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APPLICATION OF VLASOV'S METHOD

TO STATIC, DYNAMIC AND STABILITY

PROBLEMS IN PLATE STRUCTURES

BY

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A thesis submitted to the School of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of Master of Applied Science (Civil Engineering)

ISBN 0-315-36550-1

UNIVERSITÉ D'OTTAWA
UNIVERSITY OF OTTAWA

TABLE OF CONTENTS

ABSTRACT

Variational methods are widely used for the solution of boundary value problems in applied mechanics. Of all the variational methods, Galerkin's method is generally the most rapidly converging. But, the Galerkin method suffers the drawbacks that the method is usually very inefficient, as it involves the tedious and sometimes formidale task of definite integrations. It is also very difficult and sometimes impossible to find a complete deflection function to meet the geometric boundary conditions of a particular structure under consideration.

In this study, Galerkin's method is extended by the Vlasov's method in that the displacement functions are chosen with special mathematical properties and by ultilizing these mathemetical properties, Galerkin's method is greatly simplified.

To demonstrate the simplicity and accuracy of Vlasov's method, typical applied mechanics problems such as bending, vibration and buckling of plates are used as illustrative examples. The Vlasov's method is further applied to the deflection of plates on elastic foundations and sandwich plate problems. The results obtained are presented in tabular and graphical forms, and whenever possible, are compared with existing solutions based on much more tedious and lengthly methods of analysis and close agreements are found. No comparison of the results for the deflections of rectangular plates with one side fixed and three sides simply supported resting on an elastic foundation in table 4 nor the free vibration of one side fixed and three sides simply supported rectangular sandwich plate with various shear rigidities and aspect ratios in table 18 are made as no results by other investigators are readily available in the technical literatures.

The definite integrals involved in the formation of Vlasov's algebraic equations from the governing differential equations were evaluated by trapezoidal rule.

Comparing with the finite element method, the Vlasov method is more simple in for-

mation and computing time and memory requirements are small, thus making it ideally suited for computers with relatively smaller capacity and speed. In contrast, the finite element method requires the use of computers of considerable storage capacity and preparation of data for each element can be time consuming. The Vlasov method is also easier to apply in sandwich plate problems compared to the finite element method.

ACKNOWLEDGMENT

The writer wishes to express his genuine gratitude to his supervisor, Dr. S.F.NG for his constructive guidance and advice throughout the investigation of this work. The cooperation extended by the personnel of computing center, University of Ottawa is highly appreciated.

LIST OF FIGURES

LIST OF TABLES

page

NOMENCLATURE

x, y, z	rectangular cartesian coordinates
u, v, w	displacements in x,y, and z-direction
W	dimensionless displacement in z-direction
ϕ_{mn}, φ_{mn}	functions of x and y.
X_m	function of x only
Y_n	function of y only
Q	dimensionless parameter of load
q, p	lateral load per unit area
ρ	mass per unit area of plate
E	modulus of elasticity of isotropic material
I	moment of inertia
λ	aspect ratio of plate, (a/b)
λ_n	eigenvalue corresponding to the n^{th} mode of vibration
l	length of beam
D_x, D_y, D_{xy}	bending and twisting stiffness of orthotropic plate
k	modulus of elastic foundation
G	shear modulus of isotropic material
ν	poisson's ratio for isotropic material
ν_x , ν_y	poisson's ratios for orthotropic material
h	plate thickness

G_c	shear modulus of core layer of sandwich plates
E_c	modulus of elasticity of core layer of sandwich plates
ν_f	poisson's ratio of facing material of sandwich plates
t	thickness of the facing material of sandwich plates
D	flexural rigidity of plate
N_x, N_y	normal forces in x and y-directions
N_{xy}	shear force per unit length
K	dimensionless modulus of elastic foundation
W_{mn}	expansion coefficient
	circular frequency
$2a, 2b$	dimensions of the plate in x and y direction
ζ, η	dimensionless parameters of x and y directional coordinates
μ	t/a
θ	h/a
F	eigenvalue parameter
f	dimensionless frequency parameter
N_ζ, N_η	dimensionless parameters of inplane forces
R	constant
E_f	modulus of elastisity of face layers of sandwich plates

E_x, E_y	Moduli of elasticity of orthotropic material
G_{xy}	shear modulus of orthotropic material
w'	time dependent displacement in the z-direction
m, n, i, j	integer

CHAPTER I

INTRODUCTION

1.1 GENERAL

Rectangular plates are commonly used as components of building floors, foundations and bridge slabs. In general, the deflection of a plate in the above mentioned applications are generally small in comparsion with its thickness and a linear analysis is sufficient for the design of such structures. For aircrafts, missles and other aerospace structures, specialized laminated sandwich plates have recently found wide application. Sandwich plates are composed of two thin, stiff, strong sheets of high strength material separated by a thick layer of material with relative low average strength and density. The two thin sheets are termed face sheets or skins and the middle layer is called the core. The popularity of application of sandwich plates in the aerospace industry is due to the highly desirable thermal properties as well as the strength to weight ratio characteristics of such plates.

People are generally rather sensitive to vibrations and occupants of buildings or pedestrains on bridges become concerned for their safety when buildings or bridge structures vibrate. If the frequency of vibration matches the resonance frequency of structures, the structures may deflect excessively causing permanent damage to the structure. To avoid the plate from vibrating and deflecting excessively, a free vibration analysis should first be performed. For total design of a structure, all design limits of strength, deflection, vibration as well as stability must be investigated in order to ensure a high standard of performance of the structure.

The theoretical analysis of structures is generally expressed in the form of mathematical models and, for boundary-value problems, the governing differential equations of such mathematical models are usually rather complex, and exact solutions to these differental equation problems can only be obtained for very few simple cases. In most cases, it is almost impossible to find simple functions to satisfy both the boundary conditions and the governing differental equations. In the past twenty years, digital computers have enabled researchers to solve the differential equations by numerical methods. For design purposes where exact solutions are not available, engineers are generally satisfied with approximate numerical methods giving ± 10% of the exact solutions.

1.2 OBJECTIVE AND SCOPE

The main objective of this thesis is to investigate the application of Vlasov's method to overcome some of the difficulties encountered by the conventional Galerkin's method when applied to analyze static and dynamic plate problems. In this thesis, the Vlasov's method is applied to solve various static and dynamic plate problems with unsymmetric boundary conditions. The scope of this work covers the application of Vlasov's method to analyse the bending, vibration, and buckling of rectangular isotropic and orthotropic plates. To pursue the investigation further, the problems of bending and vibration of sandwich plates and bending of isotropic plates resting on elastic foundations are also studied.

1.3 OUTLINE OF THE THESIS

Since the problems studied in the thesis are related to the static and dynamic analysis of isotropic and orthotropic plates and sandwich plates, existing literature relating to the topic are briefly reviewed in chapter 2. In chapter 3, the Vlasov's algebric equations derived from the governing differential equation are formulated. Since Vlasov's method involves the evaluation of some definite integration, the trapezoidal rule of definite integration is used. In chapter 4, the governing differential equations for the bending, vibration and buckling of rectangular isotropic and orthotropic plates and bending and vibration of sandwich plates are investigated. In chapter 5, Vlasov's method is used to solve the small deflection of isotropic and orthotropic plates resting on elastic foundations. The same method is also applied to the small deflection problem of sandwich plates. In chapter 6, Vlasov's method is applied to analyse the free vibration of isotropic and orthotropic rectangular plates and sandwich plates. In chapter 7, the same method is further applied to solve the buckling problems of isotropic rectangular plates.

In the final chapter, some conclusions are drawn and summarized.

Numerical and graphical results of all the analyses are presented. Wherever possible, such results are compared with solutions obtained by other investigators. The computer programs are also included in the applendix.

CHAPTER II

REVIEW OF LITERATURE

The small deflection theory of plates is generally attributed to Kirchhoff, Love, Bernoulli and Navier [48]. In 1821, the french mathematician, Sophie Germain, obtained a differential equation for flexural vibration of isotropic plates using the calculus of variations. Lagrange corrected Germain's work by adding the missing terms and formulated the first correct differential equation of free vibration of plates.

The theories of sandwich plates were analysed by various authors. They have considered the effect of the core transverse deformability on the bending of sandwich plates and come to conclude that, except for very extreme conditions, this effect is negligible.

In 1947, William ,D.[58] obtained a simple approximate formula applicable to isotropic as well as orthotropic core of sandwich plates. He accounted for the transverse shear effects in the core of the sandwich plate by assuming that a linear element initially straight and normal to the middle plane of the core will remain straight after deformation. Reissner E. [45] investigated the general method in relation to isotropic panels with very thin faces. He concluded that the effect of the core flexibility in the vertical direction (z) is after all less important than effect of core shear deformation in the transverse planes. Reissner neglected the effect of direct transverse core strains and derived a relatively simple differential equation governing the small deflection of sandwich plates.

In 1951, Eringen A.C.[14] neglected the geometrical thickness of equal faces but included their local bending stiffness and the bending stiffness of the core and generalized the variational approach. The theory accounted for the flexural rigidity as well as transverse deformation of the core, including the flexural rigidities of the two faces about their own middle planes.

In 1955, Raville M.E.[44] applied the general method to the problem of a simply supported rectangular panel with a uniform transverse load.

In 1961, Falgout [15] derived the differential equations for free vibration of sandwich plates with isotropic facings and core by superposing bending deflections and deflections due to transverse shear.

An attempt is made here to review some of the research work which employs numerical schemes as a method of solution:

Ritz's method: Pikelt[42] used this method for the bending problem of uniformly loaded clamped rectangular plate. Young [60] and Maubetsch [37] used the Ritz method to find the solution to the vibration problems of isotropic plates with various boundary conditions. Hearmon [22] extended the Maubetsch treatment to find the solution of the vibration of rectangular orthotropic plates. Samuel Levy [34], Maubetsch [37] and Timoshenko [49] used the same technique to solve the buckling problems of clamped rectangular plates under compressive forces. March [35] used the Ritz method to solve the small deflection problems of the clamped rectangular sandwich plate.

Galerkin's Method: Galerkin's method was applied to the small deflection of clamped plates of various planforms resting on elastic foundations by Ng S.S.F.[40]. The method was applied to the free vibration of rectangular plates by Odman [41]. Munakata [39], used the method to analyze the free vibration and buckling problem of clamped rectangular plates. Bolton [5] used the method to find the large deflection problem of circular homogeneous plates with various boundary conditions.

Finite Element Method: Due to improvement in computing facilities in the past ten years, the finite element method has been widely used for plate problems. Tocher and Clough [8] summarized a variety of element stiffness matrices. Harty and Kapur [26] used the method to study the buckling of rectangular plates with clamped and simply supported boundary condition. Monforton [38] and Kwok [28] used the method for the static analysis of clamped skew sandwich plates. In applying this method, a large number of finite elements are often required to obtain sufficiently accurate answers.

Fourier Series Method : In 1820, Navier presented a paper to the French Academy of Science on the solution of bending of simply supported rectangular plates by double trigonometric series. He found that the convergence of the series is usually fast in the case of distributed load and slow for concentrated and discontinuous loads. In 1899, Levy [33] introduced a Single Fourier Series to solve plate problems with two opposite edges of the plate simply supported and various boundary conditions along the other two opposite sides. He found that the convergence of the results was extremely fast, even in the case of concentrated loads. Yen et al [59] used the method and solved the small deflection problems of a simply supported rectangular sandwich plate. Fletcher [15] applied the method to the corresponding vibration problem. Levy [34] used the same technique to solve the buckling of clamped rectangular plates under uniaxial compression.

Finite Difference Method : Szilard [48] solved a variety of plate problems by using this method. He has an extensive discussion of the method as he applied to the static and dynamic analysis of plates. Barton [4] used the method for the bending of the uniformly loaded rectangular and skew plates.

CHAPTER III

VLASOV'S METHOD

3.1 GENERAL

Some typical numerical methods that have been applied successfully to various plate problems by investigators in the past are :

a) Rayleigh-Ritz method: Ritz extended the Rayleigh method by including more than one parameters in the required shaped functions. Ritz's method is one of the most powerful energy methods. The Ritz's method is based on the principal of minimizing the total potential energy of a system when it is in stable equilibrium. In applying the method, an assumed shape function which satisfies the geometric boundary conditions for the plate is first chosen. Then, the unknown coefficients in the assumed function are obtained by minimizing the total potential energy of the system. In using this method, the differential equation need not be known . However, it is often difficult to select a complete displacement function which can satisfy the geometric boundary conditions of plates with either unsymmetric boundary or loading conditions. Also, when the number of undetermined parameters in the assumed solutions are increased, the amount of computational labour involved can be formidable.

b) Galerkin's Method: Galerkin's method is based on variational principles, much the same as in the Ritz method. Similar to the Ritz method, Galerkin's method also requires the selection of a shape function which satisfies the boundary conditions of the problem. Both Galerkin's method and the Ritz method require very tedious integration procedures and both suffer the drawback of not being applicable to nonsymmetric problems.

<u>c) Finite Element Method</u>: For the finite element method, a continuous system is divided into a large number of elements and all elements are connected at joints (nodal points) in such a way that they satisfy displacement compatibility as well as equilibrium conditions of the system. The most critical, and simultaneously the most difficult phase of the analysis is the evaluation of the element-stiffness coefficients. Fortunately, the stiffness properties of some of the more commonly used elements, which yield sufficiently accurate results, are readily available. Once the element stiffness coefficients have been determined, the analysis of the structural system follows the familiar procedure of matrix methods used in structural mechanics. The solution is obtained without the use of the governing differential equations, thus avoiding the mathematical analysis of the problem. Finite element method can be applied to any combinations of structural elements such as beams, plates, and shells and can be extended to cover virtually all fields of continuum mechanics.

<u>d) Fourier Series Method</u> : The Fourier Series is an essential instrument in the analytical treatment of many problems in the field of applied mechnanics. Once the governing differential equation of a problem is determined, a rigorous solution would invoive the adjusting of certain constants in order to satisfy the prescribed boundary conditions. Due to its ability to represent discontinuous loading, the Fourier Series has found application in numerous problems in structural mechanics. But it requires the use of electronic digital computers of considerable speed and storage capacity and it is difficult to ascertain the accuracy of the results when large structural systems are analyzed.

<u>e) Finite Difference Method</u>: The finite difference method is one of the most general methods among the numerical techniques presently available. It can be effectively applied to solve a wide variety of problems. Although the method has been known for a long time, it has gained considerable importance only after the invention of high-speed digital computers. In applying this method, the governing differential equations are replaced

by corresponding finite difference equations, which in turn yield a system of simultaneous algebraic equations.

The advantage of the methods are,

1) Simplicity in application

2) Versatility

3) It is easy to write complete programs to solve the numerical equations

4) Acceptable accuracy for most technical purpose with a relatively fine mesh.

Unfortunately, the method is characterized (beyond a certain mesh width) by slow convergence and a relatively fine mesh is generally required to obtain an acceptable accuracy. The accuracy deteriorates when the order of derivative is increased. Consequently, the method is not recommended when higher than fourth-order derivatives are involved or when high accuracy in the solution is required [48].

f) Navier Solution: Navier used double trigonometric series to express the lateral deflection and lateral load for the bending of simply supported rectangular plates and transformed the differential equation into an algebraic equation, thus considerably facilitating the required mathematical operations. Navier's method yields a mathematically exact solution for the bending of simply supported rectangular plates. The convergence of the resulting double fourier series depends considerably on the continuity of the loading functions. Slow convergence, created by discontinuous loading, is especially pronounced in the case of concentrated forces. For continuously distributed lateral loads, the convergence of Navier's method is satisfactory.

g) Levy's Solution: Levy used a single trigonometric series to solve plate problems with two opposite edges of the plate simply supported and various boundary conditions for the other two opposite sides and the shape of the loading function is the same for all sections parallel to the direction of the simply supported edges.

The convergence of Levy's solution is extremely fast, even in the case of concentrated or line loads. Because of the fast convergence of the solution in most cases, it is satisfactory to consider only the first few terms of the series.

The applicability of Levy's methods can be considerably extended by means of the superposition technique. That is, these results can provide a particular solution for the governing differential equation of the plate.

<u>h) Finite Strip Method</u>: The finite strip method can be regarded as a special form of the displacement formulation of the finite-element procedure in that it applies the minimum total potential energy theorem to develop the relationship between unknown nodal displacement parameters and the applied loading. Unlike the standard finite element method, which uses polynomial displacement functions in all directions, the finite strip method calls for use of simple polynomials in some directions and continuously differentiable smooth series in other directions.

The advantages of finite strip method are :

1) Usually a much smaller number of equations are required in comparison with the finite element method. This is especially true for problems with simply supported ends. Also much shorter computing time is required for solutions of comparable accuracy.

2) It requires very small amount of input data because of the small number of mesh lines involved due to the reduction in dimensional analysis.

3) It requires smaller amount of core and is easier to program. Because only the lowest few eigenfunction are required (for most cases), the first two to three terms of the series will normally yield sufficiently accurate results.

3.2 VLASOV'S METHOD

Vlasov's Method can be applied to obtained approximate solutions of complex problems in structural mechanics such as the static and dynamic analysis of plates and shells.

The critical phase of application of Vlasov' method lies in the choice of orthogonal functions. Once a satisfactory orthogal function is found, it can be used to obtain solutions to a large number of complex problems such as linear and non-linear analysis, vibration and buckling of isotropic and orthotropic plates and shells.

Consider a structural system in equilibrium, the sum of all the external and internal forces are zero. The equilibrium conditions of an infinitesimal element can be represented by the following differential equations :

$$\begin{aligned} L_1(u,v,w) - P_x &= 0 \\ L_2(u,v,w) - P_y &= 0 \\ L_3(u,v,w) - P_z &= 0 \end{aligned} \tag{3.2.1}$$

which describe the equilibrium of all forces in the X, Y, and Z directions, respectively. In the above equations, L_1, L_2, and L_3 are differential operators operating on the displacement functions, while P_x, P_y and P_z are external forces. The equilibrium of the structural system is obtained by integrating these differential equations over the entire structure.

Expressing the small arbitrary variations of the displacement functions by δu_i, δv_i, and δw_i and noting that although the displacement components are inter-related, their arbitrary variations are not inter-related, the virtual work of the external and internal forces,

$$\delta w_i + \delta w_e = \delta(w_i + w_e) = 0 \tag{3.2.2}$$

can be obtained directly from the differential equations of equilibrium without determining the actual potential energy of the system. Thus,

$$\int\int\int (L_1(u,v,w) - P_x)\delta u dV = 0$$
$$\int\int\int (L_2(u,v,w) - P_y)\delta v dV = 0$$
$$\int\int\int (L_3(u,v,w) - P_z)\delta w dV = 0 \qquad (3.2.3)$$

Strictly speaking, these variational equations are valid only if the displacement functions u, v, and w are the exact solutions of the problems under consideration. However, these equations will not be greatly violated if proper approximate expressions for the displacement functions are chosen and the variations carried out accordingly.

The total work performed by all these forces during a small virtual displacement δw in the Z-direction can be expressed by :

$$\int\int (L_3(u,v,w) - p_z(x,y))\delta w dx dy = 0 \qquad (3.2.4)$$

This equation represents the basic variational equation of plate bending.

The lateral deflection function can be expressed by an infinite series of the form :

$$w(x,y) = \sum_m \sum_n w_{mn}\phi_{mn}(x,y) \qquad (3.2.5)$$

similarly, the lateral load can be expressed as :

$$p_z(x,y) = \sum_m \sum_n p_{mn}\varphi_{mn}(x,y) \qquad (3.2.6)$$

These functions $\phi_{mn}(x,y)$ and $\varphi_{mn}(x,y)$ are the product of two functions, each of which depends on a single argument, that is,

$$\phi_{mn}(x,y) = X_m(x) * Y_n(y) \tag{3.2.7}$$

and

$$\varphi_{mn}(x,y) = X_m(x) * Y_n(y) \tag{3.2.8}$$

Thus, by separating the variables, the variational problem is reduced to the selection of two linearly independent sets of functions $X_m(x)$ and $Y_n(y)$, which satisfy all the boundary conditions. For these functions, Vlasov used the eigenfunctions of vibrating beams with identical boundary conditions as those of a plate.

The partial differential equations of the free vibrations of a single- span beam with uniform cross section is :

$$\frac{\partial^4 w(x,t)}{\partial x^4} = -\frac{\overline{\rho}}{EI} * \frac{\partial^2 w(x,t)}{\partial t^2} \tag{3.2.9}$$

where,

$w(x,t)$, is the time-dependent lateral deflection

$\overline{\rho}$ is the mass per unit length.

The solution is in the form :

$$w(x,t) = X(x) sin\omega t \tag{3.2.10}$$

where ω is the circular frequency of the free vibration beam.

Substitution of Equation (3.2.10) into Equation (3.2.9) results in,

$$\frac{d^4X(x)}{dx^4} = \bar{\rho}\frac{\omega^2}{EI}X(x) \tag{3.2.11}$$

By taking,

$$\frac{\lambda^4}{l^4} = \bar{\rho}\frac{\omega^2}{EI} \tag{3.2.12}$$

and substituting Equation (3.2.12) into Equation (3.2.11)

$$\frac{d^4X(x)}{dx^4} = \frac{\lambda^4}{l^4}X(x) \tag{3.2.13}$$

where :

λ is the shape parameter

l is the span length

The general solution of Equation (3.2.13) is :

$$X(x) = C_1 sin\frac{\lambda x}{l} + C_2 cos\frac{\lambda x}{l} + C_3 sinh\frac{\lambda x}{l} + C_4 cosh\frac{\lambda x}{l} \tag{3.2.14}$$

Where C_1, C_2, C_3 and C_4 are the constants which can be determined from the boundary conditions, λ is a root of the characteristic equation. This equation is derived by equating the determinants of the homogeneous equations to zero.

