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The Bitcoin Boom: An In Depth Analysis Of The Price Of Bitcoins

Major Research Paper

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Abstract

The extreme volatility of Bitcoin prices has garnered some serious attention from the media and the academic community. Academics have flocked to the crypto currency and conducted empirical analyses. Unfortunately, the results of these empirical works have been inconsistent, which makes it difficult to draw definitive conclusions regarding the factors that affect the price fluctuations of Bitcoins. This research project has two main purposes. Firstly, this paper re-examines the works done by the previous researchers and assesses if their findings remain valid ex-post. Secondly, this paper investigates the long term factors that are affecting the price of Bitcoins. Our findings show that several previous results are drastically different when they are re-tested after these results have been published . In addition, our findings show that the majority of the volatility of Bitcoins came from unexpected shocks. We were able to capture the effect of these unexpected shocks by using a Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model. The GARCH model transforms the standard OLS residuals into an endogenous process that allows its variance to vary across periods. We conclude that these unexpected shocks are by far the largest contributor to the price fluctuations of Bitcoins.

1. Introduction

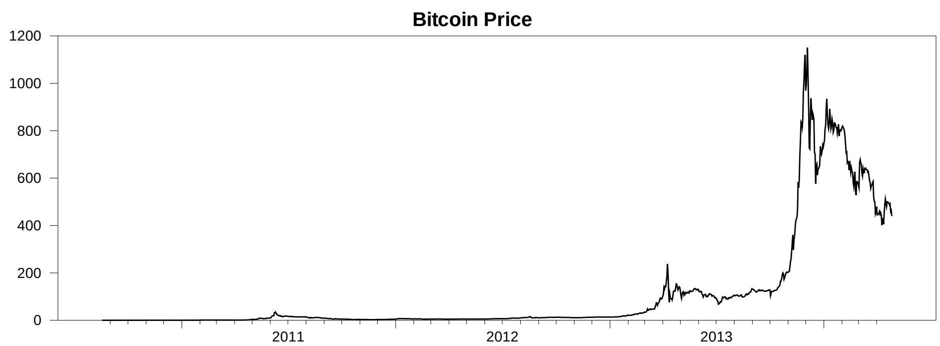
In 2008 the programmer(s) using the pseudo name of Satoshi Nakamoto wrote a document called Bitcoin: A Peer-to-Peer Electronic Cash System. In this document the author(s) give a detailed description of a new form of electronic currency. The currency, according to the authors, is said to be completely independent of any third party financial intuition, and has a built in peer to peer system that replaces all checks and balances that a typical financial institution would provide. In addition, Bitcoin production is based on a fixed rate, controlled by an algorithm that is set to expire in 2040. The algorithm is programmed to create a complex problem every ten minutes, approximately. Individuals, who are called Miners, use their personal computers to solve these complex problems. Solving this problem does two very important tasks. First, it sorts all the transactions done during a specific time frame in a meaningful manner. Solving the problem and sorting the transactions is not the same thing however, for simplicity we will say that they are. Secondly, it rewards the Miner (or Miners) who solve the problem with a set amount of Bitcoins. The algorithm controls the supply of Bitcoins by adjusting the level of difficulty associated with each problem. If Miners are solving the problems too quickly, thus increasing the supply of Bitcoins too quickly, then the algorithm will increase the level of difficulty of the proceeding problem(s).

It is now 2015 and the popularity of this seemingly ambiguous invention has taken leaps and bounds. Websites like Bitcoin.org, coindesk.com and blockchain.info are all completely devoted to the crypto currency. Financial markets trade the crypto currency alongside other reputable national currencies. Some nations, like Germany and India, have even declared Bitcoins to be a legal currency.

Along the way from its inception to now, the price of Bitcoins has fluctuated tremendously. The graphical representation below would resemble the blue print of a roller coaster more than a financial asset. To give some perspective lets highlight some key points over the years.

Figure 1

Bitcoin Prices Over Time



In 2012 the currency was trading for 5 USD$. By the end of the 2013 it exploded, and was being traded for nearly 1200 USD$, a value that was higher than gold at the time. To put it into perspective, the percentage increase for Bitcoin owners was approximately 22,000%. According fortune.com the closet asset that achieved a similar return during that time were Fannie Mae and Freddie Mac stocks. Fannie Mae and Freddie Mac yielded 1,080.4% and 1002.7% respectfully. However, since its peak the currency has been declining slowly, it now trades at around 240 USD$.

These remarkable returns and price fluctuations have garnered some attention from the academic community. Several researchers have done very insightful work to help explain the phenomenon, researchers such as Ciaian, Rajcaniova and Kancs (2014), Bouoiyour and Selmi (2014) and

Kristoufek (2015), Yelowitz and Wilson (2014) Badev and Chen (2014) to name a few. This paper continues the work done by these researchers in an attempt to close the knowledge gaps, clear up some the inconsistencies, and more importantly assess if their results are a good representative of the long term trends affecting Bitcoins.

To give a small preview of our findings, we noticed that many of the studies done were written during or right after the "Bitcoin Boom", a 53 day period in which the value of the currency increased tenfold from 100 USD$ to 1200 USD$. As such, many of the authors had models that were very good at decomposing this particular period but were not as detailed in determining some of the long term variables that affected the price.

To correctly model this series we will use a GARCH specification. The GARCH model transforms the exogenous time varying volatility of Bitcoins into an endogenous process and allows us to relax some of the Gauss Markov assumptions, specifically relating to homoscedastic time variance. Our results indicate that Bitcoin prices were mainly affected by exogenous shocks relating to this endogenous process, and more interestingly, that our independent variables had little explanatory power.

The remainder of this paper will examine, in detail, multiple aspects of the crypto currency. The second section examines the existing academic literature. The following section presents all the explanatory variables that we have tested in our econometric model. Each presentation will include a brief description of the variable and our hypothesis of its relationship with the price of Bitcoins. The fourth section provides a detailed description of our methodology. The main goal of this section is to correctly specify our empirical model. The fifth section of this paper examines the results from econometric testing. We compare and contrast our findings with our hypotheses and with the findings of different academic studies. The final section summarizes and concludes the paper.

2. Literature review

Several authors have done both empirical and theoretical work to help explain the fluctuations in the price of Bitcoins. However, the results of these academic studies are somewhat inconsistent. One simple example of these inconsistent results is seen in the studies done by Ciaian, Rajcaniova and Kancs (2014), Bouoiyour and Selmi (2014) and Kristoufek (2015). All of the researchers look at the Supply and Demand influences, macro-economic conditions and the speculative attractiveness for investors. All three authors have come to different conclusions about their significance. To look at a specific example let's examine the number of daily transactions; the authors look at this variable as a measurement for the demand of Bitcoins. Basic economic theory would tell us that increasing the number of transactions involving Bitcoins would cause an increase in the demand for the currency and a subsequent increase in its relative price.

Interestingly enough, Ciaian, Rajcaniova and Kancs (2014) find that the relationship between the variables is negative. Using a Vector Auto Regressive (VAR) model they test the relationship twice. The first time is in a sub model focusing on supply and demand factors. In this sub model the number of transactions had a negative impact on the price of Bitcoins. On the other hand, when they tested the same variables, in a larger, more inclusive model they found the relationship was insignificant.

Kristoufek (2015) used a more complex wavelet model to show that the in the long run, an increase in the number of transactions tends to drive the price up, with the caveat that the effect tends to diminish later in the sample. In the short run the author argues that a sudden spike in the relative price of Bitcoins can cause a speculative bubble. In these periods of "high uncertainty and volatility" a negative relationship can occur between the number of transactions and the price level. It is not stated explicitly but the idea here is that a speculative bubble may "burst" if we have a sudden increase in transactions. Individuals may develop a scared herd mentality and quickly try and dumb the asset on the market causing a downward pressure on the price.

Bouoiyour and Selmi (2014) use a method of cointergration to test the relationship between the number of transactions and the price of Bitcoins. This method is called the ARDL bounds testing approach. They perform several estimations, across multiple samples, and find that the relationship between the number of transactions and the price of Bitcoins is significant and positive across all periods. It should be noted that the authors use the exchange- trade ratio (ETR, the daily volumes of Bitcoins on the currency Exchange markets over the daily volume of trade) as a proxy to the number of transactions. The authors do not give any in depth explanation as to why they do not use the actual number of transactions versus the ETR. They simply state (page 5) "The exchange-trade ratio is measured as a ratio between volumes on the currency exchange market and trade. It can be considered as transactions proxy (Kristoufek, 2014), or to address whether Bitcoin is business income (Bouoiyour and Selmi. 2014)." In addition, they find that the

ETR is a driver in the price of Bitcoins, where the reverse is not true. Bitcoins do not have a significant effect on the movement in the ETR.

The lack of consistency does not end there. Other indicators like the Hash rate, defined as level of computing power needed to solve the current block chain algorithm), monetary velocity of Bitcoins, and the estimated output volume all have various level of significance and impact between the three aforementioned articles.

Even if we were to take a broader point of view, the authors seem to have different views on what grouping of variables has the greatest impact on the price of Bitcoins. Ciaian, Rajcaniova and Kancs (2014) states that the strongest impact on the price of Bitcoins is derived from individuals desire to use Bitcoins as a means of investment. In contrast, using a variance decomposition method, Bouoiyour and Selmi (2014) show that investor attractiveness contributes 20.34% to the price fluctuations of Bitcoins. Some authors feel that Bitcoins can be explained solely by speculative bubbles. Kristoufek's (2013) article is one of the first to bring up this point. More recently, Tuck and Fry (2015) published a paper that claims that the fluctuations in Bitcoin prices can be explained entirely by speculative bubbles. Their paper develops a more classical model that uses advanced calculus techniques to solve a system of equations. They then use empirical methods to test the validity of their model against the actual data.

Unfortunately, there are not as many academic papers that use models based on micro foundations to help explain the price of Bitcoins. Tuck and Fry (2015), the paper mentioned above, and Morten, Molnár, Vagstad, and Valstad (2015) are two models that use micro foundations to help explain the price fluctuations of Bitcoins. We put particular attention to the article done by Tuck and Fry (2015) because it gives some strong insight in the formation of speculative bubbles. Other articles do not give much explanation as to how these bubbles are created. We give the article done by Morten, Molnár, Vagstad, and Valstad (2015) a great deal of attention as well because the micro foundations presented in this article can be used to capture the effect Baidu had on the price of Bitcoins. The next few paragraphs are devoted to summarizing their models.

Tuck and Fry (2015) model begins with the following function.

2.1) $P(t) = P_1(t)(1-\kappa)^{j(t)}$

P(t) is the price of an asset at time t, $P_1(t)$ is the price of the asset before a crash, κ is the percentage that is wiped of the price off the asset if a crash occurs and j(t) is a jump process that takes on the value of 0 or 1. Through a series of calculations the authors come to two first order conditions. The first F.O.C. is based on the assumption that the intrinsic rate of return of the asset is μ and is constant. This gives an actual rate of return equal to the following

2.2) $\mu(t) = \mu + \nu h(t)$

2.3) $\nu = -\ln[(1-\kappa)] > 0$

Where $h(t)$ is the hazard rate. The second F.O.C. reflects the level of risk. Using the assumption that the intrinsic level of risk is σ^2 they find that the actual risk is.

2.4) $\sigma(t)^2 = \sigma^2 - \upsilon^2 h(t)$

Both 2.2) and 2.4) include an intrinsic value and a value associated to some level of risk (hazard rate). For 2.2), the model suggests that an investor must yield a higher rate of return for any increase in the hazard rate. Intuitively this make sense, typically riskier assets, such as low grade national bonds, will offer higher returns to compensate the investor for a potential loss. 2.4), is not as intuitive. 2.4) suggests that an increase in the hazard rate will lower the level of risk associated with Bitcoins. The authors acknowledge this counter intuitive results and explain that 2.4) represents the over confidence of the marketplace. The authors then conduct a hypothesis test, testing.

2.5) $H_0 : \upsilon = 0, H_1 : \upsilon > 0.$

Their results indicate that ν was significant over several different periods, testing the hypothesis repeatedly using a moving time window. They estimate that these speculative bubbles account for 48.7% of the observed prices.

While extremely interesting and thought provoking, these results continue the trend of inconsistency that we have pointed out. The wide range of possibilities presented in the literature is a testament to how difficult it is to fully explain what has happened. It is our belief that limited sample periods represent a significant portion of the problem but as Morten, Molnár, Vagstad, and Valstad (2015) have shown it may be simply caused by poor data sources.

Morten, Molnár, Vagstad, and Valstad (2015) looked at the price of Bitcoins across different Exchanges. The authors observed that for any given time t the price of Bitcoins varied significantly. At times these variations were as large as 20%. Focusing on the seven largest Exchanges Mtgox, Virtex, Btcn, Btce, Bitstamp, Bitfinex and Bitcurex, they developed a theoretical model to explain these observed differences. In this model the observed price of a particular market i at time t was composed of two parts

2.6) p* a common price that is said to follow a random walk

2.7) u^e an idiosyncratic component.

p* is called the fundamental news component or the efficient price. They define the change in the efficient price as r_t

2.8) $r_t = p_t^* - p_{t-1}^*$

Similarly the change in the observed price (y_{it}) is as such

2.9) $y_{it} = p_{it} - p_{it-1}$

2.10) $p_{it} = p_t^* + u_{it}$

To solve the model the authors make a series of assumptions, regarding the covariance and variance of each parameter. They then use these assumptions to find the following.

2.11) $\mathrm{cov}(y_{it}, r_t) = \sigma^2 + \psi_i$,

where σ^2 is the variance of the efficient price, while ψ_i is the idiosyncratic component that determines how a particular market and Exchange react to a change in the efficient price. The idea is that higher values of ψ_i imply a stronger signal to the marketplace for a given change in p_t^*. It also means that markets with a higher value of ψ will make more informed decisions. In addition they, define π_i as the share of transactions that the i^{th} Exchange has in comparison to the entire market. Multiplying this term with the covariance in (2.11) and then diving by σ^2 gives what the authors call the information share of a particular Exchange.

2.12) $IS_i = \frac{(\sigma^2+\psi_i)\pi_i}{\sigma^2}$

This information share IS_i represents the impact that a particular Exchange i has on the entire industry (industry meaning the sum of all the Exchanges). The authors were able to test their model and found that the majority of the share of information came from MTgox and Btce. Together the two accounted for more than 70% of the total information. However, this figure is not static, as its value is different in different periods. This study was thus able to show that some Exchanges were pushing the price of Bitcoins while other Exchanges were being more reactive to the changes. What is important to note about this study is that researches must be prudent about their data sources. Only using the values from one Exchange may give an incorrect image of the entire industry. They do explain that if one were to use an aggregate value of all the Exchanges, then in theory, all the idiosyncratic terms could cancel each other out and we would be left with just the efficient price* $\sum_{i=1}^{n}(\pi_i\psi_i) = 0$. Later we will revisit this study and we will see how the information share distribution can help explain some major shocks that happened to the price of Bitcoins.

