Capital Regulation, Heterogeneous Loans and Monetary Policy: a Representative Bank Model

by

Yan Lyesin

University of Ottawa

#3146875

Major Paper presented to the Department of Economics of the University of Ottawa in partial fulfillment of the requirements of the M.A. Degree

Supervisor: Professor Lilia Karnizova
ECO 7997

Ottawa, Ontario
April 2006

ABSTRACT

This paper studies how the capital regulation affects banks' reaction to changes in monetary policy. This paper modifies an existing model of a representative bank. The modified model introduces heterogeneous loans which have different riskiness and, consequently, different weights in the determination of the regulatory capital. The model studies bank's profit maximizing behavior without capital requirements and with risk-based capital requirements. I obtained and analyzed short-run and long-run solutions under both types of capital regulation. To assess the impact of capital requirements, I study the response of equilibrium values to changes in the monetary policy (monetary tightening) and what factors affect the magnitude of this response. I found that without capital requirements all loan classes and their interest rates exhibit the same behavior. However, with the introduction of capital requirements, loan classes exhibit different behavior. The reaction of loan classes differs in magnitude and direction. I also study how the introduction of the capital standards influences the response of a bank's equity to changes in monetary policy. These results suggest that capital standards affect bank's response to changes in monetary policy, and this response is not uniform. The riskiest loan class reacts to changes in monetary policy differently from less risky loan classes. Regulators have to consider these facts because the consequences of regulatory action can affect the efficiency of the monetary policy action.

CONTENTS

TABLES

1. Introduction

Capital requirements, as an instrument of banking regulation, were adopted in almost every country since their introduction in 1988. A lot of work has been done to study the impact of capital regulation, but only a few of them study the impact on the transmission of monetary policy (as mentioned by Kopecky, VanHoose (2004B)). Typically, the literature considers either how regulation affects bank portfolios or how monetary policy affects bank lending. What I want to analyze is the interaction between regulation and monetary policy. I am particularly interested in:

1) How banks respond to changes in monetary policy with and without regulatory capital requirements.
2) How bank's response changes with an introduction of capital requirements.

Different modeling approaches are used to study the impact of the capital regulation on the monetary transmission mechanism. The majority of models uses the framework of a representative bank, either static or dynamic, with the introduction of capital requirements.

This paper modifies the model of a representative bank introduced by Kopecky and VanHoose (2004B) by introducing loans which have different levels of risk and, consequently, different weights in the determination of the regulatory capital. The model studies the bank's profit maximizing behavior without capital requirements and with risk-based capital requirements. I obtained and analyzed short- and long-run solutions under both types of capital regulation. Short and long time horizons are distinguished by the cost of equity adjustment. In the short run, the bank's equity cannot be adjusted and, therefore, fixed. In the long run, banks can optimally adjust equity taking into account required return on equity.

To assess the impact of capital requirements, I study the response of equilibrium values to changes in the monetary policy. In addition, I consider factors that affect the magnitude of this response. I found that without capital requirements, all loan classes and their interest rates exhibit the same behavior. However, with the introduction of capital requirements, loan classes exhibit different behavior. The reaction of loan classes differs in magnitude and direction. I also study the implication of the introduction of the capital standards for the bank's equity. Profit-maximizing behavior makes bank equity responsive to changes in monetary policy. These results suggest that capital standards affect a bank's response to changes in monetary policy, and this response is not uniform. The riskiest loan class reacts to changes in monetary policy differently from less risky loan classes. Monetary authorities and capital regulators have to consider the fact that regulatory action can affect the efficiency of the monetary policy action.

This paper is organized as follows. Section 2 presents a brief review of the issues and literature related to rationales for regulation of banks, contemporary capital regulation, and models used to study the impact of capital regulation on the monetary transmission mechanism. Section 3 describes the model of a representative bank used to study the impact of capital regulation. Section 4 contains solutions of the model and a short discussion of the obtained results. Section 5 gives a detailed analysis of the monetary transmission mechanism under both cases of capital requirements, and discusses the impact of capital regulation on the transmission of the monetary policy. Section 6 provides results of the simulation of equations of the responses to change in monetary policy. Section 7 summarizes the main findings of the paper. The Appendix contains tables, *Mathematica* code with solutions of the model of a representative bank, and a proof of the negative sign of expression (17) under the assumption about risk-weights of loan classes.

2. Literature and Issue Review

2.1. Rationales for regulation of banks

This paper studies the impact of capital regulation, one instrument of banking regulation, on the transmission of the monetary policy using the model of a representative bank. However, before I move to the discussion of the environment and to the specific model used to estimate the impact, I want to review briefly the rationales for and main instruments of banking regulation.

Academic literature gives no uniform answer to the question: "Why should banks be regulated?" Dewatripont and Tirole (1994) state that when answering this question, usually the main emphasis is put on bank functions and features, such as the transformation function, participation in the payment system, and degree of leverage. The approach that Dewatripont and Tirole suggest is to find what, in general, motivates banking regulation and then move to specific features, functions of banks and regulatory instruments.

Dewatripont and Tirole illustrate this point with the following examples. First, for instance, any nonfinancial company can also perform the transformation function performed by banks, by investing in a project that is financed with funds that have shorter maturity. However, this nonfinancial company will not be regulated as a bank. Second, banks are active and central participants in the payment system, but are not the only institutions that are also regulated. Other exists, such as pension funds, insurance companies, etc. Third, the degree of leverage is not a prerequisite for regulation by itself. We can see a lot of leverage in different nonfinancial companies and industries that are not regulated like banks. The level of leverage defines the amount of risk on the balance sheet, and no regulation is necessary in order to reduce risk if external agents or partners require this.

Further, deposit insurance cannot be viewed as a motivation for regulation. We cannot regulate banks just because we use deposit insurance as a regulatory instrument. Dewatripont and Tirole call this "putting the cart before the horse".

According to Dewatripont and Tirole, the real reason for banking regulation is the need to protect small depositors. This necessity arises from the fact that banks are subject to moral hazard and adverse selection, and depositors must constantly monitor activities. However, monitoring is a quite complex, expensive, and lengthy process. Furthermore, it is not rational and efficient to duplicate this process with a number of different agents. Often, small depositors do not possess either enough qualification or incentives to perform monitoring of very complex bank activities. These arguments raise a need for private or public representatives for depositors. Dewatripont and Tirole call this the "representation hypothesis".

Further, Dewatripont and Tirole address three potential concerns about the representation hypothesis: 1) "Why protect depositors, and not dispersed bondholders or shareholders?", 2) "Why do small investors invest primarily in financial intermediaries?", and 3) "How extensive should the regulation of the financial sector be?"

Addressing this first concern, Dewatripont and Tirole mention that small and dispersed stock and bond investors face the same problem as bank depositors. However, their investments into stocks and bonds are significantly less regulated. In the case of bond investors, who have access to the public debt market, the availability of bond rating systems significantly limits risk for the investor, and bond issuers are safe enough – they are large and well-capitalized. In the case of stock investors, the representation mechanism exists, but it has a different form. For instance, large shareholders and boards of directors represent minority shareholders.

Addressing the third concern, Dewatripont and Tirole do not advocate a comprehensive regulation. They say, "Households should have the choice to use unregulated financial institutions as long as they are made aware of the risk they face and minimal disclosure and antifraud devices are put in place."[1]

Summarizing the circumstances under which financial institutions tend to be regulated, Dewatripont and Tirole say: "First, the claimholders are somewhat unsophisticated or free-riding (small depositors, insurance policyholders, pensioners) and/or cannot conceivably thoroughly monitor

[1] Dewatripont and Tirole (1994), page 34

their counterparts due to the number of such parties and to the time scale involved (interbank depositors, traders on securities markets). Second, no mechanism of private representation is set up that would dispense the claimholders from having to monitor, write covenants, and interfere"[2].

Freixas and Rochet (1997) say that in general, public regulation is justified by market failures that arise from "(1) the presence of market power, (2) the importance of externalities, or (3) asymmetric information between buyers and sellers."[3] In the case of banking industry, the "importance of externalities" becomes the main argument. It is necessary to create a safe banking system, because bank failure has negative consequences for its depositors and other customers, and can generate systemic crisis.

Santos (2001) outlines the risk of systemic crisis, in line with Freixas and Rochet (2001), and the inability of depositors to monitor banks, which was discussed before as "representation hypothesis", as two main arguments that are used for the justification of the regulation of banks.

According to Freixas and Rochet (1997), the instruments that are used to make the banking system safe could be classified into six types:

"1. Deposit interest rate ceilings

2. Entry, branching, network, and merger restrictions

3. Portfolio restrictions, including reserve requirements and even, as an extreme case, narrow banking

4. Deposit insurance

5. Capital requirements

6. Regulatory monitoring."[4]

Saidenberg and Schuermann (2004), in their support for systemic risk argument and the use of capital requirements to mitigate it, stated: "The assumption is that [bank] shareholders will not

[2] Dewatripont , Tirole (1994), page 44

[3] Freixas, Rochet (1997), page 257.

[4] Freixas, Rochet (1997), page 259.

take into account the social costs of systemic risk in their capital decisions and so will tend to hold less capital than if these spillover costs were considered." [5]

The model, which will be described in the section 3, deals with the fifth instrument – capital requirements and their impact on the transmission of monetary policy. It is necessary to mention that Basel I defines the framework for capital requirements only. Basel II defines the framework for both capital requirements (Pillar 1) and regulatory monitoring (Pillar 2). More detailed discussion on Basel I and Basel II is presented in the section 2.2.

2.2. Contemporary Banking Regulation: Basel I and Basel II

The framework currently used for banking regulation – Basel Capital Accord (Basel I) – was introduced in 1988 by the Bank for International Settlements' (BIS) Basel Committee on Banking Supervision. Later, central bankers in many countries endorsed their modification of the Basel Capital Accord. The main goal of the Basel I is to set the uniform capital standards for international banks in order to strengthen the stability of the banking system. Basel I provided one single measure of risk and capital for banks. This single capital standard did not take into account differences in risk from bank to bank that arise from the variety of sources.

An 8% minimum capital requirement for internationally active banks is a foundation of the Basel I framework that ensures an adequate level of capital. Under Basel I, bank capital accounted separately for two tiers. Tier 1, which includes shareholders equity and retained earnings, is 4%. Tier 2, which includes additional internal and external sources of capital available to banks, is 4%. Tier 2 is limited to 100% of Tier 1 capital. It is obvious with this structure that Basel I is focused primarily on credit risk. Table 1 in Appendix 1 summarizes risk weights for different categories of assets.

Unfortunately, this simple regulatory structure shows significant drawbacks, which undermine the purpose of the regulation to increase the strength of the banking system. One of these drawbacks comes from the capital arbitrage. It is possible by using securitization techniques and off-

[5] Saidenberg, Schuermann (2004), page 98.

balance sheet activities to move out high-quality assets with lower returns from banks' balance sheets. At the same time, it is possible to increase exposure in the loans with lowest quality and the highest expected returns in a risk class. This phenomenon was noticed by Jones (2000) and Hawke (2002). Of course, this set of actions reduces the overall quality of the bank's assets.

Another problem is that the risk weights did not match realized losses. This makes it impossible for the capital requirements, as a regulatory instrument, to accomplish their ultimate goals – to prevent systemic crisis and to protect small depositors (see section 2.1 with rationales for regulation). Using regulatory data for U.S banks from 1984 until 1999, Flood (2001) found "that the Basel risk weights do not accurately track the historical credit experience of U.S. loan portfolios." Some types of loans appear to be "under-burned" by Basel I risk-weights and some were "over-burned." Flood's findings, especially with respect to mortgage loans and commercial loans, demonstrated the following: mortgage loans, which are collateralized, generate a smaller share of losses for banks, than their respective risk-weights (over-burned). On the other hand, commercial loans generate greater losses than the risk-weighted share of the capital associated with them (under-burned).

The use of the single uniform capital leverage ratio also appears to be a misappropriate measure of the capital adequacy because it is unrelated to bank-specific risks. Again, this fact does not facilitate the accomplishment of regulatory goals (section 2.1). For instance, it is possible that a bank with fulfilled uniform capital requirements could have significant losses simply because it tended to supply loans to the borrowers with the highest risk within its risk class. In addition, modern financial organizations with global operations are so large and complex, that it is hard to assess the risk management process in these institutions without an appropriate standard.

These issues were behind the introduction of the New Basel Capital Accord, also known as Basel II, which was proposed in January 2001. Basel II is expected to be implemented starting from 2007.

Saidenberg and Schuermann (2004), describing the objectives of Basel II, noted that the foundation of Basel II is a focus on risk differentiation and the need for enhanced approaches in credit risk assessment. Basel II encourages bankers, supervisors, and other market participants to "be-come better attuned to risk and better able to act on those risk assessments at the appropriate time."[6] It is much more efficient for supervisors to be able to address issues in advance using risk management techniques, rather than dealing with them under pressure of crisis.

The major difference between Basel I and Basel II is that the latter provides more flexibility and risk sensitivity than Basel I. Basel II consists of three pillars: Pillar 1, minimum capital requirements, Pillar 2, supervisory review process, and Pillar 3, market discipline.

Pillar 1 not only incorporates Basel I, but also extends it with a more complex definition of credit risk and with the introduction of operating risk. Pillar 1 also provides a framework that allows recognition of the differences in risk (using Internal Rating-Based approaches). According to Basel II, credit risk can be measured by one of the following options:

1. Standardized approach: a modified version of the existing one under Basel I, based on external credit ratings when available.
2. Foundation Internal Rating-Based (IRB) approach: based on the bank's estimates of default probability for each borrower. Other estimates are provided by the supervisor.
3. Advanced IRB approach (A-IRB) where the bank provides all necessary inputs.

Market risk can be measured by either: 1) a standardized approach, or 2) an internal models approach. Operational risk can be measured by: 1) a basic indicator approach, 2) a standardized approach, or 3) an advanced measurement approach.

Credit Risk

According to BIS, credit risk is the risk of loss arising from default by a creditor or counterparty. In order to produce a sum of risk-weighted asset values, banks must allocate risk weights to both on- and off-balance sheet items. In contrast to Basel I, banks use internal and external credit ratings and associated probabilities of default to determine the risk weights.

The credit risk weights apply to bank assets in six categories: corporate, banks, equity, retail, project finance, and sovereigns; and they depend, in part, on credit ratings. The summary of risk-

[6] Saidenberg, Schuermann (2004), page 101.

weights for different asset categories is presented in Table 2 in Appendix 1.

Market Risk

The definition of market risk, i.e. risk that arises from market price changes on bank's positions, is unchanged from Basel I. Market risks are most commonly associated with risks in trading activities for debt and equity securities, foreign exchange, and commodity risks. Market risk is typically measured by value-at-risk (VAR).

Operational Risk

BIS defines operational risk as "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events." It includes, but is not limited to, the risk of inadequate or failed computer systems, such as computer failures or fraud, compliance issues, as well as external events, including lawsuits.

Under the basic indicator approach, a bank uses one indicator to represent the operational risk for the entire institution, and it ties its capital to a single measure of business activity, such as Gross Income. Under the standardized method, different indicators are used for different lines of business. The advanced measurement approach (AMA) uses internal loss data and the bank's own methodology to estimate the amount of capital required.

Before I move to the analysis of the impact of capital regulation on the transmission of monetary policy (sections 3-6), in section 2.3 I briefly review models and findings used to study the impact of capital regulation on the monetary transmission mechanism.

2.3. Capital regulation and monetary policy transmission: model overview

There are a number of possibilities for how regulatory capital standards may affect the transmission of monetary policy. In this section, I outline different modeling approaches used to study the impact of capital regulation on the monetary policy transmission and their results.

Thakor (1996) examines the impact of risk-based capital requirements on banks' lending. In his model, banks perform two lending functions: pre-loan screening and post-lending monitoring, and can invest into government securities. Each borrower with different creditworthiness can approach multiple banks looking for funding or refinancing. Banks determine whether to screen or monitor a loan based on information about how many other banks the borrower contacted. A bank's ability to supply loans is limited by regulatory capital requirements, which take into account the amount of credit risk (Basel framework). This model generates many results, but one of them is of particular interest for my topic: an increase in risk-based capital requirements increases the chance that a borrower will be denied credit. As a result, aggregate lending will be reduced.

Kishan and Opiela (2000) present theoretical and empirical analysis of how banks respond to changes in monetary policy depending on the capital classification of the bank. They found that depending on the asset size and capitalization, banks respond to monetary policy changes differently. Small undercapitalized banks, which have less ability to raise funds and maintain loan portfolios during the monetary tightening, are likely to be more responsive to monetary policy. These results have important implications for the interaction between regulatory and stabilization policy. Increases in capital requirements, aiming to build a stronger banking system, may lead to a banking system that is less responsive to stabilization policy. They also found that a consolidation in the banking industry, which leads to greater efficiency, might also lead to a weaker response to monetary policy in the bank-lending channel. Kishan and Opiela (2000) were the first, among models I mentioned in this section, to identify a conflict between regulation and stabilization. However, the nature of the conflict there is different from what I am interested in. The conflict in Kishan and Opiela (2000) stems not from the banking behavior itself, which is affected by capital regulation, but from different abilities to adapt to changes in monetary policy. These different abilities depend on the bank's capitalization.

Jacques and Schirm (2004) modified the theoretical model used by Kishan and Opiela (2000) to incorporate risk-based capital standards in which credit ratings determine capital requirements. Their findings were consistent with the previous model: a bank's response to monetary policy depends on bank capitalization. In addition, a model extension allowed them to notice a different effectiveness, in both magnitude and direction, of the monetary policy, depending on the balance sheet liquidity and the credit quality of the bank's assets. Jacques and Schirm found that if a bank's portfolio consists of low rated loans below a certain category, the effectiveness of monetary policy, aiming to change lending, under Basel II will be less than under Basel I. Therefore, the introduction of risk-based capital requirements, again, reveals a conflict between regulation and stabilization. However, similarly to Kishan and Opiela (2000), this conflict arises not from banking behavior, but from the change in risk weights following the transition from Basel I to Basel II.

Chami and Cosimano (2001) develop a dynamic model where the banking industry is characterized by oligopolistic behavior, with banks having market power. Banks maximize the present value of future profits, subject to a capital constraint. Chami and Cosimano analyze the "bank balance-sheet channel" of monetary policy, which can be described as the following: monetary policy affects the costs of deposits and total capital, and will result in a changed loan supply in the next period. Capital requirements in this model, whether they are binding or non-binding, introduce asymmetry in the response to monetary policy – a binding capital constraint will lead to a smaller increase in the quantity of loans and smaller decrease in the loan rates following monetary easing.

Zicchino (2005) extends the framework of Chami and Cosimano (2001) by introducing a capital requirement framework consistent with Basel II, where the risk-weight of a loan is linked to a borrower's creditworthiness. Loan defaults, and consequently risk-weights, are dependent on the macroeconomic conditions. As a result, macroeconomic conditions, which affect the creditworthiness of the borrower, will define the amount of credit risk. This model modification introduces two channels through which macroeconomic conditions affect capital and lending: 1) changes in macroeconomic conditions affect default rates and hence loan marginal costs, which in turn affect banks' capital, and 2) changes in macroeconomic conditions also affect the risk weights of loans. For instance, positive macroeconomic shock will reduce loans' marginal cost through a decrease in default rate. Simultaneously, it will decrease the risk weight of the loan. These two effects combined will give banks more opportunities to increase loans even under binding capital constraint. On the other hand, a negative shock not only increases the costs of loans, but also increases risk weights, intensifying the reduction in the loan supply.

Van den Hauvel (2002A, 2002B) presents a dynamic model of optimal bank lending under risk-based capital requirements, assuming an imperfect market for a bank's equity. This model provides a framework for analyzing the impact of capital regulation on bank lending and the response of lending to changes in monetary policy. The level of the bank's capital adequacy affects the timing and magnitude of the bank's response to changes in monetary policy; banks with low levels of capital have delayed and amplified reactions compared to well-capitalized banks. Van den Hauvel defines "bank capital channel" analogously to "bank balance-sheet channel" in Chami and Cosimano (2001). Through this channel, bank lending is affected by monetary policy, which has an impact on banks' equity.

Kopecky and VanHoose (2004A) use the model of a representative bank to study the effect of the binding capital requirement on the loan expansion process. In their model, a bank facing the costs of adjustment quantities of assets and liabilities maximizes profits resulting from the difference between interest income and resource costs. The monetary authority in this model influences reserves. The regulatory authority sets capital requirements as a certain share of the loan portfolio. Kopecky and VanHoose examine two cases – with binding capital requirements and without them. Their results indicate that the initial conflict between monetary authority and regulatory authority is resolved through adjustment in equity. However, this adjustment imposes costs on society, because it drains funds from other sectors of the economy and bears adjustment costs.

In the next paper, Kopecky and VanHoose (2004B) also use the model of a representative bank, solving an analogous profit maximization problem with a similar cost structure. Now, they define the external environment as represented by a demand for banking products, that is, loans and deposits. They also separate two time horizons for the model: short run with a fixed amount of the bank's equity, and long run where equity is optimally adjusted. The model introduces three cases of capital requirements: no capital requirements, risk-based, and non-risk-based. Within this framework, Kopecky and VanHoose demonstrate that the introduction of risk-based capital requirements dramatically changes the equilibrium outcome both in the short and in the long run, and affects the transmission mechanism of the monetary policy.

In Kopecky and VanHoose (2006), the authors extend the previous framework by introducing heterogeneous loan monitoring costs. These costs arise as a bank's response to the moral hazard behavior of borrowers. As in the previous model, Kopecky and VanHoose analyze cases with no capital requirements and with risk-based capital requirements. Within this framework, they found that binding capital requirements increase the market loan rate and reduce equilibrium loan quantities. In addition, capital requirements misallocate monitoring activity within the banking system, increasing total monitoring costs incurred by the bank.

All the papers that I mentioned here, except for Thakor (1996), point out on either conflict between regulator and monetary authority, or negative aspects associated with capital requirements (Kopecky and VanHoose (2004A) and (2006)). However, all these papers identify different sources of the conflict within the homogeneous loans framework.

In the next section, I introduce a model of a representative bank, modified from Kopecky and VanHoose (2004B). The purpose of this model is to find out how a bank responds to changes in monetary policy with and without risk-based capital requirements. In contrast to the baseline model, a bank's balance sheet in my model contains three loan classes. These loan classes are distinguished by their level of risk and, consequently, by their risk weights in the regulatory capital. By introducing heterogeneous loans in this model, I can track more precisely the effect of risk-based capital requirements on the transmission of monetary policy. This modification allows me not only to confirm the existence of the conflict between regulator and monetary authority, but also to identify one of its manifestations.

3. Model of a Representative Bank: Description

This section describes a model of a representative bank that is used to study the impact of capital requirements on the bank's reaction to changes in monetary policy. This section is organized as follows. Subsection 3.1 gives a description of the structure of the bank's balance sheet, and one of its components – required reserves. Subsections 3.2 and 3.3 introduce interest rates that are used in the model. Subsection 3.4 defines the demand for loans and deposits. Subsection 3.5 introduces the model's capital requirements framework. Subsection 3.6 defines a component of the bank's required return on equity: the market risk premium. Subsection 3.7 distinguishes timings (short and long run) in a bank's decision. Subsection 3.8 concludes section 3 with a formulation of a bank's objectives for both the short and long run.

3.1. Balance sheet identity and required reserve component

In order to study an influence of the monetary transmission mechanism and the impact of risk-based capital regulation on the bank's decision, I introduce a bank's simple balance, just as in the baseline model introduced by Kopecky and VanHoose (2004B)[7]. This balance sheet contains seven essential items. In assets, there are three different loan classes (L_1, L_2, L_3), default-risk-free government securities (G), and required reserves (R). In liabilities, there are insured deposits (D), and equity (E). Balance sheet identity has the following form:

$$R+G+L_1+L_2+L_3=D+E \tag{1}$$

The difference in the loan classes L_1, L_2, L_3 is in their level of risk, which will affect capital requirements. The higher (lower) the risk of the loan class, the greater (smaller) the weight this

[7] In the baseline model, Kopecky and VanHoose consider two possible cases that differ with respect to the type of deposit insurance. In the first case, banks issue only insured deposits. In the second case, they issue both insured and uninsured deposits. This separation helps to explain adjustments in the liability side of the bank's balance sheet to the change in capital requirements. In the modified model, I will study only the case where all deposits are insured – my objective is to study the risks that arise in the asset side of a bank's balance sheet, and the impact of risk-based regulation on monetary transmission mechanism and the asset composition.

loan class will have in the determination of the regulatory capital. I assume that the first loan class, L_1, has below average default risk, the second class, L_2, average default risk, and the third class, L_3, above average default risk. Given this assumption and risk-return tradeoff, I assume that the higher the risk of the loan class, the higher the loan rate for this loan class will be.

Reserve requirements are determined as a fraction of the insured deposit component of a bank's liabilities, D:

$$R \geq \rho D \tag{2}$$

where ρ is the effective required reserve ratio.

As in the baseline model, and for simplicity, I assume that the demand for excess reserve is zero. Therefore, equation (2) is an equality.

3.2. Interest rates: monetary policy

The next important question is how monetary policy is conducted. According to recent experience, authors of the baseline model employ the following scheme in their model: the monetary authority, which uses an interest-rate-targeting policy procedure, determines the default-risk-free interest rate (r). I will follow the same approach. In the modified model, changes in the monetary policy will occur through changes in the risk-free interest rate r.

3.3. Interest rates: balance sheet components and non-bank debt

Rates for government securities (r_G) and required rate of return on equity (r_E) are assumed to equal default-risk-free rate r plus an appropriate risk premium:

$$r_G = r + \psi \tag{3}$$

$$r_E = r + \beta \tag{4}$$

where ψ is a market premium on government securities, and β is a market premium on bank equity[8].

Authors assume that the market risk premium on government securities ψ reflects both the term-structure and liquidity risks, and these risks are determined outside the banking sector.

The interest rates on other balance sheet components, three loan classes (r_{Li}), and insured deposits (r_D), are determined in the market equilibrium of the public demand for loans and banks' supply of loans, i.e. banks' optimal behavior that takes into account adjustment costs.

Non-bank debt is an alternative source of financing for different types of borrowers. The interest rate for non-bank debt is one of the determinants of the public's loan demand. The public's loan demand is determined by the default-risk-free rate r plus a market risk premium on non-bank debt, σ_i, which reflects the default risk for this type of borrower and its financial obligations:

$$r_{B_i} = r + \sigma_i \tag{5}$$

As in the baseline model, I assume that the market risk premium on non-bank debt, σ_i, is determined in the real sector of the economy and exogenous to the model.

3.4. Demand equations

The public's loan demand for each loan class L_i is assumed to be a linear function of two components, which reflects the existence of two different types of borrowers, distinguished by the extent of their access to the debt market:

$$L_i = l_0 - l_1 r_{L_i} - l_2 (r_{L_i} - r_{B_i}) \tag{6}$$

where l_0, l_1, and l_2 are parameters of the linear loan demand function.

[8] Market premium on bank equity β is defined and explained in section 3.6.

The first type of borrower, which is represented by term $l_1 r_{L_i}$ in equation (6), is a borrower who has bank loans as their only source of financing. These borrowers cannot participate in the market of publicly traded debt instruments as issuers. According to equation (6), the increase in the loan rate r_{Li} will result in a decrease in the public's demand for loans. The second type of borrower is represented by term $l_2(r_{L_i} - r_{B_i})$ in equation (6). This borrower has access to both loan and publicly traded debt markets. According to equation (6), the increase in the loan rate r_{Li} relative to r_{Bi} will result in a decrease in the public's demand for loans, that is belongs to the second category of borrowers. This borrower will prefer to issue new publicly traded debt.

The assumption that parameters of the loan demand function (l_0, l_1, l_2) for each loan class are the same significantly simplifies expressions for the equilibrium solutions, and provides the possibility for direct comparison of the modified model with the baseline model.

The public's demand for insured deposits D is also assumed to be a linear function of two components. It has the following form:

$$D = d_0 + d_1 r_D + d_2(r_D - r_G) \tag{7}$$

where d_0, d_1, and d_2 are parameters of the insured deposits demand function.

The first term in equation (7) refers to the demand component, which is dependent on insured deposit interest rate r_D offered by the bank. The higher this interest rate, the higher the demand for insured deposits is. The second term reflects the effect of substitution from the other product available for depositors, i.e. government securities. The higher the spread between the interest rate on insured deposits r_D and the interest rate on government securities r_G, the more attractive the insured deposits are and the higher the demand.

3.5. Capital requirements

Capital requirements in this model are introduced in two different forms: non-risk based and risk-based capital requirements. Risk based capital requirements arise when banks' capital is constrained by the risk-weighted asset capital requirement ratio (θ_1) of banks' assets. It has the following form:

$$E \geq \theta_1 (\sum_{i=1}^{3} w_i L_i + w_R R + w_G G), \tag{8a}$$

where w_i, w_R, w_G are risk-based weights of the loans, required reserves, and government securities respectively. Currently, $w_R = w_G = 0$, i.e. these types of assets in the bank's balance contain no risk from regulatory point of view. w_i depends on the extent of default risk of the loan class. Given the assumed level of risk of loan classes L_1, L_2, and L_3, it is possible to give the following relationship between the magnitudes of loan weights: $w_1 < w_2 < w_3$.

Non-risk based capital requirement arises when banks' capital is constrained by the total asset capital ratio (θ_2). It has the following form:

$$E \geq \theta_2 (L + R + G). \tag{8b}$$

Combining equations (8a) and (8b) and given the fact that under present regulation $\theta_1 > \theta_2$, we arrive at the following equation for capital requirements:

$$E \geq \theta_1 \sum_{i=1}^{3} w_i L_i + \theta_2 (R + G). \tag{8}$$

3.6. Market risk premium on bank equity

The market risk premium on bank equity β reflects non-diversifiable risks. These consist of different types of risk that arise from certain components of banks' assets and liabilities. The market risk premium on bank equity has the following form:

$$\beta = \beta_0 + \beta_{1,1}\frac{L_1}{E} + \beta_{1,2}\frac{L_2}{E} + \beta_{1,3}\frac{L_3}{E} + \beta_2\frac{G}{E} \qquad (9)$$

where β_0 is the coefficient that determines the base level of market risk premium, and $\beta_{1,i}$, and β_2 are the coefficients that determine the contribution of different types of risk to the market risk premium.

The first risk that arises from banks' balance is a default risk, which is represented by $\frac{L_i}{E}$ - the ratio of the loan class size to the capital. The higher this ratio, the greater the default risk, transferred to bank's activity from the loans' ability to default. Consequently, a greater market risk premium is required for this bank, compared to a bank with the lower ratio. Negative macroeconomic shocks, for instance, affect production activity and the profitability of borrowers. These factors contribute to an increase in loan defaults. Further, this leads to an increase in the default risk in banks' balances. Given the assumed level of risk of loan classes L_1, L_2, and L_3, it is possible to state the following relationship between the magnitudes of the contribution of the loans' default risk to market risk premium: $\beta_{1,1} < \beta_{1,2} < \beta_{1,3}$.

The second type of risk is an interest rate risk and a term-structure risk. This risk arises from the banks' holdings of government securities, which is represented by $\frac{G}{E}$ - the ratio of government securities holdings to capital. The higher this ratio, the greater banks' exposure to the interest rate and term-structure risk. Consequently, a higher risk premium is required for the bank's equity.

3.7. Timing of banks' decisions

The decision to adjust banks' equity, following imposed or changed capital requirements or changed monetary policy, cannot be implemented immediately. Usually, this process requires a regulatory approval, and an interaction with an investment bank. These counterparties require significant legal, marketing and processing costs. In contrast, the deposit market can be accessed by banks quickly and inexpensively.

These factors lie behind the different timings in banks' decisions. Authors of the baseline model study the effect of capital requirements on a bank's optimal decision for two time horizons: short and long run. I will follow the same approach. In the short run, banks cannot adjust the book value of equity E, but can adjust loans L_i, government security holdings G, and insured deposit D. In the long run, banks can adjust their equity, in addition to loans, government securities, and both types of deposits, taking into account the required return on equity.

3.8. Banks' objectives

A bank maximizes the expected value of an infinite stream of discounted profits. These profits are discounted by the required return on equity r_E. Like in the baseline model, I "assume that expected future interest rates are constant at stationary-state values," and the sum of discounted profits can be represented by the following equation:

$$E\left(\sum_{i=0}^{\infty}\frac{\pi_{t+i}}{(1+r_E)^i}\right)=\pi_t+\frac{\pi_{SS}}{r_E} \tag{10}$$

where π_t is a bank's profit in period t, and π_{SS} is an expected stationary state profit. The term $\frac{\pi_{SS}}{r_E}$ represents constant, infinite stream of expected future profits, discounted at the required rate of return on equity r_E.

Like in the baseline model, the "short run" for the banking sector is defined to be the current period. This period is too short to permit the bank to adjust the equity capital. Therefore, the short-run objective of the bank is to maximize the short-run profit function, given that equity is fixed. In this case profit function (π) has the following form[9]:

$$\pi = r_{L_1}L_1 + r_{L_2}L_2 + r_{L_3}L_3 + r_G G - r_D D - \frac{b_L}{2}(L_1^2 + L_2^2 + L_3^2) - \frac{b_D}{2}D^2 \tag{11}$$

where b_L, and b_D are positive parameters that describe the magnitudes of quadratic adjustment costs for loans and deposits respectively.

In the long run, banks can adjust their equity capital incurring equity adjustment costs. The expression for the long-run profit function (π_{LR}) has the following form:

$$\pi_{LR} = \hat{r}_{L_1}L_1 + \hat{r}_{L_2}L_2 + \hat{r}_{L_3}L_3 + \hat{r}_G G - \hat{r}_D D - \frac{b_L}{2}(L_1^2 + L_2^2 + L_3^2) - \frac{b_D}{2}D^2 - \frac{b_E}{2}E^2 - r_E E \tag{12}$$

where b_E is a positive parameter that describes the cost of adjusting equity. The symbol "^" marks a stationary state value of the parameter. The additional component of the profit function, compared to the short-run profit function, is a required return paid on equity capital ($r_E E$). This component reflects the fact that long-run profits come when all relevant costs have occurred, including the required return on equity.

The formulation of banks' objectives finishes the section that describes the model used to study the impact of capital requirements to the transmission of monetary policy.

[9] Kopecky and VanHoose (2004B) assume that in the model portfolio separation holds in both short-run and long run. This assumption is necessary to derive the impact of capital requirements. This assumption is implemented by assuming that the magnitude of the government securities adjustment cost b_G is equal to zero.

4. Model of a Representative Bank: Solution

4.1. Model solution in the short run

In the short run, a bank maximizes short-run profit function (11) subject to the balance sheet constraint (1) and the capital requirements constraint (8), given predetermined capital ($E = \overline{E}$):

$$\underset{L_1,L_2,L_3,G,D}{Max} \quad \pi = r_{L_1}L_1 + r_{L_2}L_2 + r_{L_3}L_3 + r_G G - r_D D - \frac{b_L}{2}(L_1^2 + L_2^2 + L_3^2) - \frac{b_D}{2}D^2$$

Subject to:

$$\overline{E} = \theta_1 \sum_{i=1}^{3} w_{L_i} L_i + \theta_2 (R + G) \qquad (13)$$

$$R + G + L_1 + L_2 + L_3 = D + \overline{E}$$

The necessary conditions for profit maximization are given by the following system of first-order conditions:

$$\begin{aligned}
&r_{L_1} - b_L L_1 - \lambda_1 - \lambda_2 \theta_1 w_{L_1} = 0 \\
&r_{L_2} - b_L L_2 - \lambda_1 - \lambda_2 \theta_1 w_{L_2} = 0 \\
&r_{L_3} - b_L L_3 - \lambda_1 - \lambda_2 \theta_1 w_{L_3} = 0 \\
&r_G - \lambda_1 - \lambda_2 \theta_2 = 0 \\
&-r_D - b_D D + \lambda_1 (1 - \rho) - \lambda_2 \theta_2 \rho = 0 \\
&L_1 + L_2 + L_3 + G - (1 - \rho) D - \overline{E} = 0 \\
&\theta_1 (w_{L_1} L_1 + w_{L_2} L_2 + w_{L_3} L_3) + \theta_2 (G + \rho D) - \overline{E} = 0
\end{aligned} \qquad (14)$$

where λ_1 is a Lagrange multiplier associated with the balance sheet constraint (1), and λ_2 is a Lagrange multiplier associated with the capital requirements constraint (8).

The solution of the system of first-order conditions (14) gives us short-run profit-maximizing quantities for loans L_i, government securities holdings G, and insured deposits D.

Market equilibrium conditions are defined as a simultaneous equality of 1) the bank's supply of each loan class and the public's demand for them, and 2) the bank's supply of deposits and the public's demand for them[10]. The bank's supply is derived from the system of equations (14), and the public's demand from equations (6) and (7). The solution of these market equilibrium conditions allows us to derive the short-run equilibrium loan rates r_{Li}, and insured deposit rate r_D. Substituting these equilibrium loan rates into the solutions of a bank's profit maximization problem (14) allows the derivation of the short-run equilibrium values for loans L_i, deposits D, government securities G, and equity E. These equilibrium values are functions of the model parameters, exogenous interest rates, and risk premiums.

4.2. Model solution in the long run

In the long run, the bank maximizes the long-run profit function (12), subject to the balance sheet constraint (1) and the capital requirements constraint (8). Equity E is a choice variable now:

$$\underset{L_1,L_2,L_3,G,D,E}{Max} \quad \pi_{LR} = \hat{r}_{L_1} L_1 + \hat{r}_{L_2} L_2 + \hat{r}_{L_3} L_3 + \hat{r}_G G - \hat{r}_D D - \frac{b_L}{2}(L_1^2 + L_2^2 + L_3^2) - \frac{b_D}{2} D^2 - \frac{b_E}{2} E^2 - r_E E$$

Subject to:

$$E = \theta_1 \sum_{i=1}^{3} w_{L_i} L_i + \theta_2 (R+G) \qquad (15)$$

$$R + G + L_1 + L_2 + L_3 = D + E$$

The necessary conditions for profit maximization are given by the following system of first-order conditions:

[10] In this case, a bank's supply of liability such as deposit is a bank's demand for it. Similarly, a public's demand for deposits (which are assets for depositors and liabilities for bank) is a public's supply of deposits.

$$
\begin{aligned}
&\hat{r}_{L_1} - b_L L_1 - \lambda_1 - \lambda_2\theta_1 w_{L_1} - \beta_{1,1} = 0 \\
&\hat{r}_{L_2} - b_L L_2 - \lambda_1 - \lambda_2\theta_1 w_{L_2} - \beta_{1,2} = 0 \\
&\hat{r}_{L_3} - b_L L_3 - \lambda_1 - \lambda_2\theta_1 w_{L_3} - \beta_{1,3} = 0 \\
&\hat{r}_G - \lambda_1 - \lambda_2\theta_2 - \beta_2 = 0 \\
&-\hat{r}_D - b_D D + \lambda_1(1-\rho) - \lambda_2\theta_2\rho = 0 \\
&L_1 + L_2 + L_3 + G - (1-\rho)D - E = 0 \\
&\theta_1(w_{L_1}L_1 + w_{L_2}L_2 + w_{L_3}L_3) + \theta_2(G + \rho D) - E = 0 \\
&-(r+\beta_0) - b_E E + \lambda_1 + \lambda_2 = 0
\end{aligned}
\qquad (16)
$$

The solution of system of the first-order conditions (16) gives us the long-run profit-maximizing quantities of loans L_i, government securities holdings G, insured deposits D, and equity E.

Market equilibrium conditions are defined as a simultaneous equality of 1) the bank's supply of each loan class and the public's demand for them, and 2) the bank's supply of deposits and the public's demand for them. The bank's supply is derived from the system of equations (16), and the public's demand from equations (6) and (7). The solution of these market equilibrium conditions allows us to derive the long-run equilibrium loan rates r_{Li}, and insured deposit rate r_D. Substituting these equilibrium loan rates into solutions of the bank's profit maximization problem (16) allows us to derive the long-run equilibrium values for loans L_i, deposits D, government securities G, and equity E. These equilibrium values are functions of the model parameters, exogenous interest rates and risk premiums.

In the baseline model, Kopecky and VanHoose examine the short- and long-run solutions for three cases: (i) $\theta_1 = \theta_2 = 0$ (no capital requirements), (ii) $\theta_1 > 0$ and $\theta_2 = 0$ (risk-based capital requirements), and (iii) $\theta_1 = \theta_2 = \theta$ (non-risk-based capital requirements). It was shown that the solution for "non-risk based capital requirements" case is identical to the "no capital standard" solution. Therefore, in the modified model, I will focus on the first two cases only – "no capital standard" and "risk based capital requirements".

Solutions for a bank's profit-maximization problem in the modified model (systems of equations (14) and (16)) are given in Tables 3-6 in Appendix 1.

4.3. Analysis of short- and long-term solutions for the bank's profit maximization problem and equilibrium loan rates and quantities

4.3.1. Loans

All solutions for the bank's profit-maximization problem for loan quantities[11] for all four cases (short and long run, no capital requirements and risk-based capital requirements) have the same structure. The basic idea behind this structure is the following: the loan quantity is equal to the spread between loan rate and the rate of return on alternative assets adjusted by the loan adjustment costs.

So, the solution of the simplest case (short-run and no capital requirements) gives us loan quantity L_1 as the spread between the rate of return for this class of loan (r_{L1}) and the rate on government securities (r_G) divided by the loan adjustment cost b_L (see Table 3 in Appendix 1).
In the long run, the solution for the "no capital requirements" case has the same structure. However, this time the loan rate for this asset class r_{L1} is adjusted by its contribution to default risk component $\beta_{1,1}$ of the bank's required return on equity. Similarly, the rate on government securities r_G is adjusted by its contribution to the interest rate and term structure risk component β_2 of the required rate of return on equity.

These two solutions are almost identical to the baseline model. However, in the modified model, they define the quantity of the single loan class, L_i, in a bank's balance sheet.

When we consider risk-based capital requirements, the structure of the solution remains the same. It can be described as a spread adjusted by the cost of loan adjustment. Now, a bank's optimal solution for the loan quantities of each loan class is dependent not only on the rate of return on government securities, but also on the rates of return on other loan classes. The higher the spread for this loan class, the higher the demand for this loan class is. Spreads for other loan classes negatively affect the loan demand for this class. The higher the adjusted spread for the alternative loan classes, the smaller the demand for this loan class will be. The effect of each

[11] In the section 4.3, I will refer a bank's solution of the profit-maximization problem to as solution.

spread under the risk-based capital requirements in the modified model is now altered by the impact of risk weights of this and other loan classes.

To be able to analyze the transmission of monetary policy and the impact of capital requirements, it is necessary to obtain equilibrium values, which take into account the public's demand for loans and deposits. Substituting solutions for the bank's profit maximization problem into demand equations (6), I derive the equilibrium loan rate and loan quantities for each class of loans.

Tables 7 and 8 in Appendix 1 contain equilibrium loan rates and loan quantities respectively for the first loan class. Equations for other loan classes are symmetrical (see Appendix 2 with detailed solutions for the model). The pure expressions for the equilibrium values are massive and complicated, especially in the case of the long-run solution with risk-based capital requirements. Therefore, in the analysis of the monetary transmission mechanism, I will use the equations of the reaction of the equilibrium value to the change in monetary policy. These reaction equations are first-order derivatives of the equilibrium values with respect to the risk-free rate r. The risk-free rate r serves as a proxy for monetary policy changes (section 3.2). The reaction equations are presented in Tables 9 and 10 in Appendix 1.

4.3.2. Deposits

The profit-maximizing quantities of the deposits, as in the baseline model, are independent from the type of capital requirements: the solutions for all four cases bear the same structure. This structure can be described as a spread between the rate of return on government securities holdings, adjusted by required reserve ratio, and the deposit rate (see Table 4 in Appendix 1). In the long run, the rate of return on government securities holdings is adjusted by the contribution of the interest rate and term structure risk component β_2 to the required rate of return on a bank's equity.

Substituting the solution for a bank's profit maximization problem into the demand equation (7), I derive the equilibrium deposit rate.

4.3.3. Equity

As I mentioned in section 3.7, in the short run, a bank's equity is fixed. However, in the long run, equity is another choice variable. The long-run solution for the bank's profit-maximization problem contains the equilibrium expressions for the bank's equity (see Table 5 in Appendix 1).
With no capital requirements, the profit-maximizing value of a bank's equity is a spread between the rate of return on government securities and two components of the required return to equity ($r + \beta_0$) divided by the cost of equity adjustment. The rate of return on government securities is adjusted by the contribution of the interest rate and term structure component β_2 to the required rate of return on a bank's equity.

With risk-based capital requirements, the equilibrium value of a bank's equity additionally contains the term that takes into account a bank's decision to adjust loans. This decision is based on the spread between the risk-adjusted returns on the loan portfolio and government securities.

4.3.4. Government securities holdings

Solutions for government securities holdings (see Table 6 in Appendix 1) are obtained from the balance sheet constraint (1) given the profit-maximizing values for loans, deposits, and a bank's equity.

5. Analysis of the monetary transmission mechanism in the model

After deriving the equilibrium rate and quantities for loans and deposits, I substitute expressions for the interest rates (r_{Bi}, r_G). After differentiating with respect to the proxy of monetary policy – risk-free interest rate r, I can analyze how changes in monetary policy will translate into banks' optimal decisions. Particularly, I am interested in how the bank adjusts its assets and liabilities, and what implications this will have on the overall level of risk of the bank's assets.

Tables 9 and 10 in Appendix 1 contain equations for the loan rate and loan quantity reaction to the changes in monetary policy for the modified model. The last row of Tables 9 and 10 contains the reaction equations for the baseline model.[12].

5.1. No capital requirements

In cases where there are no capital requirements, the modified model shows the same reaction as the baseline model. The only difference is that equations for the modified model explain the behavior related to particular loan class.

In the modified model, contractionary monetary policy is implemented through an increase in the risk-free rate r. The modified model shows that in response to the contractionary monetary policy, the loan rate for each asset class (r_{Li}) will rise, and the loan quantity (L_i) will decline.

5.1.1. Loan rate

The equilibrium loan rate for each loan class r_{Li}, in both the short and long run, positively relates to changes in the interest rate on the non-bank debt for this loan class r_{Bi}. Furthermore, the interest rate on government securities r_G has the same impact on the equilibrium loan rate. Both r_{Bi} and r_G could change because of the monetary authority's action affecting risk-free rate r. Changes in the appropriate risk premiums, on the non-market debt σ and on government securi-

[12] The reaction equations for both loan rate and loan demand under "No capital standards" cases in the baseline model coincide with one in the modified model.

ties ψ, also affect both r_{Bi} and r_G. This means that both the contractionary monetary policy (see Table 9 in Appendix 1) and an increase in the risk premiums that occur outside the banking sector, specifically, in the real and financial sectors, tend to increase the equilibrium loan rate r_{Li} in both the short and long run.

In addition to the short run, the long-run decision involves equity adjustments. A bank has to earn enough profits to cover the required rate of return on equity. Two components of the risk premium on a bank's equity now affect the equilibrium loan rate r_{Li}. The default risk contribution margin of the loan class, $\beta_{1,i}$, has a positive effect. The interest rate and term structure risk contribution margin, β_2, has a negative effect. The default risk arises from the loan portfolio. The interest rate and term structure risk arises from the government securities holdings. The higher the spread between these margins, the higher the contribution of the default risk relative to the interest rate and term structure risk to a bank's equity risk premium is. As a result, a higher equilibrium loan rate will be required to deal with an increased required return on the bank's equity.

5.1.2. Loan quantity

The equilibrium loan quantity for loan class L_i positively reacts to changes in the non-bank debt rate r_{Bi}. Higher r_{Bi} translates into higher amount of loans L_i. This results from the fact that non-bank debt is a substitute for loans, and borrowers make their decisions on where to borrow taking into account rates in both markets. The same reaction is observed for a non-bank debt risk premium σ_i. The higher the risk premium on a non-bank debt for the appropriate loan class, the higher the equilibrium loan amount of this class will be.

The equilibrium loan quantities L_i are negatively affected by a change in the rate on government securities r_G and the risk premium on government securities ψ. The higher the rate on government securities and the risk premium, the more attractive, relative to loans, government securities holdings will be, and the lower the L_i will be.

Another factor that negatively affects equilibrium loan quantity L_i in both the short and long run, is a marginal cost of adjusting loans b_L. The higher the adjustment cost b_L, the more expensive it is for a bank to adjust loan quantities. As a result, L_i will be lower.

In addition, in the long run, two components of the risk premium on a bank's equity affect the equilibrium loan amount L_i. The default risk contribution margin $\beta_{1,i}$ has a negative effect. The interest rate and term structure risk contribution margin β_2 has a positive effect. The higher the spread between these margins, the higher the contribution of the default risk relative to the interest rate and term structure risk to the bank's equity risk premium. As a result, the equilibrium loan amount of each asset class will be lower.

The response of the equilibrium loan quantity of each loan class L_i to changes in monetary policy is negative (see Table 10 in Appendix 1). The contractionary monetary policy leads to a decrease in the loan quantity for each loan class.

The fact that this discussion is almost identical to the "No capital requirements" case in the baseline model is obvious. When I replaced one single loan component in the baseline model with three loan classes in the modified model, I just increased the dimensions of the solution. The most important modification to the baseline model, changing the capital requirements equation, is not made in the "No capital requirements" case.

5.2. Risk-based capital requirements

5.2.1. Short-run solution

Now, under the risk-based capital requirements, we can see the difference between the baseline and the modified models. In the baseline model in the short run, the level of the bank's lending is constrained by the book value of equity: $L = \dfrac{\overline{E}}{\theta_1}$. As a result, a contractionary monetary policy action will increase the equilibrium loan rate, but will not affect the constrained volume of the bank's lending.

In the modified model in the short-run, the risk-weighted loan portfolio is constrained by the book value of equity: $w_1L_1 + w_2L_2 + w_3L_3 = \dfrac{\overline{E}}{\theta_1}$. As a response to a contractionary monetary policy, the size of the *risk-weighted* loan portfolio will not be affected. However, the behavior of the equilibrium loan quantity of each individual loan class L_i and equilibrium loan rate r_{Li} will now be different (see Tables 9 and 10 in Appendix 1).

It is necessary to mention, that the modified model can be viewed as more general than the baseline model. The baseline model is now just a case where the risk-weighted loan portfolio is equal to the total loan portfolio. This generalization helps us to better explain a bank's behavior with the real-life complexity of a bank's loan portfolio.

Loan quantity

The response of the equilibrium loan quantity of the first loan class L_1[13] to the changes in monetary policy is no longer zero. As was mentioned before, the risk-weighted loan portfolio is not affected by the change in risk-free rate r. The response is a function of the parameters of the loan demand function l_1 and l_2, the marginal cost of loan adjustment b_L, and the risk weights of all loan classes w_1, w_2, and w_3. This response has the same structure as under the "no capital requirements" case: the contractionary monetary policy leads to a contraction in the loan quantity. However, the response is attenuated and, for the riskiest loan class, modified by the risk weights.

Indeed, the expression

$$\frac{w_1^2 + w_2^2 - w_3(w_1 + w_2)}{w_1^2 + w_2^2 + w_3^2} \tag{17}$$

from the reaction of the equilibrium loan quantity of the third riskiest class L_3, becomes negative under the initial assumption (section 3.5) about the level of risk of the loan class and its corresponding risk-weight in the regulatory capital: $w_1 < w_2 < w_3$ (see proof in Appendix 3). Also, equation (17) and its symmetric equivalents for other loan classes, under assumed values for risk weights (see Table 12 in Appendix 1), has an absolute value of less than unity (see Table 13 in Appendix 1 with simulated response equations). These values are 0.58, 0.02, and 0.22[14] for the first, second, and third loan classes, respectively.