Consider, for example, a clamped-clamped beam for which the boundary conditions are $X(0) = X'(0) = X(l) = X'(l) = 0$. From the first two boundary conditions, it is found that

$$C_4 = -C_2, \quad C_3 = -C_1 \tag{3.2.15}$$

Substituting these relations into equation (3.2.14), and then applying the boundary conditions at $x = l$, results in,

$$C_1(sin\lambda - sinh\lambda) + C_2(cos\lambda - cosh\lambda) = 0 \tag{3.2.16}$$

$$C_1(cos\lambda - cosh\lambda) + C_2(-sin\lambda - sinh\lambda) = 0 \tag{3.2.17}$$

Equating to zero the determinant of these equations (3.2.16) and (3.2.17) yields the frequency equation,

$$cos\lambda cosh\lambda - 1 = 0 \tag{3.2.18}$$

The roots of equation (3.2.18) are found by numerical trial-and-error calculations. It can be shown that the first four roots are $\lambda_1 = 4.7300$, $\lambda_2 = 7.8532$, $\lambda_3 = 10.9956$, $\lambda_4 = 14.1372$, and that, for higher values of m, the roots are approximately, $\lambda_m = (2m+1)\pi/2$.

The eigenfunction $X_m(x)$, for the m^{th} mode is found by substituting λ_m into equation (3.2.14) and using the relations between equation (3.2.15) and either equation (3.2.16) or equation (3.2.17). Thus,

$$X_m(x) = cosh\frac{\lambda_m x}{l} - cos\frac{\lambda_m x}{l} - \alpha_m(sinh\frac{\lambda_m x}{l} - sin\frac{\lambda_m x}{l}) \tag{3.2.19}$$

Where $\alpha_m = -\frac{C_2}{C_1}$, C_1 and C_2 are defined in equation (3.2.14) and using equation (3.2.16), α_m is given by:

$$\alpha_m = \frac{(cosh\lambda_m - cos\lambda_m)}{(sinh\lambda_m - sin\lambda_m)}$$

The above solution for a clamped-clamped beam is given to illustrate the method of solution. Solutions for beams with other boundary conditions are found in a similar manner. A table of frequencies and eigenfunctions for uniform beams from reference [48] is shown on the next page.

The eigenfunctions and their second derivatives satisfy certain important mathematical relations. Let $X_m(x)$ and $X_n(x)$ be any two eigenfunctions of a vibrating beam of length l, corresponding to circular frequencies ω_m and ω_n respectively. Then for different modes ($m \neq n$), the following relations hold:

$$\int_0^l X_m(x) * X_n(x) dx = 0$$
$$\int_0^l X_m''(x) * X_n''(x) dx = 0 \tag{3.2.20}$$

Hence, the eigenfunctions and their second derivatives are said to be orthogonal for some boundary conditions. Strictly speaking, these functions are only quasi-orthogonal. These orthogonality conditions, however, do not hold for free and guided, or elastically supported edges. The same holds for their fourth-order derivatives, while the desirable property

$$\int_0^l X_m''(x) * X_n(x) dx = 0$$

which plays a role in the solution, is slightly violated.

A similar approach can be taken with the eigenfunctions to analyze column buckling problems. The governing differential equation of column buckling is :

$$EI \frac{d^4 w}{dx^4} + P \frac{d^2 w}{dx^2} = 0 \tag{3.2.21}$$

and by introducing an expression,

TYPE	BOUNDARY CONDITIONS	FREQUENCY EQUATION	EIGENFUNCTION $X_m(x)$	ROOTS OF FREQUENCY EQUATION λ_m
Clamped-clamped	$X(0) = X'(0) = 0$ $X(l) = X'(l) = 0$	$\cos\lambda\cosh\lambda = 1$	$J\left(\frac{\lambda_m x}{l}\right) - \frac{J(\lambda_m)}{H(\lambda_m)} H\left(\frac{\lambda_m x}{l}\right)$	$\lambda_1 = 4.7300$ $\lambda_2 = 7.8532$ $\lambda_3 = 10.9956$ $\lambda_4 = 14.1372$ For m large, $\lambda_m \approx (2m+1)\pi/2$
Clamped-hinged	$X(0) = X'(0) = 0$ $X(l) = X''(0) = 0$	$\tan\lambda = \tanh\lambda$	$J\left(\frac{\lambda_m x}{l}\right) - \frac{J(\lambda_m)}{H(\lambda_m)} H\left(\frac{\lambda_m x}{l}\right)$	$\lambda_1 = 3.9266$ $\lambda_2 = 7.0686$ $\lambda_3 = 10.2102$ $\lambda_4 = 13.3518$ For m large, $\lambda_m \approx (4m+1)\pi/4$
Clamped-free	$X(0) = X'(0) = 0$ $X''(l) = X'''(l) = 0$	$\cos\lambda\cosh\lambda = -1$	$J\left(\frac{\lambda_m x}{l}\right) - \frac{G(\lambda_m)}{F(\lambda_m)} H\left(\frac{\lambda_m x}{l}\right)$	$\lambda_1 = 1.8751$ $\lambda_2 = 4.6941$ $\lambda_3 = 7.8548$ $\lambda_4 = 10.9955$ For m large, $\lambda_m \approx (2m-1)\pi/2$
Clamped-guided	$X(0) = X'(0) = 0$ $X'(l) = X'''(l) = 0$	$\tan\lambda = -\tanh\lambda$	$J\left(\frac{\lambda_m x}{l}\right) - \frac{F(\lambda_m)}{J(\lambda_m)} H\left(\frac{\lambda_m x}{l}\right)$	$\lambda_1 = 2.3650$ $\lambda_2 = 5.4978$ $\lambda_3 = 8.6394$ $\lambda_4 = 11.7810$ For m large, $\lambda_m \approx (4m-1)\pi/4$
Hinged-hinged	$X(0) = X''(0) = 0$ $X(l) = X''(l) = 0$	$\sin\lambda = 0$	$\sin\frac{m\pi x}{l}$	$\lambda_m = m\pi$
Hinged-guided	$X(0) = X''(0) = 0$ $X'(l) = X'''(l) = 0$	$\cos\lambda = 0$	$\sin\frac{(2m-1)\pi x}{2l}$	$\lambda_m \approx (2m-1)\pi/2$
Guided-guided	$X'(0) = X'''(0) = 0$ $X'(l) = X'''(l) = 0$	$\sin\lambda = 0$	$\cos\frac{m\pi x}{l}$	$\lambda_m = m\pi$
Free-free	$X''(0) = X'''(0) = 0$ $X''(l) = X'''(l) = 0$	$\cos\lambda\cosh\lambda = 1$	$G\left(\frac{\lambda_m x}{l}\right) - \frac{J(\lambda_m)}{H(\lambda_m)} F\left(\frac{\lambda_m x}{l}\right)$	Same as for clamped-clamped beam
Free-hinged	$X''(0) = X'''(0) = 0$ $X(l) = X''(l) = 0$	$\tan\lambda = \tanh\lambda$	$G\left(\frac{\lambda_m x}{l}\right) - \frac{G(\lambda_m)}{F(\lambda_m)} F\left(\frac{\lambda_m x}{l}\right)$	Same as for clamped-hinged beam
Free-guided	$X''(0) = X'''(0) = 0$ $X'(l) = X'''(l) = 0$	$\tan\lambda = -\tanh\lambda$	$G\left(\frac{\lambda_m x}{l}\right) - \frac{H(\lambda_m)}{F(\lambda_m)} F\left(\frac{\lambda_m x}{l}\right)$	Same as for clamped-guided beam

(1) The circular frequency is

$$\omega_m = \frac{\lambda_m^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

where
EI = bending stiffness
$\bar{m}$ = mass per unit length
l = length of beam

(2) Notation used in expressions for the eigenfunctions:

$F(u) = \sinh u + \sin u$
$G(u) = \cosh u + \cos u$
$H(u) = \sinh u - \sin u$
$J(u) = \cosh u - \cos u$
$u = \frac{\lambda_m x}{l}$

$$\left(\frac{\lambda}{l}\right)^2 = \frac{P}{EI} \tag{3.2.22}$$

and by substituting Equation (3.2.22) into Equation (3.2.21) ,

$$\frac{\partial^4 X(x)}{\partial x^4} + \left(\frac{\lambda}{l}\right)^2 \frac{\partial^2 X(x)}{\partial x^2} = 0 \tag{3.2.23}$$

and the general solution of equation (3.2.23) is :

$$X_m(x) = C_1 sin\frac{\lambda_m x}{l} + C_2 cos\frac{\lambda_m x}{l} + \frac{C_3 x}{l} + C_4 \tag{3.2.24}$$

Where C_1, C_2, C_3 and C_4 are the constants which can be found from the boundary conditions of the column.

λ_1, λ_2, λ_3, λ_4... are the roots of characteristic equation (3.2.24) for the first , second, third, fourth, etc., buckling modes.

These functions are also quasi-orthogonal. For $m \neq n$, they satisfy the following conditions :

$$\int_0^l X_m''''(x) X_n(x)\, dx = 0$$
$$\int_0^l X_m'''(x) X_n(x)\, dx = 0 \tag{3.2.25}$$

while

$$\int_0^l X_m(x) X_n(x)\, dx \not\approx 0 \qquad for \quad m \neq n \tag{3.2.26}$$

The violation of the orthogonality requirement in equation (3.2.26) is again of negligible order of magnitude.

CHAPTER IV

DIFFERENTIAL EQUATION OF PLATES

The classicial theory of bending of a thin elastic plate expresses the relation between the transverse deflection of middle surface of the plate w and the lateral loading q. When the deflection of the thin plate is small compared with the thickness, the membrance effect on the curvature may be neglected. The governing differential equation of the thin plate undgoing small deflection is well known [48], [51]. The differential equations governing the small deflection problem of rectangular plates on the elastic foundations, the deflections of the sandwich plates, the vibration of rectangular and sandwich plates and the buckling of rectangular plates are presented in this chapter. The basic assumptions governing the validity of these equations are first briefly stated.

4.1 BASIC ASSUMPTIONS FOR THE SMALL DEFLECTION OF PLATES

1) The deflection is small in comparison to the plate thickness. The limit for the deflection is usually one-tenth to one-fifth of the thickness of the plates.

2) Stress normal to the mid-plane of the plate arising from the applied loading is negligible in comparison with the stresses in the plane of the plates.

3) The mid-plane stresses arising from the deflection of the plate can be neglected, i.e. the middle surface can be regarded as a neutral plane.

4) The deformations are such that straight lines, initially normal to the middle surface, remain straight lines and normal to the middle surface; i.e., deformation due to transverse shear will be neglected.

5) The slope of the middle surface of the plate is small compared to unity.

6) The thickness of the plate is small compared with its other dimensions. The lateral dimension of the plate is at least ten times larger than its thickness.

4.2 THE BENDING OF RECTANGULAR PLATES ON ELASTIC FOUNDATIONS

Consider a thin elastic plate of arbitrary planform resting on a supporting medium that is isotropic, homogeneous and linearly elastic (Winkler-type foundation). Adopting a rectangular cartesian coordinate system with the origin located at some arbitrary point at the corner of plate, let the axes of the principal stiffness coincide with the x and y directions. Applying an arbitrary distributed load q(x,y) acting normal to the plane of the plate will cause a displacement in the vertical (z) direction. The governing differential equation of the out-of plane displacement, w can be expressed as [48] :

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + kw = q \tag{4.2.1}$$

where,

k is the modulus of elastic foundation

$D_x = E_x h^3/12(1 - \nu_x \nu_y)$

$D_y = E_y h^3/12(1 - \nu_x \nu_y)$

$H = D' + 2D_{xy}$

$D' = \nu_x D_y = \nu_y D_x$

$D_{xy} = G_{xy} h^3/12$

E_x = modulus of elasticity in X-direction

E_y = modulus of elasticity in Y-direction

G_{xy} = shear modulus in X-Y plane

ν_x = ratio of strain in the Y-direction to strain in X-direction due to uniaxial stress in the X-direction

ν_y = ratio of strain in the X-direction to strain in Y-direction

due to uniaxial stress in the Y-direction .

4.3 BENDING OF SANDWICH PLATES

Sandwich plate is defined as a three layers type of construction , consisting of two thin, stiff, strong sheets of high strength material separated by a thick layer of material of low average strength and density. The two thin sheets are called the faces or skins and the intermediate layer is called the core. An efficient sandwich plate is obtained when the weight of the core is roughly equal to the combined weight of the faces and generally, sandwich plates are light-weight structures with high outstanding strength and stiffness characteristics. Such structures are particularly useful in the aerospace industry.

In sandwich plate construction, the materials of the faces are usually made of a light metal with high strength such as aluminum alloy, reinforced plastic, titanum, or heat resistant steel. For the core materials, the materials and the geometric shape can vary greatly. A popular type of the core material is 'honeycomb' which consists of thin foils in the form of hexagonal cells perpendicular to the faces. Other types of core materials are corrugated sheets, with the corrugation running parallel to the faces. The main purposes for the core are to ensure that the faces are the correct distance apart and the cores must also be stiff enough in shear so as to ensure that when the panel is bent, the faces do not slide over each other.

The elastic modulus of the core materials is usually much smaller than the facing material. Therefore, the core material contributes very little in resisting the bending in comparison with the facings even though the usual ratio of the thickness of the facing to the core materials is about one-tenth to one-hundredth.

THE GOVERNING DIFFERENTIAL EQUATION OF BENDING OF SANDWICH PLATES

The governing coupled non-linear differential equations were first formulated by Reissner [45]. Reissner derived the two coupled equations by considering the equilibrium and compatibility of an infinitesimal element of the sandwich plate. The two coupled non-linear equations are as follows :

$$\nabla^2\nabla^2 F = 2tE_f\left(\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right) \tag{4.3.1}$$

$$\begin{aligned} D\nabla^2\nabla^2 w = &\left(1 - \frac{thE_f}{2(1-\nu_f^2)G_c}\nabla^2\right) \\ &\left(q + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right) \end{aligned} \tag{4.3.2}$$

where,

∇ = Laplacian operator

F = membrance stress function

w = out of plane displacement

D = flexural rigidity of the sandwich plate

$= th^2E_f/2(1-\nu_f^2)$

E_f = modulus of elasticity of facing material

t = thickness of the facings

h = the thickness of the sandwich plate

ν_f = poisson's ratio of facing material

G_c = modulus of core material

q = lateral load

When the sandwich plate deflection to thickness ratio (w/h) is small, the sandwich plate can be analysed by the linear theory . In the present study, only small deflection to thickness ratio (w/h) is investigated.

The differential equation governing the small deflection of sandwich plates can be obtained by dropping the non-linear terms in the Reissner's equations (4.3.1) and equation (4.3.2), the resulting expression is :

$$\nabla^2\nabla^2 w = \frac{q}{D} - \frac{1}{S}\nabla^2 q \tag{4.3.3}$$

where,

$$S = hG_c$$

4.4 THE GOVERNING DIFFERENTIAL EQUATION FOR THE FREE VIBRATION OF RECTANGULAR PLATES

The governing differential equation of free vibration of rectangular plates can be expressed in the same rectangular cartesian coordinates system and is expressed as follows [48] :

$$D_x\frac{\partial^4 w'}{\partial x^4} + 2H\frac{\partial^4 w'}{\partial x^2\partial y^2} + D_y\frac{\partial^4 w'}{\partial y^4} - \rho\frac{\partial^2 w'}{\partial t^2} = 0 \tag{4.4.1}$$

where,

$w' =$ $w'(x, y, t)$ is the time dependent displacement function

$t =$ time variable

$\rho =$ mass per unit area of the plate.

For free vibration, the motion of the plate is assumed to be harmonic, we can write,

$$w'(x,y,t) = w(x,y) sin\omega t \qquad (4.4.2)$$

where ω is the circular frequency of the motion.

Substituting equation(4.4.2) into equation (4.4.1) results in,

$$D_x \frac{\partial^4 w(x,y)}{\partial x^4} + 2H \frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w(x,y)}{\partial y^4} - \rho\omega^2 w(x,y) = 0 \qquad (4.4.3)$$

4.5 THE GOVERNING DIFFERENTIAL EQUATION FOR FREE VIBRATION OF SANDWICH PLATES

The well-known governing differential equation for free vibration of rectangular sandwich plates was derived by Falgout[14] who obtained the differential equation by superposing the bending deflection and the deflection due to transverse shear. The governing differential equation is as follows :

$$\frac{\partial^4 w'}{\partial x^4} + 2\frac{\partial^4 w'}{\partial x^2 \partial y^2} + \frac{\partial^4 w'}{\partial y^4} = \rho\left(\frac{1}{u'}\frac{\partial^4 w'}{\partial x^2 \partial t^2} + \frac{1}{u'}\frac{\partial^4 w'}{\partial y^2 \partial t^2} - \frac{1}{D'}\frac{\partial^2 w'}{\partial t^2}\right) \qquad (4.5.1)$$

Where,

$\rho =$ mass per unit area

$u' = G_c C$

$$D' = \frac{E_f}{(1-\nu_f^2)}(t^3/6 + th^2/2)$$

$w' = w(x,y,t)$

Assuming the sandwich plate vibrates in harmonic motion, we can write,

$$w'(x,y,t) = w(x,y) sin\omega t \tag{4.5.2}$$

Substitution of equation (4.5.2) into equation (4.5.1) results in,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{\rho\omega^2}{D'}\left(-\frac{D'}{u'}(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) + w\right) \tag{4.5.3}$$

4.6 THE GOVERNING DIFFERENTIAL EQUATION FOR STABILITY OF PLATES

The governing differential equation for buckling of elastic plates subjected to forces acting in the plane of plate can be expressed as follows [48] :

$$D_x\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w}{\partial y^4} = N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2} \tag{4.6.1}$$

Where,

$N_x =$ normal force in the X-direction per unit length

$N_y =$ normal force in the Y-direction per unit length

$N_{xy} =$ shear force in the X-Y plane per unit length.

CHAPTER V

BENDING OF PLATES

In this chapter, Vlasov's Method is used to analyse uniformly loaded rectangular isotropic plates with various boundary conditions resting on elastic foundations, rectangular orthotropic plates and the uniformly loaded rectangular sandwich plates by using the governing differential equations (4.2.1) and (4.3.3).

5.1 RECTANGULAR ISOTROPIC PLATES RESTING ON AN ELASTIC FOUNDATION

The differential equation governing the bending of a plate resting on an elastic foundations was shown in equation (4.2.1). For the isotropic plate, the flexural rigidity of the plate is the same in X and Y directions. Substituting equation (3.2.5) , equation (3.2.6) and equation (4.2.1) into equation (3.2.4) results in,

$$
\begin{aligned}
&D\sum_m\sum_n w_{mn}\int_0^a\int_0^b \phi_{ik}\nabla^4\phi_{mn}\,dxdy \\
&-\sum_m\sum_n q_{mn}\int_o^a\int_0^b \phi_{ik}\varphi_{mn}\,dxdy \\
&+k\sum_m\sum_n\int_0^a\int_0^b \phi_{ik}\phi_{mn}\,dxdy = 0 \qquad (5.1.1)
\end{aligned}
$$

Substituting equation (3.2.7) and equation (3.2.8) into equation (5.1.1), the variational equation (5.1.1) can be expressed as :

$$
\begin{aligned}
& D\int_0^a\int_0^b [X_m''''(x)Y_n(y)X_i(x)Y_k(y) + 2X_m''(x)Y_n''(y)X_i(x)Y_k(y) \\
& \quad + Y_n''''(y)X_m(x)X_i(x)Y_k(y)]dxdy \\
& - \int_0^a\int_0^b q_{mn}X_m(x)Y_n(y)X_i(x)Y_k(y)dxdy \\
& \quad + k\int_0^a\int_0^b X_m(x)Y_n(y)X_i(x)Y_k(y)dxdy = 0
\end{aligned} \tag{5.1.2}
$$

Where,

$$q_{mn} = \frac{\int_0^a\int_0^b q_z(x,y)X_m(x)Y_n(y)dxdy}{\int_0^a\int_0^b X_m^2(x)Y_n^2(y)dxdy}$$

By neglecting the terms with non-identical subscripts mi and nk, the error induced is zero or negligible. By introducing the following notations :

$$
\begin{aligned}
I_1 &= \int_0^a X_m''''(x)X_m(x)dx \\
I_2 &= \int_0^b Y_n(y)Y_n(y)dy \\
I_3 &= \int_0^a X_m''(x)X_m(x)dx \\
I_4 &= \int_0^b Y_n''(y)Y_n(y)dy \\
I_5 &= \int_0^b Y_n''''(y)Y_n(y)dy \\
I_6 &= \int_0^a X_m(x)X_m(x)dx
\end{aligned} \tag{5.1.3}
$$

and by substituting equation (5.1.3) into the first term of equation (5.1.2) results in,

$$\int_0^a\int_0^b \phi_{ik}\nabla^4\phi_{mn}dxdy = I_1I_2 + 2I_3I_4 + I_5I_6 \tag{5.1.4}$$

Similarly, by introducing the following notations,

$$I_7 = \int_0^a X_m^2(x)dx$$
$$I_8 = \int_0^b Y_n^2(y)dy \tag{5.1.5}$$

and substituting equation (5.1.5) into the second term of equation (5.1.2) results in,

$$\int_0^a \int_0^b q_{mn} X_m(x) Y_n(y) X_i(x) Y_k(y) dx dy = q_{mn} I_7 I_8 \tag{5.1.6}$$

For a particular set of m, n, the variational equation of the rectangular isotropic plate resting on an elastic foundations can be reduced to :

$$D w_{mn}[I_1 I_2 + 2 I_3 I_4 + I_5 I_6 + k I_2 I_6] = q_{mn} I_7 I_8 \tag{5.1.7}$$

Consequently, the undetermined expansion coefficients w_{mn} can be calaulated from

$$w_{mn} = \frac{q_{mn} I_7 I_8}{D[I_1 I_2 + 2 I_3 I_4 + I_5 I_6 + k I_2 I_6]} \tag{5.1.8}$$

For ease of computation, the above definite integrals are transformed into non dimensional form: Let

$$\lambda = a/b, \qquad \varsigma = x/2a, \qquad \eta = y/2b$$

$$W = w/h, \qquad Q = qa^4/Dh, \qquad K = ka^4/D$$

Substituting of the above dimensionless ratios into equations (5.1.8) results in,

$$W_{mn} = \frac{QI_7I_8}{(I_1I_2 + 2\lambda^2 I_3I_4 + \lambda^4 I_5I_6 + KI_2I_6)} \tag{5.1.9}$$

I. THE DEFLECTION OF A CLAMPED RECTANGULAR ISOTROPIC PLATE RESTING ON AN ELASTIC FOUNDATIONS

The coordinate system for the rectangular plate is shown in figure 1.

