are repeated here with exactly the similar set of numerical plots and are displayed for time varying loads.

3.3.2.1 At different Nodes: plots of D.C line currents, node voltages, and load currents

The next three Figures 3.3 to 3.4 display state variables at three nodes. We will note in every experiment a similarity between the behavior of currents and voltages of different nodes i.e. a gradual transient to steady state; here also, they differ in amplitude from node to node because of the loads. Their amplitudes also vary with time-varying loads (e.g. at $t = 40$, and at $t = 80$).

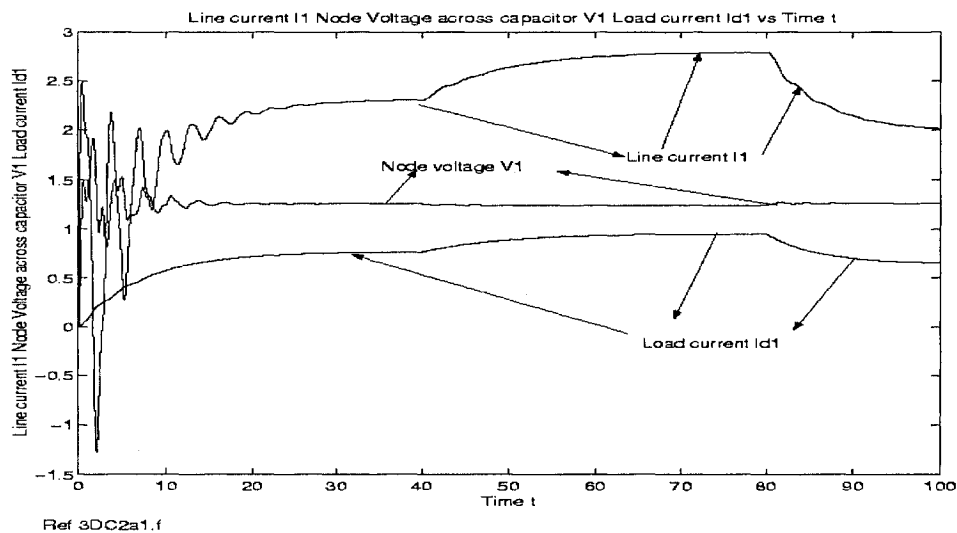


Figure 3.3- Node 1: Changes of D.C state variables (I_1 , V_1 , I_{d1}) with time.

The behaviour of state variables at Node 2 is related to those at Node 1.

In Figure 3.4, all of the state variables initially show temporary instability then become stable and steady and continue with constant amplitude because of the constant loads over time; they vary again only when there occurs a change in magnitude of loads. They are even more apparent as steady conditions vary along with loads (in time). Also, the line current amplitudes are seen to be as $I_1 > I_2 > I_3$.

The behaviors of the state variables at node2 are seemingly related to those at Node 1. They all vary in their magnitude with the time varying loads.

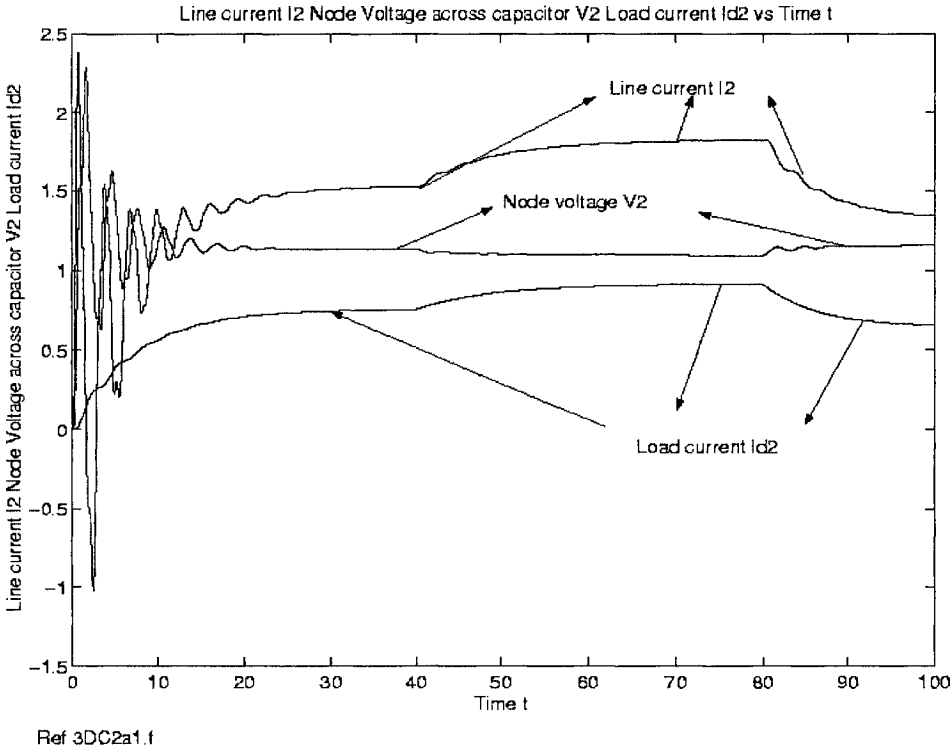


Figure 3.4- At Node 2: Changes of D.C state variables (I_2 , V_2 , I_{d2} .) with time.

In Figure 3.5, all of the state variables initially show temporary instability then become stable and steady and continue with constant amplitude because of the constant loads over time. The same remark as in Node 2 applies to those at Node 3. Since Node 3 is the last node, the load current amplitude presumably equals to that of the line current.

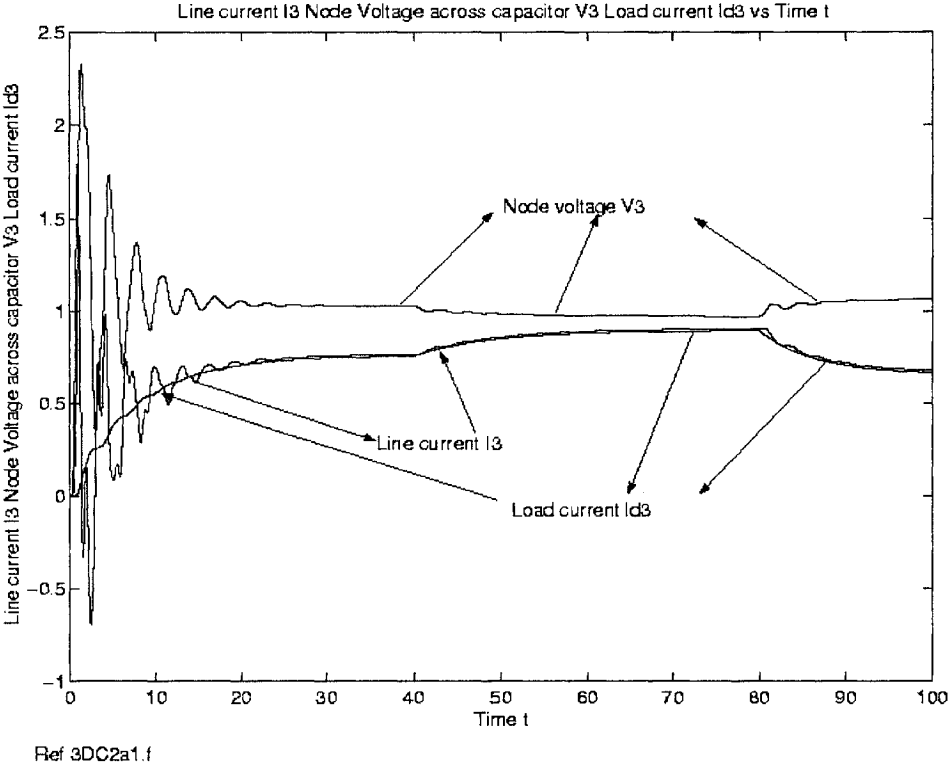


Figure 3.5- At Node 3: Changes of D.C state variables (I_3 , V_2 , I_{d2} .) with time.

3.3.2.2 Plots of only different D.C line currents or node voltages, or load currents with time. To show their behavioral changes with time.

Repeating part of the above information, the three line currents are now re-grouped by type of state variables, emphasizing the similarity in behavior of each line current. Note that their amplitudes differ (because of the loads at different nodes plus total effect of loads up to the node concerned). The results of the preceding sub-section may also be re-grouped by type of state variables emphasizing on the similarity in the behavior of voltage at each node and of each load current. Note that the load currents behave in a very different way from line currents. Immediately after load is put on the system, load current jumps up to peak and keeps on flowing with constant amplitudes over the time.

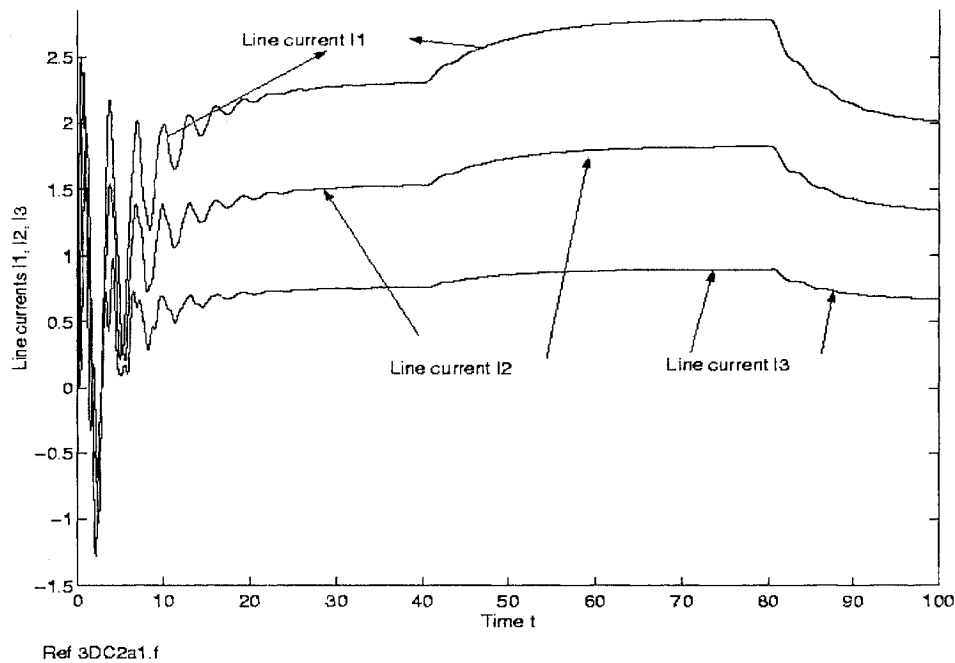


Figure 3.6- Changes of D.C currents I_1 , I_2 , I_3 . at line 1, 2, 3 with time t

Here, because of time-varying loads, amplitudes of line currents vary at time t_1 , t_2 and t_3 as shown in table below: -

	$t_1=40$	$t_2=80$	$t_3=100$
I_1	2.313	2.792	1.998
I_2	1.532	1.828	1.333
I_3	0.7648	0.9025	0.6706

In Figure 3.6, while the detailed behavior of different line currents differs appreciably from those for the time constant loads, each of the current in three lines is strongly related to that of the other lines. By re-grouping the line currents in one graph, one can clearly see how they vary in amplitudes as a group with the time varying loads (in respect of their pattern of changes).

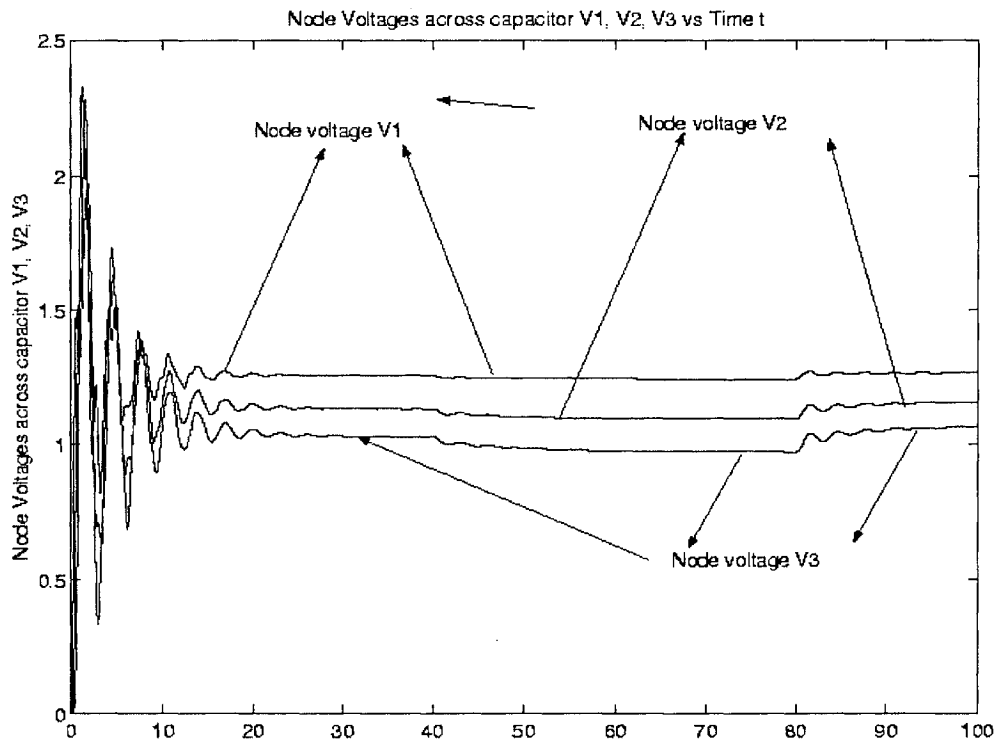


Figure 3.7- Changes of D.C voltages V_1 , V_2 , V_3 . at Node 1, 2, 3 with time.

Also, because of time-varying loads amplitudes of node voltages vary at time t_1 , t_2 and t_3 as

shown below: -	$t_1=40$	$t_2=80$	$t_3=100$
V_1	1.257	1.244	1.266
V_2	1.132	1.095	1.158
V_3	1.028	0.9721	1.067

In Figure 3.7, the node voltages show similar behavior, but varying in amplitude with time varying loads. Initially, all of them starting from zero and highly fluctuate during transient time and then become stable and steady. There are similarity in the pattern of their changes in amplitudes (amplitudes vary at $t = 40$, and at $t = 80$)

Figure 3.8 displays load currents at all 3 nodes. In every successive experiment involving time-varying loads one will notice a similarity between the behavior of load currents at different nodes i.e. a gradual transient to steady state; they differ only in amplitudes from node to node because of the time varying loads. Their amplitudes are controlled here with the load resistance (keeping load inductance and stay capacitors constant over time).

