

The Transmission Effects of Population Ageing
across Regions: A Computable Overlapping
Generations General Equilibrium Analysis

By

Hong Ding

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Student number: 2868702

Supervisor: Professor Marcel Mérette

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ABSTRACT

Demographic projections indicate that the dependency ratios of many parts of the world will increase considerably in the next several decades. While the patterns of population ageing are similar in most countries, timing and initial conditions differ substantially. To the extent that goods and financial funds are internationally mobile, international trade is expected to act as an important channel for transmission effects of population ageing. To address this issue, our paper presents a quantitative analysis by considering two hypothetical regions that are identical in every respect except for the population growth rates in a dynamic, overlapping generations, general equilibrium framework. The results of our simulation exercises indicate that both the ageing region and the fast-growing region could benefit from free trade.

Keywords: population ageing, overlapping-generations, computable general equilibrium model, unequal population growth rates

¹ I would like to thank professor Marcel Mérette for giving me support and guidance.

1. INTRODUCTION

OVER THE COMING DECADES many parts of the world, especially industrialized countries, will experience population ageing. This is primarily because the post-war "baby-boom" generation (those born between 1946 and 1966) is moving through the age structure, but also because people are living longer and fertility rates have fallen. Consequently, the old age dependency ratios, defined as the number of individuals aged 65 and over divided by those aged between 15-64, is expected to rise dramatically.

Since population ageing and the associated increases in dependency ratios have important micro and macroeconomic implications, an extensive literature investigating various aspects of the problem has emerged, including numerous studies that employ overlapping generations (OGs), general equilibrium models developed to analyze the effects of ageing within an economy-wide framework. However, the effects of increasing dependency ratios in major economies for other countries maintaining strong economic ties with them have been largely left unexplored. While the same general ageing pattern is expected to hold across most economies soon or later, there are considerable differences in the speed with which such changes are likely to occur. This contrast is even more marked comparing the economies in Organization for Economic Cooperation and Development (OECD) to non-OECD countries. The latter will typically

experience ageing much later and the dependency ratios of many of them are even projected to fall over the next few decades.

It is well known that within each country, demographic change alters the time path of aggregate variables as well as relative factor endowments. To the extent that goods and financial funds are internationally mobile, differences in population growth rates potentially emerge as one of the major determinants of commodity and financial assets flows across national borders. Our objective is to present a quantitative analysis of potential general equilibrium effects that population ageing in one region may cause domestically and within its trading partner. For this purpose, we have built a stylized computable general equilibrium model that comprises three regions including two regions within one country and the rest of the world over a 45-period horizon. The two regions of the country are identical in every respect except for the differing population growth rates and there are six different age groups and two industrial sectors within each.

Our simulation experiments are based on three scenarios concerning the effects of demographic changes: In the first scenario (reference scenario), 0 per cent population growth rate is assumed for both regions throughout the horizon. In the second scenario, we assume, for both regions, 1 per cent population growth rate from period 1 to period 6 and 0 per cent for the rest of the time path. By comparing scenario 2 with the reference scenario, we examine the effects of ageing. In the third scenario, we assume different population growth rates for the two regions to investigate inter-regional interactions.

The remainder of this paper is organized as follows. The review of previous literature is presented in Section 2. Section 3 sets out the model, explicitly showing the assumptions about households, firms, governments and investors. Section 4 describes the data and calibration procedures of the model as well as the simulation analysis and results. In Section 5, a brief conclusion is given.

2. THE LITERATURE

Numerous economic implications are expected from the upcoming demographic shift because individual's economic behavior changes with age and many governments' spending programs are sensitive to the age composition of the population (Mérette, 2002). Typically, people have the following life-cycle pattern: The young invest in human capital by devoting a good portion of his time to education; in the middle age, an individual works a lot, acquires assets to secure future consumption; the elderly has stronger preference for leisure. He works less, decreases the physical and financial assets and consumes considerable health-care services. Given the lifecycle behaviors, government expenditures and transfer programs such as health care, public pensions and education, are sensitive to the demographic structure. For instance, health care benefits mostly seniors, whereas public education expenditure targets the young generation. Consequently, the predicted ageing process, via microeconomic mechanism, may induce considerable macroeconomic impacts on issues such as fiscal balances, aggregate consumption and national savings, output and economic growth, and welfare.

Demographic Trends

The growth rate of world population, which is currently about 1½ per cent per annum, has fallen by more than ¼ per cent per annum since the early 1980s and is likely to fall steadily over the coming decades to about ½ per cent per annum by 2050 (United Nations, 1996). Assuming that age-specific labor force participation rates remain constant, the lower rates of population growth imply a smaller labor force. The OECD (1998) estimates that barring increases in participation rates and immigration levels, labor force growth could come to a halt in a few years in many industrial countries. Moreover, the general trend towards increasing participation rates of the female and older men would not even come close to reversing the current trend. According to this OECD study, despite higher labor force participation rates, the average annual labor-force growth rate in 1995 to 2020 would still decline significantly, but at a somewhat slower speed: 0.4 per cent instead of 0.9 per cent, compared to 2.4 per cent in 1970-95. OECD (2001) also predicts a fall in the working age population (age of 20 to 64) except in Australia, Canada, New Zealand, Norway, and the United States, and rapid increases in the number of elderly and, particularly, in those over 80.

Along with the shrinking labor force, a lot of countries will experience a rising share of the elderly. The OECD (1998) estimates that the dependency ratio rises to 86 per cent for Japan, 78 per cent for European Union and 65 per cent for USA by 2050.² Robson's projection of Canadian population (2003) shows a decreasing share of the young population, with sharply declining numbers in the east of Ontario; deceleration in the working population nationwide with declines in the East; and continued growth in the population aged 65 and over, the share of which in total population increases everywhere.

² The "dependency ratio" here is defined to be the ratio of the population who are not working age (both young and old) to those who are, where the population of working age is defined to be those between 15 and 64.

Impact on Private Savings

If savings behavior conforms to the traditional “ life-cycle “ hypotheses, by which households are assumed to save to secure future consumption until retirement after which they dis-save, the private savings rate of a country could rise for a while as the baby-boom cohorts move through the highest-saving stages of their life cycle. Subsequently, surges in the proportion of the elderly in the population could reduce the private savings rate as the elderly starts to deplete their stocks of accumulated assets. A study of seven major industrial countries by Heller (1989) concludes that ageing will lead to a significant decline in private savings rate after 2000. The study of Germany in different capital mobility scenarios by Börsch-Supan (2003) concludes that the aggregate savings rate decreases throughout the entire observation period in all scenarios. The decrease of the savings rate caused by population ageing is about 4.7 percentage points in the closed-economy and EU 14 (all countries in the European Union except for Germany) scenarios and 4.3 percentage points if allowing for capital mobility within the OECD region.

Fiscal Balances

As the baby-boom generation progressively reaches retirement age over the next few decades, population ageing will significantly affect public finances. Extensive research undertaken by the OECD (1998 and 2001), the International Monetary fund (IMF) (1998), the European Commission (1999) and National authorities points to the longer-term unsustainability of current fiscal policies in a majority of countries as the number of pension and health care beneficiaries relative to the number of contributors is pushed up.

The study of Economics Department of OECD (2001) shows that, based on assumptions of unchanged policy—though taking into account legislated but-not-yet

implemented reforms—old-age pension spending will rise on average by around 3 to 4 percentage points of GDP in the period to 2050. Barring the effects of the changes in public system generosity and increasing employment ratios resulted from higher female participation rates, better market structure and increased average retirement ages, increased ageing / dependency is the key factor pushing up pension spending over the period and the average impact of ageing taken alone is around 5 percentage points of GDP. The average increase in public health and long-term care spending over the 2000-2050 period for the 14 countries where this information is available is 3 to 3.5 percentage points of GDP. Overall, the projection predicts a decline in the primary surplus or increase in the deficit of 6 to 7 percentage points of GDP over the period 2000-2050. The total impact on the fiscal situation of these economies will depend on the cumulated change in the primary balance over the projection period, coupled with the associated change in debt-interest payments. Even though cyclically adjusted primary balances have improved in most OECD countries, further reforms to age-related programs are recommended by the study.

On the other hand, the decreasing proportion of the working age population is also expected to lead to an erosion of the overall tax base. This in turn will put upward pressure on government deficits or the debt-to-GDP ratio. If the contribution rates were raised only when shortfalls in the pension accounts start to occur, they would have to be raised significantly each year for many years and the distorting effect on the economy would be substantial.

Looking at the positive side, some economies have market and institutional mechanisms that respond to - and help offset- potentially negative trends. Mérette (2002) shows that Canadian governments' revenues will be bolstered by taxable withdrawals from Registered Retirement Savings Plans (RRSPs) and other tax-deferred private pension

programs, and this will happen just when upward pressure of public expenditure related to old-age spending, particularly on health care and public pensions, are expected to be the most acute. Fougère and Mérette (2000) estimates that under a pure “pay-as-you-go” system, health care spending in Canada increases by 2.4 percentage points of GDP between 1994 and 2050, while the Old Age Security (OAS), Guaranteed Income Supplement (GIS) and Spouse's Allowance (SPA) payments increase by 2.3 percentage points of GDP over the same period. The increase in government revenue from net private withdrawals more than offsets the increase in spending. Such programs also exist in the United States such as Individual Retirement Accounts, 401(K)s. The total in such accounts is roughly \$11 trillion today, with hundreds of billions in new contributions pouring in each year. In a new, as-yet-unpublished paper, Michael J. Boskin estimates that the value of these deferred taxes through 2040 is roughly \$12 trillion in today's dollars.³ By comparison, the official estimate of the unfounded Social Security liability is \$3.5 trillion, and the unfounded Medicare liability for hospital insurance \$5.9 trillion in today's dollars. He concludes that much of the anticipated long-term budget gap may not exist – if current tax laws remain unchanged.

Interest Rates versus Wages

Movements in interest rates, exchange rates and capital flows across countries reflect various conflicting forces on aggregate savings and investment. Generally speaking, to the extent that imbalances between the two occur at a global level, they are likely to be reflected in movements in real interest rates, whereas to the extent that they occur in a certain region, they will be reflected in changes in exchange rates and net foreign asset positions.

³ See BusinessWeek, June 30, 2003, pp.34 –36.

Under "life-cycle" hypotheses, the combined influence of ageing and unchanged public policies would likely lead to a fall in both private and public savings in the future. Meanwhile, ageing may also influence the optimal size of the capital stock, and hence desired investment. With relatively fewer consumers and workers, capital requirements would rise less quickly, or even fall in some countries even though the scarcity of workers could induce some substitution of capital for labor. Whether the interest rate will rise or not depends on which of the two is likely to fall the most. The OECD (1998) predicts in the reference scenario a gradual rise in world interest rates, which rises by up to $\frac{1}{2}$ percentage point by 2050. However, most of the studies using an overlapping generations structure, which better captures the upcoming demographic shift, predict a fall in world interest rate.

The reduced growth of labor supply resulting from reduced population growth may lead to lower unemployment rates. The impact on wages depends in part on whether the labor market is competitive and efficient. If the real wage increases, it will provide strong incentives for the young and future generations to invest in human capital formation, increasing the labor productivity and capital intensity, hereby boosting the living standards. However, such gains may not materialize if the young and future generations do not anticipate the upward trend in real wage caused by ageing process.

Economic Growth and Living Standard

The impact of population ageing on the future levels of economic output depends on three principal factors: (1) the growth of the effective labor supply and the rate of utilization of the labor; (2) the stock of capital; (3) the total factor productivity.

A smaller labor force, other things being equal, results in lower level of output. A rising capital stock can potentially offset the effect of reduced growth rate of the labor supply

by increasing productivity, thereby helping to sustain or even push up living standards. However, ageing populations may place downward pressure on both private and government savings, consequently crowd out the capital stock by higher interest rates. Assuming "life cycle" behavior, the study of the OECD (1998) shows that such effects of capital intensity are virtually absent. The adverse effect on Japan is the most striking, with GDP growth falling below 1 per cent per annum before 2010 and falling further to only $\frac{1}{4}$ per cent per annum by 2040. The growth rate of the European Union falls to less than 1 per cent per annum by 2020 and averages less than $\frac{1}{2}$ per cent per annum between 2030 and 2040.