The boundary conditions are :

in X-direction,

$$w(0) = w'(0) = 0, \qquad w(2a) = w'(2a) = 0 \tag{5.1.10}$$

in Y-direction,

$$w(0) = w'(0) = 0, \qquad w(2b) = w'(2b) = 0 \tag{5.1.11}$$

By selecting the eigenfunctions of $X_m(x)$ and $Y_n(y)$ to satisfy the above boundary conditions. $X_m(x)$, $Y_n(y)$ are chosen as follows :

$$\begin{aligned} X_m(x) = & \left(cosh\frac{\lambda_m x}{2a} - cos\frac{\lambda_m x}{2a}\right) \\ & - \left(\frac{cosh\lambda_m - cos\lambda_m}{sinh\lambda_m - sin\lambda_m}\right) \text{x} \left(sinh\frac{\lambda_m x}{2a} - sin\frac{\lambda_m x}{2a}\right) \end{aligned} \tag{5.1.12}$$

$$(m = 1, 3, 5, 7, ...)$$

$$\begin{aligned} Y_n(y) = & \left(cosh\frac{\lambda_n y}{2b} - cos\frac{\lambda_n y}{2b}\right) \\ & - \left(\frac{cosh\lambda_n - cos\lambda_n}{sinh\lambda_n - sin\lambda_n}\right) \text{x} \left(sinh\frac{\lambda_n y}{2b} - sin\frac{\lambda_n y}{2b}\right) \end{aligned} \tag{5.1.13}$$

$$(n = 1, 3, 5, 7, ...)$$

where,

$\lambda_1 = 4.7300$

$\lambda_2 = 7.8532$

$\lambda_3 = 10.9956$

$\lambda_4 = 14.1372$

For large values of m, n,

λ_m or $\lambda_n \approx (2m+1)\pi/2$

II. THE DEFLECTION OF ONE SIDE CLAMPED AND THREE SIDES SIMPLY SUPPORTED RECTANGULAR ISOTROPIC PLATE RESTING ON AN ELASTIC FOUNDATIONS

The coordinate system for the rectangular plate is shown in figure 2.

The boundary conditions are :

In X-direction ,

$$w(0) = w''(0) = 0$$
$$w(2a) = w''(2a) = 0 \tag{5.1.14}$$

In Y-direction,

$$w(0) = w'(0) = 0$$
$$w(2b) = w''(2b) = 0 \tag{5.1.15}$$

By selecting the eigenfunctions of $X_m(x)$ and $Y_n(y)$ to satisfy the above boundary conditions. $X_m(x)$ and $Y_n(y)$ are chosen as follows :

In X-direction (ss-ss side),

$$X_m(x) = sin\frac{m\pi x}{2a} \tag{5.1.16}$$

$$(m = 1, 3, 5..........)$$

In Y-direction (c-ss side) ,

$$Y_n(y) = sin\frac{\lambda_n y}{2b} - \frac{sin\lambda_n}{sinh\lambda_n} sinh\frac{\lambda_n y}{2b} \tag{5.1.17}$$

where,

$\lambda_1 = 3.9266$

$\lambda_2 = 7.0688$

$\lambda_3 = 10.2102$

$\lambda_4 = 13.3518$

For large values of n,

$\lambda_n \approx (4n + 1)\pi/4$

III THE DEFLECTION OF SIMPLY SUPPORTED RECTANGULAR ISOTROPIC PLATES RESTING ON AN ELASTIC FOUNDATIONS

The coordinate system for the rectangular plate geometry is shown in figure 3.

The boundary conditions are :

In X-direction,

$$w(0) = w''(0) = 0$$

$$w(2a) = w''(2a) = 0 \tag{5.1.18}$$

In Y-direction,

$$w(0) = w''(0) = 0$$

$$w(2b) = w''(2b) = 0 \qquad (5.1.19)$$

By selecting the eigenfunctions of $X_m(x)$ and $Y_n(y)$ to satisfy the above boundary conditions, $X_m(x)$ and $Y_n(y)$ are chosen as follows :

In X-direction (ss-ss side),

$$X_m(x) = sin\frac{m\pi x}{2a} \qquad (5.1.20)$$

$$(m = 1, 3, 5.......)$$

In Y-direction (ss-ss side),

$$Y_n(y) = sin\frac{n\pi y}{2b} \qquad (5.1.21)$$

$$(n = 1, 3, 5.......)$$

COMPARISON AND DISCUSSION OF RESULTS

The solutions of the deflections of all sides clamped, one side fixed and three sides simply supported and simply supported rectangular isotropic plates resting on elastic foundations are obtained with aspect ratios from 1/2 to 1 and the dimensionless foundation modulus varying from 0 to 200. A convergence study has been made using different number of terms for solving the same aspect ratio for the case of zero foundation modulus. The number of terms used are 1,4,9 and 16. The results are given in table 1, 3 and 5.

Comparing the results obtained in tables 1, 3 and 5 with those obtained by Timoshenko[51], where much more laborious computation methods are used to analyse the same problem, it can be seen that the present solutions are in an excellent agreement with the values reported by Timoshenko. The maximun deviation is less than 1.8 %.

From table 1, 3 and 5, it can be seen that the solutions obtained by using 9 terms and 16 terms deviated less than 2%. Consequently, the remaining results of rectangular plates on elastic foundations are solved using 16 terms.

In general, the results are reasonably consistent though slight deviations are observed for very small aspect ratios. This discrepancy may be due to the inability of the assumed displacement functions to represent the actual deflected shape of a plate when the aspect ratios are small.

The maximun center deflection for various foundations moduli and aspect ratios are given in table 2, 4 and 6. Plots of the maximun small deflections vs the plate aspect ratios for various foundation moduli are shown in figure 5, 6 and 7. Results obtained by Ng S.F.[39] are drawn for comparison in figure 5. From figure 5, the present results are slightly greater than those obtained by Ng S.F. The slight overestimation of the plate stiffness may be due to the limited number of terms used in the present solutions.

From this investigation, the following results are observed,

1) From figure 5, it can be seen that Vlasov's method provides accurates results which are comparable to those obtained by much more lengthy computational methods. Although the number of terms used in Vlasov's method is only 16, it provides sufficiently acccurate results. No comparison of the results for deflection of rectangular plates with one side clamped and three sides simply supported resting on an elastic foundation is made as no results by other investigators are readily available in the technical literature.

2) From table 2, 4 and 6, it can be seen that for any aspect ratio, the maximum center deflection of the plate decreases with increasing values of the foundation modulus. This is

expected because the object of the elastic foundation is to reduce the lateral pressure.

3) The effectiveness of the elastic foundation in reducing the maximum center deflection of the plate is more pronounced for small aspect ratios than it is for aspect ratios approaching unity. This is because the deflection of plates with small aspect ratios are greater than those with aspect ratios approaching one, and since the foundation reaction is proportional to the deflection, the reduction in deflections is more significant for long rectangular plates than for plates approaching a square planform.

4) From table 2, 4 and 6, it can be seen that the deflections of the simply supported plates on the elastic foundations are much higher than those of clamped plates for very low elastic foundation support. But, when the foundation modulus K approaches 200, the deflections of the plates with the two different types of boundary conditions become essentially the same. This is so because, when K approaches to higher values, the denominator of equation (5.1.9) for two different boundary conditions approach closely.

5) Vlasov's Method may also be applied to unsymmetric plate deflection problems such as plates with one side clamped and three sides simply supported. In contrast, it would be very difficult to select a complete deflection function to meet the geometric boundary conditions when using the Galerkin's method.

5.2 BENDING OF SIMPLY SUPPORTED RECTANGULAR ORTHOTROPIC PLATES

Anisotropic plates are plates in which resistance to mechanical action is different for different directions. Examples of anisotropic plates are wood, delta wood, plywood and fibre-reinforced plastic. Such materials have found wide applications in modern technology.

The well-known governing differential equations for orthotropic plates subjected to lateral loads $q_z(x,y)$ can be expressed similar to equation (4.2.1) by dropping the foundation modulus term,

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q_z(x, y) \tag{5.2.1}$$

For advantage of computation and comparison, it is convenient to convert the differential equation (5.2.1) into non-dimensional form: Let

$$H = \sqrt{D_x D_y}, \qquad \epsilon = a/b \sqrt[4]{D_x/D_y}$$

$$\varsigma = x/2a, \qquad \eta = y/2b, \qquad W = w/h$$

$$Q = \frac{qb^4}{D_y h}$$

Substituting the resulting expression into the equation (3.2.4) and using the the notations (5.1.3) results in,

$$W_{mn} = \frac{Q I_7 I_8}{(I_1 I_2 + 2\epsilon^2 I_3 I_4 + \epsilon^4 I_5 I_6)} \tag{5.2.2}$$

The coordinate system for the rectangular geometry is shown in figure 3.

The boundary conditions are,

In X-direction,

$$w(0) = w''(0) = 0$$
$$w(2a) = w''(2a) = 0 \tag{5.2.3}$$

In Y-direction,

$$w(0) = w''(0) = 0$$

$$w(2b) = w''(2b) = 0 \tag{5.2.4}$$

By selecting the eigenfunctions of $X_m(x)$ and $Y_n(y)$ to satisfy the above boundary conditions. $X_m(x)$ and $Y_n(y)$ are chosen as follows :

In X-direction (ss-ss side)

$$X_m(x) = sin\frac{m\pi x}{2a} \tag{5.2.5}$$

$$(m = 1, 3, 5.......)$$

In Y-direction (ss-ss side)

$$Y_n(y) = sin\frac{n\pi y}{2b} \tag{5.2.6}$$

$$(n = 1, 3, 5.......)$$

COMPARISON AND DISCUSSION OF RESULTS

Results of the analysis are shown on table 7. The present solutions are in excellent agreement with the accurate values reported by Timoshenko[51] and Lekhnitskii[32]. From table 7, it can be seen that as the value ϵ increases, the deflection coefficient increases correspondingly. The present results are slightly below those reported by Timoshenko. The maximum percentage difference between Timoshenko and the present results is less than 1%. If the present solution is in error at all, the error is on the safe side.

It can be further seen that Vlasov's method provides accurate results which are comparable to those obtained by much more lengthy computational methods.

5.3 BENDING OF ALL SIDES CLAMPED RECTANGULAR SANDWICH PLATE

The coordinate system for the sandwich plate geometry is shown in figure 1 and 4. The differential equation governing for sandwich plate is equation (4.3.1) and equation (4.3.2). By considering the linear analysis of a all sides clamped sandwich plate subjected to a uniformly distributed load with intensity q, the governing differential equation can be obtained by dropping the non-lineat terms in equation (4.3.1) and equation (4.3.2). The resulting expression is equation (4.3.3).

Substituting equation (3.2.5), (3.2.6) and (4.3.3) into the variational equation (3.2.4) and using the notations (5.1.3) results in,

$$w_{mn} = q_{mn}\frac{[I_7 I_8 - thE_f(I_2 I_3 + I_4 I_6)/2(1-\nu_f^2)G_c]}{D(I_1 I_2 + 2I_3 I_4 + I_5 I_6)} \tag{5.3.1}$$

For ease of computation, equation (5.3.1) is converted into dimensionless form with the following dimensionless ratios :

$$\lambda = a/b, \qquad \varsigma = x/2a, \qquad \eta = y/2b$$

$$W = w/h, \qquad \mu = t/a \qquad \theta = h/a$$

$$Q = qa^4/Dh = 2(1-\nu_f^2)qa^4/Eth^3$$

Substituting to equation (5.3.1) results in,

$$W_{mn} = Q\left(I_7 I_8 - \frac{\mu\theta}{2(1-\nu_f^2)G_c 2^2}(I_2 I_3 + I_4 I_6)\right)$$
$$\text{x}\left(\frac{1}{I_1 I_2 + 2\lambda^2 I_3 I_4 + \lambda^4 I_5 I_6}\right) \qquad (5.3.2)$$

The following numerical examples are considered in the linear analysis for the purpose of comparison of results :

Plate no. 1

$E_f = 10\text{x}10^6 psi, \qquad G_c = 500 psi$

$\nu_f = 0.32, \qquad \mu = 0.00125$

$\theta = 0.05125$

Plate no. 2

$E_f = 10\text{x}10^6 psi, \qquad G_c = 100,000 psi$

$\nu_f = 0.32, \qquad \mu = 0.00125$

$\theta = 0.05125$

Plate no. 3

$E_f = 10.5\text{x}10^6 psi, \qquad G_c = 50,000 psi$

$\nu_f = 0.30, \qquad \mu = 0.0006$

$\theta = 0.04$

The boundary conditions for the all sides clamped sandwich plates are the same as equation (5.1.10) and (5.1.11). The eigenfunctions of $X_m(x)$ and $Y_n(y)$ are the same as equation (5.1.12) and (5.1.13)

COMPARISON AND DISCUSSION OF RESULTS

Solutions are obtained for sandwich plate of aspect ratio ranging from 1/2 to 1. The convegence study has been made using different number of terms for solving the same aspect ratio. The number of terms used are 1,4,9,16 and 25. The convergence of the results can be seen as the number of terms is increased. There are no significant difference in results by using 16 and 25 terms. Consequently, the remaining results of sandwich plates are solved using 25 terms. The results are given in table 8 to table 13.

The present solutions of sandwich plates are compared with the solutions reported by March[35]. The first two sandwich plate problems were solved by Monforton et al.[38] for a square size using finite element method. The ~~solutions~~ solving by March consists of two formulae, one for $0.7 \leq b/a \leq 1.4$ and the other for large values of b/a. Therefore, comparison with ref.[35] is limited to plates of aspect ratios greater than 2/3. It can be seen that the results of the present solutions are generally comparable with those obtained by March [35].

For the first two problems solved by Monforton et al.[38] the present results are in excellent agreement with those obtain by the accurate finite element methods. The results for plate no. 3 are also in good agreement with the solutions by Kan et al [24].

By comparing with the three sandwich plates results, it can be seen that the deflection of the sandwich plates decreases with the increases in the shear rigidity, G_c, or E_f. As is expected, it is found that the more rigid is the core, the less is the deflection.

From the present study, it can be further seen that Vlasov's method is found to be powerful and at the same time very simple to apply for the deflections of sandwich plates.

CHAPTER VI

VIBRATION OF PLATES

In this chapter, Vlasov's method is used to analyse the free vibration problems of rectangular isotropic and orthotropic plates. The method is further applied to the free vibration of sandwich plates. The governing differential equations are given in chapter IV (4.4).

6.1 THE FREE VIBRATION OF RECTANGULAR ISOTROPIC PLATES

The differential equation governing for the free vibration of a rectangular plate is shown in equation (4.4.3). For an isotropic plate, the flexural rigidities of the plate are the same in both the X and Y directions. Substituting equation (4.4.3) into equation (3.2.4) and using notations (5.1.3):

$$\omega_{mn} = \sqrt{\frac{I_1 I_2 + 2I_3 I_4 + I_5 I_6}{I_7 I_8}}\sqrt{\frac{D}{\rho}} \tag{6.1.1}$$

For computational advantage, equation (6.1.1) is transformed into dimensionless forms using the following dimensionless ratios :

$$\varsigma = x/2a, \qquad \eta = y/2b, \qquad \lambda = a/b$$

$$F = \omega^2 a^4 \rho / D$$

Substituting these dimensionless ratios into equation (6.1.1) results in,

$$F = \frac{I_1 I_2 + 2\lambda^2 I_3 I_4 + \lambda^4 I_5 I_6}{I_7 I_8} \tag{6.1.2}$$

I THE FREE VIBRATION OF A CLAMPED RECTANGULAR ISOTROPIC PLATE

The coordinate system for the rectangular geometry is shown in figure 1.

The boundary conditions are the same as equation (5.1.10) and equation (5.1.11). The eigenfunctions $X_m(x)$, and $Y_n(y)$ are the same as equation (5.1.12) and equation (5.1.13). The circular frequency parameter of free vibration of rectangular isotropic plates is shown in equation (6.1.2).

II THE FREE VIBRATION OF ONE SIDE CLAMPED AND THREE SIDES SIMPLY SUPPORTED RECTANGULAR ISOTROPIC PLATE

The coordinate system for the rectangular geometry is shown in figure 2. The boundary conditions are the same as equation (5.1.14) and equation (5.1.15). The eigenfunctions $X_m(x)$ and $Y_n(y)$ are the same as equation (5.1.16) and equation (5.1.17). The circular frequency of free vibration of rectangular isotropic plates is shown in equation (6.1.2).

COMPARISON AND DISCUSSION OF RESULTS

The results of the free vibration of all sides clamped rectangular isotropic plate with aspect ratio from 0.1 to 1 are given in table 14. The results of the free vibration of one side fixed and three sides simply supported rectangular isotropic plates with aspect ratio from 1/3 to 1 are given in table 15. The first sixteen lowest frequencies (modes) are obtained for both rectangular plates. The modes are numbered in ascending order of frequencies. Plots showing the variation of frequencies vs aspect ratios are shown in figure 8 and 9.

From table 14, it can be seen that the values obtained by the present analysis are in excellent agreement with those obtained by other investigators[30]. The maximum difference is within 1 %. From table 15, the present analysis are in excellent agreement with the available data obtained from Leisser[30]. Because of the lack of data, no comparison can be made for higher frequencies in mode category for aspect ratio smaller than 1. From table 14, the mode no. 2 and 3 are the same frequencies in the case of square plate. They are called degenerate modes. As the aspect ratio λ deviates from one, this degeneracy disappears, and they become modes with distinct frequencies.

In the conventional Galerkin method, four polynomial functions which describe the mode shapes of plate have to be assumed. The four vibration modes are :

(1) Symmetric about the X- and Y- axis ,

(2) Symmetric about the X-axis and antisymmetric about Y- axis

(3) Antisymmetric about the X-axis and symmetric about the Y-axis,

(4) Antisymmetric about the X- and Y- axis.

Then, substituting each polynomial function in the governing differential equation (4.4.3). The amount of arithmetic work is quite lengthy. It is also difficult to assume the vibration mode polynomial function for an unsymmetric plate such as one side clamped and three sides simply supported. For Vlasov's method, the choice of a suitable expression to meet the prescribed boundary conditions is a matter of choosing the approximate beam functions.

From the above solved plate problems, Vlasov's method has been proven to very simple in application for the free vibration of plate problems compared with the conventional Galerkin method. Vlasov's method also provides an efficient and accurate solution for free vibration rectangular plate problems.

6.2 THE FREE VIBRATION OF ONE SIDE CLAMPED AND THREE SIDES SIMPLY SUPPORTED RECTANGULAR ORTHOTROPIC PLATES.

The coordinate system for the rectangular geometry is shown in figure 3. The differential equation governing for free vibrations of rectangular orthotropic plate is shown in equation (4.4.3). Substituting equation (4.4.3) into equation (3.2.4) and using notations

(5.1.3) results in,

$$\omega_{mn}^2 = \left[\frac{D_x/D_y I_1 I_2 + 2H/D_y I_3 I_4 + I_5 I_6}{I_7 I_8}\right]\frac{D_y}{\rho} \tag{6.2.1}$$

For computational advantage, equation (6.2.1) is transformed into dimensionless form and using the following dimensionless ratios,

$$\varsigma = x/2a, \qquad \eta = y/2b, \qquad F = \omega^2 \rho a^4 / D_y$$

Substituting to equation (6.2.1) results in,

$$F = \left[\frac{D_x/D_y I_1 I_2 + 2H/D_y \lambda^2 I_3 I_4 + \lambda^4 I_5 I_6}{I_7 I_8}\right] \tag{6.2.2}$$

In order to compare the results with other investigators, two orthotropic materials are considered in the thesis, they are Maple 5-plywood and Afara 3-plywood.

The elastic constants are as follows :

material	$E_x(psi.)$	$E_y(psi.)$	$G_{xy}(psi.)$	ν_x	ν_y
Maple 5-ply.	$1.87 * 10^6$	$0.60 * 10^6$	$0.159 * 10^6$	0.12	0.039
Afara 3-ply.	$1.96 * 10^6$	$0.165 * 10^6$	$0.110 * 10^6$	0.26	0.022

The boundary conditions for one side clamped and three sides simply supported plate are shown in equation (5.1.14) and equation (5.1.15). The eigenfunctions of $X_m(x)$ and $Y_n(y)$ are the same as equation (5.1.16) and equation (5.1.17).

COMPARISON AND DISCUSSION OF RESULTS

The results of the free vibration of one side clamped and three sides simply supported rectangular orthotropic plate with fundamental frequencies and aspect ratio from 1.0 to

2.0 are given in table 16. Comparisons of results are made with the results obtained by Hearmon [22] . It can be seen that the agreement is excellent. The maximum difference is less than 0.5 %.

Plots of frequencies vs aspect ratios for the two orthotropic plates and isotropic plates are shown in figure 10. From figure 10, it can be observed that the frequency parameter f increases with the degree of material orthotropy. As the aspect ratio λ increases, the orthotropic effect diminishes and the frequencies of orthotropic plates converge to those of isotropic plates. This can be explained by the fact that the first two terms in equation (6.2.1) which account for material orthotropy become less significant as the aspect ratio λ increases.

