

# **Optimum Decision Policy for Gradual Replacement of Conventional Power Sources by Clean Power Sources**

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# Abstract

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With the increase of world population and industrial growth of developing countries, demand for energy, in particular electric power, has gone up at an unprecedented rate over the last decades. To meet the demand, electric power generation by use of fossil fuel has increased enormously thereby producing increased quantity of greenhouse gases. This contributes more and more to atmospheric pollution, which climate scientists believe can adversely affect the global climate, as well as health and the welfare of the world population. In view of these issues, there is global awareness to look for alternate sources of energy such as natural gas, hydropower, wind, solar, geothermal and biomass. It is recognized that this requires replacement of existing infrastructure with new systems, which cannot be achieved overnight.

Optimal control theory has been widely used in diverse areas of physical sciences, medicine, engineering and economics. The main motivation of this thesis is to use this theory to find the optimum strategy for integration of all currently available renewable energy sources with the existing electric power generating systems. The ultimate goal is to eliminate fossil fuels. Eight main energy sources namely, Coal, Petroleum, Natural Gas, Conventional Hydro, Wind, Solar, Geothermal and Biomass are considered in a dynamic model. The state of the dynamic model represents the level of energy generation from each of the sources.

Different objective functions are proposed in this thesis. These range from meeting the desired target level of power generation from each of the available sources at the end of a given plan period, to reducing the implementation and investment costs; from minimizing the production from polluted energy sources to meeting the electricity demand during a whole plan period. Official released data from the U.S. Energy Information Administration have been used as a case study. Based on real life data and the mathematics of optimal control theory, we present an optimal policy for integration of renewable energy sources to the national power grid.

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## List of Acronyms

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EIA	Energy Information Administration
GHG	Green House Gas
kWh	Kilo Watt Hours
MOP	Multiple Objective Programming
REM	Renewable Energy Management

## List of Variables

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$\beta_{ij}$	Impact of the electricity generation level from energy source $j$ on the level of electricity generation from energy source $i$
$\delta$	Acceptable Tolerance
$\delta_i$	Weighting value in terminal cost problem for energy source $i$
$\varepsilon$	Step-size
$\psi(t)$	Lagrange Multiplier, Costate variable
$\psi_n$	Costate variable in $n^{\text{th}}$ iteration
$\Gamma$	Weighting value in demand problem
$q_i$	Weighting value in implementation cost problem for energy source $i$
$t_f$	Final time (end of the plan period)
$u(t)$	Control policy
$u^*$	Optimal control policy
$u_n$	Control policy in $n^{\text{th}}$ iteration
$u_i(t)$	Percentage growth rate of the level of electricity generation for energy source $i$
$v_i$	Weighting value in environmental cost problem for energy source $i$
$x(t)$	State variable
$x^*$	Optimal trajectory for state variables
$x_n$	State variable in $n^{\text{th}}$ iteration
$x_i(t)$	Level of electricity generation from energy source $i$
$x_i^d$	Desired level of electricity generation from energy source $i$ according to U.S. projection

$\mathbf{X}^d$	Total electricity demand at the end of the plan period
$\mathbf{y}_i(\mathbf{t})$	Historical data for energy source $i$
$\mathbf{D}(\mathbf{t})$	Electricity demand during the plan period

# Chapter 1. Introduction

---

The substantial increase in fuel prices, the rapid growth of the level of Green House Gas (GHG) emissions, the environmental impacts of the conventional methods of power generation, and the considerable augment in the energy demand due to the populated industrial cities, produce predilection among every society and government to improve the existing methods of energy generation.

Improving the energy system in a given place is possible through various ways. These range from integrating new energy sources, such as biomass, solar, wind and geothermal into the electricity generation system, to upgrading the existing sources to cost efficient and environment friendly ones. Finding the optimum decision policies to integrate the clean power sources into the electricity generation is a prominent field of research and the main concern of this thesis.

## 1.1. Motivation

With the increase of world population and unprecedented industrial growth in many populous countries, demand for energy, in particular electricity, has been steadily increasing over last 30-40 years.

To meet this ever-increasing demand, the use of fossil fuel for generation of electricity has been rising and along with that, the production of toxic gasses potentially threatening the climate and public health.

On the other hand, conventional energy sources such as Coal and Petroleum are becoming more and more scarce. Although there are scientists who believe that there are still ample supplies of these sources [1], most of the researchers believe that these supplies are transient and will deplete within the next sixty years [2].

In any case, considering just the adverse effects of pollution on health and global climate, it has become mandatory for industrial countries to seek for alternate sources and

follow a policy of gradual replacement of polluting energy sources by clean and renewable ones, without adversely affecting the economy [3].

In this thesis we consider all eight main energy sources used for generation of electricity. These are Coal, Petroleum, Natural Gas, Hydro, Wind, Solar, Geothermal and Biomass. Our choice of these sources is based on the availability of comprehensive official released data from the U.S. Energy Information Administration [4]. These data are used along with optimal control decision theory [5] to develop a methodology for optimal reduction or replacement of fossil fuel by renewable energy sources.

In a recent paper by Miah, Ahmed and Chowdhury [6], the Lotka-Volterra model was used as a dynamic model of power generation involving only two sources of renewable and conventional. We first extended this concept by considering eight different sources using available official data [4] to validate our proposed model and then, developed an optimal generation policy based on the Pontryagin minimum principle [5].

## **1.2. Thesis Objective**

The main objective of this thesis was to define the optimum decision policy for the gradual replacement of conventional energy sources by renewable and clean ones in various energy problems using optimal control theory. This control policy was determined through the Pontryagin minimum principle approach of the optimal control theory. The non-linear dynamic Lotka-Volterra model was utilized for modeling the generation level for each of the energy sources of our problem.

Eight main energy sources namely, Coal, Petroleum, Natural Gas, Hydro, Wind, Solar, Geothermal and Biomass have been considered. In addition to finding the optimum decision policy for each of the selected energy sources, system identification was also presented through minimizing the identification error using the historical data of the selected energy sources.

In this work, different energy problems were introduced and utilized to define the optimum decision policies in various problems. These problems vary from considering the implementation and environmental costs to meeting the electricity demand and the desired levels of electricity generation from each of the eight energy sources.

### 1.3. Thesis Contributions

Although Miah M.S., Ahmed N.U., Chowdhury M. [6] was the first publication that utilized the Lotka-Volterra model for defining the energy problems, in this thesis, and *for the first time* to the best of our knowledge, all main eight energy sources are considered, simultaneously. Also, system identification, as well as finding the optimum decision policies in various combinations of energy problems have been investigated *for the first time*.

The official released data of U.S. Energy Information Administration [4] has been used as a case study for validating our results through the real life data. The proposed mathematical method can easily be used in different energy problems with slight modifications according to the objectives of planner.

Results of this thesis are summarized in three different publications:

1. Submitted to European Control Conf. (ECC 13) [7].
2. Submitted to Int. J. on Energy Science (IJES) [8].
3. In progress to be submitted to Energy Strategy J. of Elsevier [9].

### 1.4. Thesis Outline

This thesis is organized as follows: the literature review is presented in Chapter two and followed in Chapter three by the necessary mathematical background for system identification and optimal control theory.

The mathematical model and problem formulation are presented in Chapter four. Chapter five includes the results of system identification for different systems based on the optimal control theory. Optimum decision policies for different energy problems are presented in Chapter six followed by conclusion Chapter. In Appendix A, the official released data from U.S. Energy Information Administration website [4] are presented.

## Chapter 2. Literature Review

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Transition from existing conventional methods of electricity generation to renewable ones would be costly; besides most of renewable energy sources are intermittent and need huge financial supports for their construction as well as their Operation and Maintenance. Choosing among existing renewable energy sources and deciding whether to construct a renewable power plant in a specific given place or not, are the foremost issues in the case of Renewable Energy Management (REM) [10].

Several authors developed different strategies in the field of REM and presented new methods for integration of renewable energy sources to the power grid. Evans A. et al. [11] compared the wind, hydropower, photovoltaic and geothermal energy sources based on price, availability, amount of GHG emissions, efficiency and social impacts. Wind power source was the prominent and the most suitable among all other energy sources from their point of view. Similarly, wind and small hydropower were the selected sustainable energy sources by Varun et al. [12], who graded different sustainable and renewable energy sources based on the amount of GHG emissions, energy payback time and the cost of the electricity generation.

Any decision policy in the gradual replacement of energy sources would be fruitless without accurate and complete energy modeling for each of the energy sources. Therefore, in a following section, different selected energy models are referred based on the year they have released. Then, we propose an energy model based on the Lotka-Volterra model. In the last section of this chapter, various selected optimization algorithms are also listed, and among all of them, optimal control theory is chosen according to the objective of this thesis.

### 2.1. Energy Models

Industrial countries are in demand of more and more energy. To supply energy, there should be precise and reliable energy models describing different types of energy

sources. A synthetic model based on the mineral supplies and the social efficiency was proposed in 1977 by Marchetti C. [13], as the starting point of energy modeling. In this model, he tried to solve the primary energy substitution problem. His dynamic model was based on historical data. The main problem was the necessity of providing his model with several economic and market parameters and factors.

Collier P.I. and Ornek A. [14] (1983) proposed a forecasting model for energy to determine the minimum fuel requirement while minimizing the operating costs. Several constraints and factors were considered in the formulation of their model, e.g., the level of customer service that system should provide and so on. In 1984, Musgrove A.R. [15] introduced a linear programming model named MARKAL and tried to minimize the discounted cost of energy system in Australia during the plan period of 1980-2020. MARKAL model was widely used during decades for energy systems modeling. The main drawback of this model is that it suffers from linearization. Later in 1987, Hsu G. et al [16] presented the integrated, energy planning, Leontief input-output model with the aim of multi-objective programming technique. Their model was based on the availability of each of the resources, the inter-industry interactions, the capacity limits for each of the power source, the labour costs and the Gross Domestic Product (GDP).

In 1989, Kaboudan M. [17] developed a non-linear model using the historical data of 20 years (1965-1984) to predict the electricity consumption for year 2010 in Zimbabwe. Their model was based on macroeconomic and demographic variables and tried to forecast the electricity demand of a country. A year after that, Weyant J.P. [18] reviewed various policy modeling and demonstrated the efficiency of data analysis in the environmental energy planning. Developer of the model PURHAPS for the Energy Information Administration (EIA) [4], Werbos P.J. [19] also investigated differences between econometric modeling and other types of energy modeling. Econometric models, similar to forecasting models, are made up of any set of equations to predict the future.

In 1990, Poch L.A. and Jenkins R.T. [20] reviewed different dynamic programming models and demonstrated the results derived from their model for electric system expansion planning. Dynamic programming technique was utilized to solve optimization problems based on optimal control theory. The method we used in this thesis can be categorized in this type of models.

In 1992, Tzeng G.H. et al. [21] proposed a multi-criteria evaluation method considering both conventional and renewable energy sources. The 1970s oil crisis and the increase in the environmental protections were the two main inspiring factors for them to introduce new energy systems and techniques to supply the energy demand in Taiwan. Environmental issues, economic factors and technology developments were evaluated in their plan. They considered several economic and environment factors in their model, such as supply stability, operational efficiency of system, popularity of use and so on. These huge weighting values made their approach difficult to use.

Simultaneously, Joshi A. et al. [22] developed a linear model for a sample village in India and minimized the cost function for an energy supply system. They considered some of the energy sources and conversion devices in their objective function. Biomass was the main source of energy in subsistence economy from their point of view. Furthermore, they concluded that electricity from the grid is the best choice and the most cost effective option for illumination of a typical village. Using a linear model is the main shortcoming of their method.

In 1993, Hammond G.P. and Mackay R.M. [23] proposed a forecasting model to project both supply and demand of oil and gas for the period of 1993 to 2010 within the UK. The main drawback of their model was that all their results were completely based on the estimation of the remaining oil and gas and were valid only for specific time duration.

Concurrently, Luhanga M.L. et al. [24] developed a LEAP model based on optimization models, as well as forecasting models. They demonstrated their approach based on two objective functions: Minimizing the total cost while finding the optimum combination of energy sources and minimizing the wood fuel deficit while defining the optimum numbers of end-use biomass devices. The energy demand constraints and the capacity constraints for the first approach, the constraints in wood fuel savings, the demand constraints, the afforestation constraints, and the transformation constraints for the second one, were their model constraints.

In 1995, Ramanathan R. and Ganesh L.S. [25] used the Linear Programming and the Analytic Hierarchy Process (AHP) to light urban households through seven selected energy sources namely, kerosene, biogas, solar photovoltaic, wood fuel-electricity, bio-

gas-electricity, diesel-electricity, and grid-electricity. They developed a structured technique including different quantitative and qualitative parameters. Existence of several constraints and use of linearized model were the two main drawbacks of their model.

In 1999, Chedid R. et al. [26] presented a multi-objective linear programming based on fuzzy logic. Their objective was energy management for nine selected energy sources. Rozakis S. et al. [27] presented in 2001 the integrated micro-economic multi-level mixed-integer linear programming for energy supply at a national level in France. Simultaneously, Persaud A.J. and Kumar U. [28] developed an Oil and Gas Supply Model (OGSM) for Canada to forecast the supply and the demand of oil and natural gas in the year 2020. The concerns regarding the GHG emissions and the Kyoto protocol were the main driving forces for this research. The production of oil and gas will exceed the domestic energy demand in Canada by year 2020 based on their model and Canada will be a net exporter by then. Their method was not general enough to be extended to other cases.

In 2001, Agrawal R.K. and Singh S.P. [29] proposed a fuzzy-based multi-objective model for energy allocation for cooking in households in India. Invariance of data, simplified fuzzification, single decision-making and symmetry of spaces were their assumptions. They considered nine different factors in their objective function: Minimizing the life cycle cost, the production of polluted power sources, the use of wood fuel products, and the GHG emissions while maximizing the usefully available energies, the use of locally available energy sources, the comfort of operations, the safety, and the predictability of performance. Besides the assumptions and the objectives, total cooking energy demand, limitations on solar thermal cooking, limitations on the total availability of biogas and limitations on the upper bounds for some of the variables, were the constraints in their model.

In 2006, Gupta A. et al. [30] presented a linear programming model for a hybrid energy system and minimized the cost function based on the capital cost and the cost for each type of power sources using the LINDO optimization software. Estimations and constraints of linearized models were the main drawback of their model.

Jebaraj S. et al. [31] introduced in 2008 an Optimal Electricity Allocation Model (OEAM) based on fuzzy linear programming. Their objectives were to find the optimum allocation of various energy sources while minimizing the cost and GHG emissions with

focus on biofuels. Concurrently, Gupta A. et al. [32] released their results on the reliable and effective modeling of hybrid energy systems. They proposed the mixed integer linear mathematical programming model with the objective of minimizing the total cost. Photovoltaic array, biomass, biogas, small hydro, battery bank, and fossil fuel energy sources were considered in their hybrid model.

In 2010, Mirzaesmaeeli H. et al. [33] developed a multi-period mixed-integer linear programming model for allocation of energy sources with the objective of minimum cost, minimum CO<sub>2</sub> emissions while reaching the desired demand level. The predicted energy demand and the change in the price of the fuel were some of their time-dependant parameters.

In 2011, Chu B. et al. [34] proposed a linearized state space model by taking advantage of the control theory. The optimum decision policy in their model was to reduce the emissions of GHG with the capability to anticipate the future emissions of GHG through various energy policies.

In this section, most existing reliable and effective energy models were presented, covering the period from 1977 to 2011. As reported in —Table 1 [35], these energy models can be summarized in four main categories: energy planning models, energy forecasting models, energy optimization models, and neural- or fuzzy logic-based energy models.

**Table 1** Energy Models Classification

	Energy Planning Models	Forecasting Models	Optimization Models	Neural Networks Models
Papers	[6], [13], [16], [18], [19], [20], [21], [22], [25], [27], [32], [33], [34]	[14], [17], [23], [28]	[15], [24], [30]	[26], [29], [31]

Note that, although these energy models have been widely used in different countries for different purposes, most of them have been established based on several assumptions and constraints, mainly due to the two following reasons:

- Involvement of so many factors in the energy management field such as estimations in the amount of available sources, in actual level of pollution produced by each energy source, in actual level of the energy demand due to the increasing population and industries, etc.
- Use of various linearized models that need mathematical assumptions and simplifications.

In this work, we selected the non-linear Lotka-Volterra model, categorized as a dynamic programming energy planning model. This model has been widely used for the biological systems [5], [36] for demonstrating the population dynamics.

Lotka-Volterra model is a mathematical model, which contains a pair of non-linear differential equations to describe and predict the variation of two interacting populations, especially predator and prey species. In fact, this model can be used for modeling cooperative and competing agents in an ecological environment as well as in economy [5]. For the purpose of this thesis, we developed a Lotka-Volterra model for the level of electricity generation from different competing energy sources.

In 2012, Wu L. et al. [37] proposed a Grey Lotka-Volterra model (combination of Grey Model and Lotka-Volterra Model) and proved that this model is quite effective to analyze long-term relationship of any two competitive variables in an economic system. This model was able to predict the values of the competitive economic variables effectively and can be categorized in forecasting models.

Simultaneously, Miah M.S., Ahmed N.U., Chowdhury M. [6] utilized the Lotka-Volterra model for the purpose of integrating the renewable energy sources to the power grid. The levels of electricity generation from two competitive energy sources of conventional and renewable were the variables they considered in their Lotka-Volterra predator-prey equations.

To the best of our knowledge, only Wu L. et al. [37] and Miah M.S., Ahmed N.U., Chowdhury M. [6] developed their energy models based on the Lotka-Volterra equations. However, both of these recent publications discussed only two-variable non-linear differential equations in their modelling. For the first time, we solved a set of eight non-linear Lotka-Volterra differential equations including all main energy sources, i.e. coal, petroleum, natural gas, hydro, wind, solar, geothermal, and biomass.

After presenting different energy models from 1977 till present time, and proposing our selected energy model (Lotka-Volterra), we will investigate the different available optimization methods that can be used to optimize our objectives.

## **2.2. Optimization Methods**

After defining the model for the energy systems, the next step would be defining the optimization method applied to this field. The objective of these methods varies from integration of renewable energy sources into electricity generation to the different areas of finance, economics and environments.

Optimization is minimizing or maximizing an objective function that may include some constraints. Different optimization methods are widely used for decades, in different fields of engineering, science, economic, environment and finance. In addition, specific characteristics of a problem may produce unfeasible solutions that inspired scientists to develop approximate optimization methods.

While Linear Programming (LP), Nelder-Mead Simplex (NSM) method, Lagrangian Relaxation (LR) and Quadratic Programming (QP) are samples of a traditional optimization approach, heuristic techniques and Artificial Neural Networks (ANN) can be categorized in the approximate optimization method field. Metaheuristics methods are the generalized heuristics methods that can be applied to a broad range of problems and are classified based on trajectory methods and population-based methods [10].

In 2010, Gendreau M. and Potvin J.Y. [38] presented a complete review on different types of metaheuristics methods, their definitions and applications. Metaheuristics methods are widely used in the optimization of energy systems [10].

Trajectory metaheuristics methods are according to the single optimized based solutions and include various methods such as Iterated Local Search (ILS), Tabu Search

(TS), Variable Neighbourhood Search (VNS), Hill Climbing (HC) and Simulated Annealing (SA). Population based metaheuristics methods are according to the population of solution and consist of, but not limited to, the following methods: Genetic Algorithms (GA), Scatter Search (SS), Particle Swarm Optimization (PSO), Memetic Algorithms (MA) and Ant Colony Optimization (ACO) [38].

In the field of renewable and sustainable energy system, an important problem is defining the allocation patterns of different energy sources, formulating the local energy policies, energy reliability and security [10]. Various optimization methods, which listed above, are proposed for this purpose. For example, Soroudi A. et al. [39] developed an immune Genetic Algorithm-based method for distribution network expansion, AlRashidi M.R. and El-Naggar K.M. [40] presented a PSO algorithm for forecasting the electricity annual pick load and Kahraman C. et al. [41] used a fuzzy-based algorithm to find the most suitable renewable energy alternative for a sample system.

Choosing the appropriate optimization algorithm among these vast types of optimization methods is widely based on the selected energy model for the system. Other factors, such as available data, system constraints, planners' goal(s), and available computational tools, are also important for this selection.

Optimal control theory is the mathematical method we selected for the purpose of this thesis. This optimization method has been widely used for the last five decades in diverse areas of physical sciences, medicine, engineering and economics. It defines the optimum solution of a problem without any approximations and constraints.

Miah M.S., Ahmed N.U., Chowdhury M. [6] and Fekih A. [42] (2012) developed their optimization method for energy systems through optimal control theory. The former utilized the Pontryagin Minimum Principle approach of optimal control theory for the purpose of integrating the renewable energy sources into the electricity generation; and the latter applied optimal control theory to a specific type of wind turbine and improved the operation of the turbine while minimizing its mechanical stress.

For the purpose of this thesis, Pontryagin Minimum Principle of the optimal control theory was used for system identification and for defining the optimum decision policy. This quite mathematical method assisted us to determine the absolute minimum solu-

tion for our optimization problem, which is finding the optimum decision policy for gradual replacement of conventional energy sources by clean ones.

### **2.3. Summary**

In this chapter, we started with a review on different energy models used in the literature during the period 1977-2012. Then, the mathematical Lotka-Volterra model was chosen for this work for the purpose of integration of renewable and clean energy sources in to the electricity system.

A list of different optimization methods that are widely used in the energy systems was then presented. Optimal control theory was our selected mathematical optimization method to find the optimum decision policy for allocation of renewable energy sources to the power grid.

## Chapter 3. Mathematical Background

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The necessary mathematical background for this thesis is presented in this chapter. We start with the Pontryagin Minimum Principle and continue with the Gradient method, which are the main mathematical tools for this thesis. The gradient method section covers details of the mathematical procedure to find the optimum decision policy and to identify the unknown parameters of the system to be solved.

### 3.1. Pontryagin Minimum Principle

In 1962, “The mathematical theory of optimal processes” published by Lev Pontryagin was the main starting point for the exciting topic of optimal control theory, derived from the calculus of variations [43]; however, the author basically presented the formulation of the Pontryagin Minimum Principle in 1956 [44].

Finding the optimal control policy for a given system based on the minimization or maximization of an objective function is the main problem that can be efficiently solved through optimal control theory. The Pontryagin Minimum Principle defines the optimal control solution of a dynamic system problem and finds the best trajectory for transferring from one state to another state of the system.

The basic problem of the Pontryagin Minimum Principle can be described as follows [5]:

Consider the system

$$\dot{x} = f(t, x(t), u(t)), t \in [t_1, t_2] \equiv I \quad (3.1)$$

and the general cost function

$$J(u) = \int_{t_1}^{t_2} \ell(t, x(t), u(t)) dt + \Phi(x(t_f)) \quad (3.2)$$

where  $u \in \mathcal{U} \equiv \{u \text{ measurable on } I \text{ to } R^m \text{ with values } u(t) \in U\}$ . The control set  $U$  is a closed subset of  $R^m$ , and  $\mathcal{U}$  is the feasible region for the control decision policy of  $u$  (in other words,  $\mathcal{U}$  is the class of admissible controls).

The problem is to find the control policy  $u(t)$  that minimizes the cost function (3.2) subject to the dynamic constraints of system (3.1) while transferring from the state  $x_1$  at time  $t_1$  to the state  $x_2$  at time  $t_2$  [5].

The cost function consists of two main parts namely, the running cost  $\ell(t, x(t), u(t))$  and the terminal cost  $\Phi(x(t_f))$ . The terminal cost part of the cost function has the responsibility of minimizing the gap between the desired values for the system and the obtained values from the system model at the end of the plan period. The running cost is responsible for minimizing the costs during the plan period. Different types of costs may be considered for the running cost according to the main concern of the problem.

Pontryagin introduced a function called Hamiltonian that plays an important role in his principle and described as follows:

$$H(t, x, \psi, u) \equiv \langle f(t, x, u), \psi \rangle + \ell(t, x, u) \quad (3.3)$$

where  $\langle \cdot, \cdot \rangle$  denotes the dot product.  $f(t, x, u)$  is the function that describes the system model according to equation (3.1),  $\ell(t, x, u)$  is the running cost part of the cost function and  $\psi$  is the Lagrange multiplier in calculus of variations and described in the adjoint or costate equation (3.5). In fact, the Hamiltonian function is used to introduce a new set of canonical differential equations through the following expressions:

$$\dot{x} = H_{\psi} = f(t, x, u) \quad (3.4)$$

$$\dot{\psi} = -H_x = -f_x^T(t, x, u)\psi - \ell_x(t, x, u) \quad (3.5)$$

Here, equation (3.4) is called the “State Equation”. It is the main equation of the system previously described by (3.1). Equation (3.5) is known as the “Adjoint or Costate

Equation". In this equation,  $f_x$  is the Jacobian matrix of vector  $f$  and the transposition of this matrix is denoted  $f_x^T$  [6].

The initial conditions for the state and costate equations are described as:

$$x(0) = x_0 \quad (3.6)$$

$$\psi(t_f) = \Phi_x^T(x(t_f)) \quad (3.7)$$

After specifying the main equations of the Pontryagin Minimum Principle, necessary conditions of the optimality can be introduced. In fact, to find the optimal control policy  $u^* \in \mathcal{U}$ , for the optimal trajectory of  $x^*$  while transferring from the state at time  $t_1$  to the state at time  $t_2$ , there should exist a costate  $\psi^*$  that satisfies the following equations [6], [5]:

$$H(t, x^*(t), \psi^*(t), u^*(t)) \leq H(t, x^*(t), \psi^*(t), u) \text{ for all } u \in \mathcal{U}, t \in I \quad (3.8)$$

$$\dot{x}^* = H_{\psi}(t, x^*, \psi^*(t), u^*(t)) = f(t, x^*(t), u^*(t)) \quad (3.9)$$

$$\begin{aligned} \dot{\psi}^* &= -H_x(t, x^*(t), \psi^*(t), u^*(t)) \\ &= -f_x^T(t, x^*(t), \psi^*(t), u^*(t)) - \ell_x(t, x^*(t), u^*(t)) \end{aligned} \quad (3.10)$$

$$x^*(0) = x_0, \psi^*(t_f) = \Phi_x(x^*(t_f)) \quad (3.11)$$

In order to find the optimum decision policy and to perform the system identification, an efficient method should be used.

## 3.2. Gradient Method

The gradient method is one of the most effective iterative computational methods that best determine the optimal solution of a problem. In fact, this iterative technique computes the

optimal solution using the necessary conditions of optimality and the initial conditions given in equations (3.8) to (3.11) with slight modifications according to the needs of the planner.

The Gradient iterative method, as given in [5], uses the gradient vectors and updates the control variables  $u$  in a way that these iterations minimize the cost function of the problem. It was chosen to find the optimal control solution and to define the unknown parameters of the model of our system using a MALTAB code [45].

### 3.2.1 Methodology for Computation of Optimum Decision Policy

Consider the following state and costate equations [5]:

$$\dot{x} = f(t, x(t), u(t)), u \in \mathcal{U}, x(0) = x_0 \quad (3.12)$$

$$\dot{\psi} = -f_x^T(t, x(t), u(t))\psi - \ell_x(t, x(t), u(t)), \psi(t_f) = \frac{\partial \Phi}{\partial x}(x(t_f)) \quad (3.13)$$

The problem is to minimize the cost function:

$$J(u) = \Phi(x(t_f)) + \int_{t_0}^{t_f} \ell(t, x(t), u(t))dt \quad (3.14)$$

while the Hamiltonian function is:

$$H(t, x, \psi, u) \equiv \langle f(t, x(t), u(t)), \psi(t) \rangle + \ell(t, x(t), u(t)) \quad (3.15)$$

Based on the necessary conditions of optimality, given in equations (3.8)-(3.11), the iterative algorithm developed to find the optimal control  $u$  is detailed through the following procedure:

- Step 1. Assume a piece-wise constant control while dividing the interval  $I = [t_0, t_f]$  into  $N$  equal sub-intervals:

$$u_n(t) = u_n^k, t \in [t_k, t_k + 1] \quad (3.16)$$

Step 2. Integrate the state equations (3.12) for the whole plan period using the assumed  $u_n$  and the initial condition  $x(0) = x_0$ . Store the state trajectory  $x_n$ .

Step 3. Integrate the costate equations (3.13) backward in time using the assumed  $u_n$  and stored  $x_n$  starting from terminal time  $t_f$ . This will give the costate  $\psi_n$ .

Step 4. Compute the gradient vector using  $\{x_n, \psi_n, u_n\}$

$$g_n(t) = H_u(x_n(t), \psi_n(t), u_n(t)) \quad (3.17)$$

Step 5. Determine if the equality

$$\begin{aligned} H(x_n(t), \psi_n(t), u_{n+1}(t)) = \\ H(x_n(t), \psi_n(t), u_n(t)) - \varepsilon \|H_u(x_n(t), \psi_n(t), u_n(t))\|^2 + o(\varepsilon) \end{aligned} \quad (3.18)$$

is satisfied with  $\{x_n, \psi_n, u_n\}$  replacing  $\{x^*, \psi^*, u^*\}$ . Stop the iteration if it is satisfied (because the triple  $\{x_n, \psi_n, u_n\}$  is optimal). If not, check the inequality

$$\int_{t_0}^{t_f} \|H_u(x_n(t), \psi_n(t), u_n(t))\|^2 \leq \delta \quad (3.19)$$

is satisfied, with  $\delta$  the acceptable tolerance. Stop the iteration if it is satisfied. If not, replace  $u_n$  for the next iteration using  $u_{n+1}$  given by

$$u_{n+1}(t) = u_n(t) - \varepsilon g_n(t), \quad 0 < \varepsilon < 1 \quad (3.20)$$

where  $\varepsilon$  is the step-size. Continue the search from Step 2.

The procedure mentioned in these five steps is used in this thesis to find the optimum decision policy for integration of renewable energy systems into the electricity generation. However, finding the optimal control decision is impossible without a well-defined model for the system. A procedure should be then developed for efficient system identification.

### 3.2.2 Methodology for System Identification

Consider the following state and costate equations for the system [5]:

$$\dot{x} = f(t, x, \beta), t \in [t_0, t_f] \equiv I, x(0) = x_0 \quad (3.21)$$

$$\dot{\psi} = -f_x^T(t, x, \beta)\psi + (y(t) - x(t)), \psi(t_f) = 0 \quad (3.22)$$

where  $x(t) \in R^n$  is the state vector and  $\beta$  is the unknown parameter vector, which takes values from a closed bounded subset  $\mathcal{P}$  of  $R^m$ . It is assumed that the response of the physical system is given by  $y(t), t \in I$ . The goal of the system identification problem is to find a parameter  $\beta^* \in \mathcal{P} \subset R^m$  that minimizes the identification error

$$J(\beta) = \frac{1}{2} \int_{t_0}^{t_f} |x(t, \beta) - y(t)|^2 dt \quad (3.23)$$

where  $J(\beta^*) \leq J(\beta)$  and  $x(\cdot, \beta)$  are the response of the model corresponding to the parameter  $\beta$  and the initial condition  $x_0$ , respectively.

It can be noted that this system identification problem is a special case of an optimal control problem with some modifications in the procedure. In this case the Hamiltonian function will be:

$$H(t, x, \psi, \beta) = \langle f(t, x, \beta), \psi \rangle + \frac{1}{2} |x(t, \beta) - y(t)|^2 \quad (3.24)$$

The optimal parameter  $\beta^*$  and the corresponding pair of  $\{x^*, \psi^*\}$  will satisfy the set of equations (3.21) and (3.22) and the inequality

$$\int_{t_0}^{t_f} H(t, x^*(t), \psi^*(t), \beta^*) dt \leq \int_{t_0}^{t_f} H(t, x^*(t), \psi^*(t), \beta) dt \quad (3.25)$$

As for the algorithm of the system identification, the five-step procedure discussed earlier for the optimal control is used with some modifications [5]:

Step 1. Subdivide the plan period into  $N$  equal sub-intervals

Step 2. Integrate the system equation (3.21) for the plan period with any assumed  $\beta \equiv \{\beta_{ij}\}$  and the given initial conditions at  $t_0$ . Let  $\{\beta_n, x_n\}$  denotes the values of  $\beta$  and the state  $x$  at the  $n^{\text{th}}$  iteration.

Step 3. Integrate the costate equation (3.22) backward in time with the pair  $\{\beta_n, x_n\}$  replacing  $\{\beta^*, x^*\}$  with the terminal conditions at  $t_f$  as given in equation (3.22). This will give the costate  $\psi_n$ .

Step 4. Determine the gradient vector using the data  $\{\beta_n, x_n, \psi_n\}$  :

$$g_n = \int_{t_0}^{t_f} \frac{\partial H}{\partial \beta}(t, x_n, \psi_n, \beta_n) dt = \int_{t_0}^{t_f} f_{\beta}^T(t, x_n, \beta_n) \psi_n dt \quad (3.26)$$

Step 5. Determine if the inequality (3.25) is satisfied with  $\{\beta_n, x_n, \psi_n\}$  replacing  $\{\beta^*, x^*, \psi^*\}$ . Stop the iteration if it is satisfied (because the triple  $\{\beta_n, x_n, \psi_n\}$  is optimal). If not, check if the inequality

$$\int_{t_0}^{t_f} \|H_\beta(x_n(t), \psi_n(t), \beta_n)\|^2 dt \leq \delta \quad (3.27)$$

is satisfied, with  $\delta$  the acceptable tolerance. Stop the iteration if it is satisfied. If not, replace  $\beta_n$  for the next iteration using  $\beta_{n+1}$  given by

$$\beta_{n+1} = \beta_n - \varepsilon g_n \quad (3.28)$$

where  $\varepsilon$  is the step-size. Continue the search from Step 2.

