

Identifying Common Trends and Common Cycles. The  
Case of Colombian Sectoral Output and US Regional  
Income

by

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### **Abstract**

This paper presents evidence on common cycles and common trends in real sectoral output in Colombia for the period 1926 - 1997 and in real per-capita personal income for the eight major regions of the United States for the period 1950 - 1990, using annual data in both cases. For the Colombian case, evidence of one cointegrating relationship and four common cycles is found among the five sectors. Agriculture is found to be correlated contemporaneously and at lag one to the sector of Construction. In the variance decomposition, the transitory shocks seem to be more important than permanent shocks. In the US case, the tests find two cointegrating relationships and six common cycles among the eight regions. The regions New England and Plains and Great Lakes and Rocky Mountains are highly correlated. In the variance decomposition, the transitory shocks are again observed to be more important than permanent shocks and decreasing in a horizon of four years.

**Keywords:** Common Cycles, Common Trends, Permanent and Transitory Components, Sectoral Output, Regional Income, Variance Decomposition.

**JEL Classification:** C32, C52, E32

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# 1 Introduction

Often data is characterized by features such as trends, cycles and serial correlation. As it is defined by Engle and Kozicki (1993), a feature is common if a linear combination of the series fails to have the feature even though each of the series individually has the feature.

Common trends and common cycles appear in the data as restrictions placed upon the VARs that contain the variables for each case. Interactions between long-run and short-run can then be modeled. Furthermore, under special cases, this approach provides a unique trend-cycle decomposition that allows to measure the relative importance of the permanent and transitory shock on the variation of the data.

The concept of common cofeatures was first proposed by Engle and Kozicki (1993) and further developed by Vahid and Engle (1993) and by Engle and Issler (1995). We investigated evidence of the features of common cycles and common trends in real sectoral output in Colombia for the period 1926 - 1997 and in real per-capita personal income for the eight major regions of the United States for the period 1950 - 1990, using annual data in both cases.

For the Colombian case we find one cointegrating relationship and four common cycles among the five sectors, while in the variance decomposition we find that transitory shocks are more important than permanent shocks. In this aspect, a similar result is obtained in the case of the US regions. However, the order of the innovations in the process of orthogonalization in the variance decomposition affects the relative importance of permanent and transitory components in the variation of per capita income among the eight regions.

The paper is organized as follows. Section 1 presents a summary of previous literature on the topic. Section 2 reviews the theoretical issues related to stationarity, cointegration and common trends and cycles. Section 3 discusses the empirical evidence divided into two cases, starting with the analysis of common trends and cycles in Colombian sectoral output, followed by the analysis in the US per capita regional income. Section 4 concludes. Tables and figures are presented at the end of the paper.

# 2 Previous Literature

The notion of common features was introduced by Engle and Kozicki (1993) and an application to the US sectoral output was performed by Vahid and Engle (1993). They analyzed the existence of common trends and common cycles among per-capita income of four US regions: New England, Mideast, Great Lakes and Far West. The data consisted of annual per-capita income for the period of 1948 to 1990 at constant prices of 1985. Using the methodology proposed by Johansen (1988, 1995), they found one cointegrating relationship among these regions which is equivalent to find three common trends. For the identification of common cycles, they followed the suggestion of Engle and Kozicki (1993) finding two cofeatures vectors or two common cycles among the

four regions.

On the other hand, Engle and Issler (1995) investigated the degree of short-run and long-run comovements in the US sectoral output data by estimating sectoral common trends and common cycles, conditional upon a theoretical model developed by Long and Plosser (1993). The data consisted of sectoral per-capita GNP series for the period of 1947 to 1989 at the single-digit SIC level. The sectors taken into account were Agriculture, Forestry and Fisheries; Construction; Mining; Wholesale and Retail Trade; Manufacturing; Transportation and Public Utilities; Finance, Insurance and Real Estate; and Services. Two cointegrating vectors were found in the data. Furthermore, two idiosyncratic serially correlated cycles and six idiosyncratic trends were found to be shared by the eight sectors studied.

A trend-cycle decomposition of the data was performed finding that sectors had very similar cyclical behavior and very different trend behaviors. Therefore, sectors were divided into two sub-groups according to similarity of their cycles: five out of eight sectoral cycles are in accordance to NBER recessions and were called pro-cyclical (Construction; Mining; Wholesale and Retail Trade; Manufacturing; Finance; and Insurance and Real Estate). The other three (Agriculture, Forestry and Fisheries; Transportation and Public Utilities; and Services) were called counter-cyclical since they were not in accordance to NBER recession because of upward movements during those periods. In the first group, Construction had the highest amplitude cycle, which makes economic sense. For the second group, the sectors had very similar cycles in shape, duration and amplitude. Moreover, it seemed that the shape of these cycles was just an upside down version of the pro-cyclical ones. However, in contrast to the results, the behavior of the Agriculture was odd since its plot is almost flat and increased in four out of seven recessions.

According to the above results, trend fluctuations were found to be closely related to productivity shocks, that is, they were a consequence of the impulse mechanism at work. Finally, in the variance decomposition analysis, the Wholesale and Retail Trade sector; and Manufacturing sector presented relatively important transitory shocks.

In other context, Pain and Thomas (1997) studied the existence of common trends and common cycles in the movements of real interest rates of industrial countries. Therefore, real interest rate movements were decomposed into a trend (random walk) element and a cyclical (stationary moving average) element using the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981). A common trends and cycles representation was also derived using the theory of cointegration and the theory of cofeatures developed by Vahid and Engle (1993), considering linkages between European short-term real interest rates.

First, short-term real interest rates in the three major European economies (Germany, France and the United Kingdom) were examined, observing linkages between short-term nominal interest rates. The study also investigated whether German interest rates tend to drive common movements among other European rates, that is, if the German rate represented the single common trend on which the other rates depend in the long run. In addition, they tested how the addition

of the United States to this European system affects the robustness of the results. Furthermore, the linkages between long-term real interest rates of the major G3 economies (the United States, Germany and Japan) was analyzed

Pain and Thomas (1997) interpreted the existence of a single common trend among the three rates as a common world real interest rate. As a measure of short-term European nominal interest rates, quarterly averages of three-month Euromarket rates from 1968:1 to 1994:3 are used, except for France, where a three-month inter-bank rate was taken into account. The use of Euromarket rates was intending to avoid any problems associated with periods when exchange controls operate. A ten-year government bond yields were used for long-term nominal interest rates in the G3 countries.

Using the approach of Johansen (1988, 1995), the existence of two cointegrating vectors between the European interest rates showed the existence of a single common trend. Testing for common features found that the data supported one common trend. By normalizing the cofeature vector to one, Germany's interest rate was observed to have the dominant "share" of the common trend. When the United States was introduced to the data as representative of overseas rates, a single common trend remained. However, German leadership was rejected in favor of the US (overseas) leadership.

Furthermore, the study found that linkages between long-term rates among the G3 appear stronger in the post-1980 period. This would be consistent with the effect of financial liberalization in increasing capital market integration. But there is little evidence that this common trend is some weighted average of real interest rates.

In another study, Breitung and Candelon (2000) used the serial correlation common feature procedure proposed by Engle and Kozicki (1993). They applied this approach to study the degree of synchronization between three European countries (Germany, Austria and the United Kingdom). The data used was monthly seasonally unadjusted industrial production indices for the period ranging from 1975:1-1997:4 extracted from the data-stream database. Two systems were analyzed: the first one is conformed by Germany and Austria and the second one is conformed by Germany and the United Kingdom. Surprisingly, the evidence supported that for both cases the tests for the presence of common cycle was rejected, stressing the surprisingly low degree of synchronization between European economies.

In the regional context, Carlino and Sill (2001) investigated trends and cycles dynamics in per-capita income for six regions of the US (New England, Midwest, Great Lakes, Plains, Southeast, Southwest and Far West), covering the period of 1956 to 1995. They used the same methodology used by Vahid and Engle (1993). The results of the likelihood ratio test for cointegration suggested that the series presented two cointegrating relations and five common trends, which differ from the analysis of Vahid and Engle (1993). This can be due to the fact that Carlino and Sill (2001) included seven regions in their data, while Vahid and Engle (1993) included only four. Moreover, the system was characterized by four canonical correlations meaning that it contained four independent cofeature vectors or four common cycles. Therefore, they concluded that the seven regions

do share common synchronous cycles.

One of the interesting aspects analyzed by Carlino and Sill (2001) is the volatility of the regional cycles. In fact, the variance analysis showed considerable differences in the volatility of regional cycles, finding that the standard deviation of the cyclical component of South East (the most volatile region) is almost five times as great as the one of Far West (the less volatile). The regions of New England, Mideast, Southeast and Southwest had cyclical components more volatile than the national average, while cycles in the Far West tended to be less volatile than the national average. Far West region tended to commove with the other six regions and with the nation, but the extent of that correlation was found much weaker than it was for all other regions.

Recently, Harvey and Mills (2002) have shown evidence for common features in UK sectoral output data, estimating a pseudo-structural model and performing a trend-cycle decomposition in order to analyze more deeply UK's output components. They used the concept of cofeatures (Engle and Kozicki, 1993; Vahid and Engle, 1993) following closely the approach of Engle and Issler (1995) in their study of US sectoral output. Harvey and Mills (2002) used indexes of gross value added to output at constant 1995 prices for six sectors in UK: agriculture, forestry and fishing, mining and quarrying, including oil and gas extraction, manufacturing, electricity, gas and water supply, construction, and services; covering the period of 1948 to 1998. They found strong evidence of common trends and cycles among the six sectors considered. According to their results, the six series were generated by three common trends and three common cycles.

The sectors manufacturing; electricity, gas and water supply; and Services followed almost identically shaped cycles. As a particular characteristic, they found that the same sectors (manufacturing; electricity, gas and water supply; and Services), present steadily increasing trends while the other three showed that, even though still increasing, had more variable trends. Another issue was the importance of transitory and permanent shocks for the variation of sectoral output. Following the approach suggested by Engle and Issler (1995), they found that agriculture and service sectors were both eventually dominated by permanent shocks, whereas construction and manufacturing are more influenced by transitory shocks.

In the empirical section, we closely follow the approach used by Harvey and Mills (2002) using sectoral output data for Colombia and per-capita income data for the eight US regions.

### 3 Theoretical Issues

In this section we review the concepts of stationarity, cointegration, common trends and common cycles within a VAR framework.

### 3.1 Stationarity

Let  $x_t$  be a time series where  $t = 0, 1, 2, \dots, T$ . Strict stationarity requires that the probability distribution  $F(x_t)$  be invariant with respect to displacement in time, which formally means that

$$F(x_t, \dots, x_{t+k}) = F(x_{t+m}, \dots, x_{t+k+m})$$

and

$$F(x_t) = F(x_{t+m})$$

for any  $t$ ,  $k$  and  $m$ . This requirement is, in general, too strong. For most of the standard developments, we only need a weak definition of stationarity. This definition, frequently called second order stationarity, requires that 1) the mean of the series, defined as  $\mu_x = E(x_t)$ , is time independent, that is,  $E(x_t) = E(x_{t+m})$  for any  $t$  and  $m$ ; 2) the variance of the series, define as  $\sigma^2 = E[(x_t - \mu_x)^2]$  does not vary with time, that is,  $E[(x_t - \mu_x)^2] = E[(x_{t+m} - \mu_x)^2]$  for any  $t$  and  $m$ ; and 3) the covariance of the series, define as  $\gamma_k = E[(x_t - \mu_x)(x_{t+k} - \mu_x)]$  is also independent of time, that is,  $Cov(x_t, x_{t+k}) = Cov(x_{t+m}, x_{t+m+k})$ .

If the series is stationary, it is said to be integrated of order zero, denoted as  $I(0)$ . If the series is nonstationary, first-differencing has to be applied to the series to make it stationary. Using the lag operator, this means that  $(1 - L)x_t = \Delta x_t$ , is a stationary series. If a series needs one differencing, the series is said to be integrated of order one, denoted as  $I(1)$ . In general if a time series needs  $d$  differences  $((1 - L)^d)$  in order to be stationary, it will be integrated of order  $d$ , denoted as  $I(d)$ .

### 3.2 Unit Root Tests

A unit root test is frequently used to determine the order of integration of the series. In this paper we use the augmented Dickey and Fuller test (Dickey and Fuller, 1979; Said and Dickey, 1984) and the Phillips and Perron test (Phillips and Perron, 1988). The Augmented Dickey-Fuller (referred as *ADF*) unit root test consists to estimate the following equation by OLS:

$$x_t = d_t + \rho x_{t-1} + \sum_{i=1}^k b_i \Delta x_{t-1} + \epsilon_t \quad (1)$$

where  $\epsilon_t \sim i.i.d N(0, \sigma_\epsilon^2)$  and  $d_t = \psi' z_t$ , where  $z_t$  is a set of deterministic components, that is  $z_t = \{0\}$ ,  $\{1\}$  or  $\{1, t\}$ . If  $|\rho| < 1$ ,  $x_t$  is said to be a stationary series. However, if  $\rho = 1$ ,  $x_t$  is a nonstationary series and in this case, the variance of  $x_t$  increases steadily with time and goes to infinity. If  $|\rho| > 1$ , the series has an explosive root, which is excluded in the analysis. The

null hypothesis is that the time series is non stationary ( $H_0 : \rho = 1$ ) and the alternative hypothesis is that the time series is stationary ( $H_1 : |\rho| < 1$ ).

