EQUILIBRIA "A LA" HOTELLING: a survey

by

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CONTENT

INTRODUCTION

Harold Hotelling was a pioneer of mathematical economics and statistical theory. Although the bulk of his research pertained to the latter field[1], three published articles dealing with economic theory were of pathbreaking importance. THE ECONOMICS OF EXHAUSTIBLE RESOURCES (1931), and THE GENERAL WELFARE IN RELATION TO PROBLEMS OF TAXATION AND RAILWAY AND UTILITY RATES (1938), are still today of theoretical and practical relevancy. Of at least the same importance is his STABILITY IN COMPETITION paper which appeared in The Economic Journal, march, 1929.

In this seminal article, Hotelling studies a spatial non-cooperative duopoly equilibrium in the spirit of Cournot (1838). As is well known, Bertrand (1883), in a review of Cournot's work, suggested that prices[2], not quantities should be the independant variable. This had the troublesome result of making any stable equilibrium so obtained coincide with the competitive one. Edgeworth (1925) introduced, some years later, the notion of limited capacity (steeply rising average costs equivalent to a limited capacity). In this case, the firm charging the highest price is still able to sell to those customers who have not been able to satisfy their demand from the limited capacity of the lowest price firm.

Following this, Edgeworth concluded that there would be no stable equilibrium for the bertrannesque duopoly case, because of the fluctuation between the competitive equilibrium price and the monopoly price.

Hotelling (1929) believed that by having an even number of customers strung out along a road represented by a line segment, a stable equilibrium could be obtained if firms produce commodities which, unlike the above models, are differentiated by location. Demand being assumed perfectly inelastic (each consumer purchases the same amount of the good whatever the price), the consumer makes the decision on which firm to patronize on the basis of the lowest delivered price (transport cost plus the cost at the mill). As one firm lowers his mill-price, certain customers who previously made their purchases from the other supplier, will find it more attractive to shift their purchases to the former supplier. Hotelling demonstrates that a certain firm's demand does not, as in the case described by Bertrand above, become a discrete function in the difference in prices but a continuous function in the difference in the prices between the two firms. By virtue of this, a slight decrease in price by one duopolist does not necessarily guarantee the market in its entirety. Consequently, the rival's profits are not decreased to zero and the continuous existence of positive pure profits for both firms in the industry are all but assured in equilibrium.

From this simple analysis Hotelling drew two important facts. Firstly, by considering the line segment to represent, say, the sweetness/sourness of cider, it is possible that predictions in regard to spatial effects can be extended to cover and study industries characterized by certain degrees of product differentiation. Given costly transportation, undifferentiated products sold at different locations can be regarded as differentiated goods, and as the distance from the customer to the firm is increased, so is the degree of "disutility" faced by the customer by not being able to purchase the good he would consider to be ideal. Secondly, by letting firm's freely compete for location, Hotelling arrived at his most consequential result: that firms in their pursuit to maximize profits, would tend to cluster at the center of the line segment or market. From the viewpoint of product differentiation this would come to mean that the qualities of two competing products would tend to converge towards a higher level of similarity. Thus would emerge THE PRINCIPLE OF MINIMUM DIFFERENTIATION, a term first coined by Kenneth Boulding (1966).

Since the publication of Hotelling's result, many researchers have found it to be a valuable analytical device insomuch it provides new insights in diverse fields such as political science[3], location theory[4], oligopoly theory and its logical extension, monopolistic competition. Notwithstanding, it is this latter field which has attracted, thus far, the core of the research and the most attention in regard to the range of applicability of spatial competition analysis.

In view of the central role of differentiated oligopoly theory[5], and the recent surge of interest in the study of entry-deterring strategies in the field of industrial organisation, theorists have attempted to construct a firm theoretical foundation on which to base their predictions. For instance, recent developments in spatial oligopoly theory have shown that an entry-deterring strategy such as product proliferation can be very profitable for the concerned firms. Conceivably, it is here that Hotelling's spatial model has found its most intense following amongst the specialists [see, for example, Hay (1976), Eaton and Eaton (1979), and Schmalensee (1978)].

Albeit no less important and certainly not unrelated to its contribution on the oligopoly front, Hotelling's model has also played an important role in the development of mainstream micro-economic analysis. Recently, a number of papers reporting research into spatial micro-economics have claimed equilibrium solutions quite different or even reversed from those resulting in spaceless neo-classical models of perfect competition [see, for example, Greenhut (1974), Capozza and Van Order (1978, 1980), Eaton and Lipsey (1977,1978)]. For example, a free-entry equilibrium in the spatial competition model can be consistent with positive pure profits. There are two reasons for this reasult. In the first instance, by introducing transportation costs to the spatial dimension of general equilibrium models, firms no longer see themselves as facing horizontal demand curves. This is by virtue of a certain degree of monopoly power faced by each and every firm located in a spatial market. Secondly, unlike the spaceless model which does not assume increasing returns to scale, the very existence of the firm in economic space necessitates those increasing returns. Furthermore, these increasing returns need not preclude an equilibrium.

However, except for a series of articles which appeared in the 30's and 40's [see, for example, Copeland (1940), Lerner and Singer (1937), Smithies (1941), and Zeuthen (1933)], research into the theoretical aspect behind spatial competition lay mainly dormant until the 1970's. At time, Eaton (1972), and Eaton and Lipsey (1975) wrote two very elaborate articles in support of Hotelling's center clustering result for the case of two firms competing on a linear market. These were inspired by the above-mentionned sudden interest in oligopoly theory and the need for a firmer and more rigorous approach to the spatial model.

Since the publication of the above two articles and more importantly, following the publication of an article by d'Aspremont, Gabszewicz and Thisse (1979), challenging the validity of the above mentionned PRINCIPLE OF MINIMUM DIFFERENTIATION, an explosion of articles have been flooding the economic literature criticizing and extending Hotelling's original model.

The purpose of this paper will be to review those developments in spatial economic theory which have the potential to impact considerably on micro-economic theory in general and industrial organisation in particular. In addition, it also examines how the criticisms revolving around the center clustering result have affected the predictibility and applications of spatial competition analysis.

This paper is divided into four main sections. The first one deals with the presentation of the original Hotelling model and it's notations. In the second one, the emerging equilibrium when the number of firms is fixed exogenously is examined; most work in this area has been done in the context of competing duopolists. The third one focus on a review of the articles dealing with entry into the spatial model and the resulting equilibrium; this area is of particular interest for students of industrial organisation, because of the recent advances in the theory of entry and barriers to entry. As an addendum to this paper the last section will deal with the relationship between welfare economics and spatial economics; Hotelling himself showed us that welfare is decreased as the quality of the products diverge from the desired quality of the consumer, a certain body of literature is concerned specifically with this area. Some insight into directions of further research as well as some comments will conclude this paper.

SECTION I

Hotelling's Model

On a linear market of lenght L, two sellers supply a homogeneous product with zero production costs. Transportation costs are paid by the consumers and are assumed a linear function of distance[7]. Consumers are evenly spread along the market (or street) and purchase a unit of the good over one unit of time, irrespective of it's price, making the purchase from the seller who quotes the lowest delivered price[8]. By letting firms choose their location and price so as to maximize profits, Hotelling arrived at the far-reaching conclusion that the long-run Cournot-Nash non-cooperative equilibrium[9] would be characterized by the clustering of two firms at the center of the market and separated by an infinitesimaly small distance ß, (i.e., THE PRINCIPLE OF MINIMUM DIFFERENTIATION).

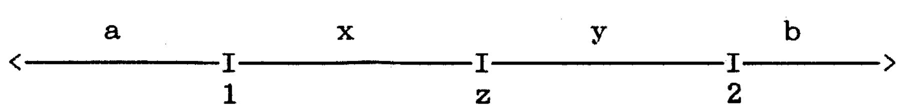
As emphasized by Hotelling, the model is capable of a variety of alternative interpretations involving forms of differentiation other than location. The street may represent a spectrum of possible colors, and distance the gap between the color offered by a supplier. In Hotelling's example, the line may be interpretated as a measure of the degree of sweetness or sourness of cider. Other examples could include the position of the American Democratic and Republican parties on the political spectrum, or even the position of religious denominations in the continuum from the fundamentalist to free-thinking.

With respect to the question of the street as a proxy for a continuum of qualities, Lovell (1970, p. 121-22) writes:

> The street is rather a restrictive environment, however, it is sufficiently rich to enable us to explore how the degree of product differentiation may be influenced by market structure.

Hotelling went to point out that this profit maximizing solution for the concerned firms would not lead to the social optimum. Social optimality implies that sellers establish themselves at the quartilles of the market so as to minimize transportation costs to the consumers.

With the above assumptions, the algebraic and graphical representation of the model[10] can now be discussed.



<u>Figure 1</u>

In figure 1, point 1 is the location of the first firm, while point 2 is the location of the second firm. The lenght of the line segment is equal to $L = a + x + y + b$. Demand being assumed perfectly inelastic, total sales can be defined as $L = q_1 + q_2$. Letting P_i denote the mill price $(i = 1,2)$, and c the transportation rate; consumers are then faced with choosing the supplier with the lowest delivered price, $P_i + cx$. For example, the customer located at point z is x units distant from firm 1, and y units distant from firm 2; so that he faces an effective delivered price of $P_1 + cx$ if he buys from firm 1, and $P_2 + cy$ if he buys from firm 2. Moving from z towards the left, it becomes gradually cheaper to make purchases from the first supplier,and more expensive to buy from the the second; z being the location of the marginal consumer, that is the customer who is indifferent from making his purchases either from firm 1 or firm 2 when $P_1+cx = P_2+cy$, every consumer to the left of Z purchases from firm 1, and every customer to the right of Z purchases from firm 2.

Given that Z is at the margin between the two suppliers, we can verify that:

$$P_1+cx = P_2+cy \qquad (1$$

$$L = a + b + x + y \qquad (2$$

By expressing locations as a function of prices and the parameters L, a, and b the following obtains

$$x = 1/2(L - a - b - \frac{P_1 - P_2}{c}) \quad (3$$

$$y = 1/2(L - a - b - \frac{P_2 - P_1}{c}) \quad (4$$

We can then derive the profit functions as a function of price as well as location for each firm. Given that the first supplier sells $q_1 = a + x$, and the second sells $q_2 = b + y$,

$$\pi_1 = P_1 q_1 = P_1 (a+x) = 1/2(L+a-b)P_1 - \frac{P_1{}^2}{2c} + \frac{P_1 P_2}{2c} \quad (5$$

$$\pi_2 = P_2 q_2 = P_2 (b+y) = 1/2(L-a+b)P_2 - \frac{P_2{}^2}{2c} + \frac{P_2 P_1}{2c} \quad (6$$

Profits can be viewed in figure 2 which is the graphical representation of the model. In this figure the slopes of the lines AB, BC, CD, DE are equal and represent the cost of transportation per mile. The area OP_1Z_1Z constitutes the profits of firm 1 and the area ZZ_2P_2L the profits of firm 2. 1B and 2D are their respective mill-prices and the line ABCDE represents the effective delivered price faced by the consumers at every point.

Figure 2 can also be used to better visualize the relationship which exist between the quality-space and the location-space. As the distance from the firm and the consumer is increased, so is the degree of "disutility" faced by the consumer by not being able to purchase his ideal color. This can be seen by the higher cost of transportation a consumer faces as his distance from the firm is increased. Because firm 1 and firm 2 offer only the colors white and orange the consumer located at point Z which prefers the color orange must face the highest cost of transportation. This is by virtue of being located at the margin between the two firms.

