Analyzing Business Cycles in G-7 Countries Using Logistic Smooth Transition Autoregressive (LSTAR) Models

by

Nodir Abduraimov

Major paper presented to the Department of Economics of the University of Ottawa in partial fulfillment of the requirements of the M. A. Degree

Supervisor
Professor Gabriel Rodríguez
ECO 7997

Ottawa, Canada
May 2002

Acknowledgments

I would like to express my thanks to Professor Gabriel Rodríguez for introducing me the topic of nonlinearity in economics and for his supervision. This research was possible with his help and support. I am also grateful to Professor Marcel Mérette for his support as an advisor during my MA program. All errors are mine.

Abstract

This paper estimates Logistic Smooth Transition Autoregressive (LSTAR) models to analyze nonlinearities and asymmetries in business cycles using quarterly data of real GDP for the G-7 countries. The modelling cycle for nonlinear models proposed by Granger and Teräsvirta (1993) and Teräsvirta (1994) is used to model the growth rates of real GDP. Null hypothesis of linearity $AR(p)$ model is strongly rejected for all seven countries. Thereafter, we estimate a LSTAR model for each country. Diagnostic tests for residual autocorrelation, parameter constancy and remaining nonlinearity are applied and almost all models passed these tests successfully.. Estimated transition function and estimates parameters show particular dynamics in the business cycles for each country. Furthermore, evolution of the estimated transition function over time allow us to identify principal peaks and troughs in G-7 economies.

Contents

1 Introduction

Using the definition of Burns and Mitchell (1946), "business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises; a cycle consists of expansions occurring at about the same time in many economic activities followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle. (p.3)". Hence, this definition possesses two basic features. The first is the comovement among individual economic variables, taking into account possible leads and lags in timing, including the historical concordance of hundred of series such as commodity output, income, prices, interest rates, banking transactions, and transportation services. The second feature is that how business cycles are divided into separate phases or regimes. The authors treat expansions separately from contractions. For example, certain series are classified as leading or lagging indicators of the cycle, depending on the general state of business conditions.

From a related perspective, it is agreed that business cycles have two important features such as nonlinearities and asymmetries. Asymmetry implies that contractions in business cycles are on average shorter and steeper than expansions (see, for example, Zarnowitz, 1992). One of the first econometric models in measuring business cycles was Tinbergen's model (1939) who used linear difference equations as instrument of analysis. This empirical work was generally focusing on the time-series properties of just one or a few macroeconomic indicators and the structure was linear without possibility to separate between recessions and expansions. In other hand, Lucas (1976) proposed the use of dynamic models to analyze movements in the economy using autocorrelation and spectral densities functions as the two more important econometric tools.

However, there are some agreement to accept the importance of nonlinearities in the economy. For example, according to Zarnowitz (1992) nonlinear models can explain endogenously the existence and amplitude of cycles. More recently, a lot of researches have considered theoretical and empirical aspects of modeling nonlinearities and asymmetries. These type of new models allow to calculate duration of recessions and expansions, for example. Interesting contributions came from Neftci (1984) and Stock and Watson (1991) using a probabilistic and a dynamic factor models, respectively. Other type of models assumed that the transition between regimes is caused by exogenous but not observable (or unknown events) variables. This is the case of Markov Switching (MS) model, originally proposed by Hamilton (1989). He used an AR(4) model for the growth rate of US output allowing for a changing mean and a constant variance. The results allow to identify phases of recession very close to the dates identified by the NBER using a large set of leading indicators. This model has been extended in many forms allowing for changing variance, varying transition probabilities, multivariate analysis, among others. For other applications using MS models to analyze output growth, see Godwin (1993), Bodman and Crosby (2000) and the reference there mentioned.

A drawback in the MS approach and other nonlinear approaches is the step where testing for linearity is done. Because there is a problem of nuisance parameter not identified under the null hypothesis, standard limiting distributions are not available and analytical expressions are also not available in most of cases. However, it is possible to test the null hypothesis of linearity against an alternative hypothesis of Smooth Transition Autoregressive (STAR) models. In the more simple version, a STAR model allows to change from one regime to another regime according to a transition variable. Hence, the transition mechanism is endogenous and in many cases, the transition variable is the dependent variable lagged d times, where this delay is selected from the linearity tests for different values of d.

There are two types of STAR models according to the transition function used in the specification. The first STAR model include a logistic function rising the so called LSTAR model which allows to analyze for asymmetries. The other possible selection is to use an exponential function rising the so called ESTAR model. Although there is a testing procedure to decide which of both transition functions are more convenient, we will consider the LSTAR option because we are interested to analyze for asymmetries.

In the empirical section we use quarterly data for real Gross Domestic Product (GDP) for seven industrialized countries. The data covers different periods according to the country. We follow the modeling cycle proposed by Teräsvirta (1994) to estimate and to evaluate STAR models. For all countries, we strongly reject the null hypothesis of linearity and then we estimate LSTAR models. The results show the different dynamics of adjustment of each country and identification of the principal periods of peaks and troughs for each of them.

This paper is organized as follows. Section 2 presents a selected survey of nonlinear models. We concentrate the discussion on the MS and STAR models. Discussion about estimation and evaluation of STAR models is presented in section 3. Section 4, presents a brief survey of the principal empirical work realized using MS and STAR models applied to the business cycle issue. Section 5 presents the empirical analysis using G-7 data. Section 6 concludes and the appendix contains tables and figures.

2 Nonlinear Models: A Selected Survey

One possible way to classify nonlinear models is to divide them in two broad groups: parametric and nonparametric models. In this paper, we are interested in some particular type of model in the first category. In general, parametric models include specific mathematical functions of dependent and independent variables which are introduced in the model as components adding also cross products of these explanatory variables. In this class of models we find, for example, nonlinear autoregressive models, nonlinear models with a transfer function, bilinear models, nonlinear moving averages models and double stochastic models (see Teräsvirta *et al,* 1994).

In this section, we present a selected survey of nonlinear models with emphasis in methodological issues. For a more extensive survey, see Granger and Teräsvirta (1993), Teräsvirta, Tjostheim and Granger (1994) and van Dijk, Teräsvirta and Franses (2001). In the section below, we mainly focus on presentation of Markov Switching (MS) models and Smooth Transition Autoregressive (STAR) models. At the end, we briefly mention some other class of nonlinear models. Most of nonlinear models assume that regime switches are caused by certain past values of the same dependent variable itself (y_{t-d}, say). Hence, regime changes are assumed to be endogenous and it is the approach taken by STAR models. But we may assume that such shifts are exogenous and perhaps caused by external effects, as wars or oil crisis. It is the case of MS models where researcher assumes that such shifts are exogenous but at the same time the model does not assume a priori knowledge of the location of the shifts.

2.1 Markov Switching (MS) Models

Let y_t be our dependent variable in the following discussion. Using notation from Kim and Nelson (2000), in general, an autoregressive model of order p with first-order, M-state MS mean and variance may be written as[1]

$$\phi(L)(y_t - \mu_{s_t}) = e_t, \tag{1}$$

where $e_t \sim N(0, \sigma^2_{s_t})$. The model is complete specifying that $Pr[s_t = j | s_{t-1} = i] = p_{ij}$ for $i, j = 1, 2, ..., M$; $\sum_{j=1}^{M} p_{ij} = 1$ and

$$\begin{aligned} \mu_{s_t} &= \mu_1 s_{1t} + \mu_2 s_{2t} + ... + \mu_M s_{Mt}, & (2) \\ \sigma^2_{s_t} &= \sigma^2_1 s_{1t} + \sigma^2_2 s_{2t} + ... + \sigma^2_M s_{Mt}, & (3) \end{aligned}$$

[1]In the seminal paper of Hamilton (1989) a constant variance and a fourth-order autoregressive process was considered in analyzing the growth rate of US output. See section 4 for more examples of empirical aplications using MS models.

where $s_{mt} = 1$, if $s_t = m$, and $s_{mt} = 0$, otherwise. In the above model, if s_t, for $t = 1, 2, ..., T$, were known a priori, this variable could be considered as a dummy variable and the construction of the likelihood function would be straightforward. Unfortunately, s_t is an unobserved variable. Let ψ_{t-1} be the set of all past information up to time $t-1$. What we need is to write the density of y_t given past information and to do that we need s_t and s_{t-1}, which are unobserved. To solve this problem, instead of considering a joint density of y_t and s_t, we consider the joint density of y_t, s_t and s_{t-1}. Basically, this can be done in two steps as shown in Kim and Nelson (2000). In the first step, we derive the joint density of y_t, s_t and s_{t-1}, conditional on ψ_{t-1}:

$$f(y_t, s_t, s_{t-1}|\psi_{t-1}) = f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}],$$

where

$$f(y_t|\psi_{t-1}, s_t, s_{t-1}) = (2\pi\sigma_{s_t}^2)^{-1/2} \exp\{-\frac{[\phi(L)(y_t - \mu_{s_t})]^2}{2\sigma_{s_t}^2}\}.$$

In the second step, to get $f(y_t|\psi_{t-1})$, we have to integrate s_t and s_{t-1} out of the joint density by summing the joint density over all possible values of s_t and s_{t-1}:

$$\begin{aligned} f(y_t|\psi_{t-1}) &= \sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t, s_t, s_{t-1}|\psi_{t-1}) \\ &= \sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}]. \end{aligned}$$

Then, the marginal density $f(y_t|\psi_{t-1})$ is a weighted average of M^2 conditional densities. Finally, the log likelihood is given by:

$$\ln L = \sum_{t=1}^{T} \ln\{\sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}]\}.$$

To complete the above procedure, we still need to deal with the problem of calculating $\Pr[s_t = j, s_{t-1} = i|\psi_{t-1}]$, that is, the weights. This is known as the filtering procedure which consists also of two steps. In the first step, given $\Pr[s_{t-1} = i|\psi_{t-1})$, $i = 1, 2, ..., M$, at the beginning of time t, we can calculate the weights by

$$\Pr[s_t = j, s_{t-1} = i|\psi_{t-1}] = \Pr[s_t = j|s_{t-1} = i] \Pr[s_{t-1} = i|\psi_{t-1}]$$

where the first term in the last expression are the transition probabilities. In the second step, once y_t is observed at the end of time t, we can update the probability terms by

$$\begin{aligned} \Pr[s_t &= j, s_{t-1} = i|\psi_{t-1}] = \Pr[s_t = j, s_{t-1} = i|\psi_{t-1}, y_t] \\ &= \frac{f(s_t = j, s_{t-1} = i, y_t|\psi_{t-1})}{f(y_t|\psi_{t-1})} \\ &= \frac{f(y_t|s_t = j, s_{t-1} = i, \psi_{t-1}) \Pr[s_t = j, s_{t-1} = i|]}{\sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}]}, \end{aligned}$$

with $\Pr[s_t = j|\psi_t] = \sum_{s_{t-1}=1}^{M} \Pr[s_t = j, s_{t-1} = i|\psi_t]$. Iterating these two steps for $t = 1, 2, ..., T$ allows us the appropriate weighting terms to use in the log likelihood.