The technical progress may contribute to living standard. To the extent that labor scarcity might spur innovation, ageing could potentially enhance technical progress. The endogenous growth model ⁴ suggests that population ageing may increase incentives to invest in human capital, leading to reallocation of investment from physical to human capital which, in turn, would increase the rate of economic growth in the long run. However, some research suggested that there were a negative relationship between technical progress and reduced population growth.⁵ For example, an older workforce may suffer from a depreciation of its skills and the rate of industrial innovation and creation may also decline as older workers become less prepared to take risks and thus are less entrepreneurial. If so, technical progress could be slowed by the anticipated ageing populations.

The increased importance of the non-OECD in the world economy might also provide investment opportunities for OECD countries. By building up net foreign assets, the benefits from the associated future investment income flows, along with improvements

⁴ Fougère and Mérette (2000)

⁵ See Simon (1977, 1986), Wattenberg (1987), and Romer (1990)

in the terms of trade could boost OECD living standards. However, the evidence⁶ suggests that such contributions are likely to be very modest due to the downward pressure on OECD savings. The progressive rundown in OECD net foreign assets will pull down the growth of living standards. Besides, there would need to be major structural change in the product, labor and particularly, capital markets of the non-OECD countries. Overall it seems that such an investment income contribution would be very limited.

Cutler et al. (1990) assesses the impact of ageing on living standards in the United States. Their analysis concludes that per capita consumption would increase up to around 2020, after which they would decline by 4.2 - 9.4 per cent by 2060 due to the dominating impact of increased dependency. The study of IMF (1998) shows that the negative effects on the annual rate of growth of output per capita range from 0.25 to 0.6 percentage points (over a period of slightly more than three decades). By 2030, the level of output per capita would be 8-20 per cent lower as a result of ageing than would otherwise be the case, unless offsetting productivity was achieved, other things being equal.

International Implication

When assessing the effects on current balances, we should not only look at the absolute fact of ageing, but also the speed at which a given country is ageing relative to its trading partners. A study by the OECD secretariat (2001) using a globally consistent, model-based approach implies that, under "life-cycle" hypothesis and in the absence of further pension reforms or fiscal consolidation, most OECD countries would experience current account surpluses for the next decade. This is because the demographic pressures do not emerge fully in most of these countries for another decade or so, and meanwhile the

⁶ OECD (1998)

faster-growing non-OECD countries (mostly developing countries) are expected to be net capital importers. However, during the middle of the next century, OECD countries may run down most of the stock of net foreign assets built up during the period of current surpluses (which is also affected by the starting levels of net foreign assets) and would be likely running significant current account deficits. The resulting swings in trade and current account balances could exacerbate friction among nations concerning trade and foreign investment policies and even strain monetary policies and exchange-rate regimes, as evoked by Ferguson and Kotlikoff (2000).

Assuming two countries identical in every respect except for different population growth rates and allowing free trade between the two, Sayan (2001) concludes that the country that is relatively abundant in one of the factors would specialize in the production of the commodity, which uses that factor more intensively and exports that commodity. However, trade would not necessarily represent a Pareto improvement over the state of autarky (which rules out trade between the two). On the contrary, the results in this paper revealed that trade may be immiserizing for the ageing country. The labor-abundant country starts acting as a large country after a while and has a commanding share in total world supply of factors as well as output. This is in line with Sayan and Emre Uyar's study (2002), which examines the consequence of economic relations between young-population countries and their regional partners where population ageing has long been under way. He suggests that free trade may cause Pareto deterioration for a small ageing country because in the long run it is actually forced to act as a small country when population of the other country is growing more rapidly. It also suggests that, in terms of welfare effects, migration makes the host country (ageing country) worse-off while the source country (fast-growing country) remains at least as well off.

3. THE MODEL

This model specifies the preferences and technologies, as well as the full equilibrium conditions for all the regions/countries involved in the analysis. The conditions that maximize firm profits are derived in section 1. In section 2, we present the problem solved by the representative household in order to establish its optimal consumption and bequest paths, including the composition of consumption. Sections 3, 4, 5, 6 present the problems of the investors, the government, the foreign trade and the rest of the world respectively. Section 7 lays out the equilibrium conditions for each market in each period.⁷

Preliminaries on sets and indices

There are two fully endogenous and symmetric regions indexed by i or j ($i=1, 2; j=1, 2$) within a certain country and a reduced form of the rest of the world indexed by *row*. The set of all the three regions is denoted \mathcal{I} and indexed by ii or jj ($ii=1, 2, \text{row}; jj=1, 2$). Each region produces two goods, called good 1 and good 2. Each good is produced by one sector and the sector producing good s ($s=1, 2$) is called sector s . At each period in time (time is indexed by t), there are δ generations (indexed by $g, g=1, \dots, \delta$) that coexist. We indexed the working generations by gj ($gj=1, \dots, \mathcal{J}$), the first generation by gi ($gi=1$).

⁷The description of this model is mostly taken from Mercenier and Mérette (2002). I interpreted some parts and derived all the first-order-condition equations. Because some of these equations are similar, I put only those for production sectors and households in Appendix A. I also have specified each optimal equation in the text so that it is easy for the readers to derive the rest equations by referring to the procedures in Appendix A.

3.1. Sector s of region j at time t

In each region, there are 2 competitive sectors. Producers are assumed to be profit maximizing and each producer produces one product, and in only one sector. The technology of production requires that the representative firm employs capital, labor and intermediate input goods from sectors across regions.

Labor and capital are homogeneous and mobile among sectors within the region. There is no inter-regional/international labor or capital mobility. Therefore, the wage and rental rate, denoted by $w_{j,t}$ and $r_{j,t}^k$ respectively, are the same across sectors within a given region, but may differ across regions/countries. Producers behave as price takers on products as well as on primary inputs. The representative producer therefore has to solve the following problem:

$$(1.1) \quad \underset{X_{ss,j,t}, K_{j,t}^k, L_{j,t}^{dem}}{\text{Minimize}} \sum_{ss} P_{j,ss,t} X_{ss,j,t} + r_{j,t}^k K_{j,t}^{dem} + w_{j,t} L_{j,t}^{dem}$$

subject to the following set of embedded constraints that characterize the firm's technology:

$$(1.2) \quad Z_{j,t} = CD(X_{j,t}, Q_{j,t}; SC_{j,t}^Z, \alpha_{j,t}^Q) \quad (P_{j,t})$$

$$(1.3) \quad X_{j,t} = CES(X_{ss,j,t}; \alpha_{ss,j,t}^X, \sigma_{j,t}^X) \quad (P_{j,t}^X)$$

$$(1.4) \quad Q_{j,t} = CD(K_{j,t}^{dem}, L_{j,t}^{dem}; SC_{j,t}^Q, \alpha_{j,t}^K) \quad (P_{j,t}^Q)$$

where:

$Z_{j,s,t}$: amount of good s produced by sector s in region j at time t ,

$\mathcal{A}_{ss,j,s,t}$: amount of intermediate inputs purchased by sector s (to produce good s) of region j from sector ss at time t ,

$K_{j,s,t}^{dem}$: capital demanded by sector s in region j at time t ,

$L_{j,s,t}^{dem}$: quantity of labor demanded by sector s in region j at time t ;

$P_{j,ss,t}$: average price for goods ss produced in region j at time t .

We denote by CD (\cdot ; S_c, α) a Cobb-Douglas form (parameterized by the scaling parameter S_c and the expenditure shares α) and by CES(\cdot ; α, σ) a constant-elasticity-of-substitution form (with share parameters α and substitution elasticity σ). The production of output $Z_{j,s,t}$ requires combining, in fixed expenditure shares, aggregate intermediate input and value added, respectively in amount $X_{j,s,t}, Q_{j,s,t}$. The aggregate intermediate input is itself a CES mix of market goods in quantities $\mathcal{A}_{ss,j,s,t}$. Value added is produced using $K_{j,s,t}^{dem}$ and $L_{j,s,t}^{dem}$ amounts of capital and labor services with fixed expenditure shares. Associated with each constraint of the firm's cost minimization problem are shadow prices, which are indicated in brackets.

Combining equations (1.1), (1.2), (1.3), (1.4), we find:⁸

$$(1.5) \quad \begin{cases} Z_{j,s,t} = SC_{j,s}^Z X_{j,s,t}^{1-\alpha_{j,s}^Q} Q_{j,s,t}^{\alpha_{j,s}^Q} \\ P_{j,s,t}^Y X_{j,s,t} = (1-\alpha_{j,s}^Q) P_{j,s,t} Z_{j,s,t} \\ P_{j,s,t}^Q Q_{j,s,t} = \alpha_{j,s}^Q P_{j,s,t} Z_{j,s,t} \end{cases}$$

$$(1.6) \quad \begin{cases} P_{j,s,t}^Y X_{j,s,t} = (1-\alpha_{j,s}^Q) P_{j,s,t} Z_{j,s,t} \\ P_{j,s,t}^Q Q_{j,s,t} = \alpha_{j,s}^Q P_{j,s,t} Z_{j,s,t} \end{cases}$$

$$(1.7) \quad \begin{cases} P_{j,s,t}^{1-\sigma_{j,s}^X} = \sum_{ss} \alpha_{ss,j,s}^{AS} P_{j,ss,t}^{1-\sigma_{j,s}^X} \\ AS_{ss,j,s,t} = \alpha_{ss,j,s}^{AS} \left[\frac{P_{j,s,t}^X}{P_{j,ss,t}^C} \right]^{\sigma_{j,s}^X} X_{j,s,t} \end{cases}$$

$$(1.8) \quad \begin{cases} Q_{j,s,t} = SC_{j,s}^Q K_{j,s,t}^{dem} \alpha_{j,s}^K L_{j,s,t}^{dem} 1-\alpha_{j,s}^K \\ r_{j,t}^K K_{j,s,t}^{dem} = \alpha_{j,s}^K P_{j,s,t}^Q Q_{j,s,t} \\ w_{j,t} L_{j,s,t}^{dem} = (1-\alpha_{j,s}^K) P_{j,s,t}^Q Q_{j,s,t} \end{cases}$$

$$(1.9) \quad \begin{cases} r_{j,t}^K K_{j,s,t}^{dem} = \alpha_{j,s}^K P_{j,s,t}^Q Q_{j,s,t} \\ w_{j,t} L_{j,s,t}^{dem} = (1-\alpha_{j,s}^K) P_{j,s,t}^Q Q_{j,s,t} \end{cases}$$

$$(1.10) \quad \begin{cases} Q_{j,s,t} = SC_{j,s}^Q K_{j,s,t}^{dem} \alpha_{j,s}^K L_{j,s,t}^{dem} 1-\alpha_{j,s}^K \\ r_{j,t}^K K_{j,s,t}^{dem} = \alpha_{j,s}^K P_{j,s,t}^Q Q_{j,s,t} \\ w_{j,t} L_{j,s,t}^{dem} = (1-\alpha_{j,s}^K) P_{j,s,t}^Q Q_{j,s,t} \end{cases}$$

3.2. Household of region j at time t

An Allais-Samuelson overlapping generations framework characterizes households, so that the model is based on the life-cycle theory of saving behavior. Each individual lives for six periods of ten years, retiring after five periods. In each period the oldest generation dies and a new generation enters the labor force, which implies that at any point in time six generations are alive. The working life starts at the age of 15; younger children are assumed to be fully dependent on their parents to which they constitute no extra burden nor provide any felicity. Perfect foresight individuals retire from the labor market at the age of 64 and die at age 74. Production, exchange and payments are assumed to be made at the end of each period.

⁸ See Appendix A.1, A.2, A.3.

In each region, a newborn individual's problem consists of, in a first step, chooses consumption levels and bequest that maximize its inter-temporal utility subject to a lifetime budget constraint. The utility function is time-separable and of the constant elasticity of substitution type:

$$(2.1) \quad U_{j,t} = \frac{1}{1-\theta} \sum_{g=1}^6 [1+\rho]^{-g} (C_{j,g,t+g-1}^{1-\theta} + \beta_g^\theta \text{Beq}_{j,g,t+g-1}^{1-\theta})$$

$$\theta > 0, \quad \beta_{g \neq 6} = 0, \quad \beta_{g=6} > 0,$$

Where:

$C_{j,g,t}$: consumption of an individual living in region j of age group g at time t ,

ρ : pure rate of time preference,

θ : inverse of inter-temporal elasticity of substitution,

β_g : constant parameter,

$\text{Beq}_{j,g,t}$: bequests (in real terms) of an individual living in region j of age group g at time t .

