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THE IMPATT AMPLIFIER IN FIN LINE

MILES BURTON

A thesis submitted in partial fulfillment  
of the requirements for the degree  
Master of Applied Science

Department of Electrical Engineering  
University of Ottawa  
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## ABSTRACT

This thesis deals with the theoretical and practical aspects of negative-resistance amplifier design in high-impedance environments such as fin line. To support the analysis, an IMPATT amplifier in single-ridge fin line is designed, and the amplifier's performance is measured. Both analysis and experiment show that the behavior of negative-resistance amplifiers in high-impedance environments is very sensitive to changes in circuit parameters. Also, the design of such amplifiers is shown to require excessive accuracy in the characterization of the diode as the diode reflection-coefficient approaches the unit circle, and as the required gain increases.

The equalizer design for the fin-line amplifier was based on an improved characterization of series stubs in fin line. This characterization is presented in the form of empirical closed-form expressions for the scattering parameters of series stubs in single-ridge unilateral fin line with narrow gap width. These expressions show the effect of fin-line gap-width, stub length and frequency, and are suitable for computer-aided design.

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## INTRODUCTION

Activity in the millimeter-wave spectrum continues to increase to meet civilian, scientific, and military needs in such diverse fields as satellite communications, remote sensing and terminal seekers for missiles. The mm-wave circuit designer must draw from a wide range of technologies to meet specifications that are unique to each application. For applications requiring moderate power amplifiers in moderate production quantities, the IMPATT amplifier in fin line technique promises to be a light weight, low cost realization.

At mm-wave frequencies the IMPATT diode is the most powerful and most efficient solid state device. Where the benefits of solid state (low cost, light weight, and low-voltage power supply) are essential, the IMPATT amplifier is preferred if its limitations (limited bandwidth, poor linearity, high noise, inadequate efficiency, and poor temperature stability) can be tolerated.

The small dimensions and associated tight tolerances of mm-wave circuits are much costlier to realize with conventional waveguide structures than with the printed-circuit structures of fin line or microstrip. At mm-wave frequencies, fin line is often preferred over microstrip because it has less severe tolerances, it has lower losses, it is naturally shielded, and it allows straightforward transitions to waveguide.

Thus, the special features of both the IMPATT diode and the fin line should combine to make a light weight and inexpensive moderate power amplifier.

This thesis records an attempt to demonstrate the viability of IMPATT amplifiers in a fin-line environment. It describes both the theory and the experimental procedures that were used to construct an amplifier operating at 11 GHz. This low frequency allows the convenience of larger dimensions and more precise

network analyser measurements; yet it does not prevent the techniques used and the lessons learned from being applied to higher frequencies.

## 2-1 AN OVERVIEW OF NEGATIVE-RESISTANCE AMPLIFIERS

The negative-resistance (or regenerative) amplifier is commonly used at frequencies where transistors with the required output power are nonexistent. The "inherent isolation, high gain, simplicity and wide bandwidth of the transistor amplifier are sacrificed for the higher power of the negative-resistance amplifier circuit.

The negative-resistance device is usually a packaged semiconductor diode (IMPATT, Gunn, etc.) that behaves like a lumped immitance with a negative real part. For example, a typical silicon IMPATT diode operating at 11 Ghz would present about  $-2 -j12$  ohms at the package terminals when biased with a direct current of 200 mA. A good general description of negative-resistance devices as well as a comprehensive list of references is given in [1].

Fig. 2-1 shows functional diagrams for the two basic types of negative-resistance amplifiers : reflection amplifiers and transmission amplifiers. In the reflection amplifier (Fig. 2-1a), the output power is the power reflected from the equalizer and the gain is  $|\Gamma_{in}|^2$ , the squared magnitude of the reflection coefficient at the equalizer input. In the transmission amplifier (Fig. 2-1b), the output power is the transmitted power and the gain is the squared magnitude of the overall transmission coefficient.

The equalizers in both types are lossless, partially reflective circuits that are designed to increase the magnitude of the appropriate diode S-parameter (reflection coefficient or transmission coefficient) to that required for the complete amplifier. This increase is obtained at the expense of reduced useable bandwidth.

Single-diode reflection amplifiers are nonreciprocal because

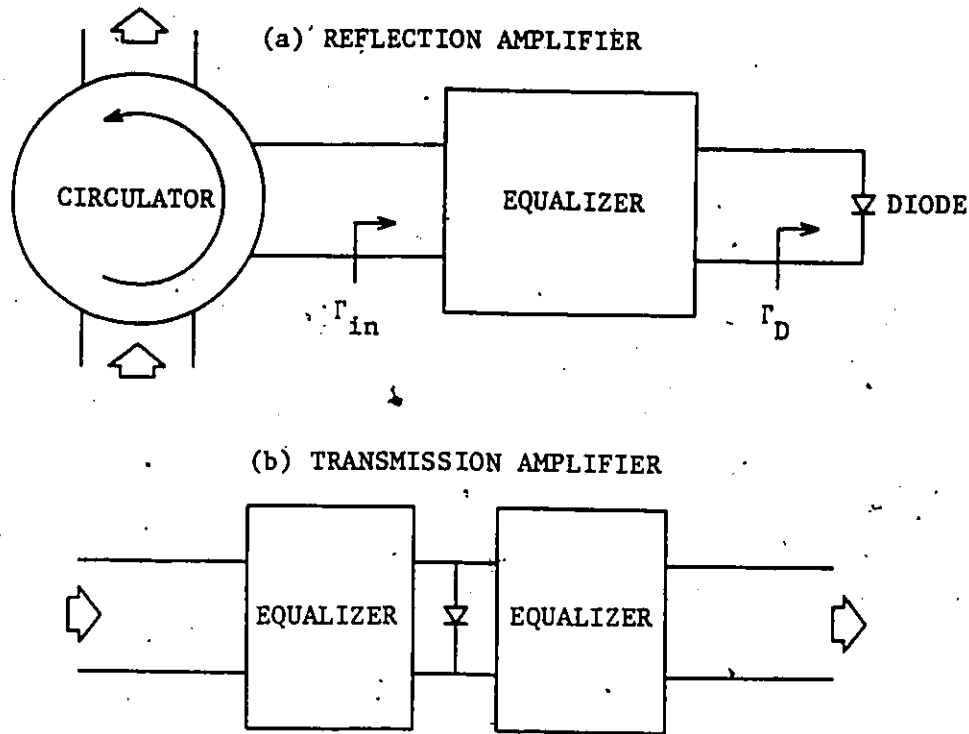


Fig. 2-1 The two basic types of reflection amplifier.

they use a circulator to separate the input and output signals. The bandwidth is usually that of the circulator, which is typically less than 5 percent for low insertion loss (less than 0.5 dB) types for mm-wave frequencies [2]. The circulator can be eliminated if two reflection amplifiers are coupled through a  $90^\circ$  hybrid circuit as shown in Fig. 2-2. The gain is the same as for either amplifier but the output power is doubled. The hybrid-coupled amplifier is reciprocal and its bandwidth is that of either individual amplifier. However, if the load and generator must be isolated, the bandwidth is reduced to that of the necessary isolator(s), which is typically the same as for the circulators mentioned above. Reference [3] contains a 1979 survey of millimeter-wave solid-state reflection amplifiers as well as a large list of references.

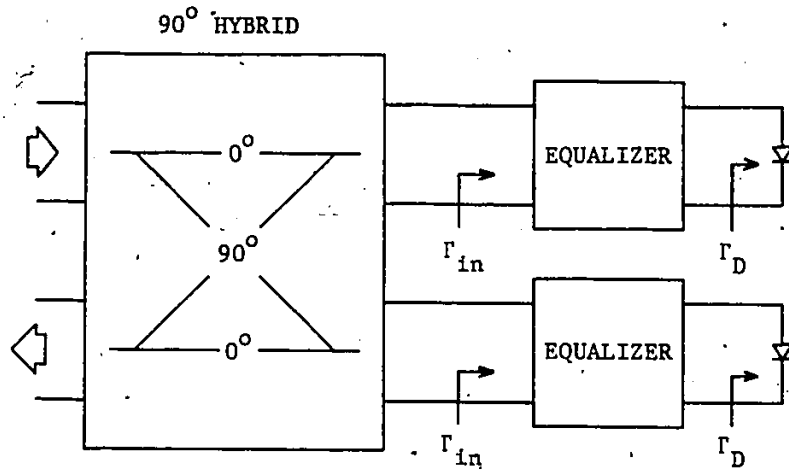


Fig. 2-2 The hybrid coupled reflection amplifier is reciprocal.

The transmission amplifier has no inherent need for a circulator. Therefore it has a greater potential bandwidth than the reflection amplifier. However, like the hybrid-coupled amplifier, the transmission amplifier is reciprocal, and the bandwidth suffers when isolators are added. Also, this amplifier type tends to exhibit reflection gain which can rise to levels significantly higher than unity at the passband edges. Three-stage transmission amplifiers without interstage isolators have been demonstrated [4].

The negative resistance of diodes decreases quickly (towards zero) as the incident power increases. In an IMPATT amplifier designed for maximum output power, the diode's negative resistance is about one-third of its small signal value. This nonlinear characteristic can cause the amplifier to oscillate when the input signal is removed. Such an amplifier is called an injection-locked oscillator because it can be considered as an oscillator whose frequency will lock to that of the "injected" input signal (when that signal is sufficiently large).

Amplifiers that don't oscillate when the input signal is removed are said to be stable. A more detailed description of the injection locked oscillator type of amplifier is given in Sec. 5-2.

IMPATT diodes are the most powerful source of solid state power but they also have the largest noise figures. Consequently, this type of diode is most often used in the final stage of a cascade of amplifiers where the additional noise is least detrimental. IMPATT amplifiers for this application are usually injection locked. See reference [5] for an example.

The fin-line amplifier that was designed for this thesis was a stable reflection-type IMPATT amplifier since it was not possible to obtain in the lab the necessary large input signal levels for injection locking. The diode was characterized and the amplifier was tested with network analyser signals of much less power ( $\sim 5 \mu\text{W}$ ) than the maximum power available from the diode ( $\sim 3 \text{ W}$ ).

2-2

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The basic reflection amplifier is a cascade connection of two circuits: the equalizer, and the negative-resistance device. The interplay of signals in such a circuit can be easily determined with the aid of a signal flow graph like the one in Fig. 3-1.

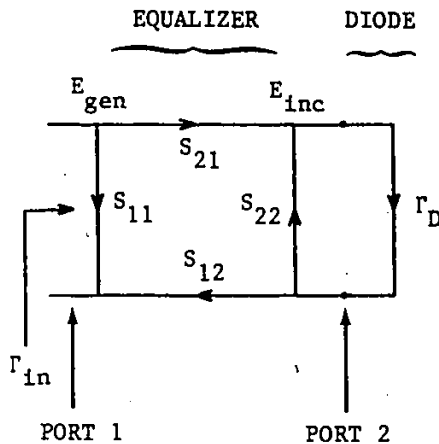


Fig. 3-1 The signal flow graph for the basic reflection amplifier.

Each of the symbols  $E_{gen}$  and  $E_{inc}$  in the figure represents the complex amplitude of the total signal at the node which is adjacent to the symbol. The magnitude is equal to the square root of the signal power. The phase is equal to the phase of the signal.

Whenever two circuits are connected in cascade, the loop equation for the signals at the interface must be satisfied. From Fig. 3-1 the loop equation for the reflection amplifier is

$$E_2 + E_{inc} \Gamma_D S_{22} = E_{inc} \quad (3-1a)$$

$$\text{where } E_2 = E_{gen} S_{21} \quad (3-1b)$$

Suppose the input signal is removed so that  $E_{gen} = 0$ . If  $E_{inc}$  is subsequently discovered to be nonzero, then the circuit is an oscillator by definition and eq. (3-1a) simplifies to

$$\Gamma_D S_{22} = 1 \quad (3-2)$$

(3-2) is notationally very simple but not as informative as the equivalent eq. (3-3).

$$\arg[S_{22}] = -\arg[\Gamma_D] \quad (3-3a)$$

$$|S_{22}| = 1/|\Gamma_D| \quad (3-3b)$$

These equations give the phase and magnitude relationships that prevail when the circuit is oscillating.

When  $E_{gen}$  is nonzero, the circuit is a reflection amplifier and (3-3b) is no longer correct. However, (3-3a) remains at least approximately correct throughout the amplifier passband. This will now be demonstrated by showing that  $|E_2|$  is much less than  $|E_{inc}|$  in a typical amplifier:

Since the equalizer in Fig. 3-1 is nearly lossless, the power added by the diode can be measured at either equalizer port as the difference between the power flowing left and the power flowing right. At port 2, the power added is

$$P_{add} = P_{inc} |\Gamma_D|^2 - P_{inc} = P_{inc} (|\Gamma_D|^2 - 1) \quad (3-4)$$

where  $P_{inc}$  is the power incident on the diode.

At port 1, the power added is

$$P_{add} = P_{gen} G - P_{gen} = P_{gen} (G - 1) \quad (3-5)$$

where  $G$  is the amplifier gain, and  $P_{gen}$  is the power incident on port 1 of the equalizer.

(3-4) and (3-5) can be combined to eliminate  $P_{add}$  and obtain a relationship between the power incident on the diode,  $P_{inc}$ , and the power incident on the amplifier,  $P_{gen}$ . The result is

$$P_{inc} = \frac{(G - 1)P_{gen}}{|\Gamma_D|^2 - 1} \quad (3-6)$$

Since the equalizer is lossless, its S-parameters obey the relationships (3-7) [1].

$$|S_{11}| = |S_{22}| \quad (3-7a)$$

$$|S_{21}| = |S_{12}| \quad (3-7b)$$

$$S_{21}S_{12} = -[1 - |S_{11}|^2] \exp(j\{\arg[S_{11}] + \arg[S_{22}]\}) \quad (3-7c)$$

(Note that in [1] the authors define the ingoing and outgoing travelling waves as voltage waves with complex amplitudes of  $a = V_{in}$  and  $b = V_{out}$ , resp. If the more popular definitions of  $a = V_{in}/\sqrt{Z_0}$  and  $b = V_{out}/\sqrt{Z_0}$  are used, then the relations given in [1] reduce to eqs. (3-7), which are independent of the characteristic impedance at either port.)