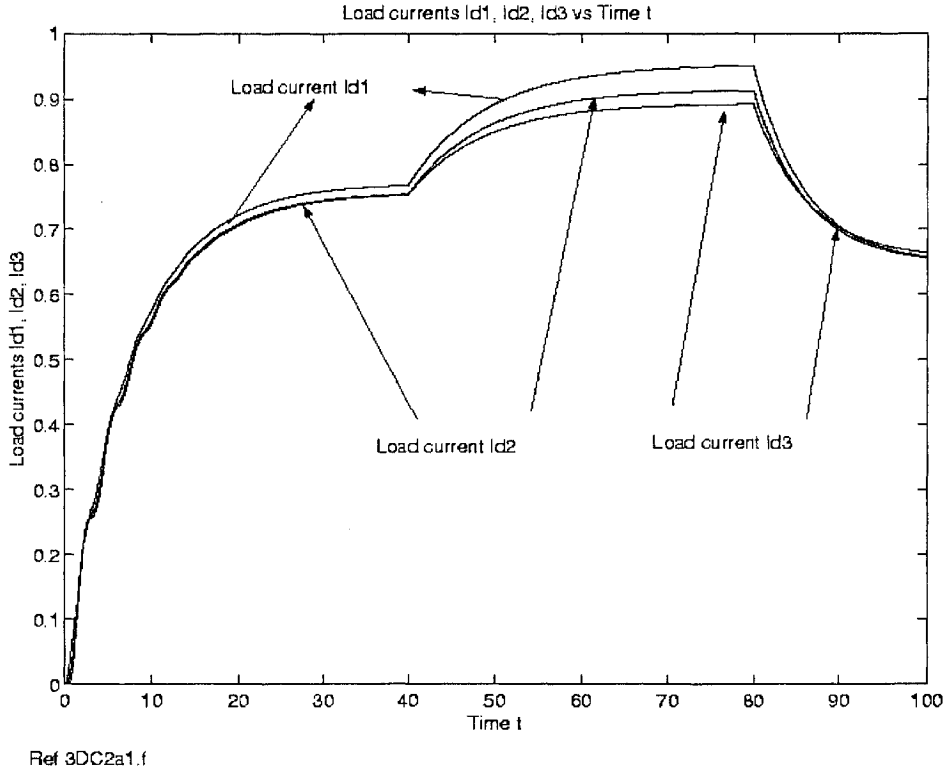


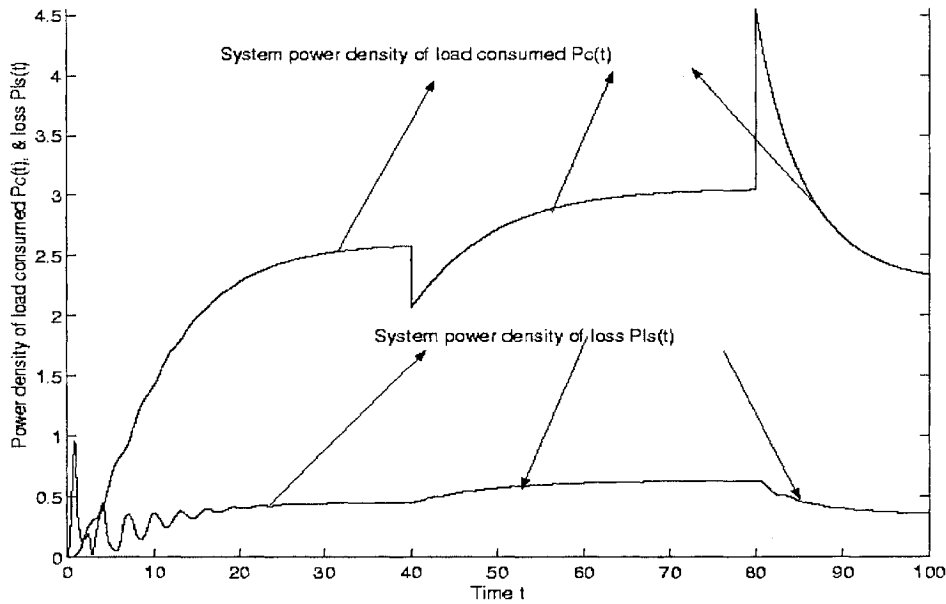
Figure 3.8- Changes of D.C loads 1, 2, 3 currents I_{d1} , I_{d2} , I_{d3} . with time.

Also, because of time-varying loads amplitudes of load currents vary at time t_1 , t_2 and t_3 as

shown in table-	$t_1= 40$	$t_2=80$	$t_3 =100$
I_{d1}	0.7622	0.9518	0.6525
I_{d2}	0.7555	0.9139	0.6510
I_{d3}	0.7545	0.8923	0.6598

3.3.2.3 Behavioral changes of D.C power (consumed and loss) intensity and Energy load (consumed and losses) with time.

Power density for load consumed as well as loss is discussed here separately. Their behavioral changes with time are emphasized to show their pattern of changes with load type and conditions. Figure 3.9- display the power density of load consumed and loss. An overall pattern of change in the behavior of power density emerges from this and related experiments. Whereas amplitudes vary from time to time because of the time varying loads, we can always see some form of discontinuity followed by a gradual transition to steady state. Initially for a shorter and transient time, loss density is very high compared to load consumed then it become low enough to an acceptable level and continue for rest of period. With time-varying loads, power density also vary (as at $t = 40$ and at $t = 80$) but for time-constant loads they remain to continue steadily at constant amplitude



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Figure 3.9- Changes of systems Intensity of D.C Power consumed $P_c(t)$ and loss $P_{ls}(t)$. with time.

Here, because of time-varying loads, max. amplitudes of power density vary at time t_1 , t_2 and t_3

as shown in table-

	$t_1= 40$	$t_2=80$	$t_3 =100$
$P_c(t)$ max	2.594	4.574	2.305
$P_{ls}(t)$ max	0.4558	0.6323	0.3557

Figure 3.10 displays energy load consumed, loss and their averages. We will note, in every experiment a similarity between the behavior of energy as an integrated value of power density; both of the graphs vary because of the loads over time (both $E_c(t)$ and $E_{ls}(t)$ take a turn at $t = 40$ and $t = 80$). As stated for the power density, energy loss also seen to be higher than energy load consumed initially and then goes down below load consumed. This may be because of the pick-up or transition time (of inductance and capacitance to get charged) of loads. Here, total energy load consumed and loss at the end of simulation $E_c(T) = 250.8$, $E_{ls}(T) = 47.76$ and their averages are $E_{ca}(T) = 2.508$, $E_{lsa}(T) = 0.4776$.

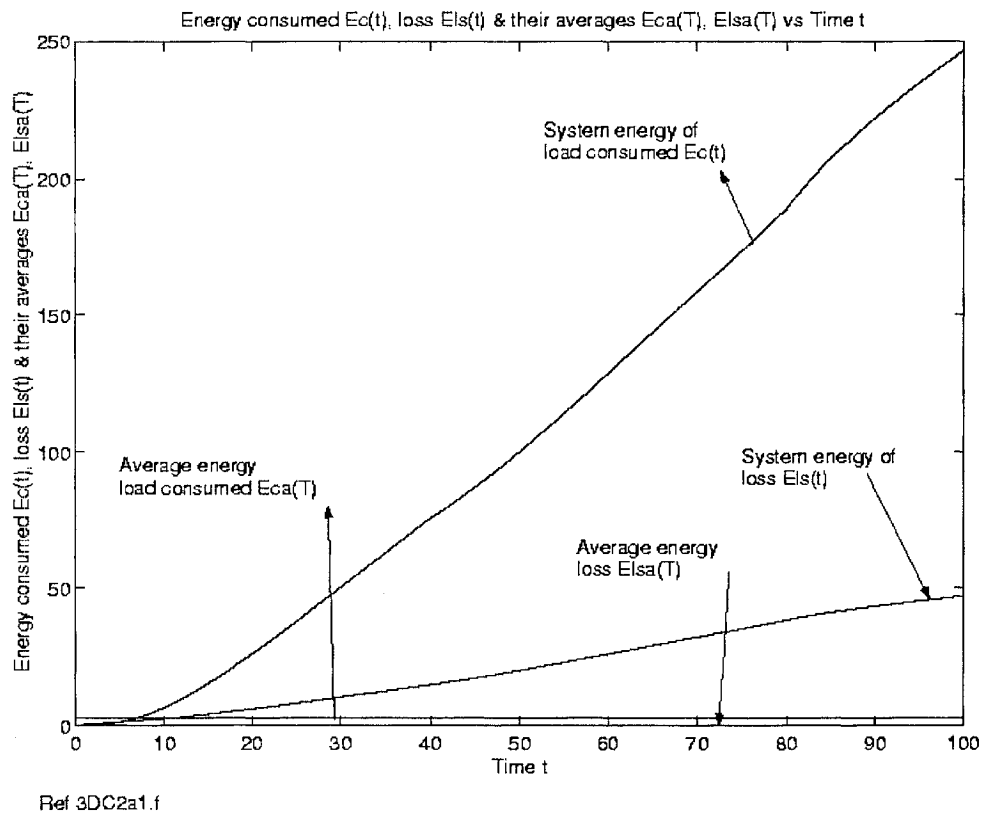


Figure 3.10: Changes of systems D.C energy consumed $E_c(t)$ and loss $E_{ls}(t)$ and their averages $E_{ca}(T)$, $E_{lsa}(T)$ with time.

3.3.3 Analysis with D.C input for the different types of loads (and conditions)

Additional patterns of variations were analyzed with respect to the cases of Sections 3.3.1 and 3.3.2 [sample data in Appendix 4). A summary of the loads (and conditions) is given in the list below.

Num ID Load type (and or conditions)

- 1 No load condition
- 2 Const. Resistive load (R_d)
- 3 Time varying resistive load ($R_d(t)$)
- 4 Const. load Resistive ($R_d, L_d=10$)
- 5 Time varying resistive load ($R_d(t)$) but const inductance ($L_d.=10$)
- 6 Const. load Resistive ($R_d, L_d=10$ & C_d)
- 7 Time varying load resistive ($R_d(t)$) but const ($L_d.=10$) & inductance ($C_d.$)
- 8 Const. load Resistive ($R_d, L_d=12$)
- 9 Time varying resistive load ($R_d(t)$) but const inductance ($L_d.=12$)
- 10 Constant load Resistive ($R_d, L_d=12$ & C_d)
- 11 Time varying load resistive ($R_d(t)$) but const ($L_d.=12$) & inductance ($C_d.$)

3.3.3.1 Variations of D.C node voltages V_1, V_2, V_3 for different types of loads (and conditions)

Note the similarity in variations of the node voltage amplitudes of each case; higher variations with load type and condition are noticed at Node 2 and 3 than at Node 1. The lowest voltage amplitude at Node 2 and 3 occurs in case of loads characterized by resistance (time varying) and series inductance (constant). On the other hand, the highest amplitudes are seen in the case of constant resistive (only) loads.

Variations of voltage amplitudes at Node 1 due to the various loads types and conditions can be seen for the time constant resistive load V_1 , which maintained maximum amplitude.

The D.C node voltage V_2 remain maximum in case of loads with time constant resistive (only); The Node 2 voltage decreases with higher combined loads (resistive, inductive and capacitive) as well as with only inductive loads.

One can observe that the variations of voltage are even more pronounced at Node 3 (the furthest node) compared to those at Node 1 and 2. Here also, the case of constant resistive loads features maximum node voltage amplitude, whereas the Node 3 voltage is the lowest for the case of time-varying resistive (plus time constant inductive) loads.

3.3.3.2 Variations of D.C power density, energy and operational factors wit type of loads (and conditions).

The variations of maximum amplitudes of power density [of load consumed $P_c(t_p)$ and loss $P_{ls}(t_p)$], average energy load [consumed $E_{ca}(T)$ and loss $E_{lsa}(T)$], load factor $F_c(T)$ and loss factor $F_{ls}(T)$ for different load types and conditions are discussed here.

The variations of the maximum amplitudes of power density are clearly different in each case because of the nature, type or conditions of applied load during whole or part of the simulation period.

Energy loads consumed vary widely between cases of loads considered. Compared with all other types and or condition of loads, the time varying resistive load with series time constant inductance clearly consumes maximum of energy, causing also maximum loss.

Factors of energy loads consumed vary widely between cases of loads considered and their duration. Maximum load factor and loss factor can be seen for the time varying (or constant) resistive load with series time constant inductance plus stay capacitor across loads; on the other hand constant resistive load gives minimum loss factor amongst all other loads here.

3.3.4 Analysis with A.C. supply and time constant load

The following sub-sections show the behavioral changes of different states with respect to time and loads for a 3-loop and node RLC circuit with constant A.C. supply voltage, load resistance (time invariant) in series with load inductance (time constant) and stray capacitors C_d (time constant) across load (e.g. motor).

Here, variations of line currents, node voltages, load currents, power density or energy (of load consumed and loss), with the time as well as with different load types (or load conditions) are shown for analysis.

3.3.4.1 Behavioral changes of state variables at each node with time:

For time constant loads, we will note in every experiment a similarity between the behaviors of currents and voltages at different nodes i.e. a gradual transition to steady-state. Again, they differ in amplitude from node to node because of the loads.

The transient behaviors of all state variables are approximately similar for Node 1, 2, and 3. In particular, the variables display the same type phase differences. As the load is constant over time, the amplitudes of the state variables are also seen to be constant during simulation period.

3.3.4.2 Behavioral changes of line currents, node voltages, load currents

Again, we note a similarity in the behavior of each type of state variables across the 3 nodes. Line currents decrease from the first line to the last line proportionate to the load currents tapped from the nodes (and leakage current which we ignored here for simplicity); thereby maximum amplitudes or instantaneous values of $i_1 > i_2 > i_3$.

One can see only little change in voltage amplitudes from node to node because of the loads (which may be controlled to maintain node voltage at the further end of the network to a desired level).

Note that load current display a very different behavior from line currents because of the series inductance and capacitance. Immediately after load is put on the system load current jumps up to maximum and keeps on flowing with constant amplitude over the time. Node voltage amplitude vary (decrease) slightly with loads at the furthest end of network.

3.3.4.3 Behavioral changes of Power density, Energy with time for A.C. supply and constant load.

Again, we notice that Loss is less than load consumed; both show initial transient unstable behavior but become stable, thus displaying steady state after a while. Loss density as well as load consumed density remain at constant level because of the time-constant loads.

The results of the power density are integrated with respect to time in order to analyze the changes of energy (of load consumed and loss and their averages) with time. It is thus found that, though initially loss is higher at the transient stage, throughout rest of the period the loss remains far less than the energy load consumed.

3.3.5 Analysis for A.C supply with time-varying loads

The analysis is now performed for a 3-loop and 3-node RLC circuit with resistance (time varying) in series, load inductance (time constant) and stay capacitors C_d . (time constant) across load (e.g. motor). (Network Figure 2.3). The analysis of Section 3.3.4 is repeated here with exactly the similar set of numerical plots displayed for time varying loads with constant A.C. supply.

3.3.5.1 Plots of state variables at different Nodes (for time varying loads)

For the 3 loop and node RLC circuit: resistance (time varying) in series with load inductance (time constant) and stay capacitors C_d . (time constant) across load (e.g. motor). [Figure 2.3]. The following are the numerical plots of state variables with time. Here, analysis of Section 3.3.2 is repeated with exactly the similar set of plots displayed for time varying A.C loads.

As in Section 3.3.2, for the display of state variables at three nodes, we will note here in every experiment a similarity between the sinusoidal behavior of currents and voltages of different nodes i.e. a gradual transient to steady state; Here, also they differ in amplitude from node to node because of the loads. Their amplitudes also vary with time-varying loads.

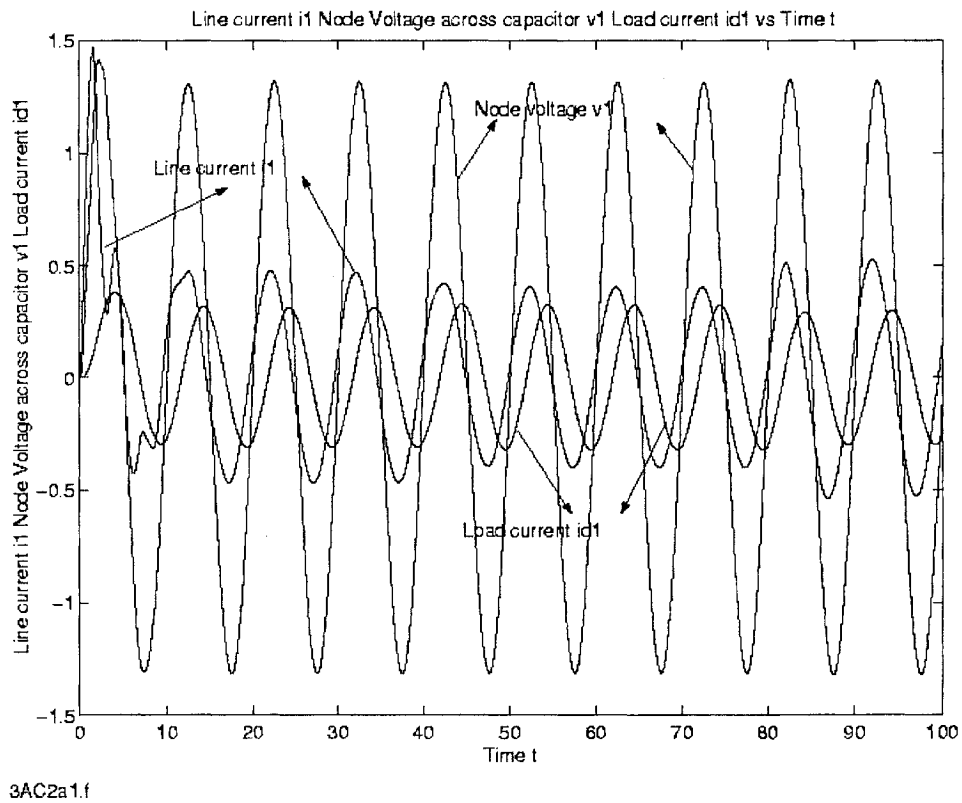


Figure 3.11- At Node 1: Changes of systems state variables (i_1 , v_1 , i_{d1} ,) with time.