This equation states that the welfare of an individual is a weighted sum of 6 periods of consumption from age group $g=1$ at period t to age group $g=6$ at $t+5$, plus the (positive) utility of bequest for $g=6$ in period $t+5$. Leisure does not enter into the utility function since individual's labor supply is assumed to be exogenous. The bequest specification follows Blinder (1974) and gives rise to inter-generational transfers in addition to public old-age pension benefits. This specification of the utility function implies that the felicity from bequest is independent of the present value of cash receipts extending beyond the death of the current generation.

Assuming no-borrowing constraints and perfect capital markets, the present value of household wealth, is the discounted sum of lifetime labor income $LInc_{j,g,t}$, public old-age pensions $Pens_{j,g,t}$ and inheritance $Inh_{j,g,t}$:

$$(2.2) \quad W_{j,t} = \sum_{g=1}^6 \left[1 + r_{t+g-1} (1 - \tau^K) \right]^{t-g} \left(LInc_{j,g,t+g-1} (1 - \tau''_{j,t+g-1} - \tau^p_{t+g-1}) \right. \\ \left. + (1 - \tau''_{j,t+g-1}) (Inh_{j,g,t+g-1} + Pens_{j,g,t+g-1}) \right)$$

Where r_t is the rate of interest at time $t+g-1$, τ^K and τ'' are the tax rates on capital and labor income respectively, and τ^p is the contribution rate on the pay-as-you-go public pension program.

The present value of household spending, is the discounted sum of lifetime consumption and the bequest:

$$(2.3) \quad \sum_{g=1}^6 \left[1 + r_{t+g-1} (1 - \tau^K) \right]^{t-g} P_{j,g,t+g-1}^{Con} (C_{j,g,t+g-1} + \mu_g Beq_{j,g,t+g-1}) \\ \mu_{g=6} = 1, \mu_{g \neq 6} = 0$$

Labor income depends on the individual's age-dependent productivity (earnings) profile that is assumed identical across region. To be more precise, the earnings profile (EP_g) is quadratic function of age g .

$$(2.4) \quad EP_g = \gamma + \lambda g - \varphi g^2, \quad \gamma, \lambda, \varphi \geq 0,$$

with parametric values chosen to ensure that the maximum is reached between mid-life and retirement.

Pension benefits of the retirees (generation 6) are proportional to their lifetime labor earnings. The fraction is determined by the pension replacement rate $PensR_j$ that applies identically everywhere within the country. Pension benefits are thus equal to:

$$(2.5) \quad Pens_{j,6,t} = PensR_j \frac{1}{5} \sum_{gj} LInc_{j,g,t-6+gj}$$

Bequests are distributed at the end of each generation's lifetime (generation 6), equally across working generations gj as inheritances:

$$(2.6) \quad Inh_{j,g,t} Pop_{j,g,t} = \frac{1}{5} P_{j,6,t}^{Con} Beq_{j,6,t} Pop_{j,6,t}$$

where $Pop_{j,g,t}$ denotes the number of people living in region j of age group g at period t .

Note that $Inh_{j,g,t}$ is defined in current prices.

The population growth rate $n_{j,t}$ is treated as exogenous:

$$(2.7) \quad Pop_{j,g,t+1} = (1+n_{j,t}) Pop_{j,g,t}$$

Differentiating the household's utility function (2.1) with respect to its lifetime budget constraint (2.2) and (2.3) yields the following first-order conditions for consumption and bequests:⁹

$$(2.8) \quad C_{j,g+1,t+g} = \left[\frac{1+r_{t+g}(1-\tau^K)}{1+\rho_j} \frac{P_{j,g,t+g-1}^{Con}}{P_{j,g+1,t+g}^{Con}} \right]^{(1/\theta)} C_{j,g,t+g-1}$$

$$(2.9) \quad Beq_{j,g,t} = \mu_g \beta_g C_{j,g,t}$$

In a second optimization step, households must allocate their consumption expenditures between the available final goods s , the preferences of the households in region j with respect to good s are represented by a constant elasticity of substitution function (CES). The optimal composition of its consumption basket in terms of different goods is given by the solution of the following optimization problem:

$$(2.10) \quad \underset{ConS_{j,s,t}}{\text{Maximize}} C_{j,g,t} = \left[\sum_s \varepsilon_{j,s,g}^{ConS} ConS_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} \right]^{\frac{\sigma_{j,s}^{Con}}{\sigma_{j,s}^{Con-1}}}$$

⁹ See Appendix A.4

subject to:

$$(2.11) \quad P_{j,g,t}^{Con} C_{j,g,t} = \sum_s P_{j,s,t}^f Cons_{j,s,g,t}$$

The first order conditions imposes that:¹⁰

$$(2.12) \quad \left\{ \begin{array}{l} P_{j,g,t}^{Con} {}^{1-\sigma_{jg}^{Con}} = \sum_s \alpha_{j,s,g}^{ConS} P_{j,s,t}^f {}^{1-\sigma_{jg}^{Con}} \\ Cons_{j,s,g,t} = \alpha_{j,s,g}^{ConS} \left[\frac{P_{j,s,t}^{Con}}{P_{j,s,t}^f} \right]^{\sigma_{jg}^{Con}} C_{j,g,t} \end{array} \right.$$

$$\alpha_{j,s,g}^{ConS} = \varepsilon_{j,s,g}^{ConS} {}^{\sigma_{jg}^{Con}}$$

where $Cons_{j,s,g,t}$ denotes the household's consumption of good s , the price of which is $P_{j,s,t}^f$. Note that, because the composition of consumption baskets varies across generations - for instance, older generations consume more health services than young ones - the aggregate consumer price index is generation dependent.

The households consider products from different regions as imperfect substitutes. After the optimal level of each commodity s consumed is determined, the households of each region need to decide the optimal composition of its consumption basket in terms of geographic origin. We will talk about this later in section 5.

¹⁰ See Appendix A.5

Households in region j invest in physical capital $K_{ij,t}$ and in bonds $B_{ij,t}$ issued by firms and regional governments across the country. It is assumed that both asset holdings by local residents are characterized by home bias (captured by calibrated portfolio shares). Because we assume that all assets are perfect substitutes and traded on fully integrated international markets, the composition of household's wealth is however without consequence, except on impact after an unexpected shock.

3.3. Investors of regions j at time t

Capital goods are built using an investment technology that also allows for substitution between different market goods and investors have to solve the following problem to choose the optimal constituting mix:

$$(3.1) \quad \underset{InvS_{j,t}}{\text{Maximize}} \quad P_{j,t}^{Inv} Inv_{j,t} = \sum_s P_{j,s,t} InvS_{j,s,t}$$

subject to:

$$(3.2) \quad Inv_{j,t} = \left[\sum_s \varepsilon_{j,s}^{InvS} InvS_{j,s,t} \frac{\sigma_j^{Inv-1}}{\sigma_j^{Inv}} \right]^{\frac{\sigma_j^{Inv}}{\sigma_j^{Inv}-1}}$$

We get the following conditions:

$$(3.3) \quad \begin{cases} P_{j,t}^{Inv} (1-\sigma_j)^{Inv} = \sum_s \alpha_{j,s}^{InvS} P_{j,s,t}^c (1-\sigma_j)^{Inv} \\ (3.4) \quad \begin{cases} InvS_{j,s,t} = \alpha_{j,s}^{InvS} \left[\frac{P_{j,t}^{Inv}}{P_{j,s,t}^c} \right]^{\sigma_j^{Inv}} Inv_{j,t} \end{cases} \end{cases}$$

where $\alpha_{j,s}^{InvS} = \varepsilon_{j,s}^{InvS \sigma_j^{Inv}}$

The regional stock of physical capital broadens with investment $Inv_{j,t}$ but narrows with depreciation at a constant rate δ_j :

$$(3.5) \quad Kstock_{j,t+1} = Inv_{j,t} + (1-\delta_j) Kstock_{j,t}$$

The one period expected rate of return on a unit of physical capital bought at time t-1, denoted $r_{j,t}^e$, is then defined by the following expression:

$$(3.6) \quad r_{j,t}^e = \frac{r_{j,t}^K + (1-\delta_j) P_{j,t}^{Inv}}{P_{j,t-1}^{Inv}}$$

i.e., as its expected real rental price net of depreciation augmented by anticipated capital gains.

3.4. The government of region j at time t

The regional government taxes labor and capital incomes, as well as consumption expenditures. Its spending includes consumption $Gov_{j,t}$ and debt interest payments.

Government consumption spending is allocated across sectors using a CES aggregator:

$$(4.1) \quad Gov_{j,t} = \left[\sum_s \varepsilon_{j,s}^{GovS} GovS_{j,s,t} \frac{\sigma_j^{Gov-1}}{\sigma_j^{Gov}} \right]^{\frac{\sigma_j^{Gov}}{\sigma_j^{Gov}-1}}$$

We can get the following conditions:

$$(4.2) \quad \left\{ \begin{array}{l} P_{j,t}^{Gov} t^{-\sigma_j^{Gov}} = \sum_s \alpha_{j,s}^{GovS} P_{j,s,t}^c t^{-\sigma_j^{Gov}} \\ GovS_{j,s,t} = \alpha_{j,s}^{GovS} \left[\frac{P_{j,t}^{Gov}}{P_{j,s,t}^c} \right]^{\sigma_j^{Gov}} Gov_{j,t} \end{array} \right.$$

where $\alpha_{j,s}^{GovS} = \varepsilon_{j,s}^{GovS} \sigma_j^{Gov}$

To satisfy its budget constraint when tax revenues come short of expenditures, the government issues new bonds. Accordingly, the budget constraint of the government is:

$$\begin{aligned}
 (4.4) \quad & P_{j,t}^{Gov} Bond_{j,t+1} + \sum_g Pop_{j,g,t} \left\{ \tau_{j,t}^w (LInc_{j,g,t} + Pens_{j,g,t}) + \tau_{j,t}^{Con} P_{j,t}^{Con} C_{j,g,t} + \right. \\
 & \left. \tau_{j,t}^K \sum_i \left[\frac{r_{i,t-1}^j P_{i,t}^{Gov}}{P_{i,t-1}^{Gov}} - 1 \right] P_{i,t-1}^{Gov} Bij_{i,j,g,t} + \tau_{j,t}^K \sum_i (r_{i,t-1}^e - 1) P_{i,t-1}^{Inv} Kij_{i,j,g,t} \right. \\
 & \left. = P_{j,t}^{Gov} Gov_{j,t} + \left[\frac{r_{i,t-1}^j P_{j,t}^{Gov}}{P_{j,t-1}^{Gov}} \right] P_{j,t-1}^{Gov} Bond_{j,t} \right.
 \end{aligned}$$

Pay-as-you-go pension benefits are financed by contribution rates on wage earnings. With population ageing, the contribution rate is expected to rise. The program is nation-wide:

$$(4.5) \quad \sum_j Pop_{j,g,t} Pens_{j,g,t} = \tau_t^p \sum_j \sum_{g'} Pop_{j,g',t} LInc_{j,g',t}$$

3.5. Foreign trade of region j at time t

All agents within region j make use of a composite good indexed s , which is priced at $P_{j,s,t}$. The aggregate demand $D_{j,s,t}$ for this good is defined by adding-up all individual demands:

$$(5.1) \quad D_{j,s,t} = \sum_{ss} AS_{s,j,ss,t} + \sum_g Pop_{j,g,t} Cons_{j,s,g,t} + InvS_{j,s,t} + GovS_{j,s,t}$$

We make use of the traditional Armington assumption to allocate this demand between regions. That is, even though individual producers are microscopic price takers, goods of sector s are assumed differentiated in demand by their geographic origin. A fictitious

importer accordingly chooses the optimal basket of domestic and interregional / international goods in each sector, assuming a $CES (E_{ii,j,s,t}; \alpha_{ii,j,s}^E, \sigma_{j,s}^E)$ aggregator.

$$(5.2) \quad D_{j,s,t} = \left[\sum_s \varepsilon_{ii,j,s}^E E_{ii,j,s,t} \frac{\sigma_{j,s}^E - 1}{\sigma_{j,s}^E} \right]^{\frac{\sigma_{j,s}^E}{\sigma_{j,s}^E - 1}}$$

The price $P_{j,s,t}^c$ can be then be expressed as a function of each supplying region's producer price $P_{j,s,t}$:

$$(5.3) \quad P_{j,s,t}^c \quad 1 - \sigma_{j,s}^E = \sum_{ii} \alpha_{ii,j,s}^E P_{ii,s,t} \quad 1 - \sigma_{j,s}^E, \quad \alpha_{i,j,s}^E = \varepsilon_{i,j,s}^E \sigma_{j,s}^E$$

and the associated demand system is :

$$(5.4) \quad E_{ii,j,s,t} = \alpha_{ii,j,s}^E \left(\frac{P_{j,s,t}^c}{P_{ii,s,t}} \right)^{\sigma_{j,s}^E} \left\{ \sum_{ss} X_{s,j,ss,t} + \sum_g Pop_{j,g,t} Cons_{j,s,g,t} + InvS_{j,s,t} + GovS_{j,s,t} \right\}$$

Where:

$E_{ij,s,t}$: demand of region j for good s produced in ii at time t,

$D_{j,s,t}$: total demand in region j for good s at time t,

$P_{j,s,t}$: aggregate consumer price index of good s in region j at time t,

$P_{j,s,t}$: price of good s produced in region j at time t.