Construction of the log likelihood and filtering steps are known as the Hamilton filter which is a modification of the Kalman filter. In fact, one of the principal outputs from Hamilton filter is the so called filtered probabilities, which are inferences about s_t using information until time t. But it is also possible to obtain smoothed probabilities which are inferences about s_t using information until time T, that is, total information. This is found using the Kim's smooth algorithm, see Kim (1994).

Other extensions to the MS models are to allow to change the autoregressive coefficients according to the unobserved variable s_t. Another possibility is to allow for time varying transition probabilities as was suggested by Filardo (1994).

2.2 Smooth Transition Autoregressive (STAR) Models

Using similar notation as in van Dijk, Teräsvirta and Franses (2001) a STAR model of order p for a univariate time series y_t, can be represented by

$$\begin{aligned} y_t &= (\phi_{1,0} + \phi_{1,1}y_{t-1} + ... + \phi_{1,p}y_{t-p})(1 - F(s_t;\gamma,c) + \\ &\quad (\phi_{2,0} + \phi_{2,1}y_{t-1} + ... + \phi_{2,p}y_{t-p})F(s_t;\gamma,c) + \varepsilon_t \end{aligned} \tag{4}$$

where ε_t is assumed to be a martingale difference sequence with respect to the history of the time series up to time $t-1$ which can be denoted as $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, ..., y_{1-(p-1)}, y_{1-p}\}$[2].

The transition function $F(.)$, to be discussed below, is a continuous function that is bounded between 0 to 1. The parameter γ determines the smoothness of the change in the value of the transition function, and thus, the smoothness of the transition from one regime to another. The parameter c can be interpreted as the threshold between the two regimes. The variable s_t is the transition variable, which as discussed in Teräsvirta (1994), is assumed to be a lagged endogenous variable, that is, $s_t = y_{t-d}$ for certain integer $d > 0$. There are also alternative choices for the transition variable as is suggested by van Dijk, Teräsvirta and Franses (2001). For example, it can be an exogenous variable ($s_t = z_t$), or a (possible nonlinear) function of lagged endogenous variables ($s_t = h(x_t;\alpha)$ for some function h, which depends on the $(q \times 1)$ parameter vector α). Finally, the transition variable can be a (function of a) linear time trend ($s_t = t$)[3].

STAR models can be interpreted in two different ways. Firstly, they can be regarded as a regime-switching model that allows for two regimes, corresponding with the endpoint values of the interval $[0, 1]$. This is the interval

[2]Most simple model assume that the conditional variance of ε_t is constant, althought it is possibly to allow for autoregressive conditional heterocedasticty (ARCH) as was analyzed by Lundbergh and Terasvirta (1998).

[3]It gives rise to a model with smoothly changing parameters, see Lin and Terasvirta (1994).

where the transition function is defined and where the transition from one regime to another is realized smoothly. Secondly, they can be interpreted as models that allow for a continuum of regimes with corresponding different values of transition function between 0 and 1. As in most papers, here, we use the two-regime interpretation.

The choice of the transition function varies with the type of time series data for which we are interested to apply a STAR model. However, in most of empirical applications, two type of transition functions have been used. A first choice is the logistic function defined as[4]

$$F(\gamma, s_t, c) = (1 + \exp[-\gamma(s_t - c)])^{-1} \tag{5}$$

where $\gamma > 0$. A STAR model with $F(.)$ like (5) is named a Logistic Smooth Transition Autoregressive (LSTAR) model. Notice that this class of model nests many other type of models; hence when $\gamma \to 0$, then LSTAR becomes a linear $AR(p)$ model; when γ becomes very large, the logistic function approaches the indicator function and consequently, the change of $F(.)$ from 0 to 1 becomes almost instantaneous at $s_t = c$. In this case LSTAR model nests a two regime threshold autoregressive (TAR) model as a special case[5].

Notice that $F(.) = 0$, when $s_t \leq c$ and $F(.) = 1$, when $s_t > c$. Then in LSTAR models the two regimes are associated with small and large changes of the transition variable with respect to the threshold parameter which rises the possibility, in the case where $s_t = y_{t-d}$, to analyze business cycle asymmetries distinguished by expansions and recessions (van Dijk, Teräsvirta and Franses, 2001). This is the case in applications proposed by Teräsvirta and Anderson (1992) and Teräsvirta, Tjostheim and Granger (1994) to analyze industrial production in OECD countries.

[4]Note that we are using s_t as the transition variable. Later and in empirical applications, we consider $s_t = y_{t-d}$.

[5]In the case where the switch becomes almost instantaneous and $s_t = y_{t-d}$, this model is called a self-exciting TAR (SETAR) model. For an extensive discussion about this type of models, see Tong (1990).

In other hand, sometimes it is convenient to have another type of regime-switching behavior as for example to assume that the behavior of some economic variable depends on the size of the deviation from the threshold parameter. In other words, it is possible to specify the transition function such as the regimes are related with small and large absolute values of the transition variable. It is possible to achieved by using the exponential function:

$$F(\gamma, s_t, c) = 1 - \exp[-\gamma(s_t - c)^2] \tag{6}$$

where $\gamma > 0$. In this case, the STAR model receives the name of Exponential Smooth Autoregressive (ESTAR) model. The parameters of ESTAR models change symmetrically about c with s_t. The ESTAR model become linear when $\gamma \to \infty$, but it also happens if $\gamma \to 0$, and then this model does not nest SETAR models. This class of models have been used in many applications related to real exchange rates (see van Dijk, Teräsvirta and Franses, 2001).

In recent years, many extensions have been proposed to the basic STAR model. One of them is the introduction of multiple regime STAR model, where is possible to use more than one transition function. For example, consider the case of the LSTAR model with m regimes

$$\begin{aligned} y_t &= \phi_1 x_t + (\phi_2 - \phi_1)' x_t F_1(s_t) + (\phi_3 - \phi_2)' x_t F_2(s_t) \\ &\quad + ... + (\phi_m - \phi_{m-1})' x_t F_{m-1}(s_t) + \varepsilon_t \end{aligned} \tag{7}$$

where $x_t = (y_{t-1}, y_{t-2}, ..., y_{t-p})'$ and $F_j(s_t) = F_j(s_t; \gamma, c)$, $j = 1, 2, ..., m-1$. This class of models are discussed in van Dijk and Franses (1999). In fact, MSTAR models also nest the flexible coefficient smooth transition time series model considered in Medeiros and Veiga (2000) which is obtained by assuming that the transition variables are linear combinations of lagged dependent variables and also imposing some restrictions between parameters.

Because nonlinearity and structural instability may be regarded as two competing alternatives to linearity, the MSTAR model have been extended to allow for time-varying characteristics given the so called time-varying MSTAR model (TVSTAR). This model is obtained when one of the transition variables in (7) is taken to be time implying that the time series y_t follows a STAR model at all times, with smooth change in the autoregressive parameters in all regimes.

Extension of univariate STAR models to the multivariate framework creates the so called vector STAR models. It is equivalent to use expression (4) but replacing scalars by vectors and the coefficients for matrices of coefficients. In this case the regimes are common to the k variables because only one and the same transition variable determines the prevailing regime and the switches between regimes in all k equations of the model. However, it is possible to allow for equation-specific regime-switching by introducing equation-specific transition functions.

Finally, another interesting extension is the smooth transition error-correction model (STECM), where the model takes into account for nonlinear or asymmetric adjustment (see Granger and Swanson, 1996). A STECM can be represented by

$$\begin{aligned}\Delta Y_t &= (\Phi_{1,0} + \alpha_1 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j}\Delta Y_{t-j})(1 - F(Y_{t-d};\gamma, c)) \\ &\quad +(\Phi_{2,0} + \alpha_2 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{2,j}\Delta Y_{t-j})F(Y_{t-d};\gamma, c)) + \varepsilon_t\end{aligned} \tag{8}$$

where $\alpha_i, i = 1, 2$ are $kx1$ vector and $z_t = \beta' Y_t$ for some $kx1$ vector β denote the error correction term, that is, z_t is the deviation from the equilibrium relationship which is given by $\beta' Y_t = 0$. An additional extension of this model is to incorporate multiple equilibrium relationships.

For a more detailed discussion of this model and other models mentioned above, see Lundbergh, Teräsvirta and van Dijk (2000) and van Dijk, Strikholm and Teräsvirta (2001). Discussion about other methodological issues as testing, estimation and evaluation of STAR models will be considered in the next section.

2.3 Some Other Nonlinear Models

Like other nonlinear models, bilinear models are composed of a linear autoregressive term, disturbance term and the cross products of residuals with the autoregressive term. In practice, bilinear models are used to illustrate processes with occasional strong disturbances. Using standard notation a general bilinear models can be represented by

$$y_t = \sum_{i=1}^{p} a_i y_{t-i} + \sum_{i=1}^{p} b_i e_{t-i} + \sum_{i=1}^{r} \sum_{j=1}^{s} c_{ij} y_{t-i} e_{t-j} + e_t \tag{9}$$

where $e_t \sim i.i.d.(0, \sigma_e^2)$. Hence, this is an Autoregressive Moving Average -*ARMA*- (p, q) with bilinear terms multiplying lagged e_t and y_t. The model has $p + q + rs + 1$ coefficients to be estimated, including variance of e_t. Although bilinear models are possible to be estimated using Newton-Raphson algorithm (see Subba, Rao and Gabr, 1984), it is difficult to estimate partial derivatives and they need to be calculated recursively. As in moving average (MA) models, verifying for invertibility is necessary. Teräsvirta (1993) suggests to check it numerically, because analytical conditions for invertibility are difficult to derive if the model is not sufficiently simple. Although bilinear models allow a certain amount of nonlinearity, according to Granger and Newbold (1986) are not so general.