The Phillips and Perron test (hereafter *PP*), is based on a non-parametric approach, where the deterministic function of the series are first removed. It consists in estimating by OLS the following equation:

$$\Delta \tilde{x}_t = \rho \tilde{x}_{t-1} + \epsilon_t$$

where  $\{\tilde{x}_t\}$  is a variable from which the deterministic function,  $d_t$ , has been removed. The *PP* test statistic, denoted by  $Z_{t_{\hat{\rho}}}$ , is calculated as follows:

$$Z_{t_{\hat{\rho}}} = \frac{\sqrt{\hat{\gamma}_0} t_{\hat{\rho}}}{\hat{\omega}} - \frac{T(\hat{\omega}^2 - \hat{\gamma}_0) \hat{\sigma}_{\hat{\rho}}}{2\hat{\omega} \hat{\sigma}_{\epsilon}}, \quad (2)$$

where  $\hat{\omega}^2$  is a consistent estimator of the spectral density at the frequency zero calculated as

$$\hat{\omega}^2 = \hat{\gamma}_0 + 2 \sum_{j=1}^q \left(1 - \frac{j}{q+1}\right) \hat{\gamma}_j$$

where  $\hat{\gamma}_0 = (1/T) \sum_{t=1}^T \hat{\epsilon}_t' \hat{\epsilon}_t$ ,  $\hat{\gamma}_j$  is the autocovariance function between  $\hat{\epsilon}_t$  and  $\hat{\epsilon}_j$ , for  $t, j = 1, 2, \dots, q$  defined by  $\hat{\gamma}_j = (1/T) \sum_{t=j+1}^T E[\hat{\epsilon}_t' \hat{\epsilon}_{t-j}]$ , and  $q$  is the truncation lag. Finally,  $\hat{\sigma}_{\hat{\rho}}$  is the standard error of  $\hat{\rho}$ .

Notice that the ADF and the PP statistics need the use of a truncation lag, which are denote by  $k$  and  $q$ , respectively. In selecting the lag length we use the sequential data-dependent rule suggested by Campbell and Perron (1991) which works as follows. The procedure starts with a maximum value of  $k$ , say  $kmax$ . The equation (1) is estimated with this lag and the t-statistic of the last lag associated to the first differences is evaluated using a 10.0% significance level of a Normal distribution. If the calculated t-statistic is larger than the critical value (1.6), the procedure ends and the  $k^* = kmax$ . If the opposite is obtained, the procedure continue using  $kmax - 1$ . The evaluation procedure is repeated until a rejection is found. If no rejection is found and  $k = 0$ , then  $k^* = 0$ . For more details about the properties of this rule in terms of size and power properties of the unit root tests, see Ng and Perron (1995). The same lag selected for the ADF is used for the PP statistic.

### 3.3 Cointegration

In order to test for cointegration, assume a VAR in the error correction model representation (VECM). Using similar notation as in Johansen (1988, 1995):

$$\Delta X_t = D_t + \Pi X_{t-1} + \Gamma_j(L) \Delta X_{t-1} + \epsilon_t \quad (3)$$

where  $X_t$  is a  $n \times 1$  vector of endogenous variables,  $\Gamma_j = -\sum_{l=j+1}^p \Phi_l$ ,  $\Pi = \sum_{j=1}^p \Phi_j - I_n = \Phi(1) - I_n$ ,  $D_t$  is a vector of deterministic components as intercepts, time trends and/or seasonal dummies, and

$$\begin{aligned}
E(\varepsilon_t) &= 0 \\
E(\varepsilon_t' \varepsilon_t) &= \begin{cases} \Omega & t = \tau \\ 0 & \text{otherwise} \end{cases}.
\end{aligned} \tag{4}$$

The equation (3) offers three possible cases according to the rank of the matrix  $\Pi$ . First, the matrix  $\Pi$  can be the zero-matrix, which implies that the rank of  $\Pi$  equals 0. Second, the matrix  $\Pi$  can have full rank  $n$ , implying all variables are stationary. Third, the matrix  $\Pi$  can have rank deficiency, that is  $0 < \text{rank}(\Pi) < n$ , in which case,  $\Pi$  can be decomposed as  $\Pi = \alpha\beta'$  where  $\beta$  and  $\alpha$  are  $n \times r$  matrices of full rank  $r$ . In the same sense,  $X_t$  has  $r$  cointegrating vectors, which are the columns of  $\beta$ , and has  $k = n - r$  unit roots, while the matrix  $\alpha$  contains the adjustment parameters.

The Johansen maximum likelihood cointegration testing method aims to test the rank of the matrix  $\Pi$ , using the reduced rank regression technique based on canonical correlations. See Franses (1999) or Johansen (1995) for a more complete description of this approach. Assuming that the intercepts are not restricted, the procedure consists of obtaining an  $n \times 1$  vector of residuals  $r_{0t}$  and  $r_{1t}$  from the so-called auxiliary regressions (regressions of  $\Delta X_t$  and  $X_{t-1}$  on a constant and the lagged  $\Delta X_{t-1}$  through  $\Delta X_{t-p+1}$ ). These residuals are used to obtain the  $(n \times n)$  residual product matrices:

$$S_{ij} = (1/T) \sum_{t=1}^T r_{it} r_{jt}' \tag{5}$$

for  $i, j = 0, 1$ . The next step is to solve the following eigenvalue problem

$$|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| = 0 \tag{6}$$

which gives the eigenvalues  $\hat{\lambda}_1 \geq \dots \geq \hat{\lambda}_n$  and the corresponding eigenvectors  $\hat{\beta}_1$  through  $\hat{\beta}_n$ , which are the cointegrating relations. A test for the rank of the matrix  $\Pi$  can now be performed by testing how many eigenvalues  $\lambda$  equals unity. The first test statistic for the resulting number of cointegration relations is the so-called Trace statistic (see Johansen, 1988), which is a likelihood ratio test defined by

$$\text{Trace} = -T \sum_{i=r+1}^n \log(1 - \hat{\lambda}_i). \tag{7}$$

The null hypothesis for the *Trace* test is that there are at most  $r$  cointegration relations. We begin by testing whether there is no cointegration ( $r = 0$ ) versus at most 1 such relation. If this is rejected, we test whether there are at most two cointegration relations and so on.

Another useful test is given by testing the significance of the estimated eigenvalues themselves

$$\lambda_{\max} = -T \log(1 - \hat{\lambda}_i) \tag{8}$$

which can be used to test the null hypothesis of  $r - 1$  against  $r$  cointegration relations.

Critical values for these tests have been tabulated by Osterwald-Lenum (1992). The limiting distributions depend on the set of deterministic components considered in equation and depend also on the set of deterministic components allowed in the cointegrating relations (matrix  $\Pi$ ). Given the nature of our series, we always consider an intercept in the estimation of the *VECMs*.

Another important issue in the application of the Johansen test is the specification of the lag length. Many suggestions appear in the literature. Here, we use information criterias such as Akaike criterion (*AIC*) or Schwarz criterion (*SIC*).

### 3.4 Common Trends and Common Cycles

Features are data characteristics such as trends, cycles, serial correlation, heteroscedasticity, seasonality, autoregressive conditional heteroscedasticity and excess kurtosis. An interesting use arises when features that are detected in single series processes are actually shared in common with other processes, in which case they would be called *common features*. As it is defined by Engle and Kozicki (1993), a feature will be said to be common if a linear combination of the series fails to have the feature, even though each of the series individually has the feature.

In order to carry out a test for common trends, we follow the procedure proposed by Engle and Kozicki (1993). Assume, in similar notation as them, the existence of the Wold representation from the VAR:

$$\Delta X_t = \Delta D_t + \Phi(L)\Delta X_{t-1} + \Delta \varepsilon_t \quad (9)$$

where  $\varepsilon_t \sim i.i.d N(0, \sigma_\varepsilon^2)$ . Notice also that,  $\Delta D_t = \Delta(\mu_0 + \mu_1 t) = \mu_0 + \mu_1 t - \mu_0 + \mu_1(t - 1) = \mu_1$ . Therefore, we can write the above equation as

$$\Delta X_t = \mu_1 + \Phi(L)\Delta X_{t-1} + \Delta \varepsilon_t, \quad (10)$$

where, solving for  $\Delta X_t$ , we proceed as follows:

$$\begin{aligned} \Delta X_t &= \mu_1 + \Phi(L)L\Delta X_t + \Delta \varepsilon_t \\ [I - \Phi(L)L]\Delta X_t &= \mu_1 + \Delta \varepsilon_t \\ \Delta X_t &= (I - \Phi(L)L)^{-1}\mu_1 + \Delta(I - \Phi(L)L)^{-1}\varepsilon_t. \end{aligned}$$

Now, for simplicity, and following Mills (1998), we ignore  $[I - \Phi(L)L]^{-1}\mu_1$ . Hence, we obtain a reduce equation of the form:

$$\Delta X_t = \varepsilon_t + \sum_{j=1}^{\infty} C_j \varepsilon_{t-j} = C(L)\varepsilon_t,$$

which is called the Wold representation of  $\Delta X_t$ . Furthermore, it is possible to decompose  $C(L)$  as  $C(1) + (1-L)C^*(L)$ , where  $C_i^* = \sum_{j>i} -C_j$ , for all  $i$ , and in particular  $C_0^* = I - C(1)$ . With this result, we have:

$$\Delta X_t = C(1)\varepsilon_t + (1-L)C^*(L)\varepsilon_t. \quad (11)$$

Notice that, since  $C(1)\varepsilon_t$  represents a multivariate white noise,  $(1-L)C^*(L)\varepsilon_t$  captures all the serial correlation of  $\Delta X_t$ . This enables us to find the Stock and Watson (1988) common trends representation, which is:

$$X_t = C(1) \sum_{i=0}^{\infty} \varepsilon_{t-i} + C^*(L)\varepsilon_t. \quad (12)$$

where  $C(1) \sum_{i=0}^{\infty} \varepsilon_{t-i}$  represents the  $k$  common trends and  $C^*(L)\varepsilon_t$  represents the stationary cyclical components.

If there are  $r$  linearly independent cointegrating relationships in  $C(1)$ ,  $0 \leq r \leq n$ , there exists  $n-r$  common trends. Moreover, if  $C^*(L)$  is of reduced rank, i.e. there is a set of  $s$  linearly independent vectors ( $0 \leq s \leq n$ ), the  $n$  variables will have their cyclical behavior governed by  $n-s$  common cycles. In other words, when  $C(1)$  is of reduce rank, common trends appear in  $X_t$ , in the same way that common cycles appear in  $X_t$  when  $C^*(L)$  is of reduce rank, i.e. there is a set of  $s$  linearity independent vectors, gather in the  $n \times s$  matrix  $\phi$ , such that:

$$\phi' c_t = \phi' C^*(L)\varepsilon_{t-j} = 0.$$

The number of linearly independent cofeature vectors making up  $\phi$  can be at most  $k$ , and these will be linearly independent of cointegrating vectors making up  $\alpha$  (Vahid and Engle, 1993). If  $r+s=n$ , a unique decomposition of  $X_t$  into trend and cycle components is available by defining:

$$A = \begin{Bmatrix} \phi' \\ \alpha' \end{Bmatrix}$$

where  $\phi'$  has the dimensions of  $s \times n$  and  $\alpha'$  has the dimensions of  $r \times n$ . So we can write,

$$X_t = \begin{Bmatrix} \phi' C(1) \sum_{i=0}^{\infty} \varepsilon_{t-i} \\ \alpha' C^*(L)\varepsilon_t \end{Bmatrix}$$

that is,  $X_t$  can be decomposed into the random walk trend component  $X_t^P = \phi^- \phi' X_t$  and the cyclical component  $X_t^C = \alpha^- \alpha' X_t$ . Furthermore, we can rewrite  $\phi$ , since it is an  $n \times s$  matrix of full column rank, as:

$$\phi = \begin{Bmatrix} I_s \\ \phi_{(n-s) \times s} \end{Bmatrix}$$

and  $\phi' \Delta x_t$  can be considered as a representation of an  $s$  pseudo-structural form equations for the first  $s$  elements of  $\Delta X_t$ . Therefore, the following restricted reduced-form system is estimated by using the three-stage least squares system or any other system method:

$$\begin{Bmatrix} I_s & \phi^{*'} \\ 0_{(n-s) \times s} & I_{n-s} \end{Bmatrix} \Delta x_t = \begin{Bmatrix} 0_{s \times (n(p-1)+r)} \\ \Gamma_1^* & \dots & \Gamma_{p-1}^* & \beta^* \end{Bmatrix} \begin{Bmatrix} \Delta X_{t-1} \\ \vdots \\ \Delta X_{t-p+1} \\ \alpha' X_{t-1} \end{Bmatrix} + \varepsilon_t. \quad (13)$$

Based upon the approach of Engle and Kozicki (1993) and extended by Vahid and Engle (1993), it is possible to determine the number of common cycles, conditional on a choice of the  $r$  cointegrating vectors. Following a similar notation as in Engle and Kozicki (1993), assume the following multivariate model:

$$X_t = \beta y_t' + \Gamma z_t' + \varepsilon_t \quad (14)$$

where  $\Gamma$  is an  $n \times k$  matrix that contains the features presented in individual series. In this sense, if any row of the matrix  $\Gamma$  is 0, the corresponding series does not present the feature. Moreover, if there is a vector  $\delta$  such that  $\delta' X_t$  does not show the feature, the vector  $\delta$  will be called a common feature vector or a cofeature vector. Therefore, any combination  $\delta' \Gamma$  that has the property of being equal to zero will be a cofeature vector. If  $\Gamma$  has rank  $n - s$ , where  $s$  is the rank of the left null space of  $\Gamma$ , then there will be  $s$  linearly independent cofeature vectors and  $s$  will be the common feature or common rank. The reduced rank of  $\Gamma$  is only a restriction if  $k \geq n$ , since  $\Gamma$  is an  $n \times k$  matrix.