Figure 3.12 displays, as in Section 3.3.2 but with the sinusoidal behavior of the state variables at Node2 which are related to those at Node 1 and 3;

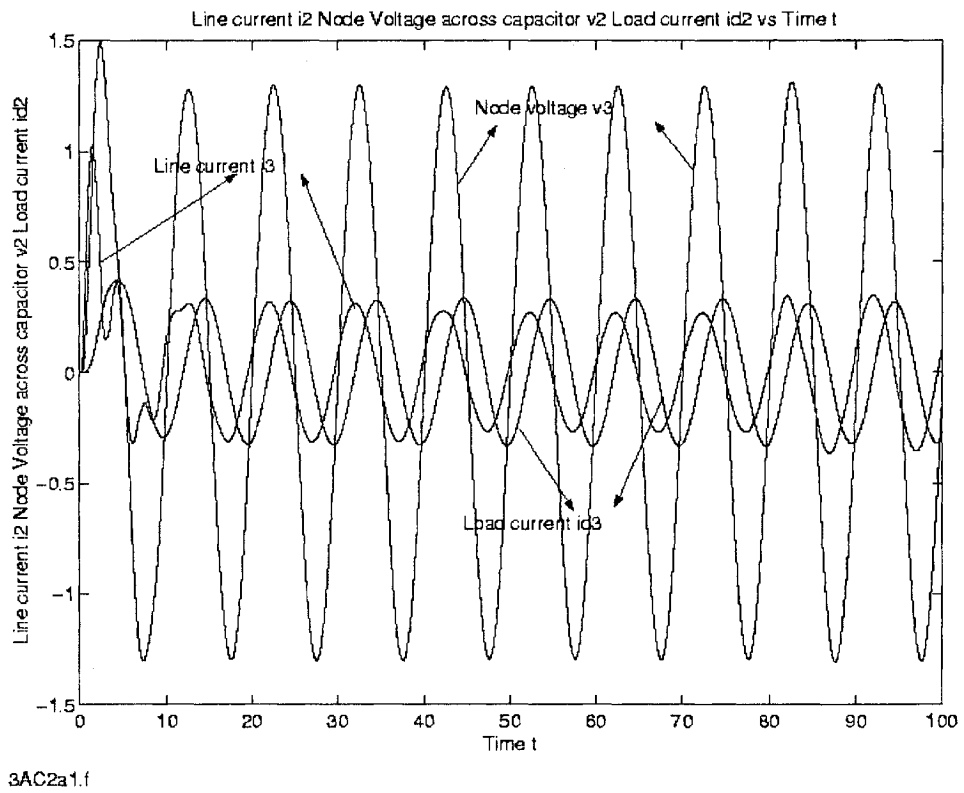


Figure 3.12- At Node 2: Changes of systems state variables (i_2 , v_2 , i_{d2} ,...) with time.

The same remark for the state-variables as in Node 2 applies to those at Node 3.

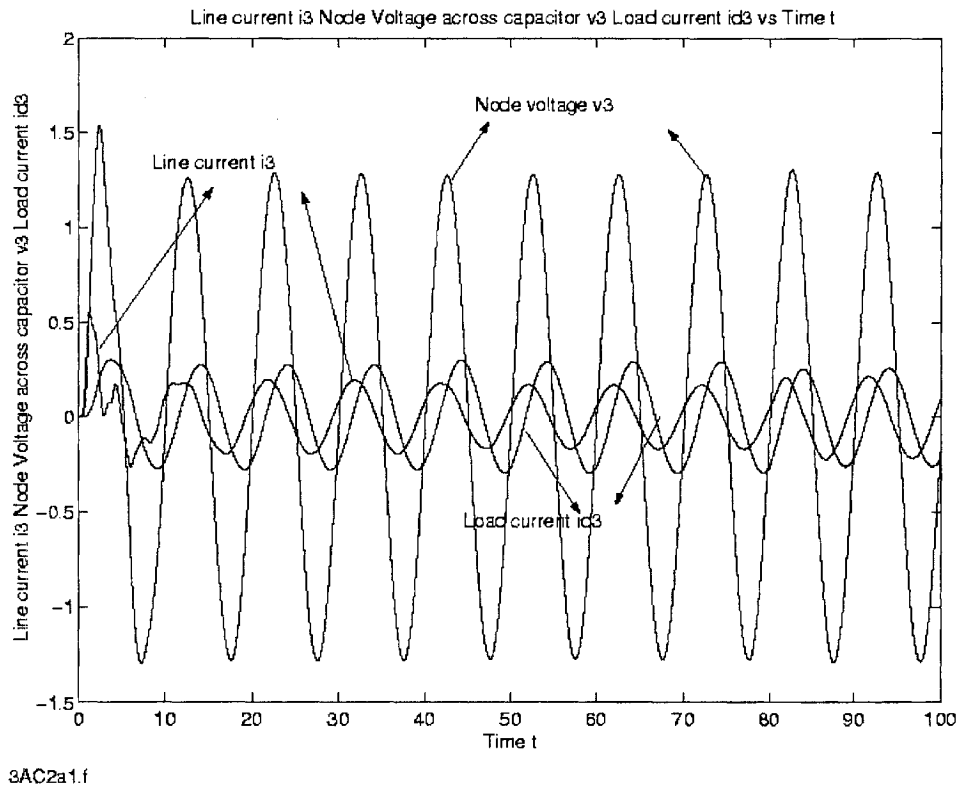


Figure 3.13- At Node 3: Changes in systems state variables (i_3 , v_3 , i_{d3} ,...) with time.

3.3.5.2 Different A.C line currents, or node voltages, or load currents at different loops and nodes (for time varying loads)

Here also the detailed behavior of currents differ appreciably from those of Section 3.3.2; each of the current in three lines is strongly related to that of the other lines

In Figure 3.14, while the detailed behavior of different line currents differs appreciably from those of Section 3.3.4.2, each of the current in three lines is strongly related to that of the other lines. By re-grouping the currents in one graph, one can clearly see their pattern how they vary in amplitudes as a group, with the time varying loads.

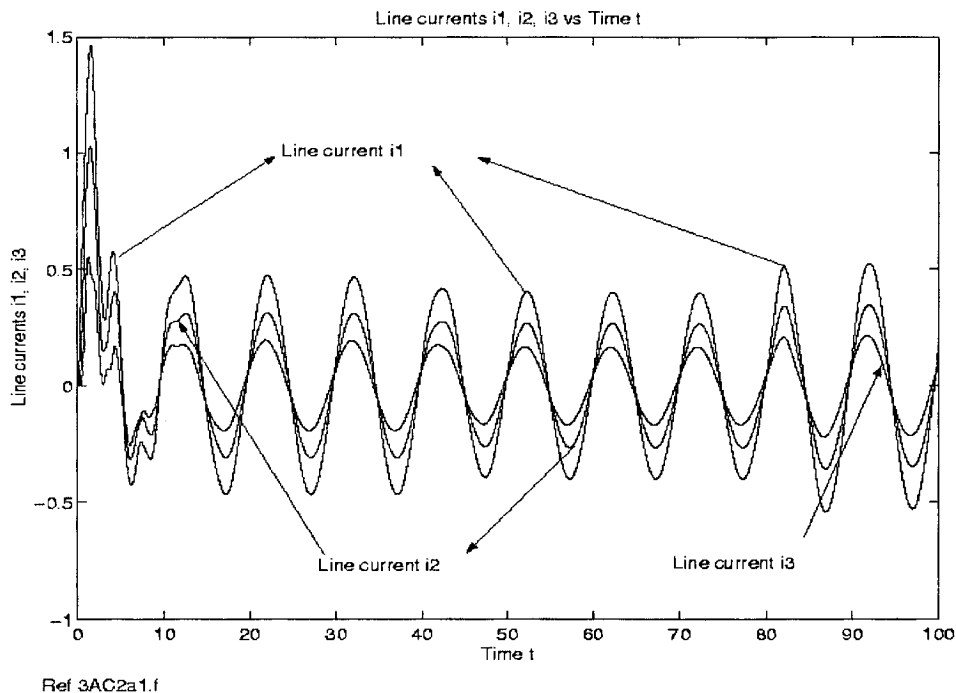


Figure 3.14- Changes in systems Line currents i_1 , i_2 , i_3 , with time.

Here, because of the time-varying loads amplitudes of line currents vary at time t_1 , t_2 and t_3 as

shown in the table:	$t_1 = 40$	$t_2 = 80$	$t_3 = 100$
I_1	0.4851	0.4185	0.4495
I_2	0.3295	0.2845	0.3056
I_3	0.1892	0.1585	0.1740

In Figure 3.15, also node voltages show similar behavior as in Section 3.3.4.3, but varying in amplitude with time varying loads. From the figure, one can see slight variations of the amplitudes of voltages at the 3 Nodes (but keeping the same phase angle).

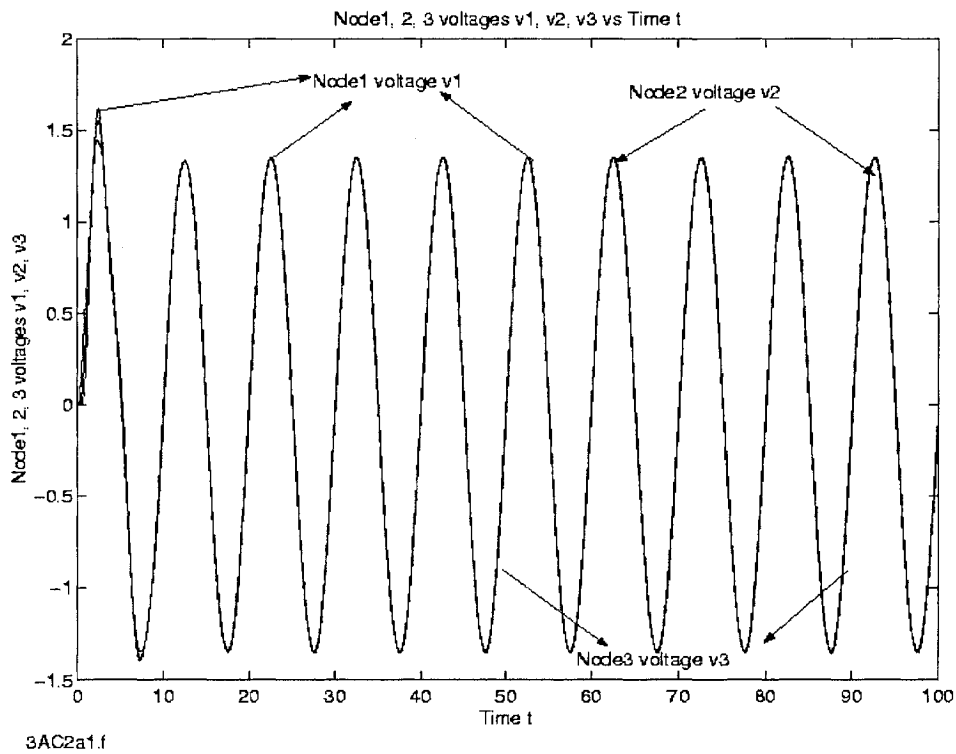


Figure 3.15- Changes of systems A.C. Node 1, 2, 3 voltages v_1, v_2, v_3 ,. with time

Also, because of the time-varying loads amplitudes of node voltages vary at time t_1, t_2 and t_3 as shown in the table below:

	$t_1=40$	$t_2=80$	$t_3=100$
V_1	1.344	1.343	1.344
V_2	1.352	1.352	1.352
V_3	1.350	1.351	1.350

Figure 3.16 display A.C. load currents at 3 nodes. We will note in every experiment a similarity between the behavior of load currents at different nodes i.e. a gradual transition to steady-state; They only differ in amplitude from node to node because of the loads. Their phase sequence also remaining same.

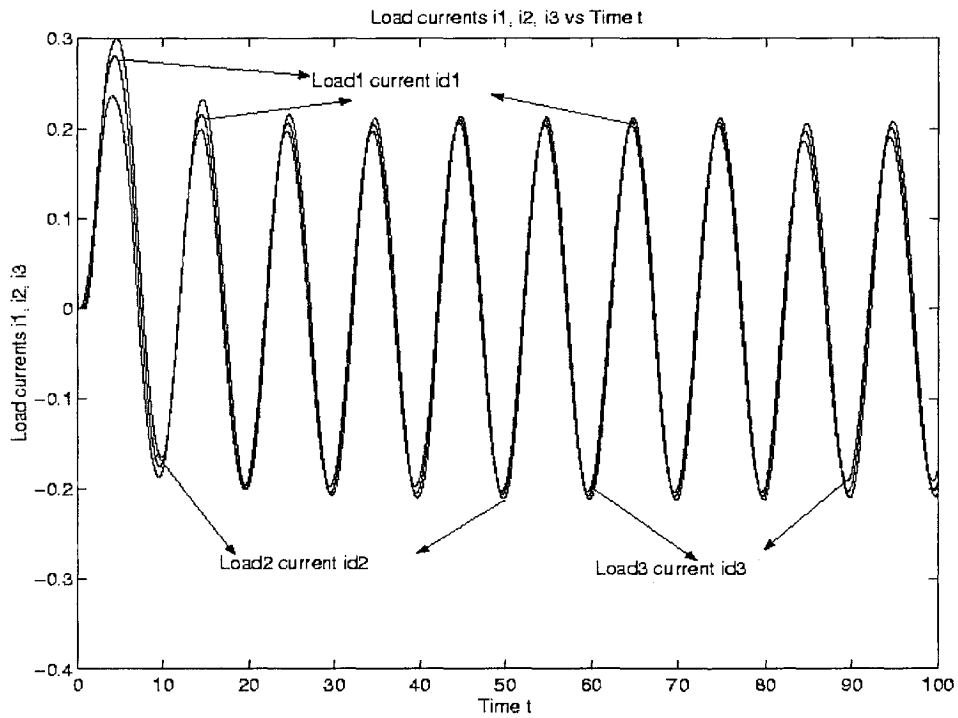


Figure 3.16- Changes of systems load currents i_{d1} , i_{d2} , i_{d3} , with time.

Also, because of time-varying loads amplitudes of load currents vary at time t_1 , t_2 and t_3 as shown

in the table below:	$t_1=40$	$t_2=80$	$t_3=100$
I_{d1}	0.2041	0.2079	0.2050
I_{d2}	0.2086	0.2121	0.2116
I_{d3}	0.1906	0.2034	0.1973

3.3.5.3 Behavioral changes of power density, energy with time-varying loads.

Figures 3.17 and 3.18 display Power density of load consumed and loss plus their respective energy. We will note in every experiment a similarity between the behavior of power density i.e. a gradual transient to steady-state condition of flow; They differ in amplitude from time to time only because of the time varying loads.

Figure 3.17 displays the power density of load consumed and loss. An overall pattern of change in the behavior of power density emerges from this and related experiments. Whereas amplitudes vary from time to time because of the time varying loads, we can always see some form of sudden rise or fall followed by a gradual transition to steady state. Initially for a shorter and transient time, loss density is higher compared to load consumed then it become low enough to an acceptable level and continue with constant level.