A neural network model can be represented by

$$y_t = \alpha + \sum_{j=1}^{q} \beta_j \Phi(\gamma_j' \mathbf{Z}_t) + u_t \tag{10}$$

where $\mathbf{Z}_t$ is the vector of explanatory variables which can be y_{t-j}, with $j = 1, ..., p$; and x_{t-j}, with $j = 0, ..., p$ for some given lag p. In (10), the function $\Phi(.)$ is named the "squashing function" and it is bounded between 0 and 1. In most of papers, a logistic function is used.

Other class of nonlinear models are the so named pursuit models. These models are discussed in Granger and Teräsvirta (1993). They consider two possible cases for these models. In this first class, the model is estimated in a two sequential step procedure using a nonparametric estimate combined with a parametric estimate. Let y_t be the dependent series and $\mathbf{Z}_t$ a vector of explanatory variables. Then the first step is to consider a model of the form

$$y_t = \Phi_1(\gamma_1'\mathbf{Z}_t) + e_{1t}$$

where $\Phi_1(.)$ is a smooth function chosen nonparametrically to minimize the mean squared residuals and γ_1 is a vector of parameters. In the second step, it is possible to construct

$$y_{1t} = Y_t - \Phi_1(\gamma_{10}'\mathbf{Z}_t)$$

where γ_{10}' is the optimal value corresponding to γ_1. Finally, a model is estimated as

$$y_{1t} = \Phi_2(\gamma_{20}'\mathbf{Z}_t) + e_{2t}$$

Notice that the model can be complemented by other forms using linear or quadratic terms in $\mathbf{Z}_t$. The second model (see Granger and Teräsvirta, 1992) is specified as

$$y_t = \alpha_0 + \boldsymbol{\alpha}'\mathbf{Z}_{1t} + \sum_{j=1}^{p} \boldsymbol{\Phi}_j(\gamma_{10}'\mathbf{Z}_{2t}) + u_t$$

where $\mathbf{Z}_{1t}$ contains x_{t-i} and y_{t-j} for $i = 1, 2, ..., k_x$ and $j = 1, 2, ..., k_y$. The vector $\mathbf{Z}_{2t}$ contains x_{t-i} and y_{t-j} for $i = 1, 2, ..., l_x$ and $j = 1, 2, ..., l_y$. In most of projection pursuit models $\boldsymbol{\Phi}_j(\gamma_t'\mathbf{Z}_t)$ is nonparametrically estimated for a given vector $\mathbf{Z}_t$. Unfortunately, these models are dense and can be approximated by a high choice of p. According to Teräsvirta *et al* (1994), the small sample sizes frequently available in economic data implies a limited p, say one or two. The goal is to keep the number of parameters to be estimated at a reasonable level. However, in theory, p should be selected using some stopping criterion or goodness of fit measure.

Other popular class of nonlinear models are the time varying parameters (TVP). In this class of model the parameters of interest depend on time and the model has to be estimated using the Kalman filter. Sometimes these models are called random coefficient models, because parameters of these models change smoothly. TVP models, as any models that include non observed components, can be represented in a space state form which consists of an observed equation and a transition equation. The first equation relates dependent variable with explanatory variables which are associated to a random coefficient. The behavior of this random coefficient is specified by the transition equation. Hence, a TVP model can be expressed by

$$\begin{aligned} y_t &= \beta_t x_{t-1} + \varepsilon_t \\ \beta_t &= m + \alpha\beta_{t-1} + \eta_t \end{aligned} \tag{11}$$

where η_t is a vector of white noise not necessarily correlated with ε_t.

3 Building, Estimating and Evaluating LSTAR Models

As it is mentioned in Granger (1993), the construction of nonlinear models consists of two steps. In the first step, a test for linearity is performed. If the null hypothesis of linearity is rejected, a second step is necessary. In this step a nonlinear model is chosen, estimated, analyzed and evaluated. Generally, at this step, a decision is done, for example between an ESTAR or a LSTAR model. However, in this paper we select directly the LSTAR model given our interest to identify asymmetries.

The linearity tests can be classified in two broad categories. In the first category, tests are derived without assuming a specific nonlinear alternative. According to Granger and Teräsvirta (1993) a few tests of this category may also be interpreted as a score or Lagrange multiplier (LM) tests. In the second category tests are derived against a specific alternative. These tests are known as LM-type tests where the specific alternative may consist of a regression where the dependent variable is regressed on its own lagged values and other explanatory variables adding nonlinear terms for which the test is applied[6,7]. A more exhaustive discussion of the different tests with comparison of size and power using Monte Carlo simulations can be found in Lee *et al.* (1993).

In most of cases, it is strongly recommended to have a specific strategy for building nonlinear time-series models. Teräsvirta (1994) proposed the following data-based modeling cycle for STAR models. In the first step, we estimate a linear AR model of order p. In general the linear model is represented in the following way:

$$A(L)y_t = u_t \tag{12}$$

where $A(L) = 1 - a_1L - a_2L^2 - .... - a_pL^p$ is a finite polynomial in the lag operator with all roots outside of the unit circle. This is customary to select p based on the Akaike Information Criteria (AIC). Sometimes, it is also possible to use the Schwarz Information Criterion (BIC) or the Ljung-Box statistic. The second step of the modelling cycle consists of testing the null hypothesis of linearity against the specific alternative of STAR nonlinearity.

[6] A classical example of a test for linearity is the statistic of Ramsey (1969), known as the *RESET* test, where the nonlinear terms of the alternative may be the powers of the lagged dependent and explanatory variables.

[7] There are also non-standard tests, for example, proposed by Brock, Deckert and Scheinkman (1987), known as the BDS test.

van Dijk, Teräsvirta and Franses (2000) suggest that if linearity is rejected in the second step of the modeling cycle, the lag order used in the AR model is not necessarily the appropriate lag order in the alternative STAR model, although it usually provides a reasonable first approximation. The third step of the modelling cycle is to choose between LSTAR and ESTAR models which allow us to define our transition function. Last steps are estimation and evaluation of the estimated model.

Let consider again the two regime STAR model presented in (4), in last section. This model can be written as

$$y_t = \phi_1' x_t (1 - F(s_t; \gamma, c)) + \phi_2' x_t F(s_t; \gamma, c) + \varepsilon_t \tag{13}$$

or

$$y_t = \phi_1' x_t + (\phi_2 - \phi_1)' x_t F(s_t; \gamma, c) + \varepsilon_t \tag{14}$$

where $x_t = (1, \widetilde{x}_t)'$ with $\widetilde{x}_t = (y_{t-1}, y_{t-2}, ..., y_{t-p})'$ and $\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})'$, for $i = 1, 2$ and s_t is a general transition variable (not necessarily at this moment y_{t-d}). Hence, the linearity hypothesis may be expressed as equality of the autoregressive parameters in the two regimes of the STAR model in (14). That is, $H_0 : \phi_1 = \phi_2$, whereas $H_1 : \phi_1 \neq \phi_2$ for at least one $j = 0, ..., p$. This testing problem is complicated because the model in (14) contains nuisance parameters which are not present under the null hypothesis. In this case, these parameters are γ and c which do not affect the likelihood under the null hypothesis. This is a problem analyzed first by Davies (1987) and further analyzed by Andrews and Ploberger (1994), Hansen (1996), among others. Notice that alternatively, H_0 can be formulated as $H_0' : \gamma = 0$ which is equivalent to the linear model, because if $\gamma = 0$, the model in (14) reduces to an AR model with a coefficient that is an average of ϕ_1 and ϕ_2. In this case, the parameters c, ϕ_1 and ϕ_2 are not identified. The main consequence of this issue is that conventional statistical theory is not available to obtain asymptotic null distributions of the classical likelihood ratio, Lagrange Multiplier and Wald statistics. These tests have non-standard distribution and in most cases analytical expressions are not available and critical values have to be calculated by simulation and for each particular case.

In the LSTAR framework, Luukkonen, Saikkonen and Teräsvirta (1988) analyzed the issue of testing for linearity[8]. They suggested to replace the transition function $F(s_t; \gamma, c)$ by a convenient Taylor series approximation. Because in the new parameterization there is no identification problems, linearity can be tested using a LM statistic with a standard asymptotic χ^2 distribution under the null hypothesis. As it is known, in a LM statistic the model under the alternative hypothesis is not needed to be calculated which gives an additional advantage to this case. To calculate the LM-type statistic, we start to estimate the following regression as suggested by Granger and Teräsvirta (1993):

$$y_t = \beta_0 + \beta_1' x_t + \sum_{j=1}^{p} \beta_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \beta_{3j} y_{t-j} y_{t-d}^2 + \sum_{j=1}^{p} \beta_{4j} y_{t-j} y_{t-d}^3 + v_t \quad (15)$$

where the null hypothesis is $H_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0, j = 1, ..., p$. In (15), the transition variable s_t has been replaced by y_{t-d}, with d as the delay parameter. In order to specify the value of d, it is appropriate to carry out the test for a range of values $1 \leqslant d \leqslant p$. If we reject H_0 for more than one value of d , then d is determined as $\widehat{d} = \arg\min\, p(d)$ for $d \in [1, p]$ where $p(d)$ is the $p-$value of the selected test[9].

Although we assume that the correct nonlinear model under the alternative hypothesis is the LSTAR model, it is interesting to explain the selection process between ESTAR and LSTAR models. This can be done by a sequence of tests within (15) as is discussed in many papers; see for example Teräsvirta (1990). It is possible to specify the following set of null hypotheses:

[8] Note that the LM type tests of testing nonlinearity differs for both LSTAR and ESTAR models. The practical ways of carrying out tests for both types of models are given in Terasvirta (1994).