If  $\Gamma$  has rank  $n - s$ , then it can be written as the product of two matrices of rank  $n - r$  with the following dimensions:

$$\begin{array}{rcl} \Gamma & = & \Lambda \quad \Phi \\ (n \times k) & = & (n \times n - s) \quad (n - s \times k) \end{array}$$

and if we define  $\Phi z_t = w_t$ , as an  $(n - s) \times 1$  vector, each element of which is a linear combination of  $X_t$ , equation (14) can be written as

$$X_t - \beta y_t' = \Lambda w_t + \varepsilon_t \quad (15)$$

which has  $n - r$  common components that exhibits the feature. This representation has been done similarly to that of Stock and Watson (1988) conventions.

Hence, the rank  $s$  can be determined by calculating the following test statistic:

$$C(s) = -T(-(p-2)) \sum_{j=1}^s \ln(1 - l_j^2) \quad (16)$$

where  $l_1^2, \dots, l_s^2$  are the  $s$  smallest estimated squared canonical correlations between  $\Delta X_t$ , and the set  $w_{t-1} = (\Delta X_{t-1}, \dots, \Delta X_{t-p+1}, e_{t-1})$ . By defining  $Y$  as the matrix of observations on  $\Delta X_t$  and  $W_{-1}$  as the matrix of observations on  $w_{t-1}$ ,  $l_1^2, \dots, l_s^2$  are obtained as the  $s$  smallest eigenvalues of  $(Y'Y)^{-1}Y'W_{-1}(W_{-1}'W_{-1})^{-1}W_{-1}Y$ . Under the null hypothesis that the rank of  $\phi$  is at least  $s$ , this statistic has a  $\chi^2$  distribution with  $s^2 + sn(p-2+r)$  degrees of freedom (see Vahid and Engle, 1993).

### 3.5 Variance Decomposition

In order to analyze the importance of transitory and permanent shocks for the variation of sectoral output, a variance decomposition is necessary. The approach followed by Engle and Issler (1995) is used. The  $h$  step-ahead trend innovation for the  $i$ -th sector is defined as  $\sum_{j=0}^{h-1} \Delta X_{i,t+j}^p$ , while the  $h$  step-ahead cycle innovation is defined as the residuals from the regression of  $\Delta X_{it}^c$  on the lagged error correction terms  $e_{j,t-k-h}$ ;  $j = 1, 2, 3$ ,  $k = 0, \dots, 3$ . Horizons  $h = 1, 2, 3, 4$  are considered. Unlike Harvey and Mills (2002), we report two sets of variance decompositions, depending on how the innovations are orthogonalised, i.e, trend innovations preceding cyclical innovations in the orthogonalisation or viceversa.

## 4 Empirical Evidence

We present 2 empirical cases. The first case analyzes and presents evidence about common features in Colombian sectoral output. In the second case, we deal with common trends and common cycles in the US regional per-capita income<sup>1</sup>.

### 4.1 Colombia

The data used consists of indices of gross value added at constant 1975 prices for five sectors of the total output. These sectors are Agriculture (referred

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<sup>1</sup>Most of the estimations were performed in Eviews. For some estimations as the identification of the cointegrating vectors, we used PcGive 10.0 which is an Ox-package. See Doornik and Hendry (2001). The test for identification of the common cycles and the variance decomposition were performed in Gauss.

as  $Agri_t$ ), Mining ( $Min_t$ ), Manufacturing ( $Man_t$ ), Construction ( $Const_t$ ) and Transportation ( $Tran_t$ ). The sector of Services was not considered due to difficulties with different sets of data provided. The period covered is 1927 - 1997, giving 71 annual observations. The data sets were obtained from the National Administrative Department of Statistics (DANE) of Colombia.

Figure 1 shows the logarithms of the five sectoral output series. We observe an upward tendency in all series, with a common negative shock in the late twenties that goes up to 1931, where all series go back to the tendency. The sector of Construction presents a very negative deep shock in the late forties, which is also present, but less persistent, in the Agriculture sector; while the other series continued growing. In the seventies, Mining entered into a very long recession that lasted about six years, which is particular, since the other series seem to keep their positive growth rates.

Figure 2 plots the growth rates of all series, where we can see the cyclical behaviour of the series. For example, the big recession in the late twenties is identified. However, the magnitude of the recession varies among sectors since the negative growth rates differ between the series by close to two points. Manufacture suffered the shortest period of recession with a -0.15% of growth rate by 1950, while the recession went deeper and lasted longer for the other sectors such as Transportation (-13.45%) and Construction (-36.82%), for which the recession lasted almost five years. The graph also shows the negative growths of the Mining sector in the seventies and a big recovery of the sector in the eighties where the annual growth rate reached 48.33% in 1986.

All series were tested for presence of unit roots in their levels. Using the ADF and the PP statistics, for all time series we cannot reject the null hypothesis of a unit root, indicating that they are integrated of order one, i.e.  $I(1)$ . The results of this test are presented in Table 1.

In selecting the lag length for the unrestricted VAR, we used the Akaike information criteria (AIC) and the Schwartz information criteria (SIC). Both criterias select  $p = 2$  which seems convenient given the frequency of our data. With five variables ( $n = 5$ ) and  $p = 2$ , the test for cointegration is performed. Results are shown in Table 3. The test rejects the null hypothesis of  $r = 0$ , but it cannot reject the null hypothesis of  $r = 1$ , which indicates that there exists one cointegrating relationship among the five variables.

Hence, a cointegrated VAR is estimated which is frequently named a Vector Error Correction Model (VECM). Following Johansen (1995), economic interpretation of the cointegrating vector is possible to be obtained towards the identification procedure. It consists to test for some (economic) restrictions on the long-run relationship. Hence, a first set of restrictions consists to exclude some variables from the cointegrating relation. Another is to specify relationships between signs of the coefficients. Finally, there is also a set of restrictions related to the short-run behavior of the variables.

The first restriction verified was the absence of a time trend in the long run relationship. The LR statistic gives a value of 3.13 with a probability of 0.209. Hence, the test cannot not reject the null hypothesis at 10% level of significance, suggesting that the trend should be dropped off the cointegrating

vector. Results of these estimations are shown in Table 4. Hence, the estimation of a cointegrated VAR where there is no time trend in the relationship was performed. Using these new results, we test for the restriction that the Mining sector is not present in the cointegrating relationship. At the same time, we impose an unity coefficient on the Agriculture sector. The LR test strongly rejected this restriction. A final set of restrictions consisted in adding short-run restrictions as the fact that Mining and Agriculture are weakly exogenous. The LR test supported this set of restrictions with a  $p$ -value = 0.70 (see Table 7). In other words, the cointegrating vector can finally be expressed as:

$$Agri_t = -29.120Man_t + 7.363Const_t + 24.752Tran_t. \quad (17)$$

With these results and knowing that  $n = 5, r = 1$  and  $p = 2$ , we can now test for the presence of common cycles among the sectoral output in the VAR framework. Table 8 presents the results of the Vahid-Engle (1993) common cycle test. The test was performed sequentially starting with the null hypothesis that there is at least one white noise linear combination of the variables contained in  $\Delta X_t$  ( $H_0 : s \geq 1$ ), against the alternative hypothesis that there are none ( $H_1 : s = 0$ ). The  $C(s)$  is calculated using equation (16). If the hypothesis is not rejected, one proceeds with testing the null hypothesis that there are at least two white noise linear combinations ( $H_0 : s \geq 2$ ) against the alternative hypothesis that there is only one ( $H_1 : s = 1$ ), and so on, up to the limit  $s = n - r$  which in this case is four. According to our findings we cannot reject the null hypothesis of four white noise linear combinations of the variables in  $\Delta X_t$  (common cycles), using 5% level of significance.

Knowing  $n, p, r$  and  $s$ , the pseudo-structural model can be now estimated. We estimated it using 3SLS. First, an unrestricted system was estimated using as instruments all the variables in first differences at lags one and two plus the cointegrating vector with only one lag. The results of this estimation are presented in Table 9. The four first columns represent the pseudo-structural form equations due that the system contains four common cycles.

Once these estimates were obtained, restrictions were imposed upon the model by taking out those coefficients that seemed not to be statistically significant. The Mining sector seems to be statistically insignificant, since for its equation only the constant is significant in the short term. After imposing restrictions for exclusion of variables, the final estimates of the pseudo-structural system are presented in Table 10.

The number of significant lagged growth rates appearing in each of the equations indicates a joint dependence of Colombian sectoral output, where most of the sectors are determined endogenously. On the other hand, the error correction  $e_{t-1}$  appears to be very significant, thereby confirming that the cointegrating rank of  $r = 1$  was appropriately chosen.

Once this model was estimated, we proceeded with the decomposition of the series into trend and cycles. Since  $r+s = n$ , an unique trend-cycle decomposition discussed in the framework is available, see equation (12). Figure 3 exposes the

cyclical components for the five sectors with their respective standard deviations, while Figure 4 presents the level and the trend components of each time series.

From Figure 3, we can see that the sectors Agriculture and Construction follow similar cycles, but Construction has greater amplitude, given by their standard deviations (0.08 for Agriculture and 0.32 for Construction). They first start with positive growth until late forties when both entered into recessions and then go back to growth in the seventies. The other pair, Manufacturing and Transportation have almost identically shaped cycles with very closed magnitudes (Manufacturing has 0.17 and Transportation has 0.20 of standard deviation). They start with recessions and go back to growth in the late thirties and early forties, lasting about forty years ending up into deep recession. Moreover, Mining becomes a mirror of the last common cycle after the late fifties. Mining presented the greatest amplitude (standard deviation of 0.36) followed by Construction (0.32), while Agriculture presented the smallest (0.08).

Table 11 presents contemporaneous correlations. Seven correlations (out of ten) are statistically significant. However, three pairs of sectors such as Mining and Agriculture, Mining and Construction and Construction and Manufacturing are not significantly correlated. Transportation is the only sector that has significant correlations with the rest of the sectors. It presents the lowest and highest correlations (in magnitude) to the others. The lowest is to Construction which is negatively correlated (-0.28). Moreover, the sector of Transportation moves counter cyclically to them, except by the sector of Manufacturing, but to which, it has the highest correlation.

Agriculture and Construction, Manufacturing and Transportation, Agriculture and Mining, Construction and Manufacturing and Construction and Mining are the pairs that have positive correlations to each other, even though the three former are not significant at any standard level of significance. On the other hand, the sectors of Agriculture and Manufacturing, Agriculture and Transportation, Construction and Transportation, Manufacturing and Mining and Mining and Transportation have negative correlations, being all statistically significant.

Table 12 contains correlations at lag one between the cyclical components of sectors. Six pairs of correlations (out of twenty) are not statistically significant at 5% level. These correlations come along with those in Table 11, except by the pair of Mining and Construction in which case the sign changes when the sector of Construction is at lag one. However, none of the three pairs of correlations seem to be significant. At lag one, all correlations have similar values as those observed in contemporaneous correlations. These correlations among the cyclical components along to the insignificance of the Mining correlations confirm the interdependencies found in the structural model fitted to the observed sectoral data.

The observed results are similar to those of Harvey and Mills (2002) for the case of UK. In their paper they observed that the sectors of Agriculture and Mining move counter cyclical to the other four sectors (Manufacturing, Electricity, gas and water supply, Construction and Services), although they were found to be essentially uncorrelated with each other and any of their correlations were

significantly different from zero at any reasonable significance level. They also found that, for UK, all the other cyclical components were positively correlated, both contemporaneously and at lag one.

However, it is essential to take into account that the two economies are very different. Colombia is not a developed country and depends mainly upon primary sectors such as Agriculture. An example of this is that we have found that Agriculture is correlated contemporaneously and at lag one to the sector of Construction, while Harvey and Mills (2002) do not find any evidence of this. A similar case is observed with the pairs of the sectors of Construction and Mining and Construction and Manufacturing that Harvey and Mills (2002) found correlated (the former with a high contemporaneous correlation of 0.79).