By applying eqs. (3-7) to (3-1b), the power leaving port 2 when the equalizer is perfectly matched can be computed as

$$\begin{aligned} P_2 &= P_{gen} |S_{21}|^2 \\ &= P_{gen} |S_{21} S_{12}| \\ &= P_{gen} (1 - |S_{11}|^2) \\ &= P_{gen} (1 - |S_{22}|^2) \end{aligned} \quad (3-8)$$

To relate the power incident on the diode to the power that would be incident on a matched load, (3-6) and (3-8) can be combined to obtain

$$\frac{P_{inc}}{P_2} = \frac{(G - 1)}{(|\Gamma_D|^2 - 1)(1 - |S_{22}|^2)} \quad (3-9)$$

An alternative and independent solution for  $P_{inc}/P_2$  can be obtained by rearranging (3-1a) to give

$$\frac{E_{inc}}{E_2} = \frac{1}{1 - \Gamma_D S_{22}} \quad (3-10)$$

The corresponding ratio of the powers is just

$$\frac{P_{inc}}{P_2} = \frac{1}{|1 - \Gamma_D S_{22}|^2} \quad (3-11)$$

This equation and (3-9) can be combined to give

$$|1 - \Gamma_D S_{22}|^2 = K(1 - |S_{22}|^2) \quad (3-12)$$

$$\text{where } K = \frac{|\Gamma_D|^2 - 1}{G - 1}$$

Although (3-12) does not uniquely determine  $S_{22}$ , it does establish upper and lower limits for  $|S_{22}|$ . (See Appendix 5.) These limits are

$$|S_{22}| = \frac{|\Gamma_D| \pm (K^2 + |\Gamma_D|^2 K - K)^{1/2}}{|\Gamma_D|^2 + K} \quad (3-13)$$

In the passband of a reflection amplifier, typical values for  $G$  and  $|\Gamma_D|$  are:

$$G = 4 \quad (3-14a)$$

$$|\Gamma_D| = 1.1 \quad (3-14b)$$

For these values,  $|S_{22}|$  must be in the range

$$0.75 \leq |S_{22}| \leq 0.96875 \quad (3-14c)$$

In (3-9) the ratio  $P_{inc}/P_2$  can be no smaller than the value set by the smallest value of  $|S_{22}|$ . Substituting  $|S_{22}| = 0.75$  and eqs. (3-14a,b) into (3-9) gives

$$P_{inc} \geq 33 P_2 \quad (3-15)$$

or, equivalently,

$$|E_2| \leq 33^{1/4} |E_{inc}| \quad (3-16)$$

Since  $|E_2|$  is much smaller than  $|E_{inc}|$  in the passband of a reflection amplifier, one can say that

$$\arg[\Gamma_D S_{22}] \approx 0 \quad (3-17)$$

Generally the phases of both  $\Gamma_D$  and  $S_{22}$  decrease monotonically with frequency so that, at some frequency  $f_c$  near the center of the passband,  $\arg[\Gamma_D S_{22}]$  is exactly zero and (3-17) is replaced by (3-3a).

Eq. (3-17) establishes  $\arg[S_{22}]$  in the passband. A formula for  $|S_{22}|$  in the passband will now be derived:

Mason's loop rule [2] can be applied to the signal flow graph in Fig. 3-1 to get the familiar relation

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_D}{1 - S_{22}\Gamma_D} \quad (3-18)$$

Since the equalizer is lossless, eqs. (3-7) can be applied to (3-18) to obtain

$$\Gamma_{in} = \frac{|S_{22}| - \Gamma_D \exp(j\arg[S_{22}])}{1 - S_{22}\Gamma_D} \exp(j\arg[S_{11}]) \quad (3-19)$$

This can be simplified with the aid of (3-17) to obtain

$$\Gamma_{in} = \frac{|S_{22}| - |\Gamma_D|}{1 - |S_{22}||\Gamma_D|} \exp(j\arg[S_{11}]) \quad (3-20)$$

which is exact when the frequency is  $f_c$ . Taking the magnitude of both sides of (3-20) and solving for  $|S_{22}|$  gives

$$|S_{22}| = \frac{|\Gamma_D| - |\Gamma_{in}|}{1 - |\Gamma_D||\Gamma_{in}|} \quad (3-21)$$

Since the gain of the amplifier is  $G = |\Gamma_{in}|^2$ , one can replace  $|\Gamma_{in}|$  in eq. (3-21) with  $G^{1/2}$  to give

$$|S_{22}| = \frac{|\Gamma_D| - G^{1/2}}{1 - |\Gamma_D|G^{1/2}} \quad (3-22)$$

which is exact at  $f_c$ . This equation is useful in equalizer

design and in the sensitivity analysis that follows next.

3-2

### SENSITIVITY ANALYSIS

The characteristics of the components of a reflection amplifier can vary from their assumed values because of ageing, environmental changes, experimental error, etc. These variations can cause the amplifier to depart significantly from its desired behavior. Thus, it is important to know in advance the sensitivity of  $\Gamma_{in}$  to changes in  $\Gamma_D$ . This section will show that the sensitivity of  $\Gamma_{in}$  to  $\Gamma_D$  is strongly dependent on the characteristic impedance of the transmission line in which the diode is mounted.

The sensitivity of  $\Gamma_{in}$  to  $\Gamma_D$  is defined as

$$S = \frac{\Gamma_D}{\Gamma_{in}} \frac{d\Gamma_{in}}{d\Gamma_D} \quad (3-23)$$

This can be evaluated by the direct substitution of (3-19), but the process is somewhat simplified by writing (3-19) as

$$\Gamma_{in} = \Gamma \exp(j\arg[S_{11}]) \quad (3-24a)$$

$$\text{where } \Gamma = \frac{|S_{22}| - \Gamma_D \exp(j\arg[S_{22}])}{1 - S_{22}\Gamma_D} \quad (3-24b)$$

Because  $\Gamma$  and  $\Gamma_{in}$  are related by a constant factor that is independent of  $\Gamma_D$ , the sensitivities of both  $\Gamma_{in}$  and  $\Gamma$  to  $\Gamma_D$  are identical. Thus  $S$  can be computed simply as

$$S = \frac{\Gamma_D}{\Gamma} \frac{d\Gamma}{d\Gamma_D} \quad (3-25)$$

Since  $\Gamma$  is an analytic function of  $\Gamma_D$ , obtaining its complex derivative is straightforward. It is

$$\frac{d\Gamma}{d\Gamma_D} = \frac{(|S_{22}|^2 - 1) \exp(j\arg[S_{22}])}{(1 - S_{22}\Gamma_D)^2} \quad (3-26)$$

Substituting (3-26) and (3-24b) into (3-25) gives

$$S = \frac{(|S_{22}|^2 - 1) \exp(j\arg[S_{22}]) \Gamma_D}{1 - S_{22}\Gamma_D \quad |S_{22}| - \Gamma_D \exp(j\arg[S_{22}])} \quad (3-27)$$

This is exact for all frequencies. In the passband, (3-17) is true and  $S$  can be approximated by

$$S \approx \frac{|S_{22}|^2 - 1 \quad |\Gamma_D|}{1 - |S_{22}| |\Gamma_D| \quad |S_{22}| - |\Gamma_D|} \quad (3-28)$$

which is exact at the frequency  $f_c$ . Thus  $S$  is real at  $f_c$  and can be expected to be nearly real elsewhere in the passband. Also, (3-22) is exact at  $f_c$ . Thus  $S$  depends only on  $|\Gamma_D|$  and  $G$  at  $f_c$  and can be expected to be relatively insensitive to  $\arg[\Gamma_D]$  and  $S_{22}$  in the rest of the passband.

The behavior of (3-28) at  $f_c$  is shown in Fig. 3-2 where  $S$  is plotted as a function of  $|\Gamma_D|$  for various passband gains. The figure shows that the sensitivity will increase as the gain increases and as  $|\Gamma_D|$  decreases. In fact, as  $|\Gamma_D|$  approaches unity, the sensitivity becomes infinite. In other words, as  $|\Gamma_D|$  approaches unity, even a very small

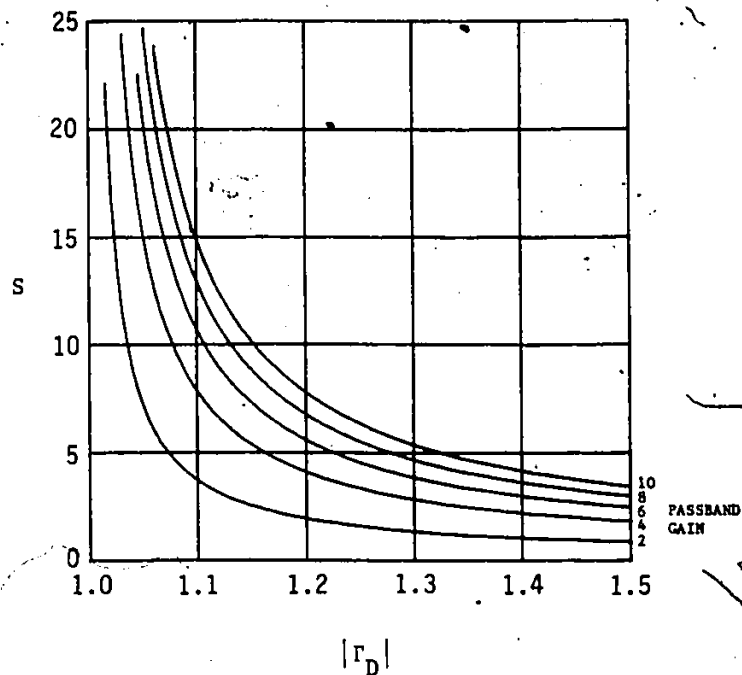


Fig. 3-2 The sensitivity  $S$  of  $\Gamma_{in}$  to  $\Gamma_D$  as a function of  $|\Gamma_D|$  for various passband gains.

uncertainty in  $\Gamma_D$  will make the design totally unpredictable.

Another way to illustrate the uncertainty in  $\Gamma_{in}$  due to a given uncertainty in  $\Gamma_D$  is based on the bilinear nature of (3-19). This equation has the property of mapping circles in the  $\Gamma_D$  plane into circles in the  $\Gamma_{in}$  plane. In particular it will map an error circle for  $\Gamma_D$  into the corresponding error circle for  $\Gamma_{in}$ .

Suppose that at the frequency  $f_c$ , an amplifier has an assumed value of  $\Gamma_D = -1.06 - j0.28$ , and that the anticipated gain is  $G = 4$ . Suppose also that the actual value of  $\Gamma_D$  lies on the circumference of a circle of radius 0.03 centered on the assumed value for  $\Gamma_D$ . The size and location of the

corresponding error circle for  $\Gamma_{in}$  can be obtained from the formulas in Appendix 2, which relate the centers and radiuses of bilinearly transformed circles. One need only use (3-17) and (3-22) to obtain a value for  $S_{22}$  in the bilinear transformation (3-19). The two circles are plotted in Fig. 3-3.

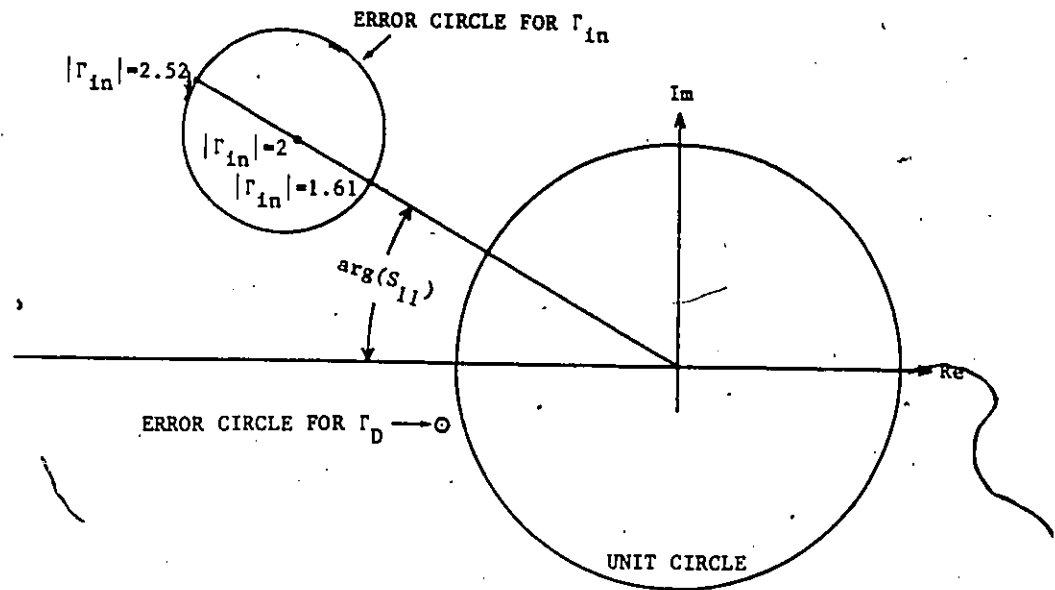


Fig. 3-3 The error circle for  $\Gamma_{in}$  when the error circle for  $\Gamma_D$  has a radius of 0.03.

The figure shows that the magnitude of  $\Gamma_{in}$  can lie anywhere between the extremes of  $|\Gamma_{in}| = 2.52$  and  $|\Gamma_{in}| = 1.61$  when  $\Gamma_D$  is allowed to move around its error circle of radius 0.03. Since  $G = |\Gamma_{in}|^2$ , the corresponding variation in gain is from  $G = 6.35$  to  $G = 2.59$ .

Naturally, knowing that the predictability will improve as  $|\Gamma_D|$  increases, the designer will want to obtain the largest possible  $|\Gamma_D|$  from the diode. One way to do this, which is applicable to IMPATT diodes, is to use the largest safe bias current. Another way, which is applicable to all negative-

resistance devices, is to use the optimum transmission line impedance. It is shown in Appendix 1 that the largest  $|\Gamma_D|$  will be measured when

$$Z_0 = |Z_D| \quad (3-29)$$

where  $Z_0$  is the characteristic impedance of the transmission line, and  $Z_D$  is the diode impedance.

Since high-power amplifiers have low  $|\Gamma_D|$ , the design of high-power, high-gain amplifiers will require precise characterization of the diode for predictable results.

3-3

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Like the reflection amplifier, the transmission amplifier has a passband in which the gain is large and relatively constant. Unlike the reflection amplifier, the transmission amplifier is transmissive rather than reflective and reciprocal rather than nonreciprocal. Hence, this type of amplifier is essentially a filter with gain. The structure of any optimum filter that operates between identical impedances (or between identical waveguide cross sections) will be symmetrical about its center. Thus, the optimum single-diode transmission amplifier will have two symmetrically identical equalizers on each side of the diode as shown in Fig. 4-1a. (Here, optimum means the largest gain-bandwidth product is obtained from a given circuit complexity.)

