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Reliability and Availability Analysis of Human-Machine Systems

Thesis submitted to the University of Ottawa
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Mechanical Engineering

By
Nianfu Yang

Ottawa-Carleton Institute for
Mechanical and Aeronautical Engineering

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ISBN 0-612-04978-7

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Acknowledgments

I would like to express my sincere gratitude to my Thesis supervisor Dr. B. S. Dhillon for his continuous encouragement, advice and supervision. I wish to thank my colleague Dr. O.C. Anude for useful discussions and helps.

I also want to seize this opportunity to thank members of my academic advisory committee Drs. D.S. Necsulescu, G. Kardos, M. Liang, and S.M. Taboun for their steadying influence and concern.

The preparation of this Thesis has been a long and arduous task, interrupted by the departures of my loved mother and father from this world. I am deeply grateful to my wife, Ning, and my family for their support and encouragement throughout this period.

Dedicated to
my wife, Ning; and my son, Chenguang;
and
to the memories of
my mother and father

Abstract

This study presents a reliability and availability analysis of human-machine systems with human errors and common-cause failures. The systems incorporate elements of several commonly used redundant configurations: standby, k-out-of-n, majority voting with imperfect voter, and parallel. The analysis considers systems with constant human error rates, increasing human error rates and general human error rates, and with arbitrarily distributed failed system repair times.

The method of linear ordinary differential equations for general system failure rates is presented to obtain the general expressions of the steady state availability for various types of system repair time distributions, such as Gamma, Weibull, lognormal, exponential and Rayleigh distributions. A method which combines the inclusion supplementary variables technique and the method of stages is developed to perform time-dependent system availability analysis for systems with both time-dependent human error rates and failed system repair rates. Generalized expressions for such relevant system performance indices as the system reliability, steady state availability, time-dependent system availability, mean time to failure and system variance of time to failure are presented. The impact of human error, common-cause failure, failed system repair policy and the elements of redundant configurations on the values of the afore-said system performance indices is demonstrated by means of plots.

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NOTATION

The following symbols are associated with the configurations in this study:

i	ith state of the system;
n	Number of active units
m	Number of standby units
k	The least number of units needed for system success (in k-out-of-n system, $k \leq n$)
r_i	Constant hardware failure rate of a unit in state i
μ_i	Constant repair rate of a failed unit in state i
$\mu_j(x)$	Time-dependent failed system repair rate when the system is in state j and has an elapsed repair time of x
λ_{ci}	Constant common-cause failure rate in state i
λ_{hi}	Constant critical human error rate in state i
λ	Unit failure rate in the operating state
λ_s	Unit failure rate in the standby state or in reserve state
s	Laplace transform variable
$P_i(t)$	The probability that the system is in state i at time t
$P_j(x,t)$	The probability that the failed system is in state j and has an elapsed repair time of x
P_i	The steady state probability that the system is in state i
$P_i(s)$	The Laplace transform of $P_i(t)$
$E_j[x]$	The mean time to system repair that the failed system is in state j and has an elapsed repair time of x
$R(t)$	System reliability
$R(s)$	The Laplace transform of $R(t)$
AV_{ss}	Steady state availability of the system
UAV_{ss}	Steady state unavailability of the system
$AV(t)$	Time-dependent system availability
$AV(s)$	The Laplace transform of $AV(t)$

α	A scale parameter for Weibull and gamma distributions.
β	A shape parameter for Weibull and gamma distributions.
σ	A standard deviation parameter of $\ln x$ for lognormal distribution
σ^2	The variance of time to failure of the system
pdf	probability density function
MTTF	System mean time to failure
$N_j(x)$	pdf of the system repair time when the system is in state j and has an elapsed repair time of x, for $j=m+n$, $m+n+1$, $m+n+2$.
$N_j(s)$	Laplace transform of $N_j(x)$.

Chapter 1

Introduction

Many engineering systems are interconnected by human links, and in most situations the human link is inevitable irrespective of the degree of automation. Human interactions with machines have long been recognized as important contributors to continued safe operation in most industries. Even when selection and training are efficiently carried out and appropriate design features are incorporated, people are not always reliable. They make mistakes, and in some cases their errors will lead to systems failure and accidents. According to Christensen [60], about 50-70% of the failures in electronic equipment were human initiated, whereas in aircraft and missile systems the human initiated failures accounted for 60-70% and 20-53% of the total failures, respectively. The nuclear power plant industry has estimated risk due to human action or inaction at 50-70% [112]. Even highly automated systems need constant monitoring and maintenance and do not totally remove human involvement. Therefore, human reliability should be an important consideration in all stages of system design and management.

Some of the human errors may occur in form of common-cause failures. A common-cause failure event occurs when multiple units or elements fail due to a single cause [75]. Taylor [347], in 1975, reporting on the frequency of common-cause failures in the United States Power Reactor Industry asserted that "of 379 component failures or groups of failures arising from independent causes, 78 involved common-mode failures of two or more components". Some of the reasons for common-cause failures include design errors and deficiencies; operations and maintenance errors, abnormal environment, external catastrophe, etc.

To ensure that the system reliability is higher than those of its sub-systems (or components), component redundancy is employed. The term *Redundant* indicates the existence of more than one means, identical or non-identical, for accomplishing a given task or mission. The parallel, k-out-of-n and standby structures are the wildly used redundant configurations. *Parallel structure* describes a system that can succeed when at least one of the components succeeds. Such a system is also known as a *fully or completely redundant* system. A *k-out-of-n system* consists of n identical independent components of which at least k (< n) of the components should succeed in order for the system to succeed. For k = 1, they become truly parallel (fully redundant) systems, and for k = n, they become series (non-redundant) systems. (k+1)-out-of-(2k+1) system is also called *majority voting* system. *Standby redundancy* is redundancy wherein a system's backup unit does not operate until it is needed and is switched on only when the main unit fails to perform its task.

For complex equipment and systems, reliability analysis is generally performed at two different levels. At subassembly level, the designer performs failure rate and failure mode analyses to check fulfillment of reliability requirements, and to detect and eliminate reliability weaknesses as early as possible in the design phase. At equipment and system level, the reliability engineer also investigates time behavior, taking into account reliability, maintainability, and logistical aspects. Depending upon the system complexity, upon the assumed distribution functions for failure-free and repair times, as well as maintenance policy, investigations are performed either analytically, making use of stochastic processes , numerical simulations, or through other means.

In usual reliability and availability analysis, the occurrence of human errors and common-cause failures is overlooked and only general failures are considered. Under such conditions, the end results may not present a true picture regarding the actual system reliability. It is widely believed that the realistic system reliability and availability analysis must include the occurrence of common-cause failures and human errors.

This study is concerned with reliability and availability analysis of human-machine systems with human errors and common-cause failures. These systems incorporate elements of several commonly used redundant configurations: parallel, k-out-of-n, majority voting and standby. Human error rates of the systems in any state are either constant, increase or general, whereas repair times follow general distribution. Some system performance indices, notably system reliability, system steady state availability and time-dependent system availability, system mean time to failure and system variance of time to failure are evaluated using stochastic process models.

1.1 Human Error and Human Reliability Analysis

Every human-machine system contains certain functions that are allocated to the person, and a failure to perform these functions correctly within prescribed limits can lead to systems failure. A *human error* is defined as a failure to perform a prescribed act (or the performance of a prohibited act), which could result in damage to equipment and property or disruption of scheduled operations [164]. There are various factors which may lead to a human error. Some of these are inadequate training of the concerned personnel, poorly written equipment maintenance and operating procedures, task complexity, inadequate lighting, high noise level, improper tools and poor equipment design [238].

The consequence of human errors may differ from one set of equipment to another or one task to another. Consequences may range from minor to severe (for example, from delay in system performance to loss of life).

A *critical human error* is one which causes the failure of the entire system, on the other hand, a *non-critical human error* does not lead to a catastrophic result.

Human reliability is the probability that a job or task will be successfully completed by personnel at any required minimum time (if the time requirement exists) [74]. Furthermore, *human reliability analysis (HRA)* is any method or approach that can be used to estimate the quantitative or qualitative contribution of human performance to engineering system reliability and safety [344]. As systems become more complex and have the potential for major economic or safety loss, there is a growing need for HRA. It is obvious that as equipment becomes more reliable, human errors contribute relatively more to system problems. In addition to safety considerations, a high level of human reliability is also essential in order to maximize the availability and productivity of such system.

1.2 A Brief History of HRA

In the earlier reliability analysis, attention was directed only to equipment, and reliability of the human element was neglected. Williams [382] recognized this shortcoming in the late 1950s and pointed out that realistic system reliability analysis must include the human aspect. In the 1960s, a number of publications relating to human reliability appeared in journals, conference proceedings and technical reports. Many of these publications are listed in Ref. [240]

However at the time, there was very little in the way of human factors data and also no accepted human performance theories or models . This state of affairs led to a research project that produced a workable collection of human reliability figures known as the AIR (American Institute for Research) Data store in 1962 [252]. In 1964, several approaches to quantifying human performance using the AIR Data Store were reported in December 1964, of *Human Factors, the journal of Human Factor Society.* In this same time frame, the well-know THERP (Technique for Human Error Rate Prediction) was developed by Swain and his collaborators [339,340].

The year 1973 may be regarded as an important milestone in the history of human reliability. It was in August of that year when *IEEE Transactions on Reliability* published a special issue devoted to human reliability [287]. A number of excellent papers appeared in this issue. HRA was initially applied to the quantitative assessment of the influence of human error in complex military systems. *The 1975 Reliability and Maintainability Symposium* [271] exhibited examples of HRA models which had been extended to different applications earlier in the 1970s. The first applications of HRA to commercial systems (nuclear reactor facilities) were done in Europe by such organizations as the Risϕ National Laboratory, Roskilde, Denmark; the United Kingdom Atomic Energy Authority, Warrington, England; and the French Atomic Energy Commission, Paris, France. The first large-scale application of HRA to a US commercial system began late in 1972, known as the WASH-1400 Reactor Safety Study [359], this study employed the THERP HRA method to assess the impact of estimated human errors in a PRA of two nuclear power plants. The experience gained in using THERP in the WASH-1400 study led to the development of the NRC (Nuclear Regulatory Commission) handbook in 1983. Ever since, researchers have been making further advances in the human reliability field. Numerous methodologies have been developed to address the HRA task. The review and criticism of most of these models may be found in [6, 8, 13, 128, 192, 209, 269, 306, 326, 367].

Although many human reliability methods and techniques have generally been developed for the use in the large scale systems such as nuclear power plants, the chemical process industry, and military systems, they also can be applied in variety of other situations. In recent years increasing attention has been paid to several different aspects of the human reliability field, namely human error analysis, data collection, quantification of human reliability, reliability evaluation of human-machine systems and so on.

1.3 Literature Review and Current Status of HRA

HRA is a developing field. Recognition of the importance of the human operator in human-machine system and attempts to quantify human reliability has been incorporated into system thinking since the late 1950s. The majority of the work has taken place within those industries that are perceived as "high risk" (for example, aerospace, chemical and nuclear industries). Filtering through the vast literature on human reliability, it is obvious that HRA could be split into follow groups:

(1) Human error classification, analysis and data bank.

(2) Human error rate prediction and human performance reliability models

(3) Human-machine system reliability--which combine human and hardware performance measures in a meaningful way to obtain a more valid system reliability measure.

(4) Applications of human reliability engineering.

A brief description of these groups is as follows.

(1) Human Error Classification, Analysis and Databank

An effective classification and analysis of human errors are necessary to develop a store of qualitative and quantitative information on the subject as well as to provide useful insights into their behaviors and preventive measures.

Dhillon [101] briefly reviewed human errors in engineering systems and Camino [51] described five human characteristics. Five ways in which human errors occur are discussed by Hammer [169] and a methodology for analysis and classification of human error is presented by Rouse [296]. Rasmussen [282] developed human error taxonomies by analyzing industrial incident reports and psychological experiments. A new human error taxonomy based on cognitive engineering, social and occupational psychology and categorization strategy of events involving 'human malfunctions' is proposed by Bagnara et al. [18]. Bellamy [27] discussed the human errors that occur because of individual, social and organizational influences on behavior while Wahlstrom [363] analyzed the influence of organization and management on human error.

Hudoklin and Rozman [191] discussed the occurrence of human errors in a human-machine system with the dependence on different stresses while Livingston-Booth [228] and Wisner [391] examined stress as one of the Performance Shaping Factors that ought to be included in human error probability. A reliability evaluation of a human operator performing his/her tasks under various levels of stress is presented by Chung [62, 63] and Park [265] developed human reliability models in discrete and continuous tasks when the human continually improves his/her performance from probabilistic learning.

Dhillon [102] discussed the guidelines for human performance reliability data system development and approaches for collecting data along with a comprehensive listing of human error data banks and sources. A detailed review of human error databases is presented by Topmiller et al. [356].

There are several means of collecting human error data and some of those are described by Carnino [49], Meister [240] and Stillwell [334]. Sources of collecting data identified by these authors are: operating experience and incident reports, published literature, simulators, expert judgment and data banks. In addition, Lucas and Embrey [231], Ridsdale et al. [291] discussed the available sources for obtaining both qualitative and quantitative data as well as proposed a classification for different types of data.

Over the years, there has been very little systematic effort for collecting human error data in industry. This question is addressed by Williams [387] and he suggests means to develop a viable data-base in the immediate future. A scheme for a quantitative probabilistic error-oriented database is presented by Meister in [241]. There appears to be no shortage of subjective estimation techniques claiming to provide estimates of human error occurrence. In this regard, Moray [249] presented a method to convert subjective estimates of human error to objective estimates and Heising and Patterson [181] reported that Bayesian methods can be used to provide probability distributions of human error occurrences.

(2) Human Error Rate Prediction and Human Performance Reliability Models

In recent years a significant amount of progress has been made in predicting human error rate and human performance reliability. There are now several models that either have been used for a formal HRA or which have been seriously proposed as viable models for HRA. Some of these models are listed in Table 1-1.

The majority of HRA techniques are based in part on behavioral psychology. These models use either directly or indirectly real-world or training simulator or expert judgment data on human performance to predict human error rate in complex systems. Embrey and Lucas [128], Apostolakis et al. [13], Spurgin and Moieni [326], Hannaman and Worldge [177], Kirwan [209] reviewed a number of available HRA techniques. Some of the more popular used techniques are described below.

Table 1-1 List of HRA Models

1. Absolute Probability Judgment (APJ) [65]
2. Apostolakis Method [12]
3. Human Cognitive Reliability (HCR) [171, 173]
4. Human Error Assessment and Reduction Technique (HEART) [386, 390]
5. Operator Action Tree (OAT) [167]
6. Success Likelihood Index Method/ Multi-Attribute Utility Decomposition (SLIM/ MAUD) [121 122]
7. Systematic Human Action Reliability Procedure (SHARP) [172, 174, 175]
8. Systematic Human Error Reduction and Prediction Approach (SHERPA) [124]
9. Technica Empirica Stima Errori Operatori (TESEO) [29]
10. Technique for Human Error Rate Prediction (THERP) [340]
11. Time Reliability Correlation (TRC) [112, 113]

i) Technique for Human Error Rate Prediction (THERP)

This Technique, developed by Swain and Guttman [340], is probably the best known of the HRA methods for discrete tasks. It has a comprehensive database human error probabilities. Mostly these are relevant to the power and process industries only. THERP is often referred to as being a decomposition approach. That is, the task to be performed is broken down into a series of elements, and a failure model (analogous to an event tree or fault tree) is drawn up which identifies the various failures that could occur and how they might be recovered.

The method is primarily used to evaluate system degradation resulting from human errors in association with factors such as system characteristics influencing people's behavior, operational procedures and the reliability of the equipment. Furthermore, the two basic measures employed by THERP are the probability that an error or group of errors will cause system failure and the probability that an operation will result in an error class. Swain and Guttman [339] suggested that THERP be regarded as a practical method of predicting human reliability, rather than as a hypothetical model.

ii) Human Cognitive Reliability (HCR)

The Major technique for quantifying human reliability using time dependent modeling is the HCR technique [171].In this technique a relationship between the time available for an operator to respond to an event and the probability that the response will be made within that time is established empirically by means of simulator experiments. Different relationships are derived for skill, rule and knowledge based behavior. The technique's steps are: determine the type of cognitive process; estimate median response time and time window; use the HCR correlation to quantify non-response probability. Normalized task performance time is obtained from the ratio of actual performance time to median task performance time (obtained from simulator measurements, task analyses, or expert judgment). The median time is modified as necessary to account for the impact of stress and human factors on crew performance. Normalized curves were based on simulator data and small scale tests. Failure probabilities can then be entered in an event or fault tree. As stated by Embrey and Lucas [128], although this method is being extensively developed in the nuclear industry it is not recommended for other industrial applications, since it is closely linked to nuclear power simulators, and is insufficiently flexible to address the range of situations that would need to be evaluated in other industrial contexts.

iii) Success Likelihood Index Method/Multi-Attribute Utility Decomposition (SLIM/MAUD)

SLIM/MAUD technique [121,122] work on the principle of experts developing a model which connects the error probability in a particular situation with the factors that influence that probability. For example, the probability that the operator would make an error in a maintenance task might be a function of his/her availability to do the task, the quality of training and procedures, the number of distractions, et al. A model is developed in which the factors influencing the set of tasks are first defined, and the tasks are then rated on these factors. Weights which define the importance of each factor are then derived, and the ratings and weights then combined to give a Success Likelihood Index (SLI) for each task. If at least two tasks with known human error probabilities are included in the set being evaluated, this allows the SLI scale to be calibrated such that the SLI for each task can be converted to a probability of error [128]. SLIM/MAUD uses an interactive computer based procedure (MAUD) for extracting and organizing expert opinion within the SLIM framework for estimating human error probabilities.

In situations where the team operates as the functional unit in a system, then either its reliability has to be assessed as different from that of the individual operators or more sophisticated mathematical models of operator reliability must be developed to account for the effects of team working as suggested by Cox and Tait [69]. Over the years, some attempts have been made to analyze human errors and human reliability of team effects. A method for describing team activities coping with abnormal conditions is presented by Sason et al. [307]. Furthermore, several human dependent failure models are developed in [223,329,381]. Issues associated with modeling of human interactions are discussed by Parry and Lydell [267] and a probabilistic approach to quantify human error dependency in performing multiple tasks is presented by Samanta [303].

(3) Human-machine System Reliability

According to the findings of some researchers, about 20-30% of system failures are directly or indirectly related to human error. Over the years, system scientists have developed several system reliability and availability models which combine human and hardware performance measures. These models are often validated through numerical analysis using data that may have been obtained via the human error rate prediction techniques. Generally, two methods are available for studying human-machine system reliability: 1) analytic method and 2) computer simulation method. In the analytic method, a model is built which reasonably idealizes the physical system and is also amenable to calculation. Human-machine system and sub-system transition phenomena are usually modeled as stochastic processes. The reliability measures are then obtained by manipulating the model. The simulation method also employs a mathematical model but proceeds by performing sampling experiments on this model. It is more flexible but is also more time consuming and less accurate.

Dhillon in Ref. [100] reported the reliability analysis of repairable and non-repairable redundant systems with human errors and common-cause failures. The parallel, k-out-of-n, and standby systems were studied. Goel et al. [140,141] presented two mathematical models to predict the performance of the human-machine system under different weather conditions while deriving various reliability measures of a repairable human-machine system. Formulas for availability and meantime to failure of three state repairable redundant electronic equipment subject to human errors are presented by Gupta et al. [156,157]. Cao [46] using renewal theory, obtained expressions for availability, failure frequency, and reliability of a human-machine system operating under fluctuating environments. Kodama and Deguchi [214] considered the case of a 2-identical-unit redundant system with Erlagian-failure and general repair distributions. The Laplace transforms of the distribution of time to system failure and the explicit formula for the mean time to system failure were derived while Dhillon and Rayapati [99], using modified diagram approach, developed three models for performing reliability analysis of redundant systems with Erlagian-distributed critical and non-critical human error times.

The same authors in Refs. [79,85,88,91,97] presented probabilistic models representing parallel, k-out-of-n and standby redundant systems with critical and non-critical human errors, developed the Laplace transform of the state probability equations and derived the expressions of system reliability, steady state availability, mean time to failure and variance of time to failure. Specific plots were shown to demonstrate the impact of human error on system reliability, availability and mean time to failure. A mathematical model of a repairable parallel system with standby units involving human error and common-cause failures were presented by Chung [61]. Lee et al. [219] developed a stochastic model of human-performance reliability and provided closed-form mathematical expressions into which multiple factors affecting the reliability of human-machine systems can be incorporated. Three elements of human-machine systems were combined to form the framework for modeling random task arrivals, transient human performance characteristics and operational requirements of the system. Stochastic effectiveness models for a multiple-component human-machine system were developed by Abbas and Kuo [1]. Performance evaluation models for a generic military system were developed and all probability measures for successful human performance were considered to be time-dependent in the models. Human-machine systems of various configurations are studied by Dhillon and Yang [103-109, 395-396] using the method of supplementary variables, Markov approach and Laplace transforms.

Hwangs [196] analyzed system reliability by considering latent human error and recovery factors. Three different models, each representing specific recovery type, were presented while Chang et al. [55] discussed a theoretical and numerical investigation of dependent human error on the failure probability of the redundant system tested periodically. Two methodologies of calculating the failure probability were presented. One was developed using the modified Karnaugh map and it can analytically compute the failure probability of the simple system. The other is developed using the minimal cut set and it gives the conservative solution.

The concept of fuzzy set into fault-tree-analysis was introduced by Terano et al. [351]. The fuzziness of human reliability through experiments especially when the information is redundant and also when the workers are redundant was studied.

Computer simulation of human-machine system is another way of gaining insight into the performance of the entire system and the interactions of the human and machine components within the system. The simulation model of mission effectiveness for military systems was developed by Tillman et al. [353] which is defined as a function of the availability of the human-machine system, its reliability, environmental effect, and human performance. Here, under the assumption of statistical independence among these elements, mission effectiveness is determined by the product of each element.

Cost issues were considered by Gupta et al. [158, 159] when they developed mathematical models to predict the expected profit and the operational availability of a 2-unit standby redundant electronic system with critical human errors and a multi-component parallel redundant complex system with overloading effect and waiting under critical human error. Using the supplementary variable technique, Laplace transforms of the probabilities of the complex systems being in various states were computed.

(4) Applications of Human Reliability Engineering

As pointed out earlier, many methods used in human reliability were developed in the context of the nuclear power industry. Nowadays, the human reliability techniques are applied across many other areas and their number is following a growing trend. Some of these application areas are discussed below.

i) Human Reliability in Maintenance and Problem Solving

It is common knowledge that a significant proportion of total human errors occurs during the product maintenance phase. Therefore, a maintenance man plays an important role in the overall reliability of system [87].

Williams and Willey [384] reviewed the problem of human error in maintenance and devised a method for assessing the likelihood of maintenance error. McWilliams [236, 237] incorporated the effects of several types of human error in models for deciding the optimal time between inspections that either maximizes the availability or minimizes the combined inspection and unavailability.

As systems are becoming increasingly complex and automated, human activity has gradually been changed to an intermittent intervening in terms of detection, diagnosis, and correction of system failures. A book based on a symposium and edited by Rasmussen and Rouse [278], provides a state-of-the-art review, assessment trends and future issues relating to human detection and diagnosis of system failures. Rouse [295] discussed the role of the human operator as a problem solver in human-machine systems, reviewed a variety of models and then proposed an overall model. The combined technologies of human factors engineering and reliability assessment is proposed by Williams and Featherstone [383] to generalize a methodology for quantifying the effectiveness of ultrasonic inspection and a fuzzy rule-based model of human problem solving is proposed by Hunt and Rouse [194].

ii) Human Reliability in Decision-Making Process

Over the last twenty years, the task of decision-making has become one of the crucial aspects of human-machine system and has been studied by many researchers [146, 148, 151, 162, 262, 280, 309]. Embrey [127] discussed the requirements for supporting analytical decision-making in various aspects of human reliability assessment. A model using fuzzy membership functions to represent features in decision-making for the human operator in a time-constrained environment have been described by Govindaraj et al. [147].

iii) Human Reliability in Computer System

Bailey [19] discussed the human error in human-computer system while Koval and Znkiwsky [216] presented the correlative aspects of computer operator errors based on computer system interruption reports complied over a ten year period. Furthermore, a non-stationary model for assessing human reliability in computer system is developed by Koval [217].

iv) Human Reliability in Control System

The problem of human error and human reliability in control system/human-machine tracking systems has received a considerable attention over the years. A variety of human operator models have been developed based on classical or modern control theories [28, 114, 294].

Kong et al. [215] developed a two-level control model of human performance representing complex tracking tasks while Greene et al. [150] proposed a sequential model that provides probabilistic descriptions of perception and control effects during manual control tracking. Time-series analysis is applied to model human operator dynamics in a pursuit and compensatory tracking model by Charles et al. [56].

v) Human Reliability in Structural Engineering

It is wildly believed that most structural failures are caused by the presence of human error. Melchers [241] reviewed some preliminary results of recent research findings relating to the assessment of the importance of human error on the reliability of engineering structures and Lind [227] discussed the causes of structural failures and compared several mathematical models of human error in structures. A probabilistic risk assessment approach used to examine the effects of human error on a typical structural engineering design task is reported by Stewart [332] while a design task model that simulates the effect of human error in design is developed by Stewart and Melchers [330].

1.4 Motivation, Objective, and Contribution of the Study

Motivation:

Past work did not address adequately the negative impact of human errors and common-cause failures on the performance of human-machine systems. Most of the research was focused on the human error rate prediction or human performance reliability. This work is one of the first bold attempting at synthesizing human reliability and equipment reliability into the reliability of the human-machine system as a whole.

Stochastic processes are powerful tools for the investigation of reliability problem, in particular for investigation of the time behavior of repairable systems. Most reliability models assume that the system failure and repair times are exponentially distributed. This assumption leads to a Markovian model. The analysis in such cases is relatively simple and the numerical results can be easily obtained. Markovian model works well as long as the failure rate and repair rate are constant. When failure rate or repair rate becomes time-dependent, the method breaks down and then the process becomes non-Markovian and different techniques are required for system solution.

In many cases while the up times (failure times) are exponentially distributed (or close to it), the down times (repair times) typically have Weibull or some other non-exponential distributions. This study attempts to generalize the failed system repair process which may be of more practical significance.

Human error rate has often been considered to be constant in human-machine system analysis. In practice, human failure has its peculiarities although there are certain similarities between human and machine in terms of their proneness to failure. Human performance is subject to environmental stresses and shocks and human continually improves his/her performance from learning. Through the analysis of raw data, it may be said that human error is more likely to be time-dependent.

The main ***objective*** of the study is, thus, to analyze the effects of human errors on system reliability, availability and system mean time to failure. To meet this objective, constant human error rates, increasing human error rates and general human error rates are considered for various system configurations.

Comparing this thesis with those of previous researchers, the main ***contributions*** of the study are:

- The models in this study integrated hardware failure, human error, common-cause failure and general repairable system.
- The models in this study considered the human errors and common-cause failures to occur from any of the operable states of the system and generalized the failed system repair time distribution.
- A new method of system steady state availability for general system repair rate was developed. Using the linear ordinary differential equations instead of complex partial differential equations and Laplace transforms, it is much easier to obtain the general expressions of the steady state availability for various repair time distributions, such as the Gamma, Weibull, exponential, Rayleigh and lognormal distributions.
- A new method which combines the inclusive supplementary variable technique and the device of stages method was developed to solve the system for both time-dependent human error and failed system repair rate.
- This study investigated not only the steady state availability and reliability, but also the point-availability of the systems with time-dependent repair rates.

1.5 Organization of Study

This dissertation is divided into five chapters:

Chapter 1 gives a brief introduction to human error and human reliability analysis and presents the literature review.

Chapter 2 is concerned with the reliability and availability analysis of systems with constant human error rates and arbitrarily distributed failed system repair times.

Chapter 3 investigates the systems with increasing human error rates and arbitrarily distributed failed system repair times.

Chapter 4 is concerned with the systems having time-dependent human error rates and general system repair rates.

In Chapters 2, 3, and 4, redundant configurations, such as standby, k-out-of-n, majority voting with imperfect voter, and parallel, are studied and the generalized expressions for system reliability, steady state availability, time-dependent availability, mean time to failure and the variance of time to failure are presented. Special case models are discussed.

Chapter 5 discusses the results of analysis conducted in the preceding chapters and also presents conclusions and recommendations for further study.

Chapter 2

Systems With Constant Human Error Rates and Arbitrary System Repair Rates

This Chapter presents mathematical models representing various types of human-machine systems, such as standby, k-out-of-n, majority voting with imperfect voter, and parallel, with constant human error rate and other failure rates. In addition, the failed system repair times are assumed arbitrarily distributed.

Markov and supplementary variable techniques were used to develop initial expressions. The method of linear ordinary differential equations is developed to obtain the general expressions for system steady state availability for various types of failed system repair time distributions, such as Gamma, Weibull, lognormal, exponential, and Rayleigh. Generalized expressions for system reliability, time-dependent availability, mean time to failure, and system variance of time to failure are presented.

2.1 General Standby System

2.1.1 The description of the system

Redundancy plays an important role in enhancing system reliability. One of the commonly used form of redundancy is the standby system. In this case, one or more unit operate and the remaining redundant units are kept in their standby mode. The standby redundancy mode is appropriate if the failure rate in the non-operating standby mode is lower than the failure rate in the continuously operating mode.

In recent years, there have been several publications on human error analysis of redundant systems [61, 85, 90, 103-106]. In most of these publications, failed system repair rates are assumed to be constant. This section overcomes this assumption and considers the case when the failed system repair rates are time-dependent. A mathematical model of a repairable parallel system with standby units involving human and common-cause failures is presented. The block diagram of the general standby system with critical human error and common-cause failure is shown in Figure 2-1a. The corresponding state transition diagram of this standby system is shown in Figure 2-1b. This system has n units in parallel with m standby units. At time t=0, all n units in parallel start operating and the remaining m units are kept in their standby mode. At least one unit is required to function normally for the system to operate successfully. As soon as one of the parallel operating unit fails, the standby unit is switched into operation. The system fails when all units (parallel plus standby) are non-operative. Furthermore, the occurrence of a common-cause failure or a critical human error causes the total system failure from any of the system operable states. The repair process starts as soon as one or more units fail.

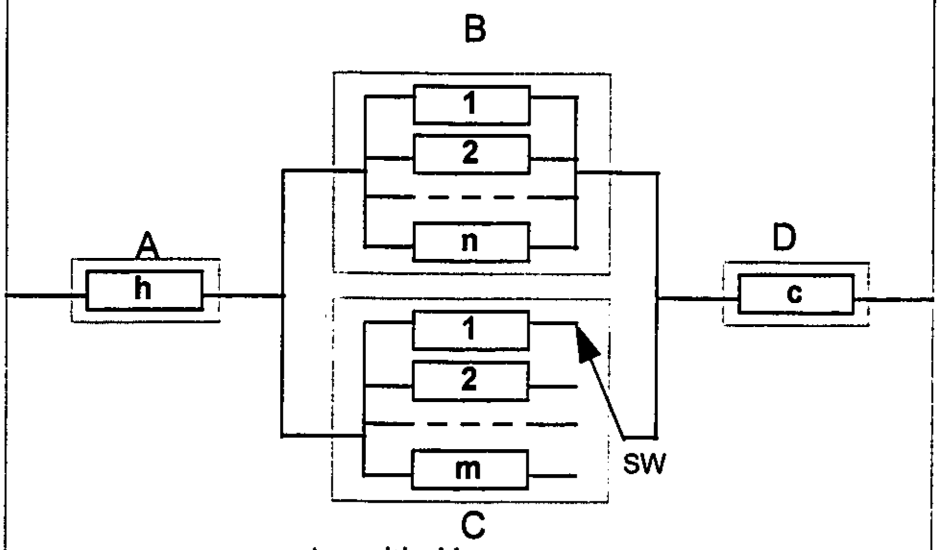
The numerals or letters (as applicable) in the boxes of Figure 2-1b denote corresponding system states. For i=0, it means all the n active units and the m standby units are in perfect working condition. For i=1, it means one operating unit has failed due to a hardware failure and a standby unit is switched into operation, i.e., n units are active and m-1 units are on standby. For i = 2, n units are active and m-2 units are on standby, and so on. In addition, for i = m, it means n units active and no standby. Similarly, for i = m+1, it means n-1 units active and no standby and so on. Explanations for the additional cases are as follows:

- i = m+n-1: only one unit active and no standby.
- i = m+n: the system has failed due to hardware failures.
- i = m+n+1: the system has failed due to a common-cause failure.
- i = m+n+2: the system has failed due to a critical human error.

Assumptions

The following assumptions are associated with all the standby configurations in this study:

1. All failures including human errors are statistically independent.
2. Common-cause and other failure rates are constant
3. A common-cause failure or a human error can trigger system failure from any of the system operable states.
4. The repaired unit or system is as good as new.
5. Switchover mechanism is perfect and instantaneous.
6. The failed system repair times are arbitrarily distributed.



A: critical human error
B: n units in parallel active
C: m units on standby
D: common-cause failure

Figure 2-1a The block diagram of the general standby system with common-cause failure and human error

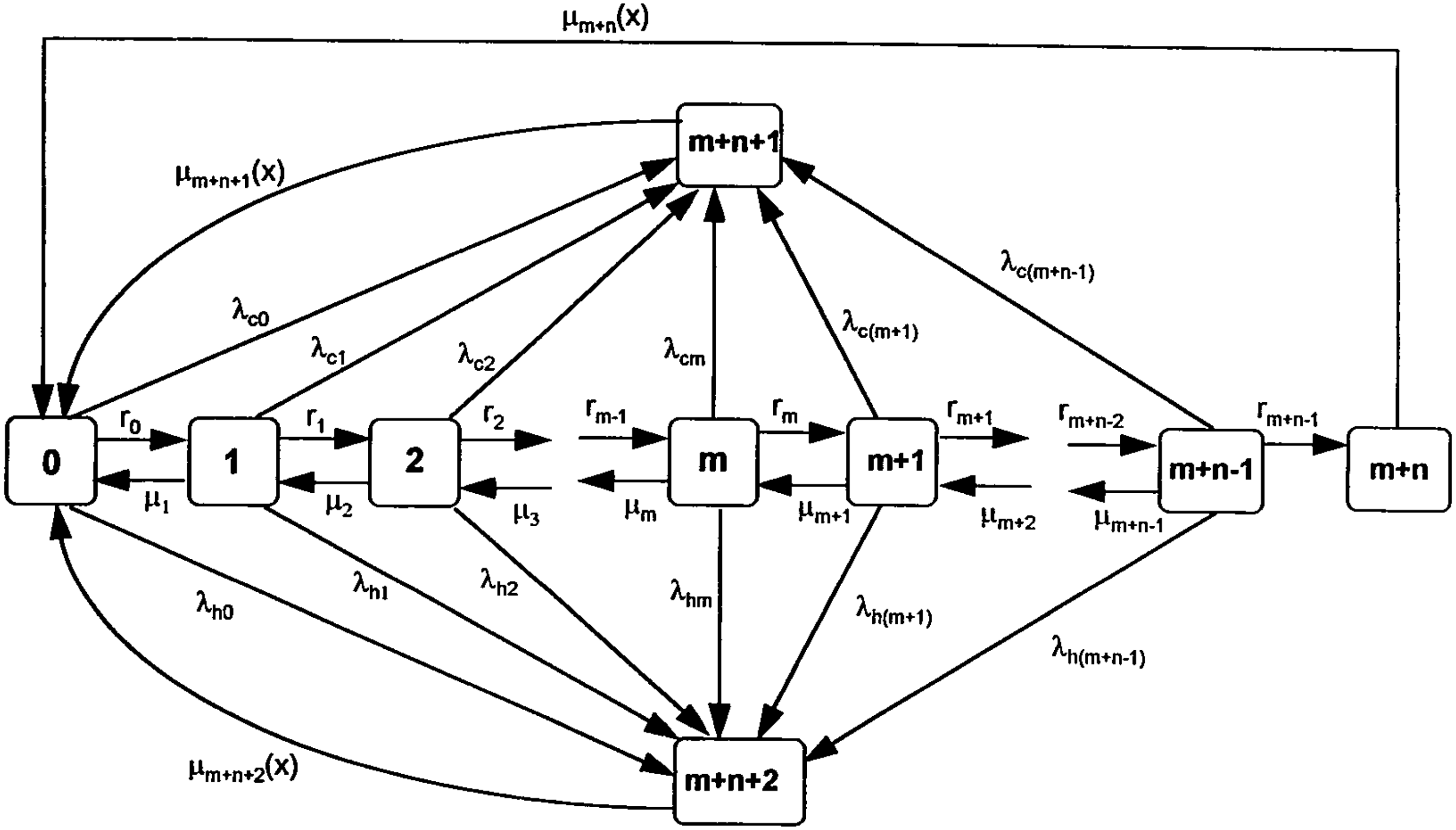


Figure 2-1b The state transition diagram of a general standby system with constant human error rates and arbitrary system repair rates

MORE GENERAL CONSIDERATION ON STANDBY CONFIGURATION

The models of all standby configurations in this study include the following situations:

- ***warm standby redundancy***

  $r_i = n\lambda + (m-i)\lambda_s \qquad (i \le m)$

  $r_i = (m+n-i)\lambda \qquad (i \ge m)$

  λ and λ_s are the same for all states.

- ***cold standby redundancy***

  $r_i = n\lambda \qquad (i \le m)$

  $r_i = (m+n-i)\lambda \qquad (i \ge m)$

  λ is the same for all states

- ***repairable system***

  $\mu_i \ne 0 \qquad (\text{for } i = 1, 2, 3, \cdots, m+n-1)$

- ***non-repairable system***

  $\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_{m+n-1} = 0$

The Descriptive Equations of the Model

The system of differential equations associated with Figure 2-1b is:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=m+n}^{m+n+2} \int_0^\infty P_j(x,t)\mu_j(x)dx \tag{2-1}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \tag{2-2}$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \tag{2-3}$$

(for i = 2, 3, ..., m+n-2)

$$\frac{dP_{m+n-1}(t)}{dt} = r_{m+n-2} P_{m+n-2}(t) - a_{m+n-1} P_{m+n-1}(t) \tag{2-4}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \tag{2-5}$$

(for j = m+n, m+n+1, m+n+2)

where

$$a_i = r_i + \lambda_{ci} + \lambda_{hi} + \mu_i$$

(for i = 0, 1, 2, 3, …, m+n-1, and assuming μ_0=0)

The associated boundary conditions are as follows:

$$P_{m+n}(0,t) = r_{m+n-1} P_{m+n-1}(t) \tag{2-6}$$

$$P_{m+n+1}(0,t) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(t) \tag{2-7}$$

$$P_{m+n+2}(0,t) = \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) \tag{2-8}$$

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

2.1.2 Steady State Availability Analysis

As time approaches infinity, Equations (2-1) - (2-5) reduce to Equations (2-9) - (2-13), respectively.

$$a_0 P_0 - \mu_1 P_1 = \sum_{j=m+n}^{m+n+2} \int_0^{\infty} P_j(x)\mu_j(x)dx \qquad (2\text{-}9)$$

$$r_0 P_0 - a_1 P_1 + \mu_2 P_2 = 0 \qquad (2\text{-}10)$$

$$r_{i-1} P_{i-1} - a_i P_i + \mu_{i+1} P_{i+1} = 0 \qquad (2\text{-}11)$$

(i = 2, 3, ..., m+n-2)

$$r_{m+n-2} P_{m+n-2} - a_{m+n-1} P_{m+n-1} = 0 \qquad (2\text{-}12)$$

$$\frac{dP_j(x)}{dx} = -\mu_j(x) P_j(x) \qquad (2\text{-}13)$$

(j = m+n, m+n+1, m+n+2)

Similarly, the boundary conditions become:

$$P_{m+n}(0) = r_{m+n-1} P_{m+n-1} \qquad (2\text{-}14)$$

$$P_{m+n+1}(0) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i \qquad (2\text{-}15)$$

$$P_{m+n+2}(0) = \sum_{i=0}^{m+n-1} \lambda_{hi} P_i \qquad (2\text{-}16)$$

where

P_i is the steady state probability that the system is in state i, for i=0, 1, 2, ..., m+n+2.

and

$$P_j = \int_0^\infty P_j(x)dx \tag{2-17}$$

(for j = m+n, m+n+1, m+n+2)

$$\sum_{i=0}^{m+n+2} P_i = 1 \tag{2-18}$$

Solving Equation (2-13), we get

$$P_j(x) = P_j(0)\exp(-\int_0^x \mu_j(\omega)d\omega) \tag{2-19}$$

(j=m+n, m+n+1, m+n+2)

Thus, from Equations (2-17) and (2-19), we have

$$\begin{aligned} P_j &= \int_0^\infty P_j(x)dx \\ &= \int_0^\infty P_j(0)\exp(-\int_0^x \mu_j(\omega)d\omega)dx \end{aligned} \tag{2-20}$$

(j = m+n, m+n+1, m+n+2)

Substituting Equations (2-14) - (2.16) into Equation (2-20), we get:

$$\begin{aligned} P_{m+n} &= \int_0^\infty r_{m+n-1}P_{m+n-1}\exp(-\int_0^x \mu_{m+n}(\omega)d\omega)dx \\ &= r_{m+n-1}P_{m+n-1}E_{m+n}[x] \end{aligned} \tag{2-21}$$

$$\begin{aligned} P_{m+n+1} &= \int_0^\infty \sum_{i=0}^{m+n-1} \lambda_{ci}P_i\exp(-\int_0^x \mu_{m+n+1}(\omega)d\omega)dx \\ &= \sum_{i=0}^{m+n-1} \lambda_{ci}P_iE_{m+n+1}[x] \end{aligned} \tag{2-22}$$

$$P_{m+n+2} = \int_0^\infty \sum_{i=0}^{m+n-1} \lambda_{hi} P_i \exp(-\int_0^x \mu_{m+n+2}(\omega) d\omega) dx$$
$$= \sum_{i=0}^{m+n-1} \lambda_{hi} P_i E_{m+n+2}[x] \qquad (2\text{-}23)$$

where

$$E_j[x] = \int_0^\infty \exp(-\int_0^x \mu_j(\omega) d\omega) dx \qquad (2\text{-}24)$$

(j=m+n, m+n+1, m+n+2)

which is the mean time to system repair when the failed system is in state j and has an elapsed repair time of x , or the expected value of x (see Appendix A for detail).

Solving the set of Equations (2-10) - (2-12), (2-18) and (2-21) - (2-23), lead to steady state probabilities, P_0, P_1, P_2, ..., P_{m+n}, P_{m+n+1}, P_{m+n+2}.

Thus, the steady state availability of the system is:

$$AV_{ss} = \sum_{i=0}^{m+n-1} P_i \qquad (2\text{-}25)$$

Similarly, the system steady state unavailability is:

$$AV_{ss} = P_{m+n} + P_{m+n+1} + P_{m+n+2} \qquad (2\text{-}26)$$

2.1.3 Time-Dependent Availability Analysis

Using Laplace transform and the initial conditions in Equations (2-1) - (2-8), we get

$$s\,P_0(s) = 1 - a_0 P_0(s) + \mu_1 P_1(s) + \sum_{j=m+n}^{m+n+2} \int_0^\infty P_j(x,s) \mu_j(x) dx \qquad (2\text{-}27)$$

$$(s + a_1) P_1(s) - r_0 P_0(s) - \mu_2 P_2(s) = 0 \qquad (2\text{-}28)$$

$$(s+a_i)P_i(s)-r_{i-1}P_{i-1}(s)-\mu_{i+1}P_{i+1}(s)=0 \tag{2-29}$$

(for i = 2, 3, ..., m+n-2)

$$(s+a_{m+n-1})P_{m+n-1}(s)-r_{m+n-2}P_{m+n-2}(s)=0 \tag{2-30}$$

$$\frac{\partial P_j(x,s)}{\partial x}+s\,P_j(x,s)+\mu_j(x)P_j(x,s)=0 \tag{2-31}$$

(for j = m+n, m+n+1, m+n+2)

and the boundary conditions:

$$P_{m+n}(0,s)=r_{m+n-1}P_{m+n-1}(s) \tag{2-32}$$

$$P_{m+n+1}(0,s)=\sum_{i=0}^{m+n-1}\lambda_{ci}P_i(s) \tag{2-33}$$

$$P_{m+n+2}(0,s)=\sum_{i=0}^{m+n-1}\lambda_{hi}P_i(s) \tag{2-34}$$

Solving differential Equation (2-31), we get the following resulting expression:

$$P_j(x,s)=P_j(0,s)e^{-sx}\exp(-\int_0^x\mu_j(\omega)d\omega) \tag{2-35}$$

(j = m+n, m+n+1, m+n+2)

Since

$$P_j(s)=\int_0^\infty P_j(x,s)dx \tag{2-36}$$

(j = m+n, m+n+1, m+n+2)

and together with Equation (2-35), we get

$$P_{m+n}(s) = r_{m+n-1}P_{m+n-1}(s)\frac{1-N_{m+n}(s)}{s} \tag{2-37}$$

$$P_{m+n+1}(s) = \sum_{i=0}^{m+n-1}\lambda_{ci}P_i(s)\frac{1-N_{m+n+1}(s)}{s} \tag{2-38}$$

$$P_{m+n+2}(s) = \sum_{i=0}^{m+n-1}\lambda_{hi}P_i(s)\frac{1-N_{m+n+2}(s)}{s} \tag{2-39}$$

where

$$\frac{1-N_j(s)}{s} = \int_0^{\infty} e^{-sx}\exp\left(-\int_0^x \mu_j(\omega)d\omega\right)dx \tag{2-40}$$

(j = m+n, m+n+1, m+n+2)

and $N_j(s)$ is the Laplace transform of $N_j(x)$.

The Laplace transform of the probabilities of all the system states adds up to 1/s, i.e.,

$$\sum_{i=0}^{m+n+2}P_i(s) = 1/s \tag{2-41}$$

Solving Equations (2-28) - (2-30), (2-37) - (2-39) and (2-41), we can obtain the Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_{m+n}(s)$, $P_{m+n+1}(s)$, and $P_{m+n+2}(s)$.

The Laplace transform of the system availability is given by

$$AV(s) = \sum_{i=0}^{m+n-1}P_i(s) \tag{2-42}$$

The steady state availability is:

$$AV_{ss} = \lim_{s\to 0} s \bullet AV(s) \tag{2-43}$$

Substituting the Laplace transform of $N_j(x)$ for different repair time distributions in Equation (2-42) and taking the inverse Laplace transform of the resulting equation, we get the time-dependent system availability:

$$AV(t) = \sum_{i=0}^{m+n-1} P_i(t) \tag{2-44}$$

2.1.4 System Reliability and MTTF With and Without Repair

Setting $\mu_{m+n}(x) = \mu_{m+n+1}(x) = \mu_{m+n+2}(x) = 0$ in Figure 2-1b, the system of differential equations becomes:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \tag{2-45}$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \tag{2-46}$$

(for i = 1, 2, 3, ..., m+n-2)

$$\frac{dP_{m+n-1}(t)}{dt} = r_{m+n-2} P_{m+n-2}(t) - a_{m+n-1} P_{m+n-1}(t) \tag{2-47}$$

$$\frac{dP_{m+n}(t)}{dt} = r_{m+n-1} P_{m+n-1}(t) \tag{2-48}$$

$$\frac{dP_{m+n+1}(t)}{dt} = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(t) \tag{2-49}$$

$$\frac{dP_{m+n+2}(t)}{dt} = \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) \tag{2-50}$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, ..., m+n+2.

Using Laplace transform in Equations (2-45) - (2-50) and solving the resulting set of Equations, leads to the following expression for the Laplace transform of the system reliability with repair:

$$R(s) = \sum_{i=0}^{m+n-1} P_i(s) \tag{2-51}$$

The time-dependent system reliability with repair can be obtained by inverting Equation (2-51):

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\sum_{i=0}^{m+n-1} P_i(s)\right] \tag{2-52}$$

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s\to 0} R(s) = \lim_{s\to 0}\left[\sum_{i=0}^{m+n-1} P_i(s)\right] \tag{2-53}$$

The system variance of time to failure with repair is expressed by

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2 \tag{2-54}$$

where R′(s) denotes the derivative of R(s) with respect to s.

When $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_{m+n-1} = 0$ in Equations (2-52) - (2-54), we get the system reliability, MTTF and the system variance of time to failure without repair , respectively.

Special Cases of Standby System with Constant Human Error Rate and General System Repair Rate

It is necessary to consider some special cases of the general standby system introduced in the previous section in order to present the procedures for assessing the impact of human errors, common-cause failures, and failed system repair policy on system reliability and availability. Therefore, this section presents several special cases resulting in the development of some general and special-case expressions for the system steady state availability, reliability, time-dependent system availability, mean time to failure, and variance of time to failure of standby systems with constant human error rate and general failed system repair time distribution.

2.2.1 One Unit Active and One Unit on Standby System

Let us consider first the case of one unit active and one on standby system, i.e., m=1 and n=1 in Figure 2-1b. Thus, the system of differential equations associated with the model is:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=2}^{4} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \qquad (2\text{-}55)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) \qquad (2\text{-}56)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \qquad (2\text{-}57)$$

where

$$a_0 = r_0 + \lambda_{c0} + \lambda_{h0}$$
$$a_1 = r_1 + \lambda_{c1} + \lambda_{h1} + \mu_1$$

The associated boundary conditions are as follows:

$$P_2(0,t) = r_1 P_1(t) \tag{2-58}$$

$$P_3(0,t) = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \tag{2-59}$$

$$P_4(0,t) = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) \tag{2-60}$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

As time t approaches infinity, Equations (2-55) - (2-57) reduce to Equations (2-61) - (2-63), respectively.

$$a_0 P_0 - \mu_1 P_1 = \sum_{j=2}^{4} \int_0^\infty P_j(x) \mu_j(x) dx \tag{2-61}$$

$$r_0 P_0 - a_1 P_1 = 0 \tag{2-62}$$

$$\frac{dP_j(x)}{dx} = \mu_j(x) P_j(x) \tag{2-63}$$

(for j = 2, 3, 4)

Similarly, the boundary conditions become:

$$P_2(0) = r_1 P_1 \tag{2-64}$$

$$P_3(0) = \lambda_{c0} P_0 + \lambda_{c1} P_1 \tag{2-65}$$

$$P_4(0) = \lambda_{h0} P_0 + \lambda_{h1} P_1 \tag{2-66}$$

Substituting Equations (2-64) - (2-66) in Equation (2-20), for j =2, 3, 4, we have

$$P_2 = r_1 P_1 E_2[x] \tag{2-67}$$

$$P_3 = (\lambda_{c0} P_0 + \lambda_{c1} P_1) E_3[x] \tag{2-68}$$

$$P_4 = (\lambda_{h0}P_0 + \lambda_{h1}P_1)E_4[x] \qquad (2\text{-}69)$$

$E_2[x]$, $E_3[x]$ and $E_4[x]$ are the mean time to system repair that the failed system is in state 2, 3 and 4, respectively.

P_i is the steady state probability that the system is in state i, for i=0, 1, 2, ..., 4, and

$$\sum_{i=0}^{4} P_i = 1 \qquad (2\text{-}70)$$

Solving the set of Equations (2-62), (2-67) - (2-69) and (2-70), we get the following steady state probabilities:

$$P_0 = b_0(1,1)/BA(1,1) \qquad (2\text{-}71)$$

$$P_1 = b_1(1,1)/BA(1,1) \qquad (2\text{-}72)$$

$$P_2 = b_2(1,1)E_2[x]/BA(1,1) \qquad (2\text{-}73)$$

$$P_3 = b_3(1,1)E_3[x]/BA(1,1) \qquad (2\text{-}74)$$

$$P_4 = b_4(1,1)E_4[x]/BA(1,1) \qquad (2\text{-}75)$$

where:

$$b_0(1,1) = a_1$$
$$b_1(1,1) = r_0$$
$$b_2(1,1) = r_0 r_1$$
$$b_3(1,1) = \sum_{i=0}^{1} b_i(1,1)\lambda_{ci}$$
$$b_4(1,1) = \sum_{i=0}^{1} b_i(1,1)\lambda_{hi}$$
$$BA(1,1) = \sum_{i=0}^{1} b_i(1,1) + \sum_{J=2}^{4} b_j(1,1)E_j[x]$$

The system steady state availability is

$$AV_{ss}(1,1) = P_0 + P_1 \\ = \sum_{i=0}^{1} b_i(1,1) \Big/ BA(1,1) \tag{2-76}$$

Similarly, the system steady state unavailability is given by

$$UAV_{ss}(1,1) = P_2 + P_3 + P_4 \\ = \sum_{j=2}^{4} b_j(1,1)E_j[x] \Big/ BA(1,1) \tag{2-77}$$

System Steady State Availability Special Cases

Case I

If the system repair time x is ***Gamma*** distributed and the probability density function (pdf) of the repair time is given by

$$N_j(x) = \frac{\mu_j^{\beta} x^{\beta-1}}{\Gamma(\beta)} \exp(-\mu_j x) \tag{2-78}$$

(for $j = 2, 3, 4, x \geq 0, \beta > 0, \mu_j > 0$)

where β and μ_j are two parameters of the Gamma distribution.

Thus, the mean time to system repair $E_j[x]$ is

$$E_j[x] = \int_0^{\infty} x N_j(x)dx = \beta \Big/ \mu_j \tag{2-79}$$

(for $j = 2, 3, 4$)

Substituting Equation (2-79) into Equation (2-76), we get the following resulting system steady state availability for the Gamma repair time distribution:

$$AV_{ss}(1,1)=\sum_{i=0}^{1}b_i(1,1)\Bigg/\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\beta/\mu_j\right] \tag{2-80}$$

If β is an integer

$$\Gamma(\beta)=(\beta-1)! \tag{2-81}$$

Thus the Gamma distribution becomes ***Erlangian distribution***.

When $\beta = 1$, the Gamma distribution reduces to ***exponential distribution,*** we have the system steady state availability

$$AV_{ss}(1,1)=\sum_{i=0}^{1}b_i(1,1)\Bigg/\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)/\mu_j\right] \tag{2-82}$$

Case II

If the system repair time x is ***Weibull*** distributed, the pdf of the repair time is expressed by

$$N_j(x)=\mu_j{}^{\beta}\beta x^{\beta-1}\exp(-\mu_j{}^{\beta}x^{\beta}) \tag{2-83}$$

(for $j = 2, 3, 4$, $x\geq 0$, $\beta>0$, $\mu_j>0$)

where β and μ_j are two parameters of the Weibull distribution.

Thus, the mean time to system repair $E_j[x]$ is given by

$$E_j[x]=\int_0^{\infty}xN_j(x)dx={\Gamma(1+1/\beta)}\Big/{\mu_j} \tag{2-84}$$

(for $j = 2, 3, 4$)

Substituting Equation (2-84) into Equation (2-76), we get the following system steady state availability for the Weibull repair time distribution:

$$AV_{ss}(1,1)=\sum_{i=0}^{1}b_i(1,1)\Bigg/\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\Gamma(1+{}^{1}\!/\!_{\beta})\Big/\mu_j\right] \qquad (2\text{-}85)$$

Case III

If the system repair time x is ***Rayleigh*** distributed and the time dependent failed system repair rate and pdf of the system repair time, respectively, are defined by

$$\mu_j(x)=\mu_j^{\,2}x \qquad (2\text{-}86)$$

(j = 2, 3, 4)

and

$$N_j(x)=\mu_j^{\,2}x\ \exp(-\mu_j^{\,2}x^2/2) \qquad (2\text{-}87)$$

(for j = 2, 3, 4, $x\geq 0$, $\mu_j>0$)

Thus, the failed system mean time to repair $E_j[x]$ is expressed by

$$E_j[x]=\int_0^\infty xN_j(x)dx=\frac{1}{\mu_j}\sqrt{\frac{\pi}{2}} \qquad (2\text{-}88)$$

(for j = 2, 3, 4)

Substituting Equation (2-88) into Equation (2-76), we get the following system steady state availability

$$AV_{ss}(1,1)=\sum_{i=0}^{1}b_i(1,1)\Bigg/\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\sqrt{\pi/2}\Big/\mu_j\right] \qquad (2\text{-}89)$$

Case IV

If the system repair time x is ***lognormally*** distributed, the pdf of the repair time is defined by

$$N_j(x) = \frac{1}{x\sigma_j\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \mu_j)^2}{2\sigma_j^{\,2}}\right] \qquad (2\text{-}90)$$

(for j = 2, 3, 4, and x≥0)

where μ_j and σ_j are the two distribution parameters (mean value and standard deviation of *ln* x, respectively).

Thus, the mean time to system repair $E_j[x]$ is given by

$$E_j[x] = \int_0^\infty xN_j(x)dx = \exp\left[\mu_j + \frac{\sigma_j^{\,2}}{2}\right] \qquad (2\text{-}91)$$

(for j = 2, 3, 4)

Substituting Equation (2-91) into Equation (2-76) yields the following system steady state availability

$$AV_{ss}(1,1) = \sum_{i=0}^{1} b_i(1,1) \Bigg/ \left[\sum_{i=0}^{1} b_i(1,1) + \sum_{j=2}^{4} b_j(1,1)\exp\left(\mu_j + \frac{\sigma_j^{\,2}}{2}\right)\right] \qquad (2\text{-}92)$$

Time-Dependent Availability Analysis

Using Laplace transform and the initial conditions given in Equations (2-55) - (2-57), we get

$$s\,P_0(s) = 1 - a_0P_0(s) + \mu_1P_1(s) + \sum_{j=2}^{4}\int_0^\infty P_j(x,s)\mu_j(x)dx \qquad (2\text{-}93)$$

$$(s + a_1)P_1(s) - r_0P_0(s) = 0 \qquad (2\text{-}94)$$

$$\frac{\partial P_j(x,s)}{\partial x} + s\,P_j(x,s) + \mu_j(x)P_j(x,s) = 0 \qquad (2\text{-}95)$$

(for j = 2, 3, 4)

and the boundary conditions:

$$P_2(0,s) = r_1 \, P_1(s) \tag{2-96}$$

$$P_3(0,s) = \lambda_{c0} P_0(s) + \lambda_{c1} P_1(s) \tag{2-97}$$

$$P_4(0,s) = \lambda_{h0} P_0(s) + \lambda_{h1} P_1(s) \tag{2-98}$$

From Equations (2-37) - (2-39), for m = 1 and n = 1, we have

$$P_2(s) = r_1 P_1(s) \frac{1 - N_2(s)}{s} \tag{2-99}$$

$$P_3(s) = \left[\lambda_{c0} P_0(s) + \lambda_{c1} P_1(S)\right] \frac{1 - N_3(s)}{s} \tag{2 -100}$$

$$P_4(s) = \left[\lambda_{h0} P_0(s) + \lambda_{h1} P_1(s)\right] \frac{1 - N_4(s)}{s} \tag{2- 101}$$

Equations (2-94), (2-99) - (2-101) together with

$$\sum_{i=0}^{4} P_i(s) = 1/s \tag{2-102}$$

can be solved to get the Laplace transform of steady state probabilities:

$$P_0(s) = \left[s + b_0(1,1)\right] / CA(1,1) \tag{2-103}$$

$$P_1(s) = b_1(1,1) / CA(1,1) \tag{2-104}$$

$$P_2(s) = b_2(1,1)[1 - N_2(s)] / [s \cdot CA(1,1)] \tag{2-105}$$

$$P_3(s) = \left[b_3(1,1) + \lambda_{c0} s\right]\left[1 - N_3(s)\right] / \left[s \cdot CA(1,1)\right] \tag{2-106}$$

$$P_4(s) = \left[b_4(1,1) + \lambda_{h0} s\right]\left[1 - N_4(s)\right] / \left[s \cdot CA(1,1)\right] \tag{2-107}$$

where

$$CA(1,1) = ca1(1,1) + ca2(1,1) \cdot s + s^2$$

$$ca1(1,1) = \sum_{j=2}^{4} b_j(1,1)\left[1 - N_j(s)\right]$$

$$ca2(1,1) = \sum_{i=0}^{1} b_i(1,1) + \lambda_{c0}\left[1 - N_3(s)\right] + \lambda_{h0}\left[1 - N_4(s)\right]$$

The Laplace transform of the system availability is:

$$\begin{aligned} AV(s) &= P_0(s) + P_1(s) \\ &= \left[s + \sum_{i=0}^{1} b_i(1,1) \right] \Big/ CA(1,1) \end{aligned} \qquad (2\text{-}108)$$

Substituting the Laplace transforms of pdf of system repair times, $N_i(s)$ for $j = 2, 3, 4$ (see Appendix A), in Equation (2-108) and taking the inverse Laplace transform of the resulting equation, we get the system time-dependent availability:

$$AV(t) = P_0(t) + P_1(t) \qquad (2\text{-}109)$$

Reliability and MTTF With and Without Repair

Setting $\mu_2(x) = \mu_3(x) = \mu_4(x) = 0$ in the model, the system of differential equations becomes:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \qquad (2\text{-}110)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) \qquad (2\text{-}111)$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) \tag{2-112}$$

$$\frac{dP_3(t)}{dt} = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \tag{2-113}$$

$$\frac{dP_4(t)}{dt} = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) \tag{2-114}$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, 3, 4.

Using Laplace transforms in Equations (2-110) - (2-114), and solving the resulting set of equations,. we can get the Laplace transforms of the state probabilities, $P_0(s)$, $P_1(s)$, ..., P4(s).

Thus, the Laplace transform of the system reliability with repair is

$$\begin{aligned} R(s) &= P_0(s) + P_1(s) \\ &= \left[\sum_{i=0}^{1} b_i(1,1) + s \right] \Big/ DA(1,1) \end{aligned} \tag{2-115}$$

where

$$DA(1,1) = da1(1,1) + da2(1,1) \bullet s + s^2$$
$$da1(1,1) = a_0 a_1 - r_0 \mu_1$$
$$da2(1,1) = a_0 + a_1$$

System reliability with repair can be obtained by inverting Equation (2-115)

$$R(t) = \frac{e^{s_1 t}(s_1 - s_a)}{s_1 - s_2} + \frac{e^{s_2 t}(s_2 - s_a)}{s_2 - s_1} \tag{2-116}$$

where

$$s_a = -a_1 - r_0$$

and s_1, s_2 are real and unique roots of the denominator of Equation (2-115).

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s\to 0} R(s) = \sum_{i=0}^{1} b_i(1,1) \Big/ da1(1,1) \qquad (2\text{-}117)$$

The variance of time to failure with repair of the system is expressed by

$$\begin{aligned}\sigma^2 &= -2\lim_{s\to 0} R'(s) - (MTTF)^2 \\ &= \frac{2da2(1,1)\sum_{i=0}^{1} b_i(1,1) - 2da1(1,1) - \left[\sum_{i=0}^{1} b_i(1,1)\right]^2}{[da1(1,1)]^2}\end{aligned} \qquad (2\text{-}118)$$

When $\mu_1 = 0$ in Equations (2-116) - (2-118), we get the system reliability, MTTF and the variance of time to failure without repair, respectively.

2.2.2 Two Units Active and One Unit on Standby System

For m = 2 and n = 1 in Figure 2-1b, the differential equations associated with the model can be written as follows:

$$\frac{dP_0(t)}{dt} = -(r_0 + \lambda_{c0} + \lambda_{h0})P_0(t) + \mu_1 P_1(t) + \int_0^\infty P_3(x,t)\mu_3(x)dx + \int_0^\infty P_4(x,t)\mu_4(x)dx + \int_0^\infty P_5(x,t)\mu_5(x)dx \tag{2-119}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - (\mu_1 + r_1 + \lambda_{c1} + \lambda_{h1})P_1(t) + \mu_2 P_2(t) \tag{2-120}$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) - (\mu_2 + r_2 + \lambda_{c2} + \lambda_{h2})P_2(t) \tag{2-121}$$

$$\frac{\partial P_3(x,t)}{\partial t} + \frac{\partial P_3(x,t)}{\partial x} = -\mu_3(x)P_3(x,t) \tag{2-122}$$

$$\frac{\partial P_4(x,t)}{\partial t} + \frac{\partial P_4(x,t)}{\partial x} = -\mu_4(x)P_4(x,t) \tag{2-123}$$

$$\frac{\partial P_5(x,t)}{\partial t} + \frac{\partial P_5(x,t)}{\partial x} = -\mu_5(x)P_5(x,t) \tag{2-124}$$

The associated boundary conditions are as follows:

$$P_3(0,t) = r_2 P_2(t) \tag{2-125}$$

$$P_4(0,t) = \lambda_{c0}P_0(t) + \lambda_{c1}P_1(t) + \lambda_{c2}P_2(t) \tag{2-126}$$

$$P_5(0,t) = \lambda_{h0}P_0(t) + \lambda_{h1}P_1(t) + \lambda_{h2}P_2(t) \tag{2-127}$$

At time $t = 0, P_0(0) = 1, P_1(0) = P_2(0) = 0, P_3(x,0) = P_4(x,0) = P_5(x,0) = 0$

Steady-State Availability Analysis

As time approaches infinity, Equations (2-119)-(2-124) reduce to Equations (2-128)-(2-131), respectively.

$$a_0P_0 - \mu_1P_1 = \int_0^\infty P_3(x,t)\mu_3(x)dx + \int_0^\infty P_4(x,t)\mu_4(x)dx + \int_0^\infty P_5(x,t)\mu_5(x)dx \quad (2\text{-}128)$$

$$r_0P_0 - a_1P + \mu_2P_2 = 0 \quad (2\text{-}129)$$

$$r_1P_1 - a_2P_2 = 0 \quad (2\text{-}130)$$

$$\frac{\partial P_i(x)}{\partial x} = -\mu_i(x)P_i(x) \qquad (for\ i = 3,4,5) \quad (2\text{-}131)$$

Similarly, the boundary conditions become:

$$P_3(0) = r_2P_2 \quad (2\text{-}132)$$

$$P_4(0) = \lambda_{c0}P_0 + \lambda_{c1}P_1 + \lambda_{c2}P_2 \quad (2\text{-}133)$$

$$P_5(0) = \lambda_{h0}P_0 + \lambda_{h1}P_1 + \lambda_{h2}P_2 \quad (2\text{-}134)$$

where

$$a_0 = r_0 + \lambda_{c0} + \lambda_{h0}$$
$$a_1 = r_1 + \lambda_{c1} + \lambda_{h1} + \mu_1$$
$$a_2 = r_2 + \lambda_{c2} + \lambda_{h2} + \mu_2$$

P_i is the steady-state probability that the system is in state i, for i = 0, 1, 2, …, 5, and

$$P_i = \int_0^\infty P_i(x)dx \qquad for\ i = 3,\ 4,\ 5 \quad (2\text{-}135)$$

Also,

$$\sum_{i=0}^{5} P_i = 1 \quad (2\text{-}136)$$

Solving differential Equation (2-131), we get

$$P_i(x) = P_i(0)\exp(-\int_0^x \mu_i(\omega)\,d\omega) \qquad (for\ \ i = 3,4,5) \qquad (2\text{-}137)$$

Thus, from Equation (2-135) and (2-137), we have

$$\begin{aligned} P_3 &= \int_0^\infty P_3(x)dx \\ &= \int_0^\infty P_3(0)\exp(-\int_0^x \mu_3(\omega)d\omega)dx \\ &= r_2 P_2 E_3[x] \end{aligned} \qquad (2\text{-}138)$$

Similarly,

$$\begin{aligned} P_4 &= \int_0^\infty P_4(x)dx \\ &= \int_0^\infty P_4(0)\exp(-\int_0^x \mu_4(\omega)d\omega)dx \\ &= (\lambda_{c0}P_0 + \lambda_{c1}P_1 + \lambda_{c2}P_2)\,E_4[x] \end{aligned} \qquad (2\text{-}139)$$

$$\begin{aligned} P_5 &= \int_0^\infty P_5(x)dx \\ &= \int_0^\infty P_5(0)\exp(-\int_0^x \mu_5(\omega)d\omega)dx \\ &= (\lambda_{h0}P_0 + \lambda_{h1}P_1 + \lambda_{h2}P_2)\,E_5[x] \end{aligned} \qquad (2\text{-}140)$$

where

$$E_i[x] = \int_0^\infty \exp(-\int_0^x \mu_i(\omega)d\omega)dx \qquad (2\text{-}141)$$

$$(\text{for } i = 3,4,5)$$

$E_3[x]$, $E_4[x]$ and $E_5[x]$ are the mean times to repair from state 3 to state 0, from state 4 to state 0, and from state 5 to state 0, respectively.

Solving the set of Equations (2-129), (2-130), (2-136) and (2-138)-(2-140), we get the following steady state probabilities:

$$P_0 = \frac{b_0(2,1)}{BA(2,1)} \qquad (2\text{-}142)$$

$$P_1 = \frac{b_1(2,1)}{BA(2,1)} \qquad (2\text{-}143)$$

$$P_2 = \frac{b_2(2,1)}{BA(2,1)} \qquad (2\text{-}144)$$

$$P_3 = \frac{b_3(2,1)E_3[x]}{BA(2,1)} \qquad (2\text{-}145)$$

$$P_4 = \frac{b_4(2,1)E_4[x]}{BA(2,1)} \qquad (2\text{-}146)$$

$$P_5 = \frac{b_5(2,1)E_5[x]}{BA(2,1)} \qquad (2\text{-}147)$$

where

$$\begin{aligned}
b_0(2,1) &= a_1a_2 - r_1\mu_2 \\
b_1(2,1) &= a_2r_0 \\
b_2(2,1) &= r_0r_1 \\
b_3(2,1) &= r_0r_1r_2 \\
b_4(2,1) &= \sum_{i=0}^{2} b_i(2,1)\lambda_{ci} \\
b_5(2,1) &= \sum_{i=0}^{2} b_i(2,1)\lambda_{hi} \\
BA(2,1) &= \sum_{i=0}^{2} b_i(2,1) + \sum_{J=3}^{5} b_j(2,1)E_j[x]
\end{aligned}$$

The system steady state availability is

$$\begin{aligned}
AV_{ss}(2,1) &= P_0 + P_1 + P_2 \\
&= \frac{\sum_{i=0}^{2} b_i(2,1)}{BA(2,1)} \qquad (2\text{-}148)
\end{aligned}$$

Similarly, the system steady state unavailability is given by

$$UAV_{ss}(2,1) = P_3 + P_4 + P_5 \\ = \frac{\sum_{j=3}^{5} b_j(2,1)E_j(x)}{BA(2,1)} \tag{2-149}$$

System Steady State Availability Special Cases

Case I

If the system repair time x is ***Gamma*** distributed, Substituting Equation (2-79) into Equation (2-148), we get the following resulting system steady state availability for the Gamma failed system repair time distribution:

$$AV_{ss}(2,1) = \frac{\sum_{i=0}^{2} b_i(2,1)}{\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\beta/\mu_j} \tag{2-150}$$

If β is an integer, the Equation (2-150) is the system steady state availability for the ***Special Erlangian*** distributed failed system repair time.

When β = 1, it is the system steady state availability for ***exponential*** distributed failed system repair time.

Case II

If the system repair time x is ***Weibull*** distributed, Substituting Equation (2-84) into Equation (2-148), we get the following resulting system steady state availability for the Weibull failed system repair time distribution:

$$AV_{ss}(2,1) = \frac{\sum_{i=0}^{2} b_i(2,1)}{\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\Gamma(1+1/\beta)/\mu_j} \qquad (2\text{-}151)$$

Case III

If the system repair time x is ***Rayleigh*** distributed, Substituting Equation (2-88) into Equation (2-148), we get the following resulting system steady state availability for the Rayleigh failed system repair time distribution:

$$AV_{ss}(2,1) = \frac{\sum_{i=0}^{2} b_i(2,1)}{\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\sqrt{\pi/2}/\mu_j} \qquad (2\text{-}152)$$

Case IV

If the system repair time x is ***lognormal*** distributed, Substituting Equation (2-91) into Equation (2-148), we get the following resulting system steady state availability for the lognormal failed system repair time distribution:

$$AV_{ss}(2,1) = \frac{\sum_{i=0}^{2} b_i(2,1)}{\sum_{i=0}^{2} b_i(2,1) + \sum_{i=3}^{5} b_i(2,1)\exp\left(\mu_i + \sigma^2/2\right)} \qquad (2\text{-}153)$$

System Steady State Availability Numerical Examples

Setting

$r_0 = r_1 = 0.0005 / hr$ $\quad r_2 = 0.0008 / hr$ $\quad \mu_1 = 0.001 / hr$ $\quad \mu_2 = 0.002 / hr$

$\lambda_{c0} = \lambda_{c1} = 0.00005 / hr$ $\quad \lambda_{c2} = 0.00002 / hr$ $\quad \lambda_{h1} = 0.0002 / hr$ $\quad \lambda_{h2} = 0.0001 / hr$

in Equation (2-148) leads to the following:

$$AV_{ss} = \frac{5.82}{[2.0E_3(x) + 2.835E_4(x) + 3.17E_5(x)] \times 10^{-4} + 5.82 + 4.11E_5(x)\lambda_{h0}}$$

The plots of system steady state availability as a function of human error rate, λ_{h0}, are shown in Figures 2-2 (Table 2-1), and 2-4 (Table 2-3) for different values of β and for Gamma and Weibull distributed repair times, respectively, for specified values of parameters $\mu_3 = 0.001/hr, \mu_4 = \mu_5 = 0.0009/hr$. The plots exhibit that the system steady state availability decreases with increasing values of human error rate, λ_{h0}. From Figure 2-2, it can be observed that the system steady state availability decreases as the value of the Gamma shape parameter, β, increases for Gamma distributed repair times. And Av_{ss} increases as the value of the Weibull shape parameter, β, increases for Weibull distributed failed system repair times.

Similarly, setting λ_{h0}=0.0002/hr and specified values of parameters μ_3=0.001/hr and μ_4=0.0009/hr for Gamma and Weibull distributed repair times, Figures 2-3 (Table 2-2) and 2-5 (Table 2-4) show the effect of increasing the human error repair parameter, μ_5, for different values of β on the system steady state availability, respectively. The plots indicate that system steady state availability increases with the increasing values of μ_5.

Figure 2-6 and Table 2-5 provide the results of the system steady state availability as a function of human error rate, λ_{h0}, for lognormally distributed failed system repair times ($\mu_3 = 1.001,\ \mu_4 = \mu_5 = 1.0009,\ \sigma_3 = \sigma_4 = \sigma_5 = \sigma$). It can be clearly seen from the plots that the system steady state availability decreases with increasing values of λ_{h0}. Furthermore, the steady state availability decreases as the value of σ increases.

Also, the plots of the system steady state availability as a function of λ_{h0} are shown in Figure 2-7 (Table 2-6) for Rayleigh distributed failed system repair times for specified values of parameters.

Figure 2-8 (Table 2-7) shows plots of system steady state availability as a function of human error rate, λ_{h0}, for specified values of parameters $\mu_3 = 0.001/hr, \mu_4 = \mu_5 = 0.0009/hr$, for different system repair time distributions: exponential, Gamma, Rayleigh, Weibull and lognormal ($\mu_3 = 1.001,\ \mu_4 = \mu_5 = 1.0009,\ \sigma_3 = \sigma_4 = \sigma_5 = \sigma = 3$).

The plots of system steady state availability effected by the parameter μ_5 are shown in Figure 2-9, for $\lambda_{h0} = 0.0002/hr$ and specified values of parameters $\mu_3 = 0.001/hr, \mu_4 = 0.0009/hr$, for different system repair time distributions: exponential, Gamma, Rayleigh, Weibull and lognormal ($\mu_3 = 1.001$, $\mu_4 = \mu_5 = 1.0009$, $\sigma_3 = \sigma_4 = \sigma_5 = \sigma = 3$).

From these plots, it can be observed that system steady state availability decreases with increasing values of λ_{h0} and Av_{ss} increases with increasing values of μ_5 regardless of the distributions of failed system repair times being considered. The system steady state availability is heavily influenced by the failed system repair time distribution. The lognormally distributed failed system repair time yielded the highest values of the system steady state availability while Gamma ($\beta \geq 2$) distributed failed system repair time produced the least values for the system steady state availability.