Table 13 presents the results obtained from the variance decomposition which is useful to investigate the relative importance of transitory and permanent shocks for the variation of sectoral output. Normally authors report two sets of variance decompositions, depending on how the innovations are orthogonalised, see Engle and Issler (1995). In the Colombian case, we find that the decomposition is hardly affected by the ordering chosen, as the covariances between the trend and cycle innovations were effectively zero for each sector. We thus, present the results for trend innovations preceding cyclical innovations in the orthogonalisation. The case where cyclical innovations preceding trend innovations in the orthogonalisation are presented in parenthesis.

The results indicate that the transitory shocks are of great importance, more than permanent shocks, for all sectors. The sector of Transportation is the less dominated by transitory shocks since 42% of output variation at a four year horizon is explained by permanent shocks. This is a reflection of how sensitive sectoral outputs in Colombia are to, presumably, macroeconomic shocks, both internal and external. Comparing these results with those of Harvey and Mills (2002), we can see several differences. They found the Agriculture and Mining in UK were characterized by being sensitive more to permanent shocks than to transitory shocks, while in Colombia all sectoral outputs were found to be explained by transitory shocks. Moreover, in our case the role played by transitory shocks in Agriculture do not diminished significantly over a four year horizon.

## 4.2 United States

The data set used is annual BEA data on real per-capita personal income (for the eight regions of the United States. The regions are Far West ( $FW_t$ ), Great Lakes ( $GL_t$ ), Mideast ( $ME_t$ ), New England ( $NE_t$ ), Plains ( $PL_t$ ), Rocky Mountains ( $RM_t$ ), Southeast ( $SE_t$ ) and South West ( $SW_t$ ). The period covered is 1929 - 1990, giving 62 annual observations. The data set is the same as used by Tomljanovich and Vogelsang (2002).

Figure 5 shows the logarithms of the eight regional per capita income series in levels and Figure 6 presents the growth rates (first differences). It seems that not all the series follow an upward trend. This is the case of the regions of Far West, Great lakes and Mideast which seemd to follow a downward pattern. The regions of Plains and Rocky Mountains follow very similar patterns and that

Mideast and New England have almost identical tendencies. From Figure 6, we observe some similarities between the regions of Mideast and New England. Nevertheless, the magnitude of the recession varies among the two regions since the negative growth rates differ between the series by almost two points. There is not a single year in which all sectors presented common negative growth rates. The closest one was 1944 in which Far West, New England, Rocky Mountains and Great Lakes presented -7%, -5.8% -5.7% and -2% respectively, while Mideast South West and South East presented positive growth rates of 1.4%, 3.5% and 3% respectively.

As in the Colombian case, all the series were tested for the presence of unit roots in their levels using the ADF and the PP tests. The results are presented in Table 14. For all regional per-capita income series, we cannot reject the null hypothesis of a one unit root, indicating that they are integrated of order one, i.e.  $I(1)$ .

In estimating an unrestricted VAR, a lag of  $p = 2$  was selected using AIC and SIC. Using this lag structure and eight variables ( $n = 8$ ), the cointegration test is performed. The results are presented in Table 16, while Table 17 contains the cointegrating vectors in their original form. The test rejects the null hypothesis of  $r = 0$  and  $r \leq 1$ , but it cannot reject the null hypothesis of  $r \leq 2$ , indicating that the system contains two cointegrating relationships. These results are in accordance with those obtained by Carlino and Sill (2001). However, they only used seven regions for their study.

Given the above results, a cointegrated VAR was estimated. In the process of identification of the two cointegrating vectors, we impose twenty-two sequential restrictions of long and short term, of which we present only five of them. The results are shown in Table 18. The first set of restrictions appearing in Table 18 consisted to restrict (in the first cointegrating vector) to zero the coefficients of the regions Plains, South East and South West are restricted to be zero, while the coefficient of the region of New England was restricted to be one. On the other hand, for the second cointegrating vector, we restricted the coefficients of the regions of Mideast, Great Lakes, Rocky Mountains and the time trend to be zero; while the coefficients of the regions of Plains is restricted to be one. Other coefficients are unrestricted. The LR test rejected the null hypothesis. After estimating several restricted cointegrated VARs, restrictions upon the short term relationships among the eight variables were imposed, ending up with the fifth VAR that appears in Table 18 where the LR test did not reject the null hypothesis. These results show that the region of Far West is present in the model as an exogenous variable respect to the long run relationship since it does not appear in any of the two cointegrating vectors. It also seems that the per-capita income of the regions of Rocky Mountains and South East are not affected by any other of the regions in the short term.

In other words, the cointegrating vector can also be expressed as:

$$\begin{aligned} SW_{1,t} &= RM_{1,t} + 0.0025863Trend_{1,t} + 3.1152PL_{1,t} \\ NE_{2,t} &= 0.33216Trend_{2,t} + 1.5357ME_{2,t} + 3.4909PL_{2,t} + -6.3963SE_{2,t}. \end{aligned}$$

Knowing that  $n = 8$ ,  $r = 2$  and  $p = 2$ , it is possible to test for the number of common cycles. Table 20 presents the results of the Vahid-Engle (1993) common cycle test. As in the Colombian case, the test was performed sequentially starting with the null hypothesis that there is at least one white noise linear combination of the variable contained in  $\Delta X_t$  ( $H_0 : s \geq 1$ ), against the alternative hypothesis that there are none ( $H_1 : s = 0$ ). The values of  $C(s)$  are calculated using equation (16). If the hypothesis is not rejected, one proceeds with testing the null hypothesis that there are at least two white noise linear combinations ( $H_0 : s \geq 2$ ) against the alternative hypothesis that there is one ( $H_1 : s = 1$ ), and so on, up to the limit  $s = n - r$  which in our case is six. According to our findings, the test cannot reject the null hypothesis of having six white noise linear combinations of the variables contained in  $\Delta X_t$  (common cycles), using a 5% level of significance.

These results contrast with those obtained by Carlino and Sill (2001). In fact, they concluded that the seven regions shared four common cycles. Our findings suggest the presence of six common cycles. The different conclusions may be caused by characteristics of the data such as the larger number of regions considered in this paper and different span of the data set<sup>2</sup>. Also notice that Carlino and Sill (2001) did not try to identify the cointegrating vectors.

We now know that  $s = 6$ . The next step consisted to estimate the pseudo-structural model (13) using 3SLS. First, an unrestricted system was estimated using as instruments all the variables in first differences at lags one and two plus the cointegrating vectors with only one lag. The results of this estimation are presented in Table 21. The six first columns represent the pseudo-estructural form equations due that the system contains six common cycles. Because there are many non significant variables, we proceeded to impose exclusion restrictions. Final estimates are shown in Table 22.

The number of significant lagged growth rates appearing in each of the equations indicates the joint dependence of regional per-capita income, with most of the regions are determined endogenously. However, as observed in the unrestricted system (Table 21), the regions of South West and South East appear in the model as exogeneous variables in the short term. The error correction term, denoted by  $e_{1,t-1}$ , appears to be very significant in both equations, while the error correction term  $e_{2,t-1}$  is only statistically significant in the second equation.

Since  $r + s = n$ , a unique trend-cycle decomposition discussed in the framework is available (see equation (12)). Figure 7 exposes the cyclical components for the five sectors with their respective standard deviations. There are six pairs of regions that share almost an identical shape of cyclical behaviour. Those are Far West and Great Lake, Far West and Rocky Mountains, Great Lakes and Rocky Mountains, Mideast and New England, Plains and Sout East and South East and South West. However, their amplitudes, measured by the standard deviations, vary among sectors. In the other hand, the region of Far West has similar cycle behaviour to the regions of Mideast and New England, which at

<sup>2</sup>The data used by Carlino and Sill goes from 1956:1 to 1995:2, for a total of 154 horizons.

the same time, are mirrors of the regions South East, New England and South West. We can also observe five pairs of regions (South East and Great Lakes, Mideast and Plains, Mideast and South East, Plains and Rocky Mountains and Rocky Mountains and Spouth East) in which, the cyclical behaviour are almost exact mirrors.

On the other hand, the regions Far West and Rocky Mountains have the highest standard deviations, indicating that they are the most sensitive to cyclical components in their regional per-capita income. In contrast, the regions of South East and South West contain the lowest standard deviations suggesting that their per-capita income are the less sensitive to cyclical components. Furthermore, volatility among regions vary dramatically. Despite the comovement of regional cyclical components, the data revealed considerable cross-regional differences in volatility. The cyclical component in the most volatile region (Rocky Mountains) is almost nine times as great as in the least volatile region (South East).

These results contrast very much to those of Carlino and Sill (2001). They found that the region of South East presents the cyclical component with higher volatility, while we have found that it is the least volatile. Besides that, the region of Far West was the second most volatile sector while they found that is the least volatile of all<sup>3</sup>.

Table 23 shows the contemporaneous correlations among the regional cycles and Table 24 presents statistics on how the cyclical components of the sectors are correlated at lag one. All correlations are statistically significant and the transitory components of the regions appear highly correlated. The lowest set of correlations are those of the pairs of Great Lakes and New England (0.54), Great Lakes and South West (-0.55) and South West and Rocky Mountains (-0.63). Notice that this is consistent with what we had already seen, in the sense that regions South West and South East seem to be determined exogenously in the short term (see Table 22) and the fact that the regions Great Lakes has coefficient of zero in both cointegrating vectors (see Table 19).

The correlations at lag one are also all statistically significant (Table 24). All regions are correlated at lag one, except by New England and Plains (-0.1) and Great Lakes and Rocky Mountains (0.1). However, notice that the regions New England and Plains and Great Lakes and Rocky Mountains are higly correlated (-0.89 and 0.78 respectively). Concerning the signs, it is very curious that the correlations of the regions Plains and South East at lag one and Plains at lag one and South East have different sign; even though, the correlation are of similar magnitude.

The results of the variance decomposition are shown in table 25. Unlike the Colombian case, in the case of regional per-capita income of US regions are affected by the ordering chosen, as the covariances between the trend and cycle innovations are not independent in some regions. Again, we present the results for trend innovations preceding cyclical innovations in the orthogonalisation and

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<sup>3</sup>Notice that Carlino and Sill (2001) did not include Rocky Mountains in their study. This is very important since we have found that the cyclical component of this region was the most volatile.

in parenthesis the results for cyclical innovations preceding trend innovations in the orthogonalisation. In the first case, the greatest contribution to the  $h$ -step-ahead forecast variance comes from the cyclical component for all the regions, specially for New England and Mideast. However, these proportions vary through horizons. For example, in the regions Great Lakes, Plains, and South East, the permanent component becomes more important at horizons 3 and 4, while at horizons 1 and 2 the transitory shocks are more important in determining the variation of the real-percapita income in these regions. For the region Rocky Mountains, the temporary shocks are very important at horizon 2. However, this is only present when the trend component is ordered first in the decomposition. Notice also that in most of the cases, when cyclical innovations precedes the trend innovations in the orthogonalisation, the values reported are higher.

## 5 Conclusions

This paper identifies common trends and common cycles in two empirical cases. In the first case the Colombian sectoral output is analyzed; while in the second case the real per-capita income of the eight US regions is studied. Identification of common trends is reached using cointegration analysis as suggested by Johansen (1988, 1995). The identification of common cycles is based on the approach suggested by Engle and Kozicki (1993).

The case of the Colombian sectoral output is closely related to the analysis developed by Harvey and Mills (2002) using UK sectoral output data. We find several differences possibly associated to the different structure of the economies of these countries. While UK is a very developed country, Colombia is just a developing country and depends mainly upon primary sectors such as Agriculture. In the case of the per-capita income of the US regions, our paper is closely related to the study of Carlino and Sill (2001) also applied to the same kind of problem, but using different number of regions and different span of data.

For the Colombian case we find one cointegrating relationship among the five sectors. The analysis of common cycles selected four common cycles. We identify similarities and differences in the behaviour of the cyclical components. The calculus of correlations allows us to determine associations between sectors; while the evaluation of the variance decomposition allows us to determine the relative contribution of the permanent and transitory shocks in each productive sector. Overall, we find that transitory shocks are very important in the sectoral variance. In the case of the US regions, a similar result is obtained although the relative importance of permanent or transitory components depends of the order of the innovations in the process of orthogonalization.

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Table 1. Unit Root Tests\*

| Sectors      | Levels     |           | First Differences |           |
|--------------|------------|-----------|-------------------|-----------|
|              | <i>ADF</i> | <i>PP</i> | <i>ADF</i>        | <i>PP</i> |
| <i>Agr</i>   | -2.107     | -2.156    | -5.296            | -9.897    |
| <i>Const</i> | -2.418     | -3.054    | -5.017            | -8.351    |
| <i>Man</i>   | -0.062     | 0.177     | -3.657            | -8.476    |
| <i>Min</i>   | -1.672     | -3.029    | -5.140            | -6.046    |
| <i>Tran</i>  | -1.993     | -1.767    | -6.625            | -6.142    |

\*Critical values for the ADF and the PP tests at 1%, 5% and 10% of significance are -4.09, -3.47, and -3.16 respectively.