Given that the majority of authors get data from blockchain.info, which provides an average value of all Exchanges, we can assume that most of the idiosyncratic components have been removed from the series. That being said, we still do not have a logical explanation as to why results from these papers differ so much. One possibility could be that authors may be miss-interpreting their findings or filling knowledge gaps with false assumptions. For example, both Ciaian, Rajcaniova and Kancs (2014) and Bouoiyour and Selmi (2014) use a previous study done by Kristoufek (2013) to proxy investor interest in Bitcoins. In his study Kristoufek (2013) tests the relationship between Bitcoin prices and virtual hits of Bitcoins on the internet. Virtual hits consist of Wikipedia and Google views. He hypothesizes that the increased interest in the virtual currency is a proxy for investor interest. Based on his research and the work done by others after him, it would appear that the literature supports the hypothesis that Wikipedia and Google hits have a significant impact on the price of Bitcoins. What is not supported is the link between those hits and the "investor interest" in Bitcoins. To clarify, Kristoufek (2013) is assuming that some fixed portion of all the individuals looking at Bitcoins on these search engines, did so with the intent of possibly investing in them. He then hypothesizes that this investor interest could have contributed to a pricing bubble. As Yelowitz and Wilson (2014) have shown, that assumption may not be correct.

Yelowitz and Wilson (2014) uses Google Trend data to test specific demographics to better understand who is using Bitcoins. This study focuses mainly on four groups, libertarians, criminals, programmers and potential investors. Potential investors refers to individuals who are profit seeking. The results are somewhat surprising. Investor interest and libertarians did not have a strong link with Bitcoin search inquires. What this implies is that individuals who were looking up Bitcoins on the internet were predominantly interested in programming or "illegal activity" in conjunction with Bitcoins. For clarification, they used word associations to determine if someone was interested in illegal activity. For example, anyone who searched for Silk Road[1] and Bitcoins was placed under the "illegal activity" category. What is very interesting here is that if we take these results as true, then Ciaian, Rajcaniova and Kancs (2014), Bouoiyour and Selmi (2014) and Kristoufek (2013) have found a link between "hackers/programmers" and Bitcoins, and "criminals" and Bitcoins, not investors.

A shortcoming of Yelowitz and Wilson (2014)'s article is the fact that it was based only on U.S. demographics. It stands to reason that the U.S. preference may not be representative of the global community and may be missing key factors, for example Satoshi Dice[2]. As Badev and Chen (2014) illustrate, from mid-2012 on more than half of the transactions done on the Bitcoin

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[1] Silk Road was an eBay like platform that sold several illicit goods on the black market. One very important aspect of Silk Road was that the only currency that was accepted was Bitcoins. Mainly because they are non-traceable by law enforcement, Bitcoins provided the underworld with a legitimate way to do business. Using a web provider that makes your movement very difficult to track, called TOR, customers and vendors were able to do business with relative anonymity. Unlike your standard .com's Silk Road was located on what is called the dark web. The dark web was originally developed by the navy and was used to communicate in secrecy. Any internet address that is located on the dark web cannot be found by your typical search engines like Google or Yahoo. One must know the exact location of the website and have access to TOR to travel to the location. In short, Silk Road was located on the most secretive areas on the internet, and provided anonymity for both the sellers and buyers.

[2] Satoshi Dice is an online gambling website, based off an Irish server, created by Erik Tristan Voorhees. The game works as such. An individual will send a Bitcoin to one of the IP addresses associated to the game (located on the Satoshi Dice website). Then, a random number generator will decide if that individual is a winner or a loser. The premise is nearly identical to a slot machine. For our purpose it is only important to know that individuals who have access to the Bitcoin network can gamble with the push of a few buttons at home. Not surprisingly, Satoshi Dice ID's are by far the most active on the network and have accounted for nearly half of the transactions ever done, see Badev and Chen (2014).

network were related to Satoshi dice. Furthermore, Satoshi Dice banned U.S. IP address in May of 2013 in fear of legal action against them. This implies that any study done based on U.S. demographics may be missing key information.

One more interesting aspect of Yelowitz and Wilson (2014)'s article, which multiple articles do not discuss, is the established connection between the criminal world and Bitcoins. To understand this link we can look to a study done by Nicolas (2012). In this study, the author dives head first into the underworld and takes detailed notes on everything that transpires on Silk Road.

Interesting enough, with all its secrecy, the creator of Silk Road did keep a public log of all the transactions that were done. He kept the identities of the sellers and the buyers secrete but did list all the goods, including prices (in Bitcoins) and the time of sale. This meant that individuals could record all the transactions directly from the site. This is precisely what Nicolas (2012) did. He found that, overwhelmingly, most of the vendors came from the U.S., accounting for 43.83% of his total sample. In addition he found that the majority of the items sold were drug related, mainly Cannabis. He estimated that by the end of 2012 the Silk Road economy had a trading volume of approximately 7000 BTC/Day and yielded 1.2 million dollars in revenue per month for all sellers. Given that the U.S. accounted for a majority of the sales done on Silk Road, Yelowitz and Wilson (2014) results seem to makes sense.

To recap, Nicolas (2012) forces us to question the arguments presented by Kristoufek (2013) that online interest in Bitcoins is a proxy for investor interest. We will not rule out Kristoufek (2013) hypothesis entirely; however it does require more research before any affirmative statement can be made, on either side. In addition, the general inconsistency in the academic research, coupled with the relatively short life span of Bitcoins, suggests that more work needs to be done. Understandably, testing everything mentioned in this literature review into one article would create something cumbersome. Nonetheless, it is important for both writers and readers to at least exhibit a simple understanding of the Bitcoin universe. If not, writers and readers may make false conclusions and assumptions, like the ones we have mentioned above.

In regards to this study, we will recreate a model similar to the ones presented by Ciaian, Rajcaniova and Kancs (2014), Bouoiyour and Selmi (2014) and Kristoufek (2015). Retesting hypothesis across different sample periods and using different methods is essential to the academic process. As such, we will extend on the works done by these three authors and retest different aspects of their results, changing the econometric methodology, sample size, and timeframe.

The next section will look at our data set. We will examine each series that we introduce into our model and discuss their expected relationship with the price of Bitcoins. We will also list our data sources at the end of the section.

3. Data

One of the most impressive aspects about the Bitcoin network is its data archives. As it stands now, anyone can go to https://blockchain.info and collect and analyze multiple data sets, such as the number of transactions per day, daily volume traded on the network, daily Bitcoin prices measured in USD$, and so on. Below, we list all of our independent variables that we have collected and include a description explaining our expectations of its effect on the fluctuations of Bitcoin prices. All of our data sets are transformed into first log differentials at a daily frequency unless specified otherwise.

3.1 Variables

3.1.1 Dependent variable

BitP: This is our dependent variable and is the value of one Bitcoin in terms of USD$.

This series is pulled from blockchain.info. It is developed by taking the average price of Bitcoins in USD$ from the largest Bitcoin Exchanges. Using an average price taken from the largest Exchanges will reduce the idiosyncratic effects created by each individual Exchange.

3.1.2 Bitcoin Exchanges and networks

Bitcoin Exchanges and Networks are composed of three series VolUSEX, CNYEX and VolBitNet. All three of these series capture the daily volume of transactions done their respective platforms.

VolUSEX: The daily volume of Bitcoins traded on the markets using USD$. This series captures any sale or purchase of Bitcoins that is done on any of the Exchanges using USD$. We expect the relationship between VolUSEX and BitP to be positive: an increase in the amount traded should have an upward push on the price of the currency caused by an increase in its demand.

CNYEX: The daily volume of Bitcoins traded on btcnCNY, a Chinese based Exchange. All transaction done on btcnCNY is done in Chinese Yuan. Recall that in our literature review we stated that Morten, Molnár, Vagstad, and Valstad (2015) information share calculations could be used to help explain some of the major shocks that occurred in the Bitcoin market. They found that from April 2013 through December 2013 the information share of btcnCNY jumped from .040 to .325. What those figures mean is that in April 2013 the Exchange accounted for roughly 4% of the information of the market and by December 2013 it was 32.5%. What is more interesting is that in the following month (January 2013) the share drops 20 percentage points to about 12.5%. As evidence that their calculations are a good representation of actual markets, they state that during the same period the Chinese Exchange (page 33) "went from being a minor regional player the world's largest Exchange in terms of volume in November and December 2013". The authors correlated this sharp increase in the information share of btcnCNY with the announcement of Baidu accepting Bitcoins as a method of payment. Baidu is a Chinese based web provider that has an estimated net worth of 50 billion dollars. Many have called it the Chinese Google.

To highlight the importance of Baidu on the Bitcoin market we conducted a very quick and rough analysis. Baidu officially accepted Bitcoins for 53 days, from October 15th to December 6th. In the 53 day priors to the Baidu announcement the currency appreciated 30%. During the 53 period when Baidu accepted Bitcoins the price of Bitocins increased by 542%. In the 53 days after it dropped the currency, the price rose by a 10%. Clearly this is not enough evidence to make an affirmative statement, however it does give us grounds to develop some theories.

If we accept the finding from Morten, Molnár, Vagstad, and Valstad (2015)'s paper and attribute the increase in the volume of the btcnCNY Exchange to Baidu, then we can use the volume exchanged on the network as proxy for the "Baidu effect". If we then find that the volume on that Exchange is able to explain some of the fluctuations in the price of Bitcoins then we can infer that Baidu has a significant impact.

Of the studies we looked at, Badev Anton, and Matthew Chen (2014), Kristoufeg et al (2014) and Bouoiyour and Selmi (2014) all show that either Baidu or the Chinese markets have had a significant impact on the price of Bitcoins. As such we feel that this series will play a key role and be highly significant.

VolBitNet: The daily volume of Bitcoins traded on the Bitcoin network, measured in units. This series includes all the Bitcoins that were sent directly from person to person. These transactions are conducted independently from any Exchange and are akin to wire transfers from persons to persons. The transactions do not imply any sale/purchase but can be used to do so. This variable may be positively or negatively related to BitP. If we expect that the increase in the daily traded volume of Bitcoins on its own network is more closely related to an increase in the number of transactions, then we can imagine that this will have a positive correlation with its price. However if we feel that the volume of traded Bitcoins has stronger relationship with the total supply of Bitcoins in existence, then we can predict a negative relationship.

Both Mitsuru, Kitamura, Matsumoto and Hayek (2014) and Kristoufek (2015) have the opinion that the total supply of Bitcoins is insignificant. Both authors feel that because the supply is always known and fixed, consumers and producers will be more likely to react to demand side factors. We concur with that belief. It is more plausible that the daily volume of Bitcoins traded on the block chain network is more closely related to the number of transactions and as such should have a positive relationship with the price of Bitcoins.

3.1.3 Bitcoin supply and demand indicators

ID: The number of unique Bitcoin ID's. The number of unique ID's is not a true representation of the number of users on the Bitcoin network. As Satoshi Nakamoto (2008) explains, one PC has one wallet which contains multiple ID's. That is to say that one person may own multiple ID's. Unfortunately it is much easier to get a total number of ID's on the network than it is to get a total number of unique end users. However, that being said, it is not an unreasonable expectation to assume that the number of unique ID's is highly correlated with the number of end users. As such, using this value should still yield a fairly accurate relationship. As far as the relationship itself, it stands to reason that an increase in the number of ID's/Users of a currency can only increase the demand for it and, as such, cause an upward push on its price.

Trans100: The total number of unique transactions excluding the top 100 most popular ID's. This variable is very interesting in its interpretation. The block chain website provides this variable, without a doubt, to remove the effect of Satoshi Dice. As we explained prior, Satoshi Dice accounts for a very large portion of the total number of daily transactions. Removing

Satoshi Dice from the analysis will eliminate many small transactions that may not be significant. This is due to two main reasons. The first reason is that the transactions done with Satoshi Dice ID's are not done to facilitate the purchase of any goods, services or investments. Any transactions done using Satoshi Dice ID's represent individuals who are playing a game. If the game was constructed in a different manner we could have far fewer transactions done. For example, if individuals bought tokens using Bitcoins and gambled with these tokens rather than Bitcoins itself, the demand for Bitcoins would be unchanged but the number of transactions would be lower. Thus, the number of transaction recorded may not be reflective of the demand. The second reason we wish to remove the top 100 ID's is because we wish to capture the effect of a growing market place. The Satoshi Dice forces individuals who are playing their game to conduct multiple transactions per day, in order to play. This large number of transactions will drown out the effect of any small increase in the demand for Bitcoins. To put it into perspective, imagine if the U.S. were trying to estimate the number of transactions that took place in a day within its borders. Would they include every spin of a slot machine as a transaction? The answer is most likely no, and as such including every turn on Satoshi Dice may not be a good indicator to the real number of transactions. We hypothesize that this variable should have a positive correlation with the price of Bitcoins.

Hash: The estimated number of giga hashes per second (billions of hashes per second) the Bitcoin network is performing. As Kristoufek (2015) explains, the hash rate is the measurement of the processing power needed to solve the mathematical problem associated with the next block chain. In order to maintain a steady flow of Bitcoins the algorithm will increase or decrease the difficulty of the mathematical problem that is presented to the miners. With this built mechanism, the supply of Bitcoins can be controlled and easily predicted.

An increase in the Hash rate can be caused by two different situations. First we may have an influx of miners working on the algorithm which, if not corrected, will increase the supply of Bitcoins too quickly. Meaning that the algorithm will increases the difficulty of the problems, and as such increase the aggregate processing power needed to solve the next chain. Another possible case is that computer processing power dramatically improves. This will cause the Hash rate to initially drop since computers have become more efficient. Following that the system will have to react and increase the difficulty to some new level that may or may not increase the hash rate to a level higher than the rate prior to the technology boost. How the hash rate is related to the price of Bitcoins is not so clear. If the Hash rate increases too high then we could see some miners exiting the market because the cost associated to mining is too high. This is the argument presented by Mitsuru, Kitamura, Matsumoto and Hayek (2014) where they explains that increased competition of Bitcoins mining will push miners to "dig" for other crypto currencies instead of Bitcoins. This exodus of miners will lower the demand for Bitcoins and as such lower the price. Their argument is simple: the presence of other crypto currencies creates a monopolistic competitive market that will have miners seeking out the most profitable product. On the other hand, as we explained above, the increase in the Hash rate could be caused by an influx of miners seeking to attain some profits. As Ciaian, Rajcaniova and Kancs (2014) explain, the number of active users can be employed as a proxy to measure the size of the Bitcoin economy. Increasing the economy, while maintaining a fixed supply of a good, will most likely drive the price up. In this case the Hash rate could be positively related with the price of Bitcoins.

Trans: The total number of unique transactions. Regardless of the effect that Satoshi dice has on this series, which we explained above, we expect a positive relationship with the price of Bitcoins. Please refer to literature review section where we analyze this series in detail.

Dif: This series records that level of difficulty of the Bitcoin algorithm over time. As we have explained in our introduction, the algorithm controls the supply of Bitcoins by increasing or decreasing the level of difficulty for each problem. This variable is used as a proxy for the hash rate; refer to our hypothesis on the hash rate.