It can be further found that Vlasov's method is easily applicable to the free vibration of unsymmetric rectangular orthotropic plates and provides very accurate results.

6.3 THE FREE VIBRATION OF SANDWICH PLATES

The differential equation governing the free vibration of a sandwich plate is shown in equation (4.5.3).

For ease in computation, equation (4.5.3) is transformed into dimensionless form using the following dimensionless ratios :

$$\varsigma = x/2a, \qquad \eta = y/2b, \qquad \lambda = a/b$$

and substituting equation (4.5.3) into equation (3.2.4) results in,

$$\int\int\Big(\frac{\partial^4 w}{(2a)^4\partial\varsigma^4} + \frac{2\lambda^2}{(2a)^4}\frac{\partial^4 w}{\partial\varsigma^2\partial\eta^2} + \lambda^4\frac{\partial^4 w}{(2a)^4\partial\eta^4}$$
$$- \frac{\rho\omega^2}{D'}[\frac{-D'}{u'}(\frac{\partial^2 w}{(2a)^2\partial\varsigma^2} + \frac{D'}{u'}\frac{\partial^2 w}{(2a)^2\partial\eta^2}\lambda^2) + w]\Big)$$
$$X_m(x)Y_n(y)dxdy = 0 \qquad (6.3.1)$$

Substituting the notations (5.1.3) into equation (6.3.1) result in,

$$\omega_{mn}^2 = \frac{[I_1 I_2 + 2I_3 I_4 + I_5 I_6](1/a)^4}{(\rho/D')[-D'/u'(I_2 I_3 (1/a)^2 + D'/u' I_4 I_6 (1/a)^2) + I_7 I_8]} \tag{6.3.2}$$

I THE FREE VIBRATION OF A CLAMPED RECTANGULAR SANDWICH PLATES

The coordinate system for rectangular sandwich plate geometry is shown in figure 1 and 4.

The boundary conditions are the same as equation (5.1.10) and equation (5.1.11). The eigenfunctions $X_m(x)$ and $Y_n(y)$ are the same as equation (5.1.12) and equation (5.1.13). The circular frequency ω is shown in equation (6.3.2).

The following numerical examples are considered in the linear analysis of free vibration of sandwich plate for the purpose of the comparison of the results with other investigators :

$$E_f = 10^7\ psi. \qquad \nu = 0.34 \qquad \rho = 500.02 * 10^{-6}\ lb - sec^2/in^4$$

$$c = 0.25\ in. \qquad t = 0.016\ in. \qquad b = 20\ in. \qquad G_c = 1000\ psi.$$

II THE FREE VIBRATION OF ONE SIDE CLAMPED AND THREE SIDES SIMPLY SUPPORTED RECTANGULAR SANDWICH PLATE

The coordinate system for the rectangular sandwich plate geometry is shown in figure 2 and 4. The boundary conditions are the same as equation (5.1.14) and equation (5.1.15). The eigenfunctuions $X_m(x)$ and $Y_n(y)$ are the same as equation (5.1.16) and equation (5.1.17). The circular frequencies ω is shown in equation (6.3.2).

The following numerical examples are considered in the linear analysis of free vibration of sandwich plate for the purpose of the comparison of different core rigidities and aspect ratios :

$$E_f = 10^7\ psi. \quad \nu = 0.34 \quad \rho = 500.02 * 10^{-6}\ lb - sec^2/in^4$$

$$c = 0.25\ in. \quad t = 0.016\ in. \quad b = 20\ in.$$

COMPARISON AND DISCUSSION OF RESULTS

The results of the first 16 modes of the free vibration of all sides fixed rectangular sandwich plate with aspect ratio 1, 1.5 and 2 are given in table 17. The results of the free vibration of one side fixed and three sides simply supported rectangular sandwich plate are given in table 18. Plots showing the variations of frequencies vs aspect ratios are shown in figure 11.

From table 17, it can be seen that the present solutions are in good agreement with results obtained by Lam[29] in the first four modes of frequencies. Because of lack of available data, no comparison is possible for modes higher than four for each of the aspect ratios.

From table 18, it can be seen that the frequencies increase with an increase in the core rigidity due to increasing stiffness of the plate. Due to lack of data in hand, no comparsion of these results is possible.

It can be seen again that Vlasov's method is an excellent numerical method for the free vibration analysis of sandwich plates.

CHAPTER VII

THE BUCKLING OF RECTANGULAR PLANE UNDER INPLANE FORCES

In this chapter, Vlasov's method is further applie~~d to analyse the buckl~~ing problems of rectangular isotropic plates under inplane forces. The differential equation governing for buckling plate problem is shown in chapter IV (4.6).

7.1 THE BUCKLING OF RECTANGULAR ISOTROPIC PLATE

The differential equation governing for the buckling of a rectangular plate is shown in equation (4.6.1). For an isotropic plate, the flexural rigidities of the plate in X and Y directions are the same. Since the shear forces are not considered, N_{xy} is assumed zero in equation (4.6.1).

Substituting equation (4.6.1) into equation(3.2.4) and the resulting expression is converted into dimensionless form with the following dimensionless ratios :

$$\varsigma = x/2a, \qquad \eta = y/2b, \qquad N_\varsigma = N_x a^2/D$$

$$N_\eta = N_y a^2/D, \qquad \lambda = a/b, \qquad W = w/h, \qquad N_y = RN_x$$

result in,

$$\int\int\left(\left(\frac{\partial^4 w}{\partial\varsigma^4} + 2\lambda^2\frac{\partial^2 w}{\partial\varsigma^2\partial\eta^2} + \lambda^4\frac{\partial^4 w}{\partial\eta^4}\right) - N_\varsigma\left(\frac{\partial^2 w}{\partial\varsigma^2} + \lambda^2 R\frac{\partial^2 w}{\partial\eta^2}\right)\right) * X_m Y_n dxdy = 0 \qquad (7.1.1)$$

Using the notations (5.3.1) and substituting into equation (7.1.1) result in,

$$N_{\varsigma} = \frac{I_1 I_2 + 2\lambda^2 I_3 I_4 + \lambda^4 I_5 I_6}{I_2 I_3 + \lambda^2 R I_4 I_6} \tag{7.1.2}$$

7.2 THE BUCKLING OF A CLAMPED RECTANGULAR ISOTROPIC PLATE

The coordinate system for the rectangular plate geometry is shown in figure 1.

The boundary conditions are :

In X-direction,

$$w(0) = w'(0) = 0, \qquad w(2a) = w'(2a) = 0 \tag{7.2.1}$$

In Y-direction,

$$w(0) = w'(0) = 0, \qquad w(2b) = w'(2b) = 0 \tag{7.2.2}$$

By selecting the eigenfunctions of $X_m(x)$ and $Y_n(y)$ to satsify the above boundary conditions,. $X_m(x)$ and $Y_n(y)$ are chosen as follows :

$$X_m(x) = 1 - cos\frac{xxm * x}{2a} \tag{7.2.3}$$

$$xxm = 2m\pi, \qquad m = 1, 2, 3......$$

$$Y_n(y) = 1 - cos\frac{yyn * y}{2b} \tag{7.2.4}$$

$$yyn = 2n\pi, \qquad n = 1, 2, 3......$$

COMPARISON AND DISCUSSION OF RESULTS

The results of the lowest buckling load coefficient Cr with aspect ratio from 1 to 10 are shown in table 19. A plot showing the variation of buckling load coefficient vs aspect ratio is shown in figure 12. From table 19, it can be seen that the present solutions are in excellent agreement with those obtained by Das [24] and Timoshenko [50]. The maximum percentage of difference is less than 2 %.

Again, it can be further seen that Vlasov's method is accurate and easily applied to buckling plate problems.

CHAPTER VIII

SUMMARY OF CONCLUSIONS

THE VLASOV METHOD

(1) By choosing eigenfunctions of a vibrating beam or column buckling, the amount of numerical work required in the conventional Galerkin's method can be greatly reduced as a result of the orthogonality conditions of these functions.

(2) The choice of an appropriate expression for the assumed solution is simply a matter of choosing beam functions with identical boundary conditions as those of the plate.

(3) It is easy to apply both for unsymmetric and sandwich plate problems.

(4) The accuracy of the method is excellent in spite of the relatively little amount of calculation involved.

(5) The solution becomes increasingly tedious as the number of terms is increased. However, from the results shown in the previous chapters, it seems that in general, only a few terms are required to produce a solution accurate enough for all practical purposes.

(6) This method definitely has its merits in the manual solution of plate problems.

(7) The Vlasov method is a simple powerful tool for the static and dynamic analysis of plates. The versatility and simplicity of this method was demonstrated by solving a wide variety of problems of significant complexity in applied mechanics. The determination of complex plate problems are reduced to the evaluation of definite integrals of simple functions.

(8) Vlasov's method is highly recommended for the solution of complex plate problems where computer facilities are not readily available.

(9) Vlasov's method represents a great saving in human and machine efforts because of the relatively small amount of computer time and storage space required for a solution.

(10) For a given aspect ratio λ, the maximum center deflection of the plate decreases with increasing values of the foundation modulus.

(11) The effectiveness of the elastic foundation in reducing the maximum center deflection of the plate is more pronounced for small aspect ratios than it is for aspect ratios approaching unity.

(12) The maximum center deflection of the sandwich plate structure decreases with increase in core rigidity. This is due to overall stiffness increases of the sandwich plate.

Finally, from the present study, it can be concluded that Vlasov's method is extremely versatile and sufficiently accurate for all types of analysis considered. The amount of arithmetic work required in the method is a mere fraction of that required in the conventional Galerkin method. Computing time and memory requirements are small, thus making it ideally suited for computers with relatively small capacity and speed.

APPENDIX A FIGURES

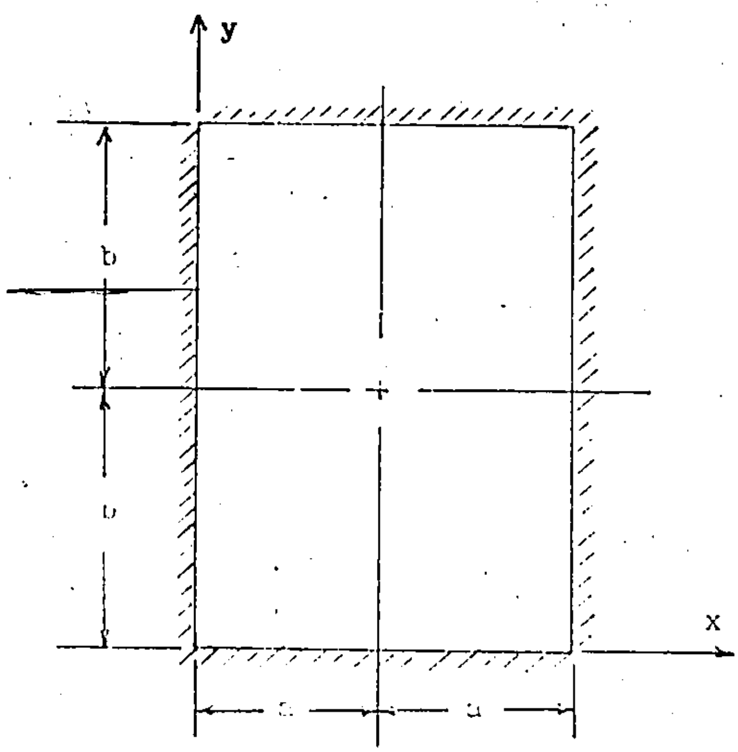

Figure 1 - A Clamped Rectangular Plate Geometry Defined by a Cartesian Coordinate System.

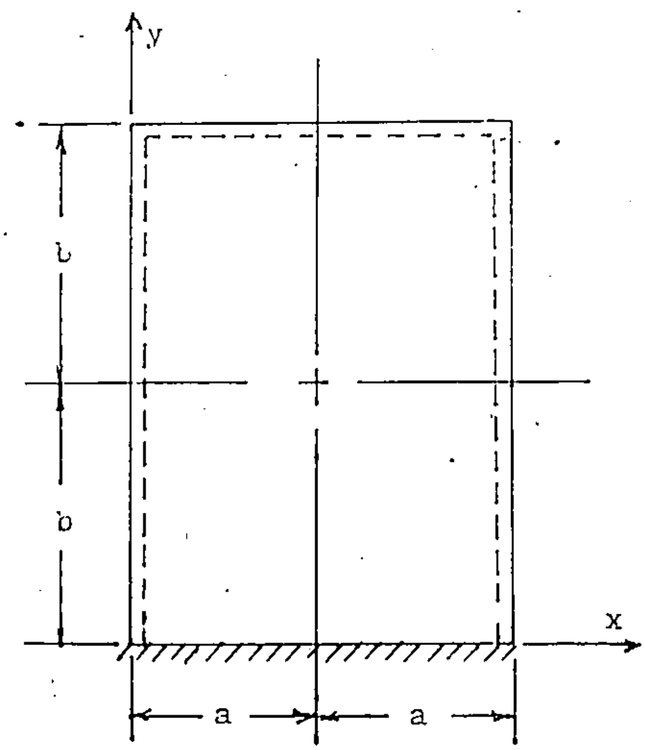

Figure 2 - A One Side Clamped and Three Sides Simply Supported Rectangular Plate Geometry Defined by a Cartesian Coordinate System.

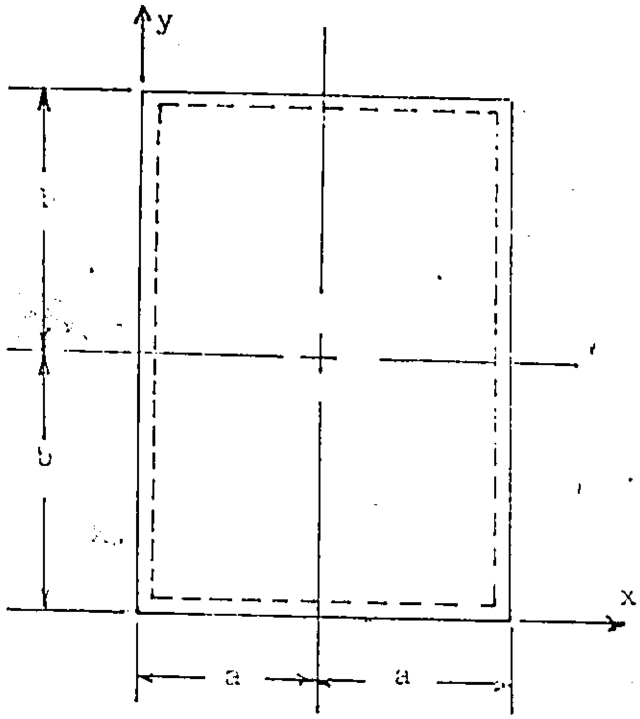

Figure 3 - A Simply Supported Rectangular Plate Geometry Defined by a Cartesian Coordinate System.

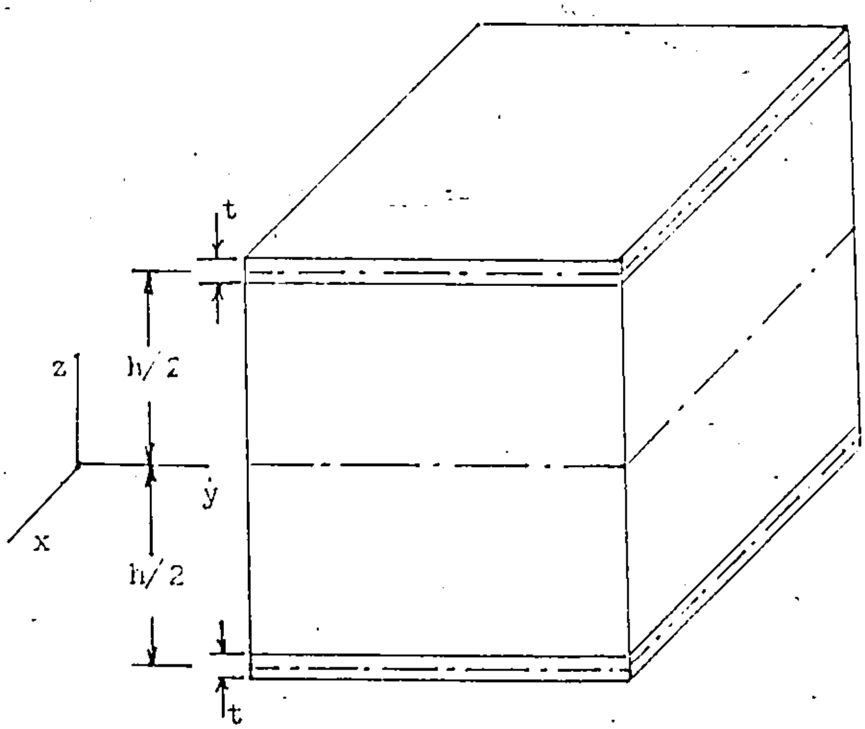

Figure 4 - Element of a Sandwich Plate.

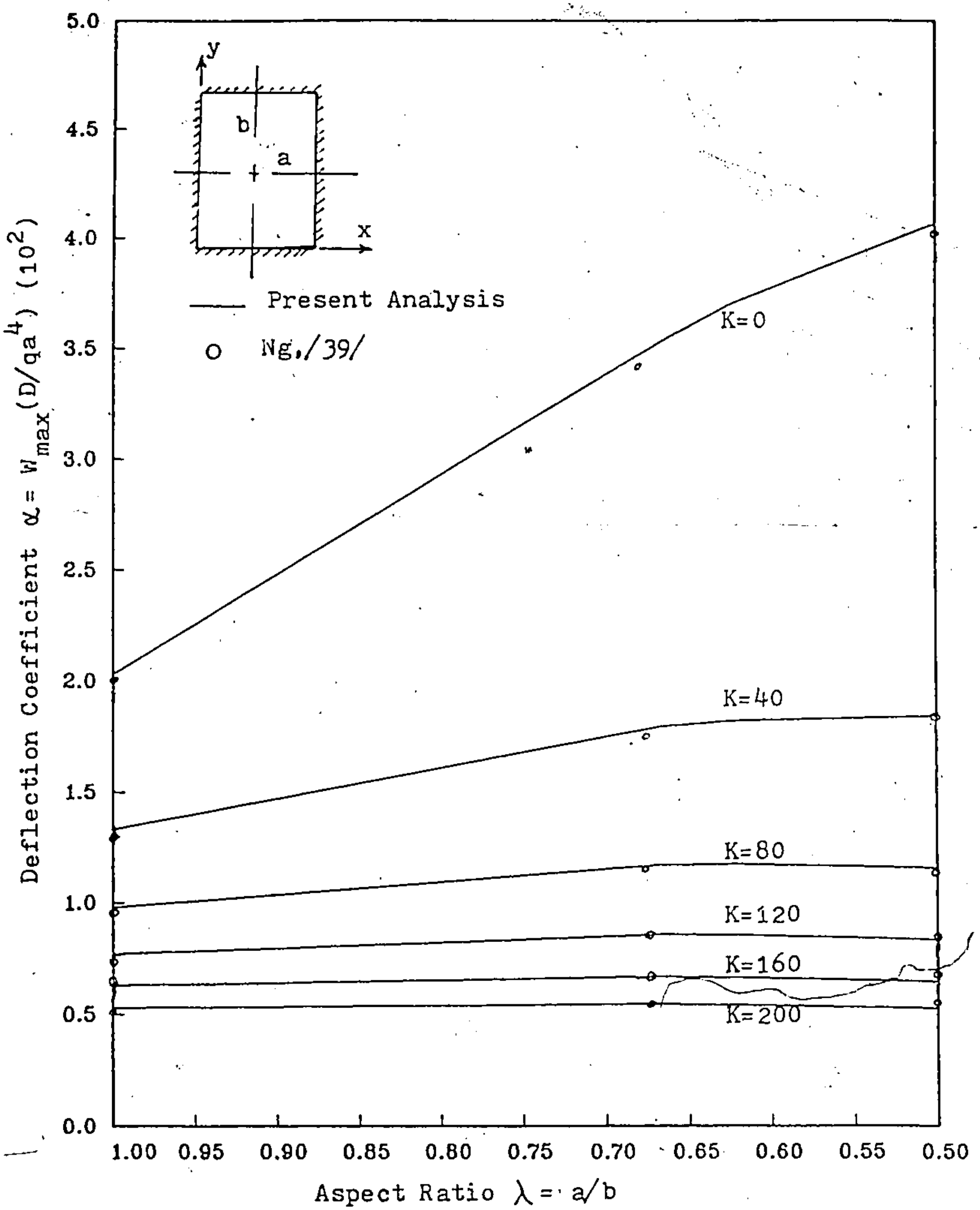

Figure 5 - Variation of Central Deflection with Lateral Pressure and Foundation Modulus for Clamped Rectangular Plate.