In section 6.1, I applied assumed risk weights, consistent with the initial assumptions about level of risk of each loan class, to reaction equations. As a result, the sign of the loan quantity reaction for riskiest loan class (L_3) is negative. In other words, contractionary monetary policy will lead to an increase in the equilibrium value of the loan class with the highest risk weight! At the same time, the equilibrium loan quantities of other loan classes will decrease, and aggregate loan portfolio will decrease. However, the risk-weighted size of the loan portfolio will remain unchanged.

This type of behavior seems strange. However, it comes from the optimization of the loan portfolio: an increase in the risk-free rate makes holdings of government securities more. Moreover, as soon as the bank's equity is fixed in the short run and constrains the size of the risk-weighted loan portfolio, the bank decides to decrease the loan portfolio and invest into government securities such that the risk-weighted loan portfolio will not be affected. This is possible, if the bank increases the loan class that has the highest risk weight at the expense of the low risk-weighted classes. The highest risk weight class is also expected to have the highest loan rate, and the bank's decision to increase the loan class with the highest return can be seen as an optimization of the loan portfolio.

This behavior is also consistent with the empirical findings presented by Denhaan, Sumner, and Yamashiro (2004). Using the aggregate balance sheet data of US banks published by Federal Reserve (H8 statistical release) from January 1963 to February 2003, they found that after monetary tightening not all loan classes exhibit the same behavior. Following the contractionary monetary

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[13] The equilibrium loan quantities for other loan classes bear the same structure and this discussion is appropriate for them.

[14] These numbers represent the share of the impact of particular loan class in the total loan portfolio. The sum of these numbers (with their respective signs) represents the reaction of the aggregate loan portfolio.

policy, banks prefer to invest into loan classes that earn a higher rate of return at the expense of the low-return loan classes.

Loan rate

Analogously to the discussion about equilibrium loan quantities, the existence of three different loan classes changes the reaction equation of the equilibrium loan rate. In the baseline model, the reaction of the equilibrium loan rate is a function of the parameters of the loan demand function l_1 and l_2. In the modified model, it is additionally a function of the marginal cost of loan adjustment b_L, and the risk weights of all loan classes w_1, w_2, and w_3. As with loan quantities, the impact of the risk weights attenuates or intensifies this reaction, depending on the risk weight of the loan class.

In the baseline model under the contractionary monetary policy, the equilibrium loan rate for loans will undoubtedly increase. However, in the modified model for the riskiest loan class, this increase will be less than in the baseline model, and for the less risky loan classes this increase will be larger than in the baseline model. This occurs due to the sign of equation (17) which influences the deviation of the response. This reaction is consistent with the behavior of the equilibrium loan quantity that increases for the riskiest loan class and decreases for less risky loan classes.

5.2.2. Long-run solution

In the long run, a bank can optimally manage the equity capital, which allows them to achieve the optimal long-run scale of operations and asset composition.

Loan quantity

The long-run equation for the equilibrium loan quantity reaction of the loan class in the modified model has, basically, the same structure as in the baseline model. It is a function of the parameters of the loan demand function l_1 and l_2, the marginal cost of the loan and equity adjustment b_L

and b_E respectively, and the risk weights of all loan classes w_1, w_2, and w_3. These risk weights either attenuate or intensify the reaction, depending on the risk level of the loan class.

Following the monetary contraction, the loan class with the highest risk weight will contract less than less risky loan classes (see section 6.1 with simulated equations).

It is necessary to mention, that the simulation undertaken in section 6.2 shows that under the set of reasonable parameter values, the long-run response of the equilibrium loan quantity to a changes in monetary policy could not be positive, i.e. the loan quantity cannot increase after monetary tightening.

Loan rate

Analogously to the discussion about equilibrium loan quantities in the long run, the existence of three different loan classes changes the equation of the reaction of the equilibrium loan rate to changes in monetary policy. In the modified model, it is a function of the parameters of the loan demand function l_1 and l_2, the marginal cost of loan and equity adjustment b_L and b_E, respectively. Additionally, it is a function of the risk weights of all loan classes w_1, w_2, and w_3, which either attenuate or intensify, depending on the level of risk of the loan class. The equilibrium loan rate for the loan class with the highest risk weight will increase less than the average or below-average risk loan class following the monetary tightening (see section 6.1 with simulated equations).

5.3. The long-run influence of risk-based capital requirements ratio on the transmission of monetary policy

The following equations allow estimation of the impact of the risk-based capital requirements on the monetary transmission mechanism in the long run. It shows how an increase in the risk-based capital ratio will affect the response of the equilibrium loan rate (equation (18)) and equilibrium loan quantity (equation (19)) to a change in monetary policy.

$$\frac{\partial\left(\frac{\partial r_{L_i}}{\partial r}\right)}{\partial \theta_1} = -\frac{2b_E w_i (w_1 + w_2 + w_3) l_1 \theta_1}{[1 + (l_1 + l_2)(b_L + b_E (w_1^2 + w_2^2 + w_3^2)\theta_1^2)]^2} \tag{18}$$

$$\frac{\partial\left(\frac{\partial L_i}{\partial r}\right)}{\partial \theta_1} = \frac{2b_E w_i (w_1 + w_2 + w_3) l_1 (l_1 + l_2)\theta_1}{[1 + (l_1 + l_2)(b_L + b_E (w_1^2 + w_2^2 + w_3^2)\theta_1^2)]^2} \tag{19}$$

The signs of equations (18) and (19) are opposite to the signs of the corresponding response equations (see Tables 9 and 10 in Appendix 1). Thus, an increase in the risk-based capital ratio reduces the impact of the contractionary monetary policy on the loan rate and equilibrium loan quantities. Moreover, the degree of this reduction is not uniform. The higher the risk weight of the loan class, the greater the reduction of the impact will be. This is true for both the equilibrium loan rate and the equilibrium loan quantity of the loan class.

5.4. The impact of the risk-based capital requirements on a bank's capital and its response to changes in monetary policy

Imposition of risk-based capital requirements dramatically changes the response of a bank's equity to changes in monetary policy in the long run (see Table 11 in Appendix 1).

A bank's equity is not affected by the changes in monetary policy without capital standards. A bank's equity is a function of the risk premium on government securities ψ, the marginal cost of equity adjustment b_E, the base level of market risk premium for bank's equity β_0, and the parameter of the interest rate and term structure risk contribution margin β_2.

With risk-based capital standards, a bank's equity response to changes in monetary policy becomes negative. Under the contractionary monetary policy, a bank's equity will decrease. The reaction of a bank's equity is now a function of the parameters of the public loan demand l_1 and l_2, the capital requirements ratio θ_1, the marginal costs of loan and equity adjustment b_L and b_E respectively, and the risk weights of all loan classes w_1, w_2, and w_3.

6. Model Simulation

The equations of the reaction of the equilibrium loan quantities and the loan rates contain many parameters. Many of them could be simulated in order to refine the equations. This allows making better judgments about the magnitudes and the signs of the expressions discussed before.

Following one of my initial assumptions about different level of risk for loan classes in section 3.1, the risk weights are obtained from the results of the Quantitative Impact Study #3 in Basel Committee on Banking Supervision (2003) (see Table 12 in Appendix 1). This study reports quality distribution for the various components of 365 banks' portfolios. Banks in this study were split into two groups depending on their size. I use the data for the quality of corporate exposures, which are one of the largest segments of the loan portfolio. For both groups of banks, I calculated the average risk weight, and the risk weight for below- and above-average exposures. Then, I averaged results across both groups. The resulting risk-weights are 0.3451, 0.8200, and 1.0203 for below average, average, and above average risk class respectively. The risk weighted asset capital requirement ratio (8% or 0.08) is also assigned according to present regulatory standard in Basel Committee on Banking Supervision (2004).

In order to perform simulation for the modified model, I will follow the methodology outlined and used by Kopecky and VanHoose (2004B) for quite a similar model. Based on results obtained from an empirical data by Elyasiani, Kopecky, VanHoose (1995), Kopecky and VanHoose use either fixed values or the range of values for the marginal costs of adjustment for loans, equity, and deposits. It is necessary to stress, that Elyasiani, Kopecky, VanHoose (1995) derived econometric estimates for the deposit adjustment costs only (0.10). Based on this range, Kopecky and VanHoose (2004B) adopted a range for equity (0.001 and 0.04) and loan (from 0.001 to 0.8) adjustment costs, which I will use in my simulation.

Values used for simulation are presented in Table 13 in Appendix 1. This table contains three groups of parameter. The first group contains the risk-weights for loan classes, the second contains the capital requirement ratio, and the third contains the adjustment costs for loans, equity, and deposits.

6.1. The response of the equilibrium loan rate, loan quantity and aggregate loan portfolio to the changes in monetary policy with risk-based capital requirements

In section 5.2, I stated that response equations for loans and loan rates are attenuated by risk-weights. In addition, for the riskiest loan class, risk weights modify the sign of the reaction equation. To support my statement about the impact of the risk weights on the magnitudes and signs of the response equations, using the risk weights for loan classes from Table 13 in Appendix 1, I simulate the following equations for the long-run reaction of the equilibrium loan rate and quantities for three loan classes to changes in monetary policy:

$$\frac{\partial r_{L_1}^{SR}}{\partial r} = \frac{l_2}{l_1 + l_2} + \frac{l_1}{(l_1 + l_2)(1 + b_L(l_1 + l_2))} * \frac{w_2^2 + w_3^2 - w_1(w_2 + w_3)}{w_1^2 + w_2^2 + w_3^2} \tag{20}$$

$$\frac{\partial r_{L_1}^{LR}}{\partial r} = \frac{1 + l_2(b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2)}{1 + (l_1 + l_2)(b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2)} + \frac{l_1 b_E \theta_1^2 (w_2^2 + w_3^2 - w_1(w_2 + w_3))}{[1 + b_L(l_1 + l_2)][1 + (l_1 + l_2)(b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2)]} \tag{21}$$

$$\frac{\partial L_1^{SR}}{\partial r} = -\frac{l_1}{1 + b_L(l_1 + l_2)} \frac{w_2^2 + w_3^2 - w_1(w_2 + w_3)}{w_1^2 + w_2^2 + w_3^2} \tag{22}$$

$$\frac{\partial L_1^{LR}}{\partial r} = -\frac{l_1}{1 + (l_1 + l_2)[b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2]} * \frac{1 + (l_1 + l_2)[b_L + b_E(w_2^2 + w_3^2 - w_1(w_2 + w_3))\theta_1^2]}{1 + b_L(l_1 + l_2)} \tag{23}$$

These results are summarized in Table 14 in Appendix 1. It is clearly visible now how risk-weights attenuate or change the direction of the impact for a particular loan class. For instance, in the short run, the equilibrium loan quantity for the third and most risky class increases following the contractionary monetary policy. Simultaneously, the other loan classes, which have less risk, contract. However, this contraction is weakened now. For the equilibrium loan rates and the long-run equilibrium loan quantities, I detect that the reaction for the riskiest class will have a smaller magnitude.

These simulation results illustrate and support previous discussion about the magnitudes and the signs of the responses to changes in monetary policy. The reaction of the riskiest loan class rate and the long-run loan quantity to changes in monetary policy will be attenuated. Short-run equilibrium loan quantity will have an opposite reaction to that of the less risky loan classes.

How important are these findings? The contractionary monetary policy will be effective in the contraction of the loan classes with least risk. However, the response is now weakened. Simultaneously, the contractionary monetary policy will be ineffective in the reduction of the riskiest loan class. This clearly demonstrates the conflict between the regulator and the monetary authority.

6.2. Is it possible that in the long run, the high-risk loan class will expand following the contractionary monetary policy?

The structure of equation (23) simulated in the previous section for the high-risk loan class response to changes in monetary policy, allows the possibility that this equation could change its sign to the opposite. This will mean that in the long run, the high-risk loan class will expand following the contractionary monetary policy. We have already seen such behavior in the short-run solution. As soon as I do not have any estimates of the loan demand function in its specific form (equation (6)), and to check the possibility of such an outcome, I undertake the simulation of the component of equation (23) for the riskiest loan class (L_3)

$$1+(l_1+l_2)[b_L-0.40b_E\theta_1^2]<0 \qquad (24)$$

with regard to parameters of the loan demand function l_1 and l_2. In other words, given possible values of the marginal cost of adjustment for loans and equity b_L and b_E respectively, the risk-based capital ratio θ_1, I simulate the sum of parameters of the loan demand function l_1+l_2 defined with the following equation:

$$l_1 + l_2 < -\frac{1}{b_L - 0.4 b_E \theta_1^2} \tag{25}$$

Table 15 in Appendix 1 contains the values of l_1+l_2 that change the sign of the equation (24) – the parameters of the loan demand function that will make the high-risk loan class expand following the contractionary monetary policy.

Elyasiani, Kopecky, and VanHoose (1995) estimated parameters of the loan demand functions with different spreads as independent variables[15]. They produced values for four groups of banks depending on their size. These values ranged from -5.315 to 102.80[16]. My simulation produces values of the parameters for the loan demand function that are either inconsistent with the initial assumptions of the model (negative values) or have incorrect magnitudes (954.198 or above).

The results of this simulation suggest, that under the assumed range of parameter values, we should rule out the possibility that in the long run high-risk loan class will expand, following the contractionary monetary policy, as we saw in the short-run.

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15 It is necessary to mention, that I do not found the estimates of the parameters of the loan demand function in exactly the same form (equation (6)) as I use. In addition, the estimates did not distinguish difference in the loan level of risk. I consider this fact as a weak point in the section 6.2 of the simulation.

16 Elyasiani, Kopecky, VanHoose (1995), page 13.

7. Conclusion

This paper has examined the effect of risk-based capital requirements on the monetary transmission mechanism – how monetary policy action, which changes the risk-free rate, affects a bank's decision to supply loans. I introduced three different loan classes that have different levels of risk and, hence, different risk weights in the determination of the regulatory capital.

In the modified framework, I confirmed previous findings that the imposition of risk-based capital requirements changes the short and the long-term equilibrium outcomes. I also found that the introduction of heterogeneous loans instead of a single loan class reveals additional aspects of a bank's response to changes in monetary policy. In this case, loan classes and loan rates for the different loan classes respond differently. The higher the risk of the loan class, the less responsive to the changes in monetary policy it will be. In the short-run, it is also possible that the riskiest loan class will expand following the contractionary monetary policy.

I also found that an increase in the risk-based capital requirements ratio attenuates the effect of monetary policy, and again, this effect is different for the different loan classes. The higher the risk-weight of the loan class, the greater the reduction of the monetary policy impact will be. This is true for both the equilibrium loan rate and the equilibrium loan quantity of a loan class.

My analysis points out that capital regulation, aiming to increase the soundness of the banking system, contends with stabilization policy, which aims to influence interest rates and loans through bank lending. Capital regulation introduces an asymmetrical response in the bank's loan portfolio with some undesirable effects, which affects the level of risk for the bank.

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Appendix 1 – Table 1

Risk weights for banks' assets (Basel I)

Category of assets	Risk weight
1. Cash and claims on Organization for Economic Cooperation and Development (OECD) central banks and national governments	0%
2. Claims on private sector OECD banks, OECD subnational government entities, and cash items in the process of collection	20%
3. First mortgages on 1-4 family real estate, and local government project finance in OECD countries	50%
4. Commercial and consumer loans, and loans to non-OECD governments	100%

Appendix 1 – Table 2

Risk weights for banks' assets (Basel II)

Corporate

Credit Assessment	AAA to AA-	A+ to A-	BBB+ to BB-	Below BB-	Unrated
Risk Weight	20%	50%	100%	150%	100%

Banks

Credit assessment of Sovereign	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to B-	Below B-	Unrated
Risk weight under Option 1[15]	20%	50%	100%	100%	150%	100%
Risk weight under Option 2[16]	20%	50%	50%	100%	150%	50%
Risk weight for short-term claims under Option 2	20%	20%	20%	50%	150%	20%

Retail

Type of Claim	Risk Weight
Claims included in the regulatory retail portfolio	75%
Claims secured by residential property	35%
Claims secured by commercial real estate	100%

Sovereigns

Credit Assessment	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to B-	Below B-	Unrated
Risk Weight	0%	20%	50%	100%	150%	100%

High Risk Categories

Type of Claim	Risk Weight
Claims on sovereigns, PSEs, banks, and securities firms rated below B-	150%
Claims on corporates rated below BB-	150%
Past due loans	100% - 150%
Securitization tranches that are rated between BB+ and BB-	350%

[15] Under the first option, all banks incorporated in a given country will be assigned a risk weight one category less favorable than that assigned to claims on the sovereign of that country. However, for claims on banks in countries with sovereigns rated BB+ to B- and on banks in unrated countries the risk weight will be capped at 100%.

[16] The second option bases the risk weighting on the external credit assessment of the bank itself with claims on unrated banks being risk-weighted at 50%. Under this option, a preferential risk weight that is one category more favorable may be applied to claims with an original maturity of three months or less, subject to a floor of 20%. This treatment will be available to both rated and unrated banks, but not to banks risk weighted at 150%.

Appendix 1 – Table 3

Solutions of the bank's profit maximization problem for loans L_1

Capital requirements	Short-run solution	Long-run solution
No capital standards	$L_1 = \dfrac{r_{L_1} - r_G}{b_L}$	$L_1 = \dfrac{(r_{L_1} - \beta_{1,1}) - (r_G - \beta_2)}{b_L}$
Risk-based capital requirements	$L_1 = \dfrac{\bar{E}w_1/\theta_1}{w_1^2 + w_2^2 + w_3^2} + \dfrac{(r_{L_1} - r_G)(w_2^2 + w_3^2)}{b_L(w_1^2 + w_2^2 + w_3^2)\theta_1} - \dfrac{(r_{L_2} - r_G)w_1 w_2}{b_L(w_1^2 + w_2^2 + w_3^2)\theta_1} - \dfrac{(r_{L_3} - r_G)w_1 w_3}{b_L(w_1^2 + w_2^2 + w_3^2)\theta_1}$	$L_1 = \dfrac{w_1\theta_1[(r_G - \beta_2) - (r + \beta_0)]}{b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2} + \dfrac{[b_L + b_E\theta_1^2(w_2^2 + w_3^2)][(r_{L_1} - \beta_{1,1}) - (r_G - \beta_2)]}{b_L(b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2)} - \dfrac{b_E\theta_1^2 w_1 w_2[(r_{L_2} - \beta_{1,2}) - (r_G - \beta_2)]}{b_L(b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2)} - \dfrac{b_E\theta_1^2 w_1 w_3[(r_{L_3} - \beta_{1,3}) - (r_G - \beta_2)]}{b_L(b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2)}$

Appendix 1 – Table 4

Solutions of the bank's profit maximization problem for deposits D

Capital requirements	Short-run solution	Long-run solution
No capital standards	$D=\frac{r_G(1-\rho)-r_D}{b_D}$	$D=\frac{(r_G-\beta_2)(1-\rho)-r_D}{b_D}$
Risk-based capital requirements	$D=\frac{r_G(1-\rho)-r_D}{b_D}$	$D=\frac{(r_G-\beta_2)(1-\rho)-r_D}{b_D}$

Appendix 1 – Table 5

Solutions of the bank's profit maximization problem for equity E

Capital requirements	Short-run solution	Long-run solution
No capital standards	$\overline{E}$	$E = \dfrac{(r_G - \beta_2) - (r + \beta_0)}{b_E}$
Risk-based[17] capital requirements	$\overline{E}$	$E = \dfrac{(w_1 + w_2 + w_3)\theta_1[(r_L - \beta_1) - (r_G - \beta_2)]}{b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2} + \dfrac{(w_1^2 + w_2^2 + w_3^2)\theta_1^2[(r_G - \beta_2) - (r + \beta_0)]}{b_L + b_E(w_1^2 + w_2^2 + w_3^2)\theta_1^2}$

[17] Expression for the profit-maximizing value of a bank's equity contains terms r_L and β_1 which are weighted (by the risk weight w_i) averages of the rates of return for the each class of loans r_{Li}, and the contribution of the each loan class to default risk $\beta_{1,i}$.

Appendix 1 – Table 6

Solutions of the bank's profit maximization problem for government securities holdings G

Capital requirements	Short-run solution	Long-run solution
No capital standards	$G=\bar{E}-$ $-\frac{(r_{L_1}-r_G)}{b_L}-\frac{(r_{L_2}-r_G)}{b_L}-\frac{(r_{L_3}-r_G)}{b_L}+$ $\frac{(r_G(1-\rho)-r_D)(1-\rho)}{b_D}$	$G=\frac{r_G-r-(\beta_0+\beta_2)}{b_E}-$ $-\frac{(r_{L_1}-\beta_{1,1})-(r_G-\beta_2)}{b_L}-$ $-\frac{(r_{L_2}-\beta_{1,2})-(r_G-\beta_2)}{b_L}-$ $-\frac{(r_{L_3}-\beta_{1,3})-(r_G-\beta_2)}{b_L}+$ $+\frac{[(r_G-\beta_2)(1-\rho)-r_D](1-\rho)}{b_D}$
Risk-based capital requirements	$G=\bar{E}+$ $\frac{-b_L\bar{E}w_1/\theta_1-(r_{L_1}-r_G)(w_2^2+w_3^2-w_1(w_2+w_3))}{b_L(w_1^2+w_2^2+w_3^2)}+$ $\frac{-b_L\bar{E}w_2/\theta_1-(r_{L_2}-r_G)(w_1^2+w_3^2-w_2(w_1+w_3))}{b_L(w_1^2+w_2^2+w_3^2)}+$ $\frac{-b_L\bar{E}w_3/\theta_1-(r_{L_3}-r_G)(w_1^2+w_2^2-w_3(w_1+w_2))}{b_L(w_1^2+w_2^2+w_3^2)}+$ $\frac{(r_G(1-\rho)-r_D)(1-\rho)}{b_D}$	The expression for G in this case is quite massive, but from balance sheet constraint (1): $G=E-L_1-L_2-L_3+D(1-\rho)$ Using the appropriate values for balance sheet components, the expression for government securities holdings derived (see Appendix 2 with detailed solutions of the modified model).

Appendix 1 – Table 7

Model solution: Equilibrium loan rate r_{L_1}

Capital requirements	Short-run solution	Long-run solution
No capital standards	$\dfrac{r+\psi+b_L l_0+b_L l_2(r+\sigma_1)}{1+b_L(l_1+l_2)}$	$\dfrac{r+\psi+b_L l_0+b_L l_2(r+\sigma_1)+\beta_{1,1}-\beta_2}{1+b_L(l_1+l_2)}$
Risk-based capital requirements [18]	$-\dfrac{\overline{E}w_1}{\theta_1}\dfrac{1}{(w_1^2+w_2^2+w_3^2)(l_1+l_2)}+$ $\dfrac{w_1}{w_1^2+w_2^2+w_3^2}\dfrac{l_0(w_1+w_2+w_3)+l_2\overline{\sigma}}{(l_1+l_2)(1+b_L(l_1+l_2))}+$ $\dfrac{l_0 b_L}{1+b_L(l_1+l_2)}+\dfrac{l_2}{l_1+l_2}\dfrac{b_L(r+\sigma_1)(1+l_2)+r}{1+b_L(l_1+l_2)}+$ $\dfrac{w_2^2+w_3^2-w_1(w_2+w_3)}{w_1^2+w_2^2+w_3^2}*$ $\left(\dfrac{l_1 r}{(l_1+l_2)(1+b_L(l_1+l_2))}+\dfrac{\psi}{1+b_L(l_1+l_2)}\right)$	The expression for r_{L_1} in this case is quite massive. In the analysis of this expression it is possible, to define the similar structure as in other cases (see Appendix 2 with detailed solutions of the modified model).

[18] Expression for equilibrium loan rate and loan quantity contains term $\overline{\sigma}$ which is weighted (by the risk weight w_i) averages of the risk premium for non-bank debt σ_i for each loan class.

Appendix 1 – Table 8

Model solution: Equilibrium loan quantity L_1

Capital requirements	Short-run solution	Long-run solution
No capital standards	$\frac{l_0 + l_2(\sigma_1 - \psi) - l_1(r+\psi)}{1 + b_L(l_1 + l_2)}$	$\frac{l_0 + l_2(\sigma_1 - \psi + \beta_2 - \beta_{1,1}) - l_1(r + \psi - \beta_2 + \beta_{1,1})}{1 + b_L(l_1 + l_2)}$
Risk-based capital requirements	$\frac{\overline{E}w_1}{\theta_1}\frac{1}{w_1^2 + w_2^2 + w_3^2} + \frac{w_2^2 + w_3^2 - w_1(w_2 + w_3)}{w_1^2 + w_2^2 + w_3^2} * \frac{l_0 - l_1(r+\psi) - l_2\psi}{1 + b_L(l_1 + l_2)} + \frac{l_2\sigma_1}{1 + b_L(l_1 + l_2)} - \frac{w_1}{w_1^2 + w_2^2 + w_3^2}\frac{l_2\overline{\sigma}}{1 + b_L(l_1 + l_2)}$	The expression for L_1 in this case is quite massive. In the analysis of this expression, it is possible to define the similar structure as in other cases (see Appendix 2 with detailed solutions of the modified model).

Appendix 1 – Table 9

Response of the equilibrium loan rate r_{L_1} to changes in monetary policy ($\frac{\partial r_{L_1}}{\partial r}$)

Capital requirements	Short-run solution	Long-run solution
No capital standards	$\frac{1+b_L l_2}{1+b_L(l_1+l_2)}$	$\frac{1+b_L l_2}{1+b_L(l_1+l_2)}$
Risk-based capital requirements	$\frac{l_2}{l_1+l_2}+$ $\frac{l_1}{(l_1+l_2)(1+b_L(l_1+l_2))}*$ $\frac{w_2^2+w_3^2-w_1(w_2+w_3)}{w_1^2+w_2^2+w_3^2}$	$\frac{1+l_2(b_L+b_E(w_1^2+w_2^2+w_3^2)\theta_1^2)}{1+(l_1+l_2)(b_L+b_E(w_1^2+w_2^2+w_3^2)\theta_1^2)}+$ $\frac{l_1 b_E\theta_1^2(w_2^2+w_3^2-w_1(w_2+w_3))}{[1+b_L(l_1+l_2)][1+(l_1+l_2)(b_L+b_E(w_1^2+w_2^2+w_3^2)\theta_1^2)]}$
Risk-based capital requirements (baseline model)	$\frac{l_2}{l_1+l_2}$	$\frac{1+l_2(b_L+b_E\theta_1^2)}{1+(l_1+l_2)(b_L+b_E\theta_1^2)}$

Appendix 1 – Table 10

Response of the equilibrium loan quantity L_1 to changes in monetary policy ($\frac{\partial L_1}{\partial r}$)

Capital requirements	Short-run solution	Long-run solution
No capital standards	$-\frac{l_1}{1+b_L(l_1+l_2)}$	$-\frac{l_1}{1+b_L(l_1+l_2)}$
Risk-based capital requirements	$-\frac{l_1}{1+b_L(l_1+l_2)} * \frac{w_2^2+w_3^2-w_1(w_2+w_3)}{w_1^2+w_2^2+w_3^2}$	$-\frac{l_1}{1+(l_1+l_2)[b_L+b_E(w_1^2+w_2^2+w_3^2)\theta_1^2]} * \frac{1+(l_1+l_2)[b_L+b_E(w_2^2+w_3^2-w_1(w_2+w_3))\theta_1^2]}{1+b_L(l_1+l_2)}$
Risk-based capital requirements (baseline model)	0	$-\frac{l_1}{1+(l_1+l_2)(b_L+b_E\theta_1^2)}$

Appendix 1 – Table 11

Bank's equity response to changes in monetary policy ($\frac{\partial E}{\partial r}$) in the long run

Capital requirements	$\frac{\partial E}{\partial r}$	Sign of the response
No capital standards	$0 \left(E = \frac{\psi - \beta_0 - \beta_2}{b_E} \right)$	
Risk-based capital requirements	$-\frac{(w_1 + w_2 + w_3) l_1 \theta_1}{1 + (l_1 + l_2)(b_L + b_E (w_1^2 + w_2^2 + w_3^2) \theta_1^2)}$	Negative
Risk-based capital requirements (baseline model)	$-\frac{l_1 \theta_1}{1 + (l_1 + l_2)(b_L + b_E \theta_1^2)}$	Negative

Appendix 1 – Table 12

Risk-weights for loan classes

Rating[19]	AAA to AA-	A+ to A-	BBB+ to BB-	Below BB-	Unrated	Past due
Risk-weight	20%	50%	100%	150%	100%	150%
Group 1[20]	11%	9%	15%	2%	62%	2%
Group 2	14%	15%	15%	1%	46%	1%

	Below-average risk-weight	Average risk-weight	Above average risk-weight
Group 1	33.50%	89.70%	102.47%
Group 2	35.52%	74.30%	101.59%
Average for both groups	**34.51%**	**82.00%**	**102.03%**

[19] This part of table contains quality distribution for corporate exposures (Basel Committee on Banking Supervision (2003), page 18).

[20] Group 1 consists of large, diversified, and internationally active banks with a Tier 1 capital in excess of €3bln. Group 2 consists of smaller, often specialized, banks.

Appendix 1 – Table 13

Values of parameters used for simulation

Name of Parameter	Symbol	Value or Range of Values
Risk weight for first loan class	w_1	0.3451[21] (below average risk)
Risk weight for second loan class	w_2	0.8200 (average risk)
Risk weight for third loan class	w_3	1.0203 (above average risk)
Risk-weighted asset capital requirement ratio	θ_1	0.08
Marginal cost of loan adjustment	b_L	0.001, 0.04
Marginal cost of equity adjustment	b_E	0.001, 0.01, 0.02, 0.04, 0.08, 0.10, 0.40, 0.80
Marginal cost of deposit adjustment	b_D	0…0.10

[21] In the simulation I used decimal representation of percentage parameters

Appendix 1 – Table 14

Simulated equations of the response to changes in monetary policy

Equation		L_1 (low risk loan class)	L_2 (average risk loan class)	L_3 (high risk loan class)
20	$\frac{\partial r_{L_i}^{SR}}{\partial r}$	$\frac{l_2(1+b_L(l_1+l_2))+0.59l_1}{(l_1+l_2)(1+b_L(l_1+l_2))}$	$\frac{l_2(1+b_L(l_1+l_2))+0.02l_1}{(l_1+l_2)(1+b_L(l_1+l_2))}$	$\frac{l_2(1+b_L(l_1+l_2))-0.22l_1}{(l_1+l_2)(1+b_L(l_1+l_2))}$
21	$\frac{\partial r_{L_i}^{LR}}{\partial r}$	$\frac{1+l_2(b_L+1.83b_E\theta_1^2)}{1+(l_1+l_2)(b_L+1.83b_E\theta_1^2)}+$ $\frac{1.08l_1b_E\theta_1^2}{1+b_L(l_1+l_2)}*$ $\frac{1}{1+(l_1+l_2)(b_L+1.83b_E\theta_1^2)}$	$\frac{1+l_2(b_L+1.83b_E\theta_1^2)}{1+(l_1+l_2)(b_L+1.83b_E\theta_1^2)}+$ $\frac{0.04l_1b_E\theta_1^2}{[1+b_L(l_1+l_2)]}*$ $\frac{1}{1+(l_1+l_2)(b_L+1.83b_E\theta_1^2)}$	$\frac{1+l_2(b_L+1.83b_E\theta_1^2)}{1+(l_1+l_2)(b_L+1.83b_E\theta_1^2)}-$ $\frac{0.40l_1b_E\theta_1^2}{[1+b_L(l_1+l_2)]}*$ $\frac{1}{1+(l_1+l_2)(b_L+1.83b_E\theta_1^2)}$
22	$\frac{\partial L_i^{SR}}{\partial r}$	$-\frac{0.59l_1}{1+b_L(l_1+l_2)}$	$-\frac{0.02l_1}{1+b_L(l_1+l_2)}$	$\frac{0.22l_1}{1+b_L(l_1+l_2)}$
23	$\frac{\partial L_i^{LR}}{\partial r}$	$-\frac{l_1}{1+(l_1+l_2)[b_L+1.83b_E\theta_1^2]}$ $*\frac{1+(l_1+l_2)[b_L+1.08b_E\theta_1^2]}{1+b_L(l_1+l_2)}$	$-\frac{l_1}{1+(l_1+l_2)[b_L+1.83b_E\theta_1^2]}$ $*\frac{1+(l_1+l_2)[b_L+0.04b_E\theta_1^2]}{1+b_L(l_1+l_2)}$	$-\frac{l_1}{1+(l_1+l_2)[b_L+1.83b_E\theta_1^2]}$ $*\frac{1+(l_1+l_2)[b_L-0.40b_E\theta_1^2]}{1+b_L(l_1+l_2)}$

Appendix 1 – Table 15

Simulation of equation (25)

b_L	b_E							
	0.001	0.01	0.02	0.04	0.08	0.10	0.40	0.80
0.001	-1002.57	-1026.27	-1053.96	-1114.08	-1257.55	-1344.09	41666.7	954.198
0.04	-25.0016	-25.016	-25.032	-25.0642	-25.1287	-25.161	-25.6568	-26.3491

Short-run solution with no capital requirements

In[1]:= $\mathcal{L} = \text{rL1 L1} + \text{rL2 L2} + \text{rL3 L3} + \text{rg}\,G - \text{rd}\,D - \frac{\text{bL}}{2}\,(\text{L1}^2 + \text{L2}^2 + \text{L3}^2) - \frac{\text{bd}}{2}\,D^2 - \lambda_1\,(\text{L1} + \text{L2} + \text{L3} + G - (1-\rho)\,D - \text{Ebar})$

Out[1]= $-\frac{\text{bd}\,D^2}{2} - \frac{1}{2}\,\text{bL}\,(\text{L1}^2 + \text{L2}^2 + \text{L3}^2) - \text{D rd} + \text{G rg} + \text{L1 rL1} + \text{L2 rL2} + \text{L3 rL3} - (-\text{Ebar} + \text{G} + \text{L1} + \text{L2} + \text{L3} - \text{D}\,(1-\rho))\,\lambda_1$

In[2]:= $\text{eq1} = \text{D}[\mathcal{L}, \text{L1}]$

Out[2]= $-\text{bL L1} + \text{rL1} - \lambda_1$

In[3]:= $\text{eq2} = \text{D}[\mathcal{L}, \text{L2}]$

Out[3]= $-\text{bL L2} + \text{rL2} - \lambda_1$

In[4]:= $\text{eq3} = \text{D}[\mathcal{L}, \text{L3}]$

Out[4]= $-\text{bL L3} + \text{rL3} - \lambda_1$

In[5]:= $\text{eq4} = \text{D}[\mathcal{L}, \text{D}]$

Out[5]= $-\text{bd D} - \text{rd} - (-1+\rho)\,\lambda_1$

In[6]:= $\text{eq5} = \text{D}[\mathcal{L}, \text{G}]$

Out[6]= $\text{rg} - \lambda_1$

In[7]:= $\text{eq6} = \text{D}[\mathcal{L}, \lambda_1]$

Out[7]= $\text{Ebar} - \text{G} - \text{L1} - \text{L2} - \text{L3} + \text{D}\,(1-\rho)$

In[8]:= $\text{sol} = \text{Solve}[\{\text{eq1} = 0, \text{eq2} = 0, \text{eq3} = 0, \text{eq4} = 0, \text{eq5} = 0, \text{eq6} = 0\}, \{\text{L1}, \text{L2}, \text{L3}, \text{G}, \text{D}, \lambda_1\}]\ //\ \text{Flatten}\ //\ \text{FullSimplify}$

Out[8]= $\left\{\text{G} \to \frac{\text{bd}\,(\text{bL Ebar} + 3\,\text{rg} - \text{rL1} - \text{rL2} - \text{rL3}) + \text{bL}\,(\text{rd} + \text{rg}\,(-1+\rho))\,(-1+\rho)}{\text{bd bL}},\ \text{L3} \to \frac{-\text{rg} + \text{rL3}}{\text{bL}},\ \text{L1} \to \frac{-\text{rg} + \text{rL1}}{\text{bL}},\ \text{L2} \to \frac{-\text{rg} + \text{rL2}}{\text{bL}},\ \text{D} \to -\frac{\text{rd} - \text{rg} + \text{rg}\,\rho}{\text{bd}},\ \lambda_1 \to \text{rg}\right\}$

In[9]:= $\text{G} = \text{G}\ /.\ \text{sol}$

Out[9]= $\frac{\text{bd}\,(\text{bL Ebar} + 3\,\text{rg} - \text{rL1} - \text{rL2} - \text{rL3}) + \text{bL}\,(\text{rd} + \text{rg}\,(-1+\rho))\,(-1+\rho)}{\text{bd bL}}$

```
In[10]:= L1 = L1 /. sol
         L2 = L2 /. sol
         L3 = L3 /. sol
```

Out[10]= $\frac{-rg + rL1}{bL}$

Out[11]= $\frac{-rg + rL2}{bL}$

Out[12]= $\frac{-rg + rL3}{bL}$

```
In[13]:= Deposits = D /. sol
```

Out[13]= $-\frac{rd - rg + rg\,\rho}{bd}$

```
In[14]:= LoanDemand1 = l0 - l1 rL1 - l2 (rL1 - rB1)
         LoanDemand2 = l0 - l1 rL2 - l2 (rL2 - rB2)
         LoanDemand3 = l0 - l1 rL3 - l2 (rL3 - rB3)
```

Out[14]= $l_0 - rL1\, l_1 - (-rB1 + rL1)\, l_2$

Out[15]= $l_0 - rL2\, l_1 - (-rB2 + rL2)\, l_2$

Out[16]= $l_0 - rL3\, l_1 - (-rB3 + rL3)\, l_2$

```
In[17]:= solL1 = Solve[L1 == LoanDemand1, rL1] // Flatten // FullSimplify
         solL2 = Solve[L2 == LoanDemand2, rL2] // Flatten // FullSimplify
         solL3 = Solve[L3 == LoanDemand3, rL3] // Flatten // FullSimplify
```

Out[17]= $\left\{rL1 \to \frac{rg + bL\, l_0 + bL\, rB1\, l_2}{1 + bL\,(l_1 + l_2)}\right\}$

Out[18]= $\left\{rL2 \to \frac{rg + bL\, l_0 + bL\, rB2\, l_2}{1 + bL\,(l_1 + l_2)}\right\}$

Out[19]= $\left\{rL3 \to \frac{rg + bL\, l_0 + bL\, rB3\, l_2}{1 + bL\,(l_1 + l_2)}\right\}$

```
In[20]:= LoanRate1 = rL1 /. solL1
         LoanRate2 = rL2 /. solL2
         LoanRate3 = rL3 /. solL3
```

Out[20]= $\frac{rg + bL\, l_0 + bL\, rB1\, l_2}{1 + bL\,(l_1 + l_2)}$

Out[21]= $\frac{rg + bL\, l_0 + bL\, rB2\, l_2}{1 + bL\,(l_1 + l_2)}$

Out[22]= $\frac{rg + bL\, l_0 + bL\, rB3\, l_2}{1 + bL\,(l_1 + l_2)}$

```
In[23]:= LoanRate1 = LoanRate1 /. {rg → r + ψ, rB1 → r + σ1}
         LoanRate2 = LoanRate2 /. {rg → r + ψ, rB2 → r + σ2}
         LoanRate3 = LoanRate3 /. {rg → r + ψ, rB3 → r + σ3}
```

Out[23]= $\frac{r + \psi + bL\, l_0 + bL\, l_2\, (r + \sigma_1)}{1 + bL\, (l_1 + l_2)}$

Out[24]= $\frac{r + \psi + bL\, l_0 + bL\, l_2\, (r + \sigma_2)}{1 + bL\, (l_1 + l_2)}$

Out[25]= $\frac{r + \psi + bL\, l_0 + bL\, l_2\, (r + \sigma_3)}{1 + bL\, (l_1 + l_2)}$

```
In[26]:= D[LoanRate1, bL] // FullSimplify
```

Out[26]= $\frac{l_0 - (r + \psi)\, l_1 + l_2\, (-\psi + \sigma_1)}{(1 + bL\, (l_1 + l_2))^2}$

```
In[27]:= D[LoanRate1, r]
         D[LoanRate2, r]
         D[LoanRate3, r]
```

Out[27]= $\frac{1 + bL\, l_2}{1 + bL\, (l_1 + l_2)}$

Out[28]= $\frac{1 + bL\, l_2}{1 + bL\, (l_1 + l_2)}$

Out[29]= $\frac{1 + bL\, l_2}{1 + bL\, (l_1 + l_2)}$

```
In[30]:= LoanDemand1 = LoanDemand1 /. rL1 → LoanRate1 // FullSimplify
         LoanDemand2 = LoanDemand2 /. rL2 → LoanRate2 // FullSimplify
         LoanDemand3 = LoanDemand3 /. rL3 → LoanRate3 // FullSimplify
```

Out[30]= $\frac{l_0 - l_1\, (r + \psi + bL\, l_2\, (r - rB1 + \sigma_1)) - l_2\, (r - rB1 + \psi + bL\, l_2\, (r - rB1 + \sigma_1))}{1 + bL\, (l_1 + l_2)}$

Out[31]= $\frac{l_0 - l_1\, (r + \psi + bL\, l_2\, (r - rB2 + \sigma_2)) - l_2\, (r - rB2 + \psi + bL\, l_2\, (r - rB2 + \sigma_2))}{1 + bL\, (l_1 + l_2)}$

Out[32]= $\frac{l_0 - l_1\, (r + \psi + bL\, l_2\, (r - rB3 + \sigma_3)) - l_2\, (r - rB3 + \psi + bL\, l_2\, (r - rB3 + \sigma_3))}{1 + bL\, (l_1 + l_2)}$

```
In[33]:= LoanDemand1 = LoanDemand1 /. rB1 → r + σ1 // FullSimplify
         LoanDemand2 = LoanDemand2 /. rB2 → r + σ2 // FullSimplify
         LoanDemand3 = LoanDemand3 /. rB3 → r + σ3 // FullSimplify
```

Out[33]= $\frac{l_0 - (r + \psi)\, l_1 + l_2\, (-\psi + \sigma_1)}{1 + bL\, (l_1 + l_2)}$

Out[34]= $\frac{l_0 - (r + \psi)\, l_1 + l_2\, (-\psi + \sigma_2)}{1 + bL\, (l_1 + l_2)}$

Out[35]= $\frac{l_0 - (r + \psi)\, l_1 + l_2\, (-\psi + \sigma_3)}{1 + bL\, (l_1 + l_2)}$

In[36]:= **D[LoanDemand1, bL]**

Out[36]= $-\frac{(l_1 + l_2)\ (l_0 - (r + \psi)\ l_1 + l_2\ (-\psi + \sigma_1))}{(1 + bL\ (l_1 + l_2))^2}$

In[37]:= **Loans = LoanDemand1 + LoanDemand2 + LoanDemand3 // FullSimplify**

Out[37]= $\frac{3\ l_0 - 3\ (r + \psi)\ l_1 + l_2\ (-3\ \psi + \sigma_1 + \sigma_2 + \sigma_3)}{1 + bL\ (l_1 + l_2)}$

In[38]:= **D[LoanDemand1, r]**
D[LoanDemand2, r]
D[LoanDemand3, r]

Out[38]= $-\frac{l_1}{1 + bL\ (l_1 + l_2)}$

Out[39]= $-\frac{l_1}{1 + bL\ (l_1 + l_2)}$

Out[40]= $-\frac{l_1}{1 + bL\ (l_1 + l_2)}$

In[41]:= **DepositsDemand = d_0 + d_1 rd + d_2 (rd - rg)**

Out[41]= $d_0 + rd\ d_1 + (rd - rg)\ d_2$

In[42]:= **Solve[Deposits == DepositsDemand, rd] // Flatten // FullSimplify**

Out[42]= $\left\{rd \to \frac{rg - rg\ \rho - bd\ d_0 + bd\ rg\ d_2}{1 + bd\ (d_1 + d_2)}\right\}$

In[43]:= **DepositRate = rd /. %**

Out[43]= $\frac{rg - rg\ \rho - bd\ d_0 + bd\ rg\ d_2}{1 + bd\ (d_1 + d_2)}$

In[44]:= **DepositRate = DepositRate /. rg → r + ψ**

Out[44]= $\frac{r + \psi - \rho\ (r + \psi) - bd\ d_0 + bd\ (r + \psi)\ d_2}{1 + bd\ (d_1 + d_2)}$

In[45]:= **D[DepositRate, r]**

Out[45]= $\frac{1 - \rho + bd\ d_2}{1 + bd\ (d_1 + d_2)}$

In[46]:= **DepositsDemand = DepositsDemand /. rd → DepositRate // FullSimplify**

Out[46]= $(r - rg + \psi)\ d_2 + \frac{d_0 - (r + \psi)\ ((-1 + \rho)\ d_1 + \rho\ d_2)}{1 + bd\ (d_1 + d_2)}$

In[47]:= **DepositsDemand = DepositsDemand /. rg → r + ψ // FullSimplify**

Out[47]= $\frac{d_0 - (r + \psi)\ ((-1 + \rho)\ d_1 + \rho\ d_2)}{1 + bd\ (d_1 + d_2)}$

In[48]:= **D[DepositsDemand, r]**

Out[48]= $\frac{-(-1 + \rho)\ d_1 - \rho\ d_2}{1 + bd\ (d_1 + d_2)}$

```
In[49]:= G = G /. {rd → DepositRate, rL1 → LoanRate1, rL2 → LoanRate2, rL3 → LoanRate3, rg → r + ψ} //
           FullSimplify
```

Out[49]=
$$\frac{(-1+\rho)\ (-d_0+(r+\psi)\ ((-1+\rho)\ d_1+\rho\, d_2))}{1+bd\ (d_1+d_2)}+\frac{Ebar-3\ l_0+(bL\ Ebar+3\ (r+\psi))\ l_1+l_2\ (bL\ Ebar+3\ \psi-\sigma_1-\sigma_2-\sigma_3)}{1+bL\ (l_1+l_2)}$$

```
In[50]:= D[G, r] // FullSimplify
```

Out[50]=
$$\frac{(-1+\rho)\ ((-1+\rho)\ d_1+\rho\, d_2)}{1+bd\ (d_1+d_2)}+\frac{3\ l_1}{1+bL\ (l_1+l_2)}$$

```
In[51]:= Assets = Loans + G // FullSimplify
```

Out[51]=
$$\frac{Ebar-(-1+\rho)\ d_0+(bd\ Ebar+(-1+\rho)^2\ (r+\psi))\ d_1+(bd\ Ebar+(-1+\rho)\ \rho\ (r+\psi))\ d_2}{1+bd\ (d_1+d_2)}$$

```
In[52]:= D[Assets, r]
```

Out[52]=
$$\frac{(-1+\rho)^2\ d_1+(-1+\rho)\ \rho\, d_2}{1+bd\ (d_1+d_2)}$$

Short-run solution with risk-based capital requirements

In[1]:= $\mathcal{L} = \text{rL1 L1} + \text{rL2 L2} + \text{rL3 L3} + \text{rg } G - \text{rd } D - \frac{\text{bL}}{2}\left(\text{L1}^2 + \text{L2}^2 + \text{L3}^2\right) - \frac{\text{bd}}{2} D^2 - \lambda_1 \left(\text{L1} + \text{L2} + \text{L3} + G - (1-\rho) D - \text{Ebar}\right) - \lambda_2 \left(\Theta_1 (\text{w1 L1} + \text{w2 L2} + \text{w3 L3}) - \text{Ebar}\right)$

Out[1]= $-\frac{\text{bd } D^2}{2} - \frac{1}{2}\text{bL}\left(\text{L1}^2 + \text{L2}^2 + \text{L3}^2\right) - \text{D rd} + \text{G rg} + \text{L1 rL1} + \text{L2 rL2} + \text{L3 rL3} - (-\text{Ebar} + \text{G} + \text{L1} + \text{L2} + \text{L3} - \text{D}(1-\rho))\lambda_1 - (-\text{Ebar} + (\text{L1 w1} + \text{L2 w2} + \text{L3 w3})\Theta_1)\lambda_2$

In[2]:= **eq1 = D[$\mathcal{L}$, L1]**

Out[2]= $-\text{bL L1} + \text{rL1} - \lambda_1 - \text{w1 } \Theta_1 \lambda_2$

In[3]:= **eq2 = D[$\mathcal{L}$, L2]**

Out[3]= $-\text{bL L2} + \text{rL2} - \lambda_1 - \text{w2 } \Theta_1 \lambda_2$

In[4]:= **eq3 = D[$\mathcal{L}$, L3]**

Out[4]= $-\text{bL L3} + \text{rL3} - \lambda_1 - \text{w3 } \Theta_1 \lambda_2$

In[5]:= **eq4 = D[$\mathcal{L}$, D]**

Out[5]= $-\text{bd D} - \text{rd} - (-1+\rho)\lambda_1$

In[6]:= **eq5 = D[$\mathcal{L}$, G]**

Out[6]= $\text{rg} - \lambda_1$

In[7]:= **eq6 = D[$\mathcal{L}$, λ_1]**

Out[7]= $\text{Ebar} - \text{G} - \text{L1} - \text{L2} - \text{L3} + \text{D}(1-\rho)$

In[8]:= **eq7 = D[$\mathcal{L}$, λ_2]**

Out[8]= $\text{Ebar} - (\text{L1 w1} + \text{L2 w2} + \text{L3 w3})\Theta_1$

```
In[9]:= solutionSR = Solve[{eq1 = 0, eq2 = 0, eq3 = 0, eq4 = 0, eq5 = 0, eq6 = 0, eq7 = 0},
           {L1, L2, L3, G, D, λ1, λ2}] // Flatten // FullSimplify
```

Out[9]= $\{G \to (-bd\,bL\,Ebar\,(w1+w2+w3) + (bd\,(-rL2\,w1^2 - rL3\,w1^2 + rL1\,w1\,w2 + rL2\,w1\,w2 - rL1\,w2^2 - rL3\,w2^2 + ((rL1+rL3)\,w1 + (rL2+rL3)\,w2)\,w3 - (rL1+rL2)\,w3^2 + bL\,Ebar\,(w1^2+w2^2+w3^2) + 2\,rg\,(w1^2 - w1\,w2 + w2^2 - (w1+w2)\,w3 + w3^2)) + bL\,(w1^2+w2^2+w3^2)\,(rd + rg\,(-1+\rho))\,(-1+\rho))\,\theta_1) / (bd\,bL\,(w1^2+w2^2+w3^2)\,\theta_1),\ D \to -\frac{rd - rg + rg\,\rho}{bd},\ L1 \to \frac{bL\,Ebar\,w1 + (w2\,(-rL2\,w1 + rg\,(w1-w2) + rL1\,w2) + (rg - rL3)\,w1\,w3 - (rg - rL1)\,w3^2)\,\theta_1}{bL\,(w1^2+w2^2+w3^2)\,\theta_1},$

$L2 \to \frac{1}{bL\,(w1^2+w2^2+w3^2)\,\theta_1}(bL\,Ebar\,w2 + (w1\,(rL2\,w1 - rL1\,w2 + rg\,(-w1+w2)) + (rg - rL3)\,w2\,w3 + (-rg + rL2)\,w3^2)\,\theta_1),$