Table 2-1 Av_{ss} vs. λ_{h0} for (2,1) standby system with constant human error rates and Gamma distributed repair times

λ_{h0}	$AV_{ss}(\beta=0.5)$	$AV_{ss}(\beta=1)$	$AV_{ss}(\beta=2)$	$AV_{ss}(\beta=3)$
0.000000	0.930662	0.870317	0.770407	0.691075
0.000100	0.897879	0.814682	0.687311	0.594383
0.000200	0.867326	0.765733	0.620396	0.521428
0.000300	0.838785	0.722333	0.565353	0.464423
0.000400	0.812062	0.683589	0.519282	0.418655
0.000500	0.786989	0.648789	0.480154	0.381098
0.000600	0.763418	0.617361	0.446509	0.349725
0.000700	0.741218	0.588837	0.417271	0.323124
0.000800	0.720272	0.562832	0.391626	0.300284
0.000900	0.700478	0.539028	0.368951	0.280459
0.001000	0.681741	0.527152	0.348764	0.263091

Table 2-2 Av_{ss} vs. μ_5 for (2,1) standby system with constant human error rates and Gamma distributed repair times

μ_5	$AV_{ss}(\beta=1)$	$AV_{ss}(\beta=2)$	$AV_{ss}(\beta=3)$	$AV_{ss}(\beta=0.5)$
0.000100	0.328350	0.196423	0.140123	0.494372
0.000200	0.483791	0.319079	0.238037	0.652101
0.000300	0.574437	0.402954	0.310317	0.729704
0.000400	0.633814	0.463930	0.365865	0.775871
0.000500	0.675723	0.510258	0.409888	0.806485
0.000600	0.706883	0.546650	0.445636	0.828273
0.000700	0.730959	0.575993	0.475241	0.844571
0.000800	0.750121	0.600155	0.500161	0.857222
0.000900	0.765733	0.620396	0.521428	0.867326
0.001000	0.778699	0.637599	0.539789	0.875583

Table 2-3 Av_{ss} vs. λ_{h0} for (2,1) standby system with constant human error rates and Weibull distributed repair times

λ_{h0}	$AV_{ss}(\beta=1)$	$AV_{ss}(\beta=2)$	$AV_{ss}(\beta=3)$	$AV_{ss}(\beta=4)$
0.000000	0.870317	0.883350	0.882565	0.881010
0.000100	0.814682	0.832229	0.831167	0.829063
0.000200	0.765733	0.786702	0.785425	0.782900
0.000300	0.722333	0.745897	0.744455	0.741607
0.000400	0.683589	0.709116	0.707548	0.704452
0.000500	0.648789	0.675793	0.674127	0.670842
0.000600	0.617361	0.645461	0.643721	0.640293
0.000700	0.588837	0.617734	0.615940	0.612405
0.000800	0.562832	0.592292	0.590457	0.586845
0.000900	0.539028	0.568862	0.566999	0.563333
0.001000	0.517155	0.547216	0.545334	0.541633

Table 2-4 Av_{ss} vs. μ_5 for (2,1) standby system with constant human error rates and Weibull distributed repair times

μ_5	$AV_{ss}(\beta=1)$	$AV_{ss}(\beta=2)$	$AV_{ss}(\beta=3)$	$AV_{ss}(\beta=4)$
0.000100	0.328350	0.355517	0.353779	0.350376
0.000200	0.483791	0.513977	0.512080	0.508352
0.000300	0.574437	0.603665	0.601847	0.598267
0.000400	0.633814	0.661368	0.659666	0.656309
0.000500	0.675723	0.701608	0.700016	0.696874
0.000600	0.706883	0.731270	0.729775	0.726823
0.000700	0.730959	0.754040	0.752629	0.749841
0.000800	0.750121	0.772070	0.770732	0.768085
0.000900	0.765733	0.786702	0.785425	0.782900
0.001000	0.778699	0.798812	0.797589	0.795170

Table 2-5 Av_{ss} vs. λ_{h0} for (2,1) standby system with constant human error rates and lognormally distributed repair times

λ_{h0}	$AV_{ss}(\sigma=0.3)$	$AV_{ss}(\sigma=1)$	$AV_{ss}(\beta=1.5)$
0.000000	0.999609	0.999383	0.998849
0.000100	0.999408	0.999067	0.998259
0.000200	0.999207	0.998751	0.997669
0.000300	0.999007	0.998435	0.997080
0.000400	0.998806	0.998119	0.996492
0.000500	0.998606	0.997804	0.995905
0.000600	0.998405	0.997489	0.995318
0.000700	0.998205	0.997174	0.994732
0.000800	0.998005	0.996859	0.994147
0.000900	0.997805	0.996544	0.993563
0.001000	0.997605	0.996230	0.992979

Table 2-6 Av_{ss} vs. λ_{h0} for (2,1) standby system with constant human error rates and Rayleigh distributed repair times

λ_{h0}	$AV_{ss}(\mu_5=0.0002)$	$AV_{ss}(\mu_5=0.0004)$	$AV_{ss}(\mu_5=0.0006)$	$AV_{ss}(\mu_5=0.0008)$
0.0000	0.688598	0.780296	0.816541	0.835957
0.0001	0.527771	0.665410	0.728762	0.765188
0.0002	0.427844	0.580012	0.658024	0.705466
0.0003	0.359734	0.514041	0.599803	0.654392
0.0004	0.310330	0.461545	0.551047	0.610214
0.0005	0.272858	0.418777	0.509622	0.571623
0.0006	0.243460	0.383263	0.473990	0.537623
0.0007	0.219781	0.353302	0.443015	0.507441
0.0008	0.200300	0.327685	0.415839	0.480467
0.0009	0.183991	0.305532	0.391806	0.456217
0.0010	0.170138	0.286185	0.370398	0.434296

Table 2-7 The comparison of Av_{ss} vs. λ_{h0} for (2,1) standby system with constant human error rates and different repair time distributions

λ_{h0}	exponential	Gamma(β=2)	Rayleigh	Weibull(β=3)	lognormal
0.00000	0.870317	0.770407	0.842635	0.882565	0.967411
0.00010	0.814682	0.687311	0.778153	0.831167	0.951491
0.00020	0.765733	0.620396	0.722838	0.785425	0.936086
0.00030	0.722333	0.565353	0.674865	0.744456	0.921173
0.00040	0.683589	0.519282	0.632864	0.707548	0.906727
0.00050	0.648789	0.480154	0.595784	0.674128	0.892727
0.00060	0.617361	0.446509	0.562809	0.643722	0.879153
0.00070	0.588837	0.417271	0.533293	0.615940	0.865985
0.00080	0.562832	0.391626	0.506718	0.590458	0.853206
0.00090	0.539028	0.368951	0.482666	0.567000	0.840799
0.00100	0.517155	0.348758	0.460794	0.545334	0.828747

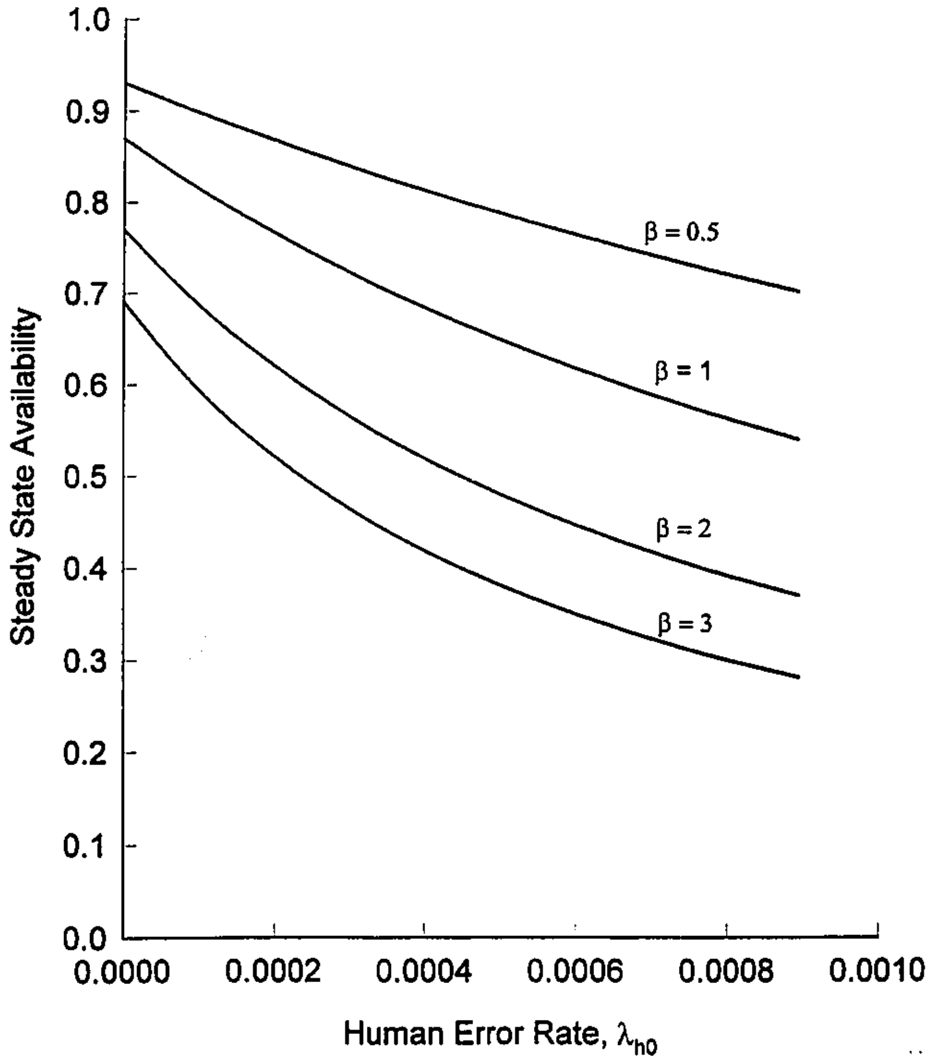


Figure 2-2 System steady state availability vs. λ_{h0} plots for two units active and one on standby system with constant human error rates and the Gamma distributed failed system repair times

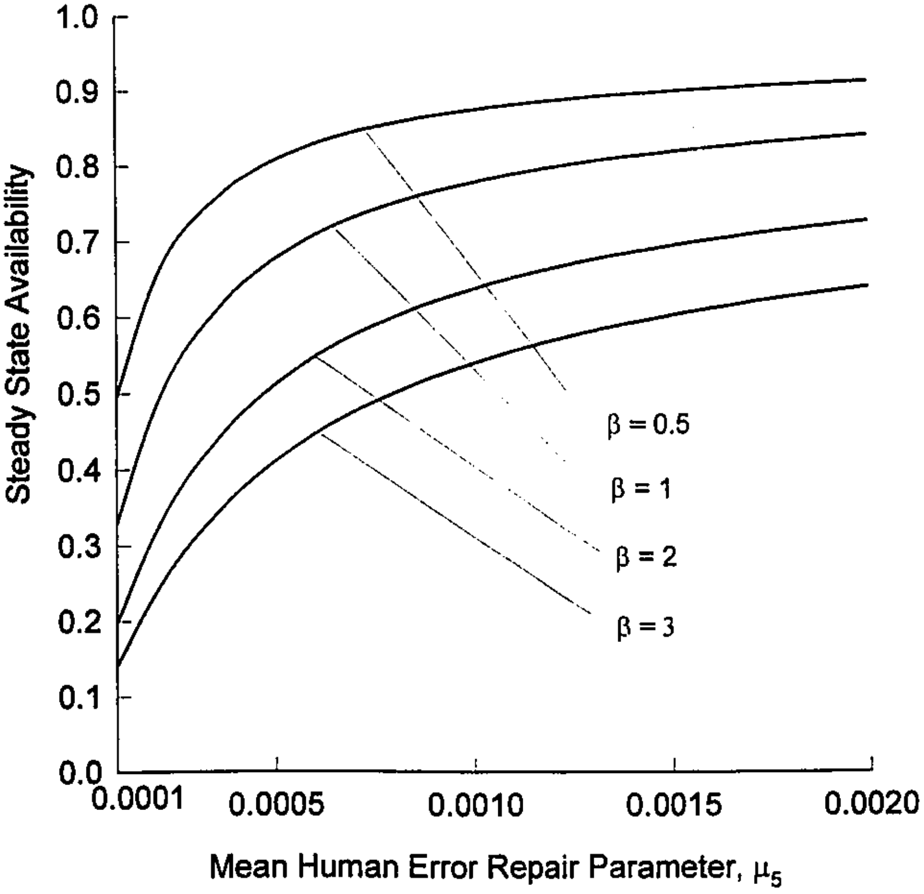


Figure 2-3 System steady state availability vs. μ_5 plots for two units active and one on standby system with constant human error rates and the Gamma distributed system repair times

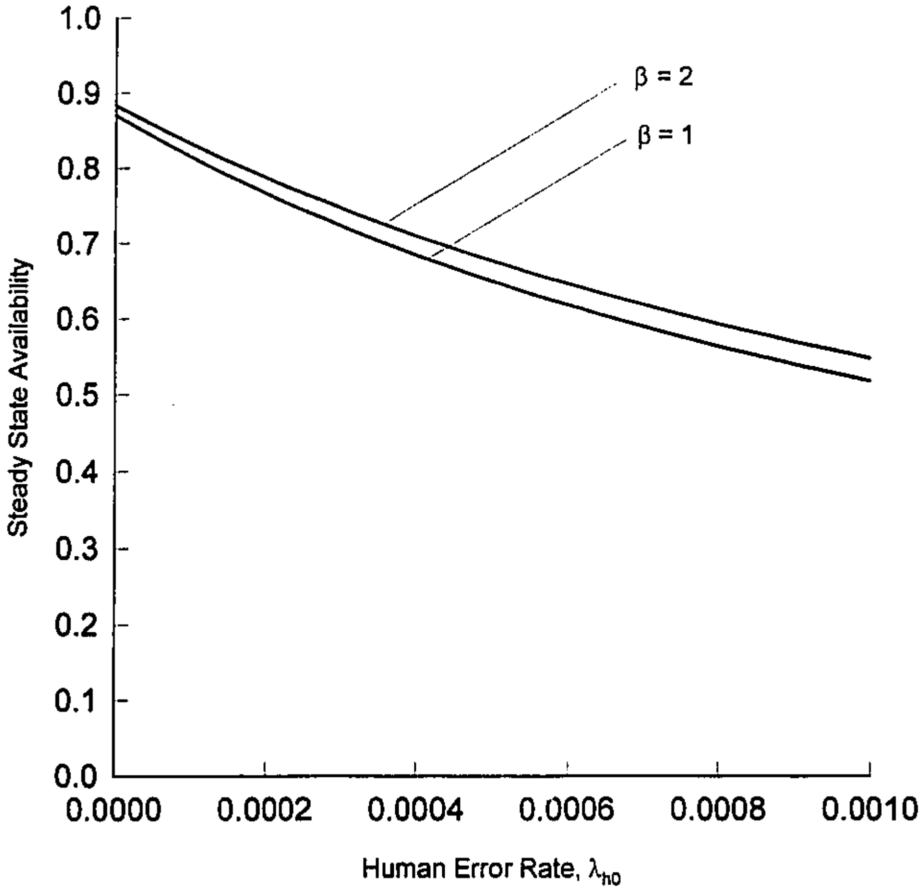


Figure 2-4 System steady state availability vs. λ_{h0} plots for two units active and one on standby system with constant human error rates and the Weibull distributed system repair times

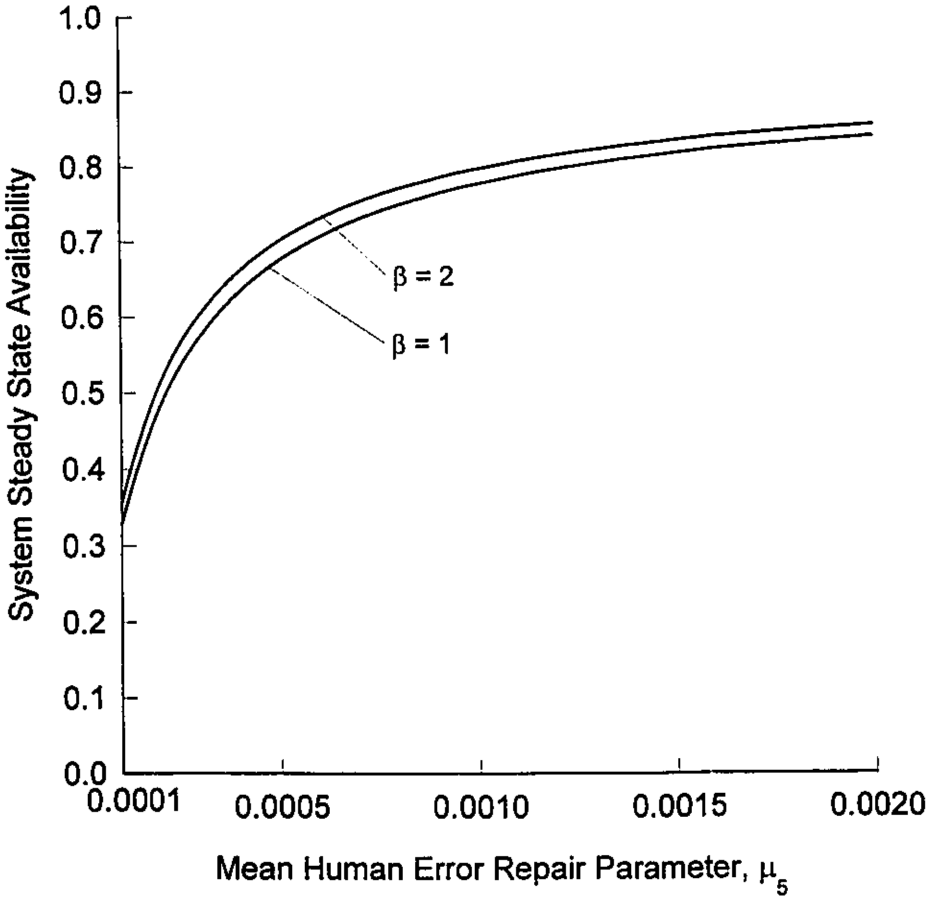


Figure 2-5 AV_{ss} vs. μ_5 plots for two units active and one on standby system with constant human error rates and the Weibull distributed system repair times

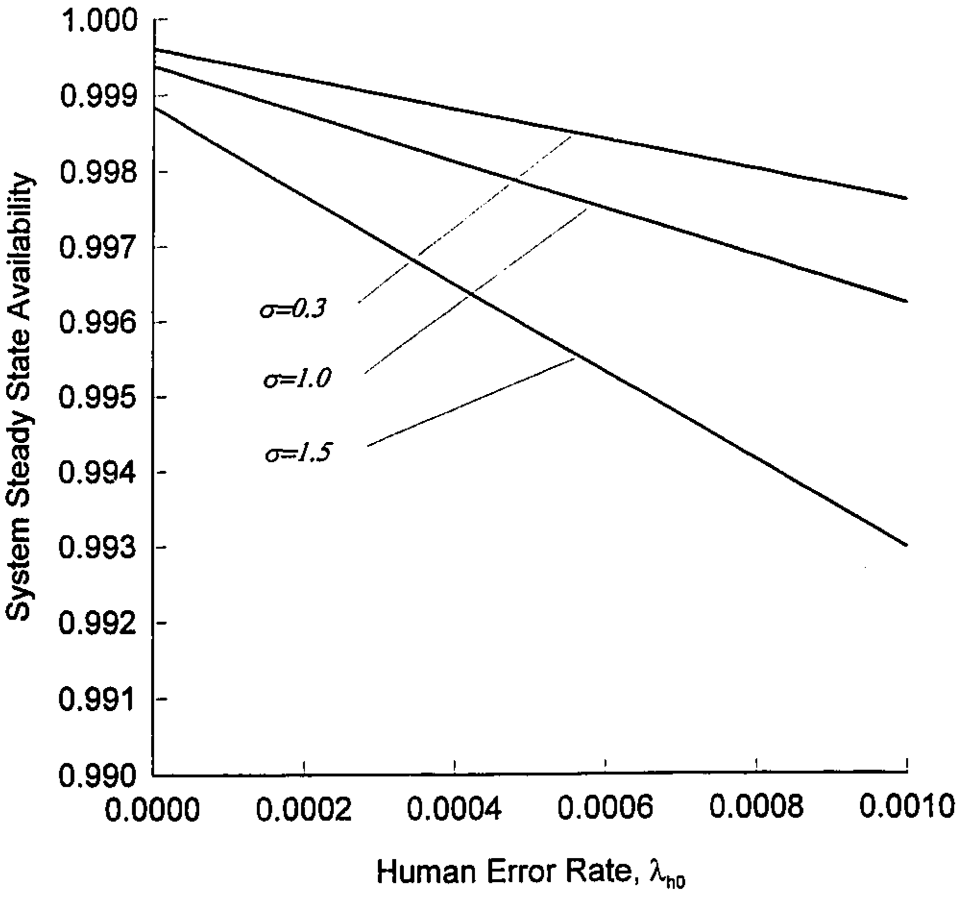


Figure 2-6 AV_{ss} vs. λ_{h0} plots for two units active and one on standby system with constant human error rates and the lognormally distributed system repair times

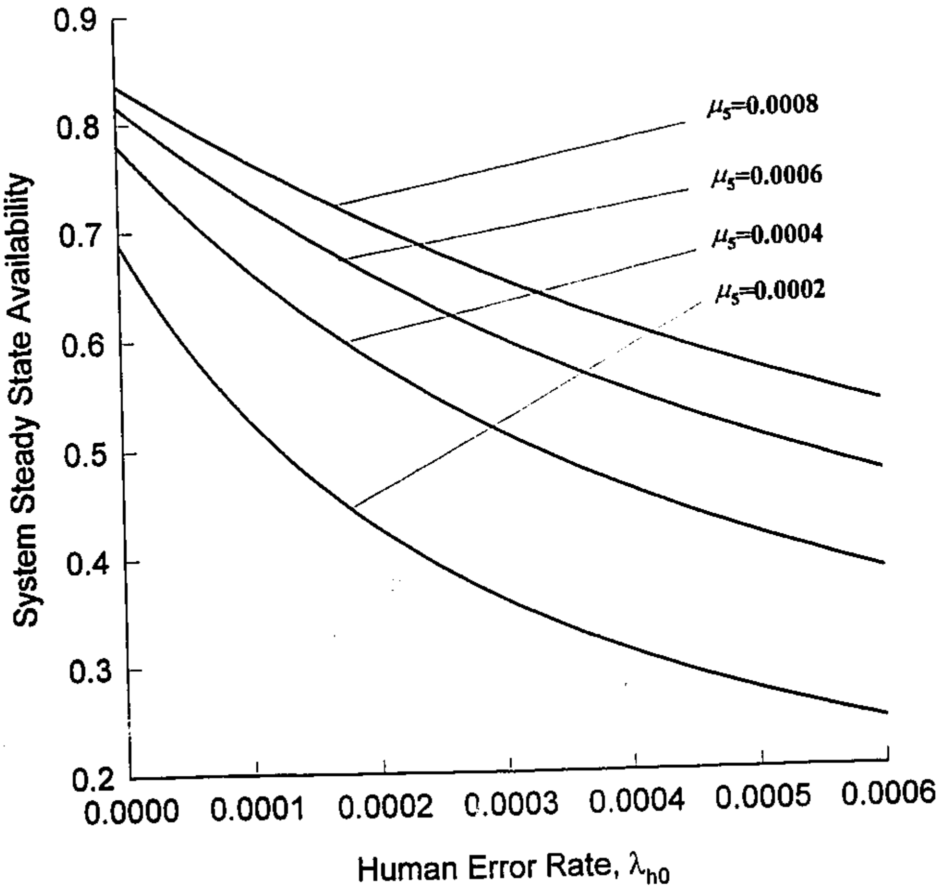


Figure 2-7 AV_{ss} vs. λ_{h0} plots for two units active and one on standby system with constant human error rates and the Rayleigh distributed system repair times

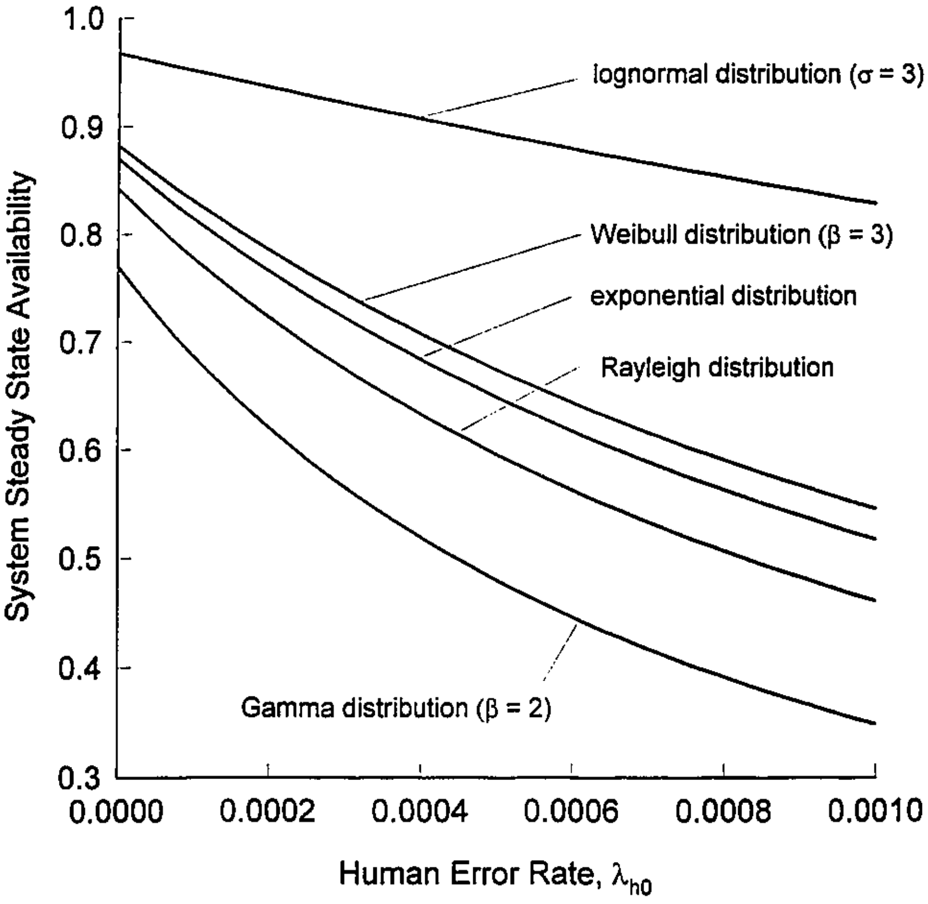


Figure 2-8 The comparison of AV_{ss} vs. λ_{h0} for two units active and one on standby system with constant human error rates and different system repair time distributions

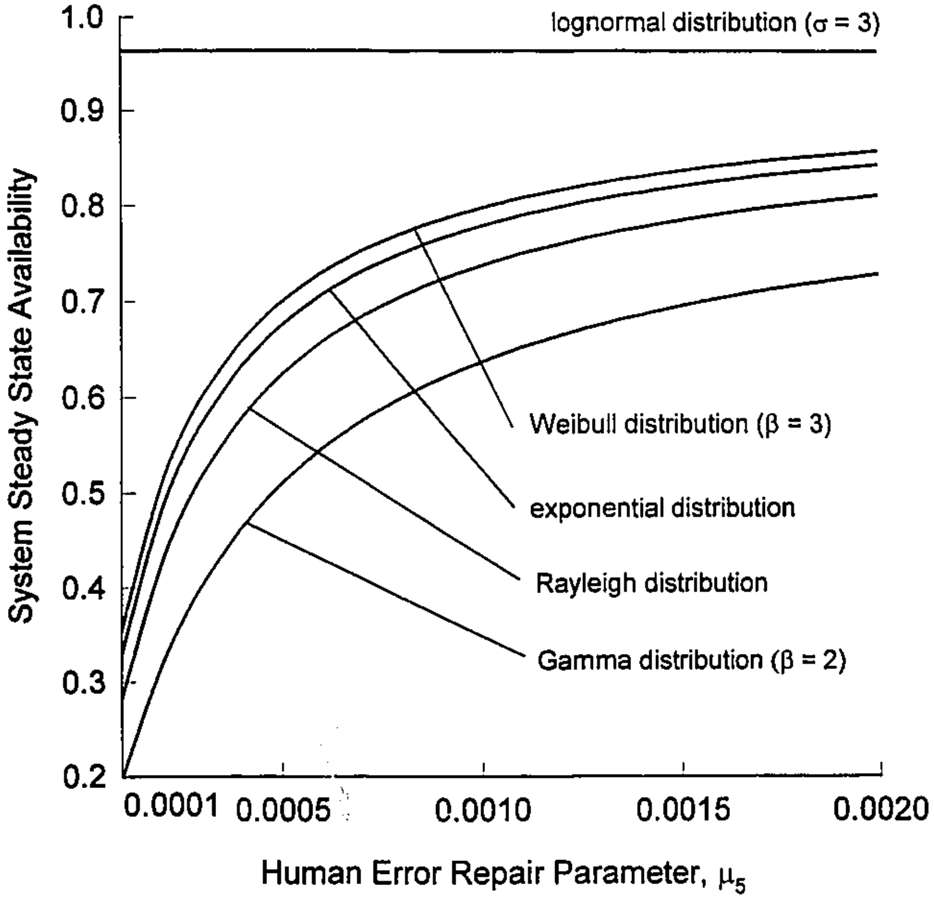


Figure 2-9 The comparision of AV_{ss} vs. μ_5 for two units active and one on standby system with constant human error rates and different system repair time distributions

Time-Dependent System Availability Analysis

Using Laplace transforms and the initial conditions in Equations (2-119) - (2-127), we get

$$\begin{aligned} sP_0(s) = 1 - a_0P_0(s) + \mu_1P_1(s) + \int_0^\infty P_3(x,s)\mu_3(x)dx + \\ \int_0^\infty P_4(x,s)\mu_4(x)dx + \int_0^\infty P_5(x,s)\mu_5(x)dx \end{aligned} \qquad (2\text{-}154)$$

$$(s + a_1)P_1(s) - r_0P_0(s) - \mu_2P_2(s) = 0 \qquad (2\text{-}155)$$

$$(s + a_2)P_2(s) - r_1P_1(s) = 0 \qquad (2\text{-}156)$$

$$\frac{\partial P_i(x,s)}{\partial x} + s\,P_i(x,s) + \mu_i(x)P_i(x,s) = 0 \qquad (\mathit{for}\ \ i = 3,4,5) \qquad (2\text{-}157)$$

and the boundary conditions:

$$\begin{aligned} P_3(0,s) &= r_2P_2(s) \\ P_4(0,s) &= \lambda_{c0}P_0(s) + \lambda_{c1}P_1(s) + \lambda_{c2}P_2(s) \\ P_5(0,s) &= \lambda_{h0}P_0(s) + \lambda_{h1}P_1(s) + \lambda_{h2}P_2(s) \end{aligned} \qquad (2\text{-}158)$$

Solving differential equation (2-157), we get

$$P_i(x,s) = P_i(0,s)e^{-sx}\exp(-\int_0^x \mu_i(\omega)d\omega) \qquad (\mathit{for}\ \ i = 3,4,5) \qquad (2\text{-}159)$$

Substituting Equations (2-158) and (2-159) in Equation (2-154), lead to

$$\begin{aligned} &[s + a_0 - \lambda_{c0}N_4(s) - \lambda_{h0}N_5(s)]P_0(s) - \\ &\quad [\mu_1 + \lambda_{c1}N_4(s) - \lambda_{h1}N_5(s)]P_1(s) - \\ &\quad [r_2N_3(s) + \lambda_{c2}N_4(s) - \lambda_{h2}N_5(s)]P_2(s) = 1 \end{aligned} \qquad (2\text{-}160)$$

where $N_i(x)$ is pdf of repair time and $N_i(s)$ is the Laplace transform of $N_i(x)$

$$N_i(s) = \int_0^\infty \exp(-sx)N_i(x)dx$$

$$N_i(x) = \mu_i(x)\exp[-\int_0^x \mu_i(\omega)d\omega] \qquad (\mathit{for}\ i = 3,4,5)$$

Since we know

$$P_i(s) = \int_0^\infty P_i(x,s)dx \qquad (for\ \ i = 3,4,5) \qquad (2\text{-}161)$$

Thus, utilizing Equation (2-159), we get

$$P_3(s) = r_2 P_2(s)\frac{1 - N_3(s)}{s} \qquad (2\text{-}162)$$

$$P_4(s) = [\lambda_{c0}P_0(s) + \lambda_{c1}P_1(s) + \lambda_{c2}P_2(s)]\frac{1 - N_4(s)}{s} \qquad (2\text{-}163)$$

$$P_5(s) = [\lambda_{h0}P_0(s) + \lambda_{h1}P_1(s) + \lambda_{h2}P_2(s)]\frac{1 - N_5(s)}{s} \qquad (2\text{-}164)$$

where

$$\frac{1 - N_i(s)}{s} = \int_0^\infty e^{-sx}\exp(-\int_0^x \mu_i(\omega)d\omega)dx \qquad (for\ \ i = 3,4,5) \qquad (2\text{-}165)$$

Solving Equations (2-155), (2-156), (2-160) and (2-162) - (2-165), we get the following Laplace transforms of state probabilities $P_0(s), P_1(s), \ldots, and\ \ P_5(s)$:

$$P_0(s) = \frac{b_0(2,1) + (a_1 + a_2)s + s^2}{CA(2,1)} \qquad (2\text{-}166)$$

$$P_1(s) = \frac{b_1(2,1) + r_0 s}{CA(2,1)} \qquad (2\text{-}167)$$

$$P_2(s) = \frac{b_2(2,1)}{CA(2,1)} \qquad (2\text{-}168)$$

$$P_3(s) = \frac{b_3(2,1)[1 - N_3(s)]}{s \cdot CA(2,1)} \qquad (2\text{-}169)$$

$$P_4(s) = \frac{[1 - N_4(s)]\left[b_4(2,1) + lc(2,1)\cdot s + \lambda_{c0}s^2\right]}{s \cdot CA(2,1)} \qquad (2\text{-}170)$$

$$P_5(s)=\frac{\left[1-N_5(s)\right]\left[b_5(2{,}1)+lh(2{,}1)\cdot s+\lambda_{h0}s^2\right]}{s\cdot CA(2{,}1)} \qquad (2\text{-}171)$$

where

$$\begin{aligned}
CA(2{,}1)&=ca1(2{,}1)+ca2(2{,}1)\cdot s+ca3(2{,}1)\cdot s^2+s^3\\
ca1(2{,}1)&=\sum_{j=3}^{5}\left[1-N_j(s)\right]b_j(2{,}1)\\
ca2(2{,}1)&=\sum_{i=0}^{2}b_i(2{,}1)+lc(2{,}1)\left[1-N_4(s)\right]+lh(2{,}1)\left[1-N_5(s)\right]\\
ca3(2{,}1)&=l0(2{,}1)+\lambda_{c0}\left[1-N_4(s)\right]+\lambda_{h1}\left[1-N_5(s)\right]\\
lc(2{,}1)&=(a_1+a_2)\lambda_{c0}+r_0\lambda_{c1}\\
lh(2{,}1)&=(a_1+a_2)\lambda_{h0}+r_0\lambda_{h1}\\
l0(2{,}1)&=a_1+a_2+r_0
\end{aligned}$$

The Laplace transform of the system availability is :

$$AV(s)=\frac{\sum_{i=0}^{2}b_i(2{,}1)+l0(2{,}1)\cdot s+s^2}{CA(2{,}1)} \qquad (2\text{-}172)$$

Taking the inverse Laplace transforms of Equation (2-172), we can obtain the time-dependent system availability.

Time-Dependent System Availability Special Cases

Substituting the Laplace transforms of the pdf of the system repair times, $N_3(s)$, $N_4(s)$, and $N_5(s)$, in Equation (2-172) and then substituting the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting μ_3=0.001/hr and μ_4=μ_5=0.0009/hr and taking inverse Laplace transform), we get the system time-dependent availability.

For ***exponential*** failed system repair time distribution and $\lambda_{h0} = 0.0002\,/\,hr$ we get

$$
\begin{aligned}
AV(t) = {} & 0.765733 + 0.0115416\exp(-0.00355207t) + \\
& 0.00738703\exp(-0.00096036t) + \\
& 2.1631\sin(0.000186153t)\exp(-0.00140379t) + \\
& 0.215338[\cos(0.000186153t)\exp(-0.00140379t) - \\
& 7.54103\sin(0.000186153t)\exp(-0.00140379t)]
\end{aligned}
$$

For different values of λ_{h0}, the system time-dependent availability plots are shown in Figure 2-10 and in Table 2-8.

For ***Gamma*** distributed failed system repair time, the plots of the time-dependent system availability for varying human error rate, λ_{h0}, are shown in Figures 2-11 (Table 2-9), for $\beta = 2$, and 2-12 (Table 2-10), for $\beta = 3$, respectively.

The plots in Figures 2-10, 2-11 and 2-12 indicate that time-dependent system availability decreases with increasing values of human error rate. The comparison of tabular time-dependent availability values in Table 2-8 with those of Tables 2-9 and 2-10 clearly shows that time-dependent availability decreases with increasing value of the Gamma shape parameter β.

Table 2-8 Time-dependent system availability for (2,1) standby system with constant human error rates and failed system repair rates

time	AV(t)(λ_{h0}=0)	AV(t)(λ_{h0}=0.0002)	AV(t)(λ_{h0}=0.0004)
0.00	1.0000	1.0000	1.0000
500.00	0.9704	0.9034	0.8420
1000.00	0.9397	0.8454	0.7644
1500.00	0.9151	0.8108	0.7250
2000.00	0.8978	0.7908	0.7049
2500.00	0.8866	0.7794	0.6945
3000.00	0.8797	0.7731	0.6892
3500.00	0.8756	0.7697	0.6865
4000.00	0.8732	0.7678	0.6851
4500.00	0.8719	0.7668	0.6844
5000.00	0.8712	0.7663	0.6840

Table 2-9 Time-dependent system availability for (2,1) standby system with constant human error rates and Gamma ($\beta = 2$) distributed system repair times

time	AV(t)(λ_{h0}=0)	AV(t)(λ_{h0}=0.0002)	AV(t)(λ_{h0}=0.0004)
0.00	1.0000	0.9998	0.9996
500.00	0.9649	0.8851	0.8195
1000.00	0.9197	0.7907	0.6933
1500.00	0.8765	0.7216	0.6121
2000.00	0.8411	0.6757	0.5640
2500.00	0.8149	0.6477	0.5379
3000.00	0.7967	0.6319	0.5249
3500.00	0.7850	0.6238	0.5194
4000.00	0.7778	0.6202	0.5175
4500.00	0.7736	0.6189	0.5173
5000.00	0.7714	0.6188	0.5177

Table 2-10 Time-dependent system availability for (2,1) standby system with constant human error rates and Gamma ($\beta = 3$) distributed system repair times

time	AV(t)(λ_{h0}=0)	AV(t)(λ_{h0}=0.0002)	AV(t)(λ_{h0}=0.0004)
0.0000	1.0000	0.9997	0.9971
500.0000	0.9648	0.8820	0.8066
1000.0000	0.9152	0.7764	0.6610
1500.0000	0.8627	0.6883	0.5548
2000.0000	0.8147	0.6209	0.4834
2500.0000	0.7746	0.5732	0.4398
3000.0000	0.7435	0.5424	0.4166
3500.0000	0.7209	0.5245	0.4068
4000.0000	0.7056	0.5155	0.4047
4500.0000	0.6960	0.5123	0.4065
5000.0000	0.6905	0.5123	0.4097

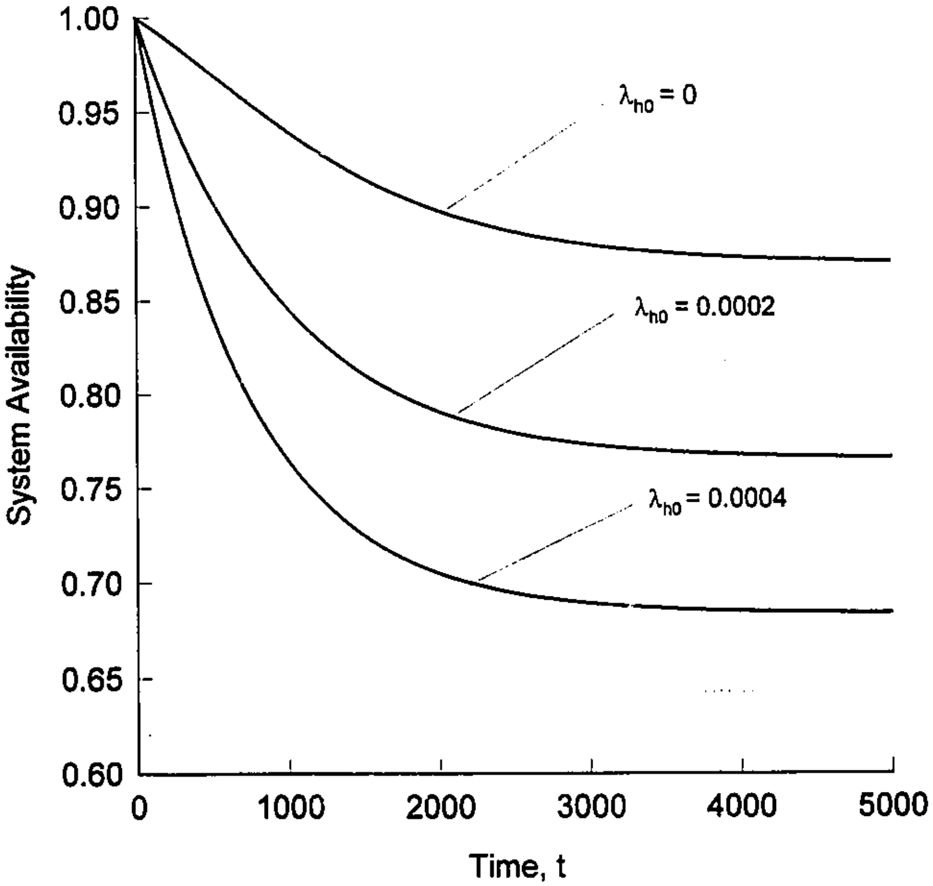


Figure 2-10 Time-dependent system availability plots for two units active and one on standby system with constant human error rates and system repair rates

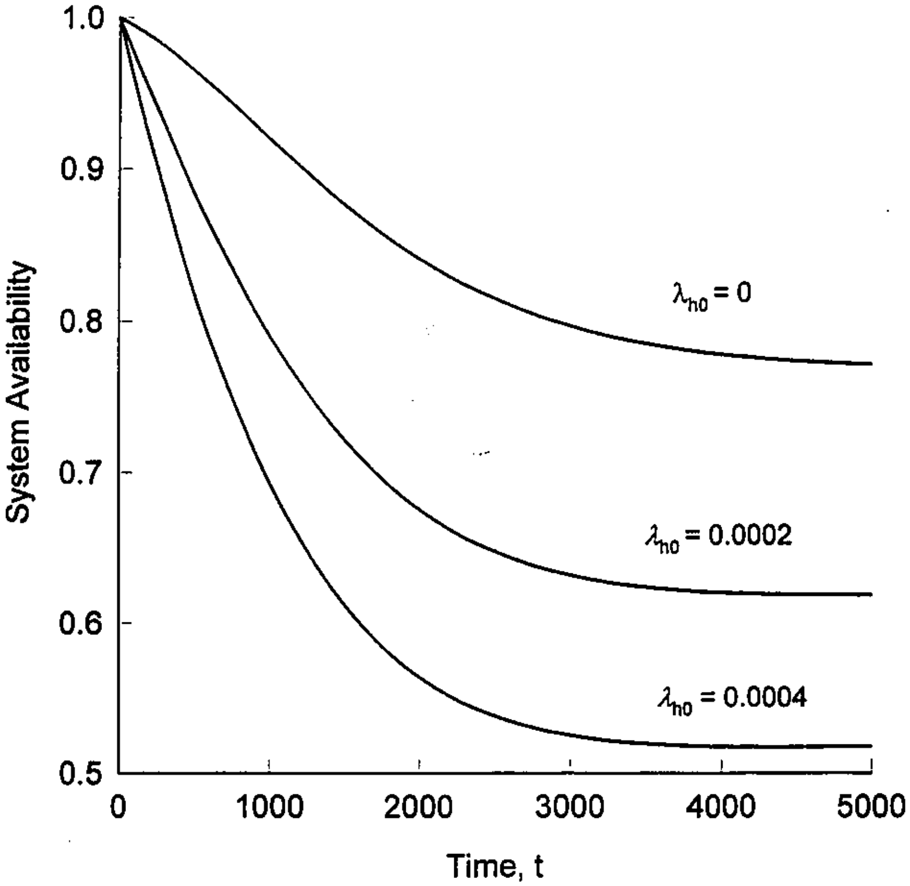


Figure 2-11 Time-dependent system availability plots for two units active and one on standby system with constant human error rates and the Gamma ($\beta = 2$) distributed system repair times

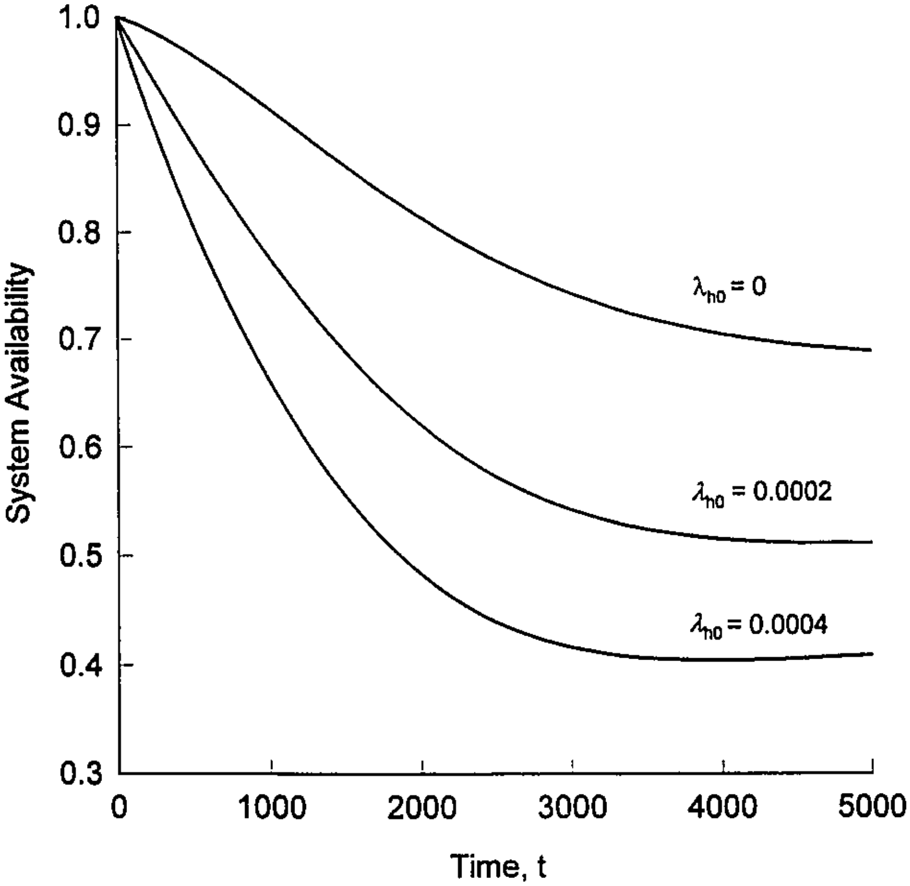


Figure 2-12 Time-dependent system availability plots for two units active and one on standby system with constant human error rates and the Gamma (β = 3) distributed system repair times

System Reliability and MTTF With and Without Repair

Setting $\mu_3(x) = \mu_4(x) = \mu_5(x) = 0$ in the model, the system of differential equations becomes:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) \tag{2-173}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \tag{2-174}$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) - a_2 P_2(t) \tag{2-175}$$

$$\frac{dP_3(t)}{dt} = r_2 P_2(t) \tag{2-176}$$

$$\frac{dP_4(t)}{dt} = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) + \lambda_{c2} P_2(t) \tag{2-177}$$

$$\frac{dP_5(t)}{dt} = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) + \lambda_{h2} P_2(t) \tag{2-178}$$

At time $t = 0$, $P_0(0) = 1$ and $P_i(0) = 0$ for $i = 1,2,3,4,5$.

Using Laplace transforms in Equations (2-173) - (2-178), and solving the resulting set of equations, we can get the Laplace transforms of the state probabilities, $P_0(s)$, $P_1(s)$, …, $P_5(s)$.

Thus, the Laplace transform of the system reliability with repair is

$$R(s) = \frac{\sum_{i=0}^{2} b_i(2,1) + l0(2,1) \cdot s + s^2}{DA(2,1)} \tag{2-179}$$

where

$$DA(2,1) = da1(2,1) + da2(2,1)\cdot s + da3(2,1)\cdot s^2 + s^3$$
$$da1(2,1) = a_0 r_0 r_1 - a_2 r_0 \mu_1$$
$$da2(2,1) = a_0(a_1 + a_2) + r_0 r_1 - r_0 \mu_1$$
$$da3(2,1) = a_0 + a_1 + a_2$$

System reliability with repair can be obtained by inverting Equation (2-179)

$$R(t) = \frac{e^{s_1 t}(s_a s_b - s_a s_1 - s_b s_1 + s_1^{\,2})}{(s_1 - s_2)(s_1 - s_3)} + \frac{e^{s_2 t}(s_a s_b - s_a s_2 - s_b s_2 + s_2^{\,2})}{(s_2 - s_1)(s_2 - s_3)} + \frac{e^{s_3 t}(s_a s_b - s_a s_3 - s_b s_3 + s_3^{\,2})}{(s_3 - s_2)(s_3 - s_1)} \qquad (2\text{-}180)$$

where s_a and s_b are the roots of the numerator of Equation (2-179), and s_1, s_2 and s_3 are the real and unique roots of the polynomial function DA(2,1).

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s\to 0} R(s) = \frac{\sum_{i=0}^{2} b_i(2,1)}{da1(2,1)} \qquad (2\text{-}181)$$

The variance of time to failure with repair of the system is expressed by

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2$$
$$= \frac{2da2(2,1)\sum_{i=0}^{2} b_i(2,1) - 2\cdot l0(2,1)\cdot da1(2,1) - \left[\sum_{i=0}^{2} b_i(2,1)\right]^2}{[da1(2,1)]^2} \qquad (2\text{-}182)$$

Plots of the system reliability with repair are shown in Figure 2-13 (Table 2-11) for same applicable data of the system steady state availability numerical examples. These plots show that R(t) decreases as λ_{h0} increases. Figure 2-14 (Table 2-12) shows the plots of system MTTF with repair. The plots indicate that MTTF decrease with the increasing values of λ_{h0}.

Setting $\mu_1 = \mu_2 = 0$ in Equations (2-180) - (2-182), we can get the system reliability, MTTF and the variance of time to failure without repair, respectively.

Table 2-11 System reliability with repair for (2,1) standby system with constant human error rates

Time	R(t) ($\lambda_{h0} = 0$)	R(t) ($\lambda_{h0} = 0.0002$)	R(t) ($\lambda_{h0} = 0.0004$)
0.00	1.0000	1.0000	1.0000
1000.00	0.9133	0.7709	0.6524
2000.00	0.8000	0.5833	0.4302
3000.00	0.6930	0.4385	0.2839
4000.00	0.5985	0.3290	0.1874
5000.00	0.5166	0.2467	0.1237
6000.00	0.4459	0.1850	0.0816
7000.00	0.3848	0.1387	0.0539
8000.00	0.3320	0.1040	0.0356
9000.00	0.2865	0.0780	0.0235
10000.00	0.2473	0.0585	0.0155

Table 2-12 System MTTF with repair for (2,1) standby system with constant human error rates

λ_{h0}	MTTF ($\lambda_{c0} = 0$)	MTTF ($\lambda_{c0} = 0.00005$)	MTTF ($\lambda_{c0} = 0.0001$)
0.0000	9781.5126	9149.5048	5785.2883
0.0001	5785.2883	5558.2084	4107.2689
0.0002	4107.2689	3991.4958	3183.8074
0.0003	3183.8074	3113.7981	2599.3747
0.0004	2599.3747	2552.5196	2196.2264
0.0005	2196.2264	2162.6844	1901.3394
0.0006	1901.3394	1876.1484	1676.2673
0.0007	1676.2673	1656.6565	1498.8411
0.0008	1498.8411	1483.1426	1355.3796
0.0009	1355.3796	1342.5296	1236.9819
0.0010	1236.9819	1226.2700	1137.6075

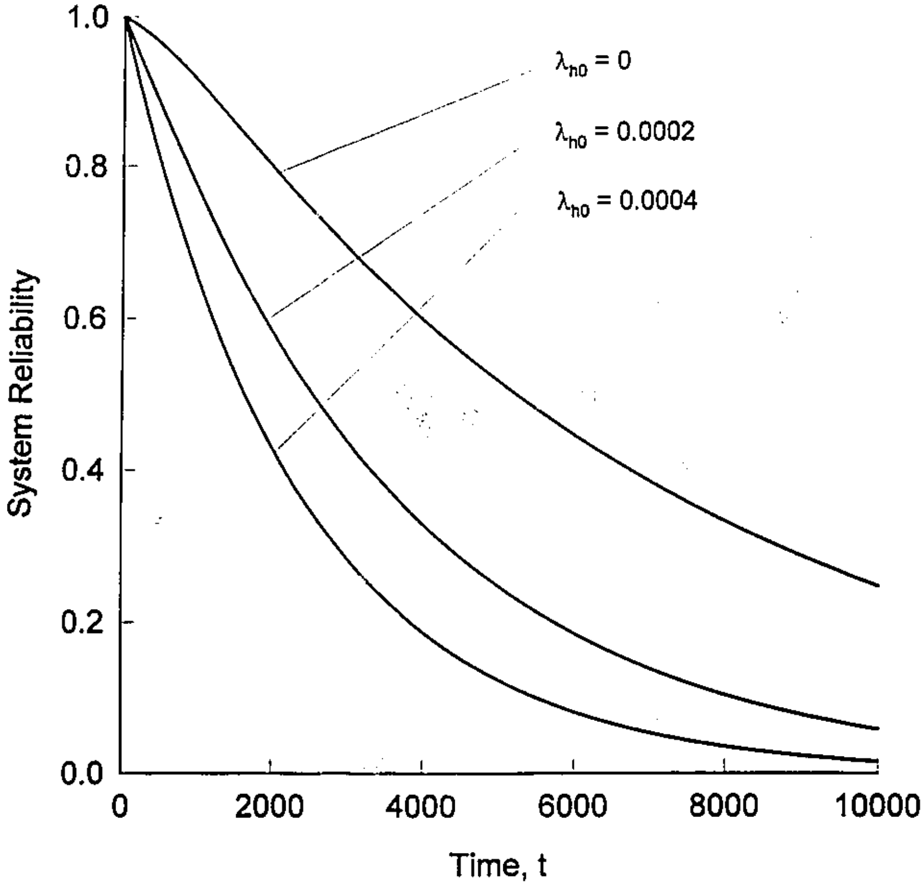


Figure 2-13 System reliability with repair plots for two units active and one on standby system with constant human error rates

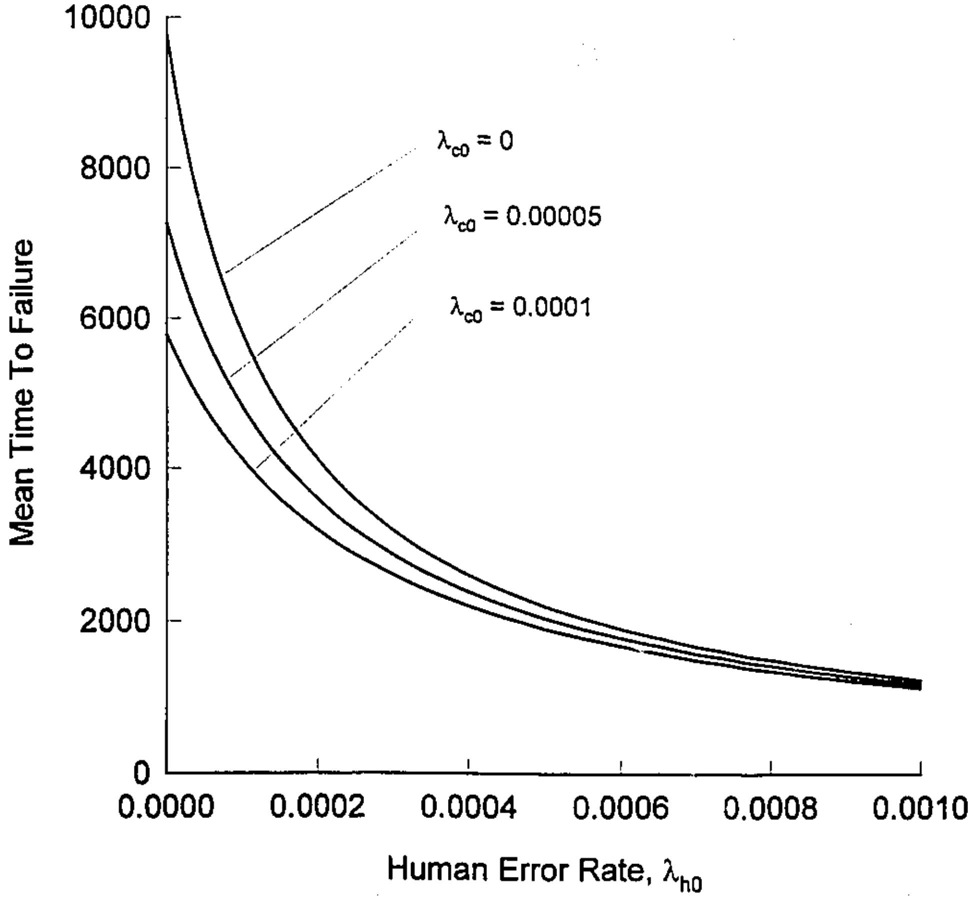


Figure 2-14 System mean time to failure with repair for two units active and one on standby system with constant human error rates

2.2.3. Three Units Active and One Unit on Standby System

For m = 3 and n = 1 in Figure 2-1b, the system of differential equations associated with the model can be written as follows:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=4}^{6} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \tag{2-183}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \tag{2-184}$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) - a_2 P_2(t) + \mu_3 P_3(t) \tag{2-185}$$

$$\frac{dP_3(t)}{dt} = r_2 P_2(t) - a_3 P_3(t) \tag{2-186}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \tag{2-187}$$

(for j = 4, 5, 6)

where

$$a_0 = r_0 + \lambda_{c0} + \lambda_{h0}$$
$$a_1 = r_1 + \lambda_{c1} + \lambda_{h1} + \mu_1$$
$$a_2 = r_2 + \lambda_{c2} + \lambda_{h2} + \mu_2$$
$$a_3 = r_3 + \lambda_{c3} + \lambda_{h3} + \mu_3$$

The associated boundary conditions are as follows:

$$P_4(0,t) = r_3 P_3(t) \tag{2-188}$$

$$P_5(0,t) = \sum_{i=0}^{3} \lambda_{ci} P_i(t) \tag{2-189}$$

$$P_6(0,t)=\sum_{i=0}^{3}\lambda_{hi}P_i(t) \tag{2-190}$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

The steady state probabilities are as follows:

$$P_0=\frac{b_0(3,1)}{BA(3,1)} \tag{2-191}$$

$$P_1=\frac{b_1(3,1)}{BA(3,1)} \tag{2-192}$$

$$P_2=\frac{b_2(3,1)}{BA(3,1)} \tag{2-193}$$

$$P_3=\frac{b_3(3,1)}{BA(3,1)} \tag{2-194}$$

$$P_4=\frac{b_4(3,1)E_4[x]}{BA(3,1)} \tag{2-195}$$

$$P_5=\frac{b_5(3,1)E_5[x]}{BA(3,1)} \tag{2-196}$$

$$P_6=\frac{b_6(3,1)E_6[x]}{BA(3,1)} \tag{2-197}$$

where:

$$b_0(3,1) = a_1a_2a_3 - a_3r_1\mu_2 - a_1r_2\mu_3$$
$$b_1(3,1) = (a_2a_3 - r_2\mu_3)r_0$$
$$b_2(3,1) = a_3r_0r_1$$
$$b_3(3,1) = r_0r_1r_2$$
$$b_4(3,1) = r_0r_1r_2r_3$$
$$b_5(3,1) = \sum_{i=0}^{3} b_i(3,1)\lambda_{ci}$$
$$b_6(3,1) = \sum_{i=0}^{3} b_i(3,1)\lambda_{hi}$$
$$BA(3,1) = \sum_{i=0}^{3} b_i(3,1) + \sum_{J=4}^{6} b_j(3,1)E_j[x]$$

The steady state availability of the system is

$$AV_{ss}(3,1) = P_0 + P_1 + P_2 + P_3$$
$$= \frac{\sum_{i=0}^{3} b_i(3,1)}{BA(3,1)} \qquad (2\text{-}198)$$

Similarly, the steady state unavailability of the system is given by

$$UAV_{ss}(3,1) = P_4 + P_5 + P_6$$
$$= \frac{\sum_{j=4}^{6} b_j(3,1)E_j[x]}{BA(3,1)} \qquad (2\text{-}199)$$

Special Cases of System Steady State Availability

Case I

If the system repair time x is ***Gamma*** distributed, the system steady state availability is given by

$$AV_{ss}(3,1)=\frac{\sum_{i=0}^{3}b_i(3,1)}{\sum_{i=0}^{3}b_i(3,1)+\sum_{j=4}^{6}b_j(3,1)\beta/\mu_j} \qquad (2\text{-}200)$$

If β is an integer, the Gamma distribution becomes ***Erlangian*** distribution. For β = 1, the Gamma distribution reduce to ***exponential*** distribution, and Equation (2-200) is the system steady state availability for the exponential repair time distribution.

Case II

If the system repair time x is ***Weibull*** distributed, the system steady state availability is given by

$$AV_{ss}(3,1)=\frac{\sum_{i=0}^{3}b_i(3,1)}{\sum_{i=0}^{3}b_i(3,1)+\sum_{j=4}^{6}b_j(3,1)\Gamma(1+1/\beta)/\mu_j} \qquad (2\text{-}201)$$

Case III

If the system repair time x is ***Rayleigh*** distributed, the system steady state availability is given by

$$AV_{ss}(3,1)=\frac{\sum_{i=0}^{3}b_i(3,1)}{\sum_{i=0}^{3}b_i(3,1)+\sum_{j=4}^{6}b_j(3,1)\sqrt{\pi/2}/\mu_j} \qquad (2\text{-}202)$$

Case IV

If the system repair time x is ***lognormally*** distributed, the system steady state availability is given by

$$AV_{ss}(3,1)=\frac{\sum_{i=0}^{3}b_i(3,1)}{\sum_{i=0}^{3}b_i(3,1)+\sum_{i=4}^{6}b_i(3,1)\exp\left(\mu_i+\sigma^2/2\right)} \tag{2-203}$$

Time-Dependent Availability Analysis

Using Laplace transform and the initial conditions in Equations (2-183) - (2-190) and solving the resulting equations, we get the following Laplace transform of state probabilities The Laplace Transform of state probabilities for this model is given by:

$$P_0(s)=\frac{b_0(3,1)+c0(3,1)\cdot s+(a_1+a_2+a_3)\cdot s^2+s^3}{CA(3,1)} \tag{2-204}$$

$$P_1(s)=\frac{b_1(3,1)+r_0(a_2+a_3)\cdot s+r_0\cdot s^2}{CA(3,1)} \tag{2-205}$$

$$P_2(s)=\frac{b_2(3,1)+r_0r_1\cdot s}{CA(3,1)} \tag{2-206}$$

$$P_3(s)=\frac{b_3(3,1)}{CA(3,1)} \tag{2-207}$$

$$P_4(s)=\frac{b_4(2,1)[1-N_4(s)]}{s\cdot CA(3,1)} \tag{2-208}$$

$$P_5(s)=\frac{\left[1-N_5(s)\right]\cdot\left[b_5(3,1)+mc(3,1)\cdot s+lc(3,1)\cdot s^2+\lambda_{c0}\cdot s^3\right]}{s\cdot CA(3,1)} \tag{2-209}$$

$$P_6(s)=\frac{\left[1-N_6(s)\right]\cdot\left[b_6(3,1)+mh(3,1)\cdot s+lh(3,1)\cdot s^2+\lambda_{h0}\cdot s^3\right]}{s\cdot CA(3,1)} \tag{2-210}$$

where:

$$CA(3,1) = ca1(3,1) + ca2(3,1)\cdot s + ca3(3,1)\cdot s^2 + ca4(3,1)\cdot s^3 + s^4$$

$$ca1(3,1) = \sum_{j=4}^{6}\left[1 - N_j(s)\right] b_j(3,1)$$

$$ca2(3,1) = \sum_{i=0}^{3} b_i(3,1) + mc(3,1)\left[1 - N_5(s)\right] + mh(3,1)\left[1 - N_6(s)\right]$$

$$ca3(3,1) = m0(3,1) + lc(3,1)\left[1 - N_5(s)\right] + lh(3,1)\left[1 - N_6(s)\right]$$

$$ca4(3,1) = l0(3,1) + \lambda_{c0}\left[1 - N_5(s)\right] + \lambda_{h0}\left[1 - N_6(s)\right]$$

$$l0(3,1) = a_0 + a_2 + a_3 + r_0$$

$$lc(3,1) = (a_1 + a_2 + a_3)\lambda_{c0} + r_0\lambda_{c1}$$

$$lh(3,1) = (a_1 + a_2 + a_3)\lambda_{h0} + r_0\lambda_{h1}$$

$$m0(3,1) = c0(3,1) + r_0(a_2 + a_3) + r_0 r_1$$

$$mc(3,1) = c0(3,1)\lambda_{c0} + r_0(a_2 + a_3)\lambda_{c1} + r_0 r_1\lambda_{c2}$$

$$mh(3,1) = c0(3,1)\lambda_{h0} + r_0(a_2 + a_3)\lambda_{h1} + r_0 r_1\lambda_{h2}$$

$$c0(3,1) = a_1a_2 + a_1a_3 + a_2a_3 - r_1\mu_2 - r_2\mu_3$$

The Laplace transform of the system availability is :

$$AV(s) = \sum_{i=0}^{3} P_i(s) = \frac{\sum_{i=0}^{3} b_i(3,1) + m0(3,1)\cdot s + l0(3,1)\cdot s^2 + s^3}{CA(3,1)} \qquad (2\text{-}211)$$

Substituting the Laplace transform of pdf of system repair distribution, $N_4(s)$, $N_5(s)$, and $N_6(s)$, in the above Equation, and taking the inverse Laplace transform of the resulting equations, we can get the time-dependent system availability.

Reliability and MTTF With and Without Repair

Setting $\mu_4(x) = \mu_5(x) = \mu_6(x) = 0$ in the model, using Laplace transforms and solving the resulting set of equations, we can get the Laplace transforms of the state probabilities. Thus, the Laplace transform of the system reliability with repair is

$$R(s) = P_0(s) + P_1(s) + P_2(s) + P_3(s)$$
$$= \frac{\sum_{i=0}^{3} b_i(3,1) + m0(3,1) \cdot s + l0(3,1) \cdot s^2 + s^3}{DA(3,1)} \tag{2-212}$$

where

$$DA(3,1) = da1(3,1) + da2(3,1) \cdot s + da3(3,1) \cdot s^2 + da4(3,1) \cdot s^3 + s^4$$
$$da1(3,1) = a_0 b_0(3,1) - \mu_1 b_1(3,1)$$
$$da2(3,1) = a_0 \cdot c0(3,1) + b_0(3,1) - r_0 \mu_1 (a_2 + a_3)$$
$$da3(3,1) = a_0(a_1 + a_2 + a_3) - r_0 \mu_1 + c0(3,1)$$
$$da4(3,1) = a_0 + a_1 + a_2 + a_3$$

The system reliability with repair can be obtained by inverting Equation (2-212).

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s \to 0} R(s) = \frac{\sum_{i=0}^{3} b_i(3,1)}{da1(3,1)} \tag{2-213}$$

The variance of time to failure with repair of the system is expressed by

$$\sigma^2 = -2 \lim_{s \to 0} R'(s) - (MTTF)^2$$
$$= \frac{2 \cdot da2(3,1) \sum_{i=0}^{3} b_i(3,1) - 2 \cdot m0(3,1) \cdot da1(3,1) - \left[\sum_{i=0}^{3} b_i(3,1) \right]^2}{[da1(3,1)]^2} \tag{2-214}$$

When $\mu_1 = \mu_2 = \mu_3 = 0$ in the above Equations, we can get the system reliability, MTTF and the variance of time to failure of the system without repair, respectively.

2.3 K-out-of-n System

This is another type of redundancy. The system consists of n identical units, k of which must be operative to ensure that the system functions normally. It is used where a specified number of units must be good for the system success. The series and parallel configuration are special cases of this configuration, that is, k = n and k = 1, respectively. Assuming ideal failure detection and switching, the reliability block diagram can be represented as in figure 2-15a.

The state space diagram for k-out-of-n system with common-cause failures and human errors is shown in Figure 2-15b. The numerals plus a letter in the boxes of the figure denote the system state numbers. This system has n units in parallel, at least k of which must function normally to ensure the overall system success. In state i, there are i units failed and (n-i) units are in working condition. Furthermore, the occurrence of a common-cause failure or a critical human error can cause the total system to fail. A common-cause failure or a critical human error can occur when the system is in any one of its operating states. In addition to the partially failed system being repaired to its previous state, the completely failed system is also repaired back to state 0.

Assumptions

The following assumptions are associated with the model:

1. The system has n identical and active units
2. All failures including human errors, common-cause and hardware failures are statistically independent.
3. Human error, common-cause and other failure rates are constant.
4. As long as at least k units are functional, the system is operable
5. The repair rates of the failed units are constant.

6. The failed system repair times are arbitrarily distributed
7. A common-cause failure or a human error can occur and trigger system failure from any of its operable states.
8. Repair is unrestricted for both units and system.
9. The repaired unit or system is as good as new.

More General Consideration on K-out-of-n Configuration

The models of all k-out-of-n configurations in this study include the following situations:

- At time t=0, k units start working, n-k units in reserve state.

  $$r_i = k\lambda + (n-k-i)\lambda_s$$

- At time t=0, all n units start working simultaneously.

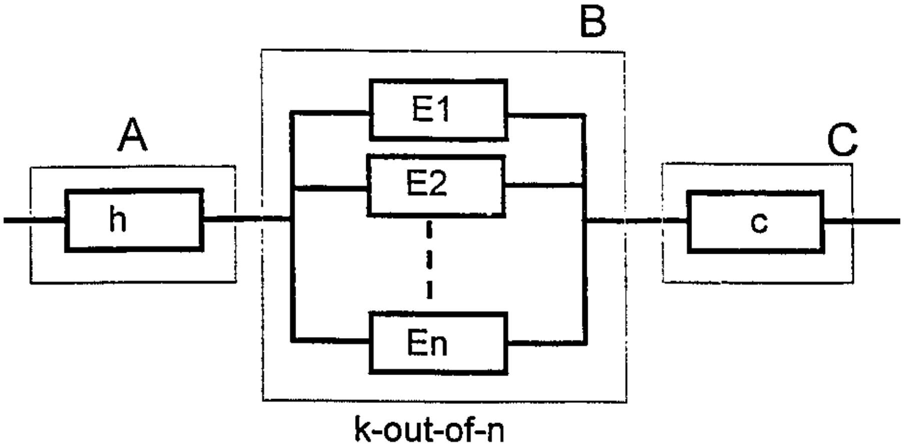
  $$r_i = (n-i)\lambda$$

- Repairable system

  At least one of the system repair rates, μ_1, μ_2, ..., $\mu_{n\text{-}k}$, is non-zero value

- Non-repairable system

$$\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_{n-k} = 0$$



A: critical human error
B: k-out-of-n
C: common-cause failure

Figure 2-15a The block diagram of a k-out-of-n redundancy with common-cause failure and human error

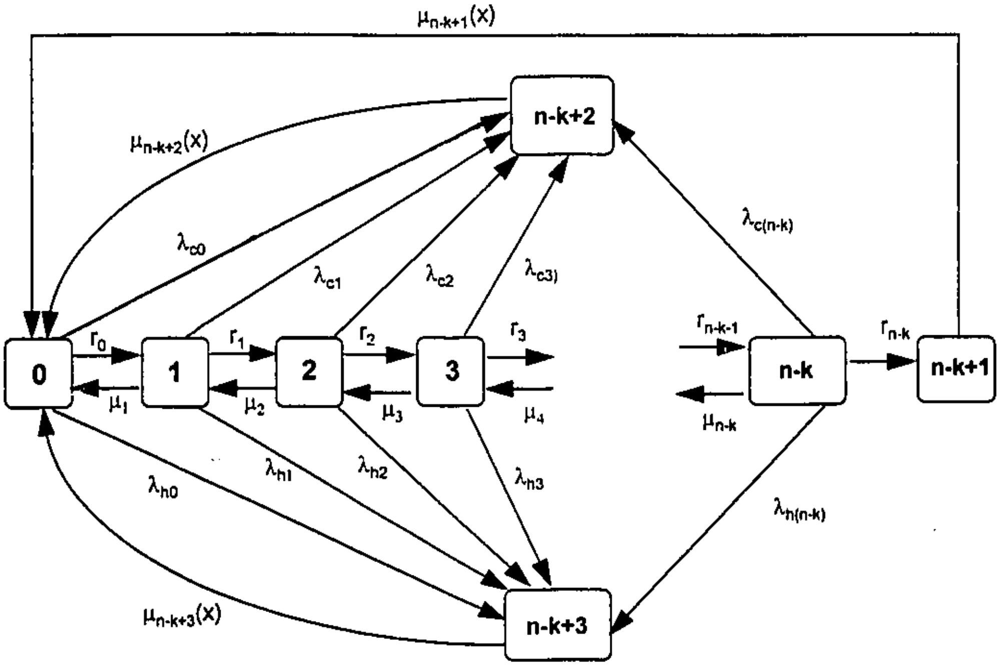


Figure 2-15b The state transition diagram of k-out-of-n redundancy with constant human error rates and arbitrary system repair rates

The corresponding differential equations for the model described in Figure 2-15b are:

$$\frac{dP_0(t)}{dt}=-a_0P_0(t)+\mu_1P_1(t)+\sum_{j=n-k+1}^{n-k+3}\int_0^{\infty}P_j(x,t)\mu_j(x)dx \tag{2-215}$$

$$\frac{dP_1(t)}{dt}=r_0P_0(t)-(r_1+\lambda_{h1}+\lambda_{c1}+\mu_1)P_1(t)+\mu_2P_2(t) \tag{2-216}$$

$$\frac{dP_2(t)}{dt}=r_1P_1(t)-(r_2+\lambda_{h2}+\lambda_{c2}+\mu_2)P_2(t)+\mu_3P_3(t) \tag{2-217}$$

$$\frac{dP_i(t)}{dt}=r_{i-1}P_{i-1}(t)-a_iP_i(t)+\mu_{i+1}P_{i+1}(t) \tag{2-218}$$

(for i = 0, 1, 2, 3, ..., n-k-1)

$$\frac{dP_{n-k}(t)}{dt}=r_{n-k-1}P_{n-k-1}(t)-a_{n-k}P_{n-k}(t) \tag{2-219}$$

$$\frac{\partial P_j(x,t)}{\partial t}+\frac{\partial P_j(x,t)}{\partial x}=-\mu_j(x)P_j(x,t) \tag{2-220}$$

(for j = n-k+1, n-k+2, n-k+3)

The associated boundary conditions are as follows:

$$P_{n-k+1}(0,t)=r_{n-k}P_{n-k}(t) \tag{2-221}$$

$$P_{n-k+2}(0,t)=\sum_{i=0}^{n-k}\lambda_{ci}P_i(t) \tag{2-222}$$

$$P_{n-k+3}(0,t)=\sum_{i=0}^{n-k}\lambda_{hi}P_i(t) \tag{2-223}$$

where

$$r_i=k\lambda+(n-k-i)\lambda_s \tag{2-224}$$

$$a_i=r_i+\lambda_{hi}+\lambda_{ci}+\mu_i \tag{2-225}$$

(for i = 0, 1, 2, ..., n-k)

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

Comparing the state space diagram for k-out-of-n with the state space diagram for standby system (Fig. 2-1b), we can see that the two systems are similar, but the division of system working or failure states is different. The general analysis of k-out-of-n is the same as that of standby system. We consider some special cases of k-out-of-n system to present the impact of human errors, common-cause failures, and failed system repair policy on system reliability, availability and mean time to failure.

2.3.1 K-out-of-n (n-k=1) System

The investigation is similar to that of the model of one unit active and one on standby. As an example, let's first consider a 1-out-of-2 active redundancy system.

For 1-out-of-2 system

$$r_0 = \lambda + \lambda_s$$
$$r_1 = \lambda$$

Substituting r_0 and r_1 in Equation (2-76) and in the symbols $b_0(1,1)$ - $b_4(1,1)$, we have the general expression of steady state availability for 1-out-of-2 system:

$$\begin{aligned} AV_{ss}(1/2) = & \left[2\lambda + \lambda_{c1} + \lambda_{h1} + \lambda_s + \mu_1\right] / \left[2\lambda + \lambda_{c1} + \lambda_{h1} + \mu_1 + \lambda(\lambda + \lambda_s)E_2[x]\right. \\ & + \left(\lambda_{c1}(\lambda + \lambda_s) + \lambda_{c0}(\lambda + \lambda_{c1} + \lambda_{h1} + \mu_1)\right)E_3[x] \\ & \left. + \left(\lambda_{h1}(\lambda + \lambda_s) + \lambda_{h0}(\lambda + \lambda_{c1} + \lambda_{h1} + \mu_1)\right)E_4[x]\right] \end{aligned} \tag{2-226}$$

Substituting the expressions of mean time to repair $E_j(x)$ for j=2, 3, 4, Equations (2-79), (2-84), (2-88) and (2-91) into Equation (2-226), we get the expressions of system steady state availability for Gamma, Weibull, Rayleigh and lognormal system repair time distributions, respectively.

The general expression of the Laplace transform of the system availability, for 1-out-of-2, is

$$\begin{aligned} AV\,(s,1/2) = {} & \left[s+2\lambda+\lambda_{c1}+\lambda_{h1}+\lambda_s+\mu_1\right]/\left[\ \lambda(\lambda+\lambda_s)(1-N_2(s))\right. \\ & +\left(\lambda_{c1}(\lambda+\lambda_s)+\lambda_{c0}(\lambda+\lambda_{c1}+\lambda_{h1}+\mu_1)\right)(1-N_3(s)) \\ & +\left(\lambda_{h1}(\lambda+\lambda_s)+\lambda_{h0}(\lambda+\lambda_{c1}+\lambda_{h1}+\mu_1)\right)(1-N_4(s)) \\ & +[2\lambda+\lambda_{c1}+\lambda_{h1}+\lambda_s+\mu_1+\lambda_{c0}(1-N_3(s)) \\ & \left.+\lambda_{h0}(1-N_4(s))]\cdot s+s^2\right] \end{aligned} \qquad (2\text{-}227)$$

Substituting the Laplace transform of the pdf of system repair times, $N_j(s)$ for j=2, 3, 4, in Equation (2-227) and then taking the inverse Laplace transform of the resulting equation, we can obtain the time-dependent system availability.

Setting $\mu_2(x) = \mu_3(x) = \mu_4(x) = 0$ in the model, the general expression of the Laplace transform of the system reliability with repair is the same as Equation (2-115). Substituting r_0, r_1, a_0, and a_1 in the Equation, we have the Laplace transform reliability for 1-out-of-2 system:

$$\begin{aligned} R\,(s,1/2) = {} & \left[s+2\lambda+\lambda_{c1}+\lambda_{h1}+\lambda_s+\mu_1\right]/\left[(\lambda+\lambda_s+\lambda_{c0}+\lambda_{h0})\bullet\right. \\ & (\lambda+\lambda_{c1}+\lambda_{h1}+\mu_1)-(\lambda+\lambda_s)\mu_1+ \\ & \left.(2\lambda+\lambda_{c0}+\lambda_{c1}+\lambda_{h0}+\lambda_{h1}+\lambda_s+\mu_1)\cdot s+s^2\right] \end{aligned} \qquad (2\text{-}228)$$

The system reliability with repair can be obtained by inverting Equation (2-228)

$$R(t,1/2) = \frac{e^{f_1 t}(f_1+f_2)+(f_1-f_2)}{2f_1\cdot\exp\left[(f_1+da2(1,1))t/2\right]} \qquad (2\text{-}229)$$

where

$$f_1 = \left[-4\cdot da1(1,1)+\left(da2(1,1)\right)^2\right]^{1/2}$$

$$f_2 = 2\sum_{i=0}^{1} b_i(1,1) - da2(1,1)$$

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \frac{2\lambda + \lambda_s + \lambda_{c1} + \lambda_{h1} + \mu_1}{(\lambda + \lambda_s + \lambda_{c0} + \lambda_{h0})(\lambda + \lambda_{c1} + \lambda_{h1} + \mu_1) - (\lambda + \lambda_s)\mu_1} \tag{2-230}$$

When $\mu_1=0$ in Equations (2-229) and (2-230), we get the system reliability and MTTF without repair, respectively.

For the special case 2-out-of-3 system

$$r_0 = 2\lambda + \lambda_s$$
$$r_1 = 2\lambda$$

Substituting 2λ instead of λ into the equations for 1-out-of-2, we can get the results for the system of 2-out-of-3.

For the special case 3-out-of-4 system

$$r_0 = 3\lambda + \lambda_s$$
$$r_1 = 3\lambda$$

Substituting 3λ instead of λ into the equations for 1-out-of-2, we can get the results for the system of 3-out-of-4, and so on.

Numerical examples

Setting

$\lambda = 0.0002 / hr$	$\lambda_s = 0.00002 / hr$	$\mu_1 = 0.001 / hr$
$\lambda_{c0} = 0.00005 / hr$	$\lambda_{c1} = 0.00005 / hr$	$\lambda_{h1} = 0.0001 / hr$

in Equation (2-226), for Gamma, exponential, Weibull and Rayleigh distributed failed system repair times, the plots of system steady state availability as a function of human error rate, λ_{h0}, are shown in Figures 2-16, 2-18, 2-20 and 2-22, respectively, for specified values of parameters $\mu_2 = 0.001 / hr, \mu_3 = \mu_4 = 0.0009 / hr$, if applicable.

Similarly, setting λ_{h0}=0.0002/hr and specified values of parameters $\mu_2 = 0.001 / hr, \mu_3 = \mu_4 = 0.0009 / hr$, if applicable, for Gamma, exponential and Weibull distributed system repair times, Figures 2-17, 2-19 and 2-21 show the effect of increasing the human error repair distribution parameter, μ_4, for k-out-of-n (n-k=1) system steady state availability, respectively.

Figure 2-23 provides the results of the system steady state availability as a function of human error rate, λ_{h0}, for lognormally distributed failed system repair times $(\mu_2 = \mu_3 = \mu_4 = 1.0009 / hr, \quad \sigma_2 = \sigma_3 = \sigma_4 = \sigma)$.

These plots exhibit that the system steady state availability decreases with increasing values of human error rate, λ_{h0}. And AV_{ss} increases with the increasing values of μ_5. Furthermore, the system steady state availability decreases as the value of k, the least number of units needed for system operating, increases.

Substituting the Laplace transforms of the pdf of the system repair times $N_2(s)$, $N_3(s)$ and $N_4(s)$ in Equation (2-108) and then substituting r_0, r_1, and the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting $\mu_2 = 0.001 / hr, \mu_3 = \mu_4 = 0.0009 / hr$ and taking inverse Laplace transform), we get the system time-dependent availability. Figure 2-24 and 2-25 show the time-dependent system availability plots for k-out-of-n (n-k=1) system with Gamma failed system repair time distribution for human error rates λ_{h0}=0 and λ_{h0}=0.0002/hr, respectively. For exponential failed system repair time distribution, the plots of the time-dependent system availability for varying human error rate, λ_{h0}, are shown in Figure 2-26.