### 3.3. Summary

In this chapter, we presented details of the numerical procedure to be followed for our optimal control problem and the system identification problem. In the following chapter, we will continue with the details of the model used to develop the energy system. Also we will introduce different important energy problems that are solved in this thesis.

## Chapter 4. Mathematical Model and Problem Formulation

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Details of the Lotka-Volterra dynamic non-linear model used for the purpose of this thesis is presented in this chapter, followed by the different energy problems of terminal cost, implementation cost, environmental cost, and demand cost.

### 4.1. Mathematical Model

To formulate our problem, let us introduce eight first order non-linear differential equations, based on the dynamic Lotka-Volterra model, as follows:

$$\dot{x}_i = f_i(x, u, \beta) = u_i x_i - \sum_{j=1, j \neq i}^8 \beta_{ij} x_i x_j \quad (4.1)$$

where  $i$  varies from 1 to 8.

The state vector  $[x_1(t), x_2(t), x_3(t), x_4(t), x_5(t), x_6(t), x_7(t), x_8(t)]^T$  demonstrates the level of electricity generation from the respective eight targeted energy sources, i.e., Coal, Petroleum, Natural Gas, Hydropower, Wind, Solar, Geothermal, and Biomass. It also indicates the trajectory that the system takes during the plan period. The rate of change in the level of generation from each of the mentioned energy sources is denoted by  $\dot{x}_i$ , where  $i$  varies from 1 to 8.

The parameter  $u_i$  is the percentage growth rate of the level of electricity generation for each of the energy sources. The penetration of each energy source into the electricity generation will increase if the corresponding coefficient  $u_i$  increases. In this point of view, these coefficients can be seen as the control variables due to the fact that they demonstrate the percentage growth for each of the energy sources. In addition, and as required for control variables, they vary with time [6]. We restrict the control variables of  $u_i$

to the closed bounded interval  $[-1, 1]$  where “1” means a hundred percent increase and “-1” a hundred percent decrease.

The impact of the electricity generation level from the energy source  $j$  on the level of electricity generation from the energy source  $i$  is represented by  $\beta_{ij}$ . For example  $\beta_{36}$  is the impact of the electricity generation level from the solar energy source ( $j=6$ ) on the level of electricity generation from the natural gas energy source ( $i=3$ ). The fifty-six unknown  $\beta$ -parameters (in the system equation (4.1)) will be determined through the gradient method for system identification discussed in the previous chapter. It is worth to mention that instead of assuming constant values for  $u$  variables, we will determine all eight variables of  $u$  through the mentioned system identification method to get more precise results.

## 4.2. Energy Problems

After defining the model (equation (4.1)) for our energy system, different energy problems should now be introduced. Here we present details of these problems including their formulations and objectives. In the numerical results of our thesis, we will use different combinations of these problems and then we will find the optimum decision policies in each case.

### 4.2.1 Terminal Cost Problem

Minimizing the difference between the desired values for the level of generation from each of the eight energy sources and the actual level of generation reached from the model of the system defined in equation (4.1) at the end of the plan period is the main concern of this type of problem. The cost function of the terminal cost problem can be defined as

$$J(u) = \frac{1}{2} \left\{ \sum_{i=1}^8 \delta_i (x_i(t_f) - x_i^d)^2 \right\} \quad (4.2)$$

Here  $\delta_i$  is the weight for the energy source  $i$ , i.e., its relative importance in the global system,  $x_i(t_f)$  its response at the end of the plan period, and  $x_i^d$  its desired value according to the clean future plan adopted in this work, i.e., that of the United States, given in appendix A. This type of problem is effective only at the end of the plan period and is shown as  $\Phi(x(t_f))$  in equation (3.2).

#### 4.2.2 Implementation Cost Problem

The implementation cost, namely control cost, represents the level of investment toward the integration of renewable and clean energy sources into the electricity generation. Capital costs include the costs of the main components required for each of the energy sources, such as turbine for wind power source, grid connection costs, civil works costs and the variable costs related to Operation and Maintenance, insurance and taxes, management and administration [46]. The objective function for the implementation cost can be expressed as:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2 \right\} dt \quad (4.3)$$

The weight  $q_i$  is the relative importance given to the implementation cost for each of the energy sources and is defined based on the available data and the needs of the planner. This type of problem is effective through the whole plan period and is considered as the running cost,  $\ell(t, x(t), u(t))$ , in equation (3.2).

#### 4.2.3 Environmental Cost Problem

The main concern of this type of problem is minimizing the emissions of GHG and the level of generation from polluted energy sources. Among coal, petroleum, natural gas, hydropower, wind, solar, geothermal and biomass energy sources, the first two are the main concerns of this type of problem and produce considerable emissions of GHG.

Defining the relationship between the level of electricity generated from the polluted energy source and the amount of GHG emissions is an inevitable part of this problem [6]. We therefore considered this relationship in the following expression:

$$F(x) = v|x|^q, q \geq 1, v > 0 \quad (4.4)$$

The above non-negative non-decreasing function is an assumption used to demonstrate the amount of GHG emissions from the polluted energy sources. Through experimentation, engineers can determine the two parameters  $\{v, q\}$  for each of the conventional sources. It is known that petroleum produces 1.23 times more CO<sub>2</sub> than coal for the same level of electric power generation in year 2011 [47]. This relationship is considered as a reference in our problem for numerical results.

The objective function for this kind of energy problem for coal and petroleum energy sources is considered as follows:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^2 F(x_i(t)) \right\} dt \quad (4.5)$$

#### 4.2.4 Demand Problem

Any energy decision planning without taking the energy demand into account in the specific time intervals defined within the whole targeted plan period will be futile. Minimizing the gap between the level of electricity generation and the level of electricity demand during the plan period is the main concern of the demand problem.

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \Gamma(D(t) - \sum_{i=1}^8 x_i(t))^2 \right\} dt \quad (4.6)$$

Here  $D(t)$  stands for the electricity demand during the plan period while the responsibility to fulfil the electricity demand by each of the energy sources is shown by  $\Gamma$

as the weighting value. Expression (4.6) defines the objective function considered for the demand problem.

### **4.3. Summary**

In this chapter, we first presented the system of equations to model the level of electricity generated from eight selected energy sources. Then, we defined the different problems of energy systems that are the main concerns in the field of energy planning.

In the following chapters, we will first identify unknown parameters of our model and then present the numerical results for different energy problems. All the numerical results are based on the official data provided in the U.S. Energy Information Administration website [4]. These data are presented in the appendix A of the thesis.

## Chapter 5. System Identification

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Numerical results of our thesis are divided into two main categories, system identification and optimum decision policy. In this chapter, we present the numerical results of the system identification, using the methodology presented in chapter 3. Based on the literature review, and to the best of our knowledge, an eight-state time-variable Lotka-Volterra system for energy problems have never been developed before; Therefore, in order to have more reliable code for our eight-state system, we started with a two-state time variable system (as presented by [6]) and validated our code through the results proposed in their research. The first step for promoting the basic code is transferring from two-state time-variable system to three-state time-variable system, which is also presented in this chapter. Finally after passing these two steps successfully, we present our results for the system identification of the main problem of this thesis, which is the eight-state time-variable Lotka-Volterra system.

### 5.1. Two-State Variable System

Miah M.S., Ahmed N.U., Chowdhury M. [6] developed a method to find the optimum decision policy for gradual replacement of conventional power sources by renewable ones. In their research, they considered a system where  $0.8(\times P)$ MW of the electricity generation is initially from conventional power sources and  $0.2(\times P)$ MW from renewable ones.  $P$  is any positive number suitable in planner's point of view, in other words, they solved a normalized problem and considered  $P=1$ . Then, they proposed a method to determine the optimum decision policy for changing the main source of the electricity production from conventional to renewable ones. For different plan periods of 15, 20 and 30 years they found the optimum decision policy to produce  $0.8(\times P)$ MW electricity from renewable power sources and  $0.2(\times P)$ MW from conventional ones.

The main shortcoming of their research was that they neither presented the system identification nor stated the existing unknown parameters in their model.

In addition to that, the levels of electricity generations are only considered as assumptions, which may not be much applicable in real life. In this chapter, we will then introduce the system identification of their model while how to define the optimum decision policies will be investigated later.

With the purpose of connecting Miah M.S., Ahmed N.U., Chowdhury M. [6] model to real life data, after defining the unknown parameters of their model, we also present a similar model consisting of two main energy sources of fossil and renewable based on the official released data of U.S EIA [48] website. The next step would be to determine the unknown parameters of the related system.

With the mentioned goal in mind, let us consider the state equations below for a two-state time-variable system [6]:

$$\begin{cases} \dot{x}_1(t) = u_1 x_1(t) - \beta_{12} x_1(t) x_2(t) \\ \dot{x}_2(t) = u_2 x_2(t) - \beta_{21} x_1(t) x_2(t) \end{cases} \quad (5.1)$$

where  $x_1(t)$  is the level of installed capacity of renewable power sources and  $x_2(t)$  the installed capacity from conventional power sources.  $u_1$  and  $u_2$  are the control policies for the conventional power sources and the renewable ones, respectively; defining them throughout the whole plan period is the main goal of the optimum decision policies. However in the case of system identification, these parameters are assumed as constant values due to the fact that finding the unknown  $\beta$ -parameters is the only concern of the system identification procedure. Whatever the  $u$  parameters are considered, it will not affect the calculated values for the  $\beta$ -parameters and may only affect the numbers of iterations.  $\beta_{12}$  shows the impact of level of conventional power on the growth rate of the clean power sources and vice versa for  $\beta_{21}$  [6].

In this section, the problem is to find the unknown parameters of  $\beta_{12}$  and  $\beta_{21}$  for the system (5.1) while moving from the initial values of  $x_1(0) = 0.2$  and  $x_2(0) = 0.8$  (which represent the current situation) to desired values  $x_1(t_f) = 0.8$  and  $x_2(t_f) = 0.2$  (where  $t_f$  is the final time of the plan period).

Availability of historical data are an inevitable part of the system identification procedure, and this is due to the fact that in our mathematical method, the unknown parameters are identified through minimizing the gap between the response of the modeled system and the actual physical results of the system. In our case, in order to be able to identify Miah M.S., Ahmed N.U., Chowdhury M. [6] unknown parameters, we used their projection for the level of installed electricity capacity. We therefore retained their plan period of fifteen years as historical data to find the unknown parameters of  $\beta_{12}$  and  $\beta_{21}$ . Thus, the response of the system (5.1) is our selected historical data,  $y_1(t)$  is interpolated data from 0.2 to 0.8 and vice versa for  $y_2(t)$ .

Using the numerical methodology for system identification presented in the gradient method section of chapter 3, we developed a MATLAB code. This code consists of four main functions

- One function for the two state {equations (5.1)}
- One function for the two costate {equations (3.22)}
- One function for the gradient value {equation (3.26)}
- One function for the updating  $\beta$  {equation (3.28)}

The step-size is considered 0.04 and the acceptable tolerance is  $\delta = 0.005$ . This means that based on the methodology presented in chapter 3, we will stop the iterations when the expression below is satisfied:

$$\int_{t_0}^{t_f} \|H_{\beta}(x_n(t), \psi_n(t), \beta_n)\|^2 \leq 0.005 \quad (5.2)$$

In this problem, 2000 iterations were enough to reach the acceptable margin of error for expression (5.2). As mentioned above, the initial values for state variables  $x_1(0)$  and  $x_2(0)$  are 0.2 and 0.8, respectively. Based on the mathematical procedure mentioned in chapter 3, the initial value for the costate variables  $\psi_1$  and  $\psi_2$  are zero (equation (3.22)).

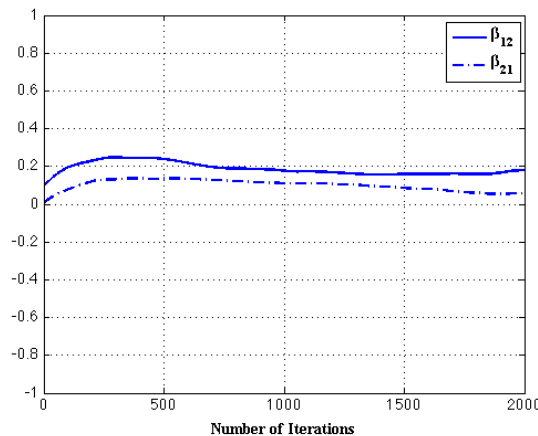
The goal is to find the parameters  $\beta_{12}$  and  $\beta_{21}$  through minimizing the identification error expressed by equation (3.23). In this case, we assumed that parameters  $u$  are constant and we expected that the unknown parameters of  $\beta_{12}$  and  $\beta_{21}$  converge to the specific values after 2000 iterations regardless of the initial values of  $\beta$ .

The calculated values for the unknown parameters of  $\beta_{12}$  and  $\beta_{21}$  are given below in four different cases. The only difference between these cases is the assumed initial values for  $\beta_{12}$  and  $\beta_{21}$ . We are able to rely on the identified values of  $\beta_{12}$  and  $\beta_{21}$  only if they satisfy the following conditions:

- The identification error (equation (3.23)) is constantly minimized,
- The converged values are approximately close to each other with different initial values,
- The number of iterations is large enough to satisfy equation (5.2).

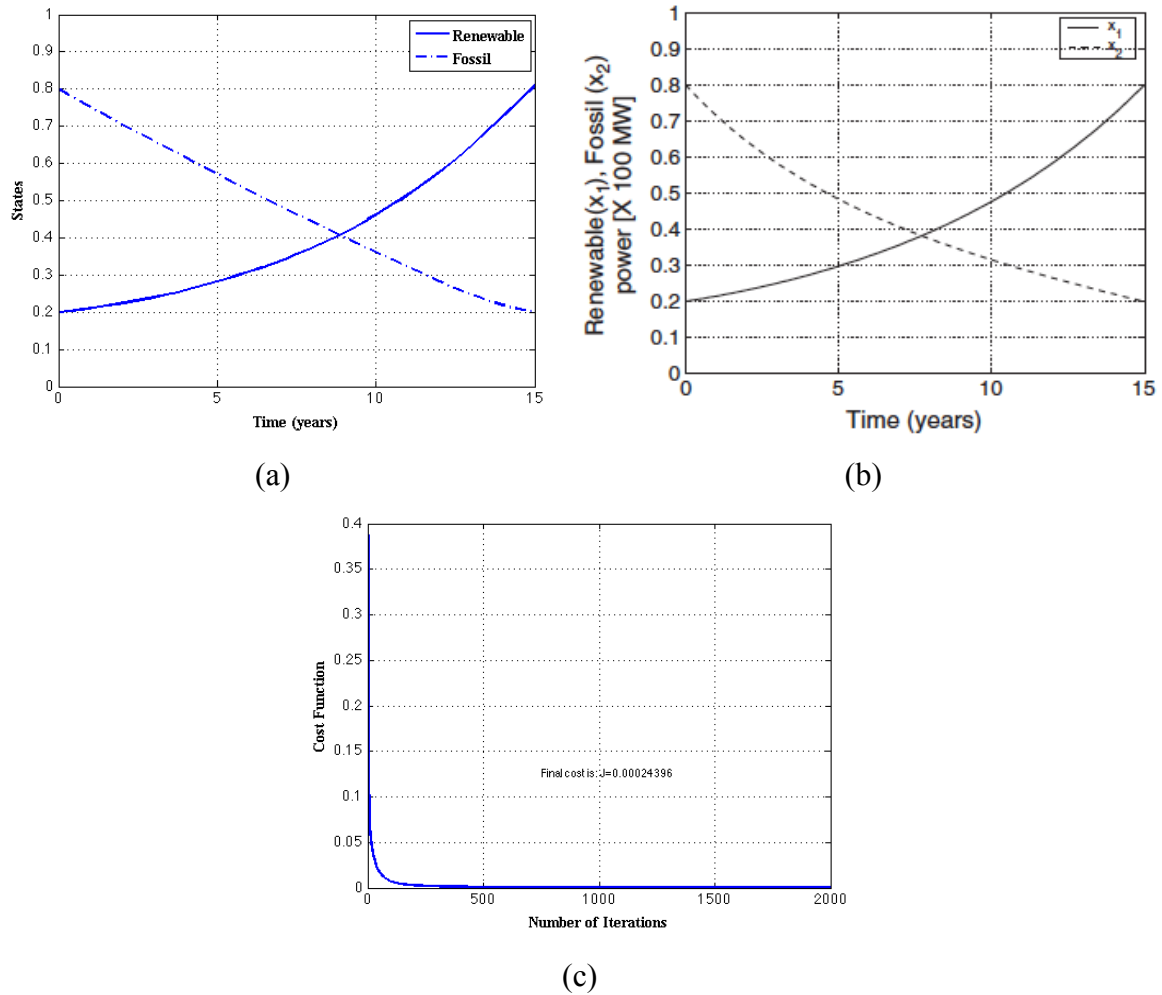
It is worth to mention that regardless of what the initial values are, parameters  $\beta_{12}$  and  $\beta_{21}$  will converge to approximately the same values; however, the number of iterations required to reach the final solution will vary in accordance with the initial selected values.

- Case 1: Initial values for  $\beta_{12} = 0.1$  and  $\beta_{21} = 0.01$



**Figure 1** Case 1- Converging trajectory of  $\beta_{12}$  and  $\beta_{21}$  after 2000 iterations

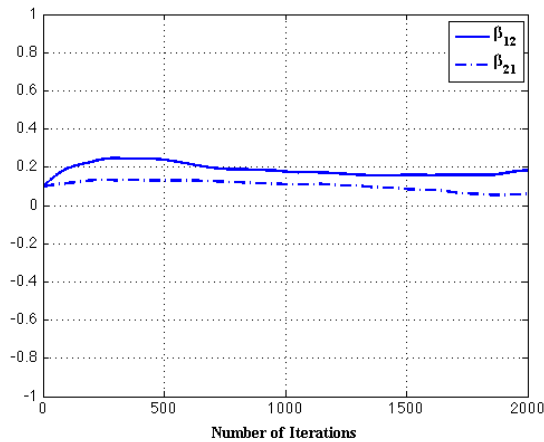
As shown in Figure 1,  $\beta_{12}$  and  $\beta_{21}$  converged to the respective values of 0.1805 and 0.0564 after 2000 iterations with a final identification error value of 0.00024396. The corresponding state trajectories and the constantly decreasing identification error are shown in Figure 2-a and Figure 2-c, respectively. Figure 2-a and Figure 2-b show the compatibility of our results for the level of installed electricity and the results presented in [6].



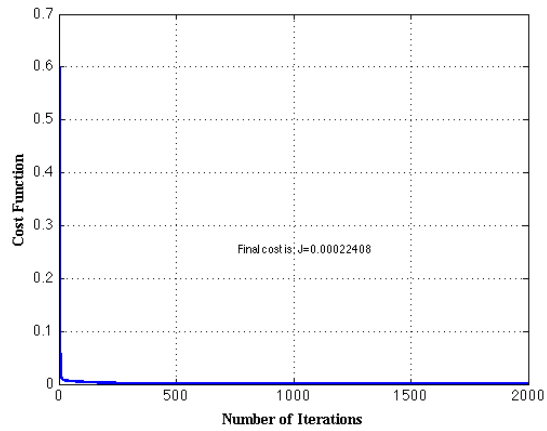
**Figure 2** Case 1- State trajectories for 15 year plan period (a) from our model, (b) from model given in [6] and (c) Identification error (equation 3.23) after 2000 iterations

To lighten the document, we will not include anymore the state trajectory, for other three cases of this problem, due to the fact that it is very similar in all cases.

- Case 2: Initial values for  $\beta_{12} = 0.1$  and  $\beta_{21} = 0.1$



(a)

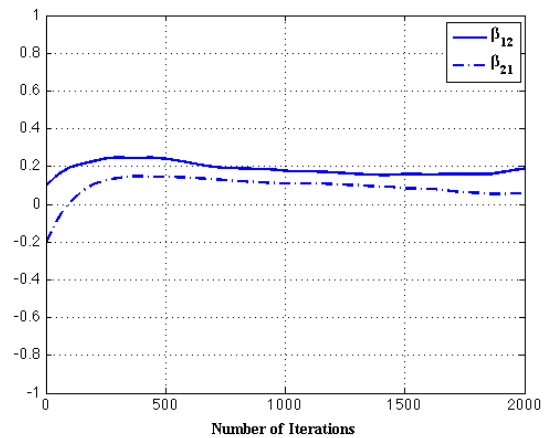


(b)

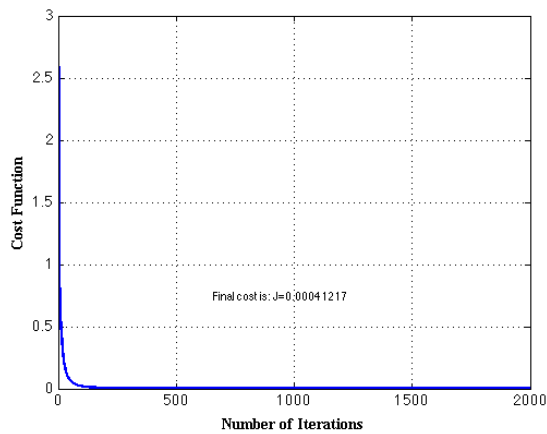
**Figure 3** Case 2- (a) Converging trajectory of  $\beta_{12}$  and  $\beta_{21}$  after 2000 iterations, (b) Identification error (equation 3.23) after 2000 iterations

In this case the converged values are 0.1840 for  $\beta_{12}$  and 0.0589 for  $\beta_{21}$ , as shown in Figure 3-a. The identification error is minimized through 2000 iterations with a final value of 0.00022408 as shown in Figure 3-b.

- Case 3: Initial values for  $\beta_{12} = 0.1$  and  $\beta_{21} = -0.2$



(a)

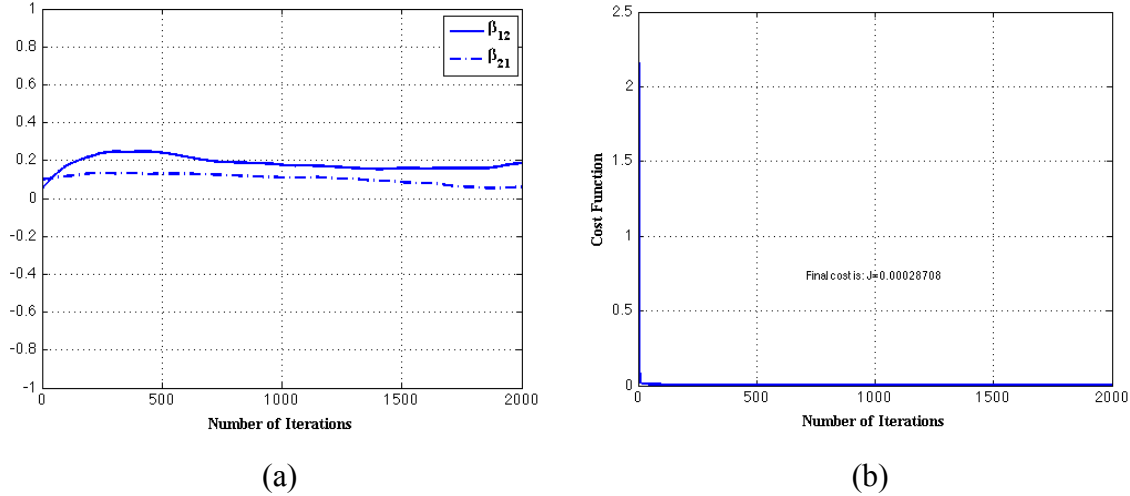


(b)

**Figure 4** Case 3- (a) Converging trajectory of  $\beta_{12}$  and  $\beta_{21}$  after 2000 iterations, (b) Identification error (equation 3.23) after 2000 iterations

As shown in Figure 4, the converged values are 0.1878 for  $\beta_{12}$  and 0.561 for  $\beta_{21}$ . The final cost of identification error is 0.00041217 after 2000 iterations.

- Case 4: Initial values for  $\beta_{12} = 0.05$  and  $\beta_{21} = 0.1$



**Figure 5** Case 4- (a) Converging trajectory of  $\beta_{12}$  and  $\beta_{21}$  after 2000 iterations, (b) Identification error (equation 3.23) after 2000 iterations

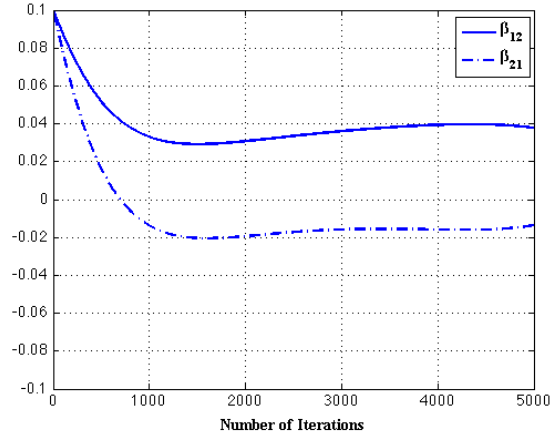
In this case the identification error final cost after 2000 iterations is 0.00028708 and the converged values are 0.1863 for  $\beta_{12}$  and 0.0591 for  $\beta_{21}$  (Figure 5).

So, we can conclude that regardless of what the initial values for the  $\beta$ -parameters are, the converged values can be considered as 0.18 for  $\beta_{12}$  and 0.05 for  $\beta_{21}$ . These values being identified, the optimum decision policy can be investigated to obtain compatible results with those given in [6]. This issue will be addressed in the next chapter. For this section, the above step helped us to confirm that our code is working properly and can be successfully extended to more variables.

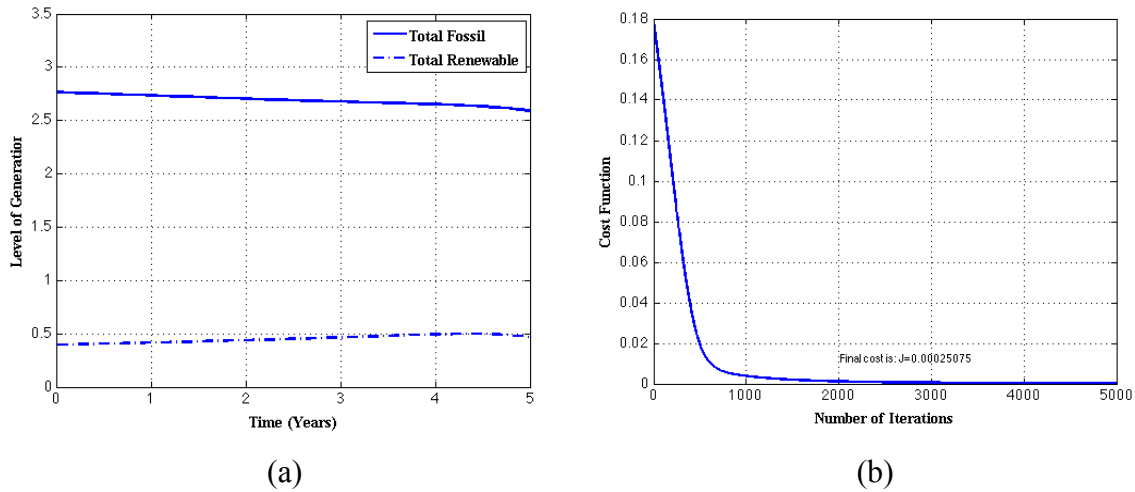
For the case study of two-state variable system, we used the same procedure for the official released data of U.S. EIA annual outlook 2012 [48] for total renewable and total fossil power sources. According to their official data, if we consider coal, petroleum and natural gas as the total fossil power sources and conventional hydropower, geothermal, wood and biomass, solar and wind as the total renewable power sources, the historical data for these two main categories are as given in Table 1-A.

Using the historical data for system equation (5.1) with step-size 0.005, acceptable tolerance of  $\delta = 0.0005$ , and initial values of 2.764 for  $x_1(0)$ , 0.39482 for  $x_2(0)$ , and zero for both  $\psi_1(t_f)$  and  $\psi_2(t_f)$ , we developed a code based on the methodology mentioned in chapter 3 for system identification for two different cases:

- Case 1: Initial values for  $\beta_{12} = 0.1$  and  $\beta_{21} = 0.1$



**Figure 6** Case 1- Converging trajectory of  $\beta_{12}$  and  $\beta_{21}$  after 5000 iterations

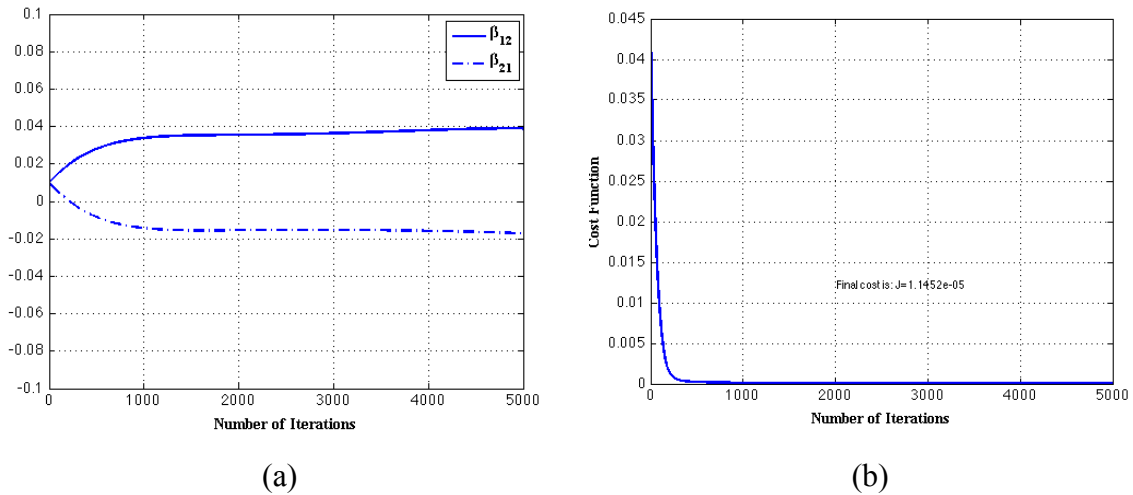


**Figure 7** Case 1- (a) State trajectories for 5 year plan period, (b) Identification error (equation 3.23) after 5000 iterations

Starting from 0.1 for both parameters of  $\beta_{12}$  and  $\beta_{21}$ , the first one converged to 0.0378 and the latter converged to -0.0156 (Figure 6). The identification error reached an acceptable value of 0.00025075 after 5000 iterations (Figure 7).

The state trajectories for this plan period are compatible with what we expected based on the historical data.

- Case 2: Initial values for  $\beta_{12} = 0.01$  and  $\beta_{21} = 0.01$



**Figure 8** Case 2- (a) Converging trajectory of  $\beta_{12}$  and  $\beta_{21}$  after 2000 iterations, (b) Identification error (equation 3.23) after 5000 iterations

In this case again, it is clear that while the identification error is minimized through 5000 iterations with a final value of  $1.1452e-5$ , the parameters  $\beta_{12}$  and  $\beta_{21}$  converged to similar values as in case 1. Here  $\beta_{12}$  converged to 0.0389 and  $\beta_{21}$  to -0.0161. Therefore in this case of problem we assume  $\beta_{12} = 0.03$  and  $\beta_{21} = -0.01$ .

The optimum decision policies for these two state variables will be also presented in the next chapter.