Suppose a symmetrical single-diode transmission amplifier is driven at each port by signals that are equal in magnitude and phase. The circuit will then be structurally and electrically symmetrical and therefore indistinguishable from two identical and simultaneously operating reflection amplifiers. See Fig. 4-1b. This means that each equalizer can be designed as if it were meant for a reflection amplifier. In other words, the design of a transmission amplifier can be reduced to the design of a reflection amplifier. The apparent diode reflection coefficient for the reflection amplifiers can be derived from the actual diode S-parameters as follows:

When the transmission amplifier is symmetrically driven, the incident and reflected waves on each side of the diode must satisfy eqs. (4-1). (See Fig. 4-1b.)

$$E_{1\text{inc}} = E_{2\text{inc}} \quad (4-1a)$$

$$E_{1\text{ref}} = E_{2\text{ref}} \quad (4-1b)$$

This means that the reflection coefficients seen looking toward

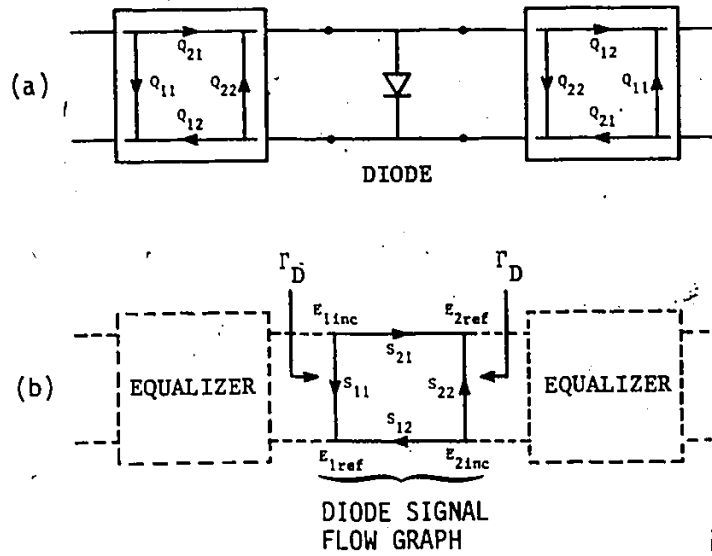


Fig. 4-1. (a) The symmetrical transmission amplifier.  
 (b) The symmetrical transmission amplifier during simultaneous excitation. This amplifier behaves like two independent reflection amplifiers whose diode reflection coefficients are  $\Gamma_D$ .

the diode from either side, are identical and equal to

$$\Gamma_D = E_{1ref}/E_{1inc} = E_{2ref}/E_{2inc} \quad (4-2)$$

Similarly, the reflection coefficients seen looking away from the diode are identical and equal to

$$E_{1inc}/E_{1ref} = E_{2inc}/E_{2ref} = 1/\Gamma_D \quad (4-3)$$

Applying eq. (3-18) to the diode gives

$$\Gamma_D = S_{11} + \frac{S_{21}S_{12}(1/\Gamma_D)}{1 - S_{22}(1/\Gamma_D)} \quad (4-4)$$

where  $S_{11}$ ,  $S_{22}$ ,  $S_{21}$ , and  $S_{12}$  are the diode S-parameters.

Because the diode is symmetrical and reciprocal

$$S_{11} = S_{22} \text{ and } S_{21} = S_{12} \quad (4-5)$$

The diode is shunt connected and electrically small. Thus the total voltage across the diode is just the sum of the incident and reflected waves or  $1 + S_{11}$  (for unity incident voltage). Since the total voltage across the diode is also the value of the signal that is transmitted, one can immediately write

$$S_{21} = 1 + S_{11} \quad (4-6)$$

Substituting (4-5) and (4-6) into (4-4) gives

$$\Gamma_D^2 - 2S_{11}\Gamma_D - 1 - 2S_{11} = 0 \quad (4-7)$$

for which there are two solutions:

$$\Gamma_D = -1 \quad (4-8)$$

and

$$\Gamma_D = 2S_{11} + 1 \quad (4-9)$$

The first solution is a special case of the second solution ( $S_{11} = -1$ ). The second solution gives the diode reflection coefficient to be used when designing a transmission amplifier by using methods appropriate for reflection amplifiers.

During simultaneous two way amplification, the diode must supply twice as much power as during one way amplification. (Equal input powers are assumed in both cases.) Since the diode is nonlinear, its parameters will be significantly different in

both cases. The optimum equalizers for simultaneous two-way transmission will not be same as those for one-way amplification. Therefore, equalizers designed by the method just described must use  $S_{11}$  as measured when the diode was adding the appropriate power for the application.

The following example illustrates the design and behavior of the simple high-power reflection amplifier shown in Fig. 5-1. The

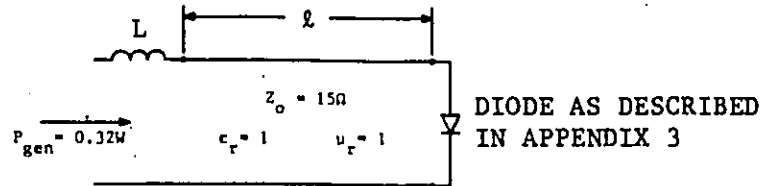


Fig. 5-1 A simple reflection amplifier design example. It is required to find the transmission line length  $l$  and the inductance  $L$  so that there is a peak gain of 10 at 11 GHz.

diode is the fictitious X-band IMPATT diode described in Appendix 3. Its characteristic curves are repeated here in Fig. 5-2 for easy reference. It is required to obtain a peak gain of  $G = 10$  at a frequency of  $f = 11$  GHz when the input power is  $P_{gen} = 0.32$  W.

The design of the equalizer rests on a knowledge of the power that the diode must add to the circuit to meet the specifications given above. From (3-5) this can be computed as

$$P_{add} = P_{gen}(G - 1) = 2.9 \text{ W} \quad (5-1)$$

Now that  $P_{add}$  is known, it is used to determine the incident power on the diode,  $P_{inc}$ , and the diode reflection coefficient,  $\Gamma_D$ . From (3-4) the incident power is

$$P_{inc} = P_{add} / (|\Gamma_D|^2 - 1) \quad (5-2)$$

Since  $P_{inc}$  and  $\Gamma_D$  are both unknown, a second relation involving them is required. It is represented by the characteristic curves for the diode. See Fig. 5-2. Usually

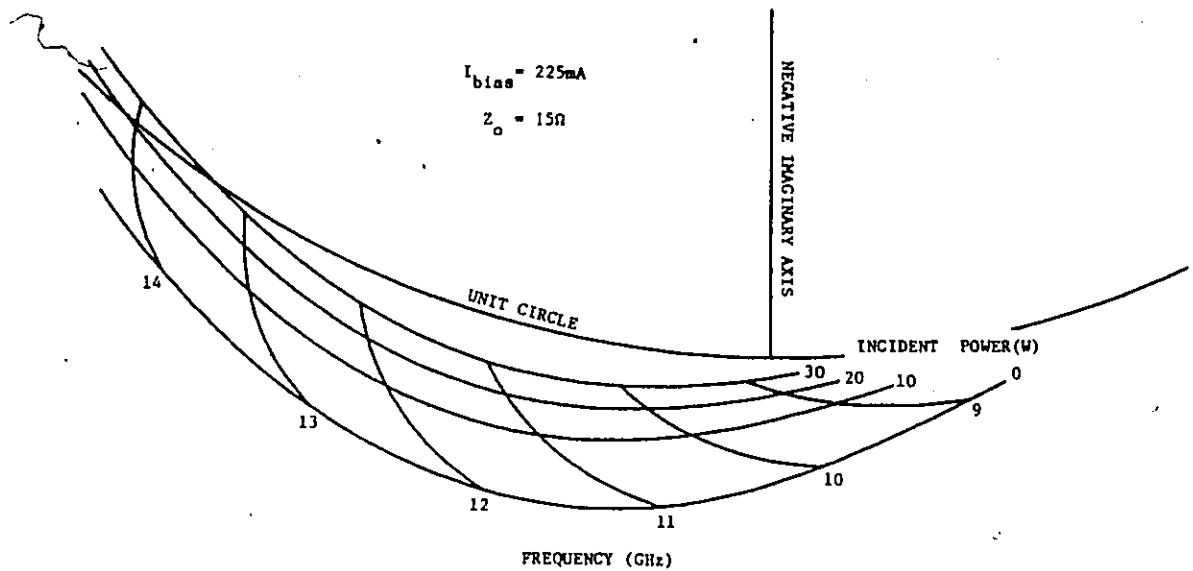


Fig. 5-2 Characteristic curves for the diode described in Appendix 3.

these curves are obtained by measurement, and then an explicit expression of the form

$$\Gamma_D = F(P_{inc}, f) \tag{5-3}$$

is fitted to the results. For the purpose of this design example the function F is simulated by the Fortran program QWIKDIOD1 described in Appendix 3. This program returns  $\Gamma_D$  for a given  $P_{inc}$  and frequency f.

(5-2) and (5-3) can be solved for  $\Gamma_D$  by using the following recursion formula:

$$(P_{inc})_{n+1} = \frac{P_{add}}{|F[(P_{inc})_n, f]|^2 - 1}, \quad (P_{inc})_0 = 0 \tag{5-4}$$

$\Gamma_D$  is the value of F when the change in  $P_{inc}$  becomes negligible. For this example,  $\Gamma_D = -0.26 - j1.05$  when  $P_{add}$

= 2.9 W and  $f = 11$  GHz.

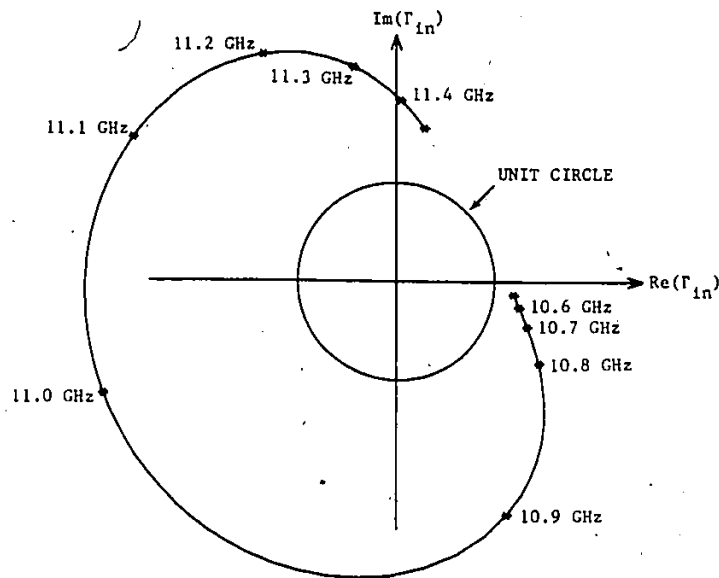
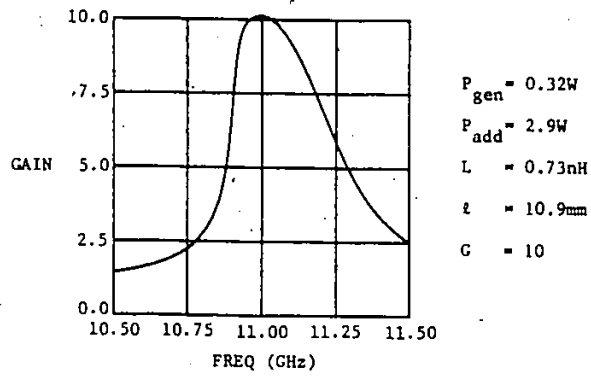


Fig. 5-3 Optimized gain and  $\Gamma_{in}$  functions for the example amplifier in Fig. 5-1.

Now that  $\Gamma_D$  is known, it can be used to determine  $S_{22}$  of

the equalizer. At frequencies near the center of the passband the phase of  $S_{22}$  can be found from (3-3a) and the magnitude of  $S_{22}$  can be found from (3-22).

Smith-chart techniques can be used to compute the size of the inductor that will present an equalizer reflection coefficient of  $S_{22}$  to the diode at 11 GHz. The results are

$$L = 0.7 \text{ nH} \quad (5-5)$$

and

$$l = 11 \text{ mm} \quad (5-6)$$

These are used as the starting point in an optimization routine to find the best fit to a flat gain of 10 between 10.9 GHz and 11.1 GHz. The optimized gain and  $\Gamma_{in}$  functions are plotted in Fig. 5-3. The optimum inductor size and position are also given.

## 5-2 THE INJECTION-LOCKED OSCILLATOR

The gain specification for this example was chosen so that the amplifier would behave as an injection-locked oscillator. The diode used in this example is somewhat more linear than a typical diode so a larger than typical gain was required to make the circuit oscillate when the input signal was removed.

As shown in Sec. 3-1, the condition for oscillation is that the loop gain in Fig. 3-1 is unity. This is written as

$$\Gamma_D S_{22} = 1 \quad (5-7a)$$

or, equivalently, as

$$S_{22} = 1/\Gamma_D \quad (5-7b)$$

Fig. 5-4 illustrates the behavior of an injection-locked oscillator by plotting the frequency dependence of both sides of (5-7b) in the complex plane.

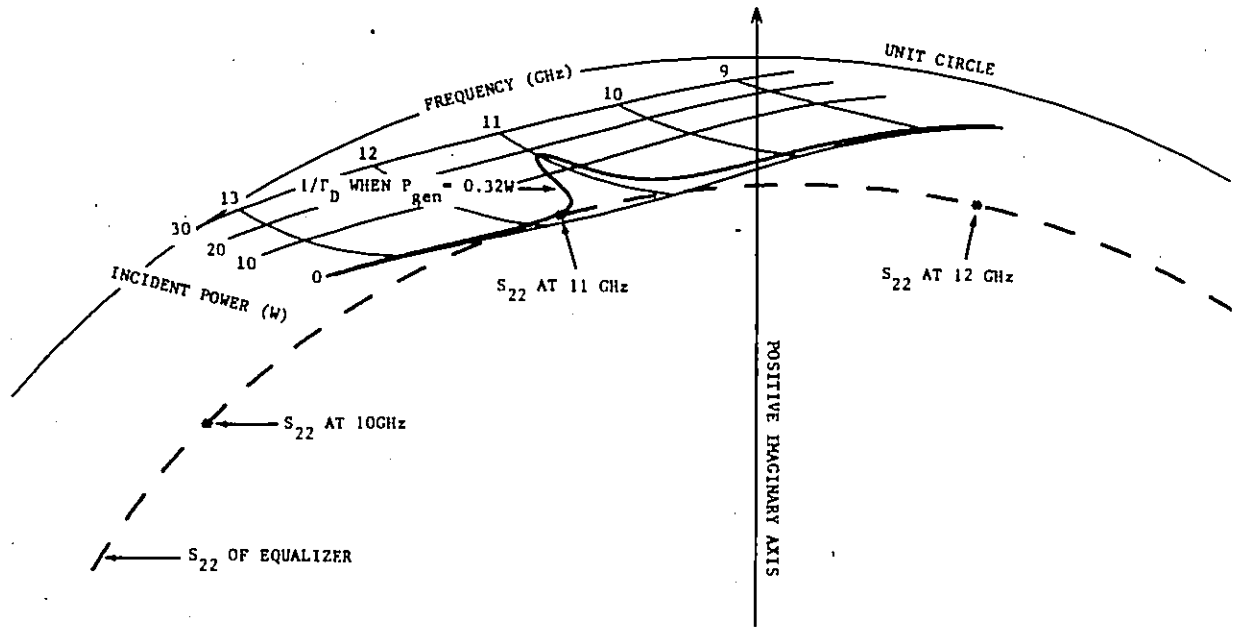


Fig. 5-4 The frequency dependent behavior of  $1/\Gamma_D$  and  $S_{22}$  in the injection-locked oscillator/amplifier of Fig. 5-1.