Table 2. Lag order selection

| No. of Lags | AIC     | SIC     |
|-------------|---------|---------|
| 1           | -12.170 | -11.183 |
| 2           | -12.504 | -10.696 |
| 3           | -12.457 | -9.823  |
| 4           | -12.286 | -8.831  |
| 5           | -12.097 | -7.819  |

Table 3. Johansen Cointegration Test

| Null Hypothesis | Treace Statistic | Probability |
|-----------------|------------------|-------------|
| $r = 0$         | 116.950*         | 0.000*      |
| $r \leq 1$      | 59.290           | 0.113       |
| $r \leq 2$      | 32.694           | 0.357       |
| $r \leq 3$      | 14.752           | 0.601       |
| $r \leq 4$      | 5.411            | 0.547       |

Table 4. Restricted Cointegrating Vector

| Variables    | $\hat{\beta}$ |
|--------------|---------------|
| <i>Agr</i>   | 1.000*        |
| <i>Min</i>   | 0.000*        |
| <i>Man</i>   | 29.120        |
| <i>Const</i> | -7.363        |
| <i>Tran</i>  | -24.752       |
| <i>Trend</i> | 0.000*        |

\* Restricted coefficients

$$H_0: \beta_{trend} = \beta_{Min} = 0; \beta_{Agr} = 1$$

Table 5. Johansen Cointegration Test

| Nul Hypothesis | Trace Statistic | Probability |
|----------------|-----------------|-------------|
| $r = 0$        | 89.327*         | 0.000*      |
| $r \leq 1$     | 34.051          | 0.504       |
| $r \leq 2$     | 15.982          | 0.719       |
| $r \leq 3$     | 6.406           | 0.652       |
| $r \leq 4$     | 0.593           | 0.441       |

Table 6. Restrictions to Cointegrating Vector

| VAR No. | Null Hypothesis                                                                    | LR statistic<br>(p-value) |
|---------|------------------------------------------------------------------------------------|---------------------------|
| 1       | $H_0: \beta_{Min} = 0;$<br>$\beta_{Agr} = 1$                                       | 0.747<br>(0.388)          |
| 2       | $H_0: \beta_{Min} = 0;$<br>$\beta_{Agr} = 1;$<br>$\beta_{Man} = \beta_{Tran}$      | 11.272<br>(0.004)         |
| 3       | $H_0: \beta_{Min} = 0;$<br>$\beta_{Agr} = 1;$<br>$\alpha_{Min} = \alpha_{Agr} = 0$ | 1.407<br>(0.704)          |

Table 7. Final Restricted Cointegrating Vector

| Variables                | Cointegrating vector ( $\hat{\beta}$ )<br>(Standard Error) | Short term coefficients ( $\hat{\alpha}$ )<br>(Standard Error) |
|--------------------------|------------------------------------------------------------|----------------------------------------------------------------|
| <i>Agr<sub>t</sub></i>   | 1.000*<br>(0.000)                                          | 0.000*<br>(0.000)                                              |
| <i>Min<sub>t</sub></i>   | 0.000*<br>(0.000)                                          | 0.000*<br>(0.000)                                              |
| <i>Man<sub>t</sub></i>   | 29.120<br>(17.268)                                         | 0.013<br>(0.003)                                               |
| <i>Const</i>             | -7.363<br>(-4.671)                                         | 0.038<br>(0.008)                                               |
| <i>Trans<sub>t</sub></i> | -24.752<br>(-14.684)                                       | 0.026<br>(0.004)                                               |

\*Restricted Coefficients

Table 8. Vahid and Engle common cycles test

| H <sub>0</sub> | H <sub>1</sub> | C(s) statistic | Probability |
|----------------|----------------|----------------|-------------|
| $s \geq 1$     | $s = 0$        | 0.507          | 0.776       |
| $s \geq 2$     | $s = 1$        | 4.998          | 0.544       |
| $s \geq 3$     | $s = 2$        | 15.390         | 0.221       |
| $s \geq 4$     | $s = 3$        | 29.694         | 0.075       |

Table 9. Unrestricted pseudo-structural system using 3SLS, p-values in parenthesis

| Regressors           | Dependent Variable |                  |                  |                   |                   |
|----------------------|--------------------|------------------|------------------|-------------------|-------------------|
|                      | $\Delta Agri_t$    | $\Delta Min_t$   | $\Delta Man_t$   | $\Delta Const_t$  | $\Delta Tran_t$   |
| Constant             | 0.018<br>(0.001)   | 0.039<br>(0.058) | 0.034<br>(0.000) | -0.019<br>(0.390) | -0.313<br>(0.000) |
| $\Delta Tran_t$      | 0.153<br>(0.016)   | 0.064<br>(0.792) | 0.384<br>(0.000) | 1.052<br>(0.000)  |                   |
| $\Delta Agri_{t-1}$  |                    |                  |                  |                   | -0.039<br>(0.813) |
| $\Delta Agri_{t-2}$  |                    |                  |                  |                   | 0.100<br>(0.543)  |
| $\Delta Min_{t-1}$   |                    |                  |                  |                   | 0.051<br>(0.298)  |
| $\Delta Min_{t-2}$   |                    |                  |                  |                   | 0.051<br>(0.221)  |
| $\Delta Man_{t-1}$   |                    |                  |                  |                   | -0.541<br>(0.001) |
| $\Delta Man_{t-2}$   |                    |                  |                  |                   | 0.021<br>(0.887)  |
| $\Delta Const_{t-1}$ |                    |                  |                  |                   | 0.233<br>(0.000)  |
| $\Delta Const_{t-2}$ |                    |                  |                  |                   | -0.112<br>(0.025) |
| $\Delta Tran_{t-1}$  |                    |                  |                  |                   | 0.463<br>(0.000)  |
| $\Delta Tran_{t-2}$  |                    |                  |                  |                   | 0.041<br>(0.648)  |
| $e_{t-1}$            |                    |                  |                  |                   | 0.029<br>(0.000)  |
| $\bar{R}^2$          | 0.095              | 0.001            | 0.212            | 0.090             | 0.503             |

Table 10. Restricted pseudo-structural system using 3SLS, p-values in parenthesis

| Regressors           | Dependent Variables |                  |                  |                  |                   |
|----------------------|---------------------|------------------|------------------|------------------|-------------------|
|                      | $\Delta Agr_t$      | $\Delta Min_t$   | $\Delta Man_t$   | $\Delta Const_t$ | $\Delta Tran_t$   |
| Constant             | 0.019<br>(0.001)    | 0.041<br>(0.011) | 0.035<br>(0.000) | -                | -0.303<br>(0.000) |
| $\Delta Tran_t$      | 0.147<br>(0.020)    | -                | 0.386<br>(0.000) | 0.934<br>(0.000) | -                 |
| $\Delta Agr_{t-1}$   |                     |                  |                  |                  | -                 |
| $\Delta Agr_{t-2}$   |                     |                  |                  |                  | -                 |
| $\Delta Min_{t-1}$   |                     |                  |                  |                  | -                 |
| $\Delta Min_{t-2}$   |                     |                  |                  |                  | 0.072<br>(0.059)  |
| $\Delta Man_{t-1}$   |                     |                  |                  |                  | -0.532<br>(0.000) |
| $\Delta Man_{t-2}$   |                     |                  |                  |                  | -                 |
| $\Delta Const_{t-1}$ |                     |                  |                  |                  | 0.218<br>(0.000)  |
| $\Delta Const_{t-2}$ |                     |                  |                  |                  | -0.103<br>(0.019) |
| $\Delta Tran_{t-1}$  |                     |                  |                  |                  | 0.500<br>(0.000)  |
| $\Delta Tran_{t-2}$  |                     |                  |                  |                  | -                 |
| $e_{t-1}$            |                     |                  |                  |                  | 0.029<br>(0.000)  |
| $\bar{R}^2$          | 0.095               | 0.000            | 0.211            | 0.120            | 0.512             |

Table 11. Contemporaneous correlations between cyclical components of sectors

| Variables | $Agr_t$<br>(p-values) | $Const_t$<br>(p-values) | $Man_t$<br>(p-values) | $Min_t$<br>(p-values) | $Tran_t$<br>(p-values) |
|-----------|-----------------------|-------------------------|-----------------------|-----------------------|------------------------|
| $Agr_t$   | 1.000<br>(0.000)      |                         |                       |                       |                        |
| $Const_t$ | 0.590<br>(0.000)      | 1.000<br>(0.000)        |                       |                       |                        |
| $Man_t$   | -0.367<br>(0.002)     | 0.203<br>(0.089)        | 1.000<br>(0.000)      |                       |                        |
| $Min_t$   | 0.121<br>(0.312)      | 0.017<br>(0.884)        | -0.530<br>(0.000)     | 1.000<br>(0.000)      |                        |
| $Tran_t$  | 0.634<br>(0.000)      | -0.286<br>(0.016)       | 0.880<br>(0.000)      | -0.531<br>(0.000)     | 1.000<br>(0.000)       |

Table 12. Lag one correlations between cyclical components of sectors

| Variables     | $Agr_t$<br>(p-values) | $Const_t$<br>(p-values) | $Man_t$<br>(p-values) | $Min_t$<br>(p-values) | $Tran_t$<br>(p-values) |
|---------------|-----------------------|-------------------------|-----------------------|-----------------------|------------------------|
| $Agr_{t-1}$   | -                     | 0.530<br>(0.000)        | -0.342<br>(0.004)     | 0.004<br>(0.973)      | -0.599<br>(0.000)      |
| $Const_{t-1}$ | 0.465<br>(0.000)      | -                       | 0.164<br>(0.172)      | -0.158<br>(0.189)     | -0.222<br>(0.062)      |
| $Man_{t-1}$   | -0.457<br>(0.000)     | 0.101<br>(0.383)        | -                     | -0.530<br>(0.000)     | 0.869<br>(0.000)       |
| $Min_{t-1}$   | 0.190<br>(0.112)      | 0.046<br>(0.701)        | -0.590<br>(0.000)     | -                     | -0.610<br>(0.000)      |
| $Tran_{t-1}$  | -0.658<br>(0.000)     | -0.255<br>(0.032)       | 0.835<br>(0.000)      | -0.446<br>(0.000)     | -                      |

Table 13. Percentage of the variance of the sectoral output innovation attributed to cyclical shocks at horizon  $h$

| Horizons | <i>Agr</i><br>(p-values) | <i>Const</i><br>(p-values) | <i>Man</i><br>(p-values) | <i>Min*</i><br>(p-values) | <i>Tran</i><br>(p-values) |
|----------|--------------------------|----------------------------|--------------------------|---------------------------|---------------------------|
| $h = 1$  | 97.570<br>(97.583)       | 89.118<br>(91.251)         | 96.797<br>(96.903)       | 100<br>(100)              | 87.635<br>(89.638)        |
| $h = 2$  | 94.240<br>(94.395)       | 78.049<br>(86.177)         | 92.778<br>(93.709)       | 100<br>(100)              | 74.612<br>(74.862)        |
| $h = 3$  | 91.140<br>(91.652)       | 70.032<br>(82.216)         | 88.603<br>(90.052)       | 100<br>(100)              | 64.300<br>(64.437)        |
| $h = 4$  | 89.434<br>(90.009)       | 65.441<br>(80.929)         | 85.371<br>(87.291)       | 100<br>(100)              | 58.389<br>(58.436)        |

\*Mining was found to be an exogenous variable in the pseudo-structural model estimated in Table 10

Table 14. Unit Root Tests\*

| Sectors | Levels |        | First Differences |         |
|---------|--------|--------|-------------------|---------|
|         | ADF    | PP     | ADF               | PP      |
| FW      | -3.292 | -3.283 | -4.458            | -9.680  |
| GL      | -3.107 | -2.955 | -5.703            | -8.486  |
| ME      | -1.705 | -0.758 | -5.011            | -5.253  |
| NE      | -1.683 | -1.199 | -5.180            | -3.924  |
| PL      | -2.034 | -2.770 | -5.084            | -12.129 |
| RM      | -1.949 | -2.350 | -5.001            | -8.572  |
| SE      | -1.200 | -1.789 | -2.764            | -6.110  |
| SW      | -2.048 | -1.124 | -4.516            | -7.122  |

Critical values for ADF and PP at 1%, 5% and 10% of significance are -3.17, -3.49 and -4.13 respectively

Table 15. Lag order selection

| No. of Lags | AIC     | SIC     |
|-------------|---------|---------|
| 1           | -45.664 | -43.106 |
| 2           | -45.533 | 40.702  |
| 3           | -44.428 | -37.323 |
| 4           | -44.533 | -35.155 |

Table 16. Johansen Cointegration Test

| Null Hypothesis | Treace Statistic | Probability |
|-----------------|------------------|-------------|
| $r = 0$         | 255.590          | 0.000       |
| $r \leq 1$      | 159.230          | 0.014       |
| $r \leq 2$      | 114.110          | 0.081       |
| $r \leq 3$      | 75.490           | 0.309       |
| $r \leq 4$      | 49.650           | 0.433       |
| $r \leq 5$      | 27.760           | 0.640       |
| $r \leq 6$      | 14.080           | 0.656       |
| $r \leq 7$      | 5.540            | 0.530       |