$L3 \to \frac{bL\,Ebar\,w3 + (-(rg - rL3)\,(w1^2+w2^2) + (-rL1\,w1 - rL2\,w2 + rg\,(w1+w2))\,w3)\,\theta_1}{bL\,(w1^2+w2^2+w3^2)\,\theta_1},$

$\lambda_2 \to \frac{-bL\,Ebar + (rL1\,w1 + rL2\,w2 + rL3\,w3 - rg\,(w1+w2+w3))\,\theta_1}{(w1^2+w2^2+w3^2)\,\theta_1^2},\ \lambda_1 \to rg\}$

```
In[10]:= L1 = L1 /. solutionSR
         L2 = L2 /. solutionSR
         L3 = L3 /. solutionSR
```

Out[10]= $\frac{bL\,Ebar\,w1 + (w2\,(-rL2\,w1 + rg\,(w1-w2) + rL1\,w2) + (rg - rL3)\,w1\,w3 - (rg - rL1)\,w3^2)\,\theta_1}{bL\,(w1^2+w2^2+w3^2)\,\theta_1}$

Out[11]= $\frac{bL\,Ebar\,w2 + (w1\,(rL2\,w1 - rL1\,w2 + rg\,(-w1+w2)) + (rg - rL3)\,w2\,w3 + (-rg + rL2)\,w3^2)\,\theta_1}{bL\,(w1^2+w2^2+w3^2)\,\theta_1}$

Out[12]= $\frac{bL\,Ebar\,w3 + (-(rg - rL3)\,(w1^2+w2^2) + (-rL1\,w1 - rL2\,w2 + rg\,(w1+w2))\,w3)\,\theta_1}{bL\,(w1^2+w2^2+w3^2)\,\theta_1}$

```
In[13]:= G = G /. solutionSR
         Deposits = D /. solutionSR
```

Out[13]= $(-bd\,bL\,Ebar\,(w1+w2+w3) + (bd\,(-rL2\,w1^2 - rL3\,w1^2 + rL1\,w1\,w2 + rL2\,w1\,w2 - rL1\,w2^2 - rL3\,w2^2 + ((rL1+rL3)\,w1 + (rL2+rL3)\,w2)\,w3 - (rL1+rL2)\,w3^2 + bL\,Ebar\,(w1^2+w2^2+w3^2) + 2\,rg\,(w1^2 - w1\,w2 + w2^2 - (w1+w2)\,w3 + w3^2)) + bL\,(w1^2+w2^2+w3^2)\,(rd + rg\,(-1+\rho))\,(-1+\rho))\,\theta_1) / (bd\,bL\,(w1^2+w2^2+w3^2)\,\theta_1)$

Out[14]= $-\frac{rd - rg + rg\,\rho}{bd}$

In[15]:= **LoanDemand1 = $l_0 - l_1$ rL1 $- l_2$ (rL1 - rB1)**
LoanDemand2 = $l_0 - l_1$ rL2 $- l_2$ (rL2 - rB2)
LoanDemand3 = $l_0 - l_1$ rL3 $- l_2$ (rL3 - rB3)

Out[15]= $l_0 - \mathrm{rL1}\, l_1 - (-\mathrm{rB1} + \mathrm{rL1})\, l_2$

Out[16]= $l_0 - \mathrm{rL2}\, l_1 - (-\mathrm{rB2} + \mathrm{rL2})\, l_2$

Out[17]= $l_0 - \mathrm{rL3}\, l_1 - (-\mathrm{rB3} + \mathrm{rL3})\, l_2$

In[18]:= **eq8 = L1 == LoanDemand1**
eq9 = L2 == LoanDemand2
eq10 = L3 == LoanDemand3

Out[18]=
$$\frac{\mathrm{bL\,Ebar\,w1} + (\mathrm{w2}\,(-\mathrm{rL2\,w1} + \mathrm{rg}\,(\mathrm{w1} - \mathrm{w2}) + \mathrm{rL1\,w2}) + (\mathrm{rg} - \mathrm{rL3})\,\mathrm{w1\,w3} - (\mathrm{rg} - \mathrm{rL1})\,\mathrm{w3}^2)\,\theta_1}{\mathrm{bL}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1} == l_0 - \mathrm{rL1}\, l_1 - (-\mathrm{rB1} + \mathrm{rL1})\, l_2$$

Out[19]=
$$\frac{\mathrm{bL\,Ebar\,w2} + (\mathrm{w1}\,(\mathrm{rL2\,w1} - \mathrm{rL1\,w2} + \mathrm{rg}\,(-\mathrm{w1} + \mathrm{w2})) + (\mathrm{rg} - \mathrm{rL3})\,\mathrm{w2\,w3} + (-\mathrm{rg} + \mathrm{rL2})\,\mathrm{w3}^2)\,\theta_1}{\mathrm{bL}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1} == l_0 - \mathrm{rL2}\, l_1 - (-\mathrm{rB2} + \mathrm{rL2})\, l_2$$

Out[20]=
$$\frac{\mathrm{bL\,Ebar\,w3} + (-(\mathrm{rg} - \mathrm{rL3})\,(\mathrm{w1}^2 + \mathrm{w2}^2) + (-\mathrm{rL1\,w1} - \mathrm{rL2\,w2} + \mathrm{rg}\,(\mathrm{w1} + \mathrm{w2}))\,\mathrm{w3})\,\theta_1}{\mathrm{bL}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1} == l_0 - \mathrm{rL3}\, l_1 - (-\mathrm{rB3} + \mathrm{rL3})\, l_2$$

In[21]:= **solLoanRates = Solve[{eq8, eq9, eq10}, {rL1, rL2, rL3}] // Flatten // FullSimplify**

Out[21]=
$$\begin{aligned}
&\{\mathrm{rL1} \to \\
&(-\mathrm{Ebar\,w1}\,(1 + \mathrm{bL}\,(l_1 + l_2)) + (\mathrm{rg}\,(\mathrm{w2}^2 + \mathrm{w3}^2 - \mathrm{w1}\,(\mathrm{w2} + \mathrm{w3})))\,l_1 + (\mathrm{rB1\,w1}^2 + \mathrm{rB2\,w1\,w2} - \mathrm{rg\,w1\,w2} + \\
&\quad \mathrm{rg\,w2}^2 + \mathrm{rB3\,w1\,w3} - \mathrm{rg\,w1\,w3} + \mathrm{rg\,w3}^2 + \mathrm{bL\,rB1}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,l_1)\,l_2 + \\
&\quad \mathrm{bL\,rB1}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,l_2^2 + l_0\,(\mathrm{w1}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3}) + \mathrm{bL}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,(l_1 + l_2)))\,\theta_1)\,/ \\
&((\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,(l_1 + l_2)\,(1 + \mathrm{bL}\,(l_1 + l_2))\,\theta_1),\ \mathrm{rL2} \to \\
&(-\mathrm{Ebar\,w2}\,(1 + \mathrm{bL}\,(l_1 + l_2)) + \\
&\quad (\mathrm{rg}\,(\mathrm{w1}^2 - \mathrm{w1\,w2} + \mathrm{w3}\,(-\mathrm{w2} + \mathrm{w3})))\,l_1 + (\mathrm{w2}\,(\mathrm{rB1\,w1} + \mathrm{rB2\,w2} + \mathrm{rB3\,w3}) + \\
&\quad \mathrm{rg}\,(\mathrm{w1}^2 - \mathrm{w1\,w2} + \mathrm{w3}\,(-\mathrm{w2} + \mathrm{w3})) + \mathrm{bL\,rB2}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,l_1)\,l_2 + \\
&\quad \mathrm{bL\,rB2}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,l_2^2 + l_0\,(\mathrm{w2}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3}) + \mathrm{bL}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,(l_1 + l_2)))\,\theta_1)\,/ \\
&((\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,(l_1 + l_2)\,(1 + \mathrm{bL}\,(l_1 + l_2))\,\theta_1),\ \mathrm{rL3} \to \\
&(-\mathrm{Ebar\,w3}\,(1 + \mathrm{bL}\,(l_1 + l_2)) + (\mathrm{rg}\,(\mathrm{w1}^2 + \mathrm{w2}^2 - (\mathrm{w1} + \mathrm{w2})\,\mathrm{w3})\,l_1 + \\
&\quad (\mathrm{w3}\,(\mathrm{rB1\,w1} + \mathrm{rB2\,w2} + \mathrm{rB3\,w3}) + \mathrm{rg}\,(\mathrm{w1}^2 + \mathrm{w2}^2 - (\mathrm{w1} + \mathrm{w2})\,\mathrm{w3}) + \mathrm{bL\,rB3}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,l_1) \\
&\quad l_2 + \mathrm{bL\,rB3}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,l_2^2 + l_0\,(\mathrm{w3}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3}) + \mathrm{bL}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,(l_1 + l_2))) \\
&\quad \theta_1)\,/\,((\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,(l_1 + l_2)\,(1 + \mathrm{bL}\,(l_1 + l_2))\,\theta_1)\}
\end{aligned}$$

```
In[22]:= LoanRate1 = rL1 /. solLoanRates
         LoanRate2 = rL2 /. solLoanRates
         LoanRate3 = rL3 /. solLoanRates
```

Out[22]= $\left(-\text{Ebar}\,w1\,(1+bL\,(l_1+l_2)) + \left(rg\,(w2^2+w3^2-w1\,(w2+w3))\,l_1 + (rB1\,w1^2+rB2\,w1\,w2-rg\,w1\,w2+rg\,w2^2+rB3\,w1\,w3-rg\,w1\,w3+rg\,w3^2+bL\,rB1\,(w1^2+w2^2+w3^2)\,l_1)\,l_2 + bL\,rB1\,(w1^2+w2^2+w3^2)\,l_2^2+l_0\,(w1\,(w1+w2+w3)+bL\,(w1^2+w2^2+w3^2)\,(l_1+l_2))\right)\theta_1\right) / \left((w1^2+w2^2+w3^2)\,(l_1+l_2)\,(1+bL\,(l_1+l_2))\,\theta_1\right)$

Out[23]= $\left(-\text{Ebar}\,w2\,(1+bL\,(l_1+l_2)) + \left(rg\,(w1^2-w1\,w2+w3\,(-w2+w3))\,l_1 + (w2\,(rB1\,w1+rB2\,w2+rB3\,w3)+rg\,(w1^2-w1\,w2+w3\,(-w2+w3))+bL\,rB2\,(w1^2+w2^2+w3^2)\,l_1)\,l_2+bL\,rB2\,(w1^2+w2^2+w3^2)\,l_2^2+l_0\,(w2\,(w1+w2+w3)+bL\,(w1^2+w2^2+w3^2)\,(l_1+l_2))\right)\theta_1\right) / \left((w1^2+w2^2+w3^2)\,(l_1+l_2)\,(1+bL\,(l_1+l_2))\,\theta_1\right)$

Out[24]= $\left(-\text{Ebar}\,w3\,(1+bL\,(l_1+l_2)) + \left(rg\,(w1^2+w2^2-(w1+w2)\,w3)\,l_1 + (w3\,(rB1\,w1+rB2\,w2+rB3\,w3)+rg\,(w1^2+w2^2-(w1+w2)\,w3)+bL\,rB3\,(w1^2+w2^2+w3^2)\,l_1)\,l_2+bL\,rB3\,(w1^2+w2^2+w3^2)\,l_2^2+l_0\,(w3\,(w1+w2+w3)+bL\,(w1^2+w2^2+w3^2)\,(l_1+l_2))\right)\theta_1\right) / \left((w1^2+w2^2+w3^2)\,(l_1+l_2)\,(1+bL\,(l_1+l_2))\,\theta_1\right)$

In[25]:= **LoanRate1 = LoanRate1 /. {rB1 → r + σ_1, rB2 → r + σ_2, rB3 → r + σ_3, rg → r + ψ} // FullSimplify**
LoanRate2 = LoanRate2 /. {rB1 → r + σ_1, rB2 → r + σ_2, rB3 → r + σ_3, rg → r + ψ} // FullSimplify
LoanRate3 = LoanRate3 /. {rB1 → r + σ_1, rB2 → r + σ_2, rB3 → r + σ_3, rg → r + ψ} // FullSimplify

Out[25]= $\left(-\text{Ebar}\,w1\,(1+bL\,(l_1+l_2))+\theta_1\,\left(l_0\,(w1\,(w1+w2+w3)+bL\,(w1^2+w2^2+w3^2)\,(l_1+l_2))+l_1\,((w2^2+w3^2-w1\,(w2+w3))\,(r+\psi)+bL\,(w1^2+w2^2+w3^2)\,l_2\,(r+\sigma_1))+l_2\,(r\,(w1^2+w2^2+w3^2)+(w2^2+w3^2-w1\,(w2+w3))\,\psi+bL\,(w1^2+w2^2+w3^2)\,l_2\,(r+\sigma_1)+w1\,(w1\,\sigma_1+w2\,\sigma_2+w3\,\sigma_3))\right)\right) / \left((w1^2+w2^2+w3^2)\,(l_1+l_2)\,(1+bL\,(l_1+l_2))\,\theta_1\right)$

Out[26]= $\left(-\text{Ebar}\,w2\,(1+bL\,(l_1+l_2))+\theta_1\,\left((w1^2-w1\,w2+w3\,(-w2+w3))\,(r+\psi)\,l_1+l_0\,(w2\,(w1+w2+w3)+bL\,(w1^2+w2^2+w3^2)\,(l_1+l_2))+bL\,(w1^2+w2^2+w3^2)\,l_2^2\,(r+\sigma_2)+l_2\,((w1^2-w1\,w2+w3\,(-w2+w3))\,(r+\psi)+bL\,(w1^2+w2^2+w3^2)\,l_1\,(r+\sigma_2)+w2\,(r\,(w1+w2+w3)+w1\,\sigma_1+w2\,\sigma_2+w3\,\sigma_3))\right)\right) / \left((w1^2+w2^2+w3^2)\,(l_1+l_2)\,(1+bL\,(l_1+l_2))\,\theta_1\right)$

Out[27]= $\left(-\text{Ebar}\,w3\,(1+bL\,(l_1+l_2))+\theta_1\,\left((w1^2+w2^2-(w1+w2)\,w3)\,(r+\psi)\,l_1+l_0\,(w3\,(w1+w2+w3)+bL\,(w1^2+w2^2+w3^2)\,(l_1+l_2))+bL\,(w1^2+w2^2+w3^2)\,l_2^2\,(r+\sigma_3)+l_2\,((w1^2+w2^2-(w1+w2)\,w3)\,(r+\psi)+bL\,(w1^2+w2^2+w3^2)\,l_1\,(r+\sigma_3)+w3\,(r\,(w1+w2+w3)+w1\,\sigma_1+w2\,\sigma_2+w3\,\sigma_3))\right)\right) / \left((w1^2+w2^2+w3^2)\,(l_1+l_2)\,(1+bL\,(l_1+l_2))\,\theta_1\right)$

In[28]:= **D[LoanRate1, θ_1] // FullSimplify**
D[LoanRate2, θ_1] // FullSimplify
D[LoanRate3, θ_1] // FullSimplify

Out[28]= $\dfrac{\text{Ebar}\,w1}{(w1^2+w2^2+w3^2)\,(l_1+l_2)\,\theta_1^2}$

Out[29]= $\dfrac{\text{Ebar}\,w2}{(w1^2+w2^2+w3^2)\,(l_1+l_2)\,\theta_1^2}$

Out[30]= $\dfrac{\text{Ebar}\,w3}{(w1^2+w2^2+w3^2)\,(l_1+l_2)\,\theta_1^2}$

```
In[31]:= LoanDemand1 = LoanDemand1 /. {rL1 → LoanRate1, rB1 → r + σ1} // FullSimplify
         LoanDemand2 = LoanDemand2 /. {rL2 → LoanRate2, rB2 → r + σ2} // FullSimplify
         LoanDemand3 = LoanDemand3 /. {rL3 → LoanRate3, rB3 → r + σ3} // FullSimplify
```

Out[31]= $(\text{Ebar}\,w1 + (w2^2 + w3^2 - w1\,(w2 + w3))\,l_0\,\Theta_1 + l_1\,(\text{bL Ebar}\,w1 + ((w1 - w2)\,w2 + w1\,w3 - w3^2)\,(r + \psi)\,\Theta_1) + l_2\,(\text{bL Ebar}\,w1 + \Theta_1\,(((w1 - w2)\,w2 + w1\,w3 - w3^2)\,\psi + (w2^2 + w3^2)\,\sigma_1 - w1\,(w2\,\sigma_2 + w3\,\sigma_3)))) / ((w1^2 + w2^2 + w3^2)\,(1 + bL\,(l_1 + l_2))\,\Theta_1)$

Out[32]= $(\text{Ebar}\,w2 + (w1^2 - w1\,w2 + w3\,(-w2 + w3))\,l_0\,\Theta_1 + l_1\,(\text{bL Ebar}\,w2 - (w1^2 - w1\,w2 - w2\,w3 + w3^2)\,(r + \psi)\,\Theta_1) + l_2\,(\text{bL Ebar}\,w2 + \Theta_1\,(-(w1^2 - w1\,w2 - w2\,w3 + w3^2)\,\psi - w1\,w2\,\sigma_1 + (w1^2 + w3^2)\,\sigma_2 - w2\,w3\,\sigma_3))) / ((w1^2 + w2^2 + w3^2)\,(1 + bL\,(l_1 + l_2))\,\Theta_1)$

Out[33]= $(\text{Ebar}\,w3 + (w1^2 + w2^2 - (w1 + w2)\,w3)\,l_0\,\Theta_1 + l_1\,(\text{bL Ebar}\,w3 - (w1^2 + w2^2 - (w1 + w2)\,w3)\,(r + \psi)\,\Theta_1) + l_2\,(\text{bL Ebar}\,w3 + \Theta_1\,(-(w1^2 + w2^2 - (w1 + w2)\,w3)\,\psi - w3\,(w1\,\sigma_1 + w2\,\sigma_2) + (w1^2 + w2^2)\,\sigma_3))) / ((w1^2 + w2^2 + w3^2)\,(1 + bL\,(l_1 + l_2))\,\Theta_1)$

```
In[34]:= D[LoanDemand1, r] // FullSimplify
```

Out[34]= $\dfrac{(-w2^2 - w3^2 + w1\,(w2 + w3))\,l_1}{(w1^2 + w2^2 + w3^2)\,(1 + bL\,(l_1 + l_2))}$

```
In[35]:= % /. {w1 → .3451, w2 → .8200, w3 → 1.0203}
```

Out[35]= $-\dfrac{0.588443\,l_1}{1 + bL\,(l_1 + l_2)}$

```
In[36]:= D[LoanDemand2, r] // FullSimplify
         % /. {w1 → .3451, w2 → .8200, w3 → 1.0203}
```

Out[36]= $-\dfrac{(w1^2 - w1\,w2 + w3\,(-w2 + w3))\,l_1}{(w1^2 + w2^2 + w3^2)\,(1 + bL\,(l_1 + l_2))}$

Out[37]= $-\dfrac{0.0220889\,l_1}{1 + bL\,(l_1 + l_2)}$

```
In[38]:= D[LoanDemand3, r] // FullSimplify
         % /. {w1 → .3451, w2 → .8200, w3 → 1.0203}
```

Out[38]= $-\dfrac{(w1^2 + w2^2 - (w1 + w2)\,w3)\,l_1}{(w1^2 + w2^2 + w3^2)\,(1 + bL\,(l_1 + l_2))}$

Out[39]= $\dfrac{0.216784\,l_1}{1 + bL\,(l_1 + l_2)}$

```
In[40]:= D[LoanRate1, r] // FullSimplify
         % /. {w1 → .3451, w2 → .8200, w3 → 1.0203} // Expand
```

Out[40]= $\dfrac{(w1^2 + w2^2 + w3^2)\,l_2\,(1 + bL\,l_2) + l_1\,(w2^2 + w3^2 - w1\,(w2 + w3) + bL\,(w1^2 + w2^2 + w3^2)\,l_2)}{(w1^2 + w2^2 + w3^2)\,(l_1 + l_2)\,(1 + bL\,(l_1 + l_2))}$

Out[41]= $\dfrac{0.588443\,l_1}{(l_1 + l_2)\,(1 + bL\,(l_1 + l_2))} + \dfrac{1.\,l_2}{(l_1 + l_2)\,(1 + bL\,(l_1 + l_2))} + \dfrac{1.\,bL\,l_1\,l_2}{(l_1 + l_2)\,(1 + bL\,(l_1 + l_2))} + \dfrac{1.\,bL\,l_2^2}{(l_1 + l_2)\,(1 + bL\,(l_1 + l_2))}$

```
In[42]:= D[LoanRate2, r] // FullSimplify
         % /. {w1 → .3451, w2 → .8200, w3 → 1.0203} // Expand
```

Out[42]= $$\frac{(w1^2+w2^2+w3^2)\ l_2\ (1+bL\ l_2)+l_1\ (w1^2-w1\ w2-w2\ w3+w3^2+bL\ (w1^2+w2^2+w3^2)\ l_2)}{(w1^2+w2^2+w3^2)\ (l_1+l_2)\ (1+bL\ (l_1+l_2))}$$

Out[43]= $$\frac{0.0220889\ l_1}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}+\frac{1.\ l_2}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}+\frac{1.\ bL\ l_1\ l_2}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}+\frac{1.\ bL\ l_2^2}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}$$

```
In[44]:= D[LoanRate3, r] // FullSimplify
         % /. {w1 → .3451, w2 → .8200, w3 → 1.0203} // Expand
```

Out[44]= $$\frac{(w1^2+w2^2+w3^2)\ l_2\ (1+bL\ l_2)+l_1\ (w1^2+w2^2-(w1+w2)\ w3+bL\ (w1^2+w2^2+w3^2)\ l_2)}{(w1^2+w2^2+w3^2)\ (l_1+l_2)\ (1+bL\ (l_1+l_2))}$$

Out[45]= $$-\frac{0.216784\ l_1}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}+\frac{1.\ l_2}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}+\frac{1.\ bL\ l_1\ l_2}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}+\frac{1.\ bL\ l_2^2}{(l_1+l_2)\ (1+bL\ (l_1+l_2))}$$

In[46]:= **DepositsDemand = $d_0 + d_1$ rd + d_2 (rd - rg)**

Out[46]= $d_0 + rd\ d_1 + (rd - rg)\ d_2$

In[47]:= **Solve[Deposits == DepositsDemand, rd] // Flatten // FullSimplify**

Out[47]= $$\left\{rd \to \frac{rg - rg\,\rho - bd\ d_0 + bd\ rg\ d_2}{1 + bd\ (d_1 + d_2)}\right\}$$

In[48]:= **DepositRate = rd /. %**

Out[48]= $$\frac{rg - rg\,\rho - bd\ d_0 + bd\ rg\ d_2}{1 + bd\ (d_1 + d_2)}$$

In[49]:= **DepositRate = DepositRate /. rg → r + ψ**

Out[49]= $$\frac{r + \psi - \rho\ (r + \psi) - bd\ d_0 + bd\ (r + \psi)\ d_2}{1 + bd\ (d_1 + d_2)}$$

In[50]:= **D[DepositRate, r]**

Out[50]= $$\frac{1 - \rho + bd\ d_2}{1 + bd\ (d_1 + d_2)}$$

In[51]:= **G = G /. {rg → r + ψ, rd → DepositRate} // FullSimplify**

Out[51]= $$\Bigg(-bd\ bL\ Ebar\ (w1+w2+w3)+bd\ \Bigg(-rL2\ w1^2-rL3\ w1^2+rL1\ w1\ w2+rL2\ w1\ w2- rL1\ w2^2-rL3\ w2^2+((rL1+rL3)\ w1+(rL2+rL3)\ w2)\ w3-(rL1+rL2)\ w3^2+ bL\ Ebar\ (w1^2+w2^2+w3^2)+2\ (w1^2+w2^2-w2\ w3+w3^2-w1\ (w2+w3))\ (r+\psi)+ \frac{bL\ (w1^2+w2^2+w3^2)\ (-1+\rho)\ (-d_0+(r+\psi)\ ((-1+\rho)\ d_1+\rho\ d_2))}{1+bd\ (d_1+d_2)}\Bigg)\ \theta_1\Bigg)\Big/\ (bd\ bL\ (w1^2+w2^2+w3^2)\ \theta_1)$$

```
In[52]:= G = G /. {rL1 → LoanRate1, rL2 → LoanRate2, rL3 → LoanRate3} // FullSimplify
```

Out[52]=

$$
\begin{aligned}
&(-\text{Ebar}\,(\text{w1}+\text{w2}+\text{w3}) - \text{bL Ebar w1}\,l_1 - \text{bL Ebar w2}\,l_1 - \text{bL Ebar w3}\,l_1 - \text{bL Ebar w1}\,l_2 - \\
&\quad \text{bL Ebar w2}\,l_2 - \text{bL Ebar w3}\,l_2 + \text{Ebar w1}^2\,\theta_1 + \text{Ebar w2}^2\,\theta_1 + \text{Ebar w3}^2\,\theta_1 + \text{w1}^2\,d_0\,\theta_1 + \text{w2}^2\,d_0\,\theta_1 + \\
&\quad \text{w3}^2\,d_0\,\theta_1 - \text{w1}^2\,\rho\,d_0\,\theta_1 - \text{w2}^2\,\rho\,d_0\,\theta_1 - \text{w3}^2\,\rho\,d_0\,\theta_1 - 2\,\text{w1}^2\,l_0\,\theta_1 + 2\,\text{w1 w2}\,l_0\,\theta_1 - 2\,\text{w2}^2\,l_0\,\theta_1 + \\
&\quad 2\,\text{w1 w3}\,l_0\,\theta_1 + 2\,\text{w2 w3}\,l_0\,\theta_1 - 2\,\text{w3}^2\,l_0\,\theta_1 + \text{bL Ebar w1}^2\,l_1\,\theta_1 + 2\,r\,\text{w1}^2\,l_1\,\theta_1 - 2\,r\,\text{w1 w2}\,l_1\,\theta_1 + \\
&\quad \text{bL Ebar w2}^2\,l_1\,\theta_1 + 2\,r\,\text{w2}^2\,l_1\,\theta_1 - 2\,r\,\text{w1 w3}\,l_1\,\theta_1 - 2\,r\,\text{w2 w3}\,l_1\,\theta_1 + \text{bL Ebar w3}^2\,l_1\,\theta_1 + \\
&\quad 2\,r\,\text{w3}^2\,l_1\,\theta_1 + 2\,\text{w1}^2\,\psi\,l_1\,\theta_1 - 2\,\text{w1 w2}\,\psi\,l_1\,\theta_1 + 2\,\text{w2}^2\,\psi\,l_1\,\theta_1 - 2\,\text{w1 w3}\,\psi\,l_1\,\theta_1 - 2\,\text{w2 w3}\,\psi\,l_1\,\theta_1 + \\
&\quad 2\,\text{w3}^2\,\psi\,l_1\,\theta_1 + \text{bL w1}^2\,d_0\,l_1\,\theta_1 + \text{bL w2}^2\,d_0\,l_1\,\theta_1 + \text{bL w3}^2\,d_0\,l_1\,\theta_1 - \text{bL w1}^2\,\rho\,d_0\,l_1\,\theta_1 - \\
&\quad \text{bL w2}^2\,\rho\,d_0\,l_1\,\theta_1 - \text{bL w3}^2\,\rho\,d_0\,l_1\,\theta_1 + \text{bL Ebar w1}^2\,l_2\,\theta_1 + \text{bL Ebar w2}^2\,l_2\,\theta_1 + \text{bL Ebar w3}^2\,l_2\,\theta_1 + \\
&\quad 2\,\text{w1}^2\,\psi\,l_2\,\theta_1 - 2\,\text{w1 w2}\,\psi\,l_2\,\theta_1 + 2\,\text{w2}^2\,\psi\,l_2\,\theta_1 - 2\,\text{w1 w3}\,\psi\,l_2\,\theta_1 - 2\,\text{w2 w3}\,\psi\,l_2\,\theta_1 + 2\,\text{w3}^2\,\psi\,l_2\,\theta_1 + \\
&\quad \text{bL w1}^2\,d_0\,l_2\,\theta_1 + \text{bL w2}^2\,d_0\,l_2\,\theta_1 + \text{bL w3}^2\,d_0\,l_2\,\theta_1 - \text{bL w1}^2\,\rho\,d_0\,l_2\,\theta_1 - \text{bL w2}^2\,\rho\,d_0\,l_2\,\theta_1 - \\
&\quad \text{bL w3}^2\,\rho\,d_0\,l_2\,\theta_1 + \text{w1 w2}\,l_2\,\theta_1\,\sigma_1 - \text{w2}^2\,l_2\,\theta_1\,\sigma_1 + \text{w1 w3}\,l_2\,\theta_1\,\sigma_1 - \text{w3}^2\,l_2\,\theta_1\,\sigma_1 - \text{w1}^2\,l_2\,\theta_1\,\sigma_2 + \\
&\quad \text{w1 w2}\,l_2\,\theta_1\,\sigma_2 + \text{w2 w3}\,l_2\,\theta_1\,\sigma_2 - \text{w3}^2\,l_2\,\theta_1\,\sigma_2 - (\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3})\,l_2\,\theta_1\,\sigma_3 + \\
&\quad d_1\,(-\text{bd Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + ((\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(\text{bd Ebar} + (-1+\rho)^2\,(r+\psi)) - \\
&\qquad 2\,\text{bd}\,(\text{w1}^2-\text{w1 w2}+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3}+\text{w3}^2)\,l_0)\,\theta_1 + \\
&\qquad l_1\,(-\text{bd bL Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + (\text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(-1+\rho)^2\,(r+\psi) + \text{bd}\,(\text{bL Ebar} \\
&\qquad\quad (\text{w1}^2+\text{w2}^2+\text{w3}^2) + 2\,(\text{w1}^2-\text{w1 w2}+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3}+\text{w3}^2)\,(r+\psi)))\,\theta_1) + l_2 \\
&\qquad (-\text{bd bL Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + \theta_1\,(\text{bd bL Ebar}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2) + 2\,\text{bd}\,(\text{w1}^2+\text{w2}^2-\text{w2 w3}+\text{w3}^2 - \\
&\qquad\quad \text{w1}\,(\text{w2}+\text{w3}))\,\psi + \text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(-1+\rho)^2\,(r+\psi) + \text{bd}\,(((\text{w1}-\text{w2})\,\text{w2}+\text{w1 w3} - \\
&\qquad\quad \text{w3}^2)\,\sigma_1 - (\text{w1}^2-\text{w1 w2}-\text{w2 w3}+\text{w3}^2)\,\sigma_2 - (\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3})\,\sigma_3)))) + \\
&\quad d_2\,(-\text{bd Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + ((\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(\text{bd Ebar} + (-1+\rho)\,\rho\,(r+\psi)) - \\
&\qquad 2\,\text{bd}\,(\text{w1}^2-\text{w1 w2}+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3}+\text{w3}^2)\,l_0)\,\theta_1 + \\
&\qquad l_1\,(-\text{bd bL Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + (\text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(-1+\rho)\,\rho\,(r+\psi) + \\
&\qquad\quad \text{bd}\,(\text{bL Ebar}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2) + 2\,(\text{w1}^2-\text{w1 w2}+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3}+\text{w3}^2)\,(r+\psi)))\,\theta_1) + \\
&\qquad l_2\,(-\text{bd bL Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + \theta_1\,(\text{bd bL Ebar}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2) + \\
&\qquad\quad 2\,\text{bd}\,(\text{w1}^2+\text{w2}^2-\text{w2 w3}+\text{w3}^2-\text{w1}\,(\text{w2}+\text{w3}))\,\psi + \\
&\qquad\quad \text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(-1+\rho)\,\rho\,(r+\psi) + \text{bd}\,(((\text{w1}-\text{w2})\,\text{w2}+\text{w1 w3}-\text{w3}^2) \\
&\qquad\quad \sigma_1 - (\text{w1}^2-\text{w1 w2}-\text{w2 w3}+\text{w3}^2)\,\sigma_2 - (\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3})\,\sigma_3))))) \,/ \\
&((\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(1+\text{bd}\,(d_1+d_2))\,(1+\text{bL}\,(l_1+l_2))\,\theta_1)
\end{aligned}
$$

```
In[53]:= D[G, r] // FullSimplify
```

Out[53]=

$$\frac{(-1+\rho)\,((-1+\rho)\,d_1+\rho\,d_2)}{1+\text{bd}\,(d_1+d_2)} + \frac{2\,(\text{w1}^2+\text{w2}^2-\text{w2 w3}+\text{w3}^2-\text{w1}\,(\text{w2}+\text{w3}))\,l_1}{(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(1+\text{bL}\,(l_1+l_2))}$$

```
In[54]:= Assets = G + LoanDemand1 + LoanDemand2 + LoanDemand3 // FullSimplify
```

Out[54]=

$$\frac{\text{Ebar} - (-1+\rho)\,d_0 + (\text{bd Ebar} + (-1+\rho)^2\,(r+\psi))\,d_1 + (\text{bd Ebar} + (-1+\rho)\,\rho\,(r+\psi))\,d_2}{1+\text{bd}\,(d_1+d_2)}$$

```
In[55]:= D[Assets, r]
```

Out[55]=

$$\frac{(-1+\rho)^2\,d_1 + (-1+\rho)\,\rho\,d_2}{1+\text{bd}\,(d_1+d_2)}$$

```
In[56]:= Loans = LoanDemand1 + LoanDemand2 + LoanDemand3 // FullSimplify
```

Out[56]=

$$
\begin{aligned}
&(\text{Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + 2\,(\text{w1}^2+\text{w2}^2-\text{w2 w3}+\text{w3}^2-\text{w1}\,(\text{w2}+\text{w3}))\,l_0\,\theta_1 + \\
&\quad l_1\,(\text{bL Ebar}\,(\text{w1}+\text{w2}+\text{w3}) - 2\,(\text{w1}^2-\text{w1 w2}+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3}+\text{w3}^2)\,(r+\psi)\,\theta_1) + \\
&\quad l_2\,(\text{bL Ebar}\,(\text{w1}+\text{w2}+\text{w3}) + \theta_1\,(-2\,(\text{w1}^2+\text{w2}^2-\text{w2 w3}+\text{w3}^2-\text{w1}\,(\text{w2}+\text{w3}))\,\psi + \\
&\qquad (\text{w2}^2+\text{w3}^2-\text{w1}\,(\text{w2}+\text{w3}))\,\sigma_1 + (\text{w1}^2-\text{w1 w2}+\text{w3}\,(-\text{w2}+\text{w3}))\,\sigma_2 + \\
&\qquad (\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3})\,\sigma_3)))\,/\,((\text{w1}^2+\text{w2}^2+\text{w3}^2)\,(1+\text{bL}\,(l_1+l_2))\,\theta_1)
\end{aligned}
$$

```
In[57]:= LoansReaction = D[Loans, r] // FullSimplify
```

Out[57]= $-\frac{2\ (w1^2 + w2^2 - w2\ w3 + w3^2 - w1\ (w2 + w3))\ l_1}{(w1^2 + w2^2 + w3^2)\ (1 + bL\ (l_1 + l_2))}$

```
In[58]:= LoanDemand1Reaction = D[LoanDemand1, r] // FullSimplify
```

Out[58]= $\frac{(-w2^2 - w3^2 + w1\ (w2 + w3))\ l_1}{(w1^2 + w2^2 + w3^2)\ (1 + bL\ (l_1 + l_2))}$

```
In[59]:= D[LoansReaction, w1] // FullSimplify
         D[LoansReaction, w2] // FullSimplify
         D[LoansReaction, w3] // FullSimplify
```

Out[59]= $-\frac{2\ (w1 + w2 + w3)\ (-w2^2 - w3^2 + w1\ (w2 + w3))\ l_1}{(w1^2 + w2^2 + w3^2)^2\ (1 + bL\ (l_1 + l_2))}$

Out[60]= $\frac{2\ (w1 + w2 + w3)\ (w1^2 - w1\ w2 + w3\ (-w2 + w3))\ l_1}{(w1^2 + w2^2 + w3^2)^2\ (1 + bL\ (l_1 + l_2))}$

Out[61]= $\frac{2\ (w1 + w2 + w3)\ (w1^2 + w2^2 - (w1 + w2)\ w3)\ l_1}{(w1^2 + w2^2 + w3^2)^2\ (1 + bL\ (l_1 + l_2))}$

```
In[62]:= RiskWLoans = w1 LoanDemand1 + w2 LoanDemand2 + w3 LoanDemand3 // FullSimplify
```

Out[62]= $\frac{Ebar}{\theta_1}$

Long-run solution with no capital requirements

In[1]:= $\mathbf{re = r + \beta_0 + \beta_{1,1} \frac{L1}{Eq} + \beta_{1,2} \frac{L2}{Eq} + \beta_{1,3} \frac{L3}{Eq} + \beta_2 \frac{G}{Eq}}$

Out[1]= $r + \beta_0 + \frac{G\,\beta_2}{Eq} + \frac{L1\,\beta_{1,1}}{Eq} + \frac{L2\,\beta_{1,2}}{Eq} + \frac{L3\,\beta_{1,3}}{Eq}$

In[2]:= $\mathcal{L} = rL1\,L1 + rL2\,L2 + rL3\,L3 + rg\,G - rd\,D - \frac{bL}{2}\left(L1^2 + L2^2 + L3^2\right) - \frac{bd}{2} D^2 - \frac{be}{2} Eq^2 - re\,Eq - \lambda_1 \left(L1 + L2 + L3 + G - (1-\rho)\,D - Eq\right)$ // FullSimplify

Out[2]= $-\frac{bd\,D^2}{2} - \frac{be\,Eq^2}{2} - \frac{1}{2} bL\left(L1^2 + L2^2 + L3^2\right) - Eq\,r - D\,rd + G\,rg + L1\,rL1 + L2\,rL2 + L3\,rL3 - Eq\,\beta_0 - G\,\beta_2 - (-Eq + G + L1 + L2 + L3 + D\,(-1+\rho))\,\lambda_1 - L1\,\beta_{1,1} - L2\,\beta_{1,2} - L3\,\beta_{1,3}$

In[3]:= **eq1 = D[$\mathcal{L}$, L1]**

Out[3]= $-bL\,L1 + rL1 - \lambda_1 - \beta_{1,1}$

In[4]:= **eq2 = D[$\mathcal{L}$, L2]**

Out[4]= $-bL\,L2 + rL2 - \lambda_1 - \beta_{1,2}$

In[5]:= **eq3 = D[$\mathcal{L}$, L3]**

Out[5]= $-bL\,L3 + rL3 - \lambda_1 - \beta_{1,3}$

In[6]:= **eq4 = D[$\mathcal{L}$, D]**

Out[6]= $-bd\,D - rd - (-1+\rho)\,\lambda_1$

In[7]:= **eq5 = D[$\mathcal{L}$, G]**

Out[7]= $rg - \beta_2 - \lambda_1$

In[8]:= **eq6 = D[$\mathcal{L}$, Eq]**

Out[8]= $-be\,Eq - r - \beta_0 + \lambda_1$

In[9]:= **eq7 = D[$\mathcal{L}$, λ_1]**

Out[9]= $Eq - G - L1 - L2 - L3 - D\,(-1+\rho)$

In[10]:= **solution = Solve[{eq1 = 0, eq2 = 0, eq3 = 0, eq4 = 0, eq5 = 0, eq6 = 0, eq7 = 0}, {L1, L2, L3, G, D, Eq, λ_1}] // Flatten // FullSimplify**

Out[10]= $\Big\{G \to \frac{1}{bd\,be\,bL}\big(-bd\,(bL\,(r - rg) + be\,(-3\,rg + rL1 + rL2 + rL3)) + be\,bL\,(rd + rg\,(-1+\rho))\,(-1+\rho) - bd\,bL\,\beta_0 - (bd\,(3\,be + bL) + be\,bL\,(-1+\rho)^2)\,\beta_2 + bd\,be\,(\beta_{1,1} + \beta_{1,2} + \beta_{1,3})\big),$
$L1 \to \frac{-rg + rL1 + \beta_2 - \beta_{1,1}}{bL},\ L2 \to \frac{-rg + rL2 + \beta_2 - \beta_{1,2}}{bL},\ L3 \to \frac{-rg + rL3 + \beta_2 - \beta_{1,3}}{bL},$
$Eq \to -\frac{r - rg + \beta_0 + \beta_2}{be},\ D \to \frac{-rd + rg - rg\,\rho + (-1+\rho)\,\beta_2}{bd},\ \lambda_1 \to rg - \beta_2\Big\}$

```
In[11]:= L1 = L1 /. solution
         L2 = L2 /. solution
         L3 = L3 /. solution
```

Out[11]= $\frac{-\mathrm{rg} + \mathrm{rL1} + \beta_2 - \beta_{1,1}}{\mathrm{bL}}$

Out[12]= $\frac{-\mathrm{rg} + \mathrm{rL2} + \beta_2 - \beta_{1,2}}{\mathrm{bL}}$

Out[13]= $\frac{-\mathrm{rg} + \mathrm{rL3} + \beta_2 - \beta_{1,3}}{\mathrm{bL}}$

```
In[14]:= G = G /. solution
```

Out[14]= $\frac{1}{\mathrm{bd\,be\,bL}} (-\mathrm{bd}\,(\mathrm{bL}\,(\mathrm{r} - \mathrm{rg}) + \mathrm{be}\,(-3\,\mathrm{rg} + \mathrm{rL1} + \mathrm{rL2} + \mathrm{rL3})) + \mathrm{be\,bL}\,(\mathrm{rd} + \mathrm{rg}\,(-1 + \rho))\,(-1 + \rho) - \mathrm{bd\,bL}\,\beta_0 - (\mathrm{bd}\,(3\,\mathrm{be} + \mathrm{bL}) + \mathrm{be\,bL}\,(-1 + \rho)^2)\,\beta_2 + \mathrm{bd\,be}\,(\beta_{1,1} + \beta_{1,2} + \beta_{1,3}))$

```
In[15]:= Deposits = D /. solution
```

Out[15]= $\frac{-\mathrm{rd} + \mathrm{rg} - \mathrm{rg}\,\rho + (-1 + \rho)\,\beta_2}{\mathrm{bd}}$

```
In[16]:= Eq = Eq /. solution
```

Out[16]= $-\frac{\mathrm{r} - \mathrm{rg} + \beta_0 + \beta_2}{\mathrm{be}}$

```
In[17]:= Eq = Eq /. {rg → r + ψ}
```

Out[17]= $-\frac{-\psi + \beta_0 + \beta_2}{\mathrm{be}}$

In[18]:= **DepositsDemand =** $\mathbf{d_0 + d_1\,rd + d_2\,(rd - rg)}$

Out[18]= $\mathrm{d}_0 + \mathrm{rd}\,\mathrm{d}_1 + (\mathrm{rd} - \mathrm{rg})\,\mathrm{d}_2$

```
In[19]:= Solve[Deposits == DepositsDemand, rd] // Flatten // FullSimplify
```

Out[19]= $\left\{\mathrm{rd} \to \frac{\mathrm{rg} - \mathrm{rg}\,\rho - \mathrm{bd}\,\mathrm{d}_0 + \mathrm{bd\,rg}\,\mathrm{d}_2 + (-1 + \rho)\,\beta_2}{1 + \mathrm{bd}\,(\mathrm{d}_1 + \mathrm{d}_2)}\right\}$

```
In[20]:= DepositRate = rd /. %
```

Out[20]= $\frac{\mathrm{rg} - \mathrm{rg}\,\rho - \mathrm{bd}\,\mathrm{d}_0 + \mathrm{bd\,rg}\,\mathrm{d}_2 + (-1 + \rho)\,\beta_2}{1 + \mathrm{bd}\,(\mathrm{d}_1 + \mathrm{d}_2)}$

```
In[21]:= DepositsDemand = DepositsDemand /. {rd → DepositRate} // FullSimplify
```

Out[21]= $\frac{\mathrm{d}_0 + (\mathrm{rg} - \mathrm{rg}\,\rho)\,\mathrm{d}_1 - \mathrm{rg}\,\rho\,\mathrm{d}_2 + (-1 + \rho)\,(\mathrm{d}_1 + \mathrm{d}_2)\,\beta_2}{1 + \mathrm{bd}\,(\mathrm{d}_1 + \mathrm{d}_2)}$

```
In[22]:= DepositsDemand = DepositsDemand /. {rg → r + ψ} // FullSimplify
```

Out[22]= $\frac{\mathrm{d}_0 - (\mathrm{r} + \psi)\,((-1 + \rho)\,\mathrm{d}_1 + \rho\,\mathrm{d}_2) + (-1 + \rho)\,(\mathrm{d}_1 + \mathrm{d}_2)\,\beta_2}{1 + \mathrm{bd}\,(\mathrm{d}_1 + \mathrm{d}_2)}$

In[23]:= **D[DepositsDemand, r]**

Out[23]= $\frac{-(-1+\rho)\, d_1 - \rho\, d_2}{1 + bd\,(d_1 + d_2)}$

In[24]:= **LoanDemand1 = $l_0 - l_1$ rL1 $- l_2$ (rL1 - rB1)**
LoanDemand2 = $l_0 - l_1$ rL2 $- l_2$ (rL2 - rB2)
LoanDemand3 = $l_0 - l_1$ rL3 $- l_2$ (rL3 - rB3)

Out[24]= $l_0 - rL1\, l_1 - (-rB1 + rL1)\, l_2$

Out[25]= $l_0 - rL2\, l_1 - (-rB2 + rL2)\, l_2$

Out[26]= $l_0 - rL3\, l_1 - (-rB3 + rL3)\, l_2$

In[27]:= **sollr1 = Solve[L1 == LoanDemand1, rL1] // Flatten // FullSimplify**
sollr2 = Solve[L2 == LoanDemand2, rL2] // Flatten // FullSimplify
sollr3 = Solve[L3 == LoanDemand3, rL3] // Flatten // FullSimplify

Out[27]= $\left\{rL1 \to \frac{rg + bL\, l_0 + bL\, rB1\, l_2 - \beta_2 + \beta_{1,1}}{1 + bL\,(l_1 + l_2)}\right\}$

Out[28]= $\left\{rL2 \to \frac{rg + bL\, l_0 + bL\, rB2\, l_2 - \beta_2 + \beta_{1,2}}{1 + bL\,(l_1 + l_2)}\right\}$

Out[29]= $\left\{rL3 \to \frac{rg + bL\, l_0 + bL\, rB3\, l_2 - \beta_2 + \beta_{1,3}}{1 + bL\,(l_1 + l_2)}\right\}$

In[30]:= **LoanRate1 = rL1 /. sollr1**
LoanRate2 = rL2 /. sollr2
LoanRate3 = rL3 /. sollr3

Out[30]= $\frac{rg + bL\, l_0 + bL\, rB1\, l_2 - \beta_2 + \beta_{1,1}}{1 + bL\,(l_1 + l_2)}$

Out[31]= $\frac{rg + bL\, l_0 + bL\, rB2\, l_2 - \beta_2 + \beta_{1,2}}{1 + bL\,(l_1 + l_2)}$

Out[32]= $\frac{rg + bL\, l_0 + bL\, rB3\, l_2 - \beta_2 + \beta_{1,3}}{1 + bL\,(l_1 + l_2)}$

In[33]:= **LoanRate1 = LoanRate1 /. {rg → r + ψ, rB1 → r + σ_1}**
LoanRate2 = LoanRate2 /. {rg → r + ψ, rB2 → r + σ_2}
LoanRate3 = LoanRate3 /. {rg → r + ψ, rB3 → r + σ_3}

Out[33]= $\frac{r + \psi + bL\, l_0 - \beta_2 + bL\, l_2\,(r + \sigma_1) + \beta_{1,1}}{1 + bL\,(l_1 + l_2)}$

Out[34]= $\frac{r + \psi + bL\, l_0 - \beta_2 + bL\, l_2\,(r + \sigma_2) + \beta_{1,2}}{1 + bL\,(l_1 + l_2)}$

Out[35]= $\frac{r + \psi + bL\, l_0 - \beta_2 + bL\, l_2\,(r + \sigma_3) + \beta_{1,3}}{1 + bL\,(l_1 + l_2)}$

In[36]:= **DepositRate = DepositRate /. {rg -> r + ψ}**

Out[36]= $\frac{r + \psi - \rho\,(r + \psi) - bd\, d_0 + bd\,(r + \psi)\, d_2 + (-1 + \rho)\, \beta_2}{1 + bd\,(d_1 + d_2)}$

```
In[37]:= D[DepositRate, r]
```

Out[37]= $\frac{1-\rho+\mathrm{bd}\,d_2}{1+\mathrm{bd}\,(d_1+d_2)}$

```
In[38]:= D[LoanRate1, r]
         D[LoanRate2, r]
         D[LoanRate3, r]
```

Out[38]= $\frac{1+\mathrm{bL}\,l_2}{1+\mathrm{bL}\,(l_1+l_2)}$

Out[39]= $\frac{1+\mathrm{bL}\,l_2}{1+\mathrm{bL}\,(l_1+l_2)}$

Out[40]= $\frac{1+\mathrm{bL}\,l_2}{1+\mathrm{bL}\,(l_1+l_2)}$

```
In[41]:= D[LoanRate1, β2] // FullSimplify
```

Out[41]= $-\frac{1}{1+\mathrm{bL}\,(l_1+l_2)}$

```
In[42]:= LoanDemand1 = LoanDemand1 /. {rL1 → LoanRate1, rB1 → r + σ1} // FullSimplify
         LoanDemand2 = LoanDemand2 /. {rL2 → LoanRate2, rB2 → r + σ2} // FullSimplify
         LoanDemand3 = LoanDemand3 /. {rL3 → LoanRate3, rB3 → r + σ3} // FullSimplify
```

Out[42]= $\frac{l_0+l_2\,(-\psi+\beta_2+\sigma_1-\beta_{1,1})-l_1\,(r+\psi-\beta_2+\beta_{1,1})}{1+\mathrm{bL}\,(l_1+l_2)}$

Out[43]= $\frac{l_0+l_2\,(-\psi+\beta_2+\sigma_2-\beta_{1,2})-l_1\,(r+\psi-\beta_2+\beta_{1,2})}{1+\mathrm{bL}\,(l_1+l_2)}$

Out[44]= $\frac{l_0+l_2\,(-\psi+\beta_2+\sigma_3-\beta_{1,3})-l_1\,(r+\psi-\beta_2+\beta_{1,3})}{1+\mathrm{bL}\,(l_1+l_2)}$

```
In[45]:= D[LoanDemand1, r]
         D[LoanDemand2, r]
         D[LoanDemand3, r]
```

Out[45]= $-\frac{l_1}{1+\mathrm{bL}\,(l_1+l_2)}$

Out[46]= $-\frac{l_1}{1+\mathrm{bL}\,(l_1+l_2)}$

Out[47]= $-\frac{l_1}{1+\mathrm{bL}\,(l_1+l_2)}$

```
In[48]:= D[LoanDemand1, bL] // FullSimplify
```

Out[48]= $\frac{(l_1+l_2)\,(-l_0+l_1\,(r+\psi-\beta_2+\beta_{1,1})+l_2\,(\psi-\beta_2-\sigma_1+\beta_{1,1}))}{(1+\mathrm{bL}\,(l_1+l_2))^2}$

In[49]:= **G = G /. {rg → r + ψ, rB1 → r + σ₁, rB2 → r + σ₂, rB3 → r + σ₃, rL1 → LoanRate1, rL2 → LoanRate2, rL3 → LoanRate3, rd → DepositRate} // FullSimplify**