The plots of the system reliability with repair when λ_{h0}=0 and λ_{h0}=0.0002/hr are shown in Figures 2-27 and 2-28, respectively, for same applicable data of the system steady state availability numerical examples. Figure 2-29 shows the plots of system mean time to failure with repair for k-out-of-n (n-k=1) system.

The plots shows that an increase in the critical human error rate λ_{h0} invariably reduces the time-dependent system availability, the system reliability and the system mean time to failure. A decrease in the value of k, the least number of units needed for system operation improves the system performance characteristics.

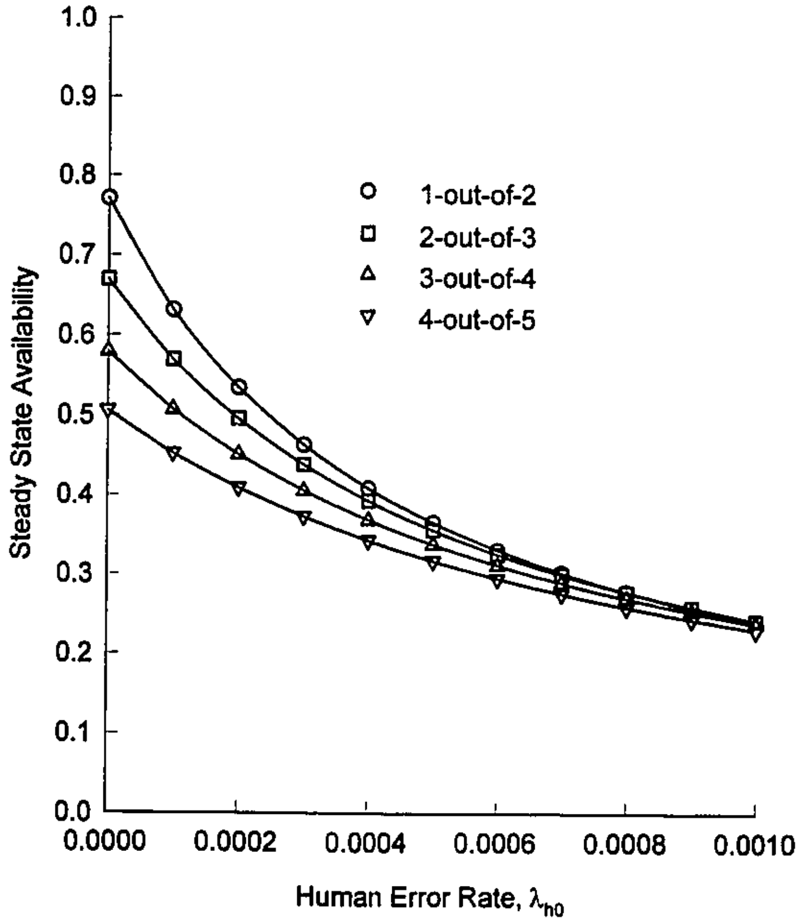


Figure 2-16 System steady state availability vs. λ_{h0} plots for k-ou-of-n (n-k = 1) system with constant human error rates and the Gamma ($\beta = 3$) distributed system repair times

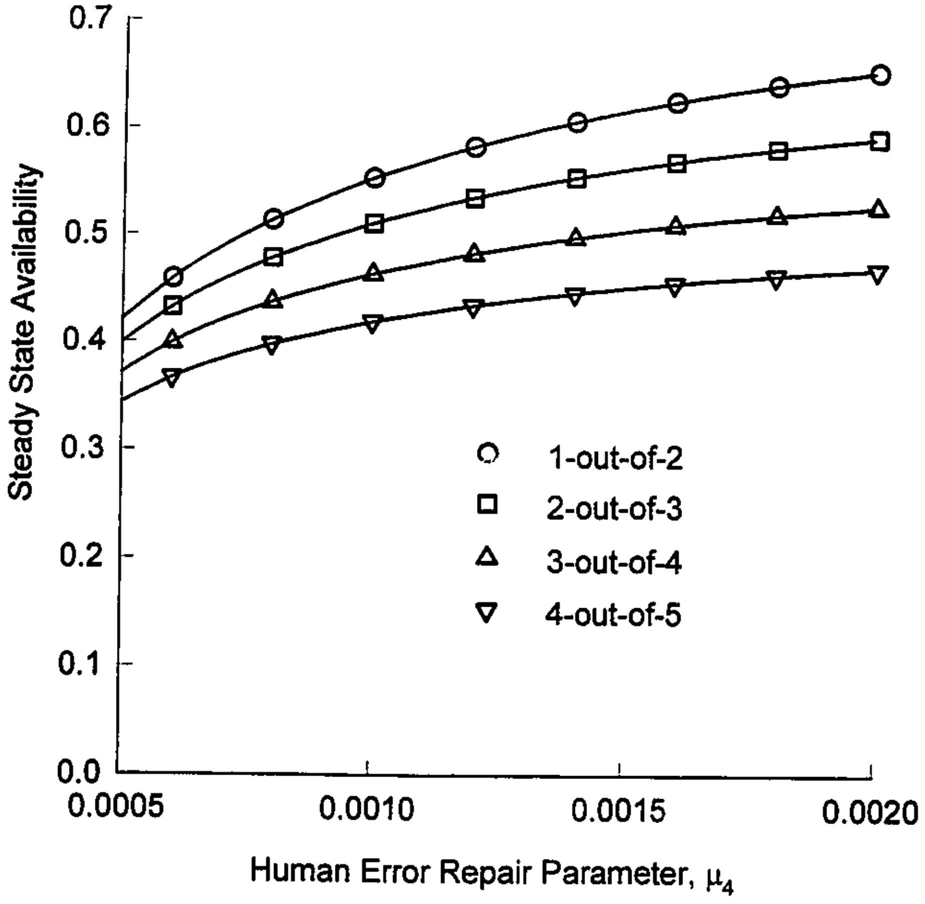


Figure 2-17 System steady state availability vs. μ_4 plots for k-out-of-n (n-k = 1) system with constant human error rates and the Gamma (β=3) distributed repair times

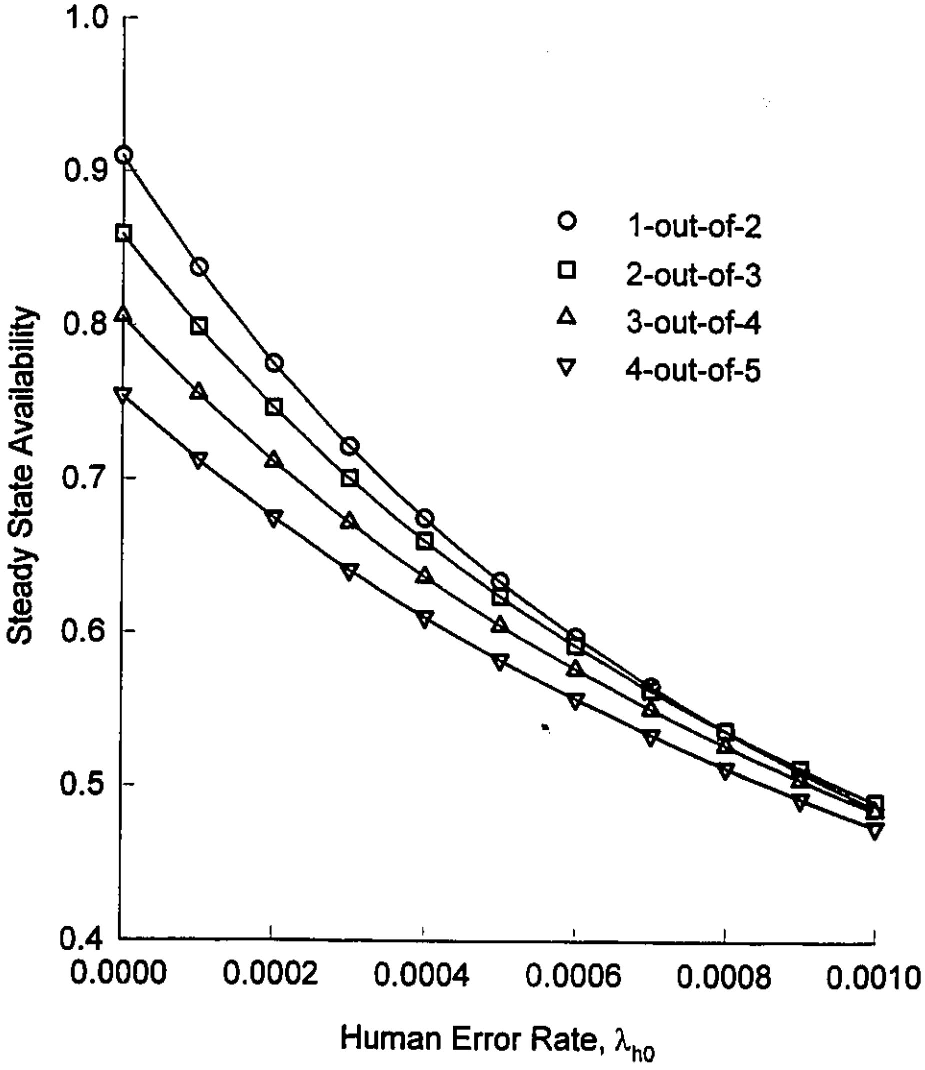


Figure 2-18 System steady state availability vs. λ_{h0} plots for k-out-of-n (n-k = 1) system with constant human error rates and system repair rates

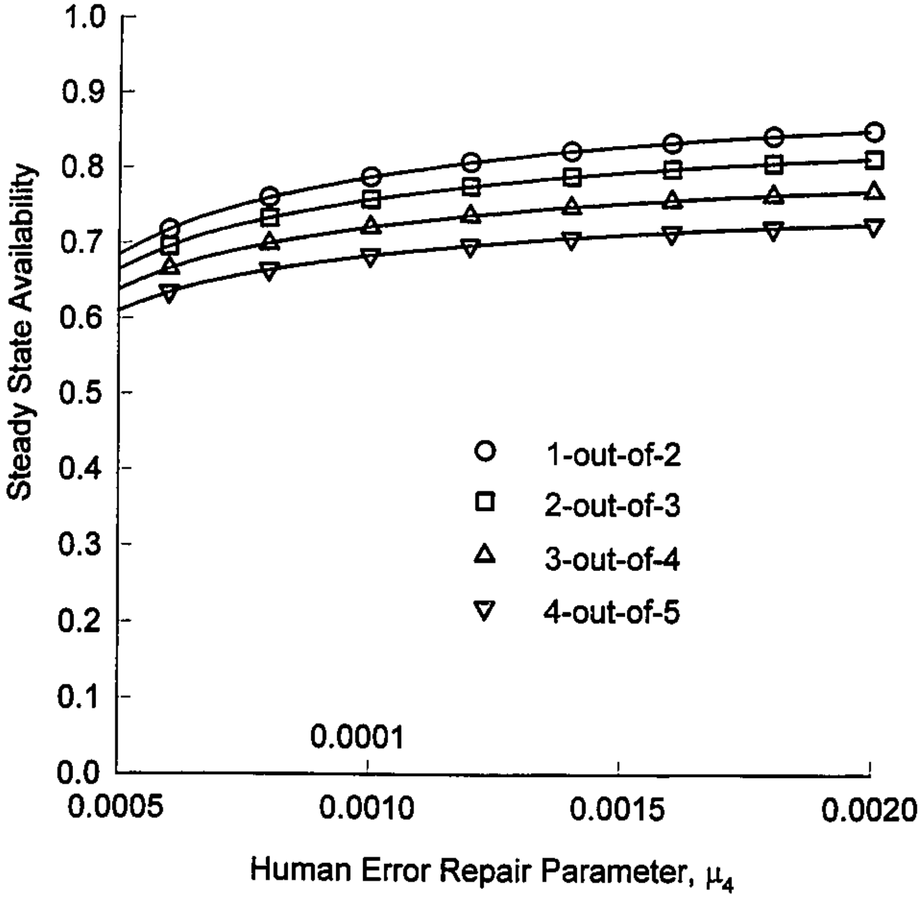


Figure 2-19 System steady state availability vs. μ_4 plots for k-out-of-n (n-k = 1) system with constant human error rates and system repair rates

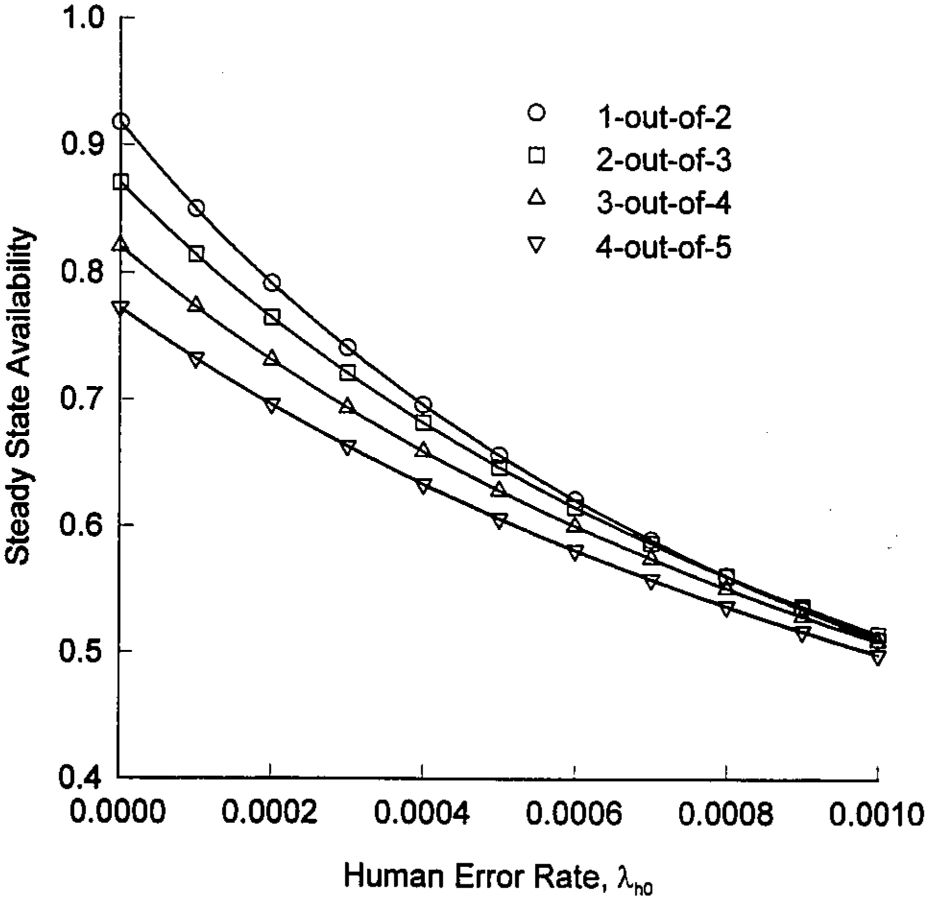


Figure 2-20 AV_{ss} vs. λ_{h0} plots for k-out-of-n (n-k = 1) system with constant human error rates and the Weibull ($\beta = 3$) distributed system repair times

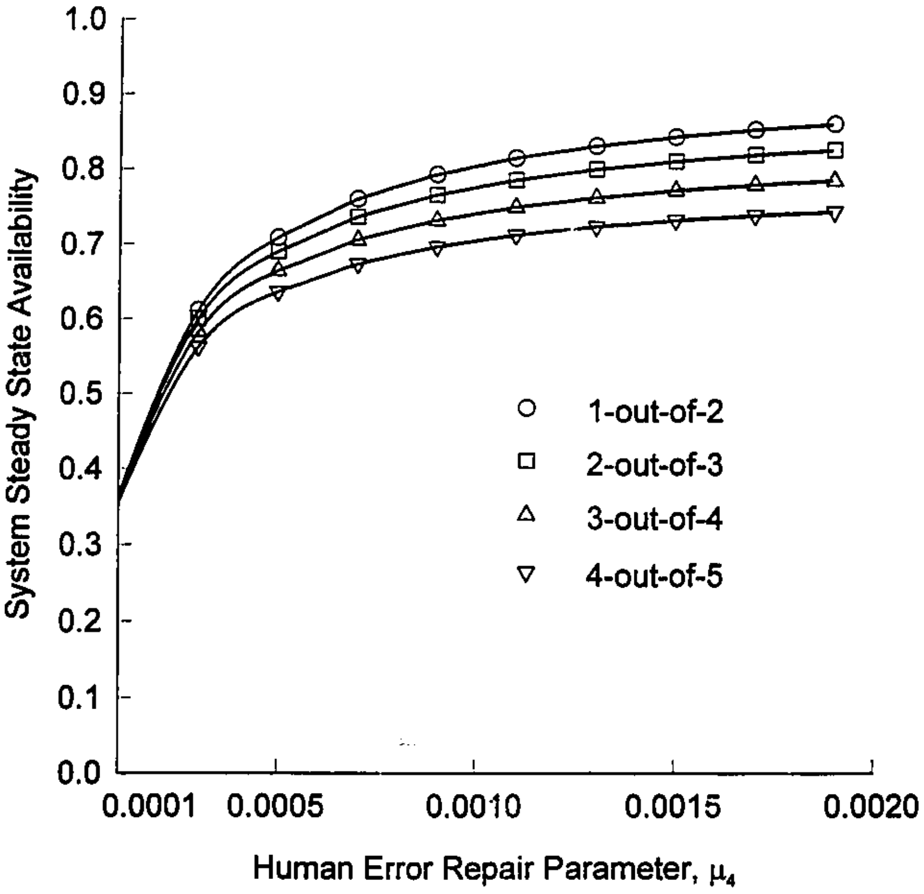


Figure 2-21 AV_{ss} vs. μ_4 plots for k-out-of-n (n-k=1) system with constant human error rates and the Weibull ($\beta = 3$) distributed system repair times

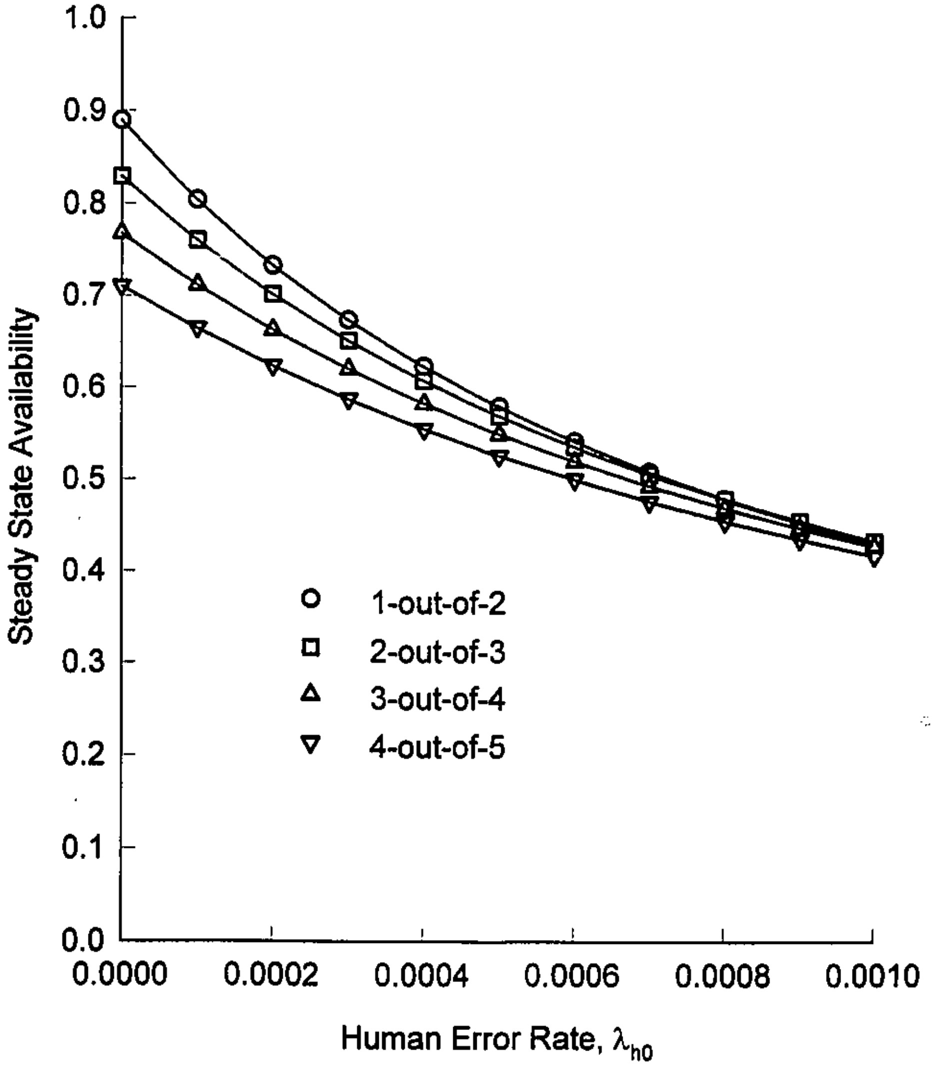


Figure 2-22 AV_{ss} vs. λ_{h0} plots for k-out-of-n (n-k = 1) system with constant human error rate and the Rayleigh distributed failed system repair times

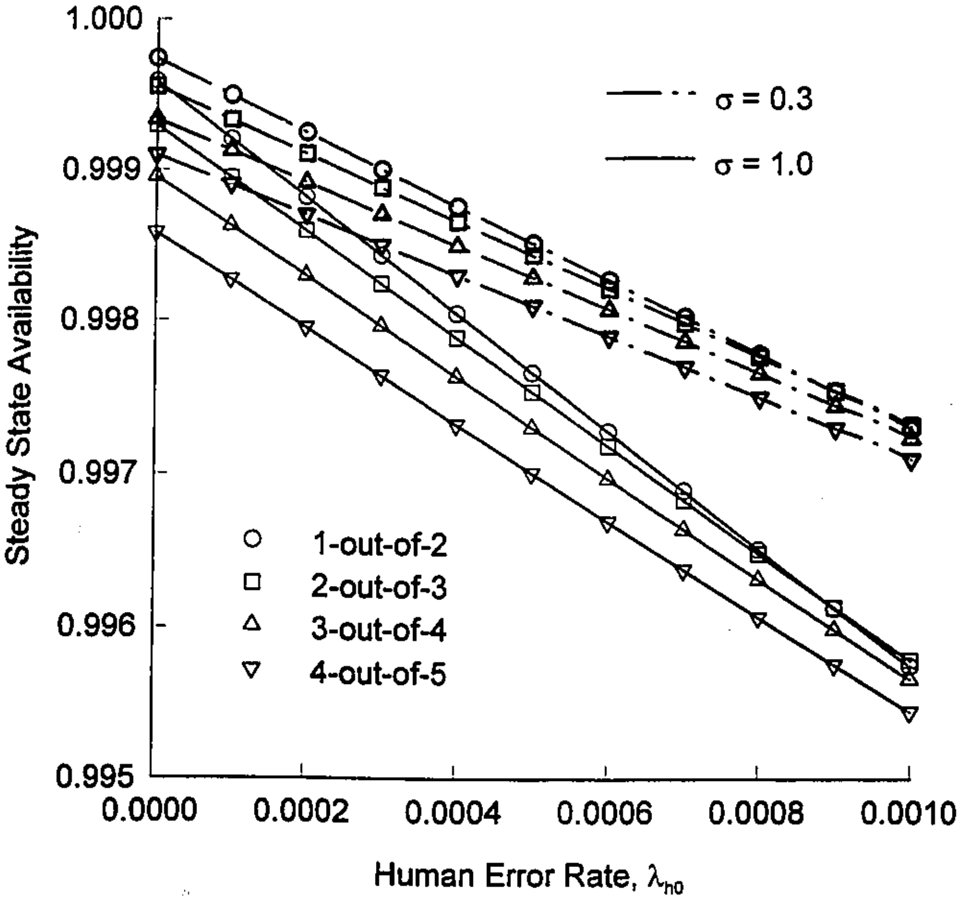


Figure 2-23 AV_{ss} vs. λ_{h0} plots for k-out-of-n (n-k = 1) system with constant human error rates and the lognormally distributed failed system repair times

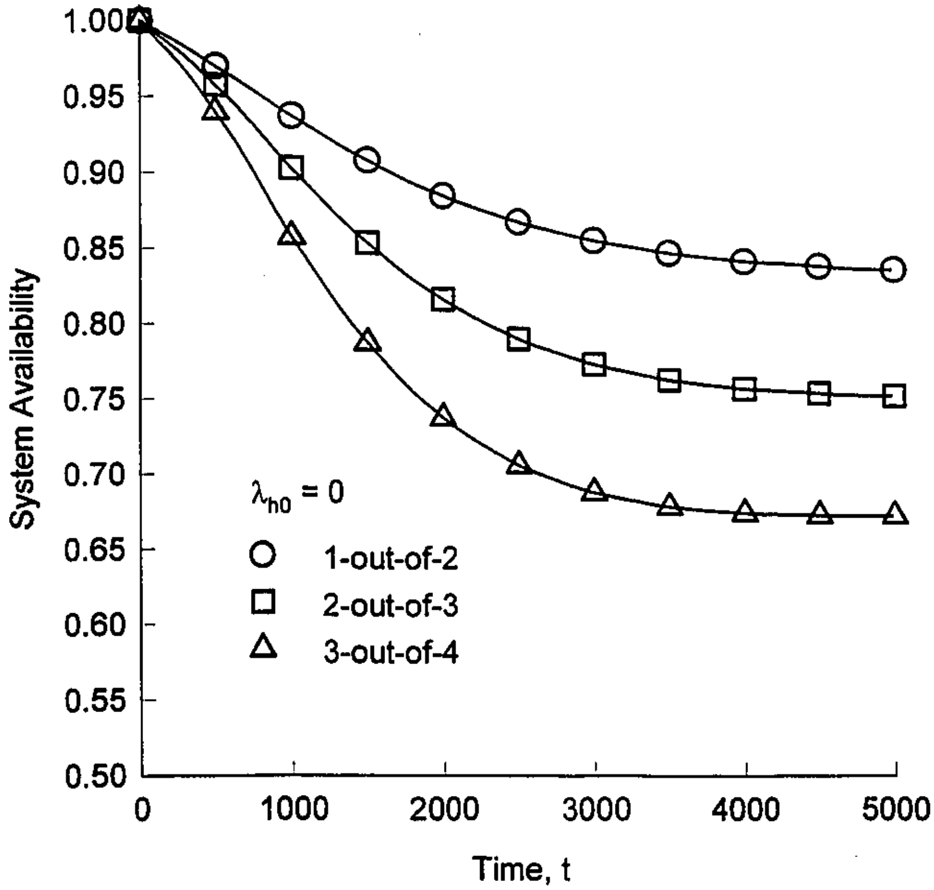


Figure 2-24 AV(t) plots for k-out-of-n (n-k=1) system with constant human error rates and the Gamma (β=2) distributed system repair times

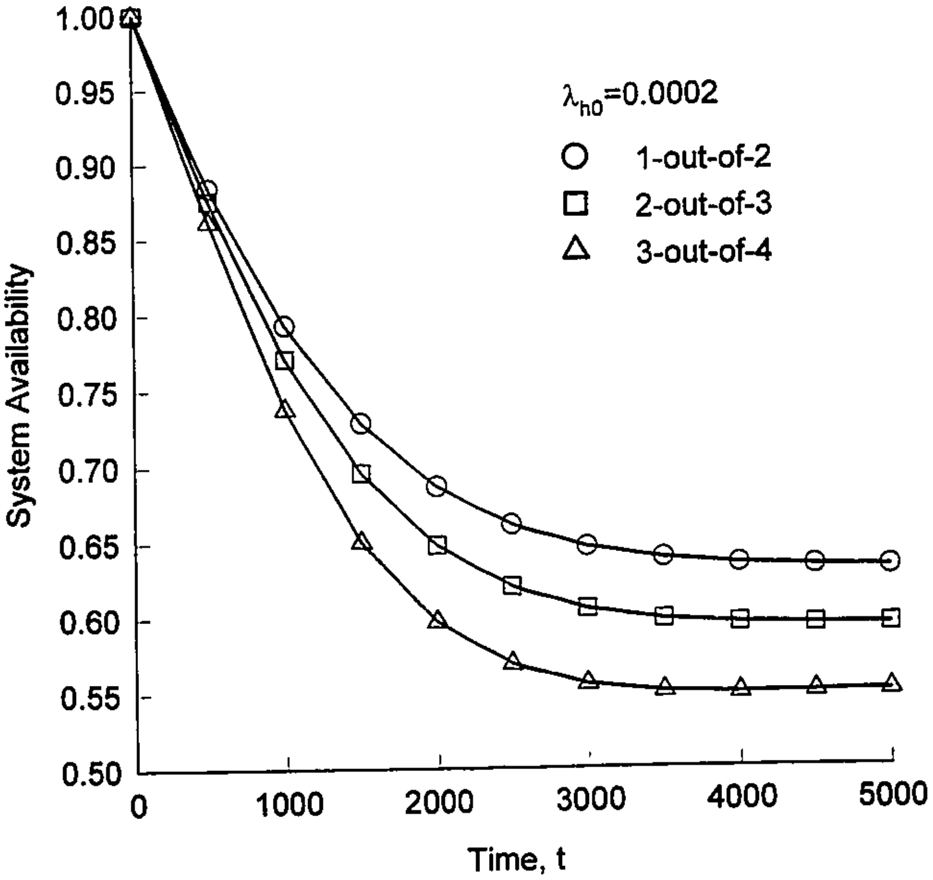


Figure 2-25 AV(t) plots for k-out-of-n (n-k=1) system with constant human error rates and the Gamma (β=2) distributed system repair times

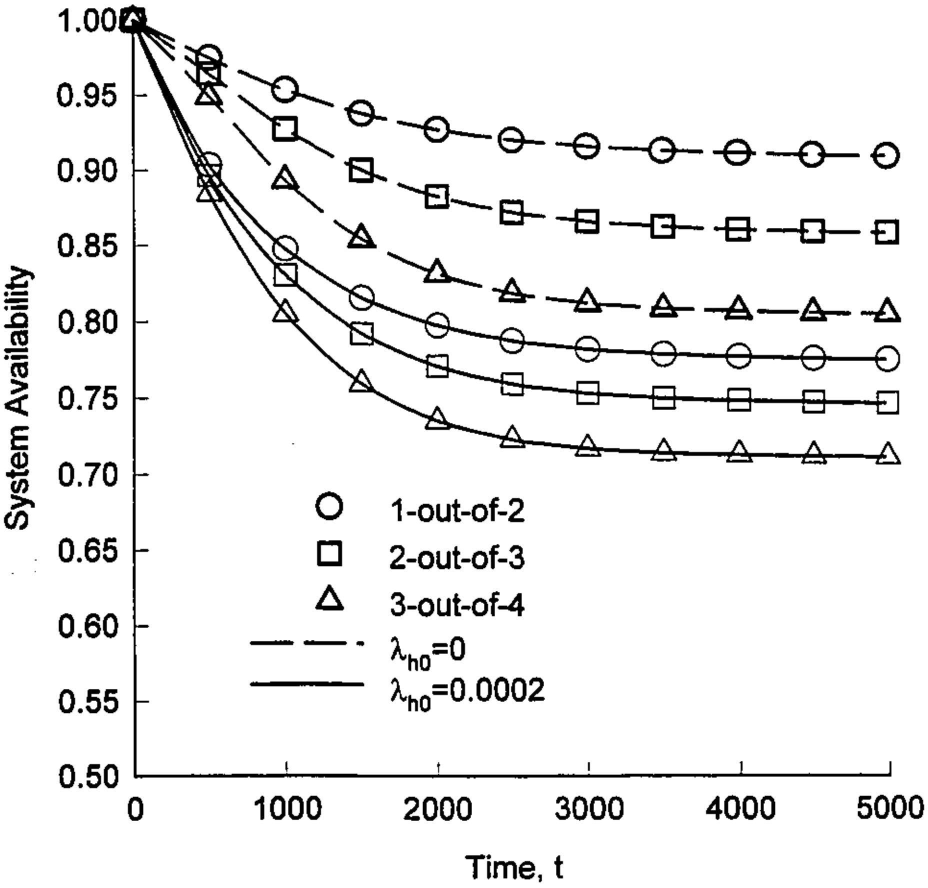


Figure 2-26 Time-dependent system availability plots for k-out-of-n (n-k=1) system with constant human error rates and system repair rates

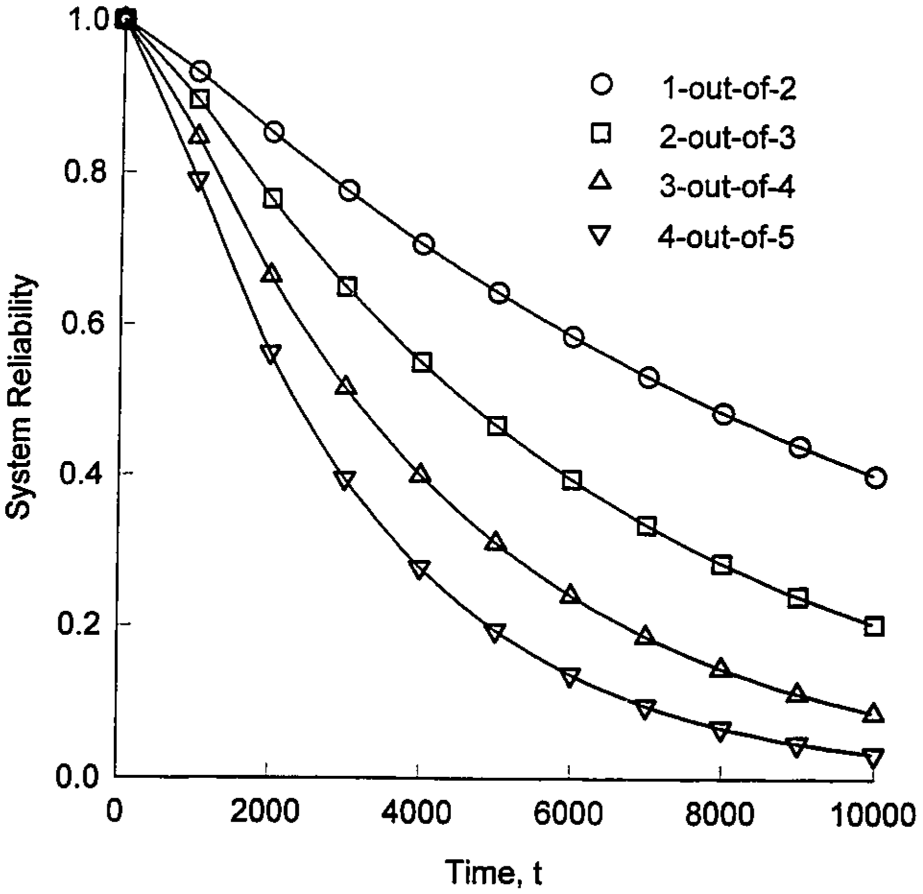


Figure 2-27 System reliability with repair plots for k-out-of-n (n-k=1) system with constant human error rates ($\lambda_{h0} = 0$)

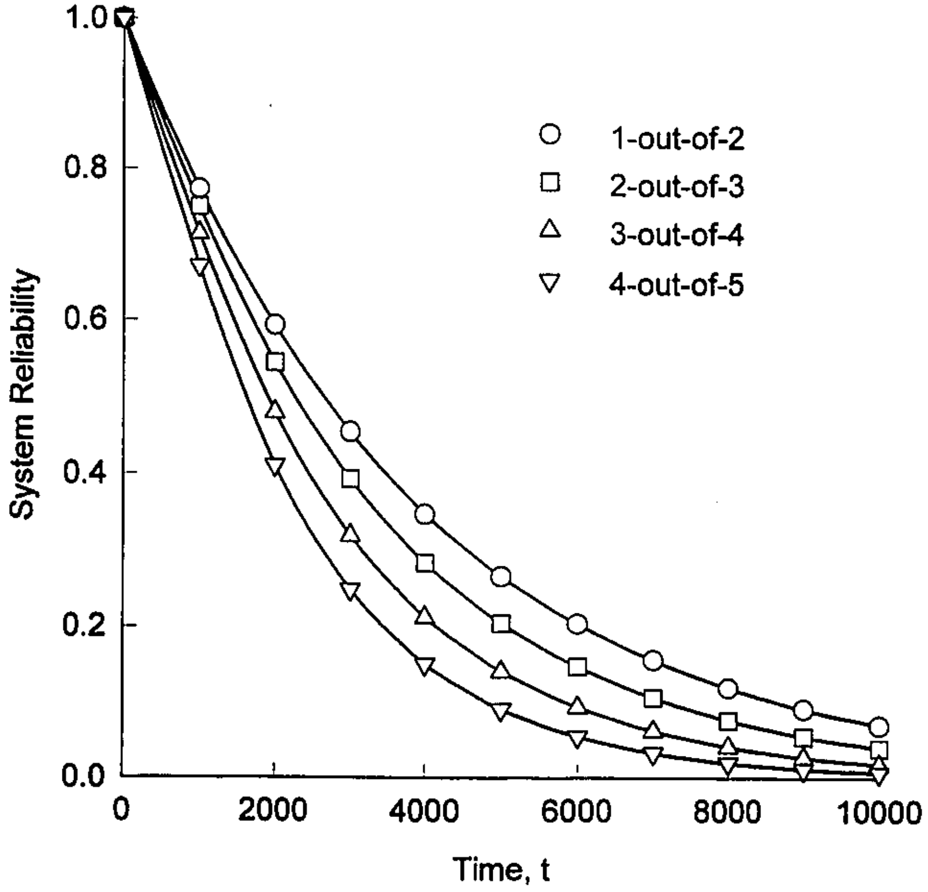


Figure 2-28 System reliability with repair plots for k-out-of-n (n-k=1) system with constant human error rates ($\lambda_{h0} = 0.0002$)

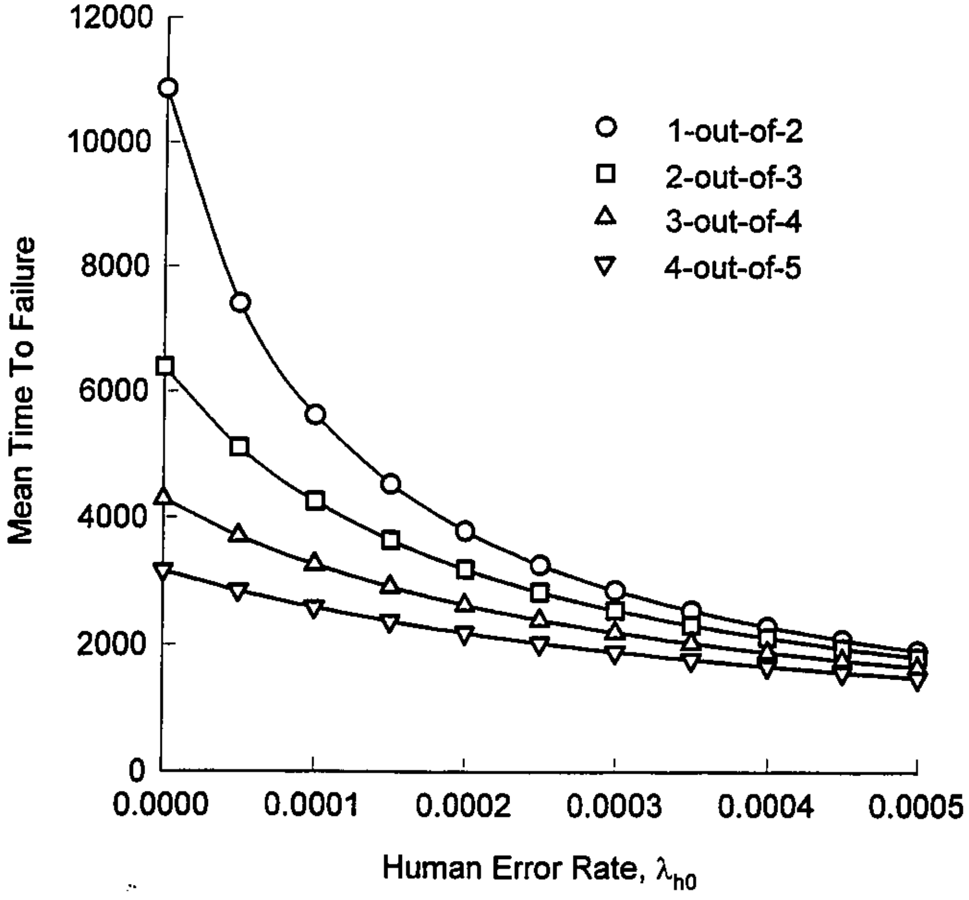


Figure 2-29 System mean time to failure with repair for k-out-of-n (n-k=1) system with constant human error rates

2.3.2 K-out-of-n (n-k=2) System

For 1-out-of-3 system

$$r_0 = \lambda + 2\lambda_s$$
$$r_1 = \lambda + \lambda_s$$
$$r_2 = \lambda$$

For 2-out-of-4 system

$$r_0 = 2\lambda + 2\lambda_s$$
$$r_1 = 2\lambda + \lambda_s$$
$$r_2 = 2\lambda$$

For 3-out-of-5 system

$$r_0 = 3\lambda + 2\lambda_s$$
$$r_1 = 3\lambda + \lambda_s$$
$$r_2 = 3\lambda$$

and so on.

The investigation and the results for k-out-of-n (n-k=2) system are similar to that of the model of two units active and one on standby system. The general expression of the steady state availability, for n-k=2, is the same as Equation (2-148).

Substituting r_0, r_1 and r_2 in Equations (2-150)-(2-153), we have the expressions for steady state availability of k-out-of-n (n-k=2) systems when the failed system repair time distributions are Gamma, Weibull, Rayleigh and lognormal, respectively.

Setting

$\lambda = 0.0002 / hr$ $\quad$ $\lambda_s = 0.00002 / hr$ $\quad$ $\mu_1 = 0.001 / hr$

$\lambda_{c0} = 0.00005 / hr$ $\quad$ $\lambda_{c1} = 0.00005 / hr$ $\quad$ $\lambda_{c2} = 0.00005 / hr$

$\mu_2 = 0.001 / hr$ $\quad$ $\lambda_{h1} = 0.0001 / hr$ $\quad$ $\lambda_{h2} = 0.0001 / hr$

in Equations (2-150)-(2-153), the steady state availabilities as a function of human error rate, λ_{h0}, are shown in Figures 2-30, 2-32, 2-34 and 2-36, for Gamma, exponential, Weibull and Rayleigh distributed failed system repair times, respectively, for specified values of failed system repair time distribution parameters $\mu_3 = 0.001 / hr, \mu_4 = \mu_5 = 0.0009 / hr$, if applicable.

Similarly, setting λ_{h0}=0.0002/hr and specified values of parameters μ_3=0.001/hr and μ_4=0.0009/hr for Gamma, exponential and Weibull distributed failed system repair times, if applicable, Figures 2-31, 2-33 and 2-35 show the effect of increasing the parameter of human error repair time distribution, μ_5, on the system steady state availability, respectively.

Figure 2-37 provides the results of the system steady state availability as a function of human error rate, λ_{h0}, for lognormally distributed failed system repair times ($\mu_3 = \mu_4 = \mu_5 = 1.0009 / hr, \quad \sigma_3 = \sigma_4 = \sigma_5 = \sigma$).

The general expression of the Laplace transform of the system availability, for n-k=2, is the same as Equation (2-172).

Substituting the Laplace transforms of the pdf of the system repair time distributions $N_j(s)$ for j=3, 4, 5, in Equation (2-172) and taking inverse Laplace transform, we can get the system time-dependent availability. Using the same data as that in the earlier system steady state availability plots, Figure 2-38 and 2-39 provide the results of the time-dependent availability plots for k-out-of-n (n-k=2) system with Gamma (β=2) failed system repair time distribution when λ_{h0}=0 and λ_{h0}=0.0002/hr, respectively. Figure 2-40 provide the time-dependent availability plots for the system with exponentially distributed failed system repair times.

Setting $\mu_3(x) = \mu_4(x) = \mu_5(x) = 0$ in the model, the general expression of the Laplace transform of the system reliability with repair is the same as Equation (2-179). Substituting r_0, r_1, and r_2 in the equation and taking inverse Laplace transform on the resulting equation, we can get the system reliability with repair. System mean time to failure with repair is given by equation (2-180). The plots of the system reliability with repair when λ_{h0}=0 and λ_{h0}=0.0002/hr are shown in Figures 2-38 and 2-39, respectively, for same applicable data of the system steady state availability numerical examples. Figure 2-43 shows the plots of system mean time to failure with repair.

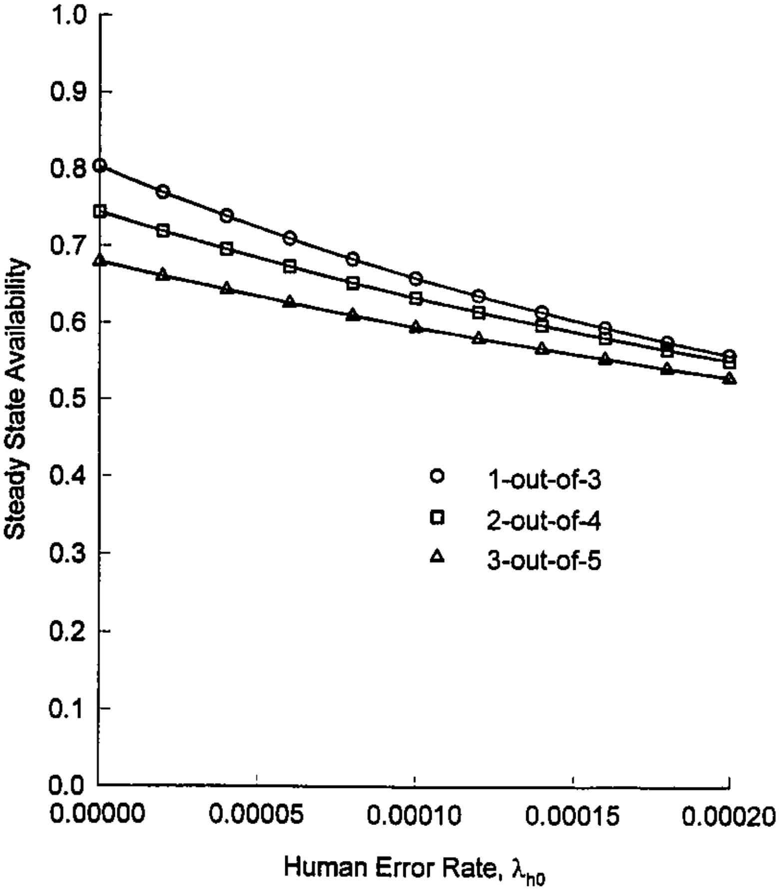


Figure 2-30 System steady state availability vs. λ_{h0} plots for k-ou-of-n (n-k = 2) system with constant human error rates and Gamma (β = 3) distributed failed system repair times

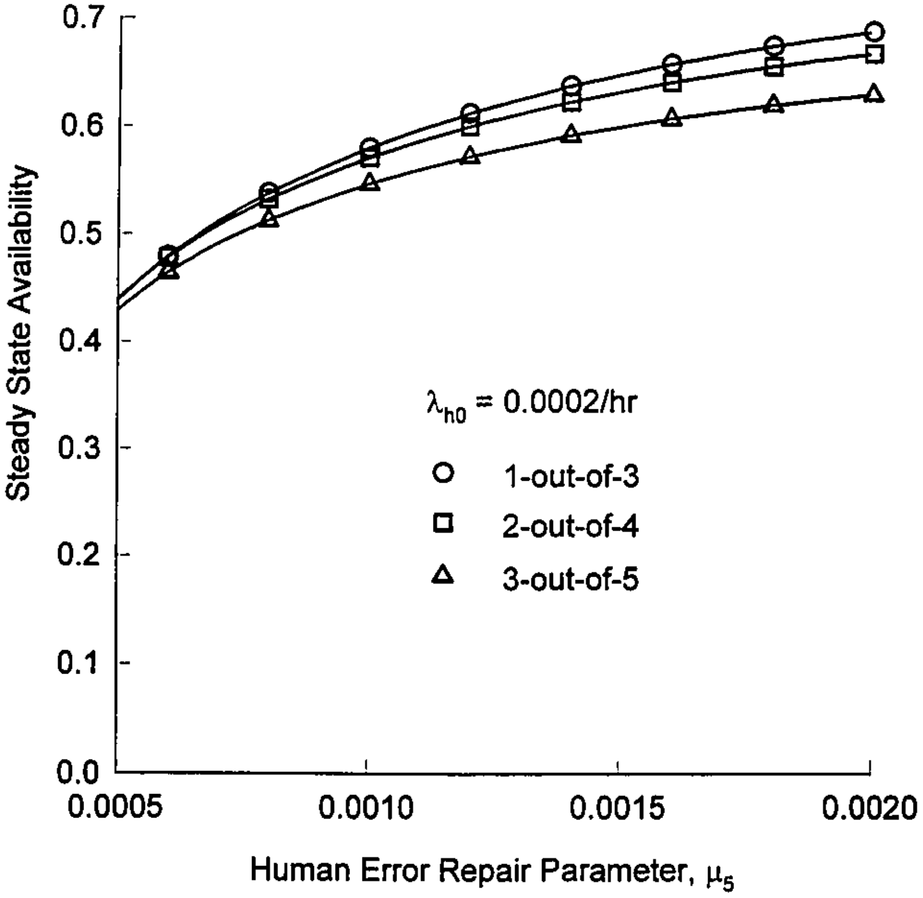


Figure 2-31 AV_{ss} vs. μ_5 plots for k-out-of-n (n-k = 2) system with constant human error rates and the Gamma (β=3) distributed system repair times

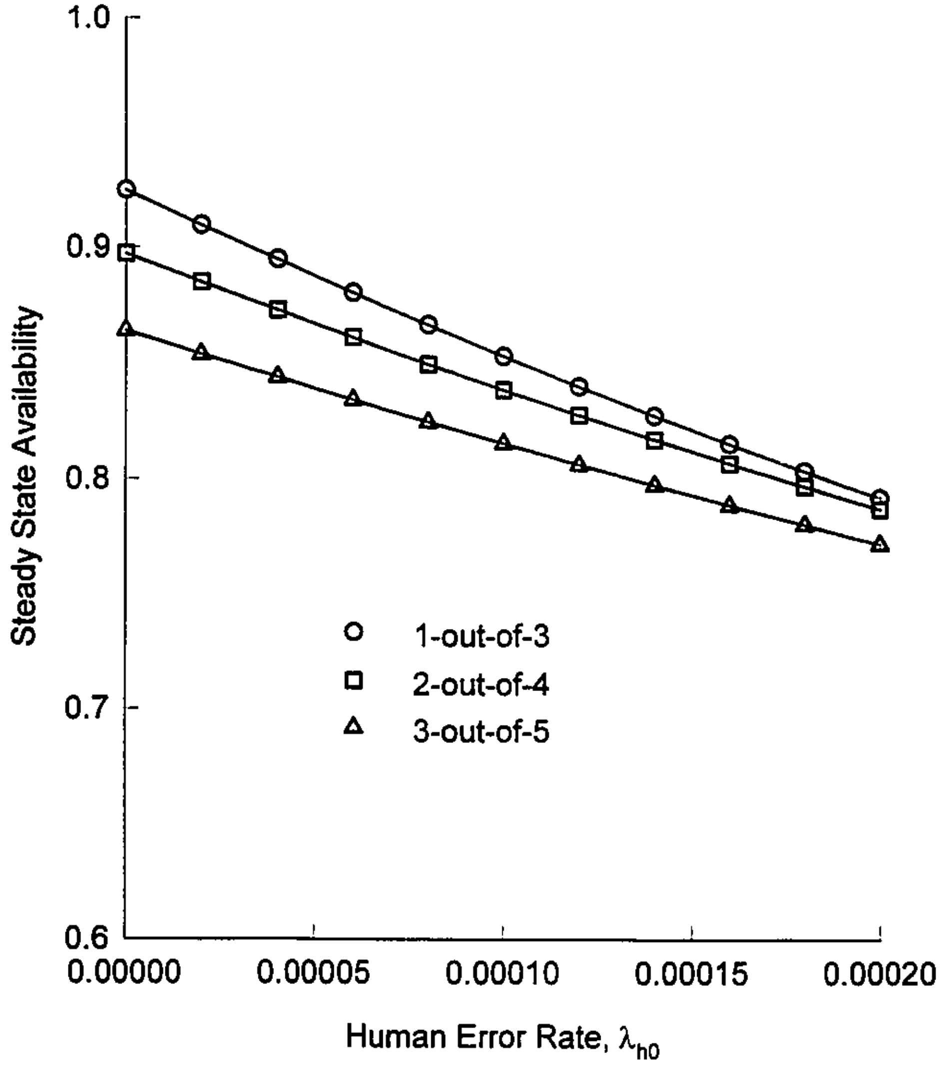


Figure 2-32 System steady state availability vs. λ_{h0} plots for k-out-of-n (n-k = 2) system with constant human error rates and system repair rates

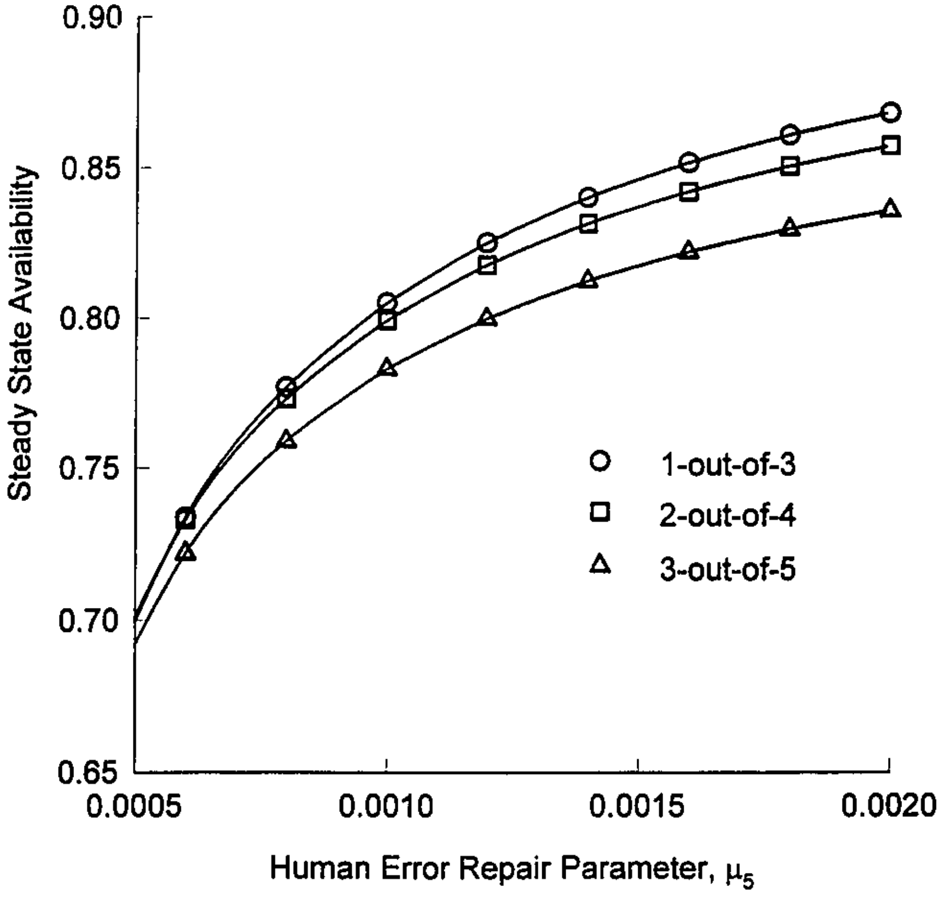


Figure 2-33 System steady state availability vs. μ_5 plots for k-out-of-n (n-k = 2) system with constant human error rates and system repair rates

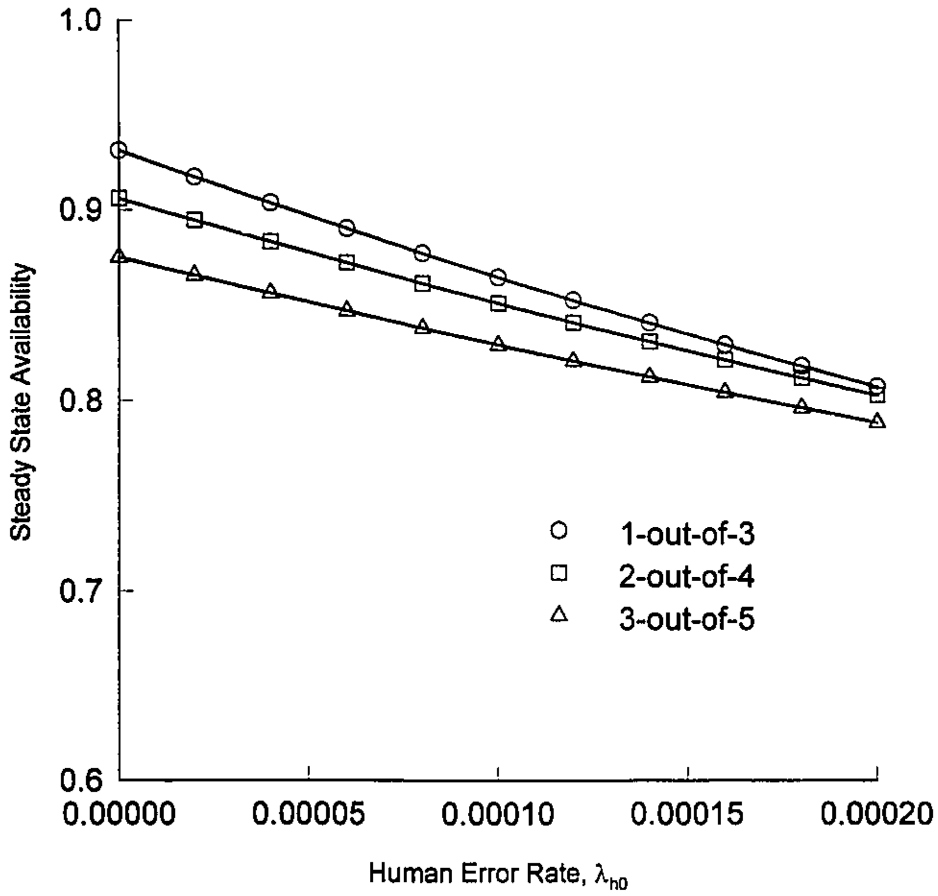


Figure 2-34 AV_{ss} vs. λ_{h0} plots for k-out-of-n (n-k = 2) system with constant human error rates and the Weibull ($\beta = 3$) distributed failed system repair times

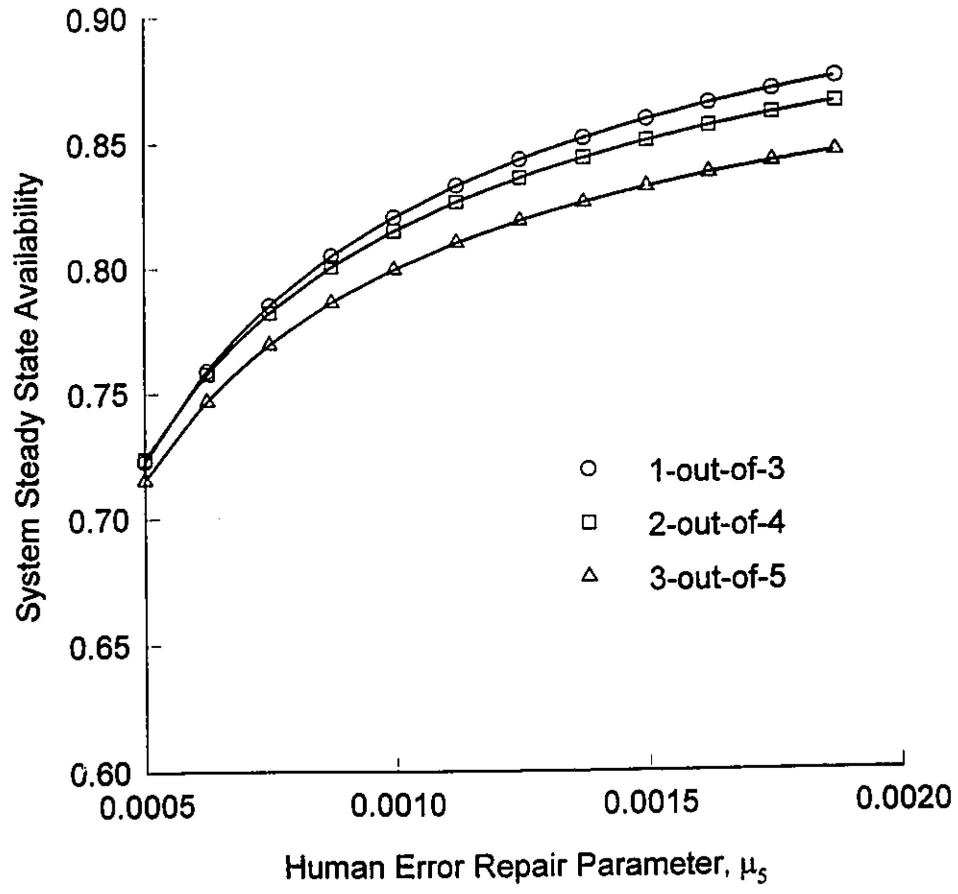


Figure 2-35 AV_{ss} vs. μ_5 plots for k-out-of-n (n-k=2) system with constant human error rates and the Weibull ($\beta = 3$) distributed system repair times

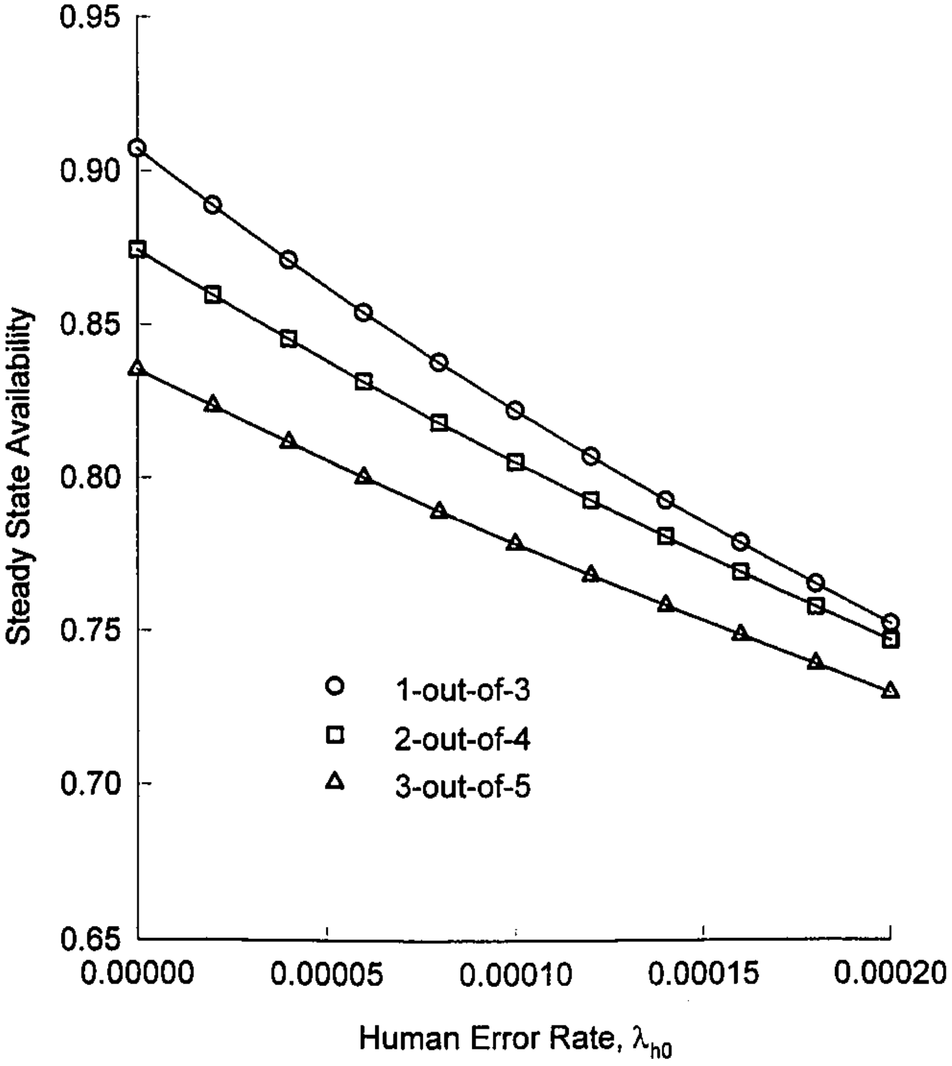


Figure 2-36 AV_{ss} vs. λ_{h0} plots for k-out-of-n (n-k = 2) system with constant human error rates and the Rayleigh distributed failed system repair times

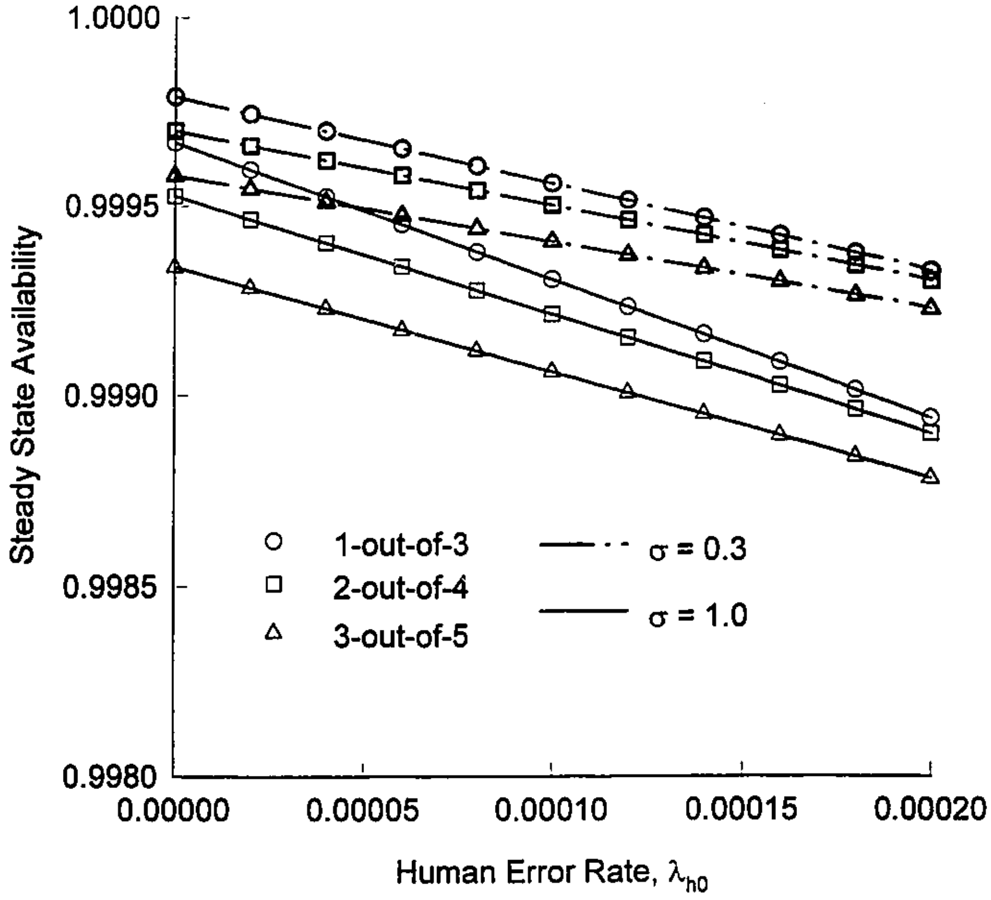


Figure 2-37 AV_{ss} vs. λ_{h0} plots for k-out-of-n (n-k = 2) system with constant human error rates and the lognormally distributed failed system repair times

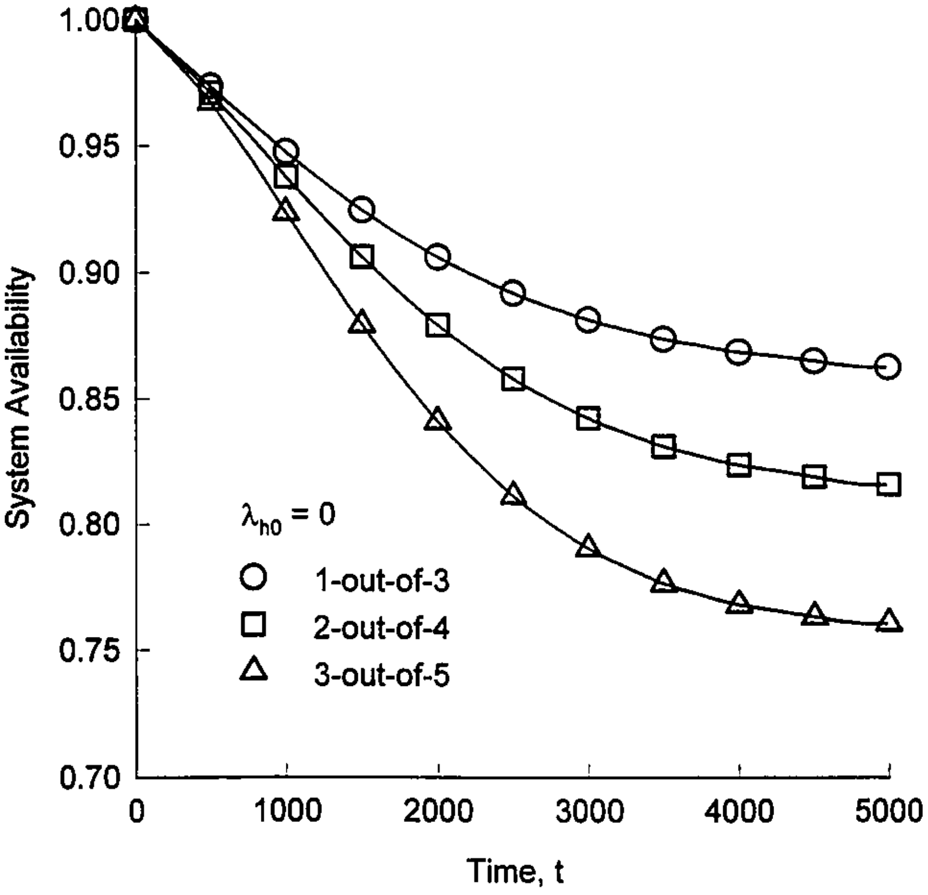


Figure 2-38 AV(t) plots for k-out-of-n (n-k=2) system with constant human error rates and the Gamma (β=2) distributed system repair times

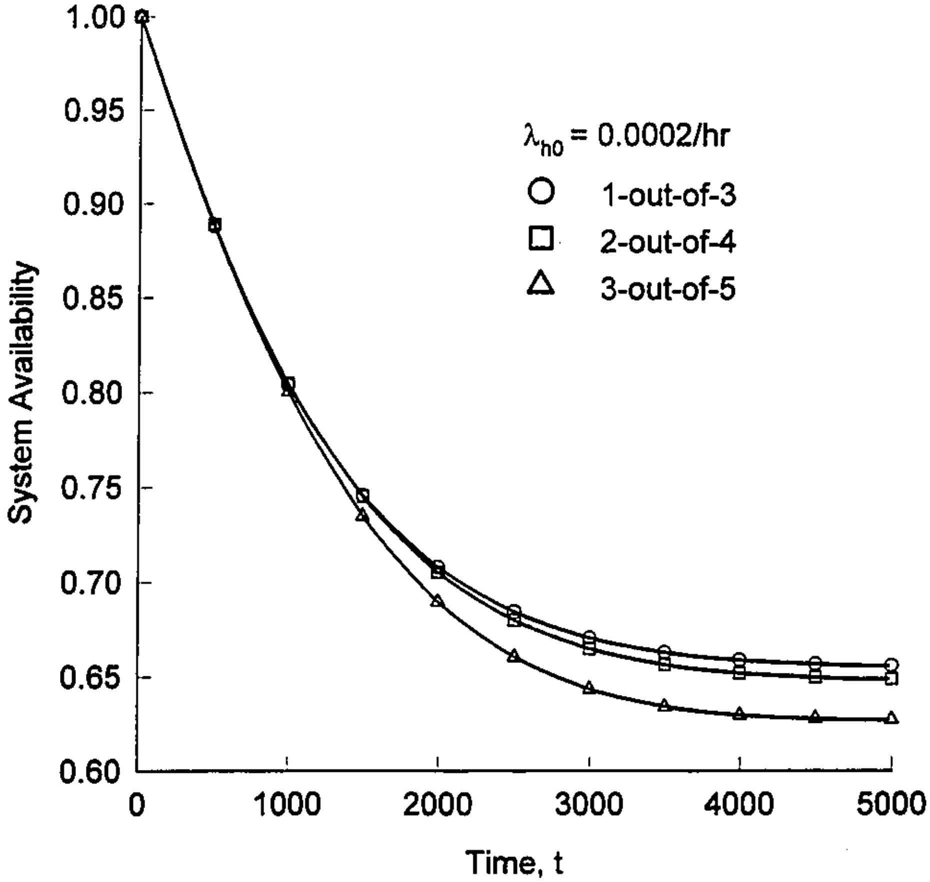


Figure 2-39 AV(t) plots for k-out-of-n (n-k=2) system with constant human error rates and the Gamma (β=2) distributed system repair times (λ_{h0}=0.0002)

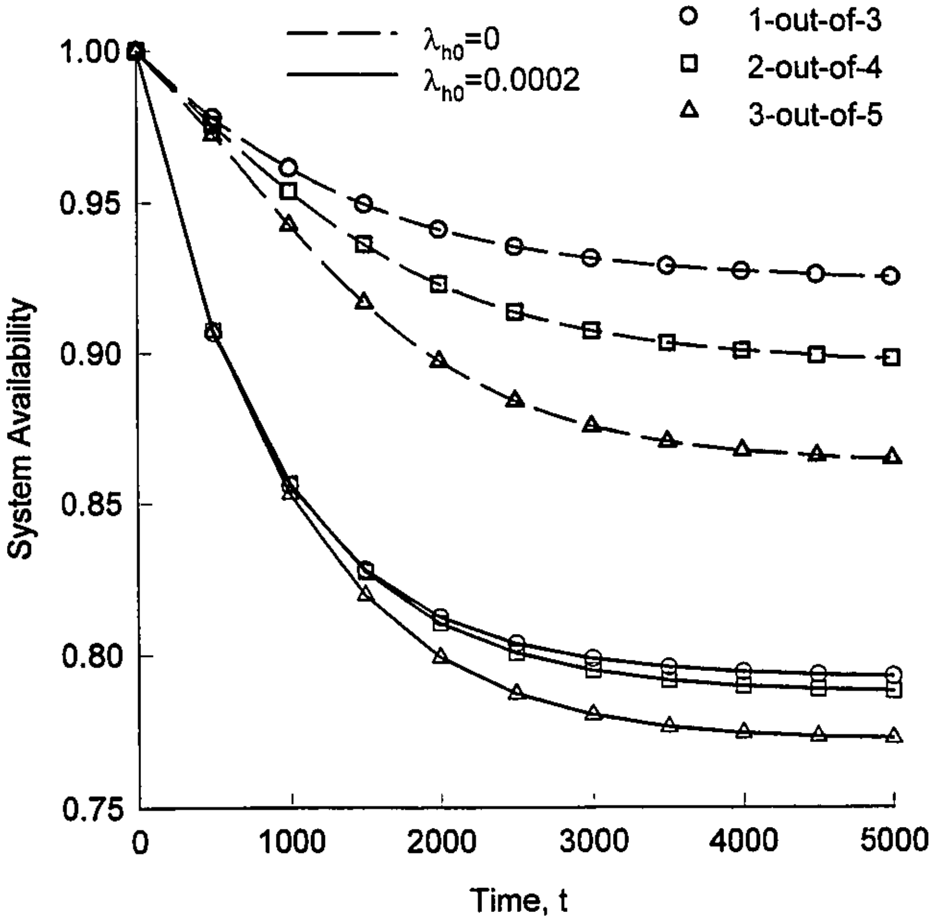


Figure 2-40 Time-dependent system availability plots for k-out-of-n (n-k=2) system with constant human error rates and system repair rates

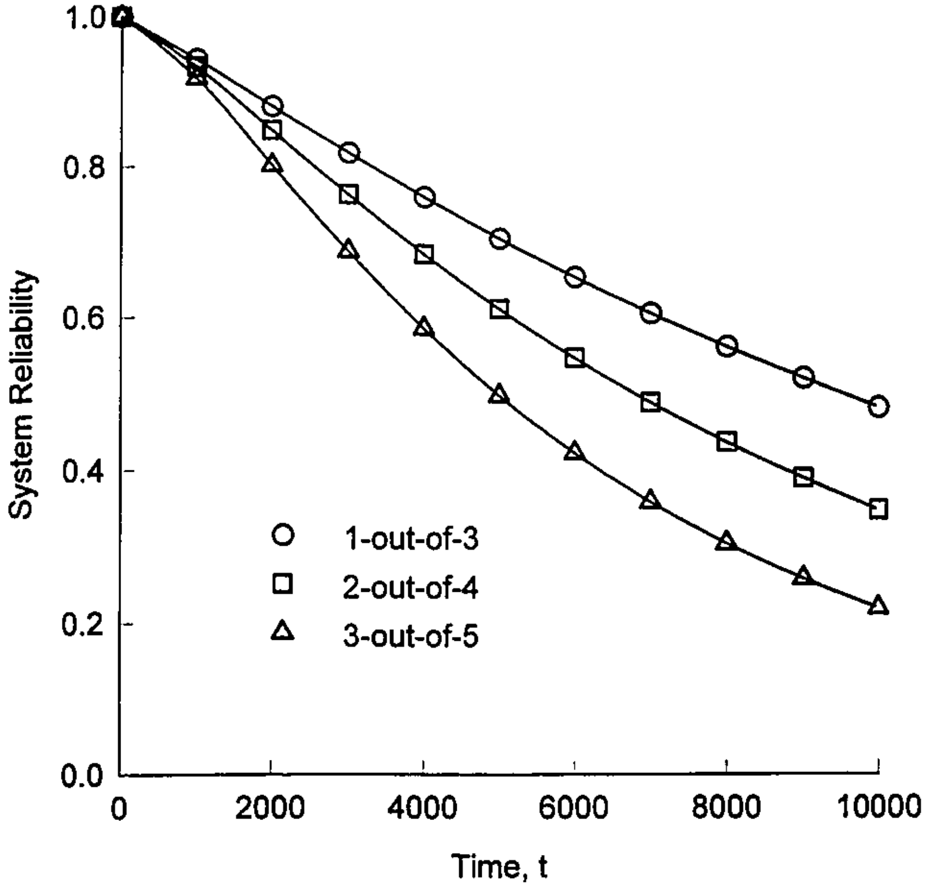


Figure 2-41 System reliability with repair plots for k-out-of-n (n-k=2) system with constant human error rates ($\lambda_{h0} = 0$)

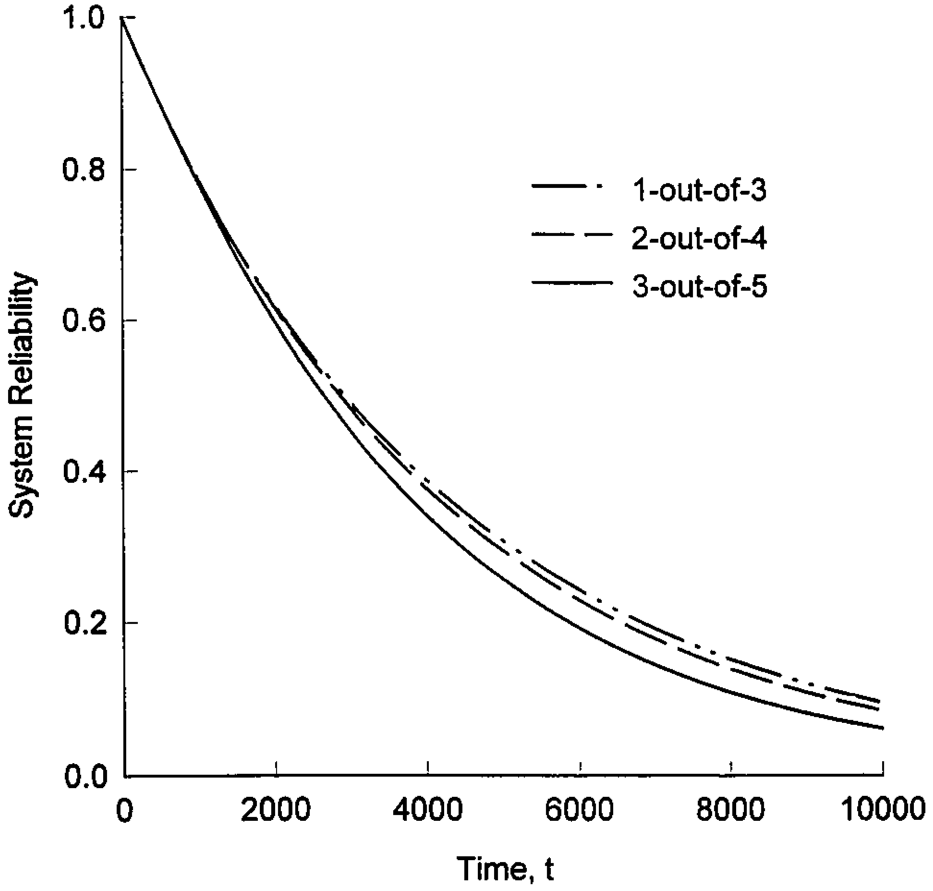


Figure 2-42 System reliability with repair plots for k-out-of-n (n-k=2) system with constant human error rates ($\lambda_{h0} = 0.0002$)

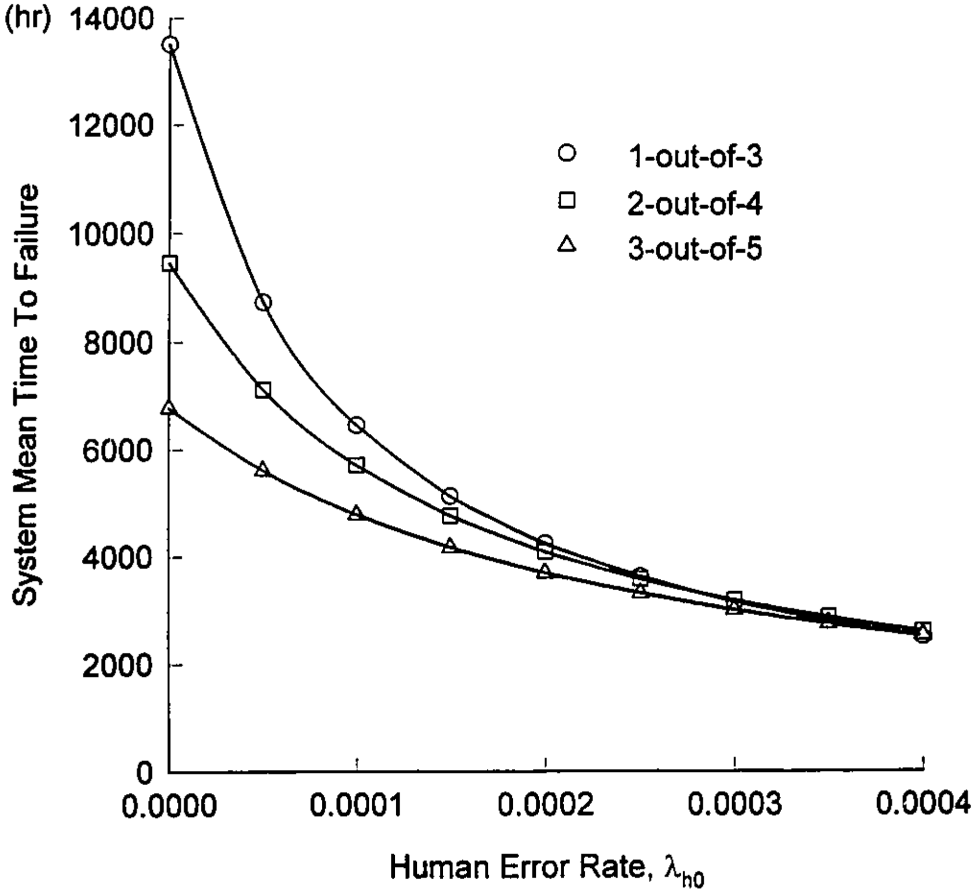


Figure 2-43 System mean time to failure with repair for k-out-of-n (n-k=2) system with constant human error rates

2.4 Majority Voting System With Imperfect Voter

In Section 2.3 we discussed k-out-of-n (k/n) system. A special case of k/n system, assuming a perfect voter, is majority voting system. Majority system is (k+1)/(2k+1) system. It is often called as Triple-Modular Redundant (TMR) System. TMR is probably the most tossed around term in computer redundancy reliability. The system consists of (2k+1) identical units feeding into a majority voter system, as shown in Figure 2-44. Typically, the (2k+1) identical units represent a single logic (binary) variable, and the output logic variable is determined on the basis of majority voting. The corresponding voter output table is illustrated in Table 2-13 [75]. The advantage of using (k+1)/(2k+1) is that if the outputs of k or fewer units of the (2k+1) units are in error, they are masked and the output remains correct.