3.6. *The rest of the world at time t*

The rest of the world only serves to close the model, and is described by a reduced form: its prices and income are exogenously held constant. Its demand for region j 's good s therefore depends on the region's sectorial competitiveness:

$$(6.1) \quad E_{j,row,s,t} = SC_{j,row,s}^E \left[\frac{P_{row,s,t}}{P_{j,s,t}} \right]^{\eta_s} \quad \eta_s > 0$$

where:

$E_{j,row,s,t}$: demand of the rest of the world for the good produced in region j at time t

$SC_{j,row,s}^E$: share parameter,

$P_{row,s,t}$: price of good s produced in the rest of the world,

η_s : substitution elasticity parameter.

Consistent with the reduced form description of the rest of the world, we assume that it does neither borrow nor lend internationally, so that its trade with the country as a whole is calibrated as balanced, and remains such at all time:

$$(6.2) \quad \sum_{ii} \sum_s P_{ii,s,t} E_{ii,rows,t} = \sum_s P_{rows,t} \sum_{ii} E_{rows,t}$$

3.7. Equilibrium conditions

Market clearing for goods:

$$(7.1) \quad Z_{j,s,t} = \sum_{ii} E_{j,ii,s,t}$$

Full employment of capital:

$$(7.2) \quad Kstock_{j,t} = \sum_s K_{j,s,t}^{dem}$$

Fully integrated asset markets:

$$(7.3) \quad \frac{r_{j,t}^j P_{j,t+1}^{Gov}}{P_{j,t}^{Gov}} = r_{j,t+1}^e$$

$$(7.4) \quad r_t = \frac{r_{j,t}^j P_{j,t+1}^{Gov}}{P_{j,t}^{Gov}}$$

This completes the model description. It is easy to check that the model implies asset market clearing at each t .

$$(7.5) \quad \sum_j \sum_g Pop_{j,g,t} Lend_{j,g+1,t+1} = \sum_j P_{j,t}^{Gov} Bond_{j,t+1} + P_{j,t}^{hw} Kstock_{j,t+1}$$

Prices of the rest of the world are chosen as numeraire. A static version of the model is easily found by setting $t-1 = t = t+1$ in all equations.

4. THE RESULTS

Value of parameters and Calibration

The model comprises three regions: region 1, region 2 which are within a certain country and the rest of the world. In each region, there are two production sectors and six generations. We simulate the model with the configuration of parameters and initial conditions given in Table 4.1, Table 4.2 and Table 4.3. Many parameters are assumed identical across the country. This is, in particular, the case for household preferences so that both intertemporal and inter-regional elasticity of substitution, as well as bequest rates are common to both regions. The common parameters are reported in Table 4.1 below. The value of the inter-temporal elasticity of substitution $1/\theta$ (0.15) is slightly lower than the 0.25 used by Auerbach and Kotlikoff (1987). No estimates of the inter-regional elasticities of substitution are available in the literature; we chose values of 2.50, 3.00, and 3.50 for consumption, investment and government respectively. Note, however, that terms of trade effects will strongly depend on these values, so that later work should assess the sensitivity of the results with respect to the chosen value of this parameter. The pension replacement rate ($PensR_j$) was set to 0.497, the bequest parameter (β_g) to 0.50 whereas the inheritance rate ($InhR$) equals 0.20 as private bequests are assumed equally distributed among the working age groups. The elasticity of substitution for intermediate goods ($\sigma_{j,s}^x$) is set equal to 3.00, also in all regions and for all sectors. Finally, the calibrated depreciation rate equals 0.30, whereas the export price elasticity (η_s) to the rest of the world are set to 3.00 in all regions.

Table 4.1 Parameters Common to both Regions

ρ	-0.181139	$\frac{I}{\theta}$	0.15
$\sigma_{j,g}^{Con}$	2.50	σ_j^{Inv}	3.00
σ_j^{Gov}	3.50	$\sigma_{j,s}^E$	1.50
$PensR_j$	0.496550	β_g	0.50
$InhR$	0.20		
δ_j	0.30	$\sigma_{j,s}^X$	3.00
$\tau_{j,t}^K$	0.25	$\tau_{j,t}^{Con}$	0.25
η_s	3.00		

Preferences are assumed perfectly identical across the country. Hence, the shares of income allocated to the various industrial goods (spending shares) are assumed identical for both regions. Consumption share parameters ($\alpha_{j,s,g}^{ConS}$), which may alter along the lifecycle, are also assumed to be the same across generations for simplicity. Later work should differentiate the spending shares for the six age groups. Though individual producers are microscopic price takers, goods of sector s are assumed differentiated in demand by their geographic origin. It is assumed that the demand is characterized by

home bias, which is captured by the calibrated portfolio shares ($\alpha_{ii,j,s}^E$). Once a balanced social accounting matrix has been constructed, the calibration of the share parameters of the technology and utility functions is a trivial matter. These parameters are reported in Table 4.2.

Table 4.2 Share parameters

$\alpha_{j,s,g}^{Cons}$		0.50
$\alpha_{j,s}^{Inv}$		0.50
$\alpha_{j,s}^{Govt}$		0.50
Value added share ($\alpha_{j,s}^Q$)		0.50
Intermediate input share ($\alpha_{ss,j,s}^{IS}$)		0.50
Capital income share ($\alpha_{j,s}^K$)		0.40
$\alpha_{ii,j,s}^E$	$\alpha_{1,1,s}^E \quad \alpha_{2,2,s}^E$	0.85
	$\alpha_{2,1,s}^E \quad \alpha_{1,2,s}^E$	0.066667
	$\alpha_{3,1,s}^E \quad \alpha_{3,2,s}^E$	0.083333

Both countries start with a total population of 1 and the same level of capital stock. Thus, the relative price of commodity 2 in terms of commodity 1 is the same and equal to 1. As financial assets are perfectly substitutable and mobile across regions, a single interest rate applies in all regions. The calibrated rate is 1.3 percent.

Table 4.3 Initial Conditions

Initial Price	Population	Dependency ratio	Interest rate	Wage
1	1	0.2	1.3	1

Simulation Analysis

In this section, we present estimates and analysis of economic effects of ageing for 45 periods within three simulation scenarios. We first present “reference scenario” with 0 per cent population growth rate for both regions. In the second scenario, we introduce a demographic shock and, by comparing with the reference scenario, present the macroeconomic effects of ageing. Finally, in order to investigate possible general equilibrium effects that population ageing in one region might inflict upon the partner region and own economy, we assume different population growth rates for the two regions for certain periods in our third scenario. The details are given in Table 4.4.

Table 4.4 Population growth rates in different scenarios

Periods	Scenario 1		Scenario 2		Scenario 3	
	n_A	n_B	n_A	n_B	n_A	n_B
T1-T6	0.00	0.00	0.01	0.01	0.01	0.06
T7-T45	0.00	0.00	0.00	0.00	0.00	0.00

1. *Comparisons between Scenario 1 and Scenario 2.*

Since in these two scenarios, the regions considered (region A and region B) are assumed to be exactly the same in every respect, our outcome is similar for both regions.

Even though both regions start with the same initial population and endowments of capital and labor under the two scenarios, increases in population growth rates under scenario 2 make them less capital abundant. This is because of two reasons: First, The labor supply under scenario 2 increases faster than scenario 1 (Figure 1). Secondly, since only the young generations bear children, the share of youngs in the entire population under scenario 2 exceeds that under scenario 1 after the first period (Figure 2)¹¹. This, in turn, implies that a greater portion of the population under scenario 2 contributes to the capital accumulation process, which further changes the relative endowments.

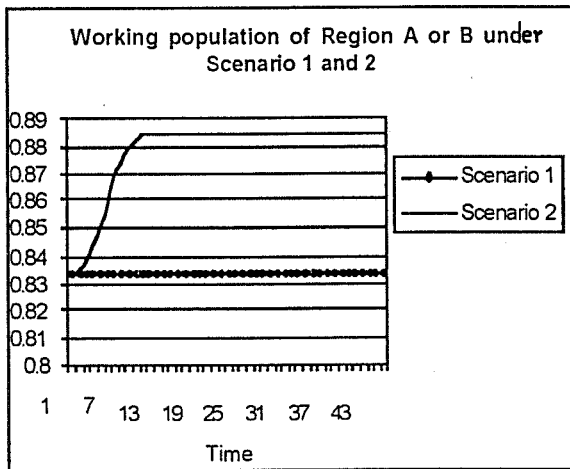


Figure 1

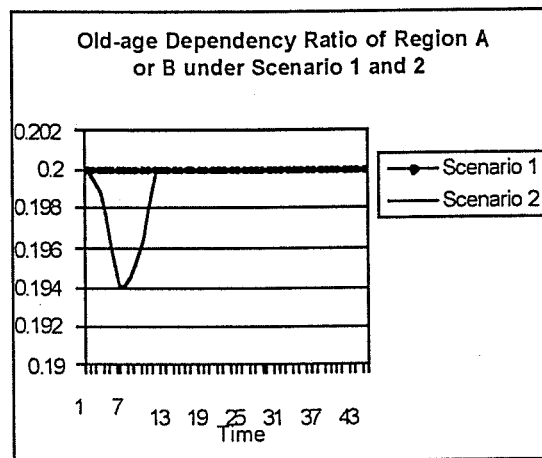


Figure 2

¹¹ Old-age dependency ratio in our model is defined as the ratio of the population who are the sixth generation (retirees) to those who are the first five generations (working generations).

As it is seen in Figure 3, starting from the first period until the steady state (period 43), capital stock per capita is greater under scenario 1 than scenario 2, at any point in time on the transition path as well as at the steady state.

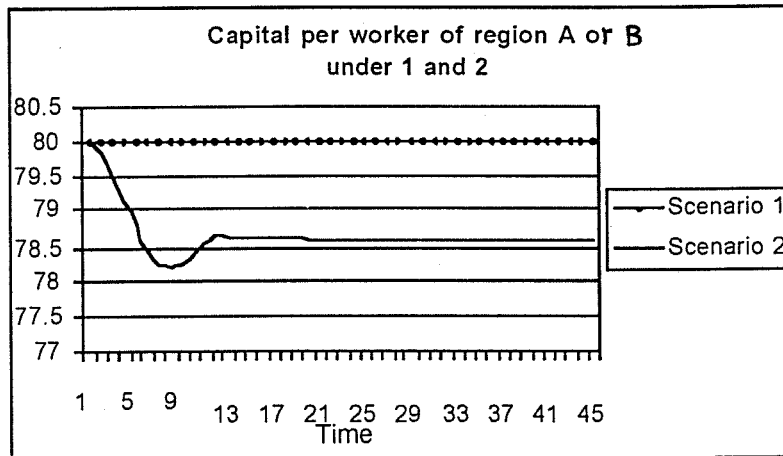


Figure 3

While scenario 1 becomes advantageous in terms of per capita variables, the aggregate production remains smaller compared to aggregate production under scenario 2. The slowdown in output growth reduces the growth in the capital stock and so the proportion of output which needs to be devoted to investment. On the other hand, if savings behavior conforms to the traditional “life-cycle” hypotheses, then ageing population (scenario 1) is likely to lead to a lower private savings ratio. The corresponding graphs for capital stock and savings under both scenarios are given in figure 4.