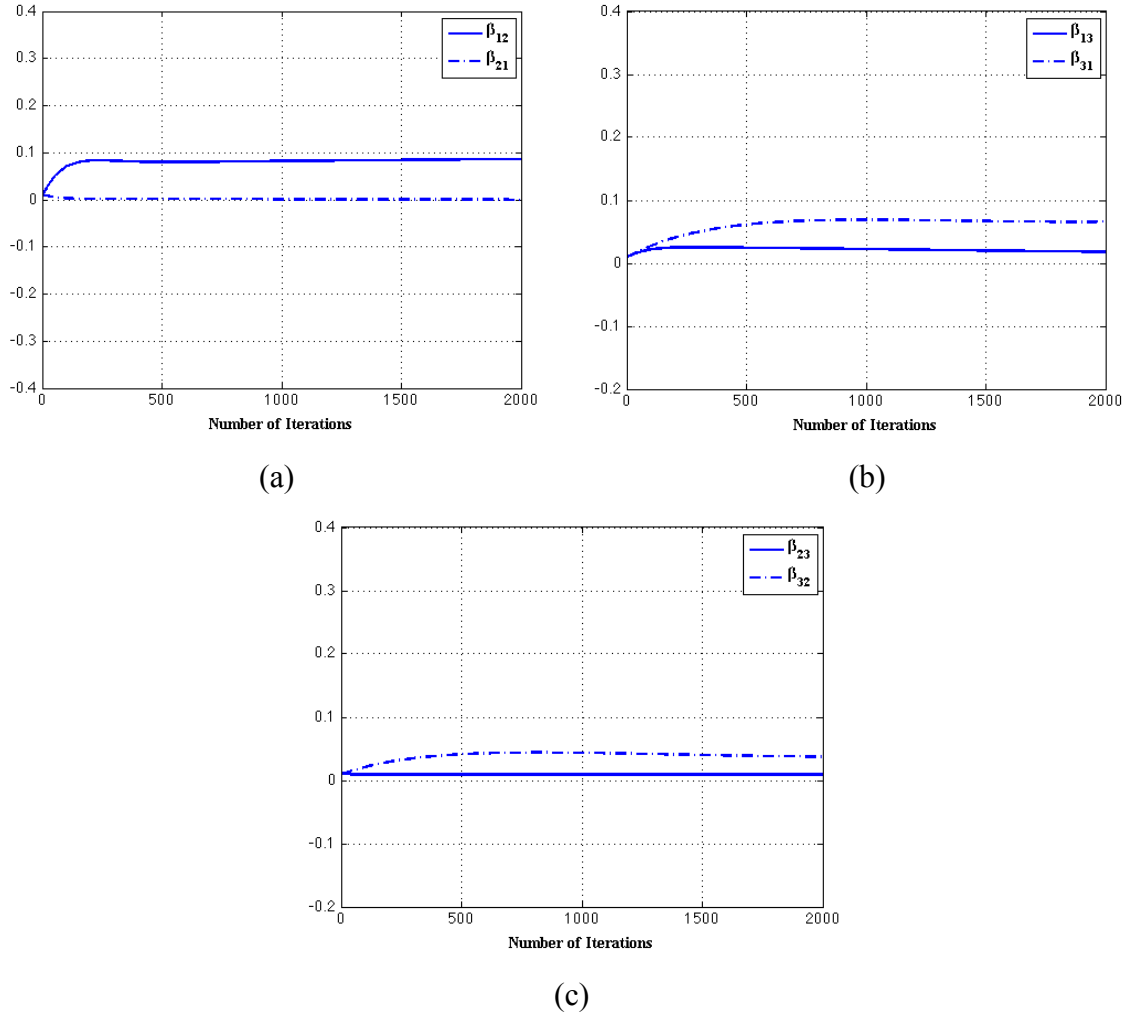
## 5.2. Three-State Variable System

In this problem we extended our code from two-state variables to three-state variables such as

$$\begin{aligned}\dot{x}_1(t) &= u_1 x_1(t) - \beta_{12} x_1(t) x_2(t) - \beta_{13} x_1(t) x_3(t) \\ \dot{x}_2(t) &= u_2 x_2(t) - \beta_{21} x_2(t) x_1(t) - \beta_{23} x_2(t) x_3(t) \\ \dot{x}_3(t) &= u_3 x_3(t) - \beta_{31} x_3(t) x_1(t) - \beta_{32} x_3(t) x_2(t)\end{aligned}\tag{5.3}$$

where  $x_1$ ,  $x_2$ , and  $x_3$  denote the level of electricity generation from coal, natural gas and wind energy sources, respectively. The control policies for different energy sources of coal, natural gas and wind are represented by  $u_1$ ,  $u_2$  and  $u_3$ , respectively (assumed to be constant for the purpose of system identification, as already mentioned). The parameter  $\beta_{ij}$  is the impact of the level of electricity generated from the energy source  $j$  on the level of electricity generated from the energy source  $i$ . The historical data for these three energy sources ( $y(t)$  in identification error equation (3.23)) are based on the released official data of U.S. EIA annual outlook 2012 [48] and given in Table 1-A.

The step-size is set to 0.04, the acceptable tolerance is  $\delta = 0.0005$  and the initial values for  $x_i$  parameters for year 2010 are  $x_1(0) = 1.831$ ,  $x_2(0) = 0.898$  and  $x_3(0) = 0.09449$  for coal, natural gas and wind energy sources, respectively. According to the historical data, we expect each of these energy sources to reach the desired values mentioned in Table 1-A in year 2015, i.e.,  $x_1^d(t_f) = 1.562$  for coal,  $x_2^d(t_f) = 1.028$  for natural gas, and  $x_3^d(t_f) = 0.15097$  for wind energy source.



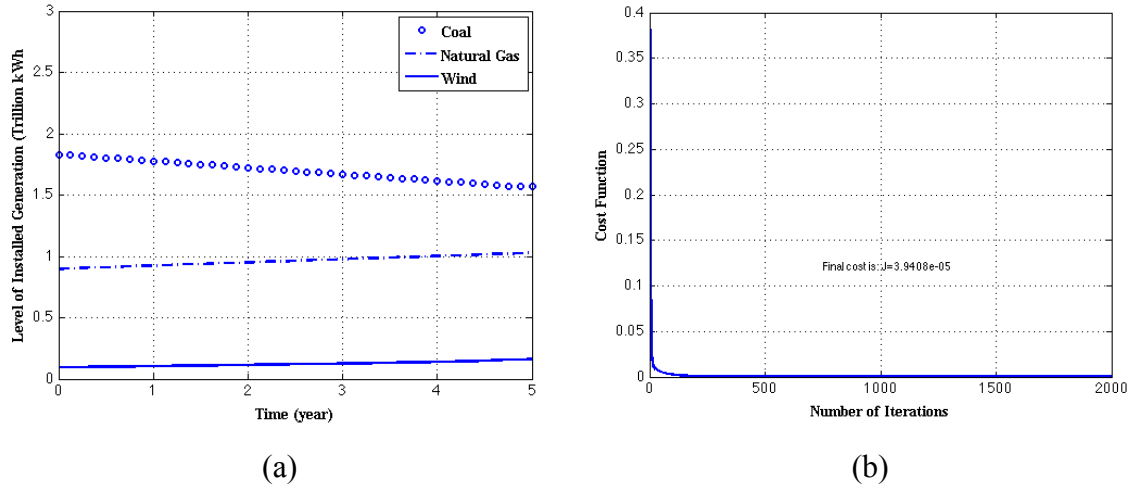
**Figure 9** Three-State System, Converging trajectory after 2000 iterations for (a)  $\beta_{12}$  and  $\beta_{21}$ , (b)  $\beta_{13}$  and  $\beta_{31}$  and (c)  $\beta_{23}$  and  $\beta_{32}$

As shown in Figure 9, all six unknown  $\beta$ -parameters ( $\beta_{12}$ ,  $\beta_{21}$ ,  $\beta_{13}$ ,  $\beta_{31}$ ,  $\beta_{23}$ , and  $\beta_{32}$ ), converged to the values reported in Table 2. Similar results were obtained assuming different initial values for these parameters.

**Table 2** Calculated  $\beta$ -parameters for the three-state system

$\beta_{ij}$	Value	$\beta_{ij}$	Value
$\beta_{12}$	0.08	$\beta_{31}$	0.06
$\beta_{21}$	0.0005	$\beta_{23}$	0.01
$\beta_{13}$	0.01	$\beta_{32}$	0.03

Figure 10-a shows the state trajectories for all three energy sources during the plan period (2010-2015) while Figure 10-b shows the converging identification error with final value of  $3.9408e-5$ .



**Figure 10** (a) State trajectories for 2010-2015 plan period, (b) Identification error (equation 3.23) after 2000 iterations

With the aim of the identified six  $\beta$ -parameters, the next step will be to apply the optimal control theory to the model of system (5.3) and find the optimum decision policies for different energy problems. However, as already mentioned, this step will be covered in the following chapter. In fact, we need first to extend the three-state system to eight variables.

### 5.3. Eight-State Variable System

In this type of system, which is the main concern of our thesis, we have considered all eight main energy sources of coal, petroleum, natural gas, hydropower, wind, solar, geothermal and biomass.

We thus considered an eight-state variable dynamic Lotka-Volterra model. All related fifty-six unknown  $\beta$ -parameters in equation (4.1) were then identified through the methodology presented for system identification in chapter 3 and a code written to take into account the above eight variables.

To validate it, we considered a step-size of 0.005 with an acceptable tolerance of  $\delta = 0.0005$ . The initial values for all costate variables were equal to zero at the final time according to equation (3.22) while the initial values for state variables of  $x_i(0)$  ( $i = 1, \dots, 8$ ) were stated as [1.831, 0.034, 0.898, 0.25532, 0.09449, 0.00128, 0.01567, 0.01151]<sup>t</sup> as given in Table 1-A.

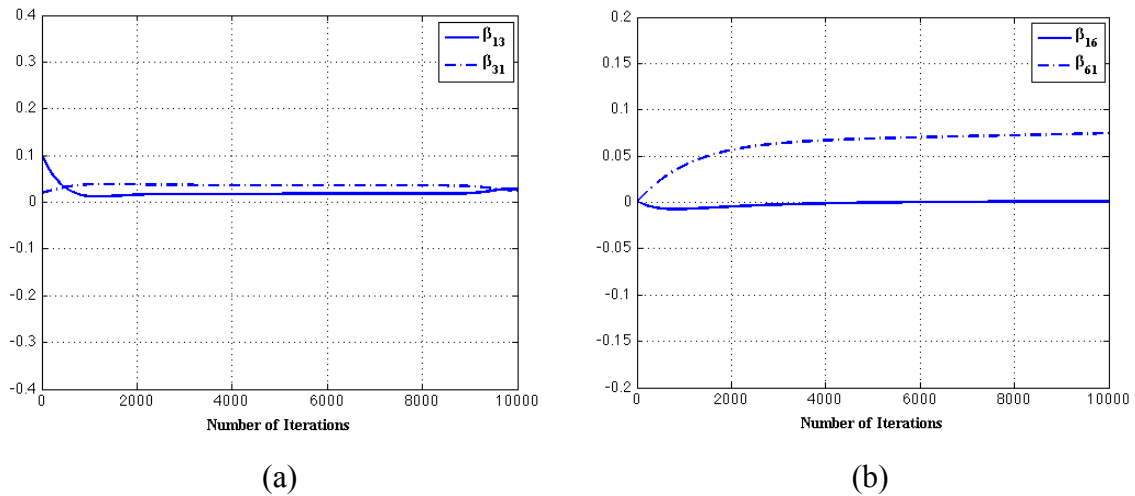
We interpolated the given data (in Table 1-A) between the years 2010 and 2015 and considered them as historical data, actual physical response of the system, for each of the energy sources. That is,  $y_1(t)$  is between 1.831 and 1.562 Trillion kWh for coal energy source,  $y_2(t)$  is between 0.034 and 0.026 Trillion kWh for petroleum energy source, etc.

The converged values for the fifty-six unknown  $\beta$ -parameters after 10,000 iterations are shown in Table 3. Note that with different initial values for these parameters, they still converge to almost close values to those given in this table.

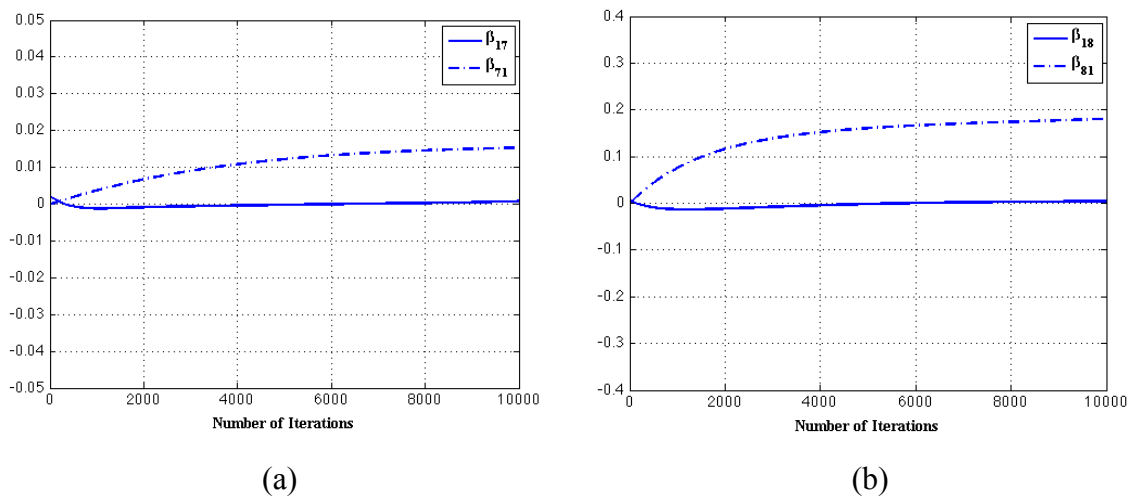
**Table 3** Converged values for fifty-six unknown  $\beta$ -parameters of the eight-state variable system

$\beta_{ij}$	Value	$\beta_{ij}$	Value	$\beta_{ij}$	Value	$\beta_{ij}$	Value
$\beta_{12}$	0.00226	$\beta_{23}$	0.00495	$\beta_{35}$	0.00676	$\beta_{48}$	0.00024
$\beta_{21}$	0.00971	$\beta_{32}$	0.00109	$\beta_{53}$	0.08901	$\beta_{84}$	0.02516
$\beta_{13}$	0.02771	$\beta_{24}$	0.00356	$\beta_{36}$	3.55e-6	$\beta_{56}$	0.00010
$\beta_{31}$	0.02508	$\beta_{42}$	0.00072	$\beta_{63}$	0.03664	$\beta_{65}$	0.00385
$\beta_{14}$	0.00944	$\beta_{25}$	0.00046	$\beta_{37}$	4.35e-5	$\beta_{57}$	0.00027
$\beta_{41}$	0.03931	$\beta_{52}$	0.00176	$\beta_{73}$	0.00750	$\beta_{75}$	0.00078
$\beta_{15}$	0.00460	$\beta_{26}$	9.28e-5	$\beta_{38}$	3.20e-5	$\beta_{58}$	0.00093
$\beta_{51}$	0.14887	$\beta_{62}$	0.00238	$\beta_{83}$	0.08850	$\beta_{85}$	0.00931
$\beta_{16}$	0.00089	$\beta_{27}$	1.20e-5	$\beta_{45}$	0.00202	$\beta_{67}$	0.00063
$\beta_{61}$	0.07470	$\beta_{72}$	0.00038	$\beta_{54}$	0.02075	$\beta_{76}$	1.07e-5
$\beta_{17}$	0.00073	$\beta_{28}$	3.53e-5	$\beta_{46}$	2.75e-5	$\beta_{68}$	0.00046
$\beta_{71}$	0.01529	$\beta_{82}$	0.00735	$\beta_{64}$	0.01041	$\beta_{86}$	0.00012
$\beta_{18}$	0.00407	$\beta_{34}$	0.00070	$\beta_{47}$	0.00033	$\beta_{78}$	0.00009
$\beta_{81}$	0.18045	$\beta_{43}$	0.01928	$\beta_{74}$	0.00213	$\beta_{87}$	0.00054

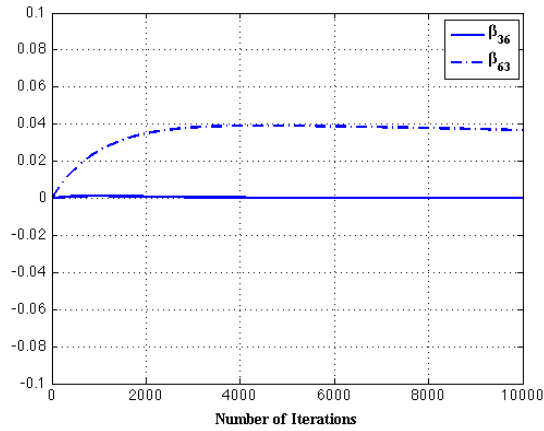
Figure 11 to Figure 15 are samples of the converging trends for the parameters  $\beta$ , demonstrating that 10,000 iterations assure a good convergence to the specific values. The response of the modeled system using these values, quite close to the historical data and the minimized identification error given in Figure 16 with the final value of 0.0037504, is the main proof for this claim.



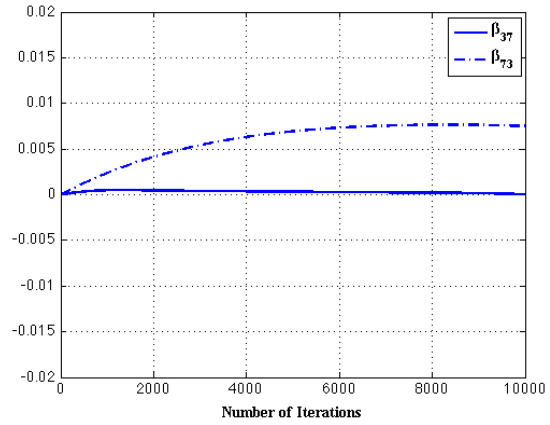
**Figure 11** Eight-State System, Converging trajectory after 10,000 iterations for (a)  $\beta_{13}$  and  $\beta_{31}$ , (b)  $\beta_{16}$  and  $\beta_{61}$



**Figure 12** Eight-State System, Converging trajectory after 10,000 iterations for (a)  $\beta_{17}$  and  $\beta_{71}$ , (b)  $\beta_{18}$  and  $\beta_{81}$

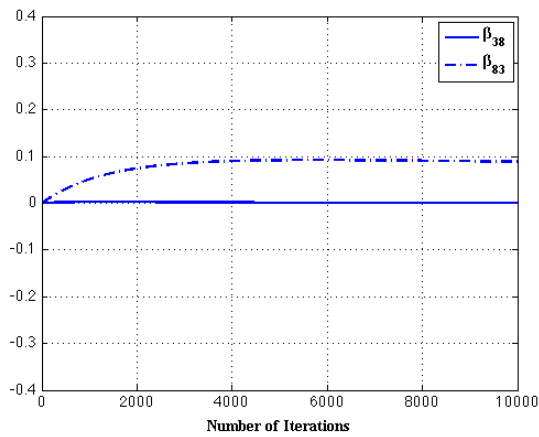


(a)

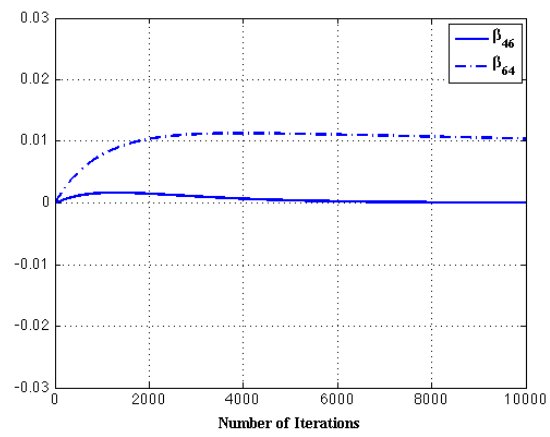


(b)

**Figure 13** Eight-State System, Converging trajectory after 10,000 iterations for (a)  $\beta_{36}$  and  $\beta_{63}$ , (b)  $\beta_{37}$  and  $\beta_{73}$

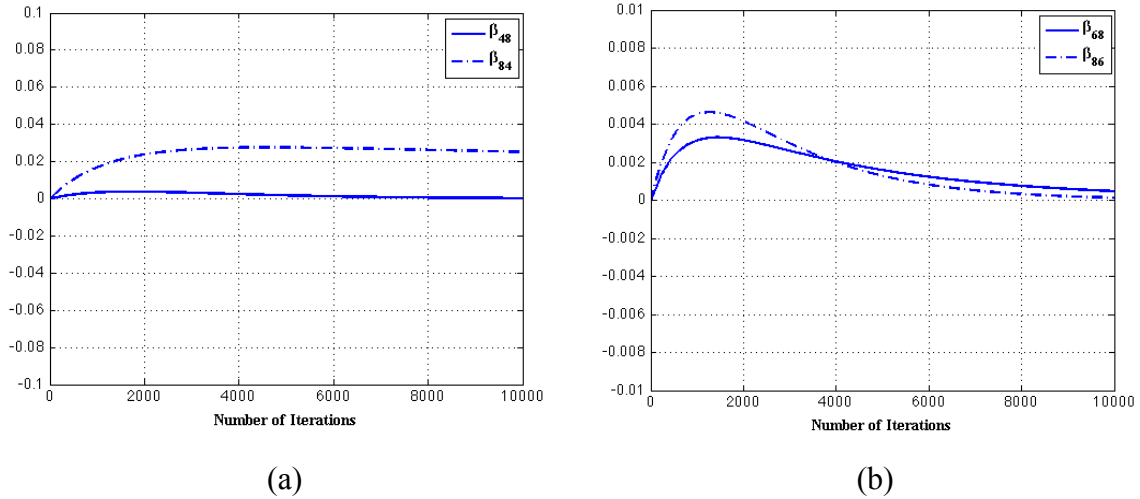


(a)

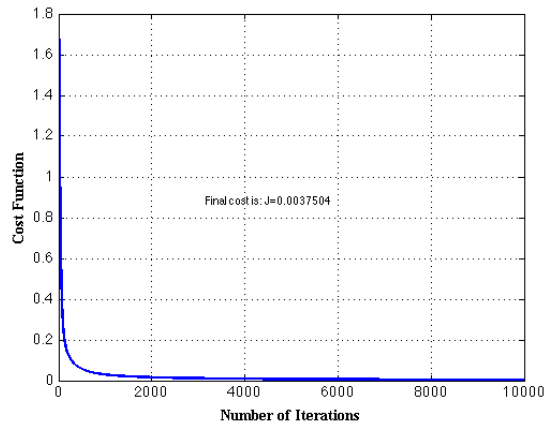


(b)

**Figure 14** Eight-State System, Converging trajectory after 10,000 iterations for (a)  $\beta_{38}$  and  $\beta_{83}$ , (b)  $\beta_{46}$  and  $\beta_{64}$



**Figure 15** Eight-State System, Converging trajectory after 10,000 iterations for (a)  $\beta_{48}$  and  $\beta_{84}$ , (b)  $\beta_{68}$  and  $\beta_{86}$

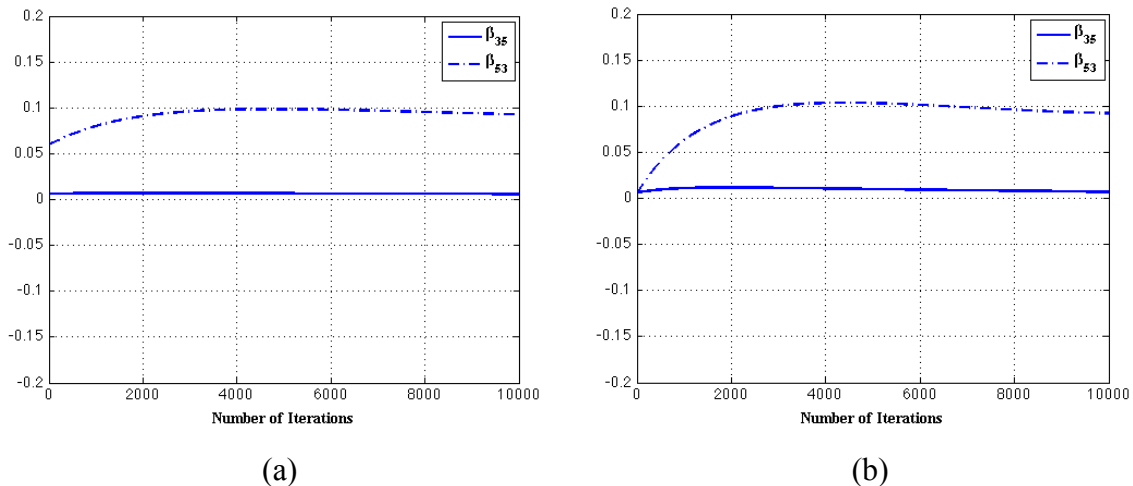


**Figure 16** Eight-State System, Identification error (equation 3.23) after 10,000 iterations with final value of 0.0037504

The minimized identification error given in Figure 16 verifies that if we substitute the  $\beta$ -parameters given in Table 3 in expression (4.1), the modeled system can adequately fit the actual trend for the level of electricity generation during the plan period of 2010 to 2015.

To validate the above calculated results for the parameters  $\beta$ , we present two different scenarios:

- Scenario I: Consider a fact that the interactions between two competitive sources depend on the authority and the power of each of the sources. In our case, we expect that the dominant energy sources have more influence on other energy sources and any changes in their capacity should have more impact on the other sources. According to Table 1-A, at year 2010, the level of electricity generated from coal energy source is equal to 1.831 Trillion kWh, while this amount is only 0.00128 Trillion kWh for solar energy source. Therefore, as we expected the impact of coal on solar ( $\beta_{61} = 0.07470$ ) is much greater than the impact of solar on coal ( $\beta_{16} = 0.00089$ ). The same algorithm is valid for all other similar cases.
- Scenario II: We expect that the converged values for the  $\beta$ -parameters should stay at approximately the same value if we change the initial values. We successfully performed this test as shown in the sample Figure 17. If the initial value for  $\beta_{53}$  changes from 0.06 to 0.005, the final converged value will change from 0.09180 to 0.08901.



**Figure 17** Eight-State System, Verifying the converging trends of  $\beta$ -parameters with different initial values

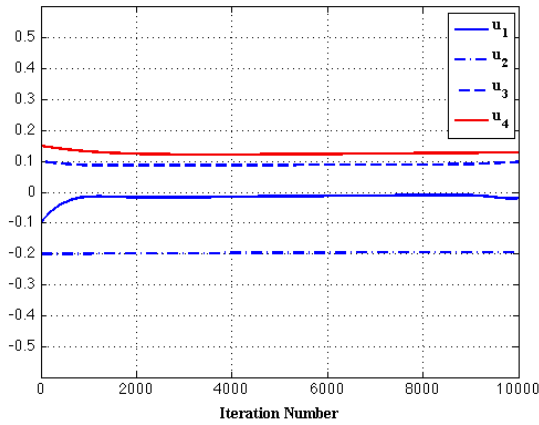
In order to have more precise values for our main unknown  $\beta$ , instead of assuming constant values for the parameters  $u_i$  ( $i = 1$  to  $8$ ) in expression (3.1), we also added them to the identification procedure and identified all eight  $u$  parameters with the same procedure used for the  $\beta$ -parameters.

The results are shown in Table 4. As expected, for the first two energy sources of coal and petroleum, the parameter  $u$  is negative due to the fact that according to the historical data given in Table 1-A, the level of electricity generated from only these two energy sources will decrease during the plan period 2010 to 2015. This amount is indeed decreasing from 1.831 to 1.562 Trillion kWh for coal and from 0.034 to 0.026 Trillion kWh for petroleum. In addition to that, the highest value of parameter  $u$  according to Table 4 is for solar energy source with the value of 0.959 and this is because the greatest increase is for this energy source based on data given in Table 1-A.

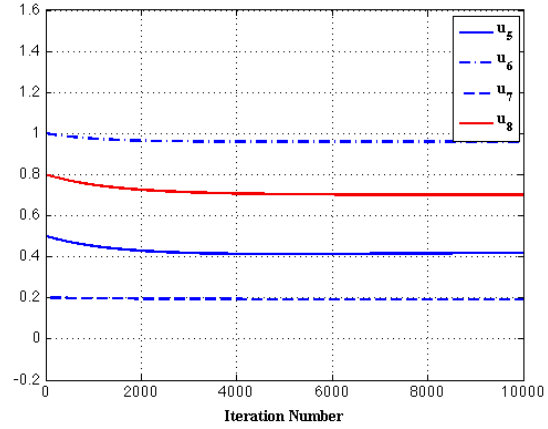
While Figure 18 shows the converging trends for all eight parameters of  $u$ , Figure 19 demonstrates the way that the sample  $u$  parameters converge to their desired values after 10,000 iterations from a closer view.

**Table 4** Calculated values for parameters of  $u$  after 10,000 iterations

$U_i$	Value	$U_i$	Value
$U_1$	-0.019	$U_5$	0.418
$U_2$	-0.194	$U_6$	0.959
$U_3$	0.097	$U_7$	0.191
$U_4$	0.128	$U_8$	0.701

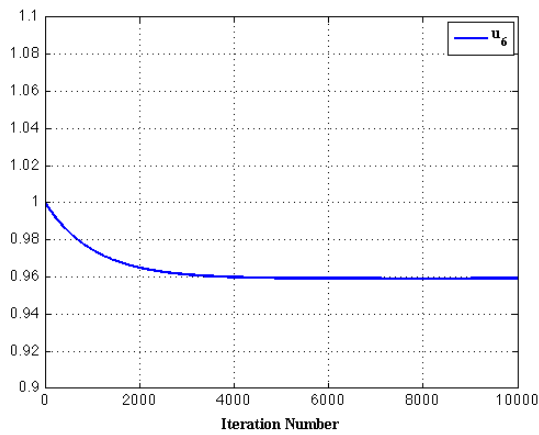


(a)

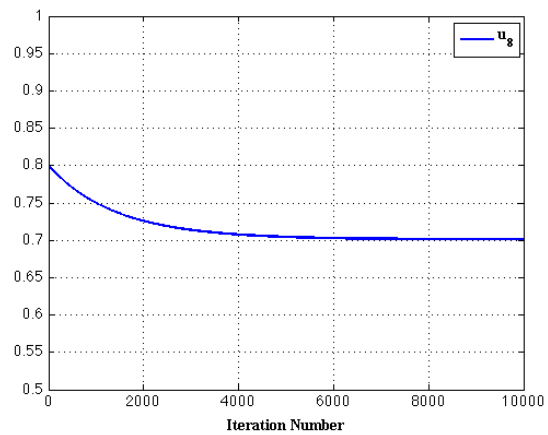


(b)

**Figure 18** Eight-State System, Converging trajectory after 10,000 iterations for (a)  $u_1$ ,  $u_2$ ,  $u_3$ ,  $u_4$  and (b)  $u_5$ ,  $u_6$ ,  $u_7$ ,  $u_8$



(a)



(b)

**Figure 19** Eight-State System, Converging trajectories after 10,000 iterations for (a)  $u_6$  and (b)  $u_8$

## 5.4. Summary

In this chapter, we presented the system identification for three different scenarios of two-state variable system, three-state variable system and eight-state variable system. The first scenario was used to validate our code with the aim of results given in [6].

The second scenario was the initial extension of the existing two-state variable system to three-state variable system, while the latter contains all existing energy sources in the eight-state variable system, i.e., the objective of the present work.

All the identified values for  $\beta$ -parameters for the above scenarios will be used in the next chapter to find the optimum decision policies in different energy problems.

## Chapter 6. Optimum Decision Policy

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This chapter includes the foremost goal of this thesis, which is finding the optimum decision policy for integration of renewable and clean energy sources into an existing electricity system from different aspects of energy problems. In the last chapter, we identified the unknown parameters to consider in the different systems. We started with a two-state time-variable model from [6] and identified the two unknown parameters  $\beta_{12}$  and  $\beta_{21}$  in expression (5.1). Then, we identified the corresponding unknown parameters of the same model using real U.S. official released data [48]. The next step was to extend the procedure to a three-state time-variable model to further demonstrate our approach, to finally setup the targeted eight-state time-variable system.

Despite the previous chapter, which was divided into sections based on the number of state variables, this chapter is divided according to different energy problems we considered.

The first problem we will introduce in this chapter is “meeting the target”, namely (P1). Minimizing the difference between the response of the modeled system and the desired values for each of the energy sources at the end of the plan period is the only concern of this problem.

“Meeting the target while minimizing the implementation cost (investment cost)”, namely (P2), will be the second problem presented in this chapter. (P2) is responsible for satisfying (P1) as well as taking into account the implementation cost.

Satisfying problems (P1) and (P2), as well as reducing the level of electricity generation from polluted energy sources is the foremost goal of the third problem, (P3). Finally, we will introduce the last problem, namely (P4), which in addition to minimizing the implementation and environmental costs, also satisfies the electricity demand during the whole plan period.

For the state variables, the initial values (year 2015) for the model given in [6] are assumed as  $x_1(0) = 0.2$  and  $x_2(0) = 0.8$  while all other models are based on Table 2-A.

The initial values for all costate variables are considered according to expression (3.7). For the model given in [6],  $x_1(t_f) = 0.8$  and  $x_2(t_f) = 0.2$  and for all other models, data given in Table 2-A for year 2035 are chosen as the desired values. For all models, the acceptable tolerance is set to  $\delta = 0.005$  (equations (3.19)).

## 6.1. (P1) Meeting the Target

The general cost function we considered in our approach can be expressed as

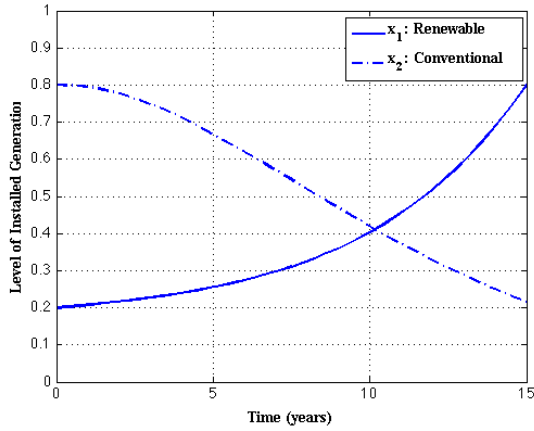
$$J(u) = \Phi(x(t_f)) = \sum_{i=1}^n \delta_i (x_i(t_f) - x_i^d)^2 \quad (6.1)$$

where  $\Phi(x(t_f))$  is the terminal cost part of the main cost functional given in equation (3.2),  $t_f$  is the final year of plan (i.e., 2035),  $n$  is the number of states for the modeled system,  $\delta_i$  is the weight given to the  $i^{\text{th}}$  energy source based on its priority to meet the target from the planner's point of view,  $x_i(t_f)$  is the response of the  $i^{\text{th}}$  energy source from the model at the end of the plan period, and  $x_i^d$  is its desired value based on our projection. The objective of our problem is to find an optimal policy  $u^*$  for the system (4.1) that minimizes the cost function given in (6.1).