CumuBitVol: The cumulated volume of total Bitocins in existence. Like Mitsuru, Kitamura, Matsumoto and Hayek (2014) and Kristoufek (2015) we believe that the cumulated volume of Bitcoins will be insignificant in the short run. Given that the supply increases at some known constant rate, we do not suspect it will have any impact on the price fluctuations. What we predict will happen, which we cannot test, is that as we approach the final block in the chain, we will see a significant drop in the price. We imagine that miners will slowly move to other crypto currencies in search of higher profits, as Mitsuru, Kitamura, Matsumoto and Hayek (2014) predict.

3.1.4 Public interest

Google Trend is used to proxy public interest in Bitcoins.

Google Trend: Google offers a very unique service in which they anonymously track user's activity. With this massive database they can summarize all the search inquires done for a particular topic over a particular time. In addition they scale all the information over the entire sample set, 100 are the highest points of interest and 0 are the lowest. This is the only series that we did not transform into logarithmic differences. Another important note, we tested this series over two different samples. We expect that the two different samples will have different results. Incongruent with all the previous studies done, we expect that this series will be very significant when tested on smaller sample sets that end around 2014 but will drop out of the significance range when tested on larger samples. Based on the previous work done we assume that this series captures the effect of a speculative price bubble, such as the one done described in the model created by Tuck and Fry (2015). However, given that the price has dramatically dropped over the past few months we speculate that these speculative price bubbles no longer exist. For this reason we expect that the results of the estimation done on a shorter sample size will show a significant correlation with the price of Bitcoins and this series. However, we expect that the correlation will diminish as the estimation sample size is extended to include more data points past 2014.

3.1.5 Macroeconomic indicators

FSI: The financial stress index. This series is an index that is computed by the Cleveland Federal Reserve. It measure the financial stress felt by households. Our belief is that this series will have a negative, if any, correlation with Bitcoins. Most the studies have found that large macro-economic indicators have little influence on the price Bitocins.

S&P 500 index, Gold Price (XAU) and SSE (Shanghai Stock Exchange) index are all macro variables that we tested. We do not expect any significance for these series.

3.2 Data sources

Our entire data set is composed of daily data from August 19th 2010 to May 27th 2015. In all our entire data set is made of 1743 observations and consists of 16 testable independent variables. Our data sources are as follows. The majority of our data comes from blockchain.info, with the exception of the following. The financial stress index (FSI) comes from the Cleveland Federal Reserve website. All of our macroeconomic variables, such as gold prices, S&P 500, etc. come from the stlouisfed.org. Lastly the google trend series comes from google itself. In addition, we provide a graphical representation of all our dependant and independent variables in the appendix.

1) https://www.clevelandfed.org/
2) https://blockchain.info/charts
3) https://www.google.ca/trends/explore#q=Bitcoins

4) http://Bitcoincharts.com

5) https://www.stlouisfed.org

4. Methodology

4.1 Why GARCH is a superior method for studying Bitcoins

Robert F. Engle (1982) presented a new method of estimation that allows for time varying volatility, called autoregressive conditional heteroscedasticity (ARCH) model. In his introduction, he explains that econometricians have found that their ability to forecast financial data can vary from period to period. He also explains that errors or residuals tend to be clustered together when extremely large or small. This implies that if a regression that is based on financial data does not allow for some time varying volatility, it could have poor predictive capabilities. The core concept of his model is that the variance of a particular series will be more precise if it is conditioned on sets of information based on previous periods.

To allow for this heteroscedastic volatility Engle transforms the error term of a standard OLS regression into an endogenous process which allows its variance to vary across periods. To assist the reader, we provide a brief description of the model.

Consider some AR(1) process, with $|\gamma| < 1$

4.1) $Y_t = \gamma Y_{t-1} + z_t$

In a standard OLS regression we assume that

4.2) $E(z_t) = 0$

4.3) $Var\ (z_t) = \sigma^2$ where σ^2 is not a function of time t.

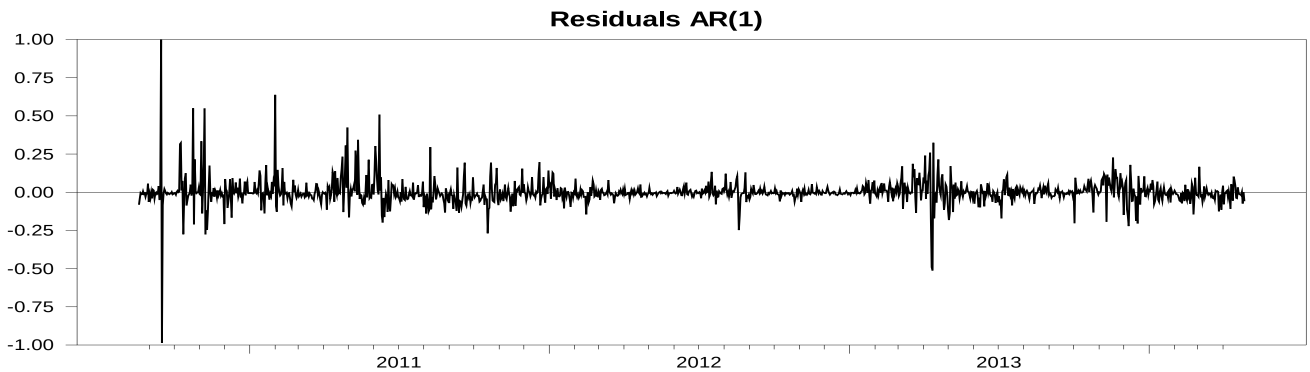
As one can see, this model does not allow for time variation, furthermore any heteroscedaticity will make hypothesis testing invalid because we would not satistify the Gauss Markov assumptions and, as such, we would no longer have the best linear unbiased estimator. If our data suffered from some sort of heteroscedastic time varying volatility then our expected value would not change, thus our estimated would not be biased i.e. $E(\varepsilon_t) = 0$ would still hold. However, any assumption made about our variance would not be satisfied i.e. $Var\ (\varepsilon_t) = \sigma^2$ would not hold. Instead we would have a non-spherical disturbance in our errors that would cause the following.

4.4) $E(\varepsilon\varepsilon'|X) = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{pmatrix} = \Omega\sigma^2$, with $\Omega \neq I, I =$ identity matrix

Using our data we ran a simple AR(1) regression on the price of Bitcoin and plotted the residuals, Figure 2.

Figure 2

Residuals From AR(1) Regression Of BitP Using OLS



As we can clearly see our errors do have an expected value of zero but their variance is most certainly not constant. This time varying volatility can make it difficult to construct a model that satisfies all the necessary Gauss Markov assumptions. To correct this series and others like it Engle does the following.

Consider the same AR(1) process as above (equation 4.1). We say that this AR(1) process is split into two parts, the first being the mean model (γY_{t-1}) and the second (z_t) is a stochastic component following the endogenous process:

4.5) $z_t = \varepsilon_t h_t^{\frac{1}{2}}$

4.6) $h_t = \alpha_0 + \alpha_1 z_{t-1}^2$

where ε_t satisfies all the Gauss Markov assumptions and $\varepsilon_t \backsim N(0,1)$. Combining 4.5) and 4.6) gives the following.

4.7) $z_t = \varepsilon_t\sqrt{\alpha_0 + \alpha_1 z_{t-1}^2}$

4.8) $E(z_t|z_{t-1}) = 0$

4.9) $Var(z_t|z_{t-1}) = \sigma_t^2$

Because ε_t is independent of z_{t-1} we can conclude that

4.10) $\sigma_t^2 = \alpha_0 + \alpha_1 z_{t-1}^2$

This is what is called an ARCH (1) process. The ARCH process can be of order p in which case is denoted, ARCH(p).

4.11) $\sigma_t^2 = \alpha_0 + \alpha_1 z_{t-1}^2 + \cdots + \alpha_p z_{t-p}^2$

Furthermore the model has been expanded to a Generalized ARCH process, GARCH (p, q). A GARCH (1, 1) process will look as follows.

4.12) $z_t = \varepsilon_t \sigma_t$

4.13) $\sigma_t^2 \quad = \alpha_0 + \alpha_1 z_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

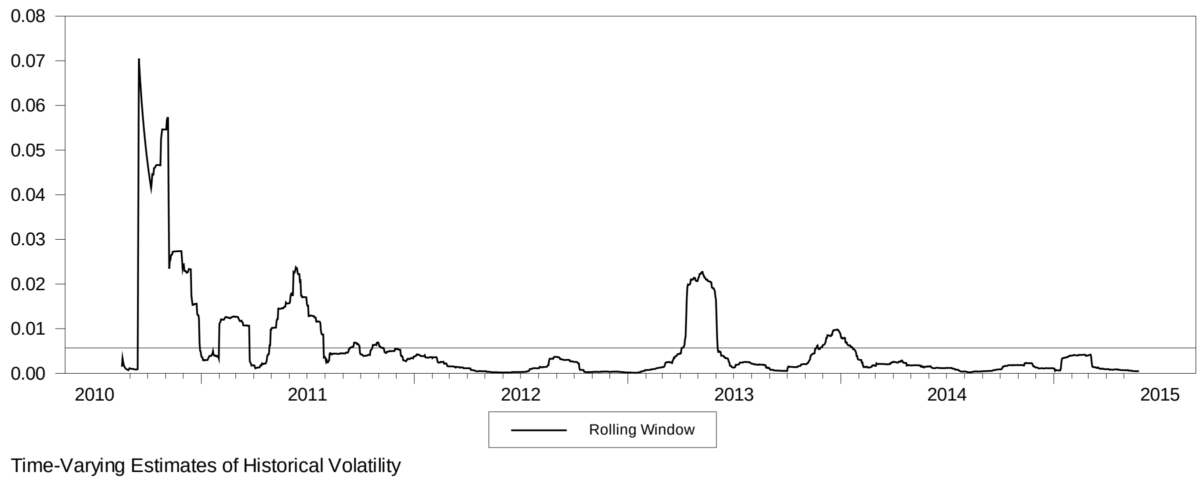
A GARCH (p,q) process will look as follows

4.14) $\sigma_t^2 \quad = \alpha_0 + \alpha_1 z_{t-1}^2 + \cdots + \alpha_p z_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2$

In both the ARCH (p) and GARCH (p, q) models, the mean model and the new error process are estimated simultaneously.

Figure 3

Time Varying Volatilty Of AR(1) Residuals



Why is this particular model process a good fit for Bitocoins? Clearly the price of Bitcoins has huge, cluttering, volatility over time, which can make it difficult to correctly specify an empirical model. Figure (3) plots the volatility of our AR (1) model, computed using a rolling window of the residuals obtained from the estimated model. We can clearly see that our volatility is not constant and is time varying. Using a simple OLS or any other methodology that does not allow for this time varying volatility will suffer from non-spherical disturbances, as we showed above. Robert F. Engle (1982) additionally points out that using robust errors are also not sufficient because they do not allow for the conditional mean and the variance to jointly evolve over time.

Furthermore recall the study done by Bouoiyour and Selmi (2014), in which they found that nearly 70% of the fluctuations in the price of Bitcoins were explained by its own “innovative shocks”. If we interpret ε_t as these innovative shocks, then we can model these stochastic shocks into an endogenous process that can be estimated and, as such, used to help understand what is transpiring in our data.

Prior to our decision of using a GARCH process, our train of thought was to use a vector auto regression (VAR) process. However we ultimately moved away from this model specification because the GARCH process provided us with a very unique asset. We have stated, multiple times, that Bitcoins are still young. Researchers are still unclear what is relevant, or irrelevant, when it comes to understanding this crypto currency. This knowledge gap could create some biases in our estimations due to miss specification. The official term is omitted variable bias. Models like OLS and VAR's cannot correct for this issue by itself, the GARCH process partially can. Robert F. Engle (1982) states (page 990)

"The ARCH specification might then be picking up the effect of variables omitted from the estimated model. The existence of an ARCH effect would be interpreted as evidence of misspecification, either by omitted variables or through structural change. If this is the case, ARCH may be a better approximation to reality than making standard assumptions about the disturbances, but trying to find the omitted variable or determine the nature of the structural change would be even better."

We have shown in our literature review that the academic community has not yet come a general consensus on the factors that are affecting the price of Bitcoins and, more importantly, have missed significant factors that could be affecting the price of Bitcoins. As such, using a GARCH process helps eliminate the concern of developing a biased model from potentially missing a significant factor affecting the variable under study.

4.2 Model Specification

To establish our correct GARCH (p, q) process, we followed the RATS Handbook for ARCH/GARCH and Volatility Models by Thomas Doan (2014). The next section will explain the specification process in detail.

Similar to the test created by Dickey-Fuller (1979), Phillips-Perron (1988) created a test that checks for a unit root in a series. Suppose we have an AR(1) process, similar to equation 4.1. If we subtract y_{t-1} from both sides we have the following.

4.15) $\Delta y_t = \rho y_{t-1} + u_t,\ \rho = \gamma - 1$ and $\rho = 0$ implies $\gamma = 1$.

The Dickey Fuller test for non-stationarity tests the null

$H_o: \rho = 0$

$H_a: \rho < 0$

The test can be extended to an AR (p) where, p is some positive value. The difference between the two test statistics is derived from Phillips-Perron (1988) choice of using the Newey–West (1987) heteroscedasticity- and autocorrelation-consistent covariance matrix estimator, which is robust to serial correlation. Given that we are working under the assumption that our data is suffering from heteroscedasticity and possibly serial correlation, the Phillips – Perron test is preferred to the standard Dickey-Fuller test.

Should $\rho = 0$ then we would have a unit root and our AR (p) processes would be non-stationary. This occurs if our t statistic is smaller than the critical value and we are not able to reject our null hypothesis. The importance of working with stationary data came about after an article was published by Hendry, David F. (2000) which discusses spurious regressions. To correct this issue we can transform our data into logarithmic scales or into logarithmic first differences. We tested all three possibilities of our dependent variable i.e. level, log level and first log differences. Our results indicate that our series are stationary after transforming them into log differences. We see that at the level and log level, the value of our test statistic is not

smaller than the critical value (-3.12929<-1.98099), (-3.12929<-1.41643) meaning we cannot reject our null hypothesis. When transformed into log differences we find that at all lags we can reject the null at the 1% level ($-44.3 < -3.96$). The results of the Philips- Perron Unit Root Test are reported in Table 1. This means our data would be best examined at the log differences. Unless otherwise specified, all the data presented in this paper have been transformed into log differences.

We mentioned earlier that our GARCH (p, q) model is made of two parts 1) the mean model and 2) the endogenous process for the error term. Before we try to specify our error term process i.e. find p and q, we should specify our mean model. In the mean model we include all our explanatory variables and any lagged dependent variables. We should note that this is not a

Multivariate GARCH model. Multivariate GARCH models are a more complex version of a GARCH model that incorporates the explanatory variables into the endogenous process of the error term through a system of equations. In this model we will simply include our explanatory variables in the mean model.