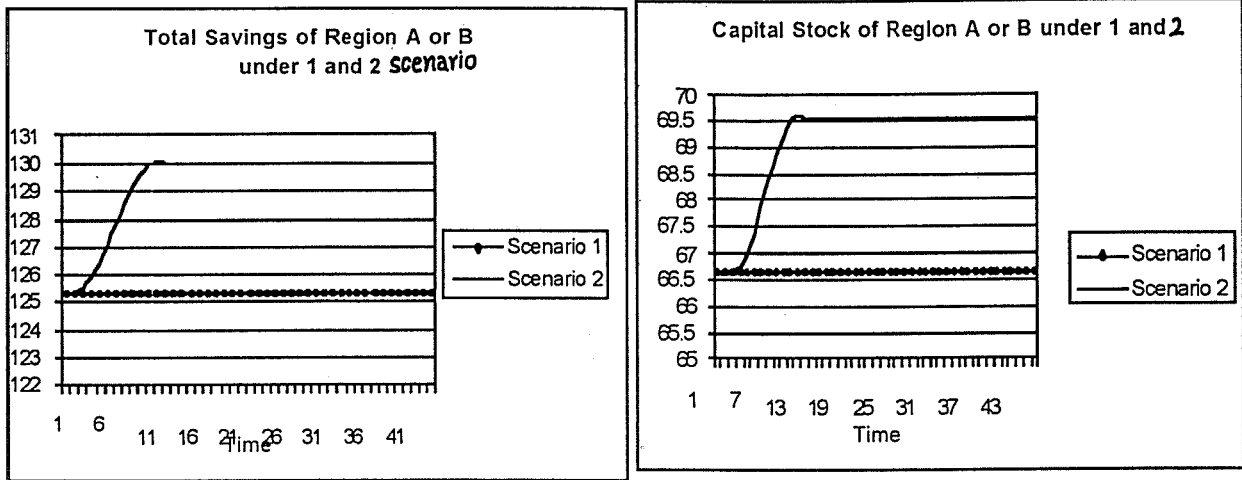


Figure 4

As shown, in the figure, both values of savings and capital stock under scenario 2 exceed those in scenario 1 immediately after the initial period, and remain higher until steady state is achieved, due to the faster population growth under scenario 2. We can see also that the downward pressures on investment exceed those on savings, as reflected in a relatively lower interest rates in scenario 1 (Figure 5).

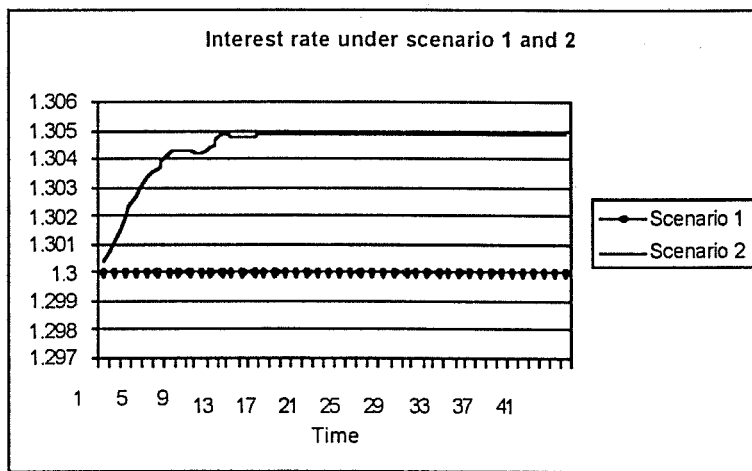


Figure 5

Figure 6, 7 contain the corresponding graphs for wage and price in both scenarios. Because in each region, the sectors are symmetric, reporting changes in sector 1 would be sufficient.

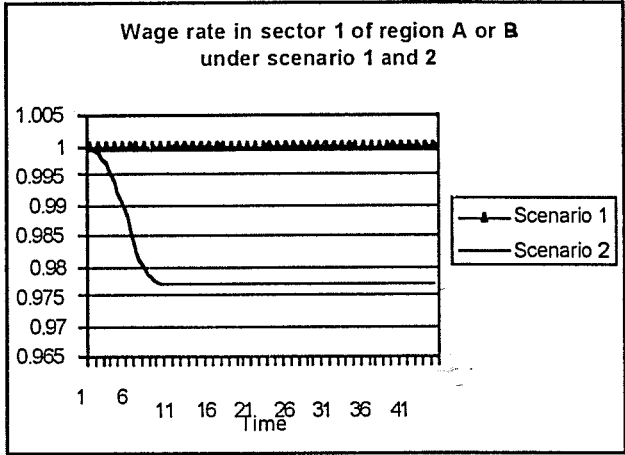


Figure 6

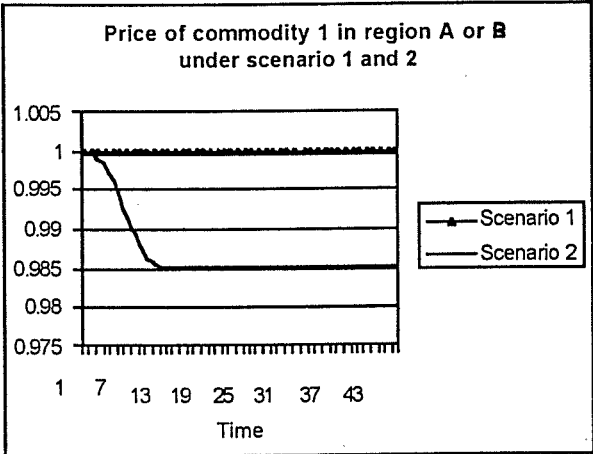


Figure 7

As expected, both wage rate and price in the low population scenario (scenario 1) are always greater than those in the high population growth scenario at any time.

As a result of the changes in relative commodity, factor prices as well as relative endowments, per capita GNP under scenario 1 is greater than that under scenario 2 in each period (Figure 8). This in turn affects consumption patterns. As seen in figure 9, per capita consumption of both commodities is higher in scenario 1 than in 2.

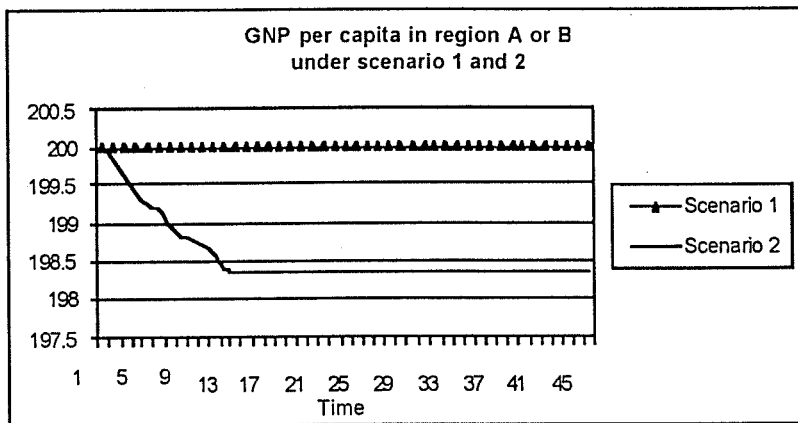


Figure 8

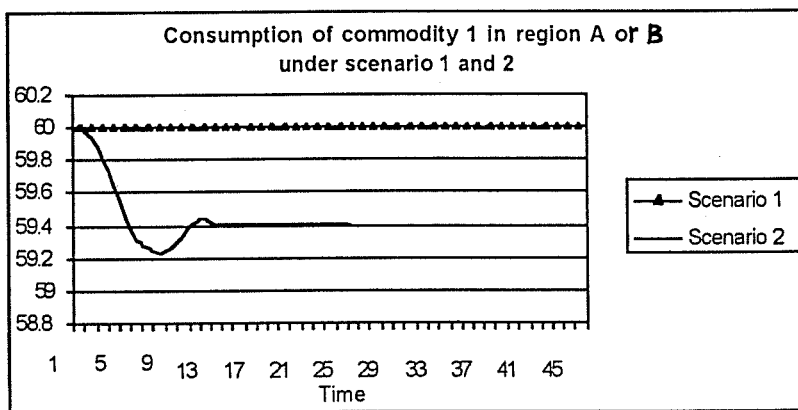
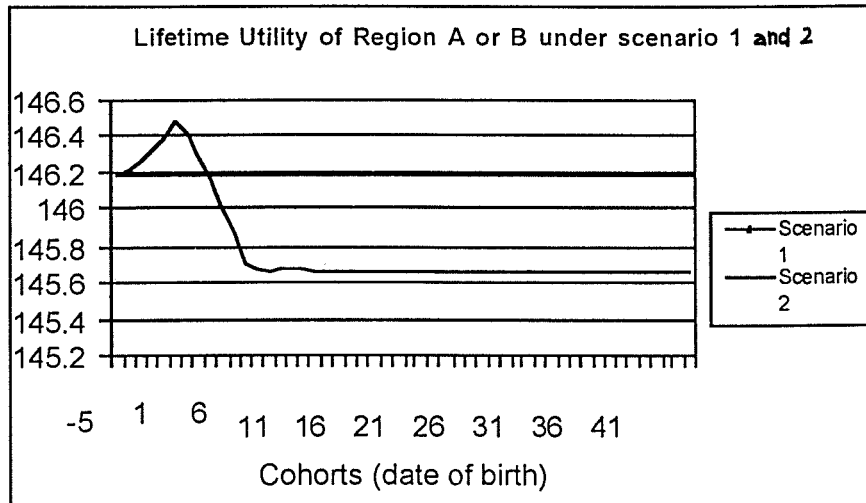


Figure 9

The issue, which is perhaps of the most concern, is the effect which ageing may ultimately have on living standard. In terms of lifetime utility, Figure 10 shows comparison of time paths of U_t ¹² (defined as the sum of six periods' utility of a person born in period t) under the two scenarios.

¹² U_t of negative period (from -1 to -5) refers to the lifetime utility of a person who is born at a period before our starting point (period 1).

Figure 10



In the initial periods, where there is a demographic shock, people under scenario 2 actually benefit from increases of the population growth rate. A larger labor force implies the scarcity of physical capital relative to labor and leads to a higher capital return and lower wage rate. Since the last period of life is assumed to be retirement time, those born before period 1 do not suffer much from lower wages and benefit from higher capital return from their savings because of the increasing interest rates. Consequently, the increased population growth rate raises the value of lifetime utility for those born around period 1. After that, the effects of lower wages result in a slower capital accumulation process. The effect gets stronger as time passes and quickly dominates the positive effect coming from higher interest rates, eventually causing the lifetime utility to drop below that under scenario 1.

2. Comparisons between Scenario 2 and Scenario 3.

In this section, we consider the results of different population growth rates for the two regions. Assuming 1 per cent population growth rate for region A, but 6 per cent for region B, we compare region A under scenario 2 with that under scenario 3 and observe the paths that endogenous variables of region A would follow when A has a fast-growing economic partner (region B).

Under scenario 3, resulting from the high population growth rate in region B from period 1 to period 6, the population of the two regions diverge until period 11 and the population of region B remains higher after that (Figure 11).