As for the dynamics of the model, Hotelling assumed that the firms would compete for price and location as to maximize profits. To describe the way in which a Cournot-Nash non-cooperative equilibrium results, firm i, i = 1, 2, would find in the first stage equilibrium price P_i^* assuming Cournot's "zero conjectural variation[11]",i.e., firm 1 sets his profit maximizing price on the assumption that firm 2 keeps his price unchanged, and vice-versa. The price, P_i^*, is found by solving the following set of simultaneous symmetrical equations

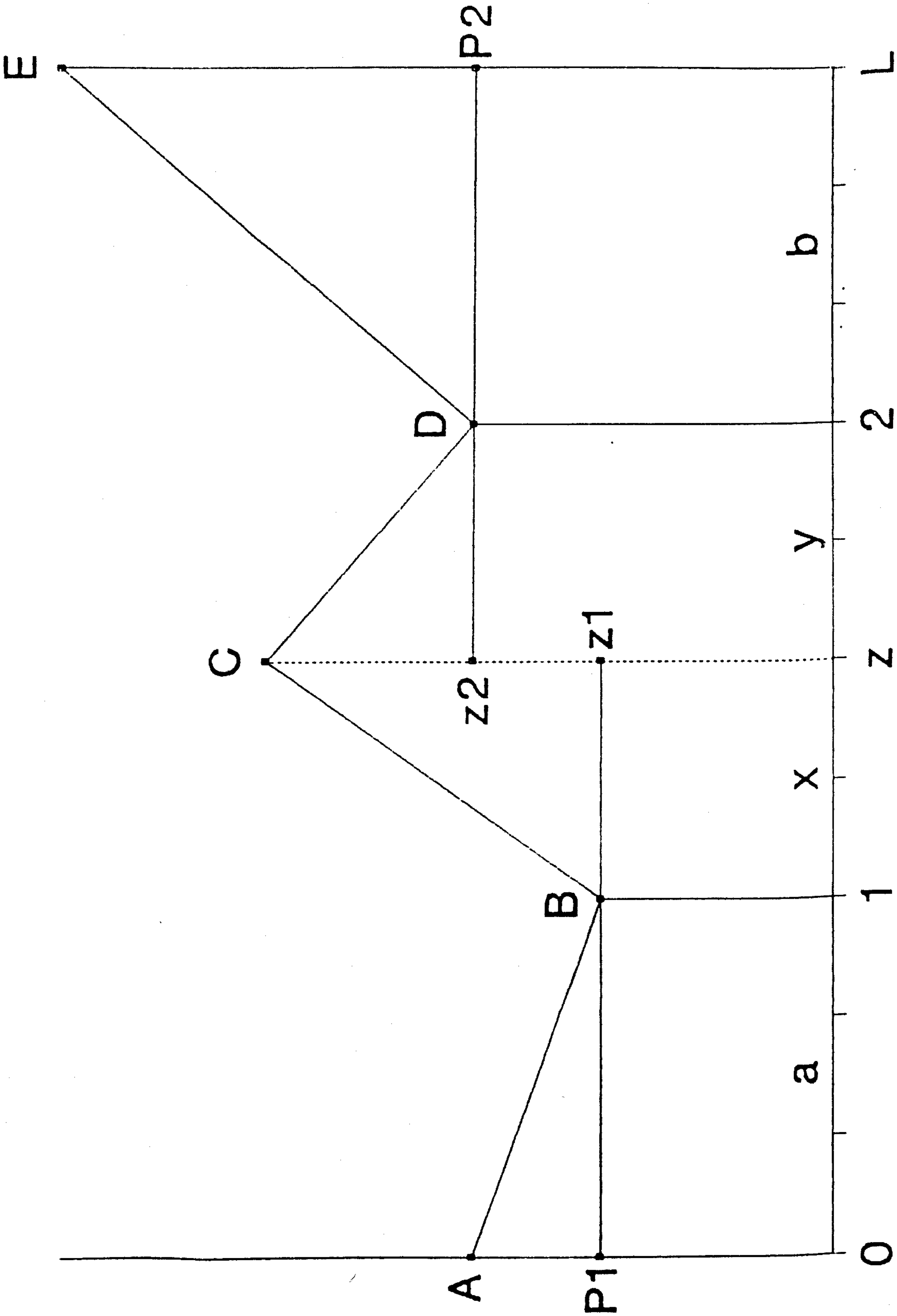


FIGURE 2

$$\frac{\partial \pi_i}{\partial P_i} = 1/2(-1)^i(L-a+b) - \frac{P_i}{c} + \frac{P_j}{2c} = 0 \qquad (7$$

The equilibrium pair of prices are then found:

$$P_i = c(L - (-1)^i \frac{a-b}{3}) \qquad (8$$

With corresponding output levels:

$$q_1 = a + x = 1/2(L + \frac{a-b}{3}) \qquad (9$$

$$q_2 = b + y = 1/2(L - \frac{a-b}{3}) \qquad (10$$

Firms are then in a position to choose their locations in order to maximize profits. Substituting the equilibrium prices into the profit functions will yield:

$$\pi_i = \frac{c}{2} (L- (-1)^i \frac{a-b}{3})^2 \qquad (11$$

From the above profit functions, it can easily be verified that the profits of firm 1 increase as the value of a increases, and that the profits of firm 2 increase as the value of b increases ($\partial\pi_1/\partial a > 0$; $\pi_2/\partial b > 0$). By making the same Cournot conjectures as before but this time in regard to location, firms will move towards the center of the market in an attempt to increase their market share and thus their profits. This action will eventually lead to them being located back to back at the center of the line segment and charging the same mill-price, hence yielding a stable equilibrium. It is easily verifiable, by the above equations, that this location move will increase the rate of profit of the duopolists.

Hotelling demonstrates here that profits are a continuous function of the difference in prices of the two firms; by lowering his prices ever so slightly, a firm will not capture the entire market and hence all of the sales, but will only entice a certain number of customers to purchase his goods. This is opposed to the case of Bertrand with homogeneous commodities, where the entire market was monopolized folowing a price reduction on the part of any of the two firms.

Nevertheless, a quick glance at the above equations will verify that it is not entirely inconceivable for a firm to capture the whole market of a competitor for certain values of a and b by simply lowering his mill-price far enough so as to make the difference in prices smaller than the total cost of transportation, $P_1 - P_2 < c(L-a-b)$. The short run decrease in profits for such a move would more than be off-set by subsequent monopoly profits once the rival is driven out of the market.

By examining figure 3, one can see how the above situation can emerge. As firm 2 lowers its price at the mill from 2D to 2D', consumers located at every point to the left of firm 1 are indifferent from making their purchases from any one of the two firms. By lowering its price slightly below 2D', firm 2 can capture the whole market and increase its sales and profits.

Friedman (1983), draws on an example to show how the above scenario can become unstable and lead to the emergence of a monopoly. By letting L=100, c=5, a=b=40, and P_2=500, firm 1 reaches a profit maximum when he charges a price P_1=500, and π_1=25000. However, at a price just below P_1=400 firm 1 can take over the whole market, thus generating profits just below π_1=40000, making such a price change a very profitable one. We will examine in the next section how this interior solution affects equilibrium in the spatial model and hence the center clustering result.

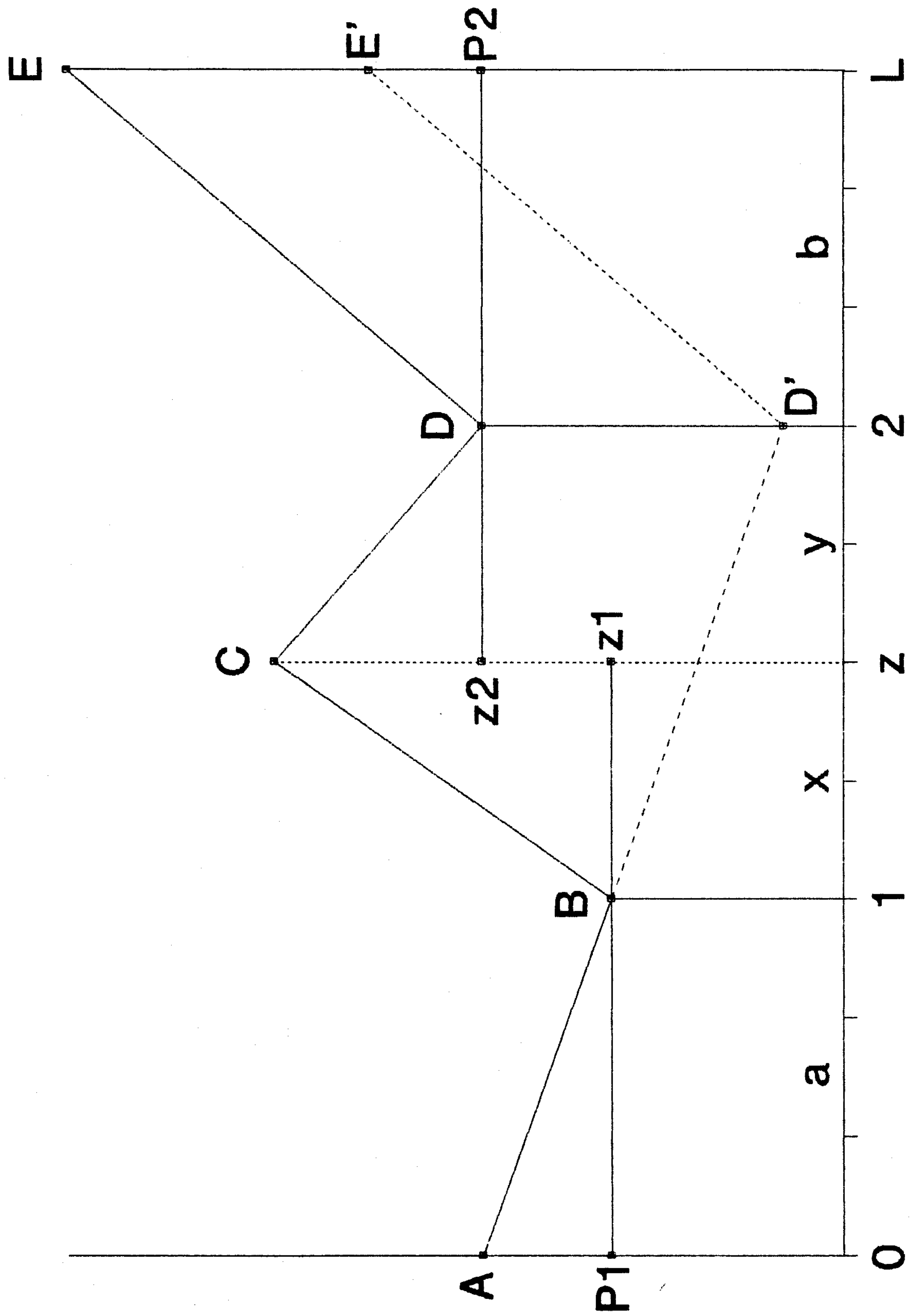


FIGURE 3

Section II

Fixed-numbers equilibrium.

This section deals with the existence and nature of an equilibrium in a spatial competition model when the number of firms is given. As we saw in the previous section with the example by Friedman, equilibrium in Hotelling's model may be undefined for certain values of a and b. However, the location of one firm in relation to the other is not the only factor affecting equilibrium. It will be shown in the following pages, that the nature of the equilibrium can also be altered by relaxing some of the assumptions concerning the exogenous environment within which the firms operate. These include: i) changing the nature of the consumer demand function from inelastic to elastic; ii) changing the shape and length of the market; and iii) increasing the number of firms involved. The degree of interdependence between firms and their products, and the conjectural variations in regard to firms' reactions to one another can also play an important role in determining the stability of equilibrium.

2.1 The Assumption of No-Mill-Price Undercutting

As seen above, mill-price undercutting is a cause of the non-existence of equilibrium in the spatial competition model. It will be instructive, therefore, to examine this more closely.

In Hotelling's model a firm would discover that by lowering its price beyond the point where the difference in prices equals the cost of transportation from its location to that of the opposing firm, the entire market can conceivably be monopolized. This happens because the special advantage that both firms enjoyed by offering differentiated products is rendered mute once undercutting becomes an optimal strategy. Thus, the other competitor by following suit and lowering its price would discover that this continued action would eventually lead to the Bertrand solution, that is, prices equaling average cost and profits equaling zero. In Salop's terms [see Salop (1979)], these prices are called ZAP PRICES, and are characterized by the discontinuity of the demand function at the point where $P_1=P_2-c(b-a)$. It is important to note that the consumer at this point is indifferent as to making his purchases from any of the two sellers (the marginal consumer).

Hotelling[12] himself recognized this contingency and it is for this reason that he frequently refers to various collusive measures that arise in which firms might cooperate in an effort to avoid this zero profit solution. These measures would act as a buffer between competing firms in order for them to avoid such mutually damaging action. In this case competitors would agree not to shift their price and location at any time in such a way that the price differential exceeds the cost of transportation between both firms location.

Smithies (1941, p. 428) was also aware of the possible non-existence problem. In order to avoid this eventuality, he invokes a sufficient assumption so as no competitor attempts to undercut his rival out of the market. He observes that competitors' willingness to survive in the market place is insured by their desire to achieve a <u>Lebenstraume</u> satisfactory to both before resorting to policies of extermination. Chamberlin (1949, p. 228) concords with the source of the possible non-existence of equilibrium. However, he adds that in order for a firm to be able to drive a rival completely out of the market, it must have sufficient production capacity to supply the entire market. In the absence of such a condition an unserved portion of the demand can always be satisfied by a second firm. In addition, given the assumption of inelastic demand, this second firm can charge any price without limit for a good. In cases where demand is price elastic, Chamberlin finds that a result very similar to the Edgeworth cycle would emerge, where prices would oscillate between the monopoly and competitive price levels.