Imagine that the circuit is amplifying according to the specifications given above. Also imagine that the frequency is being swept from below the passband to above the passband.  $S_{22}$  is seen to travel in a clockwise direction along the path indicated. Conversely,  $1/\Gamma_D$  travels in a generally counterclockwise direction. Below the passband the path of  $1/\Gamma_D$  approximates the characteristic curve for zero incident power. As the passband is traversed, the incident power seen by the diode increases and  $1/\Gamma_D$  veers sharply away from the oncoming  $S_{22}$  (when  $P_{gen} = 0.32$  W). At 11 GHz,  $S_{22}$  and  $1/\Gamma_D$  have almost exactly the same phase, and the incident power is very near to maximum. Above the passband, the path of  $1/\Gamma_D$  returns to the characteristic curve for zero incident

power.

This type of behavior can be seen in all reflection amplifiers, whether stable or injection locked.

That this amplifier might oscillate when there is no input signal is indicated by the presence of a portion of the path of  $S_{22}$  inside the region of operation of the diode. When there is no input signal, the path that would be followed by  $1/\Gamma_D$  is the characteristic curve for  $P_{inc} = 0$ . That the amplifier will oscillate is indicated by the phase equality of  $S_{22}$  and  $1/\Gamma_D$  when the frequency is  $\approx 11.15$  GHz. At this frequency,  $|1/\Gamma_D| < |S_{22}|$ . Therefore the total gain around the loop in Fig. 3-1 is real and greater than one. The loop signal, which is initially just a noise voltage, will increase continuously as it travels around the loop. At the same time  $P_{inc}$  will also increase since it is the power of the loop signal.

The incident power cannot increase forever. There must be some opposing mechanism that will eventually limit  $P_{inc}$  to some finite value. This mechanism reveals itself in the nonlinear behavior of the diode:  $\Gamma_D$  depends on  $P_{inc}$  in such a way that  $1/|\Gamma_D|$  increases as  $P_{inc}$  increases. Thus, with each increase in  $P_{inc}$ , there is an opposing increase in  $1/|\Gamma_D|$ . This reduces the loop gain, causing  $P_{inc}$  to asymptotically approach a finite final value where the loop gain is one. At this value, eqs. (5-7) are satisfied and the circuit oscillates. From Fig. 5-4, the final value for  $P_{inc}$  in this example is about 1 W.

As long as there is no input signal, the circuit is a free-running oscillator. The output power of this oscillator is just the power added by the diode as calculated by (3-4).

$$P_{osc} = P_{add} = P_{inc} (|\Gamma_D|^2 - 1) \quad (5-8)$$

To design a free-running oscillator, use the recursion formula (5-4) to find the  $\Gamma_D$  that occurs at the required power and frequency. Then, determine the necessary equalizer  $S_{22}$  from (5-7b). The rate at which  $S_{22}$  changes with frequency near the point  $1/\Gamma_D$  determines the  $Q$  of the oscillator.

This amplifier design example was created to illustrate the behavior of a reflection amplifier operating under large signal conditions. The fin-line amplifier designed for this thesis was a small-signal amplifier because it was not possible to characterize the diode for large signal operation.

Today, computer assisted design (CAD) is widely used in microwave circuit design. An important part of CAD is computer optimization. This is the minimization of a function  $C$  that describes the closeness of fit of another function  $F$  to a desired function  $D$ . In the case of computer assisted equalizer design,  $D$  is the desired gain response of the amplifier and  $F$  is the calculated gain response for a particular set of variables. These variables can be lumped element values or the physical dimensions of a distributed element circuit.

There are many optimization routines in use today, each with its own unique set of advantages and disadvantages. Fortunately for the microwave circuit designer, the merits of each are relatively unimportant and any one routine will be adequate. One method of optimization proceeds by computing the gradient at some starting point and then following, more or less accurately, the path of steepest descent to the minimum. The Fortran code to implement this method for this thesis was adapted from the version of the Marquardt-Levenberg algorithm given in [1]. For a recent review of optimization techniques with a good list of references, refer to [2]

Obviously, it is desirable to select that starting point from which a steepest descent will lead to a global rather than a local minimum. It is the technique of selecting the starting point that distinguishes the various computer assisted equalizer design methods.

For example, those techniques [3], [4], [5] based on Carlin's resistance-excursion method [6] use a theorem from circuit theory to obtain the starting point for the optimization of lumped-element equalizers. (Carlin's method is also known as the real-frequency technique. An alternative description of the method can be found in [7].) The circuit can then be

transformed to an approximately equivalent distributed-element circuit appropriate for microwave frequencies.

Another technique, which is purely numerical, obtains a set of candidate starting points from a systematic search over a coarse n-dimensional grid defined by the n variables in F. Each of the candidates is then given to the optimization routine. The smallest of the minimums thus obtained is considered to be the global minimum. This approach is used in [8] and in this thesis. The listing of a Fortran subprogram for such a systematic search is given in [9].

This method's advantage is that it can give starting points directly in the form of microwave circuit dimensions; there is no need for an intermediate step involving lumped-element circuits and no need to learn techniques for transforming lumped circuits to distributed circuits.

For this thesis, the function to be minimized was defined as

$$C = \sum_{f=f_1}^{f_n} [F(f, d_1, d_2, \dots, d_m) - D(f)]^2 \quad (6-1)$$

where  $f_i$ ,  $i = 1, n$  are frequencies equally spaced across the passband.

$d_i$ ,  $i = 1, m$  are the fin line circuit dimensions.

F is the computed gain function of the amplifier.

D is the desired flat gain function.

The gain and bandwidth of an amplifier cannot be selected independently of each other. That is, for any given passband

gain,  $G$ , the width of the passband cannot exceed a limit set by the diode reflection-coefficient function,  $\Gamma_D(f)$ , and the generator reflection-coefficient function,  $\Gamma_G(f)$ , [10], [11], [12]. For this reason the function  $D(f)$  must be chosen carefully so that the optimization routine is not asked to design an impossible equalizer. The feasibility of  $D(f)$  can be confirmed in advance by the techniques used in [10], [11], [12], [13], and [14], but the computations involved are intractable for all but the simplest  $\Gamma_D$  and  $\Gamma_G$  functions.

An alternative technique begins by optimizing an equalizer for a  $D(f)$  with a narrow and therefore feasible passband. The closeness of fit to  $D(f)$  will be better than required. The technique proceeds by repeating the optimization with progressively wider passbands until the closeness of fit is marginal. Typically three or four such iterations are all that are required. The bandwidth thus obtained is the largest realizable bandwidth for the chosen circuit complexity and type.

6-2

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USING A NETWORK ANALYSER TO CHARACTERIZE  
STRUCTURES IN FIN LINE

Network analysers can directly measure the S-parameters of only those structures that are embedded in transmission media for which standard loads are available for calibration. Although there are no readily available standard loads for fin line, there is an indirect method that can be used to characterize fin-line structures.

The method begins by deducing the S-parameters of a transition to the transmission medium for which there are no standard loads. (In this thesis the transition was from rectangular waveguide to fin line.) Then, the structure to be characterized is placed between two such transitions, and the S-parameters of the combined circuit are measured. Finally, the S-parameters of the embedded structure are obtained by using deembedding formulas such as those given in Appendix 4.

For this thesis, the transition was characterized by a method similar to that described by Souza and Talboys [1] and by Gupta et al [2]. In their method, a set of 3 equations in  $S_{11}$ ,  $S_{22}$ , and  $S_{21}S_{12}$  are derived and then linearized by restricting the magnitude of  $S_{22}$  to nearly zero. (Although the solution of a linear set of equations is simple and straightforward, the restriction to  $S_{22} \approx 0$  can be avoided by solving the original nonlinear equations with an iterative technique.)

The method developed here obtains explicit (noniterative) expressions for  $S_{11}$ ,  $S_{22}$ , and  $S_{21}S_{12}$  without invoking the restriction  $|S_{22}| \approx 0$ .

The derivation of this method begins by applying eq. (3-18) to the case of a 2-port terminated with a reflection coefficient of  $\Gamma_L$ . See Fig. 7-1. The result is

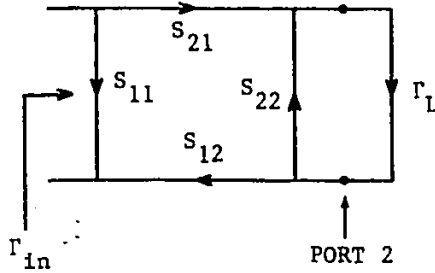


Fig. 7-1 The signal flow graph for a 2-port terminated with a reflection coefficient of  $\Gamma_L$ .

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \quad (7-1)$$

It is a simple although somewhat tedious task to obtain from (7-1) explicit formulas for  $S_{11}$ ,  $S_{22}$ , and  $S_{21}S_{12}$  in terms of 3 known pairs of reflection coefficients ( $\Gamma_{inm}$ ,  $\Gamma_{Lm}$ ),  $m = 1, 3$ . These formulas are

$$S_{11} = \frac{FL - HJ}{HI - FK} \quad (7-2a)$$

$$S_{22} = \frac{GI - EK}{HI - FK} \quad (7-2b)$$

$$S_{21}S_{12} = S_{11}S_{22} + \frac{EL - GJ}{HI - FK} \quad (7-2c)$$

where

$$\begin{aligned}
 E &= \Gamma_{L2} - \Gamma_{L3} \\
 F &= \Gamma_{L1}(-E) \\
 G &= \Gamma_{L2} - \Gamma_{L1} \\
 H &= \Gamma_{L3}(-G) \\
 I &= \Gamma_{in2} - \Gamma_{in3} \\
 J &= \Gamma_{in1}(-I) \\
 K &= \Gamma_{in2} - \Gamma_{in1} \\
 L &= \Gamma_{in3}(-K)
 \end{aligned}$$

Eqs. (7-2) would appear to be inapplicable when the transmission medium at port 2 has no standard loads available for it. Fortunately, it is possible to induce known reflection coefficients without using a known physical termination:

A reflection coefficient of +1 appears at the axis of symmetry of a symmetrical 2-port when it is simultaneously driven at both ports by signals of equal magnitude and phase. A reflection coefficient of -1 appears when the driving signals are equal in magnitude but opposite in phase. Let each half of this 2-port be identical to the 2-port described in the previous paragraph so that ports 2 face each other. See Fig. 7-2a. There are now two pairs of  $(\Gamma_{in}, \Gamma_L)$  available for the characterization. They are obtained by measuring  $\Gamma_{in}$  during each type of excitation.

The third pair of  $(\Gamma_{in}, \Gamma_L)$  can be obtained after inserting a length of uniform transmission-line with the same cross-sectional geometry as that at port 2 of the original 2-port. See Fig. 7-2b. The resulting composite-2-port is still symmetrical, and reflection coefficients of +1 and -1 can be induced at the axis of symmetry as before. The reflection coefficient that terminates the original 2-port is just

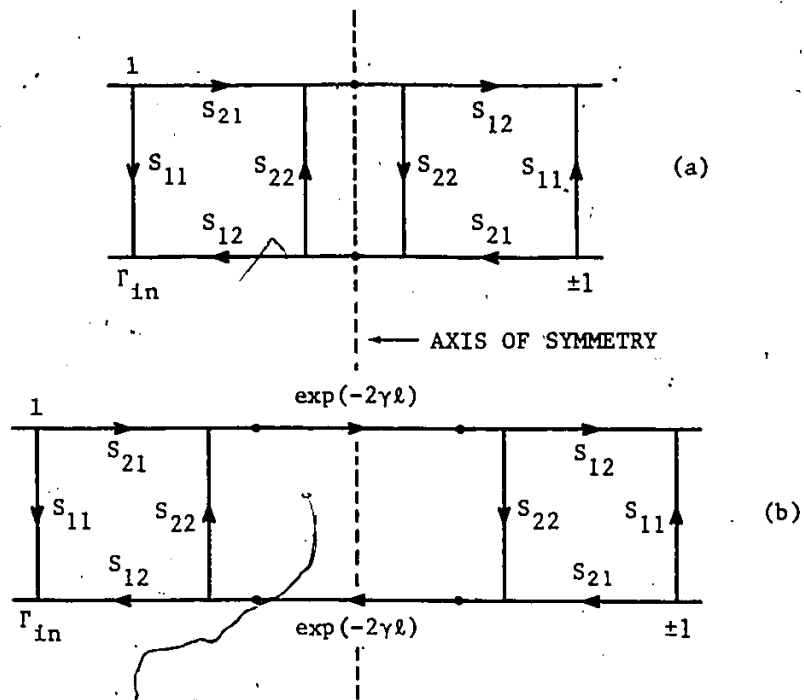


Fig. 7-2 (a) Two identical 2-ports symmetrically connected and driven so as to induce a reflection coefficient of  $\pm 1$  at the axis of symmetry. (b) A uniform transmission-line is inserted at the axis of symmetry. The reflection coefficient at the axis of symmetry is still  $\pm 1$  but the reflection coefficients at ports 2 have changed.

$$\Gamma_{L3} = \rho \exp(-2\gamma\ell) \quad (7-3)$$

where

$\gamma$  is the complex propagation constant of the transmission line.

$\ell$  is half the length of the transmission line.

$\rho$  is the reflection coefficient induced at the axis of symmetry. The choice of  $\rho = 1$  or  $\rho = -1$  is arbitrary.

The pair  $(\Gamma_{in3}, \Gamma_{L3})$  is obtained by measuring  $\Gamma_{in3}$  during either type of excitation. Eqs. (7-2) are then used to complete the characterization of the 2-port. Best experimental accuracy is obtained when  $2l$  is about one-quarter wavelength.

If the 2-port is known to be linear then the circuits of figure 2a,b need not be driven simultaneously. Instead, simultaneous excitation can be simulated by a computation based on the measured S-parameters of those circuits. The input reflection-coefficients that would have been measured during simultaneous in-phase and out-of-phase excitation are

$$\Gamma_{in} = R + T \quad (7-4a)$$

and

$$\Gamma_{in} = R - T \quad (7-4b)$$

respectively. R and T are the measured  $S_{11}$  and  $S_{21}$ , resp. of the symmetric circuit.