Table 17. Cointegrating vectors with no restrictions

| Variables    | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|--------------|-----------------|-----------------|
| <i>FW</i>    | -0.283          | -2.833          |
| <i>GL</i>    | 0.162           | -4.554          |
| <i>ME</i>    | -1.364          | 1.000           |
| <i>NE</i>    | 1.000           | -6.061          |
| <i>PL</i>    | -4.122          | -6.795          |
| <i>RM</i>    | 5.204           | 8.947           |
| <i>SE</i>    | 8.309           | -14.991         |
| <i>SW</i>    | -4.167          | -4.765          |
| <i>Trend</i> | -0.029          | 0.097           |

Table 18. Restrictions to Cointegrating Vectors

| VAR No. | Null Hypothesis                                                                                                                                                                                                                                                                                                    | LR statistic<br>(p-values) |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|
| 1       | $H_0: \beta_{1PI} = \beta_{1SE} = \beta_{1SW} = 0;$<br>$\beta_{1NE} = 1;$<br>$\beta_{2Me} = \beta_{2GL} = \beta_{2RM} = \beta_{2FW} = \beta_{2trend} = 0;$<br>$\beta_{2PI} = 1$                                                                                                                                    | 31.624<br>(0.000)          |
| 2       | $H_0: \beta_{1SE} = \beta_{1FW} = 0;$<br>$\beta_{1NE} = 1;$<br>$\beta_{2Me} = \beta_{2GL} = \beta_{2RM} = \beta_{2FW} = 0;$<br>$\beta_{2PL} = 1$                                                                                                                                                                   | 8.055<br>(0.090)           |
| 3       | $H_0: \beta_{1NE} = \beta_{1FW} = \beta_{1GL} = \beta_{1ME} = \beta_{1SE} = 0;$<br>$\beta_{1SW} = \beta_{1RO} = 1;$<br>$\beta_{2SW} = \beta_{2RM} = \beta_{2FW} = 0;$<br>$\beta_{2NE} = 1$                                                                                                                         | 11.019<br>(0.138)          |
| 4       | $H_0: \beta_{1NE} = \beta_{FW1} = \beta_{1GL} = \beta_{1ME} = \beta_{1SE} = 0;$<br>$\beta_{1SW} = \beta_{1RO} = 1;$<br>$\beta_{2SW} = \beta_{2RM} = \beta_{2FW} = 0;$<br>$\beta_{2NE} = 1;$<br>$\alpha_{1PL} = \alpha_{1SE} = \alpha_{1RM} = \alpha_{1FW} = 0;$<br>$\alpha_{2SE} = \alpha_{2RO} = 0$               | 21.257<br>(0.680)          |
| 5       | $H_0: \beta_{1NE} = \beta_{1FW} = \beta_{1GL} = \beta_{1ME} = \beta_{1SE} = 0;$<br>$\beta_{1SW} = \beta_{1RO} = 1;$<br>$\beta_{2SW} = \beta_{2RM} = \beta_{2FW} = 0;$<br>$\beta_{2NE} = \beta_{2GL} = 1;$<br>$\alpha_{1PL} = \alpha_{1SE} = \alpha_{1RM} = \alpha_{1FW} = 0;$<br>$\alpha_{2SE} = \alpha_{2RO} = 0$ | 21.419<br>(0.091)          |

Table 19. Restricted Cointegrating Vectors

| Variables    | $\hat{\beta}_1$<br>(SD) | $\hat{\beta}_2$<br>(SD) | $\hat{\alpha}_1$<br>(SD) | $\hat{\alpha}_2$<br>(SD) |
|--------------|-------------------------|-------------------------|--------------------------|--------------------------|
| <i>FW</i>    | 0.000*<br>(0.000)       | 0.000*<br>(0.000)       | 0.0000*<br>(0.000)       | -0.089<br>(0.017)        |
| <i>GL</i>    | 0.000*<br>(0.000)       | 0.000*<br>(0.000)       | -0.214<br>(0.066)        | 0.063<br>(0.182)         |
| <i>ME</i>    | 0.000*<br>(0.000)       | -1.536<br>(0.301)       | 0.234<br>(0.041)         | -0.061<br>(0.013)        |
| <i>NE</i>    | 0.000*<br>(0.000)       | 1.000*<br>(0.000)       | 0.466<br>(0.039)         | -0.159<br>(0.011)        |
| <i>PL</i>    | -3.115<br>(0.097)       | -3.491<br>(0.415)       | 0.0000*<br>(0.000)       | 0.0000*<br>(0.000)       |
| <i>RM</i>    | -1.000*<br>(0.000)      | 0.000*<br>(0.000)       | 0.0000*<br>(0.000)       | 0.0000*<br>(0.000)       |
| <i>SE</i>    | 0.000*<br>(0.000)       | 6.396<br>(0.614)        | 0.0000*<br>(0.000)       | 0.0000*<br>(0.000)       |
| <i>SW</i>    | 1.000*<br>(0.000)       | 0.000*<br>(0.000)       | -0.354<br>(0.066)        | 0.033<br>(0.017)         |
| <i>Trend</i> | -0.003<br>(0.000)       | -0.332<br>(0.003)       | -<br>-                   | -<br>-                   |

\*Restricted coefficients

Table 20. Vahid and Engle common cycle test

| $H_0$      | $H_1$   | C(s) statistic | Probability |
|------------|---------|----------------|-------------|
| $s \geq 1$ | $s = 0$ | 1.135          | 0.769       |
| $s \geq 2$ | $s = 1$ | 5.589          | 0.693       |
| $s \geq 3$ | $s = 2$ | 14.333         | 0.500       |
| $s \geq 4$ | $s = 3$ | 28.089         | 0.256       |
| $s \geq 5$ | $s = 4$ | 43.979         | 0.142       |
| $s \geq 6$ | $s = 5$ | 63.124         | 0.070       |

Table 21. Unrestricted pseudo-structural system using 3SLS, p-values in parenthesis

| Regressors        | Dependent Variable |                   |                   |                   |                   |                  |                   |                   |
|-------------------|--------------------|-------------------|-------------------|-------------------|-------------------|------------------|-------------------|-------------------|
|                   | $\Delta NE_t$      | $\Delta ME_t$     | $\Delta GL_t$     | $\Delta PL_t$     | $\Delta SE_t$     | $\Delta SW_t$    | $\Delta RM_t$     | $\Delta FW_t$     |
| Constant          | 0.0004<br>(0.901)  | -0.003<br>(0.211) | -0.003<br>(0.199) | -0.003<br>(0.454) | 0.006<br>(0.000)  | 0.004<br>(0.146) | 0.125<br>(0.005)  | 0.057<br>(0.163)  |
| $\Delta RM_t$     | -0.457<br>(0.020)  | -0.445<br>(0.003) | 0.101<br>(0.509)  | 0.939<br>(0.000)  | -0.047<br>(0.627) | 0.019<br>(0.916) |                   |                   |
| $\Delta FW_t$     | 0.723<br>(0.000)   | 0.313<br>(0.059)  | -0.423<br>(0.012) | -1.171<br>(0.000) | 0.047<br>(0.662)  | 0.049<br>(0.809) |                   |                   |
| $\Delta NE_{t-1}$ |                    |                   |                   |                   |                   |                  | -0.140<br>(0.616) | 0.545<br>(0.031)  |
| $\Delta NE_{t-2}$ |                    |                   |                   |                   |                   |                  | -0.239<br>(0.192) | -0.186<br>(0.261) |
| $\Delta ME_{t-1}$ |                    |                   |                   |                   |                   |                  | -1.511<br>(0.007) | -1.003<br>(0.047) |
| $\Delta ME_{t-2}$ |                    |                   |                   |                   |                   |                  | 1.926<br>(0.000)  | 1.047<br>(0.033)  |
| $\Delta GL_{t-1}$ |                    |                   |                   |                   |                   |                  | -0.413<br>(0.392) | -0.066<br>(0.881) |
| $\Delta GL_{t-2}$ |                    |                   |                   |                   |                   |                  | 0.850<br>(0.055)  | 0.461<br>(0.250)  |
| $\Delta PL_{t-1}$ |                    |                   |                   |                   |                   |                  | -0.505<br>(0.054) | -0.047<br>(0.843) |
| $\Delta PL_{t-2}$ |                    |                   |                   |                   |                   |                  | 0.145<br>(0.517)  | 0.030<br>(0.881)  |

Table 21 (continues). Unrestricted pseudo-structural system using 3SLS,  
p-values in parenthesis

| Regressors        | Dependent Variable |               |               |               |               |               |                   |                   |
|-------------------|--------------------|---------------|---------------|---------------|---------------|---------------|-------------------|-------------------|
|                   | $\Delta NE_t$      | $\Delta ME_t$ | $\Delta GL_t$ | $\Delta PL_t$ | $\Delta SE_t$ | $\Delta SW_t$ | $\Delta RM_t$     | $\Delta FW_t$     |
| $\Delta SE_{t-1}$ |                    |               |               |               |               |               | -0.859<br>(0.132) | -0.091<br>(0.860) |
| $\Delta SE_{t-2}$ |                    |               |               |               |               |               | 1.215<br>(0.013)  | 0.475<br>(0.280)  |
| $\Delta SW_{t-1}$ |                    |               |               |               |               |               | 0.041<br>(0.868)  | 0.153<br>(0.496)  |
| $\Delta SW_{t-2}$ |                    |               |               |               |               |               | 0.252<br>(0.295)  | 0.297<br>(0.172)  |
| $\Delta RM_{t-1}$ |                    |               |               |               |               |               | 0.066<br>(0.514)  | -0.186<br>(0.041) |
| $\Delta RM_{t-2}$ |                    |               |               |               |               |               | -0.007<br>(0.944) | -0.136<br>(0.141) |
| $\Delta FW_{t-1}$ |                    |               |               |               |               |               | -0.491<br>(0.029) | -0.117<br>(0.564) |
| $\Delta FW_{t-1}$ |                    |               |               |               |               |               | 0.205<br>(0.333)  | -0.137<br>(0.473) |
| $e_{1,t-1}$       |                    |               |               |               |               |               | 0.044<br>(0.006)  | 0.029<br>(0.052)  |
| $e_{2,t-1}$       |                    |               |               |               |               |               | -0.005<br>(0.891) | -0.091<br>(0.003) |
| $\bar{R}^2$       | 0.075              | 0.105         | 0.144         | 0.174         | -0.068        | -0.015        | 0.267             | 0.207             |

Table 22. Restricted pseudo-structural system using 3SLS,  $p$ -values in parenthesis

| Regressor         | Dependent Variable |                   |                   |                   |                  |                  |                   |                   |
|-------------------|--------------------|-------------------|-------------------|-------------------|------------------|------------------|-------------------|-------------------|
|                   | $\Delta NE_t$      | $\Delta ME_t$     | $\Delta GL_t$     | $\Delta PL_t$     | $\Delta SE_t$    | $\Delta SW_t$    | $\Delta RM_t$     | $\Delta FW_t$     |
| Constant          | -                  | -0.005<br>(0.000) | -                 | -                 | 0.005<br>(0.000) | 0.002<br>(0.256) | 0.128<br>(0.003)  | 0.067<br>(0.069)  |
| $\Delta RM_t$     | -0.427<br>(0.000)  | -0.385<br>(0.000) | -                 | 0.886<br>(0.000)  | -                | -                |                   |                   |
| $\Delta FW_t$     | 0.734<br>(0.000)   | 0.260<br>(0.040)  | -0.310<br>(0.008) | -1.119<br>(0.000) | -                | -                |                   |                   |
| $\Delta ME_{t-1}$ |                    |                   |                   |                   |                  |                  | -1.463<br>(0.000) | -0.897<br>(0.000) |
| $\Delta ME_{t-2}$ |                    |                   |                   |                   |                  |                  | 1.397<br>(0.000)  | 1.105<br>(0.000)  |
| $\Delta GL_{t-1}$ |                    |                   |                   |                   |                  |                  | -0.379<br>(0.389) | -                 |
| $\Delta GL_{t-2}$ |                    |                   |                   |                   |                  |                  | 0.435<br>(0.000)  | 0.490<br>(0.000)  |
| $\Delta PL_{t-1}$ |                    |                   |                   |                   |                  |                  | -0.400<br>(0.001) | -                 |
| $\Delta SE_{t-1}$ |                    |                   |                   |                   |                  |                  | -0.714<br>(0.011) | -                 |
| $\Delta SE_{t-2}$ |                    |                   |                   |                   |                  |                  | 0.899<br>(0.000)  | 0.524<br>(0.003)  |
| $\Delta SW_{t-1}$ |                    |                   |                   |                   |                  |                  | -                 | 0.159<br>(0.076)  |
| $\Delta SW_{t-2}$ |                    |                   |                   |                   |                  |                  | -                 | 0.270<br>(0.009)  |
| $\Delta RM_{t-1}$ |                    |                   |                   |                   |                  |                  | -                 | -0.166<br>(0.013) |
| $\Delta FW_{t-1}$ |                    |                   |                   |                   |                  |                  | -0.503<br>(0.000) | -                 |
| $\Delta FW_{t-2}$ |                    |                   |                   |                   |                  |                  | -                 | -0.204<br>(0.023) |
| $\Delta NE_{t-1}$ |                    |                   |                   |                   |                  |                  | -                 | 0.5846<br>(0.000) |
| $\Delta NE_{t-2}$ |                    |                   |                   |                   |                  |                  | -0.377<br>(0.000) | -0.226<br>(0.036) |
| $e_{t-1}$         |                    |                   |                   |                   |                  |                  | 0.045<br>(0.002)  | 0.031<br>(0.019)  |
| $e_{t-2}$         |                    |                   |                   |                   |                  |                  | -                 | -0.089<br>(0.000) |
| $\bar{R}^2$       | 0.094              | 0.117             | 0.179             | 0.203             | -0.005           | -0.008           | 0.390             | 0.296             |