Fellner (1949, pp. 87-88), in a footnote dealing with the spatial model, states that Hotelling's equilibrium solution can be obtained only in cases where no price reduction by any of the duopolists is capable of reaching over to the other side of its rivals' market area. However, his analysis does not go so far as to examine the effects of such an action and thus the question of equilibrium is completely overlooked. Other studies have generally neglected the possible non-existence of equilibrium [see, for example, Alonso (1964), and Miernyk (1971)].

In order to remove this difficulty from the spatial model, Eaton (1972) assumes what he believes is a more realistic agent's behavior, by invoking the MODIFIED ZERO CONJECTURAL VARIATION RULE. This means that any competitor believes that his action will not affect his rival's behavior subject to the qualification that the action of one producer will not completely elliminate the other from the market. Eaton and Lipsey (1976; 1978) justify the use of such a rule, appropriately termed the NO-MILL PRICE UNDERCUTTING ASSUMPTION, by arguing that the most elementary foresight will allow any firm to conclude that it cannot drive a competitor out of the market by charging a f.o.b. price lower than a competitor's mill-price. Hartwick and Hartwick (1971) introduce what they term as the HOTELLING STABILITY CONDITION in order to avoid any possibility of mill-price undercutting. In their model this becomes a necessary condition for a stable equilibrium to emerge in the duopoly case. Novshek (1980) points out that in the absence of such a behavioral assumption, equilibrium will generally not exist in spatial models.

A well behaved profit function is continuous in its vector of prices [see, for example, Varian (1984), p.46]. An elaborate proof by d'Aspremont, Gabszewikcs and Thisse (1979) demonstrates that the non-existence of equilibrium in the one-dimensional model of spatial competition is due to a discontinuity in the profit function faced by the firms.

From this observation the authors define two categories of equilibria[13]. The first is obtained when both firms are back to back at the center of the market (a+b=1) and an equilibrium of the type described by Bertrand emerges, where prices and profits are equal to zero. In the second case, equilibrium is arrived at by showing that the equation $P_1 - P_2 < c(L-a-b)$ must be satisfied. This results in the following set of equations:

$$\left(L + \frac{a-b}{3}\right)^2 \geq \frac{4}{3}L(a+2b) \qquad (12$$

$$\left(L + \frac{b-a}{3}\right)^2 \geq \frac{4}{3}L(b+2a) \qquad (13$$

If we assume symmetrical locations around the market area (a=b), these conditions reduce to

$$a \leq \frac{L}{4} \qquad (14$$

The above result demonstrates firms should position themselves outside the quantilles of the market for Cournot-Nash equilibrium prices to hold. If firms were to locate within the bounds dictated by the quartilles, except where a+b=1, then the possibility for mill-price undercutting would exist. As examined above, this causes the demand curve to suddenly jump, consequently, the profit function is characterized by a discountinuity at the ZAP price level; it is then impossible for a stable equilibrium to emerge. D'Aspremont et al. (1979) illustrate that the use of quadratic transportation costs will lead to an equilibrium vector of prices wherever firms may decide to locate. However, this result yields negative profits all along the linear market, negative profits which are reduced as firms move away from one another.

Prescott and Visscher (1977), and Shaked and Sutton (1982) have interpreted the above result in the context of product differentiation. They illustrated that producers of differentiated commodities would increase their monopoly power by diverging the quality of their products as much as possible. Hence, the lessening of price competition would increase profits to the firms. Conversely, as both qualities converge, the increased price competition between the increasingly similar commodities would reduce the profits to both firms. It is also interesting to note that this view is not totally new, Palander (1935)[14] and Hadar (1966) found that if the interdependence between products is sufficiently weak, or to put it another way, if the cross-price elasticities between products are small, then the system will be stable. Clark (1940), Triffin (1940) and Adelman (1948), all believed that under the assumption of homogeneous products, price competition can be unworkable; quality difference can ease such a situation and bring about price fluidity.

With respect to this problem Dorfman and Steiner(1954) wrote:

> "If a market consists of a number of consumers having identical demand curves, but differing in their responsiveness to quality change it will pay to provide different qualities."

As an added note to the subject of maximum product differentiation, it is interesting to draw attention to the fact that fields other than economic science have studied the same question. Micheal Porter, a pioneering specialist in the field of corporate strategy and a faculty member of the Harvard Business School, strongly believes that firms can increase their profitability by offering dissimilar products. In his best selling book COMPETITIVE ADVANTAGE (1984, p. 14) the author writes:

> " The second generic strategy is differentiation. In a differentiation strategy, a firm seeks to be unique in its industry along some dimensions that are widely valued by buyers. It selects one or more attributes that many buyers in an industry perceive as important, and uniquely positions itself to meet those needs. It is rewarded for its uniqueness with a premium price.
>
> The means of differentiation are pecular to each industry. Differentiation can be based on the product itself, the dilivery system by which it is sold, the marketing approach, and a broad range of other factors. In construction equipment, for example, Caterpillar Tractor's diferentiation is based on product durability, service, spare parts availability, and an excellent dealer network. In cosmetics, differentiation tends to be based more on product image and the positioning of counters in the store...
>
> A firm that can achieve and sustain differentiation will be an above average performer in its industry if its price premium exceeds the extra costs incurred by being unique..."

We have just examined in the previous paragraphs that no Cournot-Nash equilibrium prices exist if firms are close enough to one another, by virtue of discountinuity of the demand or profit function. In view of this, Hotelling's center clustering result or PRINCIPLE OF MINIMUM DIFFERENTIATION does not hold under this condition. However, attempts at solving this problem have been suggested by a number of authors. For example, Gabszewicks and Thisse(1979) have offered a solution for the existence problem. In their paper income disparities are privileged over taste differentiation. When consumers are located along the linear market according to a given income distribution, it is possible for the price of one producer to move down so as to make customers gradually move their purchases from the highest priced competitor to the lowest. This has the effect of making the demand and profit functions smooth by virtue of the smooth income distribution.

The above authors make the implicit assumption that the distinction between goods which are differentiated vertically or horizontally[15] is of no consequence in spatial competition models. However, Neven (1985) demonstrated that the extension of Hotelling's geographic model cannot be generalized to cover classes of goods which are differentiated on the basis of some quality characteristic (vertically differentiated). The model can only be applied to cases where goods are differentiated on the basis of some variety characteristic (horizontally differentiated). In order to make certain that the model is used for the purpose of analysing the appropriate markets, the author makes certain assumptions in regard to consumer preferences. This is to guarantee that consumers make choices in the space of horizontally differentiated products and not vertically differentiated products.

Following this, Neven demonstrates that a pure strategy Cournot-Nash equilibrium exists for every pair of products. The location tendencies of the firms are then examined and shown to corroborate most recent findings. That is that firms tend to differentiate their products as much as possible.

In a different vein than the above authors, Gal-OR (1982), and Shaked (1982), borrowing from an approach pioneered by Dasgupta and Maskin (1982), introduce mixed strategies in order to re-establish the existence of equilibrium. The authors claim that firms by moving towards one another pick their prices randomly instead of deterministicaly so as to arrive at a mixed strategy

Cournot-Nash equilibrium. However, Gal-Or (1982) states that mixed strategies are not sufficient in order to ascertain THE PRINCIPLE OF MINIMUM DIFFERENTIATION. This is because the end result will still be an equilibrium that converges to the Bertrand solution ($P_i = P_j = 0$), when firms are located back to back at the center of the market.

So far all of the above authors, have made use of price as the strategic variable of the firm. However, Salant (1986) deviates from this standard by assuming that quantities should be the independant variable in spatial models[8]. The use of inverse demand functions in spatial models leads to continuous profit functions in location and price. Hence, equilibrium can be established anywhere along the market. Gal-OR (1987) in the same spirit as Salant above, arrives at a strikingly different conclusion than d'Aspremont et al.(1979). Making use of a simple two-stage duopoly model, where the first stage is characterized by firms choosing their level of quality strategically, and then choosing the level of output in the second stage, the author is able to demonstrate that the degree of differentiation is reduced when seeking a Cournot-type equilibria vs. a Bertrand-type equilibria.

As examined above, equilibrium does not exist when the optimal strategic choice of the firm is to undercut its rival. It has even been demonstrated that firms can in many situations move away from one another (or increase the degree of product differentiation) in order to maximize profits, although some suggestions have been fowarded by new resarch in order to overcome this result. In the following sub-section the effects of the assumption of inelastic demand on the nature of equilibrium will be elaborated upon.

2.2 THE ASSUMPTION OF INELASTIC DEMAND

If an inelastic demand is assumed on the model, as Hotelling did in his original article, then consumers will purchase the good irrespective of its price. In this case, there is no reservation price for which consumers are unwilling to purchase the good. In a spatial context this fact can be linked to the lenght of the market. With an inelastic demand curve and an unlimited market lenght, consumers will purchase the good at any distance they may find themselves from the firm. By contrast, once the restrictive assumption of inelastic demand is relaxed, the lenght of the market will becomes limited. This is illustrated in figure 4.

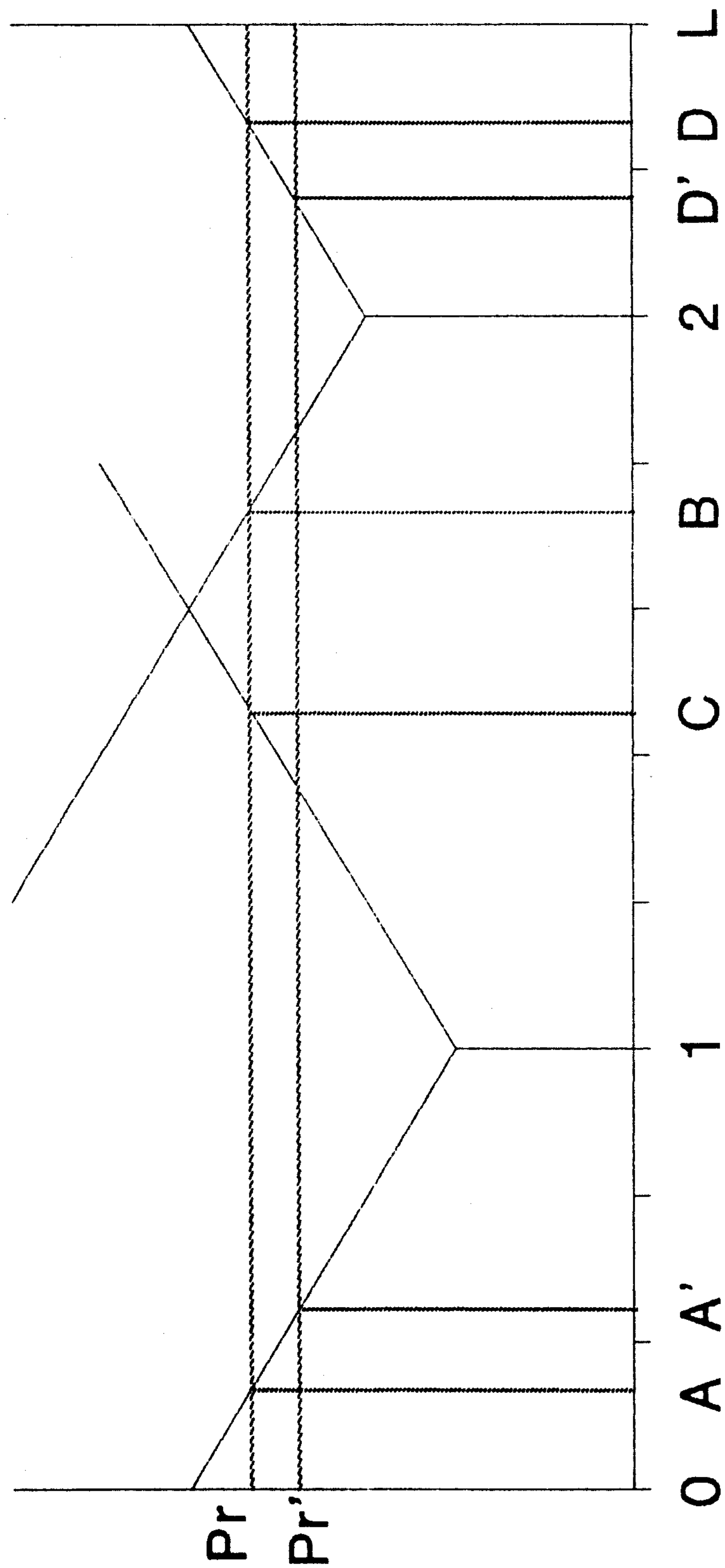


FIGURE 4

A Consumer's reservation price (P_r) is indicated in figure 4. The lenght of firm 1's market is confined to the interval AC while firm 2's is similarly confined to BD. They are only competitors for those consumers located in the interval BC. In this case as the reservation price (P^r) is decreased from P^r to $P^{r\prime}$, so is the lenght of the market, from AD to A'D'. In addition, one can also notice that firms can increase their market share by moving away from each other. If the reservation price is lowered to the point where delivered price exceeds the reservation price, demand will fall to zero.