Out[49]=
$$\frac{1}{\text{bd be bL}}\Bigg(-\text{bd bL}\,\beta_0 - (\text{bd}\,(3\,\text{be} + \text{bL}) + \text{be bL}\,(-1+\rho)^2)\,\beta_2 + \frac{\text{be bL}\,(-1+\rho)\,(-\text{bd}\,\text{d}_0 + \text{bd}\,(\text{r}+\psi)\,((-1+\rho)\,\text{d}_1 + \rho\,\text{d}_2) + (-1+\rho)\,\beta_2)}{1+\text{bd}\,(\text{d}_1+\text{d}_2)} + \text{bd be}\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3}) + \frac{1}{1+\text{bL}\,(\text{l}_1+\text{l}_2)}(\text{bd}\,(\text{bL}\,\psi - 3\,\text{be bL}\,\text{l}_0 + \text{bL}\,(\text{bL}\,\psi + 3\,\text{be}\,(\text{r}+\psi))\,\text{l}_1 + 3\,\text{be bL}\,\psi\,\text{l}_2 + \text{bL}^2\,\psi\,\text{l}_2 + 3\,\text{be}\,\beta_2 - \text{be bL}\,\text{l}_2\,\sigma_1 - \text{be bL}\,\text{l}_2\,\sigma_2 - \text{be bL}\,\text{l}_2\,\sigma_3 - \text{be}\,\beta_{1,1} - \text{be}\,\beta_{1,2} - \text{be}\,\beta_{1,3}))\Bigg)$$

In[50]:= **D[G, r] // FullSimplify**

Out[50]=
$$\frac{(-1+\rho)\,((-1+\rho)\,\text{d}_1+\rho\,\text{d}_2)}{1+\text{bd}\,(\text{d}_1+\text{d}_2)} + \frac{3\,\text{l}_1}{1+\text{bL}\,(\text{l}_1+\text{l}_2)}$$

In[51]:= **Assets = LoanDemand1 + LoanDemand2 + LoanDemand3 + G // FullSimplify**

Out[51]=
$$\frac{1}{\text{be}\,(1+\text{bd}\,(\text{d}_1+\text{d}_2))}(\psi + (\text{be} - \text{be}\,\rho)\,\text{d}_0 - \beta_0 - \beta_2 + \text{d}_1\,(\text{bd}\,\psi + \text{be}\,(-1+\rho)^2\,(\text{r}+\psi) - \text{bd}\,\beta_0 - (\text{bd} + \text{be}\,(-1+\rho)^2)\,\beta_2) + \text{d}_2\,(\text{bd}\,\psi + \text{be}\,(-1+\rho)\,\rho\,(\text{r}+\psi) - \text{bd}\,\beta_0 - (\text{bd}+\text{be}\,(-1+\rho)^2)\,\beta_2))$$

In[52]:= **D[Assets, r] // FullSimplify**

Out[52]=
$$\frac{(-1+\rho)\,((-1+\rho)\,\text{d}_1+\rho\,\text{d}_2)}{1+\text{bd}\,(\text{d}_1+\text{d}_2)}$$

```
In[53]:= renew = re /. {L1 → LoanDemand1, L2 → LoanDemand2, L3 → LoanDemand3}
```

Out[53]=

$$\begin{aligned}
&r + \beta_0 - \frac{be\,\beta_{1,1}\,(l_0 + l_2\,(-\psi + \beta_2 + \sigma_1 - \beta_{1,1}) - l_1\,(r + \psi - \beta_2 + \beta_{1,1}))}{(1 + bL\,(l_1 + l_2))\,(-\psi + \beta_0 + \beta_2)} - \\
&\frac{be\,\beta_{1,2}\,(l_0 + l_2\,(-\psi + \beta_2 + \sigma_2 - \beta_{1,2}) - l_1\,(r + \psi - \beta_2 + \beta_{1,2}))}{(1 + bL\,(l_1 + l_2))\,(-\psi + \beta_0 + \beta_2)} - \\
&\frac{be\,\beta_{1,3}\,(l_0 + l_2\,(-\psi + \beta_2 + \sigma_3 - \beta_{1,3}) - l_1\,(r + \psi - \beta_2 + \beta_{1,3}))}{(1 + bL\,(l_1 + l_2))\,(-\psi + \beta_0 + \beta_2)} - \\
&\frac{1}{bd\,bL\,(-\psi + \beta_0 + \beta_2)}\Big(\beta_2\,\Big(-bd\,bL\,\beta_0 - (bd\,(3\,be + bL) + be\,bL\,(-1 + \rho)^2)\,\beta_2 + \\
&\quad \frac{be\,bL\,(-1 + \rho)\,(-bd\,d_0 + bd\,(r + \psi)\,((-1 + \rho)\,d_1 + \rho\,d_2) + (-1 + \rho)\,\beta_2)}{1 + bd\,(d_1 + d_2)} + \\
&\quad bd\,be\,(\beta_{1,1} + \beta_{1,2} + \beta_{1,3}) + \frac{1}{1 + bL\,(l_1 + l_2)} \\
&\quad (bd\,(bL\,\psi - 3\,be\,bL\,l_0 + bL\,(bL\,\psi + 3\,be\,(r + \psi))\,l_1 + 3\,be\,bL\,\psi\,l_2 + bL^2\,\psi\,l_2 + \\
&\qquad 3\,be\,\beta_2 - be\,bL\,l_2\,\sigma_1 - be\,bL\,l_2\,\sigma_2 - be\,bL\,l_2\,\sigma_3 - be\,\beta_{1,1} - be\,\beta_{1,2} - be\,\beta_{1,3})))\Big)\Big)
\end{aligned}$$

$$\begin{aligned}
&r + \beta_0 - \frac{be\,\beta_{1,1}\,(l_0 + l_2\,(-\psi + \beta_2 + \sigma_1 - \beta_{1,1}) - l_1\,(r + \psi - \beta_2 + \beta_{1,1}))}{(1 + bL\,(l_1 + l_2))\,(-\psi + \beta_0 + \beta_2)} - \\
&\frac{be\,\beta_{1,2}\,(l_0 + l_2\,(-\psi + \beta_2 + \sigma_2 - \beta_{1,2}) - l_1\,(r + \psi - \beta_2 + \beta_{1,2}))}{(1 + bL\,(l_1 + l_2))\,(-\psi + \beta_0 + \beta_2)} - \\
&\frac{be\,\beta_{1,3}\,(l_0 + l_2\,(-\psi + \beta_2 + \sigma_3 - \beta_{1,3}) - l_1\,(r + \psi - \beta_2 + \beta_{1,3}))}{(1 + bL\,(l_1 + l_2))\,(-\psi + \beta_0 + \beta_2)} - \\
&\frac{1}{bd\,bL\,(-\psi + \beta_0 + \beta_2)}\Big(\beta_2\,\Big(-bd\,bL\,\beta_0 - (bd\,(3\,be + bL) + be\,bL\,(-1 + \rho)^2)\,\beta_2 + \\
&\quad \frac{be\,bL\,(-1 + \rho)\,(-bd\,d_0 + bd\,(r + \psi)\,((-1 + \rho)\,d_1 + \rho\,d_2) + (-1 + \rho)\,\beta_2)}{1 + bd\,(d_1 + d_2)} + \\
&\quad bd\,be\,(\beta_{1,1} + \beta_{1,2} + \beta_{1,3}) + \\
&\quad \frac{1}{1 + bL\,(l_1 + l_2)}\,(bd\,(bL\,\psi - 3\,be\,bL\,l_0 + bL\,(bL\,\psi + 3\,be\,(r + \psi))\,l_1 + 3\,be\,bL\,\psi\,l_2 + bL^2\,\psi\,l_2 + \\
&\qquad 3\,be\,\beta_2 - be\,bL\,l_2\,\sigma_1 - be\,bL\,l_2\,\sigma_2 - be\,bL\,l_2\,\sigma_3 - be\,\beta_{1,1} - be\,\beta_{1,2} - be\,\beta_{1,3})))\Big)\Big)
\end{aligned}$$


```
In[54]:= D[renew, r] // FullSimplify
```

Out[54]=

$$\frac{\frac{be\,(-1+\rho)^2\,d_1\,\beta_2}{1+bd\,(d_1+d_2)} + \frac{be\,(-1+\rho)\,\rho\,d_2\,\beta_2}{1+bd\,(d_1+d_2)} + \frac{(1+bL\,l_2)\,(\psi-\beta_0-\beta_2)+l_1\,(bL\,\psi+3\,be\,\beta_2-bL\,(\beta_0+\beta_2)-be\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3}))}{1+bL\,(l_1+l_2)}}{\psi - \beta_0 - \beta_2}$$

Long-run solution with risk-based capital requirements

In[1]:= $\mathbf{re = r + \beta_0 + \beta_{1,1} \frac{L1}{Eq} + \beta_{1,2} \frac{L2}{Eq} + \beta_{1,3} \frac{L3}{Eq} + \beta_2 \frac{G}{Eq}}$

Out[1]= $r + \beta_0 + \frac{G\,\beta_2}{Eq} + \frac{L1\,\beta_{1,1}}{Eq} + \frac{L2\,\beta_{1,2}}{Eq} + \frac{L3\,\beta_{1,3}}{Eq}$

In[2]:= $\mathcal{L} = rL1\,L1 + rL2\,L2 + rL3\,L3 + rg\,G - rd\,D - \frac{bL}{2}(L1^2 + L2^2 + L3^2) - \frac{bd}{2} D^2 - \frac{be}{2} Eq^2 - re\,Eq - \lambda_1 (L1 + L2 + L3 + G - (1-\rho) D - Eq) - \lambda_2 (\theta_1 (w1\,L1 + w2\,L2 + w3\,L3) - Eq)$ // FullSimplify

Out[2]= $-\frac{bd\,D^2}{2} - \frac{be\,Eq^2}{2} - \frac{1}{2} bL\,(L1^2 + L2^2 + L3^2) - Eq\,r - D\,rd + G\,rg + L1\,rL1 + L2\,rL2 + L3\,rL3 - Eq\,\beta_0 - G\,\beta_2 - (-Eq + G + L1 + L2 + L3 + D\,(-1+\rho))\,\lambda_1 - (-Eq + (L1\,w1 + L2\,w2 + L3\,w3)\,\theta_1)\,\lambda_2 - L1\,\beta_{1,1} - L2\,\beta_{1,2} - L3\,\beta_{1,3}$

In[3]:= **eq1 = D[$\mathcal{L}$, L1]**

Out[3]= $-bL\,L1 + rL1 - \lambda_1 - w1\,\theta_1\,\lambda_2 - \beta_{1,1}$

In[4]:= **eq2 = D[$\mathcal{L}$, L2]**

Out[4]= $-bL\,L2 + rL2 - \lambda_1 - w2\,\theta_1\,\lambda_2 - \beta_{1,2}$

In[5]:= **eq3 = D[$\mathcal{L}$, L3]**

Out[5]= $-bL\,L3 + rL3 - \lambda_1 - w3\,\theta_1\,\lambda_2 - \beta_{1,3}$

In[6]:= **eq4 = D[$\mathcal{L}$, D]**

Out[6]= $-bd\,D - rd - (-1+\rho)\,\lambda_1$

In[7]:= **eq5 = D[$\mathcal{L}$, G]**

Out[7]= $rg - \beta_2 - \lambda_1$

In[8]:= **eq6 = D[$\mathcal{L}$, Eq]**

Out[8]= $-be\,Eq - r - \beta_0 + \lambda_1 + \lambda_2$

In[9]:= **eq7 = D[$\mathcal{L}$, λ_1]**

Out[9]= $Eq - G - L1 - L2 - L3 - D\,(-1+\rho)$

In[10]:= **eq8 = D[$\mathcal{L}$, λ_2]**

Out[10]= $Eq - (L1\,w1 + L2\,w2 + L3\,w3)\,\theta_1$

In[11]:=
```
solutionLR =
  Solve[{eq1 = 0, eq2 = 0, eq3 = 0, eq4 = 0, eq5 = 0, eq6 = 0, eq7 = 0, eq8 = 0},
    {L1, L2, L3, G, D, Eq, λ1, λ2}] // Flatten // FullSimplify
```

Out[11]=

$$
\begin{aligned}
\{ & G \to (\beta_2\,(-bL\,(3\,bd + bL\,(-1+\rho)^2) + 2\,bd\,bL\,(w1+w2+w3)\,\theta_1 - \\
& \quad (bd\,(bL\,(w1^2+w2^2+w3^2) + 2\,be\,(w1^2 - w1\,w2 + w2^2 - (w1+w2)\,w3 + w3^2)) + \\
& \quad be\,bL\,(w1^2+w2^2+w3^2)\,(-1+\rho)^2)\,\theta_1^2) + \\
& \quad bd\,bL\,\theta_1\,(rL1\,w1 + rL2\,w2 + rL3\,w3 + r\,(w1+w2+w3) - 2\,rg\,(w1+w2+w3) + \\
& \quad (w1+w2+w3)\,\beta_0 - w1\,\beta_{1,1} - w2\,\beta_{1,2} - w3\,\beta_{1,3}) + bL \\
& \quad (bd\,(3\,rg - rL1 - rL2 - rL3) + bL\,(rd + rg\,(-1+\rho))\,(-1+\rho) + bd\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3})) + \\
& \quad \theta_1^2\,(bd\,(-bL\,(r - rg)\,(w1^2+w2^2+w3^2) + be\,(-rL2\,w1^2 - rL3\,w1^2 + rL1\,w1\,w2 + \\
& \quad rL2\,w1\,w2 - rL1\,w2^2 - rL3\,w2^2 + ((rL1+rL3)\,w1 + (rL2+rL3)\,w2)\,w3 - \\
& \quad (rL1+rL2)\,w3^2 + 2\,rg\,(w1^2 - w1\,w2 + w2^2 - (w1+w2)\,w3 + w3^2))) + \\
& \quad be\,bL\,(w1^2+w2^2+w3^2)\,(rd + rg\,(-1+\rho))\,(-1+\rho) - bd\,bL\,(w1^2+w2^2+w3^2)\,\beta_0 + \\
& \quad bd\,be\,((w2^2+w3^2 - w1\,(w2+w3))\,\beta_{1,1} + \\
& \quad (w1^2 - w1\,w2 + w3\,(-w2+w3))\,\beta_{1,2} + (w1^2+w2^2 - (w1+w2)\,w3)\,\beta_{1,3}))) / \\
& \quad (bd\,bL\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)),\ D \to \frac{-rd + rg - rg\,\rho + (-1+\rho)\,\beta_2}{bd}, \\
& L1 \to (-bL\,w1\,(r - rg + \beta_0)\,\theta_1 + \\
& \quad \beta_2\,(bL - bL\,w1\,\theta_1 + be\,(w2^2+w3^2 - w1\,(w2+w3))\,\theta_1^2) - \\
& \quad bL\,(rg - rL1 + \beta_{1,1}) + \\
& \quad be\,\theta_1^2\,(w2\,(rg\,w1 - rL2\,w1 - rg\,w2 + rL1\,w2) + (rg - rL3)\,w1\,w3 - (rg - rL1)\,w3^2 - \\
& \quad (w2^2+w3^2)\,\beta_{1,1} + w1\,w2\,\beta_{1,2} + w1\,w3\,\beta_{1,3}))\,/\,(bL\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)), \\
& L2 \to (-bL\,w2\,(r - rg + \beta_0)\,\theta_1 + \beta_2\,(bL - bL\,w2\,\theta_1 + be\,(w1^2 - w1\,w2 - w2\,w3 + w3^2)\,\theta_1^2) - \\
& \quad bL\,(rg - rL2 + \beta_{1,2}) + \\
& \quad be\,\theta_1^2\,(w1\,(rL2\,w1 - rL1\,w2 + rg\,(-w1+w2)) + (rg - rL3)\,w2\,w3 + (-rg + rL2)\,w3^2 + \\
& \quad w1\,w2\,\beta_{1,1} - (w1^2+w3^2)\,\beta_{1,2} + w2\,w3\,\beta_{1,3}))\,/\,(bL\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)), \\
& L3 \to (-bL\,w3\,(r - rg + \beta_0)\,\theta_1 + \beta_2\,(bL - bL\,w3\,\theta_1 + be\,(w1^2+w2^2 - (w1+w2)\,w3)\,\theta_1^2) - \\
& \quad bL\,(rg - rL3 + \beta_{1,3}) + be\,\theta_1^2\,(-(rg - rL3)\,(w1^2+w2^2) + \\
& \quad (-rL1\,w1 - rL2\,w2 + rg\,(w1+w2))\,w3 + w1\,w3\,\beta_{1,1} + w2\,w3\,\beta_{1,2} - (w1^2+w2^2)\,\beta_{1,3}))\,/ \\
& \quad (bL\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)),\ Eq \to \frac{1}{bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2} \\
& \quad (\theta_1\,(rL1\,w1 + rL2\,w2 + rL3\,w3 - rg\,(w1+w2+w3) - (w1^2+w2^2+w3^2)\,(r - rg + \beta_0)\,\theta_1 + \\
& \quad \beta_2\,(w1+w2+w3 - (w1^2+w2^2+w3^2)\,\theta_1) - w1\,\beta_{1,1} - w2\,\beta_{1,2} - w3\,\beta_{1,3})), \\
& \lambda_2 \to \frac{1}{bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2}\,(bL\,(r - rg) + bL\,\beta_0 + \\
& \quad \beta_2\,(bL + be\,(w1+w2+w3)\,\theta_1) - \\
& \quad be\,\theta_1\,(-rL1\,w1 - rL2\,w2 - rL3\,w3 + rg\,(w1+w2+w3) + \\
& \quad w1\,\beta_{1,1} + w2\,\beta_{1,2} + w3\,\beta_{1,3})),\ \lambda_1 \to rg - \beta_2 \}
\end{aligned}
$$

In[12]:=
```
Eq = Eq /. solutionLR
```

Out[12]=

$$
\begin{aligned}
& \frac{1}{bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2} \\
& (\theta_1\,(rL1\,w1 + rL2\,w2 + rL3\,w3 - rg\,(w1+w2+w3) - (w1^2+w2^2+w3^2)\,(r - rg + \beta_0)\,\theta_1 + \\
& \quad \beta_2\,(w1+w2+w3 - (w1^2+w2^2+w3^2)\,\theta_1) - w1\,\beta_{1,1} - w2\,\beta_{1,2} - w3\,\beta_{1,3}))
\end{aligned}
$$

In[13]:=
```
L1 = L1 /. solutionLR
```

Out[13]=

$$
\begin{aligned}
& (-bL\,w1\,(r - rg + \beta_0)\,\theta_1 + \\
& \quad \beta_2\,(bL - bL\,w1\,\theta_1 + be\,(w2^2+w3^2 - w1\,(w2+w3))\,\theta_1^2) - bL\,(rg - rL1 + \beta_{1,1}) + \\
& \quad be\,\theta_1^2\,(w2\,(rg\,w1 - rL2\,w1 - rg\,w2 + rL1\,w2) + (rg - rL3)\,w1\,w3 - (rg - rL1)\,w3^2 - \\
& \quad (w2^2+w3^2)\,\beta_{1,1} + w1\,w2\,\beta_{1,2} + w1\,w3\,\beta_{1,3}))\,/\,(bL\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2))
\end{aligned}
$$

In[14]:= **L2 = L2 /. solutionLR**

Out[14]= $(-\mathrm{bL}\,\mathrm{w2}\,(\mathrm{r}-\mathrm{rg}+\beta_0)\,\theta_1 + \beta_2\,(\mathrm{bL}-\mathrm{bL}\,\mathrm{w2}\,\theta_1+\mathrm{be}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}-\mathrm{w2}\,\mathrm{w3}+\mathrm{w3}^2)\,\theta_1^2) - \mathrm{bL}\,(\mathrm{rg}-\mathrm{rL2}+\beta_{1,2}) + \mathrm{be}\,\theta_1^2\,(\mathrm{w1}\,(\mathrm{rL2}\,\mathrm{w1}-\mathrm{rL1}\,\mathrm{w2}+\mathrm{rg}\,(-\mathrm{w1}+\mathrm{w2})) + (\mathrm{rg}-\mathrm{rL3})\,\mathrm{w2}\,\mathrm{w3} + (-\mathrm{rg}+\mathrm{rL2})\,\mathrm{w3}^2 + \mathrm{w1}\,\mathrm{w2}\,\beta_{1,1} - (\mathrm{w1}^2+\mathrm{w3}^2)\,\beta_{1,2} + \mathrm{w2}\,\mathrm{w3}\,\beta_{1,3}))\,/\,(\mathrm{bL}\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2))$

In[15]:= **L3 = L3 /. solutionLR**

Out[15]= $(-\mathrm{bL}\,\mathrm{w3}\,(\mathrm{r}-\mathrm{rg}+\beta_0)\,\theta_1 + \beta_2\,(\mathrm{bL}-\mathrm{bL}\,\mathrm{w3}\,\theta_1+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\theta_1^2) - \mathrm{bL}\,(\mathrm{rg}-\mathrm{rL3}+\beta_{1,3}) + \mathrm{be}\,\theta_1^2\,(-(\mathrm{rg}-\mathrm{rL3})\,(\mathrm{w1}^2+\mathrm{w2}^2) + (-\mathrm{rL1}\,\mathrm{w1}-\mathrm{rL2}\,\mathrm{w2}+\mathrm{rg}\,(\mathrm{w1}+\mathrm{w2}))\,\mathrm{w3} + \mathrm{w1}\,\mathrm{w3}\,\beta_{1,1} + \mathrm{w2}\,\mathrm{w3}\,\beta_{1,2} - (\mathrm{w1}^2+\mathrm{w2}^2)\,\beta_{1,3}))\,/\,(\mathrm{bL}\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2))$

In[16]:= **G = G /. solutionLR**

Out[16]= $(\beta_2\,(-\mathrm{bL}\,(3\,\mathrm{bd}+\mathrm{bL}\,(-1+\rho)^2) + 2\,\mathrm{bd}\,\mathrm{bL}\,(\mathrm{w1}+\mathrm{w2}+\mathrm{w3})\,\theta_1 - (\mathrm{bd}\,(\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2) + 2\,\mathrm{be}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3}+\mathrm{w3}^2)) + \mathrm{be}\,\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(-1+\rho)^2)\,\theta_1^2) + \mathrm{bd}\,\mathrm{bL}\,\theta_1\,(\mathrm{rL1}\,\mathrm{w1}+\mathrm{rL2}\,\mathrm{w2}+\mathrm{rL3}\,\mathrm{w3}+\mathrm{r}\,(\mathrm{w1}+\mathrm{w2}+\mathrm{w3}) - 2\,\mathrm{rg}\,(\mathrm{w1}+\mathrm{w2}+\mathrm{w3}) + (\mathrm{w1}+\mathrm{w2}+\mathrm{w3})\,\beta_0 - \mathrm{w1}\,\beta_{1,1} - \mathrm{w2}\,\beta_{1,2} - \mathrm{w3}\,\beta_{1,3}) + \mathrm{bL}\,(\mathrm{bd}\,(3\,\mathrm{rg}-\mathrm{rL1}-\mathrm{rL2}-\mathrm{rL3}) + \mathrm{bL}\,(\mathrm{rd}+\mathrm{rg}\,(-1+\rho))\,(-1+\rho) + \mathrm{bd}\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3})) + \theta_1^2\,(\mathrm{bd}\,(-\mathrm{bL}\,(\mathrm{r}-\mathrm{rg})\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2) + \mathrm{be}\,(-\mathrm{rL2}\,\mathrm{w1}^2-\mathrm{rL3}\,\mathrm{w1}^2+\mathrm{rL1}\,\mathrm{w1}\,\mathrm{w2} + \mathrm{rL2}\,\mathrm{w1}\,\mathrm{w2}-\mathrm{rL1}\,\mathrm{w2}^2-\mathrm{rL3}\,\mathrm{w2}^2+((\mathrm{rL1}+\mathrm{rL3})\,\mathrm{w1}+(\mathrm{rL2}+\mathrm{rL3})\,\mathrm{w2})\,\mathrm{w3} - (\mathrm{rL1}+\mathrm{rL2})\,\mathrm{w3}^2 + 2\,\mathrm{rg}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3}+\mathrm{w3}^2))) + \mathrm{be}\,\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(\mathrm{rd}+\mathrm{rg}\,(-1+\rho))\,(-1+\rho) - \mathrm{bd}\,\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\beta_0 + \mathrm{bd}\,\mathrm{be}\,((\mathrm{w2}^2+\mathrm{w3}^2-\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,\beta_{1,1} + (\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w3}\,(-\mathrm{w2}+\mathrm{w3}))\,\beta_{1,2} + (\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\beta_{1,3})))\,/\,(\mathrm{bd}\,\mathrm{bL}\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2))$

In[17]:= **Deposits = D /. solutionLR**

Out[17]= $\frac{-\mathrm{rd}+\mathrm{rg}-\mathrm{rg}\,\rho+(-1+\rho)\,\beta_2}{\mathrm{bd}}$

In[18]:= **LoanDemand1 = $\mathbf{l_0 - l_1\,rL1 - l_2\,(rL1 - rB1)}$**
LoanDemand2 = $\mathbf{l_0 - l_1\,rL2 - l_2\,(rL2 - rB2)}$
LoanDemand3 = $\mathbf{l_0 - l_1\,rL3 - l_2\,(rL3 - rB3)}$

Out[18]= $\mathrm{l}_0 - \mathrm{rL1}\,\mathrm{l}_1 - (-\mathrm{rB1}+\mathrm{rL1})\,\mathrm{l}_2$

Out[19]= $\mathrm{l}_0 - \mathrm{rL2}\,\mathrm{l}_1 - (-\mathrm{rB2}+\mathrm{rL2})\,\mathrm{l}_2$

Out[20]= $\mathrm{l}_0 - \mathrm{rL3}\,\mathrm{l}_1 - (-\mathrm{rB3}+\mathrm{rL3})\,\mathrm{l}_2$

```
In[21]:= eq9 = L1 = LoanDemand1
         eq10 = L2 = LoanDemand2
         eq11 = L3 = LoanDemand3
```

Out[21]=

$$\left(-bL\,w1\,(r - rg + \beta_0)\,\theta_1 + \beta_2\,(bL - bL\,w1\,\theta_1 + be\,(w2^2 + w3^2 - w1\,(w2 + w3))\,\theta_1^2) - bL\,(rg - rL1 + \beta_{1,1}) + be\,\theta_1^2\,(w2\,(rg\,w1 - rL2\,w1 - rg\,w2 + rL1\,w2) + (rg - rL3)\,w1\,w3 - (rg - rL1)\,w3^2 - (w2^2 + w3^2)\,\beta_{1,1} + w1\,w2\,\beta_{1,2} + w1\,w3\,\beta_{1,3})\right) / \left(bL\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\right) == l_0 - rL1\,l_1 - (-rB1 + rL1)\,l_2$$

Out[22]=

$$\left(-bL\,w2\,(r - rg + \beta_0)\,\theta_1 + \beta_2\,(bL - bL\,w2\,\theta_1 + be\,(w1^2 - w1\,w2 - w2\,w3 + w3^2)\,\theta_1^2) - bL\,(rg - rL2 + \beta_{1,2}) + be\,\theta_1^2\,(w1\,(rL2\,w1 - rL1\,w2 + rg\,(-w1 + w2)) + (rg - rL3)\,w2\,w3 + (-rg + rL2)\,w3^2 + w1\,w2\,\beta_{1,1} - (w1^2 + w3^2)\,\beta_{1,2} + w2\,w3\,\beta_{1,3})\right) / \left(bL\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\right) == l_0 - rL2\,l_1 - (-rB2 + rL2)\,l_2$$

Out[23]=

$$\left(-bL\,w3\,(r - rg + \beta_0)\,\theta_1 + \beta_2\,(bL - bL\,w3\,\theta_1 + be\,(w1^2 + w2^2 - (w1 + w2)\,w3)\,\theta_1^2) - bL\,(rg - rL3 + \beta_{1,3}) + be\,\theta_1^2\,(-(rg - rL3)\,(w1^2 + w2^2) + (-rL1\,w1 - rL2\,w2 + rg\,(w1 + w2))\,w3 + w1\,w3\,\beta_{1,1} + w2\,w3\,\beta_{1,2} - (w1^2 + w2^2)\,\beta_{1,3})\right) / \left(bL\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\right) == l_0 - rL3\,l_1 - (-rB3 + rL3)\,l_2$$

```
In[24]:= solLoanRates = Solve[{eq9, eq10, eq11}, {rL1, rL2, rL3}] // Flatten // FullSimplify
```

Out[24]=

$$
\begin{aligned}
\{ & \mathrm{rL1} \to (\mathrm{rg} + \mathrm{bL}\,\mathrm{rB1}\,l_2 + \mathrm{bL}\,\mathrm{rg}\,l_2 + \mathrm{bL}^2\,\mathrm{rB1}\,l_2^2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,\mathrm{w1}\,\theta_1 - \\
&\quad \mathrm{rg}\,\mathrm{w1}\,\theta_1 + \mathrm{bL}\,r\,\mathrm{w1}\,l_2\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w1}\,l_2\,\theta_1 + \mathrm{w1}\,\beta_0\,\theta_1 + \mathrm{bL}\,\mathrm{w1}\,l_2\,\beta_0\,\theta_1 + \mathrm{w1}\,\beta_2\,\theta_1 + \\
&\quad \mathrm{bL}\,\mathrm{w1}\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,\mathrm{rB1}\,\mathrm{w1}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB2}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB3}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{bL}\,\mathrm{rB1}\,\mathrm{w1}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB1}\,\mathrm{w2}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB1}\,\mathrm{w3}^2\,l_2^2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w2}^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w3}^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_0\,(\mathrm{bL} + \mathrm{be}\,\mathrm{w1}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3})\,\theta_1^2 + \mathrm{bL}\,(l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)) + \beta_{1,1} + \\
&\quad \mathrm{bL}\,l_2\,\beta_{1,1} + \mathrm{be}\,\mathrm{w2}^2\,l_2\,\theta_1^2\,\beta_{1,1} + \mathrm{be}\,\mathrm{w3}^2\,l_2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,3} + \\
&\quad l_1\,(\mathrm{bL}\,\mathrm{rg} + \mathrm{bL}\,r\,\mathrm{w1}\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w1}\,\theta_1 + \mathrm{bL}\,\mathrm{w1}\,\beta_0\,\theta_1 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}^2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1} \\
&\qquad \mathrm{w3}\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,\theta_1^2 + \beta_2\,(-\mathrm{bL} + \mathrm{bL}\,\mathrm{w1}\,\theta_1 + \mathrm{be}\,((\mathrm{w1} - \mathrm{w2})\,\mathrm{w2} + \mathrm{w1}\,\mathrm{w3} - \mathrm{w3}^2)\,\theta_1^2) + \\
&\qquad \mathrm{bL}\,\mathrm{rB1}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2) + \mathrm{bL}\,\beta_{1,1} + \mathrm{be}\,\mathrm{w2}^2\,\theta_1^2\,\beta_{1,1} + \\
&\qquad \mathrm{be}\,\mathrm{w3}^2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,\theta_1^2\,\beta_{1,3}))\ / \\
&\quad ((1 + \mathrm{bL}\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2))), \\
& \mathrm{rL2} \to (\mathrm{rg} + \mathrm{bL}\,\mathrm{rB2}\,l_2 + \mathrm{bL}\,\mathrm{rg}\,l_2 + \mathrm{bL}^2\,\mathrm{rB2}\,l_2^2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,\mathrm{w2}\,\theta_1 - \mathrm{rg}\,\mathrm{w2}\,\theta_1 + \\
&\quad \mathrm{bL}\,r\,\mathrm{w2}\,l_2\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w2}\,l_2\,\theta_1 + \mathrm{w2}\,\beta_0\,\theta_1 + \mathrm{bL}\,\mathrm{w2}\,l_2\,\beta_0\,\theta_1 + \mathrm{w2}\,\beta_2\,\theta_1 + \\
&\quad \mathrm{bL}\,\mathrm{w2}\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB1}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{rB2}\,\mathrm{w2}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB3}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{bL}\,\mathrm{rB2}\,\mathrm{w1}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB2}\,\mathrm{w2}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB2}\,\mathrm{w3}^2\,l_2^2\,\theta_1^2 - \\
&\quad \mathrm{be}\,\mathrm{w1}^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w3}^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_0\,(\mathrm{bL} + \mathrm{be}\,\mathrm{w2}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3})\,\theta_1^2 + \mathrm{bL}\,(l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)) - \\
&\quad \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2\,\beta_{1,1} + \beta_{1,2} + \mathrm{bL}\,l_2\,\beta_{1,2} + \mathrm{be}\,\mathrm{w1}^2\,l_2\,\theta_1^2\,\beta_{1,2} + \\
&\quad \mathrm{be}\,\mathrm{w3}^2\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,3} + \\
&\quad l_1\,(\mathrm{bL}\,\mathrm{rg} + \mathrm{bL}\,r\,\mathrm{w2}\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w2}\,\theta_1 + \mathrm{bL}\,\mathrm{w2}\,\beta_0\,\theta_1 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}^2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2 - \\
&\qquad \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}\,\mathrm{w3}\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,\theta_1^2 + \mathrm{bL}\,\mathrm{rB2}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2) + \\
&\qquad \beta_2\,(-\mathrm{bL} + \mathrm{bL}\,\mathrm{w2}\,\theta_1 - \mathrm{be}\,(\mathrm{w1}^2 - \mathrm{w1}\,\mathrm{w2} - \mathrm{w2}\,\mathrm{w3} + \mathrm{w3}^2)\,\theta_1^2) - \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2\,\beta_{1,1} + \\
&\qquad \mathrm{bL}\,\beta_{1,2} + \mathrm{be}\,\mathrm{w1}^2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,\mathrm{w3}^2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,\theta_1^2\,\beta_{1,3}))\ / \\
&\quad ((1 + \mathrm{bL}\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2))), \\
& \mathrm{rL3} \to (\mathrm{rg} + \mathrm{bL}\,\mathrm{rg}\,l_1 + \mathrm{bL}\,\mathrm{rB3}\,l_2 + \mathrm{bL}\,\mathrm{rg}\,l_2 + \mathrm{bL}^2\,\mathrm{rB3}\,l_2^2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,\mathrm{w3}\,\theta_1 - \\
&\quad \mathrm{rg}\,\mathrm{w3}\,\theta_1 + \mathrm{bL}\,r\,\mathrm{w3}\,l_2\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w3}\,l_2\,\theta_1 + \mathrm{w3}\,\beta_0\,\theta_1 + \mathrm{bL}\,\mathrm{w3}\,l_2\,\beta_0\,\theta_1 + \mathrm{w3}\,\beta_2\,\theta_1 + \\
&\quad \mathrm{bL}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB1}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 - \\
&\quad \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB2}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB3}\,\mathrm{w3}^2\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{bL}\,\mathrm{rB3}\,\mathrm{w1}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB3}\,\mathrm{w2}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB3}\,\mathrm{w3}^2\,l_2^2\,\theta_1^2 - \\
&\quad \mathrm{be}\,\mathrm{w1}^2\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w2}^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_1\,\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,\mathrm{w3}\,\theta_1 - \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 - (\mathrm{w1} + \mathrm{w2})\,\mathrm{w3})\,\theta_1^2) + \\
&\quad \mathrm{bL}\,\mathrm{rB3}\,l_1\,l_2\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2) + \\
&\quad l_0\,(\mathrm{bL} + \mathrm{be}\,\mathrm{w3}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3})\,\theta_1^2 + \mathrm{bL}\,(l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)) - \\
&\quad \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,2} - l_1\,\theta_1\,(\mathrm{bL}\,(-r + \mathrm{rg})\,\mathrm{w3} - \mathrm{bL}\,\mathrm{w3}\,\beta_0 + \\
&\qquad \mathrm{be}\,\theta_1\,(-\mathrm{rg}\,(\mathrm{w1}^2 + \mathrm{w2}^2) + \mathrm{rg}\,(\mathrm{w1} + \mathrm{w2})\,\mathrm{w3} + \mathrm{w1}\,\mathrm{w3}\,\beta_{1,1} + \mathrm{w2}\,\mathrm{w3}\,\beta_{1,2})) + \\
&\quad \beta_{1,3} + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2)\,\theta_1^2)\,\beta_{1,3})\ / \\
&\quad ((1 + \mathrm{bL}\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)))\}
\end{aligned}
$$

In[25]:=
```
LoanRate1 = rL1 /. solLoanRates
LoanRate2 = rL2 /. solLoanRates
LoanRate3 = rL3 /. solLoanRates
```

Out[25]=

$$\begin{aligned}&(\mathrm{rg} + \mathrm{bL}\,\mathrm{rB1}\,l_2 + \mathrm{bL}\,\mathrm{rg}\,l_2 + \mathrm{bL}^2\,\mathrm{rB1}\,l_2^2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,\mathrm{w1}\,\theta_1 - \mathrm{rg}\,\mathrm{w1}\,\theta_1 + \mathrm{bL}\,r\,\mathrm{w1}\,l_2\,\theta_1 - \\
&\quad \mathrm{bL}\,\mathrm{rg}\,\mathrm{w1}\,l_2\,\theta_1 + \mathrm{w1}\,\beta_0\,\theta_1 + \mathrm{bL}\,\mathrm{w1}\,l_2\,\beta_0\,\theta_1 + \mathrm{w1}\,\beta_2\,\theta_1 + \mathrm{bL}\,\mathrm{w1}\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,\mathrm{rB1}\,\mathrm{w1}^2\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{rB2}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB3}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 - \\
&\quad \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB1}\,\mathrm{w1}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB1}\,\mathrm{w2}^2\,l_2^2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{bL}\,\mathrm{rB1}\,\mathrm{w3}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w2}^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w3}^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_0\,(\mathrm{bL} + \mathrm{be}\,\mathrm{w1}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3})\,\theta_1^2 + \mathrm{bL}\,(l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)) + \beta_{1,1} + \\
&\quad \mathrm{bL}\,l_2\,\beta_{1,1} + \mathrm{be}\,\mathrm{w2}^2\,l_2\,\theta_1^2\,\beta_{1,1} + \mathrm{be}\,\mathrm{w3}^2\,l_2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,3} + \\
&\quad l_1\,(\mathrm{bL}\,\mathrm{rg} + \mathrm{bL}\,r\,\mathrm{w1}\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w1}\,\theta_1 + \mathrm{bL}\,\mathrm{w1}\,\beta_0\,\theta_1 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}^2\,\theta_1^2 - \\
&\qquad \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w3}\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,\theta_1^2 + \beta_2\,(-\mathrm{bL} + \mathrm{bL}\,\mathrm{w1}\,\theta_1 + \mathrm{be}\,((\mathrm{w1} - \mathrm{w2})\,\mathrm{w2} + \mathrm{w1}\,\mathrm{w3} - \mathrm{w3}^2)\,\theta_1^2) + \\
&\qquad \mathrm{bL}\,\mathrm{rB1}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2) + \mathrm{bL}\,\beta_{1,1} + \mathrm{be}\,\mathrm{w2}^2\,\theta_1^2\,\beta_{1,1} + \\
&\qquad \mathrm{be}\,\mathrm{w3}^2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,\theta_1^2\,\beta_{1,3}))\;/ \\
&\ ((1 + \mathrm{bL}\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)))\end{aligned}$$

Out[26]=

$$\begin{aligned}&(\mathrm{rg} + \mathrm{bL}\,\mathrm{rB2}\,l_2 + \mathrm{bL}\,\mathrm{rg}\,l_2 + \mathrm{bL}^2\,\mathrm{rB2}\,l_2^2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,\mathrm{w2}\,\theta_1 - \mathrm{rg}\,\mathrm{w2}\,\theta_1 + \mathrm{bL}\,r\,\mathrm{w2}\,l_2\,\theta_1 - \\
&\quad \mathrm{bL}\,\mathrm{rg}\,\mathrm{w2}\,l_2\,\theta_1 + \mathrm{w2}\,\beta_0\,\theta_1 + \mathrm{bL}\,\mathrm{w2}\,l_2\,\beta_0\,\theta_1 + \mathrm{w2}\,\beta_2\,\theta_1 + \mathrm{bL}\,\mathrm{w2}\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}^2\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{rB1}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB2}\,\mathrm{w2}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB3}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 - \\
&\quad \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB2}\,\mathrm{w1}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB2}\,\mathrm{w2}^2\,l_2^2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{bL}\,\mathrm{rB2}\,\mathrm{w3}^2\,l_2^2\,\theta_1^2 - \mathrm{be}\,\mathrm{w1}^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w3}^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_0\,(\mathrm{bL} + \mathrm{be}\,\mathrm{w2}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3})\,\theta_1^2 + \mathrm{bL}\,(l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)) - \\
&\quad \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,l_2\,\theta_1^2\,\beta_{1,1} + \beta_{1,2} + \mathrm{bL}\,l_2\,\beta_{1,2} + \mathrm{be}\,\mathrm{w1}^2\,l_2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,\mathrm{w3}^2\,l_2\,\theta_1^2\,\beta_{1,2} - \\
&\quad \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,3} + l_1\,(\mathrm{bL}\,\mathrm{rg} + \mathrm{bL}\,r\,\mathrm{w2}\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w2}\,\theta_1 + \mathrm{bL}\,\mathrm{w2}\,\beta_0\,\theta_1 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}^2\,\theta_1^2 - \\
&\qquad \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}\,\mathrm{w3}\,\theta_1^2 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w3}^2\,\theta_1^2 + \mathrm{bL}\,\mathrm{rB2}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2) + \\
&\qquad \beta_2\,(-\mathrm{bL} + \mathrm{bL}\,\mathrm{w2}\,\theta_1 - \mathrm{be}\,(\mathrm{w1}^2 - \mathrm{w1}\,\mathrm{w2} - \mathrm{w2}\,\mathrm{w3} + \mathrm{w3}^2)\,\theta_1^2) - \mathrm{be}\,\mathrm{w1}\,\mathrm{w2}\,\theta_1^2\,\beta_{1,1} + \\
&\qquad \mathrm{bL}\,\beta_{1,2} + \mathrm{be}\,\mathrm{w1}^2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,\mathrm{w3}^2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,\theta_1^2\,\beta_{1,3}))\;/ \\
&\ ((1 + \mathrm{bL}\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)))\end{aligned}$$

Out[27]=

$$\begin{aligned}&(\mathrm{rg} + \mathrm{bL}\,\mathrm{rg}\,l_1 + \mathrm{bL}\,\mathrm{rB3}\,l_2 + \mathrm{bL}\,\mathrm{rg}\,l_2 + \mathrm{bL}^2\,\mathrm{rB3}\,l_2^2 - \\
&\quad \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,\mathrm{w3}\,\theta_1 - \mathrm{rg}\,\mathrm{w3}\,\theta_1 + \mathrm{bL}\,r\,\mathrm{w3}\,l_2\,\theta_1 - \mathrm{bL}\,\mathrm{rg}\,\mathrm{w3}\,l_2\,\theta_1 + \\
&\quad \mathrm{w3}\,\beta_0\,\theta_1 + \mathrm{bL}\,\mathrm{w3}\,l_2\,\beta_0\,\theta_1 + \mathrm{w3}\,\beta_2\,\theta_1 + \mathrm{bL}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}^2\,l_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB1}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 - \mathrm{be}\,\mathrm{rg}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB2}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 - \\
&\quad \mathrm{be}\,\mathrm{rg}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{rB3}\,\mathrm{w3}^2\,l_2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB3}\,\mathrm{w1}^2\,l_2^2\,\theta_1^2 + \mathrm{be}\,\mathrm{bL}\,\mathrm{rB3}\,\mathrm{w2}^2\,l_2^2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{bL}\,\mathrm{rB3}\,\mathrm{w3}^2\,l_2^2\,\theta_1^2 - \mathrm{be}\,\mathrm{w1}^2\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,\mathrm{w2}^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\beta_2\,\theta_1^2 + l_1\,\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,\mathrm{w3}\,\theta_1 - \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 - (\mathrm{w1} + \mathrm{w2})\,\mathrm{w3})\,\theta_1^2) + \\
&\quad \mathrm{bL}\,\mathrm{rB3}\,l_1\,l_2\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2) + \\
&\quad l_0\,(\mathrm{bL} + \mathrm{be}\,\mathrm{w3}\,(\mathrm{w1} + \mathrm{w2} + \mathrm{w3})\,\theta_1^2 + \mathrm{bL}\,(l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)) - \\
&\quad \mathrm{be}\,\mathrm{w1}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,\mathrm{w2}\,\mathrm{w3}\,l_2\,\theta_1^2\,\beta_{1,2} - l_1\,\theta_1\,(\mathrm{bL}\,(-r + \mathrm{rg})\,\mathrm{w3} - \mathrm{bL}\,\mathrm{w3}\,\beta_0 + \\
&\qquad \mathrm{be}\,\theta_1\,(-\mathrm{rg}\,(\mathrm{w1}^2 + \mathrm{w2}^2) + \mathrm{rg}\,(\mathrm{w1} + \mathrm{w2})\,\mathrm{w3} + \mathrm{w1}\,\mathrm{w3}\,\beta_{1,1} + \mathrm{w2}\,\mathrm{w3}\,\beta_{1,2})) + \\
&\quad \beta_{1,3} + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2)\,\theta_1^2)\,\beta_{1,3})\;/ \\
&\ ((1 + \mathrm{bL}\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(\mathrm{bL} + \mathrm{be}\,(\mathrm{w1}^2 + \mathrm{w2}^2 + \mathrm{w3}^2)\,\theta_1^2)))\end{aligned}$$

In[28]:=
```
LoanRate1 =
  LoanRate1 /. {rB1 → r + σ1, rB2 → r + σ2, rB3 → r + σ3, rg → r + ψ} // FullSimplify
LoanRate2 = LoanRate2 /. {rB1 → r + σ1, rB2 → r + σ2, rB3 → r + σ3, rg → r + ψ} //
  FullSimplify
LoanRate3 = LoanRate3 /. {rB1 → r + σ1, rB2 → r + σ2, rB3 → r + σ3, rg → r + ψ} //
  FullSimplify
```

Out[28]=

$$\begin{aligned}
&(r + \psi + bL\,(r + \psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w1\,\theta_1 - w1\,(r + \psi)\,\theta_1 + bL\,r\,w1\,l_2\,\theta_1 - \\
&\quad bL\,w1\,(r + \psi)\,l_2\,\theta_1 + w1\,\beta_0\,\theta_1 + bL\,w1\,l_2\,\beta_0\,\theta_1 + w1\,\beta_2\,\theta_1 + bL\,w1\,l_2\,\beta_2\,\theta_1 - \\
&\quad be\,w1\,w2\,(r + \psi)\,l_2\,\theta_1^2 + be\,w2^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1\,w3\,(r + \psi)\,l_2\,\theta_1^2 + be\,w3^2\,(r + \psi)\,l_2\,\theta_1^2 + \\
&\quad be\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_0\,(bL + be\,w1\,(w1 + w2 + w3)\,\theta_1^2 + bL\,(l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)) + \\
&\quad bL\,l_2\,(r + \sigma_1) + bL^2\,l_2^2\,(r + \sigma_1) + be\,w1^2\,l_2\,\theta_1^2\,(r + \sigma_1) + be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r + \sigma_1) + \\
&\quad be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r + \sigma_1) + be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r + \sigma_1) + be\,w1\,w2\,l_2\,\theta_1^2\,(r + \sigma_2) + \\
&\quad be\,w1\,w3\,l_2\,\theta_1^2\,(r + \sigma_3) + \beta_{1,1} + bL\,l_2\,\beta_{1,1} + be\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1} + be\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1} - \\
&\quad be\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3} + l_1\,(bL\,(r + \psi) + bL\,r\,w1\,\theta_1 - bL\,w1\,(r + \psi)\,\theta_1 + \\
&\qquad bL\,w1\,\beta_0\,\theta_1 - be\,w1\,w2\,(r + \psi)\,\theta_1^2 + be\,w2^2\,(r + \psi)\,\theta_1^2 - be\,w1\,w3\,(r + \psi)\,\theta_1^2 + \\
&\qquad be\,w3^2\,(r + \psi)\,\theta_1^2 + \beta_2\,(-bL + bL\,w1\,\theta_1 + be\,(-w2^2 - w3^2 + w1\,(w2 + w3))\,\theta_1^2) + \\
&\qquad bL\,l_2\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r + \sigma_1) + bL\,\beta_{1,1} + \\
&\qquad be\,w2^2\,\theta_1^2\,\beta_{1,1} + be\,w3^2\,\theta_1^2\,\beta_{1,1} - be\,w1\,w2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,\theta_1^2\,\beta_{1,3}))\,/ \\
&\ ((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)))
\end{aligned}$$

Out[29]=

$$\begin{aligned}
&(r + \psi + bL\,(r + \psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w2\,\theta_1 - w2\,(r + \psi)\,\theta_1 + bL\,r\,w2\,l_2\,\theta_1 - \\
&\quad bL\,w2\,(r + \psi)\,l_2\,\theta_1 + w2\,\beta_0\,\theta_1 + bL\,w2\,l_2\,\beta_0\,\theta_1 + w2\,\beta_2\,\theta_1 + bL\,w2\,l_2\,\beta_2\,\theta_1 + \\
&\quad be\,w1^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1\,w2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w2\,w3\,(r + \psi)\,l_2\,\theta_1^2 + be\,w3^2\,(r + \psi)\,l_2\,\theta_1^2 - \\
&\quad be\,w1^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 + be\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad l_0\,(bL + be\,w2\,(w1 + w2 + w3)\,\theta_1^2 + bL\,(l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)) + \\
&\quad be\,w1\,w2\,l_2\,\theta_1^2\,(r + \sigma_1) + bL\,l_2\,(r + \sigma_2) + bL^2\,l_2^2\,(r + \sigma_2) + be\,w2^2\,l_2\,\theta_1^2\,(r + \sigma_2) + \\
&\quad be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r + \sigma_2) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r + \sigma_2) + be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r + \sigma_2) + \\
&\quad be\,w2\,w3\,l_2\,\theta_1^2\,(r + \sigma_3) - be\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,1} + \beta_{1,2} + bL\,l_2\,\beta_{1,2} + be\,w1^2\,l_2\,\theta_1^2\,\beta_{1,2} + \\
&\quad be\,w3^2\,l_2\,\theta_1^2\,\beta_{1,2} - be\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,3} + l_1\,(bL\,(r + \psi) + bL\,r\,w2\,\theta_1 - bL\,w2\,(r + \psi)\,\theta_1 + \\
&\qquad bL\,w2\,\beta_0\,\theta_1 + be\,w1^2\,(r + \psi)\,\theta_1^2 - be\,w1\,w2\,(r + \psi)\,\theta_1^2 - be\,w2\,w3\,(r + \psi)\,\theta_1^2 + \\
&\qquad be\,w3^2\,(r + \psi)\,\theta_1^2 + \beta_2\,(-bL + bL\,w2\,\theta_1 - be\,(w1^2 - w1\,w2 + w3\,(-w2 + w3))\,\theta_1^2) + \\
&\qquad bL\,l_2\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r + \sigma_2) - be\,w1\,w2\,\theta_1^2\,\beta_{1,1} + \\
&\qquad bL\,\beta_{1,2} + be\,w1^2\,\theta_1^2\,\beta_{1,2} + be\,w3^2\,\theta_1^2\,\beta_{1,2} - be\,w2\,w3\,\theta_1^2\,\beta_{1,3}))\,/ \\
&\ ((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)))
\end{aligned}$$