Table 23. Contemporaneous correlations

| Variables | $NE_t$ | $ME_t$ | $GL_t$ | $PL_t$ | $SE_t$ | $SW_t$ | $RM_t$ | $FW_t$ |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|
| $NE_t$    | 1.000  |        |        |        |        |        |        |        |
| $ME_t$    | 0.894  | 1.000  |        |        |        |        |        |        |
| $GL_t$    | 0.541  | 0.764  | 1.000  |        |        |        |        |        |
| $PL_t$    | -0.833 | -0.970 | -0.878 | 1.000  |        |        |        |        |
| $SE_t$    | -0.868 | -0.923 | -0.860 | 0.974  | 1.000  |        |        |        |
| $SW_t$    | -0.877 | -0.825 | -0.549 | 0.762  | 0.767  | 1.000  |        |        |
| $RM_t$    | 0.750  | 0.929  | 0.897  | -0.983 | -0.950 | -0.629 | 1.000  |        |
| $FW_t$    | 0.799  | 0.924  | 0.801  | -0.937 | -0.906 | -0.715 | -0.746 | 1.000  |

Table 24. Lag one correlations

| Variables  | $NE_t$ | $ME_t$ | $GL_t$ | $PL_t$ | $SE_t$ | $SW_t$ | $RM_t$ | $FW_t$ |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|
| $NE_{t-1}$ | -      | 0.786  | 0.418  | -0.103 | -0.730 | -0.842 | 0.601  | 0.747  |
| $ME_{t-1}$ | 0.940  | -      | 0.597  | -0.866 | -0.853 | -0.875 | 0.789  | 0.859  |
| $GL_{t-1}$ | 0.652  | 0.733  | -      | -0.792 | -0.803 | -0.599 | 0.780  | 0.786  |
| $PL_{t-1}$ | -0.891 | -0.901 | -0.688 | -      | 0.889  | 0.813  | -0.831 | -0.887 |
| $SE_{t-1}$ | -0.870 | -0.838 | -0.673 | -0.840 | -      | 0.779  | -0.785 | -0.871 |
| $SW_{t-1}$ | -0.809 | -0.727 | -4.295 | 0.654  | 0.664  | -      | -0.524 | -0.656 |
| $RM_{t-1}$ | 0.835  | 0.871  | 0.101  | -0.873 | -0.874 | -0.715 | -      | 0.874  |
| $FW_{t-1}$ | 0.856  | 0.843  | 0.599  | -0.806 | -0.806 | -0.773 | 0.746  | -      |

Table 25. Percentage of the variance of the regional per-capita income innovation attributed to cyclical shocks at horizon  $h$ 

| Horizons | NE                 | ME                 | GL                 | PL                 | SE                   | SW                   | RM                 | FW                 |
|----------|--------------------|--------------------|--------------------|--------------------|----------------------|----------------------|--------------------|--------------------|
| $h = 1$  | 87.475<br>(88.112) | 91.112<br>(91.286) | 57.805<br>(66.835) | 59.473<br>(60.411) | 100.000<br>(100.000) | 100.000<br>(100.000) | 57.614<br>(62.886) | 74.511<br>(76.416) |
| $h = 2$  | 84.923<br>(85.802) | 82.764<br>(84.740) | 37.598<br>(56.375) | 52.144<br>(55.770) | 100.000<br>(100.000) | 100.000<br>(100.000) | 98.597<br>(51.347) | 57.862<br>(63.052) |
| $h = 3$  | 84.719<br>(95.487) | 79.405<br>(83.068) | 33.369<br>(53.471) | 51.889<br>(56.784) | 100.000<br>(100.000) | 100.000<br>(100.000) | 33.411<br>(47.994) | 50.822<br>(58.124) |
| $h = 4$  | 82.488<br>(82.582) | 73.423<br>(78.831) | 28.499<br>(49.541) | 43.809<br>(48.324) | 100.000<br>(100.000) | 100.000<br>(100.000) | 27.347<br>(45.408) | 45.888<br>(56.447) |

Numbers in parenthesis are for trend component ordered first, in variance decomposition

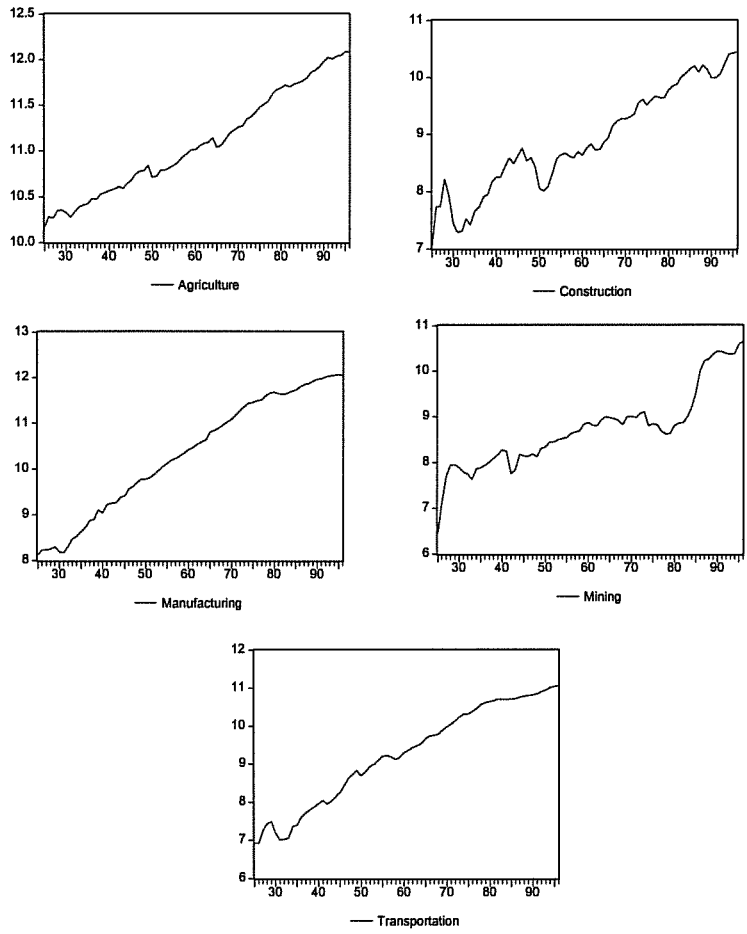


Figure 1. Logarithms of the Colombian sectoral real output, 1927 - 1997

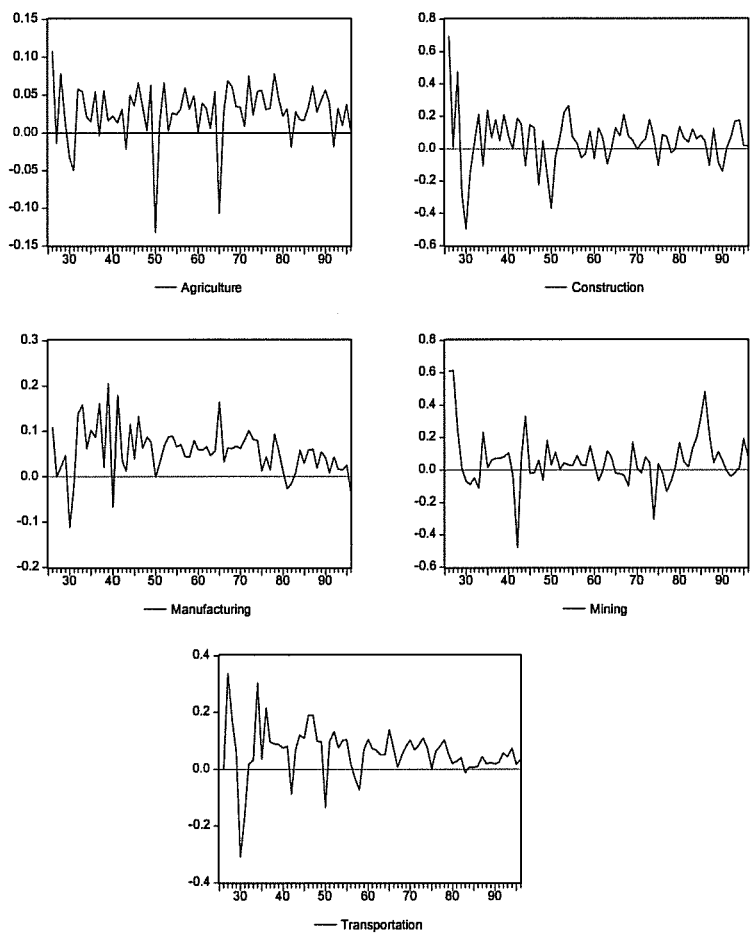


Figure 2. Growth rate of the Colombian sectoral real output, 1927 - 1997

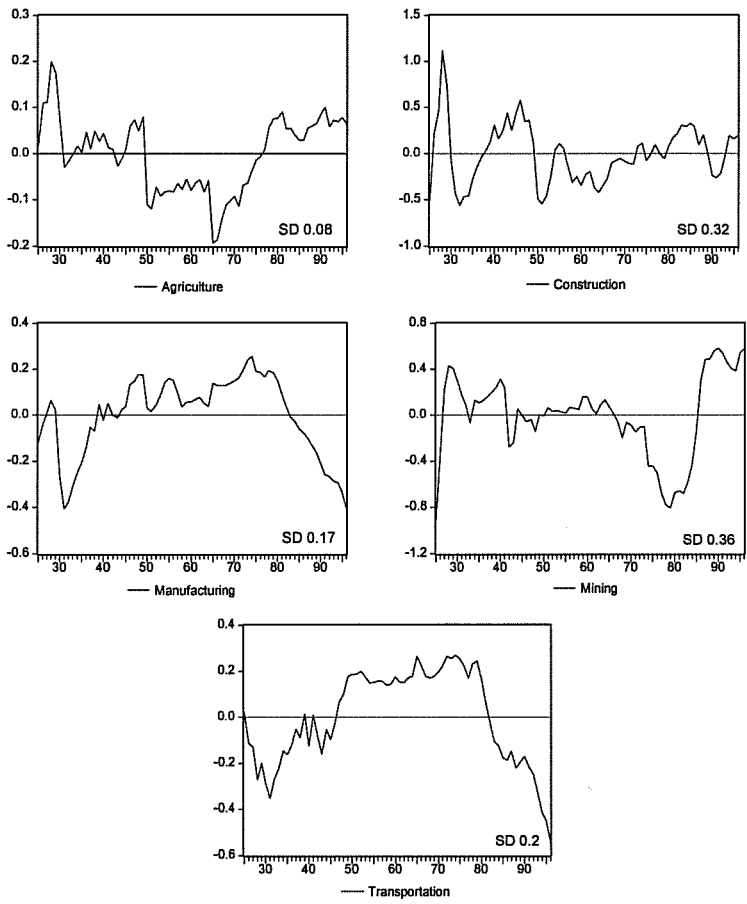


Figure 3. Cyclical components in the Colombian sectoral real output, 1927 - 1997

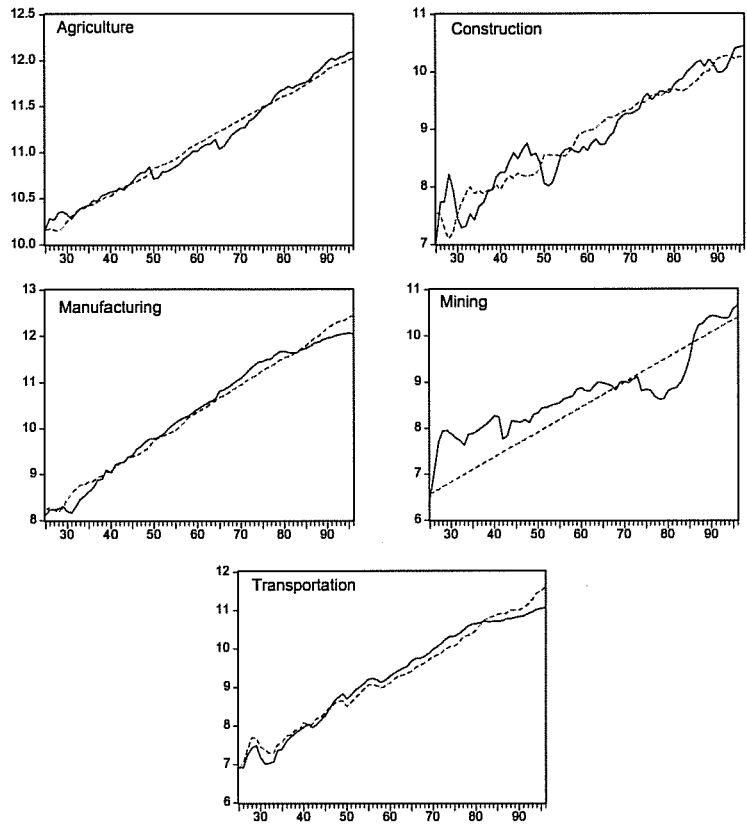


Figure 4. Levels (solid line) and Trend (dashed line) components in Colombian sectoral real output, 1927-1997