Smithies (1941), Hartwick and Hartwick (1972), and Eaton (1972) making use of price elastic demand functions show that the PRINCIPLE OF MINIMUM DIFFERENTIATION is very sensitive to changes in the slope of the demand curve. This was also confirmed by Lerner and Singer (1937), Beckman (1972), and Graitson (1982), when using "rectangular" demand functions. The consensus that arose was that firms facing "rectangular" or price elastic demands would tend to move away from one another so as to capture the consumers located at the end points of the market: consumers who would normally find the price of the comodity prohibitive if firms were located at the center of the market.

Hotelling realized this and mentioned that the introduction of a more plausible downward sloping demand function would tend to mitigate center clustering:

> "The other important modification has to do with the elasticity of demand. The problem of the two merchants on a linear market might be varied by supposing that each consumer buys an amount of the commodity in question which depends the on the delivered price. If one tries a particular demand function the mathematical complications will now be considerable, but for the most general problems elasticity must be assumed." Hotelling (p. 56, 1929)

Yet, as Eaton (1972) pointed out, the tendency towards an equilibrium characterized by center clustering can be observed if firms face downwards sloping demand curves, the market lenght is relatively short, and price undercutting is banned. Then duopolists face no trade-off between moving to the center and loosing the customers located at the end-points of the market.

Economides (1984) studies the effects of the shape of the demand curve on the nature of the equilibrium in the spatial competition model. He claims that the analysis of d'Aspremont et al.(1979) missed the point by concluding that the non-existence problem is caused by the discountinuities of the demand or profit function. According to him the source of the problem is more closely related to the infinite "reservation" price (or perfectly inelastic demand) which was originally assumed by Hotelling and other subsequent researchers. This type of demand and the fact that mill-price undercutting is the optimal strategy for profit maximizing firms generate a non-quasiconcave profit function with respect to prices. In such models, the presence of a guaranteed demand all through the commodity space creates the incentives for firms to undercut each other and shatters the possibility of ever obtaining a Cournot-Nash equilibrium.

Economides also shows that when the reservation price is lowered, there will be some consumers located at the end of the market which face a "utility cost" higher than the reservation price, and as specified above, do not purchase the differentiated good. This will induce firms to relocate towards the edges. Furthermore, it is demonstrated that firms, by lowering the reservation price low enough, move towards the direction of increased profits until each one becomes a local monopolist.

This subsection has showed that THE PRINCIPLE OF MINIMUM DIFFERENTIATION does not hold when the assumption of inelastic demand is relaxed. As a matter of fact, a slight modification of the above assumption will rather enforce THE PINCIPLE OF "MAXIMUM DIFFERENTIATION".

2.3 MULTI-FIRM EQUILIBRIA

Up to this point the paper has been confined to duopolistic competition in spatial market. This section relaxes this assumption, keeping the number of firms fixed.

Although research in this area has remained limited as opposed to the case where the number of firms is endogenously determined (this is reviewed in the next section), it is important for many reasons. Firstly, most studies in this field have tended to assume fixed prices (parametric prices). This feature avoids the price-location (or quality) interaction which is so relevant in spatial analysis. This would seem to restrict the fruitfullness of the model. However, many economists have found applications in markets where price competition is ruled out or where price has little consequence on the decisions of the consumers. Secondly, the results obtained in the following models contribute to a better understanding of the mechanics behind their natural extension into models where price is considered a decision variable of the firm.

In their pathbreaking article dealing with many aspects of the spatial competition model, Eaton and Lipsey (1975) defined the following necessary conditions* (i and ii) for the existence of a Cournot-Nash equilibrium for an arbitrary number of firms. Their analysis uses some basic assumptions of the original Hotelling model. As was previously emphasized: firms face inelastic demand, compete on a one-dimensional bounded linear market, prices are fixed, and there are no relocation costs.

The necessary conditions are:

i) No firm's whole market is smaller than any other firm's half market. Indeed, any seller can obtain a market equal to either half market of any other seller by pairing with him. This can be seen in figure 5, where firm 1's market (X) is smaller than firm 2's half market (Y). By moving towards 2, firm 1 increases his market share to X'.

```
|------|-------|---------------------------|
       1       2
|----X-----|   |-------------Y-------------|
|------X'------|
```

Figure 5

* Proof of sufficiency can be found in Graitson (1979).

ii) Two peripheral firms are paired. This condition is necessary because any firm whose market boundary is at the edge of the line segment can increase its market share by moving towards the center of the market. In fact, each movement towards its competitor increases its market share, so that each location move is prefered to the previous one. This can be seen in <u>figure 6</u> where firm 1 moves in towards firm 2, its market share is increased from X^i to X^1 after each move.

```
|-----|--------------|---------------------|
      1              2
|----X^i -|
|-------X^i-1 --|
|----------X^1 ------|
```

<u>Figure 6</u>

The results obtained from the above conditions are shown in the following proposition where n is the number of firms.

<u>Proposition 1</u>

i) when n = 1, any location is a Cournot-Nash equilibrium;

ii) when n = 2, by condition (i), each firm will increase its market share by moving towards the center of the market and by condition (ii), both firms are paired at the center of the market. The equilibrium is characterized by center clustering, and this translates into the famous PRINCIPLE OF MINIMUM DIFFERENTIATIION;

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|--->-->-->-->-->---->----||---<--<--<--<--<---<----|
                          12
```

iii) when n = 3, by attempting to fulfil the requirements of condition (ii), the center firm (interior firm) will find itself with no market whatsoever. So that one of the conditions for a stable equilibrium cannot be satisfied;

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|---------------->|||<----------------------|
                  132
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iv) When n = 4, there exist an equilibrium characterized by two firms located at the first quartilles, and two others at the third;

```
|--------||-------------------------||--------|
         12                         34
```

v) if n=5, at equilibrium, two firms are located at the first sextile, two others at the fifth one and one firm is isolated in the middle;