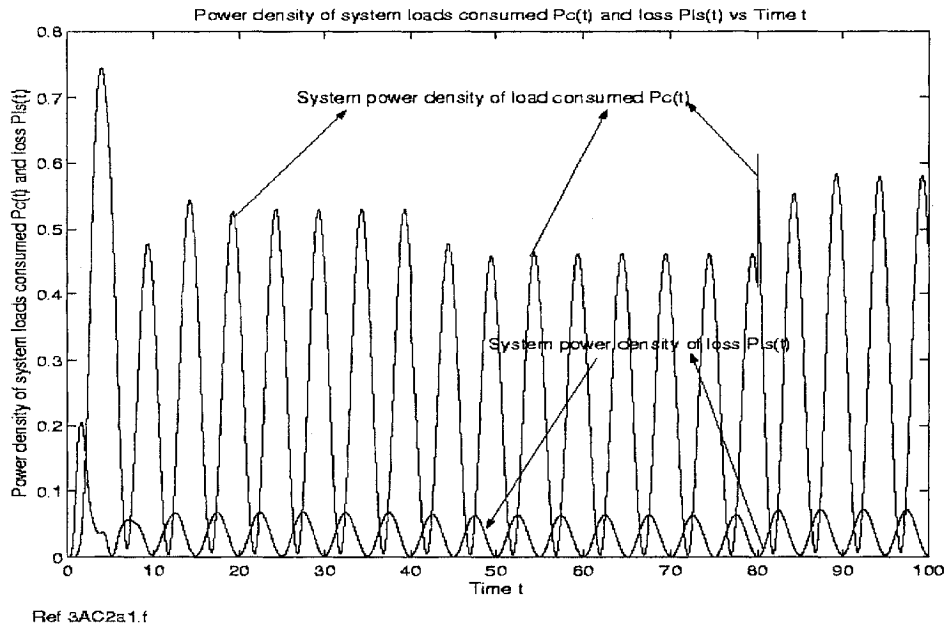


Figure 3.17- Changes in systems Intensity of Power consumed $P_c(t)$ and loss $P_l(t)$. with time.

Here, amplitudes of power density of loads consumed vary with the change in time-varying loads but for a time constant load its amplitude remains at a constant level over time. In this case, maximum amplitudes of power density at time t_1 , t_2 and t_3 , are as the bellow:

	$t_1 = 40$	$t_2 = 80$	$t_3 = 100$
$P_c(t_p)_{max.}$	0.2808	0.2040	0.2457
$P_l(t_p)_{max.}$	0.06186	0.05873	0.06033

Figure 3.18 displays energy load consumed, loss and their averages. We will note in every experiment a similarity between the behavior of energy as an integrated value of power density because of the loads. Energy consumed curve make a change with the systems time-varying loads (e.g. at $t = 40$ and at $t = 80$) but energy loss curve almost uniformly increases with time. Here, $E_c(T) = 6.130$, $E_{ca}(T) = 0.0613$, $E_{ls}(T) = 1.871$, $E_{lsa}(T) = 0.01871$

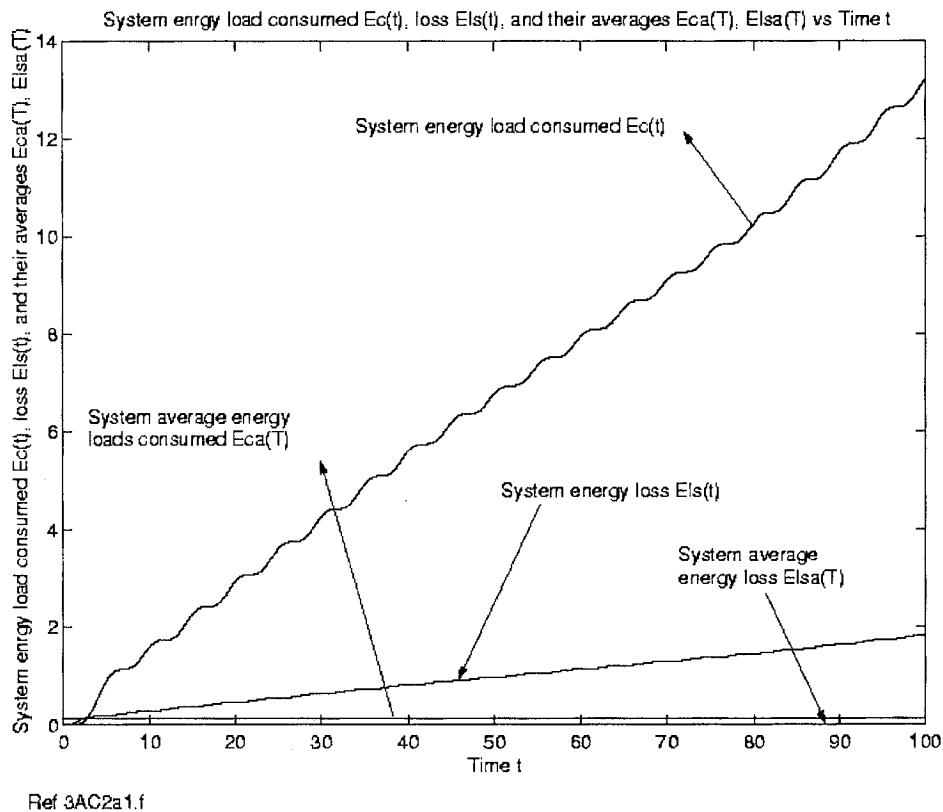


Figure 3.18- Changes of systems energy consumed $E_c(t)$ and loss $E_{ls}(t)$ and their averages $E_{ca}(T)$, $E_{lsa}(T)$, with time.

Figure 3.19 displays the behavioral responses of the reactive power of loads consumed $P_{cr}(t)$ and loss $P_{lsr}(t)$ with time. The amplitude of the reactive power load consumed also vary with time change of load (load currents); whereas, reactive power loss remain and continue with constant amplitude during steady state condition.

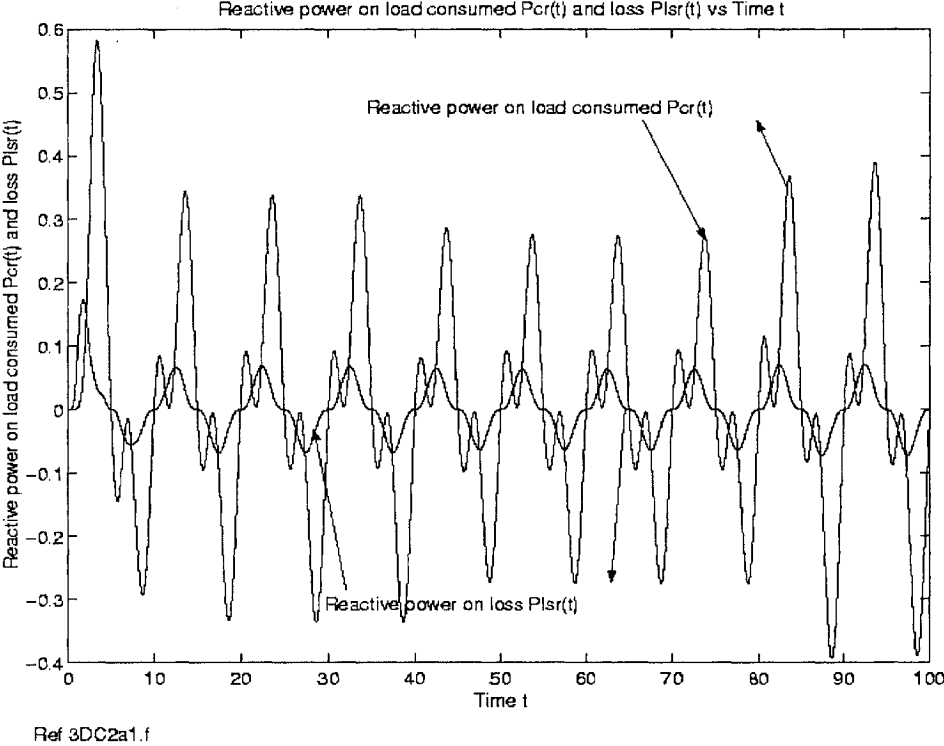


Figure 3.19- Reactive power on loads consumed $P_{cr}(t)$ and loss $P_{lsr}(t)$ response with time

3.3.6 Analysis with A.C. supply and different load types and conditions

Additional variations were analyzed with repeat to the cases of Sections 3.3.4 and 3.3.5 (sample data in Appendix 4). A summary of the loads (and conditions) is given in Section 3.3.3.

We can note a similarity in the variations of the node voltage amplitudes of each case. Higher variations are noticed at Node 2 and 3 than at Node 1. Voltage at Node 1 is the highest for combined (resistive, inductive, capacitive) loads but the lowest for the time-varying resistive loads. Node2 voltage remain the highest when suitably selected constant or time varying load (comprised of resistance, inductance and capacitance) is connected to the system; whereas, lowest node2 voltage can be seen when time-varying resistive loads are tapped. The variations of voltage are even more pronounced at Node 3.

The variations of the maximum amplitudes of power density are clearly different in each case because of the nature, type and or conditions of load during whole or part of the simulation period (based on applied loads).

One can see that energy load consumed vary widely between cases of loads considered. The time varying resistive load with series time constant inductance consumes maximum energy causing similar loss compared to other types and conditions of loads.

Factors for energy loads consumed also vary widely between cases of loads considered and their duration. For the time varying (or constant) resistive load with series time constant inductance plus stray capacitor across loads show maximum load factor and loss factor; on the other hand constant resistive load (operating during the same time) gives minimum loss factor. For all load condition a suitable selection of load parameters as well as node capacitor (i.e. through tuning-up process) maximum energy load can be tapped keeping loss minimum.

The above simulation allows us to conclude that the systems of power network can be comfortably operated keeping its state variables stable at some desired level through a manipulation (tune-up) of systems frequency, node capacitors, loads, (i.e. the through control of resistance, inductance or capacitance individually or all of them together).

We delay a synthesis of the experimental observations to Chapter 4.

CHAPTER 4

4.1 SYNTHESIS OF EXPERIMENTAL OBSERVATIONS

When analyzing the numerical plots and graphs of state variables (Figures 3.3 to 3.10 and Figures 3.11 to 3.19), the following observations were made.

i) Initially, the line currents in loops increased gradually from zero to the peak, then dropped down a little and again became steady for rest of the period, depending on the applied load current (through application of their controls like load resistance, inductance and stray capacitors, or node capacitance).

Voltages at nodes initially incurred a drastic fall then increased sharply to the peak value (because of the effect of capacitor put across the node as well as across the load) and then gradually become steady at certain values for rest of the period. They could be brought back to a desired level either with regulation (control) of load current, through application of variable resistance and inductance put in series at the load end or regulation by a capacitor bank placed at the nodes. Node capacitors also picked-up the node voltage (at the end of initial transition time) after getting charged.

ii) With the application of load current (and or changing load current parameters i.e. regulators), it was observed that with the increase of load current, node voltages decreased, sometimes even far below desired level (of $\pm 5\%$).

iii) Loss factors as well as load factors were also calculated and studied in parallel (with the help of some output data on the state variables). It was observed that through control of the load current and their duration of use, these factors could be kept at desired level (within $F_c \leq 1$, or $0 \leq F_{ls} \leq 1$) in view of keeping the overall system loss at a minimum level. Variations of node voltages, power density, energy, load consumption and loss factors with different types (or conditions of loads have also been tested and analyzed).

Overall, the above numerical (simulated) results and their numerical plots and graphs enable us to conclude that typical power network systems can be operated efficiently by maintaining suitable input voltage, controlling the node voltages and their frequency, with the help of node capacitors, and or load parameters (resistive and or inductive, and or capacitive).

Finally, the following the iterative process of testing I have used the network parameters as given below,----

Line parameters:

$R_1=0.2R$, $R_2=0.6R$, $R_3=R=0.136$ pu Ohm; $L_1=L_2=L_3=L=0.124$ pu Henry; $C_1=C=0.2$ pu Farad, $C_2=1.2C$, $C_3=1.4C$;

$R_{g1}=R_{g2}=R_{g3}=R_g=99.9$ pu ohms

Load parameters:

At all nodes selecting for load with $L_d=10.00$ Henry (or in other case $L_d=12.0$), $R_d=1.36$ Ohms (or $R_d(t)$ & $L_d(t)$ in time varying case; it was possible to use $R_d=1.36=R_{d3}$, $R_{d2}=1.1*R_d$, $R_{d1}=1.2*R_d$ as shown in Section 2.3.1). Stray capacitor across the load with (time constant) capacitance C_d . (we used $C_d=2.0$, $C_{d1}=C_d$, $C_{d2}=1.2*C_d$, $C_{d3}=1.4*C_d$) An input voltage amplitude $V=1.32$ pu volts D.C or (in case of A.C with frequency $f=0.1$). Initially, the system is seemingly unstable because of the combined effect of the system parameters (during the warming up period). However, after 15-20% time of the experiment, it became more or less stable for the rest of the time.

In most of the cases, we have also found that the intensity of power consumed is much more than the systems power loss; the energy consumed is also much more than the energy loss as well. Fine tuning with the load parameters and or node parameters (i.e. node capacitor) can bring further acceptable control over the system loss.

A set of experiments also demonstrated that load consumption and loss factors (i.e. $F_c(T)$ & $F_{ls}(T)$) can be kept close to 1. Also, the use of a stray capacitor across the load with (time constant) capacitance C_d . can save the load to a great extent from accidental damage out of short circuit condition; it can also help keep the load voltage to a desired level to some extent.

The initial experimental results enable us to suggest a more extensive suite of experiments. Subsequently, different values of load currents, considering D.C, as well as A.C supply (with different frequency, e.g., $f=0.1, 0.5, 1.0, 10.0, 30.0, 60.0$) were taken. Also taken different periods of simulation time and are reported in Sections 4.1.1 and 4.1.2. The values, $T=1.0, 5.0, 10.0, 20, 50.0, 100$ units of time (e.g., milli-seconds) were selected and, output data files were saved loaded and repeatedly plotted in Matlab environment.

The results would require too much space to document in as much detail as in the preceding chapter. Sections 4.1.1 & 4.1.2 contains a global description of the conditions under study and therefore only sample results are given here.

4.1.1 Load current changes during the simulation time period T

For the constant load inductance $L_d= 10$ or 12 pu Henry, and capacitance $C_d=0.2$ pu farad

Time period	Load currents from different nodes			Node voltages			Remarks
	u_1	u_2	u_3	v_1	v_2	v_3	
0 to 0.4T	I_{d11a}	I_{d21a}	I_{d31a}	V_{11a}	V_{11a}	V_{11a}	$R_d=1.36*1.2$
0.4T to 0.8T	I_{d12a}	I_{d22a}	I_{d32a}	V_{11a}	V_{11a}	V_{11a}	$R_d=1.36*1.1$
0.8T to T	I_{d13a}	I_{d23a}	I_{d33a}	V_{11a}	V_{11a}	V_{11a}	$R_d=1.36$

4.1.2 Load currents (using variable resistance and inductance, stray capacitance, also with varying node capacitor) change during the Simulation time period T

For the constant load inductance $L_d= 10$ or 12 pu Henry, bu variable load resistance and capacitance

Time period	Load currents from different nodes			Node voltages			Remarks
	u_1	u_2	u_3	v_1	v_2	v_3	
0 to 0.4T	i_{d11b}	i_{d21b}	i_{d31b}	V_{11b}	V_{11b}	V_{11b}	$R_d=1.36*1.2$
0.4T to 0.8T	i_{d12b}	i_{d22b}	i_{d32b}	V_{11b}	V_{11b}	V_{11b}	$R_d=1.36*1.1$
0.8T to T	i_{d13b}	i_{d23b}	i_{d33b}	V_{11b}	V_{11b}	V_{11b}	$R_d=1.36$

Based on this, the characteristic behaviors of the power network systems (i.e. other state variables like the line currents and the nodes voltages, systems power consumed and loss.), factors of energy consumed and loss plus their operating factors were analyzed with the types of loads (and load conditions).

In the experiment, the state-variables shot-up and fluctuated during an initial transition time, showing an unstable behavior (sharp fluctuations) but with time they were found to become gradually stable during rest of the experiment (while using a time-constant load current). But while using a time varying load current, they again showed some sort of unstable fluctuations initially and shot-up at each of the change-over of load currents, then gradually become stable again for rest of the experiment. With the slow increase of node capacitance, the node voltages increase after having dropped due to the high magnitude of

load. But an opposite behavior was found in case of load capacitance. Of course, with increase of load inductance the node voltage increased as well. On the other hand, with an increase (or respectively decrease) of load resistance the node voltage increased (or respectively decreased). By tuning the load current or the load resistance and inductance, capacitance, we can find out nominal values at which operation of the systems causes least energy loss during the period of the day. This can be seen from the calculations of the load factor and loss factor of corresponding period, as shown in (chapter 3), which subsequently can help systems managers in their operational decisions.

From the above analysis, it shows that a frequent change in loads (i.e. load parameters) keep the system more and more unstable, thereby incurring more loss; because of the instability in the system, node voltages fluctuate beyond the desired level, incurring more loss and more importantly, they create a cause for equipment damage.