[9] The test has maximum power if d is chosen correctly, otherwise an incorrect choice of d weakens the power of the test.

$$\begin{aligned} H_{04} &: \beta_{4j} = 0 \\ H_{03} &: \beta_{3j} = 0 | \beta_{4j} = 0 \\ H_{02} &: \beta_{2j} = 0 | \beta_{3j} = \beta_{4j} = 0 \end{aligned}$$

where for all cases, $j = 0, 1, 2, ..., p$. The logic behind this sequence is based on interpreting the coefficients β_{ij} as functions of the parameters of the original model, that is, the expression (4); see Teräsvirta and Anderson (1992). If the model follows an ESTAR specification, then $\beta_{4j} = 0$, for $j = 1, ..., p$, but $\beta_{3j} \neq 0$ for at least one j if $\phi_2 \neq 0$. Furthermore, if the model follows a LSTAR specification, $\beta_{2j} \neq 0$ for at least one j if $\phi_2 \neq 0$. Thus, if H_{04} is rejected we choose the LSTAR model. If H_{04} is not rejected and H_{03} is rejected, the ESTAR model is selected. If H_{04} and H_{03} are not rejected and H_{02} is rejected, it leads to a LSTAR model. The inconclusive case is the one in which both H_{03} and H_{02} are rejected. Details about how to proceed in this case are given in Teräsvirta and Anderson (1992).

After that transition variable s_t and the transition function $F(s_t; \gamma, c)$ have been selected, estimation of the model is the next step. Estimation of a model given by (13) may considered as an application of nonlinear least squares (NLS). Because we are assuming a two regime STAR model, the set of parameters $\theta = (\phi_1, \phi_2, \gamma, c)'$ may be estimated as

$$\widehat{\theta} = \arg\min_{\theta} Q_T(\theta) = \arg\min_{\theta} \sum_{t=1}^{T} (y_t - G(x_t; \theta))^2, \tag{16}$$

where $G(x_t; \theta) = \phi_1' x_t (1 - F(s_t; \gamma, c)) + \phi_2' x_t F(s_t; \gamma, c)$[10]. Assuming that errors are normally distributed, then NLS estimates are equivalent to those obtained from maximum likelihood. If this is not the case, NLS estimates are interpreted as quasi maximum likelihood estimates. Some details about

[10] This term is known as the skeleton of the model. See van Dijk, Terasvirta and Franses (2001).

the limiting distribution of θ and estimation of the covariance-matrix are available in Wooldridge (1994). Notice that the estimation procedure can be simplified by concentrating the sum of squares function. More details are contained in van Dijk, Teräsvirta and Franses (2001).

In the evaluation step, the residuals from the estimated model are submitted to some diagnostic tests. The first is a LM test to verify for $q-th$ order serial dependence in ε_t which is a generalization of the LM test for serial correlation in an $AR(p)$ model by Godfrey (1979). It is distributed asymptotically as a χ^2 with q degrees of freedom. If we consider the model given by (13), this test consists to do a regression of $\widehat{\varepsilon}_t$ on $\partial G(x_t;\theta)/\partial\theta$ and q lagged estimated residuals with $\theta = (\phi_1, \phi_2, \gamma, c)'$, all values calculated under the null hypothesis.

A second diagnostic test is a LM test to verify the hypothesis of no remaining nonlinearity. It was proposed by Eitheim and Teräsvirta (1996) to test the two regime LSTAR model (expression (14), for example) against the alternative of an additive STAR model which can consists of a three-regime LSTAR with an additional function $F_2(s_t;\gamma_2,c)$; hence, $H_0:\gamma_2=0$. Because in this case we have again identification problems under the null hypothesis, the new transition function $F_2(s_t;\gamma_2,c)$ is replaced by a Taylor series approximation around $\gamma_2=0$. Residuals from this extended model are regressed against the parameters in the two-regime model (evaluated under H_0) and auxiliary regressors (power of products between x_t and s_t). The statistic constructed as nR^2 has an χ^2 distribution with $3(p+1)$ degrees of freedom.

Another diagnostic test is to verify the hypothesis of parameter constancy. The null hypothesis is constancy of parameters in the two-regime STAR model (expression (13)) and the alternative hypothesis is smoothly changing parameters, that is $s_t=t$. The limiting distribution is the same as for the tests of remaining nonlinearity.

4 Business Cycles and Nonlinear Models

One of the first econometric models in measuring business cycles was proposed by Tinbergen (1939) who used linear difference equations as instrument of analysis, for some macroeconomic indicators. Unfortunately, the linear structure of the model did not consider any nonlinearities of the business cycles. Recently there has been a growing interest in applying nonlinear models to many macroeconomic and financial time series. The idea that the growth rates of output is affected in a different way and magnitude from negative and positive shocks (asymmetries) has supported the interest of application of nonlinear models to analyze the business cycle issue. Additionally, researchers used these tools to calculate the average time of a recession and expansion, dates and the adjustment process. Many models have been used to pursue these goals, but here, we select some of them, especially those related to the use of MS models and STAR models.

Empirical analysis of asymmetries in business cycles has greatly developed during 1980's with new contributions in nonlinear modeling. Most empirical business cycle studies focuses on the growth rate of output of the United States. By estimating an univariate nonlinear model for US GNP, Potter (1995) found evidence in favor of a SETAR model. His results prove that linear methods can hide much interesting economic structure. Estimating nonlinear models for post-1945 US GNP, Potter (1995) suggests that even if the economy was under the influence of negative shocks such as the Great Depression, the output level returns to its trend quickly.

One of the most applied approaches to analyze business cycles in empirical work is MS models. It was proposed by Hamilton (1989) when he analyzed the growth rates of the US output allowing for a changing mean which was dependent on a state variable (s_t). However, one of the shortcomings of his work was that he did not tested the linearity of the data. This was done later by Hansen (1992) who developed appropriate tests which have non standard limiting distribution. Other applications have followed in the literature as for example Godwin (1993) who analyze business cycles for growth rates of output in G7 countries using a MS model with two regimes. Bodman and Crosby (2000) analyzed business cycle for Canada also using a MS model imposing two different regimes. The results of this model show that the Canadian economy was in recession in 1951:3, even though negative growth was not recorded in the period. A quarter of well below-average growth after a period of very large quarterly decline (greater than minus 4 percent) was followed during the recession of 1953:3-1954:4. It is important to note that the MS model with two regimes could not find the periods of small negative growth in 1956:4 and 1957:1 as a recession and instead identifies the quarter 1957:2 as a peak followed by two quarters of negative growth as a recession.

Bodman and Crosby (2000) extended the model to allow for three regimes in the growth rate of the output of Canada. All three states of this model can be explained as a low-growth (recession) state, a high-growth state, and an intermediate state. The estimates show that the annualized average growth rate in the second regime ("normal times"), $\mu_2 = 4.62$ percent while the recession state is characterized by a negative annual growth rate of -2.77 percent. The growth rate in the recovery phase, μ_1, is estimated to be around 7.82 percent. The expected duration for each period are around 2.5 quarters, 2.7 quarters and 19.2 quarters, respectively.

Some other empirical applications are mentioned by Kim and Nelson (2000). The more important shortcoming in MS models is the test for linearity. Traditional tests, such as Wald statistics, are not available because nuisance parameters are not present under the null hypothesis as we explained in the last section. Overall, the decision of how many regimes or states exist in the growth rates of output (or any other variable) is imposed arbitrarily. In some other cases, this choice is done using a statistic proposed by Davies (1987)[11].

Examples of applications of STAR models can be found in Granger and Teräsvirta (1993), Teräsvirta and Anderson (1992) and Anderson and Teräsvirta (1992). In most of these cases, the dependent variable is the quarterly differences of the logarithm of the industrial production for 13 OECD countries. They reject the null hypothesis of linearity for most of these series and ESTAR and LSTAR models are chosen for the different variables.

Another application of STAR model but with two additive smooth transition function was estimated by Öcal and Osborn (2000). In this model, with two thresholds, they found that industrial production, contrary to consumption, is explained by three regimes of recession, normal growth and high growth. The transitions from recession to expansion for both variables are similar showing possibility that asymmetries are not very important.

5 Empirical Results

Quarterly data of growth rates of real GDP for G-7 countries is used. For Canada and United Kingdom, data spans from 1957:1 until 2000:1. For U.S. the data covers period 1957:1 until 2000:2. For Japan, data spans from 1957:1 to 1999:4. Data for Germany and Italy span from 1960:1 to 2000:1. For France, we use data from 1970:1 to 2000:1. All data was obtained from the CD ROM of International Financial Statistics of International Monetary Fund.

The application of standard statistics proposed by Dickey and Fuller (1979) and Said and Dickey (1984) shows that all variables are non stationary in logarithm levels but stationary in first differences. In consequence, subsequent analysis is performed using the first differences of the logarithm of real GDP. Evolution of the series over time are presented in Figures 1.1-

[11] See Garcia and Perron (1996) for an example of the use of this statistic to choose between two and three regimes for the real interest rate and inflation rates of US. Another example can be found in Demers and Rodríguez (2001) who applied this test to inflation and growth output rates of Canada.

1.7. For Germany rapid increase in growth rate can be observed in early 90's, perhaps as a result of merging of two Germany.

We follow the nonlinear modelling cycle proposed by Granger and Teräsvirta (1993) but if we reject the null hypothesis of linearity we proceed to estimate a LSTAR model. In applying the LM test for linearity, we used a maximum lag $p = 8$. Table 1 presents the minimum p-value obtained from the application of linearity tests using $d \in [1, 8]$. Overall, the evidence suggests strong rejection of the null hypothesis of linearity. Table 1 also presents the corresponding delay selected for each of the seven countries. In most of cases, $d = 2$ is chosen, while $d = 1$ is selected only for France $d = 3$ is chosen for Japan and Germany. According to Teräsvirta(1994) if d is selected correctly, auxiliary regression is the appropriate auxiliary regression against the nonlinear alternative. If we by mistake choose another d then auxiliary regression is misspecified. The power of the corresponding test against the misspecified nonlinear model thus can hardly be expected to be systematically higher than the power of the test based on the correctly specified auxiliary regression.

Results of estimation of LSTAR models for the seven countries are shown in Tables 2.1 until 2.7. For all cases the ratio of variances from a nonlinear models with respect to a linear model is less than unity which means that estimating nonlinear models allows us to achieve efficiency gains. Most of the coefficients are significant using customary levels of significance. As we saw before, parameter γ indicates the smoothness degree and the parameter c is the threshold parameter. These parameters are significant and show different patterns in the behavior of the growth rates for each country. Countries where the transition from one to other regime is very smooth are Germany and France. Italy presents an abrupt transition. Intermediate cases are Canada, United States, United Kingdom and Japan.