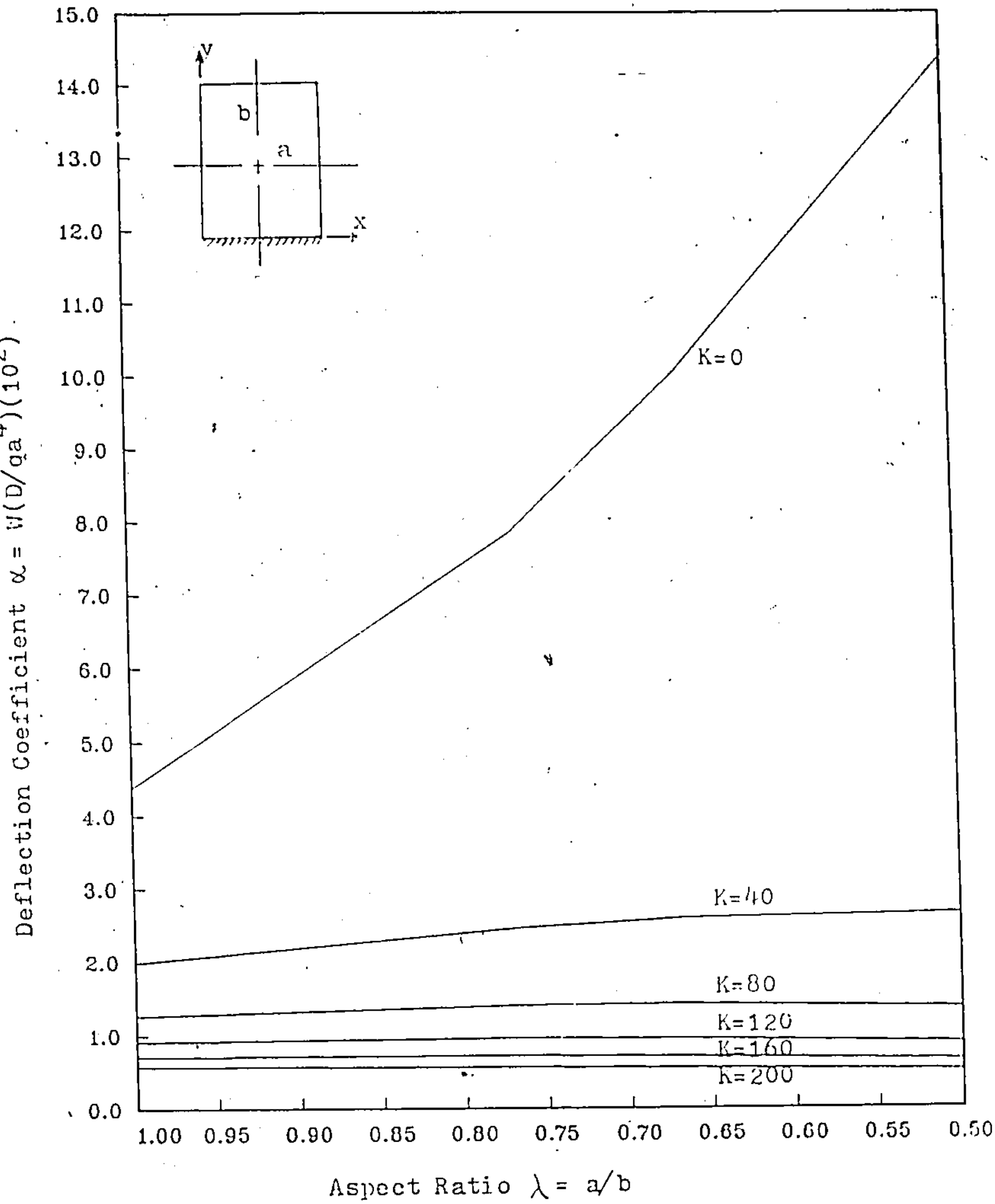

Figure 6 - Variation of Central Deflection with Lateral Pressure and Foundation Modulus for One Side Clamped and Three Sides Simply Supported Rectangular Plate.

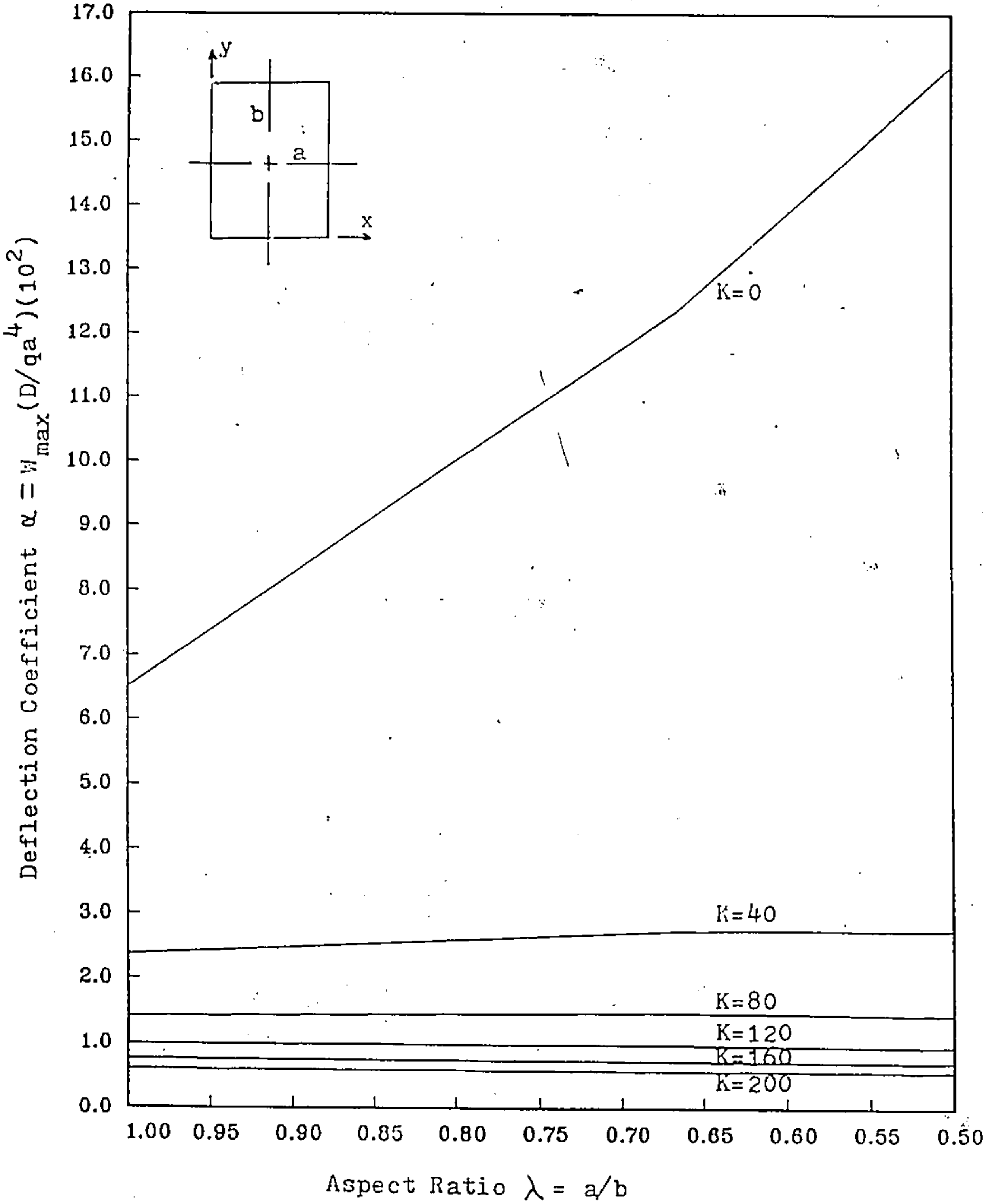

Figure 7 - Variation of Central Deflection with Lateral Pressure and Foundation Modulus for Simply Supported Rectangular Plate.

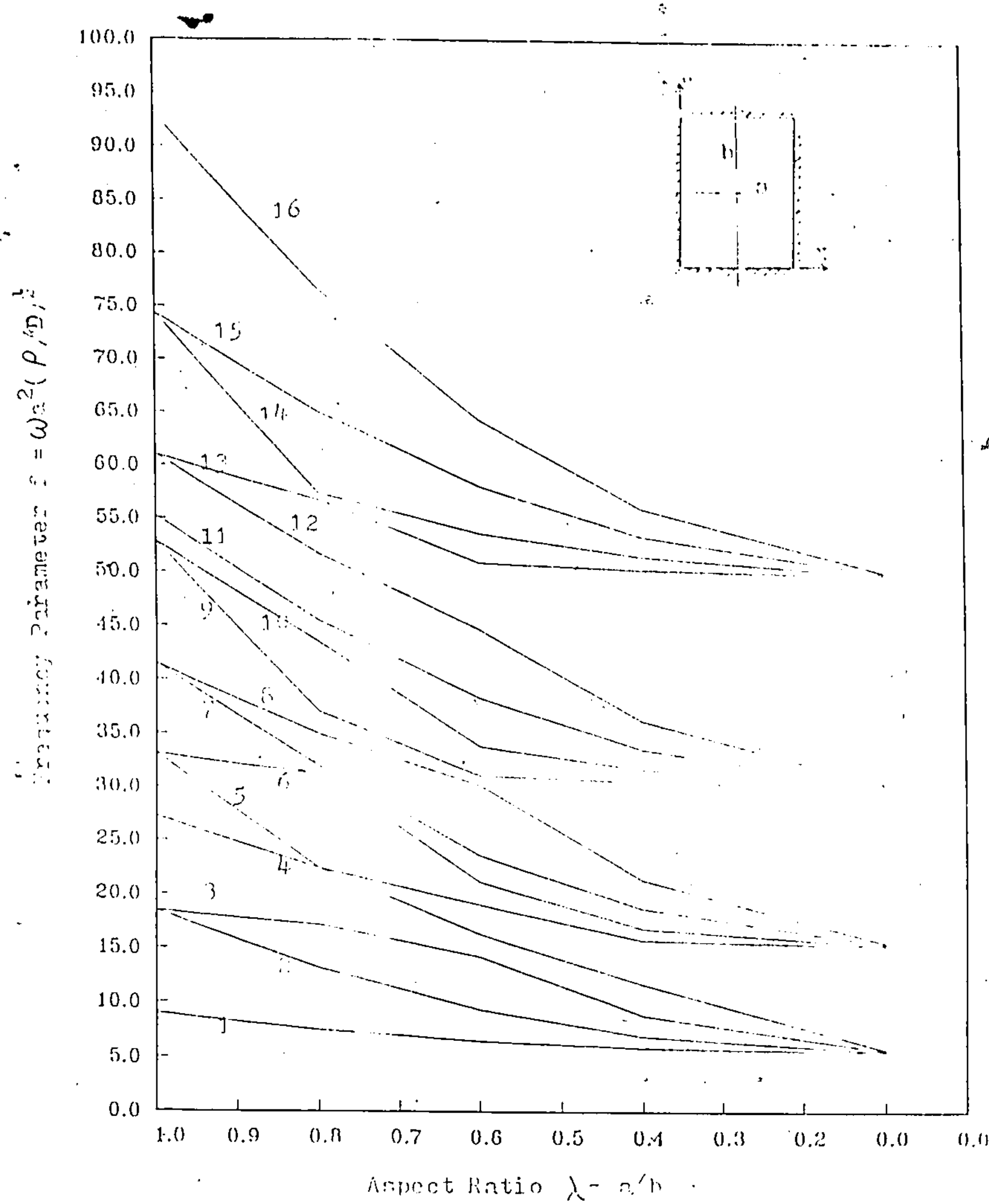

Figure 8 - Variation of Natural Frequencies with Aspect Ratio and Nodal No. for Clamped Rectangular Plate.

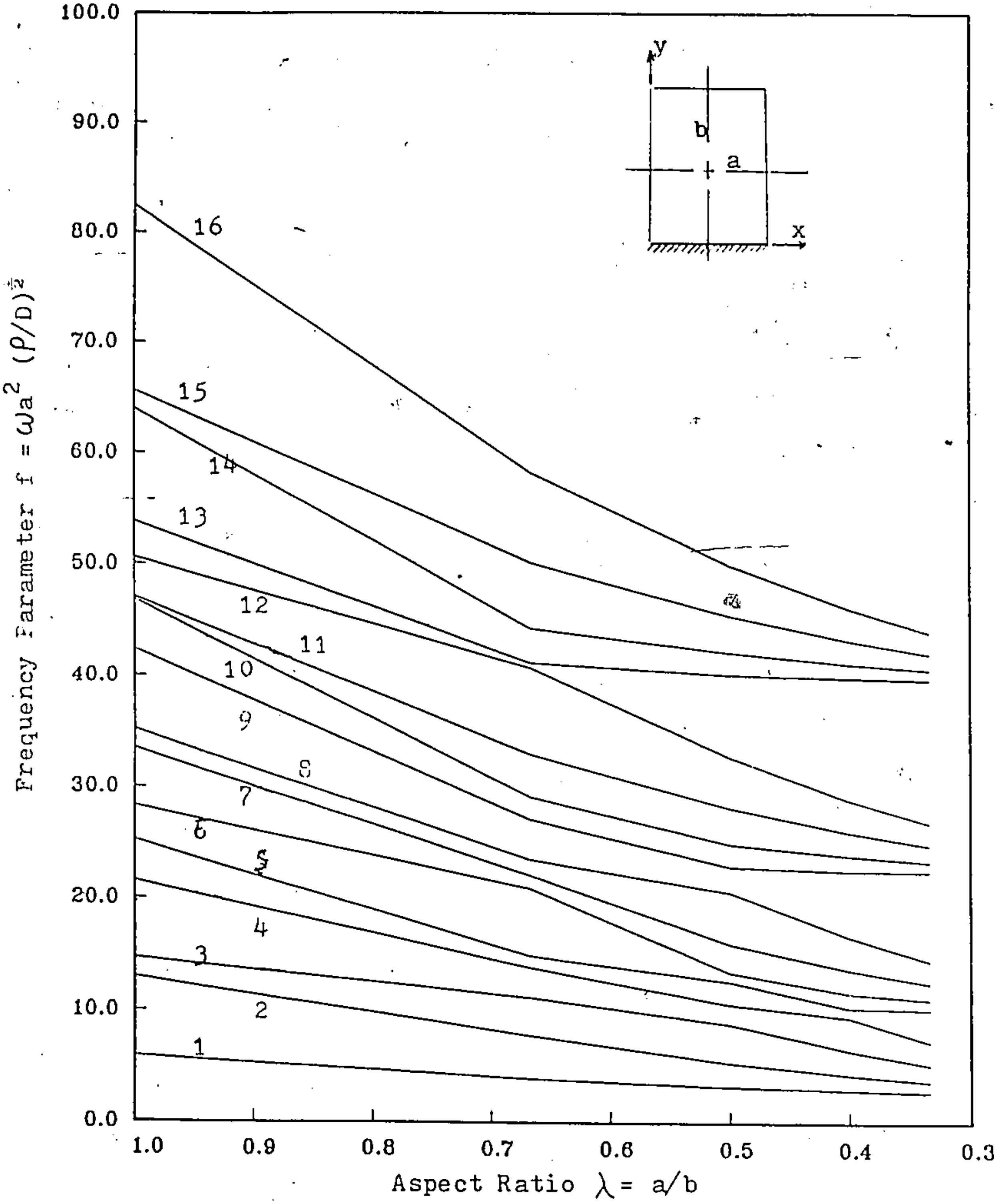

Figure 9. - Variation of Natural Frequencies with Aspect Ratio and Nodal No. for One Side Clamped and Three Sides Simply Supported Rectangular Plate.

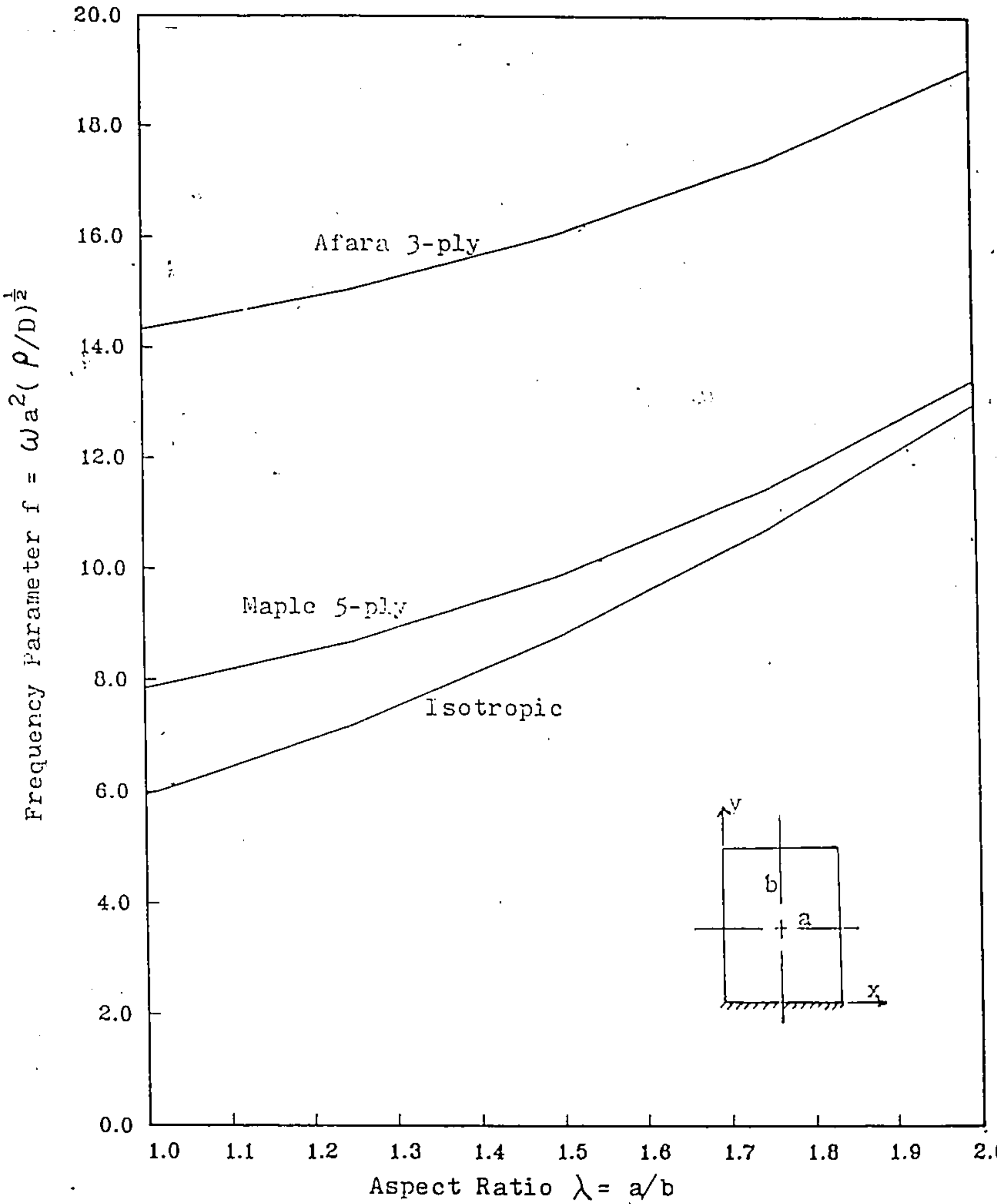

Figure 10 - Variation of Fundamental Frequency with Aspect Ratio for One Side Clamped and Three Sides Simply Supported Rectangular Orthotropic Plates.

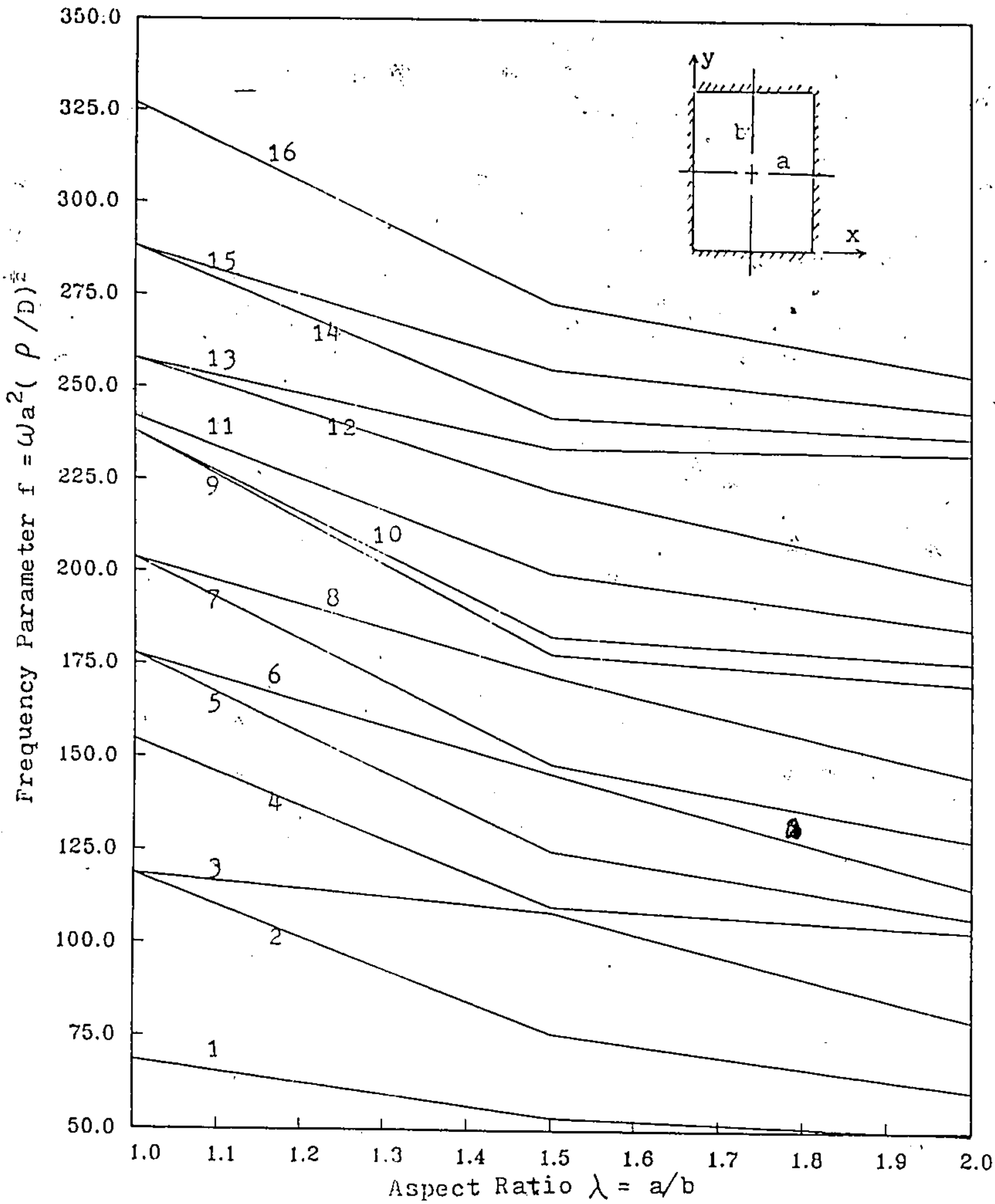

Figure 11 - Variation of Natural Frequencies with Aspect Ratio and Nodal No. for One Side Clamped and Three Sides Simply Supported Sandwich Plate.