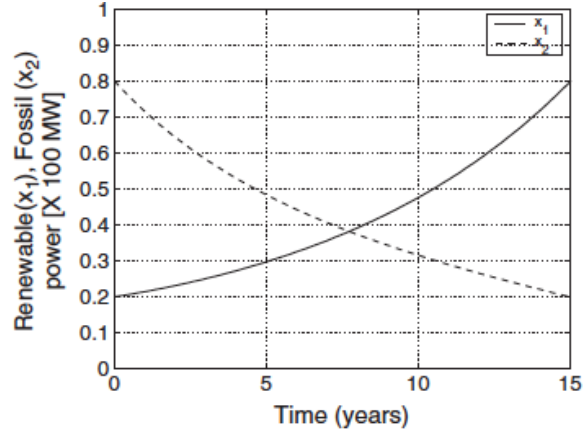
### 6.1.1 Two-State Variable System

Let us consider the two-state time-variable system introduced in expression (5.1). In the previous chapter, we determined the unknown parameters as  $\beta_{12}=0.18$  and  $\beta_{21}=0.05$ .

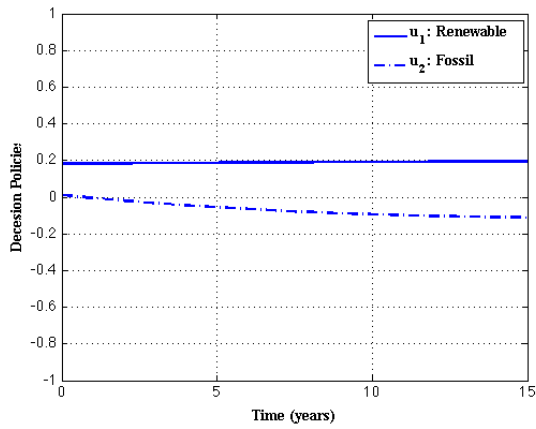
Based on [6], the weighting values have been set to  $\delta_1 = 20$  and  $\delta_2 = 5$ . The step-size is assumed as 0.00005. For different plan periods of 15 and 20 years, we found the optimum decision policy and validated our code by comparing our results to those published in [6]. The optimal path of generation level for each of the renewable and conventional power sources is shown in Figure 20-a and Figure 20-b. The optimal decision policy corresponding to each of the power sources is shown in Figure 20-c and Figure 20-d while the convergence of the cost function is demonstrated in Figure 20-e and Figure 20-f.



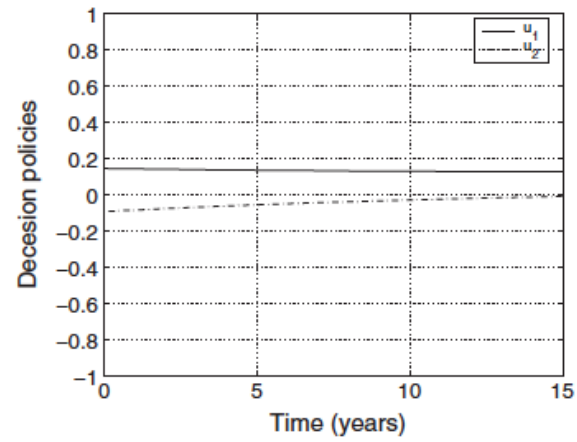
(a)



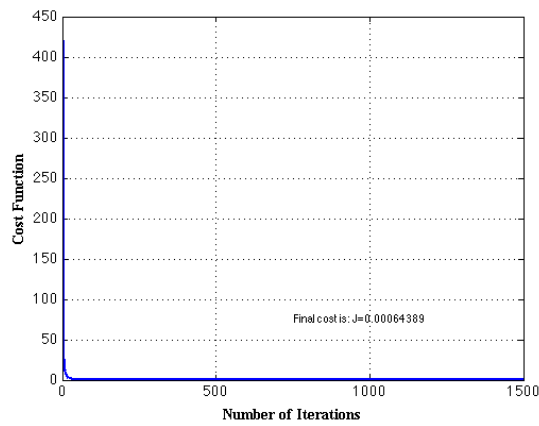
(b)



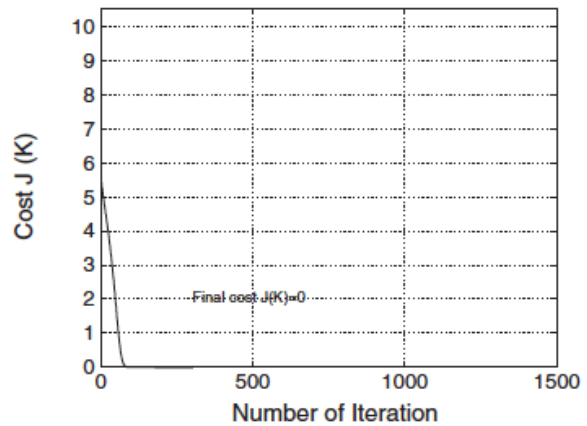
(c)



(d)



(e)



(f)

**Figure 20** (P1) Two-State system, 15 years plan period. Our model: (a) state trajectories, (c) decision policies, (e) cost function. Model given in [6]: (b) state trajectories, (d) decision policies, and (f) cost function

The above figures show slight differences between our results and the results given in [6]. This was expected because we were not able to use the exact same input data as in [6] due a lack of information in their paper. In particular, Miah M.S., Ahmed N.U., Chowdhury M. [6] did not mention some key values in their paper such as the step-size, the acceptable tolerance and the  $\beta$ -parameters they used.

So, because the values of the step-size and the acceptable tolerance can significantly affect the accuracy of the system, and the values of  $\beta_{12}$  and  $\beta_{21}$  can affect the state trajectories, it is expected to do not reach the same exact responses. However, with a final value of “0.00064389”, we can conclude that our code is working and reliable.

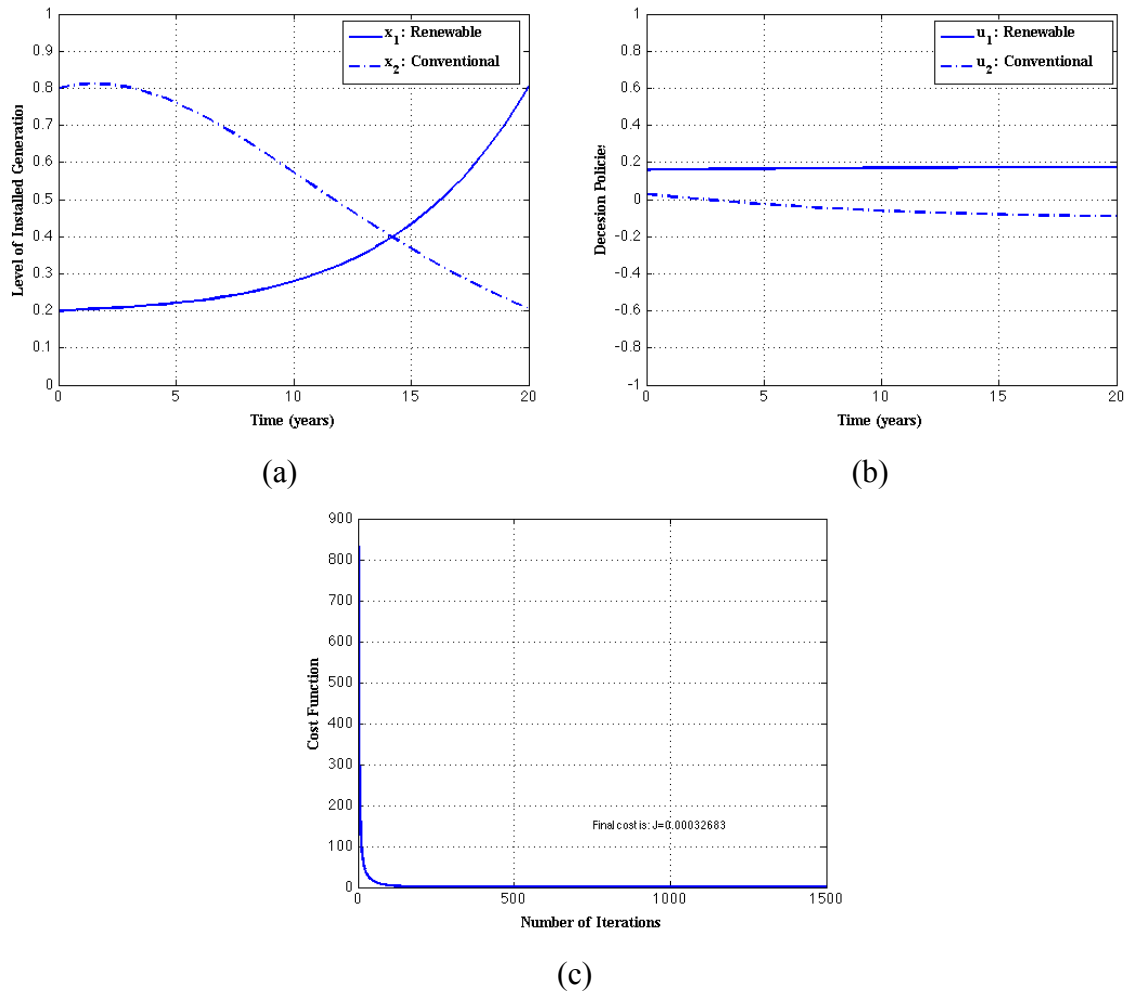
Furthermore, we reached the value of 0.8 for  $x_1(t)$  from 0.2 and vice versa for  $x_2(t)$  during the plan period of fifteen years, thus fully satisfied the targeted objective.

The decreasing trend for the conventional power sources ( $x_1(t)$ ) will be satisfied only through the negative decision policy during the plan period. As discussed in chapter 4, we restricted the control variables of  $u_1$  and  $u_2$  to the closed bounded interval  $[-1, 1]$  where 1 means a hundred percent increase and -1 is a hundred percent decrease. Figure 20-c shows details of the decision policies for each of the renewable and conventional power sources during each time interval of the plan period. Decision policy for renewable power source is constantly positive and this is due to the fact that this power source has increasing trend (from 0.2 to 0.8) during the fifteen years and the decision policy for conventional power source is negative due to the decreasing trend of it (from 0.8 to 0.2). The non-increasing trend of the cost function equation (6.1) is shown in Figure 20-e, highlighting reliable results.

The results for the same problem with different plan period (20 years) are also presented in Figure 21. Figure 21-a shows the state trajectories for renewable and conventional power sources during the plan period of twenty years. After a slight increase in the level of electricity production from conventional power sources  $x_2$ , it starts decreasing and reaches the final value of 0.2 at the end of the plan period, as expected. Figure 21-b demonstrates the decision policies for this case. The decision policy for the conventional power sources ( $u_2$ ) starts from a positive value for the increasing part of the level of electricity production shown in Figure 21-a until approximately year 2. The negative

value for the decision policy  $u_2$  after year 2 is the consequence of the decreasing trend of the state variable  $x_2$  between years 2 and 20.

The minimized cost functional is shown in Figure 21-c with final value of 0.00032683 after 1500 iterations. This final value for our cost function is acceptable considering the final value of 0.0001 given in [6]. It is worth to mention that after 1500 iterations the margin error is  $\varepsilon = 0.005$ .



**Figure 21** (P1) Two-State system [6], 20 years plan period, (a) state trajectories, (b) decision policies and (c) minimized cost function

After validating the proposed code for a two-state time-variable system through the results given by [6], we extended it to a three-state system.

### 6.1.2 Three-State Variable System

Let us consider now the model for three-state variables (equation (5.3)). In the previous chapter, we identified all six unknown parameters, i.e.,  $\beta_{12}$ ,  $\beta_{21}$ ,  $\beta_{13}$ ,  $\beta_{31}$ ,  $\beta_{23}$ , and  $\beta_{32}$ .

In this problem, the step size is assumed 0.00005 and the weighting values in expression (6.1) are assumed as  $\delta_1=10$ ,  $\delta_2=5$ , and  $\delta_3=10$ . The planner can easily change these values according to his goals and find the new decision policies corresponding to the new weighting values.

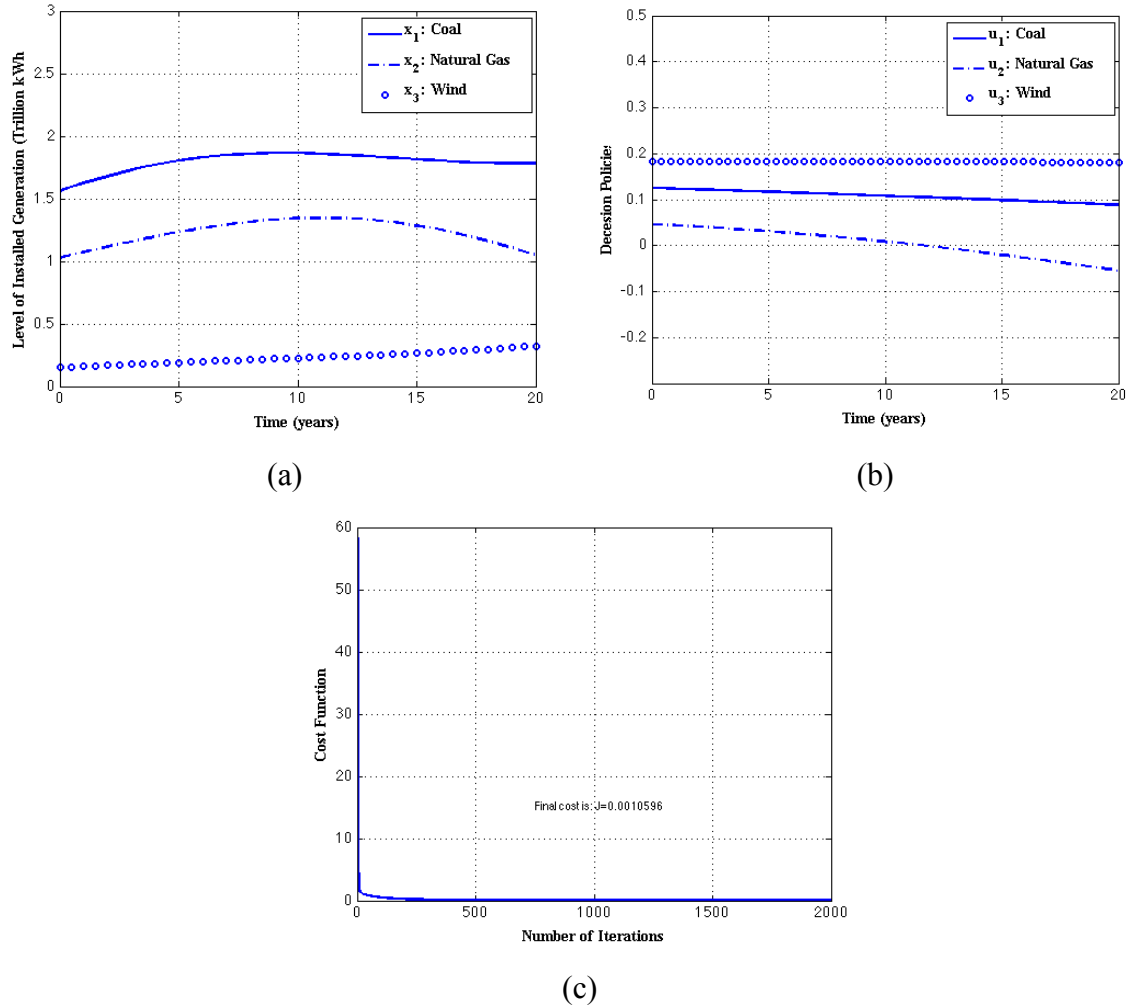
Figure 22-a shows the state trajectories for each of the energy sources of coal, natural gas, and wind, during the twenty years plan period (2015-2035). This figure shows that for coal energy source, the level of electricity generation started from 1.562 Trillion kWh in year 2015 and reached the value of 1.7829 Trillion kWh in year 2035. The desired value for this energy source according to U.S. projection [48] is 1.780 Trillion kWh, which is quite close to the value we reached. As for the level of electricity generated from natural gas energy source, the U.S. projection is a desired value of 1.037 Trillion kWh from a starting value of 1.028 Trillion kWh, while we reached 1.0465. Finally for the wind energy source, we reached a value of 0.3231 Trillion kWh, quite close to the desired one of 0.31055 Trillion kWh.

The corresponding decision policies are shown in Figure 22-b; we obtained an acceptable final value of 0.0010596 for the cost functional given in equation (6.1) after 2000 iterations (Figure 22-c).

Now, we can extend it to the targeted number of variables, i.e., an eight-state system.

### 6.1.3 Eight-State Variable System

The main goal of this thesis is finding the optimum decision policies for different energy problems for a system given in expression (4.1) and containing the eight main energy sources namely, coal, petroleum, natural gas, hydropower, wind, solar, geothermal, and biomass. This model contains fifty-six unknown  $\beta$ -parameters, identified in the previous chapter and summarized in Table 3.



**Figure 22** (P1) Three-State system, 20 years plan period, (a) state trajectories, (b) decision policies and (c) minimized cost function

After the system identification, and after defining all unknown parameters, the next step was to find the optimum decision policies to meet the desired values at the end of the plan period, for each of the eight energy sources.

The initial values and the desired values for the energy sources were considered according to the official released data of U.S. Energy Information Administration website [4] and the latest released U.S. annual projections [48] (Table 2-A).

These values are reproduced here for the convenience of the reader:

- Coal:  $x_1(0) = 1.562, x_1(t_f) = 1.780$  Trillion kWh
- Petroleum:  $x_2(0) = 0.026, x_2(t_f) = 0.028$  Trillion kWh
- Natural Gas:  $x_3(0) = 1.028, x_3(t_f) = 1.037$  Trillion kWh
- Hydropower:  $x_4(0) = 0.29543, x_4(t_f) = 0.32178$  Trillion kWh
- Wind:  $x_5(0) = 0.15097, x_5(t_f) = 0.31055$  Trillion kWh
- Solar:  $x_6(0) = 0.00647, x_6(t_f) = 0.0869$  Trillion kWh
- Geothermal:  $x_7(0) = 0.01868, x_7(t_f) = 0.05089$  Trillion kWh
- Wood and Biomass:  $x_8(0) = 0.02128, x_8(t_f) = 0.07841$  Trillion kWh

The step-size was 0.00005 and the weighting values for each of the selected energy sources were assumed as:  $\delta_1=8, \delta_2=5, \delta_3=3, \delta_4=3, \delta_5=4, \delta_6=20, \delta_7=5, \delta_8=10$ . The planner may easily change these values according to his goal/priorities.

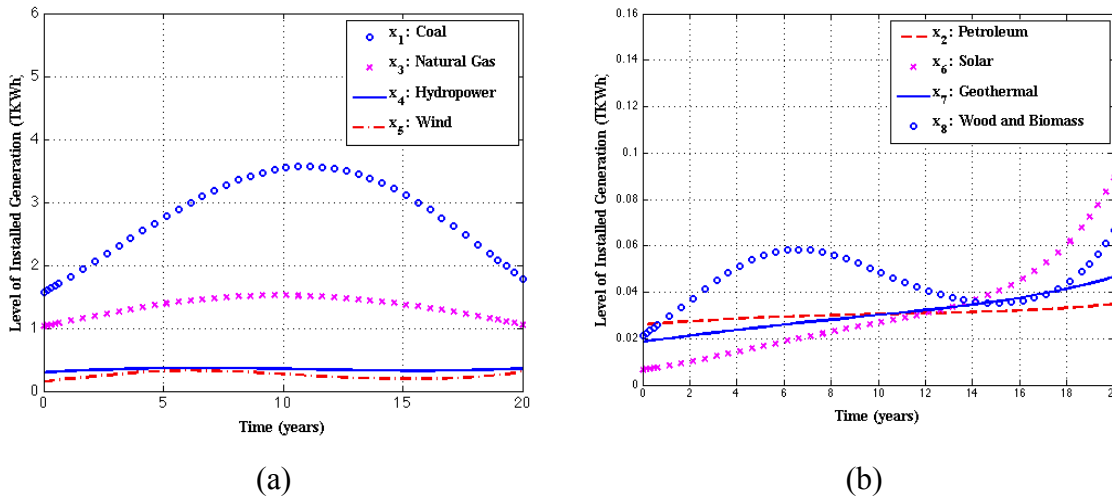
In our problem, we assumed that meeting the target for the renewable energy sources is more important for the planner rather than the conventional ones, especially for solar and biomass energy sources.

Starting from 1.562 Trillion kWh in year 2015 for the level of electricity generation from coal energy source, we reached the value of 1.7885 Trillion kWh by the year 2035 (Figure 23-a). This value is quite close to the desired value of this energy source (1.780 Trillion kWh) in the U.S. projections [48]. Starting from 0.026 and reaching the target of 0.028 Trillion kWh is the desired trend for the petroleum energy source based on [48]. As shown in Figure 23-b, we reached the value of 0.0348 Trillion kWh. For natural gas energy source, we reached the value of 1.0509 Trillion kWh while the desired value was 1.037. For hydropower energy source, we reached the value of 0.3590 while the desired value was 0.32178 Trillion kWh.

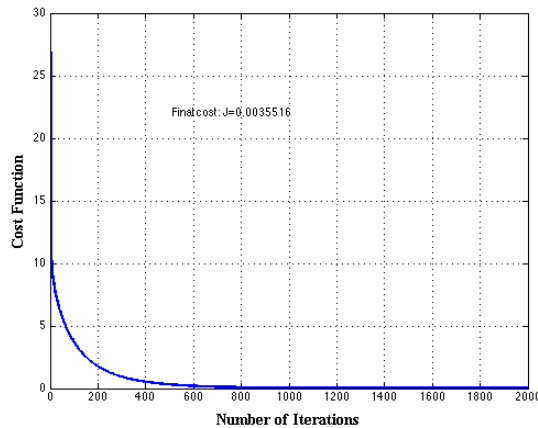
For wind energy source, at the end of the plan period the desired value was 0.31055 Trillion kWh and we reached the value of 0.3078. A desired value of 0.0869 Trillion kWh for solar energy was targeted and we reached the value of 0.0894. For geothermal energy source, the desired value by year 2035 was 0.05089 Trillion kWh and, as

shown in Figure 23-b, we reached the value of 0.0466. Finally for wood and biomass energy sources, we reached the value of 0.0669 Trillion kWh while the desired value was 0.07841.

In conclusion, for all eight selected energy sources, we reached to almost close values as expected for year 2035. The minimized trend for cost function given in equation (6.1) (when  $n=8$ ) is shown in Figure 24. After 2000 iterations the final value for this cost function is 0.0035516.

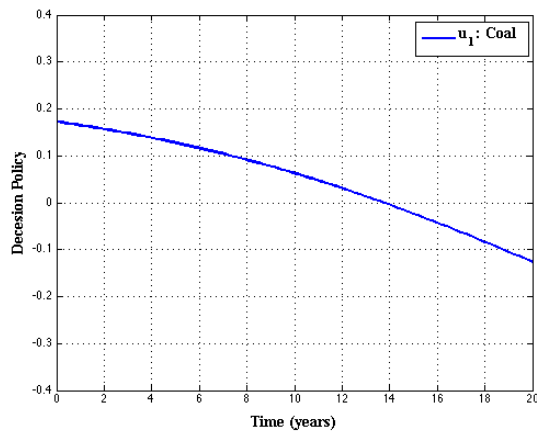


**Figure 23** (P1) Eight-State system, 20 years plan period, state trajectories for (a) coal, natural gas, hydropower, wind and (b) petroleum, solar, geothermal, wood and biomass energy sources

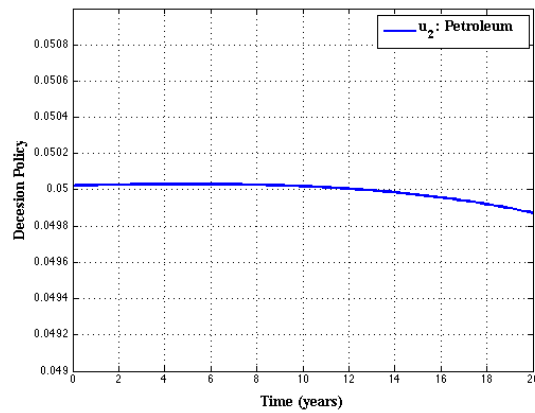


**Figure 24** (P1) Eight-State system, 20 years plan period, minimized cost function

Figure 25 shows the decision policies for each time interval during the plan period of 2015-2035 for the first two energy sources of coal and petroleum. We expected to have both positive and negative values for the decision policy regarding the coal energy source, mainly because the state trajectory for this energy source is increased at the beginning of the plan period and then decreased to reach the final desired value. This expectation is satisfied and shown in Figure 25-a.



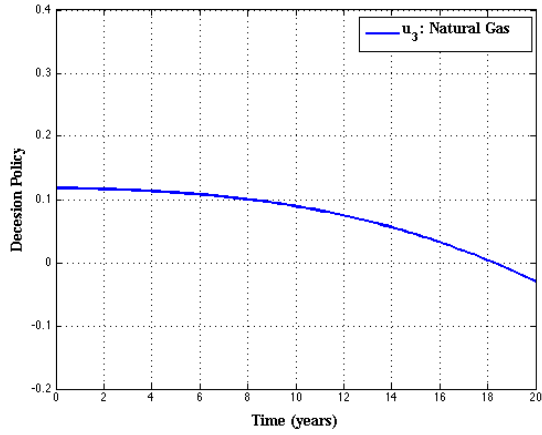
(a)



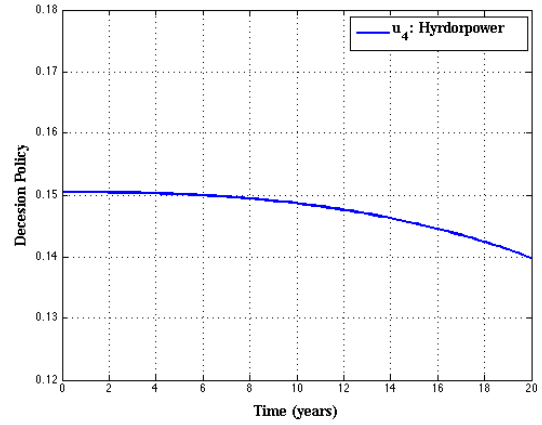
(b)

**Figure 25** (P1) Eight-State system, 20 years plan period, decision policies for (a) coal and (b) petroleum energy sources

Decision policies for natural gas, hydropower, wind, solar, geothermal and biomass energy sources are shown in Figure 26, Figure 27 and Figure 28, respectively, for each time interval of the plan period of twenty years from 2015 to 2035.

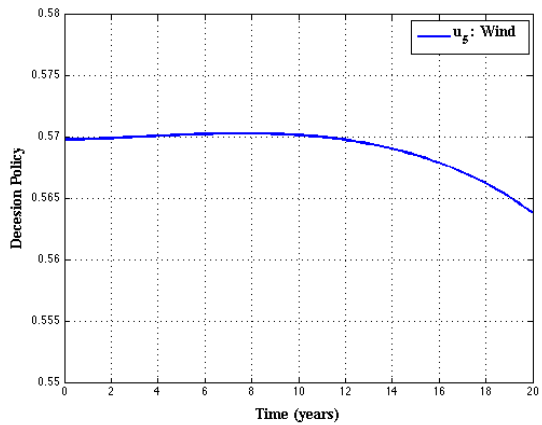


(a)

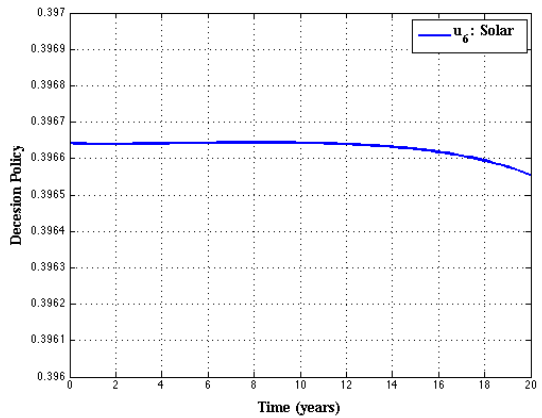


(b)

**Figure 26** (P1) Eight-State system, 20 years plan period, decision policies for (a) natural gas and (b) hydropower energy sources

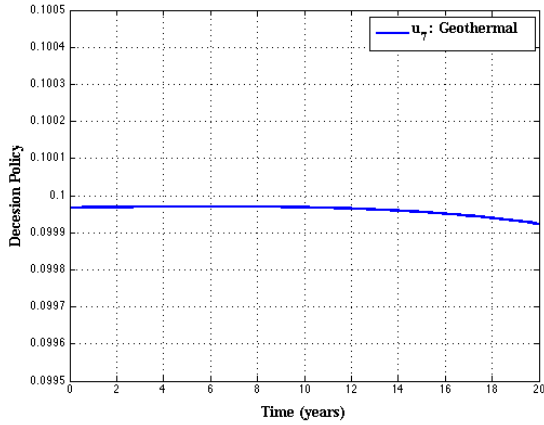


(a)

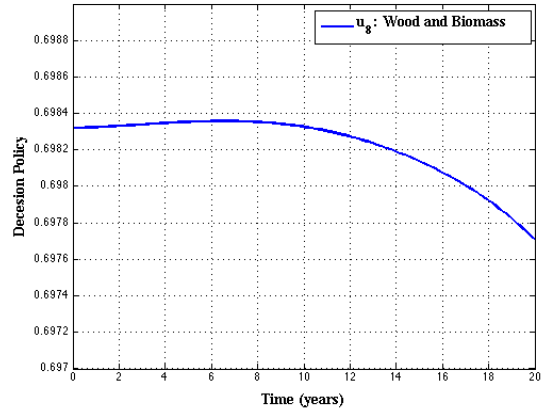


(b)

**Figure 27** (P1) Eight-State system, 20 years plan period, decision policies for (a) wind and (b) solar energy sources



(a)



(b)

**Figure 28** (P1) Eight-State system, 20 years plan period, decision policies for (a) geothermal and (b) wood and biomass energy sources

It is worth to mention that the rate of change for state trajectories of the energy sources (level of installed electricity generation) is dependent on the decision policy for each energy source and the level of installed electricity from all other energy sources. This fact is clear from the modeled system given in expression (4.1) where  $\dot{x}_1$ , for instance, not only depends on the decision policy  $u_1$  but also on the level of installed electricity generation from other seven energy sources, i.e.  $x_2$  to  $x_8$ ; similar conclusions can be applied to all other energy sources

For dominant energy sources, the effect of corresponding control policy is more prominent than for other energy sources. This can be obtained from control policy curves plotted for each of the energy sources shown in Figure 25 to Figure 28. For example, for coal and natural gas, which are dominant energy sources with higher level of installed electricity generation, the control policy term in the equation is the most effective term and the effects of other terms are negligible. Therefore, the control policy determines the trend of the state trajectories. On the other hand, for biomass energy source, which is not a dominant source, the control policy is not dominant.

## 6.2. (P2) Meeting the Target with Implementation Cost

In the previous section, we assumed that the cost of implementation of the control policy is negligible. Here, we remove this assumption and introduce the control implementation cost.

The implementation cost may involve both capital cost and variable cost. Capital cost contains the main costs of the project such as equipment, grid connections and fuels. The variable costs are mainly the operation and maintenance costs of the power plant.

### 6.2.1 Eight-State Variable System

The objective function in this problem is given by the expression below:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) \right\} dt + \sum_{i=1}^8 \delta_i (x_i(t_f) - x_i^d)^2 \quad (6.2)$$

where  $q_i \geq 0, i = 1 \dots 8$ , are suitable weights representing the unit cost of control policies. It is generally expected that the values for these weights are dependent on the source type and may be comparatively large for certain sources than others. The values of  $\{q_i\}$  and  $\{\delta_i\}$  are given in Table 5. According to what given in appendix A, the desired values of U.S. Annual Outlook 2012 [48] are based on the “Low Renewable Technology Cost Case”; therefore, here (in Table 5) we assumed that the implementation costs for conventional methods of energy generations are higher than the costs for renewable ones. The values of  $\{\delta_i\}$  are chosen exactly the same as what we had assumed in the previous problem (P1).

**Table 5** (P2) Assumed values for  $q_i$  and  $\delta_i$  in equation (6.2)

$\delta_i$	Value	$\delta_i$	Value	$q_i$	Value	$q_i$	Value
$\delta_1$	8	$\delta_5$	4	$q_1$	10	$q_5$	1
$\delta_2$	5	$\delta_6$	20	$q_2$	5	$q_6$	0.59
$\delta_3$	3	$\delta_7$	5	$q_3$	3	$q_7$	1.8
$\delta_4$	3	$\delta_8$	10	$q_4$	3	$q_8$	0.68

The step-size was assumed 0.00005. Note that reducing the step-size will reach to more accurate results, but at the same time will increase the numbers of iterations. Therefore, there should be a trade-off between accuracy, running time, and numbers of iterations to reach to the acceptable margin of error.

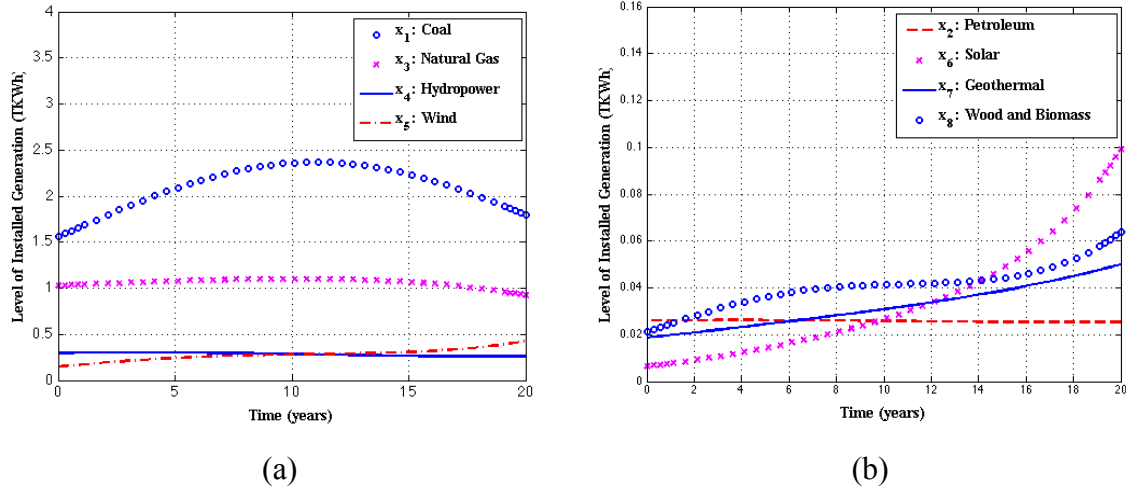
Using the methodology given in chapter 3, we found the optimum decision policies for all eight energy sources. In this problem, we try to meet the target for each of the energy sources at the end of the plan period while considering the implementation costs. The planner may easily change the weighting values according to his objectives.