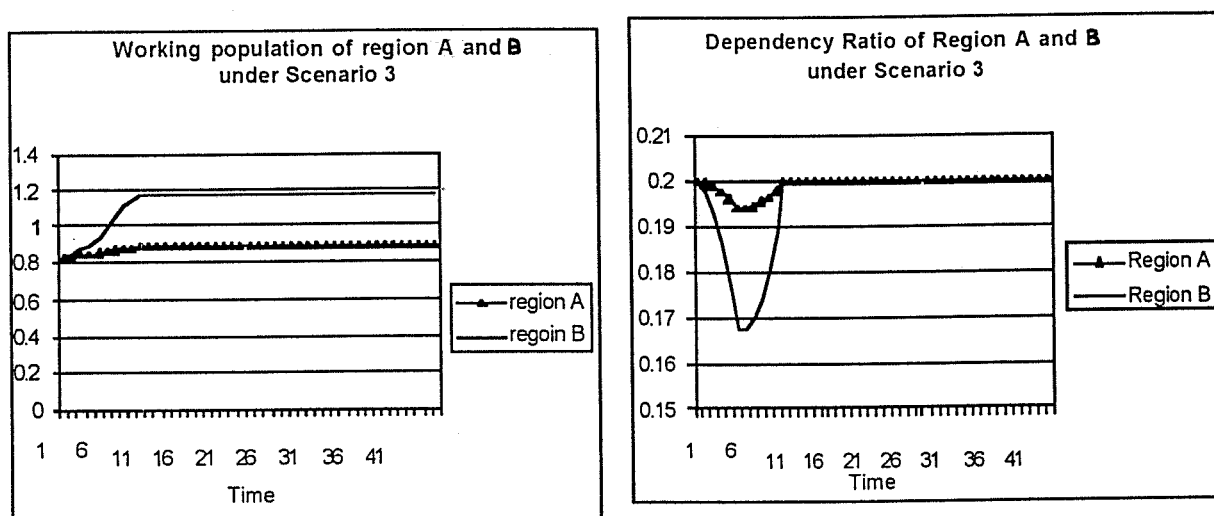


Figure 11

As a consequence, capital for the whole country (including region A and region B) is relative scarce and thus the interest rate (Figure 12) is higher compared to that under scenario 2. Figure 13 plots the difference in the interest rates between the two scenarios: The difference reaches its maximum of 0.015 in period 21 and then slightly decreases.

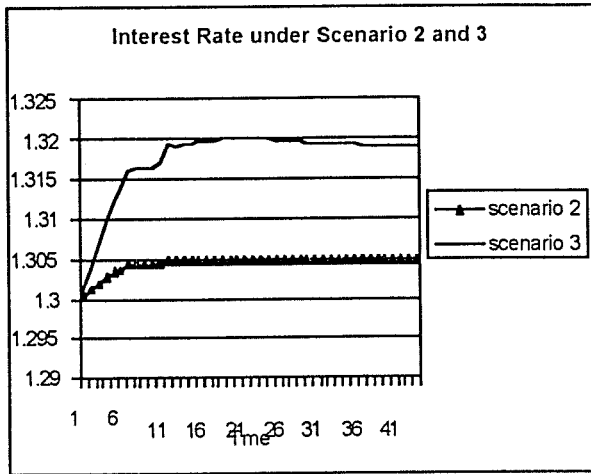


Figure 12

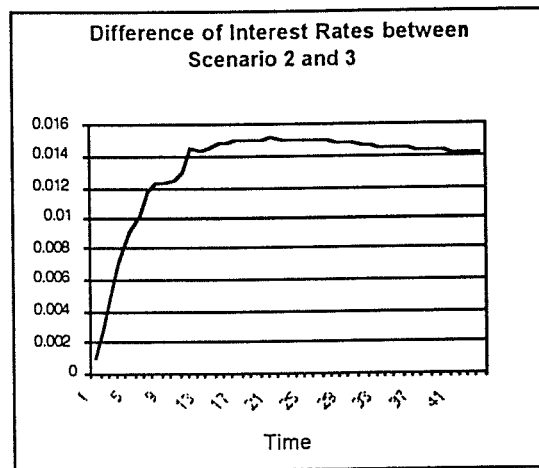


Figure 13

Now, focusing on region A, we find that under scenario 3, although A is advantageous in terms of per capita variables, its aggregate production and demand becomes smaller when compared to region B. As time passes, its contribution to the world economy becomes less important (Figure 14).

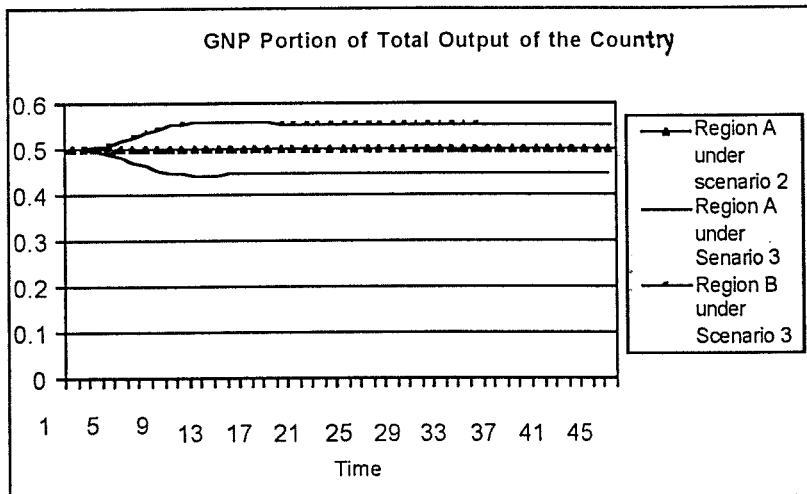


Figure 14

As shown in Figure 15, under scenario 3, capital per worker in region A is higher than that in region B in any period, but it is smaller than the value it would have attained under scenario 2 until period 18, and then becomes greater than scenario 2 value. The difference in capital per worker of region A under the two scenarios is plot in Figure 16.

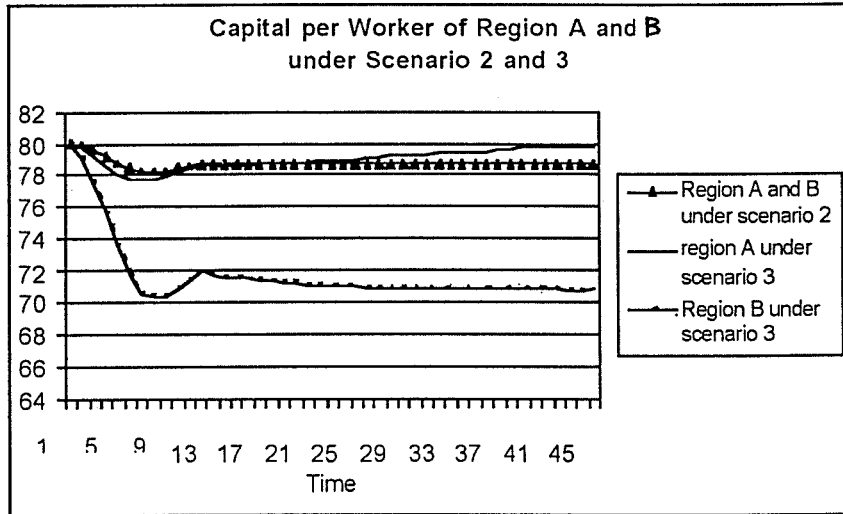


Figure 15

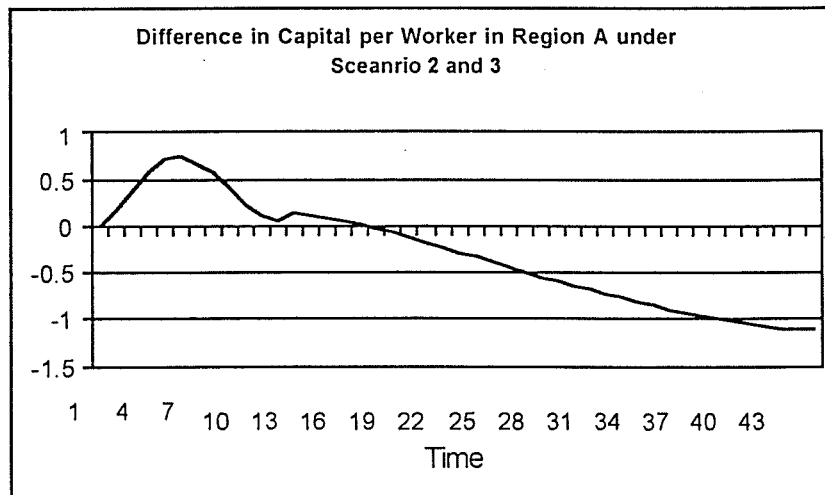
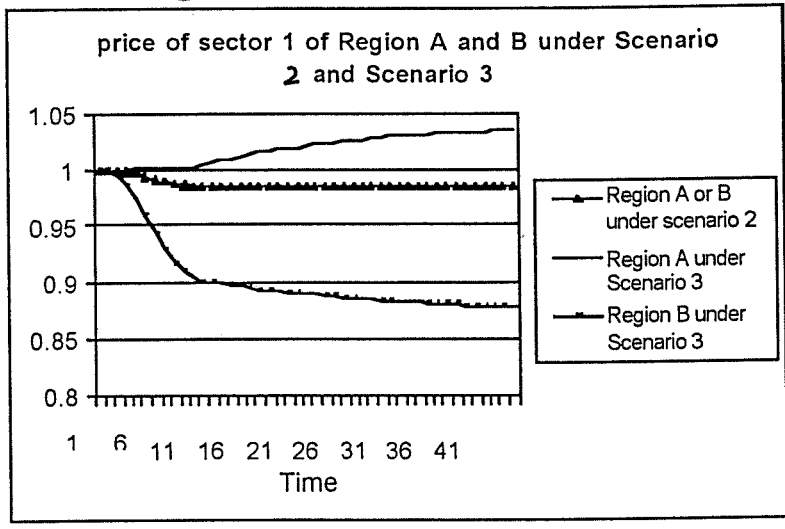


Figure 16

The reasons are as following: before period 22, higher interest rate under scenario 3 compared to scenario 2 increase the cost of the production sector and thus the optimal

size of the capital stock becomes smaller. However, the subsequent decrease in the difference in interest rates under the two scenarios weakens this effect. On the other hand, price of region B under scenario 3 is lower than that under scenario 2, whereas price of region A under scenario 3 is greater than scenario 2 (Figure 18). By trading with region B and rest of the world, region A has higher output under scenario 3 than that under scenario 2, which in turn leads to higher capital stock. This effect gets stronger as time passes and eventually dominates the process, causing the value of capital per capita of region A under scenario 3 to exceed that under scenario 2.

Figure 17



As a result of higher per capita GNP of region A under scenario 3 compared to scenario 2, per capita consumption of both commodities is higher under scenario 3 than under scenario 2. Figure 18 shows the graphs of lifetime utility under the two scenarios. Under scenario 3, the utility of a person in region A keeps increasing and is always higher than

that under scenario 2 throughout the time path. The benefits that region A gains from trading with a fast-growing region (region B) are clearly visible.

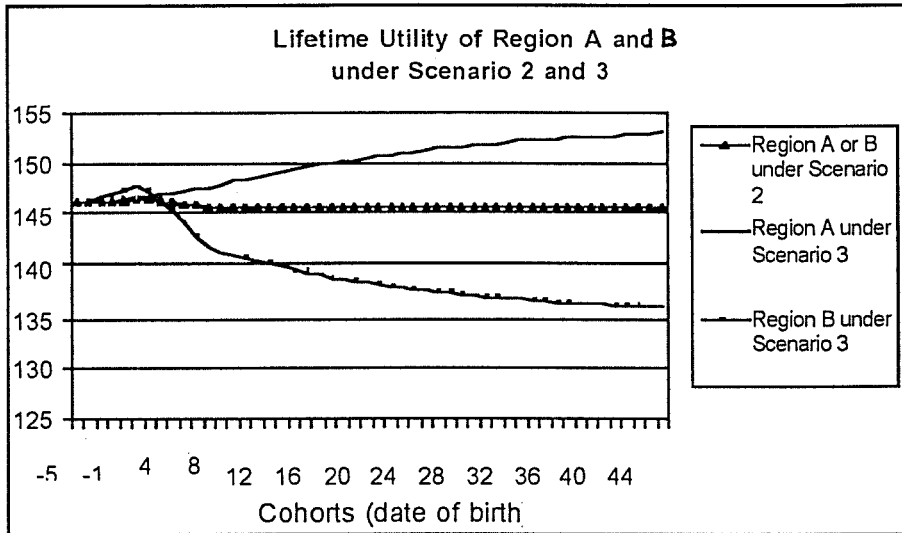


Figure 18

The changes in lifetime utility of region B under scenario 3 are affected by two factors: One is the higher population growth rate, which will finally pull down the utility below the value of scenario 2, as shown in previous section; the other is the trade between the two regions, which increases the benefits of region B (see Appendix B).