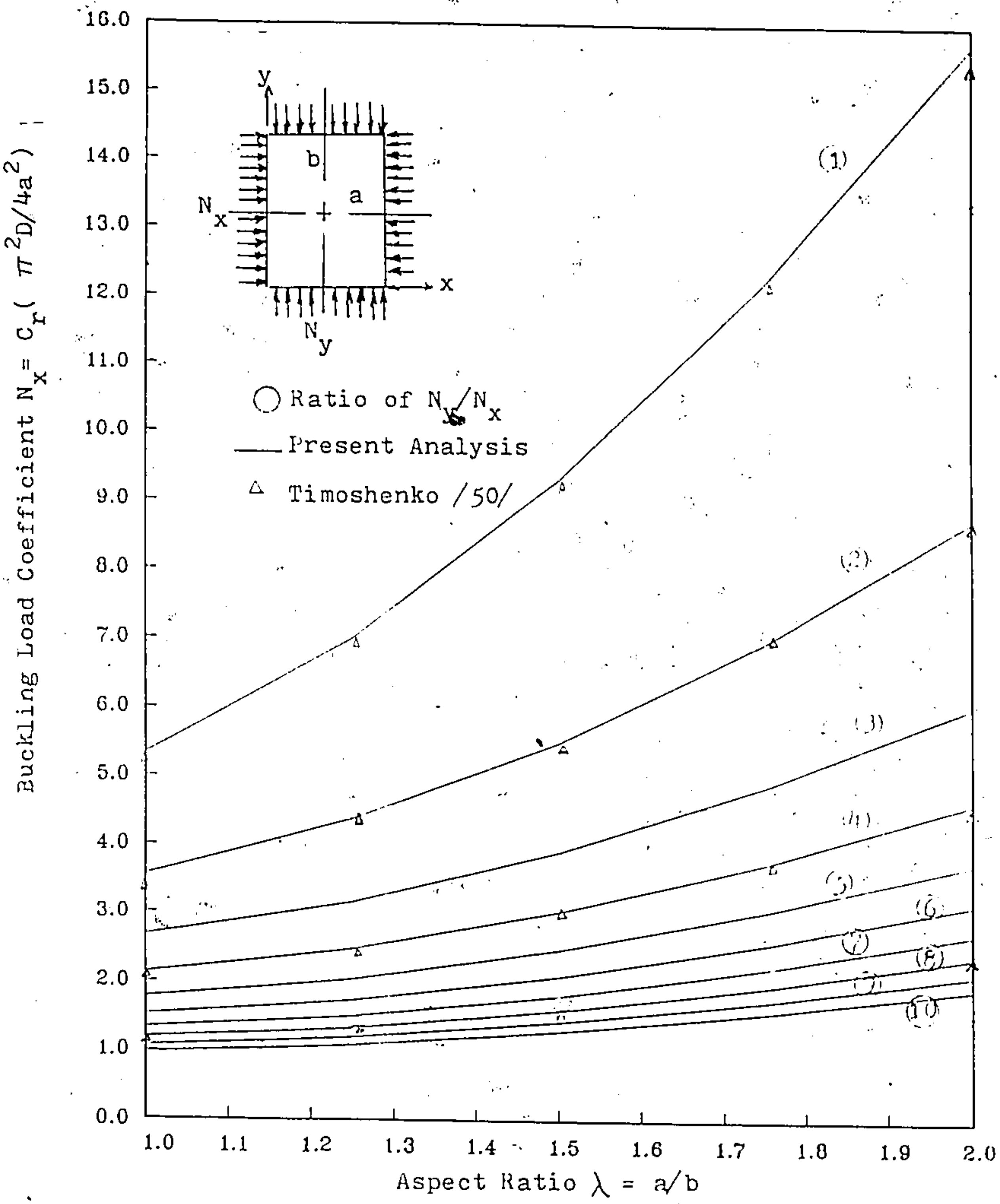

Figure 12 - Variation of Buckling Load Coefficient with Aspect Ratio and N_y/N_x Ratio for Clamped Rectangular Plate.

APPENDIX B TABLES

Aspect Ratio $\lambda = a/b$	No. of terms used				Timoshenko Ref./51/
	1	4	9	16	
1	0.02137	0.02022	0.02034	0.02031	0.02016
2/3	0.03796	0.03502	0.03536	0.03528	0.03520
5/8	0.04007	0.03662	0.03704	0.03695	0.03680
1/2	0.04586	0.04001	0.04090	0.04069	0.04000

$$w_{max.} = \alpha q a^4 / D$$

Table (1) Variation of the maximum deflection coefficient α with the number of terms used in the solution for clamped rectangular isotropic plate.

Dimensionless Foundation Modulus K	Aspect Ratio $\lambda = a/b$			
	1	5/8	2/3	1/2
0	0.0203115	0.0369494	0.0352828	0.0406882
20	0.0161220	0.0245568	0.0239276	0.0255761
40	0.0133181	0.0181840	0.0179196	0.0184070
60	0.0113119	0.0143266	0.0142204	0.0142664
80	0.0098067	0.0117537	0.0117242	0.0115889
100	0.0086368	0.0099228	0.0099325	0.0097244
120	0.0077022	0.0085580	0.0085880	0.0083560
140	0.0069389	0.0075042	0.0075445	0.0073116
160	0.0063044	0.0066680	0.0067129	0.0064898
180	0.0057689	0.0059895	0.0060358	0.0058272
200	0.0053112	0.0054289	0.0054748	0.0052822

$$W_{max.} = \alpha\, qa^4/D$$

Table (2) Variation of the maximum deflection coefficients α of clamped rectangular isotropic plate with the dimensionless foundation modulus K.

Aspect Ratio $\lambda = a/b$	number of terms used				Timoshenko Ref./51/
	1	4	9	16	
1	0.02401	0.04528	0.04383	0.04382	0.0448
1/1.3	0.08032	0.08097	0.07840	0.07837	0.0800
1/1.5	0.10315	0.10415	0.10045	0.10039	0.1024
1/2	0.14969	0.15176	0.14382	0.14366	0.1488

$$W = \alpha q a^4/D$$

Table (3) Variation of the central deflection coefficient α with the number of terms used in the solution for one side clamped and three sides simply supported rectangular plate.

Dimensionless Foundation Modulus K	Aspect Ratio $\lambda = a/b$			
	1	1/1.3	1/1.5	1/2
0	0.04382	0.07837	0.10039	0.14366
20	0.02752	0.03792	0.04207	0.04639
40	0.01990	0.02461	0.02600	0.02658
60	0.01550	0.01801	0.01852	0.01824
80	0.01263	0.01408	0.01424	0.01371
100	0.01062	0.01148	0.01147	0.01089
120	0.00913	0.00964	0.00955	0.00898
140	0.00799	0.00828	0.00814	0.00761
160	0.00708	0.00723	0.00707	0.00658
180	0.00635	0.00639	0.00622	0.00578
200	0.00574	0.00572	0.00555	0.00514

$$W = \alpha q a^4 / D$$

Table (4) Variation of the central deflection coefficient α of one side clamped and three sides simply supported rectangular plate with dimensionless foundation modulus K.

Aspect Ratio $\lambda = a/b$	number of terms used				Timoshenko Ref/51/
	1	4	9	16	
1	0.06656	0.06488	0.06501	0.06498	0.06496
2/3	0.12761	0.12324	0.12363	0.12355	0.12352
5/8	0.13767	0.13251	0.13299	0.13289	0.13280
1/2	0.17039	0.16119	0.16220	0.16199	0.16208

$$w_{max.} = \alpha q a^4 / D$$

Table (5) Variation of the maximum deflection coefficient α with the number of terms used in the solution of simply supported isotropic plate.

Dimensionless Foundation Modulus K	Aspect Ratio $\lambda = a/b$			
	1	2/3	5/8	1/2
0	0.06498	0.12355	0.13269	0.16199
20	0.03502	0.04595	0.04686	0.04849
40	0.02372	0.02749	0.02762	0.02752
60	0.01779	0.01931	0.01927	0.01891
80	0.01416	0.01474	0.01466	0.01429
100	0.01170	0.01184	0.01175	0.01143
120	0.00993	0.00985	0.00976	0.00949
140	0.00861	0.00840	0.00832	0.00810
160	0.00757	0.00730	0.00723	0.00705
180	0.00675	0.00644	0.00638	0.00623
200	0.00607	0.00576	0.00570	0.00558

$$W_{max.} = \alpha q a^4/D$$

Table (6) Variation of the maximum deflection coefficient α of simply supported rectangular isotropic plate with dimensionless foundation modulus K.

ϵ	α	Timoshenko Ref./51/	Lekhnitskii Ref./32/
1	0.06498	0.06512	0.06512
1.1	0.07789	0.07808	-
1.2	0.09039	0.09040	-
1.3	0.10225	0.10224	-
1.4	0.11333	0.11344	-
1.5	0.12355	0.12352	0.12352
1.6	0.13289	0.13296	-
1.7	0.14136	0.14144	-
1.8	0.14901	0.14912	-
1.9	0.15586	0.15584	-
2.0	0.16199	0.16208	-
2.5	0.18381	0.18400	0.18400
3	0.19551	0.19568	-
4	0.20453	0.20512	-
5	0.20640	0.20752	-
∞	0.20719	0.20832	0.20832

$$\epsilon = \frac{a}{b}\sqrt{\frac{D_y}{D_x}} \quad , \qquad W = \frac{\alpha q b^4}{D_y}$$

Table (7) Variation of the maximum deflection coefficient α of simply supported orthotropic rectangular plate with the variation of ϵ .

Sandwich Plate # 1

$\nu_1 = 0.32$ $\mu = 0.00125$ $\theta = 0.05125$

$E_f = 10 \times 10^6$ psi. $G_c = 500$ psi.

No. of terms used	The maximum small deflection coefficient			
	Aspect Ratio a/b			
	$\lambda = 1$	$\lambda = 3/4$	$\lambda = 2/3$	$\lambda = 1/2$
1	0.255878	0.321672	0.337493	0.357187
4	0.201264	0.247060	0.252351	0.246956
9	0.214816	0.266027	0.274622	0.280986
16	0.209046	0.256904	0.265229	0.266529
25	0.211858	0.260819	0.268567	0.273643

Table (8) Variation of maximum small deflection coefficient with the no. of terms used in the linear analysis of the sandwich plate # 1.

No. of terms used	Maximum center small deflection (in.)			
	Sandwich plate dimension 2b x 2a in inches			
	40 x 40	$53\frac{1}{3}$ x 40	60 x 40	80 x 40
1	0.279820	0.351770	0.369071	0.390608
4	0.220096	0.270176	0.275963	0.270063
9	0.234916	0.290918	0.300317	0.307277
16	0.228606	0.280942	0.290045	0.291467
25	0.231681	0.285223	0.293696	0.299247
March /35/	0.228190	0.298742	---	---
Monforton /38/	0.2483	---	---	---

Table (9) Comparison of the maximum centre small deflection of the sandwich plate # 1.

Sandwich plate # 2

$\nu_f = 0.32$ $\mu = 0.00125$ $\theta = 0.05125$

$E_f = 10 \times 10^6$ psi. $G_c = 100{,}000$ psi.

No. of terms used	The maximum small deflection coefficient			
	Aspect Ratio, $\lambda = a/b$			
	$\lambda = 1$	$\lambda = 3/4$	$\lambda = 2/3$	$\lambda = 1/2$
1	0.022539	0.035037	0.039460	0.047429
4	0.021127	0.032481	0.036109	0.041041
9	0.021309	0.032813	0.036561	0.042088
16	0.021255	0.032721	0.036439	0.041810
25	0.021277	0.032756	0.036485	0.041910

Table (10) Variation of maximum small deflection coefficient with the number of terms used in the linear analysis of sandwich plate # 2.

No. of terms used	maximum centre small deflection (in.)			
	sandwich plate dimension 2b x 2a in inches			
	40 x 40	$53\frac{1}{3}$ x 40	60 x 40	80 x 40
1	0.024648	0.038315	0.043152	0.051867
4	0.023104	0.035520	0.039488	0.044881
9	0.023303	0.035872	0.039982	0.046026
16	0.023244	0.035783	0.039848	0.045722
25	0.023268	0.035821	0.039899	0.045831
March /35/	0.023482	0.036716	----	----
Monforton /38/	0.023480	----	----	----

Table (11) Comparison of the maximum center small deflection of the sandwich plate 2.

Sandwich plate # 3

$\nu_f = 0.30$ $\mu = 0.0006$ $\theta = 0.04$

$E_f = 10.5 \times 10^6$ psi. $G_c = 50{,}000$ psi.

No. of terms used	The maximum small deflection coefficient			
	Aspect Ratio, $\lambda = a/b$			
	$\lambda = 1$	$\lambda = 3/4$	$\lambda = 2/3$	$\lambda = 1/2$
1	0.022277	0.034714	0.039123	0.047076
4	0.020924	0.032240	0.035864	0.040811
9	0.021091	0.032550	0.036292	0.041821
16	0.021044	0.032467	0.036178	0.041559
25	0.021062	0.032498	0.036221	0.041651
Kan et al /24/	0.021039	----	0.036249	0.041746

Table (12) Variation of the maximum small deflection coefficient with the number of terms used in the linear analysis of sandwich plate # 3.

No. of terms used	Maximum center small deflection (in.)			
	sandwich plate dimension 2b x 2a in inches			
	20 x 20	$26\frac{2}{3}$ x 20	30 x 20	40 x 20
1	0.040222	0.062678	0.070639	0.084990
4	0.037779	0.058363	0.064754	0.073687
9	0.038081	0.058771	0.065527	0.075510
16	0.037996	0.058621	0.065321	0.075037
25	0.038029	0.058677	0.065399	0.075203
March /35/	0.038402	0.060150	----	----
Kan et al. /24/	0.037987	----	0.065496	0.075375

<u>Table (13)</u> Comparison of the maximum center small deflection of sandwich plate # 3.

Mode number	Aspect Ratio $\lambda = a/b$									
	$\lambda = 1$		$\lambda = 0.9$		$\lambda = 0.8$		$\lambda = 0.7$		$\lambda = 0.6$	
	present values	ref. /39/	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/
1	9.0271	8.9963	8.1937	8.1658	7.4972	7.4720	6.9331	6.9105	6.4924	6.4723
2	18.4343	18.3485	15.6402	15.5661	13.1901	13.1274	11.0934	11.0414	9.3589	9.3165
3	18.4343	18.3485	17.7654	17.6853	17.2005	17.1269	16.7312	16.6651	14.2984	14.2285
4	27.2125	27.0541	24.6658	24.5227	22.4241	22.3138	18.0325	17.9424	16.3484	16.2910
5	33.1208	32.8952	27.4573	27.3270	22.4664	22.3375	20.6079	20.4925	19.0777	18.9758
6	33.1208	33.0512	32.5166	32.3826	31.2459	31.0814	27.1302	26.9947	21.2092	21.0985
7	41.4808	41.2501	36.0318	35.8354	31.9963	31.8760	27.5681	27.4299	23.6872	23.5765
8	41.4808	41.2501	39.1245	38.9053	34.9677	34.8021	31.5583	31.4495	30.2795	29.8378
9	52.8363	52.6305	43.3934	43.2093	37.0856	36.8856	35.3504	35.1661	31.1835	30.126
10	52.8363	52.6305	50.0348	51.4292	43.5021	43.2543	36.3472	36.1484	33.9024	33.7398
11	55.2268	---	51.7275	---	45.5017	---	41.6189	---	38.3694	---
12	61.0078	---	52.2568	---	51.7514	---	50.5223	---	44.7310	---
13	61.0078	---	58.7583	---	56.7981	---	51.3159	---	50.9467	---
14	74.3742	---	65.3445	---	57.3923	---	55.1142	---	53.6927	---
15	74.3742	---	69.3850	---	65.0264	---	61.2810	---	58.1269	---
16	93.1313	---	84.3541	---	76.6456	---	69.9952	---	64.3817	---

$$\omega = [f/a^2]\,[D/m]^{\frac{1}{2}}$$

Table (14) Variation of natural frequency parameter f of a clamped rectangular isotropic plate.

Mode number	Aspect Ratio $\lambda = a/b$									
	$\lambda = 0.5$		$\lambda = 0.4$		$\lambda = 0.3$		$\lambda = 0.2$		$\lambda = 0.1$	
	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/
1	6.1620	6.1444	5.9256	5.9110	5.7655	5.7544	5.6650	5.6581	5.6105	5.6081
2	7.9904	7.9565	6.9787	6.9517	6.2916	6.2709	5.8735	5.8597	5.6584	5.6528
3	11.2432	11.1924	8.8887	8.8543	7.2370	7.2146	6.2315	6.2170	5.7356	5.7289
4	15.9150	15.8327	11.7225	11.6678	8.6750	8.6438	6.7701	6.7540	5.8458	5.8384
5	16.0431	15.9958	15.8067	15.7707	15.6316	13.0251	15.5114	---	15.4412	---
6	17.8563	17.7691	16.9180	16.8477	16.2328	15.6074	15.7704	---	15.5047	---
7	20.9082	20.8182	18.7670	18.6945	17.2151	16.1819	16.1850	---	15.6047	---
8	25.3117	25.1980	21.4398	19.9406	18.6236	17.1598	16.7697	---	15.7432	---
9	30.8801	29.0893	30.6388	---	30.4556	---	30.3271	---	30.2510	---
10	32.7228	---	31.7928	---	31.0929	---	30.6065	---	30.3023	---
11	35.7268	---	33.6547	---	32.1100	---	31.0480	---	30.4290	---
12	39.9998	---	36.2897	---	33.5373	---	31.6615	---	30.5788	---
13	50.6403	---	50.3938	---	50.2047	---	50.0711	---	49.9915	---
14	52.5195	---	51.5813	---	50.8658	---	50.3627	---	50.0641	---
15	55.5335	---	53.4727	---	51.9118	---	50.8216	---	50.1779	---
16	59.7716	---	56.1185	---	53.3664	---	51.4560	---	50.3345	---

$$\omega = [f/a^2]\,[D/m]^{\frac{1}{2}}$$

Table (14) Variation of natural frequency parmeter f of a clamped rectangular isotropic plate.

Mode number	Aspect Ratio $\lambda = a / b$									
	$\lambda = 1$		$\lambda = 1/1.5$		$\lambda = 1/2$		$\lambda = 1/2.5$		$\lambda = 1/3$	
	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/	present values	ref. /30/
1	5.9286	5.9115	3.9160	3.8933	3.2508	3.2295	2.9565	2.9385	2.8017	2.7855
2	13.0030	12.9185	7.7737	---	5.3917	---	4.3070	---	3.7275	---
3	14.6643	14.6603	11.2069	---	8.8059	---	6.4837	---	5.2311	---
4	21.5668	21.5315	13.8499	---	10.6085	---	9.4599	---	7.2949	---
5	25.2154	25.0648	14.9102	---	12.6524	---	10.3382	---	10.1934	---
6	28.3078	28.3043	20.9243	---	13.4571	---	11.6307	---	11.0840	---
7	33.5479	33.4460	22.1102	---	15.9961	---	13.7494	---	12.5442	---
8	35.2235	35.2100	23.5134	---	20.6158	---	16.6875	---	14.5725	---
9	42.4341	---	27.1227	---	22.9351	---	22.6708	---	22.5282	---
10	46.8594	---	29.1798	---	24.9391	---	23.9444	---	23.4090	---
11	47.0797	---	33.0358	---	28.2244	---	26.0304	---	24.8502	---
12	50.6095	---	40.7735	---	32.7881	---	28.9303	---	26.8536	---
13	53.8285	---	41.2228	---	40.2031	---	39.9410	---	39.7992	---
14	63.9845	---	44.3362	---	42.1899	---	41.2070	---	40.6762	---
15	65.6546	---	50.1768	---	45.4429	---	43.2774	---	42.1091	---
16	82.4630	---	58.2899	---	49.9652	---	46.1546	---	44.0996	---

$$\omega = [f/a^2]\,[D/m]$$

Table (15) Variation of natural frequency parameter f of one side clamped and three sides simply supported rectangular plate.

Aspect Ratio a/b	Afara 3-Ply.		Maple 5-Ply.	
	present solution	Hearmon /22/	present solution	Hearmon /22/
1.0	14.32192	14.32254	7.84798	7.84822
1.25	15.05136	15.05190	8.69135	8.69108
1.50	16.06191	16.06157	9.89039	9.88964
1.75	17.39433	17.39330	11.47044	11.46927
2.00	19.07844	19.07665	13.43514	13.45123

$$\omega = [f/a^2][D/m]^{\frac{1}{2}}$$

Table (16) Variation of natural frequency parameter f of one side clamped and three sides simply supported rectangular orthotropic plate with aspect ratio $\lambda = a/b$.

Mode Number	Aspect Ratio $\lambda = a/b$					
	$\lambda = 1$		$\lambda = 1.5$		$\lambda = 2.0$	
	present solution	Lam /29/	present solution	Lam /29/	present solution	Lam /29/
1	68.422	70.29	53.479	55.67	49.397	51.18
2	118.608	123.87	75.912	81.84	60.841	65.21
3	118.608	123.87	108.482	114.65	79.719	89.62
4	154.792	165.54	109.926	121.49	103.762	111.67
5	177.732	---	124.929	---	107.408	---
6	177.732	---	145.772	---	115.267	---
7	203.677	---	148.379	---	128.098	---
8	203.677	---	172.039	---	145.472	---
9	238.049	---	177.938	---	170.295	---
10	238.049	---	182.694	---	175.998	---
11	242.072	---	199.706	---	185.204	---
12	257.772	---	222.361	---	197.916	---
13	257.772	---	233.880	---	232.553	---
14	288.224	---	242.070	---	237.002	---
15	288.224	---	255.166	---	244.120	---
16	326.980	---	273.010	---	253.983	---

Table (17) Variation of natural frequency ω of all sides fixed sandwich plate.