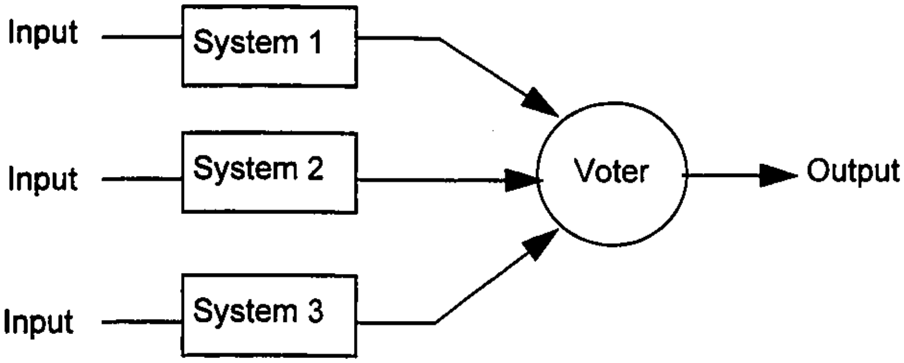


Figure 2-44 Triple-modular redundant system with voting

Table 2-13 TMR Voting

Output of Units			Voter Output
#1	#2	#3	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Now consider the situation in which the voter has a probability of failing to work properly. For the investigation, we put the imperfect voter in series with the (k+1)/(2k+1) redundancy in the reliability block diagram as shown in Figure 2-45. The corresponding transition probabilities diagram is shown in Figure 2-46. The system has only one repair crew and no further failure can occur at system down. At time t = 0, all the units start working simultaneously. In state m, there are m units failed and (2k+1-m) units are in working condition. Furthermore, the occurrence of the voter failure or a common-cause failure or a critical human error can cause the total system to fail. The voter failure or a common-cause failure or a critical human error can occur when the system is in any one of its operating states. Explanations for the state of the system are as follows:

i = 0: all (2k+1) units and voter in perfect working condition.

i = 1: one operating unit has failed due to a hardware failure and 2k units working

i = 2: two units have failed due to hardware failures and (2k-1) units working.

i = m: m units have failed due to hardware failures and (2k+1-m) units working.

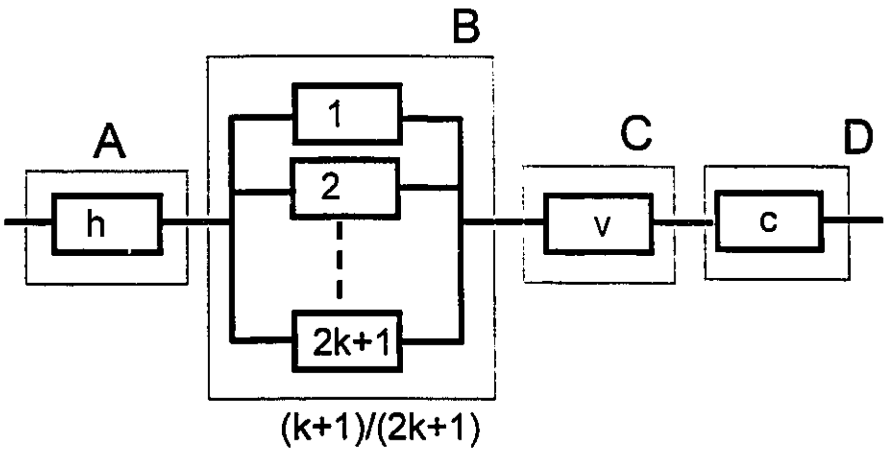
i = k: k units have failed due to hardware failures and (k+1) units working.

i = k+1: the system has failed due to hardware failures.

i = k+2: the system has failed due to a common-cause failure.

i = k+3: the system has failed due to a critical human error.

i = k+4: the system has failed due to the voter failure.



A: critical human error
B: (k+1)-out-of-(2k+1)
C: voter
D: common-cause failure

Figure 2-45 The block diagram of a (k+1)/(2k+1) majority voting system with imperfect voter, common-cause failure and human error

The following repair policies are considered in the analysis of majority voting with imperfect voter system:

I. $\mu_j(x) = 0$ (for $j=k+1, k+2, k+3, k+4$) and $\mu_i \neq 0$ (for $i=1, 2, \ldots, k$)

   The partially failed system (k or fewer units failed, k+1 or more units operating) is repaired back to its previous state, but the completely failed system is never repaired.

II. $\mu_j(x) \neq 0$ (for $j=k+1, k+2, k+3, k+4$) and $\mu_i \neq 0$ (for $i=1, 2, \ldots, k$)

   In addition to the partially failed system being repaired to its previous state, the completely failed system is also repaired back to its original state (state 0).

III. $\mu_j(x) \neq 0$ (for $j=k+1, k+2, k+3, k+4$) and $\mu_i = 0$ (for $i=1, 2, \ldots, k$)

   The completely failed system is repaired to state 0, but no repair on the partially failed system.

Repair policies I and III are the special cases of repair policy II.

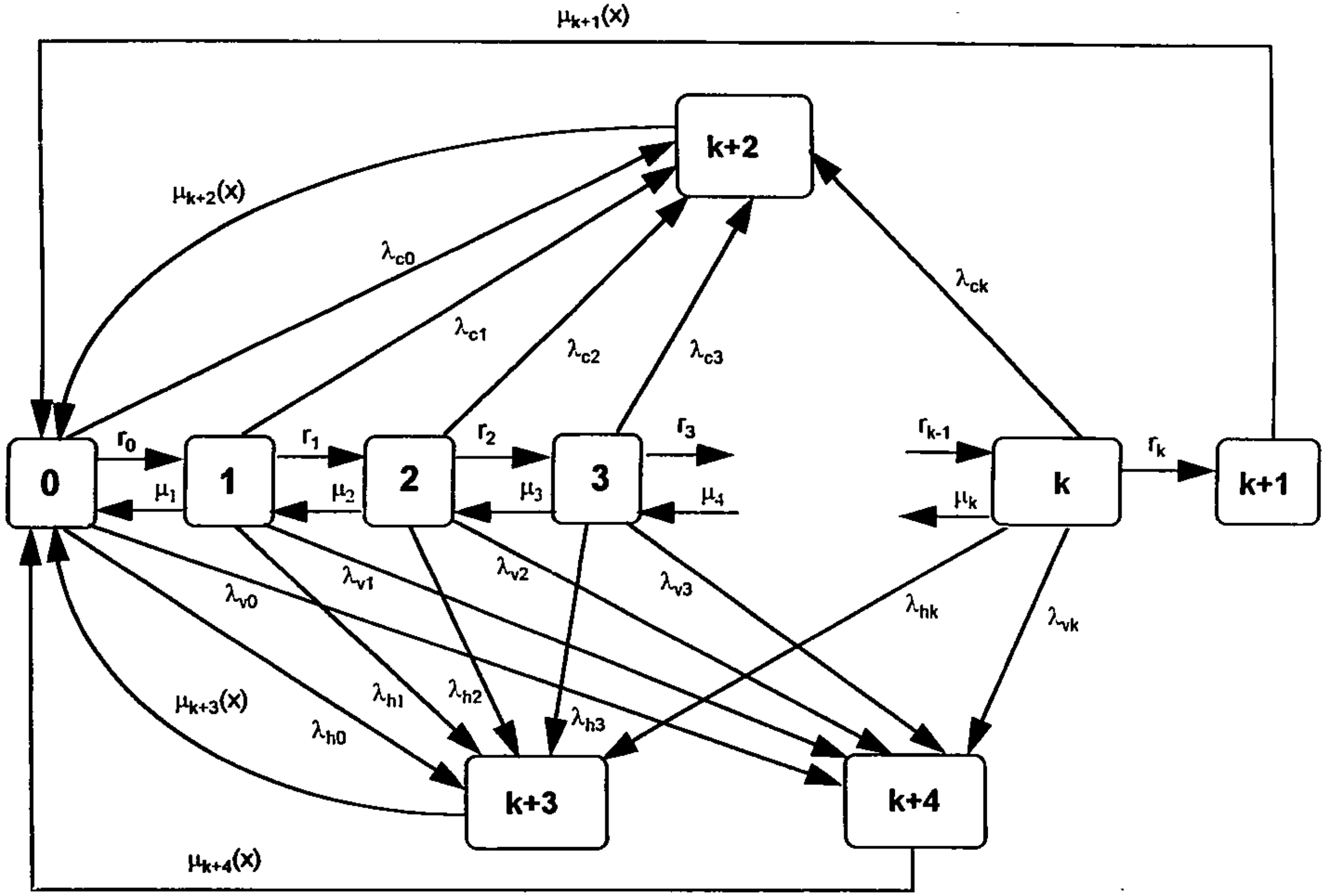


Figure 2-46 The state transition diagram of majority voting system with imperfect voter, common-cause failure and human error

The Descriptive Equations of the Model

The differential equations associated with Figure 2-46 (under type II repair policy) is:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=k+1}^{k+4} \int_0^\infty P_j(x,t)\mu_j(x)dx \qquad (2\text{-}231)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \qquad (2\text{-}232)$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \qquad (2\text{-}233)$$

(for i = 1, 2, 3, ..., k-1)

$$\frac{dP_k(t)}{dt} = r_{k-1} P_{k-1}(t) - a_k P_k(t) \qquad (2\text{-}234)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \qquad (2\text{-}235)$$

(for j = k+1, k+2, k+3, k+4)

The associated boundary conditions are as follows:

$$P_{k+1}(0,t) = r_k P_k(t) \qquad (2\text{-}236)$$

$$P_{k+2}(0,t) = \sum_{i=0}^{k} \lambda_{ci} P_i(t) \qquad (2\text{-}237)$$

$$P_{k+3}(0,t) = \sum_{i=0}^{k} \lambda_{hi} P_i(t) \qquad (2\text{-}238)$$

$$P_{k+4}(0,t) = \sum_{i=0}^{k} \lambda_{vi} P_i(t) \qquad (2\text{-}239)$$

where

$$a_i = r_i + \mu_i + \lambda_{ci} + \lambda_{hi} + \lambda_{vi}$$

$$r_i = (2k + 1 - i)\lambda \qquad \text{(for identical unit system)}$$

(for i=0, 1, 2, ..., k and assuming μ_i=0)

At time t=0, $P_0(0)$=1, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

As time approaches infinity, Equations (2-231) - (2-235) reduce to Equations (2-240)-(2-244), respectively.

$$a_0 P_0 - \mu_1 P_1 = \sum_{j=k+1}^{k+4} \int_0^{\infty} P_j(x)\mu_j(x)dx \qquad (2\text{-}240)$$

$$r_0 P_0 - a_1 P_1 + \mu_2 P_2 = 0 \qquad (2\text{-}241)$$

$$r_{i-1} P_{i-1} - a_i P_i + \mu_{i+1} P_{i+1} = 0 \qquad (2\text{-}242)$$

(i = 1, 2, 3, ..., k-1)

$$r_{k-1} P_{k-1} - a_k P_k = 0 \qquad (2\text{-}243)$$

$$\frac{\partial P_j(x)}{\partial x} = -\mu_j(x) P_j(x) \qquad (2\text{-}244)$$

(j = k+1, k+2, k+3, k+4)

Similarly, the boundary conditions become:

$$P_{k+1}(0) = r_k P_k \qquad (2\text{-}245)$$

$$P_{k+2}(0) = \sum_{i=0}^{k} \lambda_{ci} P_i \qquad (2\text{-}246)$$

$$P_{k+3}(0) = \sum_{i=0}^{k} \lambda_{hi} P_i \qquad (2\text{-}247)$$

$$P_{k+4}(0) = \sum_{i=0}^{k} \lambda_{vi} P_i \qquad (2\text{-}248)$$

Substituting Equations (2-245)-(2-248) in Equation (2-20), we have the steady state probabilities for state k+1, k+2, k+3, and k+4

$$P_{k+1} = r_k P_k E_{k+1}[x] \qquad (2\text{-}249)$$

$$P_{k+2} = \sum_{i=0}^{k} \lambda_{ci} P_i E_{k+2}[x] \qquad (2\text{-}250)$$

$$P_{k+3} = \sum_{i=0}^{k} \lambda_{hi} P_i E_{k+3}[x] \qquad (2\text{-}251)$$

$$P_{k+4} = \sum_{i=0}^{k} \lambda_{vi} P_i E_{k+4}[x] \qquad (2\text{-}252)$$

The system repair mean times, $E_{k+1}[x]$, $E_{k+2}[x]$, $E_{k+3}[x]$, and $E_{k+4}[x]$, are given by Equation (2-24).

Solving the set of Equations (2-241) - (2-243), and (2-249) - (2-252), together with

$$\sum_{i=0}^{k+4} P_i = 1 \qquad (2\text{-}253)$$

we can get steady state probabilities, P_0, P_1, P_2, ..., P_k, P_{k+1}, P_{k+2}, P_{k+3}, and P_{k+4}.

Thus, the steady state availability of the system is

$$AV_{ss} = \sum_{i=0}^{k} P_i \qquad (2\text{-}254)$$

Time-Dependent Availability Analysis

Using Laplace transforms and the initial conditions in Equations (2-231) - (2-239), we have

$$s\,P_0(s) = 1 - a_0 P_0(s) + \mu_1 P_1(s) + \sum_{j=k+1}^{k+4} \int_0^{\infty} P_j(x,s)\mu_j(x)dx \tag{2-255}$$

$$(s + a_1)P_1(s) - r_0 P_0(s) - \mu_2 P_2(s) = 0 \tag{2-256}$$

$$(s + a_i)P_i(s) - r_{i-1} P_{i-1}(s) - \mu_{i+1} P_{i+1}(s) = 0 \tag{2-257}$$

(for i = 1, 2, 3, ..., k-1)

$$(s + a_k)P_k(s) - r_{k-1} P_{k-1}(s) = 0 \tag{2-258}$$

$$\frac{\partial P_j(x,s)}{\partial x} + s\,P_j(x,s) + \mu_j(x)P_j(x,s) = 0 \tag{2-259}$$

(for j = k+1, k+2, k+3, k+4)

and the boundary conditions:

$$P_{k+1}(0,s) = r_k P_k(s) \tag{2-260}$$

$$P_{k+2}(0,s) = \sum_{i=0}^{k} \lambda_{ci} P_i(s) \tag{2-261}$$

$$P_{k+3}(0,s) = \sum_{i=0}^{k} \lambda_{hi} P_i(s) \tag{2-262}$$

$$P_{k+4}(0,s) = \sum_{i=0}^{k} \lambda_{vi} P_i(s) \tag{2-263}$$

Substituting the boundary conditions in Equation (2-35) and (2-36), we get

$$P_{k+1}(s) = r_k P_k(s) \frac{1 - N_{k+1}(s)}{s} \tag{2-264}$$

$$P_{k+2}(s) = \sum_{i=0}^{k} \lambda_{ci} P_i(s) \frac{1 - N_{k+2}(s)}{s} \tag{2-265}$$

$$P_{k+3}(s) = \sum_{i=0}^{k} \lambda_{hi} P_i(s) \frac{1 - N_{k+3}(s)}{s} \tag{2-266}$$

$$P_{k+4}(s) = \sum_{i=0}^{k} \lambda_{vi} P_i(s) \frac{1 - N_{k+4}(s)}{s} \tag{2-267}$$

The Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_k(s)$, $P_{k+1}(s)$, $P_{k+2}(s)$, $P_{k+3}(s)$, and $P_{k+4}(s)$, can be obtained by solving Equations (2-256) - (2-258), (2-264) - (2-267) and the following equation

$$\sum_{i=0}^{k+4} P_i(s) = \frac{1}{s} \tag{2-268}$$

The Laplace transform of the system availability is:

$$AV(s) = \sum_{i=0}^{k} P_i(s) \tag{2-269}$$

Substituting the Laplace transform of $N_i(x)$ for different repair time distributions in Equation (2-269) and taking the inverse Laplace transform of the resulting equation, we can obtain the time-dependent system availability

$$AV(t) = \sum_{i=0}^{k} P_i(t) \tag{2-270}$$

Reliability and MTTF With and Without Repair

Setting $\mu_{k+1}(x) = \mu_{k+2}(x) = \mu_{k+3}(x) = \mu_{k+4}(x) = 0$ in Figure 2-46, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \tag{2-271}$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \tag{2-272}$$

(for i = 1, 2, 3, ..., k-1)

$$\frac{dP_k(t)}{dt} = r_{k-1} P_{k-1}(t) - a_k P_k(t) \tag{2-273}$$

$$\frac{dP_{k+1}(t)}{dt} = r_k P_k(t) \tag{2-274}$$

$$\frac{dP_{k+2}(t)}{dt} = \sum_{i=0}^{k} \lambda_{ci} P_i(t) \tag{2-275}$$

$$\frac{dP_{k+3}(t)}{dt} = \sum_{i=0}^{k} \lambda_{hi} P_i(t) \tag{2-276}$$

$$\frac{dP_{k+4}(t)}{dt} = \sum_{i=0}^{k} \lambda_{vi} P_i(t) \tag{2-277}$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, ..., k, k+1, k+2, k+3, k+4.

Using Laplace transform in Equations (2-271) - (2-277) and solving the resulting set of equations, we have the Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_{k+1}(s)$, $P_{k+2}(s)$, $P_{k+3}(s)$, and $P_{k+4}(s)$.

The Laplace transform of the system reliability with repair is:

$$R(s) = \sum_{i=0}^{k} P_i(s) \tag{2-278}$$

The system reliability with repair can be obtained by inverting above Equation

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\sum_{i=0}^{k} P_i(s)\right] \tag{2-279}$$

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s \to 0} R(s) = \lim_{s \to 0} \left[\sum_{i=0}^{k} P_i(s) \right] \tag{2-280}$$

Setting $\mu_1 = \mu_2 = \ldots = \mu_k = 0$ in Equations (2-279) - (2-280), we can get the system reliability, MTTF without repair, respectively.

2.4.1 Special Case Model With Type I Repair

For 2-out-of-3 redundancy system with type I repair policy, that is $\mu_j(x) = 0$ (for j=2, 3, 4, 5) and $\mu_1 \neq 0$, the system of differential equations associated with the model is:

$$\frac{dP_0(t)}{dt} = -(r_0 + \lambda_{c0} + \lambda_{h0} + \lambda_{v0})P_0(t) + \mu_1 p_1(t) \tag{2-281}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - (r_1 + \lambda_{c1} + \lambda_{h1} + \lambda_{v1} + \mu_1)P_1(t) \tag{2-282}$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) \tag{2-283}$$

$$\frac{dP_3(t)}{dt} = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \tag{2-284}$$

$$\frac{dP_4(t)}{dt} = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) \tag{2-285}$$

$$\frac{dP_5(t)}{dt} = \lambda_{v0} P_0(t) + \lambda_{v1} P_1(t) \tag{2-286}$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, 3, 4, 5.

Solving Equations (2-281) - (2-286) with the aid of Laplace transforms, we can get the Laplace transforms of the state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_5(s)$. The Laplace transform of the system reliability with repair is

$$\begin{aligned} R(s) &= P_0(s) + P_1(s) \\ &= \frac{r_0 + a_1 + s}{(a_0 a_1 - r_0 \mu_1) + (a_0 + a_1)\cdot s + s^2} \end{aligned} \qquad (2\text{-}287)$$

where

$$\begin{aligned} a_0 &= r_0 + \lambda_{c0} + \lambda_{h0} + \lambda_{v0} \\ a_1 &= r_1 + \lambda_{c1} + \lambda_{h1} + \lambda_{v1} + \mu_1 \end{aligned}$$

The system reliability with repair is given by

$$R(t) = \frac{e^{s_1 t}(s_1 - s_a)}{s_1 - s_2} + \frac{e^{s_2 t}(s_2 - s_a)}{s_2 - s_1} \qquad (2\text{-}288)$$

where

$$s_a = -a_1 - r_0$$

s_1, and s_2 are real and unique roots of the denominator of Equation (2-287).

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s \to 0} R(s) = \frac{r_0 + a_1}{a_0 a_1 - r_0 \mu_1} \qquad (2\text{-}289)$$

Setting $\mu_1 = 0$ in Equations (2-281) - (2-286) and solving the resulting equations, we get the system reliability for a majority voting system without repair

$$R(t) = \frac{r_0}{a_0 - a_1} e^{-a_1 t} + \frac{a_0 - a_1 - r_0}{a_0 - a_1} e^{-a_0 t} \qquad (2\text{-}290)$$

The system mean time to failure without repair is expressed by

$$MTTF = \int_0^{\infty} R(t)dt = \frac{a_1 + r_0}{a_0 a_1} \qquad (2\text{-}291)$$

2.4.2 Special Case Model With Type II Repair

For 2-out-of-3 redundancy system with type II repair policy, that is $\mu_j(x) \neq 0$ (for j=2, 3, 4, 5) and $\mu_1 \neq 0$, the system of differential equations associated with the model is:

$$\frac{dP_0(t)}{dt} = -(r_0 + \lambda_{c0} + \lambda_{h0} + \lambda_{v0})P_0(t) + \mu_1 p_1(t) + \sum_{i=2}^{5} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \qquad (2\text{-}292)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - (r_1 + \lambda_{c1} + \lambda_{h1} + \lambda_{v1} + \mu_1)P_1(t) \qquad (2\text{-}293)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x)P_j(x,t) \qquad (2\text{-}294)$$

(for j = 2, 3, 4, 5)

The associated boundary conditions are as follows:

$$P_2(0,t) = r_1 P_1(t) \qquad (2\text{-}295)$$

$$P_3(0,t) = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \qquad (2\text{-}296)$$

$$P_4(0,t) = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) \qquad (2\text{-}297)$$

$$P_5(0,t) = \lambda_{v0} P_0(t) + \lambda_{v1} P_1(t) \qquad (2\text{-}298)$$

where

$$a_0 = r_0 + \lambda_{c0} + \lambda_{h0} + \lambda_{v0}$$
$$a_1 = r_1 + \lambda_{c1} + \lambda_{h1} + \lambda_{v1} + \mu_1$$

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

As time t approaches infinity, the derivatives with respect to t of the above equations equal to zero. Solving the resulting equations, we get the following steady state probabilities:

$$P_0 = \frac{b_0(1,1)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}299)$$

$$P_1 = \frac{b_1(1,1)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}300)$$

$$P_2 = \frac{b_2(1,1)E_2(x)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}301)$$

$$P_3 = \frac{b_3(1,1)E_3(x)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}302)$$

$$P_4 = \frac{b_4(1,1)E_4(x)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}303)$$

$$P_5 = \frac{b_5(1,1)E_5(x)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}304)$$

where

$$b_5(1,1) = a_1\lambda_{v0} + r_0\lambda_{v1} = \sum_{i=0}^{1} b_i(1,1)\lambda_{vi}$$

$b_i(1,1)$ (for i=0, 1, 2, 3, 4) and BA(1,1) are given in Section 2.2.1

$E_j(x)$ (for j=2, 3, 4, 5) is given in Equation (2-24)

The steady state availability of the system is:

$$AV_{ss} = P_0 + P_1 = \frac{b_0(1,1) + b_1(1,1)}{BA(1,1) + b_5(1,1)E_5(x)} \qquad (2\text{-}305)$$

Substituting the expressions of the failed system repair mean times $E_j(x)$ (for j=2, 3, 4, 5), for different repair time distributions Equations (2-79), (2-84), (2-88) and (2-91), into Equation (2-305), we can have the expressions of system steady state availability for majority voting system with imperfect voter for Gamma, Weibull, Rayleigh and lognormal distributed failed system repair times, respectively.

Setting

$r_0 = 0.0006 / hr$ $\quad r_1 = 0.0004 / hr$ $\quad \mu_1 = 0.001 / hr$

$\lambda_{c0} = 0.00005 / hr$ $\quad \lambda_{c1} = 0.00002 / hr$ $\quad \lambda_{v0} = 0.00008 / hr$

$\lambda_{v1} = 0.00008 / hr$ $\quad \lambda_{h1} = 0.0001 / hr$

in Equation (2-305), for Gamma and Weibull distributed failed system repair times, the plots of system steady state availability as a function of critical human error rate, λ_{h0}, are shown in Figures 2-47 and 2-48, respectively, for specified values of system repair time distribution parameters $\mu_2 = 0.001 / hr, \mu_3 = \mu_4 = 0.0008 / hr, \ \mu_5 = 0.0005 / hr$. Figure 2-49 provides the results of the system steady state availability as a function of human error rate, λ_{h0}, for lognormally distributed failed system repair times (μ_2=1.001, μ_3=μ_4=1.0008, μ_5=1.0005, $\sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = \sigma$).

Time-dependent Availability Analysis

Using Laplace transform and the initial conditions in Equations (2-292) - (2-298), we have

$$s\, P_0(s) = 1 - a_0 P_0(s) + \mu_1 P_1(s) + \sum_{j=2}^{5} \int_0^{\infty} P_j(x,s) \mu_j(x) dx \qquad (2\text{-}306)$$

$$(s + a_1) P_1(s) - r_0 P_0(s) = 0 \qquad (2\text{-}307)$$

$$\frac{\partial P_j(x,s)}{\partial x} + s P_j(x,s) + \mu_j(x) P_j(x,s) = 0 \qquad (2\text{-}308)$$

(for j = 2, 3, 4, 5)

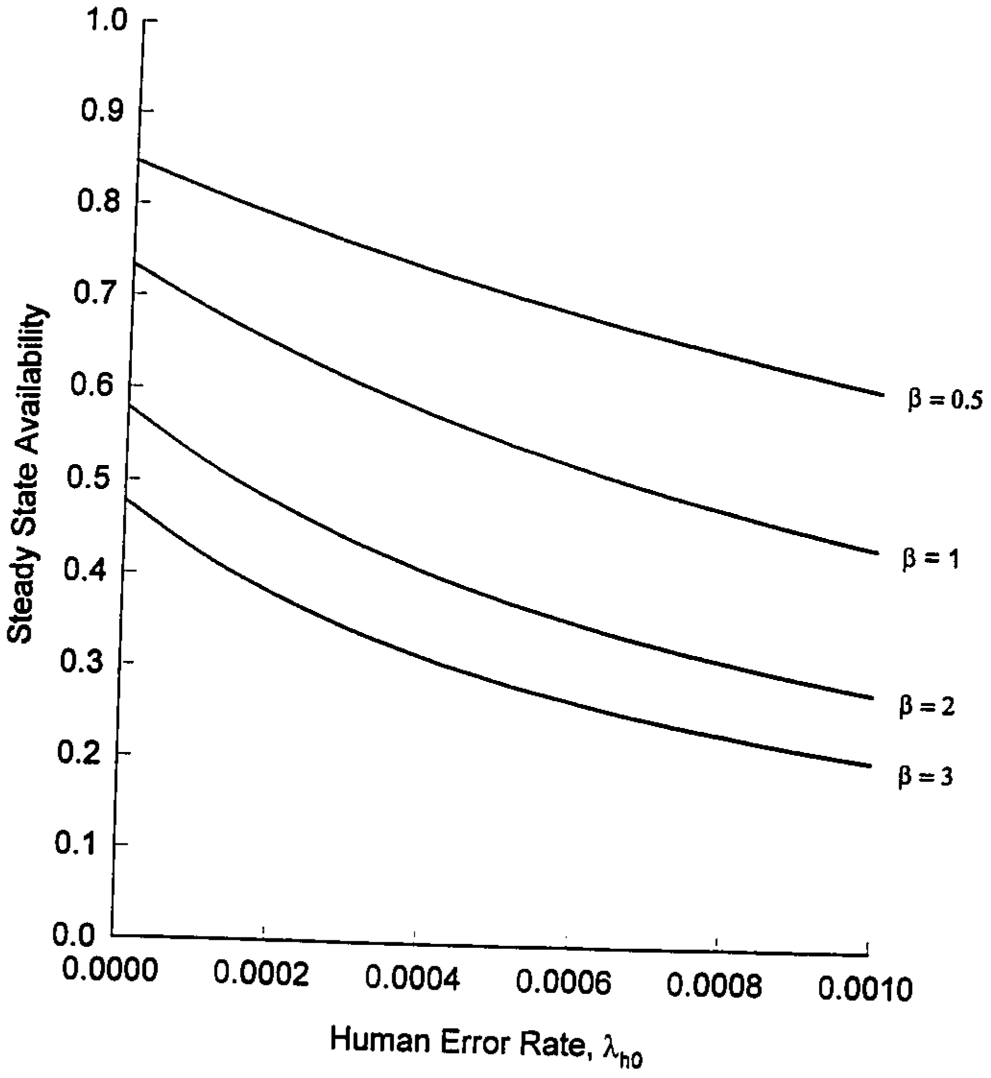


Figure 2-47 AV_{ss} vs. λ_{h0} plots for majority voting system with imperfect voter and the Gamma distributed failed system repair times

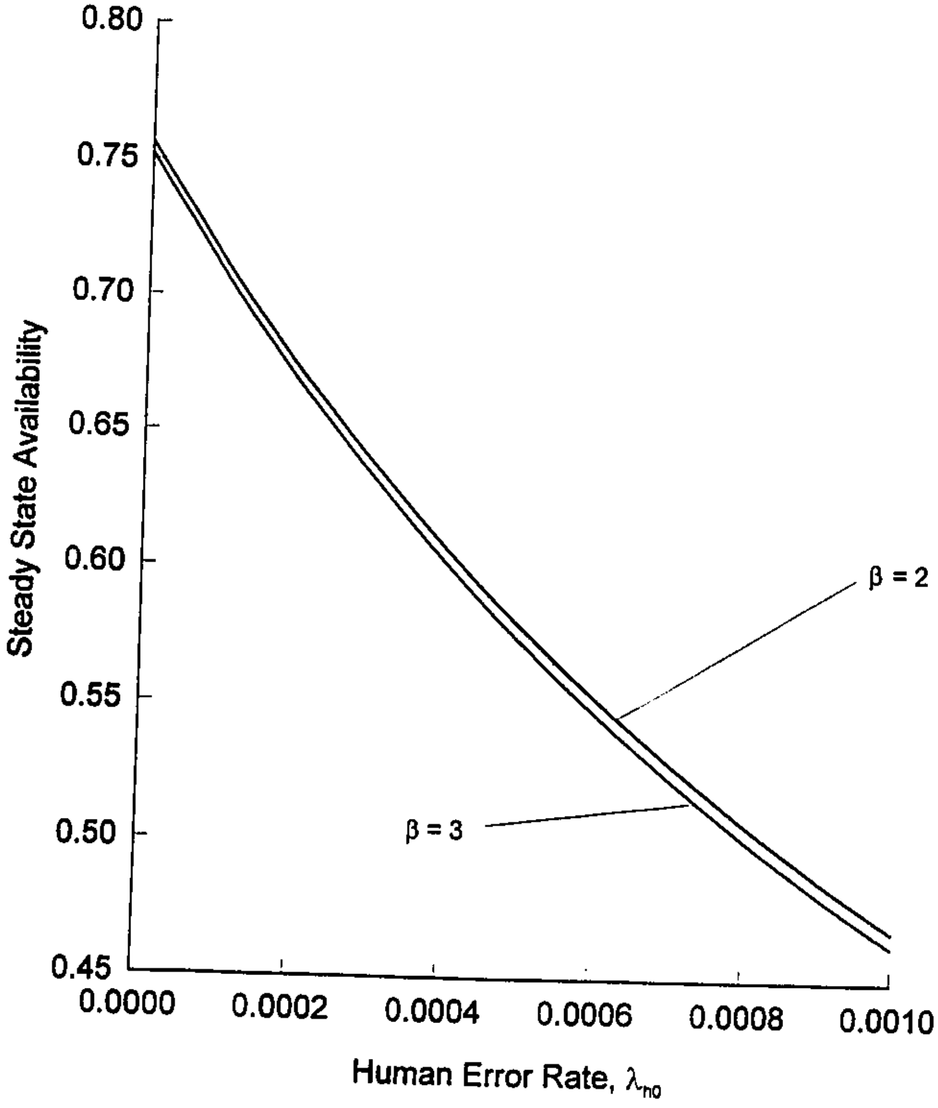


Figure 2-48 AVss vs. λ_{h0} plots for majority voting system with imperfect voter and the Weibull distributed failed system repair times

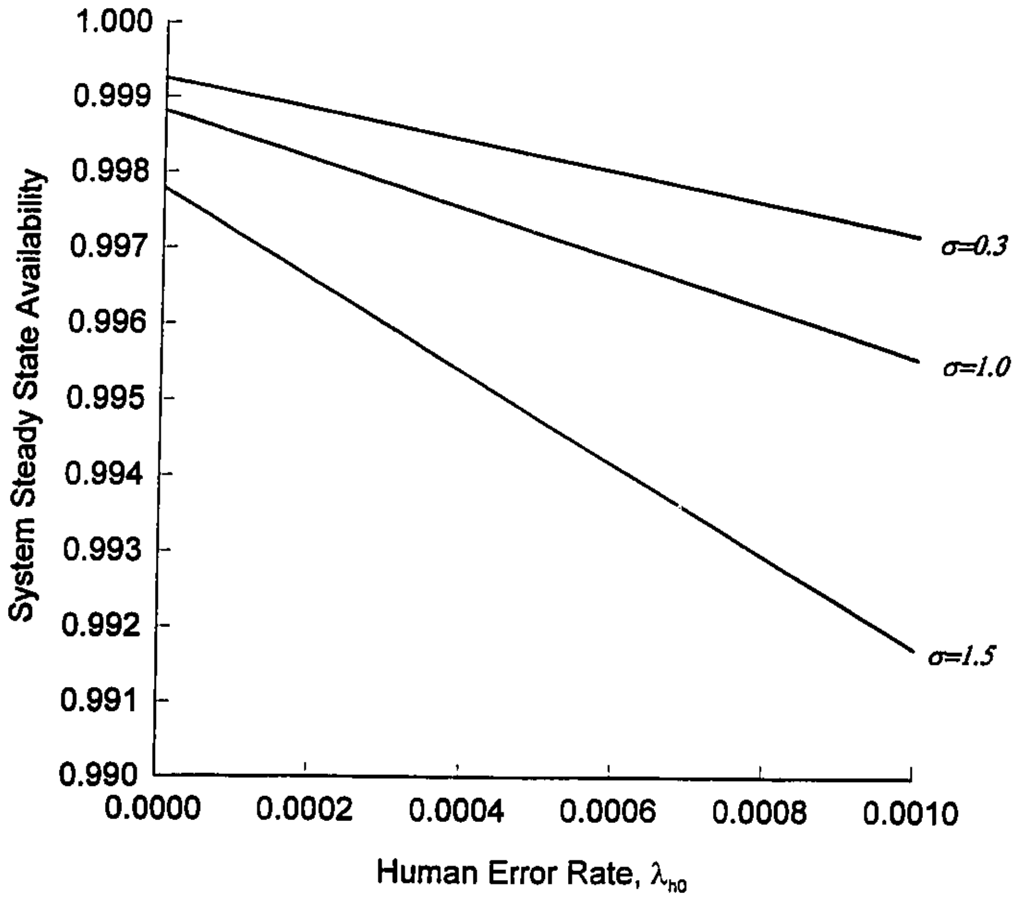


Figure 2-49 Steady state availability vs. λ_{h0} plots for majority voting system with imperfect voter and lognormally distributed failed system repair times

and the boundary conditions:

$$P_2(0,s) = r_1 P_1(s) \tag{2-309}$$

$$P_3(0,s) = \lambda_{c0} P_0(s) + \lambda_{c1} P_1(s) \tag{2-310}$$

$$P_4(0,s) = \lambda_{h0} P_0(s) + \lambda_{h1} P_1(s) \tag{2-311}$$

$$P_5(0,s) = \lambda_{v0} P_0(s) + \lambda_{v1} P_1(s) \tag{2-312}$$

Solving Equations (2-306) - (2-312) we obtained the following Laplace transforms of the state probability expressions:

$$P_0(s) = [s + b_0(1,1)] / CAM(2/3) \tag{2-313}$$

$$P_1(s) = b_1(1,1) / CAM(2/3) \tag{2-314}$$

$$P_2(s) = b_2(1,1)[1 - N_2(s)] / [s \cdot CAM(2/3)] \tag{2-315}$$

$$P_3(s) = [b_3(1,1) + \lambda_{c0}][1 - N_3(s)] / [s \cdot CAM(2/3)] \tag{2-316}$$

$$P_4(s) = [b_4(1,1) + \lambda_{h0}][1 - N_4(s)] / [s \cdot CAM(2/3)] \tag{2-317}$$

$$P_5(s) = [b_5(1,1) + \lambda_{v0}][1 - N_5(s)] / [s \cdot CAM(2/3)] \tag{2-318}$$

where

$$CAM(2/3) = cam1(2/3) + cam2(2/3) \cdot s + s^2$$

$$cam1(2/3) = \sum_{j=2}^{5} b_j(1,1)[1 - N_j(s)]$$

$$cam2(2/3) = \sum_{i=0}^{1} b_i(1,1) + \lambda_{c0}[1 - N_3(s)] + \lambda_{h0}[1 - N_4(s)] + \lambda_{v0}[1 - N_5(s)]$$

$b_i(1,1)$ (for i=0, 1, 2, 3, 4) is given in Section 2.2.1

$$a_0 = r_0 + \lambda_{c0} + \lambda_{h0} + \lambda_{v0}$$

$$a_1 = r_1 + \lambda_{c1} + \lambda_{h1} + \lambda_{v1} + \mu_1$$

The Laplace transform of the system availability is:

$$\begin{aligned} AV(s) &= P_0(s) + P_1(s) \\ &= \left[s + \sum_{i=0}^{1} b_i(1,1) \right] \Big/ CAM(2/3) \end{aligned} \qquad (2\text{-}319)$$

Substituting the Laplace transform of pdf of system repair distribution, $N_j(s)$ for j = 2, 3, 4, 5, in the above Equation and then substituting r_0, r_1 and the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting $\mu_2 = 0.001/hr,\ \mu_3 = \mu_4 = 0.0008/hr,\ \mu_5 = 0.0005/hr$ and taking inverse Laplace transform), we get the time-dependent system availability:

$$AV(t) = P_0(t) + P_1(t) \qquad (2\text{-}320)$$

Figure 2-50 shows the time-dependent system availability plots for the majority voting system with imperfect voter and with exponentially distributed failed system repair times.

For Gamma distributed failed system repair time, the plots of the time-dependent system availability for varying human error rate, λ_{h0}, are shown in Figure 2-51.

Reliability and MTTF Analysis

Setting $\mu_j(x) = 0$ (for j = 2, 3, 4, 5) in the model, and solving the resulting equations, the Laplace transform of the system reliability is given by

$$\begin{aligned} R(s) &= P_0(s) + P_1(s) \\ &= \left[\sum_{i=0}^{1} b_i(1,1) + s \right] \Big/ DA(1,1) \end{aligned} \qquad (2\text{-}321)$$

where

DA(1,1) is shown in Chapter 2.2.1, but note that here

$$a_0 = r_0 + \lambda_{c0} + \lambda_{h0} + \lambda_{v0}$$
$$a_1 = r_1 + \lambda_{c1} + \lambda_{h1} + \lambda_{v1} + \mu_1$$

The system mean time to failure (MTTF) is expressed by

$$MTTF = \lim_{s \to 0} R(s) = \sum_{i=0}^{1} b_i(1,1) \Big/ da1(1,1) \qquad (2\text{-}322)$$

The plots of the system reliability and MTTF are shown in Figures 2-52 and 2-53, respectively, for the same applicable data of the earlier system steady state availability plots.

Setting $\mu_1 = 0$ in Equations (2-305), (2-319), (2-321) and (2-322), we can have the steady state availability, the Laplace transform of time-dependent system availability, reliability and MTTF, respectively, for the majority voting system with type III repair policy ($\mu_j(x) \neq 0$ (for j=2, 3, 4, 5) and $\mu_1 = 0$).

2.5 Summary

This Chapter presents various mathematical models for performing reliability and availability analysis of systems with constant human errors and common-cause failures. The repair times of the failed systems are assumed arbitrarily distributed for all the models. These models are associated with i) standby systems, ii) k-out-of-n systems, and iii) majority voting systems with imperfect voter.

A new method of linear ordinary differential equations system steady state is developed. Using the linear ordinary differential equations instead of complex partial differential equations and Laplace transforms, it is much easier to obtain the general expressions of the system steady state availability for generalized failed system repair times. The system steady state availability expressions are developed for all these models for various repair time distributions, such as the Gamma, Weibull, exponential, Rayleigh and lognormal distributions.

With the aid of the method of supplementary variables, the Markov approach and Laplace transforms, the time-dependent availability of the systems are presented for Gamma distributed failed system repair times for all these models.

The analyses performed in this Chapter indicate that the values of systems performance indices (such as, system steady state availability, time-dependent system availability, system reliability and system mean time to failure) decrease with either increasing values of human error rates or decreasing values of the human error repair rates. For all the special cases considered in this Chapter, the steady state system availability and time-dependent system availability drop as the value of the Gamma shape parameter β increases for Gamma distributed failed system repair times. And those values increase as the value of Weibull shape parameter β increases for Weibull distributed failed system repair times.

For standby redundant configurations, an increase in either the number of standby units for a fixed number of active units or the number of active units for a given number of standby units would improve system performance indices significantly. For k-out-of-n redundant configurations, an increase in the value of k, the least number of units needed for system operation, for a given value of n, total number of units in the system, (or for given value of n-k), would decrease system performance indices.

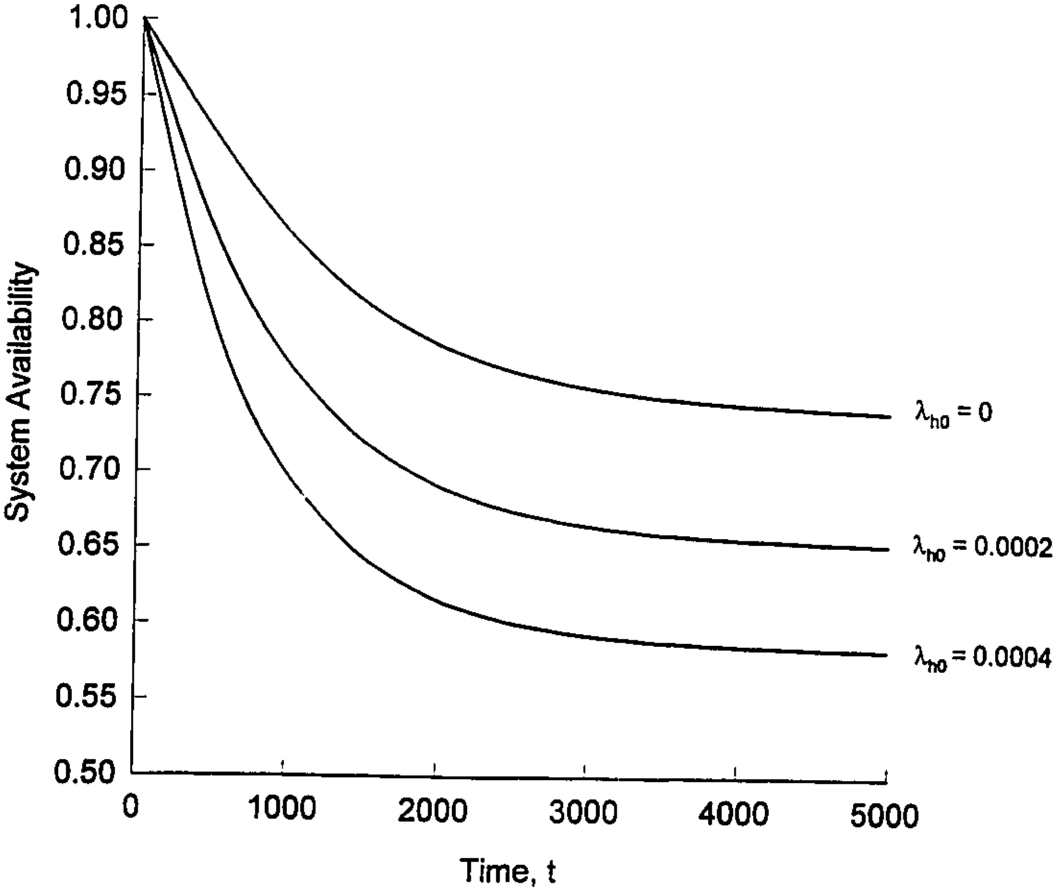


Figure 2-50 Time-dependent system availability plots for a majority voting system with imperfect voter and exponentially distributed failed system repair times

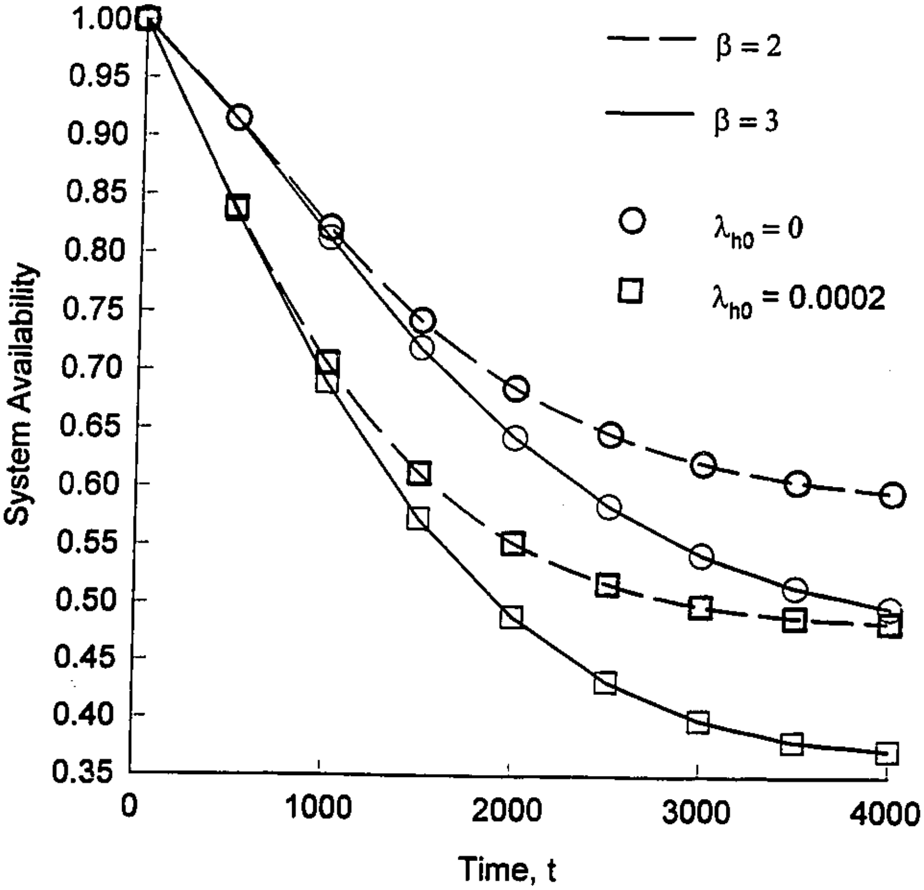


Figure 2-51 Time-dependent system availability plots for a majority voting system with imperfect voter and the Gamma distributed failed system repair times

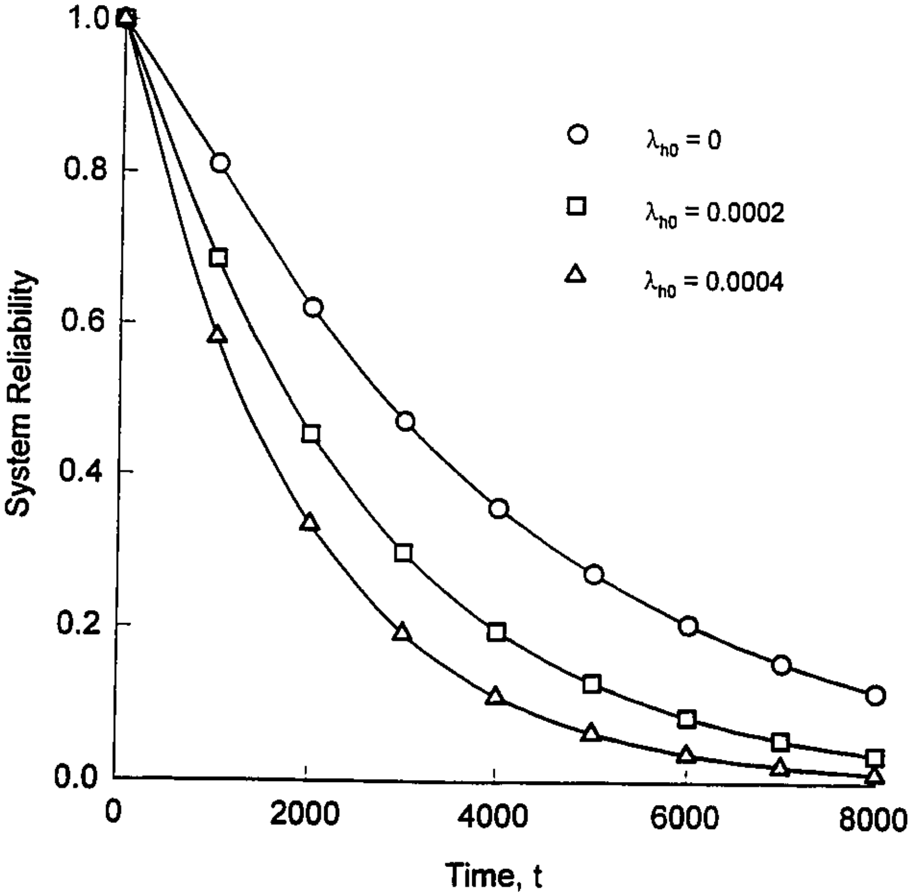


Figure 2-52 System reliability with repair plots for different human error rates

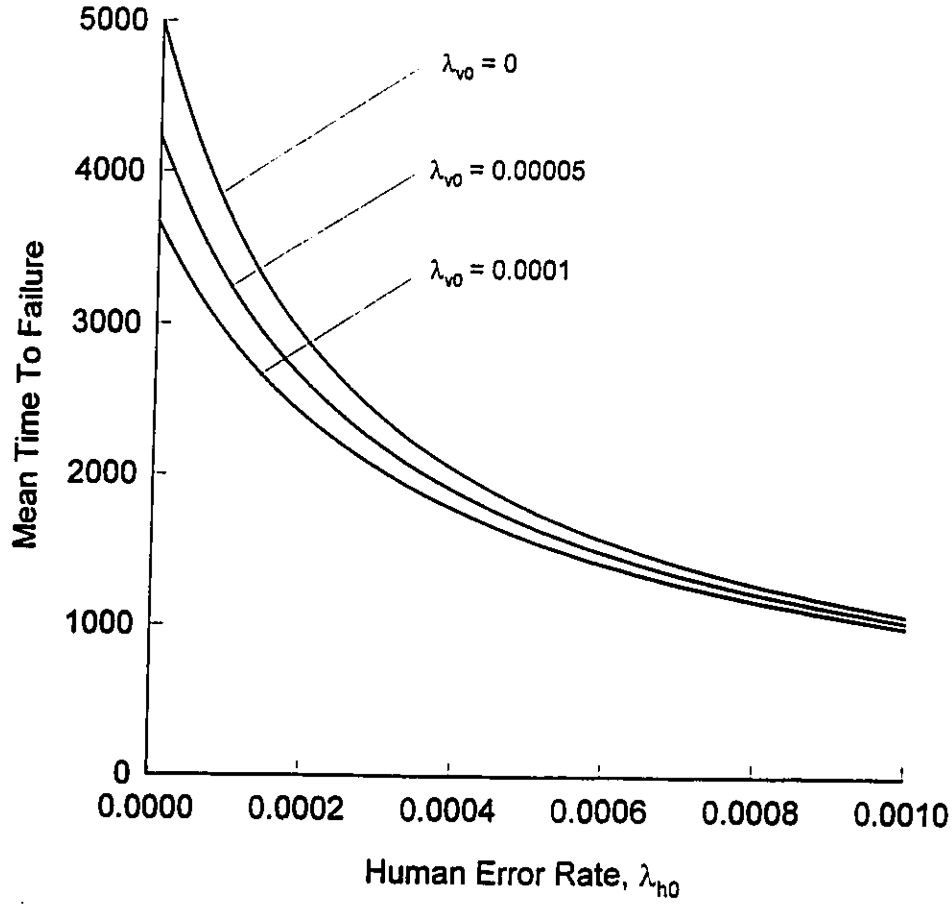


Figure 2-53 System mean time to failure with repair for different values of human error rates and voter failure rates

Chapter 3

Systems With Increasing Human Error Rates and Arbitrary System Repair Rates

Human error rate has often been considered to be constant in human-machine system analysis. But according to various studies it is not always the case. In fact it increases during the fatigue period or under stress and it decreases during learning period.

This Chapter presents mathematical models to perform reliability and availability analysis of various types of system with increasing human error rates and arbitrary failed system repair rates. Furthermore, the system can fail due to a common-cause failure.

Method of stages and supplementary variable techniques [314, 317] were used to formulate initial expressions. The general expressions of the steady state availability for various types of system repair time distributions were obtained using the method of linear ordinary differential equations. The Laplace transform technique was applied for obtaining expressions for system time-dependent availability, reliability, and mean time to failure.

3.1. Time-Dependent Failure Rate

In preceding Sections, it was assumed that the component exists in one of two states, the operating, or up state and the failed, or down state. In some practical situations, a single component may be represented by more than two states. A component may be in full operating state, derated state (partial operating state) and failed state. This therefore gives three states as shown in Figure 3-1. In practical application, additional derated states may exist. Actually, this gives a process of the component failure, that is, we can divide a failure into several stages.

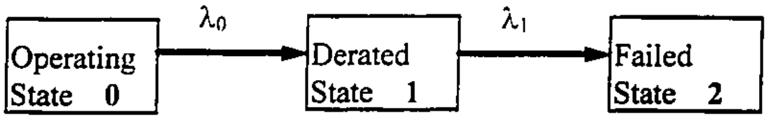


Figure 3-1 State space diagram of component with derated state

The associated equations with Figure 3-1 are

$$\frac{dP_0(t)}{dt} = -\lambda_0 P_0(t) \tag{3-1}$$

$$\frac{dP_1(t)}{dt} = \lambda_0 P_0(t) - \lambda_1 P_1(t) \tag{3-2}$$

At t=0, $P_0(0)=1$ and $P_1(0)=P_2(0)=0$

Solving the differential equations, we have the reliability of the component

$$R(t) = P_0(t) + P_1(t) = e^{-\lambda_0 t} + \frac{\lambda_0}{\lambda_1 - \lambda_0}\left(e^{-\lambda_0 t} - e^{-\lambda_1 t}\right) \tag{3-3}$$

The probability density function is

$$f(t) = -\frac{dR(t)}{dt} = \frac{\lambda_0 \lambda_1}{\lambda_1 - \lambda_0}\left(e^{-\lambda_0 t} - e^{-\lambda_1 t}\right) \tag{3-4}$$

Thus, the failure rate of the component, from state 0 to state 2, is

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\lambda_0 \lambda_1 \left[\exp(-\lambda_0 t) - \exp(-\lambda_1 t)\right]}{\lambda_1 \exp(-\lambda_0 t) - \lambda_0 \exp(-\lambda_1 t)} \tag{3-5}$$

Where $\lambda(t)$ is time-dependent failure rate shown in Figure 3-2. λ_0 and λ_1 are two parameters of $\lambda(t)$.

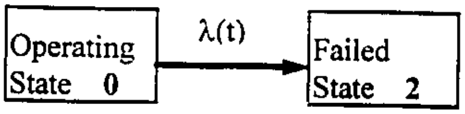


Figure 3-2 State space diagram of component with time-dependent failure rate

3.2. General Standby System

3.2.1 Description of the System

The system is the same general standby system of Chapter 2 but with one exception (i.e., the human error rates in this system increase with time).

The system state-space diagram is shown in Figure 3-3. The system has n units in parallel with m standby units. At time t = 0, all n units in parallel start operating and the remaining m units are in standby mode. The system is considered to be in an up-state as long as one unit is operating normally. As soon as one of the parallel operating unit fails, the standby unit is switched into operation. The system can fail due to a common-cause failure or a critical human error from the normal working condition as well as due to hardware failures. The repair begins soon after a unit failure. Human error rates, $\lambda_{hi}(t)$, are assumed to be time-dependent. The numerals or letters (as applicable) in the boxes of Figure 3-3 denote corresponding system states. The explanations for the states are as follows:

- i = 0: n units active and m units on standby.
- i = 1: one active unit failed due to hardware failure, a standby unit switched into operation; i.e. n units active and m-1 units on standby.
- i = 2: n units active and m-2 units on standby.
- i = 3: n units active and m-3 units on standby. ....

- i = m: n units active and no standby.
- i = m+1: n-1 units active and no standby. ....
- i = m+n-1: only one unit active and no standby.
- i = m+n: the system has failed due to hardware failures.
- i = m+n+1: the system has failed due to a common-cause failure.
- i = m+n+2: the system has failed due to a critical human error.

Considering time-dependent human error rates, $\lambda_{hi}(t)$ (for i=0, 1, 2, ..., m+n-1), in Figure 3-3, as two failure stages and having constant failure rates, λ_h and λ_{hi} , (for i=0, 1, 2, ..., m+n-1), respectively, we have the corresponding state transition diagram of the system as shown in Figure 3-4. As discussed in Section 3.1, the human error probability density function is

$$f_{hi}(t) = \frac{\lambda_{hi}\lambda_h}{\lambda_h - \lambda_{hi}}\left[\exp(-\lambda_{hi}t) - \exp(-\lambda_h t)\right] \tag{3-6}$$

(for i = 0, 1, 2, ..., m+n-1)

The human error rates, from state i to state m+n+2, for i = 0, 1, 2, ..., m+n-1, are

$$\begin{aligned}\lambda_{hi}(t) &= \frac{f_{hi}(t)}{1 - \int_0^t f_{hi}(\xi)d\xi} \\ &= \frac{\lambda_h\lambda_{hi}\left[\exp(-\lambda_{hi}t) - \exp(-\lambda_h t)\right]}{\lambda_h \exp(-\lambda_{hi}t) - \lambda_{hi}\exp(-\lambda_h t)}\end{aligned} \tag{3-7}$$

The human error rate distributions associated with Equation (3-7) are shown in Figure 3-5 for different values of parameters, λ_h and λ_{hi}..

The Descriptive Equations of the Model

The differential equations associated with Figure 3-4 can be written as:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=m+n}^{m+n+2}\int_0^\infty P_j(x,t)\mu_j(x)dx \tag{3-8}$$

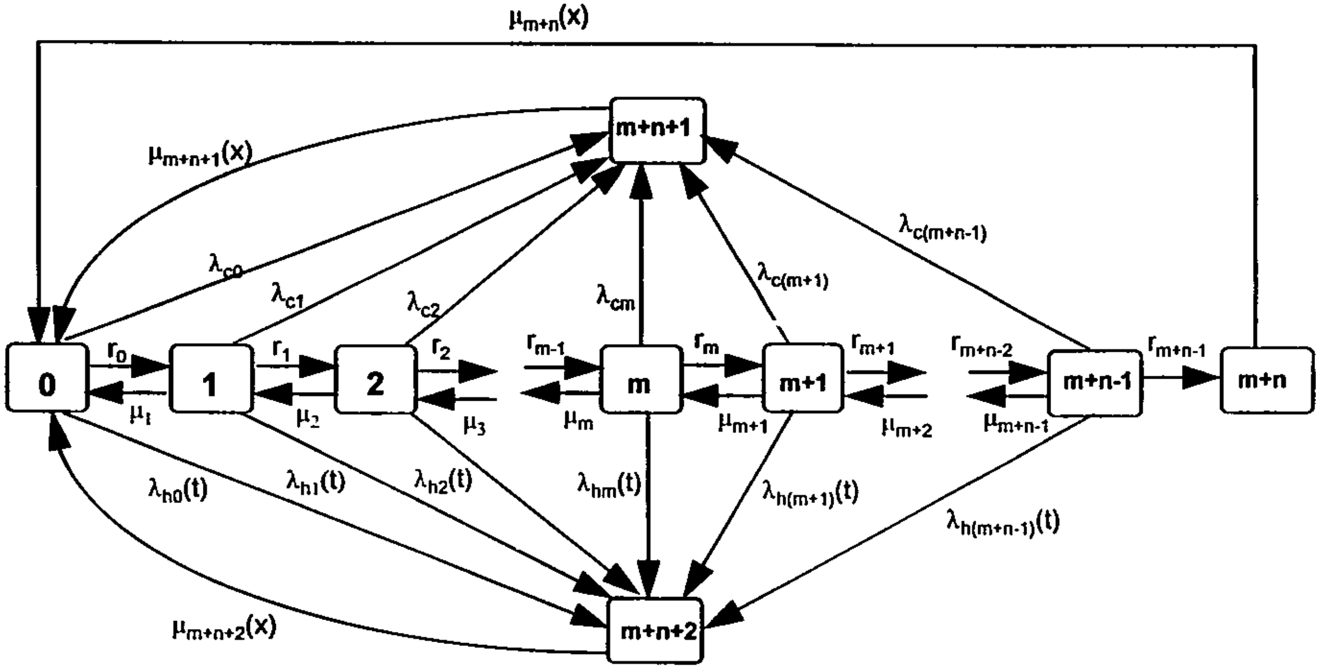


Figure 3-3 The state transition diagram of a general standby system with time-dependent human error rates and failed system repair rates

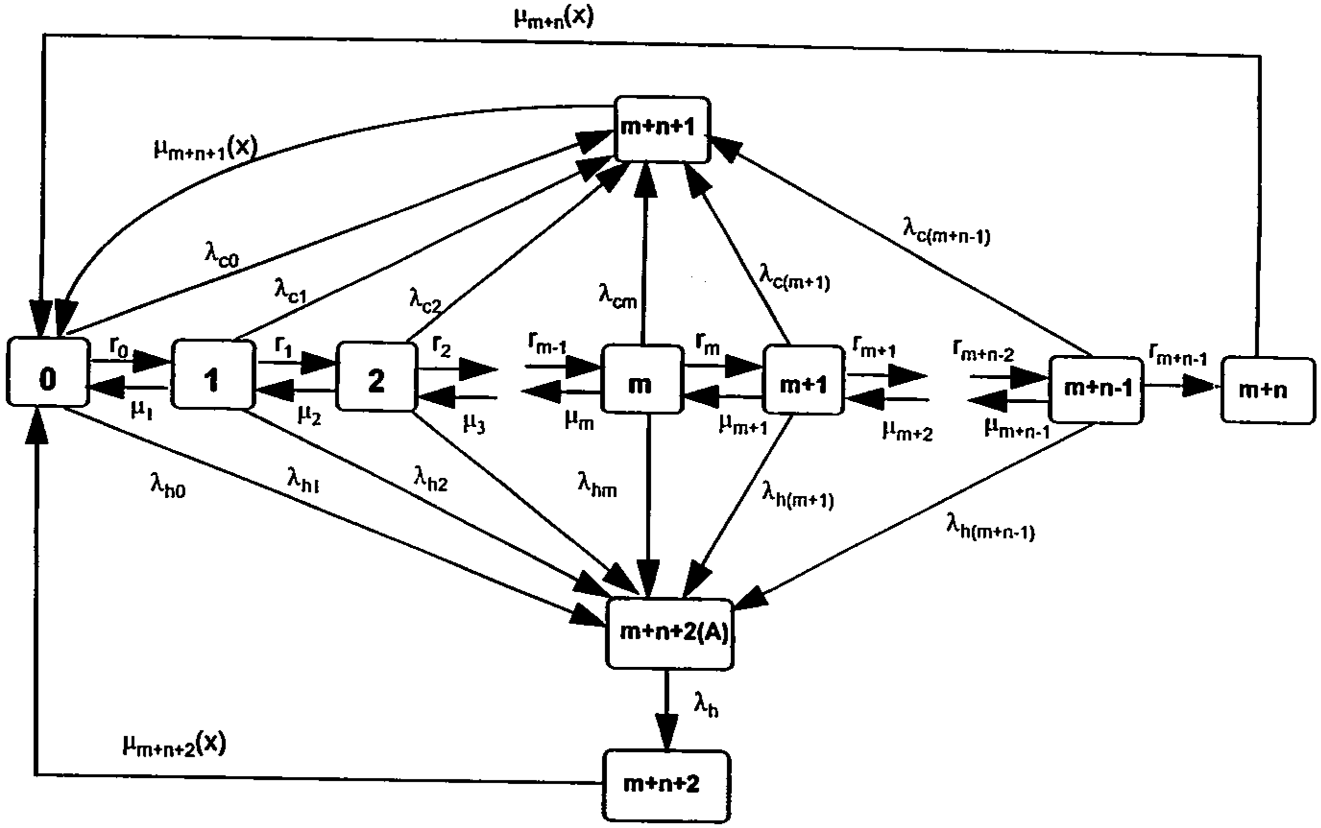


Figure 3-4 The state transition diagram of a general standby system with critical human error state represented by a device of two stages

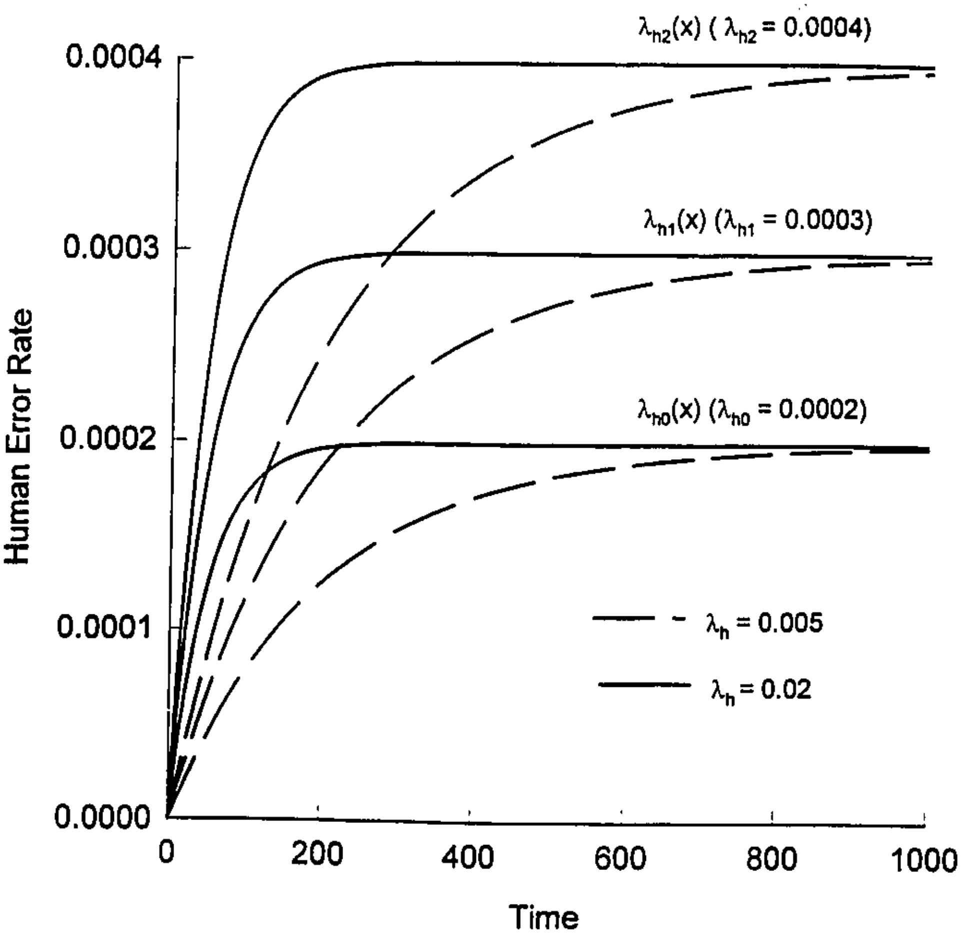


Figure 3-5 Human error rate plots associated with Equation (3-7)

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \qquad (3\text{-}9)$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) - a_2 P_2(t) + \mu_3 P_3(t) \qquad (3\text{-}10)$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \qquad (3\text{-}11)$$

(for i = 2, 3, ..., m+n-2)

$$\frac{dP_{m+n-1}(t)}{dt} = r_{m+n-2} P_{m+n-2}(t) - a_{m+n-1} P_{m+n-1}(t) \qquad (3\text{-}12)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \qquad (3\text{-}13)$$

(for j = m+n, m+n+1, m+n+2)

$$\frac{dP_{m+n+2(A)}(t)}{dt} = \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) - \lambda_h P_{m+n+2(A)}(t) \qquad (3\text{-}14)$$

The associated boundary conditions are as follows:

$$P_{m+n}(0,t) = r_{m+n-1} P_{m+n-1}(t) \qquad (3\text{-}15)$$

$$P_{m+n+1}(0,t) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(t) \qquad (3\text{-}16)$$

$$P_{m+n+2}(0,t) = \lambda_h P_{m+n+2(A)}(t) \qquad (3\text{-}17)$$

where

$$a_i = r_i + \lambda_{ci} + \lambda_{hi} + \mu_i$$

(for i = 0, 1, 2, 3, …, m+n-1, and assuming $\mu_0=0$)

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

3.2.2 Steady State Availability Analysis

As time approaches infinity, Equations (3-8) - (3-14) reduce to following Equations:

$$a_0P_0 - \mu_1P_1 = \sum_{j=m+n}^{m+n+2}\int_0^{\infty} P_j(x)\mu_j(x)dx \qquad (3\text{-}18)$$

$$r_0P_0 - a_1P_1 + \mu_2P_2 = 0 \qquad (3\text{-}19)$$

$$r_{i-1}P_{i-1} - a_iP_i + \mu_{i+1}P_{i+1} = 0 \qquad (3\text{-}20)$$

(i = 2, 3, ..., m+n-2)

$$r_{m+n-2}P_{m+n-2} - a_{m+n-1}P_{m+n-1} = 0 \qquad (3\text{-}21)$$

$$\frac{dP_j(x)}{dx} = -\mu_j(x)P_j(x) \qquad (3\text{-}22)$$

(j = m+n, m+n+1, m+n+2)

$$\sum_{i=0}^{m+n-1}\lambda_{hi}P_i - \lambda_h P_{m+n+2(A)} = 0 \qquad (3\text{-}23)$$

Similarly, the boundary conditions become:

$$P_{m+n}(0) = r_{m+n-1}P_{m+n-1} \qquad (3\text{-}24)$$

$$P_{m+n+1}(0) = \sum_{i=0}^{m+n-1}\lambda_{ci}P_i \qquad (3\text{-}25)$$

$$P_{m+n+2}(0) = \lambda_h P_{m+n+2(A)} \qquad (3\text{-}26)$$

Solving Equation (3-22), we get

$$P_j(x) = P_j(0)\exp(-\int_0^x \mu_j(\omega)d\omega) \qquad \text{(for } j = m+n, m+n+1,\ m+n+2\text{)} \quad (3\text{-}27)$$

The steady state probabilities are:

$$P_j = \int_0^{\infty} P_j(x)dx; \qquad (\text{for } j = m+n,\ m+n+1,\ m+n+2) \tag{3-28}$$

Substituting Equations (3-27) into Equation (3-28), we get:

$$\begin{aligned} P_j &= \int_0^{\infty} P_j(x)dx \\ &= \int_0^{\infty} P_j(0)\exp\left(-\int_0^x \mu_j(\omega)d\omega\right)dx \qquad (\text{for j} = \text{m} + \text{n},\ \text{m} + \text{n} + 1,\ \text{m} + \text{n} + 2) \end{aligned} \tag{3-29}$$

Substituting Equations (3-24)-(3-26) in Equation (3-29), we obtain the steady state probabilities for state m+n, m+n+1, and m+n+2:

$$P_{m+n} = r_{m+n-1} P_{m+n-1} E_{m+n}[x] \tag{3-30}$$

$$P_{m+n+1} = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i E_{m+n+1}[x] \tag{3-31}$$

$$P_{m+n+2} = \lambda_h P_{m+n+2(A)} E_{m+n+2}[x] \tag{3-32}$$

where

$$\begin{aligned} &E_j[x] = \int_0^{\infty} \exp\left(-\int_0^x \mu_j(\omega)d\omega\right)dx \\ &(\text{for j} = \text{m} + \text{n},\ \text{m} + \text{n} + 1,\ \text{m} + \text{n} + 2) \end{aligned} \tag{3-33}$$

which is the mean time to system repair when the failed system is in state j and has an elapsed repair time of x.

Solving the set of Equations (3-19) - (3-21), (3-23) and (3-30) - (3-32), together with

$$\sum_{i=0}^{m+n+2} P_i + P_{m+n+2(A)} = 1 \tag{3-34}$$

we can obtain steady state probabilities, P_0, P_1, P_2, ..., P_{m+n}, P_{m+n+1}, P_{m+n+2}, $P_{m+n+2(A)}$.

Thus, the steady state availability of the system is

$$AV_{ss} = \sum_{i=0}^{m+n-1} P_i + P_{m+n+2(A)} \tag{3-35}$$

Similarly, the steady state unavailability of the system is given by

$$UAV_{ss} = P_{m+n} + P_{m+n+1} + P_{m+n+2} \tag{3-36}$$

3.2.3. Time-Dependent Availability Analysis

Using Laplace Transform technique and the initial conditions in Equations (3-8) - (3-14), we get

$$s\,P_0(s) = 1 - a_0 P_0(s) + \mu_1 P_1(s) + \sum_{j=m+n}^{m+n+2} \int_0^{\infty} P_j(x,s)\mu_j(x)dx \tag{3-37}$$

$$(s+a_1)P_1(s) - r_0 P_0(s) - \mu_2 P_2(s) = 0 \tag{3-38}$$

$$(s+a_i)P_i(s) - r_{i-1}P_{i-1}(s) - \mu_{i+1}P_{i+1}(s) = 0 \qquad (\text{for i} = 2, 3, \ldots, \text{m}+\text{n}-2) \tag{3-39}$$

$$(s+a_{m+n-1})P_{m+n-1}(s) - r_{m+n-2}P_{m+n-2}(s) = 0 \tag{3-40}$$

$$\frac{\partial P_j(x,s)}{\partial x} + s\,P_j(x,s) + \mu_j(x)P_j(x,s) = 0 \tag{3-41}$$

(for j = m+n, m+n+1, m+n+2)

$$(s+\lambda_h)P_{m+n+2(A)}(s) - \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(s) = 0 \tag{3-42}$$

and the boundary conditions:

$$P_{m+n}(0,s) = r_{m+n-1}P_{m+n-1}(s) \tag{3-43}$$

$$P_{m+n+1}(0,s) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(s) \tag{3-44}$$

$$P_{m+n+2}(0,s) = \lambda_h P_{m+n+2(A)}(s) \tag{3-45}$$

Solving differential Equation (3-41), we get:

$$P_j(x,s) = P_j(0,s)e^{-sx}\exp(-\int_0^x \mu_j(\omega)d\omega) \qquad (3\text{-}46)$$

(for j = m+n, m+n+1, m+n+2)

Since

$$P_j(s) = \int_0^\infty P_j(x,s)dx \qquad (\text{for } j = m+n,\ m+n+1, m+n+2) \qquad (3\text{-}47)$$

substituting the boundary conditions in Equation (3-46) and (3-47), lead to

$$P_{m+n}(s) = r_{m+n-1}P_{m+n-1}(s)\frac{1-N_{m+n}(s)}{s} \qquad (3\text{-}48)$$

$$P_{m+n+1}(s) = \sum_{i=0}^{m+n-1}\lambda_{ci}P_i(s)\frac{1-N_{m+n+1}(s)}{s} \qquad (3\text{-}49)$$

$$P_{m+n+2}(s) = \lambda_h P_{m+n+2(A)}(s)\frac{1-N_{m+n+2}(s)}{s} \qquad (3\text{-}50)$$

where

$$\frac{1-N_j(s)}{s} = \int_0^\infty e^{-sx}\exp(-\int_0^x \mu_j(\omega)d\omega)dx \qquad (3\text{-}51)$$

(for j = m+n, m+n+1, m+n+2)

and $N_j(s)$ is the Laplace transform of $N_j(x)$.

The Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_{m+n}(s)$, $P_{m+n+1}(s)$, $P_{m+n+2}(s)$, $P_{m+n+2(A)}(s)$, were obtained by solving Equations (3-38) - (3-40), (3-48) - (3-50) and the following equation:

$$\sum_{i=0}^{m+n+2} P_i(s) + P_{m+n+2(A)}(s) = \frac{1}{s} \qquad (3\text{-}52)$$

The Laplace Transform of the system availability is

$$AV(s) = \sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s) \tag{3-53}$$

Substituting the Laplace transform of $N_i(x)$ for different repair time distributions in Equation (3-53) and taking the inverse Laplace transform of the resulting equation, we get the following time-dependent system availability:

$$AV(t) = \sum_{i=0}^{m+n-1} P_i(t) + P_{m+n+2(A)}(t) \tag{3-54}$$

3.2.4 System Reliability and MTTF With and Without Repair

Setting $\mu_{m+n}(x) = \mu_{m+n+1}(x) = \mu_{m+n+2}(x) = 0$ in Figure 3-4, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \tag{3-55}$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \tag{3-56}$$

(for i = 1, 2, 3, ..., m+n-2)

$$\frac{dP_{m+n-1}(t)}{dt} = r_{m+n-2} P_{m+n-2}(t) - a_{m+n-1} P_{m+n-1}(t) \tag{3-57}$$

$$\frac{dP_{m+n}(t)}{dt} = r_{m+n-1} P_{m+n-1}(t) \tag{3-58}$$

$$\frac{dP_{m+n+1}(t)}{dt} = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(t) \tag{3-59}$$

$$\frac{dP_{m+n+2(A)}(t)}{dt} = \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) - \lambda_h P_{m+n+2(A)}(t) \tag{3-60}$$

$$\frac{dP_{m+n+2}(t)}{dt} = \lambda_h P_{m+n+2(A)}(t) \tag{3-61}$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, ..., m+n+2, m+n+2(A).

Using Laplace transforms in Equations (3-55) - (3-61) and solving the resulting set of Equations. we get the Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_{m+n+2}(s)$, $P_{m+n+2(A)}(s)$.

Thus, the Laplace transform of the system reliability with repair is:

$$R(s) = \sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s) \qquad (3\text{-}62)$$

The time-dependent system reliability with repair may be obtained by inverting the above equation:

$$R(t) = L^{-1}[R(s)] = L^{-1}\left[\sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s)\right] \qquad (3\text{-}63)$$

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s\to 0} R(s) = \lim_{s\to 0}\left[\sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s)\right] \qquad (3\text{-}64)$$

The system variance of time to failure with repair is expressed by

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2 \qquad (3\text{-}65)$$

where R′(s) denotes the derivative of R(s) with respect to s.

For $\mu_1 = \mu_2 = \ldots = \mu_{m+n-1} = 0$ in Equations (3-63) - (3-65), we can get the system reliability, MTTF and the system variance of time to failure without repair, respectively.

3.3. Special Cases of Standby System With Increasing Human Error Rates

3.3.1. One Unit Active and One Unit on Standby System

For m = 1 and n = 1 in Figure 3-4, the system of differential equations associated with the model can be written as follows:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=2}^{4} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \qquad (3\text{-}66)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) \qquad (3\text{-}67)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \qquad (3\text{-}68)$$

(j=2, 3, 4)

$$\frac{dP_{4A}(t)}{dt} = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) - \lambda_h P_{4A}(t) \qquad (3\text{-}69)$$

The associated boundary conditions are as follows:

$$P_2(0,t) = r_1 P_1(t) \qquad (3\text{-}70)$$

$$P_3(0,t) = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \qquad (3\text{-}71)$$

$$P_4(0,t) = \lambda_h P_{4A}(t) \qquad (3\text{-}72)$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

As time approaches infinity, Equations (3-66) - (3-72) reduce to Equations (3-73) - (3-76), respectively.

$$a_0 P_0 - \mu_1 P_1 = \sum_{j=2}^{4} \int_0^\infty P_j(x)\mu_j(x)dx \qquad (3\text{-}73)$$

$$r_0 P_0 - a_1 P_1 = 0 \qquad (3\text{-}74)$$

$$\frac{dP_j(x)}{dx} = -\mu_j(x)P_j(x) \qquad (3\text{-}75)$$

(for $j = 2, 3, 4$)

$$\lambda_{h0} P_0 + \lambda_{h1} P_1 - \lambda_h P_{4A} = 0 \qquad (3\text{-}76)$$

Similarly, the boundary conditions become:

$$P_2(0) = r_1 P_1 \qquad (3\text{-}77)$$

$$P_3(0) = \lambda_{c0} P_0 + \lambda_{c1} P_1 \qquad (3\text{-}78)$$

$$P_4(0) = \lambda_h P_{4A} \qquad (3\text{-}79)$$

Substituting Equations (3-77) - (3-79) in Equation (2-20), for j =2, 3, 4, we have

$$P_2 = r_1 P_1 E_2[x] \qquad (3\text{-}80)$$

$$P_3 = (\lambda_{c0} P_0 + \lambda_{c1} P_1) E_3[x] \qquad (3\text{-}81)$$

$$P_4 = \lambda_h P_{4A} E_4[x] \qquad (3\text{-}82)$$

Solving the set of Equations (3-74), (3-76), (3-80) - (3-82) together with

$$\sum_{i=0}^{4} P_i + P_{4A} = 1 \tag{3-83}$$

we get the following steady state probabilities:

$$P_0 = \frac{b_0(1,1)\lambda_h}{BA(1,1)\lambda_h + b_4(1,1)} \tag{3-84}$$

$$P_1 = \frac{b_1(1,1)\lambda_h}{BA(1,1)\lambda_h + b_4(1,1)} \tag{3-85}$$

$$P_2 = \frac{b_2(1,1)\lambda_h E_2[x]}{BA(1,1)\lambda_h + b_4(1,1)} \tag{3-86}$$

$$P_3 = \frac{b_3(1,1)\lambda_h E_3[x]}{BA(1,1)\lambda_h + b_4(1,1)} \tag{3-87}$$

$$P_4 = \frac{b_4(1,1)\lambda_h E_4[x]}{BA(1,1)\lambda_h + b_4(1,1)} \tag{3-88}$$

$$P_{4A} = \frac{b_4(1,1)}{BA(1,1)\lambda_h + b_4(1,1)} \tag{3-89}$$

where, $b_0(1,1)$ - $b_4(1,1)$ and BA(1,1) are as shown in Chapter 2.2.1.

Thus, the steady state availability of the system is

$$\begin{aligned} AV_{ss}(1,1) &= P_0 + P_1 + P_{4A} \\ &= \frac{\sum_{i=0}^{1} b_i(1,1)\lambda_h + b_4(1,1)}{BA(1,1)\lambda_h + b_4(1,1)} \end{aligned} \tag{3-90}$$

Similarly, the system steady state unavailability is given by

$$UAV_{ss}(1,1) = P_2 + P_3 + P_4 = \frac{\sum_{j=2}^{4} b_j(1,1)E_j[x]\lambda_h}{BA(1,1)\lambda_h + b_4(1,1)} \qquad (3\text{-}91)$$

System Steady State Availability Special Cases

Case I

If the system repair time x is ***Gamma*** distributed, the system steady state availability is given by

$$AV_{ss}(1,1) = \frac{\sum_{i=0}^{1} b_i(1,1)\lambda_h + b_4(1,1)}{\left[\sum_{i=0}^{1} b_i(1,1) + \sum_{j=2}^{4} b_j(1,1)\beta/\mu_j\right]\lambda_h + b_4(1,1)} \qquad (3\text{-}92)$$

Case II

If the system repair time x is ***Weibull*** distributed, the system steady state availability is given by

$$AV_{ss}(1,1) = \frac{\sum_{i=0}^{1} b_i(1,1)\lambda_h + b_4(1,1)}{\left[\sum_{i=0}^{1} b_i(1,1) + \sum_{j=2}^{4} b_j(1,1)\Gamma(1+1/\beta)/\mu_j\right]\lambda_h + b_4(1,1)} \qquad (3\text{-}93)$$

Case III

If the system repair time x is ***Rayleigh*** distributed, the system steady state availability is given by

$$AV_{ss}(1,1)=\frac{\sum_{i=0}^{1}b_i(1,1)\lambda_h+b_4(1,1)}{\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\sqrt{\pi/2}\big/\mu_j\right]\lambda_h+b_4(1,1)} \qquad (3\text{-}94)$$

Case IV

If the system repair time x is ***lognormal*** distributed, the system steady state availability is given by

$$AV_{ss}(1,1)=\frac{\sum_{i=0}^{1}b_i(1,1)\lambda_h+b_4(1,1)}{\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\exp(\mu_j+\sigma_j^{\,2}/2)\right]\lambda_h+b_4(1,1)} \qquad (3\text{-}95)$$

Time-Dependent Availability Analysis

Using Laplace transform and the initial conditions in Equations (3-66) - (3-72), we get

$$s\,P_0(s)=1-a_0P_0(s)+\mu_1P_1(s)+\sum_{j=2}^{4}\int_0^{\infty}P_j(x,s)\mu_j(x)dx \qquad (3\text{-}96)$$

$$(s+a_1)P_1(s)-r_0P_0(s)=0 \qquad (3\text{-}97)$$

$$\frac{\partial P_j(x,s)}{\partial x}+s\,P_j(x,s)+\mu_j(x)P_j(x,s)=0 \qquad (3\text{-}98)$$

(for j = 2, 3, 4)

$$(s+\lambda_h)P_{4A}(s)-\sum_{i=0}^{1}\lambda_{hi}P_i(s)=0 \qquad (3\text{-}99)$$

and the boundary conditions:

$$P_2(0,s) = r_1\, P_1(s) \qquad (3\text{-}100)$$

$$P_3(0,s) = \lambda_{c0} P_0(s) + \lambda_{c1} P_1(s) \qquad (3\text{-}101)$$

$$P_4(0,s) = \lambda_h P_{4A}(s) \qquad (3\text{-}102)$$

Substituting the boundary conditions in Equation (2-36), we have:

$$P_2(s) = r_1 P_1(s) \frac{1 - N_2(s)}{s} \qquad (3\text{-}103)$$

$$P_3(s) = \left[\lambda_{c0} P_0(s) + \lambda_{c1} P_1(S)\right] \frac{1 - N_3(s)}{s} \qquad (3\text{-}104)$$

$$P_4(s) = \lambda_h P_{4A}(s) \frac{1 - N_4(s)}{s} \qquad (3\text{-}105)$$

Solving Equations (3-97), (3-99), (3-103) - (3-105) together with

$$\sum_{i=0}^{4} P_i(s) + P_{4A}(s) = \frac{1}{s} \qquad (3\text{-}106)$$

we get the following Laplace transforms of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_4(s)$, $P_{4A}(s)$:

$$P_0(s) = \frac{[s + b_0(1,1)](s + \lambda_h)}{CB(1,1)} \qquad (3\text{-}107)$$

$$P_1(s) = \frac{b_1(1,1)(s + \lambda_h)}{CB(1,1)} \qquad (3\text{-}108)$$

$$P_2(s) = \frac{\left[b_2(1,1)\lambda_h + s\right]\left[1 - N_2(s)\right]}{s \cdot CB(1,1)} \qquad (3\text{-}109)$$

$$P_3(s) = \frac{\left[b_3(1,1) + \lambda_{c0} s\right]\left[s + \lambda_h\right]\left[1 - N_3(s)\right]}{s \cdot CB(1,1)} \qquad (3\text{-}110)$$

$$P_4(s) = \frac{\left[b_4(1,1) + \lambda_{h0} s\right]\left[1 - N_4(s)\right]\lambda_h}{s \cdot CB(1,1)} \qquad (3\text{-}111)$$

$$P_{4A}(s) = \frac{b_4(1,1) + \lambda_{h0}s}{CB(1,1)} \tag{3-112}$$

where

$$CB(1,1) = cb1(1,1) + cb2(1,1) \cdot s + cb3(1,1) \cdot s^2 + s^3$$

$$cb1(1,1) = \lambda_h \cdot ca1(1,1)$$

$$cb2(1,1) = \sum_{j=2}^{3} b_j(1,1)\left[1 - N_j(s)\right] + b_4(1,1) + \lambda_h \cdot ca2(1,1)$$

$$cb3(1,1) = \sum_{i=0}^{1} b_i(1,1) + \lambda_{c0}\left[1 - N_3(s)\right] + \lambda_{h0} + \lambda_h$$

The Laplace transform of the system availability is

$$\begin{aligned} AV(s) &= P_0(s) + P_1(s) + P_{4A}(s) \\ &= \frac{\left[s + \sum_{i=0}^{1} b_i(1,1)\right] \cdot \left[s + \lambda_h\right] + \lambda_{h0} \cdot s + b_4(1,1)}{CB(1,1)} \end{aligned} \tag{3-113}$$

Substituting the Laplace transform of pdf of system repair distribution, $N_i(s)$ for j = 2, 3, 4, in the above Equation and taking the inverse Laplace transform of the resulting equation, we get the time-dependent system availability:

$$AV(t) = P_0(t) + P_1(t) + P_{4A}(t) \tag{3-114}$$

System Reliability and MTTF With and Without Repair

Setting $\mu_2(x) = \mu_3(x) = \mu_4(x) = 0$ in the model, the system of differential equations becomes:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \tag{3-115}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) \tag{3-116}$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) \tag{3-117}$$

$$\frac{dP_3(t)}{dt} = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \tag{3-118}$$

$$\frac{dP_{4A}(t)}{dt} = \lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) - \lambda_h P_{4A}(t) \tag{3-119}$$

$$\frac{dP_4(t)}{dt} = \lambda_h P_{4A}(t) \tag{3-120}$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, 3, 4.

Using Laplace transform in Equations (3-115) - (3-120), and solving the resulting set of equations. we can get the Laplace transforms of the state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_4(s)$.

Thus, the Laplace transform of the system reliability with repair is

$$\begin{aligned} R(s) &= P_0(s) + P_1(s) + P_{4A}(s) \\ &= \frac{s^2 + (a_1 + r_0 + \lambda_{h0} + \lambda_h)s + b_4(1,1) + (a_1 + r_0)\lambda_h}{(s + \lambda_h) \cdot DA(1,1)} \end{aligned} \tag{3-121}$$

The system reliability with repair can be obtained by inverting Equation (3-121):

$$\begin{aligned} R(t) = {} & \frac{e^{s_1 t}(s_a s_b - s_a s_1 - s_b s_1 + s_1{}^2)}{(s_1 - s_2)(s_1 - s_3)} + \frac{e^{s_2 t}(s_a s_b - s_a s_2 - s_b s_2 + s_2{}^2)}{(s_2 - s_1)(s_2 - s_3)} + \\ & \frac{e^{s_3 t}(s_a s_b - s_a s_3 - s_b s_3 + s_3{}^2)}{(s_3 - s_2)(s_3 - s_1)} \end{aligned} \tag{3-122}$$

where s_a and s_b are the roots of the numerator of Equation (3-121), s_1, s_2 and s_3 are real and unique roots of the denominator of Equation (3-121).

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s \to 0} R(s) = \frac{\lambda_h (a_1 + r_0) + b_4(1,1)}{\lambda_h \cdot da1(1,1)} \tag{3-123}$$

The variance of time to failure with repair of the system is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \to 0} R'(s) - (MTTF)^2 \\ &= -2 \frac{\lambda_h{}^2 \left[da2(1,1) \left(a_1 + r_0 \right) + da1(1,1) \right] + da1(1,1) \left[b_4(1,1) + \lambda_h \lambda_{h0} \right] + da2(1,1) \cdot b_4(1,1) \lambda_h}{\left[da1(1,1) \cdot \lambda_h \ \right]^2} - \\ &\quad - \frac{\left[(a_1 + r_0) \lambda_h + b_4(1,1) \right]^2}{\lambda_h{}^2 \left[da1(1,1) \right]^2} \end{aligned} \tag{3-124}$$

Setting $\mu_1 = 0$ in Equations (3-122) - (3-124), we can get the system reliability, MTTF and the variance of time to failure without repair, respectively.

3.3.2. Two Units Active and One Unit on Standby System

For m = 2 and n = 1 in Figure 3-4, the system of differential equations associated with the model can be written as follows:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=3}^{5} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \qquad (3\text{-}125)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \qquad (3\text{-}126)$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) - a_2 P_2(t) \qquad (3\text{-}127)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \qquad (3\text{-}128)$$

(for j = 3, 4, 5)

$$\frac{dP_{5A}(t)}{dt} = \sum_{i=0}^{2} \lambda_{h_i} P_i(t) - \lambda_{h_t} P_{5A}(t) \qquad (3\text{-}129)$$

The associated boundary conditions are as follows:

$$P_3(0,t) = r_2 P_2(t) \qquad (3\text{-}130)$$

$$P_4(0,t) = \sum_{i=0}^{2} \lambda_{ci} P_i(t) \qquad (3\text{-}131)$$

$$P_5(0,t) = \lambda_{h_t} P_{5A}(t) \qquad (3\text{-}132)$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

The steady state probabilities are as follows:

$$P_0 = \frac{b_0(2,1)\lambda_h}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}133)$$

$$P_1 = \frac{b_1(2,1)\lambda_h}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}134)$$

$$P_2 = \frac{b_2(2,1)\lambda_h}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}135)$$

$$P_3 = \frac{b_3(2,1)\lambda_h E_3[x]}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}136)$$

$$P_4 = \frac{b_4(2,1)\lambda_h E_4[x]}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}137)$$

$$P_5 = \frac{b_5(2,1)\lambda_h E_5[x]}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}138)$$

$$P_{5A} = \frac{b_5(2,1)}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}139)$$

Thus, the steady state availability of the system is

$$\begin{aligned} AV_{ss}(2,1) &= P_0 + P_1 + P_2 + P_{5A} \\ &= \frac{\sum_{i=0}^{2} b_i(2,1)\lambda_h + b_5(2,1)}{BA(2,1)\lambda_h + b_5(2,1)} \end{aligned} \qquad (3\text{-}140)$$

Similarly, the steady state unavailability of the system is given by

$$UAV_{ss}(2,1) = P_3 + P_4 + P_5$$

$$= \frac{\sum_{j=3}^{5} b_j(2,1) E_j[x] \lambda_h}{BA(2,1)\lambda_h + b_5(2,1)} \qquad (3\text{-}141)$$

System Steady State Availability Special Cases

Case I

If the system repair time x is ***Gamma*** distributed, the system steady state availability is given by

$$AV_{ss}(2,1) = \frac{\sum_{i=0}^{2} b_i(2,1)\lambda_h + b_5(2,1)}{\left[\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\beta/\mu_j\right]\lambda_h + b_5(2,1)} \qquad (3\text{-}142)$$

Case II

If the system repair time x is ***Weibull*** distributed, the system steady state availability is given by

$$AV_{ss}(2,1) = \frac{\sum_{i=0}^{2} b_i(2,1)\lambda_h + b_5(2,1)}{\left[\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\Gamma(1+1/\beta)/\mu_j\right]\lambda_h + b_5(2,1)} \qquad (3\text{-}143)$$

Case III

If the system repair time x is ***Rayleigh*** distributed, the system steady state availability is given by

$$AV_{ss}(2,1)=\frac{\sum_{i=0}^{2}b_i(2,1)\lambda_h+b_5(2,1)}{\left[\sum_{i=0}^{2}b_i(2,1)+\sum_{j=3}^{5}b_j(2,1)\sqrt{\pi/2}\Big/\mu_j\right]\lambda_h+b_5(2,1)} \qquad (3\text{-}144)$$

Case IV

If the system repair time x is ***lognormal*** distributed, the system steady state availability is given by

$$AV_{ss}(2,1)=\frac{\sum_{i=0}^{2}b_i(2,1)\lambda_h+b_5(2,1)}{\left[\sum_{i=0}^{2}b_i(2,1)+\sum_{i=3}^{5}b_i(2,1)\exp\left(\mu_i+\sigma^2/2\right)\right]\lambda_h+b_5(2,1)} \qquad (3\text{-}145)$$

System Steady State Availability Numerical Examples

Setting

$r_0 = r_1 = 0.0005\,/\,hr$ $\quad r_2 = 0.0008\,/\,hr$ $\quad \mu_1 = 0.001\,/\,hr$ $\quad \mu_2 = 0.002\,/\,hr$

$\lambda_{c0} = \lambda_{c1} = 0.00005\,/\,hr$ $\quad \lambda_{c2} = 0.00002\,/\,hr$ $\quad \lambda_{h1} = 0.0002\,/\,hr$ $\quad \lambda_{h2} = 0.0001\,/\,hr$

in above equations, the plots of system steady state availability as a function of human error rate parameters, λ_{h0} and λ_h, are shown in Figures 3-6 and 3-7 for different values of β and for Gamma and Weibull distributed repair times, respectively, for specified values of parameters $\mu_3 = 0.001\,/\,hr$, $\mu_4 = \mu_5 = 0.0009\,/\,hr$.

The plots of the system steady state availability as a function of human error rate parameters, λ_{h0} and λ_h, are shown in Figure 3-8 for Rayleigh distributed failed system repair times for specified values of parameters (μ_3=0.001/hr, μ_4=0.0009/hr and different values of μ_5).

Also, Figure 3-9 provides the results of the system steady state availability as a function of human error rate parameters, λ_{ho} and λ_h, for lognormally distributed failed system repair times ($\mu_3 = 1.001$, $\mu_4 = \mu_5 = 1.0009$, $\sigma_3 = \sigma_4 = \sigma_5 = \sigma$).

Figure 3-10 shows plots of system steady state availability as a function of human error rate parameters, λ_{ho} and λ_h, for specified values of repair parameters, $\mu_3 = 0.001 / hr, \mu_4 = \mu_5 = 0.0009 / hr$, for different system repair time distributions: exponential, Gamma, Rayleigh, Weibull and lognormal ($\mu_3 = 1.001$, $\mu_4 = \mu_5 = 1.0009$, $\sigma_3 = \sigma_4 = \sigma_5 = \sigma = 3$).

Time-Dependent System Availability Analysis

Using Laplace transform and the initial conditions in Equations (3-125) - (3-129) and solving the resulting equations, we get the following Laplace transform of state probabilities:

$$P_0(s) = \frac{\left[b_0(2,1) + (a_1 + a_2)s + s^2\right](s + \lambda_h)}{CB(2,1)} \tag{3-146}$$

$$P_1(s) = \frac{\left[b_1(2,1) + r_0 s\right](s + \lambda_h)}{CB(2,1)} \tag{3-147}$$

$$P_2(s) = \frac{(s + \lambda_h) b_2(2,1)}{CB(2,1)} \tag{3-148}$$

$$P_3(s) = \frac{(s + \lambda_h) b_3(2,1)[1 - N_3(s)]}{s \cdot CB(2,1)} \tag{3-149}$$

$$P_4(s) = \frac{(s + \lambda_h)[1 - N_4(s)]\left[b_4(2,1) + lc(2,1) \cdot s + \lambda_{c0} \cdot s^2\right]}{s \cdot CB(2,1)} \tag{3-150}$$

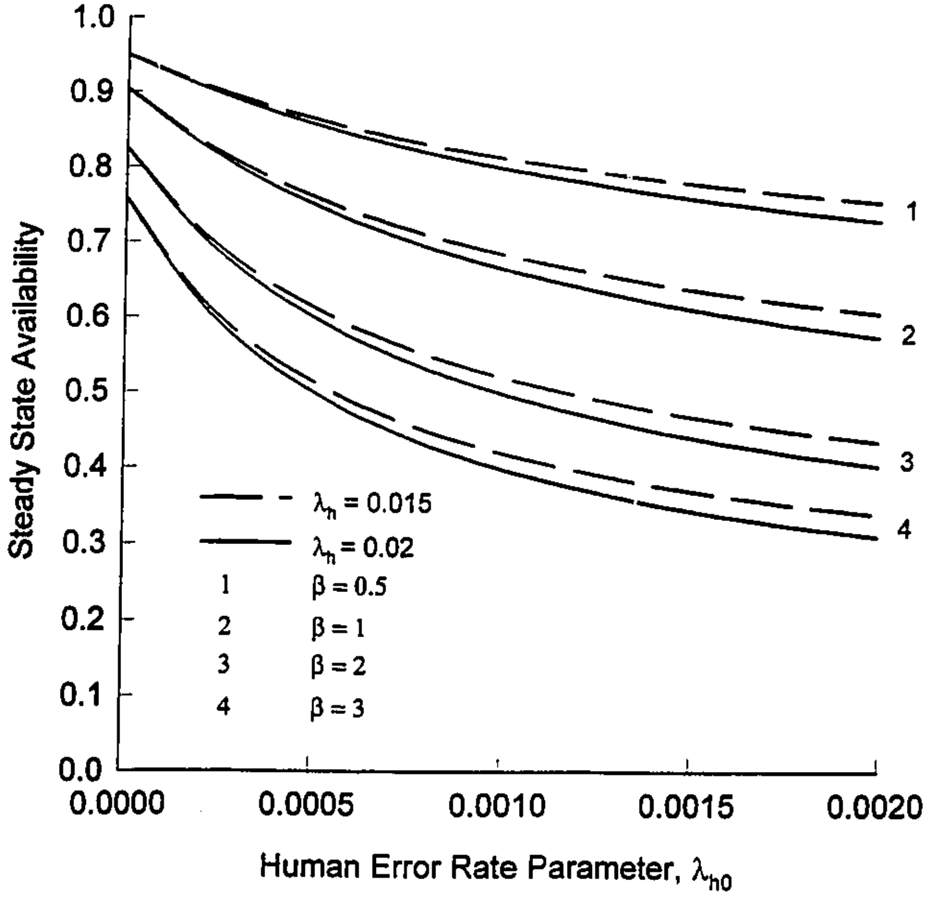


Figure 3-6 System steady state availability plots for two units active and one on standby system with increasing human error rates and the Gamma distributed failed system repair times

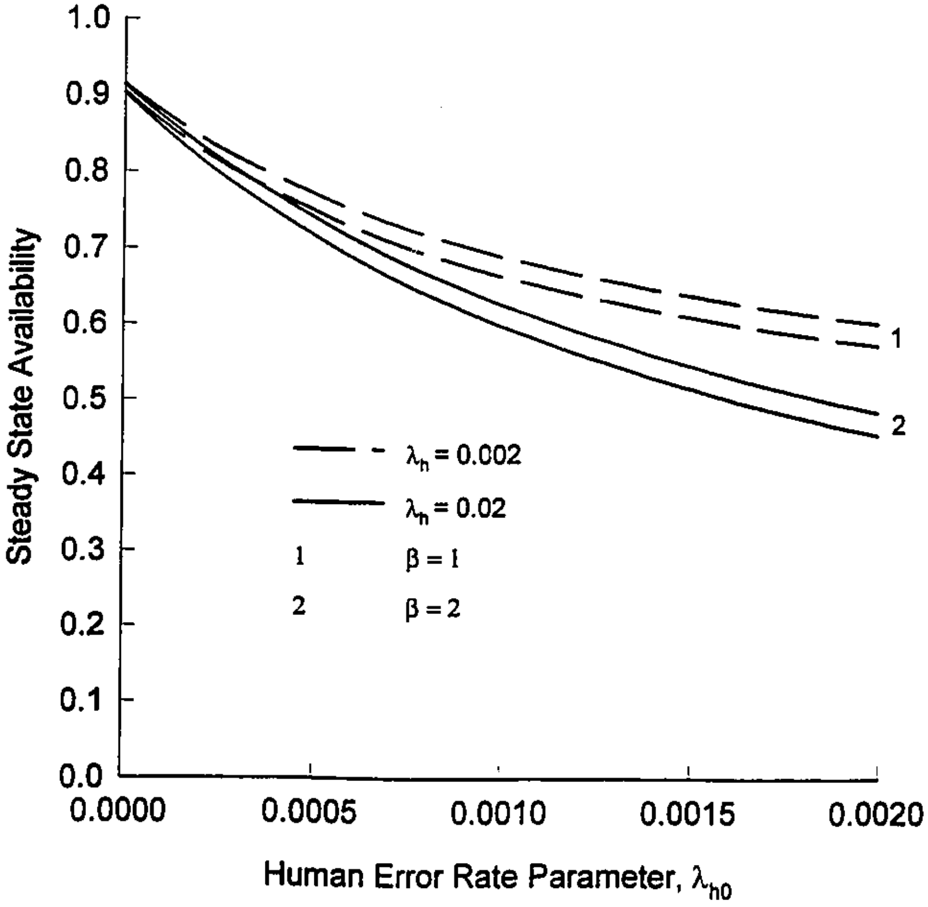


Figure 3-7 System steady state availability plots for two units active and one on standby system with increasing human error rates and the Weibull distributed failed system repair times

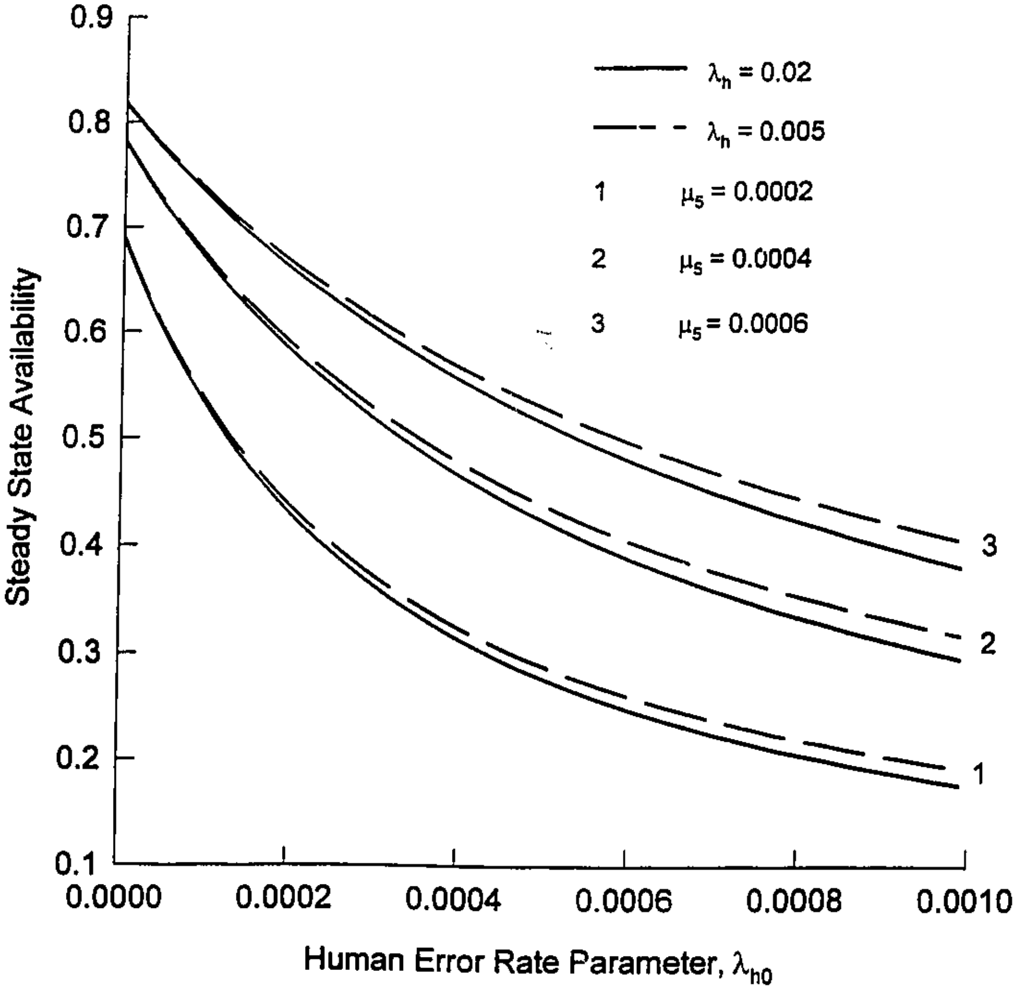


Figure 3-8 System steady state availability plots for two units active and one on standby system with increasing human error rates and the Rayleigh distributed failed system repair times

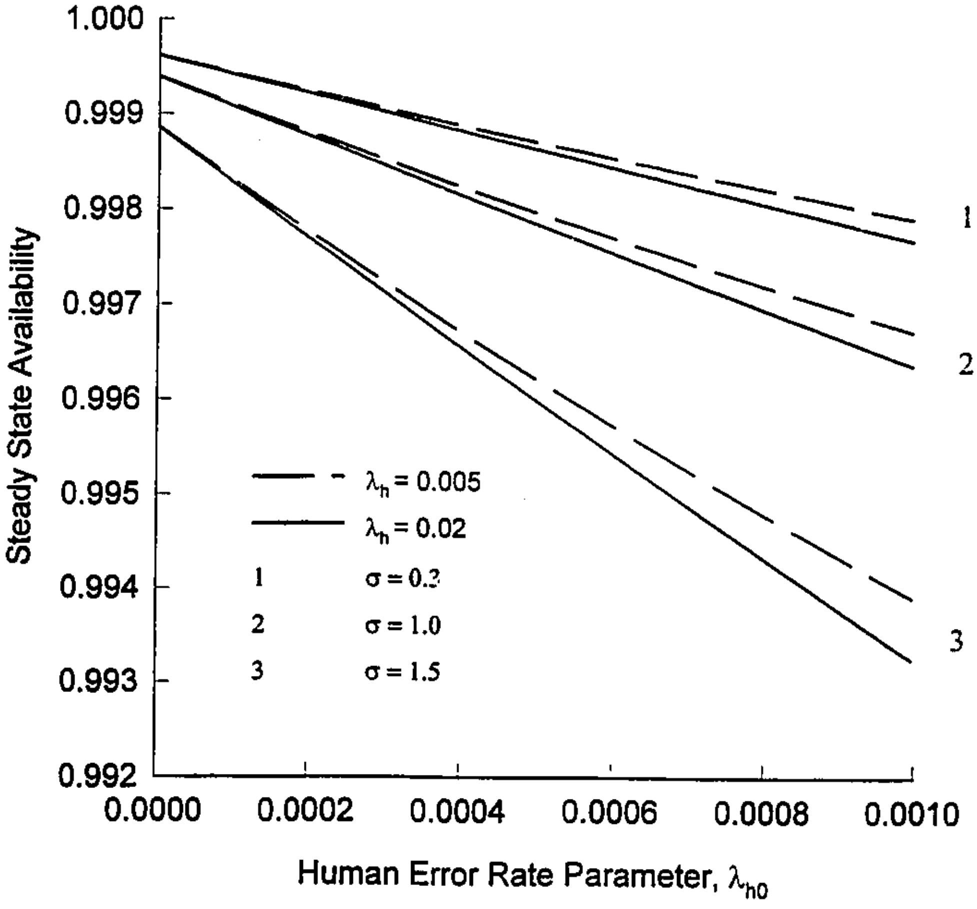


Figure 3-9 System steady state availability plots for two units active and one on standby system with increasing human error rates and the lognormally distributed failed system repair times

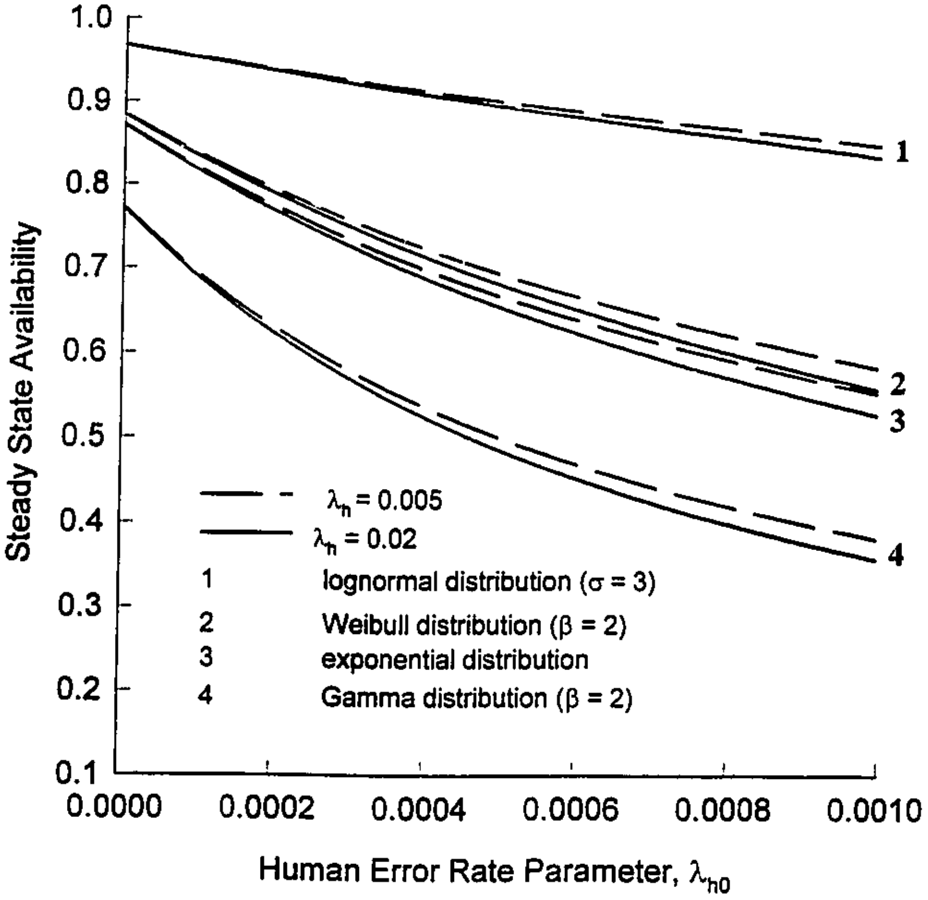


Figure 3-10 The comparison of the System steady state availability for two units active and one on standby system with increasing human error rates and various different system repair time distributions

$$P_5(s) = \frac{\lambda_h[1-N_5(s)]\left[b_5(2,1)+lh(2,1)\cdot s+\lambda_{h0}\cdot s^2\right]}{s\cdot CB(2,1)} \qquad (3\text{-}151)$$

$$P_{5A}(s) = \frac{b_5(2,1)+lh(2,1)\cdot s+\lambda_{h0}\cdot s^2}{CB(2,1)} \qquad (3\text{-}152)$$

where:

$$CB(2,1) = cb1(2,1)+cb2(2,1)\cdot s+cb3(2,1)\cdot s^2+cb4(2,1)\cdot s^3+s^4$$
$$cb1(2,1) = \lambda_h\cdot ca1(2,1)$$
$$cb2(2,1) = \sum_{j=3}^{4}\left[1-N_j(s)\right]b_j(2,1)+\lambda_h\cdot ca2(2,1)+b_5(2,1)$$
$$cb3(2,1) = \sum_{i=0}^{2}b_i(2,1)+lc(2,1)[1-N_4(s)]+\lambda_h\cdot ca3(2,1)+lh(2,1)$$
$$cb4(2,1) = l0(2,1)+\lambda_{c0}[1-N_4(s)]+\lambda_h+\lambda_{h0}$$

The Laplace transform of the system availability is

$$AV(s) = P_0(s)+P_1(s)+P_2(s)+P_{5A}(s)$$
$$= \frac{\left[\sum_{i=0}^{2}b_i(2,1)+l0(2,1)\cdot s+s^2\right](s+\lambda_h)+\left[b_5(2,1)+lh(2,1)\cdot s+\lambda_{h0}\cdot s^2\right]}{CB(2,1)} \qquad (3\text{-}153)$$

Substituting the Laplace transforms of the pdf of the system repair times, $N_3(s)$, $N_4(s)$, and $N_5(s)$, in Equation (3-153) and then substituting the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting μ_3=0.001/hr and μ_4=μ_5=0.0009/hr and taking inverse Laplace transform), we get the system time-dependent availability.

For different values of the human error rate parameters, λ_h and λ_{h0}, the time-dependent availabilities are shown in Figure 3-11 and Figure 3-12, for ***exponential*** and ***Gamma*** (β = 2) distributed failed system repair times, respectively.

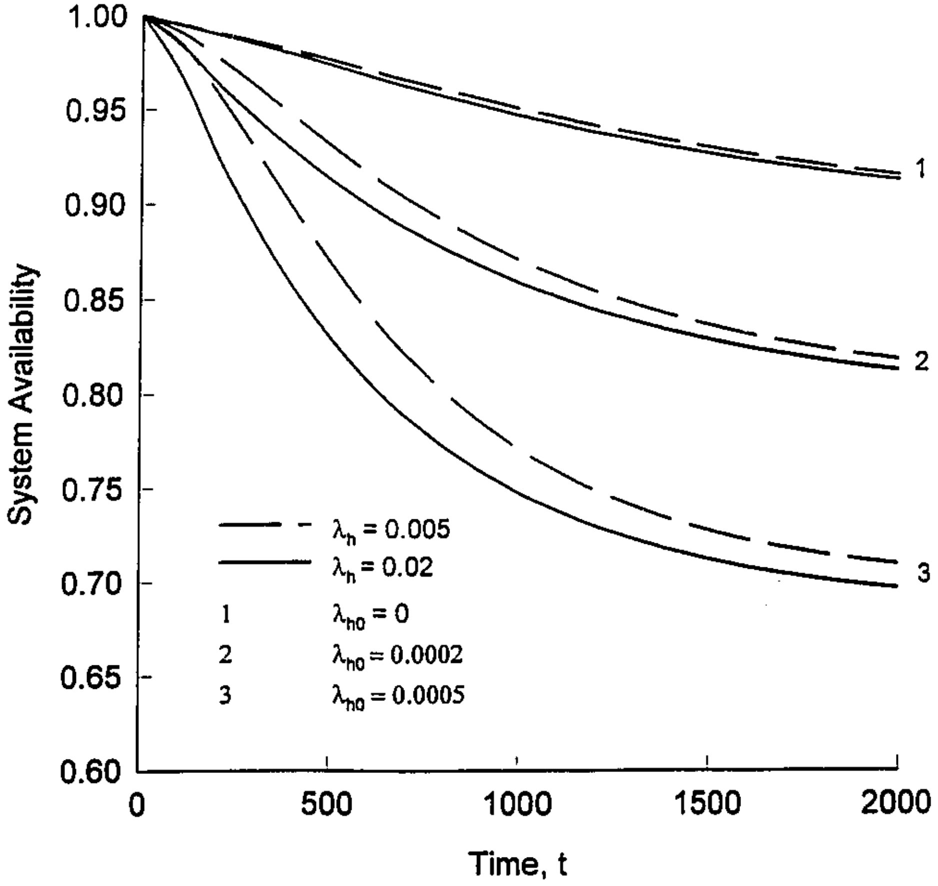


Figure 3-11 Time-dependent system availability plots for two units active and one on standby system with increasing human error rates and the exponentially distributed failed system repair times

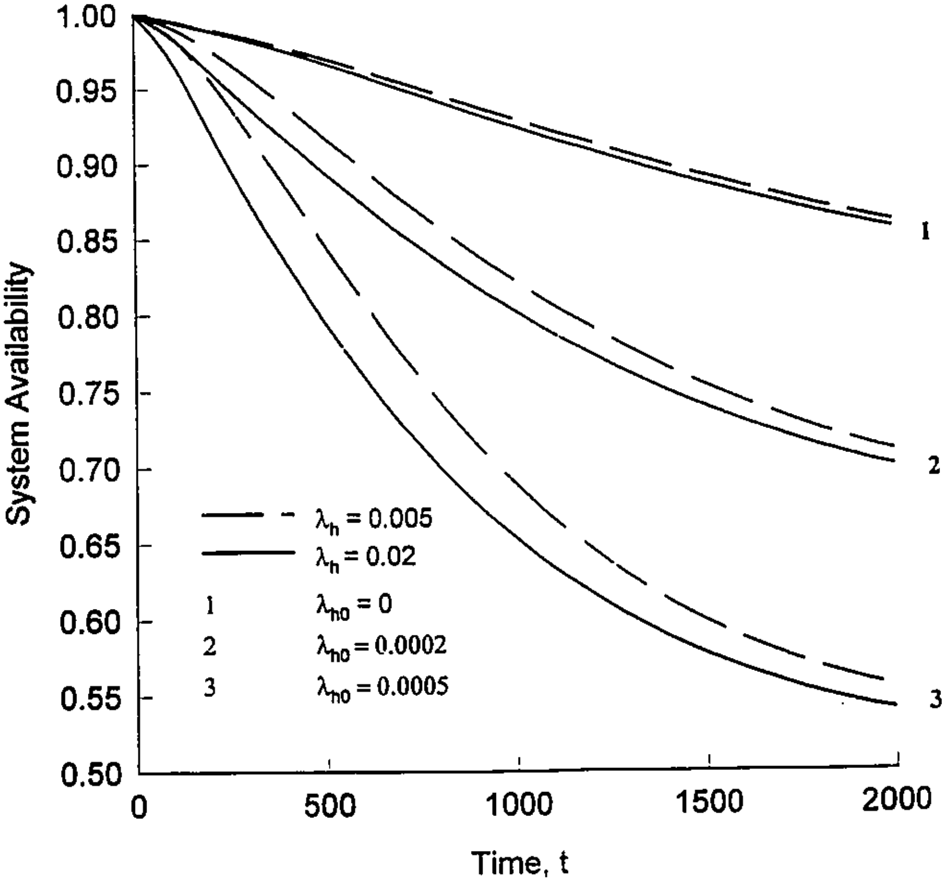


Figure 3-12 Time-dependent system availability plots for two units active and one on standby system with increasing human error rates and the Gamma ($\beta = 2$) distributed failed system repair times

Reliability and MTTF With and Without Repair

Setting $\mu_3(x) = \mu_4(x) = \mu_5(x) = 0$ in the model, using Laplace transforms and solving the resulting set of equations, we can get the Laplace transforms of the state probabilities. Thus, the Laplace transform of the system reliability with repair is

$$R(s) = \frac{b_5(2,1) + \lambda_h \sum_{i=0}^{2} b_i(2,1) + \left[\sum_{i=0}^{2} b_i(2,1) + lh(2,1) + \lambda_h \cdot l0(2,1) \right] s + \left(l0(2,1) + \lambda_h + \lambda_{h0} \right) s^2 + s^3}{DB(2,1)\left(s + \lambda_h\right)} \tag{3-154}$$

where

$$DB(2,1) = db1(2,1) + db2(2,1) \cdot s + db3(2,1) \cdot s^2 + s^3$$
$$db1(2,1) = b_0(2,1)\, a_0 - a_2 r_0 \mu_1$$
$$db2(2,1) = b_0(2,1) + a_0(a_1 + a_2) - r_0 \mu_1$$
$$db3(2,1) = a_0 + a_1 + a_2$$

The system reliability with repair can be obtained by inverting Equation (3-154)

$$\begin{aligned} R(t) = {} & \frac{e^{s_1 t}(s_a - s_1)(s_b - s_1)(s_c - s_1)}{(\lambda_h + s_1)(s_1 - s_2)(s_3 - s_1)} + \frac{e^{s_2 t}(s_a - s_2)(s_b - s_2)(s_c - s_2)}{(\lambda_h + s_2)(s_2 - s_1)(s_3 - s_2)} + \\ & \frac{e^{s_3 t}(s_a - s_3)(s_b - s_3)(s_c - s_3)}{(\lambda_h + s_3)(s_3 - s_1)(s_2 - s_3)} + \frac{e^{-\lambda_h t}(\lambda_h + s_a)(\lambda_h + s_b)(\lambda_h + s_c)}{(\lambda_h + s_1)(\lambda_h + s_2)(\lambda_h + s_3)} \end{aligned} \tag{3-155}$$

where s_a s_b s_c are the roots of the numerator of Equation (3-154), s_1, s_2 and s_3 are real and unique roots of the denominator of the Equation.

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \lim_{s \to 0} R(s) = \frac{\lambda_h \sum_{i=0}^{2} b_i(2,1) + b_5(2,1)}{\lambda_h \cdot db1(2,1)} \tag{3-156}$$

The variance of time to failure with repair of the system is expressed by

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2$$

$$= \frac{2\cdot b_5(2,1)\left[db1(2,1) - \sum_{i=0}^{2} b_i(2,1)\lambda_h + db2(2,1)\cdot\lambda_h\right] - \left[b_5(2,1)\right]^2}{\left[\lambda_h \cdot db1(2,1)\right]^2} - \frac{\lambda_h{}^2\sum_{i=0}^{2} b_i(2,1)\left[\sum_{i=0}^{2} b_i(2,1) - 2\cdot db2(2,1)\right] + 2\cdot\lambda_h\cdot db1(2,1)\left[lh(2,1) - \lambda_h\cdot l0(2,1)\right]}{\left[\lambda_h\cdot db1(2,1)\right]^2} \tag{3-157}$$

Plots of the system reliability with repair are shown in Figure 3-13 for same applicable data of the system steady state availability numerical examples. Figure 3-14 shows the plots of system MTTF with repair.

When $\mu_1 = \mu_2 = 0$ in Equations (3-155) - (3-157), we can get the system reliability, MTTF and the variance of time to failure of the system without repair, respectively.

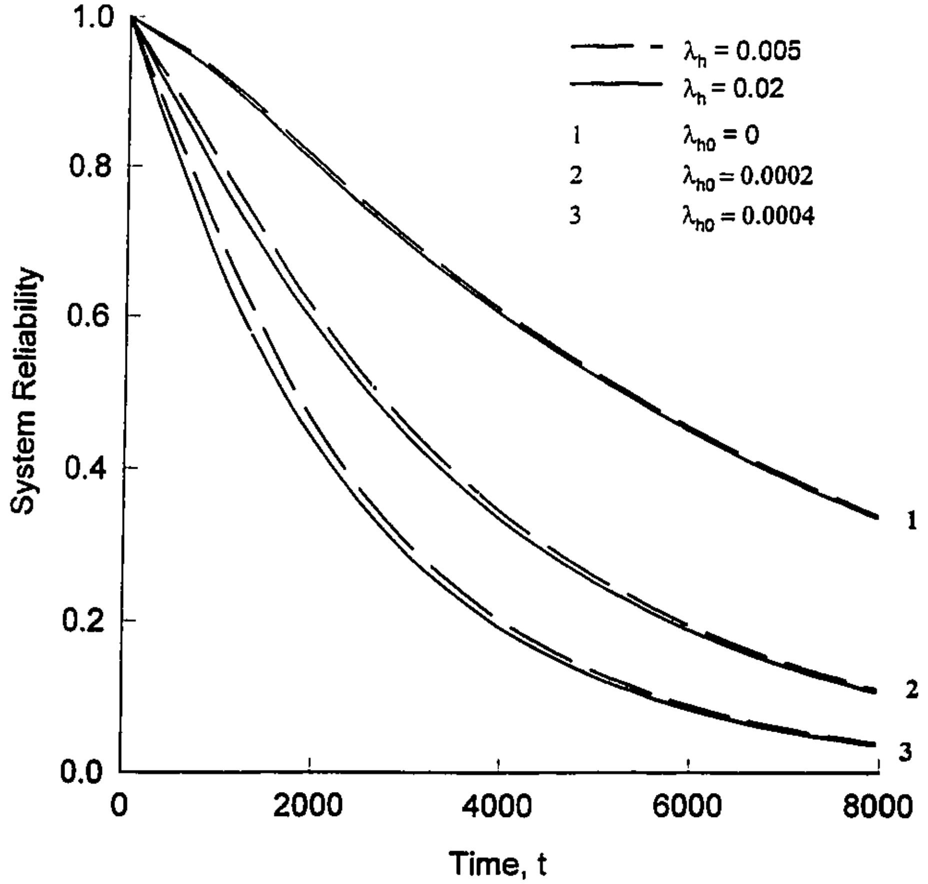


Figure 3-13 System reliability with repair plots for two units active and one on standby system with increasing human error rates

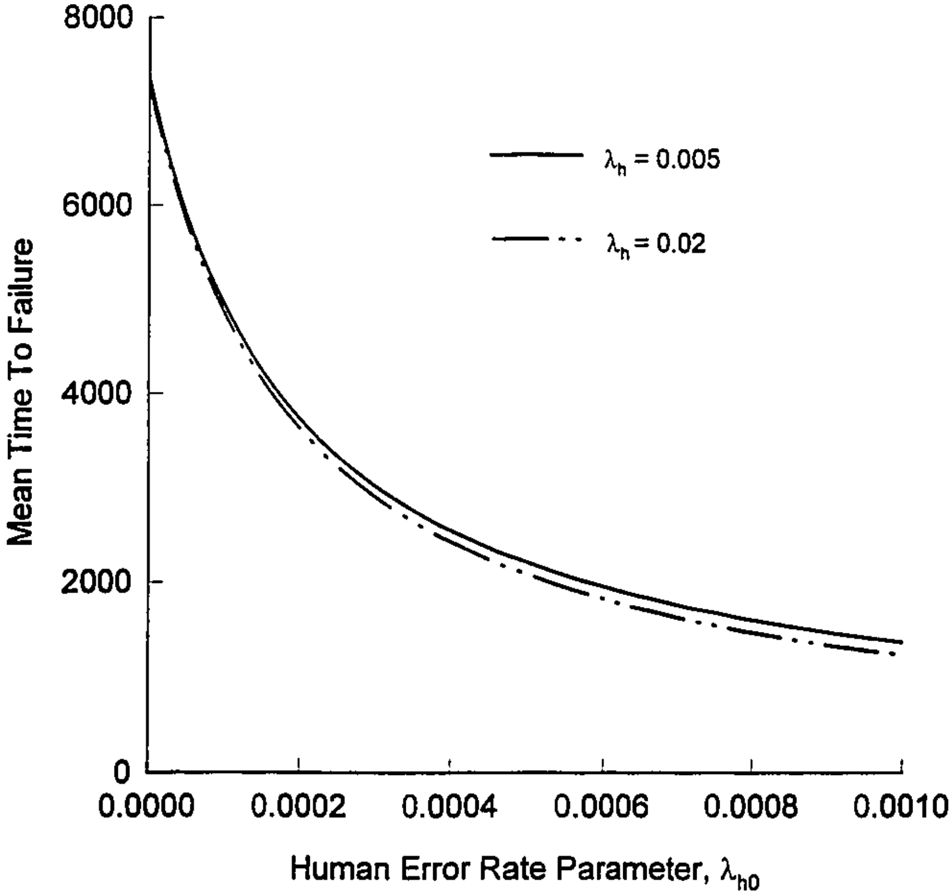


Figure 3-14 System mean time to failure with repair for two units active and one on standby system with increasing human error rates

3.4 K-out-of-n System

Similar to the standby system, the time-dependent human error in a k-out-of-n system can be devised into two stages so that human error rate is constant in each of stages. At time t = 0, all the units start working simultaneously. At least k units must operate normally for the system to function successfully. The state space diagram for k-out-of-n unit system with common-cause failures and increasing human error rates is shown in Figure 3-15. It is similar to that of standby system (Figure 3-4) but the division of system working or failure states is different. The comparison of k-out-of-n unit system and standby system is shown as follows:

Standby System		K-out-of-n System	
State	System Status	State	System Status
0	Up	0	Up
1	Up	1	Up
2	Up	2	Up
⋮	⋮	⋮	⋮
m+n-2	Up	n-k-1	Up
m+n-1	Up	n-k	Up
m+n	Down	n-k+1	Down
m+n+1	Down	n-k+2	Down
m+n+2	Down	n-k+3	Down
m+n+2(A)	Derated	n-k+3(A)	Derated

The numerals or letters (as applicable) in the boxes of Figure 3-15 denote corresponding system states. The explanations for the states are as follows:

- i = 0: all n units working normally.
- i = 1: one unit failed due to hardware failure, (n-1) units working.
- i = 2: two units failed due to hardware failures, (n-2) units working
- i = n-k: (n-k) units failed due to hardware failures, k units working.
- i = n-k+1: the system has failed due to hardware failures.

- i = n-k+2: the system has failed due to a common-cause failure.
- i = n-k+3: the system has failed due to a critical human error.

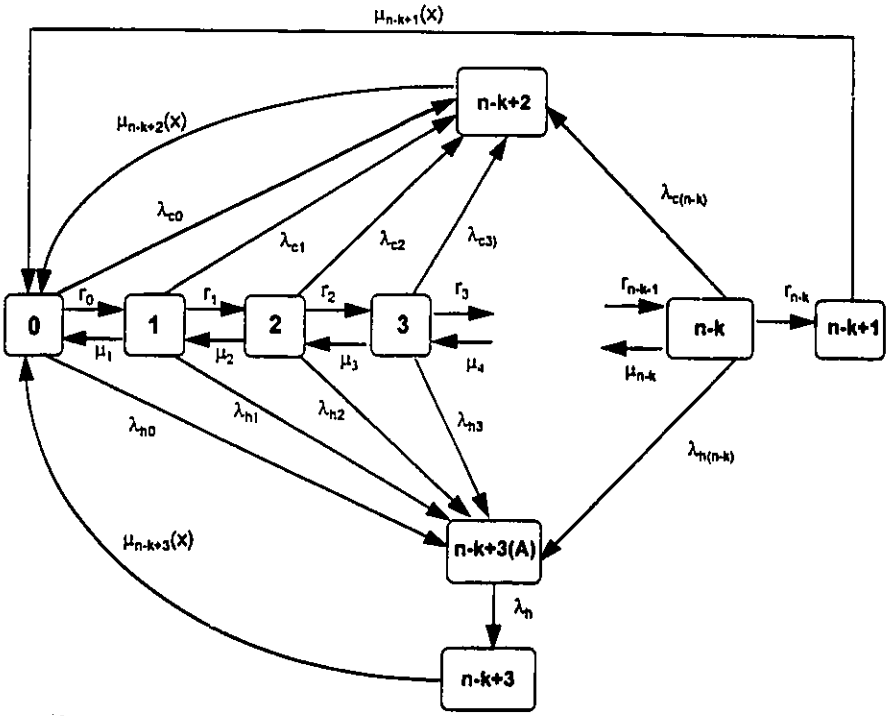


Figure 3-15 The state transition diagram of k-out-of-n redundancy with increasing human error represented by a device of two stages and arbitrary system repair rates

The corresponding differential equations for the model described in Figure 3-15 are:

$$\frac{dP_0(t)}{dt}=-a_0P_0(t)+\mu_1P_1(t)+\sum_{j=n-k+1}^{n-k+3}\int_0^\infty P_j(x,t)\mu_j(x)dx \tag{3-158}$$

$$\frac{dP_1(t)}{dt}=r_0P_0(t)-(r_1+\lambda_{h1}+\lambda_{c1}+\mu_1)P_1(t)+\mu_2P_2(t) \tag{3-159}$$

$$\frac{dP_2(t)}{dt}=r_1P_1(t)-(r_2+\lambda_{h2}+\lambda_{c2}+\mu_2)P_2(t)+\mu_3P_3(t) \tag{3-160}$$

$$\frac{dP_i(t)}{dt}=r_{i-1}P_{i-1}(t)-a_iP_i(t)+\mu_{i+1}P_{i+1}(t) \tag{3-161}$$

(for i = 0, 1, 2, 3, ..., n-k-1)

$$\frac{dP_{n-k}(t)}{dt} = r_{n-k-1}P_{n-k-1}(t) - a_{n-k}P_{n-k}(t) \quad (3\text{-}162)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x)P_j(x,t) \quad (3\text{-}163)$$

(for j = n-k+1, n-k+2, n-k+3)

$$\frac{dP_{n-k+3(A)}(t)}{dt} = \sum_{i=0}^{n-k} \lambda_{hi}P_i(t) - \lambda_h P_{n-k+3(A)}(t) \quad (3\text{-}164)$$

The associated boundary conditions are as follows:

$$P_{n-k+1}(0,t) = r_{n-k}P_{n-k}(t) \quad (3\text{-}165)$$

$$P_{n-k+2}(0,t) = \sum_{i=0}^{n-k} \lambda_{ci}P_i(t) \quad (3\text{-}166)$$

$$P_{n-k+3}(0,t) = \lambda_h P_{n-k+3(A)}(t) \quad (3\text{-}167)$$

where

$$r_i = k\lambda + (n-k-i)\lambda_s$$

$$a_i = r_i + \lambda_{hi} + \lambda_{ci} + \mu_i$$

(for i = 0, 1, 2, …, n-k)

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

3.4.1 K-out-of-n (n-k=1) System

For k-out-of-n (n-k=1) system, it is similar to the model of one unit active and one on standby system. Using the same method and procedure as that of the standby system, we can get the system steady-state availability, Laplace transform of system availability, Laplace transform of system reliability and system MTTF expressed by Equations (3-90), (3-113), (3-121) and (3-123), respectively.

Setting

$\lambda = 0.0002 / hr$ $\quad \lambda_s = 0.00002 / hr$ $\quad \mu_1 = 0.001 / hr$

$\lambda_{c0} = 0.00005 / hr$ $\quad \lambda_{c1} = 0.00005 / hr$ $\quad \lambda_{h1} = 0.0001 / hr$

in Equation (3-90), for Gamma, exponential, Weibull and Rayleigh distributed failed system repair times, the plots of system steady-state availability as a function of human error rate parameters, λ_{h0} and λ_h, are shown in Figures 3-16, 3-17, 3-18 and 3-19, respectively, for specified values of parameters $\mu_2 = 0.001 / hr, \mu_3 = \mu_4 = 0.0009 / hr$, if applicable.

Figure 3-20 provides the results of the system steady state availability as a function of human error rate parameters, λ_{h0} and λ_h, for lognormally distributed failed system repair times ($\mu_2 = \mu_3 = \mu_4 = 1.0009 / hr, \quad \sigma_2 = \sigma_3 = \sigma_4 = \sigma$).

Substituting the Laplace transforms of the pdf of the system repair times $N_2(s)$, $N_3(s)$ and $N_4(s)$ in Equation (3-113) and then substituting r_0, r_1, and the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting $\mu_2 = 0.001 / hr, \mu_3 = \mu_4 = 0.0009 / hr$ and taking inverse Laplace transform), we get the system time-dependent availability.

For 2-out-of-3 system with ***Gamma*** (β=2) distributed failed system repair time and λ_{h0}=0.0001, λ_h=0.001/hr we get

$$\begin{aligned}
AV(t) = {} & 0.6858318 + 0.00203948/\exp(0.0019749\,t) + \\
& (6.69842\times10^{-10})/\exp(0.001t) + (0.270793 \sin(8.23525\times10^{-6}\,t))/\exp(0.001072\,t) \\
& + 0.001924[(\cos(8.235254\times10^{-6}\,t))/\exp(0.001072\,t) \\
& - (130.1939 \sin(8.235254\times10^{-6}\,t))/\exp(0.0010722\,t)] \\
& +(0.914448 \sin(0.0005626\,t))/\exp(0.0009\,t) \\
& +0.31020\,[(\cos(0.0005626t))/\exp(0.0009\,t) \\
& - (1.6002 \sin(0.0005626t))/\exp(0.0009\,t)] \qquad (3\text{-}168)
\end{aligned}$$

For exponentially distributed failed system repair time, the plots of the time-dependent system availability for 1-out-of-2, 2-out-of-3 and 3-out-of-4 unit system for varying human error rate parameters, λ_{h0} and λ_h, are shown in Figures 3-21, 3-22 and 3-23, respectively.

For different values of human error rate parameters, λ_{h0} and λ_h, the system time-dependent availability plots for 1-out-of-2, 2-out-of-3 and 3-out-of-4 unit system with Gamma (β=2) distributed failed system repair times are shown in Figures 3-24, 3-25 and 3-26, respectively.

System reliability with repair for 2-out-of-3 system, when λ_{h0}=0.0001, λ_h=0.001/hr is

$$\begin{aligned} R(t) = &- 0.053144/\exp(0.00187246\ t) - 0.14775323/\exp(0.001\ t) \\ &+ 1.200897/\exp(0.000247534\ t) \end{aligned} \tag{3-169}$$

The plots of the system reliability with repair for specified values of parameters are shown in Figures 3-27 (parameter λ_h=0.001/hr) and 3-28 (parameter λ_h=0.01/hr).

The system mean time to failure (MTTF) with repair is given by

$$MTTF = \frac{0.00115(\lambda_h + \lambda_{h0}) + r_0(0.0001 + \lambda_h) + r_1(\lambda_h + \lambda_{h0})}{[5.75 \times 10^{-8} + 0.00115\lambda_{h0} + 0.00015r_0 + r_1(0.00005 + \lambda_{h0} + r_0)] \cdot \lambda_h} \tag{3-170}$$

Figure 3-29 shows the plots of system mean time to failure with repair for k-out-of-n (n-k=1) system for varying human error rate parameters, λ_{h0} and λ_h.

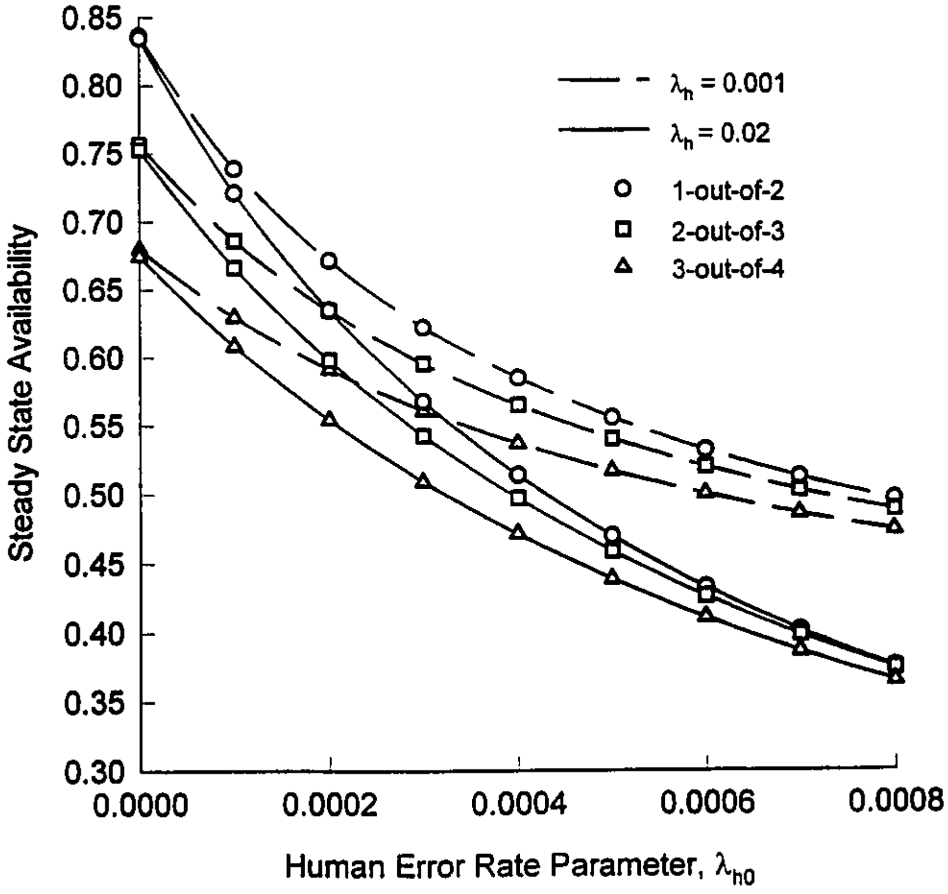


Figure 3-16 System steady state availability plots for k-out-of-n (n-k=1) system with increasing human error rates and the Gamma ($\beta=2$) distributed failed system repair times

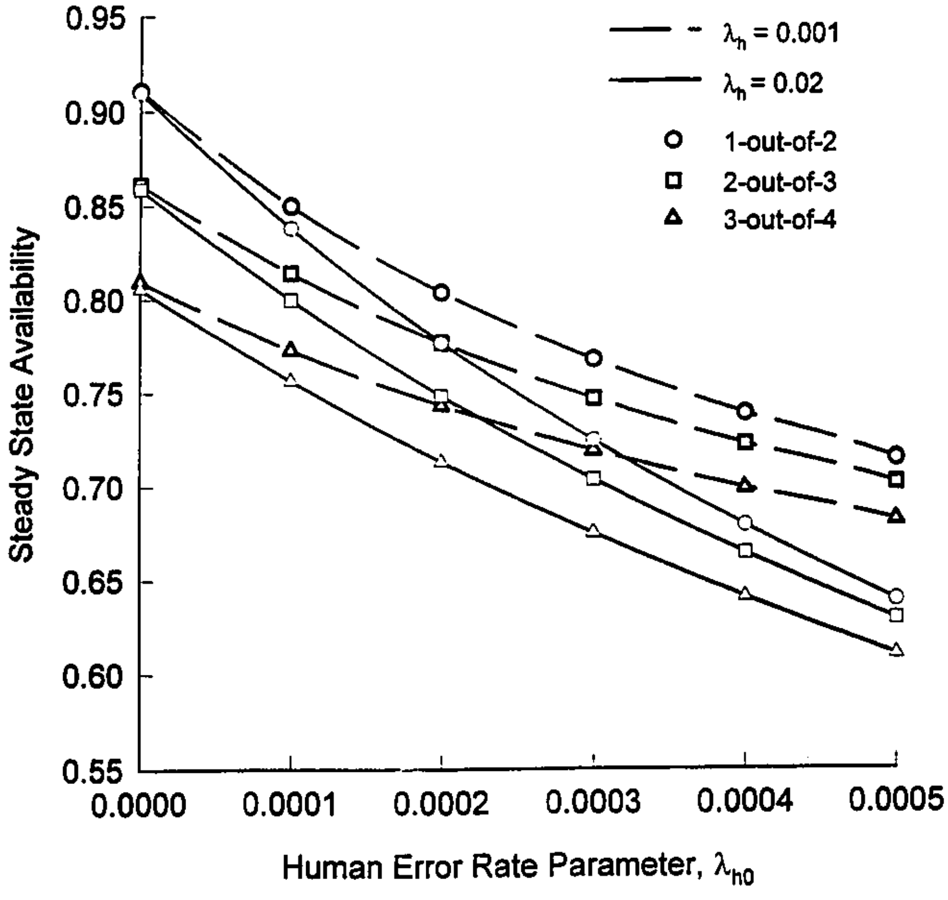


Figure 3-17 System steady state availability plots for k-out-of-n (n-k=1) system with increasing human error rates and constant system repair rates

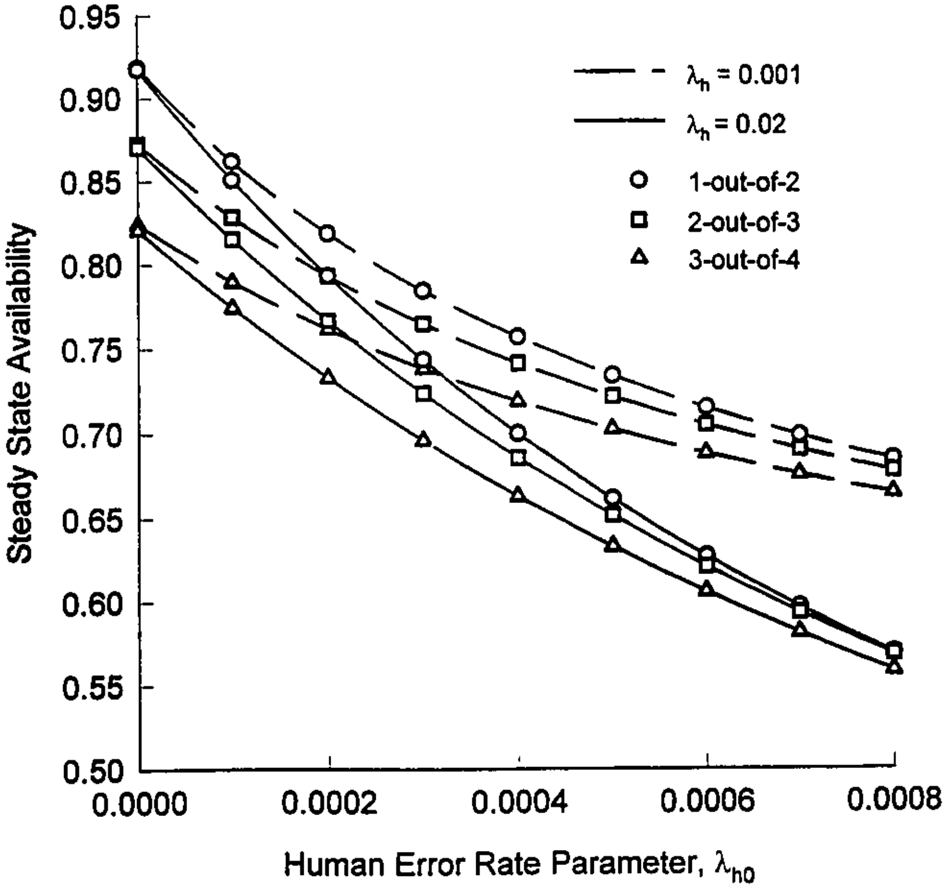


Figure 3-18 System steady state availability plots for k-out-of-n (n-k=1) system with increasing human error rates and the Weibull (β=3) distributed failed system repair times

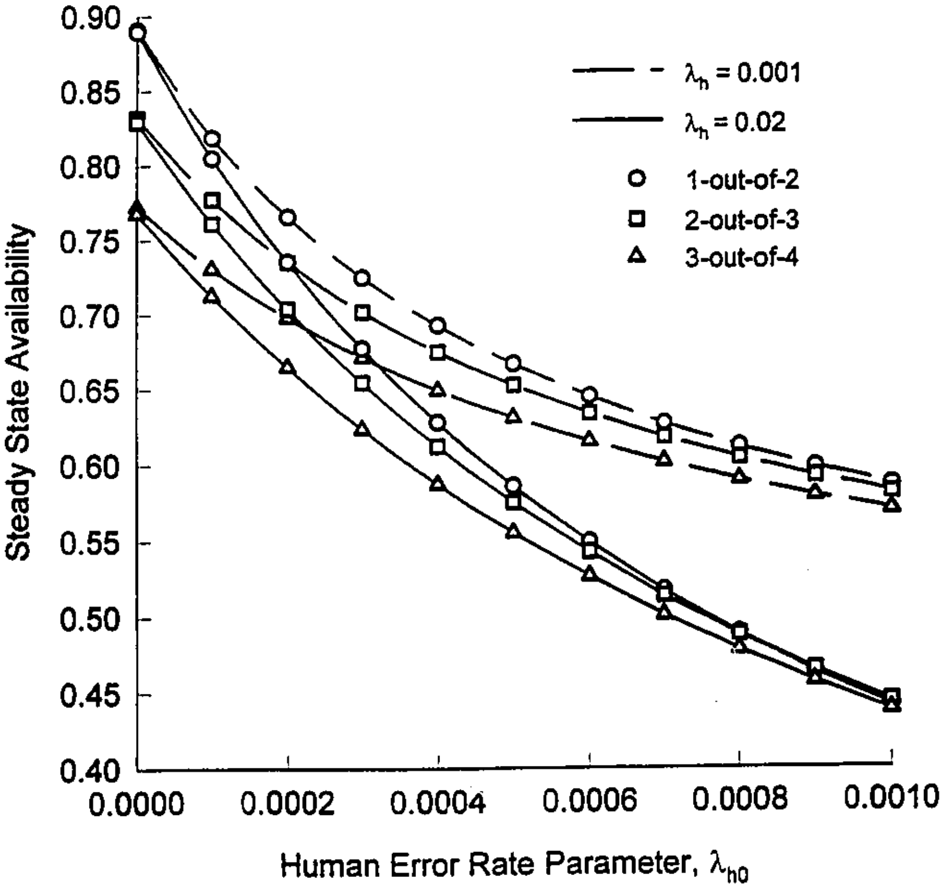


Figure 3-19 System steady state availability plots for k-out-of-n (n-k=1) system with increasing human error rates and the Rayleigh distributed failed system repair times

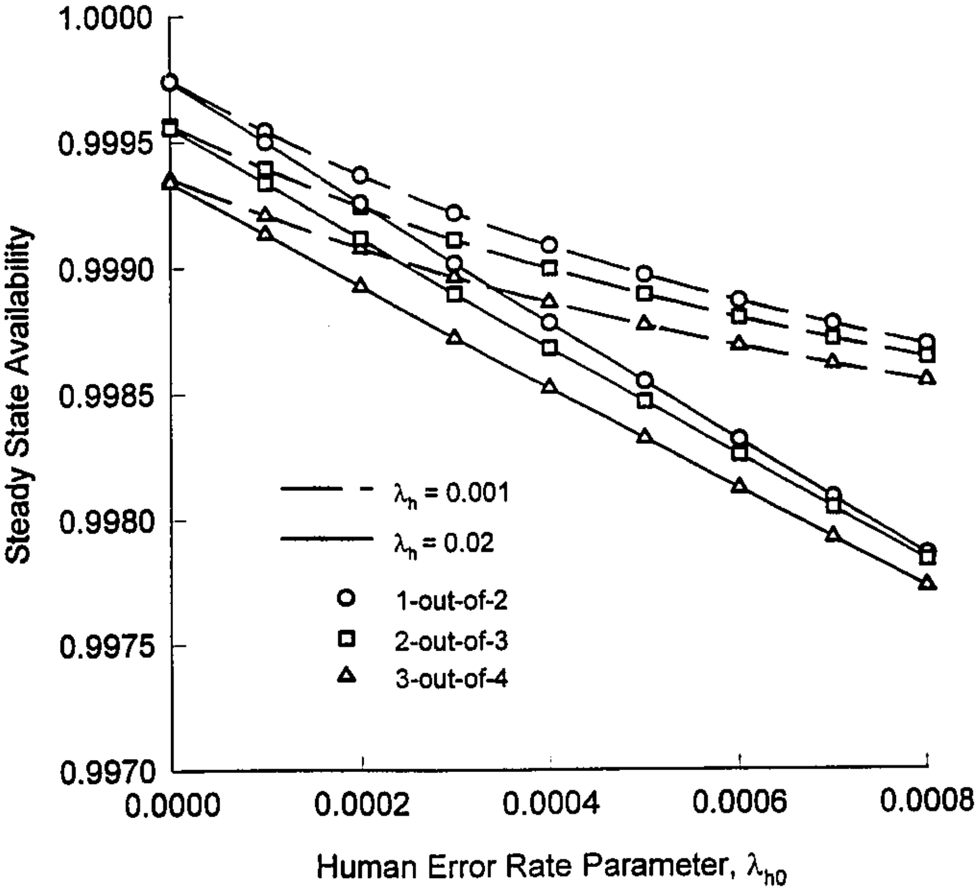


Figure 3-20 System steady state availability plots for k-out-of-n (n-k=1) system with increasing human error rates and the lognormally (σ=0.3) distributed failed system repair times