In this method, four measurements (two sets of R and T) are used to obtain the three parameters for the 2-port. This redundancy permits a reduction in experimental error. As shown in [3], each of the 2-port parameters can be determined as the average of four answers derived from the four possible combinations of the four pairs of  $(\Gamma_{in}, \Gamma_L)$  taken three at a time.

When an automatic network analyser is used, the extra measurement is obtained with no additional effort since this instrument normally gives its results in pairs of R and T.

Here is a summary of the procedure for characterizing a linear 2-port that has no standard loads available for one of its ports:

1.....Measure the S-parameters ( $R_a$  and  $T_a$ ) of the circuit in Fig. 7-2a.

2.....Compute the first two pairs of ( $\Gamma_L$ ,  $\Gamma_{in}$ ) from

$$\Gamma_{in1} = R_a + T_a, \quad \Gamma_{L1} = +1 \quad (7-5a)$$

$$\Gamma_{in2} = R_a - T_a, \quad \Gamma_{L2} = -1 \quad (7-5b)$$

3.....Measure the S-parameters ( $R_b$  and  $T_b$ ) of the circuit in Fig. 7-2b.

4.....Compute the third pair of ( $\Gamma_L$ ,  $\Gamma_{in}$ ) from

$$\Gamma_{in3} = R_b + T_b, \quad \Gamma_{L3} = \rho = 1 \exp(-2\gamma l) \quad (7-6a)$$

or from

$$\Gamma_{in3} = R_b - T_b, \quad \Gamma_{L3} = \rho = -1 \exp(-2\gamma l) \quad (7-6b)$$

5.....Use eqs. (7-2) to obtain  $S_{11}$ ,  $S_{22}$ , and  $S_{21}S_{12}$  with or without the averaging process described above.

## 7-2 CHARACTERIZING A WAVEGUIDE-TO-FIN-LINE TRANSITION

Fig. 7-3 contains diagrams of the two symmetrical circuits that were used to characterize the waveguide-to-fin-line transition. The inserted length of uniform fin-line was assumed to be lossless, hence  $\gamma$  in (7-3) became

$$\gamma = j(2\pi/\lambda_g) \quad (7-7)$$

$\lambda_g$ , the guided wavelength in the uniform fin line, was computed from

$$\lambda_g/s = a_1 + a_2(s/\lambda) + a_3(s/\lambda)^2 + a_4(s/\lambda)^3 \quad (7-8)$$

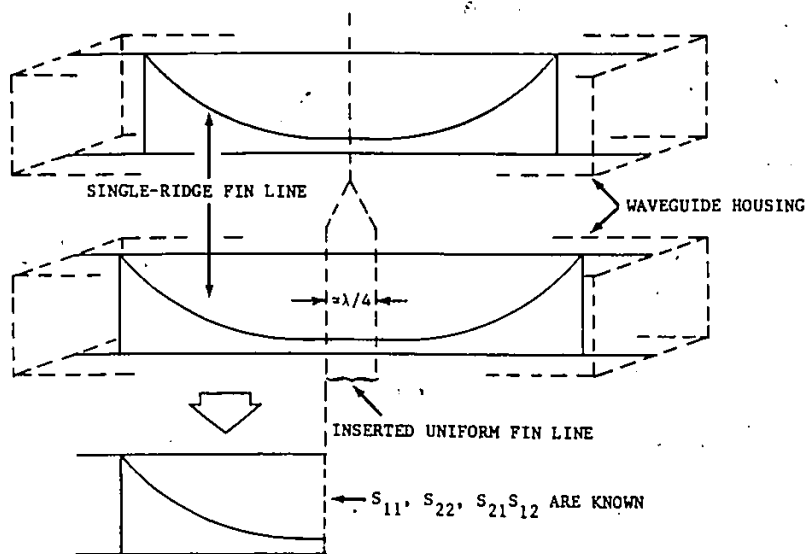


Fig. 7-3 The symmetrical circuits that were used to characterize the waveguide-to-fin-line transitions.

where  $a_1 = 150.4 + 24.29 (d/s)$  (7-9a)

$a_2 = -9092 - 2098 (d/s)$  (7-9b)

$a_3 = 236500 + 64900 (d/s)$  (7-9c)

$a_4 = -2269000 - 690800 (d/s)$  (7-9d)

$d$  is the gap width in the uniform fin line,  
 $s$  is the fin line substrate thickness and  
 $\lambda$  is the wavelength in free space.

This formula was obtained by fitting a polynomial to data obtained from a spectral domain program. The formula is within 1% of the data over the following ranges:

$s/a < 0.1$  (7-10a)

$0.6 < d/s < 2.7$  (7-10b)

$32 < \lambda/s < 49$  (7-10c)

where  $a$  is the broad dimension of the waveguide housing.

Having found the characteristics of the waveguide-to-fin line transition, the diode S-parameters were obtained next by placing the diode between two such transitions. The S-parameters of the composite circuit were measured, and the diode S-parameters were deduced by using the deembedding formulas of Appendix 4.

$S_{11}$  and  $S_{21}$  of the diode were used in (7-1) to obtain  $\Gamma_D$ , the reflection coefficient of the open-circuit terminated diode. The results are plotted in Fig. 7-4. The plot shows that the diode reflection coefficient lies very close to the unit circle. As was shown in Sec. 3-2, this is a very undesirable region for predictable amplifier design. This is a direct result of the large difference between the fin line impedance and the diode impedance. ( $Z_0$  of the fin line was about 100 ohms; the magnitude of the diode impedance was about 12 ohms.)

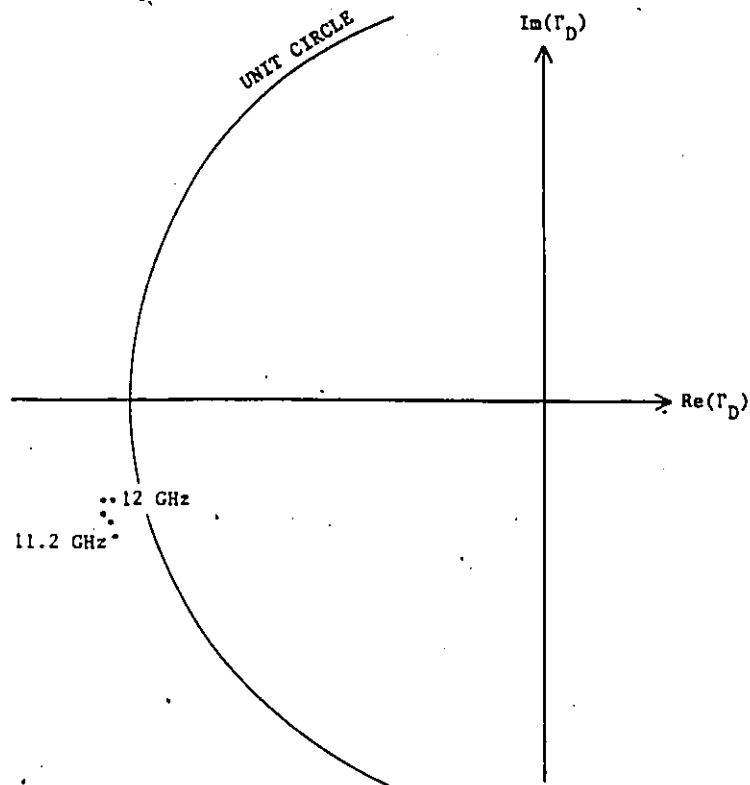


Fig. 7-4 Measured reflection coefficients of the open-circuit terminated diode in single-ridge unilateral fin line.

7-4

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## 8-1 THE CHARACTERIZATION OF SERIES STUBS IN FIN LINE

The series stub has been a popular circuit element in the design of oscillators and narrow-band filters in fin line [1], [2]. However, until this thesis work, the available information did not have sufficient accuracy or flexibility for most other design problems. Before using the series stub in the fin line equalizer, it was necessary to obtain an improved description. Measurements were made at several frequencies on stubs of various lengths in unilateral fin lines with various gap widths to obtain a set of explicit expressions for stub S-parameters. The results represent a significant increase in the available design information for short- and open-circuited series stubs in fin line. (They have been published at the 1984 IEEE International Microwave Symposium [3].)

## 8-2 THE STUB MODEL

A fin line stub can be described either by its S-parameters or by an equivalent circuit with lumped elements. The latter has two advantages. First, it directly depicts the circuit behavior of the discontinuity. Second, it separately describes the various field disturbances at the branching points and at the far end of the stub.

These effects were incorporated into the equivalent transmission line circuits shown in Fig. 8-1. Then, measurements were made with an automatic network analyser to determine the dependence of the equivalent circuit parameters on frequency, stub length, and fin line gap width.

Like the diode, the series stubs were characterized by placing them between two waveguide-to-fin-line transitions as described earlier in Sec. 7-1. The transitions and stubs were etched on

0.76 mm thick RT/Duroid® ( $\epsilon_r = 2.22$ ) and mounted in WR-90 waveguide. See Fig. 8-2. All stubs had a width of 1 mm.

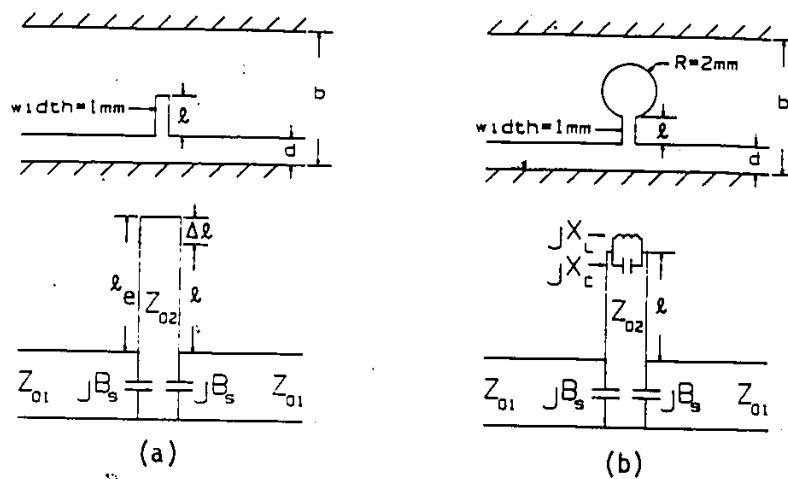


Fig. 8-1 Equivalent transmission line circuits for the short-(a) and open-(b) circuited series stubs in unilateral single-ridge fin line.

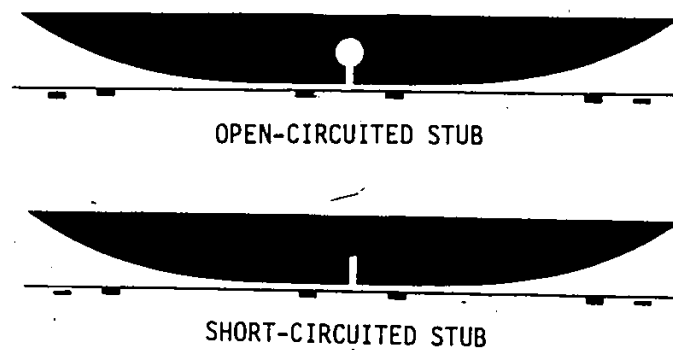


Fig. 8-2 Metalization patterns used to measure the scattering parameters of the stub discontinuities in unilateral fin line (WR-90,  $\epsilon_r = 2.22$ , substrate thickness was  $s = 0.762$  mm).

The scattering parameters of an ideal series stub in a transmission line (that is, one with no losses and zero width and hence no branching points and end effects) are

$$S_{11s} = \frac{z_{in}}{2 + z_{in}} \quad (8-1)$$

$$S_{21s} = \frac{2}{2 + z_{in}} \quad (8-2)$$

where  $z_{in}$  is the impedance seen looking into the stub normalized to the characteristic impedance of the main transmission line.  $z_{in}$  can be computed from

$$z_{in} = z \frac{z_t + \tanh \gamma l_e}{1 + z_t \tanh \gamma l_e} \quad (8-3)$$

where  $z$  is the characteristic impedance of the stub divided by the characteristic impedance of the main transmission line, and  $\gamma = \alpha + j\beta$  is the propagation constant in the stub, and  $l_e$  is the length of the stub in the equivalent circuit, and  $z_t$  is the terminating impedance of the stub divided by the characteristic impedance of the stub.

When a shunt admittance  $jB_s$  is added as shown in Fig. 8-1, the overall scattering parameters can be found from Mason's loop rule [4]. They are

$$S_{11} = S_{11b} + \frac{S_{21b}^2 (S_{11s} - \Delta S_s S_{11b})}{1 - 2S_{11s} S_{11b} + \Delta S_s S_{11b}^2} \quad (8-4)$$

$$S_{21} = \frac{S_{21b}^2 S_{21s}}{1 - 2S_{11s} S_{11b} + \Delta S_s S_{11b}^2} \quad (8-5)$$

where  $S_{11b} = (-jb_s)/(2 + jb_s)$  is the reflection coefficient of a shunt admittance of  $jb_s = jB_s Z_{01}$  (normalized to the characteristic admittance of the main transmission line, and

$S_{21b} = 1 + S_{11b}$  is the transmission coefficient of the shunt admittance, and

$$\Delta S_s = S_{11s}^2 - S_{21s}^2$$

The following formulas were obtained by optimizing the fit of (8-4) and (8-5) to the experimental data for the series stubs. For convenience of scaling, all variables were normalized to the substrate thickness  $s$ .

$$b_s = 7.56(s/\lambda) \exp[-0.3117(d/s)] \quad (8-6)$$

$$z = 2[1.524(d/s)]^{-0.594} \quad (8-7)$$

$$\alpha = 0.08(s/\lambda) \text{ nepers/mm} \quad (8-8)$$

where  $\lambda$  is the free space wavelength.

The current flowing around the end of a short-circuited stub follows a path whose length is finite. This finite path length for the current gives the stub an inductive behavior which is modelled here by an excess length of line  $\Delta l$  defined by

$$\Delta l/s = 0.1082(d/s)^2 \{1 - \exp[-0.762(l/s)]\} \quad (8-9)$$

Open-circuited stubs are terminated by a parallel combination of an inductive and capacitive reactance  $x_l$  and  $x_c$ . See Fig. 8-1b. These reactances are normalized to the characteristic

impedance of the stub and are given by (8-10) and (8-11).

$$x_l = (s/\lambda)[84.12 - 2.474(l/s)] \quad (8-10)$$

$$1/x_c = (s/\lambda)[30.93 - 7.422(l/s)][0.026 - 0.17(d/s)] \quad (8-11)$$

(8-10) and (8-11) can be used to find the normalized terminating impedance

$$z_t = \frac{x_l x_c}{x_l + x_c} \quad (8-12)$$

Expressions (8-6) to (8-11) are valid in the ranges given by (7-10) and in the range

$$1.6 < l/s < 5.3 \quad (8-13)$$

When the error is computed as the magnitude of the difference between the fitted and measured scattering parameters, the standard deviation for the short-circuit fit is 0.03 and that for the open-circuit fit is 0.045. The reference planes of the scattering parameters are coincident with the stub edges.