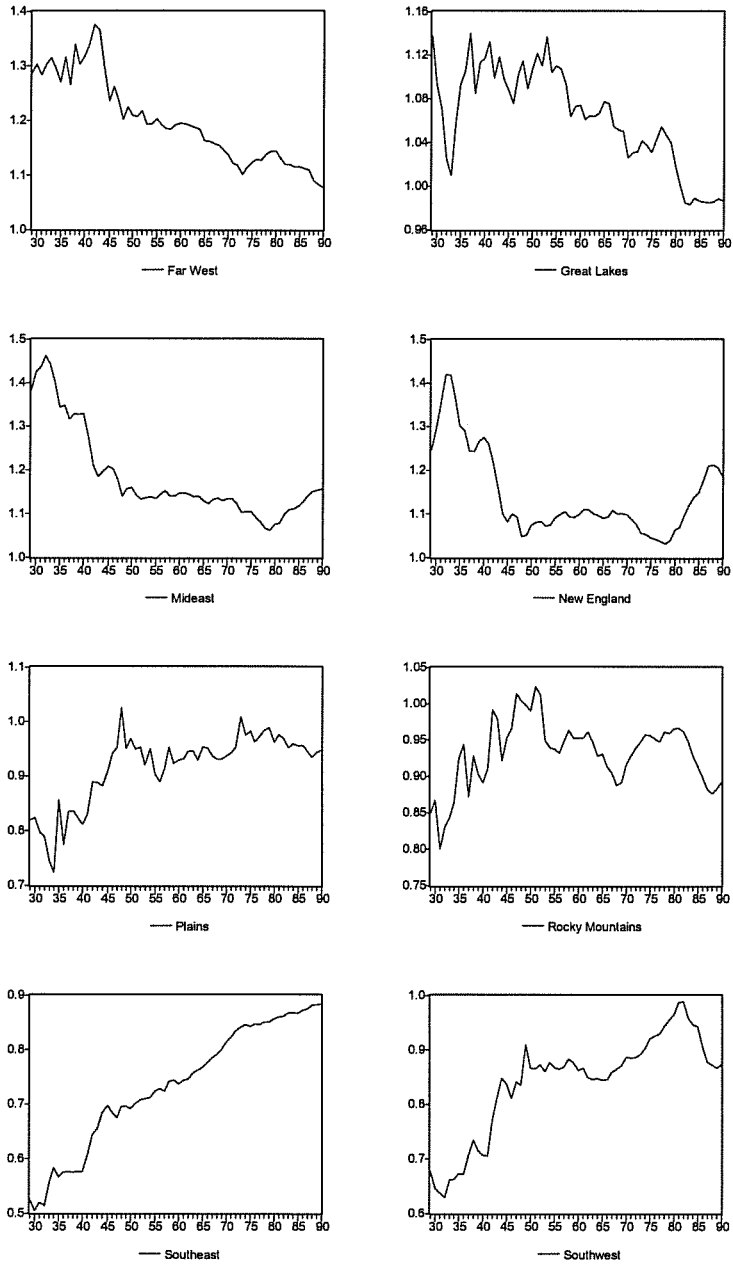


Figure 5. Logs of the US regional real per-capita income, 1929 - 1990

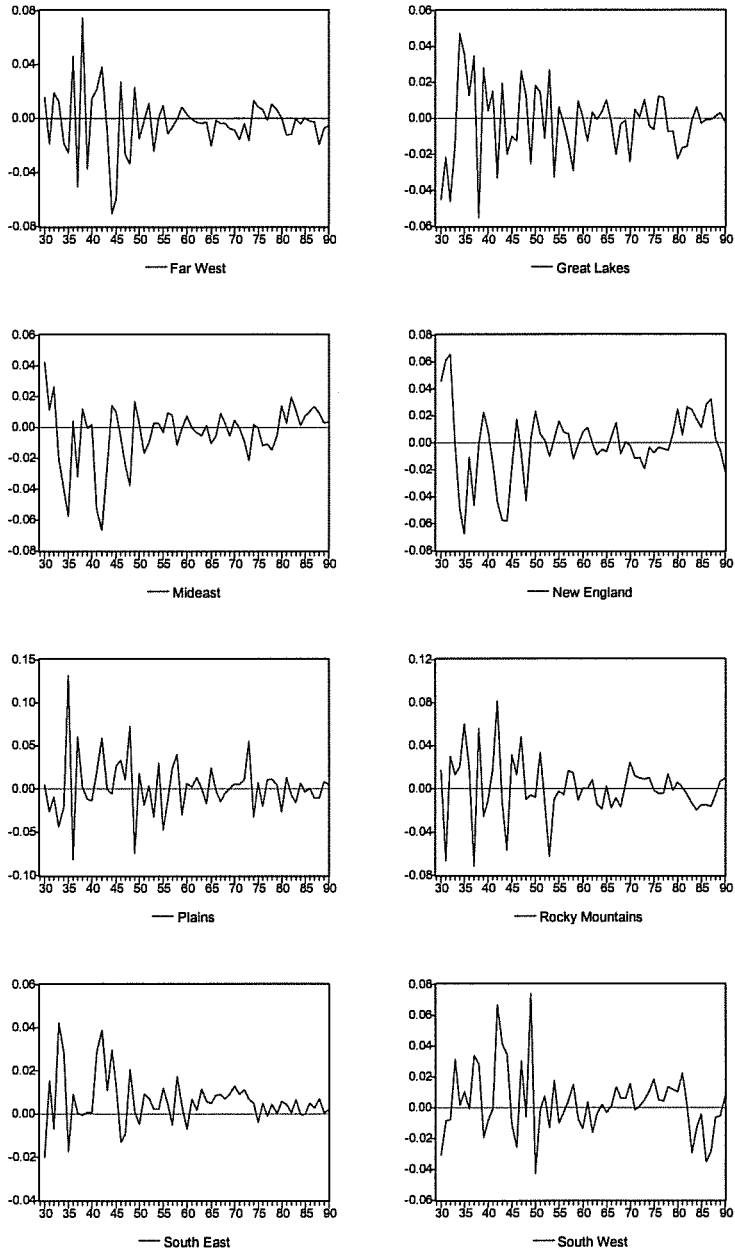


Figure 6. US Regional real per-capita income growth, 1929 - 1990

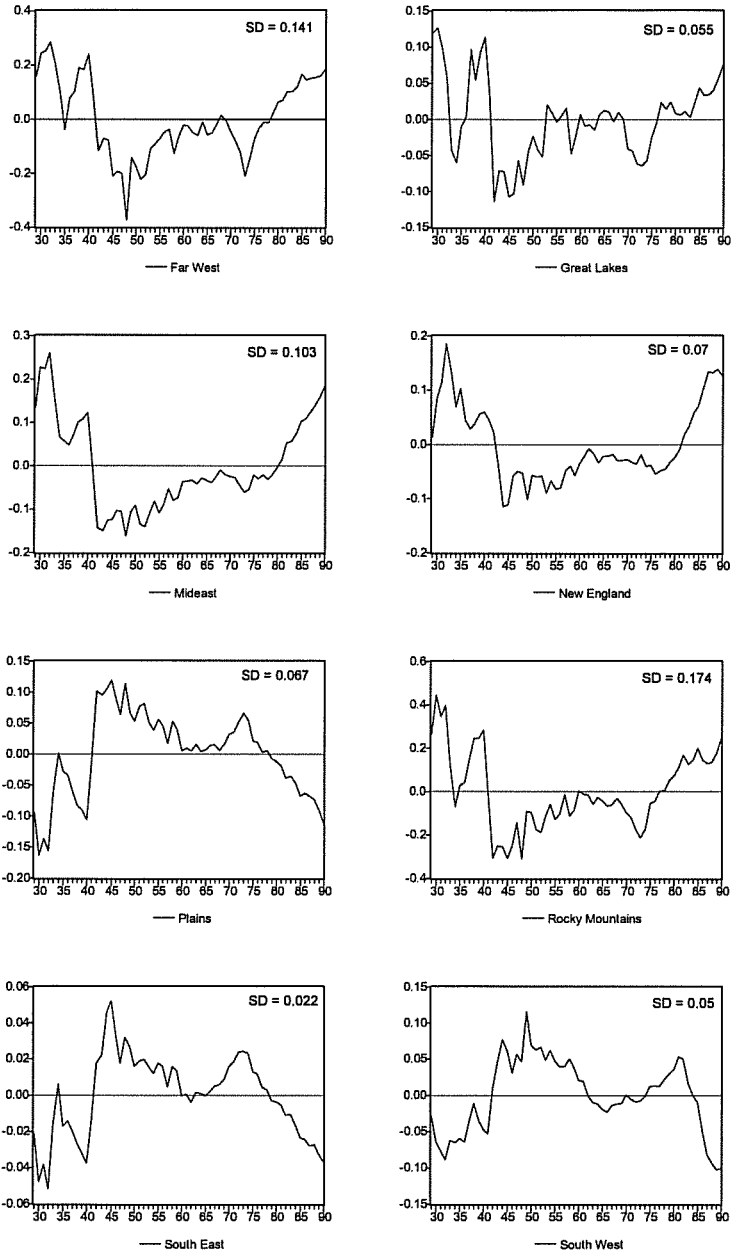


Figure 7. Cyclical components in the US regional real per-capita income, 1929-1990

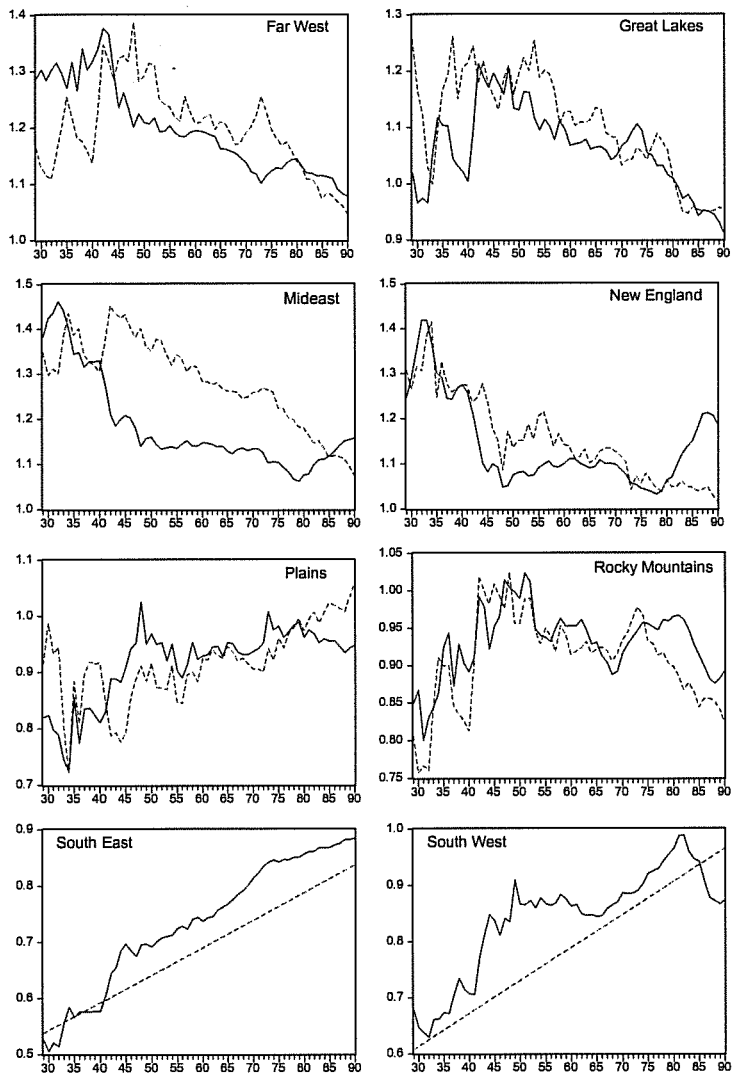


Figure 8. Levels (solid lines) and Trend (dashed lines) components in the US regional real per-capita income, 1929-1990