5. CONCLUSION

Demographic projections indicate that the dependency ratio of most industrial countries will increase considerably in the next several decades as a result of the fertility shock of the late 1940s and the 1950s. This prospective dramatic shift in the age composition of these countries will lead to profound macroeconomic as well as microeconomic consequences. Since countries become more and more interdependent, trade must be expected to act as an important channel for transmission of the effects of demographic shocks onto other countries through changing relative prices. Similarly, the changes in interest rates in major suppliers of foreign capital to the rest of the world are also likely to serve to this transmission. While the implications of the countries' own demographic transition have been investigated somewhat thoroughly in the literature, the spillover effects arising from trade and capital flows have been largely overlooked so far.

To this end, our paper aims to address the implications of ageing as well as the transmission issue by examining the effects of differing population growth rates across two regions. The magnitude and direction of these effects were investigated within a dynamic, OGS, general equilibrium framework that comprises three regions and two industrial sectors.

Simulation results reported in this paper suggest that the demographic developments leading to population ageing could have significant effects on economies. As discussed in part one of our result section, where we assume 0 per cent population growth rate for both regions under scenario 1 (ageing scenario) while 1 per cent under scenario 2 (fast-growing scenario), population ageing slows down the output growth despite the capital-abundance advantage in terms of capital per worker. There is a reduction in both investment and savings and our results suggest that the downward pressures on

investment exceed those on savings, causing lower interest rates relative to scenario 2. As a result of higher capital/population ratio, we have higher wage rate in ageing scenario. Moreover, the results report a higher level of lifetime utility for the representative consumer in ageing scenario.

Moving on to the results regarding transmission issue, we find that for a region where gradual ageing of population is well under way, trading with a fast-growing region could have important implications for the economy. In scenario 3, we assume different population growth rates for the two regions and interpret region A as ageing region and B fast-growing region. The slowdown in the growth rate of working-age population and, thereby, a shrinking labor force in region A contributes to a decline in economic growth compared to region B. This leads to changes in the relative sizes of the two regions in the world economy. As clearly shown in our analysis, region A's contribution to the world economy becomes less important as time passes.

Our results also suggest that ageing region (region A) will be better off trading with the fast-growing region (region B). Compared to the economic variables under scenario 2, region A has higher output per capita and member of this region has a higher lifetime utility under scenario 3. At a first glance, region B under scenario 3 generally has a lower lifetime utility than scenario 2. However, aside the population effect (which finally pull down the utility), we find that, in line with the static Heckscher-Ohlin (HO) theory, which rules out welfare losses for either of the countries involved in free trade, region B actually also benefits from free trade (as shown in Appendix B). This points to a need on both the part of developing countries (usually fast-growing countries) and industrial countries to further the structural changes in their product, labor and capital markets and facilitate the process of globalization.

Further study on the issue may consist of considering the investment in human capital. Population ageing may result in an increase in wages relative to interest rates, which would promote an accumulation and deepening of human capital. Moreover, our analysis is based on full employment assumption. That is, the increased population can be fully absorbed by the labor market. Later research should take into account unemployment issue.

REFERENCES

AUERBACH, ALAN J. AND LAURENCE J. KOTLIKOFF (1987): *Dynamic Fiscal Policy*, Cambridge University Press.

BÖRSCH-SUPAN, AXEL, ALEXANDER LUDWIG AND JOACHIM WINTER (2003): "Ageing, Pension Reform and Capital Flows: A Multi-country Simulation Model," *Discussion Paper 01-08*, University of Mannheim, January.

GROUP OF TEN (1998): "The Macroeconomic and Financial Implications of Ageing Populations," *Working Paper*, IMF, April.

HAGEMANN, ROBERT AND GUISEPPE NICOLETTI (1989), "Population Ageing: Economic Effects and Some Policy Implications for Financing Public Pensions," *OECD Economic Studies* No. 12, Spring.

KOHL, RICHARD AND PAUL O'BRIEN (1998): "The Macroeconomics of Ageing, Pensions and Savings: A Survey," *Economics Department Working Papers* NO. 200, June.

LAVOIE, CLAUDE, MARCEL MÉRETTE AND MOKHTAR SOUISSI (2001): "A Multi-Sector Multi-Country Dynamic General Equilibrium Model with Imperfect Competition," *Department of Finance Working Paper*, Ottawa, October.

LINDEBOOM, MAARTEN (1998): "Microeconomic Analysis of the Retirement Decision: the Netherlands," *Economics Department Working Papers*, NO. 207, June.

MARCEL MÉRETTE (2002): "The Bright Side: A Positive View on the Economics of Ageing," *IRPP*, Vol.8, no.1 March. Available at www.irpp.org

MAXIME FOUGÈRE AND MARCEL MÉRETTE (2000): "Population Ageing, Intergenerational Equity, and Growth: Analysis with an Endogenous Growth Overlapping-Generations Model," *Contribution to Economic Analysis*, North-Holland, pp. 325-363.

MERCENIER, JEAN AND MARCEL MÉRETTE (2002): "The Microeconomic Impact of Ageing on Industrial Sectors and Occupations in Canada," *Manuscript*, August.

MILES, DAVID (1999): "Modeling the Impact of Demographic Change upon the Economy," *Economic Journal*, Vol. 109, pp.1-36.

MYLONAS, PAUL AND CHRISTINE D. MAISONNEUVE (1999): "The Problems and Prospects Faced by Pay-as-you-go Pension Systems: A Case Study of Greece," *Economics Department Working Papers*, NO. 215, May.

NIALL, FERGUSON, AND LAURENCE J. KOTLIKOFF (2000): "The Degeneration of EMU," *Foreign Affairs*, Vol. 79, No. 2. March.

ROBSON, WILLIAM B.O. (2003): "Time and Money: The Fiscal Impact of Demographic Change in Canada," *C.D Howe Institute Commentary*, July.

ROMER, PAUL M. (1990): "Capital, Labor and Productivity," *Brookings Papers on Economic Activity*, pp. 337 – 347.

SAYAN, SERDAR (2001): "Dynamic Heckscher-Ohlin Results from a 2x2x2 Overlapping Generations Model with Unequal Population Growth Rates," *Economic Department Discussion Paper*, Bilkent University, Turkey.

SAYAN, SERDAR AND ALI E. UYAR (2002): "Directions of Trade Flows and Labor Movements Between High-and Low-Population Growth Countries: An Overlapping Generations General Equilibrium Analysis," *Economic Department Working Paper*, Bilkent University.

SCHIEBER, S. AND P. HEWITT (2000): "Demographic Risk in Industrial Societies, Independent Population Forecasts for The G7 Countries", *World Economics*, Vol. 1, No. 4.

THAN DANG, THAI, P. ANTOLIN AND HOWARD OXLEY (2001): "Fiscal Implications of Ageing: Projections of Age-Related Spending," *Economics Department Working Papers* No. 305, OECD, September.

TURALAY KENC AND SERDAR SAYAN (1999): "Demographic Shock Transmission from Large to Small Countries – An Overlapping Generations CGE analysis," *Journal of Policy Modeling*.

TURNER, DAVE, CLAUDE GIORNO, ALAIN D. SERRES, ANN VOUREH AND PETE RICHARDSON (1998): "The Macroeconomic Implications of Ageing in a Global Context," *Economics Department Working Papers* No. 193, OECD, March.

WANG, YAN, DIANQING XU, ZHI WANG AND FAN ZHAI (2001): "Implicit Pension Debt, Transition Cost, Options, and Impact of China's Pension Reform: A Computable General Equilibrium Analysis," *Policy Research Working Paper 2555*, February.

WATTENBERG, BEN J. (1987): "The Saving of the Elderly in Micro and Macro data," *Quarterly Journal of Economics*, Vol. 109.

APPENDIX A

A.1. Allocation between intermediate input $X_{j,s,t}$ and value added $Q_{j,s,t}$

$$(A.1.1) \quad \text{Minimize} \quad \sum_{ss} P_{j,ss,t} X_{ss,j,s,t} + r_{j,t}^K K_{j,s,t}^{dem} + w_{j,t} L_{j,s,t}$$

subject to:

$$(A.1.2) \quad Z_{j,s,t} = SC_{j,s}^Z X_{j,s,t}^{1-\alpha_{j,s}^Q} Q_{j,s,t}^{\alpha_{j,s}^Q}$$

$$(A.1.3) \quad P_{j,s,t}^X X_{j,s,t} = \sum_{ss} P_{j,ss,t} X_{ss,j,s,t}$$

$$(A.1.4) \quad P_{j,s,t}^Q Q_{j,s,t} = r_{j,t}^K K_{j,s,t}^{dem} + w_{j,t} L_{j,s,t}$$

The first order condition using equations (A.1.1), (A.1.2) and (A.1.3) for a given good s takes the following form:

$$P_{j,s,t}^X = \lambda_l SC_{j,s,t}^Z (1-\alpha_{j,s}^Q) X_{j,s,t}^{-\alpha_{j,s}^Q} Q_{j,s,t}^{\alpha_{j,s}^Q}$$

Multiplying both sides by $X_{j,s,t}$:

$$(A.1.5) \quad P_{j,s,t}^X X_{j,s,t} = \lambda_l (1-\alpha_{j,s}^Q) Z_{j,s,t}$$

λ_l is the shadow price of good s , $\lambda_l = P_{j,s,t}$

Replace λ_1 in equation (A.1.5), we have:

$$P_{j,s,t}^r X_{j,s,t} = (1 - \alpha_{j,s}^\theta) P_{j,s,t} Z_{j,s,t}$$

The first order condition using equations (A.1.1), (A.1.2) and (A.1.4) for a given good s takes the following form:

$$P_{j,s,t}^\theta = \lambda_1 S C_{j,s,t}^z \alpha_{j,s}^\theta X_{j,s,t}^{1-\alpha_{j,s}^\theta} Q_{j,s,t}^{\alpha_{j,s}^\theta - 1}$$

Multiplying both sides by $Q_{j,s,t}$:

$$(A.1.6) \quad P_{j,s,t}^r Q_{j,s,t} = \lambda_2 \alpha_{j,s}^\theta Z_{j,s,t}$$

λ_1 is the shadow price of good s and $\lambda_1 = P_{j,s,t}$

Replace λ_2 in equation (A.1.6)

$$P_{j,s,t}^r X_{j,s,t} = (1 - \alpha_{j,s}^\theta) P_{j,s,t} Z_{j,s,t}$$

A.2. Allocation of the intermediate inputs across commodities

$$(A.2.1) \quad \underset{\mathcal{X}_{ss,j,s,t}}{\text{Minimize}} P_{j,s,t}^r X_{j,s,t} = \sum_{ss} P_{j,ss,t} \mathcal{X}_{ss,j,s,t}$$

subject to:

$$(A.2.2) \quad X_{j,s,t} = \left[\sum_{SS} \varepsilon_{SS,j,s}^{\mathcal{K}} \mathcal{K}_{SS,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} \right]^{\frac{\sigma_{j,s}^{\mathcal{X}}}{\sigma_{j,s}^{\mathcal{X}-1}}}$$

yields:

$$(A.2.3) \quad \lambda_2 \left[\sum_{SS} \varepsilon_{SS,j,s}^{\mathcal{K}} \mathcal{K}_{SS,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} \right]^{\frac{\sigma_{j,s}^{\mathcal{X}}}{\sigma_{j,s}^{\mathcal{X}-1}}} \varepsilon_{SS,j,s}^{\mathcal{K}} \mathcal{K}_{SS,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} = P_{j,ss,t}^{\mathcal{K}}$$

Multiplying both sides by $\mathcal{K}_{SS,j,s,t}$, we have:

$$(A.2.4) \quad \lambda_2 \left[\sum_{SS} \varepsilon_{SS,j,s}^{\mathcal{K}} \mathcal{K}_{SS,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} \right]^{\frac{1}{\sigma_{j,s}^{\mathcal{X}-1}}} \varepsilon_{SS,j,s}^{\mathcal{K}} \mathcal{K}_{SS,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} = P_{j,ss,t}^{\mathcal{K}} \mathcal{K}_{SS,j,s,t}$$