The restriction to constant stub width allows simple expressions and should have little effect on the flexibility of practical designs. Also, expressions (8-6) to (8-11) do not depend on the waveguide dimensions because small gap ratios ( $d/b \ll 1$ ) and narrow substrates ( $s/a \ll 1$ ) are assumed.

The expressions operate on normalized dimensions to allow convenient scaling to frequencies higher than X-band. Losses are expected to increase linearly with frequency as indicated by (8-8).

The wavelength in a stub can be computed from formula (7-8) for

the guided wavelength in a single ridge unilateral fin line on substrates with  $\epsilon_r = 2.22$ . The wavelength is the same as that in single-ridge fin line when the fin-line gap is equal to half the stub width.

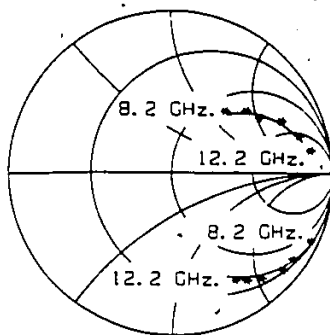


Fig. 8-3 Comparison of measured data to fitted curves for a short-circuited stub (inductive data) and an open-circuited stub (capacitive data). Each stub is 4 mm long and in a single-ridge unilateral fin line with fin line gap of 0.85 mm.

8-4

#### REFERENCES

- [1] H.Hofmann, "MM-wave Gunn oscillator with distributed feedback fin-line circuit," Proc. 1980 MTT-S International Microwave Conference, pp. 59-61.
- [2] P.J.Meier, "Wideband subharmonically pumped W-band Mixer in single-ridge fin-line," 1982 IEEE MTT-S Digest, pp. 201-203.
- [3] M.Burton and W.J.R.Hoefer, "An Improved Model for Short- and Open-Circuited Series Stubs in Fin Line," 1984 IEEE MTT-S Int'l Microwave Symp. Digest, pp. 330-332.
- [4] S.J.Mason, "Feedback theory-further properties of signal flow graphs," Proc. IRE, vol. 44, no. 7, pp. 920-926, Jul. 1956.

## 9-1. A GENERAL DESCRIPTION OF THE FIN-LINE AMPLIFIER

The salient features of the fin-line amplifier that was designed for this thesis are shown in Fig. 9-1. Going from left to right there is a waveguide-to-fin-line transition, a fin line equalizer, the IMPATT diode, and a final series stub that presents an open circuit to the right side of the diode.

The waveguide-to-fin-line transition is not essential to the operation of the amplifier. Its purpose here is to permit characterization by a network analyser with waveguide test-ports. The fin line is single-ridged and unilateral with a teflon-fiberglass substrate (RT/Duroid®,  $\epsilon_r = 2.22$ ). There is a 1 mm. gap between the edge of the fin and the floor of the housing. Single-ridged fin line was used because it is the only type of fin line that allows one terminal of the diode to be mounted in the bulk metal of the housing for adequate heat sinking.

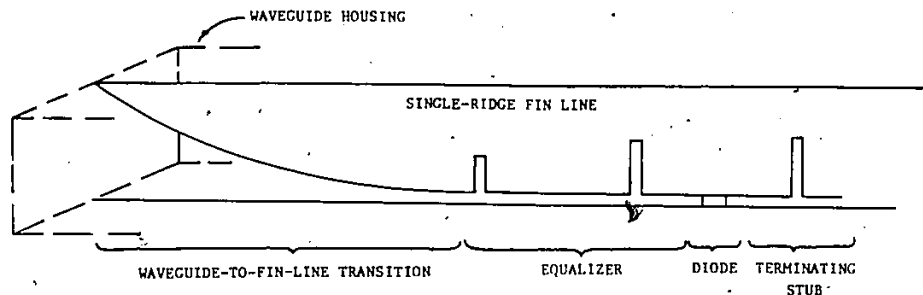


Fig. 9-1 The salient features of the fin-line amplifier.

The equalizer consists of two short-circuited stubs in series with the fin line. Their widths are 1 mm. Empirical results were used to characterize the stubs. Numerical results were used to characterize the lengths of uniform fin line that connect the stubs to each other and to the diode. Explicit formulas for the behavior of the stubs and of the uniform fin lines are presented

in Sec. 8-3 and Sec. 7-2, respectively.

The IMPATT diode is made by NEC (part no. ND8T11W-5H). This is a silicon diode capable of adding 3 W of power at 11 GHz. Its best dc-to-RF conversion efficiency is about 12%. Although the diode is capable of supplying high levels of RF power, the power levels prevailing in this amplifier were in the microwatt range. This is because both the amplifier and the diode were characterized with a network analyser whose test-port signal power was about 5 $\mu$ W. Such a small-signal amplifier is sufficient for the purpose of this thesis : to determine the feasibility of constructing an IMPATT amplifier in a fin line environment.

The series stub to the right of the diode serves to open-circuit terminate the right side of the diode. This method of termination allowed the local environment of the diode to be the same during the operation of the amplifier as during the characterization of the diode.

9-2

#### AMPLIFIER PERFORMANCE

The predicted and measured gain-curves for the amplifier are given in Fig. 9-2. There is agreement in the shape and location of the two curves, but there is a 3 dB difference in midband gain that needs to be explained. Since the measured and predicted passbands are in good agreement with respect to shape and location, it is unlikely that the gain error is due to phase errors in  $S_{22}$  or in  $\Gamma_D$ . (Refer to Sec 3-1 and the amplifier design example in Sec. 5-2.) Another possible explanation for the error in gain is that  $|\Gamma_D|$  was smaller than the measured value. The sensitivity analysis of Sec. 3-2 shows that this explanation is likely given the uncertainty in  $\Gamma_D$  and the proximity of  $\Gamma_D$  to the unit circle:

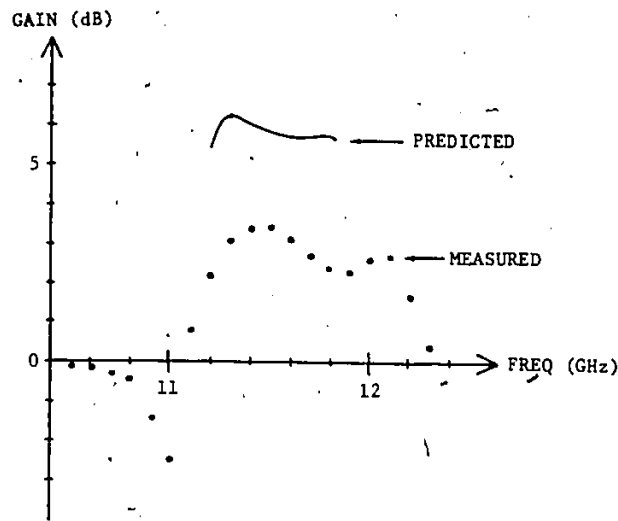


Fig. 9-2 Measured and predicted gain curves for the IMPATT amplifier in fin line.

A reasonable measure of the uncertainty in  $\Gamma_D$  comes from the results of the characterization of the fin-line stubs described in Chapt. 8. There, the magnitude of error between fitted and measured S-parameters for the shorted stubs had a standard deviation of 0.03. Since the same method and equipment were used to characterize the diode, the same standard deviation in magnitude of error can be assumed.

As shown in Fig. 7-4, the measured value for  $\Gamma_D$  is very close to the unit circle. This design used a midband value for  $\Gamma_D$  of  $-1.06 - j0.28$ .

Fig. 3-3 was created to illustrate the effects of exactly the conditions just described. It shows the circular path that  $\Gamma_{in}$  would follow as  $\Gamma_D$  is made to vary around its assumed value at a distance of 0.03. When  $|\Gamma_D|$  is 0.03 less than its assumed value ( $\arg[\Gamma_D]$  unchanged), the gain drops from 4 to 2.6. Thus, most of the error in gain can be

attributed to the small, but likely, magnitude error of 0.03 in  $|\Gamma_D|$ .

The poor predictability of this amplifier's performance is mostly due to the proximity of  $\Gamma_D$  to the unit circle. This in turn is due to the difference between the fin-line characteristic impedance and the magnitude of the diode impedance. (See Sec. 3-2.) The impedance of single-ridge fin line with a gap width of 1mm. is  $Z_0 \approx 100$  ohms. The magnitude of the diode impedance is  $Z_D \approx 15$  ohms. In theory, the fin-line impedance can be reduced to the optimum value of  $Z_D = |Z_D|$ .  $\Gamma_D$  would then be about four times further away from the unit circle (see Appendix 1), and the expected error in midband gain would be only 0.1 (assuming an experimental error of 0.03 in  $\Gamma_D$ ). However, the appropriate fin-line gap is much too small for practical implementation even at such low frequencies as 11 GHz.

Another way to improve the predictability is to increase the accuracy of the diode characterization. Suppose the amplifier described here was allowed to have a 10% error in passband gain. That is, the gain is allowed to vary from as little as 3.6 to as much as 4.4. The corresponding variation in  $|\Gamma_{in}|$  is from  $\sqrt{3.6} = 1.9$  to  $\sqrt{4.4} = 2.1$ . Thus the error circle around  $\Gamma_{in}$  in Fig. 3-3 is allowed to have a radius of as much as 0.1. Theoretically an error circle of radius 0.1 around  $\Gamma_{in}$  would require an error-circle radius of only 0.007 around  $\Gamma_D$ . This requires a four-fold improvement in the accuracy of the characterization technique. Such accuracy would be very difficult to achieve at mm-wave frequencies. Also, such accuracy may become lost in the diode mounting uncertainties. The use of a low-impedance type of transmission line (such as microstrip) would allow  $|\Gamma_D|$  to be maximized, and partially alleviates the need for excessive experimental accuracy.

The fin-line amplifier described here was a small-signal

amplifier; when the IMPATT amplifier is used in its normal function as a power amplifier the quantity  $1 - |\Gamma_D|$  drops to about one-third of its small signal value. Hence, the requirement for precise characterization becomes difficult for microstrip and practically impossible for fin line.

For these reasons it can be concluded that the IMPATT diode and the fin line are an incompatible combination for negative resistance amplifiers.

In this thesis it was demonstrated that symmetrical single-diode transmission amplifiers can be designed by the same methods that are used to design reflection amplifiers.

An improved method is described for measuring the S-parameters of structures in transmission media such as fin line for which there are no standard loads.

Empirical expressions have been presented for the S-parameters of open- and short-circuited series stubs in single-ridge unilateral fin line. This is a significant contribution to the available design information for these structures because the effects of frequency, substrate thickness, fin line gap and stub length are incorporated.

The predictability of a negative-resistance amplifier improves as the required gain decreases and as the diode reflection coefficient moves away from the unit circle. This behavior is illustrated by the sensitivity curves in Fig. 3-2. The IMPATT diode and the fin line are not a suitable combination for negative-resistance amplification because the large fin-line impedance causes the diode reflection coefficient to reside very close to the unit circle. The maximum distance from  $\Gamma_D$  to the unit circle is obtained when the transmission-line impedance is equal to the magnitude of the diode impedance. It is not feasible to produce the small fin-line gap width required to match the impedance of the IMPATT diode.

## APPENDIX 1

### THE REFLECTION COEFFICIENT OF A DIODE AS A FUNCTION OF CHARACTERISTIC IMPEDANCE

The reflection coefficient of a diode at the end of a transmission line is

$$\Gamma_D = \frac{-Z_0 + Z_D}{Z_0 + Z_D} \quad (A1-1)$$

where  $Z_0$  = transmission line characteristic impedance  
 $Z_D$  = diode impedance

The purpose of this appendix is to show how  $\Gamma_D$  varies with  $Z_0$  for a given  $Z_D = R_D + jX_D$ . This can be done by computing the mapping of the real axis of the  $Z_0$  plane into the  $\Gamma_D$  plane.

Eq. (A1-1) is a mapping of the form

$$z = \frac{aw + b}{cw + d} \quad (A1-2)$$

where  $w = Z_0$   
 $a = -1$   
 $b = Z_D$   
 $c = 1$   
 $d = Z_D$

Finding the mapping of the real axis of the  $Z_0$  plane is equivalent to finding the locus of  $w$  in the  $z$  plane by varying  $|w|$  and keeping  $\arg[w]$  constant. It is shown in [1] that the locus is a circle with a radius of

$$R = \frac{|p|}{2|d/c| \sin(\arg[d] - \arg[c] - \arg[w])} \quad (\text{A1-3})$$

and center at

$$C = a/c + R \exp(j\{\arg[p] - \arg[w] + \pi/2\}) \quad (\text{A1-4})$$

where

$$p = (ad - bc)/c^2 \quad (\text{A1-5})$$

(Note : eq. 2.157 in [1] contains a miss printed sign.)

Using the equivalences given with (A1-2) gives

$$R = [1 + (R_D/X_D)^2]^{1/2} \quad (\text{A1-6})$$

and

$$C = -jR_D/X_D \quad (\text{A1-7})$$

Since the center of the circle in the  $\Gamma_D$  plane lies on the imaginary axis, the point where  $|\Gamma_D|$  is largest must also lie on the imaginary axis. Thus, in (A1-1),

$$\arg[Z_D - Z_0] = \arg[Z_D + Z_0] \pm \pi/2 \quad (\text{A1-8})$$

when  $|\Gamma_D|$  is largest. This means that the origin and the two points  $Z_D - Z_0$  and  $Z_D + Z_0$  form the vertices of a right triangle. (See Fig. A1-1.) The hypotenuse is the magnitude of the difference between these two vertices.

$$|(Z_D + Z_0) - (Z_D - Z_0)| = 2 Z_0 \quad (\text{A1-9})$$

Application of the Pythagorean theorem to the triangle in Fig. A1-1 gives

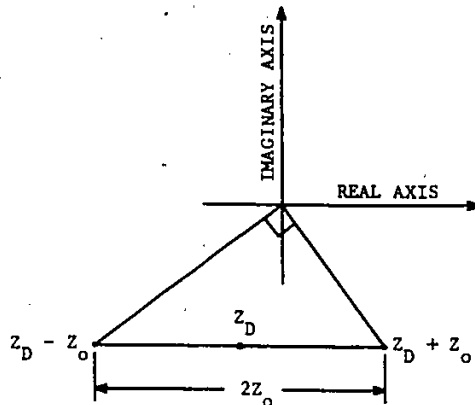


Fig. A1-1. The origin and the points  $z_D + z_0$  and  $z_D - z_0$  form a right triangle whose hypotenuse is  $2z_0$  in length.

$$|2z_0|^2 = |z_D + z_0|^2 + |z_D - z_0|^2 \quad (\text{A1-10})$$

Inserting  $z_D = R_D + jX_D$  into (A1-10) and solving for  $z_0$  gives

$$z_0^2 = R_D^2 + X_D^2 \quad (\text{A1-11})$$

or

$$z_0 = |z_D| \quad (\text{A1-12})$$

Thus for maximum  $|\Gamma_D|$ ,  $z_0$  must equal  $|z_D|$ .