Tables 3.1 until 3.7 show results from diagnostic tests applied to each LSTAR model. The first panel in each table tests the null hypothesis of non autocorrelation using $q = 1, 2, ..., 8$. What these results inform is that in all cases, we can't reject the null hypothesis and then there is no autocorrelation in the residuals obtained from each LSTAR model. The second test verifies the null hypothesis of parameter constancy. Using 5% of significance, we can't reject the null hypothesis of parameter constancy but for Canada and Japan. The third test verify the null hypothesis of remaining non linearity in the residuals of each LSTAR models. The results show that it is possible to reject the null hypothesis in United States, Japan and United Kingdom.

The estimated parameter c is also different for all of the seven countries. For most of countries this parameter is around 1.0%. Cases where this parameter is negative are United States and United Kingdom. Given that the threshold parameter indicates the point where the economy passes from one regime (recession) to another regime (expansion), it is possible to see that from the first set of the mentioned countries (where $c < 0$) this passage is done before the other countries. This is shown in Figures 2.1 until 2.7, where the transition function $\widehat{F}(.)$ is graphed against the transition delay variable. In these figures, each point corresponds to each observation and then it is possible to see their distribution. For example, look at the Figure 2.3, the transition function for United Kingdom, most observations are concentrated after the threshold parameter indicating that there are not many observations in the regime zero, that is, in a recession. Japan (Figure 2.4) shows a similar behavior. Figures for countries such as Canada, France and United States show a more concentrated distribution of observations between each regime.

A final output from the estimation of LSTAR models is identification of the periods of recession and recovery for the seven countries. This is done from the estimates of the transition function $F(.)$ over time (quarters) which is shown in Figures 3.1 until 3.7. Notice that when the transition variable is larger than the threshold the transition function is closer to the value of one. In the converse case ($y_{t-d} < c$), the value of the transition function $\widehat{F}(.)$ is close to zero. This case corresponds to the periods where the economy is in a recession. Following other studies (see for example Franses 2000), we proceed to identify peaks and troughs for each of seven countries. There is no only one 'practical' definition to use to identify duration of a recession. For example in Franses and Paap (1998), they consider that a trough consists of two consecutive quarters where $F(.)$ is below the value of the threshold parameter. Other practical guide can be to account as a trough two consecutive quarters where economy experienced negative growth rates (see Bodman and Crosby, 2000). From our results (Figures 3.1 until 3.7) there are many countries where it is difficult to have two consecutive quarters where $\widehat{F}(.) \leq 0.5$ because the changes are faster.. However, we will account as a trough the observations closer to $\widehat{F}(.) = 0$. The results are shown in Table 4.1-4.2. It is possible to see that troughs are distributed in different ways in each country. Overall, periods of recession are very similar for most of countries. However, there are some periods where we can observe that United Kingdom and Italy have different dates. In general, the dates of peaks and troughs are similar to those found by Goodwin (1993) using different span of data. Similarities can be observed for United States, United Kingdom and Germany. Some difference in dating occurs for Canada, Japan and France. For Canada, Goodwin (1993) identifies only few sets of peaks and troughs in early 80's as well as for Japan in mid 70's. For France he finds only two sets of peaks and troughs in late 60's and mid 70's.

6 Conclusions

The goal of this paper is the estimation and the analysis of business cycles in seven industrialized countries (the G-7 group). Because we are interested in identifying asymmetries in business cycles, we used a LSTAR model. Quarterly data for growth rates of the real output is used. By testing for linearity we are able to reject the null hypothesis of linearity. Given this result, we proceed to estimate a LSTAR model for each country allowing for one threshold in the specification. Different degrees of adjustment and threshold parameters are found showing the different dynamics in each economy. Transition function are calculated to show the speed of adjustment and the evolution of this estimations over time allows us to estimate the principal periods of recession and recovery for each economy. Multiple threshold analysis has not been performed for these data spans but it would be interesting to extend the analysis in such a way.

References

[1] Andrews, D.W.K. and W. Proberger (1994), "Optimal Tests When a Nuisance Parameter is Present only Under the Alternative", *Econometrica* **62**, 1383-1414

[2] Bodman, P.M. and M. Crosby (2000), "Phases of the Canadian Business Cycle", *Canadian Journal of Economics* **33**, 618-633.

[3] Burns, A. F., and W. C. Mitchell (1946), Measuring Business Cycles, New York, NBER.

[4] Davies, R. B. (1987), "Hypothesis Testing When a Nuisance Parameter is Present Only Under the Alternative," *Biometrika* **74**, 33-43.

[5] Demers, F. and G. Rodríguez (2001), "Estimation of the Taylor Rule for Canada under Multiple Structural Changes," Working Paper No 0107E, University of Ottawa.

[6] Dickey, D.A. and W.A. Fuller (1979): "Distribution of the Estimator for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association* **74**, 427-431.

[7] Eitrheim, Ø. T. Terasvirta (1996), "Testing The Adequacy of STAR models", *Journal of Econometrics* **74**, 59-76.

[8] Filardo, A. J. (1994), "Business Cycle Phases and Their Transitional Dynamics," *Journal of Business & Economics Statistics* **12**, 299-308.

[9] Franses, P. H. (1999), Time Series Models for Business and Economic Forecasting, Cambridge University Press.

[10] Franses, P. H. and D. van Dijk (2000), Non-Linear Time Series Models in Empirical Finance, Cambridge University Press.

[11] Franses, P. H. and R. Paap (1999), "Does Seasonality Influence the Dating of Business Cycle Turning Points," *Journal of Macroeconomics* **21(1)**, 79-92.

[12] Garcia, R. and P. Perron (1996), "An Analysis of the Real Interest Rate Under Regime Shifts," *Journal of Business and Economic Statistics* **78**, 111-125.

[13] Granger, C.W.J. (1993), "Strategies for Modelling Nonlinear Time-Series Relationships," *The Economic Record* **69**, 233-238.

[14] Granger, C.W.J. and N.R. Swanson (1996), "Future Developments in the Study of Cointegrated Variables," Oxford Bulletin of Economics and Statistics, 58, 537-553.

[15] Granger, C.W.J. and T. Teräsvirta. (1993), Modelling Nonlinear Economic Relationships, Oxford University Press.

[16] Goodwin T.H. (1993), "Business-Cycle Analysis With a Markov-Switching Model" *Journal of Business and Economic Statistics* **11**, 331-339.

[17] Hamilton, J. D. (1989), "A New Approach to the Econometric Analysis of Nonstationary Time Series and Business Cyces," *Econometrica* **57**, 357-384.

[18] Hansen B.E. (1996), "Inference When a Nuisance Parameter is not Identified Under the Null Hypothesis," *Econometica* **64**, 413-430.

[19] Hansen B.E. (1996), "The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov Switching Model of GNP, *Journal of Applied Econometrics* **7**, S61-S82; erratum, **11**, 195-198.

[20] Kim, C-J. (1994), "Dynamic Linear Models with Markov-Switching," *Journal of Econometrics* **60**, 1-22.

[21] Kim, C-J. and C. Nelson (1999), *State-Space Models with Regime Switching*, The MIT Press: Cambridge, Mass..

[22] Kuan, C. M. and T. Liu (1995), "Forecasting Exchange Rates using Feedforward and Recurrent Neural Networks," *Journal of Applied Econometrics* **10**, 347-364.

[23] Kuan, C. M. and H. White (1994), "Artificial Neural Networks: an Econometric Perspective," (with discussion), *Econometric Reviews* **13**, 1-91.

[24] Lee, T.-H., H. White and C. W. J. Granger (1993), "Testing for Neglected Nonlinearity in Time Series Models. A comparison of Neural Network Methods and Alternative Tests," *Journal of Econometrics* **56,** 269-290.

[25] Lin, C.F.J. and T. Teräsvirta (1994), "Testing the Constancy of Regression Parameters Against Continuous Structural Change," *Journal of Econometrics* **62,** 211-228.

[26] Lucas, R. E., "Understanding Business Cycles", in K.Brunner and A.Meltzer (eds), *Stabilization of the Domestic and International Economy, Carnegie-Rochester Series on Public Policy* **5** (Amsterdam: North-Holland, 1976), 7-29.

[27] Lundbergh, S. and T. Teräsvirta (1998), "Modelling Economic High-Frequency Time Series with STAR-STGARCH Models", Working Papers Series in Economics and Finance N0. 291, Stockolm School of Economics.

[28] Lundbergh, S. (1998), "A Simple Package for Estimating STAR and GARCH Models," manuscript, Stockolm School of Economics.

[29] Lundbergh, S., T. Teräsvirta and D. van Dijk (2000), "Time-Varying Smooth Transition Autoregressive Models," Working Paper Series in Economics and Finance No.376, Stockholm School of Economics.

[30] Luukkonen, R., P. Saikkonen, and T. Teräsvirta (1988), "Testing Linearity Against Smooth Transition Autoregressive Models," *Biometrika* **75**, 491-499.

[31] Medeiros,M.C. and Veiga (2000), "A flexible Coefficient Smooth Transition Time Series Model," Working Paper Series in Economics and Finance No.360, Stockholm Scool of Economics.

[32] Subba Rao and Garb,M.M. (1984), Intoduction to Bispectral Analysis and Bilinear Time Series Models, Leture Notes in Statistics No. 24, New York, Springer-Verlag.

[33] Neftci, S. N. (1984), "Are Economic Time series Asymmetric over the Business Cycle?," *Journal of Political Economy* **92,** 409-432.

[34] Ocal, N. and D. R. Osborn (2000), "Business Cycle Nonlinearities in UK Consumption and Production," *Journal of Applied Econometrics* **15**, 27-43.

[35] Potter, S. M. (1995), "A Nonlinear Approach to US GNP," *Journal of Applied Econometrics*, **10**, 109-125.

[36] Said, S.E. and D.A. Dickey (1984): "Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order," *Biometrika* **71**, 599-608.

[37] Stock, James H. and Mark W. Watson (1991), "A Probability Model of the Coincedent Economic Indicators." *In Learning Economic Indicators: New Approach and Forecasting Record*, ed. K Lahiri and G. H. Moore. Cambridge: Cambridge Universty Press 63-89.

[38] Teräsvirta, T. and H. M. Anderson (1992), "Characterizing Nonlinearities in Business Cycles using Smooth Transition Autoregressive Models," *Journal of Applied Econometrics*, **7**, S119-S136.