Figure 29 shows the state trajectories for eight energy sources during the plan period of twenty years. As reported in Table 6, all energy sources reached values quite close to the desired ones. The converging trend of cost function (6.2) is shown in Figure 30.

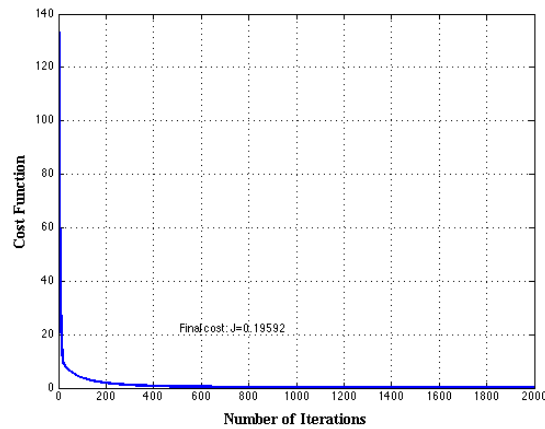
The corresponding control policies for each of the energy sources are given in Figure 31 to Figure 34. Figure 31-a shows that the decision policy for coal energy source, starts from a positive value and ends with a negative one. From this Figure, we expect that the level of electricity generation from coal energy source ( $x_1$ ) will increase in first years of the plan period and then decrease. Figure 29-a demonstrates this claim.

**Table 6** (P2) Comparison between the desired values and the reached values from our model

	Desired Values [48]	Reached Values
Coal	1.780	1.7936
Petroleum	0.028	0.0254
Natural Gas	1.037	0.9219
Conventional Hydropower	0.32178	0.2592
Wind	0.31055	0.4268
Solar	0.0869	0.0991
Geothermal	0.05089	0.0500
Wood and Biomass	0.07841	0.0640

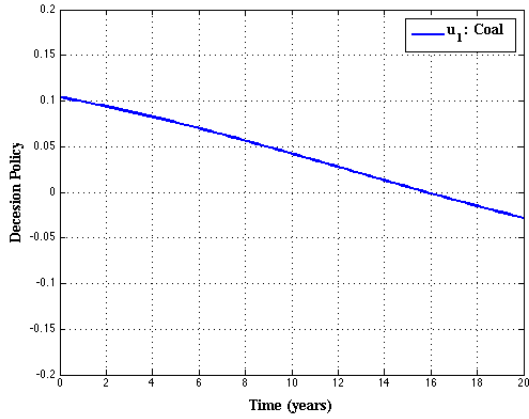


**Figure 29** (P2) Eight-State system, 20 years plan period, state trajectories for (a) coal, natural gas, hydropower, wind and (b) petroleum, solar, geothermal, wood and biomass energy sources

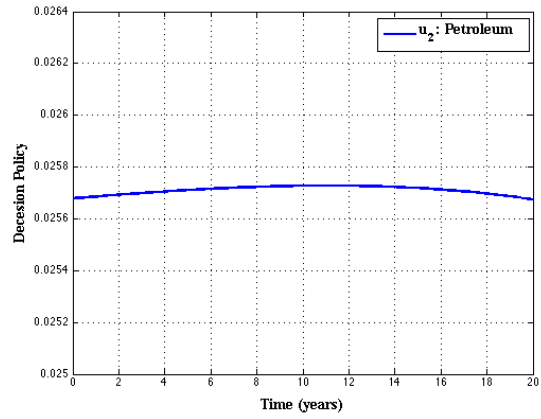


**Figure 30** (P2) Eight-State system, 20 years plan period, minimized cost function

Based on the clean future plan, the amount of change in the level of electricity generations from each of the renewable energy sources is significant. This in turn will result in a greater growth rate for renewable energy sources, which can be seen in Figure 31 to Figure 34. For example, around 45% growth rate for wind energy source for the whole plan period of 20 years with around 33% for solar and 53% for biomass energy sources.

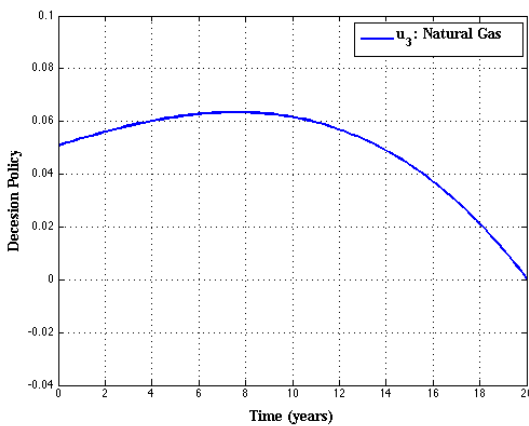


(a)

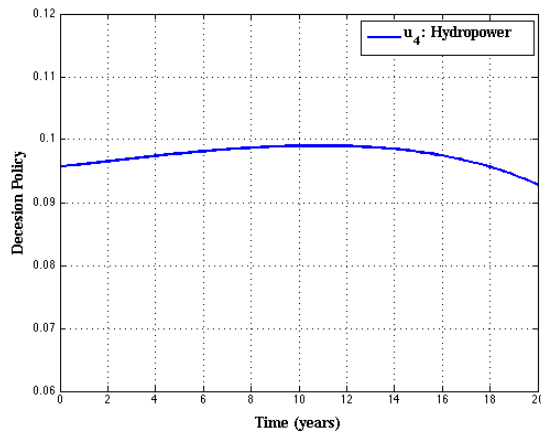


(b)

**Figure 31** (P2) Eight-State system, 20 years plan period, decision policies for (a) coal and (b) petroleum energy sources



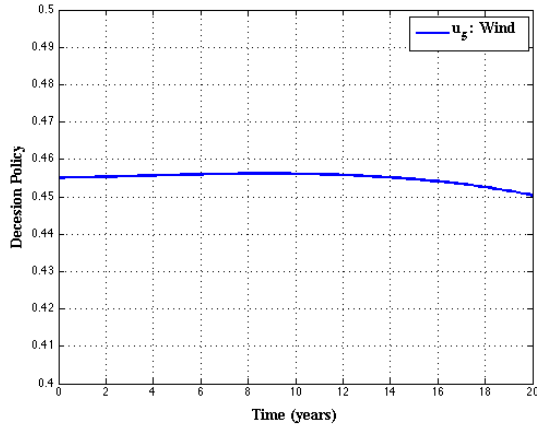
(a)



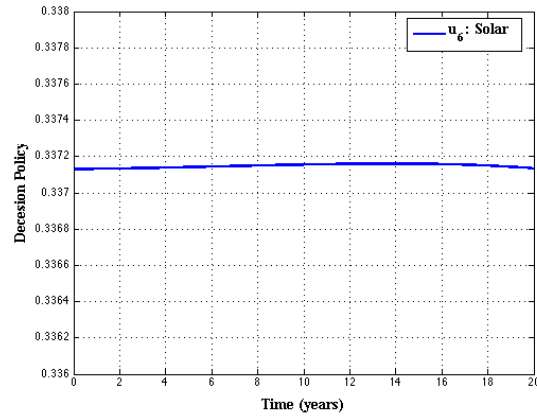
(b)

**Figure 32** (P2) Eight-State system, 20 years plan period, decision policies for (a) natural gas and (b) hydropower energy sources

These values are around 2% for petroleum, 10% for hydropower and 3% for geothermal energy sources.

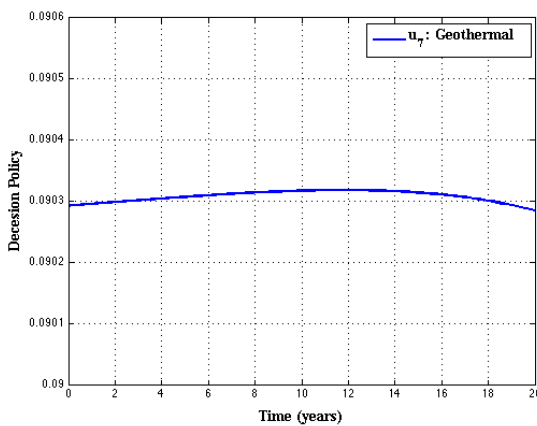


(a)

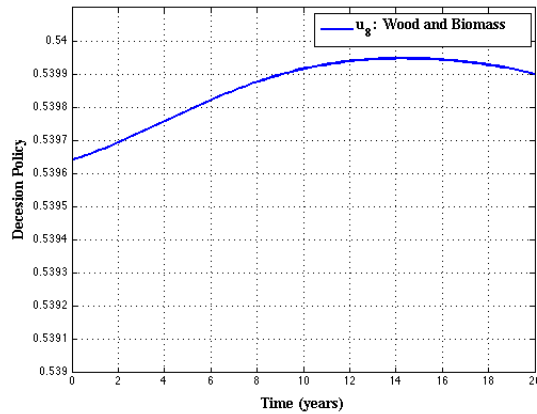


(b)

**Figure 33** (P2) Eight-State system, 20 years plan period, decision policies for (a) wind and (b) solar energy sources



(a)



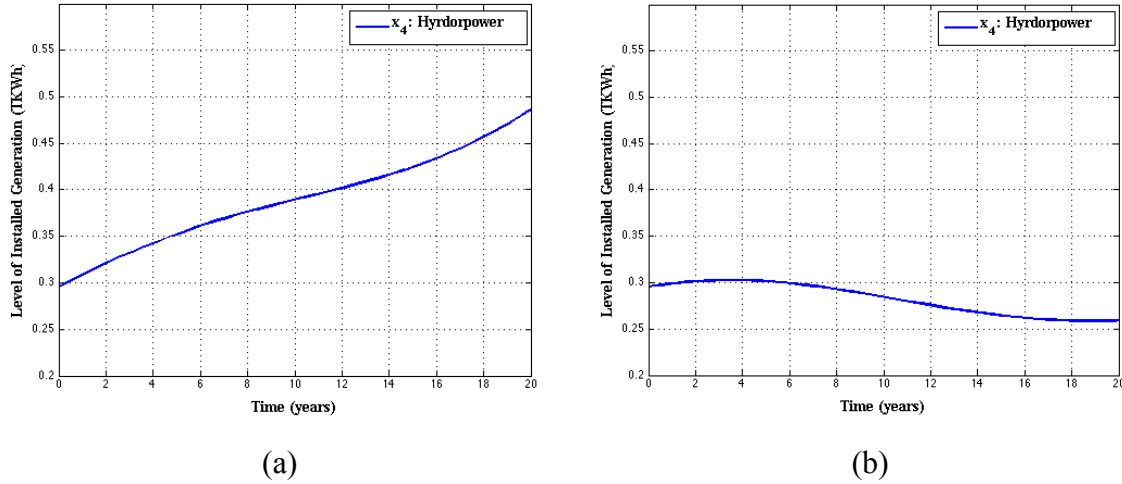
(b)

**Figure 34** (P2) Eight-State system, 20 years plan period, decision policies for (a) geothermal and (b) wood and biomass energy sources

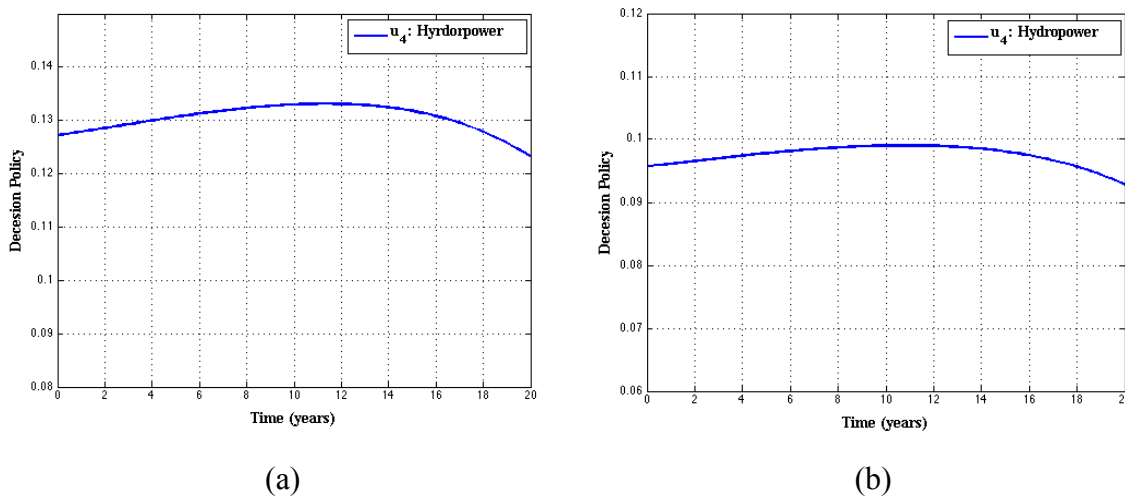
In order to validate our results, we changed some of the weighting values and observed whether the new results are compatibles with our expectations or not.

*Example 1:* reduce the weighting value of the implementation cost for hydropower energy source ( $q_4$ ), from 3 to 0.5, and keep all other weighting values of implementation cost ( $q$  in Table 5) constant: with that, we are assuming that this energy source is less

expensive than before. Therefore, we expect higher level of electricity generation from this energy source compared to the previous case. Figure 35 is a proof for this claim. In addition, higher level of electricity generation needs greater decision policy. Figure 36 shows that around 9.6% growth rate for hydropower when  $q_4=3$  is changed to around 12.8% growth rate for this energy source when  $q_4=0.5$ .

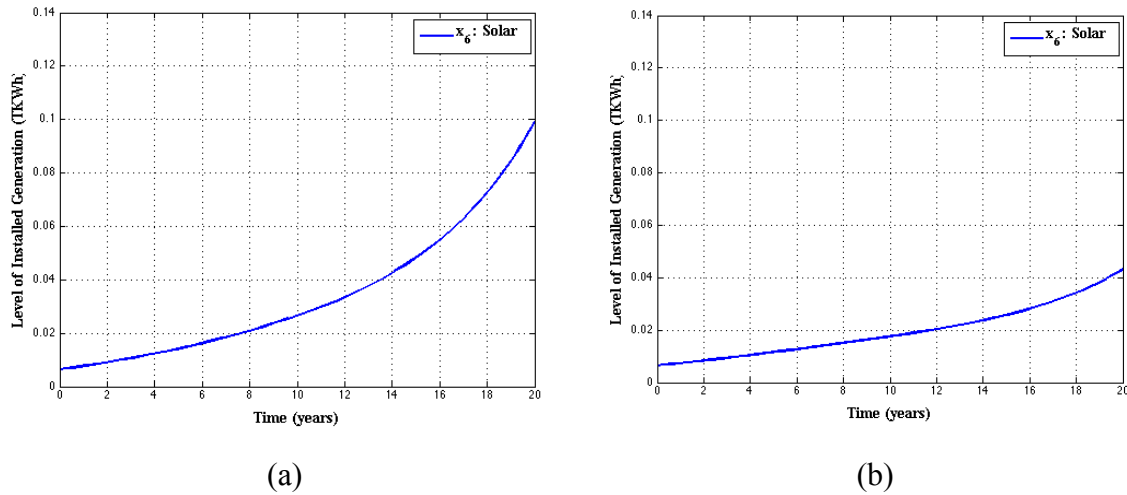


**Figure 35** (P2) Comparison between level of electricity generated from hydropower, (a) low implementation cost,  $q_4=0.5$ , (b) high implementation cost,  $q_4=3$



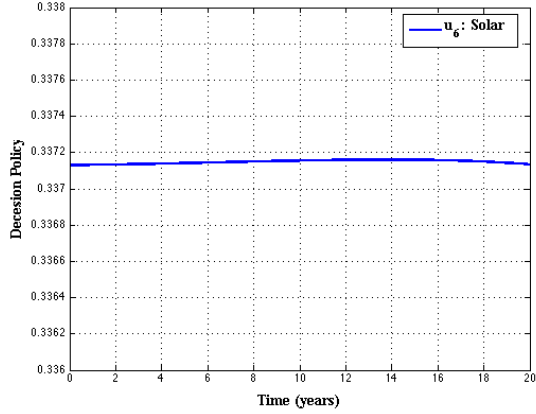
**Figure 36** (P2) Comparison between decision policies for hydropower, (a) low implementation cost,  $q_4=0.5$ , (b) high implementation cost,  $q_4=3$

*Example 2:* In this case, we assume that all given weighting values of  $q$  are constant (as given in Table 5), except  $q_6$ . If we consider higher implementation cost for solar energy source and thus, change the corresponding weighting value ( $q_6$ ) from 0.59 to 1.2, we expect lower level of electricity generation from this energy source. Figure 37 shows the level of electricity generation from solar energy source in the two cases of low and high implementation costs.

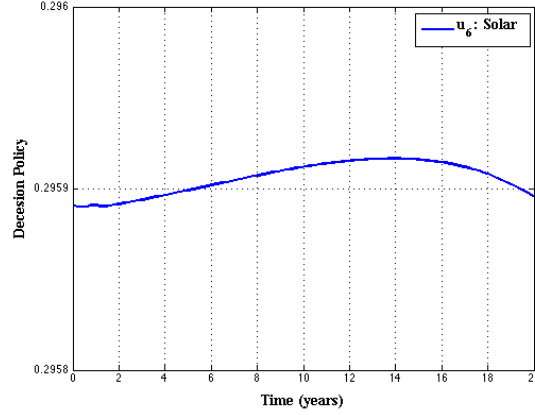


**Figure 37** (P2) Comparison between level of electricity generated from solar, (a) low implementation cost,  $q_6=0.59$ , (b) high implementation cost,  $q_6=1.2$

The corresponding decision policies are shown in Figure 38. As expected the case with lower implementation cost and higher level of electricity generation has greater decision policy (33%) compared to the second one (29%).



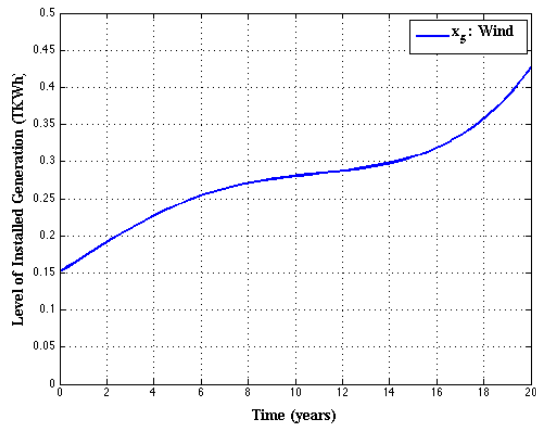
(a)



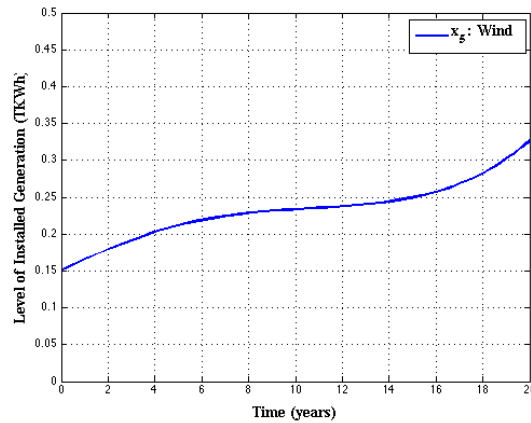
(b)

**Figure 38** (P2) Comparison between decision policies for solar, (a) low implementation cost,  $q_6=0.59$ , (b) high implementation cost,  $q_6=1.2$

*Example 3:* In this case, all weighting values for the terminal cost  $\delta$  (given in Table 5) are kept constant, except  $\delta_5$ ; by increasing  $\delta_5$ , we are increasing the importance of reaching the exact desired value for wind energy source. Table 6 shows that for wind energy source, the desired value is 0.31055. When  $\delta_5=4$ , we reached the value of 0.4268. If we increase  $\delta_5$ , we expect reaching a closer value to 0.31055. In fact, with  $\delta_5=25$ , the level of electricity generation from wind energy source will be 0.3256 (Figure 39).



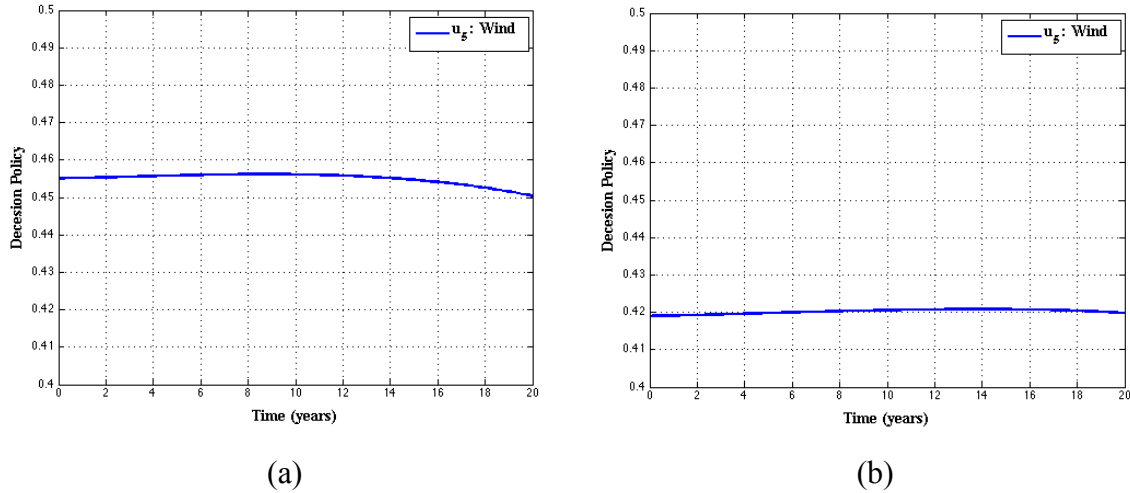
(a)



(b)

**Figure 39** (P2) Comparison between level of electricity generated from wind, (a)  $\delta_5=4$ , (b)  $\delta_5=25$

The corresponding decision policies are shown in Figure 40. The growth rate of around 45.5% for wind energy source when  $\delta_5=4$ , is changed to the growth rate of around 42% when  $\delta_5=25$ , is a proof for a claim that lower electricity production needs a lower decision policy effort.



**Figure 40** (P2) Comparison between decision policies for wind, (a)  $\delta_5=4$ , (b)  $\delta_5=25$

In (P1), we solved the energy problem for a system where meeting the desired levels for each of the energy sources at the end of the plan period is the main goal of the planner. Following that, in (P2) we added the implementation costs for each energy source and demonstrated the decision policies where minimizing the implementation costs are also important for the planner. In addition, we validated our results using different weighting values. Now, the next step is to minimize the electricity production from the polluted energy sources, which is also an important factor in planner's point of view.

### 6.3. (P3) Meeting the Target with Implementation and Environmental Costs

In this problem, we tried to minimize the terminal cost, the implementation cost and the environmental cost. The environmental cost, introduced in chapter 4, is responsible for minimizing the electricity production from polluted energy sources, i.e. coal and petroleum in our problem.

To solve this problem, we started with the simplified two-state time-variable system form of (P3) namely the “Bolza problem” discussed by [6]. Then, we extended it to three-state variables. Finally, we addressed the eight-state time variable system for the problem (P3) including all three parts of terminal cost, implementation cost and environmental cost.

### 6.3.1 Two-State Variable System

Consider the cost function expressed below [6]:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} F(x_2(t))dt + \sum_{i=1}^2 \delta_i (x_i(t_f) - x_i^d)^2 \quad (6.3)$$

The first term in equation (6.3) is responsible for the polluted conventional power sources and is discussed in equation (4.4), which is also reproduced here for the convenience of the reader:

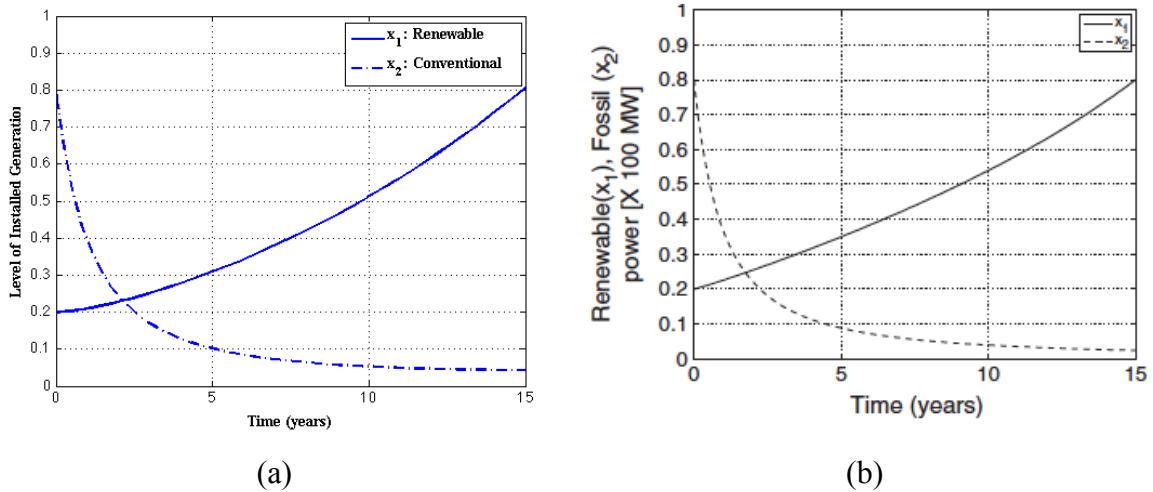
$$F(x_2(t)) = v_2 |x_2|^{q_2} \quad (6.4)$$

For this problem, Miah M.S., Ahmed N.U., Chowdhury M. [6] only considered terminal cost and environmental cost and did not consider implementation cost. Therefore, in order to validate our code, we first removed the implementation cost term to reliably compare our results with those obtained in [6]. After validation, we returned back to the original problem by adding the implementation cost.

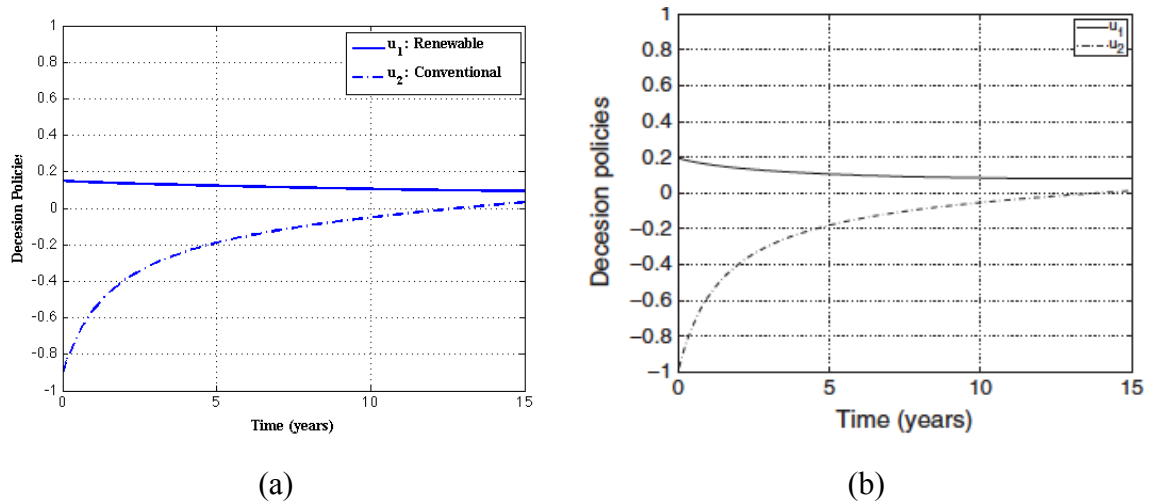
According to [6],  $v_2 = 3, q_2 = 2, \delta_1 = 20, \delta_2 = 5, x_1(0) = 0.2, x_2(0) = 0.8, x_1(t_f) = 0.8, x_2(t_f) = 0.2$ . We solved the problem for two different plan periods of 15 and 20 years and compared our results with those given in [6]. We assumed a step-size of 0.0001.

Figure 41 to Figure 43 show the state trajectories, decision policies and minimized cost function for our model during the plan period of fifteen years. The obtained results are quite compatible with those given by [6].

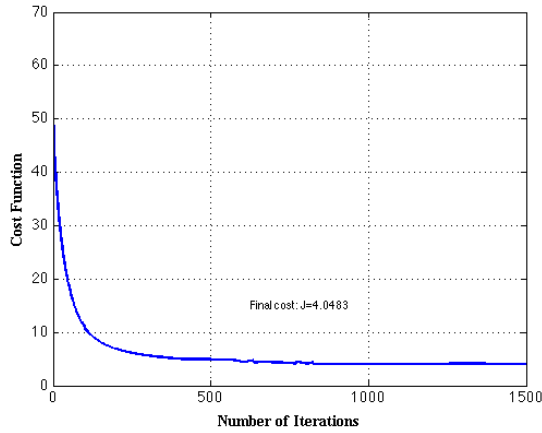
The major difference is in the final value of the cost function, possibly due to different step-sizes and acceptable tolerances. Slight differences in decision policies (Figure 42) are mainly due to different parameters  $\beta_{12}$  and  $\beta_{21}$ . We defined these parameters in the previous chapter while Miah M.S., Ahmed N.U., Chowdhury M. [6] did not mention their values in their paper.



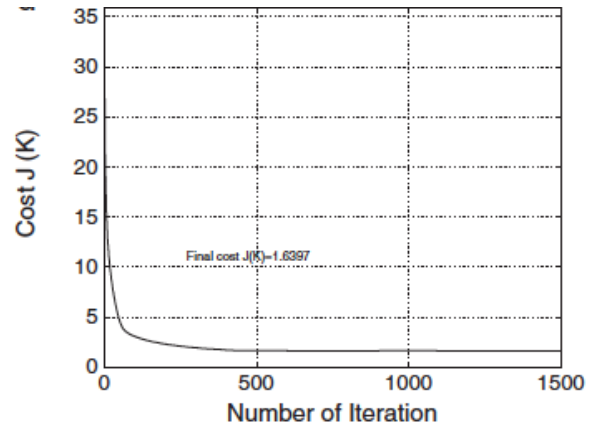
**Figure 41** (P3) Two-State system, 15 years plan period, comparison between state trajectories from (a) our model and (b) Miah M.S., Ahmed N.U., Chowdhury M. [6] model



**Figure 42** (P3) Two-State system, 15 years plan period, comparison between decision policies from (a) our model and (b) model given in [6]



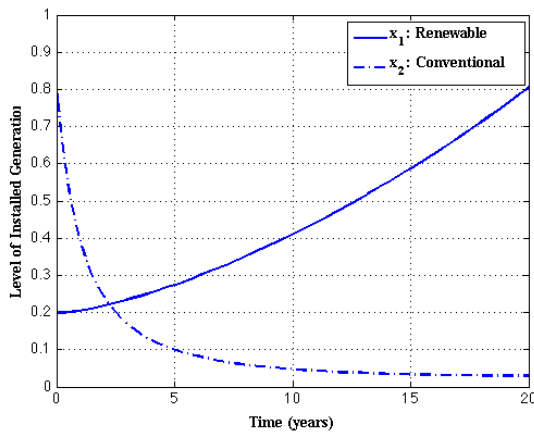
(a)



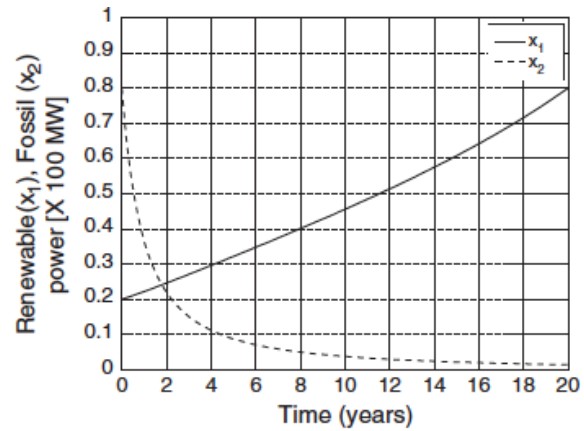
(b)

**Figure 43** (P3) Two-State system, 15 years plan period, comparison between the cost function from (a) our model and (b) model given in [6]

The state trajectories, decision policies and minimized cost function, expressed by equation (6.3) for twenty years plan period, for our model as well as the Miah M.S., Ahmed N.U., Chowdhury M. model [6] are displayed in Figure 44 to Figure 46, respectively.