```
|---||--------------|------------------||---|
    12              5                  34
```

vi) when more than five firms are competing on the market, there exist an infinite number of equilibrium configurations.

The existence of an indeterminate solution for n=3 can be explained by the incentive of the middle firm to increase his trivial market share by "leapfrogging" outside the peripheral firms. This can be resolved in two ways:
A) circular markets, and B) mixed strategies.

A) Eaton and Lipsey suggest that the problem can be resolved if, instead of assuming a linear bounded market, the analysis is extended to a market which is unbounded such as a circle. By placing the sellers on a circle the necessary conditions for equilibrium reduce to (i), thus making every firm an interior one. The following proposition presents the results of this latest assumption.

<u>Proposition 2</u>

i) there exists an infinite number of equilibrium, whatever the number of firms may be;

ii) at equilibrium, all firms may be isolated.

Notwithstanding, [see, for example, Archibald, Eaton and Lipsey (1985), Novshek (1980),Salop (1979), Schmalensee (1978), and Vickrey (1964), analyse the circular case], most papers dealing with spatial product differentiation use a linear market.

As pointed out by Kolberg and Novshek (1982), and Eaton and Eaton (1988), the circle is not appropriate for the analysis of many interesting market structure.

> "...the PRINCIPLE OF MINIMUM DIFFENTIATION does not arise in a circular market. The division of customer patronage does not change when the location of the two stores are changed.In order for our original model's predictions to hold, we need market boundaries -a beginning and an end to the market for customers. In the circular case, the market is unbounded; we cannot fix points at which the market begins and ends. The assumption of boundaries is therefore critical, and the theory is not robust when the assumption is altered." Eaton and Eaton (1988, p. 17).

B) Equilibrium can also be reestablished in the three firm model by assuming mixed strategies. According to Shubik (1980, p.117), if each firm (i), instead of choosing specific prices were permitted to select a probability distribution of the form

$F_i = \text{prob } p_i \leq p$

A mixed strategy equilibrium will exist if each firm can find a distribution such that if one knew the other's strategy neither would be motivated to change. In spite of the discontinuities of the payoff (or profit function), Shaked (1982) is able to demonstrate that a Cournot-Nash equilibrium in mixed strategies always exists in the 3-firm Hotelling problem. According to de Palma, Ginsburgh, and Thisse (1987), despite the mathematically elegant exposition of Shaked, they doubt the predictive power of his model. They arrive at a solution whereby uncertainties in the form of unobservable variables, in addition to price, affect the purchasing decisions of the consumers. This removes from the original model the assumption that consumers necessarily purchase from the nearest supplier. Following this de Palma et al. are able to demonstrate that equilibrium can best be characterized by firms being either centrally agglomerated, or by firms being symmetrically dispersed.

This interesting result is elaborated in the following proposition.

Proposition

a) As u/c increases, the distance between the two peripheral firms decreases.

b) For $u/c \geq 0.5$, the three firms are agglomerated.

c) When $u/c = 0$, the optimal locations for the three firms are at the first, second, and third sextille respectively.

Where u is the utility associated with non-observable and/or non-measurable characteristics, and c is transportation cost. The ratio u/c is the degree of variation in consumers' tastes and the transportation rate.

The authors use these results to explain a wider variety selection in the group of luxury cars. For example, Ferrari, Mercedes-Benz,and BMW (u/c is "small"). Which is compared to the opposing case where popular cars look so much alike (u/c is "large"). This is explained by the fact that wealthy consumers have very specific tastes in regard to the type of car they purchase, but less wealthy consumers care less about such things.

As was previously mentionned, price competition was ruled out in the above case. Although probably limiting the scope of the predictive nature of the model for a certain range of economic events, the simpler model does present some general results for cases where price competition is non-relevant. Some are introduced in the next paragraghs.

As exposed by Downs (1957), political parties in their bid to win an election will tend to have very similar political platforms so as to capture the mass of voters' opinions. Candidates in this case will be close to each other and also close to the "center". Although the author does present a refreshing new insight into the intricacies behind the workings of political strategies in a two party system, he did not go as far as to cover the case of equilibrium when the number of parties vying for votes is greater than two. By extending the Hotelling geographic space to a policy space, Palfrey (1984) shows that introduction of a third party into the political race against the two "established" parties will inevitably lead the entrant to always lose the election (for example, one has only to look at the Canadian Neo-Democratic Party or the Liberal Party of England).

The case of contemporary television programming is yet another example of a form of non-price competition. As quoted in this article from PLAYBOY magazine (Feburary, 1988, p. 34)

> "...each one (new script) is a slightly reworked version of the ODD COUPLE, with teenagers, or THE MARY TYLER MOORE SHOW, only set in a air force radar station, or I LOVE LUCY, only with sisters instead of a husband and wife, or THE COSBY SHOW , only with a black woman and mother raising the family instead of a black husband and wife..." "...Every show will be very close to half a dozen basic templates that have succeeded in the recent past.
>
> Why? Money, the network is really looking to attract a big audience..."

In the last sentence one can notice the centralist or sameness tendancy dictated by the PRINCIPLE OF MINIMUM DIFFERENTIATION. In this case "money" and "big audience" can easily be translated into the more common jargon of profits and large market.

Eaton and Eaton(1988, p.16) use an example whereby a certain airport scheduled 45 commercial flights to land between 8 a.m. and 8:10 a.m.. This was used to demonstrate that competitors will locate at the center of the market for morning air travel. In addition, the spatial is extended this time from a geographic dimension to one that is temporal.

This sub-section examined how the nature of equilibrium is characterized when the number of competing firms is arbitrarily fixed. Although it was shown in the previous sections how the PRINCIPLE OF MINIMUM DIFFERENTIATION is void when equilibrium cannot be defined, once price undercutting is ruled out by virtue of non-price competition, the result is some degree of clustering. Another important assumption in spatial analysis is localized competition.

2.4 LOCALIZED COMPETITION

Another feature of spatial economics that plays an important role in the determination of equilibrium is the implicit assumption that competition is localized. As shall be examined in the following paragraphs, this is directly related to the fact that the Hotelling model incorporates transportation costs.

Competition is said to be localized when the cross-price elasticities of a good and its neighbouring goods, (of which there are at most two on a one dimensional linear market) are non-zero, e.g., every product competes directly with its neighbouring product(s). This is in contrast with Chamberlin's symmetry assumption, where competition between differentiated commodities is generalized. Therefore, the introduction of a new product, will attract an equal number of consumers from all the previous existing firms.

The assumption that competiton is indeed localized can be reinforced by straight forward observation. In fact, the analysis of certain markets is definitly better served if one were to make such an assumption. The ice-cream vendor at the University of Ottawa, for instance, is clearly not in competition with with the ice-cream vendor located at Carleton University. This is clearly because of the significant transportation costs that each ice-cream buying consumer must face when shopping for the product. Thus the local ice-cream vendor is more likely to be in competition with a rival in his direct vicinity.

To remain in the spirit of the spatial model, one also finds examples where localized competiton can be applied to different product lines. The makers of the Volkswagen "Jetta" did not see themselves as competing against the likes of Mercedes-Benz or the Jaguar, but more likely against a specific neighbouring variety, such as the Mazda "626" or the Buick "Century".

So as to insure that competion is localized, one must specify the type of transportation cost function to be used in the analysis. According to Archibald, Eaton, and Lipsey(1986, p. 13)

> "If transportation costs are identical for all consumers and convex in distance, then competiton is clearly local in nature..." "...Suppose, however, that transportation costs are subjective and differ among consumers. Then a low price firm could be in direct competition with a large number of high price firms selling identical products. Its market area would no longer be a connected subset of the entire market..."

This concurs with Economides (1984), who by assuming that the transportation cost function is concave in distance, shows that consumers sometimes make their purchases from the farthest of the two suppliers. This phenomenon can be viewed in figure 7, where the transport cost function is concave in distance. In this case, firm 1 is in direct competion for the consumers to the right of firm 2.

Furthermore, there are consumers with similar preferences who cannot buy same product. This can be seen in figure 7, where consumers w,x,y,z are all characterized as having similar tastes. However, x and y purchase two completely different goods. In other words, the ice-cream vendor from the University of Ottawa can be in direct competition with the ice-cream vendor from the Carleton University. According to Economides, in order to be able to define the demand function for a "single-peaked" utility

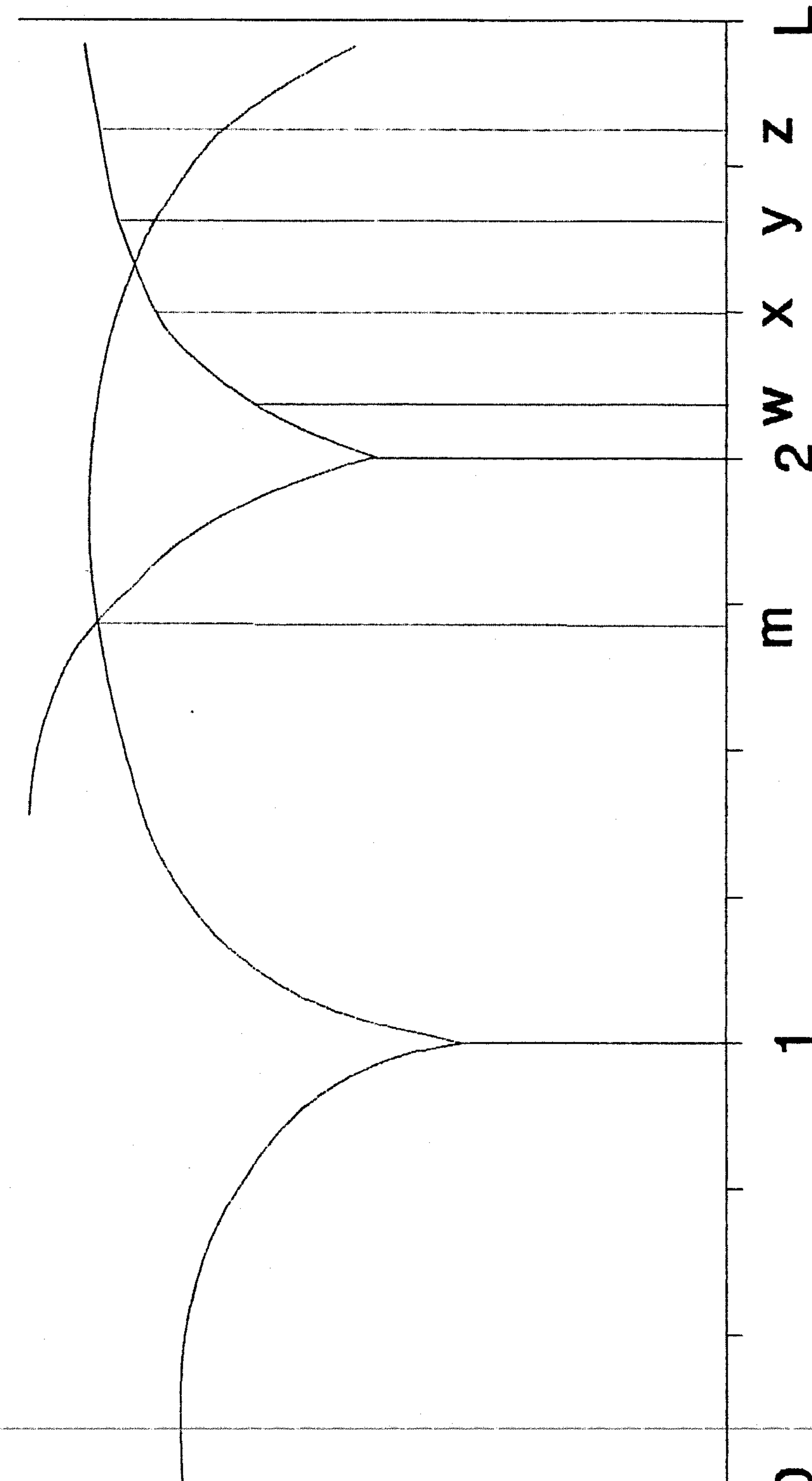

FIGURE 7

function, the set of consumers that prefer to buy the good from firm 1 rather than firm 2 must be connected. If the transportation cost function is concave, then the set of consumers that buy the same differentiated product is disconnected.

Since competition is localized, the price taking behavior that is assumed for the existence of a Cournot-Nash equilibrium is probably too myiopic. The next section examines different types of conjectures that firms make about other's reactions.

2.5 OTHER TYPES OF CONJECTURAL VARIATIONS

All of the above models assumed the Cournot-Nash equilibrium concept which is also found in most spaceless models. This equilibrium is obtained by assuming that economic agents react to one another's actions according to Cournot's zero conjectural variations. Specifically in the spatial economics context, this signifies that a firm chooses a price and location conjecturing that the other firms will keep their price and location fixed. However, for non-zero conjectural variations it is assumed that each firm will adjust its price and location on the belief that the other firm will respond in some way. For example, an exact matching in price and location by a rival will translate into a conjectural variation equal to unity, i.e., the reaction will exactly match the action.

As was exposed in the last section, spatial models are characterized by localized competition. Whereby assumption of price taking behavior is probably too restrictive. This is because in small group competition, rational firms know that their competitors will respond to any initiative that they take [see, for example, Lipsey, Purvis, and Steiner, 1988, p. 276]. According to Gannon (1972), the Cournot hypothesis is very restrictive since it implies that all the firms attribute a persistent, and complacent attitude towards each other. And this is in spite of the fact that they both recognize the rival nature of their interdependence and actual behavior. According to Archibald et al. (1986, p. 23):

> "The argument against the Nash concept is that rational firms in small-group competition know that their competitors will respond to any initiative that they take. Choice of a more plausible concept will, of course, have to be made in the end on empirical grounds."

Certain authors have constructed models in which firms choose strategies that deviate from the mainstream of

Cournot-Nash conjectural variation. Graitson (1980), for one, has found that competitors, by adopting a maximin strategy (i.e., a competitor will quote a price which maximizes its minimum profits for the worst situation taking the rivals price as equal to zero), will locate at the first and third quartilles of the linear market. A minimax strategy is studied by Eaton and Lipsey (1975). Here a firm i chooses a location which minimizes the damage that may be caused by firm j. The resulting equilibrium configuration is similar to the one obtained by Graitson. Needless to say that both results are compatible with the social optimal, i.e., the costs of transportation are minimized.