[39] Teräsvirta, T. (1994), "Specification Estimation and Evaluation of Smooth Transition Autoregressive Models," *Journal of the American Statistical Association* **89**, 208-218.

[40] Teräsvirta, T., D. Tjostheim and C. W. J. Granger (1994), "Aspects of Modelling Nonlinear Time Series," *Handbook of Econometrics* Vol. **4**, Chapter 48.

[41] Tinbergen, J. (1939), Business Cycles in the United States of America, 1919-1932, Geneva: League of Nations.

[42] van Dijk, T. Teräsvirta and P. H. Franses (2000), "Smooth Transition Autoregressive Models. A Survey of Recent Developments," Stockholm School of Economics, Working Paper Series in Economics and Finance No. 380.

[43] van Dijk, D., B. Strikholm and T. Teräsvirta (2001), "The Effects of Institutional and Technological Change and Business Cycle Fluctuations on Seasonal Patterns in Quarterly Industrial Production Series", Working Paper Series in Economics and Finance No. 429, Stockholm School of Economics.

[44] Tong, H. (1990), Non-Linear Time Series. A Dynamical System Approach, Oxford University Press.

[45] Wooldridge, J.M. (1994), "Estimation and Inference for Dependent Processes," in R.F. Engel and D.L. McFadden (eds.), *Handbook of econometrics, vol. IV*, Amsterdam :Elsevier Sience , pp.2639-2738.

[46] Zarnowitz, V., (1992), "Business Cycles: Theory, History, Indicators and Forecasting," The University of Chicago Press, Chicago.

Table 1. Linearity Test

Country	Minimum p-value	Corresponding delay	k- order
Canada	0.008	2	2
USA	0.000	2	3
United Kingdom	0.000	4	3
Japan	0.002	3	3
Germany	0.018	3	2
France	0.041	1	2
Italy	0.000	5	2

Table 2.1. STAR model for growth rate of output of Canada

$$\begin{aligned}\triangle y_t =& \underset{(0.000)}{0.004} + \underset{(0.066)}{0.388}\triangle y_{t-1} + \underset{(0.104)}{0.253}\triangle y_{t-2} + \underset{(0.075)}{0.203}\triangle y_{t-3} - \underset{(0.085)}{0.228}\triangle y_{t-4} \\ & - \underset{(0.053)}{0.104}\triangle y_{t-5} + (\underset{(0.004)}{0.007} - \underset{(0.143)}{0.489}\triangle y_{t-1} - \underset{(0.183)}{0.281}\triangle y_{t-2} - \underset{(0.135)}{0.297}\triangle y_{t-3} \\ & + \underset{(0.142)}{0.643}\triangle y_{t-4}) \times [1 + \exp(-\underset{(4.315)}{7.710}(\triangle y_{t-2} - \underset{(0.001)}{0.014})/\hat{\sigma}(\triangle y_{t-2}))]^{-1} + \hat{u}_t\end{aligned}$$

$S^2/S_L^2 = 0.92$, $AIC = -9.14, SBIC = -8.90, HQ = -9.04, \bar{R}^2 = 0.26, S = 0.0099$

Table 2.2. STAR model for growth rate of output of United States

$$\begin{aligned}\triangle y_t =& \underset{(0.001)}{0.007} - \underset{(0.168)}{0.379}\triangle y_{t-3} + \underset{(0.177)}{0.593}\triangle y_{t-5} + \underset{(0.190)}{0.785}\triangle y_{t-6} - \underset{(0.171)}{1.094}\triangle y_{t-7} \\ & - \underset{(0.187)}{0.516}\triangle y_{t-8} + (\underset{(0.053)}{0.184}\triangle y_{t-1} + \underset{(0.179)}{0.634}\triangle y_{t-3} - \underset{(0.190)}{0.752}\triangle y_{t-5} \\ & - \underset{(0.203)}{0.839}\triangle y_{t-6} + \underset{(0.185)}{1.221}\triangle y_{t-7} + \underset{(0.191)}{0.487}\triangle y_{t-8}) \\ & \times[1 + \exp(-\underset{(7.908)}{8.768}(\triangle y_{t-2} + \underset{(0.001)}{0.006})/\hat{\sigma}(\triangle y_{t-2}))]^{-1} + \hat{u}_t\end{aligned}$$

$S^2/S_L^2 = 0.78, AIC = -8.92$, $SBIC = -8.66, HQ = -8.81$, $\bar{R}^2 = 0.32, S = 0.011$

Table 2.3. STAR model for growth rate of output of United Kingdom

$$\triangle y_t = \underset{(0.003)}{-0.004} + \underset{(0.210)}{0.530} \triangle y_{t-5} - \underset{(0.189)}{0.398} \triangle y_{t-7} + (\underset{(0.003)}{0.012}$$

$$- \underset{(0.222)}{0.496} \triangle y_{t-5} + \underset{(0.197)}{0.321} \triangle y_{t-7} \times [1+\exp(-\underset{(7.646)}{10.581} (\triangle y_{t-4} + \underset{(0.002)}{0.009}) / \hat{\sigma} (\triangle y_{t-4}))]^{-1} + \hat{u}_t$$

$$S^2/S_L^2 = 0.95, AIC = -8.45, SBIC = -8.30, HQ = -8.3930, \bar{R}^2 = 0.052, S = 0.014$$

Table 2.4. STAR model for growth rate of output of Japan

$$\triangle y_t = \underset{(0.002)}{0.004} + \underset{(0.092)}{0.529} \triangle y_{t-2} + \underset{(0.051)}{0.373} \triangle y_{t-3} + \underset{(0.052)}{0.161} \triangle y_{t-4} - (\underset{(0.002)}{0.004} + \underset{(0.110)}{0.369} \triangle y_{t-2})$$

$$\times [1+\exp(-\underset{(11.611)}{13.278} (\triangle y_{t-3} - \underset{(0.002)}{0.009}) / \hat{\sigma} (\triangle y_{t-3}))]^{-1} + \hat{u}_t$$

$$S^2/S_L^2 =, AIC = -8.249, SBIC = -8.118, HQ = -8.196, \bar{R}^2 = 0.263, S = 0.015$$

Table 2.5. STAR model for growth rate of output of Germany

$$\triangle y_t = \underset{(0.076)}{0.246}\triangle y_{t-1} + \underset{(0.049)}{0.336}\triangle y_{t-4} + (\underset{(0.026)}{0.045} - \underset{(0.769)}{1.320}\triangle y_{t-1} - \underset{(0.241)}{0.395}\triangle y_{t-3})$$

$$\times[1+\exp(-\underset{(0.745)}{1.739}(\triangle y_{t-3} - \underset{(0.015)}{0.035})/\hat{\sigma}(\triangle y_{t-3}))]^{-1} + \hat{u}_t$$

$$S^2/S_L^2 = 0.86, AIC = -8.12, SBIC = -7.99, HQ = -8.07, \bar{R}^2 = 0.20, S = 0.016$$

.Table 2.6. STAR model for growth rate of output of France

$$\triangle y_t = -\underset{(0.148)}{0.709}\triangle y_{t-1} - \underset{(0.143)}{0.307}\triangle y_{t-5} + (\underset{(0.002)}{0.014} + \underset{(0.101)}{0.448}\triangle y_{t-2}$$

$$+\underset{(0.110)}{0.296}\triangle y_{t-3} + \underset{(0.222)}{0.320}\triangle y_{t-5}) \times [1+\exp(-\underset{(0.413)}{1.979}(\triangle y_{t-1} - \underset{(0.001)}{0.005})/\hat{\sigma}(\triangle y_{t-1}))]^{-1} + \hat{u}_t$$

$$S^2/S_L^2 = 0.70, AIC = -9.75, SBIC = -9.56, HQ = -9.67, \bar{R}^2 = 0.18, S = 0.007$$

Table 2.7. STAR model for growth rate of output of Italy

$$\triangle y_t = \underset{(0.001)}{0.009} - \underset{(0.045)}{0.131}\triangle y_{t-2} - \underset{(0.046)}{0.126}\triangle y_{t-4} + (\underset{(0.012)}{0.038} - \underset{(0.287)}{1.933}\triangle y_{t-1}$$

$$+\underset{(0.362)}{3.409}\triangle y_{t-2} - \underset{(0.731)}{2.324}\triangle y_{t-3} - \underset{(0.225)}{0.892}\triangle y_{t-4})$$

$$\times[1+\exp(-\underset{(18507.03)}{267.552}(\triangle y_{t-5} - \underset{(0.083)}{0.029})/\hat{\sigma}(\triangle y_{t-5}))]^{-1} + \hat{u}_t$$

$$S^2/S_L^2 = 0.64, AIC = -8.12, SBIC = -7.92, HQ = -8.04, \bar{R}^2 = 0.41, S = 0.016$$

Table 3.1. Diagnostic tests of LSTAR model for Canada

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.448	0.578	0.813	0.734	0.768	0.665	0.599	0.477
Test for parameter constancy								
	All model		Linear			Non-linear		
p-value	0.031		0.090			0.585		
Test for remaining non-linearity								
	p-value							
$\triangle y_{t-1}$	0.149							
$\triangle y_{t-2}$	0.450							
$\triangle y_{t-3}$	0.725							
$\triangle y_{t-4}$	0.307							
$\triangle y_{t-5}$	0.367							
$\triangle y_{t-6}$	0.771							
$\triangle y_{t-7}$	0.700							
$\triangle y_{t-8}$	0.452							

Table 3.2. Diagnostic tests of LSTAR model for USA

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.988	0.892	0.147	0.274	0.221	0.325	0.445	0.572
Test for parameter constancy								
	All model		Linear			Non-linear		
p-value	0.129		0.906			0.997		
Test for remaining non-linearity								
	p-value							
$\triangle y_{t-1}$	0.180							
$\triangle y_{t-2}$	0.327							
$\triangle y_{t-3}$	0.005							
$\triangle y_{t-4}$	0.028							
$\triangle y_{t-5}$	0.375							
$\triangle y_{t-6}$	0.780							
$\triangle y_{t-7}$	0.474							
$\triangle y_{t-8}$	0.535							