Mode No.	λ = 1.0		λ = 1.5		λ = 2.0	
	G_c=5000	G_c=10000	G_c=5000	G_c=10000	G_c=5000	G_c=10000
1	51.3316	52.1747	34.2360	34.6350	28.5175	28.8015
2	107.8958	111.8958	66.5570	68.0152	46.7381	47.4786
3	121.3899	126.0257	93.8217	96.8992	74.9090	76.7866
4	171.6895	181.4216	114.8175	119.1251	89.0926	91.8796
5	196.3136	209.4692	122.4537	127.5957	105.1004	108.9420
6	219.4464	234.5828	166.8090	176.1556	111.6472	115.7943
7	251.7847	272.8413	175.8395	185.8891	130.5925	136.4463
8	263.8300	286.1320	184.5014	196.1982	164.4666	173.6267
9	305.7551	336.8141	209.2581	224.1279	180.4618	191.6717
10	334.5940	370.5664	223.8312	240.3474	194.3509	207.2865
11	335.7773	370.6895	248.2507	268.8737	216.6157	232.5487
12	353.3298	393.9905	295.7608	324.9456	246.5461	266.9570
13	374.4962	418.1423	299.3062	328.7163	292.3123	320.8559
14	425.9770	483.3595	316.9843	350.2491	304.2445	335.0420
15	436.4016	495.4683	350.6864	390.8839	323.431	358.0009
16	517.4464	599.1852	395.4918	445.6459	349.4003	389.3688

Table (18) Variation of natural frequencies ω of one side fixed and three sides simply supported sandwich plates with various aspect ratios and shear rigidities.

N_y/N_x	$\lambda = 1$		$\lambda = 1.25$		$\lambda = 1.50$		$\lambda = 1.75$		$\lambda = 2.0$	
	Pres. soln.	Das. /11/	Pres. soln.	Das. /11/	Pres. soln.	Das. /11/	Pres. soln.	Das. /11/	Pres. soln.	Das. /11/
1	5.3333	5.304 5.33*	6.9980	6.864	9.3077	9.273	12.2295	12.191	15.7333	15.694
2	3.5556	3.515 3.56*	4.3472	4.307	5.5000	5.459	6.9730	6.933	8.7407	8.701
3	2.6667	---	3.1529	---	3.9032	---	4.8768	---	6.0513	---
4	2.1333	2.082 2.13*	2.4734	2.430	3.0250	2.983	3.7496	3.707	4.6275	4.584
5	1.7778	---	2.0349	---	2.4694	---	3.0457	---	3.7460	---
6	1.5238	---	1.7284	---	2.0862	---	2.5642	---	3.1467	---
7	1.333	---	1.5022	---	1.8060	---	2.2143	---	2.7126	---
8	1.1852	1.142	1.3283	1.296	1.5921	1.562	1.9483	1.919	2.3838	2.353
9	1.0667	---	1.1905	---	1.4235	---	1.7394	---	2.1261	---
10	0.9697	---	1.0786	---	1.2872	---	1.5710	---	1.9187	---

$N_x = C_r\,(\pi^2 D/4a^2)$; $\lambda = a/b$, * Timoshenko, Ref. /50/

Table (19) Buckling load coefficient C_r for clamped rectangular plate

APPENDIX C COMPUTER PROGRAMMES

85

```
$JOB
        ***********************************************************
        *                                                         *
        *     PROGRAM FOR SOLVING THE SMALL DEFLECTION OF         *
        *     RECTANGULAR PLATE RESTING ON AN ELASTIC             *
        *     FOUNDATION BY THE VLASOV'S METHOD.                  *
        *                                                         *
        ***********************************************************

        W(25,25), WW(25,25) ARE THE DISPLACEMENT COEFFICIENT OF PLATE.
        XM(10),YN(10) ARE THE EIGENVALUES OF THE EIGENFUNCTION.
        AREA1(10),AREA2(10),AREA3(10),AREA4(10),AREA5(10),AREA6(10),
        AREA7(10),AREA8(10) ARE THE NOTATION OF I1,I2,I3,I4,I5,I6,I7
        I8.

      IMPLICIT REAL *8 (A-H,O-Z)
      DIMENSION W(25,25),WW(25,25),XM(10),YN(10),AREA1(10),AREA2(10),
     +AREA3(10),AREA4(10),AREA5(10),AREA6(10),AREA7(10),AREA8(10)
      READ (5,5) M,N,NN,B,K
 5    FORMAT(3I5,F5.1,1I5)
      READ 100,(XM(I),I=1,M)
100   FORMAT(6F12.8)
      READ 101,(YN(I),I=1,N)
101   FORMAT(6F12.8)
      DO 110 I=1,M
      DO 110 J=1,N
      A=0.
      XXM=XM(I)
      YYN=YN(J)

     USING THE TRAPEZOIDAL RULE OF DEFINITE INTEGRATION TO FINE I1,I2,
     I3,I4,I5,I6,I7,I8.

      CALL TRAP1(A,B,NN,XXM   ,AREA    )
      AREA1(I)=AREA
      CALL TRAP2(A,B,NN,YYN   ,AREA    )
      AREA2(J)=AREA
      CALL TRAP3(A,B,NN,XXM   ,AREA    )
      AREA3(I)=AREA
      CALL TRAP4(A,B,NN,YYN   ,AREA    )
      AREA4(J)=AREA
      CALL TRAP5(A,B,NN,YYN   ,AREA    )
      AREA5(J)=AREA
      CALL TRAP6(A,B,NN,XXM   ,AREA    )
      AREA6(I)=AREA
        CALL TRAP7(A,B,NN,XXM   ,AREA    )
        AREA7(I)=AREA
        CALL TRAP8(A,B,NN,YYN   ,AREA    )
        AREA8(J)=AREA
110     CONTINUE
        PRINT 120,(AREA1(I),I=1,M)
```

```
120    FORMAT(///,6F12.6)
       PRINT 120,(AREA2(J),J=1,N)
       PRINT 120,(AREA3(I),I=1,M)
       PRINT 120,(AREA4(J),J=1,N)
       PRINT 120,(AREA5(J),J=1,N)
       PRINT 120,(AREA6(I),I=1,M)
       PRINT 120,(AREA7(I),I=1,M)
       PRINT 120,(AREA8(J),J=1,N)
       DO 130 I=1,M
       DO 130 J=1,N

    TO FIND THE DEFLECTION EXPANSION COEFFICIENT W.

       W(I,J)=AREA7(I)*AREA8(J)/(AREA1(I)*AREA2(J)+2*AREA3(I)*AREA4(J)+
     + AREA5(J)*AREA6(I)+K*AREA2(J)*AREA6(I))
130    CONTINUE
       DO 140 I=1,M
140    PRINT 141,(W(I,J),J=1,N)
141    FORMAT(///,6F12.7)
       SUMY=0.0
       DO 150 I=1,M
       DO 150 J=1,N
       X=0.5
       Y=0.5

    THE VALUES OF THE DEFLECTION COEFFICIENT AT THE CENTER OF THE
    PLATE.

       WW(I,J)=W(I,J)*F1(XM(I),X)*F10(YN(J),Y)
       PRINT 142, WW(I,J)
142    FORMAT(/,6F12.7)
       SUMY=SUMY+WW(I,J)
150    CONTINUE
       WW(M,N)=SUMY
       PRINT 151,WW(M,N)
151    FORMAT(F12.7)
       STOP
       END
       FUNCTION F1(XXM,    X)
       IMPLICIT REAL *8 (A-H,O-Z)
       F1=DSIN(XXM*X)
       RETURN
       END
       FUNCTION F10(YYN,    Y)
       IMPLICIT REAL *8(A-H,O-Z)
       F10=DSIN(YYN*Y)- DSIN(YYN)/ DSINH(YYN)*DSINH(YYN*Y)
       RETURN
       END
       FUNCTION F20(YYN , Y)
       IMPLICIT REAL *8 (A-H,O-Z)
       DIMENSION YN(10)
       F20=-DSIN(YYN*Y)-DSIN(YYN)/DSINH(YYN)*DSINH(YYN*Y)
```

```
      RETURN
      END
      FUNCTION F2(XXM   ,X)
      IMPLICIT REAL *8 (A-H,O-Z)
      B=1.0000000
      F2=-DSIN(XXM*X)
      RETURN
      END
      FUNCTION F3(XXM   ,X)
      IMPLICIT REAL *8 (A-H,O-Z)
      F3=DCOSH(XXM   *X)-DCOS(XXM   *X)-0.98250*(DSINH(XXM   *X)
     + -DSIN(XXM   *X))
      RETURN
      END
      FUNCTION F30(YYN   ,Y)
      IMPLICIT REAL *8 (A-H,O-Z)
      B=1.0
      F30=DCOSH(YYN   *Y)-DCOS(YYN   *Y)-0.98250*(DSINH(YYN   *Y)
     + -DSIN(YYN   *Y))
      RETURN
      END
      FUNCTION F4(X)
      IMPLICIT REAL *8 (A-H,O-Z)
      XM=4.7300
      B=1.0
      F4=-DSIN(XM*X/B)*DSIN(XM*X/B)+(0.0177*DSINH(XM*X/B))**2
      RETURN
      END
      SUBROUTINE TRAP1(A,B,NN,XXM,    AREA     )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 10 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM    ,X)*F1(XXM    ,Y)*(XXM    /2)**4
 10   CONTINUE
      AREA    =H/2*(F1(XXM    ,A)*F1(XXM,   A)*(XXM/2   )**4       +2*SUMY
     + +F1(XXM   ,B)*F1(XXM,   B)*(XXM   /2)**4)
      RETURN
      END
      SUBROUTINE TRAP5(A,B,NN,YYN   ,AREA     )
      IMPLICIT REAL *8 (A-H,O-Z )
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 11 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN   ,Y)*F10(YYN   ,Y)*(YYN   /2)**4
 11   CONTINUE
     AREA     =H/2*(F10(YYN   ,A)*F10(YYN   ,A)*(YYN   /2)**4+2*SUMY
    *+F10(YYN   ,B)*F10(YYN   ,B)*(YYN   /2)**4)
      RETURN
      END
      SUBROUTINE TRAP6(A,B,NN,XXM   ,AREA     )
```