Carruthers (1981), and Ali and Greenbaum (1977), extend the Hotelling model to the case of n firms facing an inelastic demand function. Although their analysis is in the same spirit than the case studied earlier by the seminal Eaton and Lipsey (1975), the above authors assume that both prices and location are decision variables available to the firms (recall that Eaton and Lipsey assumed parametric prices in their analysis). Recognizing that the Cournot-Nash assumption is probably too passive and restrictive to represent the interdependence between producers, Carruthers, and Ali and Greenbaum endow firms with a certain degree of foresight with regard to each other's price-location decisions. The new necessary conditions for equilibrium that must now be satisfied are:

i) the price charged by an exterior firm must be double its market share and for interior firms, price equals market share;

ii) the exterior firms have a one sided market;

iii) the penult firms (the next to the last) must be paired with a more interior firm.

iv) no firm's whole market is less than one half another firm's market.

A major difference in the case where price is also a decision variable of the firm, is when the equilibrium configuration corresponding to chosen and parametric prices are indeed different. In the case specified above, the exterior firm is not paired, has no interior market area, and benefits from its location by charging a higher price to the outside consumers than was the case when price competition is ruled out.

Gannon (1972) dscribed different types of locational equilibria associated with a variety of conjectural variations in duopoly models. In the first case, it is assumed that the locational response of each duopolist are assumed to be a function of its location. Where the one firm expects the other firm to become less aggressive as it encroaches on the latter firm's market area. This weak locational response on the part of each duopolist has the effect of attracting the firms towards the center of the market. A second case, where each competitor has identical expectations, and each firms expects aggressive behavior from the others, drives the duopolists towards the outside of the quartilles of the market. Gannon shows that this aggressive response even holds true when firms face perfectly inelastic demand curves; no degree of price inelasticity is attractive enough for firms to cluster at the center of the market.

Eaton and Lipsey (1975), assumed that tastes are located on a continuum. This obviously leads to problems when attempting to characterize tastes other than uniform. Eaton and Kierkowski (1984) demonstrate that by having consumer preferences located at discrete points along a line segment it becomes easier to study the effects on firm's location choice in accordance with variations in consumer demand. As an added dimension to their work, the authors borrow the concept of "consistent conjectures" by Breshanan (1981). In this case, each firm incorporates the other firms' responses to its own price. This leads to a pure strategy Cournot-Nash price equilibrium which is independant of the distance seperating the firms. Thus, leading Eaton and Kierkowski to conclude that firms that are selling their products in a large market must charge a lower price in order to avoid entry. On the other hand, a firm serving a smaller market can sell its product at a relatively higher price without attracting entry. This is obvious because the potential entrants cannot benefit from any scale economies. The question of entry will be dealt in the following section.

This section has examined the literature with respect to the nature of equilibrium when an exogenously fixed number of firms compete in a spatial market. The major point of interest from the above review is that Hotelling's famous PRINCIPLE OF MINIMUM DIFFERENTIATION is not as general as once believed. As was said once one begins to relax some of the original assumptions of the model, tend toward maximum differentiation in order to increase their monopoly power and thus increase profits. Although the conclusions of Hotelling have been challenged, refined, and extended in a variety of ways, his basic approach has still been applied with success when analyzing the properties of a free-entry equilibrium, which is the subject of the next section.

SECTION III

FREE-ENTRY EQUILIBRIUM

This section will be devoted to the dynamic process of entry into spatially differentiated markets. As a consequence of the similarities between location and product differentiation, the literature has naturally evolved in the area of monopolistic competition.

The concept of free-entry equilibrium is characterized by two behavioral assumptions:

i) no existing firm perceives that a change in any of the variables under its control will increase profits.

ii) no potential entrant perceives that entry is profitable.

Models dealing with free-entry generally assume that location changes or changes in a product's characteristics are costly propositions. Because of this, several models dealing with free-entry [see, for example, Hay (1976), and Archibald et al. (1986)], firm location is given and therefore the question concerning the PRINCIPLE OF MINIMUM DIFFERENTIATION is not relevant. As opposed to the previous section, the number of firms is determined endogenously. This enables the theory to answer several questions in regard to the resulting market structure (ex post). As discussed further, and this is probably the most important contribution from the theory of spatial competition; a long-run free-entry equilibrium will result in firms earning supra-normal profits as opposed to the true spaceless neoclassical model zero-profit solution, given industries with "constant cost" [see, for example, Fergurson and Gould (1975), pp. 249-50].

3.1 THE SPATIAL MODEL WITHIN THE ARROW-DEBREU GENERAL EQUILIBRIUM MODEL

It was assumed in the first section that firms faced zero production costs and that there was no loss of generality. However, when dealing with the question of entry, it becomes necessary to incorporate into the models certain assumptions regarding the cost-technology dual. The zero cost assumptiom of the previous section was obviously of no theoretical importance in the original Cournot model. In neo-classical general equilibrium models [see, for example, Arrow-Debreu (1954)], the number of firms is fixed, and profits are in general strictly positive or zero according to whether strict or weak convexity is assumed.

In the case of decreasing returns to scale, average costs are increasing, marginal costs will exceed average costs and the firm will make positive profits [see, for example, Varian (1984), p.86]. On the other hand, by assuming free-entry in this circumstance, the smaller the scale of production the lower the average costs, and the size of the firm will tend to zero [see, for example, Eaton and Lipsey (1977,1978)].

Conversely, if transport costs are strictly positive and the production technology exhibits constant returns to scale, free-entry would imply that production would take place at the point of consumption in order to eliminate transportation costs. Thus, the minimization of transport costs requires that production be spread evenly over space and no economic unit reconizable as a firm would emerge [see Koopmans (1957)].

3.2 CAPITAL AND INDIVISIBILITIES

In order to overcome this difficulty and explain the existence of the firm, it is necessary to introduce into the model increasing returns. The conventional models are ill-equiped to deal with such a contingency. In addition, real world observations and empirical studies seem to agree with this assertion. These increasing returns are due in large part to indivisibilities such as the fixed locations of the plants. The non-convexity of the production technology associated with these increasing returns to scale does not pose any real difficulties in the spatial model; as a matter of fact, the non-convexities are related to production using capital and are the <u>sine qua non</u> behind the geographical concentration of production plants.

Furthermore, indivisibilities are the reason behind the existence of the degree of monopoly power which each firm posseses in the spatial market. According to Kaldor (1935), the existence of economies of scale in production, precludes a potential producer from enterring a field profitably with less than a certain level of production. Additional production may reduce demand, either for it or/and its neigbours to the extent that the demand curves will lie below the cost curves resulting in losses for all involved.

Despite Kaldor's intuitively appealling argument, the great majority of authors, in their analysis of spatially extended markets, have either rejected or ignored it, and continue on imposing zero-profits as a condition and even a property of free-entry equilibrium. [see, for example, Beckman (1971,1974), Chamberlin (1933), Denike and Pan (1970), M.Greenhut (1952), M.L.Greenhut (1975), Greenhut and Otha (1975), Losche (1954), Mills and

and Lav (1964), Southey (1974), and Telser (1969)].

In Mills and Lav (1964), it is assumed that firms will continue to enter the industry as long as there exists some pattern of market areas (two dimensional market) that will permit more firms per square miles, with all firms making at least zero profits. The authors assume in their model that moving plants, or changing corresponding characteristics of a good, is a costless procedure; equilibrium in this case is obtained by a <u>tâtonnement</u> process for which no specific capital is commited to the market until equilibrium is reached. Archibald, Eaton, and Lipsey (1986) stipulate that if firms really could relocate cost-free they would move continuously, so to avoid the transport costs the consumers face. This way firms produce at each point in space.

Eaton (1976) shows an example of the relationship between the amount of initial capital commited by a firm and the rate of return; as the amount of capital is incremented, the rate of profit steadily declines until all possibility of profit is eliminated. He also shows that the size or lenght of the market will determine to what degree a given amount of capital will allow entry of a given number of firms. Eaton and Lipsey (1978) claim that the rate of return consistent with free-entry equilibrium can be as much as twice the normal rate of return on capital.

3.3 SPATIAL COMPETITION AND NATURAL MONOPOLY

This approach to monopolistic competition can also have some bearings on the theory of natural monopoly. If a firm strategically commits a quantity of capital to a market which is sufficiently important to produce a negative flow of profits to a new entrant, then he cannot hope to enter this field profitably. In addition, a natural monopoly must also be sustainable. According to Sharkey (1982), a natural monopoly is said to be sustainable when a single firm can satisfy all of a single market demand and revenues cover total cost of production. Notice that a sufficient condition for sustainability is that average costs of production must fall as output expands. As seen above, the spatial model is well suited for studying such a situation.

Eaton and Lipsey (1980) distinguish between a TYPE-A and a TYPE-B natural monopolies. A TYPE-A natural monopoly is as described above, characterized by profit maximizing behavior involving the commitment of enough capital which produces losses for an entrant. Whilst in the latter case a natural monopoly is said to be of the temporal type when the cost minimizing decisions which are made with respect to durability, replacement, and maintenance of capital imply that there is a point in time when entry can be profitable. The authors make clear that it is not necessarily indivisibilities or decreasing costs <u>per se</u> which create an entry barrier, but an intertemporal commitment of specific-capital to a market which create the entry barrier. In order for a natural monopoly to be insulated it must simultaneously be of TYPE-A and TYPE-B. If it only has the former, it is possible to use specific-capital to create a TYPE-B natural monopoly, because specific-capital is a sign of commitment which, and as is well known, inhibits entry [see, for example, Caves and Porter (1977), and Dixit (1980)]. Archibald, Eaton, and Lipsey (1986) add that capital in spatially differentiated markets is firm-specific[17] as well as product-specific[18], and that set-up costs, relocation costs and the cost of training specialized labor contribute to this specificity. Product-specific inputs will ensure increasing returns to scale in the production of goods. As was seen earlier, these increasing returns do not pose any problems for the spatial model and they even justify the existence of plants.

Having examined how the cost-technology dual plays an important role in explaining the spatial existence of the firm and the resulting equilibrium, we now turn our attention to certain spatial models which explicitly deal with entry.