From Figures 3.10 and 3.18 containing a plot of instantaneous as well as total average energy consumed and loss versus time, it was observed that initially energy loss is more than energy consumed. This is because of the fact that initially *it took time for the node capacitors to get charged and help boost node voltage.*

The analysis of the plot for dynamic case, is somewhat identical to that of sinusoidal case, and is therefore not duplicated. The effect of the shortened time period is illustrated in the plots for D.C case contained in Chapter 3. The absence of perceptible behavioral changes is caused by the fact that the loads are drawn for a shorter period of time and consequently the systems do not have as much time to shoot up and become stable (as measured by systems state variables like line currents, node voltages) to incur more losses. Because of the loads, the node voltages fall below the desired level, but the drop is compensated to some extent by the capacitors placed at the nodes. This is why systems node voltages may also be regulated with use of capacitor bank at the load centers/nodes. In parallel, loads can nicely be controlled (regulated) through use of variable load resistance and inductance plus placing a capacitor across the load. This load capacitor can also protect the motor loads from the danger of burning due to major short circuit conditions as in an accidental case of operations.

4.2 THE POWER-FLOW PROBLEM: CONCLUSION

The effects of inputs on the system were analyzed in the simulation experiments. The first case was for the system operated in cylindrical co-ordinates; the second case was an artificial operation of the systems.

In both cases, a computer simulation was run which solved the various differential equations that represented the systems dynamics. The states of the system were plotted against time or against type (and or condition) of loads in all the cases. First, a series of general comments are given, followed by overall recommendations highlighted here.

4.2.1 Observations

Analysis of the simulated system model in dynamic case, allow us to confirm the results as expected from theoretical calculations, aided by a suitable selection of line, load parameters, such as those of the controllers, in particular the node capacitors, the load resistance, or the inductance and stay capacitance. The analysis of the system model in sinusoidal case, revealed that with the regulation (i.e. controlling the load current), the node voltage can be maintained at or near the desired level. This control allows us to minimize the line and leakage losses of power and keeping the load factors equal 1 or less than 1 and loss factors less than 1 (but preferably near zero) for the period of systems operations. Thus, as observed at the end of Chapter 3, the thesis demonstrates that power network systems can be operated keeping their state variables stable at some desired level.

In addition, this experimental simulation gave us some practical insight into the systems power flow control; the behavioral changes of the systems state variables with the type of load and their conditions (*useful for systems operation*) or with line parameters (*useful for designers*).

To find feasible controls and state variables, it was sufficient to develop a state space model, using a 4th order Runge-Kutta method in order to obtain the desired state variables.

The simulated results may now be applied to the adjustment of the original base values, thus obtain a practical set of system parameters. As an overall benefit of the simulation, designers can then easily study and design a new power system network,

modify or extend an existing network according to the new needs arising in time. Operation managers can also calibrate their system control equipment according to the simulated results of the system concerned.

Other theoretical methods (such as Pontryagin's maximum-minimum principle) and a more powerful solution method may be needed to optimize the systems variables. This part of the problem is left for future research.

4.2.2 Recommendations for practical power-flow problems

An interconnected power system represents an electric network with a multitude of branches and nodes / load centers, where the transmission lines typically contribute the branches. Nodes are referred to as "buses".

In practice, at some of the buses power is injected into the network, where as at most of the buses the system loads are tapping it. *A given set of loads can be served from a given set of generators in an infinite number of "power-flow" configurations.*

The Power-flow analysis may then be summarized as:

1. Generation must be equal to demand at each moment; but demand undergoes *slow but wide changes* through the 24-hour of the day. This is why an economically acceptable load flow configuration that fits the demand of a certain hour of the day may be formulated based on the economic generation & transmission limits (with minimum loss of power) and considering suitably acceptable load and loss factors.
2. It is necessary to keep voltage levels of certain buses within close tolerances. This can be achieved by proper scheduling of reactive powers [this part, I did not analyze much].
3. The disturbances following massive network fault can cause system outages, the effects of which can be minimized by proper pre-fault power-flow strategies [like load balancing between phases, system voltage as well as frequency control].
4. The Power-flow analysis is very important in the planning stages of network or additions to existing ones. It is also very important for use in the efficient operation of the power system and losses management.

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APPENDIX 1

RUNGE-KUTTA METHOD FOR SIMULATED COMPUTATIONS

This section is extracted from [Jain, 1979].

We first explain the principle involved in Runge-Kutta methods.

By the Mean -Value Theorem any solution of

$$y' = f(t, y), \quad y(t_0) = y_0, \quad t \in [t_0, b]$$

satisfies $y(t_{n+1}) = y(t_n) + h y'(\xi_n) = y(t_n) + h f(\xi_n, y(\xi_n))$,

where $\xi_n = t_n + \theta_n h, \quad 0 < \theta_n < 1$

We put $\theta_n = 1/2$. By Euler's method with spacing $h/2$, we get

$$y(t_n + h/2) \cong y_n + (h/2)f(t_n, y_n)$$

Thus we have the approximation

$$y_{n+1} = y_n + h f(t_n + h/2, y_n + (h/2) f(t_n, y_n)) \quad \dots \quad (A.1)$$

Alternatively, again using Euler's method, we proceed as follows

$$y'(t_n + h/2) \cong (1/2)[y'(t_n) + y'(t_{n+1})] \cong 1/2 [f(t_n, y_n) + f(t_{n+1}, y_n + h f_n)]$$

and thus we have the approximation

$$y_{n+1} = y_n + (h/2)[f(t_n, y_n) + f(t_{n+1}, y_n + h f(t_n, y_n))] \quad \dots \quad (A.2)$$

Either (2.1) or (2.2) can be regarded as

$$y_{n+1} = y_n + h(\text{average slope}) \quad \dots \quad (A.3)$$

This is the Runge-Kutta approach or the underlying idea. In general, we find the slope at t_n and at several other points: average these slopes, multiply by h , and get the result to y_n .

Thus the Runge-Kutta method with v slopes can be written as

$$K_i = h f(t_n + c_i h, y_n + \sum_{j=1}^{i-1} a_{ij} K_j), \quad c_1 = 0, \quad i = 1, 2, \dots, v \quad \dots \quad (A.4)$$

or

$$K_1 = h f(t_n, y_n)$$

$$K_2 = h f(t_n + c_2 h, y_n + a_{21} K_1)$$

$$K_3 = h f(t_n + c_3 h, y_n + a_{31} K_1 + a_{32} K_2)$$

$$K_4 = h f(t_n + c_4 h, y_n + a_{41} K_1 + a_{42} K_2 + a_{43} K_3)$$

... ..

and

$$y_{n+1} = y_n + \sum_{i=1}^v w_i K_i$$

where the parameters $c_2, c_3, \dots, c_v, a_{2j}, \dots, a_{v(v-1)}$ and w_i are arbitrary.

From (2.3), we may interpret the increment function as the linear combination of the slopes at t_n and several other points between t_n and t_{n+1} . To obtain specific values for the parameters,

we expand y_{n+1} in powers of h such that it agrees with the Taylor's series expansion of the solution of the differential equation to a specified number of terms.

Let us study this approach with just two slopes.

A1.1 Second order methods

$$\begin{aligned}
 \text{Define} \quad K_1 &= h f(t_n, y_n) \\
 K_2 &= h f(t_n + c_2 h, y_n + a_{21} K_1) \\
 K_3 &= h f(t_n + c_3 h, y_n + a_{31} K_1 + a_{32} K_2) \\
 \text{and} \quad y_{n+1} &= y_n + w_1 K_1 + w_2 K_2. \quad \dots \quad \dots \quad (A1.5)
 \end{aligned}$$

where, the parameters $c_2, a_{21}, w_1,$ and w_2 are chosen to make y_{n+1} closer $y(t_{n+1})$.

Now Taylor's series gives

$$y(t_{n+1}) = y(t_n) + h y'(t_n) + (h^2/2!) y''(t_n) + (h^3/2!) y'''(t_n) + \dots \quad (A1.6)$$

where, $y' = f(t, y)$

$$y'' = f_t + f_y f$$

$$y''' = f_{tt} + 2f_{ty} + f_{yy} f^2 + f_y (f_t + f_y f)$$

The values of $y'(t_n), y''(t_n), \dots$ are obtained by substituting $t = t_n$.

We expand K_1, K_2 and finally $y(t_{n+1})$ about the point (t_n, y_n) .

$$\begin{aligned}
 K_1 &= h f_n \\
 K_2 &= h f_n ((t_n + c_2 h, y_n + a_{21} h f_n) = h [f_n(t_n, y_n) + (c_2 h f_t + a_{21} h f_n f_y) + \\
 &\quad (1/2!)(c_2^2 h^2 f_{tt} + 2 c_2 a_{21} h^2 f_n f_{ty} + a_{21} h^2 f_n^2 f_{yy}) + \dots] \\
 &= h f_n + h^2 (c_2 f_t + a_{21} f_n f_y) + (1/2) h^3 (c_2^2 f_{tt} + 2 c_2 a_{21} f_n f_{ty} + a_{21}^2 f_n^2 f_{yy}) + \dots \quad (A1.7)
 \end{aligned}$$

Substituting the values of K_1 and K_2 in (5), we get

$$w_1 + w_2 = 1, \quad c_2 w_2 = 1/2, \quad a_{21} w_2 = 1/2 \quad \dots \quad \dots \quad (A1.8)$$

From these equations, we see that if c_2 is chosen arbitrarily (nonzero), then

$$a_{21} = c_2, \quad w_2 = 1/(2c_2), \quad w_1 = 1 - 1/(2c_2), \quad \dots \quad \dots \quad (A1.9)$$

Substituting (2.9) in (2.6) we obtain the local truncation error

$$\begin{aligned}
 T_n = y(t_{n+1}) - y_{n+1} &= h^3 [(1/6 - c_2/4)(f_{tt} + 2f_n f_{ty} + f_n^2 f_{yy}) + 1/6 f_y (f_t + f_n f_y)] + \dots \\
 &= (h^2/12) [(2 - 3 c_2) y'' + 3 c_2 f_y y_n''] + \dots
 \end{aligned}$$

we observe that no choice of the parameter c_2 will make the leading term of T_n vanish for all $f(t, y)$. The local truncation error depends not only on derivative of the solution $y(t)$ but also on the function f . This is typical of all the Runge-Kutta methods; in most other methods the truncation error depends only on certain derivatives of $y(t)$. Generally, c_2 would be chosen between 0 and 1.

A1.3 Taylor's series method

We write

$$y_{n+1} = y_i + h y_i' + (h^2/2!) y_i'' + \dots + (h^p/p!) y_i^{(p)}, \quad i = 0, 1, 2, \dots, N-1 \quad \dots \quad (A1.12)$$

where $y_i^{(k)} = [y_{1i}^{(k)}; y_{2i}^{(k)}; \dots; y_{ni}^{(k)}]$

$$= [d^{k-1} f_1(t_i, y_{1i}, y_{2i}, y_{3i}, \dots, y_{ni})/dt^{k-1}; d^{k-1} f_2(t_i, y_{1i}, y_{2i}, y_{3i}, \dots, y_{ni})/dt^{k-1}; \dots; d^{k-1} f_n(t_i, y_{1i}, y_{2i}, y_{3i}, \dots, y_{ni})/dt^{k-1}]$$

A1.4 Fourth order Runge-Kutta methods

The classical fourth order Runge-Kutta formula becomes

$$y_{i+1} = y_i + 1/6 (K_1 + 2 K_2 + 2 K_3 + K_4) \quad \dots \quad \dots \quad (A1.13)$$

where $K_1 = [K_{11}; K_{21}; \dots; K_{n1}]$, $K_2 = [K_{12}; K_{22}; \dots; K_{n2}]$,

$$K_3 = [K_{13}; K_{23}; \dots; K_{n3}], \quad K_n = [K_{1n}; K_{2n}; \dots; K_{nn}]$$

and $K_{j1} = h f_j(t_i, y_{1i}, y_{2i}, y_{3i}, \dots, y_{ni})$,

$$K_{j2} = h f_j(t_i + 1/2 h, y_{1i} + 1/2 K_{11}, y_{2i} + 1/2 K_{21}, \dots, y_{ni} + 1/2 K_{n1}),$$

$$K_{j3} = h f_j(t_i + 1/2 h, y_{1i} + 1/2 K_{12}, y_{2i} + 1/2 K_{22}, \dots, y_{ni} + 1/2 K_{n2}),$$

$$K_{j4} = h f_j(t_i + h, y_{1i} + K_{13}, y_{2i} + K_{23}, \dots, y_{ni} + K_{n3}), \quad j = 1, 2, \dots, n$$

In an explicit form (3.04) may be expressed as

$$[y_{1, i+1}; y_{2, i+1}; \dots; y_{n, i+1}] = [y_{1, i}; y_{2, i}; \dots; y_{n, i}] + 1/6 \{ [K_{11}; K_{21}; \dots; K_{n1}] + 2[K_{12}; K_{22}; \dots; K_{n2}] + 2[K_{13}; K_{23}; \dots; K_{n3}] + [K_{14}; K_{24}; \dots; K_{n4}] \}$$

A1.5 Simulated Computations: numerical analysis using Runge-Kutta Method:

Let $[t_0, T]$ be an interval of time over which we want the solution $x(t)$ of

$$dx/dt = f(t, x), \quad x(0) = x_0. \quad \dots \quad \dots \quad (A1.14)$$

We can sub-divide $[t_0, T]$ into n intervals using a step size $h = (T - t_0)/n$,

We get time point's t_1, t_2, \dots, t_{n-1} .

A numerical method [Euler's method or any of the Runge-Kutta methods] for solving equations (8) will start with $x_0 = x(t_0)$ and then generate values $x_1, x_2, x_3, x_4, x_5, x_6, \dots, x_n$ such that x_j approximates the exact value $x(t_j)$ for $j = 1, 2, \dots, n$.

For small h the solution curve $x = x(t)$ can be approximated within the interval $[t_j, t_{j+1}]$ by the tangent line at $(t_j, x(t_j))$.

Using Euler's method we can select our control u_j arbitrarily & find out the suitable constant line parameters (i.e. co-efficient) of the model equations for the system. For that purpose, taking any two or more points on the plotted curve of the equations and comparing their slopes as

$$(x_{n+1} - x_n) / (u_{n+1} - u_n) = f(u_n, x_n); \text{ where, } h = (u_{n+1} - u_n) \forall n$$

or, $x_{n+1} = x_n + h f(u_n, x_n);$ [is also called Runge-Kutta method of order two] (A1.15)

If we can put the values of x_n in the above equation, we can solve the equation for x_{n+1} .

More accurate form of the Range-Kutta formulae [called Runge-Kutta method of order four] is given as –

let $K_1 = f(u_n, x_n); K_2 = f(u_n+h, x_n+h K_1)$

then $x_{n+1} = x_n + h/2 (K_1 + K_2);$

$x(t_0)$, incremental time interval h .

Sample Table: for calculations of Numerical Solutions.				
Arbitrarily chosen values of u_j	Corresponding values of x_j	Exact solution of the equation x_{Sj+1}	Difference of values, $x_{n+1} - x_n$	Remarks
u_0	x_0	x_{S1}		$x_{n+1} = x_n + h f(u_n, x_n)$
u_1	x_1	x_{Sj+1}		
u_2	x_2			
....
u_n	x_n	x_{Sj+1}		

Sample Table for computations: with arbitrarily chosen input values of u & initial values of $u(t_0)$,

Hence, following this, Runge-Kutta method of order-4 will be--

$$K_1 = f(u_n, x_n); K_2 = f(u_n+h/2, x_n+(K_1)/2)$$

$$K_3 = f(u_n+h/2, x_n+(K_2)/2); K_4 = f(u_n+h, x_n+ (K_3))$$

From these,

$$x_{j+1} = x_j + h/6 \{ K_1 + 2K_2 + 2K_3 + K_4 \} \quad \dots \quad \dots \quad (A1.16)$$

A1.5.1 Problem statement

To determine the states of power flows through a distribution network system, and losses there on, loss factor, load demand factor at time T, [for our computation purpose] let us take the power distribution system with T-equivalent circuit with 3 nodes / load centers only. Based on this we can have the state of the system as: -

If we take $x = [x_1, x_2, x_3, x_4, x_5, x_6] = [i_1, v_1, i_2, v_2, i_3, v_3] \in \xi^6$.