Taking sum over ss gives

$$\lambda_2 \left[\sum_{ss} \varepsilon_{ss,j,s}^{\mathcal{K}} \mathcal{K}_{ss,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} \right] \left(\sum_{ss} \varepsilon_{ss,j,s}^{\mathcal{K}} \mathcal{K}_{ss,j,s,t} \frac{\sigma_{j,s}^{\mathcal{X}-1}}{\sigma_{j,s}^{\mathcal{X}}} \right) = \sum_{ss} P_{j,ss,t}^{\mathcal{K}} \mathcal{K}_{ss,j,s,t}$$

or:

$$\lambda_2 X_{j,t} = \sum_{ss} P_{j,ss,t}^{\mathcal{K}} \mathcal{K}_{ss,j,s,t}$$

Using (A.2.1) in the above equation yields an explicit expression for the Lagrange multiplier:

$$\lambda_2 X_{j,t} = P_{j,t}^{\mathcal{K}} X_{j,t} \Rightarrow \lambda_2 = P_{j,t}^{\mathcal{K}}$$

Replacing λ_2 in (A.2.3) yields the following intermediate input demand function for good ss produced in region j at time t .

$$X_{j,t} \varepsilon_{ss,j,s}^{\mathcal{K}} \sigma_{j,s}^{\mathcal{X}} \mathcal{K}_{ss,j,s,t}^{-1} = \left(\frac{P_{j,ss,t}^{\mathcal{K}}}{P_{j,s,t}^{\mathcal{X}}} \right)^{\sigma_{j,s}^{\mathcal{X}}}$$

or:

$$\mathcal{A}_{ss,j,s,t} = \alpha_{ss,j,s}^{AS} \left(\frac{P_{j,s,t}^X}{P_{j,ss,t}^c} \right)^{\sigma_{j,s}^X} X_{j,s,t}$$

where $\alpha_{ss,j,s}^{AS} = \varepsilon_{ss,j,s}^{AS} \sigma_{j,s}^X$

Combing (A.2.4) with (A.2.1) gives the explicit form of the aggregate price:

$$\begin{aligned} P_{j,s,t}^X &= \frac{\sum_{ss} P_{j,ss,t}^c \mathcal{A}_{ss,j,s,t}}{X_{j,s,t}} \\ &= \frac{P_{j,s,t}^X \sigma_{j,s}^X X_{j,s,t} \sum_{ss} \alpha_{ss,j,s}^{AS} P_{j,ss,t}^c {}^{1-\sigma_{j,s}^X}}{X_{j,s,t}} \end{aligned}$$

$$P_{j,s,t}^X {}^{1-\sigma_{j,s}^X} = \sum_{ss} \alpha_{ss,j,s}^{AS} P_{j,ss,t}^c {}^{1-\sigma_{j,s}^X}$$

A.3. Composition of the value added

$$(A.3.1) \quad \underset{K_{j,s,t}^{dem} L_{j,s,t}^{dem}}{\text{Minimize}} P_{j,s,t}^Q Q_{j,s,t} = r_{j,t}^K K_{j,s,t}^{dem} + w_{j,t} L_{j,s,t}$$

subject to:

$$(A.3.2) \quad Q_{j,s,t} = SC_{j,s}^{\rho} K_{j,s,t}^{dem \alpha_{j,s}^K} L_{j,s,t}^{dem 1-\alpha_{j,s}^K}$$

The first-order condition with respect to $K_{j,s,t}^{dem}$ takes the following form:

$$\lambda \alpha_{j,s}^K SC_{j,s}^{\rho} K_{j,s,t}^{dem \alpha_{j,s}^K - 1} L_{j,s,t}^{dem 1-\alpha_{j,s}^K} = r_{j,t}^K$$

Multiplying both sides by $K_{j,s,t}^{dem}$:

$$(A.3.3) \quad \lambda \alpha_{j,s}^K Q_{j,s,t} = r_{j,t}^K K_{j,s,t}^{dem}$$

λ is the shadow price of the value added and $\lambda = P_{j,s,t}^{\rho}$

Replace λ in equation (A.1.5)

$$r_{j,t}^K K_{j,s,t}^{dem} = P_{j,s,t}^{\rho} \alpha_{j,s}^K Q_{j,s,t}$$

In the same way, we have

$$w_{j,t} L_{j,s,t}^{dem} = (1 - \alpha_{j,s}^K) P_{j,s,t}^{\rho} Q_{j,s,t}$$

A.4. Derivation of the intertemporal decision problem

(A.4.1) *Maximize*
 $C_{j,g}, Beq_{j,g,t}$

$$U_{j,t} = \frac{1}{1-\theta} \sum_{g=1}^6 [1+\rho]^{-g} (C_{j,g,t+g-1}^{1-\theta} + \beta_g^\theta Beq_{j,g,t+g-1}^{1-\theta})$$

$$\theta > 0, \beta_{g \neq 6} = 0, \beta_{g=6} > 0,$$

subject to:

$$(A.4.2) \sum_{g=1}^6 [1+r_{t+g-1}(1-\tau^K)]^{-g} (LInc_{j,g,t+g-1} (1-\tau_{j,t+g-1}^w - \tau_{t+g-1}^p) \\ (1-\tau_{j,t+g-1}^w)(Inh_{j,g,t+g-1} + Pens_{j,g,t+g-1})) = \\ \sum_{g=1}^6 [1+r_{t+g-1}(1-\tau^K)]^{-g} P_{j,g,t+g-1}^{Con} (C_{j,g,t+g-1} + \mu_g Beq_{j,g,t+g-1})$$

$$\mu_{g=6} = 1, \mu_{g \neq 6} = 0$$

The first-order condition with respect to the consumption takes the following form:

$$(A.4.3) [1+\rho]^{-g} C_{j,g,t+g-1}^{-\theta} = \lambda [1+r_{t+g-1}(1-\tau^K)]^{-g} P_{j,g,t+g-1}^{Con}$$

Forward one period, we have:

$$(A.4.4) \quad [1+\rho]^{-1-g} C_{j,g+1,t+g}^{-\theta} = \lambda \left[1+r_{t+g-1}(1-\tau^K) \right]^g P_{j,g+1,t+g}^{Con}$$

Divide (A.4.4) by (A.4.3), we have:

$$C_{j,g+1,t+g} = \left[\frac{1+r_{t+g}(1-\tau^K)}{1+\rho_j} \frac{P_{j,g,t+g-1}^{Con}}{P_{j,g+1,t+g}^{Con}} \right]^{(1/\theta)} C_{j,g,t+g-1}$$

The first-order condition with respect to the bequest takes the following form:

$$(A.4.5) \quad [1+\rho]^{-g} \beta_g^\theta Beq_{j,g,t+g-1}^{-\theta} = \lambda \left[1+r_{t+g-1}(1-\tau^K) \right]^{1-g} \mu_g P_{j,g,t+g-1}^{Con}$$

Divide (A.4.4) by (A.4.3), we have:

$$Beq_{j,g,t} = \mu_g \beta_g C_{j,g,t}$$

A.5. Consumption allocation across commodities

$$(A.5.1) \quad \underset{ConS_{j,s,t}}{\text{Maximize}} C_{j,g,t} = \left[\sum_s \varepsilon_{j,s,g}^{ConS} ConS_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} \right]^{\frac{\sigma_{j,g}^{Con}}{\sigma_{j,g}^{Con-1}}}$$

subject to:

$$(A.5.2) \quad P_{j,g,t}^{Con} C_{j,g,t} = \sum_s P_{j,s,t} ConS_{j,s,g,t}$$

yields:

$$(A.5.3) \quad \left[\sum_s \varepsilon_{j,s,g}^{ConS} ConS_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} \right]^{\frac{\sigma_{j,g}^{Con}}{\sigma_{j,g}^{Con-1}}} \varepsilon_{j,s,g}^{ConS} ConS_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} = \lambda P_{j,s,t}$$

Multiplying both sides by $ConS_{j,s,g,t}$, we have:

$$(A.5.4) \quad \left[\sum_s \varepsilon_{j,s,g}^{ConS} \text{Cons}_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} \right] \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con-1}} \varepsilon_{j,s,g}^{ConS} \text{Cons}_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} = \lambda P_{j,ss,t}^{ConS} \text{Cons}_{j,s,g,t}$$

Taking sum over s gives:

$$\left[\sum_s \varepsilon_{j,s,g}^{ConS} \text{Cons}_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} \right] \left(\sum_s \varepsilon_{j,s,g}^{ConS} \text{Cons}_{j,s,g,t} \frac{\sigma_{j,g}^{Con-1}}{\sigma_{j,g}^{Con}} \right) = \lambda \sum_i P_{j,ss,t}^{ConS} X_{ss,j,s,t}$$

or:

$$C_{j,g,t} = \lambda \sum_s P_{j,ss,t}^{ConS} \text{Cons}_{j,s,g,t}$$

Using (A.5.2) in the above equation yields an explicit expression for the Lagrange multiplier:

$$C_{j,g,t} = \lambda P_{j,g,t}^{Con} C_{j,g,t} \Rightarrow \lambda = \frac{1}{P_{j,g,t}^{Con}}$$

Replacing λ in (A.6.3) yields the following intermediate input demand function for good s produced in region j at time t :

$$C_{j,g,t} \varepsilon_{j,s,g}^{ConS \sigma_{j,s}^{Con}} Cons_{j,s,g,t}^{-1} = \left(\frac{P_{j,s,t}^c}{P_{j,g,t}^{Con}} \right)^{\sigma_{j,s}^{Con}}$$

or:

$$Cons_{j,s,g,t} = \alpha_{j,s,g}^{ConS} \left(\frac{P_{j,g,t}^{Con}}{P_{j,s,t}^c} \right)^{\sigma_{j,s}^{Con}} C_{j,g,t}$$

where

$$\alpha_{j,s,g}^{ConS} = \varepsilon_{j,s,g}^{ConS \sigma_{j,s}^{Con}}$$

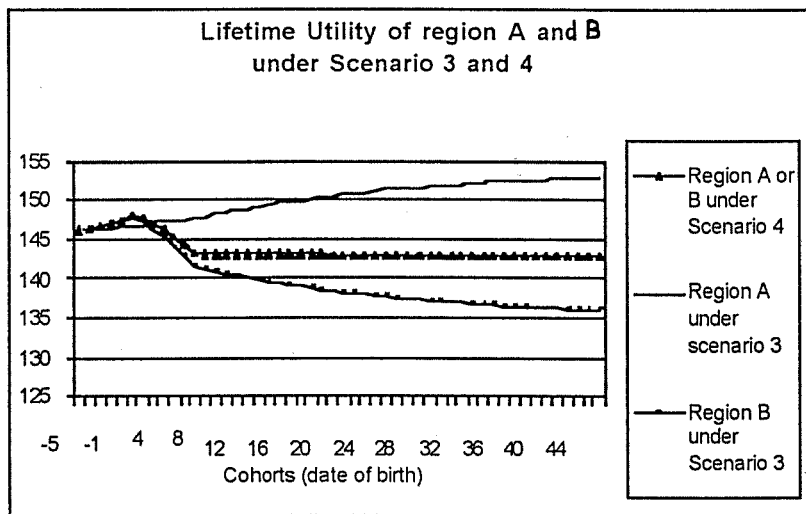
Combing (A.5.4) with (A.5.2) gives the explicit form of the aggregate price:

$$\begin{aligned} P_{j,g,t}^{Con} &= \frac{\sum_s P_{j,s,t}^c Cons_{j,s,g,t}}{C_{j,g,t}} \\ &= \frac{P_{j,g,t}^{Con \sigma_{j,s}^{Con}} C_{j,g,t} \sum_s \alpha_{j,s,g}^{Con} P_{j,s,t}^{c 1-\sigma_{j,s}^{Con}}}{C_{j,g,t}} \end{aligned}$$

$$P_{j,g,t}^{Con 1-\sigma_{j,s}^{Con}} = \sum_s \alpha_{j,s,g}^{Con} P_{j,s,t}^{c 1-\sigma_{j,s}^{Con}}$$

APPENDIX B

In order to separate the population effect from the trade effect and investigate how trade between the two regions affects region B, we assume 6 per cent population growth rate for both regions from period 1 to period 6. The following graph shows the lifetime utility under scenario 3 and scenario 4.



The lifetime utility of region B under scenario 4 is always higher than that under scenario 3 throughout the time path, clearly showing that with the same population growth rate as that under scenario 3, region B does gain benefits from trading with the ageing region (region A).

The Transmission Effects of Population Ageing
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By

Hong Ding

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Student number: 2868702

Supervisor: Professor Marcel Mérette