Out[30]=

$$\begin{aligned}
&(r + \psi + bL\,(r + \psi)\,l_1 + bL\,(r + \psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w3\,\theta_1 - w3\,(r + \psi)\,\theta_1 + \\
&\quad bL\,r\,w3\,l_2\,\theta_1 - bL\,w3\,(r + \psi)\,l_2\,\theta_1 + w3\,\beta_0\,\theta_1 + bL\,w3\,l_2\,\beta_0\,\theta_1 + w3\,\beta_2\,\theta_1 + \\
&\quad bL\,w3\,l_2\,\beta_2\,\theta_1 + be\,w1^2\,(r + \psi)\,l_2\,\theta_1^2 + be\,w2^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1\,w3\,(r + \psi)\,l_2\,\theta_1^2 - \\
&\quad be\,w2\,w3\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1^2\,l_2\,\beta_2\,\theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 + \\
&\quad be\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 + l_1\,\beta_2\,(-bL + bL\,w3\,\theta_1 - be\,(w1^2 + w2^2 - (w1 + w2)\,w3)\,\theta_1^2) + \\
&\quad l_0\,(bL + be\,w3\,(w1 + w2 + w3)\,\theta_1^2 + bL\,(l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)) + \\
&\quad be\,w1\,w3\,l_2\,\theta_1^2\,(r + \sigma_1) + be\,w2\,w3\,l_2\,\theta_1^2\,(r + \sigma_2) + bL\,l_2\,(r + \sigma_3) + bL^2\,l_2^2\,(r + \sigma_3) + \\
&\quad be\,w3^2\,l_2\,\theta_1^2\,(r + \sigma_3) + be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r + \sigma_3) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r + \sigma_3) + \\
&\quad be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r + \sigma_3) + bL\,l_1\,l_2\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r + \sigma_3) - \\
&\quad be\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,1} - be\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,2} - l_1\,\theta_1 \\
&\quad\ (bL\,w3\,\psi - bL\,w3\,\beta_0 + be\,\theta_1\,(-(w1^2 + w2^2 - (w1 + w2)\,w3)\,(r + \psi) + w1\,w3\,\beta_{1,1} + w2\,w3\,\beta_{1,2})) + \\
&\quad \beta_{1,3} + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2)\,\theta_1^2)\,\beta_{1,3})\,/ \\
&\ ((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)))
\end{aligned}$$

In[31]:=

```
In[32]:= SimLoanRateReaction1 = D[LoanRate1, r] // FullSimplify
         SimLoanRateReaction2 = D[LoanRate2, r] // FullSimplify
         SimLoanRateReaction3 = D[LoanRate3, r] // FullSimplify

Out[32]= (l1 (bL (1 + bL l2) + be (w2^2 + w3^2 - w1 (w2 + w3) + bL (w1^2 + w2^2 + w3^2) l2) θ1^2) +
           (1 + bL l2) (1 + l2 (bL + be (w1^2 + w2^2 + w3^2) θ1^2))) /
          ((1 + bL (l1 + l2)) (1 + (l1 + l2) (bL + be (w1^2 + w2^2 + w3^2) θ1^2)))

Out[33]= (l1 (bL (1 + bL l2) + be (w1^2 - w1 w2 - w2 w3 + w3^2 + bL (w1^2 + w2^2 + w3^2) l2) θ1^2) +
           (1 + bL l2) (1 + l2 (bL + be (w1^2 + w2^2 + w3^2) θ1^2))) /
          ((1 + bL (l1 + l2)) (1 + (l1 + l2) (bL + be (w1^2 + w2^2 + w3^2) θ1^2)))

Out[34]= (l1 (bL (1 + bL l2) + be (w1^2 + w2^2 - (w1 + w2) w3 + bL (w1^2 + w2^2 + w3^2) l2) θ1^2) +
           (1 + bL l2) (1 + l2 (bL + be (w1^2 + w2^2 + w3^2) θ1^2))) /
          ((1 + bL (l1 + l2)) (1 + (l1 + l2) (bL + be (w1^2 + w2^2 + w3^2) θ1^2)))

In[35]:= SimLoanRateReaction1 /. {w1 → .3451, w2 → .8200, w3 → 1.0203} // FullSimplify
         SimLoanRateReaction2 /. {w1 → .3451, w2 → .8200, w3 → 1.0203} // FullSimplify
         SimLoanRateReaction3 /. {w1 → .3451, w2 → .8200, w3 → 1.0203} // FullSimplify

Out[35]= ((1 + bL l2) (1 + l2 (bL + 1.83251 be θ1^2)) +
           l1 (bL + 1.07832 be θ1^2 + bL l2 (bL + 1.83251 be θ1^2))) /
          ((1 + bL (l1 + l2)) (1 + (l1 + l2) (bL + 1.83251 be θ1^2)))

Out[36]= ((1 + bL l2) (1 + l2 (bL + 1.83251 be θ1^2)) +
           l1 (bL + 0.0404781 be θ1^2 + bL l2 (bL + 1.83251 be θ1^2))) /
          ((1 + bL (l1 + l2)) (1 + (l1 + l2) (bL + 1.83251 be θ1^2)))

Out[37]= ((1 + bL l2) (1 + l2 (bL + 1.83251 be θ1^2)) +
           l1 (bL - 0.397258 be θ1^2 + bL l2 (bL + 1.83251 be θ1^2))) /
          ((1 + bL (l1 + l2)) (1 + (l1 + l2) (bL + 1.83251 be θ1^2)))
```

In[38]:=
```
LoanDemand1 = LoanDemand1 /. {rL1 → LoanRate1, rB1 → r + σ1} // FullSimplify
LoanDemand2 = LoanDemand2 /. {rL2 → LoanRate2, rB2 → r + σ2} // FullSimplify
LoanDemand3 = LoanDemand3 /. {rL3 → LoanRate3, rB3 → r + σ3} // FullSimplify
```

Out[38]=

$$\begin{aligned}
&-(l_0\,(-1+(l_1+l_2)\,(-bL+be\,(-w2^2-w3^2+w1\,(w2+w3))\,\Theta_1^2))+l_1^2\,(bL\,w1\,(-\psi+\beta_0)\,\Theta_1+\\
&\quad \beta_2\,(-bL+bL\,w1\,\Theta_1+be\,(-w2^2-w3^2+w1\,(w2+w3))\,\Theta_1^2)+bL\,(r+\psi+\beta_{1,1})+\\
&\quad be\,\Theta_1^2\,((w2^2+w3^2-w1\,(w2+w3))\,(r+\psi)+(w2^2+w3^2)\,\beta_{1,1}-w1\,(w2\,\beta_{1,2}+w3\,\beta_{1,3})))+\\
&\quad l_1\,(r+\psi-\beta_2+w1\,(-\psi+\beta_0+\beta_2)\,\Theta_1+\beta_{1,1}+l_2\,(-2\,bL\,w1\,(\psi-\beta_0)\,\Theta_1+2\,\beta_2\\
&\qquad (-bL+bL\,w1\,\Theta_1+be\,((w1-w2)\,w2+w1\,w3-w3^2)\,\Theta_1^2)+bL\,(r+2\,\psi-\sigma_1+2\,\beta_{1,1})+\\
&\qquad be\,\Theta_1^2\,((w2^2+w3^2-w1\,(w2+w3))\,(r+2\,\psi)-(w2^2+w3^2)\,\sigma_1+w1\,w2\,\sigma_2+\\
&\qquad w1\,w3\,\sigma_3+2\,w2^2\,\beta_{1,1}+2\,w3^2\,\beta_{1,1}-2\,w1\,w2\,\beta_{1,2}-2\,w1\,w3\,\beta_{1,3})))+\\
&\quad l_2\,(\psi-\beta_2+w1\,(-\psi+\beta_0+\beta_2)\,\Theta_1-\sigma_1+\beta_{1,1}+l_2\,(-bL\,w1\,(\psi-\beta_0)\,\Theta_1+\\
&\qquad \beta_2\,(-bL+bL\,w1\,\Theta_1+be\,((w1-w2)\,w2+w1\,w3-w3^2)\,\Theta_1^2)+bL\,(\psi-\sigma_1+\beta_{1,1})+\\
&\qquad be\,\Theta_1^2\,((w2^2+w3^2-w1\,(w2+w3))\,\psi-(w2^2+w3^2)\,\sigma_1+w1\,w2\,\sigma_2+\\
&\qquad w1\,w3\,\sigma_3+w2^2\,\beta_{1,1}+w3^2\,\beta_{1,1}-w1\,w2\,\beta_{1,2}-w1\,w3\,\beta_{1,3}))))\,/\\
&\quad ((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))
\end{aligned}$$

Out[39]=

$$\begin{aligned}
&-(l_0\,(-1+(l_1+l_2)\,(-bL-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2))+l_1^2\,(bL\,w2\,(-\psi+\beta_0)\,\Theta_1+\\
&\quad \beta_2\,(-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2)+bL\,(r+\psi+\beta_{1,2})+be\,\Theta_1^2\\
&\quad ((w1^2-w1\,w2-w2\,w3+w3^2)\,(r+\psi)-w1\,w2\,\beta_{1,1}+(w1^2+w3^2)\,\beta_{1,2}-w2\,w3\,\beta_{1,3}))+\\
&\quad l_1\,(r+\psi-\beta_2+w2\,(-\psi+\beta_0+\beta_2)\,\Theta_1+\beta_{1,2}+l_2\,(-2\,bL\,w2\,(\psi-\beta_0)\,\Theta_1+2\,\beta_2\\
&\qquad (-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2)+bL\,(r+2\,\psi-\sigma_2+2\,\beta_{1,2})+\\
&\qquad be\,\Theta_1^2\,((w1^2-w1\,w2-w2\,w3+w3^2)\,(r+2\,\psi)+w1\,w2\,\sigma_1-(w1^2+w3^2)\,\sigma_2+\\
&\qquad w2\,w3\,\sigma_3-2\,w1\,w2\,\beta_{1,1}+2\,w1^2\,\beta_{1,2}+2\,w3^2\,\beta_{1,2}-2\,w2\,w3\,\beta_{1,3})))+\\
&\quad l_2\,(\psi-\beta_2+w2\,(-\psi+\beta_0+\beta_2)\,\Theta_1-\sigma_2+\beta_{1,2}+l_2\,(-bL\,w2\,(\psi-\beta_0)\,\Theta_1+\\
&\qquad \beta_2\,(-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2)+bL\,(\psi-\sigma_2+\beta_{1,2})+\\
&\qquad be\,\Theta_1^2\,((w1^2-w1\,w2-w2\,w3+w3^2)\,\psi+w1\,w2\,\sigma_1-(w1^2+w3^2)\,\sigma_2+\\
&\qquad w2\,w3\,\sigma_3-w1\,w2\,\beta_{1,1}+w1^2\,\beta_{1,2}+w3^2\,\beta_{1,2}-w2\,w3\,\beta_{1,3}))))\,/\\
&\quad ((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))
\end{aligned}$$

Out[40]=

$$\begin{aligned}
&-(l_0\,(-1+(l_1+l_2)\,(-bL-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2))+l_1^2\,(bL\,w3\,(-\psi+\beta_0)\,\Theta_1+\\
&\quad \beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+bL\,(r+\psi+\beta_{1,3})+\\
&\quad be\,\Theta_1^2\,((w1^2+w2^2-(w1+w2)\,w3)\,(r+\psi)-w3\,(w1\,\beta_{1,1}+w2\,\beta_{1,2})+(w1^2+w2^2)\,\beta_{1,3}))+\\
&\quad l_2\,(\psi-\beta_2+w3\,(-\psi+\beta_0+\beta_2)\,\Theta_1-\sigma_3+\beta_{1,3}+\\
&\qquad l_2\,(-bL\,w3\,(\psi-\beta_0)\,\Theta_1+\beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+\\
&\qquad bL\,(\psi-\sigma_3+\beta_{1,3})+be\,\Theta_1^2\,((w1^2+w2^2-(w1+w2)\,w3)\,\psi+w1\,w3\,\sigma_1+\\
&\qquad w2\,w3\,\sigma_2-w1^2\,\sigma_3-w2^2\,\sigma_3-w1\,w3\,\beta_{1,1}-w2\,w3\,\beta_{1,2}+(w1^2+w2^2)\,\beta_{1,3})))+\\
&\quad l_1\,(r+\psi-\beta_2+w3\,(-\psi+\beta_0+\beta_2)\,\Theta_1+\beta_{1,3}+l_2\,(-2\,bL\,w3\,(\psi-\beta_0)\,\Theta_1+\\
&\qquad 2\,\beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+bL\,(r+2\,\psi-\sigma_3+2\,\beta_{1,3})+\\
&\qquad be\,\Theta_1^2\,((w1^2+w2^2-(w1+w2)\,w3)\,(r+2\,\psi)+w1\,w3\,\sigma_1+w2\,w3\,\sigma_2-\\
&\qquad w1^2\,\sigma_3-w2^2\,\sigma_3-2\,w1\,w3\,\beta_{1,1}-2\,w2\,w3\,\beta_{1,2}+2\,(w1^2+w2^2)\,\beta_{1,3}))))\,/\\
&\quad ((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))
\end{aligned}$$

In[41]:=
```
r1 = D[LoanDemand1, r] // FullSimplify
r2 = D[LoanDemand2, r] // FullSimplify
r3 = D[LoanDemand3, r] // FullSimplify
```

Out[41]= $-\frac{l_1 (1 + (l_1 + l_2) (bL + be (w2^2 + w3^2 - w1 (w2 + w3)) \theta_1^2))}{(1 + bL (l_1 + l_2)) (1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 + w3^2) \theta_1^2))}$

Out[42]= $-\frac{l_1 (1 + (l_1 + l_2) (bL + be (w1^2 - w1\, w2 + w3 (-w2 + w3)) \theta_1^2))}{(1 + bL (l_1 + l_2)) (1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 + w3^2) \theta_1^2))}$

Out[43]= $-\frac{l_1 (1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 - (w1 + w2) w3) \theta_1^2))}{(1 + bL (l_1 + l_2)) (1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 + w3^2) \theta_1^2))}$

In[44]:=
```
D[r1, θ1] // FullSimplify
D[r2, θ1] // FullSimplify
D[r3, θ1] // FullSimplify
```

Out[44]= $\frac{2\, be\, w1 (w1 + w2 + w3)\, l_1 (l_1 + l_2) \theta_1}{(1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 + w3^2) \theta_1^2))^2}$

Out[45]= $\frac{2\, be\, w2 (w1 + w2 + w3)\, l_1 (l_1 + l_2) \theta_1}{(1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 + w3^2) \theta_1^2))^2}$

Out[46]= $\frac{2\, be\, w3 (w1 + w2 + w3)\, l_1 (l_1 + l_2) \theta_1}{(1 + (l_1 + l_2) (bL + be (w1^2 + w2^2 + w3^2) \theta_1^2))^2}$

In[47]:=
```
r1 /. {w1 → .3451, w2 → .8200, w3 → 1.0203}
r2 /. {w1 → .3451, w2 → .8200, w3 → 1.0203}
r3 /. {w1 → .3451, w2 → .8200, w3 → 1.0203}
```

Out[47]= $-\frac{l_1 (1 + (l_1 + l_2) (bL + 1.07832\, be\, \theta_1^2))}{(1 + bL (l_1 + l_2)) (1 + (l_1 + l_2) (bL + 1.83251\, be\, \theta_1^2))}$

Out[48]= $-\frac{l_1 (1 + (l_1 + l_2) (bL + 0.0404781\, be\, \theta_1^2))}{(1 + bL (l_1 + l_2)) (1 + (l_1 + l_2) (bL + 1.83251\, be\, \theta_1^2))}$

Out[49]= $-\frac{l_1 (1 + (l_1 + l_2) (bL - 0.397258\, be\, \theta_1^2))}{(1 + bL (l_1 + l_2)) (1 + (l_1 + l_2) (bL + 1.83251\, be\, \theta_1^2))}$

In[50]:=
```
DepositsDemand = d0 + d1 rd + d2 (rd - rg)
```

Out[50]= $d_0 + rd\, d_1 + (rd - rg)\, d_2$

In[51]:=
```
Solve[Deposits = DepositsDemand, rd] // Flatten // FullSimplify
```

Out[51]= $\left\{rd \to \frac{rg - rg\,\rho - bd\, d_0 + bd\, rg\, d_2 + (-1 + \rho) \beta_2}{1 + bd (d_1 + d_2)}\right\}$

In[52]:=
```
DepositRate = rd /. %
```

Out[52]= $\frac{rg - rg\,\rho - bd\, d_0 + bd\, rg\, d_2 + (-1 + \rho) \beta_2}{1 + bd (d_1 + d_2)}$

In[53]:=
```
DepositRate = DepositRate /. rg → r + ψ
```

Out[53]= $\frac{r + \psi - \rho (r + \psi) - bd\, d_0 + bd (r + \psi) d_2 + (-1 + \rho) \beta_2}{1 + bd (d_1 + d_2)}$

```
In[54]:= D[DepositRate, r]
```

Out[54]=
$$\frac{1-\rho+\text{bd}\ d_2}{1+\text{bd}\ (d_1+d_2)}$$

```
In[55]:= Eq =
           Eq /. {rL1 → LoanRate1, rL2 → LoanRate2, rL3 → LoanRate3, rg → r + ψ} // FullSimplify
```

Out[55]=
$$\begin{aligned}&(\theta_1\ ((\text{w1}+\text{w2}+\text{w3})\ l_0-l_1\ ((\text{w1}+\text{w2}+\text{w3})\ (r+\psi)-(\text{w1}+\text{w2}+\text{w3})\ \beta_2-\\&\qquad(\text{w1}^2+\text{w2}^2+\text{w3}^2)\ (\psi-\beta_0)\ \theta_1+(\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \beta_2\ \theta_1+\text{w1}\ \beta_{1,1}+\text{w2}\ \beta_{1,2}+\text{w3}\ \beta_{1,3})-\\&\quad l_2\ ((\text{w1}+\text{w2}+\text{w3})\ \psi-(\text{w1}+\text{w2}+\text{w3})\ \beta_2-(\text{w1}^2+\text{w2}^2+\text{w3}^2)\ (\psi-\beta_0)\ \theta_1+\\&\qquad(\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \beta_2\ \theta_1+\text{w1}\ (-\sigma_1+\beta_{1,1})+\text{w2}\ (-\sigma_2+\beta_{1,2})+\text{w3}\ (-\sigma_3+\beta_{1,3}))))\ /\\&\ (1+(l_1+l_2)\ (\text{bL}+\text{be}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \theta_1^2))\end{aligned}$$

```
In[56]:= D[Eq, r] // FullSimplify
```

Out[56]=
$$-\frac{(\text{w1}+\text{w2}+\text{w3})\ l_1\ \theta_1}{1+(l_1+l_2)\ (\text{bL}+\text{be}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \theta_1^2)}$$

```
In[57]:= G = G /. {rg → r + ψ, rd → DepositRate} // FullSimplify
```

Out[57]=
$$\begin{aligned}&\Big(\beta_2\ (-\text{bL}\ (3\ \text{bd}+\text{bL}\ (-1+\rho)^2)+2\ \text{bd}\ \text{bL}\ (\text{w1}+\text{w2}+\text{w3})\ \theta_1-\\&\qquad(\text{bd}\ (\text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)+2\ \text{be}\ (\text{w1}^2-\text{w1}\ \text{w2}+\text{w2}^2-(\text{w1}+\text{w2})\ \text{w3}+\text{w3}^2))+\\&\qquad\quad\text{be}\ \text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ (-1+\rho)^2)\ \theta_1^2)+\\&\quad\text{bd}\ \text{bL}\ \theta_1\ (\text{rL1}\ \text{w1}+\text{rL2}\ \text{w2}+\text{rL3}\ \text{w3}-r\ (\text{w1}+\text{w2}+\text{w3})-2\ (\text{w1}+\text{w2}+\text{w3})\ \psi+\\&\qquad(\text{w1}+\text{w2}+\text{w3})\ \beta_0-\text{w1}\ \beta_{1,1}-\text{w2}\ \beta_{1,2}-\text{w3}\ \beta_{1,3})+\text{bL}\ \Big(-\text{bd}\ (-3\ r+\text{rL1}+\text{rL2}+\text{rL3}-3\ \psi)+\\&\qquad\frac{\text{bL}\ (-1+\rho)\ (-\text{bd}\ d_0+\text{bd}\ (r+\psi)\ ((-1+\rho)\ d_1+\rho\ d_2)+(-1+\rho)\ \beta_2)}{1+\text{bd}\ (d_1+d_2)}+\\&\qquad\text{bd}\ (\beta_{1,1}+\beta_{1,2}+\beta_{1,3})\Big)+\\&\quad\theta_1^2\ \Big(\text{bd}\ (\text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \psi+\text{be}\ (-\text{rL2}\ \text{w1}^2-\text{rL3}\ \text{w1}^2+\text{rL1}\ \text{w1}\ \text{w2}+\text{rL2}\ \text{w1}\ \text{w2}-\\&\qquad\quad\text{rL1}\ \text{w2}^2-\text{rL3}\ \text{w2}^2+((\text{rL1}+\text{rL3})\ \text{w1}+(\text{rL2}+\text{rL3})\ \text{w2})\ \text{w3}-(\text{rL1}+\text{rL2})\ \text{w3}^2+\\&\qquad\quad2\ (\text{w1}^2+\text{w2}^2-\text{w2}\ \text{w3}+\text{w3}^2-\text{w1}\ (\text{w2}+\text{w3}))\ (r+\psi)))-\text{bd}\ \text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \beta_0+\\&\qquad\frac{\text{be}\ \text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ (-1+\rho)\ (-\text{bd}\ d_0+\text{bd}\ (r+\psi)\ ((-1+\rho)\ d_1+\rho\ d_2)+(-1+\rho)\ \beta_2)}{1+\text{bd}\ (d_1+d_2)}\\&\qquad+\text{bd}\ \text{be}\ ((\text{w2}^2+\text{w3}^2-\text{w1}\ (\text{w2}+\text{w3}))\ \beta_{1,1}+(\text{w1}^2-\text{w1}\ \text{w2}-\text{w2}\ \text{w3}+\text{w3}^2)\ \beta_{1,2}+\\&\qquad\quad(\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\ \text{w3})\ \beta_{1,3})\Big)\Big)\Big/\ (\text{bd}\ \text{bL}\ (\text{bL}+\text{be}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \theta_1^2))\end{aligned}$$

```
In[58]:= G = G /. rL1 → LoanRate1
```

Out[58]=
$$\begin{aligned}&\Big(\beta_2\ (-\text{bL}\ (3\ \text{bd}+\text{bL}\ (-1+\rho)^2)+2\ \text{bd}\ \text{bL}\ (\text{w1}+\text{w2}+\text{w3})\ \theta_1-\\&\qquad(\text{bd}\ (\text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)+2\ \text{be}\ (\text{w1}^2-\text{w1}\ \text{w2}+\text{w2}^2-(\text{w1}+\text{w2})\ \text{w3}+\text{w3}^2))+\\&\qquad\quad\text{be}\ \text{bL}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ (-1+\rho)^2)\ \theta_1^2)+\\&\quad\text{bd}\ \text{bL}\ \theta_1\ (\text{rL2}\ \text{w2}+\text{rL3}\ \text{w3}-r\ (\text{w1}+\text{w2}+\text{w3})-2\ (\text{w1}+\text{w2}+\text{w3})\ \psi+\\&\qquad(\text{w1}+\text{w2}+\text{w3})\ \beta_0-\text{w1}\ \beta_{1,1}-\text{w2}\ \beta_{1,2}-\text{w3}\ \beta_{1,3}+\\&\qquad(\text{w1}\ (r+\psi+\text{bL}\ (r+\psi)\ l_2-\beta_2-\text{bL}\ l_2\ \beta_2+r\ \text{w1}\ \theta_1-\text{w1}\ (r+\psi)\ \theta_1+\text{bL}\ r\ \text{w1}\ l_2\ \theta_1-\\&\qquad\quad\text{bL}\ \text{w1}\ (r+\psi)\ l_2\ \theta_1+\text{w1}\ \beta_0\ \theta_1+\text{bL}\ \text{w1}\ l_2\ \beta_0\ \theta_1+\text{w1}\ \beta_2\ \theta_1+\text{bL}\ \text{w1}\ l_2\ \beta_2\ \theta_1-\text{be}\ \text{w1}\ \text{w2}\\&\qquad\quad\ (r+\psi)\ l_2\ \theta_1^2+\text{be}\ \text{w2}^2\ (r+\psi)\ l_2\ \theta_1^2-\text{be}\ \text{w1}\ \text{w3}\ (r+\psi)\ l_2\ \theta_1^2+\text{be}\ \text{w3}^2\ (r+\psi)\ l_2\ \theta_1^2+\\&\qquad\quad\text{be}\ \text{w1}\ \text{w2}\ l_2\ \beta_2\ \theta_1^2-\text{be}\ \text{w2}^2\ l_2\ \beta_2\ \theta_1^2+\text{be}\ \text{w1}\ \text{w3}\ l_2\ \beta_2\ \theta_1^2-\text{be}\ \text{w3}^2\ l_2\ \beta_2\ \theta_1^2+\\&\qquad\quad l_0\ (\text{bL}+\text{be}\ \text{w1}\ (\text{w1}+\text{w2}+\text{w3})\ \theta_1^2+\text{bL}\ (l_1+l_2)\ (\text{bL}+\text{be}\ (\text{w1}^2+\text{w2}^2+\text{w3}^2)\ \theta_1^2))+\\&\qquad\quad\text{bL}\ l_2\ (r+\sigma_1)+\text{bL}^2\ l_2^2\ (r+\sigma_1)+\text{be}\ \text{w1}^2\ l_2\ \theta_1^2\ (r+\sigma_1)+\end{aligned}$$

$$\mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+$$
$$\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_2)+\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_3)+\beta_{1,1}+\mathrm{bL}\,l_2\,\beta_{1,1}+$$
$$\mathrm{be}\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1}+\mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1}-\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2}-\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3}+$$
$$l_1\,(\mathrm{bL}\,(r+\psi)+\mathrm{bL}\,r\,w1\,\theta_1-\mathrm{bL}\,w1\,(r+\psi)\,\theta_1+\mathrm{bL}\,w1\,\beta_0\,\theta_1-\mathrm{be}\,w1\,w2\,(r+\psi)\,\theta_1^2+$$
$$\mathrm{be}\,w2^2\,(r+\psi)\,\theta_1^2-\mathrm{be}\,w1\,w3\,(r+\psi)\,\theta_1^2+\mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2+$$
$$\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,w1\,\theta_1+\mathrm{be}\,(-w2^2-w3^2+w1\,(w2+w3))\,\theta_1^2)+$$
$$\mathrm{bL}\,l_2\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_1)+\mathrm{bL}\,\beta_{1,1}+$$
$$\mathrm{be}\,w2^2\,\theta_1^2\,\beta_{1,1}+\mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,1}-\mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,2}-\mathrm{be}\,w1\,w3\,\theta_1^2\,\beta_{1,3})))\,/$$
$$((1+\mathrm{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2))))+$$
$$\mathrm{bL}\,\Bigg(\frac{\mathrm{bL}\,(-1+\rho)\,(-\mathrm{bd}\,d_0+\mathrm{bd}\,(r+\psi)\,((-1+\rho)\,d_1+\rho\,d_2)+(-1+\rho)\,\beta_2)}{1+\mathrm{bd}\,(d_1+d_2)}+$$
$$\mathrm{bd}\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3})-$$
$$\mathrm{bd}\,(-3\,r+rL2+rL3-3\,\psi+(r+\psi+\mathrm{bL}\,(r+\psi)\,l_2-\beta_2-\mathrm{bL}\,l_2\,\beta_2+r\,w1\,\theta_1-$$
$$w1\,(r+\psi)\,\theta_1+\mathrm{bL}\,r\,w1\,l_2\,\theta_1-\mathrm{bL}\,w1\,(r+\psi)\,l_2\,\theta_1+w1\,\beta_0\,\theta_1+$$
$$\mathrm{bL}\,w1\,l_2\,\beta_0\,\theta_1+w1\,\beta_2\,\theta_1+\mathrm{bL}\,w1\,l_2\,\beta_2\,\theta_1-\mathrm{be}\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2+$$
$$\mathrm{be}\,w2^2\,(r+\psi)\,l_2\,\theta_1^2-\mathrm{be}\,w1\,w3\,(r+\psi)\,l_2\,\theta_1^2+\mathrm{be}\,w3^2\,(r+\psi)\,l_2\,\theta_1^2+$$
$$\mathrm{be}\,w1\,w2\,l_2\,\beta_2\,\theta_1^2-\mathrm{be}\,w2^2\,l_2\,\beta_2\,\theta_1^2+\mathrm{be}\,w1\,w3\,l_2\,\beta_2\,\theta_1^2-\mathrm{be}\,w3^2\,l_2\,\beta_2\,\theta_1^2+$$
$$l_0\,(\mathrm{bL}+\mathrm{be}\,w1\,(w1+w2+w3)\,\theta_1^2+\mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2))+$$
$$\mathrm{bL}\,l_2\,(r+\sigma_1)+\mathrm{bL}^2\,l_2^2\,(r+\sigma_1)+\mathrm{be}\,w1^2\,l_2\,\theta_1^2\,(r+\sigma_1)+$$
$$\mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+$$
$$\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_2)+\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_3)+\beta_{1,1}+\mathrm{bL}\,l_2\,\beta_{1,1}+$$
$$\mathrm{be}\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1}+\mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1}-\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2}-\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3}+$$
$$l_1\,(\mathrm{bL}\,(r+\psi)+\mathrm{bL}\,r\,w1\,\theta_1-\mathrm{bL}\,w1\,(r+\psi)\,\theta_1+\mathrm{bL}\,w1\,\beta_0\,\theta_1-\mathrm{be}\,w1\,w2$$
$$(r+\psi)\,\theta_1^2+\mathrm{be}\,w2^2\,(r+\psi)\,\theta_1^2-\mathrm{be}\,w1\,w3\,(r+\psi)\,\theta_1^2+\mathrm{be}\,w3^2\,(r+\psi)$$
$$\theta_1^2+\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,w1\,\theta_1+\mathrm{be}\,(-w2^2-w3^2+w1\,(w2+w3))\,\theta_1^2)+\mathrm{bL}$$
$$l_2\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_1)+\mathrm{bL}\,\beta_{1,1}+\mathrm{be}\,w2^2$$
$$\theta_1^2\,\beta_{1,1}+\mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,1}-\mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,2}-\mathrm{be}\,w1\,w3\,\theta_1^2\,\beta_{1,3}))\,/$$
$$((1+\mathrm{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2))))\Bigg)+$$
$$\theta_1^2\,\Bigg(-\mathrm{bd}\,\mathrm{bL}\,(w1^2+w2^2+w3^2)\,\beta_0+$$
$$\frac{\mathrm{be}\,\mathrm{bL}\,(w1^2+w2^2+w3^2)\,(-1+\rho)\,(-\mathrm{bd}\,d_0+\mathrm{bd}\,(r+\psi)\,((-1+\rho)\,d_1+\rho\,d_2)+(-1+\rho)\,\beta_2)}{1+\mathrm{bd}\,(d_1+d_2)}$$
$$+\mathrm{bd}\,\mathrm{be}\,((w2^2+w3^2-w1\,(w2+w3))\,\beta_{1,1}+$$
$$(w1^2-w1\,w2-w2\,w3+w3^2)\,\beta_{1,2}+(w1^2+w2^2-(w1+w2)\,w3)\,\beta_{1,3})+$$
$$\mathrm{bd}\,(\mathrm{bL}\,(w1^2+w2^2+w3^2)\,\psi+\mathrm{be}\,(-rL2\,w1^2-rL3\,w1^2+rL2\,w1\,w2-rL3\,w2^2+$$
$$2\,(w1^2+w2^2-w2\,w3+w3^2-w1\,(w2+w3))\,(r+\psi)+$$
$$(w1\,w2\,(r+\psi+\mathrm{bL}\,(r+\psi)\,l_2-\beta_2-\mathrm{bL}\,l_2\,\beta_2+r\,w1\,\theta_1-w1\,(r+\psi)\,\theta_1+$$
$$\mathrm{bL}\,r\,w1\,l_2\,\theta_1-\mathrm{bL}\,w1\,(r+\psi)\,l_2\,\theta_1+w1\,\beta_0\,\theta_1+\mathrm{bL}\,w1\,l_2\,\beta_0\,\theta_1+w1\,\beta_2\,\theta_1+$$
$$\mathrm{bL}\,w1\,l_2\,\beta_2\,\theta_1-\mathrm{be}\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2+\mathrm{be}\,w2^2\,(r+\psi)\,l_2\,\theta_1^2-\mathrm{be}\,w1\,w3$$
$$(r+\psi)\,l_2\,\theta_1^2+\mathrm{be}\,w3^2\,(r+\psi)\,l_2\,\theta_1^2+\mathrm{be}\,w1\,w2\,l_2\,\beta_2\,\theta_1^2-\mathrm{be}\,w2^2\,l_2\,\beta_2\,\theta_1^2+$$
$$\mathrm{be}\,w1\,w3\,l_2\,\beta_2\,\theta_1^2-\mathrm{be}\,w3^2\,l_2\,\beta_2\,\theta_1^2+l_0\,(\mathrm{bL}+\mathrm{be}\,w1\,(w1+w2+w3)\,\theta_1^2+$$
$$\mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2))+\mathrm{bL}\,l_2\,(r+\sigma_1)+$$
$$\mathrm{bL}^2\,l_2^2\,(r+\sigma_1)+\mathrm{be}\,w1^2\,l_2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+$$
$$\mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_1)+\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_2)+$$
$$\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_3)+\beta_{1,1}+\mathrm{bL}\,l_2\,\beta_{1,1}+\mathrm{be}\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1}+$$
$$\mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1}-\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2}-\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3}+$$
$$l_1\,(\mathrm{bL}\,(r+\psi)+\mathrm{bL}\,r\,w1\,\theta_1-\mathrm{bL}\,w1\,(r+\psi)\,\theta_1+\mathrm{bL}\,w1\,\beta_0\,\theta_1-\mathrm{be}\,w1\,w2$$
$$(r+\psi)\,\theta_1^2+\mathrm{be}\,w2^2\,(r+\psi)\,\theta_1^2-\mathrm{be}\,w1\,w3\,(r+\psi)\,\theta_1^2+\mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2+$$
$$\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,w1\,\theta_1+\mathrm{be}\,(-w2^2-w3^2+w1\,(w2+w3))\,\theta_1^2)+$$
$$\mathrm{bL}\,l_2\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_1)+\mathrm{bL}\,\beta_{1,1}+$$
$$\mathrm{be}\,w2^2\,\theta_1^2\,\beta_{1,1}+\mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,1}-\mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,2}-\mathrm{be}\,w1\,w3\,\theta_1^2\,\beta_{1,3})))\,/$$
$$((1+\mathrm{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)))-$$

$$
\begin{aligned}
&(w2^2\ (r + \psi + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + r\ w1\ \Theta_1 - w1\ (r + \psi)\ \Theta_1 + bL\ r\ w1\ l_2\ \Theta_1 - \\
&\quad bL\ w1\ (r + \psi)\ l_2\ \Theta_1 + w1\ \beta_0\ \Theta_1 + bL\ w1\ l_2\ \beta_0\ \Theta_1 + w1\ \beta_2\ \Theta_1 + bL\ w1\ l_2\ \beta_2\ \Theta_1 - \\
&\quad be\ w1\ w2\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w2^2\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ l_2\ \Theta_1^2 + \\
&\quad be\ w3^2\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w1\ w2\ l_2\ \beta_2\ \Theta_1^2 - be\ w2^2\ l_2\ \beta_2\ \Theta_1^2 + \\
&\quad be\ w1\ w3\ l_2\ \beta_2\ \Theta_1^2 - be\ w3^2\ l_2\ \beta_2\ \Theta_1^2 + l_0\ (bL + be\ w1\ (w1 + w2 + w3)\ \Theta_1^2 + \\
&\qquad bL\ (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)) + bL\ l_2\ (r + \sigma_1) + \\
&\quad bL^2\ l_2^2\ (r + \sigma_1) + be\ w1^2\ l_2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w1^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + \\
&\quad be\ bL\ w2^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w3^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ w1\ w2\ l_2\ \Theta_1^2\ (r + \sigma_2) + \\
&\quad be\ w1\ w3\ l_2\ \Theta_1^2\ (r + \sigma_3) + \beta_{1,1} + bL\ l_2\ \beta_{1,1} + be\ w2^2\ l_2\ \Theta_1^2\ \beta_{1,1} + \\
&\quad be\ w3^2\ l_2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ l_2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ l_2\ \Theta_1^2\ \beta_{1,3} + \\
&\quad l_1\ (bL\ (r + \psi) + bL\ r\ w1\ \Theta_1 - bL\ w1\ (r + \psi)\ \Theta_1 + bL\ w1\ \beta_0\ \Theta_1 - be\ w1\ w2 \\
&\qquad (r + \psi)\ \Theta_1^2 + be\ w2^2\ (r + \psi)\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ \Theta_1^2 + \\
&\qquad \beta_2\ (-bL + bL\ w1\ \Theta_1 + be\ (-w2^2 - w3^2 + w1\ (w2 + w3))\ \Theta_1^2) + \\
&\qquad bL\ l_2\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)\ (r + \sigma_1) + bL\ \beta_{1,1} + \\
&\qquad be\ w2^2\ \Theta_1^2\ \beta_{1,1} + be\ w3^2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ \Theta_1^2\ \beta_{1,3})))\ / \\
&\ ((1 + bL\ (l_1 + l_2))\ (1 + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)))) - \\
&w3^2\ (rL2 + (r + \psi + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + r\ w1\ \Theta_1 - w1\ (r + \psi)\ \Theta_1 + \\
&\quad bL\ r\ w1\ l_2\ \Theta_1 - bL\ w1\ (r + \psi)\ l_2\ \Theta_1 + w1\ \beta_0\ \Theta_1 + bL\ w1\ l_2\ \beta_0\ \Theta_1 + \\
&\quad w1\ \beta_2\ \Theta_1 + bL\ w1\ l_2\ \beta_2\ \Theta_1 - be\ w1\ w2\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w2^2\ (r + \psi)\ l_2\ \Theta_1^2 - \\
&\quad be\ w1\ w3\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w1\ w2\ l_2\ \beta_2\ \Theta_1^2 - \\
&\quad be\ w2^2\ l_2\ \beta_2\ \Theta_1^2 + be\ w1\ w3\ l_2\ \beta_2\ \Theta_1^2 - be\ w3^2\ l_2\ \beta_2\ \Theta_1^2 + l_0\ (bL + \\
&\qquad be\ w1\ (w1 + w2 + w3)\ \Theta_1^2 + bL\ (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)) + \\
&\quad bL\ l_2\ (r + \sigma_1) + bL^2\ l_2^2\ (r + \sigma_1) + be\ w1^2\ l_2\ \Theta_1^2\ (r + \sigma_1) + be\ bL \\
&\quad w1^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w2^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w3^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + \\
&\quad be\ w1\ w2\ l_2\ \Theta_1^2\ (r + \sigma_2) + be\ w1\ w3\ l_2\ \Theta_1^2\ (r + \sigma_3) + \beta_{1,1} + bL\ l_2\ \beta_{1,1} + \\
&\quad be\ w2^2\ l_2\ \Theta_1^2\ \beta_{1,1} + be\ w3^2\ l_2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ l_2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ l_2 \\
&\quad \Theta_1^2\ \beta_{1,3} + l_1\ (bL\ (r + \psi) + bL\ r\ w1\ \Theta_1 - bL\ w1\ (r + \psi)\ \Theta_1 + bL\ w1\ \beta_0\ \Theta_1 - \\
&\qquad be\ w1\ w2\ (r + \psi)\ \Theta_1^2 + be\ w2^2\ (r + \psi)\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ \Theta_1^2 + be\ w3^2 \\
&\qquad (r + \psi)\ \Theta_1^2 + \beta_2\ (-bL + bL\ w1\ \Theta_1 + be\ (-w2^2 - w3^2 + w1\ (w2 + w3))\ \Theta_1^2) + \\
&\qquad bL\ l_2\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)\ (r + \sigma_1) + bL\ \beta_{1,1} + \\
&\qquad be\ w2^2\ \Theta_1^2\ \beta_{1,1} + be\ w3^2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ \Theta_1^2\ \beta_{1,3}))\ / \\
&\quad ((1 + bL\ (l_1 + l_2))\ (1 + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2))))) + \\
&w3\ ((rL2 + rL3)\ w2 + w1\ (rL3 + (r + \psi + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + \\
&\qquad r\ w1\ \Theta_1 - w1\ (r + \psi)\ \Theta_1 + bL\ r\ w1\ l_2\ \Theta_1 - bL\ w1\ (r + \psi)\ l_2\ \Theta_1 + w1\ \beta_0\ \Theta_1 + \\
&\qquad bL\ w1\ l_2\ \beta_0\ \Theta_1 + w1\ \beta_2\ \Theta_1 + bL\ w1\ l_2\ \beta_2\ \Theta_1 - be\ w1\ w2\ (r + \psi)\ l_2\ \Theta_1^2 + \\
&\qquad be\ w2^2\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ l_2\ \Theta_1^2 + \\
&\qquad be\ w1\ w2\ l_2\ \beta_2\ \Theta_1^2 - be\ w2^2\ l_2\ \beta_2\ \Theta_1^2 + be\ w1\ w3\ l_2\ \beta_2\ \Theta_1^2 - \\
&\qquad be\ w3^2\ l_2\ \beta_2\ \Theta_1^2 + l_0\ (bL + be\ w1\ (w1 + w2 + w3)\ \Theta_1^2 + bL\ (l_1 + l_2)\ (bL + \\
&\qquad\qquad be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)) + bL\ l_2\ (r + \sigma_1) + bL^2\ l_2^2\ (r + \sigma_1) + \\
&\qquad be\ w1^2\ l_2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w1^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w2^2 \\
&\qquad l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w3^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ w1\ w2\ l_2\ \Theta_1^2\ (r + \sigma_2) + \\
&\qquad be\ w1\ w3\ l_2\ \Theta_1^2\ (r + \sigma_3) + \beta_{1,1} + bL\ l_2\ \beta_{1,1} + be\ w2^2\ l_2\ \Theta_1^2\ \beta_{1,1} + be\ w3^2 \\
&\qquad l_2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ l_2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ l_2\ \Theta_1^2\ \beta_{1,3} + l_1\ (bL\ (r + \psi) + \\
&\qquad\quad bL\ r\ w1\ \Theta_1 - bL\ w1\ (r + \psi)\ \Theta_1 + bL\ w1\ \beta_0\ \Theta_1 - be\ w1\ w2\ (r + \psi)\ \Theta_1^2 + \\
&\qquad\quad be\ w2^2\ (r + \psi)\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ \Theta_1^2 + \\
&\qquad\quad \beta_2\ (-bL + bL\ w1\ \Theta_1 + be\ (-w2^2 - w3^2 + w1\ (w2 + w3))\ \Theta_1^2) + \\
&\qquad\quad bL\ l_2\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)\ (r + \sigma_1) + bL\ \beta_{1,1} + be\ w2^2 \\
&\qquad\quad \Theta_1^2\ \beta_{1,1} + be\ w3^2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ \Theta_1^2\ \beta_{1,3}))\ / \\
&\qquad ((1 + bL\ (l_1 + l_2))\ (1 + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2) \\
&\qquad\qquad\quad \Theta_1^2)))))))\Big)\Big) \Big/ (bd\ bL\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2))
\end{aligned}
$$

```
In[59]:= G = G /. rL2 → LoanRate2
```

Out[59]=

$$
\begin{aligned}
&\Big(\beta_2\,(-bL\,(3\,bd + bL\,(-1+\rho)^2) + 2\,bd\,bL\,(w1+w2+w3)\,\Theta_1 - \\
&\quad (bd\,(bL\,(w1^2+w2^2+w3^2) + 2\,be\,(w1^2 - w1\,w2 + w2^2 - (w1+w2)\,w3 + w3^2)) + \\
&\quad\quad be\,bL\,(w1^2+w2^2+w3^2)\,(-1+\rho)^2)\,\Theta_1^2) + \\
&bd\,bL\,\Theta_1\,(rL3\,w3 - r\,(w1+w2+w3) - 2\,(w1+w2+w3)\,\psi + (w1+w2+w3)\,\beta_0 - \\
&\quad w1\,\beta_{1,1} - w2\,\beta_{1,2} - w3\,\beta_{1,3} + \\
&\quad (w1\,(r+\psi+bL\,(r+\psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w1\,\Theta_1 - w1\,(r+\psi)\,\Theta_1 + bL\,r\,w1\,l_2\,\Theta_1 - \\
&\quad\quad bL\,w1\,(r+\psi)\,l_2\,\Theta_1 + w1\,\beta_0\,\Theta_1 + bL\,w1\,l_2\,\beta_0\,\Theta_1 + w1\,\beta_2\,\Theta_1 + bL\,w1\,l_2\,\beta_2\,\Theta_1 - be\,w1\,w2 \\
&\quad\quad (r+\psi)\,l_2\,\Theta_1^2 + be\,w2^2\,(r+\psi)\,l_2\,\Theta_1^2 - be\,w1\,w3\,(r+\psi)\,l_2\,\Theta_1^2 + be\,w3^2\,(r+\psi)\,l_2\,\Theta_1^2 + \\
&\quad\quad be\,w1\,w2\,l_2\,\beta_2\,\Theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\Theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\Theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\Theta_1^2 + \\
&\quad\quad l_0\,(bL + be\,w1\,(w1+w2+w3)\,\Theta_1^2 + bL\,(l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)) + \\
&\quad\quad bL\,l_2\,(r+\sigma_1) + bL^2\,l_2^2\,(r+\sigma_1) + be\,w1^2\,l_2\,\Theta_1^2\,(r+\sigma_1) + \\
&\quad\quad be\,bL\,w1^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1) + be\,bL\,w2^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1) + be\,bL\,w3^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1) + \\
&\quad\quad be\,w1\,w2\,l_2\,\Theta_1^2\,(r+\sigma_2) + be\,w1\,w3\,l_2\,\Theta_1^2\,(r+\sigma_3) + \beta_{1,1} + bL\,l_2\,\beta_{1,1} + \\
&\quad\quad be\,w2^2\,l_2\,\Theta_1^2\,\beta_{1,1} + be\,w3^2\,l_2\,\Theta_1^2\,\beta_{1,1} - be\,w1\,w2\,l_2\,\Theta_1^2\,\beta_{1,2} - be\,w1\,w3\,l_2\,\Theta_1^2\,\beta_{1,3} + \\
&\quad\quad l_1\,(bL\,(r+\psi) + bL\,r\,w1\,\Theta_1 - bL\,w1\,(r+\psi)\,\Theta_1 + bL\,w1\,\beta_0\,\Theta_1 - be\,w1\,w2\,(r+\psi)\,\Theta_1^2 + \\
&\quad\quad\quad be\,w2^2\,(r+\psi)\,\Theta_1^2 - be\,w1\,w3\,(r+\psi)\,\Theta_1^2 + be\,w3^2\,(r+\psi)\,\Theta_1^2 + \\
&\quad\quad\quad \beta_2\,(-bL + bL\,w1\,\Theta_1 + be\,(-w2^2 - w3^2 + w1\,(w2+w3))\,\Theta_1^2) + \\
&\quad\quad\quad bL\,l_2\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)\,(r+\sigma_1) + bL\,\beta_{1,1} + \\
&\quad\quad\quad be\,w2^2\,\Theta_1^2\,\beta_{1,1} + be\,w3^2\,\Theta_1^2\,\beta_{1,1} - be\,w1\,w2\,\Theta_1^2\,\beta_{1,2} - be\,w1\,w3\,\Theta_1^2\,\beta_{1,3}))) / \\
&\quad ((1 + bL\,(l_1+l_2))\,(1 + (l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2))) + \\
&\quad (w2\,(r+\psi+bL\,(r+\psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w2\,\Theta_1 - w2\,(r+\psi)\,\Theta_1 + \\
&\quad\quad bL\,r\,w2\,l_2\,\Theta_1 - bL\,w2\,(r+\psi)\,l_2\,\Theta_1 + w2\,\beta_0\,\Theta_1 + bL\,w2\,l_2\,\beta_0\,\Theta_1 + \\
&\quad\quad w2\,\beta_2\,\Theta_1 + bL\,w2\,l_2\,\beta_2\,\Theta_1 + be\,w1^2\,(r+\psi)\,l_2\,\Theta_1^2 - be\,w1\,w2\,(r+\psi)\,l_2\,\Theta_1^2 - \\
&\quad\quad be\,w2\,w3\,(r+\psi)\,l_2\,\Theta_1^2 + be\,w3^2\,(r+\psi)\,l_2\,\Theta_1^2 - be\,w1^2\,l_2\,\beta_2\,\Theta_1^2 + \\
&\quad\quad be\,w1\,w2\,l_2\,\beta_2\,\Theta_1^2 + be\,w2\,w3\,l_2\,\beta_2\,\Theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\Theta_1^2 + \\
&\quad\quad l_0\,(bL + be\,w2\,(w1+w2+w3)\,\Theta_1^2 + bL\,(l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)) + \\
&\quad\quad be\,w1\,w2\,l_2\,\Theta_1^2\,(r+\sigma_1) + bL\,l_2\,(r+\sigma_2) + bL^2\,l_2^2\,(r+\sigma_2) + \\
&\quad\quad be\,w2^2\,l_2\,\Theta_1^2\,(r+\sigma_2) + be\,bL\,w1^2\,l_2^2\,\Theta_1^2\,(r+\sigma_2) + be\,bL\,w2^2\,l_2^2\,\Theta_1^2\,(r+\sigma_2) + \\
&\quad\quad be\,bL\,w3^2\,l_2^2\,\Theta_1^2\,(r+\sigma_2) + be\,w2\,w3\,l_2\,\Theta_1^2\,(r+\sigma_3) - be\,w1\,w2\,l_2\,\Theta_1^2\,\beta_{1,1} + \\
&\quad\quad \beta_{1,2} + bL\,l_2\,\beta_{1,2} + be\,w1^2\,l_2\,\Theta_1^2\,\beta_{1,2} + be\,w3^2\,l_2\,\Theta_1^2\,\beta_{1,2} - be\,w2\,w3\,l_2\,\Theta_1^2\,\beta_{1,3} + \\
&\quad\quad l_1\,(bL\,(r+\psi) + bL\,r\,w2\,\Theta_1 - bL\,w2\,(r+\psi)\,\Theta_1 + bL\,w2\,\beta_0\,\Theta_1 + be\,w1^2\,(r+\psi)\,\Theta_1^2 - \\
&\quad\quad\quad be\,w1\,w2\,(r+\psi)\,\Theta_1^2 - be\,w2\,w3\,(r+\psi)\,\Theta_1^2 + be\,w3^2\,(r+\psi)\,\Theta_1^2 + \\
&\quad\quad\quad \beta_2\,(-bL + bL\,w2\,\Theta_1 - be\,(w1^2 - w1\,w2 + w3\,(-w2+w3))\,\Theta_1^2) + \\
&\quad\quad\quad bL\,l_2\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)\,(r+\sigma_2) - be\,w1\,w2\,\Theta_1^2\,\beta_{1,1} + \\
&\quad\quad\quad bL\,\beta_{1,2} + be\,w1^2\,\Theta_1^2\,\beta_{1,2} + be\,w3^2\,\Theta_1^2\,\beta_{1,2} - be\,w2\,w3\,\Theta_1^2\,\beta_{1,3}))) / \\
&\quad ((1 + bL\,(l_1+l_2))\,(1 + (l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))) + \\
&bL\,\Big(\frac{bL\,(-1+\rho)\,(-bd\,d_0 + bd\,(r+\psi)\,((-1+\rho)\,d_1 + \rho\,d_2) + (-1+\rho)\,\beta_2)}{1 + bd\,(d_1+d_2)} + \\
&\quad bd\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3}) - \\
&\quad bd\,(-3\,r + rL3 - 3\,\psi + (r+\psi+bL\,(r+\psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w1\,\Theta_1 - \\
&\quad\quad w1\,(r+\psi)\,\Theta_1 + bL\,r\,w1\,l_2\,\Theta_1 - bL\,w1\,(r+\psi)\,l_2\,\Theta_1 + w1\,\beta_0\,\Theta_1 + \\
&\quad\quad bL\,w1\,l_2\,\beta_0\,\Theta_1 + w1\,\beta_2\,\Theta_1 + bL\,w1\,l_2\,\beta_2\,\Theta_1 - be\,w1\,w2\,(r+\psi)\,l_2\,\Theta_1^2 + \\
&\quad\quad be\,w2^2\,(r+\psi)\,l_2\,\Theta_1^2 - be\,w1\,w3\,(r+\psi)\,l_2\,\Theta_1^2 + be\,w3^2\,(r+\psi)\,l_2\,\Theta_1^2 + \\
&\quad\quad be\,w1\,w2\,l_2\,\beta_2\,\Theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\Theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\Theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\Theta_1^2 + \\
&\quad\quad l_0\,(bL + be\,w1\,(w1+w2+w3)\,\Theta_1^2 + bL\,(l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)) + \\
&\quad\quad bL\,l_2\,(r+\sigma_1) + bL^2\,l_2^2\,(r+\sigma_1) + be\,w1^2\,l_2\,\Theta_1^2\,(r+\sigma_1) + \\
&\quad\quad be\,bL\,w1^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1) + be\,bL\,w2^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1) + be\,bL\,w3^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1) + \\
&\quad\quad be\,w1\,w2\,l_2\,\Theta_1^2\,(r+\sigma_2) + be\,w1\,w3\,l_2\,\Theta_1^2\,(r+\sigma_3) + \beta_{1,1} + bL\,l_2\,\beta_{1,1} + \\
&\quad\quad be\,w2^2\,l_2\,\Theta_1^2\,\beta_{1,1} + be\,w3^2\,l_2\,\Theta_1^2\,\beta_{1,1} - be\,w1\,w2\,l_2\,\Theta_1^2\,\beta_{1,2} - be\,w1\,w3\,l_2\,\Theta_1^2\,\beta_{1,3} + \\
&\quad\quad l_1\,(bL\,(r+\psi) + bL\,r\,w1\,\Theta_1 - bL\,w1\,(r+\psi)\,\Theta_1 + bL\,w1\,\beta_0\,\Theta_1 - be\,w1\,w2
\end{aligned}
$$