Table 1 **Unit Root Tests**		
Phillips-Perron Test for a Unit Root	Phillips-Perron Test for a Unit Root	Phillips-Perron Test for a Unit Root
Regression Run From 2010:08:19 to 2015:05:27	Regression Run From 2010:08:19 to 2015:05:27	Regression Run From 2010:08:19 to 2015:05:27
Bitcoin	Log Bitcoin	Log Differences Bitcoin
Sig Level Crit Value 1%(**) -3.96861 5%(*) -3.41492 10% -3.12929	Sig Level Crit Value 1%(**) -3.96861 5%(*) -3.41492 10% -3.12929	Sig Level Crit Value 1%(**) -3.96861 5%(*) -3.41492 10% -3.12929
Lags Statistic 0 -1.75702 1 -1.90891 2 -1.89877 3 -1.82735 4 -1.79153 5 -1.83807 6 -1.92305 7 -1.98099	Lags Statistic 0 -1.47954 1 -1.43082 2 -1.40232 3 -1.37897 4 -1.37403 5 -1.37826 6 -1.40216 7 -1.41643	Lags Statistic 0 -44.3009** 1 -44.3035** 2 -44.3306** 3 -44.3649** 4 -44.3686** 5 -44.3524** 6 -44.3056** 7 -44.2822**

4.2.1 Specifying the mean model

As a starting point we begin with an AR (1) model which looks as follows.

4.14) $BitP_t = \alpha_0 + \alpha_1 BitP_{t-1} + \varepsilon_t$

Our decision to use and AR(1) model to explain the price of bitcoins came about very simply. When using a standard multivariate OLS regression we found that none of the explanatory variables had any significance. However, we did find that an AR(1) regression did have some explanatory power.

When assessing the mean model, we are trying to determine if the mean model is stable in its moments. As Thomas Doan (2014) explains, we can use the West-Cho test, to determine if our model suffers from serial correlation. We feel that this test is appropriate because of the grouping and clustering of error terms that noted in Figure 2. The West-Cho test is robust to heteroscedasticity and, as such, an appropriate test. The test is a adaptation of the The Ljung–Box test (1978) which tests the following.

H_o: Independently distributed data

H_a: The null is false

We reject the null if our test statistic is greater that the critical value based on a chi-squared distribution. With a p-value of (.0521), Table 2, we say that at the 5% significance level we do not reject the null.

Table 2 West-Cho Tests	
Linear Regression - Estimation by Least Squares With Heteroscedasticity- Consistent (Eicker-White) Standard Errors Daily(7) Data From 2010:08:19 To 2015:05:27	
Usable Observations	1743
Degrees of Freedom	1743
Log Likelihood	2036.8131

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.004887	0.001725381	2.83243	0.00461955
2. BITP{1}	-0.056647	0.110373944	-0.51322	0.60779406

West-Cho Modified Q Test, Series %RESIDS	
Q(10)	18.18
Signif.	0.0521

We used an AR (1) as our starting point. However, we did not give any justification as to why we chose this format. To determine how many lags are needed in an autoregressive process we use three criteria.

1) Bayesian information criterion (BIC)

2) Hannan–Quinn information criterion (HQC)

3) Akaike information criterion (AIC)

All three are used for model selection but differ in the manner in which they calculate their criterion. Each method is based on a variation of some likelihood function that will vary depending on the number of lags/parameters, number of observations and residual sum of squares. In addition, all three follow the same decision rule. The lowest criterion achieved represents the optimal model specification. For example, the BIC results indicate we should be using 0 lags (-5.168). On the other hand, the HQC state that the model should be specified with one lag (-5.171) and the AIC shows that the model should include six lags (-5.178). All three criterion results can be seen in Table 3. Given that we have already tested our AR (1) model and we were not able to reject our null hypothesis of serial independence in the residuals found, we will just continue with this specification.

Table 3
Lag Specification

AIC BitP		HQC BitP		BIC BitP	
Lags	IC	Lags	IC	Lags	IC
0	-5.171	0	-5.171	0	-5.171
1	-5.173	1	-5.173	1	-5.173
2	-5.172	2	-5.172	2	-5.172
3	-5.172	3	-5.172	3	-5.172
4	-5.171	4	-5.171	4	-5.171
5	-5.171	5	-5.171	5	-5.171
6	-5.178	6	-5.178	6	-5.178
7	-5.177	7	-5.177	7	-5.177
8	-5.177	8	-5.177	8	-5.177
9	-5.176	9	-5.176	9	-5.176
10	-5.177	10	-5.177	10	-5.177
11	-5.176	11	-5.176	11	-5.176
12	-5.175	12	-5.175	12	-5.175

Continuing with the process, we perform a Hansen stability test, Table 4. This test has a fairly simple in its interpretation. The null hypothesis is that we have stability in the coefficient, variance and constant. The stability test will examine each of these indiviualy and jointly. The joint value is a calculated value that represents the overall stability of the model. The results can be seen in Table 4. We find that our coefficient (p-value = .28) and our constant (p-value = .04) are fairly stable, implying that our fist moments are correctly specified.

The issue, which we expected, is that our variance is not stable (p-value = 0.00), i.e. second moment. What we may infer from this, is that our errors might be centered around zero over time, however they may be clustering at different periods of time. Looking at figure (2) we can clearly see that the errors are grouping together at different points in time, but are still centered around zero. The introduction of the ARCH and GARCH effects, if properly specified, will capture the time-varying volatility that we have seen in the price of Bitcoins and, as such, allow for proper estimation.

Table 4
Hansen Stability Test

Test	Statistic	P-Value
Joint	2.60520783	0.00
Variance	2.27734109	0.00
Constant	0.50111261	0.04
BITP{1}	0.19088924	0.28

4.2.2 Specifying the ARCH/GARCH process

In this section we are aiming to specify our GRACH (p, q) process i.e. find a p and q that will best fit our data. To do this we can run an ARCH LM test. This begins by regressing a simple AR(1) model, as stated in equation 4.14, and extracting the residuals. Once extracted, the program squares the residuals and then proceeds to run a new regression using the squared residuals as the dependent variable. This new regression will also follow an AR process however, this model will have multiple lags. The number of lags is chosen by the researcher. In this context we chose to include 10 lags into the AR process. At this point in time, one could conduct a series of F test on the AR(10) regression of the squared residuals to assess the significance of the lags. The remaining significant lags would represent the number ARCH effects that should be included in the model. To clarify, if we found that we had three significant lags in our AR (10) estimation then we would use an ARCH (3) specification to model the price of Bitcoins. This test can be done manually. However, most software packages offer a built in one. Our test results indicated that all the first three lags are highly significant (chi-squared 386.041, 226.073, and 158.529) and that all ten lags are significant, Table 5. We determine the significance of each lag by subtracting the chi squared statistic of each lag by the proceeding chi squared statistic and then assess its significance on standard chi – squared distribution table. Common practice suggests that introducing a GARCH term is appropriate when this occurs, which we have done.

Table 5
Arch LM Test

Lags	Statistic	Signif. Level
1	386.041	0.00
2	226.073	0.00
3	158.529	0.00
4	119.326	0.00
5	95.42	0.00
6	79.423	0.00
7	67.998	0.00
8	59.461	0.00
9	52.793	0.00
10	47.553	0.00

Our base model will thus have three ARCH lags and one GARCH specification.

4.15) $Y_t = \gamma Y_{t-1} + z_t$

4.16) $z_t = \varepsilon_t \sigma_t$

4.17) $\sigma_t^2 = \alpha_0 + \alpha_1 z_{t-1}^2 + \alpha_2 z_{t-2}^2 + \alpha_3 z_{t-3}^2 + \beta_1 \sigma_{t-1}^2$

4.2.3 Diagnostics test

The last, but very important, part of this section is to perform diagnostics on our Base Model. Our goal is to ensure that the residuals (ε_t), from equation 4.16), are serial uncorrelated and homoscedastic. To test the new residuals (ε_t), one must divide the original residuals by the square root of the ARCH/GARCH process. Equation 4.18 illustrates how to isolate ε_t.

4.18) $$\varepsilon_t = \frac{z_t}{\sqrt{\alpha_0+\alpha_1 z_{t-1}^2+\alpha_2 z_{t-2}^2+\alpha_3 z_{t-3}^2+\beta_1 \sigma_{t-1}^2}}$$

Once we have isolated our new residuals we must test for serial correlation, normality and stationarity of the moments.

Using the McLeod and Li diagnostic test we can test our new residuals for serial correlation. The McLeod and Li test is virtually the same as the Ljung–Box test. However, in this instance we are testing the squared residuals (ε_t^2). Since we have introduced our ARCH/GARCH terms we no longer need to use a test which is robust to heteroskedasticity. For this reason the McLeod and Li test is more appropriate than the West-Cho test, which we used prior. The test statistic used is usually denoted by a Q and is tested against a chi-squared distribution. In addition, we provide the standard statistics of our estimation to ensure that our mean is centered around zero and that our distribution is normal. The results of the McLeodLi test, all the standard statistics and the estimation of the Base Model can be seen in Table 6 of the appendix. To assess normality we use the Jarque-Bera test statistic. This test tests the null hypothesis of normal distribution, skewness is zero and excess kurtosis is zero, against a non-normal distribution. Using a chi squared distribution with two degrease of freedom we overwhelmingly reject the null JB stat $904004.87 > CHI_{\alpha=10\%}4.61$. We also note that our mean is 0, (Signif Level (Mean = 0) p-value = .01). This implies that our estimates are unbiased. The test for serial correlation passes at the 1% mark, (Q stat = 17.523, P-value = 0.041). In this particular test a significant Q stat implies that our residuals are serially correlated. As such, we aim to reject at the highest level of significance. As Thomas Doan (2014) explains, achieving white noise in the error term in high frequency financial data is extremely difficult and as such anything that passes at 1% level of significance is sufficient. Given these results we can conclude that our model is correctly specified. In addition, we repeat these model specification tests for all models that are estimated in this paper. The results of these model specification tests are presented under the estimated results of each model in the appendix.

For the remainder of this paper we will refer to

4.19) $Y_t = \gamma Y_{t-1} + z_t$

4.20) $z_t = \varepsilon_t \sigma_t$

4.21) $\sigma_t^2 \quad = \alpha_0 + \alpha_1 z_{t-1}^2 + \alpha_2 z_{t-2}^2 + \alpha_3 z_{t-3}^2 + \beta_1 \sigma_{t-1}^2$

as the Base Model, (Table 6 reports the estimation results)

5. Estimation results

5.1 Assessing significance

In this section we will take a look at the results of our GARCH model. Using the Base Model that we established in the econometrics section (equation 4.19-4.21) as our foundation, we test multiple augmented models. Each augmented model includes an additional series in our mean model. We will use the log likelihood of the model (measure of fit) and the p-value to determine if the addition series has any significant explanatory power on the fluctuations of Bitcoin prices. We always keep the augmented models consistent, in that it will contain three ARCH terms and one GARCH term.

In total we estimated 16 different series, each one with a different set of independent variables. Each these estimations are represented in tables 6-22 in the Appendix. From the results of the regression analysis done on the augmented models we were able to create a model that best fits the historical data (model 1). We ran an additional model, model 2, that had smaller sample set. The sample set of model 2 ends in 2014 whereas the sample set model 1 is run until 2015. The discussion of these results follows suit.

5.2 Results discussion

Some of the most interesting aspects of our model do not involve what we have found to be significant, but rather what we have found to be insignificant. The Google trend series is one such series that had a surprising estimation result. In all major econometrics studies on Bitcoins that we cited in our literature review, this series represented a major contribution to the price movement. Recall that Bouoiyour and Selmi (2014) found that 20% of the fluctuations in the price of Bitcoin could be attributed to this series. Ciaian, Rajcaniova and Kancs (2014) state (page 13): "The strongest and most significant impact on Bitcoin price is estimated for variables capturing the impact of Bitcoins attractiveness for investors". As we explained prior Ciaian, Rajcaniova and Kancs (2014) use Google Trend to make this estimation. When tested in our model, which ran till 2015, Google Trend was not significant at a 10% level (table 17 p-value = .105). As such, the Google Trend series was not included in our Final Model that ran until 2015 (Final Model 1).

One logical explanation for the differences between this study and the others could stem from Kristoufek's (2013) results that Bitcoin prices are based on a speculative bubble. His results indicated that not only did the increase in the virtual hits of Bitcoins increase its price, but that the opposite was also true. Increases in prices of Bitcoins led to an increase in Bitcoin related searches. He explained that this caused an upward force on the price of Bitcoins that he speculated would ultimately collapse. Given that his paper was published in mid-2013, well before the Bitcoin's price collapse, the author does indeed deserve some praise. The likely reason why this relationship no longer holds in our study is because the speculative bubble has already "burst". Given the steady price decline that Bitcoins have experienced in the last two years, it is difficult to imagine that this relationship still holds. Admittedly, this theory contradicts the work done by Tuck and Fry (2015) (see literature section) but the author's data set ends in mid-2014, so is still much shorter than ours.

To test our hypothesis we re-estimated our model using data points only from 2010/08/19 – 2014/09/06 and a second time using data from 2010/08/19 – 2014/04/27 (we refer to this period as the bubble period). If Bitcoins did increase because of some speculative bubble, which appears to be the case, then once that bubble bursts we would imagine that the factors that contributed to this bubble would slowly dissipate over time. Our results do in fact substantiate this hypothesis. If we look at table 19 which represents the sample set that ends in April of 2014, we can see that, the Google Trend data series is very much significant even at the 1% level (p-value= 0.006). This regression only included our Google Trend series and our baseline model. As we stated, Engle explains this should not be an issue as the error process could capture the effect of any omitted variable, removing any concern of biasness in our coefficient. Regardless, we felt that it would be wise to test the series again with multiple variables, which is what we did in Final Model 2. The results were the same (p-value =0.002). We can safely say that our model supports the results from Kristoufek (2013) and Bouoiyour and Selmi (2014). We did not run any rolling estimates to find when the Google Trend series moves out of significance, however, we wanted to repeat the test again at a later date to see how much the results had changed. We thus ran the regression till September 2014, table 18. We found that, when tested alone, the series was only significant at the 10% level (p-value = 0.064), which is a large change from five months earlier.

These results seem to fit our hypothesis. Clearly the relationship between the public interest and the price of Bitcoins is fading. Recall that both Kristoufek (2013) and Tuck and Fry (2015) use the significance of this relationship as evidence of a speculative bubble. Tuck and Fry (2015) states (page 34): "Google Trends search index for the term "Bitcoin" shows a notable peak in late 2013 reinforcing an important social dimension to bubbles". Kristoufek (2013) stated (page 3), "Note that it is quite easy to invest into Bitcoin as the currency does not need to be traded in large bundles. This evidently forms a potential for a bubble development". The point of stressing these authors' point of view is that if both Kristoufek (2013) and Tuck and Fry (2015) are using the significance of public interest as argument to support the existence of price bubbles, then the opposite should also be true. The absence of a significant relationship between public interest and the price movement could indicate the absence of any price bubbles.