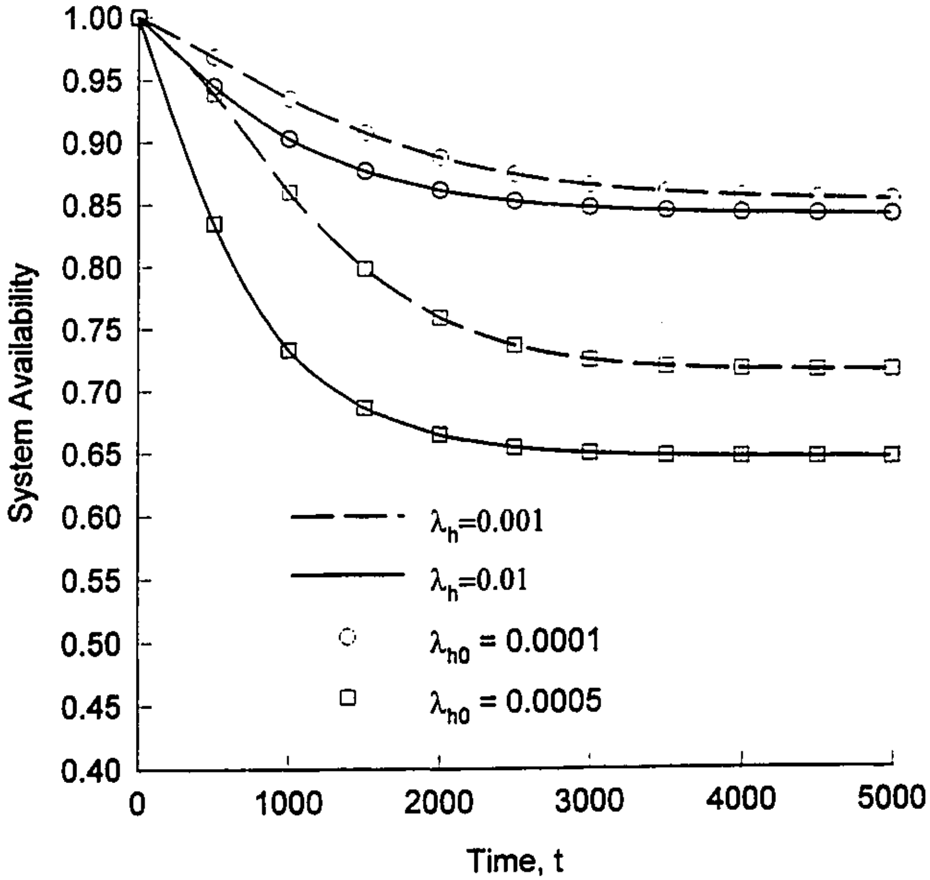


Figure 3-21 Time-dependent system availability plots for 1-out-of-2 system with increasing human error rates and the exponentially distributed system repair times

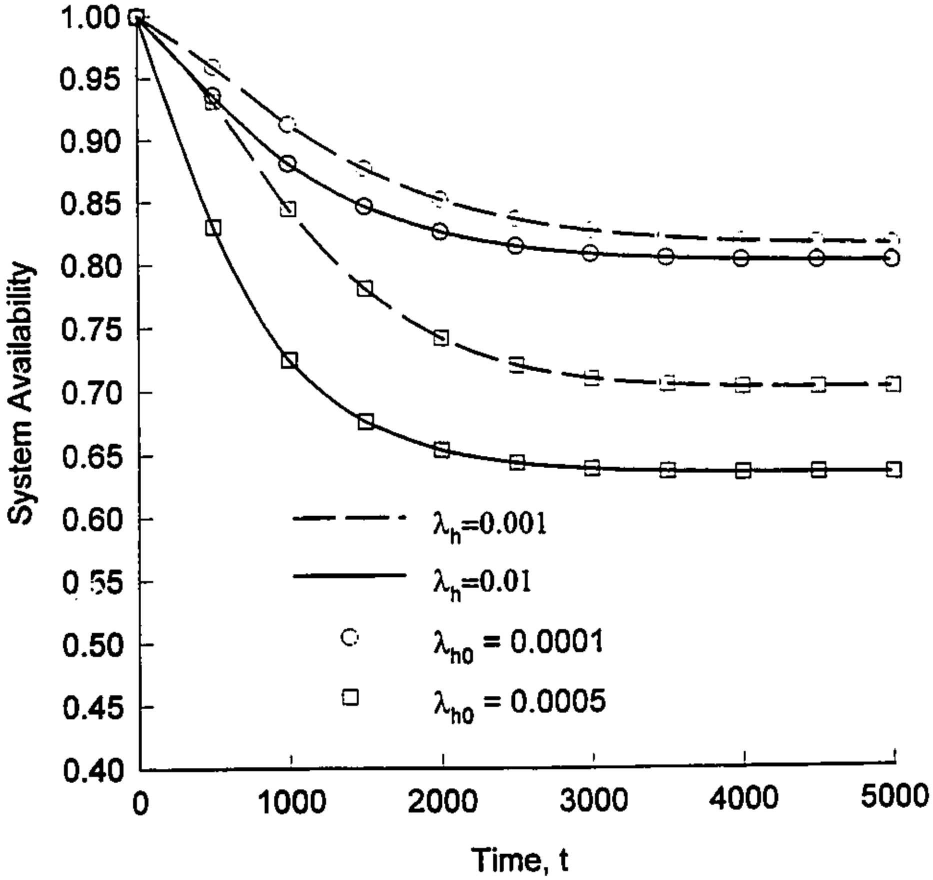


Figure 3-22 Time-dependent system availability plots for 2-out-of-3 system with increasing human error rates and the exponentially distributed system repair times

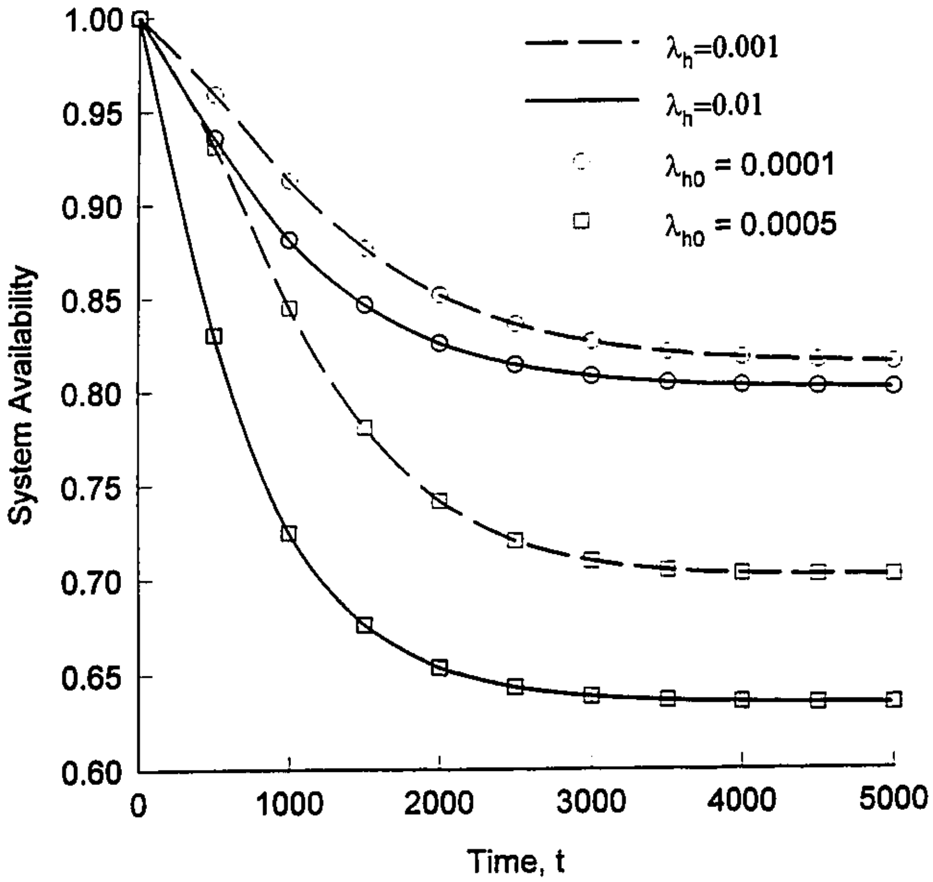


Figure 3-23 Time-dependent system availability plots for 3-out-of-4 system with increasing human error rates and the exponentially distributed system repair times

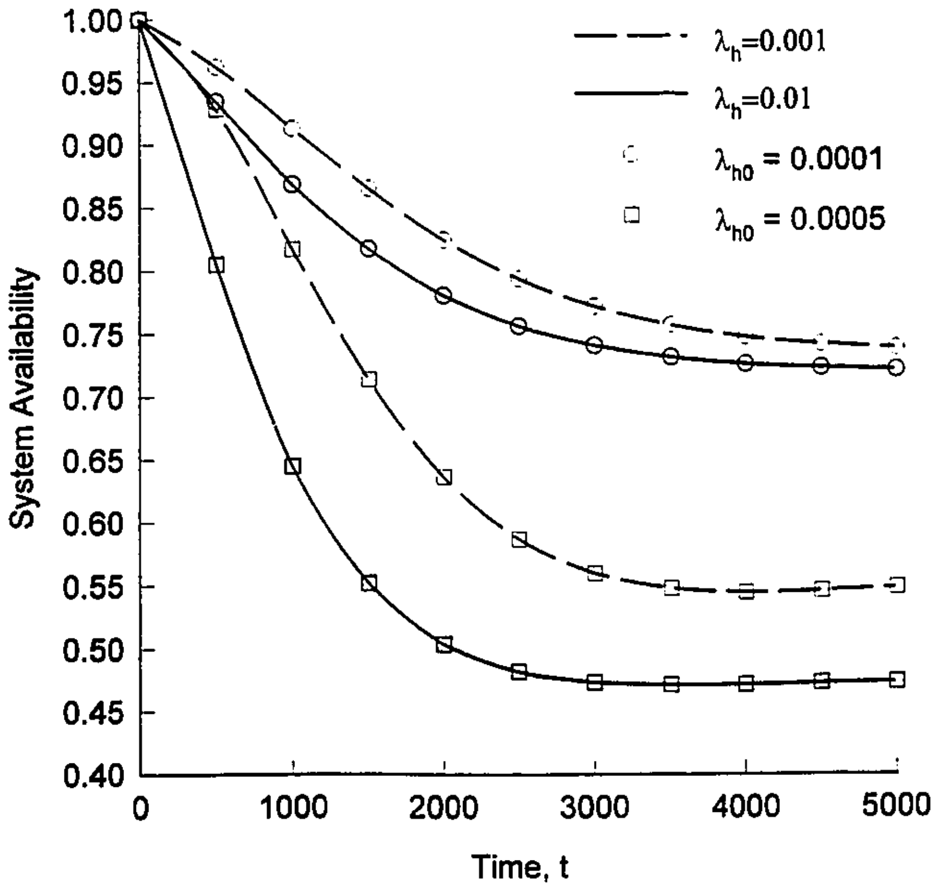


Figure 3-24 Time-dependent system availability plots for 1-out-of-2 system with increasing human error rates and the Gamma (β=2) distributed system repair times

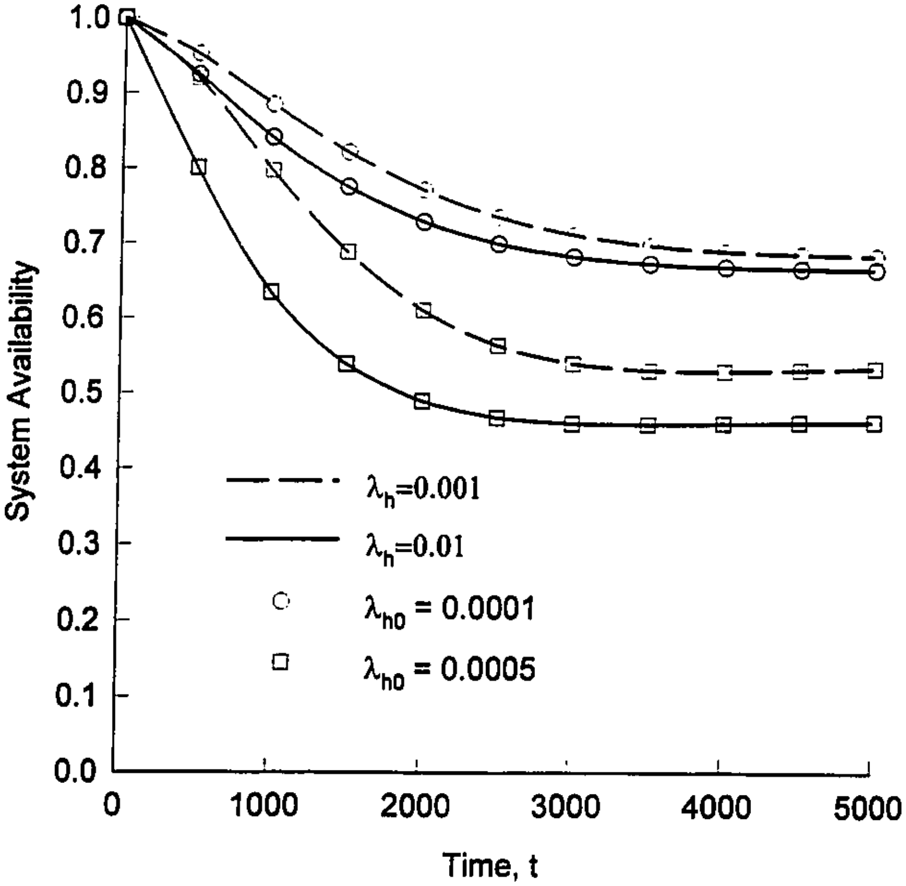


Figure 3-25 Time-dependent system availability plots for 2-out-of-3 system with increasing human error rates and the Gamma (β=2) distributed system repair times

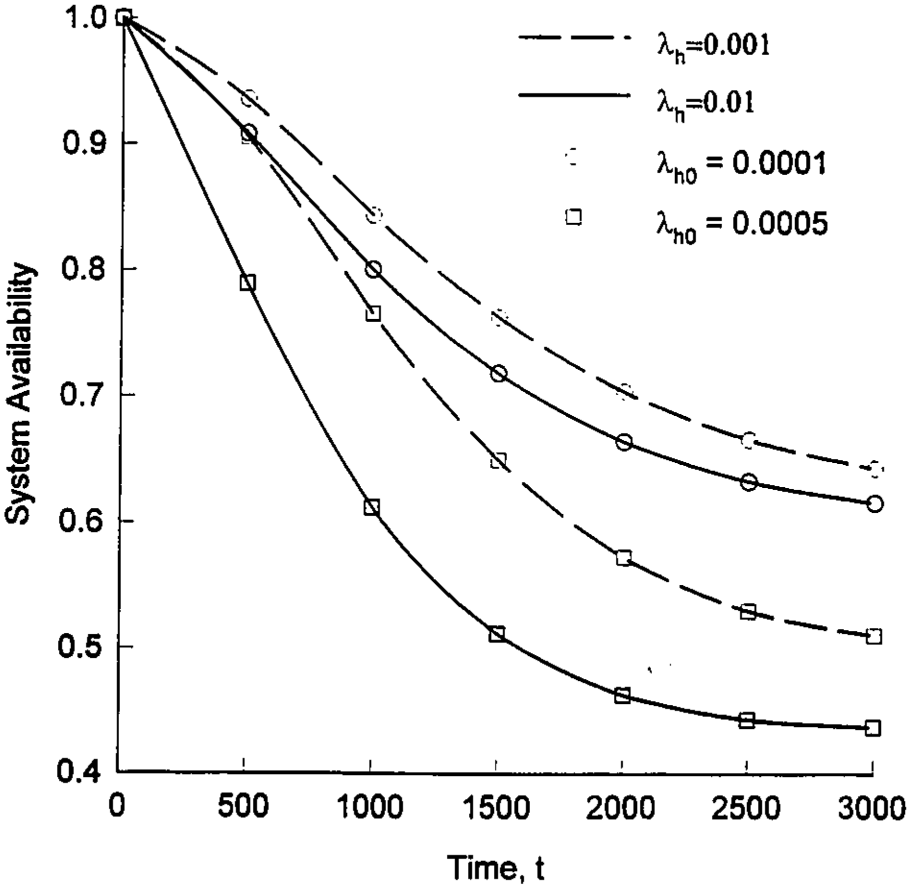


Figure 3-26 Time-dependent system availability plots for 3-out-of-4 system with increasing human error rates and the Gamma (β=2) distributed system repair times

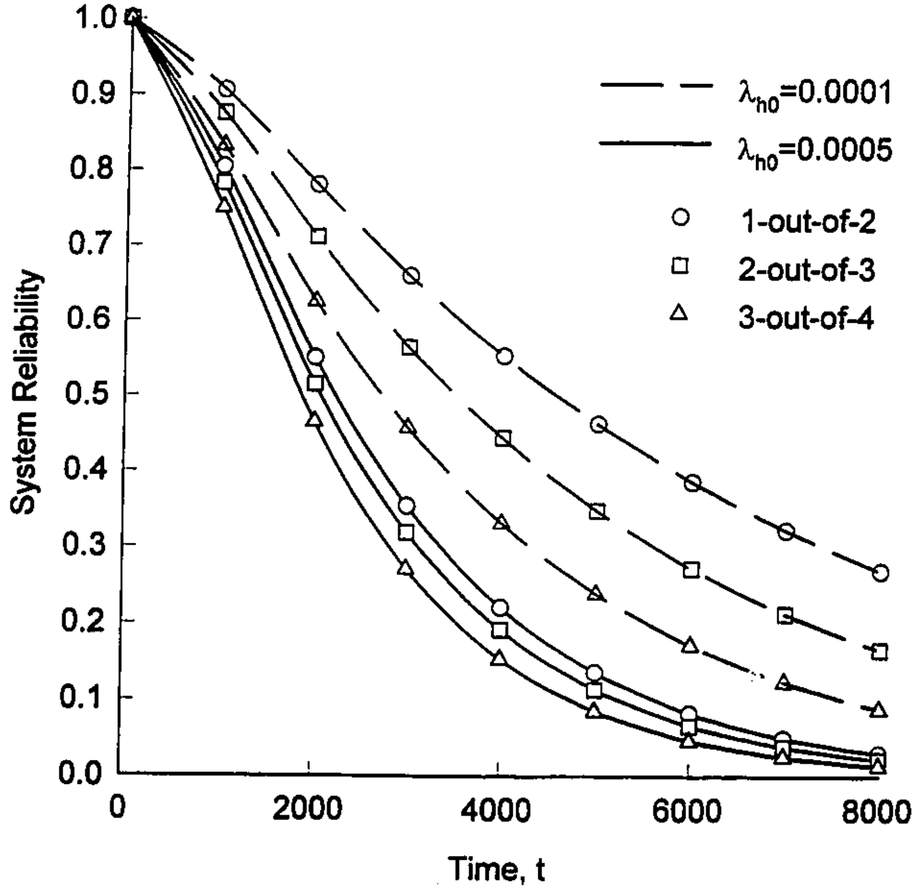


Figure 3-27 System reliability with repair plots for k-out-of-n (n-k=1) system with increasing human error rates (parameter $\lambda_h = 0.001$)

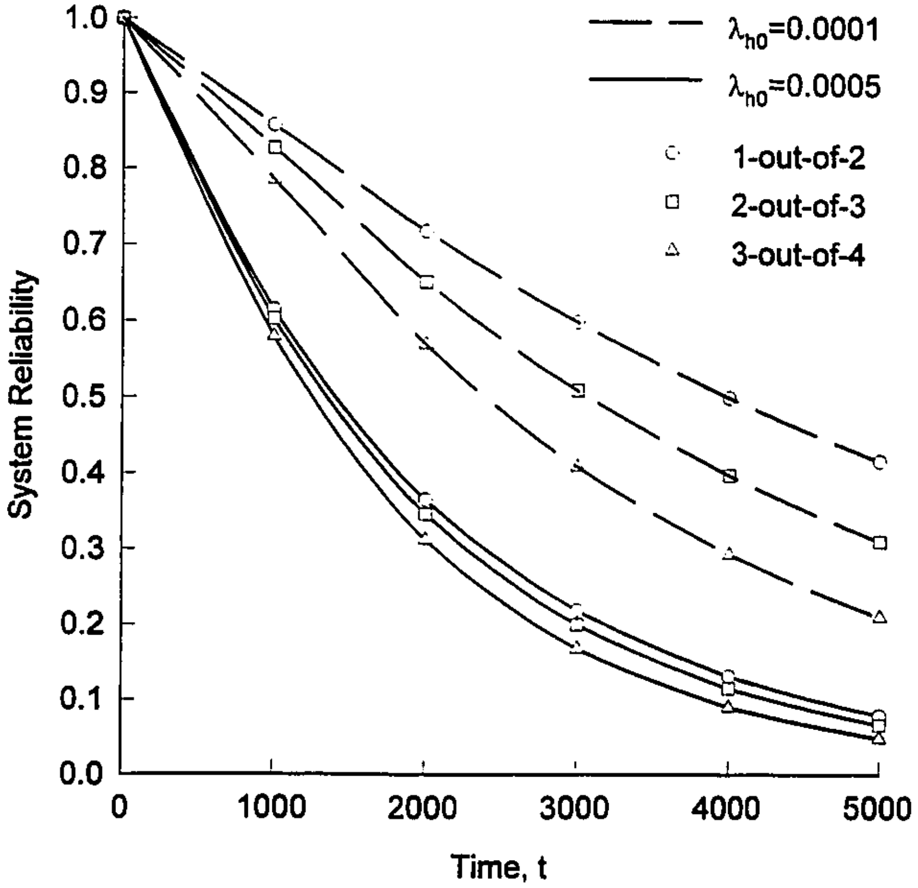


Figure 3-28 System reliability with repair plots for k-out-of-n (n-k=1) system with increasing human error rates (parameter λ_h = 0.01)

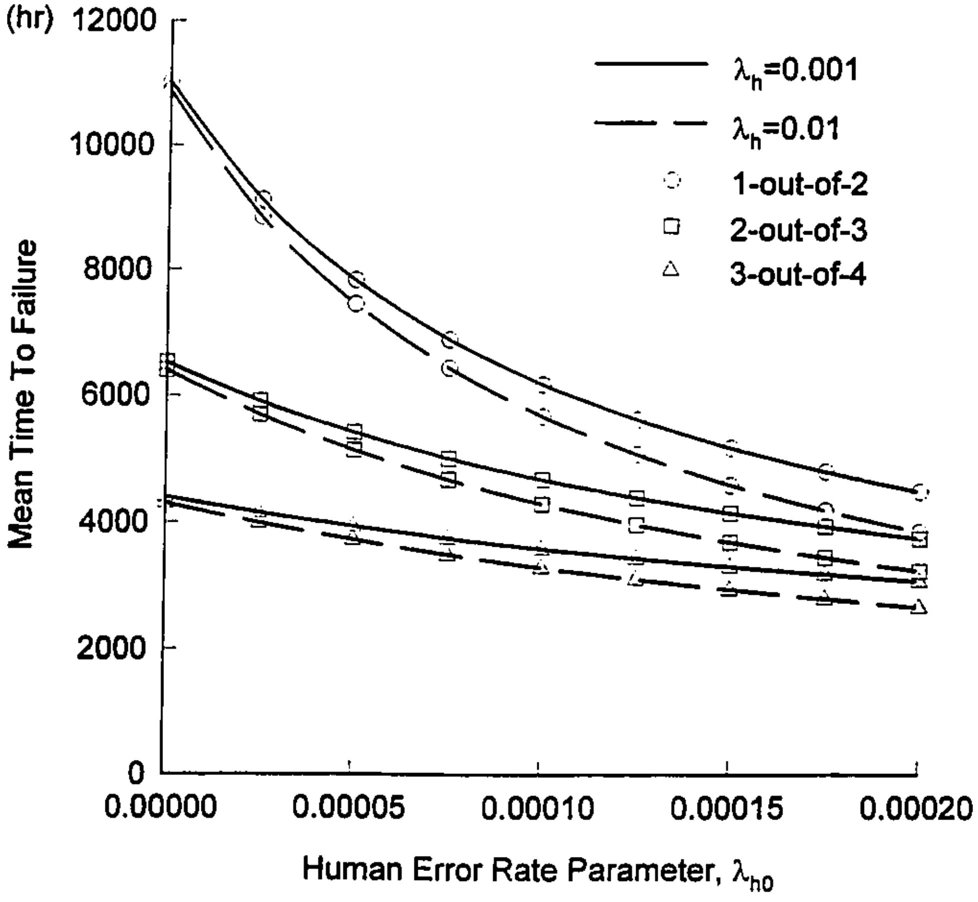


Figure 3-29 System mean time to failure with repair for k-out-of-n (n-k=1) system with increasing human error rates

3.4.2 K-out-of-n (n-k=2) System

For k-out-of-n (n-k=2) system, it is similar to the model of two units active and one unit on standby system. Using the same method and procedure as that of the standby system, we can get the system steady-state availability, Laplace transform of system availability, Laplace transform of system reliability and system MTTF expressed by Equations (3-140), (3-153), (3-154) and (3-156), respectively.

Setting

$\lambda = 0.0002 / hr$ $\quad$ $\lambda_s = 0.00002 / hr$ $\quad$ $\mu_1 = 0.001 / hr$
$\mu_2 = 0.001 / hr$ $\quad$ $\lambda_{h1} = 0.0001 / hr$ $\quad$ $\lambda_{h2} = 0.0001 / hr$
$\lambda_{c0} = 0.00005 / hr$ $\quad$ $\lambda_{c1} = 0.00005 / hr$ $\quad$ $\lambda_{c2} = 0.00005 / hr$

in Equation (3-140), for Gamma, exponential, Weibull and Rayleigh distributed failed system repair times, the plots of system steady-state availability as a function of human error rate parameters, λ_{h0} and λ_h, are shown in Figures 3-30, 3-31, 3-32 and 3-33, respectively, for specified values of parameters $\mu_3 = 0.001 / hr, \mu_4 = \mu_5 = 0.0009 / hr$, if applicable.

Figure 3-34 provides the results of the system steady state availability as a function of human error rate parameters, λ_{h0} and λ_h, for lognormally distributed failed system repair times ($\mu_3 = \mu_4 = \mu_5 = 1.0009 / hr, \ \ \sigma_3 = \sigma_4 = \sigma_5 = 0.3$).

Substituting the Laplace transforms of the pdf of the system repair times $N_3(s)$, $N_4(s)$ and $N_5(s)$ in Equation (3-153) and then substituting r_0, r_1, r_2 and the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting $\mu_3 = 0.001 / hr, \mu_4 = \mu_5 = 0.0009 / hr$ and taking inverse Laplace transform), we get the system time-dependent availability.

For 2-out-of-4 system with ***Gamma*** (β=2) distributed failed system repair time and λ_{h0}=0.0001, λ_h=0.001/hr we get

$$\begin{aligned} AV(t) = {} & 0.7409097 + 0.0058512/\exp(0.00233037\ t) + 0.0723221/\exp(0.00150399\ t) \\ & - (0.042748386 \sin(0.00005354739\ t))/\exp(0.001025683258\ t) \\ & - 0.00230048\ ((\cos(0.00005354739\ t))/\exp(0.001025683\ t) \\ & - (19.154681 \sin(0.00005354\ t))/\exp(0.0010256832\ t)) \\ & + (0.69803 \sin(0.00052658\ t))/\exp(0.00081213\ t) \\ & + 0.1832174((\cos(0.0005265835\ t))/\exp(0.00081213\ t) \\ & - (1.5422696 \sin(0.00052658\ t))/\exp(0.00081213\ t)) \qquad (3\text{-}171) \end{aligned}$$

For exponentially distributed failed system repair time, the plots of the time-dependent system availability for 1-out-of-3 and 2-out-of-4 unit systems for varying human error rate parameters, λ_{h0} and λ_h, are shown in Figures 3-35 and 3-36, respectively.

For different values of human error rate parameters, λ_{h0} and λ_h, the system time-dependent availability plots for 1-out-of-3 and 2-out-of-4 unit systems with Gamma (β=2) distributed failed system repair times are shown in Figures 3-37 and 3-38, respectively.

System reliability with repair for 3-out-of-5 system, when λ_{h0}=0.0001, λ_h=0.001/hr is

$$\begin{aligned} R(t) = {} & 0.02515027/\exp(0.002731405\ t) - 0.091352994/\exp(0.001351856\ t) - \\ & 0.177105556/\exp(0.001\ t) + 1.2433082/\exp(0.00022673\ t) \qquad (3\text{-}172) \end{aligned}$$

The plots of the system reliability with repair for specified values of parameters and the same applicable data of the earlier system steady state availability numerical examples are shown in Figures 3-39 (parameters λ_h=0.001/hr, λ_{h0}=0.0001/hr) and 3-40 (parameters λ_h=0.01/hr, λ_{h0}=0.0001/hr).

The system mean time to failure (MTTF) with repair for the same data is given by

$$
\begin{aligned}
\mathrm{MTTF} = {} & [1.3225\times10^{-6} (\lambda_h + \lambda_{h0}) + 1.15\times10^{-7} r_0 + 0.00115\lambda_h r_0 \\
& + 0.00015 r_1(\lambda_h + \lambda_{h0}) + r_0 r_1 (0.0001 + \lambda_h) \\
& + 0.00115 r_2 (\lambda_h + \lambda_{h0}) + r_0 r_2 (0.0001 + \lambda_h) \\
& + r_1 r_2 (\lambda_h + \lambda_{h0})] / [6.6125\times10^{-11} \lambda_h + 1.3225\times10^{-6} \lambda_h \lambda_{h0} \\
& + 1.725\times10^{-7} \lambda_h r_0 + 7.5\times10^{-9} \lambda_h r_1 + 0.00015 \lambda_h r_1 (\lambda_{h0} + r_0) \\
& + 5.75\times10^{-8} \lambda_h r_2 + 0.00115 \lambda_h \lambda_{h0} r_2 + 0.00015 \lambda_h r_0 r_2 \\
& + 0.00005 \lambda_h r_1 r_2 + \lambda_h \lambda_{h0} r_1 r_2 + \lambda_h r_0 r_1 r_2] \qquad (3\text{-}173)
\end{aligned}
$$

Figure 3-41 shows the plots of system mean time to failure (MTTF) with repair for k-out-of-n (n-k=1) system for varying human error rate parameters, λ_{h0} and λ_h.

3.5 Summary

This Chapter presents various mathematical models for performing reliability and availability analysis of standby systems and k-out-of-n systems with increasing human errors and arbitrarily distributed failed system repair times.

Device of stages each of which is exponentially distributed is introduced to represent a non-exponential state. A series stages combination is used to represent increasing human error rates. The method which combines the inclusion supplementary variables method and the device of stages method is developed to perform time-dependent system availability analysis for the system with both time-dependent system failure rates and failed system repair rates.

The method of linear ordinary differential equations is used to obtain the general expressions of the system steady state availability for various types of failed system repair time distributions, such as the Gamma, Weibull, exponential, Rayleigh and lognormal distributions.

With the aid of the method of supplementary variables, device of stages, the Markov approach and Laplace transforms, the time-dependent availability of the systems are presented for Gamma distributed failed system repair times for all these models.

The analyses performed in this Chapter indicate that the values of systems performance indices (such as, system steady state availability, time-dependent system availability, system reliability and system mean time to failure) decrease with either increasing values of human error rates or decreasing values of the human error repair rates. For all the special cases considered in the Chapter, the steady state system availability and time-dependent system availability drop as the value of the Gamma shape parameter β increases for Gamma distributed failed system repair times.

A close observation of the plots in this Chapter also reveals that for standby redundant configurations, an increase in either the number of standby units for a fixed number of active units or the number of active units for a given number of standby units would improve system performance indices. For k-out-of-n redundant configurations, an increase in the value of k, the least number of units needed for system operation, for a given value of n, total number of units in the system, (or for given value of n-k), would decrease system performance indices.

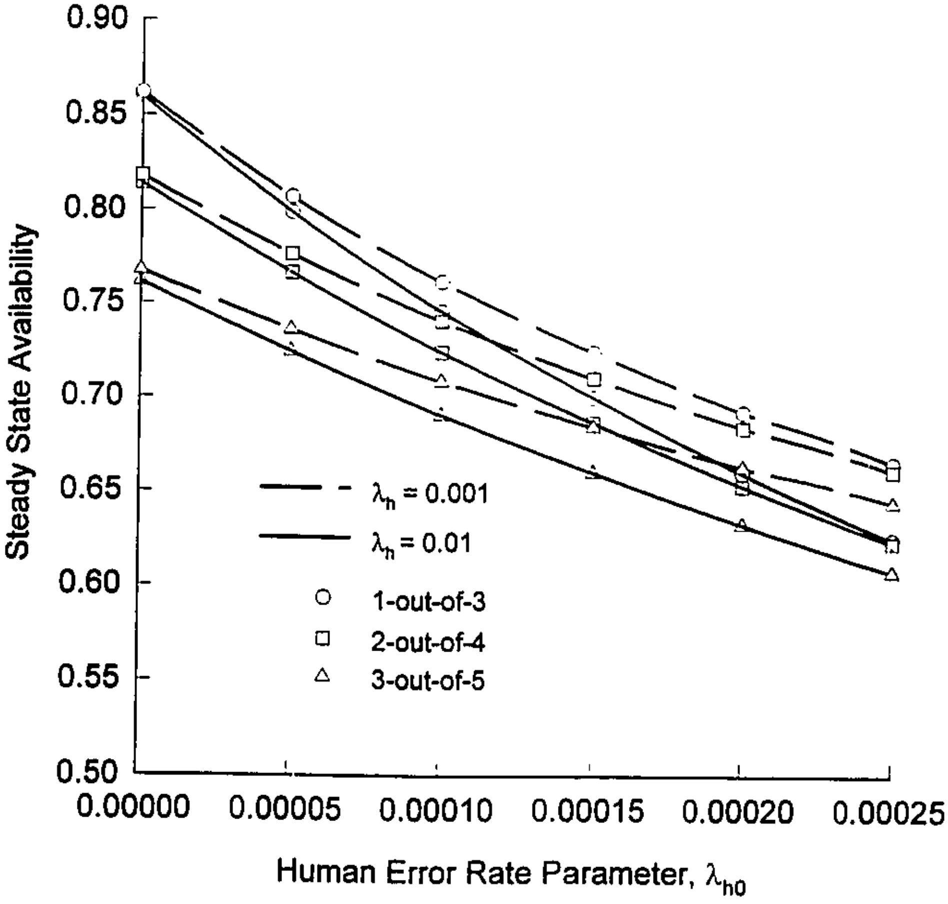


Figure 3-30 AV_{ss} plots for k-out-of-n (n-k=2) system with increasing human error rates and the Gamma (β=2) distributed failed system repair times

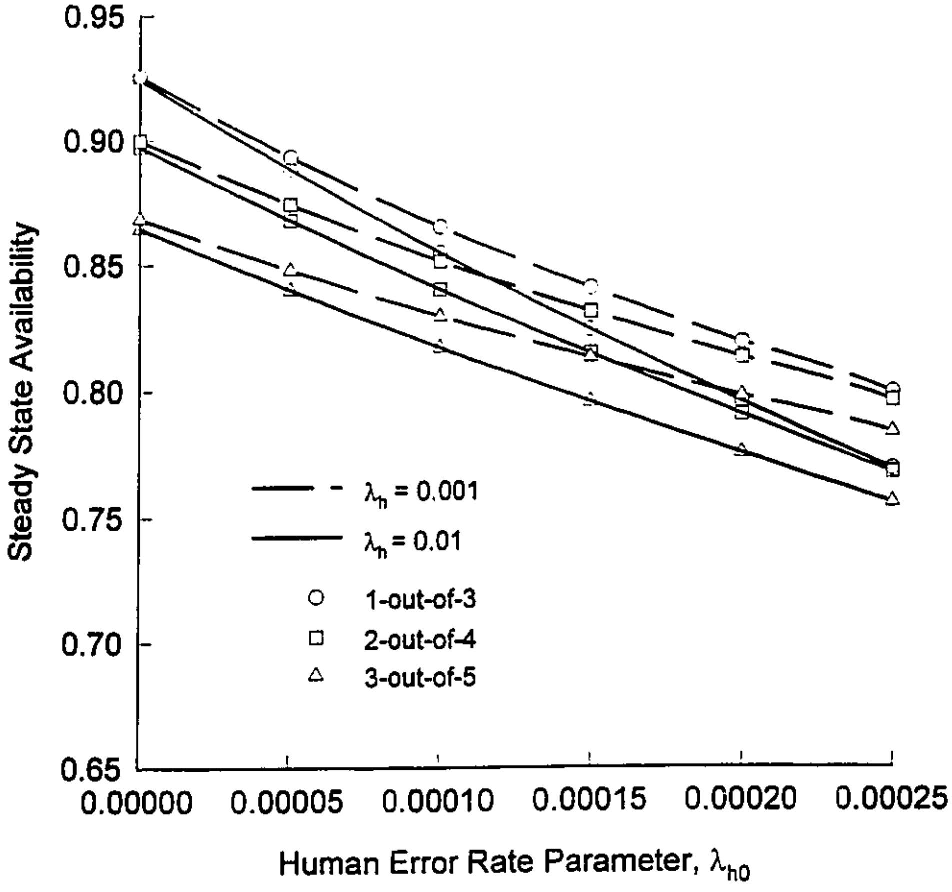


Figure 3-31 System steady state availability plots for k-out-of-n (n-k=2) system with increasing human error rates and constant system repair rates

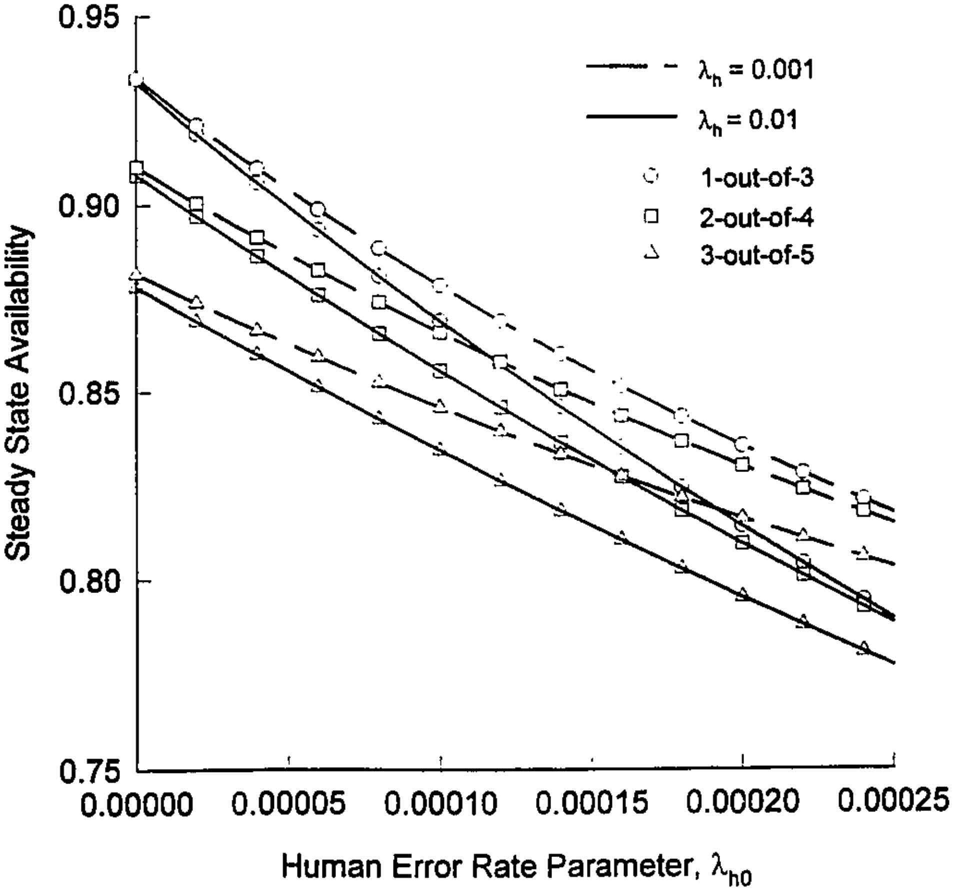


Figure 3-32 System steady state availability plots for k-out-of-n (n-k=2) system with increasing human error rates and the Weibull (β=2) distributed failed system repair times

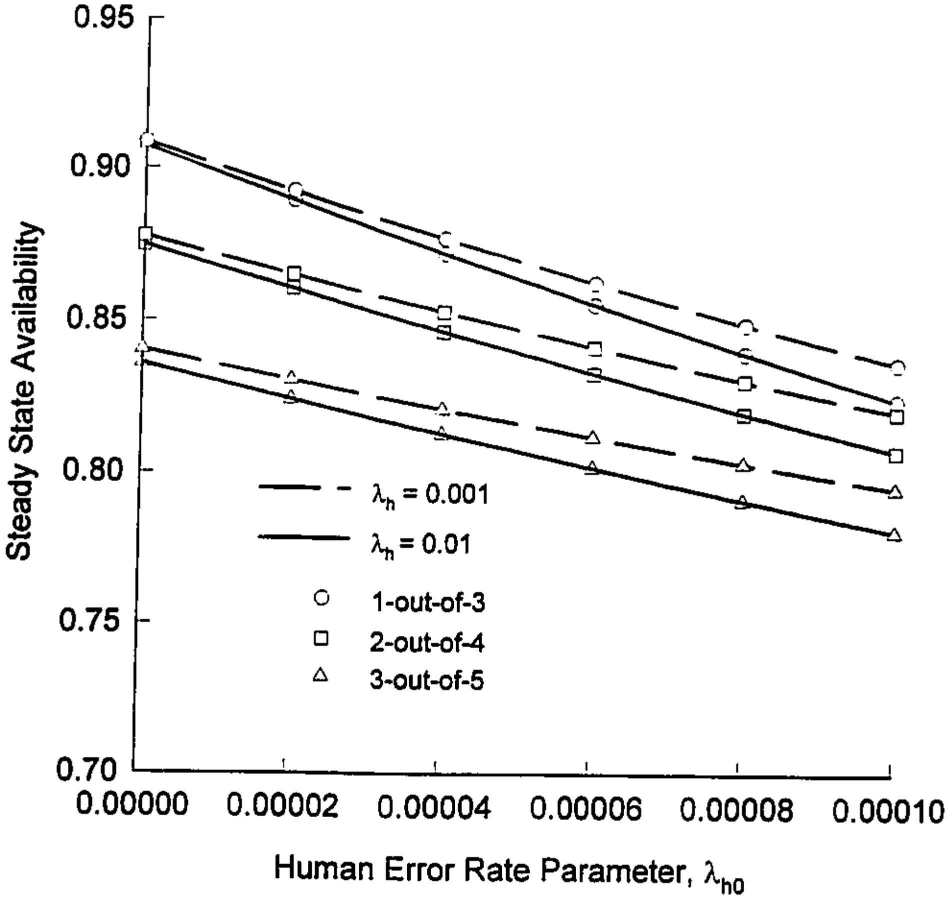


Figure 3-33 AV_{ss} plots for k-out-of-n (n-k=2) system with increasing human error rates and the Rayleigh distributed failed system repair times

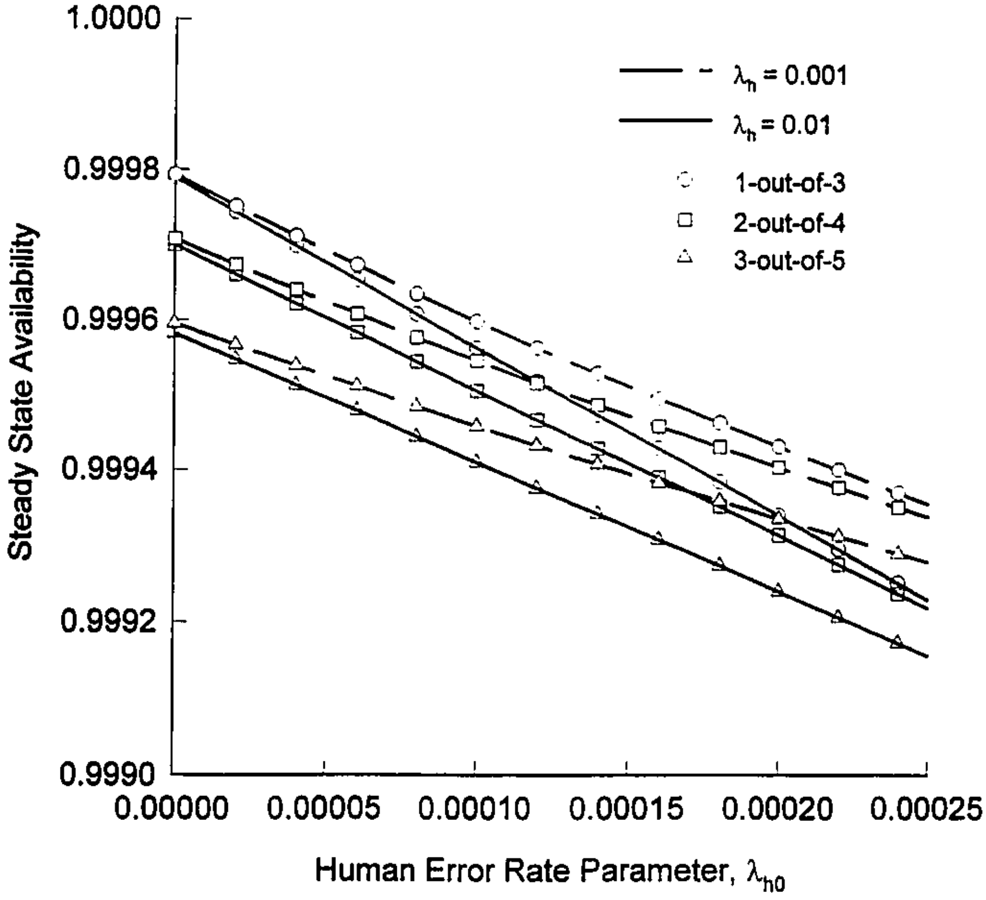


Figure 3-34 System steady state availability plots for k-out-of-n (n-k=2) system with increasing human error rates and the lognormally (σ=0.3) distributed system repair times

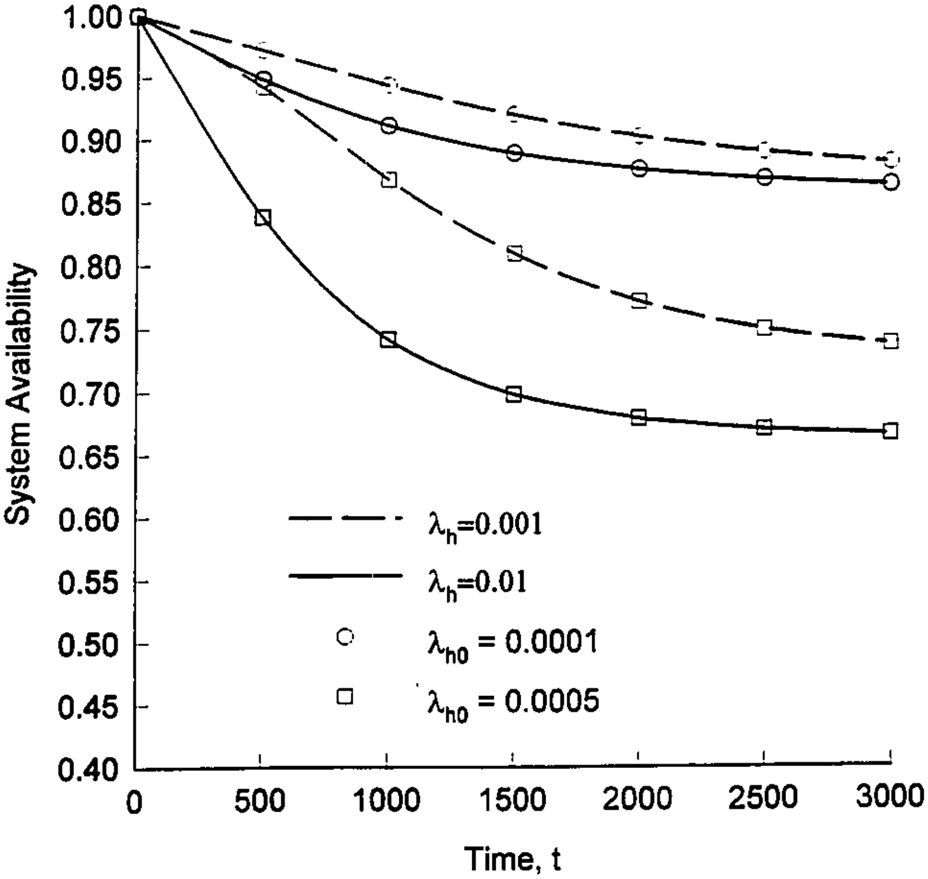


Figure 3-35 Time-dependent system availability plots for 1-out-of-3 system with increasing human error rates and exponentially distributed system repair times

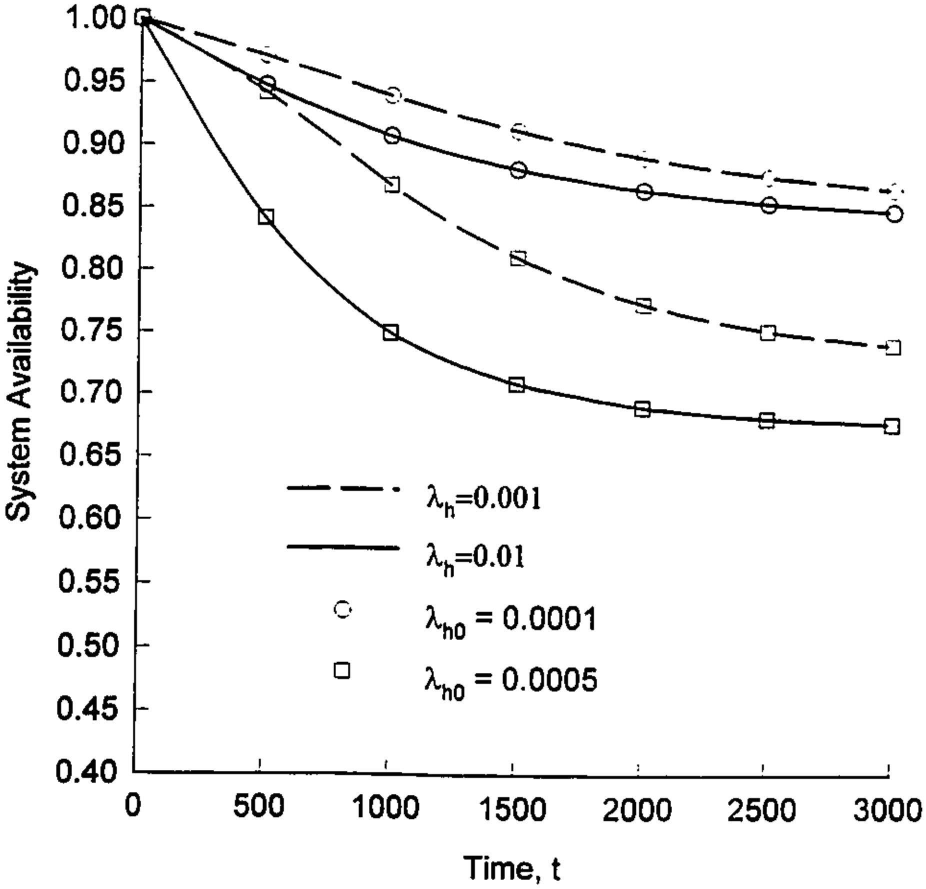


Figure 3-36 Time-dependent system availability plots for 2-out-of-4 system with increasing human error rates and exponentially distributed system repair times

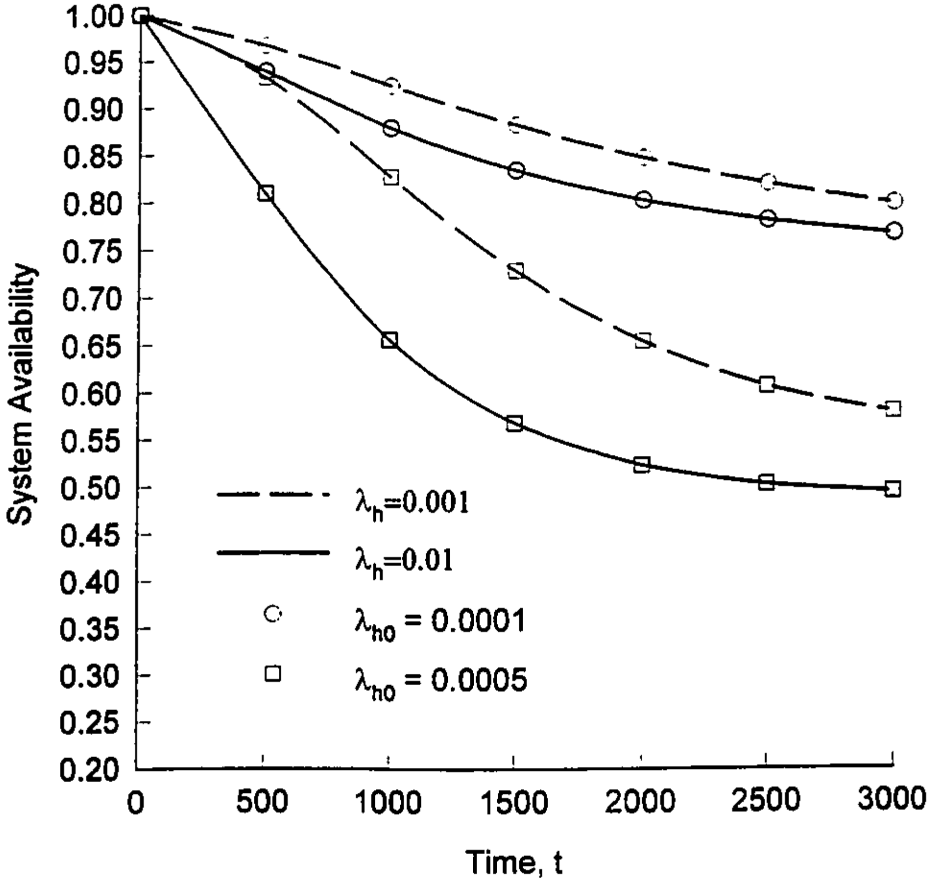


Figure 3-37 Time-dependent system availability plots for 1-out-of-3 system with increasing human error rates and the Gamma (β=2) distributed system repair times

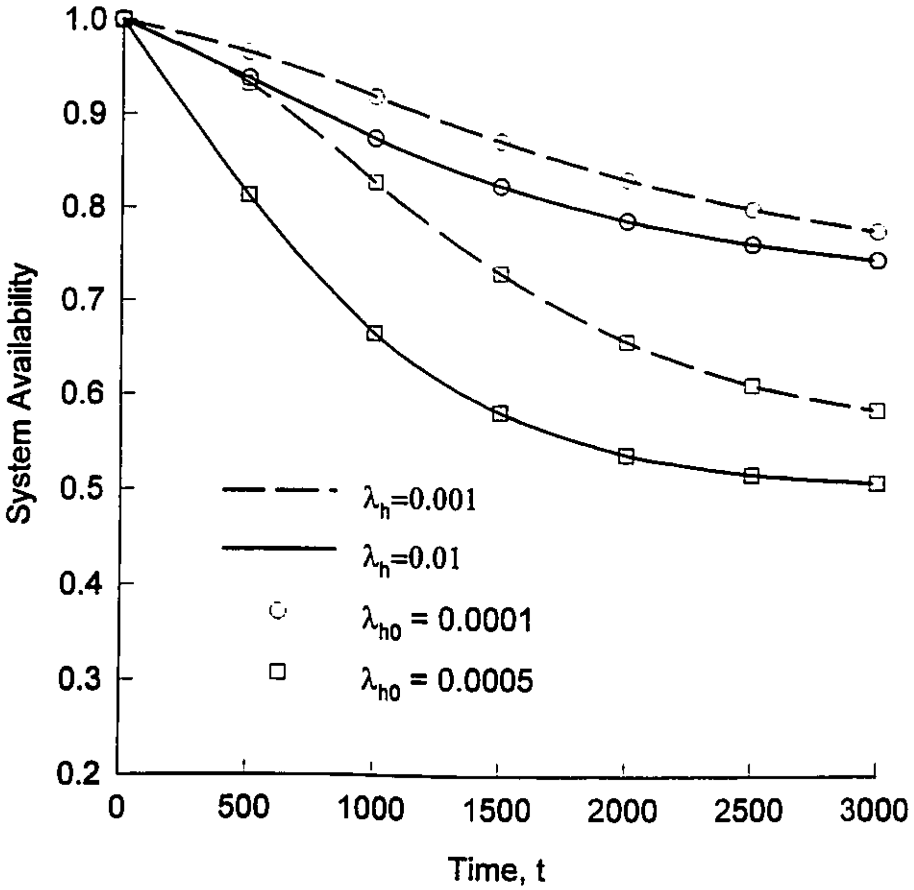


Figure 3-38 Time-dependent system availability plots for 2-out-of-4 system with increasing human error rates and the Gamma (β=2) distributed system repair times

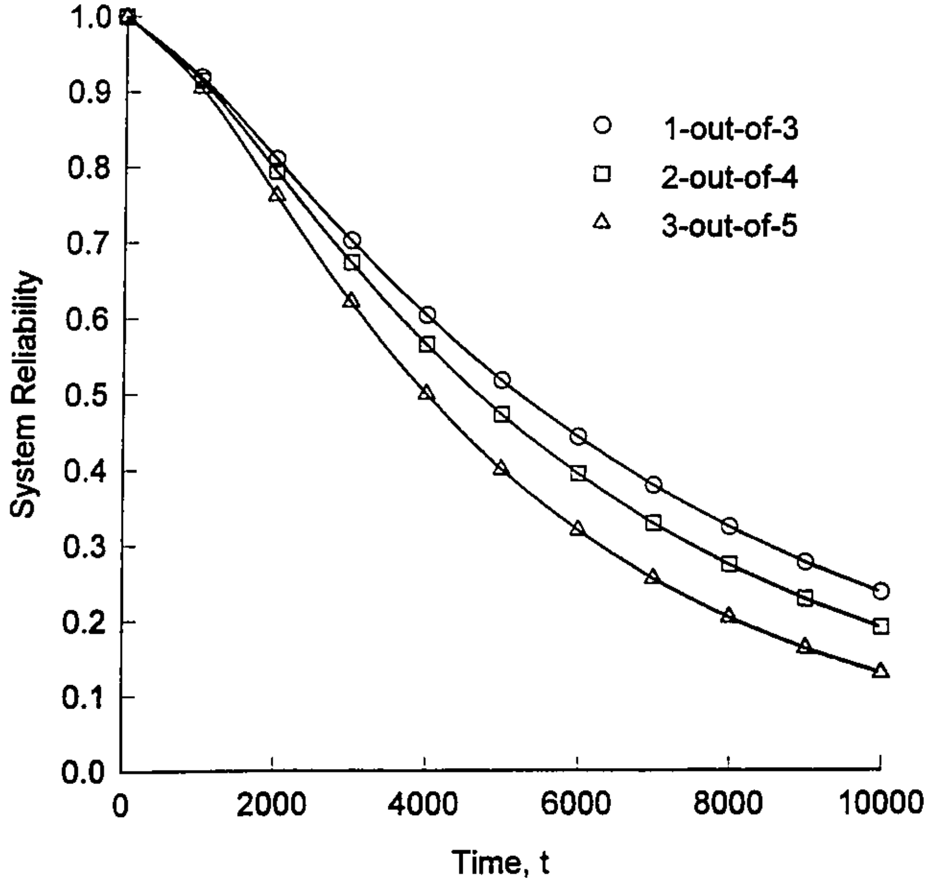


Figure 3-39 System reliability with repair plots for k-out-of-n (n-k=2) system with increasing human error rates (parameters $\lambda_h = 0.001$, λ_{h0}=0.0001)

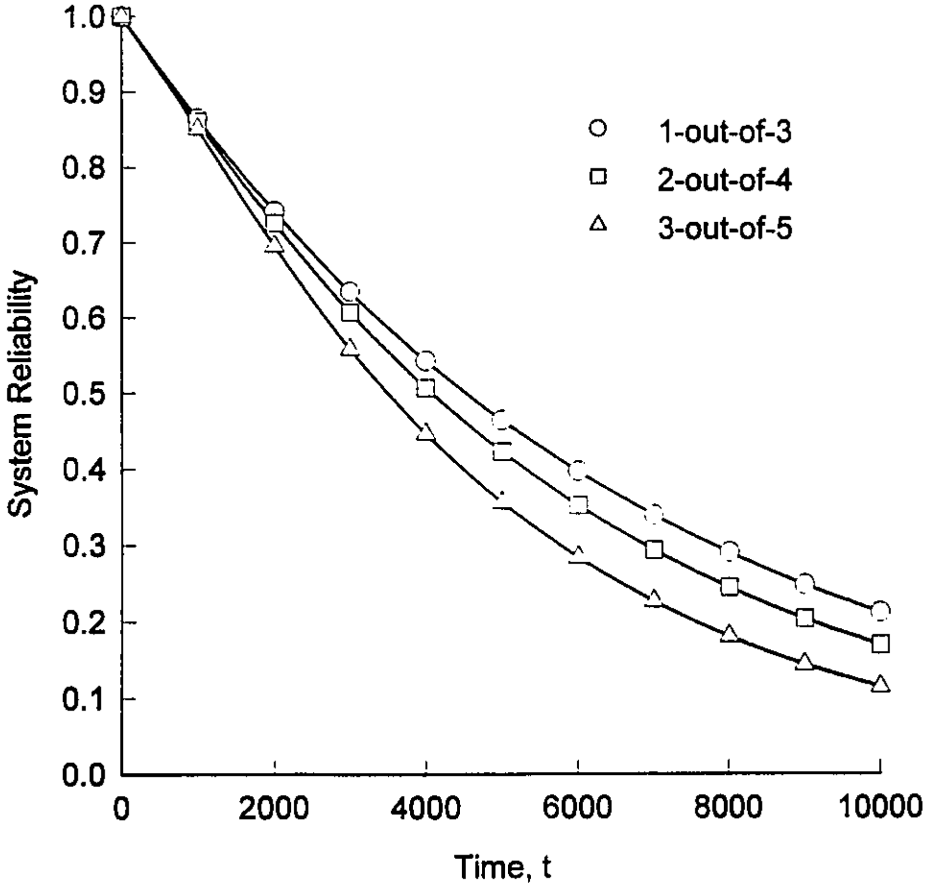


Figure 3-40 System reliability with repair plots for k-out-of-n (n-k=2) system with increasing human error rates (parameters $\lambda_h = 0.01$, $\lambda_{h0}=0.0001$)

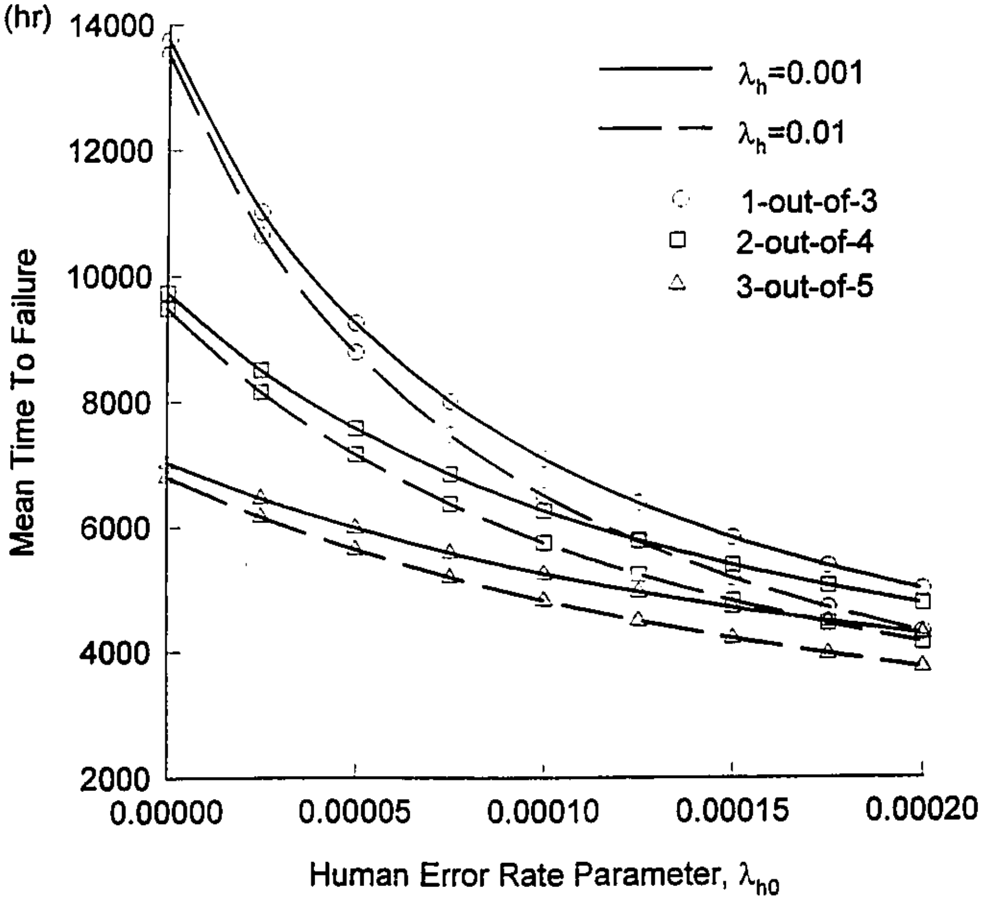


Figure 3-41 System mean time to failure with repair for k-out-of-n (n-k=2) system with increasing human error rates

Chapter 4

Systems With General Human Error and System Repair Time Distributions

With systems that are governed by the exponential distribution, the transition rate from one state to another state of a system is constant and this leads to a Markovian model. If the distribution is not exponential, then the process becomes non-Markovian. There are several techniques available for dealing with non-Markovian processes [68, 317].

One approach to modeling non-exponential distributions is by the inclusion of sufficient supplementary variables (such as the times expended in the repair process) in the specification of the state of the system to make the process Markovian.

In the method of stages, the non-exponential state is divided into a number of sub-states called stages each of which is exponentially distributed. The sequence of stages can be in series or in parallel or a combination thereof. Depending on the sequence and the rates of departure between them, the total time spent in the stages can assume a variety of distributions.

The approach that combine the inclusion supplementary variable method and the method of stages was developed in the previous chapter to solve the problem of time-dependent both system failure and repair rates. This chapter discusses a combination of parallel and series stages to represent a more general and flexible distribution for human error being considered. The inclusion supplementary variable method is still used to deal with the non-exponential failed system repair times.

4.1. The Description of Human Error Rate

We consider two series stages in parallel to represent human error distribution as shown in Figure 4-1.

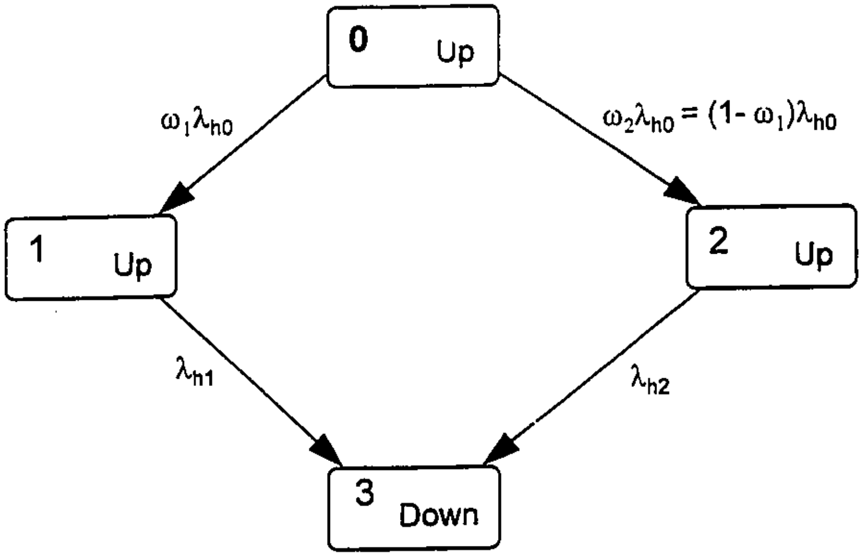


Figure 4-1 State space diagram for human error rate represented by two series stages in parallel

The associated equations with Figure 4-1 are

$$\frac{dP_0(t)}{dt} = -\omega_1\lambda_{h0}P_0(t) - \omega_2\lambda_{h0}P_0(t) \tag{4-1}$$

$$\frac{dP_1(t)}{dt} = \omega_1\lambda_{h0}P_0(t) - \lambda_{h1}P_1(t) \tag{4-2}$$

$$\frac{dP_2(t)}{dt} = \omega_2\lambda_{h0}P_0(t) - \lambda_{h2}P_2(t) \tag{4-3}$$

$$\frac{dP_3(t)}{dt} = \lambda_{h1}P_1(t) + \lambda_{h2}P_2(t) \tag{4-4}$$

At t=0, $P_0(0)=1$ and $P_1(0) = P_2(0) = P_3(0) = 0$.

Where ω_1 and ω_2 are the probabilities of beginning on the *i*th stage, the life thereafter consisting of a single stage, that, only a single stage is used in any one realization of time and we have the following relationship:

$$\omega_1 + \omega_2 = 1 \tag{4-5}$$

Solving Equations (4-1)-(4-4) we get the Laplace transform of the stage combination (from state 0 to state 3)

$$R(s) = \frac{1}{s+\lambda_{h0}} + \frac{\omega_1 \lambda_{h0}}{(s+\lambda_{h0})(s+\lambda_{h1})} + \frac{\omega_2 \lambda_{h0}}{(s+\lambda_{h0})(s+\lambda_{h2})} \tag{4-6}$$

The reliability of the stage combination can be obtained by inverting Equation (4-6)

$$\begin{aligned} R(t) = \exp(-\lambda_{h0}t) - \frac{\omega_1 \lambda_{h0}}{\lambda_{h0} - \lambda_{h1}}\left[\exp(-\lambda_{h0}t) - \exp(-\lambda_{h1}t)\right] \\ - \frac{\omega_2 \lambda_{h0}}{\lambda_{h0} - \lambda_{h2}}\left[\exp(-\lambda_{h0}t) - \exp(-\lambda_{h2}t)\right] \end{aligned} \tag{4-7}$$

The probability density function of the given stage combination is

$$\begin{aligned} f(t) &= -\frac{dR(t)}{dt} \\ &= \frac{\omega_1 \lambda_{h0} \lambda_{h1}}{\lambda_{h1} - \lambda_{h0}}[\exp(-\lambda_{h0}t) - \exp(-\lambda_{h1}t)] + \\ &\frac{\omega_2 \lambda_{h0} \lambda_{h2}}{\lambda_{h2} - \lambda_{h0}}[\exp(-\lambda_{h0}t) - \exp(-\lambda_{h2}t)] \end{aligned} \tag{4-8}$$

Thus, human error density function (the equivalent transfer rate from state 0 to state 3) is

$$\lambda_h(t) = \frac{f(t)}{R(t)} \tag{4-9}$$

There are four independent parameters, λ_{h0}, λ_{h1}, λ_{h2}, and ω_1, in the human error density function we can evaluate to match a particular distribution being modeled.

Figure 4-2 shows the time-dependent human error rate, $\lambda_h(t)$, for different values of parameters. When $\lambda_{h0} >> \lambda_{h1}$ and $\lambda_{h0} >> \lambda_{h2}$, $\lambda_h(t)$ is illustrated as plots 3 and 4, that is, $\lambda_h(t)$ decrease with time.

4.2. General Standby System

4.2.1. Description of the System

Using a combination of two series stages in parallel to represent general human error time distribution, for general standby system with both time-dependent human error rate and failed system repair rate, we have the system state-space diagram as shown in Figure 4-3.

The differential equations associated with Figure 4-3 is:

$$\frac{dP_0(t)}{dt} = -(r_0 + \lambda_{c0} + \omega_1\lambda_{h0} + \omega_2\lambda_{h0})P_0(t) + \mu_1 P_1(t) + \sum_{j=m+n}^{m+n+2} \int_0^\infty P_j(x,t)\mu_j(x)dx \quad (4\text{-}10)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - (\mu_1 + r_1 + \lambda_{c1} + \omega_1\lambda_{h1} + \omega_2\lambda_{h1})P_1(t) + \mu_2 P_2(t) \quad (4\text{-}11)$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - (\mu_i + r_i + \lambda_{ci} + \omega_1\lambda_{hi} + \omega_2\lambda_{hi})P_i(t) + \mu_{i+1} P_{i+1}(t) \quad (4\text{-}12)$$

(for i = 2, 3, ..., m+n-2)

$$\begin{aligned}\frac{dP_{m+n-1}(t)}{dt} = {} & r_{m+n-2} P_{m+n-2}(t) - (\mu_{m+n-1} + r_{m+n-1} + \lambda_{c(m+n-1)} \\ & + \omega_1\lambda_{h(m+n-1)} + \omega_2\lambda_{h(m+n-1)})P_{m+n-1}(t)\end{aligned} \quad (4\text{-}13)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x)P_j(x,t) \quad (4\text{-}14)$$

(for j = m+n, m+n+1, m+n+2)

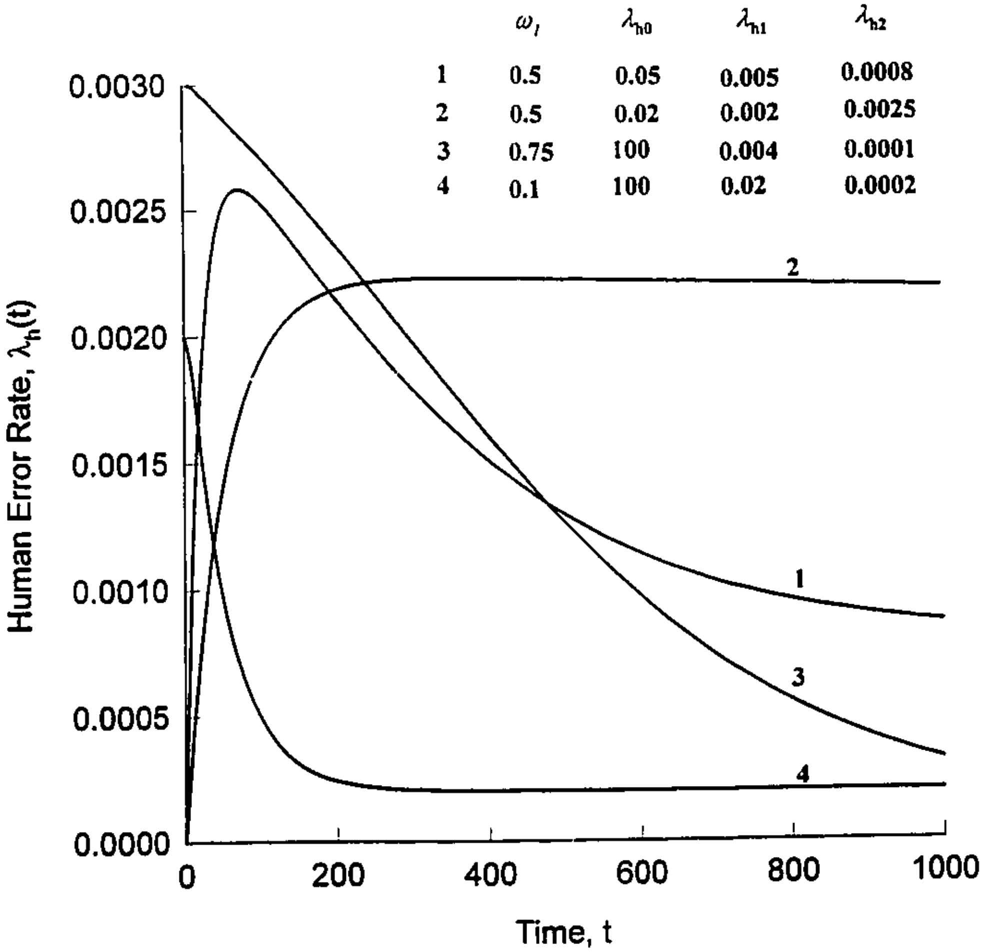


Figure 4-2 Human error rate distribution represented by two series stages in parallel for different values of parameters

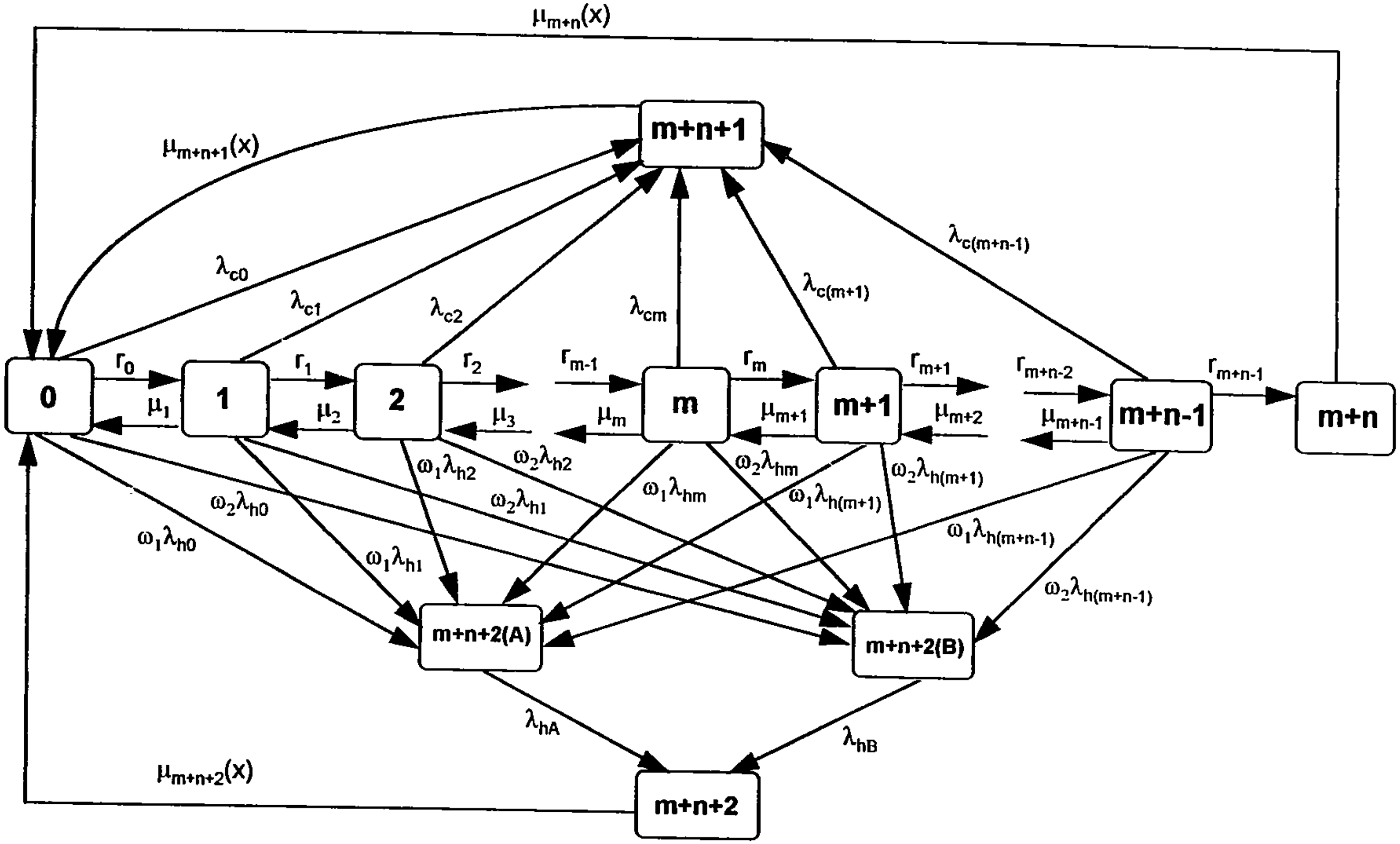


Figure 4-3 The state transition diagram of a general standby system with critical human error state represented by two series stages in parallel and general system repair rates

$$\frac{dP_{m+n+2(A)}(t)}{dt} = \omega_1 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) - \lambda_{hA} P_{m+n+2(A)}(t) \tag{4-15}$$

$$\frac{dP_{m+n+2(B)}(t)}{dt} = \omega_2 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) - \lambda_{hB} P_{m+n+2(B)}(t) \tag{4-16}$$

The associated boundary conditions are as follows:

$$P_{m+n}(0,t) = r_{m+n-1} P_{m+n-1}(t) \tag{4-17}$$

$$P_{m+n+1}(0,t) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(t) \tag{4-18}$$

$$P_{m+n+2}(0,t) = \lambda_{hA} P_{m+n+2(A)}(t) + \lambda_{hB} P_{m+n+2(B)}(t) \tag{4-19}$$

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

4.2.2 Steady State Availability Analysis

As time approaches infinity, Equations (4-10) - (4-16) reduce to following Equations:

$$a_0 P_0 - \mu_1 P_1 = \sum_{j=m+n}^{m+n+2} \int_0^{\infty} P_j(x) \mu_j(x) dx \tag{4-20}$$

$$r_0 P_0 - a_1 P_1 + \mu_2 P_2 = 0 \tag{4-21}$$

$$r_1 P_1 - a_2 P_2 + \mu_3 P_3 = 0 \tag{4-22}$$

$$r_{i-1} P_{i-1} - a_i P_i + \mu_{i+1} P_{i+1} = 0 \tag{4-23}$$

(i = 2, 3, ..., m+n-2)

$$r_{m+n-2} P_{m+n-2} - a_{n.+n-1} P_{m+n-1} = 0 \tag{4-24}$$

$$\frac{\partial P_j(x)}{\partial x} = -\mu_j(x) P_j(x) \qquad (4\text{-}25)$$

(j = m+n, m+n+1, m+n+2)

$$\omega_1 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i - \lambda_{hA} P_{m+n+2(A)} = 0 \qquad (4\text{-}26)$$

$$\omega_2 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i - \lambda_{hB} P_{m+n+2(B)} = 0 \qquad (4\text{-}27)$$

where

$$a_i = r_i + \lambda_{ci} + \lambda_{hi} + \mu_i$$

(for i = 0, 1, 2, 3, ..., m+n-1, and assuming μ_0=0)

Similarly, the boundary conditions become:

$$P_{m+n}(0) = r_{m+n-1} P_{m+n-1} \qquad (4\text{-}28)$$

$$P_{m+n+1}(0) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i \qquad (4\text{-}29)$$

$$P_{m+n+2}(0) = \lambda_{hA} P_{m+n+2(A)} + \lambda_{hB} P_{m+n+2(B)} \qquad (4\text{-}30)$$

Substituting Equations (4-28)-(4-30) in Equation (2-20), we obtain the steady state probabilities for state m+n, m+n+1, and m+n+2:

$$P_{m+n} = r_{m+n-1} P_{m+n-1} E_{m+n}[x] \qquad (4\text{-}31)$$

$$P_{m+n+1} = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i E_{m+n+1}[x] \qquad (4\text{-}32)$$

$$P_{m+n+2} = \left[\lambda_{hA} P_{m+n+2(A)} + \lambda_{hB} P_{m+n+2(B)}\right] E_{m+n+2}[x] \qquad (4\text{-}33)$$

The mean time to system repair, $E_{m+n}[x]$, $E_{m+n+1}[x]$, and $E_{m+n+2}[x]$, are still given by Equation (2-24).