$$
\begin{array}{l}
(r+\psi)\,\theta_1^2 + be\,w2^2\,(r+\psi)\,\theta_1^2 - be\,w1\,w3\,(r+\psi)\,\theta_1^2 + be\,w3^2\,(r+\psi) \\
\theta_1^2 + \beta_2\,(-bL + bL\,w1\,\theta_1 + be\,(-w2^2 - w3^2 + w1\,(w2+w3))\,\theta_1^2) + bL \\
l_2\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r+\sigma_1) + bL\,\beta_{1,1} + be\,w2^2 \\
\theta_1^2\,\beta_{1,1} + be\,w3^2\,\theta_1^2\,\beta_{1,1} - be\,w1\,w2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,\theta_1^2\,\beta_{1,3}))\,/ \\
((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2))) + \\
(r + \psi + bL\,(r+\psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w2\,\theta_1 - w2\,(r+\psi)\,\theta_1 + bL\,r\,w2\,l_2\,\theta_1 - \\
bL\,w2\,(r+\psi)\,l_2\,\theta_1 + w2\,\beta_0\,\theta_1 + bL\,w2\,l_2\,\beta_0\,\theta_1 + w2\,\beta_2\,\theta_1 + bL\,w2\,l_2\,\beta_2\,\theta_1 + be\,w1^2 \\
(r+\psi)\,l_2\,\theta_1^2 - be\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2 - be\,w2\,w3\,(r+\psi)\,l_2\,\theta_1^2 + be\,w3^2\,(r+\psi) \\
l_2\,\theta_1^2 - be\,w1^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 + be\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 + \\
l_0\,(bL + be\,w2\,(w1+w2+w3)\,\theta_1^2 + bL\,(l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)) + \\
be\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_1) + bL\,l_2\,(r+\sigma_2) + bL^2\,l_2^2\,(r+\sigma_2) + \\
be\,w2^2\,l_2\,\theta_1^2\,(r+\sigma_2) + be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \\
be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + be\,w2\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) - be\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,1} + \\
\beta_{1,2} + bL\,l_2\,\beta_{1,2} + be\,w1^2\,l_2\,\theta_1^2\,\beta_{1,2} + be\,w3^2\,l_2\,\theta_1^2\,\beta_{1,2} - be\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,3} + \\
l_1\,(bL\,(r+\psi) + bL\,r\,w2\,\theta_1 - bL\,w2\,(r+\psi)\,\theta_1 + bL\,w2\,\beta_0\,\theta_1 + be\,w1^2 \\
(r+\psi)\,\theta_1^2 - be\,w1\,w2\,(r+\psi)\,\theta_1^2 - be\,w2\,w3\,(r+\psi)\,\theta_1^2 + be\,w3^2\,(r+\psi) \\
\theta_1^2 + \beta_2\,(-bL + bL\,w2\,\theta_1 - be\,(w1^2 - w1\,w2 + w3\,(-w2+w3))\,\theta_1^2) + bL \\
l_2\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_2) - be\,w1\,w2\,\theta_1^2\,\beta_{1,1} + bL \\
\beta_{1,2} + be\,w1^2\,\theta_1^2\,\beta_{1,2} + be\,w3^2\,\theta_1^2\,\beta_{1,2} - be\,w2\,w3\,\theta_1^2\,\beta_{1,3}))\,/ \\
((1 + bL\,(l_1+l_2))\,(1 + (l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2))))\Big) + \\
\theta_1^2\,\Big(-bd\,bL\,(w1^2+w2^2+w3^2)\,\beta_0 + \\
\dfrac{be\,bL\,(w1^2+w2^2+w3^2)\,(-1+\rho)\,(-bd\,d_0 + bd\,(r+\psi)\,((-1+\rho)\,d_1 + \rho\,d_2) + (-1+\rho)\,\beta_2)}{1 + bd\,(d_1+d_2)} \\
+ bd\,be\,((w2^2 + w3^2 - w1\,(w2+w3))\,\beta_{1,1} + \\
(w1^2 - w1\,w2 - w2\,w3 + w3^2)\,\beta_{1,2} + (w1^2 + w2^2 - (w1+w2)\,w3)\,\beta_{1,3}) + \\
bd\,(bL\,(w1^2+w2^2+w3^2)\,\psi + be\,(-rL3\,w1^2 - rL3\,w2^2 + \\
2\,(w1^2 + w2^2 - w2\,w3 + w3^2 - w1\,(w2+w3))\,(r+\psi) + (w1\,w2\,(r + \psi + bL\,(r+\psi)\,l_2 - \\
\beta_2 - bL\,l_2\,\beta_2 + r\,w1\,\theta_1 - w1\,(r+\psi)\,\theta_1 + bL\,r\,w1\,l_2\,\theta_1 - bL\,w1\,(r+\psi)\,l_2\,\theta_1 + \\
w1\,\beta_0\,\theta_1 + bL\,w1\,l_2\,\beta_0\,\theta_1 + w1\,\beta_2\,\theta_1 + bL\,w1\,l_2\,\beta_2\,\theta_1 - be\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2 + \\
be\,w2^2\,(r+\psi)\,l_2\,\theta_1^2 - be\,w1\,w3\,(r+\psi)\,l_2\,\theta_1^2 + be\,w3^2\,(r+\psi)\,l_2\,\theta_1^2 + be\,w1 \\
w2\,l_2\,\beta_2\,\theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(bL + \\
be\,w1\,(w1+w2+w3)\,\theta_1^2 + bL\,(l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)) + \\
bL\,l_2\,(r+\sigma_1) + bL^2\,l_2^2\,(r+\sigma_1) + be\,w1^2\,l_2\,\theta_1^2\,(r+\sigma_1) + be\,bL \\
w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \\
be\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_2) + be\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) + \beta_{1,1} + bL\,l_2\,\beta_{1,1} + \\
be\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1} + be\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1} - be\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,l_2 \\
\theta_1^2\,\beta_{1,3} + l_1\,(bL\,(r+\psi) + bL\,r\,w1\,\theta_1 - bL\,w1\,(r+\psi)\,\theta_1 + bL\,w1\,\beta_0\,\theta_1 - \\
be\,w1\,w2\,(r+\psi)\,\theta_1^2 + be\,w2^2\,(r+\psi)\,\theta_1^2 - be\,w1\,w3\,(r+\psi)\,\theta_1^2 + be\,w3^2 \\
(r+\psi)\,\theta_1^2 + \beta_2\,(-bL + bL\,w1\,\theta_1 + be\,(-w2^2 - w3^2 + w1\,(w2+w3))\,\theta_1^2) + \\
bL\,l_2\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_1) + bL\,\beta_{1,1} + \\
be\,w2^2\,\theta_1^2\,\beta_{1,1} + be\,w3^2\,\theta_1^2\,\beta_{1,1} - be\,w1\,w2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,\theta_1^2\,\beta_{1,3})))\,/ \\
((1 + bL\,(l_1+l_2))\,(1 + (l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2))) - \\
(w2^2\,(r + \psi + bL\,(r+\psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w1\,\theta_1 - w1\,(r+\psi)\,\theta_1 + bL\,r\,w1\,l_2\,\theta_1 - \\
bL\,w1\,(r+\psi)\,l_2\,\theta_1 + w1\,\beta_0\,\theta_1 + bL\,w1\,l_2\,\beta_0\,\theta_1 + w1\,\beta_2\,\theta_1 + bL\,w1\,l_2\,\beta_2\,\theta_1 - \\
be\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2 + be\,w2^2\,(r+\psi)\,l_2\,\theta_1^2 - be\,w1\,w3\,(r+\psi)\,l_2\,\theta_1^2 + \\
be\,w3^2\,(r+\psi)\,l_2\,\theta_1^2 + be\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\theta_1^2 + \\
be\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(bL + be\,w1\,(w1+w2+w3)\,\theta_1^2 + \\
bL\,(l_1+l_2)\,(bL + be\,(w1^2+w2^2+w3^2)\,\theta_1^2)) + bL\,l_2\,(r+\sigma_1) + \\
bL^2\,l_2^2\,(r+\sigma_1) + be\,w1^2\,l_2\,\theta_1^2\,(r+\sigma_1) + be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \\
be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + be\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_2) + \\
be\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) + \beta_{1,1} + bL\,l_2\,\beta_{1,1} + be\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1} +
\end{array}
$$

$$\mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3} +$$
$$l_1\,(\mathrm{bL}\,(r+\psi) + \mathrm{bL}\,r\,w1\,\theta_1 - \mathrm{bL}\,w1\,(r+\psi)\,\theta_1 + \mathrm{bL}\,w1\,\beta_0\,\theta_1 - \mathrm{be}\,w1\,w2$$
$$(r+\psi)\,\theta_1^2 + \mathrm{be}\,w2^2\,(r+\psi)\,\theta_1^2 - \mathrm{be}\,w1\,w3\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2 +$$
$$\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,w1\,\theta_1 + \mathrm{be}\,(-w2^2 - w3^2 + w1\,(w2+w3))\,\theta_1^2) +$$
$$\mathrm{bL}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_1) + \mathrm{bL}\,\beta_{1,1} +$$
$$\mathrm{be}\,w2^2\,\theta_1^2\,\beta_{1,1} + \mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w1\,w3\,\theta_1^2\,\beta_{1,3}))) \,/$$
$$((1 + \mathrm{bL}\,(l_1+l_2))\,(1 + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)))) -$$
$$(w1^2\,(r + \psi + \mathrm{bL}\,(r+\psi)\,l_2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,w2\,\theta_1 - w2\,(r+\psi)\,\theta_1 +$$
$$\mathrm{bL}\,r\,w2\,l_2\,\theta_1 - \mathrm{bL}\,w2\,(r+\psi)\,l_2\,\theta_1 + w2\,\beta_0\,\theta_1 + \mathrm{bL}\,w2\,l_2\,\beta_0\,\theta_1 +$$
$$w2\,\beta_2\,\theta_1 + \mathrm{bL}\,w2\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2 -$$
$$\mathrm{be}\,w2\,w3\,(r+\psi)\,l_2\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1^2\,l_2\,\beta_2\,\theta_1^2 +$$
$$\mathrm{be}\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,w3^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(\mathrm{bL} +$$
$$\mathrm{be}\,w2\,(w1+w2+w3)\,\theta_1^2 + \mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)) +$$
$$\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_1) + \mathrm{bL}\,l_2\,(r+\sigma_2) + \mathrm{bL}^2\,l_2^2\,(r+\sigma_2) +$$
$$\mathrm{be}\,w2^2\,l_2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) +$$
$$\mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) - \mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,1} +$$
$$\beta_{1,2} + \mathrm{bL}\,l_2\,\beta_{1,2} + \mathrm{be}\,w1^2\,l_2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,3} +$$
$$l_1\,(\mathrm{bL}\,(r+\psi) + \mathrm{bL}\,r\,w2\,\theta_1 - \mathrm{bL}\,w2\,(r+\psi)\,\theta_1 + \mathrm{bL}\,w2\,\beta_0\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)$$
$$\theta_1^2 - \mathrm{be}\,w1\,w2\,(r+\psi)\,\theta_1^2 - \mathrm{be}\,w2\,w3\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2 +$$
$$\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,w2\,\theta_1 - \mathrm{be}\,(w1^2 - w1\,w2 + w3\,(-w2+w3))\,\theta_1^2) +$$
$$\mathrm{bL}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_2) - \mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,1} +$$
$$\mathrm{bL}\,\beta_{1,2} + \mathrm{be}\,w1^2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w2\,w3\,\theta_1^2\,\beta_{1,3}))) \,/$$
$$((1 + \mathrm{bL}\,(l_1+l_2))\,(1 + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)))) +$$
$$(w1\,w2\,(r + \psi + \mathrm{bL}\,(r+\psi)\,l_2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,w2\,\theta_1 - w2\,(r+\psi)\,\theta_1 +$$
$$\mathrm{bL}\,r\,w2\,l_2\,\theta_1 - \mathrm{bL}\,w2\,(r+\psi)\,l_2\,\theta_1 + w2\,\beta_0\,\theta_1 + \mathrm{bL}\,w2\,l_2\,\beta_0\,\theta_1 +$$
$$w2\,\beta_2\,\theta_1 + \mathrm{bL}\,w2\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2 -$$
$$\mathrm{be}\,w2\,w3\,(r+\psi)\,l_2\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1^2\,l_2\,\beta_2\,\theta_1^2 +$$
$$\mathrm{be}\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,w3^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(\mathrm{bL} +$$
$$\mathrm{be}\,w2\,(w1+w2+w3)\,\theta_1^2 + \mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)) +$$
$$\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_1) + \mathrm{bL}\,l_2\,(r+\sigma_2) + \mathrm{bL}^2\,l_2^2\,(r+\sigma_2) +$$
$$\mathrm{be}\,w2^2\,l_2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) +$$
$$\mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) - \mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,1} +$$
$$\beta_{1,2} + \mathrm{bL}\,l_2\,\beta_{1,2} + \mathrm{be}\,w1^2\,l_2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,3} +$$
$$l_1\,(\mathrm{bL}\,(r+\psi) + \mathrm{bL}\,r\,w2\,\theta_1 - \mathrm{bL}\,w2\,(r+\psi)\,\theta_1 + \mathrm{bL}\,w2\,\beta_0\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)$$
$$\theta_1^2 - \mathrm{be}\,w1\,w2\,(r+\psi)\,\theta_1^2 - \mathrm{be}\,w2\,w3\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2 +$$
$$\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,w2\,\theta_1 - \mathrm{be}\,(w1^2 - w1\,w2 + w3\,(-w2+w3))\,\theta_1^2) +$$
$$\mathrm{bL}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_2) - \mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,1} +$$
$$\mathrm{bL}\,\beta_{1,2} + \mathrm{be}\,w1^2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w2\,w3\,\theta_1^2\,\beta_{1,3}))) \,/$$
$$((1 + \mathrm{bL}\,(l_1+l_2))\,(1 + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)))) -$$
$$w3^2\,((r + \psi + \mathrm{bL}\,(r+\psi)\,l_2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,w1\,\theta_1 - w1\,(r+\psi)\,\theta_1 + \mathrm{bL}\,r\,w1\,l_2\,\theta_1 -$$
$$\mathrm{bL}\,w1\,(r+\psi)\,l_2\,\theta_1 + w1\,\beta_0\,\theta_1 + \mathrm{bL}\,w1\,l_2\,\beta_0\,\theta_1 + w1\,\beta_2\,\theta_1 + \mathrm{bL}\,w1\,l_2\,\beta_2\,\theta_1 -$$
$$\mathrm{be}\,w1\,w2\,(r+\psi)\,l_2\,\theta_1^2 + \mathrm{be}\,w2^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1\,w3\,(r+\psi)\,l_2\,\theta_1^2 +$$
$$\mathrm{be}\,w3^2\,(r+\psi)\,l_2\,\theta_1^2 + \mathrm{be}\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,w2^2\,l_2\,\beta_2\,\theta_1^2 +$$
$$\mathrm{be}\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,w3^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(\mathrm{bL} + \mathrm{be}\,w1\,(w1+w2+w3)\,\theta_1^2 +$$
$$\mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)) + \mathrm{bL}\,l_2\,(r+\sigma_1) +$$
$$\mathrm{bL}^2\,l_2^2\,(r+\sigma_1) + \mathrm{be}\,w1^2\,l_2\,\theta_1^2\,(r+\sigma_1) + \mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) +$$
$$\mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \mathrm{be}\,w1\,w2\,l_2\,\theta_1^2$$
$$(r+\sigma_2) + \mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) + \beta_{1,1} + \mathrm{bL}\,l_2\,\beta_{1,1} + \mathrm{be}\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1} +$$
$$\mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3} +$$
$$l_1\,(\mathrm{bL}\,(r+\psi) + \mathrm{bL}\,r\,w1\,\theta_1 - \mathrm{bL}\,w1\,(r+\psi)\,\theta_1 + \mathrm{bL}\,w1\,\beta_0\,\theta_1 -$$
$$\mathrm{be}\,w1\,w2\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w2^2\,(r+\psi)\,\theta_1^2 - \mathrm{be}\,w1\,w3\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w3^2$$
$$(r+\psi)\,\theta_1^2 + \beta_2\,(-\mathrm{bL} + \mathrm{bL}\,w1\,\theta_1 + \mathrm{be}\,(-w2^2 - w3^2 + w1\,(w2+w3))\,\theta_1^2) +$$

$$
\begin{aligned}
&bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} + \\
&be\, w2^2\, \theta_1^2\, \beta_{1,1} + be\, w3^2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \theta_1^2\, \beta_{1,3}))\, / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) + \\
&(r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 + bL\, r\, w2\, l_2\, \theta_1 - \\
&bL\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + bL\, w2\, l_2\, \beta_0\, \theta_1 + w2\, \beta_2\, \theta_1 + bL\, w2\, l_2\, \beta_2\, \theta_1 + \\
&be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + \\
&be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + \\
&be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL + be\, w2\, (w1 + w2 + w3)\, \theta_1^2 + \\
&bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + \\
&bL\, l_2\, (r + \sigma_2) + bL^2\, l_2^2\, (r + \sigma_2) + be\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, bL \\
&w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + \\
&be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} + \beta_{1,2} + bL\, l_2\, \beta_{1,2} + \\
&be\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,3} + \\
&l_1\, (bL\, (r + \psi) + bL\, r\, w2\, \theta_1 - bL\, w2\, (r + \psi)\, \theta_1 + bL\, w2\, \beta_0\, \theta_1 + be\, w1^2\, (r + \psi) \\
&\theta_1^2 - be\, w1\, w2\, (r + \psi)\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)\, \theta_1^2 + \\
&\beta_2\, (-bL + bL\, w2\, \theta_1 - be\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) + \\
&bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} + \\
&bL\, \beta_{1,2} + be\, w1^2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, \theta_1^2\, \beta_{1,3}))\, / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)))) + \\
&w3\, (w1\, (rL3 + (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r + \psi)\, \theta_1 + \\
&bL\, r\, w1\, l_2\, \theta_1 - bL\, w1\, (r + \psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + bL\, w1\, l_2\, \beta_0\, \theta_1 + \\
&w1\, \beta_2\, \theta_1 + bL\, w1\, l_2\, \beta_2\, \theta_1 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w2^2\, (r + \psi)\, l_2 \\
&\theta_1^2 - be\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 - \\
&be\, w2^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL + be\, w1 \\
&(w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + \\
&bL\, l_2\, (r + \sigma_1) + bL^2\, l_2^2\, (r + \sigma_1) + be\, w1^2\, l_2\, \theta_1^2\, (r + \sigma_1) + \\
&be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w3^2\, l_2^2 \\
&\theta_1^2\, (r + \sigma_1) + be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) + \\
&\beta_{1,1} + bL\, l_2\, \beta_{1,1} + be\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} - \\
&be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} + l_1\, (bL\, (r + \psi) + \\
&bL\, r\, w1\, \theta_1 - bL\, w1\, (r + \psi)\, \theta_1 + bL\, w1\, \beta_0\, \theta_1 - be\, w1\, w2\, (r + \psi)\, \theta_1^2 + \\
&be\, w2^2\, (r + \psi)\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)\, \theta_1^2 + \\
&\beta_2\, (-bL + bL\, w1\, \theta_1 + be\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \theta_1^2) + \\
&bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} + be\, w2^2 \\
&\theta_1^2\, \beta_{1,1} + be\, w3^2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \theta_1^2\, \beta_{1,3}))\, / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)))) + \\
&w2\, (rL3 + (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 + \\
&bL\, r\, w2\, l_2\, \theta_1 - bL\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + bL\, w2\, l_2\, \beta_0\, \theta_1 + \\
&w2\, \beta_2\, \theta_1 + bL\, w2\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, l_2 \\
&\theta_1^2 - be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 + \\
&be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL + be\, w2 \\
&(w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + \\
&be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + bL\, l_2\, (r + \sigma_2) + bL^2\, l_2^2\, (r + \sigma_2) + \\
&be\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w2^2\, l_2^2\, \theta_1^2 \\
&(r + \sigma_2) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - \\
&be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} + \beta_{1,2} + bL\, l_2\, \beta_{1,2} + be\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + \\
&be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,3} + l_1\, (bL\, (r + \psi) + \\
&bL\, r\, w2\, \theta_1 - bL\, w2\, (r + \psi)\, \theta_1 + bL\, w2\, \beta_0\, \theta_1 + be\, w1^2\, (r + \psi)\, \theta_1^2 - \\
&be\, w1\, w2\, (r + \psi)\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)\, \theta_1^2 + \\
&\beta_2\, (-bL + bL\, w2\, \theta_1 - be\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) + \\
&bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} +
\end{aligned}
$$

$$
\begin{aligned}
&bL\,\beta_{1,2} + be\,w1^2\,\theta_1^2\,\beta_{1,2} + be\,w3^2\,\theta_1^2\,\beta_{1,2} - be\,w2\,w3\,\theta_1^2\,\beta_{1,3}))\,/\\
&((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\\
&\theta_1^2)))))))\Big)\Big)\Big/\,(bd\,bL\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2))
\end{aligned}
$$

```
In[60]:= G = G /. rL3 → LoanRate3
```

Out[60]=

$$
\begin{aligned}
&\Big(\beta_2\,(-bL\,(3\,bd + bL\,(-1 + \rho)^2) + 2\,bd\,bL\,(w1 + w2 + w3)\,\theta_1 -\\
&(bd\,(bL\,(w1^2 + w2^2 + w3^2) + 2\,be\,(w1^2 - w1\,w2 + w2^2 - (w1 + w2)\,w3 + w3^2)) +\\
&be\,bL\,(w1^2 + w2^2 + w3^2)\,(-1 + \rho)^2)\,\theta_1^2) + bd\,bL\,\theta_1\\
&(-r\,(w1 + w2 + w3) - 2\,(w1 + w2 + w3)\,\psi + (w1 + w2 + w3)\,\beta_0 - w1\,\beta_{1,1} - w2\,\beta_{1,2} - w3\,\beta_{1,3} +\\
&(w3\,(r + \psi + bL\,(r + \psi)\,l_1 + bL\,(r + \psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w3\,\theta_1 - w3\,(r + \psi)\,\theta_1 +\\
&bL\,r\,w3\,l_2\,\theta_1 - bL\,w3\,(r + \psi)\,l_2\,\theta_1 + w3\,\beta_0\,\theta_1 + bL\,w3\,l_2\,\beta_0\,\theta_1 + w3\,\beta_2\,\theta_1 +\\
&bL\,w3\,l_2\,\beta_2\,\theta_1 + be\,w1^2\,(r + \psi)\,l_2\,\theta_1^2 + be\,w2^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1\,w3\,(r + \psi)\,l_2\,\theta_1^2 -\\
&be\,w2\,w3\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1^2\,l_2\,\beta_2\,\theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 +\\
&be\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 + l_1\,\beta_2\,(-bL + bL\,w3\,\theta_1 - be\,(w1^2 + w2^2 - (w1 + w2)\,w3)\,\theta_1^2) +\\
&l_0\,(bL + be\,w3\,(w1 + w2 + w3)\,\theta_1^2 + bL\,(l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)) +\\
&be\,w1\,w3\,l_2\,\theta_1^2\,(r + \sigma_1) + be\,w2\,w3\,l_2\,\theta_1^2\,(r + \sigma_2) + bL\,l_2\,(r + \sigma_3) + bL^2\,l_2^2\,(r + \sigma_3) +\\
&be\,w3^2\,l_2\,\theta_1^2\,(r + \sigma_3) + be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r + \sigma_3) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r + \sigma_3) +\\
&be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r + \sigma_3) + bL\,l_1\,l_2\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r + \sigma_3) -\\
&be\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,1} - be\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,2} - l_1\,\theta_1\,(bL\,w3\,\psi - bL\,w3\,\beta_0 +\\
&be\,\theta_1\,(-(w1^2 + w2^2 - (w1 + w2)\,w3)\,(r + \psi) + w1\,w3\,\beta_{1,1} + w2\,w3\,\beta_{1,2})) +\\
&\beta_{1,3} + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2)\,\theta_1^2)\,\beta_{1,3}))\,/\\
&((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2))) +\\
&(w1\,(r + \psi + bL\,(r + \psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w1\,\theta_1 - w1\,(r + \psi)\,\theta_1 + bL\,r\,w1\,l_2\,\theta_1 -\\
&bL\,w1\,(r + \psi)\,l_2\,\theta_1 + w1\,\beta_0\,\theta_1 + bL\,w1\,l_2\,\beta_0\,\theta_1 + w1\,\beta_2\,\theta_1 + bL\,w1\,l_2\,\beta_2\,\theta_1 - be\,w1\,w2\\
&(r + \psi)\,l_2\,\theta_1^2 + be\,w2^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1\,w3\,(r + \psi)\,l_2\,\theta_1^2 + be\,w3^2\,(r + \psi)\,l_2\,\theta_1^2 +\\
&be\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 - be\,w2^2\,l_2\,\beta_2\,\theta_1^2 + be\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 +\\
&l_0\,(bL + be\,w1\,(w1 + w2 + w3)\,\theta_1^2 + bL\,(l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)) +\\
&bL\,l_2\,(r + \sigma_1) + bL^2\,l_2^2\,(r + \sigma_1) + be\,w1^2\,l_2\,\theta_1^2\,(r + \sigma_1) +\\
&be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r + \sigma_1) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r + \sigma_1) + be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r + \sigma_1) +\\
&be\,w1\,w2\,l_2\,\theta_1^2\,(r + \sigma_2) + be\,w1\,w3\,l_2\,\theta_1^2\,(r + \sigma_3) + \beta_{1,1} + bL\,l_2\,\beta_{1,1} +\\
&be\,w2^2\,l_2\,\theta_1^2\,\beta_{1,1} + be\,w3^2\,l_2\,\theta_1^2\,\beta_{1,1} - be\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,3} +\\
&l_1\,(bL\,(r + \psi) + bL\,r\,w1\,\theta_1 - bL\,w1\,(r + \psi)\,\theta_1 + bL\,w1\,\beta_0\,\theta_1 -\\
&be\,w1\,w2\,(r + \psi)\,\theta_1^2 + be\,w2^2\,(r + \psi)\,\theta_1^2 - be\,w1\,w3\,(r + \psi)\,\theta_1^2 +\\
&be\,w3^2\,(r + \psi)\,\theta_1^2 + \beta_2\,(-bL + bL\,w1\,\theta_1 + be\,(-w2^2 - w3^2 + w1\,(w2 + w3))\,\theta_1^2) +\\
&bL\,l_2\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r + \sigma_1) + bL\,\beta_{1,1} +\\
&be\,w2^2\,\theta_1^2\,\beta_{1,1} + be\,w3^2\,\theta_1^2\,\beta_{1,1} - be\,w1\,w2\,\theta_1^2\,\beta_{1,2} - be\,w1\,w3\,\theta_1^2\,\beta_{1,3})))\,/\\
&((1 + bL\,(l_1 + l_2))\,(1 + (l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2))) +\\
&(w2\,(r + \psi + bL\,(r + \psi)\,l_2 - \beta_2 - bL\,l_2\,\beta_2 + r\,w2\,\theta_1 - w2\,(r + \psi)\,\theta_1 +\\
&bL\,r\,w2\,l_2\,\theta_1 - bL\,w2\,(r + \psi)\,l_2\,\theta_1 + w2\,\beta_0\,\theta_1 + bL\,w2\,l_2\,\beta_0\,\theta_1 +\\
&w2\,\beta_2\,\theta_1 + bL\,w2\,l_2\,\beta_2\,\theta_1 + be\,w1^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1\,w2\,(r + \psi)\,l_2\,\theta_1^2 -\\
&be\,w2\,w3\,(r + \psi)\,l_2\,\theta_1^2 + be\,w3^2\,(r + \psi)\,l_2\,\theta_1^2 - be\,w1^2\,l_2\,\beta_2\,\theta_1^2 +\\
&be\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 + be\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 - be\,w3^2\,l_2\,\beta_2\,\theta_1^2 +\\
&l_0\,(bL + be\,w2\,(w1 + w2 + w3)\,\theta_1^2 + bL\,(l_1 + l_2)\,(bL + be\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)) +\\
&be\,w1\,w2\,l_2\,\theta_1^2\,(r + \sigma_1) + bL\,l_2\,(r + \sigma_2) + bL^2\,l_2^2\,(r + \sigma_2) +\\
&be\,w2^2\,l_2\,\theta_1^2\,(r + \sigma_2) + be\,bL\,w1^2\,l_2^2\,\theta_1^2\,(r + \sigma_2) + be\,bL\,w2^2\,l_2^2\,\theta_1^2\,(r + \sigma_2) +\\
&be\,bL\,w3^2\,l_2^2\,\theta_1^2\,(r + \sigma_2) + be\,w2\,w3\,l_2\,\theta_1^2\,(r + \sigma_3) - be\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,1} +\\
&\beta_{1,2} + bL\,l_2\,\beta_{1,2} + be\,w1^2\,l_2\,\theta_1^2\,\beta_{1,2} + be\,w3^2\,l_2\,\theta_1^2\,\beta_{1,2} - be\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,3} +\\
&l_1\,(bL\,(r + \psi) + bL\,r\,w2\,\theta_1 - bL\,w2\,(r + \psi)\,\theta_1 + bL\,w2\,\beta_0\,\theta_1 + be\,w1^2\,(r + \psi)\,\theta_1^2 -\\
&be\,w1\,w2\,(r + \psi)\,\theta_1^2 - be\,w2\,w3\,(r + \psi)\,\theta_1^2 + be\,w3^2\,(r + \psi)\,\theta_1^2 +\\
&\beta_2\,(-bL + bL\,w2\,\theta_1 - be\,(w1^2 - w1\,w2 + w3\,(-w2 + w3))\,\theta_1^2) +
\end{aligned}
$$

$$\mathrm{bL}\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - \mathrm{be}\, w1\, w2\, \theta_1^2\, \beta_{1,1} +$$
$$\mathrm{bL}\, \beta_{1,2} + \mathrm{be}\, w1^2\, \theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, \theta_1^2\, \beta_{1,3}))) \;/$$
$$((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)))) +$$
$$\mathrm{bL} \left(\frac{\mathrm{bL}\, (-1 + \rho)\, (-\mathrm{bd}\, d_0 + \mathrm{bd}\, (r + \psi)\, ((-1 + \rho)\, d_1 + \rho\, d_2) + (-1 + \rho)\, \beta_2)}{1 + \mathrm{bd}\, (d_1 + d_2)} + \right.$$
$$\mathrm{bd}\, (\beta_{1,1} + \beta_{1,2} + \beta_{1,3}) -$$
$$\mathrm{bd}\, (-3\, r - 3\, \psi + (r + \psi + \mathrm{bL}\, (r + \psi)\, l_1 + \mathrm{bL}\, (r + \psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w3\, \theta_1 - w3\, (r + \psi)$$
$$\theta_1 + \mathrm{bL}\, r\, w3\, l_2\, \theta_1 - \mathrm{bL}\, w3\, (r + \psi)\, l_2\, \theta_1 + w3\, \beta_0\, \theta_1 + \mathrm{bL}\, w3\, l_2\, \beta_0\, \theta_1 + w3\, \beta_2\, \theta_1 + \mathrm{bL}$$
$$w3\, l_2\, \beta_2\, \theta_1 + \mathrm{be}\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$\mathrm{be}\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1^2\, l_2\, \beta_2\, \theta_1^2 - \mathrm{be}\, w2^2\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 +$$
$$\mathrm{be}\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 + l_1\, \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w3\, \theta_1 - \mathrm{be}\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \theta_1^2) +$$
$$l_0\, (\mathrm{bL} + \mathrm{be}\, w3\, (w1 + w2 + w3)\, \theta_1^2 + \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$\mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_1) + \mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_2) + \mathrm{bL}\, l_2\, (r + \sigma_3) +$$
$$\mathrm{bL}^2\, l_2^2\, (r + \sigma_3) + \mathrm{be}\, w3^2\, l_2\, \theta_1^2\, (r + \sigma_3) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$\mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$\mathrm{bL}\, l_1\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_3) - \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,1} - \mathrm{be}\, w2\, w3\, l_2$$
$$\theta_1^2\, \beta_{1,2} - l_1\, \theta_1\, (\mathrm{bL}\, w3\, \psi - \mathrm{bL}\, w3\, \beta_0 + \mathrm{be}\, \theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) +$$
$$w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) + \beta_{1,3} + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2)\, \theta_1^2)\, \beta_{1,3}) \;/$$
$$((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$
$$(r + \psi + \mathrm{bL}\, (r + \psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r + \psi)\, \theta_1 + \mathrm{bL}\, r\, w1\, l_2\, \theta_1 -$$
$$\mathrm{bL}\, w1\, (r + \psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + \mathrm{bL}\, w1\, l_2\, \beta_0\, \theta_1 + w1\, \beta_2\, \theta_1 + \mathrm{bL}\, w1\, l_2\, \beta_2\, \theta_1 - \mathrm{be}\, w1$$
$$w2\, (r + \psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w3^2\, (r + \psi)$$
$$l_2\, \theta_1^2 + \mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 - \mathrm{be}\, w2^2\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$l_0\, (\mathrm{bL} + \mathrm{be}\, w1\, (w1 + w2 + w3)\, \theta_1^2 + \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$\mathrm{bL}\, l_2\, (r + \sigma_1) + \mathrm{bL}^2\, l_2^2\, (r + \sigma_1) + \mathrm{be}\, w1^2\, l_2\, \theta_1^2\, (r + \sigma_1) +$$
$$\mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + \mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) +$$
$$\mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_2) + \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) + \beta_{1,1} + \mathrm{bL}\, l_2\, \beta_{1,1} +$$
$$\mathrm{be}\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} + \mathrm{be}\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} - \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +$$
$$l_1\, (\mathrm{bL}\, (r + \psi) + \mathrm{bL}\, r\, w1\, \theta_1 - \mathrm{bL}\, w1\, (r + \psi)\, \theta_1 + \mathrm{bL}\, w1\, \beta_0\, \theta_1 - \mathrm{be}\, w1\, w2$$
$$(r + \psi)\, \theta_1^2 + \mathrm{be}\, w2^2\, (r + \psi)\, \theta_1^2 - \mathrm{be}\, w1\, w3\, (r + \psi)\, \theta_1^2 + \mathrm{be}\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w1\, \theta_1 + \mathrm{be}\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \theta_1^2) + \mathrm{bL}$$
$$l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + \mathrm{bL}\, \beta_{1,1} + \mathrm{be}\, w2^2$$
$$\theta_1^2\, \beta_{1,1} + \mathrm{be}\, w3^2\, \theta_1^2\, \beta_{1,1} - \mathrm{be}\, w1\, w2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, \theta_1^2\, \beta_{1,3})) \;/$$
$$((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$
$$(r + \psi + \mathrm{bL}\, (r + \psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 + \mathrm{bL}\, r\, w2\, l_2\, \theta_1 -$$
$$\mathrm{bL}\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_0\, \theta_1 + w2\, \beta_2\, \theta_1 +$$
$$\mathrm{bL}\, w2\, l_2\, \beta_2\, \theta_1 + \mathrm{be}\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$\mathrm{be}\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$\mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$l_0\, (\mathrm{bL} + \mathrm{be}\, w2\, (w1 + w2 + w3)\, \theta_1^2 + \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$\mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + \mathrm{bL}\, l_2\, (r + \sigma_2) + \mathrm{bL}^2\, l_2^2\, (r + \sigma_2) +$$
$$\mathrm{be}\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + \mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) +$$
$$\mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + \mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} +$$
$$\beta_{1,2} + \mathrm{bL}\, l_2\, \beta_{1,2} + \mathrm{be}\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +$$
$$l_1\, (\mathrm{bL}\, (r + \psi) + \mathrm{bL}\, r\, w2\, \theta_1 - \mathrm{bL}\, w2\, (r + \psi)\, \theta_1 + \mathrm{bL}\, w2\, \beta_0\, \theta_1 + \mathrm{be}\, w1^2$$
$$(r + \psi)\, \theta_1^2 - \mathrm{be}\, w1\, w2\, (r + \psi)\, \theta_1^2 - \mathrm{be}\, w2\, w3\, (r + \psi)\, \theta_1^2 + \mathrm{be}\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w2\, \theta_1 - \mathrm{be}\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) + \mathrm{bL}$$
$$l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - \mathrm{be}\, w1\, w2\, \theta_1^2\, \beta_{1,1} + \mathrm{bL}$$
$$\beta_{1,2} + \mathrm{be}\, w1^2\, \theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, \theta_1^2\, \beta_{1,3})) \;/$$
$$\left. ((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)))) \right) +$$
$$\theta_1^2 \left(-\mathrm{bd}\, \mathrm{bL}\, (w1^2 + w2^2 + w3^2)\, \beta_0 + \right.$$

$$bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} +$$
$$bL\, \beta_{1,2} + be\, w1^2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, \theta_1^2\, \beta_{1,3}))) \,/$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)))) +$$
$$bL\, \Bigl(\frac{bL\, (-1 + \rho)\, (-bd\, d_0 + bd\, (r + \psi)\, ((-1 + \rho)\, d_1 + \rho\, d_2) + (-1 + \rho)\, \beta_2)}{1 + bd\, (d_1 + d_2)} +$$
$$bd\, (\beta_{1,1} + \beta_{1,2} + \beta_{1,3}) -$$
$$bd\, (-3\, r - 3\, \psi + (r + \psi + bL\, (r + \psi)\, l_1 + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w3\, \theta_1 - w3\, (r + \psi)$$
$$\theta_1 + bL\, r\, w3\, l_2\, \theta_1 - bL\, w3\, (r + \psi)\, l_2\, \theta_1 + w3\, \beta_0\, \theta_1 + bL\, w3\, l_2\, \beta_0\, \theta_1 + w3\, \beta_2\, \theta_1 + bL$$
$$w3\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 +$$
$$be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 + l_1\, \beta_2\, (-bL + bL\, w3\, \theta_1 - be\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \theta_1^2) +$$
$$l_0\, (bL + be\, w3\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_1) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_2) + bL\, l_2\, (r + \sigma_3) +$$
$$bL^2\, l_2^2\, (r + \sigma_3) + be\, w3^2\, l_2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$bL\, l_1\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_3) - be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w2\, w3\, l_2$$
$$\theta_1^2\, \beta_{1,2} - l_1\, \theta_1\, (bL\, w3\, \psi - bL\, w3\, \beta_0 + be\, \theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) +$$
$$w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) + \beta_{1,3} + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2)\, \theta_1^2)\, \beta_{1,3}) \,/$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$
$$(r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r + \psi)\, \theta_1 + bL\, r\, w1\, l_2\, \theta_1 -$$
$$bL\, w1\, (r + \psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + bL\, w1\, l_2\, \beta_0\, \theta_1 + w1\, \beta_2\, \theta_1 + bL\, w1\, l_2\, \beta_2\, \theta_1 - be\, w1$$
$$w2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)$$
$$l_2\, \theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$l_0\, (bL + be\, w1\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$bL\, l_2\, (r + \sigma_1) + bL^2\, l_2^2\, (r + \sigma_1) + be\, w1^2\, l_2\, \theta_1^2\, (r + \sigma_1) +$$
$$be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) +$$
$$be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) + \beta_{1,1} + bL\, l_2\, \beta_{1,1} +$$
$$be\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +$$
$$l_1\, (bL\, (r + \psi) + bL\, r\, w1\, \theta_1 - bL\, w1\, (r + \psi)\, \theta_1 + bL\, w1\, \beta_0\, \theta_1 - be\, w1\, w2$$
$$(r + \psi)\, \theta_1^2 + be\, w2^2\, (r + \psi)\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-bL + bL\, w1\, \theta_1 + be\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \theta_1^2) + bL$$
$$l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} + be\, w2^2$$
$$\theta_1^2\, \beta_{1,1} + be\, w3^2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \theta_1^2\, \beta_{1,3})) \,/$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$
$$(r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 + bL\, r\, w2\, l_2\, \theta_1 -$$
$$bL\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + bL\, w2\, l_2\, \beta_0\, \theta_1 + w2\, \beta_2\, \theta_1 +$$
$$bL\, w2\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$l_0\, (bL + be\, w2\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + bL\, l_2\, (r + \sigma_2) + bL^2\, l_2^2\, (r + \sigma_2) +$$
$$be\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) +$$
$$be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} +$$
$$\beta_{1,2} + bL\, l_2\, \beta_{1,2} + be\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +$$
$$l_1\, (bL\, (r + \psi) + bL\, r\, w2\, \theta_1 - bL\, w2\, (r + \psi)\, \theta_1 + bL\, w2\, \beta_0\, \theta_1 + be\, w1^2$$
$$(r + \psi)\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-bL + bL\, w2\, \theta_1 - be\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) + bL$$
$$l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} + bL$$
$$\beta_{1,2} + be\, w1^2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, \theta_1^2\, \beta_{1,3})) \,/$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))))\Bigr) +$$
$$\theta_1^2\, \Bigl(-bd\, bL\, (w1^2 + w2^2 + w3^2)\, \beta_0 +$$

$$\frac{be\, bL\, (w1^2 + w2^2 + w3^2)\, (-1 + \rho)\, (-bd\, d_0 + bd\, (r + \psi)\, ((-1 + \rho)\, d_1 + \rho\, d_2) + (-1 + \rho)\, \beta_2)}{1 + bd\, (d_1 + d_2)}$$

$$+ bd\, be\, ((w2^2 + w3^2 - w1\, (w2 + w3))\, \beta_{1,1} +$$
$$(w1^2 - w1\, w2 - w2\, w3 + w3^2)\, \beta_{1,2} + (w1^2 + w2^2 - (w1 + w2)\, w3)\, \beta_{1,3}) +$$
$$bd\, (bL\, (w1^2 + w2^2 + w3^2)\, \psi + be\, (2\, (w1^2 + w2^2 - w2\, w3 + w3^2 - w1\, (w2 + w3))\, (r + \psi) -$$
$$(w1^2\, (r + \psi + bL\, (r + \psi)\, l_1 + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w3\, \theta_1 - w3\, (r + \psi)\, \theta_1 +$$
$$bL\, r\, w3\, l_2\, \theta_1 - bL\, w3\, (r + \psi)\, l_2\, \theta_1 + w3\, \beta_0\, \theta_1 + bL\, w3\, l_2\, \beta_0\, \theta_1 +$$
$$w3\, \beta_2\, \theta_1 + bL\, w3\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$be\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2$$
$$l_2\, \beta_2\, \theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 + l_1\, \beta_2\, (-bL + bL\, w3\, \theta_1 -$$
$$be\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \theta_1^2) + l_0\, (bL + be\, w3\, (w1 + w2 + w3)\, \theta_1^2 +$$
$$bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_1) +$$
$$be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_2) + bL\, l_2\, (r + \sigma_3) + bL^2\, l_2^2\, (r + \sigma_3) +$$
$$be\, w3^2\, l_2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + bL\, l_1\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_3) -$$
$$be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,2} - l_1\, \theta_1\, (bL\, w3\, \psi - bL\, w3\, \beta_0 +$$
$$be\, \theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) + w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) +$$
$$\beta_{1,3} + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2)\, \theta_1^2)\, \beta_{1,3}))\, /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) -$$
$$(w2^2\, (r + \psi + bL\, (r + \psi)\, l_1 + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w3\, \theta_1 -$$
$$w3\, (r + \psi)\, \theta_1 + bL\, r\, w3\, l_2\, \theta_1 - bL\, w3\, (r + \psi)\, l_2\, \theta_1 + w3\, \beta_0\, \theta_1 +$$
$$bL\, w3\, l_2\, \beta_0\, \theta_1 + w3\, \beta_2\, \theta_1 + bL\, w3\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 +$$
$$be\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, l_2\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$be\, w1^2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 +$$
$$l_1\, \beta_2\, (-bL + bL\, w3\, \theta_1 - be\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \theta_1^2) + l_0\, (bL +$$
$$be\, w3\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_1) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_2) + bL\, l_2\, (r + \sigma_3) +$$
$$bL^2\, l_2^2\, (r + \sigma_3) + be\, w3^2\, l_2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) +$$
$$bL\, l_1\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_3) -$$
$$be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,2} - l_1\, \theta_1\, (bL\, w3\, \psi - bL\, w3\, \beta_0 +$$
$$be\, \theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) + w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) +$$
$$\beta_{1,3} + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2)\, \theta_1^2)\, \beta_{1,3}))\, /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$
$$(w1\, w2\, (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r + \psi)\, \theta_1 +$$
$$bL\, r\, w1\, l_2\, \theta_1 - bL\, w1\, (r + \psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + bL\, w1\, l_2\, \beta_0\, \theta_1 + w1\, \beta_2\, \theta_1 +$$
$$bL\, w1\, l_2\, \beta_2\, \theta_1 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w3$$
$$(r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL + be\, w1\, (w1 + w2 + w3)\, \theta_1^2 +$$
$$bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + bL\, l_2\, (r + \sigma_1) +$$
$$bL^2\, l_2^2\, (r + \sigma_1) + be\, w1^2\, l_2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) +$$
$$be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_2) +$$
$$be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) + \beta_{1,1} + bL\, l_2\, \beta_{1,1} + be\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} +$$
$$be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +$$
$$l_1\, (bL\, (r + \psi) + bL\, r\, w1\, \theta_1 - bL\, w1\, (r + \psi)\, \theta_1 + bL\, w1\, \beta_0\, \theta_1 - be\, w1\, w2$$
$$(r + \psi)\, \theta_1^2 + be\, w2^2\, (r + \psi)\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)\, \theta_1^2 +$$
$$\beta_2\, (-bL + bL\, w1\, \theta_1 + be\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \theta_1^2) +$$
$$bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} +$$
$$be\, w2^2\, \theta_1^2\, \beta_{1,1} + be\, w3^2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \theta_1^2\, \beta_{1,3})))\, /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) -$$
$$(w2^2\, (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r + \psi)\, \theta_1 + bL\, r\, w1\, l_2\, \theta_1 -$$
$$bL\, w1\, (r + \psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + bL\, w1\, l_2\, \beta_0\, \theta_1 + w1\, \beta_2\, \theta_1 + bL\, w1\, l_2\, \beta_2\, \theta_1 -$$