As expected, the daily volume of Bitcoins traded on the markets using USD$, (VolUSex) was significant in all periods, (see Table 7 & model 1&2). As we stated, our data was estimated using the logarithmic first differences. This implies that we can interpret our coefficients as a close approximation to a percentage change in the price of Bitcoins. In our first Final Model we had to use VolUSex_{t-1} because VolUSex_t was affecting the residuals of our model in a manner that shifted the distribution around a value that was not zero. We find that in a one unit increase in the daily volume of Bitcoins traded on these Exchanges (USD$) will cause a 0.27 % drop in the %Δ of the price of Bitcoins tomorrow. Final Model 2, which is based on a sample that ends in mid 2014 did not have any specification problems. As such, we tested this series using VolUSex_t. For this model we noticed that our coefficient flipped signs. The results of Final Model 2 states that a one unit increase in the daily volume of Bitcoins traded on these Exchanges (USD$) will cause a 1.00 % increase in the %Δ of the price of Bitcoins today. Given that nearly 80% of all trading done on all Bitcoin Exchanges were done in USD$, (Morten, Molnár, Vagstad, and Valstad, (2015)), we are not surprised that VolUSex was significant. Nearly all the articles that we examined in the production of this paper showed significance of this variable. Interestingly enough we found that the number of unique transactions (Trans100) was not always significant. In Final Model 2 we found that the variable Trans100 was not significant (Final Model 2, P-value = 0.87). This implies that during the bubble period, the bubble effect most likely dominated the price movement. When estimated in the model that was run on a longer time span (Final Model 1) we found that the variable was significant (Final Model 1, P-value = 0.09) however, it was only significant at the 10% level.

In regards to the Chinese Exchange (CNYEX), our results are somewhat surprising. We expected that the Exchange would be significant in the bubble period; however, we did not expect for it to remain so in a model estimated using a longer sample (Final Model 1, P-valeu = 0.003). Given that it is significant in the bubble period (Final Model 2, P-value = 0.02), we can say that our hypothesis regarding Baidu cannot be disproven at this time. In addition, we find that the sign of the coefficient is positive (.002), which is in line with our hypothesis. Clearly, we have a strong link between the BtcnCNY Exchange and the price of Bitcoins. Whether BtcnCNY is a good proxy for capturing the "Baidu effect" may require more in depth testing. However, until it is disproven we will continue to use Morten, Molnár, Vagstad, and Valstad's (2015) conclusion, that the 53 day acceptance window in which Baidu accepted Bitcoins for payment clearly had a significant effect.

The cumulated volume and all our macroeconomic variables (FSI, XAU, SSE, S&P 500) were found to be insignificant, (tables 14, 16, 21, 22 and 20). Nearly all macro-economic indicators have universally been rejected by all studies. Factors like gold prices, Dow Jones index and many others have had little impact on the trading price of Bitcoins. We suspect that the Bitcoin economy is too isolated and small to be seriously affected by larger Macroeconomic factors. The fact that the cumulated volume is insignificant is expected. Mitsuru, Kitamura, Matsumoto and Hayek (2014) and Ciaian, Rajcaniova and Kancs (2014) give both theoretical and empirical evidence, respectively, that this series has little impact on the price level of Bitcoins.

If we look at Final Model 1, find that our demand side factors results are consistent with Ciaian, Rajcaniova and Kancs (2014). Both the number of transactions, Trans100 (Model 1, P-value = 0.09) and the number of unique Bitcoins account, ID (Model 1, P-value = 0.02) have a positive relationship with the price of Bitcoins. These factors can be used as a measure for the demand of Bitcoins. Our results indicate that the long run factors that affect Bitcoins may be more associated with their use as a facilitator of daily transactions as opposed to their use as an investment or safe haven from poor economic conditions.

While we did find some significance in a few independent variables, the real strength in our model comes from the addition of the GARCH processes for the error term process. In both Final Model 1& 2 all three ARCH's and the GARCH specifications are significant. This indicates that our errors do suffer from some time varying volatility, which is supported by nearly all of the analysis we have presented in the previous sections of this paper.

In a more simplified manner, what seems to have the largest impact on the price of Bitcoins is the process that is being created by some unknown shock to the currency. If we look more closely we see that the first ARCH specification has a coefficient of 0.501, (Final Model 1). This implies that any shock experienced in this current (ε_t) will cause a 50% increase in the volatility of the price of Bitcoins today. This massive increase in the volatility is reduced in the second and third period. This is determined by the coefficients of the second and third period of Final Model 1, (ARCH {2} = -0.314, ARCH {3} = -0.127). If we net out the three ARCH terms we see that any shock will cause a 6% change in the volatility of the price of Bitcoins after three periods. Furthermore, we see that our GARCH term has a large coefficient (Final Model 1, Garch{1} = .94) This implies that the volatility is highly correlated with itself over time. Admittedly Bouoiyour and Selmi (2014) came to this conclusion prior to our study. In a variance decomposition for the price of Bitcoins, they found that nearly 70% of the fluctuations were caused by Bitcoins own shocks. Our model seems to follow suit. In summary, our results show that the shocks (ε_t) in our error term (z_t) are the main drivers in the price volatility of Bitcoins.

6. Discussions and conclusions

Given its short life span it may take some time before we can determine what the most significant factors affecting the price of Bitcoins and other crypto currencies are. This paper has taken some steps in the right direction by using a GARCH model which can account for the time-varying volatility of the data, and by employing a sample that spans a greater time frame than other works.

We were able to identify some of the inconsistencies, and more importantly, some of the miss-conceptions that many readers and writers had. With our own research we were able to show that the majority of the fluctuations in the price of Bitcoins can be accounted for by unknown shocks which are modeled by an endogenous process, and not by the impact of specific variables, as suggested by the majority of the works done to date. This endogenous process was estimated in this paper by a GARCH specification.

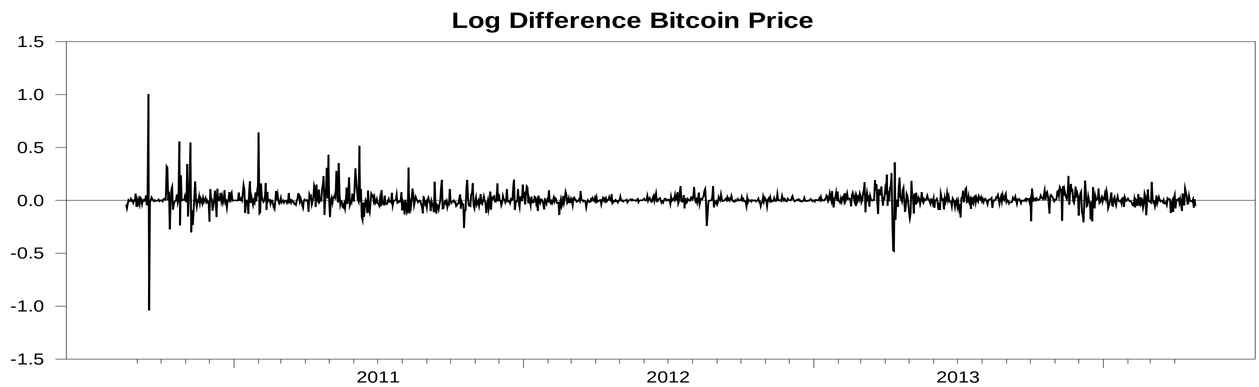
In regards to the predicting the future of Bitcoins we must be cautious. We constructed our model with the intention of fitting it to the historical data, not to forecast. In addition, our ARCH/GARCH specifications were shown to have a significant impact on the fluctuations of the price of Bitcoins which, as Engle (1982) explains, could indicate that we have omitted some explanatory variables. However, if we were to speculate based on the wealth of knowledge we have attained through our research we would say the following.

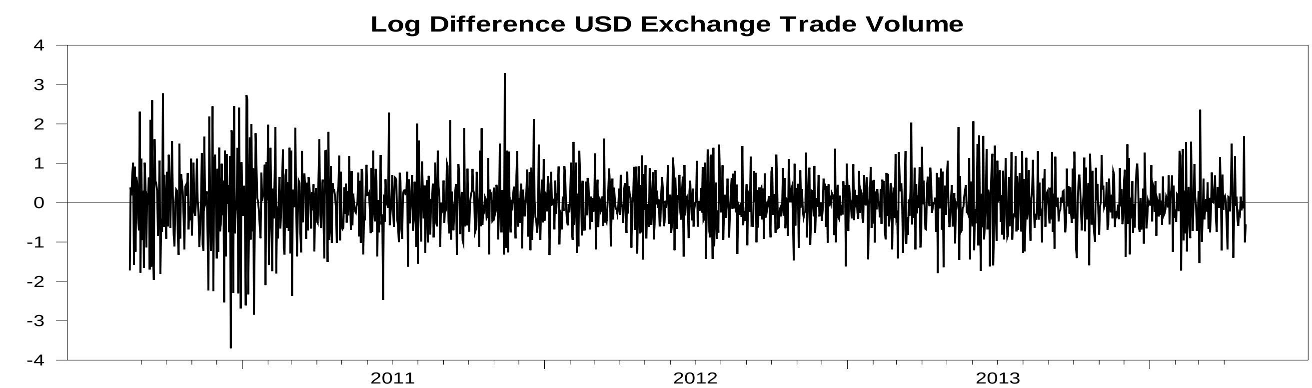
We feel that it is unlikely that we will see another “Boom” in the price of Bitcoin in the near future. Much of the hype around Bitcoins came about because it was so remarkable, we do not expect that to repeat. A specific event like Baidu accepting the crypto currency is more akin to a onetime event as opposed to something more recurring. In addition we showed that speculative bubbles were more significant in the past and were not significant when tested in a larger sample size. This leads us to believe that large spikes in the price of Bitcoins, such as the ones that took place at the end 2013, are less likely to occur again. On the other hand, we do not think the value will drop to zero. As time passes more and more establishments are beginning to accept Bitcoins as a method payment. Paypal has begun a partnership with Bitcoins and, as such, has begun accepting it as a method of payments. So long as large establishments develop systems that involve the use of Bitcoins it is hard to imagine that its worth will drop to zero. In fact, our research supports this hypothesis. We noted that demand side factors like the number of transactions and the number of unique Bitcoin accounts were positively related to the price of

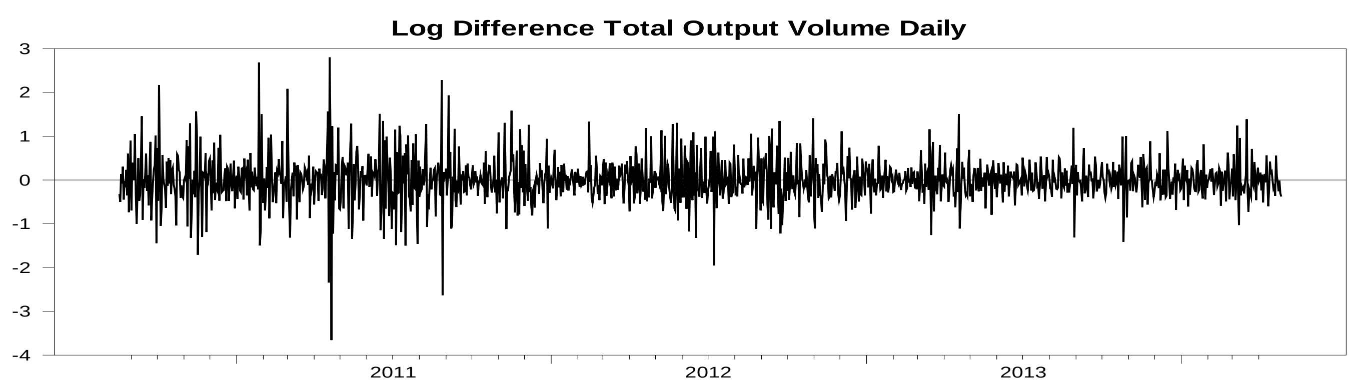
Bitcoins. It stands to reason that increasing the number of vendors that accept the crypto currency will increase the number of transactions involving Bitcoins and ultimately increase the price. Again, this is speculative since our model is based on historical data and is not a forecasting model. It is clear that Bitcoins have many variables, or factors, that are difficult to quantify. Factors like Satoshi dice, Silk Road (other illegal vendors), its disconnect from any central bank even the ability to send money across the globe for zero costs, all play a big role. These factors, and others that we may not have discovered yet, can potentially affect the value of this currency. Ultimately our research has indicated that we still have a great deal of investigation that needs to be done. However, we feel that we have added new and important information to the discussion which we feel will aid in the understanding of this phenomenon.

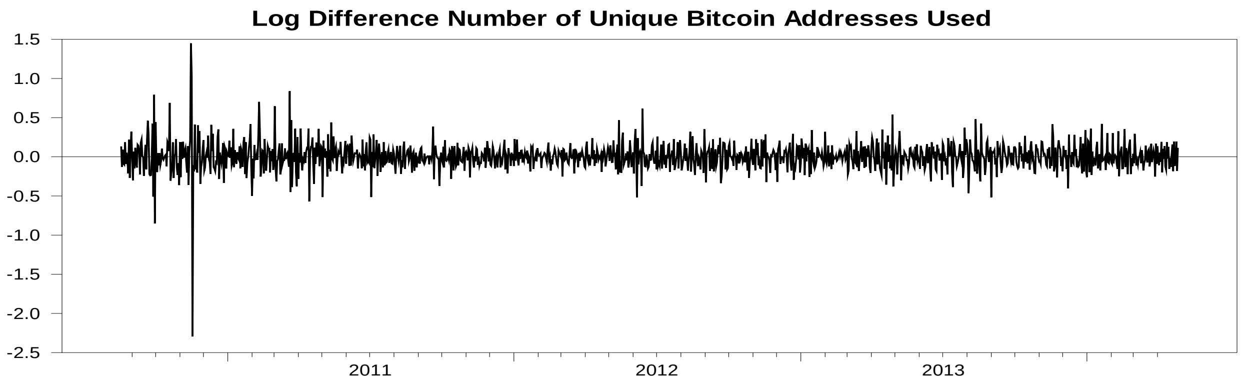
Appendix

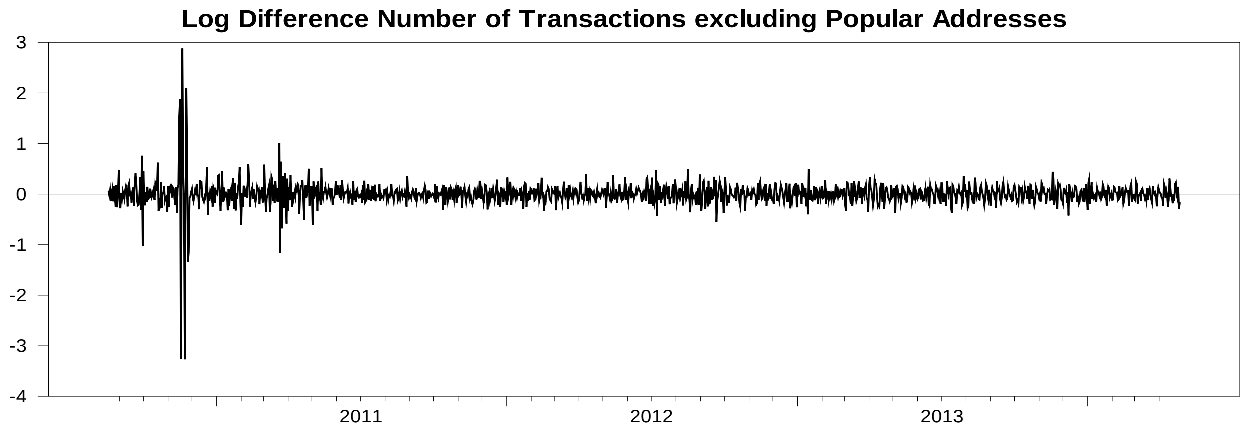
A. Graphical representation of all tested series.