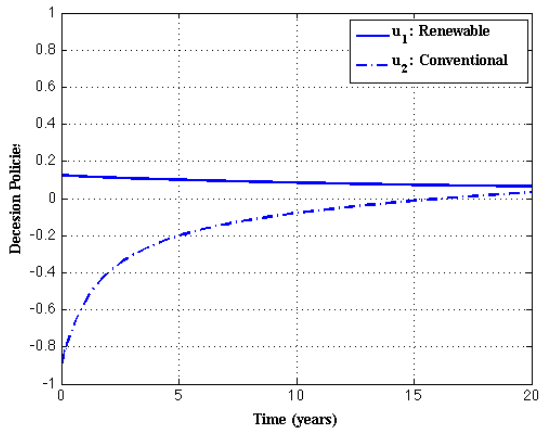


(a)

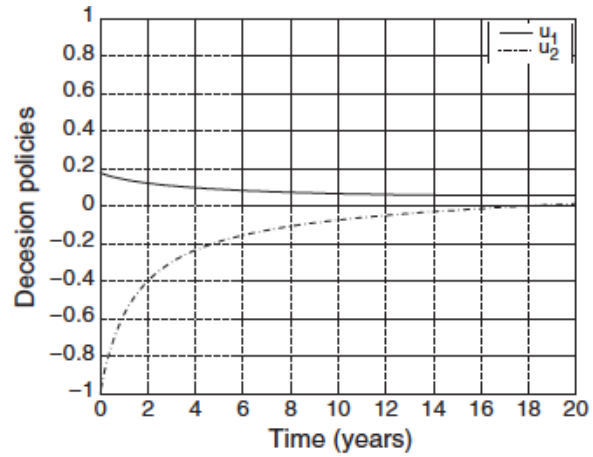


(b)

**Figure 44** (P3) Two-State system, 20 years plan period, comparison between the state trajectories from (a) our model and (b) model given in [6]

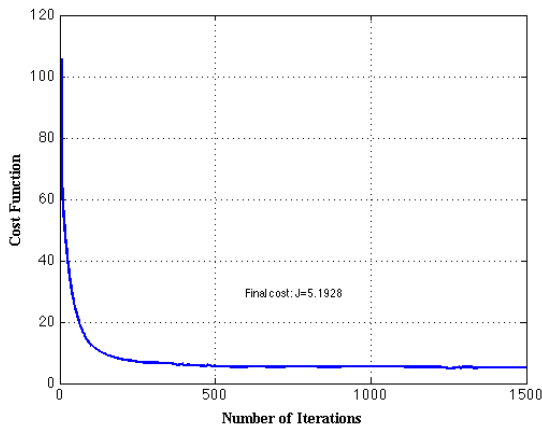


(a)

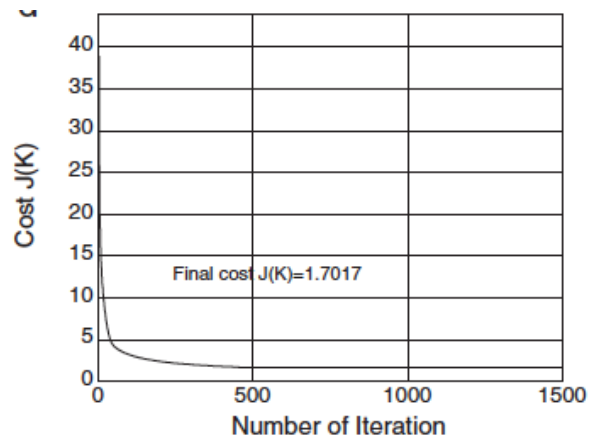


(b)

**Figure 45** (P3) Two-State system, 20 years plan period, comparison between decision policies from (a) our model and (b) model given in [6]



(a)



(b)

**Figure 46** (P3) Two-State system, 20 years plan period, comparison between the cost function from (a) our model and (b) model given in [6]

After validating our code through the results given in [6] for terminal cost and environmental cost, we extended it to a three-state time-variable system.

### 6.3.2 Three-State Variable System

Let us consider the three-state model given by expression (5.3), where all six parameters  $\beta$  have been identified in the previous chapter and calculated values given in Table 2. Here, the problem is to find the optimum decision policy for minimizing the terminal cost and the environmental cost given by

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} F(x_1(t)) dt + \sum_{i=1}^3 \delta_i (x_i(t_f) - x_i^d)^2 \quad (6.5)$$

where

$$F(x_1(t)) = v_1 |x_1|^{q_1} \quad (6.6)$$

$x_1$ ,  $x_2$  and  $x_3$  are the level of electricity generated from three main energy sources of coal, natural gas and wind, respectively. Among these three energy sources, coal is the polluted one. So, by using equation (6.6) in our cost functional, we wanted to reduce the production from this energy source to protect the environment.

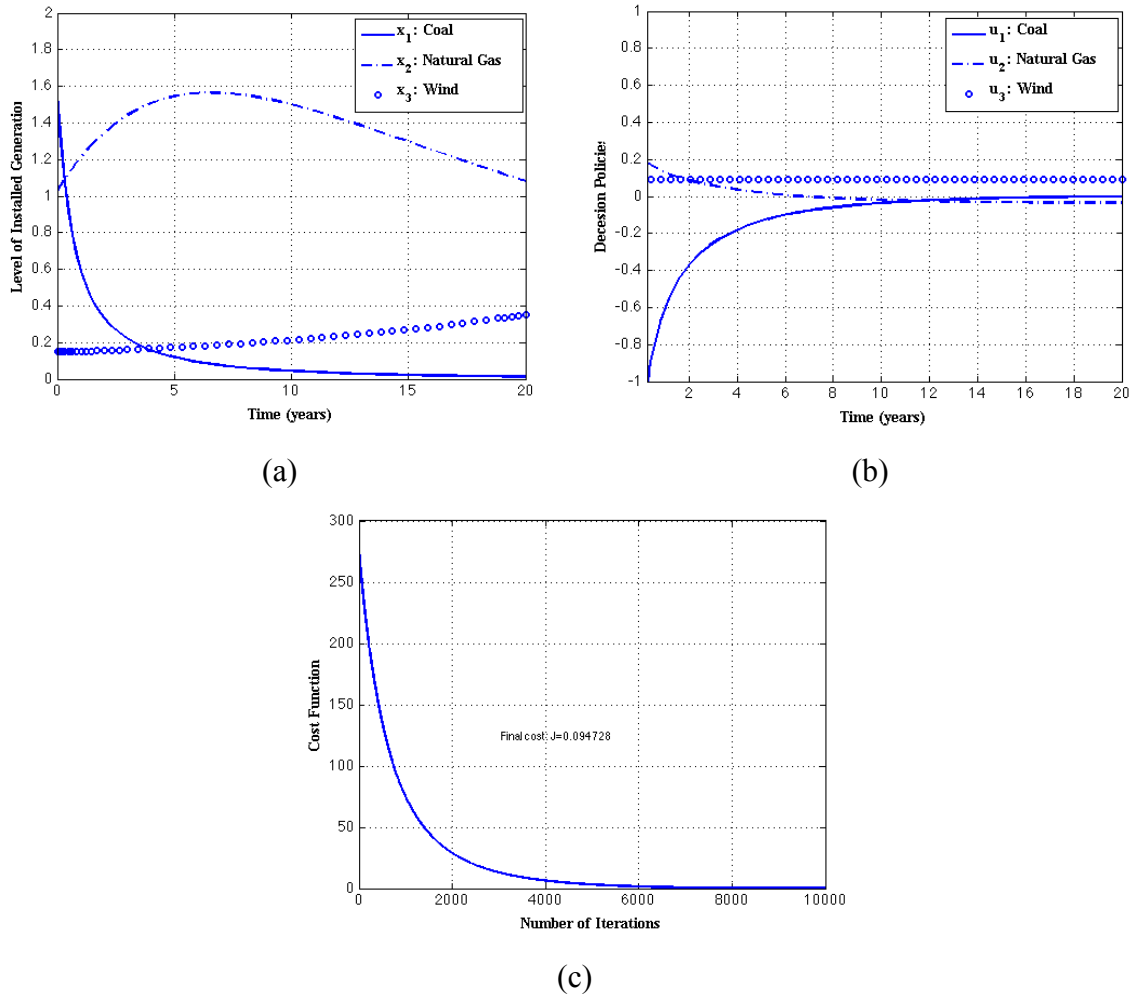
All the data used in this problem are based on the official released data of U.S. energy annual outlook 2012 [48] given in Table 2-A (appendix A), also presented here for the convenience of the reader:

- Coal energy source:  $x_1(0) = 1.562, x_1(t_f) = 1.780$  Trillion kWh
- Natural Gas energy source:  $x_2(0) = 1.028, x_2(t_f) = 1.037$  Trillion kWh
- Wind energy source:  $x_3(0) = 0.15097, x_3(t_f) = 0.31055$  Trillion kWh

where  $t_f = 20$  for the plan period between year 2015 and year 2035. The step-size is assumed as 0.0000005. In equation (6.6), we assumed that  $v_1 = 5, q_1 = 2$  and in equation (6.5),  $\delta_1 = 0, \delta_2 = 5, \delta_3 = 10$ . In this problem, minimizing the electricity generation from polluted power source of coal was our main goal; therefore, we disregarded the

desired value for this power source given in Table 2-A ( $x_1(t_f) = 1.780$ ) by selecting  $\delta_1$  equal to zero.

Figure 47 shows the state trajectories, decision policies and minimized cost function with final value of 0.094728. It is clear that for reducing the level of electricity generation from coal energy source ( $x_1$ ) (Figure 47(a)), great negative decision policy is needed Figure 47(b).



**Figure 47** (P3) Three-State system, 20 years plan period, (a) state trajectories, (b) decision policies and (c) minimized cost function

So far, we validated our code for two-state and three-state time-variable systems. In the next section, we extended our code so that we can account for eight-state time-variable system.

### 6.3.3 Eight-State Variable System

The main goal in this problem (P3) is to minimize the level of electricity production from polluted energy sources of coal and petroleum. According to U.S. official released data (given in appendix A) [48] by year 2035, the level of electricity production for coal and petroleum energy sources is projected to be 1.780 Trillion kWh and 0.028 Trillion kWh, respectively.

In previous problems, we considered the individual target level for each energy source. However, this is not feasible in this problem, as we would like to reduce electricity production from the coal and petroleum energy sources. In this way, their level of electricity production would be much lower than the projected level and the electricity demand would not be satisfied at the end of the plan period. Therefore, instead of considering the target level individually for each energy source, we considered the total electricity demand by year 2035.

In this problem (P3), we considered implementation, environmental and terminal costs. The latter should reach the total demand at the end of the plan period. One should note that we were able to consider the total demand in this problem since all eight main energy sources were taken into account in the problem. The cost function is given as

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) + \sum_{i=1}^8 v_i x_i^2(t) \right\} dt + \frac{1}{2} \Gamma (X^d(t_f) - \sum_{i=1}^8 x_i(t_f))^2 \quad (6.7)$$

where the first term represents the implementation cost while the second term represents the penalty for environmental damage due to usage of coal and petroleum. The third term represents the mismatch between the total energy demand  $X^d$  and the sum of the energies produced from each of the sources at the end of the plan (year 2035).

The values of  $\{q_i\}$  are exactly the same as what we assumed in previous problem (P2). The value of  $\{v_1\}$  is the same as what we considered above for the (P3) three-state

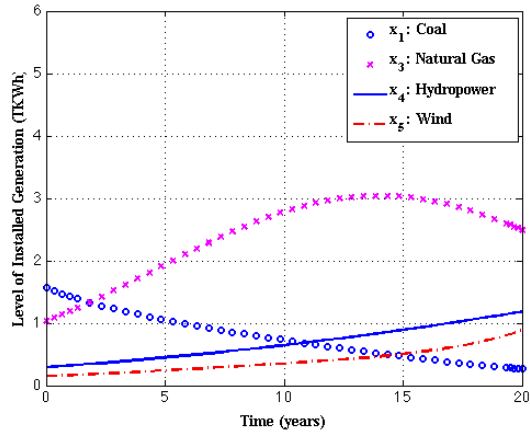
system, for coal energy source. As in chapter 4,  $\{v_2\}$  is chosen based on the relationship between the amount of GHG emissions from coal and petroleum energy sources, i.e. petroleum produces 1.23 times more CO<sub>2</sub> than coal for the same level of electric power generation [47]. Finally the value of  $\{X^d\}$  is assumed based on the official released data of U.S. [48] for the electricity demand by year 2035 (Table 7).

**Table 7** (P3) Assumed values for  $\{q_i\}$ ,  $\{v_i\}$ ,  $\{\Gamma\}$  and  $\{X^d\}$  given in equation (6.7)

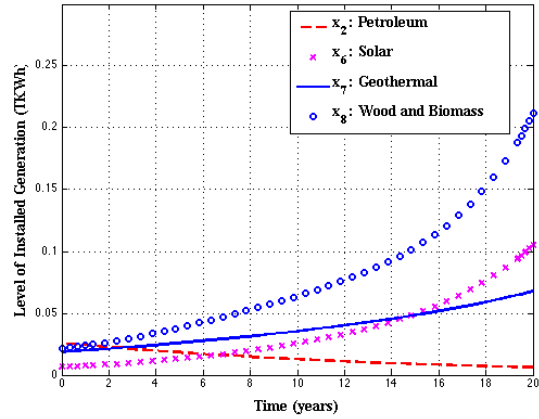
$q_i$	Value	$q_i$	Value	$v_i, \Gamma, X^d$	Value
$q_1$	10	$q_5$	1	$v_1$	5
$q_2$	5	$q_6$	0.59	$v_2$	5*1.23
$q_3$	3	$q_7$	1.8	$\Gamma$	10
$q_4$	3	$q_8$	0.68	$X^d$	4.7

The initial values for the level of electricity generation for each energy source ( $x_i$ ) are given in Table 2-A and the step-size is assumed as 0.0000005. The results are shown in Figure 48 to Figure 53.

It is clear from Figure 48 that, due to the introduction of environmental cost, the usage of polluting sources have been substantially cut down while production from clean sources such as wind, solar and biomass have increased sufficiently to meet the demand. The control policies are shown in Figure 49 to Figure 52. The convergence of the cost function expressed by equation (6.7) is shown in Figure 53-a. This minimized trend is a confirmation for minimizing the terminal demand, implementation and environmental costs, simultaneously.

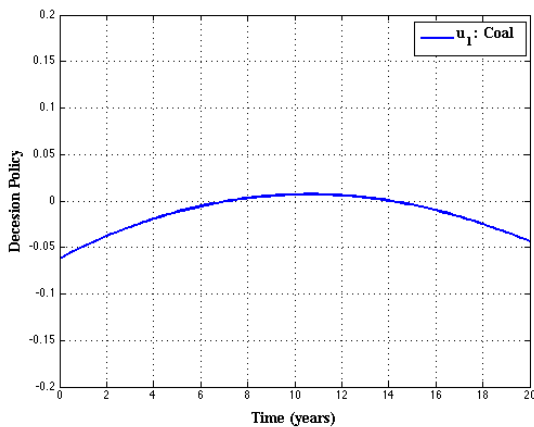


(a)

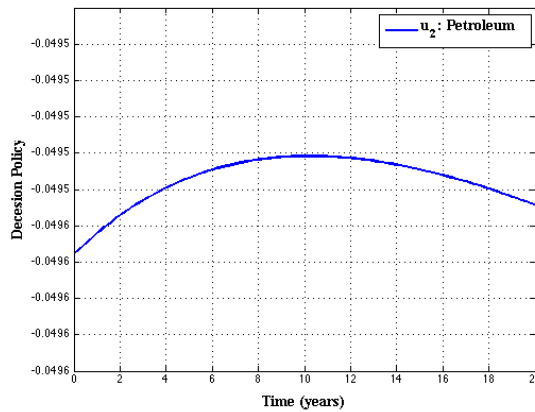


(b)

**Figure 48** (P3) Eight-State system, 20 years plan period, state trajectories for (a) coal, natural gas, hydropower, wind and (b) petroleum, solar, geothermal, wood and biomass energy sources

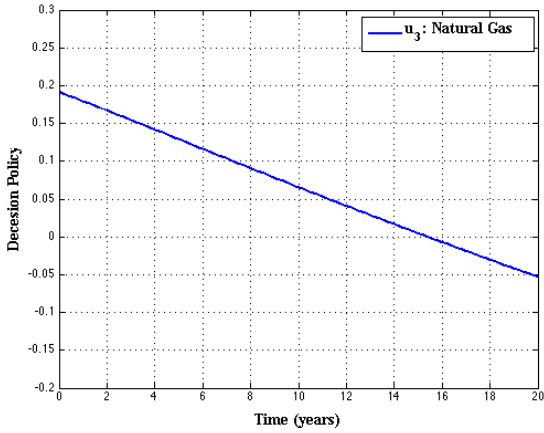


(a)

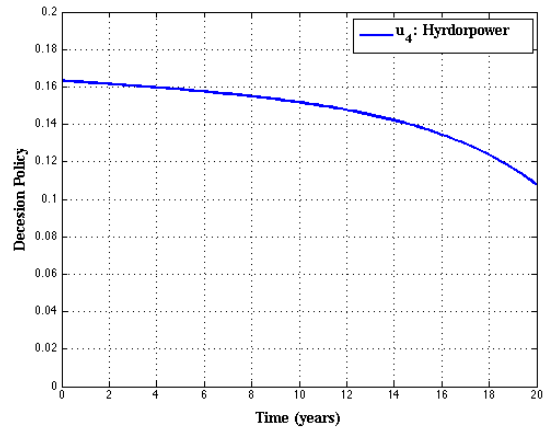


(b)

**Figure 49** (P3) Eight-State system, 20 years plan period, decision policies for (a) coal and (b) natural gas energy sources

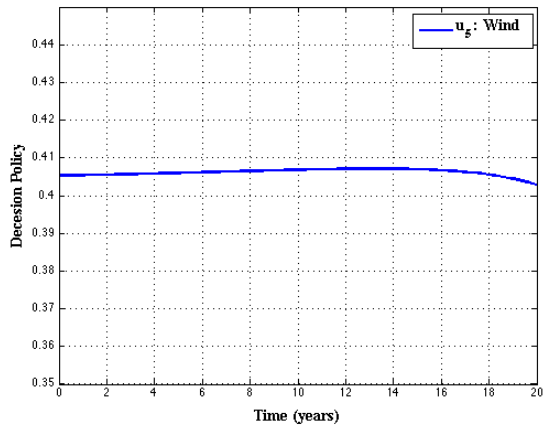


(a)

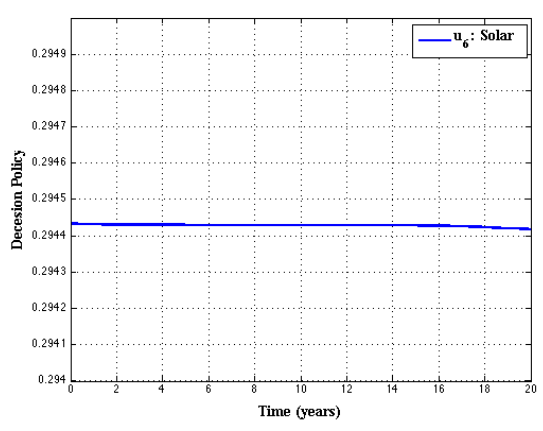


(b)

**Figure 50** (P3) Eight-State system, 20 years plan period, decision policies for (a) natural gas and (b) hydropower energy sources

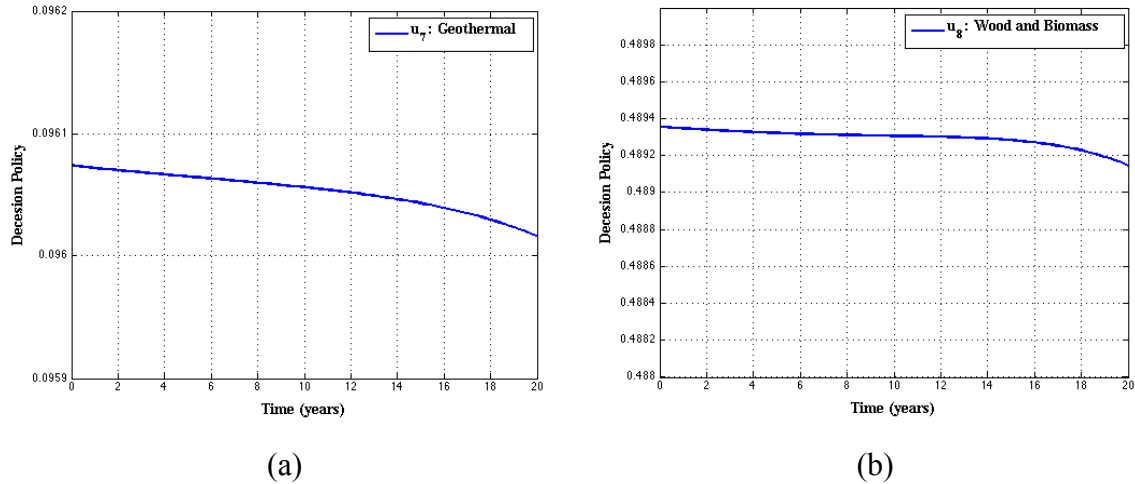


(a)



(b)

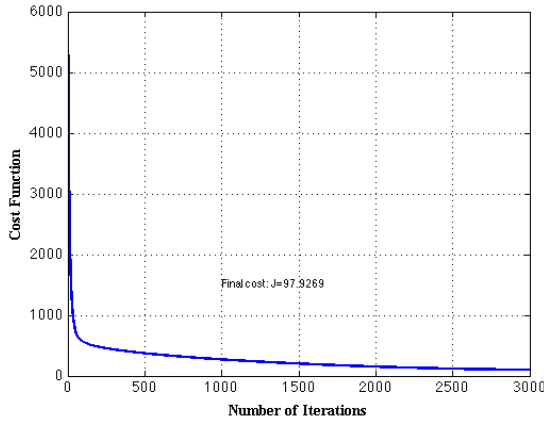
**Figure 51** (P3) Eight-State system, 20 years plan period, decision policies for (a) wind and (b) solar energy sources



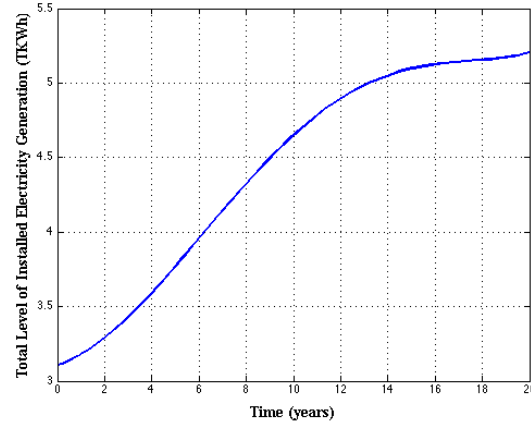
**Figure 52** (P3) Eight-State system, 20 years plan period, decision policies for (a) geothermal and (b) wood and biomass energy sources

The total level of electricity generated from the eight main energy sources during the plan period of twenty years, from 2015 to 2035, is shown in Figure 53-b. This non-decreasing trend demonstrates that we solved this problem (P3) and minimized the electricity production from polluted energy sources of coal and petroleum, without reducing the total electricity production. This proves that the electricity demand will certainly be satisfied in each time interval of the plan period.

The electricity demand in year 2035, as projected by the U.S. EIA [48] is 4.7 Trillion kWh, while according to our numerical results it is 5.2045 Trillion kWh. According to Figure 53-b we observe that the generation level reached at the end of the plan period exceeds the estimated target only by 6.15%.



(a)



(b)

**Figure 53** (P3) Eight-State system, 20 years plan period, (a) minimized cost function and (b) total level of electricity generation during the plan period

The previous problem (P2) involved terminal and implementation costs. In this problem (P3), we extended the previous problem by adding the environmental cost. For the first time, instead of taking into account individual desired levels of production for each energy source, we introduced the concept of “meeting the electricity demand by the end of the plan period”. In the following section, we will improve this concept by targeting not only the end of a given plan period, but also during any predefined time interval of the plan period.

#### 6.4. (P4) Meeting the Demand with Implementation and Environmental Costs

In this section, we started again with the two-state time-variable system introduced by [6] as a simplified form of our problem (P4), since the environmental costs was not considered in their paper.

Then, we extended our code by taking into account all three terms of demand, implementation and environmental cost, to conclude with solving the above (P4) problem for eight-state time-variable system.

### 6.4.1 Two-State Variable System

The objective functional for this system is given by expression below [6]:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^2 q_i u_i^2(t) + \sum_{i=1}^2 \Gamma_i (x_i(t) - x_i^d(t))^2 \right\} dt \quad (6.8)$$

where the first term represents the implementation costs and the second term stands for meeting the electricity demand in each predefined time interval of the plan period.

It is obvious that the summation of the two main categories of renewable and conventional energy sources should meet the electricity demand, therefore:

$$D(t) = x_1^d(t) + x_2^d(t) \quad (6.9)$$

In order to solve this problem, Miah M.S., Ahmed N.U., Chowdhury M. [6] specified the functions  $x_1^d(t)$ ,  $x_2^d(t)$  and  $D(t)$  in their paper as follows:

$$x_2^d(t) = x_2^d(0)e^{-\gamma t} \quad (6.10)$$

$$D(t) = D(0)e^{\lambda t} \quad (6.11)$$

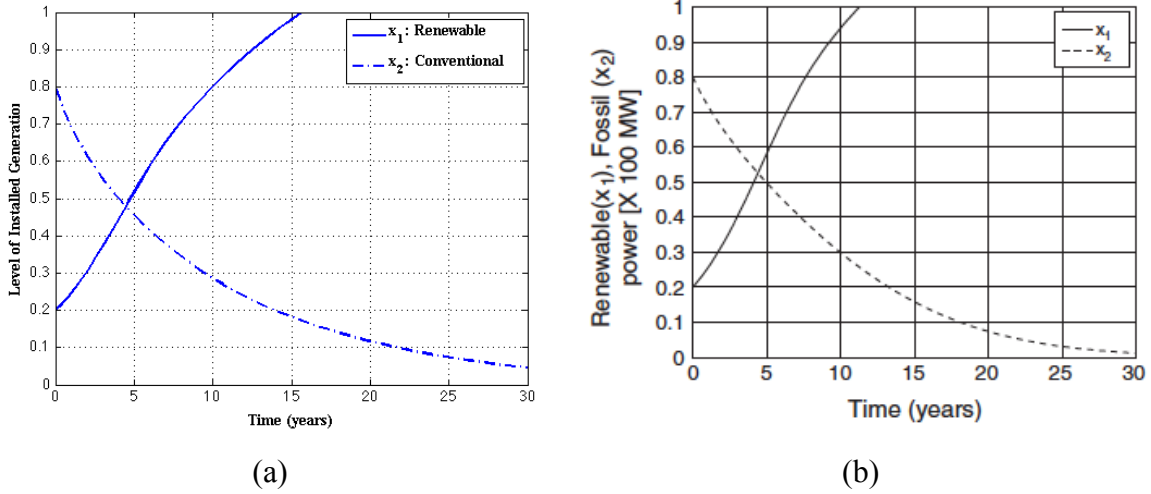
$$x_1^d(t) = D(t) - x_2^d(t) \quad (6.12)$$

For their numerical experiment, they chose  $q_1 = 1.1$ ,  $q_2 = 0.75$ ,  $\lambda = 0.01$ ,  $\gamma = 0.1$ ,  $\Gamma_1 = 1.5$ , and  $\Gamma_2 = 1.2$  [6].

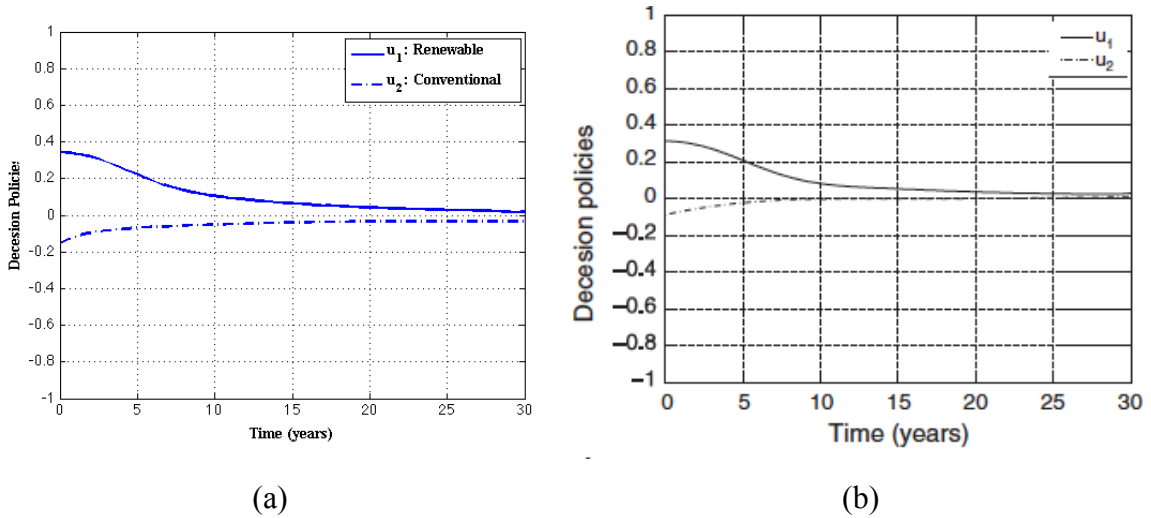
We solve their problem using  $\beta_{12}=0.18$  and  $\beta_{21}=0.05$ . The step-size is assumed 0.005 and thirty years plan period is chosen for the purpose of this problem.

State trajectories for the level of electricity generated from renewable and conventional energy sources, the decision policies for each of them as well as the minimized cost function are shown in Figure 54 to Figure 56.

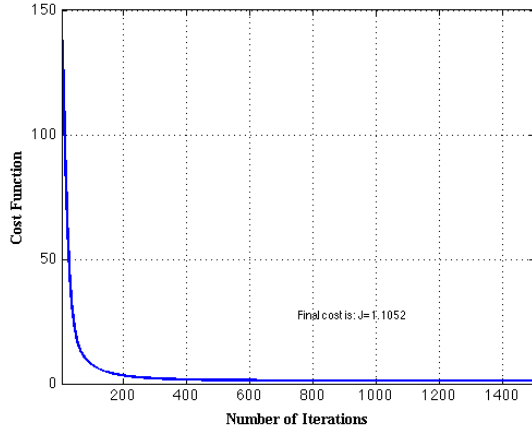
Results derived from our model are quite compatible with those given in [6]. The slight dissimilarities could be explained by using values of parameters  $\beta$ , step-size and acceptable tolerance different from those used in [6].



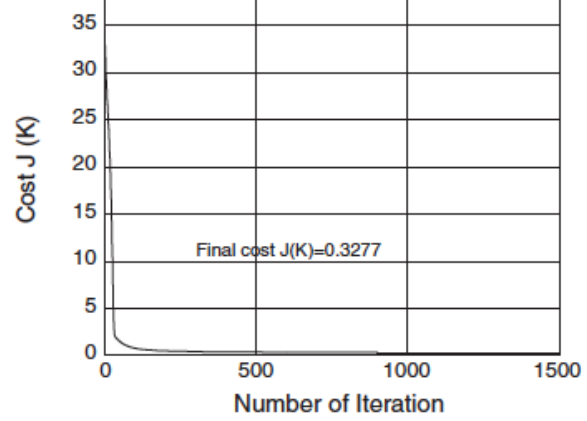
**Figure 54** (P4) Two-State system, 30 years plan period, comparison between state trajectories from (a) our model and (b) model given in [6]



**Figure 55** (P4) Two-State system, 30 years plan period, comparison between decision policies from (a) our model and (b) model given in [6]



(a)



(b)

**Figure 56** (P4) Two-State system, 30 years plan period, comparison between cost function from (a) our model and (b) model given in [6]

After validating our code for a two-state time-variable system, we applied it based to the official released data of U.S. EIA [48] (given in appendix A) for total renewable and total conventional energy sources with more improved cost function.

Consider a two-state time-variable system given by equation (5.1). In the system identification chapter, we identified  $\beta$ -parameters as  $\beta_{12}=0.03$  and  $\beta_{21}=-0.01$ . The cost functional given by expression below is selected for this problem:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^2 q_i u_i^2(t) + v_1 |x_1|^2 + \Gamma (D(t) - \sum_{i=1}^2 x_i(t))^2 \right\} dt \quad (6.13)$$

where the first term represents the investment cost (implementation cost) and the second term accounts for the pollution cost of the polluting energy sources. The last term is responsible for minimizing the difference between the electricity demand and the summation of total renewable and total conventional energy sources in each time interval during the plan period. The level of installed electricity generation from conventional energy sources and renewable energy sources is represented by  $x_1$  and  $x_2$ , respectively. The decision policy for each of the energy sources of conventional and renewable are

designated by  $u_1$  and  $u_2$ , respectively. Finally  $D(t)$  denotes the electricity demand as a function of time during the plan period.