Table 3.3. Diagnostic tests of LSTAR model for United Kingdom

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.156	0.330	0.487	0.314	0.397	0.539	0.582	0.518
Test for parameter constancy								
	All model		Linear			Non-linear		
p-value	0.487		0.782			0.976		
Test for remaining non-linearity								
				p-value				
$\triangle y_{t-1}$				0.033				
$\triangle y_{t-2}$				0.442				
$\triangle y_{t-3}$				0.000				
$\triangle y_{t-4}$				0.000				
$\triangle y_{t-5}$				0.067				
$\triangle y_{t-6}$				0.177				
$\triangle y_{t-7}$				0.041				
$\triangle y_{t-8}$				0.020				

Table 3.4. Diagnostic tests of LSTAR model for Japan

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.830	0.949	0.972	0.915	0.753	0.470	0.554	0.272

Test for parameter constancy			
	All model	Linear	Non-linear
p-value	0.004	0.017	0.026

Test for remaining non-linearity	
	p-value
$\triangle y_{t-1}$	0.112
$\triangle y_{t-2}$	0.149
$\triangle y_{t-3}$	0.189
$\triangle y_{t-4}$	0.084
$\triangle y_{t-5}$	0.453
$\triangle y_{t-6}$	0.016
$\triangle y_{t-7}$	0.173
$\triangle y_{t-8}$	0.360

Table 3.5. Diagnostic tests of LSTAR model for Germany

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.806	0.772	0.647	0.526	0.196	0.273	0.283	0.229
Test for parameter constancy								
	All model		Linear		Non-linear			
p-value	0.452		0.312		0.882			
Test for remaining non-linearity								
	p-value							
$\triangle y_{t-1}$	0.799							
$\triangle y_{t-2}$	0.160							
$\triangle y_{t-3}$	0.775							
$\triangle y_{t-4}$	0.080							
$\triangle y_{t-5}$	0.980							
$\triangle y_{t-6}$	0.945							
$\triangle y_{t-7}$	0.974							
$\triangle y_{t-8}$	0.868							

Table 3.6. Diagnostic tests of LSTAR model for France

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.648	0.690	0.770	0.880	0.897	0.857	0.891	0.931

Test for parameter constancy			
	All model	Linear	Non-linear
p-value	0.659	0.952	0.693

Test for remaining non-linearity	
	p-value
$\triangle y_{t-1}$	0.242
$\triangle y_{t-2}$	0.925
$\triangle y_{t-3}$	0.367
$\triangle y_{t-4}$	0.627
$\triangle y_{t-5}$	0.993
$\triangle y_{t-6}$	0.884
$\triangle y_{t-7}$	0.106
$\triangle y_{t-8}$	0.946

Table 3.7. Diagnostic tests of LSTAR model for Italy

Test for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.095	0.249	0.438	0.574	0.700	0.488	0.589	0.659

Test for parameter constancy			
	All model	Linear	Non-linear
p-value	0.999	0.635	0.999

Test for remaining non-linearity	
	p-value
$\triangle y_{t-1}$	0.708
$\triangle y_{t-2}$	0.995
$\triangle y_{t-3}$	0.891
$\triangle y_{t-4}$	0.808
$\triangle y_{t-5}$	0.886
$\triangle y_{t-6}$	0.819
$\triangle y_{t-7}$	0.569
$\triangle y_{t-8}$	0.922

Table 4.1. Business cycles peaks and troughs for Canada, USA,UK and Japan*

Canada		USA		UK		Japan	
Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
1958.4	1959.2	1958.1	1958.3	1962.1	1962.4	1958.2	1958.4
1963.1	1964.1	1960.3	1961.2	1974.1	1975.1	1962.4	1963.2
1969.3	1971.3	1970.1	1970.3	1980.4	1981.1	1971.2	1972.1
1974.4	1976.1	1973.2	1975.3	1991.1	1992.2	1975.2	1976.1
1977.1	1979.1	1979.2	1981.1			1979.4	1981.1
1980.1	1981.1	1982.1	1983.2			1982.4	1983.3
1981.4	1983.2	1988.4	1989.4			1991.4	1995.4
1985.1	1986.1	1990.1	1991.3			1996.4	1997.2
1989.2	1993.3	1996.2	1997.1			1997.4	1999.3
1995.1	1996.4	1999.3	2000.1				
1997.2	1998.4						

*A peak is defined as the last observation before a recession. A trough is the last quarter of a recession. A recession is defined when $\widehat{F}(\Delta y_{t-d}) \leq 0.5$

Table 4.2. Business cycles peaks and troughs for Germany, France and Italy*

Germany		France		Italy	
Peak	Trough	Peak	Trough	Peak	Trough
1961.2	1962.1	1974.1	1974.4	1962.1	1964.2
1964.2	1968.4	1979.1	1981.4	1964.4	1969.2
1969.2	1969.4	1983.1	1983.4	1971.2	1974.2
1971.1	1973.2	1984.2	1985.2	1975.1	1977.3
1975.1	1976.2	1990.3	1991.2	1978.3	1979.4
1980.1	1983.3	1992.2	1994.1	1981.1	1998.2
1989.3	1990.2	1996.2	1997.1		
1993.4	1994.3				
1999.3	2000.1				

*A peak is defined as the last observation before a recession. A trough is the last quarter of a recession. A recession is defined when $\widehat{F}(\Delta y_{t-d}) \leq 0.5$