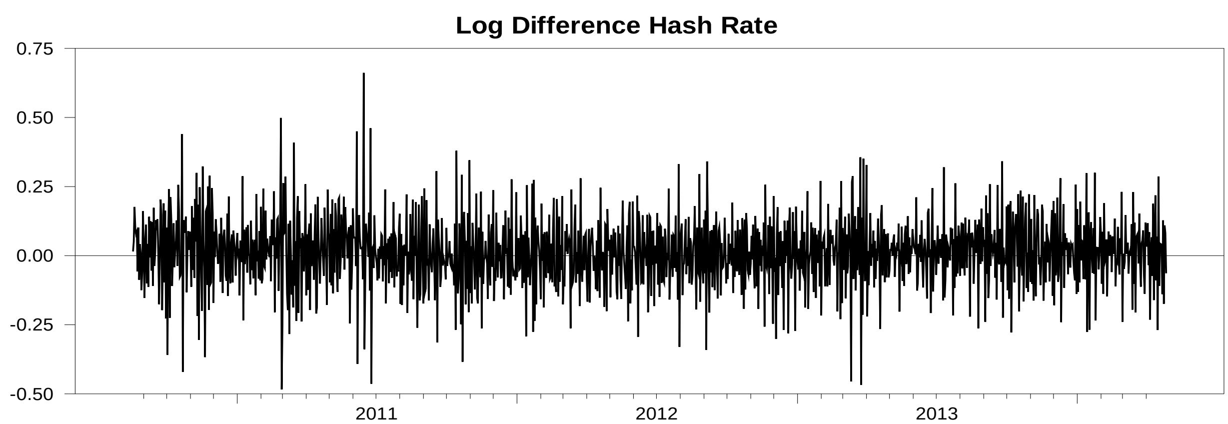


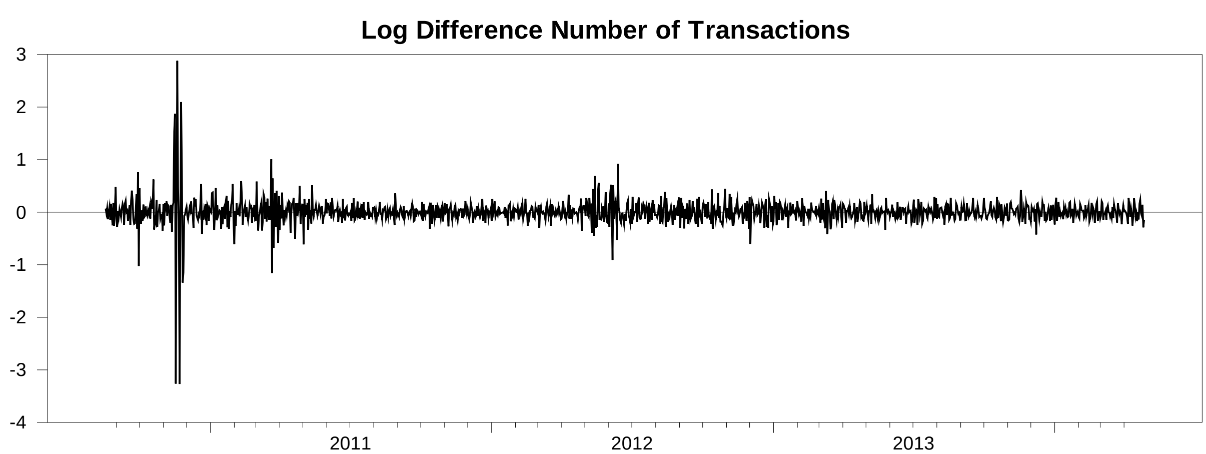


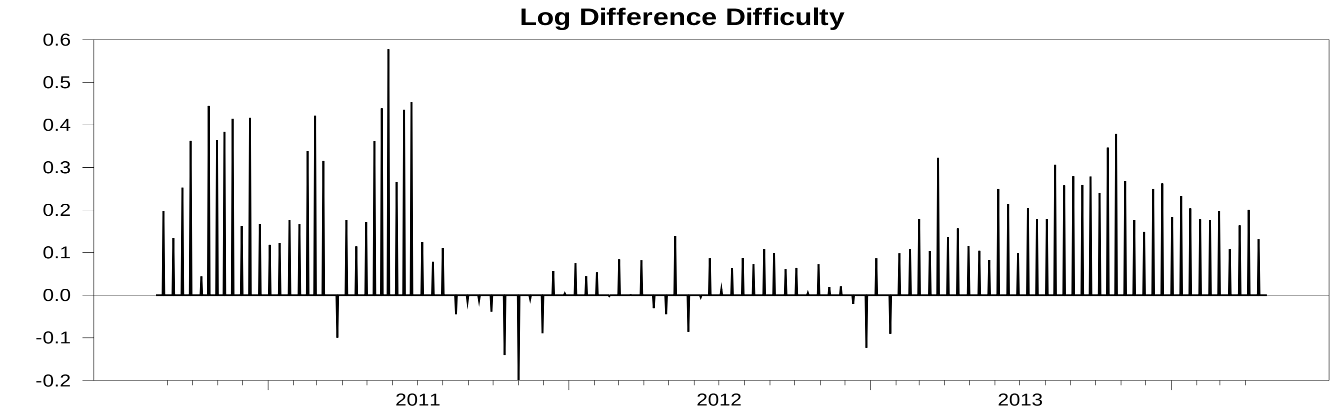


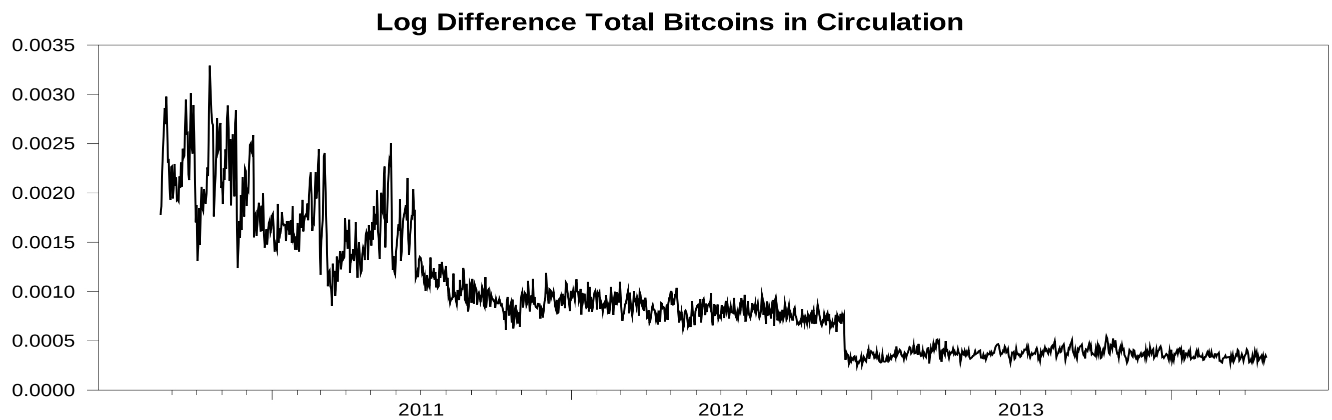

Log Difference Number of Unique Bitcoin Addresses Used
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
2011
2012
2013

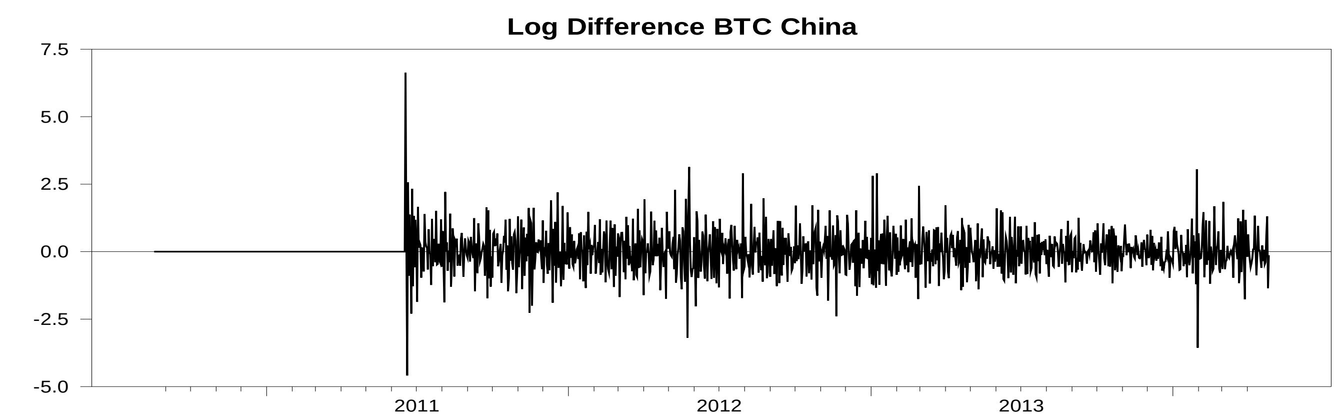

Log Difference Number of Transactions excluding Popular Addresses
3
2
1
0
-1
-2
-3
-4
2011
2012
2013

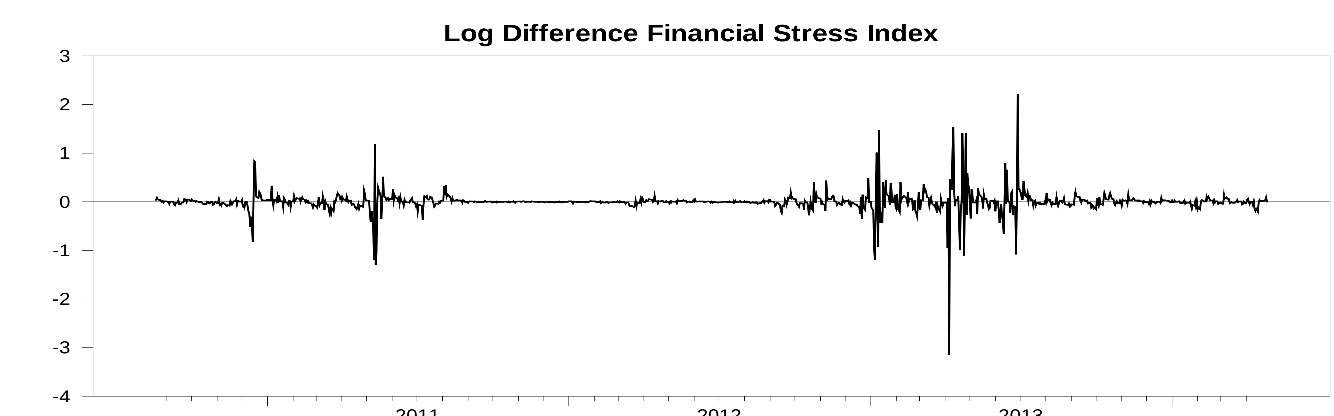

Log Difference Hash Rate
0.75
0.50
0.25
0.00
-0.25
-0.50
2011
2012
2013

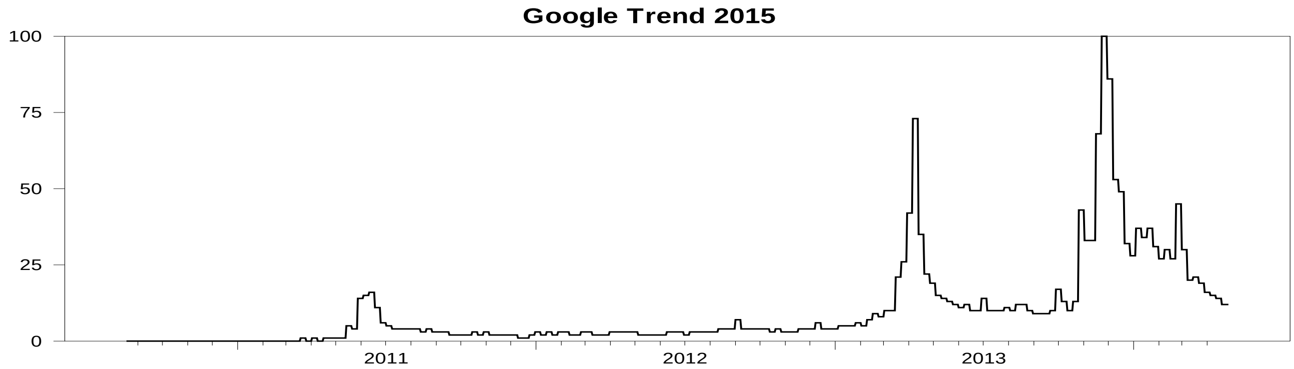

Log Difference Number of Transactions
3
2
1
0
-1
-2
-3
-4
2011
2012
2013

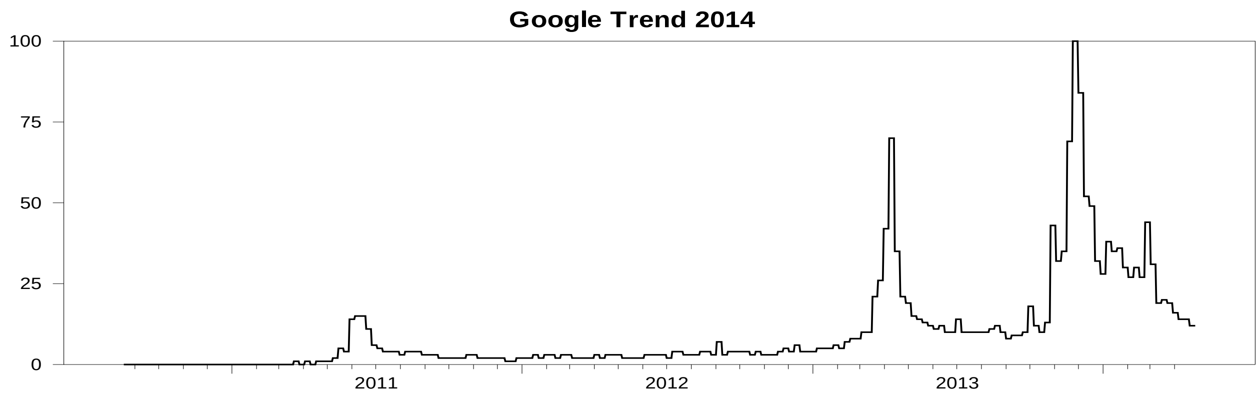

Log Difference Difficulty
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
2011
2012
2013