The electricity demand in the United States between years 2010 to 2035 is shown in Figure 7-A [48]. We used these data and fitted them to obtain an equation for  $D(t)$ , which is expressed below:

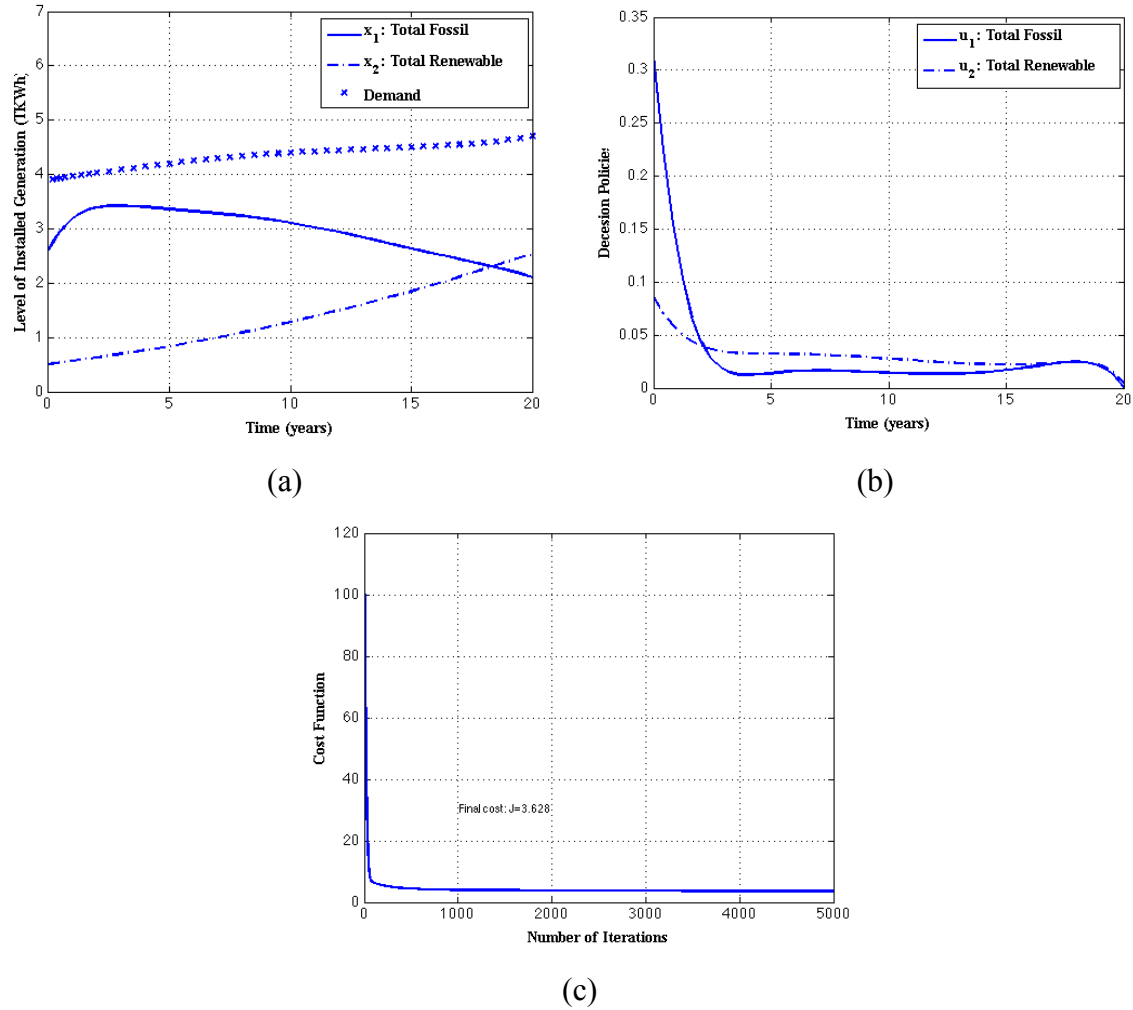
$$D(t) = a_1 t^4 + a_2 t^3 + a_3 t^2 + a_4 t + a_5 \quad (6.14)$$

where  $a_1 = 1.33e^{-5}$ ,  $a_2 = -0.0004$ ,  $a_3 = 0.001667$ ,  $a_4 = 0.06$ , and  $a_5 = 3.9$ . We choose  $q_1 = 1.1$ ,  $q_2 = 1.5$ ,  $v_1 = 0.01$ , and  $\Gamma = 5$ . The initial value for state variables are  $x_1(0) = 2.616$  and  $x_2(0) = 0.50749$  Trillion kWh using the official data of U.S. EIA [48] for total fossil and total renewable energy sources for year 2015, respectively (Table 2-A). The step-size is assumed 0.000005.

The state trajectories for total fossil and total renewable energy sources are shown in Figure 57-a. The level of electricity generation from fossil fuels is increased at the beginning of the plan period to satisfy the electricity demand in that time interval and then decreased due to the environmental cost term. Therefore, the level of polluted energy sources is reduced at the end of the plan period and the electricity demand is satisfied through the whole plan period.

As illustration, Figure 57-a shows that after five years, i.e., in year 2020, the electricity demand would be 4.2 Trillion kWh, while this value for total fossil energy source would be 3.362 Trillion kWh and 0.835 Trillion kWh for total renewable energy source. This indicates that the summation of total fossil and total renewable energy sources in year 2020 would be 4.197 Trillion kWh. This value is quite close to the expected value of 4.2 Trillion kWh (electricity demand by year 2020). We can explain the rest of the plan period with the same procedure.

Figure 57-b shows the decision policies regarding each of the selected energy sources during the plan period and the converging cost function with final value of 3.628 is shown in Figure 57-c.



**Figure 57** (P4) Two-State system, 20 years plan period, real-life data, (a) demand trend and state trajectories, (b) decision policies and (c) minimized cost function

So far, we presented the results for different two-state time-variable systems with different cost functions. The demand and implementation costs were considered in the first problem (given by [6]) and the demand, implementation and environment costs were taken into account in the second one.

Now we continue with the final goal, which is finding the optimum decision policy for integrating the renewable energy sources into the electricity generation system with eight main energy sources. All three energy problems of demand, implementation and environment are considered in this problem.

### 6.4.2 Eight-State Variable System

Consider the eight-state time-variable model expressed by equation (4.1), where all fifty-six unknown  $\beta$ -parameters are given in Table 3. For this system, we introduced an objective functional that includes three main parts of demand cost, implementation cost and environmental cost, as

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) + \sum_{i=1}^2 v_i |x_i|^2 + \Gamma(D(t) - \sum_{i=1}^8 x_i(t))^2 \right\} dt \quad (6.15)$$

where the first term is the implementation cost, the second term is the environmental cost and the last term is the demand cost. We developed three different scenarios for the electricity demand  $\{D(t)\}$ . First, using the electricity demand predicted by U.S. EIA [48] and expressed by (6.14) for 20 years plan period. Second, assuming 2% annual growth for the electricity demand for 20 years plan period, and finally 2% annual growth rate for the electricity demand for 30 years plan period.

The level of electricity production is shown by  $x_i, i = 1 \dots 8$ , and the decision policy is given by  $u_i, i = 1 \dots 8$ , for eight energy sources of coal, petroleum, natural gas, hydropower, wind, solar, geothermal and biomass, respectively. The weighting values of  $\{q_i\}$  and  $\{v_i\}$  are kept exactly the same as in previous problems (given in Table 8 for the convenience of the reader).

**Table 8** (P4) Assumed values for  $\{q_i\}$  and  $\{v_i\}$  given in equation (6.15)

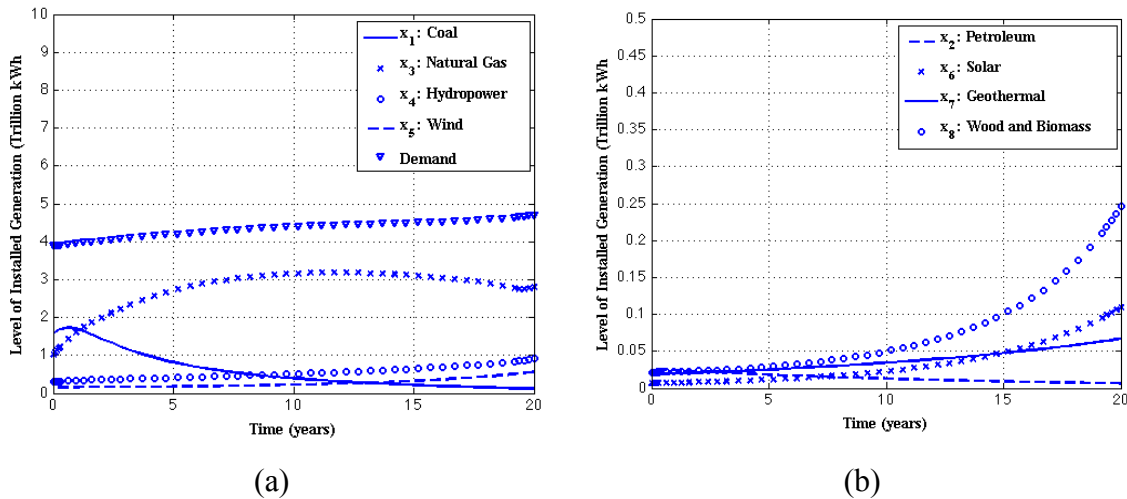
$q_i$	Value	$q_i$	Value	$v_i$	Value
$q_1$	10	$q_5$	1	$v_1$	5
$q_2$	5	$q_6$	0.59	$v_2$	$5 \cdot 1.23$
$q_3$	3	$q_7$	1.8		
$q_4$	3	$q_8$	0.68		

### Scenario 1: Electricity Demand Based on U.S. Projection, 20 Years Plan Period

In this scenario, we considered the electricity demand based on the U.S. projection [48] expressed by (6.14). The weighting value  $\Gamma$  in equation (6.15) is assumed as 150. This assumption was based on the high priority of meeting the electricity demand in our problem. The planner may easily reduce this value if, in his point of view, the priority of implementation or environmental costs are higher than the electricity demand.

The state trajectories representing the level of electricity generation from each of the energy sources are shown in Figure 58. As expected, the level of electricity production from polluted energy sources of coal and petroleum is decreased by the end of the plan period. The level of production from coal energy source is increased in the beginning of the plan period to satisfy the demand term in the cost functional given by equation (6.15).

It is expected that the summation of all eight energy sources meet the electricity demand in each time interval during the plan period. Table 9 is a proof for this claim.



**Figure 58** (P4) Eight-State system, scenario 1, 20 years plan period, (a) demand trend and state trajectories for coal, natural gas, hydropower, wind and (b) state trajectories for petroleum, solar, geothermal and biomass

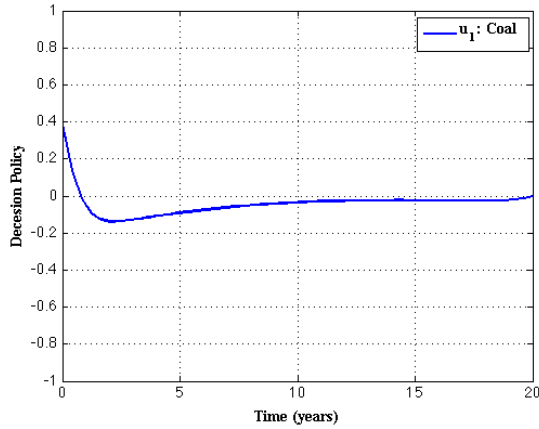
**Table 9** (P4) Scenario 1, 20 years plan period, Comparison between the electricity demand and the electricity generated from each energy source during the plan period (Trillion kWh)

$x_i$	year 2015	year 2020	year 2025	year 2030	year 2035
$x_1$	1.5620	0.7909	0.3833	0.2095	0.1211
$x_2$	0.0260	0.0180	0.0127	0.0090	0.0065
$x_3$	1.0280	2.7430	3.1580	3.0980	2.7992
$x_4$	0.2954	0.4181	0.5171	0.6563	0.9110
$x_5$	0.1510	0.1760	0.2264	0.3301	0.5632
$x_6$	0.0065	0.0118	0.0235	0.0495	0.1098
$x_7$	0.0187	0.0255	0.0348	0.0481	0.0669
$x_8$	0.0213	0.0295	0.0512	0.1039	0.2461
$\sum_{i=1}^8 x_i$	3.1089	4.2128	4.4070	4.5044	4.8238
<i>Demand</i>	3.9000	4.2100	4.4050	4.5040	4.700

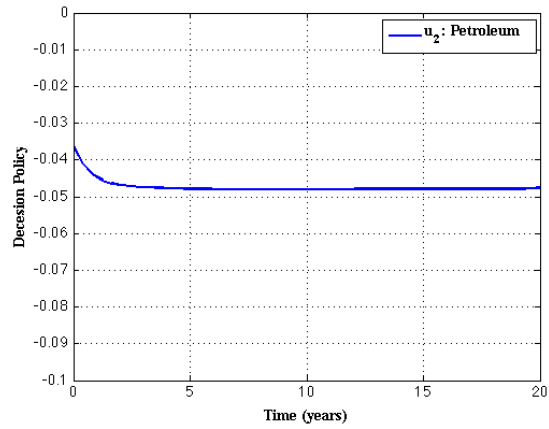
Note the difference between the level of electricity generation, at the beginning of the plan (year 2015), from the eight energy sources and the electricity demand. This can be explained by the lack of electricity generation from these energy sources in 2015. In fact, in 2015, we can avoid this shortage in electricity generation and meet the demand by using other energy sources [48]. However, as shown in Table 9, our proposed procedure will satisfy the electricity demand during the entire plan period from only these eight energy sources, and it does not need to use any other energy sources.

Decision policies for all eight energy sources during the plan period are shown in Figure 59 to Figure 62. For example, for coal energy source, the increasing and then decreasing trend of the state trajectory ( $x_1$ ) (Figure 58) is a sign for having positive and negative values for the decision policy ( $u_1$ ) shown in Figure 59.

The minimized cost function after 100,000 iterations is shown in Figure 63-a. Figure 63-b demonstrates the converging trend of this cost function from a closer view in the first 1000 iterations.

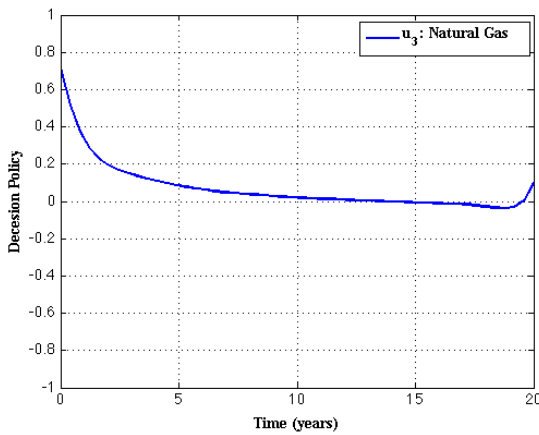


(a)

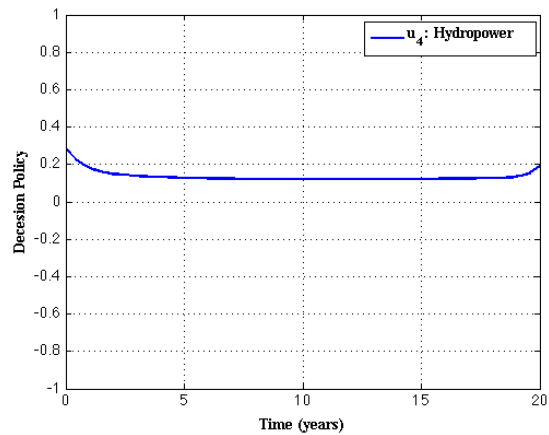


(b)

**Figure 59** (P4) Eight-State system, scenario 1, 20 years plan period, decision policies for (a) coal and (b) petroleum energy sources

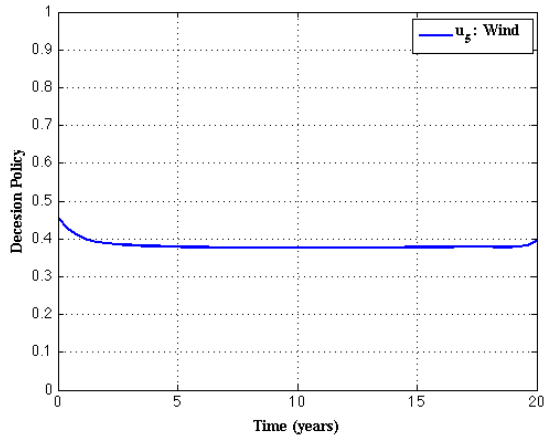


(a)

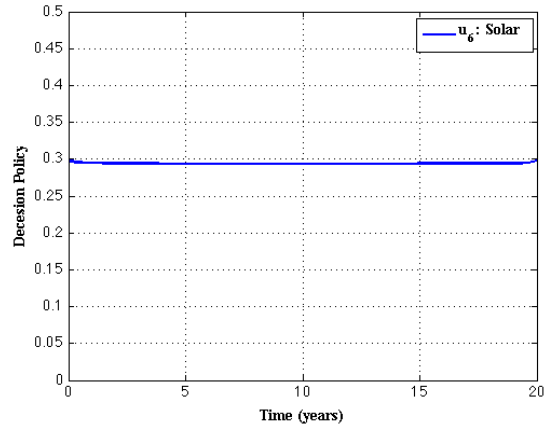


(b)

**Figure 60** (P4) Eight-State system, scenario 1, 20 years plan period, decision policies for (a) natural gas and (b) hydropower energy sources

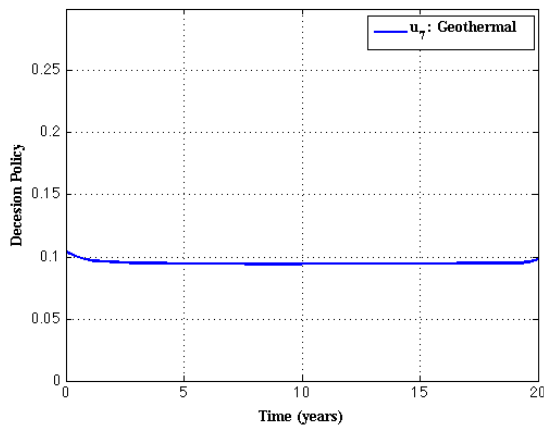


(a)

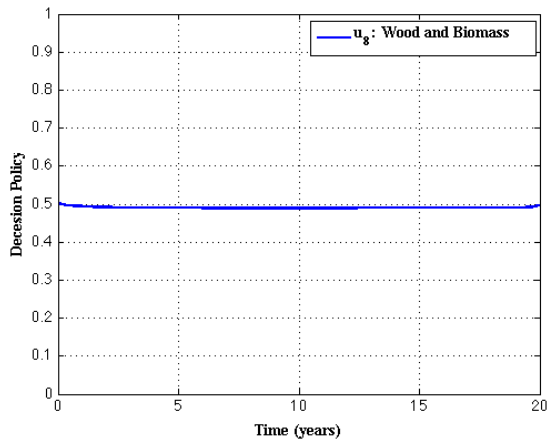


(b)

**Figure 61** (P4) Eight-State system, scenario 1, 20 years plan period, decision policies for (a) wind and (b) solar energy sources

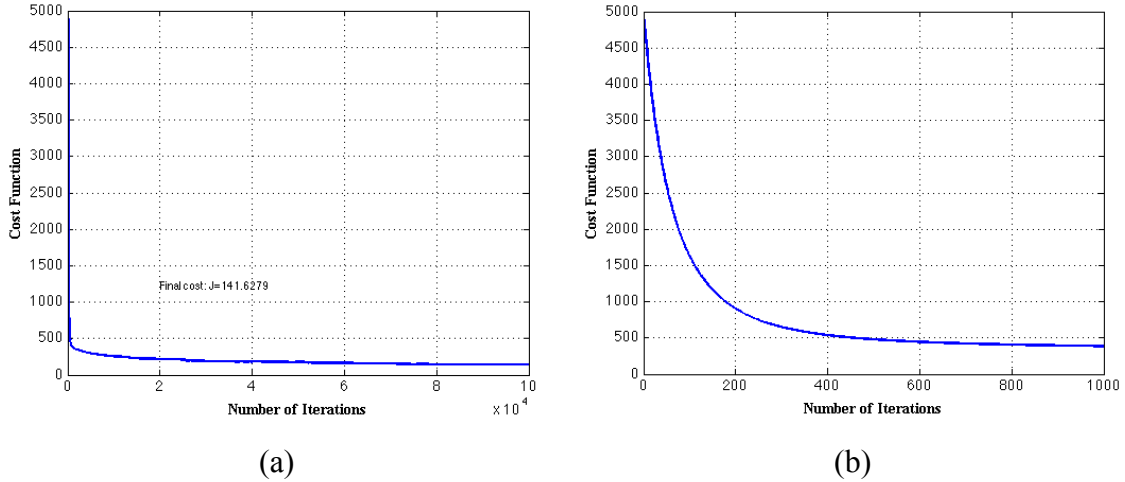


(a)



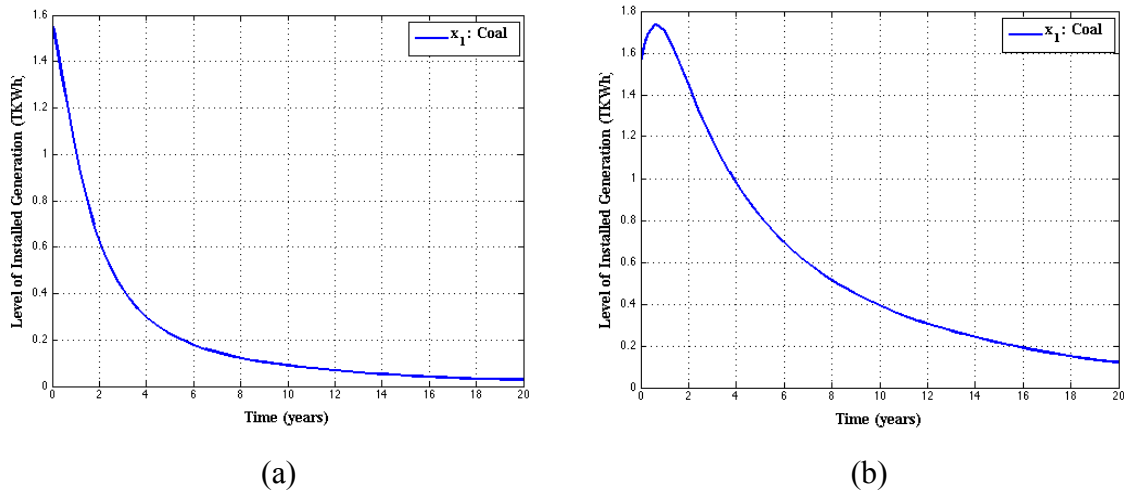
(b)

**Figure 62** (P4) Eight-State system, scenario 1, 20 years plan period, decision policies for (a) geothermal and (b) wood and biomass energy sources



**Figure 63** (P4) Eight-State system, scenario 1, 20 years plan period, (a) cost function after 100,000 iterations, (b) zoomed cost function after first 1000 iterations

If we increase the weighting value for the environmental cost ( $v_1$ ), we expect to reach to the lower level of electricity generation from polluted energy source of coal. This expectation is satisfied and shown in Figure 64. If we change  $v_1$  from 5 to 50, the final level of electricity generation from coal energy source will be 0.0297 instead of 0.1211.



**Figure 64** (P4) Comparison between level of electricity generated from coal, (a) high environmental cost,  $v_1=50$ , (b) low environmental cost,  $v_1=5$

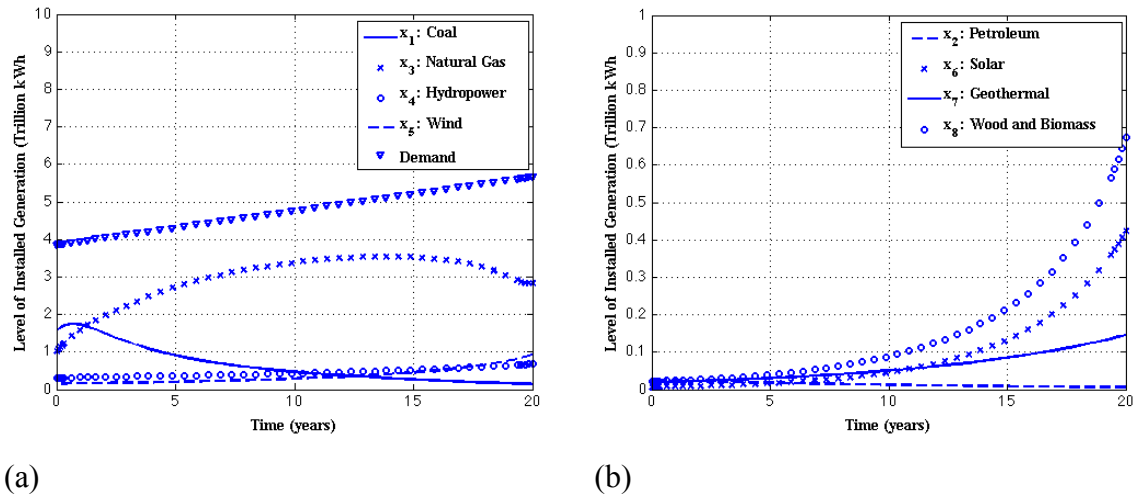
**Scenario 2: Electricity Demand Based on 2% Annual Growth, 20 Years Plan Period**

Let us consider a system where the annual growth of electricity demand is 2%. In this case, we expect higher decision policies to satisfy this demand, in comparison to the previous scenario, where the mean annual growth of electricity demand was 0.958%.

The weighting values of the selected cost function (6.15) are given in Table 8.  $\Gamma$  is set to 150 and the electricity demand  $\{D(t)\}$  is assumed as

$$D(t) = 0.09038t + 3.858 \tag{6.16}$$

This expression satisfied the 2% annual growth for U.S. electricity demand, starting from 3.9 Trillion kWh in year 2015. Figure 65 shows the electricity demand and the state trajectories for all energy sources during the twenty year plan period. This electricity demand is satisfied by increasing the level of electricity generated from renewable energy sources due to lower implementation costs (Table 8).



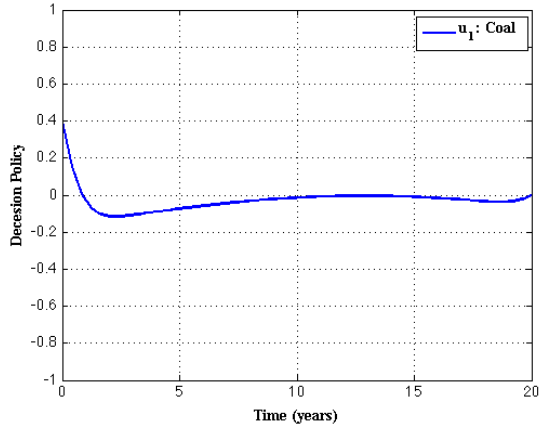
**Figure 65** (P4) Eight-State system, scenario 2, 20 years plan period, (a) demand trend and state trajectories for coal, natural gas, hydropower, wind and (b) state trajectories for petroleum, solar, geothermal and biomass

Table 10 shows that during the plan period of twenty years, the summation of all eight energy sources satisfies the electricity demand. This claim is proved, and shown in this table, after each five years of the plan period.

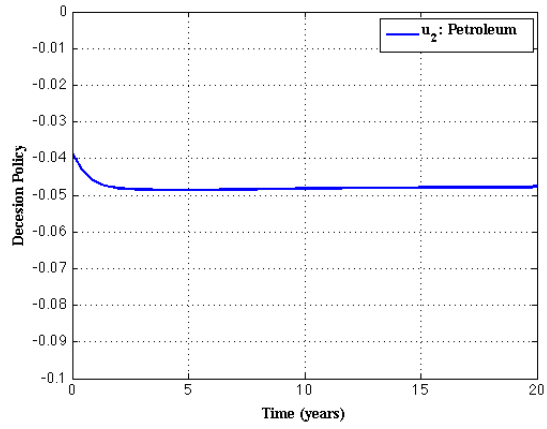
**Table 10** (P4) Scenario 2, 20 years plan period, Comparison between the electricity demand and the electricity generated from each energy source during the plan period (Trillion kWh)

$x_i$	year 2015	year 2020	year 2025	year 2030	year 2035
$x_1$	1.5620	0.9266	0.4772	0.2738	0.1443
$x_2$	0.0260	0.0183	0.0128	0.0089	0.0063
$x_3$	1.0280	2.7020	3.3700	3.5240	2.8355
$x_4$	0.2954	0.3694	0.4292	0.5263	0.6956
$x_5$	0.1510	0.1996	0.2844	0.4475	0.9262
$x_6$	0.0065	0.0162	0.0448	0.1287	0.4239
$x_7$	0.0187	0.0301	0.0498	0.0835	0.1461
$x_8$	0.0213	0.0394	0.0874	0.2118	0.6746
$\sum_{i=1}^8 x_i$	3.1089	4.3016	4.7556	5.2045	5.8525
<i>Demand</i>	3.8580	4.2980	4.7500	5.2020	5.6660

Figure 66 to Figure 69 show the decision policies regarding each of the energy sources. In comparison to scenario 1, for some of the energy sources, higher decision policies are expected to satisfy the greater electricity demand in scenario 2. This expectation is satisfied, i.e. for biomass energy source around 50% growth rate in scenario 1 is changed to around 58% in scenario 2. Similarly, for geothermal energy source, around 10% growth rate in scenario 1 is changed to around 15% in the second scenario. The minimized cost function is shown in Figure 70.

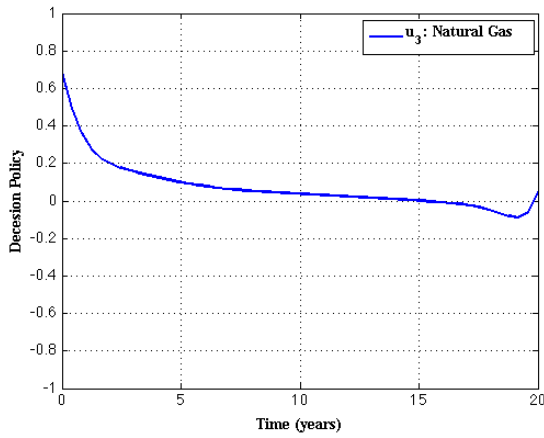


(a)

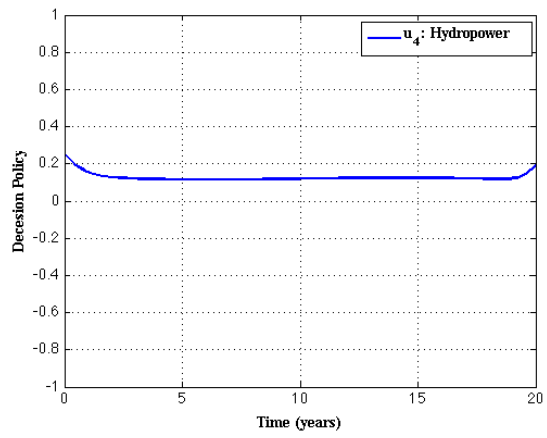


(b)

**Figure 66** (P4) Eight-State system, scenario 2, 20 years plan period, decision policies for (a) coal and (b) petroleum energy sources

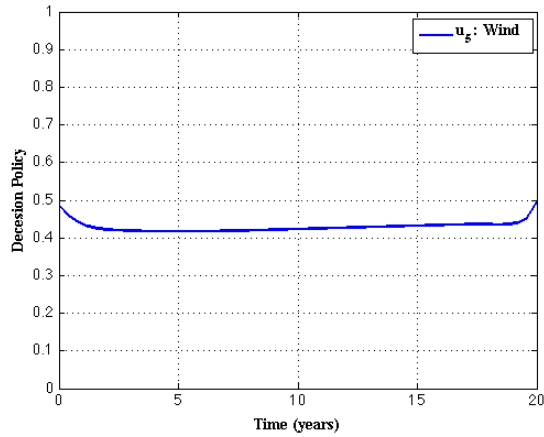


(a)

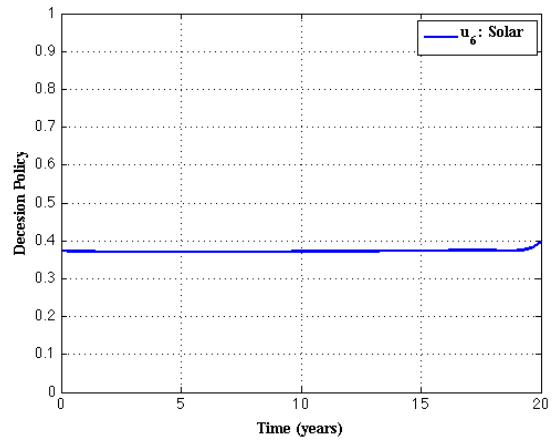


(b)

**Figure 67** (P4) Eight-State system, scenario 2, 20 years plan period, decision policies for (a) natural gas and (b) hydropower energy sources

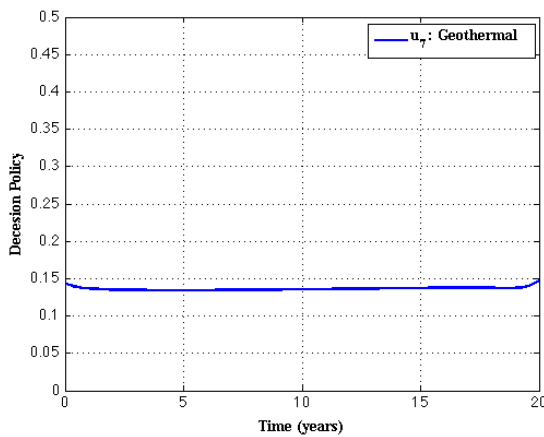


(a)

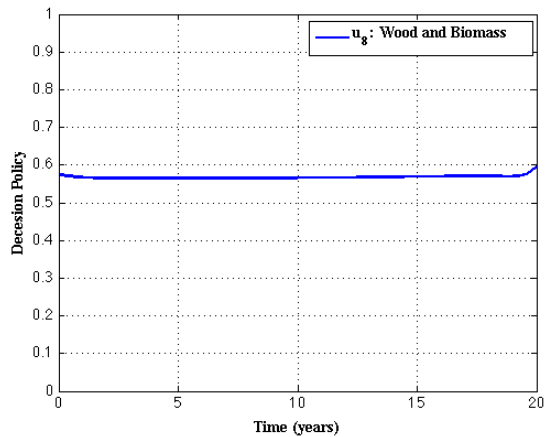


(b)

**Figure 68** (P4) Eight-State system, scenario 2, 20 years plan period, decision policies for (a) wind and (b) solar energy sources

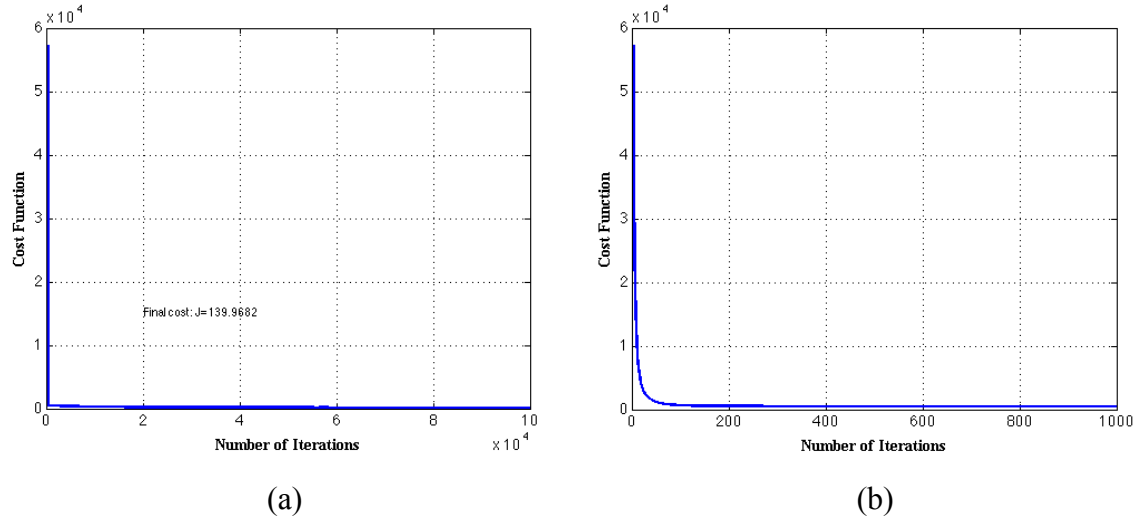


(a)



(b)

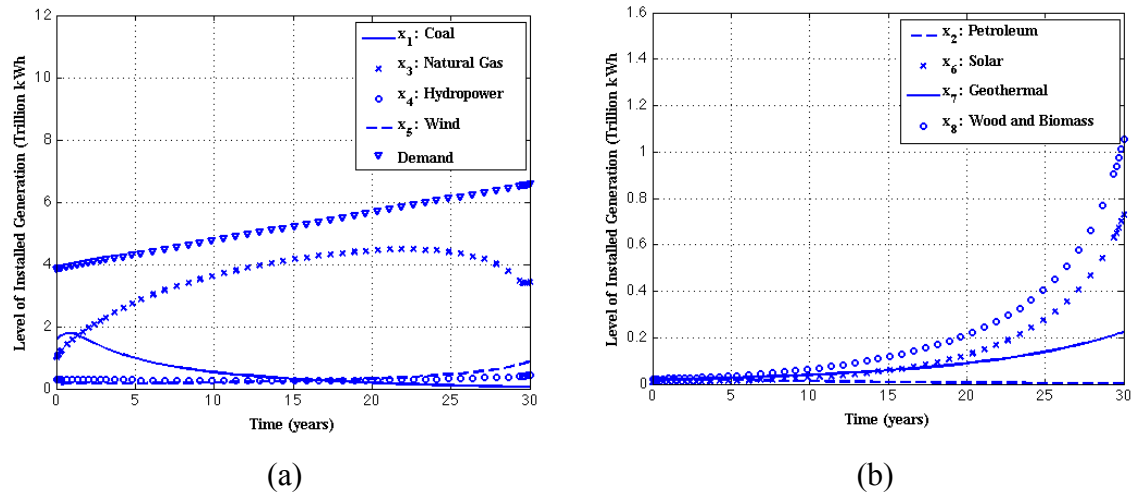
**Figure 69** (P4) Eight-State system, scenario 2, 20 years plan period, decision policies for (a) geothermal and (b) wood and biomass energy sources



**Figure 70** (P4) Eight-State system, scenario 2, 20 years plan period, (a) cost function after 100,000 iterations, (b) zoomed cost function after first 1000 iterations

**Scenario 3: Electricity Demand Based on 2% Annual Growth, 30 Years Plan Period**

In this scenario, we demonstrated that our method will also satisfy different plan periods. Let us consider the previous scenario ( $\Gamma = 150$ ), while the plan period is 30 years.