The following control is utilized in order to maintain the systems losses factor (F_{LS}) and load demand factor (F_D) at a particular level [i.e. ≤ 1]

$u(t) = \max \{g(x_1, x_2, x_3, x_4, x_5, x_6, 0)\}$ for $0 \leq t \leq T$ where T is the time period

$u = [i_{d1}, i_{d2}, i_{d3}] \in \xi^3$.

A1.5.2 Defining the dynamics of power systems:

The set of state equations (for 3-nodes system only) may then be written as: - set of equations 2.26 (shown in Chapter 2).

Similarly, Power intensity or energy load demand or consumed & loss and their factors of averages with peak value can be computed for analysis (using the relevant equations cited earlier in chapter 1).

For numerical solution of our problem, we may arbitrarily choose magnitudes of load currents at different hours of the day, ($u_k = i_{dk}$) and from amongst them the peak load current, (i_{dkPeak}); we can plot their curves with respect to time and find out acceptable values of the line parameters. Integrating their products we can also compute the total energy demand and average load demand, and losses thereof for the period of the day concerned.

Now, after playing with arbitrary control parameters (i.e. systems load currents or else) at different hours of time we can solve for the states of the system at different time period.

Similarly, numerical solutions on Power intensity (of loss and consumption) as well as total Energy (of loss and consumption) and their operational factors can be found using equations 2.13 to 2.23,

For these solutions & computations, the programs (as shown in Appendix 2) have been written in Fortran, compiled with f77 compiler and ran for the simulated outputs; All above mentioned information was entered in the program and a simulation was performed. Based on the derived numerical solutions curves were plotted using Mat lab and an analysis were. The program solved for $x_1, x_2, x_3, x_4, x_5, x_6 \dots x_{16}$.

NB: Results of my simulation and analytical data are given in Chapter 3 as well as in appendix 4 and copy of the developed software /program is given in Appendix 2.

APPENDIX 2

FORTRAN PROGRAM: DEVELOPED FOR THE SIMULATION AND NUMERICAL ANALYSIS.

Program: 3AC2a1.1.f

```
C-----
C--- THIS PROGRAM IS CALLED THE POWER FLOW PROBLEM
C-----
C---
C--- INTEGRATION USING CLASSIC FORTH ORDER RUNGE-KUTTA METHOD
C-----
C--- PROGRAM DONE FOR MASTERS THESIS ON "THE MODELING SIMULATION &
C   NUMERICAL ANALYSIS OF THE TRANSIENT CHARACTERISTICS OF UN-
C   REGULATED POWER FLOW SYTEM NETWORK PROBLEM (FOR OPERATIONAL LOSSES
C-- MANAGEMENT)"
C-- [3 nodes & loops RLC ckt having grounding / insulator resistance
C   Rg in parallel with node capacitor, and with load resistance
C-- (time variant) in series with self-inductance (time variant) plus
C-- stay capacitor(time invariant) Cd across load(motor);
C- computation of energy consumed & loss and their time averages,
C   Fc(T) & Fls(T)]; Testing with different system frequency &
C-- applying constant A.C supply.
C-----
C   DONE BY: MOHAMED ABUL BASHER, STUDENT # 1654766, SYSTEMS SCIENCE
C--- PROGRAM, UNIVERSITY OF OTTAWA, OTTAWA, ONTARIO, CANADA
C-----
      PROGRAM ON THE POWER FLOW PROBLEM
      IMPLICIT REAL*8(A-H,O-Z)
C--- Y IS THE STATE VECTOR
C--- DIMENSION = NUMBER OF EQUATIONS AND OR STATE VARIABLES
      DIMENSION Y(15)
C--- Assign a value to N , the number of first-order Differential
C--- Equations to be solved in this Problem.
      N = 15
      OPEN(2,FILE='FLOW', STATUS='UNKNOWN')
      WRITE(*,2)
2     FORMAT('ENTER VALUES FOR DT , DXPR, XT (FREE FORMAT)')
      READ(*,*) DT, DXPR, XT
      WRITE(*,3)
3     FORMAT('ENTER THE INITIAL VALUES OF DISPLACEMENT Y(I)',
*         '(FREE FORMAT)')
      READ(*,*) Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7), Y(8), Y(9),
*         Y(10), Y(11), Y(12), Y(13), Y(14), Y(15)
C--- initialize time and print time
      X = 0.0
      XPR = DXPR
C--- column headings.
      WRITE(2,5)
5     FORMAT(4x,'TIME',2x,'Y(1)',2x,'Y(2)',2x,'Y(3)',2x,'Y(4)',2x,'Y(5)',
*         2x,'Y(6)',2x,'Y(7)',2x,'Y(8)',2x,'Y(9)',2x,'Y(10)',2x,'Y(11)',
*         2x,'Y(12)',2x,'Y(13)',2x,'Y(14)',2x,'Y(15)')
      WRITE(2,7) X, Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7), Y(8),
*         Y(9), Y(10), Y(11), Y(12), Y(13), Y(14), Y(15)
C--- call Subroutine Subprogram RK4
6     CALL RK4(N,Y,X,DT)
C--- Check to see if it is Time to Print.
      IF(DABS(X-XPR).LT. .00001) THEN
```

```

WRITE(2,7) X, Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7), Y(8),
* Y(9), Y(10), Y(11), Y(12), Y(13), Y(14), Y(15)
7 FORMAT(F9.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,
* 3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,
* 3x,E10.4,3x,E10.4)
C--- Increment Print Time
XPR = XPR + DT
ENDIF
C--- Check to see if XT has been reached.
IF(DABS(X-XT).LT. .00001) THEN
STOP
ENDIF
GO TO 6
END

C--- -----
C-- System Equations
SUBROUTINE FUN(X,Y,YPRIME)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION Y(15), YPRIME(11)

C--
C--- System Line parameters, Power Loads are given as below
C-- Y(1)=i1
C-- Y(2)=v1
C-- Y(3)=id1=u1
C-- Y(4)=i2
C-- Y(5)=v2
C-- Y(6)=id2
C-- Y(7)=i3
C-- Y(8)=v3
C-- Y(9)=id3
C-- Intensity of systems instantaneous consumed power load Y(10)=Pc(t)
C-- Intensity of systems instantaneous power loss Y(11)=Pl(t)
C-- Energy consumed during time t is Ec(t), dEc/dt=YPRIME(12)=Y(10)
C-- Energy loss during time t is Els(t), dEl/dt=YPRIME(13)=Y(11)
C-- Avg. Energy Consumed Power Load Eca(X)=Y(14) & Loss Els(X)=Y(15)
C and at T=XT take Y(12)=Y(12,T)=Y12T & Y(13)=Y(13,T)=Y13T
C-- Total time period =T
T=100.00

C-- ALd(t)= time varying, Load INDUCTANCE pu HENRY/Km
IF (X.GE.0.0.AND.X.LT.0.4*T) ALd=1.0D+01
IF (X.GE.0.0.AND.X.LT.0.4*T) ALdPRIME=0

C
IF (X.GE.0.40*T.AND.X.LT.0.42*T) ALd=(5*T-10*X)*(1./T)*1.0D01
IF (X.GE.0.40*T.AND.X.LT.0.42*T) ALdPRIME= -(10./T)*1.0D+01

C
IF (X.GE.0.42*T.AND.X.LT.0.8*T) ALd=0.8*1.0D+01
IF (X.GE.0.42*T.AND.X.LT.0.8*T) ALdPRIME=0

C
IF (X.GE.0.80*T.AND.X.LT.0.82*T) ALd=(8.8*T-10*X)*(1./T)*1.0D+01
IF (X.GE.0.80*T.AND.X.LT.0.82*T) ALdPRIME= -(10./T)*1.0D01

C
If (X.GE.0.82*T.AND.X.LE.1.0*T) ALd=0.6*1.0D+01
If (X.GE.0.82*T.AND.X.LE.1.0*T) ALdPRIME=0
ALd1=ALd
ALd2=ALd
ALd3=ALd

C-- Rd(t)=time varying, Load RESISTANCES PU Ohms/Km
IF (X.GE.0.0.AND.X.LT.0.4*T) Rd=1.36D+00
IF (X.GE.0.4*T.AND.X.LT.0.8*T) Rd=0.6*1.36D+00

```

```

If (X.GE.0.8*T.AND.X.LE.1.0*T) Rd=1.2*1.36D+00
Rd1=1.2*Rd
Rd2=1.1*Rd
Rd3=Rd
C-- Cd=Time constant stay capacitance across loads (e.g. motors)
Cd=2.0
Cd1=Cd
Cd2=0.8*Cd
Cd3=0.6*Cd
C--- AL=time constant, LINE INDUCTANCES pu HENRY/Km
AL1= 1.24D-01
AL2=AL1
AL3=AL1
C C=time constant, LINE CAPACITORS CAPACITANCE PU FARADS/Km
C1= 2.00D-01
C2=1.2*C1
C3=1.4*C1
C R=time constant, LINE RESISTANCE PU OHMS/Km
R3=1.36D-01
R2=0.6*R3
R1=0.2*R3
C RG1=time constant, INSULATOR/GROUNDING LEAKAGE RESISTANCE pu Ohms/Km
RG1= 9.99D+01
RG2=RG1
RG3=RG1
C at terminal time T take Y(12)=Y(12, T)=Y12T, Y(13)=Y(13, T)=Y13T
Y12T=0.3251E-04
Y13T=0.1084E-01
C--- SIN=Trigonometric Sine function
C--- v=SYSTEMS SUPPLY VOLTAGE in pu volts A.C.
V=1.32D+00
PI=3.14
C--- f=Systems Frequency in cps (i.e. Herz)
f=0.10
C-----
v=V*DSIN(2*PI*f*x)
YPRIME(1)=(1./AL1)*(v-Y(2)-R1*Y(1))
YPRIME(2)=(1./(C1+Cd1))*(Y(1)-(1./RG1)*Y(2)-Y(3)-Y(4))
YPRIME(3)=(1./ALd1)*(Y(2)-Rd1*Y(3)-Y(3)*ALd1PRIME)
YPRIME(4)=(1./AL2)*(Y(2)-Y(5) -R2*Y(4))
YPRIME(5)=(1./(C2+Cd2))*(Y(4) - (1./RG2)*Y(5) - Y(6)-Y(7))
YPRIME(6)=(1./ALd2)*(Y(5) - Rd2*Y(6)-Y(6)*ALd2PRIME)
YPRIME(7)=(1./AL3)*(Y(5)-Y(8) -R3*Y(7))
YPRIME(8)=(1./(C3+Cd3))*(Y(7) - (1./RG3)*Y(8) - Y(9))
YPRIME(9)=(1./ALd3)*(Y(8) - Rd3*Y(9)-Y(9)*ALd3PRIME)
Y(10)=Rd1*Y(3)**2 +Rd2*Y(6)**2 +Rd3*Y(9)**2
Y(11)=R1*Y(1)**2 +R2*Y(4)**2 +R3*Y(7)**2 +(1./RG1)*Y(2)**2
* +(1./RG2)*Y(5)**2 +(1./RG3)*Y(8)**2
YPRIME(12)=(1./2)*Y(10)
YPRIME(13)=(1./2)*Y(11)
Y(14)=(1./T)*Y12T
Y(15)=(1./T)*Y13T
RETURN
END
C-----
C--- RK4 SUBROUTINE
C--- order Differential Equations of the Form DY(I)/DX = F(X,Y(I)),
C--- Where I = 1,2,.....,N.
C---

```

```

SUBROUTINE RK4(N,Y,X,DT)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION Y(11), YTEMP(20), YPRIME(20), AK1(20), AK2(20),
*      AK3(20), AK4(20)
CALL FUN(X,Y,YPRIME)
C--- Calculate the AK1(I)'s and determine the Y values (called
C--- YTEMP(I)'s) for the AK2(I)'s.
DO 2 I = 1,N
    AK1(I) = DT*YPRIME(I)
    YTEMP(I) = Y(I) + AK1(I)/2.
2    CONTINUE
C--- Calculate the AK2(I)'s and determine Y values for the AK3(I)'s.
DO 3 I = 1,N
    AK2(I) = DT*YPRIME(I)
    YTEMP(I) = Y(I) + AK2(I)/2.
3    CONTINUE
CALL FUN(X+DT/2.,YTEMP,YPRIME)
C--- Calculate the AK3(I)'s and determine Y values for the AK4(I)'s.
DO 4 I = 1,N
    AK3(I) = DT*YPRIME(I)
    YTEMP(I) = Y(I) + AK3(I)
4    CONTINUE
CALL FUN(X+DT,YTEMP,YPRIME)
C--- Calculate the AK4(I)'s.
DO 5 I = 1,N
    AK4(I) = DT*YPRIME(I)
5    CONTINUE
C--- Calculate the Y(I) values at the end of the step of magnitude DT.
DO 6 I = 1,N
    Y(I) = Y(I) + (AK1(I) + 2.*AK2(I) + 2.*AK3(I) + AK4(I))/6.
6    CONTINUE
C--- Increment X to its value at the end of the step.
X = X + DT
C--- RETURN TO THE CALLING PROGRAM.
RETURN
END
C-----

```

APPENDIX 3

RESONANCE FREQUENCY

If the system is

$$\dot{x} = Ax + Bu + Gv$$

For the system to be stable and steady, ----- eigenvalue of det ('A') < 0 i.e

$$\det(\lambda I - A) = 0$$

or

$$(\lambda I - A)e = 0$$

Where, λ is the eigenvalue of matrix 'A'.

Again, for the system

$$\ddot{\xi} + a\dot{\xi} + b\xi = e$$

In Laplace transform it can be written as,

$$s^2 \hat{\xi} + as\hat{\xi} + b\hat{\xi} = \hat{e}$$

$$(s^2 + as + b) = \hat{e}$$

$$\hat{\xi} = (1/(s^2 + as + b))\hat{e},$$

$$H(s) = 1/(s^2 + as + b) = 1/(-\omega^2 + ja\omega + b) = 1/((b - \omega^2) + ja\omega)$$

$$|H(j\omega)| = 1/\sqrt{(b - \omega^2)^2 + a^2\omega^2}$$

$$\hat{e}(\omega) = (\Gamma)(\omega / (\omega^2 + \dots))$$

$$= \int_0^{\infty} e^{-st} \sin(\omega_0 t) dt = \int_0^{\infty} \{e^{(-s+j\omega_0)t} - e^{(-s-j\omega_0)t}\} dt$$

and

$$\sin(\omega_0 t) = \{e^{j\omega_0 t} - e^{-j\omega_0 t}\} / 2j$$

Here, for $s = j\omega$, $H(j\omega) = |H(j\omega)|$, $\omega = 2\pi f$

In this case, there will be resonance frequency if $\omega = \pm\sqrt{b}$ or $f_r = \pm\sqrt{b}/(2*\pi)$

For series RLC circuits:

Series RLC circuits respond to alternating currents at various frequencies in a unique way. A specific RLC combination exhibits resonance at a point,

where, $X_L = X_C$. [i.e. $2\pi fL = \frac{1}{2\pi fC}$], or

$$4\pi^2 f^2 LC = 1, \text{ i.e. } f_0 = \frac{1}{2\pi\sqrt{LC}}, \text{ where } f_0 \text{ is the resonance frequency.}$$

At resonance the phase angle is zero and the impedance is minimum. As the frequency drops below resonant frequency, the circuit becomes capacitive; above, it becomes inductive.

For parallel RLC circuits:

Parallel RLC circuits have a specific resonant frequency. At resonance the impedance across parallel circuit approaches L / RC ohms.

Here, resonance frequency $f_0 = \frac{1}{2\pi} \sqrt{\frac{L - CR^2}{L^2C}}$. The phase angle is zero and below resonance the parallel circuit appears inductive; above, it appears capacitive.

The reciprocal impedance is admittance. $Y = 1/Z$. the conductance is the reciprocal of resistance, and susceptance is the reciprocal of reactance. A series R and X circuit must be transformed into conductance and susceptance by,

-first, converting impedance to polar form,

-second, taking the reciprocal, and then,

-third, converting the polar admittance to the equivalent rectangular form. The result is a parallel equivalent circuit.

The network theorem may be applied to A.C circuits provided taking care of all the proper polar and rectangular conversions. When theorems such as Thevenin's is used, the Thevenin impedance should be calculated using V_{oc} / I_{sc} .