To illustrate the dependence of  $\Gamma_D$  on  $z_0$ , the mapping in Fig. A1-2 was drawn. The diode impedance is the small signal impedance of the fictitious diode described in Appendix 3 (freq. = 11.6 GHz,  $I_{\text{bias}} = 225$  mA). The quantity  $1 - |\Gamma_D|$  diminishes by a factor of about four as  $z_0$  goes from  $15\Omega$  to  $100\Omega$ .

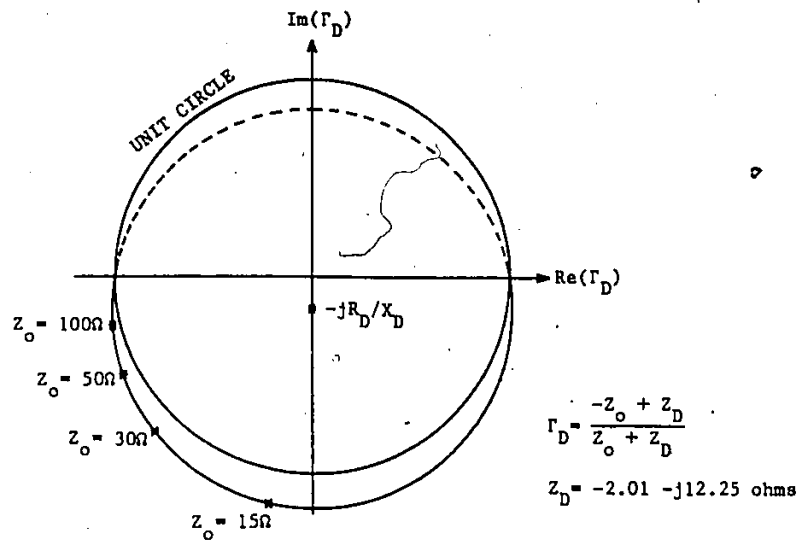


Fig. A1-2 A plot of  $\Gamma_D$  as function of  $Z_o$ .  $|\Gamma_D|$  is largest when  $Z_o = |Z_D|$ .

#### REFERENCE

- [1] D.M.Kerns and R.W.Beatty, Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis. Oxford: Pergamon Press, 1967, pp. 96-97.

## APPENDIX 2

### GENERAL FORMULAS FOR THE RADIUS AND CENTER OF A BILINEARLY TRANSFORMED CIRCLE

Eq. (A1-2) is known as the bilinear transformation. It has the property of mapping circles in the w-plane into circles in the z-plane. Whereas Appendix 1 gives the radius and center of the z-circle for the special case that the w-circle is a straight line through the origin (infinite radius and center at infinity), this appendix gives formulas for the radius and center in the general case.

The mapping (A1-2) is equivalent to

$$z = E + \frac{F}{w + G} \quad (A2-1)$$

where  $E = a/c$

$$F = \frac{bc - ad}{c^2}$$

$$G = d/c$$

Thus, to obtain the z-circle, the w-circle is first translated by G, then inverted, then stretched and rotated by F, and finally given another translation by E. Applying these operations to the w-circle with radius  $R_w$  and center at  $C_w$  leads to the following formulas for the radius  $R_z$  and the center  $C_z$  of the z-circle:

$$C_z = E + F \frac{H}{I} \quad (A2-2)$$

$$R_z = |F| \frac{|J|}{|I|} \quad (A2-3)$$

where  $H = K + L$

$$I = 2KL$$

$$J = K - L$$

$$K = M + \frac{R_w M}{|M|}$$

$$L = M - \frac{R_w M}{|M|}$$

$$M = C_w + G$$

APPENDIX 3

FORMULAS FOR THE IMPEDANCE OF A FICTITIOUS BUT TYPICAL  
IMPATT DIODE

The following two expressions describe the chip impedance of a typical silicon IMPATT DDR diode. They were created to aid the author's understanding of negative resistance amplifiers and oscillators. The expressions were derived by fitting to a variety of published experimental and theoretical curves for this type of diode. These expressions are believed to show the correct qualitative dependence on bias current, RF current, and frequency. Appropriate constants in the expressions were adjusted to give agreement with the manufacturer's data for the diode that was used in this thesis work (NEC ND8T11W-5H).

Chip reactance (ohms) :

$$X_D = -330/f \{ 1 - [1 - \exp(-I_{RF}/2.5)] [\exp((5-f)/4)] \} \quad (A3-1)$$

Chip resistance (ohms) :

$$R_D = R_1 R_2 + 0.24 I_{RF} \quad (A3-2a)$$

$$\text{where } R_1 = 5 \frac{9.4}{[1 + B^2]^{1/2}} \quad (A3-2b)$$

$$R_2 = \frac{I_{BIAS}}{225} \left[ 1 - \frac{I_{RF}}{3.7} \right] \quad (A3-2c)$$

$$B = \frac{A - 11}{3} \quad (A3-2d)$$

$$A = f - 2\exp(6 - f) \quad (\text{A3-2e})$$

$f$  = frequency in GHz.

$I_{RF}$  = peak RF current through the chip in A.

$I_{BIAS}$  = dc bias current through the chip in mA. The maximum safe bias current is 225 mA.

The maximum RF power available from this diode is 3.09 W at 11 GHz. This is also the maximum safe power for an IMPATT diode; driving a diode harder (increasing  $I_{RF}$ ) actually reduces the power added and drastically increases the risk of failure. When (A3-2) is used in

$$\text{Power added} = -\frac{1}{2}(I_{RF})^2 R_D \quad , \quad I_{BIAS} = 225 \quad , \quad f = 11 \quad (\text{A3-3})$$

the power added will increase with  $I_{RF}$  up to 3.09 W at  $I_{RF} = 2.05$  A and then decrease beyond this point.

(A3-1) and (A3-2) can be computed by the Fortran subroutine Z\_OF\_IMPATT the listing for which appears at the end of this appendix.

The IMPATT diode chip is normally placed in a hermetically sealed package that adds additional parasitic reactances to form the total impedance of the packaged diode. The equivalent circuit for the packaged diode is shown in Fig. A3-1. The subroutine PACKAGED\_DIODE computes the impedance of the packaged diode from the parasitics given in Fig. A3-1 and from the results of Z\_OF\_IMPATT. Also computed are the right and left side admittances and the diode reflection coefficient. These can be used to find the dependence of reflection coefficient on incident power. The listing for PACKAGED\_DIODE appears at the end of this appendix.

At microwave frequencies, a useful method of characterizing

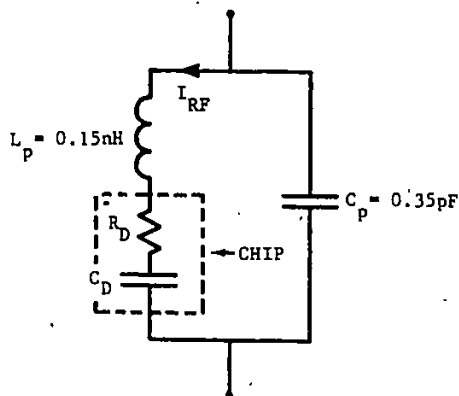


Fig. A3-1 The equivalent circuit for the fictitious diode. The package parasitics are the same as the published values for diode used in the fin line amplifier (NEC ND8T11W-5H).

nonlinear devices is to obtain their reflection coefficients for different incident power levels. To simulate this, a third program called QWIKDIOD1 was written to convert the impedance versus RF current description of the diode to a reflection coefficient versus incident power description. Because of the nonlinear interdependence of RF current, reflection coefficient and incident power, the reflection coefficient cannot be written as an explicit function of incident power. Instead, the program uses a method of successive approximations that becomes sufficiently accurate after less than six iterations. The results of this simulated characterization are presented in Fig. A3-2 as characteristic curves of  $\Gamma_D$  versus frequency for various incident power levels.

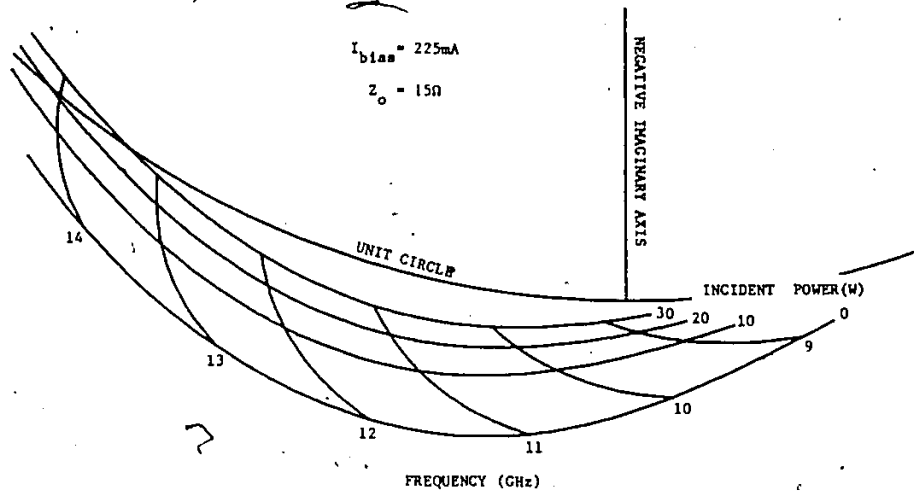


Fig. A3-2 Characteristic curves for the fictitious IMPATT diode.

The program listings for QWIKDIOD1, PACKAGED\_DIODE, and Z\_OF\_IMPATT appear next:

4-May-1985 15:29:38  
4-May-1985 15:29:09  
SYSUSER:EBURTONJRHJKI001.FORJ3

0001 C This program was written to convert the impedance versus  
0002 C RF current description of the diode to a reflection coefficient  
0003 C versus incident power description. It computes single values of peak rf  
0004 C current in the chip IRF and of power added by the diode PADDED when  
0005 C the diode is packaged and mounted at the end of a transmission line.  
0006 C The values of characteristic impedance Z0, bias current IBIAS,  
0007 C frequency FREQ and incident power PINC are manually entered at the  
0008 C program runs. Diode refl. coeff. is also computed.  
0009 C The program is essentially a rapidly converging iteration  
0010 C that makes successively better approximations of the peak rf current  
0011 C in the chip after an initial guess of IRF = 0. Convergence to a  
0012 C tolerance TOLERANCE of .0001 in IRF typically occurs before the  
0013 C sixth iteration.

REAL LAST\_IRF\_CHIP, IRF\_PKG, IRF\_CHIP, IBIAS  
COMPLEX REF\_COEFF, Z\_PKG, YLEFT, YRIGHT, Y\_PKG  
INTEGER COUNTER

1 TYPE \*  
10 TYPE 10  
FORMAT ('ENTER Z0, IBIAS, FREQ, PINC, Tolerance I')  
ACCEPT \*, Z0, IBIAS, FREQ, PINC, TOLERANCE

4 ROOT2 = SQRT(2.0)  
ROOTZO = SQRT(Z0)  
C Compute the magnitude of the incident voltage wave.  
AI = SQR1(PINC)

LAST\_IRF\_CHIP = 0.0  
COUNTER = 0  
DELTA\_IRF\_CHIP = TOLERANCE + 1.0  
C Make successive approximations of the diode behavior until the  
C change in peak rf current in the chip is less than TOLERANCE.  
DO WHILE (DELTA\_IRF\_CHIP .GT. TOLERANCE)

C Use the last rf chip current to compute a better approximation of  
C the diode behavior.  
CALL PACKAGED\_DIODE (Z0, FREQ, LAST\_IRF\_CHIP, IBIAS,  
1 REF\_COEFF, YLEFT, YRIGHT, Y\_PKG, Z\_PKG)

C Use the peak rf current at package terminals IRF\_PKG to compute the  
C the peak rf current through the chip IRF\_CHIP.  
C IRF\_PKG = ROOT12 \* AI/ROOTZO \* CABS(1.0 - REF\_COEFF)  
C IRF\_CHIP = IRF\_PKG \* CABS(YLEFT/Y\_PKG)  
C Compute the difference between the chip current just computed and  
C the chip current from the last iteration.  
DELTA\_IRF\_CHIP = ABS(LAST\_IRF\_CHIP - IRF\_CHIP)

LAST\_IRF\_CHIP = IRF\_CHIP  
COUNTER = COUNTER + 1

END III

WIKIDJ01\$MAIN

```

0058 C Compute the magnitude of the reflection coefficient.
0059 GAMMA = CABS( REF_COEFF )
0060
0061 C Compute the power added by the diode.
0062 PADDED = PINC * (GAMMA$GAMMA - 1.0)
0063
0064 C Print the results at the terminal.
0065 TYPE *, 'Reflection coefficient =', REF_COEFF
0066 TYPE *, 'Padded =', PADDED
0067 TYPE *, 'Irf =', IRF_CHIP, ' Number of iterations is', COUNTER
0068
0069
0070
0071 C Repeat for a new set of values for Z0, frequency, etc.
0072 GOTO 1
0073 END

```

PROGRAM SECTIONS

Name	Bytes	Attributes
0 \$CODE	451	PIC CUM REL LCL SHR EXE RD NOMRT LONG
1 \$DATA	311	PIC CUM REL LCL SHR NOEXE RD NOMRT LONG
2 \$LOCAL	152	PIC CUM REL LCL NOSHR NOEXE RD WRT QUAD
Total Space Allocated		714

ENTRY POINTS

Address	Type	Name
0-00000000		(WIKIDJ01\$MAIN

VARIABLES

Address	Type	Name
\$*	R24	AI
\$*	R24	DELTA_IRF_CHIP
\$*	R24	GAMMA
\$*	R24	IRF_CHIP
2-0000002B	R24	LAST_IRF_CHIP
2-0000003B	R24	PINC
\$*	R24	R00T2
2-0000003C	R24	TOLERANCE
2-0000001B	C28	YRIGHT
2-00000030	R24	Z0
\$*	C28	COUNTER
2-00000034	R24	FREQ
2-0000002C	R24	IBIAS
\$*	R24	IRF_PKG
\$*	R24	PADDED
2-00000000	C28	REF_COEFF
\$*	R24	R00T20
2-00000010	C28	YLEFT
2-00000020	C28	Y_PKG
2-00000008	C28	Z_PKG