Hay (1976), and Prescott and Visscher (1977) assume that entry is a process taking place over time and in sequence. They analyse the case where single plant firms using parametric prices will follow an entry preventing strategy and locate themselves so as to leave an insufficient profitable gap in the market. A free-entry equilibrium is realized when firms are equally spaced along a market of infinite lenght so that the relevant market share will prevent a new entrant to cover its specific-capital costs. In this case, firms make positive pure profits.

Lane (1980) extends the analysis to endogenous prices for characteristic models and finds that the free-entry equilibrium is unique when the firm makes the largest possible profits without attracting entry. Archibald and Rosenbluth (1975) have demonstrated that the possibility of supernormal profits is consistent with free-entry equilibrium.

3.4 ONE EXAMPLE OF ENTRY

To get a better feel for the issue at hand we borrow an example from Schmalensee (1978).

Letting $C = F+cq$ be the cost function, where C is the total cost made up of a fixed (F) and variable (cq) component; and $q_i(p,N) = f(p)g(N)$ be the demand function, where N is the number of firms (or brands) and p is the parametric price charged by the firms. With fixed locations, the N established brands are a distance 1/N apart on a line of unit lenght. Profits are to be expressed in terms of prices and distance served.

$$\pi(p,N)=(p-c)F(p)g(N)-F \; ; \; p>c \qquad (12$$

Now by assuming that a certain value N^* is the solution for $\pi(p,N^*)=0$, then if $N^*>N$, all established firms are profitable.

If entry is allowed and the potential entrant charges the same parametric price, he is forced to locate between two established firms. He then can expect to sell to half the customers between him and his neighbours, since the locations are fixed, the market lenght is $(1/4N) + (1/4N) = 1/2N$. The profits will be given by $\pi(p,2N)$. This value will be positive only if $N^*>2N$, so entry occurs only if $N^*/2>N$.

Hence, established firms will earn positive pure profits without attracting entry as long as $(N^*/2) < N < N^*$.

By relaxing the restrictive assumption of parametric prices, Eaton and Lipsey (1977,1978,1979) have concluded that profits are also positive in free-entry equilibrium. However, the equilibrium configuration, unlike the example above, is not necessarily unique. The lenght of the market a competitor can hope to capture will depend on his own price as well as the price charged by the two neighbours. As a matter of fact, by letting each firm set its own price the spacing of the firms will not be symmetrical but will vary in accordance with their own prices.

3.5 PRODUCT PROLIFERATION

According to Hay (1976), and Peles (1974), firms in a differentiated industry do not respond to a threat of entry by lowering their prices, but rather seek to proliferate products into the market in order to fill up sections or parts of the quality space where there is sufficient consumer demand to attract new entry. This type of strategy is deemed more profitable for the concerned firms. An important paper by Schmalensee (1978) dealing with "THE READY-TO-EAT BREAKFAST CEREAL" industry confirms the above predictions of the theory. He shows that firms always have an incentive to pre-empt entry by way of brand proliferation and in a way this action tends to inhibit true competitive behavior. The same result was established by Shaw (1982) in a study of the UK fertilizer industry. More recent examples can be found in EPSON'S expansion from four to seven models of computer printers under the banner "A PRINTER FOR ANY APPLICATION"; in PC WORLD (March 1988, p. 64) magazine where we find this quote a reference to IBM corporation states that "seperate divisions were formed to target the markets for mainframe systems".

In a recent paper Bonanno (1987), claims that the analysis of markets where a certain degree of brand proliferation is known to exist, the use of parametric prices can alter the conclusions. By studying the yet unexplored strategy of product specification, where firms try to locate at certain points along the market and charge prices that would make entry unprofitable for the potential competitor, Bonanno shows that such an action can be more attractive for the incumbent than brand proliferation. This is proved by way of a three staged game, where in stage 1 the incumbent decides on how many brands to offer and how they should be characterized. In stage 2 the potential entrant (having observed the previous action) decides to enter or not, and if he decides to enter, where to locate. In stage 3, the incumbent and entrant compete in price. The notion behind all of this is that prices can be charged freely and at will, while entry into an industry requires the construction of one or two plants.

Schmalensee's (1982) view that pioneering brands hold an advantage when it comes to market share had been formulated in the spatial competition model by Rothschild (1979). Firms, by having perfect foresight in regard to the number of anticipated entrants, will locate themselves assymmetrically along the market in order to capture the largest share of that market, at least for an initial period.

3.6 CRITIC OF GENERAL EQUILIBRIUM MODELS OF PRODUCT DIFFERENTIATION

As was discussed previously, free-entry equilibrium can be characterized by the existence and even the persistence of positive pure profits. However, models by Hart (1979), drawing from general equilibrium analysis, obtains an equilibrium in a product differentiated market similar to the competitive one where price approaches marginal cost

cost when the number of firms is increased. This we believe is an attempt to answer a fundamental question raised by Gabszewics and Thisse(1980): if by analogy with the homogenous case, the increase in the number of substitutes in an industry induces pure competition? In non-adress or spaceless models such as the ones by Jones (1982), and Mas-Collel (1975), the answer is usually yes. This can be explained by the fact that those authors borrow from Chamberlin's monopolistic competition approach where products are symmetric or equally differentiated and competition is generalized (as opposed to localized), in the sense that every product competes with every other in the product group. Here, in the long-run, equilibrium requires that profits reduce to zero. Conversely, in spatial models competition is restricted by some degree of localisation, so that firms are in competition with at most two other firms and therefore zero profits are not a condition of equilibrium.

In addition, two other factors influence the conclusions of Hart's model. In the first instance, the author expands the market by the technics of replication a la Debreu-Scarf (1963). As Archibald and al. (1986) point out, this does not clone the representive consumer, and does not necessarily reduce the degree of localization or ensure that pure profits are zero in equilibrium. In the second instance, the authors make the strong assumption that every firm's output is bounded. This guarantees that if output expands as fast as demand, the number of firms must get large, so that competitive prices must result.

This section has investigated the literature in regard to free-entry equilibrium. The major consensus emerging with regard to entry in the spatial competition model was that, contrary the neo-classical model of competition, zero profits does not have to be a property of equilibrium [see, also Capozza and Van Order (1980)].

Each firm by providing a differentiated product has a certain monopoly power which is guaranteed in large part by the fact that competition is localized. This localized competition is in turn due to capital indivisibities which create the necessary ~~decreasing~~ increasing returns to scale that explain the existence of firms in spatial markets. By increasing the product-specific capital the monopoly power of a firm is increased up to the point where entry becomes unprofitable for a potential competitor so that only that firm can offer the good. The size of the sunk cost or irrecoverable cost in relation to the size of the market justifies the existence of a natural monopoly.

SECTION IV

WELFARE

This section deals with welfare and spatial economics. Since the seminal work of Chamberlin (1933), [see also, Chamberlin (1950), Arrow (1971), Houthakker (1952), and Fisher, Grilliches, and Kaysen (1962)] economists have dwelled upon the question of product variety and welfare. It is well known that if constant or decreasing returns to scale exist, it is socially efficient (or optimal) to produce up to the point where outlay price equals marginal cost. However, when a firm is faced with increasing returns to scale or more general non-convex technology, marginal cost pricing will induce losses. Then, as in most of spatial models, high non-efficient prices will be charged.

4.1 SPATIAL MODELS AND SOCIAL EFFICIENCY

A consequence of monopolistic competition is that prices are indeed higher than the social optimum. This is related to the well known EXCESS CAPACITY THEOREM. The earlier belief was that this result suggested that differentated product industries were inefficient because of the higher of cost of variety. However, recent studies have shown that this is not necessarily true [see Eaton, Purvis, and Steiner (1988, p. 259)]. Consumers do not only take price as the only exogenous variable on which to base their purchases. They also consider the brands offered by the market. Monopolistically competitive markets offer a broader range of commodities to satisfy individual consumer's tastes.

Variety becomes wastefull only to the extent that the extra cost associated with variety exceed the benefits. The number of differentiated products should be increased until the gain in consumer surplus from adding one additional commodity is equal to the loss resulting from the higher cost of producing a smaller quantity of each commodity.

Modelling such welfare losses or gains by varying the number of products in a market have been attempted by several researchers. Many approaches have been used. The most notable ones have been Lancaster's (1975) product characteristics approach, Stiglitz's (1975) mean-variance portfolio selection model, and Dixit and Stiglitz's (1977) "direct route", i.e., the consumers' utility fuction embodies the desirability of variety. In addition, several economists have applied the spatial framework to the question of welfare. Their results are consistent with those mentioned above but in a fashion which is more intuitively appealing. [see, for example, Lancaster (1979), Meade (1974), Sherer(1979), Stern (1972), Rothschild (1976)].

Stern (1972, p.156) stated the problematic:

> " it is not to difficult to imagine how one would go about dealing with the question at hand in the contextof spatial analysis. In this case, products are differentiated by their location; variety is increased by increasing the number of points of productionand reducing each producers market area;the degree of substitubility is increased by decreasing the cost of transportation from point to point."

Hotelling (1929) noticed that his model could be used to analyze relations between variety and welfare. As we saw in the first section, equilibrium in this case, was characterized by two firms clustered at the center of the linear market. However, this equilibrium was not compatible with the social optimal. According to Hotelling, welfare would be maximized when firms located themselves at the quartilles of the linear market, so that, total transportation costs were minimized.

In this case, if firms were to move away from the center of the market, would this be Pareto Optimal? According to Rothschild (1976), a move by any producer will result in a Pareto superior state. This is because even if some buyers are found to be better off, others located at the center of the market, will be worse off. To counter this problem the author uses a criteria whereby a certain Social Welfare Function ($W = U(s) f(s)$), takes higher values the more equal the distribution of incomes. This is done by applying the Atkinson (1970)[19] condition on the net income ($f(s)$) (individual income minus transport costs). When transport costs are minimized the distribution is socially preferred.

Rothschild also shows that when the demand is perfectly inelastic and free-entry is permitted, the sellers' equilibrium will not coincide with the socially optimal outcome. However, in a later paper, by relaxing the restrictive assumption of inelastic demand, Rothschild (1979) demonsrates that consumer surplus is maximized when the number of competing firms is a critical number ($N = 2^n-1$). In this case, firms locate equidistantly from one another, thus minimizing transport costs.

4.2 AN ILLUSTATIVE EXAMPLE

Sherer (1979) illustrates well the issue at hand. His model is an extension of Schmalensee's (1978) work concerning brand proliferation " THE READY-TO-EAT BREAKFAST CEREAL ". In his analysis, Sherer goes one step further by specifically studying the welfare effects of such action.