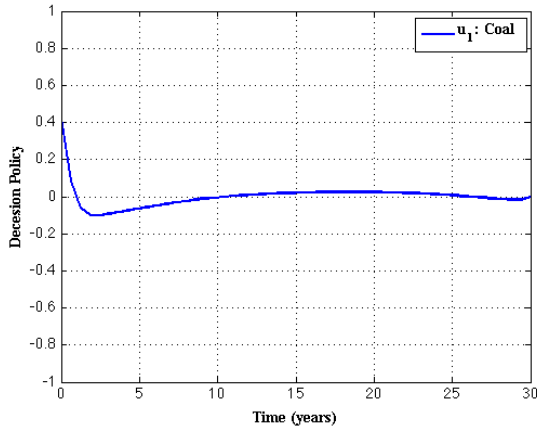
**Figure 71** (P4) Eight-State system, scenario 3, 30 years plan period, (a) demand trend and state trajectories for coal, natural gas, hydropower, wind and (b) state trajectories for petroleum, solar, geothermal and biomass.

The fact that the electricity demand is satisfied during the whole thirty years plan period is shown in Table 11.

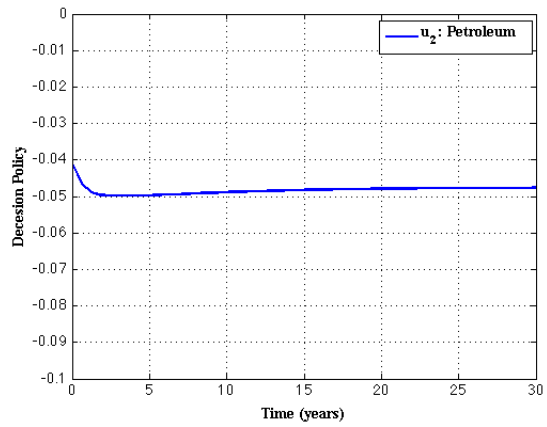
**Table 11** (P4) Scenario 3, 30 years plan period, Comparison between the electricity demand and the electricity generated from each energy source during the plan period (Trillion kWh)

$x_i$	year 2015	year 2020	year 2025	year 2030	year 2035	year 2040	year 2045
$x_1$	1.5620	1.0130	0.5352	0.3134	0.1847	0.1112	0.0579
$x_2$	0.0260	0.0182	0.0125	0.0085	0.0058	0.0042	0.0029
$x_3$	1.0280	2.7410	3.6200	4.1590	4.4530	4.3980	3.4192
$x_4$	0.2954	0.2808	0.2583	0.2534	0.2792	0.3220	0.4300
$x_5$	0.1510	0.1697	0.2017	0.2534	0.3321	0.4573	0.8745
$x_6$	0.0065	0.0127	0.0270	0.0596	0.1325	0.2734	0.7289
$x_7$	0.0187	0.0267	0.0393	0.0596	0.0921	0.1372	0.2255
$x_8$	0.0213	0.0341	0.0626	0.1188	0.2229	0.4035	1.0555
$\sum_{i=1}^8 x_i$	3.1089	4.2962	4.7566	5.2257	5.7023	6.1068	6.7944
<i>Demand</i>	3.8580	4.2930	4.7500	5.2250	5.6990	6.1060	6.5690

The optimum decision policies for all eight energy sources are shown in Figure 72 to Figure 75. Figure 76 shows the minimized cost function.

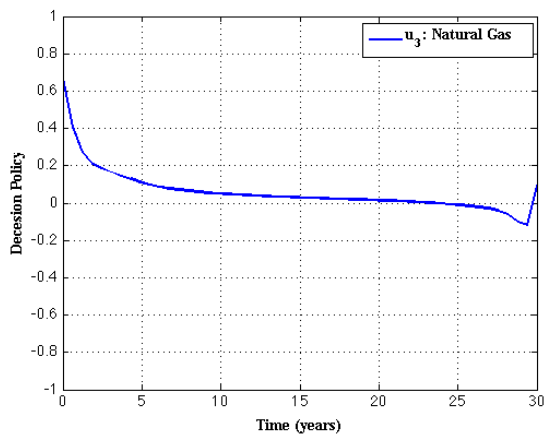


(a)

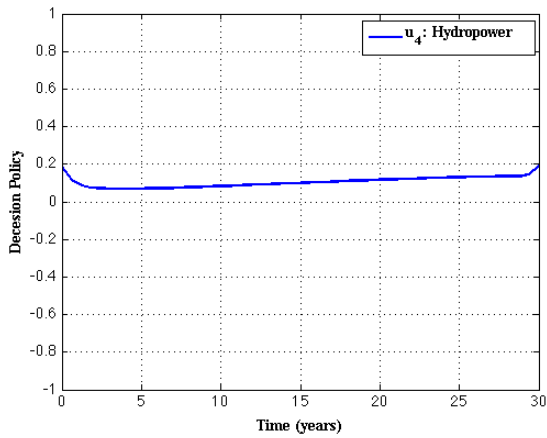


(b)

**Figure 72** (P4) Eight-State system, scenario 3, 30 years plan period, decision policies for (a) coal and (b) petroleum energy sources

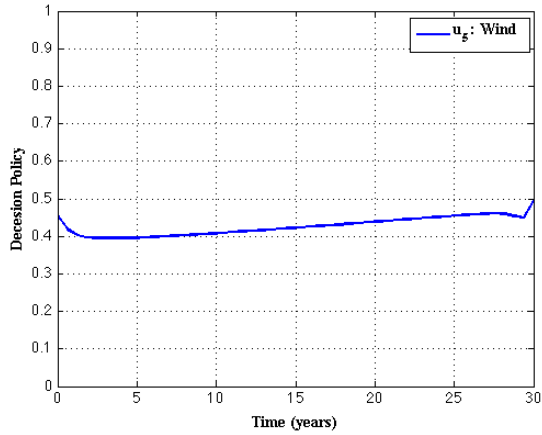


(a)

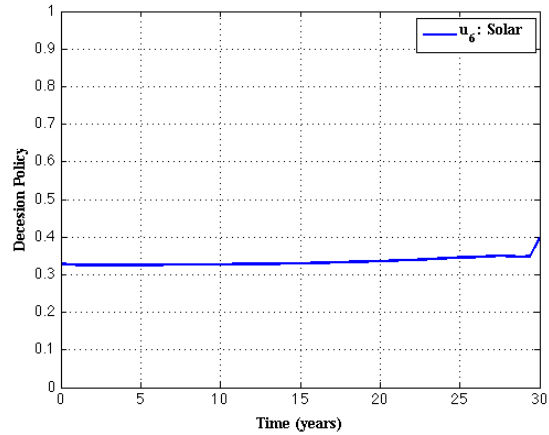


(b)

**Figure 73** (P4) Eight-State system, scenario 3, 30 years plan period, decision policies for (a) natural gas and (b) hydropower energy sources

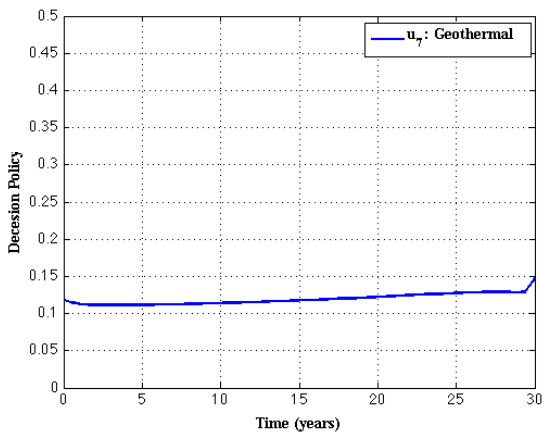


(a)

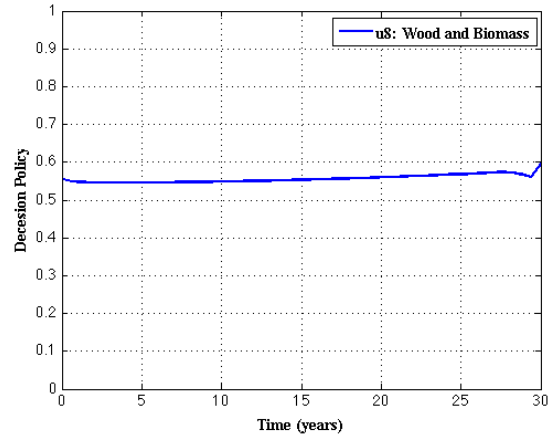


(b)

**Figure 74** (P4) Eight-State system, scenario 3, 30 years plan period, decision policies for (a) wind and (b) solar energy sources

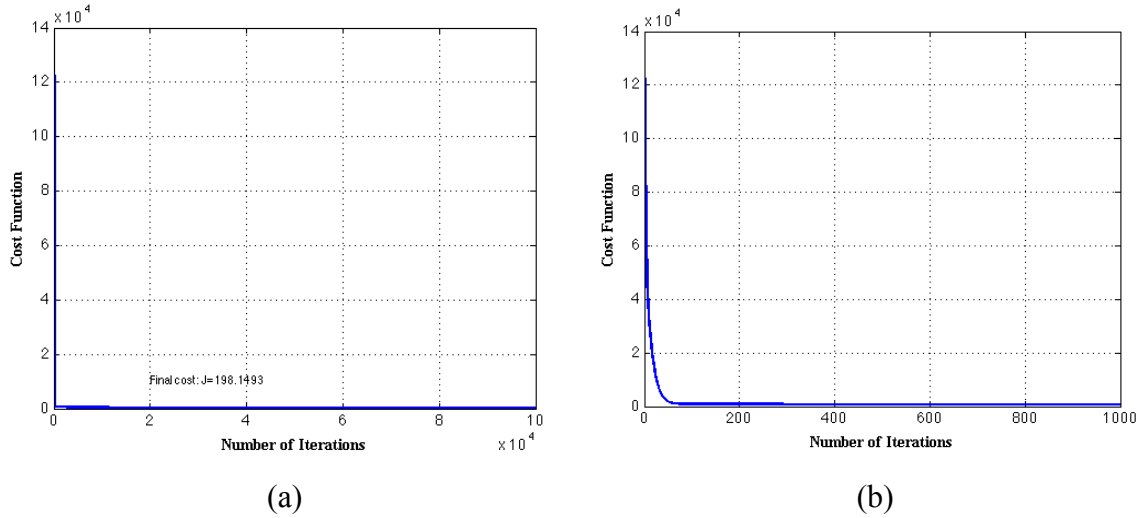


(a)



(b)

**Figure 75** (P4) Eight-State system, scenario 3, 30 years plan period, decision policies for (a) geothermal and (b) wood and biomass energy sources



**Figure 76** (P4) Eight-State system, scenario 3, 30 years plan period, (a) cost function after 100,000 iterations, (b) zoomed cost function after first 1000 iterations

The problem of meeting the demand in each time interval of the plan period while minimizing the implementation costs and the environmental costs, for eight-state time-variable system, is successfully resolved. The presented results are completely compatible with what expected. The converging trends of cost functions were the proof for minimizing the three terms of demand, implementation and environmental costs. The level of electricity generation from polluted energy sources of coal and petroleum is reduced, while the electricity demand is satisfied through the whole plan period.

## 6.5. Summary

In this chapter, we solved different types of energy problems using the identified parameters presented in the previous chapter. Four main energy problems were successfully addressed namely, (P1) meeting the target, (P2) meeting the target with implementation cost, (P3) meeting the target with implementation and environmental costs, and (P4) meeting the demand during the plan period, with implementation and environmental costs. The obtained results for two-state, three-state, and eight-state time-variable models are satisfactory. Note that none of the data used in this thesis were based on assumptions.

# Chapter 7. Conclusions

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## 7.1. Summary

The optimum decision policies based on the official released data from the U.S. Energy Information Administration official website for eight main energy sources namely, Coal, Petroleum, Natural Gas, Hydro, Wind, Solar, Geothermal and Biomass have been presented in this thesis.

A dynamic model representing the level of electricity generation from each of the selected energy sources and the economic interactions between them was considered. Pontryagin minimum principle was chosen as the selected method from optimal control theory.

Based on the available historical data, all unknown parameters of the modeled system were identified. For the twenty years plan period (2015-2035), the optimum decision policies were determined for four different problems. In the first problem (P1), the optimal policy successfully minimized the gap between the target and the actual levels of generation for the eight energy sources. In the second problem (P2), the optimum policy minimized the gap between the target and the actual levels of generation while keeping the implementation cost as low as possible. In the third problem (P3), the optimum policy satisfied the total demand at the end of the plan period while keeping implementation cost and environmental damage as low as possible. Finally in the last problem (P4), the optimum policy efficiently fulfilled the electricity demand during the entire plan period while keeping the implementation and environmental costs as low as possible.

## 7.2. Future work

The work presented in this thesis is based on the U.S. projection during the plan period of 2015-2035. However, this projection may change throughout this period due to uncertainties such as, economic growth, population increase, retirement of existing power

plants, etc. Such uncertainties could affect the estimation of the amount of electricity demand and in turn might change the predicted projection.

This point could be addressed as future work through the stochastic optimal control theory. In this case, the electricity demand will be considered as a stochastic function. The optimum decision policies will be developed based on considering all the uncertainties that affect the estimation of electricity demand.

Also, the Lotka-Volterra model using the optimal control theory, presented in this thesis, can be extended to analyze long-term relationship between different variables and to predict their trends in economic or social systems. These variables could be the Gross Domestic Products (GDPs), energy consumptions by different users (residential, commercial and industrial), and the fuel consumption such as oil, petroleum, and natural gas. Selecting any of these variables, defining their relationships using the historical data and predicting their future trends with the aim of optimal control theory, is indeed a challenging research topic.

The impact of Green House Gas emissions in the environment, and how to efficiently reduce it, is recognized as one of the main environmental priorities in most countries. So, another interesting research topic would be finding the precise relationship between the amount of GHG emissions and the electricity generated from polluted energy sources and then, defining an optimum plan period for the gradual replacement of polluted energy sources by employing the procedure used in this thesis.

Finally, it is worth to mention that the model and the mathematical approach we developed in the present work is generic. So, it can be used not only in the field of energy systems, but also in a wide panel of engineering applications involving optimal control theory, from biological and social systems to robotics and microwaves. Future researchers would have an efficient tool to optimize potential problems they may face during their research, only by applying slight modifications in our code to adapt it to their particular needs.

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## **Appendix A: U.S. EIA Official Data and Projections**

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Increasing energy demand and environmental concerns must be addressed in a way so that demand is met while impacts on environment are kept to a minimum. With this goal in mind, we have chosen the available data from the U.S. Energy Information Administration website [4] as the case study for this thesis. These data, available online, are precise, up to date, and complete.

This appendix includes three main sections: “Historical Data” that are necessary for system identification, “Desired Data” that are important to find the optimum decision policies when minimizing the terminal cost is the main goal, and “Electricity Demand” which is the main issue in any energy system and any decision policy without fulfilling the energy demand would be useless. So, the next step will be to identify these issues.

### ***U.S. Historical Data***

Historically, different types of energy sources are used in different countries, from wood and coal to petroleum and natural gas. In this work, historical data for all the chosen energy sources are taken from the latest released data of EIA website [48]. This is given in Table 1-A (in Trillion kiloWatt-hours (kWh)).

The available data for the plan period of six years from 2010 till 2015 is considered as the historical data for the purpose of system identification. Due to the insignificant level of generation from most of the renewable energy sources before year 2010, best available time period was the above six years; however, the same procedure we proposed in this thesis can easily be extended as time passes with more available historical data.

**Table 1-A** Generation Level of Electric Power Sectors, Trillion kWh, Historical Data [48]

	Year 2010	Year 2015
Coal	1.831	1.562
Petroleum	0.034	0.026
Natural Gas	0.898	1.028
Total Fossil	2.764	2.616
Conventional Hydropower	0.25532	0.29543
Wind	0.09449	0.15097
Solar	0.00128	0.00647
Geothermal	0.01567	0.01868
Wood and Biomass	0.01151	0.02128
Total Renewable	0.39482	0.50749

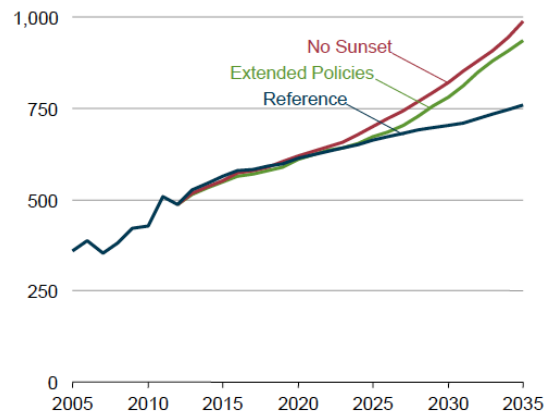
### ***U.S. Desired Data***

After determining the unknown parameters of the system model using the historical data of Table 1-A, one can focus on optimization based on this model. For optimum energy policy, it is necessary to define the desired values for each selected energy source. Different projections can be proposed for different energy sources based on various scenarios.

Increasing the level of generation from renewable energy sources is an inevitable part of any scenario. This fact is shown in Figure 1-A, which demonstrates the renewable electricity generation projection in the United States for three different cases between 2005 and 2035.

In this figure, “Reference” case can be considered as a “current laws and regulations” case. This means that during the projection period for energy planning, in reference case, existing laws and regulations in the United States will not change. The “No Sunset” case assumes that none of the policies will decline during the plan period and the

“Extended Policies” case includes additional updates that were not considered in previous cases. In this thesis, we always use the “Reference” case data.



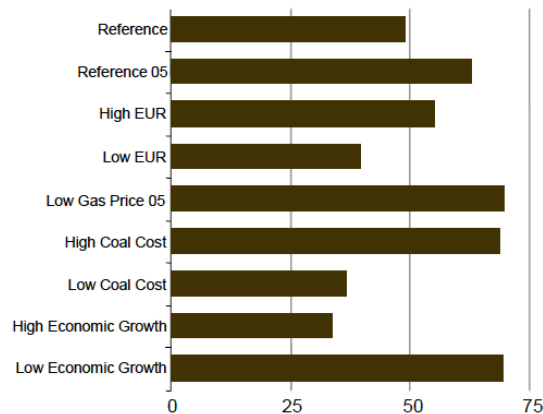
**Figure 1-A** U.S. Renewable Electricity Generation (Billion kWh), Period 2005-2035, Compiled in 2012 [48]

Decisions to add capacity to the electricity generations depend on various factors, such as retirement of certain of the existing power plants, availability of new sources of renewable energies and financial supports. Figure 2-A shows that in the reference case, 49 GigaWatts (GW) of coal-fired generation will retire during the plan period of 2011-2035. The amount of this retirement completely depends on the various cases shown in this figure [48].

In this figure, different economic terms are presented. We will briefly introduce each of them here. The “Reference” case is “current laws and regulations” case. If we reduce the cost recovery period for investments, in comparison with the reference case, from 20 years to 5 years, we are in the “Reference 05” case. The “Low Estimated Ultimate Recovery (EUR)” case is based on assuming that the oil or gas price is 50% lower than in reference case. If the oil or gas price is 50% higher than in reference case, we are in the “High EUR” case. The more optimistic assumptions about the level of gas production with 5-year recovery period for investments are considered as “Low Gas Price 05”. While the “High Coal Cost” case assumes lower mining productivity and higher costs for equipment, transportation and labour, the vice versa is the “Low Coal Cost” case. Finally, the “Low Economic Growth” case assumes lower growth rates for the productivity and population with higher interest rates, which reduce the electricity demand. The vice versa

case is the “High Economic Growth”. All these terms are completely defined in U.S. Annual Energy Outlook 2012 with Projections [48].

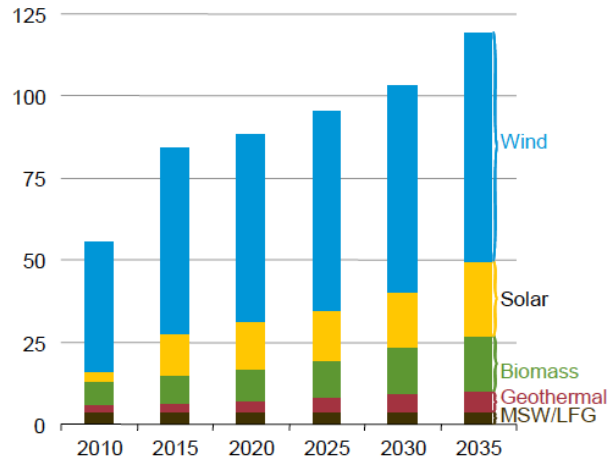
As shown in “Low Gas Price 05” case in Figure 2-A, the lower the price of natural gas is, the higher the amount of the retirement in coal-fired generation.



**Figure 2-A** Cumulative Retirements of Coal-Fired Generating Capacity (in GW), Period 2011-2035, Compiled in 2012 [48]

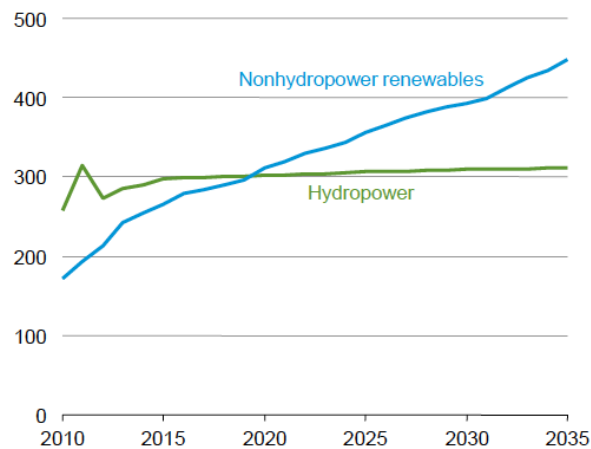
The desired values for the level of electricity production from each of the energy sources in this thesis are based on the U.S. projections for 2035. In the reference case, due to the expected retirement of around 88GW of existing capacity by the end of 2035 and the day-by-day increasing in the electricity demand, 235GW of new generation capacity is planned to be added to the U.S. electricity generation system by 2035. Renewable energy sources account for 29% of this capacity addition, while natural gas energy source is responsible for 60%, the coal energy source for 7% and the remaining 4% for other energy sources [48]. In contrast, due to the future limits on the amount of GHG emissions and the environmental impacts of coal energy source, addition of around 16GW as new generation from this energy source is under question.

The U.S. non-hydropower renewable energy generations are given Figure 3-A. During the period of 2010-2035, the capacity generation from renewable energy sources will be almost doubled. The fastest growth rate is for solar and biomass energy sources while the largest share of the new capacity is for wind energy source [48]. In this figure, MSW and LFG stand for “Municipal Solid Waste” and “Landfill Gas”, respectively.



**Figure 3-A** U.S. Non-hydropower Renewable Electricity Generation (in GW) [48]

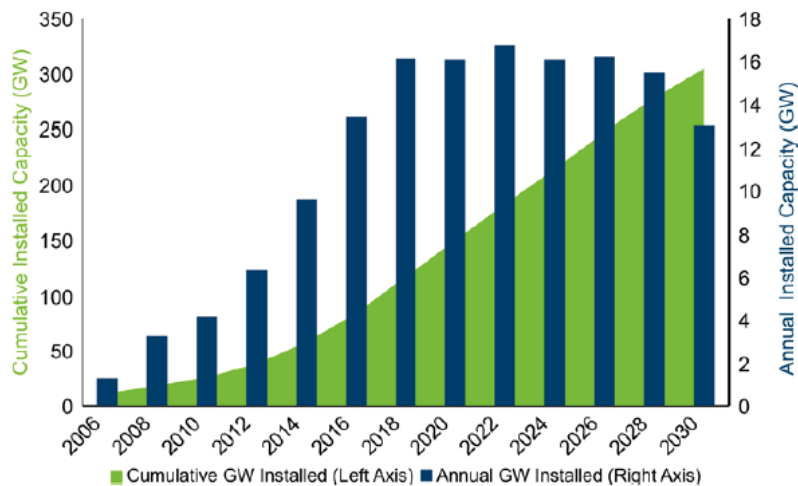
The comparison between the level of electricity generation from hydropower energy source and other renewable energy sources is shown in Figure 4-A. It is obvious that non-hydro renewable generation is expected to almost triple between 2010 and 2035 [48]. Therefore in the desired value we will consider, we should take into account that the level of electricity production from hydropower energy source should be approximately constant, while this level from non-hydro energy sources should constantly increase.



**Figure 4-A** U.S. Hydropower and Other Renewable Electricity Generation, (in Billion kWh) [48]

Several projections and scenarios exist for the portion of the electricity generation that each renewable energy source should produce to meet the future electricity demand.

In June 2008, the Energy Efficiency and Renewable Energy (EERE) network news released an article estimating that solar energy source could provide 10% of the U.S. electricity by year 2025 [49]. In July 2008, the U.S. department of energy released an article providing details of the scenario that 20% of the U.S. electricity demand by year 2030 could be provided by the wind energy source [50]. Figure 5-A shows the U.S. annual and cumulative wind energy source installation based on this projection [50].



**Figure 5-A** U.S. Annual and Cumulative Wind Installations by 2030 [50]

On the other hand, based on the article released by the Guardian on March 27, 2012, the Obama administration blocked any new construction of coal power plants [51]. For the purpose of this thesis, the official released data of EIA Annual Energy Outlook 2012 [48] is used based on the “Low Renewable Technology Cost Case” given in Table 2-A. The planner can easily modify these data according to any updated projection and follow the same procedure used in this thesis.

**Table 2-A** Generation Level of Electric Power Sectors (Trillion kWh)-Projection based on “Low Renewable Technology Cost Case” [48]

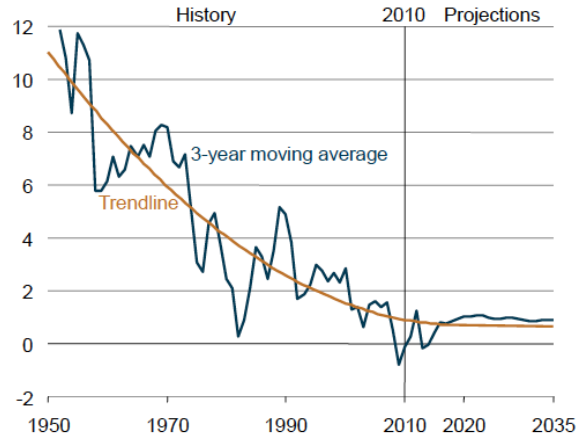
	Year 2015	Year 2035
Coal	1.562	1.780
Petroleum	0.026	0.028
Natural Gas	1.028	1.037
Total Fossil	2.616	2.846
Conventional Hydropower	0.29543	0.32178
Wind	0.15097	0.31055
Solar	0.00647	0.0869
Geothermal	0.01868	0.05089
Wood and Biomass	0.02128	0.07841
Total Renewable	0.50749	0.86320

### ***U.S. Electricity Demand***

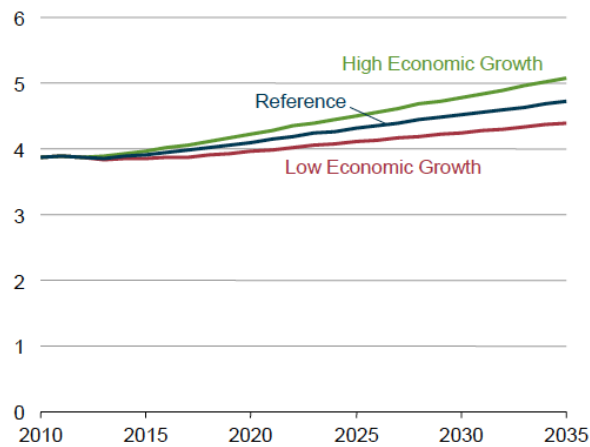
Any decision policy for integration of renewable energy sources into the electricity generation system would be futile without meeting the electricity demand in each predefined time interval. In order to be able to fulfill this need, we used the electricity demand predicted by the U.S. Annual Outlook 2012 [48].

Figure 6-A shows that the growth rate of the U.S. electricity demand has reduced since 1950. This decreasing rate was about 9.8% per year between 1949 and 1959; however it has reduced to 0.7% annually during the first decade of the 21<sup>st</sup> century. The main reason for this decreasing is the introduction of energy efficient appliances [48].

The predicted electricity demand for United States from 2010 to 2035 for three different cases of “High Economic Growth”, “Reference” and “Low Economic Growth” is shown in Figure 7-A. For the purpose of this thesis, the “Reference Case” data in Figure 7-A is used for the U.S. electricity demand.



**Figure 6-A** U.S. Electricity Demand Growth (in %), [48]



**Figure 7-A** U.S. Electricity Demand, Trillion kWh [48]

### ***Summary***

In this appendix, we defined three main necessary data for the numerical section of the thesis. First, the historical data, which will help us to identify the unknown parameters of the model of our system.

Second, the desired values, which are defined according to the U.S. projections for 2035 and will be used in different energy problems.

Finally, the predicted electricity demand, which will help us to evaluate precise decision policies that fulfill the electricity demand during the plan period.