QWIKDIOD1:MAIN

LABELS

Address	Label	Address	Label
0-0000001H	1	1-00000042	10'

FUNCTIONS AND SUBROUTINES REFERENCED

Type Name	Type Name
R04 MTHOCABS	R04 MTHASORT

COMMAND QUALIFIERS

```

F/LIS QWIKDIOD1
/CHECK=(NOROUNDS,OVERFLOW,NUNDERFLOW)
/DEBUG=(NOSTACKS,TRACEBACK)
/STANDARD=(NOSYNTAX,NOSOURCE,FORH)
/SHOW=(NODIRECTOR,NODICTIONARY,STIMBLE)
/WARNINGS=(GENERAL,NODECLARATIONS)
/CONTINUATIONS=19 /NOCROSS-REFERENCE /NOI.LINES /NOEXTEND.SOURCE /F77
/NOG.FLOATING /I4 /NOMACHINE.CODE /OPTIMIZE

```

COMPILATION STATISTICS

```

Run Time: 3.12 seconds
Elapsed Time: 9.57 seconds
Page Faults: 439
Dynamic Memory: 344 pages

```

SUBROUTINE PACKAGED\_DIODE (Z0, F, IRF, IRIAS,  
REF\_COEFF, YLEFT, YRIGHT, Y\_PKG, Z\_PKG)

1  
C This subroutine computes the typical behavior of a packaged IMPATT  
C diode designed for X-band. The diode is assumed to be mounted at  
C the end of a transmission line of characteristic impedance Z0  
C and the package capacitance CP incorporates the effect of the local  
C geometry outside of the package. The package inductance LP is assumed  
C to be unaffected by the external geometry.  
C The behavior is given in terms of the reflection coefficient  
C REF\_COEFF, the left-hand (active) admittance YLEFT of the equivalent  
C circuit, the right-hand admittance YRIGHT of the equivalent circuit,  
C the total admittance at the package terminals Y\_PKG, and the  
C corresponding impedance at the package terminals Z\_PKG.  
C The behavior is a function of the characteristic impedance Z0,  
C the frequency F, the peak rf current in the chip IRF, and the bias  
C current IRIAS. Although the peak rf current in the chip is assumed  
C to be already known, it is actually a function of the characteristic  
C impedance and the incident power. Thus, to compute the diode behavior  
C as a function of incident power, this subroutine can be used in an  
C iteration that makes successively better approximations of the diode  
C behavior by repeatedly revising the RF chip current from an initial  
C guess of IRF = 0.  
C F is in GHz, IRF is in A, and IRIAS is in mA.

COMPLEX REF\_COEFF, ZLEFT, YLEFT, YRIGHT, Y\_PKG, Z\_PKG  
COMPLEX Z\_OF\_IMPATT  
REAL LP, IRF, IRIAS  
PARAMETER (TWO\_PI=6.2832, PI=3.1416, CP=0.35E-3, LP=0.15)

C Compute the characteristic admittance Y0.  
Y0 = 1.0 / Z0

C Compute the left-hand admittance from the left-hand impedance ZLEFT.  
ZLEFT is the sum of the chip impedance and the parasitic inductive  
C reactance of the package.  
ZLEFT = Z\_OF\_IMPATT (F, IRF, IRIAS) + CMF1 X (0.0, TWO\_PI\*PI\*LP)  
YLEFT = 1.0 / ZLEFT

C Compute the right hand admittance.  
YRIGHT = CMPLX( 0.0, TWO\_PI\*PI\*CP )

C Compute the total admittance and the corresponding impedance.  
Y\_PKG = YRIGHT + YLEFT  
Z\_PKG = 1.0 / Y\_PKG

C Compute the reflection coefficient.  
REF\_COEFF = (Y0 - Y\_PKG) / (Y0 + Y\_PKG)

END

PACKAGED\_DIODE

PROGRAM SECTIONS

Name	Bytes	Attributes
0 \$CODE	213	PIC COM REL LCL SHR EXE RD NDKMT LONG
2 \$LDCAL	28	PIC COM REL LCL NDSHR MEXE RD MKT QUAD
Total Space Allocated		241

ENTRY POINTS

Address	Type	Name
0-00000000		PACKAGED_DIODE

VARIABLES

Address	Type	Name	Address	Type	Name
AP-00000008	R#4 F	IRJAS	AP-00000010	R#4 IRF	REF_COEFF
2-00000008	R#4 Y0	YLEFT	AP-0000001C	C#8 YRIGHT	Y_PKG
AP-00000004	R#4 Z0	ZLEFT	AP-00000024	C#8 Z_PKG	

FUNCTIONS AND SUBROUTINES REFERENCED

Type	Name
C#8	Z-DF-IMPATT

COMMAND QUALIFIERS

/LIS/NDOP PKGDIODE  
 /CHECK=(NOBONDS,OVERFLOW,NOUNDERFLOW)  
 /DEBUG=(NOSYMBOLS,TRACEBACK)  
 /STANDARD=(NOSYNTAX,NO\_SOURCE\_FORM)  
 /SHOW=(NDPREPROCESSOR,NOINCLUDE,MAP,NOINDICATORY,SINGLE)  
 /WARNINGS=(GENERAL,NODECLARATIONS)  
 /CONTINUATIONS=19 /NOCKROSS.REFERENCE /NOO.LINES /NOEXTEND\_SOURCE /F77  
 /NOG-FLOATING /14 /NOMACHINE\_CODE /NOOPTIMIZE

COMPILATION STATISTICS

Run Time:	1.81 seconds
Elapsed Time:	4.04 seconds
Page Faults:	274
Dynamic Memory:	328 pages

VAX FORTRAN V4.0-2  
SYS\$USER:ERUKTONJZIMPATT.FOR:1

13-Feb-1985 17:28:49  
10-Jun-1984 11:00:49

COMPLEX FUNCTION Z\_OF\_IMPATT (FREQ, IRF, IRIAS)

C This subroutine computes the impedance of an IMPATT diode chip  
C designed for operation at X-band. The formulas for chip resistance RD  
C and chip reactance XD show a dependence on frequency FREQ, bias  
C current IRIAS, and peak rf current IRF that is typical of a silicon  
C DBR diode. The maximum power generated by the diode is 3.09 W, at  
C 11 GHz, with peak rf current of 2.051 A. The corresponding chip  
C resistance is -j.47 ohms.

C This subroutine is called by subroutine PACKAGED\_DIODE to  
C compute the behavior of the packaged chip when it is mounted at the end of  
C a transmission line.

C Frequency is in GHz, bias current is in mA, and the peak  
C rf current is in A.

REAL IRIAS, IBIAS\_OP, IRF

PARAMETER (IBIAS\_OP = 225.0)

XD = -330.0 / FREQ \* (1.0 - (1.0 - EXP(-IRF/2.5)) \*  
1 EXP((5.0 - FREQ) / 4.0))

FF = FREQ - 2.0 \* EXP(6.0 - FREQ)

PP = (FF - 11.0) / 3.0

R1 = -9.4 / SQRT(1.0 + PP\*PP) + 5.0

K2 = IBIAS/IRIAS\_OP \* (1.0 - IRF/3.7)

RD = R1\*R2 + 0.24\*IRF

Z\_OF\_IMPATT = CMPLX( RD, XD )

END

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13-Feb-1985 17:28:49 VAX FORTKAM V4.0.0  
 10-Jun-1984 11:00:49 SYSUSER:(BURTON)ZIMFALL.FOR:1

Z\_OF\_IMPATT

PROGRAM SECTIONS

Name	Bytes	Attributes
0 \$CODE	180	PIC CON REL LCL SHR EXE RD NOWRT LONG
2 \$LOCAL	32	PIC CON REL LCL NDSHR NOEXE RD WRT QUAD
Total Space Allocated	212	

ENTRY POINTS

Address	Type	Name
0-00000000	C#B	Z_OF_IMPATT

VARIABLES

Address	Type	Name	Address	Type	Name
2-00000000	R#4	FF	AP-00000000	R#4	IRIAS
2-00000010	R#4	FP	2-0000001B	R#4	R?
2-00000008	R#4	XI	2-0000001C	R#4	RD

FUNCTIONS AND SUBROUTINES REFERENCED

Type	Name
R#4	MHS\$EXP
R#4	MHS\$SORT

COMMAND QUALIFIERS

```

F/LIS/NOOP ZIMPATT
/CHECK=(NOROUNDS,OVERFLOW,NOUNDERFLOW)
/DEBUG=(NOSYMBOLS,TRACEBACK)
/STANDARD=(NOSYNTAX,NOSOURCE_FORM)
/SHOW=(NOPREPROCESSOR,NOINCLUDE,MAP,NOINDICATORY,SINGLE)
/WARNING=(GENERAL,NODECLARATIONS)
/CONTINUATIONS=19 /NOCROSS-REFERENCE /NO_LINES /NOEXTEND_SOURCE /F77
/NOB_FLOATING /I4 /NOMACHINE_CODE /NOOPTIMIZE
  
```

COMPILATION STATISTICS

```

Run Time: 1.73 seconds
Elapsed Time: 4.11 seconds
Page Faults: 188
Dynamic Memory: 320 Pages
  
```

APPENDIX 4

THE DE-EMBEDDING FORMULAS OF KRUPPA AND SODOMSKY

The S-parameters of the unknown 2-port in Fig. A4-1 can be determined from the S-parameters of the embedding 2-ports and from the measured S-parameters of the cascade of 2-ports. Explicit formulas for the S-parameters of the embedded 2-port are [1]

$$S_{11} = \frac{(R_F - S_{11}^{(1)})[S_{22}^{(2)}(R_R - S_{11}^{(2)}) + S_{21}^{(2)}S_{12}^{(2)}] - S_{22}^{(2)}T_R T_F}{D} \quad (A4-1)$$

$$S_{12} = \frac{T_R S_{21}^{(1)} S_{12}^{(2)}}{D} \quad (A4-2)$$

$$S_{21} = \frac{T_F S_{21}^{(2)} S_{12}^{(1)}}{D} \quad (A4-3)$$

$$S_{22} = \frac{(R_R - S_{11}^{(2)})[S_{22}^{(1)}(R_F - S_{11}^{(1)}) + S_{21}^{(1)}S_{12}^{(1)}] - S_{22}^{(1)}T_R T_F}{D} \quad (A4-4)$$

where  $D = UV - W$

$$U = S_{12}^{(1)}S_{21}^{(1)} + S_{22}^{(1)}(R_F - S_{11}^{(1)})$$

$$V = S_{21}^{(2)}S_{12}^{(2)} + S_{22}^{(2)}(R_R - S_{11}^{(2)})$$

$$W = S_{22}^{(2)}S_{22}^{(1)}T_R T_F$$

$R_F$ ,  $T_F$ ,  $R_R$ , and  $T_R$  are the measured S-parameters of the composite circuit in Fig. A4-1.  $R_F$  and  $T_F$  are the reflection

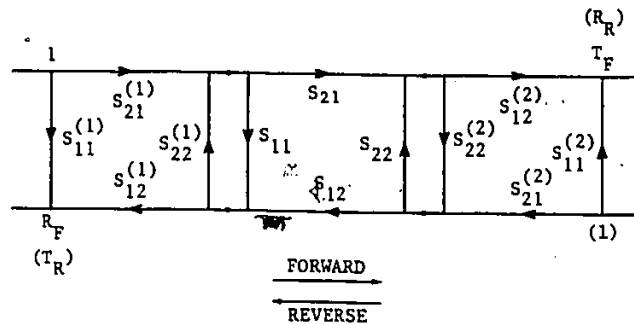


Fig. A4-1 An unknown 2-port embedded between two known 2-ports. The forward S-parameters of the entire cascade are  $R_F$  and  $T_F$ . The reverse S-parameters are  $R_R$  and  $T_R$ .

and transmission S-parameters, resp. measured in the forward direction (left to right in the figure).  $R_R$  and  $T_R$  are the reflection and transmission S-parameters, resp. measured in the reverse direction (right to left in the figure).

#### REFERENCE

- [1] W.Kruppa and K.F.Sodomsy, "An explicit solution for the scattering parameters of a linear two-port measured with an imperfect test set," IEEE Trans. Microwave Theory Tech., vol. MTT-19, pp. 122-123, Jan. 1971.

APPENDIX 5

A DETERMINATION OF THE RANGE OF  $S_{22}$  FOR A LOSSLESS EQUALIZER THAT WILL OBTAIN THE REQUIRED GAIN FROM A NEGATIVE-RESISTANCE DEVICE

In Sec. 3-1 it is shown that, when a lossless equalizer is connected to a negative-resistance device (N.R.D.), the circuit must satisfy the the following restriction:

$$|1 - \Gamma_D S_{22}|^2 = K(1 - |S_{22}|^2) \quad (A5-1)$$

$$\text{where } K = \frac{|\Gamma_D|^2 - 1}{G - 1}$$

$G$  = amplifier gain

$\Gamma_D$  = reflection coefficient of the N.R.D.

$S_{22}$  = match terminated reflection coefficient of the lossless equalizer as seen by the N.R.D.

The squared magnitude of  $1 - \Gamma_D S_{22}$  can be obtained by a direct application of the cosine law. The result is

$$1 + |\Gamma_D S_{22}|^2 - 2|\Gamma_D S_{22}|\cos(\alpha) = K(1 - |S_{22}|^2) \quad (A5-2)$$

where  $\alpha = \arg[\Gamma_D] + \arg[S_{22}]$

Isolating  $\cos(\alpha)$  and simplifying gives

$$\cos(\alpha) = \frac{1 - K}{2|\Gamma_D||S_{22}|} + \frac{(|\Gamma_D|^2 + K)|S_{22}|}{2|\Gamma_D|} \quad (A5-3)$$

Since  $\cos(\alpha)$  must be less than or equal to one, the limits for  $|S_{22}|$  can be found by solving

$$1 = \frac{1 - K}{2|\Gamma_D||S_{22}|} + \frac{(|\Gamma_D|^2 + K)|S_{22}|}{2|\Gamma_D|} \quad (\text{A5-4})$$

for  $|S_{22}|$ . The two solutions are

$$|S_{22}| = \frac{|\Gamma_D| \pm (K^2 + |\Gamma_D|^2 K - K)^{1/2}}{|\Gamma_D|^2 + K} \quad (\text{A5-5})$$

When  $S_{22}$  is within these limits, its argument can be found from

$$\arg[S_{22}] = \alpha - \arg[\Gamma_D] \quad (\text{A5-6})$$