Sherer's model is a hybrid of Hotelling's model and of Spence's area-under-the-demand function model [see Spence (1976, a, b)]. In the former, the horizontal axis represents a one-dimensional space over which a producer's characteristics might vary. The vertical axis, which is Spence's contribution, measures the integral under the inverse demand function for all consumers whose preferences match that point. Thus by examining figure 8 we notice two tent like functions.

The upper function shows the surplus derived for a consumers whose preferences are defined at any given point. As one can see in figure 8, consumers located at point A and point B have the highest consumer surplus. Those located away from any of these two points, let say at boundary AB, must incur higher transportation costs, and thus a loss in consumer surplus. The downward sloping

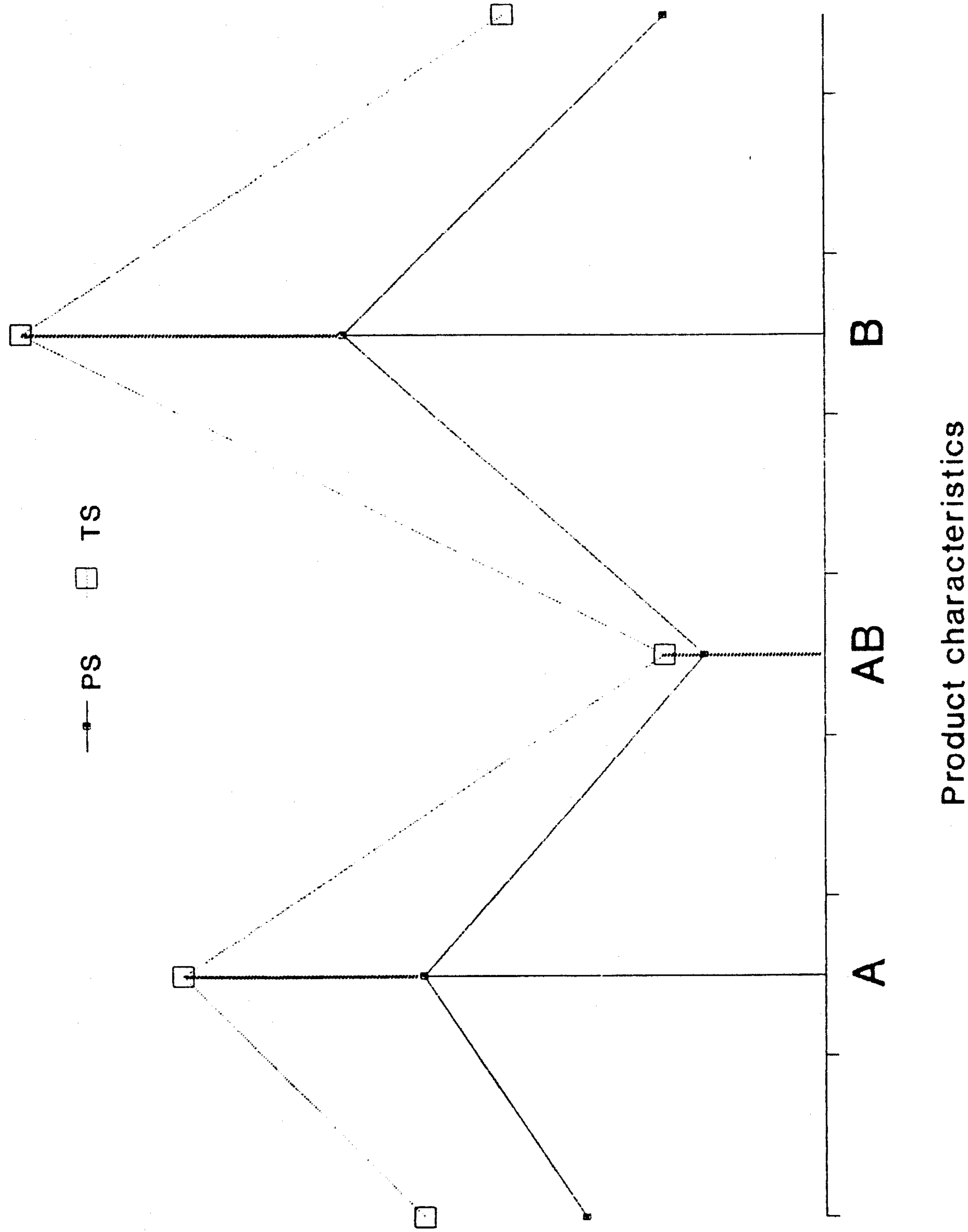

FIGURE 8

demand function faced by the producer translates into the tent-like-shape of both the producer and consumer surplus', since consumers who move away from the point of production must pay a higher price.

Sherer shows when differentiated products supplied under monopolistic competition are optimal. Here, a new product should be introduced if the increase in total surplus, following entry into the industry, exceeds the fixed costs of the introduction. This can be illusrated by an example. Looking at <u>figure 9</u>, one can notice a new firm offering a product C. Under Cournot's conjectural variation, its gain will be the area defined by $T_a+P_a+P_b+T_b+K_a+K_b$. If launching costs are smaller than this sum, production should go ahead. Social welfare will be improsed if the fixed costs of introduction are smaller than the sum of the pure producer surplus increase P_a+P_b and the consumer surplus gain S_a+S_b.

<u>Figure 10</u> shows another situation where the the slopes of the surplus funtions are much gentler, meaning a higher degree of substitubility between products. In this case, it is entirely possible that excessive product proliferation will occur. In the more hypothetical case where the surplus

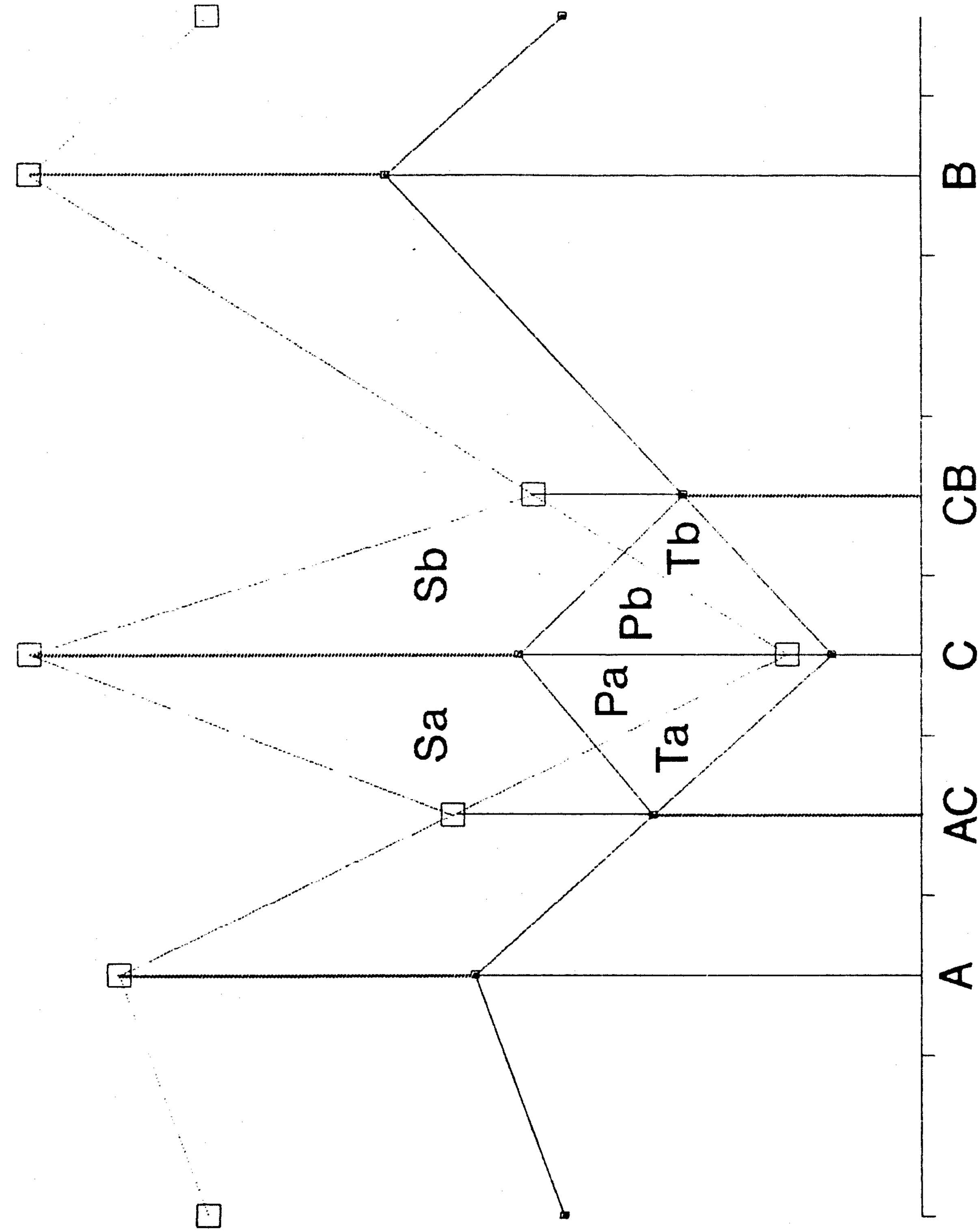

FIGURE 9

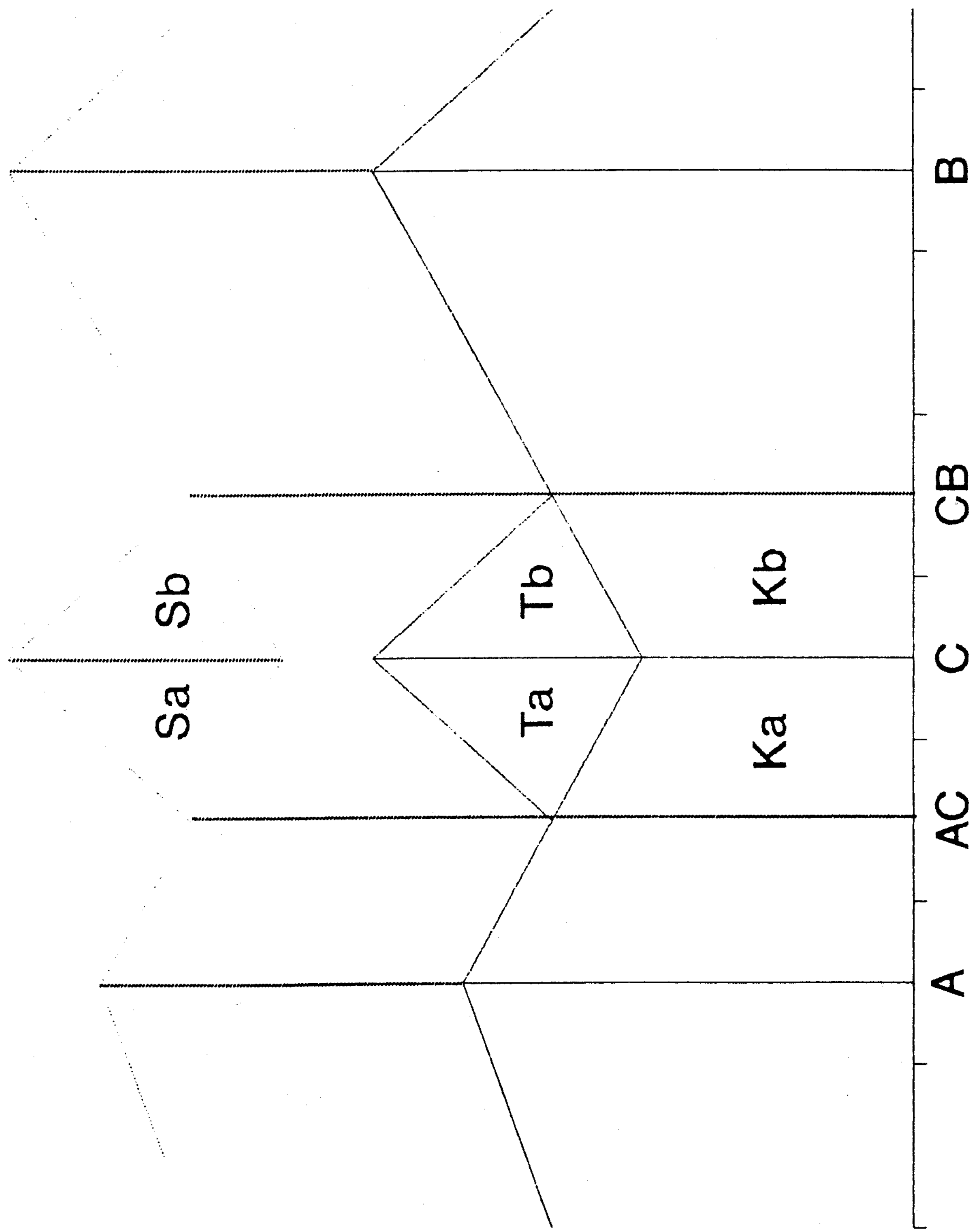

FIGURE 10

functions are horizontal lines (goods are perfect substitutes). The introduction of a new product does not expand demand. There is nothing but "cannibalization" by the new entrant and thus excessive product proliferation.

Sherer's results agree with those of Meade (1974), Stern (1972), Spence (1976a), and Lancaster (1975). Meade found that the optimal variety was influenced by two factors. Firstly, the desirability consumers seek when products are not "very good" substitutes for one another. As Spence (1976a) noted, if goods are "not very" good substitutes the cross-price elasticities are low, so that the welfare is augmented when variety is increased. Secondly, variety was costlier the greater the economies of scale. This view was also formulated by Lancaster (1975). He stated that there is a degree of increasing returns to scale, sufficiently large, for which the socially optimal number of goods is one. Stern (1972) found that a high level of fixed costs may create market areas which are larger than the optimal. Salop (1979a) constructs a spatial model whereby monopolistic competition can yield either too much or too little variety. By relaxing the assumption that the transport cost function is linear, he finds that a concave function in distance results in the market offering an abundance of variety, whilst a convex function yields too little variety.

As Meade (1974) pointed out, the interaction between those two forces, the need for variety on one hand and the cost advantage from limiting the number of products on the other, pull in the opposite direction. Spence (1976a) supports this view and finds that high fixed costs tend to reduce product variety and low cross elasticities tend to augment it. The net effect will at the end have to be determined on empirical grounds.

5. SUMMARY AND CONCLUDING REMARKS

This section underlines some of the questions raised by the literature concerning the theory of spatial competition and gives some direction which may be the basis of future research.

Hotelling's important result, THE PRINCIPLE OF MINIMUM DIFFERENTIATION is not as general as believed. As a matter of fact, when some of the assumptions of the model are relaxed, firms tend instead towards some form of MAXIMUM DIFFERENTIATION. A downward sloping demand function and/or mill-price undercutting would result in firms moving away form one another. To overcome this problem a number of solution criteria were examined such as mixed strategies.

The introduction of uncertainty opens up new areas of research. Stahl and Varaiya (1979) show that clustering can still occur if firms face imperfect information about the spatial distribution of demand [see, Stahl and Varaiya (1979)]. On the consumer side, Stuart (1979) shows that economic agents minimize the cost associated with search. Consequently, firms tend to agglomarate. According to de Palma, Ginsburgh, Papageorgiou, and Thisse (1985), minimum differentiation is obtained when variations in consumer tastes are sufficiently large, and consumers' choice is stochastic. In fact, Lewis (1945), notices that the retail trade has a certain tendancy to cluster. Von Ungern-Sternberg (1988), points out that the tendency of firms to offer similar products is exemplified by the introduction of general purpose products such as multi-optional computers by IBM. In this case, "transport cost" is a decision instead of an exogenous variable.

The consensus in the literature is that minimum differentiation is very sensitive to the charateristics of a particular economic environment. The right model is the one which fits the given set of circumstances.

When Hotelling's model is extended to the dynamics of entry, the results become quite interesting. As opposed to the conventional model, the spatial model makes the more realistic assumption of capital indivisibilities and thus non-convex technologies. Casual empiricism is consistent with this insight. In this case, firms can exhibit long-run positive pure-profits without attracting entry. The reason for this, is that competition is localized. A simple conceptual excercise illustrates this. Imagine lowering the price of any product. If this price reduction decreases the quantities demanded of all other products, then competition is generalized and models such as Hart's apply and prices can tend towards average cost. On the other hand, if the price cut reduces the quantity demanded of a small subset of those products, then competition is localized. The spatial competition model is better adapted to deal with those latter real world cases.

Free-entry equilibrium can lead to the existence of natural monopolies. Although it is simple to understand why natural monopolies are public utilities (an efficient scale of production that is large relative to the market demand). It is not so easy to explain why a small town should only have one movie theater. Spatial competition theory supplies a simple answer to such questions.

Spatial competion theory contributes in an important way to the understanding of some non-standard market structures. Given a certain cost function, it is possible to define the number of competing firms in equilibrium. By assuming constant or increasing average costs there is no reason why a firm should exist in economic space. As the product-specific costs are increased the number of potential competing firms that can operate without attracting entry is decreased up to the point where monopoly prevails.

Another important fact is that by relaxing the assumption of the single-firm single-plant case, firms can increase profits by proliferating the market with other brands. By positioning select goods in the market a firm can avoid the threat of entry in that unserved part of the market and so secure higher returns.

Spatial competition models are of theoretical importance to study optimal product ranges. Although Archibald et al. (1986) stipulate that partial-equilibrium analysis of single markets poses second-best problems, indeed models of overlapping oligopolies such as the spatial model, prices do not equal marginal costs. Nevertheless, Lancaster (1975, 1979) shows that in spatial models the combination of scale effects and diversity of tastes assumed for distributional considerations makes of spatial models a questionable approach for measuring the optimal product range.

New research into the field of spatial competition analysis can be productive in many areas. For instance, most models assume, implicitly or explicitly, that competition is localized. There is no real reason why firms should be in competition with at most two firms. A spatial model could probably be developed incorporating a certain extended cross-over effect that would carry the competition process over to more than two firms.

According to Archibald et al. (1986), one of the most rewarding areas of reasearch would probably be some empirical evaluation of the endogenous market structure which arises in free-entry equilibrium. This would necessitate an elaborate study of the dynamic process of entry and entry preventing strategies which are related to the nature of the firm-specific, and group-specific capital.

NOTES

1-For a complete bibliography of the works of Harold Hotelling, one can consult Ghurye, Hoeffding, Madow, Mann, Henny, and Oklin, eds. in Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling (1960).

2-For a more formal and complete analysis of the case of quantity versus price as the decision variable, see, Klemper and Meyer (1986), Okuguchi (1987), Vives (1985).

3-See, for example, Davis, Hinnich, and Odershook (1970), Downs (1957), Krammer (1977).

4- See, for example, Losch (1954).

5- See, Friedman (1983).

6-A slightly more generalized set of assumptions can be found in Graitson (1982).

7-In regard to this particular, Davletogloo (1965) introduces what he terms a "doubtfull area" into the spatial model. This area represents consumer uncertainty in regard to the delivered prices. This has the effect of creating a distinct area from which consumers are indifferent between patronizing one seller over another. This makes the degree of uncertainty an inverse function of the distance seperating both firms. Hence, suppliers will tend to move away from each other.

8-This particular example of the model can be found in Vickrey (1964). While the notations can be found in Hotelling (1929), and Friedman (1983).

9- Cournot-Nash equilibrium is a set of actions such that no player taking his opponent's actions as given, wishes to change his own action.

Let a_i and a_{-i} the actions chosen by the i^{th} player, and by all the other players, respectively, and $\Pi^i(a_i, a_{-i})$ the utility of the payoff corresponding to the i^{th} player's action given the actions of all the other players; a Cournot-Nash equilibrium will then satisfy $\Pi^i(a_i^*, a_{-i}^*) \geq \Pi^i(a_i, a_{-i})$ where a_i^* and a_{-i}^* indicates utility maximizing choices.

10-This particular example of the model can be found in Vickrey (1964). While the notations can be found in Hotelling (1929), and Friedman (1983).

11-Algebraically, "zero conjectural variation" is defined in the following way

Let $Q = Q(Q1, Q2(Q1))$
"zero conjectural variation" is defined by
$dQ2/dQ1 = 0$ (see, Friedman (1983))
or
$dQ/dQ1 = 1$ (see, Varian (1984))
However, both definitions are equivalent:
$dQ/dQ2 = \partial Q/\partial Q1 \; dQ1 + \partial Q/\partial Q2 \; dQ2/dQ1 \; dQ1$
but $\partial Q/\partial Q1 = 1$ so that $dQ/dQ1 = 1 + \partial Q/\partial Q2 \; dQ2/dQ1$
if $dQ2/dQ1 = 0$ then $dQ/dQ1 = 1$

12- See Hotelling (1929, pp.47-8).

13-The authors do not go on to prove their findings, although proof can be found in Bonanno (1985a, b).

14-An English summary of Palander's work can be found in Isard(1956).

15-According to Phlips and Thisse (1982) , differentiation is said to be horizontal when the commodity space comprises products that are in the following manner: between two products, the level of some characteristics is augmented while it is lowered for some others, as in the case of different versions (called varieties) of a car. Differentiation is called vertical when the commodity refers to products that are differentiated in time or quality in the following manners: between two products the level of all characteristics is augmented or lowered (called quality), as in the case of cars of a different series.

16-Vickrey (1964) states that a change in certain characteristics can imply negligeable amounts of capital; in this case relocation costs tend to zero and the analysis becomes similar to the one in the previous section. However, Eaton and Lipsey (1978) find that the assumption of fixed locations is appropriate when half the firm's fixed costs are "location specific". Schmalensee (1978) argues that repositionnig costs can be substantial in a product development context.

17-Product-specific capital goods are ones that are useful in the production of a particular good and which have no alternative uses.

18-Firm-specific capital goods are simply product-specific capital goods for which there exist no other use that will increase the value of the firm.

19-Atkinson has shown that a distribution f(s) will be preferred to another distribution f*(s), when the given social welfare function is superior for more equal income distributions.

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