APPENDIX 4

SAMPLE OUTPUT DATA FOR ANALYSIS

A4.1 ANALYSIS OF THE BEHAVIOR OF AN EQUIVALENT ELECTRICAL POWER NETWORK

Analysis on the Power Systems:

Consideration of totally resistive circuit at no load at zero sequence operation of time

For the initial condition with load current (control) $U=0$;

$$dv_1/dt = dv_2/dt = dv_3/dt = 0; \text{ also } di_1/dt = di_2/dt = di_3/dt = 0;$$

line resistance per unit length $R_1 = R_2 = R_3 = 0.13572 \text{ ohms} = R$,

Leakage i.e. grounding resistance per unit length $R_{g1} = R_{g2} = R_{g3} = 99.9 = R_g$.

Input voltage $e = v = 1.0 \text{ kv}$

Equivalent resistive circuit at no load and at zero sequence of time may becomes like the one shown in Fig. A4.1. below.---

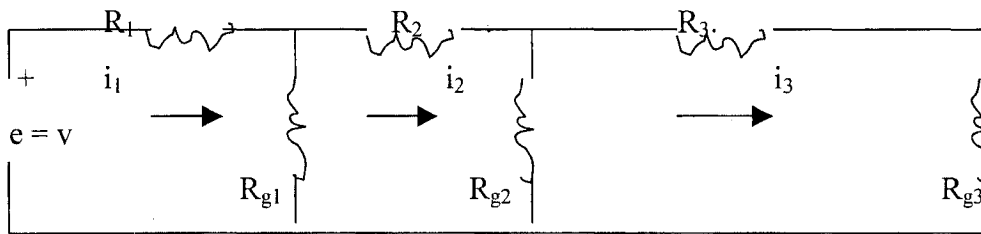


Fig. A4.1

At node 1 & loop1:

$$\begin{aligned} v &= i_1 R + i_{g1} R_g; & \& \quad i_1 = i_2 + i_{g1}, \\ \text{or, } v &= (i_2 + i_{g1}) R + i_{g1} R_g. \\ \text{or, } i_2 &= (v - i_{g1} R - i_{g1} R_g) / R, & \dots & \dots \end{aligned} \quad (A4.1)$$

At node2 & loop2:

$$\begin{aligned} i_{g1} R_g &= i_2 R + i_{g2} R_g; & \& \quad i_2 = i_3 + i_{g2}, \\ \text{or, } i_{g1} R_g &= (i_3 + i_{g2}) R + i_{g2} R_g. \\ \text{or, } i_3 &= (i_{g1} R_g - i_{g2} R_g - i_{g2} R) / R & \dots & \dots \end{aligned} \quad (A4.2)$$

At node3 & loop3:

$$i_{g2} R_g = i_3 R + i_{g3} R_g; \& i_3 = i_4 + i_{g3},$$

but in our case $i_4 = 0$ since node3 is the last node, hence $i_3 = i_{g3}$,

$$\text{or, } i_{g3} = R_g i_{g2} / (R + R_g) \quad \dots \quad \dots \quad (A4.3)$$

but from (1) $i_2 = (v - i_{g2} R - i_{g1} R_g) / R = i_3 + i_{g2}$.

$$\text{or, } i_3 = (v - i_{g2} R - i_{g1} R_g) / R - i_{g2} \quad \dots \quad \dots \quad (A4.4)$$

also from (2) & (4) we have,

$$i_3 = (i_{g1} R_g - i_{g2} R_g - i_{g2} R) / R = (v - i_{g2} R - i_{g1} R_g) / R - i_{g2}.$$

$$\text{or, } i_{g1} R_g - i_{g2} R_g - i_{g2} R = v - i_{g2} R - i_{g1} R_g - i_{g2} R$$

$$\text{or, } (R_g - R) i_{g2} = (2i_{g1} R_g - v)$$

$$\text{or, } i_{g2} = (2i_{g1} R_g - v) / (R_g - R) \quad \dots \quad \dots \quad (A4.5)$$

putting (5) in (3) we get,

$$i_{g3} = R_g (2i_{g1} R_g - v) / (R_g - R) \quad \dots \quad \dots \quad (A4.6)$$

Again we have ,

$$i_1 = i_2 + i_{g1} = (i_3 + i_{g2}) + i_{g1}, \quad // \text{ putting value of } i_2.$$

$$= (i_{g1} R_g - i_{g2} R_g - i_{g2} R) / R + i_{g2} + i_{g1}, \quad // \text{ putting value of } i_3.$$

$$= 1/R \{ i_{g1} R_g - i_{g2} (R_g - v + R) + R i_{g1} \}$$

$$= 1/R \{ i_{g1} (R_g - R) - (R_g - R + v) (2i_{g1} R_g - v) / (R_g - R) \},$$

// putting value of i_{g2} .

$$= 1/R [i_{g1} \{ (R_g - R) - 2(R_g^2 - R R_g + R_g) / (R_g - R) \} + (R_g - R + v) / (R_g - R)] \quad \dots \quad (A4.7)$$

also, we have,

$$i_1 R + i_{g1} R_g = v,$$

$$\text{or, } 1/R [i_{g1} \{ (R_g - R) - 2(R_g^2 - R R_g + R_g) / (R_g - R) \} + (R_g - R + v) / (R_g - R)] R + i_{g1} R_g.$$

$$= v, \quad // \text{ putting the value of } i_1 \text{ from (7).}$$

$$\text{or, } i_{g1} = [\{ v (R_g - R) - (R_g - R + v) \} / (R_g - R)] [(R_g - R) / \{ (R_g - R)^2 + R_g (R_g - R) - 2 (R_g^2 - R R_g + R_g) \}]$$

$$\text{or, } i_{g1} = v / (R_g^2 - 5R R_g - 2R_g) \quad \dots \quad \dots \quad (A4.8)$$

Check: for the voltages and currents at nodes and loops.

From the simulated outputs some interesting data are placed below in the following tables:

A4.2 DYNAMIC STUDY: NUMERICAL OUTPUTS [SAMPLE DATA]

A4.2.1 Steady state analysis with D.C supply

Steady states with D.C supply in an electrical network (3nodes) with line R and L in series, node C in parallel to Rg, taking initial conditions zero.

(a) For the line parameters: $R_1=0.2R_3$, $R_2=0.6R_3$, $R_3=0.136$, at all loops and node $R_g=99.9$, $L=0.124$, $C_1=0.2$, $C_2=1.2*C_1$, $C_3=1.4*C_1$, time constant D.C input voltage amplitude $V=1.32$ pu volts,

For the load parameters: $R_{d1}=1.36=R_{d3}$, $R_{d1}=1.2*R_d$, $R_{d2}=1.1*R_d$, $L_d=10.0$; and stray capacitors $C_{d1}=C_d$, $C_{d2}=0.8*C_d$, $C_{d3}=0.6*C_d$.

(i) In case of load $R_d(t)=\text{constant over time}$

(ii) In case of load $R_d=1.36$ for $t=[0, 0.4T]$, $R_d=0.8*1.36$ for $t=[0.4T, 0.8T]$, $R_d=1.2*1.36$ for $t=[0.8T, T]$, but $C_d=0.2$ & $L_d=10.0$ for $t=[0, T]$

Table 1:

System	V ₁ .	V ₂ .	V ₃ .	P _c (t _p)	P _{ls} (t _p)	E _{ca} (T)	E _{lsa} (T)	F _c (T)	F _{ls} (T)
Load type									
No load condition	1.319	1.317	1.315	0.00	0.0520	0.00	0.0261	0.00	0.50
For const load R _d .	1.270	1.185	1.151	2.901	0.2320	1.448	0.1169	0.25	0.50
For time variable load R _d .	1.270	1.185	1.152	2.903	0.2320	1.522	0.1297	0.44	0.40
	1.259	1.156	1.116	3.474	0.3212				
	1.278	1.205	1.177	2.492	0.1806				
For const load R _d . & L _d .	1.251	1.121	1.018	0.2833	0.4975	1.122	0.2088	0.40	0.42
For variable load R _d . but const L _d .	1.256	1.123	1.028	0.2519	0.4603	2.516	0.5970	0.67	0.40
	1.219	1.035	0.8919	3.544	0.9482	[3DC2			
	1.236	1.075	0.9492	3.736	0.7607	.f]			
For const load R _d ., L _d . & C _d .	1.266	1.158	1.067	2.305	0.3557	2.508	0.4776	0.44	0.50
For variable load R _d ., but const L _d . & C _d .	1.257	1.132	1.028	2.580	0.4539	2.463	0.5035	0.95	0.72
	1.244	1.095	0.9724	3.044	0.6311	[3DC2			
	1.266	1.156	1.065	2.339	0.3605	a1.f]			
For const load R _d . & L _d .(=12);	1.257	1.132	1.028	2.601	0.4569	2.516	0.418	0.967	0.61
For variable load R _d . but const	1.245	1.098	0.9778	2.913	0.6075	4.492	0.7784	0.77	0.64
	1.266	1.158	1.068	2.267	0.3507				

$L_d(=12)$	1.266	1.158	1.068	2.265	0.3503				
For const load R_d , C_d & $L_d(=12)$	1.1.257	1.132	1.028	2.601	0.4569	1.158	0.2108	0.445	0.46
For variable load R_d , but const C_d & $L_d(=12)$;	1.257	1.132	1.028	2.580	0.4535	2.467	0.4705	0.956	0.745
	1.244	1.095	0.9724	3.043	0.6311				
	1.266	1.157	1.065	2.339	0.3601				

(b) For the line parameters: $R_1=0.2R_3$, $R_2=0.6R_3$, $R_3=0.03$, at all loops & node $R_g=99.9$,

$L=0.12$, $C_1=0.2$, $C_2=0.6*C_1$, $C_3=0.4*C_1$, time constant D.C input voltage amplitude

$V=1.32$ pu volts,

For the load parameters: $R_{d1}=R_d$, $R_{d3}=0.5*R_d$, $R_{d2}=0.7*R_d$,

$L_d=L_{d1}=L_{d2}=L_{d3}$; & stay capacitors $C_{d1}=C_d$, $C_{d2}=0.8*C_d$, $C_{d3}=0.6*C_d$.

(i) In case of load $R_d(t)=\text{constant}$ over time

(ii) In case of load $R_d=1.2$ for $t=[0, 0.4T]$, $R_d=0.8*1.2$ for $t=[0.4T, 0.8T]$,

$R_d=1.2*1.2$ for $t=[0.8T, T]$, but $C_d=2.0$ & $L_d=2.0$ for $t=[0, T]$

Table 1a:

System	V_1	V_2	V_3	$P_c(t_p)$	$P_{ls}(t_p)$	$E_{ca}(T)$	$E_{lsa}(T)$	$F_c(T)$	$F_{ls}(T)$
Load type									
For const load R_d , L_d , & C_d .	1.293	1.231	1.171	5.484	0.4984	2.626	0.2463	0.48	0.49
For variable load R_d , but const L_d , & C_d .	1.295	1.256	1.203	5.617	0.4814	2.789	0.2776	0.42	0.38
	1.289	1.213	1.141	6.600	0.7227				
	1.286	1.228	1.175	4.721	0.3602				

For the D.C supply:

From the numerical data (given in table 2 above) obtained through simulation of the system it is observed that we can maintain the node voltages within our desired level by manipulating the systems load parameters as well as the node capacitors. This can help us operate the power network systems keeping load factor nearest to but less than 1; while the loss factor nearest to but more than zero during that period.

The characteristic behavior of the systems state variables, Power load consumed intensity & loss, total Energy consumed & lost during the period of operation can be seen below from their numerical plots (Figure 3.3 to 3.16) with time.

A4.2.2 Steady-state analysis with A.C supply

In an electrical network (3nodes) with line R & L in series, C in parallel to R_g , taking initial conditions of states to be zero.

N.B--, our targets were that,

_Node voltages be maintained within $\pm 5\%$ of rated voltage;

_Stability of the system must be maintained;

_For each systems operation,

- (1) load factor, $F_c \leq 1$ or preferably be $\cong 1$, so that the plant capacity does not remain idle for longer time period and thereby minimize capital (establishment) cost;
- (2) loss factor $F_{ls} \leq 1$ or preferably be $\cong 0$, so that there may be some loss which is not continued for longer duration of time.

(a) For the line parameters:

$$R_1=0.2R_3, R_2=0.6R_3, R_3=0.136, \text{ at all loops \& node } R_g=99.9, L=0.124, C_1=0.2,$$

$$C_2=1.2*C_1, C_3=1.4*C_1, \text{ time constant A.C input voltage amplitude}$$

$$V=1.32 \text{ pu volts, with } f=0.1$$

For the load parameters:

$$L_d=10.0 \text{ or } 12.0; \& C_d=2.0D-01, C_{d1}=C_d, C_{d2}=0.8*C_d, C_{d3}=0.6*C_d.$$

$$R_d=1.36=R_{d3}, R_{d1}=1.2*R_d, R_{d2}=1.1*R_d,$$

$$R_d=1.36 \text{ for } t=[0, 0.4T], R_d=0.6*1.36 \text{ for } t=[0.4T, 0.8T],$$

$$R_d=1.2*1.36 \text{ for } t=[0.8T, T], \text{ but } C_d=2.0D-01 \& L_d=10.00 \text{ for } t=[0, T]$$

Table 2:

System	$\pm V_1$.	$\pm V_2$.	$\pm V_3$.	$\pm P_c(t_p)$	$\pm P_{ls}(t_p)$	$E_{ca}(T)$	$E_{lsa}(T)$	$F_c(T)$	$F_{ls}(T)$
With									
No load condition	1.319	1.317	1.315	0.00	0.0522	0.00	0.01306	0.00	0.25
Const load R_d .	1.293	1.228	1.210	3.099	0.2721	0.7771	0.06949	0.25	0.26
Time variable	1.293	1.228	1.210	3.099	0.2721	0.8118	0.0757	0.24	0.21

load R_d .	1.273	1.188	1.162	3.646	0.3624		5		
	1.306	1.256	1.243	2.690	0.2185				
Const load R_d . & $L_d=10$;	1.315	1.308	1.303	0.1843	0.05380	0.04838	0.01415	0.26	0.26
Variable load R_d . but const $L_d=10$	1.315	1.309	1.303	0.1527	0.05331	0.05144	0.01423	0.24	0.26
	1.315	1.308	1.301	0.2158	0.05447				
	1.315	1.308	1.301	0.2158	0.05447				
Const load R_d ., $L_d=10$, C_d ;	1.355	1.373	1.379	0.1422	0.05878	0.03754	0.02043	0.26	0.35
Variable load R_d ., but const $L_d=10$ & C_d ;	1.346	1.356	1.359	0.1965	0.05839	0.04962	0.01858	0.21	0.31
	1.346	1.358	1.361	0.1610	0.05757				
	1.346	1.356	1.358	0.2311	0.0544				
Const load R_d . & $L_d(=12)$;	1.323	1.323	1.322	0.1329	0.05403	0.03524	0.01444	0.27	0.27
Variable load R_d . but const $L_d(=12)$;	1.323	1.326	1.320	0.1063	0.05364	0.03758	0.0145	0.35	0.27
	1.323	1.322	1.320	0.08126	0.05441				
	1.323	1.322	1.321	0.08125	0.05441				
For Const load R_d ., $L_d=12$, C_d .	1.460	1.500	1.497	2.309	0.1253	0.5998	0.04198	0.26	0.34
For variable load R_d ., const C_d . $L_d=12$	1.478	1.542	1.552	2.368	0.1391	0.582	0.0407	0.23	0.26
	1.438	1.468	1.461	2.034	0.1046				
	1.492	1.553	1.559	2.545	0.1547				

For the A.C supply:

From the numerical data (given in table 2 above) obtained through simulation of the system it is observed that we can maintain the node voltages within our desired level by manipulating the systems load parameters as well as the node capacitors. This can help us operate the power network systems keeping load factor nearest to but less than 1; while the loss factor nearest to but more than zero during that period.

The characteristic behavior of the systems state variables, Power intensity of load consumed & loss, total Energy consumed & lost as well as their operating factors during the period of operation can be seen from their numerical plots (Figures not shown here) with time.

APPENDIX 5

POWER SYSTEMS LOSSES

[Ref: Internet sources < <http://www.powersystemlosses/> >]

The appendix contributes information from the following:

09-Jun-99: Publications. Petter Krus. H Svensson, J Andersson, K-E Rydberg,0

'Modelling of Losses and Temperature Calculations in Fluid Power Systems'

27-Apr-99: **ABB Power Systems - Reactive Power Compensation**

Reactive power appears in every AC power system and causes instability, unbalance and high losses of the active power.