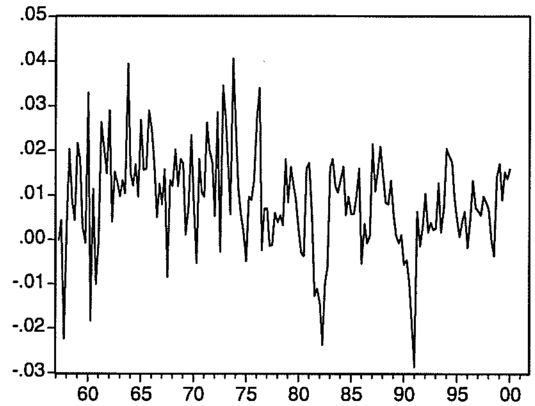

Figure 1.1. Growth Rates of Output for Canada

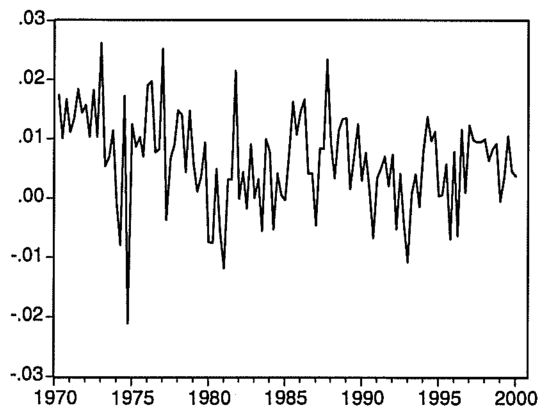

Figure 1.2. Growth Rates of Output for France

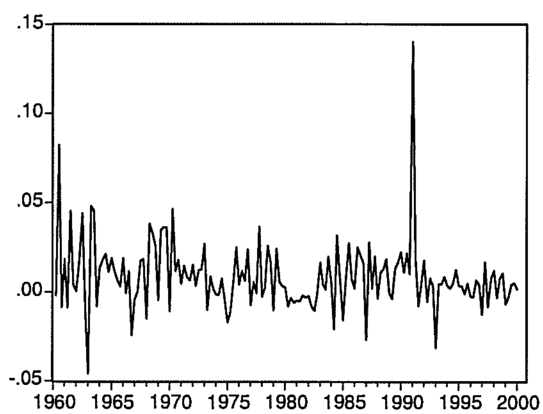

Figure 1.3. Growth Rates of Output for Germany

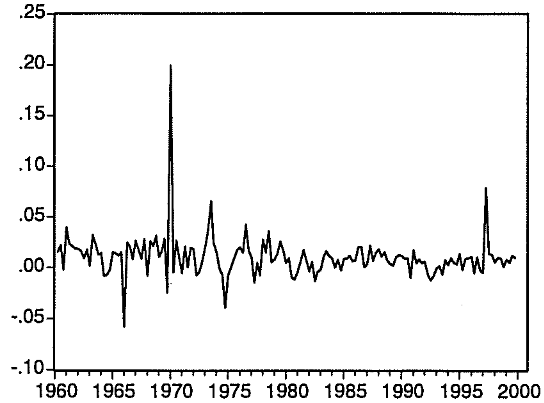

Figure 1.4. Growth Rates of Output for Italy

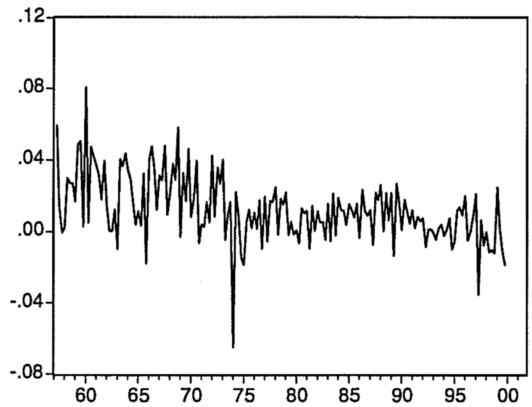

Figure 1.5. Growth Rates of Output for Japan

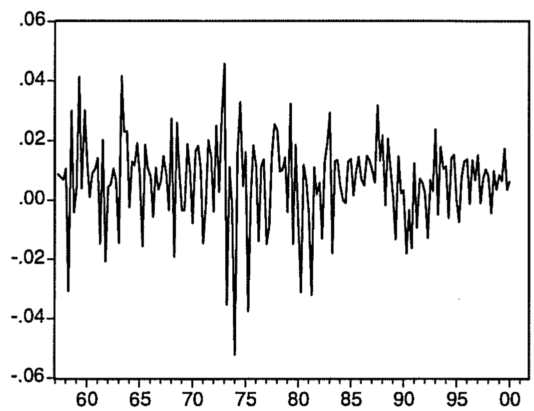

Figure 1.6. Growth Rates of Output for United Kindom

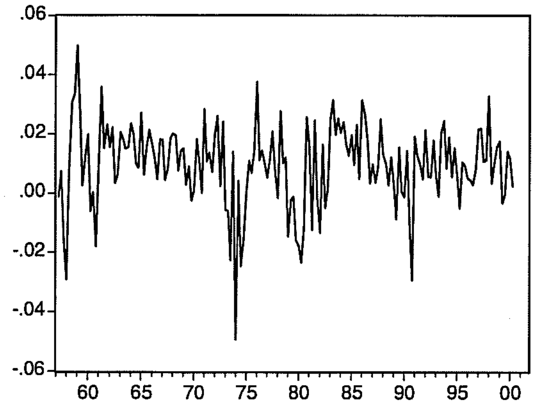

Figure 1.7. Growth Rates of Output for United States

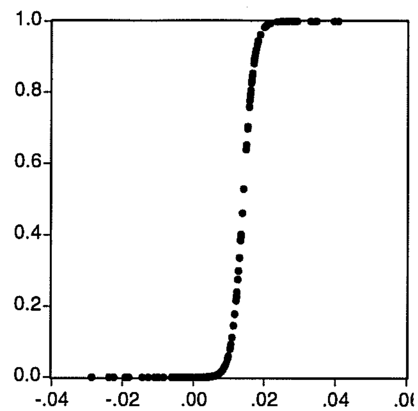

Figure 2.1. Transition Function against Δy_{t-2} for Canada

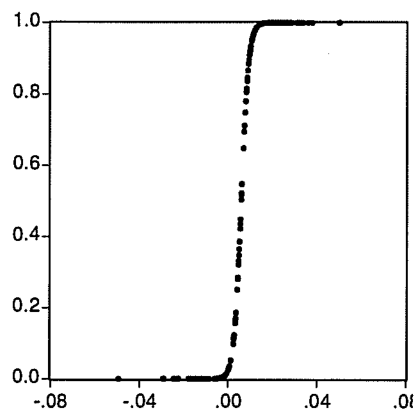

Figure 2.2. Transition Function against Δy_{t-2} for United States

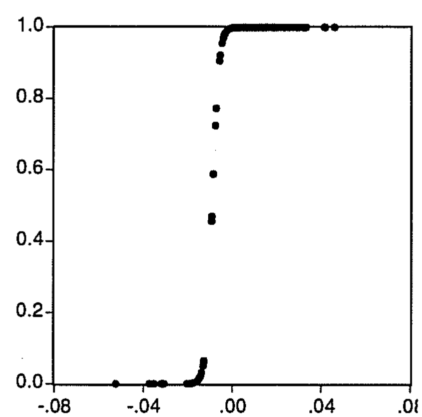

Figure 2.3. Transition Function against Δy_{t-4} for United Kindom

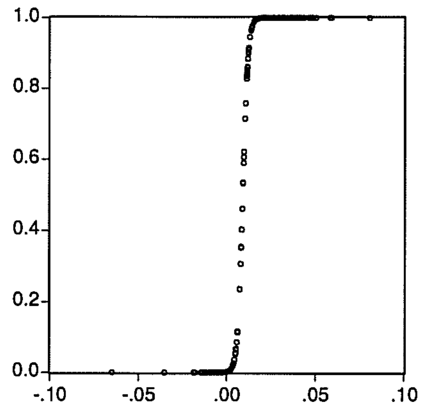

Figure 2.4. Transition Function against Δy_{t-3} for Japan

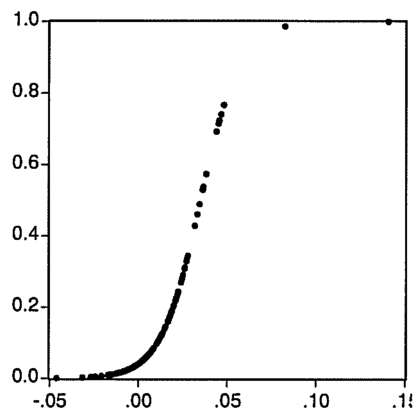

Figure 2.5. Transition Function against Δy_{t-3} for Germany

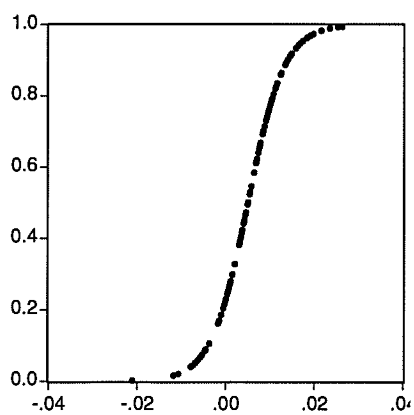

Figure 2.6. Transition Function against $\Delta y_{t-1}$for France

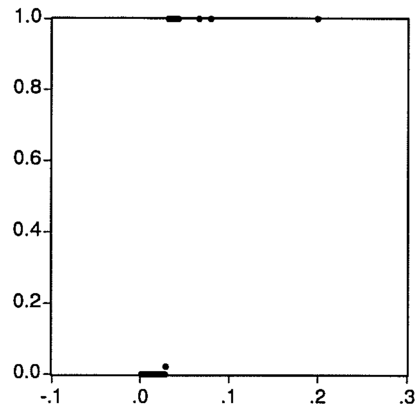

Figure 2.7. Transition Function against Δy_{t-5} for Italy

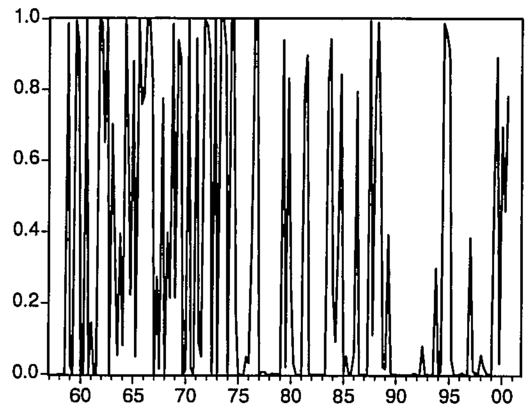

Figure 3.1. Transition Function against Time for Canada

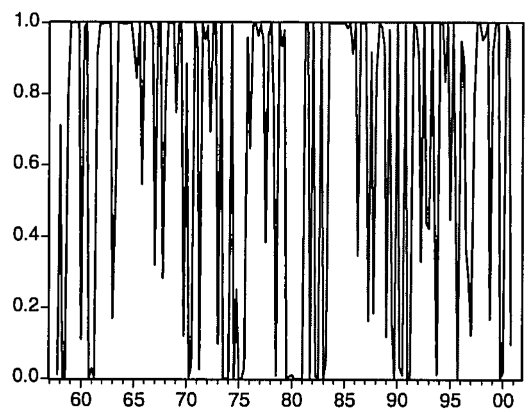

Figure 3.2. Transition Function against Time for United States

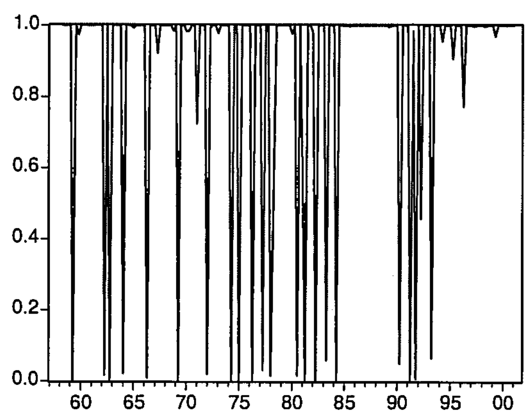

Figure 3.3. Transition Function against Time for United Kindom

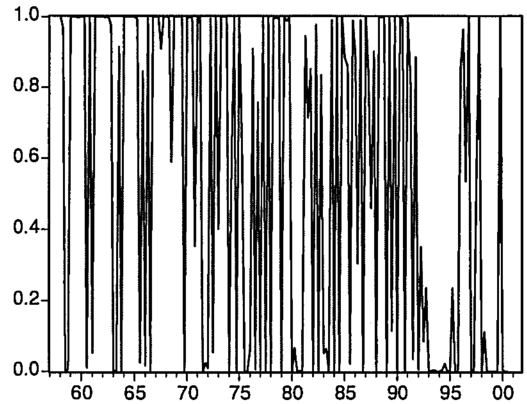

Figure 3.4. Transition Function against Time for Japan

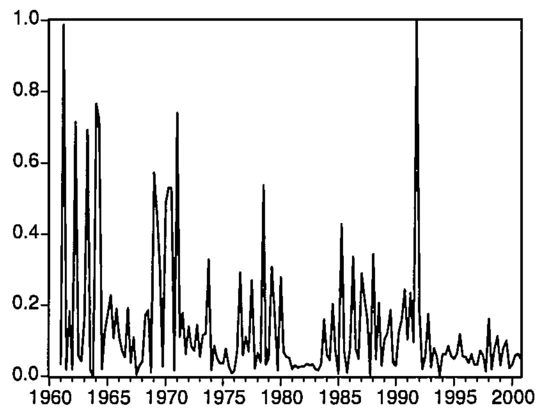

Figure 3.5. Transition Function against Time for Germany

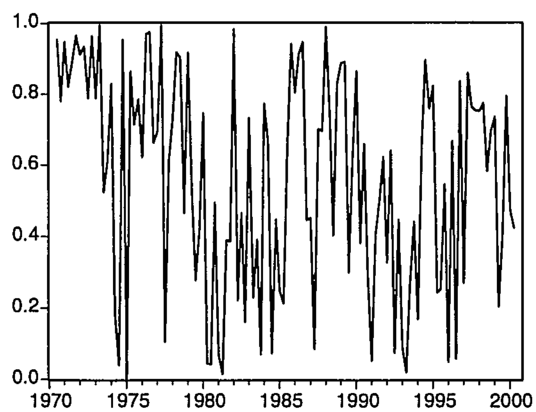

Figure 3.6. Transition Function against Time for France

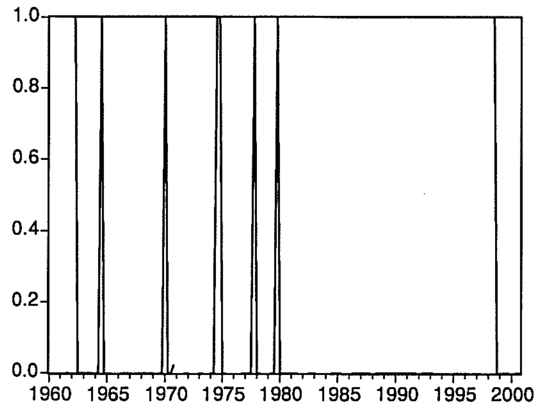

Figure 3.7. Transition Function against Time for Italy