$$
\begin{aligned}
&\mathrm{be}\, w1\, w2\, (r+\psi)\, l_2\, \Theta_1^2 + \mathrm{be}\, w2^2\, (r+\psi)\, l_2\, \Theta_1^2 - \mathrm{be}\, w1\, w3\, (r+\psi)\, l_2\, \Theta_1^2 +\\
&\mathrm{be}\, w3^2\, (r+\psi)\, l_2\, \Theta_1^2 + \mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \Theta_1^2 - \mathrm{be}\, w2^2\, l_2\, \beta_2\, \Theta_1^2 +\\
&\mathrm{be}\, w1\, w3\, l_2\, \beta_2\, \Theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \Theta_1^2 + l_0\, (\mathrm{bL} + \mathrm{be}\, w1\, (w1 + w2 + w3)\, \Theta_1^2 +\\
&\quad \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) + \mathrm{bL}\, l_2\, (r + \sigma_1) +\\
&\mathrm{bL}^2\, l_2^2\, (r + \sigma_1) + \mathrm{be}\, w1^2\, l_2\, \Theta_1^2\, (r + \sigma_1) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) +\\
&\mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + \mathrm{be}\, w1\, w2\, l_2\, \Theta_1^2\, (r + \sigma_2) +\\
&\mathrm{be}\, w1\, w3\, l_2\, \Theta_1^2\, (r + \sigma_3) + \beta_{1,1} + \mathrm{bL}\, l_2\, \beta_{1,1} + \mathrm{be}\, w2^2\, l_2\, \Theta_1^2\, \beta_{1,1} +\\
&\mathrm{be}\, w3^2\, l_2\, \Theta_1^2\, \beta_{1,1} - \mathrm{be}\, w1\, w2\, l_2\, \Theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, l_2\, \Theta_1^2\, \beta_{1,3} +\\
&l_1\, (\mathrm{bL}\, (r+\psi) + \mathrm{bL}\, r\, w1\, \Theta_1 - \mathrm{bL}\, w1\, (r+\psi)\, \Theta_1 + \mathrm{bL}\, w1\, \beta_0\, \Theta_1 - \mathrm{be}\, w1\, w2\\
&\qquad (r+\psi)\, \Theta_1^2 + \mathrm{be}\, w2^2\, (r+\psi)\, \Theta_1^2 - \mathrm{be}\, w1\, w3\, (r+\psi)\, \Theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, \Theta_1^2 +\\
&\quad \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w1\, \Theta_1 + \mathrm{be}\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \Theta_1^2) +\\
&\quad \mathrm{bL}\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)\, (r + \sigma_1) + \mathrm{bL}\, \beta_{1,1} +\\
&\quad \mathrm{be}\, w2^2\, \Theta_1^2\, \beta_{1,1} + \mathrm{be}\, w3^2\, \Theta_1^2\, \beta_{1,1} - \mathrm{be}\, w1\, w2\, \Theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, \Theta_1^2\, \beta_{1,3}))) /\\
&((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2))) -\\
&(w1^2\, (r + \psi + \mathrm{bL}\, (r+\psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w2\, \Theta_1 - w2\, (r+\psi)\, \Theta_1 +\\
&\mathrm{bL}\, r\, w2\, l_2\, \Theta_1 - \mathrm{bL}\, w2\, (r+\psi)\, l_2\, \Theta_1 + w2\, \beta_0\, \Theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_0\, \Theta_1 +\\
&w2\, \beta_2\, \Theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_2\, \Theta_1 + \mathrm{be}\, w1^2\, (r+\psi)\, l_2\, \Theta_1^2 - \mathrm{be}\, w1\, w2\, (r+\psi)\, l_2\, \Theta_1^2 -\\
&\mathrm{be}\, w2\, w3\, (r+\psi)\, l_2\, \Theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, l_2\, \Theta_1^2 - \mathrm{be}\, w1^2\, l_2\, \beta_2\, \Theta_1^2 +\\
&\mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \Theta_1^2 + \mathrm{be}\, w2\, w3\, l_2\, \beta_2\, \Theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \Theta_1^2 + l_0\, (\mathrm{bL} +\\
&\quad \mathrm{be}\, w2\, (w1 + w2 + w3)\, \Theta_1^2 + \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) +\\
&\mathrm{be}\, w1\, w2\, l_2\, \Theta_1^2\, (r + \sigma_1) + \mathrm{bL}\, l_2\, (r + \sigma_2) + \mathrm{bL}^2\, l_2^2\, (r + \sigma_2) +\\
&\mathrm{be}\, w2^2\, l_2\, \Theta_1^2\, (r + \sigma_2) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_2) + \mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_2) +\\
&\mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_2) + \mathrm{be}\, w2\, w3\, l_2\, \Theta_1^2\, (r + \sigma_3) - \mathrm{be}\, w1\, w2\, l_2\, \Theta_1^2\, \beta_{1,1} +\\
&\beta_{1,2} + \mathrm{bL}\, l_2\, \beta_{1,2} + \mathrm{be}\, w1^2\, l_2\, \Theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, l_2\, \Theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, l_2\, \Theta_1^2\, \beta_{1,3} +\\
&l_1\, (\mathrm{bL}\, (r+\psi) + \mathrm{bL}\, r\, w2\, \Theta_1 - \mathrm{bL}\, w2\, (r+\psi)\, \Theta_1 + \mathrm{bL}\, w2\, \beta_0\, \Theta_1 + \mathrm{be}\, w1^2\, (r+\psi)\\
&\qquad \Theta_1^2 - \mathrm{be}\, w1\, w2\, (r+\psi)\, \Theta_1^2 - \mathrm{be}\, w2\, w3\, (r+\psi)\, \Theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, \Theta_1^2 +\\
&\quad \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w2\, \Theta_1 - \mathrm{be}\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \Theta_1^2) +\\
&\quad \mathrm{bL}\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)\, (r + \sigma_2) - \mathrm{be}\, w1\, w2\, \Theta_1^2\, \beta_{1,1} +\\
&\quad \mathrm{bL}\, \beta_{1,2} + \mathrm{be}\, w1^2\, \Theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, \Theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, \Theta_1^2\, \beta_{1,3}))) /\\
&((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2))) +\\
&(w1\, w2\, (r + \psi + \mathrm{bL}\, (r+\psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w2\, \Theta_1 - w2\, (r+\psi)\, \Theta_1 +\\
&\mathrm{bL}\, r\, w2\, l_2\, \Theta_1 - \mathrm{bL}\, w2\, (r+\psi)\, l_2\, \Theta_1 + w2\, \beta_0\, \Theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_0\, \Theta_1 +\\
&w2\, \beta_2\, \Theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_2\, \Theta_1 + \mathrm{be}\, w1^2\, (r+\psi)\, l_2\, \Theta_1^2 - \mathrm{be}\, w1\, w2\, (r+\psi)\, l_2\, \Theta_1^2 -\\
&\mathrm{be}\, w2\, w3\, (r+\psi)\, l_2\, \Theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, l_2\, \Theta_1^2 - \mathrm{be}\, w1^2\, l_2\, \beta_2\, \Theta_1^2 +\\
&\mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \Theta_1^2 + \mathrm{be}\, w2\, w3\, l_2\, \beta_2\, \Theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \Theta_1^2 + l_0\, (\mathrm{bL} +\\
&\quad \mathrm{be}\, w2\, (w1 + w2 + w3)\, \Theta_1^2 + \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) +\\
&\mathrm{be}\, w1\, w2\, l_2\, \Theta_1^2\, (r + \sigma_1) + \mathrm{bL}\, l_2\, (r + \sigma_2) + \mathrm{bL}^2\, l_2^2\, (r + \sigma_2) +\\
&\mathrm{be}\, w2^2\, l_2\, \Theta_1^2\, (r + \sigma_2) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_2) + \mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_2) +\\
&\mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_2) + \mathrm{be}\, w2\, w3\, l_2\, \Theta_1^2\, (r + \sigma_3) - \mathrm{be}\, w1\, w2\, l_2\, \Theta_1^2\, \beta_{1,1} +\\
&\beta_{1,2} + \mathrm{bL}\, l_2\, \beta_{1,2} + \mathrm{be}\, w1^2\, l_2\, \Theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, l_2\, \Theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, l_2\, \Theta_1^2\, \beta_{1,3} +\\
&l_1\, (\mathrm{bL}\, (r+\psi) + \mathrm{bL}\, r\, w2\, \Theta_1 - \mathrm{bL}\, w2\, (r+\psi)\, \Theta_1 + \mathrm{bL}\, w2\, \beta_0\, \Theta_1 + \mathrm{be}\, w1^2\, (r+\psi)\\
&\qquad \Theta_1^2 - \mathrm{be}\, w1\, w2\, (r+\psi)\, \Theta_1^2 - \mathrm{be}\, w2\, w3\, (r+\psi)\, \Theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, \Theta_1^2 +\\
&\quad \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w2\, \Theta_1 - \mathrm{be}\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \Theta_1^2) +\\
&\quad \mathrm{bL}\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)\, (r + \sigma_2) - \mathrm{be}\, w1\, w2\, \Theta_1^2\, \beta_{1,1} +\\
&\quad \mathrm{bL}\, \beta_{1,2} + \mathrm{be}\, w1^2\, \Theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, \Theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, \Theta_1^2\, \beta_{1,3}))) /\\
&((1 + \mathrm{bL}\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2))) -\\
&w3^2\, ((r + \psi + \mathrm{bL}\, (r+\psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w1\, \Theta_1 - w1\, (r+\psi)\, \Theta_1 + \mathrm{bL}\, r\, w1\, l_2\, \Theta_1 -\\
&\quad \mathrm{bL}\, w1\, (r+\psi)\, l_2\, \Theta_1 + w1\, \beta_0\, \Theta_1 + \mathrm{bL}\, w1\, l_2\, \beta_0\, \Theta_1 + w1\, \beta_2\, \Theta_1 + \mathrm{bL}\, w1\, l_2\, \beta_2\, \Theta_1 -\\
&\quad \mathrm{be}\, w1\, w2\, (r+\psi)\, l_2\, \Theta_1^2 + \mathrm{be}\, w2^2\, (r+\psi)\, l_2\, \Theta_1^2 - \mathrm{be}\, w1\, w3\, (r+\psi)\, l_2\, \Theta_1^2 +\\
&\quad \mathrm{be}\, w3^2\, (r+\psi)\, l_2\, \Theta_1^2 + \mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \Theta_1^2 - \mathrm{be}\, w2^2\, l_2\, \beta_2\, \Theta_1^2 +\\
&\quad \mathrm{be}\, w1\, w3\, l_2\, \beta_2\, \Theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \Theta_1^2 + l_0\, (\mathrm{bL} + \mathrm{be}\, w1\, (w1 + w2 + w3)\, \Theta_1^2 +\\
&\qquad \mathrm{bL}\, (l_1 + l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) + \mathrm{bL}\, l_2\, (r + \sigma_1) +
\end{aligned}
$$

$$\begin{aligned}
&\mathrm{bL}^2\, l_2^2\, (r+\sigma_1) + \mathrm{be}\, w1^2\, l_2\, \theta_1^2\, (r+\sigma_1) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \theta_1^2\, (r+\sigma_1) +\\
&\mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r+\sigma_1) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \theta_1^2\, (r+\sigma_1) + \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\\
&(r+\sigma_2) + \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, (r+\sigma_3) + \beta_{1,1} + \mathrm{bL}\, l_2\, \beta_{1,1} + \mathrm{be}\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} +\\
&\mathrm{be}\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} - \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +\\
&l_1\, (\mathrm{bL}\, (r+\psi) + \mathrm{bL}\, r\, w1\, \theta_1 - \mathrm{bL}\, w1\, (r+\psi)\, \theta_1 + \mathrm{bL}\, w1\, \beta_0\, \theta_1 -\\
&\mathrm{be}\, w1\, w2\, (r+\psi)\, \theta_1^2 + \mathrm{be}\, w2^2\, (r+\psi)\, \theta_1^2 - \mathrm{be}\, w1\, w3\, (r+\psi)\, \theta_1^2 + \mathrm{be}\, w3^2\\
&(r+\psi)\, \theta_1^2 + \beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w1\, \theta_1 + \mathrm{be}\, (-w2^2 - w3^2 + w1\, (w2+w3))\, \theta_1^2) +\\
&\mathrm{bL}\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)\, (r+\sigma_1) + \mathrm{bL}\, \beta_{1,1} +\\
&\mathrm{be}\, w2^2\, \theta_1^2\, \beta_{1,1} + \mathrm{be}\, w3^2\, \theta_1^2\, \beta_{1,1} - \mathrm{be}\, w1\, w2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, \theta_1^2\, \beta_{1,3})) /\\
&((1+\mathrm{bL}\, (l_1+l_2))\, (1+(l_1+l_2)\, (\mathrm{bL}+\mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2))) +\\
&(r+\psi+\mathrm{bL}\, (r+\psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r+\psi)\, \theta_1 + \mathrm{bL}\, r\, w2\, l_2\, \theta_1 -\\
&\mathrm{bL}\, w2\, (r+\psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_0\, \theta_1 + w2\, \beta_2\, \theta_1 + \mathrm{bL}\, w2\, l_2\, \beta_2\, \theta_1 +\\
&\mathrm{be}\, w1^2\, (r+\psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1\, w2\, (r+\psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w2\, w3\, (r+\psi)\, l_2\, \theta_1^2 +\\
&\mathrm{be}\, w3^2\, (r+\psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1^2\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 +\\
&\mathrm{be}\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (\mathrm{bL} + \mathrm{be}\, w2\, (w1+w2+w3)\, \theta_1^2 +\\
&\mathrm{bL}\, (l_1+l_2)\, (\mathrm{bL}+\mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)) + \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, (r+\sigma_1) +\\
&\mathrm{bL}\, l_2\, (r+\sigma_2) + \mathrm{bL}^2\, l_2^2\, (r+\sigma_2) + \mathrm{be}\, w2^2\, l_2\, \theta_1^2\, (r+\sigma_2) + \mathrm{be}\, \mathrm{bL}\\
&w1^2\, l_2^2\, \theta_1^2\, (r+\sigma_2) + \mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r+\sigma_2) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \theta_1^2\, (r+\sigma_2) +\\
&\mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, (r+\sigma_3) - \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} + \beta_{1,2} + \mathrm{bL}\, l_2\, \beta_{1,2} +\\
&\mathrm{be}\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +\\
&l_1\, (\mathrm{bL}\, (r+\psi) + \mathrm{bL}\, r\, w2\, \theta_1 - \mathrm{bL}\, w2\, (r+\psi)\, \theta_1 + \mathrm{bL}\, w2\, \beta_0\, \theta_1 + \mathrm{be}\, w1^2\, (r+\psi)\\
&\theta_1^2 - \mathrm{be}\, w1\, w2\, (r+\psi)\, \theta_1^2 - \mathrm{be}\, w2\, w3\, (r+\psi)\, \theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, \theta_1^2 +\\
&\beta_2\, (-\mathrm{bL} + \mathrm{bL}\, w2\, \theta_1 - \mathrm{be}\, (w1^2 - w1\, w2 + w3\, (-w2+w3))\, \theta_1^2) +\\
&\mathrm{bL}\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)\, (r+\sigma_2) - \mathrm{be}\, w1\, w2\, \theta_1^2\, \beta_{1,1} +\\
&\mathrm{bL}\, \beta_{1,2} + \mathrm{be}\, w1^2\, \theta_1^2\, \beta_{1,2} + \mathrm{be}\, w3^2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w2\, w3\, \theta_1^2\, \beta_{1,3})) /\\
&((1+\mathrm{bL}\, (l_1+l_2))\, (1+(l_1+l_2)\, (\mathrm{bL}+\mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)))) +\\
&w3\, (w1\, ((r+\psi+\mathrm{bL}\, (r+\psi)\, l_1 + \mathrm{bL}\, (r+\psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w3\, \theta_1 - w3\, (r+\psi)\\
&\theta_1 + \mathrm{bL}\, r\, w3\, l_2\, \theta_1 - \mathrm{bL}\, w3\, (r+\psi)\, l_2\, \theta_1 + w3\, \beta_0\, \theta_1 + \mathrm{bL}\, w3\, l_2\, \beta_0\, \theta_1 +\\
&w3\, \beta_2\, \theta_1 + \mathrm{bL}\, w3\, l_2\, \beta_2\, \theta_1 + \mathrm{be}\, w1^2\, (r+\psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w2^2\, (r+\psi)\, l_2\, \theta_1^2 -\\
&\mathrm{be}\, w1\, w3\, (r+\psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w2\, w3\, (r+\psi)\, l_2\, \theta_1^2 - \mathrm{be}\, w1^2\, l_2\, \beta_2\, \theta_1^2 -\\
&\mathrm{be}\, w2^2\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 + l_1\, \beta_2\\
&(-\mathrm{bL} + \mathrm{bL}\, w3\, \theta_1 - \mathrm{be}\, (w1^2 + w2^2 - (w1+w2)\, w3)\, \theta_1^2) + l_0\, (\mathrm{bL} + \mathrm{be}\, w3\\
&(w1+w2+w3)\, \theta_1^2 + \mathrm{bL}\, (l_1+l_2)\, (\mathrm{bL}+\mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)) +\\
&\mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, (r+\sigma_1) + \mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, (r+\sigma_2) + \mathrm{bL}\, l_2\, (r+\sigma_3) +\\
&\mathrm{bL}^2\, l_2^2\, (r+\sigma_3) + \mathrm{be}\, w3^2\, l_2\, \theta_1^2\, (r+\sigma_3) + \mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \theta_1^2\, (r+\sigma_3) +\\
&\mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r+\sigma_3) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\, \theta_1^2\, (r+\sigma_3) +\\
&\mathrm{bL}\, l_1\, l_2\, (\mathrm{bL} + \mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)\, (r+\sigma_3) - \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,1} -\\
&\mathrm{be}\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,2} - l_1\, \theta_1\, (\mathrm{bL}\, w3\, \psi - \mathrm{bL}\, w3\, \beta_0 + \mathrm{be}\, \theta_1\\
&(-(w1^2 + w2^2 - (w1+w2)\, w3)\, (r+\psi) + w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) +\\
&\beta_{1,3} + (l_1+l_2)\, (\mathrm{bL} + \mathrm{be}\, (w1^2+w2^2)\, \theta_1^2)\, \beta_{1,3}) /\\
&((1+\mathrm{bL}\, (l_1+l_2))\, (1+(l_1+l_2)\, (\mathrm{bL}+\mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2))) +\\
&(r+\psi+\mathrm{bL}\, (r+\psi)\, l_2 - \beta_2 - \mathrm{bL}\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r+\psi)\, \theta_1 +\\
&\mathrm{bL}\, r\, w1\, l_2\, \theta_1 - \mathrm{bL}\, w1\, (r+\psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + \mathrm{bL}\, w1\, l_2\, \beta_0\, \theta_1 +\\
&w1\, \beta_2\, \theta_1 + \mathrm{bL}\, w1\, l_2\, \beta_2\, \theta_1 - \mathrm{be}\, w1\, w2\, (r+\psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w2^2\, (r+\psi)\, l_2\\
&\theta_1^2 - \mathrm{be}\, w1\, w3\, (r+\psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w3^2\, (r+\psi)\, l_2\, \theta_1^2 + \mathrm{be}\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 -\\
&\mathrm{be}\, w2^2\, l_2\, \beta_2\, \theta_1^2 + \mathrm{be}\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 - \mathrm{be}\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (\mathrm{bL} + \mathrm{be}\, w1\\
&(w1+w2+w3)\, \theta_1^2 + \mathrm{bL}\, (l_1+l_2)\, (\mathrm{bL}+\mathrm{be}\, (w1^2+w2^2+w3^2)\, \theta_1^2)) +\\
&\mathrm{bL}\, l_2\, (r+\sigma_1) + \mathrm{bL}^2\, l_2^2\, (r+\sigma_1) + \mathrm{be}\, w1^2\, l_2\, \theta_1^2\, (r+\sigma_1) +\\
&\mathrm{be}\, \mathrm{bL}\, w1^2\, l_2^2\, \theta_1^2\, (r+\sigma_1) + \mathrm{be}\, \mathrm{bL}\, w2^2\, l_2^2\, \theta_1^2\, (r+\sigma_1) + \mathrm{be}\, \mathrm{bL}\, w3^2\, l_2^2\\
&\theta_1^2\, (r+\sigma_1) + \mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, (r+\sigma_2) + \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, (r+\sigma_3) +\\
&\beta_{1,1} + \mathrm{bL}\, l_2\, \beta_{1,1} + \mathrm{be}\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} + \mathrm{be}\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} -\\
&\mathrm{be}\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - \mathrm{be}\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} + l_1\, (\mathrm{bL}\, (r+\psi) +
\end{aligned}$$

$$
\begin{aligned}
&\mathrm{bL}\,r\,w1\,\theta_1 - \mathrm{bL}\,w1\,(r+\psi)\,\theta_1 + \mathrm{bL}\,w1\,\beta_0\,\theta_1 - \mathrm{be}\,w1\,w2\,(r+\psi)\,\theta_1^2 + \\
&\mathrm{be}\,w2^2\,(r+\psi)\,\theta_1^2 - \mathrm{be}\,w1\,w3\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2 + \\
&\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,w1\,\theta_1 + \mathrm{be}\,(-w2^2 - w3^2 + w1\,(w2+w3))\,\theta_1^2) + \\
&\mathrm{bL}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(w1^2 + w2^2 + w3^2)\,\theta_1^2)\,(r+\sigma_1) + \mathrm{bL}\,\beta_{1,1} + \mathrm{be}\,w2^2 \\
&\theta_1^2\,\beta_{1,1} + \mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,1} - \mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w1\,w3\,\theta_1^2\,\beta_{1,3}))\,/ \\
&((1 + \mathrm{bL}\,(l_1+l_2))\,(1 + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)))) + \\
&w2\,((r + \psi + \mathrm{bL}\,(r+\psi)\,l_1 + \mathrm{bL}\,(r+\psi)\,l_2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,w3\,\theta_1 - w3\,(r+\psi) \\
&\theta_1 + \mathrm{bL}\,r\,w3\,l_2\,\theta_1 - \mathrm{bL}\,w3\,(r+\psi)\,l_2\,\theta_1 + w3\,\beta_0\,\theta_1 + \mathrm{bL}\,w3\,l_2\,\beta_0\,\theta_1 + \\
&w3\,\beta_2\,\theta_1 + \mathrm{bL}\,w3\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)\,l_2\,\theta_1^2 + \mathrm{be}\,w2^2\,(r+\psi)\,l_2\,\theta_1^2 - \\
&\mathrm{be}\,w1\,w3\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w2\,w3\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1^2\,l_2\,\beta_2\,\theta_1^2 - \\
&\mathrm{be}\,w2^2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,w1\,w3\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 + l_1\,\beta_2 \\
&(-\mathrm{bL} + \mathrm{bL}\,w3\,\theta_1 - \mathrm{be}\,(w1^2 + w2^2 - (w1+w2)\,w3)\,\theta_1^2) + l_0\,(\mathrm{bL} + \mathrm{be}\,w3 \\
&(w1+w2+w3)\,\theta_1^2 + \mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)) + \\
&\mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,(r+\sigma_1) + \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,(r+\sigma_2) + \mathrm{bL}\,l_2\,(r+\sigma_3) + \\
&\mathrm{bL}^2\,l_2^2\,(r+\sigma_3) + \mathrm{be}\,w3^2\,l_2\,\theta_1^2\,(r+\sigma_3) + \mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_3) + \\
&\mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2\,(r+\sigma_3) + \mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_3) + \\
&\mathrm{bL}\,l_1\,l_2\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_3) - \mathrm{be}\,w1\,w3\,l_2\,\theta_1^2\,\beta_{1,1} - \\
&\mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,2} - l_1\,\theta_1\,(\mathrm{bL}\,w3\,\psi - \mathrm{bL}\,w3\,\beta_0 + \mathrm{be}\,\theta_1 \\
&(-(w1^2 + w2^2 - (w1+w2)\,w3)\,(r+\psi) + w1\,w3\,\beta_{1,1} + w2\,w3\,\beta_{1,2})) + \\
&\beta_{1,3} + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2)\,\theta_1^2)\,\beta_{1,3})\,/ \\
&((1 + \mathrm{bL}\,(l_1+l_2))\,(1 + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2))) + \\
&(r + \psi + \mathrm{bL}\,(r+\psi)\,l_2 - \beta_2 - \mathrm{bL}\,l_2\,\beta_2 + r\,w2\,\theta_1 - w2\,(r+\psi)\,\theta_1 + \\
&\mathrm{bL}\,r\,w2\,l_2\,\theta_1 - \mathrm{bL}\,w2\,(r+\psi)\,l_2\,\theta_1 + w2\,\beta_0\,\theta_1 + \mathrm{bL}\,w2\,l_2\,\beta_0\,\theta_1 + \\
&w2\,\beta_2\,\theta_1 + \mathrm{bL}\,w2\,l_2\,\beta_2\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1\,w2\,(r+\psi)\,l_2 \\
&\theta_1^2 - \mathrm{be}\,w2\,w3\,(r+\psi)\,l_2\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,l_2\,\theta_1^2 - \mathrm{be}\,w1^2\,l_2\,\beta_2\,\theta_1^2 + \\
&\mathrm{be}\,w1\,w2\,l_2\,\beta_2\,\theta_1^2 + \mathrm{be}\,w2\,w3\,l_2\,\beta_2\,\theta_1^2 - \mathrm{be}\,w3^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(\mathrm{bL} + \mathrm{be}\,w2 \\
&(w1+w2+w3)\,\theta_1^2 + \mathrm{bL}\,(l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)) + \\
&\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,(r+\sigma_1) + \mathrm{bL}\,l_2\,(r+\sigma_2) + \mathrm{bL}^2\,l_2^2\,(r+\sigma_2) + \\
&\mathrm{be}\,w2^2\,l_2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w1^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w2^2\,l_2^2\,\theta_1^2 \\
&(r+\sigma_2) + \mathrm{be}\,\mathrm{bL}\,w3^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,(r+\sigma_3) - \\
&\mathrm{be}\,w1\,w2\,l_2\,\theta_1^2\,\beta_{1,1} + \beta_{1,2} + \mathrm{bL}\,l_2\,\beta_{1,2} + \mathrm{be}\,w1^2\,l_2\,\theta_1^2\,\beta_{1,2} + \\
&\mathrm{be}\,w3^2\,l_2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w2\,w3\,l_2\,\theta_1^2\,\beta_{1,3} + l_1\,(\mathrm{bL}\,(r+\psi) + \\
&\mathrm{bL}\,r\,w2\,\theta_1 - \mathrm{bL}\,w2\,(r+\psi)\,\theta_1 + \mathrm{bL}\,w2\,\beta_0\,\theta_1 + \mathrm{be}\,w1^2\,(r+\psi)\,\theta_1^2 - \\
&\mathrm{be}\,w1\,w2\,(r+\psi)\,\theta_1^2 - \mathrm{be}\,w2\,w3\,(r+\psi)\,\theta_1^2 + \mathrm{be}\,w3^2\,(r+\psi)\,\theta_1^2 + \\
&\beta_2\,(-\mathrm{bL} + \mathrm{bL}\,w2\,\theta_1 - \mathrm{be}\,(w1^2 - w1\,w2 + w3\,(-w2+w3))\,\theta_1^2) + \\
&\mathrm{bL}\,l_2\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2)\,(r+\sigma_2) - \mathrm{be}\,w1\,w2\,\theta_1^2\,\beta_{1,1} + \\
&\mathrm{bL}\,\beta_{1,2} + \mathrm{be}\,w1^2\,\theta_1^2\,\beta_{1,2} + \mathrm{be}\,w3^2\,\theta_1^2\,\beta_{1,2} - \mathrm{be}\,w2\,w3\,\theta_1^2\,\beta_{1,3}))\,/ \\
&((1 + \mathrm{bL}\,(l_1+l_2))\,(1 + (l_1+l_2)\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2) \\
&\theta_1^2)))))))\Big)\Big) \Big/ (\mathrm{bd}\,\mathrm{bL}\,(\mathrm{bL} + \mathrm{be}\,(w1^2+w2^2+w3^2)\,\theta_1^2))
\end{aligned}
$$

In[61]:= **D[%, r] // FullSimplify**

Out[61]=

$$\begin{aligned}
&((1+\mathrm{bL}\,(l_1+l_2))\,(3\,l_1+d_2\,((3\,\mathrm{bd}+\mathrm{bL}\,(-1+\rho)\,\rho)\,l_1+(-1+\rho)\,\rho\,(1+\mathrm{bL}\,l_2)))-\\
&\quad(\mathrm{w1}+\mathrm{w2}+\mathrm{w3})\,(1+\mathrm{bd}\,d_2)\,l_1\,(1+\mathrm{bL}\,(l_1+l_2))\,\theta_1+\\
&\quad\mathrm{be}\,(l_1+l_2)\,(2\,(\mathrm{w1}^2+\mathrm{w2}^2-\mathrm{w2}\,\mathrm{w3}+\mathrm{w3}^2-\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,l_1+\\
&\qquad d_2\,((2\,\mathrm{bd}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3}+\mathrm{w3}^2)+\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(-1+\rho)\,\rho)\,l_1+\\
&\qquad\quad(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(-1+\rho)\,\rho\,(1+\mathrm{bL}\,l_2)))\,\theta_1^2+\\
&\quad d_1\,(l_1^2\,(\mathrm{bL}\,(3\,\mathrm{bd}+\mathrm{bL}\,(-1+\rho)^2)-\mathrm{bd}\,\mathrm{bL}\,(\mathrm{w1}+\mathrm{w2}+\mathrm{w3})\,\theta_1+\mathrm{be}\,(2\,\mathrm{bd}\\
&\qquad\quad(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3}+\mathrm{w3}^2)+\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(-1+\rho)^2)\,\theta_1^2)+\\
&\qquad l_1\,((3\,\mathrm{bd}+2\,\mathrm{bL}\,(-1+\rho)^2)\,(1+\mathrm{bL}\,l_2)-\mathrm{bd}\,(\mathrm{w1}+\mathrm{w2}+\mathrm{w3})\,(1+\mathrm{bL}\,l_2)\,\theta_1+\\
&\qquad\quad\mathrm{be}\,((\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(-1+\rho)^2+2\,(\mathrm{bd}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3}+\mathrm{w3}^2)+\\
&\qquad\qquad\mathrm{bL}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,(-1+\rho)^2)\,l_2)\,\theta_1^2)+\\
&\qquad(-1+\rho)^2\,(1+\mathrm{bL}\,l_2)\,(1+l_2\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2))))\,/\\
&\ ((1+\mathrm{bd}\,(d_1+d_2))\,(1+\mathrm{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2)))
\end{aligned}$$

In[62]:= **Assets = LoanDemand1 + LoanDemand2 + LoanDemand3 + G**

Out[62]=

$$\begin{aligned}
&-(l_0\,(-1+(l_1+l_2)\,(-\mathrm{bL}-\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\theta_1^2))+\\
&\qquad l_1^2\,(\mathrm{bL}\,\mathrm{w3}\,(-\psi+\beta_0)\,\theta_1+\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w3}\,\theta_1-\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\theta_1^2)+\\
&\qquad\quad\mathrm{bL}\,(r+\psi+\beta_{1,3})+\mathrm{be}\,\theta_1^2\,((\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,(r+\psi)-\mathrm{w3}\,(\mathrm{w1}\,\beta_{1,1}+\mathrm{w2}\,\beta_{1,2})+\\
&\qquad\qquad(\mathrm{w1}^2+\mathrm{w2}^2)\,\beta_{1,3}))+l_2\,(\psi-\beta_2+\mathrm{w3}\,(-\psi+\beta_0+\beta_2)\,\theta_1-\sigma_3+\beta_{1,3}+\\
&\qquad\quad l_2\,(-\mathrm{bL}\,\mathrm{w3}\,(\psi-\beta_0)\,\theta_1+\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w3}\,\theta_1-\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\theta_1^2)+\\
&\qquad\qquad\mathrm{bL}\,(\psi-\sigma_3+\beta_{1,3})+\mathrm{be}\,\theta_1^2\,((\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\psi+\mathrm{w1}\,\mathrm{w3}\,\sigma_1+\\
&\qquad\qquad\quad\mathrm{w2}\,\mathrm{w3}\,\sigma_2-\mathrm{w1}^2\,\sigma_3-\mathrm{w2}^2\,\sigma_3-\mathrm{w1}\,\mathrm{w3}\,\beta_{1,1}-\mathrm{w2}\,\mathrm{w3}\,\beta_{1,2}+(\mathrm{w1}^2+\mathrm{w2}^2)\,\beta_{1,3})))+\\
&\qquad l_1\,(r+\psi-\beta_2+\mathrm{w3}\,(-\psi+\beta_0+\beta_2)\,\theta_1+\beta_{1,3}+l_2\,(-2\,\mathrm{bL}\,\mathrm{w3}\,(\psi-\beta_0)\,\theta_1+\\
&\qquad\qquad 2\,\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w3}\,\theta_1-\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,\theta_1^2)+\\
&\qquad\qquad\mathrm{bL}\,(r+2\,\psi-\sigma_3+2\,\beta_{1,3})+\mathrm{be}\,\theta_1^2\,((\mathrm{w1}^2+\mathrm{w2}^2-(\mathrm{w1}+\mathrm{w2})\,\mathrm{w3})\,(r+2\,\psi)+\mathrm{w1}\,\mathrm{w3}\,\sigma_1+\\
&\qquad\qquad\quad\mathrm{w2}\,\mathrm{w3}\,\sigma_2-\mathrm{w1}^2\,\sigma_3-\mathrm{w2}^2\,\sigma_3-2\,\mathrm{w1}\,\mathrm{w3}\,\beta_{1,1}-2\,\mathrm{w2}\,\mathrm{w3}\,\beta_{1,2}+2\,(\mathrm{w1}^2+\mathrm{w2}^2)\,\beta_{1,3}))))\,/\\
&\quad((1+\mathrm{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2)))-\\
&\ (l_0\,(-1+(l_1+l_2)\,(-\mathrm{bL}+\mathrm{be}\,(-\mathrm{w2}^2-\mathrm{w3}^2+\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,\theta_1^2))+\\
&\quad l_1^2\,(\mathrm{bL}\,\mathrm{w1}\,(-\psi+\beta_0)\,\theta_1+\\
&\qquad\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w1}\,\theta_1+\mathrm{be}\,(-\mathrm{w2}^2-\mathrm{w3}^2+\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,\theta_1^2)+\mathrm{bL}\,(r+\psi+\beta_{1,1})+\\
&\qquad\mathrm{be}\,\theta_1^2\,((\mathrm{w2}^2+\mathrm{w3}^2-\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,(r+\psi)+(\mathrm{w2}^2+\mathrm{w3}^2)\,\beta_{1,1}-\mathrm{w1}\,(\mathrm{w2}\,\beta_{1,2}+\mathrm{w3}\,\beta_{1,3})))+\\
&\quad l_1\,(r+\psi-\beta_2+\mathrm{w1}\,(-\psi+\beta_0+\beta_2)\,\theta_1+\beta_{1,1}+l_2\,(-2\,\mathrm{bL}\,\mathrm{w1}\,(\psi-\beta_0)\,\theta_1+2\,\beta_2\\
&\qquad\qquad(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w1}\,\theta_1+\mathrm{be}\,((\mathrm{w1}-\mathrm{w2})\,\mathrm{w2}+\mathrm{w1}\,\mathrm{w3}-\mathrm{w3}^2)\,\theta_1^2)+\mathrm{bL}\,(r+2\,\psi-\sigma_1+2\,\beta_{1,1})+\\
&\qquad\quad\mathrm{be}\,\theta_1^2\,((\mathrm{w2}^2+\mathrm{w3}^2-\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,(r+2\,\psi)-(\mathrm{w2}^2+\mathrm{w3}^2)\,\sigma_1+\mathrm{w1}\,\mathrm{w2}\,\sigma_2+\\
&\qquad\qquad\mathrm{w1}\,\mathrm{w3}\,\sigma_3+2\,\mathrm{w2}^2\,\beta_{1,1}+2\,\mathrm{w3}^2\,\beta_{1,1}-2\,\mathrm{w1}\,\mathrm{w2}\,\beta_{1,2}-2\,\mathrm{w1}\,\mathrm{w3}\,\beta_{1,3})))+\\
&\quad l_2\,(\psi-\beta_2+\mathrm{w1}\,(-\psi+\beta_0+\beta_2)\,\theta_1-\sigma_1+\beta_{1,1}+l_2\,(-\mathrm{bL}\,\mathrm{w1}\,(\psi-\beta_0)\,\theta_1+\\
&\qquad\quad\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w1}\,\theta_1+\mathrm{be}\,((\mathrm{w1}-\mathrm{w2})\,\mathrm{w2}+\mathrm{w1}\,\mathrm{w3}-\mathrm{w3}^2)\,\theta_1^2)+\mathrm{bL}\,(\psi-\sigma_1+\beta_{1,1})+\\
&\qquad\quad\mathrm{be}\,\theta_1^2\,((\mathrm{w2}^2+\mathrm{w3}^2-\mathrm{w1}\,(\mathrm{w2}+\mathrm{w3}))\,\psi-(\mathrm{w2}^2+\mathrm{w3}^2)\,\sigma_1+\mathrm{w1}\,\mathrm{w2}\,\sigma_2+\\
&\qquad\qquad\mathrm{w1}\,\mathrm{w3}\,\sigma_3+\mathrm{w2}^2\,\beta_{1,1}+\mathrm{w3}^2\,\beta_{1,1}-\mathrm{w1}\,\mathrm{w2}\,\beta_{1,2}-\mathrm{w1}\,\mathrm{w3}\,\beta_{1,3}))))\,/\\
&\quad((1+\mathrm{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\mathrm{bL}+\mathrm{be}\,(\mathrm{w1}^2+\mathrm{w2}^2+\mathrm{w3}^2)\,\theta_1^2)))-\\
&\ (l_0\,(-1+(l_1+l_2)\,(-\mathrm{bL}-\mathrm{be}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w3}\,(-\mathrm{w2}+\mathrm{w3}))\,\theta_1^2))+\\
&\quad l_1^2\,(\mathrm{bL}\,\mathrm{w2}\,(-\psi+\beta_0)\,\theta_1+\\
&\qquad\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w2}\,\theta_1-\mathrm{be}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w3}\,(-\mathrm{w2}+\mathrm{w3}))\,\theta_1^2)+\mathrm{bL}\,(r+\psi+\beta_{1,2})+\mathrm{be}\,\theta_1^2\\
&\qquad\quad((\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}-\mathrm{w2}\,\mathrm{w3}+\mathrm{w3}^2)\,(r+\psi)-\mathrm{w1}\,\mathrm{w2}\,\beta_{1,1}+(\mathrm{w1}^2+\mathrm{w3}^2)\,\beta_{1,2}-\mathrm{w2}\,\mathrm{w3}\,\beta_{1,3}))+\\
&\quad l_1\,(r+\psi-\beta_2+\mathrm{w2}\,(-\psi+\beta_0+\beta_2)\,\theta_1+\beta_{1,2}+l_2\,(-2\,\mathrm{bL}\,\mathrm{w2}\,(\psi-\beta_0)\,\theta_1+2\,\beta_2\\
&\qquad\qquad(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w2}\,\theta_1-\mathrm{be}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w3}\,(-\mathrm{w2}+\mathrm{w3}))\,\theta_1^2)+\mathrm{bL}\,(r+2\,\psi-\sigma_2+2\,\beta_{1,2})+\\
&\qquad\quad\mathrm{be}\,\theta_1^2\,((\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}-\mathrm{w2}\,\mathrm{w3}+\mathrm{w3}^2)\,(r+2\,\psi)+\mathrm{w1}\,\mathrm{w2}\,\sigma_1-(\mathrm{w1}^2+\mathrm{w3}^2)\,\sigma_2+\\
&\qquad\qquad\mathrm{w2}\,\mathrm{w3}\,\sigma_3-2\,\mathrm{w1}\,\mathrm{w2}\,\beta_{1,1}+2\,\mathrm{w1}^2\,\beta_{1,2}+2\,\mathrm{w3}^2\,\beta_{1,2}-2\,\mathrm{w2}\,\mathrm{w3}\,\beta_{1,3})))+\\
&\quad l_2\,(\psi-\beta_2+\mathrm{w2}\,(-\psi+\beta_0+\beta_2)\,\theta_1-\sigma_2+\beta_{1,2}+l_2\,(-\mathrm{bL}\,\mathrm{w2}\,(\psi-\beta_0)\,\theta_1+\\
&\qquad\quad\beta_2\,(-\mathrm{bL}+\mathrm{bL}\,\mathrm{w2}\,\theta_1-\mathrm{be}\,(\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}+\mathrm{w3}\,(-\mathrm{w2}+\mathrm{w3}))\,\theta_1^2)+\mathrm{bL}\,(\psi-\sigma_2+\beta_{1,2})+\\
&\qquad\quad\mathrm{be}\,\theta_1^2\,((\mathrm{w1}^2-\mathrm{w1}\,\mathrm{w2}-\mathrm{w2}\,\mathrm{w3}+\mathrm{w3}^2)\,\psi+\mathrm{w1}\,\mathrm{w2}\,\sigma_1-(\mathrm{w1}^2+\mathrm{w3}^2)\,\sigma_2+\\
&\qquad\qquad\mathrm{w2}\,\mathrm{w3}\,\sigma_3-\mathrm{w1}\,\mathrm{w2}\,\beta_{1,1}+\mathrm{w1}^2\,\beta_{1,2}+\mathrm{w3}^2\,\beta_{1,2}-\mathrm{w2}\,\mathrm{w3}\,\beta_{1,3}))))\,/
\end{aligned}$$

$$((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))+$$
$$\Big(\beta_2\,(-bL\,(3\,bd+bL\,(-1+\rho)^2)+2\,bd\,bL\,(w1+w2+w3)\,\Theta_1-$$
$$(bd\,(bL\,(w1^2+w2^2+w3^2)+2\,be\,(w1^2-w1\,w2+w2^2-(w1+w2)\,w3+w3^2))+$$
$$be\,bL\,(w1^2+w2^2+w3^2)\,(-1+\rho)^2)\,\Theta_1^2)+bd\,bL\,\Theta_1$$
$$(-r\,(w1+w2+w3)-2\,(w1+w2+w3)\,\psi+(w1+w2+w3)\,\beta_0-w1\,\beta_{1,1}-w2\,\beta_{1,2}-w3\,\beta_{1,3}+$$
$$(w3\,(r+\psi+bL\,(r+\psi)\,l_1+bL\,(r+\psi)\,l_2-\beta_2-bL\,l_2\,\beta_2+r\,w3\,\Theta_1-w3\,(r+\psi)\,\Theta_1+$$
$$bL\,r\,w3\,l_2\,\Theta_1-bL\,w3\,(r+\psi)\,l_2\,\Theta_1+w3\,\beta_0\,\Theta_1+bL\,w3\,l_2\,\beta_0\,\Theta_1+w3\,\beta_2\,\Theta_1+$$
$$bL\,w3\,l_2\,\beta_2\,\Theta_1+be\,w1^2\,(r+\psi)\,l_2\,\Theta_1^2+be\,w2^2\,(r+\psi)\,l_2\,\Theta_1^2-be\,w1\,w3\,(r+\psi)\,l_2$$
$$\Theta_1^2-be\,w2\,w3\,(r+\psi)\,l_2\,\Theta_1^2-be\,w1^2\,l_2\,\beta_2\,\Theta_1^2-be\,w2^2\,l_2\,\beta_2\,\Theta_1^2+be\,w1\,w3\,l_2\,\beta_2\,\Theta_1^2+$$
$$be\,w2\,w3\,l_2\,\beta_2\,\Theta_1^2+l_1\,\beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+$$
$$l_0\,(bL+be\,w3\,(w1+w2+w3)\,\Theta_1^2+bL\,(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2))+$$
$$be\,w1\,w3\,l_2\,\Theta_1^2\,(r+\sigma_1)+be\,w2\,w3\,l_2\,\Theta_1^2\,(r+\sigma_2)+bL\,l_2\,(r+\sigma_3)+$$
$$bL^2\,l_2^2\,(r+\sigma_3)+be\,w3^2\,l_2\,\Theta_1^2\,(r+\sigma_3)+be\,bL\,w1^2\,l_2^2\,\Theta_1^2\,(r+\sigma_3)+be\,bL\,w2^2\,l_2^2$$
$$\Theta_1^2\,(r+\sigma_3)+be\,bL\,w3^2\,l_2^2\,\Theta_1^2\,(r+\sigma_3)+bL\,l_1\,l_2\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)$$
$$(r+\sigma_3)-be\,w1\,w3\,l_2\,\Theta_1^2\,\beta_{1,1}-be\,w2\,w3\,l_2\,\Theta_1^2\,\beta_{1,2}-l_1\,\Theta_1\,(bL\,w3\,\psi-bL\,w3$$
$$\beta_0+be\,\Theta_1\,(-(w1^2+w2^2-(w1+w2)\,w3)\,(r+\psi)+w1\,w3\,\beta_{1,1}+w2\,w3\,\beta_{1,2}))+$$
$$\beta_{1,3}+(l_1+l_2)\,(bL+be\,(w1^2+w2^2)\,\Theta_1^2)\,\beta_{1,3}))\,/$$
$$((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))+$$
$$(w1\,(r+\psi+bL\,(r+\psi)\,l_2-\beta_2-bL\,l_2\,\beta_2+r\,w1\,\Theta_1-w1\,(r+\psi)\,\Theta_1+bL\,r\,w1\,l_2\,\Theta_1-$$
$$bL\,w1\,(r+\psi)\,l_2\,\Theta_1+w1\,\beta_0\,\Theta_1+bL\,w1\,l_2\,\beta_0\,\Theta_1+w1\,\beta_2\,\Theta_1+bL\,w1\,l_2\,\beta_2\,\Theta_1-be\,w1$$
$$w2\,(r+\psi)\,l_2\,\Theta_1^2+be\,w2^2\,(r+\psi)\,l_2\,\Theta_1^2-be\,w1\,w3\,(r+\psi)\,l_2\,\Theta_1^2+be\,w3^2\,(r+\psi)$$
$$l_2\,\Theta_1^2+be\,w1\,w2\,l_2\,\beta_2\,\Theta_1^2-be\,w2^2\,l_2\,\beta_2\,\Theta_1^2+be\,w1\,w3\,l_2\,\beta_2\,\Theta_1^2-be\,w3^2\,l_2\,\beta_2\,\Theta_1^2+$$
$$l_0\,(bL+be\,w1\,(w1+w2+w3)\,\Theta_1^2+bL\,(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2))+$$
$$bL\,l_2\,(r+\sigma_1)+bL^2\,l_2^2\,(r+\sigma_1)+be\,w1^2\,l_2\,\Theta_1^2\,(r+\sigma_1)+$$
$$be\,bL\,w1^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1)+be\,bL\,w2^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1)+be\,bL\,w3^2\,l_2^2\,\Theta_1^2\,(r+\sigma_1)+$$
$$be\,w1\,w2\,l_2\,\Theta_1^2\,(r+\sigma_2)+be\,w1\,w3\,l_2\,\Theta_1^2\,(r+\sigma_3)+\beta_{1,1}+bL\,l_2\,\beta_{1,1}+$$
$$be\,w2^2\,l_2\,\Theta_1^2\,\beta_{1,1}+be\,w3^2\,l_2\,\Theta_1^2\,\beta_{1,1}-be\,w1\,w2\,l_2\,\Theta_1^2\,\beta_{1,2}-be\,w1\,w3\,l_2\,\Theta_1^2\,\beta_{1,3}+$$
$$l_1\,(bL\,(r+\psi)+bL\,r\,w1\,\Theta_1-bL\,w1\,(r+\psi)\,\Theta_1+bL\,w1\,\beta_0\,\Theta_1-be\,w1\,w2$$
$$(r+\psi)\,\Theta_1^2+be\,w2^2\,(r+\psi)\,\Theta_1^2-be\,w1\,w3\,(r+\psi)\,\Theta_1^2+be\,w3^2\,(r+\psi)$$
$$\Theta_1^2+\beta_2\,(-bL+bL\,w1\,\Theta_1+be\,(-w2^2-w3^2+w1\,(w2+w3))\,\Theta_1^2)+bL$$
$$l_2\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)\,(r+\sigma_1)+bL\,\beta_{1,1}+be\,w2^2\,\Theta_1^2$$
$$\beta_{1,1}+be\,w3^2\,\Theta_1^2\,\beta_{1,1}-be\,w1\,w2\,\Theta_1^2\,\beta_{1,2}-be\,w1\,w3\,\Theta_1^2\,\beta_{1,3})))\,/$$
$$((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))+$$
$$(w2\,(r+\psi+bL\,(r+\psi)\,l_2-\beta_2-bL\,l_2\,\beta_2+r\,w2\,\Theta_1-w2\,(r+\psi)\,\Theta_1+bL\,r\,w2\,l_2\,\Theta_1-$$
$$bL\,w2\,(r+\psi)\,l_2\,\Theta_1+w2\,\beta_0\,\Theta_1+bL\,w2\,l_2\,\beta_0\,\Theta_1+w2\,\beta_2\,\Theta_1+bL\,w2\,l_2\,\beta_2\,\Theta_1+be\,w1^2$$
$$(r+\psi)\,l_2\,\Theta_1^2-be\,w1\,w2\,(r+\psi)\,l_2\,\Theta_1^2-be\,w2\,w3\,(r+\psi)\,l_2\,\Theta_1^2+be\,w3^2\,(r+\psi)$$
$$l_2\,\Theta_1^2-be\,w1^2\,l_2\,\beta_2\,\Theta_1^2+be\,w1\,w2\,l_2\,\beta_2\,\Theta_1^2+be\,w2\,w3\,l_2\,\beta_2\,\Theta_1^2-be\,w3^2\,l_2\,\beta_2\,\Theta_1^2+$$
$$l_0\,(bL+be\,w2\,(w1+w2+w3)\,\Theta_1^2+bL\,(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2))+$$
$$be\,w1\,w2\,l_2\,\Theta_1^2\,(r+\sigma_1)+bL\,l_2\,(r+\sigma_2)+bL^2\,l_2^2\,(r+\sigma_2)+be\,w2^2\,l_2\,\Theta_1^2\,(r+\sigma_2)+$$
$$be\,bL\,w1^2\,l_2^2\,\Theta_1^2\,(r+\sigma_2)+be\,bL\,w2^2\,l_2^2\,\Theta_1^2\,(r+\sigma_2)+be\,bL\,w3^2\,l_2^2\,\Theta_1^2\,(r+\sigma_2)+$$
$$be\,w2\,w3\,l_2\,\Theta_1^2\,(r+\sigma_3)-be\,w1\,w2\,l_2\,\Theta_1^2\,\beta_{1,1}+\beta_{1,2}+bL\,l_2\,\beta_{1,2}+be\,w1^2\,l_2\,\Theta_1^2\,\beta_{1,2}+$$
$$be\,w3^2\,l_2\,\Theta_1^2\,\beta_{1,2}-be\,w2\,w3\,l_2\,\Theta_1^2\,\beta_{1,3}+l_1\,(bL\,(r+\psi)+bL\,r\,w2\,\Theta_1-bL\,w2\,(r+\psi)$$
$$\Theta_1+bL\,w2\,\beta_0\,\Theta_1+be\,w1^2\,(r+\psi)\,\Theta_1^2-be\,w1\,w2\,(r+\psi)\,\Theta_1^2-be\,w2\,w3\,(r+\psi)$$
$$\Theta_1^2+be\,w3^2\,(r+\psi)\,\Theta_1^2+\beta_2\,(-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))$$
$$\Theta_1^2)+bL\,l_2\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)\,(r+\sigma_2)-be\,w1\,w2\,\Theta_1^2$$
$$\beta_{1,1}+bL\,\beta_{1,2}+be\,w1^2\,\Theta_1^2\,\beta_{1,2}+be\,w3^2\,\Theta_1^2\,\beta_{1,2}-be\,w2\,w3\,\Theta_1^2\,\beta_{1,3})))\,/$$
$$((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2))))+$$
$$bL\,\Big(\frac{bL\,(-1+\rho)\,(-bd\,d_0+bd\,(r+\psi)\,((-1+\rho)\,d_1+\rho\,d_2)+(-1+\rho)\,\beta_2)}{1+bd\,(d_1+d_2)}+$$
$$bd\,(\beta_{1,1}+\beta_{1,2}+\beta_{1,3})-$$
$$bd\,(-3\,r-3\,\psi+(r+\psi+bL\,(r+\psi)\,l_1+bL\,(r+\psi)\,l_2-\beta_2-bL\,l_2\,\beta_2+$$
$$r\,w3\,\Theta_1-w3\,(r+\psi)\,\Theta_1+bL\,r\,w3\,l_2\,\Theta_1-bL\,w3\,(r+\psi)\,l_2\,\Theta_1+w3\,\beta_0\,\Theta_1+$$