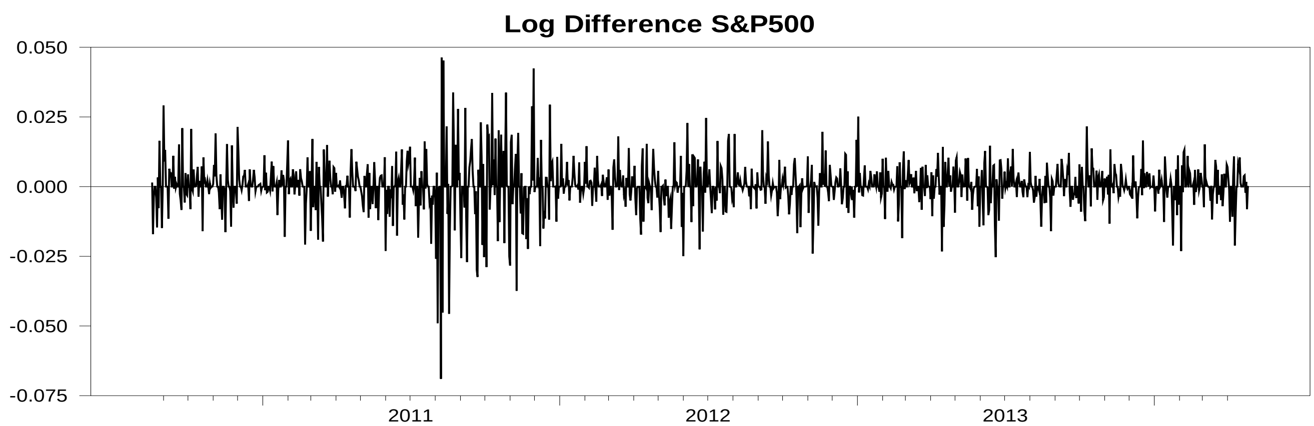

Log Difference Total Bitcoins in Circulation
0.0035
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
2011
2012
2013

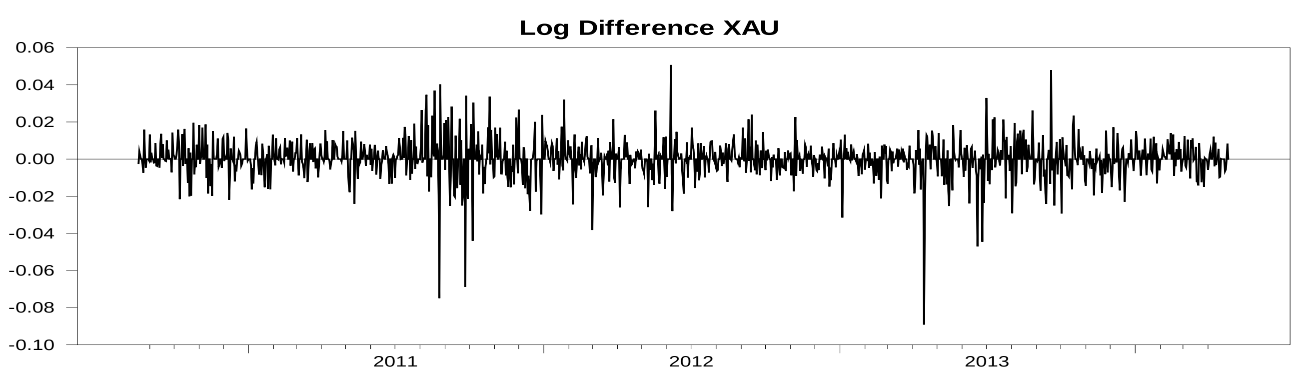

Log Difference BTC China
7.5
5.0
2.5
0.0
-2.5
-5.0
2011
2012
2013


Log Difference Financial Stress Index
3
2
1
0
-1
-2
-3
-4
2011
2012
2013


Google Trend 2015
100
75
50
25
0
2011
2012
2013


Google Trend 2014
100
75
50
25
0
2011
2012
2013


Log Difference S&P500
0.050
0.025
0.000
-0.025
-0.050
-0.075
2011
2012
2013


Log Difference XAU
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
2011
2012
2013

B. Regression results

Table 6 Base Model
GARCH Model - Estimation by BFGS
Convergence in 40 Iterations. Final criterion was 0.0000017 <= 0.0000100
Dependent Variable BITP
Daily(7) Data From 2010:08:19 To 2015:05:27
Log Likelihood 3059.0156

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.0002	0.0005	0.31284	0.75
2. BITP{1}	0.0297	0.0227	1.31013	0.19
3. C	0.0000	0.0000	2.08892	0.04
4. ARCH {1}	0.4304	0.0577	7.46061	0.00
5. ARCH {2}	-0.2432	0.0782	-3.10829	0.00
6. ARCH {3}	-0.0841	0.0586	-1.43395	0.15
7. GARCH{1}	0.8968	0.0392	22.88081	0.00
8. Shape	2.8160	0.1002	28.0994	0.00

Daily(7) Data From 2010:08:19 To 2015:05:27			
Observations	1743		
Sample Mean	0.072252	Variance	1.396588
Standard Error	1.181773	SE of Sample Mean	0.028306
t-Statistic (Mean=0)	2.552476	Signif Level (Mean=0)	0.010781
Skewness	5.473929	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	111.030247	Signif Level (Ku=0)	0.000000
Jarque-Bera	904004.8708	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.52289	significance level	0.04113
McLeod-Li for Residual ARCH=	0.1949	significance level	0.99986

Table 7

Augmented Model VOLUSEX

GARCH Model - Estimation by BFGS

Convergence in 44 Iterations. Final criterion was 0.0000005 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3093.3679

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.001465245	0.000660179	2.21946	0.02645512
2. BITP{1}	0.064372689	0.021470508	2.99819	0.00271587
3. VOLUSEX	0.008828083	0.001030663	8.56544	0.00000000
4. C	0.000019863	0.000009851	2.01635	0.04376322
5. ARCH {1}	0.494260778	0.059855005	8.25763	0.00000000
6. ARCH {2}	-0.315609578	0.080934425	-3.89957	0.00009636
7. ARCH {3}	-0.119341471	0.054682571	-2.18244	0.02907702
8. GARCH{1}	0.940690270	0.024703153	38.07976	0.00000000
9. Shape	2.870353169	0.107661426	26.66092	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.050797	Variance	1.575097
Standard Error	1.255029	SE of Sample Mean	0.030061
t-Statistic (Mean=0)	1.689789	Signif Level (Mean=0)	0.091247
Skewness	8.313969	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	195.555393	Signif Level (Ku=0)	0.000000
Jarque-Bera	2797398.791187	Signif Level (JB=0)	0.000000
Q for Residual Serial Correlation	14.44025	significance level	0.07099
McLeod-Li for Residual ARCH=	0.10236	significance level	0.99998

Table 8
Augmented Model VOLBITNET
GARCH Model - Estimation by BFGS Convergence in 39 Iterations. Final criterion was 0.0000029 <= 0.0000100 Dependent Variable BITP Daily(7) Data From 2010:08:19 To 2015:05:27 Log Likelihood 3059.6378

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000196071	0.000589716	0.33248	0.73952425
2. BITP{1}	0.030428303	0.022477399	1.35373	0.17582291
3. VOLBITNET	0.001467031	0.001278531	1.14743	0.25120204
4. C	0.000041196	0.000020784	1.14743	0.25120204
5. ARCH {1}	0.434212763	0.056080432	7.74268	0.00000000
6. ARCH {2}	-0.247985700	0.076809187	-3.22859	0.00124400
7. ARCH {3}	-0.081078409	0.059305209	-1.36714	0.17158203
8. GARCH{1}	0.894851346	0.039755124	22.50908	0.00000000
9. Shape	2.818095526	0.102405077	27.51910	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27			
Observations	1743		
Sample Mean	0.071158	Variance	1.385417
Standard Error	1.177037	SE of Sample Mean	0.028193
t-Statistic (Mean=0)	2.523941	Signif Level (Mean=0)	0.011693
Skewness	5.271082	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	105.492757	Signif Level (Ku=0)	0.000000
Jarque-Bera	816294.761031	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.95903	significance level	0.02154
McLeod-Li for Residual ARCH=	0.20768	significance level	0.99983

Table 9

Augmented Model ID

GARCH Model - Estimation by BFGS

Convergence in 39 Iterations. Final criterion was 0.0000029 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3066.9036

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000160811	0.000601406	0.26739	0.78916820
2. BITP{1}	0.035271609	0.024018129	1.46854	0.14195730
3. ID	0.014900270	0.003692358	4.03543	0.00005450
4. C	0.000041262	0.000021909	1.88333	0.05965537
5. ARCH {1}	0.442018525	0.056896473	7.76882	0.00000000
6. ARCH {2}	-0.256463522	0.077410470	-3.31303	0.00092290
7. ARCH {3}	-0.083324904	0.059467744	-1.40118	0.16116080
8. GARCH{1}	0.897769901	0.041482392	21.64219	0.00000000
9. Shape	2.806852417	0.096814541	28.99205	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.071236	Variance	1.380876
Standard Error	1.175107	SE of Sample Mean	0.028147
t-Statistic (Mean=0)	2.530871	Signif Level (Mean=0)	0.011465
Skewness	5.346664	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	107.150863	Signif Level (Ku=0)	0.000000
Jarque-Bera	842134.4240	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.39571	significance level	0.02624
McLeod-Li for Residual ARCH=	0.20401	significance level	0.99984

Table 10

Augmented Model TRANS100

GARCH Model - Estimation by BFGS

Convergence in 39 Iterations. Final criterion was 0.0000029 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3065.4751

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000267485	0.000637660	0.41948	0.67486614
2. BITP{1}	0.032366020	0.022701914	1.42570	0.15395621
3. TRANS100	0.013143092	0.003712021	3.54068	0.00039909
4. C	0.000038375	0.000016790	2.28553	0.02228176
5. ARCH {1}	0.433290086	0.057596429	7.52286	0.00000000
6. ARCH {2}	-0.254189612	0.077624192	-3.27462	0.00105805
7. ARCH {3}	-0.078704711	0.053192444	-1.47962	0.13897415
8. GARCH{1}	0.899604237	0.034608554	25.99370	0.00000000
9. Shape	2.829507687	0.109565506	25.82480	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations 1743

Sample Mean	0.069574	Variance	1.388232
Standard Error	1.178233	SE of Sample Mean	0.028222
t-Statistic (Mean=0)	2.465255	Signif Level (Mean=0)	0.013787
Skewness	5.393895	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	108.815834	Signif Level (Ku=0)	0.000000
Jarque-Bera	868396.1685	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.60047	significance level	0.02443
McLeod-Li for Residual ARCH=	0.20021	significance level	0.99984

Table 11

Augmented Model HASH

GARCH Model - Estimation by BFGS

Convergence in 39 Iterations. Final criterion was 0.0000029 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3059.3010

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000200090	0.000597300	0.33499	0.73763246
2. BITP{1}	0.029316216	0.023550104	1.24484	0.21318895
3. HASH	-0.003651443	0.004861441	-0.75110	0.45259071
4. C	0.000040579	0.000020660	1.96418	0.04950971
5. ARCH {1}	0.429344181	0.057504543	7.46627	0.00000000
6. ARCH {2}	-0.243517722	0.078003801	-3.12187	0.00179706
7. ARCH {3}	-0.080526319	0.057506543	-1.40030	0.16142398
8. GARCH{1}	0.894699860	0.041318281	21.65385	0.00000000
9. Shape	2.818104833	0.103938201	27.11327	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.072056	Variance	1.396176
Standard Error	1.181599	SE of Sample Mean	0.028302
t-Statistic (Mean=0)	2.545938	Signif Level (Mean=0)	0.010984
Skewness	5.421732	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	109.603368	Signif Level (Ku=0)	0.000000
Jarque-Bera	880976.033906	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.70052	significance level	0.02359
McLeod-Li for Residual ARCH=	0.19798	significance level	0.99985

Table 12

Augmented Model TRANS

GARCH Model - Estimation by BFGS

Convergence in 40 Iterations. Final criterion was 0.0000024 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3061.2661

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000183822	0.000586883	0.31322	0.75411505
2. BITP{1}	0.032428746	0.023681415	1.36938	0.17088197
3. TRANS	0.006660697	0.002859817	2.32906	0.01985570
4. C	0.000039588	0.000019888	1.99058	0.04652747
5. ARCH {1}	0.435829846	0.052606915	8.28465	0.00000000
6. ARCH {2}	-0.252645693	0.075946971	-3.32661	0.00087910
7. ARCH {3}	-0.081990444	0.053509408	-1.53226	0.12545774
8. GARCH{1}	0.898806291	0.037443073	24.00461	0.00000000
9. Shape	2.819261912	0.103033171	27.36266	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations 1743

Sample Mean	0.071552	Variance	1.388449
Standard Error	1.178325	SE of Sample Mean	0.028224
t-Statistic (Mean=0)	2.535170	Signif Level (Mean=0)	0.011326
Skewness	5.406338	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	109.081990	Signif Level (Ku=0)	0.000000
Jarque-Bera	872647.07687	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.43454	significance level	0.02589
McLeod-Li for Residual ARCH=	0.19901	significance level	0.99985

Table 13

Augmented Model DIF

GARCH Model - Estimation by BFGS

Convergence in 41 Iterations. Final criterion was 0.0000066 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3059.8380

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000057144	0.000626007	0.09128	0.92726736
2. BITP{1}	0.029805230	0.022017570	1.35370	0.17583144
3. DIF	0.020772224	0.015182757	1.36815	0.17126649
4. C	0.000038343	0.000018932	2.02533	0.04283331
5. ARCH {1}	0.424613261	0.051595072	8.22973	0.00000000
6. ARCH {2}	-0.233705447	0.059260384	-3.94370	0.00008023
7. ARCH {3}	-0.090417120	0.051077411	-1.77020	0.07669420
8. GARCH{1}	0.899509307	0.037343904	24.08718	0.00000000
9. Shape	2.819261912	0.103033171	27.77169	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.071222	Variance	1.392664
Standard Error	1.180112	SE of Sample Mean	0.028267
t-Statistic (Mean=0)	2.519652	Signif Level (Mean=0)	0.011836
Skewness	5.427646	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	109.873271	Signif Level (Ku=0)	0.000000
Jarque-Bera	885296.789578	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.62624	significance level	0.02421
McLeod-Li for Residual ARCH=	0.19822	significance level	0.99985

Table 14

Augmented Model CUMUBITVOL

GARCH Model - Estimation by BFGS

Convergence in 38 Iterations. Final criterion was 0.0000056 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3059.6853

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.001147059	0.001035275	1.10798	0.26787246
2. BITP{1}	0.029279198	0.022725966	1.28836	0.19762109
3. CUMUBITVOL	-1.645271247	1.402729317	-1.17291	0.24083304
4. C	0.000044680	0.000024286	1.83972	0.06580991
5. ARCH {1}	0.429389190	0.056324613	7.62347	0.00000000
6. ARCH {2}	-0.238765228	0.077457940	-3.08251	0.00205260
7. ARCH {3}	-0.080086756	0.057584335	-1.39077	0.16429418
8. GARCH{1}	0.889462794	0.044454338	20.00846	0.00000000
9. Shape	2.804118793	0.100127893	28.00537	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations 1743