Solving the set of Equations (4-21) - (4-24), (4-26), (4-27) and (4-31) - (4-33), together with:

$$\sum_{i=0}^{m+n+2} P_i + P_{m+n+2(A)} + P_{m+n+2(B)} = 1 \qquad (4\text{-}34)$$

we can obtain steady state probabilities, P_0, P_1, P_2, ..., P_{m+n}, P_{m+n+1}, P_{m+n+2}, $P_{m+n+2(A)}$ and $P_{m+n+2(B)}$.

Thus, the steady state availability of the system is

$$AV_{ss} = \sum_{i=0}^{m+n-1} P_i + P_{m+n+2(A)} + P_{m+n+2(B)} \qquad (4\text{-}35)$$

Similarly, the steady state unavailability of the system is given by

$$AV_{ss} = P_{m+n} + P_{m+n+1} + P_{m+n+2} \qquad (4\text{-}36)$$

4.2.3. Time-Dependent Availability Analysis

Using Laplace transform technique and the initial conditions in Equations (4-10) - (4-16), we get

$$s\,P_0(s) = 1 - a_0 P_0(s) + \mu_1 P_1(s) + \sum_{j=m+n}^{m+n+2} \int_0^\infty P_j(x,s)\mu_j(x)dx \qquad (4\text{-}37)$$

$$(s + a_i)P_i(s) - r_{i-1}P_{i-1}(s) - \mu_{i+1}P_{i+1}(s) = 0 \qquad (4\text{-}38)$$

(for i =1, 2, 3, ..., m+n-2)

$$(s + a_{m+n-1})P_{m+n-1}(s) - r_{m+n-2}P_{m+n-2}(s) = 0 \qquad (4\text{-}39)$$

$$\frac{\partial P_j(x,s)}{\partial x} + s\,P_j(x,s) + \mu_j(x)P_j(x,s) = 0 \qquad (4\text{-}40)$$

(for j = m+n, m+n+1, m+n+2)

$$(s+\lambda_{hA})P_{m+n+2(A)}(s) - \omega_1 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(s) = 0 \qquad (4\text{-}41)$$

$$(s+\lambda_{hB})P_{m+n+2(B)}(s) - \omega_2 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(s) = 0 \qquad (4\text{-}42)$$

and the boundary conditions:

$$P_{m+n}(0,s) = r_{m+n-1} P_{m+n-1}(s) \qquad (4\text{-}43)$$

$$P_{m+n+1}(0,s) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(s) \qquad (4\text{-}44)$$

$$P_{m+n+2}(0,s) = \lambda_{hA} P_{m+n+2(A)}(s) + \lambda_{hB} P_{m+n+2(B)}(s) \qquad (4\text{-}45)$$

Substituting the boundary conditions in Equation (2-35) and (2-36), we have:

$$P_{m+n}(s) = r_{m+n-1} P_{m+n-1}(s) \frac{1 - N_{m+n}(s)}{s} \qquad (4\text{-}46)$$

$$P_{m+n+1}(s) = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(s) \frac{1 - N_{m+n+1}(s)}{s} \qquad (4\text{-}47)$$

$$P_{m+n+2}(s) = \left[\lambda_{hA} P_{m+n+2(A)}(s) + \lambda_{hB} P_{m+n+2(B)}(s)\right] \frac{1 - N_{m+n+2}(s)}{s} \qquad (4\text{-}48)$$

The Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_{m+n}(s)$, $P_{m+n+1}(s)$, $P_{m+n+2}(s)$, $P_{m+n+2(A)}(s)$ and $P_{m+n+2(B)}$, were obtained by solving Equations (4-38), (4-39), (4-41), (4-42), (4-46) - (4-48) and the following equation:

$$\sum_{i=0}^{m+n+2} P_i(s) + P_{m+n+2(A)}(s) + P_{m+n+2(B)}(s) = \frac{1}{s} \qquad (4\text{-}49)$$

The Laplace transform of the system availability is

$$AV(s) = \sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s) + P_{m+n+2(B)}(s) \tag{4-50}$$

Substituting the Laplace transform of $N_i(x)$ for different repair time distributions in Equation (4-50) and taking the inverse Laplace transform of the resulting equation, we get the following time-dependent system availability:

$$AV(t) = \sum_{i=0}^{m+n-1} P_i(t) + P_{m+n+2(A)}(t) + P_{m+n+2(B)}(t) \tag{4-51}$$

4.2.4. System Reliability and MTTF With and Without Repair

Setting $\mu_{m+n}(x) = \mu_{m+n+1}(x) = \mu_{m+n+2}(x) = 0$ in Figure 4-3, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \tag{4-52}$$

$$\frac{dP_i(t)}{dt} = r_{i-1} P_{i-1}(t) - a_i P_i(t) + \mu_{i+1} P_{i+1}(t) \tag{4-53}$$

(for $i = 1, 2, 3, \ldots, m+n-2$)

$$\frac{dP_{m+n-1}(t)}{dt} = r_{m+n-2} P_{m+n-2}(t) - a_{m+n-1} P_{m+n-1}(t) \tag{4-54}$$

$$\frac{dP_{m+n}(t)}{dt} = r_{m+n-1} P_{m+n-1}(t) \tag{4-55}$$

$$\frac{dP_{m+n+1}(t)}{dt} = \sum_{i=0}^{m+n-1} \lambda_{ci} P_i(t) \tag{4-56}$$

$$\frac{dP_{m+n+2(A)}(t)}{dt} = \omega_1 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) - \lambda_{hA} P_{m+n+2(A)}(t) \tag{4-57}$$

$$\frac{dP_{m+n+2(B)}(t)}{dt} = \omega_2 \sum_{i=0}^{m+n-1} \lambda_{hi} P_i(t) - \lambda_{hB} P_{m+n+2(B)}(t) \qquad (4\text{-}58)$$

$$\frac{dP_{m+n+2}(t)}{dt} = \lambda_{hA} P_{m+n+2(A)}(t) + \lambda_{hB} P_{m+n+2(B)}(t) \qquad (4\text{-}59)$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for $i = 1, 2, \ldots, m+n+2, m+n+2(A)$ *and* $m+n+2(B)$.

Using Laplace transform in Equations (4-52) - (4-59) and solving the resulting set of Equations. we get the Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_{m+n+2}(s)$, $P_{m+n+2(A)}(s)$ and $P_{m+n+2(B)}$.

Thus, the Laplace transform of the system reliability with repair is:

$$R(s) = \sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s) + P_{m+n+2(B)}(s) \qquad (4\text{-}60)$$

The time-dependent system reliability with repair may be obtained by inverting above Equation:

$$R(t) = L^{-1}\left[R(s)\right] = L^{-1}\left[\sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s) + P_{m+n+2(B)}(s)\right] \qquad (4\text{-}61)$$

The system mean time to failure (MTTF) with repair is given by:

$$MTTF = \lim_{s \to 0}\left[\sum_{i=0}^{m+n-1} P_i(s) + P_{m+n+2(A)}(s) + P_{m+n+2(B)}(s)\right] \qquad (4\text{-}62)$$

The system variance of time to failure with repair is expressed by:

$$\sigma^2 = -2 \lim_{s \to 0} R'(s) - (MTTF)^2 \qquad (4\text{-}63)$$

Setting $\mu_1 = \mu_2 = \ldots = \mu_{m+n-1} = 0$ in Figure 4-3 and in Equations (4-61) - (4-63), we get the system reliability, MTTF and the system variance of time to failure without repair, respectively.

4.3. Special Cases of Standby Systems With General Human Errors and System Repair Time Distributions

4.3.1. One Unit Active and One Unit on Standby System

For m = 1 and n = 1 in Figure 4-3, the system of differential equations associated with the model can be written as follows:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=2}^{4} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \tag{4-64}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) \tag{4-65}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \tag{4-66}$$

$(j=2,\ 3,\ 4)$

$$\frac{dP_{4A}(t)}{dt} = \omega_1 \left[\lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) \right] - \lambda_{hA} P_{4A}(t) \tag{4-67}$$

$$\frac{dP_{4B}(t)}{dt} = \omega_2 \left[\lambda_{h0} P_0(t) + \lambda_{h1} P_1(t) \right] - \lambda_{hB} P_{4B}(t) \tag{4-68}$$

The associated boundary conditions are as follows:

$$P_2(0,t) = r_1 P_1(t) \tag{4-69}$$

$$P_3(0,t) = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \tag{4-70}$$

$$P_4(0,t) = \lambda_{hA} P_{4A}(t) + \lambda_{hB} P_{4B}(t) \tag{4-71}$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

As t approaches infinity, from Equations (4-64) - (4-71), we have:

$$a_0 P_0 - \mu_1 P_1 = \sum_{j=2}^{4} \int_0^{\infty} P_j(x) \mu_j(x) dx \tag{4-72}$$

$$r_0 P_0 - a_1 P_1 = 0 \tag{4-73}$$

$$\frac{dP_j(x)}{dx} = -\mu_j(x) P_j(x) \tag{4-74}$$

$(j = 2, 3, 4)$

$$\omega_1 (\lambda_{h0} P_0 + \lambda_{h1} P_1) - \lambda_{hA} P_{4A} = 0 \tag{4-75}$$

$$\omega_2 (\lambda_{h0} P_0 + \lambda_{h1} P_1) - \lambda_{hB} P_{4B} = 0 \tag{4-76}$$

The boundary conditions become:

$$P_2(0) = r_1 P_1 \tag{4-77}$$

$$P_3(0) = \lambda_{c0} P_0 + \lambda_{c1} P_1 \tag{4-78}$$

$$P_4(0) = \lambda_{hA} P_{4A} + \lambda_{hB} P_{4B} \tag{4-79}$$

Substituting Equations (4-77) - (4-79) in Equation (2-20), for j =2, 3, 4, we have:

$$P_2 = r_1 P_1 E_2[x] \tag{4-80}$$

$$P_3 = (\lambda_{c0} P_0 + \lambda_{c1} P_1) E_3[x] \tag{4-81}$$

$$P_4 = (\lambda_{hA} P_{4A} + \lambda_{hB} P_{4B}) E_4[x] \tag{4-82}$$

Solving the set of Equations (4-73), (4-75), (4-76), (4-80) - (4-82), together with

$$\sum_{i=0}^{4} P_i + P_{4A} + P_{4B} = 1 \qquad (4\text{-}83)$$

The steady state probabilities are:

$$P_0 = \frac{b_0(1,1)\lambda_{hA}\lambda_{hB}}{BC(1,1)} \qquad (4\text{-}84)$$

$$P_1 = \frac{b_1(1,1)\lambda_{hA}\lambda_{hB}}{BC(1,1)} \qquad (4\text{-}85)$$

$$P_2 = \frac{b_2(1,1)\lambda_{hA}\lambda_{hB}E_2[x]}{BC(1,1)} \qquad (4\text{-}86)$$

$$P_3 = \frac{b_3(1,1)\lambda_{hA}\lambda_{hB}E_3[x]}{BC(1,1)} \qquad (4\text{-}87)$$

$$P_4 = \frac{b_4(1,1)\lambda_{hA}\lambda_{hB}E_4[x]}{BC(1,1)} \qquad (4\text{-}88)$$

$$P_{4A} = \frac{\omega_1\lambda_{hB}b_4(1,1)}{BC(1,1)} \qquad (4\text{-}89)$$

$$P_{4B} = \frac{\omega_2\lambda_{hA}b_4(1,1)}{BC(1,1)} \qquad (4\text{-}90)$$

where:

$$BC(1,1) = \lambda_{hA}\lambda_{hB} \cdot BA(1,1) + bc1(1,1)$$

The steady state availability of the system is:

$$\begin{aligned} AV_{ss}(1,1) &= P_0 + P_1 + P_{4A} + P_{4B} \\ &= \frac{bc2(1,1) + bc1(1,1)}{BC(1,1)} \end{aligned} \qquad (4\text{-}91)$$

where:

$$bc1(1,1) = (\omega_1\lambda_{hB} + \omega_2\lambda_{hA})b_4(1,1)$$

$$bc2(1,1) = \lambda_{hA}\lambda_{hB}\sum_{i=0}^{1} b_i(1,1)$$

The steady state unavailability of the system is:

$$UAV_{ss}(1,1) = P_2 + P_3 + P_4 = \frac{\sum_{j=2}^{4} b_j(1,1)E_j[x]\lambda_{hA}\lambda_{hB}}{BC(1,1)} \tag{4-92}$$

Special cases of steady state availability

Case I

for the *Gamma* repair time distribution, the steady state availability is:

$$AV_{ss}(1,1) = \frac{bc1(1,1) + bc2(1,1)}{\lambda_{hA}\lambda_{hB}\left[\sum_{i=0}^{1} b_i(1,1) + \sum_{j=2}^{4} b_j(1,1)\,\beta\mu_j\right] + bc1(1,1)} \tag{4-93}$$

Case II

For the ***Weibull*** repair time distribution, the steady state availability is:

$$AV_{ss}(1,1) = \frac{bc1(1,1) + bc2(1,1)}{\lambda_{hA}\lambda_{hB}\left[\sum_{i=0}^{1} b_i(1,1) + \sum_{j=2}^{4} b_j(1,1)\,\Gamma(1 + 1/\beta)\right] + bc1(1,1)} \tag{4-94}$$

Case III

If the system repair time is the ***Rayleigh*** distribution the steady state availability is:

$$AV_{ss}(1,1)=\frac{bc1(1,1)+bc2(1,1)}{\lambda_{hA}\lambda_{hB}\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\ \sqrt{\tfrac{\pi}{2}}\Big/\mu_j\right]+bc1(1,1)} \tag{4-95}$$

Case IV

If the system repair time is the ***lognormal*** distribution the steady state availability is:

$$AV_{ss}(1,1)=\frac{bc1(1,1)+bc2(1,1)}{\lambda_{hA}\lambda_{hB}\left[\sum_{i=0}^{1}b_i(1,1)+\sum_{j=2}^{4}b_j(1,1)\ \exp(\mu_j+\sigma_j^2/2)\right]+bc1(1,1)} \tag{4-96}$$

Time-Dependent Availability Analysis

By using Laplace transform and the initial conditions in Equations (4-64) - (4-71), we have:

$$s\,P_0(s)=1-a_0P_0(s)+\mu_1P_1(s)+\sum_{j=2}^{4}\int_0^{\infty}P_j(x,s)\mu_j(x)dx \tag{4-97}$$

$$(s+a_1)P_1(s)-r_0P_0(s)=0 \tag{4-98}$$

$$\frac{\partial P_j(x,s)}{\partial x}+s\,P_j(x,s)+\mu_j(x)P_j(x,s)=0 \tag{4-99}$$

(for j = 2, 3, 4)

$$(s+\lambda_{hA})P_{4A}(s)-\omega_1\sum_{i=0}^{1}\lambda_{hi}P_i(s)=0 \tag{4-100}$$

$$(s+\lambda_{hB})P_{4B}(s)-\omega_2\sum_{i=0}^{1}\lambda_{hi}P_i(s)=0 \tag{4-101}$$

and the boundary conditions:

$$P_2(0,s) = r_1\, P_1(s) \tag{4-102}$$

$$P_3(0,s) = \lambda_{c0} P_0(s) + \lambda_{c1} P_1(s) \tag{4-103}$$

$$P_4(0,s) = \lambda_{hA} P_{4A}(s) + \lambda_{hB} P_{4B}(s) \tag{4-104}$$

Substituting the boundary conditions in Equation (2-36), we have:

$$P_2(s) = r_1 P_1(s) \frac{1 - N_2(s)}{s} \tag{4-105}$$

$$P_3(s) = \left[\lambda_{c0} P_0(s) + \lambda_{c1} P_1(S)\right] \frac{1 - N_3(s)}{s} \tag{4-106}$$

$$P_4(s) = \left[\lambda_{4A} P_{4A}(s) + \lambda_{4B} P_{4B}(s)\right] \frac{1 - N_4(s)}{s} \tag{4-107}$$

The Laplace transform of state probabilities, $P_0(s)$, $P_1(s)$, ..., $P_4(s)$, $P_{4A}(s)$ and $P_{4B}(s)$, are obtained by solving Equations (4-98), (4-100), (4-101), (4-105) - (4-107) together with

$$\sum_{i=0}^{4} P_i(s) + P_{4A}(s) + P_{4B}(s) = \frac{1}{s} \tag{4-108}$$

and the Laplace transform of state probabilities of the system is:

$$P_0(s) = \frac{[s + b_0(1,1)](s + \lambda_{hA})(s + \lambda_{hB})}{CC(1,1)} \tag{4-109}$$

$$P_1(s) = \frac{b_1(1,1)(s + \lambda_{hA})(s + \lambda_{hB})}{CC(1,1)} \tag{4-110}$$

$$P_2(s) = \frac{b_2(1,1)(s + \lambda_{hA})(s + \lambda_{hB})[1 - N_2(s)]}{s \cdot CC(1,1)} \tag{4-111}$$

$$P_3(s) = \frac{[\lambda_{c0} \cdot s + b_3(1,1)](s + \lambda_{hA})(s + \lambda_{hB})[1 - N_3(s)]}{s \cdot CC(1,1)} \tag{4-112}$$

$$P_4(s) = \frac{[\lambda_{h0} \cdot s + b_4(1,1)]\left[(\omega_1 \lambda_{hA} + \omega_2 \lambda_{hB}) \cdot s + \lambda_{hA} \lambda_{hB}\right][1 - N_4(s)]}{s \cdot CC(1,1)} \qquad (4\text{-}113)$$

$$P_{4A}(s) = \frac{\omega_1 [\lambda_{h0} \cdot s + b_4(1,1)](s + \lambda_{hB})}{CC(1,1)} \qquad (4\text{-}114)$$

$$P_{4B}(s) = \frac{\omega_2 [\lambda_{h0} \cdot s + b_4(1,1)](s + \lambda_{hA})}{CC(1,1)} \qquad (4\text{-}115)$$

where

$$CC(1,1) = cc1(1,1) + cc2(1,1) \cdot s + cc3(1,1) \cdot s^2 + cc4(1,1) \cdot s^3 + s^4$$

$$cc1(1,1) = \lambda_{hA} \lambda_{hB} \cdot ca1(1,1)$$

$$\begin{aligned} cc2(1,1) = {} & \lambda_{hA} \left[\sum_{j=2}^{3} b_j(1,1)\left[1 - N_j(s)\right] + b_4(1,1)\left(1 - \omega_1 N_4(s)\right) \right] \\ & + \lambda_{hB} \left[\sum_{j=2}^{3} b_j(1,1)\left[1 - N_j(s)\right] + b_4(1,1)\left(1 - \omega_2 N_4(s)\right) \right] + \lambda_{hA} \lambda_{hB} \cdot ca2(1,1) \end{aligned}$$

$$\begin{aligned} cc3(1,1) = {} & \lambda_{hA} \left[\sum_{i=0}^{1} b_i(1,1) + \lambda_{c0}\left[1 - N_3(s)\right] + \lambda_{h0}\left[1 - \omega_1 N_4(s)\right] \right] \\ & + \lambda_{hB} \left[\sum_{i=0}^{1} b_i(1,1) + \lambda_{c0}\left[1 - N_3(s)\right] + \lambda_{h0}\left[1 - \omega_2 N_4(s)\right] \right] \\ & + \lambda_{hA} \lambda_{hB} + \sum_{j=2}^{3} b_j(1,1)\left[1 - N_j(s)\right] + b_4(1,1) \end{aligned}$$

$$cc4(1,1) = \lambda_{hA} + \lambda_{hB} + \lambda_{h0} + \sum_{i=0}^{1} b_i(1,1) + \lambda_{c0}\left[1 - N_3(s)\right]$$

The Laplace transform of the system availability is :

$$AV(s) = P_0(s) + P_1(s) + P_{4A}(s) + P_{4B}(s)$$

$$= \frac{\left[s + \sum_{i=0}^{1} b_i(1,1)\right] \cdot (s + \lambda_{hA})(s + \lambda_{hB}) + (\lambda_{h0} \cdot s + b_4(1,1))\left[\omega_1 (s + \lambda_{hB}) + \omega_2 (s + \lambda_{hA})\right]}{CC(1,1)}$$

(4-116)

Substituting the Laplace transform of pdf of system repair time distribution, $N_i(s)$ for $j = 2, 3, 4$, in the above equation and taking the inverse Laplace transform of the resulting equation, we get the time-dependent system availability:

$$AV(t) = P_0(t) + P_1(t) + P_{4A}(t) + P_{4B}(t) \qquad (4\text{-}117)$$

Reliability and MTTF With and Without Repair

Setting $\mu_2(x) = \mu_3(x) = \mu_4(x) = 0$ in the model, the system of differential equations becomes:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 p_1(t) \qquad (4\text{-}118)$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) \qquad (4\text{-}119)$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) \qquad (4\text{-}120)$$

$$\frac{dP_3(t)}{dt} = \lambda_{c0} P_0(t) + \lambda_{c1} P_1(t) \qquad (4\text{-}121)$$

$$\frac{dP_{4A}(t)}{dt} = \omega_1 \left[\lambda_{h0} P_0(t) + \lambda_{h1} P_1(t)\right] - \lambda_{hA} P_{4A}(t) \qquad (4\text{-}122)$$

$$\frac{dP_{4B}(t)}{dt} = \omega_2 \left[\lambda_{h0} P_0(t) + \lambda_{h1} P_1(t)\right] - \lambda_{hB} P_{4B}(t) \qquad (4\text{-}124)$$

$$\frac{dP_4(t)}{dt} = \lambda_{hA} P_{4A}(t) + \lambda_{hB} P_{4B}(t) \qquad (4\text{-}125)$$

At time t=0, $P_0(0) = 1$, and $P_i(0) = 0$, for i = 1, 2, 3, 4, 4A and 4B

Using Laplace transform in Equations (4-118) - (4-124) and solving the resulting set of Equations. we have the Laplace transform of the state probabilities, and the Laplace transform of the system reliability with repair:

$$R(s) = \frac{s + a_1 + r_0}{DA(1,1)} + \frac{\omega_1\left[\lambda_{h0} s + b_4(1,1)\right]}{\left(s - \lambda_{hA}\right)\cdot DA(1,1)} + \frac{\omega_2\left[\lambda_{h0} s + b_4(1,1)\right]}{\left(s - \lambda_{hB}\right)\cdot DA(1,1)} \qquad (4\text{-}125)$$

The system reliability with repair can be obtained by inverting Equation (4-125).

The system mean time to failure (MTTF) with repair is given by:

$$MTTF = \lim_{s\to 0} R(s) = \frac{bc2(1,1) - bc1(1,1)}{\lambda_{hA}\lambda_{hB}\cdot da1(1,1)} \qquad (4\text{-}126)$$

The system variance of time to failure with repair is:

$$\begin{aligned}\sigma^2 &= -2\lim_{s\to 0} R'(s) - (MTTF)^2\\ &= \frac{\left[bc1(1,1) - bc2(1,1)\right]\left[bc2(1,1) + 2\lambda_{hA}\lambda_{hB}\cdot da1(1,1) - bc1(1,1)\right]}{\left[\lambda_{hA}\lambda_{hB}\cdot da1(1,1)\right]^2}\end{aligned} \qquad (4\text{-}127)$$

When $\mu_1 = 0$ in Equations (4-125) - (4-127), we get the system reliability, MTTF and the system variance of time to failure without repair for the one unit active and one unit on standby system with general human error rates and system repair rates, respectively.

4.3.2. Two Units Active and One Unit on Standby System

For m = 2 and n = 1 in Figure 4-3, the system of differential equations associated with the model is:

$$\frac{dP_0(t)}{dt} = -a_0 P_0(t) + \mu_1 P_1(t) + \sum_{j=3}^{5} \int_0^{\infty} P_j(x,t)\mu_j(x)dx \tag{4-128}$$

$$\frac{dP_1(t)}{dt} = r_0 P_0(t) - a_1 P_1(t) + \mu_2 P_2(t) \tag{4-129}$$

$$\frac{dP_2(t)}{dt} = r_1 P_1(t) - a_2 P_2(t) \tag{4-130}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x) P_j(x,t) \tag{4-131}$$

(*for j = 3, 4, 5*)

$$\frac{dP_{5A}(t)}{dt} = \omega_1 \sum_{i=0}^{2} \lambda_{hi} P_i(t) - \lambda_{hA} P_{5A}(t) \tag{4-132}$$

$$\frac{dP_{5B}(t)}{dt} = \omega_2 \sum_{i=0}^{2} \lambda_{hi} P_i(t) - \lambda_{hB} P_{5B}(t) \tag{4-133}$$

The boundary conditions are:

$$P_3(0,t) = r_2 P_2(t) \tag{4-134}$$

$$P_4(0,t) = \sum_{i=0}^{2} \lambda_{ci} P_i(t) \tag{4-135}$$

$$P_5(0,t) = \lambda_{hA} P_{5A}(t) + \lambda_{hB} P_{5B}(t) \tag{4-136}$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

Steady State Availability Analysis

For m = 2 and n = 1, we have the following steady state probabilities:

$$P_0 = \frac{b_0(2,1)\lambda_{hA}\lambda_{hB}}{BC(2,1)} \tag{4-137}$$

$$P_1 = \frac{b_1(2,1)\lambda_{hA}\lambda_{hB}}{BC(2,1)} \tag{4-138}$$

$$P_2 = \frac{b_2(2,1)\lambda_{hA}\lambda_{hB}}{BC(2,1)} \tag{4-139}$$

$$P_3 = \frac{b_3(2,1)\lambda_{hA}\lambda_{hB}E_3[x]}{BC(2,1)} \tag{4-140}$$

$$P_4 = \frac{b_4(2,1)\lambda_{hA}\lambda_{hB}E_4[x]}{BC(2,1)} \tag{4-141}$$

$$P_5 = \frac{b_5(2,1)\lambda_{hA}\lambda_{hB}E_5[x]}{BC(2,1)} \tag{4-142}$$

$$P_{5A} = \frac{\omega_1\lambda_{hB}b_5(2,1)}{BC(2,1)} \tag{4-143}$$

$$P_{5B} = \frac{\omega_2\lambda_{hA}b_5(2,1)}{BC(2,1)} \tag{4-144}$$

where

$$BC(2,1) = \lambda_{hA}\lambda_{hB} \cdot BA(2,1) + bc1(2,1)$$
$$bc1(2,1) = b_5(2,1)\left[\omega_1\lambda_{hB} + \omega_2\lambda_{hA}\right]$$

The steady state availability of the system is:

$$AV_{ss}(2,1) = P_0 + P_1 + P_2 + P_{5A} + P_{5B}$$
$$= \frac{bc1(2,1) + bc2(2,1)}{BC(2,1)} \tag{4-145}$$

where

$$bc2(2,1) = \lambda_{hA}\lambda_{hB}\sum_{i=0}^{2} b_i(2,1)$$

The steady state unavailability of the system is:

$$UAV_{ss}(2,1) = P_3 + P_4 + P_5$$
$$= \frac{\lambda_{hA}\lambda_{hB}\sum_{j=3}^{5} b_j(2,1)E_j[x]}{BC(2,1)} \tag{4-146}$$

Special cases of steady state availability

Case I

For the *Gamma* repair time distribution the steady state availability is:

$$AV_{ss}(2,1) = \frac{bc1(2,1) + bc2(2,1)}{\left[\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\beta/\mu_j\right]\lambda_{hA}\lambda_{hB} + bc1(2,1)} \tag{4-147}$$

If β is an integer, the Equation (4-147) is the steady state availability for the ***Special Erlangian*** distribution and When $\beta = 1$, it is the steady state availability for the ***exponential*** distribution.

Case II

For the ***Weibull*** repair time distribution, the steady state availability is:

$$AV_{ss}(2,1) = \frac{bc1(2,1) + bc2(2,1)}{\left[\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\Gamma(1 + 1/\beta)/\mu_j\right]\lambda_{hA}\lambda_{hB} + bc1(2,1)} \qquad (4\text{-}148)$$

When $\beta = 1$, the Weibull distribution represents an ***exponential*** distribution.

Case III

If the system repair time is ***Rayleigh*** distribution, the steady state availability is:

$$AV_{ss}(2,1) = \frac{bc1(2,1) + bc2(2,1)}{\left[\sum_{i=0}^{2} b_i(2,1) + \sum_{j=3}^{5} b_j(2,1)\sqrt{\pi/2}/\mu_j\right]\lambda_{hA}\lambda_{hB} + bc1(2,1)} \qquad (4\text{-}149)$$

Case IV

If the system repair time is the ***lognormal*** distribution, the system steady state availability is:

$$AV_{ss}(2,1) = \frac{bc1(2,1) + bc2(2,1)}{\left[\sum_{i=0}^{2} b_i(2,1) + \sum_{i=3}^{5} b_i(2,1)\exp\left(\mu_i + \sigma^2/2\right)\right]\lambda_{hA}\lambda_{hB} + bc1(2,1)} \qquad (4\text{-}150)$$

Numerical examples of steady state availability

Setting the same data as that in the numerical examples of steady state availability of Section 2.2.2 and the parameters, λ_{h0}, λ_{h1}, and λ_{h2}, very large values, for different values of λ_{hA} and ω_1, Figure 4-4 shows the system steady state availability as a function of human error rate parameter, λ_{hB}, when the system repair time is the ***Gamma*** distribution.

Similarly, Figure 4-5 sketches the system steady-state availability against human error rate parameter, λ_{hB}, when the system repair time distribution is the ***Weibull.***

Figure 4-6 illustrates the variation of the steady-state availability with human error rate parameters, λ_{hA}, λ_{hB} and ω_1 , when the repair time is ***lognormal*** distribution and $\mu_3 = 1.001$, $\mu_4 = \mu_5 = 1.0009$, $\sigma_3 = \sigma_4 = \sigma_5 = \sigma$

The comparison of the steady-state availability for different repair time distributions, such as exponential distribution, Gamma distribution, Weibull distribution and lognormal distribution, and setting the same data as that above, is shown in Figure 4-7.

Time-Dependent Availability Analysis

The Laplace transform of state probabilities for this model is given by:

$$P_0(s) = \frac{\left[b_0(2,1) + (a_1 + a_2)s + s^2\right](s + \lambda_{hA})(s + \lambda_{hB})}{CC(2,1)} \qquad (4\text{-}151)$$

$$P_1(s) = \frac{\left[b_1(2,1) + r_0 s\right](s + \lambda_{hA})(s + \lambda_{hB})}{CC(2,1)} \qquad (4\text{-}152)$$

$$P_2(s) = \frac{(s + \lambda_{hA})(s + \lambda_{hB})b_2(2,1)}{CC(2,1)} \qquad (4\text{-}153)$$

$$P_3(s) = \frac{(s + \lambda_{hA})(s + \lambda_{hB})b_3(2,1)[1 - N_3(s)]}{s \cdot CC(2,1)} \qquad (4\text{-}154)$$

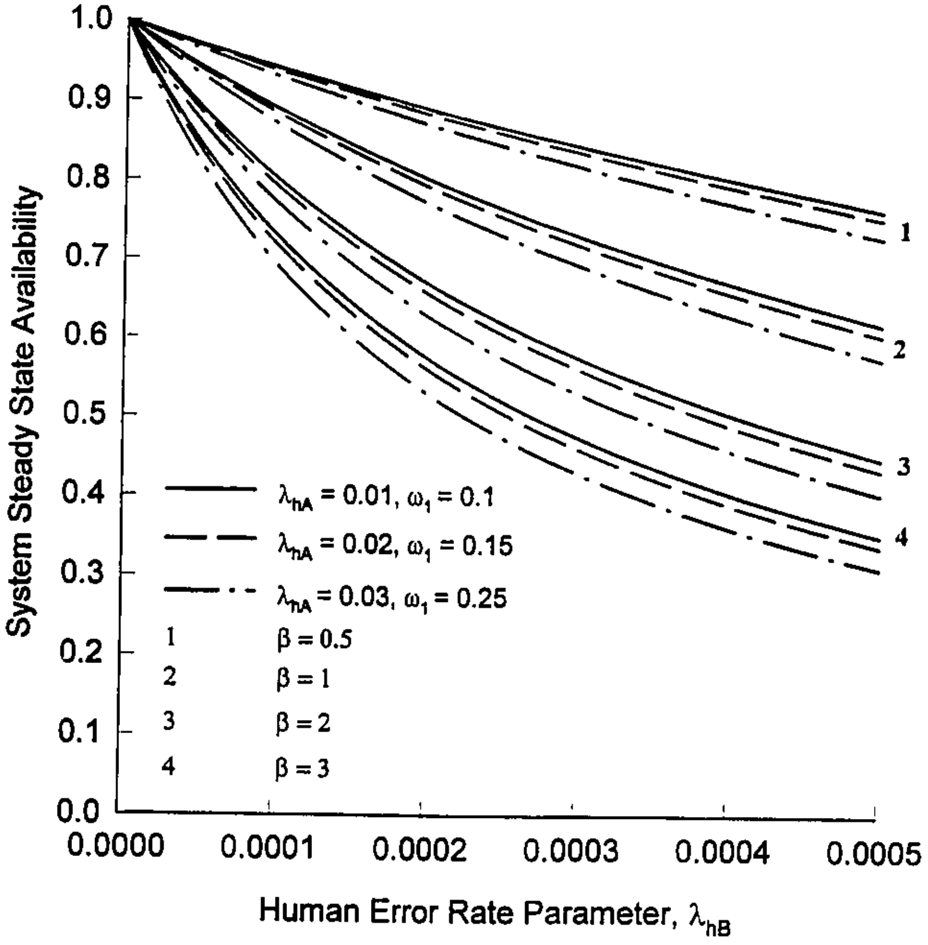


Figure 4-4 System steady state availability for (2,1) standby system with general human error rates and the Gamma distributed failed system repair times

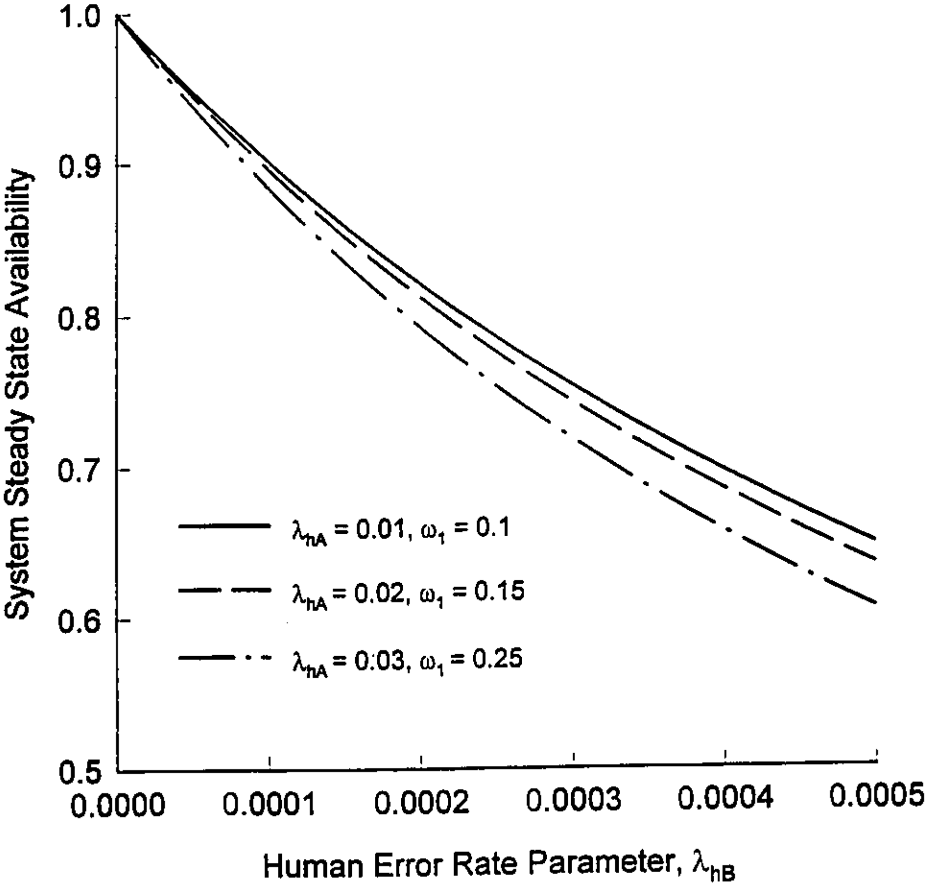


Figure 4-5 System steady state availability for (2,1) standby system with general human error rates and the Weibull ($\beta = 2$) distributed failed system repair times

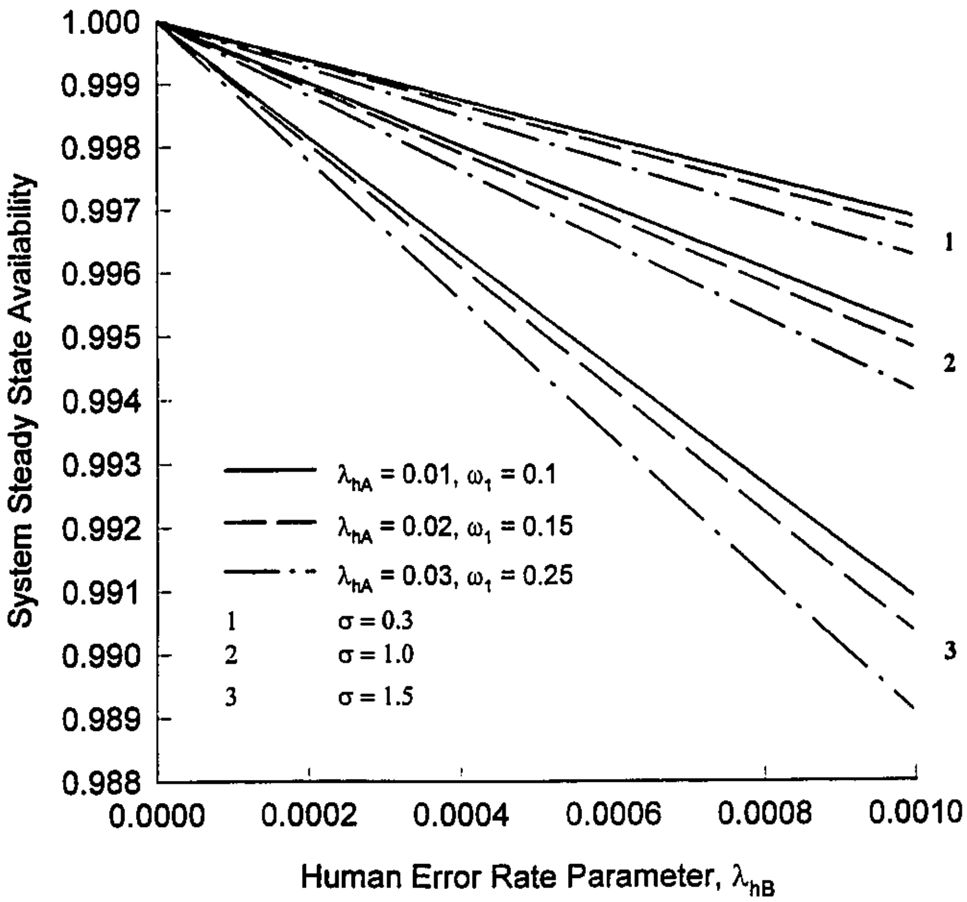


Figure 4-6 System steady state availability for (2,1) standby system with general human error rates and the lognormally distributed system repair times

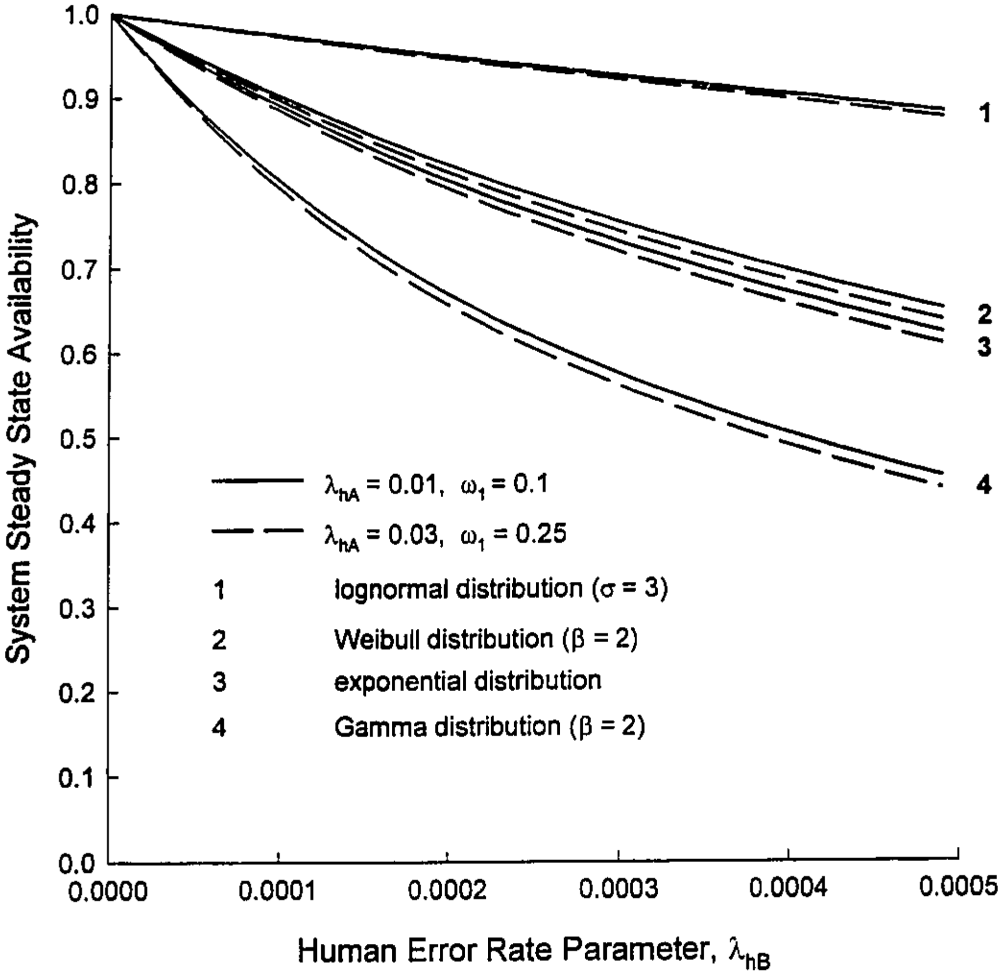


Figure 4-7 The comparison of the System steady state availability for (2,1) standby system with general human error rates and different system repair time distributions

$$P_4(s)=\frac{\left(s+\lambda_{hA}\right)\left(s+\lambda_{hB}\right)\left[1-N_4(s)\right]\left[b_4(2,1)+lc(2,1)\cdot s+\lambda_{c0}\cdot s^2\right]}{s\cdot CC(2,1)} \qquad (4\text{-}155)$$

$$P_5(s)=\frac{\left[1-N_5(s)\right]\left[bc2(2,1)+\left(lh(2,1)+bc1(2,1)\right)\cdot s+\lambda_{h0}\cdot s^2\right]}{s\cdot CC(2,1)} \qquad (4\text{-}156)$$

$$P_{5A}(s)=\frac{\omega_1\left(s+\lambda_{hB}\right)\left[b_5(2,1)+lh(2,1)\cdot s+\lambda_{h0}\cdot s^2\right]}{CC(2,1)} \qquad (4\text{-}157)$$

$$P_{5B}(s)=\frac{\omega_2\left(s+\lambda_{hA}\right)\left[b_5(2,1)+lh(2,1)\cdot s+\lambda_{h0}\cdot s^2\right]}{CC(2,1)} \qquad (4\text{-}158)$$

where

$$CC(2,1)=cc1(2,1)+cc2(2,1)\cdot s+cc3(2,1)\cdot s^2+cc4(2,1)\cdot s^3+cc5(2,1)\cdot s^4+s^5$$

$$cc1(2,1)=\lambda_{hA}\lambda_{hB}\cdot ca1(2,1)$$

$$\begin{aligned}cc2(2,1)=\lambda_{hA}&\left[\sum_{j=3}^{4}\left[1-N_j(s)\right]b_j(2,1)+b_5(2,1)\left(1-\omega_1N_5(s)\right)\right]\\&+\lambda_{hB}\left[\sum_{j=3}^{4}\left[1-N_j(s)\right]b_j(2,1)+b_5(2,1)\left(1-\omega_2N_5(s)\right)\right]+\lambda_{hA}\lambda_{hB}\cdot ca2(2,1)\end{aligned}$$

$$\begin{aligned}cc3(2,1)=\lambda_{hA}&\left[\sum_{i=0}^{2}b_i(2,1)+lc(2,1)\left(1-N_4(s)\right)+lh(2,1)\left(1-\omega_1N_5(s)\right)\right]\\&+\lambda_{hB}\left[\sum_{i=0}^{2}b_i(2,1)+lc(2,1)\left(1-N_4(s)\right)+lh(2,1)\left(1-\omega_2N_5(s)\right)\right]\\&+\sum_{j=3}^{4}b_j(2,1)\left(1-N_j(s)\right)+b_5(2,1)+\lambda_{hA}\lambda_{hB}\cdot ca3(2,1)\end{aligned}$$

$$\begin{aligned}cc4(2,1)=\lambda_{hA}&\left[a_1+a_2+r_0+\lambda_{c0}\left(1-N_4(s)\right)+\lambda_{h0}\left(1-\omega_1N_5(s)\right)\right]\\&+\lambda_{hB}\left[a_1+a_2+r_0+\lambda_{c0}\left(1-N_4(s)\right)+\lambda_{h0}\left(1-\omega_2N_5(s)\right)\right]\\&+\sum_{i=0}^{2}b_i(2,1)+lc(2,1)\left(1-N_4(s)\right)+\lambda_{hA}\lambda_{hB}+lh(2,1)\end{aligned}$$

$$cc5(2,1) = a_1 + a_2 + r_0 + \lambda_{c0}\left(1 - N_4(s)\right) + \lambda_{h0} + \lambda_{hA} + \lambda_{hB}$$

The Laplace transform of the system availability is :

$$\begin{aligned} AV(s) &= P_0(s) + P_1(s) + P_2(s) + P_{5A}(s) + P_{5B}(s) \\ &= \frac{(s+\lambda_{hA})(s+\lambda_{hB})\left[\sum_{i=0}^{2} b_i(2,1) + l0(2,1)\cdot s + s^2\right]}{CC(2,1)} \\ &+ \frac{\left[\omega_1(s+\lambda_{hB}) + \omega_2(s+\lambda_{hA})\right]\left[b_5(2,1) + lh(2,1)\cdot s + \lambda_{h0}\cdot s^2\right]}{CC(2,1)} \end{aligned} \qquad (4\text{-}159)$$

Substituting the Laplace transform of pdf of system repair distribution, $N_3(s)$, $N_4(s)$, and $N_5(s)$, in the above Equation, and taking the inverse Laplace transform of the resulting equations, we can get the time-dependent system availability.

$$AV(t) = P_0(t) + P_1(t) + P_2(t) + P_{5A}(t) + P_{5B}(s) \qquad (4\text{-}160)$$

Setting the same data as that in the numerical examples of steady-state availability, for the different values of the human error rate parameters, ω_1, λ_{h0}, λ_{hA} and λ_{hB}, the time-dependent availability, when the system repair times are ***exponential*** distributions and the ***Gamma*** distributions ($\beta = 2$), are shown in Figure 4-8 and Figure 4-9, respectively.

Reliability and MTTF With and Without Repair

Setting $\mu_3(x) = \mu_4(x) = \mu_5(x) = 0$ in the model, the Laplace transform of the system reliability with repair is:

$$\begin{aligned} R(s) &= \frac{\sum_{i=0}^{2} b_i(2,1) + l0(2,1)\cdot s + s^2}{DB(2,1)} + \frac{\omega_1\left[b_5(2,1) + lh(2,1)\cdot s + \lambda_{h0}\cdot s^2\right]}{(s+\lambda_{hA})\cdot DB(2,1)} \\ &+ \frac{\omega_2\left[b_5(2,1) + lh(2,1)\cdot s + \lambda_{h0}\cdot s^2\right]}{(s+\lambda_{hB})\cdot DB(2,1)} \end{aligned} \qquad (4\text{-}161)$$

The system mean time to failure (MTTF) with repair is given by:

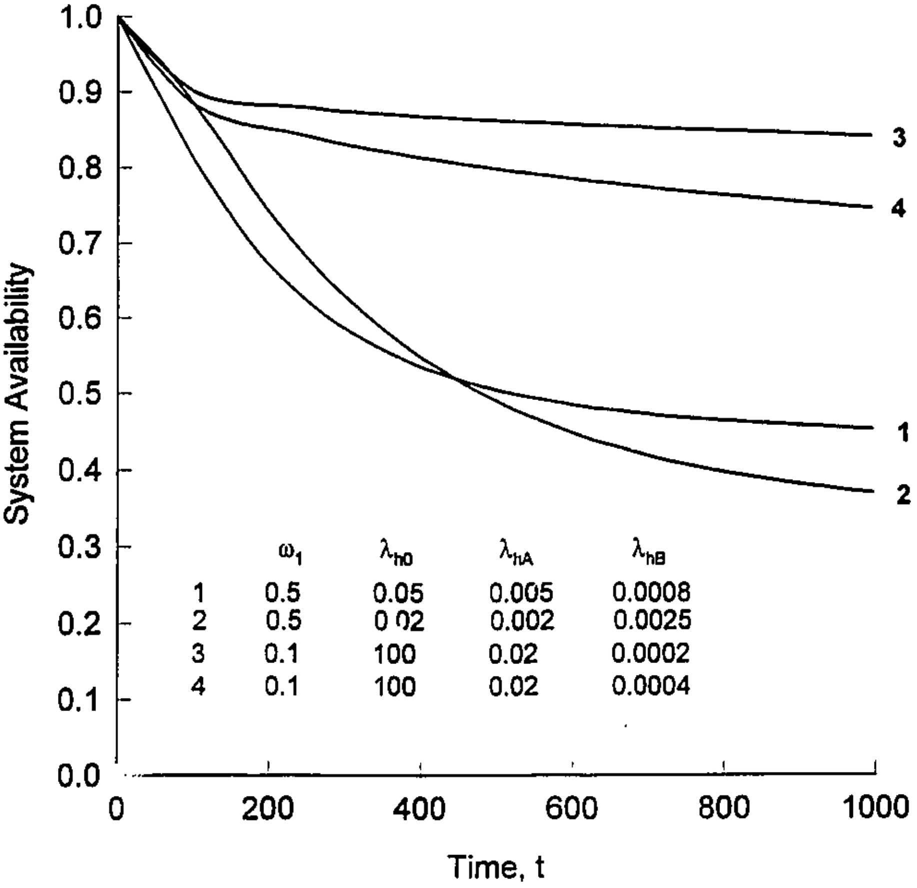
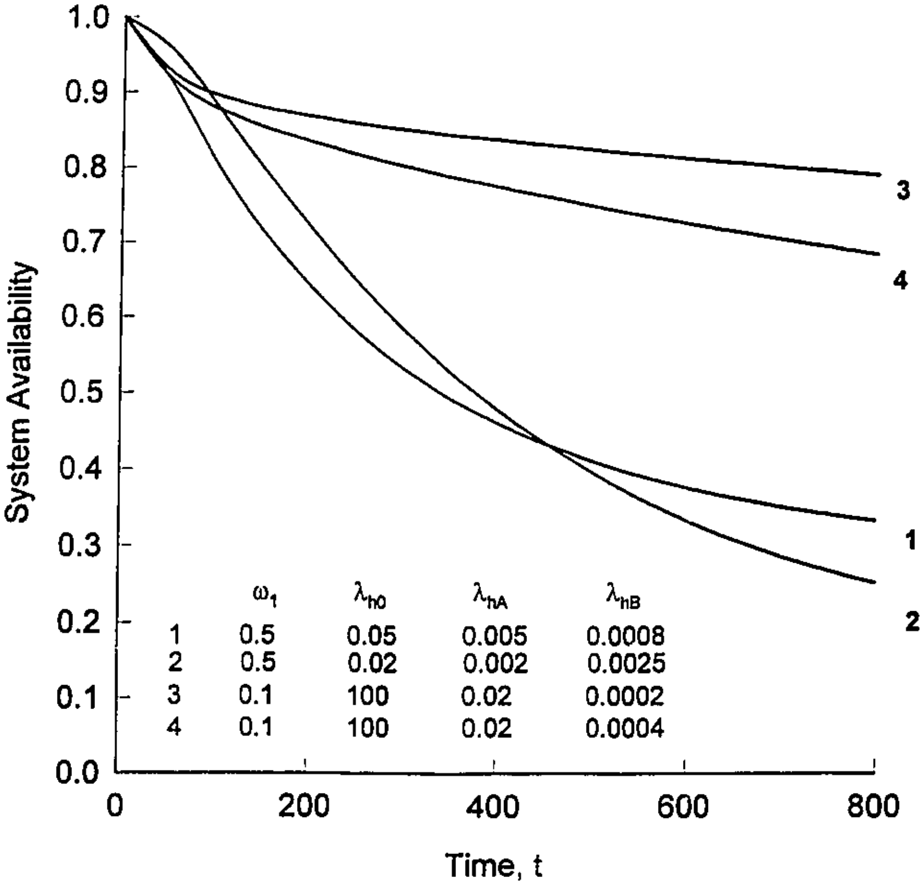


Figure 4-8 Time-dependent system availability plots for (2,1) standby system with general human error rates and the exponentially distributed system repair times



	ω_1	λ_{h0}	λ_{hA}	λ_{hB}
1	0.5	0.05	0.005	0.0008
2	0.5	0.02	0.002	0.0025
3	0.1	100	0.02	0.0002
4	0.1	100	0.02	0.0004

Figure 4-9 Time-dependent system availability plots for (2,1) standby system with general human error rates and the Gamma (β=2) distributed system repair times

$$MTTF = \lim_{s\to 0} R(s) = \frac{\sum_{i=0}^{2} b_i(2,1)}{db1(2,1)} + \frac{\omega_1 \cdot b_5(2,1)}{\lambda_{hA} \cdot db1(2,1)} + \frac{\omega_2 \cdot b_5(2,1)}{\lambda_{hB} \cdot db1(2,1)} \qquad (4\text{-}162)$$

The variance of time to failure with repair of the system is expressed by:

$$\begin{aligned}
\sigma^2 &= -2\lim_{s\to 0} R'(s) - (MTTF)^2 \\
&= -2\left[\frac{a_1 + a_2 + r_0}{db1(2,1)} - \frac{\sum_{i=0}^{2} b_i(2,1) \cdot db2(2,1)}{[db1(2,1)]^2} + \frac{\omega_1 \cdot lh(2,1)}{\lambda_{hA} db1(2,1)} + \frac{\omega_2 \cdot lh(2,1)}{\lambda_{hB} db1(2,1)}\right] \\
&\quad +2\left[\frac{\omega_1 b_5(2,1)[db1(2,1) + \lambda_{hA} db2(2,1)]}{[\lambda_{hA} db1(2,1)]^2} + \frac{\omega_2 b_5(2,1)[db1(2,1) + \lambda_{hB} db2(2,1)]}{[\lambda_{hB} db1(2,1)]^2}\right] \qquad (4\text{-}163) \\
&\quad -\left[\frac{\sum_{i=0}^{2} b_i(2,1)}{db1(2,1)} + \frac{\omega_1 \cdot b_5(2,1)}{\lambda_{hA} db1(2,1)} + \frac{\omega_2 \cdot b_5(2,1)}{\lambda_{hB} db1(2,1)}\right]^2
\end{aligned}$$

Setting the same data as that in the numerical examples of steady-state availability in Equation (4-161) and taking inverse Laplace transform, for the different values of the human error rate parameters, ω_1, λ_{h0}, λ_{hA} and λ_{hB}, the plots of the system reliability with repair are shown in Figure 4-10. The plots of MTTF with repair are given in Figure 4-11.

When $\mu_1 = \mu_2 = 0$ in Equations (4-161) - (4-163), we can get the system reliability, MTTF and the variance of time to failure of the system without repair, respectively.

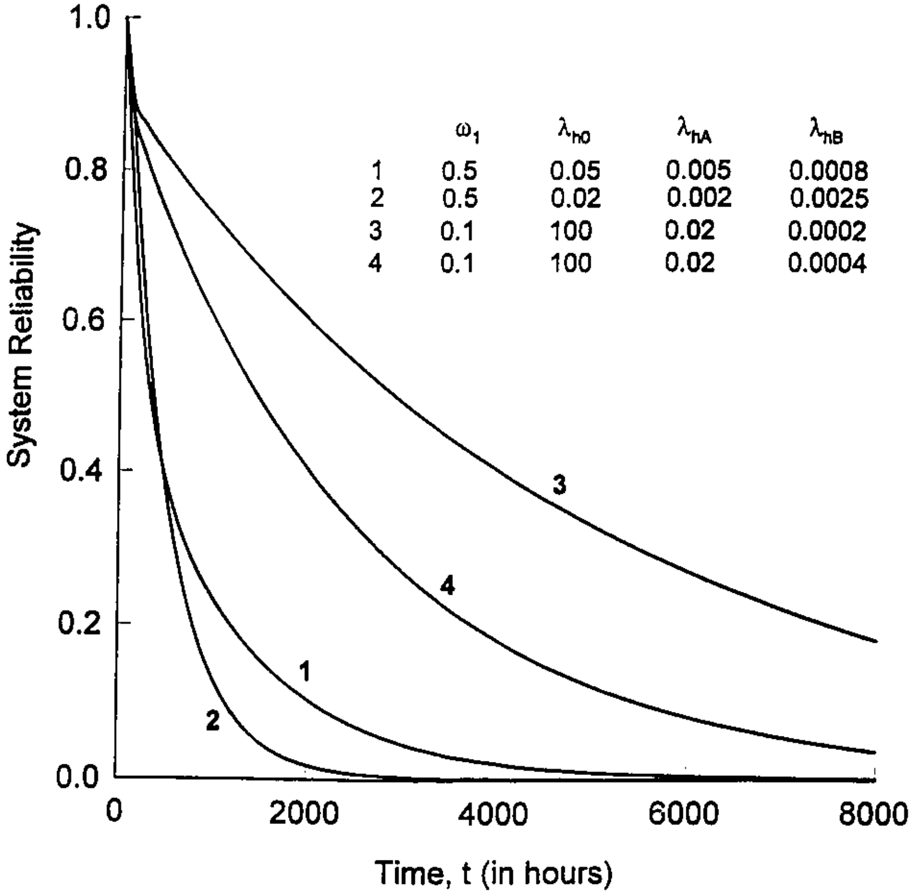


Figure 4-10 System reliability with repair plots for (2,1) standby system with general human error rates

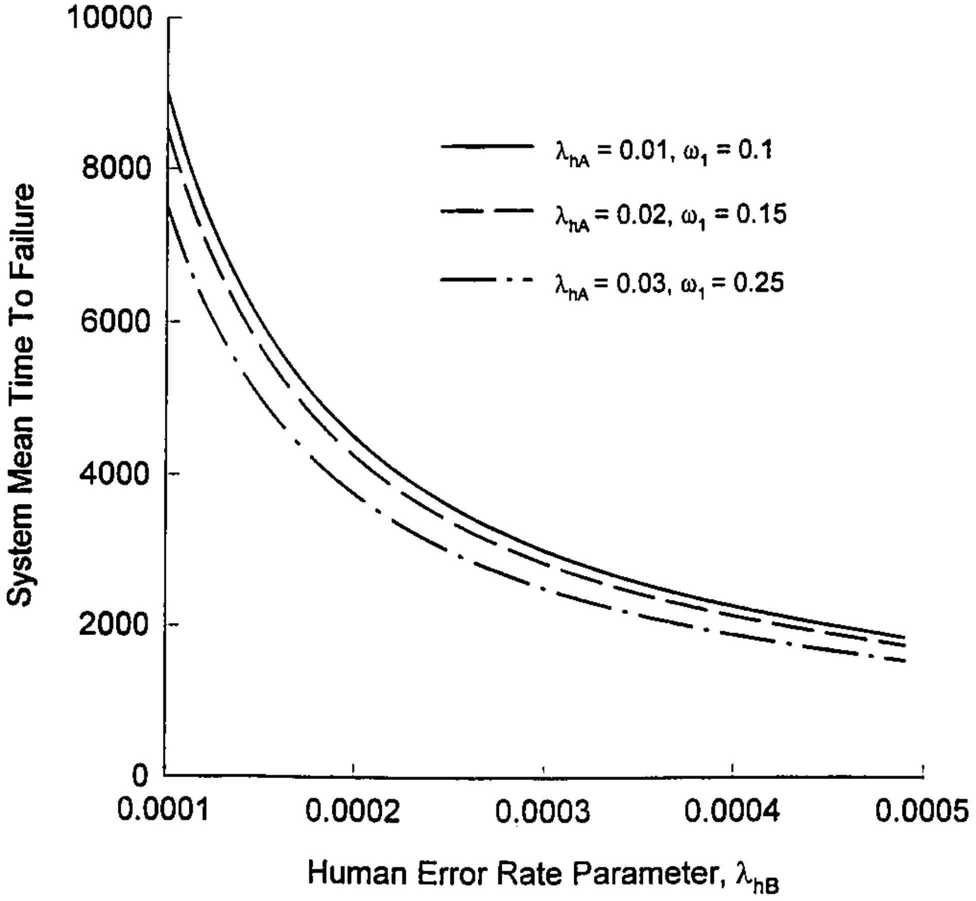


Figure 4-11 System mean time to failure with repair for (2,1) standby system with general human error rates

4.4 K-out-of-n System

4.4.1 Description of the System

Similar to the standby system, the time-dependent human error in k-out-of-n system is represented by a combination of two series stages in parallel. The inclusion supplementary variable method is used to deal with the non-exponential failed system repair times. The state space diagram for k-out-of-n unit system with general human error and general failed system repair distributions is shown in Figure 4-12. It is similar to that of standby system (Figure 4-3), but the division of system working or failure states is different.

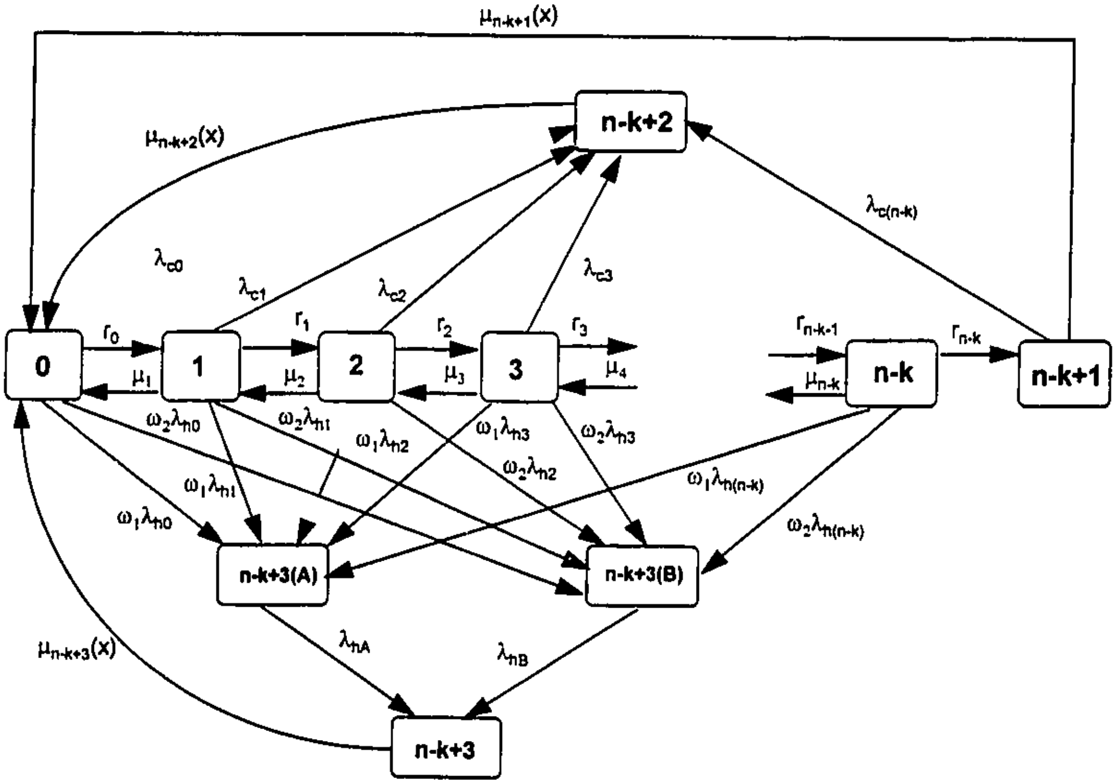


Figure 4-12 The state transition diagram of k-out-of-n system with critical human error state represented by two series stages in parallel and general system repair rates

The corresponding differential equations for the model described in Figure 4-12 are:

$$\frac{dP_0(t)}{dt} = -a_0P_0(t) + \mu_1P_1(t) + \sum_{j=n-k+1}^{n-k+3}\int_0^{\infty}P_j(x,t)\mu_j(x)dx \tag{4-164}$$

$$\frac{dP_1(t)}{dt} = r_0P_0(t) - (r_1 + \lambda_{h1} + \lambda_{c1} + \mu_1)P_1(t) + \mu_2P_2(t) \tag{4-165}$$

$$\frac{dP_2(t)}{dt} = r_1P_1(t) - (r_2 + \lambda_{h2} + \lambda_{c2} + \mu_2)P_2(t) + \mu_3P_3(t) \tag{4-166}$$

$$\frac{dP_i(t)}{dt} = r_{i-1}P_{i-1}(t) - a_iP_i(t) + \mu_{i+1}P_{i+1}(t) \tag{4-167}$$

(for i = 0, 1, 2, 3, ..., n-k-1)

$$\frac{dP_{n-k}(t)}{dt} = r_{n-k-1}P_{n-k-1}(t) - a_{n-k}P_{n-k}(t) \tag{4-168}$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} = -\mu_j(x)P_j(x,t) \tag{4-169}$$

(for j = n-k+1, n-k+2, n-k+3)

$$\frac{dP_{n-k+3(A)}(t)}{dt} = \omega_1\sum_{i=0}^{n-k}\lambda_{hi}P_i(t) - \lambda_{hA}P_{n-k+3(A)}(t) \tag{4-170}$$

$$\frac{dP_{n-k+3(B)}(t)}{dt} = \omega_2\sum_{i=0}^{n-k}\lambda_{hi}P_i(t) - \lambda_{hB}P_{n-k+3(A)}(t) \tag{4-171}$$

The associated boundary conditions are as follows:

$$P_{n-k+1}(0,t) = r_{n-k}P_{n-k}(t) \tag{4-172}$$

$$P_{n-k+2}(0,t) = \sum_{i=0}^{n-k}\lambda_{ci}P_i(t) \tag{4-173}$$

$$P_{n-k+3}(0,t) = \lambda_{hA}P_{n-k+3(A)}(t) + \lambda_{hB}P_{n-k+3(B)}(t) \tag{4-174}$$

where

$$r_i = k\lambda + (n-k-i)\lambda_s$$

$$a_i = r_i + \lambda_{hi} + \lambda_{ci} + \mu_i$$

(for i = 0, 1, 2, …, n-k)

and

$$\omega_1 + \omega_2 = 1$$

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

4.4.2 K-out-of-n (n-k=1) System

For k-out-of-n (n-k=1) system with both time-dependent human error rate and failed system repair rate, the analysis is similar to the model of one unit active and one on standby system discussed in Section 4.3.1. Using the same method and procedure as that of standby system, we can get the system steady-state availability, Laplace transform of system availability, Laplace transform of system reliability and system MTTF expressed by Equations (4-91), (4-116), (4-125) and (4-126), respectively.

Setting

$\lambda = 0.0002/hr$	$\lambda_s = 0.00002/hr$	$\mu_1 = 0.001/hr$
$\lambda_{c0} = 0.00005/hr$	$\lambda_{c1} = 0.00005/hr$	$\lambda_{h1} = 0.0001/hr$

in Equation (4-91), for exponential, Gamma (β = 3), and Weibull (β = 2) distributed failed system repair times, the plots of system steady-state availability as a function of human error rate parameter, λ_{hB}, are shown in Figures 4-13, 4-14 and 4-15, respectively, for specified values of parameters λ_{hA}, λ_{h0}, ω_1, and $\mu_2 = 0.001/hr$, $\mu_3 = \mu_4 = 0.0009/hr$, if applicable.

Figure 4-16 provides the results of the system steady state availability as a function of human error rate parameter, λ_{hB}, for specified values of parameters λ_{hA}, λ_{h0}, ω_1, for lognormally distributed failed system repair times ($\mu_2 = 1.001/hr$, $\mu_3 = \mu_4 = 1.0009/hr$, $\sigma_2 = \sigma_3 = \sigma_4 = \sigma = 0.3$)

Substituting the Laplace transforms of the pdf of the system repair times $N_2(s)$, $N_3(s)$ and $N_4(s)$ in Equation (4-116) and then substituting r_0, r_1, and the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting $\mu_2 = 0.001 / hr$, $\mu_3 = \mu_4 = 0.0009 / hr$ and taking inverse Laplace transform), we get the system time-dependent availability.

For 3-out-of-4 system with ***Gamma*** (β=2) distributed failed system repair times and λ_{h0}=0.001/hr, λ_{hA}=0.005/hr, λ_{hB}=0.0002/hr, and ω_1=0.75 we get

$$\begin{aligned} AV(t) = {} & 0.493602 - 0.208574/\exp(0.005054\ t) + 0.08122/\exp(0.002418\ t) \\ & - 0.076633/\exp(0.000278\ t) -(0.006481 \sin(0.0000472\ t))/\exp(0.000963\ t) \\ & - 0.0002936\ ((\cos(0.0000472\ t))/\exp(0.000963\ t) \\ & - (20.39 \sin(0.0000472\ t))/\exp(0.000963\ t)) \\ & + (1.2506 \sin(0.0008156\ t))/\exp(0.001371\ t) \\ & + 0.710678\ ((\cos(0.0008156\ t))/\exp(0.001371\ t) \\ & - (1.681543 \sin(0.0008156\ t))/\exp(0.001371\ t)\ ) \end{aligned} \qquad (4\text{-}175)$$

The plots of the time-dependent system availability of 2-out-of-3 system with exponentially distributed failed system repair times for specified human error rate parameters, λ_{hA}, λ_{hB}, λ_{h0}, and ω_1, are shown in Figure 4-17.

For specified values of human error rate parameters, λ_{hA}, λ_{hB}, λ_{h0}, and ω_1, the system time-dependent availability plots of 3-out-of-4 system with Gamma (β=2) distributed failed system repair times are shown in Figure 4-18.

For specified values of human error rate parameters, λ_{hA}=0.005/hr, λ_{hB}=0.0002/hr, λ_{h0}=0.001/hr, and ω_1=0.75, system reliability with repair for 2-out-of-3 system is given by

$$\begin{aligned} R(t) = {} & 0.205374/\exp(0.002159\ t) + 0.743171/\exp(0.0008607\ t) \\ & +0.268829/\exp(0.0002\ t) -0.217374/\exp(0.005\ t) \end{aligned} \qquad (4\text{-}176)$$

The plots of the system reliability with repair for specified values of human error parameters for k-out-of-n (n-k=1) system are shown in Figure 4-19.

The system mean time to failure (MTTF) with repair is given by

$$\begin{aligned} MTTF = [&0.00115\, \lambda_{hA} \lambda_{hB} + \lambda_{hA} \lambda_{hB} r_0 + \lambda_{hA} \lambda_{hB} r_1 + 0.00115\, \lambda_{hB} \lambda_{h0} \omega_1 \\ &+ 0.0001\, \lambda_{hB} r_0 \omega_1 + \lambda_{hB} \lambda_{h0} r_1 \omega_1 + 0.00115\, \lambda_{hA} \lambda_{h0} \omega_2 \\ &+ 0.0001\, \lambda_{hA} r_0 \omega_2 + \lambda_{hA} \lambda_{h0} r_1 \omega_2] \,/\, [5.75\; 10^{-8} \lambda_{hA} \lambda_{hB} \\ &+ 0.00115\, \lambda_{hA} \lambda_{hB} \lambda_{h0} + 0.00015\, \lambda_{hA} \lambda_{hB} r_0 + 0.00005\, \lambda_{hA} \lambda_{hB} r_1 \\ &+ \lambda_{hA} \lambda_{hB} \lambda_{h0}\, r_1 + \lambda_{hA} \lambda_{hB} r_0 r_1] \end{aligned} \qquad (4\text{-}177)$$

Figure 4-20 shows the plots of system mean time to failure with repair for k-out-of-n (n-k=1) system for specified values of human error parameters.

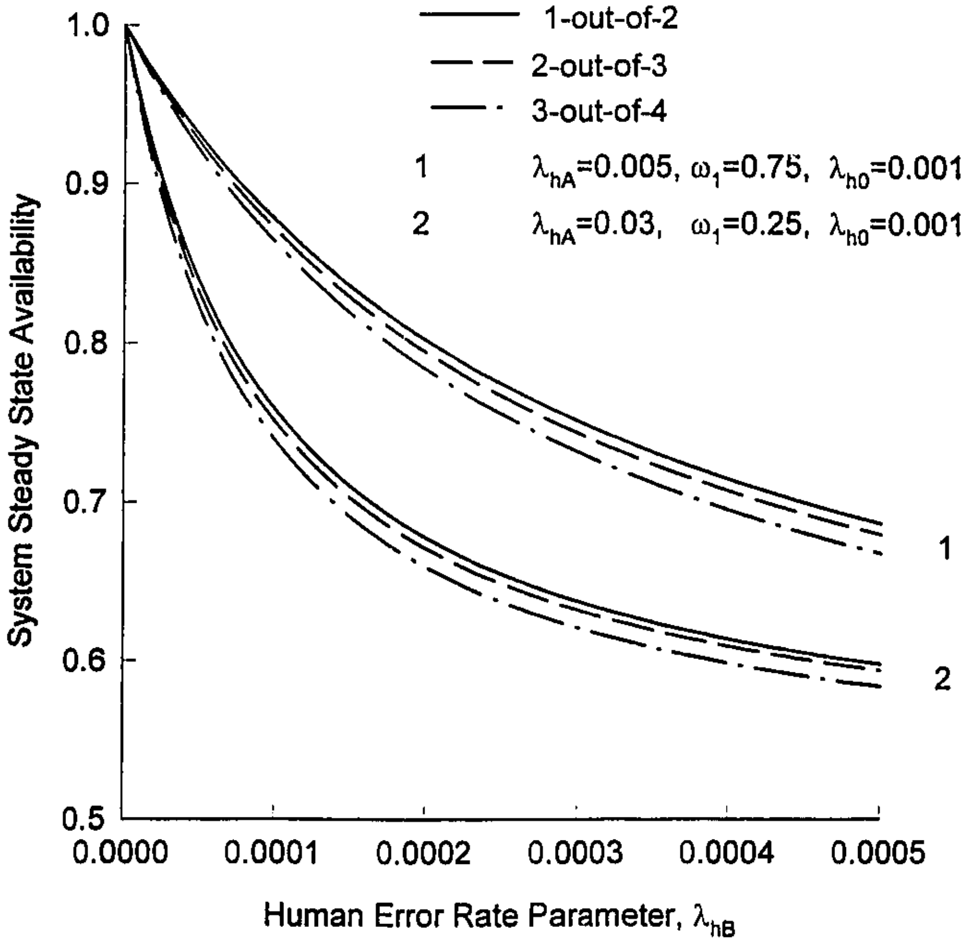


Figure 4-13 System steady state availability for k-out-of-n (n-k=1) system with general human error rates and the exponentially distributed failed system repair times

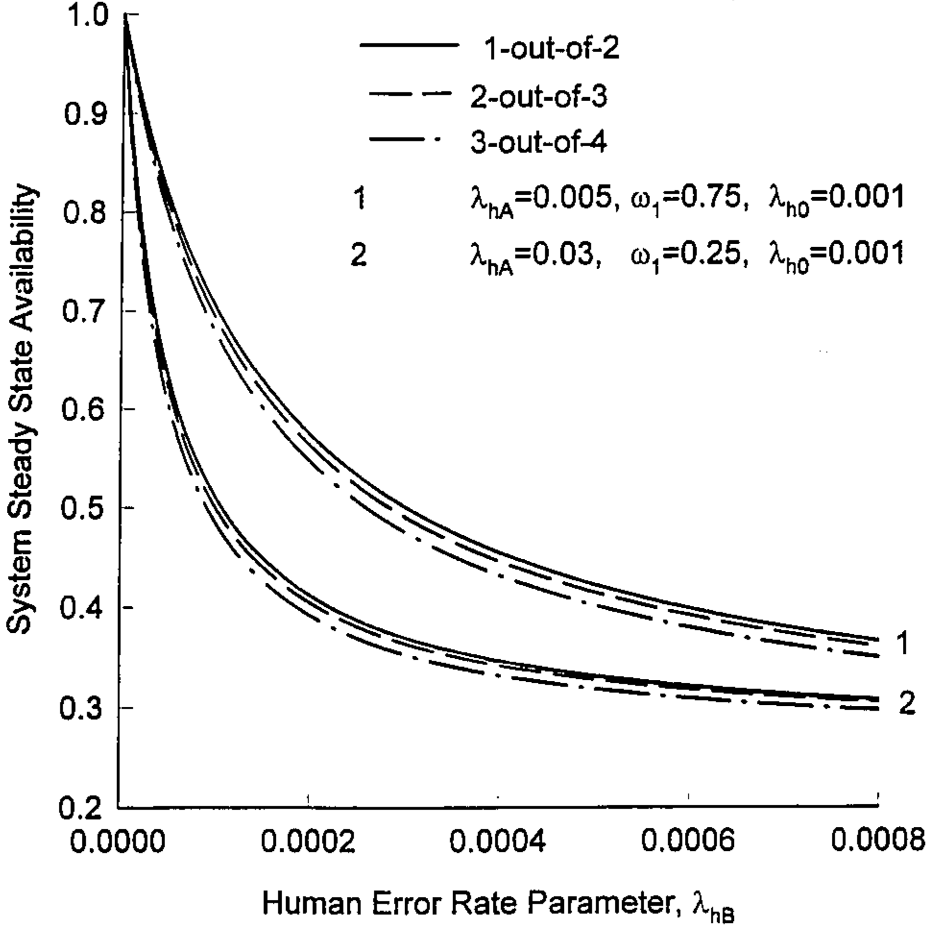


Figure 4-14 System steady state availability for k-out-of-n (n-k=1) system with general human error rates and the Gamma (β=3) distributed failed system repair times

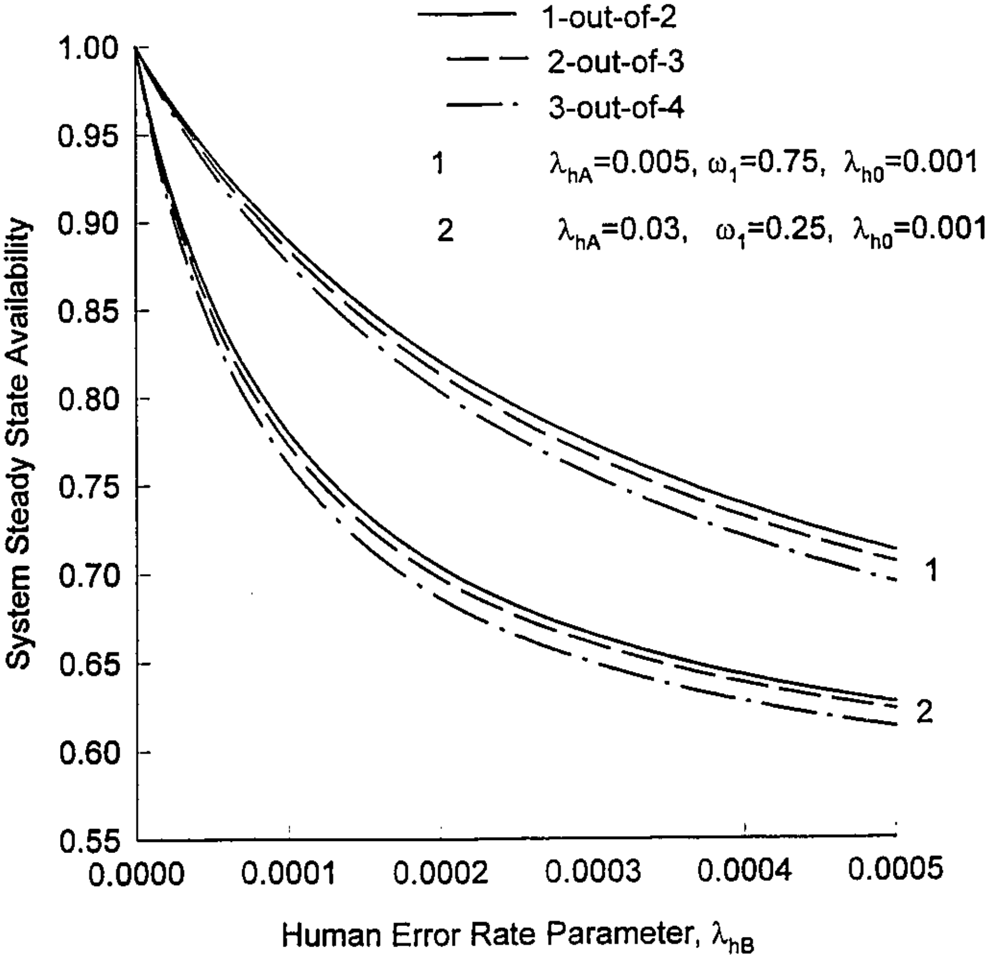


Figure 4-15 System steady state availability for k-out-of-n (n-k=1) system with general human error rates and the Weibull (β=2) distributed system repair times.