10-Jul-97: **Economic Analysis of Power Systems.** Conventional and renewable energy sources. Analysis and forecasting of loads. Power generation stations load scheduling.

A. How to take care of power losses in a network

When more than one utility are participating in the network, allocating the losses is no longer as straight forward as it is for a vertically integrated utility. When multiple generators are involved, each of the generators has an influence on the losses incurred by other generators. The current **way to determine these losses** is to conduct a load flow study and to determine the incremental losses. However, with a large number of buses, such study on a regular basis may be impractical. Therefore, several approximate methods have been proposed.

1) Megawatt-Mileage Method: This method requires conducting a base load flow study to determine the distribution of the actual power flow and distribution of the losses from each generator and then uses this information on a pro-rated basis. A variation of this method has been proposed by United Illuminating Company for the NEPOOL. Their proposal can be found in the web site (energyonline.com) under Blueprints: New England - Regional Transmission Group Pricing Proposal.

2) A second method has been proposed by Professors Felix Wu and Pravin Varaiya of University of California at Berkeley. Their proposal requires computing load flow, congestion and incremental losses for each new transaction using load flow methodology. The load flow program is used to calculate a loss factor for each transaction which can be used for determining

the incremental loss for each new trade. A description of this method can be found in our Blueprints article: California - Coordinated Multilateral Trades for Electric Power Networks.

3) The third method which also requires load flow analysis is used in the Poolco context by determining losses from and to designated hubs. These losses can be calculated by the pool operator and used for the settlement process to determine the transmission cost.

Responses

1. How accurately can a trader's risk to losses be assessed before transmission? (Kevin Schuyler)
 1. Untitled ()
 2. Energy project at INSEAD (i nemeth)
2. Practicality of charging methods; what about system upgrading? (John Brooks)
3. Cost ()

B. [Ref: Base: The Restructuring Forum

Previous: Quality and Reliability Issues (Jeffrey T. Nodland)

Next: Short term Forward and Futures contracts (Frank Ashe)

Date: Sat, 16 Sep 1995 12:13:40 GMT

From: (@ix-atl8-24.ix.netcom.com)]

Power Losses:

When only one utility's power is on transmission and distribution systems the power losses, I assume, are included in power bills. What if multiple power providers supply power to the system. How are the line losses to be split?

Responses

1. How to take care of power losses in a network (anindo roy)
 1. How accurately can a trader's risk to losses be assessed before transmission? (Kevin Schuyler)
 1. Untitled ()
 2. Energy project at INSEAD (i nemeth)
 2. Practicality of charging methods; what about system upgrading? (John Brooks)
 3. Cost ()

MODELING POWER SYSTEMS:

A **Power System Blockset model** of a motor-generator uninterruptable power supply (UPS).

Using two machines connected together, this system models the mechanical and electrical transients following the fault and isolation of the motor/generator system (Click on the diagram for a larger view).

Simulate it

The Power System Blockset is completely integrated with Simulink at the block level. Combining Power System and other Simulink blocks creates a unique environment for multidomain modeling and controller design. This environment allows the combination of electrical, power-electronic, mechanical, hydraulic, and other systems models.

For **time-domain simulation**, the Power System Blockset takes advantage of Simulink's powerful variable-step integrators and zero-crossing detection capabilities to produce highly accurate simulations of power system models. In addition, we have access to all of the block building and masking features, allowing you to build more complex components from electrical primitives.

Powerful analysis methods at our fingertips

The new blockset contains features that allow us to specifically analyze the electrical portions of your Simulink model. We can extract the linear portions of the electrical model into the MATLAB workspace as a state-space system for further analysis. This capability is useful if, for example, you want to examine the impedance frequency characteristics of our system.

You can work with the **electrical state-space system** using any of the tools from The MathWorks. If we are developing a control system for an electrical subsystem, for example, we can use one of the Control System Toolboxes to design controllers around the plant model you have extracted from the Simulink diagram.

The **blockset also provides a GUI** that allows we to interact directly with the blocks and states of the electrical system. We can set the state variables of the electrical system through the GUI. For example, we may want to set the states of the system (voltages or currents) to simulate some

transients. The GUI also provides the steady-state values of voltage and current so that we can return to a steady-state simulation easily for other types of analysis.

Finally, to aid in working with the blockset library of machines and drives, the Power System Blockset provides a function that performs load flow analysis. Load flow analysis is done to balance the electrical and mechanical loads between machines in our model.

Conclusion:

Whether we are a systems engineer working on a controller interfacing with electrical subsystems or a power utilities engineer designing a fault-tolerant power distribution network, we can now make use of the powerful Simulink environment. By using the Power System Blockset, we reach into the electrical domain with all the ease of use and extensibility of open systems that we have come to expect of tools from The MathWorks.

APPENDIX 6

SAMPLE OF CHARACTERISTICS USED BY OTTAWA HYDRO

Reference (web page: <http://www.ottawahydro.on.ca/>)

TABLE 1
SERVICES AVAILABLE FROM DIFFERENT SUPPLY POINTS

Supply Point	Maximum kW	Amp	Maximum Main Switch Secondary Voltage
Pole 75	400	120/240V	
Pole 75	100	600Y/347V	
Pedestal 46	3 x 200	120/240V	
Transformer Enclosure	100	600	120/240V
4.16/2.4 kV Padmount Transformer	300	1200 400	208Y/120V 600Y/347V
13.2/7.6 kV Padmount Transformer	500	1600 600	208Y/120V 600Y/347V
2.4/7.6 kV 1 Vault	167	800	120/240V
4.16 kV 3, 4W Vault	300	1200 400	208Y/120V 600Y/347V
13.2/7.6 kV 3, 4W Vault	1000	3000 1200	208Y/120V 600Y/347V
13.2 kV 3 Vault			

See Primary Voltage Service Specifications <primary.html> for various scenarios up to 27000 kVA.

APPENDIX 7

SOME REFERENCE WEBSITES

The appendix contributes information from the following websites.

1. **Sparsity Technology, power system analysis, power system programming,**
load flow, ... -
<http://www.geocities.com/SiliconValley/Lab/4223/>
2. **Power System Analysis - Power System Analysis 1 The Power System: An**
Overview 2 Basic Principles 3 Generator - Transformer Models and the Per Unit
System 4 Transmission Line Parameters 5 Line Model and Performance 6 Power ..
http://preview.mcgraw-hill.com/info/com.mcgrawhill_books_0075616343.html?
3. **Introducing EIFFEL: an object-technology approach to power system analysis - Resource**
Locator
http://www.progsoc.uts.edu.au/~geldridg/psa-objects/ppr_9601.html
4. **Solar power system sizing for your solar electric project.** - Solar4Power provides
complete assistance and equipment from leading manufacturers for your solar power
project. <http://www.solar4power.com/solar-power-sizing.html>
6. **Power System Analysis -** <http://power.mines.edu/PowSys/> 7. **Power System Analysis -**
<http://egweb.mines.edu/EGGN484/> Fast Forward »
8. **Electric Power System in Czech Republic: A Strategic Entry Report, 1996** – Electric
Power System in Czech Republic: A Strategic Entry Report, 1996 Click here for more
information about buying this research report. This report puts executives and strategic
planners on the fast.
http://preview.marketresearch.com/info/com.marketresearch_reports_233494.ht...
10. **MSB: Power System Planning** - MSB Energy Associates Professionals specializing in
energy regulation and policy design. <http://www.msbnrg.com/exp-power.shtml>

APPENDIX 8

POWER SYSTEM ANALYSIS HINTS

[Reference: Internet resource <http://www.powersystemsanalysis/>]

Objective:

The objective of this is to take in the study of the largest machine ever built: the integrated power grid. I have been introduced to the broad range of theory and methods related to power system analysis and design. In the process, I tried to:

1. Understand the use of transmission lines as a means of conveyance of energy.
2. Learn tools for the analysis of power systems.
3. Understand the purpose and application of protective devices.
4. Be able to determine the flow of power through a transmission line.
5. Understand voltage regulation, efficiency, practical stability limits, etc.

Our goal is to provide with a positive, effective learning environment in which we will learn the fundamentals of electric power systems and their effective regulation through study of the states & their behavioral changes with different line parameters as well as loads.

Engineering philosophy: of the Power Systems.

Power System Components		
Electrical Components		
Light bulb	Socket	Wire to switch
Switch	Wire to circuit box	Circuit breaker
Watt-hour-meter	Connection to distribution system	Distribution transformer
Distribution system	Substation	Capacitors
Circuit breakers	Disconnects	Buses
Transformers	Sub-transmission system	Capacitor banks
Tap changers	Current transformers	Potential transformers
Protective relaying	Reactors	Metal-oxide varistors
Transmission system	Suspension insulators	Lightning arrestors
Generator step-up transformers	Iso-phase bus	Generators

Non-Electrical Components		
Glass for bulbs	Manufacture of bulbs	Sockets
Switches	Circuit box	Steel for circuit box
Copper for wire	Aluminum for wire	Poles for overhead lines
Transmission towers	Maintenance	Plastics for capacitor insulation
Controls for protective relaying schemes	Communications for data and protection	Fiber optics for communications
Foundations for substation equipment	Excavation equipment and crews	Ceramics and polymers for suspension insulators
Oil for transformers and circuit breakers	Sulfur-hexa-flouride for gas insulated substations	Springs for circuit breakers
Process control for component manufacturing	Computers for process control	Computers for generation control and dispatch
Turbines for turning generator	Coal for making steam to turn turbine	Trains for hauling coal
Cars	Bridges	People

Typical voltages for different parts of the American power system:

System Type	From	To
Residential	110 V	220 V (split single Phase)
Commercial	480 V	(three Phase)
Industrial	480 V	4160 V.
Distribution	2300 V	32000 V
Sub-transmission	25 kV	130 kV
Transmission	115 kV	765 kV
Generation	13.2 kV	36 kV

Current Issues of Power Systems

Extensive outages in 1996 , and in 1998 in Canada as well as in year 2001 in the USA.

Issues are:

- (1)_ Combined issues of Power system stability

- (2)_Protective Relaying
- (3)_System Planning
- (4)_Two million customers affected in 14 states, Canada and Mexico
- (5)_Initiating event related to power line touching a tree

Note: This is about the flow of energy. How electrical energy gets from one place to another, and what are some of the issues that affect that flow.

Relationship between Energy, Work and Power

$$\text{Mechanical Energy, } W = \int F \cdot dl = Dd \cos \theta$$

$$\text{Rotational Energy, } W = \int \tau d\theta = \tau \theta$$

$$\text{Electrical Energy, } W = \int qdV = qV$$

$$\text{Power, } P = \frac{dW}{dt}$$

$$\text{Electrical Power, } P = V \frac{d}{dt} q = VI$$

Phasors

A phasor is a representation of a sinusoidal voltage or current as a vector rotating about the origin of the complex plane. This is an area with which we are already familiar. Nevertheless, because of the degree to which we depend upon our understanding of this principle, we may review it in detail.

Example of Voltage and current calculations without phasors:

$$v(t) = V_{\max} \cos(\omega t + \theta)$$

For a simple RL circuit with the above excitation voltage, find the current:

$$v(t) = Ri(t) + L \frac{d}{dt} i(t)$$

This becomes a very difficult problem to solve, with the solution:

$$i(t) = I_{\max} \cos(\omega t + \beta)$$

$$I_{\max} = \frac{V_{\max}}{\sqrt{R^2 + (\omega L)^2}}$$

$$\beta = \arctan\left(\frac{\omega L}{R}\right)$$

Euler's Equation, $e^{\theta} = \cos \theta + j \sin \theta$

Representation of voltages and currents as complex numbers:

$$V_{\max} \cos(\omega t + \theta) = \operatorname{Re}(V_{\max} e^{j(\omega t + \theta)})$$

We then shorten the notation, assuming that all phasors that will be used in a system are at the same frequency, the $(ej\omega t)$ term is implicit in all references to the value. Another assumption that is made is that the magnitude of any voltage or current as a function of time is the real part of its complex representation. Hence, $v(t) = V_{\max} \cos(\omega t + \theta)$ may be represented in any of the following ways:

$$V_{\max} \cos(\omega t + \theta) \Leftrightarrow V e^{j\theta} = V \angle \theta = V(\cos \theta + j \sin \theta)$$

being called the exponential, polar, and rectangular forms respectively,

where, $V = \frac{V_{\max}}{\sqrt{2}}$ is the root mean square (rms) of the voltage wave form.

Definition of RMS

$$V_{rms} = V = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

Phasor representation of Resistance, Inductance, Capacitance

$$R \Rightarrow R$$

$$L \Rightarrow X_L = j\omega L$$

$$C \Rightarrow X_C = \frac{1}{j\omega C}$$

Advantages of Phasors

_ Less Cumbersome (short hand notation)

_ Simpler Calculations (complex arithmetic, calculators can do),

generally less need for integration and differentiation

_ Additional insights may be obtained about relations between currents, voltages, and power

Limitations:

Applies only to sinusoidal steady-state systems;

Power Calculated using phasors is only the time average.

Instantaneous Power

$$\begin{aligned} P(t) &= v(t)I(t) = V_{\max} \cos(\omega t + \theta) I_{\max} \cos(\omega t + \beta) \\ &= \frac{1}{2} V_{\max} I_{\max} \cos(\theta - \beta) + \frac{1}{2} V_{\max} I_{\max} \cos(2\omega t + \theta + \beta) \end{aligned}$$

Average Power

$$P = \frac{1}{T} \int_0^T v(t)i(t)dt = \frac{1}{2}V_{\max} I_{\max} \cos(\theta - \beta) = VI \cos(\theta - \beta)$$

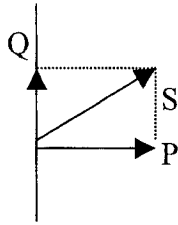
Complex Power

$$S = VI = P + jQ \quad \text{=Real power + Imaginary power}$$

$$\text{Real Power, } P = \text{Re}[S] = |I|^2 R = \frac{|V|^2}{R}$$

$$\text{Reactive Power, } Q = \text{Im}[S] = |I|^2 X = \frac{|V|^2}{X}$$

Power Triangle



Note that the real and reactive power of parallel components add. That is, if a capacitor were placed in parallel with an inductive load, the net reactive power for the combination would be the sum of the inductive (positive) reactive power and the capacitive (negative) reactive power.

Power Factor Correction

Determine uncorrected real and reactive power

Determine level of correction desired

Determine resultant required reactive power

Subtract uncompensated reactive power to determine required VARs added.

Network Equations: KCL and KVL in phasor domain, Formulation of mesh equations

Formulation of nodal equations, Conversion of system of equations to matrices

Matrix operations: Use of Inverse, Transpose, Conjugate, and

Solution of matrix equations

Review of principles of 3 phase

Why Three phase?

_ Three single phase circuits

_ Neutrals joined

_ Phase shifted sources

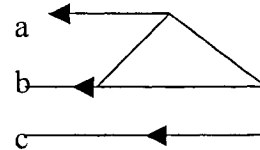
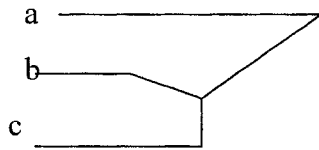
_ Power in three phase systems

Advantages of three phase

- _ Reduced cost (same power less wire or more power same wire)
- _ Constant torque (reduced vibrations)
- _ Greater "power per pound" in motors, generators, and transformers.

Conventions in 3 phase systems

- * Line to line voltage vs. line to neutral voltage
 - _ Neutral reference may not be accessible
- * Wye connections vs. Delta connections



$$P_{3\phi} = V_a I_a \cos(\theta_{V_a} - \theta_{I_a}) + V_b I_b \cos(\theta_{V_b} - \theta_{I_b}) + V_c I_c \cos(\theta_{V_c} - \theta_{I_c})$$
$$= 3V_a I_a \cos(\theta) = 3V_{ab} I_{ab} \cos(\theta) = \sqrt{3} V_{ab} I_a \cos(\theta)$$

* Power delivered to a 3 phase load

- * Delta -Wye conversions

$$Z_Y = \frac{Z_{\Delta}}{3}$$

Per-phase equivalency (one-line diagrams)