Sample Mean	0.073400	Variance	1.383095
Standard Error	1.176050	SE of Sample Mean	0.028169
t-Statistic (Mean=0)	2.605660	Signif Level (Mean=0)	0.009248
Skewness	5.320740	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	106.848422	Signif Level (Ku=0)	0.000000
Jarque-Bera	837353.6418	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	19.45149	significance level	0.01262
McLeod-Li for Residual ARCH=	0.20823	significance level	0.99983

Table 15

Augmented Model CNYEX

GARCH Model - Estimation by BFGS

Convergence in 38 Iterations. Final criterion was 0.0000056 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3069.1543

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000569893	0.000628960	0.90609	0.36488926
2. BITP{1}	0.044088364	0.023644244	1.86466	0.06222974
3. CNYEX	0.004232381	0.000959514	4.41096	0.00001029
4. C	0.000030554	0.000019549	1.56297	0.11805932
5. ARCH {1}	0.470576685	0.064668328	7.27677	0.00000000
6. ARCH {2}	-0.292078503	0.082895099	-3.52347	0.00042593
7. ARCH {3}	-0.100155582	0.066707391	-1.50142	0.13324785
8. GARCH{1}	0.921657400	0.044678098	20.62884	0.00000000
9. Shape	2.819999843	0.111809001	25.22158	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.064412	Variance	1.360419
Standard Error	1.166370	SE of Sample Mean	0.027938
t-Statistic (Mean=0)	2.305587	Signif Level (Mean=0)	0.021251
Skewness	5.089652	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	101.080016	Signif Level (Ku=0)	0.000000
Jarque-Bera	749547.217632	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	18.14780	significance level	0.02015
McLeod-Li for Residual ARCH=	0.21060	significance level	0.99982

Table 16

Augmented Model FSI

GARCH Model - Estimation by BFGS

Convergence in 40 Iterations. Final criterion was 0.0000011 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3059.3043

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000136183	0.000587816	0.23168	0.81678925
2. BITP{1}	0.029470337	0.022742076	1.29585	0.19502691
3. FSI	-0.001664117	0.002444098	-0.68087	0.49595268
4. C	0.000038824	0.000021452	1.80981	0.07032513
5. ARCH {1}	0.431998553	0.053310390	8.10346	0.00000000
6. ARCH {2}	-0.243822794	0.072838056	-3.34746	0.00081554
7. ARCH {3}	-0.087532502	0.057631904	-1.51882	0.12880775
8. GARCH{1}	0.899356743	0.040604886	22.14898	0.00000000
9. Shape	2.814166582	0.103604941	27.16247	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.073088	Variance	1.394432
Standard Error	1.180861	SE of Sample Mean	0.028285
t-Statistic (Mean=0)	2.584018	Signif Level (Mean=0)	0.009846
Skewness	5.439556	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	110.154907	Signif Level (Ku=0)	0.000000
Jarque-Bera	889834.806982	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.61401	significance level	0.02431
McLeod-Li for Residual ARCH=	0.19592	significance level	0.99985

Table 17

Augmented Model TREND2015

GARCH Model - Estimation by BFGS

Convergence in 41 Iterations. Final criterion was 0.0000054 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3060.2282

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	-0.000846197	0.000901294	-0.93887	0.34779834
2. BITP{1}	0.029877198	0.023530049	1.26975	0.20417493
3. TREND2015	0.000142033	0.000087710	1.61934	0.10537306
4. C	0.000043906	0.000022243	1.97393	0.07032513
5. ARCH {1}	0.427567956	0.055359730	7.72345	0.00000000
6. ARCH {2}	-0.239272972	0.075596368	-3.16514	0.00155009
7. ARCH {3}	-0.080882847	0.058513031	-1.38230	0.16687811
8. GARCH{1}	0.892587862	0.040416039	22.08499	0.00000000
9. Shape	2.791821932	0.098732597	28.27660	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.066737	Variance	1.384065
Standard Error	1.176463	SE of Sample Mean	0.028179
t-Statistic (Mean=0)	2.368290	Signif Level (Mean=0)	0.017979
Skewness	5.440733	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	110.381045	Signif Level (Ku=0)	0.000000
Jarque-Bera	893460.451602	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	18.94139	significance level	0.01518
McLeod-Li for Residual ARCH=	0.19804	significance level	0.99985

Table 18

Augmented Model TREND2014 2014:09:06

GARCH Model - Estimation by BFGS

Convergence in 43 Iterations. Final criterion was 0.0000037 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2014:09:06

Log Likelihood 2527.3744

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	-0.000514896	0.000849513	-0.60611	0.54444347
2. BITP{1}	0.047318370	0.025331390	1.86797	0.06176574
3. TREND2014	0.000160218	0.000086589	1.85032	0.06426786
4. C	0.000032386	0.000019023	1.70250	0.08866194
5. ARCH {1}	0.399990503	0.058141667	6.87958	0.00000000
6. ARCH {2}	-0.187375677	0.079422063	-2.35924	0.01831243
7. ARCH {3}	-0.113336127	0.066279223	-1.70998	0.08726963
8. GARCH{1}	0.900721301	0.041815767	21.54023	0.00000000
9. Shape	2.820949263	0.107661096	26.20212	0.00000000

Daily(7) Data From 2010:08:19 To 2014:09:06

Observations	1480		
Sample Mean	0.086389	Variance	1.507052
Standard Error	1.227620	SE of Sample Mean	0.031910
t-Statistic (Mean=0)	2.707232	Signif Level (Mean=0)	0.006863
Skewness	5.433252	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	104.513841	Signif Level (Ku=0)	0.000000
Jarque-Bera	680875.470565	Signif Level (JB=0)	0.000000
Q for Residual Serial Correlation	15.26709	significance level	0.05416
McLeod-Li for Residual ARCH=	0.21576	significance level	0.99981

Table 19

Augmented Model TREND2014 2014:04:27

GARCH Model - Estimation by BFGS

Convergence in 29 Iterations. Final criterion was 0.0000037 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2014:04:27

Log Likelihood 2527.3744

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	-0.000554183	0.000889438	-0.62307	0.53323832
2. BITP{1}	0.041932511	0.025099754	1.67183	0.09455794
3. TREND2014	0.000251148	0.000092163	2.72505	0.00642915
4. C	0.000264538	0.000128330	2.06139	0.03926628
5. ARCH {1}	0.418207894	0.061768489	6.77057	0.00000000
6. ARCH {2}	-0.074674216	0.088492961	-0.84384	0.39875682
7. ARCH {3}	0.034115773	0.069519154	0.49074	0.62361093
8. GARCH{1}	0.622350549	0.111499751	5.58163	0.00000002
9. Shape	2.750233030	0.105032509	26.18459	0.00000000

Daily(7) Data From 2010:08:19 To 2014:09:06

Observations	1348		
Sample Mean	0.091909	Variance	1.688322
Standard Error	1.299354	SE of Sample Mean	0.035390
t-Statistic (Mean=0)	2.597020	Signif Level (Mean=0)	0.009506
Skewness	5.222263	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	77.347198	Signif Level (Ku=0)	0.000000
Jarque-Bera	342149.204743	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	15.96514	significance level	0.04288
McLeod-Li for Residual ARCH=	0.53662	significance level	0.99981

Table 20
Augmented Model SP500
GARCH Model - Estimation by BFGS
Convergence in 43 Iterations. Final criterion was 0.0000037 <= 0.0000100
Dependent Variable BITP
Daily(7) Data From 2010:08:19 To 2015:05:27
Log Likelihood 3059.3456

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.002706935	0.002970496	0.91127	0.36215122
2. BITP{1}	0.028918535	0.023078880	1.25303	0.21019468
3. SP500	-0.000001567	0.000001826	-0.85825	0.39075417
4. C	0.000037895	0.000019499	1.94345	0.08866194
5. ARCH {1}	0.429959394	0.057464459	7.48218	0.00000000
6. ARCH {2}	-0.187375677	0.076865079	-3.17498	0.00149846
7. ARCH {3}	-0.085099654	0.058886352	-1.44515	0.14841555
8. GARCH{1}	0.899185465	0.039699946	22.64954	0.00000000
9. Shape	2.825372708	0.106108739	26.62714	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27			
Observations	1743		
Sample Mean	0.072503	Variance	1.403056
Standard Error	1.184507	SE of Sample Mean	0.028372
t-Statistic (Mean=0)	2.555452	Signif Level (Mean=0)	0.010689
Skewness	5.500839	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	111.839676	Signif Level (Ku=0)	0.000000
Jarque-Bera	917192.017610	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	16.58364	significance level	0.03475
McLeod-Li for Residual ARCH=	0.19248	significance level	0.99986

Table 21

Augmented Model XAU

GARCH Model - Estimation by BFGS

Convergence in 43 Iterations. Final criterion was 0.0000037 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3059.4449

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	-0.003692413	0.004049541	-0.91181	0.36186855
2. BITP{1}	0.028529560	0.021434777	1.33099	0.18319099
3. XAU	0.000002626	0.000002730	0.96166	0.33622035
4. C	0.000040020	0.000019028	2.10324	0.03544493
5. ARCH {1}	0.430383706	0.052593741	8.18317	0.00000000
6. ARCH {2}	-0.243270953	0.066693528	-3.64759	0.00026471
7. ARCH {3}	-0.082687888	0.054832209	-1.50802	0.13155024
8. GARCH{1}	0.895575135	0.038808546	23.07675	0.00000000
9. Shape	2.825132070	0.102617903	27.53060	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27			
Observations	1743		
Sample Mean	0.073776	Variance	1.399454
Standard Error	1.182985	SE of Sample Mean	0.028335
t-Statistic (Mean=0)	2.603664	Signif Level (Mean=0)	0.009302
Skewness	5.471600	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	110.982476	Signif Level (Ku=0)	0.000000
Jarque-Bera	903227.214715	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.03414	significance level	0.02976
McLeod-Li for Residual ARCH=	0.19594	significance level	0.99985

Table 22

Augmented Model SSE

GARCH Model - Estimation by BFGS

Convergence in 43 Iterations. Final criterion was 0.0000037 <= 0.0000100

Dependent Variable BITP

Daily(7) Data From 2010:08:19 To 2015:05:27

Log Likelihood 3059.7832

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.003143663	0.002445337	1.28557	0.19859172
2. BITP{1}	0.028303043	0.021776048	1.29973	0.19369258
3. SSE	-0.000001210	0.000000969	-1.24883	0.21172583
4. C	0.000042636	0.000021710	1.96395	0.04953631
5. ARCH {1}	0.427238519	0.057281767	7.45854	0.00000000
6. ARCH {2}	-0.238961598	0.077577593	-3.08029	0.00206798
7. ARCH {3}	-0.079487644	0.056833430	-1.39861	0.16193081
8. GARCH{1}	0.891210723	0.040564200	21.97038	0.00000000
9. Shape	2.815786287	0.095935200	29.35092	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27

Observations	1743		
Sample Mean	0.072366	Variance	1.400944
Standard Error	1.183615	SE of Sample Mean	0.028351
t-Statistic (Mean=0)	2.552554	Signif Level (Mean=0)	0.010778
Skewness	5.510049	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	112.190469	Signif Level (Ku=0)	0.000000
Jarque-Bera	922928.942977	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.40514	significance level	0.02616
McLeod-Li for Residual ARCH=	0.19517	significance level	0.99986

C. Final Models

Final Model 1
Convergence in 39 Iterations. Final criterion was 0.0000029 <= 0.0000100 Dependent Variable BITP Daily(7) Data From 2010:08:19 To 2015:05:27 Log Likelihood 3080.4165

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000344941	0.000612213	0.56343	0.57314041
2. BITP{1}	0.064454766	0.024966608	2.58164	0.00983324
3. VOLUSEX{1}	-0.002727207	0.001024773	-2.66128	0.00778445
4. TRANS100	0.006954639	0.004091693	1.69970	0.08918788
5. ID	0.010103667	0.004498241	2.24614	0.02469522
6. CNYEX	0.003624108	0.001019785	3.55380	0.00037971
7. C	0.000023865	0.000016706	1.42847	0.15315799
8. ARCH {1}	0.501278552	0.068043916	7.36699	0.00000000
9. ARCH {2}	-0.314960421	0.083845066	-3.75646	0.00017234
10. ARCH {3}	-0.127512377	0.069029483	-1.84722	0.06471582
11. GARCH{1}	0.941194246	0.038863014	24.21825	0.00000000
12. Shape	2.812362257	0.101615629	27.67647	0.00000000

Daily(7) Data From 2010:08:19 To 2015:05:27			
Observations	1743		
Sample Mean	0.070079	Variance	1.354102
Standard Error	1.163659	SE of Sample Mean	0.027873
t-Statistic (Mean=0)	2.514264	Signif Level (Mean=0)	0.012018
Skewness	5.385247	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	106.063767	Signif Level (Ku=0)	0.000000
Jarque-Bera	825421.338246	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	17.20144	significance level	0.01614
McLeod-Li for Residual ARCH=	0.18895	significance level	0.99987

Final Model 2

Convergence in 51 Iterations. Final criterion was 0.0000076 <= 0.0000100
Dependent Variable BITP
Daily(7) Data From 2010:08:19 To 2014:04:27
Log Likelihood 2279.4044

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.000781677	0.000919155	0.85043	0.39508584
2. BITP{1}	0.105688646	0.025728838	4.10779	0.00003995
3. VOLUSEX	0.010500704	0.001183690	8.87116	0.00000000
4. ID	0.008761999	0.006513510	1.34520	0.17855942
5. CNYEX	0.002061222	0.000900905	2.28794	0.02214073
6. TREND 2014	0.000275242	0.000091714	3.00109	0.00269015
7. Trans 100	0.000784251	0.005013086	0.15644	0.87568554
8. C	0.000012781	0.000006331	2.01883	0.04350513
9. ARCH {1}	0.510344698	0.060154610	8.48388	0.00000000
10. ARCH {2}	-0.299165793	0.083855713	-3.56763	0.00036023
11. ARCH {3}	-0.169138317	0.049069661	-3.44690	0.00056705
12. GARCH{1}	0.957959413	0.016186697	59.18190	0.00000000
13. Shape	2.842908315	0.108005940	26.32178	0.00000000

Daily(7) Data From 2010:08:19 To 2015:04:27			
Observations	1348		
Sample Mean	0.061080	Variance	1.711679
Standard Error	1.308311	SE of Sample Mean	0.035634
t-Statistic (Mean=0)	1.714079	Signif Level (Mean=0)	0.086744
Skewness	7.503280	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	153.287757	Signif Level (Ku=0)	0.000000
Jarque-Bera	1332404.3877	Signif Level (JB=0)	0.000000

Q for Residual Serial Correlation	12.36998	significance level	0.03006
McLeod-Li for Residual ARCH=	0.13766	significance level	0.99995

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