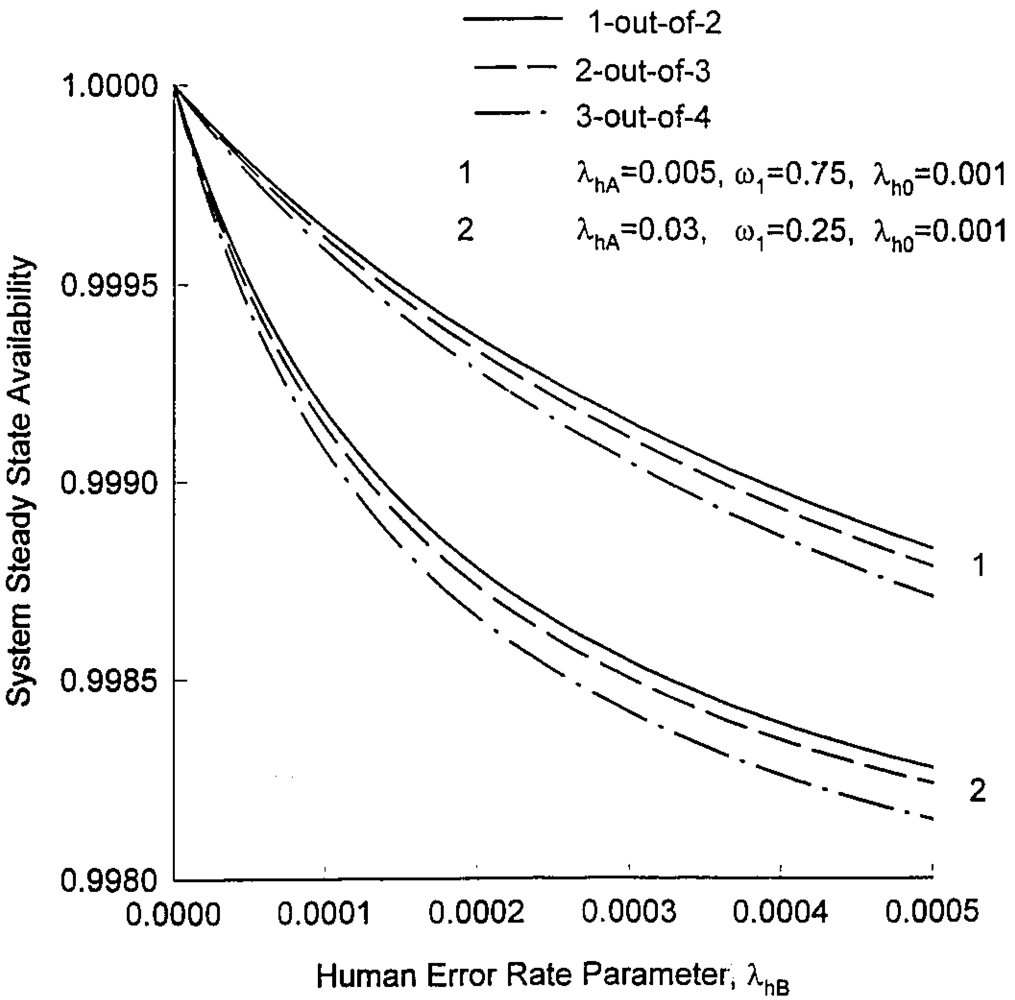


Figure 4-16 System steady state availability for k-out-of-n (n-k=1) system with general human error rates and the lognormally (σ=0.3) distributed system repair times

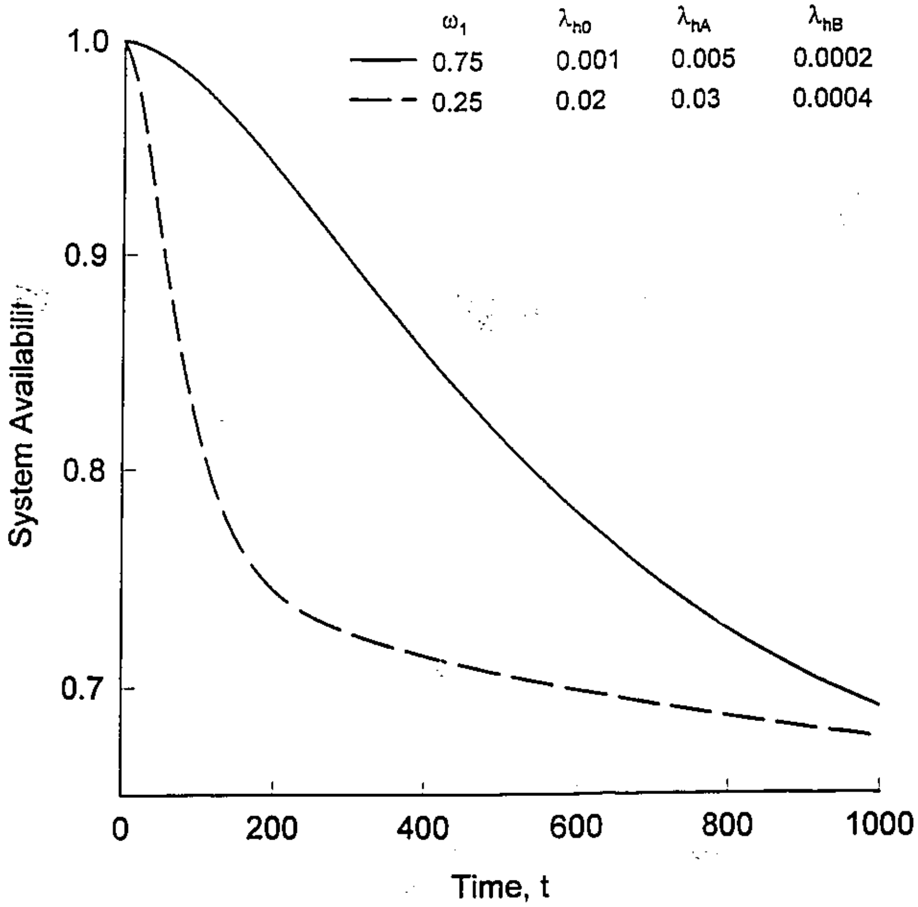


Figure 4-17 Time-dependent system availability plots for 2-out-of-3 system with general human error rates and the exponentially distributed system repair times

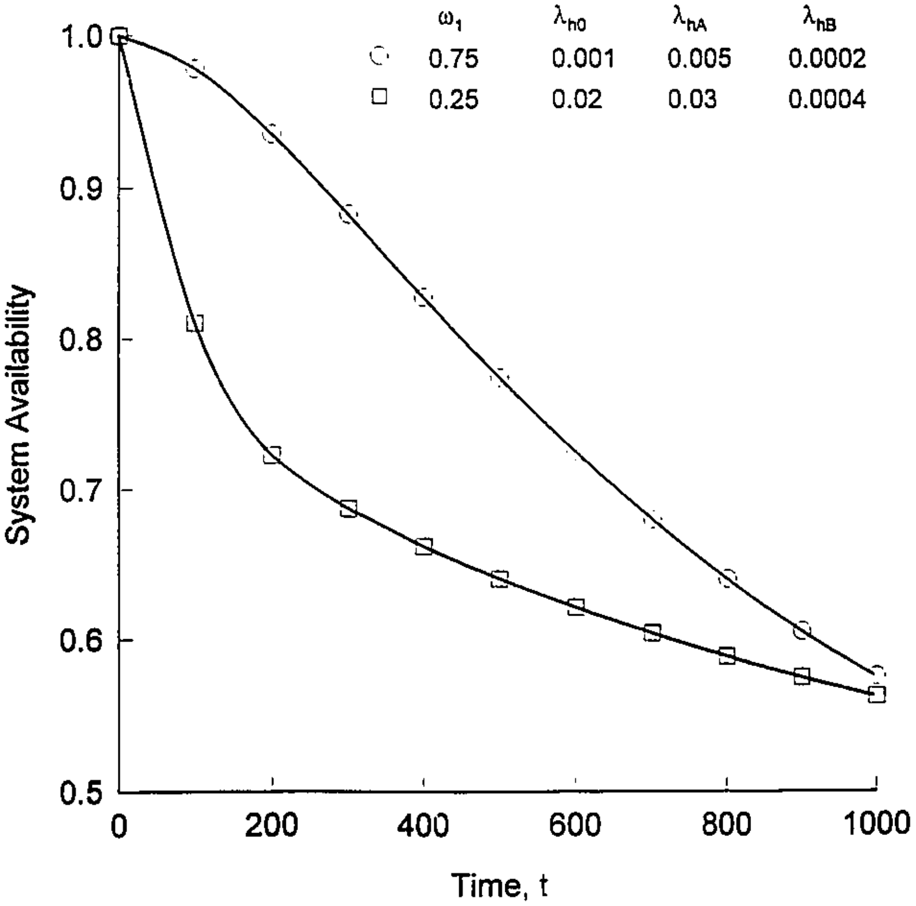


Figure 4-18 Time-dependent system availability plots for 3-out-of-4 system with general human error rates and the Gamma ($\beta=2$) distributed system repair times

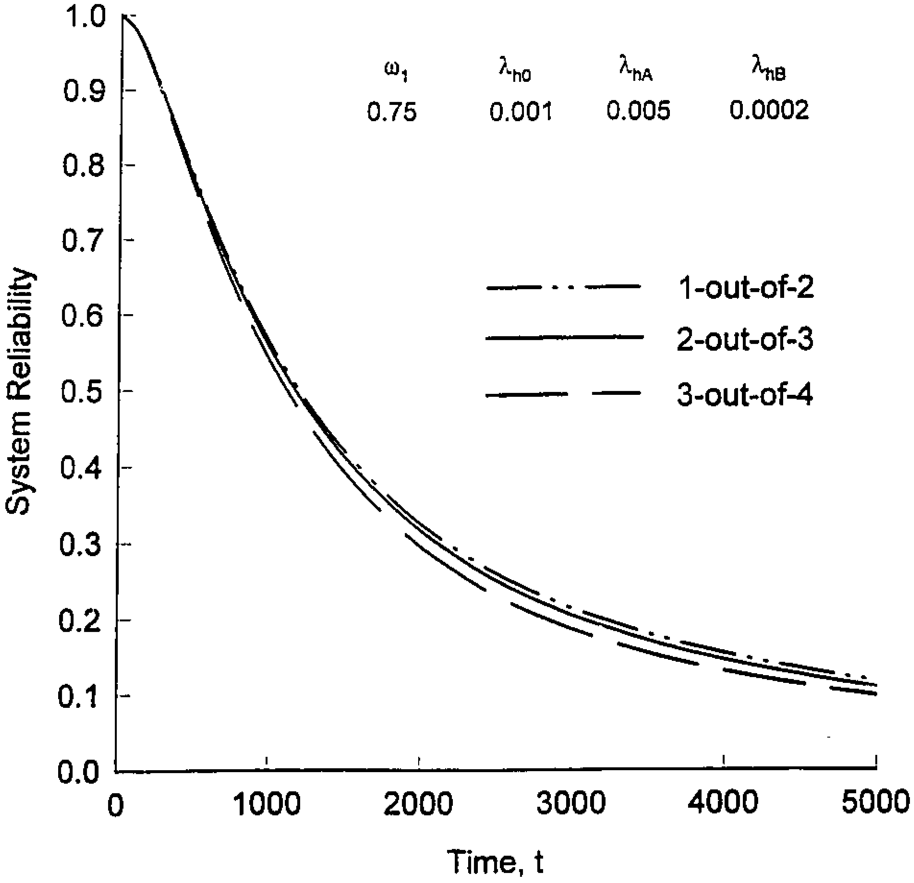


Figure 4-19 System reliability with repair plots for k-out-of-n (n-k=1) system with general human error rates

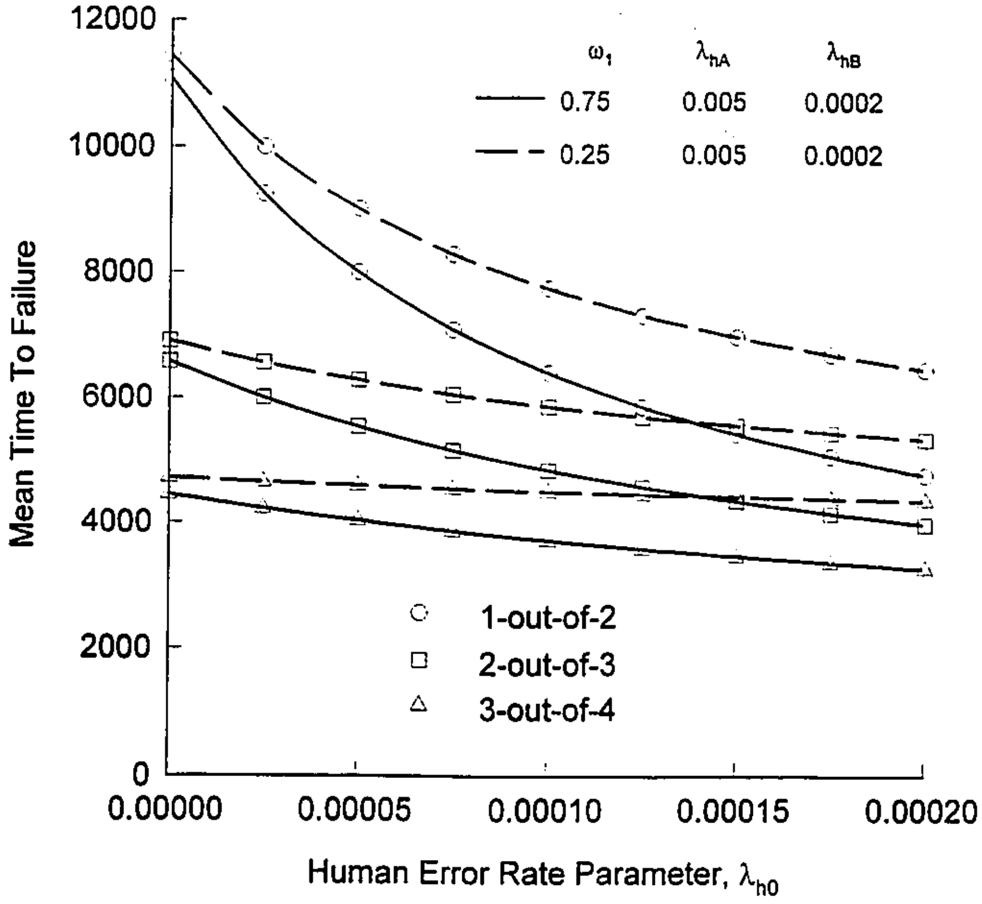


Figure 4-20 System mean time to failure with repair for k-out-of-n (n-k=1) system with general human error rates

4.4.3 K-out-of-n (n-k=2) System

For k-out-of-n (n-k=2) system, it is similar to the model of two units active and one unit on standby system. Using the same method and procedure as that of standby system, we can get the system steady-state availability, Laplace transform of system availability, Laplace transform of system reliability and system MTTF expressed by Equations (4-145), (4-159), (4-161) and (4-162), respectively.

Setting

$\lambda = 0.0002 / hr$	$\lambda_s = 0.00002 / hr$	$\mu_1 = 0.001 / hr$
$\mu_2 = 0.001 / hr$	$\lambda_{h1} = 0.0001 / hr$	$\lambda_{h2} = 0.0001 / hr$
$\lambda_{c0} = 0.00005 / hr$	$\lambda_{c1} = 0.00005 / hr$	$\lambda_{c2} = 0.00005 / hr$

in Equation (4-145), for exponential, Gamma (β = 3), Weibull (β = 2), and Rayleigh distributed failed system repair times, the plots of system steady-state availability as a function of human error rate parameter, λ_{h0}, are shown in Figures 4-21, 4-22, 4-23 and 4-24, respectively, for specified values of parameters λ_{hA}, λ_{hB}, ω_1, and $\mu_3 = 0.001 / hr$, $\mu_4 = \mu_5 = 0.0009 / hr$, if applicable.

Figure 4-25 provides the results of the system steady state availability as a function of human error rate parameter, λ_{h0}, for specified values of parameters λ_{hA}, λ_{hB}, ω_1, for lognormally distributed failed system repair times ($\mu_3 = 1.001 / hr$, $\mu_4 = \mu_5 = 1.0009 / hr$, $\sigma_3 = \sigma_4 = \sigma_5 = \sigma = 0.3$).

Substituting the Laplace transforms of the pdf of the system repair times $N_3(s)$, $N_4(s)$ and $N_5(s)$ in Equation (4-159) and then substituting r_0, r_1, r_2 and the same applicable data of the earlier system steady state availability numerical examples into the resulting expression (also setting $\mu_3 = 0.001 / hr$, $\mu_4 = \mu_5 = 0.0009 / hr$ and taking inverse Laplace transform), we get the system time-dependent availability.

For 2-out-of-4 system with ***Gamma*** (β=2) distributed failed system repair times and λ_{h0}=0.001/hr, λ_{hA}=0.005/hr, λ_{hB}=0.0002/hr, and ω_1=0.75 we get

$$
\begin{aligned}
AV(t) = {} & 0.531291 - 0.195910/\exp(0.005051\,t) + 0.050284/\exp(0.002369\,t) \\
& - 0.004953/\exp(0.001196\,t) - 0.0651366/\exp(0.000284\,t) \\
& +(0.009028 \sin(0.000031\,t))/\exp(0.0009385\,t) \\
& + 0.000318\,[(\cos(0.0000308\,t))/\exp(0.0009385\,t) \\
& - (30.43 \sin(0.0000308\,t))/\exp(0.0009385\,t)] \\
& + (1.454378 \sin(0.000685\,t))/\exp(0.0014168\,t) \\
& + 0.685107\,[(\cos(0.000685\,t))/\exp(0.0014168\,t) \\
& - (2.067653 \sin(0.000685\,t))/\exp(0.0014168\,t)]
\end{aligned}
\tag{4-178}
$$

For exponentially distributed failed system repair times, the plots of the time-dependent system availability for specified values of human error rate parameters, λ_{hA}, λ_{hB}, λ_{h0}, and ω_1, are shown in Figure 4-26.

For specified values of human error rate parameters, λ_{hA}, λ_{hB}, λ_{h0}, and ω_1, the system time-dependent availability plots of k-out-of-n (n-k=2) system with Gamma (β=2) distributed failed system repair times are shown in Figure 4-27.

For specified values of human error rate parameters, λ_{hA}=0.005/hr, λ_{hB}=0.0002/hr, λ_{h0}=0.001/hr, and ω_1=0.75, system reliability with repair for 2-out-of-4 system is given by

$$
\begin{aligned}
R(t) = {} & -0.2194964/\exp(0.005\,t) + 0.0660009/\exp(0.00247287\,t) \\
& - 1657.81947/\exp(0.00247279\,t) + 1609.04382/\exp(0.00247279\,t) \\
& + 5.570955/\exp(0.0015207\,t) - 6.324713/\exp(0.0015207\,t) \\
& + 2.1464969/\exp(0.0015207\,t) + 2076.831374/\exp(0.0006164\,t) \\
& + 5406.85476/\exp(0.0006164\,t) + 0.3014627/\exp(0.0002\,t)
\end{aligned}
\tag{4-179}
$$

The plots of the system reliability with repair for specified values of human error parameters for k-out-of-n (n-k=2) system are shown in Figure 4-28.

Figure 4-29 shows the plots of system mean time to failure with repair as a function of λ_{h0} for k-out-of-n (n-k=2) system for specified values of human error parameters.

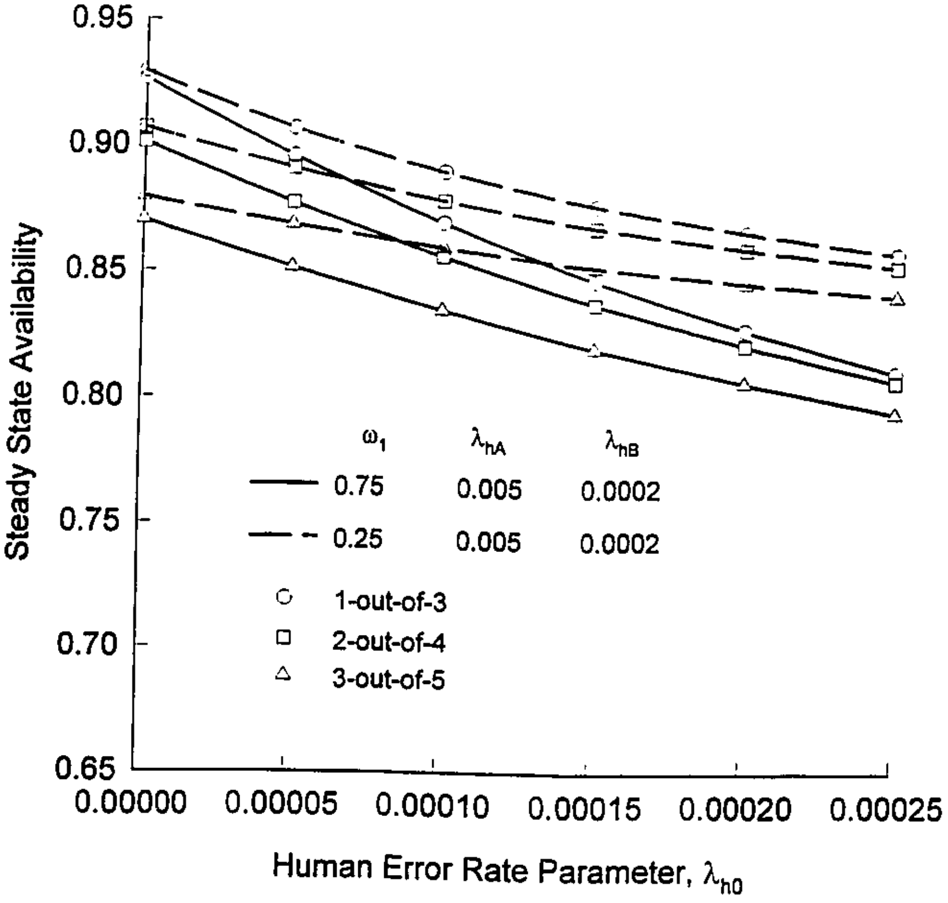


Figure 4-21 System steady state availability plots for k-out-of-n (n-k=2) system with general human error rates and constant system repair rates

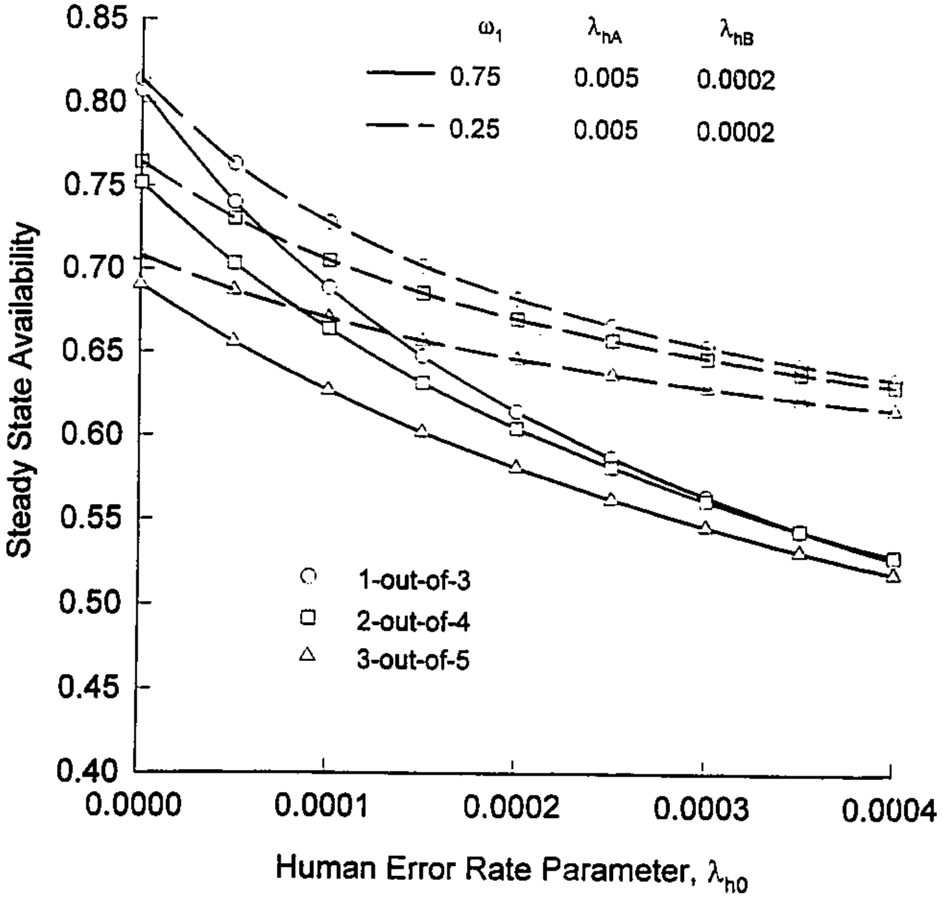


Figure 4-22 System steady state availability plots for k-out-of-n (n-k=2) system with general human error rates and the Gamma (β=3) distributed system repair times

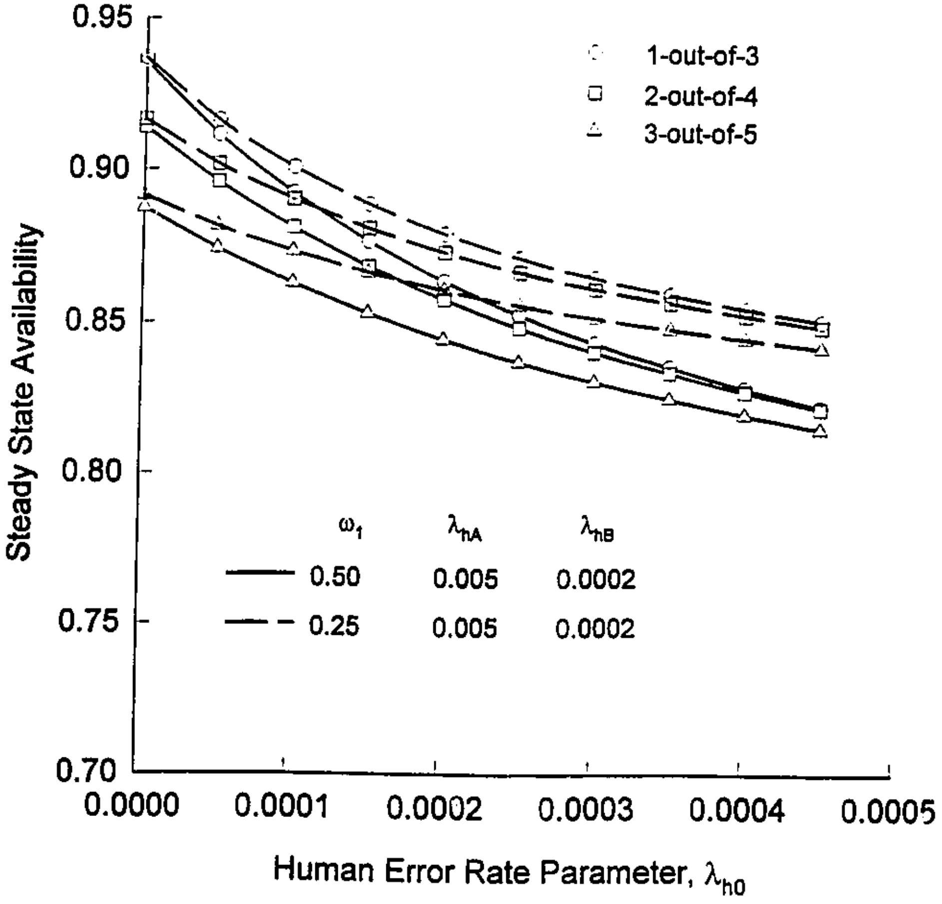


Figure 4-23 System steady state availability plots for k-out-of-n (n-k=2) system with general human error rates and the Weibull (β=2) distributed system repair times

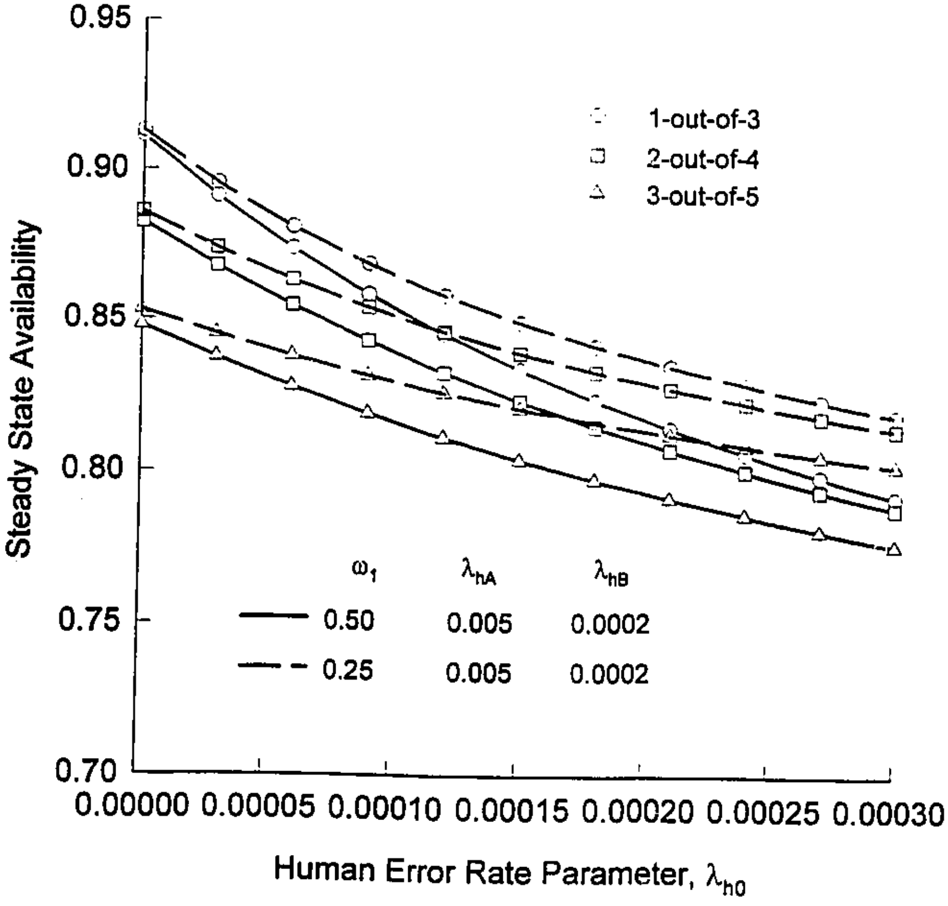


Figure 4-24 System steady state availability plots for k-out-of-n (n-k=2) system with general human error rates and the Rayleigh distributed system repair times

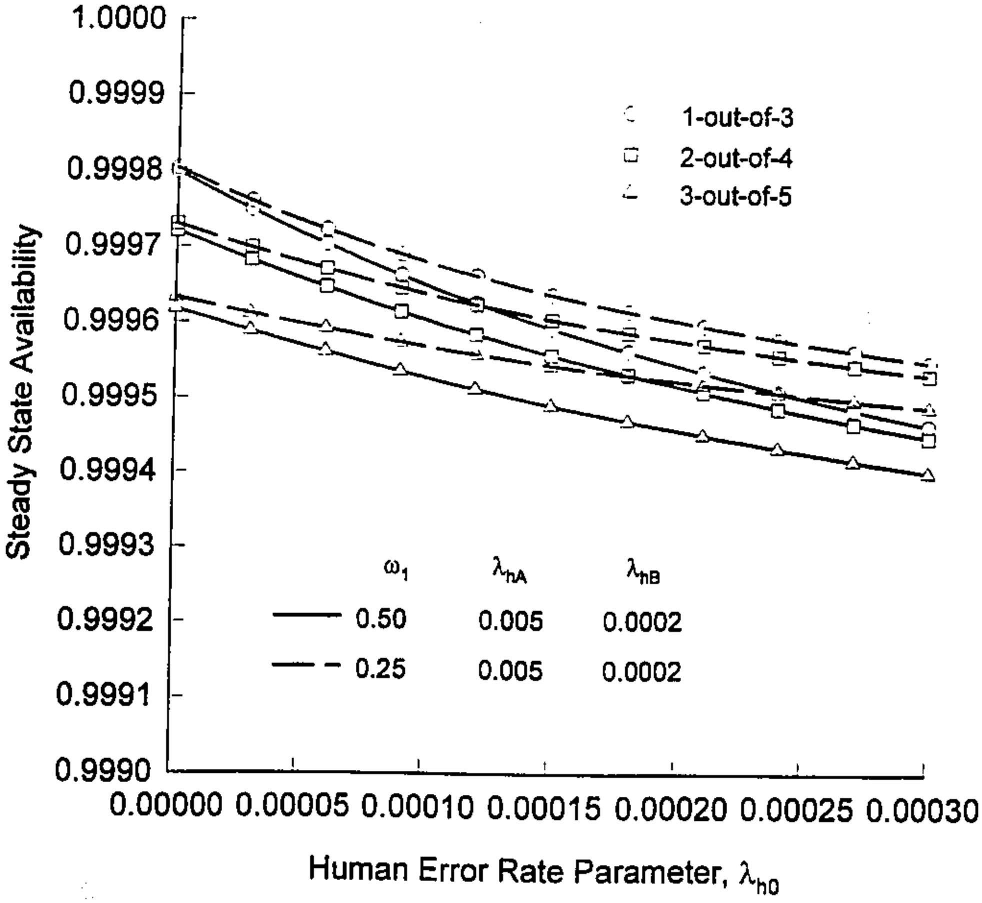


Figure 4-25 System steady state availability plots for k-out-of-n (n-k=2) system with general human error rates and the lognormally (σ=0.3) distributed system repair times

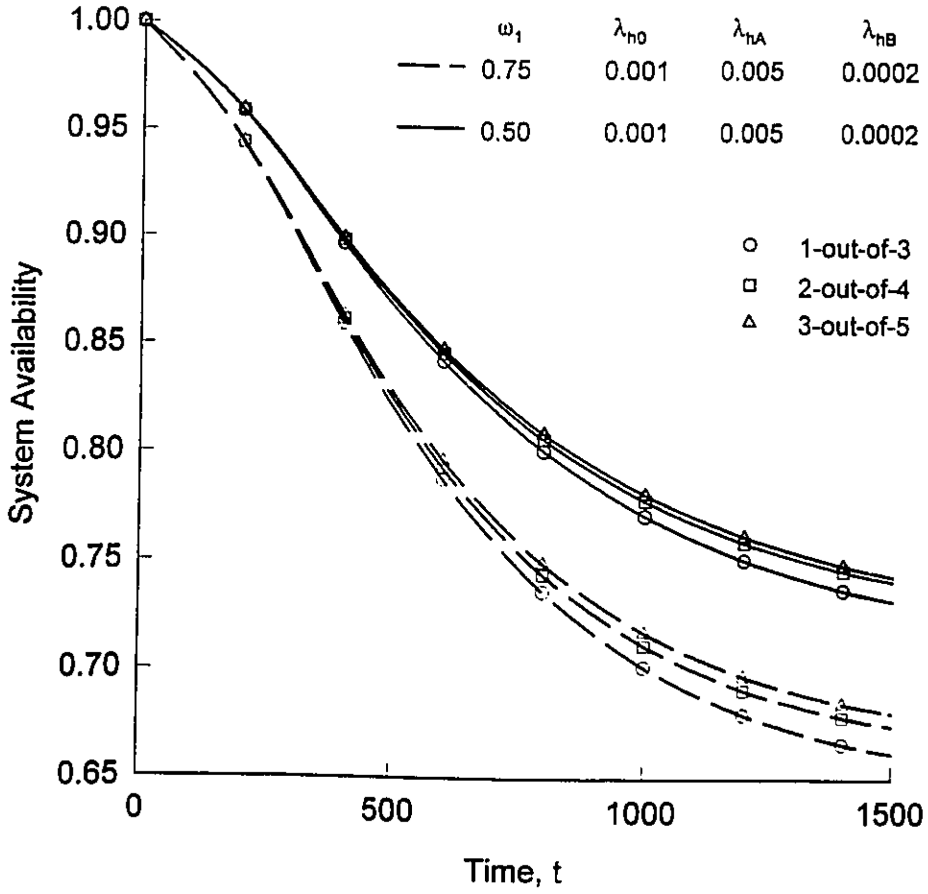


Figure 4-26 Time-dependent system availability plots for k-out-of-n (n-k=2) system with general human error rates and constant failed system repair rates

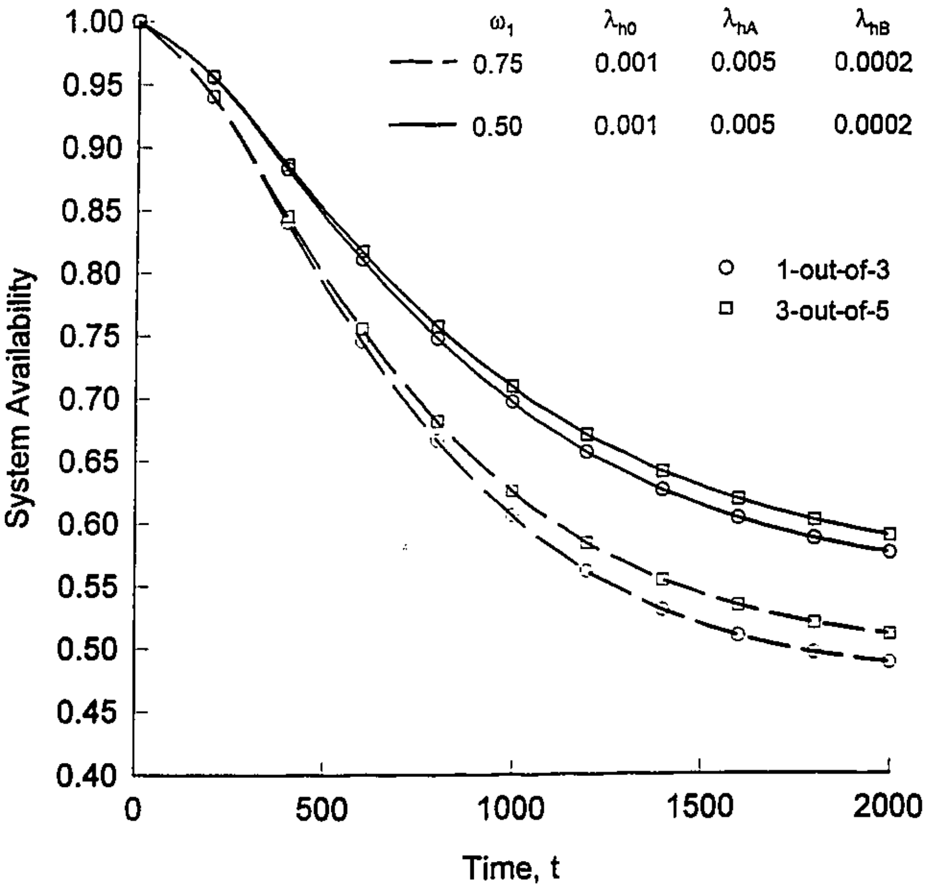


Figure 4-27 Time-dependent system availability plots for k-out-of-n (n-k=2) system with general human error rates and the Gamma (β=2) distributed system repair times

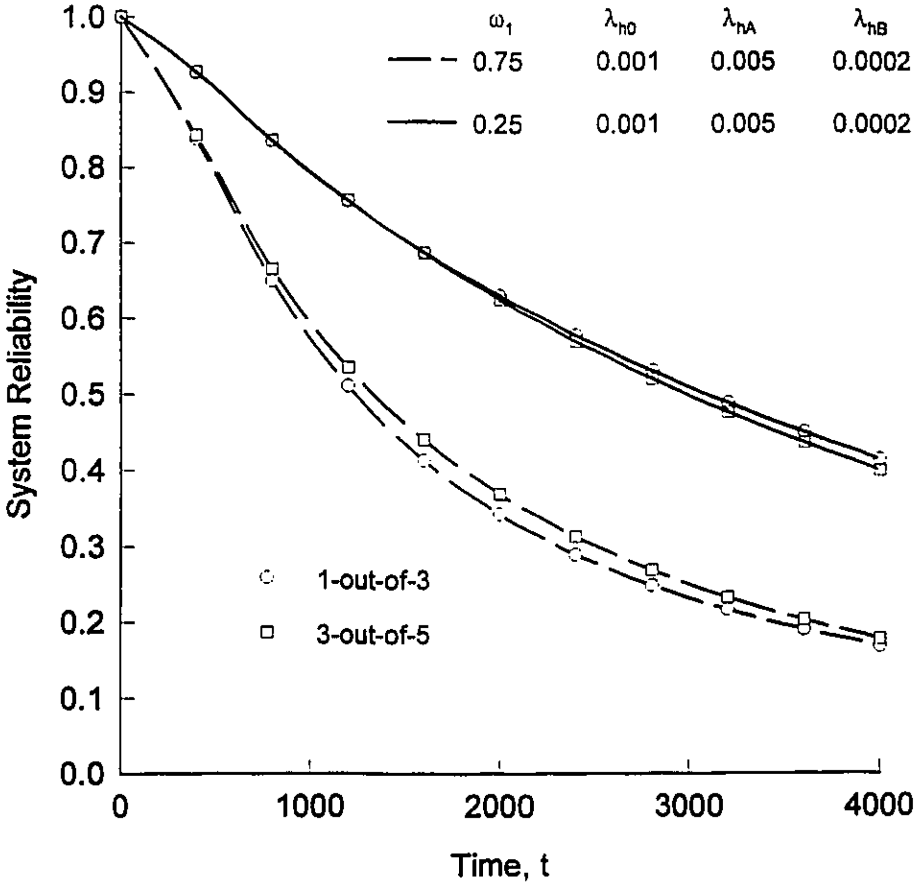


Figure 4-28 System reliability with repair plots for k-out-of-n (n-k=2) system with general human error rates

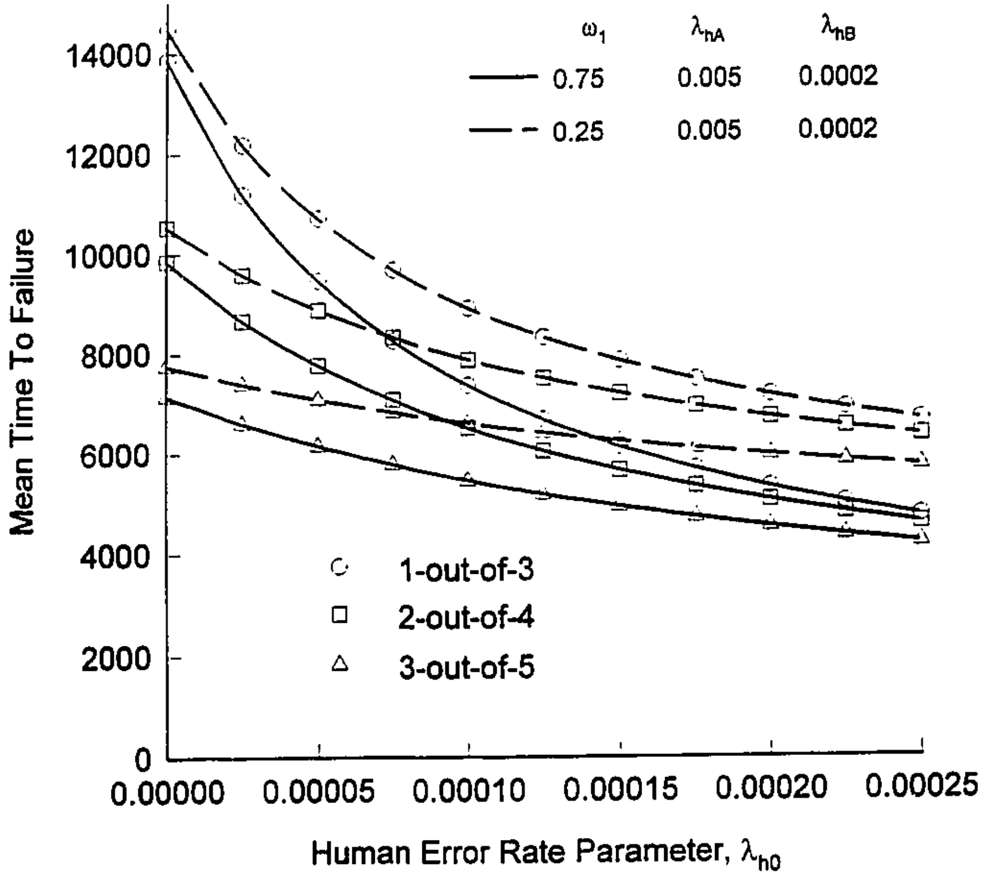


Figure 4-29 System mean time to failure with repair for k-out-of-n (n-k=2) system with general human error rates

4.5 Summary

This Chapter presents various mathematical models for performing reliability and availability analysis of standby systems and k-out-of-n systems with general human error rates and arbitrarily distributed failed system repair times.

A combination of two series stages in parallel is used to represent human error time distribution. There are four independent parameters, λ_{h0}, λ_{h1}, λ_{h2}, and ω_1, in the human error density function we can evaluate to match a particular distribution being considered.

A general standby system and a general k-out-of-n system with both time-dependent human error rate and failed system repair rate are discussed. The method of linear ordinary differential equations is used to obtain the general expressions of the system steady state availability for various types of failed system repair time distributions. The method which combines the inclusion supplementary variables method and the device of stages method is developed to perform time-dependent system availability analysis.

The analyses performed in this Chapter indicate that the steady state system availability, time-dependent system availability, system reliability and system mean time to failure decrease with a increase of human error rates parameters. An observation of the plots shows clearly that for standby redundant configurations, an increase in either the number of standby units for a fixed number of active units or the number of active units for a given number of standby units would improve system performance indices. For k-out-of-n redundant configurations, an increase in the value of k, the least number of units needed for system operation, for a given value of n, total number of units in the system, (or for given value of n-k), would decrease system performance indices.

Chapter 5

Discussion, Conclusions and Recommendations

5.1 Discussion

This study has presented a reliability and availability analysis of human-machine systems with human errors and common-cause failures. The systems incorporate elements of several commonly used redundant configurations: standby, k-out-of-n and majority voting with imperfect voter. The analysis considers system with constant human error rates, increasing human error rates and general human error rates, with arbitrarily distributed failed system repair times. Generalized expressions for such relevant system performance indices as the system reliability, steady state availability, time-dependent system availability, mean time to failure and system variance of time to failure are presented. The impact of human error, common-cause failure, failed system repair policy and the elements of redundant configurations on the values of the afore-said system performance indices is demonstrated by means of plots for the above configurations.

The steady state availability of repairable redundant systems appears to be one of the index of system performance most concern. The general concept of the mean time to system repair for generally distributed repair times and the method of linear ordinary differential equations are introduced in this study. Using the linear ordinary differential equations instead of complex partial differential equations and Laplace transforms, it is much easier to obtain the general expressions of steady state system availability for various types of failed system repair time distributions, such as the Gamma, Weibull, exponential, Rayleigh and lognormal distributions.

With systems that are governed by the exponential distribution, the transition rate from one state to another state of a system is constant and does not depend on how long the system spends in a given state nor does it depend on how it arrived at a particular state. This assumption leads to Markovian process. If the distribution is not exponential, then the process becomes non-Markovian. There are several techniques available for solving this problem, one of which is known as the method of stages and another one is the method of supplementary variables. In the method of stages, the non-exponential state is divided into a number of sub-states called stages, these stages are traversed in a given sequence, and all transfers between them are assumed to be made at constant rates. Depending on the sequence and the rates of departure between them, the total time spent in the stages can assume a variety of distributions.

The method which combines the inclusion supplementary variables technique and the method of stages is developed in this study for the systems with both time-dependent system failure rates and failed system repair rates. Generalized expressions for system reliability, steady state system availability, time-dependent system availability, system mean time to failure and system variance of time to failure are presented.

This study investigated not only the steady state availability, system reliability and system mean time to failure but also the time-dependent system availability for Gamma distributed failed system repair times whilst the human error rate assumed constant, increasing or general. The research indicates that the systems performance indices generally depend upon the system configuration, upon the distribution functions of failure and repair times, and upon the maintenance policy.

The influence of human errors on the systems is clearly evident as one goes through the various tables and plots. The general picture that emerges is that an increase in the critical human error rate invariably reduces the steady state system availability, time-dependent system availability, system reliability, and the system mean time to failure. Generalization of the failed systems repair time distribution reveals that the system performance indices are influenced by the systems repair time distributions. It is clearly conclusive that the steady state system availability and time-dependent system availability drop as the value of the Gamma shape parameter β increases for Gamma distributed failed system repair times and those values increase as the value of Weibull shape parameter β increases for Weibull distributed failed system repair times. This trend runs through the various special cases considered regardless of the system configuration and the type of failed system repair.

The influence of system configuration on system performance indices can be observed by comparing the plots for various systems. For standby redundant configurations, an increase in either the number of standby units for a fixed number of active units or the number of active units for a given number of standby units would improve system performance indices significantly. For k-out-of-n redundant configurations, an increase in the value of k, the least number of units needed for system operation, for a given value of n, total number of units in the system, (or for given value of n-k), would decrease system performance indices.

5.2 Conclusions

The main results obtained in this study can be summarized as follows:

1. The system steady state availability, time-dependent system availability, system reliability and system mean time to failure (MTTF) decrease with increasing values of the human errors and common-cause failures. This is true regardless of whether the human error rates are time-dependent or constant, the system units are identical or non-identical, the system is repairable or non-repairable.

2. The steady state system availability and time-dependent system availability are heavily influenced by the failed system repair time distributions. AV_{ss} and AV(t) decrease as the value of the Gamma shape parameter β increases for Gamma distributed failed system repair times. And those values increase as the value of Weibull shape parameter β increases for Weibull distributed failed system repair times. Furthermore, AV_{ss} and AV(t) increase with increasing values of failed system repair rates regardless of which types of distributions of failed system repair times are used. The lognormally distributed failed system repair time yielded the highest values of the steady state system availability while Gamma ($\beta \geq 2$) distributed failed system repair time produced the least values for the steady state system availability.

3. The method of linear algebraic equations is developed in this study. Using the linear ordinary differential equations instead of complex partial differential equations and Laplace transforms, it is much easier to obtain the general expressions of steady state system availability for various types of failed system repair time distributions, such as the Gamma, Weibull, exponential, Rayleigh and lognormal distributions.

4. The method which combines the inclusion supplementary variables method and the method of stages is developed in this study to perform system steady state availability and time-dependent system availability analysis. This method overcomes the difficulties that arise when the system has both time-dependent system failure rates and failed system repair rates.

5.3. Recommendations for Further Study

1. Critical human errors which cause the entire system failure were considered in this thesis. The models could be studied further by considering both critical and non-critical human errors.

2. The time-dependent system availability was studied for special cases when failed system repair times were assumed Gamma distributed. It is still difficult to get the time-dependent system availability expressions for lognormally or Weibull distributed failed system repair times due to difficulty in taking Laplace transforms of pdf of the system repair time, $N_j(s)$ (see details in Appendix A). For these distributions, it is very cumbersome to invert the Laplace transform expressions from s-domain to time-domain in order to compute time-dependent system availability. Different techniques need to be developed in further studies to solve this problem. Simulation can be considered a technique of last resort, maybe it is the only practical solution to a real problem despite its difficulties and the costs and time required [212].

Complex structures, such as bridges, multistate models and the influence of preventive maintenance need to be considered in further studies.

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Appendix A

Reliability and Maintainability Characteristics

A.1. Reliability Characteristics

The following four functions are commonly used in reliability analysis:

- f(t) — probability density function (pdf) of failure time distribution

$$f(t) = \frac{dF(t)}{dt}$$

- F(t) — cumulative failure function (failure probability at time t)

$$F(t) = \int_0^t f(\xi)d\xi$$

- R(t) — reliability function (cumulative unfailure probability at time t)

$$R(t) = 1 - F(t)$$

- λ(t) — hazard function (failure rate at time t)

$$\lambda(t) = \frac{f(t)}{R(t)}$$

The relationships between the four functions are summarized in Table A-1.

Table A-1 Relationships between different reliability functions

	f(t)	λ(t)	F(t)	R(t)
f(t) =	f(t)	$\lambda(t)\exp[-\int_0^t \lambda(\xi)d\xi\,]$	$\frac{dF(t)}{dt}$	$-\frac{dR(t)}{dt}$
λ(t) =	$\frac{f(t)}{1-\int_0^t f(\xi)d\xi}$	λ(t)	$\frac{1}{1-F(t)}\frac{dF(t)}{dt}$	$-\frac{d}{dt}[\ln R(t)]$
F(t) =	$\int_0^t f(\xi)d\xi$	$1\text{-}\exp[-\int_0^t f(\xi)d\xi\,]$	F(t)	1- F(t)
R(t) =	$1\text{-}\int_0^t f(\xi)d\xi$	$\exp[-\int_0^t f(\xi)d\xi\,]$	1- F(t)	R(t)

There are two basic parameters that can be used to describe a probability distribution, i.e., expected value and variance. In reliability analysis, the expected value is known as the mean time to failure or simply, MTTF. MTTF mathematically is the first moment of failure time distribution

$$MTTF = E[t] = \int_0^\infty t\, f(t)dt$$

Alternatively, MTTF can be expressed in various different forms:

$$MTTF = \int_0^\infty t\, f(t)dt = \int_0^\infty t\lambda(t)\exp[-\int_0^t \lambda(\xi)d\xi]dt$$

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty \exp[-\int_0^t \lambda(\xi)d\xi]dt$$

$$MTTF = \lim_{s\to 0} R(s)$$

where R(s) is the Laplace transform of system reliability R(t).

Variance of time to failure is the second central moment of a distribution

$$\begin{aligned}\sigma^2 &= E[t-E[t]]^2\\ &= \int_0^\infty [t-E[t]]^2 f(t)dt\end{aligned}$$

Alternatively, it can also be obtained from the following expression:

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2$$

where R′(s) denotes the derivative of R(s) with respect to s.

A.2. Maintainability Characteristics

The most frequently used four functions of maintainability are as follows:

- N(x) — probability density function (pdf) of repair time distribution

$$N(x) = \frac{dM(x)}{dx}$$

- M(x) — cumulative repair function (repair probability at time x)

$$M(x) = \int_0^x N(\xi)d\xi$$

- G(x) — cumulative unrepair function

$$G(x) = 1 - M(x)$$

- μ(x) — repair rate at time x

$$\mu(x) = \frac{N(x)}{G(x)}$$

The relationships between the four functions are summarized in Table A-2.

Table A-2 Relationships between different maintainability functions

	N(x)	μ(x)	M(x)	G(x)
N(x) =	N(x)	$\mu(x)\exp[-\int_0^x \mu(\xi)d\xi]$	$\frac{dM(x)}{dx}$	$-\frac{dG(x)}{dx}$
μ(x) =	$\frac{N(x)}{1-\int_0^x N(\xi)d\xi}$	μ(x)	$\frac{1}{1-M(x)}\frac{dM(x)}{dx}$	$-\frac{d}{dx}[\ln G(x)$
M(x) =	$\int_0^x N(\xi)d\xi$	$1-\exp[-\int_0^x \mu(\xi)d\xi]$	M(x)	1- G(x)
G(x) =	$1-\int_0^x N(\xi)d\xi$	$\exp[-\int_0^x \mu(\xi)d\xi]$	1- M(x)	G(x)

In maintainability analysis, the expected value is known as the mean time to repair or simply, MTTR. MTTR mathematically is the first moment of repair time distribution

$$MTTR = E[x] = \int_0^\infty x\, N(x)dx$$

Alternatively, MTTR can be expressed in various different forms:

$$MTTR = \int_0^\infty x\, N(x)dx = \int_0^\infty x\mu(x)\exp[-\int_0^x \mu(\xi)d\xi]dx$$

$$MTTR = \int_0^\infty G(x)dx = \int_0^\infty \exp[-\int_0^x \mu(\xi)d\xi]dx$$

$$MTTR = \lim_{s\to 0} G(s)$$

where G(s) is the Laplace transform of system cumulative unrepair function G(x).

Variance of time to repair is the second central moment of a distribution

$$\begin{aligned}\sigma^2 &= E[x-E[x]]^2\\ &= \int_0^\infty [x-E[x]]^2 N(x)dx\end{aligned}$$

A.3. N(x), E(x) and N(s) For Various Different Distributions

The probability density function, N(x), expected values of time to repair, E[x], and the Laplace transforms of probability density function, N(s), for various different distributions are given in Table A-3.

Table A-3 N(x), E(x) and N(s) for different distributions

distribution	N(x)	E[x]	N(s)
Gamma	$\frac{\mu^{\beta} x^{\beta-1}}{\Gamma(\beta)} \exp(-\mu x)$	$\frac{\beta}{\mu}$	$\left(\frac{\mu}{s+\mu}\right)^{\beta}$
Weibull	$\mu^{\beta} \beta x^{\beta-1} \exp(-\mu^{\beta} x^{\beta})$	$\frac{\Gamma(1+1/\beta)}{\mu}$	$\frac{-s\, e^{(s^2/4\mu)}}{4\sqrt{\mu}}\left[2\sqrt{\pi}+\Gamma\left(-\frac{1}{2},0,\frac{s^2}{4\mu}\right)\right]$ (when $\beta = 2$)
lognormal	$\frac{1}{x\sigma\sqrt{2\pi}} \exp(-\frac{(\ln x-\mu)^2}{2\sigma^2})$	$\exp(\mu+\sigma^2/2)$	
exponential	$\mu \exp(-\mu x)$	$\frac{1}{\mu}$	$\frac{\mu}{s+\mu}$
Rayleigh	$\mu^2 x \exp(-\mu^2 x^2/2)$	$\frac{1}{\mu}\sqrt{\frac{\pi}{2}}$	$\frac{-s\, e^{(s^2/2\mu^2)}}{4\mu}\left[4\sqrt{\frac{\pi}{2}}+\sqrt{2}\,\Gamma\left(-\frac{1}{2},0,\frac{s^2}{2\mu^2}\right)\right]$

Appendix B

Comparisons of Device of Stages - Markov Method Results With That of Block Diagram Technique

B.1. Two-Unit Parallel System With Critical Human Error

In order to compare results obtained through the device of stages-Markov method with those obtained by the block diagram approach, the block diagram of a two-unit parallel system with critical human error is shown in Figure B-1.

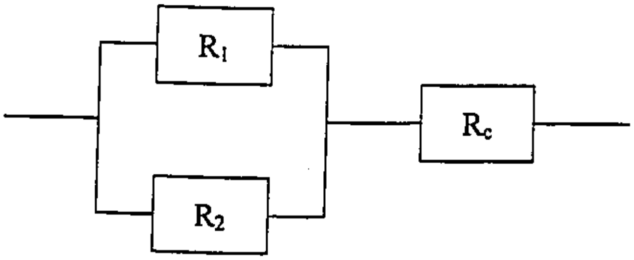

Figure B-1 Two-unit parallel system with critical human error

If the two units are identical and the reliability of each unit is

$$R_1(t) = R_2(t) = e^{-\lambda t} \tag{B-1}$$

and the reliability of human is

$$R_c(t) = e^{-\lambda_{c0} t} + \frac{\lambda_{c0}}{\lambda_c - \lambda_{c0}}\left(e^{-\lambda_{c0} t} - e^{-\lambda_c t}\right) \tag{B-2}$$

where λ_{c0} and λ_c are two parameters of the distribution.

Using block diagram method, the reliability of the system can be expressed as

$$R_{sA}(t) = \left(2e^{-\lambda t} - e^{-2\lambda t}\right) R_c(t) \tag{B-3}$$

The system mean time to failure (MTTF) is given by

$$\begin{aligned} MTTF_A &= \int_0^{\infty} R_{sA}(t)\,dt \\ &= \frac{-2\lambda_{c0}}{(1+\lambda_c)(\lambda_c - \lambda_{c0})} + \frac{\lambda_{c0}}{(2\lambda+\lambda_c)(\lambda_c - \lambda_{c0})} + \frac{2}{(\lambda + \lambda_{c0})} \\ &\quad + \frac{2\lambda_{c0}}{(\lambda_c - \lambda_{c0})(\lambda + \lambda_{c0})} - \frac{1}{(2\lambda + \lambda_{c0})} - \frac{\lambda_{c0}}{(\lambda_c - \lambda_{c0})(2\lambda + \lambda_{c0})} \end{aligned} \tag{B-4}$$

Also, the system reliability and MTTF for Figure B-1 using the device of stages-Markov method are presented as follows:

The state transition diagram of the system is shown in Figure B-2.

The differential equations associated with Figure B-2 is

$$\frac{dP_0(t)}{dt} = -2\lambda P_0(t) - \lambda_{c0} P_0(t) \tag{B-5}$$

$$\frac{dP_1(t)}{dt} = 2\lambda P_0(t) - (\lambda + \lambda_{c0})\, P_1(t) \tag{B-6}$$

$$\frac{dP_2(t)}{dt} = \lambda P_1(t) \tag{B-7}$$

$$\frac{dP_{3A}(t)}{dt} = \lambda_{c0} P_0(t) + \lambda_{c0} P_1(t) - \lambda_c P_{3A}(t) \tag{B-8}$$

$$\frac{dP_3(t)}{dt} = \lambda_c P_{3A}(t) \tag{B-9}$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

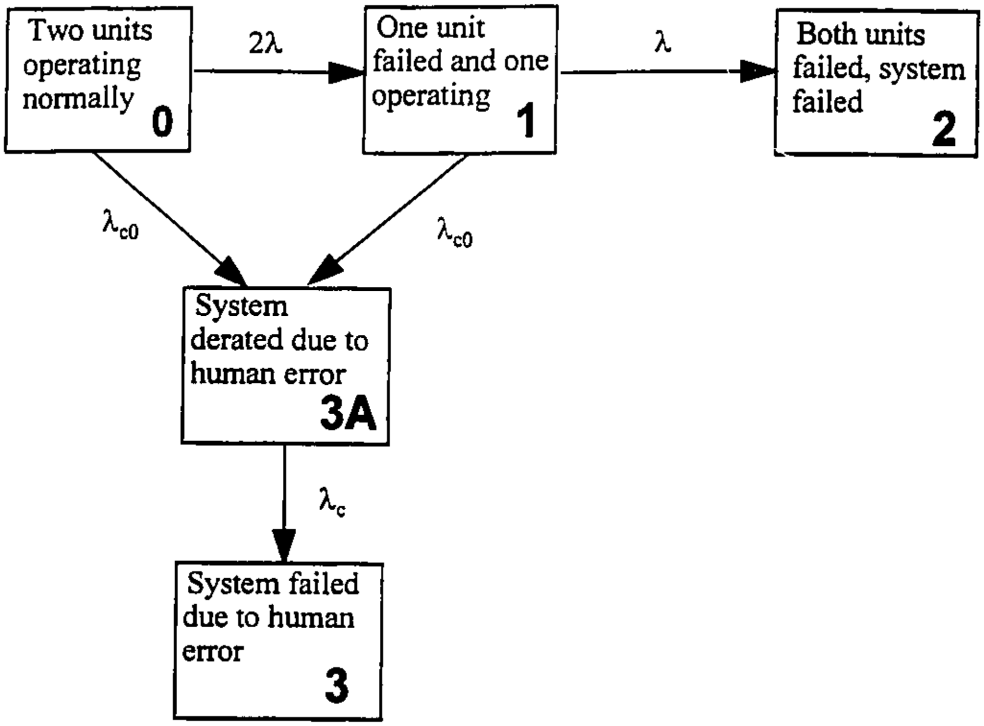

Figure B-2 The state transition diagram of the system

Using Laplace transforms and the initial conditions in Equations (B-5)-(B-9), we get

$$sP_0(s)-1=-(2\lambda+\lambda_{c0})\,P_0(s)$$

$$sP_1(s)=2\lambda P_0(s)-(\lambda+\lambda_{c0})\,P_1(s)$$

$$sP_2(s)=\lambda P_1(s)$$

$$sP_{3A}(s)=\lambda_{c0}P_0(s)+\lambda_{c0}P_1(s)-\lambda_c P_{3A}(s)$$

$$sP_3(s)=\lambda_c P_{3A}(s)$$

The Laplace transform of the system reliability is:

$$R_{sB}(s) = P_0(s) + P_1(s) + P_{3A}(s)$$
$$= \frac{(s+\lambda_c+\lambda_{c0})(s+3\lambda+\lambda_{c0})}{(s+\lambda_c)(s+\lambda+\lambda_{c0})(s+2\lambda+\lambda_{c0})} \qquad \text{(B-10)}$$

The system reliability can be obtained by inverting Equation (B-10)

$$R_{sB}(t) = \frac{2(\lambda-\lambda_c)}{(\lambda-\lambda_c+\lambda_{c0})}e^{-(\lambda+\lambda_{c0})t} + \frac{\lambda_c-2\lambda}{(2\lambda-\lambda_c+\lambda_{c0})}e^{-(2\lambda+\lambda_{c0})t}$$
$$+\frac{\lambda_{c0}{}^2+3\lambda\lambda_{c0}-\lambda_c\lambda_{c0}}{(\lambda-\lambda_c+\lambda_{c0})(2\lambda-\lambda_c+\lambda_{c0})}e^{-\lambda_c t} \qquad \text{(B-11)}$$

Setting λ=0.00002 and λ_c=0.005 in Equations (B-3) and (B-11), the reliability of the system $R_{sA}(t)$ and $R_{sB}(t)$ are shown in Table B-1 for different values of λ_{c0}. The results clearly indicate that $R_{sA}(t)$ and $R_{sB}(t)$ are nearly equal to each other.

Table B-1 The comparison of $R_{sA}(t)$ and $R_{sB}(t)$ for two units in parallel system with critical human error

Time	$R_{sA}(t)$			$R_{sB}(t)$		
	λ_{c0}=0.00005	λ_{c0}=0.0001	λ_{c0}=0.0002	λ_{c0}=0.00005	λ_{c0}=0.0001	λ_{c0}=0.0002
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1000	0.96039	0.92280	0.85223	0.96039	0.92281	0.85223
2000	0.91257	0.83415	0.69717	0.91257	0.83416	0.69718
3000	0.86645	0.75337	0.56974	0.86646	0.75338	0.56975
4000	0.82211	0.67996	0.46528	0.82212	0.67996	0.46529
5000	0.77954	0.61330	0.37974	0.77955	0.61331	0.37975

The system mean time to failure (MTTF) is given by

$$MTTF_B = \lim_{s\to 0} R_{sB}(s) = \frac{(\lambda_c+\lambda_{c0})(3\lambda+\lambda_{c0})}{\lambda_c(\lambda+\lambda_{c0})(2\lambda+\lambda_{c0})} \qquad \text{(B-12)}$$

Setting λ=0.0002 in Equations (B-4) and (B-12), for different values of λ_{c0} and λ_c, system mean time to failure $MTTF_A$ and $MTTF_B$ are shown in Table B-2. The result is clearly indicated that $MTTF_A$ and $MTTF_B$ are quite close to each other.

Table B-2 The comparison of $MTTF_A$ and $MTTF_B$ for two units in parallel system with critical human error

λ_{c0}	$MTTF_A$			$MTTF_B$		
	λ_c=0.005	λ_c=0.008	λ_c=0.01	λ_c=0.005	λ_c=0.008	λ_c=0.01
0.0000	7500.00	7500.00	7500.00	7500.00	7500.00	7500.00
0.0001	4757.83	4724.16	4712.80	4760.00	4725.00	4713.33
0.0002	3463.91	3415.60	3399.32	3466.67	3416.67	3400.00
0.0003	2722.83	2666.75	2647.87	2725.71	2667.86	2648.57
0.0004	2247.15	2186.41	2165.98	2250.00	2187.50	2166.67
0.0005	1917.88	1854.11	1832.67	1920.63	1855.16	1833.33
0.0006	1677.35	1611.50	1589.37	1680.00	1612.50	1590.00
0.0007	1494.43	1427.08	1404.45	1496.97	1428.03	1405.05
0.0008	1350.90	1282.42	1259.43	1353.33	1283.33	1260.00
0.0009	1235.43	1166.09	1142.81	1237.76	1166.96	1143.36
0.0010	1140.62	1070.60	1047.10	1142.86	1071.43	1047.62

The plots of system reliability $R_{sA}(t)$ and $R_{sB}(t)$, and the plots of system mean time to failure $MTTF_A$ and $MTTF_B$ for various different values of distribution parameters are shown in Figures B-3 and B-4, respectively. It can be clearly seen from the plots that $R_{sA}(t)$ and $R_{sB}(t)$, and $MTTF_A$ and $MTTF_B$ are identical, respectively.

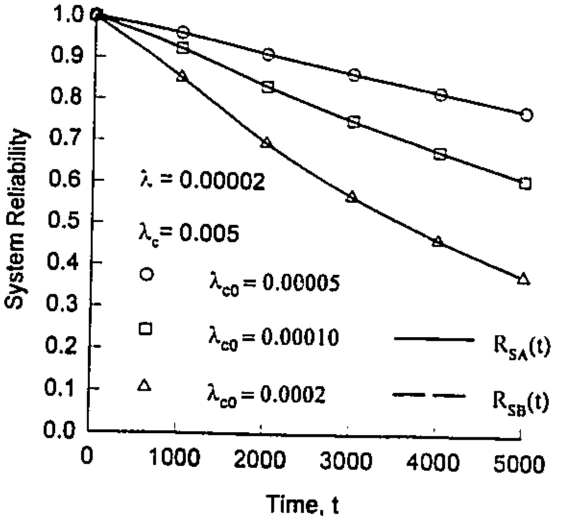

Figure B-3 Plots of $R_{SA}(t)$ and $R_{SB}(t)$ for two units in parallel system with critical human error

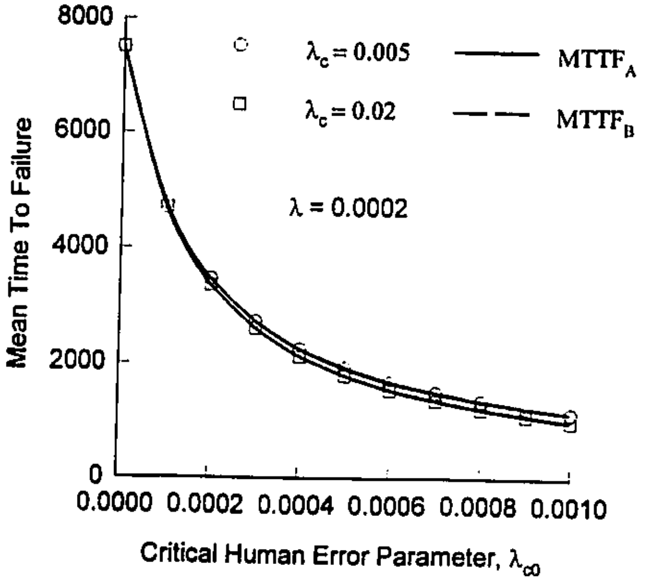

Figure B-4 $MTTF_A$ and $MTTF_B$ plots for two units in parallel system with critical human error

B.2. Three-Unit Parallel System With Critical Human Error

When $\lambda_{c0} = \lambda_c$, we obtain a Gamma distribution with a density function of the human operating

$$f_c(t) = \lambda_c^2 t\, e^{-\lambda_c t} \tag{B-13}$$

and the associated human operating reliability is

$$R_c(t) = e^{-\lambda_c t}(1+\lambda_c t) \tag{B-14}$$

For three identical units in parallel system with critical human error, using block diagram method, the reliability of the system can be expressed as

$$R_{sA}(t) = \left(3e^{-\lambda t} - 3e^{-2\lambda t} + e^{-3\lambda t}\right) e^{-\lambda_c t}(1+\lambda_c t) \tag{B-15}$$

The system mean time to failure (MTTF) is given by

$$\begin{aligned} MTTF_A &= \int_0^\infty R_{sA}(t)\,dt \\ &= \frac{3\lambda_c}{(\lambda+\lambda_c)^2} + \frac{3}{\lambda+\lambda_c} - \frac{3\lambda_c}{(2\lambda+\lambda_c)^2} - \frac{3}{2\lambda+\lambda_c} + \frac{\lambda_c}{(3\lambda+\lambda_c)^2} + \frac{1}{3\lambda+\lambda_c} \end{aligned} \tag{B-16}$$

Using device of stages-Markov technique, the system reliability and mean time to failure are expressed as follows:

$$\begin{aligned} R_{sB}(t) &= \frac{3(\lambda-\lambda_c)}{\lambda} e^{-(\lambda+\lambda_c)t} + \frac{3\lambda-\lambda_c}{3\lambda} e^{-(3\lambda+\lambda_c)t} \\ &\quad + \frac{11\lambda_c}{6\lambda} e^{-\lambda_c t} + \frac{3(\lambda_c-2\lambda)}{2\lambda} e^{-(2\lambda+\lambda_c)t} \end{aligned} \tag{B-17}$$

and

$$MTTF_B = \int_0^\infty R_{sB}(t)\,dt = \frac{2(11\lambda^2 + 6\lambda\lambda_c + \lambda_c^2)}{(\lambda+\lambda_c)(2\lambda+\lambda_c)(3\lambda+\lambda_c)} \tag{B-18}$$

Setting $\lambda=0.00001$ in Equations (B-15) and (B-17), the reliability of the system $R_{sA}(t)$ and $R_{sB}(t)$ are shown in Table B-3 for different values of λ_c. The results indicate that $R_{sA}(t)$ and $R_{sB}(t)$ are nearly equal to each other.

Table B-3 The comparison of $R_{sA}(t)$ and $R_{sB}(t)$

Time (t)	$R_{sA}(t)$			$R_{sB}(t)$		
	$\lambda_c=0.00005$	$\lambda_c=0.0001$	$\lambda_c=0.0005$	$\lambda_c=0.00005$	$\lambda_c=0.0001$	$\lambda_c=0.0005$
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
500	0.99969	0.99879	0.99532	0.99969	0.99879	0.99532
1000	0.99879	0.99532	0.98248	0.99879	0.99532	0.98248
1500	0.99732	0.98981	0.96306	0.99732	0.98981	0.96306
2000	0.99531	0.98247	0.93844	0.99531	0.98247	0.93844
2500	0.99279	0.97349	0.90978	0.99280	0.97349	0.90979
3000	0.98979	0.96304	0.87808	0.98979	0.96304	0.87808

System mean time to failure $MTTF_A$ and $MTTF_B$ of Equations (B-16) and (B-18), for different values of λ and λ_c, are shown in Table B-4. These results indicate that $MTTF_A$ and $MTTF_B$ are close to each other.

Similarly, one can extend device of stages-Markov method to n-unit parallel system. The evidence available to the author was sufficient to justify the claim that the non-exponential distribution could be quite appropriately presented by stages models with constant transition rates, and thereby allowing the construction of Markov models.

Table B-4 The comparison of $MTTF_A$ and $MTTF_B$

λ	$MTTF_A$			$MTTF_B$		
	$\lambda_c=0.005$	$\lambda_c=0.008$	$\lambda_c=0.01$	$\lambda_c=0.005$	$\lambda_c=0.008$	$\lambda_c=0.01$
0.00000	400	250	200	400	250	200
0.00002	399.9996	250	200	399.9998	250	200
0.00004	399.9971	249.9996	199.9998	399.9988	249.9998	199.9999
0.00006	399.9905	249.9985	199.9994	399.9961	249.9994	199.9997
0.00008	399.9781	249.9965	199.9986	399.9910	249.9986	199.9994
0.00010	399.9583	249.9933	199.9972	399.9829	249.9972	199.9988
0.00012	399.9299	249.9886	199.9953	399.9711	249.9954	199.9981
0.00014	399.8916	249.9822	199.9925	399.9552	249.9927	199.9969
0.00016	399.8425	249.9739	199.9890	399.9346	249.9893	199.9955
0.00018	399.7816	249.9635	199.9846	399.9090	249.9850	199.9937
0.00020	399.7082	249.9508	199.9792	399.8779	249.9797	199.9914