87

```
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 20 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM   ,X)*F1(XXM   ,X)
 20   CONTINUE
      AREA     =H/2*(F1(XXM   ,A)**2+2*SUMY+F1(XXM   ,B)**2)
      RETURN
      END
      SUBROUTINE TRAP2(A,B,NN,YYN   ,AREA     )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0.0
      H=(B-A)/NN
      N1=NN-1
      DO 21 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN   ,Y)*F10(YYN   ,Y)
 21   CONTINUE
      AREA     =H/2*(F10(YYN   , A)**2+2*SUMY+F10(YYN   ,B)**2)
      RETURN
      END
      SUBROUTINE TRAP3(A,B,NN,XXM   ,AREA     )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 30 I=1,N1
      X=A+I*H
      SUMY=SUMY+(XXM   /2)**2*F1(XXM   ,X)*F2(XXM   ,X)
 30   CONTINUE
      AREA     =H/2*(F2(XXM   ,A)*F1(XXM   ,A)*(XXM   /2)**2+2*SUMY
     + +F1(XXM   ,B)*F2(XXM   ,B)*(XXM   /2)**2)
      RETURN
      END
      SUBROUTINE TRAPA(A,B,NN,YYN   ,AREA     )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 31 I=1,N1
      Y=A+I*H
      SUMY=SUMY+(YYN   /2)**2*F10(YYN   ,Y)*F20(YYN   ,Y)
 31   CONTINUE
      AREA     =H/2*(F20(YYN   ,   A)*F10(YYN   ,A)*(YYN   /2)**2+2*SUMY
     ++F10(YYN   ,B)*F20(YYN   ,B)*(YYN   /2)**2)
      RETURN
      END
      SUBROUTINE TRAP7(A,B,NN,XXM   ,AREA     )
      IMPLICIT REAL*8 (A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 70 I=1,N1
```

```
      X=A+I*H
      SUMY=SUMY+F1(XXM   ,X)
 70   CONTINUE
      AREA      =H/2*(F1(XXM   ,A)+2*SUMY+F1(XXM   ,B))
      RETURN
      END
      SUBROUTINE TRAP8(A,B,NN,YYN   ,AREA       )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 71  I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN   ,Y)
 71   CONTINUE
      AREA      =H/2*(F10(YYN   , A)+2*SUMY+F10(YYN   ,B))
      RETURN
      END

           50  1.0      0
```

```
$JOB
C
C        *****************************************************
C        *                                                   *
C        *     PROGRAM FOR SOLVING THE SMALL DEFLECTION OF   *
C        *     RECTANGULAR SANDWICH PLATE BY USING THE       *
C        *     VLASOV'S METHOD.                              *
C        *                                                   *
C        *****************************************************
C
C
C
      IMPLICIT REAL *8 (A-H,O-Z)
      DIMENSION W(25,25),WW(25,25),XM(10),YN(10),AREA1(10),AREA2(10),
     +AREA3(10),AREA4(10),AREA5(10),AREA6(10),AREA7(10),AREA8(10),
     +FRE(30,30)
      READ (5,5) M,N,NN,B,C,U,Q, VF,GC,EF
    5 FORMAT(3I5,1F5.1,4F8.6,1F8.1,1F10.1)
      READ 100,(XM(I),I=1,M)
  100 FORMAT(5F15.8)
      READ 101,(YN(I),I=1,N)
  101 FORMAT(5F15.8)
      DO 110 I=1,M
      DO 110 J=1,N
      A=0.
      XXM=XM(I)
      YYN=YN(J)
      CALL TRAP1(A,B,NN,XXM    ,AREA    )
      AREA1(I)=AREA
      CALL TRAP2(A,B,NN,YYN,C   ,AREA     )
      AREA2(J)=AREA
      CALL TRAP3(A,B,NN,XXM   ,AREA     )
      AREA3(I)=AREA
      CALL TRAP4(A,B,NN,YYN,C,AREA     )
      AREA4(J)=AREA
      CALL TRAP5(A,B,NN,YYN,C,AREA     )
      AREA5(J)=AREA
      CALL TRAP6(A,B,NN,XXM   ,AREA     )
      AREA6(I)=AREA
       CALL TRAP7(A,B,NN,XXM   ,AREA     )
       AREA7(I)=AREA
       CALL TRAP8(A,B,NN,YYN,C,AREA     )
       AREA8(J)=AREA
  110  CONTINUE
      PRINT 120,(AREA1(I),I=1,M)
  120 FORMAT(///,6F12.6)
       PRINT 120,(AREA2(J),J=1,N)
       PRINT 120,(AREA3(I),I=1,M)
       PRINT 120,(AREA4(J),J=1,N)
       PRINT 120,(AREA5(J),J=1,N)
       PRINT 120,(AREA6(I),I=1,M)
       PRINT 120,(AREA7(I),I=1,M)
       PRINT 120,(AREA8(J),J=1,N)
       DO 130 I=1,M
       DO 130 J=1,N
       W(I,J)= (AREA7(I)*AREA8(J)-U*Q*EF*AREA7(I)*AREA8(J)/(AREA2(J)*
```

```
     + AREA6(I))/(2*(1-VF**2)*GC*4)*(AREA2(J)*AREA3(I)+AREA4(J)*
     + AREA6(I))/(AREA1(I)*AREA2(J)+2*AREA3(I)*AREA4(J)+AREA5(J)*
     + AREA6(I))
 130   CONTINUE
       SUMY=0.0
       DO 150  I=1,M
       DO 150  J=1,N
       X=0.5
       Y=0.5
       WW(I,J)=WPI,J)*F1(XM(I),X)*F10(YN(J),Y)
       SUMY=SUMY+WW(I,J)
 150   CONTINUE
       WW(M,N)=SUMY
       PRINT 151,WW(M,N)
 151   FORMAT(F12.7)
       STOP
       END
       FUNCTION F1(XXM,    X)
       IMPLICIT REAL *8 (A-H,O-Z)
       F1=DCOSH(XXM    *X)-DCOS(XXM    *X)
     + -((DCOSH(XXM)-DCOS(XXM))/(DSINH(XXM)-DSIN(XXM)))*
     + (DSINH(XXM*X)-DSIN(XXM*X))
       RETURN
       END
       FUNCTION F10(YYN,    Y)
       IMPLICIT REAL *8(A-H,O-Z)
       F10=DCOSH(YYN   *Y)-DCOS(YYN   *Y)
     + -((DCOSH(YYN)-DCOS(YYN))/(DSINH(YYN)-DSIN(YYN)))*
     + (DSINH(YYN*Y)-DSIN(YYN   *Y))
       RETURN
       END
       FUNCTION F20(YYN  ,  Y)
       IMPLICIT REAL *8  (A-H,O-Z)
       DIMENSION YN(10)
       F20=DCOSH(YYN   *Y)+DCOS(YYN   *Y)
     + -((DCOSH(YYN)-DCOS(YYN))/(DSINH(YYN)-DSIN(YYN)))*
     + (DSINH(YYN*Y)+DSIN(YYN*Y))
       RETURN
       END
       FUNCTION F2(XXM   ,X)
       IMPLICIT REAL *8  (A-H,O-Z)
       B=1.0000000
       F2=DCOSH(XXM   *X)+DCOS(XXM   *X)
     + -((DCOSH(XXM)-DCOS(XXM))/(DSINH(XXM)-DSIN(XXM)))*
     + (DSINH(XXM*X)+DSIN(XXM    *X))
       RETURN
       END
       FUNCTION F3(XXM   ,X)
       IMPLICIT REAL *8  (A-H,O-Z)
       F3=DCOSH(XXM   *X)-DCOS(XXM   *X)-0.33250*(DSINH(XXM   *X)
     + -DSIN(XXM   *X))
       RETURN
       END
       FUNCTION F30(YYN   ,Y)
       IMPLICIT REAL *8(A-H,O-Z)
```

```
      B=1.0
      F30=DCOSH(YYN   *Y)-DCOS(YYN   *Y)-0.9825*(DSINH(YYN   *Y)
     *-DSIN(YYN   *Y))
      RETURN
      END
      FUNCTION F4(X)
      IMPLICIT REAL *8 (A-H,O-Z)
      XM=4.7300
      B=1.0
      F4=-DSIN(XM*X/B)*DSIN(XM*X/B)+(0.0177*DSINH(XM*X/B))**2
      RETURN
      END
      SUBROUTINE TRAP1(A,B,NN,XXM,     AREA     )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 10 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM    ,X)*F1(XXM    ,X)*(XXM    /2)**4
   10 CONTINUE
      AREA    =H/2*(F1(XXM    ,A)*F1(XXM,   A)*(XXM/2   )**4       +2*SUMY
     *+F1(XXM    ,B)*F1(XXM,   B)*(XXM   /2)**4)
      RETURN
      END
      SUBROUTINE TRAPS(A,B,NN,YYN,C,AREA     )
      IMPLICIT REAL *8 (A-H,O-Z )
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 11 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN    ,Y)*F10(YYN    ,Y)*(YYN*C/2)**4
   11 CONTINUE
      AREA     =H/2*(F10(YYN    ,A)*F10(YYN    ,A)*(YYN*C/2)**4+2*SUMY
     *+F10(YYN    ,B)*F10(YYN    ,B)*(YYN*C/2)**4)
      RETURN
      END
      SUBROUTINE TRAP6(A,B,NN,XXM    ,AREA     )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 20 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM    ,X)*F1(XXM    ,X)
   20 CONTINUE
      AREA     =H/2*(F1(XXM    ,A)**2+2*SUMY+F1(XXM    ,B)**2)
      RETURN
      END
      SUBROUTINE TRAP2(A,B,NN,YYN,C,AREA     )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0.0
      H=(B-A)/NN
      N1=NN-1
```

```
      DO 21 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN   ,Y)*F10(YYN   ,Y)
 21   CONTINUE
      AREA     =H/2*(F10(YYN   , A)**2+2*SUMY+F10(YYN   ,B)**2)
      RETURN
      END
      SUBROUTINE TRAP3(A,B,NN,XXM   ,AREA    )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 30 I=1,N1
      X=A+I*H
      SUMY=SUMY+(XXM   /2)**2*F1(XXM   ,X)*F2(XXM   ,X)
 30   CONTINUE
      AREA     =H/2*(F2(XXM   ,A)*F1(XXM   ,A)*(XXM   /2)**2+2*SUMY
     ++F1(XXM   ,B)*F2(XXM   ,B)*(XXM   /2)**2)
      RETURN
      END
      SUBROUTINE TRAP4(A,B,NN,YYN,C,AREA       )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 31 I=1,N1
      Y=A+I*H
      SUMY=SUMY+(YYN*C/2)**2*F10(YYN   ,Y)*F20(YYN   ,Y)
 31   CONTINUE
      AREA     =H/2*(F20(YYN   ,    A)*F10(YYN   ,A)*(YYN*C/2)**2+2*SUMY
     ++F10(YYN   ,B)*F20(YYN   ,B)*(YYN*C/2)**2)
      RETURN
      END
      SUBROUTINE TRAP7(A,B,NN,XXM   ,AREA       )
      IMPLICIT REAL*8 (A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 70 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM   ,X)
 70   CONTINUE
      AREA     =H/2*(F1(XXM   ,A)+2*SUMY+F1(XXM   ,B))
      RETURN
      END
      SUBROUTINE TRAP8(A,B,NN,YYN,C,AREA       )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 71 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN   ,Y)
 71   CONTINUE
      AREA     =H/2*(F10(YYN   , A)+2*SUMY+F10(YYN   ,B))
```

```
      RETURN
      END
```

```
C     ************************************************************
C     *                                                          *
C     *  PROGRAM FOR SOLVING THE FREE VIBRATION OF THE RECTANGULAR *
C     *    PLATES AND SANDWICH PLATE WITH VARIOUS BOUNDARY CONDITIONS *
C     *    BY USING VLASOV'S METHOD.                             *
C     *                                                          *
C     ************************************************************
C
C
C     FRE(30,30) IS THE FREQUENCY PARAMETER OF THE FREE VIBRATION OF
C     PLATE.
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION W(25,25),WW(25,25),XM(10),YN(10),AREA1(10),AREA2(10),
     +AREA3(10),AREA4(10),AREA5(10),AREA6(10),AREA7(10),AREA8(10),
     +FRE(30,30)
      READ (2,5) M,N,NN,B,C,K
    5 FORMAT(3I5,1F5.1,1F10.5,1I5)
      READ (2,100) (XM(I),I=1,M)
  100 FORMAT(5F12.8)
      READ (2,101) (YN(I),I=1,N)
  101 FORMAT(5F12.8)
      DO 110 I=1,M
      DO 110 J=1,N
      A=0.
      XXM=XM(I)
      YYN=YN(J)
      CALL TRAP1(A,B,NN,XXM    ,AREA     )
      AREA1(I)=AREA
      CALL TRAP2(A,B,NN,YYN,C  ,AREA     )
      AREA2(J)=AREA
      CALL TRAP3(A,B,NN,XXM   ,AREA      )
      AREA3(I)=AREA
      CALL TRAP4(A,B,NN,YYN,C,AREA       )
      AREA4(J)=AREA
      CALL TRAP5(A,B,NN,YYN,C,AREA       )
      AREA5(J)=AREA
      CALL TRAP6(A,B,NN,XXM   ,AREA      )
      AREA6(I)=AREA
        CALL TRAP7(A,B,NN,XXM   ,AREA    )
        AREA7(I)=AREA
        CALL TRAP8(A,B,NN,YYN,C,AREA     )
        AREA8(J)=AREA
  110   CONTINUE
        WRITE(3,120) (AREA1(I),I=1,M)
  120   FORMAT(///,6F12.6)
        WRITE(3,120) (AREA2(J),J=1,N)
        WRITE(3,120) (AREA3(I),I=1,M)
        WRITE(3,120) (AREA4(J),J=1,N)
        WRITE(3,120) (AREA5(J),J=1,N)
        WRITE(3,120) (AREA6(I),I=1,M)
        WRITE(3,120) (AREA7(I),I=1,M)
        WRITE(3,120) (AREA8(J),J=1,N)
        DO 130 I=1,M
```

```
      DO 130  J=1,N                                                      VIB00560
      W(I,J)=AREA7(I)*AREA8(J)/(AREA1(I)*AREA2(J)+2*AREA3(I)*AREA4(J)+  VIB00570
     + AREA5(J)*AREA6(I)+K*AREA2(J)*AREA5(I))                           VIB00580
      FRE(I,J)=((          1   *AREA2(J)*AREA1(I)+2*1        *AREA3(I)  VIB00590
     + *AREA4(J)*AREA6(I)   *AREA5(J))/(AREA6(I)*AREA2(J)))             VIB00600
130   CONTINUE                                                           VIB00610
      DO 140  I=1,M                                                      VIB00620
140   WRITE(3,141)  (FRE(I,J),J=1,N)                                     VIB00630
141   FORMAT(///,6F12.5)                                                 VIB00640
      SUMY=0.0                                                           VIB00650
      DO 150  I=1,M                                                      VIB00660
      DO 150  J=1,N                                                      VIB00670
      X=0.5                                                              VIB00680
      Y=0.5                                                              VIB00690
      WW(I,J)=W(I,J)*F1(XM(I),X)*F10(YN(J),Y)                            VIB00700
      SUMY=SUMY+WW(I,J)                                                  VIB00710
150   CONTINUE                                                           VIB00720
      WW(M,N)=SUMY                                                       VIB00730
      WRITE(3,151)  WW(M,N)                                              VIB00740
151   FORMAT(F12.7)                                                      VIB00750
      STOP                                                               VIB00760
      END                                                                VIB00770
      FUNCTION F10(YYN,       Y)                                         VIB00780
      IMPLICIT REAL *8  (A-H,O-Z)                                        VIB00790
      F10=DSIN(YYN*Y)                                                    VIB00800
      RETURN                                                             VIB00810
      END                                                                VIB00820
      FUNCTION F1  (XXM,    X)                                           VIB00830
      IMPLICIT REAL *8(A-H,O-Z)                                          VIB00840
      F1  =DSIN(XXM*X)-DSIN(XXM)/DSINH(XXM)*DSINH(XXM*X)                 VIB00850
      RETURN                                                             VIB00860
      END                                                                VIB00870
      FUNCTION F2  (XXM   , X)                                           VIB00880
      IMPLICIT REAL *8  (A-H,O-Z)                                        VIB00890
      DIMENSION YN(10)                                                   VIB00900
      F2  =-DSIN(XXM*X)-DSIN(XXM)/DSINH(XXM)*DSINH(XXM*X)                VIB00910
      RETURN                                                             VIB00920
      END                                                                VIB00930
      FUNCTION F20(YYN   ,Y)                                             VIB00940
      IMPLICIT REAL *8  (A-H,O-Z)                                        VIB00950
      B=1.0000000                                                        VIB00960
      F20=-DSIN(YYN*Y)                                                   VIB00970
      RETURN                                                             VIB00980
      END                                                                VIB00990
      FUNCTION F3  (YYN   ,Y)                                            VIB01000
      IMPLICIT REAL *8  (A-H,O-Z)                                        VIB01010
      F3=DCOSH(YYN   *Y)-DCOS(YYN   *Y)-0.9*250*(DSINH(XXM   *X)         VIB01020
     + -DSIN(XXM   *X))                                                  VIB01030
      RETURN                                                             VIB01040
      END                                                                VIB01050
      FUNCTION F30(YYN   ,Y)                                             VIB01060
      IMPLICIT REAL *8(A-H,O-Z)                                          VIB01070
      B=1.0                                                              VIB01080
      F30=DCOSH(YYN   *Y)-DCOS(YYN   *Y)-0.9825*(DSINH(YYN   *Y)         VIB01090
     + -DSIN(YYN   *Y))                                                  VIB01100
```

```
      RETURN                                                            VIB01110
      END                                                               VIB01120
      FUNCTION F4(X)                                                    VIB01130
      IMPLICIT REAL *8 (A-H,O-Z)                                        VIB01140
      XM=4.7300                                                         VIB01150
      B=1.0                                                             VIB01160
      F4=-DSIN(XM*X/B)*DSIN(XM*X/B)+(0.0177*DSINH(XM*X/B))**2           VIB01170
      RETURN                                                            VIB01180
      END                                                               VIB01190
      SUBROUTINE TRAP1(A,B,NN,XXM,  AREA     )                          VIB01200
      IMPLICIT REAL *8 (A-H,O-Z)                                        VIB01210
      SUMY=0.                                                           VIB01220
      H=(B-A)/NN                                                        VIB01230
      N1=NN-1                                                           VIB01240
      DO 10 I=1,N1                                                      VIB01250
      X=A+I*H                                                           VIB01260
      SUMY=SUMY+F1(XXM    ,X)*F1(XXM    ,X)*(XXM    /2)**4              VIB01270
   10 CONTINUE                                                          VIB01280
      AREA     =H/2*(F1(XXM    ,A)*F1(XXM,   A)*(XXM/2   )**4     +2*SUMY VIB01290
     * +F1(XXM    ,B)*F1(XXM,   B)*(XXM   /2)**4)                       VIB01300
      RETURN                                                            VIB01310
      END                                                               VIB01320
      SUBROUTINE TRAP5(A,B,NN,YYN,C,AREA     )                          VIB01330
      IMPLICIT REAL *8 (A-H,O-Z )                                       VIB01340
      SUMY=0.                                                           VIB01350
      H=(B-A)/NN                                                        VIB01360
      N1=NN-1                                                           VIB01370
      DO 11 I=1,N1                                                      VIB01380
      Y=A+I*H                                                           VIB01390
      SUMY=SUMY+F10(YYN    ,Y)*F10(YYN    ,Y)*(YYN*C/2)**4              VIB01400
   11 CONTINUE                                                          VIB01410
     AREA      =H/2*(F10(YYN    ,A)*F10(YYN    ,A)*(YYN*C/2)**4+2*SUMY  VIB01420
    **F10(YYN    ,B)*F10(YYN    ,B)*(YYN*C/2)**4)                       VIB01430
      RETURN                                                            VIB01440
      END                                                               VIB01450
      SUBROUTINE TRAP6(A,B,NN,XXM    ,AREA     )                        VIB01460
      IMPLICIT REAL *8 (A-H,O-Z)                                        VIB01470
      SUMY=0.                                                           VIB01480
      H=(B-A)/NN                                                        VIB01490
      N1=NN-1                                                           VIB01500
      DO 20 I=1,N1                                                      VIB01510
      X=A+I*H                                                           VIB01520
      SUMY=SUMY+F1(XXM    ,X)*F1(XXM    ,X)                             VIB01530
   20 CONTINUE                                                          VIB01540
      AREA     =H/2*(F1(XXM    ,A)**2+2*SUMY+F1(XXM    ,B)**2)          VIB01550
      RETURN                                                            VIB01560
      END                                                               VIB01570
      SUBROUTINE TRAP2(A,B,NN,YYN,C,AREA     )                          VIB01580
      IMPLICIT REAL *8(A-H,O-Z)                                         VIB01590
      SUMY=0.0                                                          VIB01600
      H=(B-A)/NN                                                        VIB01610
      N1=NN-1                                                           VIB01620
      DO 21 I=1,N1                                                      VIB01630
      Y=A+I*H                                                           VIB01640
      SUMY=SUMY+F10(YYN    ,Y)*F10(YYN    ,Y)                           VIB01650
```

```
21    CONTINUE                                                          VIB01660
      AREA    =H/2*(F10(YYN   , A)**2+2*SUMY+F10(YYN   ,B)**2)          VIB01670
      RETURN                                                            VIB01680
      END                                                               VIB01690
      SUBROUTINE TRAP3(A,B,NN,XXM   ,AREA    )                          VIB01700
      IMPLICIT REAL *8 (A-H,O-Z)                                        VIB01710
      SUMY=0                                                            VIB01720
      H=(B-A)/NN                                                        VIB01730
      N1=NN-1                                                           VIB01740
      DO 30 I=1,N1                                                      VIB01750
      X=A+I*H                                                           VIB01760
      SUMY=SUMY+(XXM   /2)**2*F1(XXM   ,X)*F2(XXM   ,X)                 VIB01770
30    CONTINUE                                                          VIB01780
      AREA    =H/2*(F2(XXM   ,A)*F1(XXM   ,A)*(XXM   /2)**2+2*SUMY      VIB01790
     * +F1(XXM   ,B)*F2(XXM   ,B)*(XXM   /2)**2)                        VIB01800
      RETURN                                                            VIB01810
      END                                                               VIB01820
      SUBROUTINE TRAP4(A,B,NN,YYN,C,AREA     )                          VIB01830
      IMPLICIT REAL *8(A-H,O-Z)                                         VIB01840
      SUMY=0                                                            VIB01850
      H=(B-A)/NN                                                        VIB01860
      N1=NN-1                                                           VIB01870
      DO 31 I=1,N1                                                      VIB01880
      Y=A+I*H                                                           VIB01890
      SUMY=SUMY+(YYN*C/2)**2*F10(YYN   ,Y)*F20(YYN   ,Y)                VIB01900
31    CONTINUE                                                          VIB01910
      AREA    =H/2*(F20(YYN   ,    A)*F10(YYN   ,A)*(YYN*C/2)**2+2*SUMY VIB01920
     ++F10(YYN   ,B)*F20(YYN   ,B)*(YYN*C/2)**2)                        VIB01930
      RETURN                                                            VIB01940
      END                                                               VIB01950
      SUBROUTINE TRAP7(A,B,NN,XXM   ,AREA    )                          VIB01960
      IMPLICIT REAL*8 (A-H,O-Z)                                         VIB01970
      SUMY=0                                                            VIB01980
      H=(B-A)/NN                                                        VIB01990
      N1=NN-1                                                           VIB02000
      DO 70 I=1,N1                                                      VIB02010
      X=A+I*H                                                           VIB02020
      SUMY=SUMY+F1(XXM   ,X)                                            VIB02030
70    CONTINUE                                                          VIB02040
      AREA    =H/2*(F1(XXM   ,A)+2*SUMY+F1(XXM   ,B))                   VIB02050
      RETURN                                                            VIB02060
      END                                                               VIB02070
      SUBROUTINE TRAP8(A,B,NN,YYN,C,AREA     )                          VIB02080
      IMPLICIT REAL *8(A-H,O-Z)                                         VIB02090
      SUMY=0                                                            VIB02100
      H=(B-A)/NN                                                        VIB02110
      N1=NN-1                                                           VIB02120
      DO 71 I=1,N1                                                      VIB02130
      Y=A+I*H                                                           VIB02140
      SUMY=SUMY+F10(YYN   ,Y)                                           VIB02150
71    CONTINUE                                                          VIB02160
      AREA    =H/2*(F10(YYN   , A)+2*SUMY+F10(YYN   ,B))                VIB02170
      RETURN                                                            VIB02180
      END                                                               VIB02190
```

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FILE: BUC1     WATFIV     *    UNIV O'/OF OTTAWA CMS REL   E 4                    PAGE 00001

$JOB
C     *************************************************************
C     *                                                           *
C     *   PROGRAM FOR SOLVING THE BUCKLING PROBLEMS OF RECTANGULAR *
C     *   PLATE BY USING THE VLASOV'S METHOD.                     *
C     *                                                           *
C     *************************************************************
C
C     CR IS THE BUSKLING LOAD COEFFICIENTS.
C
      IMPLICIT REAL *8 (A-H,O-Z)
      DIMENSION W(25,25),WW(25,25),XM(10),YN(10),AREA1(10),AREA2(10),
     +AREA3(10),AREA4(10),AREA5(10),AREA6(10),AREA7(10),AREA8(10),
     +CR(25,25)
      READ (5,5) M,N,NN,B,C,XX
    5 FORMAT(3I5,1F5.1,1F10.6,1F5.4)
      READ 100,(XM(I),I=1,M)
  100 FORMAT(5F12.8)
      READ 101,(YN(I),I=1,N)
  101 FORMAT(5F12.8)
      DO 110 I=1,M
      DO 110 J=1,N
      A=0.
      XXM=XM(I)
      YYN=YN(J)
      CALL TRAP1(A,B,NN,XXM    ,AREA    )
      AREA1(I)=AREA
      CALL TRAP2(A,B,NN,YYN    ,AREA    )
      AREA2(J)=AREA
      CALL TRAP3(A,B,NN,XXM    ,AREA    )
      AREA3(I)=AREA
      CALL TRAP4(A,B,NN,YYN    ,AREA    )
      AREA4(J)=AREA
      CALL TRAP5(A,B,NN,YYN    ,AREA    )
      AREA5(J)=AREA
      CALL TRAP6(A,B,NN,XXM    ,AREA    )
      AREA6(I)=AREA
        CALL TRAP7(A,B,NN,XXM    ,AREA    )
        AREA7(I)=AREA
        CALL TRAP8(A,B,NN,YYN    ,AREA    )
        AREA8(J)=AREA
  110   CONTINUE
        PRINT 120,(AREA1(I),I=1,M)
  120   FORMAT(///,6F12.6)
        PRINT 120,(AREA2(J),J=1,N)
        PRINT 120,(AREA3(I),I=1,M)
        PRINT 120,(AREA4(J),J=1,N)
        PRINT 120,(AREA5(J),J=1,N)
        PRINT 120,(AREA6(I),I=1,M)
        PRINT 120,(AREA7(I),I=1,M)
        PRINT 120,(AREA8(J),J=1,N)
        DO 130  I=1,M
        DO 130  J=1,N
        CR(I,J)=(AREA1(I)*AREA2(J)+2*C**2*AREA3(I)*AREA4(J)+C**4*
```

```
     +   AREA5(J)*AREA6(I))/(AREA2(J)*AREA3(I)+XX*C**2*AREA4(J)*AREA8(I))
     +   *4/9.8696
 130     CONTINUE
         DO 140 I=1,M
 140     PRINT 141,(CR(I,J),J=1,N)
 141     FORMAT(///,6F12.7)
         STOP
         END
         FUNCTION F1(XXM,      X)
         IMPLICIT REAL *8  (A-H,O-Z)
         F1=1-DCOS(XXM*X)
         RETURN
         END
         FUNCTION F10(YYN,     Y)
         IMPLICIT REAL *8(A-H,O-Z)
         F10=(1-DCOS(YYN*Y))
         RETURN
         END
         FUNCTION F20(YYN  ,  Y)
         IMPLICIT REAL *8  (A-H,O-Z)
         DIMENSION YN(10)
         F20=DCOS(YYN*Y)
         RETURN
         END
         FUNCTION F2(XXM   ,X)
         IMPLICIT REAL *8  (A-H,O-Z)
         B=1.0000000
         F2=DCOS(XXM*X)
         RETURN
         END
         FUNCTION F3(XXM   ,X)
         IMPLICIT REAL *8  (A-H,O-Z)
         F3=-DCOS(XXM*X)
         RETURN
         END
         FUNCTION F30(YYN   ,Y)
         IMPLICIT REAL *8(A-H,O-Z)
         B=1.0
         F30=-DCOS (YYN   *Y)
         RETURN
         END
         FUNCTION F4(X)
         IMPLICIT REAL *8  (A-H,O-Z)
         XM=4.7300
         B=1.0
         F4=-DSIN(XM*X/B)*DSIN(XM*X/B)+(0.0177*DSINH(XM*X/B))**2
         RETURN
         END
         SUBROUTINE TRAP1(A,B,NN,XXM,    ,AREA    )
         IMPLICIT REAL *8  (A-H,O-Z)
         SUMY=0.
         H=(B-A)/NN
         N1=NN-1
         DO 10 I=1,N1
         X=A+I*H
```

```
      SUMY=SUMY+F1(XXM    ,X)*F3(XXM    ,X)*(XXM    /2)**4
10    CONTINUE
      AREA     =H/2*(F1(XXM    ,A)*F3(XXM,   A)*(XXM/2    )**4       +2*SUMY
    + +F1(XXM   ,B)*F3(XXM,   B)*(XXM   /2)**4)
      RETURN
      END
      SUBROUTINE TRAP5(A,B,NN,YYN    ,AREA     )
      IMPLICIT REAL *8 (A-H,O-Z )
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 11 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN    ,Y)*F30(YYN    ,Y)*(YYN    /2)**4
11    CONTINUE
     AREA     =H/2*(F10(YYN    ,A)*F30(YYN    ,A)*(YYN    /2)**4+2*SUMY
    ++F10(YYN    ,B)*F30(YYN    ,B)*(YYN    /2)**4)
      RETURN
      END
      SUBROUTINE TRAP5(A,B,NN,XXM    ,AREA      )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0.
      H=(B-A)/NN
      N1=NN-1
      DO 20 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM    ,X)*F1(XXM    ,X)
20    CONTINUE
      AREA     =H/2*(F1(XXM    ,A)**2+2*SUMY+F1(XXM    ,B)**2)
      RETURN
      END
      SUBROUTINE TRAP2(A,B,NN,YYN    ,AREA      )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0.0
      H=(B-A)/NN
      N1=NN-1
      DO 21 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN    ,Y)*F10(YYN    ,Y)
21    CONTINUE
      AREA     =H/2*(F10(YYN    ,A)**2+2*SUMY+F10(YYN    ,B)**2)
      RETURN
      END
      SUBROUTINE TRAP3(A,B,NN,XXM    ,AREA      )
      IMPLICIT REAL *8 (A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 30 I=1,N1
      X=A+I*H
      SUMY=SUMY+(XXM    /2)**2*F1(XXM    ,X)*F2(XXM    ,X)
30    CONTINUE
      AREA     =H/2*(F2(XXM    ,A)*F1(XXM    ,A)*(XXM    /2)**2+2*SUMY
    + +F1(XXM    ,B)*F2(XXM    ,B)*(XXM    /2)**2)
      RETURN
```

```
      END
      SUBROUTINE TRAP4(A,B,NN,YYN   ,AREA     )
      IMPLICIT REAL*8(A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 31 I=1,N1
      Y=A+I*H
      SUMY=SUMY+(YYN   /2)**2*F10(YYN   ,Y)*F20(YYN   ,Y)
 31   CONTINUE
      AREA    =H/2*(F20(YYN   ,    A)*F10(YYN   ,A)*(YYN   /2)**2+2*SUMY
     *+F10(YYN   ,B)*F20(YYN   ,B)*(YYN   /2)**2)
      RETURN
      END
      SUBROUTINE TRAP7(A,B,NN,XXM   ,AREA     )
      IMPLICIT REAL*8 (A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 70 I=1,N1
      X=A+I*H
      SUMY=SUMY+F1(XXM   ,X)
 70   CONTINUE
      AREA    =H/2*(F1(XXM   ,A)+2*SUMY+F1(XXM   ,B))
      RETURN
      END
      SUBROUTINE TRAP8(A,B,NN,YYN   ,AREA      )
      IMPLICIT REAL *8(A-H,O-Z)
      SUMY=0
      H=(B-A)/NN
      N1=NN-1
      DO 71 I=1,N1
      Y=A+I*H
      SUMY=SUMY+F10(YYN   ,Y)
 71   CONTINUE
      AREA    =H/2*(F10(YYN   ,  A)+2*SUMY+F10(YYN   ,B))
      RETURN
      END
```

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