$$
\begin{aligned}
&\text{bL}\,\text{w3}\,l_2\,\beta_0\,\theta_1 + \text{w3}\,\beta_2\,\theta_1 + \text{bL}\,\text{w3}\,l_2\,\beta_2\,\theta_1 + \text{be}\,\text{w1}^2\,(r+\psi)\,l_2\,\theta_1^2 + \\
&\text{be}\,\text{w2}^2\,(r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w1}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w2}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1^2 - \\
&\text{be}\,\text{w1}^2\,l_2\,\beta_2\,\theta_1^2 - \text{be}\,\text{w2}^2\,l_2\,\beta_2\,\theta_1^2 + \text{be}\,\text{w1}\,\text{w3}\,l_2\,\beta_2\,\theta_1^2 + \text{be}\,\text{w2}\,\text{w3}\,l_2\,\beta_2\,\theta_1^2 + \\
&l_1\,\beta_2\,(-\text{bL} + \text{bL}\,\text{w3}\,\theta_1 - \text{be}\,(\text{w1}^2 + \text{w2}^2 - (\text{w1}+\text{w2})\,\text{w3})\,\theta_1^2) + \\
&l_0\,(\text{bL} + \text{be}\,\text{w3}\,(\text{w1}+\text{w2}+\text{w3})\,\theta_1^2 + \text{bL}\,(l_1+l_2)\,(\text{bL} + \text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)) + \\
&\text{be}\,\text{w1}\,\text{w3}\,l_2\,\theta_1^2\,(r+\sigma_1) + \text{be}\,\text{w2}\,\text{w3}\,l_2\,\theta_1^2\,(r+\sigma_2) + \text{bL}\,l_2\,(r+\sigma_3) + \\
&\text{bL}^2\,l_2^2\,(r+\sigma_3) + \text{be}\,\text{w3}^2\,l_2\,\theta_1^2\,(r+\sigma_3) + \text{be}\,\text{bL}\,\text{w1}^2\,l_2^2\,\theta_1^2\,(r+\sigma_3) + \text{be}\,\text{bL}\,\text{w2}^2\,l_2^2 \\
&\quad \theta_1^2\,(r+\sigma_3) + \text{be}\,\text{bL}\,\text{w3}^2\,l_2^2\,\theta_1^2\,(r+\sigma_3) + \text{bL}\,l_1\,l_2\,(\text{bL} + \text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2) \\
&\quad (r+\sigma_3) - \text{be}\,\text{w1}\,\text{w3}\,l_2\,\theta_1^2\,\beta_{1,1} - \text{be}\,\text{w2}\,\text{w3}\,l_2\,\theta_1^2\,\beta_{1,2} - l_1\,\theta_1\,(\text{bL}\,\text{w3}\,\psi - \text{bL}\,\text{w3}\,\beta_0 + \\
&\quad\quad \text{be}\,\theta_1\,(-(\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3})\,(r+\psi) + \text{w1}\,\text{w3}\,\beta_{1,1} + \text{w2}\,\text{w3}\,\beta_{1,2})) + \\
&\beta_{1,3} + (l_1+l_2)\,(\text{bL} + \text{be}\,(\text{w1}^2+\text{w2}^2)\,\theta_1^2)\,\beta_{1,3})\,/ \\
&((1+\text{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)))\, + \\
&(r+\psi+\text{bL}\,(r+\psi)\,l_2 - \beta_2 - \text{bL}\,l_2\,\beta_2 + r\,\text{w1}\,\theta_1 - \text{w1}\,(r+\psi)\,\theta_1 + \text{bL}\,r\,\text{w1}\,l_2\,\theta_1 - \\
&\text{bL}\,\text{w1}\,(r+\psi)\,l_2\,\theta_1 + \text{w1}\,\beta_0\,\theta_1 + \text{bL}\,\text{w1}\,l_2\,\beta_0\,\theta_1 + \text{w1}\,\beta_2\,\theta_1 + \text{bL}\,\text{w1}\,l_2\,\beta_2\,\theta_1 - \\
&\text{be}\,\text{w1}\,\text{w2}\,(r+\psi)\,l_2\,\theta_1^2 + \text{be}\,\text{w2}^2\,(r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w1}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1^2 + \\
&\text{be}\,\text{w3}^2\,(r+\psi)\,l_2\,\theta_1^2 + \text{be}\,\text{w1}\,\text{w2}\,l_2\,\beta_2\,\theta_1^2 - \text{be}\,\text{w2}^2\,l_2\,\beta_2\,\theta_1^2 + \text{be}\,\text{w1}\,\text{w3}\,l_2\,\beta_2\,\theta_1^2 - \\
&\text{be}\,\text{w3}^2\,l_2\,\beta_2\,\theta_1^2 + l_0\,(\text{bL} + \text{be}\,\text{w1}\,(\text{w1}+\text{w2}+\text{w3})\,\theta_1^2 + \text{bL}\,(l_1+l_2) \\
&\quad\quad (\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)) + \text{bL}\,l_2\,(r+\sigma_1) + \text{bL}^2\,l_2^2\,(r+\sigma_1) + \\
&\text{be}\,\text{w1}^2\,l_2\,\theta_1^2\,(r+\sigma_1) + \text{be}\,\text{bL}\,\text{w1}^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \text{be}\,\text{bL}\,\text{w2}^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \\
&\text{be}\,\text{bL}\,\text{w3}^2\,l_2^2\,\theta_1^2\,(r+\sigma_1) + \text{be}\,\text{w1}\,\text{w2}\,l_2\,\theta_1^2\,(r+\sigma_2) + \text{be}\,\text{w1}\,\text{w3}\,l_2\,\theta_1^2\,(r+\sigma_3) + \\
&\beta_{1,1} + \text{bL}\,l_2\,\beta_{1,1} + \text{be}\,\text{w2}^2\,l_2\,\theta_1^2\,\beta_{1,1} + \text{be}\,\text{w3}^2\,l_2\,\theta_1^2\,\beta_{1,1} - \text{be}\,\text{w1}\,\text{w2}\,l_2\,\theta_1^2\,\beta_{1,2} - \\
&\text{be}\,\text{w1}\,\text{w3}\,l_2\,\theta_1^2\,\beta_{1,3} + l_1\,(\text{bL}\,(r+\psi) + \text{bL}\,r\,\text{w1}\,\theta_1 - \text{bL}\,\text{w1}\,(r+\psi)\,\theta_1 + \\
&\quad \text{bL}\,\text{w1}\,\beta_0\,\theta_1 - \text{be}\,\text{w1}\,\text{w2}\,(r+\psi)\,\theta_1^2 + \text{be}\,\text{w2}^2\,(r+\psi)\,\theta_1^2 - \text{be}\,\text{w1}\,\text{w3}\,(r+\psi)\,\theta_1^2 + \\
&\quad \text{be}\,\text{w3}^2\,(r+\psi)\,\theta_1^2 + \beta_2\,(-\text{bL} + \text{bL}\,\text{w1}\,\theta_1 + \text{be}\,(-\text{w2}^2 - \text{w3}^2 + \text{w1}\,(\text{w2}+\text{w3}))\,\theta_1^2) + \\
&\quad \text{bL}\,l_2\,(\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)\,(r+\sigma_1) + \text{bL}\,\beta_{1,1} + \\
&\quad \text{be}\,\text{w2}^2\,\theta_1^2\,\beta_{1,1} + \text{be}\,\text{w3}^2\,\theta_1^2\,\beta_{1,1} - \text{be}\,\text{w1}\,\text{w2}\,\theta_1^2\,\beta_{1,2} - \text{be}\,\text{w1}\,\text{w3}\,\theta_1^2\,\beta_{1,3}))\,/ \\
&((1+\text{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)))\, + \\
&(r+\psi+\text{bL}\,(r+\psi)\,l_2 - \beta_2 - \text{bL}\,l_2\,\beta_2 + r\,\text{w2}\,\theta_1 - \text{w2}\,(r+\psi)\,\theta_1 + \text{bL}\,r\,\text{w2}\,l_2\,\theta_1 - \\
&\text{bL}\,\text{w2}\,(r+\psi)\,l_2\,\theta_1 + \text{w2}\,\beta_0\,\theta_1 + \text{bL}\,\text{w2}\,l_2\,\beta_0\,\theta_1 + \text{w2}\,\beta_2\,\theta_1 + \text{bL}\,\text{w2}\,l_2\,\beta_2\,\theta_1 + \text{be}\,\text{w1}^2 \\
&\quad (r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w1}\,\text{w2}\,(r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w2}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1^2 + \text{be}\,\text{w3}^2\,(r+\psi) \\
&\quad l_2\,\theta_1^2 - \text{be}\,\text{w1}^2\,l_2\,\beta_2\,\theta_1^2 + \text{be}\,\text{w1}\,\text{w2}\,l_2\,\beta_2\,\theta_1^2 + \text{be}\,\text{w2}\,\text{w3}\,l_2\,\beta_2\,\theta_1^2 - \text{be}\,\text{w3}^2\,l_2\,\beta_2\,\theta_1^2 + \\
&l_0\,(\text{bL} + \text{be}\,\text{w2}\,(\text{w1}+\text{w2}+\text{w3})\,\theta_1^2 + \text{bL}\,(l_1+l_2)\,(\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)) + \\
&\text{be}\,\text{w1}\,\text{w2}\,l_2\,\theta_1^2\,(r+\sigma_1) + \text{bL}\,l_2\,(r+\sigma_2) + \text{bL}^2\,l_2^2\,(r+\sigma_2) + \\
&\text{be}\,\text{w2}^2\,l_2\,\theta_1^2\,(r+\sigma_2) + \text{be}\,\text{bL}\,\text{w1}^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \text{be}\,\text{bL}\,\text{w2}^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \\
&\text{be}\,\text{bL}\,\text{w3}^2\,l_2^2\,\theta_1^2\,(r+\sigma_2) + \text{be}\,\text{w2}\,\text{w3}\,l_2\,\theta_1^2\,(r+\sigma_3) - \text{be}\,\text{w1}\,\text{w2}\,l_2\,\theta_1^2\,\beta_{1,1} + \\
&\beta_{1,2} + \text{bL}\,l_2\,\beta_{1,2} + \text{be}\,\text{w1}^2\,l_2\,\theta_1^2\,\beta_{1,2} + \text{be}\,\text{w3}^2\,l_2\,\theta_1^2\,\beta_{1,2} - \text{be}\,\text{w2}\,\text{w3}\,l_2\,\theta_1^2\,\beta_{1,3} + \\
&l_1\,(\text{bL}\,(r+\psi) + \text{bL}\,r\,\text{w2}\,\theta_1 - \text{bL}\,\text{w2}\,(r+\psi)\,\theta_1 + \text{bL}\,\text{w2}\,\beta_0\,\theta_1 + \text{be}\,\text{w1}^2\,(r+\psi)\,\theta_1^2 - \\
&\quad \text{be}\,\text{w1}\,\text{w2}\,(r+\psi)\,\theta_1^2 - \text{be}\,\text{w2}\,\text{w3}\,(r+\psi)\,\theta_1^2 + \text{be}\,\text{w3}^2\,(r+\psi)\,\theta_1^2 + \\
&\quad \beta_2\,(-\text{bL} + \text{bL}\,\text{w2}\,\theta_1 - \text{be}\,(\text{w1}^2 - \text{w1}\,\text{w2} + \text{w3}\,(-\text{w2}+\text{w3}))\,\theta_1^2) + \\
&\quad \text{bL}\,l_2\,(\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2)\,(r+\sigma_2) - \text{be}\,\text{w1}\,\text{w2}\,\theta_1^2\,\beta_{1,1} + \\
&\quad \text{bL}\,\beta_{1,2} + \text{be}\,\text{w1}^2\,\theta_1^2\,\beta_{1,2} + \text{be}\,\text{w3}^2\,\theta_1^2\,\beta_{1,2} - \text{be}\,\text{w2}\,\text{w3}\,\theta_1^2\,\beta_{1,3}))\,/ \\
&((1+\text{bL}\,(l_1+l_2))\,(1+(l_1+l_2)\,(\text{bL}+\text{be}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\theta_1^2))))\Big) + \\
&\theta_1^2\,\Big(-\text{bd}\,\text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\beta_0 + \frac{1}{1+\text{bd}\,(d_1+d_2)}\,(\text{be}\,\text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2) \\
&\quad (-1+\rho)\,(-\text{bd}\,d_0 + \text{bd}\,(r+\psi)\,((-1+\rho)\,d_1 + \rho\,d_2) + (-1+\rho)\,\beta_2)) + \\
&\quad \text{bd}\,\text{be}\,((\text{w2}^2+\text{w3}^2 - \text{w1}\,(\text{w2}+\text{w3}))\,\beta_{1,1} + (\text{w1}^2 - \text{w1}\,\text{w2} - \text{w2}\,\text{w3} + \text{w3}^2)\,\beta_{1,2} + \\
&\quad\quad (\text{w1}^2+\text{w2}^2-(\text{w1}+\text{w2})\,\text{w3})\,\beta_{1,3}) + \\
&\quad \text{bd}\,(\text{bL}\,(\text{w1}^2+\text{w2}^2+\text{w3}^2)\,\psi + \text{be}\,(2\,(\text{w1}^2+\text{w2}^2-\text{w2}\,\text{w3}+\text{w3}^2-\text{w1}\,(\text{w2}+\text{w3}))\,(r+\psi) - \\
&\quad\quad (\text{w1}^2\,(r+\psi+\text{bL}\,(r+\psi)\,l_1 + \text{bL}\,(r+\psi)\,l_2 - \beta_2 - \text{bL}\,l_2\,\beta_2 + r\,\text{w3}\,\theta_1 - \text{w3}\,(r+\psi) \\
&\quad\quad\quad \theta_1 + \text{bL}\,r\,\text{w3}\,l_2\,\theta_1 - \text{bL}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1 + \text{w3}\,\beta_0\,\theta_1 + \text{bL}\,\text{w3}\,l_2\,\beta_0\,\theta_1 + \\
&\quad\quad\quad \text{w3}\,\beta_2\,\theta_1 + \text{bL}\,\text{w3}\,l_2\,\beta_2\,\theta_1 + \text{be}\,\text{w1}^2\,(r+\psi)\,l_2\,\theta_1^2 + \text{be}\,\text{w2}^2\,(r+\psi)\,l_2\,\theta_1^2 - \\
&\quad\quad\quad \text{be}\,\text{w1}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w2}\,\text{w3}\,(r+\psi)\,l_2\,\theta_1^2 - \text{be}\,\text{w1}^2\,l_2\,\beta_2\,\theta_1^2 - \text{be}\,\text{w2}^2
\end{aligned}
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\begin{aligned}
&l_2\, \beta_2\, \Theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \Theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \Theta_1^2 + l_1\, \beta_2\, (-bL + bL\, w3\, \Theta_1 - \\
&\quad be\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \Theta_1^2) + l_0\, (bL + be\, w3\, (w1 + w2 + w3)\, \Theta_1^2 + \\
&\quad bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) + be\, w1\, w3\, l_2\, \Theta_1^2\, (r + \sigma_1) + \\
&be\, w2\, w3\, l_2\, \Theta_1^2\, (r + \sigma_2) + bL\, l_2\, (r + \sigma_3) + bL^2\, l_2^2\, (r + \sigma_3) + be\, w3^2 \\
&\; l_2\, \Theta_1^2\, (r + \sigma_3) + be\, bL\, w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_3) + be\, bL\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_3) + be \\
&\; bL\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_3) + bL\, l_1\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)\, (r + \sigma_3) - \\
&be\, w1\, w3\, l_2\, \Theta_1^2\, \beta_{1,1} - be\, w2\, w3\, l_2\, \Theta_1^2\, \beta_{1,2} - l_1\, \Theta_1\, (bL\, w3\, \psi - bL\, w3\, \beta_0 + \\
&\quad be\, \Theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) + w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) + \\
&\beta_{1,3} + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2)\, \Theta_1^2)\, \beta_{1,3})) \, / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2))) - \\
&(w2^2\, (r + \psi + bL\, (r + \psi)\, l_1 + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w3\, \Theta_1 - w3 \\
&\quad (r + \psi)\, \Theta_1 + bL\, r\, w3\, l_2\, \Theta_1 - bL\, w3\, (r + \psi)\, l_2\, \Theta_1 + w3\, \beta_0\, \Theta_1 + bL\, w3\, l_2\, \beta_0\, \Theta_1 + \\
&\; w3\, \beta_2\, \Theta_1 + bL\, w3\, l_2\, \beta_2\, \Theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \Theta_1^2 - \\
&\; be\, w1\, w3\, (r + \psi)\, l_2\, \Theta_1^2 - be\, w2\, w3\, (r + \psi)\, l_2\, \Theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \Theta_1^2 - be\, w2^2 \\
&\; l_2\, \beta_2\, \Theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \Theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \Theta_1^2 + l_1\, \beta_2\, (-bL + bL\, w3\, \Theta_1 - \\
&\quad be\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \Theta_1^2) + l_0\, (bL + be\, w3\, (w1 + w2 + w3)\, \Theta_1^2 + \\
&\quad bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) + be\, w1\, w3\, l_2\, \Theta_1^2\, (r + \sigma_1) + \\
&\; be\, w2\, w3\, l_2\, \Theta_1^2\, (r + \sigma_2) + bL\, l_2\, (r + \sigma_3) + bL^2\, l_2^2\, (r + \sigma_3) + be\, w3^2 \\
&\; l_2\, \Theta_1^2\, (r + \sigma_3) + be\, bL\, w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_3) + be\, bL\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_3) + be \\
&\; bL\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_3) + bL\, l_1\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)\, (r + \sigma_3) - \\
&\; be\, w1\, w3\, l_2\, \Theta_1^2\, \beta_{1,1} - be\, w2\, w3\, l_2\, \Theta_1^2\, \beta_{1,2} - l_1\, \Theta_1\, (bL\, w3\, \psi - bL\, w3\, \beta_0 + \\
&\quad be\, \Theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) + w1\, w3\, \beta_{1,1} + w2\, w3\, \beta_{1,2})) + \\
&\; \beta_{1,3} + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2)\, \Theta_1^2)\, \beta_{1,3})) \, / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2))) + \\
&(w1\, w2\, (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \Theta_1 - w1\, (r + \psi)\, \Theta_1 + \\
&\; bL\, r\, w1\, l_2\, \Theta_1 - bL\, w1\, (r + \psi)\, l_2\, \Theta_1 + w1\, \beta_0\, \Theta_1 + bL\, w1\, l_2\, \beta_0\, \Theta_1 + \\
&\; w1\, \beta_2\, \Theta_1 + bL\, w1\, l_2\, \beta_2\, \Theta_1 - be\, w1\, w2\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \Theta_1^2 - \\
&\; be\, w1\, w3\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \Theta_1^2 - \\
&\; be\, w2^2\, l_2\, \beta_2\, \Theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \Theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \Theta_1^2 + l_0\, (bL + \\
&\quad be\, w1\, (w1 + w2 + w3)\, \Theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) + \\
&\; bL\, l_2\, (r + \sigma_1) + bL^2\, l_2^2\, (r + \sigma_1) + be\, w1^2\, l_2\, \Theta_1^2\, (r + \sigma_1) + be\, bL \\
&\; w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + be\, bL\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + be\, bL\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + \\
&\; be\, w1\, w2\, l_2\, \Theta_1^2\, (r + \sigma_2) + be\, w1\, w3\, l_2\, \Theta_1^2\, (r + \sigma_3) + \beta_{1,1} + bL\, l_2\, \beta_{1,1} + \\
&\; be\, w2^2\, l_2\, \Theta_1^2\, \beta_{1,1} + be\, w3^2\, l_2\, \Theta_1^2\, \beta_{1,1} - be\, w1\, w2\, l_2\, \Theta_1^2\, \beta_{1,2} - be\, w1\, w3\, l_2 \\
&\; \Theta_1^2\, \beta_{1,3} + l_1\, (bL\, (r + \psi) + bL\, r\, w1\, \Theta_1 - bL\, w1\, (r + \psi)\, \Theta_1 + bL\, w1\, \beta_0\, \Theta_1 - \\
&\quad be\, w1\, w2\, (r + \psi)\, \Theta_1^2 + be\, w2^2\, (r + \psi)\, \Theta_1^2 - be\, w1\, w3\, (r + \psi)\, \Theta_1^2 + be\, w3^2 \\
&\qquad (r + \psi)\, \Theta_1^2 + \beta_2\, (-bL + bL\, w1\, \Theta_1 + be\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \Theta_1^2) + \\
&\quad bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} + \\
&\quad be\, w2^2\, \Theta_1^2\, \beta_{1,1} + be\, w3^2\, \Theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \Theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \Theta_1^2\, \beta_{1,3}))) \, / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2))) - \\
&(w2^2\, (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \Theta_1 - w1\, (r + \psi)\, \Theta_1 + \\
&\; bL\, r\, w1\, l_2\, \Theta_1 - bL\, w1\, (r + \psi)\, l_2\, \Theta_1 + w1\, \beta_0\, \Theta_1 + bL\, w1\, l_2\, \beta_0\, \Theta_1 + \\
&\; w1\, \beta_2\, \Theta_1 + bL\, w1\, l_2\, \beta_2\, \Theta_1 - be\, w1\, w2\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \Theta_1^2 - \\
&\; be\, w1\, w3\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \Theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \Theta_1^2 - \\
&\; be\, w2^2\, l_2\, \beta_2\, \Theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \Theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \Theta_1^2 + l_0\, (bL + \\
&\quad be\, w1\, (w1 + w2 + w3)\, \Theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \Theta_1^2)) + \\
&\; bL\, l_2\, (r + \sigma_1) + bL^2\, l_2^2\, (r + \sigma_1) + be\, w1^2\, l_2\, \Theta_1^2\, (r + \sigma_1) + be\, bL \\
&\; w1^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + be\, bL\, w2^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + be\, bL\, w3^2\, l_2^2\, \Theta_1^2\, (r + \sigma_1) + \\
&\; be\, w1\, w2\, l_2\, \Theta_1^2\, (r + \sigma_2) + be\, w1\, w3\, l_2\, \Theta_1^2\, (r + \sigma_3) + \beta_{1,1} + bL\, l_2\, \beta_{1,1} + \\
&\; be\, w2^2\, l_2\, \Theta_1^2\, \beta_{1,1} + be\, w3^2\, l_2\, \Theta_1^2\, \beta_{1,1} - be\, w1\, w2\, l_2\, \Theta_1^2\, \beta_{1,2} - be\, w1\, w3\, l_2 \\
&\; \Theta_1^2\, \beta_{1,3} + l_1\, (bL\, (r + \psi) + bL\, r\, w1\, \Theta_1 - bL\, w1\, (r + \psi)\, \Theta_1 + bL\, w1\, \beta_0\, \Theta_1 - \\
&\quad be\, w1\, w2\, (r + \psi)\, \Theta_1^2 + be\, w2^2\, (r + \psi)\, \Theta_1^2 - be\, w1\, w3\, (r + \psi)\, \Theta_1^2 + be\, w3^2
\end{aligned}
$$

$$(r + \psi)\, \theta_1^2 + \beta_2\, (-bL + bL\, w1\, \theta_1 + be\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \theta_1^2) +$$
$$bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} +$$
$$be\, w2^2\, \theta_1^2\, \beta_{1,1} + be\, w3^2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \theta_1^2\, \beta_{1,3}))) /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) -$$
$$(w1^2\, (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 +$$
$$bL\, r\, w2\, l_2\, \theta_1 - bL\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + bL\, w2\, l_2\, \beta_0\, \theta_1 +$$
$$w2\, \beta_2\, \theta_1 + bL\, w2\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL +$$
$$be\, w2\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + bL\, l_2\, (r + \sigma_2) + bL^2\, l_2^2\, (r + \sigma_2) +$$
$$be\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) +$$
$$be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} +$$
$$\beta_{1,2} + bL\, l_2\, \beta_{1,2} + be\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, l_2\, \theta_1^2$$
$$\beta_{1,3} + l_1\, (bL\, (r + \psi) + bL\, r\, w2\, \theta_1 - bL\, w2\, (r + \psi)\, \theta_1 + bL\, w2\, \beta_0\, \theta_1 + be\, w1^2$$
$$(r + \psi)\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-bL + bL\, w2\, \theta_1 - be\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) +$$
$$bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} +$$
$$bL\, \beta_{1,2} + be\, w1^2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, \theta_1^2\, \beta_{1,3}))) /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$
$$(w1\, w2\, (r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 +$$
$$bL\, r\, w2\, l_2\, \theta_1 - bL\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + bL\, w2\, l_2\, \beta_0\, \theta_1 +$$
$$w2\, \beta_2\, \theta_1 + bL\, w2\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 -$$
$$be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL +$$
$$be\, w2\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) +$$
$$be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + bL\, l_2\, (r + \sigma_2) + bL^2\, l_2^2\, (r + \sigma_2) +$$
$$be\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) +$$
$$be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} +$$
$$\beta_{1,2} + bL\, l_2\, \beta_{1,2} + be\, w1^2\, l_2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, l_2\, \theta_1^2$$
$$\beta_{1,3} + l_1\, (bL\, (r + \psi) + bL\, r\, w2\, \theta_1 - bL\, w2\, (r + \psi)\, \theta_1 + bL\, w2\, \beta_0\, \theta_1 + be\, w1^2$$
$$(r + \psi)\, \theta_1^2 - be\, w1\, w2\, (r + \psi)\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-bL + bL\, w2\, \theta_1 - be\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) +$$
$$bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} +$$
$$bL\, \beta_{1,2} + be\, w1^2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, \theta_1^2\, \beta_{1,3}))) /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) -$$
$$w3^2\, ((r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w1\, \theta_1 - w1\, (r + \psi)\, \theta_1 +$$
$$bL\, r\, w1\, l_2\, \theta_1 - bL\, w1\, (r + \psi)\, l_2\, \theta_1 + w1\, \beta_0\, \theta_1 + bL\, w1\, l_2\, \beta_0\, \theta_1 + w1\, \beta_2\, \theta_1 +$$
$$bL\, w1\, l_2\, \beta_2\, \theta_1 - be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w2^2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w1\, w3$$
$$(r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2\, l_2\, \beta_2\, \theta_1^2 +$$
$$be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 - be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL + be\, w1\, (w1 + w2 + w3)\, \theta_1^2 +$$
$$bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + bL\, l_2\, (r + \sigma_1) +$$
$$bL^2\, l_2^2\, (r + \sigma_1) + be\, w1^2\, l_2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) +$$
$$be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_1) + be\, w1\, w2\, l_2\, \theta_1^2$$
$$(r + \sigma_2) + be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) + \beta_{1,1} + bL\, l_2\, \beta_{1,1} + be\, w2^2\, l_2\, \theta_1^2\, \beta_{1,1} +$$
$$be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,3} +$$
$$l_1\, (bL\, (r + \psi) + bL\, r\, w1\, \theta_1 - bL\, w1\, (r + \psi)\, \theta_1 + bL\, w1\, \beta_0\, \theta_1 - be\, w1\, w2$$
$$(r + \psi)\, \theta_1^2 + be\, w2^2\, (r + \psi)\, \theta_1^2 - be\, w1\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)$$
$$\theta_1^2 + \beta_2\, (-bL + bL\, w1\, \theta_1 + be\, (-w2^2 - w3^2 + w1\, (w2 + w3))\, \theta_1^2) +$$
$$bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_1) + bL\, \beta_{1,1} +$$
$$be\, w2^2\, \theta_1^2\, \beta_{1,1} + be\, w3^2\, \theta_1^2\, \beta_{1,1} - be\, w1\, w2\, \theta_1^2\, \beta_{1,2} - be\, w1\, w3\, \theta_1^2\, \beta_{1,3})) /$$
$$((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) +$$

$$(r + \psi + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + r\ w2\ \Theta_1 - w2\ (r + \psi)\ \Theta_1 + bL\ r\ w2\ l_2\ \Theta_1 -$$
$$bL\ w2\ (r + \psi)\ l_2\ \Theta_1 + w2\ \beta_0\ \Theta_1 + bL\ w2\ l_2\ \beta_0\ \Theta_1 + w2\ \beta_2\ \Theta_1 + bL\ w2\ l_2\ \beta_2\ \Theta_1 +$$
$$be\ w1^2\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w1\ w2\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w2\ w3\ (r + \psi)\ l_2\ \Theta_1^2 +$$
$$be\ w3^2\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w1^2\ l_2\ \beta_2\ \Theta_1^2 + be\ w1\ w2\ l_2\ \beta_2\ \Theta_1^2 +$$
$$be\ w2\ w3\ l_2\ \beta_2\ \Theta_1^2 - be\ w3^2\ l_2\ \beta_2\ \Theta_1^2 + l_0\ (bL + be\ w2\ (w1 + w2 + w3)\ \Theta_1^2 + bL$$
$$(l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)) + be\ w1\ w2\ l_2\ \Theta_1^2\ (r + \sigma_1) +$$
$$bL\ l_2\ (r + \sigma_2) + bL^2\ l_2^2\ (r + \sigma_2) + be\ w2^2\ l_2\ \Theta_1^2\ (r + \sigma_2) + be\ bL$$
$$w1^2\ l_2^2\ \Theta_1^2\ (r + \sigma_2) + be\ bL\ w2^2\ l_2^2\ \Theta_1^2\ (r + \sigma_2) + be\ bL\ w3^2\ l_2^2\ \Theta_1^2\ (r + \sigma_2) +$$
$$be\ w2\ w3\ l_2\ \Theta_1^2\ (r + \sigma_3) - be\ w1\ w2\ l_2\ \Theta_1^2\ \beta_{1,1} + \beta_{1,2} + bL\ l_2\ \beta_{1,2} +$$
$$be\ w1^2\ l_2\ \Theta_1^2\ \beta_{1,2} + be\ w3^2\ l_2\ \Theta_1^2\ \beta_{1,2} - be\ w2\ w3\ l_2\ \Theta_1^2\ \beta_{1,3} + l_1$$
$$(bL\ (r + \psi) + bL\ r\ w2\ \Theta_1 - bL\ w2\ (r + \psi)\ \Theta_1 + bL\ w2\ \beta_0\ \Theta_1 + be\ w1^2\ (r + \psi)$$
$$\Theta_1^2 - be\ w1\ w2\ (r + \psi)\ \Theta_1^2 - be\ w2\ w3\ (r + \psi)\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ \Theta_1^2 +$$
$$\beta_2\ (-bL + bL\ w2\ \Theta_1 - be\ (w1^2 - w1\ w2 + w3\ (-w2 + w3))\ \Theta_1^2) +$$
$$bL\ l_2\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)\ (r + \sigma_2) - be\ w1\ w2\ \Theta_1^2\ \beta_{1,1} +$$
$$bL\ \beta_{1,2} + be\ w1^2\ \Theta_1^2\ \beta_{1,2} + be\ w3^2\ \Theta_1^2\ \beta_{1,2} - be\ w2\ w3\ \Theta_1^2\ \beta_{1,3}))\ /$$
$$((1 + bL\ (l_1 + l_2))\ (1 + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2))))) +$$
$$w3\ (w1\ ((r + \psi + bL\ (r + \psi)\ l_1 + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + r\ w3\ \Theta_1 -$$
$$w3\ (r + \psi)\ \Theta_1 + bL\ r\ w3\ l_2\ \Theta_1 - bL\ w3\ (r + \psi)\ l_2\ \Theta_1 + w3\ \beta_0\ \Theta_1 + bL\ w3$$
$$l_2\ \beta_0\ \Theta_1 + w3\ \beta_2\ \Theta_1 + bL\ w3\ l_2\ \beta_2\ \Theta_1 + be\ w1^2\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w2^2$$
$$(r + \psi)\ l_2\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w2\ w3\ (r + \psi)\ l_2\ \Theta_1^2 - be$$
$$w1^2\ l_2\ \beta_2\ \Theta_1^2 - be\ w2^2\ l_2\ \beta_2\ \Theta_1^2 + be\ w1\ w3\ l_2\ \beta_2\ \Theta_1^2 + be\ w2\ w3\ l_2\ \beta_2\ \Theta_1^2 +$$
$$l_1\ \beta_2\ (-bL + bL\ w3\ \Theta_1 - be\ (w1^2 + w2^2 - (w1 + w2)\ w3)\ \Theta_1^2) + l_0\ (bL +$$
$$be\ w3\ (w1 + w2 + w3)\ \Theta_1^2 + bL\ (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)$$
$$\Theta_1^2)) + be\ w1\ w3\ l_2\ \Theta_1^2\ (r + \sigma_1) + be\ w2\ w3\ l_2\ \Theta_1^2\ (r + \sigma_2) +$$
$$bL\ l_2\ (r + \sigma_3) + bL^2\ l_2^2\ (r + \sigma_3) + be\ w3^2\ l_2\ \Theta_1^2\ (r + \sigma_3) +$$
$$be\ bL\ w1^2\ l_2^2\ \Theta_1^2\ (r + \sigma_3) + be\ bL\ w2^2\ l_2^2\ \Theta_1^2\ (r + \sigma_3) + be\ bL\ w3^2\ l_2^2$$
$$\Theta_1^2\ (r + \sigma_3) + bL\ l_1\ l_2\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)\ (r + \sigma_3) -$$
$$be\ w1\ w3\ l_2\ \Theta_1^2\ \beta_{1,1} - be\ w2\ w3\ l_2\ \Theta_1^2\ \beta_{1,2} - l_1\ \Theta_1\ (bL\ w3\ \psi - bL\ w3\ \beta_0 +$$
$$be\ \Theta_1\ (-(w1^2 + w2^2 - (w1 + w2)\ w3)\ (r + \psi) + w1\ w3\ \beta_{1,1} +$$
$$w2\ w3\ \beta_{1,2})) + \beta_{1,3} + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2)\ \Theta_1^2)\ \beta_{1,3})\ /$$
$$((1 + bL\ (l_1 + l_2))\ (1 + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2))) +$$
$$(r + \psi + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + r\ w1\ \Theta_1 - w1\ (r + \psi)\ \Theta_1 +$$
$$bL\ r\ w1\ l_2\ \Theta_1 - bL\ w1\ (r + \psi)\ l_2\ \Theta_1 + w1\ \beta_0\ \Theta_1 + bL\ w1\ l_2\ \beta_0\ \Theta_1 +$$
$$w1\ \beta_2\ \Theta_1 + bL\ w1\ l_2\ \beta_2\ \Theta_1 - be\ w1\ w2\ (r + \psi)\ l_2\ \Theta_1^2 +$$
$$be\ w2^2\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ l_2\ \Theta_1^2 +$$
$$be\ w1\ w2\ l_2\ \beta_2\ \Theta_1^2 - be\ w2^2\ l_2\ \beta_2\ \Theta_1^2 + be\ w1\ w3\ l_2\ \beta_2\ \Theta_1^2 -$$
$$be\ w3^2\ l_2\ \beta_2\ \Theta_1^2 + l_0\ (bL + be\ w1\ (w1 + w2 + w3)\ \Theta_1^2 + bL\ (l_1 + l_2)\ (bL +$$
$$be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)) + bL\ l_2\ (r + \sigma_1) + bL^2\ l_2^2\ (r + \sigma_1) +$$
$$be\ w1^2\ l_2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w1^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ bL\ w2^2\ l_2^2$$
$$\Theta_1^2\ (r + \sigma_1) + be\ bL\ w3^2\ l_2^2\ \Theta_1^2\ (r + \sigma_1) + be\ w1\ w2\ l_2\ \Theta_1^2\ (r + \sigma_2) + be$$
$$w1\ w3\ l_2\ \Theta_1^2\ (r + \sigma_3) + \beta_{1,1} + bL\ l_2\ \beta_{1,1} + be\ w2^2\ l_2\ \Theta_1^2\ \beta_{1,1} + be\ w3^2\ l_2$$
$$\Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ l_2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ l_2\ \Theta_1^2\ \beta_{1,3} + l_1\ (bL\ (r + \psi) +$$
$$bL\ r\ w1\ \Theta_1 - bL\ w1\ (r + \psi)\ \Theta_1 + bL\ w1\ \beta_0\ \Theta_1 - be\ w1\ w2\ (r + \psi)\ \Theta_1^2 +$$
$$be\ w2^2\ (r + \psi)\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ \Theta_1^2 + be\ w3^2\ (r + \psi)\ \Theta_1^2 +$$
$$\beta_2\ (-bL + bL\ w1\ \Theta_1 + be\ (-w2^2 - w3^2 + w1\ (w2 + w3))\ \Theta_1^2) +$$
$$bL\ l_2\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)\ (r + \sigma_1) + bL\ \beta_{1,1} + be\ w2^2$$
$$\Theta_1^2\ \beta_{1,1} + be\ w3^2\ \Theta_1^2\ \beta_{1,1} - be\ w1\ w2\ \Theta_1^2\ \beta_{1,2} - be\ w1\ w3\ \Theta_1^2\ \beta_{1,3}))\ /$$
$$((1 + bL\ (l_1 + l_2))\ (1 + (l_1 + l_2)\ (bL + be\ (w1^2 + w2^2 + w3^2)\ \Theta_1^2)))) +$$
$$w2\ ((r + \psi + bL\ (r + \psi)\ l_1 + bL\ (r + \psi)\ l_2 - \beta_2 - bL\ l_2\ \beta_2 + r\ w3\ \Theta_1 -$$
$$w3\ (r + \psi)\ \Theta_1 + bL\ r\ w3\ l_2\ \Theta_1 - bL\ w3\ (r + \psi)\ l_2\ \Theta_1 + w3\ \beta_0\ \Theta_1 + bL\ w3$$
$$l_2\ \beta_0\ \Theta_1 + w3\ \beta_2\ \Theta_1 + bL\ w3\ l_2\ \beta_2\ \Theta_1 + be\ w1^2\ (r + \psi)\ l_2\ \Theta_1^2 + be\ w2^2$$
$$(r + \psi)\ l_2\ \Theta_1^2 - be\ w1\ w3\ (r + \psi)\ l_2\ \Theta_1^2 - be\ w2\ w3\ (r + \psi)\ l_2\ \Theta_1^2 - be$$

$$
\begin{aligned}
&w1^2\, l_2\, \beta_2\, \theta_1^2 - be\, w2^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w3\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 + \\
&l_1\, \beta_2\, (-bL + bL\, w3\, \theta_1 - be\, (w1^2 + w2^2 - (w1 + w2)\, w3)\, \theta_1^2) + l_0\, (bL + \\
&\quad be\, w3\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2) \\
&\qquad \theta_1^2)) + be\, w1\, w3\, l_2\, \theta_1^2\, (r + \sigma_1) + be\, w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_2) + \\
&bL\, l_2\, (r + \sigma_3) + bL^2\, l_2^2\, (r + \sigma_3) + be\, w3^2\, l_2\, \theta_1^2\, (r + \sigma_3) + \\
&be\, bL\, w1^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_3) + be\, bL\, w3^2\, l_2^2 \\
&\theta_1^2\, (r + \sigma_3) + bL\, l_1\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_3) - \\
&be\, w1\, w3\, l_2\, \theta_1^2\, \beta_{1,1} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,2} - l_1\, \theta_1\, (bL\, w3\, \psi - bL\, w3\, \beta_0 + \\
&\quad be\, \theta_1\, (-(w1^2 + w2^2 - (w1 + w2)\, w3)\, (r + \psi) + w1\, w3\, \beta_{1,1} + \\
&\qquad w2\, w3\, \beta_{1,2})) + \beta_{1,3} + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2)\, \theta_1^2)\, \beta_{1,3}) / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))) + \\
&(r + \psi + bL\, (r + \psi)\, l_2 - \beta_2 - bL\, l_2\, \beta_2 + r\, w2\, \theta_1 - w2\, (r + \psi)\, \theta_1 + \\
&\quad bL\, r\, w2\, l_2\, \theta_1 - bL\, w2\, (r + \psi)\, l_2\, \theta_1 + w2\, \beta_0\, \theta_1 + bL\, w2\, l_2\, \beta_0\, \theta_1 + \\
&\quad w2\, \beta_2\, \theta_1 + bL\, w2\, l_2\, \beta_2\, \theta_1 + be\, w1^2\, (r + \psi)\, l_2\, \theta_1^2 - \\
&\quad be\, w1\, w2\, (r + \psi)\, l_2\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, l_2\, \theta_1^2 + be\, w3^2\, (r + \psi)\, l_2\, \theta_1^2 - \\
&\quad be\, w1^2\, l_2\, \beta_2\, \theta_1^2 + be\, w1\, w2\, l_2\, \beta_2\, \theta_1^2 + be\, w2\, w3\, l_2\, \beta_2\, \theta_1^2 - \\
&\quad be\, w3^2\, l_2\, \beta_2\, \theta_1^2 + l_0\, (bL + be\, w2\, (w1 + w2 + w3)\, \theta_1^2 + bL\, (l_1 + l_2) \\
&\qquad (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)) + be\, w1\, w2\, l_2\, \theta_1^2\, (r + \sigma_1) + bL\, l_2 \\
&\quad (r + \sigma_2) + bL^2\, l_2^2\, (r + \sigma_2) + be\, w2^2\, l_2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w1^2\, l_2^2 \\
&\quad \theta_1^2\, (r + \sigma_2) + be\, bL\, w2^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be\, bL\, w3^2\, l_2^2\, \theta_1^2\, (r + \sigma_2) + be \\
&\quad w2\, w3\, l_2\, \theta_1^2\, (r + \sigma_3) - be\, w1\, w2\, l_2\, \theta_1^2\, \beta_{1,1} + \beta_{1,2} + bL\, l_2\, \beta_{1,2} + be\, w1^2 \\
&\quad l_2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, l_2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, l_2\, \theta_1^2\, \beta_{1,3} + l_1\, (bL\, (r + \psi) + \\
&\qquad bL\, r\, w2\, \theta_1 - bL\, w2\, (r + \psi)\, \theta_1 + bL\, w2\, \beta_0\, \theta_1 + be\, w1^2\, (r + \psi)\, \theta_1^2 - \\
&\qquad be\, w1\, w2\, (r + \psi)\, \theta_1^2 - be\, w2\, w3\, (r + \psi)\, \theta_1^2 + be\, w3^2\, (r + \psi)\, \theta_1^2 + \\
&\qquad \beta_2\, (-bL + bL\, w2\, \theta_1 - be\, (w1^2 - w1\, w2 + w3\, (-w2 + w3))\, \theta_1^2) + \\
&\qquad bL\, l_2\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)\, (r + \sigma_2) - be\, w1\, w2\, \theta_1^2\, \beta_{1,1} + \\
&\qquad bL\, \beta_{1,2} + be\, w1^2\, \theta_1^2\, \beta_{1,2} + be\, w3^2\, \theta_1^2\, \beta_{1,2} - be\, w2\, w3\, \theta_1^2\, \beta_{1,3})) / \\
&((1 + bL\, (l_1 + l_2))\, (1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2) \\
&\qquad \theta_1^2))))))) \Big) \Big/ (bd\, bL\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2))
\end{aligned}
$$

In[63]:= **D[Assets, r] // FullSimplify**

Out[63]=
$$\frac{(-1 + \rho)\, ((-1 + \rho)\, d_1 + \rho\, d_2)}{1 + bd\, (d_1 + d_2)} - \frac{(w1 + w2 + w3)\, l_1\, \theta_1}{1 + (l_1 + l_2)\, (bL + be\, (w1^2 + w2^2 + w3^2)\, \theta_1^2)}$$

In[64]:= **Loans = LoanDemand1 + LoanDemand2 + LoanDemand3**

Out[64]=

$$
\begin{aligned}
&-(l_0\,(-1+(l_1+l_2)\,(-bL-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2))+\\
&\quad l_1^2\,(bL\,w3\,(-\psi+\beta_0)\,\Theta_1+\beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+\\
&\qquad bL\,(r+\psi+\beta_{1,3})+be\,\Theta_1^2\,((w1^2+w2^2-(w1+w2)\,w3)\,(r+\psi)-w3\,(w1\,\beta_{1,1}+w2\,\beta_{1,2})+\\
&\qquad\quad (w1^2+w2^2)\,\beta_{1,3}))+l_2\,(\psi-\beta_2+w3\,(-\psi+\beta_0+\beta_2)\,\Theta_1-\sigma_3+\beta_{1,3}+\\
&\qquad l_2\,(-bL\,w3\,(\psi-\beta_0)\,\Theta_1+\beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+\\
&\qquad\quad bL\,(\psi-\sigma_3+\beta_{1,3})+be\,\Theta_1^2\,((w1^2+w2^2-(w1+w2)\,w3)\,\psi+w1\,w3\,\sigma_1+\\
&\qquad\qquad w2\,w3\,\sigma_2-w1^2\,\sigma_3-w2^2\,\sigma_3-w1\,w3\,\beta_{1,1}-w2\,w3\,\beta_{1,2}+(w1^2+w2^2)\,\beta_{1,3})))+\\
&\quad l_1\,(r+\psi-\beta_2+w3\,(-\psi+\beta_0+\beta_2)\,\Theta_1+\beta_{1,3}+l_2\,(-2\,bL\,w3\,(\psi-\beta_0)\,\Theta_1+\\
&\qquad\quad 2\,\beta_2\,(-bL+bL\,w3\,\Theta_1-be\,(w1^2+w2^2-(w1+w2)\,w3)\,\Theta_1^2)+\\
&\qquad\quad bL\,(r+2\,\psi-\sigma_3+2\,\beta_{1,3})+be\,\Theta_1^2\,((w1^2+w2^2-(w1+w2)\,w3)\,(r+2\,\psi)+w1\,w3\,\sigma_1+\\
&\qquad\qquad w2\,w3\,\sigma_2-w1^2\,\sigma_3-w2^2\,\sigma_3-2\,w1\,w3\,\beta_{1,1}-2\,w2\,w3\,\beta_{1,2}+2\,(w1^2+w2^2)\,\beta_{1,3}))))\,/\\
&\quad ((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))-\\
&(l_0\,(-1+(l_1+l_2)\,(-bL+be\,(-w2^2-w3^2+w1\,(w2+w3))\,\Theta_1^2))+\\
&\quad l_1^2\,(bL\,w1\,(-\psi+\beta_0)\,\Theta_1+\\
&\qquad \beta_2\,(-bL+bL\,w1\,\Theta_1+be\,(-w2^2-w3^2+w1\,(w2+w3))\,\Theta_1^2)+bL\,(r+\psi+\beta_{1,1})+\\
&\qquad be\,\Theta_1^2\,((w2^2+w3^2-w1\,(w2+w3))\,(r+\psi)+(w2^2+w3^2)\,\beta_{1,1}-w1\,(w2\,\beta_{1,2}+w3\,\beta_{1,3})))+\\
&\quad l_1\,(r+\psi-\beta_2+w1\,(-\psi+\beta_0+\beta_2)\,\Theta_1+\beta_{1,1}+l_2\,(-2\,bL\,w1\,(\psi-\beta_0)\,\Theta_1+2\,\beta_2\\
&\qquad\quad (-bL+bL\,w1\,\Theta_1+be\,((w1-w2)\,w2+w1\,w3-w3^2)\,\Theta_1^2)+bL\,(r+2\,\psi-\sigma_1+2\,\beta_{1,1})+\\
&\qquad\quad be\,\Theta_1^2\,((w2^2+w3^2-w1\,(w2+w3))\,(r+2\,\psi)-(w2^2+w3^2)\,\sigma_1+w1\,w2\,\sigma_2+\\
&\qquad\qquad w1\,w3\,\sigma_3+2\,w2^2\,\beta_{1,1}+2\,w3^2\,\beta_{1,1}-2\,w1\,w2\,\beta_{1,2}-2\,w1\,w3\,\beta_{1,3})))+\\
&\quad l_2\,(\psi-\beta_2+w1\,(-\psi+\beta_0+\beta_2)\,\Theta_1-\sigma_1+\beta_{1,1}+l_2\,(-bL\,w1\,(\psi-\beta_0)\,\Theta_1+\\
&\qquad\quad \beta_2\,(-bL+bL\,w1\,\Theta_1+be\,((w1-w2)\,w2+w1\,w3-w3^2)\,\Theta_1^2)+bL\,(\psi-\sigma_1+\beta_{1,1})+\\
&\qquad\quad be\,\Theta_1^2\,((w2^2+w3^2-w1\,(w2+w3))\,\psi-(w2^2+w3^2)\,\sigma_1+w1\,w2\,\sigma_2+\\
&\qquad\qquad w1\,w3\,\sigma_3+w2^2\,\beta_{1,1}+w3^2\,\beta_{1,1}-w1\,w2\,\beta_{1,2}-w1\,w3\,\beta_{1,3}))))\,/\\
&\quad ((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))-\\
&(l_0\,(-1+(l_1+l_2)\,(-bL-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2))+\\
&\quad l_1^2\,(bL\,w2\,(-\psi+\beta_0)\,\Theta_1+\\
&\qquad \beta_2\,(-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2)+bL\,(r+\psi+\beta_{1,2})+be\,\Theta_1^2\\
&\qquad\quad ((w1^2-w1\,w2-w2\,w3+w3^2)\,(r+\psi)-w1\,w2\,\beta_{1,1}+(w1^2+w3^2)\,\beta_{1,2}-w2\,w3\,\beta_{1,3}))+\\
&\quad l_1\,(r+\psi-\beta_2+w2\,(-\psi+\beta_0+\beta_2)\,\Theta_1+\beta_{1,2}+l_2\,(-2\,bL\,w2\,(\psi-\beta_0)\,\Theta_1+2\,\beta_2\\
&\qquad\quad (-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2)+bL\,(r+2\,\psi-\sigma_2+2\,\beta_{1,2})+\\
&\qquad\quad be\,\Theta_1^2\,((w1^2-w1\,w2-w2\,w3+w3^2)\,(r+2\,\psi)+w1\,w2\,\sigma_1-(w1^2+w3^2)\,\sigma_2+\\
&\qquad\qquad w2\,w3\,\sigma_3-2\,w1\,w2\,\beta_{1,1}+2\,w1^2\,\beta_{1,2}+2\,w3^2\,\beta_{1,2}-2\,w2\,w3\,\beta_{1,3})))+\\
&\quad l_2\,(\psi-\beta_2+w2\,(-\psi+\beta_0+\beta_2)\,\Theta_1-\sigma_2+\beta_{1,2}+l_2\,(-bL\,w2\,(\psi-\beta_0)\,\Theta_1+\\
&\qquad\quad \beta_2\,(-bL+bL\,w2\,\Theta_1-be\,(w1^2-w1\,w2+w3\,(-w2+w3))\,\Theta_1^2)+bL\,(\psi-\sigma_2+\beta_{1,2})+\\
&\qquad\quad be\,\Theta_1^2\,((w1^2-w1\,w2-w2\,w3+w3^2)\,\psi+w1\,w2\,\sigma_1-(w1^2+w3^2)\,\sigma_2+\\
&\qquad\qquad w2\,w3\,\sigma_3-w1\,w2\,\beta_{1,1}+w1^2\,\beta_{1,2}+w3^2\,\beta_{1,2}-w2\,w3\,\beta_{1,3}))))\,/\\
&\quad ((1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2)))
\end{aligned}
$$

In[65]:= **D[Loans, r] // FullSimplify**

Out[65]=

$$
\frac{l_1\,(-3+(l_1+l_2)\,(-3\,bL-2\,be\,(w1^2+w2^2-w2\,w3+w3^2-w1\,(w2+w3))\,\Theta_1^2))}{(1+bL\,(l_1+l_2))\,(1+(l_1+l_2)\,(bL+be\,(w1^2+w2^2+w3^2)\,\Theta_1^2))}
$$

Appendix 3

Proof that expression (17) has a negative sign under the assumption about risk-weights of loan classes ($w_1 < w_2 < w_3$)

In the expression (17)

$$\frac{w_1^2 + w_2^2 - w_3(w_1 + w_2)}{w_1^2 + w_2^2 + w_3^2}$$

denominator has no influence on the sign of this expression because $w_1^2 + w_2^2 + w_3^2$ is always positive.

So, the sign of expression (17) depends on the sign of the numerator:

$w_1^2 + w_2^2 - w_3(w_1 + w_2)$.

As soon as $w_1 < w_2 < w_3$:

$$w_1^2 < w_1 w_3$$
$$w_2^2 < w_2 w_3$$
$$w_1^2 + w_2^2 < w_3(w_1 + w_2)$$
$$w_1^2 + w_2^2 - w_3(w_1 + w_2) < 0$$

Hence, under the initial assumption about risk-weights of loan class ($w_1 < w_2 < w_3$), expression (17) is negative.